# STATA BASE REFERENCE MANUAL RELEASE 13 



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## Contents

intro Introduction to base reference manual ..... 1
about Display information about your Stata ..... 7
adoupdate Update user-written ado-files ..... 8
ameans Arithmetic, geometric, and harmonic means ..... 12
anova Analysis of variance and covariance ..... 16
anova postestimation Postestimation tools for anova ..... 57
areg Linear regression with a large dummy-variable set ..... 74
areg postestimation Postestimation tools for areg ..... 80
asclogit Alternative-specific conditional logit (McFadden's choice) model ..... 84
asclogit postestimation Postestimation tools for asclogit ..... 94
asmprobit Alternative-specific multinomial probit regression ..... 101
asmprobit postestimation Postestimation tools for asmprobit ..... 126
asroprobit Alternative-specific rank-ordered probit regression ..... 136
asroprobit postestimation Postestimation tools for asroprobit ..... 149
BIC note Calculating and interpreting BIC ..... 157
binreg Generalized linear models: Extensions to the binomial family ..... 162
binreg postestimation Postestimation tools for binreg ..... 175
biprobit Bivariate probit regression ..... 178
biprobit postestimation Postestimation tools for biprobit ..... 185
bitest Binomial probability test ..... 188
bootstrap Bootstrap sampling and estimation ..... 193
bootstrap postestimation Postestimation tools for bootstrap ..... 215
boxcox Box-Cox regression models ..... 219
boxcox postestimation Postestimation tools for boxcox ..... 230
brier Brier score decomposition ..... 235
bsample Sampling with replacement ..... 241
bstat Report bootstrap results ..... 249
centile Report centile and confidence interval ..... 256
ci Confidence intervals for means, proportions, and counts ..... 262
clogit Conditional (fixed-effects) logistic regression ..... 274
clogit postestimation Postestimation tools for clogit ..... 290
cloglog Complementary log-log regression ..... 295
cloglog postestimation Postestimation tools for cloglog ..... 304
cls Clear Results window ..... 307
cnsreg Constrained linear regression ..... 308
cnsreg postestimation Postestimation tools for cnsreg ..... 314
constraint Define and list constraints ..... 317
contrast Contrasts and linear hypothesis tests after estimation ..... 320
contrast postestimation Postestimation tools for contrast ..... 383
copyright Display copyright information ..... 385
copyright apache Apache copyright notification ..... 386
copyright boost Boost copyright notification ..... 390
copyright freetype FreeType copyright notification ..... 391
copyright icu ICU copyright notification ..... 394
copyright jagpdf JagPDF copyright notification ..... 395
copyright lapack LAPACK copyright notification ..... 396
copyright libpng libpng copyright notification ..... 397
copyright miglayout MiG Layout copyright notification ..... 399
copyright scintilla Scintilla copyright notification ..... 400
copyright ttf2pt1 ttf2pt1 copyright notification ..... 401
copyright zlib zlib copyright notification ..... 403
correlate Correlations (covariances) of variables or coefficients ..... 404
cumul Cumulative distribution ..... 412
cusum Cusum plots and tests for binary variables ..... 416
db Launch dialog ..... 420
diagnostic plots Distributional diagnostic plots ..... 422
display Substitute for a hand calculator ..... 434
do Execute commands from a file ..... 435
doedit Edit do-files and other text files ..... 436
dotplot Comparative scatterplots ..... 437
dstdize Direct and indirect standardization ..... 444
dydx Calculate numeric derivatives and integrals ..... 463
eform_option Displaying exponentiated coefficients ..... 469
eivreg Errors-in-variables regression ..... 471
eivreg postestimation Postestimation tools for eivreg ..... 476
error messages Error messages and return codes ..... 478
esize Effect size based on mean comparison ..... 479
estat Postestimation statistics ..... 490
estat classification Classification statistics and table ..... 491
estat gof Pearson or Hosmer-Lemeshow goodness-of-fit test ..... 494
estat ic Display information criteria ..... 503
estat summarize Summarize estimation sample ..... 507
estat vce Display covariance matrix estimates ..... 510
estimates Save and manipulate estimation results ..... 513
estimates describe Describe estimation results ..... 517
estimates for Repeat postestimation command across models ..... 519
estimates notes Add notes to estimation results ..... 521
estimates replay Redisplay estimation results ..... 523
estimates save Save and use estimation results ..... 526
estimates stats Model-selection statistics ..... 530
estimates store Store and restore estimation results ..... 532
estimates table Compare estimation results ..... 535
estimates title Set title for estimation results ..... 541
estimation options Estimation options ..... 542
exit Exit Stata ..... 545
exlogistic Exact logistic regression ..... 546
exlogistic postestimation Postestimation tools for exlogistic ..... 564
expoisson Exact Poisson regression ..... 569
expoisson postestimation Postestimation tools for expoisson ..... 581
fp Fractional polynomial regression ..... 583
fp postestimation Postestimation tools for fp ..... 607
frontier Stochastic frontier models ..... 616
frontier postestimation Postestimation tools for frontier ..... 631
fvrevar Factor-variables operator programming command ..... 635
fvset Declare factor-variable settings ..... 638
gllamm Generalized linear and latent mixed models ..... 643
glm Generalized linear models ..... 645
glm postestimation Postestimation tools for glm ..... 679
glogit Logit and probit regression for grouped data ..... 685
glogit postestimation Postestimation tools for glogit, gprobit, blogit, and bprobit ..... 696
gmm Generalized method of moments estimation ..... 698
gmm postestimation Postestimation tools for gmm ..... 760
grmeanby Graph means and medians by categorical variables ..... 764
hausman Hausman specification test ..... 767
heckman Heckman selection model ..... 776
heckman postestimation Postestimation tools for heckman ..... 794
heckoprobit Ordered probit model with sample selection ..... 800
heckoprobit postestimation Postestimation tools for heckoprobit ..... 809
heckprobit Probit model with sample selection ..... 814
heckprobit postestimation Postestimation tools for heckprobit ..... 822
help Display help in Stata ..... 827
hetprobit Heteroskedastic probit model ..... 829
hetprobit postestimation Postestimation tools for hetprobit ..... 836
histogram Histograms for continuous and categorical variables ..... 839
icc Intraclass correlation coefficients ..... 850
inequality Inequality measures ..... 872
intreg Interval regression ..... 875
intreg postestimation Postestimation tools for intreg ..... 885
ivpoisson Poisson regression with endogenous regressors ..... 890
ivpoisson postestimation Postestimation tools for ivpoisson ..... 905
ivprobit Probit model with continuous endogenous regressors ..... 910
ivprobit postestimation Postestimation tools for ivprobit ..... 923
ivregress Single-equation instrumental-variables regression ..... 927
ivregress postestimation Postestimation tools for ivregress ..... 943
ivtobit Tobit model with continuous endogenous regressors ..... 961
ivtobit postestimation Postestimation tools for ivtobit ..... 971
jackknife Jackknife estimation ..... 975
jackknife postestimation Postestimation tools for jackknife ..... 987
kappa Interrater agreement 988
kdensity Univariate kernel density estimation 1002
ksmirnov Kolmogorov-Smirnov equality-of-distributions test 1012
kwallis Kruskal-Wallis equality-of-populations rank test 1016
ladder Ladder of powers 1019
level Set default confidence level 1026
limits Quick reference for limits 1028
lincom Linear combinations of estimators 1033
linktest Specification link test for single-equation models 1041
lnskew0 Find zero-skewness log or Box-Cox transform 1047
log Echo copy of session to file 1051
logistic Logistic regression, reporting odds ratios 1055
logistic postestimation Postestimation tools for logistic 1067
logitLogistic regression, reporting coefficients 1077
oprobit postestimationPostestimation tools for oprobit 1560
orthog
Orthogonalize variables and compute orthogonal polynomials ..... 1564
pcorr Partial and semipartial correlation coefficients ..... 1570
permute Monte Carlo permutation tests ..... 1573
pk Pharmacokinetic (biopharmaceutical) data 1583
pkcollapse Generate pharmacokinetic measurement dataset 1591
pkcross Analyze crossover experiments ..... 1594
pkequiv Perform bioequivalence tests 1603
pkexamine Calculate pharmacokinetic measures 1610
pkshape Reshape (pharmacokinetic) Latin-square data 1616
pksumm Summarize pharmacokinetic data 1624
poisson Poisson regression 1629
poisson postestimation Postestimation tools for poisson 1639
predict Obtain predictions, residuals, etc., after estimation 1645
predictnl Obtain nonlinear predictions, standard errors, etc., after estimation 1656
probitProbit regression 1668
probit postestimation Postestimation tools for probit 1681
proportion Estimate proportions 1685
proportion postestimation Postestimation tools for proportion 1691
prtest Tests of proportions 1693
pwcompare Pairwise comparisons 1698
pwcompare postestimation Postestimation tools for pwcompare 1730
pwmean Pairwise comparisons of means 1732
pwmean postestimation Postestimation tools for pwmean 1744
qc Quality control charts 1746
qregQuantile regression 1761
qreg postestimation Postestimation tools for qreg, iqreg, sqreg, and bsqreg 1791
queryDisplay system parameters 1795
ranksum Equality tests on unmatched data 1802
ratio Estimate ratios 1809
ratio postestimation Postestimation tools for ratio 1818
reg3 Three-stage estimation for systems of simultaneous equations 1819
reg3 postestimation Postestimation tools for reg3 1840
regress Linear regression 1845
regress postestimation Postestimation tools for regress 1870
regress postestimation diagnostic plots Postestimation plots for regress 1905
regress postestimation time series Postestimation tools for regress with time series 1924
\#review Review previous commands 1934
roc Receiver operating characteristic (ROC) analysis ..... 1935
roccomp Tests of equality of ROC areas 1937
rocfit Parametric ROC models 1949
rocfit postestimation Postestimation tools for rocfit 1956
rocreg Receiver operating characteristic (ROC) regression 1960
rocreg postestimation Postestimation tools for rocreg 2013
rocregplot Plot marginal and covariate-specific ROC curves after rocreg 2028
roctabNonparametric ROC analysis 2048
rologit Rank-ordered logistic regression 2058
rologit postestimation Postestimation tools for rologit 2075
rreg
Robust regression 2077
rreg postestimation
runtest
Postestimation tools for rreg 2084
scobit . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . Skewed logistic regression 2092
scobit postestimation . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . Postestimation tools for scobit 2101
sdtest . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . Variance-comparison tests 2104
search ...................................... . . Search Stata documentation and other resources 2110
serrbar ......................................................... . . . . Graph standard error bar chart 2116
set . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . Overview of system parameters 2119
set cformat .............................................. . . Format settings for coefficient tables 2131
set_defaults . . . . . . . . . . . . . . . . . . . . . . Reset system parameters to original Stata defaults 2134
set emptycells . . . . . . . . . . . . . . . . . . . . . . Set what to do with empty cells in interactions 2136
set seed ........................................ . . Specify initial value of random-number seed 2137
set showbaselevels . . . . . . . . . . . . . . . . . . . . . . . . . . . . . Display settings for coefficient tables 2142
signrank
simulate
sj .................. . . . . . . . . . . . . . . . . . . . . . Stata Journal and STB installation instructions 2164
sktest . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . Skewness and kurtosis test for normality 2167
slogit ............................................................ . . . Stereotype logistic regression 2172
slogit postestimation . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . Postestimation tools for slogit 2185
smooth ................................................................ . . Robust nonlinear smoother 2189
spearman . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . Spearman's and Kendall's correlations 2197
spikeplot . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . Spike plots and rootograms 2206
ssc ................................................. . . Install and uninstall packages from SSC 2210
stem ....................................................................... . . . Stem-and-leaf displays 2218
stepwise ............................................. . . . . . . . . . . . . . . . . . . . . . Stepwise estimation 2222
stored results ........................................................................ . . . Stored results 2232

summarize ..................................................................... . . Summary statistics 2255
sunflower . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . Density-distribution sunflower plots 2265
sureg . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . Zellner's seemingly unrelated regression 2271
sureg postestimation . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . Postestimation tools for sureg 2279
swilk . . . . . . . . . . . . . . . . . . . . . . . . . Shapiro-Wilk and Shapiro-Francia tests for normality 2282
symmetry . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . Symmetry and marginal homogeneity tests 2286

tabstat . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . Compact table of summary statistics 2305
tabulate oneway . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . One-way table of frequencies 2310
tabulate twoway . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . Two-way table of frequencies 2318
tabulate, summarize() . . . . . . . . . . . . . . . . . . One- and two-way tables of summary statistics 2335
test ..................................................... . . Test linear hypotheses after estimation 2340
testnl . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . Test nonlinear hypotheses after estimation 2359
tetrachoric . . . . . . . . . . . . . . . . . . . . . . . . . . . . Tetrachoric correlations for binary variables 2368
tnbreg .............................................. . . Truncated negative binomial regression 2378
tnbreg postestimation . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . Postestimation tools for tnbreg 2387
tobit ................................................................................... . . Tobit regression 2391
tobit postestimation . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . Postestimation tools for tobit 2398
total . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . Estimate totals 2403
total postestimation . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . Postestimation tools for total 2409
tpoisson ........................................................... . . . Truncated Poisson regression 2410
tpoisson postestimation . . . . . . . . . . . . . . . . . . . . . . . . . . . . Postestimation tools for tpoisson 2418
translate
truncregTruncated regression 2431
truncreg postestimation Postestimation tools for truncreg 2438
ttest $t$ tests (mean-comparison tests) 2441
update Check for official updates 2451
vce_option Variance estimators 2454
view View files and logs 2459
vwls Variance-weighted least squares 2462
vwls postestimation Postestimation tools for vwls 2468
which Display location and version for an ado-file 2470
xiInteraction expansion 2472
zinb Zero-inflated negative binomial regression 2482
zinb postestimation Postestimation tools for zinb 2489
zip Zero-inflated Poisson regression 2492
zip postestimation Postestimation tools for zip 2499
Author index ..... 2503
Subject index ..... 2519

## Cross-referencing the documentation

When reading this manual, you will find references to other Stata manuals. For example,
[U] 26 Overview of Stata estimation commands
[XT] xtabond
[D] reshape
The first example is a reference to chapter 26, Overview of Stata estimation commands, in the User's Guide; the second is a reference to the xtabond entry in the Longitudinal-Data/Panel-Data Reference Manual; and the third is a reference to the reshape entry in the Data Management Reference Manual.

All the manuals in the Stata Documentation have a shorthand notation:

| [GSM] | Getting Started with Stata for Mac |
| :--- | :--- |
| [GSU] | Getting Started with Stata for Unix |
| [GSW] | Getting Started with Stata for Windows |
| [U] | Stata User's Guide |
| [R] | Stata Base Reference Manual |
| [D] | Stata Data Management Reference Manual |
| [G] | Stata Graphics Reference Manual |
| [XT] | Stata Longitudinal-Data/Panel-Data Reference Manual |
| [ME] | Stata Multilevel Mixed-Effects Reference Manual |
| [MI] | Stata Multiple-Imputation Reference Manual |
| [MV] | Stata Multivariate Statistics Reference Manual |
| [PSS] | Stata Power and Sample-Size Reference Manual |
| [P] | Stata Programming Reference Manual |
| [SEM] | Stata Structural Equation Modeling Reference Manual |
| [SVY] | Stata Survey Data Reference Manual |
| [ST] | Stata Survival Analysis and Epidemiological Tables Reference Manual |
| [TS] | Stata Time-Series Reference Manual |
| [TE] | Stata Treatment-Effects Reference Manual: |
|  | Potential Outcomes/Counterfactual Outcomes |
| [I] | Stata Glossary and Index |
| [M] | Mata Reference Manual |

## Title

intro - Introduction to base reference manual
Description Remarks and examples Also see

## Description

This entry describes the organization of the reference manuals.

## Remarks and examples

The complete list of reference manuals is as follows:
[R] Stata Base Reference Manual
[D] Stata Data Management Reference Manual
[G] Stata Graphics Reference Manual
[XT] Stata Longitudinal-Data/Panel-Data Reference Manual
[ME] Stata Multilevel Mixed-Effects Reference Manual
[MI] Stata Multiple-Imputation Reference Manual
[MV] Stata Multivariate Statistics Reference Manual
[PSS] Stata Power and Sample-Size Reference Manual
[P] Stata Programming Reference Manual
[SEM] Stata Structural Equation Modeling Reference Manual
[SVY] Stata Survey Data Reference Manual
[ST] Stata Survival Analysis and Epidemiological Tables Reference Manual
[TS] Stata Time-Series Reference Manual
[TE] Stata Treatment-Effects Reference Manual:
Potential Outcomes/Counterfactual Outcomes
[I] Stata Glossary and Index
[M] Mata Reference Manual

When we refer to "reference manuals", we mean all manuals listed above.
When we refer to the specialty manuals, we mean all the manuals listed above except $[\mathrm{R}]$ and [I].

## Arrangement of the reference manuals

Each manual contains the following sections:

- Contents.

A table of contents can be found at the beginning of each manual.

- Cross-referencing the documentation.

This entry lists all the manuals and explains how they are cross-referenced.

- Introduction.

This entry-usually called intro-provides an overview of the manual. In the specialty manuals, this introduction suggests entries that you might want to read first and provides information about new features.

Each specialty manual contains an overview of the commands described in it.

- Entries.

Entries are arranged in alphabetical order. Most entries describe Stata commands, but some entries discuss concepts, and others provide overviews.
Entries that describe estimation commands are followed by an entry discussing postestimation commands that are available for use after the estimation command. For example, the xtlogit entry in the [XT] manual is followed by the xtlogit postestimation entry.

- Index.

An index can be found at the end of each manual.
The Glossary and Index, [I], contains a subject table of contents for all the reference manuals and the User's Guide, a combined acronym glossary, a combined glossary, a vignette index, a combined author index, and a combined subject index for all the manuals.

To find information and commands quickly, use Stata's search command; see [R] search (see the entry search in the [R] manual).

## Arrangement of each entry

Entries in the Stata reference manuals, except the [M] and [SEM] manuals, generally contain the following sections, which are explained below:

Syntax<br>Menu<br>Description<br>Options<br>Remarks and examples<br>Stored results<br>Methods and formulas<br>References<br>Also see

## Syntax

A command's syntax diagram shows how to type the command, indicates all possible options, and gives the minimal allowed abbreviations for all the items in the command. For instance, the syntax diagram for the summarize command is
summarize $[$ varlist $][$ if $][$ in $][$ weight $][$, options $]$
options
Description
Main
detail
meanonly
format
separator (\#)
display_options
display additional statistics
suppress the display; calculate only the mean; programmer's option use variable's display format
draw separator line after every \# variables; default is separator (5)
control spacing and base and empty cells
varlist may contain factor variables; see [U] 11.4.3 Factor variables.
varlist may contain time-series operators; see [U] 11.4.4 Time-series varlists.
by is allowed; see [D] by.
aweights, fweights, and iweights are allowed. However, iweights may not be used with the detail option; see
[U] 11.1.6 weight.

Items in the typewriter-style font should be typed exactly as they appear in the diagram, although they may be abbreviated. Underlining indicates the shortest abbreviations where abbreviations are allowed. For instance, summarize may be abbreviated su, sum, summ, etc., or it may be spelled out completely. Items in the typewriter font that are not underlined may not be abbreviated.

Square brackets denote optional items. In the syntax diagram above, varlist, if, in, weight, and the options are optional.

The options are listed in a table immediately following the diagram, along with a brief description of each.

Items typed in italics represent arguments for which you are to substitute variable names, observation numbers, and the like.
The diagrams use the following symbols:
\# Indicates a literal number, for example, 5; see [U] 12.2 Numbers.
[ ] Anything enclosed in brackets is optional.
\{ \} At least one of the items enclosed in braces must appear.
| The vertical bar separates alternatives.
$\%$ Any Stata format, for example, \%8.2f; see [U] 12.5 Formats: Controlling how data are displayed.
depvar The dependent variable in an estimation command; see [U] 20 Estimation and postestimation commands.
$\exp \quad$ Any algebraic expression, for example, (5+myvar)/2; see [U] 13 Functions and expressions.
filename Any filename; see [U] 11.6 Filenaming conventions.
indepvars The independent variables in an estimation command; see [U] 20 Estimation and postestimation commands.
newvar A variable that will be created by the current command; see [U] 11.4.2 Lists of new variables.
numlist A list of numbers; see [U] 11.1.8 numlist.
oldvar A previously created variable; see [U] 11.4.1 Lists of existing variables.
options A list of options; see [U] 11.1.7 options.
range An observation range, for example, 5/20; see [U] 11.1.4 in range.
"string" Any string of characters enclosed in double quotes; see [U] 12.4 Strings.
varlist A list of variable names; see [U] 11.4 varlists. If varlist allows factor variables, a note to that effect will be shown below the syntax diagram; see [U] 11.4.3 Factor variables. If varlist allows time-series operators, a note to that effect will be shown below the syntax diagram; see [U] 11.4.4 Time-series varlists.
varname A variable name; see [U] 11.3 Naming conventions.
weight A [wgttype $=\exp ]$ modifier; see [U] 11.1.6 weight and [U] 20.23 Weighted estimation.
xvar The variable to be displayed on the horizontal axis.
$y v a r \quad$ The variable to be displayed on the vertical axis.
The Syntax section will indicate whether factor variables or time-series operators may be used with a command. summarize allows factor variables and time-series operators.

If a command allows prefix commands, this will be indicated immediately following the table of options. summarize allows by.

If a command allows weights, the types of weights allowed will be specified, with the default weight listed first. summarize allows aweights, fweights, and iweights, and if the type of weight is not specified, the default is aweights.

## Menu

A menu indicates how the dialog box for the command may be accessed using the menu system.

## Description

Following the syntax diagram is a brief description of the purpose of the command.

## Options

If the command allows any options, they are explained here, and for dialog users the location of the options in the dialog is indicated. For instance, in the logistic entry in this manual, the Options section looks like this:

$\square$
$\qquad$

## Remarks and examples

The explanations under Description and Options are exceedingly brief and technical; they are designed to provide a quick summary. The remarks explain in English what the preceding technical jargon means. Examples are used to illustrate the command.

## Stored results

Commands are classified as e-class, r-class, s-class, or n -class, according to whether they store calculated results in $e(), r(), s()$, or not at all. These results can then be used in subroutines by other programs (ado-files). Such stored results are documented here; see [U] 18.8 Accessing results calculated by other programs and [U] 18.9 Accessing results calculated by estimation commands.

## Methods and formulas

The techniques and formulas used in obtaining the results are described here as tersely and technically as possible.

## References

Published sources are listed that either were directly referenced in the preceding text or might be of interest.

## Also see <br> Other manual entries relating to this entry are listed that might also interest you.

Elizabeth L. ("Betty") Scott (1917-1988) was an astronomer and mathematician trained at the University of California at Berkeley. She published her first paper when she was just 22 years old, and her work was focused on comets for much of her early academic career.

During World War II, Scott began working at the statistical laboratory at Berkeley, which had recently been established by Jerzy Neyman, sparking what would be a long and fruitful collaboration with him. After the war, she shifted her focus toward mathematics and statistics, partly because of limited career opportunities as an astronomer, though she still applied her research to astronomical topics. For example, in 1949 she published a paper using statistical techniques to analyze the distribution of binary star systems. She also published papers examining the distribution of galaxies, and she is the name behind the "Scott effect", which helps determine the distances to galaxies. Later in her career, Scott applied her statistical knowledge to problems associated with ozone depletion and its effects on the incidence of skin cancer as well as weather modification. She was also a champion of equality for women graduate students and faculty.

Among Scott's many awards and accomplishments, she was elected an honorary fellow of the Royal Statistical Society and was a fellow of the American Association for the Advancement of Science. In 1992, the Committee of Presidents of Statistical Societies established the Elizabeth L. Scott Award, a biannual award to recognize those who have strived to enhance the status of women within the statistics profession.

## Also see

[U] 1.1 Getting Started with Stata

## Title

about - Display information about your Stata
Syntax Menu Description Remarks and examples Also see

## Syntax

about

## Menu

Help > About Stata

## Description

about displays information about your version of Stata.

## Remarks and examples

If you are running Stata for Windows, information about memory is also displayed:

```
. about
Stata/MP 13 for Windows (64-bit x86-64)
Revision date
Copyright 1985-2013 StataCorp LP
    Total physical memory: }8388608\mathrm{ KB
    Available physical memory: 937932 KB
10-user 32-core Stata network perpetual license:
    Serial number: 5013041234
        Licensed to: Alan R. Riley
        StataCorp
```


## Also see

[R] which - Display location and version for an ado-file
[U] 3 Resources for learning and using Stata
[U] 5 Flavors of Stata

## Title

adoupdate - Update user-written ado-files
Syntax Description Options Remarks and examples

Stored results Also see

## Syntax

```
    adoupdate [pkglist] [, options]
options Description
update perform update; default is to list packages that have updates, but not to
                                update them
all include packages that might have updates; default is to list or update
                                only packages that are known to have updates
ssconly check only packages obtained from SSC; default is to check all installed packages
dir(dir) check packages installed in dir; default is to check those installed in PLUS
verbose provide output to assist in debugging network problems
```


## Description

User-written additions to Stata are called packages. These packages can add remarkable abilities to Stata. Packages are found and installed by using ssc, search, and net; see $[R]$ ssc, $[R]$ search, and $[R]$ net.

User-written packages are updated by their developers, just as official Stata software is updated by StataCorp.

To determine whether your official Stata software is up to date, and to update it if it is not, you use update; see [R] update.

To determine whether your user-written additions are up to date, and to update them if they are not, you use adoupdate.

## Options

update specifies that packages with updates be updated. The default is simply to list the packages that could be updated without actually performing the update.

The first time you adoupdate, do not specify this option. Once you see adoupdate work, you will be more comfortable with it. Then type

```
. adoupdate, update
```

The packages that can be updated will be listed and updated.
all is rarely specified. Sometimes, adoupdate cannot determine whether a package you previously installed has been updated. adoupdate can determine that the package is still available over the web but is unsure whether the package has changed. Usually, the package has not changed, but if you want to be certain that you are using the latest version, reinstall from the source.

Specifying all does this. Typing
. adoupdate, all
adds such packages to the displayed list as needing updating but does not update them. Typing . adoupdate, update all
lists such packages and updates them.
ssconly is a popular option. Many packages are available from the Statistical Software Components (SSC) archive-often called the Boston College Archive-which is provided at http://repec.org. Many users find most of what they want there. See [R] ssc for more information on the SSC.
ssconly specifies that adoupdate check only packages obtained from that source. Specifying this option is popular because SSC always provides distribution dates, and so adoupdate can be certain whether an update exists.
$\operatorname{dir}(d i r)$ specifies which installed packages be checked. The default is $\operatorname{dir}$ (PLUS), and that is probably correct. If you are responsible for maintaining a large system, however, you may have previously installed packages in dir (SITE), where they are shared across users. See [P] sysdir for an explanation of these directory codewords. You may also specify an actual directory name, such as C: \mydir.
verbose is specified when you suspect network problems. It provides more detailed output that may help you diagnose the problem.

## Remarks and examples

Do not confuse adoupdate with update. Use adoupdate to update user-written files. Use update to update the components (including ado-files) of the official Stata software. To use either command, you must be connected to the Internet.

Remarks are presented under the following headings:

> Using adoupdate
> Possible problem the first time you run adoupdate and the solution Notes for developers

## Using adoupdate

The first time you try adoupdate, type

```
. adoupdate
```

That is, do not specify the update option. adoupdate without update produces a report but does not update any files. The first time you run adoupdate, you may see messages such as

```
. adoupdate
    (note: package utx was installed more than once; older copy removed)
    (remaining output omitted)
```

Having the same packages installed multiple times is common; adoupdate cleans that up.
The second time you run adoupdate, pick one package to update. Suppose that the report indicates that package st0008 has an update available. Type

```
. adoupdate st0008, update
```

You can specify one or many packages after the adoupdate command. You can even use wildcards such as st* to mean all packages that start with st or st*8 to mean all packages that start with st and end with 8 . You can do that with or without the update option.

Finally, you can let adoupdate update all your user-written additions:

```
. adoupdate, update
```


## Possible problem the first time you run adoupdate and the solution

The first time you run adoupdate, you might get many duplicate messages:

```
- adoupdate
(note: package ___ installed more than once; older copy removed)
(note: package ___ installed more than once; older copy removed)
(note: package ___ installed more than once; older copy removed)
(note: package __- installed more than once; older copy removed)
(remaining output omitted)
```

Some users have hundreds of duplicates. You might even see the same package name repeated more than once:

```
(note: package stylus installed more than once; older copy removed)
(note: package stylus installed more than once; older copy removed)
```

That means that the package was duplicated twice.
Stata tolerates duplicates, and you did nothing wrong when you previously installed and updated packages. adoupdate, however, needs the duplicates removed, mainly so that it does not keep checking the same files.

The solution is to just let adoupdate run. adoupdate will run faster next time, when there are no (or just a few) duplicates.

## Notes for developers

adoupdate reports whether an installed package is up to date by comparing its distribution date with that of the package available over the web.

If you are distributing software, include the line
d Distribution-Date: date
somewhere in your .pkg file. The capitalization of Distribution-Date does not matter, but include the hyphen and the colon as shown. Code the date in either of two formats:
all numeric:
Stata standard:
yyyymmdd, for example, 20120701
ddMONyyyy, for example, 01jul2012

## Stored results

adoupdate stores the following in r() :
Macros
r(pkglist) a space-separated list of package names that need updating (update not specified) or that were updated (update specified)

## Also see

$[\mathrm{R}]$ net - Install and manage user-written additions from the Internet
$[R]$ search - Search Stata documentation and other resources
[R] ssc - Install and uninstall packages from SSC
[R] update - Check for official updates

## Title

ameans - Arithmetic, geometric, and harmonic means

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | Acknowledgments |
| References | Also see |  |  |

## Syntax

$$
\text { ameans }[\text { varlist }][\text { if }][\text { in }][\text { weight }][\text {, options }]
$$

options Description
Main
add (\#) add \# to each variable in varlist
only add \# only to variables with nonpositive values
level(\#) set confidence level; default is level (95)
by is allowed; see [D] by.
aweights and fweights are allowed; see [U] 11.1.6 weight.

## Menu

Statistics $>$ Summaries, tables, and tests $>$ Summary and descriptive statistics $>$ Arith./geometric/harmonic means

## Description

ameans computes the arithmetic, geometric, and harmonic means, with their corresponding confidence intervals, for each variable in varlist or for all the variables in the data if varlist is not specified. gmeans and hmeans are synonyms for ameans.

If you simply want arithmetic means and corresponding confidence intervals, see $[\mathrm{R}]$ ci.

## Options

[^0]
## Remarks and examples

## > Example 1

We have a dataset containing 8 observations on a variable named x . The eight values are 5,4 , $-4,-5,0,0$, missing, and 7 .

| . ameans x <br> Variable | Type | Obs | Mean | [95\% Conf. Interval] |  |
| ---: | :---: | ---: | ---: | ---: | ---: |
| x | Arithmetic | 7 | 1 | -3.204405 | 5.204405 |
|  | Geometric | 3 | 5.192494 | 2.57899 | 10.45448 |
|  | Harmonic | 3 | 5.060241 | 3.023008 | 15.5179 |


| . ameans x, add(5) |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: |
| Variable | Type | Obs | Mean | [95\% Conf. Interval] |  |
| x | Arithmetic | 7 | 6 | 1.795595 | $10.2044 *$ |
|  | Geometric | 6 | 5.477226 | 2.1096 | $14.22071 *$ |
|  | Harmonic | 6 | 3.540984 | . | $*$ |

(*) 5 was added to the variables prior to calculating the results.
Missing values in confidence intervals for harmonic mean indicate that confidence interval is undefined for corresponding variables. Consult Reference Manual for details.

The number of observations displayed for the arithmetic mean is the number of nonmissing observations. The number of observations displayed for the geometric and harmonic means is the number of nonmissing, positive observations. Specifying the add (5) option produces 3 more positive observations. The confidence interval for the harmonic mean is not reported; see Methods and formulas below.

## Video example

Descriptive statistics in Stata

## Stored results

ameans stores the following in r() :
Scalars

| r (N) | number of nonmissing observations; used for arithmetic mean |
| :---: | :---: |
| r(N_pos) | number of nonmissing positive observations; used for geometric and harmonic means |
| r (mean) | arithmetic mean |
| r(lb) | lower bound of confidence interval for arithmetic mean |
| r (ub) | upper bound of confidence interval for arithmetic mean |
| r (Var) | variance of untransformed data |
| r(mean_g) | geometric mean |
| r (lb_g) | lower bound of confidence interval for geometric mean |
| r (ub_g) | upper bound of confidence interval for geometric mean |
| $r$ (Var_g) | variance of $\ln x_{i}$ |
| $r$ (mean_h) | harmonic mean |
| $\mathrm{r}(\mathrm{lb}$-h) | lower bound of confidence interval for harmonic mean |
| r (ub_h) | upper bound of confidence interval for harmonic mean |
| r Var $^{\text {d }}$ ) | variance of $1 / x_{i}$ |

## Methods and formulas

See Armitage, Berry, and Matthews (2002) or Snedecor and Cochran (1989). For a history of the concept of the mean, see Plackett (1958).

When restricted to the same set of values (that is, to positive values), the arithmetic mean $(\bar{x})$ is greater than or equal to the geometric mean, which in turn is greater than or equal to the harmonic mean. Equality holds only if all values within a sample are equal to a positive constant.

The arithmetic mean and its confidence interval are identical to those provided by ci; see $[\mathrm{R}]$ ci.
To compute the geometric mean, ameans first creates $u_{j}=\ln x_{j}$ for all positive $x_{j}$. The arithmetic mean of the $u_{j}$ and its confidence interval are then computed as in ci. Let $\bar{u}$ be the resulting mean, and let $[L, U]$ be the corresponding confidence interval. The geometric mean is then $\exp (\bar{u})$, and its confidence interval is $[\exp (L), \exp (U)]$.

The same procedure is followed for the harmonic mean, except that then $u_{j}=1 / x_{j}$. The harmonic mean is then $1 / \bar{u}$, and its confidence interval is $[1 / U, 1 / L]$ if $L$ is greater than zero. If $L$ is not greater than zero, this confidence interval is not defined, and missing values are reported.

When weights are specified, ameans applies the weights to the transformed values, $u_{j}=\ln x_{j}$ and $u_{j}=1 / x_{j}$, respectively, when computing the geometric and harmonic means. For details on how the weights are used to compute the mean and variance of the $u_{j}$, see [R] summarize. Without weights, the formula for the geometric mean reduces to

$$
\exp \left\{\frac{1}{n} \sum_{j} \ln \left(x_{j}\right)\right\}
$$

Without weights, the formula for the harmonic mean is

$$
\frac{n}{\sum_{j} \frac{1}{x_{j}}}
$$

## Acknowledgments

This improved version of ameans is based on the gmci command (Carlin, Vidmar, and Ramalheira 1998) and was written by John Carlin of the Murdoch Children's Research Institute and the University of Melbourne; Suzanna Vidmar of the University of Melbourne; and Carlos Ramalheira of Coimbra University Hospital, Portugal.

## References

Armitage, P., G. Berry, and J. N. S. Matthews. 2002. Statistical Methods in Medical Research. 4th ed. Oxford: Blackwell.

Carlin, J. B., S. Vidmar, and C. Ramalheira. 1998. sg75: Geometric means and confidence intervals. Stata Technical Bulletin 41: 23-25. Reprinted in Stata Technical Bulletin Reprints, vol. 7, pp. 197-199. College Station, TX: Stata Press.
Keynes, J. M. 1911. The principal averages and the laws of error which lead to them. Journal of the Royal Statistical Society 74: 322-331.

Plackett, R. L. 1958. Studies in the history of probability and statistics: VII. The principle of the arithmetic mean. Biometrika 45: 130-135.
Snedecor, G. W., and W. G. Cochran. 1989. Statistical Methods. 8th ed. Ames, IA: Iowa State University Press.
Stigler, S. M. 1985. Arithmetric means. In Vol. 1 of Encyclopedia of Statistical Sciences, ed. S. Kotz and N. L. Johnson, 126-129. New York: Wiley.

## Also see

$[R]$ ci - Confidence intervals for means, proportions, and counts
[R] mean - Estimate means
$[R]$ summarize - Summary statistics
[SVY] svy estimation - Estimation commands for survey data

## Title

anova - Analysis of variance and covariance

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | References | Also see |

## Syntax

anova varname [termlist] [if] [in] [weight] [, options]
where termlist is a factor-variable list (see [U] 11.4.3 Factor variables) with the following additional features:

- Variables are assumed to be categorical; use the c. factor-variable operator to override this.
- The \| symbol (indicating nesting) may be used in place of the \# symbol (indicating interaction).
- The / symbol is allowed after a term and indicates that the following term is the error term for the preceding terms.
options
Description
Model
repeated (varlist) variables in terms that are repeated-measures variables
partial
sequential
noconstant
dropemptycells
use partial (or marginal) sums of squares
use sequential sums of squares
suppress constant term
drop empty cells from the design matrix
Adv. model
bse (term)
bseunit (varname)
grouping (varname)
between-subjects error term in repeated-measures ANOVA variable representing lowest unit in the between-subjects error term grouping variable for computing pooled covariance matrix
bootstrap, by, fp, jackknife, and statsby are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the bootstrap prefix; see $[R]$ bootstrap.
aweights are not allowed with the jackknife prefix; see $[R]$ jackknife.
aweights and fweights are allowed; see [U] 11.1.6 weight.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.


## Menu

Statistics $>$ Linear models and related $>$ ANOVA/MANOVA $>$ Analysis of variance and covariance

## Description

The anova command fits analysis-of-variance (ANOVA) and analysis-of-covariance (ANCOVA) models for balanced and unbalanced designs, including designs with missing cells; for repeated-measures ANOVA; and for factorial, nested, or mixed designs.

The regress command (see $[R]$ regress) will display the coefficients, standard errors, etc., of the regression model underlying the last run of anova.

If you want to fit one-way ANOVA models, you may find the oneway or loneway command more convenient; see [R] oneway and [R] loneway. If you are interested in MANOVA or MANCOVA, see [MV] manova.

## Options

$\qquad$ Model
repeated (varlist) indicates the names of the categorical variables in the terms that are to be treated as repeated-measures variables in a repeated-measures ANOVA or ANCOVA.
partial presents the ANOVA table using partial (or marginal) sums of squares. This setting is the default. Also see the sequential option.
sequential presents the ANOVA table using sequential sums of squares.
noconstant suppresses the constant term (intercept) from the ANOVA or regression model.
dropemptycells drops empty cells from the design matrix. If c (emptycells) is set to keep (see $[\mathrm{R}]$ set emptycells), this option temporarily resets it to drop before running the ANOVA model. If $c$ (emptycells) is already set to drop, this option does nothing.

Adv. model
bse (term) indicates the between-subjects error term in a repeated-measures ANOVA. This option is needed only in the rare case when the anova command cannot automatically determine the between-subjects error term.
bseunit (varname) indicates the variable representing the lowest unit in the between-subjects error term in a repeated-measures ANOVA. This option is rarely needed because the anova command automatically selects the first variable listed in the between-subjects error term as the default for this option.
grouping (varname) indicates a variable that determines which observations are grouped together in computing the covariance matrices that will be pooled and used in a repeated-measures ANOVA. This option is rarely needed because the anova command automatically selects the combination of all variables except the first (or as specified in the bseunit () option) in the between-subjects error term as the default for grouping observations.

## Remarks and examples

Remarks are presented under the following headings:

[^1]
## Introduction

anova uses least squares to fit the linear models known as ANOVA or ANCOVA (henceforth referred to simply as ANOVA models).

If your interest is in one-way ANOVA, you may find the oneway command to be more convenient; see $[R]$ oneway.

Structural equation modeling provides a more general framework for fitting ANOVA models; see the Stata Structural Equation Modeling Reference Manual.

ANOVA was pioneered by Fisher. It features prominently in his texts on statistical methods and his design of experiments $(1925,1935)$. Many books discuss ANOVA; see, for instance, Altman (1991); van Belle et al. (2004); Cobb (1998); Snedecor and Cochran (1989); or Winer, Brown, and Michels (1991). For a classic source, see Scheffé (1959). Kennedy and Gentle (1980) discuss ANOVA's computing problems. Edwards (1985) is concerned primarily with the relationship between multiple regression and ANOVA. Acock (2014, chap. 9) illustrates his discussion with Stata output. Repeated-measures anOva is discussed in Winer, Brown, and Michels (1991); Kuehl (2000); and Milliken and Johnson (2009). Pioneering work in repeated-measures ANOVA can be found in Box (1954); Geisser and Greenhouse (1958); Huynh and Feldt (1976); and Huynh (1978). For a Stata-specific discussion of ANOVA contrasts, see Mitchell (2012, chap. 7-9).

## One-way ANOVA

anova, entered without options, performs and reports standard ANOVA. For instance, to perform a one-way layout of a variable called endog on exog, you would type anova endog exog.
> Example 1: One-way ANOVA
We run an experiment varying the amount of fertilizer used in growing apple trees. We test four concentrations, using each concentration in three groves of 12 trees each. Later in the year, we measure the average weight of the fruit.

If all had gone well, we would have had 3 observations on the average weight for each of the four concentrations. Instead, two of the groves were mistakenly leveled by a confused man on a large bulldozer. We are left with the following data:

```
. use http://www.stata-press.com/data/r13/apple
(Apple trees)
. list, abbrev(10) sepby(treatment)
```

|  | treatment | weight |
| :---: | ---: | ---: |
| 1. | 1 | 117.5 |
| 2. | 1 | 113.8 |
| 3. | 1 | 104.4 |
| 4. | 2 | 48.9 |
| 5. | 2 | 50.4 |
| 6. | 2 | 58.9 |
| 7. | 3 | 70.4 |
| 8. | 3 | 86.9 |
| 9. | 4 | 87.7 |
| 10. | 4 | 67.3 |

To obtain one-way ANOVA results, we type

```
. anova weight treatment
```



We find significant (at better than the $1 \%$ level) differences among the four concentrations.
Although the output is a usual ANOVA table, let's run through it anyway. Above the table is a summary of the underlying regression. The model was fit on 10 observations, and the root mean squared error (Root MSE) is 9.07 . The $R^{2}$ for the model is 0.9147 , and the adjusted $R^{2}$ is 0.8721 .

The first line of the table summarizes the model. The sum of squares (Partial SS) for the model is 5295.5 with 3 degrees of freedom (df). This line results in a mean square (MS) of 5295.5/3 $\approx 1765.2$. The corresponding $F$ statistic is 21.46 and has a significance level of 0.0013 . Thus the model appears to be significant at the $0.13 \%$ level.

The next line summarizes the first (and only) term in the model, treatment. Because there is only one term, the line is identical to that for the overall model.

The third line summarizes the residual. The residual sum of squares is 493.59 with 6 degrees of freedom, resulting in a mean squared error of 82.27 . The square root of this latter number is reported as the Root MSE.

The model plus the residual sum of squares equals the total sum of squares, which is reported as 5789.1 in the last line of the table. This is the total sum of squares of weight after removal of the mean. Similarly, the model plus the residual degrees of freedom sum to the total degrees of freedom, 9. Remember that there are 10 observations. Subtracting 1 for the mean, we are left with 9 total degrees of freedom.

## - Technical note

Rather than using the anova command, we could have performed this analysis by using the oneway command. Example 1 in $[\mathrm{R}]$ oneway repeats this same analysis. You may wish to compare the output.

Type regress to see the underlying regression model corresponding to an ANOVA model fit using the anova command.

## > Example 2: Regression table from a one-way ANOVA

Returning to the apple tree experiment, we found that the fertilizer concentration appears to significantly affect the average weight of the fruit. Although that finding is interesting, we next want to know which concentration appears to grow the heaviest fruit. One way to find out is by examining the underlying regression coefficients.
. regress, baselevels

| Source | SS | df | MS |
| ---: | ---: | ---: | :---: |
| Model | 5295.54433 | 3 | 1765.18144 |
| Residual | 493.591667 | 6 | 82.2652778 |
| Total | 5789.136 | 9 | 643.237333 |


| Number of obs | $=$ | 10 |
| :--- | ---: | ---: |
| $\mathrm{~F}(3$, | $6)$ | $=21.46$ |
| Prob $>$ F | $=$ | 0.0013 |
| R-squared | $=$ | 0.9147 |
| Adj R-squared | $=$ | 0.8721 |
| Root MSE | $=$ | 9.07 |


| weight | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | :---: |
| treatment |  |  |  |  |  |  |
| 1 | 0 | (base) |  |  |  |  |
| 2 | -59.16667 | 7.405641 | -7.99 | 0.000 | -77.28762 | -41.04572 |
| 3 | -33.25 | 8.279758 | -4.02 | 0.007 | -53.50984 | -12.99016 |
| 4 | -34.4 | 8.279758 | -4.15 | 0.006 | -54.65984 | -14.14016 |
|  |  | 111.9 | 5.236579 | 21.37 | 0.000 | 99.08655 |
| _cons |  |  |  |  |  |  |

See $[R]$ regress for an explanation of how to read this table. The baselevels option of regress displays a row indicating the base category for our categorical variable, treatment. In summary, we find that concentration 1, the base (omitted) group, produces significantly heavier fruits than concentration 2, 3 , and 4 ; concentration 2 produces the lightest fruits; and concentrations 3 and 4 appear to be roughly equivalent.

## > Example 3: ANOVA replay

We previously typed anova weight treatment to produce and display the ANOVA table for our apple tree experiment. Typing regress displays the regression coefficients. We can redisplay the ANOVA table by typing anova without arguments:


## Two-way ANOVA

You can include multiple explanatory variables with the anova command, and you can specify interactions by placing '\#' between the variable names. For instance, typing anova y a b performs a two-way layout of y on a and b . Typing anova $\mathrm{y} a \mathrm{~b} \mathrm{a}$ \#b performs a full two-way factorial layout. The shorthand anova y a\#\#b does the same.

With the default partial sums of squares, when you specify interacted terms, the order of the terms does not matter. Typing anova y a b a\#b is the same as typing anova y bab\#a.

## > Example 4: Two-way factorial ANOVA

The classic two-way factorial ANOVA problem, at least as far as computer manuals are concerned, is a two-way ANOVA design from Afifi and Azen (1979).

Fifty-eight patients, each suffering from one of three different diseases, were randomly assigned to one of four different drug treatments, and the change in their systolic blood pressure was recorded. Here are the data:

|  | Disease 1 | Disease 2 | Disease 3 |
| :--- | :--- | :--- | :--- |
| Drug 1 | $42,44,36$ | $33,26,33$ | $31,-3,25$ |
|  | $13,19,22$ | 21 | 25,24 |
| Drug 2 | $28,23,34$ | $34,33,31$ | $3,26,28$ |
|  | 42,13 | 36 | $32,4,16$ |
| Drug 3 | $1,29,19$ | $11,9,7$ | $21,1,9$ |
|  |  | $1,-6$ | 3 |
| Drug 4 | $24,9,22$ | $27,12,12$ | $22,7,25$ |
|  | $-2,15$ | $-5,16,15$ | 5,12 |

Let's assume that we have entered these data into Stata and stored the data as systolic.dta. Below we use the data, list the first 10 observations, summarize the variables, and tabulate the control variables:

```
. use http://www.stata-press.com/data/r13/systolic
(Systolic Blood Pressure Data)
. list in 1/10
```

|  | drug | disease | systolic |
| ---: | ---: | ---: | ---: |
| 1. | 1 | 1 | 42 |
| 2. | 1 | 1 | 44 |
| 3. | 1 | 1 | 36 |
| 4. | 1 | 1 | 13 |
| 5. | 1 | 1 | 19 |
| 6. | 1 | 1 | 22 |
| 7. | 1 | 2 | 33 |
| 8. | 1 | 2 | 26 |
| 9. | 1 | 2 | 33 |
| 10. | 1 | 2 | 21 |

. summarize

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| drug | 58 | 2.5 | 1.158493 | 1 | 4 |
| disease | 58 | 2.017241 | .8269873 | 1 | 3 |
| systolic | 58 | 18.87931 | 12.80087 | -6 | 44 |

. tabulate drug disease

|  | Patient's Disease |  |  |  |
| ---: | :---: | :---: | :---: | ---: |
| Drug Used | 1 | 2 | 3 | Total |
| 1 | 6 | 4 | 5 | 15 |
| 2 | 5 | 4 | 6 | 15 |
| 3 | 3 | 5 | 4 | 12 |
| 4 | 5 | 6 | 5 | 16 |
| Total | 19 | 19 | 20 | 58 |

Each observation in our data corresponds to one patient, and for each patient we record drug, disease, and the increase in the systolic blood pressure, systolic. The tabulation reveals that the data are not balanced-there are not equal numbers of patients in each drug-disease cell. Stata does not require that the data be balanced. We can perform a two-way factorial ANOVA by typing


Although Stata's table command does not perform ANOVA, it can produce useful summary tables of your data (see [R] table):

```
. table drug disease, c(mean systolic) row col f(%8.2f)
```

|  | Patient's |  |  |  |
| ---: | ---: | ---: | ---: | ---: |
|  | 1 | 2 | 3 | Total |
|  |  |  |  |  |
| 1 | 29.33 | 28.25 | 20.40 | 26.07 |
| 2 | 28.00 | 33.50 | 18.17 | 25.53 |
| 3 | 16.33 | 4.40 | 8.50 | 8.75 |
| 4 | 13.60 | 12.83 | 14.20 | 13.50 |
|  |  |  |  |  |
| Total | 22.79 | 18.21 | 15.80 | 18.88 |

These are simple means and are not influenced by our anova model. More useful is the margins command (see [R] margins) that provides marginal means and adjusted predictions. Because drug is the only significant factor in our ANOVA, we now examine the adjusted marginal means for drug.

| . margins drug, asbalanced |  |  |
| :--- | :--- | :--- |
| Adjusted predictions | Number of obs |  |
| Expression Linear prediction, predict()  <br> at (asbalanced)  <br>  drug (asbalanced) |  |  |


|  | Delta-method |  |  |  |  |  |
| :---: | ---: | :---: | :---: | :---: | :---: | ---: |
|  | Margin | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| drug |  |  |  |  |  |  |
| 1 | 25.99444 | 2.751008 | 9.45 | 0.000 | 20.45695 | 31.53194 |
| 2 | 26.55556 | 2.751008 | 9.65 | 0.000 | 21.01806 | 32.09305 |
| 3 | 9.744444 | 3.100558 | 3.14 | 0.003 | 3.503344 | 15.98554 |
| 4 | 13.54444 | 2.637123 | 5.14 | 0.000 | 8.236191 | 18.8527 |

These adjusted marginal predictions are not equal to the simple drug means (see the total column from the table command); they are based upon predictions from our ANOVA model. The asbalanced option of margins corresponds with the interpretation of the $F$ statistic produced by ANOVA-each cell is given equal weight regardless of its sample size (see the following three technical notes). You
can omit the asbalanced option and obtain predictive margins that take into account the unequal sample sizes of the cells.

| . margins drug <br> Predictive margins <br> Expression <br> : Linear prediction, predict() |
| :--- |

## - Technical note

How do you interpret the significance of terms like drug and disease in unbalanced data? If you are familiar with SAS, the sums of squares and the $F$ statistic reported by Stata correspond to SAS type III sums of squares. (Stata can also calculate sequential sums of squares, but we will postpone that topic for now.)

Let's think in terms of the following table:

|  | Disease 1 | Disease 2 | Disease 3 |  |
| :--- | :---: | :---: | :---: | :---: |
| Drug 1 | $\mu_{11}$ | $\mu_{12}$ | $\mu_{13}$ | $\mu_{1}$. |
| Drug 2 | $\mu_{21}$ | $\mu_{22}$ | $\mu_{23}$ | $\mu_{2}$. |
| Drug 3 | $\mu_{31}$ | $\mu_{32}$ | $\mu_{33}$ | $\mu_{3 .}$ |
| Drug 4 | $\mu_{41}$ | $\mu_{42}$ | $\mu_{43}$ | $\mu_{4 .}$ |
|  | $\mu_{\cdot 1}$ | $\mu_{\cdot 2}$ | $\mu_{\cdot 3}$ | $\mu_{\cdot .}$ |

In this table, $\mu_{i j}$ is the mean increase in systolic blood pressure associated with drug $i$ and disease $j$, while $\mu_{i}$. is the mean for drug $i, \mu_{\cdot j}$ is the mean for disease $j$, and $\mu$.. is the overall mean.

If the data are balanced, meaning that there are equal numbers of observations going into the calculation of each mean $\mu_{i j}$, the row means, $\mu_{i}$, are given by

$$
\mu_{i .}=\frac{\mu_{i 1}+\mu_{i 2}+\mu_{i 3}}{3}
$$

In our case, the data are not balanced, but we define the $\mu_{i}$. according to that formula anyway. The test for the main effect of drug is the test that

$$
\mu_{1 .}=\mu_{2 .}=\mu_{3 .}=\mu_{4}
$$

To be absolutely clear, the $F$ test of the term drug, called the main effect of drug, is formally equivalent to the test of the three constraints:

$$
\begin{aligned}
& \frac{\mu_{11}+\mu_{12}+\mu_{13}}{3}=\frac{\mu_{21}+\mu_{22}+\mu_{23}}{3} \\
& \frac{\mu_{11}+\mu_{12}+\mu_{13}}{3}=\frac{\mu_{31}+\mu_{32}+\mu_{33}}{3} \\
& \frac{\mu_{11}+\mu_{12}+\mu_{13}}{3}=\frac{\mu_{41}+\mu_{42}+\mu_{43}}{3}
\end{aligned}
$$

In our data, we obtain a significant $F$ statistic of 9.05 and thus reject those constraints.

## - Technical note

Stata can display the symbolic form underlying the test statistics it presents, as well as display other test statistics and their symbolic forms; see Obtaining symbolic forms in $[\mathrm{R}]$ anova postestimation. Here is the result of requesting the symbolic form for the main effect of drug in our data:

```
. test drug, symbolic
drug
        1 -(r2+r3+r4)
        r2
        r3
        r4
disease
        1 0
        2 0
        3 0
drug#disease
    1 1 -1/3 (r2+r3+r4)
        1 2 -1/3 (r2+r3+r4)
        1 3 -1/3 (r2+r3+r4)
        2 1 1/3 r2
        2 2 1/3 r2
        2 3 1/3 r2
        3 1 1/3 r3
        3 2 1/3 r3
        3 3 1/3 r3
        4 1 1/3 r4
        4 2 1/3 r4
        4 3 1/3 r4
_cons 0
```

This says exactly what we said in the previous technical note.

## - Technical note

Saying that there is no main effect of a variable is not the same as saying that it has no effect at all. Stata's ability to perform ANOVA on unbalanced data can easily be put to ill use.

For example, consider the following table of the probability of surviving a bout with one of two diseases according to the drug administered to you:

|  | Disease 1 | Disease 2 |
| :--- | :---: | :---: |
| Drug 1 | 1 | 0 |
| Drug 2 | 0 | 1 |

If you have disease 1 and are administered drug 1, you live. If you have disease 2 and are administered drug 2, you live. In all other cases, you die.

This table has no main effects of either drug or disease, although there is a large interaction effect. You might now be tempted to reason that because there is only an interaction effect, you would be indifferent between the two drugs in the absence of knowledge about which disease infects you. Given an equal chance of having either disease, you reason that it does not matter which drug is administered to you-either way, your chances of surviving are 0.5 .

You may not, however, have an equal chance of having either disease. If you knew that disease 1 was 100 times more likely to occur in the population, and if you knew that you had one of the two diseases, you would express a strong preference for receiving drug 1.

When you calculate the significance of main effects on unbalanced data, you must ask yourself why the data are unbalanced. If the data are unbalanced for random reasons and you are making predictions for a balanced population, the test of the main effect makes perfect sense. If, however, the data are unbalanced because the underlying populations are unbalanced and you are making predictions for such unbalanced populations, the test of the main effect may be practically-if not statistically -meaningless.

## > Example 5: ANOVA with missing cells

Stata can perform ANOVA not only on unbalanced populations, but also on populations that are so unbalanced that entire cells are missing. For instance, using our systolic blood pressure data, let's refit the model eliminating the drug 1 -disease 1 cell. Because anova follows the same syntax as all other Stata commands, we can explicitly specify the data to be used by typing the if qualifier at the end of the anova command. Here we want to use the data that are not for drug 1 and disease 1 :


Here we used drug\#\#disease as a shorthand for drug disease drug\#disease.

## - Technical note

The test of the main effect of drug in the presence of missing cells is more complicated than that for unbalanced data. Our underlying tableau now has the following form:

|  | Disease 1 | Disease 2 | Disease 3 |  |
| :--- | :---: | :---: | :---: | :---: |
| Drug 1 |  | $\mu_{12}$ | $\mu_{13}$ |  |
| Drug 2 | $\mu_{21}$ | $\mu_{22}$ | $\mu_{23}$ | $\mu_{2}$. |
| Drug 3 | $\mu_{31}$ | $\mu_{32}$ | $\mu_{33}$ | $\mu_{3}$. |
| Drug 4 | $\mu_{41}$ | $\mu_{42}$ | $\mu_{43}$ | $\mu_{4}$. |
|  |  | $\mu_{\cdot 2}$ | $\mu_{\cdot 3}$ |  |

The hole in the drug 1 -disease 1 cell indicates that the mean is unobserved. Considering the main effect of drug, the test is unchanged for the rows in which all the cells are defined:

$$
\mu_{2}=\mu_{3 .}=\mu_{4}
$$

The first row, however, requires special attention. Here we want the average outcome for drug 1, which is averaged only over diseases 2 and 3, to be equal to the average values of all other drugs averaged over those same two diseases:

$$
\frac{\mu_{12}+\mu_{13}}{2}=\frac{\left(\mu_{22}+\mu_{23}\right) / 2+\left(\mu_{32}+\mu_{33}\right) / 2+\left(\mu_{42}+\mu_{43}\right) / 2}{3}
$$

Thus the test contains three constraints:

$$
\begin{array}{rcc}
\frac{\mu_{21}+\mu_{22}+\mu_{23}}{3} & = & \frac{\mu_{31}+\mu_{32}+\mu_{33}}{3} \\
\frac{\mu_{21}+\mu_{22}+\mu_{23}}{3} & = & \frac{\mu_{41}+\mu_{42}+\mu_{43}}{3} \\
\frac{\mu_{12}+\mu_{13}}{2} & = & \frac{\mu_{22}+\mu_{23}+\mu_{32}+\mu_{33}+\mu_{42}+\mu_{43}}{6}
\end{array}
$$

Stata can calculate two types of sums of squares, partial and sequential. If you do not specify which sums of squares to calculate, Stata calculates partial sums of squares. The technical notes above have gone into great detail about the definition and use of partial sums of squares. Use the sequential option to obtain sequential sums of squares.

## - Technical note

Before we illustrate sequential sums of squares, consider one more feature of the partial sums. If you know how such things are calculated, you may worry that the terms must be specified in some particular order, that Stata would balk or, even worse, produce different results if you typed, say, anova drug\#disease drug disease rather than anova drug disease drug\#disease. We assure you that is not the case.

When you type a model, Stata internally reorganizes the terms, forms the cross-product matrix, inverts it, converts the result to an upper-Hermite form, and then performs the hypothesis tests. As a final touch, Stata reports the results in the same order that you typed the terms.

## > Example 6: Sequential sums of squares

We wish to estimate the effects on systolic blood pressure of drug and disease by using sequential sums of squares. We want to introduce disease first, then drug, and finally, the interaction of drug and disease:


The $F$ statistic on disease is now 2.21 . When we fit this same model by using partial sums of squares, the statistic was 1.88 .

## N-way ANOVA

You may include high-order interaction terms, such as a third-order interaction between the variables $\mathrm{A}, \mathrm{B}$, and C, by typing A\#B\#C.

## > Example 7: Three-way factorial ANOVA

We wish to determine the operating conditions that maximize yield for a manufacturing process. There are three temperature settings, two chemical supply companies, and two mixing methods under investigation. Three observations are obtained for each combination of these three factors.

| . use http://www.stata-press.com/data/r13/manuf |
| :--- |
| (manufacturing process data) |
| . describe |
| Contains data <br> obs: <br> vars: <br> size: |

We wish to perform a three-way factorial ANOVA. We could type

```
. anova yield temp chem temp#chem meth temp#meth chem#meth temp#chem#meth
```

but prefer to use the \#\# factor-variable operator for brevity.
anova yield temp\#\#chem\#\#meth


The interaction between temperature and method appears to be the important story in these data. A table of means for this interaction is given below.
. table method temp, c(mean yield) row col f(\%8.2f)

| mixing <br> method | machine temperature setting |  |  |  |
| ---: | ---: | ---: | ---: | ---: |
|  | low | medium | high | Total |
| stir | 7.50 | 6.00 | 6.00 | 6.50 |
| fold | 5.50 | 9.00 | 11.50 | 8.67 |
| Total | 6.50 | 7.50 | 8.75 | 7.58 |

Here our ANOVA is balanced (each cell has the same number of observations), and we obtain the same values as in the table above (but with additional information such as confidence intervals) by using the margins command. Because our ANOVA is balanced, using the asbalanced option with margins would not produce different results. We request the predictive margins for the two terms that appear significant in our ANOVA: temperature\#method and method.

| Predictive margins |  |  |  | Number of obs |  | 36 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Expression : Linear prediction, predict() |  |  |  |  |  |  |
|  | Delta-method |  |  | $p>\|t\|$ | [95\% Conf | Interval] |
|  | Margin | Std. Err. | t |  |  |  |
| temperature\# method |  |  |  |  |  |  |
| low\#stir | 7.5 | 1.073675 | 6.99 | 0.000 | 5.284044 | 9.715956 |
| low\#fold | 5.5 | 1.073675 | 5.12 | 0.000 | 3.284044 | 7.715956 |
| medium\#stir | 6 | 1.073675 | 5.59 | 0.000 | 3.784044 | 8.215956 |
| medium\#fold | 9 | 1.073675 | 8.38 | 0.000 | 6.784044 | 11.21596 |
| high\#stir | 6 | 1.073675 | 5.59 | 0.000 | 3.784044 | 8.215956 |
| high\#fold | 11.5 | 1.073675 | 10.71 | 0.000 | 9.284044 | 13.71596 |
| method |  |  |  |  |  |  |
| stir | 6.5 | . 6198865 | 10.49 | 0.000 | 5.220617 | 7.779383 |
| fold | 8.666667 | . 6198865 | 13.98 | 0.000 | 7.387284 | 9.946049 |

We decide to use the folding method of mixing and a high temperature in our manufacturing process.

## Weighted data

Like all estimation commands, anova can produce estimates on weighted data. See [U] 11.1.6 weight for details on specifying the weight.

## > Example 8: Three-way factorial ANOVA on grouped data

We wish to investigate the prevalence of byssinosis, a form of pneumoconiosis that can afflict workers exposed to cotton dust. We have data on 5,419 workers in a large cotton mill. We know whether each worker smokes, his or her race, and the dustiness of the work area. The variables are

| smokes | smoker or nonsmoker in the last five years |
| :--- | :--- |
| race | white or other |
| workplace | 1 (most dusty), 2 (less dusty), 3 (least dusty) |

We wish to fit an ANOVA model explaining the prevalence of byssinosis according to a full factorial model of smokes, race, and workplace.

The data are unbalanced. Moreover, although we have data on 5,419 workers, the data are grouped according to the explanatory variables, along with some other variables, resulting in 72 observations. For each observation, we know the number of workers in the group (pop), the prevalence of byssinosis (prob), and the values of the three explanatory variables. Thus we wish to fit a three-way factorial model on grouped data.

We begin by showing a bit of the data, which are from Higgins and Koch (1977).

```
. use http://www.stata-press.com/data/r13/byssin
(Byssinosis incidence)
. describe
Contains data from http://www.stata-press.com/data/r13/byssin.dta
    obs: 72 Byssinosis incidence
    vars: 5 19 Dec 2012 07:04
    size: 864
```

| variable name | storage <br> type | display <br> format | value <br> label | variable label |
| :--- | :--- | :--- | :--- | :--- |
| smokes | int | $\% 8.0 \mathrm{~g}$ | smokes | Smokes |
| race | int | $\% 8.0 \mathrm{~g}$ | race | Race |
| workplace | int | $\% 8.0 \mathrm{~g}$ | workplace |  |
| pop | int | $\% 8.0 \mathrm{~g}$ |  | Dustiness of workplace <br> prob |
| float | $\% 9.0 \mathrm{~g}$ |  | Prevalence of byssinosis |  |

Sorted by:
. list in 1/5, abbrev(10) divider

| smokes | race | workplace | pop | prob |
| :---: | :---: | ---: | ---: | ---: |
| yes | white | most | 40 | .075 |
| yes | white | less | 74 | 0 |
| yes | white | least | 260 | .0076923 |
| yes | other | most | 164 | .152439 |
| yes | other | less | 88 | 0 |

The first observation in the data represents a group of 40 white workers who smoke and work in a "most" dusty work area. Of those 40 workers, $7.5 \%$ have byssinosis. The second observation represents a group of 74 white workers who also smoke but who work in a "less" dusty environment. None of those workers has byssinosis.

Almost every Stata command allows weights. Here we want to weight the data by pop. We can, for instance, make a table of the number of workers by their smoking status and race:
. tabulate smokes race [fw=pop]

| Race |  |  |  |
| ---: | ---: | ---: | ---: |
| Smokes | Rather | white | Total |
| no | 799 | 1,431 | 2,230 |
| yes | 1,104 | 2,085 | 3,189 |
| Total | 1,903 | 3,516 | 5,419 |

The [ $f w=p o p$ ] at the end of the tabulate command tells Stata to count each observation as representing pop persons. When making the tally, tabulate treats the first observation as representing 40 workers, the second as representing 74 workers, and so on.

Similarly, we can make a table of the dustiness of the workplace:

| Dustiness of workplace | Freq. | Percent | Cum. |
| :---: | :---: | :---: | :---: |
| least | 3,450 | 63.66 | 63.66 |
| less | 1,300 | 23.99 | 87.65 |
| most | 669 | 12.35 | 100.00 |
| Total | 5,419 | 100.00 |  |

We can discover the average incidence of byssinosis among these workers by typing

| . summarize prob [fw=pop] |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Variable | Obs | Mean | Std. Dev. | Min | Max |
| prob | 5419 | .0304484 | .0567373 | 0 | .287037 |

We discover that $3.04 \%$ of these workers have byssinosis. Across all cells, the byssinosis rates vary from 0 to $28.7 \%$. Just to prove that there might be something here, let's obtain the average incidence rates according to the dustiness of the workplace:
. table workplace smokes race [fw=pop], c(mean prob)

| Dustiness of workplace | Race and Smokes <br> other $\qquad$ $\qquad$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| least | . 0107527 | . 0101523 | . 0081549 | . 0162774 |
| less | . 02 | . 0081633 | . 0136612 | . 0143149 |
| most | . 0820896 | . 1679105 | . 0833333 | . 2295082 |

Let's now fit the ANOVA model.


Of course, if we want to see the underlying regression, we could type regress.
Above we examined simple means of the cells of workplace\#smokes\#race. Our ANOVA shows workplace, smokes, and their interaction as being the only significant factors in our model. We now examine the predictive marginal mean byssinosis rates for these terms.

| Predictive margins |  |  |  | Number of obs |  | 65 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Expression : Linear prediction, predict() |  |  |  |  |  |  |
|  | Delta-method |  |  | $P>\|t\|$ | [95\% Conf. Interval] |  |
|  | Margin | Std. Err. | t |  |  |  |  |
| workplace\# smokes |  |  |  |  |  |  |
| least\#no | . 0090672 | . 0062319 | 1.45 | 0.152 | -. 0034323 | . 0215667 |
| least\#yes | . 0141264 | . 0053231 | 2.65 | 0.010 | . 0034497 | . 0248032 |
| less\#no | . 0158872 | . 009941 | 1.60 | 0.116 | -. 0040518 | . 0358263 |
| less\#yes | . 0121546 | . 0087353 | 1.39 | 0.170 | -. 0053662 | . 0296755 |
| most\#no | . 0828966 | . 0182151 | 4.55 | 0.000 | . 0463617 | . 1194314 |
| most\#yes | . 2078768 | . 012426 | 16.73 | 0.000 | . 1829533 | . 2328003 |
| workplace |  |  |  |  |  |  |
| least | . 0120701 | . 0040471 | 2.98 | 0.004 | . 0039526 | . 0201875 |
| less | . 0137273 | . 0065685 | 2.09 | 0.041 | . 0005526 | . 0269019 |
| most | . 1566225 | . 0104602 | 14.97 | 0.000 | . 1356419 | . 177603 |
| smokes |  |  |  |  |  |  |
| no | . 0196915 | . 0050298 | 3.91 | 0.000 | . 0096029 | . 02978 |
| yes | . 0358626 | . 0041949 | 8.55 | 0.000 | . 0274488 | . 0442765 |

Smoking combined with the most dusty workplace produces the highest byssinosis rates.

Ronald Aylmer Fisher (1890-1962) (Sir Ronald from 1952) studied mathematics at Cambridge. Even before he finished his studies, he had published on statistics. He worked as a statistician at Rothamsted Experimental Station (1919-1933), as professor of eugenics at University College London (1933-1943), as professor of genetics at Cambridge (1943-1957), and in retirement at the CSIRO Division of Mathematical Statistics in Adelaide. His many fundamental and applied contributions to statistics and genetics mark him as one of the greatest statisticians of all time, including original work on tests of significance, distribution theory, theory of estimation, fiducial inference, and design of experiments.

## ANCOVA

You can include multiple explanatory variables with the anova command, but unless you explicitly state otherwise by using the c . factor-variable operator, all the variables are interpreted as categorical variables. Using the c. operator, you can designate variables as continuous and thus perform ANCOVA.
> Example 9: ANCOVA (ANOVA with a continuous covariate)
We have census data recording the death rate (drate) and median age (age) for each state. The dataset also includes the region of the country in which each state is located (region):

```
. use http://www.stata-press.com/data/r13/census2
(1980 Census data by state)
. summarize drate age region
```

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | :---: | :---: | :---: | :---: | ---: |
| drate | 50 | 84.3 | 13.07318 | 40 | 107 |
| age | 50 | 29.5 | 1.752549 | 24 | 35 |
| region | 50 | 2.66 | 1.061574 | 1 | 4 |

age is coded in integral years from 24 to 35 , and region is coded from 1 to 4 , with 1 standing for the Northeast, 2 for the North Central, 3 for the South, and 4 for the West.

When we examine the data more closely, we discover large differences in the death rate across regions of the country:

| . tabulate region, summarize(drate) |  |  |  |
| ---: | ---: | ---: | ---: |
| Census <br> region | Summary of Death Rate <br> Mean <br> Std. Dev. |  | Freq. |
| NE | 93.444444 | 7.0553368 | 9 |
| N Cntrl | 88.916667 | 5.5833899 | 12 |
| South | 88.3125 | 8.5457104 | 16 |
| West | 68.769231 | 13.342625 | 13 |
| Total | 84.3 | 13.073185 | 50 |

Naturally, we wonder if these differences might not be explained by differences in the median ages of the populations. To find out, we fit a regression model (via anova) of drate on region and age. In the anova example below, we treat age as a categorical variable.

```
. anova drate region age
```

|  | Number of obs Root MSE | 50 R-squared |  |  | $\begin{array}{ll} = & 0.7927 \\ = & 0.7328 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Source | Partial SS | df | MS | F | Prob > F |
| Model | 6638.86529 | 11 | 603.533208 | 13.21 | 0.0000 |
| region | 1320.00973 | 3 | 440.003244 | 9.63 | 0.0001 |
| age | 2237.24937 | 8 | 279.656171 | 6.12 | 0.0000 |
| Residual | 1735.63471 | 38 | 45.6745977 |  |  |
| Total | 8374.5 | 49 | 170.908163 |  |  |

We have the answer to our question: differences in median ages do not eliminate the differences in death rates across the four regions. The ANOVA table summarizes the two terms in the model, region and age. The region term contains 3 degrees of freedom, and the age term contains 8 degrees of freedom. Both are significant at better than the $1 \%$ level.

The age term contains 8 degrees of freedom. Because we did not explicitly indicate that age was to be treated as a continuous variable, it was treated as categorical, meaning that unique coefficients were estimated for each level of age. The only clue of this labeling is that the number of degrees of freedom associated with the age term exceeds 1 . The labeling becomes more obvious if we review the regression coefficients:
. regress, baselevels

| Source | SS | df | MS |
| ---: | ---: | :---: | :---: |
| Model | 6638.86529 | 11 | 603.533208 |
| Residual | 1735.63471 | 38 | 45.6745977 |
| Total | 8374.5 | 49 | 170.908163 |

Number of obs $=50$
$F(11,38)=13.21$
Prob $>\mathrm{F}=0.0000$
$R$-squared $=0.7927$
Adj R-squared $=0.7328$
Root MSE $=6.7583$

| drate | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| :---: | ---: | :---: | ---: | :---: | ---: | ---: |
| region |  |  |  |  |  |  |
| NE | 0 | (base) |  |  |  |  |
| N Cntrl | .4428387 | 3.983664 | 0.11 | 0.912 | -7.621668 | 8.507345 |
| South | -.2964637 | 3.934766 | -0.08 | 0.940 | -8.261981 | 7.669054 |
| West | -13.37147 | 4.195344 | -3.19 | 0.003 | -21.8645 | -4.878439 |
|  |  |  |  |  |  |  |
| age | 0 | (base) |  |  |  |  |
| 24 | -15 | 9.557677 | -1.57 | 0.125 | -34.34851 | 4.348506 |
| 26 | 14.30833 | 7.857378 | 1.82 | 0.076 | -1.598099 | 30.21476 |
| 27 | 12.66011 | 7.495513 | 1.69 | 0.099 | -2.51376 | 27.83399 |
| 28 | 18.861 | 7.28918 | 2.59 | 0.014 | 4.104825 | 33.61717 |
| 29 | 20.87003 | 7.210148 | 2.89 | 0.006 | 6.273847 | 35.46621 |
| 30 | 29.91307 | 8.242741 | 3.63 | 0.001 | 13.22652 | 46.59963 |
| 31 | 27.02853 | 8.509432 | 3.18 | 0.003 | 9.802089 | 44.25498 |
| 32 | 38.925 | 9.944825 | 3.91 | 0.000 | 18.79275 | 59.05724 |
| 35 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| cons | 68.37147 | 7.95459 | 8.60 | 0.000 | 52.26824 | 84.47469 |

The regress command displayed the anova model as a regression table. We used the baselevels option to display the dropped level (or base) for each term.

If we want to treat age as a continuous variable, we must prepend c. to age in our anova.

|  | Number of obs Root MSE | $\begin{aligned} & = \\ & =7 . \end{aligned}$ | $\begin{array}{r} 50 \\ 21483 \end{array}$ | R-squared <br> Adj R-squared | $\begin{aligned} & =0.7203 \\ & =0.6954 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Source | Partial SS | df | MS | F | Prob > F |
| Model | 6032.08254 | 4 | 1508.02064 | $4 \quad 28.97$ | 0.0000 |
| region | 1645.66228 | 3 | 548.554092 | $2 \quad 10.54$ | 0.0000 |
| age | 1630.46662 | 1 | 1630.46662 | 231.32 | 0.0000 |
| Residual | 2342.41746 | 45 | 52.0537213 |  |  |
| Total | 8374.5 | 49 | 170.908163 |  |  |

The age term now has 1 degree of freedom. The regression coefficients are
. regress, baselevels

| Source | SS | df | MS |
| ---: | ---: | ---: | :---: |
| Model | 6032.08254 | 4 | 1508.02064 |
| Residual | 2342.41746 | 45 | 52.0537213 |
| Total | 8374.5 | 49 | 170.908163 |


| Number of obs | $=$ | 50 |
| :--- | ---: | ---: |
| $\mathrm{~F}(4, \quad 45)$ | $=$ | 28.97 |
| Prob $>$ F | $=$ | 0.0000 |
| R-squared | $=$ | 0.7203 |
| Adj R-squared | $=$ | 0.6954 |
| Root MSE | $=$ | 7.2148 |


| drate | Coef. | Std. Err. | t | $\mathrm{P}>\mid \mathrm{tl}$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| region |  |  |  |  |  |  |
| NE | 0 | (base) |  |  |  |  |
| N Cntrl | 1.792526 | 3.375925 | 0.53 | 0.598 | -5.006935 | 8.591988 |
| South | .6979912 | 3.18154 | 0.22 | 0.827 | -5.70996 | 7.105942 |
| West | -13.37578 | 3.723447 | -3.59 | 0.001 | -20.87519 | -5.876377 |
|  |  |  |  |  |  |  |
| age | 3.922947 | .7009425 | 5.60 | 0.000 | 2.511177 | 5.334718 |
| _cons | -28.60281 | 21.93931 | -1.30 | 0.199 | -72.79085 | 15.58524 |

Although we started analyzing these data to explain the regional differences in death rate, let's focus on the effect of age for a moment. In our first model, each level of age had a unique death rate associated with it. For instance, the predicted death rate in a north central state with a median age of 28 was

$$
0.44+12.66+68.37 \approx 81.47
$$

whereas the predicted death rate from our current model is

$$
1.79+3.92 \times 28-28.60 \approx 82.95
$$

Our previous model had an $R^{2}$ of 0.7927 , whereas our current model has an $R^{2}$ of 0.7203 . This "small" loss of predictive power accompanies a gain of 7 degrees of freedom, so we suspect that the continuous-age model is as good as the discrete-age model.

## - Technical note

There is enough information in the two ANOVA tables to attach a statistical significance to our suspicion that the loss of predictive power is offset by the savings in degrees of freedom. Because the continuous-age model is nested within the discrete-age model, we can perform a standard Chow test. For those of us who know such formulas off the top of our heads, the $F$ statistic is

$$
\frac{(2342.41746-1735.63471) / 7}{45.6745977}=1.90
$$

There is, however, a better way.
We can find out whether our continuous model is as good as our discrete model by putting age in the model twice: once as a continuous variable and once as a categorical variable. The categorical variable will then measure deviations around the straight line implied by the continuous variable, and the $F$ test for the significance of the categorical variable will test whether those deviations are jointly zero.
. anova drate region c.age age
Number of obs $=\quad 50 \quad$ R-squared $=0.7927$
Root MSE $=6.7583 \quad$ Adj R -squared $=0.7328$

| Source | Partial SS | df | MS | F | Prob > F |
| ---: | ---: | ---: | :---: | ---: | ---: |
| Model | 6638.86529 | 11 | 603.533208 | 13.21 | 0.0000 |
|  |  |  |  |  |  |
| region | 1320.00973 | 3 | 440.003244 | 9.63 | 0.0001 |
| age | 699.74137 | 1 | 699.74137 | 15.32 | 0.0004 |
| age | 606.782747 | 7 | 86.6832496 | 1.90 | 0.0970 |
| Residual | 1735.63471 | 38 | 45.6745977 |  |  |
| Total | 8374.5 | 49 | 170.908163 |  |  |

We find that the $F$ test for the significance of the (categorical) age variable is 1.90 , just as we calculated above. It is significant at the $9.7 \%$ level. If we hold to a $5 \%$ significance level, we cannot reject the null hypothesis that the effect of age is linear.

## > Example 10: Interaction of continuous and categorical variables

In our census data, we still find significant differences across the regions after controlling for the median age of the population. We might now wonder whether the regional differences are differences in level-independent of age - or are instead differences in the regional effects of age. Just as we can interact categorical variables with other categorical variables, we can interact categorical variables with continuous variables.

|  | Number of obs Root MSE | $\begin{aligned} & = \\ & =7 . \end{aligned}$ | $\begin{array}{rl} 50 & \mathrm{R} \\ 24852 & \mathrm{~A} \end{array}$ | R-squared <br> Adj R-squared | $\begin{aligned} & =0.7365 \\ & =0.6926 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Source | Partial SS | df | MS | F | Prob > F |
| Model | 6167.7737 | 7 | 881.110529 | $9 \quad 16.77$ | 0.0000 |
| region | 188.713602 | 3 | 62.9045339 | $9 \quad 1.20$ | 0.3225 |
| age | 873.425599 | 1 | 873.425599 | $9 \quad 16.62$ | 0.0002 |
| region\#age | 135.691162 | 3 | 45.2303874 | $4 \quad 0.86$ | 0.4689 |
| Residual | 2206.7263 | 42 | 52.5411023 |  |  |
| Total | 8374.5 | 49 | 170.908163 |  |  |

The region\#c. age term in our model measures the differences in slopes across the regions. We cannot reject the null hypothesis that there are no such differences. The region effect is now "insignificant". This status does not mean that there are no regional differences in death rates because each test is a marginal or partial test. Here, with region\#c. age included in the model, region is being tested at the point where age is zero. Apart from this value not existing in the dataset, it is also a long way from the mean value of age, so the test of region at this point is meaningless (although it is valid if you acknowledge what is being tested).

To obtain a more sensible test of region, we can subtract the mean from the age variable and use this in the model.

```
. anova drate region c.mage region#c.mage
```

| Source | $\begin{array}{llr} \text { Number of obs } & = & 50 \\ \text { Root MSE } & =7.24852 \end{array}$ |  |  | R -squared | $\begin{aligned} & =0.7365 \\ & =0.6926 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Partial SS | df | MS | F | Prob > F |
| Model | 6167.7737 | 7 | 881.110529 | 16.77 | 0.0000 |
| region | 1166.14735 | 3 | 388.715783 | 7.40 | 0.0004 |
| mage | 873.425599 | 1 | 873.425599 | 16.62 | 0.0002 |
| region\#mage | 135.691162 | 3 | 45.2303874 | 0.86 | 0.4689 |
| Residual | 2206.7263 | 42 | 52.5411023 |  |  |
| Total | 8374.5 | 49 | 170.908163 |  |  |

region is significant when tested at the mean of the age variable.

Remember that we can specify interactions by typing varname\#varname. We have seen examples of interacting categorical variables with categorical variables and, in the examples above, a categorical variable (region) with a continuous variable (age or mage).

We can also interact continuous variables with continuous variables. To include an age ${ }^{2}$ term in our model, we could type c.age\#c.age. If we also wanted to interact the categorical variable region with the age ${ }^{2}$ term, we could type region\#c.age\#c.age (or even c.age\#region\#c.age).

## Nested designs

In addition to specifying interaction terms, nested terms can also be specified in an ANOVA. A vertical bar is used to indicate nesting: A|B is read as A nested within B. A|B|C is read as A nested within $B$, which is nested within $C . A \mid B \# C$ is read as $A$ is nested within the interaction of $B$ and $C$. $A \# B \mid C$ is read as the interaction of $A$ and $B$, which is nested within $C$.

Different error terms can be specified for different parts of the model. The forward slash is used to indicate that the next term in the model is the error term for what precedes it. For instance, anova y $\mathrm{A} / \mathrm{B} \mid \mathrm{A}$ indicates that the $F$ test for A is to be tested by using the mean square from $\mathrm{B} \mid \mathrm{A}$ in the denominator. Error terms (terms following the slash) are generally not tested unless they are themselves followed by a slash. Residual error is the default error term.

For example, consider $A / B / C$, where $A, B$, and $C$ may be arbitrarily complex terms. Then anova will report $A$ tested by $B$ and $B$ tested by $C$. If we add one more slash on the end to form $A / B / C /$, then anova will also report $C$ tested by the residual error.

## $\lambda$ Example 11: Simple nested ANOVA

We have collected data from a manufacturer that is evaluating which of five different brands of machinery to buy to perform a particular function in an assembly line. Twenty assembly-line employees were selected at random for training on these machines, with four employees assigned to learn a particular machine. The output from each employee (operator) on the brand of machine for which he trained was measured during four trial periods. In this example, the operator is nested within machine. Because of sickness and employee resignations, the final data are not balanced. The following table gives the mean output and sample size for each machine and operator combination.

```
. use http://www.stata-press.com/data/r13/machine, clear
(machine data)
```



| five brands of machine | operator nested in machine |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | Total |
| 1 | 9.15 | 9.48 | 8.27 | 8.20 | 8.75 |
|  | 2 | 4 | 3 | 4 | 13 |
| 2 | 15.03 | 11.55 | 11.45 | 11.52 | 12.47 |
|  | 3 | 2 | 2 |  | 11 |
| 3 | 11.27 | 10.13 | 11.13 |  | 10.84 |
|  | 3 | 3 | 3 |  | 9 |
| 4 | 16.10 | 18.97 | 15.35 | 16.60 | 16.65 |
|  | 3 | 3 | 4 | 3 | 13 |
| 5 | 15.30 | 14.35 | 10.43 |  | 13.63 |
|  | 4 | 4 | 3 |  | 11 |

Assuming that operator is random (that is, we wish to infer to the larger population of possible operators) and machine is fixed (that is, only these five machines are of interest), the typical test for machine uses operator nested within machine as the error term. operator nested within machine can be tested by residual error. Our earlier warning concerning designs with either unplanned missing cells or unbalanced cell sizes, or both, also applies to interpreting the ANOVA results from this unbalanced nested example.

|  | $\begin{array}{llr} \text { Number of obs } & = & 57 \\ \text { Root MSE } & =1.47089 \end{array}$ |  |  | R-squared <br> Adj R-squared | $\begin{aligned} & =0.8661 \\ & =0.8077 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Source | Partial SS | df | MS | F | Prob > F |
| Model | 545.822288 | 17 | 32.1071934 | $4 \quad 14.84$ | 0.0000 |
| machine | 430.980792 | 4 | 107.745198 | $8 \quad 13.82$ | 0.0001 |
| operator\|machine | 101.353804 | 13 | 7.79644648 |  |  |
| operator\|machine | 101.353804 | 13 | 7.79644648 | $8 \quad 3.60$ | 0.0009 |
| Residual | 84.3766582 | 39 | 2.16350406 |  |  |
| Total | 630.198947 | 56 | 11.2535526 |  |  |

operator|machine is preceded by a slash, indicating that it is the error term for the terms before it (here machine). operator|machine is also followed by a slash that indicates it should be tested with residual error. The output lists the operator|machine term twice, once as the error term for machine, and again as a term tested by residual error. A line is placed in the ANOVA table to separate the two. In general, a dividing line is placed in the output to separate the terms into groups that are tested with the same error term. The overall model is tested by residual error and is separated from the rest of the table by a blank line at the top of the table.

The results indicate that the machines are not all equal and that there are significant differences between operators.

## > Example 12: ANOVA with multiple levels of nesting

Your company builds and operates sewage treatment facilities. You want to compare two particulate solutions during the particulate reduction step of the sewage treatment process. For each solution, two area managers are randomly selected to implement and oversee the change to the new treatment process in two of their randomly chosen facilities. Two workers at each of these facilities are trained to operate the new process. A measure of particulate reduction is recorded at various times during the month at each facility for each worker. The data are described below.

```
. use http://www.stata-press.com/data/r13/sewage
(Sewage treatment)
. describe
Contains data from http://www.stata-press.com/data/r13/sewage.dta
    obs: 64 Sewage treatment
    vars: 5 9 May 2013 12:43
    size: 320
```

| variable name | storage <br> type | display <br> format | value <br> label | variable label |
| :--- | :---: | :--- | :--- | :--- |
| particulate | byte | $\% 9.0 \mathrm{~g}$ |  | particulate reduction |
| solution | byte | $\% 9.0 \mathrm{~g}$ |  | 2 particulate solutions |
| manager | byte | $\% 9.0 \mathrm{~g}$ |  | 2 managers per solution |
| facility | byte | $\% 9.0 \mathrm{~g}$ |  | 2 facilities per manager |
| worker | byte | $\% 9.0 \mathrm{~g}$ |  | 2 workers per facility |

Sorted by: solution manager facility worker
You want to determine if the two particulate solutions provide significantly different particulate reduction. You would also like to know if manager, facility, and worker are significant effects. solution is a fixed factor, whereas manager, facility, and worker are random factors.

In the following anova command, we use abbreviations for the variable names, which can sometimes make long ANOVA model statements easier to read.
. anova particulate $s / m|s / f| m|s / w| f|m| s /, ~ d r o p e m p t y c e l l s$

|  | Number of obs Root MSE | 64 R-squared |  |  | $\begin{aligned} & =0.6338 \\ & =0.5194 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Source | Partial SS | df | MS | F | Prob > F |
| Model | 13493.6094 | 15 | 899.573958 | 5.54 | 0.0000 |
| solution | 7203.76563 | 1 | 7203.76563 | 17.19 | 0.0536 |
| manager\|solution | 838.28125 | 2 | 419.140625 |  |  |
| manager\|solution facility|manager| | 838.28125 | 2 | 419.140625 | 0.55 | 0.6166 |
| facility\|manager| solution | 3064.9375 | 4 | 766.234375 |  |  |
| facility\|manager| |  |  |  |  |  |
| solution | 3064.9375 | 4 | 766.234375 | 2.57 | 0.1193 |
| worker\|facility| manager|solution | 2386.625 | 8 | 298.328125 |  |  |
| worker\|facilityl |  |  |  |  |  |
| manager\|solution | 2386.625 | 8 | 298.328125 | 1.84 | 0.0931 |
| Residual | 7796.25 | 48 | 162.421875 |  |  |
| Total | 21289.8594 | 63 | 337.934276 |  |  |

While solution is not declared significant at the $5 \%$ significance level, it is near enough to that threshold to warrant further investigation (see example 3 in $[R]$ anova postestimation for a continuation of the analysis of these data).

## - Technical note

Why did we use the dropemptycells option with the previous anova? By default, Stata retains empty cells when building the design matrix and currently treats $\mid$ and \# the same in how it determines the possible number of cells. Retaining empty cells in an ANOVA with nested terms can cause your design matrix to become too large. In example 12, there are $1024=2 \times 4 \times 8 \times 16$ cells that are considered possible for the worker|facility|manager|solution term because the worker, facility, and manager variables are uniquely numbered. With the dropemptycells option, the worker|facility|manager|solution term requires just 16 columns in the design matrix (corresponding to the 16 unique workers).

Why did we not use the dropemptycells option in example 11, where operator is nested in machine? If you look at the table presented at the beginning of that example, you will see that operator is compactly instead of uniquely numbered (you need both operator number and machine number to determine the operator). Here the dropemptycells option would have only reduced our design matrix from 26 columns down to 24 columns (because there were only 3 operators instead of 4 for machines 3 and 5).

We suggest that you specify dropemptycells when there are nested terms in your ANOVA. You could also use the set emptycells drop command to accomplish the same thing; see [R] set.

## Mixed designs

An ANOVA can consist of both nested and crossed terms. A split-plot ANOVA design provides an example.

## > Example 13: Split-plot ANOVA

Two reading programs and three skill-enhancement techniques are under investigation. Ten classes of first-grade students were randomly assigned so that five classes were taught with one reading program and another five classes were taught with the other. The 30 students in each class were divided into six groups with 5 students each. Within each class, the six groups were divided randomly so that each of the three skill-enhancement techniques was taught to two of the groups within each class. At the end of the school year, a reading assessment test was administered to all the students. In this split-plot ANOVA, the whole-plot treatment is the two reading programs, and the split-plot treatment is the three skill-enhancement techniques.

```
. use http://www.stata-press.com/data/r13/reading
(Reading experiment data)
```

| ```Contains data obs: vars: size:``` | $\begin{gathered} \text { from htt } \\ 300 \\ 5 \\ 1,500 \end{gathered}$ | //www.st | a-press | data/r13/reading.dta <br> Reading experiment data <br> 9 Mar 2013 18:57 <br> (_dta has notes) |
| :---: | :---: | :---: | :---: | :---: |
| variable name | storage type | display <br> format | value <br> label | variable label |
| score | byte | $\% 9.0 \mathrm{~g}$ |  | reading score |
| program | byte | \% 9.0g |  | reading program |
| class | byte | \% 9.0g |  | class nested in program |
| skill | byte | \%9.0g |  | skill enhancement technique |
| group | byte | \%9.0g |  | group nested in class and skill |

Sorted by:
In this split-plot ANOVA, the error term for program is class nested within program. The error term for skill and the program by skill interaction is the class by skill interaction nested within program. Other terms are also involved in the model and can be seen below.

Our anova command is too long to fit on one line of this manual. Where we have chosen to break the command into multiple lines is arbitrary. If we were typing this command into Stata, we would just type along and let Stata automatically wrap across lines, as necessary.


The program\#skill term is significant, as is the program term. Let's look at the predictive margins for these two terms and at a marginsplot for the first term.

| Predictive margins |  |  |  | Number of obs |  | 300 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Expression : Linear prediction, predict() <br> within : program skill <br> Empty cells : reweight |  |  |  |  |  |  |
|  | Delta-method |  |  | $P>\|t\|$ | [95\% Conf. | Interval] |
| program\#skill |  |  |  |  |  |  |
| 11 | 68.16 | 2.068542 | 32.95 | 0.000 | 64.08518 | 72.23482 |
| 12 | 52.86 | 2.068542 | 25.55 | 0.000 | 48.78518 | 56.93482 |
| 13 | 61.54 | 2.068542 | 29.75 | 0.000 | 57.46518 | 65.61482 |
| 21 | 50.7 | 2.068542 | 24.51 | 0.000 | 46.62518 | 54.77482 |
| 22 | 56.54 | 2.068542 | 27.33 | 0.000 | 52.46518 | 60.61482 |
| 23 | 52.1 | 2.068542 | 25.19 | 0.000 | 48.02518 | 56.17482 |

```
. marginsplot, plot2opts(lp(dash) m(D)) plot3opts(lp(dot) m(T))
    Variables that uniquely identify margins: program skill
```




Because our ANOVA involves nested terms, we used the within() option of margins; see [R] margins.
skill 2 produces a low score when combined with program 1 and a high score when combined with program 2, demonstrating the interaction between the reading program and the skill-enhancement
technique. You might conclude that the first reading program and the first skill-enhancement technique perform best when combined. However, notice the overlapping confidence interval for the first reading program and the third skill-enhancement technique.

## - Technical note

There are several valid ways to write complicated anova terms. In the reading experiment example (example 13), we had a term grouplclass\#skill|program. This term can be read as group nested within both class and skill and further nested within program. You can also write this term as grouplclass\#skill\#program or grouplprogram\#class\#skill or grouplskill\#class|program, etc. All variations will produce the same result. Some people prefer having only one ' 1 ' in a term and would use grouplclass\#skill\#program, which is read as group nested within class, skill, and program.

Gertrude Mary Cox (1900-1978) was born on a farm near Dayton, Iowa. Initially intending to become superintendent of an orphanage, she enrolled at Iowa State College. There she majored in mathematics and attained the college's first Master's degree in statistics. After working on her PhD in psychological statistics for two years at the University of California-Berkeley, she decided to go back to Iowa State to work with George W. Snedecor. There she pursued her interest in and taught a course in design of experiments. That work led to her collaboration with W. G. Cochran, which produced a classic text. In 1940, when Snedecor shared with her his list of men he was nominating to head the statistics department at North Carolina State College, she wanted to know why she had not been included. He added her name, she won the position, and she built an outstanding department at North Carolina State. Cox retired early so she could work at the Research Triangle Institute in North Carolina. She consulted widely, served as editor of Biometrics, and was elected to the National Academy of Sciences.

## Latin-square designs

You can use anova to analyze a Latin-square design. Consider the following example, published in Snedecor and Cochran (1989).
> Example 14: Latin-square ANOVA
Data from a Latin-square design are as follows:

| Row | Column 1 | Column 2 | Column 3 | Column 4 | Column 5 |
| ---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $257(\mathrm{~B})$ | $230(\mathrm{E})$ | $279(\mathrm{~A})$ | $287(\mathrm{C})$ | $202(\mathrm{D})$ |
| 2 | $245(\mathrm{D})$ | $283(\mathrm{~A})$ | $245(\mathrm{E})$ | $280(\mathrm{~B})$ | $260(\mathrm{C})$ |
| 3 | $182(\mathrm{E})$ | $252(\mathrm{~B})$ | $280(\mathrm{C})$ | $246(\mathrm{D})$ | $250(\mathrm{~A})$ |
| 4 | $203(\mathrm{~A})$ | $204(\mathrm{C})$ | $227(\mathrm{D})$ | $193(\mathrm{E})$ | $259(\mathrm{~B})$ |
| 5 | $231(\mathrm{C})$ | $271(\mathrm{D})$ | $266(\mathrm{~B})$ | $334(\mathrm{~A})$ | $338(\mathrm{E})$ |

In Stata, the data might appear as follows:
. use http://www.stata-press.com/data/r13/latinsq
. list

|  | row | $c 1$ | $c 2$ | $c 3$ | $c 4$ | $c 5$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
|  | c4 | 1 | 257 | 230 | 279 | 287 |
| 1. | 2 | 245 | 283 | 245 | 280 | 260 |
| 2. | 3 | 182 | 252 | 280 | 246 | 250 |
| 4. | 4 | 203 | 204 | 227 | 193 | 259 |
| 5. | 5 | 231 | 271 | 266 | 334 | 338 |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |

Before anova can be used on these data, the data must be organized so that the outcome measurement is in one column. reshape is inadequate for this task because there is information about the treatments in the sequence of these observations. pkshape is designed to reshape this type of data; see $[R]$ pkshape.

```
. pkshape row row c1-c5, order(beacd daebc ebcda acdeb cdbae)
. list
```

|  | sequence | outcome | treat | carry | period |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | 1 | 257 | 1 | 0 | 1 |
| 2. | 2 | 245 | 5 | 0 | 1 |
| 3. | 3 | 182 | 2 | 0 | 1 |
| 4. | 4 | 203 | 3 | 0 | 1 |
| 5. | 5 | 231 | 4 | 0 | 1 |
| 6. | 1 | 230 | 2 | 1 | 2 |
| 7. | 2 | 283 | 3 | 5 | 2 |
| 8. | 3 | 252 | 1 | 2 | 2 |
| 9. | 4 | 204 | 4 | 3 | 2 |
| 10. | 5 | 271 | 5 | 4 | 2 |
| 11. | 1 | 279 | 3 | 2 | 3 |
| 12. | 2 | 245 | 2 | 3 | 3 |
| 13. | 3 | 280 | 4 | 1 | 3 |
| 14. | 4 | 227 | 5 | 4 | 3 |
| 15. | 5 | 266 | 1 | 5 | 3 |
| 16. | 1 | 287 | 4 | 3 | 4 |
| 17. | 2 | 280 | 1 | 2 | 4 |
| 18. | 3 | 246 | 5 | 4 | 4 |
| 19. | 4 | 193 | 2 | 5 | 4 |
| 20. | 5 | 334 | 3 | 1 | 4 |
| 21. | 1 | 202 | 5 | 4 | 5 |
| 22. | 2 | 260 | 4 | 1 | 5 |
| 23. | 3 | 250 | 3 | 5 | 5 |
| 24. | 4 | 259 | 1 | 2 | 5 |
| 25. | 5 | 338 | 2 | 3 | 5 |

. anova outcome sequence period treat


These methods will work with any type of Latin-square design, including those with replicated measurements. For more information, see $[R] \mathbf{p k},[R]$ pkcross, and $[R]$ pkshape.

## Repeated-measures ANOVA

One approach for analyzing repeated-measures data is to use multivariate ANOVA (MANOVA); see [MV] manova. In this approach, the data are placed in wide form (see [D] reshape), and the repeated measures enter the MANOVA as dependent variables.

A second approach for analyzing repeated measures is to use anova. However, one of the underlying assumptions for the $F$ tests in ANOVA is independence of observations. In a repeated-measures design, this assumption is almost certainly violated or is at least suspect. In a repeated-measures ANOVA, the subjects (or whatever the experimental units are called) are observed for each level of one or more of the other categorical variables in the model. These variables are called the repeated-measure variables. Observations from the same subject are likely to be correlated.

The approach used in repeated-measures ANOVA to correct for this lack of independence is to apply a correction to the degrees of freedom of the $F$ test for terms in the model that involve repeated measures. This correction factor, $\epsilon$, lies between the reciprocal of the degrees of freedom for the repeated term and 1 . Box (1954) provided the pioneering work in this area. Milliken and Johnson (2009) refer to the lower bound of this correction factor as Box's conservative correction factor. Winer, Brown, and Michels (1991) call it simply the conservative correction factor.

Geisser and Greenhouse (1958) provide an estimate for the correction factor called the GreenhouseGeisser $\epsilon$. This value is estimated from the data. Huynh and Feldt (1976) show that the GreenhouseGeisser $\epsilon$ tends to be conservatively biased. They provide a revised correction factor called the Huynh-Feldt $\epsilon$. When the Huynh-Feldt $\epsilon$ exceeds 1, it is set to 1 . Thus there is a natural ordering for these correction factors:

Box's conservative $\epsilon \leq$ Greenhouse-Geisser $\epsilon \leq$ Huynh-Feldt $\epsilon \leq 1$
A correction factor of 1 is the same as no correction.
anova with the repeated() option computes these correction factors and displays the revised test results in a table that follows the standard ANOVA table. In the resulting table, H-F stands for Huynh-Feldt, G-G stands for Greenhouse-Geisser, and Box stands for Box's conservative $\epsilon$.

## > Example 15: Repeated-measures ANOVA

This example is taken from table 4.3 of Winer, Brown, and Michels (1991). The reaction time for five subjects each tested with four drugs was recorded in the variable score. Here is a table of the data (see $[\mathrm{P}]$ tabdisp if you are unfamiliar with tabdisp):

```
. use http://www.stata-press.com/data/r13/t43, clear
(T4.3 -- Winer, Brown, Michels)
. tabdisp person drug, cellvar(score)
```

|  | drug |  |  |  |
| ---: | ---: | ---: | ---: | ---: |
| person | 1 | 2 | 3 | 4 |
| 1 | 30 | 28 | 16 | 34 |
| 2 | 14 | 18 | 10 | 22 |
| 3 | 24 | 20 | 18 | 30 |
| 4 | 38 | 34 | 20 | 44 |
| 5 | 26 | 28 | 14 | 30 |

drug is the repeated variable in this simple repeated-measures ANOVA example. The ANOVA is specified as follows:


Here the Huynh-Feldt $\epsilon$ is 1.0789 , which is larger than 1. It is reset to 1 , which is the same as making no adjustment to the standard test computed in the main ANOVA table. The Greenhouse-Geisser $\epsilon$ is 0.6049 , and its associated $p$-value is computed from an $F$ ratio of 24.76 using $1.8147(=3 \epsilon)$ and $7.2588(=12 \epsilon)$ degrees of freedom. Box's conservative $\epsilon$ is set equal to the reciprocal of the degrees of freedom for the repeated term. Here it is $1 / 3$, so Box's conservative test is computed using 1 and 4 degrees of freedom for the observed $F$ ratio of 24.76.

Even for Box's conservative $\epsilon$, drug is significant with a $p$-value of 0.0076 . The following table gives the predictive marginal mean score (that is, response time) for each of the four drugs:

| . margins drug <br> Predictive margins <br> Expression <br> : Linear prediction, predict() |
| :--- |

The ANOVA table for this example provides an $F$ test for person, but you should ignore it. An appropriate test for person would require replication (that is, multiple measurements for person and drug combinations). Also, without replication there is no test available for investigating the interaction between person and drug.

## > Example 16: Repeated-measures ANOVA with nesting

Table 7.7 of Winer, Brown, and Michels (1991) provides another repeated-measures ANOVA example. There are four dial shapes and two methods for calibrating dials. Subjects are nested within calibration method, and an accuracy score is obtained. The data are shown below.

```
. use http://www.stata-press.com/data/r13/t77
(T7.7 -- Winer, Brown, Michels)
. tabdisp shape subject calib, cell(score)
```

| 4 dial shapes | 2 methods for calibrating dials and subject nested in calib |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 1 | 2 | 3 |
| 1 | 0 | 3 | 4 | 4 | 5 | 7 |
| 2 | 0 | 1 | 3 | 2 | 4 | 5 |
| 3 | 5 | 5 | 6 | 7 | 6 | 8 |
| 4 | 3 | 4 | 2 | 8 | 6 | 9 |

The calibration method and dial shapes are fixed factors, whereas subjects are random. The appropriate test for calibration method uses the nested subject term as the error term. Both the dial shape and the interaction between dial shape and calibration method are tested with the dial shape by subject interaction nested within calibration method. Here we drop this term from the anova command, and it becomes residual error. The dial shape is the repeated variable because each subject is tested with all four dial shapes. Here is the anova command that produces the desired results:

```
. anova score calib / subject|calib shape calib#shape, repeated(shape)
```

Number of obs $=\quad 24 \quad$ R-squared $=0.8925$
Root MSE $=1.11181 \quad$ Adj R-squared $=0.7939$

| Source | Partial SS | df | MS | F | Prob > F |
| ---: | ---: | ---: | :---: | :---: | ---: |
| Model | 123.125 | 11 | 11.1931818 | 9.06 | 0.0003 |
| calib | 51.0416667 | 1 | 51.0416667 | 11.89 | 0.0261 |
| subject\|calib | 17.1666667 | 4 | 4.29166667 |  |  |
| shape | 47.4583333 | 3 | 15.8194444 | 12.80 | 0.0005 |
| calib\#shape | 7.45833333 | 3 | 2.48611111 | 2.01 | 0.1662 |
| Residual | 14.8333333 | 12 | 1.23611111 |  |  |
| Total | 137.958333 | 23 | 5.99818841 |  |  |

Between-subjects error term: subject|calib
Levels: 6 (4 df)
Lowest b.s.e. variable: subject
Covariance pooled over: calib (for repeated variable)
Repeated variable: shape
Huynh-Feldt epsilon $=0.8483$ Greenhouse-Geisser epsilon $=0.4751$ Box's conservative epsilon $=0.3333$

| Source | df | F | Regular | $\mathrm{H}-\mathrm{F}$ | G-G | Box |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| shape | 3 | 12.80 | 0.0005 | 0.0011 | 0.0099 | 0.0232 |
| calib\#shape | 3 | 2.01 | 0.1662 | 0.1791 | 0.2152 | 0.2291 |
| Residual | 12 |  |  |  |  |  |

The repeated-measure $\epsilon$ corrections are applied to any terms that are tested in the main ANOVA table and have the repeated variable in the term. These $\epsilon$ corrections are given in a table below the main ANOVA table. Here the repeated-measures tests for shape and calib\#shape are presented.

Calibration method is significant, as is dial shape. The interaction between calibration method and dial shape is not significant. The repeated-measure $\epsilon$ corrections do not change these conclusions, but they do change the significance level for the tests on shape and calib\#shape. Here, though, unlike in the example 15, the Huynh-Feldt $\epsilon$ is less than 1.

Here are the predictive marginal mean scores for calibration method and dial shapes. Because the interaction was not significant, we request only the calib and shape predictive margins.

```
. margins, within(calib)
Predictive margins Number of obs = 24
Expression : Linear prediction, predict()
within : calib
Empty cells : reweight
```

|  | MarginDelta-method <br> Std. Err.$\quad \mathrm{t}$ | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |  |  |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| calib |  |  |  |  |  |  |
| 1 | 3 | .3209506 | 9.35 | 0.000 | 2.300709 | 3.699291 |
| 2 | 5.916667 | .3209506 | 18.43 | 0.000 | 5.217375 | 6.615958 |



## Technical note

The computation of the Greenhouse-Geisser and Huynh-Feldt epsilons in a repeated-measures ANOVA requires the number of levels and degrees of freedom for the between-subjects error term, as well as a value computed from a pooled covariance matrix. The observations are grouped based on all but the lowest-level variable in the between-subjects error term. The covariance over the repeated variables is computed for each resulting group, and then these covariance matrices are pooled. The dimension of the pooled covariance matrix is the number of levels of the repeated variable (or combination of levels for multiple repeated variables). In example 16, there are four levels of the repeated variable (shape), so the resulting covariance matrix is $4 \times 4$.

The anova command automatically attempts to determine the between-subjects error term and the lowest-level variable in the between-subjects error term to group the observations for computation of the pooled covariance matrix. anova issues an error message indicating that the bse() or bseunit () option is required when anova cannot determine them. You may override the default selections of anova by specifying the bse(), bseunit(), or grouping() option. The term specified in the bse() option must be a term in the ANOVA model.

The default selection for the between-subjects error term (the bse() option) is the interaction of the nonrepeated categorical variables in the ANOVA model. The first variable listed in the between-subjects error term is automatically selected as the lowest-level variable in the between-subjects error term but can be overridden with the bseunit (varname) option. varname is often a term, such as subject or subsample within subject, and is most often listed first in the term because of the nesting notation of ANOVA. This term makes sense in most repeated-measures ANOVA designs when the terms of the model are written in standard form. For instance, in example 16, there were three categorical variables (subject, calib, and shape), with shape being the repeated variable. Here anova looked for a term involving only subject and calib to determine the between-subjects error term. It found subject|calib as the term with six levels and 4 degrees of freedom. anova then picked subject as the default for the bseunit() option (the lowest variable in the between-subjects error term) because it was listed first in the term.

The grouping of observations proceeds, based on the different combinations of values of the variables in the between-subjects error term, excluding the lowest level variable (as found by default or as specified with the bseunit() option). You may specify the grouping() option to change the default grouping used in computing the pooled covariance matrix.

The between-subjects error term, number of levels, degrees of freedom, lowest variable in the term, and grouping information are presented after the main ANOVA table and before the rest of the repeated-measures output.

## > Example 17: Repeated-measures ANOVA with two repeated variables

Data with two repeated variables are given in table 7.13 of Winer, Brown, and Michels (1991). The accuracy scores of subjects making adjustments to three dials during three different periods are recorded. Three subjects are exposed to a certain noise background level, whereas a different set of three subjects is exposed to a different noise background level. Here is a table of accuracy scores for the noise, subject, period, and dial variables:

| noise <br> background and subject nested in noise | 1 | 1 | $\frac{0 \mathrm{mi}}{3}$ | tin 1 | $\begin{gathered} \text { peri } \\ 2 \\ 2 \end{gathered}$ | $\frac{3}{3}$ | ${ }^{1}$ | $\begin{array}{r} 3 \\ 2 \end{array}$ | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |  |  |  |  |  |
| 1 | 45 | 53 | 60 | 40 | 52 | 57 | 28 | 37 | 46 |
| 2 | 35 | 41 | 50 | 30 | 37 | 47 | 25 | 32 | 41 |
| 3 | 60 | 65 | 75 | 58 | 54 | 70 | 40 | 47 | 50 |
| 2 |  |  |  |  |  |  |  |  |  |
| 1 | 50 | 48 | 61 | 25 | 34 | 51 | 16 | 23 | 35 |
| 2 | 42 | 45 | 55 | 30 | 37 | 43 | 22 | 27 | 37 |
| 3 | 56 | 60 | 77 | 40 | 39 | 57 | 31 | 29 | 46 |

noise, period, and dial are fixed, whereas subject is random. Both period and dial are repeated variables. The ANOVA for this example is specified next.
. anova score noise / subject|noise period noise\#period /
> period\#subject|noise dial noise\#dial /
> dial\#subject|noise period\#dial noise\#period\#dial, repeated(period dial)

| Number of obs | $=54$ |  |
| :--- | :--- | :--- |
| R-squared | $=0.9872$ |  |
| Root MSE | $=2.81859$ |  |
| Adj R-squared | $=0.9576$ |  |


| Source | Partial SS | df | MS | F | Prob > F |
| ---: | ---: | ---: | ---: | ---: | ---: |
| Model | 9797.72222 | 37 | 264.803303 | 33.33 | 0.0000 |
| noise | 468.166667 | 1 | 468.166667 | 0.75 | 0.4348 |
| subject\|noise | 2491.11111 | 4 | 622.777778 |  |  |
| period | 3722.33333 | 2 | 1861.16667 | 63.39 | 0.0000 |
| 333 | 2 | 166.5 | 5.67 | 0.0293 |  |
| period\#subject\|noise | 234.888889 | 8 | 29.3611111 |  |  |
| dial | 2370.33333 | 2 | 1185.16667 | 89.82 | 0.0000 |
| noise\#period |  |  |  |  |  |
| dial\#subject\|noise | 105.555556 | 8 | 13.1944444 | 1.91 | 0.2102 |
| noise\#dial | 50.3333333 | 2 | 25.1666667 |  |  |
| period\#dial | 10.6666667 | 4 | 2.66666667 | 0.34 | 0.8499 |
| noise\#period\#dial | 11.3333333 | 4 | 2.83333333 | 0.36 | 0.8357 |
| Residual | 127.111111 | 16 | 7.94444444 |  |  |
| Total | 9924.83333 | 53 | 187.261006 |  |  |

Between-subjects error term: subject|noise
Levels: 6 (4 df)
Lowest b.s.e. variable: subject
Covariance pooled over: noise (for repeated variables)
Repeated variable: period
Huynh-Feldt epsilon $=1.0668$
*Huynh-Feldt epsilon reset to 1.0000
Greenhouse-Geisser epsilon $=0.6476$
Box's conservative epsilon $=0.5000$

| Source | df | F | - Prob > F |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Regular | H-F | G-G | Box |
| period | 2 | 63.39 | 0.0000 | 0.0000 | 0.0003 | 0.0013 |
| noise\#period | 2 | 5.67 | 0.0293 | 0.0293 | 0.0569 | 0.0759 |
| period\#subject\|noise | 8 |  |  |  |  |  |

Repeated variable: dial
Huynh-Feldt epsilon $=2.0788$
*Huynh-Feldt epsilon reset to 1.0000
Greenhouse-Geisser epsilon $=0.9171$
Box's conservative epsilon $=0.5000$

|  | Source |  |  |  |  |  |  | df | F | Regular | H-F | G-G | Box |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| dial | 2 | 89.82 | 0.0000 | 0.0000 | 0.0000 | 0.0007 |  |  |  |  |  |  |  |
| noise\#dial | 2 | 1.91 | 0.2102 | 0.2102 | 0.2152 | 0.2394 |  |  |  |  |  |  |  |
| dial\#subject\|noise | 8 |  |  |  |  |  |  |  |  |  |  |  |  |



For each repeated variable and for each combination of interactions of repeated variables, there are different $\epsilon$ correction values. The anova command produces tables for each applicable combination.

The two most significant factors in this model appear to be dial and period. The noise by period interaction may also be significant, depending on the correction factor you use. Below is a table of predictive margins for the accuracy score for dial, period, and noise by period.

```
. margins, within(dial)
Predictive margins Number of obs = 54
Expression : Linear prediction, predict()
within : dial
Empty cells : reweight
```

|  | Delta-method <br> Margin <br> Std. Err. |  |  |  |  |  |  |  | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| dial |  |  |  |  |  |  |  |  |  |  |  |
| 1 | 37.38889 | .6643478 | 56.28 | 0.000 | 35.98053 | 38.79724 |  |  |  |  |  |
| 2 | 42.22222 | .6643478 | 63.55 | 0.000 | 40.81387 | 43.63058 |  |  |  |  |  |
| 3 | 53.22222 | .6643478 | 80.11 | 0.000 | 51.81387 | 54.63058 |  |  |  |  |  |

. margins, within(period)
Predictive margins $\quad$ Number of obs $=54$
Expression : Linear prediction, predict()
within : period
Empty cells : reweight

|  | Delta-method |  |  |  |  |  |
| ---: | ---: | :---: | :---: | :---: | :---: | :---: |
|  | Margin | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| period |  |  |  |  |  |  |
| 1 | 54.33333 | .6643478 | 81.78 | 0.000 | 52.92498 | 55.74169 |
| 2 | 44.5 | .6643478 | 66.98 | 0.000 | 43.09165 | 45.90835 |
| 3 | 34 | .6643478 | 51.18 | 0.000 | 32.59165 | 35.40835 |


| Predictive margins |  |  |  | Number of obs |  | 54 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ```Expression : Linear prediction, predict() within : noise period Empty cells : reweight``` |  |  |  |  |  |  |
|  | Delta-method |  |  | $P>\|t\|$ | [95\% Conf. | Interval] |
| noise\#period |  |  |  |  |  |  |
| 11 | 53.77778 | . 9395297 | 57.24 | 0.000 | 51.78606 | 55.76949 |
| 12 | 49.44444 | . 9395297 | 52.63 | 0.000 | 47.45273 | 51.43616 |
| 13 | 38.44444 | . 9395297 | 40.92 | 0.000 | 36.45273 | 40.43616 |
| 21 | 54.88889 | . 9395297 | 58.42 | 0.000 | 52.89717 | 56.8806 |
| 22 | 39.55556 | . 9395297 | 42.10 | 0.000 | 37.56384 | 41.54727 |
| 23 | 29.55556 | . 9395297 | 31.46 | 0.000 | 27.56384 | 31.54727 |

Dial shape 3 produces the highest score, and scores decrease over the periods.

Example 17 had two repeated-measurement variables. Up to four repeated-measurement variables may be specified in the anova command.

## Video examples

Analysis of covariance in Stata
Two-way ANOVA in Stata

## Stored results

anova stores the following in e() :

Scalars
e(N)
e(mss)
e(df_m)
e(rss)
e(df_r)
e(r2)
e(r2_a)
e(F)
e(rmse)
e(11)
e(11_0)
e(ss_\#)
e(df_\#)
e(ssdenom_\#)
e(dfdenom_\#)
e(F_\#)
e(N_bse)
e(df_bse)
e(box\#)
e(gg\#)
e(hf\#)
$e$ (rank)
Macros
e(cmd)
e(cmdline)
e(depvar)
$e$ (varnames)
e(term_\#)
e(errorterm_\#)
e(sstype)
e(repvars)
e(repvar\#)
e(model)
e(wtype)
e(wexp)
e(properties)
e(estat_cmd)
$e$ (predict)
$e$ (asbalanced)
e(asobserved)
Matrices
e(b)
e(V)
e(Srep)
Functions
e(sample)
number of observations
model sum of squares
model degrees of freedom
residual sum of squares
residual degrees of freedom
$R$-squared
adjusted $R$-squared
$F$ statistic
root mean squared error
log likelihood
$\log$ likelihood, constant-only model
sum of squares for term \#
numerator degrees of freedom for term \#
denominator sum of squares for term \# (when using nonresidual error)
denominator degrees of freedom for term \# (when using nonresidual error)
$F$ statistic for term \# (if computed)
number of levels of the between-subjects error term
degrees of freedom for the between-subjects error term
Box's conservative epsilon for a particular combination of repeated variables (repeated() only)
Greenhouse-Geisser epsilon for a particular combination of repeated variables (repeated() only)
Huynh-Feldt epsilon for a particular combination of repeated variables
(repeated() only)
rank of $e(V)$
anova
command as typed
name of dependent variable
names of the right-hand-side variables
term \#
error term for term \# (when using nonresidual error)
type of sum of squares; sequential or partial
names of repeated variables (repeated() only)
names of repeated variables for a particular combination (repeated() only)
ols
weight type
weight expression
b V
program used to implement estat
program used to implement predict
factor variables fvset as asbalanced
factor variables fvset as asobserved
coefficient vector
variance-covariance matrix of the estimators
covariance matrix based on repeated measures (repeated() only)
marks estimation sample

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## Also see

[R] anova postestimation - Postestimation tools for anova
[R] contrast - Contrasts and linear hypothesis tests after estimation
[R] icc - Intraclass correlation coefficients
[R] loneway - Large one-way ANOVA, random effects, and reliability
[R] oneway - One-way analysis of variance
$[R]$ regress - Linear regression
[MV] manova - Multivariate analysis of variance and covariance
[PSS] power oneway - Power analysis for one-way analysis of variance
[PSS] power repeated - Power analysis for repeated-measures analysis of variance
[PSS] power twoway - Power analysis for two-way analysis of variance
Stata Structural Equation Modeling Reference Manual

## Title

anova postestimation - Postestimation tools for anova

| Description | Syntax for predict | Syntax for test after anova |
| :--- | :--- | :--- |
| Menu for test after anova | Options for test after anova | Remarks and examples |
| References | Also see |  |

## Description

The following postestimation commands are of special interest after anova:

| Command | Description |
| :--- | :--- |
| dfbeta | DFBETA influence statistics |
| estat hettest | tests for heteroskedasticity |
| estat imtest | information matrix test |
| estat ovtest | Ramsey regression specification-error test for omitted variables |
| estat szroeter | Szroeter's rank test for heteroskedasticity |
| estat vif | variance inflation factors for the independent variables |
| estat esize | $\eta^{2}$ and $\omega^{2}$ effect sizes |
| rvfplot | residual-versus-fitted plot |
| avplot | added-variable plot |
| avplots | all added-variables plots in one image |
| cprplot | component-plus-residual plot |
| acprplot | augmented component-plus-residual plot |
| rvpplot | residual-versus-predictor plot |
| lvr2plot | leverage-versus-squared-residual plot |

The following standard postestimation commands are also available:

| Command | Description |
| :--- | :--- |
| contrast | contrasts and ANOVA-style joint tests of estimates |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample |
| estat vce | variance-covariance matrix of the estimators (VCE) <br> estimates <br> hausman |
| lincom | Hausman's specification test <br> point estimates, standard errors, testing, and inference for linear <br> combinations of coefficients |
| linktest | link test for model specification |
| lrtest | likelihood-ratio test |
| margins | marginal means, predictive margins, marginal effects, and average marginal <br> effects |
| marginsplot | graph the results from margins (profile plots, interaction plots, etc.) <br> point estimates, standard errors, testing, and inference for nonlinear <br> combinations of coefficients |
| nlcom | predictions, residuals, influence statistics, and other diagnostic measures <br> point estimates, standard errors, testing, and inference for generalized predictions |
| predict | pairwise comparisons of estimates <br> predictnl <br> swcompare |
| suest | Wald tests of simple and composite linear hypotheses |
| test | Wald tests of nonlinear hypotheses |
| testnl |  |

## Special-interest postestimation commands

In addition to the common estat commands (see [R] estat), estat hettest, estat imtest, estat ovtest, estat szroeter, and estat vif are also available. dfbeta is also available. The syntax for dfbeta and these estat commands is the same as after regress; see [R] regress postestimation.

For information on the plot commands, see $[R]$ regress postestimation diagnostic plots.
In addition to the standard syntax of test (see [R] test), test after anova has three additionally allowed syntaxes; see below. test performs Wald tests of expressions involving the coefficients of the underlying regression model. Simple and composite linear hypotheses are possible.

## Syntax for predict

predict after anova follows the same syntax as predict after regress and can provide predictions, residuals, standardized residuals, Studentized residuals, the standard error of the residuals, the standard error of the prediction, the diagonal elements of the projection (hat) matrix, and Cook's $D$. See [R] regress postestimation for details.

## Syntax for test after anova

In addition to the standard syntax of test (see [R] test), test after anova also allows the following:

| test, test(matname) $[\underline{m t e s t}[(o p t)]$ matvlc(matname) $]$ | syntax a |
| :--- | :---: |
| test, showorder | syntax b |
| $\underline{\text { test }[\text { term }[\text { term } . . .]][/ \operatorname{term}[\text { term } . . .]][\text {, symbolic }]}$ | syntax c |

syntax a test expression involving the coefficients of the underlying regression model; you provide information as a matrix
syntax b show underlying order of design matrix, which is useful when constructing matname argument of the test () option
syntax c test effects and show symbolic forms

## Menu for test after anova

Statistics $>$ Linear models and related $>$ ANOVA/MANOVA $>$ Test linear hypotheses after anova

## Options for test after anova

test (matname) is required with syntax a of test. The rows of matname specify linear combinations of the underlying design matrix of the ANOVA that are to be jointly tested. The columns correspond to the underlying design matrix (including the constant if it has not been suppressed). The column and row names of matname are ignored.
A listing of the constraints imposed by the test () option is presented before the table containing the tests. You should examine this table to verify that you have applied the linear combinations you desired. Typing test, showorder allows you to examine the ordering of the columns for the design matrix from the ANOVA.
mtest $[(o p t)]$ specifies that tests are performed for each condition separately. opt specifies the method for adjusting $p$-values for multiple testing. Valid values for opt are

| bonferroni | Bonferroni's method |
| :--- | :--- |
| holm | Holm's method |
| sidak | Šidák's method |
| noadjust | no adjustment is to be made |

Specifying mtest with no argument is equivalent to mtest (noadjust).
matvlc(matname), a programmer's option, saves the variance-covariance matrix of the linear combinations involved in the suite of tests. For the test $\mathbf{L b}=\mathbf{c}$, what is returned in matname is $\mathbf{L} V \mathbf{L}^{\prime}$, where $V$ is the estimated variance-covariance matrix of $\mathbf{b}$.
showorder causes test to list the definition of each column in the design matrix. showorder is not allowed with any other option.
symbolic requests the symbolic form of the test rather than the test statistic. When this option is specified with no terms (test, symbolic), the symbolic form of the estimable functions is displayed.

## Remarks and examples

Remarks are presented under the following headings:

> Testing effects
> Obtaining symbolic forms
> Testing coefficients and contrasts of margins Video example

See examples $4,7,8,13,15,16$, and 17 in $[R]$ anova for examples that use the margins command.

## Testing effects

After fitting a model using anova, you can test for the significance of effects in the ANOVA table, as well as for effects that are not reported in the ANOVA table, by using the test or contrast command. You follow test or contrast by the list of effects that you wish to test. By default, these commands use the residual mean squared error in the denominator of the $F$ ratio. You can specify other error terms by using the slash notation, just as you would with anova. See [R] contrast for details on this command.

## > Example 1: Testing effects

Recall our byssinosis example (example 8) in [R] anova:

|  | Number of obs Root MSE |  | $\begin{array}{rr} 65 & R- \\ 25902 & \text { Ad } \end{array}$ | R-squared <br> Adj R-squared | $\begin{aligned} & =0.8300 \\ & =0.7948 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Source | Partial SS | df | MS | F | Prob > F |
| Model | . 173646538 | 11 | . 015786049 | 923.53 | 0.0000 |
| workplace | . 097625175 | 2 | . 048812588 | $8 \quad 72.76$ | 0.0000 |
| smokes | . 013030812 | 1 | . 013030812 | 219.42 | 0.0001 |
| race | . 001094723 | 1 | . 001094723 | -1.63 | 0.2070 |
| workplace\#smokes | . 019690342 | 2 | . 009845171 | 14.67 | 0.0000 |
| workplace\#race | . 001352516 | 2 | . 000676258 | - 1.01 | 0.3718 |
| smokes\#race | . 001662874 | 1 | . 001662874 | $4 \quad 2.48$ | 0.1214 |
| workplace\#smokes\#race | . 000950841 | 2 | . 00047542 | $2 \quad 0.71$ | 0.4969 |
| Residual | . 035557766 | 53 | . 000670901 |  |  |
| Total | . 209204304 | 64 | . 003268817 |  |  |

We can easily obtain a test on a particular term from the ANOVA table. Here are two examples:

```
. test smokes
```

| Source | Partial SS | df | MS | F | Prob > F |
| ---: | :---: | ---: | :---: | :---: | ---: |
| smokes | .013030812 | 1 | .013030812 | 19.42 | 0.0001 |
| Residual | .035557766 | 53 | .000670901 |  |  |

. test smokes\#race

| Source | Partial SS | df | MS | F | Prob > F |
| ---: | :---: | ---: | :---: | :---: | :---: |
| smokes\#race | .001662874 | 1 | .001662874 | 2.48 | 0.1214 |
| Residual | .035557766 | 53 | .000670901 |  |  |

Both of these tests use residual error by default and agree with the ANOVA table produced earlier.

We could have performed these same tests with contrast:

```
. contrast smokes
Contrasts of marginal linear predictions
Margins : asbalanced
```

|  | df | F | $\mathrm{P}>\mathrm{F}$ |
| ---: | :---: | :---: | ---: |
| smokes | 1 | 19.42 | 0.0001 |
| Denominator | 53 |  |  |

. contrast smokes\#race
Contrasts of marginal linear predictions
Margins : asbalanced

|  | df | F | $\mathrm{P}>\mathrm{F}$ |
| :--- | :---: | :---: | ---: |
| smokes\#race | 1 | 2.48 | 0.1214 |
| Denominator | 53 |  |  |

## - Technical note

After anova, you can use the ' $/$ ' syntax in test or contrast to perform tests with a variety of non $-\sigma^{2} \mathbf{I}$ error structures. However, in most unbalanced models, the mean squares are not independent and do not have equal expectations under the null hypothesis. Also, be warned that you assume responsibility for the validity of the test statistic.

## > Example 2: Testing effects with different error terms

We return to the nested ANOVA example (example 11) in [R] anova, where five brands of machinery were compared in an assembly line. We can obtain appropriate tests for the nested terms using test, even if we had run the anova command without initially indicating the proper error terms.


In this ANOVA table, machine is tested with residual error. With this particular nested design, the appropriate error term for testing machine is operator nested within machine, which is easily obtained from test.
. test machine / operator|machine

| Source | Partial SS | df | MS | F | Prob > F |
| ---: | :---: | ---: | :---: | :---: | ---: |
| machine | 430.980792 | 4 | 107.745198 | 13.82 | 0.0001 |
| operator machine | 101.353804 | 13 | 7.79644648 |  |  |

This result from test matches what we obtained from our anova command.

## > Example 3: Pooling terms when testing effects

The other nested ANOVA example (example 12) in [R] anova was based on the sewage data. The ANOVA table is presented here again. As before, we will use abbreviations of variable names in typing the commands.


In practice, it is often beneficial to pool nonsignificant nested terms to increase the power of tests on remaining terms. One rule of thumb is to allow the pooling of a term whose $p$-value is larger than 0.25 . In this sewage example, the $p$-value for the test of manager is 0.6166 . This value indicates that the manager effect is negligible and might be ignored. Currently, solution is tested by manager|solution, which has only 2 degrees of freedom. If we pool the manager and facility terms and use this pooled estimate as the error term for solution, we would have a term with 6 degrees of freedom.

Below are two tests: a test of solution with the pooled manager and facility terms and a test of this pooled term by worker.

```
. test s / m|s f|m|s
```

| Source | Partial SS | df | MS | F | Prob > F |
| ---: | :---: | :---: | :---: | :---: | :---: |
| solution | 7203.76563 | 1 | 7203.76563 | 11.07 | 0.0159 |
| manager\|solution <br> facility manager <br> solution | 3903.21875 | 6 | 650.536458 |  |  |

. test $\mathrm{m}|\mathrm{s} \mathrm{f}| \mathrm{m}|\mathrm{s} / \mathrm{w}| \mathrm{f}|\mathrm{m}| \mathrm{s}$

| Source | Partial SS | df | MS | F | Prob > F |
| :---: | :---: | :---: | :---: | :---: | :---: |
| manager\|solution facility|manager| |  |  |  |  |  |
| solution | 3903.21875 | 6 | 650.536458 | 2.18 | 0.1520 |
| worker\|facility|manager| |  |  |  |  |  |
| solution | 2386.625 | 8 | 298.328125 |  |  |

In the first test, we included two terms after the forward slash ( $\mathrm{m} \mid \mathrm{s}$ and $\mathrm{f}|\mathrm{m}| \mathrm{s}$ ). test after anova allows multiple terms both before and after the slash. The terms before the slash are combined and are then tested by the combined terms that follow the slash (or residual error if no slash is present).

The $p$-value for solution using the pooled term is 0.0159 . Originally, it was 0.0536 . The increase in the power of the test is due to the increase in degrees of freedom for the pooled error term.

We can get identical results if we drop manager from the anova model. (This dataset has unique numbers for each facility so that there is no confusion of facilities when manager is dropped.)


This output agrees with our earlier test results.

In the following example, two terms from the anova are jointly tested (pooled).
> Example 4: Obtaining overall significance of a term using contrast
In example 10 of $[\mathrm{R}]$ anova, we fit the model anova drate region c .mage region\#c.mage. Now we use the contrast command to test for the overall significance of region.

| . contrast region region\#c.mage, overall <br> Contrasts of marginal linear predictions <br> Margins <br> : asbalanced |
| :--- |
| region |
| df |
| region\#c.mage |

The overall $F$ statistic associated with the region and region\#c.mage terms is 5.65 , and it is significant at the $0.02 \%$ level.

In the ANOVA output, the region term, by itself, had a sum of squares of 1166.15 , which, based on 3 degrees of freedom, yielded an $F$ statistic of 7.40 and a significance level of 0.0004 . This is the same test that is reported by contrast in the row labeled region. Likewise, the test from the ANOVA output for the region\#c.mage term is reproduced in the second row of the contrast output.

## Obtaining symbolic forms

test can produce the symbolic form of the estimable functions and symbolic forms for particular tests.

## > Example 5: Symbolic form of the estimable functions

After fitting an ANOVA model, we type test, symbolic to obtain the symbolic form of the estimable functions. For instance, returning to our blood pressure data introduced in example 4 of $[R]$ anova, let's begin by reestimating systolic on drug, disease, and drug\#disease:

|  | Number of obs Root MSE | $\begin{aligned} & = \\ & =10 \end{aligned}$ | $\begin{array}{rr} 58 & \mathrm{R} \\ .5096 & \mathrm{~A} \end{array}$ | R-squared <br> Adj R-squared | $\begin{aligned} & =0.4560 \\ & =0.3259 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Source | Partial SS | df | MS | F | Prob > F |
| Model | 4259.33851 | 11 | 387.212591 | $1 \quad 3.51$ | 0.0013 |
| drug | 2997.47186 | 3 | 999.157287 | $7 \quad 9.05$ | 0.0001 |
| disease | 415.873046 | 2 | 207.936523 | 31.88 | 0.1637 |
| drug\#disease | 707.266259 | 6 | 117.87771 | $1 \quad 1.07$ | 0.3958 |
| Residual | 5080.81667 | 46 | 110.452536 |  |  |
| Total | 9340.15517 | 57 | 163.862371 |  |  |

To obtain the symbolic form of the estimable functions, type

```
. test, symbolic
drug
    - (r2+r3+r4-r0)
    r2
    ra
    r4
disease
    1 -(r6+r7-r0)
    r6
    r7
drug#disease
    1 1 -(r2+r3+r4+r6+r7-r12-r13-r15-r16-r18-r19-r0)
    1 2 r6 - (r12+r15+r18)
    1 3 r7 - (r13+r16+r19)
    2 1 r2 - (r12+r13)
    2r12
    r13
    1 r3 - (r15+r16)
    r r15
    rr16
        r4 - (r18+r19)
        r18
        r19
_cons
        r0
```


## > Example 6: Symbolic form for a particular test

To obtain the symbolic form for a particular test, we type test term [term ...], symbolic. For instance, the symbolic form for the test of the main effect of drug is

```
. test drug, symbolic
drug
    1 -(r2+r3+r4)
    2 r2
    3 r3
    r4
disease
    0
    2 0
    3 0
drug#disease
    1 1 -1/3 (r2+r3+r4)
    1 2 -1/3 (r2+r3+r4)
    1 3-1/3 (r2+r3+r4)
    2 1 1/3 r2
    2 2 1/3 r2
    2 3 1/3 r2
    3 1 1/3 r3
    3 2 1/3 r3
    3 3 1/3 r3
    4 1 1/3 r4
    4 2 1/3 r4
    4 3 1/3 r4
_cons 0
```

If we omit the symbolic option, we instead see the result of the test:

| Source | Partial SS | df | MS | F | Prob > F |
| :---: | :---: | :---: | :---: | :---: | :---: |
| drug | 2997.47186 | 3 | 999.157287 | 9.05 | 0.0001 |
| Residual | 5080.81667 | 46 | 110.452536 |  |  |

## Testing coefficients and contrasts of margins

The test command allows you to perform tests directly on the coefficients of the underlying regression model. For instance, the coefficient on the third drug and the second disease is referred to as 3.drug\#2.disease. This could also be written as i3.drug\#i2.disease, or _b[3.drug\#2.disease], or even _coef[i3.drug\#i2.disease]; see [U] 13.5 Accessing coefficients and standard errors.
> Example 7: Testing linear combinations of coefficients
Let's begin by testing whether the coefficient on the third drug is equal to the coefficient on the fourth in our blood pressure data. We have already fit the model anova systolic drug\#\#disease (equivalent to anova systolic drug disease drug\#disease), and you can see the results of that estimation in example 5. Even though we have performed many tasks since we fit the model, Stata still remembers, and we can perform tests at any time.

```
. test 3.drug = 4.drug
( 1) 3.drug - 4.drug = 0
    F(1, 46) = 0.13
        Prob > F = 0.7234
```

We find that the two coefficients are not significantly different, at least at any significance level smaller than $73 \%$.

For more complex tests, the contrast command often provides a more concise way to specify the test we are interested in and prevents us from having to write the tests in terms of the regression coefficients. With contrast, we instead specify our tests in terms of differences in the marginal means for the levels of a particular factor. For example, if we want to compare the third and fourth drugs, we can test the difference in the mean impact on systolic blood pressure separately for each disease using the @ operator. We also use the reverse adjacent operator, ar., to compare the fourth level of drug with the previous level.


None of the individual contrasts shows significant differences between the third drug and the fourth drug. Likewise, the overall $F$ statistic is 0.85 , which is hardly significant. We cannot reject the hypothesis that the third drug has the same effect as the fourth drug.

## Technical note

Alternatively, we could have specified these tests based on the coefficients of the underlying regression model using the test command. We would have needed to perform tests on the coefficients for drug and for the coefficients on drug interacted with disease in order to test for differences in the means mentioned above. To do this, we start with our previous test command:

```
. test 3.drug = 4.drug
```

Notice that the $F$ statistic for this test is equivalent to the test labeled ( 4 vs 3 ) 1 in the contrast output. Let's now add the constraint that the coefficient on the third drug interacted with the third disease is equal to the coefficient on the fourth drug, again interacted with the third disease. We do that by typing the new constraint and adding the accumulate option:

```
. test 3.drug#3.disease = 4.drug#3.disease, accumulate
( 1) 3.drug - 4.drug = 0
( 2) 3.drug#3.disease - 4.drug#3.disease = 0
    F( 2, 46) = 0.39
        Prob > F = 0.6791
```

So far, our test includes the equality of the two drug coefficients, along with the equality of the two drug coefficients when interacted with the third disease. Now we add two more equations, one for each of the remaining two diseases:

```
. test 3.drug#2.disease = 4.drug#2.disease, accumulate
( 1) 3.drug - 4.drug = 0
( 2) 3.drug#3.disease - 4.drug#3.disease = 0
( 3) 3.drug#2.disease - 4.drug#2.disease = 0
    F( 3, 46) = 0.85
    Prob > F = 0.4761
. test 3.drug#1.disease = 4.drug#1.disease, accumulate
( 1) 3.drug - 4.drug = 0
( 2) 3.drug#3.disease - 4.drug#3.disease = 0
( 3) 3.drug#2.disease - 4.drug#2.disease = 0
( 4) 3o.drug#1b.disease - 4o.drug#1b.disease = 0
        Constraint 4 dropped
        F( 3, 46) = 0.85
            Prob > F = 0.4761
```

The overall $F$ statistic reproduces the one from the joint test in the contrast output.
You may notice that we also got the message "Constraint 4 dropped". For the technically inclined, this constraint was unnecessary, given the normalization of the model. If we specify all the constraints involved in our test or use contrast, we need not worry about the normalization because Stata handles this automatically.

The test () option of test provides another alternative for testing coefficients. Instead of spelling out each coefficient involved in the test, a matrix representing the test provides the needed information. test, showorder shows the order of the terms in the ANOVA corresponding to the order of the columns for the matrix argument of test ().

## > Example 8: Another way to test linear combinations of coefficients

We repeat the last test of example 7 above with the test () option. First, we view the definition and order of the columns underlying the ANOVA performed on the systolic data.

```
. test, showorder
Order of columns in the design matrix
        (drug==1)
    (drug==2)
    (drug==3)
    (drug==4)
    (disease==1)
    (disease==2)
    (disease==3)
    (drug==1)*(disease==1)
    (drug==1)*(disease==2)
    (drug==1)*(disease==3)
    (drug==2)*(disease==1)
    (drug==2)*(disease==2)
    (drug==2)*(disease==3)
    (drug==3)*(disease==1)
    (drug==3)*(disease==2)
    (drug==3)*(disease==3)
    (drug==4)*(disease==1)
    (drug==4)*(disease==2)
    (drug==4)*(disease==3)
    _cons
```

Columns 1-4 correspond to the four levels of drug. Columns 5-7 correspond to the three levels of disease. Columns $8-19$ correspond to the interaction of drug and disease. The last column corresponds to _cons, the constant in the model.

We construct the matrix dr3vs4 with the same four constraints as the last test shown in example 7 and then use the test (dr3vs4) option to perform the test.

```
. matrix dr3vs4 = (0,0,1,-1, 0,0,0, 0,0,0,0,0,0,0,0,0, 0, 0, 0, 0 \
> 0,0,0,0, 0,0,0, 0,0,0,0,0,0,0,0,1,0,0,-1,0\
    0,0,0, 0, 0,0,0, 0,0,0,0,0,0,0,1,0, 0,-1, 0, 0\
    0,0,0, 0, 0,0,0, 0,0,0,0,0,0,1,0,0,-1, 0, 0, 0)
. test, test(dr3vs4)
( 1) 3.drug - 4.drug = 0
( 2) 3.drug#3.disease - 4.drug#3.disease = 0
( 3) 3.drug#2.disease - 4.drug#2.disease = 0
( 4) 3o.drug#1b.disease - 4o.drug#1b.disease = 0
    Constraint 4 dropped
    F( 3, 46) = 0.85
        Prob > F = 0.4761
```

Here the effort involved with spelling out the coefficients is similar to that of constructing a matrix and using it in the test () option. When the test involving coefficients is more complicated, the test () option may be more convenient than specifying the coefficients directly in test. However, as previously demonstrated, contrast may provide an even simpler method for testing the same hypothesis.

After fitting an ANOVA model, various contrasts (1-degree-of-freedom tests comparing different levels of a categorical variable) are often of interest. contrast can perform each 1-degree-of-freedom test in addition to the combined test, even in cases in which the contrasts do not correspond to one of the contrast operators.

## > Example 9: Testing particular contrasts of interest

Rencher and Schaalje (2008) illustrate 1-degree-of-freedom contrasts for an ANOVA comparing the net weight of cans filled by five machines (labeled A-E). The data were originally obtained from Ostle and Mensing (1975). Rencher and Schaalje use a cell-means ANOVA model approach for this problem. We could do the same by using the noconstant option of anova; see [R] anova. Instead, we obtain the same results by using the standard overparameterized ANOVA approach (that is, we keep the constant in the model).

```
. use http://www.stata-press.com/data/r13/canfill
(Can Fill Data)
. list, sepby(machine)
\begin{tabular}{|c|c|c|}
\hline & machine & weight \\
\hline 1. & A & 11.95 \\
\hline 2. & A & 12.00 \\
\hline 3. & A & 12.25 \\
\hline 4. & A & 12.10 \\
\hline 5. & B & 12.18 \\
\hline 6. & B & 12.11 \\
\hline 7. & C & 12.16 \\
\hline 8. & C & 12.15 \\
\hline 9. & C & 12.08 \\
\hline 10. & D & 12.25 \\
\hline 11. & D & 12.30 \\
\hline 12. & D & 12.10 \\
\hline 13. & E & 12.10 \\
\hline 14. & E & 12.04 \\
\hline 15. & E & 12.02 \\
\hline 16. & E & 12.02 \\
\hline
\end{tabular}
```

. anova weight machine

|  | $\begin{array}{llr} \text { Number of obs } & = & 16 \\ \text { Root MSE } & =.087758 \end{array}$ |  |  | R-squared <br> Adj R-squared | $\begin{aligned} & =0.4123 \\ & =0.1986 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Source | Partial SS | df | MS | F | Prob > F |
| Model | . 059426993 | 4 | . 014856748 | 1.93 | 0.1757 |
| machine | . 059426993 | 4 | . 014856748 | 1.93 | 0.1757 |
| Residual | . 084716701 | 11 | . 007701518 |  |  |
| Total | . 144143694 | 15 | . 00960958 |  |  |

The four 1-degree-of-freedom tests of interest among the five machines are $A$ and $D$ versus $B, C$, and E; B and E versus C; A versus D; and B versus E. We can specify these tests as user-defined contrasts by placing the corresponding contrast coefficients into positions related to the five levels of machine as described in User-defined contrasts of $[\mathrm{R}]$ contrast.

| . contrast \{machine 3-2-2 3-2\} |  |  |  |
| :---: | :---: | :---: | :---: |
| $>\quad\left\{\begin{array}{l}\text { machine }\end{array} 0 \begin{array}{lllll} & -2 & 0 & 1\}\end{array}\right.$ |  |  |  |
|  |  |  |  |
| > |  |  |  |
| Contrasts of marginal linear predictions |  |  |  |
| Margins |  |  |  |
|  | df | F | $\mathrm{P}>\mathrm{F}$ |
| machine |  |  |  |
| (1) | 1 | 0.75 | 0.4055 |
| (2) | 1 | 0.31 | 0.5916 |
| (3) | 1 | 4.47 | 0.0582 |
| (4) | 1 | 1.73 | 0.2150 |
| Joint | 4 | 1.93 | 0.1757 |
| Denominator | 11 |  |  |

contrast produces a 1-degree-of-freedom test for each of the specified contrasts as well as a joint test. We included the noeffects option so that the table displaying the values of the individual contrasts with their confidence intervals was suppressed.

The significance values above are not adjusted for multiple comparisons. We could have produced the Bonferroni-adjusted significance values by using the mcompare (bonferroni) option.

```
. contrast {machine 3-2 -2 3-2}
> {machine 0 1 -2 0
> {machine 1 0 0 -1 0}
> {machine 0 1 0 0 -1}, mcompare(bonferroni) noeffects
Contrasts of marginal linear predictions
Margins : asbalanced
```

|  |  |  | Bonferroni <br> $\mathrm{P}>\mathrm{F}$ |  |
| ---: | :---: | :---: | :---: | ---: |
| machine | df |  | $\mathrm{F}>\mathrm{F}$ |  |
| (1) | 1 | 0.75 | 0.4055 | 1.0000 |
| $(2)$ | 1 | 0.31 | 0.5916 | 1.0000 |
| (3) | 1 | 4.47 | 0.0582 | 0.2329 |
| (4) | 1 | 1.73 | 0.2150 | 0.8601 |
| Joint | 4 | 1.93 | 0.1757 |  |
| Denominator |  |  |  |  |

Note: Bonferroni-adjusted p-values are reported for tests
on individual contrasts only.

## $>$ Example 10: Linear and quadratic contrasts

Here there are two factors, A and B, each with three levels. The levels are quantitative so that linear and quadratic contrasts are of interest.
. use http://www.stata-press.com/data/r13/twowaytrend
. anova Y A B A\#B

|  | $\begin{array}{ll} \text { Number of obs } & = \\ \text { Root MSE } & = \end{array}$ | 36 R-squared |  |  | $\begin{aligned} & =0.9304 \\ & =0.9097 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Source | Partial SS | df | MS | F | Prob > F |
| Model | 2578.55556 | 8 | 322.319444 | 45.09 | 0.0000 |
| A | 2026.72222 | 2 | 1013.36111 | 141.77 | 0.0000 |
| B | 383.722222 | 2 | 191.861111 | 26.84 | 0.0000 |
| A\#B | 168.111111 | 4 | 42.0277778 | 5.88 | 0.0015 |
| Residual | 193 | 27 | 7.14814815 |  |  |
| Total | 2771.55556 | 35 | 79.1873016 |  |  |

We can use the p. contrast operator to obtain the 1-degree-of-freedom tests for the linear and quadratic effects of $A$ and $B$.

```
. contrast p.A p.B, noeffects
Contrasts of marginal linear predictions
Margins : asbalanced
```

|  | df | F | $\mathrm{P}>\mathrm{F}$ |
| :---: | :---: | ---: | ---: |
| A |  |  |  |
| (linear) | 1 | 212.65 | 0.0000 |
| (quadratic) | 1 | 70.88 | 0.0000 |
| Joint | 2 | 141.77 | 0.0000 |
| B |  |  |  |
| (linear) | 1 | 26.17 | 0.0000 |
| (quadratic) | 1 | 27.51 | 0.0000 |
| Joint | 2 | 26.84 | 0.0000 |
| Denominator |  |  |  |

All the above tests appear to be significant. In addition to presenting the 1-degree-of-freedom tests, the combined tests for A and B are produced and agree with the original ANOVA results.

Now we explore the interaction between A and B.

| . contrast p.A\#p1.B, noeffects <br> Contrasts of marginal linear predictions <br> Margins <br> : asbalanced |
| :--- |
| A\#B |
| (linear) (linear) |

The 2-degrees-of-freedom test of the interaction of A with the linear components of B is significant at the 0.0011 level. But, when we examine the two 1 -degree-of-freedom tests that compose this result,
the significance is due to the linear A by linear B contrast (significance level of 0.0003). A significance value of 0.7893 for the quadratic A by linear B indicates that this factor is not significant for these data.

| Contrasts of marginal linear predictions Margins : asbalanced |  |  |  |
| :---: | :---: | :---: | :---: |
|  | df | F | P>F |
| A\#B |  |  |  |
| (linear) (quadratic) | 1 | 2.80 | 0.1058 |
| (quadratic) (quadratic) | 1 | 2.94 | 0.0979 |
| Joint | 2 | 2.87 | 0.0741 |
| Denominator | 27 |  |  |

The test of A with the quadratic components of B does not fall below the 0.05 significance level.

## Video example

Introduction to contrasts in Stata: One-way ANOVA

## References

Mitchell, M. N. 2012. Interpreting and Visualizing Regression Models Using Stata. College Station, TX: Stata Press. Ostle, B., and R. W. Mensing. 1975. Statistics in Research. 3rd ed. Ames, IA: Iowa State University Press. Rencher, A. C., and G. B. Schaalje. 2008. Linear Models in Statistics. 2nd ed. New York: Wiley.

## Also see

[R] anova - Analysis of variance and covariance
$[R]$ regress postestimation - Postestimation tools for regress
[R] regress postestimation diagnostic plots - Postestimation plots for regress
[U] 20 Estimation and postestimation commands

## Title

areg - Linear regression with a large dummy-variable set

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

Also see

## Syntax

$$
\text { areg depvar }[\text { indepvars }][\text { if }][\text { in }][\text { weight }] \text {, absorb (varname) }[\text { options }]
$$

options
Description

## Model

* absorb (varname) categorical variable to be absorbed

SE/Robust
vce(vcetype) vcetype may be ols, robust, cluster clustvar, bootstrap, or jackknife

Reporting
level (\#)
display_options
coeflegend
set confidence level; default is level (95)
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
display legend instead of statistics
*absorb (varname) is required.
indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
depvar and indepvars may contain time-series operators; see [U] 11.4.4 Time-series varlists.
bootstrap, by, fp, jackknife, mi estimate, rolling, and statsby are allowed; see [U] 11.1.10 Prefix commands. vce(bootstrap) and vce (jackknife) are not allowed with the mi estimate prefix; see [MI] mi estimate.
Weights are not allowed with the bootstrap prefix; see $[R]$ bootstrap.
aweights are not allowed with the jackknife prefix; see $[R]$ jackknife.
aweights, fweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Linear models and related $>$ Other $>$ Linear regression absorbing one cat. variable

## Description

areg fits a linear regression absorbing one categorical factor. areg is designed for datasets with many groups, but not a number of groups that increases with the sample size. See the xtreg, fe command in [XT] xtreg for an estimator that handles the case in which the number of groups increases with the sample size.

## Options


#### Abstract

Model absorb(varname) specifies the categorical variable, which is to be included in the regression as if it were specified by dummy variables. absorb() is required.


SE/Robust
vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (ols), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
vce(ols), the default, uses the standard variance estimator for ordinary least-squares regression. Exercise caution when using the vce (cluster clustvar) option with areg. The effective number of degrees of freedom for the robust variance estimator is $n_{g}-1$, where $n_{g}$ is the number of clusters. Thus the number of levels of the absorb() variable should not exceed the number of clusters.

## Reporting

level (\#); see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\%fmt), pformat(\%fmt), sformat (\%fmt), and nolstretch; see [R] estimation options.

The following option is available with areg but is not shown in the dialog box:
coeflegend; see [R] estimation options.

## Remarks and examples

Suppose that you have a regression model that includes among the explanatory variables a large number, $k$, of mutually exclusive and exhaustive dummies:

$$
\mathbf{y}=\mathbf{X} \boldsymbol{\beta}+\mathbf{d}_{1} \gamma_{1}+\mathbf{d}_{2} \gamma_{2}+\cdots+\mathbf{d}_{k} \gamma_{k}+\boldsymbol{\epsilon}
$$

For instance, the dummy variables, $\mathbf{d}_{i}$, might indicate countries in the world or states of the United States. One solution would be to fit the model with regress, but this solution is possible only if $k$ is small enough so that the total number of variables (the number of columns of $\mathbf{X}$ plus the number of $\mathbf{d}_{i}$ 's plus one for $\mathbf{y}$ ) is sufficiently small-meaning less than matsize (see [R] matsize). For problems with more variables than the largest possible value of matsize ( 100 for Small Stata, 800 for Stata/IC, and 11,000 for Stata/SE and Stata/MP), regress will not work. areg provides a way of obtaining estimates of $\boldsymbol{\beta}$ —but not the $\gamma_{i}$ 's -in these cases. The effects of the dummy variables are said to be absorbed.
> Example 1
So that we can compare the results produced by areg with Stata's other regression commands, we will fit a model in which $k$ is small. areg's real use, however, is when $k$ is large.

In our automobile data, we have a variable called rep78 that is coded $1,2,3,4$, and 5 , where 1 means poor and 5 means excellent. Let's assume that we wish to fit a regression of mpg on weight, gear_ratio, and rep78 (parameterized as a set of dummies).

```
. use http://www.stata-press.com/data/r13/auto2
(1978 Automobile Data)
. regress mpg weight gear_ratio b5.rep78
```

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | 1575.97621 | 6 | 262.662702 |
| Residual | 764.226686 | 62 | 12.3262369 |
| Total | 2340.2029 | 68 | 34.4147485 |


| Number of obs | $=$ | 69 |
| :--- | ---: | ---: |
| F 6, | $62)$ | $=$ |
| Prob $>$ F | $=$ | 0.0000 |
| R-squared | $=$ | 0.6734 |
| Adj R-squared | $=$ | 0.6418 |
| Root MSE | $=$ | 3.5109 |


| mpg | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| weight | -.0051031 | .0009206 | -5.54 | 0.000 | -.0069433 | -.003263 |
| gear_ratio | .901478 | 1.565552 | 0.58 | 0.567 | -2.228015 | 4.030971 |
| rep78 |  |  |  |  |  |  |
| Poor | -2.036937 | 2.740728 | -0.74 | 0.460 | -7.515574 | 3.4417 |
| Fair | -2.419822 | 1.764338 | -1.37 | 0.175 | -5.946682 | 1.107039 |
| Average | -2.557432 | 1.370912 | -1.87 | 0.067 | -5.297846 | .1829814 |
| Good | -2.788389 | 1.395259 | -2.00 | 0.050 | -5.577473 | .0006939 |
| _cons | 36.23782 | 7.01057 | 5.17 | 0.000 | 22.22389 | 50.25175 |

To fit the areg equivalent, we type
. areg mpg weight gear_ratio, absorb(rep78)
Linear regression, absorbing indicators

| Number of obs | $=$ | 69 |
| :--- | :--- | ---: |
| F ( 2, 62) | $=$ | 41.64 |
| Prob > F | $=$ | 0.0000 |
| R-squared | $=$ | 0.6734 |
| Adj R-squared | $=$ | 0.6418 |
| Root MSE | $=$ | 3.5109 |


| mpg | Coef. | Std. Err. | t | P>\|t| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | ---: | :---: | ---: | ---: |
| weight | -.0051031 | .0009206 | -5.54 | 0.000 | -.0069433 | -.003263 |
| gear_ratio | .901478 | 1.565552 | 0.58 | 0.567 | -2.228015 | 4.030971 |
| _cons | 34.05889 | 7.056383 | 4.83 | 0.000 | 19.95338 | 48.1644 |
| rep78 | $\mathrm{F}(4,62)=$ | 1.117 | 0.356 |  | (5 categories) |  |

Both regress and areg display the same $R^{2}$ values, root mean squared error, and-for weight and gear_ratio-the same parameter estimates, standard errors, $t$ statistics, significance levels, and confidence intervals. areg, however, does not report the coefficients for rep78, and, in fact, they are not even calculated. This computational trick makes the problem manageable when $k$ is large. areg reports a test that the coefficients associated with rep78 are jointly zero. Here this test has a significance level of $35.6 \%$. This $F$ test for rep78 is the same that we would obtain after regress if we were to specify test 1. rep78 2.rep78 3.rep78 4.rep78; see $[R]$ test.

The model $F$ tests reported by regress and areg also differ. The regress command reports a test that all coefficients except that of the constant are equal to zero; thus, the dummies are included in this test. The areg output shows a test that all coefficients excluding the dummies and the constant are equal to zero. This is the same test that can be obtained after regress by typing test weight gear_ratio.

## - Technical note

areg is designed for datasets with many groups, but not a number that grows with the sample size. Consider two different samples from the U.S. population. In the first sample, we have 10,000 individuals and we want to include an indicator for each of the 50 states, whereas in the second sample we have 3 observations on each of 10,000 individuals and we want to include an indicator for each individual. areg was designed for datasets similar to the first sample in which we have a fixed number of groups, the 50 states. In the second sample, the number of groups, which is the number of individuals, grows as we include more individuals in the sample. For an estimator designed to handle the case in which the number of groups grows with the sample size, see the xtreg, fe command in [XT] xtreg.

Although the point estimates produced by areg and xtreg, fe are the same, the estimated VCEs differ when vce (cluster clustvar) is specified because the commands make different assumptions about whether the number of groups increases with the sample size.

- Technical note

The intercept reported by areg deserves some explanation because, given $k$ mutually exclusive and exhaustive dummies, it is arbitrary. areg identifies the model by choosing the intercept that makes the prediction calculated at the means of the independent variables equal to the mean of the dependent variable: $\overline{\mathbf{y}}=\overline{\mathbf{x}} \widehat{\boldsymbol{\beta}}$.

```
. predict yhat
(option xb assumed; fitted values)
. summarize mpg yhat if rep78 != .
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline mpg & 69 & 21.28986 & 5.866408 & 12 & 41 \\
yhat & 69 & 21.28986 & 4.383224 & 11.58643 & 28.07367
\end{tabular}
```

We had to include if rep78 < . in our summarize command because we have missing values in our data. areg automatically dropped those missing values (as it should) in forming the estimates, but predict with the xb option will make predictions for cases with missing rep78 because it does not know that rep78 is really part of our model.

These predicted values do not include the absorbed effects (that is, the $\mathbf{d}_{i} \gamma_{i}$ ). For predicted values that include these effects, use the xbd option of predict (see [R] areg postestimation) or see [XT] xtreg.

## > Example 2

areg, vce (robust) is a Huberized version of areg; see [P] _robust. Just as areg is equivalent to using regress with dummies, areg, vce (robust) is equivalent to using regress, vce (robust) with dummies. You can use areg, vce(robust) when you expect heteroskedastic or nonnormal errors. areg, vce(robust), like ordinary regression, assumes that the observations are independent, unless the vce(cluster clustvar) option is specified. If the vce(cluster clustvar) option is specified, this independence assumption is relaxed and only the clusters identified by equal values of clustvar are assumed to be independent.

Assume that we were to collect data by randomly sampling 10,000 doctors (from 100 hospitals) and then sampling 10 patients of each doctor, yielding a total dataset of 100,000 patients in a cluster sample. If in some regression we wished to include effects of the hospitals to which the doctors belonged, we would want to include a dummy variable for each hospital, adding 100 variables to our model. areg could fit this model by
. areg depvar patient_vars, absorb(hospital) vce(cluster doctor)

## Stored results

areg stores the following in e():
Scalars
e(N)
e(tss)
e(df_m)
e(rss)
e(df_r)
e(r2)
e(r2_a)
e(df_a)
e(rmse)
e(11)
e(11_0)
e(N_clust)
e(F)
e(F_absorb)
$e$ (rank)
Macros
e(cmd)
e(cmdline)
e(depvar)
e(absvar)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(vce)
e(vcetype)
e(datasignature)
e(datasignaturevars)
e(properties)
e(predict)
$e$ (footnote)
e(marginsnotok)
e(asbalanced)
e(asobserved)
Matrices
e(b)
e(Cns)
e(V)
e(V_modelbased)
Functions
$e$ (sample) marks estimation sample

## Methods and formulas

areg begins by recalculating depvar and indepvars to have mean 0 within the groups specified by absorb(). The overall mean of each variable is then added back in. The adjusted depvar is then regressed on the adjusted indepvars with regress, yielding the coefficient estimates. The degrees of freedom of the variance-covariance matrix of the coefficients is then adjusted to account for the absorbed variables - this calculation yields the same results (up to numerical roundoff error) as if the matrix had been calculated directly by the formulas given in [ $R$ ] regress.
areg with vce(robust) or vce(cluster clustvar) works similarly, calling _robust after regress to produce the Huber/White/sandwich estimator of the variance or its clustered version. See [P] _robust, particularly Introduction and Methods and formulas. The model $F$ test uses the robust variance estimates. There is, however, no simple computational means of obtaining a robust test of the absorbed dummies; thus this test is not displayed when the vce (robust) or vce (cluster clustvar) option is specified.

The number of groups specified in absorb() are included in the degrees of freedom used in the finite-sample adjustment of the cluster-robust VCE estimator. This statement is only valid if the number of groups is small relative to the sample size. (Technically, the number of groups must remain fixed as the sample size grows.) For an estimator that allows the number of groups to grow with the sample size, see the xtreg, fe command in [XT] xtreg.

## References

Blackwell, J. L., III. 2005. Estimation and testing of fixed-effect panel-data systems. Stata Journal 5: 202-207.
McCaffrey, D. F., K. Mihaly, J. R. Lockwood, and T. R. Sass. 2012. A review of Stata commands for fixed-effects estimation in normal linear models. Stata Journal 12: 406-432.

## Also see

[R] areg postestimation - Postestimation tools for areg
$[R]$ regress - Linear regression
[MI] estimation - Estimation commands for use with mi estimate
[XT] xtreg - Fixed-, between-, and random-effects and population-averaged linear models
[U] 20 Estimation and postestimation commands

## Title

areg postestimation - Postestimation tools for areg

| Description | Syntax for predict | Menu for predict | Options for predict |
| :--- | :--- | :--- | :--- |
| Remarks and examples | References | Also see |  |

## Description

The following postestimation commands are available after areg:

| Command | Description |
| :--- | :--- |
| contrast | contrasts and ANOVA-style joint tests of estimates |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample |
| estat vce | variance-covariance matrix of the estimators (VCE) <br> cataloging estimation results |
| estimates |  |
| forecast |  |
| lincom | dynamic forecasts and simulations <br> point estimates, standard errors, testing, and inference for linear <br> combinations of coefficients |
| linktest | link test for model specification |
| lrtest | likelihood-ratio test <br> marginal means, predictive margins, marginal effects, and average marginal <br> margins |
| effects |  |

[^2]
## Syntax for predict

predict [type] newvar [if] [in] [, statistic]
where $y_{j}=\mathbf{x}_{j} \mathbf{b}+d_{\text {absorbvar }}+e_{j}$ and statistic is
statistic
Description
Main
$\mathrm{xb} \quad \mathbf{x}_{j} \mathbf{b}$, fitted values; the default
stdp standard error of the prediction
dresiduals $\quad d_{\text {absorbvar }}+e_{j}=y_{j}-\mathbf{x}_{j} \mathbf{b}$
*xbd
*d

* residuals
$\mathbf{x}_{j} \mathbf{b}+d_{\text {absorbvar }}$
*score
$d_{\text {absorbvar }}$
residual
score; equivalent to residuals
Unstarred statistics are available both in and out of sample; type predict . . . if e (sample) ... if wanted only for the estimation sample. Starred statistics are calculated only for the estimation sample, even when if e(sample) is not specified.


## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
xb , the default, calculates the prediction of $\mathbf{x}_{j} \mathbf{b}$, the fitted values, by using the average effect of the absorbed variable. Also see xbd below.
stdp calculates the standard error of $\mathbf{x}_{j} \mathbf{b}$.
dresiduals calculates $y_{j}-\mathbf{x}_{j} \mathbf{b}$, which are the residuals plus the effect of the absorbed variable.
xbd calculates $\mathbf{x}_{j} \mathbf{b}+d_{\text {absorbvar }}$, which are the fitted values including the individual effects of the absorbed variable.
d calculates $d_{\text {absorbvar }}$, the individual coefficients for the absorbed variable.
residuals calculates the residuals, that is, $y_{j}-\left(\mathbf{x}_{j} \mathbf{b}+d_{\text {absorbvar }}\right)$.
score is a synonym for residuals.

## Remarks and examples

$>$ Example 1
Continuing with example 1 of $[R]$ areg, we refit the model with robust standard errors and then obtain linear predictions and standard errors for those linear predictions.

```
. use http://www.stata-press.com/data/r13/auto2
(1978 Automobile Data)
```

```
. areg mpg weight gear_ratio, absorb(rep78) vce(robust)
    (output omitted)
. predict xb_ar
(option xb assumed; fitted values)
. predict stdp_ar, stdp
```

We can obtain the same linear predictions by fitting the model with xtreg, fe, but we would first need to specify the panel structure by using xtset.

```
. xtset rep78
    panel variable: rep78 (unbalanced)
. xtreg mpg weight gear_ratio, fe vce(robust)
    (output omitted)
. predict xb_xt
(option xb assumed; fitted values)
. predict stdp_xt, stdp
. summarize xb_ar xb_xt stdp*
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline xb_ar & 74 & 21.36805 & 4.286788 & 11.58643 & 28.07367 \\
xb_xt & 74 & 21.36805 & 4.286788 & 11.58643 & 28.07367 \\
stdp_ar & 74 & .7105649 & .1933936 & .4270821 & 1.245179 \\
stdp_xt & 74 & .8155919 & .4826332 & .0826999 & 1.709786
\end{tabular}
```

The predicted xb values above are the same for areg and xtreg, fe, but the standard errors for those linear predictions are different. The assumptions for these two estimators lead to different formulations for their standard errors. The robust variance estimates with areg are equivalent to the robust variance estimates using regress, including the panel dummies. The consistent robust variance estimates with xtreg are equivalent to those obtained by specifying vce (cluster panelvar) with that estimation command. For a theoretical discussion, see Wooldridge (2013), Stock and Watson (2008), and Arellano (2003); also see the technical note after example 3 of [XT] xtreg.

## > Example 2

We would like to use linktest to check whether the dependent variable for our model is correctly specified:

```
. use http://www.stata-press.com/data/r13/auto2, clear
(1978 Automobile Data)
```

. areg mpg weight gear_ratio, absorb(rep78)
(output omitted)
. linktest, absorb(rep78)
Linear regression, absorbing indicators Number of obs $=\quad 69$
F 2 , 62) $=46.50$
Prob > F $=0.0000$
R-squared $=0.6939$
Adj R-squared $=0.6643$
Root MSE $=3.3990$

| mpg | Coef. | Std. Err. | t | $\mathrm{P}>\mid \mathrm{tI}$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| _hat | -.9305602 | .9537856 | -0.98 | 0.333 | -2.83715 | .9760302 |
| _hatsq | .0462785 | .0227219 | 2.04 | 0.046 | .0008582 | .0916989 |
| _cons | 19.24899 | 9.725618 | 1.98 | 0.052 | -.1922457 | 38.69022 |
| rep78 | $\mathrm{F}(4,62)=$ | 1.278 | 0.288 |  | (5 categories) |  |

The squared residuals are significant in the regression for mpg on the linear and squared residuals; therefore, the test indicates that our dependent variable does not seem to be well specified. Let's transform the dependent variable into energy consumption, gallons per mile, fit the alternative model, and check the link test again.

```
. generate gpm = 1/mpg
. areg gpm weight gear_ratio, absorb(rep78)
(output omitted)
. linktest, absorb(rep78)
```

Linear regression, absorbing indicators Number of obs $=\quad 69$
F ( 2, 62) $=\quad 72.60$
Prob > F $=0.0000$
R-squared $=0.7436$
Adj R-squared $=0.7187$
Root MSE $=0.0068$

| gpm | Coef. | Std. Err. | t | P>\|t| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| _hat | .2842582 | .7109124 | 0.40 | 0.691 | -1.136835 | 1.705352 |
| _hatsq | 6.956965 | 6.862439 | 1.01 | 0.315 | -6.760855 | 20.67478 |
| _cons | .0175457 | .0178251 | 0.98 | 0.329 | -.0180862 | .0531777 |
| rep78 | $F(4,62)=$ | 0.065 | 0.992 |  | (5 categories) |  |

The link test supports the use of the transformed dependent variable.

## References

Arellano, M. 2003. Panel Data Econometrics. Oxford: Oxford University Press.
Stock, J. H., and M. W. Watson. 2008. Heteroskedasticity-robust standard errors for fixed effects panel data regression. Econometrica 76: 155-174.

Wooldridge, J. M. 2013. Introductory Econometrics: A Modern Approach. 5th ed. Mason, OH: South-Western.

## Also see

$[R]$ areg - Linear regression with a large dummy-variable set
[U] 20 Estimation and postestimation commands

## Title

asclogit - Alternative-specific conditional logit (McFadden's choice) model

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

```
asclogit depvar [indepvars] [if] [in] [weight], case(varname)
alternatives(varname) [options]
```

options
Model

* case (varname)
* alternatives (varname)
casevars (varlist)
basealternative (\#|lbl|str) noconstant
altwise
offset (varname)
constraints (constraints)
collinear
SE/Robust
vce (vcetype)

Reporting
level(\#)
or
noheader
nocnsreport
display_options
Maximization
maximize_options
coeflegend
use varname to identify cases
use varname to identify the alternatives available for each case case-specific variables
alternative to normalize location
suppress alternative-specific constant terms
use alternativewise deletion instead of casewise deletion include varname in model with coefficient constrained to 1 apply specified linear constraints
keep collinear variables
vcetype may be oim, robust, cluster clustvar, bootstrap, or jackknife
set confidence level; default is level (95)
report odds ratios
do not display the header on the coefficient table
do not display constraints
control column formats and line width
control the maximization process; seldom used
display legend instead of statistics

[^3]bootstrap, by, fp, jackknife, statsby, and xi are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the bootstrap prefix; see $[R]$ bootstrap.
fweights, iweights, and pweights are allowed (see [U] 11.1.6 weight), but they are interpreted to apply to cases as a whole, not to individual observations. See Use of weights in $[R]$ clogit.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Categorical outcomes $>$ Alternative-specific conditional logit

## Description

asclogit fits McFadden's choice model, which is a specific case of the more general conditional logistic regression model (McFadden 1974). asclogit requires multiple observations for each case (individual or decision), where each observation represents an alternative that may be chosen. The cases are identified by the variable specified in the case () option, whereas the alternatives are identified by the variable specified in the alternatives () option. The outcome or chosen alternative is identified by a value of 1 in depvar, whereas zeros indicate the alternatives that were not chosen. There can be multiple alternatives chosen for each case.
asclogit allows two types of independent variables: alternative-specific variables and case-specific variables. Alternative-specific variables vary across both cases and alternatives and are specified in indepvars. Case-specific variables vary only across cases and are specified in the casevars () option.

See [R] clogit for a more general application of conditional logistic regression. For example, clogit would be used when you have grouped data where each observation in a group may be a different individual, but all individuals in a group have a common characteristic. You may use clogit to obtain the same estimates as asclogit by specifying the case() variable as the group() variable in clogit and generating variables that interact the casevars() in asclogit with each alternative (in the form of an indicator variable), excluding the interaction variable associated with the base alternative. asclogit takes care of this data management burden for you. Also, for clogit, each record (row in your data) is an observation, whereas in asclogit each case, consisting of several records (the alternatives) in your data, is an observation. This last point is important because asclogit will drop observations, by default, in a casewise fashion. That is, if there is at least one missing value in any of the variables for each record of a case, the entire case is dropped from estimation. To use alternativewise deletion, specify the altwise option and only the records with missing values will be dropped from estimation.

## Options

$\xlongequal{\text { Model }}$
case (varname) specifies the numeric variable that identifies each case. case() is required and must be integer valued.
alternatives (varname) specifies the variable that identifies the alternatives for each case. The number of alternatives can vary with each case; the maximum number of alternatives cannot exceed the limits of tabulate oneway; see $[R]$ tabulate oneway. alternatives () is required and may be a numeric or a string variable.
casevars (varlist) specifies the case-specific numeric variables. These are variables that are constant for each case. If there are a maximum of $J$ alternatives, there will be $J-1$ sets of coefficients associated with the casevars().
basealternative (\# $|l b l| s t r)$ specifies the alternative that normalizes the latent-variable location (the level of utility). The base alternative may be specified as a number, label, or string depending on the storage type of the variable indicating alternatives. The default is the alternative with the highest frequency.
If vce(bootstrap) or vce (jackknife) is specified, you must specify the base alternative. This is to ensure that the same model is fit with each call to asclogit.
noconstant suppresses the $J-1$ alternative-specific constant terms.
altwise specifies that alternativewise deletion be used when marking out observations due to missing values in your variables. The default is to use casewise deletion; that is, the entire group of observations making up a case is deleted if any missing values are encountered. This option does not apply to observations that are marked out by the if or in qualifier or the by prefix.
offset (varname), constraints(numlist|matname), collinear; see [R] estimation options.

SE/Robust
vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

Reporting
level (\#); see [R] estimation options.
or reports the estimated coefficients transformed to odds ratios, that is, $e^{b}$ rather than $b$. Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated or may be specified at estimation or when replaying previously estimated results.
noheader prevents the coefficient table header from being displayed.
nocnsreport; see [R] estimation options.
display_options: cformat (\%fmt), pformat (\%fmt), sformat (\% fint), and nolstretch; see [R] estimation options.
$\qquad$ Maximization
maximize_options: difficult, technique (algorithm_spec), iterate(\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance(\#), nonrtolerance, and from (init_specs); see [R] maximize. These options are seldom used.
technique (bhhh) is not allowed.
The initial estimates must be specified as from(matname [, copy]), where matname is the matrix containing the initial estimates and the copy option specifies that only the position of each element in matname is relevant. If copy is not specified, the column stripe of matname identifies the estimates.

The following option is available with asclogit but is not shown in the dialog box: coeflegend; see $[R]$ estimation options.

## Remarks and examples

asclogit fits McFadden's choice model (McFadden [1974]; for a brief introduction, see Greene [2012, sec. 18.2] or Cameron and Trivedi [2010, sec. 15.5]). In this model, we have a set of unordered alternatives indexed by $1,2, \ldots, J$. Let $y_{i j}, j=1, \ldots, J$, be an indicator variable for the alternative actually chosen by the $i$ th individual (case). That is, $y_{i j}=1$ if individual $i$ chose alternative $j$ and $y_{i j}=0$ otherwise. The independent variables come in two forms: alternative specific and case
specific. Alternative-specific variables vary among the alternatives (as well as cases), and case-specific variables vary only among cases. Assume that we have $p$ alternative-specific variables so that for case $i$ we have a $J \times p$ matrix, $\mathbf{X}_{i}$. Further, assume that we have $q$ case-specific variables so that we have a $1 \times q$ vector $\mathbf{z}_{i}$ for case $i$. Our random-utility model can then be expressed as

$$
\mathbf{u}_{i}=\mathbf{X}_{i} \boldsymbol{\beta}+\left(\mathbf{z}_{i} \mathbf{A}\right)^{\prime}+\boldsymbol{\epsilon}_{i}
$$

Here $\boldsymbol{\beta}$ is a $p \times 1$ vector of alternative-specific regression coefficients and $\mathbf{A}=\left(\boldsymbol{\alpha}_{1}, \ldots, \boldsymbol{\alpha}_{J}\right)$ is a $q \times J$ matrix of case-specific regression coefficients. The elements of the $J \times 1$ vector $\epsilon_{i}$ are independent Type I (Gumbel-type) extreme-value random variables with mean $\gamma$ (the Euler-Mascheroni constant, approximately 0.577 ) and variance $\pi^{2} / 6$. We must fix one of the $\boldsymbol{\alpha}_{j}$ to the constant vector to normalize the location. We set $\boldsymbol{\alpha}_{k}=0$, where $k$ is specified by the basealternative() option. The vector $\mathbf{u}_{i}$ quantifies the utility that the individual gains from the $J$ alternatives. The alternative chosen by individual $i$ is the one that maximizes utility.

## > Example 1

We have data on 295 consumers and their choice of automobile. Each consumer chose among an American, Japanese, or European car; the variable car indicates the nationality of the car for each alternative. We want to explore the relationship between the choice of car to the consumer's sex (variable sex) and income (variable income in thousands of dollars). We also have information on the number of dealerships of each nationality in the consumer's city in the variable dealer that we want to include as a regressor. We assume that consumers' preferences are influenced by the number of dealerships in an area but that the number of dealerships is not influenced by consumer preferences (which we admit is a rather strong assumption). The variable dealer is an alternative-specific variable ( $\mathbf{X}_{i}$ is a $3 \times 1$ vector in our previous notation), and sex and income are case-specific variables ( $\mathbf{z}_{i}$ is a $1 \times 2$ vector). Each consumer's chosen car is indicated by the variable choice.

Let's list some of the data.

```
. use http://www.stata-press.com/data/r13/choice
. list id car choice dealer sex income in 1/12, sepby(id)
```

|  | id | car | choice | dealer | sex | income |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| 1. | 1 | American | 0 | 18 | male | 46.7 |
| 2. | 1 | Japan | 0 | 8 | male | 46.7 |
| 3. | 1 | Europe | 1 | 5 | male | 46.7 |
| 4. | 2 | American | 1 | 17 | male | 26.1 |
| 5. | 2 | Japan | 0 | 6 | male | 26.1 |
| 6. | 2 | Europe | 0 | 2 | male | 26.1 |
| 7. | 3 | American | 1 | 12 | male | 32.7 |
| 8. | 3 | Japan | 0 | 6 | male | 32.7 |
| 9. | 3 | Europe | 0 | 2 | male | 32.7 |
| 10. | 4 | American | 0 | 18 | female | 49.2 |
| 11. | 4 | Japan | 1 | 7 | female | 49.2 |
| 12. | 4 | Europe | 0 | 4 | female | 49.2 |

We see, for example, that the first consumer, a male earning $\$ 46,700$ per year, chose to purchase a European car even though there are more American and Japanese car dealers in his area. The fourth consumer, a female earning \$49,200 per year, purchased a Japanese car.

We now fit our model.

| Iteration 0: | log likelihood $=-273.55685$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Iteration 1: | log likelihood $=-252.75109$ |  |  |  |  |  |
| Iteration 2: | log likelihood $=-250.78555$ |  |  |  |  |  |
| Iteration 3: | log likelihood $=-250.7794$ |  |  |  |  |  |
| Iteration 4: | log likelihood $=-250.7794$ |  |  |  |  |  |
| Alternative-specific conditional logit |  |  |  | Number | obs | 885 |
| Case variable: id |  |  |  | Number | cases | 295 |
| Alternative variable: car |  |  |  | Alts per case: min $=$ |  |  |
|  |  |  |  |  | avg | 3.0 |
|  |  |  |  |  | max | 3 |
|  |  |  |  | Wald | hi2(5) | 15.86 |
| Log likelihood = -250.7794 |  |  |  | Prob > chi2 |  | 0.0072 |
| choice | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Con | Interval] |
| car |  |  |  |  |  |  |
| dealer | . 0680938 | . 0344465 | 1.98 | 0.048 | . 00058 | . 1356076 |
| American | (base alternative) |  |  |  |  |  |
| Japan |  |  |  |  |  |  |
| sex | -. 5346039 | . 3141564 | -1.70 | 0.089 | -1.150339 | . 0811314 |
| income | . 0325318 | . 012824 | 2.54 | 0.011 | . 0073973 | . 0576663 |
| _cons | -1.352189 | . 6911829 | -1.96 | 0.050 | -2.706882 | . 0025049 |
| Europe |  |  |  |  |  |  |
| sex | . 5704109 | . 4540247 | 1.26 | 0.209 | -. 3194612 | 1.460283 |
| income | . 032042 | . 0138676 | 2.31 | 0.021 | . 004862 | . 0592219 |
| _cons | -2.355249 | . 8526681 | -2.76 | 0.006 | -4.026448 | -. 6840501 |

Displaying the results as odds ratios makes interpretation easier.
. asclogit, or noheader

| choice | Odds Ratio | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| car |  |  |  |  |  |  |
| dealer | 1.070466 | . 0368737 | 1.98 | 0.048 | 1.00058 | 1.145232 |
| American | (base alternative) |  |  |  |  |  |
| Japan |  |  |  |  |  |  |
| sex | . 5859013 | . 1840647 | -1.70 | 0.089 | . 3165294 | 1.084513 |
| income | 1.033067 | . 013248 | 2.54 | 0.011 | 1.007425 | 1.059361 |
| _cons | . 2586735 | . 1787907 | -1.96 | 0.050 | . 0667446 | 1.002508 |
| Europe |  |  |  |  |  |  |
| sex | 1.768994 | . 8031669 | 1.26 | 0.209 | . 7265404 | 4.307178 |
| income | 1.032561 | . 0143191 | 2.31 | 0.021 | 1.004874 | 1.061011 |
| _cons | . 0948699 | . 0808925 | -2.76 | 0.006 | . 0178376 | . 5045693 |

These results indicate that men (sex $=1$ ) are less likely to pick a Japanese car over an American car than women (odds ratio 0.59) but that men are more likely to choose a European car over an American car (odds ratio 1.77). Raising a person's income increases the likelihood that he or she
purchases a Japanese or European car; interestingly, the effect of higher income is about the same for these two types of cars.

Daniel Little McFadden was born in 1937 in North Carolina. He studied physics, psychology, and economics at the University of Minnesota and has taught economics at Pittsburgh, Berkeley, miT, and the University of Southern California. His contributions to logit models were triggered by a student's project on freeway routing decisions, and his work consistently links economic theory and applied problems. In 2000, he shared the Nobel Prize in Economics with James J. Heckman.

## Technical note

McFadden's choice model is related to multinomial logistic regression (see [R] mlogit). If all the independent variables are case specific, then the two models are identical. We verify this supposition by running the previous example without the alternative-specific variable, dealer.

| Alternative-specific conditional logit Case variable: id |  |  |  | Number | obs cases | 885 295 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Alternative variable: car |  |  |  | Alts pe | case: min | 3 |
|  |  |  |  |  | avg | 3.0 |
|  |  |  |  |  | max | 3 |
|  |  |  |  | Wald | hi2(4) | 12.53 |
| Log likelihood = -252.72012 |  |  |  | Prob | chi2 | 0.0138 |
| choice | Coef. | Std. Err. | z | $P>\|z\|$ | [95\% Conf | Interval] |
| American | (base alternative) |  |  |  |  |  |
| Japan |  |  |  |  |  |  |
| sex | -. 4694799 | . 3114939 | -1.51 | 0.132 | -1.079997 | . 141037 |
| income | . 0276854 | . 0123666 | 2.24 | 0.025 | . 0034472 | . 0519236 |
| _cons | -1.962652 | . 6216804 | -3.16 | 0.002 | -3.181123 | -. 7441807 |
| Europe |  |  |  |  |  |  |
| sex | . 5388441 | . 4525279 | 1.19 | 0.234 | -. 3480942 | 1.425782 |
| income | . 0273669 | . 013787 | 1.98 | 0.047 | . 000345 | . 0543889 |
| _cons | -3.180029 | . 7546837 | -4.21 | 0.000 | -4.659182 | -1.700876 |

To run mlogit, we must rearrange the dataset. mlogit requires a dependent variable that indicates the choice-1, 2 , or 3 -for each individual. We will use car as our dependent variable for those observations that represent the choice actually chosen.

| ```. keep if choice == 1 (590 observations deleted) . mlogit car sex income``` |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Iteration 0: log likelihood = -259.1712 |  |  |  |  |  |  |
| Iteration 1: log likelihood $=-252.81165$ |  |  |  |  |  |  |
| Iteration 2: log likelihood $=-252.72014$ |  |  |  |  |  |  |
| Iteration 3: log likelihood $=-252.72012$ |  |  |  |  |  |  |
| Multinomial logistic regression |  |  |  | Numb | of obs | 295 |
|  |  |  |  | LR | (4) | 12.90 |
|  |  |  |  | Prob | chi2 | 0.0118 |
| Log likelihood = -252.72012 |  |  |  | Pse |  | 0.0249 |
| car | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Con | Interval] |
| American | (base outcome) |  |  |  |  |  |
| Japan |  |  |  |  |  |  |
| sex | -. 4694798 | . 3114939 | -1.51 | 0.132 | -1.079997 | . 1410371 |
| income | . 0276854 | . 0123666 | 2.24 | 0.025 | . 0034472 | . 0519236 |
| _cons | -1.962651 | . 6216803 | -3.16 | 0.002 | -3.181122 | -. 7441801 |
| Europe |  |  |  |  |  |  |
| sex | . 5388443 | . 4525278 | 1.19 | 0.234 | -. 348094 | 1.425783 |
| income | . 027367 | . 013787 | 1.98 | 0.047 | . 000345 | . 0543889 |
| _cons | -3.18003 | . 7546837 | -4.21 | 0.000 | -4.659182 | -1.700877 |

The results are the same except for the model statistic: asclogit uses a Wald test and mlogit uses a likelihood-ratio test. If you prefer the likelihood-ratio test, you can fit the constant-only model for asclogit followed by the full model and use [R] lrtest. The following example will carry this out.

```
. use http://www.stata-press.com/data/r13/choice, clear
```

. asclogit choice, case(id) alternatives(car)
. estimates store null
. asclogit choice, case(id) alternatives(car) casevars(sex income)
. lrtest null .

## Technical note

We force you to explicitly identify the case-specific variables in the casevars() option to ensure that the program behaves as you expect. For example, an if or in qualifier may drop observations in such a way that (what was expected to be) an alternative-specific variable turns into a case-specific variable. Here you would probably want asclogit to terminate instead of interacting the variable with the alternative indicators. This situation could also occur if asclogit drops cases, or observations if you use the altwise option, because of missing values.

## Stored results

asclogit stores the following in e():
Scalars

```
e(N)
    e(N_case)
    e(k)
    e(k_alt)
    e(k_indvars)
    e(k_casevars)
    e(k_eq)
    e(k_eq_model)
    e(df_m)
    e(ll)
    e(N_clust)
    e(const)
    e(i_base)
    e(chi2)
    e(F)
    e(p)
    e(alt_min)
    e(alt_avg)
    e(alt_max)
    e(rank)
    e(ic)
    e(rc)
    e(converged)
```

Macros
$e$ (cmd)
e(cmdline)
e(depvar)
e(indvars)
e(casevars)
e(case)
e(altvar)
e(alteqs)
e(alt\#)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(offset)
e(chi2type)
e(vce)
e(vcetype)
e(opt)
e(which)
e(ml_method)
e(user)
e(technique)
$e$ (datasignature)
e(datasignaturevars)
e(properties)
e(estat_cmd)
$e$ (predict)
e(marginsnotok)
number of observations
number of cases
number of parameters
number of alternatives
number of alternative-specific variables
number of case-specific variables
number of equations in $e(b)$
number of equations in overall model test
model degrees of freedom
log likelihood
number of clusters
constant indicator
base alternative index
$\chi^{2}$
$F$ statistic
significance
minimum number of alternatives
average number of alternatives
maximum number of alternatives
rank of e(V)
number of iterations
return code
1 if converged, 0 otherwise
asclogit
command as typed
name of dependent variable
alternative-specific independent variable
case-specific variables
variable defining cases
variable defining alternatives
alternative equation names
alternative labels
weight type
weight expression
title in estimation output
name of cluster variable
linear offset variable
Wald, type of model $\chi^{2}$ test
vcetype specified in vce()
title used to label Std. Err.
type of optimization
max or min; whether optimizer is to perform maximization or minimization
type of ml method
name of likelihood-evaluator program
maximization technique
the checksum
variables used in calculation of checksum
b V
program used to implement estat
program used to implement predict
predictions disallowed by margins

```
Matrices
    e(b)
    e(stats)
    e(altvals)
    e(altfreq)
    e(alt_casevars)
    e(ilog)
    e(gradient)
    e(V)
    e(V_modelbased)
```

Functions
e(sample)
coefficient vector alternative statistics alternative values alternative frequencies
indicators for estimated case-specific coefficients-e(k_alt) $\times e\left(k \_c a s e v a r s\right)$
iteration $\log$ (up to 20 iterations)
gradient vector
variance-covariance matrix of the estimators
model-based variance
marks estimation sample

## Methods and formulas

In this model, we have a set of unordered alternatives indexed by $1,2, \ldots, J$. Let $y_{i j}, j=1, \ldots, J$, be an indicator variable for the alternative actually chosen by the $i$ th individual (case). That is, $y_{i j}=1$ if individual $i$ chose alternative $j$ and $y_{i j}=0$ otherwise. The independent variables come in two forms: alternative specific and case specific. Alternative-specific variables vary among the alternatives (as well as cases), and case-specific variables vary only among cases. Assume that we have $p$ alternative-specific variables so that for case $i$ we have a $J \times p$ matrix, $\mathbf{X}_{i}$. Further, assume that we have $q$ case-specific variables so that we have a $1 \times q$ vector $\mathbf{z}_{i}$ for case $i$. The deterministic component of the random-utility model can then be expressed as

$$
\begin{aligned}
\boldsymbol{\eta}_{i} & =\mathbf{X}_{i} \boldsymbol{\beta}+\left(\mathbf{z}_{i} \mathbf{A}\right)^{\prime} \\
& =\mathbf{X}_{i} \boldsymbol{\beta}+\left(\mathbf{z}_{i} \otimes \mathbf{I}_{J}\right) \operatorname{vec}\left(\mathbf{A}^{\prime}\right) \\
& =\left(\mathbf{X}_{i}, \mathbf{z}_{i} \otimes \mathbf{I}_{J}\right)\binom{\boldsymbol{\beta}}{\operatorname{vec}\left(\mathbf{A}^{\prime}\right)} \\
& =\mathbf{X}_{i}^{*} \boldsymbol{\beta}^{*}
\end{aligned}
$$

As before, $\boldsymbol{\beta}$ is a $p \times 1$ vector of alternative-specific regression coefficients, and $\mathbf{A}=\left(\boldsymbol{\alpha}_{1}, \ldots, \boldsymbol{\alpha}_{J}\right)$ is a $q \times J$ matrix of case-specific regression coefficients; remember that we must fix one of the $\boldsymbol{\alpha}_{j}$ to the constant vector to normalize the location. Here $\mathbf{I}_{J}$ is the $J \times J$ identity matrix, vec() is the vector function that creates a vector from a matrix by placing each column of the matrix on top of the other (see $[\mathrm{M}-5] \operatorname{vec}()$ ), and $\otimes$ is the Kronecker product (see [M-2] op_kronecker).

We have rewritten the linear equation so that it is a form that can be used by clogit, namely, $\mathbf{X}_{i}^{*} \boldsymbol{\beta}^{*}$, where

$$
\begin{gathered}
\mathbf{X}_{i}^{*}=\left(\mathbf{X}_{i}, \mathbf{z}_{i} \otimes \mathbf{I}_{J}\right) \\
\boldsymbol{\beta}^{*}=\binom{\boldsymbol{\beta}}{\operatorname{vec}\left(\mathbf{A}^{\prime}\right)}
\end{gathered}
$$

With this in mind, see Methods and formulas in [R] clogit for the computational details of the conditional logit model.

This command supports the clustered version of the Huber/White/sandwich estimator of the variance using vce(robust) and vce(cluster clustvar). See [ P ] _robust, particularly Maximum likelihood estimators and Methods and formulas. Specifying vce (robust) is equivalent to specifying vce(cluster casevar), where casevar is the variable that identifies the cases.

## References

Cameron, A. C., and P. K. Trivedi. 2010. Microeconometrics Using Stata. Rev. ed. College Station, TX: Stata Press. Greene, W. H. 2012. Econometric Analysis. 7th ed. Upper Saddle River, NJ: Prentice Hall.

McFadden, D. L. 1974. Conditional logit analysis of qualitative choice behavior. In Frontiers in Econometrics, ed. P. Zarembka, 105-142. New York: Academic Press.

## Also see

$[\mathrm{R}]$ asclogit postestimation - Postestimation tools for asclogit
[R] asmprobit - Alternative-specific multinomial probit regression
[R] asroprobit - Alternative-specific rank-ordered probit regression
$[\mathrm{R}]$ clogit - Conditional (fixed-effects) logistic regression
[ $R$ ] logistic - Logistic regression, reporting odds ratios
[R] logit - Logistic regression, reporting coefficients
[R] nlogit — Nested logit regression
$[R]$ ologit - Ordered logistic regression
[U] 20 Estimation and postestimation commands

## Title

asclogit postestimation - Postestimation tools for asclogit

| Description | Syntax for predict | Menu for predict |
| :--- | :--- | :--- |
| Options for predict | Syntax for estat | Menu for estat |
| Options for estat mfx | Remarks and examples | Stored results |
| Methods and formulas | Also see |  |

## Description

The following postestimation commands are of special interest after asclogit:

| Commands | Description |
| :--- | :--- |
| estat alternatives | alternative summary statistics |
| estat mfx | marginal effects |

The following standard postestimation commands are also available:
Commands Description

| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| :--- | :--- |
| estat summarize | summary statistics for the estimation sample <br> estat vce <br> estimates <br> hausman <br> lincom |
| cataloging estimation results |  |
| lrtest | Hausman's specification test <br> point estimates, standard errors, testing, and inference for linear <br> combinations of coefficients |
| nlcom | likelihood-ratio test <br> point estimates, standard errors, testing, and inference for nonlinear <br> combinations of coefficients |
| predictnl | predicted probabilities, estimated linear predictor and its standard error <br> point estimates, standard errors, testing, and inference for generalized <br> predictions |
| test | Wald tests of simple and composite linear hypotheses <br> testnl |

## Special-interest postestimation commands

estat alternatives displays summary statistics about the alternatives in the estimation sample. estat mfx computes probability marginal effects.

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic options]
predict [type] {stub*|newvarlist } [if] [in], scores
```

statistic Description

## Main

| $\frac{\mathrm{pr}}{\mathrm{xb}}$ | probability that each alternative is chosen; the default |
| :--- | :--- |
| stdp | linear prediction |
| standard error of the linear prediction |  |

options Description

## Main

* $k$ (\#| observed) condition on \# alternatives per case or on observed number of alternatives altwise use alternativewise deletion instead of casewise deletion when computing probabilities
nooffset ignore the offset() variable specified in asclogit
* k (\#| observed) may be used only with pr.

These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
pr computes the probability of choosing each alternative conditioned on each case choosing k() alternatives. This is the default statistic with default $\mathrm{k}(1)$; one alternative per case is chosen.
xb computes the linear prediction.
stdp computes the standard error of the linear prediction.
k (\#|observed) conditions the probability on \# alternatives per case or on the observed number of alternatives. The default is $\mathrm{k}(1)$. This option may be used only with the pr option.
altwise specifies that alternativewise deletion be used when marking out observations due to missing values in your variables. The default is to use casewise deletion. The xb and stdp options always use alternativewise deletion.
nooffset is relevant only if you specified offset (varname) for asclogit. It modifies the calculations made by predict so that they ignore the offset variable; the linear prediction is treated as $\mathbf{x} \boldsymbol{\beta}$ rather than as $\mathbf{x} \boldsymbol{\beta}+$ offset.
scores calculates the scores for each coefficient in $e(b)$. This option requires a new variable list of length equal to the number of columns in e(b). Otherwise, use the stub* option to have predict generate enumerated variables with prefix stub.

## Syntax for estat

## Alternative summary statistics

estat alternatives

Marginal effects
estat $\operatorname{mfx}[$ if $][$ in $][$, options $]$
options
Description
Main
varlist(varlist)
at(mean $[$ atlist $] \mid$ median $[$ atlist $]$ )
k(\#)
Options
level(\#)
nodiscrete
noesample
nowght
display marginal effects for varlist
calculate marginal effects at these values
condition on the number of alternatives chosen to be \#
set confidence interval level; default is level(95)
treat indicator variables as continuous
do not restrict calculation of means and medians to the estimation sample
ignore weights when calculating means and medians

## Menu for estat

Statistics $>$ Postestimation $>$ Reports and statistics

## Options for estat mfx

$\qquad$
varlist (varlist) specifies the variables for which to display marginal effects. The default is all variables.
at (mean $[$ atlist $] \mid$ median $[$ atlist $]$ ) specifies the values at which the marginal effects are to be calculated. atlist is

$$
[[\text { alternative }: \text { variable }=\#][\text { variable }=\#][\text { alternative }: \text { offset }=\#][\ldots]]
$$

The default is to calculate the marginal effects at the means of the independent variables by using the estimation sample, at (mean). If offset () is used during estimation, the means of the offsets (by alternative) are computed by default.
After specifying the summary statistic, you can specify a series of specific values for variables. You can specify values for alternative-specific variables by alternative, or you can specify one value for all alternatives. You can specify only one value for case-specific variables. You specify values for the offset () variable (if present) the same way as for alternative-specific variables. For example, in the choice dataset (car choice), income is a case-specific variable, whereas dealer is an alternative-specific variable. The following would be a legal syntax for estat mfx:

```
. estat mfx, at(mean American:dealer=18 income=40)
```

When nodiscrete is not specified, at (mean [atlist]) or at (median [atlist]) has no effect on computing marginal effects for indicator variables, which are calculated as the discrete change in the simulated probability as the indicator variable changes from 0 to 1 .

The mean and median computations respect any if or in qualifiers, so you can restrict the data over which the statistic is computed. You can even restrict the values to a specific case, for example,
. estat $m f x$ if case==21
$\mathrm{k}(\#)$ computes the probabilities conditioned on \# alternatives chosen. The default is one alternative chosen.

Options
level(\#) sets the confidence level; default is level(95).
nodiscrete specifies that indicator variables be treated as continuous variables. An indicator variable is one that takes on the value 0 or 1 in the estimation sample. By default, the discrete change in the simulated probability is computed as the indicator variable changes from 0 to 1.
noesample specifies that the whole dataset be considered instead of only those marked in the e(sample) defined by the asclogit command.
nowght specifies that weights be ignored when calculating the medians.

## Remarks and examples

Remarks are presented under the following headings:
Predicted probabilities
Obtaining estimation statistics

## Predicted probabilities

After fitting a McFadden's choice model with alternative-specific conditional logistic regression, you can use predict to obtain the estimated probability of alternative choices given case profiles.

## > Example 1

In example 1 of $[\mathrm{R}]$ asclogit, we fit a model of consumer choice of automobile. The alternatives are nationality of the automobile manufacturer: American, Japanese, or European. There is one alternativespecific variable in the model, dealer, which contains the number of dealerships of each nationality in the consumer's city. The case-specific variables are sex, the consumer's sex, and income, the consumer's income in thousands of dollars.

```
. use http://www.stata-press.com/data/r13/choice
. asclogit choice dealer, case(id) alternatives(car) casevars(sex income)
(output omitted)
- predict p
(option pr assumed; Pr(car))
. predict p2, k(2)
(option pr assumed; Pr(car))
. format p p2 %6.4f
```

. list car choice dealer sex income p p2 in 1/9, sepby(id)

|  | car | choice | dealer | sex | income | p | p2 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | 1. | American | 0 | 18 | male | 46.7 | 0.6025 |
| 2. | Japan | 0 | 8 | male | 46.7 | 0.2112 | 0.5979 |
| 3. | Europe | 1 | 5 | male | 46.7 | 0.1863 | 0.5437 |
|  | American | 1 | 17 | male | 26.1 | 0.7651 | 0.9293 |
| 5. | Japan | 0 | 6 | male | 26.1 | 0.1282 | 0.5778 |
| 6. | Europe | 0 | 2 | male | 26.1 | 0.1067 | 0.4929 |
| 7. | American | 1 | 12 | male | 32.7 | 0.6519 | 0.8831 |
| 8. | Japan | 0 | 6 | male | 32.7 | 0.1902 | 0.5995 |
| 9. | Europe | 0 | 2 | male | 32.7 | 0.1579 | 0.5174 |

## Obtaining estimation statistics

Here we will demonstrate the specialized estat subcommands after asclogit. Use estat alternatives to obtain a table of alternative statistics. The table will contain the alternative values, labels (if any), the number of cases in which each alternative is present, the frequency that the alternative is selected, and the percent selected.

Use estat mfx to obtain marginal effects after asclogit.

## $>$ Example 2

We will continue with the automobile choice example, where we first list the alternative statistics and then compute the marginal effects at the mean income in our sample, assuming that there are five automobile dealers for each nationality. We will evaluate the probabilities for females because sex is coded 0 for females, and we will be obtaining the discrete change from 0 to 1 .
. estat alternatives
Alternatives summary for car

| index | Alternative <br> value | label | Cases <br> present | Frequency <br> selected | Percent <br> selected |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 1 | American | 295 | 192 | 65.08 |
| 2 | 2 | Japan | 295 | 64 | 21.69 |
| 3 | 3 | Europe | 295 | 39 | 13.22 |

. estat mfx, at (dealer $=0$ sex=0) varlist(sex income)
$\operatorname{Pr}($ choice $=$ Americanl 1 selected $)=.41964329$
$\operatorname{Pr}($ choice $=$ American|1 selected) $=.41964329$

| variable | dp/dx | Std. Err. | z | $P>\|z\|$ | [ 95\% | C.I. ] | X |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| casevars |  |  |  |  |  |  |  |
| sex* | . 026238 | . 068311 | 0.38 | 0.701 | -. 107649 | . 160124 | 0 |
| income | -. 007891 | . 002674 | -2.95 | 0.003 | -. 013132 | -. 00265 | 42.097 |

(*) $d p / d x$ is for discrete change of indicator variable from 0 to 1
$\operatorname{Pr}($ choice $=$ Japan 11 selected $)=.42696187$

| variable | $\mathrm{dp} / \mathrm{dx}$ | Std. Err | z | $\mathrm{P}>\|\mathrm{z}\|$ | [ 95\% | C.I. ] | X |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| casevars |  |  |  |  |  |  |  |
| sex* | -. 161164 | . 079238 | -2.03 | 0.042 | -. 316468 | -. 005859 | 0 |
| income | . 005861 | . 002997 | 1.96 | 0.051 | -. 000014 | . 011735 | 42.097 |

(*) $\mathrm{dp} / \mathrm{dx}$ is for discrete change of indicator variable from 0 to 1
$\operatorname{Pr}($ choice $=$ Europel1 selected $)=.15339484$

| variable | dp/dx | Std. Err. | $z$ | P>\|z| | $\left[\begin{array}{ll}95 \% & \text { C.I. }\end{array}\right]$ | $X$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| casevars |  |  |  |  |  |  |  |
| sex* | .134926 | .076556 | 1.76 | 0.078 | -.015122 | .284973 | 0 |
| income | .00203 | .001785 | 1.14 | 0.255 | -.001469 | .00553 | 42.097 |

(*) dp/dx is for discrete change of indicator variable from 0 to 1
The marginal effect of income indicates that there is a lower chance for a consumer to buy American automobiles with an increase in income. There is an indication that men have a higher preference for European automobiles than women but a lower preference for Japanese automobiles. We did not include the marginal effects for dealer because we view these as nuisance parameters, so we adjusted the probabilities by fixing dealer to a constant, 0 .

## Stored results

estat mfx stores the following in r() :
Scalars
$r$ (pr_alt) scalars containing the computed probability of each alternative evaluated at the value that is labeled X in the table output. Here alt are the labels in the macro e(alteqs).
Matrices
r(alt)
matrices containing the computed marginal effects and associated statistics. There is one matrix for each alternative, where alt are the labels in the macro e(alteqs). Column 1 of each matrix contains the marginal effects; column 2, their standard errors; column 3, their $z$ statistics; and columns 4 and 5, the confidence intervals. Column 6 contains the values of the independent variables used to compute the probabilities $r\left(p r \_a l t\right)$.

## Methods and formulas

The deterministic component of the random-utility model can be expressed as

$$
\begin{aligned}
\boldsymbol{\eta} & =\mathbf{X} \boldsymbol{\beta}+(\mathbf{z A})^{\prime} \\
& =\mathbf{X} \boldsymbol{\beta}+\left(\mathbf{z} \otimes \mathbf{I}_{J}\right) \operatorname{vec}\left(\mathbf{A}^{\prime}\right) \\
& =\left(\mathbf{X}, \mathbf{z} \otimes \mathbf{I}_{J}\right)\binom{\boldsymbol{\beta}}{\operatorname{vec}\left(\mathbf{A}^{\prime}\right)} \\
& =\mathbf{X}^{*} \boldsymbol{\beta}^{*}
\end{aligned}
$$

where $\mathbf{X}$ is the $J \times p$ matrix containing the alternative-specific covariates, $\mathbf{z}$ is a $1 \times q$ vector of case-specific variables, $\boldsymbol{\beta}$ is a $p \times 1$ vector of alternative-specific regression coefficients, and $\mathbf{A}=\left(\boldsymbol{\alpha}_{1}, \ldots, \boldsymbol{\alpha}_{J}\right)$ is a $q \times J$ matrix of case-specific regression coefficients (with one of the $\boldsymbol{\alpha}_{j}$ fixed to the constant). Here $\mathbf{I}_{J}$ is the $J \times J$ identity matrix, vec() is the vector function that creates a vector from a matrix by placing each column of the matrix on top of the other (see [M-5] vec()), and $\otimes$ is the Kronecker product (see [M-2] op_kronecker).

We have rewritten the linear equation so that it is a form that we all recognize, namely, $\boldsymbol{\eta}=\mathbf{X}^{*} \boldsymbol{\beta}^{*}$, where

$$
\begin{aligned}
\mathbf{X}^{*} & =\left(\mathbf{X}, \mathbf{z} \otimes \mathbf{I}_{J}\right) \\
\boldsymbol{\beta}^{*} & =\binom{\boldsymbol{\beta}}{\operatorname{vec}\left(\mathbf{A}^{\prime}\right)}
\end{aligned}
$$

To compute the marginal effects, we use the derivative of the $\log$ likelihood $\partial \ell(\mathbf{y} \mid \boldsymbol{\eta}) / \partial \boldsymbol{\eta}$, where $\ell(\mathbf{y} \mid \boldsymbol{\eta})=\log \operatorname{Pr}(\mathbf{y} \mid \boldsymbol{\eta})$ is the $\log$ of the probability of the choice indicator vector $\mathbf{y}$ given the linear predictor vector $\boldsymbol{\eta}$. Namely,

$$
\begin{aligned}
\frac{\partial \operatorname{Pr}(\mathbf{y} \mid \boldsymbol{\eta})}{\partial \operatorname{vec}\left(\mathbf{X}^{*}\right)^{\prime}} & =\operatorname{Pr}(\mathbf{y} \mid \boldsymbol{\eta}) \frac{\partial \ell(\mathbf{y} \mid \boldsymbol{\eta})}{\partial \boldsymbol{\eta}^{\prime}} \frac{\partial \boldsymbol{\eta}}{\partial \operatorname{vec}\left(\mathbf{X}^{*}\right)^{\prime}} \\
& =\operatorname{Pr}(\mathbf{y} \mid \boldsymbol{\eta}) \frac{\partial \ell(\mathbf{y} \mid \boldsymbol{\eta})}{\partial \boldsymbol{\eta}^{\prime}}\left(\boldsymbol{\beta}^{* \prime} \otimes \mathbf{I}_{J}\right)
\end{aligned}
$$

The standard errors of the marginal effects are computed using the delta method.

## Also see

[R] asclogit - Alternative-specific conditional logit (McFadden's choice) model
[U] 20 Estimation and postestimation commands

## Title

asmprobit - Alternative-specific multinomial probit regression

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

```
asmprobit depvar [indepvars] [if] [in] [weight], case(varname)
alternatives(varname) [options]
```

options
Model

* case (varname)
* alternatives (varname)
casevars (varlist)
constraints (constraints)
collinear
Model 2
correlation(correlation)
stddev (stddev)
structural
factor (\#)
noconstant
basealternative(\#|lbl|str) scalealternative(\#|lbl|str) altwise

SE/Robust
vce (vcetype)

Reporting
level(\#)
notransform
nocnsreport
display_options
use varname to identify cases
use varname to identify the alternatives available for each case case-specific variables
apply specified linear constraints
keep collinear variables
correlation structure of the latent-variable errors
variance structure of the latent-variable errors
use the structural covariance parameterization; default is the differenced covariance parameterization
use the factor covariance structure with dimension \#
suppress the alternative-specific constant terms
alternative used for normalizing location
alternative used for normalizing scale
use alternativewise deletion instead of casewise deletion
vcetype may be oim, robust, cluster clustvar, opg, bootstrap, or jackknife
set confidence level; default is level (95)
do not transform variance-covariance estimates to the standard deviation and correlation metric
do not display constraints
control column formats and line width

| Integration |  |
| :---: | :---: |
| intmethod (seqtype) | type of quasi- or pseudouniform point set |
| intpoints(\#) | number of points in each sequence |
| intburn(\#) | starting index in the Hammersley or Halton sequence |
| intseed(code\|\#) | pseudouniform random-number seed |
| antithetics | use antithetic draws |
| nopivot | do not use integration interval pivoting |
| initbhhh(\#) | use the BHHH optimization algorithm for the first \# iterations |
| favor (speed $\mid$ space) | favor speed or space when generating integration points |
| Maximization |  |
| maximize_options | control the maximization process |
| coeflegend | display legend instead of statistics |
| correlation | Description |
| unstructured | one correlation parameter for each pair of alternatives; correlations with the basealternative() are zero; the default |
| exchangeable | one correlation parameter common to all pairs of alternatives; correlations with the basealternative() are zero |
| independent | constrain all correlation parameters to zero |
| pattern matname | user-specified matrix identifying the correlation pattern |
| fixed matname | user-specified matrix identifying the fixed and free correlation parameters |


| stddev | Description |
| :--- | :--- |
| heteroskedastic | estimate standard deviation for each alternative; standard deviations <br> for basealternative() and scalealternative() set to one |
| homoskedastic | all standard deviations are one |
| pattern matname <br> $\underline{\text { fixed }}$ matname | user-specified matrix identifying the standard deviation pattern <br> deviations |

seqtype Description
hammersley
halton
random

Hammersley point set
Halton point set
uniform pseudorandom point set

* case(varname) and alternatives(varname) are required.
bootstrap, by, jackknife, statsby, and xi are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the bootstrap prefix; see [R] bootstrap.
fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.


## Menu

Statistics $>$ Categorical outcomes $>$ Alternative-specific multinomial probit

## Description

asmprobit fits multinomial probit (MNP) models by using maximum simulated likelihood (MSL) implemented by the Geweke-Hajivassiliou-Keane (GHK) algorithm. By estimating the variancecovariance parameters of the latent-variable errors, the model allows you to relax the independence of irrelevant alternatives (IIA) property that is characteristic of the multinomial logistic model.
asmprobit requires multiple observations for each case (decision), where each observation represents an alternative that may be chosen. The cases are identified by the variable specified in the case() option, whereas the alternatives are identified by the variable specified in the alternative() option. The outcome (chosen alternative) is identified by a value of 1 in depvar, with 0 indicating the alternatives that were not chosen; only one alternative may be chosen for each case.
asmprobit allows two types of independent variables: alternative-specific variables and casespecific variables. Alternative-specific variables vary across both cases and alternatives and are specified in indepvars. Case-specific variables vary only across cases and are specified in the casevars() option.

## Options

$\qquad$ Model
case (varname) specifies the variable that identifies each case. This variable identifies the individuals or entities making a choice. case() is required.
alternatives(varname) specifies the variable that identifies the alternatives for each case. The number of alternatives can vary with each case; the maximum number of alternatives is 20. alternatives() is required.
casevars (varlist) specifies the case-specific variables that are constant for each case(). If there are a maximum of $J$ alternatives, there will be $J-1$ sets of coefficients associated with casevars(). constraints (constraints), collinear; see [R] estimation options.

## Model 2

correlation(correlation) specifies the correlation structure of the latent-variable errors.
correlation(unstructured) is the most general and has $J(J-3) / 2+1$ unique correlation parameters. This is the default unless stdev() or structural are specified.
correlation(exchangeable) provides for one correlation coefficient common to all latent variables, except the latent variable associated with the basealternative() option.
correlation(independent) assumes that all correlations are zero.
correlation(pattern matname) and correlation(fixed matname) give you more flexibility in defining the correlation structure. See Variance structures later in this entry for more information.
stddev (stddev) specifies the variance structure of the latent-variable errors.
stddev (heteroskedastic) is the most general and has $J-2$ estimable parameters. The standard deviations of the latent-variable errors for the alternatives specified in basealternative() and scalealternative() are fixed to one.
stddev(homoskedastic) constrains all the standard deviations to equal one.
stddev (pattern matname) and stddev (fixed matname) give you added flexibility in defining the standard deviation parameters. See Variance structures later in this entry for more information. structural requests the $J \times J$ structural covariance parameterization instead of the default $J-1 \times J-1$ differenced covariance parameterization (the covariance of the latent errors differenced with that of the base alternative). The differenced covariance parameterization will achieve the same MSL regardless of the choice of basealternative() and scalealternative(). On the other hand, the structural covariance parameterization imposes more normalizations that may bound the model away from its maximum likelihood and thus prevent convergence with some datasets or choices of basealternative() and scalealternative().
factor (\#) requests that the factor covariance structure of dimension \# be used. The factor () option can be used with the structural option but cannot be used with stddev() or correlation(). A $\# \times J$ ( or $\# \times J-1$ ) matrix, $\mathbf{C}$, is used to factor the covariance matrix as $I+\mathbf{C}^{\prime} \mathbf{C}$, where $I$ is the identity matrix of dimension $J$ (or $J-1$ ). The column dimension of $\mathbf{C}$ depends on whether the covariance is structural or differenced. The row dimension of $\mathbf{C}$, \#, must be less than or equal to floor $((J(J-1) / 2-1) /(J-2)$ ), because there are only $J(J-1) / 2-1$ identifiable variance-covariance parameters. This covariance parameterization may be useful for reducing the number of covariance parameters that need to be estimated.
If the covariance is structural, the column of $\mathbf{C}$ corresponding to the base alternative contains zeros. The column corresponding to the scale alternative has a one in the first row and zeros elsewhere. If the covariance is differenced, the column corresponding to the scale alternative (differenced with the base) has a one in the first row and zeros elsewhere.
noconstant suppresses the $J-1$ alternative-specific constant terms.
basealternative (\#|lbl|str) specifies the alternative used to normalize the latent-variable location (also referred to as the level of utility). The base alternative may be specified as a number, label, or string. The standard deviation for the latent-variable error associated with the base alternative is fixed to one, and its correlations with all other latent-variable errors are set to zero. The default is the first alternative when sorted. If a fixed or pattern matrix is given in the stddev() and correlation() options, the basealternative() will be implied by the fixed standard deviations and correlations in the matrix specifications. basealternative() cannot be equal to scalealternative().
scalealternative (\# $|l b l|$ str) specifies the alternative used to normalize the latent-variable scale (also referred to as the scale of utility). The scale alternative may be specified as a number, label, or string. The default is to use the second alternative when sorted. If a fixed or pattern matrix is given in the stddev() option, the scalealternative() will be implied by the fixed standard deviations in the matrix specification. scalealternative() cannot be equal to basealternative().
If a fixed or pattern matrix is given for the stddev() option, the base alternative and scale alternative are implied by the standard deviations and correlations in the matrix specifications, and they need not be specified in the basealternative() and scalealternative() options.
altwise specifies that alternativewise deletion be used when marking out observations due to missing values in your variables. The default is to use casewise deletion; that is, the entire group of observations making up a case is deleted if any missing values are encountered. This option does not apply to observations that are marked out by the if or in qualifier or the by prefix.
vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
If specifying vce(bootstrap) or vce(jackknife), you must also specify basealternative() and scalealternative().

## Reporting

level(\#); see [R] estimation options.
notransform prevents retransforming the Cholesky-factored variance-covariance estimates to the correlation and standard deviation metric.

This option has no effect if structural is not specified because the default differenced variancecovariance estimates have no interesting interpretation as correlations and standard deviations. notransform also has no effect if the correlation() and stddev() options are specified with anything other than their default values. Here it is generally not possible to factor the variancecovariance matrix, so optimization is already performed using the standard deviation and correlation representations.
nocnsreport; see [R] estimation options.
display_options: cformat (\%fmt), pformat (\%fmt), sformat (\%fmt), and nolstretch; see [R] estimation options.

## Integration

intmethod(hammersley $\mid$ halton $\mid$ random) specifies the method of generating the point sets used in the quasi-Monte Carlo integration of the multivariate normal density. intmethod (hammersley), the default, uses the Hammersley sequence; intmethod (halton) uses the Halton sequence; and intmethod (random) uses a sequence of uniform random numbers.
intpoints(\#) specifies the number of points to use in the quasi-Monte Carlo integration. If this option is not specified, the number of points is $50 \times J$ if intmethod(hammersley) or intmethod(halton) is used and $100 \times J$ if intmethod (random) is used. Larger values of intpoints() provide better approximations of the log likelihood, but at the cost of added computation time.
intburn(\#) specifies where in the Hammersley or Halton sequence to start, which helps reduce the correlation between the sequences of each dimension. The default is 0 . This option may not be specified with intmethod (random).
intseed (code $\mid \#$ ) specifies the seed to use for generating the uniform pseudorandom sequence. This option may be specified only with intmethod (random). code refers to a string that records the state of the random-number generator runiform(); see [R] set seed. An integer value \# may be used also. The default is to use the current seed value from Stata's uniform random-number generator, which can be obtained from c (seed).
antithetics specifies that antithetic draws be used. The antithetic draw for the $J-1$ vector uniform-random variables, $\mathbf{x}$, is $1-\mathbf{x}$.
nopivot turns off integration interval pivoting. By default, asmprobit will pivot the wider intervals of integration to the interior of the multivariate integration. This improves the accuracy of the quadrature estimate. However, discontinuities may result in the computation of numerical secondorder derivatives using finite differencing (for the Newton-Raphson optimize technique, tech (nr))
when few simulation points are used, resulting in a non-positive-definite Hessian. asmprobit uses the Broyden-Fletcher-Goldfarb-Shanno optimization algorithm, by default, which does not require computing the Hessian numerically using finite differencing.
initbhhh (\#) specifies that the Berndt-Hall-Hall-Hausman (BHHH) algorithm be used for the initial \# optimization steps. This option is the only way to use the BHHH algorithm along with other optimization techniques. The algorithm switching feature of ml's technique() option cannot include bhhh.
favor (speed $\mid$ space) instructs asmprobit to favor either speed or space when generating the integration points. favor (speed) is the default. When favoring speed, the integration points are generated once and stored in memory, thus increasing the speed of evaluating the likelihood. This speed increase can be seen when there are many cases or when the user specifies a large number of integration points, intpoints (\#). When favoring space, the integration points are generated repeatedly with each likelihood evaluation.
For unbalanced data, where the number of alternatives varies with each case, the estimates computed using intmethod (random) will vary slightly between favor (speed) and favor (space). This is because the uniform sequences will not be identical, even when initiating the sequences using the same uniform seed, intseed (code $\mid \#$ ). For favor (speed), ncase blocks of intpoints (\#) $\times$ $J-2$ uniform points are generated, where $J$ is the maximum number of alternatives. For favor(space), the column dimension of the matrices of points varies with the number of alternatives that each case has.

## Maximization

maximize_options: difficult, technique(algorithm_spec), iterate(\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance(\#), nonrtolerance, and from(init_specs); see [R] maximize.
The following options may be particularly useful in obtaining convergence with asmprobit: difficult, technique(algorithm_spec), nrtolerance(\#), nonrtolerance, and from(init_specs).
If technique() contains more than one algorithm specification, bhhh cannot be one of them. To use the BHHH algorithm with another algorithm, use the initbhhh() option and specify the other algorithm in technique().
Setting the optimization type to technique (bhhh) resets the default vcetype to vce (opg).
The following option is available with asmprobit but is not shown in the dialog box: coeflegend; see $[R]$ estimation options.

## Remarks and examples

Remarks are presented under the following headings:

[^4]
## Introduction

The MNP model is used with discrete dependent variables that take on more than two outcomes that do not have a natural ordering. The stochastic error terms are assumed to have a multivariate normal distribution that is heteroskedastic and correlated. Say that you have a set of $J$ unordered alternatives that are modeled by a regression of both case-specific and alternative-specific covariates. A "case" refers to the information on one decision maker. Underlying the model is the set of $J$ latent variables (utilities),

$$
\begin{equation*}
\eta_{i j}=\mathbf{x}_{i j} \boldsymbol{\beta}+\mathbf{z}_{i} \boldsymbol{\alpha}_{j}+\xi_{i j} \tag{1}
\end{equation*}
$$

where $i$ denotes cases and $j$ denotes alternatives. $\mathbf{x}_{i j}$ is a $1 \times p$ vector of alternative-specific variables, $\boldsymbol{\beta}$ is a $p \times 1$ vector of parameters, $\mathbf{z}_{i}$ is a $1 \times q$ vector of case-specific variables, $\boldsymbol{\alpha}_{j}$ is a $q \times 1$ vector of parameters for the $j$ th alternative, and $\boldsymbol{\xi}_{i}=\left(\xi_{i 1}, \ldots, \xi_{i J}\right)$ is distributed multivariate normal with mean zero and covariance matrix $\boldsymbol{\Omega}$. The decision maker selects the alternative whose latent variable is highest.

Because the MNP model allows for a general covariance structure in $\xi_{i j}$, it does not impose the IIA property inherent in multinomial logistic and conditional logistic models. That is, the MNP model permits the odds of choosing one alternative over another to depend on the remaining alternatives. For example, consider the choice of travel mode between two cities: air, train, bus, or car, as a function of the travel mode cost, travel time (alternative-specific variables), and an individual's income (a case-specific variable). The odds of choosing air travel over a bus may not be independent of the train alternative because both bus and train travel are public ground transportation. That is, the probability of choosing air travel is $\operatorname{Pr}\left(\eta_{\text {air }}>\eta_{\text {bus }}, \eta_{\text {air }}>\eta_{\text {train }}, \eta_{\text {air }}>\eta_{\text {car }}\right)$, and the two events $\eta_{\text {air }}>\eta_{\text {bus }}$ and $\eta_{\text {air }}>\eta_{\text {train }}$ may be correlated.

An alternative to MNP that will allow a nested correlation structure in $\xi_{i j}$ is the nested logit model (see [R] nlogit).

The added flexibility of the MNP model does impose a significant computation burden because of the need to evaluate probabilities from the multivariate normal distribution. These probabilities are evaluated using simulation techniques because a closed-form solution does not exist. See Methods and formulas for more information.

Not all the $J$ sets of regression coefficients $\boldsymbol{\alpha}_{j}$ are identifiable, nor are all $J(J+1) / 2$ elements of the variance-covariance matrix $\boldsymbol{\Omega}$. As described by Train (2009, sec. 2.5), the model requires normalization because both the location (level) and scale of the latent variable are irrelevant. Increasing the latent variables by a constant does not change which $\eta_{i j}$ is the maximum for decision maker $i$, nor does multiplying them by a constant. To normalize location, we choose an alternative, indexed by $k$, say, and take the difference between the latent variable $k$ and the $J-1$ others,

$$
\begin{align*}
v_{i j k} & =\eta_{i j}-\eta_{i k} \\
& =\left(\mathbf{x}_{i j}-\mathbf{x}_{i k}\right) \boldsymbol{\beta}+\mathbf{z}_{i}\left(\boldsymbol{\alpha}_{j}-\boldsymbol{\alpha}_{k}\right)+\xi_{i j}-\xi_{i k}  \tag{2}\\
& =\boldsymbol{\delta}_{i j^{\prime}} \boldsymbol{\beta}+\mathbf{z}_{i} \boldsymbol{\gamma}_{j^{\prime}}+\epsilon_{i j^{\prime}} \\
& =\lambda_{i j^{\prime}}+\epsilon_{i j^{\prime}}
\end{align*}
$$

where $j^{\prime}=j$ if $j<k$ and $j^{\prime}=j-1$ if $j>k$, so that $j^{\prime}=1, \ldots, J-1$. One can now work with the $(J-1) \times(J-1)$ covariance matrix $\boldsymbol{\Sigma}_{(k)}$ for $\boldsymbol{\epsilon}_{i}^{\prime}=\left(\epsilon_{i 1}, \ldots, \epsilon_{i, J-1}\right)$. The $k$ th alternative here is the basealternative() in asmprobit. From (2), the probability that decision maker $i$ chooses alternative $k$, for example, is

$$
\begin{aligned}
\operatorname{Pr}(i \text { chooses } k) & =\operatorname{Pr}\left(v_{i 1 k} \leq 0, \ldots, v_{i, J-1, k} \leq 0\right) \\
& =\operatorname{Pr}\left(\epsilon_{i 1} \leq-\lambda_{i 1}, \ldots, \epsilon_{i, J-1} \leq-\lambda_{i, J-1}\right)
\end{aligned}
$$

To normalize for scale, one of the diagonal elements of $\boldsymbol{\Sigma}_{(k)}$ must be fixed to a constant. In asmprobit, this is the error variance for the alternative specified by scalealternative(). Thus there are a total of, at most, $J(J-1) / 2-1$ identifiable variance-covariance parameters. See Variance structures below for more on this issue.

In fact, the model is slightly more general in that not all cases need to have faced all $J$ alternatives. The model allows for situations in which some cases chose among all possible alternatives, whereas other cases were given a choice among a subset of them, and perhaps other cases were given a choice among a different subset. The number of observations for each case is equal to the number of alternatives faced.

The MNP model is often motivated using a random-utility consumer-choice framework. Equation (1) represents the utility that consumer $i$ receives from good $j$. The consumer purchases the good for which the utility is highest. Because utility is ordinal, all that matters is the ranking of the utilities from the alternatives. Thus one must normalize for location and scale.

## > Example 1

Application of MNP models is common in the analysis of transportation data. Greene (2012, sec. 18.2.9) uses travel-mode choice data between Sydney and Melbourne to demonstrate estimating parameters of various discrete-choice models. The data contain information on 210 individuals' choices of travel mode. The four alternatives are air, train, bus, and car, with indices $1,2,3$, and 4 , respectively. One alternative-specific variable is travelcost, a measure of generalized cost of travel that is equal to the sum of in-vehicle cost and a wagelike measure times the amount of time spent traveling. A second alternative-specific variable is the terminal time, termtime, which is zero for car transportation. Household income, income, is a case-specific variable.

```
. use http://www.stata-press.com/data/r13/travel
. list id mode choice travelcost termtime income in 1/12, sepby(id)
```

|  | id | mode | choice | travel $\sim$ t | termtime | income |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | 1 | air | 0 | 70 | 69 | 35 |
| 2. | 1 | train | 0 | 71 | 34 | 35 |
| 3. | 1 | bus | 0 | 70 | 35 | 35 |
| 4. | 1 | car | 1 | 30 | 0 | 35 |
| 5. | 2 | air | 0 | 68 | 64 | 30 |
| 6. | 2 | train | 0 | 84 | 44 | 30 |
| 7. | 2 | bus | 0 | 85 | 53 | 30 |
| 8. | 2 | car | 1 | 50 | 0 | 30 |
| 9. | 3 | air | 0 | 129 | 69 | 40 |
| 10. | 3 | train | 0 | 195 | 34 | 40 |
| 11. | 3 | bus | 0 | 149 | 35 | 40 |
| 12. | 3 | car | 1 | 101 | 0 | 40 |

The model of travel choice is

$$
\eta_{i j}=\beta_{1} \text { travelcost }_{i j}+\beta_{2} \text { termtime }_{i j}+\alpha_{1 j} \text { income }_{i}+\alpha_{0 j}+\xi_{i j}
$$

The alternatives can be grouped as air and ground travel. With this in mind, we set the air alternative to be the basealternative() and choose train as the scaling alternative. Because these are the first and second alternatives in the mode variable, they are also the defaults.


| choice | Coef . | Std. Err | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Con | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| mode <br> travelcost termtime | $\begin{array}{r} -.00977 \\ -.0377095 \end{array}$ | $\begin{aligned} & .0027834 \\ & .0094088 \end{aligned}$ | $\begin{aligned} & -3.51 \\ & -4.01 \end{aligned}$ | $\begin{aligned} & 0.000 \\ & 0.000 \end{aligned}$ | $\begin{aligned} & -.0152253 \\ & -.0561504 \end{aligned}$ | $\begin{aligned} & -.0043146 \\ & -.0192686 \end{aligned}$ |
| air | (base alternative) |  |  |  |  |  |
| train |  |  |  |  |  | $-.0117052$ |
| income _cons | $\begin{array}{r} -.0291971 \\ .5616376 \end{array}$ | $\begin{aligned} & .0089246 \\ & .3946551 \end{aligned}$ | $\begin{array}{r} -3.27 \\ 1.42 \end{array}$ | $\begin{aligned} & 0.001 \\ & 0.155 \end{aligned}$ | $\begin{array}{r} -.046689 \\ -.2118721 \end{array}$ |  |
| bus |  |  |  |  |  | . 0027857 |
| income _cons | -. 0127503 | . 0079267 | -1.61 | 0.108 | -. 0282863 |  |
|  | -. 0571364 | . 4791861 | -0.12 | 0.905 | -. 9963239 | . 882051 |
| car | $\begin{aligned} & -.0049086 \\ & -1.833393 \end{aligned}$ | $\begin{aligned} & .0077486 \\ & .8186156 \end{aligned}$ | $\begin{aligned} & -0.63 \\ & -2.24 \end{aligned}$ | $\begin{aligned} & 0.526 \\ & 0.025 \end{aligned}$ | $\begin{array}{r} -.0200957 \\ -3.43785 \end{array}$ | $\begin{array}{r} .0102784 \\ -.2289357 \end{array}$ |
| income <br> _cons |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| /lnl2_2 | $\begin{aligned} & -.5502039 \\ & -.6005552 \end{aligned}$ | $\begin{aligned} & .3905204 \\ & .3353292 \end{aligned}$ | $\begin{aligned} & -1.41 \\ & -1.79 \end{aligned}$ | 0.159 | -1.31561 | . 2152021 |
| /lnl3_3 |  |  |  | 0.073 | -1. 257788 | . 0566779 |
| /12_1 | 1.131518 | . 2124817 | 5.33 | 0.000 | . 7150612 | 1.547974 |
| /13_1 | . 9720669 | . 2352116 | 4.13 | 0.000 | . 5110606 | 1.433073 |
| /13_2 | . 5197214 | . 2861552 | 1.82 | 0.069 | -. 0411325 | 1.080575 |

(mode=air is the alternative normalizing location)
(mode=train is the alternative normalizing scale)
. estimates store full
By default, the differenced covariance parameterization is used, so the covariance matrix for this model is $3 \times 3$. There are two free variances to estimate and three correlations. To help ensure that the covariance matrix remains positive definite, asmprobit uses the square root transformation, where it optimizes on the Cholesky-factored variance-covariance. To ensure that the diagonal elements of the Cholesky estimates remain positive, we use the log transformation. The estimates labeled /lnl2_2 and /lnl3_3 in the coefficient table are the log-transformed diagonal elements of the Cholesky matrix. The estimates labeled /12_1, /13_1, and /13_2 are the off-diagonal entries for elements $(2,1),(3,1)$, and $(3,2)$ of the Cholesky matrix.

Although the transformed parameters of the differenced covariance parameterization are difficult to interpret, you can view them untransformed by using the estat command. Typing
. estat correlation

|  | train | bus | car |
| ---: | :---: | :---: | :---: |
| train | 1.0000 |  |  |
| bus | 0.8909 | 1.0000 |  |
| car | 0.7895 | 0.8951 | 1.0000 |

Note: correlations are for alternatives differenced with air
gives the correlations, and typing
. estat covariance

|  | train | bus | car |
| ---: | ---: | ---: | ---: |
| train | 2 |  |  |
| bus | 1.600208 | 1.613068 |  |
| car | 1.37471 | 1.399703 | 1.515884 |

Note: covariances are for alternatives differenced with air
gives the (co)variances.
We can reduce the number of covariance parameters in the model by using the factor model by Cameron and Trivedi (2005). For large models with many alternatives, the parameter reduction can be dramatic, but for our example we will use factor(1), a one-dimension factor model, to reduce by 3 the number of parameters associated with the covariance matrix.

```
. asmprobit choice travelcost termtime, case(id) alternatives(mode)
> casevars(income) factor(1)
```

    (output omitted)
    | Alternative-specific multinomial probit | Number of obs | $=$ | 840 |
| :--- | :--- | ---: | :--- |
| Case variable: id | Number of cases | $=$ | 210 |
| Alternative variable: mode | Alts per case: min | $=$ | 4 |
|  |  | avg | $=$ |
|  |  | max | $=$ |


| Integration sequence: | Hammersley |  |  |  |
| :--- | ---: | :--- | :--- | :--- |
| Integration points: | 200 | Wald chi2(5) | $=$ | 107.85 |
| Log simulated-likelihood $=$ | -196.85094 | Prob > chi2 | $=$ | 0.0000 |


(mode=air is the alternative normalizing location)
(mode=train is the alternative normalizing scale)
The estimates labeled /c1_2 and /c1_3 in the coefficient table are the factor loadings. These factor loadings produce the following differenced covariance estimates:
. estat covariance

|  | train | bus | car |
| ---: | ---: | ---: | ---: |
| train | 2 |  |  |
| bus | 1.182805 | 2.399027 |  |
| car | 1.227705 | 1.452135 | 2.507259 |

Note: covariances are for alternatives differenced with air

## Variance structures

The matrix $\boldsymbol{\Omega}$ has $J(J+1) / 2$ distinct elements because it is symmetric. Selecting a base alternative, normalizing its error variance to one, and constraining the correlations between its error and the other errors reduces the number of estimable parameters by $J$. Moreover, selecting a scale alternative and normalizing its error variance to one reduces the number by one, as well. Hence, there are at most $m=J(J-1) / 2-1$ estimable parameters in $\boldsymbol{\Omega}$.

In practice, estimating all $m$ parameters can be difficult, so one must often place more restrictions on the parameters. The asmprobit command provides the correlation() option to specify restrictions on the $J(J-3) / 2+1$ correlation parameters not already restricted as a result of choosing the base alternatives, and it provides stddev() to specify restrictions on the $J-2$ standard deviations not already restricted as a result of choosing the base and scale alternatives.

When the structural option is used, asmprobit fits the model by assuming that all $m$ parameters can be estimated, which is equivalent to specifying correlation(unstructured) and stddev(heteroskedastic). The unstructured correlation structure means that all $J(J-3) / 2+1$ of the remaining correlation parameters will be estimated, and the heteroskedastic specification means that all $J-2$ standard deviations will be estimated. With these default settings, the log likelihood is maximized with respect to the Cholesky decomposition of $\boldsymbol{\Omega}$, and then the parameters are transformed to the standard deviation and correlation form.

The correlation (exchangeable) option forces the $J(J-3) / 2+1$ correlation parameters to be equal, and correlation(independent) forces all the correlations to be zero. Using the stddev (homoskedastic) option forces all $J$ standard deviations to be one. These options may help in obtaining convergence for a model if the default options do not produce satisfactory results. In fact, when fitting a complex model, it may be advantageous to first fit a simple one and then proceed with removing the restrictions one at a time.

Advanced users may wish to specify alternative variance structures of their own choosing, and the next few paragraphs explain how to do so.
correlation(pattern matname) allows you to give the name of a $J \times J$ matrix that identifies a correlation structure. Sequential positive integers starting at 1 are used to identify each correlation parameter: if there are three correlation parameters, they are identified by 1,2 , and 3 . The integers can be repeated to indicate that correlations with the same number should be constrained to be equal. A zero or a missing value (.) indicates that the correlation is to be set to zero. asmprobit considers only the elements of the matrix below the main diagonal.

Suppose that you have a model with four alternatives, numbered $1-4$, and alternative 1 is the base. The unstructured and exchangeable correlation structures identified in the $4 \times 4$ lower triangular matrices are

| unstructured |  |  |  |  | exchangeable |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 |  | 1 | 2 | 2 | 3 | 4 |
| 2 3 4 | $\left(\begin{array}{l}\text { b } \\ 0 \\ 0 \\ 0\end{array}\right.$ | 1 | 3 | . | 1 2 3 4 | $\left(\begin{array}{l}\text { b } \\ 0 \\ 0 \\ 0\end{array}\right.$ |  | 1 | 1 | . $)$ |

asmprobit labels these correlation structures unstructured and exchangeable, even though the correlations corresponding to the base alternative are set to zero. More formally: these terms are appropriate when considering the $(J-1) \times(J-1)$ submatrix $\boldsymbol{\Sigma}_{(k)}$ defined in the Introduction above.

You can also use the correlation(fixed matname) option to specify a matrix that specifies fixed and free parameters. Here the free parameters (those that are to be estimated) are identified by a missing value, and nonmissing values represent correlations that are to be taken as given. Below is a correlation structure that would set the correlations of alternative 1 to be 0.5 :

$$
\left.\begin{array}{llll}
1 \\
1 \\
2 \\
3 \\
3 & 2 & 3 & 4 \\
0.5 & . & & \\
40.5 & \cdot & . & \\
0.5 & . & . & .
\end{array}\right)
$$

The order of the elements of the pattern or fixed matrices must be the same as the numeric order of the alternative levels.

To specify the structure of the standard deviations-the diagonal elements of $\Omega$-you can use the stddev(pattern matname) option, where matname is a $1 \times J$ matrix. Sequential positive integers starting at 1 are used to identify each standard deviation parameter. The integers can be repeated to indicate that standard deviations with the same number are to be constrained to be equal. A missing value indicates that the corresponding standard deviation is to be set to one. In the four-alternative example mentioned above, suppose that you wish to set the first and second standard deviations to one and that you wish to constrain the third and fourth standard deviations to be equal; the following pattern matrix will do that:

$$
\left.\begin{array}{c} 
\\
1
\end{array} \begin{array}{cccc}
1 & 2 & 3 & 4 \\
(\cdot & \cdot & 1 & 1
\end{array}\right)
$$

Using the stddev(fixed matname) option allows you to identify the fixed and free standard deviations. Fixed standard deviations are entered as positive real numbers, and free parameters are identified with missing values. For example, to constrain the first and second standard deviations to equal one and to allow the third and fourth to be estimated, you would use this fixed matrix:

$$
\left.\begin{array}{ccccc} 
& 1 & 2 & 3 & 4 \\
1 & (1 & 1 & \cdot & \cdot
\end{array}\right)
$$

When supplying either the pattern or the fixed matrices, you must ensure that the model is properly scaled. At least two standard deviations must be constant for the model to be scaled. A warning is issued if asmprobit detects that the model is not scaled.

The order of the elements of the pattern or fixed matrices must be the same as the numeric order of the alternative levels.

## > Example 2

In example 1, we used the differenced covariance parameterization, the default. We now use the structural option to view the $J-2$ standard deviation estimates and the $(J-1)(J-2) / 2$ correlation estimates. Here we will fix the standard deviations for the air and train alternatives to 1 and the correlations between air and the rest of the alternatives to 0 .


| choice | Coef. | Std. Err. | z | $P>\|z\|$ | [95\% Con | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| mode travelcost termtime | $\begin{aligned} & -.0097703 \\ & -.0377103 \end{aligned}$ | $\begin{aligned} & .0027834 \\ & .0094092 \end{aligned}$ | $\begin{aligned} & -3.51 \\ & -4.01 \end{aligned}$ | $\begin{aligned} & 0.000 \\ & 0.000 \end{aligned}$ | $\begin{array}{r} -.0152257 \\ -.056152 \end{array}$ | $\begin{aligned} & -.0043149 \\ & -.0192687 \end{aligned}$ |
| air | (base alternative) |  |  |  |  |  |
| train |  |  |  |  |  |  |
| income | -. 0291975 | . 0089246 | -3.27 | 0.001 | -. 0466895 | -. 0117055 |
| _cons | . 5616448 | . 3946529 | 1.42 | 0.155 | -. 2118607 | 1.33515 |
| bus |  |  |  |  |  |  |
| income | -. 01275 | . 0079266 | -1.61 | 0.108 | -. 0282858 | . 0027858 |
| _cons | -. 0571664 | . 4791996 | -0.12 | 0.905 | -. 9963803 | . 8820476 |
| car |  |  |  |  |  |  |
| income | -. 0049085 | . 0077486 | -0.63 | 0.526 | -. 0200955 | . 0102785 |
| _cons | -1.833444 | . 8186343 | -2.24 | 0.025 | -3.437938 | -. 22895 |
| /lnsigma3 | -. 2447428 | . 4953363 | -0.49 | 0.621 | -1.215584 | . 7260985 |
| /lnsigma4 | -. 3309429 | . 6494493 | -0.51 | 0.610 | -1.60384 | . 9419543 |
| /atanhr3_2 | 1.01193 | . 3890994 | 2.60 | 0.009 | . 249309 | 1.774551 |
| /atanhr4_2 | . 5786576 | . 3940461 | 1.47 | 0.142 | -. 1936586 | 1.350974 |
| /atanhr4_3 | . 8885204 | . 5600561 | 1.59 | 0.113 | -. 2091693 | 1.98621 |
| sigma1 |  | (base alternative) |  |  |  |  |
| sigma2 | 1 | (scale alternative) |  |  |  |  |
| sigma3 | . 7829059 | . 3878017 |  |  | . 2965368 | 2.067 |
| sigma4 | . 7182462 | . 4664645 |  |  | . 2011227 | 2.564989 |
| rho3_2 | . 766559 | . 1604596 |  |  | . 244269 | . 9441061 |
| rho4_2 | . 5216891 | . 2868027 |  |  | -. 1912734 | . 874283 |
| rho4_3 | . 7106622 | . 277205 |  |  | -. 2061713 | . 9630403 |

(mode=air is the alternative normalizing location)
(mode=train is the alternative normalizing scale)
When comparing this output to that of example 1 , we see that we have achieved the same log likelihood. That is, the structural parameterization using air as the base alternative and train as the scale alternative applied no restrictions on the model. This will not always be the case. We leave it up to you to try different base and scale alternatives, and you will see that not all the different combinations will achieve the same log likelihood. This is not true for the differenced covariance parameterization: it will always achieve the same log likelihood (and the maximum possible likelihood) regardless of the base and scale alternatives. This is why it is the default parameterization.

For an exercise, we can compute the differenced covariance displayed in example 1 by using the following ado-code.

```
. estat covariance
\begin{tabular}{|r|rrrr|}
\hline & air & train & bus & car \\
\hline air & 1 & & & \\
train & 0 & 1 & & \\
bus & 0 & .6001436 & .6129416 & \\
car & 0 & .3747012 & .399619 & .5158776 \\
\hline
\end{tabular}
. return list
matrices:
            r(cov) : 4 x 4
. matrix cov = r(cov)
. matrix M = (1,-1,0,0 \ 1,0,-1,0 \ 1,0,0,-1)
. matrix cov1 = M*cov*M'
. matrix list cov1
symmetric cov1[3,3]
    r1 r2 r3
r1 rrr
r3 1.3747012 1.399619 1.5158776
```

The slight difference in the regression coefficients between the example 1 and example 2 coefficient tables reflects the accuracy of the [M-5] ghk() algorithm using 200 points from the Hammersley sequence.

We now fit the model using the exchangeable correlation matrix and compare the models with a likelihood-ratio test.


| choice | Coef . | Std. Err. | z | $P>\|z\|$ | [95\% Con | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| mode travelcost termtime | $\begin{aligned} & -.0084636 \\ & -.0345394 \end{aligned}$ | $\begin{aligned} & .0020452 \\ & .0072812 \end{aligned}$ | $\begin{aligned} & -4.14 \\ & -4.74 \end{aligned}$ | $\begin{aligned} & 0.000 \\ & 0.000 \end{aligned}$ | $\begin{array}{r} -.012472 \\ -.0488103 \end{array}$ | $\begin{aligned} & -.0044551 \\ & -.0202684 \end{aligned}$ |
| air | (base alternative) |  |  |  |  |  |
| train |  |  |  |  |  |  |
| income | -. 0290357 | . 0083226 | -3.49 | 0.000 | -. 0453477 | -. 0127237 |
| _cons | . 5517445 | . 3719913 | 1.48 | 0.138 | -. 177345 | 1.280834 |
| bus |  |  |  |  |  |  |
| income | -. 0132562 | . 0074133 | -1.79 | 0.074 | -. 0277859 | . 0012735 |
| _cons | -. 0052517 | . 4337932 | -0.01 | 0.990 | -. 8554708 | . 8449673 |
| car |  |  |  |  |  |  |
| income | -. 0060878 | . 006638 | -0.92 | 0.359 | -. 0190981 | . 0069224 |
| _cons | -1.565918 | . 6633007 | -2.36 | 0.018 | -2.865964 | -. 265873 |
| /lnsigmaP1 | -. 3557589 | . 1972809 | -1.80 | 0.071 | -. 7424222 | . 0309045 |
| /lnsigmaP2 | -1.308596 | . 8872957 | -1.47 | 0.140 | -3.047663 | . 4304719 |
| /atanhrP1 | 1.116589 | . 3765488 | 2.97 | 0.003 | . 3785667 | 1.854611 |
| sigma1 | 1 | (base alternative) <br> (scale alternative) |  |  |  |  |
| sigma2 |  |  |  |  |  |  |
| sigma3 | . 7006416 | . 1382232 |  |  | . 4759596 | 1.031387 |
| sigma4 | . 2701992 | . 2397466 |  |  | . 0474697 | 1.537983 |
| rho3_2 | . 8063791 | . 131699 |  |  | . 3614621 | . 9521783 |
| rho4_2 | . 8063791 | . 131699 |  |  | . 3614621 | . 9521783 |
| rho4_3 | . 8063791 | . 131699 |  |  | . 3614621 | . 9521783 |

(mode=air is the alternative normalizing location)
(mode=train is the alternative normalizing scale)
. lrtest full .
Likelihood-ratio test LR chi2(2) = 0.75
(Assumption: . nested in full) Prob > chi2 = 0.6882
The likelihood-ratio test suggests that a common correlation is a plausible hypothesis, but this could be an artifact of the small sample size. The labeling of the standard deviation and correlation estimates has changed from /lnsigma and /atanhr, in the previous example, to /lnsigmaP and /atanhrP. The " P " identifies the parameter's index in the pattern matrices used by asmprobit. The pattern matrices are stored in e(stdpattern) and e(corpattern).

- Technical note

Another way to fit the model with the exchangeable correlation structure in example 2 is to use the constraint command to define the constraints on the rho parameters manually and then apply those.

```
. constraint 1 [atanhr3_2]_cons = [atanhr4_2]_cons
. constraint 2 [atanhr3_2]_cons = [atanhr4_3]_cons
. asmprobit choice travelcost termtime, case(id) alternatives(mode)
> casevars(income) constraints(1 2) structural
```

With this method, however, we must keep track of what parameterization of the rhos is used in estimation, and that depends on the options specified.

## > Example 3

In the last example, we used the correlation(exchangeable) option, reducing the number of correlation parameters from three to one. We can explore a two-correlation parameter model by specifying a pattern matrix in the correlation() option. Suppose that we wish to have the correlation between train and bus be equal to the correlation between bus and car and to have the standard deviations for the bus and car equations be equal. We will use air as the base category and train as the scale category.

```
. matrix define corpat = J(4, 4, .)
. matrix corpat[3,2] = 1
. matrix corpat[4,3] = 1
. matrix corpat[4,2] = 2
. matrix define stdpat = J(1, 4, .)
. matrix stdpat[1,3] = 1
. matrix stdpat[1,4] = 1
. asmprobit choice travelcost termtime, case(id) alternatives(mode)
> casevars(income) correlation(pattern corpat) stddev(pattern stdpat)
    (output omitted)
Alternative-specific multinomial probit Number of obs = 840
Case variable: id Number of cases = 210
Alternative variable: mode
Integration sequence: Hammersley
Integration points: 200
Log simulated-likelihood = -190.12871
\begin{tabular}{rrr} 
Number of obs & \(=\) & 840 \\
Number of cases & \(=\) & 210 \\
Alts per case: min & \(=\) & 4 \\
\(\operatorname{avg}\) & \(=\) & 4.0 \\
\(\max\) & \(=\) & 4 \\
Wald chi2(5) & \(=\) & 41.67 \\
Prob \(>\) chi2 & \(=\) & 0.0000
\end{tabular}
```


(mode=air is the alternative normalizing location)
(mode=train is the alternative normalizing scale)
In the call to asmprobit, we did not need to specify the basealternative() and scalealternative() options because they are implied by the specifications of the pattern matrices.

- Technical note

If you experience convergence problems, try specifying nopivot, increasing intpoints(), specifying antithetics, specifying technique(nr) with difficult, or specifying a switching algorithm in the technique() option. As a last resort, you can use the nrtolerance() and showtolerance options. Changing the base and scale alternative in the model specification can also affect convergence if the structural option is used.

Because simulation methods are used to obtain multivariate normal probabilities, the estimates obtained have a limited degree of precision. Moreover, the solutions are particularly sensitive to the starting values used. Experimenting with different starting values may help in obtaining convergence, and doing so is a good way to verify previous results.

If you wish to use the BHHH algorithm along with another maximization algorithm, you must specify the initbhhh (\#) option, where \# is the number of BHHH iterations to use before switching to the algorithm specified in technique(). The BHHH algorithm uses an outer-product-of-gradients approximation for the Hessian, and asmprobit must perform the gradient calculations differently than for the other algorithms.

## - Technical note

If there are no alternative-specific variables in your model, the variance-covariance matrix parameters are not identifiable. For such a model to converge, you would therefore need to use correlation(independent) and stddev(homoskedastic). A better alternative is to use mprobit, which is geared specifically toward models with only case-specific variables. See $[R]$ mprobit.

## Stored results

asmprobit stores the following in e():
Scalars

```
e(N)
e(N_case)
e(k)
e(k_alt)
e(k_indvars)
e(k_casevars)
e(k_sigma)
e(k_rho)
e(k_eq)
e(k_eq_model)
e(df_m)
e(ll)
e(N_clust)
e(const)
e(i_base)
e(i_scale)
e(mc_points)
e(mc_burn)
e(mc_antithetics)
e(chi2)
e(p)
e(fullcov)
e(structcov)
e(cholesky)
e(alt_min)
e(alt_avg)
e(alt_max)
e(rank)
e(ic)
e(rc)
e(converged)
number of observations
number of cases
number of parameters
number of alternatives
number of alternative-specific variables
number of case-specific variables
number of variance estimates
number of correlation estimates
number of equations in e(b)
number of equations in overall model test
model degrees of freedom
log simulated-likelihood
number of clusters
constant indicator
base alternative index
scale alternative index
number of Monte Carlo replications
starting sequence index
antithetics indicator
\chi
significance
unstructured covariance indicator
1 \text { if structured covariance; 0 otherwise}
Cholesky-factored covariance indicator
minimum number of alternatives
average number of alternatives
maximum number of alternatives
rank of e(V)
number of iterations
return code
1 if converged, 0 otherwise
```

Macros
e (cmd)
e(cmdline)
e(depvar)
e(indvars)
e(casevars)
e(case)
e(altvar)
e(alteqs)
e(alt\#)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(correlation)
e(stddev)
e(cov_class)
e(chi2type)
e(vce)
e(vcetype)
e(opt)
e(which)
e(ml_method)
e(mc_method)
e(mc_seed)
e(user)
e(technique)
e(datasignature)
e(datasignaturevars)
e(properties)
e(estat_cmd)
e(mfx_dlg)
e(predict)
e(marginsnotok)
Matrices
e(b)
e(Cns)
e(stats)
e(stdpattern)
e(stdfixed)
e(altvals)
e(altfreq)
e(alt_casevars)
e(corpattern)
e(corfixed)
e(ilog)
e(gradient)
e(V)
e(V_modelbased)
Functions
e(sample)
asmprobit
command as typed
name of dependent variable
alternative-specific independent variable
case-specific variables
variable defining cases
variable defining alternatives
alternative equation names
alternative labels
weight type
weight expression
title in estimation output
name of cluster variable
correlation structure
variance structure
class of the covariance structure
Wald, type of model $\chi^{2}$ test
vcetype specified in vce()
title used to label Std. Err.
type of optimization
$\max$ or min; whether optimizer is to perform maximization or minimization
type of ml method
technique used to generate sequences
random-number generator seed
name of likelihood-evaluator program
maximization technique
the checksum
variables used in calculation of checksum
b V
program used to implement estat
program used to implement estat mfx dialog
program used to implement predict
predictions disallowed by margins
coefficient vector
constraints matrix
alternative statistics
variance pattern
fixed and free standard deviations
alternative values
alternative frequencies
indicators for estimated case-specific coefficients-e (k_alt) $\times e\left(k_{\text {_ }}\right.$ casevars $)$
correlation structure
fixed and free correlations
iteration $\log$ (up to 20 iterations)
gradient vector
variance-covariance matrix of the estimators
model-based variance
marks estimation sample

## Methods and formulas

The simulated maximum likelihood estimates for the MNP are obtained using ml ; see $[\mathrm{R}] \mathrm{ml}$. The likelihood evaluator implements the GHK algorithm to approximate the multivariate distribution function (Geweke 1989; Hajivassiliou and McFadden 1998; Keane and Wolpin 1994). The technique is also described in detail by Genz (1992), but Genz describes a more general algorithm where both
lower and upper bounds of integration are finite. We briefly describe the GHK simulator and refer you to Bolduc (1999) for the score computations.

As discussed earlier, the latent variables for a $J$-alternative model are $\eta_{i j}=\mathbf{x}_{i j} \boldsymbol{\beta}+\mathbf{z}_{i} \boldsymbol{\alpha}_{j}+\xi_{i j}$, for $j=1, \ldots, J, i=1, \ldots, n$, and $\boldsymbol{\xi}_{i}^{\prime}=\left(\xi_{i, 1}, \ldots, \xi_{i, \mathrm{~J}}\right) \sim \operatorname{MVN}(\mathbf{0}, \boldsymbol{\Omega})$. The experimenter observes alternative $k$ for the $i$ th observation if $k=\arg \max \left(\eta_{i j}, j=1, \ldots, J\right)$. Let

$$
\begin{aligned}
v_{i j^{\prime}} & =\eta_{i j}-\eta_{i k} \\
& =\left(\mathbf{x}_{i j}-\mathbf{x}_{i k}\right) \boldsymbol{\beta}+\mathbf{z}_{i}\left(\boldsymbol{\alpha}_{j}-\boldsymbol{\alpha}_{k}\right)+\xi_{i j}-\xi_{i k} \\
& =\boldsymbol{\delta}_{i j^{\prime}} \boldsymbol{\beta}+\mathbf{z}_{i} \boldsymbol{\gamma}_{j^{\prime}}+\epsilon_{i j^{\prime}}
\end{aligned}
$$

where $j^{\prime}=j$ if $j<k$ and $j^{\prime}=j-1$ if $j>k$, so that $j^{\prime}=1, \ldots, J-1$. Further, $\boldsymbol{\epsilon}_{i}=$ $\left(\epsilon_{i 1}, \ldots, \epsilon_{i, \mathrm{~J}-1}\right) \sim \operatorname{MVN}\left(\mathbf{0}, \boldsymbol{\Sigma}_{(k)}\right) . \boldsymbol{\Sigma}$ is indexed by $k$ because it depends on the choice made. We denote the deterministic part of the model as $\lambda_{i j^{\prime}}=\boldsymbol{\delta}_{i j^{\prime}} \boldsymbol{\beta}+\mathbf{z}_{j} \gamma_{j^{\prime}}$, and the probability of this event is

$$
\begin{align*}
\operatorname{Pr}\left(y_{i}\right. & =k)=\operatorname{Pr}\left(v_{i 1} \leq 0, \ldots, v_{i, J-1} \leq 0\right) \\
& =\operatorname{Pr}\left(\epsilon_{i 1} \leq-\lambda_{i 1}, \ldots, \epsilon_{i, J-1} \leq-\lambda_{i, J-1}\right)  \tag{3}\\
& =(2 \pi)^{-(J-1) / 2}\left|\boldsymbol{\Sigma}_{(k)}\right|^{-1 / 2} \int_{-\infty}^{-\lambda_{i 1}} \cdots \int_{-\infty}^{-\lambda_{i, J-1}} \exp \left(-\frac{1}{2} \mathbf{z}^{\prime} \boldsymbol{\Sigma}_{(k)}^{-1} \mathbf{z}\right) d \mathbf{z}
\end{align*}
$$

## Simulated likelihood

For clarity in the discussion that follows, we drop the index denoting case so that for an arbitrary observation $\boldsymbol{v}^{\prime}=\left(v_{1}, \ldots, v_{J-1}\right), \boldsymbol{\lambda}^{\prime}=\left(\lambda_{1}, \ldots, \lambda_{J-1}\right)$, and $\boldsymbol{\epsilon}^{\prime}=\left(\epsilon_{1}, \ldots, \epsilon_{J-1}\right)$.

The Cholesky-factored variance-covariance, $\boldsymbol{\Sigma}=\mathbf{L} \mathbf{L}^{\prime}$, is lower triangular,

$$
\mathbf{L}=\left(\begin{array}{cccc}
l_{11} & 0 & \ldots & 0 \\
l_{21} & l_{22} & \ldots & 0 \\
\vdots & \vdots & & \vdots \\
l_{J-1,1} & l_{J-1,2} & \ldots & l_{J-1, J-1}
\end{array}\right)
$$

and the correlated latent-variable errors can be expressed as linear functions of uncorrelated normal variates, $\boldsymbol{\epsilon}=\mathbf{L} \boldsymbol{\zeta}$, where $\boldsymbol{\zeta}^{\prime}=\left(\zeta_{1}, \ldots, \zeta_{J-1}\right)$ and $\zeta_{j} \sim \operatorname{iid} \mathrm{~N}(0,1)$. We now have $\boldsymbol{v}=\boldsymbol{\lambda}+\mathbf{L} \boldsymbol{\zeta}$, and by defining

$$
z_{j}= \begin{cases}-\frac{\lambda_{1}}{l_{11}} & \text { for } j=1  \tag{4}\\ -\frac{\lambda_{j}+\sum_{i=1}^{j-1} l_{j i} \zeta_{i}}{l_{j j}} & \text { for } j=2, \ldots, J-1\end{cases}
$$

we can express the probability statement (3) as the product of conditional probabilities

$$
\begin{aligned}
& \operatorname{Pr}\left(y_{i}=k\right)=\operatorname{Pr}\left(\zeta_{1} \leq z_{1}\right) \operatorname{Pr}\left(\zeta_{2} \leq z_{2} \mid \zeta_{1} \leq z_{1}\right) \cdots \\
& \operatorname{Pr}\left(\zeta_{J-1} \leq z_{J-1} \mid \zeta_{1} \leq z_{1}, \ldots, \zeta_{J-2} \leq z_{J-2}\right)
\end{aligned}
$$

because

$$
\begin{aligned}
\operatorname{Pr}\left(v_{1} \leq 0\right) & =\operatorname{Pr}\left(\lambda_{1}+l_{11} \zeta_{1} \leq 0\right) \\
& =\operatorname{Pr}\left(\zeta_{1} \leq-\frac{\lambda_{1}}{l_{11}}\right) \\
\operatorname{Pr}\left(v_{2} \leq 0\right) & =\operatorname{Pr}\left(\lambda_{2}+l_{21} \zeta_{1}+l_{22} \zeta_{2} \leq 0\right) \\
& =\operatorname{Pr}\left(\left.\zeta_{2} \leq-\frac{\lambda_{2}+l_{21} \zeta_{1}}{l_{22}} \right\rvert\, \zeta_{1} \leq-\frac{\lambda_{1}}{l_{11}}\right)
\end{aligned}
$$

The Monte Carlo algorithm then must make draws from the truncated standard normal distribution. It does so by generating $J-1$ uniform variates, $\delta_{j}, j=1, \ldots, J-1$, and computing

$$
\widetilde{\zeta}_{j}= \begin{cases}\Phi^{-1}\left\{\delta_{1} \Phi\left(-\frac{\lambda_{1}}{l_{11}}\right)\right\} & \text { for } j=1 \\ \Phi^{-1}\left\{\delta_{j} \Phi\left(\frac{-\lambda_{j}-\sum_{i=1}^{j-1} l_{j i} \widetilde{\zeta}_{i}}{l_{j j}}\right)\right\} & \text { for } j=2, \ldots, J-1\end{cases}
$$

Define $\widetilde{z}_{j}$ by replacing $\widetilde{\zeta}_{i}$ for $\zeta_{i}$ in (4) so that the simulated probability for the $l$ th draw is

$$
p_{l}=\prod_{j=1}^{J-1} \Phi\left(\widetilde{z}_{j}\right)
$$

To increase accuracy, the bounds of integration, $\lambda_{j}$, are ordered so that the largest integration intervals are on the inside. The rows and columns of the variance-covariance matrix are pivoted accordingly (Genz 1992).

For a more detailed description of the GHK algorithm in Stata, see Gates (2006).
Repeated draws are made, say, $N$, and the simulated likelihood for the $i$ th case, denoted $\widehat{L}_{i}$, is computed as

$$
\widehat{L}_{i}=\frac{1}{N} \sum_{l=1}^{N} p_{l}
$$

The overall simulated $\log$ likelihood is $\sum_{i} \log \widehat{L}_{i}$.
If the true likelihood is $L_{i}$, the error bound on the approximation can be expressed as

$$
\left|\widehat{L}_{i}-L_{i}\right| \leq V\left(L_{i}\right) D_{N}\left\{\left(\delta_{i}\right)\right\}
$$

where $V\left(L_{i}\right)$ is the total variation of $L_{i}$ and $D_{N}$ is the discrepancy, or nonuniformity, of the set of abscissas. For the uniform pseudorandom sequence, $\delta_{i}$, the discrepancy is of order $O\left\{(\log \log N / N)^{1 / 2}\right\}$. The order of discrepancy can be improved by using quasirandom sequences.

Quasi-Monte Carlo integration is carried out by asmprobit by replacing the uniform deviates with either the Halton or the Hammersley sequences. These sequences spread the points more evenly than the uniform random sequence and have a smaller order of discrepancy, $O\left[\left\{(\log N)^{J-1}\right\} / N\right]$ and $O\left[\left\{(\log N)^{J-2}\right\} / N\right]$, respectively. The Halton sequence of dimension $J-1$ is generated from the first $J-1$ primes, $p_{k}$, so that on draw $l$ we have $\mathbf{h}_{l}=\left\{r_{p_{1}}(l), r_{p_{2}}(l), \ldots, r_{p_{J-1}}(l)\right\}$, where

$$
r_{p_{k}}(l)=\sum_{j=0}^{q} b_{j k}(l) p_{k}^{-j-1} \in(0,1)
$$

is the radical inverse function of $l$ with base $p_{k}$ so that $\sum_{j=0}^{q} b_{j k}(l) p_{k}^{j}=l$, where $p_{k}^{q} \leq l<p_{k}^{q+1}$ (Fang and Wang 1994).

This function is demonstrated with base $p_{3}=5$ and $l=33$, which generates $r_{5}(33)$. Here $q=2$, $b_{0,3}(33)=3, b_{1,5}(33)=1$, and $b_{2,5}(33)=1$, so that $r_{5}(33)=3 / 5+1 / 25+1 / 625$.

The Hammersley sequence uses an evenly spaced set of points with the first $J-2$ components of the Halton sequence

$$
\mathbf{h}_{l}=\left\{\frac{2 l-1}{2 N}, r_{p_{1}}(l), r_{p_{2}}(l), \ldots, r_{p_{J-2}}(l)\right\}
$$

for $l=1, \ldots, N$.
For a more detailed description of the Halton and Hammersley sequences, see Drukker and Gates (2006).

Computations for the derivatives of the simulated likelihood are taken from Bolduc (1999). Bolduc gives the analytical first-order derivatives for the $\log$ of the simulated likelihood with respect to the regression coefficients and the parameters of the Cholesky-factored variance-covariance matrix. asmprobit uses these analytical first-order derivatives and numerical second-order derivatives.

This command supports the clustered version of the Huber/White/sandwich estimator of the variance using vce (robust) and vce (cluster clustvar). See [ P ] _robust, particularly Maximum likelihood estimators and Methods and formulas. Specifying vce (robust) is equivalent to specifying vce(cluster casevar), where casevar is the variable that identifies the cases.

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## Also see

[R] asmprobit postestimation - Postestimation tools for asmprobit
[R] asclogit - Alternative-specific conditional logit (McFadden's choice) model
[R] asroprobit - Alternative-specific rank-ordered probit regression
[R] mlogit - Multinomial (polytomous) logistic regression
[R] mprobit - Multinomial probit regression
[U] 20 Estimation and postestimation commands

## Title

asmprobit postestimation - Postestimation tools for asmprobit

| Description | Syntax for predict | Menu for predict | Options for predict |
| :--- | :--- | :--- | :--- |
| Syntax for estat | Menu for estat | Options for estat | Remarks and examples |
| Stored results | Methods and formulas | Also see |  |

## Description

The following postestimation commands are of special interest after asmprobit:
Command Description
estat alternatives alternative summary statistics
estat covariance
estat correlation
estat facweights
estat mfx
covariance matrix of the latent-variable errors for the alternatives
correlation matrix of the latent-variable errors for the alternatives covariance factor weights matrix
marginal effects

The following standard postestimation commands are also available:
Command Description
estat ic
estat summarize
estat vce
estimates
lincom
lrtest
nlcom
predict
predictnl
test
testnl

Description
Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) summary statistics for the estimation sample variance-covariance matrix of the estimators (VCE) cataloging estimation results
point estimates, standard errors, testing, and inference for linear combinations of coefficients
likelihood-ratio test
point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients
predicted probabilities, estimated linear predictor and its standard error point estimates, standard errors, testing, and inference for generalized predictions
Wald tests of simple and composite linear hypotheses
Wald tests of nonlinear hypotheses

## Special-interest postestimation commands

estat alternatives displays summary statistics about the alternatives in the estimation sample and provides a mapping between the index numbers that label the covariance parameters of the model and their associated values and labels for the alternative variable.
estat covariance computes the estimated variance-covariance matrix of the latent-variable errors for the alternatives. The estimates are displayed, and the variance-covariance matrix is stored in $r$ (cov).
estat correlation computes the estimated correlation matrix of the latent-variable errors for the alternatives. The estimates are displayed, and the correlation matrix is stored in $r$ (cor).
estat facweights displays the covariance factor weights matrix and stores it in $r(C)$. estat mfx computes the simulated probability marginal effects.

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic altwise]
predict [type] {stub*|newvarlist } [if] [in], scores
```

statistic Description
Main
$\mathrm{pr} \quad$ probability alternative is chosen; the default
xb
linear prediction
stdp
standard error of the linear prediction
These statistics are available both in and out of sample; type predict . . . if e (sample) ... if wanted only for the estimation sample.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
pr , the default, calculates the probability that alternative $j$ is chosen in case $i$.
xb calculates the linear prediction $\mathbf{x}_{i j} \boldsymbol{\beta}+\mathbf{z}_{i} \boldsymbol{\alpha}_{j}$ for alternative $j$ and case $i$.
stdp calculates the standard error of the linear predictor.
altwise specifies that alternativewise deletion be used when marking out observations due to missing values in your variables. The default is to use casewise deletion. The xb and stdp options always use alternativewise deletion.
scores calculates the scores for each coefficient in e(b). This option requires a new variable list of length equal to the number of columns in $\mathrm{e}(\mathrm{b})$. Otherwise, use the $s t u b *$ option to have predict generate enumerated variables with prefix stub.

## Syntax for estat

Alternative summary statistics
estat alternatives

Covariance matrix of the latent-variable errors for the alternatives

```
estat covariance [, format(%fmt) border(bspec) left(#)]
```

Correlation matrix of the latent-variable errors for the alternatives

```
estat correlation [, format(%fmt) border(bspec) left(#)]
```

Covariance factor weights matrix

```
estat facweights [, format(%fmt) border(bspec) left(#)]
```

Marginal effects

```
estat mfx [if][in] [, estat_mfx_options]
```

estat_mfx_options Description
Main
varlist (varlist) display marginal effects for varlist
at(mean $[$ atlist $] \mid$ median $[$ atlist $]$ ) calculate marginal effects at these values
Options
level(\#)
nodiscrete
noesample
nowght
set confidence interval level; default is level (95)
treat indicator variables as continuous
do not restrict calculation of means and medians to the estimation sample
ignore weights when calculating means and medians

## Menu for estat

Statistics $>$ Postestimation $>$ Reports and statistics

## Options for estat

Options for estat are presented under the following headings:
Options for estat covariance, estat correlation, and estat facweights Options for estat mfx

## Options for estat covariance, estat correlation, and estat facweights

format (\%fmt) sets the matrix display format. The default for estat covariance and estat facweights is format ( $\% 9.0 \mathrm{~g}$ ) ; the default for estat correlation is format (\%9.4f).
border (bspec) sets the matrix display border style. The default is border (all). See [P] matlist. left(\#) sets the matrix display left indent. The default is left(2). See [P] matlist.

## Options for estat mfx

$\square$ Main
varlist(varlist) specifies the variables for which to display marginal effects. The default is all variables.
at (mean $[$ atlist $] \mid$ median $[$ atlist $]$ ) specifies the values at which the marginal effects are to be calculated. atlist is
$[[$ alternative $:$ variable $=\#][$ variable $=\#][\ldots]]$
The default is to calculate the marginal effects at the means of the independent variables at the estimation sample, at (mean).
After specifying the summary statistic, you can specify a series of specific values for variables. You can specify values for alternative-specific variables by alternative, or you can specify one value for all alternatives. You can specify only one value for case-specific variables. For example, in travel.dta, income is a case-specific variable, whereas termtime and travelcost are alternative-specific variables. The following would be a legal syntax for estat mfx :

```
. estat mfx, at(mean air:termtime=50 travelcost=100 income=60)
```

When nodiscrete is not specified, at (mean [atlist]) or at (median [atlist]) has no effect on computing marginal effects for indicator variables, which are calculated as the discrete change in the simulated probability as the indicator variable changes from 0 to 1 .

The mean and median computations respect any if and in qualifiers, so you can restrict the data over which the means or medians are computed. You can even restrict the values to a specific case; for example,

```
. estat mfx if case==21
```

level(\#) specifies the confidence level, as a percentage, for confidence intervals. The default is level (95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals. nodiscrete specifies that indicator variables be treated as continuous variables. An indicator variable is one that takes on the value 0 or 1 in the estimation sample. By default, the discrete change in the simulated probability is computed as the indicator variable changes from 0 to 1 .
noesample specifies that the whole dataset be considered instead of only those marked in the e(sample) defined by the asmprobit command.
nowght specifies that weights be ignored when calculating the means or medians.

## Remarks and examples

Remarks are presented under the following headings:
Predicted probabilities
Obtaining estimation statistics
Obtaining marginal effects

## Predicted probabilities

After fitting an alternative-specific multinomial probit model, you can use predict to obtain the simulated probabilities that an individual will choose each of the alternatives. When evaluating the multivariate normal probabilities via Monte Carlo simulation, predict uses the same method to generate the random sequence of numbers as the previous call to asmprobit. For example, if you specified intmethod(Halton) when fitting the model, predict also uses the Halton sequence.

## > Example 1

In example 1 of [R] asmprobit, we fit a model of individuals' travel-mode choices. We can obtain the simulated probabilities that an individual chooses each alternative by using predict:

```
. use http://www.stata-press.com/data/r13/travel
. asmprobit choice travelcost termtime, case(id) alternatives(mode)
> casevars(income)
    (output omitted)
. predict prob
(option pr assumed; Pr(mode))
. list id mode prob choice in 1/12, sepby(id)
```

| id | mode | prob | choice |
| ---: | ---: | ---: | ---: |
| 1 | air | .1494137 | 0 |
| 1 | train | .329167 | 0 |
| 1 | bus | .1320298 | 0 |
| 1 | car | .3898562 | 1 |
| 2 | air | .2565875 | 0 |
| 2 | train | .2761054 | 0 |
| 2 | bus | .0116135 | 0 |
| 2 | car | .4556921 | 1 |
| 3 | air | .2098406 | 0 |
| 3 | train | .1081824 | 0 |
| 3 | bus | .1671841 | 0 |
| 3 | car | .5147822 | 1 |

## Obtaining estimation statistics

Once you have fit a multinomial probit model, you can obtain the estimated variance or correlation matrices for the model alternatives by using the estat command.
> Example 2
To display the correlations of the errors in the latent-variable equations, we type
. estat correlation

|  | train | bus | car |
| ---: | ---: | ---: | ---: |
| train | 1.0000 |  |  |
| bus | 0.8909 | 1.0000 |  |
| car | 0.7895 | 0.8951 | 1.0000 |

Note: correlations are for alternatives differenced with air

The covariance matrix can be displayed by typing

```
. estat covariance
\begin{tabular}{|r|rrr|}
\hline & train & bus & car \\
\hline train & 2 & & \\
bus & 1.600208 & 1.613068 & \\
car & 1.37471 & 1.399703 & 1.515884 \\
\hline
\end{tabular}
```

Note: covariances are for alternatives differenced with air

## Obtaining marginal effects

The marginal effects are computed as the derivative of the simulated probability for an alternative with respect to an independent variable. A table of marginal effects is displayed for each alternative, with the table containing the marginal effect for each case-specific variable and the alternative for each alternative-specific variable.

By default, the marginal effects are computed at the means of each continuous independent variable over the estimation sample. For indicator variables, the difference in the simulated probability evaluated at 0 and 1 is computed by default. Indicator variables will be treated as continuous variables if the nodiscrete option is used.

## > Example 3

Continuing with our model from example 1, we obtain the marginal effects for alternatives air, train, bus, and car evaluated at the mean values of each independent variable. Recall that the travelcost and termtime variables are alternative specific, taking on different values for each alternative, so they have a separate marginal effect for each alternative.
. estat mfx
$\operatorname{Pr}($ choice $=$ air $)=.29434926$

| variable | dp/dx | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [ | $95 \%$ |  | C.I. |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| travelcost |  |  |  |  |  |  | X |  |
| air | -.002688 | .000677 | -3.97 | 0.000 | -.004015 | -.001362 | 102.65 |  |
| train | .0009 | .000436 | 2.07 | 0.039 | .000046 | .001755 | 130.2 |  |
| bus | .000376 | .000271 | 1.39 | 0.166 | -.000155 | .000908 | 115.26 |  |
| car | .001412 | .00051 | 2.77 | 0.006 | .000412 | .002412 | 95.414 |  |
| termtime |  |  |  |  |  |  |  |  |
| air | -.010376 | .002711 | -3.83 | 0.000 | -.015689 | -.005063 | 61.01 |  |
| train | .003475 | .001639 | 2.12 | 0.034 | .000264 | .006687 | 35.69 |  |
| bus | .001452 | .001008 | 1.44 | 0.150 | -.000523 | .003427 | 41.657 |  |
| car | .005449 | .002164 | 2.52 | 0.012 | .001209 | .00969 | 0 |  |
| casevars |  |  |  |  |  |  |  |  |
| income | .003891 | .001847 | 2.11 | 0.035 | .000271 | .007511 | 34.548 |  |

$\operatorname{Pr}($ choice $=$ train $)=.29531182$

| variable | dp/dx | Std. Err. | z | $P>\|z\|$ | [ 95\% | C.I. ] | X |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| travelcost |  |  |  |  |  |  |  |
| air | . 000899 | . 000436 | 2.06 | 0.039 | . 000045 | . 001753 | 102.65 |
| train | -. 004081 | . 001466 | -2.78 | 0.005 | -. 006953 | -. 001208 | 130.2 |
| bus | . 001278 | . 00063 | 2.03 | 0.042 | . 000043 | . 002513 | 115.26 |
| car | . 001904 | . 000887 | 2.15 | 0.032 | . 000166 | . 003641 | 95.414 |
| termtime |  |  |  |  |  |  |  |
| air | . 003469 | . 001638 | 2.12 | 0.034 | . 000258 | . 00668 | 61.01 |
| train | -. 01575 | . 00247 | -6.38 | 0.000 | -. 020591 | -. 010909 | 35.69 |
| bus | . 004934 | . 001593 | 3.10 | 0.002 | . 001812 | . 008056 | 41.657 |
| car | . 007348 | . 002228 | 3.30 | 0.001 | . 00298 | . 011715 | 0 |
| casevars |  |  |  |  |  |  |  |
| income | -. 00957 | . 002223 | -4.31 | 0.000 | -. 013927 | -. 005214 | 34.548 |

$\operatorname{Pr}($ choice $=$ bus $)=.08880039$

| variable | dp/dx | Std. Err. | $z$ | $\mathrm{P}>\|\mathrm{z}\|$ | [ | $95 \%$ |  | C.I. |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| travelcost |  |  |  |  |  |  | X |  |
| air | .00038 | .000274 | 1.39 | 0.165 | -.000157 | .000916 | 102.65 |  |
| train | .001279 | .00063 | 2.03 | 0.042 | .000044 | .002514 | 130.2 |  |
| bus | -.003182 | .001175 | -2.71 | 0.007 | -.005485 | -.00088 | 115.26 |  |
| car | .001523 | .000675 | 2.26 | 0.024 | .0002 | .002847 | 95.414 |  |
| termtime |  |  |  |  |  |  |  |  |
| air | .001466 | .001017 | 1.44 | 0.149 | -.000526 | .003459 | 61.01 |  |
| train | .004937 | .001591 | 3.10 | 0.002 | .001819 | .008055 | 35.69 |  |
| bus | -.012283 | .002804 | -4.38 | 0.000 | -.017778 | -.006788 | 41.657 |  |
| car | .00588 | .002255 | 2.61 | 0.009 | .001461 | .010299 | 0 |  |
| casevars |  |  |  |  |  |  |  |  |
| income | .000435 | .001461 | 0.30 | 0.766 | -.002428 | .003298 | 34.548 |  |

$\operatorname{Pr}($ choice $=$ car $)=.32168607$

| variable | dp/dx | Std. Err. | $z$ | $\mathrm{P}>\|\mathrm{z}\|$ | [ | 95\% C.I. | ] | X |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| travelcost |  |  |  |  |  |  |  |  |
| air | .00141 | .000509 | 2.77 | 0.006 | .000411 | .002408 | 102.65 |  |
| train | .001903 | .000886 | 2.15 | 0.032 | .000166 | .003641 | 130.2 |  |
| bus | .001523 | .000675 | 2.25 | 0.024 | .000199 | .002847 | 115.26 |  |
| car | -.004836 | .001539 | -3.14 | 0.002 | -.007853 | -.001819 | 95.414 |  |
| termtime |  |  |  |  |  |  |  |  |
| air | .005441 | .002161 | 2.52 | 0.012 | .001205 | .009677 | 61.01 |  |
| train | .007346 | .002228 | 3.30 | 0.001 | .00298 | .011713 | 35.69 |  |
| bus | .005879 | .002256 | 2.61 | 0.009 | .001456 | .010301 | 41.657 |  |
| car | -.018666 | .003938 | -4.74 | 0.000 | -.026385 | -.010948 | 0 |  |
| casevars |  |  |  |  |  |  |  |  |
| income | .005246 | .002166 | 2.42 | 0.015 | .001002 | .00949 | 34.548 |  |

First, we note that there is a separate marginal effects table for each alternative and that table begins by reporting the overall probability of choosing the alternative, for example, 0.2944 for air travel. We see in the first table that a unit increase in terminal time for air travel from 61.01 minutes will result in a decrease in probability of choosing air travel (when the probability is evaluated at the mean of all variables) by approximately 0.01 , with a $95 \%$ confidence interval of about -0.016 to -0.005 . Travel cost has a less negative effect of choosing air travel (at the average cost of 102.65). Alternatively, an increase in terminal time and travel cost for train, bus, or car from these mean values will increase the chance for air travel to be chosen. Also, with an increase in income from 34.5, it would appear that an individual would be more likely to choose air or automobile travel over bus or train. (While the marginal effect for bus travel is positive, it is not significant.)

## > Example 4

Plotting the simulated probability marginal effect evaluated over a range of values for an independent variable may be more revealing than a table of values. Below are the commands for generating the simulated probability marginal effect of air travel for increasing air travel terminal time. We fix all other independent variables at their medians.

```
. qui gen meff = .
. qui gen tt = .
. qui gen lb = .
. qui gen ub = .
. forvalues i=0/19 {
    2. local termtime = 5+5*'i'
    3. qui replace tt = 'termtime' if _n == 'i'+1
    4. qui estat mfx, at(median air:termtime='termtime') var(termtime)
    5. mat air = r(air)
    6. qui replace meff = air[1,1] if _n == 'i'+1
    7. qui replace lb = air[1,5] if _n == 'i'+1
    8. qui replace ub = air[1,6] if _n == 'i'+1
    9. qui replace prob = r(pr_air) if _n == 'i'+1
10. }
. label variable tt "terminal time"
```

```
. twoway (rarea lb ub tt, pstyle(ci)) (line meff tt, lpattern(solid)), name(meff)
> legend(off) title(" marginal effect of air travel" "terminal time and"
> "95% confidence interval", position(3))
. twoway line prob tt, name(prob) title(" probability of choosing" "air travel",
> position(3)) graphregion(margin(r+9)) ytitle("") xtitle("")
. graph combine prob meff, cols(1) graphregion(margin(l+5 r+5))
```



From the graphs, we see that the simulated probability of choosing air travel decreases in an sigmoid fashion. The marginal effects display the rate of change in the simulated probability as a function of the air travel terminal time. The rate of change in the probability of choosing air travel decreases until the air travel terminal time reaches about 45; thereafter, it increases.

## Stored results

estat mfx stores the following in $r()$ :
Scalars
$r$ (pr_alt) scalars containing the computed probability of each alternative evaluated at the value that is labeled X in the table output. Here alt are the labels in the macro e(alteqs).
Matrices
$r($ alt $) \quad$ matrices containing the computed marginal effects and associated statistics. There is one matrix for each alternative, where alt are the labels in the macro e(alteqs). Column 1 of each matrix contains the marginal effects; column 2, their standard errors; columns 3 and 4, their $z$ statistics and the $p$-values for the $z$ statistics; and columns 5 and 6 , the confidence intervals. Column 7 contains the values of the independent variables used to compute the probabilities $r$ (pr_alt).

## Methods and formulas

## Marginal effects

The marginal effects are computed as the derivative of the simulated probability with respect to each independent variable. A set of marginal effects is computed for each alternative; thus, for $J$ alternatives, there will be $J$ tables. Moreover, the alternative-specific variables will have $J$ entries, one for each alternative in each table. The details of computing the effects are different for alternative-specific variables and case-specific variables, as well as for continuous and indicator variables.

We use the latent-variable notation of asmprobit (see [R] asmprobit) for a $J$-alternative model and, for notational convenience, we will drop any subscripts involving observations. We then have the following linear functions $\eta_{j}=\mathbf{x}_{j} \boldsymbol{\beta}+\mathbf{z} \boldsymbol{\alpha}_{j}$, for $j=1, \ldots, J$. Let $k$ index the alternative of interest, and then

$$
\begin{aligned}
v_{j^{\prime}} & =\eta_{j}-\eta_{k} \\
& =\left(\mathbf{x}_{j}-\mathbf{x}_{k}\right) \boldsymbol{\beta}+\mathbf{z}\left(\boldsymbol{\alpha}_{j}-\boldsymbol{\alpha}_{k}\right)+\boldsymbol{\epsilon}_{j^{\prime}}
\end{aligned}
$$

where $j^{\prime}=j$ if $j<k$ and $j^{\prime}=j-1$ if $j>k$, so that $j^{\prime}=1, \ldots, J-1$ and $\boldsymbol{\epsilon}_{j^{\prime}} \sim \operatorname{mVN}(\mathbf{0}, \boldsymbol{\Sigma})$. Denote $p_{k}=\operatorname{Pr}\left(v_{1} \leq 0, \ldots, v_{J-1} \leq 0\right)$ as the simulated probability of choosing alternative $k$ given profile $\mathbf{x}_{k}$ and $\mathbf{z}$. The marginal effects are then $\partial p_{k} / \partial \mathbf{x}_{k}, \partial p_{k} / \partial \mathbf{x}_{j}$, and $\partial p_{k} / \partial \mathbf{z}$, where $k=1, \ldots, J, j \neq k$. asmprobit analytically computes the first-order derivatives of the simulated probability with respect to the $v$ 's, and the marginal effects for $\mathbf{x}$ 's and $\mathbf{z}$ are obtained via the chain rule. The standard errors for the marginal effects are computed using the delta method.

## Also see

[R] asmprobit - Alternative-specific multinomial probit regression
[U] 20 Estimation and postestimation commands

## Title

asroprobit - Alternative-specific rank-ordered probit regression

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | Reference |
| Also see |  |  |  |

## Syntax

```
asroprobit depvar [indepvars] [if] [in] [weight], case(varname)
alternatives(varname) [options]
```

options
Model

* case (varname)
* alternatives (varname)
casevars (varlist)
constraints (constraints)
collinear
Model 2
correlation(correlation)
stddev (stddev)
structural
factor (\#)
noconstant
basealternative(\#|lbl|str) scalealternative(\#|lbl|str)
altwise
reverse

SE/Robust
vce (vcetype)

Reporting
level(\#)
notransform
nocnsreport
display_options
use varname to identify cases
use varname to identify the alternatives available for each case case-specific variables
apply specified linear constraints
keep collinear variables
correlation structure of the latent-variable errors
variance structure of the latent-variable errors
use the structural covariance parameterization; default is the differenced covariance parameterization
use the factor covariance structure with dimension \#
suppress the alternative-specific constant terms
alternative used for normalizing location
alternative used for normalizing scale
use alternativewise deletion instead of casewise deletion interpret the lowest rank in depvar as the best; the default is the highest rank is the best
vcetype may be oim, robust, cluster clustvar, opg, bootstrap, or jackknife
set confidence level; default is level (95)
do not transform variance-covariance estimates to the standard deviation and correlation metric
do not display constraints
control column formats and line width

| Integration |  |
| :---: | :---: |
| intmethod (seqtype) | type of quasi- or pseudouniform sequence |
| intpoints (\#) | number of points in each sequence |
| intburn(\#) | starting index in the Hammersley or Halton sequence |
| intseed (code \| \#) | pseudouniform random-number seed |
| antithetics | use antithetic draws |
| nopivot | do not use integration interval pivoting |
| initbhhh(\#) | use the BHHH optimization algorithm for the first \# iterations |
| favor (speed \| space) | favor speed or space when generating integration points |
| Maximization |  |
| maximize_options | control the maximization process |
| coeflegend | display legend instead of statistics |
| correlation | Description |
| unstructured | one correlation parameter for each pair of alternatives; correlations with the basealternative() are zero; the default |
| exchangeable | one correlation parameter common to all pairs of alternatives; correlations with the basealternative() are zero |
| independent | constrain all correlation parameters to zero |
| pattern matname | user-specified matrix identifying the correlation pattern |
| fixed matname | user-specified matrix identifying the fixed and free correlation parameters |

stddev

## Description

heteroskedastic
homoskedastic
pattern matname
fixed matname
estimate standard deviation for each alternative; standard deviations for basealternative() and scalealternative() set to one all standard deviations are one user-specified matrix identifying the standard deviation pattern user-specified matrix identifying the fixed and free standard deviations
seqtype Description

| hammersley | Hammersley point set |
| :--- | :--- |
| $\underline{\text { halton }}$ | Halton point set |
| random | uniform pseudorandom point set |

* case(varname) and alternatives(varname) are required.
bootstrap, by, jackknife, statsby, and xi are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the bootstrap prefix; see [R] bootstrap.
fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.


## Menu

Statistics $>$ Ordinal outcomes $>$ Rank-ordered probit regression

## Description

asroprobit fits rank-ordered probit (ROP) models by using maximum simulated likelihood (MSL). The model allows you to relax the independence of irrelevant alternatives (IIA) property that is characteristic of the rank-ordered logistic model by estimating the variance-covariance parameters of the latent-variable errors. Each unique identifier in the case() variable has multiple alternatives identified in the alternatives() variable, and depvar contains the ranked alternatives made by each case. Only the order in the ranks, not the magnitude of their differences, is assumed to be relevant. By default, the largest rank indicates the more desirable alternative. Use the reverse option if the lowest rank should be interpreted as the more desirable alternative. Tied ranks are allowed, but they increase the computation time because all permutations of the tied ranks are used in computing the likelihood for each case. asroprobit allows two types of independent variables: alternative-specific variables, in which the values of each variable vary with each alternative, and case-specific variables, which vary with each case.

The estimation technique of asroprobit is nearly identical to that of asmprobit, and the two routines share many of the same options; see [R] asmprobit.

## Options

## Model

case (varname) specifies the variable that identifies each case. This variable identifies the individuals or entities making a choice. case() is required.
alternatives (varname) specifies the variable that identifies the alternatives available for each case.
The number of alternatives can vary with each case; the maximum number of alternatives is 20 . alternatives() is required.
casevars (varlist) specifies the case-specific variables that are constant for each case(). If there are
a maximum of $J$ alternatives, there will be $J-1$ sets of coefficients associated with casevars ().
constraints (constraints), collinear; see [R] estimation options.

## Model 2 <br> correlation(correlation) specifies the correlation structure of the latent-variable errors.

correlation(unstructured) is the most general and has $J(J-3) / 2+1$ unique correlation parameters. This is the default unless stddev() or structural are specified.
correlation(exchangeable) provides for one correlation coefficient common to all latent variables, except the latent variable associated with the basealternative().
correlation(independent) assumes that all correlations are zero.
correlation(pattern matname) and correlation(fixed matname) give you more flexibility in defining the correlation structure. See Variance structures in [R] asmprobit for more information.
stddev (stddev) specifies the variance structure of the latent-variable errors.
stddev (heteroskedastic) is the most general and has $J-2$ estimable parameters. The standard deviations of the latent-variable errors for the alternatives specified in basealternative() and scalealternative() are fixed to one.
stddev (homoskedastic) constrains all the standard deviations to equal one.
stddev (pattern matname) and stddev (fixed matname) give you added flexibility in defining the standard deviation parameters. See Variance structures in $[\mathrm{R}]$ asmprobit for more information.
structural requests the $J \times J$ structural covariance parameterization instead of the default $J-1 \times J-1$ differenced covariance parameterization (the covariance of the latent errors differenced with that of the base alternative). The differenced covariance parameterization will achieve the same maximum simulated likelihood regardless of the choice of basealternative() and scalealternative(). On the other hand, the structural covariance parameterization imposes more normalizations that may bound the model away from its maximum likelihood and thus prevent convergence with some datasets or choices of basealternative() and scalealternative().
factor (\#) requests that the factor covariance structure of dimension \# be used. The factor() option can be used with the structural option but cannot be used with stddev() or correlation(). A $\# \times J$ (or $\# \times J-1$ ) matrix, $\mathbf{C}$, is used to factor the covariance matrix as $I+\mathbf{C}^{\prime} \mathbf{C}$, where $I$ is the identity matrix of dimension $J$ (or $J-1$ ). The column dimension of $\mathbf{C}$ depends on whether the covariance is structural or differenced. The row dimension of $\mathbf{C}$, \#, must be less than or equal to floor $((J(J-1) / 2-1) /(J-2)$ ), because there are only $J(J-1) / 2-1$ identifiable variance-covariance parameters. This covariance parameterization may be useful for reducing the number of covariance parameters that need to be estimated.
If the covariance is structural, the column of $\mathbf{C}$ corresponding to the base alternative contains zeros. The column corresponding to the scale alternative has a one in the first row and zeros elsewhere. If the covariance is differenced, the column corresponding to the scale alternative (differenced with the base) has a one in the first row and zeros elsewhere.
noconstant suppresses the $J-1$ alternative-specific constant terms.
basealternative (\# $|l b l| s t r)$ specifies the alternative used to normalize the latent-variable location (also referred to as the level of utility). The base alternative may be specified as a number, label, or string. The standard deviation for the latent-variable error associated with the base alternative is fixed to one, and its correlations with all other latent-variable errors are set to zero. The default is the first alternative when sorted. If a fixed or pattern matrix is given in the stddev() and correlation() options, the basealternative() will be implied by the fixed standard deviations and correlations in the matrix specifications. basealternative() cannot be equal to scalealternative().
scalealternative (\# $|l b l|$ str) specifies the alternative used to normalize the latent-variable scale (also referred to as the scale of utility). The scale alternative may be specified as a number, label, or string. The default is to use the second alternative when sorted. If a fixed or pattern matrix is given in the stddev() option, the scalealternative() will be implied by the fixed standard deviations in the matrix specification. scalealternative() cannot be equal to basealternative().

If a fixed or pattern matrix is given for the stddev() option, the base alternative and scale alternative are implied by the standard deviations and correlations in the matrix specifications, and they need not be specified in the basealternative() and scalealternative() options.
altwise specifies that alternativewise deletion be used when marking out observations due to missing values in your variables. The default is to use casewise deletion; that is, the entire group of observations making up a case is deleted if any missing values are encountered. This option does not apply to observations that are marked out by the if or in qualifier or the by prefix.
reverse directs asroprobit to interpret the rank in depvar that is smallest in value as the preferred alternative. By default, the rank that is largest in value is the favored alternative.
vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
If specifying vce (bootstrap) or vce(jackknife), you must also specify basealternative() and scalealternative().

Reporting
level(\#); see [R] estimation options.
notransform prevents retransforming the Cholesky-factored variance-covariance estimates to the correlation and standard deviation metric.

This option has no effect if structural is not specified because the default differenced variancecovariance estimates have no interesting interpretation as correlations and standard deviations. notransform also has no effect if the correlation() and stddev() options are specified with anything other than their default values. Here it is generally not possible to factor the variancecovariance matrix, so optimization is already performed using the standard deviation and correlation representations.
nocnsreport; see [R] estimation options.
display_options: cformat ( $\%$ fmt $)$, pformat ( $\% f m t$ ), sformat ( $\% f m t$ ), and nolstretch; see [R] estimation options.

## Integration

intmethod(hammersley $\mid$ halton $\mid$ random) specifies the method of generating the point sets used in the quasi-Monte Carlo integration of the multivariate normal density. intmethod (hammersley), the default, uses the Hammersley sequence; intmethod (halton) uses the Halton sequence; and intmethod (random) uses a sequence of uniform random numbers.
intpoints(\#) specifies the number of points to use in the quasi-Monte Carlo integration. If this option is not specified, the number of points is $50 \times J$ if intmethod(hammersley) or intmethod(halton) is used and $100 \times J$ if intmethod (random) is used. Larger values of intpoints() provide better approximations of the log likelihood, but at the cost of added computation time.
intburn(\#) specifies where in the Hammersley or Halton sequence to start, which helps reduce the correlation between the sequences of each dimension. The default is 0 . This option may not be specified with intmethod (random).
intseed (code $\mid \#$ ) specifies the seed to use for generating the uniform pseudorandom sequence. This option may be specified only with intmethod (random). code refers to a string that records the state of the random-number generator runiform (); see [R] set seed. An integer value \# may be used also. The default is to use the current seed value from Stata's uniform random-number generator, which can be obtained from c (seed).
antithetics specifies that antithetic draws be used. The antithetic draw for the $J-1$ vector uniform-random variables, $\mathbf{x}$, is $1-\mathbf{x}$.
nopivot turns off integration interval pivoting. By default, asroprobit will pivot the wider intervals of integration to the interior of the multivariate integration. This improves the accuracy of the quadrature estimate. However, discontinuities may result in the computation of numerical secondorder derivatives using finite differencing (for the Newton-Raphson optimize technique, tech (nr))
when few simulation points are used, resulting in a non-positive-definite Hessian. asroprobit uses the Broyden-Fletcher-Goldfarb-Shanno optimization algorithm, by default, which does not require computing the Hessian numerically using finite differencing.
initbhhh (\#) specifies that the Berndt-Hall-Hall-Hausman (BHHH) algorithm be used for the initial \# optimization steps. This option is the only way to use the BHHH algorithm along with other optimization techniques. The algorithm switching feature of ml's technique() option cannot include bhhh.
favor (speed $\mid$ space) instructs asroprobit to favor either speed or space when generating the integration points. favor (speed) is the default. When favoring speed, the integration points are generated once and stored in memory, thus increasing the speed of evaluating the likelihood. This speed increase can be seen when there are many cases or when the user specifies a large number of integration points, intpoints(\#). When favoring space, the integration points are generated repeatedly with each likelihood evaluation.
For unbalanced data, where the number of alternatives varies with each case, the estimates computed using intmethod(random) will vary slightly between favor (speed) and favor (space). This is because the uniform sequences will not be identical, even when initiating the sequences using the same uniform seed, intseed (code $\mid \#$ ). For favor (speed), ncase blocks of intpoints (\#) $\times$ $J-2$ uniform points are generated, where $J$ is the maximum number of alternatives. For favor (space), the column dimension of the matrices of points varies with the number of alternatives that each case has.
$\sqrt{\text { Maximization }}$
maximize_options: difficult, technique (algorithm_spec), iterate(\#), [no $]$ log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance(\#), nonrtolerance, and from(init_specs); see [R] maximize.
The following options may be particularly useful in obtaining convergence with asroprobit: difficult, technique(algorithm_spec), nrtolerance(\#), nonrtolerance, and from (init_specs).

If technique() contains more than one algorithm specification, bhhh cannot be one of them. To use the BHHH algorithm with another algorithm, use the initbhhh() option and specify the other algorithm in technique().
Setting the optimization type to technique (bhhh) resets the default vcetype to vce (opg).
When specifying from (matname [, copy]), the values in matname associated with the latentvariable error variances must be for the log-transformed standard deviations and inverse-hyperbolic tangent-transformed correlations. This option makes using the coefficient vector from a previously fitted asroprobit model convenient as a starting point.

The following option is available with asroprobit but is not shown in the dialog box:
coeflegend; see [R] estimation options.

## Remarks and examples

The mathematical description and numerical computations of the rank-ordered probit model are similar to that of the multinomial probit model. The only difference is that the dependent variable of the rank-ordered probit model is ordinal, showing preferences among alternatives, as opposed to the binary dependent variable of the multinomial probit model, indicating a chosen alternative. We will describe how the likelihood of a ranking is computed using the latent-variable framework here,
but for details of the latent-variable parameterization of these models and the method of maximum simulated likelihood, see $[R]$ asmprobit.

Consider the latent-variable parameterization of a $J$ alternative rank-ordered probit model. Using the notation from asmprobit, we have variables $\eta_{i j}, j=1, \ldots, J$, such that

$$
\eta_{i j}=\mathbf{x}_{i j} \boldsymbol{\beta}+\mathbf{z}_{i} \boldsymbol{\alpha}_{j}+\xi_{i j}
$$

Here the $\mathbf{x}_{i j}$ are the alternative-specific independent variables, the $\mathbf{z}_{i}$ are the case-specific variables, and the $\xi_{i j}$ are multivariate normal with mean zero and covariance $\boldsymbol{\Omega}$. Without loss of generality, assume that individual $i$ ranks the alternatives in order of the alternative indices $j=1,2, \ldots, J$, so the alternative $J$ is the preferred alternative and alternative 1 is the least preferred alternative. The probability of this ranking given $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}_{j}$ is the probability that $\eta_{i, J-1}-\eta_{i, J} \leq 0$ and $\eta_{i, J-2}-\eta_{i, J-1} \leq 0, \ldots$, and $\eta_{i, 1}-\eta_{i, 2} \leq 0$.

## > Example 1

Long and Freese $(2014,477)$ provide an example of a rank-ordered logit model with alternativespecific variables. We use this dataset to demonstrate asroprobit. The data come from the Wisconsin Longitudinal Study. This is a study of 1957 Wisconsin high school graduates that were asked to rate their relative preference of four job characteristics: esteem, a job other people regard highly; variety, a job that is not repetitive and allows you to do a variety of things; autonomy, a job where your supervisor does not check on you frequently; and security, a job with a low risk of being laid off. The case-specific covariates are gender, female, an indicator variable for females, and score, a score on a general mental ability test measured in standard deviations. The alternative-specific variables are high and low, which indicate whether the respondent's current job is high or low in esteem, variety, autonomy, or security. This approach provides three states for a respondent's current job status for each alternative, $(1,0),(0,1)$, and $(0,0)$, using the notation (high, low). The score $(1,1)$ is omitted because the respondent's current job cannot be considered both high and low in one of the job characteristics. The $(0,0)$ score would indicate that the respondent's current job does not rank high or low (is neutral) in a job characteristic. The alternatives are ranked such that 1 is the preferred alternative and 4 is the least preferred.

```
. use http://www.stata-press.com/data/r13/wlsrank
(1992 Wisconsin Longitudinal Study data on job values)
. list id jobchar rank female score high low in 1/12, sepby(id)
```

|  | id | jobchar | rank | female | score | high | low |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | 1. | 1 | security | 1 | 1 | .0492111 | 0 |
| 2. | 1 | autonomy | 4 | 1 | .0492111 | 0 | 0 |
| 3. | 1 | variety | 1 | 1 | .0492111 | 0 | 0 |
| 4. | 1 | esteem | 3 | 1 | .0492111 | 0 | 0 |
| 5. | 5 | security | 2 | 1 | 2.115012 | 1 | 0 |
| 6. | 5 | variety | 2 | 1 | 2.115012 | 1 | 0 |
| 7. | 5 | esteem | 2 | 1 | 2.115012 | 1 | 0 |
| 8. | 5 | autonomy | 1 | 1 | 2.115012 | 0 | 0 |
| 9. | 7 | autonomy | 1 | 0 | 1.701852 | 1 | 0 |
| 10. | 7 | variety | 1 | 0 | 1.701852 | 0 | 1 |
| 11. | 7 | esteem | 4 | 0 | 1.701852 | 0 | 0 |
| 12. | 7 | security | 1 | 0 | 1.701852 | 0 | 0 |

The three cases listed have tied ranks. asroprobit will allow ties, but at the cost of increased computation time. To evaluate the likelihood of the first observation, asroprobit must compute

$$
\begin{aligned}
& \operatorname{Pr}(\text { esteem }=3, \text { variety }=1, \text { autonomy }=4, \text { security }=2)+ \\
& \operatorname{Pr}(\text { esteem }=3, \text { variety }=2, \text { autonomy }=4, \text { security }=1)
\end{aligned}
$$

and both of these probabilities are estimated using simulation. In fact, the full dataset contains 7,237 tied ranks and asroprobit takes a great deal of time to estimate the parameters. For exposition, we estimate the rank-ordered probit model by using the cases without ties. These cases are marked in the variable noties.

The model of job preference is

$$
\eta_{i j}=\beta_{1} \operatorname{high}_{i j}+\beta_{2} \text { low }_{i j}+\alpha_{1 j} \text { female }_{i}+\alpha_{2 j} \text { score }_{i}+\alpha_{0 j}+\xi_{i j}
$$

for $j=1,2,3,4$. The base alternative will be esteem, so $\alpha_{01}=\alpha_{11}=\alpha_{21}=0$.

```
. asroprobit rank high low if noties, case(id) alternatives(jobchar)
> casevars(female score) reverse
note: variable high has 107 cases that are not alternative-specific: there is
    no within-case variability
note: variable low has 193 cases that are not alternative-specific: there is
    no within-case variability
Iteration 0: log simulated-likelihood = -1103.2768
Iteration 1: log simulated-likelihood = -1089.3361 (backed up)
    (output omitted)
Alternative-specific rank-ordered probit Number of obs = 1660
Case variable: id
    Number of cases = 415
Alternative variable: jobchar
Integration sequence: Hammersley
Integration points: 200
Wald chi2(8) = 34.01
Log simulated-likelihood = -1080.2206
Prob > chi2 = 0.0000
```


(jobchar=esteem is the alternative normalizing location)
(jobchar=variety is the alternative normalizing scale)
We specified the reverse option because a rank of 1 is the highest preference. The variancecovariance estimates are for the Cholesky-factored variance-covariance for the latent-variable errors differenced with that of alternative esteem. We can view the estimated correlations by entering

```
. estat correlation
```

|  | variety | autonomy | security |
| ---: | ---: | ---: | ---: |
| variety | 1.0000 |  |  |
| autonomy | 0.4516 | 1.0000 |  |
| security | 0.2652 | 0.2399 | 1.0000 |

Note: correlations are for alternatives differenced with esteem
and typing
. estat covariance

|  | variety | autonomy | security |
| ---: | ---: | ---: | ---: |
| variety <br> autonomy <br> security | 2 |  |  |

Note: covariances are for alternatives differenced with esteem
gives the (co)variances. [R] mprobit explains that if the latent-variable errors are independent, then the correlations in the differenced parameterization should be $\sim 0.5$ and the variances should be $\sim 2.0$, which seems to be the case here.

The coefficient estimates for the probit models can be difficult to interpret because of the normalization for location and scale. The regression estimates for the case-specific variables will be relative to the base alternative and the regression estimates for both the case-specific and alternativespecific variables are affected by the scale normalization. The more pronounced the heteroskedasticity and correlations, the more pronounced the resulting estimate differences when choosing alternatives to normalize for location and scale. However, when using the differenced covariance structure, you will obtain the same model likelihood regardless of which alternatives you choose as the base and scale alternatives. For model interpretation, you can examine the estimated probabilities and marginal effects by using postestimation routines predict and estat mfx . See $[\mathrm{R}]$ asroprobit postestimation.

## Stored results

asroprobit stores the following in e() :
Scalars

```
e(N)
e(N_case)
e(N_ties)
e(k)
e(k_alt)
e(k_indvars)
e(k_casevars)
e(k_sigma)
e(k_rho)
e(k_eq)
e(k_eq_model)
e(df_m)
e(ll)
e(N_clust)
e(const)
e(i_base)
e(i_scale)
e(mc_points)
e(mc_burn)
e(mc_antithetics)
e(reverse)
e(chi2)
e(p)
e(fullcov)
e(structcov)
e(cholesky)
e(alt_min)
e(alt_avg)
e(alt_max)
e(rank)
e(ic)
e(rc)
e(converged)
number of observations
number of cases
number of ties
number of parameters
number of alternatives
number of alternative-specific variables
number of case-specific variables
number of variance estimates
number of correlation estimates
number of equations in e(b)
number of equations in overall model test
model degrees of freedom
log simulated-likelihood
number of clusters
constant indicator
base alternative index
scale alternative index
number of Monte Carlo replications
starting sequence index
antithetics indicator
1 if minimum rank is best, 0 if maximum rank is best
\chi
significance
unstructured covariance indicator
1 if structured covariance; 0 otherwise
Cholesky-factored covariance indicator
minimum number of alternatives
average number of alternatives
maximum number of alternatives
rank of e(V)
number of iterations
return code
1 if converged, 0 otherwise
```

Macros
e(cmd)
e(cmdline)
e(depvar)
e(indvars)
e(casevars)
e(case)
e(altvar)
e(alteqs)
e(alt\#)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(correlation)
e(stddev)
e(chi2type)
e(vce)
e(vcetype)
e(opt)
e(which)
e(ml_method)
e(mc_method)
e(mc_seed)
e(user)
e(technique)
e(datasignature)
e(datasignaturevars)
e(properties)
e(estat_cmd)
e(mfx_dlg)
e(predict)
e(marginsnotok)
Matrices
e(b)
e(Cns)
e(stats)
e(stdpattern)
e(stdfixed)
e(altvals)
e(altfreq)
e(alt_casevars)
e(corpattern)
e(corfixed)
e(ilog)
e(gradient)
e(V)
e(V_modelbased)
Functions
e(sample)
asroprobit
command as typed
name of dependent variable
alternative-specific independent variable
case-specific variables
variable defining cases
variable defining alternatives
alternative equation names
alternative labels
weight type
weight expression
title in estimation output
name of cluster variable
correlation structure
variance structure
Wald, type of model $\chi^{2}$ test
vcetype specified in vce()
title used to label Std. Err.
type of optimization
max or min; whether optimizer is to perform maximization or minimization type of ml method
Hammersley, Halton, or uniform random; technique to generate sequences random-number generator seed
name of likelihood-evaluator program
maximization technique
the checksum
variables used in calculation of checksum
b V
program used to implement estat
program used to implement estat mfx dialog
program used to implement predict
predictions disallowed by margins
coefficient vector
constraints matrix
alternative statistics
variance pattern
fixed and free standard deviations
alternative values
alternative frequencies
indicators for estimated case-specific coefficients—e (k_alt) $\times e$ (k_casevars)
correlation structure
fixed and free correlations
iteration $\log$ (up to 20 iterations)
gradient vector
variance-covariance matrix of the estimators
model-based variance
marks estimation sample

## Methods and formulas

From a computational perspective, asroprobit is similar to asmprobit and the two programs share many numerical tools. Therefore, we will use the notation from Methods and formulas in $[\mathrm{R}]$ asmprobit to discuss the rank-ordered probit probability model.

The latent variables for a $J$-alternative model are $\eta_{i j}=\mathbf{x}_{i j} \boldsymbol{\beta}+\mathbf{z}_{i} \boldsymbol{\alpha}_{j}+\xi_{i j}$, for $j=1, \ldots, J$, $i=1, \ldots, n$, and $\boldsymbol{\xi}_{i}^{\prime}=\left(\xi_{i, 1}, \ldots, \xi_{i, \mathrm{~J}}\right) \sim \operatorname{MVN}(\mathbf{0}, \boldsymbol{\Omega})$. Without loss of generality, assume for
the $i$ th observation that an individual ranks the alternatives in the order of their numeric indices, $\mathbf{y}_{i}=(J, J-1, \ldots, 1)$, so the first alternative is the most preferred and the last alternative is the least preferred. We can then difference the latent variables such that

$$
\begin{aligned}
v_{i k} & =\eta_{i, k+1}-\eta_{i, k} \\
& =\left(\mathbf{x}_{i, k+1}-\mathbf{x}_{i, k}\right) \boldsymbol{\beta}+\mathbf{z}_{i}\left(\boldsymbol{\alpha}_{k+1}-\boldsymbol{\alpha}_{k}\right)+\xi_{i, k+1}-\xi_{i k} \\
& =\boldsymbol{\delta}_{i k} \boldsymbol{\beta}+\mathbf{z}_{i} \boldsymbol{\gamma}_{k}+\epsilon_{i k}
\end{aligned}
$$

for $k=1, \ldots, J-1$ and where $\boldsymbol{\epsilon}_{i}=\left(\epsilon_{i 1}, \ldots, \epsilon_{i, \mathrm{~J}-1}\right) \sim \operatorname{MVN}\left(\mathbf{0}, \boldsymbol{\Sigma}_{(i)}\right) . \boldsymbol{\Sigma}$ is indexed by $i$ because it is specific to the ranking of individual $i$. We denote the deterministic part of the model as $\lambda_{i k}=\delta_{i k} \boldsymbol{\beta}+\mathbf{z}_{j} \gamma_{k}$, and the probability of this event is

$$
\begin{aligned}
\operatorname{Pr}\left(\mathbf{y}_{i}\right) & =\operatorname{Pr}\left(v_{i 1} \leq 0, \ldots, v_{i, J-1} \leq 0\right) \\
& =\operatorname{Pr}\left(\epsilon_{i 1} \leq-\lambda_{i 1}, \ldots, \epsilon_{i, J-1} \leq-\lambda_{i, J-1}\right) \\
& =(2 \pi)^{-(J-1) / 2}\left|\boldsymbol{\Sigma}_{(i)}\right|^{-1 / 2} \int_{-\infty}^{-\lambda_{i 1}} \cdots \int_{-\infty}^{-\lambda_{i, J-1}} \exp \left(-\frac{1}{2} \mathbf{z}^{\prime} \boldsymbol{\Sigma}_{(i)}^{-1} \mathbf{z}\right) d \mathbf{z}
\end{aligned}
$$

The integral has the same form as (3) of Methods and formulas in [R] asmprobit. See [R] asmprobit for details on evaluating this integral numerically by using simulation.
asroprobit handles tied ranks by enumeration. For $k$ tied ranks, it will generate $k$ ! rankings, where! is the factorial operator $k!=k(k-1)(k-2) \cdots(2)(1)$. For two sets of tied ranks of size $k_{1}$ and $k_{2}$, asroprobit will generate $k_{1}!k_{2}$ ! rankings. The total probability is the sum of the probability of each ranking. For example, if there are two tied ranks such that $\mathbf{y}_{i}=(J, J, J-2, \ldots, 1)$, then asroprobit will evaluate $\operatorname{Pr}\left(\mathbf{y}_{i}\right)=\operatorname{Pr}\left(\mathbf{y}_{i}^{(1)}\right)+\operatorname{Pr}\left(\mathbf{y}_{i}^{(2)}\right)$, where $\mathbf{y}_{i}^{(1)}=(J, J-1, J-2, \ldots, 1)$ and $\mathbf{y}_{i}^{(2)}=(J-1, J, J-2, \ldots, 1)$.

This command supports the clustered version of the Huber/White/sandwich estimator of the variance using vce(robust) and vce(cluster clustvar). See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas. Specifying vce (robust) is equivalent to specifying vce(cluster casevar), where casevar is the variable that identifies the cases.

## Reference

Long, J. S., and J. Freese. 2014. Regression Models for Categorical Dependent Variables Using Stata. 3rd ed. College Station, TX: Stata Press.

## Also see

[R] asroprobit postestimation - Postestimation tools for asroprobit
$[\mathrm{R}]$ asmprobit - Alternative-specific multinomial probit regression
[R] mlogit - Multinomial (polytomous) logistic regression
[R] mprobit - Multinomial probit regression
[R] oprobit - Ordered probit regression
[U] 20 Estimation and postestimation commands

## Title

asroprobit postestimation - Postestimation tools for asroprobit

Description Syntax for predict Menu for predict Options for predict
Syntax for estat
Stored results
Menu for estat Options for estat
Remarks and examples

## Description

The following postestimation commands are of special interest after asroprobit:
Command Description
estat alternatives
estat covariance
estat correlation
estat facweights
estat mfx
alternative summary statistics covariance matrix of the latent-variable errors for the alternatives correlation matrix of the latent-variable errors for the alternatives covariance factor weights matrix marginal effects

The following standard postestimation commands are also available:
Command Description
estat ic
estat summarize
estat vce
estimates
lincom
lrtest
nlcom
predict
predictnl
test
testnl

## Description

Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) summary statistics for the estimation sample variance-covariance matrix of the estimators (VCE) cataloging estimation results
point estimates, standard errors, testing, and inference for linear combinations of coefficients
likelihood-ratio test
point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients
predicted probabilities, estimated linear predictor and its standard error point estimates, standard errors, testing, and inference for generalized predictions
Wald tests of simple and composite linear hypotheses
Wald tests of nonlinear hypotheses

## Special-interest postestimation commands

estat alternatives displays summary statistics about the alternatives in the estimation sample. The command also provides a mapping between the index numbers that label the covariance parameters of the model and their associated values and labels for the alternative variable.
estat covariance computes the estimated variance-covariance matrix of the latent-variable errors for the alternatives. The estimates are displayed, and the variance-covariance matrix is stored in $r$ (cov).
estat correlation computes the estimated correlation matrix of the latent-variable errors for the alternatives. The estimates are displayed, and the correlation matrix is stored in $r$ (cor).
estat facweights displays the covariance factor weights matrix and stores it in $r(C)$.
estat mfx computes marginal effects of a simulated probability of a set of ranked alternatives. The probability is stored in $r(p r)$, the matrix of rankings is stored in $r$ (ranks), and the matrix of marginal-effect statistics is stored in $r(m f x)$.

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic altwise]
predict [type] {stub*|newvarlist } [if] [in], scores
```


## statistic Description

Main

| pr | probability of each ranking, by case; the default |
| :--- | :--- |
| pr 1 | probability that each alternative is preferred |
| xb | linear prediction |
| stdp | standard error of the linear prediction |

These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

Main
pr , the default, calculates the probability of each ranking. For each case, one probability is computed for the ranks in e(depvar). pr1 calculates the probability that each alternative is preferred. xb calculates the linear prediction $\mathbf{x}_{i j} \boldsymbol{\beta}+\mathbf{z}_{i} \boldsymbol{\alpha}_{j}$ for alternative $j$ and case $i$. stdp calculates the standard error of the linear predictor.
altwise specifies that alternativewise deletion be used when marking out observations due to missing values in your variables. The default is to use casewise deletion. The xb and stdp options always use alternativewise deletion.
scores calculates the scores for each coefficient in e(b). This option requires a new variable list of length equal to the number of columns in e(b). Otherwise, use the stub* option to have predict generate enumerated variables with prefix stub.

## Syntax for estat

Alternative summary statistics
estat alternatives

Covariance matrix of the latent-variable errors for the alternatives

```
estat covariance [, format(%fmt) border(bspec) left(#)]
```

Correlation matrix of the latent-variable errors for the alternatives

```
estat correlation [, format(%fmt) border(bspec) left(#)]
```

Covariance factor weights matrix

```
estat facweights [, format(%fmt) border(bspec) left(#)]
```

Marginal effects

```
estat mfx [if] [in] [, estat_mfx_options]
```

estat_mfx_options Description
Main
varlist (varlist) display marginal effects for varlist at (median $[$ atlist $]$ ) calculate marginal effects at these values
rank(ranklist) calculate marginal effects for the simulated probability of these ranked alternatives

Options
level(\#)
set confidence interval level; default is level (95)
nodiscrete
noesample
nowght
treat indicator variables as continuous
do not restrict calculation of the medians to the estimation sample ignore weights when calculating medians

## Menu for estat

Statistics $>$ Postestimation $>$ Reports and statistics

## Options for estat

Options for estat are presented under the following headings:
Options for estat covariance, estat correlation, and estat facweights Options for estat mfx

## Options for estat covariance, estat correlation, and estat facweights

format (\% fint) sets the matrix display format. The default for estat covariance and estat facweights is format ( $\% 9.0 \mathrm{~g}$ ). The default for estat correlation is format (\% $\% .4 \mathrm{f}$ ).
border (bspec) sets the matrix display border style. The default is border (all). See [P] matlist. left(\#) sets the matrix display left indent. The default is left(2). See [P] matlist.

## Options for estat mfx

## Main

varlist (varlist) specifies the variables for which to display marginal effects. The default is all variables.
at (median [atlist]) specifies the values at which the marginal effects are to be calculated. atlist is

$$
[[\text { alternative }: \text { variable }=\#][\text { variable }=\#][\ldots]])
$$

The marginal effects are calculated at the medians of the independent variables.
After specifying the summary statistic, you can specify specific values for variables. You can specify values for alternative-specific variables by alternative, or you can specify one value for all alternatives. You can specify only one value for case-specific variables. For example, in the wlsrank dataset, female and score are case-specific variables, whereas high and low are alternative-specific variables. The following would be a legal syntax for estat mfx:
. estat mfx, at(median high=0 esteem:high=1 low=0 security:low=1 female=1)
When nodiscrete is not specified, at (median [atlist]) has no effect on computing marginal effects for indicator variables, which are calculated as the discrete change in the simulated probability as the indicator variable changes from 0 to 1 .

The median computations respect any if or in qualifiers, so you can restrict the data over which the medians are computed. You can even restrict the values to a specific case, for example,
. estat mfx if case==13
rank (ranklist) specifies the ranks for the alternatives. ranklist is

$$
\text { alternative }=\# \text { alternative }=\#[\ldots])
$$

The default is to rank the calculated latent variables. Alternatives excluded from rank() are omitted from the analysis. You must therefore specify at least two alternatives in rank(). You may have tied ranks in the rank specification. Only the order in the ranks is relevant.

Options
level(\#) specifies the confidence level, as a percentage, for confidence intervals. The default is level (95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals. nodiscrete specifies that indicator variables be treated as continuous variables. An indicator variable is one that takes on the value 0 or 1 in the estimation sample. By default, the discrete change in the simulated probability is computed as the indicator variable changes from 0 to 1 .
noesample specifies that the whole dataset be considered instead of only those marked in the e(sample) defined by the asroprobit command.
nowght specifies that weights be ignored when calculating the medians.

## Remarks and examples

Remarks are presented under the following headings:
Predicted probabilities
Obtaining estimation statistics

## Predicted probabilities

After fitting an alternative-specific rank-ordered probit model, you can use predict to obtain the probabilities of alternative rankings or the probabilities of each alternative being preferred. When evaluating the multivariate normal probabilities via (quasi) Monte Carlo, predict uses the same method to generate the (quasi) random sequence of numbers as the previous call to asroprobit. For example, if you specified intmethod(halton) when fitting the model, predict also uses Halton sequences.
> Example 1
In example 1 of $[R]$ asroprobit, we fit a model of job characteristic preferences. This is a study of 1957 Wisconsin high school graduates that were asked to rate their relative preference of four job characteristics: esteem, a job other people regard highly; variety, a job that is not repetitive and allows you to do a variety of things; autonomy, a job where your supervisor does not check on you frequently; and security, a job with a low risk of being laid off. The case-specific covariates are gender, female, an indicator variable for females, and score, a score on a general mental ability test measured in standard deviations. The alternative-specific variables are high and low, which indicate whether the respondent's current job is high or low in esteem, variety, autonomy, or security. This approach provides three states for a respondent's current job status for each alternative, $(1,0),(0,1)$, and $(0,0)$, using the notation (high, low). The score ( 1,1 ) is omitted because the respondent's current job cannot be considered both high and low in one of the job characteristics. The $(0,0)$ score would indicate that the respondent's current job does not rank high or low (is neutral) in a job characteristic. The alternatives are ranked such that 1 is the preferred alternative and 4 is the least preferred.

We can obtain the probabilities of the observed alternative rankings, the pr option, and the probability of each alternative being preferred, the pr1 option, by using predict:

```
. use http://www.stata-press.com/data/r13/wlsrank
(1992 Wisconsin Longitudinal Study data on job values)
. asroprobit rank high low if noties, case(id) alternatives(jobchar)
> casevars(female score) reverse
(output omitted)
. keep if e(sample)
(11244 observations deleted)
. predict prob, pr
. predict prob1, pr1
. list id jobchar prob prob1 rank female score high low in 1/12
```

| id | jobchar | prob | prob1 | rank | female | score | high | low |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 13 | security | .0421807 | .2784269 | 3 | 0 | .3246512 | 0 | 1 |
| 13 | autonomy | .0421807 | .1029036 | 1 | 0 | .3246512 | 0 | 0 |
| 13 | variety | .0421807 | .6026725 | 2 | 0 | .3246512 | 1 | 0 |
| 13 | esteem | .0421807 | .0160111 | 4 | 0 | .3246512 | 0 | 1 |
| 19 | autonomy | .0942025 | .1232488 | 4 | 1 | .0492111 | 0 | 0 |
| 19 | esteem | .0942025 | .0140261 | 3 | 1 | .0492111 | 0 | 0 |
| 19 | security | .0942025 | .4601368 | 1 | 1 | .0492111 | 1 | 0 |
| 19 | variety | .0942025 | .4025715 | 2 | 1 | .0492111 | 0 | 0 |
| 22 | esteem | .1414177 | .0255264 | 4 | 1 | 1.426412 | 1 | 0 |
| 22 | variety | .1414177 | .4549441 | 1 | 1 | 1.426412 | 0 | 0 |
| 22 | security | .1414177 | .2629494 | 3 | 1 | 1.426412 | 0 | 0 |
| 22 | autonomy | .1414177 | .2566032 | 2 | 1 | 1.426412 | 1 | 0 |

The prob variable is constant for each case because it contains the probability of the ranking in the rank variable. On the other hand, the prob1 variable contains the estimated probability of each alternative being preferred. For each case, the sum of the values in prob1 will be approximately 1.0. They do not add up to exactly 1.0 because of approximations due to the GHK algorithm.

## Obtaining estimation statistics

For examples of the specialized estat subcommands covariance and correlation, see $[R]$ asmprobit postestimation. The entry also has a good example of computing marginal effects after asmprobit that is applicable to asroprobit. Below we will elaborate further on marginal effects after asroprobit where we manipulate the rank() option.

## > Example 2

We will continue with the preferred job characteristics example where we first compute the marginal effects for case $i d=13$.
. estat mfx if id==13, rank(security=3 autonomy=1 variety=2 esteem=4)
$\operatorname{Pr}($ esteem $=4$ variety $=2$ autonomy=1 security=3) $=.04218068$

| variable | dp/dx | Std. Err. | z | $P>\|z\|$ | [ 95\% | C.I. ] | X |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| high* |  |  |  |  |  |  |  |
| esteem | -. 008713 | . 001964 | -4.44 | 0.000 | -. 012562 | -. 004864 | 0 |
| variety | -. 009102 | . 003127 | -2.91 | 0.004 | -. 015231 | -. 002973 | 1 |
| autonomy | . 025535 | . 007029 | 3.63 | 0.000 | . 011758 | . 039313 | 0 |
| security | -. 003745 | . 001394 | -2.69 | 0.007 | -. 006477 | -. 001013 | 0 |
| low* |  |  |  |  |  |  |  |
| esteem | . 001614 | . 002646 | 0.61 | 0.542 | -. 003572 | . 0068 | 1 |
| variety | . 001809 | . 003012 | 0.60 | 0.548 | -. 004094 | . 007712 | 0 |
| autonomy | -. 003849 | . 006104 | -0.63 | 0.528 | -. 015813 | . 008115 | 0 |
| security | . 000582 | . 000985 | 0.59 | 0.554 | -. 001348 | . 002513 | 1 |
| casevars |  |  |  |  |  |  |  |
| female* | . 009767 | . 009064 | 1.08 | 0.281 | -. 007998 | . 027533 | 0 |
| score | . 008587 | . 004488 | 1.91 | 0.056 | -. 00021 | . 017384 | . 32465 |

(*) $\mathrm{dp} / \mathrm{dx}$ is for discrete change of indicator variable from 0 to 1
Next we compute the marginal effects for the probability that autonomy is preferred given the profile of case $i d=13$.

(*) $\mathrm{dp} / \mathrm{dx}$ is for discrete change of indicator variable from 0 to 1
The probability computed by estat mfx matches the probability computed by predict, pr1 only within three digits. This outcome is because of how the computation is carried out and the numeric inaccuracy of the GHK simulator using a Hammersley point set of length 200. The computation carried out by estat mfx literally computes all six probabilities listed in the header of the MFX table and sums them. The computation by predict, pr1 is the same as predict after asmprobit (multinomial probit): it computes the probability that autonomy is chosen, thus requiring only one
call to the GHK simulator. Hence, there is a difference in the reported values even though the two probability statements are equivalent.

## Stored results

estat mfx stores the following in r() :
Scalars
$r(\mathrm{pr}) \quad$ scalar containing the computed probability of the ranked alternatives.
Matrices
$r$ (ranks) column vector containing the alternative ranks. The rownames identify the alternatives.
$r(m f x) \quad$ matrix containing the computed marginal effects and associated statistics. Column 1 of the matrix contains the marginal effects; column 2, their standard errors; column 3, their $z$ statistics; and columns 4 and 5, the confidence intervals. Column 6 contains the values of the independent variables used to compute the probabilities $\mathrm{r}(\mathrm{pr})$.

## Also see

[R] asroprobit - Alternative-specific rank-ordered probit regression
[R] asmprobit - Alternative-specific multinomial probit regression
[U] 20 Estimation and postestimation commands

## Title

BIC note - Calculating and interpreting BIC

Description Remarks and examples Methods and formulas References
Also see

## Description

This entry discusses a statistical issue that arises when using the Bayesian information criterion (BIC) to compare models.

Stata calculates BIC, assuming $N=\mathrm{e}(\mathrm{N})$ —we will explain—but sometimes it would be better if a different $N$ were used. Commands that calculate BIC have an n() option, allowing you to specify the $N$ to be used.

In summary,

1. If you are comparing results estimated by the same estimation command, using the default BIC calculation is probably fine. There is an issue, but most researchers would ignore it.
2. If you are comparing results estimated by different estimation commands, you need to be on your guard.
a. If the different estimation commands share the same definitions of observations, independence, and the like, you are back in case 1.
b. If they differ in these regards, you need to think about the value of $N$ that should be used. For example, logit and xtlogit differ in that the former assumes independent observations and the latter, independent panels.
c. If estimation commands differ in the events being used over which the likelihood function is calculated, the information criteria may not be comparable at all. We say information criteria because this would apply equally to the Akaike information criterion (AIC), as well as to BIC. For instance, streg and stcox produce such incomparable results. The events used by streg are the actual survival times, whereas the events used by stcox are failures within risk pools, conditional on the times at which failures occurred.

## Remarks and examples

Remarks are presented under the following headings:

[^5]
## Background

The AIC and the BIC are two popular measures for comparing maximum likelihood models. AIC and BIC are defined as

$$
\begin{aligned}
& \text { AIC }=-2 \times \ln (\text { likelihood })+2 \times k \\
& \text { BIC }=-2 \times \ln (\text { likelihood })+\ln (N) \times k
\end{aligned}
$$

where

$$
\begin{aligned}
k & =\text { number of parameters estimated } \\
N & =\text { number of observations }
\end{aligned}
$$

We are going to discuss AIC along with BIC because AIC has some of the problems that BIC has, but not all.

AIC and BIC can be viewed as measures that combine fit and complexity. Fit is measured negatively by $-2 \times \ln$ (likelihood); the larger the value, the worse the fit. Complexity is measured positively, either by $2 \times k$ (AIC) or $\ln (N) \times k$ (BIC).

Given two models fit on the same data, the model with the smaller value of the information criterion is considered to be better.

There is substantial literature on these measures: see Akaike (1974); Raftery (1995); Sakamoto, Ishiguro, and Kitagawa (1986); and Schwarz (1978).

When Stata calculates the above measures, it uses the rank of $e(V)$ for $k$ and it uses $e(N)$ for $N . e(V)$ and $e(N)$ are Stata notation for results stored by the estimation command. e(V) is the variance-covariance matrix of the estimated parameters, and $e(N)$ is the number of observations in the dataset used in calculating the result.

## The problem of determining $\mathbf{N}$

The difference between AIC and BIC is that AIC uses the constant 2 to weight $k$, whereas BIC uses $\ln (N)$.

Determining what value of $N$ should be used is problematic. Despite appearances, the definition " $N$ is the number of observations" is not easy to make operational. $N$ does not appear in the likelihood function itself, $N$ is not the output of a standard statistical formula, and what is an observation is often subjective.

## > Example 1

Often what is meant by $N$ is obvious. Consider a simple logit model. What is meant by $N$ is the number of observations that are statistically independent and that corresponds to $M$, the number of observations in the dataset used in the calculation. We will write $N=M$.

But now assume that the same dataset has a grouping variable and the data are thought to be clustered within group. To keep the problem simple, let's pretend that there are $G$ groups and $m$ observations within group, so that $M=G \times m$. Because you are worried about intragroup correlation, you fit your model with xtlogit, grouping on the grouping variable. Now you wish to calculate BIC. What is the $N$ that should be used? $N=M$ or $N=G$ ?

That is a deep question. If the observations really are independent, then you should use $N=M$. If the observations within group are not just correlated but are duplicates of one another, and they had to be so, then you should use $M=G$. Between those two extremes, you should probably use a number between $N$ and $G$, but determining what that number should be from measured correlations is difficult. Using $N=M$ is conservative in that, if anything, it overweights complexity. Conservativeness, however, is subjective, too: using $N=G$ could be considered more conservative in that fewer constraints are being placed on the data.

When the estimated correlation is high, our reaction would be that using $N=G$ is probably more reasonable. Our first reaction, however, would be that using BIC to compare models is probably a misuse of the measure.

Stata uses $N=M$. An informal survey of web-based literature suggests that $N=M$ is the popular choice.

There is another reason, not so good, to choose $N=M$. It makes across-model comparisons more likely to be valid when performed without thinking about the issue. Say that you wish to compare the logit and xtlogit results. Thus you need to calculate

$$
\begin{aligned}
\operatorname{BIC}_{p} & =-2 \times \ln \left(\text { likelihood }_{p}\right)+\ln \left(N_{p}\right) \times k \\
\operatorname{BIC}_{x} & =-2 \times \ln \left(\text { likelihood }_{x}\right)+\ln \left(N_{x}\right) \times k
\end{aligned}
$$

Whatever $N$ you use, you must use the same $N$ in both formulas. Stata's choice of $N=M$ at least meets that test.

## > Example 2

In the above example, using $N=M$ is reasonable. Now let's look at when using $N=M$ is wrong, even if popular.

Consider a model fit by stcox. Using $N=M$ is certainly wrong if for no other reason than $M$ is not even a well-defined number. The same data can be represented by different datasets with different numbers of observations. For example, in one dataset, there might be 1 observation per subject. In another, the same subjects could have two records each, the first recording the first half of the time at risk and the second recording the remaining part. All statistics calculated by Stata on either dataset would be the same, but $M$ would be different.

Deciding on the right definition, however, is difficult. Viewed one way, $N$ in the Cox regression case should be the number of risk pools, $R$, because the Cox regression calculation is made on the basis of the independent risk pools. Viewed another way, $N$ should be the number of subjects, $N_{\text {subj }}$, because, even though the likelihood function is based on risk pools, the parameters estimated are at the subject level.

You can decide which argument you prefer.
For parametric survival models, in single-record data, $N=M$ is unambiguously correct. For multirecord data, there is an argument for $N=M$ and for $N=N_{\text {subj }}$.

## The problem of conformable likelihoods

The problem of conformable likelihoods does not concern $N$. Researchers sometimes use information criteria such as BIC and AIC to make comparisons across models. For that to be valid, the likelihoods must be conformable; that is, the likelihoods must all measure the same thing.

It is common to think of the likelihood function as the $\operatorname{Pr}$ (data|parameters), but in fact, the likelihood is

$$
\operatorname{Pr}(\text { particular events in the data } \mid \text { parameters })
$$

You must ensure that the events are the same.
For instance, they are not the same in the semiparametric Cox regression and the various parametric survival models. In Cox regression, the events are, at each failure time, that the subjects observed to fail in fact failed, given that failures occurred at those times. In the parametric models, the events are that each subject failed exactly when the subject was observed to fail.

The formula for AIC and BIC is

$$
\text { measure }=-2 \times \ln (\text { likelihood })+\text { complexity }
$$

When you are comparing models, if the likelihoods are measuring different events, even if the models obtain estimates of the same parameters, differences in the information measures are irrelevant.

## The first problem does not arise with AIC; the second problem does

Regardless of model, the problem of defining $N$ never arises with AIC because $N$ is not used in the AIC calculation. AIC uses a constant 2 to weight complexity as measured by $k$, rather than $\ln (N)$.

For both AIC and BIC, however, the likelihood functions must be conformable; that is, they must be measuring the same event.

## Calculating BIC correctly

When using BIC to compare results, and especially when using BIC to compare results from different models, you should think carefully about how $N$ should be defined. Then specify that number by using the n() option:

```
. estimates stats full sub, n(74)
Akaike's information criterion and Bayesian information criterion
```

| Model | Obs | ll(null) | ll(model) | df | AIC | BIC |
| ---: | :---: | :---: | :---: | :---: | :---: | ---: |
| full | 74 | -45.03321 | -20.59083 | 4 | 49.18167 | 58.39793 |
| sub | 74 | -45.03321 | -27.17516 | 3 | 60.35031 | 67.26251 |

Note: $N=74$ used in calculating BIC
Both estimates stats and estat ic allow the $n()$ option; see $[R]$ estimates stats and $[R]$ estat ic.

## Methods and formulas

AIC and BIC are defined as

$$
\begin{aligned}
& \text { AIC }=-2 \times \ln (\text { likelihood })+2 \times k \\
& \text { BIC }=-2 \times \ln (\text { likelihood })+\ln (N) \times k
\end{aligned}
$$

where $k$ is the model degrees of freedom calculated as the rank of variance-covariance matrix of the parameters e(V) and $N$ is the number of observations used in estimation or, more precisely, the number of independent terms in the likelihood. Operationally, $N$ is defined as e( N ) unless the n () option is specified.

## References

Akaike, H. 1974. A new look at the statistical model identification. IEEE Transactions on Automatic Control 19: 716-723.

Raftery, A. E. 1995. Bayesian model selection in social research. In Vol. 25 of Sociological Methodology, ed. P. V. Marsden, 111-163. Oxford: Blackwell.

Sakamoto, Y., M. Ishiguro, and G. Kitagawa. 1986. Akaike Information Criterion Statistics. Dordrecht, The Netherlands: Reidel.

Schwarz, G. 1978. Estimating the dimension of a model. Annals of Statistics 6: 461-464.

## Also see

[R] estat ic - Display information criteria
[R] estimates stats - Model-selection statistics

## Title

binreg - Generalized linear models: Extensions to the binomial family

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

```
binreg depvar [indepvars] [if] [in] [weight] [, options]
```

options Description
Model
noconstant suppress constant term
or use logit link and report odds ratios
rr use log link and report risk ratios
$\mathrm{hr} \quad$ use log-complement link and report health ratios
rd use identity link and report risk differences
n (\#|varname) use \# or varname for number of trials
exposure (varname)
offset (varname)
include $\ln$ (varname) in model with coefficient constrained to 1
constraints (constraints)
collinear
mu(varname)
init (varname) include varname in model with coefficient constrained to 1
apply specified linear constraints
keep collinear variables
use varname as the initial estimate for the mean of depvar synonym for mu(varname)

SE/Robust
vce (vcetype)
t(varname)
vfactor(\#)
disp(\#)
scale(x2|dev|\#)
vcetype may be eim, robust, cluster clustvar, oim, opg,
bootstrap, jackknife, hac kernel, jackknife1, or unbiased
variable name corresponding to time
multiply variance matrix by scalar \#
quasi-likelihood multiplier
set the scale parameter; default is scale(1)
Reporting
level(\#)
coefficients
nocnsreport
display_options
set confidence level; default is level (95)
report nonexponentiated coefficients
do not display constraints
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling

Maximization
irls
ml
maximize_options
fisher (\#)
search
coeflegend
use iterated, reweighted least-squares optimization; the default use maximum likelihood optimization
control the maximization process; seldom used
Fisher scoring steps
search for good starting values
display legend instead of statistics
indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
depvar and indepvars may contain time-series operators; see [U] 11.4.4 Time-series varlists.
bootstrap, by, fp, jackknife, mi estimate, rolling, and statsby are allowed; see [U] 11.1.10 Prefix commands.
vce(bootstrap), vce(jackknife), and vce(jackknife1) are not allowed with the mi estimate prefix; see
[MI] mi estimate.
Weights are not allowed with the bootstrap prefix; see [R] bootstrap.
aweights are not allowed with the jackknife prefix; see [R] jackknife.
fweights, aweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Generalized linear models $>$ GLM for the binomial family

## Description

binreg fits generalized linear models for the binomial family. It estimates odds ratios, risk ratios, health ratios, and risk differences. The available links are

| Option | Implied link | Parameter |
| ---: | ---: | ---: |
| or | logit | odds ratios $=\exp (\beta)$ |
| rr | log | risk ratios $=\exp (\beta)$ |
| hr | log complement | health ratios $=\exp (\beta)$ |
| rd | identity | risk differences $=\beta$ |

Estimates of odds, risk, and health ratios are obtained by exponentiating the appropriate coefficients. The or option produces the same results as Stata's logistic command, and or coefficients yields the same results as the logit command. When no link is specified, or is assumed.

## Options

noconstant; see $[R]$ estimation options.
or requests the logit link and results in odds ratios if coefficients is not specified.
rr requests the log link and results in risk ratios if coefficients is not specified.
hr requests the log-complement link and results in health ratios if coefficients is not specified. rd requests the identity link and results in risk differences.
$\mathrm{n}(\# \mid$ varname ) specifies either a constant integer to use as the denominator for the binomial family or a variable that holds the denominator for each observation.
exposure (varname), offset (varname), constraints (constraints), collinear; see [R] estimation options. constraints (constraints) and collinear are not allowed with irls.
mu (varname) specifies varname containing an initial estimate for the mean of depvar. This option can be useful if you encounter convergence difficulties. init(varname) is a synonym.

SE/Robust
vce(vcetype) specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), that are derived from asymptotic theory (oim, opg), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
vce (eim), the default, uses the expected information matrix (EIM) for the variance estimator.
binreg also allows the following:
vce (hac kernel [\#]) specifies that a heteroskedasticity- and autocorrelation-consistent (HAC) variance estimate be used. HAC refers to the general form for combining weighted matrices to form the variance estimate. There are three kernels built into binreg. kernel is a user-written program or one of
nwest | gallant | anderson
If \# is not specified, $N-2$ is assumed.
vce (jackknife1) specifies that the one-step jackknife estimate of variance be used.
vce (unbiased) specifies that the unbiased sandwich estimate of variance be used.
t (varname) specifies the variable name corresponding to time; see [TS] tsset. binreg does not always need to know $t$ (), though it does if vce (hac ...) is specified. Then you can either specify the time variable with $t$ (), or you can tsset your data before calling binreg. When the time variable is required, binreg assumes that the observations are spaced equally over time.
vfactor(\#) specifies a scalar by which to multiply the resulting variance matrix. This option allows users to match output with other packages, which may apply degrees of freedom or other small-sample corrections to estimates of variance.
disp(\#) multiplies the variance of depvar by \# and divides the deviance by \#. The resulting distributions are members of the quasilikelihood family.
scale( $\mathrm{x} 2|\mathrm{dev}| \#$ ) overrides the default scale parameter. This option is allowed only with Hessian (information matrix) variance estimates.

By default, scale(1) is assumed for the discrete distributions (binomial, Poisson, and negative binomial), and scale(x2) is assumed for the continuous distributions (Gaussian, gamma, and inverse Gaussian).
scale (x2) specifies that the scale parameter be set to the Pearson chi-squared (or generalized chi-squared) statistic divided by the residual degrees of freedom, which was recommended by McCullagh and Nelder (1989) as a good general choice for continuous distributions.
scale(dev) sets the scale parameter to the deviance divided by the residual degrees of freedom. This option provides an alternative to scale(x2) for continuous distributions and overdispersed or underdispersed discrete distributions.
scale(\#) sets the scale parameter to \#.
level(\#), noconstant; see [R] estimation options.
coefficients displays the nonexponentiated coefficients and corresponding standard errors and confidence intervals. This option has no effect when the rd option is specified, because it always presents the nonexponentiated coefficients.
nocnsreport; see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% $f m t$ ), pformat (\% fmt), sformat (\%fmt), and nolstretch; see [R] estimation options.

Maximization
irls requests iterated, reweighted least-squares (IRLS) optimization of the deviance instead of Newton-Raphson optimization of the log likelihood. This option is the default.
ml requests that optimization be carried out by using Stata's ml command; see [R] ml.
maximize_options: technique (algorithm_spec), [no] log, trace, gradient, showstep, hessian, showtolerance, difficult, iterate(\#), tolerance(\#), ltolerance(\#),
nrtolerance(\#), nonrtolerance, and from(init_specs); see [R] maximize. These options are seldom used.

Setting the optimization method to ml, with technique() set to something other than BHHH, changes the vcetype to vce (oim). Specifying technique (bhhh) changes vcetype to vce (opg).
fisher (\#) specifies the number of Newton-Raphson steps that should use the Fisher scoring Hessian or EIM before switching to the observed information matrix (OIM). This option is available only if ml is specified and is useful only for Newton-Raphson optimization.
search specifies that the command search for good starting values. This option is available only if ml is specified and is useful only for Newton-Raphson optimization.

The following option is available with binreg but is not shown in the dialog box:
coeflegend; see $[R]$ estimation options.

## Remarks and examples

Wacholder (1986) suggests methods for estimating risk ratios and risk differences from prospective binomial data. These estimates are obtained by selecting the proper link functions in the generalized linear-model framework. (See Methods and formulas for details; also see [R] glm.)
> Example 1
Wacholder (1986) presents an example, using data from Wright et al. (1983), of an investigation of the relationship between alcohol consumption and the risk of a low-birthweight baby. Covariates examined included whether the mother smoked (yes or no), mother's social class (three levels), and drinking frequency (light, moderate, or heavy). The data for the 18 possible categories determined by the covariates are illustrated below.

Let's first describe the data and list a few observations.
. use http://www.stata-press.com/data/r13/binreg
. list

|  | category | n_lbw_~s | n_women | alcohol | smokes | social |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | 1 | 11 | 84 | heavy | nonsmoker | 1 |
| 2. | 2 | 5 | 79 | moderate | nonsmoker | 1 |
| 3. | 3 | 11 | 169 | light | nonsmoker | 1 |
| 4. | 4 | 6 | 28 | heavy | smoker | 1 |
| 5. | 5 | 3 | 13 | moderate | smoker | 1 |
| 6. | 6 | 1 | 26 | light | smoker | 1 |
| 7. | 7 | 4 | 22 | heavy | nonsmoker | 2 |
| 8. | 8 | 3 | 25 | moderate | nonsmoker | 2 |
| 9. | 9 | 12 | 162 | light | nonsmoker | 2 |
| 10. | 10 | 4 | 17 | heavy | smoker | 2 |
| 11. | 11 | 2 | 7 | moderate | smoker | 2 |
| 12. | 12 | 6 | 38 | light | smoker | 2 |
| 13. | 13 | 0 | 14 | heavy | nonsmoker | 3 |
| 14. | 14 | 1 | 18 | moderate | nonsmoker | 3 |
| 15. | 15 | 12 | 91 | light | nonsmoker | 3 |
| 16. | 16 | 7 | 19 | heavy | smoker | 3 |
| 17. | 17 | 2 | 18 | moderate | smoker | 3 |
| 18. | 18 | 8 | 70 | light | smoker | 3 |

Each observation corresponds to one of the 18 covariate structures. The number of low-birthweight babies from n_women in each category is given by the n_lbw_babies variable.

We begin by estimating risk ratios:

```
. binreg n_lbw_babies i.soc i.alc i.smo, n(n_women) rr
Iteration 1: deviance = 14.2879
Iteration 2: deviance = 13.607
Iteration 3: deviance = 13.60503
Iteration 4: deviance = 13.60503
Generalized linear models No. of obs = 18
Optimization : MQL Fisher scoring Residual df = 12
```



```
Pearson = 11.51517095
Variance function: V(u) = u*(1-u/n_women)
Link function : g(u) = ln(u/n_women)
\begin{tabular}{lrr} 
No. of obs & \(=\) & 18 \\
Residual df & \(=\) & 12 \\
Scale parameter & \(=\) & 1 \\
(1/df) Deviance & \(=\) & 1.133752 \\
(1/df) Pearson & \(=\) & .9595976 \\
[Binomial] & & \\
[Log] & &
\end{tabular}
```

BIC = -21.07943

| n_lbw_babies | Risk Ratio | $\begin{gathered} \text { EIM } \\ \text { Std. Err. } \end{gathered}$ | z | $P>\|z\|$ | [95\% Conf. | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| social |  |  |  |  |  |  |
| 2 | 1.340001 | . 3127382 | 1.25 | 0.210 | . 848098 | 2.11721 |
| 3 | 1.349487 | . 3291488 | 1.23 | 0.219 | . 8366715 | 2.176619 |
| alcohol |  |  |  |  |  |  |
| moderate | 1.191157 | . 3265354 | 0.64 | 0.523 | . 6960276 | 2.038503 |
| heavy | 1.974078 | . 4261751 | 3.15 | 0.002 | 1.293011 | 3.013884 |
| smokes |  |  |  |  |  |  |
| smoker | 1.648444 | . 332875 | 2.48 | 0.013 | 1.109657 | 2.448836 |
| _cons | . 0630341 | . 0128061 | -13.61 | 0.000 | . 0423297 | . 0938656 |

By default, Stata reports the risk ratios (the exponentiated regression coefficients) estimated by the model. We can see that the risk ratio comparing heavy drinkers with light drinkers, after adjusting for smoking and social class, is 1.974078 . That is, mothers who drink heavily during their pregnancy have approximately twice the risk of delivering low-birthweight babies as mothers who are light drinkers.

The nonexponentiated coefficients can be obtained with the coefficients option:

```
. binreg n_lbw_babies i.soc i.alc i.smo, n(n_women) rr coefficients
Iteration 1: deviance = 14.2879
Iteration 2: deviance = 13.607
Iteration 3: deviance = 13.60503
Iteration 4: deviance = 13.60503
Generalized linear models No. of obs = 18
\begin{tabular}{|c|c|c|c|}
\hline Optimization & \begin{tabular}{l}
MQL Fisher scoring \\
(IRLS EIM)
\end{tabular} & Residual df Scale parameter & 12 \\
\hline Deviance & \(=13.6050268\) & (1/df) Deviance & 1.133752 \\
\hline Pearson & \(=11.51517095\) & (1/df) Pearson & . 9595976 \\
\hline Variance function & \(\mathrm{V}(\mathrm{u})=\mathrm{u} *\left(1-\mathrm{u} / \mathrm{n}_{\text {_ }}\right.\) women \()\) & [Binomial] & \\
\hline Link function & \(\mathrm{g}(\mathrm{u})=\ln \left(\mathrm{u} / \mathrm{n}_{-}\right.\)women) & [Log] & \\
\hline
\end{tabular}
```

BIC = -21.07943

| n_lbw_babies | Coef. | $\begin{gathered} \text { EIM } \\ \text { Std. Err. } \end{gathered}$ | z | $\mathrm{P}>\|z\|$ | [95\% Conf. | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| social |  |  |  |  |  |  |
| 2 | . 2926702 | . 2333866 | 1.25 | 0.210 | -. 1647591 | . 7500994 |
| 3 | . 2997244 | . 2439066 | 1.23 | 0.219 | -. 1783238 | . 7777726 |
| alcohol |  |  |  |  |  |  |
| moderate | . 1749248 | . 274133 | 0.64 | 0.523 | -. 362366 | . 7122156 |
| heavy | . 6801017 | . 2158856 | 3.15 | 0.002 | . 2569737 | 1.10323 |
| smokes |  |  |  |  |  |  |
| smoker | . 4998317 | . 2019329 | 2.48 | 0.013 | . 1040505 | . 8956129 |
| _cons | -2.764079 | . 2031606 | -13.61 | 0.000 | -3.162266 | -2.365891 |

Risk differences are obtained with the rd option:

| . binreg n_lbw_babies i.soc i.alc i. |  |  |
| :--- | :--- | ---: |
| Iteration 1: | deviance $=$ | 18.67277 |
| Iteration 2: | deviance $=$ | 14.94364 |
| Iteration 3: | deviance $=$ | 14.9185 |
| Iteration 4: | deviance $=$ | 14.91762 |
| Iteration 5: | deviance $=$ | 14.91758 |
| Iteration 6: | deviance $=$ | 14.91758 |
| Iteration 7: | deviance $=$ | 14.91758 |


| Generalized linear models | No. of obs | $=$ | 18 |
| :--- | :--- | :--- | :--- |
| Optimization | $:$ MQL Fisher scoring | (IRLS EIM) | Residual df |


|  | EIM |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| n_lbw_babies | Risk Diff. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| social |  |  |  |  |  |  |
| 2 | .0263817 | .0232124 | 1.14 | 0.256 | -.0191137 | .0718771 |
| 3 | .0365553 | .0268668 | 1.36 | 0.174 | -.0161026 | .0892132 |
| alcohol |  |  |  |  |  |  |
| moderate <br> heavy | .0122539 | .0257713 | 0.48 | 0.634 | -.0382569 | .0627647 |
| smokes | .0801291 | .0302878 | 2.65 | 0.008 | .020766 | .1394921 |
| smoker | .0542415 | .0270838 | 2.00 | 0.045 | .0011582 | .1073248 |
| _cons | .059028 | .0160693 | 3.67 | 0.000 | .0275327 | .0905232 |

The risk difference between heavy drinkers and light drinkers is 0.0801291 . Because the risk differences are obtained directly from the coefficients estimated by using the identity link, the coefficients option has no effect here.

Health ratios are obtained with the hr option. The health ratios (exponentiated coefficients for the log-complement link) are reported directly.

```
. binreg n_lbw_babies i.soc i.alc i.smo, n(n_women) hr
Iteration 1: deviance = 21.15233
Iteration 2: deviance = 15.16467
Iteration 3: deviance = 15.13205
Iteration 4: deviance = 15.13114
Iteration 5: deviance = 15.13111
Iteration 6: deviance = 15.13111
Iteration 7: deviance = 15.13111
Generalized linear models No. of obs = 18
Optimization : MQL Fisher scoring Residual df =
Deviance = 15.13110545 (1/df) Deviance = 1.260925
Pearson = 12.84203917 (1/df) Pearson = 1.07017
Variance function: V(u) = u*(1-u/n_women) [Binomial]
Link function : g(u) = ln(1-u/n_women) [Log complement]
BIC = -19.55336
```

| n_lbw_babies | EIM |  |  | $P>\|z\|$ | [95\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| social |  |  |  |  |  |  |
| 2 | . 9720541 | . 024858 | -1.11 | 0.268 | . 9245342 | 1.022017 |
| 3 | . 9597182 | . 0290412 | -1.36 | 0.174 | . 9044535 | 1.01836 |
| alcohol |  |  |  |  |  |  |
| moderate | . 9871517 | . 0278852 | -0.46 | 0.647 | . 9339831 | 1.043347 |
| heavy | . 9134243 | . 0325726 | -2.54 | 0.011 | . 8517631 | . 9795493 |
| smokes |  |  |  |  |  |  |
| smoker | . 9409983 | . 0296125 | -1.93 | 0.053 | . 8847125 | 1.000865 |
| _cons | . 9409945 | . 0163084 | -3.51 | 0.000 | . 9095674 | . 9735075 |

(HR) Health ratios
To see the nonexponentiated coefficients, we can specify the coefficients option.

## Stored results

binreg, irls stores the following in e():
Scalars

```
e(N)
e(k)
e(k_eq_model)
e(df_m)
e(df)
e(phi)
e(disp)
e(bic)
e(N_clust)
e(deviance)
e(deviance_s)
e(deviance_p)
e(deviance_ps)
e(dispers)
e(dispers_s)
e(dispers_p)
e(dispers_ps)
e(vf)
e(rank)
e(rc)
```

Macros
e(cmd)
e(cmdline)
e(depvar)
e(eform)
e(varfunc)
e(varfunct)
e(varfuncf)
e(link)
e(linkt)
e(linkf)
e(m)
e(wtype)
e(wexp)
e(title_fl)
e(clustvar)
e(offset)
e(cons)
e(hac_kernel)
e(hac_lag)
e(vce)
e(vcetype)
e(opt)
e(opt1)
e(opt2)
e(properties)
e(predict)
e(marginsok)
e(marginsnotok)
e(asbalanced)
e(asobserved)
number of observations
number of parameters
number of equations in overall model test
model degrees of freedom
residual degrees of freedom
model scale parameter
dispersion parameter
model BIC
number of clusters
deviance
scaled deviance
Pearson deviance
scaled Pearson deviance
dispersion
scaled dispersion
Pearson dispersion
scaled Pearson dispersion
factor set by vfactor(), 1 if not set
rank of e(V)
return code

## binreg

command as typed
name of dependent variable
eform() option implied by or, $r$, hr , or rd
program to calculate variance function
variance title
variance function
program to calculate link function
link title
link function
number of binomial trials
weight type
weight expression
family-link title
name of cluster variable
linear offset variable
noconstant or not set
HAC kernel
HAC lag
vcetype specified in vce()
title used to label Std. Err.
type of optimization
optimization title, line 1
optimization title, line 2
b V
program used to implement predict
predictions allowed by margins
predictions disallowed by margins
factor variables fvset as asbalanced
factor variables fvset as asobserved

Matrices

```
    e(b)
    e(Cns)
    e(V)
    e(V_modelbased)
```

Functions
e(sample) marks estimation sample
binreg, ml stores the following in e():
Scalars
e(N)
e(k)
e(k_eq)
e(k_eq_model)
e(k_dv)
$e\left(d f \_m\right)$
e(df)
e(phi)
e(aic)
e(bic)
e(ll)
e(N_clust)
e(chi2)
e(p)
e(deviance)
e(deviance_s)
e(deviance_p)
e(deviance_ps)
e(dispers)
e(dispers_s)
e(dispers_p)
e(dispers_ps)
e(vf)
e(rank)
e(ic)
e(rc)
e(converged)
Macros
e(cmd)
e(cmdline)
e(depvar)
e(eform)
e(varfunc)
e(varfunct)
e(varfuncf)
e(link)
e(linkt)
e(linkf)
e(m)
e(wtype)
e(wexp)
e(title)
e(title_fl)
e(clustvar)
e(offset)
e(cons)
e(hac_kernel)
e(hac_lag)
e(chi2type)
e(vce)
coefficient vector
constraints matrix
variance-covariance matrix of the estimators
model-based variance

```
    e(vcetype)
    e(opt)
    e(opt1)
    e(which)
    e(ml_method)
    e(user)
    e(technique)
    e(properties)
    e(predict)
    e(marginsok)
    e(marginsnotok)
    e(asbalanced)
    e(asobserved)
Matrices
    e(b)
    e(Cns)
    e(ilog)
    e(gradient)
    e(V)
    e(V_modelbased)
```

Functions
e(sample)
title used to label Std. Err.
type of optimization
optimization title, line 1
max or min; whether optimizer is to perform maximization or minimization
type of ml method
name of likelihood-evaluator program
maximization technique
b V
program used to implement predict
predictions allowed by margins
predictions disallowed by margins
factor variables fvset as asbalanced
factor variables fvset as asobserved
coefficient vector
constraints matrix
iteration $\log$ (up to 20 iterations)
gradient vector
variance-covariance matrix of the estimators
model-based variance
marks estimation sample

## Methods and formulas

Let $\pi_{i}$ be the probability of success for the $i$ th observation, $i=1, \ldots, N$, and let $X \beta$ be the linear predictor. The link function relates the covariates of each observation to its respective probability through the linear predictor.

In logistic regression, the logit link is used:

$$
\ln \left(\frac{\pi}{1-\pi}\right)=X \beta
$$

The regression coefficient $\beta_{k}$ represents the change in the logarithm of the odds associated with a one-unit change in the value of the $X_{k}$ covariate; thus $\exp \left(\beta_{k}\right)$ is the ratio of the odds associated with a change of one unit in $X_{k}$.

For risk differences, the identity link $\pi=X \beta$ is used. The regression coefficient $\beta_{k}$ represents the risk difference associated with a change of one unit in $X_{k}$. When using the identity link, you can obtain fitted probabilities outside the interval $(0,1)$. As suggested by Wacholder, at each iteration, fitted probabilities are checked for range conditions (and put back in range if necessary). For example, if the identity link results in a fitted probability that is smaller than $1 \mathrm{e}-4$, the probability is replaced with $1 \mathrm{e}-4$ before the link function is calculated.

A similar adjustment is made for the logarithmic link, which is used for estimating the risk ratio, $\ln (\pi)=X \beta$, where $\exp \left(\beta_{k}\right)$ is the risk ratio associated with a change of one unit in $X_{k}$, and for the log-complement link used to estimate the probability of no disease or health, where $\exp \left(\beta_{k}\right)$ represents the "health ratio" associated with a change of one unit in $X_{k}$.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce(robust) and vce(cluster clustvar), respectively. See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas.

## References

Cummings, P. 2009. Methods for estimating adjusted risk ratios. Stata Journal 9: 175-196.
Hardin, J. W., and M. A. Cleves. 1999. sbe29: Generalized linear models: Extensions to the binomial family. Stata Technical Bulletin 50: 21-25. Reprinted in Stata Technical Bulletin Reprints, vol. 9, pp. 140-146. College Station, TX: Stata Press.

Kleinbaum, D. G., and M. Klein. 2010. Logistic Regression: A Self-Learning Text. 3rd ed. New York: Springer.
McCullagh, P., and J. A. Nelder. 1989. Generalized Linear Models. 2nd ed. London: Chapman \& Hall/CRC.
Wacholder, S. 1986. Binomial regression in GLIM: Estimating risk ratios and risk differences. American Journal of Epidemiology 123: 174-184.

Wright, J. T., I. G. Barrison, I. G. Lewis, K. D. MacRae, E. J. Waterson, P. J. Toplis, M. G. Gordon, N. F. Morris, and I. M. Murray-Lyon. 1983. Alcohol consumption, pregnancy and low birthweight. Lancet 1: 663-665.

[^6]
## Title

binreg postestimation - Postestimation tools for binreg

Description Syntax for predict Menu for predict Options for predict
References Also see

## Description

The following postestimation commands are available after binreg:

| Command | Description |
| :--- | :--- |
| contrast | contrasts and ANOVA-style joint tests of estimates |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample <br> estat vce <br> estimates <br> foriance-covariance matrix of the estimators (VCE) |
| lincom | cataloging estimation results <br> dynamic forecasts and simulations <br> point estimates, standard errors, testing, and inference for linear combinations <br> of coefficients |
| linktest | link test for model specification <br> marginal means, predictive margins, marginal effects, and average marginal <br> effects |
| margins | graph the results from margins (profile plots, interaction plots, etc.) |
| marginsplot | point estimates, standard errors, testing, and inference for nonlinear combinations <br> of coefficients |
| nlcom | predictions, residuals, influence statistics, and other diagnostic measures <br> point estimates, standard errors, testing, and inference for generalized predictions |
| predictnl | pairwise comparisons of estimates |
| pwcompare | Wald tests of simple and composite linear hypotheses |
| test | Wald tests of nonlinear hypotheses |
| testnl |  |

[^7]
## Syntax for predict

predict [type] newvar [if] [in] [, statistic options]
statistic Description
Main
mu expected value of $y$; the default
$\mathrm{xb} \quad$ linear prediction $\eta=\mathbf{x} \widehat{\boldsymbol{\beta}}$
eta
synonym for $x b$
stdp standard error of the linear prediction
anscombe Anscombe (1953) residuals
cooksd
deviance
hat
likelihood
pearson
response
score
working

Cook's distance
deviance residuals
diagonals of the "hat" matrix
weighted average of the standardized deviance and standard Pearson residuals Pearson residuals
differences between the observed and fitted outcomes
first derivative of the $\log$ likelihood with respect to $\mathbf{x}_{j} \boldsymbol{\beta}$ working residuals
options
Description
Options
nooffset
adjusted
standardized
studentized
modified
modify calculations to ignore the offset variable
adjust deviance residual to speed up convergence
multiply residual by the factor $(1-h)^{1 / 2}$
multiply residual by one over the square root of the estimated scale parameter modify denominator of residual to be a reasonable estimate of the variance of depvar

These statistics are available both in and out of sample; type predict ... if e(sample)... if wanted only for the estimation sample.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

[^8]anscombe calculates the Anscombe (1953) residuals to produce residuals that closely follow a normal distribution.
cooksd calculates Cook's distance, which measures the aggregate change in the estimated coefficients when each observation is left out of the estimation.
deviance calculates the deviance residuals, which are recommended by McCullagh and Nelder (1989) and others as having the best properties for examining goodness of fit of a GLM. They are approximately normally distributed if the model is correct and may be plotted against the fitted values or against a covariate to inspect the model's fit. Also see the pearson option below.
hat calculates the diagonals of the "hat" matrix, analogous to linear regression.
likelihood calculates a weighted average of the standardized deviance and standardized Pearson (described below) residuals.
pearson calculates the Pearson residuals, which often have markedly skewed distributions for nonnormal family distributions. Also see the deviance option above.
response calculates the differences between the observed and fitted outcomes.
score calculates the equation-level score, $\partial \ln L / \partial\left(\mathbf{x}_{j} \boldsymbol{\beta}\right)$.
working calculates the working residuals, which are response residuals weighted according to the derivative of the link function.

Options
nooffset is relevant only if you specified offset (varname) for binreg. It modifies the calculations made by predict so that they ignore the offset variable; the linear prediction is treated as $\mathbf{x}_{j} \mathbf{b}$ rather than as $\mathbf{x}_{j} \mathbf{b}+$ offset $_{j}$.
adjusted adjusts the deviance residual to make the convergence to the limiting normal distribution faster. The adjustment deals with adding to the deviance residual a higher-order term depending on the variance function family. This option is allowed only when deviance is specified.
standardized requests that the residual be multiplied by the factor $(1-h)^{-1 / 2}$, where $h$ is the diagonal of the hat matrix. This step is done to take into account the correlation between depvar and its predicted value.
studentized requests that the residual be multiplied by one over the square root of the estimated scale parameter.
modified requests that the denominator of the residual be modified to be a reasonable estimate of the variance of depvar. The base residual is multiplied by the factor $(k / w)^{-1 / 2}$, where $k$ is either one or the user-specified dispersion parameter and $w$ is the specified weight (or one if left unspecified).

## References

Anscombe, F. J. 1953. Contribution of discussion paper by H. Hotelling "New light on the correlation coefficient and its transforms". Journal of the Royal Statistical Society, Series B 15: 229-230.
McCullagh, P., and J. A. Nelder. 1989. Generalized Linear Models. 2nd ed. London: Chapman \& Hall/CRC.

## Also see

[R] binreg - Generalized linear models: Extensions to the binomial family
[U] 20 Estimation and postestimation commands

## Title

biprobit - Bivariate probit regression

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

Also see

## Syntax

Bivariate probit regression
biprobit depvar $_{1}$ depvar $_{2}$ [indepvars] [if] [in] [weight] [, options]
Seemingly unrelated bivariate probit regression

where equation $_{1}$ and equation ${ }_{2}$ are specified as
( [eqname: ] depvar $[=]$ [indepvars] [, noconstant offset(varname)])
options
Description
Model
noconstant
partial
offset 1 (varname)
offset2 (varname)
constraints (constraints)
collinear
SE/Robust
vce(vcetype)

Reporting
level(\#)
noskip
nocnsreport
display_options

Maximization
maximize_options
coeflegend
suppress constant term
fit partial observability model
offset variable for first equation
offset variable for second equation
apply specified linear constraints
keep collinear variables
vcetype may be oim, robust, cluster clustvar, opg, bootstrap, or jackknife
set confidence level; default is level (95)
perform likelihood-ratio test
do not display constraints
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
control the maximization process; seldom used
display legend instead of statistics

## su_options

Description

## Model

partial
constraints (constraints)
collinear

## SE/Robust

vce (vcetype)

Reporting
level (\#)
noskip
nocnsreport
display_options

Maximization
maximize_options
coeflegend
fit partial observability model
apply specified linear constraints
keep collinear variables
vcetype may be oim, robust, cluster clustvar, opg, bootstrap, or jackknife
set confidence level; default is level (95)
perform likelihood-ratio test
do not display constraints
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
depvar ${ }_{1}$, depvar ${ }_{2}$, indepvars, and depvar may contain time-series operators; see [U] 11.4.4 Time-series varlists. bootstrap, by, fp, jackknife, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands. Weights are not allowed with the bootstrap prefix; see $[R]$ bootstrap.
vce(), noskip, and weights are not allowed with the svy prefix; see [SVY] svy.
pweights, fweights, and iweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

biprobit
Statistics $>$ Binary outcomes $>$ Bivariate probit regression

## seemingly unrelated biprobit

Statistics $>$ Binary outcomes $>$ Seemingly unrelated bivariate probit regression

## Description

biprobit fits maximum-likelihood two-equation probit models-either a bivariate probit or a seemingly unrelated probit (limited to two equations).

## Options

$\qquad$ Model
noconstant; see [R] estimation options.
partial specifies that the partial observability model be fit. This particular model commonly has poor convergence properties, so we recommend that you use the difficult option if you want to fit the Poirier partial observability model; see $[\mathrm{R}]$ maximize.

This model computes the product of the two dependent variables so that you do not have to replace each with the product.
offset1(varname), offset2(varname), constraints(constraints), collinear; see [R] estimation options.

SE/Robust
vce (vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

## Reporting

level (\#); see [R] estimation options.
noskip specifies that a full maximum-likelihood model with only a constant for the regression equation be fit. This model is not displayed but is used as the base model to compute a likelihood-ratio test for the model test statistic displayed in the estimation header. By default, the overall model test statistic is an asymptotically equivalent Wald test of all the parameters in the regression equation being zero (except the constant). For many models, this option can substantially increase estimation time.
nocnsreport; see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% fmt), pformat (\%fmt), sformat(\%fmt), and nolstretch; see [R] estimation options.

Maximization
maximize_options: difficult, technique (algorithm_spec), iterate(\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance (\#), nonrtolerance, and from(init_specs); see [R] maximize. These options are seldom used.

Setting the optimization type to technique (bhhh) resets the default vcetype to vce (opg).
The following option is available with biprobit but is not shown in the dialog box: coeflegend; see $[R]$ estimation options.

## Remarks and examples

For a good introduction to the bivariate probit models, see Greene $(2012,738-752)$ and Pindyck and Rubinfeld (1998). Poirier (1980) explains the partial observability model. Van de Ven and Van Pragg (1981) explain the probit model with sample selection; see [R] heckprobit for details.

## > Example 1

We use the data from Pindyck and Rubinfeld (1998, 332). In this dataset, the variables are whether children attend private school (private), number of years the family has been at the present residence (years), log of property tax (logptax), $\log$ of income (loginc), and whether the head of the household voted for an increase in property taxes (vote).

We wish to model the bivariate outcomes of whether children attend private school and whether the head of the household voted for an increase in property tax based on the other covariates.

```
. use http://www.stata-press.com/data/r13/school
. biprobit private vote years logptax loginc
Fitting comparison equation 1:
Iteration 0: log likelihood = -31.967097
Iteration 1: log likelihood = -31.452424
Iteration 2: log likelihood = -31.448958
Iteration 3: log likelihood = -31.448958
Fitting comparison equation 2:
Iteration 0: log likelihood = -63.036914
Iteration 1: log likelihood = -58.534843
Iteration 2: log likelihood = -58.497292
Iteration 3: log likelihood = -58.497288
Comparison: log likelihood = -89.946246
```

Fitting full model:
Iteration 0: log likelihood $=-89.946246$
Iteration 1: log likelihood $=-89.258897$
Iteration 2: log likelihood $=-89.254028$
Iteration 3: log likelihood $=-89.254028$

| Bivariate probit regression | Number of obs | = | 95 |
| :---: | :---: | :---: | :---: |
|  | Wald chi2(6) |  | 9.59 |
| Log likelihood = -89.254028 | Prob > chi2 |  | 0.1431 |


|  | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| private |  |  |  |  |  |  |
| years | -.0118884 | .0256778 | -0.46 | 0.643 | -.0622159 | .0384391 |
| logptax | -.1066962 | .6669782 | -0.16 | 0.873 | -1.413949 | 1.200557 |
| loginc | .3762037 | .5306484 | 0.71 | 0.478 | -.663848 | 1.416255 |
| _cons | -4.184694 | 4.837817 | -0.86 | 0.387 | -13.66664 | 5.297253 |
| vote |  |  |  |  |  |  |
| years | -.0168561 | .0147834 | -1.14 | 0.254 | -.0458309 | .0121188 |
| logptax | -1.288707 | .5752266 | -2.24 | 0.025 | -2.416131 | -.1612839 |
| loginc | .998286 | .4403565 | 2.27 | 0.023 | .1352031 | 1.861369 |
| _cons | -.5360573 | 4.068509 | -0.13 | 0.895 | -8.510188 | 7.438073 |
| /athrho | -.2764525 | .2412099 | -1.15 | 0.252 | -.7492153 | .1963102 |
| rho | -.2696186 | .2236753 |  |  | -.6346806 | .1938267 |

Likelihood-ratio test of rho=0: chi2(1) = 1.38444 Prob > chi2 = 0.2393

The output shows several iteration logs. The first iteration $\log$ corresponds to running the univariate probit model for the first equation, and the second $\log$ corresponds to running the univariate probit for the second model. If $\rho=0$, the sum of the log likelihoods from these two models will equal the $\log$ likelihood of the bivariate probit model; this sum is printed in the iteration log as the comparison log likelihood.

The final iteration $\log$ is for fitting the full bivariate probit model. A likelihood-ratio test of the $\log$ likelihood for this model and the comparison log likelihood is presented at the end of the output. If we had specified the vce (robust) option, this test would be presented as a Wald test instead of as a likelihood-ratio test.

We could have fit the same model by using the seemingly unrelated syntax as

```
. biprobit (private=years logptax loginc) (vote=years logptax loginc)
```


## Stored results

biprobit stores the following in e() :

```
Scalars
        e(N)
    e(k)
    e(k_eq)
    e(k_aux)
    e(k_eq_model)
    e(k_dv)
    e(df_m)
    e(11)
    e(ll_0)
    e(ll_c)
    e(N_clust)
    e(chi2)
    e(chi2_c)
    e(p)
    e(rho)
    e(rank)
    e(rank0)
    e(ic)
    e(rc)
    e(converged)
Macros
    e(cmd)
    e(cmdline)
    e(depvar)
    e(wtype)
    e(wexp)
    e(title)
    e(clustvar)
    e(offset1)
    e(offset2)
    e(chi2type)
    e(chi2_ct)
    e(vce)
    e(vcetype)
    e(opt)
    e(which)
    e(ml_method)
    e(user)
    e(technique)
    e(properties)
    d(predict)
    e(asbalanced)
    e(asobserved)
    number of observations
    number of parameters
    number of equations in e(b)
    number of auxiliary parameters
    number of equations in overall model test
    number of dependent variables
    model degrees of freedom
    log likelihood
    log likelihood, constant-only model (noskip only)
    log likelihood, comparison model
    number of clusters
    \chi
    \chi ^ { 2 } \text { for comparison test}
    significance
    \rho
    rank of e(V)
    rank of e(V) for constant-only model
    number of iterations
    return code
    1 if converged, 0 otherwise
    biprobit
command as typed
names of dependent variables
weight type
weight expression
title in estimation output
name of cluster variable
offset for first equation
offset for second equation
Wald or LR; type of model }\mp@subsup{\chi}{}{2}\mathrm{ test
Wald or LR; type of model }\mp@subsup{\chi}{}{2}\mathrm{ test corresponding to e(chi2_c)
vcetype specified in vce()
title used to label Std. Err.
type of optimization
max or min; whether optimizer is to perform maximization or minimization
type of ml method
name of likelihood-evaluator program
maximization technique
b V
program used to implement predict
factor variables fvset as asbalanced
factor variables fvset as asobserved
```

Matrices

```
    e(b)
    e(Cns)
    e(ilog)
    e(gradient)
    e(V)
    e(V_modelbased)
```

coefficient vector
constraints matrix
iteration $\log$ (up to 20 iterations)
gradient vector
variance-covariance matrix of the estimators
model-based variance
Functions
e(sample) marks estimation sample

## Methods and formulas

The $\log$ likelihood, $\ln L$, is given by

$$
\begin{aligned}
\xi_{j}^{\beta} & =x_{j} \beta+\text { offset }_{j}^{\beta} \\
\xi_{j}^{\gamma} & =z_{j} \gamma+\text { offset }_{j}^{\gamma} \\
q_{1 j} & = \begin{cases}1 & \text { if } y_{1 j} \neq 0 \\
-1 & \text { otherwise }\end{cases} \\
q_{2 j} & = \begin{cases}1 & \text { if } y_{2 j} \neq 0 \\
-1 & \text { otherwise }\end{cases} \\
\rho_{j}^{*} & =q_{1 j} q_{2 j} \rho \\
\ln L & =\sum_{j=1}^{n} w_{j} \ln \Phi_{2}\left(q_{1 j} \xi_{j}^{\beta}, q_{2 j} \xi_{j}^{\gamma}, \rho_{j}^{*}\right)
\end{aligned}
$$

where $\Phi_{2}()$ is the cumulative bivariate normal distribution function (with mean $\left[\begin{array}{ll}0 & 0\end{array}\right]^{\prime}$ ) and $w_{j}$ is an optional weight for observation $j$. This derivation assumes that

$$
\begin{aligned}
y_{1 j}^{*} & =x_{j} \beta+\epsilon_{1 j}+\operatorname{offset}_{j}^{\beta} \\
y_{2 j}^{*} & =z_{j} \gamma+\epsilon_{2 j}+\operatorname{offset}_{j}^{\gamma} \\
E\left(\epsilon_{1}\right) & =E\left(\epsilon_{2}\right)=0 \\
\operatorname{Var}\left(\epsilon_{1}\right) & =\operatorname{Var}\left(\epsilon_{2}\right)=1 \\
\operatorname{Cov}\left(\epsilon_{1}, \epsilon_{2}\right) & =\rho
\end{aligned}
$$

where $y_{1 j}^{*}$ and $y_{2 j}^{*}$ are the unobserved latent variables; instead, we observe only $y_{i j}=1$ if $y_{i j}^{*}>0$ and $y_{i j}=0$ otherwise (for $i=1,2$ ).

In the maximum likelihood estimation, $\rho$ is not directly estimated, but atanh $\rho$ is

$$
\operatorname{atanh} \rho=\frac{1}{2} \ln \left(\frac{1+\rho}{1-\rho}\right)
$$

From the form of the likelihood, if $\rho=0$, then the log likelihood for the bivariate probit models is equal to the sum of the log likelihoods of the two univariate probit models. A likelihood-ratio test may therefore be performed by comparing the likelihood of the full bivariate model with the sum of the $\log$ likelihoods for the univariate probit models.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce (robust) and vce (cluster clustvar), respectively. See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas.
biprobit also supports estimation with survey data. For details on VCEs with survey data, see [SVY] variance estimation.

## References

De Luca, G. 2008. SNP and SML estimation of univariate and bivariate binary-choice models. Stata Journal 8: 190-220.
Greene, W. H. 2012. Econometric Analysis. 7th ed. Upper Saddle River, NJ: Prentice Hall.
Hardin, J. W. 1996. sg61: Bivariate probit models. Stata Technical Bulletin 33: 15-20. Reprinted in Stata Technical Bulletin Reprints, vol. 6, pp. 152-158. College Station, TX: Stata Press.

Heckman, J. 1979. Sample selection bias as a specification error. Econometrica 47: 153-161.
Lokshin, M., and Z. Sajaia. 2011. Impact of interventions on discrete outcomes: Maximum likelihood estimation of the binary choice models with binary endogenous regressors. Stata Journal 11: 368-385.

Pindyck, R. S., and D. L. Rubinfeld. 1998. Econometric Models and Economic Forecasts. 4th ed. New York: McGraw-Hill.
Poirier, D. J. 1980. Partial observability in bivariate probit models. Journal of Econometrics 12: 209-217.
Van de Ven, W. P. M. M., and B. M. S. Van Pragg. 1981. The demand for deductibles in private health insurance: A probit model with sample selection. Journal of Econometrics 17: 229-252.

## Also see

[R] biprobit postestimation - Postestimation tools for biprobit
$[\mathrm{R}]$ mprobit - Multinomial probit regression
[R] probit - Probit regression
[SVY] svy estimation - Estimation commands for survey data
[U] 20 Estimation and postestimation commands

## Title

biprobit postestimation - Postestimation tools for biprobit
Description Syntax for predict Menu for predict Options for predict Also see

## Description

The following postestimation commands are available after biprobit:

| Command | Description |
| :---: | :---: |
| contrast | contrasts and ANOVA-style joint tests of estimates |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample |
| estat vce | variance-covariance matrix of the estimators (VCE) |
| estat (svy) | postestimation statistics for survey data |
| estimates | cataloging estimation results |
| lincom | point estimates, standard errors, testing, and inference for linear combinations of coefficients |
| lrtest ${ }^{1}$ | likelihood-ratio test |
| margins | marginal means, predictive margins, marginal effects, and average marginal effects |
| marginsplot | graph the results from margins (profile plots, interaction plots, etc.) |
| nlcom | point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients |
| predict | predictions, residuals, influence statistics, and other diagnostic measures |
| predictnl | point estimates, standard errors, testing, and inference for generalized predictions |
| pwcompare | pairwise comparisons of estimates |
| suest | seemingly unrelated estimation |
| test | Wald tests of simple and composite linear hypotheses |
| testnl | Wald tests of nonlinear hypotheses |

[^9]
## Syntax for predict

predict [type] newvar [if][in][, statistic nooffset]
predict $[$ type $]\left\{\right.$ stub* $\mid$ newvar $_{\text {eq1 }}$ newvar $_{\text {eq2 }}$ newvar $\left._{\text {athrho }}\right\}[$ if $][$ in $]$, scores
statistic Description
Main
p11
$\Phi_{2}\left(\mathbf{x}_{j} \mathbf{b}, \mathbf{z}_{j} \mathbf{g}, \rho\right)$, predicted probability $\operatorname{Pr}\left(y_{1 j}=1, y_{2 j}=1\right)$; the default
p10
$\Phi_{2}\left(\mathbf{x}_{j} \mathbf{b},-\mathbf{z}_{j} \mathbf{g},-\rho\right)$, predicted probability $\operatorname{Pr}\left(y_{1 j}=1, y_{2 j}=0\right)$
p01
$\Phi_{2}\left(-\mathbf{x}_{j} \mathbf{b}, \mathbf{z}_{j} \mathbf{g},-\rho\right)$, predicted probability $\operatorname{Pr}\left(y_{1 j}=0, y_{2 j}=1\right)$
p00
$\Phi_{2}\left(-\mathbf{x}_{j} \mathbf{b},-\mathbf{z}_{j} \mathbf{g}, \rho\right)$, predicted probability $\operatorname{Pr}\left(y_{1 j}=0, y_{2 j}=0\right)$
pmarg1
$\Phi\left(\mathbf{x}_{j} \mathbf{b}\right)$, marginal success probability for equation 1
pmarg2
$\Phi\left(\mathbf{z}_{j} \mathbf{g}\right)$, marginal success probability for equation 2
pcond1
$\Phi_{2}\left(\mathbf{x}_{j} \mathbf{b}, \mathbf{z}_{j} \mathbf{g}, \rho\right) / \Phi\left(\mathbf{z}_{j} \mathbf{g}\right)$, conditional probability of success for equation 1
pcond2
xb1
$\Phi_{2}\left(\mathbf{x}_{j} \mathbf{b}, \mathbf{z}_{j} \mathbf{g}, \rho\right) / \Phi\left(\mathbf{x}_{j} \mathbf{b}\right)$, conditional probability of success for equation 2
xb2
$\mathbf{x}_{j} \mathbf{b}$, linear prediction for equation 1
stdp1
stdp2
$\mathbf{z}_{j} \mathbf{g}$, linear prediction for equation 2
standard error of the linear prediction for equation 1
standard error of the linear prediction for equation 2
where $\Phi()$ is the standard normal-distribution function and $\Phi_{2}()$ is the bivariate standard normal-distribution function.

These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

p11, the default, calculates the bivariate predicted probability $\operatorname{Pr}\left(y_{1 j}=1, y_{2 j}=1\right)$.
p10 calculates the bivariate predicted probability $\operatorname{Pr}\left(y_{1 j}=1, y_{2 j}=0\right)$.
p01 calculates the bivariate predicted probability $\operatorname{Pr}\left(y_{1 j}=0, y_{2 j}=1\right)$.
p00 calculates the bivariate predicted probability $\operatorname{Pr}\left(y_{1 j}=0, y_{2 j}=0\right)$.
pmarg1 calculates the univariate (marginal) predicted probability of success $\operatorname{Pr}\left(y_{1 j}=1\right)$.
pmarg2 calculates the univariate (marginal) predicted probability of success $\operatorname{Pr}\left(y_{2 j}=1\right)$.
pcond1 calculates the conditional (on success in equation 2) predicted probability of success
$\operatorname{Pr}\left(y_{1 j}=1, y_{2 j}=1\right) / \operatorname{Pr}\left(y_{2 j}=1\right)$.
pcond2 calculates the conditional (on success in equation 1) predicted probability of success
$\operatorname{Pr}\left(y_{1 j}=1, y_{2 j}=1\right) / \operatorname{Pr}\left(y_{1 j}=1\right)$.
xb 1 calculates the probit linear prediction $\mathbf{x}_{j} \mathbf{b}$.
xb 2 calculates the probit linear prediction $\mathbf{z}_{j} \mathbf{g}$.
stdp1 calculates the standard error of the linear prediction for equation 1.
stdp2 calculates the standard error of the linear prediction for equation 2.
nooffset is relevant only if you specified offset1 (varname) or offset2 (varname) for biprobit. It modifies the calculations made by predict so that they ignore the offset variables; the linear predictions are treated as $\mathbf{x}_{j} \mathbf{b}$ rather than as $\mathbf{x}_{j} \mathbf{b}+$ offset $_{1 \mathrm{j}}$ and $\mathbf{z}_{j} \gamma$ rather than as $\mathbf{z}_{j} \gamma+$ offset $_{2 j}$. scores calculates equation-level score variables.

The first new variable will contain $\partial \ln L / \partial\left(\mathbf{x}_{j} \boldsymbol{\beta}\right)$.
The second new variable will contain $\partial \ln L / \partial\left(\mathbf{z}_{j} \gamma\right)$.
The third new variable will contain $\partial \ln L / \partial(\operatorname{atanh} \rho)$.

## Also see

[R] biprobit - Bivariate probit regression
[U] 20 Estimation and postestimation commands

## Title

bitest - Binomial probability test

| Syntax | Menu | Description | Option |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

Binomial probability test
bitest varname $=\#_{p}[$ if $][$ in $][$ weight $][$, detail $]$

Immediate form of binomial probability test
bitesti $\#_{N} \#_{\text {succ }} \#_{p}[$, detail $]$
by is allowed with bitest; see [D] by.
fweights are allowed with bitest; see [U] 11.1.6 weight.

## Menu

bitest
Statistics $>$ Summaries, tables, and tests $>$ Classical tests of hypotheses $>$ Binomial probability test
bitesti
Statistics $>$ Summaries, tables, and tests $>$ Classical tests of hypotheses $>$ Binomial probability test calculator

## Description

bitest performs exact hypothesis tests for binomial random variables. The null hypothesis is that the probability of a success on a trial is $\#_{p}$. The total number of trials is the number of nonmissing values of varname (in bitest) or $\#_{N}$ (in bitesti). The number of observed successes is the number of 1 s in varname (in bitest) or $\#_{\text {succ }}$ (in bitesti). varname must contain only $0 \mathrm{~s}, 1 \mathrm{~s}$, and missing.
bitesti is the immediate form of bitest; see [U] 19 Immediate commands for a general introduction to immediate commands.

## Option

$\qquad$ Advanced
detail shows the probability of the observed number of successes, $k_{\text {obs }}$; the probability of the number of successes on the opposite tail of the distribution that is used to compute the two-sided $p$-value, $k_{\mathrm{opp}}$; and the probability of the point next to $k_{\mathrm{opp}}$. This information can be safely ignored. See the technical note below for details.

## Remarks and examples

Remarks are presented under the following headings:
bitest
bitesti

## bitest

$>$ Example 1
We test 15 university students for high levels of one measure of visual quickness which, from other evidence, we believe is present in $30 \%$ of the nonuniversity population. Included in our data is quick, taking on the values 1 ("success") or 0 ("failure") depending on the outcome of the test.

```
. use http://www.stata-press.com/data/r13/quick
. bitest quick == 0.3
\begin{tabular}{r|rcccr} 
Variable & N & Observed k & Expected k & Assumed p & Observed p \\
\hline quick & 15 & 7 & 4.5 & 0.30000 & 0.46667 \\
\(\operatorname{Pr}(\mathrm{k}>=7)\) & \(=0.131143\) & (one-sided test) & & \\
\(\operatorname{Pr}(\mathrm{k}<=7)\) & \(=0.949987\) & (one-sided test) & & \\
\(\operatorname{Pr}(\mathrm{k}<=1\) or \(\mathrm{k}>=7)\) & \(=0.166410\) & (two-sided test) & &
\end{tabular}
```

The first part of the output reveals that, assuming a true probability of success of 0.3 , the expected number of successes is 4.5 and that we observed seven. Said differently, the assumed frequency under the null hypothesis $H_{0}$ is 0.3 , and the observed frequency is 0.47 .

The first line under the table is a one-sided test; it is the probability of observing seven or more successes conditional on $p=0.3$. It is a test of $H_{0}: p=0.3$ versus the alternative hypothesis $H_{\mathrm{A}}: p>0.3$. Said in English, the alternative hypothesis is that more than $30 \%$ of university students score at high levels on this test of visual quickness. The $p$-value for this hypothesis test is 0.13 .

The second line under the table is a one-sided test of $H_{0}$ versus the opposite alternative hypothesis $H_{\mathrm{A}}: p<0.3$.

The third line is the two-sided test. It is a test of $H_{0}$ versus the alternative hypothesis $H_{\mathrm{A}}: p \neq 0.3$.

## - Technical note

The $p$-value of a hypothesis test is the probability (calculated assuming $H_{0}$ is true) of observing any outcome as extreme or more extreme than the observed outcome, with extreme meaning in the direction of the alternative hypothesis. In example 1 , the outcomes $k=8,9, \ldots, 15$ are clearly "more extreme" than the observed outcome $k_{\text {obs }}=7$ when considering the alternative hypothesis $H_{\mathrm{A}}: p \neq 0.3$. However, outcomes with only a few successes are also in the direction of this alternative hypothesis. For two-sided hypotheses, outcomes with $k$ successes are considered "as extreme or more extreme" than the observed outcome $k_{\text {obs }}$ if $\operatorname{Pr}(k) \leq \operatorname{Pr}\left(k_{\text {obs }}\right)$. Here $\operatorname{Pr}(k=0)$ and $\operatorname{Pr}(k=1)$ are both less than $\operatorname{Pr}(k=7)$, so they are included in the two-sided $p$-value.

The detail option allows you to see the probability (assuming that $H_{0}$ is true) of the observed successes $(k=7)$ and the probability of the boundary point $(k=1)$ of the opposite tail used for the two-sided $p$-value.


Also shown is the probability of the point next to the boundary point. This probability, namely, $\operatorname{Pr}(k=2)=0.092$, is certainly close to the probability of the observed outcome $\operatorname{Pr}(k=7)=0.081$, so some people might argue that $k=2$ should be included in the two-sided $p$-value. Statisticians (at least some we know) would reply that the $p$-value is a precisely defined concept and that this is an arbitrary "fuzzification" of its definition. When you compute exact $p$-values according to the precise definition of a $p$-value, your type I error is never more than what you say it is -so no one can criticize you for being anticonservative. Including the point $k=2$ is being overly conservative because it makes the $p$-value larger yet. But it is your choice; being overly conservative, at least in statistics, is always safe. Know that bitest and bitesti always keep to the precise definition of a $p$-value, so if you wish to include this extra point, you must do so by hand or by using the $r$ () stored results; see Stored results below.

## bitesti

$>$ Example 2
The binomial test is a function of two statistics and one parameter: $N$, the number of observations; $k_{\text {obs }}$, the number of observed successes; and $p$, the assumed probability of a success on a trial. For instance, in a city of $N=2,500,000$, we observe $k_{\text {obs }}=36$ cases of a particular disease when the population rate for the disease is $p=0.00001$.
. bitesti 250000036 . 00001

| N | Observed k | Expected k | Assumed p | Observed p |
| :--- | :---: | :---: | :---: | :---: |
| 2500000 | 36 | 25 | 0.00001 | 0.00001 |
| $\operatorname{Pr}(\mathrm{k}>=36)$ | $=0.022458$ | (one-sided test) |  |  |
| $\operatorname{Pr}(\mathrm{k}<=36)$ | $=0.985448$ | (one-sided test) |  |  |
| $\operatorname{Pr}(\mathrm{k}\langle=14$ or $\mathrm{k}>=36)$ | $=0.034859$ | (two-sided test) |  |  |

## > Example 3

Boice and Monson (1977) present data on breast cancer cases and person-years of observations for women with tuberculosis who were repeatedly exposed to multiple x-ray fluoroscopies and for women with tuberculosis who were not. The data are

|  | Exposed | Not exposed | Total |
| :--- | ---: | ---: | ---: |
| Breast cancer | 41 | 15 | 56 |
| Person-years | 28,010 | 19,017 | 47,027 |

We can thus test whether x-ray fluoroscopic examinations are associated with breast cancer; the assumed rate of exposure is $p=28010 / 47027$.
. bitesti 5641 28010/47027

| N | Observed k | Expected k | Assumed p | Observed p |
| :---: | :---: | :---: | :---: | :---: |
| 56 | 41 | 33.35446 | 0.59562 | 0.73214 |
| $\operatorname{Pr}(\mathrm{k}>=41)$ | $=0.023830$ | (one-sided test) |  |  |
| $\operatorname{Pr}(\mathrm{k}<=41)$ | $=0.988373$ | (one-sided test) |  |  |
| $\operatorname{Pr}(\mathrm{k}<=25$ or $\mathrm{k}>=41)$ | $=0.040852$ | (two-sided test) |  |  |

## Stored results

bitest and bitesti store the following in $r()$ :
Scalars

| r (N) | number $N$ of trials | r(k_opp) | opposite extreme $k$ |
| :---: | :---: | :---: | :---: |
| r (P_p) | assumed probability $p$ of success | r(P_k) | probability of observed $k$ (detail only) |
| $\mathrm{r}(\mathrm{k})$ | observed number $k$ of successes | r(P_oppk) | probability of opposite extreme $k$ (detail only) |
| $r\left(p \_1\right)$ | lower one-sided $p$-value | r(k_nopp) | $k$ next to opposite extreme (detail only) |
| $r(p-u)$ | upper one-sided $p$-value | $r$ (P_noppk) | probability of $k$ next to opposite extreme |
| $r$ (p) | two-sided $p$-value |  | (detail only) |

## Methods and formulas

Let $N, k_{\text {obs }}$, and $p$ be, respectively, the number of observations, the observed number of successes, and the assumed probability of success on a trial. The expected number of successes is $N p$, and the observed probability of success on a trial is $k_{\text {obs }} / N$.
bitest and bitesti compute exact $p$-values based on the binomial distribution. The upper one-sided $p$-value is

$$
\operatorname{Pr}\left(k \geq k_{\mathrm{obs}}\right)=\sum_{m=k_{\mathrm{obs}}}^{N}\binom{N}{m} p^{m}(1-p)^{N-m}
$$

The lower one-sided $p$-value is

$$
\operatorname{Pr}\left(k \leq k_{\mathrm{obs}}\right)=\sum_{m=0}^{k_{\mathrm{obs}}}\binom{N}{m} p^{m}(1-p)^{N-m}
$$

If $k_{\text {obs }} \geq N p$, the two-sided $p$-value is

$$
\operatorname{Pr}\left(k \leq k_{\text {opp }} \text { or } k \geq k_{\text {obs }}\right)
$$

where $k_{\text {opp }}$ is the largest number $\leq N p$ such that $\operatorname{Pr}\left(k=k_{\text {opp }}\right) \leq \operatorname{Pr}\left(k=k_{\text {obs }}\right)$. If $k_{\text {obs }}<N p$, the two-sided $p$-value is

$$
\operatorname{Pr}\left(k \leq k_{\mathrm{obs}} \text { or } k \geq k_{\mathrm{opp}}\right)
$$

where $k_{\text {opp }}$ is the smallest number $\geq N p$ such that $\operatorname{Pr}\left(k=k_{\text {opp }}\right) \leq \operatorname{Pr}\left(k=k_{\text {obs }}\right)$.

## References

Boice, J. D., Jr., and R. R. Monson. 1977. Breast cancer in women after repeated fluoroscopic examinations of the chest. Journal of the National Cancer Institute 59: 823-832.
Hoel, P. G. 1984. Introduction to Mathematical Statistics. 5th ed. New York: Wiley.

## Also see

$[\mathrm{R}]$ ci - Confidence intervals for means, proportions, and counts
[R] prtest - Tests of proportions

## Title

bootstrap - Bootstrap sampling and estimation

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

bootstrap exp_list $[$, options eform_option $]$ : command

## options

Description
Main
reps (\#) perform \# bootstrap replications; default is reps(50)
Options
strata(varlist) variables identifying strata
size(\#) draw samples of size \#; default is $\_$N
cluster (varlist) variables identifying resampling clusters
idcluster(newvar)
saving (filename, ...)
bca
ties
mse
Reporting
level(\#)
notable
noheader
nolegend
verbose
nodots
noisily
trace
title(text)
display_options
eform_option
Advanced
nodrop do not drop observations
nowarn
force
create new cluster ID variable
save results to filename; save statistics in double precision;
save results to filename every \# replications
compute acceleration for $\mathrm{BC}_{a}$ confidence intervals
adjust $\mathrm{BC} / \mathrm{BCa}$ confidence intervals for ties
use MSE formula for variance estimation
set confidence level; default is level(95)
suppress table of results
suppress table header
suppress table legend
display the full table legend
suppress replication dots
display any output from command
trace command
use text as title for bootstrap results
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling display coefficient table in exponentiated form
do not warn when e(sample) is not set
do not check for weights or svy commands; seldom used

```
reject(exp) identify invalid results
seed(#)
```

group (varname)
jackknifeopts(jkopts)
coeflegend
identify invalid results set random-number seed to \#

ID variable for groups within cluster () options for jackknife; see [R] jackknife display legend instead of statistics
weights are not allowed in command.
group(), jackknifeopts(), and coeflegend do not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

| exp_list contains | (name: elist) |
| :---: | :---: |
|  | elist |
|  | eexp |
| elist contains | newvar $=(\exp )$ |
|  | (exp) |
| eexp is | specname |
|  | [eqno]specname |
| specname is | _b |
|  | _b [] |
|  | -se |
|  | _se [] |
| eqno is | \# \# |
|  | name |

$\exp$ is a standard Stata expression; see [U] 13 Functions and expressions.
Distinguish between [], which are to be typed, and [], which indicate optional arguments.

## Menu

Statistics $>$ Resampling $>$ Bootstrap estimation

## Description

bootstrap performs bootstrap estimation. Typing

```
. bootstrap exp_list, reps(#): command
```

executes command multiple times, bootstrapping the statistics in exp_list by resampling observations (with replacement) from the data in memory \# times. This method is commonly referred to as the nonparametric bootstrap.
command defines the statistical command to be executed. Most Stata commands and user-written programs can be used with bootstrap, as long as they follow standard Stata syntax; see [U] 11 Language syntax. If the bca option is supplied, command must also work with jackknife; see $[\mathrm{R}]$ jackknife. The by prefix may not be part of command.
exp_list specifies the statistics to be collected from the execution of command. If command changes the contents in $\mathrm{e}(\mathrm{b})$, exp_list is optional and defaults to _b.

Because bootstrapping is a random process, if you want to be able to reproduce results, set the random-number seed by specifying the seed(\#) option or by typing

```
. set seed #
```

where \# is a seed of your choosing, before running bootstrap; see [R] set seed.
Many estimation commands allow the vce(bootstrap) option. For those commands, we recommend using vce (bootstrap) over bootstrap because the estimation command already handles clustering and other model-specific details for you. The bootstrap prefix command is intended for use with nonestimation commands, such as summarize, user-written commands, or functions of coefficients.
bs and bstrap are synonyms for bootstrap.

## Options

Main
reps (\#) specifies the number of bootstrap replications to be performed. The default is 50 . A total of $50-200$ replications are generally adequate for estimates of standard error and thus are adequate for normal-approximation confidence intervals; see Mooney and Duval (1993, 11). Estimates of confidence intervals using the percentile or bias-corrected methods typically require 1,000 or more replications.

## Options

strata (varlist) specifies the variables that identify strata. If this option is specified, bootstrap samples are taken independently within each stratum.
size(\#) specifies the size of the samples to be drawn. The default is _N, meaning to draw samples of the same size as the data. If specified, \# must be less than or equal to the number of observations within strata().

If cluster() is specified, the default size is the number of clusters in the original dataset. For unbalanced clusters, resulting sample sizes will differ from replication to replication. For cluster sampling, \# must be less than or equal to the number of clusters within strata().
cluster (varlist) specifies the variables that identify resampling clusters. If this option is specified, the sample drawn during each replication is a bootstrap sample of clusters.
idcluster (newvar) creates a new variable containing a unique identifier for each resampled cluster. This option requires that cluster () also be specified.
saving (filename [, suboptions]) creates a Stata data file (.dta file) consisting of (for each statistic in exp_list) a variable containing the replicates.
double specifies that the results for each replication be saved as doubles, meaning 8-byte reals. By default, they are saved as floats, meaning 4-byte reals. This option may be used without the saving () option to compute the variance estimates by using double precision.
every (\#) specifies that results be written to disk every \#th replication. every () should be specified only in conjunction with saving () when command takes a long time for each replication. This option will allow recovery of partial results should some other software crash your computer. See [P] postfile.
replace specifies that filename be overwritten if it exists. This option does not appear in the dialog box.
bca specifies that bootstrap estimate the acceleration of each statistic in exp_list. This estimate is used to construct $\mathrm{BC}_{a}$ confidence intervals. Type estat bootstrap, bca to display the $\mathrm{BC}_{a}$ confidence interval generated by the bootstrap command.
ties specifies that bootstrap adjust for ties in the replicate values when computing the median bias used to construct BC and BCa confidence intervals.
mse specifies that bootstrap compute the variance by using deviations of the replicates from the observed value of the statistics based on the entire dataset. By default, bootstrap computes the variance by using deviations from the average of the replicates.

## Reporting

level (\#); see [R] estimation options.
notable suppresses the display of the table of results.
noheader suppresses the display of the table header. This option implies nolegend. This option may also be specified when replaying estimation results.
nolegend suppresses the display of the table legend. This option may also be specified when replaying estimation results.
verbose specifies that the full table legend be displayed. By default, coefficients and standard errors are not displayed. This option may also be specified when replaying estimation results.
nodots suppresses display of the replication dots. By default, one dot character is displayed for each successful replication. A red ' $x$ ' is displayed if command returns an error or if one of the values in exp_list is missing.
noisily specifies that any output from command be displayed. This option implies the nodots option.
trace causes a trace of the execution of command to be displayed. This option implies the noisily option.
title (text) specifies a title to be displayed above the table of bootstrap results. The default title is the title stored in e(title) by an estimation command, or if e (title) is not filled in, Bootstrap results is used. title() may also be specified when replaying estimation results.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\%fmt), pformat (\% fmt), sformat (\%fmt), and nolstretch; see [R] estimation options.
eform_option causes the coefficient table to be displayed in exponentiated form; see [R] eform_option. command determines which of the following are allowed (eform(string) and eform are always allowed):
eform_option Description

| eform (string) | use string for the column title |
| :--- | :--- |
| eform | exponentiated coefficient, string is $\exp (b)$ <br> hr |
| har | subhazard ratio, string is Haz. Ratio |
| shr | incing is SHR |
| $\underline{\mathrm{irr}}$ | odds ratio, string is Odds Ratio |
| or | relative-risk ratio, string is RRR |

nodrop prevents observations outside e(sample) and the if and in qualifiers from being dropped before the data are resampled.
nowarn suppresses the display of a warning message when command does not set e(sample).
force suppresses the restriction that command not specify weights or be a svy command. This is a rarely used option. Use it only if you know what you are doing.
reject (exp) identifies an expression that indicates when results should be rejected. When exp is true, the resulting values are reset to missing values.
seed(\#) sets the random-number seed. Specifying this option is equivalent to typing the following command prior to calling bootstrap:
. set seed \#
The following options are available with bootstrap but are not shown in the dialog box:
group (varname) re-creates varname containing a unique identifier for each group across the resampled clusters. This option requires that idcluster() also be specified.

This option is useful for maintaining unique group identifiers when sampling clusters with replacement. Suppose that cluster 1 contains 3 groups. If the idcluster (newclid) option is specified and cluster 1 is sampled multiple times, newclid uniquely identifies each copy of cluster 1 . If group (newgroupid) is also specified, newgroupid uniquely identifies each copy of each group.
jackknifeopts(jkopts) identifies options that are to be passed to jackknife when it computes the acceleration values for the $\mathrm{BC}_{a}$ confidence intervals; see [R] jackknife. This option requires the bca option and is mostly used for passing the eclass, rclass, or $n(\#)$ option to jackknife.
coeflegend; see $[R]$ estimation options.

## Remarks and examples

Remarks are presented under the following headings:

```
Introduction
Regression coefficients
Expressions
Combining bootstrap datasets
A note about macros
Achieved significance level
Bootstrapping a ratio
Warning messages and e(sample)
Bootstrapping statistics from data with a complex structure
```


## Introduction

With few assumptions, bootstrapping provides a way of estimating standard errors and other measures of statistical precision (Efron 1979; Efron and Stein 1981; Efron 1982; Efron and Tibshirani 1986; Efron and Tibshirani 1993; also see Davison and Hinkley [1997]; Guan [2003]; Mooney and Duval [1993]; Poi [2004]; and Stine [1990]). It provides a way to obtain such measures when no formula is otherwise available or when available formulas make inappropriate assumptions. Cameron and Trivedi (2010, chap. 13) discuss many bootstrapping topics and demonstrate how to do them in Stata.

To illustrate bootstrapping, suppose that you have a dataset containing $N$ observations and an estimator that, when applied to the data, produces certain statistics. You draw, with replacement, $N$ observations from the $N$-observation dataset. In this random drawing, some of the original observations will appear once, some more than once, and some not at all. Using the resampled dataset, you apply the estimator and collect the statistics. This process is repeated many times; each time, a new random sample is drawn and the statistics are recalculated.

This process builds a dataset of replicated statistics. From these data, you can calculate the standard error by using the standard formula for the sample standard deviation

$$
\widehat{\mathrm{se}}=\left\{\frac{1}{k-1} \sum\left(\widehat{\theta}_{i}-\bar{\theta}\right)^{2}\right\}^{1 / 2}
$$

where $\widehat{\theta}_{i}$ is the statistic calculated using the $i$ th bootstrap sample and $k$ is the number of replications. This formula gives an estimate of the standard error of the statistic, according to Hall and Wilson (1991). Although the average, $\bar{\theta}$, of the bootstrapped estimates is used in calculating the standard deviation, it is not used as the estimated value of the statistic itself. Instead, the original observed value of the statistic, $\widehat{\theta}$, is used, meaning the value of the statistic computed using the original $N$ observations.

You might think that $\bar{\theta}$ is a better estimate of the parameter than $\widehat{\theta}$, but it is not. If the statistic is biased, bootstrapping exaggerates the bias. In fact, the bias can be estimated as $\bar{\theta}-\hat{\theta}$ (Efron 1982, 33). Knowing this, you might be tempted to subtract this estimate of bias from $\widehat{\theta}$ to produce an unbiased statistic. The bootstrap bias estimate has an indeterminate amount of random error, so this unbiased estimator may have greater mean squared error than the biased estimator (Mooney and Duval 1993; Hinkley 1978). Thus $\widehat{\theta}$ is the best point estimate of the statistic.

The logic behind the bootstrap is that all measures of precision come from a statistic's sampling distribution. When the statistic is estimated on a sample of size $N$ from some population, the sampling distribution tells you the relative frequencies of the values of the statistic. The sampling distribution, in turn, is determined by the distribution of the population and the formula used to estimate the statistic.

Sometimes the sampling distribution can be derived analytically. For instance, if the underlying population is distributed normally and you calculate means, the sampling distribution for the mean is also normal but has a smaller variance than that of the population. In other cases, deriving the sampling distribution is difficult, as when means are calculated from nonnormal populations. Sometimes, as in the case of means, it is not too difficult to derive the sampling distribution as the sample size goes to infinity $(N \rightarrow \infty)$. However, such asymptotic distributions may not perform well when applied to finite samples.

If you knew the population distribution, you could obtain the sampling distribution by simulation: you could draw random samples of size $N$, calculate the statistic, and make a tally. Bootstrapping does precisely this, but it uses the observed distribution of the sample in place of the true population distribution. Thus the bootstrap procedure hinges on the assumption that the observed distribution is a good estimate of the underlying population distribution. In return, the bootstrap produces an estimate, called the bootstrap distribution, of the sampling distribution. From this, you can estimate the standard error of the statistic, produce confidence intervals, etc.

The accuracy with which the bootstrap distribution estimates the sampling distribution depends on the number of observations in the original sample and the number of replications in the bootstrap. A crudely estimated sampling distribution is adequate if you are only going to extract, say, a standard error. A better estimate is needed if you want to use the 2.5 th and 97.5 th percentiles of the distribution to produce a $95 \%$ confidence interval. To extract many features simultaneously about the distribution,
an even better estimate is needed. Generally, replications on the order of 1,000 produce very good estimates, but only $50-200$ replications are needed for estimates of standard errors. See Poi (2004) for a method to choose the number of bootstrap replications.

## Regression coefficients

## Example 1

Let's say that we wish to compute bootstrap estimates for the standard errors of the coefficients from the following regression:

```
. use http://www.stata-press.com/data/r13/auto
```

(1978 Automobile Data)
. regress mpg weight gear foreign

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | 1629.67805 | 3 | 543.226016 |
| Residual | 813.781411 | 70 | 11.6254487 |
| Total | 2443.45946 | 73 | 33.4720474 |


| Number of obs | $=$ | 74 |
| :--- | ---: | ---: |
| F 3, | $70)$ | $=46.73$ |
| Prob $>$ F | $=$ | 0.0000 |
| R-squared | $=$ | 0.6670 |
| Adj R-squared | $=$ | 0.6527 |
| Root MSE | $=3.4096$ |  |


| mpg | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| weight | -.006139 | .0007949 | -7.72 | 0.000 | -.0077245 | -.0045536 |
| gear_ratio | 1.457113 | 1.541286 | 0.95 | 0.348 | -1.616884 | 4.53111 |
| foreign | -2.221682 | 1.234961 | -1.80 | 0.076 | -4.684735 | .2413715 |
| _cons | 36.10135 | 6.285984 | 5.74 | 0.000 | 23.56435 | 48.63835 |

To run the bootstrap, we simply prefix the above regression command with the bootstrap command (specifying its options before the colon separator). We must set the random-number seed before calling bootstrap.

|  |  |  |  |
| :---: | :---: | :---: | :---: |
|  | 50 |  |  |
|  | . . 100 |  |  |
| Linear regression | Number of obs | $=$ | 74 |
|  | Replications | = | 100 |
|  | Wald chi2(3) | = | 111.96 |
|  | Prob > chi2 | = | 0.0000 |
|  | R-squared | = | 0.6670 |
|  | Adj R-squared | = | 0.6527 |
|  | Root MSE | $=$ | 3.4096 |


|  | Observed <br> Coef. | Bootstrap <br> Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ |  | Normal-based <br> [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | :---: |
| weight | -.006139 | .0006498 | -9.45 | 0.000 | -.0074127 | -.0048654 |  |
| gear_ratio | 1.457113 | 1.297786 | 1.12 | 0.262 | -1.086501 | 4.000727 |  |
| foreign | -2.221682 | 1.162728 | -1.91 | 0.056 | -4.500587 | .0572236 |  |
| _cons | 36.10135 | 4.71779 | 7.65 | 0.000 | 26.85465 | 45.34805 |  |

The displayed confidence interval is based on the assumption that the sampling (and hence bootstrap) distribution is approximately normal (see Methods and formulas below). Because this confidence interval is based on the standard error, it is a reasonable estimate if normality is approximately true, even for a few replications. Other types of confidence intervals are available after bootstrap; see $[\mathrm{R}]$ bootstrap postestimation.

We could instead supply names to our expressions when we run bootstrap. For example,

```
. bootstrap diff=(_b[weight]-_b[gear]): regress mpg weight gear foreign
```

would bootstrap a statistic, named diff, equal to the difference between the coefficients on weight and gear_ratio.

## Expressions

$>$ Example 2
When we use bootstrap, the list of statistics can contain complex expressions, as long as each expression is enclosed in parentheses. For example, to bootstrap the range of a variable $x$, we could type

```
. bootstrap range=(r(max) -r(min)), reps(1000): summarize x
```

Of course, we could also bootstrap the minimum and maximum and later compute the range.

```
. bootstrap max=r(max) min=r(min), reps(1000) saving(mybs): summarize x
. use mybs, clear
(bootstrap: summarize)
. generate range = max - min
. bstat range, stat(19.5637501)
```

The difference between the maximum and minimum of x in the sample is 19.5637501 .
The stat () option to bstat specifies the observed value of the statistic (range) to be summarized. This option is useful when, as shown above, the statistic of ultimate interest is not specified directly to bootstrap but instead is calculated by other means.

Here the observed values of $r(\max )$ and $r(\min )$ are stored as characteristics of the dataset created by bootstrap and are thus available for retrieval by bstat; see [R] bstat. The observed range, however, is unknown to bstat, so it must be specified.

## Combining bootstrap datasets

You can combine two datasets from separate runs of bootstrap by using append (see [D] append) and then get the bootstrap statistics for the combined datasets by running bstat. The runs must have been performed independently (having different starting random-number seeds), and the original dataset, command, and bootstrap statistics must have been all the same.

## A note about macros

In example 2, we executed the command

```
. bootstrap max=r(max) min=r(min), reps(1000) saving(mybs): summarize x
```

We did not enclose $r(\max )$ and $r(\min )$ in single quotes, as we would in most other contexts, because it would not produce what was intended:

```
. bootstrap 'r(max)' 'r(min)', reps(1000) saving(mybs): summarize x
```

To understand why, note that ' $r(\max )$ ', like any reference to a local macro, will evaluate to a literal string containing the contents of $r$ (max) before bootstrap is even executed. Typing the command above would appear to Stata as if we had typed

```
bootstrap 14.5441234 33.4393293, reps(1000) saving(mybs): summarize x
```

Even worse, the current contents of $r$ (min) and $r$ (max) could be empty, producing an even more confusing result. To avoid this outcome, refer to statistics by name (for example, $r(\max )$ ) and not by value (for example, ' $r(\max )$ ').

## Achieved significance level

$>$ Example 3
Suppose that we wish to estimate the achieved significance level (ASL) of a test statistic by using the bootstrap. ASL is another name for $p$-value. An example is

$$
\mathrm{ASL}=\operatorname{Pr}\left(\widehat{\theta}^{*} \geq \widehat{\theta} \mid H_{0}\right)
$$

for an upper-tailed, alternative hypothesis, where $H_{0}$ denotes the null hypothesis, $\widehat{\theta}$ is the observed value of the test statistic, and $\widehat{\theta}^{*}$ is the random variable corresponding to the test statistic, assuming that $H_{0}$ is true.

Here we will compare the mean miles per gallon (mpg) between foreign and domestic cars by using the two-sample $t$ test with unequal variances. The following results indicate the $p$-value to be 0.0034 for the two-sided test using Satterthwaite's approximation. Thus assuming that mean mpg is the same for foreign and domestic cars, we would expect to observe a $t$ statistic more extreme (in absolute value) than 3.1797 in about $0.3 \%$ of all possible samples of the type that we observed. Thus we have evidence to reject the null hypothesis that the means are equal.

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. ttest mpg, by(foreign) unequal
Two-sample \(t\) test with unequal variances
```

| Group | Obs | Mean | Std. Err. | Std. Dev. | [95\% Conf | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Domestic | 52 | 19.82692 | . 657777 | 4.743297 | 18.50638 | 21.14747 |
| Foreign | 22 | 24.77273 | 1.40951 | 6.611187 | 21.84149 | 27.70396 |
| combined | 74 | 21.2973 | . 6725511 | 5.785503 | 19.9569 | 22.63769 |
| diff |  | -4.945804 | 1.555438 |  | -8.120053 | -1.771556 |
| diff $=$ mean(Domestic) mean(Foreign) $t=-3.1797$ <br> Ho: diff $=0$ Satterthwaite's degrees of freedom $=30.5463$  <br> Ha: diff < 0 Ha: diff $!=0$ Ha: diff $>0$ <br> $\operatorname{Pr}(T<t)=0.0017$ $\operatorname{Pr}(\|T\|>\|t\|)=0.0034$ $\operatorname{Pr}(T>t)=0.9983$ |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |

We also place the value of the test statistic in a scalar for later use.

```
. scalar tobs = r(t)
```

Efron and Tibshirani (1993, 224) describe an alternative to Satterthwaite's approximation that estimates the ASL by bootstrapping the statistic from the test of equal means. Their idea is to recenter the two samples to the combined sample mean so that the data now conform to the null hypothesis but that the variances within the samples remain unchanged.

```
. summarize mpg, meanonly
. scalar omean = r(mean)
. summarize mpg if foreign==0, meanonly
. replace mpg = mpg - r(mean) + scalar(omean) if foreign==0
mpg was int now float
(52 real changes made)
. summarize mpg if foreign==1, meanonly
. replace mpg = mpg - r(mean) + scalar(omean) if foreign==1
(22 real changes made)
. sort foreign
. by foreign: summarize mpg
```

| -> foreign = Domestic |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Variable | Obs | Mean | Std. Dev. | Min | Max |
| mpg | 52 | 21.2973 | 4.743297 | 13.47037 | 35.47038 |

-> foreign = Foreign

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| mpg | 22 | 21.2973 | 6.611187 | 10.52457 | 37.52457 |

Each sample (foreign and domestic) is a stratum, so the bootstrapped samples must have the same number of foreign and domestic cars as the original dataset. This requirement is facilitated by the strata() option to bootstrap. By typing the following, we bootstrap the test statistic using the modified dataset and save the values in bsauto2.dta:

```
. keep mpg foreign
. set seed 1
. bootstrap t=r(t), rep(1000) strata(foreign) saving(bsauto2) nodots: ttest mpg,
> by(foreign) unequal
Warning: Because ttest is not an estimation command or does not set
    e(sample), bootstrap has no way to determine which observations are
    used in calculating the statistics and so assumes that all
    observations are used. This means that no observations will be
    excluded from the resampling because of missing values or other
    reasons.
    If the assumption is not true, press Break, save the data, and drop
    the observations that are to be excluded. Be sure that the dataset
    in memory contains only the relevant data.
Bootstrap results
Number of strata = 2 }\quad\begin{array}{llr}{\mathrm{ Number of obs }}&{=}&{74}\\{\mathrm{ Replications }}&{=}&{1000}
    command: ttest mpg, by(foreign) unequal
        t: r(t)
\begin{tabular}{r|rlccrr}
\hline & \begin{tabular}{r} 
Observed \\
Coef.
\end{tabular} & \begin{tabular}{l} 
Bootstrap \\
Std. Err.
\end{tabular} & z & \(\mathrm{P}>|\mathrm{z}|\) & \multicolumn{2}{c}{\begin{tabular}{c} 
Normal-based \\
[95\% Conf. Interval]
\end{tabular}} \\
\hline t & \(1.75 \mathrm{e}-07\) & 1.036437 & 0.00 & 1.000 & -2.031379 & 2.031379 \\
\hline
\end{tabular}
```

We can use the data in bsauto2.dta to estimate ASL via the fraction of bootstrap test statistics that are more extreme than 3.1797.

```
. use bsauto2, clear
(bootstrap: ttest)
. generate indicator = abs(t)>=abs(scalar(tobs))
. summarize indicator, meanonly
. display "ASLboot = " r(mean)
ASLboot = . }00
```

The result is $\mathrm{ASL}_{\text {boot }}=0.005$. Assuming that the mean mpg is the same between foreign and domestic cars, we would expect to observe a $t$ statistic more extreme (in absolute value) than 3.1797 in about $0.5 \%$ of all possible samples of the type we observed. This finding is still strong evidence to reject the hypothesis that the means are equal.

## Bootstrapping a ratio

## Example 4

Suppose that we wish to produce a bootstrap estimate of the ratio of two means. Because summarize stores results for only one variable, we must call summarize twice to compute the means. Actually, we could use collapse to compute the means in one call, but calling summarize twice is much faster. Thus we will have to write a small program that will return the results we want.

We write the program below and save it to a file called ratio. ado (see [U] $\mathbf{1 7}$ Ado-files). Our program takes two variable names as input and saves them in the local macros y (first variable) and $x$ (second variable). It then computes one statistic: the mean of ' $y$ ' divided by the mean of ' $x$ '. This value is returned as a scalar in $r$ (ratio). ratio also returns the ratio of the number of observations used to the mean for each variable.

```
program myratio, rclass
    version 13
    args y x
    confirm var 'y'
    confirm var 'x'
    tempname ymean yn
    summarize 'y', meanonly
    scalar 'ymean' = r(mean)
    return scalar n_'y' = r(N)
    summarize 'x', meanonly
    return scalar n_'x' = r(N)
    return scalar ratio = 'ymean'/r(mean)
end
```

Remember to test any newly written commands before using them with bootstrap.

```
. use http://www.stata-press.com/data/r13/auto, clear
(1978 Automobile Data)
. summarize price
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline price & 74 & 6165.257 & 2949.496 & 3291 & 15906
\end{tabular}
. scalar mean1=r(mean)
. summarize weight
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline weight & 74 & 3019.459 & 777.1936 & 1760 & 4840
\end{tabular}
. scalar mean2=r(mean)
. di scalar(mean1)/scalar(mean2)
2.0418412
. myratio price weight
. return list
scalars:
```

$$
\begin{aligned}
\mathrm{r}(\text { ratio }) & =2.041841210168278 \\
\mathrm{r}(\text { n_weight }) & =74 \\
\mathrm{r}(\text { n_price }) & =74
\end{aligned}
$$

The results of running bootstrap on our program are


As mentioned previously, we should specify the saving() option if we wish to save the bootstrap dataset.

## Warning messages and e(sample)

bootstrap is not meant to be used with weighted calculations. bootstrap determines the presence of weights by parsing the prefixed command with standard syntax. However, commands like stcox and streg require that weights be specified in stset, and some user commands may allow weights to be specified by using an option instead of the standard syntax. Both cases pose a problem for bootstrap because it cannot determine the presence of weights under these circumstances. In these cases, we can only assume that you know what you are doing.
bootstrap does not know which variables of the dataset in memory matter to the calculation at hand. You can speed their execution by dropping unnecessary variables because, otherwise, they are included in each bootstrap sample.

You should thus drop observations with missing values. Leaving in missing values causes no problem in one sense because all Stata commands deal with missing values gracefully. It does, however, cause a statistical problem. Bootstrap sampling is defined as drawing, with replacement, samples of size $N$ from a set of $N$ observations. bootstrap determines $N$ by counting the number of observations in memory, not counting the number of nonmissing values on the relevant variables. The result is that too many observations are resampled; the resulting bootstrap samples, because they are drawn from a population with missing values, are of unequal sizes.

If the number of missing values relative to the sample size is small, this will make little difference. If you have many missing values, however, you should first drop the observations that contain them.

## > Example 5

To illustrate, we use the previous example but replace some of the values of price with missing values. The number of values of price used to compute the mean for each bootstrap is not constant. This is the purpose of the Warning message.
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. replace price $=$. if inlist(_n,1,3,5,7)
(4 real changes made, 4 to missing)
. set seed 1

> myratio price weight
Warning: Because myratio is not an estimation command or does not set
e(sample), bootstrap has no way to determine which observations are
used in calculating the statistics and so assumes that all
observations are used. This means that no observations will be
excluded from the resampling because of missing values or other
reasons.
If the assumption is not true, press Break, save the data, and drop
the observations that are to be excluded. Be sure that the dataset
in memory contains only the relevant data.
Bootstrap results $\quad$ Number of obs $\quad=\quad 74$
Replications $=100$
command: myratio price weight
ratio: r(ratio)
np: r(n_price)
nw: r(n_weight)

|  | Observed Coef. | Bootstrap <br> Std. Err. | z | $P>\|z\|$ | Normal-based |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ratio | 2.063051 | . 0893669 | 23.09 | 0.000 | 1.887896 | 2.238207 |
| np | 70 | 1.872178 | 37.39 | 0.000 | 66.3306 | 73.6694 |
| nw | 74 | . | . | - | . |  |

## Bootstrapping statistics from data with a complex structure

Here we describe how to bootstrap statistics from data with a complex structure, for example, longitudinal or panel data, or matched data. bootstrap, however, is not designed to work with complex survey data. It is important to include all necessary information about the structure of the data in the bootstrap syntax to obtain correct bootstrap estimates for standard errors and confidence intervals.
bootstrap offers several options identifying the specifics of the data. These options are strata(), cluster(), idcluster(), and group(). The usage of strata() was described in example 3 above. Below we demonstrate several examples that require specifying the other three options.

## Example 6

Suppose that the auto data in example 1 above are clustered by rep78. We want to obtain bootstrap estimates for the standard errors of the difference between the coefficients on weight and gear_ratio, taking into account clustering.

We supply the cluster (rep78) option to bootstrap to request resampling from clusters rather than from observations in the dataset.

```
. use http://www.stata-press.com/data/r13/auto, clear
(1978 Automobile Data)
. keep if rep78<.
(5 observations deleted)
. bootstrap diff=(_b[weight]-_b[gear]), seed(1) cluster(rep78): regress mpg
> weight gear foreign
(running regress on estimation sample)
Bootstrap replications (50)
| 1 + 2- % 3- \ 4 % 5
\begin{tabular}{llll} 
Linear regression & Number of obs & \(=\) & 69 \\
& Replications & \(=\) & 50
\end{tabular}
    command: regress mpg weight gear foreign
        diff: _b[weight]-_b[gear]
                            (Replications based on 5 clusters in rep78)
\begin{tabular}{r|rlccrr}
\hline & \begin{tabular}{r} 
Observed \\
Coef.
\end{tabular} & \begin{tabular}{l} 
Bootstrap \\
Std. Err.
\end{tabular} & z & P>|z| & \multicolumn{2}{c}{\begin{tabular}{r} 
Normal-based \\
[95\% Conf. Interval]
\end{tabular}} \\
\hline diff & -1.910396 & 1.876778 & -1.02 & 0.309 & -5.588812 & 1.768021 \\
\hline
\end{tabular}
```

We drop missing values in rep78 before issuing the command because bootstrap does not allow missing values in cluster(). See the section above about using bootstrap when variables contain missing values.

We can also obtain these same results by using the following syntax:

```
. bootstrap diff=(_b[weight]-_b[gear]), seed(1): regress mpg weight gear foreign,
> vce(cluster rep78)
```

When only clustered information is provided to the command, bootstrap can pick up the vce(cluster clustvar) option from the main command and use it to resample from clusters.

## > Example 7

Suppose now that we have matched data and want to use bootstrap to obtain estimates of the standard errors of the exponentiated difference between two coefficients (or, equivalently, the ratio of two odds ratios) estimated by clogit. Consider the example of matched case-control data on birthweight of infants described in example 2 of $[R]$ clogit.

The infants are paired by being matched on mother's age. All groups, defined by the pairid variable, have $1: 1$ matching. clogit requires that the matching information, pairid, be supplied to the group() (or, equivalently, strata()) option to be used in computing the parameter estimates. Because the data are matched, we need to resample from groups rather than from the whole dataset. However, simply supplying the grouping variable pairid in cluster() is not enough with bootstrap, as it is with clustered data.


For the syntax above, imagine that the first pair was sampled twice during a replication. Then the bootstrap sample has four subjects with pairid equal to one, which clearly violates the original 1:1 matching design. As a result, the estimates of the coefficients obtained from this bootstrap sample will be incorrect.

Therefore, in addition to resampling from groups, we need to ensure that resampled groups are uniquely identified in each of the bootstrap samples. The idcluster (newcluster) option is designed for this. It requests that at each replication bootstrap create the new variable, newcluster, containing unique identifiers for all resampled groups. Thus, to make sure that the correct matching is preserved during each replication, we need to specify the grouping variable in cluster(), supply a variable name to idcluster (), and use this variable as the grouping variable with clogit, as we demonstrate below.


Note the difference between the estimates of the bootstrap standard error for the two specifications of the bootstrap syntax.

## - Technical note

Similarly, when you have panel (longitudinal) data, all resampled panels must be unique in each of the bootstrap samples to obtain correct bootstrap estimates of statistics. Therefore, both cluster (panelvar) and idcluster(newpanelvar) must be specified with bootstrap, and i (newpanelvar) must be used with the main command. Moreover, you must clear the current xtset settings by typing xtset, clear before calling bootstrap.

## > Example 8

Continuing with our birthweight data, suppose that we have more information about doctors supervising women's pregnancies. We believe that the data on the pairs of infants from the same doctor may be correlated and want to adjust standard errors for possible correlation among the pairs. clogit offers the vce(cluster clustvar) option to do this.

Let's add a cluster variable to our dataset. One thing to keep in mind is that to use vce (cluster clustvar), groups in group() must be nested within clusters.

```
. use http://www.stata-press.com/data/r13/lowbirth2, clear
(Applied Logistic Regression, Hosmer & Lemeshow)
. set seed 12345
. by pairid, sort: egen byte doctor = total(int(2*runiform()+1)*(_n == 1))
. clogit low lwt smoke ptd ht ui i.race, group(pairid) vce(cluster doctor)
Iteration 0: log pseudolikelihood = -26.768693
Iteration 1: log pseudolikelihood = -25.810476
Iteration 2: log pseudolikelihood = -25.794296
Iteration 3: log pseudolikelihood = -25.794271
Iteration 4: log pseudolikelihood = -25.794271
Conditional (fixed-effects) logistic regression Number of obs = 112
Wald chi2(1) =
Prob > chi2 = .
Log pseudolikelihood = -25.794271 Pseudo R2 = 0.3355
```

(Std. Err. adjusted for 2 clusters in doctor)

| low | Robust |  |  |  | [95\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| lwt | -. 0183757 | . 0217802 | -0.84 | 0.399 | -. 0610641 | . 0243128 |
| smoke | 1.400656 | . 0085545 | 163.73 | 0.000 | 1.38389 | 1.417423 |
| ptd | 1.808009 | . 938173 | 1.93 | 0.054 | -. 0307765 | 3.646794 |
| ht | 2.361152 | 1.587013 | 1.49 | 0.137 | -. 7493362 | 5.47164 |
| ui | 1.401929 | . 8568119 | 1.64 | 0.102 | -. 2773913 | 3.08125 |
| race |  |  |  |  |  |  |
| black | . 5713643 | . 0672593 | 8.49 | 0.000 | . 4395385 | . 7031902 |
| other | -. 0253148 | . 9149785 | -0.03 | 0.978 | -1.81864 | 1.76801 |

To obtain correct bootstrap standard errors of the exponentiated difference between the two coefficients in this example, we need to make sure that both resampled clusters and groups within resampled clusters are unique in each of the bootstrap samples. To achieve this, bootstrap needs the information about clusters in cluster(), the variable name of the new identifier for clusters in idcluster(), and the information about groups in group(). We demonstrate the corresponding syntax of bootstrap below.
. bootstrap ratio=exp(_b[smoke]-_b[ptd]), seed(1) cluster(doctor)
> idcluster(uidoctor) group(pairid): clogit low lwt smoke ptd ht ui i.race,
> group(pairid)
(running clogit on estimation sample)

............................................................... . . . 50
$\begin{array}{lll}\text { Bootstrap results } & \begin{array}{ll}\text { Number of obs } & = \\ \text { Replications } & =\end{array} & 112 \\ & & 50\end{array}$
command: clogit low lwt smoke ptd ht ui i.race, group(pairid)
ratio: exp(_b[smoke]-_b[ptd])
(Replications based on 2 clusters in doctor)

|  | Observed <br> Coef. | Bootstrap <br> Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | Normal-based <br> [95\% Conf. Interval] |  |
| ---: | ---: | :--- | :---: | :---: | :---: | ---: |
| ratio | .6654095 | .3156251 | 2.11 | 0.035 | .0467956 | 1.284023 |

In the above syntax, although we specify group (pairid) with clogit, it is not the group identifiers of the original pairid variable that are used to compute parameter estimates from bootstrap samples. The way bootstrap works is that, at each replication, the clusters defined by doctor are resampled and the new variable, uidoctor, uniquely identifying resampled clusters is created. After that, another new variable uniquely identifying the (uidoctor, group) combination is created and renamed to have the same name as the grouping variable, pairid. This newly defined grouping variable is then used by clogit to obtain the parameter estimates from this bootstrap sample of clusters. After all replications are performed, the original values of the grouping variable are restored.

## - Technical note

The same logic must be used when running bootstrap with commands designed for panel (longitudinal) data that allow specifying the cluster (clustervar) option. To ensure that the combination of (clustervar, panelvar) values are unique in each of the bootstrap samples, cluster (clustervar), idcluster (newclustervar), and group (panelvar) must be specified with bootstrap, and i (panelvar) must be used with the main command.

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## Stored results

bootstrap stores the following in e():
Scalars

```
e(N)
e(N_reps)
    e(N_misreps)
    e(N_strata)
    e(N_clust)
    e(k_eq)
    e(k_exp)
    e(k_eexp)
    e(k_extra)
    e(level)
    e(bs_version)
    e(rank)
```

Macros
e(cmdname)
e(cmd)
e(command)
e(cmdline)
e(prefix)
e(title)
e(strata)
e(cluster)
e(seed)
e(size)
e (exp\#)
e(ties)
e(mse)
e(vce)
e(vcetype)
e(properties)
Matrices
e(b)
e(b_bs)
e(reps)
e(bias)
e(se)
e(z0)
e(accel)
e(ci_normal)
e(ci_percentile)
e(ci_bc)
e(ci_bca)
e(V)
e(V_modelbased)
sample size
number of complete replications
number of incomplete replications
number of strata
number of clusters
number of equations in e(b)
number of standard expressions
number of extended expressions (i.e., _b)
number of extra equations beyond the original ones from $e(b)$
confidence level for bootstrap CIs
version for bootstrap results
rank of $e(V)$
command name from command
same as e(cmdname) or bootstrap
command
command as typed
bootstrap
title in estimation output
strata variables
cluster variables
initial random-number seed
from the size(\#) option
expression for the \#th statistic
ties, if specified
mse, if specified
bootstrap
title used to label Std. Err.
b V
observed statistics
bootstrap estimates
number of nonmissing results
estimated biases
estimated standard errors
median biases
estimated accelerations
normal-approximation CIs
percentile CIs
bias-corrected CIs
bias-corrected and accelerated CIs
bootstrap variance-covariance matrix
model-based variance

When exp_list is _b, bootstrap will also carry forward most of the results already in e() from command.

## Methods and formulas

Let $\widehat{\theta}$ be the observed value of the statistic, that is, the value of the statistic calculated with the original dataset. Let $i=1,2, \ldots, k$ denote the bootstrap samples, and let $\widehat{\theta}_{i}$ be the value of the statistic from the $i$ th bootstrap sample.

When the mse option is specified, the standard error is estimated as

$$
\widehat{\mathrm{se}}_{\mathrm{MSE}}=\left\{\frac{1}{k} \sum_{i=1}^{k}\left(\widehat{\theta}_{i}-\widehat{\theta}\right)^{2}\right\}^{1 / 2}
$$

Otherwise, the standard error is estimated as

$$
\widehat{\mathrm{se}}=\left\{\frac{1}{k-1} \sum_{i=1}^{k}\left(\widehat{\theta}_{i}-\bar{\theta}\right)^{2}\right\}^{1 / 2}
$$

where

$$
\bar{\theta}=\frac{1}{k} \sum_{i=1}^{k} \widehat{\theta}_{i}
$$

The variance-covariance matrix is similarly computed. The bias is estimated as

$$
\widehat{\mathrm{bias}}=\bar{\theta}-\widehat{\theta}
$$

Confidence intervals with nominal coverage rates $1-\alpha$ are calculated according to the following formulas. The normal-approximation method yields the confidence intervals

$$
\left[\widehat{\theta}-z_{1-\alpha / 2} \widehat{\mathrm{se}}, \widehat{\theta}+z_{1-\alpha / 2} \widehat{\mathrm{se}}\right]
$$

where $z_{1-\alpha / 2}$ is the $(1-\alpha / 2)$ th quantile of the standard normal distribution. If the mse option is specified, bootstrap will report the normal confidence interval using $\widehat{\operatorname{se}}_{\text {MSE }}$ instead of se. estat bootstrap only uses $\widehat{\text { se }}$ in the normal confidence interval.

The percentile method yields the confidence intervals

$$
\left[\theta_{\alpha / 2}^{*}, \theta_{1-\alpha / 2}^{*}\right]
$$

where $\theta_{p}^{*}$ is the $p$ th quantile (the $100 p$ th percentile) of the bootstrap distribution $\left(\widehat{\theta}_{1}, \ldots, \widehat{\theta}_{k}\right)$.
Let

$$
z_{0}=\Phi^{-1}\left\{\#\left(\widehat{\theta}_{i} \leq \widehat{\theta}\right) / k\right\}
$$

where $\#\left(\widehat{\theta}_{i} \leq \widehat{\theta}\right)$ is the number of elements of the bootstrap distribution that are less than or equal to the observed statistic and $\Phi$ is the standard cumulative normal. $z_{0}$ is known as the median bias of $\widehat{\theta}$. When the ties option is specified, $z_{0}$ is estimated as $\#\left(\widehat{\theta}_{i}<\widehat{\theta}\right)+\#\left(\widehat{\theta}_{i}=\widehat{\theta}\right) / 2$, which is the number of elements of the bootstrap distribution that are less than the observed statistic plus half the number of elements that are equal to the observed statistic.

Let

$$
a=\frac{\sum_{i=1}^{n}\left(\bar{\theta}_{(\cdot)}-\widehat{\theta}_{(i)}\right)^{3}}{6\left\{\sum_{i=1}^{n}\left(\bar{\theta}_{(\cdot)}-\widehat{\theta}_{(i)}\right)^{2}\right\}^{3 / 2}}
$$

where $\widehat{\theta}_{(i)}$ are the leave-one-out (jackknife) estimates of $\widehat{\theta}$ and $\bar{\theta}_{(\cdot)}$ is their mean. This expression is known as the jackknife estimate of acceleration for $\widehat{\theta}$. Let

$$
\begin{aligned}
& p_{1}=\Phi\left\{z_{0}+\frac{z_{0}-z_{1-\alpha / 2}}{1-a\left(z_{0}-z_{1-\alpha / 2}\right)}\right\} \\
& p_{2}=\Phi\left\{z_{0}+\frac{z_{0}+z_{1-\alpha / 2}}{1-a\left(z_{0}+z_{1-\alpha / 2}\right)}\right\}
\end{aligned}
$$

where $z_{1-\alpha / 2}$ is the $(1-\alpha / 2)$ th quantile of the normal distribution. The bias-corrected and accelerated $\left(\mathrm{BC}_{a}\right)$ method yields confidence intervals

$$
\left[\theta_{p_{1}}^{*}, \theta_{p_{2}}^{*}\right]
$$

where $\theta_{p}^{*}$ is the $p$ th quantile of the bootstrap distribution as defined previously. The bias-corrected (but not accelerated) method is a special case of $\mathrm{BC}_{a}$ with $a=0$.

## References

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## Also see

[R] bootstrap postestimation - Postestimation tools for bootstrap
[R] jackknife - Jackknife estimation
[R] permute - Monte Carlo permutation tests
[R] simulate - Monte Carlo simulations
[SVY] svy bootstrap - Bootstrap for survey data
[U] 13.5 Accessing coefficients and standard errors
[U] 13.6 Accessing results from Stata commands
[U] 20 Estimation and postestimation commands

## Title

bootstrap postestimation - Postestimation tools for bootstrap

| Description | Syntax for predict | Syntax for estat bootstrap |
| :--- | :--- | :--- |
| Menu for estat | Options for estat bootstrap | Remarks and examples |
| Also see |  |  |

## Description

The following postestimation command is of special interest after bootstrap:
Command Description
estat bootstrap percentile-based and bias-corrected CI tables

The following standard postestimation commands are also available:

Command Description

| * contrast | contrasts and ANOVA-style joint tests of estimates |
| :---: | :---: |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample |
| estat vce | variance-covariance matrix of the estimators (VCE) |
| estimates | cataloging estimation results |
| *hausman | Hausman's specification test |
| *lincom | point estimates, standard errors, testing, and inference for linear combinations of coefficients |
| *margins | marginal means, predictive margins, marginal effects, and average marginal effects |
| *marginsplot | graph the results from margins (profile plots, interaction plots, etc.) |
| * nl com | point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients |
| * predict | predictions, residuals, influence statistics, and other diagnostic measures |
| *predictnl | point estimates, standard errors, testing, and inference for generalized predictions |
| * pwcompare | pairwise comparisons of estimates |
| * test | Wald tests of simple and composite linear hypotheses |
| * testnl | Wald tests of nonlinear hypotheses |

* This postestimation command is allowed if it may be used after command.


## Special-interest postestimation command

estat bootstrap displays a table of confidence intervals for each statistic from a bootstrap analysis.

## Syntax for predict

The syntax of predict (and even if predict is allowed) following bootstrap depends upon the command used with bootstrap. If predict is not allowed, neither is predictnl.

## Syntax for estat bootstrap

estat bootstrap [, options]
options Description
bc bias-corrected CIs; the default
bca bias-corrected and accelerated ( $\mathrm{BC}_{a}$ ) CIs
normal normal-based CIs
percentile percentile CIs
all all available CIs
noheader suppress table header
nolegend suppress table legend
verbose display the full table legend
bc, bca, normal, and percentile may be used together.

## Menu for estat

Statistics $>$ Postestimation $>$ Reports and statistics

## Options for estat bootstrap

bc is the default and displays bias-corrected confidence intervals.
bca displays bias-corrected and accelerated confidence intervals. This option assumes that you also specified the bca option on the bootstrap prefix command.
normal displays normal approximation confidence intervals.
percentile displays percentile confidence intervals.
all displays all available confidence intervals.
noheader suppresses display of the table header. This option implies nolegend.
nolegend suppresses display of the table legend, which identifies the rows of the table with the expressions they represent.
verbose requests that the full table legend be displayed.

## Remarks and examples

## > Example 1

The estat bootstrap postestimation command produces a table containing the observed value of the statistic, an estimate of its bias, the bootstrap standard error, and up to four different confidence intervals.

If we were interested merely in getting bootstrap standard errors for the model coefficients, we could use the bootstrap prefix with our estimation command. If we were interested in performing a thorough bootstrap analysis of the model coefficients, we could use the estat bootstrap postestimation command after fitting the model with the bootstrap prefix.

Using example 1 from [R] bootstrap, we need many more replications for the confidence interval types other than the normal based, so let's rerun the estimation command. We will reset the randomnumber seed-in case we wish to reproduce the results-increase the number of replications, and save the bootstrap distribution as a dataset called bsauto.dta.

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
set seed 1
. bootstrap _b, reps(1000) saving(bsauto) bca: regress mpg weight gear foreign
    (output omitted)
. estat bootstrap, all
```

| Linear regression | Number of obs $=$ | 74 |  |
| :--- | :--- | ---: | ---: |
|  | Replications | $=$ | 1000 |


| mpg | Observed <br> Coef. | Bias | Bootstrap <br> Std. Err. | [95\% Conf. Interval] |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |


| (N) | normal confidence interval |
| :--- | :--- |
| (P) | percentile confidence interval |
| (BC) | bias-corrected confidence interval |
| (BCa) | bias-corrected and accelerated confidence interval |

The estimated standard errors here differ from our previous estimates using only 100 replications by, respectively, $8 \%, 3 \%, 11 \%$, and $6 \%$; see example 1 of [R] bootstrap. So much for our advice that $50-200$ replications are good enough to estimate standard errors. Well, the more replications the better-that advice you should believe.

Which of the methods to compute confidence intervals should we use? If the statistic is unbiased, the percentile ( P ) and bias-corrected ( BC ) methods should give similar results. The bias-corrected confidence interval will be the same as the percentile confidence interval when the observed value of the statistic is equal to the median of the bootstrap distribution. Thus, for unbiased statistics, the two methods should give similar results as the number of replications becomes large. For biased statistics, the bias-corrected method should yield confidence intervals with better coverage probability (closer to the nominal value of $95 \%$ or whatever was specified) than the percentile method. For statistics with variances that vary as a function of the parameter of interest, the bias-corrected and accelerated method $\left(\mathrm{BC}_{a}\right)$ will typically have better coverage probability than the others.

When the bootstrap distribution is approximately normal, all of these methods should give similar confidence intervals as the number of replications becomes large. If we examine the normality of these bootstrap distributions using, say, the pnorm command (see [R] diagnostic plots), we see that they closely follow a normal distribution. Thus here, the normal approximation would also be a valid choice. The chief advantage of the normal-approximation method is that it (supposedly) requires fewer replications than the other methods. Of course, it should be used only when the bootstrap distribution exhibits normality.

We can load bsauto.dta containing the bootstrap distributions for these coefficients:


We can now run other commands, such as pnorm, on the bootstrap distributions. As with all standard estimation commands, we can use the bootstrap command to replay its output table. The default variable names assigned to the statistics in exp_list are _bs_1, _bs_2, ..., and each variable is labeled with the associated expression. The naming convention for the extended expressions _b and _se is to prepend _ $\mathrm{b}_{-}$and _se_, respectively, onto the name of each element of the coefficient vector. Here the first coefficient is _b [weight], so bootstrap named it _b_weight.

## Also see

[R] bootstrap - Bootstrap sampling and estimation
[U] 20 Estimation and postestimation commands

## Title

boxcox - Box-Cox regression models

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

boxcox depvar [indepvars] [if] [in] [weight] [, options]
options Description

Model
noconstant model(lhsonly)
model (rhsonly)
model (lambda)
model(theta)
notrans (varlist)
Reporting
level(\#)
lrtest
Maximization
nolog suppress full-model iteration log
nologlr
maximize_options
suppress constant term
left-hand-side Box-Cox model; the default right-hand-side Box-Cox model
both sides Box-Cox model with same parameter
both sides Box-Cox model with different parameters nontransformed independent variables
set confidence level; default is level (95)
perform likelihood-ratio test
suppress restricted-model lrtest iteration log
control the maximization process; seldom used
depvar and indepvars may contain time-series operators; see [U] 11.4.4 Time-series varlists.
bootstrap, by, jackknife, rolling, statsby, and xi are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the bootstrap prefix; see [R] bootstrap.
fweights and iweights are allowed; see [U] 11.1.6 weight.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Linear models and related $>$ Box-Cox regression

## Description

boxcox finds the maximum likelihood estimates of the parameters of the Box-Cox transform, the coefficients on the independent variables, and the standard deviation of the normally distributed errors for a model in which depvar is regressed on indepvars. You can fit the following models:

| Option | Estimates |
| :--- | :--- |
| lhsonly | $y_{j}^{(\theta)}=\beta_{1} x_{1 j}+\beta_{2} x_{2 j}+\cdots+\beta_{k} x_{k j}+\epsilon_{j}$ |
| rhsonly | $y_{j}=\beta_{1} x_{1 j}^{(\lambda)}+\beta_{2} x_{2 j}^{(\lambda)}+\cdots+\beta_{k} x_{k j}^{(\lambda)}+\epsilon_{j}$ |
| rhsonly notrans ( ) | $y_{j}=\beta_{1} x_{1 j}^{(\lambda)}+\beta_{2} x_{2 j}^{(\lambda)}+\cdots+\beta_{k} x_{k j}^{(\lambda)}+\gamma_{1} z_{1 j}+\cdots+\gamma_{l} z_{l j}+\epsilon_{j}$ |
| lambda | $y_{j}^{(\lambda)}=\beta_{1} x_{1 j}^{(\lambda)}+\beta_{2} x_{2 j}^{(\lambda)}+\cdots+\beta_{k} x_{k j}^{(\lambda)}+\epsilon_{j}$ |
| lambda notrans() | $y_{j}^{(\lambda)}=\beta_{1} x_{1 j}^{(\lambda)}+\beta_{2} x_{2 j}^{(\lambda)}+\cdots+\beta_{k} x_{k j}^{(\lambda)}+\gamma_{1} z_{1 j}+\cdots+\gamma_{l} z_{l j}+\epsilon_{j}$ |
| theta | $y_{j}^{(\theta)}=\beta_{1} x_{1 j}^{(\lambda)}+\beta_{2} x_{2 j}^{(\lambda)}+\cdots+\beta_{k} x_{k j}^{(\lambda)}+\epsilon_{j}$ |
| theta notrans() | $y_{j}^{(\theta)}=\beta_{1} x_{1 j}^{(\lambda)}+\beta_{2} x_{2 j}^{(\lambda)}+\cdots+\beta_{k} x_{k j}^{(\lambda)}+\gamma_{1} z_{1 j}+\cdots+\gamma_{l} z_{l j}+\epsilon_{j}$ |

Any variable to be transformed must be strictly positive.

## Options

## Model

noconstant; see $[R]$ estimation options.
model (lhsonly $\mid$ rhsonly $\mid$ lambda $\mid$ theta) specifies which of the four models to fit.
model (lhsonly) applies the Box-Cox transform to depvar only. model (lhsonly) is the default. model (rhsonly) applies the transform to the indepvars only.
model (lambda) applies the transform to both depvar and indepvars, and they are transformed by the same parameter.
model(theta) applies the transform to both depvar and indepvars, but this time, each side is transformed by a separate parameter.
notrans (varlist) specifies that the variables in varlist be included as nontransformed independent variables.
$\qquad$ Reporting
level (\#); see [R] estimation options.
lrtest specifies that a likelihood-ratio test of significance be performed and reported for each independent variable.
$\qquad$ Maximization
nolog suppresses the iteration $\log$ when fitting the full model.
nologlr suppresses the iteration $\log$ when fitting the restricted models required by the lrtest option. maximize_options: iterate(\#) and from(init_specs); see [R] maximize.

| Model | Initial value specification |
| :--- | :--- |
| lhsonly | from $\left(\theta_{0}\right.$, copy $)$ |
| rhsonly | from $\left(\lambda_{0}\right.$, copy $)$ |
| lambda | from $\left(\lambda_{0}\right.$, copy $)$ |
| theta | from $\left(\lambda_{0} \theta_{0}\right.$, copy $)$ |

## Remarks and examples

Remarks are presented under the following headings:

Introduction<br>Theta model<br>Lambda model<br>Left-hand-side-only model<br>Right-hand-side-only model

## Introduction

The Box-Cox transform

$$
y^{(\lambda)}=\frac{y^{\lambda}-1}{\lambda}
$$

has been widely used in applied data analysis. Box and Cox (1964) developed the transformation and argued that the transformation could make the residuals more closely normal and less heteroskedastic. Cook and Weisberg (1982) discuss the transform in this light. Because the transform embeds several popular functional forms, it has received some attention as a method for testing functional forms, in particular,

$$
y^{(\lambda)}= \begin{cases}y-1 & \text { if } \lambda=1 \\ \ln (y) & \text { if } \lambda=0 \\ 1-1 / y & \text { if } \lambda=-1\end{cases}
$$

Davidson and MacKinnon (1993) discuss this use of the transform. Atkinson (1985) also gives a good general treatment.

## Theta model

boxcox obtains the maximum likelihood estimates of the parameters for four different models. The most general of the models, the theta model, is

$$
y_{j}^{(\theta)}=\beta_{0}+\beta_{1} x_{1 j}^{(\lambda)}+\beta_{2} x_{2 j}^{(\lambda)}+\cdots+\beta_{k} x_{k j}^{(\lambda)}+\gamma_{1} z_{1 j}+\gamma_{2} z_{2 j}+\cdots+\gamma_{l} z_{l j}+\epsilon_{j}
$$

where $\epsilon \sim N\left(0, \sigma^{2}\right)$. Here the dependent variable, $y$, is subject to a Box-Cox transform with parameter $\theta$. Each of the indepvars, $x_{1}, x_{2}, \ldots, x_{k}$, is transformed by a Box-Cox transform with parameter $\lambda$. The $z_{1}, z_{2}, \ldots, z_{l}$ specified in the notrans() option are independent variables that are not transformed.

Box and Cox (1964) argued that this transformation would leave behind residuals that more closely follow a normal distribution than those produced by a simple linear regression model. Bear in mind that the normality of $\epsilon$ is assumed and that boxcox obtains maximum likelihood estimates of the $k+l+4$ parameters under this assumption. boxcox does not choose $\lambda$ and $\theta$ so that the residuals are approximately normally distributed. If you are interested in this type of transformation to normality, see the official Stata commands Inskew0 and bcskew0 in [R] Inskew0. However, those commands work on a more restrictive model in which none of the independent variables is transformed.

## Example 1

Below we fit a theta model to a nonrepresentative extract of the Second National Health and Nutrition Examination Survey (NHANES II) dataset discussed in McDowell et al. (1981).

We model individual-level diastolic blood pressure (bpdiast) as a function of the transformed variables body mass index (bmi) and cholesterol level (tcresult) and of the untransformed variables age (age) and sex (sex).

```
. use http://www.stata-press.com/data/r13/nhanes2
. boxcox bpdiast bmi tcresult, notrans(age sex) model(theta) lrtest
Fitting comparison model
Iteration 0: log likelihood = -41178.61
Iteration 1: log likelihood = -41032.51
Iteration 2: log likelihood = -41032.488
Iteration 3: log likelihood = -41032.488
Fitting full model
Iteration 0: log likelihood = -39928.606
Iteration 1: log likelihood = -39775.026
Iteration 2: log likelihood = -39774.987
Iteration 3: log likelihood = -39774.987
Fitting comparison models for LR tests
Iteration 0: log likelihood = -39947.144
Iteration 1: log likelihood = -39934.55
Iteration 2: log likelihood = -39934.516
Iteration 3: log likelihood = -39934.516
Iteration 0: log likelihood = -39906.96
Iteration 1: log likelihood = -39896.63
Iteration 2: log likelihood = -39896.629
Iteration 0: log likelihood = -40464.599
Iteration 1: log likelihood = -40459.752
Iteration 2: log likelihood = -40459.604
Iteration 3: log likelihood = -40459.604
Iteration 0: log likelihood = -39829.859
Iteration 1: log likelihood = -39815.576
Iteration 2: log likelihood = -39815.575
```

| Number of obs | $=$ | 10351 |
| :--- | :--- | ---: |
| LR chi2 $(5)$ | $=$ | 2515.00 |
| Prob $>$ chi2 | $=$ | 0.000 |


| bpdiast | Coef. | Std. Err. | $z$ | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | :---: |
| /lambda | .6383286 | .1577601 | 4.05 | 0.000 | .3291245 | .9475327 |
| /theta | .1988197 | .0454088 | 4.38 | 0.000 | .1098201 | .2878193 |

Estimates of scale-variant parameters

|  | Coef. | chi2(df) | P>chi2(df) | df of chi2 |
| :---: | :---: | :---: | :---: | :---: |
| Notrans |  |  |  |  |
| age | . 003811 | 319.060 | 0.000 | 1 |
| sex | -. 1054887 | 243.284 | 0.000 | 1 |
| _cons | 5.835555 |  |  |  |
| Trans |  |  |  |  |
| bmi | . 0872041 | 1369.235 | 0.000 | 1 |
| tcresult | . 004734 | 81.177 | 0.000 | 1 |
| /sigma | . 3348267 |  |  |  |
| Test | Res | icted |  |  |
| H0: | log 1 | elihood | chi2 | Prob > chi2 |
| theta=lambda | -1 -40 | 2.898 | 775.82 | 0.000 |
| theta=lambda | $0 \quad-39$ | 0.945 | 31.92 | 0.000 |
| theta=lambda | $1-39$ | 8.606 | 307.24 | 0.000 |

The output is composed of the iteration logs and three distinct tables. The first table contains a standard header for a maximum likelihood estimator and a standard output table for the BoxCox transform parameters. The second table contains the estimates of the scale-variant parameters. The third table contains the output from likelihood-ratio tests on three standard functional form specifications.

The right-hand-side and the left-hand-side transformations each add to the regression fit at the $1 \%$ significance level and are both positive but less than 1. All the variables have significant impacts on diastolic blood pressure, bpdiast. As expected, the transformed variables-the body mass index, bmi, and cholesterol level, tcresult-contribute to higher blood pressure. The last output table shows that the linear, multiplicative inverse, and log specifications are strongly rejected.

## Technical note

Spitzer (1984) showed that the Wald tests of the joint significance of the coefficients of the right-hand-side variables, either transformed or untransformed, are not invariant to changes in the scale of the transformed dependent variable. Davidson and MacKinnon (1993) also discuss this point. This problem demonstrates that Wald statistics can be manipulated in nonlinear models. Lafontaine and White (1986) analyze this problem numerically, and Phillips and Park (1988) analyze it by using Edgeworth expansions. See Drukker (2000b) for a more detailed discussion of this issue. Because the parameter estimates and their Wald tests are not scale invariant, no Wald tests or confidence intervals are reported for these parameters. However, when the lrtest option is specified, likelihood-ratio tests are performed and reported. Schlesselman (1971) showed that, if a constant is included in the model, the parameter estimates of the Box-Cox transforms are scale invariant. For this reason, we strongly recommend that you not use the noconstant option.

The lrtest option does not perform a likelihood-ratio test on the constant, so no value for this statistic is reported. Unless the data are properly scaled, the restricted model does not often converge. For this reason, no likelihood-ratio test on the constant is performed by the lrtest option. However, if you have a special interest in performing this test, you can do so by fitting the constrained model separately. If problems with convergence are encountered, rescaling the data by their means may help.

## Lambda model

A less general model than the one above is called the lambda model. It specifies that the same parameter be used in both the left-hand-side and right-hand-side transformations. Specifically,

$$
y_{j}^{(\lambda)}=\beta_{0}+\beta_{1} x_{1 j}^{(\lambda)}+\beta_{2} x_{2 j}^{(\lambda)}+\cdots+\beta_{k} x_{k j}^{(\lambda)}+\gamma_{1} z_{1 j}+\gamma_{2} z_{2 j}+\cdots+\gamma_{l} z_{l j}+\epsilon_{j}
$$

where $\epsilon \sim N\left(0, \sigma^{2}\right)$. Here the depvar variable, $y$, and each of the indepvars, $x_{1}, x_{2}, \ldots, x_{k}$, is transformed by a Box-Cox transform with the common parameter $\lambda$. Again the $z_{1}, z_{2}, \ldots, z_{l}$ are independent variables that are not transformed.

## Left-hand-side-only model

Even more restrictive than a common transformation parameter is transforming the dependent variable only. Because the dependent variable is on the left-hand side of the equation, this model is known as the lhsonly model. Here you are estimating the parameters of the model

$$
y_{j}^{(\theta)}=\beta_{0}+\beta_{1} x_{1 j}+\beta_{2} x_{2 j}+\cdots+\beta_{k} x_{k j}+\epsilon_{j}
$$

where $\epsilon \sim N\left(0, \sigma^{2}\right)$. Here only the depvar, $y$, is transformed by a Box-Cox transform with the parameter $\theta$.

## > Example 2

In this example, we model the transform of diastolic blood pressure as a linear combination of the untransformed body mass index, cholesterol level, age, and sex.
. boxcox bpdiast bmi tcresult age sex, model(lhsonly) lrtest nolog nologlr
Fitting comparison model
Fitting full model
Fitting comparison models for LR tests
Log likelihood = - 39777.709

Estimates of scale-variant parameters

|  | Coef. | chi2(df) | P>chi2(df) | df of chi2 |
| ---: | ---: | ---: | :---: | :---: |
| Notrans |  |  |  |  |
| bmi | .0272628 | 1375.841 | 0.000 | 1 |
| tcresult | .0006929 | 82.380 | 0.000 | 1 |
| age | .0040141 | 334.117 | 0.000 | 1 |
| sex | -.1122274 | 263.219 | 0.000 | 1 |
| _cons | 6.302855 |  |  |  |
| /sigma | .3476615 |  |  |  |


| Test <br> H0: | Restricted <br> log likelihood | LR statistic <br> chi2 | P-value <br> Prob $>$ chi2 |
| ---: | ---: | ---: | ---: |
| theta $=-1$ | -40146.678 | 737.94 | 0.000 |
| theta $=$ | 0 | -39788.241 | 21.06 |
| theta $=$ | 1 | -39928.606 | 301.79 |

The maximum likelihood estimate of the transformation parameter for this model is positive and significant. Once again, all the scale-variant parameters are significant, and we find a positive impact of body mass index (bmi) and cholesterol levels (tcresult) on the transformed diastolic blood pressure (bpdiast). This model rejects the linear, multiplicative inverse, and log specifications.

## Right-hand-side-only model

The fourth model leaves the depvar alone and transforms a subset of the indepvars using the parameter $\lambda$. This is the rhsonly model. In this model, the depvar, $y$, is given by

$$
y_{j}=\beta_{0}+\beta_{1} x_{1 j}^{(\lambda)}+\beta_{2} x_{2 j}^{(\lambda)}+\cdots+\beta_{k} x_{k j}^{(\lambda)}+\gamma_{1} z_{1 j}+\gamma_{2} z_{2 j}+\cdots+\gamma_{l} z_{l j}+\epsilon_{j}
$$

where $\epsilon \sim N\left(0, \sigma^{2}\right)$. Here each of the indepvars, $x_{1}, x_{2}, \ldots, x_{k}$, is transformed by a Box-Cox transform with the parameter $\lambda$. Again the $z_{1}, z_{2}, \ldots, z_{l}$ are independent variables that are not transformed.

## > Example 3

Now we consider a rhsonly model in which the regressors sex and age are not transformed.

```
. boxcox bpdiast bmi tcresult, notrans(sex age) model(rhsonly) lrtest nolog
> nologlr
```

Fitting full model
Fitting comparison models for LR tests

|  | Number of obs | $=$ |
| :--- | :--- | :--- |
|  | 10351 |  |
| Log likelihood $=-39928.212$ | LR chi2 $(5)$ | $=$ |


| bpdiast | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| :---: | ---: | :---: | :---: | :---: | :---: | :---: |
| /lambda | .8658841 | .1522387 | 5.69 | 0.000 | .5675018 | 1.164266 |

Estimates of scale-variant parameters

|  | Coef. | chi2(df) | P>chi2(df) | df of chi2 |
| :---: | :---: | :---: | :---: | :---: |
| Notrans |  |  |  |  |
| sex | -3.544042 | 235.020 | 0.000 | 1 |
| age | 128809 | 311.754 | 0.000 | 1 |
| _cons | 50.01498 |  |  |  |
| Trans |  |  |  |  |
| bmi | 1.418215 | 1396.709 | 0.000 | 1 |
| tcresult | . 0462964 | 78.500 | 0.000 | 1 |
| /sigma | 11.4557 |  |  |  |


| Test <br> H0: | Restricted <br> log likelihood | LR statistic <br> chi2 | P-value <br> Prob $>$ chi2 |
| ---: | :---: | :---: | :---: |
| lambda $=-1$ | -39989.331 | 122.24 | 0.000 |
| lambda $=$ | 0 | -39942.945 | 29.47 |
| lambda $=$ | -39928.606 | 0.79 | 0.000 |

The maximum likelihood estimate of the transformation parameter in this model is positive and significant at the $1 \%$ level. The transformed bmi coefficient behaves as expected, and the remaining scale-variant parameters are significant at the $1 \%$ level. This model rejects the multiplicative inverse and $\log$ specifications strongly. However, we cannot reject the hypothesis that the model is linear.

## Stored results

boxcox stores the following in e():
Scalars

| e(N) | number of observations |
| :---: | :---: |
| e(11) | log likelihood |
| e(chi2) | LR statistic of full vs. comparison |
| e (df_m) | full model degrees of freedom |
| e(ll0) | log likelihood of the restricted model |
| e(df_r) | restricted model degrees of freedom |
| e(ll_t1) | $\log$ likelihood of model $\lambda=\theta=1$ |
| e(chi2_t1) | LR of $\lambda=\theta=1 \mathrm{vs}$. full model |
| e(p_t1) | $p$-value of $\lambda=\theta=1$ vs. full model |
| e(ll_tm1) | $\log$ likelihood of model $\lambda=\theta=-1$ |
| e(chi2_tm1) | LR of $\lambda=\theta=-1$ vs. full model |
| e(p_tm1) | $p$-value of $\lambda=\theta=-1$ vs. full model |
| e(ll_t0) | $\log$ likelihood of model $\lambda=\theta=0$ |
| e(chi2_t0) | LR of $\lambda=\theta=0$ vs. full model |
| e (p_t0) | $p$-value of $\lambda=\theta=0$ vs. full model |
| e (rank) | rank of e(V) |
| e(ic) | number of iterations |
| e(rc) | return code |

Macros
e(cmd)
e(cmdline)
e(depvar)
e(model)
e(wtype)
e(wexp)
e(ntrans)
e(chi2type)
e(lrtest)
e(properties)
e(predict)
e(marginsnotok)
Matrices
e(b)
e(V)
e(pm)
e(df)
e(chi2m)
Functions
e(sample) marks estimation sample
$e(V)$ contains all zeros, except for the elements that correspond to the parameters of the Box-Cox transform.

## Methods and formulas

In the internal computations,

$$
y^{(\lambda)}= \begin{cases}\frac{y^{\lambda}-1}{\lambda} & \text { if }|\lambda|>10^{-10} \\ \ln (y) & \text { otherwise }\end{cases}
$$

The unconcentrated log likelihood for the theta model is

$$
\ln L=\left(\frac{-N}{2}\right)\left\{\ln (2 \pi)+\ln \left(\sigma^{2}\right)\right\}+(\theta-1) \sum_{i=1}^{N} \ln \left(y_{i}\right)-\left(\frac{1}{2 \sigma^{2}}\right) \mathrm{SSR}
$$

where

$$
\operatorname{SSR}=\sum_{i=1}^{N}\left(y_{i}^{(\theta)}-\beta_{0}+\beta_{1} x_{i 1}^{(\lambda)}+\beta_{2} x_{i 2}^{(\lambda)}+\cdots+\beta_{k} x_{i k}^{(\lambda)}+\gamma_{1} z_{i 1}+\gamma_{2} z_{i 2}+\cdots+\gamma_{l} z_{i l}\right)^{2}
$$

Writing the SSR in matrix form,

$$
\mathrm{SSR}=\left(\mathbf{y}^{(\theta)}-\mathbf{X}^{(\lambda)} \mathbf{b}^{\prime}-\mathbf{Z g}^{\prime}\right)^{\prime}\left(\mathbf{y}^{(\theta)}-\mathbf{X}^{(\lambda)} \mathbf{b}^{\prime}-\mathbf{Z g}^{\prime}\right)
$$

where $\mathbf{y}^{(\theta)}$ is an $N \times 1$ vector of elementwise transformed data, $\mathbf{X}^{(\lambda)}$ is an $N \times k$ matrix of elementwise transformed data, $\mathbf{Z}$ is an $N \times l$ matrix of untransformed data, $\mathbf{b}$ is a $1 \times k$ vector of coefficients, and $\mathbf{g}$ is a $1 \times l$ vector of coefficients. Letting

$$
\mathbf{W}_{\lambda}=\left(\mathbf{X}^{(\lambda)} \mathbf{Z}\right)
$$

be the horizontal concatenation of $\mathbf{X}^{(\lambda)}$ and $\mathbf{Z}$ and

$$
\mathbf{d}^{\prime}=\binom{\mathbf{b}^{\prime}}{\mathbf{g}^{\prime}}
$$

be the vertical concatenation of the coefficients yields

$$
\mathrm{SSR}=\left(\mathbf{y}^{(\theta)}-\mathbf{W}_{\lambda} \mathbf{d}^{\prime}\right)^{\prime}\left(\mathbf{y}^{(\theta)}-\mathbf{W}_{\lambda} \mathbf{d}^{\prime}\right)
$$

For given values of $\lambda$ and $\theta$, the solutions for $\mathbf{d}^{\prime}$ and $\sigma^{2}$ are

$$
\widehat{\mathbf{d}}^{\prime}=\left(W_{\lambda}^{\prime} W_{\lambda}\right)^{-1} W_{\lambda}^{\prime} y^{(\theta)}
$$

and

$$
\widehat{\sigma}^{2}=\frac{1}{N}\left(\mathbf{y}^{(\theta)}-W_{\lambda} \widehat{d}^{\prime}\right)^{\prime}\left(y^{(\theta)}-W_{\lambda}{\widehat{d^{\prime}}}^{\prime}\right)
$$

Substituting these solutions into the log-likelihood function yields the concentrated log-likelihood function

$$
\ln L_{c}=\left(-\frac{N}{2}\right)\left\{\ln (2 \pi)+1+\ln \left(\widehat{\sigma}^{2}\right)\right\}+(\theta-1) \sum_{i=1}^{N} \ln \left(y_{i}\right)
$$

Similar calculations yield the concentrated log-likelihood function for the lambda model,

$$
\ln L_{c}=\left(-\frac{N}{2}\right)\left\{\ln (2 \pi)+1+\ln \left(\widehat{\sigma}^{2}\right)\right\}+(\lambda-1) \sum_{i=1}^{N} \ln \left(y_{i}\right)
$$

the lhsonly model,

$$
\ln L_{c}=\left(-\frac{N}{2}\right)\left\{\ln (2 \pi)+1+\ln \left(\widehat{\sigma}^{2}\right)\right\}+(\theta-1) \sum_{i=1}^{N} \ln \left(y_{i}\right)
$$

and the rhsonly model,

$$
\ln L_{c}=\left(-\frac{N}{2}\right)\left\{\ln (2 \pi)+1+\ln \left(\widehat{\sigma}^{2}\right)\right\}
$$

where $\widehat{\sigma}^{2}$ is specific to each model and is defined analogously to that in the theta model.

## References

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Phillips, P. C. B., and J. Y. Park. 1988. On the formulation of Wald tests of nonlinear restrictions. Econometrica 56: 1065-1083.

Schlesselman, J. J. 1971. Power families: A note on the Box and Cox transformation. Journal of the Royal Statistical Society, Series B 33: 307-311.
Spitzer, J. J. 1984. Variance estimates in models with the Box-Cox transformation: Implications for estimation and hypothesis testing. Review of Economics and Statistics 66: 645-652.

## Also see <br> [R] boxcox postestimation - Postestimation tools for boxcox <br> [R] Inskew0 - Find zero-skewness log or Box-Cox transform <br> $[R]$ regress - Linear regression

[U] 20 Estimation and postestimation commands

## Title

boxcox postestimation - Postestimation tools for boxcox

| Description | Syntax for predict | Menu for predict | Options for predict |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Methods and formulas | References | Also see |

## Description

The following postestimation commands are available after boxcox:

Command Description
estat ic Akaike's and Schwarz's Bayesian information criteria (AIC and BIC)
estat summarize summary statistics for the estimation sample
estat vce variance-covariance matrix of the estimators (VCE)
estimates cataloging estimation results

* lincom point estimates, standard errors, testing, and inference for linear combinations of coefficients
*nlcom point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients
predict predictions, residuals, influence statistics, and other diagnostic measures
*test Wald tests of simple and composite linear hypotheses
*testnl Wald tests of nonlinear hypotheses
*Inference is valid only for hypotheses concerning $\lambda$ and $\theta$.


## Syntax for predict

predict [type] newvar [if] [in] [, statistic options]
statistic Description
Main
yhat predicted value of $y$; the default
residuals residuals
options
Description
Options
smearing
btransform
compute statistic using smearing method; the default compute statistic using back-transform method

These statistics are available both in and out of sample; type predict ... if e(sample)... if wanted only for the estimation sample.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
yhat, the default, calculates the predicted value of the dependent variable.
residuals calculates the residuals, that is, the observed value minus the predicted value.

Options
smearing calculates the statistics yhat and residuals using the smearing method proposed by Duan (1983) (see Methods and formulas for a description of this method). smearing is the default. btransform calculates the statistics yhat and residuals using the back-transform method (see Methods and formulas for a description of this method).

## Remarks and examples

Below we present two examples that illustrate how to use the smearing and btransform options.
> Example 1: Predictions with the smearing option
In this example, we calculate the predicted values of diastolic blood pressure, bpdiast, that arise from the theta model calculated in example 1 of $[R]$ boxcox.

```
. use http://www.stata-press.com/data/r13/nhanes2
. boxcox bpdiast bmi tcresult, notrans(age sex) model(theta) lrtest
(output omitted)
. predict yhat
(statistic yhat and option smearing are assumed)
```

In the expression above, yhat is the name we gave to the estimates of the conditional expectation. Given that we did not specify any statistic or option, the corresponding defaults yhat and smearing were assumed.

As the summary table below illustrates, the mean of the dependent variable is close to the mean of the predicted value yhat. This indicates that the theta model does a good job approximating the true value of diastolic blood pressure, bpdiast.
. summarize bpdiast yhat

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| bpdiast | 10351 | 81.715 | 12.92722 | 35 | 150 |
| yhat | 10351 | 81.71406 | 5.983486 | 66.93709 | 110.5283 |

Similarly, we could have asked that residuals be calculated. Here we again use the default smearing option:

```
. predict resid, residuals
(option smearing assumed to compute residuals)
```


## > Example 2: Predictions with the btransform option

In this example, we illustrate the tradeoffs involved by using the btransform option as opposed to the default smearing option. Continuing with example 1 , we compute the predicted values using the back-transform method.

```
. predict yhatb, btransform
(statistic yhat assumed)
```

We now compute the predicted values using the smearing option and summarize both computations.

```
. predict yhats
(statistic yhat and option smearing are assumed)
. summarize bpdiast yhats yhatb
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline bpdiast & 10351 & 81.715 & 12.92722 & 35 & 150 \\
yhats & 10351 & 81.71406 & 5.983486 & 66.93709 & 110.5283 \\
yhatb & 10351 & 81.08018 & 5.95549 & 66.37479 & 109.7671
\end{tabular}
```

As can be seen from the mean and the standard deviation of the summary table, the predicted values using the back-transform method give biased estimates but are less variable than those coming from the smearing method. However, the efficiency loss is small compared with the bias reduction.

## Technical note

boxcox estimates variances only for the $\lambda$ and $\theta$ parameters (see the technical note in [R] boxcox), so the extent to which postestimation commands can be used following boxcox is limited. Formulas used in lincom, nlcom, test, and testnl are dependent on the estimated variances. Therefore, the use of these commands is limited and generally applicable only to inferences on the $\lambda$ and $\theta$ coefficients.

## Methods and formulas

The computation of the expected value of the dependent variable conditional on the regressors for the Box-Cox model does not follow the logic of the standard linear regression model because the random disturbance does not vanish from the conditional expectation and must be accounted for. To show this, we will revisit the lhsonly model described by

$$
y_{j}^{(\lambda)}=\beta_{0}+\beta_{1} x_{1 j}+\beta_{2} x_{2 j}+\cdots+\beta_{(k-1)} x_{(k-1) j}+\epsilon_{j}
$$

where

$$
y^{(\lambda)}=\frac{y^{\lambda}-1}{\lambda}
$$

and

$$
y^{(\lambda)}= \begin{cases}y-1 & \text { if } \lambda=1 \\ \ln (y) & \text { if } \lambda=0 \\ 1-1 / y & \text { if } \lambda=-1\end{cases}
$$

For the presentation below, let $\mathbf{y}^{(\lambda)}$ be an $N \times 1$ vector of elementwise transformed data, $\mathbf{X}$ be an $N \times k$ matrix of regressors, $\boldsymbol{\beta}$ be a $k \times 1$ vector of parameters, and $\iota$ be an $n \times 1$ vector of ones.

If we were interested in $E\left(\mathbf{y}^{(\lambda)} \mid \mathbf{X}\right)$, then the conventional logic would follow, and we would obtain predictions as $y^{(\widehat{\lambda})}=\mathbf{X} \widehat{\boldsymbol{\beta}}$, where $\widehat{\boldsymbol{\beta}}$ is the estimate of $\boldsymbol{\beta}$. However, to estimate the conditional expectation of $\mathbf{y}$, we need to isolate it on the left-hand side of the model. In the case of the lhsonly model, this yields

$$
\mathbf{y}=\{\lambda(\mathbf{X} \widehat{\boldsymbol{\beta}}+\boldsymbol{\epsilon})+\iota\}^{1 / \lambda}
$$

The conditional expectation is then defined by

$$
E(\mathbf{y} \mid \mathbf{X})=\int\{\lambda(\mathbf{X} \boldsymbol{\beta}+\boldsymbol{\epsilon})+\boldsymbol{\iota}\}^{1 / \lambda} d F(\boldsymbol{\epsilon} \mid \mathbf{X})
$$

In the expression above, $d F(\boldsymbol{\epsilon} \mid \mathbf{X})$ corresponds to the cdf of $\boldsymbol{\epsilon}$ conditional on the regressors. It is also clear that the random disturbance does not vanish.

To address this issue, the default methodology used by predict computes this integral using the smearing method proposed by Duan (1983) to implement a two-step estimator, as was suggested by Abrevaya (2002).

In the first step, we get an estimate for $\epsilon$ defined as

$$
\widehat{\boldsymbol{\epsilon}}=\mathbf{y}^{(\widehat{\lambda})}-\mathbf{X} \widehat{\boldsymbol{\beta}}
$$

In the second step, for each $j$ we compute our predicted values as the sum:

$$
\widehat{y}_{j}=\frac{1}{N} \sum_{i=1}^{N}\left\{\widehat{\lambda}\left(\mathbf{x}_{j} \widehat{\boldsymbol{\beta}}+\widehat{\epsilon}_{i}\right)+1\right\}^{1 / \widehat{\lambda}}
$$

In the expression above, $\mathbf{x}_{j}$ is the $j$ th row of the matrix $\mathbf{X}$ (in other words, the values of the covariates for individual $j$ ), and $\widehat{\epsilon}_{i}$ is the residual for individual $i$. The result of this summation gives us the conditional expectation of the dependent variable for individual $j$. Given that this operation is performed for each individual $j$, the methodology is computationally intensive.

The back-transform method can be understood as a naïve estimate that disregards the random disturbance. The predictions using this approach are given by

$$
\widehat{y}_{j}=\left(\widehat{\lambda} \mathbf{x}_{j} \widehat{\boldsymbol{\beta}}+1\right)^{1 / \widehat{\lambda}}
$$

## References

Abrevaya, J. 2002. Computing marginal effects in the Box-Cox model. Econometric Reviews 21: 383-393.
Duan, N. 1983. Smearing estimate: A nonparametric retransformation method. Journal of the American Statistical Association 78: 605-610.

## Also see

[R] boxcox - Box-Cox regression models
[U] 20 Estimation and postestimation commands

## Title

brier - Brier score decomposition

| Syntax | Menu | Description | Option |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | Acknowledgment |
| References | Also see |  |  |

## Syntax

brier outcomevar forecastvar $[$ if $][$ in $][$, group(\#) $]$ by is allowed; see [D] by.

## Menu

Statistics $>$ Epidemiology and related $>$ Other $>$ Brier score decomposition

## Description

brier computes the Yates, Sanders, and Murphy decompositions of the Brier Mean Probability Score. outcomevar contains $0 / 1$ values reflecting the actual outcome of the experiment, and forecastvar contains the corresponding probabilities as predicted by, say, logit, probit, or a human forecaster.

## Option

Main
group(\#) specifies the number of groups that will be used to compute the decomposition. group(10) is the default.

## Remarks and examples

You have a binary ( $0 / 1$ ) response and a formula that predicts the corresponding probabilities of having observed a positive outcome (1). If the probabilities were obtained from logistic regression, there are many methods that assess goodness of fit (see, for instance, [R] estat gof). However, the probabilities might be computed from a published formula or from a model fit on another sample, both completely unrelated to the data at hand, or perhaps the forecasts are not from a formula at all. In any case, you now have a test dataset consisting of the forecast probabilities and observed outcomes. Your test dataset might, for instance, record predictions made by a meteorologist on the probability of rain along with a variable recording whether it actually rained.

The Brier score is an aggregate measure of disagreement between the observed outcome and a prediction - the average squared error difference. The Brier score decomposition is a partition of the Brier score into components that suggest reasons for discrepancy. These reasons fall roughly into three groups: 1) lack of overall calibration between the average predicted probability and the actual probability of the event in your data, 2) misfit of the data in groups defined within your sample, and 3 ) inability to match actual 0 and 1 responses.

Problem 1 refers to simply overstating or understating the probabilities.
Problem 2 refers to what is standardly called a goodness-of-fit test: the data are grouped, and the predictions for the group are compared with the outcomes.
Problem 3 refers to an individual-level measure of fit. Imagine that the grouped outcomes are predicted on average correctly but that within the group, the outcomes are poorly predicted.

Using logit or probit analysis to fit your data will guarantee that there is no lack of fit due to problem 1 , and a good model fitter will be able to avoid problem 2. Problem 3 is inherent in any prediction exercise.
$>$ Example 1
We have data on the outcomes of 20 basketball games (win) and the probability of victory predicted by a local pundit (for).
. use http://www.stata-press.com/data/r13/bball
. summarize win for
Variable

win
for

The mean probabilities of forecast and outcome are simply the mean of the predicted probabilities and the actual outcomes (wins/losses). The correlation is the product-moment correlation between them.

The Brier score measures the total difference between the event (winning) and the forecast probability of that event as an average squared difference. As a benchmark, a perfect forecaster would have a Brier score of 0 , a perfect misforecaster (predicts probability of win is 1 when loses and 0 when wins) would have a Brier score of 1 , and a fence-sitter (forecasts every game as $50 / 50$ ) would have a Brier score of 0.25 . Our pundit is doing reasonably well.

Spiegelhalter's $z$ statistic is a standard normal test statistic for testing whether an individual Brier score is extreme. The ROC area is the area under the receiver operating curve, and the associated test is a test of whether it is greater than 0.5 . The more accurate the forecast probabilities, the larger the ROC area.

The Sanders-modified Brier score measures the difference between a grouped forecast measure and the event, where the data are grouped by sorting the sample on the forecast and dividing it into
approximately equally sized groups. The difference between the modified and the unmodified score is typically minimal. For this and the other statistics that require grouping-the Sanders and Murphy resolutions and reliability-in-the-small-to be well-defined, group boundaries are chosen so as not to allocate observations with the same forecast probability to different groups. This task is done by grouping on the forecast using xtile, $\mathrm{n}(\#)$, with \# being the number of groups; see [D] pctile.

The Sanders resolution measures error that arises from statistical considerations in evaluating the forecast for a group. A group with all positive or all negative outcomes would have a Sanders resolution of 0 ; it would most certainly be feasible to predict exactly what happened to each member of the group. If the group had $40 \%$ positive responses, on the other hand, a forecast that assigned $p=0.4$ to each member of the group would be a good one, and yet, there would be "errors" in the squared difference sense. The "error" would be $(1-0.4)^{2}$ or $(0-0.4)^{2}$ for each member. The Sanders resolution is the average across groups of such "expected" errors. The 0.1400 value in our data from an overall Brier score of 0.1828 or 0.1861 suggests that a substantial portion of the "error" in our data is inherent.

Outcome index variance is just the variance of the outcome variable. This is the expected value of the Brier score if all the forecast probabilities were merely the average observed outcome. Remember that a fence-sitter has an expected Brier score of 0.25 ; a smarter fence sitter (who would guess $p=0.65$ for these data) would have a Brier score of 0.2275 .

The Murphy resolution measures the variation in the average outcomes across groups. If all groups have the same frequency of positive outcomes, little information in any forecast is possible, and the Murphy resolution is 0 . If groups differ markedly, the Murphy resolution is as large as 0.25 . The 0.0875 means that there is some variation but not a lot, and 0.0875 is probably higher than in most real cases. If you had groups in your data that varied between $40 \%$ and $60 \%$ positive outcomes, the Murphy resolution would be 0.01 ; between $30 \%$ and $70 \%$, it would be 0.04 .

Reliability-in-the-small measures the error that comes from the average forecast within group not measuring the average outcome within group-a classical goodness-of-fit measure, with 0 meaning a perfect fit and 1 meaning a complete lack of fit. The calculated value of 0.0461 shows some amount of lack of fit. Remember, the number is squared, and we are saying that probabilities could be just more than $\sqrt{0.0461}=0.215$ or $21.5 \%$ off.

Forecast variance measures the amount of discrimination being attempted-that is, the variation in the forecasted probabilities. A small number indicates a fence-sitter making constant predictions. If the forecasts were from a logistic regression model, forecast variance would tend to increase with the amount of information available. Our pundit shows considerable forecast variance of 0.0438 (standard deviation $\sqrt{0.0438}=0.2093$ ), which is in line with the reliability-in-the-small, suggesting that the forecaster is attempting as much variation as is available in these data.

Excess forecast variance is the amount of actual forecast variance over a theoretical minimum. The theoretical minimum-called the minimum forecast variance-corresponds to forecasts of $p_{0}$ for observations ultimately observed to be negative responses and $p_{1}$ for observations ultimately observed to be positive outcomes. Moreover, $p_{0}$ and $p_{1}$ are set to the average forecasts made for the ultimate negative and positive outcomes. These predictions would be just as good as the predictions the forecaster did make, and any variation in the actual forecast probabilities above this is useless. If this number is large, above $1 \%-2 \%$, then the forecaster may be attempting more than is possible. The 0.0285 in our data suggests this possibility.

Reliability-in-the-large measures the discrepancy between the mean forecast and the observed fraction of positive outcomes. This discrepancy will be 0 for forecasts made by most statistical models-at least when measured on the same sample used for estimation-because they, by design, reproduce sample means. For our human pundit, the 0.0294 says that there is a $\sqrt{0.0294}$, or 17 -percentage-point, difference. (This difference can also be found by calculating the difference in the
averages of the observed outcomes and forecast probabilities: $0.65-0.4785=0.17$.) That difference, however, is not significant, as we would see if we typed ttest win=for; see [R] ttest. If these data were larger and the bias persisted, this difference would be a critical shortcoming of the forecast.

Twice the forecast-outcome covariance is a measure of how accurately the forecast corresponds to the outcome. The concept is similar to that of $R$-squared in linear regression.

## Stored results

brier stores the following in $r()$ :
Scalars

| $r\left(p \_r o c\right)$ | significance of ROC area | $r($ murphy $)$ | Murphy resolution |
| :--- | :--- | :--- | :--- |
| $r\left(r o c \_a r e a\right)$ | ROC area | $r(r e l i n s m)$ | reliability-in-the-small |
| $r(z)$ | Spiegelhalter's $z$ statistic | $r\left(V a r \_f\right)$ | forecast variance |
| $r(p)$ | significance of $z$ statistic | $r\left(V a r \_f e x\right)$ | excess forecast variance |
| $r(b r i e r)$ | Brier score | $r\left(V a r \_f m i n\right)$ | minimum forecast variance |
| $r\left(b r i e r \_s\right)$ | Sanders-modified Brier score | $r(r e l i n l a)$ | reliability-in-the-large |
| $r($ sanders $)$ | Sanders resolution | $r\left(\operatorname{cov} \_2 f\right)$ | $2 \times$ forecast-outcome-covariance |

## Methods and formulas

See Wilks (2011) or Schmidt and Griffith (2005) for a discussion of the Brier score.
Let $d_{j}, j=1, \ldots, N$, be the observed outcomes with $d_{j}=0$ or $d_{j}=1$, and let $f_{j}$ be the corresponding forecasted probabilities that $d_{j}$ is $1,0 \leq f_{j} \leq 1$. Assume that the data are ordered so that $f_{j+1} \geq f_{j}$ (brier sorts the data to obtain this order). Divide the data into $K$ nearly equally sized groups, with group 1 containing observations 1 through $j_{2}-1$, group 2 containing observations $j_{2}$ through $j_{3}-1$, and so on.

Define

$$
\begin{aligned}
\bar{f}_{0} & =\text { average } f_{j} \text { among } d_{j}=0 \\
\bar{f}_{1} & =\text { average } f_{j} \text { among } d_{j}=1 \\
\bar{f} & =\text { average } f_{j} \\
\bar{d} & =\text { average } d_{j} \\
\widetilde{f}_{k} & =\text { average } f_{j} \text { in group } k \\
\widetilde{d}_{k} & =\text { average } d_{j} \text { in group } k \\
\widetilde{n}_{k} & =\text { number of observations in group } k
\end{aligned}
$$

The Brier score is $\sum_{j}\left(d_{j}-f_{j}\right)^{2} / N$.
The Sanders-modified Brier score is $\sum_{j}\left(d_{j}-\widetilde{f}_{k(j)}\right)^{2} / N$.
Let $p_{j}$ denote the true but unknown probability that $d_{j}=1$. Under the null hypothesis that $p_{j}=$ $f_{j}$ for all $j$, Spiegelhalter (1986) determined that the expectation and variance of the Brier score is given by the following:

$$
\begin{aligned}
E(\text { Brier }) & =\frac{1}{N} \sum_{j=1}^{N} f_{j}\left(1-f_{j}\right) \\
\operatorname{Var}(\text { Brier }) & =\frac{1}{N^{2}} \sum_{j=1}^{N} f_{j}\left(1-f_{j}\right)\left(1-2 f_{j}\right)^{2}
\end{aligned}
$$

Denoting the observed value of the Brier score by $O$ (Brier), Spiegelhalter's $z$ statistic is given by

$$
Z=\frac{O(\text { Brier })-E(\text { Brier })}{\sqrt{\operatorname{Var}(\text { Brier })}}
$$

The corresponding $p$-value is given by the upper-tail probability of $Z$ under the standard normal distribution.

The area under the ROC curve is estimated by applying the trapezoidal rule to the empirical ROC curve. This area is Wilcoxon's test statistic, so the corresponding $p$-value is just that of a one-sided Wilcoxon test of the null hypothesis that the distribution of predictions is constant across the two outcomes.

The Sanders resolution is $\sum_{k} \widetilde{n}_{k}\left\{\widetilde{d}_{k}\left(1-\widetilde{d}_{k}\right)\right\} / N$.
The outcome index variance is $\bar{d}(1-\bar{d})$.
The Murphy resolution is $\sum_{k} \widetilde{n}_{k}\left(\widetilde{d}_{k}-\bar{d}\right)^{2} / N$.
Reliability-in-the-small is $\sum_{k} \widetilde{n}_{k}\left(\widetilde{d}_{k}-\widetilde{f}_{k}\right)^{2} / N$.
The forecast variance is $\sum_{j}\left(f_{j}-\bar{f}\right)^{2} / N$.
The minimum forecast variance is $\left\{\sum_{j \in F}\left(f_{j}-\bar{f}_{0}\right)^{2}+\sum_{j \in S}\left(f_{j}-\bar{f}_{1}\right)^{2}\right\} / N$, where $F$ is the set of observations for which $d_{j}=0$ and $S$ is the complement.

The excess forecast variance is the difference between the forecast variance and the minimum forecast variance.

Reliability-in-the-large is $(\bar{f}-\bar{d})^{2}$.
Twice the outcome covariance is $2\left(\bar{f}_{1}-\bar{f}_{0}\right) \bar{d}(1-\bar{d})$.

Glenn Wilson Brier (1913-1998) was an American meteorological statistician who, after obtaining degrees in physics and statistics, was for many years head of meteorological statistics at the U.S. Weather Bureau in Washington, DC. In the latter part of his career, he was associated with Colorado State University. Brier worked especially on verification and evaluation of predictions and forecasts, statistical decision making, the statistical theory of turbulence, the analysis of weather modification experiments, and the application of permutation techniques.

## Acknowledgment

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## Also see <br> [R] logistic - Logistic regression, reporting odds ratios <br> [R] logit - Logistic regression, reporting coefficients <br> [R] probit - Probit regression

## Title

bsample - Sampling with replacement

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | References | Also see |  |

## Syntax

```
bsample [exp][if][in] [,options]
```

where exp is a standard Stata expression; see [U] 13 Functions and expressions.
options
Description
strata (varlist)
cluster(varlist)
idcluster(newvar)
weight (varname)
variables identifying strata
variables identifying resampling clusters
create new cluster ID variable
replace varname with frequency weights

## Menu

Statistics $>$ Resampling $>$ Draw bootstrap sample

## Description

bsample draws bootstrap samples (random samples with replacement) from the data in memory.
$\exp$ specifies the size of the sample, which must be less than or equal to the number of sampling units in the data. The observed number of units is the default when $\exp$ is not specified.

For bootstrap sampling of the observations, $\exp$ must be less than or equal to $\_\mathrm{N}$ (the number of observations in the data; see [U] 13.4 System variables (_variables)).
For stratified bootstrap sampling, exp must be less than or equal to $\_\mathrm{N}$ within the strata identified by the strata() option.
For clustered bootstrap sampling, exp must be less than or equal to $N_{c}$ (the number of clusters identified by the cluster() option).
For stratified bootstrap sampling of clusters, $\exp$ must be less than or equal to $N_{c}$ within the strata identified by the strata() option.
Observations that do not meet the optional if and in criteria are dropped (not sampled).

## Options

strata (varlist) specifies the variables identifying strata. If strata() is specified, bootstrap samples are selected within each stratum.
cluster (varlist) specifies the variables identifying resampling clusters. If cluster() is specified, the sample drawn during each replication is a bootstrap sample of clusters.
idcluster (newvar) creates a new variable containing a unique identifier for each resampled cluster. weight (varname) specifies a variable in which the sampling frequencies will be placed. varname must be an existing variable, which will be replaced. After bsample, varname can be used as an fweight in any Stata command that accepts fweights, which can speed up resampling for commands like regress and summarize. This option cannot be combined with idcluster().

By default, bsample replaces the data in memory with the sampled observations; however, specifying the weight () option causes only the specified varname to be changed.

## Remarks and examples

Below is a series of examples illustrating how bsample is used with various sampling schemes.

## > Example 1: Bootstrap sampling

We have data on the characteristics of hospital patients and wish to draw a bootstrap sample of 200 patients. We type

```
. use http://www.stata-press.com/data/r13/bsample1
```

. bsample 200
. count
200

## > Example 2: Stratified samples with equal sizes

Among the variables in our dataset is female, an indicator for the female patients. To get a bootstrap sample of 200 female patients and 200 male patients, we type

```
. use http://www.stata-press.com/data/r13/bsample1, clear
. bsample 200, strata(female)
. tabulate female
\begin{tabular}{r|rrr} 
female & Freq. & Percent & Cum. \\
\hline male & 200 & 50.00 & 50.00 \\
female & 200 & 50.00 & 100.00 \\
\hline Total & 400 & 100.00 &
\end{tabular}
```

> Example 3: Stratified samples with unequal sizes
To sample 300 females and 200 males, we must generate a variable that is 300 for females and 200 for males and then use this variable in $\exp$ when we call bsample.
. use http://www.stata-press.com/data/r13/bsample1, clear
. generate nsamp $=$ cond (female,300,200)
. bsample nsamp, strata(female)
. tabulate female

| female | Freq. | Percent | Cum. |
| ---: | ---: | ---: | ---: |
| male | 200 | 40.00 | 40.00 |
| female | 300 | 60.00 | 100.00 |
| Total | 500 | 100.00 |  |

> Example 4: Stratified samples with proportional sizes
Our original dataset has 2,392 males and 3,418 females.
. use http://www.stata-press.com/data/r13/bsample1, clear
. tabulate female

| female | Freq. | Percent | Cum. |
| ---: | ---: | ---: | ---: |
| male | 2,392 | 41.17 | 41.17 |
| female | 3,418 | 58.83 | 100.00 |
| Total | 5,810 | 100.00 |  |

To sample $10 \%$ from females and males, we type

```
. bsample round(0.1*_N), strata(female)
```

bsample requires that the specified size of the sample be an integer, so we use the round() function to obtain the nearest integer to $0.1 \times 2392$ and $0.1 \times 3418$. Our sample now has 239 males and 342 females:
. tabulate female

| female | Freq. | Percent | Cum. |
| ---: | ---: | ---: | ---: |
| male | 239 | 41.14 | 41.14 |
| female | 342 | 58.86 | 100.00 |
| Total | 581 | 100.00 |  |

## Example 5: Samples satisfying a condition

For a bootstrap sample of 200 female patients, we type
. use http://www.stata-press.com/data/r13/bsample1, clear
. bsample 200 if female
. tabulate female
female
female

## > Example 6: Generating frequency weights

To identify the sampled observations using frequency weights instead of dropping unsampled observations, we use the weight() option (we will need to supply it an existing variable name) and type

```
. use http://www.stata-press.com/data/r13/bsample1, clear
. set seed 1234
. generate fw = .
(5810 missing values generated)
. bsample 200 if female, weight(fw)
. tabulate fw female
\begin{tabular}{r|rr|r} 
fw & \multicolumn{2}{|c|}{ female } & female
\end{tabular} Total \begin{tabular}{rrr} 
male & fem & 3,221 \\
\hline 0 & 2,392 & 513 \\
1 & 0 & 194
\end{tabular}
```

Note that $(194 \times 1)+(3 \times 2)=200$.

## Example 7: Oversampling observations

bsample requires the expression in $\exp$ to evaluate to a number that is less than or equal to the number of observations. To sample twice as many male and female patients as there are already in memory, we must expand the data before using bsample. For example,

```
. use http://www.stata-press.com/data/r13/bsample1, clear
. set seed 1234
. expand 2
(5810 observations created)
. bsample, strata(female)
. tabulate female
\begin{tabular}{r|rrr} 
female & Freq. & Percent & Cum. \\
\hline male & 4,784 & 41.17 & 41.17 \\
female & 6,836 & 58.83 & 100.00 \\
\hline Total & 11,620 & 100.00 &
\end{tabular}
```


## > Example 8: Stratified oversampling with unequal sizes

To sample twice as many female patients as male patients, we must expand the records for the female patients because there are less than twice as many of them as there are male patients, but first put the number of observed male patients in a local macro. After expanding the female records, we generate a variable that contains the number of observations to sample within the two groups.

```
. use http://www.stata-press.com/data/r13/bsample1, clear
. set seed 1234
. count if !female
2392
. local nmale = r(N)
. expand 2 if female
(3418 observations created)
. generate nsamp = cond(female, 2*'nmale','nmale')
. bsample nsamp, strata(female)
. tabulate female
\begin{tabular}{r|rrr} 
female & Freq. & Percent & Cum. \\
\hline male & 2,392 & 33.33 & 33.33 \\
female & 4,784 & 66.67 & 100.00 \\
\hline Total & 7,176 & 100.00 &
\end{tabular}
```


## > Example 9: Oversampling of clusters

For clustered data, sampling more clusters than are present in the original dataset requires more than just expanding the data. To illustrate, suppose we wanted a bootstrap sample of eight clusters from a dataset consisting of five clusters of observations.

```
. use http://www.stata-press.com/data/r13/bsample2, clear
. tabstat x, stat(n mean) by(group)
Summary for variables: x
    by categories of: group
\begin{tabular}{r|rr} 
group & N & mean \\
\hline A & 15 & -.3073028 \\
B & 10 & -.00984 \\
C & 11 & .0810985 \\
D & 11 & -.1989179 \\
E & 29 & -.095203 \\
\hline Total & 76 & -.1153269 \\
\hline
\end{tabular}
```

bsample will complain if we simply expand the dataset.

```
. use http://www.stata-press.com/data/r13/bsample2
. expand 3
(152 observations created)
. bsample 8, cluster(group)
resample size must not be greater than number of clusters
r(498);
```

Expanding the data will only partly solve the problem. We also need a new variable that uniquely identifies the copied clusters. We use the expandcl command to accomplish both these tasks; see [D] expandcl.

```
. use http://www.stata-press.com/data/r13/bsample2, clear
. set seed 1234
. expandcl 2, generate(expgroup) cluster(group)
(76 observations created)
. tabstat x, stat(n mean) by(expgroup)
Summary for variables: x
    by categories of: expgroup
\begin{tabular}{r|rr} 
expgroup & N & mean \\
\hline 1 & \(15-.3073028\)
\end{tabular}
                        15-. 3073028
                                10-.00984
                                10-.00984
                                11 . }081098
                                11 . }081098
                                11 -. }198917
                                11-. }198917
                                29-.095203
                        29 -. 095203
\begin{tabular}{c|c} 
Total & \(152-.1153269\) \\
\hline
\end{tabular}
```

```
. generate fw = .
(152 missing values generated)
. bsample 8, cluster(expgroup) weight(fw)
. tabulate fw group
\begin{tabular}{r|rrrrr|r} 
fw & A & B & C & D & E & Total \\
\hline 0 & 15 & 10 & 0 & 0 & 29 & 54 \\
1 & 15 & 10 & 22 & 22 & 0 & 69 \\
2 & 0 & 0 & 0 & 0 & 29 & 29 \\
\hline Total & 30 & 20 & 22 & 22 & 58 & 152
\end{tabular}
```

The results from tabulate on the generated frequency weight variable versus the original cluster ID (group) show us that the bootstrap sample contains one copy of cluster A, one copy of cluster B, two copies of cluster $C$, two copies of cluster $D$, and two copies of cluster $E(1+1+2+2+2=8)$.

## Example 10: Stratified oversampling of clusters

Suppose that we have a dataset containing two strata with five clusters in each stratum, but the cluster identifiers are not unique between the strata. To get a stratified bootstrap sample with eight clusters in each stratum, we first use expandcl to expand the data and get a new cluster ID variable. We use cluster (strid group) in the call to expandcl; this action will uniquely identify the $2 * 5=10$ clusters across the strata.

```
. use http://www.stata-press.com/data/r13/bsample2, clear
. set seed 1234
. tabulate group strid
\begin{tabular}{r|cr|r} 
group & \begin{tabular}{l} 
strid \\
1
\end{tabular} & 2 & Total \\
\hline A & 7 & 8 & 15 \\
B & 5 & 5 & 10 \\
C & 5 & 6 & 11 \\
D & 5 & 6 & 11 \\
E & 14 & 15 & 29 \\
\hline Total & 36 & 40 & 76
\end{tabular}
. expandcl 2, generate(expgroup) cluster(strid group)
(76 observations created)
```

Now we can use bsample with the expanded data, stratum ID variable, and new cluster ID variable.

```
. generate fw = .
(152 missing values generated)
```

. bsample 8, cluster(expgroup) str(strid) weight(fw)
. by strid, sort: tabulate fw group
-> strid = 1

| fw | A | B | Croup | D | $E$ | Total |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0 | 0 | 5 | 0 | 5 | 14 | 24 |
| 1 | 14 | 5 | 10 | 5 | 0 | 34 |
| 2 | 0 | 0 | 0 | 0 | 14 | 14 |
| Total | 14 | 10 | 10 | 10 | 28 | 72 |

-> strid $=2$

| fw | A | Broup | C | D | E | Total |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0 | 8 | 10 | 0 | 6 | 0 | 24 |
| 1 | 8 | 0 | 6 | 6 | 15 | 35 |
| 2 | 0 | 0 | 6 | 0 | 15 | 21 |
| Total | 16 | 10 | 12 | 12 | 30 | 80 |

The results from by strid: tabulate on the generated frequency weight variable versus the original cluster ID (group) show us how many times each cluster was sampled for each stratum. For stratum 1, the bootstrap sample contains two copies of cluster A, one copy of cluster B, two copies of cluster C, one copy of cluster D, and two copies of cluster E $(2+1+2+1+2=8)$. For stratum 2, the bootstrap sample contains one copy of cluster A, zero copies of cluster B, three copies of cluster C, one copy of cluster $D$, and three copies of cluster $E(1+0+3+1+3=8)$.

## References

Gould, W. W. 2012a. Using Stata's random-number generators, part 2: Drawing without replacement. The Stata Blog: Not Elsewhere Classified.
http://blog.stata.com/2012/08/03/using-statas-random-number-generators-part-2-drawing-without-replacement/.
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## Also see

$[R]$ bootstrap - Bootstrap sampling and estimation
[R] bstat - Report bootstrap results
[R] simulate - Monte Carlo simulations
[D] sample - Draw random sample

## Title

bstat — Report bootstrap results

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | References | Also see |

## Syntax

Bootstrap statistics from variables
bstat [varlist][if][in][,options]
Bootstrap statistics from file
bstat [namelist][using filename] [if] [in] [, options]
options Description
Main
stat (vector) observed values for each statistic
accel(vector) acceleration values for each statistic
ties adjust BC/BCa confidence intervals for ties
mse use MSE formula for variance estimation
Reporting
level(\#) set confidence level; default is level(95)
n (\#) \# of observations from which bootstrap samples were taken
notable suppress table of results
noheader suppress table header
nolegend suppress table legend
verbose display the full table legend
title (text) use text as title for bootstrap results
display_options control column formats and line width
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Resampling $>$ Report bootstrap results

## Description

bstat is a programmer's command that computes and displays estimation results from bootstrap statistics.

For each variable in varlist (the default is all variables), then bstat computes a covariance matrix, estimates bias, and constructs several different confidence intervals (CIs). The following CIs are constructed by bstat:

1. Normal CIs (using the normal approximation)
2. Percentile CIs
3. Bias-corrected (BC) CIs
4. Bias-corrected and accelerated $\left(\mathrm{BC}_{a}\right)$ CIs (optional)
estat bootstrap displays a table of one or more of the above confidence intervals; see [R] bootstrap postestimation.

If there are bootstrap estimation results in e(), bstat replays them. If given the using modifier, bstat uses the data in filename to compute the bootstrap statistics while preserving the data currently in memory. Otherwise, bstat uses the data in memory to compute the bootstrap statistics.

The following options may be used to replay estimation results from bstat:
level(\#) notable noheader nolegend verbose title (text)
For all other options and the qualifiers using, if, and in, bstat requires a bootstrap dataset.

## Options

Main
stat (vector) specifies the observed value of each statistic (that is, the value of the statistic using the original dataset).
accel (vector) specifies the acceleration of each statistic, which is used to construct $\mathrm{BC}_{a}$ CIs.
ties specifies that bstat adjust for ties in the replicate values when computing the median bias used to construct BC and BCa CIs.
mse specifies that bstat compute the variance by using deviations of the replicates from the observed value of the statistics. By default, bstat computes the variance by using deviations from the average of the replicates.
level(\#); see [R] estimation options.
n (\#) specifies the number of observations from which bootstrap samples were taken. This value is used in no calculations but improves the table header when this information is not saved in the bootstrap dataset.
notable suppresses the display of the output table.
noheader suppresses the display of the table header. This option implies nolegend.
nolegend suppresses the display of the table legend.
verbose specifies that the full table legend be displayed. By default, coefficients and standard errors are not displayed.
title (text) specifies a title to be displayed above the table of bootstrap results; the default title is Bootstrap results.
display_options: cformat ( $\% f m t$ ), pformat ( $\% f m t$ ), sformat ( $\% f m t$ ), and nolstretch; see [R] estimation options.

## Remarks and examples

Remarks are presented under the following headings:
Bootstrap datasets
Creating a bootstrap dataset

## Bootstrap datasets

Although bstat allows you to specify the observed value and acceleration of each bootstrap statistic via the stat() and accel() options, programmers may be interested in what bstat uses when these options are not supplied.

When working from a bootstrap dataset, bstat first checks the data characteristics (see [P] char) that it understands:
_dta[bs_version] identifies the version of the bootstrap dataset. This characteristic may be empty (not defined), 2, or 3; otherwise, bstat will quit and display an error message. This version tells bstat which other characteristics to look for in the bootstrap dataset.
bstat uses the following characteristics from version 3 bootstrap datasets:
_dta [N]
_dta[N_strata]
_dta[N_cluster]
_dta[command]
varname [observed]
varname[acceleration]
varname [expression]
bstat uses the following characteristics from version 2 bootstrap datasets:
_dta [N]
_dta[N_strata]
_dta[N_cluster]
varname [observed]
varname [acceleration]
An empty bootstrap dataset version implies that the dataset was created by the bstrap command in a version of Stata earlier than Stata 8. Here bstat expects varname [bstrap] to contain the observed value of the statistic identified by varname (varname [observed] in version 2). All other characteristics are ignored.
_dta [ N ] is the number of observations in the observed dataset. This characteristic may be overruled by specifying the $n()$ option.
_dta[N_strata] is the number of strata in the observed dataset.
_dta[N_cluster] is the number of clusters in the observed dataset.
_dta[command] is the command used to compute the observed values of the statistics.
varname [observed] is the observed value of the statistic identified by varname. To specify a different value, use the stat() option.
varname [acceleration] is the estimate of acceleration for the statistic identified by varname. To specify a different value, use the accel() option.
varname [expression] is the expression or label that describes the statistic identified by varname.

## Creating a bootstrap dataset

Suppose that we are interested in obtaining bootstrap statistics by resampling the residuals from a regression (which is not possible with the bootstrap command). After loading some data, we run a regression, save some results relevant to the bstat command, and save the residuals in a new variable, res.

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. regress mpg weight length
```

| Source | SS | df | MS |
| ---: | :---: | ---: | ---: |
| Model | 1616.08062 | 2 | 808.040312 |
| Residual | 827.378835 | 71 | 11.653223 |
| Total | 2443.45946 | 73 | 33.4720474 |


| Number of obs | $=$ | 74 |
| :--- | ---: | ---: |
| F $(2$, | $71)$ | $=$ |
| Prob $>$ F | $=0.34$ |  |
| R-squared | $=$ | 0.0000 |
| Adj R-squared | $=$ | 0.6514 |
| Root MSE | $=3.4137$ |  |


| mpg | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| weight | -.0038515 | .001586 | -2.43 | 0.018 | -.0070138 | -.0006891 |
| length | -.0795935 | .0553577 | -1.44 | 0.155 | -.1899736 | .0307867 |
| _cons | 47.88487 | 6.08787 | 7.87 | 0.000 | 35.746 | 60.02374 |

```
. matrix b = e(b)
. local n = e(N)
. predict res, residuals
```

We can resample the residual values in res by generating a random observation ID (rid), generate a new response variable (y), and run the original regression with the new response variables.

```
. set seed 54321
. generate rid = int(_N*runiform())+1
. matrix score double y = b
. replace y = y + res[rid]
(74 real changes made)
```

. regress y weight length

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | 1773.23548 | 2 | 886.617741 |
| Residual | 608.747732 | 71 | 8.57391172 |
| Total | 2381.98321 | 73 | 32.629907 |


| Number of obs | $=$ | 74 |
| :--- | ---: | ---: |
| $\mathrm{~F}(2$, | $71)$ | $=103.41$ |
| Prob $>$ F | $=$ | 0.0000 |
| R-squared | $=$ | 0.7444 |
| Adj R-squared | $=$ | 0.7372 |
| Root MSE | $=$ | 2.9281 |


| y | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | :---: | ---: | ---: |
| weight | -.0059938 | .0013604 | -4.41 | 0.000 | -.0087064 | -.0032813 |
| length | -.0127875 | .0474837 | -0.27 | 0.788 | -.1074673 | .0818924 |
| _cons | 42.23195 | 5.22194 | 8.09 | 0.000 | 31.8197 | 52.6442 |

Instead of programming this resampling inside a loop, it is much more convenient to write a short program and use the simulate command; see [R] simulate. In the following, mysim_r requires the user to specify a coefficient vector and a residual variable. mysim_r then retrieves the list of predictor variables (removing _cons from the list), generates a new temporary response variable with the resampled residuals, and regresses the new response variable on the predictors.

```
program mysim_r
    version 13
    syntax name(name=bvector), res(varname)
    tempvar y rid
    local xvars : colnames 'bvector'
    local cons _cons
    local xvars : list xvars - cons
    matrix score double 'y' = 'bvector'
    gen long 'rid' = int(_N*runiform()) + 1
    replace 'y' = 'y' + 'res'['rid']
    regress 'y' 'xvars'
end
```

We can now give mysim_r a test run, but we first set the random-number seed (to reproduce results).

```
. set seed 54321
. mysim_r b, res(res)
(74 real changes made)
```

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | 1773.23548 | 2 | 886.617741 |
| Residual | 608.747732 | 71 | 8.57391172 |
| Total | 2381.98321 | 73 | 32.629907 |

                                    Number of obs \(=\quad 74\)
                                    \(F(2,71)=103.41\)
                                    Prob > F \(=0.0000\)
                                    \(R\)-squared \(=0.7444\)
                                    Adj \(R\)-squared \(=0.7372\)
                                    Root MSE \(=2.9281\)
    | __000000 | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | :---: | ---: | ---: | ---: |
| weight | -.0059938 | .0013604 | -4.41 | 0.000 | -.0087064 | -.0032813 |
| length | -.0127875 | .0474837 | -0.27 | 0.788 | -.1074673 | .0818924 |
| _cons | 42.23195 | 5.22194 | 8.09 | 0.000 | 31.8197 | 52.6442 |

Now that we have a program that will compute the results we want, we can use simulate to generate a bootstrap dataset and bstat to display the results.


Finally, we see that simulate created some of the data characteristics recognized by bstat. All we need to do is correctly specify the version of the bootstrap dataset, and bstat will automatically use the relevant data characteristics.

```
    char list
    _dta[seed]: X681014b5c43f462544a474abacbdd93d00042842
    _dta[command]: mysim_r b, res(res)
    _b_weight[is_eexp]:
    _b_weight[colname]:
    _b_weight[coleq]:
    _b_weight[expression]:
    _b_length[is_eexp]:
    _b_length[colname]:
    _b_length[coleq]:
    _b_length[expression]:
    _b_cons[is_eexp]:
    _b_cons[colname]:
    _b_cons[coleq]:
    _b_cons[expression]:
_b[_cons]
. char _dta[bs_version] 3
. bstat, stat(b) n('n')
Bootstrap results Number of obs = 74
Replications = 200
command: mysim_r b, res(res)
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline & Observed Coef. & \begin{tabular}{l}
Bootstrap \\
Std. Err.
\end{tabular} & z & \(P>|z|\) & \multicolumn{2}{|l|}{Normal-based} \\
\hline weight & -. 0038515 & . 0015715 & -2.45 & 0.014 & -. 0069316 & -. 0007713 \\
\hline length & -. 0795935 & . 0552415 & -1.44 & 0.150 & -. 1878649 & . 0286779 \\
\hline _cons & 47.88487 & 6.150069 & 7.79 & 0.000 & 35.83096 & 59.93879 \\
\hline
\end{tabular}
```

See Poi (2004) for another example of residual resampling.

## Stored results

bstat stores the following in e():
Scalars

$$
e(N)
$$

e(N_reps)
e(N_misreps)
e(N_strata)
e(N_clust)
e(k_aux)
e(k_eq)
e(k_exp)
e(k_eexp)
e(k_extra)
e(level)
e(bs_version)
$e$ (rank)
sample size
number of complete replications
number of incomplete replications
number of strata
number of clusters
number of auxiliary parameters
number of equations in $e(b)$
number of standard expressions
number of extended expressions (i.e., _b)
number of extra equations beyond the original ones from e(b)
confidence level for bootstrap CIs
version for bootstrap results
rank of e(V)
Macros
e(cmd)
bstat
e(command)
e(cmdline)
$e($ title)
e(exp\#)
e(prefix)
e(ties)
e(mse)
from _dta [command]
command as typed
title in estimation output
expression for the \#th statistic
bootstrap
ties, if specified
mse, if specified
e(vce)
e(vcetype)
bootstrap
title used to label Std. Err.
b V
Matrices
e(b)
e(b_bs)
e(reps)
e(bias)
e(se)
observed statistics
bootstrap estimates
number of nonmissing results
estimated biases
estimated standard errors
e(z0)
median biases
e(accel)
e(ci_normal)
e(ci_percentile)
estimated accelerations
normal-approximation CIs
percentile CIs
e(ci_bc)
bias-corrected CIs
e(ci_bca)
e(V)
bias-corrected and accelerated CIs
bootstrap variance-covariance matrix

## References

Ng, E. S.-W., R. Grieve, and J. R. Carpenter. 2013. Two-stage nonparametric bootstrap sampling with shrinkage correction for clustered data. Stata Journal 13: 141-164.

Poi, B. P. 2004. From the help desk: Some bootstrapping techniques. Stata Journal 4: 312-328.

## Also see

[R] bootstrap postestimation - Postestimation tools for bootstrap
[R] bootstrap - Bootstrap sampling and estimation
[R] bsample - Sampling with replacement

## Title

centile - Report centile and confidence interval

Syntax
Remarks and examples References

## Menu

Stored results Also see

Description
Methods and formulas

Options
Acknowledgment

## Syntax

```
centile [varlist][if][in][,options]
```

options Description

Main
centile(numlist) report specified centiles; default is centile(50)
Options
cci binomial exact; conservative confidence interval
normal
meansd
level(\#) normal, based on observed centiles normal, based on mean and standard deviation set confidence level; default is level (95)

## Menu

Statistics $>$ Summaries, tables, and tests $>$ Summary and descriptive statistics $>$ Centiles with Cls

## Description

centile estimates specified centiles and calculates confidence intervals. If no varlist is specified, centile calculates centiles for all the variables in the dataset. If centile() is not specified, medians (centile(50)) are reported.

By default, centile uses a binomial method for obtaining confidence intervals that makes no assumptions about the underlying distribution of the variable.

## Options

centile (numlist) specifies the centiles to be reported. The default is to display the 50 th centile.
Specifying centile(5) requests that the fifth centile be reported. Specifying centile( 550
95) requests that the 5 th, 50 th, and 95 th centiles be reported. Specifying centile (10(10) 90)
requests that the 10 th, 20 th, ..., 90th centiles be reported; see [U] $\mathbf{1 1 . 1 . 8}$ numlist.

Options
cci (conservative confidence interval) forces the confidence limits to fall exactly on sample values. Confidence intervals displayed with the cci option are slightly wider than those with the default (nocci) option.
normal causes the confidence interval to be calculated by using a formula for the standard error of a normal-distribution quantile given by Kendall and Stuart (1969, 237). The normal option is useful when you want empirical centiles - that is, centiles based on sample order statistics rather than on the mean and standard deviation-and are willing to assume normality.
meansd causes the centile and confidence interval to be calculated based on the sample mean and standard deviation, and it assumes normality.
level(\#) specifies the confidence level, as a percentage, for confidence intervals. The default is level (95) or as set by set level; see [R] level.

## Remarks and examples

The $q$ th centile of a continuous random variable, $X$, is defined as the value of $C_{q}$, which fulfills the condition $\operatorname{Pr}\left(X \leq C_{q}\right)=q / 100$. The value of $q$ must be in the range $0<q<100$, though $q$ is not necessarily an integer. By default, centile estimates $C_{q}$ for the variables in varlist and for the values of $q$ given in centile(numlist). It makes no assumptions about the distribution of $X$, and, if necessary, uses linear interpolation between neighboring sample values. Extreme centiles (for example, the 99th centile in samples smaller than 100) are fixed at the minimum or maximum sample value. An "exact" confidence interval for $C_{q}$ is also given, using the binomial-based method described below in Methods and formulas and in Conover (1999, 143-148). Again linear interpolation is used to improve the accuracy of the estimated confidence limits, but extremes are fixed at the minimum or maximum sample value.

You can prevent centile from interpolating when calculating binomial-based confidence intervals by specifying cci. The resulting intervals are generally wider than with the default; that is, the coverage (confidence level) tends to be greater than the nominal value (given as usual by level(\#), by default $95 \%$ ).

If the data are believed to be normally distributed (a common case), there are two alternative methods for estimating centiles. If normal is specified, $C_{q}$ is calculated, as just described, but its confidence interval is based on a formula for the standard error (se) of a normal-distribution quantile given by Kendall and Stuart $(1969,237)$. If meansd is alternatively specified, $C_{q}$ is estimated as $\bar{x}+z_{q} \times s$, where $\bar{x}$ and $s$ are the sample mean and standard deviation, and $z_{q}$ is the $q$ th centile of the standard normal distribution (for example, $z_{95}=1.645$ ). The confidence interval is derived from the se of the estimate of $C_{q}$.

## > Example 1

Using auto.dta, we estimate the 5th, 50th, and 95th centiles of the price variable:

| ```(1978 Automobile Data) . format price %8.2fc . centile price, centile(5 50 95)``` |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Variable | Obs | Percentile | Centile | [95\% Con | Interp. Interval] |
| price | 74 | 5 | 3,727.75 | 3,291.23 | 3,914.16 |
|  |  | 50 | 5,006.50 | 4,593.57 | 5,717.90 |
|  |  | 95 | 13,498.00 | 11,061.53 | 15,865.30 |

summarize produces somewhat different results from centile; see Methods and formulas.

```
. summarize price, detail
```

Price

|  | Percentiles | Smallest |  |  |
| ---: | ---: | ---: | :--- | ---: |
| $1 \%$ | 3291 | 3291 |  |  |
| $5 \%$ | 3748 | 3299 |  | 74 |
| $10 \%$ | 3895 | 3667 | Obs | 74 |
| $25 \%$ | 4195 | 3748 | Sum of Wgt. | 6165.257 |
| $50 \%$ | 5006.5 |  | Mean | 2949.496 |
|  |  | Largest | Std. Dev. |  |
| $75 \%$ | 6342 | 13466 |  | 8699526 |
| $90 \%$ | 11385 | 13594 | Variance | 1.653434 |
| $95 \%$ | 13466 | 14500 | Skewness | 4.819188 |

The confidence limits produced by using the cci option are slightly wider than those produced without this option:

```
. centile price, c(5 50 95) cci
```

| Variable | Obs | Percentile | Centile | - Binomi <br> [95\% Conf | $1 \text { Exact - }$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| price | 74 | 5 | 3,727.75 | 3,291.00 | 3,955.00 |
|  |  | 50 | 5,006.50 | 4,589.00 | 5,719.00 |
|  |  | 95 | 13,498.00 | 10,372.00 | 15,906.00 |

If we are willing to assume that price is normally distributed, we could include either the normal or the meansd option:

```
. centile price, c(5 50 95) normal
```

| Variable | Obs | Percentile | - Normal, based on observed centiles Centile [95\% Conf. Interval] |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| price | 74 | 5 | 3,727.75 | 3,211.19 | 4,244.31 |
|  |  | 50 | 5,006.50 | 4,096.68 | 5,916.32 |
|  |  | 95 | 13,498.00 | 5,426.81 | 21,569.19 |

. centile price, c(5 50 95) meansd

| Variable | Obs | Percentile | Normal, based on mean and std. dev.Centile [95\% Conf. Interval] |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| price | 74 | 5 | 1,313.77 | 278.93 | 2,348.61 |
|  |  | 50 | 6,165.26 | 5,493.24 | 6,837.27 |
|  |  | 95 | 11,016.75 | 9,981.90 | 12,051.59 |

With the normal option, the centile estimates are, by definition, the same as before. The confidence intervals for the 5th and 50th centiles are similar to the previous ones, but the interval for the 95th centile is different. The results using the meansd option also differ from both previous sets of estimates.

We can use sktest (see [R] sktest) to check the correctness of the normality assumption:

```
. sktest price
```

                    Skewness/Kurtosis tests for Normality
    | Variable | Obs | $\operatorname{Pr}($ Skewness $)$ | $\operatorname{Pr}$ (Kurtosis) | adj $\overline{c h i 2(2)}$ | joint $\overline{\text { Prob>chi2 }}$ <br> price |
| ---: | :---: | :---: | :---: | :---: | :---: |
| 74 | 0.0000 | 0.0127 | 21.77 | 0.0000 |  |

sktest reveals that price is definitely not normally distributed, so the normal assumption is not reasonable, and the normal and meansd options are not appropriate for these data. We should rely on the results from the default choice, which does not assume normality. If the data are normally distributed, however, the precision of the estimated centiles and their confidence intervals will be ordered (best) meansd $>$ normal $>$ [default] (worst). The normal option is useful when we really do want empirical centiles (that is, centiles based on sample order statistics rather than on the mean and standard deviation) but are willing to assume normality.

## Stored results

```
centile stores the following in r():
Scalars
    r(lb_#) #-requested centile lower confidence bound
    r(ub_#) #-requested centile upper confidence bound
Macros
    r(centiles) centiles requested
```


## Methods and formulas

Methods and formulas are presented under the following headings:
Default case
Normal case
meansd case

## Default case

The calculation is based on the method of Mood and Graybill $(1963,408)$. Let $x_{1} \leq x_{2} \leq \cdots \leq x_{n}$ be a sample of size $n$ arranged in ascending order. Denote the estimated $q$ th centile of the $x$ 's as $c_{q}$. We require that $0<q<100$. Let $R=(n+1) q / 100$ have integer part $r$ and fractional part $f$; that is, $r=\operatorname{int}(R)$ and $f=R-r$. (If $R$ is itself an integer, then $r=R$ and $f=0$.) Note that $0 \leq r \leq n$. For convenience, define $x_{0}=x_{1}$ and $x_{n+1}=x_{n} . C_{q}$ is estimated by

$$
c_{q}=x_{r}+f \times\left(x_{r+1}-x_{r}\right)
$$

that is, $c_{q}$ is a weighted average of $x_{r}$ and $x_{r+1}$. Loosely speaking, a (conservative) $p \%$ confidence interval for $C_{q}$ involves finding the observations ranked $t$ and $u$, which correspond, respectively, to the $\alpha=(100-p) / 200$ and $1-\alpha$ quantiles of a binomial distribution with parameters $n$ and $q / 100$, that is, $\mathrm{B}(n, q / 100)$. More precisely, define the $i$ th value $(i=0, \ldots, n)$ of the cumulative binomial distribution function as $F_{i}=\operatorname{Pr}(S \leq i)$, where $S$ has distribution $\mathrm{B}(n, q / 100)$. For convenience, let $F_{-1}=0$ and $F_{n+1}=1 . t$ is found such that $F_{t} \leq \alpha$ and $F_{t+1}>\alpha$, and $u$ is found such that $1-F_{u} \leq \alpha$ and $1-F_{u-1}>\alpha$.

With the cci option in force, the (conservative) confidence interval is $\left(x_{t+1}, x_{u+1}\right)$, and its actual coverage probability is $F_{u}-F_{t}$.

The default case uses linear interpolation on the $F_{i}$ as follows. Let

$$
\begin{aligned}
g & =\left(\alpha-F_{t}\right) /\left(F_{t+1}-F_{t}\right) \\
h & =\left\{\alpha-\left(1-F_{u}\right)\right\} /\left\{\left(1-F_{u-1}\right)-\left(1-F_{u}\right)\right\} \\
& =\left(\alpha-1+F_{u}\right) /\left(F_{u}-F_{u-1}\right)
\end{aligned}
$$

The interpolated lower and upper confidence limits $\left(c_{q L}, c_{q U}\right)$ for $C_{q}$ are

$$
\begin{aligned}
c_{q L} & =x_{t+1}+g \times\left(x_{t+2}-x_{t+1}\right) \\
c_{q U} & =x_{u+1}-h \times\left(x_{u+1}-x_{u}\right)
\end{aligned}
$$

Suppose that we want a $95 \%$ confidence interval for the median of a sample of size 13. $n=13$, $q=50, p=95, \alpha=0.025, R=14 \times 50 / 100=7$, and $f=0$. Therefore, the median is the 7th observation. Some example data, $x_{i}$, and the values of $F_{i}$ are as follows:

| $i$ | $F_{i}$ | $1-F_{i}$ | $x_{i}$ | $i$ | $F_{i}$ | $1-F_{i}$ | $x_{i}$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0 | 0.0001 | 0.9999 | - | 7 | 0.7095 | 0.2905 | 33 |
| 1 | 0.0017 | 0.9983 | 5 | 8 | 0.8666 | 0.1334 | 37 |
| 2 | 0.0112 | 0.9888 | 7 | 9 | 0.9539 | 0.0461 | 45 |
| 3 | 0.0461 | 0.9539 | 10 | 10 | 0.9888 | 0.0112 | 59 |
| 4 | 0.1334 | 0.8666 | 15 | 11 | 0.9983 | 0.0017 | 77 |
| 5 | 0.2905 | 0.7095 | 23 | 12 | 0.9999 | 0.0001 | 104 |
| 6 | 0.5000 | 0.5000 | 28 | 13 | 1.0000 | 0.0000 | 211 |

The median is $x_{7}=33$. Also, $F_{2} \leq 0.025$ and $F_{3}>0.025$, so $t=2 ; 1-F_{10} \leq 0.025$ and $1-F_{9}>0.025$, so $u=10$. The conservative confidence interval is therefore

$$
\left(c_{50 L}, c_{50 U}\right)=\left(x_{t+1}, x_{u+1}\right)=\left(x_{3}, x_{11}\right)=(10,77)
$$

with actual coverage $F_{10}-F_{2}=0.9888-0.0112=0.9776$ ( $97.8 \%$ confidence). For the interpolation calculation, we have

$$
\begin{aligned}
& g=(0.025-0.0112) /(0.0461-0.0112)=0.395 \\
& h=(0.025-1+0.9888) /(0.9888-0.9539)=0.395
\end{aligned}
$$

So,

$$
\begin{array}{r}
c_{50 L}=x_{3}+0.395 \times\left(x_{4}-x_{3}\right)=10+0.395 \times 5=11.98 \\
c_{50 U}=x_{11}-0.395 \times\left(x_{11}-x_{10}\right)=77-0.395 \times 18=69.89
\end{array}
$$

## Normal case

The value of $c_{q}$ is as above. Its se is given by the formula

$$
s_{q}=\sqrt{q(100-q)} /\left\{100 \sqrt{n} Z\left(c_{q} ; \bar{x}, s\right)\right\}
$$

where $\bar{x}$ and $s$ are the mean and standard deviation of the $x_{i}$, and

$$
Z(Y ; \mu, \sigma)=\left(1 / \sqrt{2 \pi \sigma^{2}}\right) e^{-(Y-\mu)^{2} / 2 \sigma^{2}}
$$

is the density function of a normally distributed variable $Y$ with mean $\mu$ and standard deviation $\sigma$. The confidence interval for $C_{q}$ is $\left(c_{q}-z_{100(1-\alpha)} s_{q}, c_{q}+z_{100(1-\alpha)} s_{q}\right)$.

## meansd case

The value of $c_{q}$ is $\bar{x}+z_{q} \times s$. Its se is given by the formula

$$
s_{q}^{\star}=s \sqrt{1 / n+z_{q}^{2} /(2 n-2)}
$$

The confidence interval for $C_{q}$ is $\left(c_{q}-z_{100(1-\alpha)} \times s_{q}^{\star}, c_{q}+z_{100(1-\alpha)} \times s_{q}^{\star}\right)$.

## Acknowledgment

centile was written by Patrick Royston of the MRC Clinical Trials Unit, London, and coauthor of the Stata Press book Flexible Parametric Survival Analysis Using Stata: Beyond the Cox Model.

## References

Conover, W. J. 1999. Practical Nonparametric Statistics. 3rd ed. New York: Wiley.
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## Also see

[R] ci - Confidence intervals for means, proportions, and counts
[R] summarize - Summary statistics
[D] petile - Create variable containing percentiles

## Title

ci - Confidence intervals for means, proportions, and counts

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | Acknowledgment |
| References | Also see |  |  |

## Syntax

Syntax for ci
ci $[$ varlist $][$ if $][$ in $][$ weight $][$, options $]$

Immediate command for variable distributed as normal
cii $\#_{\text {obs }} \#_{\text {mean }} \#_{\text {sd }}[$, ciin_option $]$

Immediate command for variable distributed as binomial

$$
\text { cii } \#_{\text {obs }} \#_{\text {succ }}[\text {, ciib_options }]
$$

Immediate command for variable distributed as Poisson
cii $\#_{\text {exposure }} \#_{\text {events }}$, poisson [ciip_options]
options
Description

Main
poisson
exposure (varname)
exact
wald
wilson
agresti
jeffreys
total
separator(\#)
level(\#)
binomial binomial $0 / 1$ variables; compute exact confidence intervals
Poisson variables; compute exact confidence intervals exposure variable; implies poisson
calculate exact confidence intervals; the default calculate Wald confidence intervals calculate Wilson confidence intervals calculate Agresti-Coull confidence intervals calculate Jeffreys confidence intervals add output for all groups combined (for use with by only) draw separator line after every \# variables; default is separator (5) set confidence level; default is level (95)
by is allowed with ci; see [D] by.
aweights and fweights are allowed, but aweights may not be specified with the binomial or poisson options; see [U] 11.1.6 weight.
ciin_option
Description
level(\#) set confidence level; default is level (95)
ciib_options Description

```
level(#) set confidence level; default is level(95)
exact calculate exact confidence intervals; the default
wald
wilson
agresti
jeffreys
calculate Wald confidence intervals
calculate Wilson confidence intervals
calculate Agresti-Coull confidence intervals
calculate Jeffreys confidence intervals
```


## ciip_options

Description
*poisson
level(\#)
numbers are Poisson-distributed counts
set confidence level; default is level (95)

[^10]
## Menu

Statistics $>$ Summaries, tables, and tests $>$ Summary and descriptive statistics $>$ Confidence intervals
cii for variable distributed as normal
Statistics $>$ Summaries, tables, and tests $>$ Summary and descriptive statistics $>$ Normal Cl calculator
cii for variable distributed as binomial
Statistics $>$ Summaries, tables, and tests $>$ Summary and descriptive statistics $>$ Binomial Cl calculator
cii for variable distributed as Poisson
Statistics $>$ Summaries, tables, and tests $>$ Summary and descriptive statistics $>$ Poisson Cl calculator

## Description

ci computes standard errors and confidence intervals for each of the variables in varlist.
cii is the immediate form of ci; see [U] 19 Immediate commands for a general discussion of immediate commands.

In the binomial and Poisson variants of cii, the second number specified (\# succ or $\#_{\text {events }}$ ) must be an integer or between 0 and 1. If the number is between 0 and 1 , Stata interprets it as the fraction of successes or events and converts it to an integer number representing the number of successes or events. The computation then proceeds as if two integers had been specified.

## Options

## Main

binomial tells ci that the variables are $0 / 1$ variables and that binomial confidence intervals will be calculated. (cii produces binomial confidence intervals when only two numbers are specified.) poisson specifies that the variables (or numbers for cii) are Poisson-distributed counts; exact Poisson confidence intervals will be calculated.
exposure (varname) is used only with poisson. You do not need to specify poisson if you specify exposure(); poisson is assumed. varname contains the total exposure (typically a time or an area) during which the number of events recorded in varlist were observed.
exact, wald, wilson, agresti, and jeffreys specify that variables are $0 / 1$ and specify how binomial confidence intervals are to be calculated.
exact is the default and specifies exact (also known in the literature as Clopper-Pearson [1934]) binomial confidence intervals.
wald specifies calculation of Wald confidence intervals.
wilson specifies calculation of Wilson confidence intervals.
agresti specifies calculation of Agresti-Coull confidence intervals.
jeffreys specifies calculation of Jeffreys confidence intervals.
See Brown, Cai, and DasGupta (2001) for a discussion and comparison of the different binomial confidence intervals.
total is for use with the by prefix. It requests that, in addition to output for each by-group, output be added for all groups combined.
separator(\#) specifies how often separation lines should be inserted into the output. The default is separator (5), meaning that a line is drawn after every five variables. separator (10) would draw the line after every 10 variables. separator ( 0 ) suppresses the separation line.
level(\#) specifies the confidence level, as a percentage, for confidence intervals. The default is level (95) or as set by set level; see [R] level.

## Remarks and examples

Remarks are presented under the following headings:
Ordinary confidence intervals
Binomial confidence intervals
Poisson confidence intervals
Immediate form
Video examples

## Ordinary confidence intervals

$>$ Example 1
Without the binomial or poisson options, ci produces "ordinary" confidence intervals, meaning those that are correct if the variable is distributed normally, and asymptotically correct for all other distributions satisfying the conditions of the central limit theorem.

| . use http://www.stata-press.com/data/r13/auto |
| :--- |
| (1978 Automobile Data) |
| . ci mpg price      <br> Variable Obs Mean Std. Err.   <br> [95\% Conf. Interval]      <br> price 74 21.2973 .6725511 19.9569 22.63769$\quad 74$ |

The standard error of the mean of mpg is 0.67 , and the $95 \%$ confidence interval is [19.96, 22.64]. We can obtain wider confidence intervals, $99 \%$, by typing

| . ci mpg price, level(99) |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: |
| Variable | Obs | Mean | Std. Err. | [99\% Conf. Interval] |  |
| mpg | 74 | 21.2973 | .6725511 | 19.51849 | 23.07611 |
| price | 74 | 6165.257 | 342.8719 | 5258.405 | 7072.108 |

## > Example 2

by () breaks out the confidence intervals according to by-group; total adds an overall summary. For instance,
. ci mpg, by(foreign) total

| Variable | Obs | Mean | Std. Err. | [95\% Conf. | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| mpg | 52 | 19.82692 | . 657777 | 18.50638 | 21.14747 |
| -> foreign = Foreign |  |  |  |  |  |
| Variable | Obs | Mean | Std. Err. | [95\% Conf. | Interval] |
| mpg | 22 | 24.77273 | 1.40951 | 21.84149 | 27.70396 |
| -> Total |  |  |  |  |  |
| Variable | Obs | Mean | Std. Err | [95\% Conf. | Interval] |
| mpg | 74 | 21.2973 | . 6725511 | 19.9569 | 22.63769 |

## - Technical note

You can control the formatting of the numbers in the output by specifying a display format for the variable; see [U] 12.5 Formats: Controlling how data are displayed. For instance,
. format mpg \%9.2f
. ci mpg

| Variable | Obs | Mean | Std. Err. | [95\% Conf. Interval] |  |
| ---: | :---: | :---: | :---: | ---: | ---: |
| mpg | 74 | 21.30 | 0.67 | 19.96 | 22.64 |

## Binomial confidence intervals

## > Example 3

We have data on employees, including a variable marking whether the employee was promoted last year.
. use http://www.stata-press.com/data/r13/promo
. ci promoted, binomial

| Variable | Obs | Mean | Std. Err. | — Binomial Exact — |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| [95\% Conf. Interval] |  |  |  |  |  |

The above interval is the default for binomial data, known equivalently as both the exact binomial and the Clopper-Pearson interval.

Nominally, the interpretation of a $95 \%$ confidence interval is that under repeated samples or experiments, $95 \%$ of the resultant intervals would contain the unknown parameter in question. However, for binomial data, the actual coverage probability, regardless of method, usually differs from that interpretation. This result occurs because of the discreteness of the binomial distribution, which produces only a finite set of outcomes, meaning that coverage probabilities are subject to discrete jumps and the exact nominal level cannot always be achieved. Therefore, the term exact confidence interval refers to its being derived from the binomial distribution, the distribution exactly generating the data, rather than resulting in exactly the nominal coverage.

For the Clopper-Pearson interval, the actual coverage probability is guaranteed to be greater than or equal to the nominal confidence level, here $95 \%$. Because of the way it is calculated-see Methods and formulas-it may also be interpreted as follows: If the true probability of being promoted were 0.012 , the chances of observing a result as extreme or more extreme than the result observed ( $20 \times 0.1=2$ or more promotions) would be $2.5 \%$. If the true probability of being promoted were 0.317 , the chances of observing a result as extreme or more extreme than the result observed (two or fewer promotions) would be $2.5 \%$.

## > Example 4

The Clopper-Pearson interval is desirable because it guarantees nominal coverage; however, by dropping this restriction, you may obtain accurate intervals that are not as conservative. In this vein, you might opt for the Wilson (1927) interval,

```
. ci promoted, binomial wilson
```

| Variable | Obs | Mean | Std. Err. | [95\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: | ---: |
| promoted | 20 | .1 | .067082 | .0278665 | .3010336 |

the Agresti-Coull (1998) interval,

```
. ci promoted, binomial agresti
```

| Variable | Obs | Mean | Std. Err. | [9gresti-Coull[95\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| promoted | 20 | .1 | .067082 | .0156562 | .3132439 |

or the Bayesian-derived Jeffreys interval (Brown, Cai, and DasGupta 2001), . ci promoted, binomial jeffreys

| Variable | Obs | Mean | Std. Err. | [95\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: | ---: |
| promoted | 20 | .1 | .067082 | .0213725 | .2838533 |

Picking the best interval is a matter of balancing accuracy (coverage) against precision (average interval length) and depends on sample size and success probability. Brown, Cai, and DasGupta (2001) recommend the Wilson or Jeffreys interval for small sample sizes $(\leq 40)$ yet favor the Agresti-Coull interval for its simplicity, decent performance for sample sizes less than or equal to 40 , and performance comparable to Wilson/Jeffreys for sample sizes greater than 40 . They also deem the Clopper-Pearson interval to be "wastefully conservative and [...] not a good choice for practical use", unless of course one requires, at a minimum, the nominal coverage level.

Finally, the binomial Wald confidence interval is obtained by specifying the binomial and wald options. The Wald interval is the one taught in most introductory statistics courses and for the above is simply, for level $1-\alpha$, Mean $\pm z_{\alpha}$ (Std. Err.), where $z_{\alpha}$ is the $1-\alpha / 2$ quantile of the standard normal. Because its overall poor performance makes it impractical, the Wald interval is available mainly for pedagogical purposes. The binomial Wald interval is also similar to the interval produced by treating binary data as normal data and using ci without the binomial option, with two exceptions. First, when binomial is specified, the calculation of the standard error uses denominator $n$ rather than $n-1$, used for normal data. Second, confidence intervals for normal data are based on the $t$ distribution rather than the standard normal. Of course, both discrepancies vanish as sample size increases.

## Technical note

Let's repeat example 3, but this time with data in which there are no promotions over the observed period:
. use http://www.stata-press.com/data/r13/promonone
. ci promoted, binomial

| Variable | Obs | Mean | Std. Err. | — Binomial Exact — <br> [95\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| promoted | 20 | 0 | 0 | 0 | $.1684335 *$ |
| (*) one-sided, $97.5 \%$ confidence interval |  |  |  |  |  |

The confidence interval is $[0,0.168]$, and this is the confidence interval that most books publish. It is not, however, a true $95 \%$ confidence interval because the lower tail has vanished. As Stata notes, it is a one-sided, $97.5 \%$ confidence interval. If you wanted to put $5 \%$ in the right tail, you could type ci promoted, binomial level(90).

## - Technical note

ci with the binomial option ignores any variables that do not take on the values 0 and 1 exclusively. For instance, with our automobile dataset,

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. ci mpg foreign, binomial
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Err. & — Binomial Exact - & [95\% Conf. Interval] \\
\hline foreign & 74 & .2972973 & .0531331 & .196584 & .4148353
\end{tabular}
```

We also requested the confidence interval for mpg, but Stata ignored us. It does that so you can type ci, binomial and obtain correct confidence intervals for all the variables that are $0 / 1$ in your data.

## Poisson confidence intervals

> Example 5
We have data on the number of bacterial colonies on a Petri dish. The dish has been divided into 36 small squares, and the number of colonies in each square has been counted. Each observation in our dataset represents a square on the dish. The variable count records the number of colonies in each square counted, which varies from 0 to 5 .

```
. use http://www.stata-press.com/data/r13/petri
. ci count, poisson
\begin{tabular}{r|rrrrr} 
Variable & Exposure & Mean & Std. Err. & \begin{tabular}{c} 
— Poisson Exact — \\
[95\% Conf. Interval]
\end{tabular} \\
\hline count & 36 & 2.333333 & .2545875 & 1.861158 & 2.888825
\end{tabular}
```

ci reports that the average number of colonies per square is 2.33 . If the expected number of colonies per square were as low as 1.86 , the probability of observing 2.33 or more colonies per square would be $2.5 \%$. If the expected number were as large as 2.89 , the probability of observing 2.33 or fewer colonies per square would be $2.5 \%$.

## Technical note

The number of "observations" -how finely the Petri dish is divided-makes no difference. The Poisson distribution is a function only of the count. In example 4, we observed a total of $2.33 \times 36=84$ colonies and a confidence interval of $[1.86 \times 36,2.89 \times 36]=[67,104]$. We would obtain the same $[67,104]$ confidence interval if our dish were divided into, say, 49 squares, rather than 36.

For the counts, it is not even important that all the squares be of the same size. For rates, however, such differences do matter, but in an easy-to-calculate way. Rates are obtained from counts by dividing by exposure, which is typically a number multiplied by either time or an area. For our Petri dishes, we divide by an area to obtain a rate, but if our example were cast in terms of being infected by a disease, we might divide by person-years to obtain the rate. Rates are convenient because they are easier to compare: we might have 2.3 colonies per square inch or 0.0005 infections per person-year.

So, let's assume that we wish to obtain the number of colonies per square inch, and, moreover, that not all the "squares" on our dish are of equal size. We have a variable called area that records the area of each "square":
. ci count, exposure(area)

| Variable | Exposure | Mean | Std. Err. | —Poisson Exact — <br> [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: |
| count | 3 | 28 | 3.055051 | 22.3339 | 34.66591 |

The rates are now in more familiar terms. In our sample, there are 28 colonies per square inch and the $95 \%$ confidence interval is [22.3, 34.7]. When we did not specify exposure(), ci assumed that each observation contributed 1 to exposure.

- Technical note

As with the binomial option, if there were no colonies on our dish, ci would calculate a one-sided confidence interval:
. use http://www.stata-press.com/data/r13/petrinone
. ci count, poisson

| Variable | Exposure | Mean | Std. Err. | — Poisson Exact — <br> [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| count | 36 | 0 | 0 | 0 | $.1024689 *$ |

(*) one-sided, $97.5 \%$ confidence interval

## Immediate form

$>$ Example 6
We are reading a soon-to-be-published paper by a colleague. In it is a table showing the number of observations, mean, and standard deviation of 1980 median family income for the Northeast and West. We correctly think that the paper would be much improved if it included the confidence intervals. The paper claims that for 166 cities in the Northeast, the average of median family income is $\$ 19,509$ with a standard deviation of $\$ 4,379$ :

For the Northeast:

| cii 166 <br> Variable | Obs | Mean | Std. Err. | [95\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 166 | 19509 | 339.8763 | 18837.93 | 20180.07 |

For the West:

| cii 256 <br> Variable | Obs | Mean | Std. Err. |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | 256 | 22557 | 312.6875 | 21941.22 | 23172.78 |

## > Example 7

We flip a coin 10 times, and it comes up heads only once. We are shocked and decide to obtain a $99 \%$ confidence interval for this coin:
. cii 10 1, level(99)

| Variable | Obs | Mean | Std. Err. | [99\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: | ---: |
|  | 10 | .1 | .0948683 | .0005011 | .5442871 |

## > Example 8

The number of reported traffic accidents in Santa Monica over a 24 -hour period is 27 . We need know nothing else:
. cii 127 , poisson

| Variable | Exposure | Mean | Std. Err. | — Poisson Exact — <br> [95\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 27 | 5.196152 | 17.79317 | 39.28358 |

## Video examples

Immediate commands in Stata: Confidence intervals for Poisson data
Immediate commands in Stata: Confidence intervals for binomial data
Immediate commands in Stata: Confidence intervals for normal data

## Stored results

ci and cii store the following in r() :

| Scalars |  |  |  |
| :--- | :--- | :--- | :--- |
| $r(N)$ | number of observations or exposure | $r(l b)$ | lower bound of confidence interval |
| $r(m e a n)$ | mean |  |  |
| $r(s e)$ | estimate of standard error |  |  |

## Methods and formulas

Methods and formulas are presented under the following headings:

Ordinary<br>Binomial<br>Poisson

## Ordinary

Define $n, \bar{x}$, and $s^{2}$ as, respectively, the number of observations, (weighted) average, and (unbiased) estimated variance of the variable in question; see $[R]$ summarize.

The standard error of the mean, $s_{\mu}$, is defined as $\sqrt{s^{2} / n}$.
Let $\alpha$ be $1-l / 100$, where $l$ is the significance level specified by the user. Define $t_{\alpha}$ as the two-sided $t$ statistic corresponding to a significance level of $\alpha$ with $n-1$ degrees of freedom; $t_{\alpha}$ is obtained from Stata as invttail $(n-1,0.5 * \alpha)$. The lower and upper confidence bounds are, respectively, $\bar{x}-s_{\mu} t_{\alpha}$ and $\bar{x}+s_{\mu} t_{\alpha}$.

## Binomial

Given $k$ successes of $n$ trials, the estimated probability is $\widehat{p}=k / n$ with standard error $\sqrt{\widehat{p}(1-\widehat{p}) / n}$. ci calculates the exact (Clopper-Pearson) confidence interval $\left[p_{1}, p_{2}\right]$ such that

$$
\operatorname{Pr}\left(K \geq k \mid p=p_{1}\right)=\alpha / 2
$$

and

$$
\operatorname{Pr}\left(K \leq k \mid p=p_{2}\right)=\alpha / 2
$$

where $K$ is distributed as binomial $(n, p)$. The endpoints may be obtained directly by using Stata's invbinomial() function. If $k=0$ or $k=n$, the calculation of the appropriate tail is skipped.

The Wald interval is $\widehat{p} \pm z_{\alpha} \sqrt{\widehat{p}(1-\widehat{p}) / n}$, where $z_{\alpha}$ is the $1-\alpha / 2$ quantile of the standard normal. The interval is obtained by inverting the acceptance region of the large-sample Wald test of $H_{0}: p=p_{0}$ versus the two-sided alternative. That is, the confidence interval is the set of all $p_{0}$ such that

$$
\left|\frac{\widehat{p}-p_{0}}{\sqrt{n^{-1} \widehat{p}(1-\widehat{p})}}\right| \leq z_{\alpha}
$$

The Wilson interval is a variation on the Wald interval, using the null standard error $\sqrt{n^{-1} p_{0}\left(1-p_{0}\right)}$ in place of the estimated standard error $\sqrt{n^{-1} \widehat{p}(1-\widehat{p})}$ in the above expression. Inverting this acceptance region is more complicated yet results in the closed form

$$
\frac{k+z_{\alpha}^{2} / 2}{n+z_{\alpha}^{2}} \pm \frac{z_{\alpha} n^{1 / 2}}{n+z_{\alpha}^{2} / 2}\left\{\widehat{p}(1-\widehat{p})+\frac{z_{\alpha}^{2}}{4 n}\right\}^{1 / 2}
$$

The Agresti-Coull interval is basically a Wald interval that borrows its center from the Wilson interval. Defining $\widetilde{k}=k+z_{\alpha}^{2} / 2, \widetilde{n}=n+z_{\alpha}^{2}$, and (hence) $\widetilde{p}=\widetilde{k} / \widetilde{n}$, the Agresti-Coull interval is

$$
\widetilde{p} \pm z_{\alpha} \sqrt{\widetilde{p}(1-\widetilde{p}) / \widetilde{n}}
$$

When $\alpha=0.05, z_{\alpha}$ is near enough to 2 that $\widetilde{p}$ can be thought of as a typical estimate of proportion where two successes and two failures have been added to the sample (Agresti and Coull 1998). This typical estimate of proportion makes the Agresti-Coull interval an easy-to-present alternative for introductory statistics students.

The Jeffreys interval is a Bayesian interval and is based on the Jeffreys prior, which is the $\operatorname{Beta}(1 / 2,1 / 2)$ distribution. Assigning this prior to $p$ results in a posterior distribution for $p$ that is Beta with parameters $k+1 / 2$ and $n-k+1 / 2$. The Jeffreys interval is then taken to be the $1-\alpha$ central posterior probability interval, namely, the $\alpha / 2$ and $1-\alpha / 2$ quantiles of the $\operatorname{Beta}(k+1 / 2, n-k+1 / 2)$ distribution. These quantiles may be obtained directly by using Stata's invibeta() function.

## Poisson

Given the total cases, $k$, the estimate of the expected count $\lambda$ is $k$, and its standard error is $\sqrt{k}$. ci calculates the exact confidence interval $\left[\lambda_{1}, \lambda_{2}\right]$ such that

$$
\operatorname{Pr}\left(K \geq k \mid \lambda=\lambda_{1}\right)=\alpha / 2
$$

and

$$
\operatorname{Pr}\left(K \leq k \mid \lambda=\lambda_{2}\right)=\alpha / 2
$$

where $K$ is Poisson with mean $\lambda$. Solution is by Newton's method. If $k=0$, the calculation of $\lambda_{1}$ is skipped. All values are then reported as rates, which are the above numbers divided by the total exposure.

Harold Jeffreys (1891-1989) was born near Durham, England, and spent more than 75 years studying and working at the University of Cambridge, principally on theoretical and observational problems in geophysics, astronomy, mathematics, and statistics. He developed a systematic Bayesian approach to inference in his monograph Theory of Probability.
Edwin Bidwell (E. B.) Wilson (1879-1964) majored in mathematics at Harvard and studied and taught at Yale and MIT before returning to Harvard in 1922. He worked in mathematics, physics, and statistics. His method for binomial intervals can be considered a precursor, for a particular problem, of Neyman's concept of confidence intervals.
Jerzy Neyman (1894-1981) was born in Bendery, Russia, now Moldavia. He studied and then taught at Kharkov University, moving from physics to mathematics. In 1921, Neyman moved to Poland, where he worked in statistics at Bydgoszcz and then Warsaw. Neyman received a Rockefeller Fellowship to work with Karl Pearson at University College London. There, he collaborated with Egon Pearson, Karl's son, on the theory of hypothesis testing. Life in Poland became progressively more difficult, and Neyman returned to UCL to work there from 1934 to 1938. At this time, he published on the theory of confidence intervals. He then was offered a post in California at Berkeley, where he settled. Neyman established an outstanding statistics department and remained highly active in research, including applications in astronomy, meteorology, and medicine. He was one of the great statisticians of the 20th century.

## Acknowledgment

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## Also see

$[R]$ ameans - Arithmetic, geometric, and harmonic means
[R] bitest - Binomial probability test
$[\mathrm{R}]$ centile - Report centile and confidence interval
[D] pctile - Create variable containing percentiles
[R] prtest - Tests of proportions
$[R]$ summarize - Summary statistics
$[\mathrm{R}]$ ttest $-t$ tests (mean-comparison tests)

## Title

clogit — Conditional (fixed-effects) logistic regression

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

clogit depvar [indepvars] [if] [in] [weight], group(varname) [options]
options
Description

## Model

* group (varname)
offset (varname)
constraints (constraints)
collinear
SE/Robust
vce (vcetype)
nonest
Reporting
level(\#)
or
nocnsreport
display_options

Maximization
maximize_options
coeflegend or jackknife
report odds ratios
matched group variable
include varname in model with coefficient constrained to 1
apply specified linear constraints
keep collinear variables
vcetype may be oim, robust, cluster clustvar, opg, bootstrap,
do not check that panels are nested within clusters
set confidence level; default is level (95)
do not display constraints
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
*group (varname) is required.
indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
bootstrap, by, fp, jackknife, mfp, mi estimate, nestreg, rolling, statsby, stepwise, and svy are allowed; see [U] 11.1.10 Prefix commands.
vce(bootstrap) and vce(jackknife) are not allowed with the mi estimate prefix; see [MI] mi estimate.
Weights are not allowed with the bootstrap prefix; see $[R]$ bootstrap.
vce(), nonest, and weights are not allowed with the svy prefix; see [SVY] svy.
fweights, iweights, and pweights are allowed (see [U] 11.1.6 weight), but they are interpreted to apply to groups as a whole, not to individual observations. See Use of weights below.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Categorical outcomes $>$ Conditional logistic regression

## Description

clogit fits what biostatisticians and epidemiologists call conditional logistic regression for matched case-control groups (see, for example, Hosmer, Lemeshow, and Sturdivant [2013, chap. 7]) and what economists and other social scientists call fixed-effects logit for panel data (see, for example, Chamberlain [1980]). Computationally, these models are the same. depvar equal to nonzero and nonmissing (typically depvar equal to one) indicates a positive outcome, whereas depvar equal to zero indicates a negative outcome.

See [R] asclogit if you want to fit McFadden's choice model (McFadden 1974). Also see [R] logistic for a list of related estimation commands.

## Options

Model
group (varname) is required; it specifies an identifier variable (numeric or string) for the matched groups. strata(varname) is a synonym for group().
offset (varname), constraints (constraints), collinear; see [R] estimation options.

## SE/Robust

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] ve__option.
nonest, available only with vce(cluster clustvar), prevents checking that matched groups are nested within clusters. It is the user's responsibility to verify that the standard errors are theoretically correct.
level(\#); see [R] estimation options.
or reports the estimated coefficients transformed to odds ratios, that is, $e^{b}$ rather than $b$. Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated. or may be specified at estimation or when replaying previously estimated results.
nocnsreport; see $[R]$ estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\%fmt), pformat(\%fmt), sformat(\%fmt), and nolstretch; see [R] estimation options.

Maximization
maximize_options: difficult, technique (algorithm_spec), iterate (\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance(\#), nonrtolerance, and from (init_specs); see [R] maximize. These options are seldom used.

Setting the optimization type to technique (bhhh) resets the default vcetype to vce (opg).

The following option is available with clogit but is not shown in the dialog box: coeflegend; see $[R]$ estimation options.

## Remarks and examples

Remarks are presented under the following headings:
Introduction
Matched case-control data
Use of weights
Fixed-effects logit

## Introduction

clogit fits maximum likelihood models with a dichotomous dependent variable coded as $0 / 1$ (more precisely, clogit interprets 0 and not 0 to indicate the dichotomy). Conditional logistic analysis differs from regular logistic regression in that the data are grouped and the likelihood is calculated relative to each group; that is, a conditional likelihood is used. See Methods and formulas at the end of this entry.

Biostatisticians and epidemiologists fit these models when analyzing matched case-control studies with $1: 1$ matching, $1: k_{2 i}$ matching, or $k_{1 i}: k_{2 i}$ matching, where $i$ denotes the $i$ th matched group for $i=1,2, \ldots, n$, where $n$ is the total number of groups. clogit fits a model appropriate for all of these matching schemes or for any mix of the schemes because the matching $k_{1 i}: k_{2 i}$ can vary from group to group. clogit always uses the true conditional likelihood, not an approximation. Biostatisticians and epidemiologists sometimes refer to the matched groups as "strata", but we will stick to the more generic term "group".

Economists and other social scientists fitting fixed-effects logit models have data that look exactly like the data biostatisticians and epidemiologists call $k_{1 i}: k_{2 i}$ matched case-control data. In terms of how the data are arranged, $k_{1 i}: k_{2 i}$ matching means that in the $i$ th group, the dependent variable is 1 a total of $k_{1 i}$ times and 0 a total of $k_{2 i}$ times. There are a total of $T_{i}=k_{1 i}+k_{2 i}$ observations for the $i$ th group. This data arrangement is what economists and other social scientists call "panel data", "longitudinal data", or "cross-sectional time-series data".

So no matter what terminology you use, the computation and the use of the clogit command is the same. The following example shows how your data should be arranged to use clogit.

## > Example 1

Suppose that we have grouped data with the variable id containing a unique identifier for each group. Our outcome variable, $y$, contains 0 s and 1 s . If we were biostatisticians, $\mathrm{y}=1$ would indicate a case, $\mathrm{y}=0$ would be a control, and id would be an identifier variable that indicates the groups of matched case-control subjects.

If we were economists, $y=1$ might indicate that a person was unemployed at any time during a year and $\mathrm{y}=0$, that a person was employed all year, and id would be an identifier variable for persons.

If we list the first few observations of this dataset, it looks like

```
. use http://www.stata-press.com/data/r13/clogitid
. list y x1 x2 id in 1/11
```

|  | y | x1 | x2 | id |
| :---: | :---: | :---: | :---: | :---: |
| 1. | 0 | 0 | 4 | 1014 |
| 2. | 0 | 1 | 4 | 1014 |
| 3. | 0 | 1 | 6 | 1014 |
| 4. | 1 | 1 | 8 | 1014 |
| 5. | 0 | 0 | 1 | 1017 |
| 6. | 0 | 0 | 7 | 1017 |
| 7. | 1 | 1 | 10 | 1017 |
| 8. | 0 | 0 | 1 | 1019 |
| 9. | 0 | 1 | 7 | 1019 |
| 10. | 1 | 1 | 7 | 1019 |
| 11. | 1 | 1 | 9 | 1019 |

Pretending that we are biostatisticians, we describe our data as follows. The first group (id $=1014$ ) consists of four matched persons: 1 case $(y=1)$ and three controls $(y=0)$, that is, $1: 3$ matching. The second group has $1: 2$ matching, and the third $2: 2$.

Pretending that we are economists, we describe our data as follows. The first group consists of 4 observations (one per year) for person 1014. This person had a period of unemployment during 1 year of 4 . The second person had a period of unemployment during 1 year of 3 , and the third had a period of 2 years of 4 .

Our independent variables are x 1 and x 2 . To fit the conditional (fixed-effects) logistic model, we type

| note: multiple positive outcomes within groups encountered. |  |  |  |
| :---: | :---: | :---: | :---: |
| Iteration 1: $\quad$ log likelihood $=-123.41386$ |  |  |  |
| Iteration 2: log likelihood = -123.41386 |  |  |  |
| Conditional (fixed-effects) logistic regression | Number of obs |  | 369 |
|  | LR chi2(2) |  | 9.07 |
|  | Prob > chi2 |  | 0.0107 |
| Log likelihood = -123.41386 | Pseudo R2 |  | 0.0355 |


| y | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| x 1 | .653363 | .2875215 | 2.27 | 0.023 | .0898312 | 1.216895 |
| x 2 | .0659169 | .0449555 | 1.47 | 0.143 | -.0221943 | .1540281 |

## - Technical note

The message "note: multiple positive outcomes within groups encountered" at the top of the clogit output for the previous example merely informs us that we have $k_{1 i}: k_{2 i}$ matching with $k_{1 i}>1$ for at least one group. If your data should be $1: k_{2 i}$ matched, this message tells you that there is an error in the data somewhere.

We can see the distribution of $k_{1 i}$ and $T_{i}=k_{1 i}+k_{2 i}$ for the data of the example 1 by using the following steps:

```
. by id, sort: gen k1 = sum(y)
. by id: replace k1 = . if _n < _N
(303 real changes made, 303 to missing)
. by id: gen T = sum(y<.)
. by id: replace T = . if _n < _N
(303 real changes made, 303 to missing)
. tabulate k1
```

| k1 | Freq. | Percent | Cum. |
| ---: | ---: | ---: | ---: |
| 1 | 48 | 72.73 | 72.73 |
| 2 | 12 | 18.18 | 90.91 |
| 3 | 4 | 6.06 | 96.97 |
| 4 | 2 | 3.03 | 100.00 |
| Total | 66 | 100.00 |  |

. tabulate T

| T | Freq. | Percent | Cum. |
| ---: | ---: | ---: | ---: |
| 2 | 5 | 7.58 | 7.58 |
| 3 | 5 | 7.58 | 15.15 |
| 4 | 12 | 18.18 | 33.33 |
| 5 | 11 | 16.67 | 50.00 |
| 6 | 13 | 19.70 | 69.70 |
| 7 | 8 | 12.12 | 81.82 |
| 8 | 3 | 4.55 | 86.36 |
| 9 | 7 | 10.61 | 96.97 |
| 10 | 2 | 3.03 | 100.00 |
| Total | 66 | 100.00 |  |

We see that $k_{1 i}$ ranges from 1 to 4 and $T_{i}$ ranges from 2 to 10 for these data.

## Technical note

For $k_{1 i}: k_{2 i}$ matching (and hence in the general case of fixed-effects logit), clogit uses a recursive algorithm to compute the likelihood, which means that there are no limits on the size of $T_{i}$. However, computation time is proportional to $\sum T_{i} \min \left(k_{1 i}, k_{2 i}\right)$, so clogit will take roughly 10 times longer to fit a model with $10: 10$ matching than one with $1: 10$ matching. But clogit is fast, so computation time becomes an issue only when $\min \left(k_{1 i}, k_{2 i}\right)$ is around 100 or more. See Methods and formulas for details.

## Matched case-control data

Here we give a more detailed example of matched case-control data.

## $>$ Example 2

Hosmer, Lemeshow, and Sturdivant $(2013,24)$ present data on matched pairs of infants, each pair having one with low birthweight and another with regular birthweight. The data are matched on age of the mother. Several possible maternal exposures are considered: race (three categories), smoking status, presence of hypertension, presence of uterine irritability, previous preterm delivery, and weight at the last menstrual period.

| . use http://www.stata-press.com/data/r13/lowbirth2, clear (Applied Logistic Regression, Hosmer \& Lemeshow) <br> . describe |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Contains data from http://www.stata-press.com/data/r13/lowbirth2.dta obs: 112 Applied Logistic Regression, Hosmer \& Lemeshow |  |  |  |  |
| vars: | 9 |  |  | 30 Jan 2013 08:46 |
| size: | 1,120 |  |  |  |
| variable name | storage type | display <br> format | value <br> label | variable label |
| pairid | byte | \%8.0g |  | Case-control pair ID |
| low | byte | \%8.0g |  | Baby has low birthweight |
| age | byte | \%8.0g |  | Age of mother |
| lwt | int | \%8.0g |  | Mother's last menstrual weight |
| smoke | byte | \%8.0g |  | Mother smoked during pregnancy |
| ptd | byte | \%8.0g |  | Mother had previous preterm baby |
| ht | byte | \%8.0g |  | Mother has hypertension |
| ui | byte | \%8.0g |  | Uterine irritability |
| race | byte | \%9.0g | race | ```race of mother: 1=white, 2=black, 3=other``` |

Sorted by:
We list the case-control indicator variable, low; the match identifier variable, pairid; and two of the covariates, lwt and smoke, for the first 10 observations.
. list low lwt smoke pairid in $1 / 10$

|  | low | lwt | smoke | pairid |
| ---: | ---: | ---: | ---: | ---: |
| 1. | 0 | 135 | 0 | 1 |
| 2. | 1 | 101 | 1 | 1 |
| 3. | 0 | 98 | 0 | 2 |
| 4. | 1 | 115 | 0 | 2 |
| 5. | 0 | 95 | 0 | 3 |
| 6. | 1 | 130 | 0 | 3 |
| 7. | 0 | 103 | 0 | 4 |
| 8. | 1 | 130 | 1 | 4 |
| 9. | 0 | 122 | 1 | 5 |
| 10. | 1 | 110 | 1 | 5 |

We fit a conditional logistic model of low birthweight on mother's weight, race, smoking behavior, and history.

| $\begin{aligned} & \text { Conditional (fixed-effects) } \\ & \text { Log likelihood }=-25.794271 \end{aligned}$ |  | logistic regression |  | Number of obs LR chi2(7) <br> Prob > chi2 <br> Pseudo R2 |  |  | $\begin{array}{r} 112 \\ 26.04 \\ 0.0005 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | 0.3355 |
| low | Coef. | Std. Err. | z | $P>\|z\|$ | [95\% |  | Interval] |
| lwt | -. 0183757 | . 0100806 | -1.82 | 0.068 | -. 0381 |  | . 0013819 |
| smoke | 1.400656 | . 6278396 | 2.23 | 0.026 | . 1701 |  | 2.631199 |
| ptd | 1.808009 | . 7886502 | 2.29 | 0.022 | . 26228 |  | 3.353735 |
| ht | 2.361152 | 1.086128 | 2.17 | 0.030 | . 2323 |  | 4.489924 |
| ui | 1.401929 | . 6961585 | 2.01 | 0.044 | . 03748 |  | 2.766375 |
| race |  |  |  |  |  |  |  |
| black | . 5713643 | . 689645 | 0.83 | 0.407 | -. 7803 |  | 1.923044 |
| other | -. 0253148 | . 6992044 | -0.04 | 0.971 | -1.39 |  | 1.345101 |

We might prefer to see results presented as odds ratios. We could have specified the or option when we first fit the model, or we can now redisplay results and specify or:

| . clogit, or |  |  |  |
| :--- | :--- | :--- | :--- |
| Conditional (fixed-effects) logistic regression | Number of obs | $=$ | 112 |
|  | LR chi2 (7) | $=$ | 26.04 |
|  | Prob $>$ chi2 | $=$ | 0.0005 |
| Log likelihood $=-25.794271$ | Pseudo R2 | $=$ | 0.3355 |


| low | Odds Ratio | Std. Err. | $\mathbf{z}$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| lwt | .9817921 | .009897 | -1.82 | 0.068 | .9625847 | 1.001383 |
| smoke | 4.057862 | 2.547686 | 2.23 | 0.026 | 1.185439 | 13.89042 |
| ptd | 6.098293 | 4.80942 | 2.29 | 0.022 | 1.299894 | 28.60938 |
| ht | 10.60316 | 11.51639 | 2.17 | 0.030 | 1.261599 | 89.11467 |
| ui | 4.06303 | 2.828513 | 2.01 | 0.044 | 1.038195 | 15.90088 |
| race |  |  |  |  |  |  |
| black | 1.770681 | 1.221141 | 0.83 | 0.407 | .4582617 | 6.84175 |
| other | .975003 | .6817263 | -0.04 | 0.971 | .2476522 | 3.838573 |

Smoking, previous preterm delivery, hypertension, uterine irritability, and possibly the mother's weight all contribute to low birthweight. Race of black and race of other are statistically insignificant when compared with the race of white omitted group, although the race of black effect is large. We can test the joint statistical significance of race being black (2.race) and race being other (3.race) by using test:

```
. test 2.race 3.race
( 1) [low]2.race = 0
( 2) [low]3.race = 0
    chi2( 2) = 0.88
    Prob > chi2 = 0.6436
```

For a more complete description of test, see [R] test. test presents results in coefficients rather than odds ratios. Jointly testing that the coefficients on 2 . race and 3 .race are 0 is equivalent to jointly testing that the odds ratios are 1 .

Here one case was matched to one control, that is, 1:1 matching. From clogit's point of view, that was not important - $k_{1}$ cases could have been matched to $k_{2}$ controls ( $k_{1}: k_{2}$ matching), and we would have fit the model in the same way. Furthermore, the matching can change from group
to group, which we have denoted as $k_{1 i}: k_{2 i}$ matching, where $i$ denotes the group. clogit does not care. To fit the conditional logistic regression model, we specified the group (varname) option, group(pairid). The case and control are stored in separate observations. clogit knew that they were linked (in the same group) because the related observations share the same value of pairid.

## - Technical note

clogit provides a way to extend McNemar's test to multiple controls per case ( $1: k_{2 i}$ matching) and to multiple controls matched with multiple cases ( $k_{1 i}: k_{2 i}$ matching).

In Stata, McNemar's test is calculated by the mcc command; see [ST] epitab. The mcc command, however, requires that the matched case and control appear in one observation, so the data will need to be manipulated from 1 to 2 observations per stratum before using clogit. Alternatively, if you begin with clogit's 2-observations-per-group organization, you will have to change it to 1 observation per group if you wish to use mcc. In either case, reshape provides an easy way to change the organization of the data. We will demonstrate its use below, but we direct you to [D] reshape for a more thorough discussion.

In example 2, we used clogit to analyze the relationship between low birthweight and various characteristics of the mother. Assume that we now want to assess the relationship between low birthweight and smoking, ignoring the mother's other characteristics. Using clogit, we obtain the following results:


Let's compare our estimated odds ratio and $95 \%$ confidence interval with that produced by mcc. We begin by reshaping the data:

| Data | long | -> | wide |
| :---: | :---: | :---: | :---: |
| Number of obs. | 112 | -> | 56 |
| Number of variables | 3 | -> | 3 |
| j variable (2 values) | low | -> | (dropped) |
| , | smoke | -> | smoke0 smoke1 |

We now have the variables smoke0 (formed from smoke and low $=0$ ), recording 1 if the control mother smoked and 0 otherwise; and smoke1 (formed from smoke and low $=1$ ), recording 1 if the case mother smoked and 0 otherwise. We can now use mcc:

```
. mcc smoke1 smoke0
\(\left.\begin{array}{r|rr|r}\text { Cases } & & \begin{array}{r}\text { Controls } \\ \text { Exposed }\end{array} & \text { Unexposed }\end{array}\right)\) Total 1
McNemar's chi2(1) = 6.53 Prob > chi2 = 0.0106
Exact McNemar significance probability =0.0161
Proportion with factor
    Cases . }535714
    Controls . 2857143 [95% Conf. Interval]
    difference . 25 . }0519726 . 448027
    ratio 1.875 1.148685 3.060565
    rel. diff. . 35 . }1336258 .566374
    odds ratio 2.75 1.179154 7.143667 (exact)
```

Both methods estimated the same odds ratio, and the $95 \%$ confidence intervals are similar. clogit produced a confidence interval of $[1.22,6.18]$, whereas mcc produced a confidence interval of [1.18, 7.14].

## Use of weights

With clogit, weights apply to groups as a whole, not to individual observations. For example, if there is a group in your dataset with a frequency weight of 3, there are a total of three groups in your sample with the same values of the dependent and independent variables as this one group. Weights must have the same value for all observations belonging to the same group; otherwise, an error message will be displayed.

## > Example 3

We use the example from the above discussion of the mcc command. Here we have a total of 56 matched case-control groups, each with one case matched to one control. We had 8 matched pairs in which both the case and the control are exposed, 22 pairs in which the case is exposed and the control is unexposed, 8 pairs in which the case is unexposed and the control is exposed, and 18 pairs in which they are both unexposed.

With weights, it is easy to enter these data into Stata and run clogit.

```
. clear
. input id case exposed weight
    id case exposed weight
    1. 1 1 1 8
    2. 1 0 1 8
    3. 2 1 1 22
    4. 2 0 0 22
    5. 3 1 0 8
    6. 3018
    7. 4 1 0 18
    8. 4 0 0 18
    9. end
. clogit case exposed [w=weight], group(id) or
(frequency weights assumed)
Iteration 0: log likelihood = -35.425931
Iteration 1: log likelihood = -35.419283
Iteration 2: log likelihood = -35.419282
Conditional (fixed-effects) logistic regression
\begin{tabular}{llr} 
Number of obs & \(=\) & 112 \\
LR chi2 (1) & \(=\) & 6.79 \\
Prob > chi2 & \(=\) & 0.0091 \\
Pseudo R2 & \(=\) & 0.0875
\end{tabular}
\begin{tabular}{r|rrrrrr}
\hline case & Odds Ratio & Std. Err. & z & P>|z| & [95\% Conf. Interval] \\
\hline exposed & 2.75 & 1.135369 & 2.45 & 0.014 & 1.224347 & 6.176763 \\
\hline
\end{tabular}
```


## Fixed-effects logit

The fixed-effects logit model can be written as

$$
\operatorname{Pr}\left(y_{i t}=1 \mid \mathbf{x}_{i t}\right)=F\left(\alpha_{i}+\mathbf{x}_{i t} \boldsymbol{\beta}\right)
$$

where $F$ is the cumulative logistic distribution

$$
F(z)=\frac{\exp (z)}{1+\exp (z)}
$$

$i=1,2, \ldots, n$ denotes the independent units (called "groups" by clogit), and $t=1,2, \ldots, T_{i}$ denotes the observations for the $i$ th unit (group).

Fitting this model by using a full maximum-likelihood approach leads to difficulties, however. When $T_{i}$ is fixed, the maximum likelihood estimates for $\alpha_{i}$ and $\boldsymbol{\beta}$ are inconsistent (Andersen 1970; Chamberlain 1980). This difficulty can be circumvented by looking at the probability of $\mathbf{y}_{i}=$ $\left(y_{i 1}, \ldots, y_{i T_{i}}\right)$ conditional on $\sum_{t=1}^{T_{i}} y_{i t}$. This conditional probability does not involve the $\alpha_{i}$, so they are never estimated when the resulting conditional likelihood is used. See Hamerle and Ronning (1995) for a succinct and lucid development. See Methods and formulas for the estimation equation.

## > Example 4

We are studying unionization of women in the United States by using the union dataset; see [XT] xt. We fit the fixed-effects logit model:

```
. use http://www.stata-press.com/data/r13/union, clear
(NLS Women 14-24 in 1968)
. clogit union age grade not_smsa south black, group(idcode)
note: multiple positive outcomes within groups encountered.
note: 2744 groups (14165 obs) dropped because of all positive or
    all negative outcomes.
note: black omitted because of no within-group variance.
Iteration 0: log likelihood = -4521.3385
Iteration 1: log likelihood = -4516.1404
Iteration 2: log likelihood = -4516.1385
Iteration 3: log likelihood = -4516.1385
Conditional (fixed-effects) logistic regression Number of obs = 12035
    LR chi2(4) = 68.09
    Prob > chi2 = 0.0000
Log likelihood = -4516.1385
Pseudo R2 = 0.0075
```

| union | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| age | .0170301 | .004146 | 4.11 | 0.000 | .0089042 | .0251561 |
| grade | .0853572 | .0418781 | 2.04 | 0.042 | .0032777 | .1674368 |
| not_smsa | .0083678 | .1127963 | 0.07 | 0.941 | -.2127088 | .2294445 |
| south | -.748023 | .1251752 | -5.98 | 0.000 | -.9933619 | -.5026842 |
| black | 0 | (omitted) |  |  |  |  |

We received three messages at the top of the output. The first one, "multiple positive outcomes within groups encountered", we expected. Our data do indeed have multiple positive outcomes (union $=1$ ) in many groups. (Here a group consists of all the observations for a particular individual.)

The second message tells us that 2,744 groups were "dropped" by clogit. When either union $=0$ or union $=1$ for all observations for an individual, this individual's contribution to the log-likelihood is zero. Although these are perfectly valid observations in every sense, they have no effect on the estimation, so they are not included in the total "Number of obs". Hence, the reported "Number of obs" gives the effective sample size of the estimation. Here it is 12,035 observations - only $46 \%$ of the total 26,200.

We can easily check that there are indeed 2,744 groups with union either all 0 or all 1 . We will generate a variable that contains the fraction of observations for each individual who has union $=1$.

```
. by idcode, sort: generate fraction = sum(union)/sum(union < .)
```

. by idcode: replace fraction = . if _n < _N
(21766 real changes made, 21766 to missing)
. tabulate fraction

| fraction | Freq. | Percent | Cum. |
| ---: | ---: | ---: | ---: |
| 0 | 2,481 | 55.95 | 55.95 |
| .0833333 | 30 | 0.68 | 56.63 |
| .0909091 | 33 | 0.74 | 57.37 |
| .1 | 53 | 1.20 | 58.57 |
| (output omitted) |  |  |  |
| .9 | 10 | 0.23 | 93.59 |
| .9090909 | 11 | 0.25 | 93.84 |
| .9166667 | 10 | 0.23 | 94.07 |
| 1 | 263 | 5.93 | 100.00 |
| Total | 4,434 | 100.00 |  |

Because $2481+263=2744$, we confirm what clogit did.
The third warning message from clogit said "black omitted because of no within-group variance". Obviously, race stays constant for an individual across time. Any such variables are collinear with the $\alpha_{i}$ (that is, the fixed effects), and just as the $\alpha_{i}$ drop out of the conditional likelihood, so do all variables that are unchanging within groups. Thus they cannot be estimated with the conditional fixed-effects model.

There are several other estimators implemented in Stata that we could use with these data:

```
cloglog ... , vce(cluster idcode)
logit ... , vce(cluster idcode)
probit ... , vce(cluster idcode)
scobit ... , vce(cluster idcode)
xtcloglog ...
xtgee ... , family(binomial) link(logit) corr(exchangeable)
xtlogit ...
xtprobit ...
```

See [R] cloglog, [R] logit, [R] probit, [R] scobit, [XT] xtcloglog, [XT] xtgee, [XT] xtlogit, and [XT] xtprobit for details.

## Stored results

clogit stores the following in e():
Scalars

```
e(N)
e(N_drop)
e(N_group_drop)
e(k)
e(k_eq)
e(k_eq_model)
e(k_dv)
e(df_m)
e(r2_p)
e(ll)
e(11_0)
e(N_clust)
e(chi2)
e(p)
e(rank)
e(ic)
e(rc)
e(converged)
```

Macros
e(cmd)
e(cmdline)
e(depvar)
e(group)
$e$ (multiple)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(offset)
e(chi2type)
e(vce)
e(vcetype)
e(opt)
$e$ (which)
e(ml_method)
e(user)
e(technique)
e(properties)
e(predict)
e(marginsok)
e(marginsnotok)
e(asbalanced)
e(asobserved)
number of observations
number of observations dropped because of all positive or all negative outcomes number of groups dropped because of all positive or all negative outcomes number of parameters
number of equations in $e(b)$
number of equations in overall model test
number of dependent variables
model degrees of freedom
pseudo- $R$-squared
log likelihood
log likelihood, constant-only model
number of clusters
$\chi^{2}$
significance
rank of e(V)
number of iterations
return code
1 if converged, 0 otherwise
clogit
command as typed
name of dependent variable
name of group() variable
multiple if multiple positive outcomes within group
weight type
weight expression
title in estimation output
name of cluster variable
linear offset variable
Wald or LR; type of model $\chi^{2}$ test
vcetype specified in vce()
title used to label Std. Err.
type of optimization
max or min; whether optimizer is to perform maximization or minimization
type of ml method
name of likelihood-evaluator program
maximization technique
b V
program used to implement predict
predictions allowed by margins
predictions disallowed by margins
factor variables fvset as asbalanced
factor variables fvset as asobserved

```
Matrices
    e(b)
    e(Cns)
    e(ilog)
    e(gradient)
    e(V)
    e(V_modelbased)
```

Functions
e(sample) marks estimation sample

## Methods and formulas

Breslow and Day (1980, 247-279), Collett (2003, 251-267), and Hosmer, Lemeshow, and Sturdivant (2013, 243-268) provide a biostatistical point of view on conditional logistic regression. Hamerle and Ronning (1995) give a succinct and lucid review of fixed-effects logit; Chamberlain (1980) is a standard reference for this model. Greene (2012, chap. 17) provides a straightforward textbook description of conditional logistic regression from an economist's point of view, as well as a brief description of choice models.

Let $i=1,2, \ldots, n$ denote the groups and let $t=1,2, \ldots, T_{i}$ denote the observations for the $i$ th group. Let $y_{i t}$ be the dependent variable taking on values 0 or 1 . Let $\mathbf{y}_{i}=\left(y_{i 1}, \ldots, y_{i T_{i}}\right)$ be the outcomes for the $i$ th group as a whole. Let $\mathbf{x}_{i t}$ be a row vector of covariates. Let

$$
k_{1 i}=\sum_{t=1}^{T_{i}} y_{i t}
$$

be the observed number of ones for the dependent variable in the $i$ th group. Biostatisticians would say that there are $k_{1 i}$ cases matched to $k_{2 i}=T_{i}-k_{1 i}$ controls in the $i$ th group.

We consider the probability of a possible value of $\mathbf{y}_{i}$ conditional on $\sum_{t=1}^{T_{i}} y_{i t}=k_{1 i}$ (Hamerle and Ronning 1995, eq. 8.33; Hosmer, Lemeshow, and Sturdivant 2013, eq. 7.4),

$$
\operatorname{Pr}\left(\mathbf{y}_{i} \mid \sum_{t=1}^{T_{i}} y_{i t}=k_{1 i}\right)=\frac{\exp \left(\sum_{t=1}^{T_{i}} y_{i t} \mathbf{x}_{i t} \boldsymbol{\beta}\right)}{\sum_{\mathbf{d}_{i} \in S_{i}} \exp \left(\sum_{t=1}^{T_{i}} d_{i t} \mathbf{x}_{i t} \boldsymbol{\beta}\right)}
$$

where $d_{i t}$ is equal to 0 or 1 with $\sum_{t=1}^{T_{i}} d_{i t}=k_{1 i}$, and $S_{i}$ is the set of all possible combinations of $k_{1 i}$ ones and $k_{2 i}$ zeros. Clearly, there are $\binom{T_{i}}{k_{1 i}}$ such combinations, but we need not count all of these combinations to compute the denominator of the above equation. It can be computed recursively.

Denote the denominator by

$$
f_{i}\left(T_{i}, k_{1 i}\right)=\sum_{\mathbf{d}_{i} \in S_{i}} \exp \left(\sum_{t=1}^{T_{i}} d_{i t} \mathbf{x}_{i t} \boldsymbol{\beta}\right)
$$

Consider, computationally, how $f_{i}$ changes as we go from a total of 1 observation in the group to 2 observations to 3 , etc. Doing this, we derive the recursive formula

$$
f_{i}(T, k)=f_{i}(T-1, k)+f_{i}(T-1, k-1) \exp \left(\mathbf{x}_{i T} \boldsymbol{\beta}\right)
$$

where we define $f_{i}(T, k)=0$ if $T<k$ and $f_{i}(T, 0)=1$.

The conditional log-likelihood is

$$
\ln L=\sum_{i=1}^{n}\left\{\sum_{t=1}^{T_{i}} y_{i t} \mathbf{x}_{i t} \boldsymbol{\beta}-\log f_{i}\left(T_{i}, k_{1 i}\right)\right\}
$$

The derivatives of the conditional log-likelihood can also be computed recursively by taking derivatives of the recursive formula for $f_{i}$.

Computation time is roughly proportional to

$$
p^{2} \sum_{i=1}^{n} T_{i} \min \left(k_{1 i}, k_{2 i}\right)
$$

where $p$ is the number of independent variables in the model. If $\min \left(k_{1 i}, k_{2 i}\right)$ is small, computation time is not an issue. But if it is large-say, 100 or more-patience may be required.

If $T_{i}$ is large for all groups, the bias of the unconditional fixed-effects estimator is not a concern, and we can confidently use logit with an indicator variable for each group (provided, of course, that the number of groups does not exceed matsize; see $[R]$ matsize).

This command supports the clustered version of the Huber/White/sandwich estimator of the variance using vce (robust) and vce(cluster clustvar). See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas. Specifying vce (robust) is equivalent to specifying vce(cluster groupvar), where groupvar is the variable for the matched groups.
clogit also supports estimation with survey data. For details on VCEs with survey data, see [SVY] variance estimation.

## References

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Hamerle, A., and G. Ronning. 1995. Panel analysis for qualitative variables. In Handbook of Statistical Modeling for the Social and Behavioral Sciences, ed. G. Arminger, C. C. Clogg, and M. E. Sobel, 401-451. New York: Plenum.
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McFadden, D. L. 1974. Conditional logit analysis of qualitative choice behavior. In Frontiers in Econometrics, ed. P. Zarembka, 105-142. New York: Academic Press.

## Also see

[R] clogit postestimation - Postestimation tools for clogit
[R] asclogit - Alternative-specific conditional logit (McFadden's choice) model
[R] logistic - Logistic regression, reporting odds ratios
[R] mlogit - Multinomial (polytomous) logistic regression
[R] nlogit - Nested logit regression
$[\mathrm{R}]$ ologit - Ordered logistic regression
[R] scobit - Skewed logistic regression
[MI] estimation - Estimation commands for use with mi estimate
[SVY] svy estimation - Estimation commands for survey data
[XT] xtgee - Fit population-averaged panel-data models by using GEE
[XT] xtlogit - Fixed-effects, random-effects, and population-averaged logit models
[U] 20 Estimation and postestimation commands

## Title

clogit postestimation - Postestimation tools for clogit

| Description | Syntax for predict | Menu for predict | Options for predict |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Methods and formulas | Reference | Also see |

## Description

The following standard postestimation commands are available after clogit:

| Command | Description |
| :---: | :---: |
| contrast | contrasts and ANOVA-style joint tests of estimates |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample |
| estat vce | variance-covariance matrix of the estimators (VCE) |
| estat (svy) | postestimation statistics for survey data |
| estimates | cataloging estimation results |
| hausman | Hausman's specification test |
| lincom | point estimates, standard errors, testing, and inference for linear combinations of coefficients |
| linktest | link test for model specification |
| lrtest ${ }^{1}$ | likelihood-ratio test |
| margins ${ }^{2}$ | marginal means, predictive margins, marginal effects, and average marginal effects |
| marginsplot | graph the results from margins (profile plots, interaction plots, etc.) |
| nlcom | point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients |
| predict | predictions, residuals, influence statistics, and other diagnostic measures |
| predictnl | point estimates, standard errors, testing, and inference for generalized predictions |
| pwcompare | pairwise comparisons of estimates |
| suest | seemingly unrelated estimation |
| test | Wald tests of simple and composite linear hypotheses |
| testnl | Wald tests of nonlinear hypotheses |

1 lrtest is not appropriate with svy estimation results.
${ }^{2}$ The default prediction statistic pc1 cannot be correctly handled by margins; however, margins can be used after clogit with options predict(pu0) and predict( xb ).

## Syntax for predict

predict [type] newvar [if] [in] [, statistic nooffset]

## statistic Description

Main
pc1
probability of a positive outcome; the default
pu0
xb
stdp

* dbeta
probability of a positive outcome, assuming fixed effect is zero
linear prediction
standard error of the linear prediction
* dx 2

Delta- $\boldsymbol{\beta}$ influence statistic
Delta- $\chi^{2}$ lack-of-fit statistic

* gdbeta

Delta- $\boldsymbol{\beta}$ influence statistic for each group

* gdx 2

Delta- $\chi^{2}$ lack-of-fit statistic for each group
*hat
*residuals
*rstandard
Hosmer and Lemeshow leverage
score
Pearson residuals
standardized Pearson residuals
first derivative of the $\log$ likelihood with respect to $\mathbf{x}_{j} \boldsymbol{\beta}$
Unstarred statistics are available both in and out of sample; type predict . . . if e (sample) ... if wanted only for the estimation sample. Starred statistics are calculated only for the estimation sample, even when if e(sample) is not specified.

Starred statistics are available for multiple controls per case-matching design only. They are not available if vce (robust), vce(cluster clustvar), or pweights were specified with clogit. dbeta, dx2, gdbeta, gdx2, hat, and rstandard are not available if constraints() was specified with clogit.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

pc1, the default, calculates the probability of a positive outcome conditional on one positive outcome within group.
puO calculates the probability of a positive outcome, assuming that the fixed effect is zero.
xb calculates the linear prediction.
stdp calculates the standard error of the linear prediction.
dbeta calculates the Delta- $\boldsymbol{\beta}$ influence statistic, a standardized measure of the difference in the coefficient vector that is due to deletion of the observation.
dx 2 calculates the Delta- $\chi^{2}$ influence statistic, reflecting the decrease in the Pearson chi-squared that is due to deletion of the observation.
gdbeta calculates the approximation to the Pregibon stratum-specific Delta- $\boldsymbol{\beta}$ influence statistic, a standardized measure of the difference in the coefficient vector that is due to deletion of the entire stratum.
gdx2 calculates the approximation to the Pregibon stratum-specific Delta- $\chi^{2}$ influence statistic, reflecting the decrease in the Pearson chi-squared that is due to deletion of the entire stratum.
hat calculates the Hosmer and Lemeshow leverage or the diagonal element of the hat matrix.
residuals calculates the Pearson residuals.
rstandard calculates the standardized Pearson residuals.
score calculates the equation-level score, $\partial \ln L / \partial\left(\mathbf{x}_{i t} \boldsymbol{\beta}\right)$.
nooffset is relevant only if you specified offset (varname) for clogit. It modifies the calculations made by predict so that they ignore the offset variable; the linear prediction is treated as $\mathbf{x}_{j} \mathbf{b}$ rather than as $\mathbf{x}_{j} \mathbf{b}+$ offset $_{j}$. This option cannot be specified with dbeta, dx2, gdbeta, gdx2, hat, and rstandard.

## Remarks and examples

predict may be used after clogit to obtain predicted values of the index $\mathbf{x}_{i t} \boldsymbol{\beta}$. Predicted probabilities for conditional logistic regression must be interpreted carefully. Probabilities are estimated for each group as a whole, not for individual observations. Furthermore, the probabilities are conditional on the number of positive outcomes in the group (that is, the number of cases and the number of controls), or it is assumed that the fixed effect is zero. predict may also be used to obtain influence and lack of fit statistics for an individual observation and for the whole group, to compute Pearson, standardized Pearson residuals, and leverage values.
predict may be used for both within-sample and out-of-sample predictions.
> Example 1
Suppose that we have $1: k_{2 i}$ matched data and that we have previously fit the following model:

```
. use http://www.stata-press.com/data/r13/clogitid
. clogit y x1 x2, group(id)
    (output omitted)
```

To obtain the predicted values of the index, we could type predict idx, $x b$ to create a new variable called idx. From idx, we could then calculate the predicted probabilities. Easier, however, would be to type

```
- predict phat
(option pc1 assumed; probability of success given one success within group)
```

phat would then contain the predicted probabilities.
As noted previously, the predicted probabilities are really predicted probabilities for the group as a whole (that is, they are the predicted probability of observing $y_{i t}=1$ and $y_{i t^{\prime}}=0$ for all $t^{\prime} \neq t$ ). Thus, if we want to obtain the predicted probabilities for the estimation sample, it is important that, when we make the calculation, predictions be restricted to the same sample on which we estimated the data. We cannot predict the probabilities and then just keep the relevant ones because the entire sample determines each probability. Thus, assuming that we are not attempting to make out-of-sample predictions, we type

```
. predict phat2 if e(sample)
(option pc1 assumed; probability of success given one success within group)
```


## Methods and formulas

Recall that $i=1, \ldots, n$ denote the groups and $t=1, \ldots, T_{i}$ denote the observations for the $i$ th group.
predict produces probabilities of a positive outcome within group conditional on there being one positive outcome (pc1),

$$
\operatorname{Pr}\left(y_{i t}=1 \mid \sum_{t=1}^{T_{i}} y_{i t}=1\right)=\frac{\exp \left(\mathbf{x}_{i t} \boldsymbol{\beta}\right)}{\sum_{t=1}^{T_{i}} \exp \left(\mathbf{x}_{i t} \boldsymbol{\beta}\right)}
$$

or predict calculates the unconditional pu0:

$$
\operatorname{Pr}\left(y_{i t}=1\right)=\frac{\exp \left(\mathbf{x}_{i t} \boldsymbol{\beta}\right)}{1+\exp \left(\mathbf{x}_{i t} \boldsymbol{\beta}\right)}
$$

Let $N=\sum_{j=1}^{n} T_{j}$ denote the total number of observations, $p$ denote the number of covariates, and $\widehat{\theta}_{i t}$ denote the conditional predicted probabilities of a positive outcome (pc1).

For the multiple control per case ( $1: k_{2 i}$ ) matching, Hosmer, Lemeshow, and Sturdivant (2013, 248-251) propose the following diagnostics:

The Pearson residual is

$$
r_{i t}=\frac{\left(y_{i t}-\widehat{\theta}_{i t}\right)}{\sqrt{\widehat{\theta}_{i t}}}
$$

The leverage (hat) value is defined as

$$
h_{i t}=\widehat{\theta}_{i t} \widetilde{\mathbf{x}}_{i t}^{T}\left(\widetilde{\mathbf{X}}^{T} \mathbf{U} \widetilde{\mathbf{X}}\right)^{-1} \widetilde{\mathbf{x}}_{i t}
$$

where $\widetilde{\mathbf{x}}_{i t}=\mathbf{x}_{i t}-\sum_{j=1}^{T_{i}} \mathbf{x}_{i j} \widehat{\theta}_{i j}$ is the $1 \times p$ row vector of centered by a weighted stratum-specific mean covariate values, $\mathbf{U}_{N}=\operatorname{diag}\left\{\widehat{\theta}_{i t}\right\}$, and the rows of $\widetilde{\mathbf{X}}_{N \times p}$ are composed of $\widetilde{\mathbf{x}}_{i t}$ values.

The standardized Pearson residual is

$$
r_{s i t}=\frac{r_{i t}}{\sqrt{1-h_{i t}}}
$$

The lack of fit and influence diagnostics for an individual observation are (respectively) computed as

$$
\Delta \chi_{i t}^{2}=r_{s i t}^{2}
$$

and

$$
\Delta \widehat{\beta}_{i t}=\Delta \chi_{i t}^{2} \frac{h_{i t}}{1-h_{i t}}
$$

The lack of fit and influence diagnostics for the groups are the group-specific totals of the respective individual diagnostics shown above.

## Reference

Hosmer, D. W., J.., S. A. Lemeshow, and R. X. Sturdivant. 2013. Applied Logistic Regression. 3rd ed. Hoboken, NJ: Wiley.

## Also see

[R] clogit - Conditional (fixed-effects) logistic regression
[U] 20 Estimation and postestimation commands

## Title

cloglog - Complementary log-log regression

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | Acknowledgment |
| References | Also see |  |  |

## Syntax

$$
\text { cloglog depvar }[\text { indepvars }][\text { if }][\text { in }][\text { weight }][\text {, options }]
$$

options
Description

Model
noconstant
offset (varname)
asis
constraints (constraints)
collinear
SE/Robust
vce (vcetype)

Reporting
level(\#)
eform
nocnsreport
display_options

Maximization
maximize_options
coeflegend
suppress constant term
include varname in model with coefficient constrained to 1
retain perfect predictor variables
apply specified linear constraints
keep collinear variables
vcetype may be oim, robust, cluster clustvar, opg, bootstrap, or jackknife
set confidence level; default is level (95)
report exponentiated coefficients
do not display constraints
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
depvar and indepvars may contain time-series operators; see [U] 11.4.4 Time-series varlists.
bootstrap, by, fp, jackknife, mi estimate, nestreg, rolling, statsby, stepwise, and svy are allowed; see [U] 11.1.10 Prefix commands.
vce(bootstrap) and vce(jackknife) are not allowed with the mi estimate prefix; see [MI] mi estimate.
Weights are not allowed with the bootstrap prefix; see $[\mathrm{R}]$ bootstrap.
vce() and weights are not allowed with the svy prefix; see [SVY] svy.
fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Binary outcomes $>$ Complementary log-log regression

## Description

cloglog fits maximum-likelihood complementary log-log models.
See [R] logistic for a list of related estimation commands.

## Options

$\qquad$ Model
noconstant, offset (varname); see [R] estimation options.
asis forces retention of perfect predictor variables and their associated perfectly predicted observations and may produce instabilities in maximization; see [R] probit. constraints (constraints), collinear; see [R] estimation options.

SE/Robust
vce (vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

## Reporting

level (\#); see [R] estimation options.
eform displays the exponentiated coefficients and corresponding standard errors and confidence intervals.
nocnsreport; see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% fmt), pformat (\%fmt), sformat(\%fmt), and nolstretch; see [R] estimation options.
$\qquad$ Maximization
maximize_options: difficult, technique (algorithm_spec), iterate(\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance(\#), nonrtolerance, and from(init_specs); see [R] maximize. These options are seldom used.

Setting the optimization type to technique (bhhh) resets the default vcetype to vce (opg).
The following option is available with cloglog but is not shown in the dialog box:
coeflegend; see [R] estimation options.

## Remarks and examples

Remarks are presented under the following headings:
Introduction to complementary log-log regression
Robust standard errors

## Introduction to complementary log-log regression

cloglog fits maximum likelihood models with dichotomous dependent variables coded as $0 / 1$ (or, more precisely, coded as 0 and not 0 ).

## $>$ Example 1

We have data on the make, weight, and mileage rating of 22 foreign and 52 domestic automobiles. We wish to fit a model explaining whether a car is foreign based on its weight and mileage. Here is an overview of our data:


The variable foreign takes on two unique values, 0 and 1 . The value 0 denotes a domestic car, and 1 denotes a foreign car.

The model that we wish to fit is

$$
\operatorname{Pr}(\text { foreign }=1)=F\left(\beta_{0}+\beta_{1} \text { weight }+\beta_{2} \mathrm{mpg}\right)
$$

where $F(z)=1-\exp \{-\exp (z)\}$.
To fit this model, we type

```
. cloglog foreign weight mpg
Iteration 0: log likelihood = -34.054593
Iteration 1: log likelihood = -27.869915
Iteration 2: log likelihood = -27.742997
Iteration 3: log likelihood = -27.742769
Iteration 4: log likelihood = -27.742769
```

| Complementary log-log regression | Number of obs | $=$ | 74 |
| :--- | :--- | :--- | ---: |
|  | Zero outcomes | $=$ | 52 |
|  | Nonzero outcomes | $=$ | 22 |
|  | LR chi2 (2) | $=$ | 34.58 |
| Log likelihood $=-27.742769$ | Prob $>$ chi2 | $=$ | 0.0000 |


| foreign | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | :---: | ---: | ---: |
| weight | -.0029153 | .0006974 | -4.18 | 0.000 | -.0042823 | -.0015483 |
| mpg | -.1422911 | .076387 | -1.86 | 0.062 | -.2920069 | .0074247 |
| _cons | 10.09694 | 3.351841 | 3.01 | 0.003 | 3.527448 | 16.66642 |

We find that heavier cars are less likely to be foreign and that cars yielding better gas mileage are also less likely to be foreign, at least when holding the weight of the car constant.

See [R] maximize for an explanation of the output.

## - Technical note

Stata interprets a value of 0 as a negative outcome (failure) and treats all other values (except missing) as positive outcomes (successes). Thus, if your dependent variable takes on the values 0 and 1,0 is interpreted as failure and 1 as success. If your dependent variable takes on the values 0,1 , and 2,0 is still interpreted as failure, but both 1 and 2 are treated as successes.

If you prefer a more formal mathematical statement, when you type cloglog $y x$, Stata fits the model

$$
\operatorname{Pr}\left(y_{j} \neq 0 \mid \mathbf{x}_{j}\right)=1-\exp \left\{-\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}\right)\right\}
$$

## Robust standard errors

If you specify the vce (robust) option, cloglog reports robust standard errors, as described in [U] 20.21 Obtaining robust variance estimates. For the model of foreign on weight and mpg, the robust calculation increases the standard error of the coefficient on mpg by $44 \%$ :


Without vce (robust), the standard error for the coefficient on mpg was reported to be 0.076 , with a resulting confidence interval of $[-0.29,0.01]$.

The vce(cluster clustvar) option can relax the independence assumption required by the complementary log-log estimator to being just independence between clusters. To demonstrate this ability, we will switch to a different dataset.

We are studying unionization of women in the United States by using the union dataset; see [XT] xt. We fit the following model, ignoring that women are observed an average of 5.9 times each in this dataset:

| (NLS Women 14-24 in 1968) |  |  |  |
| :---: | :---: | :---: | :---: |
| Iteration 0: log likelihood $=-13606.373$ |  |  |  |
| Iteration 1: log likelihood $=-13540.726$ |  |  |  |
| Iteration 2: log likelihood $=-13540.607$ |  |  |  |
| Iteration 3: log likelihood = -13540.607 |  |  |  |
| Complementary log-log regression | Number of obs | = | 26200 |
|  | Zero outcomes | = | 20389 |
|  | Nonzero outcomes | = | 5811 |
|  | LR chi2(6) | = | 647.24 |
| Log likelihood = -13540.607 | Prob > chi2 | = | 0.0000 |


| union | Coef. | Std. Err. | $z$ | $P>\|z\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| age | .0185346 | .0043616 | 4.25 | 0.000 | .009986 | .0270833 |
| grade | .0452772 | .0057125 | 7.93 | 0.000 | .0340809 | .0564736 |
| not_smsa | -.1886592 | .0317801 | -5.94 | 0.000 | -.2509471 | -.1263712 |
| 1.south | -1.422292 | .3949381 | -3.60 | 0.000 | -2.196356 | -.648227 |
| year | -.0133007 | .0049576 | -2.68 | 0.007 | -.0230174 | -.0035839 |
| south\#c.year |  |  |  |  |  |  |
| 1 | .0105659 | .0049234 | 2.15 | 0.032 | .0009161 | .0202157 |
| _cons | -1.219801 | .2952374 | -4.13 | 0.000 | -1.798455 | -.6411462 |

The reported standard errors in this model are probably meaningless. Women are observed repeatedly, and so the observations are not independent. Looking at the coefficients, we find a large southern effect against unionization and a different time trend for the south. The vce(cluster clustvar) option provides a way to fit this model and obtains correct standard errors:

| . cloglog union age grade not_smsa south\#\#c. year, vce (cluster id) | nolog |  |  |
| :--- | :--- | :--- | :--- |
| Complementary log-log regression | Number of obs | $=$ | 26200 |
|  | Zero outcomes | $=$ | 20389 |
|  | Nonzero outcomes | $=$ | 5811 |
|  | Wald chi2 (6) | $=$ | 160.76 |
| Log pseudolikelihood $=-13540.607$ | Prob $>$ chi2 | $=$ | 0.0000 |

(Std. Err. adjusted for 4434 clusters in idcode)

|  | Coef. | Robust |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| union | Std. Err. | $z$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |  |
| age | .0185346 | .0084873 | 2.18 | 0.029 | .0018999 | .0351694 |
| grade | .0452772 | .0125776 | 3.60 | 0.000 | .0206255 | .069929 |
| not_smsa | -.1886592 | .0642068 | -2.94 | 0.003 | -.3145021 | -.0628162 |
| 1.south | -1.422292 | .506517 | -2.81 | 0.005 | -2.415047 | -.4295365 |
| year | -.0133007 | .0090628 | -1.47 | 0.142 | -.0310633 | .004462 |
| south\#c.year |  |  |  |  |  |  |
| 1 | .0105659 | .0063175 | 1.67 | 0.094 | -.0018162 | .022948 |
| _cons | -1.219801 | .5175129 | -2.36 | 0.018 | -2.234107 | -.2054942 |

These standard errors are larger than those reported by the inappropriate conventional calculation. By comparison, another way we could fit this model is with an equal-correlation population-averaged complementary log-log model:

| GEE population-averaged model |  | Number of obs | = | 26200 |
| :---: | :---: | :---: | :---: | :---: |
| Group variable: | idcode | Number of groups | = | 4434 |
| Link: | cloglog | Obs per group: min | = | 1 |
| Family: | binomial | avg | $=$ | 5.9 |
| Correlation: | exchangeable | max | = | 12 |
|  |  | Wald chi2(6) | = | 234.66 |
| Scale parameter: | 1 | Prob > chi2 | = | 0.0000 |


| union | Coef. | Std. Err. | $z$ | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| age | .0153737 | .0081156 | 1.89 | 0.058 | -.0005326 | .03128 |
| grade | .0549518 | .0095093 | 5.78 | 0.000 | .0363139 | .0735897 |
| not_smsa | -.1045232 | .0431082 | -2.42 | 0.015 | -.1890138 | -.0200326 |
| 1.south | -1.714868 | .3384558 | -5.07 | 0.000 | -2.378229 | -1.051507 |
| year | -.0115881 | .0084125 | -1.38 | 0.168 | -.0280763 | .0049001 |
| south\#c.year |  |  |  |  |  |  |
| 1 | .0149796 | .0041687 | 3.59 | 0.000 | .0068091 | .0231501 |
| _cons | -1.488278 | .4468005 | -3.33 | 0.001 | -2.363991 | -.6125652 |

The coefficient estimates are similar, but these standard errors are smaller than those produced by cloglog, vce(cluster clustvar). This finding is as we would expect. If the within-panel correlation assumptions are valid, the population-averaged estimator should be more efficient.

In addition to this estimator, we may use the xtgee command to fit a panel estimator (with complementary log-log link) and any number of assumptions on the within-idcode correlation.
cloglog, vce (cluster clustvar) is robust to assumptions about within-cluster correlation. That is, it inefficiently sums within cluster for the standard-error calculation rather than attempting to exploit what might be assumed about the within-cluster correlation (as do the xtgee population-averaged models).

## Stored results

cloglog stores the following in e():
Scalars
e(N)
e(k)
e(k_eq)
e(k_eq_model)
e(k_dv)
e(N_f)
e(N_s)
e(df_m)
e(11)
e(ll_0)
e(N_clust)
e(chi2)
e(p)
e(rank)
e(ic)
e(rc)
e(converged)
Macros
e(cmd)
e(cmdline)
e(depvar)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(offset)
e(chi2type)
e(vce)
e(vcetype)
e(opt)
e(which)
e(ml_method)
e(user)
e(technique)
e(properties)
e(predict)
e(asbalanced)
e(asobserved)
Matrices
e(b)
e(Cns)
e(ilog)
e(gradient)
e(V)
e(V_modelbased)
Functions
e(sample) marks estimation sample

## Methods and formulas

Complementary log-log analysis (related to the gompit model, so named because of its relationship to the Gompertz distribution) is an alternative to logit and probit analysis, but it is unlike these other estimators in that the transformation is not symmetric. Typically, this model is used when the positive (or negative) outcome is rare.

The log-likelihood function for complementary $\log$-log is

$$
\ln L=\sum_{j \in S} w_{j} \ln F\left(\mathbf{x}_{j} \mathbf{b}\right)+\sum_{j \notin S} w_{j} \ln \left\{1-F\left(\mathbf{x}_{j} \mathbf{b}\right)\right\}
$$

where $S$ is the set of all observations $j$ such that $y_{j} \neq 0, F(z)=1-\exp \{-\exp (z)\}$, and $w_{j}$ denotes the optional weights. $\ln L$ is maximized as described in [ R$]$ maximize.

We can fit a gompit model by reversing the success-failure sense of the dependent variable and using cloglog.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce (robust) and vce(cluster clustvar), respectively. See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas. The scores are calculated as $\mathbf{u}_{j}=$ $\left[\exp \left(\mathbf{x}_{j} \mathbf{b}\right) \exp \left\{-\exp \left(\mathbf{x}_{j} \mathbf{b}\right)\right\} / F\left(\mathbf{x}_{j} \mathbf{b}\right)\right] \mathbf{x}_{j}$ for the positive outcomes and $\left\{-\exp \left(\mathbf{x}_{j} \mathbf{b}\right)\right\} \mathbf{x}_{j}$ for the negative outcomes.
cloglog also supports estimation with survey data. For details on VCEs with survey data, see [SVY] variance estimation.

## Acknowledgment

We thank Joseph Hilbe of Arizona State University for providing the inspiration for the cloglog command (Hilbe 1996, 1998).

## References

Clayton, D. G., and M. Hills. 1993. Statistical Models in Epidemiology. Oxford: Oxford University Press.
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Xu, J., and J. S. Long. 2005. Confidence intervals for predicted outcomes in regression models for categorical outcomes. Stata Journal 5: 537-559.

## Also see

[R] cloglog postestimation - Postestimation tools for cloglog
$[\mathrm{R}]$ clogit - Conditional (fixed-effects) logistic regression
[R] glm - Generalized linear models
$[\mathrm{R}]$ logistic - Logistic regression, reporting odds ratios
[R] scobit - Skewed logistic regression
[ME] mecloglog - Multilevel mixed-effects complementary log-log regression
[MI] estimation - Estimation commands for use with mi estimate
[SVY] svy estimation - Estimation commands for survey data
[XT] xtcloglog — Random-effects and population-averaged cloglog models
[U] 20 Estimation and postestimation commands

## Title

cloglog postestimation - Postestimation tools for cloglog

| Description | Syntax for predict |
| :--- | :--- |
| Remarks and examples | Also see for predict |$\quad$ Options for predict

## Description

The following postestimation commands are available after cloglog:

| Command | Description |
| :---: | :---: |
| contrast | contrasts and ANOVA-style joint tests of estimates |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample |
| estat vce | variance-covariance matrix of the estimators (VCE) |
| estat (svy) | postestimation statistics for survey data |
| estimates | cataloging estimation results |
| forecast ${ }^{1}$ | dynamic forecasts and simulations |
| lincom | point estimates, standard errors, testing, and inference for linear combinations of coefficients |
| linktest | link test for model specification |
| lrtest ${ }^{2}$ | likelihood-ratio test |
| margins | marginal means, predictive margins, marginal effects, and average marginal effects |
| marginsplot | graph the results from margins (profile plots, interaction plots, etc.) |
| nlcom | point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients |
| predict | predictions, residuals, influence statistics, and other diagnostic measures |
| predictnl | point estimates, standard errors, testing, and inference for generalized predictions |
| pwcompare | pairwise comparisons of estimates |
| suest | seemingly unrelated estimation |
| test | Wald tests of simple and composite linear hypotheses |
| testnl | Wald tests of nonlinear hypotheses |

[^11]
## Syntax for predict

predict [type] newvar [if] [in] [, statistic nooffset]

## statistic <br> Description

Main
$\underline{\mathrm{pr}} \quad$ probability of a positive outcome; the default
$\mathrm{xb} \quad$ linear prediction
stdp standard error of the linear prediction
score first derivative of the log likelihood with respect to $\mathbf{x}_{j} \boldsymbol{\beta}$
These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
pr , the default, calculates the probability of a positive outcome.
xb calculates the linear prediction.
stdp calculates the standard error of the linear prediction.
score calculates the equation-level score, $\partial \ln L / \partial\left(\mathbf{x}_{j} \boldsymbol{\beta}\right)$.
nooffset is relevant only if you specified offset (varname) for cloglog. It modifies the calculations made by predict so that they ignore the offset variable; the linear prediction is treated as $\mathbf{x}_{j} \mathbf{b}$ rather than as $\mathbf{x}_{j} \mathbf{b}+$ offset $_{j}$.

## Remarks and examples

Once you have fit a model, you can obtain the predicted probabilities by using the predict command for both the estimation sample and other samples; see [U] 20 Estimation and postestimation commands and $[R]$ predict. Here we will make only a few comments.
predict without arguments calculates the predicted probability of a positive outcome. With the xb option, it calculates the linear combination $\mathbf{x}_{j} \mathbf{b}$, where $\mathbf{x}_{j}$ are the independent variables in the $j$ th observation and $\mathbf{b}$ is the estimated parameter vector.

With the stdp option, predict calculates the standard error of the linear prediction, which is not adjusted for replicated covariate patterns in the data.

## $>$ Example 1

In example 1 in [R] cloglog, we fit the complementary log-log model cloglog foreign weight mpg . To obtain predicted probabilities,

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. cloglog foreign weight mpg
    (output omitted)
. predict p
(option pr assumed; Pr(foreign))
    . summarize foreign p
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline foreign & 74 & .2972973 & .4601885 & 0 & 1 \\
p & 74 & .2928348 & .29732 & .0032726 & .9446067
\end{tabular}
```


## Also see

[R] cloglog - Complementary log-log regression
[U] 20 Estimation and postestimation commands

## Title

cls - Clear Results window

Syntax Description

## Syntax

cls

## Description

cls clears the Results window, causing all text to be removed. This operation cannot be undone.

## Title

cnsreg - Constrained linear regression

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

Also see

## Syntax

cnsreg depvar indepvars [if] [in] [weight], constraints(constraints) [options]
options
Model

* constraints (constraints)
collinear
noconstant
SE/Robust
vce (vcetype)

Reporting
level(\#) nocnsreport
display_options
mse1
coeflegend

Description
apply specified linear constraints
keep collinear variables
suppress constant term
vcetype may be ols, robust, cluster clustvar, bootstrap, or jackknife

* constraints (constraints) is required.
indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
depvar and indepvars may contain time-series operators; see [U] 11.4.4 Time-series varlists.
bootstrap, by, fp, jackknife, mi estimate, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands.
vce(bootstrap) and vce(jackknife) are not allowed with the mi estimate prefix; see [MI] mi estimate.
With the $f p$ prefix (see $[R] f p$ ), constraints cannot be specified for the variable containing fractional polynomial terms.
Weights are not allowed with the bootstrap prefix; see $[\mathrm{R}]$ bootstrap.
aweights are not allowed with the jackknife prefix; see [R] jackknife.
vce(), mse1, and weights are not allowed with the svy prefix; see [SVY] svy.
aweights, fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
mse1 and coeflegend do not appear in the dialog.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.


## Menu

Statistics $>$ Linear models and related $>$ Constrained linear regression

## Description

cnsreg fits constrained linear regression models.

## Options

Model
constraints (constraints), collinear, noconstant; see [R] estimation options.

SE/Robust
vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (ols), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
vce(ols), the default, uses the standard variance estimator for ordinary least-squares regression.
$\qquad$ Reporting
level (\#); see [R] estimation options.
nocnsreport; see $[\mathrm{R}]$ estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\%fmt), pformat(\%fmt), sformat(\%fmt), and nolstretch; see [R] estimation options.

The following options are available with cnsreg but are not shown in the dialog box:
mse1 is used only in programs and ado-files that use cnsreg to fit models other than constrained linear regression. mse1 sets the mean squared error to 1 , thus forcing the variance-covariance matrix of the estimators to be $\left(\mathbf{X}^{\prime} \mathbf{D X}\right)^{-1}$ (see Methods and formulas in $[\mathrm{R}]$ regress) and affecting calculated standard errors. Degrees of freedom for $t$ statistics are calculated as $n$ rather than $n-p+c$, where $p$ is the total number of parameters (prior to restrictions and including the constant) and $c$ is the number of constraints.
mse1 is not allowed with the svy prefix.
coeflegend; see $[R]$ estimation options.

## Remarks and examples

For a discussion of constrained linear regression, see Greene (2012, 121-122); Hill, Griffiths, and Lim (2011, 231-233); or Davidson and MacKinnon (1993, 17).

## Example 1: One constraint

In principle, we can obtain constrained linear regression estimates by modifying the list of independent variables. For instance, if we wanted to fit the model

$$
\mathrm{mpg}=\beta_{0}+\beta_{1} \text { price }+\beta_{2} \text { weight }+u
$$

and constrain $\beta_{1}=\beta_{2}$, we could write

$$
\operatorname{mpg}=\beta_{0}+\beta_{1}(\text { price }+ \text { weight })+u
$$

and run a regression of mpg on price + weight. The estimated coefficient on the sum would be the constrained estimate of $\beta_{1}$ and $\beta_{2}$. Using cnsreg, however, is easier:


We define constraints by using the constraint command; see [R] constraint. We fit the model with cnsreg and specify the constraint number or numbers in the constraints() option.

Just to show that the results above are correct, here is the result of applying the constraint by hand:

```
. generate x = price + weight
```

. regress mpg x

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | 838.065767 | 1 | 838.065767 |
| Residual | 1605.39369 | 72 | 22.2971346 |
| Total | 2443.45946 | 73 | 33.4720474 |


| Number of obs | $=$ | 74 |
| :--- | ---: | ---: |
| F $(1$, | $72)$ | $=$ |
| Prob $>$ F | $=$ | 37.59 |
| R-squared | $=0000$ |  |
| Adj R-squared | $=$ | 0.3430 |
| Root MSE | $=$ | 4.722 |


| mpg | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | ---: | ---: | ---: |
| x | -.0009875 | .0001611 | -6.13 | 0.000 | -.0013086 | -.0006664 |
| _cons | 30.36718 | 1.577958 | 19.24 | 0.000 | 27.22158 | 33.51278 |

## - Example 2: Multiple constraints

Models can be fit subject to multiple simultaneous constraints. We simply define the constraints and then include the constraint numbers in the constraints() option. For instance, say that we wish to fit the model

$$
\begin{aligned}
\operatorname{mpg}= & \beta_{0}+\beta_{1} \text { price }+\beta_{2} \text { weight }+\beta_{3} \text { displ }+\beta_{4} \text { gear_ratio }+\beta_{5} \text { foreign }+ \\
& \beta_{6} \text { length }+u
\end{aligned}
$$

subject to the constraints

$$
\begin{aligned}
& \beta_{1}=\beta_{2}=\beta_{3}=\beta_{6} \\
& \beta_{4}=-\beta_{5}=\beta_{0} / 20
\end{aligned}
$$

(This model, like the one in example 1, is admittedly senseless.) We fit the model by typing

```
. constraint 1 price=weight
. constraint 2 displ=weight
. constraint 3 length=weight
. constraint 5 gear_ratio = -foreign
. constraint 6 gear_ratio = _cons/20
. cnsreg mpg price weight displ gear_ratio foreign length, c(1-3,5-6)
Constrained linear regression Number of obs =
    F( 2, 72) = 785.20
                                    Prob > F = 0.0000
                                    Root MSE = 4.6823
```

    ( 1) price - weight \(=0\)
    ( 2) - weight + displacement \(=0\)
    (3) - weight + length \(=0\)
    (4) gear_ratio + foreign \(=0\)
    (5) gear_ratio - . 05*_cons \(=0\)
    | mpg | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| price | -.000923 | .0001534 | -6.02 | 0.000 | -.0012288 | -.0006172 |
| weight | -.000923 | .0001534 | -6.02 | 0.000 | -.0012288 | -.0006172 |
| displacement | -.000923 | .0001534 | -6.02 | 0.000 | -.0012288 | -.0006172 |
| gear_ratio | 1.326114 | .0687589 | 19.29 | 0.000 | 1.189046 | 1.463183 |
| foreign | -1.326114 | .0687589 | -19.29 | 0.000 | -1.463183 | -1.189046 |
| length | -.000923 | .0001534 | -6.02 | 0.000 | -.0012288 | -.0006172 |
| _cons | 26.52229 | 1.375178 | 19.29 | 0.000 | 23.78092 | 29.26365 |

There are many ways we could have specified the constraints() option (which we abbreviated $c$ () above). We typed $c(1-3,5-6)$, meaning that we want constraints 1 through 3 and 5 and 6 ; those numbers correspond to the constraints we defined. The only reason we did not use the number 4 was to emphasize that constraints do not have to be consecutively numbered. We typed $c(1-3,5-6)$, but we could have typed $c(1,2,3,5,6)$ or $c(1-3,5,6)$ or $c(1-2,3,5,6)$ or even $c(1-6)$, which would have worked as long as constraint 4 was not defined. If we had previously defined a constraint 4 , then $c(1-6)$ would have included it.

## Stored results

cnsreg stores the following in e() :

Scalars

```
e(N)
    e(df_m)
    e(df_r)
    e(F)
    e(rmse)
    e(ll)
    e(N_clust)
    e(rank)
```

Macros
e(cmd)
e(cmdline)
e(depvar)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(vce)
e(vcetype)
e(properties)
e(predict)
e(asbalanced)
e(asobserved)
Matrices
e(b)
e(Cns)
e(V)
e(V_modelbased)

Functions
e(sample) marks estimation sample

## Methods and formulas

Let $n$ be the number of observations, $p$ be the total number of parameters (prior to restrictions and including the constant), and $c$ be the number of constraints. The coefficients are calculated as $\mathbf{b}^{\prime}=\mathbf{T}\left\{\left(\mathbf{T}^{\prime} \mathbf{X}^{\prime} \mathbf{W} \mathbf{X} \mathbf{T}\right)^{-1}\left(\mathbf{T}^{\prime} \mathbf{X}^{\prime} \mathbf{W} \mathbf{y}-\mathbf{T}^{\prime} \mathbf{X}^{\prime} \mathbf{W} \mathbf{X} \mathbf{a}^{\prime}\right)\right\}+\mathbf{a}^{\prime}$, where $\mathbf{T}$ and $\mathbf{a}$ are as defined in $[\mathrm{P}]$ makecns. $\mathbf{W}=\mathbf{I}$ if no weights are specified. If weights are specified, let $\mathbf{v}: 1 \times n$ be the specified weights. If fweight frequency weights are specified, $\mathbf{W}=\operatorname{diag}(\mathbf{v})$. If aweight analytic weights are specified, then $\mathbf{W}=\operatorname{diag}\left[\mathbf{v} /\left(\mathbf{1}^{\prime} \mathbf{v}\right)\left(\mathbf{1}^{\prime} \mathbf{1}\right)\right]$, meaning that the weights are normalized to sum to the number of observations.

The mean squared error is $s^{2}=\left(\mathbf{y}^{\prime} \mathbf{W} \mathbf{y}-2 \mathbf{b}^{\prime} \mathbf{X}^{\prime} \mathbf{W} \mathbf{y}+\mathbf{b}^{\prime} \mathbf{X}^{\prime} \mathbf{W} \mathbf{X b}\right) /(n-p+c)$. The variance covariance matrix is $s^{2} \mathbf{T}\left(\mathbf{T}^{\prime} \mathbf{X}^{\prime} \mathbf{W} \mathbf{X} \mathbf{T}\right)^{-1} \mathbf{T}^{\prime}$.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce (robust) and vce(cluster clustvar), respectively. See [P] _robust, particularly Introduction and Methods and formulas.
cnsreg also supports estimation with survey data. For details on VCEs with survey data, see [SVY] variance estimation.

## References

Davidson, R., and J. G. MacKinnon. 1993. Estimation and Inference in Econometrics. New York: Oxford University Press.
Greene, W. H. 2012. Econometric Analysis. 7th ed. Upper Saddle River, NJ: Prentice Hall.
Hill, R. C., W. E. Griffiths, and G. C. Lim. 2011. Principles of Econometrics. 4th ed. Hoboken, NJ: Wiley.

## Also see

[R] ensreg postestimation - Postestimation tools for cnsreg
$[R]$ regress - Linear regression
[MI] estimation - Estimation commands for use with mi estimate
[SVY] svy estimation - Estimation commands for survey data
[U] 20 Estimation and postestimation commands

## Title

cnsreg postestimation - Postestimation tools for cnsreg
Description Syntax for predict Menu for predict Options for predict Also see

## Description

The following postestimation commands are available after cnsreg:
Command Description

| contrast | contrasts and ANOVA-style joint tests of estimates |
| :--- | :--- |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample <br> estat vce |
| variance-covariance matrix of the estimators (VCE) |  |
| estat (svy) | postestimation statistics for survey data <br> cataloging estimation results |
| estimates $^{\text {forecast }} 1$ | dynamic forecasts and simulations <br> point estimates, standard errors, testing, and inference for linear combinations <br> lincom |
| of coefficients |  |

[^12]
## Syntax for predict

predict [type] newvar [if] [in] [, statistic]
statistic Description

| Main |  |
| :--- | :--- |
| xb | linear prediction; the default |
| residuals | residuals |
| stdp | standard error of the prediction |
| stdf | standard error of the forecast |
| $\underline{\operatorname{pr}}(a, b)$ | $\operatorname{Pr}\left(a<y_{j}<b\right)$ |
| $\mathrm{e}(a, b)$ | $E\left(y_{j} \mid a<y_{j}<b\right)$ |
| $\underline{\mathrm{ystar}(a, b)}$ | $E\left(y_{j}^{*}\right), y_{j}^{*}=\max \left\{a, \min \left(y_{j}, b\right)\right\}$ |
| $\underline{\operatorname{score}}$ | equivalent to residuals |

These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.
stdf is not allowed with svy estimation results.
where $a$ and $b$ may be numbers or variables; $a$ missing ( $a \geq$.) means $-\infty$, and $b$ missing ( $b \geq$.) means $+\infty$; see [U] 12.2.1 Missing values.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
xb , the default, calculates the linear prediction.
residuals calculates the residuals, that is, $y_{j}-\mathbf{x}_{j} \mathbf{b}$.
stdp calculates the standard error of the prediction, which can be thought of as the standard error of the predicted expected value or mean for the observation's covariate pattern. The standard error of the prediction is also referred to as the standard error of the fitted value.
stdf calculates the standard error of the forecast, which is the standard error of the point prediction for 1 observation. It is commonly referred to as the standard error of the future or forecast value. By construction, the standard errors produced by stdf are always larger than those produced by stdp; see Methods and formulas in [R] regress postestimation.
$\operatorname{pr}(a, b)$ calculates $\operatorname{Pr}\left(a<\mathbf{x}_{j} \mathbf{b}+u_{j}<b\right)$, the probability that $y_{j} \mid \mathbf{x}_{j}$ would be observed in the interval $(a, b)$.
$a$ and $b$ may be specified as numbers or variable names; $l b$ and $u b$ are variable names;
$\operatorname{pr}(20,30)$ calculates $\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$;
$\operatorname{pr}(l b, u b)$ calculates $\operatorname{Pr}\left(l b<\mathbf{x}_{j} \mathbf{b}+u_{j}<u b\right)$; and
$\operatorname{pr}(20, u b)$ calculates $\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{j}<u b\right)$.
$a$ missing ( $a \geq$. ) means $-\infty$; $\operatorname{pr}(., 30)$ calculates $\operatorname{Pr}\left(-\infty<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$;
$\operatorname{pr}(l b, 30)$ calculates $\operatorname{Pr}\left(-\infty<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$ in observations for which $l b \geq$.
and calculates $\operatorname{Pr}\left(l b<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$ elsewhere.
$b$ missing $(b \geq$.$) means +\infty$; $\operatorname{pr}(20,$.$) calculates \operatorname{Pr}\left(+\infty>\mathbf{x}_{j} \mathbf{b}+u_{j}>20\right)$; $\operatorname{pr}(20, u b)$ calculates $\operatorname{Pr}\left(+\infty>\mathbf{x}_{j} \mathbf{b}+u_{j}>20\right)$ in observations for which $u b \geq$. and calculates $\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{j}<u b\right)$ elsewhere.
$\mathrm{e}(a, b)$ calculates $E\left(\mathbf{x}_{j} \mathbf{b}+u_{j} \mid a<\mathbf{x}_{j} \mathbf{b}+u_{j}<b\right)$, the expected value of $y_{j} \mid \mathbf{x}_{j}$ conditional on $y_{j} \mid \mathbf{x}_{j}$ being in the interval $(a, b)$, meaning that $y_{j} \mid \mathbf{x}_{j}$ is truncated. $a$ and $b$ are specified as they are for pr() .
ystar $(a, b)$ calculates $E\left(y_{j}^{*}\right)$, where $y_{j}^{*}=a$ if $\mathbf{x}_{j} \mathbf{b}+u_{j} \leq a, y_{j}^{*}=b$ if $\mathbf{x}_{j} \mathbf{b}+u_{j} \geq b$, and $y_{j}^{*}=\mathbf{x}_{j} \mathbf{b}+u_{j}$ otherwise, meaning that $y_{j}^{*}$ is censored. $a$ and $b$ are specified as they are for pr() . score is equivalent to residuals for linear regression models.

## Also see

$[\mathrm{R}]$ ensreg - Constrained linear regression
[U] 20 Estimation and postestimation commands

## Title

constraint - Define and list constraints

| Syntax | Menu | Description | Remarks and examples |
| :--- | :--- | :--- | :--- |
| References | Also see |  |  |

## Syntax

Define constraints

```
constraint [\underline{define] # [exp=exp |coeflist]}
```

List constraints

```
constraint dir [numlist|_all]
constraint list [numlist|_all]
```

Drop constraints

```
constraint drop {numlist |_all }
```

Programmer's commands

```
constraint get #
constraint free
```

where coeflist is as defined in $[\mathrm{R}]$ test and \# is restricted to the range $1-1,999$, inclusive.

## Menu

Statistics $>$ Other $>$ Manage constraints

## Description

constraint defines, lists, and drops linear constraints. Constraints are for use by models that allow constrained estimation.

Constraints are defined by the constraint command. The currently defined constraints can be listed by either constraint list or constraint dir; both do the same thing. Existing constraints can be eliminated by constraint drop.
constraint get and constraint free are programmer's commands. constraint get returns the contents of the specified constraint in macro $r$ (contents) and returns in scalar $r$ (defined) 0 or $1-1$ being returned if the constraint was defined. constraint free returns the number of a free (unused) constraint in macro $r$ (free).

## Remarks and examples

Using constraints is discussed in [R] cnsreg, $[R]$ mlogit, and $[R]$ reg3; this entry is concerned only with practical aspects of defining and manipulating constraints.

## > Example 1

Constraints are numbered from 1 to 1,999 , and we assign the number when we define the constraint:

```
. use http://www.stata-press.com/data/r13/sysdsn1
(Health insurance data)
. constraint 2 [indemnity]2.site = 0
```

The currently defined constraints can be listed by constraint list:

```
. constraint list
```

    2: \(\quad\) [indemnity]2.site \(=0\)
    constraint drop drops constraints:
. constraint drop 2
. constraint list
The empty list after constraint list indicates that no constraints are defined. Below we demonstrate the various syntaxes allowed by constraint:

```
. constraint 1 [Indemnity]
. constraint 10 [Indemnity]: 1.site 2.site
. constraint 11 [Indemnity]: 3.site
. constraint 21 [Prepaid=Uninsure]: nonwhite
. constraint 30 [Prepaid]
. constraint 31 [Insure]
. constraint list
    1: [Indemnity]
    10: [Indemnity]: 1.site 2.site
    11: [Indemnity]: 3.site
    21: [Prepaid=Uninsure]: nonwhite
    30: [Prepaid]
    31: [Insure]
. constraint drop 21-25, 31
. constraint list
    1: [Indemnity]
    10: [Indemnity]: 1.site 2.site
    11: [Indemnity]: 3.site
    30: [Prepaid]
. constraint drop _all
. constraint list
```

- Technical note

The constraint command does not check the syntax of the constraint itself because a constraint can be interpreted only in the context of a model. Thus constraint is willing to define constraints that later will not make sense. Any errors in the constraints will be detected and mentioned at the time of estimation.

## References

Buis, M. L. 2012. Stata tip 108: On adding and constraining. Stata Journal 12: 342-344.
Weesie, J. 1999. sg100: Two-stage linear constrained estimation. Stata Technical Bulletin 47: 24-30. Reprinted in Stata Technical Bulletin Reprints, vol. 8, pp. 217-225. College Station, TX: Stata Press.

## Also see

[R] cnsreg - Constrained linear regression

## Title

contrast - Contrasts and linear hypothesis tests after estimation

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

Also see

## Syntax

contrast termlist [, options]
where termlist is a list of factor variables or interactions that appear in the current estimation results. The variables may be typed with or without contrast operators, and you may use any factor-variable syntax:

See the operators (op.) table below for the list of contrast operators. options Description

Main
overall
asobserved
lincom

Equations
equation (eqspec)
atequations
Advanced
emptycells (empspec) treatment of empty cells for balanced factors
noestimcheck
Reporting
level(\#)
mcompare(method)
noeffects
cieffects
pveffects
effects
nowald
noatlevels
nosvyadjust
sort
post
display_options
eform_option
df (\#)
add a joint hypothesis test for all specified contrasts treat all factor variables as observed treat user-defined contrasts as linear combinations
perform contrasts in termlist for equation eqspec perform contrasts in termlist within each equation
suppress estimability checks
confidence level; default is level (95)
adjust for multiple comparisons; default is mcompare (noadjust)
suppress table of individual contrasts
show effects table with confidence intervals
show effects table with $p$-values
show effects table with confidence intervals and $p$-values
suppress table of Wald tests
report only the overall Wald test for terms that use the within © or nested | operator
compute unadjusted Wald tests for survey results
sort the individual contrast values in each term
post contrasts and their VCEs as estimation results report exponentiated contrasts
use $t$ distribution with \# degrees of freedom for computing $p$-values
control column formats, row spacing, line width, and factor-variable labeling and confidence intervals
$\mathrm{df}(\#)$ does not appear in the dialog box.

Term Description

| Main effects |  |
| :---: | :---: |
| A | joint test of the main effects of A |
| $r$. A | individual contrasts that decompose A using $r$. |
| Interaction effects |  |
| A\#B | joint test of the two-way interaction effects of $A$ and $B$ |
| A\#B\#C | joint test of the three-way interaction effects of $\mathrm{A}, \mathrm{B}$, and C |
| $r . A \# g . \mathrm{B}$ | individual contrasts for each interaction of A and B defined by $r$. and $g$. |
| Partial interaction effects |  |
| $r . A \# B$ | joint tests of interactions of A and B within each contrast defined by $r$. A |
| A\#r.B | joint tests of interactions of A and B within each contrast defined by r.B |
| Simple effects |  |
| A@B | joint tests of the effects of A within each level of B |
| A@B\#C | joint tests of the effects of A within each combination of the levels of B and C |
| $r . A @ B$ | individual contrasts of A that decompose A@B using $r$. |
| $r . A @ B \# C$ | individual contrasts of A that decompose A@B\#C using $r$. |


| Other conditional effects |  |
| :--- | :--- |
| A\#B@C | joint tests of the interaction effects of $A$ and $B$ within each level of $C$ |
| A\#B@C\#D | joint tests of the interaction effects of $A$ and $B$ within each combination of |
| the levels of $C$ and $D$ |  |

Nested effects
A|B joint tests of the effects of A nested in each level of B
A|B\#C
A\#B|C
A\#BlC\#D
$r . \mathrm{A} \mid \mathrm{B}$
$r . \mathrm{A} \mid \mathrm{B} \# \mathrm{C}$
$r . \mathrm{A} \# g . \mathrm{BlC}$

Slope effects
A\#c. $x$
A\#c.x\#c.y
A\#B\#c. x
A\#B\#c.x\#c.y
$r . A \# c . x$
joint test of the effects of $A$ on the slopes of $x$
joint test of the effects of $A$ on the slopes of the product (interaction) of $x$ and $y$ joint test of the interaction effects of $A$ and $B$ on the slopes of $x$
joint test of the interaction effects of $A$ and $B$ on the slopes of the product (interaction) of x and y
individual contrasts of A's effects on the slopes of x using $r$.
Denominators
... / term2
use term2 as the denominator in the $F$ tests of the preceding terms
... / use the residual as the denominator in the $F$ tests of the preceding terms (the default if no other /s are specified)
$\mathrm{A}, \mathrm{B}, \mathrm{C}$, and D represent any factor variable in the current estimation results.
x and y represent any continuous variable in the current estimation results.
$r$. and $g$. represent any contrast operator. See the table below.
c. specifies that a variable be treated as continuous; see [U] 11.4.3 Factor variables.

Operators are allowed on any factor variable that does not appear to the right of @ or I. Operators decompose the effects of the associated factor variable into one-degree-of-freedom effects (contrasts).
Higher-level interactions are allowed anywhere an interaction operator (\#) appears in the table.
Time-series operators are allowed if they were used in the estimation.
_eqns designates the equations in manova, mlogit, mprobit, and mvreg and can be specified anywhere a factor variable appears.
/ is allowed only after anova, cnsreg, manova, mvreg, or regress.

## operators (op.) Description

r. differences from the reference (base) level; the default
a. differences from the next level (adjacent contrasts)
ar. differences from the previous level (reverse adjacent contrasts)
As-balanced operators
g.
h.
j.
p.
q.

As-observed operators
gw.
hw.
jw.
pw.
qW.
differences from the balanced grand mean
differences from the balanced mean of subsequent levels (Helmert contrasts)
differences from the balanced mean of previous levels (reverse Helmert contrasts)
orthogonal polynomial in the level values
orthogonal polynomial in the level sequence
differences from the observation-weighted grand mean
differences from the observation-weighted mean of subsequent levels
differences from the observation-weighted mean of previous levels observation-weighted orthogonal polynomial in the level values
observation-weighted orthogonal polynomial in the level sequence

One or more individual contrasts may be selected by using the op\#. or op (numlist) . syntax. For example, a3. A selects the adjacent contrast for level 3 of $A$, and $p(1 / 2)$. B selects the linear and quadratic effects of B. Also see Orthogonal polynomial contrasts and Beyond linear models.

## Custom contrasts Description

\{A numlist $\}$
\{A\#B numlist\}
user-defined contrast on the levels of factor A
user-defined contrast on the levels of the interaction between A and B
Custom contrasts may be part of a term, such as $\{\mathrm{A}$ numlist $\} \# \mathrm{~B},\{\mathrm{~A}$ numlist $\} @ \mathrm{~B},\{\mathrm{~A}$ numlist $\} \mid \mathrm{B},\{\mathrm{A} \# \mathrm{~B}$ numlist $\}$, and \{A numlist \}\#\{B numlist\}. The same is true of higher-order custom contrasts, such as $\{\mathrm{A} \# \mathrm{~B}$ numlist $\} @ C,\{\mathrm{~A} \# \mathrm{~B}$ numlist $\} \# r$. C , and $\{\mathrm{A} \# \mathrm{~B}$ numlist \}\#c.x.

Higher-order interactions with at most eight factor variables are allowed with custom contrasts.
method
Description
noadjust
bonferroni [adjustall]
sidak[adjustall]
scheffe
do not adjust for multiple comparisons; the default
Bonferroni's method; adjust across all terms
Šidák's method; adjust across all terms
Scheffé's method

## Menu

Statistics $>$ Postestimation $>$ Contrasts

## Description

contrast tests linear hypotheses and forms contrasts involving factor variables and their interactions from the most recently fit model. The tests include ANOVA-style tests of main effects, simple effects, interactions, and nested effects. contrast can use named contrasts to decompose these effects into comparisons against reference categories, comparisons of adjacent levels, comparisons against the grand mean, orthogonal polynomials, and such. Custom contrasts may also be specified.
contrast can be used with svy estimation results; see [SVY] svy postestimation.
Contrasts can also be computed for margins of linear and nonlinear responses; see [R] margins, contrast.

## Options

overall specifies that a joint hypothesis test over all terms be performed.
asobserved specifies that factor covariates be evaluated using the cell frequencies observed in the estimation sample. The default is to treat all factor covariates as though there were an equal number of observations in each level.
lincom specifies that user-defined contrasts be treated as linear combinations. The default is to require that all user-defined contrasts sum to zero. (Summing to zero is part of the definition of a contrast.)
equation(eqspec) specifies the equation from which contrasts are to be computed. The default is to compute contrasts from the first equation.
atequations specifies that the contrasts be computed within each equation.

## Advanced

emptycells (empspec) specifies how empty cells are handled in interactions involving factor variables that are being treated as balanced.
emptycells(strict) is the default; it specifies that contrasts involving empty cells be treated as not estimable.
emptycells (reweight) specifies that the effects of the observed cells be increased to accommodate any missing cells. This makes the contrast estimable but changes its interpretation.
noestimcheck specifies that contrast not check for estimability. By default, the requested contrasts are checked and those found not estimable are reported as such. Nonestimability is usually caused by empty cells. If noestimcheck is specified, estimates are computed in the usual way and reported even though the resulting estimates are manipulable, which is to say they can differ across equivalent models having different parameterizations.

Reporting
level(\#) specifies the confidence level, as a percentage, for confidence intervals. The default is level (95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals.
mcompare (method) specifies the method for computing $p$-values and confidence intervals that account for multiple comparisons within a factor-variable term.
Most methods adjust the comparisonwise error rate, $\alpha_{c}$, to achieve a prespecified experimentwise error rate, $\alpha_{e}$.
mcompare(noadjust) is the default; it specifies no adjustment.

$$
\alpha_{c}=\alpha_{e}
$$

mcompare (bonferroni) adjusts the comparisonwise error rate based on the upper limit of the Bonferroni inequality

$$
\alpha_{e} \leq m \alpha_{c}
$$

where $m$ is the number of comparisons within the term.
The adjusted comparisonwise error rate is

$$
\alpha_{c}=\alpha_{e} / m
$$

mcompare (sidak) adjusts the comparisonwise error rate based on the upper limit of the probability inequality

$$
\alpha_{e} \leq 1-\left(1-\alpha_{c}\right)^{m}
$$

where $m$ is the number of comparisons within the term.
The adjusted comparisonwise error rate is

$$
\alpha_{c}=1-\left(1-\alpha_{e}\right)^{1 / m}
$$

This adjustment is exact when the $m$ comparisons are independent.
mcompare (scheffe) controls the experimentwise error rate using the $F$ or $\chi^{2}$ distribution with degrees of freedom equal to the rank of the term.
mcompare(method adjustall) specifies that the multiple-comparison adjustments count all comparisons across all terms rather than performing multiple comparisons term by term. This leads to more conservative adjustments when multiple variables or terms are specified in marginslist. This option is compatible only with the bonferroni and sidak methods.
noeffects suppresses the table of individual contrasts with confidence intervals. This table is produced by default when the mcompare () option is specified or when a term in termlist implies all individual contrasts.
cieffects specifies that a table containing a confidence interval for each individual contrast be reported.
pveffects specifies that a table containing a $p$-value for each individual contrast be reported.
effects specifies that a single table containing a confidence interval and $p$-value for each individual contrast be reported.
nowald suppresses the table of Wald tests.
noatlevels indicates that only the overall Wald test be reported for each term containing within or nested (@ or I) operators.
nosvyadjust is for use with svy estimation commands. It specifies that the Wald test be carried out without the default adjustment for the design degrees of freedom. That is to say the test is carried out as $W / k \sim F(k, d)$ rather than as $(d-k+1) W /(k d) \sim F(k, d-k+1)$, where $k$ is the dimension of the test and $d$ is the total number of sampled PSUs minus the total number of strata.
sort specifies that the table of individual contrasts be sorted by the contrast values within each term.
post causes contrast to behave like a Stata estimation (e-class) command. contrast posts the vector of estimated contrasts along with the estimated variance-covariance matrix to e(), so you can treat the estimated contrasts just as you would results from any other estimation command. For example, you could use test to perform simultaneous tests of hypotheses on the contrasts, or you could use lincom to create linear combinations.
display_options: vsquish, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% fmt), pformat (\%fmt), sformat ( $\% f m t$ ), and nolstretch.
vsquish specifies that the blank space separating factor-variable terms or time-series-operated variables from other variables in the model be suppressed.
nofvlabel displays factor-variable level values rather than attached value labels. This option overrides the fvlabel setting; see [R] set showbaselevels.
fvwrap(\#) specifies how many lines to allow when long value labels must be wrapped. Labels requiring more than \# lines are truncated. This option overrides the fvwrap setting; see [R] set showbaselevels.
fvwrapon(style) specifies whether value labels that wrap will break at word boundaries or break based on available space.
fvwrapon(word), the default, specifies that value labels break at word boundaries. fvwrapon(width) specifies that value labels break based on available space.
This option overrides the fvwrapon setting; see [R] set showbaselevels.
cformat (\%fmt) specifies how to format contrasts, standard errors, and confidence limits in the table of estimated contrasts.
pformat (\% fint) specifies how to format $p$-values in the table of estimated contrasts.
sformat (\% fimt) specifies how to format test statistics in the table of estimated contrasts.
nolstretch specifies that the width of the table of estimated contrasts not be automatically widened to accommodate longer variable names. The default, lstretch, is to automatically widen the table of estimated contrasts up to the width of the Results window. To change the default, use set lstretch off. nolstretch is not shown in the dialog box.
eform_option specifies that the contrasts table be displayed in exponentiated form. $e^{\text {contrast }}$ is displayed rather than contrast. Standard errors and confidence intervals are also transformed. See $[\mathrm{R}]$ eform_option for the list of available options.

The following option is available with contrast but is not shown in the dialog box:
df (\#) specifies that the $t$ distribution with \# degrees of freedom be used for computing $p$-values and confidence intervals. The default is to use e(df_r) degrees of freedom or the standard normal distribution if e(df_r) is missing.

## Remarks and examples

Remarks are presented under the following headings:

```
Introduction
One-way models
    Estimated cell means
    Testing equality of cell means
    Reference category contrasts
    Reverse adjacent contrasts
    Orthogonal polynomial contrasts
Two-way models
    Estimated interaction cell means
    Simple effects
    Interaction effects
    Main effects
    Partial interaction effects
Three-way and higher-order models
Contrast operators
    Differences from a reference level (r.)
    Differences from the next level (a.)
    Differences from the previous level (ar.)
    Differences from the grand mean (g.)
    Differences from the mean of subsequent levels (h.)
    Differences from the mean of previous levels (j.)
    Orthogonal polynomials (p. and q.)
User-defined contrasts
Empty cells
Empty cells, ANOVA style
Nested effects
Multiple comparisons
Unbalanced data
    Using observed cell frequencies
    Weighted contrast operators
Testing factor effects on slopes
Chow tests
Beyond linear models
Multiple equations
Video example
```


## Introduction

contrast performs ANOVA-style tests of main effects, interactions, simple effects, and nested effects. It can easily decompose these tests into constituent contrasts using either named contrasts (codings) or user-specified contrasts. Comparing levels of factor variables-whether as main effects, interactions, or simple effects-is as easy as adding a contrast operator to the variable. The operators can compare each level with the previous level, each level with a reference level, each level with the mean of previous levels, and more.
contrast tests and estimates contrasts. A contrast of the parameters $\mu_{1}, \mu_{2}, \ldots, \mu_{p}$ is a linear combination $\sum_{i} c_{i} \mu_{i}$ whose $c_{i}$ sum to zero. A difference of population means that $\mu_{1}-\mu_{2}$ is a contrast, as are most other comparisons of population or model quantities (Coster 2005). Some contrasts may be estimated with lincom, but contrast is much more powerful. contrast can handle multiple contrasts simultaneously, and the command's contrast operators make it easy to specify complicated linear combinations.

Both the contrast operation and the creation of the margins for comparison can be performed as though the data were balanced (typical for experimental designs) or using the observed frequencies in the estimation sample (typical for observational studies). contrast can perform these analyses on the results of almost all of Stata's estimators, not just the linear-models estimators.

Most of contrast's computations can be considered comparisons of estimated cell means from a model fit. Tests of interactions are tests of whether the cell means for the interaction are all equal. Tests of main effects are tests of whether the marginal cell means for the factor are all equal. More focused comparisons of cell means (for example, is level 2 equal to level 1) are specified using contrast operators. More formally, all of contrast's computations are comparisons of conditional expectations; cell means are one type of conditional expectation.

All contrasts can also easily be graphed; see [R] marginsplot.
For a discussion of contrasts and testing for linear models, see Searle (1971) and Searle (1997). For discussions specifically related to experimental design, see Kuehl (2000), Winer, Brown, and Michels (1991), and Milliken and Johnson (2009). Rosenthal, Rosnow, and Rubin (2000) focus on contrasts with applications in behavioral sciences. Mitchell (2012) focuses on contrasts in Stata.
contrast is a flexible tool for understanding the effects of categorical covariates. If your model contains categorical covariates, and especially if it contains interactions, you will want to use contrast.

## One-way models

Suppose we have collected data on cholesterol levels for individuals from five age groups. To study the effect of age group on cholesterol, we can begin by fitting a one-way model using regress:

```
. use http://www.stata-press.com/data/r13/cholesterol
(Artificial cholesterol data)
. label list ages
ages:
        1 10-19
        2 20-29
        3 30-39
        40-59
        5 60-79
```

. regress chol i.agegrp

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | 14943.3997 | 4 | 3735.84993 |
| Residual | 7468.21971 | 70 | 106.688853 |
| Total | 22411.6194 | 74 | 302.859722 |

                                    Number of obs \(=\quad 75\)
                                    \(F(4,70)=35.02\)
                                    Prob > F \(=0.0000\)
                                    \(R\)-squared \(=0.6668\)
                                    Adj R-squared \(=0.6477\)
                                    Root MSE \(=10.329\)
    | chol | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| agegrp |  |  |  |  |  |  |
| $20-29$ | 8.203575 | 3.771628 | 2.18 | 0.033 | .6812991 | 15.72585 |
| $30-39$ | 21.54105 | 3.771628 | 5.71 | 0.000 | 14.01878 | 29.06333 |
| $40-59$ | 30.15067 | 3.771628 | 7.99 | 0.000 | 22.6284 | 37.67295 |
| $60-79$ | 38.76221 | 3.771628 | 10.28 | 0.000 | 31.23993 | 46.28448 |
|  |  |  |  |  |  |  |
| _cons | 180.5198 | 2.666944 | 67.69 | 0.000 | 175.2007 | 185.8388 |

## Estimated cell means

margins will show us the estimated cell means for each age group based on our fitted model:

| - margins agegrp <br> Adjusted predictions <br> Model VCE <br> : OLS <br> Expression <br> : Linear prediction, predict() |
| :--- |

We can graph those means with marginsplot:

```
. marginsplot
Variables that uniquely identify margins: agegrp
```



## Testing equality of cell means

Are all the means equal? That is to say is there an effect of age group on cholesterol level? We can answer that by asking contrast to test whether the means of the age groups are identical.

| . contrast agegrp <br> Contrasts of marginal <br> Margins <br> : asbalanced |
| :--- |
| agegrp |

The means are clearly different. We could have obtained this same test directly had we fit our model using anova rather than regress.


Achieving a more direct test result is why we recommend using anova instead of regress for models where our focus is on the categorical covariates. The models fit by anova and regress are identical; they merely parameterize the effects differently. The results of contrast will be identical regardless of which command is used to fit the model. If, however, we were fitting models whose responses are nonlinear functions of the covariates, such as logistic regression, then there would be no analogue to anova, and we would appreciate contrast's ability to quickly test main effects and interactions.

## Reference category contrasts

Now that we know that the overall effect of age group is statistically significant, we can explore the effects of each age group. One way to do that is to use the reference category operator, r.:

```
. contrast r.agegrp
Contrasts of marginal linear predictions
Margins : asbalanced
```

|  | df | $F$ | $P>F$ |
| ---: | ---: | ---: | ---: |
| agegrp |  |  |  |
| $(20-29$ vs 10-19) | 1 | 4.73 | 0.0330 |
| $(30-39$ vs 10-19) | 1 | 32.62 | 0.0000 |
| $(40-59$ vs 10-19) | 1 | 63.91 | 0.0000 |
| $(60-79$ vs 10-19) | 1 | 105.62 | 0.0000 |
| Joint | 4 | 35.02 | 0.0000 |
| Denominator | 70 |  |  |


|  | Contrast | Std. Err. | [95\% Conf. Interval] |  |
| :--- | ---: | :--- | ---: | ---: |
| agegrp |  |  |  |  |
| $(20-29$ vs 10-19) | 8.203575 | 3.771628 | .6812991 | 15.72585 |
| $(30-39$ vs 10-19) | 21.54105 | 3.771628 | 14.01878 | 29.06333 |
| $(40-59$ vs 10-19) | 30.15067 | 3.771628 | 22.6284 | 37.67295 |
| $(60-79$ vs 10-19) | 38.76221 | 3.771628 | 31.23993 | 46.28448 |

The cell mean of each age group is compared against the base age group (ages 10-19). The first table shows that each difference is significant. The second table gives an estimate and confidence interval for each contrast. These are the comparisons that linear regression gives with a factor covariate and no interactions. The contrasts are identical to the coefficients from our linear regression.

## Reverse adjacent contrasts

We have far more flexibility with contrast. Age group is ordinal, so it is interesting to compare each age group with the preceding age group (rather than against one reference group). We specify that analysis by using the reverse adjacent operator, ar.:

| Contrasts of marginal linear predictions Margins : asbalanced |  |  |  |
| :---: | :---: | :---: | :---: |
|  | df | F | $\mathrm{P}>\mathrm{F}$ |
| agegrp |  |  |  |
| (20-29 vs 10-19) | 1 | 4.73 | 0.0330 |
| (30-39 vs 20-29) | 1 | 12.51 | 0.0007 |
| (40-59 vs 30-39) | 1 | 5.21 | 0.0255 |
| (60-79 vs 40-59) | 1 | 5.21 | 0.0255 |
| Joint | 4 | 35.02 | 0.0000 |
| Denominator | 70 |  |  |


|  | Contrast | Std. Err. | [95\% Conf. Interval] |  |
| :--- | ---: | :--- | :--- | :--- |
| agegrp |  |  |  |  |
| $(20-29$ vs 10-19) | 8.203575 | 3.771628 | .6812991 | 15.72585 |
| $(30-39$ vs 20-29) | 13.33748 | 3.771628 | 5.815204 | 20.85976 |
| $(40-59$ vs $30-39)$ | 8.60962 | 3.771628 | 1.087345 | 16.1319 |
| $(60-79$ vs 40-59) | 8.611533 | 3.771628 | 1.089257 | 16.13381 |

The 20-29 age group's cholesterol level is 8.2 points higher than the $10-19$ age group's cholesterol level; the $30-39$ age group's level is 13.3 points higher than the $20-29$ age group's level; and so on. Each age group is statistically different from the preceding age group at the $5 \%$ level.

## Orthogonal polynomial contrasts

The relationship between age group and cholesterol level looked almost linear in our graph. We can examine that relationship further by using the orthogonal polynomial operator, p.:

```
. contrast p.agegrp, noeffects
Contrasts of marginal linear predictions
Margins : asbalanced
```

|  | df | F | $\mathrm{P}>\mathrm{F}$ |
| :---: | :---: | ---: | ---: |
| agegrp |  |  |  |
| (linear) | 1 | 139.11 | 0.0000 |
| (quadratic) | 1 | 0.15 | 0.6962 |
| (cubic) | 1 | 0.37 | 0.5448 |
| (quartic) | 1 | 0.43 | 0.5153 |
| Joint | 4 | 35.02 | 0.0000 |
| Denominator | 70 |  |  |

Only the linear effect is statistically significant.
We can even perform the joint test that all effects beyond linear are zero. We do that by selecting all polynomial contrasts above linear-that is, polynomial contrasts 2,3 , and 4.

| . contrast p(2 3 4). agegrp, noeffects <br> Contrasts of marginal linear predictions <br> : asbalanced |
| :--- |
| Margins |
| agegrp |

The joint test has three degrees of freedom and is clearly insignificant. A linear effect of age group seems adequate for this model.

## Two-way models

Suppose we are investigating the effects of different dosages of a blood pressure medication and believe that the effects may be different for men and women. We can fit the following ANOVA model for bpchange, the change in diastolic blood pressure. Change is defined as the after measurement minus the before measurement, so that negative values of bpchange correspond to decreases in blood pressure.


## Estimated interaction cell means

Everything is significant, including the interaction. So increasing dosage is effective and differs by gender. Let's explore the effects. First, let's look at the estimated cell mean of blood pressure change for each combination of gender and dosage.

| Adjusted predictions |  |  |  | Number of obs |  | 30 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Linear prediction, predict() |  |  |  |  |  |
|  | Delta-method |  |  |  | [95\% Conf. | Interval] |
|  | Margin | Std. Err. | t | $P>\|t\|$ |  |  |
| dose\#gender |  |  |  |  |  |  |
| 250\#male | -7.35384 | . 6563742 | -11.20 | 0.000 | -8.708529 | -5.99915 |
| 250\#female | 3.706567 | . 6563742 | 5.65 | 0.000 | 2.351877 | 5.061257 |
| 500\#male | -13.73386 | . 6563742 | -20.92 | 0.000 | -15.08855 | -12.37917 |
| 500\#female | -6.584167 | . 6563742 | -10.03 | 0.000 | -7.938857 | -5.229477 |
| 750\#male | -16.82108 | . 6563742 | -25.63 | 0.000 | -18.17576 | -15.46639 |
| 750\#female | -14.38795 | . 6563742 | -21.92 | 0.000 | -15.74264 | -13.03326 |

Our data are balanced, so these results will not be affected by the many different ways that margins can compute cell means. Moreover, because our model consists of only dose and gender, these are also the point estimates for each combination.

We can graph the results:

```
. marginsplot
    Variables that uniquely identify margins: dose gender
```



The lines are not parallel, which we expected because the interaction term is significant. Males experience a greater decline in blood pressure at every dosage level, but the effect of increasing dosage is greater for females. In fact, it is not clear if we can tell the difference between male and female response at the maximum dosage.

## Simple effects

We can contrast the male and female responses within dosage to see the simple effects of gender. Because there are only two levels in gender, the choice of contrast operator is largely irrelevant. Aside from orthogonal polynomials, all operators produce the same estimates, although the effects can change signs.

```
. contrast r.gender@dose
Contrasts of marginal linear predictions
Margins : asbalanced
```

|  | df | F | $\mathrm{P}>\mathrm{F}$ |
| ---: | ---: | ---: | ---: |
| gender@dose |  |  |  |
| (female vs male) 250 | 1 | 141.97 | 0.0000 |
| (female vs male) 500 | 1 | 59.33 | 0.0000 |
| (female vs male) 750 | 1 | 6.87 | 0.0150 |
| Joint | 3 | 69.39 | 0.0000 |
| Denominator | 24 |  |  |


|  | Contrast | Std. Err. | [95\% Conf. Interval] |  |
| :--- | ---: | ---: | ---: | ---: |
| gender@dose |  |  |  |  |
| (female vs male) 250 | 11.06041 | .9282533 | 9.144586 | 12.97623 |
| (female vs male) 500 | 7.149691 | .9282533 | 5.23387 | 9.065512 |
| (female vs male) 750 | 2.433124 | .9282533 | .5173031 | 4.348944 |

The effect for males is about 11 points higher than for females at a dosage of 250, and that shrinks to 2.4 points higher at the maximum dosage of 750 .

We can form the simple effects the other way by contrasting the effect of dose at each level of gender:

```
. contrast ar.dose@gender
Contrasts of marginal linear predictions
Margins : asbalanced
```

|  | df | F | $\mathrm{P}>\mathrm{F}$ |
| ---: | ---: | ---: | ---: |
| dose@gender |  |  |  |
| $(500$ vs 250) male | 1 | 47.24 | 0.0000 |
| $(500 \mathrm{vs} \mathrm{250)} \mathrm{female}$ | 1 | 122.90 | 0.0000 |
| $(750 \mathrm{vs} \mathrm{500)} \mathrm{male}$ | 1 | 11.06 | 0.0028 |
| $(750 \mathrm{vs} \mathrm{500)} \mathrm{female}$ | 1 | 70.68 | 0.0000 |
| Joint | 4 | 122.65 | 0.0000 |
| Denominator |  |  |  |


|  | Contrast | Std. Err. | [95\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: |
| dose@gender |  |  |  |  |
| $(500$ vs 250) male | -6.380018 | .9282533 | -8.295839 | -4.464198 |
| $(500$ vs 250) female | -10.29073 | .9282533 | -12.20655 | -8.374914 |
| $(750$ vs 500) male | -3.087217 | .9282533 | -5.003038 | -1.171396 |
| $(750$ vs 500) female | -7.803784 | .9282533 | -9.719605 | -5.887963 |

Here we use the ar. reverse adjacent contrast operator so that first we are comparing a dosage of 500 with a dosage of 250 , and then we are comparing 750 with 500 . We see that increasing the dosage has a larger effect on females- 10.3 points when going from 250 to 500 compared with 6.4 points for males, and 7.8 points when going from 500 to 750 versus 3.1 points for males.

## Interaction effects

By specifying contrast operators on both factors, we can decompose the interaction effect into separate interaction contrasts.

| Contrasts of marginal linear predictions Margins : asbalanced |  |  |  |
| :---: | :---: | :---: | :---: |
|  | df | F | P>F |
| dose\#gender |  |  |  |
| (500 vs 250) (female vs male) | 1 | 8.87 | 0.0065 |
| (750 vs 500) (female vs male) | 1 | 12.91 | 0.0015 |
| Joint | 2 | 21.66 | 0.0000 |
| Denominator | 24 |  |  |


|  | Contrast | Std. Err. | [95\% Conf. Interval] |  |
| ---: | :---: | :---: | :---: | :---: |
| dose\#gender <br> $(500$ vs 250) <br> $(\mathrm{female}$ vs male) <br> $(750$ vs 500) <br> $(\mathrm{female}$ vs male) | -3.910716 | 1.312748 | -6.620095 | -1.201336 |

Look for departures from zero to indicate an interaction effect between dose and gender. Both contrasts are significantly different from zero. Of course, we already knew the overall interaction was significant from our ANOVA results. The effect of increasing dose from 250 to 500 is 3.9 points greater in females than in males, and the effect of increasing dose from 500 to 750 is 4.7 points greater in females than in males. The confidence intervals for both estimates easily exclude zero, meaning that there is an interaction effect.

The joint test of these two interaction effects reproduces the test of interaction effects in the anova output. We can see that the $F$ statistic of 21.66 matches the statistic from our original ANOVA results.

## Main effects

We can perform tests of the main effects by listing each variable individually in contrast.

| . contrast dose gender <br> Contrasts of marginal linear predictions <br> Margins <br> : asbalanced |
| :--- |
| dose |
| gender |

The $F$ tests are equivalent to the tests of main effects in the anova output. This is true only for linear models. contrast provides an easy way to obtain main effects and other ANOVA-style tests for models whose responses are not linear in the parameters-logistic, probit, glm, etc.

If we include contrast operators on the variables, we can also decompose the main effects into individual contrasts:

| contrast ar.dose r.gender <br> Contrasts of marginal linear predictions <br> Margins <br> asbalanced |
| :--- |


|  | Contrast | Std. Err. | [95\% Conf. Interval] |  |
| ---: | ---: | :--- | ---: | ---: |
| dose |  |  |  |  |
| $(500$ vs 250) | -8.335376 | .6563742 | -9.690066 | -6.980687 |
| $(750$ vs 500) | -5.4455 | .6563742 | -6.80019 | -4.090811 |
| (female vs male) | 6.881074 | .5359273 | 5.774974 | 7.987173 |

By specifying the ar. operator on dose, we decompose the main effect for dose into two one-degree-of-freedom contrasts, comparing the marginal mean of blood pressure change for each dosage level with that of the previous level. Because gender has only two levels, we cannot decompose this main effect any further. However, specifying a contrast operator on gender allowed us to calculate the difference in the marginal means for women and men.

## Partial interaction effects

At this point, we have looked at the total interaction effects and at the main effects of each variable. The partial interaction effects are a midpoint between these two types of effects where we collect the individual interaction effects along the levels of one of the variables and perform a joint test of those interactions. If we think of the interaction effects as forming a table, with the levels of one factor variable forming the rows and the levels of the other forming the columns, partial interaction effects are joint tests of the interactions in a row or a column. To perform these tests, we specify a contrast operator on only one of the variables in our interaction. For this particular model, these are not very interesting because our variables have only two and three levels. Therefore, the tests of the partial interaction effects reproduce the tests that we obtained for the total interaction effects. We specify a contrast operator only on dose to decompose the overall test for interaction effects into joint tests for each ar.dose contrast:

| . contrast ar.dose\#gender <br> Contrasts of marginal linear predictions <br> Margins <br> : asbalanced |
| :--- |

The first row is a joint test of all the interaction effects involving the ( 500 vs 250 ) comparison of dosages. The second row is a joint test of all the interaction effects involving the ( 750 vs 500 ) comparison. If we look back at our output in Interaction effects, we can see that there was only one of each of these interaction effects. Therefore, each test labeled (joint) has only one degree-of-freedom.

We could have instead included a contrast operator on gender to compute the partial interaction effects along the other dimension:

| . contrast dose\#r.gender <br> Contrasts of marginal linear predictions <br> ( asbalanced |
| :--- |
| Margins df F $\mathrm{P}>\mathrm{F}$ <br>  2 21.66 0.0000 <br> dose\#gender 24   |

Here we obtain a joint test of all the interaction effects involving the (female vs male) comparison for gender. Because gender has only two levels, the (female vs male) contrast is the only reference category contrast possible. Therefore, we obtain a single joint test of all the interaction effects.

Clearly, the partial interaction effects are not interesting for this particular model. However, if our factors had more levels, the partial interaction effects would produce tests that are not available in the total interaction effects. For example, if our model included factors for four dosage levels and three races, then typing

```
. contrast ar.dose#race
```

would produce three joint tests, one for each of the reverse adjacent contrasts for dosage. Each of these tests would be a two-degree-of-freedom test because race has three levels.

## Three-way and higher-order models

All the contrasts and tests that we reviewed above for two-way models can be used with models that have more terms. For instance, we could fit a three-way full factorial model by using the anova command:

```
. use http://www.stata-press.com/data/r13/cont3way
. anova y race##sex##group
```

We could then test the simple effects of race within each level of the interaction between sex and group:

```
. contrast race@sex#group
```

To see the reference category contrasts that decompose these simple effects, type

```
. contrast r.race@sex#group
```

We could test the three-way interaction effects by typing

```
. contrast race#sex#group
```

or the interaction effects for the interaction of race and sex by typing

```
. contrast race#sex
```

To see the individual reference category contrasts that decompose this interaction effect, type

```
. contrast r.race#r.sex
```

We could even obtain joint tests for the interaction of race and sex within each level of group by typing

```
. contrast race#sex@group
```

For tests of the main effects of each factor, we can type

```
. contrast race sex group
```

We can calculate the individual reference category contrasts that decompose these main effects:

```
. contrast r.race r.sex r.group
```

For the partial interaction effects, we could type

```
. contrast r.race#group
```

to obtain a joint test of the two-way interaction effects of race and group for each of the individual r.race contrasts.

We could type
. contrast r.race\#sex\#group
to obtain a joint test of all the three-way interaction terms for each of the individual r.race contrasts.

## Contrast operators

contrast recognizes a set of contrast operators that are used to specify commonly used contrasts. When these operators are used, contrast will report a test for each individual contrast in addition to the joint test for the term. We have already seen a few of these, like r. and ar., in the previous examples. Here we will take a closer look at each of the unweighted operators.

Here we use the cholesterol dataset and the one-way ANOVA model from the example in One-way models:

```
. use http://www.stata-press.com/data/r13/cholesterol
(Artificial cholesterol data)
. anova chol agegrp
    (output omitted)
```

The margins command reports the estimated cell means, $\widehat{\mu}_{1}, \ldots, \widehat{\mu}_{5}$, for each of the five age groups.

```
. margins agegrp
Adjusted predictions
Number of obs = 75
Expression : Linear prediction, predict()
```

|  | Delta-method |  |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | :---: |
|  | Margin | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |  |
| agegrp |  |  |  |  |  |  |  |
| $10-19$ | 180.5198 | 2.666944 | 67.69 | 0.000 | 175.2007 | 185.8388 |  |
| $20-29$ | 188.7233 | 2.666944 | 70.76 | 0.000 | 183.4043 | 194.0424 |  |
| $30-39$ | 202.0608 | 2.666944 | 75.76 | 0.000 | 196.7418 | 207.3799 |  |
| $40-59$ | 210.6704 | 2.666944 | 78.99 | 0.000 | 205.3514 | 215.9895 |  |
| $60-79$ | 219.282 | 2.666944 | 82.22 | 0.000 | 213.9629 | 224.601 |  |

Contrast operators provide an easy way to make certain types of comparisons of these cell means. We use the ordinal factor agegrp to demonstrate these operators because some types of contrasts are only meaningful when the levels of the factor have a natural ordering. We demonstrate these contrast operators using a one-way model; however, they are equally applicable to main effects, simple effects, and interactions for more complicated models.

## Differences from a reference level (r.)

The r. operator specifies that each level of the attached factor variable be compared with a reference level. These are referred to as reference-level or reference-category contrasts (or effects), and $r$. is the reference-level operator.

In the following, we use the r. operator to test the effect of each category of age group when that category is compared with a reference category.

```
. contrast r.agegrp
Contrasts of marginal linear predictions
Margins : asbalanced
```

|  | df | $F$ | $P>F$ |
| ---: | ---: | ---: | ---: |
| agegrp |  |  |  |
| $(20-29$ vs 10-19) | 1 | 4.73 | 0.0330 |
| $(30-39$ vs 10-19) | 1 | 32.62 | 0.0000 |
| $(40-59$ vs 10-19) | 1 | 63.91 | 0.0000 |
| $(60-79$ vs 10-19) | 1 | 105.62 | 0.0000 |
| Joint | 4 | 35.02 | 0.0000 |
| Denominator | 70 |  |  |


|  | Contrast | Std. Err. | [95\% Conf. Interval] |  |
| :--- | ---: | ---: | ---: | ---: |
| agegrp |  |  |  |  |
| $(20-29$ vs 10-19) | 8.203575 | 3.771628 | .6812991 | 15.72585 |
| $(30-39$ vs 10-19) | 21.54105 | 3.771628 | 14.01878 | 29.06333 |
| $(40-59$ vs 10-19) | 30.15067 | 3.771628 | 22.6284 | 37.67295 |
| $(60-79$ vs 10-19) | 38.76221 | 3.771628 | 31.23993 | 46.28448 |

In the first table, the row labeled (20-29 vs $10-19$ ) is a test of $\mu_{2}=\mu_{1}$, a test that the mean cholesterol levels for the $10-19$ age group and the $20-29$ age group are equal. The tests in the next three rows are defined similarly. The row labeled Joint provides the joint test for these four hypotheses, which is just the test of the main effects of age group.

The second table provides the contrasts of each category with the reference category along with confidence intervals. The contrast in the row labeled ( $20-29$ vs $10-19$ ) is the difference in the cell means of the second age group and the first age group, $\widehat{\mu}_{2}-\widehat{\mu}_{1}$.

The first level of a factor is the default reference level, but we can specify a different reference level by using the b. operator; see [U] 11.4.3.2 Base levels. Here we use the last age group, (60-79), instead of the first as the reference category. We also include the nowald option so that only the table of contrasts and their confidence intervals is produced.

```
. contrast rb5.agegrp, nowald
Contrasts of marginal linear predictions
Margins : asbalanced
```

|  | Contrast | Std. Err. | [95\% Conf. Interval] |  |
| :--- | :--- | :--- | :--- | :--- |
| agegrp |  |  |  |  |
| $(10-19$ vs 60-79) | -38.76221 | 3.771628 | -46.28448 | -31.23993 |
| $(20-29$ vs 60-79) | -30.55863 | 3.771628 | -38.08091 | -23.03636 |
| $(30-39$ vs 60-79) | -17.22115 | 3.771628 | -24.74343 | -9.698877 |
| $(40-59$ vs 60-79) | -8.611533 | 3.771628 | -16.13381 | -1.089257 |

Now the first row is labeled (10-19 vs 60-79) and is the difference in the cell means of the first and fifth age groups.

## Differences from the next level (a.)

The a. operator specifies that each level of the attached factor variable be compared with the next level. These are referred to as adjacent contrasts (or effects), and a. is the adjacent operator. This operator is only meaningful with factor variables that have a natural ordering in the levels.

We can use the a. operator to perform tests that each level of age group differs from the next adjacent level.

```
. contrast a.agegrp
Contrasts of marginal linear predictions
Margins : asbalanced
```

|  | df | F | $\mathrm{P}>\mathrm{F}$ |
| ---: | ---: | ---: | ---: |
| agegrp |  |  |  |
| $(10-19$ vs 20-29) | 1 | 4.73 | 0.0330 |
| $(20-29$ vs $30-39)$ | 1 | 12.51 | 0.0007 |
| $(30-39$ vs 40-59) | 1 | 5.21 | 0.0255 |
| $(40-59$ vs 60-79) | 1 | 5.21 | 0.0255 |
| Joint | 4 | 35.02 | 0.0000 |
| Denominator | 70 |  |  |


|  | Contrast | Std. Err. | [95\% Conf. Interval] |  |
| :--- | ---: | :--- | ---: | ---: |
| agegrp |  |  |  |  |
| $(10-19$ vs 20-29) | -8.203575 | 3.771628 | -15.72585 | -.6812991 |
| $(20-29$ vs $30-39)$ | -13.33748 | 3.771628 | -20.85976 | -5.815204 |
| $(30-39$ vs $40-59)$ | -8.60962 | 3.771628 | -16.1319 | -1.087345 |
| $(40-59$ vs $60-79)$ | -8.611533 | 3.771628 | -16.13381 | -1.089257 |

In the first table, the row labeled ( $10-19$ vs $20-29$ ) tests the effect of belonging to the $10-19$ age group instead of the 20-29 age group. Likewise, the rows labeled ( $20-29$ vs 30-39) , (30-39 vs 40-59), and (40-59 vs 60-79) are tests for the effects of being in the younger of the two age groups instead of the older one.

In the second table, the contrast in the row labeled (10-19 vs $20-29$ ) is the difference in the cell means of the first and second age groups, $\widehat{\mu}_{1}-\widehat{\mu}_{2}$. The contrasts in the other rows are defined similarly.

## Differences from the previous level (ar.)

The ar. operator specifies that each level of the attached factor variable be compared with the previous level. These are referred to as reverse adjacent contrasts (or effects), and ar. is the reverse adjacent operator. As with the a. operator, this operator is only meaningful with factor variables that have a natural ordering in the levels.

In the following, we use the ar. operator to report tests for the individual reverse adjacent effects of agegrp.

| . contrast ar.agegrp <br> Contrasts of marginal linear predictions <br> Margins <br> : asbalanced |
| :--- |


|  | Contrast | Std. Err. | [95\% Conf. Interval] |  |
| ---: | ---: | :--- | ---: | ---: |
| agegrp |  |  |  |  |
| $(20-29$ vs 10-19) | 8.203575 | 3.771628 | .6812991 | 15.72585 |
| $(30-39$ vs 20-29) | 13.33748 | 3.771628 | 5.815204 | 20.85976 |
| $(40-59$ vs 30-39) | 8.60962 | 3.771628 | 1.087345 | 16.1319 |
| $(60-79$ vs 40-59) | 8.611533 | 3.771628 | 1.089257 | 16.13381 |

Here the Wald tests in the first table for the individual reverse adjacent effects are equivalent to the tests for the adjacent effects in the previous example. However, if we compare values of the contrasts in the bottom tables, we see the difference between the $r$. and the ar. operators. This time, the contrast in the first row is labeled (20-29 vs 10-19) and is the difference in the cell means of the second and first age groups, $\widehat{\mu}_{2}-\widehat{\mu}_{1}$. This is the estimated effect of belonging to the 20-29 age group instead of the 10-19 age group. The remaining rows make similar comparisons to the previous level.

## Differences from the grand mean (g.)

The $g$. operator specifies that each level of a factor variable be compared with the grand mean of all levels. For this operator, the grand mean is computed using a simple average of the cell means.

Here are the grand mean effects of agegrp:

```
. contrast g.agegrp
Contrasts of marginal linear predictions
Margins : asbalanced
```

|  | df | $F$ | $P>F$ |
| :---: | :---: | :---: | ---: |
| agegrp |  |  |  |
| $(10-19$ vs mean) | 1 | 68.42 | 0.0000 |
| $(30-29$ vs mean) | 1 | 23.36 | 0.0000 |
| $(40-59$ vs mean) | 1 | 0.58 | 0.4506 |
| $(60-79$ vs mean) | 1 | 19.08 | 0.0000 |
| Joint | 1 | 63.65 | 0.0000 |
| Denominator | 4 | 35.02 | 0.0000 |


|  | Contrast | Std. Err. | [95\% Conf. Interval] |  |
| ---: | ---: | :--- | ---: | ---: |
| agegrp |  |  |  |  |
| $(10-19$ vs mean) | -19.7315 | 2.385387 | -24.48901 | -14.974 |
| $(20-29$ vs mean) | -11.52793 | 2.385387 | -16.28543 | -6.770423 |
| $(30-39$ vs mean) | 1.809552 | 2.385387 | -2.947953 | 6.567057 |
| $(40-59$ vs mean) | 10.41917 | 2.385387 | 5.661668 | 15.17668 |
| $(60-79$ vs mean) | 19.0307 | 2.385387 | 14.2732 | 23.78821 |

There are five age groups in our estimation sample. Thus the row labeled (10-19 vs mean) tests $\mu_{1}=$ $\left(\mu_{1}+\mu_{2}+\mu_{3}+\mu_{4}+\mu_{5}\right) / 5$. The row labeled (20-29 vs mean) tests $\mu_{2}=\left(\mu_{1}+\mu_{2}+\mu_{3}+\mu_{4}+\mu_{5}\right) / 5$. The remaining rows perform similar tests for the third, fourth, and fifth age groups. In our example, the means for all age groups except the $30-39$ age group are statistically different from the grand mean.

## Differences from the mean of subsequent levels (h.)

The $h$. operator specifies that each level of the attached factor variable be compared with the mean of subsequent levels. These are referred to as Helmert contrasts (or effects), and h. is the Helmert operator. For this operator, the mean is computed using a simple average of the cell means. This operator is only meaningful with factor variables that have a natural ordering in the levels.

Here are the Helmert contrasts for agegrp:

| Contrasts of marginal linear predictions Margins : asbalanced |  |  |  |
| :---: | :---: | :---: | :---: |
|  | df | F | $\mathrm{P}>\mathrm{F}$ |
| agegrp |  |  |  |
| (10-19 vs >10-19) | 1 | 68.42 | 0.0000 |
| ( $20-29$ vs >20-29) | 1 | 50.79 | 0.0000 |
| (30-39 vs >30-39) | 1 | 15.63 | 0.0002 |
| (40-59 vs 60-79) | 1 | 5.21 | 0.0255 |
| Joint | 4 | 35.02 | 0.0000 |
| Denominator | 70 |  |  |


|  | Contrast | Std. Err. | [95\% Conf. Interval] |  |
| :--- | :--- | :--- | :--- | :--- |
| agegrp |  |  |  |  |
| $(10-19$ vs $>10-19)$ | -24.66438 | 2.981734 | -30.61126 | -18.7175 |
| $(20-29$ vs $>20-29)$ | -21.94774 | 3.079522 | -28.08965 | -15.80583 |
| $(30-39$ vs $>30-39)$ | -12.91539 | 3.266326 | -19.42987 | -6.400905 |
| $(40-59$ vs $60-79)$ | -8.611533 | 3.771628 | -16.13381 | -1.089257 |

The row labeled (10-19 vs >10-19) tests $\mu_{1}=\left(\mu_{2}+\mu_{3}+\mu_{4}+\mu_{5}\right) / 4$, that is, that the cell mean for the youngest age group is equal to the average of the cell means for the older age groups. The row labeled (20-29 vs >20-29) tests $\mu_{2}=\left(\mu_{3}+\mu_{4}+\mu_{5}\right) / 3$. The tests in the other rows are defined similarly.

## Differences from the mean of previous levels (j.)

The $j$. operator specifies that each level of the attached factor variable be compared with the mean of the previous levels. These are referred to as reverse Helmert contrasts (or effects), and $j$. is the reverse Helmert operator. For this operator, the mean is computed using a simple average of the cell means. This operator is only meaningful with factor variables that have a natural ordering in the levels.

Here are the reverse Helmert contrasts of agegrp:

| Contrasts of marginal linear predictions Margins : asbalanced |  |  |  |
| :---: | :---: | :---: | :---: |
|  | df | F | $\mathrm{P}>\mathrm{F}$ |
| agegrp |  |  |  |
| (20-29 vs 10-19) | 1 | 4.73 | 0.0330 |
| (30-39 vs <30-39) | 1 | 28.51 | 0.0000 |
| (40-59 vs <40-59) | 1 | 43.18 | 0.0000 |
| (60-79 vs <60-79) | 1 | 63.65 | 0.0000 |
| Joint | 4 | 35.02 | 0.0000 |
| Denominator | 70 |  |  |


|  | Contrast | Std. Err. | [95\% Conf. Interval] |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
|  | agegrp |  |  |  |  |
| $(20-29$ vs 10-19) | 8.203575 | 3.771628 | .6812991 | 15.72585 |  |
| $(30-39$ vs <30-39) | 17.43927 | 3.266326 | 10.92479 | 23.95375 |  |
| $(40-59$ vs <40-59) | 20.2358 | 3.079522 | 14.09389 | 26.37771 |  |
| $(60-79$ vs <60-79) | 23.78838 | 2.981734 | 17.8415 | 29.73526 |  |

The row labeled (20-29 vs 10-19) tests $\mu_{2}=\mu_{1}$, that is, that the cell means for the 20-29 and the $10-19$ age groups are equal. The row labeled (30-39 vs <30-29) tests $\mu_{3}=\left(\mu_{1}+\mu_{2}\right) / 2$, that is, that the cell mean for the $30-39$ age group is equal to the average of the cell means for the $10-19$ and $20-29$ age groups. The tests in the remaining rows are defined similarly.

## Orthogonal polynomials (p. and q.)

The p. and q. operators specify that orthogonal polynomials be applied to the attached factor variable. Orthogonal polynomial contrasts allow us to partition the effects of a factor variable into linear, quadratic, cubic, and higher-order polynomial components. The p. operator applies orthogonal polynomials using the values of the factor variable. The q. operator applies orthogonal polynomials using the level indices. If the level values of the factor variable are equally spaced, as with our agegrp variable, then the p . and q . operators yield the same result. These operators are only meaningful with factor variables that have a natural ordering in the levels.

Because agegrp has five levels, contrast can test the linear, quadratic, cubic, and quartic effects of agegrp.

| . contrast p.agegrp, noeffects <br> Contrasts of marginal linear predictions <br> Margins <br> : asbalanced |
| :--- |

The row labeled (linear) tests the linear effect of agegrp, the only effect that appears to be significant in this case.

The labels for our agegrp variable show the age ranges that correspond to each level.

```
. label list ages
ages:
    1 10-19
    2 20-29
    3 30-39
    4 40-59
    5 60-79
```

Notice that these groups do not have equal widths. Now let's refit our model using the agemidpt variable. The values of agemidpt indicate the midpoint of each age group that was defined by the agegrp variable and are, therefore, not equally spaced.


Now if we use the q. operator, we will obtain the same results as above because the level indices of agemidpt are equivalent to the values of agegrp.

```
. contrast q.agemidpt, noeffects
Contrasts of marginal linear predictions
Margins : asbalanced
```

|  | df | F | $\mathrm{P}>\mathrm{F}$ |
| :---: | :---: | ---: | ---: |
| agemidpt |  |  |  |
| (linear) | 1 | 139.11 | 0.0000 |
| (quadratic) | 1 | 0.15 | 0.6962 |
| (cubic) | 1 | 0.37 | 0.5448 |
| (quartic) | 1 | 0.43 | 0.5153 |
| Joint | 4 | 35.02 | 0.0000 |
| Denominator | 70 |  |  |

However, if we use the p . operator, we will instead fit an orthogonal polynomial to the midpoint values.

| . contrast p.agemidpt, noeffects <br> Contrasts of marginal linear predictions <br> Margins <br> : asbalanced |
| :--- |

Using the values of the midpoints, the quadratic effect is also significant at the $5 \%$ level.

## - Technical note

We used the noeffects option when working with orthogonal polynomial contrasts. Apart from perhaps the sign of the contrast, the values of the individual contrasts are not meaningful for orthogonal polynomial contrasts. In addition, many textbooks provide tables with contrast coefficients that can be used to compute orthogonal polynomial contrasts where the levels of a factor are equally spaced. If we use these coefficients and calculate the contrasts manually with user-defined contrasts, as described below, the Wald tests for the polynomial terms will be equivalent, but the values of the individual contrasts will not necessarily match those that we obtain when using the polynomial contrast operator. When we use one of these contrast operators, an algorithm is used to calculate the coefficients of the polynomial contrast that will allow for unequal spacing in the levels of the factor as well as in the weights for the cell frequencies (when using pw. or qw.), as described in Methods and formulas.

## User-defined contrasts

In the previous examples, we performed tests using contrast operators. When there is not a contrast operator available to calculate the contrast in which we are interested, we can specify custom contrasts.

Here we fit a one-way model for cholesterol on the factor race, which has three levels:

margins calculates the estimated cell mean cholesterol level for each race:

| Adjusted predictions |  |  |  | Number of obs |  | 75 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Expression | inear pred | ction, pred | ct () |  |  |  |
|  | Delta-method |  |  |  |  |  |
|  | Margin | Std. Err. | t | $P>\|t\|$ | [95\% Conf. | Interval] |
| race |  |  |  |  |  |  |
| black | 204.4279 | 3.475497 | 58.82 | 0.000 | 197.4996 | 211.3562 |
| white | 197.6132 | 3.475497 | 56.86 | 0.000 | 190.6849 | 204.5415 |
| other | 198.7127 | 3.475497 | 57.18 | 0.000 | 191.7844 | 205.6409 |

Suppose we want to test the following linear combination:

$$
\sum_{i=1}^{3} c_{i} \mu_{i}
$$

where $\mu_{i}$ is the cell mean of chol when race is equal to its $i$ th level (the means estimated using margins above). Assuming the $c_{i}$ elements sum to zero, this linear combination is a contrast. We can specify this type of custom contrast by using the following syntax:

$$
\left\{\text { race } c_{1} c_{2} c_{3}\right\}
$$

The null hypothesis for the test of the main effects of race is

$$
H_{0_{\mathrm{race}}}: \mu_{1}=\mu_{2}=\mu_{3}
$$

Although $H_{0_{\text {race }}}$ can be tested using any of several different contrasts on the cell means, we will test it by comparing the second and third cell means with the first. To test that the cell means for blacks and whites are equal, $\mu_{1}=\mu_{2}$, we can specify the contrast

$$
\{\text { race -1 } 110\}
$$

To test that the cell means for blacks and other races are equal, $\mu_{1}=\mu_{3}$, we can specify the contrast

$$
\{\text { race }-101\}
$$

We can use both in a single call to contrast.


|  | Contrast | Std. Err. | [95\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: |
| race |  |  |  |  |
| $(1)$ | -6.814717 | 4.915095 | -16.61278 | 2.983345 |
| $(2)$ | -5.715261 | 4.915095 | -15.51332 | 4.082801 |

The row labeled (1) is the test for $\mu_{1}=\mu_{2}$, the first specified contrast. The row labeled (2) is the test for $\mu_{1}=\mu_{3}$, the second specified contrast. The row labeled Joint is the overall test for the main effects of race.

Now let's fit a model with two factors, race and age group:
. anova chol race\#\#agegrp

| Source | $\begin{array}{llr} \text { Number of obs } & = & 75 \\ \text { Root MSE } & =9.61785 \end{array}$ |  |  | R-squared $=0.7524$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Partial SS | df | MS | F | Prob > F |
| Model | 16861.438 | 14 | 1204.38843 | 13.02 | 0.0000 |
| race | 669.278235 | 2 | 334.639117 | 3.62 | 0.0329 |
| agegrp | 14943.3997 | 4 | 3735.84993 | 40.39 | 0.0000 |
| race\#agegrp | 1248.76005 | 8 | 156.095006 | 1.69 | 0.1201 |
| Residual | 5550.18143 | 60 | 92.5030238 |  |  |
| Total | 22411.6194 | 74 | 302.859722 |  |  |

The null hypothesis for the test of the main effects of race is now

$$
H_{0_{\mathrm{race}}}: \mu_{1}=\mu_{2}=\mu_{3}
$$

where $\mu_{i}$. is the marginal mean of chol when race is equal to its $i$ th level.

We can use the same syntax as above to perform this test by specifying contrasts on the marginal means of race:

| Contrasts of marginal linear predictions Margins : asbalanced |  |  |  |
| :---: | :---: | :---: | :---: |
|  | df | F | $\mathrm{P}>\mathrm{F}$ |
| race |  |  |  |
| (1) | 1 | 6.28 | 0.0150 |
| (2) | 1 | 4.41 | 0.0399 |
| Joint | 2 | 3.62 | 0.0329 |
| Denominator | 60 |  |  |


|  | Contrast | Std. Err. | [95\% Conf. Interval] |  |
| :--- | ---: | ---: | ---: | ---: |
| race |  |  |  |  |
| $(1)$ | -6.814717 | 2.720339 | -12.2562 | -1.37323 |
| $(2)$ | -5.715261 | 2.720339 | -11.15675 | -.2737739 |

Custom contrasts may be specified on the cell means of interactions, too. Here we use margins to calculate the mean of chol for each cell in the interaction of race and agegrp:
. margins race\#agegrp

| Adjusted predictions | Number of obs | $=$ |
| :--- | :--- | :--- |
| Expression $\quad$ Linear prediction, predict() | 75 |  |


|  | Delta-method |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
|  | Margin | Std. Err. | t | $\mathrm{P}>\mid \mathrm{tl}$ | [95\% Conf. Interval] |  |
| race\#agegrp |  |  |  |  |  |  |
| black\#10-19 | 179.2309 | 4.301233 | 41.67 | 0.000 | 170.6271 | 187.8346 |
| black\#20-29 | 196.4777 | 4.301233 | 45.68 | 0.000 | 187.874 | 205.0814 |
| black\#30-39 | 210.6694 | 4.301233 | 48.98 | 0.000 | 202.0656 | 219.2731 |
| black\#40-59 | 214.097 | 4.301233 | 49.78 | 0.000 | 205.4933 | 222.7008 |
| black\#60-79 | 221.6646 | 4.301233 | 51.54 | 0.000 | 213.0609 | 230.2684 |
| white\#10-19 | 186.0727 | 4.301233 | 43.26 | 0.000 | 177.469 | 194.6765 |
| white\#20-29 | 184.6714 | 4.301233 | 42.93 | 0.000 | 176.0676 | 193.2751 |
| white\#30-39 | 196.2633 | 4.301233 | 45.63 | 0.000 | 187.6595 | 204.867 |
| white\#40-59 | 209.9953 | 4.301233 | 48.82 | 0.000 | 201.3916 | 218.5991 |
| white\#60-79 | 211.0633 | 4.301233 | 49.07 | 0.000 | 202.4595 | 219.667 |
| other\#10-19 | 176.2556 | 4.301233 | 40.98 | 0.000 | 167.6519 | 184.8594 |
| other\#20-29 | 185.0209 | 4.301233 | 43.02 | 0.000 | 176.4172 | 193.6247 |
| other\#30-39 | 199.2498 | 4.301233 | 46.32 | 0.000 | 190.646 | 207.8535 |
| other\#40-59 | 207.9189 | 4.301233 | 48.34 | 0.000 | 199.3152 | 216.5227 |
| other\#60-79 | 225.118 | 4.301233 | 52.34 | 0.000 | 216.5143 | 233.7218 |

Now we are interested in testing the following linear combination of these cell means:

$$
\sum_{i=1}^{3} \sum_{j=1}^{5} c_{i j} \mu_{i j}
$$

We can specify this type of custom contrast using the following syntax:

$$
\left\{\text { race\#agegrp } c_{11} c_{12} \ldots c_{15} c_{21} c_{22} \ldots c_{25} c_{31} c_{32} \ldots c_{35}\right\}
$$

Because the marginal means of chol for each level of race are linear combinations of the cell means, we can compose the test for the main effects of race in terms of the cell means directly. The constraint that the marginal means for blacks and whites are equal, $\mu_{1} .=\mu_{2}$., translates to the following constraint on the cell means:

$$
\frac{1}{5}\left(\mu_{11}+\mu_{12}+\mu_{13}+\mu_{14}+\mu_{15}\right)=\frac{1}{5}\left(\mu_{21}+\mu_{22}+\mu_{23}+\mu_{24}+\mu_{25}\right)
$$

Ignoring the common factor, we can specify this contrast as

$$
\{\text { race\#agegrp -1 -1 -1 -1 }-11111111000000\}
$$

contrast will fill in the trailing zeros for us if we neglect to specify them, so

$$
\{\text { race\#agegrp -1 -1 }-1-1-1
$$

is also allowed. The other constraint, $\mu_{1}=\mu_{3 .}$, translates to

$$
\frac{1}{5}\left(\mu_{11}+\mu_{12}+\mu_{13}+\mu_{14}+\mu_{15}\right)=\frac{1}{5}\left(\mu_{31}+\mu_{32}+\mu_{33}+\mu_{34}+\mu_{35}\right)
$$

This can be specified to contrast as

$$
\text { \{race\#agegrp -1-1-1-1-1000000111111\} }
$$

The following call to contrast yields the same test results as above.

```
. contrast {race#agegrp -1 -1 -1 -1 -1 1 1 1 1 1 1 1}
> {race#agegrp -1 -1 -1 -1 -1 0 0 0 0 0 0 1 1 1 1 1}, noeffects
Contrasts of marginal linear predictions
Margins : asbalanced
```

|  | df | F | $\mathrm{P}>\mathrm{F}$ |
| :---: | :---: | :---: | :---: |
| race\#agegrp |  |  |  |
| (1) (1) | 1 | 6.28 | 0.0150 |
| (2) (2) | 1 | 4.41 | 0.0399 |
| Joint | 2 | 3.62 | 0.0329 |
| Denominator | 60 |  |  |

The row labeled (1) (1) is the test for

$$
\mu_{11}+\mu_{12}+\mu_{13}+\mu_{14}+\mu_{15}=\mu_{21}+\mu_{22}+\mu_{23}+\mu_{24}+\mu_{25}
$$

It was the first specified contrast. The row labeled (2) (2) is the test for

$$
\mu_{11}+\mu_{12}+\mu_{13}+\mu_{14}+\mu_{15}=\mu_{31}+\mu_{32}+\mu_{33}+\mu_{34}+\mu_{35}
$$

It was the second specified contrast. The row labeled Joint tests (1) (1) and (2) (2) simultaneously.

We used the noeffects option above to suppress the table of contrasts. We can omit the $1 / 5$ from the equations for $\mu_{1}=\mu_{2}$. and $\mu_{1}=\mu_{3}$. and still obtain the appropriate tests. However, if we want to calculate the differences in the marginal means, we must include the $1 / 5=0.2$ on each of the contrast coefficients as follows:

```
. contrast {race#agegrp -0.2 -0.2 -0.2 -0.2 -0.2
```



```
    {race#agegrp -0.2 -0.2 -0.2 -0.2 -0.2
    0
    0.2 0.2 0.2 0.2 0.2}
```

So far, we have reproduced the reference category contrasts by specifying user-defined contrasts on the marginal means and then on the cell means. For this test, it would have been easier to use the r. contrast operator:

```
. contrast r.race, noeffects
Contrasts of marginal linear predictions
Margins : asbalanced
```

|  | df | F | $\mathrm{P}>\mathrm{F}$ |
| ---: | :---: | :---: | :---: |
| race |  |  |  |
| (white vs black) | 1 | 6.28 | 0.0150 |
| (other vs black) | 1 | 4.41 | 0.0399 |
| Joint | 2 | 3.62 | 0.0329 |
| Denominator | 60 |  |  |

In most cases, we can use contrast operators to perform tests. However, if we want to compare, for instance, the second and third age groups with the fourth and fifth age groups with the test

$$
\frac{1}{2}\left(\mu_{\cdot 2}+\mu \cdot 3\right)=\frac{1}{2}\left(\mu_{\cdot 4}+\mu_{\cdot 5}\right)
$$

there is not a contrast operator that corresponds to this particular contrast. A custom contrast is necessary.

| Contrasts of marginal linear predictions <br> Margins : asbalanced |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | df | F | P>F |  |
| agegrp | 1 | 62.19 | 0.0000 |  |
| Denominator | 60 |  |  |  |
|  | Contrast | Std. Err. | [95\% Con | Interval] |
| agegrp <br> (1) | 19.58413 | 2.483318 | 14.61675 | 24.5515 |

## Empty cells

An empty cell is a combination of the levels of factor variables that is not observed in the estimation sample. In the previous examples, we have seen data with three levels of race, five levels of agegrp, and all level combinations of race and agegrp present. Suppose there are no observations for white individuals in the second age group (ages 20-29).

```
. use http://www.stata-press.com/data/r13/cholesterol2
(Artificial cholesterol data, empty cells)
. label list
ages:
    1 10-19
    2 20-29
    3 30-39
    4 40-59
    5 60-79
race:
    1 \mathrm { black }
    2 white
    3 other
```

. regress chol race\#\#agegrp
note: 2.race\#2.agegrp identifies no observations in the sample

| Source | SS | df | MS | Number of obs | 70 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | F ( 13, 56) | 13.51 |
| Model | 15751.6113 | 13 | 1211.66241 | Prob > F | 0.0000 |
| Residual | 5022.71559 | 56 | 89.6913498 | R-squared | 0.7582 |
| Total | 20774.3269 | 69 | 301.077201 | Adj R-squared Root MSE | 0.7021 9.4706 |



Now let's use contrast to test the main effects of race:

| . contrast race |
| :--- |
| Contrasts of marginal linear predictions  <br> Margins : asbalanced |
| race |
| (not testable) |

By "not testable", contrast means that it cannot form a test for the main effects of race based on estimable functions of the model coefficients. agegrp has five levels, so contrast constructs an estimate of the $i$ th margin for race as

$$
\widehat{\mu}_{i} .=\frac{1}{5} \sum_{j=1}^{5} \widehat{\mu}_{i j}=\widehat{\mu}_{0}+\widehat{\alpha}_{i}+\frac{1}{5} \sum_{j=1}^{5}\left\{\widehat{\beta}_{j}+(\widehat{\alpha \beta})_{i j}\right\}
$$

but $(\widehat{\alpha \beta})_{22}$ was constrained to zero because of the empty cell, so $\widehat{\mu}_{2}$. is not an estimable function of the model coefficients.

See Estimable functions in Methods and formulas of [R] margins for a technical description of estimable functions. The emptycells (reweight) option causes contrast to estimate $\mu_{2}$. by

$$
\widehat{\mu}_{2 .}=\frac{\widehat{\mu}_{21}+\widehat{\mu}_{23}+\widehat{\mu}_{24}+\widehat{\mu}_{25}}{4}
$$

which is an estimable function of the model coefficients.

```
. contrast race, emptycells(reweight)
Contrasts of marginal linear predictions
Margins : asbalanced
Empty cells : reweight
```

|  | df | F | $\mathrm{P}>\mathrm{F}$ |
| ---: | ---: | ---: | ---: |
| race | 2 | 3.17 | 0.0498 |
| Denominator | 56 |  |  |

We can reconstruct the effect of the emptycells (reweight) option by using custom contrasts.

```
. contrast {race#agegrp -4 -4 -4 -4 -4 5 0 5 5 5 5}
> {race#agegrp -1 -1 -1 -1 -1 0 0 0 0 0 0 1 1 1 1 1}, noeffects
Contrasts of marginal linear predictions
Margins : asbalanced
```

|  | df | F | $\mathrm{P}>\mathrm{F}$ |
| :---: | :---: | :---: | :---: |
| race\#agegrp |  |  |  |
| (1) (1) | 1 | 1.06 | 0.3080 |
| (2) (2) | 1 | 2.37 | 0.1291 |
| Joint | 2 | 3.17 | 0.0498 |
| Denominator | 56 |  |  |

The row labeled (1) (1) is the test for

$$
\frac{1}{5}\left(\mu_{11}+\mu_{12}+\mu_{13}+\mu_{14}+\mu_{15}\right)=\frac{1}{4}\left(\mu_{21}+\mu_{23}+\mu_{24}+\mu_{25}\right)
$$

It was the first specified contrast. The row labeled (2) (2) is the test for

$$
\mu_{11}+\mu_{12}+\mu_{13}+\mu_{14}+\mu_{15}=\mu_{31}+\mu_{32}+\mu_{33}+\mu_{34}+\mu_{35}
$$

It was the second specified contrast. The row labeled Joint is the overall test of the main effects of race.

## Empty cells, ANOVA style

Let's refit the linear model from the previous example with anova to compare with contrast's test for the main effects of race.

contrast and anova handled the empty cell differently; the $F$ statistic reported by contrast was 3.17 , but anova reported 1.70 . To see how they differ, consider the following table of the cell means and margins for our situation.

|  |  | agegrp |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 5 |  |
|  | 1 | $\mu_{11}$ | $\mu_{12}$ | $\mu_{13}$ | $\mu_{14}$ | $\mu_{15}$ | $\mu_{1}$. |
| race | 2 | $\mu_{21}$ |  | $\mu_{23}$ | $\mu_{24}$ | $\mu_{25}$ |  |
|  | 3 | $\mu_{31}$ | $\mu_{32}$ | $\mu_{33}$ | $\mu_{34}$ | $\mu_{35}$ | $\mu_{3 .}$ |
|  |  | $\mu_{\cdot 1}$ |  | $\mu_{\cdot 3}$ | $\mu_{\cdot 4}$ | $\mu_{\cdot 5}$ |  |

For testing the main effects of race, we know that we will be testing the equality of the marginal means for rows 1 and 3 , that is, $\mu_{1}=\mu_{3}$. This translates into the following constraint:

$$
\mu_{11}+\mu_{12}+\mu_{13}+\mu_{14}+\mu_{15}=\mu_{31}+\mu_{32}+\mu_{33}+\mu_{34}+\mu_{35}
$$

Because row 2 contains an empty cell in column 2, anova dropped column 2 and tested the equality of the marginal mean for row 2 with the average of the marginal means from rows 1 and 3 , using only the remaining cell means. This translates into the following constraint:

$$
\begin{equation*}
2\left(\mu_{21}+\mu_{23}+\mu_{24}+\mu_{25}\right)=\mu_{11}+\mu_{13}+\mu_{14}+\mu_{15}+\mu_{31}+\mu_{33}+\mu_{34}+\mu_{35} \tag{1}
\end{equation*}
$$

Now that we know the constraints that anova used to test for the main effects of race, we can use custom contrasts to reproduce the anova test result.

| Contrasts of marginal linear predictions Margins : asbalanced |  |  |  |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
|  | df | F | P>F |
| race\#agegrp |  |  |  |
| (1) (1) | 1 | 2.37 | 0.1291 |
| (2) (2) | 1 | 1.03 | 0.3138 |
| Joint | 2 | 1.70 | 0.1914 |
| Denominator | 56 |  |  |

The row labeled (1) (1) is the test for $\mu_{1}=\mu_{3}$.; it was the first specified contrast. The row labeled (2) (2) is the test for the constraint in (1); it was the second specified contrast. The row labeled Joint is an overall test for the main effects of race.

## Nested effects

contrast has the $\mid$ operator for computing simple effects when the levels of one factor are nested within the levels of another. Here is a fictional example where we are interested in the effect of five methods of teaching algebra on students' scores for the math portion of the SAT. Suppose three algebra classes are randomly sampled from classes using each of the five methods so that class is nested in method as demonstrated in the following tabulation.

```
. use http://www.stata-press.com/data/r13/sat
(Artificial SAT data)
```

. tabulate class method

| class | 1 | 2 | 3 | 4 | 5 | Total |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 5 | 0 | 0 | 0 | 0 | 5 |
| 2 | 5 | 0 | 0 | 0 | 0 | 5 |
| 3 | 5 | 0 | 0 | 0 | 0 | 5 |
| 4 | 0 | 5 | 0 | 0 | 0 | 5 |
| 5 | 0 | 5 | 0 | 0 | 0 | 5 |
| 6 | 0 | 5 | 0 | 0 | 0 | 5 |
| 7 | 0 | 0 | 5 | 0 | 0 | 5 |
| 8 | 0 | 0 | 5 | 0 | 0 | 5 |
| 9 | 0 | 0 | 0 | 0 | 0 | 5 |
| 10 | 0 | 0 | 0 | 5 | 0 | 5 |
| 11 | 0 | 0 | 0 | 5 | 0 | 5 |
| 12 | 0 | 0 | 0 | 0 | 5 | 5 |
| 13 | 0 | 0 | 0 | 0 | 5 | 5 |
| 14 | 15 | 15 | 15 | 15 | 15 | 5 |
| 15 | 0 | 0 | 0 | 5 |  |  |

We will consider method as fixed and class nested in method as random. To use class nested in method as the error term for method, we can specify the following anova model:

|  | Number of obs Root MSE | $\begin{aligned} & = \\ & =71 \end{aligned}$ | $\begin{array}{rr} 75 & \mathrm{R} \\ .8517 & \mathrm{AC} \end{array}$ | R-squared <br> Adj R-squared | $\begin{aligned} & =0.7599 \\ & =0.7039 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Source | Partial SS | df | MS | F | Prob > F |
| Model | 980312 | 14 | 70022.2857 | $7 \quad 13.56$ | 0.0000 |
| method | 905872 | 4 | 226468 | $8 \quad 30.42$ | 0.0000 |
| class/method | 74440 | 10 | 7444 |  |  |
| class/method | 74440 | 10 | 7444 | $4 \quad 1.44$ | 0.1845 |
| Residual | 309760 | 60 | 5162.66667 |  |  |
| Total | 1290072 | 74 | 17433.4054 |  |  |

Like anova, contrast allows the $\mid$ operator, which specifies that one variable is nested in the levels of another. We can use contrast to test the main effects of method and the simple effects of class within method.

| Contrasts of marginal linear predictions Margins : asbalanced |  |  |  |
| :---: | :---: | :---: | :---: |
|  | df | F | $\mathrm{P}>\mathrm{F}$ |
| method | (not testable) |  |  |
| class\|method |  |  |  |
| 1 | 2 | 2.80 | 0.0687 |
| 2 | 2 | 0.91 | 0.4089 |
| 3 | 2 | 1.10 | 0.3390 |
| 4 | 2 | 0.22 | 0.8025 |
| 5 | 2 | 2.18 | 0.1221 |
| Joint | 10 | 1.44 | 0.1845 |
| Denominator | 60 |  |  |

Although contrast was able to perform the individual tests for the simple effects of class within method, empty cells in the interaction between method and class prevented contrast from testing for a main effect of method. Here we add the emptycells (reweight) option so that contrast can take the empty cells into account when computing the marginal means for method.

```
. contrast method class|method, emptycells(reweight)
Contrasts of marginal linear predictions
\begin{tabular}{ll} 
Margins & : asbalanced \\
Empty cells & : reweight
\end{tabular}
```

|  | df | F | $\mathrm{P}>\mathrm{F}$ |
| ---: | ---: | ---: | ---: |
| method | 4 | 43.87 | 0.0000 |
| class\|method |  |  |  |
| 1 | 2 | 2.80 | 0.0687 |
| 2 | 2 | 0.91 | 0.4089 |
| 3 | 2 | 1.10 | 0.3390 |
| 4 | 2 | 0.22 | 0.8025 |
| 5 | 2 | 2.18 | 0.1221 |
| Joint | 10 | 1.44 | 0.1845 |
|  |  |  |  |
| Denominator | 60 |  |  |

Now contrast does report a test for the main effects of method. However, if we compare this with the anova results, we will see that the results are different. They are different because contrast uses the residual error term to compute the $F$ test by default. Using notation similar to anova, we can use the / operator to specify a different error term for the test. Therefore, we can reproduce the test of main effects from our anova command by typing

```
. contrast method / class|method /, emptycells(reweight)
Contrasts of marginal linear predictions
Margins : asbalanced
Empty cells : reweight
```

|  | df | F | $\mathrm{P}>\mathrm{F}$ |
| ---: | ---: | :---: | ---: |
| method | 4 | 30.42 | 0.0000 |
| class\|method | 10 | (denominator) |  |
| classlmethod |  |  |  |
| 1 | 2 | 2.80 | 0.0687 |
| 2 | 2 | 0.91 | 0.4089 |
| 3 | 2 | 1.10 | 0.3390 |
| 4 | 2 | 0.22 | 0.8025 |
| 5 | 2 | 2.18 | 0.1221 |
| Joint | 10 | 1.44 | 0.1845 |
|  |  |  |  |
| Denominator | 60 |  |  |

## Multiple comparisons

We have seen that contrast can report the individual linear combinations that make up the requested effects. Depending upon the specified option, contrast will report confidence intervals, $p$-values, or both in the effects table. By default, the reported confidence intervals and $p$-values are not adjusted for multiple comparisons. Use the mcompare () option to adjust the confidence intervals and $p$-values for multiple comparisons of the individual effects.

Let's compute the grand mean effects of race using the g . operator. We also specify the mcompare (bonferroni) option to compute $p$-values and confidence intervals using Bonferroni's adjustment.

```
. use http://www.stata-press.com/data/r13/cholesterol
(Artificial cholesterol data)
. anova chol race##agegrp
    (output omitted)
. contrast g.race, mcompare(bonferroni)
Contrasts of marginal linear predictions
Margins : asbalanced
```

|  |  |  | Bonferroni <br> $P>F$ |  |
| ---: | :---: | :---: | :---: | ---: |
| race |  |  |  |  |
| (black vs mean) | 1 | 7.07 | 0.0100 | 0.0301 |
| (white vs mean) | 1 | 2.82 | 0.0982 | 0.2947 |
| (other vs mean) | 1 | 0.96 | 0.3312 | 0.9936 |
| Joint | 2 | 3.62 | 0.0329 |  |
| Denominator | 60 |  |  |  |

[^13]|  | Number of <br> Comparisons |
| ---: | ---: |
| race | 3 |


|  |  |  | Bonferroni <br> [95\% Conf. Interval] |  |
| :---: | ---: | :---: | ---: | ---: |
|  | Contrast | Std. Err. |  |  |
| race |  |  |  |  |
| (black vs mean) | 4.17666 | 1.570588 | .3083743 | 8.044945 |
| (white vs mean) | -2.638058 | 1.570588 | -6.506343 | 1.230227 |
| (other vs mean) | -1.538602 | 1.570588 | -5.406887 | 2.329684 |

The last table reports a Bonferroni-adjusted confidence interval for each individual contrast. (Use the effects option to add $p$-values to the last table.) The first table includes a Bonferroni-adjusted $p$-value for each test that is not a joint test.

Joint tests are never adjusted for multiple comparisons. For example,


Note: Bonferroni-adjusted p-values are reported for tests on individual contrasts only.

|  | Number of <br> Comparisons |
| ---: | ---: |
| race@agegrp | 10 |


|  |  |  | Bonferroni <br> [95\% Conf. Interval] |  |
| :--- | ---: | ---: | ---: | ---: |
|  | Contrast | Std. Err. |  |  |
| (white vs base) | 10-19 | 6.841855 | 6.082862 | -10.88697 |
| (white vs base) $20-29$ | -11.80631 | 6.082862 | -29.53513 | 5.922513 |
| (white vs base) | $30-39$ | -14.40607 | 6.082862 | -32.13489 |
| (white vs base) | $40-59$ | -4.101691 | 6.082862 | -21.83051 |
| (white vs base) $60-79$ | -10.60137 | 6.082862 | -28.33019 | 13.62713 |
| (other vs base) $10-19$ | -2.975244 | 6.082862 | -20.70407 | 14.75358 |
| (other vs base) $20-29$ | -11.45679 | 6.082862 | -29.18561 | 6.272031 |
| (other vs base) 30-39 | -11.41958 | 6.082862 | -29.1484 | 6.309244 |
| (other vs base) $40-59$ | -6.17807 | 6.082862 | -23.90689 | 11.55075 |
| (other vs base) $60-79$ | 3.453375 | 6.082862 | -14.27545 | 21.1822 |

Here we have five tests of simple effects with two degrees of freedom each. No Bonferroni-adjusted $p$-values are available for these tests, but the confidence intervals for the individual contrasts are adjusted.

## Unbalanced data

By default, contrast treats all factors as balanced when computing marginal means. By balanced, we mean that contrast assumes an equal number of observations in each level of each factor and an equal number of observations in each cell of each interaction. If our data are balanced, there is no issue. If, however, our data are not balanced, we might prefer that contrast use the actual cell frequencies from our data in computing marginal means. We instruct contrast to use observed frequencies by adding the asobserved option.

Even if our data are unbalanced, we might still want contrast to compute balanced marginal means. It depends on what we want to test and what our data represent. If we have data from a designed experiment that started with an equal number of males and females but the data became unbalanced because the data from a few males were unusable, we might still want our margins computed as though the data were balanced. If, however, we have a representative sample of individuals from Los Angeles with $40 \%$ of European descent, $34 \%$ African-American, $25 \%$ Hispanic, and $1 \%$ Australian, we probably want our margins computed using these representative frequencies. We do not want Australians receiving the same weight as Europeans.

The following examples will use an unbalanced version of our dataset.

```
. use http://www.stata-press.com/data/r13/cholesterol3
(Artificial cholesterol data, unbalanced)
```

. tab race agegrp

| race | $10-19$ | $20-29$ | agegrp <br> $30-39$ | $40-59$ | $60-79$ | Total |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| black | 1 | 5 | 5 | 4 | 3 | 18 |
| white | 4 | 5 | 7 | 4 | 4 | 24 |
| other | 3 | 7 | 6 | 5 | 4 | 25 |
| Total | 8 | 17 | 18 | 13 | 11 | 67 |

The row labeled Total gives observed cell frequencies for age group. These can be obtained by summing frequencies from the cells in the corresponding column. In this respect, we can also refer to them as marginal frequencies. We use the terms marginal frequencies and cell frequencies interchangeably below.

We begin by fitting the two-factor model with an interaction.

|  | Number of obs Root MSE | $\begin{aligned} & = \\ & =8 . \end{aligned}$ | $\begin{array}{r} 67 \\ 37496 \end{array}$ | $\begin{array}{ll} \text { R-squared } & = \\ \text { Adj R-squared }= \end{array}$ | $\begin{aligned} & =0.8179 \\ & =\quad 0.7689 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Source | Partial SS | df | MS | F | Prob > F |
| Model | 16379.9926 | 14 | 1169.99947 | $7 \quad 16.68$ | 0.0000 |
| race | 230.754396 | 2 | 115.377198 | $8 \quad 1.64$ | 0.2029 |
| agegrp | 13857.9877 | 4 | 3464.49693 | $3 \quad 49.39$ | 0.0000 |
| race\#agegrp | 857.815209 | 8 | 107.226901 | 11.53 | 0.1701 |
| Residual | 3647.2774 | 52 | 70.13995 |  |  |
| Total | 20027.27 | 66 | 303.443485 |  |  |

## Using observed cell frequencies

Recall that the marginal means are computed from the cell means. Treating the factors as balanced yields the following marginal means for race:

$$
\begin{aligned}
& \eta_{1 .}=\frac{1}{5}\left(\mu_{11}+\mu_{12}+\mu_{13}+\mu_{14}+\mu_{15}\right) \\
& \eta_{2 .}=\frac{1}{5}\left(\mu_{21}+\mu_{22}+\mu_{23}+\mu_{24}+\mu_{25}\right) \\
& \eta_{3 .}=\frac{1}{5}\left(\mu_{31}+\mu_{32}+\mu_{33}+\mu_{34}+\mu_{35}\right)
\end{aligned}
$$

If we have a fixed population and unbalanced cells, then the $\eta_{i}$. do not represent population means. If, however, our data are representative of the population, we can use the frequencies from our estimation sample to estimate the population marginal means, denoted $\mu_{i}$.

Here are the results of testing for a main effect of race, treating all the factors as balanced.
. contrast r.race
Contrasts of marginal linear predictions
Margins $\quad:$ asbalanced

|  | df | F | $\mathrm{P}>\mathrm{F}$ |
| ---: | :---: | :---: | ---: |
| race |  |  |  |
| (white vs black) | 1 | 3.28 | 0.0757 |
| (other vs black) | 1 | 1.50 | 0.2263 |
| Joint | 2 | 1.64 | 0.2029 |
| Denominator | 52 |  |  |


|  | Contrast | Std. Err. | [95\% Conf. Interval] |  |
| :--- | :--- | :--- | :--- | :--- |
| race |  |  |  |  |
| (white vs black) | -5.324254 | 2.93778 | -11.21934 | .5708338 |
| (other vs black) | -3.596867 | 2.93778 | -9.491955 | 2.298221 |

The row labeled (white vs black) is the test for $\eta_{2} .=\eta_{1}$. The row labeled (other vs black) is the test for $\eta_{3}=\eta_{1}$.

If the observed marginal frequencies are representative of the distribution of the levels of agegrp, we can use them to form the marginal means of chol for each of the levels of race from the cell means.

$$
\begin{aligned}
& \mu_{1 .}=\frac{1}{67}\left(8 \mu_{11}+17 \mu_{12}+18 \mu_{13}+13 \mu_{14}+11 \mu_{15}\right) \\
& \mu_{2 .}=\frac{1}{67}\left(8 \mu_{21}+17 \mu_{22}+18 \mu_{23}+13 \mu_{24}+11 \mu_{25}\right) \\
& \mu_{3 .}=\frac{1}{67}\left(8 \mu_{31}+17 \mu_{32}+18 \mu_{33}+13 \mu_{34}+11 \mu_{35}\right)
\end{aligned}
$$

Here are the results of testing for the main effects of race, using the observed marginal frequencies:


The row labeled (white vs black) is the test for $\mu_{2}=\mu_{1}$.. The row labeled (other vs black) is the test for $\mu_{3}=\mu_{1}$.. Both tests were insignificant when we tested the cell means resulting from balanced frequencies; however, when we tested the cell means from observed frequencies, the first test is significant beyond the $5 \%$ level (and the second test is nearly so).

Here we reproduce the results of the asobserved option with custom contrasts. Because we are modifying the way that the marginal means are constructed from the cell means, we will specify the contrasts on the predicted cell means. We use macro expansion, =exp, to evaluate the fractions instead of approximating them with decimals. Macro expansion guarantees that the contrast coefficients sum to zero. For more information, see Macro expansion operators and function in [P] macro.


## Weighted contrast operators

contrast provides observation-weighted versions of five of the contrast operators-gw., hw., jw., pw., and qw.. The first three of these operators perform comparisons of means across cells, and like the marginal means just discussed, these means can be computed in two ways: 1) as though the cell frequencies were equal or 2 ) using the observed cell frequencies from the estimation sample. The weighted operators provide versions of the standard (as balanced) operators that weight these means by their cell frequencies. The two orthogonal polynomial operators involve similar adjustments for weighting.

Let's examine what this means by using the gw. operator. The gw. operator is a weighted version of the g . operator. The gw. operator computes the grand mean using the cell frequencies for race obtained from the model fit.

Here we test the effects of race, comparing each level with the weighted grand mean but otherwise treating the factors as balanced in the marginal mean calculations.

| . contrast gw.race <br> Contrasts of marginal linear predictions <br> (: asbalanced |
| :--- |
| Margins |


|  | Contrast | Std. Err. | [95\% Conf. Interval] |  |
| :--- | ---: | ---: | ---: | ---: |
| race |  |  |  |  |
| (black vs mean) | 3.24931 | 1.948468 | -.6605779 | 7.159198 |
| (white vs mean) | -2.074944 | 1.44618 | -4.976915 | .8270276 |
| (other vs mean) | -.347557 | 1.414182 | -3.18532 | 2.490206 |

The observed marginal frequencies of race are 18,24 , and 25 . Thus the row labeled (black vs mean) tests $\eta_{1} .=\left(18 \eta_{1 .}+24 \eta_{2} .+25 \eta_{3 .}\right) / 67$; the row labeled (white vs mean) tests $\eta_{2}=\left(18 \eta_{1} .+\right.$ $\left.24 \eta_{2 .}+25 \eta_{3 .}\right) / 67$; and the row labeled (other vs mean) tests $\eta_{3 .}=\left(18 \eta_{1 .}+24 \eta_{2} .+25 \eta_{3 .}\right) / 67$.

Now we reproduce the above results using custom contrasts. We are weighting the calculation of the grand mean from the marginal means for each of the races, but we are not weighting the calculation of the marginal means themselves. Therefore, we can specify the custom contrast on the marginal means for race instead of on the cell means.

| contrast | \{race | $'=49 / 67$ ' | - ' $=24 / 67$ ' | $\left.-^{\prime}=25 / 67^{\prime}\right\}$ |
| :---: | :---: | :---: | :---: | :---: |
| > | \{race | - ' $=18 / 67$ ' | ' $=43 / 67$ ' | $\left.-^{\prime}=25 / 67^{\prime}\right\}$ |
| > | \{race | - '=18/67' | - ' $=24 / 67$ ' | ' $=42 / 67$ '\} |

Contrasts of marginal linear predictions
Margins : asbalanced

|  | $d f$ | $F$ | $P>F$ |
| ---: | :---: | :---: | ---: |
| race |  |  |  |
| $(1)$ | 1 | 2.78 | 0.1014 |
| $(2)$ | 1 | 2.06 | 0.1573 |
| (3) | 1 | 0.06 | 0.8068 |
| Joint | 2 | 1.64 | 0.2029 |
| Denominator | 52 |  |  |


|  | Contrast | Std. Err. | [95\% Conf. Interval] |  |
| :--- | ---: | ---: | ---: | ---: |
| race |  |  |  |  |
| $(1)$ | 3.24931 | 1.948468 | -.6605779 | 7.159198 |
| $(2)$ | -2.074944 | 1.44618 | -4.976915 | .8270276 |
| $(3)$ | -.347557 | 1.414182 | -3.18532 | 2.490206 |

Now we will test for each race the difference between the marginal mean and the weighted grand mean, treating the factors as observed in the marginal mean calculations.

```
. contrast gw.race, asobserved wald ci
Contrasts of marginal linear predictions
Margins : asobserved
```

|  | df | F | $\mathrm{P}>\mathrm{F}$ |
| ---: | :---: | :---: | :---: |
| race |  |  |  |
| (black vs mean) | 1 | 6.81 | 0.0118 |
| (white vs mean) | 1 | 3.74 | 0.0587 |
| (other vs mean) | 1 | 0.26 | 0.6099 |
| Joint | 2 | 3.74 | 0.0304 |
| Denominator | 52 |  |  |


|  | Contrast | Std. Err. | [95\% Conf. Interval] |  |
| :--- | ---: | ---: | ---: | ---: |
| race |  |  |  |  |
| (black vs mean) | 4.542662 | 1.740331 | 1.050432 | 8.034891 |
| (white vs mean) | -2.689771 | 1.39142 | -5.481859 | .1023172 |
| (other vs mean) | -.6885363 | 1.341261 | -3.379973 | 2.002901 |

The row labeled (black vs mean) tests $\mu_{1}=\left(18 \mu_{1}+24 \mu_{2}+25 \mu_{3}.\right) / 67$; the row labeled (white vs mean) tests $\mu_{2}=\left(18 \mu_{1} .+24 \mu_{2}+25 \mu_{3}.\right) / 67$; and the row labeled (other vs mean) tests $\mu_{3 .}=\left(18 \mu_{1} .+24 \mu_{2}+25 \mu_{3}.\right) / 67$.

Here we use a custom contrast to reproduce the above result testing $\mu_{1}=\left(18 \mu_{1} .+24 \mu_{2} .+\right.$ $25 \mu_{3}$.) $/ 67$. Because both the calculation of the marginal means and the calculation of the grand mean are adjusted, we specify the custom contrast on the cell means.


The Helmert and reverse Helmert contrasts also involve calculating averages of the marginal means; therefore, weighted versions of these parameters are available as well. The hw. operator is a weighted version of the $h$. operator that computes the mean of the subsequent levels using the cell frequencies obtained from the model fit. The jw . operator is a weighted version of the j . operator that computes the mean of the previous levels using the cell frequencies obtained from the model fit.

For orthogonal polynomials, we can use the pw. and qw. operators, which are the weighted versions of the p . and q . operators. In this case, the cell frequencies from the model fit are used in the calculation of the orthogonal polynomial contrast coefficients.

## Testing factor effects on slopes

For linear models where the independent variables are all factor variables, the linear prediction at fixed levels of the factor variables turns out to be a cell mean. With these models, contrast computes and tests the effects of the factor variables on the expected mean of the dependent variable. When factor variables are interacted with continuous variables, contrast distinguishes factor effects on the intercept from factor effects on the slope.

Here we have 1980 census data including information on the birth rate (brate), the median age (medage), and the region of the country (region) for each of the 50 states. We can fit an ANCOVA model for brate using main effects of the factor variable region and the continuous variable medage.

```
. use http://www.stata-press.com/data/r13/census3
(1980 Census data by state)
. label list cenreg
cenreg:
    1 NE
    2 NCentral
    3 South
    4 \text { West}
. anova brate i.region c.medage
\begin{tabular}{|c|c|c|c|c|c|}
\hline & \multicolumn{3}{|l|}{\begin{tabular}{llr} 
Number of obs & \(=50\) \\
Root MSE & \(=12.7575\)
\end{tabular}} & \multicolumn{2}{|l|}{\begin{tabular}{ll} 
R-squared & \(=0.8264\) \\
Adj R-squared & \(=0.8110\)
\end{tabular}} \\
\hline Source & Partial SS & df & MS & F & Prob > F \\
\hline Model & 34872.8589 & 4 & 8718.21473 & 53.57 & 0.0000 \\
\hline region & 2197.75453 & 3 & 732.584844 & 4.50 & 0.0076 \\
\hline medage & 15327.423 & 1 & 15327.423 & 94.18 & 0.0000 \\
\hline Residual & 7323.96108 & 45 & 162.754691 & & \\
\hline Total & 42196.82 & 49 & 861.159592 & & \\
\hline
\end{tabular}
```

For those more comfortable with linear regression, this is equivalent to the regression model

```
. regress brate i.region c.medage
```

You may use either.
We can use contrast to compute reference category effects for region. These contrasts compare the adjusted means of NCentral, South, and West regions with the adjusted mean of the NE region.

| Contrasts of marginal linear predictions <br> Margins : asbalanced |  |  |  |
| :---: | :---: | :---: | :---: |
|  | df | F | $\mathrm{P}>\mathrm{F}$ |
| region |  |  |  |
| (NCentral vs NE) | 1 | 2.24 | 0.1417 |
| (South vs NE) | 1 | 0.78 | 0.3805 |
| (West vs NE) | 1 | 10.33 | 0.0024 |
| Joint | 3 | 4.50 | 0.0076 |
| Denominator | 45 |  |  |


|  | Contrast | Std. Err. | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: |
| region |  |  |  |  |
| (NCentral vs NE) | 9.061063 | 6.057484 | -3.139337 | 21.26146 |
| (South vs NE) | 5.06991 | 5.72396 | -6.458738 | 16.59856 |
| (West vs NE) | 21.71328 | 6.755616 | 8.106774 | 35.31979 |

Let's add the interaction between region and medage to the model.

| Source | Number of obs Root MSE | $=\begin{gathered}50 \\ \text { R-squared }\end{gathered}$ |  |  | $\begin{aligned} & =0.9000 \\ & =0.8833 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Partial SS | df | MS | F | Prob > F |
| Model | 37976.3149 | 7 | 5425.18784 | 4 53.99 | 0.0000 |
| region | 3405.07044 | 3 | 1135.02348 | $8 \quad 11.30$ | 0.0000 |
| medage | 5279.71448 | 1 | 5279.71448 | 852.54 | 0.0000 |
| region\#medage | 3103.45597 | 3 | 1034.48532 | 3210.29 | 0.0000 |
| Residual | 4220.5051 | 42 | 100.488217 |  |  |
| Total | 42196.82 | 49 | 861.159592 |  |  |

The parameterization for the expected value of brate as a function of region and medage is given by

$$
E(\text { brate } \mid \text { region }=i, \text { medage })=\alpha_{0}+\alpha_{i}+\beta_{0} \text { medage }+\beta_{i} \text { medage }
$$

where $\alpha_{0}$ is the intercept and $\beta_{0}$ is the slope of medage. We are modeling the effects of region in two different ways. The $\alpha_{i}$ parameters measure the effect of region on the intercept, and the $\beta_{i}$ parameters measure the effect of region on the slope of medage.
contrast computes and tests effects on slopes separately from effects on intercepts. First, we will compute the reference category effects of region on the intercept:

| . contrast r.region |
| :--- |
| Contrasts of marginal linear predictions <br> Margins <br> : asbalanced |
| region |
|  |
| (NCentral vs NE) |
| (South vs NE) |
| (West vs NE) |
| Joint |


|  | Contrast | Std. Err. | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: |
| region |  |  |  |  |
| (NCentral vs NE) | -49.38396 | 167.1281 | -386.6622 | 287.8942 |
| (South vs NE) | -9.058983 | 117.424 | -246.0302 | 227.9123 |
| (West vs NE) | 343.0024 | 117.6547 | 105.5656 | 580.4393 |

Now we will compute the reference category effects of region on the slope of medage:

| . contrast r.region\#c.medage <br> Contrasts of marginal linear predictions <br> : asbalanced |
| :--- |
| Margins |


|  | Contrast | Std. Err. | [95\% Conf. Interval] |  |
| ---: | ---: | :--- | :--- | :--- |
| region\#c.medage |  |  |  |  |
| (NCentral vs NE) | 2.208539 | 5.530981 | -8.953432 | 13.37051 |
| (South vs NE) | .6928008 | 3.788735 | -6.953175 | 8.338777 |
| (West vs NE) | -10.94649 | 3.827357 | -18.67041 | -3.22257 |

At the 5\% level, the slope of medage for the West region differs from that of the NE region, but at that level of significance, we cannot say that the slope for the NCentral or the South region differs from that of the NE region.

This model is simple enough that the reference category contrasts reproduce the coefficients for region and for the interactions in an equivalent model fit by regress.
. regress brate region\#\#c.medage

| Source | SS | df | MS |
| ---: | ---: | ---: | :---: |
| Model | 37976.3149 | 7 | 5425.18784 |
| Residual | 4220.5051 | 42 | 100.488217 |
| Total | 42196.82 | 49 | 861.159592 |


| Number of obs | $=$ | 50 |
| :--- | ---: | ---: |
| F $(7, \quad 42)$ | $=$ | 53.99 |
| Prob $>$ F | $=$ | 0.0000 |
| R-squared | $=$ | 0.9000 |
| Adj R-squared | $=$ | 0.8833 |
| Root MSE | $=10.024$ |  |


| brate | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Con | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| region |  |  |  |  |  |  |
| NCentral | -49.38396 | 167.1281 | -0.30 | 0.769 | -386.6622 | 287.8942 |
| South | -9.058983 | 117.424 | -0.08 | 0.939 | -246.0302 | 227.9123 |
| West | 343.0024 | 117.6547 | 2.92 | 0.006 | 105.5656 | 580.4393 |
| medage | -8.802707 | 3.462865 | -2.54 | 0.015 | -15.79105 | -1.814362 |
| region\# c.medage |  |  |  |  |  |  |
| NCentral | 2.208539 | 5.530981 | 0.40 | 0.692 | -8.953432 | 13.37051 |
| South | . 6928008 | 3.788735 | 0.18 | 0.856 | -6.953175 | 8.338777 |
| West | -10.94649 | 3.827357 | -2.86 | 0.007 | -18.67041 | -3.22257 |
| _cons | 411.8268 | 108.2084 | 3.81 | 0.000 | 193.4533 | 630.2002 |

This will not be the case for models that are more complicated.

## Chow tests

Now let's suppose we are fitting a model for birth rates on median age and marriage rate. We are also interested in whether the regression coefficients differ for states in the east versus states in the west. We use census divisions to create a new variable, west, that indicates which states are in the western half of the United States.

```
. generate west = inlist(division, 4, 7, 8, 9)
```

We fit a model that includes a separate intercept for west as well as an interaction between west and each of the other variables in our model.
. regress brate i.west\#\#c.medage i.west\#\#c.mrgrate

| Source | SS | df | MS |
| ---: | ---: | ---: | :---: |
| Model | 38516.2172 | 5 | 7703.24344 |
| Residual | 3680.60281 | 44 | 83.6500639 |
| Total | 42196.82 | 49 | 861.159592 |


| Number of obs | $=$ | 50 |
| :--- | ---: | ---: |
| F $($ 5, 44$)$ | $=92.09$ |  |
| Prob $>$ F | $=$ | 0.0000 |
| R-squared | $=0.9128$ |  |
| Adj R-squared | $=$ | 0.9029 |
| Root MSE | $=$ | 9.146 |


| brate | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Conf | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.west | 327.8733 | 58.71793 | 5.58 | 0.000 | 209.5351 | 446.2115 |
| medage | -7.532304 | 1.387624 | -5.43 | 0.000 | -10.32888 | -4.735731 |
| west\# <br> c.medage | $\begin{array}{llllll} -10.11443 & 1.849103 & -5.47 & 0.000 & -13.84105 & -6.387808 \end{array}$ |  |  |  |  |  |
| 1 |  |  |  |  |  |  |
| mrgrate | 828.6813 | 643.3443 | 1.29 | 0.204 | -467.8939 | 2125.257 |
| west\# c.mrgrate |  |  |  |  |  |  |
| 1 | -800.8036 | 645.488 | -1.24 | 0.221 | -2101.699 | 500.092 |
| _cons | 366.5325 | 47.08904 | 7.78 | 0.000 | 271.6308 | 461.4343 |

We can test the effects of west on the intercept and on the slopes of medage and mrgrate. We will specify all of these effects in a single contrast command and include the overall option to obtain a joint test of effects, that is, a test that the coefficients for eastern states and for western states are equal.

| . contrast west west\#c.medage west\#c.mrgrate, overall |
| :--- |
| Contrasts of marginal linear predictions <br> Margins <br> : asbalanced |
| west |
| west\#c.medage |
| west\#c.mrgrate |
| Overall |

This overall test is referred to as a Chow test in econometrics (Chow 1960).

## Beyond linear models

contrast may be used after almost any estimation command, with the added benefit that contrast provides direct support for testing main and interaction effects that is not available in most estimation commands. To illustrate, we will use contrast with results from a logistic regression. Stata's logit command fits logistic regression models, reporting the fitted regression coefficients. The logistic command fits the same models but reports odds ratios. Although contrast can report odds ratios for the computed effects, the tests are all computed from linear combinations of the model coefficients regardless of which estimation command we used.

Suppose we have data on patient satisfaction for three hospitals in a city. Let's begin by fitting a model for satisfied, whether the patient was satisfied with his or her treatment, using the main effects of hospital:

```
. use http://www.stata-press.com/data/r13/hospital, clear
(Artificial hospital satisfaction data)
. logit satisfied i.hospital
Iteration 0: log likelihood = -393.72216
Iteration 1: log likelihood = -387.55736
Iteration 2: log likelihood = -387.4768
Iteration 3: log likelihood = -387.47679
```

Logistic regression Number of obs = 802
LR chi2(2) $=12.49$
Prob > chi2 $=0.0019$
Pseudo R2 $=0.0159$

| satisfied | Coef. | Std. Err. | $z$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: |
| hospital |  |  |  |  |  |  |
| 2 | .5348129 | .2136021 | 2.50 | 0.012 | .1161604 | .9534654 |
| 3 | .7354519 | .2221929 | 3.31 | 0.001 | .2999618 | 1.170942 |
| _cons | 1.034708 | .1391469 | 7.44 | 0.000 | .7619855 | 1.307431 |

Because there are no other independent variables in this model, the reference category effects of hospital computed by contrast will match the fitted model coefficients, assuming a common reference level.

| . contrast r.hospital <br> Contrasts of marginal linear predictions <br> Margins <br> : asbalanced |
| :--- |


|  | Contrast | Std. Err. | [95\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: |
| hospital |  |  |  |  |
| $(2$ vs 1) | .5348129 | .2136021 | .1161604 | .9534654 |
| $(3$ vs 1) | .7354519 | .2221929 | .2999618 | 1.170942 |

We see that the reference category effects are equal to the fitted coefficients. They also have the same interpretation, the difference in $\log$ odds from the reference category. The top table also provides a joint test of these effects, a test of the main effects of hospital.

We also have information on the condition for which each patient is being treated in the variable illness. Here we fit a logistic regression using a two-way crossed model of hospital and illness.


Using contrast, we can obtain an ANOVA-style table of tests for the main effects and interaction effects of hospital and illness.

```
. contrast hospital##illness
Contrasts of marginal linear predictions
Margins : asbalanced
```

|  | df | chi2 | P>chi2 |
| ---: | :---: | :---: | :---: |
| hospital | 2 | 14.92 | 0.0006 |
| illness | 4 | 4.09 | 0.3937 |
| hospital\#illness | 8 | 20.45 | 0.0088 |

Our interaction effect is significant, so we decide to evaluate the simple reference category effects of hospital within illness. We are particularly interested in patient satisfaction when being treated for a heart attack or stroke, so we will use the i. operator to limit our output to simple effects within the first two illnesses.

```
. contrast r.hospital@i(1 2).illness, nowald
Contrasts of marginal linear predictions
Margins : asbalanced
```

|  |  | Contrast | Std. Err. | [95\% Conf. Interval] |
| :--- | :--- | ---: | :--- | :--- | :--- |

The row labeled (2 vs 1) heart attack estimates simple effects on the log odds when comparing hospital 2 with hospital 1 for patients having heart attacks. These effects are differences in the cell means of the linear predictions.

We can add the or option to report an odds ratio for each of these simple effects:

| Contrasts of marginal linear predictions <br> Margins : asbalanced |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Odds Ratio | Std. Err. | [95\% Conf | Interval] |
| hospital@illness |  |  |  |  |
| (2 vs 1) heart attack | 1.226496 | . 5492177 | . 509921 | 2.950049 |
| (2 vs 1) stroke | 2.985366 | 1.708014 | . 9727486 | 9.162089 |
| (3 vs 1) heart attack | 1.711111 | . 8061016 | . 6796395 | 4.308021 |
| (3 vs 1) stroke | . 9268293 | . 4321179 | . 3716567 | 2.311306 |

These odds ratios are just the exponentiated version of the contrasts in the previous table.
For contrasts of the margins of nonlinear predictions, such as predicted probabilities, see $[\mathrm{R}]$ margins, contrast.

## Multiple equations

contrast works with models containing multiple equations. Commands such as intreg and gnbreg allow their ancillary parameters to be modeled as functions of independent variables, and contrast can compute and test effects within these equations. In addition, contrast allows a special pseudofactor for equation-called _eqns-when working with results from manova, mvreg, mlogit, and mprobit.

In example 4 of [MV] manova, we fit a two-way MANOVA model using data from Woodard (1931). Here we will fit this model using mvreg. The data represent patients with jaw fractures. y1 is the patient's age, y2 is blood lymphocytes, and y3 is blood polymorphonuclears. Two factor variables, gender and fracture, are used as independent variables.

```
. use http://www.stata-press.com/data/r13/jaw
(Table 4.6 Two-Way Unbalanced Data for Fractures of the Jaw -- Rencher (1998))
. mvreg y1 y2 y3 = gender##fracture, vsquish nofvlabel
```

| Equation | Obs | Parms | RMSE | "R-sq" | F | P |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| y1 | 27 | 6 | 10.21777 | 0.4086 | 2.902124 | 0.0382 |
| y2 | 27 | 6 | 5.268768 | 0.4743 | 3.78967 | 0.0133 |
| y3 | 27 | 6 | 4.993647 | 0.4518 | 3.460938 | 0.0195 |


contrast computes Wald tests using the coefficients from the first equation by default.

| Contrasts of marginal linear predictions Margins : asbalanced |  |  |  |
| :---: | :---: | :---: | :---: |
|  | df | F | $\mathrm{P}>\mathrm{F}$ |
| y1 |  |  |  |
| gender | 1 | 2.16 | 0.1569 |
| fracture | 2 | 2.74 | 0.0880 |
| gender\#fracture | 2 | 1.69 | 0.2085 |
| Denominator | 21 |  |  |

Here we use the equation() option to compute the Wald tests in the y2 equation:

| . contrast gender\#\#fracture, equation(y2) <br> Contrasts of marginal linear predictions <br> Margins <br> : asbalanced |  |  |  |
| :--- | :---: | :---: | ---: |
| gender | df | F | P>F |
| fracture | 1 | 5.41 | 0.0301 |
| gender\#fracture | 2 | 7.97 | 0.0027 |
| Denominator | 2 | 5.97 | 0.0088 |

Here we use the equation index to compute the Wald tests in the third equation:

| Contrasts of marginal linear predictions Margins : asbalanced |  |  |  |
| :---: | :---: | :---: | :---: |
|  | df | F | $\mathrm{P}>\mathrm{F}$ |
| y3 |  |  |  |
| gender | 1 | 2.23 | 0.1502 |
| fracture | 2 | 6.36 | 0.0069 |
| gender\#fracture | 2 | 6.66 | 0.0058 |
| Denominator | 21 |  |  |

Here we use the atequations option to compute Wald tests for each equation in the model. We also use the vsquish option to suppress the extra blank lines between terms.


Because we are investigating the results from mvreg, we can use the special _eqns factor to test for a marginal effect on the means among the dependent variables:

| . contrast _eqns <br> Contrasts of marginal linear predictions <br> Margins <br> : asbalanced |
| :--- |
| _eqns |

Here we test whether the main effects of gender differ among the dependent variables:

| . contrast gender\#_eqns <br> Contrasts of marginal linear predictions <br> Margins <br> : asbalanced |
| :--- |

Although it is not terribly interesting in this case, we can even calculate contrasts across equations:

| . contrast gender\#r._eqns <br> Contrasts of marginal linear predictions <br> : asbalanced |
| :--- |
| Margins |

## Video example

Introduction to contrasts in Stata: One-way ANOVA

## Stored results

contrast stores the following in $r()$ :
Scalars

$$
\begin{array}{ll}
\mathrm{r}\left(\mathrm{df} \_\mathrm{r}\right) & \text { variance degrees of freedom } \\
\mathrm{r}\left(\mathrm{k} \_ \text {terms }\right) & \text { number of terms in termlist } \\
\mathrm{r}(\text { level }) & \text { confidence level of confidence intervals }
\end{array}
$$

Macros

```
    r(cmd)
    r(cmdline)
    r(est_cmd)
    r(est_cmdline)
    r(title)
    r(overall)
    r(emptycells)
    r(mcmethod)
    r(mctitle)
    r(mcadjustall)
    r(margin_method)
```

contrast
command as typed
$e(\mathrm{cmd})$ from original estimation results
$e$ (cmdline) from original estimation results
title in output
overall or empty
empspec from emptycells()
method from mcompare()
title for method from mcompare()
adjustall or empty
asbalanced or asobserved

Matrices

```
r(b)
r(v)
r(error)
```

contrast estimates
variance-covariance matrix of the contrast estimates
contrast estimability codes;
0 means estimable,
8 means not estimable

```
r(L)
    r(table)
    r(F)
    r(chi2)
    r(p)
    r(df)
    r(df2)
matrix of contrasts applied to the model coefficients
matrix containing the contrasts with their standard errors,
test statistics, p-values, and confidence intervals
vector of F statistics; r(df_r) present
vector of }\mp@subsup{\chi}{}{2}\mathrm{ statistics; r(df_r) not present
vector of p-values corresponding to r(F) or r(chi2)
vector of degrees of freedom corresponding to r(p)
vector of denominator degrees of freedom corresponding to r(F)
```

contrast with the post option stores the following in e() :
Scalars
e(df_r)
e(k_terms)
Macros
e(cmd)
e(cmdline)
e(est_cmd)
e(est_cmdline)
e(title)
e(overall)
e(emptycells)
$e$ (margin_method)
$e$ (properties)
Matrices
e(b)
e(V)
e(error)
e(L)
e(F)
e(chi2)
$e(p)$
e(df)
e(df2)
variance degrees of freedom number of terms in termlist

## contrast

command as typed
$e(\mathrm{cmd})$ from original estimation results
$e$ (cmdline) from original estimation results
title in output
overall or empty
empspec from emptycells()
asbalanced or asobserved
b V

## contrast estimates

variance-covariance matrix of the contrast estimates
contrast estimability codes;
0 means estimable,
8 means not estimable
matrix of contrasts applied to the model coefficients
vector of unadjusted $F$ statistics; e(df_r) present
vector of $\chi^{2}$ statistics; e(df_r) not present
vector of unadjusted $p$-values corresponding to $e(F)$ or $e$ (chi2)
vector of degrees of freedom corresponding to $e(p)$
vector of denominator degrees of freedom corresponding to e(F)

## Methods and formulas

Methods and formulas are presented under the following headings:
Marginal linear predictions
Contrast operators
Reference level contrasts
Adjacent contrasts
Grand mean contrasts
Helmert contrasts
Reverse Helmert contrasts
Orthogonal polynomial contrasts
Contrasts within interactions
Multiple comparisons

## Marginal linear predictions

contrast treats intercept effects separately from slope effects. To illustrate, consider the following parameterization for a quadratic regression of $y$ on $x$ that also models the effects of two factor variables $A$ and $B$, where the levels of $A$ are indexed by $i=1, \ldots, k_{a}$ and the levels of $B$ are indexed by $j=1, \ldots, k_{b}$.

$$
\begin{gathered}
E(y \mid A=i, B=j, x)=\eta_{0 i j}+\eta_{1 i j} x+\eta_{2 i j} x^{2} \\
\eta_{0 i j}=\eta_{0}+\alpha_{0 i}+\beta_{0 j}+(\alpha \beta)_{0 i j}
\end{gathered}
$$

$$
\begin{aligned}
& \eta_{1 i j}=\eta_{1}+\alpha_{1 i}+\beta_{1 j}+(\alpha \beta)_{1 i j} \\
& \eta_{2 i j}=\eta_{2}+\alpha_{2 i}+\beta_{2 j}+(\alpha \beta)_{2 i j}
\end{aligned}
$$

We have partitioned the coefficients into three groups of parameters: $\eta_{0 i j}$ is a cell prediction for the intercept, $\eta_{1 i j}$ is a cell prediction for the slope on $x$, and $\eta_{2 i j}$ is a cell prediction for the slope on $x^{2}$. For the intercept parameters, $\eta_{0}$ is the intercept, $\alpha_{0 i}$ represents a main effect for factor $A$ at its $i$ th level, $\beta_{0 j}$ represents a main effect for factor $B$ at its $j$ th level, and $(\alpha \beta)_{0 i j}$ represents an effect for the interaction of $A$ and $B$ at the $i j$ th level. The individual coefficients in $\eta_{1 i j}$ and $\eta_{2 i j}$ have similar interpretations, but the effects are on the slopes of $x$ and $x^{2}$, respectively.

The marginal intercepts for $A$ are given by

$$
\eta_{0 i .}=\sum_{j=1}^{k_{b}} f_{i j} \eta_{0 i j}
$$

where $f_{i j}$ is a marginal relative frequency of the $j$ th level of $B$ and is controlled by the asobserved and emptycells (reweight) options according to

$$
f_{i j}= \begin{cases}1 / k_{b}, & \text { default } \\ w_{. j} / w_{. .}, & \text {asobserved } \\ 1 /\left(k_{b}-e_{i .}\right), & \text { emptycells(reweight) } \\ w_{i j} / w_{i .}, & \text { emptycells(reweight) and asobserved }\end{cases}
$$

Above, $w_{i j}$ is the number of individuals with $A$ at its $i$ th level and $B$ at its $j$ th,

$$
\begin{gathered}
w_{i .}=\sum_{j=1}^{k_{b}} w_{i j} \\
w_{. j}=\sum_{i=1}^{k_{a}} w_{i j} \\
w_{. .}=\sum_{i=1}^{k_{a}} \sum_{j=1}^{k_{b}} w_{i j}
\end{gathered}
$$

and $e_{i}$ is the number of empty cells where $A$ is at its $i$ th level. The marginal intercepts for $B$ and marginal slopes on $x$ and $x^{2}$ are similarly defined.

Estimates for the cell intercepts and slopes are computed using the corresponding linear combination of the coefficients from the fitted model. For example, the estimated cell intercepts are computed using

$$
\widehat{\eta}_{0 i j}=\widehat{\eta}_{0}+\widehat{\alpha}_{0 i}+\widehat{\beta}_{0 j}+(\widehat{\alpha \beta})_{0 i j}
$$

and the estimated marginal intercepts for $A$ are computed as

$$
\widehat{\eta}_{0 i .}=\sum_{j=1}^{k_{b}} f_{i j} \widehat{\eta}_{0 i j}
$$

## Contrast operators

contrast performs Wald tests using linear combinations of marginal linear predictions. For example, the following linear combination can be used to test for a specific effect of factor $A$ on the marginal intercepts.

$$
\sum_{i=1}^{k_{a}} c_{i} \eta_{0 i}
$$

If the $c_{i}$ elements sum to zero, the linear combination is called a contrast. If the factor $A$ is represented by a variable named A , then we specify this contrast using the following syntax:

$$
\left\{\mathrm{A} c_{1} c_{2} \ldots c_{k_{a}}\right\}
$$

Similarly, the following linear combination can be used to test for a specific interaction effect of factors $A$ and $B$ on the marginal slope of $x$.

$$
\sum_{i=1}^{k_{a}} \sum_{j=1}^{k_{b}} c_{i j} \eta_{1 i j}
$$

If the factor $B$ is represented by a variable named B , then we specify this contrast using the following syntax:

$$
\left\{\mathrm{A} \# \mathrm{~B} c_{11} c_{12} \ldots c_{1 k_{b}} c_{21} \ldots c_{k_{a} k_{b}}\right\}
$$

contrast has variable operators for several commonly used contrasts. Each contrast operator specifies a matrix of linear combinations that yield the requested set of contrasts to be applied to the marginal linear predictions associated with the attached factor variable.

## Reference level contrasts

The r. operator compares each level with a reference level. Let $\mathbf{R}$ be the corresponding contrast matrix for factor $A$, and then $\mathbf{R}$ is a $\left(k_{a}-1\right) \times k_{a}$ matrix with elements

$$
\mathbf{R}_{i j}=\left\{\begin{aligned}
-1, & \text { if } j \text { is the reference level } \\
1, & \text { if } i=j \text { and } j \text { is less than the reference level } \\
1, & \text { if } i+1=j \text { and } j \text { is greater than the reference level } \\
0, & \text { otherwise }
\end{aligned}\right.
$$

If $k_{a}=5$ and the reference level is the third level of $A$ (specified as $\left.\mathrm{rb}(\# 3) \cdot \mathrm{A}\right)$, then

$$
\mathbf{R}=\left(\begin{array}{lllll}
1 & 0 & -1 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 \\
0 & 0 & -1 & 1 & 0 \\
0 & 0 & -1 & 0 & 1
\end{array}\right)
$$

## Adjacent contrasts

The a. operator compares each level with the next level. Let $\mathbf{A}$ be the corresponding contrast matrix for factor $A$, and then $\mathbf{A}$ is a $\left(k_{a}-1\right) \times k_{a}$ matrix with elements

$$
\mathbf{A}_{i j}=\left\{\begin{aligned}
1, & \text { if } i=j \\
-1, & \text { if } i+1=j \\
0, & \text { otherwise }
\end{aligned}\right.
$$

If $k_{a}=5$, then

$$
\mathbf{A}=\left(\begin{array}{rrrrr}
1 & -1 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 1 & -1
\end{array}\right)
$$

The ar. operator compares each level with the previous level. If $\mathbf{A}$ is the contrast matrix for the a. operator, then $-\mathbf{A}$ is the corresponding contrast matrix for the ar. operator.

## Grand mean contrasts

The g . operator compares each level with the mean of all the levels. Let $\mathbf{G}$ be the corresponding contrast matrix for factor $A$, and then $\mathbf{G}$ is a $k_{a} \times k_{a}$ matrix with elements

$$
\mathbf{G}_{i j}=\left\{\begin{aligned}
1-1 / k_{a}, & \text { if } i=j \\
-1 / k_{a}, & \text { if } i \neq j
\end{aligned}\right.
$$

If $k_{a}=5$, then

$$
\mathbf{G}=\left(\begin{array}{rrrrr}
4 / 5 & -1 / 5 & -1 / 5 & -1 / 5 & -1 / 5 \\
-1 / 5 & 4 / 5 & -1 / 5 & -1 / 5 & -1 / 5 \\
-1 / 5 & -1 / 5 & 4 / 5 & -1 / 5 & -1 / 5 \\
-1 / 5 & -1 / 5 & -1 / 5 & 4 / 5 & -1 / 5 \\
-1 / 5 & -1 / 5 & -1 / 5 & -1 / 5 & 4 / 5
\end{array}\right)
$$

The gw. operator compares each level with the weighted mean of all the levels. The weights are taken from the observed weighted cell frequencies in the estimation sample of the fitted model. Let $\mathbf{G}_{w}$ be the corresponding contrast matrix for factor $A$, and then $\mathbf{G}_{w}$ is a $k_{a} \times k_{a}$ matrix with elements

$$
\mathbf{G}_{i j}=\left\{\begin{array}{cc}
1-w_{i} / w . & \text { if } i=j \\
-w_{j} / w . & \text { if } i \neq j
\end{array}\right.
$$

where $w_{i}$ is a marginal weight representing the number of individuals with $A$ at its $i$ th level and $w .=\sum_{i} w_{i}$.

## Helmert contrasts

The $h$. operator compares each level with the mean of the subsequent levels. Let $\mathbf{H}$ be the corresponding contrast matrix for factor $A$, and then $\mathbf{H}$ is a $\left(k_{a}-1\right) \times k_{a}$ matrix with elements

$$
\mathbf{H}_{i j}=\left\{\begin{array}{cl}
1, & \text { if } i=j \\
-1 /\left(k_{a}-i\right), & \text { if } i<j \\
0, & \text { otherwise }
\end{array}\right.
$$

If $k_{a}=5$, then

$$
\mathbf{H}=\left(\begin{array}{ccccl}
1 & -1 / 4 & -1 / 4 & -1 / 4 & -1 / 4 \\
0 & 1 & -1 / 3 & -1 / 3 & -1 / 3 \\
0 & 0 & 1 & -1 / 2 & -1 / 2 \\
0 & 0 & 0 & 1 & -1
\end{array}\right)
$$

The hw. operator compares each level with the weighted mean of the subsequent levels. Let $\mathbf{H}_{w}$ be the corresponding contrast matrix for factor $A$, and then $\mathbf{H}_{w}$ is a $\left(k_{a}-1\right) \times k_{a}$ matrix with elements

$$
\mathbf{H}_{w i j}= \begin{cases}1, & \text { if } i=j \\ -w_{j} / \sum_{l=j}^{k_{a}} w_{l}, & \text { if } i<j \\ 0, & \text { otherwise }\end{cases}
$$

## Reverse Helmert contrasts

The $\mathbf{j}$. operator compares each level with the mean of the previous levels. Let $\mathbf{J}$ be the corresponding contrast matrix for factor $A$, and then $\mathbf{J}$ is a $\left(k_{a}-1\right) \times k_{a}$ matrix with elements

$$
\mathbf{J}_{i j}=\left\{\begin{array}{cl}
1, & \text { if } i+1=j \\
-1 / i, & \text { if } j \leq i \\
0, & \text { otherwise }
\end{array}\right.
$$

If $k_{a}=5$, then

$$
\mathbf{H}=\left(\begin{array}{ccccc}
-1 & 1 & 0 & 0 & 0 \\
-1 / 2 & -1 / 2 & 1 & 0 & 0 \\
-1 / 3 & -1 / 3 & -1 / 3 & 1 & 0 \\
-1 / 4 & -1 / 4 & -1 / 4 & -1 / 4 & 1
\end{array}\right)
$$

The $j w$. operator compares each level with the weighted mean of the previous levels. Let $\mathbf{J}_{w}$ be the corresponding contrast matrix for factor $A$, and then $\mathbf{J}_{w}$ is a $\left(k_{a}-1\right) \times k_{a}$ matrix with elements

$$
\mathbf{J}_{w i j}= \begin{cases}1, & \text { if } i+1=j \\ -w_{j} / \sum_{l=1}^{i} w_{l}, & \text { if } i \leq j \\ 0, & \text { otherwise }\end{cases}
$$

## Orthogonal polynomial contrasts

The p. operator applies orthogonal polynomial contrasts using the level values of the attached factor variable. The q. operator applies orthogonal polynomial contrasts using the level indices of the attached factor variable. These two operators are equivalent when the level values of the attached factor are equally spaced. The pw. and qw. operators are weighted versions of p. and q., where the weights are taken from the observed weighted cell frequencies in the estimation sample of the fitted model. contrast uses the Christoffel-Darboux recurrence formula for computing orthogonal polynomial contrasts (Abramowitz and Stegun 1972). The elements of the contrasts are normalized such that

$$
\mathbf{Q}^{\prime} \mathbf{W Q}=\frac{1}{w .} \mathbf{I}
$$

where $\mathbf{W}$ is a diagonal matrix of the marginal cell weights $w_{1}, w_{2}, \ldots, w_{k}$ of the attached factor variable (all 1 for p . and q .), and $w$. is the sum of the weights (the number of levels $k$ for p . and q.).

## Contrasts within interactions

Contrast operators are allowed to be specified on factor variables participating in interactions. In such cases, contrast applies the proper matrix product of the contrast matrices to the cell margins of the interacted factor variables.

For example, consider the contrasts implied by specifying r.A\#h.B. Let $M$ be the matrix of estimated cell margins for the levels of $A$ and $B$, where the rows of $\mathbf{M}$ are indexed by the levels of $A$ and the columns are indexed by the levels of $B$. contrast puts the estimated cell margins in the following vector form:

$$
\mathbf{v}=\operatorname{vec}\left(\mathbf{M}^{\prime}\right)=\left(\begin{array}{c}
\mathbf{M}_{11} \\
\mathbf{M}_{12} \\
\vdots \\
\mathbf{M}_{1 k_{b}} \\
\mathbf{M}_{21} \\
\mathbf{M}_{22} \\
\vdots \\
\mathbf{M}_{2 k_{b}} \\
\vdots \\
\mathbf{M}_{k_{a} k_{b}}
\end{array}\right)
$$

The individual contrasts are then given by the elements of

$$
(\mathbf{R} \otimes \mathbf{H}) \mathbf{v}
$$

where $\otimes$ denotes the Kronecker direct product.

## Multiple comparisons

See $[R]$ pwcompare for details on the methods and formulas used to adjust $p$-values and confidence intervals for multiple comparisons. The formulas for Bonferroni's method and Šidák's method are presented with $m=k(k-1) / 2$, the number of pairwise comparisons for a factor term with $k$ levels. For contrasts, $m$ is instead the number of contrasts being performed on the factor term; often, $m=k-1$ for a term with $k$ levels.

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## Also see

[R] contrast postestimation - Postestimation tools for contrast
[R] lincom - Linear combinations of estimators
[R] margins - Marginal means, predictive margins, and marginal effects
[R] margins, contrast - Contrasts of margins
[R] pwcompare - Pairwise comparisons
[R] test - Test linear hypotheses after estimation
[U] 20 Estimation and postestimation commands

## Title

contrast postestimation - Postestimation tools for contrast

## Description Remarks and examples Also see

## Description

The following postestimation commands are available after contrast, post:
Command Description

| estat vce | variance-covariance matrix of the estimators (VCE) <br> estat (svy) <br> postestimation statistics for survey data <br> estimates <br> cataloging estimation results |
| :--- | :--- |
| lincom | point estimates, standard errors, testing, and inference for linear combinations <br> of coefficients |
| nlcom | point estimates, standard errors, testing, and inference for nonlinear combinations <br> of coefficients |
| test | Wald tests of simple and composite linear hypotheses |
| testnl | Wald tests of nonlinear hypotheses |

## Remarks and examples

In Orthogonal polynomial contrasts in [R] contrast, we used the p. operator to test the orthogonal polynomial effects of age group.

```
. contrast p.agegrp, noeffects
```

We then used a second contrast command,

```
. contrast p(2 3 4).agegrp, noeffects
```

selecting levels to test whether the quadratic, cubic, and quartic contrasts were jointly significant.
We can perform the same joint test by using the test command after specifying the post option with our first contrast command.

```
. use http://www.stata-press.com/data/r13/cholesterol
(Artificial cholesterol data)
. anova chol agegrp
    (output omitted)
. contrast p.agegrp, noeffects post
Contrasts of marginal linear predictions
Margins : asbalanced
\begin{tabular}{c|crr}
\hline & \(d f\) & \(F\) & \(P>F\) \\
\hline agegrp & & & \\
(linear) & 1 & 139.11 & 0.0000 \\
(quadratic) & 1 & 0.15 & 0.6962 \\
(qubic) & 1 & 0.37 & 0.5448 \\
Joint & 1 & 0.43 & 0.5153 \\
Denominator & 4 & 35.02 & 0.0000 \\
\hline
\end{tabular}
. test p2.agegrp p3.agegrp p4.agegrp
( 1) p2.agegrp \(=0\)
( 2) p3.agegrp \(=0\)
( 3) p4.agegrp \(=0\)
\(F(3,70)=0.32\)
Prob > F = 0.8129
```


## Also see

[R] contrast - Contrasts and linear hypothesis tests after estimation
[U] 20 Estimation and postestimation commands

## Title

copyright - Display copyright information

## Syntax Description Remarks and examples Also see

## Syntax

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Description Also see

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## Title

correlate - Correlations (covariances) of variables or coefficients

| Syntax | Menu | Description | Options for correlate |
| :--- | :--- | :--- | :--- |
| Options for pwcorr | Remarks and examples | Stored results | Methods and formulas |
| References | Also see |  |  |

## Syntax

Display correlation matrix or covariance matrix correlate [varlist] [if][in] [weight] [, correlate_options]

Display all pairwise correlation coefficients

$$
\text { pwcorr }[\text { varlist }][\text { if }][\text { in }][\text { weight }][\text {, pwcorr_options }]
$$

correlate_options Description
Options
means display means, standard deviations, minimums, and maximums with matrix
noformat ignore display format associated with variables
covariance display covariances
wrap allow wide matrices to wrap
pwcorr_options Description
Main
obs print number of observations for each entry
sig print significance level for each entry
listwise use listwise deletion to handle missing values
casewise synonym for listwise
print(\#) significance level for displaying coefficients
star (\#) significance level for displaying with a star
bonferroni use Bonferroni-adjusted significance level
sidak use Šidák-adjusted significance level
varlist may contain time-series operators; see [U] 11.4.4 Time-series varlists.
by is allowed with correlate and pwcorr; see [D] by.
aweights and fweights are allowed; see [U] 11.1.6 weight.

## Menu

correlate
Statistics $>$ Summaries, tables, and tests $>$ Summary and descriptive statistics $>$ Correlations and covariances

## pwcorr

Statistics $>$ Summaries, tables, and tests $>$ Summary and descriptive statistics $>$ Pairwise correlations

## Description

The correlate command displays the correlation matrix or covariance matrix for a group of variables. If varlist is not specified, the matrix is displayed for all variables in the dataset. Also see the estat vce command in [R] estat vce.
pwcorr displays all the pairwise correlation coefficients between the variables in varlist or, if varlist is not specified, all the variables in the dataset.

## Options for correlate

$\qquad$ Options
means displays summary statistics (means, standard deviations, minimums, and maximums) with the matrix.
noformat displays the summary statistics requested by the means option in g format, regardless of the display formats associated with the variables.
covariance displays the covariances rather than the correlation coefficients.
wrap requests that no action be taken on wide correlation matrices to make them readable. It prevents Stata from breaking wide matrices into pieces to enhance readability. You might want to specify this option if you are displaying results in a window wider than 80 characters. Then you may need to set linesize to however many characters you can display across a line; see $[\mathrm{R}] \log$.

## Options for pwcorr

$\qquad$ Main
obs adds a line to each row of the matrix reporting the number of observations used to calculate the correlation coefficient.
sig adds a line to each row of the matrix reporting the significance level of each correlation coefficient.
listwise handles missing values through listwise deletion, meaning that the entire observation is omitted from the estimation sample if any of the variables in varlist is missing for that observation. By default, pwcorr handles missing values by pairwise deletion; all available observations are used to calculate each pairwise correlation without regard to whether variables outside that pair are missing.
correlate uses listwise deletion. Thus listwise allows users of pwcorr to mimic correlate's treatment of missing values while retaining access to pwcorr's features.
casewise is a synonym for listwise.
print (\#) specifies the significance level of correlation coefficients to be printed. Correlation coefficients with larger significance levels are left blank in the matrix. Typing pwcorr, print (.10) would list only correlation coefficients significant at the $10 \%$ level or better.
star(\#) specifies the significance level of correlation coefficients to be starred. Typing pwcorr, star (.05) would star all correlation coefficients significant at the $5 \%$ level or better.
bonferroni makes the Bonferroni adjustment to calculated significance levels. This option affects printed significance levels and the print() and star() options. Thus pwcorr, print (.05) bonferroni prints coefficients with Bonferroni-adjusted significance levels of 0.05 or less.
sidak makes the Šidák adjustment to calculated significance levels. This option affects printed significance levels and the print() and star() options. Thus pwcorr, print(.05) sidak prints coefficients with Sidák-adjusted significance levels of 0.05 or less.

## Remarks and examples

Remarks are presented under the following headings:

```
correlate
pwcorr
Video example
```


## correlate

Typing correlate by itself produces a correlation matrix for all variables in the dataset. If you specify the varlist, a correlation matrix for just those variables is displayed.

## > Example 1

We have state data on demographic characteristics of the population. To obtain a correlation matrix, we type

|  | state | brate | pop | medage | division | region | mrgrate |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| state | 1.0000 |  |  |  |  |  |  |
| brate | 0.0208 | 1.0000 |  |  |  |  |  |
| pop | -0.0540 | -0.2830 | 1.0000 |  |  |  |  |
| medage | -0.0624 | -0.8800 | 0.3294 | 1.0000 |  |  |  |
| division | -0.1345 | 0.6356 | -0.1081 | -0.5207 | 1.0000 |  |  |
| region | -0.1339 | 0.6086 | -0.1515 | -0.5292 | 0.9688 | 1.0000 |  |
| mrgrate | 0.0509 | 0.0677 | -0.1502 | -0.0177 | 0.2280 | 0.2490 | 1.0000 |
| dvcrate | -0.0655 | 0.3508 | -0.2064 | -0.2229 | 0.5522 | 0.5682 | 0.7700 |
| medagesq | -0.0621 | -0.8609 | 0.3324 | 0.9984 | -0.5162 | -0.5239 | -0.0202 |
| dvcrate medagesq |  |  |  |  |  |  |  |
| dvcrate | 1.0000 |  |  |  |  |  |  |
| medagesq | -0.2192 | 1.0000 |  |  |  |  |  |

Because we did not specify the wrap option, Stata did its best to make the result readable by breaking the table into two parts.

To obtain the correlations between mrgrate, dvcrate, and medage, we type

```
. correlate mrgrate dvcrate medage
(obs=50)
```

|  | mrgrate | dvcrate | medage |
| ---: | ---: | ---: | ---: |
| mrgrate | 1.0000 |  |  |
| dvcrate | 0.7700 | 1.0000 |  |
| medage | -0.0177 | -0.2229 | 1.0000 |

## Example 2

The pop variable in example 1 represents the total population of the state. Thus, to obtain population-weighted correlations among mrgrate, dvcrate, and medage, we type

```
. correlate mrgrate dvcrate medage [w=pop]
(analytic weights assumed)
(sum of wgt is 2.2591e+08)
(obs=50)
\begin{tabular}{r|rrr} 
& mrgrate & dvcrate & medage \\
\hline mrgrate & 1.0000 & & \\
dvcrate & 0.5854 & 1.0000 & \\
medage & -0.1316 & -0.2833 & 1.0000
\end{tabular}
```

With the covariance option, correlate can be used to obtain covariance matrices, as well as correlation matrices, for both weighted and unweighted data.

## > Example 3

To obtain the matrix of covariances between mrgrate, dvcrate, and medage, we type correlate mrgrate dvcrate medage, covariance:

```
. correlate mrgrate dvcrate medage, covariance
```

(obs=50)

|  | mrgrate | dvcrate | medage |
| ---: | ---: | ---: | ---: |
| mrgrate | .000662 |  |  |
| dvcrate | .000063 | $1.0 \mathrm{e}-05$ |  |
| medage | -.000769 | -.001191 | 2.86775 |

We could have obtained the pop-weighted covariance matrix by typing correlate mrgrate dvcrate medage [w=pop], covariance.

## pwcorr

correlate calculates correlation coefficients by using casewise deletion; when you request correlations of variables $x_{1}, x_{2}, \ldots, x_{k}$, any observation for which any of $x_{1}, x_{2}, \ldots, x_{k}$ is missing is not used. Thus if $x_{3}$ and $x_{4}$ have no missing values, but $x_{2}$ is missing for half the data, the correlation between $x_{3}$ and $x_{4}$ is calculated using only the half of the data for which $x_{2}$ is not missing. Of course, you can obtain the correlation between $x_{3}$ and $x_{4}$ by using all the data by typing correlate $x_{3} x_{4}$.
pwcorr makes obtaining such pairwise correlation coefficients easier.

## Example 4

Using auto.dta, we investigate the correlation between several of the variables.

. pwcorr mpg price headroom rear_seat trunk rep78 foreign, print(.05) star(.01)

|  | mpg | price headroom rear_s $\sim t$ | trunk | rep78 foreign |  |  |  |
| ---: | ---: | :--- | :--- | :--- | :--- | :--- | :--- |
| mpg | 1.0000 |  |  |  |  |  |  |
| price | $-0.4594 *$ | 1.0000 |  |  |  |  |  |
| headroom | $-0.4220 *$ |  | 1.0000 |  |  |  |  |
| rear_seat | $-0.5213 *$ | $0.4194 *$ | $0.5238 *$ | 1.0000 |  |  |  |
| trunk | $-0.5703 *$ | $0.3143 *$ | $0.6620 *$ | $0.6480 *$ | 1.0000 |  |  |
| rep78 | $0.3739 *$ |  |  |  |  |  | 1.0000 |
| foreign | $0.3613 *$ |  | -0.2939 | -0.2409 | $-0.3594 *$ | $0.5922 *$ | 1.0000 |

. pwcorr mpg price headroom rear_seat trunk rep78 foreign, print(.05) bon

|  | mpg | price headroom rear_s~t | trunk | rep78 foreign |  |  |
| ---: | ---: | :---: | :---: | :---: | :---: | :---: |
| mpg | 1.0000 |  |  |  |  |  |
| price | -0.4594 | 1.0000 |  |  |  |  |
| headroom | -0.4220 |  | 1.0000 |  |  |  |
| rear_seat | -0.5213 | 0.4194 | 0.5238 | 1.0000 |  |  |
| trunk | -0.5703 |  | 0.6620 | 0.6480 | 1.0000 |  |
| rep78 | 0.3739 |  |  |  |  | 1.0000 |
| foreign | 0.3613 |  |  |  |  |  |
|  |  |  |  |  |  |  |

- Technical note

The correlate command will report the correlation matrix of the data, but there are occasions when you need the matrix stored as a Stata matrix so that you can further manipulate it. You can obtain the matrix by typing

```
. matrix accum R = varlist, noconstant deviations
. matrix R = corr(R)
```

The first line places the cross-product matrix of the data in matrix $R$. The second line converts that to a correlation matrix. Also see $[\mathrm{P}]$ matrix define and $[\mathrm{P}]$ matrix accum.

## Video example

Pearson's correlation coefficient in Stata

## Stored results

correlate stores the following in $r()$ :
Scalars

$$
\begin{array}{ll}
r(N) & \text { number of observations } \\
r(r h o) & \rho \text { (first and second variables) } \\
r\left(c o v \_12\right) & \text { covariance (covariance only) } \\
r(\text { Var_1) } & \text { variance of first variable (covariance only) } \\
r(\text { Var_2 }) & \text { variance of second variable (covariance only) } \\
\text { rices } & \\
r(C) & \text { correlation or covariance matrix }
\end{array}
$$

Matrices
pwcorr will leave in its wake only the results of the last call that it makes internally to correlate for the correlation between the last variable and itself. Only rarely is this feature useful.

## Methods and formulas

For a discussion of correlation, see, for instance, Snedecor and Cochran (1989, 177-195); for an introductory explanation using Stata examples, see Acock (2014, 200-206).

According to Snedecor and Cochran (1989, 180), the term "co-relation" was first proposed by Galton (1888). The product-moment correlation coefficient is often called the Pearson product-moment correlation coefficient because Pearson (1896) and Pearson and Filon (1898) were partially responsible for popularizing its use. See Stigler (1986) for information on the history of correlation.

The estimate of the product-moment correlation coefficient, $\rho$, is

$$
\widehat{\rho}=\frac{\sum_{i=1}^{n} w_{i}\left(x_{i}-\bar{x}\right)\left(y_{i}-\bar{y}\right)}{\sqrt{\sum_{i=1}^{n} w_{i}\left(x_{i}-\bar{x}\right)^{2}} \sqrt{\sum_{i=1}^{n} w_{i}\left(y_{i}-\bar{y}\right)^{2}}}
$$

where $w_{i}$ are the weights, if specified, or $w_{i}=1$ if weights are not specified. $\bar{x}=\left(\sum w_{i} x_{i}\right) /\left(\sum w_{i}\right)$ is the mean of $x$, and $\bar{y}$ is similarly defined.

The unadjusted significance level is calculated by pwcorr as

$$
p=2 * \operatorname{ttail}\left(n-2,|\widehat{\rho}| \sqrt{n-2} / \sqrt{1-\widehat{\rho}^{2}}\right)
$$

Let $v$ be the number of variables specified so that $k=v(v-1) / 2$ correlation coefficients are to be estimated. If bonferroni is specified, the adjusted significance level is $p^{\prime}=\min (1, k p)$. If sidak is specified, $p^{\prime}=\min \left\{1,1-(1-p)^{k}\right\}$. In both cases, see Methods and formulas in [R] oneway for a more complete description of the logic behind these adjustments.

Carlo Emilio Bonferroni (1892-1960) studied in Turin and taught there and in Bari and Florence. He published on actuarial mathematics, probability, statistics, analysis, geometry, and mechanics. His work on probability inequalities has been applied to simultaneous statistical inference, although the method known as Bonferroni adjustment usually relies only on an inequality established earlier by Boole.

Florence Nightingale David (1909-1993) was born in Ivington, England, to parents who were friends with Florence Nightingale, David's namesake. She began her studies in statistics under the direction of Karl Pearson at University College London and continued her studies under the direction of Jerzy Neyman. After receiving her doctorate in statistics in 1938, David became a senior statistician for various departments within the British military. She developed statistical models to forecast the toll on life and infrastructure that would occur if a large city were bombed. In 1938, she also published her book Tables of the Correlation Coefficient, dealing with the distributions of correlation coefficients. After the war, she returned to University College London, serving as a lecturer until her promotion to professor in 1962. In 1967, David joined the University of California-Riverside, eventually becoming chair of the Department of Statistics. One of her most well-known works is the book Games, Gods and Gambling: The Origins and History of Probability and Statistical Ideas from the Earliest Times to the Newtonian Era, a history of statistics. David published over 100 papers on topics including combinatorics, symmetric functions, the history of statistics, and applications of statistics, including ecological diversity. She published under the name F. N. David to avoid revealing her gender in a male-dominated profession.
Karl Pearson (1857-1936) studied mathematics at Cambridge. He was professor of applied mathematics (1884-1911) and eugenics (1911-1933) at University College London. His publications include literary, historical, philosophical, and religious topics. Statistics became his main interest in the early 1890s after he learned about its application to biological problems. His work centered on distribution theory, the method of moments, correlation, and regression. Pearson introduced the chi-squared test and the terms coefficient of variation, contingency table, heteroskedastic, histogram, homoskedastic, kurtosis, mode, random sampling, random walk, skewness, standard deviation, and truncation. Despite many strong qualities, he also fell into prolonged disagreements with others, most notably, William Bateson and R. A. Fisher.
Zbyněk Šidák (1933-1999) was a notable Czech statistician and probabilist. He worked on Markov chains, rank tests, multivariate distribution theory and multiple-comparison methods, and he served as the chief editor of Applications of Mathematics.

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## Also see

[R] esize - Effect size based on mean comparison
[R] icc - Intraclass correlation coefficients
[R] pcorr - Partial and semipartial correlation coefficients
[R] spearman - Spearman's and Kendall's correlations
[R] summarize - Summary statistics
[R] tetrachoric - Tetrachoric correlations for binary variables

## Title

cumul - Cumulative distribution

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Acknowledgment | References | Also see |

## Syntax

cumul varname $[$ if $][$ in $][$ weight $]$, generate(newvar) [options]
options
Description
Main

* generate (newvar) create variable newvar
freq use frequency units for cumulative
equal
generate equal cumulatives for tied values
*generate (newvar) is required.
by is allowed; see [D] by.
fweights and aweights are allowed; see [U] 11.1.6 weight.


## Menu

Statistics $>$ Summaries, tables, and tests $>$ Distributional plots and tests $>$ Generate cumulative distribution

## Description

cumul creates newvar, defined as the empirical cumulative distribution function of varname.

## Options

generate (newvar) is required. It specifies the name of the new variable to be created.
freq specifies that the cumulative be in frequency units; otherwise, it is normalized so that newvar is 1 for the largest value of varname.
equal requests that observations with equal values in varname get the same cumulative value in newvar.

## Remarks and examples

## > Example 1

cumul is most often used with graph to graph the empirical cumulative distribution. For instance, we have data on the median family income of 957 U.S. cities:

```
. use http://www.stata-press.com/data/r13/hsng
(1980 Census housing data)
. cumul faminc, gen(cum)
. sort cum
. line cum faminc, ylab(, grid) ytitle("") xlab(, grid)
> title("Cumulative of median family income")
> subtitle("1980 Census, 957 U.S. Cities")
```

Cumulative of median family income
1980 Census, 957 U.S. Cities


It would have been enough to type line cum faminc, but we wanted to make the graph look better; see [G-2] graph twoway line.

If we had wanted a weighted cumulative, we would have typed cumul faminc [w=pop] at the first step.
> Example 2
To graph two (or more) cumulatives on the same graph, use cumul and stack; see [D] stack. For instance, we have data on the average January and July temperatures of 956 U.S. cities:

```
. use http://www.stata-press.com/data/r13/citytemp, clear
(City Temperature Data)
. cumul tempjan, gen(cjan)
. cumul tempjuly, gen(cjuly)
. stack cjan tempjan cjuly tempjuly, into(c temp) wide clear
. line cjan cjuly temp, sort ylab(, grid) ytitle("") xlab(, grid)
> xtitle("Temperature (F)")
> title("Cumulatives:" "Average January and July Temperatures")
> subtitle("956 U.S. Cities")
```



As before, it would have been enough to type line cjan cjuly temp, sort. See [D] stack for an explanation of how the stack command works.

## Technical note

According to Beniger and Robyn (1978), Fourier (1821) published the first graph of a cumulative frequency distribution, which was later given the name "ogive" by Galton (1875).

Jean Baptiste Joseph Fourier (1768-1830) was born in Auxerre in France. As a young man, Fourier became entangled in the complications of the French Revolution. As a result, he was arrested and put into prison, where he feared he might meet his end at the guillotine. When he was not in prison, he was studying, researching, and teaching mathematics. Later, he served Napolean's army in Egypt as a scientific adviser. Upon his return to France in 1801, he was appointed Prefect of the Department of Isère. While prefect, Fourier worked on the mathematical basis of the theory of heat, which is based on what are now called Fourier series. This work was published in 1822, despite the skepticism of Lagrange, Laplace, Legendre, and others-who found the work lacking in generality and even rigor-and disagreements of both priority and substance with Biot and Poisson.

## Acknowledgment

The equal option was added by Nicholas J. Cox of the Department of Geography at Durham University, UK, and coeditor of the Stata Journal.

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## Also see

[R] diagnostic plots - Distributional diagnostic plots
[R] kdensity - Univariate kernel density estimation
[D] stack - Stack data

## Title

cusum - Cusum plots and tests for binary variables

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Acknowledgment | References |
| Also see |  |  |  |

## Syntax

cusum yvar xvar [if] [in] [, options]
options
Description
Main
generate (newvar) save cumulative sum in newvar
yfit (fitvar) calculate cumulative sum against fitvar
nograph suppress the plot
nocalc suppress cusum test statistics
Cusum plot
connect_options
affect the rendition of the plotted line
Add plots
addplot (plot) add plots to the generated graph
Y axis, X axis, Titles, Legend, Overall
twoway_options
any options other than by () documented in [G-3] twoway_options

## Menu

Statistics $>$ Other $>$ Quality control $>$ Cusum plots and tests for binary variables

## Description

cusum graphs the cumulative sum (cusum) of a binary ( $0 / 1$ ) variable, yvar, against a (usually) continuous variable, xvar.

## Options

$\qquad$ Main
generate (newvar) saves the cusum in newvar.
yfit(fitvar) calculates a cusum against fitvar, that is, the running sums of the "residuals" fitvar minus yvar. Typically, fitvar is the predicted probability of a positive outcome obtained from a logistic regression analysis.
nograph suppresses the plot. nocalc suppresses calculation of the cusum test statistics.
connect_options affect the rendition of the plotted line; see [G-3] connect_options.

Add plots
addplot (plot) provides a way to add other plots to the generated graph. See [G-3] addplot_option.

Y axis, X axis, Titles, Legend, Overall
twoway_options are any of the options documented in [G-3] twoway_options, excluding by (). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

## Remarks and examples

The cusum is the running sum of the proportion of ones in the sample, a constant number, minus yvar,

$$
c_{j}=\sum_{k=1}^{j} f-y \operatorname{var}_{(k)}, \quad 1 \leq j \leq N
$$

where $f=\left(\sum y v a r\right) / N$ and $\operatorname{yvar}_{(k)}$ refers to the corresponding value of $y v a r$ when $x v a r$ is placed in ascending order: $\operatorname{xvar}_{(k+1)} \geq \operatorname{xvar}_{(k)}$. Tied values of xvar are broken at random. If you want them broken the same way in two runs, you must set the random-number seed to the same value before giving the cusum command; see $[R]$ set seed.

A U-shaped or inverted U-shaped cusum indicates, respectively, a negative or a positive trend of $y v a r$ with xvar. A sinusoidal shape is evidence of a nonmonotonic (for example, quadratic) trend. cusum displays the maximum absolute cusum for monotonic and nonmonotonic trends of yvar on xvar. These are nonparametric tests of departure from randomness of yvar with respect to xvar. Approximate values for the tests are given.
> Example 1
For the automobile dataset, auto.dta, we wish to investigate the relationship between foreign ( $0=$ domestic, $1=$ foreign ) and car weight as follows:

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. cusum foreign weight
\begin{tabular}{l|rccccccc} 
Variable & Obs & \(\operatorname{Pr}(1)\) & CusumL & zL & \(\operatorname{Pr}>\) zL & CusumQ & zQ & \(\operatorname{Pr}>\mathrm{zQ}\) \\
\hline foreign & 74 & 0.2973 & 10.30 & 3.963 & 0.000 & 3.32 & 0.469 & 0.320
\end{tabular}
```



The resulting plot, which is U-shaped, suggests a negative monotonic relationship. The trend is confirmed by a highly significant linear cusum statistic, labeled CusumL in the output above.

Some $29.73 \%$ of the cars are foreign (coded 1). The proportion of foreign cars diminishes with increasing weight. The domestic cars are crudely heavier than the foreign ones. We could have discovered that by typing table foreign, stats (mean weight), but such an approach does not give the full picture of the relationship. The quadratic cusum (CusumQ) is not significant, so we do not suspect any tendency for the very heavy cars to be foreign rather than domestic. A slightly enhanced version of the plot shows the preponderance of domestic (coded 0) cars at the heavy end of the weight axis:

| Variable | Obs | $\operatorname{Pr}(1)$ | CusumL | zL | Pr>zL | CusumQ | zQ | Pr>zQ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| foreign | 74 | 0.2973 | 10.30 | 3.963 | 0.000 | 2.92 | 0.064 | 0.475 |



The example is, of course, artificial, because we would not really try to model the probability of a car being foreign given its weight.

## Stored results

cusum stores the following in $r()$ :
Scalars

| r (N) | number of observations | r(P_zl) | $p$-value for test (linear) |
| :---: | :---: | :---: | :---: |
| r(prop1) | proportion of positive outcomes | r(cusumq) | quadratic cusum |
| r (cusuml) | cusum | r (zq) | test (quadratic) |
| r(zl) | test (linear) | r(P_zq) | $p$-value for test (quadratic) |

## Acknowledgment

cusum was written by Patrick Royston of the MRC Clinical Trials Unit, London, and coauthor of the Stata Press book Flexible Parametric Survival Analysis Using Stata: Beyond the Cox Model.

## References

Royston, P. 1992. The use of cusums and other techniques in modelling continuous covariates in logistic regression. Statistics in Medicine 11: 1115-1129.
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## Also see

[R] logistic - Logistic regression, reporting odds ratios
[R] logit - Logistic regression, reporting coefficients
[R] probit - Probit regression

## Title

db — Launch dialog

Syntax Description Options Remarks and examples Also see

## Syntax

Syntax for $d b$
db commandname

For programmers
db commandname [, message(string) debug dryrun]
Set system parameter
set maxdb \# [, permanently]
where \# must be between 5 and 1,000 .

## Description

db is the command-line way to launch a dialog for a Stata command.
The second syntax (which is the same but includes options) is for use by programmers.
If you wish to allow the launching of dialogs from a help file, see [P] smcl for information on the dialog SMCL directive.
set maxdb sets the maximum number of dialog boxes whose contents are remembered from one invocation to the next during a session. The default value of maxdb is 50 .

## Options

message (string) specifies that string be passed to the dialog box, where it can be referred to from the __MESSAGE STRING property.
debug specifies that the underlying dialog box be loaded with debug messaging turned on.
dryrun specifies that, rather than launching the dialog, db show the commands it would issue to launch the dialog.
permanently specifies that, in addition to making the change right now, the maxdb setting be remembered and become the default setting when you invoke Stata.

## Remarks and examples

The usual way to launch a dialog is to open the Data, Graphics, or Statistics menu and to make your selection from there. When you know the name of the command that you want to run, however, db provides a way to invoke the dialog from the command line.
db follows the same abbreviation rules that Stata's command-line interface follows. So, to launch the dialog for regress, you can type
. db regress
or
. db reg
Say that you use the dialog box for regress, either by selecting
Statistics $>$ Linear models and related $>$ Linear regression
or by typing
. db regress
You fit a regression.
Much later during the session, you return to the regress dialog box. It will have the contents as you left them if 1) you have not typed clear all between the first and second invocations; 2) you have not typed discard between the two invocations; and 3) you have not used more than 50 different dialog boxes-regardless of how many times you have used each—between the first and second invocations of regress. If you use 51 or more, the contents of the regress dialog box will be forgotten.
set maxdb determines how many different dialog boxes are remembered. A dialog box takes, on average, about 20 KB of memory, so the 50 default corresponds to allowing dialog boxes to consume about 1 MB of memory.

## Also see

[R] query - Display system parameters

## Title

diagnostic plots - Distributional diagnostic plots

## Syntax

Description
Options for qnorm and pnorm
Remarks and examples
Acknowledgments
Also see

## Menu

Options for symplot, quantile, and qqplot
Options for qchi and pchi
Methods and formulas
References

## Syntax

Symmetry plot
symplot varname [if][in][, options $\left.{ }_{1}\right]$

Ordered values of varname against quantiles of uniform distribution quantile varname [if][in] [, options $\left.{ }_{1}\right]$

Quantiles of varname $1_{1}$ against quantiles of varname ${ }_{2}$ qqplot varname $_{1}$ varname $_{2}[$ if $][$ in $]\left[\right.$, options $\left.{ }_{1}\right]$

Quantiles of varname against quantiles of normal distribution qnorm varname [if] [in] [, options ${ }_{2}$ ]

Standardized normal probability plot
pnorm varname [if] [in] [, options 2 ]
Quantiles of varname against quantiles of $\chi^{2}$ distribution qchi varname $[$ if $][$ in] [, options 3$]$
$\chi^{2}$ probability plot
pchi varname $[$ if $][$ in] [, options 3$]$
options $_{1}$
Description
Plot
marker_options change look of markers (color, size, etc.)
marker_label_options
add marker labels; change look or position
Reference line
rlopts (cline_options)
affect rendition of the reference line
Add plots
addplot (plot) add other plots to the generated graph
Y axis, X axis, Titles, Legend, Overall
twoway_options any options other than by () documented in [G-3] twoway_options

## options $_{2}$ <br> Description

Main
grid add grid lines
Plot
marker_options
marker_label_options
change look of markers (color, size, etc.)
add marker labels; change look or position
Reference line rlopts (cline_options) affect rendition of the reference line

Add plots
addplot (plot) add other plots to the generated graph
Y axis, X axis, Titles, Legend, Overall
twoway_options any options other than by () documented in [G-3] twoway_options
options $_{3}$
Description
Main
grid
add grid lines
df (\#)
degrees of freedom of $\chi^{2}$ distribution; default is $\mathrm{df}(1)$
Plot
marker_options change look of markers (color, size, etc.)
marker_label_options
add marker labels; change look or position
Reference line
rlopts (cline_options) affect rendition of the reference line
Add plots
addplot (plot) add other plots to the generated graph
Y axis, X axis, Titles, Legend, Overall
twoway_options any options other than by () documented in [G-3] twoway_options

## Menu

## symplot

Statistics $>$ Summaries, tables, and tests $>$ Distributional plots and tests $>$ Symmetry plot
quantile
Statistics $>$ Summaries, tables, and tests $>$ Distributional plots and tests $>$ Quantiles plot

## qqplot

Statistics $>$ Summaries, tables, and tests $>$ Distributional plots and tests $>$ Quantile-quantile plot

## qnorm

Statistics $>$ Summaries, tables, and tests $>$ Distributional plots and tests $>$ Normal quantile plot

## pnorm

Statistics $>$ Summaries, tables, and tests $>$ Distributional plots and tests $>$ Normal probability plot, standardized

## qchi

Statistics $>$ Summaries, tables, and tests $>$ Distributional plots and tests $>$ Chi-squared quantile plot

## pchi

Statistics $>$ Summaries, tables, and tests $>$ Distributional plots and tests $>$ Chi-squared probability plot

## Description

symplot graphs a symmetry plot of varname.
quantile plots the ordered values of varname against the quantiles of a uniform distribution. qqplot plots the quantiles of varname $_{1}$ against the quantiles of varname 2 ( $\mathrm{Q}-\mathrm{Q}$ plot). qnorm plots the quantiles of varname against the quantiles of the normal distribution ( $\mathrm{Q}-\mathrm{Q}$ plot). pnorm graphs a standardized normal probability plot ( $\mathrm{P}-\mathrm{P}$ plot).
qchi plots the quantiles of varname against the quantiles of a $\chi^{2}$ distribution ( $\mathrm{Q}-\mathrm{Q}$ plot). pchi graphs a $\chi^{2}$ probability plot ( $\mathrm{P}-\mathrm{P}$ plot).
See $[R]$ regress postestimation diagnostic plots for regression diagnostic plots and $[R]$ logistic postestimation for logistic regression diagnostic plots.

## Options for symplot, quantile, and qqplot

$\qquad$ Plot marker_options affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] marker_options.
marker_label_options specify if and how the markers are to be labeled; see [G-3] marker_label_options.

addplot (plot) provides a way to add other plots to the generated graph; see [G-3] addplot_option.

Y axis, X axis, Titles, Legend, Overall
twoway_options are any of the options documented in [G-3] twoway_options, excluding by (). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

## Options for qnorm and pnorm

$\qquad$ Main
grid adds grid lines at the $0.05,0.10,0.25,0.50,0.75,0.90$, and 0.95 quantiles when specified with qnorm. With pnorm, grid is equivalent to yline (.25, .5, .75) xline (. $25, .5, .75$ ).

## $\int \sqrt{\text { Plot }}$

marker_options affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] marker_options.
marker_label_options specify if and how the markers are to be labeled; see [G-3] marker_label_options.


```
Add plots
addplot (plot) provides a way to add other plots to the generated graph; see [G-3] addplot_option.
```

$\longleftarrow \sqrt{\mathrm{Y}}$ axis, X axis, Titles, Legend, Overall
twoway_options are any of the options documented in [G-3] twoway_options, excluding by (). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

## Options for qchi and pchi

$\qquad$ Main
grid adds grid lines at the $0.05,0.10,0.25,0.50,0.75,0.90$, and .95 quantiles when specified with qchi. With pchi, grid is equivalent to yline (. $25, .5, .75$ ) xline (. $25, .5, .75$ ). $\mathrm{df}(\#)$ specifies the degrees of freedom of the $\chi^{2}$ distribution. The default is $\mathrm{df}(1)$.
$\qquad$ $\sqrt{\text { Plot }}$
marker_options affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] marker_options.
marker_label_options specify if and how the markers are to be labeled; see [G-3] marker_label_options.

Reference line
rlopts(cline_options) affect the rendition of the reference line; see [G-3] cline_options.

Add plots
addplot (plot) provides a way to add other plots to the generated graph; see [G-3] addplot_option.

Y axis, X axis, Titles, Legend, Overall
twoway_options are any of the options documented in [G-3] twoway_options, excluding by (). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

## Remarks and examples

Remarks are presented under the following headings:

```
symplot
quantile
qqplot
qnorm
pnorm
qchi
pchi
```


## symplot

$\rangle$ Example 1
We have data on 74 automobiles. To make a symmetry plot of the variable price, we type

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. symplot price
```



All points would lie along the reference line (defined as $y=x$ ) if car prices were symmetrically distributed. The points in this plot lie above the reference line, indicating that the distribution of car prices is skewed to the right - the most expensive cars are far more expensive than the least expensive cars are inexpensive.

The logic works as follows: a variable, $z$, is distributed symmetrically if

$$
\text { median }-z_{(i)}=z_{(N+1-i)}-\text { median }
$$

where $z_{(i)}$ indicates the $i$ th-order statistic of $z$. symplot graphs $y_{i}=$ median $-z_{(i)}$ versus $x_{i}=$ $z_{(N+1-i)}$ - median.

For instance, consider the largest and smallest values of price in the example above. The most expensive car costs $\$ 15,906$ and the least expensive, $\$ 3,291$. Let's compare these two cars with the typical car in the data and see how much more it costs to buy the most expensive car, and compare that with how much less it costs to buy the least expensive car. If the automobile price distribution is symmetric, the price differences would be the same.

Before we can make this comparison, we must agree on a definition for the word "typical". Let's agree that "typical" means median. The price of the median car is $\$ 5,006.50$, so the most expensive car costs $\$ 10,899.50$ more than the median car, and the least expensive car costs $\$ 1,715.50$ less than the median car. We now have one piece of evidence that the car price distribution is not symmetric. We can repeat the experiment for the second-most-expensive car and the second-least-expensive car. We find that the second-most-expensive car costs $\$ 9,494.50$ more than the median car, and the second-least-expensive car costs $\$ 1,707.50$ less than the median car. We now have more evidence. We can continue doing this with the third most expensive and the third least expensive, and so on.

Once we have all of these numbers, we want to compare each pair and ask how similar, on average, they are. The easiest way to do that is to plot all the pairs.

## quantile

## > Example 2

We have data on the prices of 74 automobiles. To make a quantile plot of price, we type

```
. use http://www.stata-press.com/data/r13/auto, clear
(1978 Automobile Data)
. quantile price, rlopts(clpattern(dash))
```



We changed the pattern of the reference line by specifying rlopts(clpattern(dash)).
In a quantile plot, each value of the variable is plotted against the fraction of the data that have values less than that fraction. The diagonal line is a reference line. If automobile prices were rectangularly distributed, all the data would be plotted along the line. Because all the points are below the reference line, we know that the price distribution is skewed right.

## qqplot

## > Example 3

We have data on the weight and country of manufacture of 74 automobiles. We wish to compare the distributions of weights for domestic and foreign automobiles:

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. generate weightd=weight if !foreign
(22 missing values generated)
. generate weightf=weight if foreign
(52 missing values generated)
. qqplot weightd weightf
```



## qnorm

## Example 4

Continuing with our price data on 74 automobiles, we now wish to compare the distribution of price with the normal distribution:
. qnorm price, grid ylabel(, angle(horizontal) axis(1))
> ylabel(, angle(horizontal) axis(2))


The result shows that the distributions are different.

## - Technical note

The idea behind qnorm is recommended strongly by Miller (1997): he calls it probit plotting. His recommendations from much practical experience should interest many users. "My recommendation for detecting nonnormality is probit plotting" (Miller 1997, 10). "If a deviation from normality cannot be spotted by eye on probit paper, it is not worth worrying about. I never use the Kolmogorov-Smirnov test (or one of its cousins) or the $\chi^{2}$ test as a preliminary test of normality. They do not tell you how the sample is differing from normality, and I have a feeling they are more likely to detect irregularities in the middle of the distribution than in the tails" (Miller 1997, 13-14).

## pnorm

## > Example 5

Quantile-normal plots emphasize the tails of the distribution. Normal probability plots put the focus on the center of the distribution:

```
. pnorm price, grid
```



## qchi

## > Example 6

Suppose that we want to examine the distribution of the sum of squares of price and mpg, standardized for their variances.

```
. egen c1 = std(price)
. egen c2 = std(mpg)
. generate ch = c1^2 + c2^2
. qchi ch, df(2) grid ylabel(, alt axis(2)) xlabel(, alt axis(2))
```



The quadratic form is clearly not $\chi^{2}$ with 2 degrees of freedom.

## pchi

## Example 7

We can focus on the center of the distribution by doing a probability plot:
. pchi ch, $\mathrm{df}(2)$ grid


## Methods and formulas

Let $x_{(1)}, x_{(2)}, \ldots, x_{(N)}$ be the data sorted in ascending order.
If a continuous variable, $x$, has a cumulative distribution function $F(x)=P(X \leq x)=p$, the quantiles $x_{p_{i}}$ are such that $F\left(x_{p_{i}}\right)=p_{i}$. For example, if $p_{i}=0.5$, then $x_{0.5}$ is the median. When we plot data, the probabilities, $p_{i}$, are often referred to as plotting positions. There are many different conventions for choice of plotting positions, given $x_{(1)} \leq \cdots \leq x_{(N)}$. Most belong to the family $(i-a) /(N-2 a+1) . a=0.5$ (suggested by Hazen) and $a=0$ (suggested by Weibull) are popular choices.

For a wider discussion of the calculation of plotting positions, see Cox (2002).
symplot plots median $-x_{(i)}$ versus $x_{(N+1-i)}$ - median.
quantile plots $x_{(i)}$ versus $(i-0.5) / N$ (the Hazen position).
qnorm plots $x_{(i)}$ against $q_{i}$, where $q_{i}=\Phi^{-1}\left(p_{i}\right), \Phi$ is the cumulative normal distribution, and $p_{i}=i /(N+1)$ (the Weibull position).
pnorm plots $\Phi\left\{\left(x_{i}-\widehat{\mu}\right) / \widehat{\sigma}\right\}$ versus $p_{i}=i /(N+1)$, where $\widehat{\mu}$ is the mean of the data and $\widehat{\sigma}$ is the standard deviation.
qchi and pchi are similar to qnorm and pnorm; the cumulative $\chi^{2}$ distribution is used in place of the cumulative normal distribution.
qqplot is just a two-way scatterplot of one variable against the other after both variables have been sorted into ascending order, and both variables have the same number of nonmissing observations. If the variables have unequal numbers of nonmissing observations, interpolated values of the variable with more data are plotted against the variable with fewer data.

Ramanathan Gnanadesikan (1932- ) was born in Madras. He obtained degrees from the Universities of Madras and North Carolina. He worked in industry at Procter \& Gamble, Bell Labs, and Bellcore, as well as in universities, retiring from Rutgers in 1998. Among many contributions to statistics he is especially well known for work on probability plotting, robustness, outlier detection, clustering, classification, and pattern recognition.
Martin Bradbury Wilk (1922-2013) was born in Montreal. He obtained degrees in chemical engineering and statistics from McGill and Iowa State Universities. After holding several statisticsrelated posts in industry and at universities (including periods at Princeton, Bell Labs, and Rutgers), Wilk was appointed Chief Statistician at Statistics Canada (1980-1986). He is especially well known for his work with Gnanadesikan on probability plotting and with Shapiro on tests for normality.

## Acknowledgments

We thank Peter A. Lachenbruch of the Department of Public Health at Oregon State University for writing the original versions of qchi and pchi. Patrick Royston of the MRC Clinical Trials Unit, London, and coauthor of the Stata Press book Flexible Parametric Survival Analysis Using Stata: Beyond the Cox Model also published a similar command in the Stata Technical Bulletin (Royston 1996).

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## Also see

[ R ] cumul - Cumulative distribution
[ R$]$ kdensity - Univariate kernel density estimation
[R] logistic postestimation - Postestimation tools for logistic
[R] lv - Letter-value displays
$[R]$ regress postestimation diagnostic plots - Postestimation plots for regress

## Title

display - Substitute for a hand calculator

## Syntax Description Remarks and examples Also see

## Syntax

```
display exp
```


## Description

display displays strings and values of scalar expressions.
display really has many more features and a more complex syntax diagram, but the diagram shown above is adequate for interactive use. For a full discussion of display's capabilities, see [P] display.

## Remarks and examples

display can be used as a substitute for a hand calculator.

## Example 1

display $2+2$ produces the output 4. Stata variables may also appear in the expression, such as in display myvar/2. Because display works only with scalars, the resulting calculation is performed only for the first observation. You could type display myvar[10]/2 to display the calculation for the 10th observation. Here are more examples:

```
. display sqrt(2)/2
.70710678
. display normal(-1.1)
.13566606
. di (57.2-3)/(12-2)
5.42
. display myvar/10
7
. display myvar[10]/2
3.5
```


## Also see

[ P ] display - Display strings and values of scalar expressions
[U] 13 Functions and expressions

## Title

do - Execute commands from a file

| Syntax | Menu | Description | Option |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Reference | Also see |  |

## Syntax

$\{\mathrm{do} \mid \underline{\mathrm{run}}\}$ filename $[$ arguments $]$ [, nostop]

## Menu

File > Do...

## Description

do and run cause Stata to execute the commands stored in filename just as if they were entered from the keyboard. do echoes the commands as it executes them, whereas run is silent. If filename is specified without an extension, .do is assumed.

## Option

nostop allows the do-file to continue executing even if an error occurs. Normally, Stata stops executing the do-file when it detects an error (nonzero return code).

## Remarks and examples

You can create filename (called a do-file) using Stata's Do-file Editor; see [R] doedit. This file will be a standard ASCII (text) file. A complete discussion of do-files can be found in [U] 16 Do-files.

You can also create filename by using a non-Stata text editor; see [D] shell for a way to invoke your favorite editor from inside Stata. Make sure that you save the file in ASCII format.

If the path or filename contains spaces, it should be enclosed in double quotes.

## Reference

Jenkins, S. P. 2006. Stata tip 32: Do not stop. Stata Journal 6: 281.

[^14]
## Title

doedit - Edit do-files and other text files
Syntax Menu Description Remarks and examples Also see

## Syntax

doedit [filename]

## Menu

Window $>$ Do-file Editor

## Description

doedit opens a text editor that lets you edit do-files and other text files.
The Do-file Editor lets you submit several commands to Stata at once.

## Remarks and examples

Clicking on the Do-file Editor button is equivalent to typing doedit.
doedit, typed by itself, invokes the Editor with an empty document. If you specify filename, that file is displayed in the Editor.

You may have more than one Do-file Editor open at once. Each time you submit the doedit command, a new window will be opened.

A tutorial discussion of doedit can be found in the Getting Started with Stata manual. Read [U] 16 Do-files for an explanation of do-files, and then read [GSW] 13 Using the Do-file Editorautomating Stata to learn how to use the Do-file Editor to create and execute do-files.

## Also see

[GSM] 13 Using the Do-file Editor-automating Stata
[GSU] 13 Using the Do-file Editor-automating Stata
[GSW] 13 Using the Do-file Editor-automating Stata
[U] 16 Do-files

## Title

dotplot - Comparative scatterplots

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Acknowledgments | References |

## Syntax

Dotplot of varname, with one column per value of groupvar

```
dotplot varname [if] [in] [, options]
```

Dotplot for each variable in varlist, with one column per variable dotplot varlist [if] [in] [, options]
options
Options
over(groupvar)
$n x$ (\#)
ny (\#)
incr (\#)
mean $\mid \underline{\text { median }}$
bounded
bar
nogroup
center
Plot
marker_options
marker_label_options
Y axis, X axis, Titles, Legend, Overall
twoway_options

Description
display one columnar dotplot for each value of groupvar horizontal dot density; default is $\mathrm{nx}(0)$ vertical dot density; default is ny (35) label every \# group; default is incr (1) plot a horizontal line of pluses at the mean or median use minimum and maximum as boundaries
plot horizontal dashed lines at shoulders of each group use the actual values of yvar center the dot for each column
change look of markers (color, size, etc.)
add marker labels; change look or position
any options other than by () documented in [G-3] twoway_options

## Menu

Graphics $>$ Distributional graphs $>$ Distribution dotplot

## Description

A dotplot is a scatterplot with values grouped together vertically ("binning", as in a histogram) and with plotted points separated horizontally. The aim is to display all the data for several variables or groups in one compact graphic.

In the first syntax, dotplot produces a columnar dotplot of varname, with one column per value of groupvar. In the second syntax, dotplot produces a columnar dotplot for each variable in varlist, with one column per variable; over (groupvar) is not allowed. In each case, the "dots" are plotted as small circles to increase readability.

## Options

Options
over (groupvar) identifies the variable for which dotplot will display one columnar dotplot for each value of groupvar.
nx (\#) sets the horizontal dot density. A larger value of \# will increase the dot density, reducing the horizontal separation between dots. This option will increase the separation between columns if two or more groups or variables are used.
ny (\#) sets the vertical dot density (number of "bins" on the $y$ axis). A larger value of \# will result in more bins and a plot that is less spread out horizontally. \# should be determined in conjunction with nx() to give the most pleasing appearance.
incr(\#) specifies how the $x$ axis is to be labeled. incr (1), the default, labels all groups. incr (2) labels every second group.
[mean $\mid$ median] plots a horizontal line of pluses at the mean or median of each group.
bounded forces the minimum and maximum of the variable to be used as boundaries of the smallest and largest bins. It should be used with one variable whose support is not the whole of the real line and whose density does not tend to zero at the ends of its support, for example, a uniform random variable or an exponential random variable.
bar plots horizontal dashed lines at the "shoulders" of each group. The shoulders are taken to be the upper and lower quartiles unless mean has been specified; here they will be the mean plus or minus the standard deviation.
nogroup uses the actual values of yvar rather than grouping them (the default). This option may be useful if yvar takes on only a few values.
center centers the dots for each column on a hidden vertical line.
marker_options affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] marker_options.
marker_label_options specify if and how the markers are to be labeled; see [G-3] marker_label_options.

Y axis, X axis, Titles, Legend, Overall
twoway_options are any of the options documented in [G-3] twoway_options, excluding by (). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

## Remarks and examples

dotplot produces a figure that has elements of a boxplot, a histogram, and a scatterplot. Like a boxplot, it is most useful for comparing the distributions of several variables or the distribution of 1 variable in several groups. Like a histogram, the figure provides a crude estimate of the density, and, as with a scatterplot, each symbol (dot) represents 1 observation.
> Example 1
dotplot may be used as an alternative to Stata's histogram graph for displaying the distribution of one variable.
. set seed 123456789
. set obs 1000
. generate norm = rnormal()
. dotplot norm, title("Normal distribution, sample size 1000")


## > Example 2

The over () option lets us use dotplot to compare the distribution of one variable within different levels of a grouping variable. The center, median, and bar options create a graph that may be compared with Stata's boxplot; see [G-2] graph box. The next graph illustrates this option with Stata's automobile dataset.

```
. use http://www.stata-press.com/data/r13/auto, clear
(1978 Automobile Data)
. dotplot mpg, over(foreign) nx(25) ny(10) center median bar
```



## Example 3

The second version of dotplot lets us compare the distribution of several variables. In the next graph, all 10 variables contain measurements on tumor volume.
. use http://www.stata-press.com/data/r13/dotgr
. dotplot g1r1-g1r10, ytitle("Tumor volume, cu mm")


## > Example 4

When using the first form with the over () option, we can encode a third dimension in a dotplot by using a different plotting symbol for different groups. The third dimension cannot be encoded with a varlist. The example is of a hypothetical matched case-control study. The next graph shows the exposure of each individual in each matched stratum. Cases are marked by the letter ' x ', and controls are marked by the letter ' $o$ '.

```
. use http://www.stata-press.com/data/r13/dotdose
. label define symbol 0 "o" 1 "x"
. label values case symbol
. dotplot dose, over(strata) m(none) mlabel(case) mlabp(0) center
```



## $>$ Example 5

dotplot can also be used with two virtually continuous variables as an alternative to jittering the data to distinguish ties. We must use the xlabel() option, because otherwise dotplot will attempt to label too many points on the $x$ axis. It is often useful in such instances to use a value of nx that is smaller than the default. That was not necessary in this example, partly because of our choice of symbols.

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. generate byte hi_price = (price>10000) if price < .
. label define symbol 0 "|" 1 "o"
. label values hi_price symbol
```

```
. dotplot weight, over(gear_ratio) m(none) mlabel(hi_price) mlabp(0) center
> xlabel(#5)
```



## > Example 6

The following figure is included mostly for aesthetic reasons. It also demonstrates dotplot's ability to cope with even very large datasets. The sample size for each variable is 10,000 , so it may take a long time to print.
. clear all
. set seed 123456789
. set obs 10000
. gen norm0 = rnormal()

- gen norm1 $=$ rnormal ()$+1$
. gen norm2 $=$ rnormal ()$+2$
. label variable norm0 "N(0,1)"
. label variable norm1 "N(1,1)"
. label variable norm2 "N(2,1)"
. dotplot norm0 norm1 norm2



## Stored results

dotplot stores the following in $r()$ :
Scalars
$r(n x) \quad$ horizontal dot density
$r$ (ny) vertical dot density

## Acknowledgments

dotplot was written by Peter Sasieni of the Wolfson Institute of Preventive Medicine, London, and Patrick Royston of the MrC Clinical Trials Unit, London, and coauthor of the Stata Press book Flexible Parametric Survival Analysis Using Stata: Beyond the Cox Model.

## References

Sasieni, P. D., and P. Royston. 1994. gr14: dotplot: Comparative scatterplots. Stata Technical Bulletin 19: 8-10. Reprinted in Stata Technical Bulletin Reprints, vol. 4, pp. 50-54. College Station, TX: Stata Press.
-_. 1996. Dotplots. Applied Statistics 45: 219-234.

## Title

dstdize - Direct and indirect standardization

| Syntax | Menu | Description | Options for dstdize |
| :--- | :--- | :--- | :--- |
| Options for istdize | Remarks and examples | Stored results | Methods and formulas |
| Acknowledgments | References | Also see |  |

## Syntax

```
Direct standardization
dstdize charvar popvar stratavars [if] [in], by(groupvars) [dstdize_options]
```

Indirect standardization
istdize casevars popvar stratavars [if] [in] using filename,
$\left\{\right.$ popvars $^{\left.\left(\text {casevar }_{p} \text { popvar }_{p}\right) \mid \operatorname{rate}\left(\text { ratevar }_{p}\left\{\# \mid \text { crudevar }_{p}\right\}\right)\right\}}$
[istdize_options]
dstdize_options
Description

## Main

* by (groupvars)
using (filename)
base (\#| string)
level(\#)
Options
saving (filename)
format ( $\%$ fmt)
print
nores
*by (groupvars) is required.
istdize_options
Description
Main
* popvars (casevar popvar $_{p}$ ) for standard population, casevar $_{p}$ is number of cases and popvar $_{p}$ is number of individuals
*rate (ratevar $\quad\left\{\# \mid\right.$ crudevar $\left._{p}\right\}$ ) ratevar $_{p}$ is stratum-specific rates and \# or crudevar ${ }_{p}$ is the crude case rate value or variable
level(\#) set confidence level; default is level (95)
Options
by (groupvars)
format ( $\%$ fmt)
print
*Either popvars $\left(\right.$ casevar $_{p}$ popvar $\left._{p}\right)$ or rate $\left(\right.$ ratevar $_{p}\left\{\# \mid\right.$ crudevar $\left.\left._{p}\right\}\right)$ must be specified.


## Menu

dstdize
Statistics $>$ Epidemiology and related $>$ Other $>$ Direct standardization
istdize
Statistics $>$ Epidemiology and related $>$ Other $>$ Indirect standardization

## Description

dstdize produces standardized rates for charvar, which are defined as a weighted average of the stratum-specific rates. These rates can be used to compare the characteristic charvar across different populations identified by groupvars. Weights used in the standardization are given by popvar; the strata across which the weights are to be averaged are defined by stratavars.
istdize produces indirectly standardized rates for a study population based on a standard population. This standardization method is appropriate when the stratum-specific rates for the population being studied are either unavailable or based on small samples and thus are unreliable. The standardization uses the stratum-specific rates of a standard population to calculate the expected number of cases in the study population(s), sums them, and then compares them with the actual number of cases observed. The standard population is in another Stata data file specified by using filename, and it must contain popvar and stratavars.

In addition to calculating rates, the indirect standardization command produces point estimates and exact confidence intervals of the study population's standardized mortality ratio (SMR), if death is the event of interest, or the standardized incidence ratio (SIR) for studies of incidence. Here we refer to both ratios as SMR.
casevar $_{s}$ is the variable name for the study population's number of cases (usually deaths). It must contain integers, and for each group, defined by groupvar, each subpopulation identified by stratavars must have the same values or missing.
popvar ${ }_{s}$ identifies the number of subjects represented by each observation in the study population. stratavars define the strata.

## Options for dstdize

by (groupvars) is required for the dstaize command; it specifies the variables identifying the study
populations. If base() is also specified, there must be only one variable in the by () group. If
you do not have a variable for this option, you can generate one by using something like gen
newvar=1 and then use newvar as the argument to this option.
using(filename) or base (\#| string) may be used to specify the standard population. You may not
specify both options. using (filename) supplies the name of a . dta file containing the standard
population. The standard population must contain the popvar and the stratavars. If using () is
not specified, the standard population distribution will be obtained from the data. base (\#| string)
lets you specify one of the values of groupvar -either a numeric value or a string-to be used
as the standard population. If neither base () nor using() is specified, the entire dataset is used
to determine an estimate of the standard population.
level(\#) specifies the confidence level, as a percentage, for a confidence interval of the adjusted rate. The default is level (95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals.
$\xrightarrow{\text { Options }}$
saving (filename) saves the computed standard population distribution as a Stata dataset that can be used in further analyses.
format (\%fmt) specifies the format in which to display the final summary table. The default is $\% 10.0 \mathrm{~g}$.
print includes a table summary of the standard population before displaying the study population results.
nores suppresses storing results in $r()$. This option is seldom specified. Some results are stored in matrices. If there are more groups than matsize, dstdize will report "matsize too small". Then you can either increase matsize or specify nores. The nores option does not change how results are calculated but specifies that results need not be left behind for use by other programs.

## Options for istdize

$\qquad$ Main
popvars (casevar popvar $_{p}$ ) or rate (ratevar ${ }_{p} \# \mid$ ratevar $_{p}$ crudevar $_{p}$ ) must be specified with istdize. Only one of these two options is allowed. These options are used to describe the standard population's data.
With popvars (casevar popvar $_{p}$ ), casevar $_{p}$ records the number of cases (deaths) for each stratum in the standard population, and popvar ${ }_{p}$ records the total number of individuals in each stratum (individuals at risk).
 specifies the crude case rate either by a variable name or by the crude case rate value. If a crude rate variable is used, it must be the same for all observations, although it could be missing for some.
level(\#) specifies the confidence level, as a percentage, for a confidence interval of the adjusted rate. The default is level (95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals.

## Options

by (groupvars) specifies variables identifying study populations when more than one exists in the data. If this option is not specified, the entire study population is treated as one group.
format (\% fint) specifies the format in which to display the final summary table. The default is $\% 10.0 \mathrm{~g}$.
print outputs a table summary of the standard population before displaying the study population results.

## Remarks and examples

Remarks are presented under the following headings:

## Direct standardization

Indirect standardization

In epidemiology and other fields, you will often need to compare rates for some characteristic across different populations. These populations often differ on factors associated with the characteristic under study; thus directly comparing overall rates may be misleading.

See van Belle et al. (2004, 642-684), Fleiss, Levin, and Paik (2003, chap. 19), or Kirkwood and Sterne (2003, chap. 25) for a discussion of direct and indirect standardization.

## Direct standardization

The direct method of adjusting for differences among populations involves computing the overall rates that would result if, instead of having different distributions, all populations had the same standard distribution. The standardized rate is defined as a weighted average of the stratum-specific rates, with the weights taken from the standard distribution. Direct standardization may be applied only when the specific rates for a given population are available.
dstdize generates adjusted summary measures of occurrence, which can be used to compare prevalence, incidence, or mortality rates between populations that may differ on certain characteristics (for example, age, gender, race). These underlying differences may affect the crude prevalence, mortality, or incidence rates.
> Example 1
We have data (Rothman 1986, 42) on mortality rates for Sweden and Panama for 1962, and we wish to compare mortality in these two countries:

```
. use http://www.stata-press.com/data/r13/mortality
(1962 Mortality, Sweden & Panama)
. describe
Contains data from http://www.stata-press.com/data/r13/mortality.dta
    obs: 6 1962 Mortality, Sweden & Panama
    vars: 4 14 Apr 2013 16:18
    size: 90
```

| variable name | storage <br> type | display <br> format | value <br> label | variable label |
| :--- | :--- | :--- | :--- | :--- |
| nation | str6 | $\% 9 \mathrm{~s}$ |  | Nation |
| age_category | byte | $\% 9.0 \mathrm{~g}$ <br> population | float | age_10.0gc |
| deaths | float | $\% 9.0 g c$ |  | Age Category <br> Population in Age Category |
|  |  |  | Deaths in Age Category |  |

Sorted by:
. list, sepby(nation) abbrev(12) divider

| nation | age_category | population | deaths |
| :--- | ---: | ---: | ---: |
| Sweden | $0-29$ | 3145000 | 3,523 |
| Sweden | $30-59$ | 3057000 | 10,928 |
| Sweden | $60+$ | 1294000 | 59,104 |
| Panama | $0-29$ | 741,000 | 3,904 |
| Panama | $30-59$ | 275,000 | 1,421 |
| Panama | $60+$ | 59,000 | 2,456 |

We divide the total number of cases in the population by the population to obtain the crude rate:

```
. collapse (sum) pop deaths, by(nation)
```

. list, abbrev(10) divider

|  | nation | population | deaths |
| :--- | :--- | ---: | ---: |
| 1. | Panama | 1075000 | 7,781 |
| 2. | Sweden | 7496000 | 73,555 |

- generate crude = deaths/pop
. list, abbrev(10) divider

| nation | population | deaths | crude |
| :---: | ---: | ---: | ---: |
| Panama | 1075000 | 7,781 | .0072381 |
| Sweden | 7496000 | 73,555 | .0098126 |

If we examine the total number of deaths in the two nations, the total crude mortality rate in Sweden is higher than that in Panama. From the original data, we see one possible explanation: Swedes are older than Panamanians, making direct comparison of the mortality rates difficult.

Direct standardization lets us remove the distortion caused by the different age distributions. The adjusted rate is defined as the weighted sum of the crude rates, where the weights are given by the standard distribution. Suppose that we wish to standardize these mortality rates to the following age distribution:

```
. use http://www.stata-press.com/data/r13/1962, clear
(Standard Population Distribution)
. list, abbrev(12) divider
```

| age_category | population |
| ---: | ---: |
| $0-29$ | .35 |
| $30-59$ | .35 |
| $60+$ | .3 |

. save 1962
file 1962.dta saved
If we multiply the above weights for the age strata by the crude rate for the corresponding age category, the sum gives us the standardized rate.

```
. use http://www.stata-press.com/data/r13/mortality
(1962 Mortality, Sweden & Panama)
. generate crude=deaths/pop
. drop pop
. merge m:1 age_cat using 1962
age_category was byte now float
    Result # of obs.
    not matched 0
    matched 6 (_merge==3)
. list, sepby(age_category) abbrev(12)
```

|  | nation | age_category | deaths | crude | population | _merge |
| :--- | :--- | ---: | ---: | ---: | ---: | ---: |
| 1. | Sweden | $0-29$ | 3,523 | .0011202 | .35 | matched (3) |
| 2. | Panama | $0-29$ | 3,904 | .0052686 | .35 | matched (3) |
|  | 3. | Panama | $30-59$ | 1,421 | .0051673 | .35 |
| 4. | Sweden | $30-59$ | 10,928 | .0035747 | .35 | matched (3) |
| 5. | Panama | $60+$ | 2,456 | .0416271 | .3 | matched (3) |
| 6. | Sweden | $60+$ | 59,104 | .0456754 | .3 | matched (3) |

. generate product = crude*pop
. by nation, sort: egen adj_rate = sum(product)
. drop _merge
. list, sepby(nation)

|  | nation | age_ca~y | deaths | crude | popula~n | product | adj_rate |
| :--- | :--- | ---: | ---: | ---: | ---: | ---: | ---: |
|  | 1. | Panama | $0-29$ | 3,904 | .0052686 | .35 | .001844 |
| 2. | Panama | $30-59$ | 1,421 | .0051673 | .35 | .0018085 | .0161407 |
| 3. | Panama | $60+$ | 2,456 | .0416271 | .3 | .0124881 | .0161407 |
|  |  | Sweden | $60+$ | 59,104 | .0456754 | .3 | .0137026 |
| 4. | Sweden | $30-59$ | 10,928 | .0035747 | .35 | .0012512 | .0153459 |
| 6. | Sweden | $0-29$ | 3,523 | .0011202 | .35 | .0003921 | .0153459 |

Comparing the standardized rates indicates that the Swedes have a slightly lower mortality rate.

To perform the above analysis with dstdize, type

```
. use http://www.stata-press.com/data/r13/mortality, clear
(1962 Mortality, Sweden & Panama)
. dstdize deaths pop age_cat, by(nation) using(1962)
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \multicolumn{7}{|l|}{-> nation= Panama} \\
\hline Stratum & Pop. & Cases & \begin{tabular}{l}
Pop. \\
Dist.
\end{tabular} & \begin{tabular}{l}
Stratum \\
Rate [s]
\end{tabular} & Pop. Dst [P] & s*P \\
\hline 0-29 & 741000 & 3904 & 0.689 & 0.0053 & 0.350 & 0.0018 \\
\hline 30-59 & 275000 & 1421 & 0.256 & 0.0052 & 0.350 & 0.0018 \\
\hline 60+ & 59000 & 2456 & 0.055 & 0.0416 & 0.300 & 0.0125 \\
\hline \multirow[t]{4}{*}{Totals:} & \multirow[t]{4}{*}{1075000} & \multirow[t]{3}{*}{7781} & \multicolumn{3}{|l|}{Adjusted Cases:} & 17351.2 \\
\hline & & & & Crude Ra & te: & 0.0072 \\
\hline & & & & justed Ra & te: & 0.0161 \\
\hline & & \% Conf & Inte & rval: [0. & 0156, & 0.0166] \\
\hline
\end{tabular}
```

| -> nation= Sweden |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Stratum | Pop. | Cases | Pop. <br> Dist. | Stratum <br> Rate[s] | Pop. Dst [P] | s*P |
| 0-29 | 3145000 | 3523 | 0.420 | 0.0011 | 0.350 | 0.0004 |
| 30-59 | 3057000 | 10928 | 0.408 | 0.0036 | 0.350 | 0.0013 |
| 60+ | 1294000 | 59104 | 0.173 | 0.0457 | 0.300 | 0.0137 |
| Totals: | 7496000 | 73555 | Adjusted Cases: 115032.5 |  |  |  |
|  |  |  |  | Crude Ra | te: | 0.0098 |
|  |  |  |  | justed Ra | te: | 0.0153 |
|  |  | 95\% Conf | Inter | rval: [0.01 | 0152, | $0.0155]$ |

Summary of Study Populations:

| nation | N | Crude | Adj_Rate |  | Confidence |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
|  |  | Interval |  |  |  |  |
| Panama | 1075000 | 0.007238 | 0.016141 | $\left[\begin{array}{lll}0.015645, & 0.016637]\end{array}\right.$ |  |  |
| Sweden | 7496000 | 0.009813 | 0.015346 | $\left[\begin{array}{llll}0.015235, & 0.015457]\end{array}\right.$ |  |  |

The summary table above lets us make a quick inspection of the results within the study populations, and the detail tables give the behavior among the strata within the study populations.

## Example 2

We have individual-level data on persons in four cities over several years. Included in the data is a variable indicating whether the person has high blood pressure, together with information on the person's age, sex, and race. We wish to obtain standardized high blood pressure rates for each city for 1990 and 1992, using, as the standard, the age, sex, and race distribution of the four cities and two years combined.

Our dataset contains

```
. use http://www.stata-press.com/data/r13/hbp
. describe
Contains data from http://www.stata-press.com/data/r13/hbp.dta
    obs: 1,130
    vars: 7 21 Feb 2013 06:42
    size: 19,210
\begin{tabular}{lllll}
\hline variable name & \begin{tabular}{c} 
storage \\
type
\end{tabular} & \begin{tabular}{l} 
display \\
format
\end{tabular} & \begin{tabular}{l} 
value \\
label
\end{tabular} & variable label \\
\hline id & str10 & \(\% 10 \mathrm{~s}\) & & Record identification number \\
city & byte & \(\% 8.0 \mathrm{~g}\) & & \\
year & int & \(\% 8.0 \mathrm{~g}\) & & \\
sex & byte & \(\% 8.0 \mathrm{~g}\) & sexfmt & \\
age_group & byte & \(\% 8.0 \mathrm{~g}\) & agefmt & \\
race & byte & \(\% 8.0 \mathrm{~g}\) & racefmt & \\
hbp & byte & \(\% 8.0 \mathrm{~g}\) & yn & high blood pressure \\
\hline
\end{tabular}
```

Sorted by:
The dstdize command is designed to work with aggregate data but will work with individuallevel data only if we create a variable recording the population represented by each observation. For individual-level data, this is one:

```
. generate pop = 1
```

On the next page, we specify print to obtain a listing of the standard population and level (90) to request $90 \%$ rather than $95 \%$ confidence intervals. Typing if year==1990| year==1992 restricts the data to the two years for both summary tables and the standard population.
. dstdize hbp pop age race sex if year==1990 | year==1992, by (city year) print
> level(90)

|  |  | Stratum | Pop. | Dist. |
| :---: | :---: | :---: | :---: | :---: |
| 15-19 | Black | Female | 35 | 0.077 |
| 15-19 | Black | Male | 44 | 0.097 |
| 15-19 | Hispanic | Female | 5 | 0.011 |
| 15-19 | Hispanic | Male | 10 | 0.022 |
| 15-19 | White | Female | 7 | 0.015 |
| 15-19 | White | Male | 5 | 0.011 |
| 20-24 | Black | Female | 43 | 0.095 |
| 20-24 | Black | Male | 67 | 0.147 |
| 20-24 | Hispanic | Female | 14 | 0.031 |
| 20-24 | Hispanic | Male | 13 | 0.029 |
| 20-24 | White | Female | 4 | 0.009 |
| 20-24 | White | Male | 21 | 0.046 |
| 25-29 | Black | Female | 17 | 0.037 |
| 25-29 | Black | Male | 44 | 0.097 |
| 25-29 | Hispanic | Female | 7 | 0.015 |
| 25-29 | Hispanic | Male | 13 | 0.029 |
| 25-29 | White | Female | 9 | 0.020 |
| 25-29 | White | Male | 16 | 0.035 |
| 30-34 | Black | Female | 16 | 0.035 |
| 30-34 | Black | Male | 32 | 0.070 |
| 30-34 | Hispanic | Female | 2 | 0.004 |
| 30-34 | Hispanic | Male | 3 | 0.007 |
| 30-34 | White | Female | 5 | 0.011 |
| 30-34 | White | Male | 23 | 0.051 |

Total:
455
(6 observations excluded because of missing values)


```
-> city year= 1 1992
```

|  |  | Stratum | Pop. | -_Unadjusted-_ |  |  |  | s*P |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Cases | Pop. <br> Dist. | Stratum <br> Rate[s] |  |  |
| 15-19 | Black | Female | 3 | 0 | 0.054 | 0.0000 | 0.077 | 0.0000 |
| 15-19 | Black | Male | 9 | 0 | 0.161 | 0.0000 | 0.097 | 0.0000 |
| 15-19 | Hispanic | Male | 1 | 0 | 0.018 | 0.0000 | 0.022 | 0.0000 |
| 20-24 | Black | Female | 7 | 0 | 0.125 | 0.0000 | 0.095 | 0.0000 |
| 20-24 | Black | Male | 9 | 0 | 0.161 | 0.0000 | 0.147 | 0.0000 |
| 20-24 | Hispanic | Female | 1 | 0 | 0.018 | 0.0000 | 0.031 | 0.0000 |
| 25-29 | Black | Female | 2 | 0 | 0.036 | 0.0000 | 0.037 | 0.0000 |
| 25-29 | Black | Male | 11 | 1 | 0.196 | 0.0909 | 0.097 | 0.0088 |
| 25-29 | Hispanic | Male | 1 | 0 | 0.018 | 0.0000 | 0.029 | 0.0000 |
| 30-34 | Black | Female | 7 | 0 | 0.125 | 0.0000 | 0.035 | 0.0000 |
| 30-34 | Black | Male | 4 | 0 | 0.071 | 0.0000 | 0.070 | 0.0000 |
| 30-34 | White | Female | 1 | 0 | 0.018 | 0.0000 | 0.011 | 0.0000 |
| Totals: |  |  | 56 | 1 | Adju | usted Ca | ses: | 0.5 |
|  |  |  |  |  |  | Crude R | Rate: | 0.0179 |
|  |  |  |  |  |  | justed R | ate: | 0.0088 |
|  |  |  | 90\% Con |  | Inter | val: [0 | .0000, | 0.0226] |



|  |  | Stratum | Pop. | Unadjusted-_ |  |  | Std. Pop. Dst [P] | s*P |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Cases |  | Pop. Dist. | Stratum <br> Rate[s] |  |  |
| 15-19 | Black |  | Female | 1 | 0 | 0.015 | 0.0000 | 0.077 | 0.0000 |
| 15-19 | Black | Male | 5 | 0 | 0.075 | 0.0000 | 0.097 | 0.0000 |
| 15-19 | Hispanic | Female | 3 | 0 | 0.045 | 0.0000 | 0.011 | 0.0000 |
| 15-19 | Hispanic | Male | 1 | 0 | 0.015 | 0.0000 | 0.022 | 0.0000 |
| 15-19 | White | Male | 1 | 0 | 0.015 | 0.0000 | 0.011 | 0.0000 |
| 20-24 | Black | Female | 8 | 0 | 0.119 | 0.0000 | 0.095 | 0.0000 |
| 20-24 | Black | Male | 11 | 0 | 0.164 | 0.0000 | 0.147 | 0.0000 |
| 20-24 | Hispanic | Female | 6 | 0 | 0.090 | 0.0000 | 0.031 | 0.0000 |
| 20-24 | Hispanic | Male | 4 | 2 | 0.060 | 0.5000 | 0.029 | 0.0143 |
| 20-24 | White | Female | 1 | 0 | 0.015 | 0.0000 | 0.009 | 0.0000 |
| 20-24 | White | Male | 2 | 0 | 0.030 | 0.0000 | 0.046 | 0.0000 |
| 25-29 | Black | Female | 2 | 0 | 0.030 | 0.0000 | 0.037 | 0.0000 |
| 25-29 | Black | Male | 3 | 0 | 0.045 | 0.0000 | 0.097 | 0.0000 |
| 25-29 | Hispanic | Female | 2 | 0 | 0.030 | 0.0000 | 0.015 | 0.0000 |
| 25-29 | Hispanic | Male | 4 | 0 | 0.060 | 0.0000 | 0.029 | 0.0000 |
| 25-29 | White | Female | 4 | 0 | 0.060 | 0.0000 | 0.020 | 0.0000 |
| 25-29 | White | Male | 2 | 0 | 0.030 | 0.0000 | 0.035 | 0.0000 |
| 30-34 | Black | Female | 1 | 0 | 0.015 | 0.0000 | 0.035 | 0.0000 |
| 30-34 | Black | Male | 2 | 0 | 0.030 | 0.0000 | 0.070 | 0.0000 |
| 30-34 | Hispanic | Male | 1 | 0 | 0.015 | 0.0000 | 0.007 | 0.0000 |
| 30-34 | White | Female | 2 | 0 | 0.030 | 0.0000 | 0.011 | 0.0000 |
| 30-34 | White | Male | 1 | 0 | 0.015 | 0.0000 | 0.051 | 0.0000 |
| Totals: |  |  | 67 | 2 | Adjusted Cases: |  |  | 1.0 |
|  |  |  |  |  | Crude Rate: |  |  | 0.0299 |
|  |  |  |  |  | Adjusted Rate: |  |  | 0.0143 |
|  |  |  |  | \% Con | In | val: | 025 | $0.0260]$ |

```
-> city year= 3 1990
```

——Unadjusted—— Std.
Pop. Stratum Pop.

|  |  | Stratum | Pop. | Cases | Dist. | Rate [s] | Dst [P] | s*P |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 15-19 | Black | Female | 3 | 0 | 0.043 | 0.0000 | 0.077 | 0.0000 |
| 15-19 | Black | Male | 1 | 0 | 0.014 | 0.0000 | 0.097 | 0.0000 |
| 15-19 | Hispanic | Female | 1 | 0 | 0.014 | 0.0000 | 0.011 | 0.0000 |
| 15-19 | White | Female | 3 | 0 | 0.043 | 0.0000 | 0.015 | 0.0000 |
| 15-19 | White | Male | 1 | 0 | 0.014 | 0.0000 | 0.011 | 0.0000 |
| 20-24 | Black | Female | 1 | 0 | 0.014 | 0.0000 | 0.095 | 0.0000 |
| 20-24 | Black | Male | 9 | 0 | 0.130 | 0.0000 | 0.147 | 0.0000 |
| 20-24 | Hispanic | Male | 3 | 0 | 0.043 | 0.0000 | 0.029 | 0.0000 |
| 20-24 | White | Female | 2 | 0 | 0.029 | 0.0000 | 0.009 | 0.0000 |
| 20-24 | White | Male | 8 | 1 | 0.116 | 0.1250 | 0.046 | 0.0058 |
| 25-29 | Black | Female | 1 | 0 | 0.014 | 0.0000 | 0.037 | 0.0000 |
| 25-29 | Black | Male | 8 | 3 | 0.116 | 0.3750 | 0.097 | 0.0363 |
| 25-29 | Hispanic | Male | 4 | 0 | 0.058 | 0.0000 | 0.029 | 0.0000 |
| 25-29 | White | Female | 1 | 0 | 0.014 | 0.0000 | 0.020 | 0.0000 |
| 25-29 | White | Male | 6 | 0 | 0.087 | 0.0000 | 0.035 | 0.0000 |
| 30-34 | Black | Male | 6 | 2 | 0.087 | 0.3333 | 0.070 | 0.0234 |
| 30-34 | White | Male | 11 | 5 | 0.159 | 0.4545 | 0.051 | 0.0230 |
| Totals: |  |  | 69 | 11 | Adjusted Cases: |  |  | 6.1 |
|  |  |  |  |  | Crude Rate: |  |  | 0.1594 |
|  |  |  |  |  | Adjusted Rate: |  |  | 0.0885 |
|  |  |  |  | \% Conf | Int | val: [0 | .0501, | 0.1268] |


| ty year= 31992 |  | Stratum | Pop. | Unadjusted-_ |  |  | Std. Pop. Dst [P] | s*P |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Cases |  | Pop. Dist. | Stratum <br> Rate[s] |  |  |
| 15-19 | Black |  | Female | 2 | 0 | 0.054 | 0.0000 | 0.077 | 0.0000 |
| 15-19 | Hispanic | Male | 3 | 0 | 0.081 | 0.0000 | 0.022 | 0.0000 |
| 15-19 | White | Female | 2 | 0 | 0.054 | 0.0000 | 0.015 | 0.0000 |
| 15-19 | White | Male | 1 | 0 | 0.027 | 0.0000 | 0.011 | 0.0000 |
| 20-24 | Black | Male | 3 | 0 | 0.081 | 0.0000 | 0.147 | 0.0000 |
| 20-24 | Hispanic | Female | 1 | 0 | 0.027 | 0.0000 | 0.031 | 0.0000 |
| 20-24 | Hispanic | Male | 3 | 0 | 0.081 | 0.0000 | 0.029 | 0.0000 |
| 20-24 | White | Female | 1 | 0 | 0.027 | 0.0000 | 0.009 | 0.0000 |
| 20-24 | White | Male | 6 | 1 | 0.162 | 0.1667 | 0.046 | 0.0077 |
| 25-29 | Hispanic | Male | 1 | 0 | 0.027 | 0.0000 | 0.029 | 0.0000 |
| 25-29 | White | Male | 5 | 1 | 0.135 | 0.2000 | 0.035 | 0.0070 |
| 30-34 | Black | Male | 1 | 0 | 0.027 | 0.0000 | 0.070 | 0.0000 |
| 30-34 | White | Male | 8 | 5 | 0.216 | 0.6250 | 0.051 | 0.0316 |
| Totals: |  |  | 37 | 7 | Adju | usted Cas | es: | 1.7 |
|  |  |  |  |  |  | Crude R |  | 0.1892 |
|  |  |  |  |  |  | justed R | 位: | 0.0463 |
|  |  |  |  | \% Con | Inte | val: [0. | 0253, | $0.0674]$ |


|  |  | Stratum | Pop. | Unadjusted |  |  | Std. Pop. Dst [P] | s*P |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Cases |  | Pop. Dist. | Stratum <br> Rate [s] |  |  |
| 15-19 | Black |  | Female | 9 | 0 | 0.196 | 0.0000 | 0.077 | 0.0000 |
| 15-19 | Black | Male | 7 | 0 | 0.152 | 0.0000 | 0.097 | 0.0000 |
| 15-19 | Hispanic | Male | 1 | 0 | 0.022 | 0.0000 | 0.022 | 0.0000 |
| 15-19 | White | Male | 1 | 0 | 0.022 | 0.0000 | 0.011 | 0.0000 |
| 20-24 | Black | Female | 4 | 0 | 0.087 | 0.0000 | 0.095 | 0.0000 |
| 20-24 | Black | Male | 6 | 0 | 0.130 | 0.0000 | 0.147 | 0.0000 |
| 20-24 | Hispanic | Female | 1 | 0 | 0.022 | 0.0000 | 0.031 | 0.0000 |
| 25-29 | Black | Female | 3 | 1 | 0.065 | 0.3333 | 0.037 | 0.0125 |
| 25-29 | Black | Male | 5 | 0 | 0.109 | 0.0000 | 0.097 | 0.0000 |
| 25-29 | Hispanic | Female | 1 | 0 | 0.022 | 0.0000 | 0.015 | 0.0000 |
| 25-29 | White | Female | 2 | 1 | 0.043 | 0.5000 | 0.020 | 0.0099 |
| 30-34 | Black | Female | 2 | 0 | 0.043 | 0.0000 | 0.035 | 0.0000 |
| 30-34 | Black | Male | 3 | 0 | 0.065 | 0.0000 | 0.070 | 0.0000 |
| 30-34 | White | Male | 1 | 0 | 0.022 | 0.0000 | 0.051 | 0.0000 |
| Totals: |  |  | 46 | 2 | Adj | usted Cas | es: | 1.0 |
|  |  |  |  |  |  | Crude R | ate: | 0.0435 |
|  |  |  |  |  |  | justed Ra | te: | 0.0223 |
|  |  |  |  | \% Conf | Inte | val: [0 | 0020, | $0.0426]$ |



Summary of Study Populations: city
year N Crude Adj_Rate Confidence Interval

| 1 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1990 | 47 | 0.063830 | 0.041758 | [ | 0.007427, | $0.076089]$ |
| 1 |  |  |  |  |  |  |
| 1992 | 56 | 0.017857 | 0.008791 | [ | 0.000000, | $0.022579]$ |
| 2 |  |  |  |  |  |  |
| 1990 | 64 | 0.046875 | 0.044898 | [ | 0.009072, | $0.080724]$ |
| 2 |  |  |  |  |  |  |
| 1992 | 67 | 0.029851 | 0.014286 | [ | 0.002537, | $0.026035]$ |
| 3 |  |  |  |  |  |  |
| 1990 | 69 | 0.159420 | 0.088453 | [ | 0.050093, | $0.126813]$ |
| 3 |  |  |  |  |  |  |
| 1992 | 37 | 0.189189 | 0.046319 | [ | 0.025271, | $0.067366]$ |
| 5 |  |  |  |  |  |  |
| 1990 | 46 | 0.043478 | 0.022344 | [ | 0.002044 , | $0.042644]$ |
| 5 |  |  |  |  |  |  |
| 1992 | 69 | 0.014493 | 0.050549 | [ | 0.050549, | $0.050549]$ |

## Indirect standardization

Standardization of rates can be performed via the indirect method whenever the stratum-specific rates are either unknown or unreliable. If the stratum-specific rates are known, the direct standardization method is preferred.

To apply the indirect method, you must have the following information:

- The observed number of cases in each population to be standardized, $O$. For example, if death rates in two states are being standardized using the U.S. death rate for the same period, you must know the total number of deaths in each state.
- The distribution across the various strata for the population being studied, $n_{1}, \ldots, n_{k}$. If you are standardizing the death rate in the two states, adjusting for age, you must know the number of individuals in each of the $k$ age groups.
- The stratum-specific rates for the standard population, $p_{1}, \ldots, p_{k}$. For example, you must have the U.S. death rate for each stratum (age group).
- The crude rate of the standard population, $C$. For example, you must have the U.S. mortality rate for the year.

The indirect adjusted rate is then

$$
R_{\text {indirect }}=C \frac{O}{E}
$$

where $E$ is the expected number of cases (deaths) in each population. See Methods and formulas for a more detailed description of calculations.

## - Example 3

This example is borrowed from Kahn and Sempos (1989, 95-105). We want to compare 1970 mortality rates in California and Maine, adjusting for age. Although we have age-specific population counts for the two states, we lack age-specific death rates. Direct standardization is not feasible here. We can use the U.S. population census data for the same year to produce indirectly standardized rates for these two states.

From the U.S. census, the standard population for this example was entered into Stata and saved in popkahn.dta.

```
. use http://www.stata-press.com/data/r13/popkahn, clear
. list age pop deaths rate, sep(4)
```

| age | population | deaths | rate |
| ---: | ---: | ---: | ---: |
| $<15$ | $57,900,000$ | 103,062 | .00178 |
| $15-24$ | $35,441,000$ | 45,261 | .00128 |
| $25-34$ | $24,907,000$ | 39,193 | .00157 |
| $35-44$ | $23,088,000$ | 72,617 | .00315 |
| $45-54$ | $23,220,000$ | 169,517 | .0073 |
| $55-64$ | $18,590,000$ | 308,373 | .01659 |
| $65-74$ | $12,436,000$ | 445,531 | .03583 |
| $75+$ | $7,630,000$ | 736,758 | .09656 |

The standard population contains for each age stratum the total number of individuals (pop) and both the age-specific mortality rate (rate) and the number of deaths. The standard population need not contain all three. If we have only the age-specific mortality rate, we can use the rate (ratevar ${ }_{p}$ crudevar $_{p}$ ) or rate (ratevar ${ }_{p}$ \#) option, where crudevar $_{p}$ refers to the variable containing the total population's crude death rate or \# is the total population's crude death rate.

Now let's look at the states' data (study population):

```
. use http://www.stata-press.com/data/r13/kahn
. list, sep(4)
```

|  | state | age | populat~n | death | st | death_~e |
| :--- | :--- | ---: | ---: | ---: | ---: | ---: |
| 1. | California | $<15$ | $5,524,000$ | 166,285 | 1 | .0016 |
| 2. | California | $15-24$ | $3,558,000$ | 166,285 | 1 | .0013 |
| 3. | California | $25-34$ | $2,677,000$ | 166,285 | 1 | .0015 |
| 4. | California | $35-44$ | $2,359,000$ | 166,285 | 1 | .0028 |
| 5. | California | $45-54$ | $2,330,000$ | 166,285 | 1 | .0067 |
| 6. | California | $55-64$ | $1,704,000$ | 166,285 | 1 | .0154 |
| 7. | California | $65-74$ | $1,105,000$ | 166,285 | 1 | .0328 |
| 8. | California | $75+$ | 696,000 | 166,285 | 1 | .0917 |
|  |  |  |  |  |  |  |
| 9. | Maine | $<15$ | 286,000 | 11,051 | 2 | .0019 |
| 10. | Maine | $15-24$ | 168,000 | . | 2 | .0011 |
| 11. | Maine | $25-34$ | 110,000 | . | 2 | .0014 |
| 12. | Maine | $35-44$ | 109,000 | . | 2 | .0029 |
| 13. | Maine | $45-54$ | 110,000 | . | 2 | .0069 |
| 14. | Maine | $55-64$ | 94,000 | . | 2 | .0173 |
| 15. | Maine | $65-74$ | 69,000 | . | 2 | .039 |
| 16. | Maine | $75+$ | 46,000 | . | 2 | .1041 |

For each state, the number of individuals in each stratum (age group) is contained in the pop variable. The death variable is the total number of deaths observed in the state during the year. It must have the same value for all observations in the group, as for California, or it could be missing in all but one observation per group, as for Maine.

To match these two datasets, the strata variables must have the same name in both datasets and ideally the same levels. If a level is missing from either dataset, that level will not be included in the standardization.

With kahn. dta in memory, we now execute the command. We will use the print option to obtain the standard population's summary table, and because we have both the standard population's age-specific count and deaths, we will specify the popvars (casevar popvar $_{p}$ ) option. Or, we could specify the rate (rate 0.00945 ) option because we know that 0.00945 is the U.S. crude death rate for 1970.
. istdize death pop age using http://www.stata-press.com/data/r13/popkahn,
> by (state) pop(deaths pop) print

| Standard Population |  |
| :---: | :---: |
| Stratum |  |$\quad$| Rate |  |
| :---: | :---: |
| $<15$ | 0.00178 |
| $15-24$ | 0.00128 |
| $25-34$ | 0.00157 |
| $35-44$ | 0.00315 |
| $45-54$ | 0.00730 |
| $55-64$ | 0.01659 |
| $65-74$ | 0.03583 |
| $75+$ | 0.09656 |

Standard population's crude rate: 0.00945

| -> state= California |  |  |  |
| :---: | :---: | :---: | :---: |
|  | Indirect <br> Standard <br> Population <br> Rate | Standardization <br> Observed | Cases <br> Expected |
| Stratum | 0.0018 | 5524000 | 9832.72 |
| $<15$ | 0.0013 | 3558000 | 4543.85 |
| $15-24$ | 0.0016 | 2677000 | 4212.46 |
| $25-34$ | 0.0031 | 2359000 | 7419.59 |
| $35-44$ | 0.0073 | 2330000 | 17010.10 |
| $45-54$ | 0.0166 | 1704000 | 28266.14 |
| $55-64$ | 0.0358 | 1105000 | 39587.63 |
| $65-74$ | 0.0966 | 696000 | 67206.23 |


| Stratum | Indirect <br> Standard <br> Population Rate | ndardization <br> Observed <br> Population | Cases <br> Expected |  |
| :---: | :---: | :---: | :---: | :---: |
| <15 | 0.0018 | 286000 | 509.08 |  |
| 15-24 | 0.0013 | 168000 | 214.55 |  |
| 25-34 | 0.0016 | 110000 | 173.09 |  |
| 35-44 | 0.0031 | 109000 | 342.83 |  |
| 45-54 | 0.0073 | 110000 | 803.05 |  |
| 55-64 | 0.0166 | 94000 | 1559.28 |  |
| 65-74 | 0.0358 | 69000 | 2471.99 |  |
| 75+ | 0.0966 | 46000 | 4441.79 |  |
| Totals: |  | 992000 | 10515.67 |  |
|  |  |  | Observed Cases: | 11051 |
|  |  |  | SMR (Obs/Exp) : | 1.05 |
|  |  | SMR exact 95\% | f. Interval: [1.0314, | 1.0707] |
|  |  |  | Crude Rate: | 0.0111 |
|  |  |  | Adjusted Rate: | 0.0099 |
|  |  | 95\% | f. Interval: [0.0097, | 0.0101] |


| Summary of <br> state | tudy Popul <br> Cases Observed | ions (Rate <br> Crude | s) : Adj_Rate | Confidence Interval |
| :---: | :---: | :---: | :---: | :---: |
| California | 166285 | 0.008334 | 0.008824 | [0.008782, 0.008866] |
| Maine | 11051 | 0.011140 | 0.009931 | [0.009747, 0.010118] |
| Summary of Study Populations (SMR) : |  |  |  |  |
| state | Cases Observed | Cases <br> Expected | SMR | Exact <br> Confidence Interval |
| California | 166285 | 178078.73 | 0.934 | [0.929290, 0.938271] |
| Maine | 11051 | 10515.67 | 1.051 | [1.031405, 1.070688] |

## Stored results

dstdize stores the following in $r()$ :
Scalars
$r(k) \quad$ number of populations
Macros

$$
\begin{array}{ll}
r(b y) & \text { variable names specified in by }() \\
r(c \#) & \text { values of } r(b y) \text { for \#th group }
\end{array}
$$

Matrices

```
    r(se) 1\timesk vector of standard errors of adjusted rates
    r(ub) 1 <k vector of upper bounds of confidence intervals for adjusted rates
    r(lb) 1 < vector of lower bounds of confidence intervals for adjusted rates
    r(Nobs) 1 1 k vector of number of observations
    r(crude) 1 1 k vector of crude rates (*)
    r(adj) 1\timesk vector of adjusted rates (*)
```

(*) If, in a group, the number of observations is 0 , then 9 is stored for the corresponding crude and adjusted rates.
istdize stores the following in $r()$ :
Scalars $r(k) \quad$ number of populations
Macros
$r$ (by) variable names specified in by ()
$r(c \#) \quad$ values of $r(b y)$ for \#th group
Matrices
r (cases_obs) $1 \times k$ vector of number of observed cases
r (cases_exp) $1 \times k$ vector of number of expected cases
$r$ (ub_adj) $1 \times k$ vector of upper bounds of confidence intervals for adjusted rates
$r$ (lb_adj) $\quad 1 \times k$ vector of lower bounds of confidence intervals for adjusted rates
$r$ (crude) $\quad 1 \times k$ vector of crude rates
$\mathrm{r}(\mathrm{adj}) \quad 1 \times k$ vector of adjusted rates
r (smr) $\quad 1 \times k$ vector of SMRs
r (ub_smr) $\quad 1 \times k$ vector of upper bounds of confidence intervals for SMRs
r(lb_smr) $1 \times k$ vector of lower bounds of confidence intervals for SMRs

## Methods and formulas

The directly standardized rate, $S_{\mathrm{R}}$, is defined by

$$
S_{\mathrm{R}}=\frac{\sum_{i=1}^{k} w_{i} R_{i}}{\sum_{i=1}^{k} w_{i}}
$$

(Rothman 1986, 44), where $R_{i}$ is the stratum-specific rate in stratum $i$ and $w_{i}$ is the weight for stratum $i$ derived from the standard population.

If $n_{i}$ is the population of stratum $i$, the standard error, $\operatorname{se}\left(S_{\mathrm{R}}\right)$, in stratified sampling for proportions (ignoring the finite population correction) is

$$
\operatorname{se}\left(S_{\mathrm{R}}\right)=\frac{1}{\sum w_{i}} \sqrt{\sum_{i=1}^{k} \frac{w_{i}^{2} R_{i}\left(1-R_{i}\right)}{n_{i}}}
$$

(Cochran 1977, 108), from which the confidence intervals are calculated.
For indirect standardization, define $O$ as the observed number of cases in each population to be standardized; $n_{1}, \ldots, n_{k}$ as the distribution across the various strata for the population being studied; $R_{1}, \ldots, R_{k}$ as the stratum-specific rates for the standard population; and $C$ as the crude rate of the standard population. The expected number of cases (deaths), $E$, in each population is obtained by applying the standard population stratum-specific rates, $R_{1}, \ldots, R_{k}$, to the study populations:

$$
E=\sum_{i=1}^{k} n_{i} R_{i}
$$

The indirectly adjusted rate is then

$$
R_{\text {indirect }}=C \frac{O}{E}
$$

and $O / E$ is the study population's SMR if death is the event of interest or the SIR for studies of disease (or other) incidence.

The exact confidence interval is calculated for each estimated SMR by assuming a Poisson process as described in Breslow and Day (1987, 69-71). These intervals are obtained by first calculating the upper and lower bounds for the confidence interval of the Poisson-distributed observed events, $O —$ say, $L$ and $U$, respectively—and then computing $\operatorname{SMR}_{L}=L / E$ and $\operatorname{SMR}_{U}=U / E$.

## Acknowledgments

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## Also see

[ST] epitab - Tables for epidemiologists
[SVY] direct standardization - Direct standardization of means, proportions, and ratios

## Title

dydx - Calculate numeric derivatives and integrals

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | Acknowledgment |
| References | Also see |  |  |

## Syntax

Derivatives of numeric functions
dydx yvar xvar [if] [in], generate(newvar) [dydx_options]

Integrals of numeric functions
integ yvar xvar [if] [in] [, integ_options]
dydx_options
Description
Main

* generate (newvar) create variable named newvar
replace overwrite the existing variable
*generate (newvar) is required.
integ_options
Main
generate (newvar) create variable named newvar
trapezoid use trapezoidal rule to compute integrals; default is cubic splines
initial(\#)
replace

Description
initial value of integral; default is initial ( 0 )
overwrite the existing variable
by is allowed with dydx and integ; see [D] by.

## Menu

## dydx

Data $>$ Create or change data $>$ Other variable-creation commands $>$ Calculate numerical derivatives

## integ

Data $>$ Create or change data $>$ Other variable-creation commands $>$ Calculate numeric integrals

## Description

dydx and integ calculate derivatives and integrals of numeric "functions".

## Options

$\qquad$ Main
generate(newvar) specifies the name of the new variable to be created. It must be specified with dydx.
trapezoid requests that the trapezoidal rule [the sum of $\left(x_{i}-x_{i-1}\right)\left(y_{i}+y_{i-1}\right) / 2$ ] be used to compute integrals. The default is cubic splines, which give superior results for most smooth functions; for irregular functions, trapezoid may give better results.
initial(\#) specifies the initial condition for calculating definite integrals; see Methods and formulas below. The default is initial(0).
replace specifies that if an existing variable is specified for generate(), it should be overwritten.

## Remarks and examples

dydx and integ lets you extend Stata's graphics capabilities beyond data analysis and into mathematics. (See Gould [1993] for another command that draws functions.)

## Example 1

We graph $y=e^{-x / 6} \sin (x)$ over the interval $[0,12.56]$ :

```
. range x 0 12.56 100
obs was 0, now 100
. generate y = exp(-x/6)*sin(x)
. label variable y "exp(-x/6)*sin(x)"
. twoway connected y x, connect(i) yline(0)
```



We estimate the derivative by using $d y d x$ and compute the relative difference between this estimate and the true derivative.

```
. dydx y x, gen(dy)
. generate dytrue = exp(-x/6)*(cos(x) - sin(x)/6)
. generate error = abs(dy - dytrue)/dytrue
```

The error is greatest at the endpoints, as we would expect. The error is approximately $0.5 \%$ at each endpoint, but the error quickly falls to less than $0.01 \%$.

```
. label variable error "Error in derivative estimate"
```

. twoway line error $x$, ylabel(0(.002).006)


We now estimate the integral by using integ:

```
. integ y x, gen(iy)
number of points = 100
integral = . }8531639
. generate iytrue = (36/37)*(1-\operatorname{exp}(-x/6)*(cos(x) + sin(x)/6))
. display iytrue[_N]
.85315901
. display abs(r(integral) - iytrue[_N])/iytrue[_N]
5.799e-06
. generate diff = iy - iytrue
```

The relative difference between the estimate [stored in r(integral)] and the true value of the integral is about $6 \times 10^{-6}$. A graph of the absolute difference (diff) is shown below. Here error is cumulative. Again most of the error is due to a relatively poorer fit near the endpoints.

```
. label variable diff "Error in integral estimate"
. twoway line diff x, ylabel(0(5.00e-06).00001)
```



## Stored results

dydx stores the following in $r()$ :
Macros

$$
\mathrm{r}(\mathrm{y}) \quad \text { name of } y \mathrm{var}
$$

integ stores the following in $r()$ :
Scalars
r (N_points) number of unique $x$ points
$r$ (integral) estimate of the integral

## Methods and formulas

Consider a set of data points, $\left(x_{1}, y_{1}\right), \ldots,\left(x_{n}, y_{n}\right)$, generated by a function $y=f(x)$. dydx and integ first fit these points with a cubic spline, which is then analytically differentiated (integrated) to give an approximation for the derivative (integral) of $f$.

The cubic spline (see, for example, Press et al. [2007]) consists of $n-1$ cubic polynomials $P_{i}(x)$, with the $i$ th one defined on the interval $\left[x_{i}, x_{i+1}\right]$,

$$
P_{i}(x)=y_{i} a_{i}(x)+y_{i+1} b_{i}(x)+y_{i}^{\prime \prime} c_{i}(x)+y_{i+1}^{\prime \prime} d_{i}(x)
$$

where

$$
\begin{aligned}
a_{i}(x) & =\frac{x_{i+1}-x}{x_{i+1}-x_{i}} & b_{i}(x) & =\frac{x-x_{i}}{x_{i+1}-x_{i}} \\
c_{i}(x) & =\frac{1}{6}\left(x_{i+1}-x_{i}\right)^{2} a_{i}(x)\left[\left\{a_{i}(x)\right\}^{2}-1\right] & d_{i}(x) & =\frac{1}{6}\left(x_{i+1}-x_{i}\right)^{2} b_{i}(x)\left[\left\{b_{i}(x)\right\}^{2}-1\right]
\end{aligned}
$$

and $y_{i}^{\prime \prime}$ and $y_{i+1}^{\prime \prime}$ are constants whose values will be determined as described below. The notation for these constants is justified because $P_{i}^{\prime \prime}\left(x_{i}\right)=y_{i}^{\prime \prime}$ and $P_{i}^{\prime \prime}\left(x_{i+1}\right)=y_{i+1}^{\prime \prime}$.

Because $a_{i}\left(x_{i}\right)=1, a_{i}\left(x_{i+1}\right)=0, b_{i}\left(x_{i}\right)=0$, and $b_{i}\left(x_{i+1}\right)=1$. Therefore, $P_{i}\left(x_{i}\right)=y_{i}$, and $P_{i}\left(x_{i+1}\right)=y_{i+1}$. Thus the $P_{i}$ jointly define a function that is continuous at the interval boundaries. The first derivative should be continuous at the interval boundaries; that is,

$$
P_{i}^{\prime}\left(x_{i+1}\right)=P_{i+1}^{\prime}\left(x_{i+1}\right)
$$

The above $n-2$ equations (one equation for each point except the two endpoints) and the values of the first derivative at the endpoints, $P_{1}^{\prime}\left(x_{1}\right)$ and $P_{n-1}^{\prime}\left(x_{n}\right)$, determine the $n$ constants $y_{i}^{\prime \prime}$.

The value of the first derivative at an endpoint is set to the value of the derivative obtained by fitting a quadratic to the endpoint and the two adjacent points; namely, we use

$$
P_{1}^{\prime}\left(x_{1}\right)=\frac{y_{1}-y_{2}}{x_{1}-x_{2}}+\frac{y_{1}-y_{3}}{x_{1}-x_{3}}-\frac{y_{2}-y_{3}}{x_{2}-x_{3}}
$$

and a similar formula for the upper endpoint.
dydx approximates $f^{\prime}\left(x_{i}\right)$ by using $P_{i}^{\prime}\left(x_{i}\right)$.
integ approximates $F\left(x_{i}\right)=F\left(x_{1}\right)+\int_{x_{1}}^{x_{i}} f(x) d x$ by using

$$
I_{0}+\sum_{k=1}^{i-1} \int_{x_{k}}^{x_{k+1}} P_{k}(x) d x
$$

where $I_{0}$ (an estimate of $F\left(x_{1}\right)$ ) is the value specified by the initial (\#) option. If the trapezoid option is specified, integ approximates the integral by using the trapezoidal rule:

$$
I_{0}+\sum_{k=1}^{i-1} \frac{1}{2}\left(x_{k+1}-x_{k}\right)\left(y_{k+1}+y_{k}\right)
$$

If there are ties among the $x_{i}$, the mean of $y_{i}$ is computed at each set of ties and the cubic spline is fit to these values.

## Acknowledgment

The present versions of dydx and integ were inspired by the dydx2 command written by Patrick Royston of the MrC Clinical Trials Unit, London, and coauthor of the Stata Press book Flexible Parametric Survival Analysis Using Stata: Beyond the Cox Model.

## References

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## Also see

[D] obs - Increase the number of observations in a dataset
[D] range - Generate numerical range

## Title

eform_option - Displaying exponentiated coefficients
Description Remarks and examples Reference Also see

## Description

An eform_option causes the coefficient table to be displayed in exponentiated form: for each coefficient, $e^{b}$ rather than $b$ is displayed. Standard errors and confidence intervals (CIs) are also transformed.

An eform_option is one of the following:

| eform_option | Description |
| :--- | :--- |
| eform(string) | use string for the column title |
| eform | exponentiated coefficient, string is exp (b) |
| hr | hazard ratio, string is Haz. Ratio |
| shr | subhazard ratio, string is SHR |
| $\underline{\mathrm{irr}}$ | incidence-rate ratio, string is IRR <br> or |
| $\underline{\mathrm{rrr}}$ | odds ratio, string is Odds Ratio <br> relative-risk ratio, string is RRR |

## Remarks and examples

## > Example 1

Here is a simple example of the or option with svy: logit. The CI for the odds ratio is computed by transforming (by exponentiating) the endpoints of the CI for the corresponding coefficient.

```
. use http://www.stata-press.com/data/r13/nhanes2d
. svy, or: logit highbp female black
(running logit on estimation sample)
    (output omitted)
```

|  | Linearized |  |  |  |  |  |
| ---: | ---: | :---: | :---: | :---: | :---: | ---: |
| highbp | Odds Ratio | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| female | .6107011 | .0326159 | -9.23 | 0.000 | .5476753 | .6809798 |
| black | 1.384865 | .1336054 | 3.37 | 0.002 | 1.137507 | 1.686011 |
| _cons | .7249332 | .0551062 | -4.23 | 0.000 | .6208222 | .8465035 |

We also could have specified the following command and received the same results as above:

## Reference

Buis, M. L. 2012. Stata tip 107: The baseline is now reported. Stata Journal 12: 165-166.

## Also see

[R] ml - Maximum likelihood estimation

## Title

eivreg - Errors-in-variables regression

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

eivreg depvar [indepvars] [if][in] [weight] [, options] options

Description
Model
reliab (indepvar \# [indepvar \# [...] ])
specify measurement reliability for each indepvar measured with error
Reporting
level(\#)
display_options
coeflegend
set confidence level; default is level (95)
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
display legend instead of statistics
indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
bootstrap, by, jackknife, rolling, and statsby are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the bootstrap prefix; see [R] bootstrap.
aweights are not allowed with the jackknife prefix; see $[R]$ jackknife.
aweights and fweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Linear models and related $>$ Errors-in-variables regression

## Description

eivreg fits errors-in-variables regression models.

## Options

## Model

reliab (indepvar \# [indepvar \# [...]]) specifies the measurement reliability for each independent variable measured with error. Reliabilities are specified as pairs consisting of an independent variable name (a name that appears in indepvars) and the corresponding reliability $r, 0<r \leq 1$. Independent variables for which no reliability is specified are assumed to have reliability 1 . If the option is not specified, all variables are assumed to have reliability 1 , and the result is thus the same as that produced by regress (the ordinary least-squares results).

Reporting
level(\#); see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% fint), pformat (\% fmt), sformat (\% fint), and nolstretch; see [R] estimation options.

The following option is available with eivreg but is not shown in the dialog box: coeflegend; see [R] estimation options.

## Remarks and examples

For an introduction to errors-in-variables regression, see Draper and Smith (1998, 89-91) or Kmenta (1997, 352-357). Treiman (2009, 258-261) compares the results of errors-in-variables regression with conventional regression.

Errors-in-variables regression models are useful when one or more of the independent variables are measured with additive noise. Standard regression (as performed by regress) would underestimate the effect of the variable, and the other coefficients in the model can be biased to the extent that they are correlated with the poorly measured variable. You can adjust for the biases if you know the reliability:

$$
r=1-\frac{\text { noise variance }}{\text { total variance }}
$$

That is, given the model $\mathbf{y}=\mathbf{X} \boldsymbol{\beta}+\mathbf{u}$, for some variable $\mathbf{x}_{i}$ in $\mathbf{X}$, the $\mathbf{x}_{i}$ is observed with error, $\mathbf{x}_{i}=\mathbf{x}_{i}^{*}+\mathbf{e}$, and the noise variance is the variance of $\mathbf{e}$. The total variance is the variance of $\mathbf{x}_{i}$.

## > Example 1

Say that in our automobile data, the weight of cars was measured with error, and the reliability of our measured weight is 0.85 . The result of this would be to underestimate the effect of weight in a regression of, say, price on weight and foreign, and it would also bias the estimate of the coefficient on foreign (because being of foreign manufacture is correlated with the weight of cars). We would ignore all of this if we fit the model with regress:

| Source | SS | df | MS |  | Number of obs | $=74$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Model | 316859273 | 215 | 29637 |  | F ( ${ }^{\text {Prob }}$, F | $=35.35$ $=0.0000$ |
| Residual | 318206123 | 71448 | 76.38 |  | R -squared | $=0.4989$ |
| Total | 635065396 | $73 \quad 869$ | 25.97 |  | Adj R-squared Root MSE | $\begin{array}{lr} = & 0.4848 \\ = & 2117 \end{array}$ |
| price | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Conf. | Interval] |
| weight | 3.320737 | . 3958784 | 8.39 | 0.000 | 2.531378 | 4.110096 |
| foreign | 3637.001 | 668.583 | 5.44 | 0.000 | 2303.885 | 4970.118 |
| _cons | -4942.844 | 1345.591 | -3.67 | 0.000 | -7625.876 | -2259.812 |

With eivreg, we can account for our measurement error:
. eivreg price weight foreign, r(weight . 85)

| variable | $\begin{aligned} & \text { assumed } \\ & \text { reliability } \end{aligned}$ | Errors-in-variables regression |
| :---: | :---: | :---: |
|  |  | Number of obs $=\quad 74$ |
| weight | 0.8500 | $F(2,71)=50.37$ |
| * | 1.0000 | Prob > F $\quad=0.0000$ |
|  |  | R -squared $=0.6483$ |
|  |  | Root MSE = 1773.54 |


| price | Coef. | Std. Err. | t | P>\|t| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| weight | 4.31985 | .431431 | 10.01 | 0.000 | 3.459601 | 5.180099 |
| foreign | 4637.32 | 624.5362 | 7.43 | 0.000 | 3392.03 | 5882.609 |
| _cons | -8257.017 | 1452.086 | -5.69 | 0.000 | -11152.39 | -5361.639 |

The effect of weight is increased - as we knew it would be - and here the effect of foreign manufacture is also increased. A priori, we knew only that the estimate of foreign might be biased; we did not know the direction.

## Technical note

Swept under the rug in our example is how we would determine the reliability, $r$. We can easily see that a variable is measured with error, but we may not know the reliability because the ingredients for calculating $r$ depend on the unobserved noise.

For our example, we made up a value for $r$, and in fact we do not believe that weight is measured with error at all, so the reported eivreg results have no validity. The regress results were the statistically correct results here.

But let's say that we do suspect that weight is measured with error and that we do not know $r$. We could then experiment with various values of $r$ to describe the sensitivity of our estimates to possible error levels. We may not know $r$, but $r$ does have a simple interpretation, and we could probably produce a sensible range for $r$ by thinking about how the data were collected.

If the reliability, $r$, is less than the $R^{2}$ from a regression of the poorly measured variable on all the other variables, including the dependent variable, the information might as well not have been collected; no adjustment to the final results is possible. For our automobile data, running a regression of weight on foreign and price would result in an $R^{2}$ of 0.6743 . Thus the reliability must be at least 0.6743 here. If we specify a reliability that is too small, eivreg will inform us and refuse to fit the model:

```
. eivreg price weight foreign, r(weight .6742)
reliability r() too small
r(399);
```

Returning to our problem of how to estimate $r$, too small or not, if the measurements are summaries of scaled items, the reliability may be estimated using the alpha command; see [MV] alpha. If the score is computed from factor analysis and the data are scored using predict's default options (see [MV] factor postestimation), the square of the standard deviation of the score is an estimate of the reliability.

## - Technical note

Consider a model with more than one variable measured with error. For instance, say that our model is that price is a function of weight, foreign, and mpg and that both weight and mpg are measured with error.


| price | Coef. | Std. Err. | t | $\mathrm{P}>\mid \mathrm{tI}$ | $[95 \%$ Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| weight | 12.88302 | .6820532 | 18.89 | 0.000 | 11.52271 | 14.24333 |
| foreign | 8268.951 | 352.8719 | 23.43 | 0.000 | 7565.17 | 8972.732 |
| mpg | 999.2043 | 73.60037 | 13.58 | 0.000 | 852.413 | 1145.996 |
| _cons | -56473.19 | 3710.015 | -15.22 | 0.000 | -63872.58 | -49073.8 |

## Stored results

eivreg stores the following in e() :
Scalars
$e(N) \quad$ number of observations
$e(d f-m) \quad$ model degrees of freedom
$e\left(d f \_r\right) \quad$ residual degrees of freedom
$\mathrm{e}(\mathrm{r} 2) \quad R$-squared
$\mathrm{e}(\mathrm{F}) \quad F$ statistic
$e$ (rmse) root mean squared error
$e$ (rank) rank of e(V)
Macros
e(cmd)
e(cmdline)
e(depvar)
eivreg
command as typed
$e$ (rellist) indepvars and associated reliabilities
e(wtype) weight type
$e$ (wexp) weight expression
e(properties)
b V
e (predict) program used to implement predict
$e$ (asbalanced) factor variables fvset as asbalanced
$e$ (asobserved) factor variables fvset as asobserved
Matrices
e(b)
coefficient vector
e(Cns)
e(V)
constraints matrix
variance-covariance matrix of the estimators
Functions
e(sample) marks estimation sample

## Methods and formulas

Let the model to be fit be

$$
\begin{aligned}
\mathbf{y} & =\mathbf{X}^{*} \boldsymbol{\beta}+\mathbf{e} \\
\mathbf{X} & =\mathbf{X}^{*}+\mathbf{U}
\end{aligned}
$$

where $\mathbf{X}^{*}$ are the true values and $\mathbf{X}$ are the observed values. Let $\mathbf{W}$ be the user-specified weights. If no weights are specified, $\mathbf{W}=\mathbf{I}$. If weights are specified, let $\mathbf{v}$ be the specified weights. If fweight frequency weights are specified, then $\mathbf{W}=\operatorname{diag}(\mathbf{v})$. If aweight analytic weights are specified, then $\mathbf{W}=\operatorname{diag}\left\{\mathbf{v} /\left(\mathbf{1}^{\prime} \mathbf{v}\right)\left(\mathbf{1}^{\prime} \mathbf{1}\right)\right\}$, meaning that the weights are normalized to sum to the number of observations.

The estimates $\mathbf{b}$ of $\boldsymbol{\beta}$ are obtained as $\mathbf{A}^{-1} \mathbf{X}^{\prime} \mathbf{W} \mathbf{y}$, where $\mathbf{A}=\mathbf{X}^{\prime} \mathbf{W} \mathbf{X}-\mathbf{S} . \mathbf{S}$ is a diagonal matrix with elements $N\left(1-r_{i}\right) s_{i}^{2}$. N is the number of observations, $r_{i}$ is the user-specified reliability coefficient for the $i$ th explanatory variable or 1 if not specified, and $s_{i}^{2}$ is the (appropriately weighted) variance of the variable.

The variance-covariance matrix of the estimators is obtained as $s^{2} \mathbf{A}^{-1} \mathbf{X}^{\prime} \mathbf{W} \mathbf{X} \mathbf{A}^{-1}$, where the root mean squared error $s^{2}=\left(\mathbf{y}^{\prime} \mathbf{W} \mathbf{y}-\mathbf{b A} \mathbf{b}^{\prime}\right) /(N-p)$, where $p$ is the number of estimated parameters.

## References

Draper, N., and H. Smith. 1998. Applied Regression Analysis. 3rd ed. New York: Wiley.
Kmenta, J. 1997. Elements of Econometrics. 2nd ed. Ann Arbor: University of Michigan Press.
Treiman, D. J. 2009. Quantitative Data Analysis: Doing Social Research to Test Ideas. San Francisco: Jossey-Bass.

## Also see

[R] eivreg postestimation - Postestimation tools for eivreg
$[R]$ regress - Linear regression
[SEM] example 24 - Reliability
[U] 20 Estimation and postestimation commands

## Title

eivreg postestimation - Postestimation tools for eivreg
Description Syntax for predict Menu for predict Options for predict
Also see

## Description

The following postestimation commands are available after eivreg:

| Command | Description |
| :--- | :--- |
| contrast | contrasts and ANOVA-style joint tests of estimates |
| estat summarize | summary statistics for the estimation sample <br> variance-covariance matrix of the estimators (VCE) |
| estat vce | cataloging estimation results |
| estimates | dynamic forecasts and simulations <br> point estimates, standard errors, testing, and inference for linear combinations <br> of coefficients |
| lincom | link test for model specification <br> marginal means, predictive margins, marginal effects, and average marginal <br> effects <br> graph the results from margins (profile plots, interaction plots, etc.) |
| linktest | point estimates, standard errors, testing, and inference for nonlinear combinations |
| margins | of coefficients |

## Syntax for predict

predict [type] newvar [if] [in] [, statistic]
statistic
Description

| Main |  |
| :--- | :--- |
| xb | linear prediction; the default |
| $\underline{\text { residuals }}$ | residuals |
| stdp | standard error of the prediction |
| stdf | standard error of the forecast |
| $\underline{\operatorname{pr}(a, b)}$ | $\operatorname{Pr}\left(a<y_{j}<b\right)$ |
| $\mathrm{e}(a, b)$ | $E\left(y_{j} \mid a<y_{j}<b\right)$ |
| $\underline{\operatorname{ystar}(a, b)}$ | $E\left(y_{j}^{*}\right), y_{j}^{*}=\max \left\{a, \min \left(y_{j}, b\right)\right\}$ |

These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.
where $a$ and $b$ may be numbers or variables; $a$ missing ( $a \geq$. ) means $-\infty$, and $b$ missing ( $b \geq$.) means $+\infty$; see [U] 12.2.1 Missing values.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
xb , the default, calculates the linear prediction.
residuals calculates the residuals, that is, $y_{j}-\mathbf{x}_{j} \mathbf{b}$.
stdp calculates the standard error of the prediction, which can be thought of as the standard error of the predicted expected value or mean for the observation's covariate pattern. The standard error of the prediction is also referred to as the standard error of the fitted value.
stdf calculates the standard error of the forecast, which is the standard error of the point prediction for 1 observation and is commonly referred to as the standard error of the future or forecast value. By construction, the standard errors produced by stdf are always larger than those produced by stdp; see Methods and formulas in [R] regress postestimation.
$\operatorname{pr}(a, b)$ calculates $\operatorname{Pr}\left(a<\mathbf{x}_{j} \mathbf{b}+u_{j}<b\right)$, the probability that $y_{j} \mid \mathbf{x}_{j}$ would be observed in the interval $(a, b)$.
$a$ and $b$ may be specified as numbers or variable names; $l b$ and $u b$ are variable names;
$\operatorname{pr}(20,30)$ calculates $\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$;
$\operatorname{pr}(l b, u b)$ calculates $\operatorname{Pr}\left(l b<\mathbf{x}_{j} \mathbf{b}+u_{j}<u b\right)$; and
$\operatorname{pr}(20, u b)$ calculates $\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{j}<u b\right)$.
$a$ missing ( $a \geq$. ) means $-\infty$; $\operatorname{pr}(., 30)$ calculates $\operatorname{Pr}\left(-\infty<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$;
$\operatorname{pr}(l b, 30)$ calculates $\operatorname{Pr}\left(-\infty<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$ in observations for which $l b \geq$.
and calculates $\operatorname{Pr}\left(l b<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$ elsewhere.
$b$ missing $(b \geq$.$) means +\infty$; $\operatorname{pr}(20,$.$) calculates \operatorname{Pr}\left(+\infty>\mathbf{x}_{j} \mathbf{b}+u_{j}>20\right)$;
$\operatorname{pr}(20, u b)$ calculates $\operatorname{Pr}\left(+\infty>\mathbf{x}_{j} \mathbf{b}+u_{j}>20\right)$ in observations for which $u b \geq$.
and calculates $\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{j}<u b\right)$ elsewhere.
$\mathrm{e}(a, b)$ calculates $E\left(\mathbf{x}_{j} \mathbf{b}+u_{j} \mid a<\mathbf{x}_{j} \mathbf{b}+u_{j}<b\right)$, the expected value of $y_{j} \mid \mathbf{x}_{j}$ conditional on $y_{j} \mid \mathbf{x}_{j}$ being in the interval $(a, b)$, meaning that $y_{j} \mid \mathbf{x}_{j}$ is truncated. $a$ and $b$ are specified as they are for pr() .
ystar $(a, b)$ calculates $E\left(y_{j}^{*}\right)$, where $y_{j}^{*}=a$ if $\mathbf{x}_{j} \mathbf{b}+u_{j} \leq a, y_{j}^{*}=b$ if $\mathbf{x}_{j} \mathbf{b}+u_{j} \geq b$, and $y_{j}^{*}=\mathbf{x}_{j} \mathbf{b}+u_{j}$ otherwise, meaning that $y_{j}^{*}$ is censored. $a$ and $b$ are specified as they are for pr() .

## Also see

[R] eivreg - Errors-in-variables regression
[U] 20 Estimation and postestimation commands

## Title

error messages - Error messages and return codes

## Description Also see

## Description

Whenever Stata detects that something is wrong-that what you typed is uninterpretable, that you are trying to do something you should not be trying to do, or that you requested the impossible - Stata responds by typing a message describing the problem, together with a return code. For instance,

```
. lsit
unrecognized command: lsit
r(199);
. list myvar
variable myvar not found
r(111);
. test a=b
last estimates not found
r(301);
```

In each case, the message is probably sufficient to guide you to a solution. When we typed lsit, Stata responded with "unrecognized command". We meant to type list. When we typed list myvar, Stata responded with "variable myvar not found". There is no variable named myvar in our data. When we typed test $\mathrm{a}=\mathrm{b}$, Stata responded with "last estimates not found". test tests hypotheses about previously fit models, and we have not yet fit a model.

The numbers in parentheses in the $r(199), r(111)$, and $r(301)$ messages are called the return codes. To find out more about these messages, type search rc \#, where \# is the number returned in the parentheses.

## > Example 1

. search rc 301
[P] error messages . . . . . . . . . . . . . . . . . . . . Return code 301
last estimates not found;
You typed an estimation command such as regress without arguments or attempted to perform a test or typed predict, but there were no previous estimation results.

Programmers should see [P] error for details on programming error messages.

## Also see

[R] search - Search Stata documentation and other resources

## Title

esize - Effect size based on mean comparison

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |
| Also see |  |  |  |

## Syntax

Effect sizes for two independent samples using groups
esize twosample varname [if] [in], by (groupvar) [options]

Effect sizes for two independent samples using variables

```
esize unpaired varname }\mp@subsup{\mp@code{1}}{1}{==}\mp@subsup{\mathrm{ varname }}{2}{[}[\mathrm{ if ] [in], [options]
```

Immediate form of effect sizes for two independent samples

$$
\text { esizei } \#_{\mathrm{obs} 1} \#_{\text {mean } 1} \#_{\mathrm{sd} 1} \#_{\mathrm{obs} 2} \#_{\mathrm{mean} 2} \#_{\mathrm{sd} 2}[\text {, options }]
$$

Immediate form of effect sizes for $F$ tests after an ANOVA

```
esizei ##df1 #df2 # F [, level(#)]
```

options
Description
Main
cohensd report Cohen's $d$ (1988)
hedgesg report Hedges's $g$ (1981)
glassdelta report Glass's $\Delta$ (Smith and Glass 1977) using each group's standard deviation
pbcorr report the point-biserial correlation coefficient (Pearson 1909)
all report all estimates of effect size
unequal use unequal variances
welch use Welch's (1947) approximation
level(\#) set confidence level; default is level(95)
by is allowed with esize; see [D] by.

## Menu

esize
Statistics $>$ Summaries, tables, and tests $>$ Classical tests of hypotheses $>$ Effect size based on mean comparison

## esizei

Statistics $>$ Summaries, tables, and tests $>$ Classical tests of hypotheses $>$ Effect-size calculator

## Description

esize calculates effect sizes for comparing the difference between the means of a continuous variable for two groups. In the first form, esize calculates effect sizes for the difference between the mean of varname for two groups defined by groupvar. In the second form, esize calculates effect sizes for the difference between varname $_{1}$ and varname ${ }_{2}$, assuming unpaired data.
esizei is the immediate form of esize; see [U] 19 Immediate commands. In the first form, esizei calculates the effect size for comparing the difference between the means of two groups. In the second form, esizei calculates the effect size for an $F$ test after an ANOVA.

## Options

> Main
> by (groupvar) specifies the groupvar that defines the two groups that esize will use to estimate the effect sizes. Do not confuse the by () option with the by prefix; you can specify both.
> cohensd specifies that Cohen's $d$ (1988) be reported.
> hedgesg specifies that Hedges's $g$ (1981) be reported.
> glassdelta specifies that Glass's $\Delta$ (Smith and Glass 1977) be reported.
> pbcorr specifies that the point-biserial correlation coefficient (Pearson 1909) be reported.
> all specifies that all estimates of effect size be reported. The default is Cohen's $d$ and Hedges's $g$. unequal specifies that the data not be assumed to have equal variances.
> welch specifies that the approximate degrees of freedom for the test be obtained from Welch's formula (1947) rather than from Satterthwaite's approximation formula (1946), which is the default when unequal is specified. Specifying welch implies unequal.
> level(\#) specifies the confidence level, as a percentage, for confidence intervals. The default is level (95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals.

## Remarks and examples

Whereas $p$-values are used to assess the statistical significance of a result, measures of effect size are used to assess the practical significance of a result. Effect sizes can be broadly categorized as "measures of group differences" (the $d$ family) and "measures of association" (the $r$ family); see Ellis (2010, table 1.1). The $d$ family includes estimators such as Cohen's $d$, Hedges's $g$, and Glass's $\Delta$. The $r$ family includes estimators such as the point-biserial correlation coefficient, $\omega^{2}$ and $\eta^{2}$ (also see estat esize in [R] regress postestimation). For an introduction to the concepts and calculation of effect sizes, see Kline (2013) and Thompson (2006). For a more detailed discussion, see Kirk (1996), Ellis (2010), Cumming (2012), Grissom and Kim (2012), and Kelley and Preacher (2012).

It should be noted that there is much variation in the definitions of measures of effect size (Kline 2013). As Ellis (2010, 27) cautions, "However, beware the inconsistent terminology. What is labeled here as $g$ was labeled by Hedges and Olkin as $d$ and vice versa. For these authors writing in the early 1980s, $g$ was the mainstream effect-size index developed by Cohen and refined by Glass (hence $g$ for Glass). However, since then $g$ has become synonymous with Hedges's equation (not Glass's) and the reason it is called Hedges's $g$ and not Hedges's $h$ is because it was originally named after Glass-even though it was developed by Larry Hedges. Confused?"

To avoid confusion, esize and esizei closely follow the notation of Hedges (1981), Smithson (2001), Kline (2013), and Ellis (2010).

## $>$ Example 1: Effect size for two independent samples using by()

Suppose we are interested in question 1 from the fictitious depression.dta: "My statistical software makes me feel sad". We might have conducted a $t$ test to test the null hypothesis that there is no difference in response by sex. We could then compute various measures of effect size to describe the magnitude of the effect of sex.

```
. use http://www.stata-press.com/data/r13/depression
(Fictitious Depression Inventory data based on the Beck Depression Inventory)
. esize twosample qu1, by(sex) all
Effect size based on mean comparison
```

    Obs per group:
    Female \(=712\)
        Male \(=288\)
    | Effect Size | Estimate | [95\% Conf. |  |
| ---: | ---: | ---: | ---: | | Interval] |
| ---: |
| Cohen's d |
| Hedges's g |
| Glass's Delta 1 |
| Glass's Delta 2 |
| Point-Biserial r |

Cohen's $d$, Hedges's $g$, and both estimates of Glass's $\Delta$ indicate that the score for females is 0.05 standard deviations lower than the score for males. The point-biserial correlation coefficient indicates that there is a small, negative correlation between the scores for females and males.

## - Technical note

Glass's $\Delta$ has traditionally been estimated for experimental studies using the control group standard deviation rather than the pooled standard deviation. Kline (2013) notes that the choice of group becomes arbitrary for data arising from observational studies and recommends the reporting of Glass's $\Delta$ using each group standard deviation.

## > Example 2: Effect size for two independent samples by a third variable

If we are interested in the same effect sizes from example 1 stratified by race, we could use the by prefix with the sort option to accomplish this task.
. by race, sort: esize twosample qu1, by(sex)

```
-> race = Hispanic
Effect size based on mean comparison
```

        Obs per group:
            Female \(=88\)
            Male \(=45\)
    | Effect Size | Estimate | $[95 \%$ Conf. Interval] |  |
| ---: | ---: | :---: | ---: |
| Cohen's d | -.1042883 | -.463503 | .2553235 |
| Hedges's g | -.1036899 | -.4608434 | .2538584 |

-> race = Black
Effect size based on mean comparison
Obs per group:
Female $=259$
Male $=\quad 95$

| Effect Size | Estimate | $[95 \%$ Conf. Interval] |  |
| ---: | :---: | :---: | ---: |
| Cohen's d | -.1720681 | -.4073814 | .063489 |
| Hedges's g | -.1717012 | -.4065128 | .0633536 |

-> race = White
Effect size based on mean comparison
Obs per group:
Female $=365$
Male $=148$

| Effect Size | Estimate | [95\% Conf. Interval] |  |
| ---: | :---: | :---: | ---: |
| Cohen's d | .0479511 | -.1430932 | .2389486 |
| Hedges's g | .0478807 | -.1428831 | .2385977 |

## Example 3: Bootstrap confidence intervals for effect sizes

Simulation studies have shown that bootstrap confidence intervals may be preferable to confidence intervals based on the noncentral $t$ distribution when the variable of interest does not have a normal distribution (Kelley 2005; Algina, Keselman, and Penfield 2006). Bootstrap confidence intervals can be easily estimated for effect sizes using the bootstrap prefix.

| ```(Fictitious Depression Inventory data based on the Beck Depression Inventory) . set seed 12345 . bootstrap r(d) r(g), reps(1000) nodots nowarn: esize twosample qu1, by(sex) Bootstrap results }\begin{array}{llll}{\mathrm{ Number of obs }}&{=}&{1000}\\{}&{\mathrm{ Replications }}&{=}&{1000} command: esize twosample qu1, by(sex) _bs_1: r(d) _bs_2: r(g)``` |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Observed Coef. | Bootstrap Std. Err. | z | $\begin{gathered} \text { Normal-based } \\ \mathrm{P}>\|z\| \quad[95 \% \text { Conf. Interval] } \end{gathered}$ |  |  |
| _bs_1 | -. 0512417 | . 07169 | -0.71 | 0.475 | -. 1917515 | . 0892682 |
| _bs_2 | -. 0512032 | . 0716361 | -0.71 | 0.475 | -. 1916074 | . 0892011 |

> Example 4: Effect sizes for two independent samples using variables
Sometimes, the data of interest are stored in two separate variables. We can calculate effect sizes for the two groups by using the unpaired version of esize.

|  |  | Number of obs $=24$ |  |
| :---: | :---: | :---: | :---: |
| Effect Size | Estimate | [95\% Conf. | Interval] |
| Cohen's d | -. 5829654 | -1.394934 | . 2416105 |
| Hedges's g | -. 5628243 | -1.34674 | . 2332631 |

> Example 5: Immediate form for effect sizes for two means
Often we do not have access to raw data, but we are given summary statistics in a report or manuscript. To calculate the effect sizes from summary statistics, we can use the immediate command esizei. For example, Kline (2013) in table 4.2 shows summary statistics for a hypothetical sample where mean $_{1}=13, \operatorname{sd}_{1}=2.74$, mean $_{2}=11$, and $\operatorname{sd}_{2}=2.24$; there are 30 people in each group. We can estimate the effect sizes from these summary data using esizei:

|  |  | $\begin{aligned} \text { Obs per group: } & \\ \text { Group } 1 & = \\ \text { Group } 2 & = \end{aligned} 30$ |  |
| :---: | :---: | :---: | :---: |
| Effect Size | Estimate | [95\% Conf. | Interval] |
| Cohen's d | . 7991948 | . 2695509 | 1.322465 |
| Hedges's g | . 7888081 | . 2660477 | 1.305277 |

> Example 6: Immediate form for effect sizes for F tests after an ANOVA
esizei can also be used to compute $\eta^{2}$ and $\omega^{2}$ for $F$ tests after an ANOVA. The following example from Smithson $(2001,623)$ illustrates the use of esizei for $\mathrm{df}_{\text {num }}=4, \mathrm{df}_{\text {den }}=50$, and $F=4.2317$.

```
. esizei 4 50 4.2317, level(90)
```

Effect sizes for linear models

| Effect Size | Estimate | $[90 \%$ Conf. Interval] |  |
| ---: | ---: | ---: | ---: |
| Eta-Squared | .2529151 | .0521585 | .3603621 |
| Omega-Squared | .1931483 | 0 | .309191 |

## Stored results

esize and esizei for comparing two means store the following in $r()$ :
Scalars

```
r(d)
    r(lb_d)
    r(ub_d)
    r(g)
    r(lb_g)
    r(ub_g)
    r(delta1)
    r(lb_delta1)
    r(ub_delta1)
    r(delta2)
    r(lb_delta2)
    r(ub_delta2)
    r(r_pb)
    r(lb_r_pb)
    r(ub_r_pb)
    r(N_1)
    r(N_2)
    r(df_t)
    r(level)
        Cohen's d
        lower confidence bound for Cohen's d
        upper confidence bound for Cohen's d
        Hedges's g
        lower confidence bound for Hedges's g
        upper confidence bound for Hedges's g
        Glass's \Delta for group 1
        lower confidence bound for Glass's }\Delta\mathrm{ for group 1
        upper confidence bound for Glass's }\Delta\mathrm{ for group 1
        Glass's }\Delta\mathrm{ for group 2
        lower confidence bound for Glass's }\Delta\mathrm{ for group 2
        upper confidence bound for Glass's }\Delta\mathrm{ for group 2
        point-biserial correlation coefficient
    lower confidence bound for the point-biserial correlation coefficient
    upper confidence bound for the point-biserial correlation coefficient
    sample size }\mp@subsup{n}{1}{
    sample size }\mp@subsup{n}{2}{
    degrees of freedom
    confidence level
```

    esizei for \(F\) tests after ANOVA stores the following in r() :
    Scalars
$r$ (eta2)
r(lb_eta2)
r(ub_eta2)
r(omega2)
r(lb_omega2)
r(ub_omega2)
r(level)

```
\eta
lower confidence bound for }\mp@subsup{\eta}{}{2
upper confidence bound for }\mp@subsup{\eta}{}{2
\omega
lower confidence bound for }\mp@subsup{\omega}{}{2
upper confidence bound for }\mp@subsup{\omega}{}{2
confidence level
```


## Methods and formulas

For the $d$ family, the effect-size parameter of interest is the scaled difference between the means given by

$$
\delta=\frac{\left(\mu_{1}-\mu_{2}\right)}{\sigma}
$$

One of the most popular estimators of effect size is Cohen's $d$, given by

$$
\text { Cohen's } d=\frac{\left(\bar{x}_{1}-\bar{x}_{2}\right)}{s^{*}}
$$

where

$$
s^{*}=\sqrt{\frac{\left(n_{1}-1\right) s_{1}^{2}+\left(n_{2}-1\right) s_{2}^{2}}{n_{1}+n_{2}-2}}
$$

Hedges (1981) showed that Cohen's $d$ is biased and proposed the unbiased estimator

$$
\text { Hedges's } g=\text { Cohen's } d \times c(m)
$$

where $m=n_{1}+n_{2}-2$ and

$$
c(m)=\frac{\Gamma\left(\frac{m}{2}\right)}{\sqrt{\frac{m}{2}} \Gamma\left(\frac{m-1}{2}\right)}
$$

Glass (Smith and Glass 1977) proposed an estimator for $\delta$ in the context of designed experiments,

$$
\text { Glass's } \Delta=\frac{\left(\bar{x}_{\text {treated }}-\bar{x}_{\text {control }}\right)}{s_{\text {control }}}
$$

where $s_{\text {control }}$ is the standard deviation for the control group.
As noted above, esize and esizei report two estimates of Glass's $\Delta$ : one using the standard deviation for group 1 and the other using the standard deviation for group 2:

$$
\text { Glass's } \Delta_{1}=\frac{\left(\bar{x}_{1}-\bar{x}_{2}\right)}{s_{1}}
$$

and

$$
\text { Glass's } \Delta_{2}=\frac{\left(\bar{x}_{1}-\bar{x}_{2}\right)}{s_{2}}
$$

For the $r$ family, the effect-size parameter of interest is the ratio of the variance attributable to an effect and the total variance:

$$
\eta^{2}=\frac{\sigma_{\text {effect }}^{2}}{\sigma_{\text {total }}^{2}}
$$

A popular estimator of $\eta$ when there are two groups is the point-biserial correlation coefficient,

$$
r_{\mathrm{PB}}=\frac{t}{\sqrt{t^{2}+\mathrm{df}}}
$$

where $t$ is the $t$ statistic for the difference between the means of the two groups, and $d f$ is the corresponding degrees of freedom. Satterthwaite's or Welch's adjustment (see [R] ttest for details) to the degrees of freedom can be used to calculate $r_{\mathrm{PB}}$ by specifying the unequal or welch option, respectively.

When more than two means are being compared, as in the case of an ANOVA with $p$ groups, a popular estimator of effect size is the correlation ratio denoted $\eta^{2}$ (Fisher 1925; Kerlinger 1964). $\eta^{2}$ can be computed directly as the ratio of the $\mathrm{SS}_{\text {effect }}$ and the $\mathrm{SS}_{\text {total }}$ or as a function of the $F$ statistic with numerator degrees of freedom equal to $\mathrm{df}_{\text {num }}$ and denominator degrees of freedom equal to $\mathrm{df}_{\text {den }}$.

$$
\widehat{\eta}^{2}=\frac{\left(F \times \mathrm{df}_{\mathrm{num}}\right)}{\left(F \times \mathrm{df}_{\mathrm{num}}\right)+\mathrm{df}_{\mathrm{den}}}
$$

Like its equivalent estimator $R^{2}, \eta^{2}$ has an upward bias. The less biased (though not unbiased) estimator $\omega^{2}$ (Hays 1963) is equivalent to the adjusted $R^{2}$ and can be estimated directly from the sums of squares, the $F$ statistic, or as a function of $\eta^{2}$; that is,

$$
\widehat{\omega}^{2}=\frac{\mathrm{SS}_{\text {between }}-(p-1) \mathrm{MS}_{\text {within }}}{\mathrm{SS}_{\mathrm{total}}+\mathrm{MS}_{\mathrm{within}}}
$$

or

$$
\widehat{\omega}^{2}=\frac{(p-1)(F-1)}{(p-1)(F-1)+(p)(n)}
$$

or

$$
\widehat{\omega}^{2}=\eta^{2}-\left(\frac{\mathrm{df}_{\mathrm{num}}}{\mathrm{df}_{\mathrm{den}}}\right) \times\left(1-\eta^{2}\right)
$$

To calculate $\widehat{\eta}^{2}$ and $\widehat{\omega}^{2}$ directly after anova or regress, see estat esize in $[\mathrm{R}]$ regress postestimation.

Cohen's $d$, Hedges's $g$, and Glass's $\Delta$ have been shown to have a noncentral $t$ distribution (Hedges 1981) with noncentrality parameter equal to

$$
\lambda=\delta \sqrt{\frac{n_{1} n_{2}}{n_{1}+n_{2}}}
$$

Confidence intervals are calculated by finding the noncentrality parameters $\lambda_{\text {lower }}$ and $\lambda_{\text {upper }}$ that correspond to

$$
\operatorname{Pr}\left(\mathrm{df}, \delta, \lambda_{\text {lower }}\right)=1-\frac{\alpha}{2}
$$

and

$$
\operatorname{Pr}\left(\mathrm{df}, \delta, \lambda_{\text {upper }}\right)=\frac{\alpha}{2}
$$

using the function npnt ( $d f, t, p$ ). The noncentrality parameters are then transformed back to the effect-size scale:

$$
\delta_{\text {lower }}=\lambda_{\text {lower }} \sqrt{\frac{n_{1}+n_{2}}{n_{1} n_{2}}}
$$

and

$$
\delta_{\text {upper }}=\lambda_{\text {upper }} \sqrt{\frac{n_{1}+n_{2}}{n_{1} n_{2}}}
$$

(see Venables [1975]; Steiger and Fouladi [1997]; Cumming and Finch [2001]; Smithson [2001]).

Confidence intervals for the point-biserial correlation coefficient are calculated similarly and transformed back to the effect-size scale as

$$
r_{\text {lower }}=\frac{\lambda_{\text {lower }}}{\sqrt{\lambda_{\text {lower }}^{2}+\mathrm{df}}}
$$

and

$$
r_{\text {upper }}=\frac{\lambda_{\text {upper }}}{\sqrt{\lambda_{\text {upper }}^{2}+\mathrm{df}}}
$$

Following Smithson's (2001) notation, the $F$ statistic is written as

$$
F_{\mathrm{df}_{\mathrm{num}}, \mathrm{df}_{\mathrm{den}}}=f^{2}\left(\mathrm{df}_{\mathrm{num}} / \mathrm{df}_{\mathrm{den}}\right)
$$

This equation has a noncentral $F$ distribution with noncentrality parameter:

$$
\lambda=f^{2}\left(\mathrm{df}_{\mathrm{num}}+\mathrm{df}_{\mathrm{den}}+1\right)
$$

where $f^{2}=R^{2} /\left(1-R^{2}\right)$.
Confidence intervals for $\widehat{\eta}^{2}$ and $\widehat{\omega}^{2}$ are calculated by finding the noncentrality parameters $\lambda_{\text {lower }}$ and $\lambda_{\text {upper }}$ for a noncentral $F$ distribution that correspond to

$$
\operatorname{Pr}\left(\mathrm{df}_{\text {num }}, \mathrm{df}_{\text {den }}, F, \lambda_{\text {lower }}\right)=1-\frac{\alpha}{2}
$$

and

$$
\operatorname{Pr}\left(\mathrm{df}_{\text {num }}, \mathrm{df}_{\text {den }}, F, \lambda_{\text {upper }}\right)=\frac{\alpha}{2}
$$

using the function $\operatorname{npnF}\left(d f_{1}, d f_{2}, f, p\right)$. The noncentrality parameters are transformed back to the $\widehat{\eta}^{2}$ scale as

$$
\widehat{\eta}_{\text {lower }}^{2}=\max \left(0, \frac{\lambda_{\text {lower }}}{\lambda_{\text {lower }}+\mathrm{df}_{\text {num }}+\mathrm{df}_{\text {den }}+1}\right)
$$

and

$$
\widehat{\eta}_{\text {upper }}^{2}=\min \left(1, \frac{\lambda_{\text {upper }}}{\lambda_{\text {upper }}+\mathrm{df}_{\text {num }}+\mathrm{df}_{\text {den }}+1}\right)
$$

The confidence limits for $\widehat{\omega}^{2}$ are then calculated as a function of $\widehat{\eta}^{2}$ :

$$
\widehat{\omega}_{\text {lower }}^{2}=\widehat{\eta}_{\text {lower }}^{2}-\left(\frac{\mathrm{df}_{\text {num }}}{\mathrm{df}_{\text {den }}}\right) \times\left(1-\widehat{\eta}_{\text {lower }}^{2}\right)
$$

and

$$
\widehat{\omega}_{\text {upper }}^{2}=\widehat{\eta}_{\text {upper }}^{2}-\left(\frac{\mathrm{df}_{\text {num }}}{\mathrm{df}_{\text {den }}}\right) \times\left(1-\widehat{\eta}_{\text {upper }}^{2}\right)
$$

See Smithson (2001) for further details.

Fred Nichols Kerlinger (1910-1991) was born in New York City. He studied music at New York University and graduated magna cum laude with a degree in education and philosophy. After graduation, he joined the U.S. Army and served as a counterintelligence officer in Japan in 1946. Kerlinger earned an MA and a PhD in educational psychology from the University of Michigan and held faculty appointments at several universities, including New York University. He was president of the American Educational Research Association and is best known for his popular and influential book Foundations of Behavioral Research (1964), which introduced Fisher's (1925) $\eta^{2}$ statistic to behavioral researchers.

William Lee Hays (1926-1995) was born in Clarksville, Texas. He studied mathematics and psychology at Paris Junior College in Paris, Texas, and at East Texas State College. He earned BS and MS degrees from North Texas State University. Upon completion of his PhD in psychology at the University of Michigan, he joined the faculty, where he eventually became associate vice president for academic affairs. In 1977, Hays accepted an appointment as vice president for academic affairs at the University of Texas at Austin, where he remained until his death in 1995. Hays is best known for his book Statistics for Psychologists (1963), which introduced the $\omega^{2}$ statistic.

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## Also see

[R] bitest - Binomial probability test
$[R]$ ci - Confidence intervals for means, proportions, and counts
[R] mean - Estimate means
[R] oneway - One-way analysis of variance
[R] prtest - Tests of proportions
[R] sdtest - Variance-comparison tests
[R] ttest - $t$ tests (mean-comparison tests)

## Title

estat - Postestimation statistics

## Syntax Description

## Syntax

## Command <br> Reference

Display information criteria

```
estat ic \([, \mathrm{n}(\#)] \quad[\mathrm{R}]\) estat ic
```

Summarize estimation sample
estat summarize [eqlist] [, estat_summ_options] [R] estat summarize

Display covariance matrix estimates

```
estat vce[, estat_vce_options] [R] estat vce
```

Command-specific
estat subcommand ${ }_{1}\left[\right.$, options $\left._{1}\right]$

## Description

estat displays scalar- and matrix-valued statistics after estimation; it complements predict, which calculates variables after estimation. Exactly what statistics estat can calculate depends on the previous estimation command.

Three sets of statistics are so commonly used that they are available after all estimation commands that store the model log likelihood. estat ic displays Akaike's and Schwarz's Bayesian information criteria. estat summarize summarizes the variables used by the command and automatically restricts the sample to e (sample); it also summarizes the weight variable and cluster structure, if specified. estat vce displays the covariance or correlation matrix of the parameter estimates of the previous model.

## Title

estat classification - Classification statistics and table

| Syntax | Menu for estat | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

estat classification $[$ if $][$ in $][$ weight $][$, options $]$
options
Description
Main
all display summary statistics for all observations in the data
cutoff(\#)
positive outcome threshold; default is cutoff (0.5)
fweights are allowed; see [U] 11.1.6 weight.
estat classification is not appropriate after the svy prefix.

## Menu for estat

Statistics $>$ Postestimation $>$ Reports and statistics

## Description

estat classification reports various summary statistics, including the classification table.
estat classification requires that the current estimation results be from logistic, logit, probit, or ivprobit; see [R] logistic, [R] logit, [R] probit, or [R] ivprobit.

## Options

$\qquad$ Main
all requests that the statistic be computed for all observations in the data, ignoring any if or in restrictions specified by the estimation command.
cutoff(\#) specifies the value for determining whether an observation has a predicted positive outcome. An observation is classified as positive if its predicted probability is $\geq \#$. The default is 0.5 .

## Remarks and examples

estat classification presents the classification statistics and classification table after logistic, logit, probit, or ivprobit.

Statistics are produced either for the estimation sample (the default) or for any set of observations. When weights, if, or in is used with the estimation command, it is not necessary to repeat the qualifier when you want statistics computed for the estimation sample. Specify if, in, or the all option only when you want statistics computed for a set of observations other than the estimation sample. Specify weights only when you want to use a different set of weights.

## > Example 1

We illustrate estat classification after logistic; see [R] logistic.

| . logistic low age lwt i.race smoke ptl ht ui (output omitted) |  |  |
| :---: | :---: | :---: |
| . estat classification |  |  |
| Logistic model for low |  |  |
| Classified D | $\sim$ D | Total |
| + 21 | 12 | 33 |
| 38 | 118 | 156 |
| Total 59 | 130 | 189 |
| Classified + if predicted $\operatorname{Pr}(\mathrm{D})$ >= . 5 |  |  |
| True D defined as low ! $=0$ |  |  |
| Sensitivity | $\operatorname{Pr}(+\mid \mathrm{D})$ | 35.59\% |
| Specificity | $\operatorname{Pr}(-\mid \sim D)$ | 90.77\% |
| Positive predictive value | $\operatorname{Pr}(\mathrm{D} \mid+)$ | 63.64\% |
| Negative predictive value | $\operatorname{Pr}(\sim D \mid-)$ | 75.64\% |
| False + rate for true $\sim$ D | $\operatorname{Pr}(+\mid \sim D)$ | 9.23\% |
| False - rate for true D | $\operatorname{Pr}(-\mid \mathrm{D})$ | 64.41\% |
| False + rate for classified + | $\operatorname{Pr}(\sim \mathrm{D} \mid+)$ | 36.36\% |
| False - rate for classified - | $\operatorname{Pr}(\mathrm{D} \mid ~-) ~$ | 24.36\% |
| Correctly classified |  | 73.54\% |

The overall rate of correct classification is estimated to be 73.54 , with $90.77 \%$ of the normal weight group correctly classified (specificity) and only $35.59 \%$ of the low weight group correctly classified (sensitivity). Classification is sensitive to the relative sizes of each component group, and always favors classification into the larger group. This phenomenon is evident here.

By default, estat classification uses a cutoff of 0.5 , although you can vary this with the cutoff() option. You can use the lsens command to review the potential cutoffs; see [R] Isens.

## Stored results

estat classification stores the following in r() :
Scalars

| $r\left(P \_c o r r\right)$ | percent correctly classified |
| :--- | :--- |
| $r(P-p 1)$ | sensitivity |
| $r(P-n 0)$ | specificity |
| $r(P-p 0)$ | false-positive rate given true negative |
| $r\left(P \_n 1\right)$ | false-negative rate given true positive |
| $r(P-1 p)$ | positive predictive value |
| $r(P-0 n)$ | negative predictive value |
| $r(P-0)$ | false-positive rate given classified positive |
| $r(P-1 n)$ | false-negative rate given classified negative |

## Methods and formulas

Let $j$ index observations. Define $c$ as the cutoff() specified by the user or, if not specified, as 0.5 . Let $p_{j}$ be the predicted probability of a positive outcome and $y_{j}$ be the actual outcome, which we will treat as 0 or 1 , although Stata treats it as 0 and non- 0 , excluding missing observations.

A prediction is classified as positive if $p_{j} \geq c$ and otherwise is classified as negative. The classification is correct if it is positive and $y_{j}=1$ or if it is negative and $y_{j}=0$.

Sensitivity is the fraction of $y_{j}=1$ observations that are correctly classified. Specificity is the percentage of $y_{j}=0$ observations that are correctly classified.

## References

Hosmer, D. W., Jr., S. A. Lemeshow, and R. X. Sturdivant. 2013. Applied Logistic Regression. 3rd ed. Hoboken, NJ: Wiley.
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[^15]
## Title

estat gof - Pearson or Hosmer-Lemeshow goodness-of-fit test

| Syntax | Menu for estat | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

estat gof $[$ if $][$ in $][$ weight $][$, options $]$

options

## Description

Main
group (\#) perform Hosmer-Lemeshow goodness-of-fit test using \# quantiles all execute test for all observations in the data
outsample adjust degrees of freedom for samples outside estimation sample table display table of groups used for test
fweights are allowed; see [U] 11.1.6 weight.
For information on using estat gof with survey data, see [SVY] estat.

## Menu for estat

Statistics $>$ Postestimation $>$ Reports and statistics

## Description

estat gof reports the Pearson goodness-of-fit test or the Hosmer-Lemeshow goodness-of-fit test.
estat gof requires that the current estimation results be from logistic, logit, or probit; see $[R]$ logistic, $[R]$ logit, or $[R]$ probit. For estat gof after poisson, see $[R]$ poisson postestimation. For estat gof after sem, see [SEM] estat gof.

## Options

group (\#) specifies the number of quantiles to be used to group the data for the Hosmer-Lemeshow goodness-of-fit test. group (10) is typically specified. If this option is not given, the Pearson goodness-of-fit test is computed using the covariate patterns in the data as groups.
all requests that the statistic be computed for all observations in the data, ignoring any if or in restrictions specified by the estimation command.
outsample adjusts the degrees of freedom for the Pearson and Hosmer-Lemeshow goodness-of-fit tests for samples outside the estimation sample. See Samples other than the estimation sample later in this entry.
table displays a table of the groups used for the Hosmer-Lemeshow or Pearson goodness-of-fit test with predicted probabilities, observed and expected counts for both outcomes, and totals for each group.

## Remarks and examples

Remarks are presented under the following headings:
Introduction
Samples other than the estimation sample

## Introduction

estat gof computes goodness-of-fit tests: either the Pearson $\chi^{2}$ test or the Hosmer-Lemeshow test.

By default, estat gof computes statistics for the estimation sample by using the last model fit by logistic, logit, or probit. However, samples other than the estimation sample can be specified; see Samples other than the estimation sample later in this entry.

## > Example 1

estat gof, typed without options, presents the Pearson $\chi^{2}$ goodness-of-fit test for the fitted model. The Pearson $\chi^{2}$ goodness-of-fit test is a test of the observed against expected number of responses using cells defined by the covariate patterns; see predict with the number option in [R] logistic postestimation for the definition of covariate patterns.

```
. use http://www.stata-press.com/data/r13/lbw
(Hosmer & Lemeshow data)
. logistic low age lwt i.race smoke ptl ht ui
    (output omitted)
. estat gof
Logistic model for low, goodness-of-fit test
    number of observations = 189
    number of covariate patterns = 182
        Pearson chi2(173) = 179.24
                Prob > chi2 = 0.3567
```

Our model fits reasonably well. However, the number of covariate patterns is close to the number of observations, making the applicability of the Pearson $\chi^{2}$ test questionable but not necessarily inappropriate. Hosmer, Lemeshow, and Sturdivant (2013, 157-160) suggest regrouping the data by ordering on the predicted probabilities and then forming, say, 10 nearly equal-sized groups. estat gof with the group() option does this:

```
. estat gof, group(10)
Logistic model for low, goodness-of-fit test
    (Table collapsed on quantiles of estimated probabilities)
            number of observations = 189
            number of groups = 10
            Hosmer-Lemeshow chi2(8) = 9.65
                        Prob > chi2 = 0.2904
```

Again we cannot reject our model. If we specify the table option, estat gof displays the groups along with the expected and observed number of positive responses (low-birthweight babies):

```
. estat gof, group(10) table
```

Logistic model for low, goodness-of-fit test
(Table collapsed on quantiles of estimated probabilities)

| Group | Prob | Obs_1 | Exp_1 | Obs_0 | Exp_0 | Total |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 0.0827 | 0 | 1.2 | 19 | 17.8 | 19 |
| 2 | 0.1276 | 2 | 2.0 | 17 | 17.0 | 19 |
| 3 | 0.2015 | 6 | 3.2 | 13 | 15.8 | 19 |
| 4 | 0.2432 | 1 | 4.3 | 18 | 14.7 | 19 |
| 5 | 0.2792 | 7 | 4.9 | 12 | 14.1 | 19 |
| 6 | 0.3138 | 7 | 5.6 | 12 | 13.4 | 19 |
| 7 | 0.3872 | 6 | 6.5 | 13 | 12.5 | 19 |
| 8 | 0.4828 | 7 | 8.2 | 12 | 10.8 | 19 |
| 9 | 0.5941 | 10 | 10.3 | 9 | 8.7 | 19 |
| 10 | 0.8391 | 13 | 12.8 | 5 | 5.2 | 18 |

        number of observations \(=\)
        189
            number of groups \(=10\)
    Hosmer-Lemeshow chi2(8) = 9.65
                        Prob \(>\) chi2 \(=0.2904\)
    
## Technical note

estat gof with the group() option puts all observations with the same predicted probabilities into the same group. If, as in the previous example, we request 10 groups, the groups that estat gof makes are $\left[p_{0}, p_{10}\right],\left(p_{10}, p_{20}\right],\left(p_{20}, p_{30}\right], \ldots,\left(p_{90}, p_{100}\right]$, where $p_{k}$ is the $k$ th percentile of the predicted probabilities, with $p_{0}$ the minimum and $p_{100}$ the maximum.

If there are many ties at the quantile boundaries, as will often happen if all independent variables are categorical and there are only a few of them, the sizes of the groups will be uneven. If the totals in some of the groups are small, the $\chi^{2}$ statistic for the Hosmer-Lemeshow test may be unreliable. In this case, fewer groups should be specified, or the Pearson goodness-of-fit test may be a better choice.

## > Example 2

The table option can be used without the group() option. We would not want to specify this for our current model because there were 182 covariate patterns in the data, caused by including the two continuous variables, age and lwt, in the model. As an aside, we fit a simpler model and specify table with estat gof:
. logistic low i.race smoke ui
Logistic regression

Log likelihood = -107.93404

| Number of obs | $=$ | 189 |
| :--- | :--- | ---: |
| LR chi2 $(4)$ | $=$ | 18.80 |
| Prob > chi2 | $=$ | 0.0009 |
| Pseudo R2 | $=$ | 0.0801 |


| low | Odds Ratio | Std. Err. | $\mathbf{z}$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :--- | :--- | :--- | :--- | :--- |
| race |  |  |  |  |  |  |
| black | 3.052746 | 1.498087 | 2.27 | 0.023 | 1.166747 | 7.987382 |
| other | 2.922593 | 1.189229 | 2.64 | 0.008 | 1.316457 | 6.488285 |
|  |  |  |  |  |  |  |
| smoke | 2.945742 | 1.101838 | 2.89 | 0.004 | 1.415167 | 6.131715 |
| ui | 2.419131 | 1.047359 | 2.04 | 0.041 | 1.035459 | 5.651788 |
| _cons | .1402209 | .0512295 | -5.38 | 0.000 | .0685216 | .2869447 |

. estat gof, table
Logistic model for low, goodness-of-fit test

| Group | Prob | Obs_1 | Exp_1 | Obs_0 | Exp_0 | Total |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 0.1230 | 3 | 4.9 | 37 | 35.1 | 40 |
| 2 | 0.2533 | 1 | 1.0 | 3 | 3.0 | 4 |
| 3 | 0.2907 | 16 | 13.7 | 31 | 33.3 | 47 |
| 4 | 0.2923 | 15 | 12.6 | 28 | 30.4 | 43 |
| 5 | 0.2997 | 3 | 3.9 | 10 | 9.1 | 13 |
| 6 | 0.4978 | 4 | 4.0 | 4 | 4.0 | 8 |
| 7 | 0.4998 | 4 | 4.5 | 5 | 4.5 | 9 |
| 8 | 0.5087 | 2 | 1.5 | 1 | 1.5 | 3 |
| 9 | 0.5469 | 2 | 4.4 | 6 | 3.6 | 8 |
| 10 | 0.5577 | 6 | 5.6 | 4 | 4.4 | 10 |
| 11 | 0.7449 | 3 | 3.0 | 1 | 1.0 | 4 |


| Group | Prob | race | smoke | ui |
| ---: | ---: | ---: | ---: | ---: |
| 1 | 0.1230 | white | nonsmoker | 0 |
| 2 | 0.2533 | white | nonsmoker | 1 |
| 3 | 0.2907 | other | nonsmoker | 0 |
| 4 | 0.2923 | white | smoker | 0 |
| 5 | 0.2997 | black | nonsmoker | 0 |
| 6 | 0.4978 | other | nonsmoker | 1 |
| 7 | 0.4998 | white | smoker | 1 |
| 8 | 0.5087 | black | nonsmoker | 1 |
| 9 | 0.5469 | other | smoker | 0 |
| 10 | 0.5577 | black | smoker | 0 |
| 11 | 0.7449 | other | smoker | 1 |

number of observations $=189$
number of covariate patterns $=11$
Pearson chi2(6) = 5.71

$$
\text { Prob }>\text { chi2 }=0.4569
$$

## - Technical note

logistic, logit, or probit and estat gof keep track of the estimation sample. If you type, for instance, logistic ... if $x==1$, then when you type estat gof, the statistics will be calculated on the $x==1$ subsample of the data automatically.

You should specify if or in with estat gof only when you wish to calculate statistics for a set of observations other than the estimation sample. See Samples other than the estimation sample later in this entry.

If the logistic model was fit with fweights, estat gof properly accounts for the weights in its calculations. (estat gof only allows fweights.) You do not have to specify the weights when you run estat gof. Weights should be specified with estat gof only when you wish to use a different set of weights.

## Samples other than the estimation sample

estat gof can be used with samples other than the estimation sample. By default, estat gof remembers the estimation sample used with the last logistic, logit, or probit command. To override this, simply use an if or in restriction to select another set of observations, or specify the all option to force the command to use all the observations in the dataset.

If you use estat gof with a sample that is completely different from the estimation sample (that is, no overlap), you should also specify the outsample option so that the $\chi^{2}$ statistic properly adjusts the degrees of freedom upward. For an overlapping sample, the conservative thing to do is to leave the degrees of freedom the same as they are for the estimation sample.

## > Example 3

We want to develop a model for predicting low-birthweight babies. One approach would be to divide our data into two groups, a developmental sample and a validation sample. See Lemeshow and Gall (1994) and Tilford, Roberson, and Fiser (1995) for more information on developing prediction models and severity-scoring systems.

We will do this with the low-birthweight data that we considered previously. First, we randomly divide the data into two samples.

```
. use http://www.stata-press.com/data/r13/lbw, clear
(Hosmer & Lemeshow data)
. set seed 1
. generate r = runiform()
. sort r
. generate group = 1 if _n <= _N/2
(95 missing values generated)
. replace group = 2 if group==.
(95 real changes made)
```

Then we fit a model using the first sample (group $=1$ ), which is our developmental sample.

| . logistic low age lwt i.race smoke ptl ht ui if group==1 |  |  |  |
| :--- | :--- | :--- | :--- |
| Logistic regression | Number of obs | $=$ | 94 |
|  | LR chi2 (8) | $=$ | 29.14 |
|  | Prob > chi2 | $=$ | 0.0003 |
| Log likelihood $=-44.293342$ | Pseudo R2 | $=$ | 0.2475 |


| low | Odds Ratio | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| age | .91542 | .0553937 | -1.46 | 0.144 | .8130414 | 1.03069 |
| lwt | .9744276 | .0112295 | -2.25 | 0.025 | .9526649 | .9966874 |
| race |  |  |  |  |  |  |
| black | 5.063678 | 3.78442 | 2.17 | 0.030 | 1.170327 | 21.90913 |
| other | 2.606209 | 1.657608 | 1.51 | 0.132 | .7492483 | 9.065522 |
|  |  |  |  |  |  |  |
| smoke | .909912 | .5252898 | -0.16 | 0.870 | .2934966 | 2.820953 |
| ptl | 3.033543 | 1.507048 | 2.23 | 0.025 | 1.145718 | 8.03198 |
| ht | 21.07656 | 22.64788 | 2.84 | 0.005 | 2.565304 | 173.1652 |
| ui | .988479 | .6699458 | -0.02 | 0.986 | .2618557 | 3.731409 |
| _cons | 30.73641 | 56.82168 | 1.85 | 0.064 | .8204589 | 1151.462 |

To test calibration in the developmental sample, we calculate the Hosmer-Lemeshow goodness-of-fit test by using estat gof.

```
. estat gof, group(10)
```

Logistic model for low, goodness-of-fit test
(Table collapsed on quantiles of estimated probabilities)
$\begin{array}{rll}\text { number of observations }= & 94 \\ \text { number of groups }= & 10 \\ \text { Hosmer-Lemeshow chi2 }(8)= & 6.67 \\ \text { Prob }>\text { chi2 } & = & 0.5721\end{array}$

We did not specify an if statement with estat gof because we wanted to use the estimation sample. Because the test is not significant, we are satisfied with the fit of our model.

Running lroc (see $[R]$ lroc) gives a measure of the discrimination:

```
. lroc, nograph
Logistic model for low
number of observations = 94
area under ROC curve = 0.8156
```

Now we test the calibration of our model by performing a goodness-of-fit test on the validation sample. We specify the outsample option so that the number of degrees of freedom is 10 rather than 8 .

```
. estat gof if group==2, group(10) table outsample
```

Logistic model for low, goodness-of-fit test
(Table collapsed on quantiles of estimated probabilities)

| Group | Prob | Obs_1 | Exp_1 | Obs_0 | Exp_0 | Total |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 0.0725 | 1 | 0.4 | 9 | 9.6 | 10 |
| 2 | 0.1202 | 4 | 0.8 | 5 | 8.2 | 9 |
| 3 | 0.1549 | 3 | 1.3 | 7 | 8.7 | 10 |
| 4 | 0.1888 | 1 | 1.5 | 8 | 7.5 | 9 |
| 5 | 0.2609 | 3 | 2.2 | 7 | 7.8 | 10 |
| 6 | 0.3258 | 4 | 2.7 | 5 | 6.3 | 9 |
| 7 | 0.4217 | 2 | 3.7 | 8 | 6.3 | 10 |
| 8 | 0.4915 | 3 | 4.1 | 6 | 4.9 | 9 |
| 9 | 0.6265 | 4 | 5.5 | 6 | 4.5 | 10 |
| 10 | 0.9737 | 4 | 7.1 | 5 | 1.9 | 9 |

number of observations $=$
number of groups $=10$
Hosmer-Lemeshow chi2(10) $=28.03$
Prob > chi2 $=0.0018$

We must acknowledge that our model does not fit well on the validation sample. The model's discrimination in the validation sample is appreciably lower, as well.

```
. lroc if group==2, nograph
Logistic model for low
number of observations = 95
area under ROC curve = 0.5839
```


## Stored results

estat gof stores the following in $r()$ :
Scalars

```
r(N) number of observations
    r(m) number of covariate patterns or groups
    r(df) degrees of freedom
    r(chi2) }\mp@subsup{\chi}{}{2
```


## Methods and formulas

Let $M$ be the total number of covariate patterns among the $N$ observations. View the data as collapsed on covariate patterns $j=1,2, \ldots, M$, and define $m_{j}$ as the total number of observations having covariate pattern $j$ and $y_{j}$ as the total number of positive responses among observations with covariate pattern $j$. Define $p_{j}$ as the predicted probability of a positive outcome in covariate pattern $j$.

The Pearson $\chi^{2}$ goodness-of-fit statistic is

$$
\chi^{2}=\sum_{j=1}^{M} \frac{\left(y_{j}-m_{j} p_{j}\right)^{2}}{m_{j} p_{j}\left(1-p_{j}\right)}
$$

This $\chi^{2}$ statistic has approximately $M-k$ degrees of freedom for the estimation sample, where $k$ is the number of independent variables, including the constant. For a sample outside the estimation sample, the statistic has $M$ degrees of freedom.

The Hosmer-Lemeshow goodness-of-fit $\chi^{2}$ (Hosmer and Lemeshow 1980; Lemeshow and Hosmer 1982; Hosmer, Lemeshow, and Klar 1988) is calculated similarly, except that rather than using the $M$ covariate patterns as the group definition, the quantiles of the predicted probabilities are used to form groups. Let $G=$ \# be the number of quantiles requested with group (\#). The smallest index $1 \leq q(i) \leq M$, such that

$$
W_{q(i)}=\sum_{j=1}^{q(i)} m_{j} \geq \frac{N}{G}
$$

gives $p_{q(i)}$ as the upper boundary of the $i$ th quantile for $i=1,2, \ldots, G$. Let $q(0)=1$ denote the first index.

The groups are then

$$
\left[p_{q(0)}, p_{q(1)}\right],\left(p_{q(1)}, p_{q(2)}\right], \ldots,\left(p_{q(G-1)}, p_{q(G)}\right]
$$

If the table option is given, the upper boundaries $p_{q(1)}, \ldots, p_{q(G)}$ of the groups appear next to the group number on the output.

The resulting $\chi^{2}$ statistic has approximately $G-2$ degrees of freedom for the estimation sample. For a sample outside the estimation sample, the statistic has $G$ degrees of freedom.

## References

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## Also see

$[\mathrm{R}]$ logistic - Logistic regression, reporting odds ratios
[R] logit — Logistic regression, reporting coefficients
[R] probit - Probit regression
[R] estat classification - Classification statistics and table
[R] Iroc - Compute area under ROC curve and graph the curve
$[R]$ Isens - Graph sensitivity and specificity versus probability cutoff
[U] 20 Estimation and postestimation commands

## Title

estat ic - Display information criteria

| Syntax | Menu for estat | Description | Option |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

Also see

## Syntax

estat ic $[, \mathrm{n}(\#)]$

## Menu for estat

Statistics $>$ Postestimation $>$ Reports and statistics

## Description

estat ic displays Akaike's and Schwarz's Bayesian information criteria.

## Option

$\mathrm{n}(\#)$ specifies the $N$ to be used in calculating BIC; see $[\mathrm{R}]$ BIC note.

## Remarks and examples

estat ic calculates two information criteria used to compare models. Unlike likelihood-ratio, Wald, and similar testing procedures, the models need not be nested to compare the information criteria. Because they are based on the log-likelihood function, information criteria are available only after commands that report the log likelihood.

In general, "smaller is better": given two models, the one with the smaller AIC fits the data better than the one with the larger AIC. As with the AIC, a smaller BIC indicates a better-fitting model. For AIC and BIC formulas, see Methods and formulas.
> Example 1
In [R] mlogit, we fit a model explaining the type of insurance a person has on the basis of age, gender, race, and site of study. Here we refit the model with and without the site dummies and compare the models.

```
. use http://www.stata-press.com/data/r13/sysdsn1
(Health insurance data)
. mlogit insure age male nonwhite
    (output omitted)
. estat ic
Akaike's information criterion and Bayesian information criterion
```

| Model | Obs | 11 (null) | 11 (model) | df | AIC | BIC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| . | 615 | -555.8545 | -545.5833 | 8 | 1107.167 | 1142.54 |
| mlogit insure age male nonwhite i.site (output omitted) |  |  |  |  |  |  |
| estat ic |  |  |  |  |  |  |

Akaike's information criterion and Bayesian information criterion

| Model | Obs | ll(null) | ll(model) | df | AIC | BIC |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| . | 615 | -555.8545 | -534.3616 | 12 | 1092.723 | 1145.783 |

Note: $N=O b s$ used in calculating BIC; see [R] BIC note
The AIC indicates that the model including the site dummies fits the data better, whereas the BIC indicates the opposite. As is often the case, different model-selection criteria have led to conflicting conclusions.

## Technical note

glm and binreg, ml report a slightly different version of AIC and BIC; see $[\mathrm{R}]$ glm for the formulas used. That version is commonly used within the GLM literature; see, for example, Hardin and Hilbe (2012). The literature on information criteria is vast; see, among others, Akaike (1973), Sawa (1978), and Raftery (1995). Judge et al. (1985) contains a discussion of using information criteria in econometrics. Royston and Sauerbrei (2008, chap. 2) examine the use of information criteria as an alternative to stepwise procedures for selecting model variables.

## Stored results

estat ic stores the following in $r()$ :
Matrices
$r(S) \quad 1 \times 6$ matrix of results:

1. sample size
2. $\log$ likelihood of null model
3. log likelihood of full model
4. degrees of freedom
5. AIC
6. BIC

## Methods and formulas

Akaike's (1974) information criterion is defined as

$$
\mathrm{AIC}=-2 \ln L+2 k
$$

where $\ln L$ is the maximized $\log$-likelihood of the model and $k$ is the number of parameters estimated. Some authors define the AIC as the expression above divided by the sample size.

Schwarz's (1978) Bayesian information criterion is another measure of fit defined as

$$
\mathrm{BIC}=-2 \ln L+k \ln N
$$

where $N$ is the sample size. See [R] BIC note for additional information on calculating and interpreting BIC.

Hirotugu Akaike (1927-2009) was born in Fujinomiya City, Shizuoka Prefecture, Japan. He was the son of a silkworm farmer. He gained BA and DSc degrees from the University of Tokyo. Akaike's career from 1952 at the Institute of Statistical Mathematics in Japan culminated in service as Director General; after 1994, he was Professor Emeritus. His best known work in a prolific career is on what is now known as the Akaike information criterion (AIC), which was formulated to help selection of the most appropriate model from a number of candidates.
Gideon E. Schwarz (1933-2007) was a professor of Statistics at the Hebrew University, Jerusalem. He was born in Salzburg, Austria, and obtained an MSc in 1956 from the Hebrew University and a PhD in 1961 from Columbia University. His interests included stochastic processes, sequential analysis, probability, and geometry. He is best known for the Bayesian information criterion (BIC).

## References

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## Also see

[R] estat - Postestimation statistics
$[R]$ estat summarize - Summarize estimation sample
[R] estat vce - Display covariance matrix estimates

## Title

estat summarize - Summarize estimation sample

| Syntax | Menu for estat | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Also see |  |

## Syntax

```
estat summarize [eqlist] [, estat_summ_options]
```

estat_summ_options Description

| equation | display summary by equation <br> display summary by group; only after sem |
| :--- | :--- |
| group | display variable labels |
| labels | suppress the header |
| noheader | ignore weights <br> control row spacing, line width, display of omitted variables <br> and base and empty cells, and factor-variable labeling |
| display_options | and |
|  |  |

[^16][U] 11.4.4 Time-series varlists.

## Menu for estat

Statistics $>$ Postestimation $>$ Reports and statistics

## Description

estat summarize summarizes the variables used by the command and automatically restricts the sample to e(sample); it also summarizes the weight variable and cluster structure, if specified.

## Options

equation requests that the dependent variables and the independent variables in the equations be displayed in the equation-style format of estimation commands, repeating the summary information about variables entered in more than one equation.
group displays summary information separately for each group. group is only allowed after sem with a group() variable specified.
labels displays variable labels.
noheader suppresses the header.
noweights ignores the weights, if any, from the previous estimation command. The default when weights are present is to perform a weighted summarize on all variables except the weight variable itself. An unweighted summarize is performed on the weight variable.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), and fvwrapon (style); see [R] estimation options.

## Remarks and examples

Often when fitting a model, you will also be interested in obtaining summary statistics, such as the sample means and standard deviations of the variables in the model. estat summarize makes this process simple. The output displayed is similar to that obtained by typing

```
. summarize varlist if e(sample)
```

without the need to type the varlist containing the dependent and independent variables.

## $>$ Example 1

Continuing with the example in $[\mathrm{R}]$ estat ic, here we summarize the variables by using estat summarize.


The output in the previous example contains all the variables in one table, though mlogit presents its results in a multiple-equation format. For models in which the same variables appear in all equations, that is fine; but for other multiple-equation models, we may prefer to have the variables separated by the equation in which they appear. The equation option makes this possible.
$>$ Example 2
Systems of simultaneous equations typically have different variables in each equation, and the equation option of estat summarize is helpful in such situations. In example 2 of $[\mathrm{R}]$ reg3, we have a model of supply and demand. We first refit the model and then call estat summarize.


The first block of the table contains statistics on the dependent (or, more accurately, left-hand-side) variables, and because we specified quantity as the left-hand-side variable in both equations, it is listed twice. The second block refers to the variables in the first equation we specified, which we labeled "Demand" in our call to reg3; and the final block refers to the supply equation.

## Stored results

estat summarize stores the following in $r()$ :
Scalars
$r$ (N_groups) number of groups (group only)
Matrices
$r$ (stats) $\quad k \times 4$ matrix of means, standard deviations, minimums, and maximums
$\mathrm{r}($ stats [ \# ] $) \quad \begin{gathered}k \times 4 \text { matrix of means, standard deviations, minimums, and maximums for group \# (group } \\ \text { only })\end{gathered}$

## Also see

[R] estat - Postestimation statistics
[R] estat ic - Display information criteria
[R] estat vce - Display covariance matrix estimates

## Title

estat vce - Display covariance matrix estimates

| Syntax | Menu for estat | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Reference | Also see |

## Syntax

estat vce [, estat_vce_options]
estat_vce_options Description

| covariance | display as covariance matrix; the default <br> display as correlation matrix |
| :--- | :--- |
| correlation | display only specified equations <br> equation $($ spec $)$ |
| display submatrices by equation |  |

## Menu for estat

Statistics $>$ Postestimation $>$ Reports and statistics

## Description

estat vce displays the covariance or correlation matrix of the parameter estimates of the previous model.

## Options

covariance displays the matrix as a variance-covariance matrix; this is the default.
correlation displays the matrix as a correlation matrix rather than a variance-covariance matrix. rho is a synonym.
equation (spec) selects part of the VCE to be displayed. If spec is eqlist, the VCE for the listed equations is displayed. If spec is eqlistl $\backslash$ eqlist 2 , the part of the VCE associated with the equations in eqlistl (rowwise) and eqlist 2 (columnwise) is displayed. If spec is $*$, all equations are displayed. equation() implies block if diag is not specified.
block displays the submatrices pertaining to distinct equations separately.
diag displays the diagonal submatrices pertaining to distinct equations separately.
format (\% firt) specifies the number format for displaying the elements of the matrix. The default is format ( $\% 10.0 \mathrm{~g}$ ) for covariances and format (\%8.4f) for correlations. See [U] 12.5 Formats: Controlling how data are displayed for more information.
nolines suppresses lines between equations.
display_options: noomitted, noemptycells, baselevels, allbaselevels; see [R] estimation options.

## Remarks and examples

estat vce allows you to display the VCE of the parameters of the previously fit model, as either a covariance matrix or a correlation matrix.

## Example 1

Returning to the example in $[\mathrm{R}]$ estat ic, here we display the covariance matrix of the parameters of the mlogit model by using estat vce.


|  | age | male | nonwhite | _cons |
| :---: | :---: | :---: | :---: | :---: |
| age | . 00001753 | -. 00007926 | -. 00004564 | -. 00076886 |
| male | -. 00007544 | . 02188398 | . 0023186 | -. 00145923 |
| nonwhite | -. 00004577 | . 00250588 | . 02813553 | -. 00263872 |
| _cons | -. 00077045 | -. 00130535 | -. 00257593 | . 03888032 |
| covariances of equation Uninsure |  |  |  |  |
|  | age | male | nonwhite | _cons |
| age | . 00013022 |  |  |  |
| male | -. 00050406 | . 13248095 |  |  |
| nonwhite | -. 00026145 | . 01505449 | . 16861327 |  |
| _cons | -. 00562159 | -. 01686629 | -. 02474852 | . 28607591 |

The block option is particularly useful for multiple-equation estimators. The first block of output here corresponds to the VCE of the estimated parameters for the first equation-the square roots of the diagonal elements of this matrix are equal to the standard errors of the first equation's parameters. Similarly, the final block corresponds to the VCE of the parameters for the second equation. The middle block shows the covariances between the estimated parameters of the first and second equations.

## Stored results

estat vce stores the following in $r()$ :
Matrices
$r(V) \quad V C E$ or correlation matrix

## Reference

Hamilton, L. C. 2013. Statistics with Stata: Updated for Version 12. 8th ed. Boston: Brooks/Cole.

## Also see <br> [R] estat - Postestimation statistics <br> [R] estat ic - Display information criteria <br> [R] estat summarize - Summarize estimation sample

## Title

estimates - Save and manipulate estimation results
Syntax Description Remarks and examples Also see

## Syntax

Command Reference
Save and use results from disk

```
estimates save filename [R] estimates save
estimates use filename [R] estimates save
```

estimates describe using filename [R] estimates describe
estimates esample: ... [R] estimates save

Store and restore estimates in memory

```
estimates store name [R] estimates store
estimates restore name [R] estimates store
estimates query [R] estimates store
estimates dir [R] estimates store
estimates drop namelist [R] estimates store
estimates clear [R] estimates store
```

Set titles and notes

```
estimates title: text
[R] estimates title
estimates title
[R] estimates title
estimates notes: text [R] estimates notes
estimates notes [R] estimates notes
estimates notes list... [R] estimates notes
estimates notes drop ... [R] estimates notes
```

Report

| estimates describe [name] | $[\mathrm{R}]$ estimates describe |
| :--- | :--- |
| $\underline{\text { estimates replay }[\text { namelist }]}$ | $[\mathrm{R}]$ estimates replay |

Tables and statistics

| estimates table [namelist] | $[\mathrm{R}]$ estimates table |
| :--- | :--- |
| estimates stats [namelist] | $[\mathrm{R}]$ estimates stats |
| estimates for namelist:... | $[\mathrm{R}]$ estimates for |

## Description

estimates allows you to store and manipulate estimation results:

- You can save estimation results in a file for use in later sessions.
- You can store estimation results in memory so that you can
a. switch among separate estimation results and
b. form tables combining separate estimation results.


## Remarks and examples

estimates is for use after you have fit a model, be it with regress, logistic, etc. You can use estimates after any estimation command, whether it be an official estimation command of Stata or a user-written one.
estimates has three separate but related capabilities:

1. You can save estimation results in a file on disk so that you can use them later, even in a different Stata session.
2. You can store up to 300 estimation results in memory so that they are at your fingertips.
3. You can make tables comparing any results you have stored in memory.

Remarks are presented under the following headings:

```
Saving and using estimation results
Storing and restoring estimation results
Comparing estimation results
Jargon
```


## Saving and using estimation results

After you have fit a model, say, with regress, type

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. regress mpg weight displ foreign
    (output omitted)
```

You can save the results in a file:
. estimates save basemodel
(file basemodel.ster saved)
Later, say, in a different session, you can reload those results:

```
. estimates use basemodel
```

The situation is now nearly identical to what it was immediately after you fit the model. You can replay estimation results:
. regress
(output omitted)
You can perform tests:

```
. test foreign==0
```

(output omitted)

And you can use any postestimation command or postestimation capability of Stata. The only difference is that Stata no longer knows what the estimation sample, e (sample) in Stata jargon, was. When you reload the estimation results, you might not even have the original data in memory. That is okay. Stata will know to refuse to calculate anything that can be calculated only on the original estimation sample.

If it is important that you use a postestimation command that can be used only on the original estimation sample, there is a way you can do that. You use the original data and then use estimates esample: to tell Stata what the original sample was.

See $[R]$ estimates save for details.

## Storing and restoring estimation results

Storing and restoring estimation results in memory is much like saving them to disk. You type

```
. estimates store base
```

to save the current estimation results under the name base, and you type
. estimates restore base
to get them back later. You can find out what you have stored by typing
. estimates dir
Saving estimation results to disk is more permanent than storing them in memory, so why would you want merely to store them? The answer is that, once they are stored, you can use other estimates commands to produce tables and reports from them.

See [R] estimates store for details about the estimates store and restore commands.

## Comparing estimation results

Let's say that you have done the following:

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. regress mpg weight displ
    (output omitted)
. estimates store base
. regress mpg weight displ foreign
    (output omitted)
. estimates store alt
```

You can now get a table comparing the coefficients:
. estimates table base alt

| Variable | base | alt |
| ---: | :---: | ---: |
| weight | -.00656711 | -.00677449 |
| displacement | .00528078 | .00192865 |
| foreign |  | -1.6006312 |
| _cons | 40.084522 | 41.847949 |

estimates table can do much more; see [R] estimates table. Also see [R] estimates stats. estimates stats works similarly to estimates table but produces model comparisons in terms of BIC and AIC.

## Jargon

You know that if you fit a model, say, by typing
. regress mpg weight displacement
then you can later replay the results by typing
. regress
and you can do tests and calculate other postestimation statistics by typing
. test displacement==0
. estat vif
. predict mpghat
As a result, we often refer to the estimation results or the current estimation results or the most recent estimation results or the last estimation results or the estimation results in memory.

With estimates store and estimates restore, you can have many estimation results in memory. One set of those, the set most recently estimated, or the set most recently restored, are the current or active estimation results, which you can replay, which you can test, or from which you can calculate postestimation statistics.

Current and active are the two words we will use interchangeably from now on.

## Also see

[P] _estimates - Manage estimation results

## Title

estimates describe - Describe estimation results

| Syntax | Menu | Description | Option |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Also see |  |

## Syntax

```
estimates describe
estimates describe name
estimates describe using filename [, number(#)]
```


## Menu

Statistics $>$ Postestimation $>$ Manage estimation results $>$ Describe results

## Description

estimates describe describes the current (active) estimates. Reported are the command line that produced the estimates, any title that was set by estimates title (see [R] estimates title), and any notes that were added by estimates notes (see [R] estimates notes).
estimates describe name does the same but reports results for estimates stored by estimates store (see [R] estimates store).
estimates describe using filename does the same but reports results for estimates saved by estimates save (see [R] estimates save). If filename contains multiple sets of estimates (saved in it by estimates save, append), the number of sets of estimates is also reported. If filename is specified without an extension, .ster is assumed.

## Option

number (\#) specifies that the \#th set of estimation results from filename be described. This assumes that multiple sets of estimation results have been saved in filename by estimates save, append. The default is number (1).

## Remarks and examples

estimates describe can be used to describe the estimation results currently in memory,

```
. estimates describe
Estimation results produced by
```

. regress mpg weight displ if foreign
or to describe results saved by estimates save in a .ster file:

```
. estimates describe using final
Estimation results "Final results" saved on 12apr2013 14:20, produced by
    . logistic myopic age sex drug1 drug2 if complete==1
Notes:
    1. Used file patient.dta
    2. "datasignature myopic age sex drug1 drug2 if complete==1"
        reports 148:5(58763):2252897466:3722318443
    3. must be reviewed by rgg
```


## Example 1

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. regress mpg weight displ if foreign
    (output omitted)
. estimates notes: file 'c(filename)'
. datasignature
    74:12(71728):3831085005:1395876116
. estimates notes: datasignature report 'r(datasignature)'
. estimates save foreign
file foreign.ster saved
. regress mpg weight displ if !foreign
    (output omitted)
estimates describe using foreign
    Estimation results saved on 02may2013 10:33, produced by
        . regress mpg weight displ if foreign
    Notes:
            1. file http://www.stata-press.com/data/r13/auto.dta
            2. datasignature report 74:12(71728):3831085005:1395876116
```


## Stored results

estimates describe and estimates describe name store the following in r() :
Macros

```
r(title) title
    r(cmdline) original command line
```

estimates describe using filename stores the above and the following in r() :
Scalars
$r$ (datetime) \%tc value of date/time file saved
$r$ (nestresults) number of sets of estimation results in file

## Also see

[R] estimates - Save and manipulate estimation results

## Title

estimates for - Repeat postestimation command across models
Syntax Description Options Remarks and examples Also see

## Syntax

estimates for namelist [, options]: postestimation_command
where namelist is a name, a list of names,__all, or *. A name may be ., meaning the current (active) estimates. _all and $*$ mean the same thing.
options
Description
noheader do not display title
nostop
do not stop if command fails

## Description

estimates for performs postestimation_command on each estimation result specified.

## Options

noheader suppresses the display of the header as postestimation_command is executed each time. nostop specifies that execution of postestimation_command is to be performed on the remaining models even if it fails on some.

## Remarks and examples

In the example that follows, we fit a model two different ways, store the results, and then use estimates for to perform the same test on both of them:
> Example 1

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. generate gpm = 1/mpg
. regress gpm i.foreign i.foreign#c.weight displ
(output omitted)
. estimates store reg
. qreg gpm i.foreign i.foreign#c.weight displ
(output omitted)
. estimates store qreg
```

estimates for reg qreg: test 0.foreign\#c.weight==1.foreign\#c.weight

Model reg
( 1) Ob.foreign\#c.weight - 1.foreign\#c.weight $=0$
$F(1,69)=4.87$
Prob $>\mathrm{F}=0.0307$

Model qreg
( 1) Ob.foreign\#c.weight - 1.foreign\#c.weight $=0$
$F(1,69)=0.03$
Prob > F = 0.8554

## Also see

[R] estimates - Save and manipulate estimation results

## Title

estimates notes - Add notes to estimation results

## Syntax Description Remarks and examples Also see

## Syntax

```
estimates notes: text
estimates notes
estimates notes list [in noterange]
estimates notes drop in noterange
```

where noterange is \# or \#/\# and where \# may be a number, the letter f (meaning first), or the letter 1 (meaning last).

## Description

estimates notes: text adds a note to the current (active) estimation results. estimates notes and estimates notes list list the current notes. estimates notes drop in noterange eliminates the specified notes.

## Remarks and examples

After adding or removing notes, if estimates have been stored, do not forget to store them again. If estimates have been saved, do not forget to save them again.

Notes are most useful when you intend to save estimation results in a file; see $[\mathrm{R}]$ estimates save. For instance, after fitting a model, you might type

```
. estimates note: I think these are final
. estimates save lock2
```

and then, later when going through your files, you could type

```
. estimates use lock2
. estimates notes
    1. I think these are final
```

Up to 9,999 notes can be attached to estimation results. If estimation results are important, we recommend that you add a note identifying the .dta dataset you used. The best way to do that is to type

```
. estimates notes: file 'c(filename)'
```

because 'c(filename)' will expand to include not just the name of the file but also its full path; see $[P]$ creturn.

If estimation results took a long time to estimate-say, they were produced by asmprobit or gllamm (see [R] asmprobit and http://www.gllamm.org) -it is also a good idea to add a data signature. A data signature takes less time to compute than reestimation when you need proof that you really have the right dataset. The easy way to do that is to type

```
. datasignature
74:12(71728):3831085005:1395876116
. estimates notes: datasignature reports 'r(datasignature)'
```

Now when you ask to see the notes, you will see

```
. estimates notes
    1. I think these are final
    2. file C:\project\one\pat4.dta
    3. datasignature reports 74:12(71728):3831085005:1395876116
```


## See [D] datasignature.

Notes need not be positive. You might set a note to be, "I need to check that age is defined correctly."
> Example 1

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. regress mpg weight displ if foreign
    (output omitted)
. estimates notes: file 'c(filename)'
. datasignature
    74:12(71728):3831085005:1395876116
. estimates notes: datasignature report 'r(datasignature)'
. estimates save foreign
file foreign.ster saved
. estimates notes list in 1/2
            1. file http://www.stata-press.com/data/r13/auto.dta
            2. datasignature report 74:12(71728):3831085005:1395876116
. estimates notes drop in 2
    (1 note dropped)
. estimates notes
    1. file http://www.stata-press.com/data/r13/auto.dta
```


## Also see

[R] estimates - Save and manipulate estimation results

## Title

estimates replay - Redisplay estimation results

## Syntax Menu Description Remarks and examples Also see

## Syntax

```
estimates replay
estimates replay namelist
```

where namelist is a name, a list of names,_all, or *. A name may be ., meaning the current (active) estimates. _all and $*$ mean the same thing.

## Menu

Statistics $>$ Postestimation $>$ Manage estimation results $>$ Redisplay estimation output

## Description

estimates replay redisplays the current (active) estimation results, just as typing the name of the estimation command would do.
estimates replay namelist redisplays each specified estimation result. The active estimation results are left unchanged.

## Remarks and examples

In the example that follows, we fit a model two different ways, store the results, use estimates for to perform the same test on both of them, and then replay the results:
$>$ Example 1

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. generate gpm = 1/mpg
. regress gpm i.foreign i.foreign#c.weight displ
(output omitted)
. estimates store reg
. qreg gpm i.foreign i.foreign#c.weight displ
(output omitted)
. estimates store qreg
```


## Model reg

( 1) Ob.foreign\#c.weight - 1.foreign\#c.weight $=0$
$F(1,69)=4.87$
Prob $>\mathrm{F}=0.0307$

Model qreg
( 1) Ob.foreign\#c.weight - 1.foreign\#c.weight $=0$
$F(1,69)=0.03$ Prob > F = 0.8554
estimates replay

Model qreg

| Median regression <br> Raw sum of deviations <br> Min sum of deviations <br> . 3201479 | Number of obs $=$ | 74 |  |
| :--- | :--- | :--- | :--- |
|  | (about .05) |  |  |


| gpm | Coef. | Std. Err. | t | $\mathrm{P}>\mid \mathrm{tI}$ | [95\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| foreign <br> Foreign | .0065352 | .0109777 | 0.60 | 0.554 | -.0153647 | .0284351 |
| foreign\# |  |  |  |  |  |  |
| c.weight |  |  |  |  |  |  |
| Domestic <br> Foreign | .0000147 | $2.93 \mathrm{e}-06$ | 5.00 | 0.000 | $8.81 \mathrm{e}-06$ | .0000205 |
| displacement | .0000155 | $4.17 \mathrm{e}-06$ | 3.71 | 0.000 | $7.16 \mathrm{e}-06$ | .0000238 |
| _cons | .0000179 | .0000239 | 0.75 | 0.457 | -.0000298 | .0000656 |

. estimates replay reg

| Model reg |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Source | SS | df | MS |  | Number of obs | $=74$ |
|  |  |  |  |  | F ( 4, 69) | 61.62 |
| Model <br> Residual | . 009342436 | 4.00 | 35609 |  | Prob > F | $=0.0000$ |
|  | . 002615192 | 69.00 | 37901 |  | R -squared | $=0.7813$ |
| Total | . 011957628 | 73.00 | 63803 |  | Adj R-squared Root MSE | $\begin{aligned} & =0.7686 \\ & =\quad .00616 \end{aligned}$ |
| gpm | Coef. | Std. Err. |  | $P>\|t\|$ | [95\% Conf. | Interval] |
| foreign | -. 0117756 | . 0086088 | -1.37 | 0.176 | -. 0289497 | . 0053986 |
| foreign\# <br> c.weight |  |  |  |  |  |  |
| Domestic | . 0000123 | $2.30 \mathrm{e}-06$ | 5.36 | 0.000 | $7.75 \mathrm{e}-06$ | . 0000169 |
| Foreign | . 00002 | $3.27 \mathrm{e}-06$ | 6.12 | 0.000 | . 0000135 | . 0000265 |
| displacement | . 0000296 | . 0000187 | 1.58 | 0.119 | -7.81e-06 | . 000067 |
| _cons | . 0053352 | . 0046748 | 1.14 | 0.258 | -. 0039909 | . 0146612 |

## Also see

[R] estimates - Save and manipulate estimation results

## Title

estimates save - Save and use estimation results

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Also see |  |

## Syntax

```
estimates save filename [, append replace]
estimates use filename [, number(#)]
estimates esample: [varlist] [if] [in] [weight]
    [, replace stringvars(varlist) zeroweight]
estimates esample
```


## Menu

## estimates save

Statistics $>$ Postestimation $>$ Manage estimation results $>$ Save to disk
estimates use
Statistics $>$ Postestimation $>$ Manage estimation results $>$ Load from disk

## Description

estimates save filename saves the current (active) estimation results in filename.
estimates use filename loads the results saved in filename into the current (active) estimation results.

In both cases, if filename is specified without an extension, .ster is assumed.
estimates esample: (note the colon) resets e(sample). After estimates use filename, $e$ (sample) is set to contain 0 , meaning that none of the observations currently in memory was used in obtaining the estimates.
estimates esample (without a colon) displays how e(sample) is currently set.

## Options

append, used with estimates save, specifies that results be appended to an existing file. If the file does not already exist, a new file is created.
replace, used with estimates save, specifies that filename can be replaced if it already exists.
number (\#), used with estimates use, specifies that the \#th set of estimation results from filename be loaded. This assumes that multiple sets of estimation results have been saved in filename by estimates save, append. The default is number (1).
replace, used with estimates esample:, specifies that e(sample) can be replaced even if it is already set.
stringvars(varlist), used with estimates esample:, specifies string variables. Observations containing variables that contain "" will be omitted from e(sample).
zeroweight, used with estimates esample:, specifies that observations with zero weights are to be included in e(sample).

## Remarks and examples

See [R] estimates for an overview of the estimates commands.
For a description of estimates save and estimates use, see Saving and using estimation results in $[\mathrm{R}]$ estimates.

The rest of this entry concerns e(sample).
Remarks are presented under the following headings:

```
Setting e(sample)
Resetting e(sample)
Determining who set e(sample)
```


## Setting e(sample)

After estimates use filename, the situation is nearly identical to what it was immediately after you fit the model. The one difference is that e (sample) is set to 0 .
$e$ (sample) is Stata's function to mark which observations among those currently in memory were used in producing the estimates. For instance, you might type

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. regress mpg weight displ if foreign
    (output omitted)
. summarize mpg if e(sample)
    (output omitted)
```

and summarize would report the summary statistics for the observations regress in fact used, which would exclude not only observations for which foreign $=0$ but also any observations for which mpg , weight, or displ was missing.

If you saved the above estimation results and then reloaded them, however, summarize mpg if e(sample) would produce

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | :---: | :---: | :---: | :---: | :---: |
| $m p g$ | 0 |  |  |  |  |

Stata thinks that none of these observations was used in producing the estimates currently loaded.

What else could Stata think? When you estimates use filename, you do not have to have the original data in memory. Even if you do have data in memory that look like the original data, they might not be. Setting e (sample) to 0 is the safe thing to do. There are some postestimation statistics, for instance, that are appropriate only when calculated on the estimation sample. Setting e (sample) to 0 ensures that if you ask for one of them, you will get back a null result.

We recommend that you leave e(sample) set to 0 . But what if you really need to calculate that postestimation statistic? Well, you can get it, but you are going to be responsible for setting $e$ (sample) correctly. Here we just happen to know that all the observations with foreign $=1$ were used, so we can type

```
. estimates esample: if foreign
```

If all the observations had been used, we could simply type

```
. estimates esample:
```

The safe thing to do, however, is to look at the estimation command-estimates describe will show it to you-and then type
. estimates esample: mpg weight displ if foreign
We include all observations with foreign $=1$, excluding any with missing values in the mpg, weight, or displ variable, that are to be treated as the estimation sample.

## Resetting e(sample)

estimates esample: will allow you to not only set but also reset e(sample). If e(sample) has already been set (say that you just fit the model) and you try to set it, you will see

```
. estimates esample: mpg weight displ if foreign
no; e(sample) already set
r(322);
```

Here you can specify the replace option:

```
. estimates esample: mpg weight displ if foreign, replace
```

We do not recommend resetting e (sample), but the situation can arise where you need to. Imagine that you estimates use filename, you set e(sample), and then you realize that you set it wrong. Here you would want to reset it.

## Determining who set e(sample)

estimates esample without a colon will report whether and how e(sample) was set. You might see

```
. estimates esample
e(sample) set by estimation command
```

or

```
. estimates esample
e(sample) set by user
```

or

```
. estimates esample
e(sample) not set (0 assumed)
```


## Stored results

estimates esample without the colon saves macro r(who), which will contain cmd, user, or zero'd.

## Also see

[R] estimates - Save and manipulate estimation results

## Title

estimates stats - Model-selection statistics

| Syntax | Menu | Description | Option |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | Also see |

## Syntax

```
estimates stats [namelist][, n(#)]
```

where namelist is a name, a list of names,_all, or *. A name may be ., meaning the current (active) estimates. _all and $*$ mean the same thing.

## Menu

Statistics $>$ Postestimation $>$ Manage estimation results $>$ Table of fit statistics

## Description

estimates stats reports model-selection statistics, including the Akaike information criterion (AIC) and the Bayesian information criterion (BIC). These measures are appropriate for maximum likelihood models.

If estimates stats is used for a non-likelihood-based model, such as qreg, missing values are reported.

## Option

$\mathrm{n}(\#)$ specifies the $N$ to be used in calculating BIC; see [R] BIC note.

## Remarks and examples

If you type estimates stats without arguments, a table for the most recent estimation results will be shown:

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. logistic foreign mpg weight displ
    (output omitted)
. estimates stats
Akaike's information criterion and Bayesian information criterion
\begin{tabular}{r|rrrrrr}
\hline Model & Obs & ll(null) & ll(model) & df & AIC & BIC \\
\hline. & 74 & -45.03321 & -20.59083 & 4 & 49.18167 & 58.39793 \\
\hline
\end{tabular}
Note: \(N=O b s\) used in calculating BIC; see [R] BIC note
```

Regarding the note at the bottom of the table, $N$ is an ingredient in the calculation of BIC; see $[\mathrm{R}]$ BIC note. The note changes if you specify the n() option, which tells estimates stats what $N$ to use. $N=0 \mathrm{bs}$ is the default.

Regarding the table itself, 11 (null) is the log likelihood for the constant-only model, 11 (model) is the $\log$ likelihood for the model, df is the number of degrees of freedom, and AIC and BIC are the Akaike and Bayesian information criteria.

Models with smaller values of an information criterion are considered preferable.
estimates stats can compare estimation results:
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. logistic foreign mpg weight displ (output omitted)
. estimates store full
. logistic foreign mpg weight (output omitted)
. estimates store sub
. estimates stats full sub
Akaike's information criterion and Bayesian information criterion

| Model | Obs | 11 (null) | 11 (model) | df | AIC | BIC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| full | 74 | -45.03321 | -20.59083 | 4 | 49.18167 | 58.39793 |
| sub | 74 | -45.03321 | -27.17516 | 3 | 60.35031 | 67.26251 |

## Stored results

estimates stats stores the following in $r()$ :
Matrices
$r(S) \quad$ matrix with 6 columns ( $\mathrm{N}, 110,11, \mathrm{df}, \mathrm{AIC}$, and BIC) and rows corresponding to models in table

## Methods and formulas

See [R] BIC note.

## Also see

[R] estimates - Save and manipulate estimation results

## Title

estimates store - Store and restore estimation results

| Syntax | Menu | Description | Option |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | References | Also see |

## Syntax

```
estimates store name [, nocopy]
estimates restore name
estimates query
estimates dir [namelist]
estimates drop namelist
estimates clear
```

where namelist is a name, a list of names, _all, or $*$. _all and $*$ mean the same thing.

## Menu

## estimates store

Statistics $>$ Postestimation $>$ Manage estimation results $>$ Store in memory

## estimates restore

Statistics $>$ Postestimation $>$ Manage estimation results $>$ Restore from memory

## estimates dir

Statistics $>$ Postestimation $>$ Manage estimation results $>$ List results stored in memory

## estimates drop

Statistics $>$ Postestimation $>$ Manage estimation results $>$ Drop from memory

## Description

estimates store name stores the current (active) estimation results under the name name.
estimates restore name loads the results stored under name into the current (active) estimation results.
estimates query tells you whether the current (active) estimates have been stored and, if so, the name.
estimates dir displays a list of the stored estimates.
estimates drop namelist drops the specified stored estimation results.
estimates clear drops all stored estimation results.
estimates clear, estimates drop _all, and estimates drop * do the same thing. estimates drop and estimates clear do not eliminate the current (active) estimation results.

## Option

nocopy, used with estimates store, specifies that the current (active) estimation results are to be moved into name rather than copied. Typing
. estimates store hold, nocopy
is the same as typing
. estimates store hold
. ereturn clear
except that the former is faster. The nocopy option is sometimes used by programmers.

## Remarks and examples

estimates store stores estimation results in memory so that you can access them later.

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. regress mpg weight displ
(output omitted)
. estimates store myreg
```

. . . . you do other things, including fitting other models . . .
. estimates restore myreg
. regress
(same output shown again)
After estimates restore myreg, things are once again just as they were, estimationwise, just after you typed regress mpg weight displ.
estimates store stores results in memory. When you exit Stata, those stored results vanish. If you wish to make a permanent copy of your estimation results, see $[R]$ estimates save.

The purpose of making copies in memory is 1 ) so that you can quickly switch between them and 2) so that you can make tables comparing estimation results. Concerning the latter, see [R] estimates table and $[R]$ estimates stats.

## Stored results

estimates dir stores the following in $r()$ :
Macros
$r$ (names) names of stored results

## References

Jann, B. 2005. Making regression tables from stored estimates. Stata Journal 5: 288-308.

- 2007. Making regression tables simplified. Stata Journal 7: 227-244.


## Also see

[R] estimates - Save and manipulate estimation results

## Title

estimates table - Compare estimation results

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | References | Also see |

## Syntax

estimates table [namelist] [, options]
where namelist is a name, a list of names, _all, or *. A name may be ., meaning the current (active) estimates. _all and $*$ mean the same thing.
options
Main
stats (scalarlist)
star[(\#1 \#2 \#3)]
Options
keep (coeflist)
drop(coeflist)
equations(matchlist)
Numerical formats
$\mathrm{b}[(\% f m t)]$
se $[(\% f m t)]$
$\mathrm{t}[(\% f m t)]$
$\mathrm{p}[(\% f m t)]$
stfmt ( $\%$ fint)
General format
varwidth(\#)
modelwidth(\#)
eform
varlabel
newpanel
style(oneline)
style(columns)
style(noline)
coded

Description
report scalarlist in table use stars to denote significance levels
report coefficients in order specified omit specified coefficients from table match equations of models as specified
how to format coefficients, which are always reported report standard errors and use optional format
report $t$ or $z$ and use optional format report $p$-values and use optional format how to format scalar statistics
use \# characters to display variable names and statistics use \# characters to display model names
display coefficients in exponentiated form display variable labels rather than variable names display statistics in separate table from coefficients
put vertical line after variable names; the default
put vertical line separating every column
suppress all vertical lines
display compact table

```
Reporting
    display_options
    title(string) title for table
```

    title() does not appear in the dialog box.
    where

- A scalarlist is a list of any or all of the names of scalars stored in e(), plus aic, bic, and rank.
- \#1 \#2 \#3 are three numbers such as . 05 . 01 . 001.
- A coeflist is a list of coefficient names, each name of which may be simple (for example, price), an equation name followed by a colon (for example, mean:), or a full name (for example, mean:price). Names are separated by blanks.
- A matchlist specifies how equations from different estimation results are to be matched. If you need to specify a matchlist, the solution is usually 1 , as in equations(1). The full syntax is

$$
\begin{aligned}
\text { matchlist }:= & \text { term }[, \text { term } \ldots] \\
\text { term }:= & {[\text { eqname }=] \#: \# . . . \# } \\
& {[\text { eqname }=] \# }
\end{aligned}
$$

See equations () under Options below.

## Menu

Statistics $>$ Postestimation $>$ Manage estimation results $>$ Table of estimation results

## Description

estimates table displays a table of coefficients and statistics for one or more sets of estimation results.

## Options



| aic | Akaike's information criterion |
| :--- | :--- |
| bic | Schwarz's Bayesian information criterion |
| rank | rank of $e(V)$ (\# of free parameters in model) |

along with the names of any scalars stored in $e()$. The specified statistics do not have to be available for all estimation results being displayed.

For example, stats ( N 1 l chi2 aic) specifies that $\mathrm{e}(\mathrm{N})$, e(ll), e(chi2), and AIC be included. In Stata, e(N) records the number of observations; e(ll), the log likelihood; and e(chi2), the chi-squared test that all coefficients in the first equation of the model are equal to zero.
star and star (\#1 \#2 \#3) specify that stars (asterisks) are to be used to mark significance. The second syntax specifies the significance levels for one, two, and three stars. If you specify simply star, that is equivalent to specifying star (.05 . 01 .001), which means one star (*) if $p<0.05$, two stars ( ${ }^{* *}$ ) if $p<0.01$, and three stars $(* * *)$ if $p<0.001$.
The star and star() options may not be combined with se, t, or poption.

## Options

keep (coeflist) and drop (coeflist) are alternatives; they specify coefficients to be included or omitted from the table. The default is to display all coefficients.
If keep () is specified, it specifies not only the coefficients to be included but also the order in which they appear.

A coeflist is a list of coefficient names, each name of which may be simple (for example, price), an equation name followed by a colon (for example, mean:), or a full name (for example, mean: price). Names are separated from each other by blanks.
When full names are not specified, all coefficients that match the partial specification are included. For instance, drop (_cons) would omit _cons for all equations.
equations (matchlist) specifies how the equations of the models in namelist are to be matched. The default is to match equations by name. Matching by name usually works well when all results were fit by the same estimation command. When you are comparing results from different estimation commands, however, specifying equations() may be necessary.
The most common usage is equations(1), which indicates that all first equations are to be matched into one equation named \#1.
matchlist has the syntax

$$
\operatorname{term}[, \text { term } \ldots]
$$

where term is

$$
\begin{array}{ll}
{[\text { eqname }=] \#: \# \ldots: \#} & (\text { syntax } 1) \\
{[\text { eqname }=] \#} & (\operatorname{syntax} 2)
\end{array}
$$

In syntax 1, each \# is a number or a period (.). If a number, it specifies the position of the equation in the corresponding model; 1:3:1 would indicate that equation 1 in the first model matches equation 3 in the second, which matches equation 1 in the third. A period indicates that there is no corresponding equation in the model; 1:.:1 indicates that equation 1 in the first matches equation 1 in the third.
In syntax 2 , you specify just one number, say, 1 or 2 , and that is shorthand for $1: 1 \ldots: 1$ or $2: 2 \ldots: 2$, meaning that equation 1 matches across all models specified or that equation 2 matches across all models specified.

Now that you can specify a term, you can put that together into a matchlist by separating one term from the other by commas. In what follows, we will assume that three names were specified,

```
. estimates table alpha beta gamma, ...
```

equations(1) is equivalent to equations(1:1:1); we would be saying that the first equations match across the board.
equations(1:.:1) would specify that equation 1 matches in models alpha and gamma but that there is nothing corresponding in model beta.
equations $(1,2)$ is equivalent to equations $(1: 1: 1,2: 2: 2)$. We would be saying that the first equations match across the board and so do the second equations.
equations (1, 2:.:2) would specify that the first equations match across the board, that the second equations match for models alpha and gamma, and that there is nothing equivalent to equation 2 in model beta.
If equations() is specified, equations not matched by position are matched by name.

## Numerical formats

$\mathrm{b}(\% f m t)$ specifies how the coefficients are to be displayed. You might specify $\mathrm{b}(\% 9.2 \mathrm{f})$ to make decimal points line up. There is also a b option, which specifies that coefficients are to be displayed, but that is just included for consistency with the $s e$, $t$, and $p$ options. Coefficients are always displayed.
$\mathrm{se}, \mathrm{t}$, and p specify that standard errors, $t$ or $z$ statistics, and significance levels are to be displayed. The default is not to display them. se $(\% f m t)$, $\mathrm{t}(\% f m t)$, and $\mathrm{p}(\% f m t)$ specify that each is to be displayed and specifies the display format to be used to format them.
stfmt ( $\%$ fmt ) specifies the format for displaying the scalar statistics included by the stats() option.

## General format

varwidth (\#) specifies the number of character positions used to display the names of the variables and statistics. The default is 12 .
modelwidth(\#) specifies the number of character positions used to display the names of the models. The default is 12 .
eform displays coefficients in exponentiated form. For each coefficient, $\exp (\boldsymbol{\beta})$ rather than $\boldsymbol{\beta}$ is displayed, and standard errors are transformed appropriately. Display of the intercept, if any, is suppressed.
varlabel specifies that variable labels be displayed instead of variable names.
newpanel specifies that the statistics be displayed in a table separated by a blank line from the table with coefficients rather than in the style of another equation in the table of coefficients.
style (stylespec) specifies the style of the coefficient table.
style(oneline) specifies that a vertical line be displayed after the variables but not between the models. This is the default.
style(columns) specifies that vertical lines be displayed after each column.
style(noline) specifies that no vertical lines be displayed.
coded specifies that a compact table be displayed. This format is especially useful for comparing variables that are included in a large collection of models.

Reporting
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), and fvwrapon(style); see [R] estimation options.

The following option is available with estimates table but is not shown in the dialog box:
title(string) specifies the title to appear above the table.

## Remarks and examples

If you type estimates table without arguments, a table of the most recent estimation results will be shown:

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. regress mpg weight displ
    (output omitted)
. estimates table
\begin{tabular}{r|r}
\hline Variable & active \\
\hline weight & -.00656711 \\
displacement & .00528078 \\
_cons & 40.084522 \\
\hline
\end{tabular}
```

The real use of estimates table, however, is for comparing estimation results, and that requires using it after estimates store:

```
- regress mpg weight displ
(output omitted)
. estimates store base
. regress mpg weight displ foreign
(output omitted)
. estimates store alt
. qreg mpg weight displ foreign
(output omitted)
- estimates store qreg
. estimates table base alt qreg, stats(r2)
```

| Variable | base | alt | qreg |
| ---: | :---: | ---: | ---: |
| weight | -.00656711 | -.00677449 | -.00595056 |
| displacement | .00528078 | .00192865 | .00018552 |
| foreign |  | -1.6006312 | -2.1326004 |
| _cons | 40.084522 | 41.847949 | 39.213348 |
| r 2 | .6529307 | .66287957 |  |

## Stored results

estimates table stores the following in r() :
Macros
$r$ (names) names of results used
Matrices

$$
\begin{array}{ll}
\mathrm{r} \text { (coef) } & \text { matrix } M: n \times 2 * m \\
& M[i, 2 j-1]=i \text { th parameter estimate for model } j ; \\
& M[i, 2 j]=\text { variance of } M[i, 2 j-1] ; i=1, \ldots, n ; j=1, \ldots, m \\
\text { r(stats) } & \text { matrix } S: k \times m \text { (if option stats } \text { ) specified) } \\
& S[i, j]=i \text { th statistic for model } j ; i=1, \ldots, k ; j=1, \ldots, m
\end{array}
$$

## References

Gallup, J. L. 2012. A new system for formatting estimation tables. Stata Journal 12: 3-28.
Weiss, M. 2010. Stata tip 90: Displaying partial results. Stata Journal 10: 500-502.

## Also see

[R] estimates - Save and manipulate estimation results

## Title

estimates title - Set title for estimation results

Syntax Menu Description Remarks and examples Also see

## Syntax

```
estimates title: [text]
estimates title
```


## Menu

Statistics $>$ Postestimation $>$ Manage estimation results $>$ Title/retitle results

## Description

estimates title: (note the colon) sets or clears the title for the current estimation results. The title is used by estimates table and estimates stats (see [R] estimates table and [R] estimates stats).
estimates title without the colon displays the current title.

## Remarks and examples

After setting the title, if estimates have been stored, do not forget to store them again:

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. regress mpg gear turn
    (output omitted)
. estimates store reg
```

Now let's add a title:

```
. estimates title: "My regression"
```

- estimates store reg


## Also see

[R] estimates - Save and manipulate estimation results

## Title

Syntax Description Options Also see

## Syntax

| estimation_cmd $\ldots[$, options $]$ |  |
| ---: | ---: |
| options | Description |

Model
noconstant
offset (varname ${ }_{o}$ )
exposure (varname $e_{e}$ )
constraints (constraints)
collinear
Reporting
level(\#)
noskip
nocnsreport
noomitted
vsquish
noemptycells
baselevels
allbaselevels
nofvlabel
fvwrap(\#)
fvwrapon(style)
cformat ( $\%$ fmt)
pformat (\%fmt)
sformat ( $\% f m t$ )
nolstretch
Integration
intmethod (intmethod)
intpoints(\#)
coeflegend
suppress constant term
include varname $_{o}$ in model with coefficient constrained to 1
include $\ln \left(\right.$ varname $\left._{e}\right)$ in model with coefficient constrained to 1
apply specified linear constraints
keep collinear variables
set confidence level; default is level (95)
perform overall model test as a likelihood-ratio test
do not display constraints
do not display omitted collinear variables
suppress blank space separating factor variables or time-series variables
do not display empty interaction cells of factor variables
report base levels whose bases cannot be inferred
display all base levels for factor variables and interactions
display factor-variable level values rather than value labels
allow \# lines when wrapping long value labels
apply style for wrapping long value labels;
style may be word or width
format for coefficients, standard errors, and confidence limits
format for $p$-values
format for test statistics
do not automatically widen coefficient table for long variable names
integration method for random-effects models
use \# integration (quadrature) points
display legend instead of statistics

## Description

This entry describes the options common to many estimation commands. Not all the options documented here work with all estimation commands. See the documentation for the particular estimation command; if an option is listed there, it is applicable.

## Options

## Model

noconstant suppresses the constant term (intercept) in the model.
offset $\left(\right.$ varname $\left._{o}\right)$ specifies that varname $_{o}$ be included in the model with the coefficient constrained to be 1 .
exposure (varname $e_{e}$ ) specifies a variable that reflects the amount of exposure over which the depvar events were observed for each observation; $\ln \left(\right.$ varname $\left._{e}\right)$ with coefficient constrained to be 1 is entered into the log-link function.
constraints (numlist $\mid$ matname) specifies the linear constraints to be applied during estimation. The default is to perform unconstrained estimation. See $[R]$ reg3 for the use of constraints in multiple-equation contexts.
constraints (numlist) specifies the constraints by number after they have been defined by using the constraint command; see [R] constraint. Some commands (for example, slogit) allow only constraints (numlist).
constraints (matname) specifies a matrix containing the constraints; see [P] makecns.
constraints (clist) is used by some estimation commands, such as mlogit, where clist has the form \# $[-\#][, \#[-\#] \ldots]$.
collinear specifies that the estimation command not omit collinear variables. Usually, there is no reason to leave collinear variables in place, and, in fact, doing so usually causes the estimation to fail because of the matrix singularity caused by the collinearity. However, with certain models, the variables may be collinear, yet the model is fully identified because of constraints or other features of the model. In such cases, using the collinear option allows the estimation to take place, leaving the equations with collinear variables intact. This option is seldom used.

## Reporting

level(\#) specifies the confidence level, as a percentage, for confidence intervals. The default is level (95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals.
noskip specifies that a full maximum-likelihood model with only a constant for the regression equation be fit. This model is not displayed but is used as the base model to compute a likelihood-ratio test for the model test statistic displayed in the estimation header. By default, the overall model test statistic is an asymptotically equivalent Wald test of all the parameters in the regression equation being zero (except the constant). For many models, this option can substantially increase estimation time.
nocnsreport specifies that no constraints be reported. The default is to display user-specified constraints above the coefficient table.
noomitted specifies that variables that were omitted because of collinearity not be displayed. The default is to include in the table any variables omitted because of collinearity and to label them as "(omitted)".
vsquish specifies that the blank space separating factor-variable terms or time-series-operated variables from other variables in the model be suppressed.
noemptycells specifies that empty cells for interactions of factor variables not be displayed. The default is to include in the table interaction cells that do not occur in the estimation sample and to label them as "(empty)".
baselevels and allbaselevels control whether the base levels of factor variables and interactions are displayed. The default is to exclude from the table all base categories.
baselevels specifies that base levels be reported for factor variables and for interactions whose bases cannot be inferred from their component factor variables.
allbaselevels specifies that all base levels of factor variables and interactions be reported.
nofvlabel displays factor-variable level values rather than attached value labels. This option overrides the fvlabel setting; see [R] set showbaselevels.
fvwrap(\#) specifies how many lines to allow when long value labels must be wrapped. Labels requiring more than \# lines are truncated. This option overrides the fvwrap setting; see [R] set showbaselevels.
fvwrapon(style) specifies whether value labels that wrap will break at word boundaries or break based on available space.
fvwrapon(word), the default, specifies that value labels break at word boundaries.
fvwrapon(width) specifies that value labels break based on available space.
This option overrides the fvwrapon setting; see [R] set showbaselevels.
cformat ( $\%$ fint) specifies how to format coefficients, standard errors, and confidence limits in the coefficient table. The maximum format width is 9 .
pformat (\% fint) specifies how to format $p$-values in the coefficient table. The maximum format width is 5 .
sformat ( $\%$ fimt) specifies how to format test statistics in the coefficient table. The maximum format width is 8 .
nolstretch specifies that the width of the coefficient table not be automatically widened to accommodate longer variable names. The default, lstretch, is to automatically widen the coefficient table up to the width of the Results window. To change the default, use set lstretch off. nolstretch is not shown in the dialog box.

[^17]The following option is not shown in the dialog box:
coeflegend specifies that the legend of the coefficients and how to specify them in an expression be displayed rather than displaying the statistics for the coefficients.

## Also see

[U] 20 Estimation and postestimation commands

## Title

exit — Exit Stata

## Syntax Description Option Remarks and examples Also see

## Syntax

```
exit [, clear]
```


## Description

Typing exit causes Stata to stop processing and return control to the operating system. If the dataset in memory has changed since the last save command, you must specify the clear option before Stata will let you exit.
exit may also be used for exiting do-files or programs; see [P] exit.
Stata for Windows users may also exit Stata by clicking on the Close button or by pressing Alt + F4.
Stata for Mac users may also exit Stata by pressing Command+Q.
Stata(GUI) users may also exit Stata by clicking on the Close button.

## Option

clear permits you to exit, even if the current dataset has not been saved.

## Remarks and examples

Type exit to leave Stata and return to the operating system. If the dataset in memory has changed since the last time it was saved, however, Stata will refuse. At that point, you can either save the dataset and then type exit, or type exit, clear:

```
. exit
no; data in memory would be lost
r(4);
. exit, clear
```


## Also see

[P] exit - Exit from a program or do-file

## Title

exlogistic - Exact logistic regression

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

```
exlogistic depvar indepvars [if] [in] [weight] [, options]
```

options Description
Model
condvars (varlist)
condition on variables in varlist
group (varname) groups/strata are stratified by unique values of varname
binomial (varname|\#) data are in binomial form and the number of trials is contained in varname or in \#
estconstant
noconstant
estimate constant term; do not condition on the number of successes suppress constant term

Terms
terms (termsdef) terms definition
Options
memory (\#[b|k|m|g]) saving (filename)

Reporting
coef report estimated coefficients
test (testopt) report significance of observed sufficient statistic, conditional scores test,
midp
nolog
level(\#) set confidence level; default is level(95) or conditional probabilities test
mue (varlist) compute the median unbiased estimates for varlist
set limit on memory usage; default is memory ( 10 m )
save the joint conditional distribution to filename
use the mid- $p$-value rule
do not display the enumeration $\log$
by, statsby, and xi are allowed; see [U] 11.1.10 Prefix commands.
fweights are allowed; see [U] 11.1.6 weight.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Exact statistics $>$ Exact logistic regression

## Description

exlogistic fits an exact logistic regression model of depvar on indepvars.
exlogistic is an alternative to logistic, the standard maximum-likelihood-based logistic regression estimator; see $[R]$ logistic. exlogistic produces more-accurate inference in small samples because it does not depend on asymptotic results and exlogistic can better deal with one-way causation, such as the case where all females are observed to have a positive outcome.
exlogistic with the group (varname) option is an alternative to clogit, the conditional logistic regression estimator; see [R] clogit. Like clogit, exlogistic conditions on the number of positive outcomes within stratum.
depvar can be specified in two ways. It can be zero/nonzero, with zero indicating failure and nonzero representing positive outcomes (successes), or if you specify the binomial (varname|\#) option, depvar may contain the number of positive outcomes within each trial.
exlogistic is computationally intensive. Unlike most estimators, rather than calculating coefficients for all independent variables at once, results for each independent variable are calculated separately with the other independent variables temporarily conditioned out. You can save considerable computer time by skipping the parameter calculations for variables that are not of direct interest. Specify such variables in the condvars() option rather than among the indepvars; see condvars() below.

Unlike Stata's other estimation commands, you may not use test, lincom, or other postestimation commands after exlogistic. Given the method used to calculate estimates, hypothesis tests must be performed during estimation by using exlogistic's terms() option; see terms() below.

## Options

## Model

condvars (varlist) specifies variables whose parameter estimates are not of interest to you. You can save substantial computer time and memory moving such variables from indepvars to condvars (). Understand that you will get the same results for x 1 and x 3 whether you type

```
. exlogistic y x1 x2 x3 x4
```

or

```
. exlogistic y x1 x3, condvars(x2 x4)
```

group (varname) specifies the variable defining the strata, if any. A constant term is assumed for each stratum identified in varname, and the sufficient statistics for indepvars are conditioned on the observed number of successes within each group. This makes the model estimated equivalent to that estimated by clogit, Stata's conditional logistic regression command (see [R] clogit). group() may not be specified with noconstant or estconstant.
binomial (varname |\#) indicates that the data are in binomial form and depvar contains the number of successes. varname contains the number of trials for each observation. If all observations have the same number of trials, you can instead specify the number as an integer. The number of trials must be a positive integer at least as great as the number of successes. If binomial () is not specified, the data are assumed to be Bernoulli, meaning that depvar equaling zero or nonzero records one failure or success.
estconstant estimates the constant term. By default, the models are assumed to have an intercept (constant), but the value of the intercept is not calculated. That is, the conditional distribution of
the sufficient statistics for the indepvars is computed given the number of successes in depvar, thus conditioning out the constant term of the model. Use estconstant if you want the estimate of the intercept reported. estconstant may not be specified with group().
noconstant; see $[R]$ estimation options. noconstant may not be specified with group().
 terms of the model on which you want exlogistic to perform joint-significance hypothesis tests. By default, exlogistic reports tests individually on each variable in indepvars. For instance, if variables $x 1$ and $x 3$ are in indepvars, and you want to jointly test their significance, specify terms ( $\mathrm{t} 1=\mathrm{x} 1 \mathrm{x} 3$ ). To also test the joint significance of x 2 and x 4 , specify terms ( $\mathrm{t} 1=\mathrm{x} 1 \mathrm{x} 3$, $\mathrm{t} 2=\mathrm{x} 2 \mathrm{x} 4$ ). Each variable can be assigned to only one term.
Joint tests are computed only for the conditional scores tests and the conditional probabilities tests. See the test () option below.

Options
memory ( $\#[\mathrm{~b}|\mathrm{k}| \mathrm{m} \mid \mathrm{g}]$ ) sets a limit on the amount of memory exlogistic can use when computing the conditional distribution of the parameter sufficient statistics. The default is memory ( 10 m ), where m stands for megabyte, or $1,048,576$ bytes. The following are also available: b stands for byte; k stands for kilobyte, which is equal to 1,024 bytes; and g stands for gigabyte, which is equal to 1,024 megabytes. The minimum setting allowed is 1 m and the maximum is 2048 m or 2 g , but do not attempt to use more memory than is available on your computer. Also see the first technical note under example 4 on counting the conditional distribution.
saving (filename [, replace]) saves the joint conditional distribution to filename. This distribution is conditioned on those variables specified in condvars(). Use replace to replace an existing file with filename. A Stata data file is created containing all the feasible values of the parameter sufficient statistics. The variable names are the same as those in indepvars, in addition to a variable named ${ }_{-} f_{-}$containing the feasible value frequencies (sometimes referred to as the condition numbers).

## Reporting

level(\#); see [R] estimation options. The level(\#) option will not work on replay because confidence intervals are based on estimator-specific enumerations. To change the confidence level, you must refit the model.
coef reports the estimated coefficients rather than odds ratios (exponentiated coefficients). coef may be specified when the model is fit or upon replay. coef affects only how results are displayed and not how they are estimated.
test (sufficient|score|probability) reports the significance level of the observed sufficient statistics, the conditional scores tests, or the conditional probabilities tests, respectively. The default is test (sufficient). If terms() is included in the specification, the conditional scores test and the conditional probabilities test are applied to each term providing conditional inference for several parameters simultaneously. All the statistics are computed at estimation time regardless of which is specified. Each statistic may thus also be displayed postestimation without having to refit the model; see $[R]$ exlogistic postestimation.
mue (varlist) specifies that median unbiased estimates (MUEs) be reported for the variables in varlist. By default, the conditional maximum likelihood estimates (CMLEs) are reported, except for those parameters for which the CMLEs are infinite. Specify mue (_all) if you want MUEs for all the indepvars.
midp instructs exlogistic to use the mid- $p$-value rule when computing the MUEs, significance levels, and confidence intervals. This adjustment is for the discreteness of the distribution and halves the value of the discrete probability of the observed statistic before adding it to the $p$-value. The mid- $p$-value rule cannot be applied to MUEs whose corresponding parameter CMLE is infinite.
nolog prevents the display of the enumeration log. By default, the enumeration $\log$ is displayed, showing the progress of computing the conditional distribution of the sufficient statistics.

## Remarks and examples

Exact logistic regression is the estimation of the logistic model parameters by using the conditional distribution of the parameter sufficient statistics. The estimates are referred to as the conditional maximum likelihood estimates (CMLEs). This technique was first introduced by Cox and Snell (1989) as an alternative to using maximum likelihood estimation, which can perform poorly for small sample sizes. For stratified data, exact logistic regression is a small-sample alternative to conditional logistic regression. See $[R]$ logit, $[R]$ logistic, and $[R]$ clogit to obtain maximum likelihood estimates (MLEs) for the logistic model and the conditional logistic model. For a comprehensive overview of exact logistic regression, see Mehta and Patel (1995).

Let $Y_{i}$ denote a Bernoulli random variable where we observe the outcome $Y_{i}=y_{i}, i=1, \ldots, n$. Associated with each independent observation is a $1 \times p$ vector of covariates, $\mathbf{x}_{i}$. We will denote $\pi_{i}=\operatorname{Pr}\left(Y_{i} \mid \mathbf{x}_{i}\right)$ and let the logit function model the relationship between $Y_{i}$ and $\mathbf{x}_{i}$,

$$
\log \left(\frac{\pi_{i}}{1-\pi_{i}}\right)=\theta+\mathbf{x}_{i} \boldsymbol{\beta}
$$

where the constant term $\theta$ and the $1 \times p$ vector of regression parameters $\boldsymbol{\beta}$ are unknown. The probability of observing $Y_{i}=y_{i}, i=1, \ldots, n$, is

$$
\operatorname{Pr}(\mathbf{Y}=\mathbf{y})=\prod_{i=1}^{n} \pi_{i}^{y_{i}}\left(1-\pi_{i}\right)^{1-y_{i}}
$$

where $\mathbf{Y}=\left(Y_{1}, \ldots, Y_{n}\right)$ and $\mathbf{y}=\left(y_{1}, \ldots, y_{n}\right)$. The MLEs for $\theta$ and $\boldsymbol{\beta}$ maximize the $\log$ of this function.

The sufficient statistics for $\theta$ and $\beta_{j}, j=1, \ldots, p$, are $M=\sum_{i=1}^{n} Y_{i}$ and $T_{j}=\sum_{i=1}^{n} Y_{i} x_{i j}$, respectively, and we observe $M=m$ and $T_{j}=t_{j}$. By default, exlogistic tallies the conditional distribution of $\mathbf{T}=\left(T_{1}, \ldots, T_{p}\right)$ given $M=m$. This distribution will have a size of $\binom{n}{m}$. (It would have a size of $2^{n}$ without conditioning on $M=m$.) Denote one of these vectors $\mathbf{T}^{(k)}=$ $\left(t_{1}^{(k)}, \ldots, t_{p}^{(k)}\right), k=1, \ldots, N$, with combinatorial coefficient (frequency) $c_{k}, \sum_{k=1}^{N} c_{k}=\binom{n}{m}$. For each independent variable $x_{j}, j=1, \ldots, p$, we reduce the conditional distribution further by conditioning on all other observed sufficient statistics $T_{l}=t_{l}, l \neq j$. The conditional probability of observing $T_{j}=t_{j}$ has the form

$$
\operatorname{Pr}\left(T_{j}=t_{j} \mid T_{l}=t_{l}, l \neq j, M=m\right)=\frac{c e^{t_{j} \beta_{j}}}{\sum_{k} c_{k} e^{t_{j}^{(k)} \beta_{j}}}
$$

where the sum is over the subset of $\mathbf{T}$ vectors such that $\left(T_{1}^{(k)}=t_{1}, \ldots, T_{j}^{(k)}=t_{j}^{(k)}, \ldots, T_{p}^{(k)}=t_{p}\right)$ and $c$ is the combinatorial coefficient associated with the observed $\mathbf{t}$. The CMLE for $\beta_{j}$ maximizes the $\log$ of this function.

Specifying nuisance variables in condvars() will reduce the size of the conditional distribution by conditioning on their observed sufficient statistics as well as conditioning on $M=m$. This reduces the amount of memory consumed at the cost of not obtaining regression estimates for those variables specified in condvars().

Inferences from MLEs rely on asymptotics, and if your sample size is small, these inferences may not be valid. On the other hand, inferences from the CMLEs are exact in the sense that they use the conditional distribution of the sufficient statistics outlined above.

For small datasets, it is common for the dependent variable to be completely determined by the data. Here the MLEs and the CMLEs are unbounded. exlogistic will instead compute the MUE, the regression estimate that places the observed sufficient statistic at the median of the conditional distribution.

## > Example 1

One example presented by Mehta and Patel (1995) is data from a prospective study of perinatal infection and human immunodeficiency virus type 1 (HIV-1). We use a variation of this dataset. There was an investigation Hutto et al. (1991) into whether the blood serum levels of glycoproteins CD4 and CD8 measured in infants at 6 months of age might predict their development of HIV infection. The blood serum levels are coded as ordinal values 0,1 , and 2 .

```
. use http://www.stata-press.com/data/r13/hiv1
(prospective study of perinatal infection of HIV-1)
. list in 1/5
```

| hiv | cd4 | cd8 |
| ---: | ---: | ---: |
| 1 | 0 | 0 |
| 0 | 0 | 0 |
| 1 | 0 | 2 |
| 1 | 1 | 0 |
| 0 | 1 | 0 |

We first obtain the MLEs from logistic so that we can compare the estimates and associated statistics with the CMLEs from exlogistic.

| . logistic hiv cd4 cd8, coef |  |  |  |
| :--- | :--- | :--- | :--- |
| Logistic regression | Number of obs | $=$ | 47 |
|  | LR chi2 (2) | $=$ | 15.75 |
|  | Prob > chi2 | $=$ | 0.0004 |
| Log likelihood $=-20.751687$ | Pseudo R2 | $=$ | 0.2751 |


| hiv | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | :---: | ---: | ---: |
| cd4 | -2.541669 | .8392231 | -3.03 | 0.002 | -4.186517 | -.8968223 |
| cd8 | 1.658586 | .821113 | 2.02 | 0.043 | .0492344 | 3.267938 |
| _cons | .5132389 | .6809007 | 0.75 | 0.451 | -.8213019 | 1.84778 |

```
. exlogistic hiv cd4 cd8, coef
Enumerating sample-space combinations:
observation 1: enumerations = 2
observation 2: enumerations = 3
    (output omitted)
observation 46: enumerations = 601
observation 47: enumerations = 326
```

Exact logistic regression Number of obs = 47
Model score $=13.34655$
$\operatorname{Pr}>=$ score $=0.0006$

| hiv | Coef. | Suff. | $2 * \operatorname{Pr}$ (Suff.) | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: |
| cd4 | -2.387632 | 10 | 0.0004 | -4.699633 | -.8221807 |
| cd8 | 1.592366 | 12 | 0.0528 | -.0137905 | 3.907876 |

exlogistic produced a log showing how many records are generated as it processes each observation. The primary purpose of the $\log$ is to provide feedback because generating the distribution can be time consuming, but we also see from the last entry that the joint distribution for the sufficient statistics for cd4 and cd8 conditioned on the total number of successes has 326 unique values (but a size of $\left.\binom{47}{14}=341,643,774,795\right)$.

The statistics for logistic are based on asymptotics: for a large sample size, each $Z$ statistic will be approximately normally distributed (with a mean of zero and a standard deviation of one) if the associated regression parameter is zero. The question is whether a sample size of 47 is large enough.

On the other hand, the $p$-values computed by exlogistic are from the conditional distributions of the sufficient statistics for each parameter given the sufficient statistics for all other parameters. In this sense, these $p$-values are exact. By default, exlogistic reports the sufficient statistics for the regression parameters and the probability of observing a more extreme value. These are singleparameter tests for $H_{0}: \beta_{\mathrm{cd} 4}=0$ and $H_{0}: \beta_{\mathrm{cd} 8}=0$ versus the two-sided alternatives. The conditional scores test, located in the coefficient table header, is testing that both $H_{0}: \beta_{\mathrm{cd} 4}=0$ and $H_{0}: \beta_{\mathrm{cd} 8}=0$. We find these $p$-values to be in fair agreement with the Wald and likelihood-ratio tests from logistic.

The confidence intervals for exlogistic are computed from the exact conditional distributions. The exact confidence intervals are asymmetrical about the estimate and are wider than the normal-based confidence intervals from logistic.

Both estimation techniques indicate that the incidence of HIV infection decreases with increasing CD4 blood serum levels and increases with increasing CD8 blood serum levels. The constant term is missing from the exact logistic coefficient table because we conditioned out its observed sufficient statistic when tallying the joint distribution of the sufficient statistics for the cd4 and cd8 parameters.

The test () option provides two other test statistics used in exact logistic: the conditional scores test, test (score), and the conditional probabilities test, test (probability). For comparison, we display the individual parameter conditional scores tests.

| . exlogistic, test(score) coef |
| :--- |
| Exact logistic regression |
|  |
|  |
| hiv |

For the probabilities test, the probability statistic is computed from (1) in Methods and formulas with $\boldsymbol{\beta}=0$. For this example, the significance of the probabilities tests matches the scores tests so they are not displayed here.

## Technical note

Typically, the value of $\theta$, the constant term, is of little interest, as well as perhaps some of the parameters in $\boldsymbol{\beta}$, but we need to include all parameters in the model to correctly specify it. By conditioning out the nuisance parameters, we can reduce the size of the joint conditional distribution that is used to estimate the regression parameters of interest. The condvars () option allows you to specify a varlist of nuisance variables. By default, exlogistic conditions on the sufficient statistic of $\theta$, which is the number of successes. You can save computation time and computer memory by using the condvars () option because infeasible values of the sufficient statistics associated with the variables in condvars () can be dropped from consideration before all $n$ observations are processed.

Specifying some of your independent variables in condvars() will not change the estimated regression coefficients of the remaining independent variables. For instance, in example 1, if we instead type

```
. exlogistic hiv cd4, condvars(cd8) coef
```

the regression coefficient for $\operatorname{cd4}$ (as well as all associated inference) will be identical.
One reason to have multiple variables in indepvars is to make conditional inference of several parameters simultaneously by using the terms () option. If you do not wish to test several parameters simultaneously, it may be more efficient to obtain estimates for individual variables by calling exlogistic multiple times with one variable in indepvars and all other variables listed in condvars (). The estimates will be the same as those with all variables in indepvars.

## - Technical note

If you fit a clogit (see [R] clogit) model to the HIV data from example 1, you will find that the estimates differ from those with exlogistic. (To fit the clogit model, you will have to create a group variable that includes all observations.) The regression estimates will be different because clogit conditions on the constant term only, whereas the estimates from exlogistic condition on the sufficient statistic of the other regression parameter as well as the constant term.

## > Example 2

The HIV data presented in table IV of Mehta and Patel (1995) are in a binomial form, where the variable hiv contains the HIV cases that tested positive and the variable $n$ contains the number of individuals with the same CD4 and CD8 levels, the binomial number-of-trials parameter. Here depvar is hiv, and we use the binomial(n) option to identify the number-of-trials variable.

```
. use http://www.stata-press.com/data/r13/hiv_n
(prospective study of perinatal infection of HIV-1; binomial form)
. list
```

|  | cd4 | cd8 | hiv | n |
| :--- | ---: | ---: | ---: | ---: |
| 1. | 0 | 2 | 1 | 1 |
| 2. | 1 | 2 | 2 | 2 |
| 3. | 0 | 0 | 4 | 7 |
| 4. | 1 | 1 | 4 | 12 |
| 5. | 2 | 2 | 1 | 3 |
| 6. | 1 | 0 | 2 | 7 |
| 7. | 2 | 0 | 0 | 2 |
| 8. | 2 | 1 | 0 | 13 |

Further, the cd4 and cd8 variables of the hiv dataset are actually factor variables, where each has the ordered levels of $(0,1,2)$. Another approach to the analysis is to use indicator variables, and following Mehta and Patel (1995), we used a $0-1$ coding scheme that will give us the odds ratio of level 0 versus 2 and level 1 versus 2.

| . generate byte cd4_0 = (cd4==0) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| . generate byte cd4_1 = (cd4==1) |  |  |  |  |  |
| . generate byte cd8_0 = (cd8==0) |  |  |  |  |  |
| . generate byte cd8_1 = (cd8==1) |  |  |  |  |  |
| . exlogistic hiv cd4_0 cd4_1 cd8_0 cd8_1, terms(cd4=cd4_0 cd4_1, <br> > cd8=cd8_0 cd8_1) binomial(n) test(probability) saving(dist, replace) nolog |  |  |  |  |  |
| note: saving distribution to file dist.dta |  |  |  |  |  |
| note: CMLE estimate for cd4_0 is +inf; computing MUE |  |  |  |  |  |
| note: CMLE estimate for cd4_1 is +inf; computing MUE |  |  |  |  |  |
| note: CMLE estimate for cd8_0 is -inf; computing MUE <br> note: CMLE estimate for cd8_1 is -inf; computing MUE |  |  |  |  |  |
|  |  |  |  |  |  |
| Exact logistic regression |  |  |  | of obs | 47 |
| Binomial variable: n |  |  |  | prob. | $3.19 \mathrm{e}-06$ |
|  |  |  |  | prob. | 0.0011 |
| hiv | Odds Ratio | Prob. | $\operatorname{Pr}<=$ Prob . | [95\% Conf | Interval] |
| cd4 |  | . 0007183 | 0.0055 |  |  |
| cd4_0 | 18.82831* | . 007238 | 0.0072 | 1.714079 | +Inf |
| cd4_1 | 11.53732* | . 0063701 | 0.0105 | 1.575285 | +Inf |
| cd8 |  | . 0053212 | 0.0323 |  |  |
| cd8_0 | .1056887* | . 0289948 | 0.0290 | 0 | 1.072531 |
| cd8_1 | .0983388* | . 0241503 | 0.0242 | 0 | . 9837203 |

(*) median unbiased estimates (MUE)
. matrix list e(sufficient)
e(sufficient) [1,4]

```
r1 cd4_0 crd4_1 cri8_0 crecd8_1
. display e(n_possible)
1091475
```

Here we used terms () to specify two terms in the model, cd4 and cd8, that make up the cd4 and cd8 indicator variables. By doing so, we obtained a conditional probabilities test for cd4, simultaneously testing both cd4_0 and cd4_1, and for cd8, simultaneously testing both cd8_0 and cd8_1. The significance levels for the two terms are 0.0055 and 0.0323 , respectively.

This example also illustrates instances where the dependent variable is completely determined by the independent variables and CMLEs are infinite. If we try to obtain MLEs, logistic will drop each variable and then terminate with a no-data error, error number 2000.

```
. use http://www.stata-press.com/data/r13/hiv_n, clear
(prospective study of perinatal infection of HIV-1; binomial form)
. generate byte cd4_0 = (cd4==0)
. generate byte cd4_1 = (cd4==1)
. generate byte cd8_0 = (cd8==0)
. generate byte cd8_1 = (cd8==1)
. expand n
(39 observations created)
. logistic hiv cd4_0 cd4_1 cd8_0 cd8_1
note: cd4_0 != 0 predicts success perfectly
    cd4_0 dropped and 8 obs not used
note: cd4_1 != 0 predicts success perfectly
    cd4_1 dropped and 21 obs not used
note: cd8_0 != 0 predicts failure perfectly
    cd8_0 dropped and 2 obs not used
outcome = cd8_1 <= 0 predicts data perfectly
r(2000);
```

In example 2, exlogistic generated the joint conditional distribution of $T_{\mathrm{cd} 4 \_0}, T_{\mathrm{cd} 4 \_1}, T_{\mathrm{cd} 8 \_0}$, and $T_{\text {cd8_1 }}$ given $M=14$ (the number of individuals that tested positive), and for reference, we listed the observed sufficient statistics that are stored in the matrix e(sufficient). Below we take that distribution and further condition on $T_{\mathrm{cd} 4 \_1}=8, T_{\mathrm{cd} 8 \_0}=6$, and $T_{\mathrm{cd} 8 \_1}=4$, giving the conditional distribution of $T_{\text {cd4_0 }}$. Here we see that the observed sufficient statistic $T_{\text {cd4_0 }}=5$ is last in the sorted listing or, equivalently, $T_{\mathrm{cd} 4 \_0}$ is at the domain boundary of the conditional probability distribution. When this occurs, the conditional probability distribution is monotonically increasing in $\beta_{\mathrm{cd} 4 \_0}$ and a maximum does not exist.

```
. use dist, clear
. keep if cd4_1==8 & cd8_0==6 & cd8_1==4
(4139 observations deleted)
. list, sep(0)
```

| -f_ | cd4_0 | cd4_1 | cd8_0 | cd8_1 |
| ---: | ---: | ---: | ---: | ---: |
| 1668667 | 0 | 8 | 6 | 4 |
| 18945542 | 1 | 8 | 6 | 4 |
| 55801053 | 2 | 8 | 6 | 4 |
| 55867350 | 3 | 8 | 6 | 4 |
| 17423175 | 4 | 8 | 6 | 4 |
| 1091475 | 5 | 8 | 6 | 4 |

When the CMLEs are infinite, the MUEs are computed (Hirji, Tsiatis, and Mehta 1989). For the cd4_0 estimate, we compute the value $\bar{\beta}_{\text {cd4_0 }}$ such that

$$
\operatorname{Pr}\left(T_{\mathrm{cd} 4 \_0} \geq 5 \mid \beta_{\mathrm{cd} 4 \_0}=\bar{\beta}_{\mathrm{cd} 4 \_0}, T_{\mathrm{cd} 4 \_1}=8, T_{\mathrm{cd} 8 \_0}=6, T_{\mathrm{cd} 8 \_1}=4, M=14\right)=1 / 2
$$

using (1) in Methods and formulas.

The output is in agreement with example 1: there is an increase in risk of HIV infection for a CD4 blood serum level of 0 relative to a level of 2 and for a level of 1 relative to a level of 2 ; there is a decrease in risk of HIV infection for a CD8 blood serum level of 0 relative to a level of 2 and for a level of 1 relative to a level of 2 .

We also displayed e(n_possible). This is the combinatorial coefficient associated with the observed sufficient statistics. The same value is found in the _f_ variable of the conditional distribution dataset listed above. The size of the distribution is $\binom{47}{14}=341,643,774,795$. This can be verified by summing the ${ }_{-} f_{-}$variable of the generated conditional distribution dataset.

```
. use dist, clear
. summarize _f_, meanonly
. di %15.1f r(sum)
341643774795.0
```


## > Example 3

One can think of exact logistic regression as a covariate-adjusted exact binomial. To demonstrate this point, we will use exlogistic to compute a binomial confidence interval for $m$ successes of $n$ trials, by fitting the constant-only model, and we will compare it with the confidence interval computed by ci (see [R] ci). We will use the saving() option to retain the dataset containing the feasible values for the constant term sufficient statistic, namely, the number of successes, $m$, given $n$ trials and their associated combinatorial coefficients $\binom{n}{m}, m=0,1, \ldots, n$.

```
. input y
            y
    1. 1
    2. 0
    3. }
    4. 0
    5. 1
    6. 1
    7. end
. ci y, binomial
```

| Variable | Obs | Mean | Std. Err. | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: |
| y | 6 | .6666667 | .1924501 | .2227781 | .9567281 |

. exlogistic y, estconstant nolog coef saving(binom)
note: saving distribution to file binom.dta
Exact logistic regression

Number of obs $=$

| $y$ | Coef. | Suff. | $2 * \operatorname{Pr}($ Suff. $)$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | ---: | ---: |
| _cons | .6931472 | 4 | 0.6875 | -1.24955 | 3.096017 |

We use the postestimation program estat predict to transform the estimated constant term and its confidence bounds by using the inverse logit function, invlogit () (see [D] functions). The standard error for the estimated probability is computed using the delta method.
. estat predict

| $y$ | Predicted | Std. Err. | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: |
| Probability | 0.6667 | 0.1925 | 0.2228 | 0.9567 |

. use binom, replace
. list, sep(0)

| $-f_{-}$ | _cons |
| ---: | ---: |
| 1 | 0 |
| 6 | 1 |
| 15 | 2 |
| 20 | 3 |
| 15 | 4 |
| 6 | 5 |
| 1 | 6 |

Examining the listing of the generated data, the values contained in the variable _cons_ are the feasible values of $M$, and the values contained in the variable ${ }_{-} \mathrm{f}_{-}$are the binomial coefficients $\binom{6}{m}$ with total $\sum_{m=0}^{6}\binom{6}{m}=2^{6}=64$. In the coefficient table, the sufficient statistic for the constant term, labeled Suff., is $m=4$. This value is located at record 5 of the dataset. Therefore, the two-tailed probability of the sufficient statistic is computed as $0.6875=2(15+6+1) / 64$.

The constant term is the value of $\theta$ that maximizes the probability of observing $M=4$; see (1) of Methods and formulas:

$$
\operatorname{Pr}(M=4 \mid \theta)=\frac{15 e^{4 \alpha}}{1+6 e^{\alpha}+15 e^{2 \alpha}+20 e^{3 \alpha}+15 e^{4 \alpha}+6 e^{5 \alpha}+e^{6 \alpha}}
$$

The maximum is at the value $\theta=\log 2$, which is demonstrated in the figure below.


The lower and upper confidence bounds are the values of $\theta$ such that $\operatorname{Pr}(M \geq 4 \mid \theta)=0.025$ and $\operatorname{Pr}(M \leq 4 \mid \theta)=0.025$, respectively. These probabilities are plotted in the figure below for $\theta \in[-2,4]$.


## > Example 4

This example demonstrates the group() option, which allows the analysis of stratified data. Here the logistic model is

$$
\log \left(\frac{\pi_{i k}}{1-\pi_{i k}}\right)=\theta_{k}+\mathbf{x}_{k i} \boldsymbol{\beta}
$$

where $k$ indexes the $s$ strata, $k=1, \ldots, s$, and $\theta_{k}$ is the strata-specific constant term whose sufficient statistic is $M_{k}=\sum_{i=1}^{n_{k}} Y_{k i}$.

Mehta and Patel (1995) use a case-control study to demonstrate this model, which is useful in comparing the estimates from exlogistic and clogit. This study was intended to determine the role of birth complications in people with schizophrenia (Garsd 1988). Siblings from seven families took part in the study, and each individual was classified as normal or schizophrenic. A birth complication index is recorded for each individual that ranges from 0 , an uncomplicated birth, to 15 , a very complicated birth. Some of the frequencies contained in variable $f$ are greater than 1 , and these count different births at different times where the individual has the same birth complications index, found in variable BCindex.
. use http://www.stata-press.com/data/r13/schizophrenia, clear (case-control study on birth complications for people with schizophrenia) . list, sepby(family)

|  | family | BCindex | schizo | f |
| :---: | :---: | :---: | :---: | :---: |
| 1. | 1 | 6 | 0 | 1 |
| 2. | 1 | 7 | 0 | 1 |
| 3. | 1 | 3 | 0 | 2 |
| 4. | 1 | 2 | 0 | 3 |
| 5. | 1 | 5 | 0 | 1 |
| 6. | 1 | 0 | 0 | 1 |
| 7. | 1 | 15 | 1 | 1 |
| 8. | 2 | 2 | 1 | 1 |
| 9. | 2 | 0 | 0 | 1 |
| 10. | 3 | 2 | 0 | 1 |
| 11. | 3 | 9 | 1 | 1 |
| 12. | 3 | 1 | 0 | 1 |
| 13. | 4 | 2 | 1 | 1 |
| 14. | 4 | 0 | 0 | 4 |
| 15. | 5 | 3 | 1 | 1 |
| 16. | 5 | 6 | 0 | 1 |
| 17. | 5 | 0 | 1 | 1 |
| 18. | 6 | 3 | 0 | 1 |
| 19. | 6 | 0 | 1 | 1 |
| 20. | 6 | 0 | 0 | 2 |
| 21. | 7 | 2 | 0 | 1 |
| 22. | 7 | 6 | 1 | 1 |

- exlogistic schizo BCindex [fw=f], group(family) test(score) coef

Enumerating sample-space combinations:
observation 1: enumerations $=$ 2
observation 2: enumerations $=\quad 3$
observation 3: enumerations $=$ 4
observation 4: enumerations $=\quad 5$
observation 5: enumerations $=\quad 6$
observation 6: enumerations $=\quad 7$
(output omitted)
observation 21: enumerations = 72
observation 22: enumerations $=\quad 40$
Exact logistic regression $\quad$ Number of obs 29
Group variable: family

| Number of obs | $=$ | 29 |
| :--- | :--- | ---: |
| Number of groups | $=$ | 7 |
| Obs per group: min | $=$ | 2 |
|  | avg | $=$ |
| $\max$ | $=$ | 4.1 |
|  | $=$ | 10 |
| Model score |  | 6.32803 |
| Pr $>=$ score | $=$ | 0.0167 |


| schizo | Coef. | Score | Pr>=Score | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | ---: | ---: |
| BCindex | .3251178 | 6.328033 | 0.0167 | .0223423 | .7408832 |

The asymptotic alternative for this model can be estimated using clogit (equivalently, xtlogit, fe ) and is listed below for comparison. We must expand the data because clogit will not accept frequency weights if they are not constant within the groups.


Both techniques compute the same regression estimate for the BCindex, which might not be too surprising because both estimation techniques condition on the total number of successes in each group. The difference lies in the $p$-values and confidence intervals. The $p$-value testing $H_{0}: \beta_{\mathrm{BCindex}}=0$ is approximately 0.0167 for the exact conditional scores test and 0.053 for the asymptotic Wald test. Moreover, the exact confidence interval is asymmetric about the estimate and does not contain zero.

## - Technical note

The memory(\#) option limits the amount of memory that exlogistic will consume when computing the conditional distribution of the parameter sufficient statistics. memory () is independent of the data maximum memory setting (see set max_memory in [D] memory), and it is possible for exlogistic to exceed the memory limit specified in set max-memory without terminating. By default, a $\log$ is provided that displays the number of enumerations (the size of the conditional distribution) after processing each observation. Typically, you will see the number of enumerations increase, and then at some point they will decrease as the multivariate shift algorithm (Hirji, Mehta, and Patel 1987) determines that some of the enumerations cannot achieve the observed sufficient statistics of the conditioning variables. When the algorithm is complete, however, it is necessary to store the conditional distribution of the parameter sufficient statistics as a dataset. It is possible, therefore, to get a memory error when the algorithm has completed if there is not enough memory to store the conditional distribution.

## - Technical note

Computing the conditional distributions and reported statistics requires data sorting and numerical comparisons. If there is at least one single-precision variable specified in the model, exlogistic will make comparisons with a relative precision of $2^{-5}$. Otherwise, a relative precision of $2^{-11}$ is used. Be careful if you use recast to promote a single-precision variable to double precision (see [D] recast). You might try listing the data in full precision (maybe $\% 20.15 \mathrm{~g}$; see [D] format) to make sure that this is really what you want. See [D] data types for information on precision of numeric storage types.

## Stored results

exlogistic stores the following in e() :
Scalars

```
e(N)
    e(k_groups)
    e(n_possible)
    e(n_trials)
    e(sum_y)
    e(k_indvars)
    e(k_terms)
    e(k_condvars)
    e(condcons)
    e(midp)
    e(eps)
```

number of observations
number of groups
number of distinct possible outcomes where sum(sufficient) equals observed e(sufficient)
binomial number-of-trials parameter
sum of depvar
number of independent variables
number of model terms
number of conditioning variables
conditioned on the constant(s) indicator
mid- $p$-value rule indicator
relative difference tolerance
Macros
e(cmd)
e(cmdline)
e(title)
e(depvar)
e(indvars)
e(condvars)
e(groupvar)
e(binomial)
e(level)
e(wtype)
e(wexp)
e(datasignature)
e(datasignaturevars)
e(properties)
e(estat_cmd)
e(marginsnotok)
Matrices
e(b)
e(mue_indicators)
e(se)
e(ci)
e(sum_y_groups)
e(N_g)
e(sufficient)
e(p_sufficient)
e(scoretest)
e(p_scoretest)
e(probtest)
e(p_probtest)
exlogistic
command as typed
title in estimation output
name of dependent variable
independent variables
conditional variables
group variable
binomial number-of-trials variable
confidence level
weight type
weight expression
the checksum
variables used in calculation of checksum
b
program used to implement estat
predictions disallowed by margins
coefficient vector
indicator for elements of $e(b)$ estimated using MUE instead of CMLE
$e$ (b) standard errors (CMLEs only)
matrix of e(level) confidence intervals for e(b)
sum of e(depvar) for each group
number of observations in each group
sufficient statistics for $e(b)$
$p$-value for e(sufficient)
conditional scores tests for indepvars
$p$-values for e(scoretest)
conditional probabilities tests for indepvars
$p$-value for e(probtest)
conditional scores tests for model terms
$p$-value for e(scoretest_m)
conditional probabilities tests for model terms
$p$-value for e (probtest_m)
Functions
e(sample)
marks estimation sample

## Methods and formulas

Methods and formulas are presented under the following headings:
Sufficient statistics
Conditional distribution and CMLE
Median unbiased estimates and exact CI
Conditional hypothesis tests
Sufficient-statistic p-value

## Sufficient statistics

Let $\left\{Y_{1}, Y_{2}, \ldots, Y_{n}\right\}$ be a set of $n$ independent Bernoulli random variables, each of which can realize two outcomes, $\{0,1\}$. For each $i=1, \ldots, n$, we observe $Y_{i}=y_{i}$, and associated with each observation is the covariate row vector of length $p, \mathbf{x}_{i}=\left(x_{i 1}, \ldots, x_{i p}\right)$. Denote $\boldsymbol{\beta}=\left(\beta_{1}, \ldots, \beta_{p}\right)^{T}$ to be the column vector of regression parameters and $\theta$ to be the constant. The sufficient statistic for $\beta_{j}$ is $T_{j}=\sum_{i=1}^{n} Y_{i} x_{i j}, j=1, \ldots, p$, and for $\theta$ is $M=\sum_{i=1}^{n} Y_{i}$. We observe $T_{j}=t_{j}, t_{j}=\sum_{i=1}^{n} y_{i} x_{i j}$, and $M=m, m=\sum_{i=1}^{n} y_{i}$. The probability of observing $\left(Y_{1}=y_{1}, Y_{2}=y_{2}, \ldots, Y_{n}=y_{n}\right)$ is

$$
\operatorname{Pr}\left(Y_{1}=y_{1}, \ldots, Y_{n}=y_{n} \mid \boldsymbol{\beta}, \mathbf{X}\right)=\frac{\exp (m \theta+\mathbf{t} \boldsymbol{\beta})}{\prod_{i=1}^{n}\left\{1+\exp \left(\theta+\mathbf{x}_{i} \boldsymbol{\beta}\right)\right\}}
$$

where $\mathbf{t}=\left(t_{1}, \ldots, t_{p}\right)$ and $\mathbf{X}=\left(\mathbf{x}_{1}^{T}, \ldots, \mathbf{x}_{n}^{T}\right)^{T}$.
The joint distribution of the sufficient statistics $\mathbf{T}$ is obtained by summing over all possible binary sequences $Y_{1}, \ldots, Y_{n}$ such that $\mathbf{T}=\mathbf{t}$ and $M=m$. This probability function is

$$
\operatorname{Pr}\left(T_{1}=t_{1}, \ldots, T_{p}=t_{p}, M=m \mid \boldsymbol{\beta}, \mathbf{X}\right)=\frac{c(\mathbf{t}, m) \exp (m \theta+\mathbf{t} \boldsymbol{\beta})}{\prod_{i=1}^{n}\left\{1+\exp \left(\theta+\mathbf{x}_{i} \boldsymbol{\beta}\right)\right\}}
$$

where $c(\mathbf{t}, m)$ is the combinatorial coefficient of $(\mathbf{t}, m)$ or the number of distinct binary sequences $Y_{1}, \ldots, Y_{n}$ such that $\mathbf{T}=\mathbf{t}$ and $M=m$ (Cox and Snell 1989).

## Conditional distribution and CMLE

Without loss of generality, we will restrict our discussion to computing the CMLE of $\beta_{1}$. If we condition on observing $M=m$ and $T_{2}=t_{2}, \ldots, T_{p}=t_{p}$, the probability function of $\left(T_{1} \mid \beta_{1}, T_{2}=\right.$ $\left.t_{2}, \ldots, T_{p}=t_{p}, M=m\right)$ is

$$
\begin{equation*}
\operatorname{Pr}\left(T_{1}=t_{1} \mid \beta_{1}, T_{2}=t_{2}, \ldots, T_{p}=t_{p}, M=m\right)=\frac{c(\mathbf{t}, m) e^{t_{1} \beta_{1}}}{\sum_{u} c\left(u, t_{2}, \ldots, t_{p}, m\right) e^{u \beta_{1}}} \tag{1}
\end{equation*}
$$

where the sum in the denominator is over all possible values of $T_{1}$ such that $M=m$ and $T_{2}=t_{2}, \ldots, T_{p}=t_{p}$ and $c\left(u, t_{2}, \ldots, t_{p}, m\right)$ is the combinatorial coefficient of $\left(u, t_{2}, \ldots, t_{p}, m\right)$ (Cox and Snell 1989). The CMLE for $\beta_{1}$ is the value $\widehat{\beta}_{1}$ that maximizes the $\log$ of (1). This optimization task is carried out by ml , using the conditional frequency distribution of $\left(T_{1} \mid T_{2}=t_{2}, \ldots, T_{p}=\right.$ $\left.t_{p}, M=m\right)$ as a dataset. Generating the joint conditional distribution is efficiently computed using the multivariate shift algorithm described by Hirji, Mehta, and Patel (1987).

Difficulties in computing $\widehat{\beta}_{1}$ arise if the observed $\left(T_{1}=t_{1}, \ldots, T_{p}=t_{p}, M=m\right)$ lies on the boundaries of the distribution of $\left(T_{1} \mid T_{2}=t_{2}, \ldots, T_{p}=t_{p}, M=m\right)$, where the conditional probability function is monotonically increasing (or decreasing) in $\beta_{1}$. Here the CMLE is plus infinity if it is on the upper boundary, $\operatorname{Pr}\left(T_{1} \leq t_{1} \mid T_{2}=t_{2}, \ldots, T_{p}=t_{p}, M=m\right)=1$, and is minus infinity if it is on the lower boundary of the distribution, $\operatorname{Pr}\left(T_{1} \geq t_{1} \mid T_{2}=t_{2}, \ldots, T_{p}=t_{p}, M=m\right)=1$. This concept is demonstrated in example 2. When infinite CMLEs occur, the MUE is computed.

## Median unbiased estimates and exact $\mathbf{C l}$

The MUE is computed using the technique outlined by Hirji, Tsiatis, and Mehta (1989). First, we find the values of $\beta_{1}^{(u)}$ and $\beta_{1}^{(l)}$ such that

$$
\begin{align*}
& \operatorname{Pr}\left(T_{1} \leq t_{1} \mid \beta_{1}=\beta_{1}^{(u)}, T_{2}=t_{2}, \ldots, T_{p}=t_{p}, M=m\right)=  \tag{2}\\
& \operatorname{Pr}\left(T_{1} \geq t_{1} \mid \beta_{1}=\beta_{1}^{(l)}, T_{2}=t_{2}, \ldots, T_{p}=t_{p}, M=m\right)=1 / 2
\end{align*}
$$

The MUE is then $\bar{\beta}_{1}=\left(\beta_{1}^{(l)}+\beta_{1}^{(u)}\right) / 2$. However, if $T_{1}$ is equal to the minimum of the domain of the conditional distribution, $\beta^{(l)}$ does not exist and $\bar{\beta}_{1}=\beta^{(u)}$. If $T_{1}$ is equal to the maximum of the domain of the conditional distribution, $\beta^{(u)}$ does not exist and $\bar{\beta}_{1}=\beta^{(l)}$.

Confidence bounds for $\beta$ are computed similarly, except that we substitute $\alpha / 2$ for $1 / 2$ in (2), where $1-\alpha$ is the confidence level. Here $\beta_{1}^{(l)}$ would then be the lower confidence bound and $\beta_{1}^{(u)}$ would be the upper confidence bound (see example 3).

## Conditional hypothesis tests

To test $H_{0}: \beta_{1}=0$ versus $H_{1}: \beta_{1} \neq 0$, we obtain the exact $p$-value from $\sum_{u \in E} f_{1}(u)-f_{1}\left(t_{1}\right) / 2$ if the mid- $p$-value rule is used and $\sum_{u \in E} f_{1}(u)$ otherwise. Here $E$ is a critical region, and we define $f_{1}(u)=\operatorname{Pr}\left(T_{1}=u \mid \beta_{1}=0, T_{2}=t_{2}, \ldots, T_{p}=t_{p}, M=m\right)$ for ease of notation. There are two popular ways to define the critical region: the conditional probabilities test and the conditional scores test (Mehta and Patel 1995). The critical region when using the conditional probabilities test is all values of the sufficient statistic for $\beta_{1}$ that have a probability less than or equal to that of the observed $t_{1}, E_{p}=\left\{u: f_{1}(u) \leq f_{1}\left(t_{1}\right)\right\}$. The critical region of the conditional scores test is defined as all values of the sufficient statistic for $\beta_{1}$ such that its score is greater than or equal to that of $t_{1}$,

$$
\left.E_{s}=\left\{u:\left(u-\mu_{1}\right)^{2} / \sigma_{1}^{2} \geq\left(t_{1}-\mu_{1}\right)^{2} / \sigma_{1}^{2}\right)\right\}
$$

Here $\mu_{1}$ and $\sigma_{1}^{2}$ are the mean and variance of $\left(T_{1} \mid \beta_{1}=0, T_{2}=t_{2}, \ldots, T_{p}=t_{p}, M=m\right)$.
The score statistic is defined as

$$
\left\{\frac{\partial \ell(\beta)}{\partial \beta}\right\}^{2}\left[-E\left\{\frac{\partial^{2} \ell(\beta)}{\partial \beta^{2}}\right\}\right]^{-1}
$$

evaluated at $H_{0}: \beta=0$, where $\ell$ is the log of (1). The score test simplifies to $(t-E[T \mid \beta])^{2} / \operatorname{var}(T \mid \beta)$ (Hirji 2006), where the mean and variance are computed from the conditional distribution of the sufficient statistic with $\beta=0$ and $t$ is the observed sufficient statistic.

## Sufficient-statistic p-value

The $p$-value for testing $H_{0}: \beta_{1}=0$ versus the two-sided alternative when $\left(T_{1}=t_{1} \mid T_{2}=\right.$ $\left.t_{2}, \ldots, T_{p}=t_{p}\right)$ is computed as $2 \times \min \left(p_{l}, p_{u}\right)$, where

$$
\begin{aligned}
p_{l} & =\frac{\sum_{u \leq t_{1}} c\left(u, t_{2}, \ldots, t_{p}, m\right)}{\sum_{u} c\left(u, t_{2}, \ldots, t_{p}, m\right)} \\
p_{u} & =\frac{\sum_{u \geq t_{1}} c\left(u, t_{2}, \ldots, t_{p}, m\right)}{\sum_{u} c\left(u, t_{2}, \ldots, t_{p}, m\right)}
\end{aligned}
$$

It is the probability of observing a more extreme $T_{1}$.

## References

Cox, D. R., and E. J. Snell. 1989. Analysis of Binary Data. 2nd ed. London: Chapman \& Hall.
Garsd, A. 1988. Schizophrenia and birth complications. Unpublished manuscript.
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Hutto, C., W. P. Parks, S. Lai, M. T. Mastrucci, C. Mitchell, J. Muñoz, E. Trapido, I. M. Master, and G. B. Scott. 1991. A hospital-based prospective study of perinatal infection with human immunodeficiency virus type 1. Journal of Pediatrics 118: 347-353.

Mehta, C. R., and N. R. Patel. 1995. Exact logistic regression: Theory and examples. Statistics in Medicine 14: 2143-2160.

## Also see

[R] exlogistic postestimation - Postestimation tools for exlogistic
[R] binreg - Generalized linear models: Extensions to the binomial family
[R] clogit - Conditional (fixed-effects) logistic regression
[R] expoisson - Exact Poisson regression
$[\mathrm{R}]$ logistic - Logistic regression, reporting odds ratios
[R] logit - Logistic regression, reporting coefficients
[U] 20 Estimation and postestimation commands

## Title

exlogistic postestimation - Postestimation tools for exlogistic

| Description | Syntax for estat | Menu for estat | Options for estat predict |
| :--- | :--- | :--- | :--- |
| Option for estat se | Remarks and examples | Stored results | Reference |

## Description

The following postestimation commands are of special interest after exlogistic:

| Command | Description |
| :--- | :--- |
| estat predict | single-observation prediction |
| estat se | report ORs or coefficients and their asymptotic standard errors |

The following standard postestimation command is also available:
Command Description
estat summarize summary statistics for the estimation sample
estat summarize is not allowed if the binomial() option was specified in exlogistic.
See [R] estat summarize for details.

## Special-interest postestimation commands

estat predict computes a predicted probability (or linear predictor), its asymptotic standard error, and its exact confidence interval for 1 observation. Predictions are carried out by estimating the constant coefficient after shifting the independent variables and conditioned variables by the values specified in the at () option or by their medians. Therefore, predictions must be done with the estimation sample in memory. If a different dataset is used or if the dataset is modified, then an error will result.
estat se reports odds ratio or coefficients and their asymptotic standard errors. The estimates are stored in the matrix $r$ (estimates).

## Syntax for estat

Single-observation prediction
estat predict [, options]

Report ORs or coefficients and their asymptotic standard errors
estat se $[$, coef $]$

## options <br> Description

$\underline{\mathrm{pr}} \quad$ probability; the default
xb linear effect
at (atspec) use the specified values for the indepvars and condvars ()
level(\#)
memory (\# $[\mathrm{b}|\mathrm{k}| \mathrm{m} \mid \mathrm{g}]$ )
set confidence level for the predicted value; default is level(95)
nolog
set limit on memory usage; default is memory ( 10 m )
do not display the enumeration $\log$

These statistics are available only for the estimation sample.

## Menu for estat

Statistics $>$ Postestimation $>$ Reports and statistics

## Options for estat predict

pr , the default, calculates the probability.
xb calculates the linear effect.
at $($ varname $=\#[[$ varname $=\#][.]]$.$) specifies values to use in computing the predicted$ value. Here varname is one of the independent variables, indepvars, or the conditioned variables, condvars(). The default is to use the median of each independent and conditioned variable.
level(\#) specifies the confidence level, as a percentage, for confidence intervals. The default is level (95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals.
memory ( $\#[\mathrm{~b}|\mathrm{k}| \mathrm{m} \mid \mathrm{g}]$ ) sets a limit on the amount of memory estat predict can use when generating the conditional distribution of the constant parameter sufficient statistic. The default is memory ( 10 m ), where m stands for megabyte, or $1,048,576$ bytes. The following are also available: b stands for byte; k stands for kilobyte, which is equal to 1,024 bytes; and g stands for gigabyte, which is equal to 1,024 megabytes. The minimum setting allowed is 1 m and the maximum is 512 m or 0.5 g , but do not attempt to use more memory than is available on your computer. Also see Remarks and examples in [R] exlogistic for details on enumerating the conditional distribution.
nolog prevents the display of the enumeration log. By default, the enumeration log is displayed showing the progress of enumerating the distribution of the observed successes conditioned on the independent variables shifted by the values specified in at() (or by their medians). See Methods and formulas in [R] exlogistic for details of the computations.

## Option for estat se

coef requests that the estimated coefficients and their asymptotic standard errors be reported. The default is to report the odds ratios and their asymptotic standard errors.

## Remarks and examples

Predictions must be done using the estimation sample. This is because the prediction is really an estimated constant coefficient (the intercept) after shifting the independent variables and conditioned variables by the values specified in at () or by their medians. The justification for this approach can be seen by rewriting the model as

$$
\log \left(\frac{\pi_{i}}{1-\pi_{i}}\right)=\left(\alpha+\mathbf{x}_{0} \boldsymbol{\beta}\right)+\left(\mathbf{x}_{i}-\mathbf{x}_{0}\right) \boldsymbol{\beta}
$$

where $\mathbf{x}_{0}$ are the specified values for the indepvars (Mehta and Patel 1995). Because the estimation of the constant term is required, this technique is not appropriate for stratified models that used the group() option.

## > Example 1

To demonstrate, we return to the example 2 in $[R]$ exlogistic using data from a prospective study of perinatal infection and HIV-1. Here there was an investigation into whether the blood serum levels of CD4 and CD8 measured in infants at 6 months of age might predict their development of HIV infection. The blood serum levels are coded as ordinal values 0 , 1 , and 2 . These data are used by Mehta and Patel (1995) as an exposition of exact logistic.

```
. use http://www.stata-press.com/data/r13/hiv_n
(prospective study of perinatal infection of HIV-1; binomial form)
. generate byte cd4_0 = (cd4==0)
. generate byte cd4_1 = (cd4==1)
. generate byte cd8_0 = (cd8==0)
. generate byte cd8_1 = (cd8==1)
. exlogistic hiv cd4_0 cd4_1 cd8_0 cd8_1, terms(cd4=cd4_0 cd4_1,
> cd8=cd8_0 cd8_1) binomial(n) test(probability) saving(dist)
    (output omitted)
```

```
. estat predict
Enumerating sample-space combinations:
observation 1: enumerations = 3
observation 2: enumerations = 12
observation 3: enumerations = 5
observation 4: enumerations = 5
observation 5: enumerations = 5
observation 6: enumerations = 35
observation 7: enumerations = 15
observation 8: enumerations = 15
observation 9: enumerations = 9
observation 10: enumerations = 9
observation 11: enumerations = 5
observation 12: enumerations = 18
note: CMLE estimate for _cons is -inf; computing MUE
Predicted value at cd4_0 = 0, cd4_1 = 0, cd8_0 = 0, cd8_1 = 1
\begin{tabular}{r|rccc}
\hline hiv & Predicted & Std. Err. & \multicolumn{2}{c}{ [95\% Conf. Interval] } \\
\hline Probability & \(0.0390 *\) & N/A & 0.0000 & 0.1962 \\
\hline
\end{tabular}
(*) identifies median unbiased estimates (MUE) ; because an MUE is computed, there is no SE estimate
```

Because we did not specify values by using the at () option, the median values of the indepvars are used for the prediction. By default, medians are used instead of means because we want to use values that are observed in the dataset. If the means of the binary variables cd4_0-cd8_1 were used, we would have created floating point variables in $(0,1)$ that not only do not properly represent the indicator variables but also would be a source of computational inefficiency in generating the conditional distribution. Because the MUE is computed for the predicted value, there is no standard-error estimate.

From the example discussions in [R] exlogistic, the infants at highest risk are those with a CD4 level of 0 and a CD8 level of 2 . Below we use the at () option to make a prediction at these blood serum levels.

```
. estat predict, at(cd4_0=1 cd4_1=0 cd8_0=0 cd8_1=0) nolog
note: CMLE estimate for _cons is +inf; computing MUE
Predicted value at cd4_0 = 1, cd4_1 = 0, cd8_0 = 0, cd8_1 = 0
```

| hiv | Predicted | Std. Err. | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: |
| Probability | $0.9063 *$ | N/A | 0.4637 | 1.0000 |

(*) identifies median unbiased estimates (MUE); because an MUE is computed, there is no SE estimate

## Stored results

estat predict stores the following in $r()$ :
Scalars
$r$ (imue) $\quad 1$ if $r$ (pred) is an MUE and 0 if a CMLE
$r$ (pred) estimated probability or the linear effect
$r$ (se) asymptotic standard error of $r$ (pred)
Macros
$r$ (estimate) prediction type: pr or xb
$r$ (level) confidence level
Matrices
$r(c i) \quad$ confidence interval
$\mathrm{r}(\mathrm{x}) \quad$ indepvars and condvars() values

## Reference

Mehta, C. R., and N. R. Patel. 1995. Exact logistic regression: Theory and examples. Statistics in Medicine 14: 2143-2160.

## Also see

[R] exlogistic - Exact logistic regression
[U] 20 Estimation and postestimation commands

## Title

expoisson - Exact Poisson regression

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

expoisson depvar indepvars [if] [in] [weight] [, options]
options Description
Model
condvars (varlist)
group (varname)
exposure ( varname $_{e}$ )
offset ( varname $_{o}$ )
condition on variables in varlist
groups/strata are stratified by unique values of varname include $\ln \left(\right.$ varname $\left._{e}\right)$ in model with coefficient constrained to 1 include varname $_{o}$ in model with coefficient constrained to 1

Options
memory (\#[b|k|m|g]) set limit on memory usage; default is memory ( 25 m )
saving(filename) save the joint conditional distribution to filename
Reporting
level(\#) set confidence level; default is level (95)
irr
test (testopt)
mue (varlist)
midp
nolog
report incidence-rate ratios
report significance of observed sufficient statistic, conditional scores test, or conditional probabilities test
compute the median unbiased estimates for varlist
use the mid- $p$-value rule
do not display the enumeration $\log$
by, statsby, and xi are allowed; see [U] 11.1.10 Prefix commands.
fweights are allowed; see [U] 11.1.6 weight.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Exact statistics $>$ Exact Poisson regression

## Description

expoisson fits an exact Poisson regression model of depvar on indepvars. Exact Poisson regression is an alternative to standard maximum-likelihood-based Poisson regression (see [R] poisson) that offers more accurate inference in small samples because it does not depend on asymptotic results. For stratified data, expoisson is an alternative to fixed-effects Poisson regression (see xtpoisson, fe in [XT] xtpoisson); like fixed-effects Poisson regression, exact Poisson regression conditions on the number of events in each stratum.

Exact Poisson regression is computationally intensive, so if you have regressors whose parameter estimates are not of interest (that is, nuisance parameters), you should specify those variables in the condvars () option instead of in indepvars.

## Options

Model

| condvars (varlist) specifies variables whose parameter estimates are not of interest to you. You |
| :--- |
| can save substantial computer time and memory by moving such variables from indepvars to |
| condvars () . Understand that you will get the same results for $x 1$ and $x 3$ whether you type | - expoisson y x1 x2 x3 x4

or

```
. expoisson y x1 x3, condvars(x2 x4)
```

group (varname) specifies the variable defining the strata, if any. A constant term is assumed for each stratum identified in varname, and the sufficient statistics for indepvars are conditioned on the observed number of successes within each group (as well as other variables in the model). The group variable must be integer valued.
exposure ( varname $_{e}$ ), offset ( varname $_{o}$ ) ; see [R] estimation options.
Options
memory ( $\#[\mathrm{~b}|\mathrm{k}| \mathrm{m} \mid \mathrm{g}]$ ) sets a limit on the amount of memory expoisson can use when computing the conditional distribution of the parameter sufficient statistics. The default is memory ( 25 m ), where m stands for megabyte, or $1,048,576$ bytes. The following are also available: b stands for byte; k stands for kilobyte, which is equal to 1,024 bytes; and g stands for gigabyte, which is equal to 1,024 megabytes. The minimum setting allowed is 1 m and the maximum is 2048 m or 2 g , but do not attempt to use more memory than is available on your computer. Also see the first technical note under example 3 on counting the conditional distribution.
saving (filename [, replace]) saves the joint conditional distribution for each independent variable specified in indepvars. There is one file for each variable, and it is named using the prefix filename with the variable name appended. For example, saving (mydata) with an independent variable named $X$ would generate a data file named mydata_X.dta. Use replace to replace an existing file. Each file contains the conditional distribution for one of the independent variables specified in indepvars conditioned on all other indepvars and those variables specified in condvars(). There are two variables in each data file: the feasible sufficient statistics for the variable's parameter and their associated weights. The weights variable is named ${ }_{-} \mathrm{w}_{-}$.

Reporting
level(\#); see [R] estimation options. The level(\#) option will not work on replay because confidence intervals are based on estimator-specific enumerations. To change the confidence level, you must refit the model.
irr reports estimated coefficients transformed to incidence-rate ratios, that is, $\exp (\beta)$ rather than $\beta$. Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated or stored. irr may be specified at estimation or when replaying previously estimated results.
test (sufficient $\mid$ score $\mid$ probability) reports the significance level of the observed sufficient statistic, the conditional scores test, or the conditional probabilities test. The default is test (sufficient). All the statistics are computed at estimation time, and each statistic may be displayed postestimation; see $[R]$ expoisson postestimation.
mue (varlist) specifies that median unbiased estimates (MUEs) be reported for the variables in varlist. By default, the conditional maximum likelihood estimates (CMLEs) are reported, except for those parameters for which the CMLEs are infinite. Specify mue (_all) if you want MUEs for all the indepvars.
midp instructs expoisson to use the mid- $p$-value rule when computing the MUEs, significance levels, and confidence intervals. This adjustment is for the discreteness of the distribution by halving the value of the discrete probability of the observed statistic before adding it to the $p$-value. The mid- $p$-value rule cannot be MUEs whose corresponding parameter CMLE is infinite.
nolog prevents the display of the enumeration log. By default, the enumeration $\log$ is displayed, showing the progress of computing the conditional distribution of the sufficient statistics.

## Remarks and examples

Exact Poisson regression estimates the model parameters by using the conditional distributions of the parameters' sufficient statistics, and the resulting parameter estimates are known as CMLEs. Exact Poisson regression is a small-sample alternative to the maximum-likelihood ML Poisson model. See $[\mathrm{R}]$ poisson and [XT] xtpoisson to obtain maximum likelihood estimates (MLEs) for the Poisson model and the fixed-effects Poisson model.

Let $Y_{i}$ denote a Poisson random variable where we observe the outcome $Y_{i}=y_{i}, i=1, \ldots, n$. Associated with each independent observation is a $1 \times p$ vector of covariates, $\mathbf{x}_{i}$. We will denote $\mu_{i}=E\left[Y_{i} \mid \mathbf{x}_{i}\right]$ and use the $\log$ linear model to model the relationship between $Y_{i}$ and $\mathbf{x}_{i}$,

$$
\log \left(\mu_{i}\right)=\theta+\mathbf{x}_{i} \boldsymbol{\beta}
$$

where the constant term, $\theta$, and the $p \times 1$ vector of regression parameters, $\boldsymbol{\beta}$, are unknown. The probability of observing $Y_{i}=y_{i}, i=1, \ldots, n$, is

$$
\operatorname{Pr}(\mathbf{Y}=\mathbf{y})=\prod_{i=1}^{n} \frac{\mu_{i}^{y_{i}} e^{-\mu_{i}}}{y_{i}!}
$$

where $\mathbf{Y}=\left(Y_{1}, \ldots, Y_{n}\right)$ and $\mathbf{y}=\left(y_{1}, \ldots, y_{n}\right)$. The MLEs for $\theta$ and $\boldsymbol{\beta}$ maximize the $\log$ of this function.

The sufficient statistics for $\theta$ and $\beta_{j}, j=1, \ldots, p$, are $M=\sum_{i=1}^{n} Y_{i}$ and $T_{j}=\sum_{i=1}^{n} Y_{i} x_{i j}$, respectively, and we observe $M=m$ and $T_{j}=t_{j}$. expoisson tallies the conditional distribution for each $T_{j}$, given the other sufficient statistics $T_{l}=t_{l}, l \neq j$ and $M=m$. Denote one of these values to be $t_{j}^{(k)}, k=1, \ldots, N$, with weight $w_{k}$ that accounts for all the generated $\mathbf{Y}$ vectors that give rise to $t_{j}^{(k)}$. The conditional probability of observing $T_{j}=t_{j}$ has the form

$$
\begin{equation*}
\operatorname{Pr}\left(T_{j}=t_{j} \mid T_{l}=t_{l}, l \neq j, M=m\right)=\frac{w e^{t_{j} \beta_{j}}}{\sum_{k} w_{k} e^{t_{j}^{(k)} \beta_{j}}} \tag{1}
\end{equation*}
$$

where the sum is over the subset of $\mathbf{T}$ vectors such that $\left(T_{1}^{(k)}=t_{1}, \ldots, T_{j}^{(k)}=t_{j}^{(k)}, \ldots, T_{p}^{(k)}=t_{p}\right)$ and $w$ is the weight associated with the observed $\mathbf{t}$. The CMLE for $\beta_{j}$ maximizes the $\log$ of this function.

Specifying nuisance variables in condvars() prevents expoisson from estimating their associated regression coefficients. These variables are still conditional variables when tallying the conditional distribution for the variables in indepvars.

Inferences from MLEs rely on asymptotics, and if your sample size is small, these inferences may not be valid. On the other hand, inferences from the CMLEs are exact in that they use the conditional distribution of the sufficient statistics outlined above.

For small datasets, the dependent variable can be completely determined by the data. Here the MLEs and the CMLEs are unbounded. When this occurs, expoisson will compute the MUE, the regression estimate that places the observed sufficient statistic at the median of the conditional distribution.

See $[R]$ exlogistic for a more thorough discussion of exact estimation and related statistics.

## > Example 1

Armitage, Berry, and Matthews (2002, 499-501) fit a log-linear model to data containing the number of cerebrovascular accidents experienced by 41 men during a fixed period, each of whom had recovered from a previous cerebrovascular accident and was hypertensive. Sixteen men received treatment, and in the original data, there are three age groups ( $40-49,50-59, \geq 60$ ), but we pool the first two age groups to simplify the example. Armitage, Berry, and Matthews point out that this was not a controlled trial, but the data are useful to inquire whether there is evidence of fewer accidents for the treatment group and if age may be an important factor. The dependent variable count contains the number of accidents, variable treat is an indicator for the treatment group $(1=$ treatment, $0=$ control), and variable age is an indicator for the age group ( $0=40-59 ; 1=\geq 60$ ).

First, we load the data, list it, and tabulate the cerebrovascular accident counts by treatment and age group.

```
. use http://www.stata-press.com/data/r13/cerebacc
(cerebrovascular accidents in hypotensive-treated and control groups)
. list
```

|  | treat | count | age |
| :--- | ---: | ---: | ---: |
| 1. | control | 0 | $40 / 59$ |
| 2. | control | 0 | $>=60$ |
| 3. | control | 1 | $40 / 59$ |
| 4. | control | 1 | $>=60$ |
| 5. | control | 2 | $40 / 59$ |
| 6. | control | 2 | $>=60$ |
| 7. | control | 3 | $40 / 59$ |


| 35. | treatment | 0 | $40 / 59$ |
| :--- | :--- | :--- | :--- |
| 36. | treatment | 0 | $40 / 59$ |
| 37. | treatment | 0 | $40 / 59$ |
| 38. | treatment | 0 | $40 / 59$ |
| 39. | treatment | 1 | $40 / 59$ |
| 40. | treatment | 1 | $40 / 59$ |
| 41. | treatment | 1 | $40 / 59$ |
|  |  |  |  |


| hypotensiv <br> e drug <br> treatment | $\begin{array}{r} \text { age } \\ 40 / 59 \end{array}$ | >=60 | Total |
| :---: | :---: | :---: | :---: |
| control | 15 | 10 | 25 |
| treatment | 4 | 0 | 4 |
| Total | 19 | 10 | 29 |

Next we estimate the CMLE with expoisson and, for comparison, the MLE with poisson.

| Estimating: treat |  |
| :---: | :---: |
| Enumerating sample-space combinations: observation 1: enumerations = | 11 |
| observation 2: enumerations = | 11 |
| observation 3: enumerations (output omitted) | 11 |
| observation 39: enumerations = | 410 |
| observation 40: enumerations | 410 |
| observation 41: enumerations | 30 |
| Estimating: age |  |
| Enumerating sample-space combinations: observation 1: enumerations = | 5 |
| observation 2: enumerations | 15 |
| observation 3: enumerations (output omitted) | 15 |
| observation 39: enumerations = | 455 |
| observation 40: enumerations | 455 |
| observation 41: enumerations | 30 |
| xact Poisson regression |  |


| count | Coef. | Suff. | $2 * \operatorname{Pr}$ (Suff.) | [95\% Conf | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| treat | -1.594306 | 4 | 0.0026 | -3.005089 | -. 4701708 |
| age | -. 5112067 | 10 | 0.2794 | -1.416179 | . 3429232 |


| - poisson count treat age, nolog |  |  |  |
| :--- | :--- | :--- | :--- |
| Poisson regression | Number of obs | $=$ | 41 |
|  | LR chi2 (2) | $=$ | 10.64 |
|  | Prob $>$ chi2 | $=$ | 0.0049 |
| Log likelihood $=-38.97981$ | Pseudo R2 | $=$ | 0.1201 |


| count | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | :---: | ---: | ---: | ---: |
| treat | -1.594306 | .5573614 | -2.86 | 0.004 | -2.686714 | -.5018975 |
| age | -.5112067 | .4043525 | -1.26 | 0.206 | -1.303723 | .2813096 |
| _cons | .233344 | .2556594 | 0.91 | 0.361 | -.2677391 | .7344271 |

expoisson generates an enumeration log for each independent variable in indepvars. The conditional distribution of the parameter sufficient statistic is tallied for each independent variable. The conditional distribution for treat, for example, has 30 records containing the weights, $w_{k}$, and feasible sufficient statistics, $t_{\text {treat }}^{(k)}$. In essence, the set of points $\left(w_{k}, t_{\text {treat }}^{(k)}\right), k=1, \ldots, 30$, tallied by expoisson now become the data to estimate the regression coefficient for treat, using (1) as the likelihood. Remember that one of the $30\left(w_{k}, t_{\text {treat }}^{(k)}\right)$ must contain the observed sufficient statistic, $t_{\text {treat }}=\sum_{i=1}^{41}$ treat $_{i} \times$ count $_{i}=4$, and its relative position in the sorted set of points (sorted by $\left.t_{\text {treat }}^{(k)}\right)$ is how the sufficient-statistic significance is computed. This algorithm is repeated for the age variable.

The regression coefficients for treat and age are numerically identical for both Poisson models. Both models indicate that the treatment is significant at reducing the rate of cerebrovascular accidents, $\approx e^{-1.59} \approx 0.204$, or a reduction of about $80 \%$. There is no significant age effect.

The $p$-value for the treatment regression-coefficient sufficient statistic indicates that the treatment effect is a bit more significant than for the corresponding asymptotic $Z$ statistic from poisson. However, the exact confidence intervals are wider than their asymptotic counterparts.

## > Example 2

Agresti $(2013,129)$ used the data from Laird and Olivier (1981) to demonstrate the Poisson model for modeling rates. The data consist of patient survival after heart valve replacement operations. The sample consists of 109 patients that are classified by type of heart valve (aortic, mitral) and by age $(<55, \geq 55)$. Follow-up observations cover lengths from 3 to 97 months, and the time at risk, or exposure, is stored in the variable TAR. The response is whether the subject died. First, we take a look at the data and then estimate the incidence rates (IRs) with expoisson and poisson.

```
. use http://www.stata-press.com/data/r13/heartvalve
(heart valve replacement data)
. list
```

| age | valve | deaths | TAR |
| :---: | :---: | :---: | :---: |
| $<55$ | aortic | 4 | 1259 |
| $<55$ | mitral | 1 | 2082 |
| $>=55$ | aortic | 7 | 1417 |
| $>=55$ | mitral | 9 | 1647 |

The age variable is coded 0 for age $<55$ and 1 for age $\geq 55$, and the valve variable is coded 0 for the aortic valve and 1 for the mitral valve. The total number of deaths, $M=21$, is small enough that enumerating the conditional distributions for age and valve type is feasible and asymptotic inferences associated with standard ML Poisson regression may be questionable.

```
. expoisson deaths age valve, exposure(TAR) irr
Estimating: age
Enumerating sample-space combinations:
observation 1: enumerations = 11
observation 2: enumerations = 11
observation 3: enumerations = 132
observation 4: enumerations = 22
Estimating: valve
Enumerating sample-space combinations:
observation 1: enumerations = 17
observation 2: enumerations = 17
observation 3: enumerations = 102
observation 4: enumerations = 22
Exact Poisson regression
```

Number of obs $=$
4

| deaths | IRR | Suff. | $2 * \operatorname{Pr}$ (Suff.) | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: |
| age | 3.390401 | 16 | 0.0194 | 1.182297 | 11.86935 |
| valve | .7190197 | 10 | 0.5889 | .2729881 | 1.870068 |
| $\ln (T A R)$ | 1 | (exposure) |  |  |  |


| - poisson deaths age valve, exposure(TAR) irr nolog |  |  |  |
| :--- | :--- | :--- | :--- |
| Poisson regression | Number of obs | $=$ | 4 |
|  | LR chi2 (2) | $=$ | 7.62 |
|  | Prob > chi2 | $=$ | 0.0222 |
| Log likelihood $=-8.1747285$ |  | Pseudo R2 | $=$ |


| deaths | IRR | Std. Err. | $z$ | $P>\|z\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | ---: |
| age | 3.390401 | 1.741967 | 2.38 | 0.017 | 1.238537 | 9.280965 |
| valve | .7190197 | .3150492 | -0.75 | 0.452 | .3046311 | 1.6971 |
| _cons | .0018142 | .0009191 | -12.46 | 0.000 | .0006722 | .0048968 |
| $\ln$ (TAR) | 1 | (exposure) |  |  |  |  |

The CMLE and the MLE are numerically identical. The death rate for the older age group is about 3.4 times higher than the younger age group, and this difference is significant at the $5 \%$ level. This means that for every death in the younger group each month, we would expect about three deaths in the older group. The IR estimate for valve type is approximately 0.72 , but it is not significantly different from one. The exact Poisson confidence intervals are a bit wider than the asymptotic CIs.

You can use ir (see [ST] epitab) to estimate IRs and exact CIs for one covariate, and we compare these CIs with those from expoisson, where we estimate the incidence rate by using age only.
. ir deaths age TAR

|  | age of patient <br> Exposed Unexposed | Total |  |
| :---: | :---: | :---: | :---: |
| number of deaths time at risk | 16 5 <br> 3064 3341 | $\begin{array}{r} 21 \\ 6405 \end{array}$ |  |
| Incidence rate | .0052219 .0014966 <br> Point estimate | .0032787 <br> [95\% Con | Interval] |
| Inc. rate diff. <br> Inc. rate ratio <br> Attr. frac. ex. <br> Attr. frac. pop | $\begin{aligned} & .0037254 \\ & 3.489295 \\ & .7134092 \\ & .5435498 \end{aligned}$ | $\begin{array}{r} .00085 \\ 1.221441 \\ .1812948 \end{array}$ | $\begin{array}{ll} .0066007 \\ 12.17875 & \text { (exact) } \\ .9178898 & \text { (exact) } \end{array}$ |
|  | $\left.\begin{array}{rl}(\text { midp }) & \operatorname{Pr}(k>=16) \\ (\text { midp }) & 2 * \operatorname{Pr}(k>=16)\end{array}\right) \quad 0.0049$ (exact) |  |  |

. expoisson deaths age, exposure(TAR) irr midp nolog
Exact Poisson regression
Number of obs $=$
4

| deaths | IRR | Suff. | $2 * \operatorname{Pr}$ (Suff.) | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: |
| age | 3.489295 | 16 | 0.0099 | 1.324926 | 10.64922 |
| $\ln (T A R)$ | 1 | (exposure) |  |  |  |

mid-p-value computed for the probabilities and CIs
Both ir and expoisson give identical IRs and $p$-values. Both report the two-sided exact significance by using the mid- $p$-value rule that accounts for the discreteness in the distribution by subtracting $p_{1 / 2}=$ $\operatorname{Pr}(T=t) / 2$ from $p_{l}=\operatorname{Pr}(T \leq t)$ and $p_{g}=\operatorname{Pr}(T \geq t)$, computing $2 \times \min \left(p_{l}-p_{1 / 2}, p_{g}-p_{1 / 2}\right)$. By default, expoisson will not use the mid- $p$-value rule (when you exclude the midp option), and here the two-sided exact significance would be $2 \times \min \left(p_{l}, p_{g}\right)=0.0158$. The confidence intervals differ because expoisson uses the mid- $p$-value rule when computing the confidence intervals, yet
ir does not. You can verify this by executing expoisson without the midp option for this example; you will get the same CIs as ir.

You can replay expoisson to view the conditional scores test or the conditional probabilities test by using the test() option.

```
. expoisson, test(score) irr
Exact Poisson regression
```

Number of obs $=$
4

| deaths | IRR | Score | Pr>=Score | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | ---: | ---: |
| age | 3.489295 | 6.76528 <br> (exposure) | 0.0113 | 1.324926 | 10.64922 |

mid-p-value computed for the probabilities and CIs
All the statistics for expoisson are defined in Methods and formulas of [R] exlogistic. Apart from enumerating the conditional distributions for the logistic and Poisson sufficient statistics, computationally, the primary difference between exlogistic and expoisson is the weighting values in the likelihood for the parameter sufficient statistics.

## > Example 3

In this example, we fabricate data that will demonstrate the difference between the CMLE and the MUE when the CMLE is not infinite. A difference in these estimates will be more pronounced when the probability of the coefficient sufficient statistic is skewed when plotted as a function of the regression coefficient.

```
. clear
. input y x
    y x
    1. 0 2
    2. 1 1
    3. 10
    4. 0 0
    5. 0. . 
    6. 1 . 5
    7. 2 . 01
    8. 3 . 001
    9. 4.0001
10. end
. expoisson y x, test(score)
Enumerating sample-space combinations:
observation 1: enumerations = 13
observation 2: enumerations = 91
observation 3: enumerations = 169
observation 4: enumerations = 169
observation 5: enumerations = 313
observation 6: enumerations = 313
observation 7: enumerations = 1469
observation 8: enumerations = 5525
observation 9: enumerations = 5479
Exact Poisson regression
```

| y | Coef. | Score | Pr>=Score | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: |
| x | -1.534468 | 2.955316 | 0.0810 | -3.761718 | .0485548 |

. expoisson $y$ x, test(score) mue(x) nolog
Exact Poisson regression

$$
\text { Number of obs }=
$$

9

| y | Coef. | Score | Pr>=Score | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: |
| x | $-1.309268 *$ | 2.955316 | 0.0810 | -3.761718 | .0485548 |

(*) median unbiased estimates (MUE)
We observe $\left(x_{i}, y_{i}\right), i=1, \ldots, 9$. If we condition on $m=\sum_{i=1}^{9} y_{i}=12$, the conditional distribution of $T_{x}=\sum_{i} Y_{i} x_{i}$ has a size of 5,479 elements. For each entry in this enumeration, a realization of $Y_{i}=y_{i}^{(k)}, k=1, \ldots, 5,479$, is generated such that $\sum_{i} y_{i}^{(k)}=12$. One of these realizations produces the observed $t_{x}=\sum_{i} y_{i} x_{i} \approx 1.5234$.

Below is a graphical display comparing the CMLE with the MUE. We plot $\operatorname{Pr}\left(T_{x}=t_{x} \mid M=12, \beta_{x}\right)$ versus $\beta_{x},-6 \leq \beta_{x} \leq 1$, in the upper panel and the cumulative probabilities, $\operatorname{Pr}\left(T_{x} \leq t_{x} \mid M=\right.$ $\left.12, \beta_{x}\right)$ and $\operatorname{Pr}\left(T_{x} \geq t_{x} \mid M=12, \beta_{x}\right)$, in the lower panel.


The location of the CMLE, indicated by the dashed line, is at the mode of the probability profile, and the MUE, indicated by the dotted line, is to the right of the mode. If we solve for the $\beta_{x}^{(u)}$ and $\beta_{x}^{(l)}$ such that $\operatorname{Pr}\left(T_{x} \leq t_{x} \mid M=12, \beta_{x}^{(u)}\right)=1 / 2$ and $\operatorname{Pr}\left(T_{x} \geq t_{x} \mid M=12, \beta_{x}^{(l)}\right)=1 / 2$, the MUE is $\left(\beta_{x}^{(u)}+\beta_{x}^{(l)}\right) / 2$. As you can see in the lower panel, the MUE cuts through the intersection of these cumulative probability profiles.

## - Technical note

The memory (\#) option limits the amount of memory that expoisson will consume when computing the conditional distribution of the parameter sufficient statistics. memory () is independent of the data maximum memory setting (see set max_memory in [D] memory), and it is possible for expoisson to exceed the memory limit specified in set max_memory without terminating. By default, a log is provided that displays the number of enumerations (the size of the conditional distribution) after processing each observation. Typically, you will see the number of enumerations increase, and then at some point they will decrease as the multivariate shift algorithm (Hirji, Mehta, and Patel 1987) determines that some of the enumerations cannot achieve the observed sufficient statistics of the conditioning variables. When the algorithm is complete, however, it is necessary to store the conditional distribution of the parameter sufficient statistics as a dataset. It is possible, therefore, to get a memory error when the algorithm has completed if there is not enough memory to store the conditional distribution.

## - Technical note

Computing the conditional distributions and reported statistics requires data sorting and numerical comparisons. If there is at least one single-precision variable specified in the model, expoisson will make comparisons with a relative precision of $2^{-5}$. Otherwise, a relative precision of $2^{-11}$ is used. Be careful if you use recast to promote a single-precision variable to double precision (see [D] recast). You might try listing the data in full precision (maybe $\% 20.15 \mathrm{~g}$; see [D] format) to make sure that this is really what you want. See [D] data types for information on precision of numeric storage types.

## Stored results

expoisson stores the following in e() :
Scalars
e(N)
e(k_groups)
$e($ relative_weight)
e(sum_y)
e(k_indvars)
e(k_condvars)
e(midp)
e(eps)
Macros
e(cmd)
e(cmdline)
e(title)
e(depvar)
e(indvars)
e(condvars)
e(groupvar)
e(exposure)
e(offset)
e(level)
e(wtype)
e(wexp)
e(datasignature)
e(datasignaturevars)
e(properties)
e(estat_cmd)
e(marginsnotok)
number of observations
number of groups
relative weight for the observed e(sufficient) and e(condvars)
sum of depvar
number of independent variables
number of conditioning variables
mid- $p$-value rule indicator
relative difference tolerance
expoisson
command as typed
title in estimation output
name of dependent variable
independent variables
conditional variables
group variable
exposure variable
linear offset variable
confidence level
weight type
weight expression
the checksum
variables used in calculation of checksum
b V
program used to implement estat
predictions disallowed by margins

```
Matrices
    e(b)
    e(mue_indicators)
    e(se)
    e(ci)
    e(sum_y_groups)
    e(N_g)
    e(sufficient)
    e(p_sufficient)
    e(scoretest)
    e(p_scoretest)
    e(probtest)
    e(p_probtest)
```

Functions
e(sample)
coefficient vector
indicator for elements of e(b) estimated using MUE instead of CMLE $e(b)$ standard errors (CMLEs only)
matrix of e(level) confidence intervals for e(b)
sum of e(depvar) for each group
number of observations in each group
sufficient statistics for e(b)
$p$-value for e(sufficient)
conditional scores tests for indepvars
$p$-values for e(scoretest)
conditional probability tests for indepvars
$p$-value for e(probtest)
marks estimation sample

## Methods and formulas

Let $\left\{Y_{1}, Y_{2}, \ldots, Y_{n}\right\}$ be a set of $n$ independent Poisson random variables. For each $i=1, \ldots, n$, we observe $Y_{i}=y_{i} \geq 0$, and associated with each observation is the covariate row vector of length $p, \mathbf{x}_{i}=\left(x_{i 1}, \ldots, x_{i p}\right)$. Denote $\boldsymbol{\beta}=\left(\beta_{1}, \ldots, \beta_{p}\right)^{T}$ to be the column vector of regression parameters and $\theta$ to be the constant. The sufficient statistic for $\beta_{j}$ is $T_{j}=\sum_{i=1}^{n} Y_{i} x_{i j}, j=1, \ldots, p$, and for $\theta$ is $M=\sum_{i=1}^{n} Y_{i}$. We observe $T_{j}=t_{j}, t_{j}=\sum_{i=1}^{n} y_{i} x_{i j}$, and $M=m, m=\sum_{i=1}^{n} y_{i}$. Let $\kappa_{i}$ be the exposure for the $i$ th observation. Then the probability of observing $\left(Y_{1}=y_{1}, Y_{2}=y_{2}, \ldots, Y_{n}=y_{n}\right)$ is

$$
\operatorname{Pr}\left(Y_{1}=y_{1}, \ldots, Y_{n}=y_{n} \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\kappa}\right)=\frac{\exp (m \theta+\mathbf{t} \boldsymbol{\beta})}{\exp \left\{\sum_{i=1}^{n} \kappa_{i} \exp \left(\theta+\mathbf{x}_{i} \boldsymbol{\beta}\right)\right\}} \prod_{i=1}^{n} \frac{\kappa_{i}^{y_{i}}}{y_{i}!}
$$

where $\mathbf{t}=\left(t_{1}, \ldots, t_{p}\right), \mathbf{X}=\left(\mathbf{x}_{1}^{T}, \ldots, \mathbf{x}_{n}^{T}\right)^{T}$, and $\boldsymbol{\kappa}=\left(\kappa_{1}, \ldots, \kappa_{n}\right)^{T}$.
The joint distribution of the sufficient statistics $(\mathbf{T}, M)$ is obtained by summing over all possible sequences $Y_{1} \geq 0, \ldots, Y_{n} \geq 0$ such that $\mathbf{T}=\mathbf{t}$ and $M=m$. This probability function is

$$
\operatorname{Pr}\left(T_{1}=t_{1}, \ldots, T_{p}=t_{p}, M=m \mid \boldsymbol{\beta}, \mathbf{X}, \boldsymbol{\kappa}\right)=\frac{\exp (m \theta+\mathbf{t} \boldsymbol{\beta})}{\exp \left\{\sum_{i=1}^{n} \kappa_{i} \exp \left(\theta+\mathbf{x}_{i} \boldsymbol{\beta}\right)\right\}}\left(\sum_{\mathbf{u}} \prod_{i=1}^{n} \frac{\kappa_{i}^{u_{i}}}{u_{i}!}\right)
$$

where the sum $\sum_{\mathbf{u}}$ is over all nonnegative vectors $\mathbf{u}$ of length $n$ such that $\sum_{i=1}^{n} u_{i}=m$ and $\sum_{i=1}^{n} u_{i} \mathbf{x}_{i}=\mathbf{t}$.

## Conditional distribution

Without loss of generality, we will restrict our discussion to the conditional distribution of the sufficient statistic for $\beta_{1}, T_{1}$. If we condition on observing $M=m$ and $T_{2}=t_{2}, \ldots, T_{p}=t_{p}$, the probability function of $\left(T_{1} \mid \beta_{1}, T_{2}=t_{2}, \ldots, T_{p}=t_{p}, M=m\right)$ is

$$
\begin{equation*}
\operatorname{Pr}\left(T_{1}=t_{1} \mid \beta_{1}, T_{2}=t_{2}, \ldots, T_{p}=t_{p}, M=m\right)=\frac{\left(\sum_{\mathbf{u}} \prod_{i=1}^{n} \frac{\kappa_{i}^{u_{i}}}{u_{i}!}\right) e^{t_{1} \beta_{1}}}{\sum_{\mathbf{v}}\left(\prod_{i=1}^{n} \frac{\kappa_{i}^{v_{i}}}{v_{i}!}\right) e^{\beta_{1} \sum_{i} v_{i} x_{i 1}}} \tag{2}
\end{equation*}
$$

where the sum $\sum_{\mathbf{u}}$ is over all nonnegative vectors $\mathbf{u}$ of length $n$ such that $\sum_{i=1}^{n} u_{i}=m$ and $\sum_{i=1}^{n} u_{i} \mathbf{x}_{i}=\mathbf{t}$, and the sum $\sum_{\mathbf{v}}$ is over all nonnegative vectors $\mathbf{v}$ of length $n$ such that $\sum_{i=1}^{n} v_{i}=m$, $\sum_{i=1}^{n=1} v_{i} x_{i 2}=t_{2}, \ldots, \sum_{i=1}^{n} v_{i} x_{i p}=t_{p}$. The CMLE for $\beta_{1}$ is the value that maximizes the log of (1). This optimization task is carried out by ml (see [R] ml ), using the conditional distribution of $\left(T_{1} \mid T_{2}=t_{2}, \ldots, T_{p}=t_{p}, M=m\right)$ as a dataset. This dataset consists of the feasible values and weights for $T_{1}$,

$$
\left\{\left(s_{1}, \prod_{i=1}^{n} \frac{\kappa_{i}^{v_{i}}}{v_{i}!}\right): \sum_{i=1}^{n} v_{i}=m, \sum_{i=1}^{n} v_{i} x_{i 1}=s_{1}, \sum_{i=1}^{n} v_{i} x_{i 2}=t_{2}, \ldots, \sum_{i=1}^{n} v_{i} x_{i p}=t_{p}\right\}
$$

Computing the CMLE, MUE, confidence intervals, conditional hypothesis tests, and sufficient statistic $p$-values is discussed in Methods and formulas of [R] exlogistic. The only difference between the two techniques is the use of the weights; that is, the weights for exact logistic are the combinatorial coefficients, $c(\mathbf{t}, m)$, in (1) of Methods and formulas in [R] exlogistic. expoisson and exlogistic use the same ml likelihood evaluator to compute the CMLEs as well as the same ado-programs and Mata functions to compute the MUEs and estimate statistics.

## References

Agresti, A. 2013. Categorical Data Analysis. 3rd ed. Hoboken, NJ: Wiley.
Armitage, P., G. Berry, and J. N. S. Matthews. 2002. Statistical Methods in Medical Research. 4th ed. Oxford: Blackwell.

Cox, D. R., and E. J. Snell. 1989. Analysis of Binary Data. 2nd ed. London: Chapman \& Hall.
Hirji, K. F., C. R. Mehta, and N. R. Patel. 1987. Computing distributions for exact logistic regression. Journal of the American Statistical Association 82: 1110-1117.
Laird, N. M., and D. Olivier. 1981. Covariance analysis of censored survival data using log-linear analysis techniques. Journal of the American Statistical Association 76: 231-240.

## Also see

$[R]$ expoisson postestimation - Postestimation tools for expoisson
[R] poisson - Poisson regression
[XT] xtpoisson - Fixed-effects, random-effects, and population-averaged Poisson models
[U] 20 Estimation and postestimation commands

## Title

expoisson postestimation - Postestimation tools for expoisson

| Description | Syntax for estat se $\quad$ Menu for estat $\quad$ Option for estat se |
| :--- | :--- |
| Remarks and examples | Also see |

## Description

The following postestimation command is of special interest after expoisson:

| Command | Description |
| :--- | :--- |
| estat se | report coefficients or IRRs and their asymptotic standard errors |

The following standard postestimation command is also available:
Command
Description
estat summarize
summary statistics for the estimation sample
See $[R]$ estat summarize for details.

## Special-interest postestimation command

estat se reports regression coefficients or incidence-rate asymptotic standard errors. The estimates are stored in the matrix $r$ (estimates).

## Syntax for estat se

estat se [, irr]

## Menu for estat

Statistics $>$ Postestimation $>$ Reports and statistics

## Option for estat se

irr requests that the incidence-rate ratios and their asymptotic standard errors be reported. The default is to report the coefficients and their asymptotic standard errors.

## Remarks and examples

## Example 1

To demonstrate estat se after expoisson, we use the British physicians smoking data.


```
. use http://www.stata-press.com/data/r13/smokes
(cigarette smoking and lung cancer among British physicians (45-49 years))
. expoisson cases smokes, exposure(peryrs) irr nolog
Exact Poisson regression
    Number of obs = 7
```

. estat se, irr

## Also see

[R] expoisson - Exact Poisson regression
[U] 20 Estimation and postestimation commands

## Title

fp - Fractional polynomial regression

| Syntax | Menu | Description |
| :--- | :--- | :--- |
| Options for fp | Options for fp generate | Remarks and examples |
| Stored results | Methods and formulas | Acknowledgment |
| References | Also see |  |

## Syntax

## Estimation

fp <term> [, est_options]: est_cmd
est_cmd may be almost any estimation command that stores the e(ll) result. To confirm whether fp works with a specific est_cmd, see the documentation for that est_cmd.

Instances of <term> (with the angle brackets) that occur within est_cmd are replaced in est_cmd by a varlist containing the fractional powers of the variable term. These variables will be named term_1, term_2, ....
fp performs est_cmd with this substitution, fitting a fractional polynomial regression in term. est_cmd in either this or the following syntax may not contain other prefix commands; see [U] 11.1.10 Prefix commands.

Estimation (alternate syntax)
fp <term>(varname) [, est_options]: est_cmd
Use this syntax to specify that fractional powers of varname are to be calculated. The fractional polynomial power variables will still be named term_1, term_2,....

Replay estimation results
fp [, replay_options]

Create specified fractional polynomial power variables
fp generate [type][newvar =] varname^(numlist) [if] [in] [, gen_options $]$

Main
Search powers (\# \# . . . \#)
powers to be searched; default is powers ( $-2-1-.50 .5123$ ) dimension(\#) maximum degree of fractional polynomial; default is dimension(2)

Or specify fp(\# \# ...\#) use specified fractional polynomial

And then specify any of these options
Options
classic perform automatic scaling and centering and omit comparison table
replace replace existing fractional polynomial power variables named term_1, term_2, ...
all generate term_1, term_2, ... in all observations; default is in observations if esample()
scale (\#_a \#_b) use $($ term $+a) / b$; default is to use variable term as is
scale specify $a$ and $b$ automatically
center $\left(\# \_c\right) \quad$ report centered-on- $c$ results; default is uncentered results
center $\quad$ specify $c$ to be the mean of (scaled) term
zero
catzero
set term_1, term_2, .. to zero if scaled term $\leq 0$; default is to issue an error message
same as zero and include term_0 $=($ term $\leq 0)$ among fractional polynomial power variables

Reporting
replay_options
specify how results are displayed
replay_options
Description
Reporting
nocompare
reporting_options
do not display model-comparison test results
any options allowed by est_cmd for replaying estimation results

Description
Main
replace
replace existing fractional polynomial power variables named term_1, term_2, ...
scale $\left(\#-a \# \_b\right) \quad$ use $($ term $+a) / b$; default is to use variable term as is
scale
center (\#_c)
center
zero
catzero
specify $a$ and $b$ automatically
report centered-on- $c$ results; default is uncentered results
specify $c$ to be the mean of (scaled) term
set term_1, term_2, .. to zero if scaled term $\leq 0$; default is to issue an error message
same as zero and include term_0 $=($ term $\leq 0)$ among fractional polynomial power variables

## Menu

fp
Statistics $>$ Linear models and related $>$ Fractional polynomials $>$ Fractional polynomial regression

## fp generate

Statistics $>$ Linear models and related $>$ Fractional polynomials $>$ Create fractional polynomial variables

## Description

fp <term>: est_cmd fits models with the "best"-fitting fractional polynomial substituted for <term> wherever it appears in est_cmd. fp <weight> : regress mpg <weight> foreign would fit a regression model of mpg on a fractional polynomial in weight and (linear) foreign.

By specifying option $f p()$, you may set the exact powers to be used. Otherwise, a search through all possible fractional polynomials up to the degree set by dimension() with the powers set by powers() is performed.
fp without arguments redisplays the previous estimation results, just as typing est_cmd would. You can type either one. fp will include a fractional polynomial comparison table.
fp generate creates fractional polynomial power variables for a given set of powers. For instance, fp <weight>: regress mpg <weight> foreign might produce the fractional polynomial weight ${ }^{(-2,-1)}$ and store weight ${ }^{-2}$ in weight_1 and weight ${ }^{-1}$ in weight_2. Typing fp generate weight ${ }^{\wedge}(-2-1)$ would allow you to create the same variables in another dataset.

See [R] mfp for multivariable fractional polynomial models.

## Options for fp

$\qquad$ Main
powers (\# \# ... \#) specifies that a search be performed and details about the search provided. powers() works with the dimension() option; see below. The default is powers ( $-2-1-.50$ .5123 ).
dimension(\#) specifies the maximum degree of the fractional polynomial to be searched. The default is dimension(2).

If the defaults for both powers() and dimension() are used, then the fractional polynomial could be any of the following 44 possibilities:

```
            term \(^{(-2)}\)
            term \({ }^{(-1)}\)
            :
            term \({ }^{(3)}\)
term \(^{(-2)}\),term \(^{(-2)}\)
term \(^{(-2)}\) term \(^{(-1)}\)
term \(^{(-2)}\),term \(^{(3)}\)
term \(^{(-1)}\), term \(^{(-2)}\)
term \(^{(3)}\) term \(^{(3)}\)
```

$\mathrm{fp}(\# \# \ldots$. \#) specifies that no search be performed and that the fractional polynomial specified be used. $f p()$ is an alternative to powers() and dimension().

## Options

classic performs automatic scaling and centering and omits the comparison table. Specifying classic is equivalent to specifying scale, center, and nocompare.
replace replaces existing fractional polynomial power variables named term_1, term_2, $\ldots$.
all specifies that term_1, term_2, .. be filled in for all observations in the dataset rather than just for those in e(sample).
scale(\#_a \#_b) specifies that term be scaled in the way specified, namely, that $($ term $+a) / b$ be calculated. All values of scaled term are required to be greater than zero unless you specify options zero or catzero. Values should not be too large or too close to zero, because by default, cubic powers and squared reciprocal powers will be considered. When scale $(a b)$ is specified, values in the variable term are not modified; fp merely remembers to scale the values whenever powers are calculated.

You will probably not use scale $(a b)$ for values of $a$ and $b$ that you create yourself, although you could. It is usually easier just to generate a scaled variable. For instance, if term is age, and age in your data is required to be greater than or equal to 20 , you might generate an age 5 variable, for use as term:

```
. generate age5 = (age-19)/5
```

scale ( $a b$ ) is useful when you previously fit a model using automatic scaling (option scale) in one dataset and now want to create the fractional polynomials in another. In the first dataset, fp with scale added notes to the dataset concerning the values of $a$ and $b$. You can see them by typing

```
    notes
```

You can then use fp generate, $\operatorname{scale}(a b)$ in the second dataset.
The default is to use term as it is used in calculating fractional powers; thus term's values are required to be greater than zero unless you specify options zero or catzero. Values should not be too large, because by default, cubic powers will be considered.
scale specifies that term be scaled to be greater than zero and not too large in calculating fractional powers. See Scaling for more details. When scale is specified, values in the variable term are not modified; $f p$ merely remembers to scale the values whenever powers are calculated.
center(\#_c) reports results for the fractional polynomial in (scaled) term, centered on $c$. The default is to perform no centering.
term ${ }^{\left(p_{1}, p_{2}, \ldots, p_{m}\right)}-c^{\left(p_{1}, p_{2}, \ldots, p_{m}\right)}$ is reported. This makes the constant coefficient (intercept) easier to interpret. See Centering for more details.
center performs center $(c)$, where $c$ is the mean of (scaled) term.
zero and catzero specify how nonpositive values of term are to be handled. By default, nonpositive values of term are not allowed, because we will be calculating natural logarithms and fractional powers of term. Thus an error message is issued.
zero sets the fractional polynomial value to zero for nonpositive values of (scaled) term.
catzero sets the fractional polynomial value to zero for nonpositive values of (scaled) term and includes a dummy variable indicating where nonpositive values of (scaled) term appear in the model.

Reporting
nocompare suppresses display of the comparison tests.
reporting_options are any options allowed by est_cmd for replaying estimation results.

## Options for fp generate

$\qquad$ Main
replace replaces existing fractional polynomial power variables named term_1, term_2, $\ldots$
scale (\#_a \#_b) specifies that term be scaled in the way specified, namely, that $($ term $+a) / b$ be calculated. All values of scaled term are required to be greater than zero unless you specify options zero or catzero. Values should not be too large or too close to zero, because by default, cubic powers and squared reciprocal powers will be considered. When scale $(a b)$ is specified, values in the variable term are not modified; $f p$ merely remembers to scale the values whenever powers are calculated.
You will probably not use scale $(a b)$ for values of $a$ and $b$ that you create yourself, although you could. It is usually easier just to generate a scaled variable. For instance, if term is age, and age in your data is required to be greater than or equal to 20 , you might generate an age 5 variable, for use as term:

```
. generate age5 = (age-19)/5
```

scale ( $a b$ ) is useful when you previously fit a model using automatic scaling (option scale) in one dataset and now want to create the fractional polynomials in another. In the first dataset, fp with scale added notes to the dataset concerning the values of $a$ and $b$. You can see them by typing

```
. notes
```

You can then use fp generate, $\operatorname{scale}(a b)$ in the second dataset.
The default is to use term as it is used in calculating fractional powers; thus term's values are required to be greater than zero unless you specify options zero or catzero. Values should not be too large, because by default, cubic powers will be considered.
scale specifies that term be scaled to be greater than zero and not too large in calculating fractional powers. See Scaling for more details. When scale is specified, values in the variable term are not modified; fp merely remembers to scale the values whenever powers are calculated.
center (\#_c) reports results for the fractional polynomial in (scaled) term, centered on $c$. The default is to perform no centering.
term ${ }^{\left(p_{1}, p_{2}, \ldots, p_{m}\right)}-c^{\left(p_{1}, p_{2}, \ldots, p_{m}\right)}$ is reported. This makes the constant coefficient (intercept) easier to interpret. See Centering for more details.
center performs center $(c)$, where $c$ is the mean of (scaled) term.
zero and catzero specify how nonpositive values of term are to be handled. By default, nonpositive values of term are not allowed, because we will be calculating natural logarithms and fractional powers of term. Thus an error message is issued.
zero sets the fractional polynomial value to zero for nonpositive values of (scaled) term.
catzero sets the fractional polynomial value to zero for nonpositive values of (scaled) term and includes a dummy variable indicating where nonpositive values of (scaled) term appear in the model.

## Remarks and examples

Remarks are presented under the following headings:
Fractional polynomial regression Scaling Centering
Examples

## Fractional polynomial regression

Regression models based on fractional polynomial functions of a continuous covariate are described by Royston and Altman (1994).

Fractional polynomials increase the flexibility afforded by the family of conventional polynomial models. Although polynomials are popular in data analysis, linear and quadratic functions are limited in their range of curve shapes, whereas cubic and higher-order curves often produce undesirable artifacts such as edge effects and waves.

Fractional polynomials differ from regular polynomials in that 1) they allow logarithms, 2) they allow noninteger powers, and 3) they allow powers to be repeated.

We will write a fractional polynomial in $x$ as

$$
x^{\left(p_{1}, p_{2}, \ldots, p_{m}\right)^{\prime}} \boldsymbol{\beta}
$$

We will write $x^{(p)}$ to mean a regular power except that $x^{(0)}$ is to be interpreted as meaning $\ln (x)$ rather than $x^{(0)}=1$.

Then if there are no repeated powers in $\left(p_{1}, p_{2}, \ldots, p_{m}\right)$,

$$
x^{\left(p_{1}, p_{2}, \ldots, p_{m}\right) \prime} \boldsymbol{\beta}=\beta_{0}+\beta_{1} x^{\left(p_{1}\right)}+\beta_{2} x^{\left(p_{2}\right)}+\cdots+\beta_{m} x^{\left(p_{m}\right)}
$$

Powers are allowed to repeat in fractional polynomials. Each time a power repeats, it is multiplied by another $\ln (x)$. As an extreme case, consider the fractional polynomial with all-repeated powers, say, $m$ of them,

$$
x^{(p, p, \ldots, p) \prime} \boldsymbol{\beta}=\beta_{0}+\beta_{1} x^{(p)}+\beta_{2} x^{(p)} \ln (x)+\cdots+\beta_{m} x^{(p)}\{\ln (x)\}^{m-1}
$$

Thus the fractional polynomial $x^{(0,0,2) \prime} \boldsymbol{\beta}$ would be

$$
\begin{aligned}
x^{(0,0,2) \prime} \boldsymbol{\beta} & =\beta_{0}+\beta_{1} x^{(0)}+\beta_{2} x^{(0)} \ln (x)+\beta_{3} x^{(2)} \\
& =\beta_{0}+\beta_{1} \ln (x)+\beta_{2}\{\ln (x)\}^{2}+\beta_{3} x^{2}
\end{aligned}
$$

With this definition, we can obtain a much wider range of shapes than can be obtained with regular polynomials. The following graphs appeared in Royston and Sauerbrei (2008, sec. 4.5). The first graph shows the shapes of differing fractional polynomials.


The second graph shows some of the curve shapes available with different $\boldsymbol{\beta}$ s for the degree-2 fractional polynomial, $x^{(-2,2)}$.


In modeling a fractional polynomial, Royston and Sauerbrei (2008) recommend choosing powers from among $\{-2,-1,-0.5,0,0.5,1,2,3\}$. By default, fp chooses powers from this set, but other powers can be explicitly specified in the powers () option.
fp <term> : est_cmd fits models with the terms of the best-fitting fractional polynomial substituted for <term> wherever it appears in est_cmd. We will demonstrate with auto.dta, which contains repair records and other information about a variety of vehicles in 1978.

We use $f p$ to find the best fractional polynomial in automobile weight (lbs.) (weight) for the linear regression of miles per gallon ( mpg ) on weight and an indicator of whether the vehicle is foreign (foreign).

By default, $f p$ will fit degree-2 fractional polynomial (FP2) models and choose the fractional powers from the set $\{-2,-1,-0.5,0,0.5,1,2,3\}$. Because car weight is measured in pounds and will have a cubic transformation applied to it, we shrink it to a smaller scale before estimation by dividing by 1,000 .

We modify the existing weight variable for conciseness and to facilitate the comparison of tables. When applying a data transformation in practice, rather than modifying the existing variables, you should create new variables that hold the transformed values.

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. replace weight = weight/1000
weight was int now float
(74 real changes made)
. fp <weight>: regress mpg <weight> foreign
(fitting 44 models)
(. . . 10% . . . 20% . . . . 30% . . . 40% . . . . 50% . . . 60% . . . 70% . . . . 80% . . . . 90% . . . . 100%)
Fractional polynomial comparisons:
```

| weight | df | Deviance | Res. s.d. | Dev. dif. | $P(*)$ | Powers |
| ---: | ---: | :---: | :---: | :---: | :---: | :--- |
| omitted | 0 | 456.347 | 5.356 | 75.216 | 0.000 |  |
| linear | 1 | 388.366 | 3.407 | 7.236 | 0.082 | 1 |
| m = 1 | 2 | 381.806 | 3.259 | 0.675 | 0.733 | -.5 |
| m $=2$ | 4 | 381.131 | 3.268 | 0.000 | -- | $-2-2$ |

(*) $\mathrm{P}=$ sig. level of model with $\mathrm{m}=2$ based on F with 68 denominator dof.

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | 1696.05949 | 3 | 565.353163 |
| Residual | 747.399969 | 70 | 10.6771424 |
| Total | 2443.45946 | 73 | 33.4720474 |


| Number of obs | $=$ | 74 |
| :--- | ---: | ---: |
| $\mathrm{~F}(3$, | $70)$ | $=$ |
| Prob $>$ F | $=$ | 0.0000 |
| R-squared | $=$ | 0.6941 |
| Adj R-squared | $=$ | 0.6810 |
| Root MSE | $=$ | 3.2676 |


| mpg | Coef. | Std. Err. | t | $\mathrm{P}>\mid \mathrm{tI}$ | [95\% Conf. Interval] |  |
| ---: | ---: | :--- | :---: | :---: | ---: | ---: |
| weight_1 | 15.88527 | 20.60329 | 0.77 | 0.443 | -25.20669 | 56.97724 |
| weight_2 | 127.9349 | 47.53106 | 2.69 | 0.009 | 33.13723 | 222.7326 |
| foreign | -2.222515 | 1.053782 | -2.11 | 0.039 | -4.324218 | -.1208131 |
| _cons | 3.705981 | 3.367949 | 1.10 | 0.275 | -3.011182 | 10.42314 |

fp begins by showing the model-comparison table. This table shows the best models of each examined degree, obtained by searching through all possible power combinations. The fractional powers of the models are shown in the Powers column. A separate row is provided for the linear fractional polynomial because it is often the default used when including a predictor in the model. The null model does not include any fractional polynomial terms for weight. The df column shows the count of the additional parameters used in each model beyond the quantity of parameters used in the null model. The model deviance, which we define as twice the negative log likelihood, is given in the Deviance column. The difference of the model deviance from the deviance of the model with the lowest deviance is given in the Dev. dif. column.

The $p$-value for the partial $F$ test comparing the models and the lowest deviance model is given in the $\mathrm{P}(*)$ column. An estimate of the residual standard error is given in the Res. Sd. column. Under linear regression, a partial $F$ test is used in the model-comparison table. In other settings, a likelihood-ratio test is performed. Then a $\chi^{2}$ statistic is reported.

Under robust variance estimation and some other cases (see [R] Irtest), the likelihood-ratio test cannot be performed. When the likelihood-ratio test cannot be performed on the model specified in est_cmd, fp still reports the model-comparison table, but the comparison tests are not performed.
fp reports the "best" model as the model with the lowest deviance; however, users may choose a more efficient model based on the comparison table. They may choose the lowest degree model that the partial $F$ test (or likelihood-ratio test) fails to reject in favor of the lowest deviance model.

After the comparison table, the results of the estimation command for the lowest deviance model are shown. Here the best model has terms weight ${ }^{(-2,-2)}$. However, based on the model-comparison
table, we can reject the model without weight and the linear model at the 0.1 significance level. We fail to reject the $m=1$ model at any reasonable level. We will choose the FP1 model, which includes weight ${ }^{(-.5)}$.

We use $f p$ again to estimate the parameters for this model. We use the $f p()$ option to specify what powers we want to use; this option specifies that we do not want to perform a search for the best powers. We also specify the replace option to overwrite the previously created fractional polynomial power variables.


Alternatively, we can use $f p$ generate to create the fractional polynomial variable corresponding to weight ${ }^{(-.5)}$ and then use regress. We store weight ${ }^{(-.5)}$ in the new variable wgt_nsqrt.
. fp generate wgt_nsqrt=weight^(-.5)
. regress mpg wgt_nsqrt foreign

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | 1689.20874 | 2 | 844.604371 |
| Residual | 754.250718 | 71 | 10.6232495 |
| Total | 2443.45946 | 73 | 33.4720474 |


| Number of obs | $=$ | 74 |
| :--- | ---: | ---: |
| $\mathrm{~F}(2$, | $71)$ | $=$ |
| Prob $>$ F | $=0.51$ |  |
| R-squared | $=$ | 0.6900 |
| Adj R-squared | $=$ | 0.6826 |
| Root MSE | $=3.2593$ |  |


| mpg | Coef. | Std. Err. | t | P>\|t| | [95\% Conf. Interval] |  |
| ---: | ---: | :--- | :---: | :---: | ---: | ---: |
| wgt_nsqrt_1 | 66.89665 | 6.021748 | 11.11 | 0.000 | 54.88963 | 78.90368 |
| foreign | -2.095622 | 1.043513 | -2.01 | 0.048 | -4.176328 | -.0149155 |
| _cons | -17.58651 | 3.397991 | -5.18 | 0.000 | -24.36191 | -10.81111 |

## Scaling

Fractional polynomials are only defined for positive term variables. By default, fp will assume that the variable $x$ is positive and attempt to compute fractional powers of $x$. If the positive value assumption is incorrect, an error will be reported and estimation will not be performed.

If the values of the variable are too large or too small, the reported results of $f p$ may be difficult to interpret. By default, cubic powers and squared reciprocal powers will be considered in the search for the best fractional polynomial in term.

We can scale the variable $x$ to 1) make it positive and 2) ensure its magnitude is not too large or too small.

Suppose you have data on hospital patients with age as a fractional polynomial variable of interest. age is required to be greater than or equal to 20 , so you might generate an age 5 variable by typing

```
. generate age5 = (age-19)/5
```

A unit change in age5 is equivalent to a five-year change in age, and the minimum value of age 5 is $1 / 5$ instead of 20 .

In the automobile example of Fractional polynomial regression, our term variable was automobile weight (lbs.). Cars weigh in the thousands of pounds, so cubing their weight figures results in large numbers. We prevented this from being a problem by shrinking the weight by 1,000 ; that is, we typed

```
. replace weight = weight/1000
```

Calendar year is another type of variable that can have a problematically large magnitude. We can shrink this by dividing by 10 , making a unit change correspond to a decade.

```
. generate decade = calendar_year/10
```

You may also have a variable that measures deviation from zero. Perhaps x has already been demeaned and is symmetrically about zero. The fractional polynomial in $x$ will be undefined for half of its domain. We can shift the location of $x$, making it positive by subtracting its minimum and adding a small number to it. Suppose x ranges from -4 to 4 ; we could use

```
. generate newx = x+5
```

Rescaling ourselves provides easily communicated results. We can tell exactly how the scaling was performed and how it should be performed in similar applications.

Alternatively, fp can scale the fractional polynomial variable so that its values are positive and the magnitude of the values are not too large. This can be done automatically or by directly specifying the scaling values.

Scaling can be automatically performed with $f p$ by specifying the scale option. If term has nonpositive values, the minimum value of term is subtracted from each observation of term. In this case, the counting interval, the minimum distance between the sorted values of term, is also added to each observation of term.

After adjusting the location of term so that its minimum value is positive, creating term*, automatic scaling will divide each observation of term by a power of ten. The exponent of this scaling factor is given by

$$
\begin{aligned}
p & =\log _{10}\left\{\max \left(\text { term }^{*}\right)-\min \left(\text { term }^{*}\right)\right\} \\
p^{*} & =\operatorname{sign}(p) \text { floor }(|p|)
\end{aligned}
$$

Rather than letting fp automatically choose the scaling of term, you may specify adjustment and scale factors $a$ and $b$ by using the scale $(a b)$ option. Fractional powers are then calculated using the $($ term $+a) / b$ values.

When scale or scale $(a b)$ is specified, values in the variable term are not modified; fp merely remembers to scale the values whenever powers are calculated.

In addition to fp , both scale and $\operatorname{scale}(a b)$ may be used with fp generate.
You will probably not use scale $(a b)$ with fp for values of $a$ and $b$ that you create yourself, although you could. As we demonstrated earlier, it is usually easier just to generate a scaled variable.
scale ( $a b$ ) is useful when you previously fit a model using scale in one dataset and now want to create the fractional polynomials in another. In the first dataset, $f p$ with scale added notes to the dataset concerning the values of $a$ and $b$. You can see them by typing

You can then use fp generate, $\operatorname{scale}(a b)$ in the second dataset.
When you apply the scaling rules of a previously fit model to new data with the scale $(a b)$ option, it is possible that the scaled term may have nonpositive values. $f p$ will be unable to calculate the fractional powers of the term in this case and will issue an error.

The options zero and catzero cause $f p$ and $f p$ generate to output zero values for each fractional polynomial variable when the input (scaled) fractional polynomial variable is nonpositive. Specifying catzero causes a dummy variable indicating nonpositive values of the (scaled) fractional polynomial variable to be included in the model. A detailed example of the use of catzero and zero is shown in example 3 below.

Using the scaling options, we can fit our previous model again using the auto.dta. We specify scale ( 0 1000) so that $f p$ will shrink the magnitude of weight in estimating the regression. This is done for demonstration purposes because our scaling rule is simple. As mentioned before, in practice, you would probably only use scale $(a b)$ when applying the scaling rules from a previous analysis. Allowing fp to scale does have the advantage of not altering the original variable, weight.

```
. use http://www.stata-press.com/data/r13/auto, clear
(1978 Automobile Data)
. fp <weight>, fp(-.5) scale(0 1000): regress mpg <weight> foreign
-> regress mpg weight_1 foreign
```

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | 1689.20861 | 2 | 844.604307 |
| Residual | 754.250846 | 71 | 10.6232514 |
| Total | 2443.45946 | 73 | 33.4720474 |

of obs $=\quad 74$
$F(2,71)=79.51$
Prob > F $=0.0000$
$R$-squared $=0.6913$
Adj R-squared $=0.6826$
Root MSE $=3.2593$

| mpg | Coef. | Std. Err. | t | P>\|t| | [95\% Conf. Interval] |  |
| ---: | ---: | :--- | :---: | :---: | ---: | ---: |
| weight_1 | 66.89665 | 6.021749 | 11.11 | 0.000 | 54.88963 | 78.90368 |
| foreign | -2.095622 | 1.043513 | -2.01 | 0.048 | -4.176329 | -.0149159 |
| _cons | -17.58651 | 3.397992 | -5.18 | 0.000 | -24.36192 | -10.81111 |

The scaling is clearly indicated in the variable notes for the generated variable weight_1.

```
. notes weight_1
weight_1:
    1. fp term 1 of }\mp@subsup{x}{}{\wedge}(-.5), where x is weight scaled
    2. Scaling was user specified: x = (weight+a)/b where a=0 and b=1000
    3. Fractional polynomial variables created by fp <weight>, fp(-.5) scale(0
        1000): regress mpg <weight> foreign
    4. To re-create the fractional polynomial variables, for instance, in another
        dataset, type fp gen double weight^(-.5), scale(0 1000)
```


## Centering

The fractional polynomial of term, centered on $c$ is

$$
\left(\operatorname{term}^{\left(p_{1}, \ldots, p_{m}\right)}-c^{\left(p_{1}, \ldots, p_{m}\right)}\right)^{\prime} \boldsymbol{\beta}
$$

The intercept of a centered fractional polynomial can be interpreted as the effect at zero for all the covariates. When we center the fractional polynomial terms using $c$, the intercept is now interpreted as the effect at term $=c$ and zero values for the other covariates.

Suppose we wanted to center the fractional polynomial of $x$ with powers $(0,0,2)$ at $x=c$.

$$
\begin{aligned}
& \left(x^{(0,0,2)}-c^{(0,0,2)}\right)^{\prime} \boldsymbol{\beta} \\
& =\beta_{0}+\beta_{1}\left(x^{(0)}-c^{(0)}\right)+\beta_{2}\left\{x^{(0)} \ln (x)-c^{(0)} \ln (c)\right\}+\beta_{3}\left(x^{(2)}-c^{(2)}\right) \\
& =\beta_{0}+\beta_{1}\{\ln (x)-\ln (c)\}+\beta_{2}\left[\{\ln (x)\}^{2}-\{\ln (c)\}^{2}\right]+\beta_{3}\left(x^{2}-c^{2}\right)
\end{aligned}
$$

When center is specified, $f p$ centers based on the sample mean of (scaled) term. A previously chosen value for centering, $c$, may also be specified in center ( $c$ ). This would be done when applying the results of a previous model fitting to a new dataset.

The center and center ( $c$ ) options may be used in fp or fp generate.
Returning to the model of mileage per gallon based on automobile weight and foreign origin, we refit the model with the fractional polynomial of weight centered at its scaled mean.


Note that the coefficients for weight_1 and foreign do not change. Only the intercept _cons changes. It can be interpreted as the estimated average miles per gallon of an American-made car of average weight.

## Examples

## > Example 1: Linear regression

Consider the serum immunoglobulin G (IgG) dataset from Isaacs et al. (1983), which consists of 298 independent observations in young children. The dependent variable sqrtigg is the square root of the IgG concentration, and the independent variable age is the age of each child. (Preliminary Box-Cox analysis shows that a square root transformation removes the skewness in IgG.)

The aim is to find a model that accurately predicts the mean of sqrtigg given age. We use fp to find the best FP2 model (the default option). We specify center for automatic centering. The age of each child is small in magnitude and positive, so we do not use the scaling options of $f p$ or scale ourselves.

```
. use http://www.stata-press.com/data/r13/igg, clear
(Immunoglobulin in children)
. fp <age>, scale center: regress sqrtigg <age>
(fitting 44 models)
(. . . . 10% . . . 20% . . . . 30% . . . 40% . . . 50% . . . . 60% . . . . 70% . . . . 80% . . . . 90% . . . 100%)
```

Fractional polynomial comparisons:

| age | df | Deviance | Res. s.d. | Dev. dif. | $P(*)$ | Powers |
| ---: | ---: | ---: | ---: | ---: | ---: | :--- |
| omitted | 0 | 427.539 | 0.497 | 108.090 | 0.000 |  |
| linear | 1 | 337.561 | 0.428 | 18.113 | 0.000 | 1 |
| m = 1 | 2 | 327.436 | 0.421 | 7.987 | 0.020 | 0 |
| m $=2$ | 4 | 319.448 | 0.416 | 0.000 | -- | -22 |

(*) $P=$ sig. level of model with $m=2$ based on $F$ with 293 denominator dof.

| Source | SS | df | MS |
| ---: | ---: | ---: | :---: |
| Model <br> Residual | 22.2846976 | 2 | 11.1423488 |
| Total | 73.2523469 | 297 | .246640898 |


| Number of obs | $=$ | 298 |
| :--- | ---: | ---: |
| $\mathrm{~F}(2$, 295) | $=64.49$ |  |
| Prob $>$ F | $=$ | 0.0000 |
| R-squared | $=$ | 0.3042 |
| Adj R-squared | $=$ | 0.2995 |
| Root MSE | $=$ | .41566 |


| sqrtigg | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| age_1 | -.1562156 | .027416 | -5.70 | 0.000 | -.2101713 | -.10226 |
| age_2 | .0148405 | .0027767 | 5.34 | 0.000 | .0093757 | .0203052 |
| _cons | 2.283145 | .0305739 | 74.68 | 0.000 | 2.222974 | 2.343315 |

The new variables created by fp contain the best-fitting fractional polynomial powers of age, as centered by fp . For example, age_1 is centered by subtracting the mean of age raised to the power -2 .

The variables created by $f p$ and $f p$ generate are centered or scaled as specified by the user, which is reflected in the estimated regression coefficients and intercept. Centering does have its advantages (see Centering earlier in this entry). By default, fp will not perform scaling or centering. For a more detailed discussion, see Royston and Sauerbrei (2008, sec. 4.11).

The fitted curve has an asymmetric $S$ shape. The best model has powers $(-2,2)$ and deviance 319.448. We reject lesser degree models: the null, linear, and natural $\log$ power models at the 0.05 level. As many as 44 models have been fit in the search for the best powers. Now let's look at models of degree $\leq 4$. The highest allowed degree is specified in dimension(). We overwrite the previously generated fractional polynomial power variables by including replace.

```
. fp <age>, dimension(4) center replace: regress sqrtigg <age>
(fitting 494 models)
(. . . . 10% . . . 20% . . . . 30% . . . 40% . . . . 50% . . . 60% . . . 70% . . . . 80% . . . 90% . . . . 100%)
```

Fractional polynomial comparisons:

| age | df | Deviance | Res. s.d. | Dev. dif. | $P(*)$ | Powers |
| ---: | ---: | ---: | ---: | ---: | ---: | :--- |
| omitted | 0 | 427.539 | 0.497 | 109.795 | 0.000 |  |
| linear | 1 | 337.561 | 0.428 | 19.818 | 0.007 | 1 |
| m = 1 | 2 | 327.436 | 0.421 | 9.692 | 0.149 | 0 |
| m $=2$ | 4 | 319.448 | 0.416 | 1.705 | 0.798 | -22 |
| $\mathrm{~m}=3$ | 6 | 319.275 | 0.416 | 1.532 | 0.476 | -211 |
| $\mathrm{~m}=4$ | 8 | 317.744 | 0.416 | 0.000 | -- | 0333 |

(*) $\mathrm{P}=$ sig. level of model with $\mathrm{m}=4$ based on F with 289 denominator dof.

| Source | SS | df | MS |
| ---: | ---: | ---: | ---: |
| Model | 22.5754541 | 4 | 5.64386353 |
| Residual | 50.6768927 | 293 | .172958678 |
| Total | 73.2523469 | 297 | .246640898 |


| Number of obs | $=$ | 298 |
| :--- | ---: | ---: |
| F $(4$, 293 $)$ | $=32.63$ |  |
| Prob $>$ F | $=$ | 0.0000 |
| R-squared | $=0.3082$ |  |
| Adj R-squared | $=0.2987$ |  |
| Root MSE | $=.41588$ |  |


| sqrtigg | Coef. | Std. Err. | t | $\mathrm{P}>\mid \mathrm{tI}$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | ---: | :---: | ---: | ---: |
| age_1 | .8761824 | .1898721 | 4.61 | 0.000 | .5024962 | 1.249869 |
| age_2 | -.1922029 | .0684934 | -2.81 | 0.005 | -.3270044 | -.0574015 |
| age_3 | .2043794 | .074947 | 2.73 | 0.007 | .0568767 | .3518821 |
| age_4 | -.0560067 | .0212969 | -2.63 | 0.009 | -.097921 | -.0140924 |
| _cons | 2.238735 | .0482705 | 46.38 | 0.000 | 2.143734 | 2.333736 |

It appears that the FP4 model is not significantly different from the other fractional polynomial models (at the 0.05 level).

Let's compare the curve shape from the $m=2$ model with that from a conventional quartic polynomial whose fit turns out to be significantly better than a cubic (not shown). We use the ability of $f p$ both to generate the required powers of age, namely, $(1,2,3,4)$ for the quartic and $(-2,2)$ for the second-degree fractional polynomial, and to fit the model. The $f p()$ option is used to specify the powers. We use predict to obtain the fitted values of each regression. We fit both models with fp and graph the resulting curves with twoway scatter.

. predict fit1
(option xb assumed; fitted values)
. label variable fit1 "Quartic"
. fp <age>, center fp(-2 2) replace: regress sqrtigg <age>
-> regress sqrtigg age_1 age_2

| Source | SS | df MS |  |  | Number of obs $=298$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | F ( 2, 295) | 64.49 |
| Model | 22.2846976 | 211. | 23488 |  | Prob > F | $=0.0000$ |
| Residual | 50.9676492 | 295.17 | 71692 |  | R -squared | $=0.3042$ |
|  |  |  |  |  | Adj R-squared | $=0.2995$ |
| Total | 73.2523469 | 297.24 | 40898 |  | Root MSE | . 41566 |
| sqrtigg | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Conf. | Interval] |
| age_1 | -. 1562156 | . 027416 | -5.70 | 0.000 | -. 2101713 | -. 10226 |
| age_2 | . 0148405 | . 0027767 | 5.34 | 0.000 | . 0093757 | . 0203052 |
| _cons | 2.283145 | . 0305739 | 74.68 | 0.000 | 2.222974 | 2.343315 |

```
. predict fit2
(option xb assumed; fitted values)
. label variable fit2 "FP 2"
. scatter sqrtigg fit1 fit2 age, c(. l l) m(o i i) msize(small)
> lpattern(. __.) ytitle("Square root of IgG") xtitle("Age, years")
```



The quartic curve has an unsatisfactory wavy appearance that is implausible for the known behavior of IgG, the serum level of which increases throughout early life. The fractional polynomial curve (FP2) increases monotonically and is therefore biologically the more plausible curve. The two models have approximately the same deviance.

## > Example 2: Cox regression

Data from Smith et al. (1992) contain times to complete healing of leg ulcers in a randomized, controlled clinical trial of two treatments in 192 elderly patients. Several covariates were available, of which an important one is mthson, the number of months since the recorded onset of the ulcer. This time is recorded in whole months, not fractions of a month; therefore, some zero values are recorded.

Because the response variable is time to an event of interest and some (in fact, about one-half) of the times are censored, using Cox regression to analyze the data is appropriate. We consider fractional polynomials in mthson, adjusting for four other covariates: age; ulcarea, the area of tissue initially affected by the ulcer; deepppg, a binary variable indicating the presence or absence of deep vein involvement; and treat, a binary variable indicating treatment type.

We fit fractional polynomials of degrees 1 and 2 with fp . We specify scale to perform automatic scaling on mthson. This makes it positive and ensures that its magnitude is not too large. (See Scaling for more details.) The display option nohr is specified before the colon so that the coefficients and not the hazard ratios are displayed.

The center option is specified to obtain automatic centering. age and ulcarea are also demeaned by using summarize and then subtracting the returned result $r$ (mean).

In Cox regression, there is no constant term, so we cannot see the effects of centering in the table of regression estimates. The effects would be present if we were to graph the baseline hazard or survival function because these functions are defined with all predictors set equal to 0 .

In these graphs, we will see the estimated baseline hazard or survival function under no deep vein involvement or treatment and under mean age, ulcer area, and number of months since the recorded onset of the ulcer.

```
. use http://www.stata-press.com/data/r13/legulcer1, clear
```

(Leg ulcer clinical trial)
. stset ttevent, fail(cens)
failure event: censored $!=0$ \& censored < .
obs. time interval: (0, ttevent]
exit on or before: failure


Fractional polynomial comparisons:

| mthson | df | Deviance | Dev. dif. | $P(*)$ | Powers |
| ---: | ---: | ---: | ---: | ---: | :--- |
| omitted | 0 | 754.345 | 17.636 | 0.001 |  |
| linear | 1 | 751.680 | 14.971 | 0.002 | 1 |
| m = 1 | 2 | 738.969 | 2.260 | 0.323 | -.5 |
| m $=2$ | 4 | 736.709 | 0.000 | -- | .5 .5 |

(*) $P=$ sig. level of model with $m=2$ based on chi^2 of dev. dif.
Cox regression -- Breslow method for ties

| No. of subjects $=$ | 192 | Number of obs | $=$ | 192 |
| :--- | ---: | :--- | ---: | :--- |
| No. of failures $=$ | 92 |  |  |  |
| Time at risk | $=$ | 13825 |  |  |
|  |  | LR chi2 (6) | $=$ | 108.59 |
| Log likelihood $=$ | -368.35446 | Prob $>$ chi2 | $=$ | 0.0000 |


| _t | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| mthson_1 | -2.81425 | .6996385 | -4.02 | 0.000 | -4.185516 | -1.442984 |
| mthson_2 | 1.541451 | .4703143 | 3.28 | 0.001 | .6196521 | 2.46325 |
| age | -.0261111 | .0087983 | -2.97 | 0.003 | -.0433556 | -.0088667 |
| ulcarea | -.0017491 | .000359 | -4.87 | 0.000 | -.0024527 | -.0010455 |
| deepppg | -.5850499 | .2163173 | -2.70 | 0.007 | -1.009024 | -.1610758 |
| treat | -.1624663 | .2171048 | -0.75 | 0.454 | -.5879838 | .2630513 |

The best-fitting fractional polynomial of degree 2 has powers $(0.5,0.5)$ and deviance 736.709 . However, this model does not fit significantly better than the fractional polynomial of degree 1 (at the 0.05 level), which has power -0.5 and deviance 738.969 . We prefer the model with $m=1$.

```
. fp <mthson>, replace center scale nohr fp(-.5): stcox <mthson> age ulcarea
> deepppg treat
-> stcox mthson_1 age ulcarea deepppg treat
Cox regression -- Breslow method for ties
\begin{tabular}{lrlll} 
No. of subjects \(=\) & 192 & Number of obs & \(=\) & 192 \\
No. of failures \(=\) & 92 & & \\
Time at risk \(=\) & 13825 & & \\
& & LR chi2(5) & \(=\) & 106.33 \\
Log likelihood \(=\) & -369.48426 & & Prob \(>\) chi2 & \(=\) \\
& & & 0.0000
\end{tabular}
```

| _t | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| mthson_1 | .1985592 | .0493922 | 4.02 | 0.000 | .1017523 | .2953662 |
| age | -.02691 | .0087875 | -3.06 | 0.002 | -.0441331 | -.0096868 |
| ulcarea | -.0017416 | .0003482 | -5.00 | 0.000 | -.0024241 | -.0010591 |
| deepppg | -.5740759 | .2185134 | -2.63 | 0.009 | -1.002354 | -.1457975 |
| treat | -.1798575 | .2175726 | -0.83 | 0.408 | -.6062921 | .246577 |

The hazard for healing is much higher for patients whose ulcer is of recent onset than for those who have had an ulcer for many months.

A more appropriate analysis of this dataset, if one wanted to model all the predictors, possibly with fractional polynomial functions, would be to use $\mathrm{mf} p$; see $[R] \mathbf{m f p}$.

## Example 3: Logistic regression

The zero option permits fitting a fractional polynomial model to the positive values of a covariate, taking nonpositive values as zero. An application is the assessment of the effect of cigarette smoking as a risk factor. Whitehall 1 is an epidemiological study, which was examined in Royston and Sauerbrei (2008), of 18,403 male British Civil Servants employed in London. We examine the data collected in Whitehall 1 and use logistic regression to model the odds of death based on a fractional polynomial in the number of cigarettes smoked.

Nonsmokers may be qualitatively different from smokers, so the effect of smoking (regarded as a continuous variable) may not be continuous between zero cigarettes and one cigarette. To allow for this possibility, we model the risk as a constant for the nonsmokers and as a fractional polynomial function of the number of cigarettes for the smokers, adjusted for age.

The dependent variable all 10 is an indicator of whether the individual passed away in the 10 years under study. cigs is the number of cigarettes consumed per day. After loading the data, we demean age and create a dummy variable, nonsmoker. We then use $f p$ to fit the model.

```
. use http://www.stata-press.com/data/r13/smoking, clear
(Smoking and mortality data)
. qui sum age
. replace age = age - r(mean)
age was byte now float
(17260 real changes made)
. generate byte nonsmoker = cond(cigs==0, 1, 0) if cigs < .
. fp <cigs>, zero: logit all10 <cigs> nonsmoker age
(fitting 44 models)
(. . . 10% . . . 20% . . . . 30% . . . 40% . . . . 50% . . . 60% . . . 70% . . . . 80% . . . . 90% . . . . 100%)
```

Fractional polynomial comparisons:

| cigs | df | Deviance | Dev. dif. | $P(*)$ | Powers |
| ---: | ---: | ---: | ---: | ---: | :--- |
| omitted | 0 | 9990.804 | 46.096 | 0.000 |  |
| linear | 1 | 9958.801 | 14.093 | 0.003 | 1 |
| m = 1 | 2 | 9946.603 | 1.895 | 0.388 | 0 |
| $\mathrm{~m}=2$ | 4 | 9944.708 | 0.000 | -- | $-1-1$ |

(*) $\mathrm{P}=$ sig. level of model with $\mathrm{m}=2$ based on chi^2 of dev. dif.

| Logistic regression | Number of obs | $=$ | 17260 |
| :--- | :--- | :--- | :--- |
|  | LR chi2 $(4)$ | $=$ | 1029.03 |
|  | Prob $>$ chi2 | $=$ | 0.0000 |
| Log likelihood $=-4972.3539$ |  | Pseudo R2 | $=$ |


| all10 | Coef. | Std. Err. | $z$ | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| cigs_1 | -1.285867 | .3358483 | -3.83 | 0.000 | -1.944117 | -.6276162 |
| cigs_2 | -1.982424 | .572109 | -3.47 | 0.001 | -3.103736 | -.8611106 |
| nonsmoker | -1.223749 | .1119583 | -10.93 | 0.000 | -1.443183 | -1.004315 |
| age | .1194541 | .0045818 | 26.07 | 0.000 | .1104739 | .1284343 |
| _cons | -1.591489 | .1052078 | -15.13 | 0.000 | -1.797693 | -1.385286 |

Omission of the zero option would cause $f p$ to halt with an error message because nonpositive covariate values (for example, values of cigs) are invalid unless the scale option is specified.

A closely related approach involves the catzero option. Here we no longer need to have nonsmoker in the model, because fp creates its own dummy variable cigs_0 to indicate whether the individual does not smoke on that day.

```
. fp <cigs>, catzero replace: logit all10 <cigs> age
(fitting 44 models)
(. . . 10% . . . 20% . . . . 30% . . . 40% . . . . 50% . . . 60% . . . 70% . . . . 80% . . . 90% . . . . 100%)
```

Fractional polynomial comparisons:

| cigs | df | Deviance | Dev. dif. | $P(*)$ | Powers |
| ---: | ---: | ---: | ---: | :--- | :--- |
| omitted | 0 | 10175.75 | 231.047 | 0.000 |  |
| linear | 2 | 9958.80 | 14.093 | 0.003 | 1 |
| m = 1 | 3 | 9946.60 | 1.895 | 0.388 | 0 |
| m $=2$ | 5 | 9944.71 | 0.000 | -- | $-1-1$ |

(*) $\mathrm{P}=$ sig. level of model with $\mathrm{m}=2$ based on chi^2 of dev. dif.

| Logistic regression | Number of obs | $=$ | 17260 |
| :--- | :--- | :--- | :--- |
|  | LR chi2 $(4)$ | $=$ | 1029.03 |
|  | Prob $>$ chi2 | $=$ | 0.0000 |
| Log likelihood $=-4972.3539$ |  | Pseudo R2 | $=$ |


| all10 | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| cigs_0 | -1.223749 | .1119583 | -10.93 | 0.000 | -1.443183 | -1.004315 |
| cigs_1 | -1.285867 | .3358483 | -3.83 | 0.000 | -1.944117 | -.6276162 |
| cigs_2 | -1.982424 | .572109 | -3.47 | 0.001 | -3.103736 | -.8611106 |
| age | .1194541 | .0045818 | 26.07 | 0.000 | .1104739 | .1284343 |
| _cons | -1.591489 | .1052078 | -15.13 | 0.000 | -1.797693 | -1.385286 |

Under both approaches, the comparison table suggests that we can accept the FP1 model instead of the FP2 model. We estimate the parameters of the accepted model-that is, the one that uses the natural logarithm of cigs-with fp.

```
. fp <cigs>, catzero replace fp(0): logit all10 <cigs> age
-> logit all10 cigs_0 cigs_1 age
```

| Logistic regression | Number of obs | $=$ | 17260 |
| :--- | :--- | :--- | :--- |
|  | LR chi2(3) | $=$ | 1027.13 |
|  | Prob > chi2 | $=$ | 0.0000 |
| Log likelihood $=-4973.3016$ |  | Pseudo R2 | $=$ |


| all10 | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| cigs_0 | .1883732 | .1553093 | 1.21 | 0.225 | -.1160274 | .4927738 |
| cigs_1 | .3469842 | .0543552 | 6.38 | 0.000 | .2404499 | .4535185 |
| age | .1194976 | .0045818 | 26.08 | 0.000 | .1105174 | .1284778 |
| _cons | -3.003767 | .1514909 | -19.83 | 0.000 | -3.300683 | -2.70685 |

The high $p$-value for cigs_0 in the output indicates that we cannot reject that there is no extra effect at zero for nonsmokers.

## Stored results

In addition to the results that est_cmd stores, fp stores the following in e():
Scalars
e(fp_dimension)
e(fp_center_mean)
e(fp_scale_a)
e(fp_scale_b)
e(fp_compare_df2)
Macros
e(fp_cmd)
e(fp_cmdline)
e(fp_variable)
e(fp_terms)
e(fp_gen_cmdline)
e(fp_catzero)
e(fp_zero)
e(fp_compare_type)
Matrices

$$
e\left(f p \_f p\right)
$$

e(fp_compare)
e(fp_compare_stat)
e(fp_compare_df1)
e(fp_compare_fp)
e(fp_compare_length)
e(fp_powers)
degree of fractional polynomial
value used for centering or .
value used for scaling or .
value used for scaling or .
denominator degree of freedom in $F$ test
fp, search(): or fp, powers():
full $f p$ command as typed
fractional polynomial variable
generated $f p$ variables
fp generate command to re-create e(fp_terms) variables
catzero, if specified
zero, if specified
F or chi2
powers used in fractional polynomial
results of model comparisons
$F$ test statistics
numerator degree of $F$ test powers of comparison models encoded string for display of row titles powers that are searched
fp generate stores the following in $r()$ :
Scalars
$r$ (fp_center_mean) value used for centering or .
$r\left(f p \_s c a l e \_a\right) \quad$ value used for scaling or .
$r\left(f p \_s c a l e \_b\right) \quad$ value used for scaling or .
Macros
$r$ (fp_cmdline)
$r\left(f p \_v a r i a b l e\right)$
$r$ (fp_terms)
r(fp_catzero)
r(fp_zero)
full $f p$ generate command as typed
fractional polynomial variable
generated $f p$ variables
catzero, if specified
zero, if specified
Matrices
$r\left(f p \_f p\right) \quad$ powers used in fractional polynomial

## Methods and formulas

The general definition of a fractional polynomial, accommodating possible repeated powers, may be written for functions $H_{1}(x), \ldots, H_{m}(x)$ of $x>0$ as

$$
\beta_{0}+\sum_{j=1}^{m} \beta_{j} H_{j}(x)
$$

where $H_{1}(x)=x^{\left(p_{1}\right)}$ and for $j=2, \ldots, m$,

$$
H_{j}(x)= \begin{cases}x^{\left(p_{j}\right)} & \text { if } p_{j} \neq p_{j-1} \\ H_{j-1}(x) \ln (x) & \text { if } p_{j}=p_{j-1}\end{cases}
$$

For example, a fractional polynomial of degree 3 with powers $(1,3,3)$ has $H_{1}(x)=x, H_{2}(x)=x^{3}$, and $H_{3}(x)=x^{3} \ln (x)$ and equals $\beta_{0}+\beta_{1} x+\beta_{2} x^{3}+\beta_{3} x^{3} \ln (x)$.

We can express a fractional polynomial in vector notation by using $\mathbf{H}(x)=\left[H_{1}(x), \ldots, H_{d}(x)\right]^{\prime}$. We define $x^{\left(p_{1}, p_{2}, \ldots, p_{m}\right)}=\left[\mathbf{H}(x)^{\prime}, 1\right]^{\prime}$. Under this notation, we can write

$$
x^{(1,3,3) \prime} \boldsymbol{\beta}=\beta_{0}+\beta_{1} x+\beta_{2} x^{3}+\beta_{3} x^{3} \ln (x)
$$

The fractional polynomial may be centered so that the intercept can be more easily interpreted. When centering the fractional polynomial of $x$ at $c$, we subtract $c^{\left(p_{1}, p_{2}, \ldots, p_{m}\right)}$ from $x^{\left(p_{1}, p_{2}, \ldots, p_{m}\right)}$, where $c^{\left(p_{1}, p_{2}, \ldots, p_{d}\right)}=\left[\mathbf{H}(x)^{\prime}, 0\right]^{\prime}$. The centered fractional polynomial is

$$
\left(x^{\left(p_{1}, \ldots, p_{d}\right)}-c^{\left(p_{1}, \ldots, p_{d}\right)}\right)^{\prime} \boldsymbol{\beta}
$$

The definition may be extended to allow $x \leq 0$ values. For these values, the fractional polynomial is equal to the intercept $\beta_{0}$ or equal to a zero-offset term $\alpha_{0}$ plus the intercept $\beta_{0}$.

A fractional polynomial model of degree $m$ is taken to have $2 m+1$ degrees of freedom (df): one for $\beta_{0}$ and one for each $\beta_{j}$ and its associated power. Because the powers in a fractional polynomial are chosen from a finite set rather than from the entire real line, the df defined in this way are approximate.

The deviance $D$ of a model is defined as -2 times its maximized log likelihood. For normal-errors models, we use the formula

$$
D=n\left(1-\bar{l}+\ln \frac{2 \pi \mathrm{RSS}}{n}\right)
$$

where $n$ is the sample size, $\bar{l}$ is the mean of the lognormalized weights ( $\bar{l}=0$ if the weights are all equal), and RSS is the residual sum of squares as fit by regress.
fp reports a table comparing fractional polynomial models of degree $k<m$ with the degree $m$ fractional polynomial model, which will have the lowest deviance.

The $p$-values reported by fp are calculated differently for normal and nonnormal regressions. Let $D_{k}$ and $D_{m}$ be the deviances of the models with degrees $k$ and $m$, respectively. For normal-errors models, a variance ratio $F$ is calculated as

$$
F=\frac{n_{2}}{n_{1}}\left\{\exp \left(\frac{D_{k}-D_{m}}{n}\right)-1\right\}
$$

where $n_{1}$ is the numerator df, the quantity of the additional parameters that the degree $m$ model has over the degree $k$ model. $n_{2}$ is the denominator df and equals the residual degrees of freedom of the degree $m$ model, minus the number of powers estimated, $m$. The $p$-value is obtained by referring $F$ to an $F$ distribution on $\left(n_{1}, n_{2}\right) \mathrm{df}$.

For nonnormal models, the $p$-value is obtained by referring $D_{k}-D_{m}$ to a $\chi^{2}$ distribution on $2 m-2 k$ df. These $p$-values for comparing models are approximate and are typically somewhat conservative (Royston and Altman 1994).

## Acknowledgment

We thank Patrick Royston of the MrC Clinical Trials Unit, London, and coauthor of the Stata Press book Flexible Parametric Survival Analysis Using Stata: Beyond the Cox Model for writing fracpoly and fracgen, the commands on which $f p$ and $f p$ generate are based. We also thank Professor Royston for his advice on and review of the new $f p$ commands.

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## Also see

[R] fp postestimation - Postestimation tools for fp
[R] mfp - Multivariable fractional polynomial models
[U] 20 Estimation and postestimation commands

## Title

fp postestimation - Postestimation tools for fp

Description
Menu for $f p$ plot and $f p$ predict
Remarks and examples
References

Syntax for predict
Options for fp plot
Methods and formulas
Also see

Syntax for fp plot and fp predict
Options for fp predict
Acknowledgment

## Description

The following postestimation commands are of special interest after fp:
Command Description
fp plot component-plus-residual plot from most recently fit fractional polynomial model
fp predict create variable containing prediction or SEs of fractional polynomials

The following standard postestimation commands are also available if available after est_cmd:

| Command | Description |
| :--- | :--- |
| contrast | contrasts and ANOVA-style joint tests of estimates |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample <br> estat vce |
| variance-covariance matrix of the estimators (VCE) |  |
| estimates | cataloging estimation results |
| hausman | Hausman's specification test <br> point estimates, standard errors, testing, and inference for linear combinations <br> of coefficients |
| lincom | link test for model specification |
| linktest | likelihood-ratio test <br> marginal means, predictive margins, marginal effects, and average marginal <br> effects |
| margins | graph the results from margins (profile plots, interaction plots, etc.) <br> point estimates, standard errors, testing, and inference for nonlinear combinations <br> of coefficients |
| marginsplot | predictions, residuals, influence statistics, and other diagnostic measures <br> point estimates, standard errors, testing, and inference for generalized predictions <br> nlcom |
| predict | peemingly unrelated estimation |
| predictnl | Wald tests of simple and composite linear hypotheses |
| pwcompare | Wald tests of nonlinear hypotheses |
| suest |  |

## Special-interest postestimation commands

fp plot produces a component-plus-residual plot. The fractional polynomial comprises the component, and the residual is specified by the user in residuals (). The component-plus-residuals are plotted against the fractional polynomial variable. If you only want to plot the component fit, without residuals, you would specify residuals(none).
fp predict generates the fractional polynomial or the standard error of the fractional polynomial. The fractional polynomial prediction is equivalent to the fitted values prediction given by predict, xb , with the covariates other than the fractional polynomial variable set to zero. The standard error may be quite large if the range of the other covariates is far from zero. In this situation, the covariates would be centered and their range would include, or come close to including, zero.

These postestimation commands can be used only when the fractional polynomial variables do not interact with other variables in the specification of est_cmd. See [U] 11.4.3 Factor variables for more information about interactions.

## Syntax for predict

The behavior of predict following fp is determined by est_cmd. See the corresponding est_cmd postestimation entry for available predict options.

Also see information on fp predict below.

## Syntax for fp plot and fp predict

Component-plus-residual plot for most recently fit fractional polynomial model fp plot [if] [in], residuals(res_option) [graph_options]

Create variable containing the prediction or SEs of fractional polynomials
fp predict [type] newvar [if] [in] [, predict_options]

## graph_options

Description

## Main

*residuals(res_option)
residual option name to use in predict after est_cmd, or residuals (none) if residuals are not to be graphed equation(eqno)
specify equation
level(\#)
set confidence level; default is level (95)
Plot
plotopts(scatter_options) affect rendition of the component-plus-residual scatter points
Fitted line
lineopts(cline_options)
CI plot
ciopts (area_options) affect rendition of the confidence bands
Add plots
addplot (plot) add other plots to the generated graph
Y axis, X axis, Titles, Legend, Overall
twoway_options
any options other than by () documented in [G-3] twoway_options

* residuals(res_option) is required.
predict_options
Description
Main
fp stdp equation(eqno)
calculate the fractional polynomial; the default calculate the standard error of the fractional polynomial specify equation


## Menu for fp plot and fp predict

## fp plot

Statistics $>$ Linear models and related $>$ Fractional polynomials $>$ Component-plus-residual plot

## fp predict

Statistics $>$ Linear models and related $>$ Fractional polynomials $>$ Fractional polynomial prediction

## Options for fp plot

[^18]equation(\#1) would mean that the calculation is to be made for the first equation, equation(\#2) would mean the second, and so on. You could also refer to the equations by their names: equation(income) would refer to the equation name income, and equation(hours) would refer to the equation named hours.
If you do not specify equation(), the results are the same as if you specified equation(\#1). level(\#); see [R] estimation options.
$\qquad$ Plot
plotopts (scatter_options) affects the rendition of the component-plus-residual scatter points; see [G-2] graph twoway scatter.
$\qquad$ Fitted line
lineopts (cline_options) affects the rendition of the fitted line; see [G-3] cline_options.
$\qquad$ $\widehat{\mathrm{Cl} \text { plot }}$
ciopts (area_options) affects the rendition of the confidence bands; see [G-3] area_options.

Add plots
addplot (plot) provides a way to add other plots to the generated graph. See [G-3] addplot_option.
$\qquad$ Y axis, X axis, Titles, Legend, Overall
twoway_options are any of the options documented in [G-3] twoway_options, excluding by (). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

## Options for fp predict

Main
fp calculates the fractional polynomial, the linear prediction with other variables set to zero. This is the default.
stdp calculates the standard error of the fractional polynomial.
equation (eqno) is relevant only when you have previously fit a multiple-equation model in est_cmd. It specifies the equation to which you are referring.
equation(\#1) would mean that the calculation is to be made for the first equation, equation (\#2) would mean the second, and so on. You could also refer to the equations by their names: equation(income) would refer to the equation name income, and equation(hours) would refer to the equation named hours.
If you do not specify equation(), the results are the same as if you specified equation(\#1).

## Remarks and examples

After a model is fit using $f p$, the estimated fractional polynomial may be of interest. This is the linear combination of the fractional polynomial terms and the constant intercept using the model coefficients estimated by $f p$. It is equivalent to the fitted values prediction given by predict, xb , with the covariates and the fractional polynomial variable set to zero. When these other covariates have been centered, the prediction is made at the centering values of the covariates.

A component-plus-residual plot is generated by fp plot. The fractional polynomial comprises the component, and the residual is specified by the user in residuals(). The residuals() option takes the same argument that would be supplied to predict after est_cmd to obtain the desired type of residuals. If you only want to plot the component fit, without residuals, you would specify residuals(none).
fp predict generates the fractional polynomial. If the stdp option is specified, the standard error of the fractional polynomial is generated instead. This standard error may be quite large if the range of the other covariates is far from zero. In this situation, the covariates would be centered and their range would include, or come close to including, zero.

These postestimation commands can be used only when the fractional polynomial terms do not interact with other variables in the specification of est_cmd. See [U] 11.4.3 Factor variables for more information about interactions.

## Examples

> Example 1: fp plot after linear regression
In example 1 of $[R] \mathbf{f p}$, we modeled the mean of the square root of a child's serum immunoglobulin G (IgG) level as a fractional polynomial function of the child's age. An FP2 model with powers $(-2,2)$ is chosen.

We load the data and then fit the model with $f p$. Then we use $f p$ plot to draw the component-plus-residual plot. A $95 \%$ confidence interval is produced for the fractional polynomial in age (the component). The residuals prediction option for regress is specified in the residuals() option in $f p$ plot so that the residuals are rendered.

```
. use http://www.stata-press.com/data/r13/igg
(Immunoglobulin in children)
. fp <age>, scale center: regress sqrtigg <age>
    (output omitted)
. fp plot, residuals(residuals)
```



## > Example 2: fp plot after Cox regression

In example 2 of $[\mathrm{R}] \mathbf{f p}$, we modeled the time to complete healing of leg ulcers for 192 elderly patients using a Cox regression. A one-degree fractional polynomial in mthson, the number of months since the onset of the ulcer, is used as a predictor in the regression. The power -0.5 is used for mthson. Other covariates are age (age), ulcer area (ulcarea), treatment type, and a binary indicator of deep vein involvement (deepppg).

We load the data and then demean ulcer area and age. Then we fit the model with fp and draw the component-plus-residual plot with fp plot. mgale is specified in the residuals() option to obtain martingale residuals. See [ST] stcox postestimation for more details.

```
. use http://www.stata-press.com/data/r13/legulcer1, clear
(Leg ulcer clinical trial)
. quietly stset ttevent, failure(cens)
. quietly summarize age
. replace age = age - r(mean)
age was byte now float
(192 real changes made)
. quietly summarize ulcarea
. replace ulcarea = ulcarea - r(mean)
ulcarea was int now float
(192 real changes made)
. fp <mthson>, replace center scale nohr fp(-.5): stcox <mthson> age ulcarea
> deepppg treat
    (output omitted)
    fp plot, residuals(mgale)
```



## > Example 3: fp plot and fp predict after logistic regression

In example 3 of $[\mathrm{R}] \mathbf{f p}$, we used logistic regression to model the odds of death for male civil servants in Britain conditional on cigarette consumption. The dependent variable all10 is an indicator of whether the individual passed away in the 10 years under study.

Nonsmokers may be qualitatively different from smokers, so the effect of smoking (regarded as a continuous variable) may not be continuous between zero cigarettes and one cigarette. To allow for this possibility, we model the risk as constant intercept for the nonsmokers and as a fractional polynomial function of the number of cigarettes for the smokers, cigs, adjusted for age. An FP1 model with power 0 is chosen.

We load the data and demean age. Then we fit the model using fp and graph the fit of the model and $95 \%$ confidence interval using fp plot. Only the component fit is graphed by specifying residuals(none).

```
. use http://www.stata-press.com/data/r13/smoking, clear
(Smoking and mortality data)
. quietly summarize age
. replace age = age - r(mean)
age was byte now float
(17260 real changes made)
. fp <cigs>, catzero replace fp(0): logit all10 <cigs> age
-> logit all10 cigs_0 cigs_1 age
```

| Logistic regression | Number of obs | $=$ | 17260 |
| :--- | :--- | :--- | :--- |
|  | LR chi2(3) | $=$ | 1027.13 |
|  | Prob $>$ chi2 | $=$ | 0.0000 |
| Log likelihood $=-4973.3016$ |  | Pseudo R2 | $=$ |


| all10 | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| cigs_0 | .1883732 | .1553093 | 1.21 | 0.225 | -.1160274 | .4927738 |
| cigs_1 | .3469842 | .0543552 | 6.38 | 0.000 | .2404499 | .4535185 |
| age | .1194976 | .0045818 | 26.08 | 0.000 | .1105174 | .1284778 |
| _cons | -3.003767 | .1514909 | -19.83 | 0.000 | -3.300683 | -2.70685 |

```
fp plot, residuals(none)
```



We see a small spike at zero for cigs because of the effect of cigs_0 on the fractional polynomial; however, the high $p$-value for cigs_0 in the model output indicates that we cannot reject that there is no extra effect at zero for nonsmokers.

We can also use $f p$ predict to predict the fractional polynomial for nonsmokers and the mean of age. This is the value at the spike. We store the result in fpO . We see it is equivalent to the sum of the constant intercept estimate and the estimate of the cigs_0 coefficient.

```
. fp predict fp0 if cigs == 0
(7157 missing values generated)
. summarize fp0
\begin{tabular}{r|rrrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline fp0 & 10103 & -2.815393 & & 0 & -2.815393 & -2.815393
\end{tabular}
. display _b[cigs_0]+_b[_cons]
-2.8153935
```


## Methods and formulas

Let the data consist of triplets $\left(y_{i}, x_{i}, \mathbf{z}_{i}\right), i=1, \ldots, n$, where $\mathbf{z}_{i}$ is the vector of covariates for the $i$ th observation and $x_{i}$ is the fractional polynomial variable.
fp predict calculates the fractional polynomial at the centering value $x_{0}, \widehat{\eta}_{i}=\left(x_{i}{ }^{\left(p_{1}, \ldots, p_{d}\right)}-\right.$ $\left.x_{0}{ }^{\left(p_{1}, \ldots, p_{m}\right)}\right)^{\prime} \widehat{\boldsymbol{\beta}}$. This is equivalent to the linear predictor of the model at $\mathbf{z}_{i}=\mathbf{0}$. The standard error is calculated from the variance-covariance matrix of $\widehat{\boldsymbol{\beta}}$, ignoring estimation of the powers. When $x_{i} \leq 0, \mathbf{H}\left(x_{i}\right)$, and thus $x_{i}{ }^{\left(p_{1}, \ldots, p_{m}\right)}$, is either undefined or zero. A zero offset term, $\alpha_{0}$, may be added to $\widehat{\eta}_{i}$ for these nonpositive $x_{i}$ values.

The values $\widehat{\eta}_{i}$ represent the behavior of the fractional polynomial model for $x$ at fixed values $\mathbf{z}=\mathbf{0}$ of the (centered) covariates. The $i$ th component-plus-residual is defined as $\widehat{\eta}_{i}+d_{i}$, where $d_{i}$ is the residual for the $i$ th observation. The definition of $d_{i}$ will change according to the type of model used and the preference of the user. fp plot plots $\widehat{\eta}_{i}+d_{i}$ versus $x_{i}$, overlaying $\widehat{\eta}_{i}$ and its confidence interval.

## Acknowledgment

We thank Patrick Royston of the MrC Clinical Trials Unit, London, and coauthor of the Stata Press book Flexible Parametric Survival Analysis Using Stata: Beyond the Cox Model for writing fracplot and fracpred, the commands on which fp plot and fp predict are based. We also thank Professor Royston for his advice on and review of $f p$ plot and fp predict.

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## Also see

[R] fp - Fractional polynomial regression
[U] 20 Estimation and postestimation commands

## Title

frontier - Stochastic frontier models

Syntax
Remarks and examples

Menu
Stored results

Description
Methods and formulas

Options
References

## Syntax

frontier depvar [indepvars] [if] [in] [weight] [, options] options

Description
Model
noconstant
distribution(hnormal)
distribution(exponential)
distribution(tnormal)
ufrom (matrix)
cm (varlist [, noconstant])

Model 2
constraints (constraints)
collinear
uhet (varlist [, noconstant]) vhet (varlist [, noconstant]) cost

SE
vce (vcetype)
Reporting
level(\#)
nocnsreport
display_options

Maximization
maximize_options
coeflegend
suppress constant term
half-normal distribution for the inefficiency term
exponential distribution for the inefficiency term
truncated-normal distribution for the inefficiency term specify untransformed log likelihood; only with d(tnormal)
fit conditional mean model; only with d(tnormal); use noconstant to suppress constant term
apply specified linear constraints
keep collinear variables
explanatory variables for technical inefficiency variance function; use noconstant to suppress constant term explanatory variables for idiosyncratic error variance function; use noconstant to suppress constant term fit cost frontier model; default is production frontier model
vcetype may be oim, opg, bootstrap, or jackknife
set confidence level; default is level (95)
do not display constraints
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
control the maximization process; seldom used
display legend instead of statistics
indepvars and varlist may contain factor variables; see [U] 11.4.3 Factor variables.
bootstrap, by, fp, jackknife, rolling, and statsby are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the bootstrap prefix; see $[R]$ bootstrap.
fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Linear models and related $>$ Frontier models

## Description

frontier fits stochastic production or cost frontier models; the default is a production frontier model. It provides estimators for the parameters of a linear model with a disturbance that is assumed to be a mixture of two components, which have a strictly nonnegative and symmetric distribution, respectively. frontier can fit models in which the nonnegative distribution component (a measurement of inefficiency) is assumed to be from a half-normal, exponential, or truncated-normal distribution. See Kumbhakar and Lovell (2000) for a detailed introduction to frontier analysis.

## Options

Model
noconstant; see [R] estimation options.
distribution(distname) specifies the distribution for the inefficiency term as half-normal (hnormal), exponential, or truncated-normal (tnormal). The default is hnormal.
ufrom (matrix) specifies a $1 \times K$ matrix of untransformed starting values when the distribution is truncated-normal (tnormal). frontier can estimate the parameters of the model by maximizing either the $\log$ likelihood or a transformed log likelihood (see Methods and formulas). frontier automatically transforms the starting values before passing them on to the transformed log likelihood. The matrix must have the same number of columns as there are parameters to estimate.
cm (varlist [, noconstant]) may be used only with distribution(tnormal). Here frontier will fit a conditional mean model in which the mean of the truncated-normal distribution is modeled as a linear function of the set of covariates specified in varlist. Specifying noconstant suppresses the constant in the mean function.

## Model 2

constraints (constraints), collinear; see [R] estimation options.
By default, when fitting the truncated-normal model or the conditional mean model, frontier maximizes a transformed log likelihood. When constraints are applied, frontier will maximize the untransformed log likelihood with constraints defined in the untransformed metric.
uhet (varlist [, noconstant ]) specifies that the technical inefficiency component is heteroskedastic, with the variance function depending on a linear combination of varlist ${ }_{u}$. Specifying noconstant suppresses the constant term from the variance function. This option may not be specified with distribution(tnormal).
vhet (varlist [, noconstant]) specifies that the idiosyncratic error component is heteroskedastic, with the variance function depending on a linear combination of varlist $t_{v}$. Specifying noconstant suppresses the constant term from the variance function. This option may not be specified with distribution(tnormal).
cost specifies that frontier fit a cost frontier model.
vce (vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg) and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

## Reporting

level(\#); see [R] estimation options.
nocnsreport; see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% fmt), pformat (\% fmt), sformat (\% fmt), and nolstretch; see [R] estimation options.

Maximization
maximize_options: difficult, technique (algorithm_spec), iterate (\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#),
nrtolerance(\#), nonrtolerance, and from (init_specs); see [R] maximize. These options are seldom used.

Setting the optimization type to technique (bhhh) resets the default vcetype to vce (opg).
The following option is available with frontier but is not shown in the dialog box:
coeflegend; see [R] estimation options.

## Remarks and examples

Stochastic production frontier models were introduced by Aigner, Lovell, and Schmidt (1977) and Meeusen and van den Broeck (1977). Since then, stochastic frontier models have become a popular subfield in econometrics. Kumbhakar and Lovell (2000) provide a good introduction.
frontier fits three stochastic frontier models with distinct parameterizations of the inefficiency term and can fit stochastic production or cost frontier models.

Let's review the nature of the stochastic frontier problem. Suppose that a producer has a production function $f\left(\mathbf{z}_{i}, \boldsymbol{\beta}\right)$. In a world without error or inefficiency, the $i$ th firm would produce

$$
q_{i}=f\left(\mathbf{z}_{i}, \boldsymbol{\beta}\right)
$$

Stochastic frontier analysis assumes that each firm potentially produces less than it might due to a degree of inefficiency. Specifically,

$$
q_{i}=f\left(\mathbf{z}_{i}, \boldsymbol{\beta}\right) \xi_{i}
$$

where $\xi_{i}$ is the level of efficiency for firm $i ; \xi_{i}$ must be in the interval $(0,1]$. If $\xi_{i}=1$, the firm is achieving the optimal output with the technology embodied in the production function $f\left(\mathbf{z}_{i}, \boldsymbol{\beta}\right)$. When $\xi_{i}<1$, the firm is not making the most of the inputs $\mathbf{z}_{i}$ given the technology embodied in the production function $f\left(\mathbf{z}_{i}, \boldsymbol{\beta}\right)$. Because the output is assumed to be strictly positive (that is, $q_{i}>0$ ), the degree of technical efficiency is assumed to be strictly positive (that is, $\xi_{i}>0$ ).

Output is also assumed to be subject to random shocks, implying that

$$
q_{i}=f\left(\mathbf{z}_{i}, \boldsymbol{\beta}\right) \xi_{i} \exp \left(v_{i}\right)
$$

Taking the natural $\log$ of both sides yields

$$
\ln \left(q_{i}\right)=\ln \left\{f\left(\mathbf{z}_{i}, \boldsymbol{\beta}\right)\right\}+\ln \left(\xi_{i}\right)+v_{i}
$$

Assuming that there are $k$ inputs and that the production function is linear in logs, defining $u_{i}=-\ln \left(\xi_{i}\right)$ yields

$$
\begin{equation*}
\ln \left(q_{i}\right)=\beta_{0}+\sum_{j=1}^{k} \beta_{j} \ln \left(z_{j i}\right)+v_{i}-u_{i} \tag{1}
\end{equation*}
$$

Because $u_{i}$ is subtracted from $\ln \left(q_{i}\right)$, restricting $u_{i} \geq 0$ implies that $0<\xi_{i} \leq 1$, as specified above.
Kumbhakar and Lovell (2000) provide a detailed version of the above derivation, and they show that performing an analogous derivation in the dual cost function problem allows us to specify the problem as

$$
\begin{equation*}
\ln \left(c_{i}\right)=\beta_{0}+\beta_{q} \ln \left(q_{i}\right)+\sum_{j=1}^{k} \beta_{j} \ln \left(p_{j i}\right)+v_{i}+u_{i} \tag{2}
\end{equation*}
$$

where $q_{i}$ is output, $z_{j i}$ are input quantities, $c_{i}$ is cost, and the $p_{j i}$ are input prices.
Intuitively, the inefficiency effect is required to lower output or raise expenditure, depending on the specification.

## Technical note

The model that frontier actually fits is of the form

$$
y_{i}=\beta_{0}+\sum_{j=1}^{k} \beta_{j} x_{j i}+v_{i}-s u_{i}
$$

where

$$
s=\left\{\begin{aligned}
1, & \text { for production functions } \\
-1, & \text { for cost functions }
\end{aligned}\right.
$$

so, in the context of the discussion above, $y_{i}=\ln \left(q_{i}\right)$, and $x_{j i}=\ln \left(z_{j i}\right)$ for a production function; and for a cost function, $y_{i}=\ln \left(c_{i}\right)$, and the $x_{j i}$ are the $\ln \left(p_{j i}\right)$ and $\ln \left(q_{i}\right)$. You must take the natural logarithm of the data before fitting a stochastic frontier production or cost model. frontier performs no transformations on the data.

Different specifications of the $u_{i}$ and the $v_{i}$ terms give rise to distinct models. frontier provides estimators for the parameters of three basic models in which the idiosyncratic component, $v_{i}$, is assumed to be independently $N\left(0, \sigma_{v}\right)$ distributed over the observations. The basic models differ in their specification of the inefficiency term, $u_{i}$, as follows:
exponential: the $u_{i}$ are independently exponentially distributed with variance $\sigma_{u}^{2}$ hnormal: the $u_{i}$ are independently half-normally $N^{+}\left(0, \sigma_{u}^{2}\right)$ distributed tnormal: the $u_{i}$ are independently $N^{+}\left(\mu, \sigma_{u}^{2}\right)$ distributed with truncation point at 0
For half-normal or exponential distributions, frontier can fit models with heteroskedastic error components, conditional on a set of covariates. For a truncated-normal distribution, frontier can also fit a conditional mean model in which the mean is modeled as a linear function of a set of covariates.

## - Example 1: The half-normal and the exponential models

For our first example, we demonstrate the half-normal and exponential models by reproducing a study found in Greene (2003, 505), which uses data originally published in Zellner and Revankar (1969). In this study of the transportation-equipment manufacturing industry, observations on value added, capital, and labor are used to estimate a Cobb-Douglas production function. The variable lnv is the log-transformed value added, lnk is the log-transformed capital, and $\operatorname{lnl}$ is the log-transformed labor. OLS estimates are compared with those from stochastic frontier models using both the half-normal and exponential distribution for the inefficiency term.
. use http://www.stata-press.com/data/r13/greene9
. regress lnv lnk lnl

| Source | SS | df MS |  |  | Number of obs $=\quad 25$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | F ( 2, 22) | $=397.54$ |
| Model | 44.1727741 | 222 | 6387 |  | Prob > F | $=0.0000$ |
| Residual | 1.22225984 | 22.05 | 7265 |  | R -squared | 0.9731 |
|  |  |  |  |  | Adj R-squared | 0.9706 |
| Total | 45.3950339 | 241.8 | 5975 |  | Root MSE | $=.23571$ |
| lnv | Coef. | Std. Err . | t | $P>\|t\|$ | [95\% Conf. | Interval] |
| lnk | . 2454281 | . 1068574 | 2.30 | 0.032 | . 0238193 | . 4670368 |
| lnl | . 805183 | . 1263336 | 6.37 | 0.000 | . 5431831 | 1.067183 |
| _cons | 1.844416 | . 2335928 | 7.90 | 0.000 | 1.359974 | 2.328858 |


| Iteration 0: log likelihood = 2.3357572 |  |  |  |
| :---: | :---: | :---: | :---: |
| Iteration 1: $\quad$ log likelihood $=2.4673009$ |  |  |  |
| Iteration 2: $\quad$ log likelihood $=2.4695125$ |  |  |  |
| Iteration 3: log likelihood $=2.4695222$ |  |  |  |
| Iteration 4: log likelihood $=2.4695222$ |  |  |  |
| Stoc. frontier normal/half-normal model | Number of obs | = | 25 |
|  | Wald chi2(2) |  | 743.71 |
| Log likelihood = 2.4695222 | Prob > chi2 | = | 0.0000 |


| lnv | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| lnk | .2585478 | .098764 | 2.62 | 0.009 | .0649738 | .4521218 |
| lnl | .7802451 | .1199399 | 6.51 | 0.000 | .5451672 | 1.015323 |
| _cons | 2.081135 | .281641 | 7.39 | 0.000 | 1.529128 | 2.633141 |
| /lnsig2v | -3.48401 | .6195353 | -5.62 | 0.000 | -4.698277 | -2.269743 |
| /lnsig2u | -3.014599 | 1.11694 | -2.70 | 0.007 | -5.203761 | -.8254368 |
| sigma_v | .1751688 | .0542616 |  |  | .0954514 | .3214633 |
| sigma_u | .2215073 | .1237052 |  |  | .074134 | .6618486 |
| sigma2 | .0797496 | .0426989 |  |  | -.0039388 | .163438 |
| lambda | 1.264536 | .1678684 |  |  | .9355204 | 1.593552 |

Likelihood-ratio test of sigma_u=0: chibar2(01) = 0.43 Prob>=chibar2 $=0.256$
. predict double u_h, u
. frontier lnv lnk lnl, distribution(exponential)
Iteration 0: log likelihood $=2.7270659$
Iteration 1: log likelihood = 2.8551532
Iteration 2: log likelihood = 2.8604815
Iteration 3: log likelihood $=2.8604897$
Iteration 4: log likelihood $=2.8604897$
Stoc. frontier normal/exponential model
Log likelihood $=2.8604897 \quad$ Prob > chi2 $=0.0000$

| lnv | Coef. | Std. Err. | $z$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | :---: |
| lnk | .2624859 | .0919988 | 2.85 | 0.004 | .0821717 | .4428002 |
| lnl | .7703795 | .1109569 | 6.94 | 0.000 | .5529079 | .9878511 |
| _cons | 2.069242 | .2356159 | 8.78 | 0.000 | 1.607444 | 2.531041 |
| /lnsig2v | -3.527598 | .4486176 | -7.86 | 0.000 | -4.406873 | -2.648324 |
| /lnsig2u | -4.002457 | .9274575 | -4.32 | 0.000 | -5.820241 | -2.184674 |
| sigma_v | .1713925 | .0384448 |  |  | .1104231 | .2660258 |
| sigma_u | .1351691 | .0626818 |  |  | .0544692 | .3354317 |
| sigma2 | .0476461 | .0157921 |  |  | .016694 | .0785981 |
| lambda | .7886525 | .087684 |  |  | .616795 | .9605101 |

Likelihood-ratio test of sigma_u=0: chibar2(01) = 1.21 Prob>=chibar2 = 0.135
. predict double u_e, u
. list state u_h u_e

| state | u_h | u_e |
| ---: | ---: | ---: |
| Alabama | .2011338 | .14592865 |
| California | .14480966 | .0972165 |
| Connecticut | .1903485 | .13478797 |
| Florida | .51753139 | .5903303 |
| Georgia | .10397912 | .07140994 |
| Illinois | .12126696 | .0830415 |
| Indiana | .21128212 | .15450664 |
| Iowa | .24933153 | .20073081 |
| Kansas | .10099517 | .06857629 |
| Kentucky | .05626919 | .04152443 |
| Louisiana | .20332731 | .15066405 |
| Maine | .22263164 | .17245793 |
| Maryland | .13534062 | .09245501 |
| Massachusetts | .15636999 | .10932923 |
| Michigan | .15809566 | .10756915 |
| Missouri | .10288047 | .0704146 |
| NewJersey | .09584337 | .06587986 |
| NewYork | .27787793 | .22249416 |
| Ohio | .22914231 | .16981857 |
| Pennsylvania | .1500667 | .10302905 |
| Texas | .20297875 | .14552218 |
| Virginia | .14000132 | .09676078 |
| Washington | .11047581 | .07533251 |
| WestVirginia | .15561392 | .11236153 |
| Wisconsin | .14067066 | .0970861 |

The parameter estimates and the estimates of the inefficiency terms closely match those published in Greene (2003, 505), but the standard errors of the parameter estimates are estimated differently (see the technical note below).

The output from frontier includes estimates of the standard deviations of the two error components, $\sigma_{v}$ and $\sigma_{u}$, which are labeled sigma_v and sigma_u, respectively. In the log likelihood, they are parameterized as $\ln \sigma_{v}^{2}$ and $\ln \sigma_{u}^{2}$, and these estimates are labeled $/ \ln s i g 2 \mathrm{v}$ and $/ \operatorname{lnsig} 2 \mathrm{u}$ in the output. frontier also reports two other useful parameterizations. The estimate of the total error variance, $\sigma_{S}^{2}=\sigma_{v}^{2}+\sigma_{u}^{2}$, is labeled sigma2, and the estimate of the ratio of the standard deviation of the inefficiency component to the standard deviation of the idiosyncratic component, $\lambda=\sigma_{u} / \sigma_{v}$, is labeled lambda.

At the bottom of the output, frontier reports the results of a test that there is no technical inefficiency component in the model. This is a test of the null hypothesis $H_{0}: \sigma_{u}^{2}=0$ against the alternative hypotheses $H_{1}: \sigma_{u}^{2}>0$. If the null hypothesis is true, the stochastic frontier model reduces to an OLS model with normal errors. However, because the test lies on the boundary of the parameter space of $\sigma_{u}^{2}$, the standard likelihood-ratio test is not valid, and a one-sided generalized likelihood-ratio test must be constructed; see Gutierrez, Carter, and Drukker (2001). For this example, the output shows LR $=0.43$ with a $p$-value of 0.256 for the half-normal model and $\mathrm{LR}=1.21$ with a $p$-value of 0.135 for the exponential model. There are several possible reasons for the failure to reject the null hypothesis, but the fact that the test is based on an asymptotic distribution and the sample size was 25 is certainly a leading candidate among those possibilities.

## Technical note

frontier maximizes the log-likelihood function of a stochastic frontier model by using the Newton-Raphson method, and the estimated variance-covariance matrix is calculated as the inverse of the negative Hessian (matrix of second partial derivatives); see $[R] \mathbf{m l}$. When comparing the results with those published using other software, be aware of the difference in the optimization methods, which may result in different, yet asymptotically equivalent, variance estimates.
> Example 2: Models with heteroskedasticity
Often the error terms may not have constant variance. frontier allows you to model heteroskedasticity in either error term as a linear function of a set of covariates. The variance of either the technical inefficiency or the idiosyncratic component may be modeled as

$$
\sigma_{i}^{2}=\exp \left(\mathbf{w}_{i} \boldsymbol{\delta}\right)
$$

The default constant included in $\mathbf{w}_{i}$ may be suppressed by appending a noconstant option to the list of covariates. Also, you can simultaneously specify covariates for both $\sigma_{u_{i}}$ and $\sigma_{v_{i}}$.

In this example, we use a sample of 756 observations of fictional firms producing a manufactured good by using capital and labor. The firms are hypothesized to use a constant returns-to-scale technology, but the sizes of the firms differ. Believing that this size variation will introduce heteroskedasticity into the idiosyncratic error term, we estimate the parameters of a Cobb-Douglas production function. To do this, we use a conditional heteroskedastic half-normal model, with the size of the firm as an explanatory variable in the variance function for the idiosyncratic error. We also perform a test of the hypothesis that the firms use a constant returns-to-scale technology.

| . frontier lnoutput lnlabor lncapital, vhet(size) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Iteration 0: log likelihood = -1508.3692 |  |  |  |  |  |  |
| Iteration 1: log likelihood = -1501.583 |  |  |  |  |  |  |
| Iteration 2: log likelihood $=-1500.3942$ |  |  |  |  |  |  |
| Iteration 3: log likelihood = -1500.3794 |  |  |  |  |  |  |
| Iteration 4: log likelihood $=-1500.3794$ |  |  |  |  |  |  |
| Stoc. frontier normal/half-normal model |  |  |  | Number of obs Wald chi2(2) |  | 756 |
|  |  |  |  | 9.68 |
| Log likelihood = -1500.3794 |  |  |  |  |  | Prob > chi2 |  | 0.0079 |
| lnoutput | Coef. | Std. Err. | z | $P>\|z\|$ | [95\% | Interval] |
| Inoutput |  |  |  |  |  |  |
| lnlabor | . 7090933 | . 2349374 | 3.02 | 0.003 | . 2486 | 1.169562 |
| lncapital | . 3931345 | . 5422173 | 0.73 | 0.468 | -. 6695 | 1.455861 |
| _cons | 1.252199 | 3.14656 | 0.40 | 0.691 | -4.914 | 7.419344 |
| lnsig2v |  |  |  |  |  |  |
| size | -. 0016951 | . 0004748 | -3.57 | 0.000 | -. 0026 | -. 0007645 |
| _cons | 3.156091 | . 9265826 | 3.41 | 0.001 | 1.340 | 4.97216 |
| lnsig2u |  |  |  |  |  |  |
| _cons | 1.947487 | . 1017653 | 19.14 | 0.000 | 1.748 | 2.146943 |
| sigma_u | 2.647838 | . 134729 |  |  | 2.396 | 2.925518 |
| . test _b[lnlabor] + _b[lncapital] = 1 |  |  |  |  |  |  |
| ( 1) [lnoutput]lnlabor + [lnoutput]lncapital = 1 |  |  |  |  |  |  |
| chi2 ( 1) = 0.03 |  |  |  |  |  |  |
| Prob > chi2 = 0.8622 |  |  |  |  |  |  |

The output above indicates that the variance of the idiosyncratic error term is a function of firm size. Also, we failed to reject the hypothesis that the firms use a constant returns-to-scale technology.

## Technical note

In small samples, the conditional heteroskedastic estimators will lack precision for the variance parameters and may fail to converge altogether.
> Example 3: The truncated-normal model
Let's turn our attention to the truncated-normal model. Once again, we will use fictional data. For this example, we have 1,231 observations on the quantity of output, the total cost of production for each firm, the prices that each firm paid for labor and capital services, and a categorical variable measuring the quality of each firm's management. After taking the natural logarithm of the costs (lncost), prices (lnp_k and lnp_l), and output (lnout), we fit a stochastic cost frontier model and specify the distribution for the inefficiency term to be truncated normal.
. use http://www.stata-press.com/data/r13/frontier2
. frontier lncost lnp_k lnp_l lnout, distribution(tnormal) cost

| Iteration 0: | log likelihood $=-2386.9523$ |
| :--- | :--- |
| Iteration 1: | log likelihood $=-2386.5146$ |
| Iteration 2: | log likelihood $=-2386.2704$ |
| Iteration 3: | log likelihood $=-2386.2504$ |
| Iteration 4: | log likelihood $=-2386.2493$ |
| Iteration 5: | log likelihood $=-2386.2493$ |


| Stoc. frontier normal/truncated-normal model | Number of obs | = | 1231 |
| :---: | :---: | :---: | :---: |
|  | Wald chi2(3) |  | 8.82 |
| Log likelihood = -2386.2493 | Prob > chi2 |  | 0.0318 |



In addition to the coefficients, the output reports estimates for several parameters. sigma_v2 is the estimate of $\sigma_{v}^{2}$. sigma_u2 is the estimate of $\sigma_{u}^{2}$. gamma is the estimate of $\gamma=\sigma_{u}^{2} / \sigma_{S}^{2}$. sigma2 is the estimate of $\sigma_{S}^{2}=\sigma_{v}^{2}+\sigma_{u}^{2}$. Because $\gamma$ must be between 0 and 1 , the optimization is parameterized in terms of the inverse logit of $\gamma$, and this estimate is reported as ilgtgamma. Because $\sigma_{S}^{2}$ must be positive, the optimization is parameterized in terms of $\ln \left(\sigma_{S}^{2}\right)$, whose estimate is reported as lnsigma2. Finally, mu is the estimate of $\mu$, the mean of the truncated-normal distribution.

In the output above, the generalized log-likelihood test for the presence of the inefficiency term has been replaced with a test based on the third moment of the OLS residuals. When $\mu=0$ and $\sigma_{u}=0$, the truncated-normal model reduces to a linear regression model with normally distributed errors. However, the distribution of the test statistic under the null hypothesis is not well established, because it becomes impossible to evaluate the $\log$ likelihood as $\sigma_{u}$ approaches zero, prohibiting the use of the likelihood-ratio test.

However, Coelli (1995) noted that the presence of an inefficiency term would negatively skew the residuals from an OLS regression. By identifying negative skewness in the residuals with the presence of an inefficiency term, Coelli derived a one-sided test for the presence of the inefficiency term. The results of this test are given at the bottom of the output. For this example, the null hypothesis of no inefficiency component is rejected.

In the example below, we fit a truncated model and detect a statistically significant inefficiency term in the model. We might question whether the inefficiency term is identically distributed over all firms or whether there might be heterogeneity across firms. frontier provides an extension to the truncated normal model by allowing the mean of the inefficiency term to be modeled as a linear function of a set of covariates. In our dataset, we have a categorical variable that measures the quality of a firm's management. We refit the model, including the cm () option, specifying a set of
binary indicator variables representing the different categories of the quality-measurement variable as covariates.

| Iteration 0: log likelihood = -2386.9523 |  |  |  |
| :---: | :---: | :---: | :---: |
| Iteration 1: $\quad$ log likelihood $=-2384.936$ |  |  |  |
| Iteration 2: $\quad$ log likelihood $=-2382.3942$ |  |  |  |
| Iteration 3: $\quad \mathrm{log}$ likelihood $=-2382.324$ |  |  |  |
| Iteration 4: log likelihood $=-2382.3233$ |  |  |  |
| Iteration 5: log likelihood = -2382.3233 |  |  |  |
| Stoc. frontier normal/truncated-normal model | Number of obs |  | 1231 |
|  | Wald chi2(3) |  | 9.31 |
| Log likelihood = -2382.3233 | Prob > chi2 | = | 0.0254 |


| lncost | Coef . | Std. Err. | z | $P>\|z\|$ | [95\% Con | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Incost |  |  |  |  |  |  |
| lnp_k | . 3611204 | . 2359749 | 1.53 | 0.126 | -. 1013819 | . 8236227 |
| lnp_l | . 680446 | . 4934935 | 1.38 | 0.168 | -. 2867835 | 1.647675 |
| lnout | . 7605533 | . 3466102 | 2.19 | 0.028 | . 0812098 | 1.439897 |
| _cons | 2.550769 | 1.078911 | 2.36 | 0.018 | . 4361417 | 4.665396 |
| mu |  |  |  |  |  |  |
| quality |  |  |  |  |  |  |
| 2 | . 5056067 | . 3382907 | 1.49 | 0.135 | -. 1574309 | 1.168644 |
| 3 | . 783223 | . 376807 | 2.08 | 0.038 | . 0446947 | 1.521751 |
| 4 | . 5577511 | . 3355061 | 1.66 | 0.096 | -. 0998288 | 1.215331 |
| 5 | . 6792882 | . 3428073 | 1.98 | 0.048 | . 0073981 | 1.351178 |
| _cons | . 6014025 | . 990167 | 0.61 | 0.544 | -1.339289 | 2.542094 |
| /lnsigma2 | 1.541784 | . 1790926 | 8.61 | 0.000 | 1.190769 | 1.892799 |
| /ilgtgamma | 1.242302 | . 2588968 | 4.80 | 0.000 | . 734874 | 1.749731 |
| sigma2 | 4.67292 | . 8368852 |  |  | 3.289611 | 6.637923 |
| gamma | . 7759645 | . 0450075 |  |  | . 6758739 | . 8519189 |
| sigma_u2 | 3.62602 | . 7139576 |  |  | 2.226689 | 5.025351 |
| sigma_v2 | 1.0469 | . 2583469 |  |  | . 5405491 | 1.553251 |

The conditional mean model was developed in the context of panel-data estimators, and we can apply frontier's conditional mean model to panel data.

## Stored results

frontier stores the following in e():
Scalars

```
e(N)
e(df_m)
e(k)
e(k_eq)
e(k_eq_model)
e(k_dv)
e(chi2)
e(1l)
e(ll_c)
e(z)
e(sigma_u)
e(sigma_v)
e(p)
e(chi2_c)
e(p_z)
e(rank)
e(ic)
e(rc)
e(converged)
```

Macros

```
e(cmd)
    e(cmdline)
    e(depvar)
    e(function)
    e(wtype)
    e(wexp)
    e(title)
    e(chi2type)
    e(dist)
    e(het)
    e(u_hetvar)
    e(v_hetvar)
    e(vce)
    e(vcetype)
    e(opt)
    e(which)
    e(ml_method)
    e(user)
    e(technique)
    e(properties)
    e(predict)
    e(asbalanced)
    e(asobserved)
```

    number of observations
    model degrees of freedom
    number of parameters
    number of equations in \(e(b)\)
    number of equations in overall model test
    number of dependent variables
    \(\chi^{2}\)
    log likelihood
    log likelihood for \(H_{0}: \sigma_{u}=0\)
    test for negative skewness of OLS residuals
    standard deviation of technical inefficiency
    standard deviation of \(v_{i}\)
    significance
    LR test statistic
    \(p\)-value for z
    rank of \(e(V)\)
    number of iterations
    return code
    1 if converged, 0 otherwise
    frontier
command as typed
name of dependent variable
production or cost
weight type
weight expression
title in estimation output
Wald; type of model $\chi^{2}$ test
distribution assumption for $u_{i}$
heteroskedastic components
varlist in uhet()
varlist in vhet()
vcetype specified in vce()
title used to label Std. Err.
type of optimization
max or min; whether optimizer is to perform maximization or minimization
type of ml method
name of likelihood-evaluator program
maximization technique
b V
program used to implement predict
factor variables fvset as asbalanced
factor variables fvset as asobserved

Matrices

```
e(b)
    e(Cns)
    e(ilog)
    e(gradient)
    e(V)
    e(V_modelbased)
```

coefficient vector
constraints matrix
iteration $\log$ (up to 20 iterations)
gradient vector
variance-covariance matrix of the estimators
model-based variance

Functions
e(sample) marks estimation sample

## Methods and formulas

Consider an equation of the form

$$
y_{i}=\mathbf{x}_{i} \boldsymbol{\beta}+v_{i}-s u_{i}
$$

where $y_{i}$ is the dependent variable, $\mathbf{x}_{i}$ is a $1 \times k$ vector of observations on the independent variables included as indent covariates, $\boldsymbol{\beta}$ is a $k \times 1$ vector of coefficients, and

$$
s=\left\{\begin{aligned}
1, & \text { for production functions } \\
-1, & \text { for cost functions }
\end{aligned}\right.
$$

The log-likelihood functions are as follows.
Normal/half-normal model:

$$
\ln L=\sum_{i=1}^{N}\left\{\frac{1}{2} \ln \left(\frac{2}{\pi}\right)-\ln \sigma_{S}+\ln \Phi\left(-\frac{s \epsilon_{i} \lambda}{\sigma_{S}}\right)-\frac{\epsilon_{i}^{2}}{2 \sigma_{S}^{2}}\right\}
$$

Normal/exponential model:

$$
\ln L=\sum_{i=1}^{N}\left\{-\ln \sigma_{u}+\frac{\sigma_{v}^{2}}{2 \sigma_{u}^{2}}+\ln \Phi\left(\frac{-s \epsilon_{i}-\frac{\sigma_{v}^{2}}{\sigma_{u}}}{\sigma_{v}}\right)+\frac{s \epsilon_{i}}{\sigma_{u}}\right\}
$$

Normal/truncated-normal model:

$$
\begin{aligned}
\ln L= & \sum_{i=1}^{N}\left\{-\frac{1}{2} \ln (2 \pi)-\ln \sigma_{S}-\ln \Phi\left(\frac{\mu}{\sigma_{S} \sqrt{\gamma}}\right)\right. \\
& \left.+\ln \Phi\left[\frac{(1-\gamma) \mu-s \gamma \epsilon_{i}}{\left\{\sigma_{S}^{2} \gamma(1-\gamma)\right\}^{1 / 2}}\right]-\frac{1}{2}\left(\frac{\epsilon_{i}+s \mu}{\sigma_{S}}\right)^{2}\right\}
\end{aligned}
$$

where $\sigma_{S}=\left(\sigma_{u}^{2}+\sigma_{v}^{2}\right)^{1 / 2}, \lambda=\sigma_{u} / \sigma_{v}, \gamma=\sigma_{u}^{2} / \sigma_{S}^{2}, \epsilon_{i}=y_{i}-\mathbf{x}_{i} \boldsymbol{\beta}$, and $\Phi()$ is the cumulative distribution function of the standard normal distribution.

To obtain estimation for $u_{i}$, you can use either the mean or the mode of the conditional distribution $f(u \mid \epsilon)$.

$$
\begin{aligned}
& E\left(u_{i} \mid \epsilon_{i}\right)=\mu_{* i}+\sigma_{*}\left\{\frac{\phi\left(-\mu_{* i} / \sigma_{*}\right)}{\Phi\left(\mu_{* i} / \sigma_{*}\right)}\right\} \\
& M\left(u_{i} \mid \epsilon_{i}\right)= \begin{cases}\mu_{* i} & \text { if } \mu_{* i} \geq 0 \\
0 & \text { otherwise }\end{cases}
\end{aligned}
$$

Then the technical efficiency $(s=1)$ or cost efficiency $(s=-1)$ will be estimated by

$$
\begin{aligned}
E_{i} & =E\left\{\exp \left(-s u_{i}\right) \mid \epsilon_{i}\right\} \\
& =\left\{\frac{1-\Phi\left(s \sigma_{*}-\mu_{* i} / \sigma_{*}\right)}{1-\Phi\left(-\mu_{* i} / \sigma_{*}\right)}\right\} \exp \left(-s \mu_{* i}+\frac{1}{2} \sigma_{*}^{2}\right)
\end{aligned}
$$

where $\mu_{* i}$ and $\sigma_{*}$ are defined for the normal/half-normal model as

$$
\begin{aligned}
\mu_{* i} & =-s \epsilon_{i} \sigma_{u}^{2} / \sigma_{S}^{2} \\
\sigma_{*} & =\sigma_{u} \sigma_{v} / \sigma_{S}
\end{aligned}
$$

for the normal/exponential model as

$$
\begin{aligned}
\mu_{* i} & =-s \epsilon_{i}-\sigma_{v}^{2} / \sigma_{u} \\
\sigma_{*} & =\sigma_{v}
\end{aligned}
$$

and for the normal/truncated-normal model as

$$
\begin{aligned}
\mu_{* i} & =\frac{-s \epsilon_{i} \sigma_{u}^{2}+\mu \sigma_{v}^{2}}{\sigma_{S}^{2}} \\
\sigma_{*} & =\sigma_{u} \sigma_{v} / \sigma_{S}
\end{aligned}
$$

In the half-normal and exponential models, when heteroskedasticity is assumed, the standard deviations, $\sigma_{u}$ or $\sigma_{v}$, will be replaced in the above equations by

$$
\sigma_{i}^{2}=\exp \left(\mathbf{w}_{i} \boldsymbol{\delta}\right)
$$

where $\mathbf{w}$ is the vector of explanatory variables in the variance function.
In the conditional mean model, the mean parameter of the truncated normal distribution, $\mu$, is modeled as a linear combination of the set of covariates, $\mathbf{w}$.

$$
\mu=\mathbf{w}_{i} \boldsymbol{\delta}
$$

Therefore, the log-likelihood function can be rewritten as

$$
\begin{aligned}
\ln L= & \sum_{i=1}^{N}\left[-\frac{1}{2} \ln (2 \pi)-\ln \sigma_{S}-\ln \Phi\left(\frac{\mathbf{w}_{i} \boldsymbol{\delta}}{\sqrt{\sigma_{S}^{2} \gamma}}\right)\right. \\
& \left.+\ln \Phi\left\{\frac{(1-\gamma) \mathbf{w}_{i} \boldsymbol{\delta}-s \gamma \epsilon_{i}}{\sqrt{\sigma_{S}^{2} \gamma(1-\gamma)}}\right\}-\frac{1}{2}\left(\frac{\epsilon_{i}+s \mathbf{w}_{i} \boldsymbol{\delta}}{\sigma_{S}}\right)^{2}\right]
\end{aligned}
$$

The $z$ test reported in the output of the truncated-normal model is a third-moment test developed by Coelli (1995) as an extension of a test previously developed by Pagan and Hall (1983). Coelli shows that under the null of normally distributed errors, the statistic

$$
z=\frac{m_{3}}{\left(\frac{6 m_{2}^{3}}{N}\right)^{1 / 2}}
$$

has a standard normal distribution, where $m_{3}$ is the third moment from the OLS regression. Because the residuals are either negatively skewed (production function) or positively skewed (cost function), a one-sided $p$-value is used.

## References

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## Also see

[R] frontier postestimation - Postestimation tools for frontier
$[\mathrm{R}]$ regress - Linear regression
[XT] xtfrontier - Stochastic frontier models for panel data
[U] 20 Estimation and postestimation commands

## Title

frontier postestimation - Postestimation tools for frontier

| Description | Syntax for predict | Menu for predict | Options for predict |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Reference | Also see |  |

## Description

The following postestimation commands are available after frontier:

| Command | Description |
| :--- | :--- |
| contrast | contrasts and ANOVA-style joint tests of estimates |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample <br> estat vce |
| variance-covariance matrix of the estimators (VCE) |  |
| estimates | cataloging estimation results <br> point estimates, standard errors, testing, and inference for linear combinations <br> of coefficients |
| lincom | link test for model specification |
| linktest | likelihood-ratio test <br> marginal means, predictive margins, marginal effects, and average marginal <br> effects |
| margins | graph the results from margins (profile plots, interaction plots, etc.) <br> point estimates, standard errors, testing, and inference for nonlinear combinations <br> of coefficients |
| marginsplot |  |
| nlcom | predictions, residuals, influence statistics, and other diagnostic measures <br> point estimates, standard errors, testing, and inference for generalized predictions |
| predict | pairwise comparisons of estimates |
| predictnl | Wald tests of simple and composite linear hypotheses |
| pwcompare | Wald tests of nonlinear hypotheses |

## Syntax for predict

predict [type] newvar $[$ if $][$ in $]$ [, statistic]
predict $[$ type $]\left\{\right.$ stub* $\mid$ newvar $_{\mathrm{xb}}$ newvar $_{v}$ newvar $\left._{u}\right\}[$ if $][$ in $]$, scores

## statistic

Description

## Main

$\mathrm{xb} \quad$ linear prediction; the default
stdp
u
m
te
standard error of the prediction
estimates of minus the natural $\log$ of the technical efficiency via $E\left(u_{i} \mid \epsilon_{i}\right)$ estimates of minus the natural $\log$ of the technical efficiency via $M\left(u_{i} \mid \epsilon_{i}\right)$ estimates of the technical efficiency via $E\left\{\exp \left(-s u_{i}\right) \mid \epsilon_{i}\right\}$
$s=\left\{\begin{aligned} 1, & \text { for production functions } \\ -1, & \text { for cost functions }\end{aligned}\right.$

These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
xb , the default, calculates the linear prediction.
stdp calculates the standard error of the linear prediction.
u produces estimates of minus the natural $\log$ of the technical efficiency via $E\left(u_{i} \mid \epsilon_{i}\right)$.
m produces estimates of minus the natural $\log$ of the technical efficiency via $M\left(u_{i} \mid \epsilon_{i}\right)$. te produces estimates of the technical efficiency via $E\left\{\exp \left(-s u_{i}\right) \mid \epsilon_{i}\right\}$.
scores calculates equation-level score variables.
The first new variable will contain $\partial \ln L / \partial\left(\mathbf{x}_{i} \boldsymbol{\beta}\right)$.
The second new variable will contain $\partial \ln L / \partial(\operatorname{lnsig} 2 \mathrm{v})$.
The third new variable will contain $\partial \ln L / \partial(\ln s i g 2 u)$.

## Remarks and examples

$>$ Example 1
In example 2 of [R] frontier, we modeled heteroskedasticity by specifying the vhet () option. We would like to compare the predicted efficiency in that case with respect to a model specification without accounting for the presence of heteroskedasticity in the error term. Kumbhakar and Lovell (2000, 117) show that failing to account for heteroskedasticity associated with firm size may lead to bias in the estimation of the technical efficiency. By incorrectly assuming homoskedasticity, the estimates for relatively small firms would be biased upward, while the estimates for relatively large firms would be biased downward. Let's refit the model and use the te option of predict:
. use http://www.stata-press.com/data/r13/frontier1
. frontier lnoutput lnlabor lncapital, vhet(size)
(output omitted)
predict te_vhet, te
Next we fit the model assuming homoskedasticity and then again predict the technical efficiency with the te option of predict:

```
. frontier lnoutput lnlabor lncapital
    (output omitted)
```

. predict te, te

The graph below shows the estimates for technical efficiency for the smaller and larger firms. The technical efficiency tends to be smaller for smaller firms when the model specification accounts for the presence of heteroskedasticity, whereas the predictions for the technical efficiency tends to be smaller for larger firms assuming homoskedasticity. These results agree with the theoretical statement in Kumbhakar and Lovell (2000) because the firm size was actually relevant to model heteroskedasticity in the idiosyncratic component of the error term.


## > Example 2

We also test in example 2 of $[\mathrm{R}]$ frontier whether the firms use constant returns to scale. We can use lincom as an alternative to perform an equivalent test based on the normal distribution.

```
. use http://www.stata-press.com/data/r13/frontier1, clear
. frontier lnoutput lnlabor lncapital, vhet(size)
(output omitted)
. lincom _b[lnlabor] + _b[lncapital]-1
( 1) [lnoutput]lnlabor + [lnoutput]lncapital = 1
```

| lnoutput | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | :---: | ---: | ---: |
| $(1)$ | .1022278 | .5888511 | 0.17 | 0.862 | -1.051899 | 1.256355 |

The $p$-value is exactly the same as the one we obtained with the test command in example 2 of $[\mathrm{R}]$ frontier. However, notice that by using lincom, we obtained an estimate of the deviation from the constant returns-to-scale assumption, which is not significantly different from zero in this case.

## Reference

Kumbhakar, S. C., and C. A. K. Lovell. 2000. Stochastic Frontier Analysis. Cambridge: Cambridge University Press.

## Also see

$[\mathrm{R}]$ frontier - Stochastic frontier models
[U] 20 Estimation and postestimation commands

## Title

fvrevar - Factor-variables operator programming command

| Syntax | Description | Options | Remarks and examples |
| :--- | :--- | :--- | :--- |
| Stored results | Also see |  |  |

## Syntax

```
fvrevar [varlist] [if][in] [, substitute tsonly list stub(stub)]
```

You must tsset your data before using fvrevar if varlist contains time-series operators; see [TS] tsset.

## Description

fvrevar creates an equivalent, temporary variable list for a varlist that might contain factor variables, interactions, or time-series-operated variables so that the resulting variable list can be used by commands that do not otherwise support factor variables or time-series-operated variables. The resulting list also could be used in a program to speed execution at the cost of using more memory.

## Options

substitute specifies that equivalent, temporary variables be substituted for any factor variables, interactions, or time-series-operated variables in varlist. substitute is the default action taken by fvrevar; you do not need to specify the option.
tsonly specifies that equivalent, temporary variables be substituted for only the time-series-operated variables in varlist.
list specifies that all factor-variable operators and time-series operators be removed from varlist and the resulting list of base variables be returned in $r$ (varlist). No new variables are created with this option.
stub (stub) specifies that fvrevar generate named variables instead of temporary variables. The new variables will be named stub\#.

## Remarks and examples

fvrevar might create no new variables, one new variable, or many new variables, depending on the number of factor variables, interactions, and time-series operators appearing in varlist. Any new variables created are temporary. The new, equivalent varlist is returned in $r$ (varlist). The new varlist corresponds one to one with the original varlist.
> Example 1
Typing

[^19]creates five temporary variables corresponding to the levels of rep78. No new variables are created for variables mpg and turn because they do not contain factor-variable or time-series operators.

The resulting variable list is

```
. display "'r(varlist)'"
__000000 __000001 __000002 __000003 __000004 mpg turn
```

(Your temporary variable names may be different, but that is of no consequence.)
Temporary variables automatically vanish when the program concludes.

## $>$ Example 2

Suppose we want to create temporary variables for specific levels of a factor variable. To do this, we can use the parenthesis notation of factor-variable syntax.

```
. fvrevar i(2,3)bn.rep78 mpg
```

creates two temporary variables corresponding to levels 2 and 3 of rep78. Notice that we specified that neither level 2 nor 3 be set as the base level by using the bn notation. If we did not specify bn, level 2 would have been treated as the base level.

The resulting variable list is

```
. display "'r(varlist)'"
__000005 __000002 mpg
```

We can see the results by listing the new variables alongside the original value of rep78.
. list rep78 'r(varlist)' in $1 / 5$

| rep78 | $\ldots 000005$ | $\ldots 000002$ | mpg |
| ---: | ---: | ---: | ---: |
| Average | 0 | 1 | 22 |
| Average | 0 | 1 | 17 |
| $\cdot$ | . | . | 22 |
| Average | 0 | 1 | 20 |
| Good | 0 | 0 | 15 |

If we had needed only the base-variable names, we could have specified

```
. fvrevar i(2,3)bn.rep78 mpg, list
. display "'r(varlist)'"
mpg rep78
```

The order of the list will probably differ from that of the original list; base variables are listed only once.

## Example 3

Now let's assume we have a varlist containing both an interaction and time-series-operated variables. If we want to create temporary variables for the entire equivalent varlist, we can specify fvrevar with no options.

```
. generate t = _n
. tsset t
    time variable: t, 1 to 74
        delta: 1 unit
    . fvrevar c.turn#i(2,3).rep78 L.mpg
```

The resulting variable list is
. display "'r(varlist)'"
__000006 __000007 __000008
If we want to create temporary variables only for the time-series-operated variables, we can specify the tsonly option.
. fvrevar c.turn\#i(2,3).rep78 L.mpg, tsonly
The resulting variable list is
. display "'r(varlist)'"
c.turn\#2b.rep78 c.turn\#3.rep78 __000008

Notice that fvrevar returned the expanded factor-variable list with the tsonly option.

## Technical note

fvrevar, substitute avoids creating duplicate variables. Consider
. fvrevar i.rep78 turn mpg i.rep78
i.rep78 appears twice in the varlist. fvrevar will create only one set of new variables for the five levels of rep78 and will use these new variables once in the resulting r(varlist). Moreover, fvrevar will do this even across multiple calls:
. fvrevar i.rep78 turn mpg
. fvrevar i.rep78
i.rep78 appears in two separate calls. At the first call, fvrevar creates five temporary variables corresponding to the five levels of rep78. At the second call, fvrevar remembers what it has done and uses the same temporary variables for i.rep78.

## Stored results

fvrevar stores the following in r() :
Macros
$r$ (varlist) the modified variable list or list of base-variable names

## Also see

[TS] tsrevar - Time-series operator programming command
[P] syntax - Parse Stata syntax
[P] unab - Unabbreviate variable list
[U] 11 Language syntax
[U] 11.4.4 Time-series varlists
[U] 18 Programming Stata

## Title

fvset - Declare factor-variable settings

Syntax Description Options Remarks and examples Stored results

## Syntax

Declare base settings fvset base base_spec varlist

Declare design settings
fvset design design_spec varlist

Clear the current settings
fvset clear varlist

Report the current settings
fvset report [varlist] [, base(base_spec) design(design_spec)]

| base_spec | Description |
| :--- | :--- |
| default | default base |
| first | lowest level value; the default |
| last | highest level value |
| frequent | most frequent level value |
| none | no base |
| $\#$ | nonnegative integer value |

design_spec Description
default default base
asbalanced
accumulate using $1 / k, k=$ number of levels
asobserved accumulate using observed relative frequencies; the default

## Description

fvset declares factor-variable settings. Factor-variable settings identify the base level and how to accumulate statistics over levels.
fvset base specifies the base level for each variable in varlist. The default for factor variables without a declared base level is first.
fvset design specifies how to accumulate over the levels of a factor variable. The margins command is the only command aware of this setting; see [R] margins. By default, margins assumes that factor variables are asobserved, meaning that they are accumulated by weighting by the number of observations or the sum of the weights if weights have been specified.
fvset clear removes factor-variable settings for each variable in varlist. fvset clear _all removes all factor-variable settings from all variables.
fvset report reports the current factor-variable settings for each variable in varlist. fvset without arguments is a synonym for fvset report.

## Options

base (base_spec) restricts fvset report to report only the factor-variable settings for variables with the specified base_spec.
design (design_spec) restricts fvset report to report only the factor-variable settings for variables with the specified design_spec.

## Remarks and examples

## $>$ Example 1

Using auto2.dta, we include factor variable i.rep78 in a regression:

```
. use http://www.stata-press.com/data/r13/auto2
(1978 Automobile Data)
. regress mpg i.rep78, baselevels
```

| Source | SS | df | MS |
| ---: | ---: | ---: | :---: |
| Model | 549.415777 | 4 | 137.353944 |
| Residual | 1790.78712 | 64 | 27.9810488 |
| Total | 2340.2029 | 68 | 34.4147485 |


| Number of obs | $=$ |
| :--- | ---: |
| F 4, | 69 |
| Prob $>$ F | $=$ |
| R-squared | $=$ |
| Adj R-squared | $=0.0016$ |
| Root MSE | 0.2348 |
|  | $=$ |
|  |  |


| mpg | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | :---: |
| rep78 |  |  |  |  |  |  |
| Poor | 0 | (base) |  |  |  |  |
| Fair | -1.875 | 4.181884 | -0.45 | 0.655 | -10.22927 | 6.479274 |
| Average | -1.566667 | 3.863059 | -0.41 | 0.686 | -9.284014 | 6.150681 |
| Good | .6666667 | 3.942718 | 0.17 | 0.866 | -7.209818 | 8.543152 |
| Excellent | 6.363636 | 4.066234 | 1.56 | 0.123 | -1.759599 | 14.48687 |
| _cons | 21 | 3.740391 | 5.61 | 0.000 | 13.52771 | 28.47229 |

We specified the baselevels option so that the base level would be included in the output. By default, the first level is the base level. We can change the base level to 2 :
. fvset base 2 rep78
. regress mpg i.rep78, baselevels

| Source | SS | df | MS |
| ---: | ---: | ---: | :---: |
| Model | 549.415777 | 4 | 137.353944 |
| Residual | 1790.78712 | 64 | 27.9810488 |
| Total | 2340.2029 | 68 | 34.4147485 |

Number of obs $=69$
$F(4,64)=4.91$
Prob > F $=0.0016$
R -squared $=0.2348$
Adj R-squared $=0.1869$
Root MSE $=5.2897$

| mpg | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| rep78 |  |  |  |  |  |  |
| Poor | 1.875 | 4.181884 | 0.45 | 0.655 | -6.479274 | 10.22927 |
| Fair | 0 | (base) |  |  |  |  |
| Average | .3083333 | 2.104836 | 0.15 | 0.884 | -3.896559 | 4.513226 |
| Good | 2.541667 | 2.247695 | 1.13 | 0.262 | -1.948621 | 7.031954 |
| Excellent | 8.238636 | 2.457918 | 3.35 | 0.001 | 3.32838 | 13.14889 |
| _cons | 19.125 | 1.870195 | 10.23 | 0.000 | 15.38886 | 22.86114 |

Let's set rep78 to have no base level and fit a cell-means regression:
. fvset base none rep78
. regress mpg i.rep78, noconstant

| Source | SS | df | MS |
| ---: | ---: | ---: | :---: |
| Model <br> Residual | 31824.2129 | 5 | 6364.84258 |
| Total | 33615 | 69 | 487.173913 |


| Number of obs | $=$ | 69 |
| :--- | ---: | ---: |
| F $($ 5, 64$)$ | $=227.47$ |  |
| Prob $>$ F | $=$ | 0.0000 |
| R-squared | $=$ | 0.9467 |
| Adj R-squared | $=$ | 0.9426 |
| Root MSE | $=5.2897$ |  |


| mpg | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| rep78 |  |  |  |  |  |  |
| Poor | 21 | 3.740391 | 5.61 | 0.000 | 13.52771 | 28.47229 |
| Fair | 19.125 | 1.870195 | 10.23 | 0.000 | 15.38886 | 22.86114 |
| Average | 19.43333 | .9657648 | 20.12 | 0.000 | 17.504 | 21.36267 |
| Good | 21.66667 | 1.246797 | 17.38 | 0.000 | 19.1759 | 24.15743 |
| Excellent | 27.36364 | 1.594908 | 17.16 | 0.000 | 24.17744 | 30.54983 |

## Example 2

By default, margins accumulates a margin by using the observed relative frequencies of the factor levels.
. regress mpg i.foreign

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | 378.153515 | 1 | 378.153515 |
| Residual | 2065.30594 | 72 | 28.6848048 |
| Total | 2443.45946 | 73 | 33.4720474 |


| Number of obs | $=$ | 74 |
| :--- | ---: | ---: |
| F 1, | 13.18 |  |
| Prob $>$ F | $=$ | 0.0005 |
| R-squared | $=$ | 0.1548 |
| Adj R-squared | $=$ | 0.1430 |
| Root MSE | $=$ | 5.3558 |


| mpg | Coef. | Std. Err. | $t$ | P>\|t| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| foreign |  |  |  |  |  |  |
| Foreign | 4.945804 | 1.362162 | 3.63 | 0.001 | 2.230384 | 7.661225 |
| _cons | 19.82692 | .7427186 | 26.70 | 0.000 | 18.34634 | 21.30751 |



Let's set foreign to always accumulate using equal relative frequencies:

```
. fvset design asbalanced foreign
```

. regress mpg i.foreign


| Adjusted predictions |  |  |  | Number of obs |  | = | 74 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Model VCE | : OLS |  |  |  |  |  |  |
| Expression at | Linear prediction, predict() <br> foreign (asbalanced) |  |  |  |  |  |  |
|  | Delta-method |  |  |  |  |  |  |
|  | Margin | Std. Err. | t | $P>\|t\|$ | [95\% | Conf | Interval] |
| _cons | 22.29983 | . 6810811 | 32.74 | 0.000 | 20.94 | 211 | 23.65754 |

Suppose that we issued the fvset design command earlier in our session and that we cannot remember which variables we set as asbalanced. We can retrieve this information by using the fvset report command:
. fvset report, design(asbalanced)

| Variable | Base | Design |
| :--- | :--- | :--- |
| foreign |  | asbalanced |

## - Technical note

margins is aware of a factor variable's design setting only through the estimation results it is working with. The design setting is stored by the estimation command; thus changing the design setting between the estimation command and margins will have no effect. For example, the output from the following two calls to margins yields the same results:
. fvset clear foreign
. regress mpg i.foreign

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | 378.153515 | 1 | 378.153515 |
| Residual | 2065.30594 | 72 | 28.6848048 |
| Total | 2443.45946 | 73 | 33.4720474 |

Number of obs $=\quad 74$
$F(1,72)=13.18$
Prob > F $\quad=0.0005$
$R$-squared $=0.1548$
Adj R-squared $=0.1430$
Root MSE $=5.3558$

| mpg | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| foreign |  |  |  |  |  |  |
| Foreign | 4.945804 | 1.362162 | 3.63 | 0.001 | 2.230384 | 7.661225 |
| _cons | 19.82692 | .7427186 | 26.70 | 0.000 | 18.34634 | 21.30751 |

. margins
Predictive margins $\quad$ Number of obs $=74$
Model VCE : OLS
Expression : Linear prediction, predict()

|  | MarginDelta-method <br> Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| _cons | 21.2973 | .6226014 | 34.21 | 0.000 | 20.05616 | 22.53843 |

. fvset design asbalanced foreign
. margins
Predictive margins $\quad$ Number of obs $=\quad 74$
Model VCE : OLS
Expression : Linear prediction, predict()

|  | Delta-method |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
|  | Margin | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| _cons | 21.2973 | .6226014 | 34.21 | 0.000 | 20.05616 | 22.53843 |

## Stored results

fvset stores the following in $r()$ :
Macros

$$
\begin{array}{ll}
\mathrm{r} \text { (varlist) } & \text { varlist } \\
\mathrm{r} \text { (baselist) } & \text { base setting for each variable in varlist } \\
\mathrm{r} \text { (designlist) } & \text { design setting for each variable in varlist }
\end{array}
$$

## Title

gllamm - Generalized linear and latent mixed models
Description Remarks and examples References Also see

## Description

GLLAMM stands for generalized linear latent and mixed models, and gllamm is a Stata command for fitting such models written by Sophia Rabe-Hesketh (University of California-Berkeley) as part of joint work with Anders Skrondal (Norwegian Institute of Public Health) and Andrew Pickles (King's College London).

## Remarks and examples

Generalized linear latent and mixed models are a class of multilevel latent variable models, where a latent variable is a factor or a random effect (intercept or coefficient), or a disturbance (residual). The gllamm command for fitting such models is not an official command of Stata; it has been independently developed by highly regarded authors and is itself highly regarded. You can learn more about gllamm by visiting http://www.gllamm.org.
gllamm is available from the Statistical Software Components (SSC) archive. To install, type
. ssc describe gllamm
. ssc install gllamm
If you later wish to uninstall gllamm, type ado uninstall gllamm.

## References

Miranda, A., and S. Rabe-Hesketh. 2006. Maximum likelihood estimation of endogenous switching and sample selection models for binary, ordinal, and count variables. Stata Journal 6: 285-308.

Rabe-Hesketh, S., and B. S. Everitt. 2007. A Handbook of Statistical Analyses Using Stata. 4th ed. Boca Raton, FL: Chapman \& Hall/CRC.

Rabe-Hesketh, S., A. Pickles, and C. Taylor. 2000. sg129: Generalized linear latent and mixed models. Stata Technical Bulletin 53: 47-57. Reprinted in Stata Technical Bulletin Reprints, vol. 9, pp. 293-307. College Station, TX: Stata Press.

Rabe-Hesketh, S., and A. Skrondal. 2012. Multilevel and Longitudinal Modeling Using Stata. 3rd ed. College Station, TX: Stata Press.

Rabe-Hesketh, S., A. Skrondal, and A. Pickles. 2002. Reliable estimation of generalized linear mixed models using adaptive quadrature. Stata Journal 2: 1-21.
——. 2003. Maximum likelihood estimation of generalized linear models with covariate measurement error. Stata Journal 3: 386-411.

Skrondal, A., and S. Rabe-Hesketh. 2004. Generalized Latent Variable Modeling: Multilevel, Longitudinal, and Structural Equation Models. Boca Raton, FL: Chapman \& Hall/CRC.
Zheng, X., and S. Rabe-Hesketh. 2007. Estimating parameters of dichotomous and ordinal item response models with gllamm. Stata Journal 7: 313-333.

The references above are restricted to works by the primary authors of gllamm. There are many other books and articles that use or discuss gllamm; see http://www.gllamm.org/pub.html for a list.

## Also see

[ME] meglm - Multilevel mixed-effects generalized linear model
[ME] mixed - Multilevel mixed-effects linear regression
[SEM] intro 2 - Learning the language: Path diagrams and command language
[SEM] intro 5 - Tour of models

## Title

glm - Generalized linear models

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | Acknowledgments |
| References | Also see |  |  |

## Syntax

$$
\text { glm depvar }[\text { indepvars }][\text { if }][\text { in }][\text { weight }][\text {,options }]
$$

options Description
Model
family (familyname) distribution of depvar; default is family (gaussian)
link(linkname) link function; default is canonical link for family() specified
Model 2
noconstant suppress constant term
exposure (varname)
offset (varname)
include $\ln$ (varname) in model with coefficient constrained to 1 include varname in model with coefficient constrained to 1
constraints (constraints) apply specified linear constraints
collinear keep collinear variables
asis
mu (varname)
init(varname)
retain perfect predictor variables
use varname as the initial estimate for the mean of depvar
synonym for mu (varname)
SE/Robust
vce (vcetype)
vfactor (\#)
disp(\#)
scale (x2| dev $\mid \#$ )
vcetype may be oim, robust, cluster clustvar, eim, opg,
bootstrap, jackknife, hac kernel, jackknife1, or unbiased multiply variance matrix by scalar \#
quasilikelihood multiplier
set the scale parameter
Reporting
level(\#)
eform
nocnsreport
display_options

Maximization
ml
irls
maximize_options
fisher (\#)
search
use maximum likelihood optimization; the default use iterated, reweighted least-squares optimization of the deviance control the maximization process; seldom used use the Fisher scoring Hessian or expected information matrix (EIM) search for good starting values
noheader
notable
nodisplay
coeflegend
suppress header table from above coefficient table suppress coefficient table suppress the output; iteration $\log$ is still displayed display legend instead of statistics
familyname
gaussian
igaussian
binomial [varname ${ }_{N} \mid \#_{N}$ ]
poisson
n̄inomial $\left[\#_{k} \mid \mathrm{ml}\right]$
gamma

## Description

Gaussian (normal)
inverse Gaussian
Bernoulli/binomial
Poisson
negative binomial
gamma

## linkname

## Description

| identity | identity |
| :--- | :--- |
| log | log |
| logit | logit |
| probit | probit |
| cloglog | cloglog |
| power \# | power |
| opower \# | odds power |
| $\underline{\text { nbinomial }}$ | negative binomial |
| loglog | log-log |

indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
depvar and indepvars may contain time-series operators; see [U] 11.4.4 Time-series varlists.
bootstrap, by, fp, jackknife, mfp, mi estimate, nestreg, rolling, statsby, stepwise, and svy are allowed; see [U] 11.1.10 Prefix commands.
vce(bootstrap), vce(jackknife), and vce(jackknife1) are not allowed with the mi estimate prefix; see [MI] mi estimate.
Weights are not allowed with the bootstrap prefix; see $[R]$ bootstrap.
aweights are not allowed with the jackknife prefix; see [R] jackknife.
vce(), vfactor(), disp(), scale(), irls, fisher(), noheader, notable, nodisplay, and weights are not allowed with the svy prefix; see [SVY] svy.
fweights, aweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
noheader, notable, nodisplay, and coeflegend do not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Generalized linear models $>$ Generalized linear models (GLM)

## Description

glm fits generalized linear models. It can fit models by using either IRLS (maximum quasilikelihood) or Newton-Raphson (maximum likelihood) optimization, which is the default.

See [U] 26 Overview of Stata estimation commands for a description of all of Stata's estimation commands, several of which fit models that can also be fit using glm.

## Options

$\leadsto \sqrt{\text { Model }}$
family (familyname) specifies the distribution of depvar; family (gaussian) is the default.
link(linkname) specifies the link function; the default is the canonical link for the family() specified (except for family (nbinomial)).

Model 2
noconstant, exposure(varname), offset(varname), constraints(constraints), collinear; see $[\mathrm{R}]$ estimation options. constraints (constraints) and collinear are not allowed with irls.
asis forces retention of perfect predictor variables and their associated, perfectly predicted observations and may produce instabilities in maximization; see $[\mathrm{R}]$ probit. This option is only allowed with option family (binomial) with a denominator of 1.
mu(varname) specifies varname as the initial estimate for the mean of depvar. This option can be useful with models that experience convergence difficulties, such as family (binomial) models with power or odds-power links. init(varname) is a synonym.

## SE/Robust

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
In addition to the standard vcetypes, glm allows the following alternatives:
vce (eim) specifies that the EIM estimate of variance be used.
vce (jackknife1) specifies that the one-step jackknife estimate of variance be used.
vce (hac kernel [\#]) specifies that a heteroskedasticity- and autocorrelation-consistent (HAC) variance estimate be used. HAC refers to the general form for combining weighted matrices to form the variance estimate. There are three kernels built into glm. kernel is a user-written program or one of

```
nwest|gallant|\underline{anderson}
```

\# specifies the number of lags. If \# is not specified, $N-2$ is assumed. If you wish to specify vce(hac...), you must tsset your data before calling glm.
vce (unbiased) specifies that the unbiased sandwich estimate of variance be used.
vfactor(\#) specifies a scalar by which to multiply the resulting variance matrix. This option allows you to match output with other packages, which may apply degrees of freedom or other small-sample corrections to estimates of variance.
disp(\#) multiplies the variance of depvar by \# and divides the deviance by \#. The resulting distributions are members of the quasilikelihood family.
scale (x2|dev|\#) overrides the default scale parameter. This option is allowed only with Hessian (information matrix) variance estimates.

By default, scale(1) is assumed for the discrete distributions (binomial, Poisson, and negative binomial), and scale(x2) is assumed for the continuous distributions (Gaussian, gamma, and inverse Gaussian).
scale ( x 2 ) specifies that the scale parameter be set to the Pearson chi-squared (or generalized chisquared) statistic divided by the residual degrees of freedom, which is recommended by McCullagh and Nelder (1989) as a good general choice for continuous distributions.
scale (dev) sets the scale parameter to the deviance divided by the residual degrees of freedom. This option provides an alternative to scale (x2) for continuous distributions and overdispersed or underdispersed discrete distributions.
scale(\#) sets the scale parameter to \#. For example, using scale(1) in family (gamma) models results in exponential-errors regression. Additional use of link(log) rather than the default link (power -1) for family (gamma) essentially reproduces Stata's streg, dist (exp) nohr command (see [ST] streg) if all the observations are uncensored.

Reporting
level (\#); see [R] estimation options.
eform displays the exponentiated coefficients and corresponding standard errors and confidence intervals. For family(binomial) link(logit) (that is, logistic regression), exponentiation results are odds ratios; for family (nbinomial) link (log) (that is, negative binomial regression) and for family (poisson) link(log) (that is, Poisson regression), exponentiated coefficients are incidence-rate ratios.
nocnsreport; see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% fmt), pformat (\% fmt), sformat (\% fmt), and nolstretch; see [R] estimation options.

Maximization
ml requests that optimization be carried out using Stata's ml commands and is the default.
irls requests iterated, reweighted least-squares (IRLS) optimization of the deviance instead of NewtonRaphson optimization of the log likelihood. If the irls option is not specified, the optimization is carried out using Stata's ml commands, in which case all options of ml maximize are also available.
maximize_options: difficult, technique (algorithm_spec), iterate(\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#),
nrtolerance(\#), nonrtolerance, and from (init_specs); see [R] maximize. These options are seldom used.

Setting the optimization type to technique (bhhh) resets the default vcetype to vce (opg).
fisher (\#) specifies the number of Newton-Raphson steps that should use the Fisher scoring Hessian or EIM before switching to the observed information matrix (OIM). This option is useful only for Newton-Raphson optimization (and not when using irls).
search specifies that the command search for good starting values. This option is useful only for Newton-Raphson optimization (and not when using irls).

The following options are available with glm but are not shown in the dialog box:
noheader suppresses the header information from the output. The coefficient table is still displayed.
notable suppresses the table of coefficients from the output. The header information is still displayed. nodisplay suppresses the output. The iteration $\log$ is still displayed.
coeflegend; see $[R]$ estimation options.

## Remarks and examples

Remarks are presented under the following headings:

General use<br>Variance estimators<br>User-defined functions

## General use

glm fits generalized linear models of $y$ with covariates $\mathbf{x}$ :

$$
g\{E(y)\}=\mathbf{x} \boldsymbol{\beta}, \quad y \sim F
$$

$g()$ is called the link function, and $F$ is the distributional family. Substituting various definitions for $g()$ and $F$ results in a surprising array of models. For instance, if $y$ is distributed as Gaussian (normal) and $g()$ is the identity function, we have

$$
E(y)=\mathbf{x} \boldsymbol{\beta}, \quad y \sim \text { Normal }
$$

or linear regression. If $g()$ is the logit function and $y$ is distributed as Bernoulli, we have

$$
\operatorname{logit}\{E(y)\}=\mathbf{x} \boldsymbol{\beta}, \quad y \sim \text { Bernoulli }
$$

or logistic regression. If $g()$ is the natural $\log$ function and $y$ is distributed as Poisson, we have

$$
\ln \{E(y)\}=\mathbf{x} \boldsymbol{\beta}, \quad y \sim \text { Poisson }
$$

or Poisson regression, also known as the log-linear model. Other combinations are possible.
Although glm can be used to perform linear regression (and, in fact, does so by default), this regression should be viewed as an instructional feature; regress produces such estimates more quickly, and many postestimation commands are available to explore the adequacy of the fit; see $[R]$ regress and $[R]$ regress postestimation.

In any case, you specify the link function by using the link() option and specify the distributional family by using family(). The available link functions are

| Link function | glm option |
| :--- | :--- |
| identity | link(identity) |
| log | link(log) |
| logit | link(logit) |
| probit | link(probit) |
| complementary log-log | link(cloglog) |
| odds power | link(opower \#) |
| power | link(power \#) |
| negative binomial | link(nbinomial) |
| log-log | link(loglog) |
| log-complement | link(logc) |

Define $\mu=E(y)$ and $\eta=g(\mu)$, meaning that $g(\cdot)$ maps $E(y)$ to $\eta=\mathbf{x} \boldsymbol{\beta}+$ offset.

Link functions are defined as follows:
identity is defined as $\eta=g(\mu)=\mu$.
$\log$ is defined as $\eta=\ln (\mu)$.
logit is defined as $\eta=\ln \{\mu /(1-\mu)\}$, the natural $\log$ of the odds.
probit is defined as $\eta=\Phi^{-1}(\mu)$, where $\Phi^{-1}()$ is the inverse Gaussian cumulative.
cloglog is defined as $\eta=\ln \{-\ln (1-\mu)\}$.
opower is defined as $\eta=\left[\{\mu /(1-\mu)\}^{n}-1\right] / n$, the power of the odds. The function is generalized so that link (opower 0 ) is equivalent to link(logit), the natural log of the odds. power is defined as $\eta=\mu^{n}$. Specifying link(power 1) is equivalent to specifying link(identity). The power function is generalized so that $\mu^{0} \equiv \ln (\mu)$. Thus link(power 0 ) is equivalent to $\operatorname{link}(\mathrm{log})$. Negative powers are, of course, allowed.
nbinomial is defined as $\eta=\ln \{\mu /(\mu+k)\}$, where $k=1$ if family (nbinomial) is specified, $k=\#_{k}$ if family (nbinomial $\#_{k}$ ) is specified, and $k$ is estimated via maximum likelihood if family (nbinomial ml) is specified.
$\log l o g$ is defined as $\eta=-\ln \{-\ln (\mu)\}$.
loge is defined as $\eta=\ln (1-\mu)$.
The available distributional families are

| Family | glm option |
| :--- | :--- |
| Gaussian (normal) | family (gaussian) |
| inverse Gaussian | family (igaussian) |
| Bernoulli/binomial | family (binomial) |
| Poisson | family (poisson) |
| negative binomial | family (nbinomial) |
| gamma | family (gamma) |

family (normal) is a synonym for family (gaussian).
The binomial distribution can be specified as 1) family (binomial), 2) family (binomial $\#_{N}$ ), or 3) family (binomial varname ${ }_{N}$ ). In case $2, \#_{N}$ is the value of the binomial denominator $N$, the number of trials. Specifying family (binomial 1) is the same as specifying family (binomial). In case 3, varname $N_{N}$ is the variable containing the binomial denominator, allowing the number of trials to vary across observations.

The negative binomial distribution can be specified as 1) family(nbinomial), 2) family (nbinomial $\#_{k}$ ), or 3) family (nbinomial ml ). Omitting $\#_{k}$ is equivalent to specifying family (nbinomial 1). In case 3, the value of $\#_{k}$ is estimated via maximum likelihood. The value $\#_{k}$ enters the variance and deviance functions. Typical values range between 0.01 and 2 ; see the technical note below.

You do not have to specify both family() and link(); the default link() is the canonical link for the specified family() (except for nbinomial):

| Family | Default link |
| :--- | :--- |
| family (gaussian) | link(identity) |
| family (igaussian) | link(power -2) |
| family (binomial) | link(logit) |
| family (poisson) | link(log) |
| family (nbinomial) | link(log) |
| family (gamma) | link(power -1) |

If you specify both family() and link(), not all combinations make sense. You may choose from the following combinations:

|  | identity | $\log$ | logit | probit | cloglog | power | opower | nbinomial | $\log \log$ | $\operatorname{logc}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Gaussian | x | x |  |  |  | x |  |  |  |  |
| inverse Gaussian | x | x |  |  |  | x |  |  |  |  |
| binomial | x | x | x | x | x | x | x |  | x | x |
| Poisson | x | x |  |  |  | x |  |  |  |  |
| negative binomial | x | x |  |  |  | x |  | x |  |  |
| gamma | x | x |  |  | x |  |  |  |  |  |

## Technical note

Some family() and link() combinations result in models already fit by Stata. These are

| family() | link() | Options | Equivalent Stata command |
| :--- | :--- | :--- | :--- |
| gaussian | identity | nothing $\mid$ irls $\mid$ irls vce (oim) | regress |
| gaussian | identity | t(var) vce(hac nwest \#) <br> vfactor (\#v) | newey, t(var) lag(\#) (see note 1) |
| binomial | cloglog | nothing $\mid$ irls vce(oim) | cloglog (see note 2) |
| binomial | probit | nothing $\mid$ irls vce(oim) | probit (see note 2) |
| binomial | logit | nothing $\mid$ irls $\mid$ irls vce(oim) | logit or logistic (see note 3) |
| poisson | log | nothing $\mid$ irls $\mid$ irls vce(oim) | poisson (see note 3) |
| nbinomial | log | nothing $\mid$ irls vce(oim) | nbreg (see note 4) |
| gamma | log | scale(1) | streg, dist (exp) nohr (see note 5) |

## Notes:

1. The variance factor $\#_{v}$ should be set to $n /(n-k)$, where $n$ is the number of observations and $k$ the number of regressors. If the number of regressors is not specified, the estimated standard errors will, as a result, differ by this factor.
2. Because the link is not the canonical link for the binomial family, you must specify the vee (oim) option if using irls to get equivalent standard errors. If irls is used without vce(oim), the regression coefficients will be the same but the standard errors will be only asymptotically equivalent. If no options are specified (nothing), glm will optimize using Newton-Raphson, making it equivalent to the other Stata command.

See $[R]$ cloglog and $[R]$ probit for more details about these commands.
3. Because the canonical link is being used, the standard errors will be equivalent whether the EIM or the OIM estimator of variance is used.
4. Family negative binomial, log-link models-also known as negative binomial regression models - are used for data with an overdispersed Poisson distribution. Although glm can be used to fit such models, using Stata's maximum likelihood nbreg command is probably better. In the GLM approach, you specify family (nbinomial $\#_{k}$ ) and then search for a $\#_{k}$ that results in the deviance-based dispersion being 1 . You can also specify family (nbinomial ml) to estimate $\#_{k}$ via maximum likelihood, which will report the same value returned from nbreg. However, nbreg also reports a confidence interval for it; see [R] nbreg and Rogers (1993). Of course, glm allows links other than log, and for those links, including the canonical nbinomial link, you will need to use glm.
5. glm can be used to estimate parameters from exponential regressions, but this method requires specifying scale(1). However, censoring is not available. Censored exponential regression may be modeled using glm with family (poisson). The $\log$ of the original response is entered into a Poisson model as an offset, whereas the new response is the censor variable. The result of such modeling is identical to the log relative hazard parameterization of streg, dist(exp) nohr. See [ST] streg for details about the streg command.
In general, where there is overlap between a capability of glm and that of some other Stata command, we recommend using the other Stata command. Our recommendation is not because of some inferiority of the GLM approach. Rather, those other commands, by being specialized, provide options and ancillary commands that are missing in the broader glm framework. Nevertheless, glm does produce the same answers where it should.
Special note. When equivalence is expected, for some datasets, you may still see very slight differences in the results, most often only in the later digits of the standard errors. When you compare glm output to an equivalent Stata command, these tiny discrepancies arise for many reasons:
a. glm uses a general methodology for starting values, whereas the equivalent Stata command may be more specialized in its treatment of starting values.
b. When using a canonical link, glm, irls should be equivalent to the maximum likelihood method of the equivalent Stata command, yet the convergence criterion is different (one is for deviance, the other for log likelihood). These discrepancies are easily resolved by adjusting one convergence criterion to correspond to the other.
c. When both glm and the equivalent Stata command use Newton-Raphson, small differences may still occur if the Stata command has a different default convergence criterion from that of glm. Adjusting the convergence criterion will resolve the difference. See $[R] \mathbf{m l}$ and $[R]$ maximize for more details.

## $>$ Example 1

In example 1 of $[\mathrm{R}]$ logistic, we fit a model based on data from a study of risk factors associated with low birthweight (Hosmer, Lemeshow, and Sturdivant 2013, 24). We can replicate the estimation by using glm:
. use http://www.stata-press.com/data/r13/lbw
(Hosmer \& Lemeshow data)
. glm low age lwt i.race smoke ptl ht ui, family (binomial) link(logit)
Iteration 0: log likelihood = -101.0213
Iteration 1: log likelihood $=-100.72519$
Iteration 2: log likelihood $=-100.724$
Iteration 3: log likelihood $=-100.724$
Generalized linear models No. of obs $=\quad 189$
Optimization : ML Residual df $=\quad 180$
Deviance $=201.4479911 \quad(1 / \mathrm{df})$ Deviance $=1.119156$

Pearson $=182.0233425 \quad(1 / \mathrm{df})$ Pearson $=1.011241$
Variance function: $V(u)=u *(1-u)$
[Bernoulli]
Link function : $g(u)=\ln (u /(1-u))$
[Logit]
AIC = 1.1611
Log likelihood $=-100.7239956$ BIC $=-742.0665$

|  | OIM |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| low | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| age | -.0271003 | .0364504 | -0.74 | 0.457 | -.0985418 | .0443412 |
| lwt | -.0151508 | .0069259 | -2.19 | 0.029 | -.0287253 | -.0015763 |
| race |  |  |  |  |  |  |
| black | 1.262647 | .5264101 | 2.40 | 0.016 | .2309024 | 2.294392 |
| other | .8620792 | .4391532 | 1.96 | 0.050 | .0013548 | 1.722804 |
| smoke | .9233448 | .4008266 | 2.30 | 0.021 | .137739 | 1.708951 |
| ptl | .5418366 | .346249 | 1.56 | 0.118 | -.136799 | 1.220472 |
| ht | 1.832518 | .6916292 | 2.65 | 0.008 | .4769494 | 3.188086 |
| ui | .7585135 | .4593768 | 1.65 | 0.099 | -.1418484 | 1.658875 |
| _cons | .4612239 | 1.20459 | 0.38 | 0.702 | -1.899729 | 2.822176 |

glm, by default, presents coefficient estimates, whereas logistic presents the exponentiated coefficients - the odds ratios. glm's eform option reports exponentiated coefficients, and glm, like Stata's other estimation commands, replays results.

| glm, eform |  |
| :---: | :---: |
| Generalized linear models |  |
| Optimization | ML |
| Deviance | 201.4479911 |
| Pearson | $=182.0233425$ |
| Variance function | $\mathrm{V}(\mathrm{u})=\mathrm{u} *(1-u)$ |
| Link function | $g(u)=\ln (u /(1-u))$ |
| Log likelihood | $=-100.7239956$ |


| No. of obs | $=$ | 189 |
| :--- | ---: | ---: |
| Residual df | $=$ | 180 |
| Scale parameter | $=$ | 1 |
| (1/df) Deviance | $=$ | 1.119156 |
| (1/df) Pearson | $=$ | 1.011241 |
| [Bernoulli] |  |  |
| [Logit] |  |  |
| AIC | $=$ | 1.1611 |
| BIC | $=$ | -742.0665 |


|  | OIM |  |  |  |  |  |
| ---: | ---: | :---: | :---: | :---: | :---: | ---: | ---: |
| low | Odds Ratio | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| age | .9732636 | .0354759 | -0.74 | 0.457 | .9061578 | 1.045339 |
| lwt | .9849634 | .0068217 | -2.19 | 0.029 | .9716834 | .9984249 |
| race |  |  |  |  |  |  |
| black | 3.534767 | 1.860737 | 2.40 | 0.016 | 1.259736 | 9.918406 |
| other | 2.368079 | 1.039949 | 1.96 | 0.050 | 1.001356 | 5.600207 |
|  |  |  |  |  |  |  |
| smoke | 2.517698 | 1.00916 | 2.30 | 0.021 | 1.147676 | 5.523162 |
| ptl | 1.719161 | .5952579 | 1.56 | 0.118 | .8721455 | 3.388787 |
| ht | 6.249602 | 4.322408 | 2.65 | 0.008 | 1.611152 | 24.24199 |
| ui | 2.1351 | .9808153 | 1.65 | 0.099 | .8677528 | 5.2534 |
| _cons | 1.586014 | 1.910496 | 0.38 | 0.702 | .1496092 | 16.8134 |

These results are the same as those reported in example 1 of $[R]$ logistic.
Included in the output header are values for the Akaike (1973) information criterion (AIC) and the Bayesian information criterion (BIC) (Raftery 1995). Both are measures of model fit adjusted for the number of parameters that can be compared across models. In both cases, a smaller value generally indicates a better model fit. AIC is based on the $\log$ likelihood and thus is available only when Newton-Raphson optimization is used. BIC is based on the deviance and thus is always available. $\triangleleft$

Technical note
The values for AIC and BIC reported in the output after glm are different from those reported by estat ic:
. estat ic
Akaike's information criterion and Bayesian information criterion

| Model | Obs | ll(null) | ll(model) | df | AIC | BIC |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| . | 189 |  | . | -100.724 | 9 | 219.448 |

Note: $N=O b s$ used in calculating BIC; see [R] BIC note
There are various definitions of these information criteria (IC) in the literature; glm and estat ic use different definitions. glm bases its computation of the BIC on deviance, whereas estat ic uses the likelihood. Both glm and estat ic use the likelihood to compute the AIC; however, the AIC from estat ic is equal to $N$, the number of observations, times the AIC from glm. Refer to Methods and formulas in this entry and $[\mathrm{R}]$ estat ic for the references and formulas used by glm and estat ic, respectively, to compute AIC and BIC. Inferences based on comparison of IC values reported by glm
for different GLM models will be equivalent to those based on comparison of IC values reported by estat ic after glm.

## > Example 2

We use data from an early insecticide experiment, given in Pregibon (1980). The variables are ldose, the log dose of insecticide; $n$, the number of flour beetles subjected to each dose; and $r$, the number killed.

```
. use http://www.stata-press.com/data/r13/ldose
. list, sep(4)
```

|  | ldose | n | r |
| :--- | ---: | ---: | ---: |
| 1. | 1.6907 | 59 | 6 |
| 2. | 1.7242 | 60 | 13 |
| 3. | 1.7552 | 62 | 18 |
| 4. | 1.7842 | 56 | 28 |
| 5. | 1.8113 | 63 | 52 |
| 6. | 1.8369 | 59 | 53 |
| 7. | 1.861 | 62 | 61 |
| 8. | 1.8839 | 60 | 60 |
|  |  |  |  |

The aim of the analysis is to estimate a dose-response relationship between $p$, the proportion killed, and $X$, the $\log$ dose.

As a first attempt, we will formulate the model as a linear logistic regression of $p$ on ldose; that is, we will take the logit of $p$ and represent the dose-response curve as a straight line in $X$ :

$$
\ln \{p /(1-p)\}=\beta_{0}+\beta_{1} X
$$

Because the data are grouped, we cannot use Stata's logistic command to fit the model. Stata does, however, already have a command for performing logistic regression on data organized in this way, so we could type

Instead, we will fit the model by using glm:

| . glm r ldose, family(binomial n) link(logit) |  |  |
| :---: | :---: | :---: |
| Iteration 0: log likelihood $=-18.824848$ |  |  |
| Iteration 1: log likelihood $=-18.715271$ |  |  |
| Iteration 2: log likelihood $=-18.715123$ |  |  |
| Iteration 3: log likelihood $=-18.715123$ |  |  |
| Generalized linear models | No. of obs | 8 |
| Optimization : ML | Residual df | 6 |
|  | Scale parameter | $=1$ |
| Deviance $=11.23220702$ | (1/df) Deviance | $=1.872035$ |
| Pearson $=10.0267936$ | (1/df) Pearson | $=1.671132$ |
| Variance function: $V(u)=u *(1-u / n)$ | [Binomial] |  |
| Link function : $\mathrm{g}(\mathrm{u})=\ln (\mathrm{u} /(\mathrm{n}-\mathrm{u})$ ) | [Logit] |  |
|  | AIC | $=5.178781$ |
| Log likelihood = -18.71512262 | BIC | $=-1.244442$ |


|  | OIM |  |  |  |  |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: | ---: |
| $r$ | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ldose | 34.27034 | 2.912141 | 11.77 | 0.000 | 28.56265 | 39.97803 |
| _cons | -60.71747 | 5.180713 | -11.72 | 0.000 | -70.87149 | -50.56346 |

The only difference between blogit and glm here is how they obtain the answer. blogit expands the data to contain 481 observations (the sum of $n$ ) so that it can run Stata's standard, individual-level logistic command. glm, on the other hand, uses the information on the binomial denominator directly. We specified family (binomial n), meaning that variable $n$ contains the denominator. Parameter estimates and standard errors from the two approaches do not differ.

An alternative model, which gives asymmetric sigmoid curves for $p$, involves the complementary log-log, or cloglog, function:

$$
\ln \{-\ln (1-p)\}=\beta_{0}+\beta_{1} X
$$

We fit this model by using glm:


|  | OIM |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $r$ | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ldose | 22.04118 | 1.793089 | 12.29 | 0.000 | 18.52679 | 25.55557 |
| _cons | -39.57232 | 3.229047 | -12.26 | 0.000 | -45.90114 | -33.24351 |

The complementary log-log model is preferred; the deviance for the logistic model, 11.23, is much higher than the deviance for the cloglog model, 3.45. This change also is evident by comparing log likelihoods, or equivalently, AIC values.

This example also shows the advantage of the glm command-we can vary assumptions easily. Note the minor difference in what we typed to obtain the logistic and cloglog models:

```
. glm r ldose, family(binomial n) link(logit)
. glm r ldose, family(binomial n) link(cloglog)
```

If we were performing this work for ourselves, we would have typed the commands in a more abbreviated form:

```
. glm r ldose, f(b n) l(l)
. glm r ldose, f(b n) l(cl)
```


## Technical note

Factor variables may be used with glm. Say that, in the example above, we had ldose, the $\log$ dose of insecticide; $n$, the number of flour beetles subjected to each dose; and $r$, the number killed-all as before-except that now we have results for three different kinds of beetles. Our hypothetical data include beetle, which contains the values 1 ("Destructive flour"), 2 ("Red flour"), and 3 ("Mealworm").

```
. use http://www.stata-press.com/data/r13/beetle
. list, sep(0)
```

|  | beetle | ldose | n | r |
| :--- | ---: | ---: | ---: | ---: |
|  | 1. | 1 | 1.6907 | 59 |
| 2. | 1 | 1.7242 | 60 | 13 |
| 3. | 1 | 1.7552 | 62 | 18 |
| 4. | 1 | 1.7842 | 56 | 28 |
| 5. | 1 | 1.8113 | 63 | 52 |

Let's assume that, at first, we wish merely to add a shift factor for the type of beetle. We could type

```
. glm r i.beetle ldose, family(bin n) link(cloglog)
Iteration 0: log likelihood = -79.012269
Iteration 1: log likelihood = -76.94951
Iteration 2: log likelihood = -76.945645
Iteration 3: log likelihood = -76.945645
Generalized linear models No. of obs = 24
Optimization : ML Residual df = 20
Scale parameter = 1
(1/df) Deviance = 3.688253
(1/df) Pearson = 3.594506
[Binomial]
[Complementary log-log]
AIC = 6.74547
Log likelihood = -76.94564525 BIC = 10.20398
```

|  | OIM |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| r | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| beetle |  |  |  |  |  |  |
| Red flour | -.0910396 | .1076132 | -0.85 | 0.398 | -.3019576 | .1198783 |
| Mealworm | -1.836058 | .1307125 | -14.05 | 0.000 | -2.09225 | -1.579867 |
|  |  |  |  |  |  |  |
| ldose | 19.41558 | .9954265 | 19.50 | 0.000 | 17.46458 | 21.36658 |
| _cons | -34.84602 | 1.79333 | -19.43 | 0.000 | -38.36089 | -31.33116 |

We find strong evidence that the insecticide works differently on the mealworm. We now check whether the curve is merely shifted or also differently sloped:

```
. glm r beetle##c.ldose, family(bin n) link(cloglog)
Iteration 0: log likelihood = -67.270188
Iteration 1: log likelihood = -65.149316
Iteration 2: log likelihood = -65.147978
Iteration 3: log likelihood = -65.147978
Generalized linear models No. of obs = 24
Optimization : ML Residual df = 18
Deviance = 50.16972096 (1/df) Deviance = 2.787207
Pearson = 49.28422567 (1/df) Pearson = 2.738013
Variance function: V (u) = u*(1-u/n)
```



```
Log likelihood = -65.14797776 隹 AIC BIC = 5.928998
```

| r | OIM |  |  |  | [95\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| beetle |  |  |  |  |  |  |
| Red flour | -. 79933 | 4.470882 | -0.18 | 0.858 | -9.562098 | 7.963438 |
| Mealworm | 17.78741 | 4.586429 | 3.88 | 0.000 | 8.798172 | 26.77664 |
| ldose | 22.04118 | 1.793089 | 12.29 | 0.000 | 18.52679 | 25.55557 |
| $\begin{aligned} & \text { beetle\# } \\ & \text { c.ldose } \end{aligned}$ |  |  |  |  |  |  |
| Red flour | . 3838708 | 2.478477 | 0.15 | 0.877 | -4.473855 | 5.241596 |
| Mealworm | -10.726 | 2.526412 | -4.25 | 0.000 | -15.67768 | -5.774321 |
| _cons | -39.57232 | 3.229047 | -12.26 | 0.000 | -45.90114 | -33.24351 |

We find that the (complementary log-log) dose-response curve for the mealworm has roughly half the slope of that for the destructive flour beetle.

See [U] 25 Working with categorical data and factor variables; what is said there concerning linear regression is applicable to any GLM model.

## Variance estimators

glm offers many variance options and gives different types of standard errors when used in various combinations. We highlight some of them here, but for a full explanation, see Hardin and Hilbe (2012).

## > Example 3

Continuing with our flour beetle data, we rerun the most recently displayed model, this time requesting estimation via IRLS.

```
. use http://www.stata-press.com/data/r13/beetle
. glm r beetle##c.ldose, f(bin n) l(cloglog) ltol(1e-13) irls
Iteration 1: deviance = 54.41414
Iteration 2: deviance = 50.19424
Iteration 3: deviance = 50.16973
    (output omitted)
```

| Generalized linear | models | No. of obs | $=\quad 24$ |
| :---: | :---: | :---: | :---: |
| Optimization | MQL Fisher scoring | Residual df | 18 |
|  | (IRLS EIM) | Scale parameter | $=1$ |
| Deviance = | 50.16972096 | (1/df) Deviance | $=2.787207$ |
| Pearson = | 49.28422567 | (1/df) Pearson | $=2.738013$ |
| Variance function: | $\mathrm{V}(\mathrm{u})=\mathrm{u} *(1-u / n)$ | [Binomial] |  |
| Link function : | $\mathrm{g}(\mathrm{u})=\ln (-\ln (1-\mathrm{u} / \mathrm{n}))$ | [Complementary | log-log] |
|  |  | BIC | $=-7.035248$ |


| r | Coef. | $\begin{gathered} \text { EIM } \\ \text { Std. Err. } \end{gathered}$ | z | $P>\|z\|$ | [95\% Conf. | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| beetle |  |  |  |  |  |  |
| Red flour | -. 79933 | 4.586649 | -0.17 | 0.862 | -9.788997 | 8.190337 |
| Mealworm | 17.78741 | 4.624834 | 3.85 | 0.000 | 8.7229 | 26.85192 |
| ldose | 22.04118 | 1.799356 | 12.25 | 0.000 | 18.5145 | 25.56785 |
| $\begin{aligned} & \text { beetle\# } \\ & \text { c.ldose } \end{aligned}$ |  |  |  |  |  |  |
| Red flour | . 3838708 | 2.544068 | 0.15 | 0.880 | -4.602411 | 5.370152 |
| Mealworm | -10.726 | 2.548176 | -4.21 | 0.000 | -15.72033 | -5.731665 |
| _cons | -39.57232 | 3.240274 | -12.21 | 0.000 | -45.92314 | -33.2215 |

Note our use of the ltol () option, which, although unrelated to our discussion on variance estimation, was used so that the regression coefficients would match those of the previous Newton-Raphson (NR) fit.

Because IRLS uses the EIM for optimization, the variance estimate is also based on EIM. If we want optimization via IRLS but the variance estimate based on OIM, we specify glm, irls vce (oim):
. glm r beetle\#\#c.ldose, f(b n) l(cl) ltol(1e-15) irls vce(oim) noheader nolog

| r | Coef. | $\begin{gathered} \text { OIM } \\ \text { Std. Err. } \end{gathered}$ | z | $P>\|z\|$ | [95\% Conf. | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| beetle |  |  |  |  |  |  |
| Red flour | -. 79933 | 4.470882 | -0.18 | 0.858 | -9.562098 | 7.963438 |
| Mealworm | 17.78741 | 4.586429 | 3.88 | 0.000 | 8.798172 | 26.77664 |
| ldose | 22.04118 | 1.793089 | 12.29 | 0.000 | 18.52679 | 25.55557 |
| $\begin{aligned} & \text { beetle\# } \\ & \text { c.ldose } \end{aligned}$ |  |  |  |  |  |  |
| Red flour | . 3838708 | 2.478477 | 0.15 | 0.877 | -4.473855 | 5.241596 |
| Mealworm | -10.726 | 2.526412 | -4.25 | 0.000 | -15.67768 | -5.774321 |
| _cons | -39.57232 | 3.229047 | -12.26 | 0.000 | -45.90114 | -33.24351 |

This approach is identical to NR except for the convergence path. Because the cloglog link is not the canonical link for the binomial family, EIM and OIM produce different results. Both estimators, however, are asymptotically equivalent.

Going back to NR, we can also specify vce(robust) to get the Huber/White/sandwich estimator of variance:
. glm r beetle\#\#c.ldose, $f(b \mathrm{n}) \mathrm{l}(\mathrm{cl})$ vce(robust) noheader nolog

| $r$ | Robust |  |  | $P>\|z\|$ | [95\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| beetle |  |  |  |  |  |  |
| Red flour | -. 79933 | 5.733049 | -0.14 | 0.889 | -12.0359 | 10.43724 |
| Mealworm | 17.78741 | 5.158477 | 3.45 | 0.001 | 7.676977 | 27.89784 |
| ldose | 22.04118 | . 8998551 | 24.49 | 0.000 | 20.27749 | 23.80486 |
| $\begin{aligned} & \text { beetle\# } \\ & \text { c.ldose } \end{aligned}$ |  |  |  |  |  |  |
| Red flour | . 3838708 | 3.174427 | 0.12 | 0.904 | -5.837892 | 6.605633 |
| Mealworm | -10.726 | 2.800606 | -3.83 | 0.000 | -16.21508 | -5.236912 |
| _cons | -39.57232 | 1.621306 | -24.41 | 0.000 | -42.75003 | -36.39462 |

The sandwich estimator gets its name from the form of the calculation-it is the multiplication of three matrices, with the outer two matrices (the "bread") set to the OIM variance matrix. When irls is used along with vce (robust), the EIM variance matrix is instead used as the bread. Using a result from McCullagh and Nelder (1989), Newson (1999) points out that the EIM and OIM variance matrices are equivalent under the canonical link. Thus if irls is specified with the canonical link, the resulting variance is labeled "Robust". When the noncanonical link for the family is used, which is the case in the example below, the EIM and OIM variance matrices differ, so the resulting variance is labeled "Semirobust".

| . glm r beetle\#\#c.ldose, f(b n) l(cl) irls ltol(1e-15) vce(robust) noheader |
| :--- |
| > nolog |
| r |

The outer product of the gradient ( OPG ) estimate of variance is one that avoids the calculation of second derivatives. It is equivalent to the "middle" part of the sandwich estimate of variance and can be specified by using glm, vce (opg), regardless of whether NR or IRLS optimization is used.
. glm r beetle\#\#c.ldose, f(b n) l(cl) vce(opg) noheader nolog

| $r$ | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |  |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| beetle <br> Red flour <br> Mealworm | -.79933 | 6.664045 | -0.12 | 0.905 | -13.86062 | 12.26196 |  |
| ldose | 22.04118 | 3.572983 | 6.17 | 0.000 | 15.03826 | 29.0441 |  |
| beetle\# |  |  |  |  |  |  |  |
| c.ldose <br> Red flour <br> Mealworm | -3838708 | 3.700192 | 0.10 | 0.917 | -6.868372 | 7.636114 |  |
| -10.726 | 3.796448 | -2.83 | 0.005 | -18.1669 | -3.285097 |  |  |
| _cons | -39.57232 | 6.433101 | -6.15 | 0.000 | -52.18097 | -26.96368 |  |

The OPG estimate of variance is a component of the BHHH (Berndt et al. 1974) optimization technique. This method of optimization is also available with glm with the technique() option; however, the technique() option is not allowed with the irls option.

## Example 4

The Newey-West (1987) estimator of variance is a sandwich estimator with the "middle" of the sandwich modified to take into account possible autocorrelation between the observations. These estimators are a generalization of those given by the Stata command newey for linear regression. See [TS] newey for more details.

For example, consider the dataset given in [TS] newey, which has time-series measurements on usr and idle. We want to perform a linear regression with Newey-West standard errors.
. use http://www.stata-press.com/data/r13/idle2
. list usr idle time

| usr idle time <br> 0 100 1 <br> 0 100 2 <br> 0 97 3 <br> 1 98 4 <br> 2 94 5 |
| :--- |
| (output omitted )   <br> 1 98 29 <br> 1 98 30 |

Examining Methods and formulas of [TS] newey, we see that the variance estimate is multiplied by a correction factor of $n /(n-k)$, where $k$ is the number of regressors. glm, vce (hac ...) does not make this correction, so to get the same standard errors, we must use the vfactor() option within glm to make the correction manually.

```
. display 30/28
1.0714286
. tsset time
        time variable: time, 1 to 30
        delta: 1 unit
. glm usr idle, vce(hac nwest 3) vfactor(1.0714286)
Iteration 0: log likelihood = -71.743396
Generalized linear models No. of obs = 30
Optimization : ML Residual df = 28
Deviance = 209.8123165 (1/df) Deviance = 7.493297
Pearson = 209.8123165 (1/df) Pearson = 7.493297
Variance function: V(u) = 1
    [Gaussian]
Link function : g(u) = u [Identity]
HAC kernel (lags): Newey-West (3)
Log likelihood = -71.74339627 BIC = 114.5788
\begin{tabular}{r|rcccrrr}
\hline \multirow{3}{*}{ usr } & Coef. & Std. Err. & \(\mathbf{z}\) & \(\mathrm{P}>|\mathrm{z}|\) & [95\% Conf. Interval] \\
\hline idle & -.2281501 & .0690928 & -3.30 & 0.001 & -.3635694 & -.0927307 \\
_cons & 23.13483 & 6.327033 & 3.66 & 0.000 & 10.73407 & 35.53558 \\
\hline
\end{tabular}
```

The glm command above reproduces the results given in [TS] newey. We may now generalize this output to models other than simple linear regression and to different kernel weights.

```
glm usr idle, fam(gamma) link(log) vce(hac gallant 3)
Iteration 0: log likelihood = -61.76593
Iteration 1: log likelihood = -60.963233
Iteration 2: log likelihood = -60.95097
Iteration 3: log likelihood = -60.950965
Generalized linear models No. of obs = 30
Optimization : ML Residual df = 28
Scale parameter = .431296
(1/df) Deviance = . 3538752
(1/df) Pearson = . 431296
[Gamma]
[Log]
AIC = 4.196731
Log likelihood = -60.95096484 BIC = -85.32502
```

|  | HAC |  |  |  |  |  |
| ---: | ---: | :---: | :---: | :---: | :---: | ---: |
| usr | Coef. | Std. Err. | $z$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| idle | -.0796609 | .0184647 | -4.31 | 0.000 | -.115851 | -.0434708 |
| _cons | 7.771011 | 1.510198 | 5.15 | 0.000 | 4.811078 | 10.73094 |

glm also offers variance estimators based on the bootstrap (resampling your data with replacement) and the jackknife (refitting the model with each observation left out in succession). Also included is the one-step jackknife estimate, which, instead of performing full reestimation when each observation is omitted, calculates a one-step NR estimate, with the full data regression coefficients as starting values.


See Hardin and Hilbe (2012) for a full discussion of the variance options that go with glm and, in particular, of how the different variance estimators are modified when vce (cluster clustvar) is specified. Finally, not all variance options are supported with all types of weights. See help glm for a current table of the variance options that are supported with the different weights.

## User-defined functions

glm may be called with a user-written link function, variance (family) function, Newey-West kernel-weight function, or any combination of the three.

## Syntax of link functions

```
program progname
    version 13
    args todo eta mu return
    if 'todo' == -1 {
            /* Set global macros for output */
            global SGLM_lt "title for link function"
            global SGLM_lf "subtitle showing link definition"
            exit
    }
    if 'todo' == 0 {
            /* set }\eta=g(\mu) *
            /* Intermediate calculations go here */
            generate double 'eta' = ...
            exit
    }
    if 'todo' == 1 {
            /* set }\mu=\mp@subsup{g}{}{-1}(\eta)*
            /* Intermediate calculations go here */
            generate double 'mu' = ...
            exit
    }
    if 'todo' == 2 {
            /* set return = \partial\mu/\partial\eta */
            /* Intermediate calculations go here */
            generate double 'return' = ...
            exit
    }
    if 'todo' == 3 {
            /* set return = \partial}\mp@subsup{}{2}{\mu}/\partial\mp@subsup{\eta}{}{2}*
            /* Intermediate calculations go here */
            generate double 'return' = ...
            exit
    }
    display as error "Unknown call to glm link function"
    exit 198
end
```


## Syntax of variance functions

```
program progname
    version 13
    args todo eta mu return
    if 'todo' == -1 {
        /* Set global macros for output */
        /* Also check that depvar is in proper range */
        /* Note: For this call, eta contains indicator for whether each obs. is in est. sample */
        global SGLM_vt "title for variance function"
        global SGLM_vf "subtitle showing function definition"
        global SGLM_mu "program to call to enforce boundary conditions on }\mu\mathrm{ "
        exit
    }
    if 'todo' == 0 {
        /* set }\eta\mathrm{ to initial value. */
        /* Intermediate calculations go here */
        generate double 'eta' = ...
        exit
    }
    if 'todo' == 1 {
            /* set return = V( }\mu)*
            /* Intermediate calculations go here */
            generate double 'return' = ...
            exit
}
if 'todo' == 2 {
            /* set return = \partialV ( }\mu)/\partial\mu *
            /* Intermediate calculations go here */
            generate double 'return' = ...
            exit
}
if 'todo' == 3 {
            /* set return = squared deviance (per observation) */
            /* Intermediate calculations go here */
            generate double 'return' = ...
            exit
}
if 'todo' == 4 {
            /* set return = Anscombe residual */
            /* Intermediate calculations go here */
            generate double 'return' = ...
            exit
}
if 'todo' == 5 {
            /* set return = log likelihood */
            /* Intermediate calculations go here */
            generate double 'return' = ...
            exit
}
if 'todo' == 6 {
            /* set return = adjustment for deviance residuals */
            /* Intermediate calculations go here */
            generate double 'return' = ...
    exit
}
display as error "Unknown call to glm variance function"
exit 198
end
```


## Syntax of Newey-West kernel-weight functions

```
program progname, rclass
    version 13
    args G j
    /* G is the maximum lag */
    /* j is the current lag */
    /* Intermediate calculations go here */
    return scalar wt = computed weight
    return local setype "Newey-West"
    return local sewtype "name of kernel"
end
```


## Global macros available for user-written programs

| Global macro | Description |
| :--- | :--- |
| SGLM_V | program name of variance (family) evaluator |
| SGLM_L | program name of link evaluator |
| dependent variable name |  |
| SGLM_y | binomial denominator |
| SGLM_m | negative binomial $k$ |
| SGLM_a | power if power () or opower() is used, or |
| SGLM_p | an argument from a user-specified link function |
|  | indicator; set to one if scale is equal to one |
| SGLM_s1 | value of scale parameter |

## > Example 5

Suppose that we wish to perform Poisson regression with a log-link function. Although this regression is already possible with standard glm, we will write our own version for illustrative purposes.

Because we want a log link, $\eta=g(\mu)=\ln (\mu)$, and for a Poisson family the variance function is $V(\mu)=\mu$.

The Poisson density is given by

$$
f\left(y_{i}\right)=\frac{e^{-\exp \left(\mu_{i}\right)} e^{\mu_{i} y_{i}}}{y_{i}!}
$$

resulting in a log likelihood of

$$
L=\sum_{i=1}^{n}\left\{-e^{\mu_{i}}+\mu_{i} y_{i}-\ln \left(y_{i}!\right)\right\}
$$

The squared deviance of the $i$ th observation for the Poisson family is given by

$$
d_{i}^{2}= \begin{cases}2 \widehat{\mu}_{i} & \text { if } y_{i}=0 \\ 2\left\{y_{i} \ln \left(y_{i} / \widehat{\mu}_{i}\right)-\left(y_{i}-\widehat{\mu}_{i}\right)\right\} & \text { otherwise }\end{cases}
$$

We now have enough information to write our own Poisson-log glm module. We create the file mylog.ado, which contains

```
program mylog
    version 13
    args todo eta mu return
    if 'todo' == -1 \{
            global SGLM_lt "My Log" // Titles for output
            global SGLM_lf "ln(u)"
            exit
    \}
    if 'todo' == 0 \{
            gen double 'eta' \(=\ln (' m u ') \quad / / \quad \eta=\ln (\mu)\)
            exit
\}
    if 'todo' == 1 \{
            gen double 'mu' \(=\exp (' e t a ') \quad / / \mu=\exp (\eta)\)
            exit
    \}
    if 'todo' == 2 \{
            gen double 'return' \(=\) 'mu' \(\quad / / \partial \mu / \partial \eta=\exp (\eta)=\mu\)
            exit
    \}
    if 'todo' == 3 \{
            gen double 'return' = 'mu' \(\quad / / \partial^{2} \mu / \partial \eta^{2}=\exp (\eta)=\mu\)
            exit
        \}
    di as error "Unknown call to glm link function"
    exit 198
end
```

and we create the file mypois.ado, which contains

```
program mypois
    version 13
    args todo eta mu return
    if 'todo' == -1 {
        local y "$SGLM_y"
        local touse "'eta'" // 'eta' marks estimation sample here
            capture assert 'y'>=0 if 'touse' // check range of y
            if _rc {
                di as error '"dependent variable 'y' has negative values"'
                exit 499
            }
            global SGLM_vt "My Poisson" // Titles for output
            global SGLM_vf "u"
            global SGLM_mu "glim_mu 0 ." // see note 1
            exit
    }
    if 'todo' == 0 { // Initialization of \eta; see note 2
            gen double 'eta' = ln('mu')
            exit
    }
```

```
if 'todo' == 1 {
    gen double 'return' = 'mu' // V( }\mu)=
    exit
}
if 'todo' == 2 { // \partial V (\mu)/\partial\mu
    gen byte 'return' = 1
    exit
}
if 'todo' == 3 { // squared deviance, defined above
    local y "$SGLM_y"
    if "'y'" == "" {
            local y "'e(depvar)'"
    }
    gen double 'return' = cond('y'==0, 2*'mu', /*
    */ 2*('y'*ln('y'/'mu')-('y'-'mu')))
    exit
}
if 'todo' == 4 { // Anscombe residual; see note 3
    local y "$SGLM_y"
    if "'y'" == "" {
                local y "'e(depvar)'"
    }
    gen double 'return' = 1.5*('y'^(2/3)-'mu'^(2/3)) / 'mu'^(1/6)
    exit
}
if 'todo' == 5 { // log likelihood; see note 4
    local y "$SGLM_y"
    if "'y'" == "" {
            local y "'e(depvar)'"
    }
    gen double 'return' = -'mu'+'y'*ln('mu')-lngamma('y'+1)
    exit
}
if 'todo' == 6 { // adjustment to residual; see note 5
    gen double 'return' = 1/(6*sqrt('mu'))
    exit
}
di as error "Unknown call to glm variance function"
error }19
```

end

Notes:

1. glim_mu is a Stata program that will, at each iteration, bring $\widehat{\mu}$ back into its plausible range, should it stray out of it. Here glim_mu is called with the arguments zero and missing, meaning that zero is the lower bound of $\widehat{\mu}$ and there exists no upper bound-such is the case for Poisson models.
2. Here the initial value of $\eta$ is easy because we intend to fit this model with our user-defined log link. In general, however, the initialization may need to vary according to the link to obtain convergence. If so, the global macro SGLM_L is used to determine which link is being utilized.
3. The Anscombe formula is given here because we know it. If we were not interested in Anscombe residuals, we could merely set 'return' to missing. Also, the local macro y is set either to SGLM_y if it is in current estimation or to e (depvar) if this function is being accessed by predict.
4. If we were not interested in ML estimation, we could omit this code entirely and just leave an exit statement in its place. Similarly, if we were not interested in deviance or IRLS optimization, we could set 'return' in the deviance portion of the code ('todo'==3) to missing.
5. This code defines the term to be added to the predicted residuals if the adjusted option is specified. Again, if we were not interested, we could set 'return' to missing.
We can now test our Poisson-log module by running it on the airline data presented in [R] poisson.
. use http://www.stata-press.com/data/r13/airline
. list airline injuries n XYZowned

|  | airline | injuries | n | XYZowned |
| :--- | ---: | ---: | ---: | ---: |
|  | 1. | 1 | 11 | 0.0950 |
| 2. | 2 | 7 | 0.1920 | 1 |
| 3. | 3 | 7 | 0.0750 | 0 |
| 4. | 4 | 19 | 0.2078 | 0 |
| 5. | 5 | 9 | 0.1382 | 0 |
|  |  | 4 | 0.0540 | 0 |
| 6. | 6 | 3 | 0.1292 | 1 |
| 7. | 7 | 1 | 0.0503 | 0 |
| 8. | 8 | 3 | 0.0629 | 0 |
| 9. | 9 |  |  | 1 |

- generate $\operatorname{lnN}=\ln (n)$
. glm injuries XYZowned lnN, f(mypois) l(mylog) scale(1)
Iteration 0: log likelihood $=-22.557572$
Iteration 1: log likelihood $=-22.332861$
Iteration 2: log likelihood $=-22.332276$
Iteration 3: log likelihood $=-22.332276$

| Generalized linear models | No. of obs | 9 |
| :---: | :---: | :---: |
| Optimization : ML | Residual df | 6 |
|  | Scale parameter | $=1$ |
| Deviance $=12.70432823$ | (1/df) Deviance | $=2.117388$ |
| Pearson $=12.7695081$ | (1/df) Pearson | $=2.128251$ |
| Variance function: V(u) = u | [My Poisson] |  |
| Link function : $\mathrm{g}(\mathrm{u})=\ln (\mathrm{u})$ | [My Log] |  |
|  | AIC | $=5.629395$ |
| Log likelihood = -22.33227605 | BIC | $=-.4790192$ |


|  | OIM |  |  |  |  |  |
| ---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: |
| injuries | Coef. | Std. Err. | $z$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| XYZowned | .6840668 | .3895877 | 1.76 | 0.079 | -.0795111 | 1.447645 |
| lnN | 1.424169 | .3725155 | 3.82 | 0.000 | .6940517 | 2.154286 |
| _cons | 4.863891 | .7090501 | 6.86 | 0.000 | 3.474178 | 6.253603 |

(Standard errors scaled using dispersion equal to square root of 1.)
These are precisely the results given in $[R]$ poisson and are those that would have been given had we run glm, family(poisson) link(log). The only minor adjustment we needed to make was to specify the scale(1) option. If scale() is left unspecified, glm assumes scale(1) for discrete distributions and scale(x2) for continuous ones. By default, glm assumes that any user-defined family is continuous because it has no way of checking. Thus we needed to specify scale(1) because our model is discrete.

Because we were careful in defining the squared deviance, we could have fit this model with IRLS. Because $\log$ is the canonical link for the Poisson family, we would not only get the same regression coefficients but also the same standard errors.

## > Example 6

Suppose now that we wish to use our log link (mylog.ado) with glm's binomial family. This task requires some modification because our current function is not equipped to deal with the binomial denominator, which we are allowed to specify. This denominator is accessible to our link function through the global macro SGLM_m. We now make the modifications and store them in mylog2.ado.

```
program mylog2 // <-- changed
    version 13
    args todo eta mu return
    if 'todo' == -1 {
        global SGLM_lt "My Log, Version 2" // <-- changed
        if "$SGLM_m" == "1" { // <-- changed
            global SGLM_lf "ln(u)" // <-- changed
            }
        else global SGLM_lf "ln(u/$SGLM_m)"
        exit
    }
    if 'todo' == 0 {
        gen double 'eta' = ln('mu'/$SGLM_m) // <-- changed
        exit
    }
    if 'todo' == 1 {
        gen double 'mu' = $SGLM_m*exp('eta') // <-- changed
        exit
    }
    if 'todo' == 2 {
        gen double 'return' = 'mu'
        exit
    }
    if 'todo' == 3 {
        gen double 'return' = 'mu'
        exit
    }
    di as error "Unknown call to glm link function"
    exit 198
end
```

We can now run our new log link with glm's binomial family. Using the flour beetle data from earlier, we have

```
. use http://www.stata-press.com/data/r13/beetle, clear
. glm r ldose, f(bin n) l(mylog2) irls
Iteration 1: deviance = 2212.108
Iteration 2: deviance = 452.9352
Iteration 3: deviance = 429.95
Iteration 4: deviance = 429.2745
Iteration 5: deviance = 429.2192
Iteration 6: deviance = 429.2082
Iteration 7: deviance = 429.2061
Iteration 8: deviance = 429.2057
Iteration 9: deviance = 429.2056
Iteration 10: deviance = 429.2056
Iteration 11: deviance = 429.2056
Iteration 12: deviance = 429.2056
\begin{tabular}{|c|c|c|c|}
\hline Generalized linear & models & No. of obs & 24 \\
\hline Optimization & MQL Fisher scoring & Residual df & 22 \\
\hline & (IRLS EIM) & Scale parameter & 1 \\
\hline Deviance & 429.205599 & (1/df) Deviance & 19.50935 \\
\hline Pearson & 413.088142 & (1/df) Pearson & 18.77673 \\
\hline Variance function: & \(\mathrm{V}(\mathrm{u})=\mathrm{u} *(1-\mathrm{u} / \mathrm{n})\) & [Binomial] & \\
\hline Link function : & \(\mathrm{g}(\mathrm{u})=\ln (\mathrm{u} / \mathrm{n})\) & [My Log, Version & \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline r & \multicolumn{3}{|c|}{EIM} & \(\mathrm{P}>|\mathrm{z}|\) & \multicolumn{2}{|l|}{[95\% Conf. Interval]} \\
\hline ldose & 8.478908 & . 4702808 & 18.03 & 0.000 & 7.557175 & 9.400642 \\
\hline _cons & -16.11006 & . 8723167 & -18.47 & 0.000 & -17.81977 & -14.40035 \\
\hline
\end{tabular}
```

For a more detailed discussion on user-defined functions, and for an example of a user-defined Newey-West kernel weight, see Hardin and Hilbe (2012).

John Ashworth Nelder (1924-2010) was born in Somerset, England. He studied mathematics and statistics at Cambridge and worked as a statistician at the National Vegetable Research Station and then Rothamsted Experimental Station. In retirement, he was actively affiliated with Imperial College London. Nelder was especially well known for his contributions to the theory of linear models and to statistical computing. He was the principal architect of generalized and hierarchical generalized linear models and of the programs GenStat and GLIM.

Robert William Maclagan Wedderburn (1947-1975) was born in Edinburgh and studied mathematics and statistics at Cambridge. At Rothamsted Experimental Station, he developed the theory of generalized linear models with Nelder and originated the concept of quasilikelihood. He died of anaphylactic shock from an insect bite on a canal holiday.

## Stored results

$\mathrm{glm}, \mathrm{ml}$ stores the following in e() :
Scalars

```
e(N)
    e(k)
    e(k_eq)
    e(k_eq_model)
    e(k_dv)
    e(df_m)
    e(df)
    e(phi)
    e(aic)
    e(bic)
    e(11)
    e(N_clust)
    e(chi2)
    e(p)
    e(deviance)
    e(deviance_s)
    e(deviance_p)
    e(deviance_ps)
    e(dispers)
    e(dispers_s)
    e(dispers_p)
    e(dispers_ps)
    e(nbml)
    e(vf)
    e(power)
    e(rank)
    e(ic)
    e(rc)
    e(converged)
number of observations
number of parameters
number of equations in e(b)
number of equations in overall model test
number of dependent variables
model degrees of freedom
residual degrees of freedom
scale parameter
model AIC
model BIC
log likelihood, if NR
number of clusters
\chi
significance
deviance
scaled deviance
Pearson deviance
scaled Pearson deviance
dispersion
scaled dispersion
Pearson dispersion
scaled Pearson dispersion
1 \text { if negative binomial parameter estimated via ML, 0 otherwise}
factor set by vfactor(), 1 if not set
power set by power(), opower()
rank of e(V)
number of iterations
return code
1 if converged, 0 otherwise
```

Macros

```
e(cmd)
    e(cmdline)
    e(depvar)
    e(varfunc)
    e(varfunct)
    e(varfuncf)
```

    e(link)
    e(linkt)
    e(linkf)
    e(m)
    e(wtype)
    e(wexp)
    e(title)
    e(clustvar)
    e(offset)
    e(chi2type)
    e(cons)
    e(hac_kernel)
    e(hac_lag)
    e(vce)
    e(vcetype)
    e(opt)
    e(opt1)
    e(which)
    e(ml_method)
    e(user)
    e(technique)
    e(properties)
    e(predict)
    e(marginsok)
    e(marginsnotok)
    e(asbalanced)
    e(asobserved)
    Matrices
e(b)
e(Cns)
e(ilog)
e(gradient)
e(V)
e(V_modelbased)

Functions
e(sample)
glm
command as typed
name of dependent variable
program to calculate variance function
variance title
variance function
program to calculate link function
link title
link function
number of binomial trials
weight type
weight expression
title in estimation output
name of cluster variable
linear offset variable
Wald; type of model $\chi^{2}$ test
set if noconstant specified
HAC kernel
HAC lag
vcetype specified in vce()
title used to label Std. Err.
ml or irls
optimization title, line 1
$\max$ or min; whether optimizer is to perform maximization or minimization
type of ml method
name of likelihood-evaluator program
maximization technique
b V
program used to implement predict
predictions allowed by margins
predictions disallowed by margins
factor variables fvset as asbalanced
factor variables fvset as asobserved
coefficient vector
constraints matrix
iteration $\log$ (up to 20 iterations)
gradient vector
variance-covariance matrix of the estimators
model-based variance
marks estimation sample
glm, irls stores the following in e():
Scalars

```
e(N)
e(k)
e(k_eq_model)
    e(df_m)
    e(df)
    e(phi)
    e(disp)
    e(bic)
    e(N_clust)
    e(deviance)
    e(deviance_s)
    e(deviance_p)
    e(deviance_ps)
    e(dispers)
    e(dispers_s)
    e(dispers_p)
    e(dispers_ps)
    e(nbml)
    e(vf)
    e(power)
    e(rank)
    e(rc)
```

Macros
e(cmd)
e(cmdline)
e(depvar)
e(varfunc)
e(varfunct)
$e$ (varfuncf)
e(link)
e(linkt)
e(linkf)
$e(m)$
e(wtype)
e(wexp)
e(clustvar)
e(offset)
e(cons)
e(hac_kernel)
e(hac_lag)
e(vce)
e(vcetype)
e(opt)
e(opt1)
e(opt2)
e(properties)
e(predict)
e(marginsok)
e(marginsnotok)
e(asbalanced)
e(asobserved)
Matrices
e(b)
e(Cns)
e(V)
e(V_modelbased)

Functions
e(sample)
number of observations
number of parameters
number of equations in overall model test
model degrees of freedom
residual degrees of freedom
scale parameter
dispersion parameter
model BIC
number of clusters
deviance
scaled deviance
Pearson deviance
scaled Pearson deviance
dispersion
scaled dispersion
Pearson dispersion
scaled Pearson dispersion
1 if negative binomial parameter estimated via ML, 0 otherwise
factor set by vfactor(), 1 if not set
power set by power(), opower()
rank of e(V)
return code
glm
command as typed
name of dependent variable
program to calculate variance function
variance title
variance function
program to calculate link function
link title
link function
number of binomial trials
weight type
weight expression
name of cluster variable
linear offset variable
set if noconstant specified
HAC kernel
HAC lag
vcetype specified in vce()
title used to label Std. Err.
ml or irls
optimization title, line 1
optimization title, line 2
b V
program used to implement predict
predictions allowed by margins
predictions disallowed by margins
factor variables fvset as asbalanced
factor variables fvset as asobserved
coefficient vector
constraints matrix
variance-covariance matrix of the estimators
model-based variance
marks estimation sample

## Methods and formulas

The canonical reference on GLM is McCullagh and Nelder (1989). The term "generalized linear model" is from Nelder and Wedderburn (1972). Many people use the acronym GLIM for GLM models because of the classic GLM software tool GLIM, by Baker and Nelder (1985). See Dobson and Barnett (2008) for a concise introduction and overview. See Rabe-Hesketh and Everitt (2007) for more examples of GLM using Stata. Hoffmann (2004) focuses on applying generalized linear models, using real-world datasets, along with interpreting computer output, which for the most part is obtained using Stata.

This discussion highlights the details of parameter estimation and predicted statistics. For a more detailed treatment, and for information on variance estimation, see Hardin and Hilbe (2012). glm supports estimation with survey data. For details on VCEs with survey data, see [SVY] variance estimation.
glm obtains results by IRLS, as described in McCullagh and Nelder (1989), or by maximum likelihood using Newton-Raphson. The implementation here, however, allows user-specified weights, which we denote as $v_{j}$ for the $j$ th observation. Let $M$ be the number of "observations" ignoring weights. Define

$$
w_{j}= \begin{cases}1 & \text { if no weights are specified } \\ v_{j} & \text { if fweights or iweights are specified } \\ M v_{j} /\left(\sum_{k} v_{k}\right) & \text { if aweights or pweights are specified }\end{cases}
$$

The number of observations is then $N=\sum_{j} w_{j}$ if fweights are specified and $N=M$ otherwise. Each IRLS step is performed by regress using $w_{j}$ as the weights.
Let $d_{j}^{2}$ denote the squared deviance residual for the $j$ th observation:
For the Gaussian family, $d_{j}^{2}=\left(y_{j}-\widehat{\mu}_{j}\right)^{2}$.
For the Bernoulli family (binomial with denominator 1),

$$
d_{j}^{2}= \begin{cases}-2 \ln \left(1-\widehat{\mu}_{j}\right) & \text { if } y_{j}=0 \\ -2 \ln \left(\widehat{\mu}_{j}\right) & \text { otherwise }\end{cases}
$$

For the binomial family with denominator $m_{j}$,

$$
d_{j}^{2}= \begin{cases}2 y_{j} \ln \left(y_{j} / \widehat{\mu}_{j}\right)+2\left(m_{j}-y_{j}\right) \ln \left\{\left(m_{j}-y_{j}\right) /\left(m_{j}-\widehat{\mu}_{j}\right)\right\} & \text { if } 0<y_{j}<m_{j} \\ 2 m_{j} \ln \left\{m_{j} /\left(m_{j}-\widehat{\mu}_{j}\right)\right\} & \text { if } y_{j}=0 \\ 2 y_{j} \ln \left(y_{j} / \widehat{\mu}_{j}\right) & \text { if } y_{j}=m_{j}\end{cases}
$$

For the Poisson family,

$$
d_{j}^{2}= \begin{cases}2 \widehat{\mu}_{j} & \text { if } y_{j}=0 \\ 2\left\{y_{j} \ln \left(y_{j} / \widehat{\mu}_{j}\right)-\left(y_{j}-\widehat{\mu}_{j}\right)\right\} & \text { otherwise }\end{cases}
$$

For the gamma family, $d_{j}^{2}=-2\left\{\ln \left(y_{j} / \widehat{\mu}_{j}\right)-\left(y_{j}-\widehat{\mu}_{j}\right) / \widehat{\mu}_{j}\right\}$.
For the inverse Gaussian, $d_{j}^{2}=\left(y_{j}-\widehat{\mu}_{j}\right)^{2} /\left(\widehat{\mu}_{j}^{2} y_{j}\right)$.

For the negative binomial,

$$
d_{j}^{2}= \begin{cases}2 \ln \left(1+k \widehat{\mu}_{j}\right) / k & \text { if } y_{j}=0 \\ 2 y_{j} \ln \left(y_{j} / \widehat{\mu}_{j}\right)-2\left\{\left(1+k y_{j}\right) / k\right\} \ln \left\{\left(1+k y_{j}\right) /\left(1+k \widehat{\mu}_{j}\right)\right\} & \text { otherwise }\end{cases}
$$

Let $\phi=1$ if the scale parameter is set to one; otherwise, define $\phi=\widehat{\phi}_{0}(n-k) / n$, where $\widehat{\phi}_{0}$ is the estimated scale parameter and $k$ is the number of covariates in the model (including intercept).
Let $\ln L_{j}$ denote the $\log$ likelihood for the $j$ th observation:
For the Gaussian family,

$$
\ln L_{j}=-\frac{1}{2}\left[\left\{\frac{\left(y_{j}-\widehat{\mu}_{j}\right)^{2}}{\phi}\right\}+\ln (2 \pi \phi)\right]
$$

For the binomial family with denominator $m_{j}$ (Bernoulli if all $m_{j}=1$ ),

$$
\ln L_{j}=\phi \times \begin{cases}\ln \left\{\Gamma\left(m_{j}+1\right)\right\}-\ln \left\{\Gamma\left(y_{j}+1\right)\right\}+\ln \left\{\Gamma\left(m_{j}-y_{j}+1\right)\right\} & \text { if } 0<y_{j}<m_{j} \\ \quad+\left(m_{j}-y_{j}\right) \ln \left(1-\widehat{\mu}_{j} / m_{j}\right)+y_{j} \ln \left(\widehat{\mu}_{j} / m_{j}\right) & \\ m_{j} \ln \left(1-\widehat{\mu}_{j} / m_{j}\right) & \text { if } y_{j}=0 \\ m_{j} \ln \left(\widehat{\mu}_{j} / m_{j}\right) & \text { if } y_{j}=m_{j}\end{cases}
$$

For the Poisson family,

$$
\ln L_{j}=\phi\left[y_{j} \ln \left(\widehat{\mu}_{j}\right)-\widehat{\mu}_{j}-\ln \left\{\Gamma\left(y_{j}+1\right)\right\}\right]
$$

For the gamma family, $\ln L_{j}=-y_{j} / \widehat{\mu}_{j}+\ln \left(1 / \widehat{\mu}_{j}\right)$.
For the inverse Gaussian,

$$
\ln L_{j}=-\frac{1}{2}\left\{\frac{\left(y_{j}-\widehat{\mu}_{j}\right)^{2}}{y_{j} \widehat{\mu}_{j}^{2}}+3 \ln \left(y_{j}\right)+\ln (2 \pi)\right\}
$$

For the negative binomial (let $m=1 / k$ ),

$$
\begin{aligned}
\ln L_{j}= & \phi\left[\ln \left\{\Gamma\left(m+y_{j}\right)\right\}-\ln \left\{\Gamma\left(y_{j}+1\right)\right\}-\ln \{\Gamma(m)\}\right. \\
& \left.-m \ln \left(1+\widehat{\mu}_{j} / m\right)+y_{j} \ln \left\{\widehat{\mu}_{j} /\left(\widehat{\mu}_{j}+m\right)\right\}\right]
\end{aligned}
$$

The overall deviance reported by glm is $D^{2}=\sum_{j} w_{j} d_{j}^{2}$. The dispersion of the deviance is $D^{2}$ divided by the residual degrees of freedom.
The Akaike information criterion (AIC) and Bayesian information criterion (BIC) are given by

$$
\begin{aligned}
& \mathrm{AIC}=\frac{-2 \ln L+2 k}{N} \\
& \mathrm{BIC}=D^{2}-(N-k) \ln (N)
\end{aligned}
$$

where $\ln L=\sum_{j} w_{j} \ln L_{j}$ is the overall log likelihood.

The Pearson deviance reported by glm is $\sum_{j} w_{j} r_{j}^{2}$. The corresponding Pearson dispersion is the Pearson deviance divided by the residual degrees of freedom. glm also calculates the scaled versions of all these quantities by dividing by the estimated scale parameter.

## Acknowledgments

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## Also see

[R] glm postestimation - Postestimation tools for glm
[R] cloglog - Complementary log-log regression
[R] logistic - Logistic regression, reporting odds ratios
[R] nbreg - Negative binomial regression
[R] poisson - Poisson regression
$[\mathrm{R}]$ regress - Linear regression
[ME] meglm - Multilevel mixed-effects generalized linear model
[MI] estimation - Estimation commands for use with mi estimate
[SVY] svy estimation - Estimation commands for survey data
[XT] xtgee - Fit population-averaged panel-data models by using GEE
[U] 20 Estimation and postestimation commands

## Title

glm postestimation - Postestimation tools for glm

| Description | Syntax for predict | Menu for predict | Options for predict |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Methods and formulas | References | Also see |

## Description

The following postestimation commands are available after glm:

| Command | Description |
| :--- | :--- |
| contrast | contrasts and ANOVA-style joint tests of estimates |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample |
| estat vce | variance-covariance matrix of the estimators (VCE) |
| estat (svy) | postestimation statistics for survey data |
| estimates | cataloging estimation results |
| forecast | dynamic forecasts and simulations <br> point estimates, standard errors, testing, and inference for linear combinations <br> of coefficients |
| lincom | link test for model specification |
| linktest | likelihood-ratio test |
| lrtest ${ }^{2}$ | marginal means, predictive margins, marginal effects, and average marginal |
| margins | effects |

[^20]
## Syntax for predict

predict [type] newvar [if] [in] [, statistic options]
statistic Description
Main
mu expected value of $y$; the default
$\mathrm{xb} \quad$ linear prediction $\eta=\mathbf{x} \widehat{\boldsymbol{\beta}}$
eta
synonym of xb
stdp standard error of the linear prediction
anscombe Anscombe (1953) residuals
cooksd
Cook's distance
deviance
deviance residuals
hat
likelihood
pearson
response
score
diagonals of the "hat" matrix
a weighted average of standardized deviance and standardized Pearson residuals
working
Pearson residuals
differences between the observed and fitted outcomes
first derivative of the $\log$ likelihood with respect to $\mathbf{x}_{j} \boldsymbol{\beta}$
working residuals

## options

Description
Options
nooffset modify calculations to ignore offset variable
adjusted
standardized
adjust deviance residual to speed up convergence
studentized
modified
multiply residual by the factor $(1-h)^{-1 / 2}$
multiply residual by one over the square root of the estimated scale parameter modify denominator of residual to be a reasonable estimate of the variance of depvar

These statistics are available both in and out of sample; type predict ... if e(sample)... if wanted only for the estimation sample.
$\mathrm{mu}, \mathrm{xb}$, stdp, and score are the only statistics allowed with svy estimation results.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
mu , the default, specifies that predict calculate the expected value of $y$, equal to $g^{-1}(\mathbf{x} \widehat{\boldsymbol{\beta}})\left[n g^{-1}(\mathbf{x} \widehat{\boldsymbol{\beta}})\right.$ for the binomial family].
xb calculates the linear prediction $\eta=\mathbf{x} \widehat{\boldsymbol{\beta}}$.
eta is a synonym for $x b$.
stdp calculates the standard error of the linear prediction.
anscombe calculates the Anscombe (1953) residuals to produce residuals that closely follow a normal distribution.
cooksd calculates Cook's distance, which measures the aggregate change in the estimated coefficients when each observation is left out of the estimation.
deviance calculates the deviance residuals. Deviance residuals are recommended by McCullagh and Nelder (1989) and by others as having the best properties for examining the goodness of fit of a GLM. They are approximately normally distributed if the model is correct. They may be plotted against the fitted values or against a covariate to inspect the model's fit. Also see the pearson option below.
hat calculates the diagonals of the "hat" matrix, analogous to linear regression.
likelihood calculates a weighted average of standardized deviance and standardized Pearson residuals.
pearson calculates the Pearson residuals. Pearson residuals often have markedly skewed distributions for nonnormal family distributions. Also see the deviance option above.
response calculates the differences between the observed and fitted outcomes.
score calculates the equation-level score, $\partial \ln L / \partial\left(\mathbf{x}_{j} \boldsymbol{\beta}\right)$.
working calculates the working residuals, which are response residuals weighted according to the derivative of the link function.

Options
nooffset is relevant only if you specified offset (varname) for glm. It modifies the calculations made by predict so that they ignore the offset variable; the linear prediction is treated as $\mathbf{x}_{j} \mathbf{b}$ rather than as $\mathbf{x}_{j} \mathbf{b}+$ offset $_{j}$.
adjusted adjusts the deviance residual to speed up the convergence to the limiting normal distribution. The adjustment deals with adding to the deviance residual a higher-order term that depends on the variance function family. This option is allowed only when deviance is specified.
standardized requests that the residual be multiplied by the factor $(1-h)^{-1 / 2}$, where $h$ is the diagonal of the hat matrix. This operation is done to account for the correlation between depvar and its predicted value.
studentized requests that the residual be multiplied by one over the square root of the estimated scale parameter.
modified requests that the denominator of the residual be modified to be a reasonable estimate of the variance of depvar. The base residual is multiplied by the factor $(k / w)^{-1 / 2}$, where $k$ is either one or the user-specified dispersion parameter and $w$ is the specified weight (or one if left unspecified).

## Remarks and examples

Remarks are presented under the following headings:

## Predictions

## > Example 1

After glm estimation, predict may be used to obtain various predictions based on the model. In example 2 of $[R]$ glm, we mentioned that the complementary log-log link seemed to fit the data better than the logit link. Now we go back and obtain the fitted values and deviance residuals:

```
. use http://www.stata-press.com/data/r13/ldose
. glm r ldose, family(binomial n) link(logit)
(output omitted)
. predict mu_logit
(option mu assumed; predicted mean r)
. predict dr_logit, deviance
. quietly glm r ldose, f(binomial n) l(cloglog)
. predict mu_cl
(option mu assumed; predicted mean r)
. predict dr_cl, d
. format mu_logit dr_logit mu_cl dr_cl %9.5f
. list r mu_logit dr_logit mu_cl dr_cl, sep(4)
```

| $r$ | mu_logit | dr_logit | mu_cl | dr_cl |
| ---: | ---: | ---: | ---: | ---: |
| 6 | 3.45746 | 1.28368 | 5.58945 | 0.18057 |
| 13 | 9.84167 | 1.05969 | 11.28067 | 0.55773 |
| 18 | 22.45139 | -1.19611 | 20.95422 | -0.80330 |
| 28 | 33.89761 | -1.59412 | 30.36942 | -0.63439 |
| 52 | 50.09584 | 0.60614 | 47.77644 | 1.28883 |
| 53 | 53.29092 | -0.12716 | 54.14273 | -0.52366 |
| 61 | 59.22216 | 1.25107 | 61.11331 | -0.11878 |
| 60 | 58.74297 | 1.59398 | 59.94723 | 0.32495 |

In six of the eight cases, $\mid \mathrm{dr} \_$logit $\left|>\left|\mathrm{dr} \_\mathrm{cl}\right|\right.$. The above represents only one of the many available options for predict. See Hardin and Hilbe (2012) for a more in-depth examination.

## Other postestimation commands

- Technical note

After glm estimation, you may perform any of the postestimation commands that you would perform after any other kind of estimation in Stata; see [U] 20 Estimation and postestimation commands. Below we test the joint significance of all the interaction terms.
. use http://www.stata-press.com/data/r13/beetle, clear
. glm r beetle\#\#c.ldose, family(binomial n) link(cloglog) (output omitted)
. testparm i.beetle beetle\#c.ldose
( 1) $[r] 2$. beetle $=0$
( 2) [r]3.beetle $=0$
( 3) [r]2.beetle\#c.ldose $=0$
( 4) [r]3.beetle\#c.ldose $=0$ chi2( 4) $=249.69$
Prob > chi2 $=0.0000$
If you wanted to print the variance-covariance matrix of the estimators, you would type estat vce.

If you use the linktest postestimation command, you must also specify the family() and link() options; see [R] linktest.

## Methods and formulas

We follow the terminology used in Methods and formulas of $[\mathrm{R}]$ glm.
The deviance residual calculated by predict following glm is $r_{j}^{D}=\operatorname{sign}\left(y_{j}-\widehat{\mu}_{j}\right) \sqrt{d_{j}^{2}}$.
The Pearson residual calculated by predict following glm is

$$
r_{j}^{P}=\frac{y_{j}-\widehat{\mu}_{j}}{\sqrt{V\left(\widehat{\mu}_{j}\right)}}
$$

where $V\left(\widehat{\mu}_{j}\right)$ is the family-specific variance function.

$$
V\left(\widehat{\mu}_{j}\right)= \begin{cases}\widehat{\mu}_{j}\left(1-\widehat{\mu}_{j} / m_{j}\right) & \text { if binomial or Bernoulli }\left(m_{j}=1\right) \\ \widehat{\mu}_{j}^{2} & \text { if gamma } \\ 1 & \text { if Gaussian } \\ \widehat{\mu}_{j}^{3} & \text { if inverse Gaussian } \\ \widehat{\mu}_{j}+k \widehat{\mu}_{j}^{2} & \text { if negative binomial } \\ \widehat{\mu}_{j} & \text { if Poisson }\end{cases}
$$

The response residuals are given by $r_{i}^{R}=y_{i}-\mu_{i}$. The working residuals are

$$
r_{i}^{W}=\left(y_{i}-\widehat{\mu}_{i}\right)\left(\frac{\partial \eta}{\partial \mu}\right)_{i}
$$

and the score residuals are

$$
r_{i}^{S}=\frac{y_{i}-\widehat{\mu}_{i}}{V\left(\widehat{\mu}_{i}\right)}\left(\frac{\partial \eta}{\partial \mu}\right)_{i}^{-1}
$$

Define $\widehat{W}=V(\widehat{\mu})$ and $X$ to be the covariate matrix. $h_{i}$, then, is the $i$ th diagonal of the hat matrix given by

$$
\widehat{H}=\widehat{W}^{1 / 2} X\left(X^{T} \widehat{W} X\right)^{-1} X^{T} \widehat{W}^{1 / 2}
$$

As a result, the likelihood residuals are given by

$$
r_{i}^{L}=\operatorname{sign}\left(y_{i}-\widehat{\mu}_{i}\right)\left\{h_{i}\left(r_{i}^{P^{\prime}}\right)^{2}+\left(1-h_{i}\right)\left(r_{i}^{D \prime}\right)^{2}\right\}^{1 / 2}
$$

where $r_{i}^{P \prime}$ and $r_{i}^{D^{\prime}}$ are the standardized Pearson and standardized deviance residuals, respectively. By standardized, we mean that the residual is divided by $\left\{1-h_{i}\right\}^{1 / 2}$.

Cook's distance is an overall measure of the change in the regression coefficients caused by omitting the $i$ th observation from the analysis. Computationally, Cook's distance is obtained as

$$
C_{i}=\frac{\left(r_{i}^{P \prime}\right)^{2} h_{i}}{k\left(1-h_{i}\right)}
$$

where $k$ is the number of regressors, including the constant.
Anscombe residuals are given by

$$
r_{i}^{A}=\frac{A\left(y_{i}\right)-A\left(\widehat{\mu}_{i}\right)}{A^{\prime}\left(\widehat{\mu}_{i}\right)\left\{V\left(\widehat{\mu}_{i}\right)\right\}^{1 / 2}}
$$

where

$$
A(\cdot)=\int \frac{d \mu}{V^{1 / 3}(\mu)}
$$

Deviance residuals may be adjusted (predict, adjusted) to make the following correction:

$$
r_{i}^{D a}=r_{i}^{D}+\frac{1}{6} \rho_{3}(\theta)
$$

where $\rho_{3}(\theta)$ is a family-specific correction. See Hardin and Hilbe (2012) for the exact forms of $\rho_{3}(\theta)$ for each family.

## References

Anscombe, F. J. 1953. Contribution of discussion paper by H. Hotelling "New light on the correlation coefficient and its transforms". Journal of the Royal Statistical Society, Series B 15: 229-230.
Hardin, J. W., and J. M. Hilbe. 2012. Generalized Linear Models and Extensions. 3rd ed. College Station, TX: Stata Press.

McCullagh, P., and J. A. Nelder. 1989. Generalized Linear Models. 2nd ed. London: Chapman \& Hall/CRC.
Newson, R. B. 2013. Attributable and unattributable risks and fractions and other scenario comparisons. Stata Journal 13: 672-698.

## Also see

[R] glm - Generalized linear models
$[R]$ regress postestimation - Postestimation tools for regress
[U] 20 Estimation and postestimation commands

## Title

glogit — Logit and probit regression for grouped data

Syntax
Options for blogit and bprobit
Stored results
Also see

## Menu

Options for glogit and gprobit Methods and formulas

## Description

Remarks and examples
References

## Syntax

Logistic regression for grouped data
blogit pos_var pop_var [indepvars] [if] [in] [, blogit_options]
Probit regression for grouped data
bprobit pos_var pop_var [indepvars] [if][in][,bprobit_options]
Weighted least-squares logistic regression for grouped data

```
glogit pos_var pop_var [indepvars] [if] [in] [,glogit_options]
```

Weighted least-squares probit regression for grouped data gprobit pos_var pop_var [indepvars] [if][in][,gprobit_options]
blogit_options Description
Model
noconstant
asis
offset (varname)
constraints (constraints)
collinear
SE/Robust
vce (vcetype)

Reporting
level(\#)
or
nocnsreport
display_options

Maximization
maximize_options
nocoef
coeflegend
suppress constant term
retain perfect predictor variables
include varname in model with coefficient constrained to 1
apply specified linear constraints
keep collinear variables
vcetype may be oim, robust, cluster clustvar, bootstrap, or jackknife
set confidence level; default is level (95)
report odds ratios
do not display constraints
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
control the maximization process; seldom used
do not display coefficient table; seldom used display legend instead of statistics
bprobit_options
Description

## Model

noconstant
asis
offset (varname)
constraints (constraints)
collinear
SE/Robust
vce (vcetype)

Reporting
level (\#)
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control the maximization process; seldom used
do not display coefficient table; seldom used display legend instead of statistics
glogit_options
SE
vce (vcetype) vcetype may be ols, bootstrap, or jackknife
Reporting
level(\#)
or
display_options
coeflegend
Description
set confidence level; default is level (95)
report odds ratios
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
display legend instead of statistics

Description
SE

Reporting
level(\#)
display_options
coeflegend
vce(vcetype) vcetype may be ols, bootstrap, or jackknife
set confidence level; default is level (95)
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
display legend instead of statistics
indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
bootstrap, by, jackknife, rolling, and statsby are allowed; see [U] 11.1.10 Prefix commands. fp is allowed with blogit and bprobit.
nocoef and coeflegend do not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

## blogit

Statistics $>$ Binary outcomes $>$ Grouped data $>$ Logit regression for grouped data

## bprobit

Statistics $>$ Binary outcomes $>$ Grouped data $>$ Probit regression for grouped data
glogit
Statistics $>$ Binary outcomes $>$ Grouped data $>$ Weighted least-squares logit regression

## gprobit

Statistics $>$ Binary outcomes $>$ Grouped data $>$ Weighted least-squares probit regression

## Description

blogit and bprobit produce maximum-likelihood logit and probit estimates on grouped ("blocked") data; glogit and gprobit produce weighted least-squares estimates. In the syntax diagrams above, pos_var and pop_var refer to variables containing the total number of positive responses and the total population.

See [R] logistic for a list of related estimation commands.

## Options for blogit and bprobit

$\qquad$ Model
noconstant; see $[R]$ estimation options.
asis forces retention of perfect predictor variables and their associated perfectly predicted observations and may produce instabilities in maximization; see [R] probit.
offset (varname), constraints (constraints), collinear; see [R] estimation options.

## SE/Robust

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
$\qquad$ Reporting
level(\#); see [R] estimation options.
or (blogit only) reports the estimated coefficients transformed to odds ratios, that is, $e^{b}$ rather than $b$. Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated. or may be specified at estimation or when replaying previously estimated results.
nocnsreport; see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% fmt), pformat (\% fmt), sformat (\% fmt), and nolstretch; see [R] estimation options.

Maximization
maximize_options: difficult, technique (algorithm_spec), iterate (\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance(\#), nonrtolerance, and from (init_specs); see [R] maximize. These options are seldom used.

The following options are available with blogit and bprobit but are not shown in the dialog box: nocoef specifies that the coefficient table not be displayed. This option is sometimes used by program writers but is useless interactively.
coeflegend; see $[R]$ estimation options.

## Options for glogit and gprobit

SE
vce (vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (ols) and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
vce(ols), the default, uses the standard variance estimator for ordinary least-squares regression.

## Reporting

level (\#); see [R] estimation options.
or (glogit only) reports the estimated coefficients transformed to odds ratios, that is, $e^{b}$ rather than $b$. Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated. or may be specified at estimation or when replaying previously estimated results.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\%fmt), pformat(\%fmt), sformat(\%fmt), and nolstretch; see [R] estimation options.

The following option is available with glogit and gprobit but is not shown in the dialog box: coeflegend; see $[R]$ estimation options.

## Remarks and examples

Remarks are presented under the following headings:
Maximum likelihood estimates
Weighted least-squares estimates

## Maximum likelihood estimates

blogit produces the same results as logit and logistic, and bprobit produces the same results as probit, but the "blocked" commands accept data in a slightly different "shape". Consider the following two datasets:

```
. use http://www.stata-press.com/data/r13/xmpl1
. list, sepby(agecat)
```

|  | agecat | exposed | died | pop |
| :--- | ---: | ---: | ---: | ---: |
| 1. | 0 | 0 | 0 | 115 |
| 2. | 0 | 0 | 1 | 5 |
| 3. | 0 | 1 | 0 | 98 |
| 4. | 0 | 1 | 1 | 8 |
| 5. | 1 | 0 | 0 | 69 |
| 6. | 1 | 0 | 1 | 16 |
| 7. | 1 | 1 | 0 | 76 |
| 8. | 1 | 1 | 1 | 22 |

. use http://www.stata-press.com/data/r13/xmpl2
. list

|  | agecat | exposed | deaths | pop |
| :--- | ---: | ---: | ---: | ---: |
| 1. | 0 | 0 | 5 | 120 |
| 2. | 0 | 1 | 8 | 106 |
| 3. | 1 | 0 | 16 | 85 |
| 4. | 1 | 1 | 22 | 98 |

These two datasets contain the same information; observations 1 and 2 of xmpl1 correspond to observation 1 of xmpl2, observations 3 and 4 of xmpl1 correspond to observation 2 of $x m p l 2$, and so on.

The first observation of xmpl1 says that for agecat==0 and exposed==0, 115 subjects did not die (died==0). The second observation says that for the same agecat and exposed groups, five subjects did die (died==1). In xmpl2, the first observation says that there were five deaths of a population of 120 in agecat $==0$ and exposed $==0$. These are two different ways of saying the same thing. Both datasets are transcriptions from the following table, reprinted in Rothman, Greenland, and Lash (2008, 260), for age-specific deaths from all causes for tolbutamide and placebo treatment groups (University Group Diabetes Program 1970):

|  | Age through 54 |  | Age 55 and above |  |
| :--- | ---: | ---: | ---: | ---: |
|  | Tolbutamide | Placebo | Tolbutamide | Placebo |
| Dead | 8 | 5 | 22 | 16 |
| Surviving | 98 | 115 | 76 | 79 |

The data in xmpl1 are said to be "fully relational", which is computer jargon meaning that each observation corresponds to one cell of the table. Stata typically prefers data in this format. The second form of storing these data in xmpl2 is said to be "folded", which is computer jargon for something less than fully relational.
blogit and bprobit deal with "folded" data and produce the same results that logit and probit would have if the data had been stored in the "fully relational" representation.

## - Example 1

For the tolbutamide data, the fully relational representation is preferred. We could then use logistic, logit, and any of the epidemiological table commands; see $[R]$ logistic, $[R]$ logit, and [ST] epitab. Nevertheless, there are occasions when the folded representation seems more natural. With blogit and bprobit, we avoid the tedium of having to unfold the data:
. use http://www.stata-press.com/data/r13/xmpl2
. blogit deaths pop agecat exposed, or
Logistic regression for grouped data

If we had not specified the or option, results would have been presented as coefficients instead of as odds ratios. The estimated odds ratio of death for tolbutamide exposure is 1.40 , although the $95 \%$ confidence interval includes 1 . (By comparison, these data, in fully relational form and analyzed using the cs command [see [ST] epitab], produce a Mantel-Haenszel weighted odds ratio of 1.40 with a $95 \%$ confidence interval of 0.76 to 2.59 .)

We can see the underlying coefficients by replaying the estimation results and not specifying the or option:


## - Example 2

bprobit works like blogit, substituting the probit for the logit-likelihood function.
. bprobit deaths pop agecat exposed
Probit regression for grouped data

Log likelihood $=-142.56478$

| Number of obs | $=$ | 409 |
| :--- | :--- | ---: |
| LR chi2 (2) | $=$ | 22.58 |
| Prob > chi2 | $=$ | 0.0000 |
| Pseudo R2 | $=$ | 0.0734 |


| _outcome | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| agecat | .7542049 | .1709692 | 4.41 | 0.000 | .4191114 | 1.089298 |
| exposed | .1906236 | .1666059 | 1.14 | 0.253 | -.1359179 | .5171651 |
| _cons | -1.673973 | .1619594 | -10.34 | 0.000 | -1.991408 | -1.356539 |

## Weighted least-squares estimates

## > Example 3

We have state data for the United States on the number of marriages (marriage), the total population aged 18 years or more (pop18p), and the median age (medage). The dataset excludes Nevada, so it has 49 observations. We now wish to estimate a logit equation for the marriage rate. We will include age squared by specifying the term c.medage\#c.medage:

| (1980 Census data by state) <br> . glogit marriage pop18p medage c.medage\#c.medage |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Weighted LS logistic regression for grouped data |  |  |  |  |  |  |
| Source | SS | df MS |  |  | $\begin{array}{lr} \text { Number of obs } & = \\ F(2, & 46) \\ F(29 \end{array}$ |  |
| Model <br> Residual | . 71598314 | 2 . | . 35799157 |  |  |  |
|  | 1.27772858 | 46.0 | . 027776708 |  | R -squared <br> Adj R-squared | 0.3591 |
|  |  |  |  |  |  | $=0.3313$ |
| Total | 1.99371172 | 48.0 | 041535661 |  | Root MSE | $=.16666$ |
|  | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Conf. | Interval] |
| medage | -. 6459349 | . 2828381 | -2.28 | 0.027 | -1.215258 | -. 0766114 |
| c.medage\# <br> c.medage | . 0095414 | . 0046608 | 2.05 | 0.046 | . 0001598 | . 0189231 |
| _cons | 6.503833 | 4.288977 | 1.52 | 0.136 | -2.129431 | 15.1371 |

## > Example 4

We could just as easily have fit a grouped-probit model by typing gprobit rather than glogit:
. gprobit marriage pop18p medage c.medage\#c.medage
Weighted LS probit regression for grouped data

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | .108222962 | 2 | .054111481 |
| Residual | .192322476 | 46 | .004180923 |
| Total | .300545438 | 48 | .006261363 |


| Number of obs | $=$ | 49 |
| :--- | ---: | ---: |
| F 2, | $46)$ | $=$ |
| Prob $>$ F | $=0.0000$ |  |
| R-squared | $=$ | 0.3601 |
| Adj R-squared | $=$ | 0.3323 |
| Root MSE | $=.06466$ |  |


|  | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t\mid}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: |
| medage | -.2755007 | .1121042 | -2.46 | 0.018 | -.5011548 | -.0498466 |
| c.medage\# <br> c.medage | .0041082 | .0018422 | 2.23 | 0.031 | .0004001 | .0078163 |
| _cons | 2.357708 | 1.704446 | 1.38 | 0.173 | -1.073164 | 5.788579 |

## Stored results

blogit and bprobit store the following in e():
Scalars

| e(N) | number of observations |
| :--- | :--- |
| e(N_cds) | number of completely determined successes |
| e(N_cdf) | number of completely determined failures |
| e(k) | number of parameters |
| e(k_eq) | number of equations in e(b) |
| e(k_eq_model) | number of equations in overall model test |
| e(k_dv) | number of dependent variables |
| e(df_m) | model degrees of freedom |
| e(r2_p) | pseudo- $R$-squared |
| e(ll) | log likelihood |
| e(ll_0) | log likelihood, constant-only model |
| e(N_clust) | number of clusters |
| e(chi2) | $\chi^{2}$ |
| e(p) | significance of model test |
| e(rank) | rank of e(V) |
| e(ic) | number of iterations |
| e(rc) | return code |
| e(converged) | 1 if converged, 0 otherwise |

Macros
e(cmd)
e(cmdline)
e(depvar)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(offset)
e(chi2type)
e(vce)
e(vcetype)
e(opt)
e(which)
e(ml_method)
e(user)
e(technique)
e(properties)
e(predict)
e(marginsok)
e(asbalanced)
e(asobserved)
Matrices
e(b)
e(Cns)
e(ilog)
e(gradient)
e(mns)
e(rules)
e(V)
e(V_modelbased)
Functions
e(sample) marks estimation sample
blogit or bprobit
command as typed
weight type
weight expression
title in estimation output
name of cluster variable
linear offset variable
Wald or LR; type of model $\chi^{2}$ test
vcetype specified in vce()
title used to label Std. Err.
type of optimization
type of ml method
name of likelihood-evaluator program
maximization technique
b V
program used to implement predict
predictions allowed by margins
factor variables fvset as asbalanced
factor variables fvset as asobserved
coefficient vector
constraints matrix
iteration $\log$ (up to 20 iterations)
gradient vector
information about perfect predictors
model-based variance
glogit and gprobit store the following in e():

Scalars
e(N)
e(mss)
$e\left(d f \_m\right)$
e(rss)
e(df_r)
e(r2)
e(r2_a)
e(F)
e(rmse)
e(rank)
Macros
e(cmd)
e(cmdline)
e(depvar)
e(model)
e(title)
e(vce)
e(vcetype)
e(properties)
e(predict)
e(marginsok)
e(asbalanced)
e(asobserved)
variable containing number of positive responses and variable containing population size
max or min; whether optimizer is to perform maximization or minimization
vector of means of the independent variables
variance-covariance matrix of the estimators

Matrices
e(b)
coefficient vector
e(V) variance-covariance matrix of the estimators
Functions
e(sample) marks estimation sample

## Methods and formulas

Methods and formulas are presented under the following headings:
Maximum likelihood estimates
Weighted least-squares estimates

## Maximum likelihood estimates

The results reported by blogit and bprobit are obtained by maximizing a weighted logit- or probit-likelihood function. Let $F()$ denote the normal- or logistic-likelihood function. The likelihood of observing each observation in the data is then

$$
F(\beta x)^{s}\{1-F(\beta x)\}^{t-s}
$$

where $s$ is the number of successes and $t$ is the population. The term above is counted as contributing $s+(t-s)=t$ degrees of freedom. All of this follows directly from the definitions of logit and probit.
blogit and bprobit support the Huber/White/sandwich estimator of the variance and its clustered version using vce (robust) and vce (cluster clustvar), respectively. See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas.

## Weighted least-squares estimates

The logit function is defined as the log of the odds ratio. If there is one explanatory variable, the model can be written as

$$
\begin{equation*}
\log \left(\frac{p_{j}}{1-p_{j}}\right)=\beta_{0}+\beta_{1} x_{j}+\epsilon_{j} \tag{1}
\end{equation*}
$$

where $p_{j}$ represents successes divided by population for the $j$ th observation. (If there is more than one explanatory variable, we simply interpret $\beta_{1}$ as a row vector and $x_{j}$ as a column vector.) The large-sample expectation of $\epsilon_{j}$ is zero, and its variance is

$$
\sigma_{j}^{2}=\frac{1}{n_{j} p_{j}\left(1-p_{j}\right)}
$$

where $n_{j}$ represents the population for observation $j$. We can thus apply weighted least squares to the observations, with weights proportional to $n_{j} p_{j}\left(1-p_{j}\right)$.

As in any feasible generalized least-squares problem, estimation proceeds in two steps. First, we fit (1) by OLS and compute the predicted probabilities as

$$
\widehat{p_{j}}=\frac{\exp \left(\widehat{\beta_{0}}+\widehat{\beta_{1}} x_{j}\right)}{1+\exp \left(\widehat{\beta_{0}}+\widehat{\beta_{1}} x_{j}\right)}
$$

In the second step, we fit (1) by using analytic weights equal to $n_{j} \widehat{p_{j}}\left(1-\widehat{p_{j}}\right)$.

For gprobit, write $\Phi(\cdot)$ for the cumulative normal distribution, and define $z_{j}$ implicitly by $\Phi\left(z_{j}\right)=p_{j}$, where $p_{j}$ is the fraction of successes for observation $j$. The probit model for one explanatory variable can be written as

$$
\Phi^{-1}\left(p_{j}\right)=\beta_{0}+\beta_{1} x_{j}+\epsilon_{j}
$$

(If there is more than one explanatory variable, we simply interpret $\beta_{1}$ as a row vector and $x_{j}$ as a column vector.)

The expectation of $\epsilon_{j}$ is zero, and its variance is given by

$$
\sigma_{j}^{2}=\frac{p_{j}\left(1-p_{j}\right)}{n_{j} \phi^{2}\left\{\Phi^{-1}\left(p_{j}\right)\right\}}
$$

where $\phi(\cdot)$ represents the normal density (Amemiya 1981, 1498). We can thus apply weighted least squares to the observations with weights proportional to $1 / \sigma_{j}^{2}$. As for grouped logit, we use a two-step estimator to obtain the weighted least-squares estimates.

## References

Amemiya, T. 1981. Qualitative response models: A survey. Journal of Economic Literature 19: 1483-1536.
Hosmer, D. W., Jr., S. A. Lemeshow, and R. X. Sturdivant. 2013. Applied Logistic Regression. 3rd ed. Hoboken, NJ: Wiley.
Judge, G. G., W. E. Griffiths, R. C. Hill, H. Lütkepohl, and T.-C. Lee. 1985. The Theory and Practice of Econometrics. 2nd ed. New York: Wiley.

Rothman, K. J., S. Greenland, and T. L. Lash. 2008. Modern Epidemiology. 3rd ed. Philadelphia: Lippincott Williams \& Wilkins.
University Group Diabetes Program. 1970. A study of the effects of hypoglycemic agents on vascular complications in patients with adult-onset diabetes, II: Mortality results. Diabetes 19, supplement 2: 789-830.

[^21]
## Title

glogit postestimation - Postestimation tools for glogit, gprobit, blogit, and bprobit
Description Syntax for predict Menu for predict Options for predict

## Description

The following postestimation commands are available after glogit, gprobit, blogit, and bprobit:

Command Description

| contrast | contrasts and ANOVA-style joint tests of estimates |
| :--- | :--- |
| * estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize |  |
| estat vce | summary statistics for the estimation sample <br> variance-covariance matrix of the estimators (VCE) <br> cataloging estimation results |
| estimates | point estimates, standard errors, testing, and inference for linear combinations <br> of coefficients |
| lincom | likelihood-ratio test <br> marginal means, predictive margins, marginal effects, and average marginal <br> effects |
| margins | graph the results from margins (profile plots, interaction plots, etc.) |
| mlcom | point estimates, standard errors, testing, and inference for nonlinear combinations <br> of coefficients |
| predict | predictions, residuals, influence statistics, and other diagnostic measures <br> predictnl <br> pwcompare estimates, standard errors, testing, and inference for generalized predictions <br> pairwise comparisons of estimates |
| test | Wald tests of simple and composite linear hypotheses |

* estat ic and lrtest are not appropriate after glogit and gprobit.


## Syntax for predict

predict [type] newvar [if] [in] [, statistic]

## statistic

## Description

## Main

predicted count; the default
$\mathrm{pr} \quad$ probability of a positive outcome
$\mathrm{xb} \quad$ linear prediction
stdp
standard error of the linear prediction
These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

Main
n , the default, calculates the expected count, that is, the estimated probability times pop_var, which is the total population.
pr calculates the predicted probability of a positive outcome.
xb calculates the linear prediction.
stdp calculates the standard error of the linear prediction.

## Also see

[R] glogit - Logit and probit regression for grouped data
[U] 20 Estimation and postestimation commands

## Title

gmm - Generalized method of moments estimation

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |
| Also see |  |  |  |

## Syntax

Interactive version
$\operatorname{gmm}\left(\left[\right.\right.$ eqname $\left.\left._{1}:\right]<\operatorname{mexp}_{1}>\right)\left(\left[\right.\right.$ eqname $\left.\left._{2}:\right]<\operatorname{mexp}_{2}>\right) \ldots[$ if $][$ in $][$ weight $][$, options $]$

Moment-evaluator program version

```
gmm moment_prog [if] [in] [weight], { equations(namelist)|\underline{nequations(#)}}
```


where
$\operatorname{mexp}_{j}$ is the substitutable expression for the $j$ th moment equation and moment_prog is a moment-evaluator program.
options
Description
Model
derivative ( $\left\langle\operatorname{dexp}_{m n}\right\rangle$ ) specify derivative of $\operatorname{mexp}_{m}$ with respect to parameter $n$; can be specified more than once (interactive version only)
*twostep use two-step GMM estimator; the default

* onestep use one-step GMM estimator
*igmm use iterative GMM estimator
Instruments
instruments ([<eqlist>:]varlist $[$, noconstant $]$ )
specify instruments; can be specified more than once
xtinstruments ([<eqlist>: $]$ varlist, lags $\left(\#_{1} / \#_{2}\right)$ )
specify panel-style instruments; can be specified more than once
Weight matrix
wmatrix (wmtype [, independent])
specify weight matrix; wmtype may be robust, cluster clustvar, hac kernel [lags], or unadjusted
center center moments in weight-matrix computation
winitial(iwtype [, independent])
specify initial weight matrix; iwtype may be identity, unadjusted, xt xtspec, or the name of a Stata matrix

Options
variables (varlist)
nocommonesample
specify variables in model
do not restrict estimation sample to be the same for all equations

SE/Robust
$\operatorname{vce}(v c e t y p e[$, independent $])$
vcetype may be robust, cluster clustvar, bootstrap, jackknife, hack kernel lags, or unadjusted
quickderivatives
use alternative method of computing numerical derivatives for VCE

Reporting
level (\#) set confidence level; default is level (95)
title (string) display string as title above the table of parameter estimates
title (string)
display_options
display string as subtitle
control column formats and line width
Optimization
from (initial_values)
$\ddagger_{\text {igmmiterate }}$ (\#)
$\ddagger$ igmmeps $(\#)$
$\ddagger_{\text {igmmweps }}(\#)$
optimization_options
coeflegend
specify initial values for parameters
specify maximum number of iterations for iterated GMM estimator
specify \# for iterated GMM parameter convergence criterion; default is igmmeps ( $1 e-6$ )
specify \# for iterated GMM weight-matrix convergence criterion; default is igmmweps (1e-6)
control the optimization process; seldom used
display legend instead of statistics

* You can specify at most one of these options.
$\ddagger$ These options may be specified only when igmm is specified.
program_options
Model evaluator_options
* hasderivatives
* haslfderivatives
$\dagger$ equations (namelist)
$\dagger$ nequations (\#)
$\ddagger$ parameters (namelist)
$\ddagger$ nparameters (\#)

Description
additional options to be passed to the moment-evaluator program moment-evaluator program can calculate parameter-level derivatives moment-evaluator program can calculate linear-form derivatives specify moment-equation names
specify number of moment equations
specify parameter names
specify number of parameters

[^22]bootstrap, by, jackknife, rolling, statsby, and xi are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the bootstrap prefix; see [R] bootstrap.
aweights are not allowed with the jackknife prefix; see [R] jackknife.
aweights, fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.
$<\operatorname{mexp}_{j}>$ and $<\operatorname{dexp}_{m n}>$ are extensions of valid Stata expressions that also contain parameters to be estimated. The parameters are enclosed in curly braces and must otherwise satisfy the naming requirements for variables; \{beta\} is an example of a parameter. Also allowed is a notation of the form $\{<$ eqname $>$ : varlist $\}$ for linear combinations of multiple covariates and their parameters. For example, $\{\mathrm{xb}$ : mpg price turn\} defines a linear combination of the variables mpg, price, and turn. See Substitutable expressions under Remarks and examples below.

## Menu

Statistics $>$ Endogenous covariates $>$ Generalized method of moments estimation

## Description

gmm performs generalized method of moments (GMM) estimation. With the interactive version of the command, you enter the moment equations directly into the dialog box or on the command line using substitutable expressions. The moment-evaluator program version gives you greater flexibility in exchange for increased complexity; with this version, you write a program in an ado-file that calculates the moments based on a vector of parameters passed to it.
gmm can fit both single- and multiple-equation models, and it allows moment conditions of the form $E\left\{\mathbf{z}_{i} u_{i}(\beta)\right\}=\mathbf{0}$, where $\mathbf{z}_{i}$ is a vector of instruments and $u_{i}(\boldsymbol{\beta})$ is often an additive regression error term, as well as more general moment conditions of the form $E\left\{\mathbf{h}_{i}\left(\mathbf{z}_{i} ; \boldsymbol{\beta}\right)\right\}=\mathbf{0}$. gmm works with cross-sectional, time-series, and longitudinal (panel) data.

## Options

$\xlongequal[{\text { derivative ( }[\text { eqname } \mid \#] / \text { name }=<\operatorname{dexp}_{m n}>\text { ) specifies the derivative of moment equation eqname }}]{\text { Model }}$ or \# with respect to parameter name. If eqname or \# is not specified, gmm assumes that the derivative applies to the first moment equation.
For a moment equation of the form $E\left\{\mathbf{z}_{m i} u_{m i}(\boldsymbol{\beta})\right\}=\mathbf{0}$, derivative $\left(m / \beta_{j}=<\operatorname{dexp}_{m n}>\right)$ is to contain a substitutable expression for $\partial u_{m i} / \partial \beta_{j}$.
For a moment equation of the form $E\left\{h_{m i}\left(\mathbf{z}_{i} ; \boldsymbol{\beta}\right)\right\}=\mathbf{0}$, derivative $\left(m / \beta_{j}=<\operatorname{dexp}_{m n}>\right)$ is to contain a substitutable expression for $\partial h_{m i} / \partial \beta_{j}$.
$<\exp _{m n}>$ uses the same substitutable expression syntax as is used to specify moment equations. If you declare a linear combination in a moment equation, you provide the derivative for the linear combination; gmm then applies the chain rule for you. See Specifying derivatives under Remarks and examples below for examples.
If you do not specify the derivative() option, gmm calculates derivatives numerically. You must either specify no derivatives or specify all the derivatives that are not identically zero; you cannot specify some analytic derivatives and have gmm compute the rest numerically.
twostep, onestep, and igmm specify which estimator is to be used. You can specify at most one of these options. twostep is the default.
twostep requests the two-step GMM estimator. gmm obtains parameter estimates based on the initial weight matrix, computes a new weight matrix based on those estimates, and then reestimates the parameters based on that weight matrix.
onestep requests the one-step GMM estimator. The parameters are estimated based on an initial weight matrix, and no updating of the weight matrix is performed except when calculating the appropriate variance-covariance (VCE) matrix.
igmm requests the iterative GMM estimator. gmm obtains parameter estimates based on the initial weight matrix, computes a new weight matrix based on those estimates, reestimates the parameters based on that weight matrix, computes a new weight matrix, and so on, to convergence. Convergence is declared when the relative change in the parameter vector is less than igmmeps (), the relative change in the weight matrix is less than igmmweps(), or igmmiterate() iterations have been completed. Hall (2005, sec. 2.4 and 3.6) mentions that there may be gains to finite-sample efficiency from using the iterative estimator.
nstruments
instruments ([<eqlist>:] varlist [, noconstant $]$ ) specifies a list of instrumental variables to be used. If you specify a single moment equation, then you do not need to specify the equations to which the instruments apply; you can omit the eqlist and simply specify instruments (varlist). By default, a constant term is included in varlist; to omit the constant term, use the noconstant suboption: instruments(varlist, noconstant).

If you specify a model with multiple moment conditions of the form

$$
E\left\{\begin{array}{c}
\mathbf{z}_{1 i} u_{1 i}(\boldsymbol{\beta}) \\
\cdots \\
\mathbf{z}_{q i} u_{q i}(\boldsymbol{\beta})
\end{array}\right\}=\mathbf{0}
$$

then you can specify the equations to indicate the moment equations for which the list of variables is to be used as instruments if you do not want that list applied to all the moment equations. For example, you might type

```
gmm (main:< mexp }>>)(<\mp@subsup{\operatorname{mexp}}{2}{}>)(<\mp@subsup{\operatorname{mexp}}{3}{}>), instruments(z1 z2) ///
    instruments(2: z3) instruments(main 3: z4)
```

Variables $z 1$ and $z 2$ will be used as instruments for all three equations, $z 3$ will be used as an instrument for the second equation, and $z 4$ will be used as an instrument for the first and third equations. Notice that we chose to supply a name for the first moment equation but not the second two.
varlist may contain factor variables and time-series operators; see [U] 11.4.3 Factor variables and [U] 11.4.4 Time-series varlists, respectively.
xtinstruments $\left([<e q l i s t>:]\right.$ varlist, $\operatorname{lags}\left(\#_{1} / \#_{2}\right)$ ) is for use with panel-data models in which the set of available instruments depends on the time period. As with instruments (), you can prefix the list of variables with equation names or numbers to target instruments to specific equations. Unlike with instruments(), a constant term is not included in varlist. You must xtset your data before using this option; see [XT] xtset.

If you specify

```
gmm ..., xtinstruments(x, lags(1/.)) ...
```

then for panel $i$ and period $t$, gmm uses as instruments $x_{i, t-1}, x_{i, t-2}, \ldots, x_{i 1}$. More generally, specifying xtinstruments ( $\mathrm{x}, \operatorname{lags}\left(\#_{1}, \#_{2}\right.$ ) ) uses as instruments $x_{i, t-\#_{1}}, \ldots, x_{i, t-\#_{2}}$; setting $\#_{2}=$. requests all available lags. $\#_{1}$ and $\#_{2}$ must be zero or positive integers.
gmm automatically excludes observations for which no valid instruments are available. It does, however, include observations for which only a subset of the lags is available. For example, if you request that lags one through three be used, then gmm will include the observations for the second and third time periods even though fewer than three lags are available as instruments.

## Weight matrix

wmatrix (wmtype [, independent]) specifies the type of weight matrix to be used in conjunction with the two-step and iterated GMM estimators.

Specifying wmatrix (robust) requests a weight matrix that is appropriate when the errors are independent but not necessarily identically distributed. wmatrix (robust) is the default.
Specifying wmatrix(cluster clustvar) requests a weight matrix that accounts for arbitrary correlation among observations within clusters identified by clustvar.
Specifying wmatrix(hac kernel \#) requests a heteroskedasticity- and autocorrelation-consistent (HAC) weight matrix using the specified kernel (see below) with \# lags. The bandwidth of a kernel is equal to the number of lags plus one.
Specifying wmatrix (hac kernel opt) requests an HAC weight matrix using the specified kernel, and the lag order is selected using Newey and West's (1994) optimal lag-selection algorithm.
Specifying wmatrix (hac kernel) requests an HAC weight matrix using the specified kernel and $N-2$ lags, where $N$ is the sample size.

There are three kernels available for HAC weight matrices, and you may request each one by using the name used by statisticians or the name perhaps more familiar to economists:
bartlett or nwest requests the Bartlett (Newey-West) kernel;
parzen or gallant requests the Parzen (Gallant) kernel; and
quadraticspectral or andrews requests the quadratic spectral (Andrews) kernel.
Specifying wmatrix (unadjusted) requests a weight matrix that is suitable when the errors are homoskedastic. In some applications, the GMM estimator so constructed is known as the (nonlinear) two-stage least-squares (2SLS) estimator.
Including the independent suboption creates a weight matrix that assumes moment equations are independent. This suboption is often used to replicate other models that can be motivated outside the GMM framework, such as the estimation of a system of equations by system-wide 2SLS. This suboption has no effect if only one moment equation is specified.
wmatrix() has no effect if onestep is also specified.
center requests that the sample moments be centered (demeaned) when computing GMM weight matrices. By default, centering is not done.
winitial(wmtype [, independent]) specifies the weight matrix to use to obtain the first-step parameter estimates.

Specifying winitial (unadjusted) requests a weight matrix that assumes the moment equations are independent and identically distributed. This matrix is of the form $\left(\mathbf{Z}^{\prime} \mathbf{Z}\right)^{-1}$, where $\mathbf{Z}$ represents all the instruments specified in the instruments () option. To avoid a singular weight matrix, you should specify at least $q-1$ moment equations of the form $E\left\{\mathbf{z}_{h i} u_{h i}(\boldsymbol{\beta})\right\}=\mathbf{0}$, where $q$ is the number of moment equations, or you should specify the independent suboption.

Including the independent suboption creates a weight matrix that assumes moment equations are independent. Elements of the weight matrix corresponding to covariances between two moment equations are set equal to zero. This suboption has no effect if only one moment equation is specified.
winitial(unadjusted) is the default.
winitial (xt xtspec) is for use with dynamic panel-data models in which one of the moment equations is specified in first-differences form. xtspec is a string consisting of the letters "L" and "D", the length of which is equal to the number of moment equations in the model. You specify "L" for a moment equation if that moment equation is written in levels, and you specify "D" for a moment equation if it is written in first-differences; xtspec is not case sensitive. When you specify this option, you can specify at most one moment equation in levels and one moment equation in first-differences. See the examples listed in Dynamic panel-data models under Remarks and examples below.
winitial (identity) requests that the identity matrix be used.
winitial (matname) requests that Stata matrix matname be used. You cannot specify the independent suboption if you specify winitial(matname).

Options
variables (varlist) specifies the variables in the model. gmm ignores observations for which any of these variables has a missing value. If you do not specify variables (), then gmm assumes all the observations are valid and issues an error message with return code 480 if any moment equations evaluate to missing for any observations at the initial value of the parameter vector.
nocommonesample requests that gmm not restrict the estimation sample to be the same for all equations. By default, gmm will restrict the estimation sample to observations that are available for all equations in the model, mirroring the behavior of other multiple-equation estimators such as nlsur, sureg, or reg3. For certain models, however, different equations can have different numbers of observations. For these models, you should specify nocommonesample. See Dynamic panel-data models below for one application of this option. You cannot specify weights if you specify nocommonesample.

SE/Robust
vce (vcetype [, independent]) specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
vce (unadjusted) specifies that an unadjusted (nonrobust) VCE matrix be used; this, along with the twostep option, results in the "optimal two-step GMM" estimates often discussed in textbooks.

The default vcetype is based on the wmtype specified in the wmatrix() option. If wmatrix() is specified but vce() is not, then vcetype is set equal to wmtype. To override this behavior and obtain an unadjusted (nonrobust) VCE matrix, specify vce (unadjusted).
Specifying vce (bootstrap) or vce (jackknife) results in standard errors based on the bootstrap or jackknife, respectively. See $[R]$ ve _option, $[R]$ bootstrap, and $[R]$ jackknife for more information on these VCEs.

The syntax for vcetypes other than bootstrap and jackknife is identical to those for watrix().
quickderivatives requests that an alternative method be used to compute the numerical derivatives for the VCE. This option has no effect if you specify the derivatives(), hasderivatives, or haslfderivatives option.

The VCE depends on a matrix of partial derivatives that gmm must compute numerically unless you supply analytic derivatives. This Jacobian matrix will be especially large if your model has many instruments, moment equations, or parameters.
By default, gmm computes each element of the Jacobian matrix individually, searching for an optimal step size each time. Although this procedure results in accurate derivatives, it is computationally taxing: gmm may have to evaluate the moments of your model five or more times for each element of the Jacobian matrix.

When you specify the quickderivatives option, gmm computes all derivatives corresponding to a parameter at once, using a fixed step size proportional to the parameter's value. This method requires just two evaluations of the model's moments to compute an entire column of the Jacobian matrix and therefore has the most impact when you specify many instruments or moment equations.

Most of the time, the two methods produce virtually identical results, but the quickderivatives method may fail if a moment equation is highly nonlinear or if instruments differ by orders of magnitude. In the rare case where you specify quickderivatives and obtain suspiciously large or small standard errors, try refitting your model without this option.
$\sqrt{\text { Reporting }}$
level(\#); see [R] estimation options.
title(string) specifies an optional title that will be displayed just above the table of parameter estimates.
title2(string) specifies an optional subtitle that will be displayed between the title specified in title() and the table of parameter estimates. If title2() is specified but title() is not, title2() has the same effect as title().
display_options: cformat (\%fmt), pformat (\%fmt), sformat (\%fmt), and nolstretch; see [R] estimation options.

Optimization
from(initial_values) specifies the initial values to begin the estimation. You can specify a $1 \times k$ matrix, where $k$ is the number of parameters in the model, or you can specify a parameter name, its initial value, another parameter name, its initial value, and so on. For example, to initialize alpha to 1.23 and delta to 4.57 , you would type
gmm ..., from(alpha 1.23 delta 4.57) ...
Initial values declared using this option override any that are declared within substitutable expressions. If you specify a parameter that does not appear in your model, gmm exits with error code 480. If you specify a matrix, the values must be in the same order in which the parameters are declared in your model. gmm ignores the row and column names of the matrix.
igmmiterate(\#), igmmeps(\#), and igmmweps(\#) control the iterative process for the iterative GMM estimator. These options can be specified only if you also specify igmm.
igmmiterate(\#) specifies the maximum number of iterations to perform with the iterative GMM estimator. The default is the number set using set maxiter (set [R] maximize), which is 16,000 by default.
igmmeps (\#) specifies the convergence criterion used for successive parameter estimates when the iterative GMM estimator is used. The default is igmmeps ( $1 \mathrm{e}-6$ ). Convergence is declared when the relative difference between successive parameter estimates is less than igmmeps () and the relative difference between successive estimates of the weight matrix is less than igmmweps ().
igmmweps (\#) specifies the convergence criterion used for successive estimates of the weight matrix when the iterative GMM estimator is used. The default is igmmweps ( $1 e-6$ ). Convergence is declared when the relative difference between successive parameter estimates is less than igmmeps() and the relative difference between successive estimates of the weight matrix is less than igmmweps().
optimization_options: technique(), conv_maxiter(), conv_ptol(), conv_vtol(),
conv_nrtol(), tracelevel(). technique() specifies the optimization technique to use; gn (the default), nr , dfp , and bfgs are allowed. conv_maxiter() specifies the maximum number of iterations; conv_ptol(), conv_vtol(), and conv_nrtol() specify the convergence criteria for the parameters, gradient, and scaled Hessian, respectively. tracelevel() allows you to obtain additional details during the iterative process. See [M-5] optimize().

The following options pertain only to the moment-evaluator program version of gmm.

Model
evaluator_options refer to any options allowed by your moment_prog.
hasderivatives and haslfderivatives indicate that you have written your moment-evaluator program to compute derivatives. You may specify one or the other but not both. If you do not specify either of these options, gmm computes the derivatives numerically.
hasderivatives indicates that your moment-evaluator program computes parameter-level derivatives.
haslfderivatives indicates that your moment-evaluator program computes equation-level derivatives and is useful only when you specify the parameters of your model using the <eqname $>$ :<varname $>$ syntax of the parameters() option.
See Details of moment-evaluator programs below for more information.
equations (namelist) specifies the names of the moment equations in the model. If you specify both equations() and nequations(), the number of names in the former must match the number specified in the latter.
nequations(\#) specifies the number of moment equations in the model. If you do not specify names with the equations() option, gmm numbers the moment equations $1,2,3, \ldots$. If you specify both equations() and nequations(), the number of names in the former must match the number specified in the latter.
parameters (namelist) specifies the names of the parameters in the model. The names of the parameters must adhere to the naming conventions of Stata's variables;
see [U] 11.3 Naming conventions.
Alternatively, you may specify a list of names in which each item in the list is of the form $<$ eqname $>:<$ varname $>$, where eqname is an equation name used to group parameters, and varname is the name of an existing variable or _cons to indicate a constant term. When you use this syntax, gmm adorns the parameter vector passed to your evaluator program with these names so that you can use matrix score (see [P] matrix score) to compute linear combinations of parameters. These equation names are not related to the names you may give to the moment equations.

If you specify both parameters() and nparameters(), the number of names in the former must match the number specified in the latter.
nparameters (\#) specifies the number of parameters in the model. If you do not specify names with the parameters() option, gmm names them b1, b2, ..., b\#. If you specify both parameters() and nparameters (), the number of names in the former must match the number specified in the latter.

The following option is available with gmm but is not shown in the dialog box:
coeflegend; see [R] estimation options.

## Remarks and examples

Remarks are presented under the following headings:

Introduction<br>Substitutable expressions<br>The weight matrix and two-step estimation Obtaining standard errors<br>Exponential (Poisson) regression models Specifying derivatives Exponential regression models with panel data Rational-expectations models<br>System estimators<br>Dynamic panel-data models<br>Details of moment-evaluator programs

## Introduction

The generalized method of moments (GMM) estimator is a workhorse of modern econometrics and is discussed in all the leading textbooks, including Cameron and Trivedi (2005, 2010), Davidson and MacKinnon (1993, 2004), Greene (2012, 468-506), Ruud (2000), Hayashi (2000), Wooldridge (2010), Hamilton (1994), and Baum (2006). An excellent treatise on GMM with a focus on time-series applications is Hall (2005). The collection of papers by Mátyás (1999) provides both theoretical and applied aspects of GMM. Here we give a brief introduction to the methodology and emphasize how the various options of gmm are used.

The starting point for the generalized method of moments (GMM) estimator is the analogy principle, which says we can estimate a parameter by replacing a population moment condition with its sample analogue. For example, the mean of an independent and identically distributed (i.i.d.) population is defined as the value $\mu$ such that the first (central) population moment is zero; that is, $\mu$ solves $E(y-\mu)=0$ where $y$ is a random draw from the population. The analogy principle tells us that to obtain an estimate, $\widehat{\mu}$, of $\mu$, we replace the population-expectations operator with its sample analogue (Manski 1988; Wooldridge 2010):

$$
E(y-\mu)=0 \longrightarrow \frac{1}{N} \sum_{i=1}^{N}\left(y_{i}-\widehat{\mu}\right)=0 \longrightarrow \widehat{\mu}=\frac{1}{N} \sum_{i=1}^{N} y_{i}
$$

where $N$ denotes sample size and $y_{i}$ represents the $i$ th observation of $y$ in our dataset. The estimator $\widehat{\mu}$ is known as the method of moments (MM) estimator, because we started with a population moment condition and then applied the analogy principle to obtain an estimator that depends on the observed data.

Ordinary least-squares (OLS) regression can also be viewed as an MM estimator. In the model

$$
y=\mathbf{x}^{\prime} \boldsymbol{\beta}+u
$$

we assume that $u$ has mean zero conditional on $\mathbf{x}: E(u \mid \mathbf{x})=0$. This conditional expectation implies the unconditional expectation $E(\mathbf{x} u)=\mathbf{0}$ because, using the law of iterated expectations,

$$
E(\mathbf{x} u)=E_{\mathbf{x}}\{E(\mathbf{x} u \mid \mathbf{x})\}=E_{\mathbf{x}}\{\mathbf{x} E(u \mid \mathbf{x})\}=\mathbf{0}
$$

(Using the law of iterated expectations to derive unconditional expectations based on conditional expectations, perhaps motivated by subject theory, is extremely common in GMM estimation.) Continuing,

$$
E(\mathbf{x} u)=E\left\{\mathbf{x}\left(y-\mathbf{x}^{\prime} \boldsymbol{\beta}\right)\right\}=\mathbf{0}
$$

Applying the analogy principle,

$$
E\left\{\mathbf{x}\left(y-\mathbf{x}^{\prime} \beta\right)\right\} \longrightarrow \frac{1}{N} \sum_{i=1}^{N} \mathbf{x}_{i}\left(y_{i}-\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}\right)=\mathbf{0}
$$

so that

$$
\widehat{\boldsymbol{\beta}}=\left(\sum_{i} \mathbf{x}_{i} \mathbf{x}_{i}^{\prime}\right)^{-1} \sum_{i} \mathbf{x}_{i} y_{i}
$$

which is just the more familiar formula $\widehat{\boldsymbol{\beta}}=\left(\mathbf{X}^{\prime} \mathbf{X}\right)^{-1} \mathbf{X}^{\prime} \mathbf{y}$ written using summation notation.
In both the previous examples, the number of parameters we were estimating equaled the number of moment conditions. In the first example, we estimated one parameter, $\mu$, and had one moment condition $E(y-\mu)=0$. In the second example, the parameter vector $\boldsymbol{\beta}$ had $k$ elements, as did the vector of regressors $\mathbf{x}$, yielding $k$ moment conditions. Ignoring peculiar cases, a model of $m$ equations in $m$ unknowns has a unique solution, and because the moment equations in these examples were linear, we were able to solve for the parameters analytically. Had the moment conditions been nonlinear, we would have had to use numerical techniques to solve for the parameters, but that is not a significant limitation with modern computers.

What if we have more moment conditions than parameters? Say we have $l$ moment conditions and $k$ parameters. A model of $l>k$ equations in $k$ unknowns does not have a unique solution. Any size- $k$ subset of the moment conditions would yield a consistent parameter estimate, though the parameter estimate so obtained would in general be different based on which $k$ moment conditions we used.

For concreteness, let's return to our regression model,

$$
y=\mathbf{x}^{\prime} \boldsymbol{\beta}+u
$$

but we no longer wish to assume that $E(\mathbf{x} u)=\mathbf{0}$; we suspect that the error term $u$ affects one or more elements of $\mathbf{x}$. As a result, we can no longer use the OLS estimator. Suppose we have a vector $\mathbf{z}$ with the properties that $E(\mathbf{z} u)=\mathbf{0}$, that the rank of $E\left(\mathbf{z}^{\prime} \mathbf{z}\right)$ equals $l$, and that the rank of $E\left(\mathbf{z}^{\prime} \mathbf{x}\right)=k$. The first assumption simply states that $\mathbf{z}$ is not correlated with the error term. The second assumption rules out perfect collinearity among the elements of $\mathbf{z}$. The third assumption, known as the rank condition in econometrics, ensures that $\mathbf{z}$ is sufficiently correlated with $\mathbf{x}$ and that the estimator is feasible. If some elements of $\mathbf{x}$ are not correlated with $u$, then they should also appear in $\mathbf{z}$.

If $l<k$, then the rank of $E\left(\mathbf{z}^{\prime} \mathbf{x}\right)<k$, violating the rank condition.

If $l=k$, then we can use the simpler MM estimator we already discussed; we would obtain what is sometimes called the simple instrumental-variables estimator $\widehat{\boldsymbol{\beta}}=\left(\sum_{i} \mathbf{z}_{i} \mathbf{x}_{i}^{\prime}\right)^{-1} \sum_{i} \mathbf{z}_{i} y_{i}$. The rank condition ensures that $\sum_{i} \mathbf{z}_{i} \mathbf{x}_{i}^{\prime}$ is invertible, at least in the population.

If $l>k$, the GMM estimator chooses the value, $\widehat{\boldsymbol{\beta}}$, that minimizes a quadratic function of the moment conditions. We could define

$$
\begin{equation*}
\widehat{\boldsymbol{\beta}} \equiv \arg \min _{\boldsymbol{\beta}}\left\{\frac{1}{N} \sum_{i} \mathbf{z}_{i} u_{i}(\boldsymbol{\beta})\right\}^{\prime}\left\{\frac{1}{N} \sum_{i} \mathbf{z}_{i} u_{i}(\boldsymbol{\beta})\right\} \tag{1}
\end{equation*}
$$

where for our linear regression example $u_{i}(\boldsymbol{\beta})=y_{i}-\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}$. This estimator tries to make the moment conditions as close to zero as possible. This simple estimator, however, applies equal weight to each of the moment conditions; and as we shall see later, we can obtain more efficient estimators by choosing to weight some moment conditions more highly than others.

Consider the quadratic function

$$
Q(\boldsymbol{\beta})=\left\{\frac{1}{N} \sum_{i} \mathbf{z}_{i} u_{i}(\boldsymbol{\beta})\right\}^{\prime} \mathbf{W}\left\{\frac{1}{N} \sum_{i} \mathbf{z}_{i} u_{i}(\boldsymbol{\beta})\right\}
$$

where $\mathbf{W}$ is a symmetric positive-definite matrix known as a weight matrix. Then we define the GMM estimator as

$$
\begin{equation*}
\widehat{\boldsymbol{\beta}} \equiv \arg \min _{\boldsymbol{\beta}} Q(\boldsymbol{\beta}) \tag{2}
\end{equation*}
$$

Continuing with our regression model example, if we choose

$$
\begin{equation*}
\mathbf{W}=\left(\frac{1}{N} \sum_{i} \mathbf{z}_{i} \mathbf{z}_{i}^{\prime}\right)^{-1} \tag{3}
\end{equation*}
$$

then we obtain

$$
\begin{aligned}
\widehat{\boldsymbol{\beta}}=\{ & \left.\left(\frac{1}{N} \sum_{i} \mathbf{x}_{i} \mathbf{z}_{i}^{\prime}\right)\left(\frac{1}{N} \sum_{i} \mathbf{z}_{i} \mathbf{z}_{i}^{\prime}\right)^{-1}\left(\frac{1}{N} \sum_{i} \mathbf{z}_{i} \mathbf{x}_{i}^{\prime}\right)\right\}^{-1} \times \\
& \left(\frac{1}{N} \sum_{i} \mathbf{x}_{i} \mathbf{z}_{i}^{\prime}\right)\left(\frac{1}{N} \sum_{i} \mathbf{z}_{i} \mathbf{z}_{i}^{\prime}\right)^{-1}\left(\frac{1}{N} \sum_{i} \mathbf{z}_{i} y_{i}\right)
\end{aligned}
$$

which is the well-known two-stage least-squares (2SLS) estimator. Our choice of weight matrix here was based on the assumption that $u$ was homoskedastic. A feature of GMM estimation is that by selecting different weight matrices, we can obtain estimators that can tolerate heteroskedasticity, clustering, autocorrelation, and other features of $u$. See [R] ivregress for more information about the 2SLS and linear GMM estimators.

Returning to the case where the model is "just identified", meaning that $l=k$, if we apply the GMM estimator, we will obtain the same estimate, $\widehat{\boldsymbol{\beta}}$, regardless of our choice of $\mathbf{W}$. Because $l=k$, if a unique solution exists, it will set all the sample moment conditions to zero jointly, so $\mathbf{W}$ has no impact on the value of $\boldsymbol{\beta}$ that minimizes the objective function.

We will highlight other features of the GMM estimator and the gmm command as we proceed through examples. First, though, we discuss how to specify moment equations by using substitutable expressions.

## Substitutable expressions

To use the interactive version of gmm, you define the moment equations by using substitutable expressions. In most applications, your moment conditions are of the form $E\left\{\mathbf{z}_{i} u_{i}(\boldsymbol{\beta})\right\}$, where $u_{i}(\boldsymbol{\beta})$ is a residual term that depends on the parameter vector $\boldsymbol{\beta}$ as well as variables in your dataset, though we suppress expressing the variables for notational simplicity; we refer to $u_{i}(\boldsymbol{\beta})$ as the moment equation to differentiate it from the moment conditions $E\left\{z_{i}^{\prime} u_{i}(\boldsymbol{\beta})\right\}=\mathbf{0}$.

Substitutable expressions in gmm work much like those used in nl and nlsur, though with one important difference. For the latter two commands, you type the name of the dependent variable, an equal sign, and then the regression function. For example, in $n l$, if you want to fit the function $y=f(\mathbf{x} ; \boldsymbol{\beta})+u$, you would type
$\mathrm{nl}(\mathrm{y}=<$ expression for $f(\mathrm{x} ; \beta)>), \ldots$
On the other hand, gmm requires you to write a substitutable expression for $u$; in this example, $u=y-f(\mathbf{x} ; \boldsymbol{\beta})$, so you would type
gmm ( $\mathrm{y}-<$ expression for $f(\mathbf{x} ; \beta)>$ ), ...
The advantage of writing the substitutable expression directly in terms of $u$ is that you are not restricted to fitting models with additive error terms as you are with nl and nlsur.

You specify substitutable expressions just like any other mathematical expression involving scalars and variables, such as those you would use with Stata's generate command, except that the parameters to be estimated are bound in braces. See [U] 13.2 Operators and [U] 13.3 Functions for more information on expressions. Parameter names must follow the same conventions as variable names. See [U] 11.3 Naming conventions.

For example, say that the $t$ th observation on a sample moment is

$$
u_{t}=1-\beta\left\{\left(1+r_{t+1}\right)\left(c_{t+1} / c_{t}\right)^{-\gamma}\right\}
$$

where $t$ denotes time period, $\beta$ and $\gamma$ are the parameters to be estimated, and $r$ and $c$ are variables in your dataset. Then you would type

$$
\operatorname{gmm}\left(1-\{\operatorname{beta}\} *\left((1+\mathrm{F} . \mathrm{r}) *(\mathrm{~F} . \mathrm{c} / \mathrm{c})^{\wedge}(-1 *\{\text { gamma }\})\right)\right), \ldots
$$

Because $\beta$ and $\gamma$ are parameters, we enclose them in braces. Also notice our use of the forward operator to refer to the values of $r$ and $c$ one period ahead; time-series operators are allowed in substitutable expressions as long as you have previously tsset (see [TS] tsset) your data. See [U] 13.9 Time-series operators for more information on time-series operators.

To specify initial values for some parameters, you can include an equal sign and the initial value after a parameter:

```
gmm (1 - {beta}*((1 + F.r)*(F.c/c)^(-1*{gamma=1}))), ...
```

would initialize $\gamma$ to be one. If you do not specify an initial value for a parameter, it is initialized to zero.

Frequently, even nonlinear functions contain linear combinations of variables. As an example, suppose you have this moment equation:

$$
u=\left\{y-\exp \left(\beta_{1} x_{1}+\beta_{2} x_{2}+\beta_{3} x_{3}\right)\right\} / \exp \left(\beta_{1} x_{1}+\beta_{2} x_{2}+\beta_{3} x_{3}\right)
$$

Instead of typing

```
gmm ((y - exp({beta1}*x1 + {beta2}*x2 + {beta3}*x3)) / ///
    exp({beta1}*x1 + {beta2}*x2 + {beta3}*x3)) ...
```

```
you can type
gmm ((y - exp({xb:x1 x2 x3})) / exp({xb:})) .....
```

The notation $\{\mathrm{xb}: \mathrm{x} 1 \mathrm{x} 2 \mathrm{x} 3\}$ tells gmm that you want a linear combination of the variables $\mathrm{x} 1, \mathrm{x} 2$, and $x 3$. We named this linear combination $x b$, so gmm will name the three parameters corresponding to the three variables $x b \_x 1, x b \_x 2$, and $x b \_x 3$. You can name the linear combination anything you wish (subject to Stata's naming conventions for variable names); gmm then names the parameter corresponding to variable $x l c \_x$, where $l c$ is the name of your linear combination. You cannot use the same name for both an individual parameter and a linear combination. You can, however, refer to one parameter in a linear combination after it has been declared as you would any other parameter by using the notation $\left\{l c \_x\right\}$. Linear combinations do not include a constant term.

Once we have declared the variables in the linear combination $x b$, we can subsequently refer to the linear combination in our substitutable expression by using the notation $\mathrm{xb}:$. The colon is not optional; it tells gmm that you are referring to a previously declared linear combination, not an individual parameter. This shorthand notation is also handy when specifying derivatives, as we will show later.

In general, there are three rules to follow when defining substitutable expressions:

1. Parameters of the model are bound in braces: $\{b 0\}$, \{param\}, etc.
2. Initial values for parameters are given by including an equal sign and the initial value inside the braces: $\{b 0=1\}$, $\{$ param $=3.571\}$, etc.
3. Linear combinations of variables can be included using the notation \{eqname: varlist $\}$ : $\{\mathrm{xb}$ : mpg price weight\}, \{score: wxz\}, etc. Parameters of linear combinations are initialized to zero.

If you specify initial values by using the from() option, they override whatever initial values are given within the substitutable expression. Substitutable expressions are so named because, once values are assigned to the parameters, the resulting expressions can be handled by generate and replace.
> Example 1: OLS regression
In Introduction, we stated that OLS is an MM estimator. Say that we want to fit the model

$$
\operatorname{mpg}=\beta_{0}+\beta_{1} \text { weight }+\beta_{2} \text { length }+u
$$

where $u$ is an i.i.d. error term. We type
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. gmm (mpg - {b1}*weight - {b2}*length - {b0}), instruments(weight length)
. gmm (mpg - {b1}*weight - {b2}*length - {b0}), instruments(weight length)
Step 1
Iteration 0: GMM criterion $Q(b)=475.4138$
Iteration 1: GMM criterion $Q(b)=3.305 \mathrm{e}-20$
Iteration 2: GMM criterion $Q(b)=3.795 \mathrm{e}-27$
Step 2
Iteration 0: GMM criterion $Q(b)=7.401 \mathrm{e}-28$
Iteration 1: GMM criterion $Q(b)=3.771 \mathrm{e}-31$
GMM estimation
Number of parameters $=3$
Number of moments $=3$
Initial weight matrix: Unadjusted $\quad$ Number of obs $=\quad 74$
GMM weight matrix: Robust

|  | Coef. | Robust |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | Std. Err. | $z$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |  |
| /b1 | -.0038515 | .0019472 | -1.98 | 0.048 | -.0076678 | -.0000351 |
| /b2 | -.0795935 | .0677528 | -1.17 | 0.240 | -.2123864 | .0531995 |
| /b0 | 47.88487 | 7.505985 | 6.38 | 0.000 | 33.17341 | 62.59633 |

Instruments for equation 1: weight length _cons
Recall that the moment condition for OLS regression is $E(\mathbf{x} u)=\mathbf{0}$, where $\mathbf{x}$, the list of instruments, is the same as the list of regressors in the model. In our command, we defined the residual term, $u$, inside parentheses by using a substitutable expression; because linear combinations declared in substitutable expressions do not include a constant term, we included our own (b0). Inside the instruments () option, we listed our instruments; by default, gmm includes a constant term among the instrument list.

Because the number of moments equals the number of parameters we are estimating, the model is said to be "just identified" or "exactly identified." Therefore, the choice of weight matrix has no impact on the solution to (2), and the criterion function $Q(\boldsymbol{\beta})$ achieves its minimum value at zero.

The OLS estimator is a one-step GMM estimator, but we did not bother to specify the onestep option because the model is just identified. Doing a second step of GMM estimation affects neither the point estimates nor the standard errors, so to keep the syntax as simple as possible, we did not include the onestep option. The first step of estimation resulted in $Q(\boldsymbol{\beta})=0$ as expected, and the second step of estimation did not change the minimized value of $Q(\boldsymbol{\beta}) .\left(4 \times 10^{-27}\right.$ and $3 \times 10^{-31}$ are both zero for all practical purposes.)

When you do not specify either the wmatrix() or the vce() option, gmm reports heteroskedasticityrobust standard errors. The parameter estimates reported here match those that we would obtain from the command

```
. regress mpg weight length, vce(robust)
```

The standard errors reported by that regress command would be larger than those reported by gmm by a factor of sqrt(74/71) because regress makes a small-sample adjustment to the estimated variance matrix while gmm does not. Likewise, had we specified the vce (unadjusted) option with our gmm command, then our standard errors would differ by a factor of $\operatorname{sqrt}(74 / 71)$ from those reported by regress without the vce(robust) option.

Using the notation for linear combinations of parameters, we could have typed

```
. gmm (mpg - {xb: weight length} - {b0}), instruments(weight length)
```

and obtained identical results. Instead of having parameters b1 and b2, with this syntax we would have parameters $\mathrm{xb} \_$weight and $\mathrm{xb} \_$length.

## > Example 2: Instrumental-variables regression

In Introduction, we mentioned that 2 SLS can be viewed as a GMM estimator. In example 1 of $[R]$ ivregress, we fit by 2SLS a model of rental rates (rent) as a function of the value of owner-occupied housing (hsngval) and the percentage of the population living in urban areas (pcturban):

$$
\text { rent }=\beta_{0}+\beta_{1} \text { hsngval }+\beta_{2} \text { pcturban }+u
$$

by 2 SLS. We argued that random shocks that affect rental rates likely also affect housing values, so we treated hsngval as an endogenous variable. As additional instruments, we used family income, faminc, and three regional dummies (reg2-reg4).

To replicate the results of ivregress 2sls by using gmm, we type

```
. use http://www.stata-press.com/data/r13/hsng2
(1980 Census housing data)
. gmm (rent - \{xb:hsngval pcturban\} - \{b0\}),
> instruments(pcturban faminc reg2-reg4) vce(unadjusted) onestep
Step 1
Iteration 0: GMM criterion \(Q(b)=56115.03\)
Iteration 1: GMM criterion \(Q(b)=110.91583\)
Iteration 2: GMM criterion \(Q(b)=110.91583\)
GMM estimation
Number of parameters \(=3\)
Number of moments \(=6\)
Initial weight matrix: Unadjusted \(\quad\) Number of obs \(=50\)
\begin{tabular}{r|rccccr}
\hline & Coef. & Std. Err. & z & P>|z| & [95\% Conf. Interval] \\
\hline /xb_hsngval & .0022398 & .0003284 & 6.82 & 0.000 & .0015961 & .0028836 \\
/xb_pcturban & .081516 & .2987652 & 0.27 & 0.785 & -.504053 & .667085 \\
/b0 & 120.7065 & 15.22839 & 7.93 & 0.000 & 90.85942 & 150.5536 \\
\hline
\end{tabular}
```

Instruments for equation 1: pcturban faminc reg2 reg3 reg4 _cons
We specified vce(unadjusted) so that we would obtain an unadjusted VCE matrix and our standard errors would match those reported in [R] ivregress.

Pay attention to how we specified the instruments() option. In Introduction, we mentioned that the moment conditions for the 2SLS estimator are $E(\mathbf{z} u)=\mathbf{0}$, and we mentioned that if some elements of $\mathbf{x}$ (the regressors) are not endogenous, then they should also appear in $\mathbf{z}$. In this model, we assume the regressor pcturban is exogenous, so we included it in the list of instrumental variables. Commands like ivregress, ivprobit, and ivtobit accept standard varlists, so they can deduce the exogenous regressors in the model. Because gmm accepts arbitrary functions in the form of substitutable expressions, it has no way of discerning the exogenous variables of the model on its own.

Also notice that we specified the onestep option. The 2SLS estimator is a one-step GMM estimator that is based on a weight matrix that assumes the error terms are i.i.d. Unlike the previous example, here we had more instruments than parameters, so the minimized value of $Q(\boldsymbol{\beta})$ is nonzero. We discuss the weight matrix and its relationship to two-step estimation next.

## The weight matrix and two-step estimation

Recall our definition of the GMM estimator given in (2). The estimator, $\widehat{\boldsymbol{\beta}}$, depends on the choice of the weight matrix, $\mathbf{W}$. Under relatively mild assumptions, our estimator, $\widehat{\boldsymbol{\beta}}$, is consistent regardless of the choice of $\mathbf{W}$, so how are we to decide what $\mathbf{W}$ to use? The most common solution is to use the two-step estimator, which we now describe.

A key result in Hansen's (1982) seminal paper is that if we denote by $\mathbf{S}$ the covariance matrix of the moment conditions, then the optimal (in a way we make precise later) GMM estimator is the one that uses a weight matrix equal to the inverse of the moment covariance matrix. That is, if we let $\mathbf{S}=\operatorname{Cov}(\mathbf{z} u)$, then we want to use $\mathbf{W}=\mathbf{S}^{-1}$. But how do we obtain $\mathbf{S}$ in the first place?

If we assume that the errors are i.i.d., then

$$
\operatorname{Cov}(\mathbf{z} u)=E\left(u^{2} \mathbf{z \mathbf { z } ^ { \prime }}\right)=\sigma^{2} E\left(\mathbf{z z}^{\prime}\right)
$$

where $\sigma^{2}$ is the variance of $u$. Because $\sigma^{2}$ is a positive scalar, we can ignore it when solving (2). Thus we compute

$$
\begin{equation*}
\widehat{\mathbf{W}}_{1}=\left(\frac{1}{N} \sum_{i} \mathbf{z}_{i} \mathbf{z}_{i}^{\prime}\right)^{-1} \tag{4}
\end{equation*}
$$

which does not depend on any unknown model parameters. (Notice that $\widehat{\mathbf{W}}_{1}$ is the same weight matrix used in 2SLS.) Given $\widehat{\mathbf{W}}_{1}$, we can solve (2) to obtain an initial estimate, say, $\widehat{\boldsymbol{\beta}}_{1}$.

Our estimate, $\widehat{\boldsymbol{\beta}}_{1}$, is consistent, so by Slutsky's theorem, the sample residuals $\widehat{u}$ computed at this value of $\boldsymbol{\beta}$ will also be consistent. Using virtually the same arguments used to justify the Huber/Eicker/White heteroskedasticity-robust VCE, if we assume that the residuals are independent though not identically distributed, we can estimate $\mathbf{S}$ as

$$
\widehat{\mathbf{S}}=\frac{1}{N} \sum_{i} \widehat{u}_{i}^{2} \mathbf{z}_{i} \mathbf{z}_{i}^{\prime}
$$

Then, in the second step, we re-solve (2), using $\widehat{\mathbf{W}}_{2}=\widehat{\mathbf{S}}^{-1}$, yielding the two-step GMM estimate $\widehat{\beta}_{2}$. If the residuals exhibit clustering, you can specify wmatrix (cluster varname) so that gmm computes a weight matrix that does not assume the $u_{i}$ 's are independent within clusters identified by varname. You can specify wmatrix (hac ...) to obtain weight matrices that are suitable for when the $u_{i}$ 's exhibit autocorrelation as well as heteroskedasticity.

We could take the point estimates from the second round of estimation and use them to compute yet another weight matrix, $\widehat{\mathbf{W}}_{3}$, say, re-solve (2) yet again, and so on, stopping when the parameters or weight matrix do not change much from one iteration to the next. This procedure is known as the iterative GMM estimator and is obtained with the igmm option. Asymptotically, the two-step and iterative GMM estimators have the same distribution. However, Hall (2005, 90) suggests that the iterative estimator may have better finite-sample properties.

Instead of computing $\widehat{\mathbf{W}}_{1}$ as in (4), we could simply choose $\widehat{\mathbf{W}}_{1}=\mathbf{I}$, the identity matrix. The initial estimate, $\widehat{\boldsymbol{\beta}}_{1}$, would still be consistent. You can request this behavior by specifying the winitial (identity) option. However, if you specify all your moment equations of the form $E(\mathbf{z u})=\mathbf{0}$, we recommend using the default winitial (unadjusted) instead; the rescaling of the moment conditions implied by using a homoskedastic initial weight matrix makes the numerical routines used to solve (2) more stable.

If you fit a model with more than one of the moment equations of the form $E\{h(\mathbf{z} ; \boldsymbol{\beta})\}=\mathbf{0}$, then you must use winitial(identity) or winitial (unadjusted, independent). With moment equations of that form, you do not specify a list of instruments, and gmm cannot evaluate (4)-the matrix expression in parentheses would necessarily be singular, so it cannot be inverted.

## $>$ Example 3: Two-step linear GMM estimator

From the previous discussion and the comments in Introduction, we see that the linear 2SLS estimator is a one-step GMM estimator where we use the weight matrix defined in (4) that assumes the errors are i.i.d. If we use the 2SLS estimate of $\boldsymbol{\beta}$ to obtain the sample residuals, compute a new weight matrix based on those residuals, and then do a second step of GMM estimation, we obtain the linear two-step GMM estimator as implemented by ivregress gmm.

In example 3 of $[R]$ ivregress, we fit the model of rental rates as discussed in example 2 above. We now allow the residuals to be heteroskedastic, though we will maintain our assumption that they are independent. We type

```
. gmm (rent - {xb:hsngval pcturban} - {b0}), inst(pcturban faminc reg2-reg4)
Step 1
Iteration 0: GMM criterion Q(b) = 56115.03
Iteration 1: GMM criterion Q(b) = 110.91583
Iteration 2: GMM criterion Q(b) = 110.91583
Step 2
Iteration 0: GMM criterion Q(b) = . }240608
Iteration 1: GMM criterion Q(b) = . }1367280
Iteration 2: GMM criterion Q(b) = . }13672801 (backed up
GMM estimation
Number of parameters = 3
Number of moments = 6
Initial weight matrix: Unadjusted Number of obs = 50
GMM weight matrix: Robust
\begin{tabular}{r|rccccr}
\hline & Coef. & Robust \\
& Std. Err. & z & P>|z| & [95\% Conf. Interval] \\
\hline /xb_hsngval & .0014643 & .0004473 & 3.27 & 0.001 & .0005877 & .002341 \\
/xb_pcturban & .7615482 & .2895105 & 2.63 & 0.009 & .1941181 & 1.328978 \\
/b0 & 112.1227 & 10.80234 & 10.38 & 0.000 & 90.95052 & 133.2949 \\
\hline
\end{tabular}
```

Instruments for equation 1: pcturban faminc reg2 reg3 reg4 _cons
By default, gmm computes a heteroskedasticity-robust weight matrix before the second step of estimation, though we could have specified wmatrix (robust) if we wanted to be explicit. Because we did not specify the vce() option, gmm used a heteroskedasticity-robust one. Our results match those in example 3 of $[R]$ ivregress. Moreover, the only substantive difference between this example and example 2 is that here we did not specify the onestep option, so we obtain the two-step estimates.

## Obtaining standard errors

This section is a bit more theoretical and can be skipped on first reading. However, the information is sufficiently important that you should return to this section at some point.

So far in our discussion, we have focused on point estimation without much mention of how we obtain the standard errors of the estimates. We also mentioned that if we choose $\mathbf{W}$ to be the inverse of the covariance matrix of the moment conditions, then we obtain the "optimal" GMM estimator. We elaborate those points now.

Using mostly standard statistical arguments, we can show that for the GMM estimator defined in (2), the variance of $\widehat{\boldsymbol{\beta}}$ is given by

$$
\begin{equation*}
\operatorname{Var}(\widehat{\boldsymbol{\beta}})=\frac{1}{N}\left\{\overline{\mathbf{G}}(\widehat{\boldsymbol{\beta}})^{\prime} \mathbf{W} \overline{\mathbf{G}}(\widehat{\boldsymbol{\beta}})\right\}^{-1} \overline{\mathbf{G}}(\widehat{\boldsymbol{\beta}})^{\prime} \mathbf{W} \mathbf{S W} \overline{\mathbf{G}}(\widehat{\boldsymbol{\beta}})\left\{\overline{\mathbf{G}}(\widehat{\boldsymbol{\beta}})^{\prime} \mathbf{W} \overline{\mathbf{G}}(\widehat{\boldsymbol{\beta}})\right\}^{-1} \tag{5}
\end{equation*}
$$

where

$$
\overline{\mathbf{G}}(\widehat{\boldsymbol{\beta}})=\left.\frac{1}{N} \sum_{i} \mathbf{z}_{i} \frac{\partial u_{i}}{\partial \boldsymbol{\beta}}\right|_{\boldsymbol{\beta}=\widehat{\boldsymbol{\beta}}} \quad \text { or } \quad \overline{\mathbf{G}}(\widehat{\boldsymbol{\beta}})=\left.\frac{1}{N} \sum_{i} \frac{\partial \mathbf{h}_{i}}{\partial \boldsymbol{\beta}}\right|_{\boldsymbol{\beta}=\widehat{\boldsymbol{\beta}}}
$$

as the case may be and $\mathbf{S}=E\left(\mathbf{z} u u^{\prime} \mathbf{z}^{\prime}\right)$.
Assuming the vce (unadjusted) option is not specified, gmm reports standard errors based on the robust variance matrix defined in (5). For the two-step estimator, $\mathbf{W}$ is the weight matrix requested using the wmatrix() option, and it is calculated based on the residuals obtained after the first estimation step. The second-step point estimates and residuals are obtained, and $\mathbf{S}$ is calculated based on the specification of the vce() option. For the iterated estimator, $\mathbf{W}$ is calculated based on the second-to-last round of estimation, while $\mathbf{S}$ is based on the residuals obtained after the last round of estimation. Computation of the covariance matrix for the one-step estimator is, perhaps surprisingly, more involved; we discuss the covariance matrix with the one-step estimator in the technical note at the end of this section.

If we choose the weight matrix to be the inverse of the covariance matrix of the moment conditions so that $\mathbf{W}=\mathbf{S}^{-1}$, then (5) simplifies substantially:

$$
\begin{equation*}
\operatorname{Var}(\widehat{\boldsymbol{\beta}})=\frac{1}{N}\left\{\overline{\mathbf{G}}(\widehat{\boldsymbol{\beta}})^{\prime} \mathbf{W} \overline{\mathbf{G}}(\widehat{\boldsymbol{\beta}})\right\}^{-1} \tag{6}
\end{equation*}
$$

The GMM estimator constructed using this choice of weight matrix along with the covariance matrix in (6) is known as the "optimal" GMM estimator. One can show that if in fact $\mathbf{W}=\mathbf{S}^{-1}$, then the variance in (6) is smaller than the variance in (5) of any other GMM estimator based on the same moment conditions but with a different choice of weight matrix. Thus the optimal GMM estimator is also known as the efficient GMM estimator, because it has the smallest variance of any estimator based on the given moment conditions.

To obtain standard errors from gmm based on the optimal GMM estimator, you specify the vce(unadjusted) option. We call that VCE unadjusted because we do not recompute the residuals after estimation to obtain the matrix $\mathbf{S}$ required in (5) or allow for the fact that those residuals may not be i.i.d. Some statistical packages by default report standard errors based on (6) and offer standard errors based on (5) only as an option or not at all. While the optimal GMM estimator is theoretically appealing, Cameron and Trivedi $(2005,177)$ suggest that in finite samples it need not perform better than the GMM estimator that uses (5) to obtain standard errors.

## - Technical note

Computing the covariance matrix of the parameters after using the one-step estimator is actually a bit more complex than after using the two-step or iterative estimators. We can illustrate most of the intricacies by using linear regression with moment conditions of the form $E\left\{\mathbf{x}\left(y-\mathbf{x}^{\prime} \boldsymbol{\beta}\right)\right\}=\mathbf{0}$.

If you specify winitial (unadjusted) and vce(unadjusted), then the initial weight matrix will be computed as

$$
\begin{equation*}
\widehat{\mathbf{W}}_{1}=\left(\frac{1}{N} \sum_{i} \mathbf{x}_{i} \mathbf{x}_{i}^{\prime}\right)^{-1} \tag{7}
\end{equation*}
$$

Moreover, for linear regression, we can show that

$$
\overline{\mathbf{G}}(\widehat{\boldsymbol{\beta}})=\frac{1}{N} \sum_{i} \mathbf{x}_{i} \mathbf{x}_{i}^{\prime}
$$

so that (6) becomes

$$
\begin{align*}
\operatorname{Var}(\widehat{\boldsymbol{\beta}}) & =\frac{1}{N}\left\{\left(\frac{1}{N} \sum_{i} \mathbf{x}_{i} \mathbf{x}_{i}^{\prime}\right)\left(\frac{1}{N} \sum_{i} \mathbf{x}_{i} \mathbf{x}_{i}^{\prime}\right)^{-1}\left(\frac{1}{N} \sum_{i} \mathbf{x}_{i} \mathbf{x}_{i}^{\prime}\right)\right\}^{-1} \\
& =\left(\sum_{i} \mathbf{x}_{i} \mathbf{x}_{i}^{\prime}\right)^{-1} \\
& =\left(\mathbf{X}^{\prime} \mathbf{X}\right)^{-1} \tag{8}
\end{align*}
$$

However, we know that the nonrobust covariance matrix for the OLS estimator is actually $\widehat{\sigma}^{2}\left(\mathbf{X}^{\prime} \mathbf{X}\right)^{-1}$. What is missing from (8) is the scalar $\widehat{\sigma}^{2}$, the estimated variance of the residuals. When you use the one-step estimator and specify winitial (unadjusted), the weight matrix (7) does not include the $\widehat{\sigma}^{2}$ term because gmm does not have a consistent estimate of $\boldsymbol{\beta}$ from which it can then estimate $\sigma^{2}$. The point estimates are still correct, because multiplying the weight matrix by a scalar factor does not affect the solution to the minimization problem.

To circumvent this issue, if you specify winitial (unadjusted) and vce (unadjusted), gmm uses the estimated $\widehat{\boldsymbol{\beta}}$ (which is consistent) to obtain a new unadjusted weight matrix that does include the term $\widehat{\sigma}^{2}$ so that evaluating (6) will yield correct standard errors.

If you use the two-step or iterated GMM estimators, this extra effort is not needed to obtain standard errors because the first-step (and subsequent steps') estimate of $\boldsymbol{\beta}$ is consistent and can be used to estimate $\sigma^{2}$ or some other weight matrix based on the wmatrix() option. Straightforward algebra shows that this extra effort is also not needed if you request any type of adjusted (robust) covariance matrix with the one-step estimator.

A similar issue arises when you specify winitial (identity) and vce(unadjusted) with the one-step estimator. Again the solution is to compute an unadjusted weight matrix after obtaining $\widehat{\beta}$ so that (6) provides the correct standard errors.

We have illustrated the problem and solution using a single-equation linear model. However, the problem arises whenever you use the one-step estimator with an unadjusted VCE, regardless of the number of equations; and gmm handles all the details automatically. Computation of Hansen's $J$ statistic presents an identical issue, and gmm takes care of that as well.

If you supply your own initial weight matrix by using winitial(matname), then the standard errors (as well as the $J$ statistic reported by estat overid) are based on that weight matrix. You should verify that the weight matrix you provide will yield appropriate statistics.

## Exponential (Poisson) regression models

Exponential regression models are frequently encountered in applied work. For example, they can be used as alternatives to linear regression models on log-transformed dependent variables, obviating the need for post-hoc transformations to obtain predicted values in the original metric of the dependent variable. When the dependent variable represents a discrete count variable, they are also known as Poisson regression models; see Cameron and Trivedi (2013).

For now, we consider models of the form

$$
\begin{equation*}
y=\exp \left(\mathbf{x}^{\prime} \boldsymbol{\beta}\right)+u \tag{9}
\end{equation*}
$$

where $u$ is a zero-mean additive error term so that $E(y)=\exp \left(\mathbf{x}^{\prime} \boldsymbol{\beta}\right)$. Because the error term is additive, if $\mathbf{x}$ represents strictly exogenous regressors, then we have the population moment condition

$$
\begin{equation*}
E\left[\mathbf{x}\left\{y-\exp \left(\mathbf{x}^{\prime} \boldsymbol{\beta}\right)\right\}\right]=\mathbf{0} \tag{10}
\end{equation*}
$$

Moreover, because the number of parameters in the model is equal to the number of instruments, there is no point to using the two-step GMM estimator.
$>$ Example 4: Exponential regression
Cameron and Trivedi (2010, 323) fit a model of the number of doctor visits based on whether the patient has private insurance, whether the patient has a chronic disease, gender, and income. Here we fit that model by using gmm. To allow for potential excess dispersion, we will obtain a robust VCE matrix, which is the default for gmm anyway. We type

| Step 1 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Iteration 0: GMM criterion Q(b) = 16.853973 |  |  |  |  |  |  |
| Iteration 1: GMM criterion Q (b) |  |  | 2706472 |  |  |  |
| Iteration 2: GMM criterion Q (b) |  |  | 9088097 |  |  |  |
| Iteration 3: GMM criterion Q (b) |  |  | 0041101 |  |  |  |
| Iteration 4: GMM criterion Q (b) |  |  | 939e-09 |  |  |  |
| Iteration 5: GMM criterion Q (b) |  |  | 572e-19 |  |  |  |
| GMM estimation |  |  |  |  |  |  |
| Number of parameters = 5 |  |  |  |  |  |  |
| Number of moments $=5$ |  |  |  |  |  |  |
| Initial weight matrix: Unadjusted |  |  |  |  | mer of obs | $=4412$ |
|  |  | Robust |  |  |  |  |
|  | Coef. | Std. Err. | z | $P>\|z\|$ | [95\% Conf | Interval] |
| /xb_private | . 7986654 | . 1089891 | 7.33 | 0.000 | . 5850507 | 1.01228 |
| /xb_chronic | 1.091865 | . 0559888 | 19.50 | 0.000 | . 9821291 | 1.201601 |
| /xb_female | . 4925481 | . 0585298 | 8.42 | 0.000 | . 3778317 | . 6072644 |
| /xb_income | . 003557 | . 0010824 | 3.29 | 0.001 | . 0014356 | . 0056784 |
| /b0 | -. 2297263 | . 1108607 | -2.07 | 0.038 | -. 4470093 | -. 0124434 |

[^23]Our point estimates agree with those reported by Cameron and Trivedi to at least six significant digits; the small discrepancies are attributable to different optimization techniques and convergence criteria being used by gmm and poisson. The standard errors differ by a factor of $\operatorname{sqrt}(4412 / 4411)$ because gmm uses $N$ in the denominator of the formula for the robust covariance matrix, while the robust covariance matrix estimator used by poisson uses $N-1$.

## - Technical note

That the GMM and maximum likelihood estimators of the exponential regression model coincide is not a general property of these two classes of estimators. The maximum likelihood estimator solves the score equations

$$
\frac{1}{N} \sum_{i=1}^{N} \frac{\partial \ln \ell_{i}}{\partial \boldsymbol{\beta}}=\mathbf{0}
$$

where $l_{i}$ is the likelihood for the $i$ th observation. These score equations can be viewed as the sample analogues of the population moment conditions

$$
E\left\{\frac{\partial \ln \ell_{i}}{\partial \boldsymbol{\beta}}\right\}=\mathbf{0}
$$

establishing that maximum likelihood estimators represent a subset of the class of GMM estimators.
For the Poisson model,

$$
\ln \ell_{i}=-\exp \left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}\right)+y_{i} \mathbf{x}_{i}^{\prime} \boldsymbol{\beta}-\ln y_{i}!
$$

so the score equations are

$$
\frac{1}{N} \sum_{i=1}^{N} \mathbf{x}_{i}\left\{y_{i}-\exp \left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}\right)\right\}=\mathbf{0}
$$

which are just the sample moment conditions implied by (10) that we used in the previous example. That is why our results using gmm match Cameron and Trivedi's results using poisson.

On the other hand, an intuitive set of moment conditions to consider for GMM estimation of a probit model is

$$
E\left[\mathbf{x}\left\{y-\Phi\left(\mathbf{x}^{\prime} \boldsymbol{\beta}\right)\right\}\right]=\mathbf{0}
$$

where $\Phi()$ is the standard normal cumulative distribution function. Differentiating the likelihood function for the maximum likelihood probit estimator, we can show that the corresponding score equations are

$$
\frac{1}{N} \sum_{i=1}^{N}\left[\mathbf{x}_{i}\left\{y_{i} \frac{\phi\left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}\right)}{\Phi\left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}\right)}-\left(1-y_{i}\right) \frac{\phi\left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}\right)}{1-\Phi\left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}\right)}\right\}\right]=\mathbf{0}
$$

where $\phi()$ is the standard normal density function. These two moment conditions are not equivalent, so the maximum likelihood and GMM probit estimators are distinct.
> Example 5: Comparison of GMM and maximum likelihood
Using the automobile dataset, here we fit a probit model of foreign on gear_ratio, length, and headroom using first the score equations and then the intuitive set of GMM equations. We type

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. global xb "{b1}*gear_ratio + {b2}*length + {b3}*headroom + {b0}"
. global phi "normalden($xb)"
. global Phi "normal($xb)"
. gmm (foreign*$phi/$Phi - (1-foreign)*$phi/(1-$Phi)),
> instruments(gear_ratio length headroom) onestep
    (output omitted)
. estimates store ml
. gmm (foreign - $Phi), instruments(gear_ratio length headroom) onestep
    (output omitted)
. estimates store gmm
. estimates table ml gmm, b se
```



The coefficients on gear_ratio and length are close for the two estimators. The GMM estimate of the coefficient on headroom is twice that of the maximum likelihood estimate, though the relatively large standard errors imply that this difference is not significant. You can verify that the coefficients in the column marked "ml" match those you would obtain using probit. We have not discussed the differences among standard errors based on the various GMM and maximum-likelihood covariance matrix estimators to avoid tedious algebra, though you can verify that the robust covariance matrix after one-step GMM estimation differs by only a finite-sample adjustment factor of $(N / N-1)$ from the robust covariance matrix reported by probit. Both the maximum likelihood and GMM probit estimators require the normality assumption, and the maximum likelihood estimator is efficient if that normality assumption is correct; therefore, in this particular example, there is no reason to prefer the GMM estimator.

We can modify (10) easily to allow for endogenous regressors. Suppose that $x_{j}$ is endogenous in the sense that $E\left(u \mid x_{j}\right) \neq 0$. Then (10) is no longer a valid moment condition. However, suppose we have some variables other than $\mathbf{x}$ such that $E(u \mid \mathbf{z})=\mathbf{0}$. We can instead use the moment conditions

$$
\begin{equation*}
E(\mathbf{z} u)=E\left[\mathbf{z}\left\{y-\exp \left(\mathbf{x}^{\prime} \boldsymbol{\beta}\right)\right\}\right]=\mathbf{0} \tag{11}
\end{equation*}
$$

As usual, if some elements of $\mathbf{x}$ are exogenous, then they should appear in $\mathbf{z}$ as well.

## > Example 6: Exponential regression with endogenous regressors

Returning to the model discussed in example 4, here we treat income as endogenous; unobservable factors that determine a person's income may also affect the number of times a person visits a doctor. We use a person's age and race as instruments. These are valid instruments if we believe that age and race influence a person's income but do not have a direct impact on the number of doctor visits. (Whether this belief is justified is another matter; we test that belief in [R] gmm postestimation.) Because we have more instruments (seven) than parameters (five), we have an overidentified model. Therefore, the choice of weight matrix does matter. We will utilize the default two-step GMM estimator. In the first step, we will use a weight matrix that assumes the errors are i.i.d. In the second step, we will use a weight matrix that assumes heteroskedasticity. When you specify twostep, these are the defaults for the first- and second-step weight matrices, so we do not have to use the winitial() or wmatrix () options. We will again obtain a robust VCE, which is also the default. We type

```
. use http://www.stata-press.com/data/r13/docvisits
. gmm (docvis - exp({xb:private chronic female income}+{b0})),
> instruments(private chronic female age black hispanic) twostep
Step 1
Iteration 0: GMM criterion Q(b) = 16.910173
Iteration 1: GMM criterion Q(b) = . }8227610
Iteration 2: GMM criterion Q(b) = . }2183203
Iteration 3: GMM criterion Q(b) = . }1268593
Iteration 4: GMM criterion Q(b) = . }1267236
Iteration 5: GMM criterion Q(b) = . }1267236
Step 2
Iteration 0: GMM criterion Q(b) = .00234641
Iteration 1: GMM criterion Q(b) = .00215957
Iteration 2: GMM criterion Q(b) = .00215911
Iteration 3: GMM criterion Q(b) = .00215911
GMM estimation
Number of parameters = 5
Number of moments = 7
Initial weight matrix: Unadjusted Number of obs = 4412
GMM weight matrix: Robust
```

|  | Robust |  |  |  |  |  |
| ---: | ---: | :---: | :---: | :---: | :---: | ---: | ---: |
|  | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| /xb_private | .535335 | .1599039 | 3.35 | 0.001 | .2219291 | .8487409 |
| /xb_chronic | 1.090126 | .0617659 | 17.65 | 0.000 | .9690668 | 1.211185 |
| /xb_female | .6636579 | .0959884 | 6.91 | 0.000 | .4755241 | .8517918 |
| /xb_income | .0142855 | .0027162 | 5.26 | 0.000 | .0089618 | .0196092 |
| /b0 | -.5983477 | .138433 | -4.32 | 0.000 | -.8696713 | -.327024 |

Instruments for equation 1: private chronic female age black hispanic _cons
Once we control for the endogeneity of income, we find that its coefficient has quadrupled in size. Additionally, access to private insurance has less of an impact on the number of doctor visits and gender has more of an impact.

## - Technical note

Although perhaps at first tempting, unlike the Poisson model, you cannot simply replace $\mathbf{x}$ in the moment conditions for the probit (or logit) model with a vector of instruments, $\mathbf{z}$, if you have endogenous regressors. See Wilde (2008).

Mullahy (1997) considers a slightly more complicated version of the exponential regression model that incorporates nonadditive unobserved heterogeneity. His model can be written as

$$
y_{i}=\exp \left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}\right) \eta_{i}+\epsilon_{i}
$$

where $\eta_{i}>0$ is an unobserved heterogeneity term that may be correlated with $\mathbf{x}_{i}$. One result from his paper is that instead of using the additive moment condition (10), we can use the multiplicative moment condition

$$
\begin{equation*}
E\left\{\mathbf{z} \frac{y-\exp \left(\mathbf{x}^{\prime} \boldsymbol{\beta}\right)}{\exp \left(\mathbf{x}^{\prime} \boldsymbol{\beta}\right)}\right\}=E\left[\mathbf{z}\left\{y \exp \left(-\mathbf{x}^{\prime} \boldsymbol{\beta}\right)-1\right\}\right]=\mathbf{0} \tag{12}
\end{equation*}
$$

Windmeijer and Santos Silva (1997) discuss the use of additive versus multiplicative moment conditions with endogenous regressors and note that a set of instruments that satisfies the additive moment conditions will not also satisfy the multiplicative moment conditions. They remark that which to use is an empirical issue that can at least partially be settled by using the test of overidentifying restrictions that is implemented by estat overid after gmm to ascertain whether the instruments for a given model are valid. See $[R]$ gmm postestimation for information on the test of overidentifying restrictions.

## Specifying derivatives

By default, gmm calculates derivatives numerically, and the method used produces accurate results for the vast majority of applications. However, if you refit the same model repeatedly or else have the derivatives available, then gmm will run more quickly if you supply it with analytic derivatives.

When you use the interactive version of gmm, you specify derivatives using substitutable expressions in much the same way you specify the moment equations. There are three rules you must follow:

1. As with the substitutable expressions that define residual equations, you bind parameters of the model in braces: \{b0\}, \{param\}, etc.
2. You must specify a derivative for each parameter that appears in each moment equation. If a parameter does not appear in a moment equation, then you do not specify a derivative for that parameter in that moment equation.
3. If you declare a linear combination in an equation, then you specify a derivative with respect to that linear combination. gmm applies the chain rule to obtain the derivatives with respect to the individual parameters encompassed by that linear combination.
We illustrate with several examples.

## > Example 7: Derivatives for a single-equation model

Consider a simple exponential regression model with one exogenous regressor and a constant term. We have

$$
u_{i}=y_{i}-\exp \left(\beta_{0}+\beta_{1} x_{i}\right)
$$

Now

$$
\frac{\partial u_{i}}{\partial \beta_{0}}=-\exp \left(\beta_{0}+\beta_{1} x_{i}\right) \quad \text { and } \quad \frac{\partial u_{i}}{\partial \beta_{1}}=-x_{i} \exp \left(\beta_{0}+\beta_{1} x_{i}\right)
$$

In Stata, we type

```
. gmm (docvis - exp({b0} + {b1}*income)), instruments(income)
> deriv(/b0 = -1*exp({b0} + {b1}*income))
> deriv(/b1 = -1*income*exp({b0}+{b1}*income)) onestep
Step 1
Iteration 0: GMM criterion Q(b) = 9.1548611
Iteration 1: GMM criterion Q(b) = 3.5146131
Iteration 2: GMM criterion Q(b) = .01344695
Iteration 3: GMM criterion Q(b) = 3.690e-06
Iteration 4: GMM criterion Q(b) = 4.606e-13
Iteration 5: GMM criterion Q(b) = 1.501e-26
GMM estimation
Number of parameters = 2
Number of moments = 2
Initial weight matrix: Unadjusted Number of obs = 4412
\begin{tabular}{l|rcccccc}
\hline & & \multicolumn{4}{c}{ Robust } & & \\
& Coef. & Std. Err. & z & P>|z| & [95\% Conf. Interval] \\
\hline /b0 & 1.204888 & .0462355 & 26.06 & 0.000 & 1.114268 & 1.295507 \\
/b1 & .0046702 & .0009715 & 4.81 & 0.000 & .0027662 & .0065743 \\
\hline
\end{tabular}
Instruments for equation 1: income _cons
```

Notice how we specified the derivative() option for each parameter. We simply specified a slash, the name of the parameter, an equal sign, then a substitutable expression that represents the derivative. Because our model has only one residual equation, we do not need to specify equation numbers in the derivative() options.

When you specify a linear combination of variables, your derivative should be with respect to the entire linear combination. For example, say we have the residual equation

$$
u_{i}=y-\exp \left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}+\beta_{0}\right)
$$

for which we would type

```
. gmm (y - exp({xb: x1 x2 x3} + {b0}) ...
```

Then in addition to the derivative $\partial u_{i} / \partial \beta_{0}$, we are to compute and specify

$$
\frac{\partial u_{i}}{\partial \mathbf{x}_{i}^{\prime} \boldsymbol{\beta}}=-\exp \left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}+\beta_{0}\right)
$$

Using the chain rule, $\partial u_{i} / \partial \beta_{j}=\partial u_{i} / \partial\left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}\right) \times \partial\left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}\right) / \partial \beta_{j}=-x_{i j} \exp \left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}+\beta_{0}\right)$. Stata does this last calculation automatically. It knows the variables in the linear combination, so all it needs is the derivative of the residual function with respect to the linear combination. This allows you to change the variables in your linear combination without having to change the derivatives.

## > Example 8: Derivatives with a linear combination

We refit the model described in the example illustrating exponential regression with endogenous regressors, now providing analytic derivatives. We type

```
. gmm (docvis - exp({xb:private chronic female income}+{b0})),
> instruments(private chronic female age black hispanic)
> derivative(/xb = -1*exp({xb:} + {b0}))
> derivative(/b0 = -1*exp({xb:} + {b0}))
Step 1
Iteration 0: GMM criterion Q(b) = 16.910173
Iteration 1: GMM criterion Q(b) = . }8227087
Iteration 2: GMM criterion Q(b) = . }2183199
Iteration 3: GMM criterion Q(b) = . }1268593
Iteration 4: GMM criterion Q(b) = . }1267236
Iteration 5: GMM criterion Q(b) = . }1267236
Step 2
Iteration 0: GMM criterion Q(b) = .00234641
Iteration 1: GMM criterion Q(b) = .00215957
Iteration 2: GMM criterion Q(b) = . 00215911
Iteration 3: GMM criterion Q(b) = .00215911
GMM estimation
Number of parameters = 5
Number of moments = 7
Initial weight matrix: Unadjusted Number of obs = 4412
GMM weight matrix: Robust
```

|  | Coef. | Robust | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| /xb_private | .535335 | .159904 | 3.35 | 0.001 | .221929 | .848741 |
| /xb_chronic | 1.090126 | .0617659 | 17.65 | 0.000 | .9690668 | 1.211185 |
| /xb_female | .6636579 | .0959885 | 6.91 | 0.000 | .475524 | .8517918 |
| /xb_income | .0142855 | .0027162 | 5.26 | 0.000 | .0089618 | .0196092 |
| /b0 | -.5983477 | .138433 | -4.32 | 0.000 | -.8696714 | -.327024 |

Instruments for equation 1: private chronic female age black hispanic _cons
In the first derivative() option, we specified the name of the linear combination, $x b$, instead of an individual parameter's name. We already declared the variables of our linear combination in the substitutable expression for the residual equation, so in our substitutable expressions for the derivatives, we can use the shorthand notation $\{\mathrm{xb}:\}$ to refer to it.

Our point estimates are identical to those we obtained earlier. The standard errors and confidence intervals differ by only trivial amounts.

## Exponential regression models with panel data

In addition to supporting cross-sectional and time-series data, gmm also works with panel-data models. Here we illustrate gmm's panel-data capabilities by expanding our discussion of exponential regression models to allow for panel data. This also provides us the opportunity to demonstrate the moment-evaluator program version of gmm. Our discussion is based on Blundell, Griffith, and Windmeijer (2002). Also see Wooldridge (1999) for further discussion of nonlinear panel-data models.

First, we expand (9) for panel data. With individual heterogeneity term $\eta_{i}$, we have

$$
E\left(y_{i t} \mid \mathbf{x}_{i t}, \eta_{i}\right)=\exp \left(\mathbf{x}_{i t}^{\prime} \boldsymbol{\beta}+\eta_{i}\right)=\mu_{i t} \nu_{i}
$$

where $\mu_{i t}=\exp \left(\mathbf{x}_{i t}^{\prime} \boldsymbol{\beta}\right)$ and $\nu_{i}=\exp \left(\eta_{i}\right)$. Note that there is no constant term in this model because its effect cannot be disentangled from $\nu_{i}$. With an additive idiosyncratic error term, we have the regression model

$$
y_{i t}=\mu_{i t} \nu_{i}+\epsilon_{i t}
$$

We do not impose the assumption $E\left(\mathbf{x}_{i t} \eta_{i}\right)=\mathbf{0}$, so $\eta_{i}$ can be considered a fixed effect in the sense that it may be correlated with the regressors.

As discussed by Blundell, Griffith, and Windmeijer (2002), if $\mathbf{x}_{i t}$ is strictly exogenous, meaning $E\left(\mathbf{x}_{i t} \epsilon_{i s}\right)=\mathbf{0}$ for all $t$ and $s$, then we can estimate the parameters of the model by using the sample moment conditions

$$
\begin{equation*}
\sum_{i} \sum_{t} \mathbf{x}_{i t}\left(y_{i t}-\mu_{i t} \frac{\bar{y}_{i}}{\bar{\mu}_{i}}\right)=\mathbf{0} \tag{13}
\end{equation*}
$$

where $\bar{y}_{i}$ and $\bar{\mu}_{i}$ are the means of $y_{i t}$ and $\mu_{i t}$ for panel $i$, respectively. Because $\bar{\mu}_{i}$ depends on the parameters of the model, it must be recomputed each time gmm needs to evaluate the residual equation. Therefore, we cannot use the substitutable expression version of gmm. Instead, we must use the moment-evaluator program version.

The moment-evaluator program version of gmm functions much like the function-evaluator program versions of nl and nl sur. The program you write is passed one or more variables to be filled in with the residuals evaluated at the parameter values specified in an option passed to your program. For the fixed-effects Poisson model with strictly exogenous regressors, our first crack at a function-evaluator program is

```
program gmm_poi
    version 13
    syntax varlist if, at(name)
    quietly {
        tempvar mu mubar ybar
        gen double 'mu' = exp(x1*'at'[1,1] + x2*'at'[1,2] ///
                                + x3*'at'[1,3]) 'if'
        egen double 'mubar' = mean('mu') 'if', by(id)
        egen double 'ybar' = mean(y) 'if', by(id)
        replace 'varlist' = y - 'mu'*'ybar'/'mubar' 'if'
    }
```

end

You can save your program in an ado-file named name. ado, where name is the name you use for your program; here we would save the program in the ado-file gmm_poi.ado. Alternatively, if you are working from within a do-file, you can simply define the program before calling gmm. The syntax statement declares that we are expecting to receive a varlist, containing the names of variables whose values we are to replace with the values of the residual equations, and an if expression that will mark the estimation sample; because our model has one residual equation, varlist will consist of one variable. at () is a required option to our program, and it will contain the name of a matrix containing the parameter values at which we are to evaluate the residual equation. All moment-evaluator programs must accept the varlist, if condition, and at () option.

The first part of our program computes $\mu_{i t}$. In the model we will fit shortly, we have three regressors, named $x 1, x 2$, and $x 3$. The 'at' vector will have three elements, one for each of those variables. Notice that we included 'if' at the end of each statement that affects variables to restrict the computations to the relevant estimation sample. The two egen statements compute $\bar{\mu}_{i}$ and $\bar{y}_{i}$; in the example dataset we will use shortly, the panel variable is named id, and for simplicity we hardcoded that variable into our program as well. Finally, we compute the residual equation, which is the portion of (13) bound in parentheses.

## > Example 9: Panel Poisson with strictly exogenous regressors

To fit our model, we type

```
. use http://www.stata-press.com/data/r13/poisson1
. gmm gmm_poi, nequations(1) parameters(b1 b2 b3)
> instruments(x1 x2 x3, noconstant) vce(cluster id) onestep
Step 1
Iteration 0: GMM criterion Q(b) = 51.99142
Iteration 1: GMM criterion Q(b) = . 04345191
Iteration 2: GMM criterion Q(b) = 8.720e-06
Iteration 3: GMM criterion Q(b) = 7.115e-13
Iteration 4: GMM criterion Q(b) = 5.130e-27
GMM estimation
Number of parameters = 3
Number of moments = 3
Initial weight matrix: Unadjusted Number of obs = 409
```

(Std. Err. adjusted for 45 clusters in id)

|  |  | Robust |  |  |  |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: | ---: |
|  | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| /b1 | 1.94866 | .1000265 | 19.48 | 0.000 | 1.752612 | 2.144709 |
| /b2 | -2.966119 | .0923592 | -32.12 | 0.000 | -3.14714 | -2.785099 |
| /b3 | 1.008634 | .1156561 | 8.72 | 0.000 | .781952 | 1.235315 |

Instruments for equation 1: x1 x2 x3
All three of our regressors are strictly exogenous, so they can serve as their own regressors. There is no constant term in the model (it would be unidentified), so we exclude a constant term from our list of instruments. We have one residual equation as indicated by nequations (1), and we have three parameters, named b1, b2, and b3. The order in which you declare parameters in the parameters() option determines the order in which they appear in the 'at' vector in the moment-evaluator program. We specified vce(cluster id) to obtain standard errors that allow for correlation among observations within each panel.

The program we just wrote is sufficient to fit the model to the poisson1 dataset, but if we want to fit that model to other datasets, we would need to change the variable names and perhaps account for having a different number of parameters as well. Despite those limitations, if you just want to fit a single model, that program is adequate.

Next we take advantage of the ability to specify full equation names in the parameters() option and rewrite our evaluator program so that we can more easily change the variables in our model. This approach is particularly useful if some of the moment equations are linear in the parameters, because then we can use matrix score (see [P] matrix score) to evaluate those moments.

Our new evaluator program is

```
program gmm_poieq
    version 13
    syntax varlist if, at(name)
    quietly {
            tempvar mu mubar ybar
            matrix score double 'mu' = 'at' 'if', eq(#1)
            replace 'mu' = exp('mu')
            egen double 'mubar' = mean('mu') 'if', by(id)
            egen double 'ybar' = mean(y) 'if', by(id)
            replace 'varlist' = y - 'mu'*'ybar'/'mubar' 'if'
    }
```

end

Rather than using generate to compute the temporary variable ' mu ', we used matrix score to obtain the linear combination $\mathbf{x}_{i t}^{\prime} \boldsymbol{\beta}$ and then called replace to compute $\exp \left(\mathbf{x}_{i t}^{\prime} \boldsymbol{\beta}\right)$.

## > Example 10: Panel Poisson using matrix score

To fit our model, we type

```
. use http://www.stata-press.com/data/r13/poisson1
. gmm gmm_poieq, nequations(1) parameters(y:x1 y:x2 y:x3)
> instruments(x1 x2 x3, noconstant) vce(cluster id) onestep
Step 1
Iteration 0: GMM criterion Q(b) = 51.99142
Iteration 1: GMM criterion Q(b) = . 04345191
Iteration 2: GMM criterion Q(b) = 8.720e-06
Iteration 3: GMM criterion Q(b) = 7.115e-13
Iteration 4: GMM criterion Q(b) = 5.106e-27
GMM estimation
Number of parameters = 3
Number of moments = 3
Initial weight matrix: Unadjusted Number of obs = 409
(Std. Err. adjusted for 45 clusters in id)
\begin{tabular}{r|rrccrrr}
\hline & \multicolumn{4}{|c}{ Robust } & & \\
& Coef. & Std. Err. & z & P>|z| & [95\% Conf. Interval] \\
\hline x1 & 1.94866 & .1000265 & 19.48 & 0.000 & 1.752612 & 2.144709 \\
x2 & -2.966119 & .0923592 & -32.12 & 0.000 & -3.14714 & -2.785099 \\
x3 & 1.008634 & .1156561 & 8.72 & 0.000 & .781952 & 1.235315 \\
\hline
\end{tabular}
```

Instruments for equation 1: x1 x2 x3
Instead of specifying simple parameter names in the parameters() option, we specified an equation name and the variables associated with that equation. We named our equation $y$, but you could use any valid Stata name. When we use this syntax, the rows of the coefficient table are grouped by the equation names.

Say we wanted to refit our model using just x1 and x3 as regressors. We do not need to make any changes to gmm_poieq. We just change the specification of the parameters() option:

```
. gmm gmm_poieq, nequations(1) parameters(y:x1 y:x3) ///
> instruments(x1 x3, noconstant) vce(cluster id) onestep
```

In this evaluator program, we have still hard-coded the name of the dependent variable. The next two examples include methods to tackle that shortcoming.

## Technical note

Say we specify the parameters() option like this:

```
. gmm ..., parameters(y1:x1 y1:x2 y1:_cons y2:_cons y3:x1 y3:_cons)
```

Then the 'at' vector passed to our program will have the following column names attached to it:

```
'at'[1,6]
    y1: y1: y1: y2: y3: y3:
```

Typing
. matrix score double eq1 = 'at', eq(\#1)
is equivalent to typing

```
. generate double eq1 = x1*'at'[1,1] + x2*'at'[1,2] + 'at'[1,3]
```

with one important difference. If we change some of the variables in the parameters() option when we call gmm, matrix score will compute the correct linear combination. If we were to use the generate statement instead, then every time we wanted to change the variables in our model, we would have to modify that statement as well.

The command
. matrix score double alpha = 'at', eq(\#2) scalar
is equivalent to

```
. scalar alpha = 'at'[1,4]
```

Thus even if you specify equation and variable names in the parameters() option, you can still have scalar parameters in your model.

When past values of the idiosyncratic error term affect the value of a regressor, we say that regressor is predetermined. When one or more regressors are predetermined, sample moment condition (10) is no longer valid. However, Chamberlain (1992) shows that a simple alternative is to consider moment conditions of the form

$$
\begin{equation*}
\sum_{i} \sum_{t=2}^{T} \mathbf{x}_{i, t-1}\left(y_{i, t-1}-\mu_{i, t-1} \frac{y_{i t}}{\mu_{i t}}\right)=\mathbf{0} \tag{14}
\end{equation*}
$$

Also see Wooldridge (1997) and Windmeijer (2000) for other moment conditions that can be used with predetermined regressors.

## > Example 11: Panel Poisson with predetermined regressors

Here we refit the previous model, treating all the regressors as predetermined and using the moment conditions in (14). Our moment-evaluator program is

```
program gmm_poipre
    version 13
    syntax varlist if, at(name) mylhs(varlist)
    quietly {
        tempvar mu
        matrix score double 'mu' = 'at' 'if', eq(#1)
        replace 'mu' = exp('mu')
    replace 'varlist' = L.'mylhs' - L.'mu'*'mylhs'/'mu' 'if'
}
end
```

To compute the moment equation, we used lag-operator notation so that Stata properly handles gaps in our panel dataset. We also made our program accept an additional option that we will use to pass in the dependent variable. When we specify this option in our gmm statement, it will get passed to our evaluator program because gmm will not recognize the option as one of its own. Equation (14) shows that we are to use the first lags of the regressors as instruments, so we type

```
. gmm gmm_poipre, mylhs(y) nequations(1) vce(cluster id) onestep
> parameters(y:x1 y:x2 y:x3) instruments(L.(x1 x2 x3), noconstant)
warning: 45 missing values returned for equation 1 at initial values
Step 1
Iteration 0: GMM criterion Q(b) = 52.288048
Iteration 1: GMM criterion Q(b) = 2.3599714
Iteration 2: GMM criterion Q(b) = . }1695173
Iteration 3: GMM criterion Q(b) = .00020399
Iteration 4: GMM criterion Q(b) = 3.392e-10
Iteration 5: GMM criterion Q(b) = 9.230e-22
GMM estimation
Number of parameters = 3
Number of moments = 3
Initial weight matrix: Unadjusted Number of obs = 319
```

(Std. Err. adjusted for 45 clusters in id)

|  | Coef. | Robust Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| x 1 | 2.025956 | . 2777156 | 7.30 | 0.000 | 1.481644 | 2.570269 |
| x 2 | -2.909646 | . 2577577 | -11.29 | 0.000 | -3.414842 | -2.404451 |
| x3 | 1.202926 | . 1873571 | 6.42 | 0.000 | . 8357131 | 1.570139 |

Instruments for equation 1: L.x1 L.x2 L.x3
Here, like earlier with strictly exogenous regressors, the number of instruments equals the number of parameters, so there is no gain to using the two-step or iterated estimators. However, if you do have more instruments than parameters, you will most likely want to use one of those other estimators instead.

Instead of making our program accept the mylhs() option, we could have used Stata's coleq extended macro function to determine the dependent variable based on the column names attached to the 'at' vector; see [P] macro. Then we could refit our model with a different dependent variable by changing the eqname used in the parameters() option. In the next example, we take this approach.

In the previous example, we used $\mathbf{x}_{i, t-1}$ as instruments. A more efficient GMM estimator would also use $\mathbf{x}_{i, t-2}, \mathbf{x}_{i, t-3}, \ldots, \mathbf{x}_{i, 1}$ as instruments in period $t$ as well. gmm's xtinstruments() option allows you to specify instrument lists that grow as $t$ increases. Later we discuss the xtinstruments() option in detail in the context of linear dynamic panel-data models.

When a regressor is contemporaneously correlated with the idiosyncratic error term, we say that regressor is endogenous. Windmeijer (2000) shows that here we can use the moment condition

$$
\sum_{i} \sum_{t=3}^{T} \mathbf{x}_{i, t-2}\left(\frac{y_{i t}}{\mu_{i t}}-\frac{y_{i, t-1}}{\mu_{i, t-1}}\right)
$$

Here we use the second lag of the endogenous regressor as an instrument. If a variable is strictly exogenous, it can of course serve as its own instrument.

## > Example 12: Panel Poisson with endogenous regressors

Here we refit the model, treating $x 3$ as endogenous and $x 1$ and $x 2$ as strictly exogenous. Our moment-evaluator program is

```
program gmm_poiend
    version 13
    syntax varlist if, at(name)
    quietly {
            tempvar mu
            matrix score double 'mu' = 'at' 'if', eq(#1)
            replace 'mu' = exp('mu')
            local mylhs : coleq 'at'
            local mylhs : word 1 of 'mylhs'
            replace 'varlist' = 'mylhs'/'mu' - L.'mylhs'/L.'mu' 'if'
    }
end
```

Now we call gmm using x1, x2, and L2.x3 as instruments:

```
. use http://www.stata-press.com/data/r13/poisson2
. gmm gmm_poiend, nequations(1) vce(cluster id) onestep
> parameters(y:x1 y:x2 y:x3) instruments(x1 x2 L2.x3, noconstant)
warning: 500 missing values returned for equation 1 at initial values
Step 1
Iteration 0: GMM criterion Q(b) = 43.799922
Iteration 1: GMM criterion Q(b) = . 06998898
Iteration 2: GMM criterion Q(b) = .04165161
Iteration 3: GMM criterion Q(b) = . 03573502
Iteration 4: GMM criterion Q(b) = .00001981
Iteration 5: GMM criterion Q(b) = 3.168e-12
Iteration 6: GMM criterion Q(b) = 1.529e-23
GMM estimation
```

Number of parameters $=3$
Number of moments $=3$
Initial weight matrix: Unadjusted $\quad$ Number of obs $=3266$
(Std. Err. adjusted for 500 clusters in id)

|  | Robust |  |  |  |  |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: | ---: |
|  | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| x1 | 1.857766 | .2631454 | 7.06 | 0.000 | 1.34201 | 2.373521 |
| x2 | -2.865858 | .2151529 | -13.32 | 0.000 | -3.28755 | -2.444167 |
| x3 | 4.961867 | 14.91462 | 0.33 | 0.739 | -24.27025 | 34.19399 |

Instruments for equation 1: x1 x2 L2.x3
The warning at the top of the output indicates that we have 500 panels in our dataset. Our moment equation includes lagged terms and therefore cannot be evaluated for the first time period within each panel. Warning messages like that can be ignored once you know why they occurred. If you receive a warning message that you were not expecting, you should first investigate the cause of the warning before trusting the results. As in the previous example, instead of using just $\mathbf{x}_{i, t-2}$ as an instrument, we could use all further lags of $\mathbf{x}_{i t}$ as instruments as well.

## Rational-expectations models

Macroeconomic models typically assume that agents' expectations about the future are formed rationally. By rational expectations, we mean that agents use all information available when forming their forecasts, so the forecast error is uncorrelated with the information available when the forecast was made. Say that at time $t$, people make a forecast, $\widehat{y}_{t+1}$, of variable $y$ in the next period. If $\Omega_{t}$ denotes all available information at time $t$, then rational expectations implies that $E\left\{\left(\widehat{y}_{t+1}-y_{t+1}\right) \mid \Omega_{t}\right\}=0$. If $\Omega_{t}$ denotes observable variables such as interest rates or prices, then this conditional expectation can serve as the basis of a moment condition for GMM estimation.

## > Example 13: Fitting a Euler equation

In a well-known article, Hansen and Singleton (1982) consider a model of portfolio decision making and discuss parameter estimation using GMM. We will consider a simple example with one asset in which the agent can invest. A consumer wants to maximize the present value of his lifetime utility derived from consuming the good. On the one hand, the consumer is impatient, so he would rather consume today than wait until tomorrow. On the other hand, if he consumes less today, he can invest more of his money, earning more interest that he can then use to consume more of the good tomorrow. Thus there is a tradeoff between having his cake today or sacrificing a bit today to have more cake tomorrow.

If we assume a specific form for the agent's utility function, known as the constant relative-risk aversion utility function, we can show that the Euler equation is

$$
E\left[\mathbf{z}_{t}\left\{1-\beta\left(1+r_{t+1}\right)\left(c_{t+1} / c_{t}\right)^{-\gamma}\right\}\right]=\mathbf{0}
$$

where $\beta$ and $\gamma$ are the parameters to estimate, $r_{t}$ is the return to the financial asset, and $c_{t}$ is consumption in period $t$. $\beta$ measures the agent's discount factor. If $\beta$ is near one, the agent is patient and is more willing to forgo consumption this period. If $\beta$ is close to zero, the agent is less patient and prefers to consume more now. The parameter $\gamma$ characterizes the agent's utility function. If $\gamma$ equals zero, the utility function is linear. As $\gamma$ tends toward one, the utility function tends toward $u=\log (c)$.

We have data on 3-month Treasury bills $\left(r_{t}\right)$ and consumption expenditures $\left(c_{t}\right)$. As instruments, we will use lagged rates of return and past growth rates of consumption. We will use the two-step estimator and a weight matrix that allows for heteroskedasticity and autocorrelation up to four lags with the Bartlett kernel. In Stata, we type

```
. use http://www.stata-press.com/data/r13/cr
. generate cgrowth = c / L.c
(1 missing value generated)
. gmm (1 - {b=1}*(1+F.r)*(F.c/c)^(-1*{gamma=1})),
> inst(L.r L2.r cgrowth L.cgrowth) wmat(hac nw 4) twostep
warning: 1 missing value returned for equation 1 at initial values
Step 1
Iteration 0: GMM criterion Q(b) = .00226482
Iteration 1: GMM criterion Q(b) = . 00054369
Iteration 2: GMM criterion Q(b) = .00053904
Iteration 3: GMM criterion Q(b) = .00053904
Step 2
Iteration 0: GMM criterion Q(b) = .0600729
Iteration 1: GMM criterion Q(b) = . }059636
Iteration 2: GMM criterion Q(b) = . 0596369
GMM estimation
Number of parameters = 2
Number of moments = 5
Initial weight matrix: Unadjusted Number of obs = 239
GMM weight matrix: HAC Bartlett 4
\begin{tabular}{r|rcccrrr}
\hline & \multicolumn{4}{c}{ HAC } & & \\
& Coef. & Std. Err. & \(z\) & \(\mathrm{P}>|\mathrm{z}|\) & [95\% Conf. Interval] \\
\hline /b & .9204617 & .0134646 & 68.36 & 0.000 & .8940716 & .9468518 \\
/gamma & -4.222361 & 1.473895 & -2.86 & 0.004 & -7.111143 & -1.333579 \\
\hline
\end{tabular}
HAC standard errors based on Bartlett kernel with 4 lags.
Instruments for equation 1: L.r L2.r cgrowth L.cgrowth _cons
```

The warning message at the top of the output appears because the forward operator in our substitutable expression says that residuals can be computed only for 239 observations; our dataset contains 240 observations. Our estimate of $\beta$ is near one, in line with expectations and published results. However, our estimate of $\gamma$ implies risk-loving behavior and therefore a poorly specified model.

## System estimators

In many economic models, two or more variables are determined jointly through a system of simultaneous equations. Indeed, some of the earliest work in econometrics, including that of the Cowles Commission, was centered around estimation of the parameters of simultaneous equations. The 2SLS and IV estimators we have already discussed are used in some circumstances to estimate such parameters. Here we focus on the joint estimation of all the parameters of systems of equations, and we begin with the well-known three-stage least-squares (3SLS) estimator.

Recall that the 2SLS estimator is based on the moment conditions $E(\mathbf{z} u)=\mathbf{0}$. The 2SLS estimator can be used to estimate the parameters of one equation of a system of structural equations. Moreover, with the 2SLS estimator, we do not even need to specify the structural relationship among all the endogenous variables; we need to specify only the equation on which interest focuses and simply assume reduced-form relationships among the endogenous regressors of the equation of interest and the exogenous variables of the model. If we are willing to specify the complete system of structural equations, then assuming our model is correctly specified, by estimating all the equations jointly, we can obtain estimates that are more efficient than equation-by-equation 2SLS.

In [R] reg3, we fit a simple two-equation macroeconomic model:

$$
\begin{align*}
\text { consump } & =\beta_{0}+\beta_{1} \text { wagepriv }+\beta_{2} \text { wagegovt }+\epsilon_{1}  \tag{15}\\
\text { wagepriv } & =\beta_{3}+\beta_{4} \text { consump }+\beta_{5} \text { govt }+\beta_{6} \text { capital1 }+\epsilon_{2} \tag{16}
\end{align*}
$$

where consump represents aggregate consumption; wagepriv and wagegovt are total wages paid by the private and government sectors, respectively; govt is government spending; and capital1 is the previous period's capital stock. We are not willing to assume that $\epsilon_{1}$ and $\epsilon_{2}$ are independent, so we must treat both consump and wagepriv as endogenous. Suppose that a random shock makes $\epsilon_{2}$ positive. Then by (16), wagepriv will be higher than it otherwise would. Moreover, $\epsilon_{1}$ will either be higher or lower, depending on the correlation between it and $\epsilon_{2}$. The shock to $\epsilon_{2}$ has made both wagepriv and $\epsilon_{1}$ move, implying that in (15) wagepriv is an endogenous regressor. A similar argument shows that consump is an endogenous regressor in the second equation. In our model, wagegovt, govt, and capital1 are all exogenous variables.

Let $\mathbf{z}_{1}$ and $\mathbf{z}_{2}$ denote the instruments for the first and second equations, respectively; we will discuss what comprises them shortly. We have two sets of moment conditions:

$$
E\left\{\begin{array}{c}
\mathbf{z}_{1}\left(\text { consump }-\beta_{0}-\beta_{1} \text { wagepriv }-\beta_{2} \text { wagegovt }\right)  \tag{17}\\
\mathbf{z}_{2}\left(\text { wagepriv }-\beta_{3}-\beta_{4} \text { consump }-\beta_{5} \text { govt }-\beta_{6} \text { capital1 }\right)
\end{array}\right\}=\mathbf{0}
$$

One of the defining characteristics of 3SLS is that the errors are homoskedastic conditional on the instrumental variables. Using this assumption, we have

$$
E\left[\left\{\begin{array}{l}
\mathbf{z}_{1} \epsilon_{1}  \tag{18}\\
\mathbf{z}_{2} \epsilon_{2}
\end{array}\right\}\left\{\begin{array}{ll}
\mathbf{z}_{1}^{\prime} \epsilon_{1} & \left.\mathbf{z}_{2}^{\prime} \epsilon_{2}\right\}
\end{array}\right]=\left\{\begin{array}{ll}
\sigma_{11} E\left(\mathbf{z}_{1} \mathbf{z}_{1}^{\prime}\right) & \sigma_{12} E\left(\mathbf{z}_{1} \mathbf{z}_{2}^{\prime}\right) \\
\sigma_{21} E\left(\mathbf{z}_{2} \mathbf{z}_{1}^{\prime}\right) & \sigma_{22} E\left(\mathbf{z}_{2} \mathbf{z}_{2}^{\prime}\right)
\end{array}\right\}\right.
$$

where $\sigma_{i j}=\operatorname{cov}\left(\epsilon_{i}, \epsilon_{j}\right)$. Let $\boldsymbol{\Sigma}$ denote the $2 \times 2$ matrix with typical element $\sigma_{i j}$.
The second defining characteristic of the 3SLS estimator is that it uses all the exogenous variables as instruments for all equations; here $\mathbf{z}_{1}=\mathbf{z}_{2}=$ (wagegovt, govt, capital1, 1 ), where the 1 indicates a constant term. From our discussion on the weight matrix and two-step estimation, we want to use the sample analogue of the matrix inverse of the right-hand side of (18) as our weight matrix.

To implement the 3SLS estimator, we apparently need to know $\boldsymbol{\Sigma}$ or at least have a consistent estimator of it. The solution is to fit (15) and (16) by 2SLS, use the sample residuals $\widehat{\epsilon_{1}}$ and $\widehat{\epsilon_{2}}$ to estimate $\boldsymbol{\Sigma}$, then estimate the parameters of (17) via GMM by using the weight matrix just discussed.

## > Example 14: 3SLS estimation

3SLS is easier to do using gmm than it sounds. The 3SLS estimator is a two-step GMM estimator. In the first step, we do the equivalent of 2SLS on each equation, and then we compute a weight matrix based on (18). Finally, we perform a second step of GMM with this weight matrix.

In Stata, we type

```
. use http://www.stata-press.com/data/r13/klein, clear
. gmm (eq1: consump - {b0} - {xb: wagepriv wagegovt})
> (eq2: wagepriv - {c0} - {xc: consump govt capital1}),
> instruments(eq1: wagegovt govt capital1)
> instruments(eq2: wagegovt govt capital1)
> winitial(unadjusted, independent) wmatrix(unadjusted) twostep
Step 1
Iteration 0: GMM criterion Q(b) = 4195.4487
Iteration 1: GMM criterion Q(b) = . 22175631
Iteration 2: GMM criterion Q(b) = . 22175631 (backed up)
Step 2
Iteration 0: GMM criterion Q(b) = .09716589
Iteration 1: GMM criterion Q(b) = .07028208
Iteration 2: GMM criterion Q(b) = .07028208
GMM estimation
Number of parameters = 7
Number of moments = 8
Initial weight matrix: Unadjusted Number of obs = 22
GMM weight matrix: Unadjusted
```

|  | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | ---: | ---: | ---: |
| /b0 | 19.3559 | 3.583772 | 5.40 | 0.000 | 12.33184 | 26.37996 |
| /xb_wagepriv | .8012754 | .1279329 | 6.26 | 0.000 | .5505314 | 1.052019 |
| /xb_wagegovt | 1.029531 | .3048424 | 3.38 | 0.001 | .432051 | 1.627011 |
| /c0 | 14.63026 | 10.26693 | 1.42 | 0.154 | -5.492552 | 34.75306 |
| /xc_consump | .4026076 | .2567312 | 1.57 | 0.117 | -.1005764 | .9057916 |
| /xc_govt | 1.177792 | .5421253 | 2.17 | 0.030 | .1152461 | 2.240338 |
| /xc_capital1 | -.0281145 | .0572111 | -0.49 | 0.623 | -.1402462 | .0840173 |

Instruments for equation 1: wagegovt govt capital1 _cons
Instruments for equation 2: wagegovt govt capital1 _cons
The independent suboption of the winitial() option tells gmm to assume that the residuals are independent across equations; this suboption sets $\sigma_{21}=\sigma_{12}=0$ in (18). Assuming both homoskedasticity and cross-equation independence is equivalent to fitting the two equations of our model independently by 2SLS. The wmatrix() option controls how the weight matrix is computed based on the first-step parameter estimates before the second step of estimation; here we request a weight matrix that assumes conditional homoskedasticity but that does not impose the cross-equation independence like the initial weight matrix we used. In this example, we also illustrated how to name equations and how equation names can be used in the instruments() option. Our results are identical to those in [ R ] reg3.

We could have specified our instruments with the syntax
instruments(wagegovt govt capital1)
because gmm uses the variables listed in the instruments() option for all equations unless you specify which equations the list of instruments is to be used with. However, we wanted to emphasize that the same instruments are being used for both equations; in a moment, we will discuss an estimator that does not use the same instruments in all equations.

In the previous example, if we omit the twostep option, the resulting coefficients will be equivalent to equation-by-equation 2SLS, which Wooldridge $(2010,216)$ calls the "system 2SLS estimator". Eliminating the twostep option makes the wmatrix() option irrelevant, so that option can be eliminated as well.

So far, we have developed the traditional 3SLS estimator. Wooldridge (2010, chap. 8) discusses the "GMM 3SLS" estimator that extends the traditional 3SLS estimator by allowing for heteroskedasticity and different instruments for different equations.

Generalizing (18) to an arbitrary number of equations, we have

$$
\begin{equation*}
E\left(\mathbf{Z}^{\prime} \boldsymbol{\epsilon} \boldsymbol{\epsilon}^{\prime} \mathbf{Z}\right)=E\left(\mathbf{Z}^{\prime} \mathbf{\Sigma} \mathbf{Z}\right) \tag{19}
\end{equation*}
$$

where

$$
\mathbf{Z}=\left[\begin{array}{cccc}
\mathbf{z}_{1} & \mathbf{0} & \cdots & \mathbf{0} \\
\mathbf{0} & \mathbf{z}_{2} & \cdots & \mathbf{0} \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{0} & \mathbf{0} & \cdots & \mathbf{z}_{m}
\end{array}\right]
$$

and $\boldsymbol{\Sigma}$ is now $m \times m$. Equation (19) is the multivariate analogue of a homoskedasticity assumption; for each equation, the error variance is constant for all observations, as is the covariance between any two equations' errors.

We can relax this homoskedasticity assumption by considering different weight matrices. For example, if we continue to assume that observations are independent but not necessarily identically distributed, then by specifying wmatrix (robust), we would obtain a weight matrix that allows for heteroskedasticity:

$$
\widehat{W}=\frac{1}{N} \sum_{i} \mathbf{Z}_{i}^{\prime} \widehat{\epsilon}_{i} \widehat{\epsilon}_{i}^{\prime} \mathbf{Z}_{i}
$$

This is the weight matrix in Wooldridge's $(2010,218)$ Procedure 8.1 , "GMM with Optimal Weighting Matrix". By default, gmm would report standard errors based on his covariance matrix (8.27); specifying vce (unadjusted) would provide the optimal GMM standard errors. If you have multiple observations for each individual or firm in your dataset, you could specify wmatrix (cluster id), where id identifies individuals or firms. This would allow arbitrary within-individual correlation, though it does not account for an individual-specific fixed or random effect. In both cases, we would continue to use winitial (unadjusted, independent) so that the first-step estimates are the system 2SLS estimates.

Wooldridge (2010, sec. 9.6) discusses instances where it is necessary to use different instruments in different equations. The GMM 3SLS estimator with different instruments in different equations but with conditional homoskedasticity is what Hayashi $(2000,275)$ calls the "full-information instrumental variables efficient" (FIVE) estimator. Implementing the FIVE estimator is easy with gmm. For example, say we have a two-equation system, where kids, age, income, and education are all valid instruments for the first equation; but education is not a valid instrument for the second equation. Then our syntax would take the form

```
gmm (<mexp_1>) (<mexp_2>), instruments(1:kids age income education)
    instruments(2:kids age income)
```

The following syntax is equivalent:

```
gmm (<mexp_1>) (<mexp_2>), instruments(kids age income)
    instruments(1:education)
```

Because we did not specify a list of equations in the second example's first instruments () option, those variables are used as instruments in both equations. You can use whichever syntax you prefer. The first requires a bit more typing but is arguably more transparent.

If all the regressors in the model are exogenous, then the traditional 3SLS estimator is the seemingly unrelated regression (SUR) estimator. Here you would specify all the regressors as instruments.

## Dynamic panel-data models

Commands in Stata that work with panel data expect the data to be in the "long" format, meaning that each row of the dataset consists of one subobservation that is a member of a logical observation (represented by the panel identifier variable). See [D] reshape for a discussion of the long versus "wide" data forms. gmm is no exception in this respect when used with panel data. From a theoretical perspective, however, it is sometimes easier to view GMM estimators for panel data as system estimators in which we have $N$ observations on a system of $T$ equations, where $N$ and $T$ are the number of observations and panels, respectively, rather than a single-equation estimator with $N T$ observations. Usually, each of the $T$ equations will in fact be the same, though we will want to specify different instruments for each of these equations.

In a dynamic panel-data model, lagged values of the dependent variable are included as regressors. Here we consider a simple model with one lag of the dependent variable $y$ as a regressor and a vector of strictly exogenous regressors, $\mathbf{x}_{i t}$ :

$$
\begin{equation*}
y_{i t}=\rho y_{i, t-1}+\mathbf{x}_{i t}^{\prime} \boldsymbol{\beta}+u_{i}+\epsilon_{i t} \tag{20}
\end{equation*}
$$

$u_{i}$ can be either a fixed- or a random-effect term, in the sense that we do not require $\mathbf{x}_{i t}$ to be independent of it. Even with the assumption that $\epsilon_{i t}$ is i.i.d., the presence of both $y_{i, t-1}$ and $u_{i}$ in (20) renders both the standard fixed- and random-effects estimators to be inconsistent because of the well-known Nickell (1981) bias. OLS regression of $y_{i t}$ on $y_{i, t-1}$ and $\mathbf{x}_{i t}$ also produces inconsistent estimates, because $y_{i, t-1}$ will be correlated with the error term.

## - Technical note

Stata has the xtabond, xtdpd, and xtdpdsys commands (see [XT] xtabond, [XT] xtdpd, and [XT] xtdpdsys) to fit equations like (20), and for everyday use those commands are preferred because they offer features such as Windmeijer (2005) bias-corrected standard errors to account for the bias of traditional two-step GMM standard errors seen in dynamic panel-data models and, being linear estimators, only require you to specify variable names instead of complete equations. However, using gmm has several pedagogical advantages, including the ability to tie those model-specific commands into a more general framework, a clear illustration of how certain types of instrument matrices for panel-data models are formed, and demonstrations of several advanced features of gmm.

First-differencing (20) removes the panel-specific $u_{i}$ term:

$$
\begin{equation*}
y_{i t}-y_{i, t-1}=\rho\left(y_{i, t-1}-y_{i, t-2}\right)+\left(\mathbf{x}_{i t}-\mathbf{x}_{i, t-1}\right)^{\prime} \boldsymbol{\beta}+\left(\epsilon_{i t}-\epsilon_{i, t-1}\right) \tag{21}
\end{equation*}
$$

However, now $\left(y_{i, t-1}-y_{i, t-2}\right)$ is correlated with $\left(\epsilon_{i t}-\epsilon_{i, t-1}\right)$. Thus we need an instrument that is correlated with the former but not the latter. The lagged variables in (21) mean that equation is not estimable for $t<3$, so consider when $t=3$. We have

$$
\begin{equation*}
y_{i 3}-y_{i 2}=\rho\left(y_{i 2}-y_{i 1}\right)+\left(\mathbf{x}_{i 3}-\mathbf{x}_{i 2}\right)^{\prime} \boldsymbol{\beta}+\left(\epsilon_{i 3}-\epsilon_{i 2}\right) \tag{22}
\end{equation*}
$$

In the Arellano-Bond (1991) estimator, lagged levels of the dependent variable are used as instruments. With our assumption that the $\epsilon_{i t}$ are i.i.d., (20) intimates that $y_{i 1}$ can serve as an instrumental variable when we fit (22).

Next consider (21) when $t=4$. We have

$$
y_{i 4}-y_{i 3}=\rho\left(y_{i 3}-y_{i 2}\right)+\left(\mathbf{x}_{i 4}-\mathbf{x}_{i 3}\right)^{\prime} \boldsymbol{\beta}+\left(\epsilon_{i 4}-\epsilon_{i 3}\right)
$$

Now (20) shows that both $y_{i 1}$ and $y_{i 2}$ are uncorrelated with the error term $\left(\epsilon_{i 4}-\epsilon_{i 3}\right)$, so we have two instruments available. For $t=5$, you can show that $y_{i 1}, y_{i 2}$, and $y_{i 3}$ can serve as instruments. As may now be apparent, one of the key features of these dynamic panel-data models is that the available instruments depend on the time period, $t$, as was the case for some of the panel Poisson models we considered earlier. Because the $\mathbf{x}_{i t}$ are strictly exogenous by assumption, they can serve as their own instruments.

The initial weight matrix that is appropriate for the GMM dynamic panel-data estimator is slightly more involved than the unadjusted matrix we have used in most of our previous examples that assumes the errors are i.i.d. First, rewrite (21) for panel $i$ as

$$
\mathbf{y}_{i}-\mathbf{y}_{i}^{L}=\rho\left(\mathbf{y}_{i}^{L}-\mathbf{y}_{i}^{L L}\right)+\left(\mathbf{X}_{i}-\mathbf{X}_{i}^{L}\right) \boldsymbol{\beta}+\left(\boldsymbol{\epsilon}_{i}-\boldsymbol{\epsilon}_{i}^{L}\right)
$$

where $\mathbf{y}_{i}=\left(y_{i 3}, \ldots, y_{i T}\right)$ and $\mathbf{y}_{i}^{L}=\left(y_{i 2}, \ldots, y_{i, T-1}\right), \mathbf{y}_{i}^{L L}=\left(y_{i 1}, \ldots, y_{i, T-2}\right)$, and $\mathbf{X}_{i}, \mathbf{X}_{i}^{L}, \boldsymbol{\epsilon}_{i}$, and $\boldsymbol{\epsilon}_{i}^{L}$ are defined analogously. Let $\mathbf{Z}$ denote the full matrix of instruments for panel $i$, including the variables specified in both the instruments() and xtinstruments() options; the exact structure is detailed in Methods and formulas.

By assumption, $\epsilon_{i t}$ is i.i.d., so the first-difference $\left(\epsilon_{i t}-\epsilon_{i, t-1}\right)$ is necessarily autocorrelated with correlation -0.5 . Therefore, we should not use a weight matrix that assumes the errors are independent. For dynamic panel-data models, we can show that the appropriate initial weight matrix is

$$
\widehat{\mathbf{W}}=\left(\frac{1}{N} \sum_{i} \mathbf{Z}_{i}^{\prime} \mathbf{H}_{D} \mathbf{Z}_{i}\right)^{-1}
$$

where

$$
\mathbf{H}_{D}=\left[\begin{array}{cccccc}
1 & -0.5 & 0 & \ldots & 0 & 0 \\
-0.5 & 1 & -0.5 & \ldots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \ldots & 1 & -0.5 \\
0 & 0 & 0 & \ldots & -0.5 & 1
\end{array}\right]
$$

We can obtain this initial weight matrix by specifying winitial (xt $D$ ). The letter $D$ indicates that the equation we are estimating is specified in first-differences.

## > Example 15: Arellano-Bond estimator

Say we want to fit the model

$$
\begin{equation*}
\mathrm{n}_{i t}=\rho \mathrm{n}_{i, t-1}+\beta_{1} \mathrm{w}_{i t}+\beta_{2} \mathrm{w}_{i, t-1}+\beta_{3} \mathrm{k}_{i t}+\beta_{4} \mathrm{k}_{i, t-1}+u_{i}+\epsilon_{i t} \tag{23}
\end{equation*}
$$

where we assume that $\mathrm{w}_{i t}$ and $\mathrm{k}_{i t}$ are strictly exogenous. First-differencing, our residual equation is

$$
\begin{align*}
\epsilon_{i t}^{*}=\left(\epsilon_{i t}-\epsilon_{i, t-1}\right)= & \mathrm{n}_{i t}-\mathrm{n}_{i, t-1}-\rho\left(\mathrm{n}_{i, t-1}-\mathrm{n}_{i, t-2}\right)-\beta_{1}\left(\mathrm{w}_{i t}-\mathrm{w}_{i, t-1}\right) \\
& -\beta_{2}\left(\mathrm{w}_{i, t-1}-\mathrm{w}_{i, t-2}\right)-\beta_{3}\left(\mathrm{k}_{i t}-\mathrm{k}_{i, t-1}\right)-\beta_{4}\left(\mathrm{k}_{i, t-1}-\mathrm{k}_{i, t-2}\right) \tag{24}
\end{align*}
$$

In Stata, we type

```
. use http://www.stata-press.com/data/r13/abdata
. gmm (D.n - {rho}*LD.n - {xb:D.w LD.w D.k LD.k}),
> xtinstruments(n, lags(2/.)) instruments(D.w LD.w D.k LD.k, noconstant)
> deriv(/rho = -1*LD.n) deriv(/xb = -1) winitial(xt D) onestep
Step 1
Iteration 0: GMM criterion Q(b) = .0011455
Iteration 1: GMM criterion Q(b) = .00009103
Iteration 2: GMM criterion Q(b) = .00009103 (backed up)
GMM estimation
Number of parameters = 5
Number of moments = 32
Initial weight matrix: XT D Number of obs = 751
```

|  | Robust |  |  |  |  |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
|  | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| /rho | .8041712 | .1199819 | 6.70 | 0.000 | .5690111 | 1.039331 |
| /xb_D_w | -.5600476 | .1619472 | -3.46 | 0.001 | -.8774583 | -.242637 |
| /xb_LD_w | .3946699 | .1092229 | 3.61 | 0.000 | .1805969 | .6087429 |
| /xb_D_k | .3520286 | .0536546 | 6.56 | 0.000 | .2468676 | .4571897 |
| /xb_LD_k | -.2160435 | .0679689 | -3.18 | 0.001 | -.3492601 | -.0828269 |

Instruments for equation 1:
XT-style: L(2/.).n
Standard: D.w LD.w D.k LD.k
Because w and k are strictly exogenous, we specified the variants of them that appear in (24) in the instruments () option; because there is no constant term in the model, we specified noconstant to omit the constant from the instrument list.

We specified xtinstruments ( $n$, lags(2/.)) to tell gmm what instruments to use for the lagged dependent variable included as a regressor in (23). Based on our previous discussion, lags two and higher of $\mathrm{n}_{i t}$ can serve as instruments. The lags (2/.) suboption tells gmm that the first available instrument for $\mathrm{n}_{i t}$ is the lag-two value $\mathrm{n}_{i, t-2}$. The "." tells gmm to use all further lags of $\mathrm{n}_{i t}$ as instruments as well. The instrument matrices in dynamic panel-data models can become large if the dataset has many time periods per panel. In those cases, you could specify, for example, lags (2/4) to use just lags two through four instead of using all available lags.

Our results are identical to those we would obtain using xtabond with the syntax

```
xtabond n L(0/1).w L(0/1).k, lags(1) noconstant vce(robust)
```

Had we left off the vce(robust) option in our call to xtabond, we would have had to specify vce (unadjusted) in our call to gmm to obtain the same standard errors.

## Technical note

gmm automatically excludes observations for which there are no valid observations for the panelstyle instruments. However, it keeps in the estimation sample those observations for which fewer than the maximum number of instruments you requested are available. For example, if you specify the lags (2/4) suboption, you have requested three instruments, but gmm will keep observations even if only one or two instruments are available.

## > Example 16: Two-step Arellano-Bond estimator

Here we refit the model from example 15, using the two-step GMM estimator.

```
. gmm (D.n - {rho}*LD.n - {xb:D.w LD.w D.k LD.k}),
> xtinstruments(n, lags(2/.)) instruments(D.w LD.w D.k LD.k, noconstant)
> deriv(/rho = -1*LD.n) deriv(/xb = -1) winitial(xt D) wmatrix(robust)
> vce(unadjusted)
Step 1
Iteration 0: GMM criterion Q(b) = .0011455
Iteration 1: GMM criterion Q(b) = .00009103
Iteration 2: GMM criterion Q(b) = .00009103 (backed up)
Step 2
Iteration 0: GMM criterion Q(b) = .44107941
Iteration 1: GMM criterion Q(b) = . 4236729
Iteration 2: GMM criterion Q(b) = . }423672
GMM estimation
Number of parameters = 5
Number of moments = 32
Initial weight matrix: XT D Number of obs = 751
GMM weight matrix: Robust
```

|  | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| /rho | .8044783 | .0534763 | 15.04 | 0.000 | .6996667 | .90929 |
| /xb_D_W | -.5154978 | .0335506 | -15.36 | 0.000 | -.5812557 | -.4497399 |
| /xb_LD_w | .4059309 | .0637294 | 6.37 | 0.000 | .2810235 | .5308384 |
| /xb_D_k | .3556204 | .0390892 | 9.10 | 0.000 | .2790071 | .4322337 |
| /xb_LD_k | -.2204521 | .046439 | -4.75 | 0.000 | -.3114709 | -.1294332 |

Instruments for equation 1:
XT-style: L(2/.).n
Standard: D.w LD.w D.k LD.k

Our results match those you would obtain with the command

```
xtabond n L(0/1).(w k), lags(1) noconstant twostep
```


## - Technical note

Had we specified vce(robust) in our call to gmm, we would have obtained the traditional sandwich-based robust covariance matrix, but our standard errors would not match those we would obtain by specifying vce (robust) with the xtabond command. The xtabond, xtdpd, and xtdpdsys commands implement a bias-corrected robust VCE for the two-step GMM dynamic panel-data estimator. Traditional VCEs computed after the two-step dynamic panel-data estimator have been shown to exhibit often-severe bias; see Windmeijer (2005).

Neither of the two dynamic panel-data examples (15 and 16) we have fit so far include a constant term. When a constant term is included, the dynamic panel-data estimator is in fact a two-equation system estimator. For notational simplicity, consider a simple model containing just a constant term and one lag of the dependent variable:

$$
y_{i t}=\alpha+\rho y_{i, t-1}+u_{i}+\epsilon_{i t}
$$

First-differencing to remove the $u_{i}$ term, we have

$$
\begin{equation*}
y_{i t}-y_{i, t-1}=\rho\left(y_{i, t-1}-y_{i, t-2}\right)+\left(\epsilon_{i t}-\epsilon_{i, t-1}\right) \tag{25}
\end{equation*}
$$

This has also eliminated the constant term. If we assume $E\left(u_{i}\right)=0$, which is reasonable if a constant term is included in the model, then we can recover $\alpha$ by including the moment condition

$$
\begin{equation*}
y_{i t}=\alpha+\rho y_{i, t-1}+\epsilon_{i t}^{\prime} \tag{26}
\end{equation*}
$$

where $\epsilon_{i t}^{\prime}=u_{i}+\epsilon_{i t}$. The parameter $\rho$ continues to be identified by (25), so the only instrument we use with (26) is a constant term. As before, the error term $\left(\epsilon_{i, t}-\epsilon_{i, t-1}\right)$ is necessarily autocorrelated with correlation coefficient -0.5 , though the error term $\epsilon_{i t}^{\prime}$ is white noise. Therefore, our initial weight matrix should be

$$
\widehat{\mathbf{W}}=\left(\frac{1}{N} \sum_{i} \mathbf{Z}_{i}^{\prime} \mathbf{H Z} \mathbf{Z}_{i}\right)^{-1}
$$

where

$$
\mathbf{H}=\left[\begin{array}{cc}
\mathbf{H}_{D} & \mathbf{0} \\
\mathbf{0} & \mathbf{I}
\end{array}\right]
$$

and $\mathbf{I}$ is a conformable identity matrix.
One complication arises concerning the relevant estimation sample. Looking at (25), we apparently lose the first two observations from each panel because of the presence of $y_{i, t-2}$, but in (26) we need only to sacrifice one observation, for $y_{i, t-1}$. For most multiple-equation models, we need to use the same estimation sample for all equations. However, in dynamic panel-data models, we can use more observations to fit the equation in level form [(26) here] than the equation in first-differences [equation (25)]. To request this behavior, we specify the nocommonesample option to gmm. That option tells gmm to use as many observations as possible for each equation, ignoring the loss of observations due to lagging or differencing.
> Example 17: Arellano-Bond estimator with constant term
Here we fit the model

$$
\mathrm{n}_{i t}=\alpha+\rho \mathrm{n}_{i, t-1}+u_{i}+\epsilon_{i t}
$$

Without specifying derivatives, our command would be

```
. gmm (D.n - {rho}*LD.n) (n - {alpha} - {rho}*L.n),
> xtinstruments(1: n, lags(2/.)) instruments(1:, noconstant) onestep
> winitial(xt DL) vce(unadj) nocommonesample
```

We would specify winitial (xt DL) to obtain the required initial weight matrix. The notation DL indicates that our first moment equation is in first-differences and the second moment equation is in levels (not first-differenced). We exclude a constant in the instrument list for the first equation, because first-differencing removed the constant term. Because we do not specify the instruments () option for the second moment equation, a constant is used by default.

This example also provides us the opportunity to illustrate how to specify derivatives for multipleequation GMM models. Within the derivative() option, instead of specifying just the parameter name, now you must specify the equation name or number, a slash, and the parameter name to which the derivative applies. In Stata, we type

```
. gmm (D.n - {rho}*LD.n) (n - {alpha} - {rho}*L.n),
> xtinstruments(1: n, lags(2/.)) instruments(1:, noconstant)
> derivative(1/rho = -1*LD.n) derivative(2/alpha = -1)
> derivative(2/rho = -1*L.n) winitial(xt DL) vce(unadj)
> nocommonesample onestep
Step 1
Iteration 0: GMM criterion Q(b) = .09894466
Iteration 1: GMM criterion Q(b) = .00023508
Iteration 2: GMM criterion Q(b) = .00023508
GMM estimation
Number of parameters = 2
Number of moments = 29
Initial weight matrix: XT DL Number of obs = *
\begin{tabular}{r|rrcccc}
\hline & Coef. & Std. Err. & z & P>|z| & [95\% Conf. Interval] \\
\hline /rho & 1.023349 & .0608293 & 16.82 & 0.000 & .9041259 & 1.142572 \\
/alpha & -.0690864 & .0660343 & -1.05 & 0.295 & -.1985112 & .0603384 \\
\hline
\end{tabular}
* Number of observations for equation 1: 751
    Number of observations for equation 2: }89
Instruments for equation 1:
    XT-style: L(2/.).n
Instruments for equation 2:
    Standard: _cons
```

These results are identical to those we would obtain by typing

```
xtabond n, lags(1)
```

Because we specified nocommonesample, gmm did not report the number of observations used in the header of the output. In this dataset, there are in fact 1,031 observations on 140 panels. In the second equation, the presence of the lagged value of $n$ reduces the sample size for that equation to $1031-140=891$. In the first equation, we lose the first two observations per panel due to lagging and differencing, leading to 751 usable observations. These tallies are listed after the coefficient table in the output.

## Technical note

Specifying
xtinstruments(x1 x2 x3, lags(1/3))
differs from
instruments(L(1/3).(x1 x2 x3))
in how observations are excluded from the estimation sample. When you use the latter syntax, gmm must exclude the first three observations from each panel when computing the moment equation: you requested three lags of each regressor be used as instruments, so the first residual that could be interacted with those instruments is the one for $t=4$. On the other hand, when you use xtinstruments (), you are telling gmm that you would like to use up to the first three lags of x1, x2, and x3 as instruments but that using just one lag is acceptable. Because most panel datasets have a relatively modest number of observations per panel, dynamic instrument lists are typically used so that the number of usable observations is maximized. Dynamic instrument lists also accommodate the fact that there are more valid instruments for later time periods than earlier time periods.

Specifying panel-style instruments using the xtinstruments () option also affects how the standard instruments specified in the instruments() option are treated. To illustrate, suppose we have a balanced panel dataset with $T=5$ observations per panel and we specify

```
. gmm ..., xtinstruments(w, lags(1/2)) instruments(x)
```

We will lose the first observation because we need at least one lag of w to serve as an instrument. Our instrument matrix for panel $i$ will therefore be

$$
\mathbf{Z}_{i}=\left[\begin{array}{cccc}
w_{i 1} & 0 & 0 & 0  \tag{27}\\
0 & w_{i 1} & 0 & 0 \\
0 & w_{i 2} & 0 & 0 \\
0 & 0 & w_{i 2} & 0 \\
0 & 0 & w_{i 3} & 0 \\
0 & 0 & 0 & w_{i 3} \\
0 & 0 & 0 & w_{i 4} \\
x_{i 2} & x_{i 3} & x_{i 4} & x_{i 5} \\
1 & 1 & 1 & 1
\end{array}\right]
$$

The vector of ones in the final row represents the constant term implied by the instruments() option. Because we lost the first observation, the residual vector $\mathbf{u}_{i}$ will be $4 \times 1$. Thus our moment conditions for the $i$ th panel can be written in matrix notation as

$$
E\left\{\mathbf{Z}_{i} \mathbf{u}_{i}(\boldsymbol{\beta})\right\}=E\left\{\mathbf{Z}_{i}\left[\begin{array}{c}
u_{i 2}(\boldsymbol{\beta}) \\
u_{i 3}(\boldsymbol{\beta}) \\
u_{i 4}(\boldsymbol{\beta}) \\
u_{i 5}(\boldsymbol{\beta})
\end{array}\right]\right\}=\mathbf{0}
$$

The moment conditions corresponding to the final two rows of (27) say that

$$
E\left\{\sum_{t=2}^{T=4} x_{i t} u_{i t}(\boldsymbol{\beta})\right\}=0 \quad \text { and } \quad E\left\{\sum_{t=2}^{T=4} u_{i t}(\boldsymbol{\beta})\right\}=0
$$

Because we specified panel-style instruments with the xtinstruments () option, gmm no longer uses moment conditions for strictly exogenous variables of the form $E\left\{x_{i t} u_{i t}(\beta)\right\}=0$ for each $t$. Instead, the moment conditions now stipulate that the average (over $t$ ) of $x_{i t} u_{i t}(\boldsymbol{\beta})$ has expectation zero. This corresponds to the approach proposed by Arellano and Bond (1991, 280) and others.

When you request panel-style instruments with the xtinstruments() option, the number of instruments in the $\mathbf{Z}_{i}$ matrix increases quadratically in the number of periods. The dynamic paneldata estimators we have discussed in this section are designed for datasets that contain a large number of panels and a modest number of time periods. When the number of time periods is large, estimators that use standard (non-panel-style) instruments are more appropriate.

We have focused on the Arellano-Bond dynamic panel-data estimator because of its relative simplicity. gmm can additionally fit any models that can be formulated using the xtdpd and xtdpdsys commands; see [XT] xtdpd and [XT] xtdpdsys. The key is to determine the appropriate instruments to use for the level and difference equations. You may find it useful to fit a version of your model with those commands to determine what instruments and XT-style instruments to use. We conclude this section with an example using the Arellano-Bover/Blundell-Bond estimator.

## > Example 18: Arellano-Bover/Blundell-Bond estimator

We fit a small model that includes one lag of the dependent variable n as a regressor as well as the contemporaneous and first lag of w , which we assume are strictly exogenous. We could fit our model using xtdpdsys using the syntax

```
xtdpdsys n L(0/1).w, lags(1) twostep
```

Applying virtually all the syntax issues we have discussed so far, the equivalent gmm command is

```
. gmm (n - {rho}*L.n - {w}*w - {lagw}*L.w - {c})
> (D.n - {rho}*LD.n - {w}*D.w - {lagw}*LD.w),
> xtinst(1: D.n, lags(1/1)) xtinst(2: n, lags(2/.))
> inst(2: D.w LD.w, noconstant)
> deriv(1/rho = -1*L.n) deriv(1/w = -1*w)
> deriv(1/lagw = -1*L.w) deriv(1/c = -1)
> deriv(2/rho = -1*LD.n) deriv(2/w = -1*D.w)
> deriv(2/lagw = -1*LD.w)
> winit(xt LD) wmatrix(robust) vce(unadjusted)
> nocommonesample
```

Step 1
Iteration 0: GMM criterion Q(b) = . 10170339
Iteration 1: GMM criterion $Q(b)=.00022772$
Iteration 2: GMM criterion $Q(b)=.00022772$
Step 2
Iteration 0: GMM criterion $Q(b)=$. 59965014
Iteration 1: GMM criterion $Q(b)=.56578186$
Iteration 2: GMM criterion $Q(b)=.56578186$
GMM estimation
Number of parameters $=4$
Number of moments $=39$
Initial weight matrix: XT LD Number of obs $=\quad *$
GMM weight matrix: Robust

|  | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| /rho | 1.122738 | .0206512 | 54.37 | 0.000 | 1.082263 | 1.163214 |
| /w | -.6719909 | .0246148 | -27.30 | 0.000 | -.7202351 | -.6237468 |
| /lagw | .571274 | .0403243 | 14.17 | 0.000 | .4922398 | .6503083 |
| /c | .154309 | .17241 | 0.90 | 0.371 | -.1836084 | .4922263 |

* Number of observations for equation 1: 891
Number of observations for equation 2: 751
Instruments for equation 1:
XT-style: LD.n
Standard: _cons
Instruments for equation 2 :
XT-style: L(2/.).n
Standard: D.w LD.w


## Details of moment-evaluator programs

In examples $9,10,11$, and 12 , we used moment-evaluator programs to evaluate moment conditions that could not be specified using the interactive version of gmm. In example 11, we also showed how to pass additional information to an evaluator program. Here we discuss how to make moment-evaluator programs provide derivatives and accept weights.

The complete specification for a moment-evaluator program's syntax statement is

```
syntax varlist if [weight], at(name) options [derivatives(varlist)]
```

The macro 'varlist' contains the list of variables that we are to fill in with the values of our residual equations. The macro 'if' represents an if condition that restricts the estimation sample. The macro 'at' represents a vector containing the parameter values at which we are to evaluate our residual equations. options represent other options that you specify in your call to gmm and want to have passed to your moment-evaluator programs. In example 11, we included the mylhs() option so that we could pass the name of the dependent variable to our evaluator program.

Two new elements of the syntax statement allow for weights and derivatives. weight specifies the types of weights your program allows. The interactive version of gmm allows for fweights, aweights, and pweights. However, unless you explicitly allow your moment evaluator program to accept weights, you cannot specify weights in your call to gmm with the moment-evaluator program version.

The derivatives() option is used to pass to your program a set of variables that you are to fill in with the derivatives of your residual equations with respect to the parameters.

To indicate that your program can calculate derivatives, you specify either the hasderivatives or the haslfderivatives option to gmm. The hasderivatives option indicates that your program calculates parameter-level derivatives; that method requires more work but can be applied to any GMM problem. The haslfderivatives option requires less work but can be used only when the model's residual equations satisfy certain restrictions and you use the $<$ eqname $\rangle$ : $<$ varname $\rangle$ syntax with the parameters() option.

We first consider how to write the derivative computation logic to work with the hasderivatives option and provide an example; then we do the same for the haslfderivatives option.

Say you specify $k$ parameters in the nparameters () or parameters () option and $q$ equations in the nequations() or equations() option and you specify hasderivatives. Then 'derivatives' will contain $k \times q$ variables. The first $k$ variables are for the derivatives of the first residual equation with respect to the $k$ parameters, the second $k$ variables are for the derivatives of the second residual equation, and so on.

## > Example 19: Specifying derivatives with simple parameter names

To focus on how to specify derivatives, we return to the simple moment-evaluator program we used in example 9 , in which we had three regressors, and extend it to supply derivatives. The residual equation corresponding to moment condition (13) is

$$
u_{i t}(\boldsymbol{\beta})=y_{i t}-\mu_{i t} \frac{\bar{y}_{i}}{\bar{\mu}_{i}}
$$

where $\mu_{i t}, \bar{\mu}_{i}$, and $\bar{y}_{i}$ were defined previously. Now

$$
\begin{equation*}
\frac{\partial}{\partial \beta_{j}} u_{i t}(\boldsymbol{\beta})=-\mu_{i t} \frac{\bar{y}_{i}}{\bar{\mu}_{i}^{2}}\left(x_{i t}^{(j)} \bar{\mu}_{i}-\frac{1}{T} \sum_{l=1}^{l=T} x_{i l}^{(j)} \mu_{i l}\right) \tag{28}
\end{equation*}
$$

where $x_{i t}^{(j)}$ represents the $j$ th element of $\mathbf{x}_{i t}$.

Our moment-evaluator program is

```
program gmm_poideriv
    version 13
    syntax varlist if, at(name) [derivatives(varlist)]
    quietly {
        // Calculate residuals as before
        tempvar mu mubar ybar
        gen double 'mu' = exp(x1*'at'[1,1] + x2*'at'[1,2] ///
                                + x3*'at'[1,3]) 'if'
        egen double 'mubar' = mean('mu') 'if', by(id)
        egen double 'ybar' = mean(y) 'if', by(id)
        replace 'varlist' = y - 'mu'*'ybar'/'mubar' 'if'
        // Did -gmm- request derivatives?
        if "'derivatives'" == "" {
                exit // no, so we are done
    }
    // Calculate derivatives
    // We need the panel means of x1*mu, x2*mu, and x x*mu
    tempvar work x1mubar x2mubar x3mubar
    generate double 'work' = x1*'mu' 'if'
    egen double 'x1mubar' = mean('work') 'if', by(id)
    replace 'work' = x2*'mu' 'if,
    egen double 'x2mubar' = mean('work') 'if', by(id)
    replace 'work' = x3*'mu' 'if'
    egen double 'x3mubar' = mean('work') 'if', by(id)
    local d1: word 1 of 'derivatives'
    local d2: word 2 of 'derivatives'
    local d3: word 3 of 'derivatives'
    replace 'd1' = -1*'mu'*'ybar'/'mubar'` 2*(x1*'mubar' - 'x1mubar')
    replace 'd2' = -1*'mu'*'ybar'/'mubar'`2*(x2*'mubar' - 'x2mubar')
    replace 'd3' = -1*'mu'*'ybar'/'mubar'` 2*(x3*'mubar' - 'x3mubar')
    }
end
```

The derivatives() option is made optional in the syntax statement by placing it in square brackets. If gmm needs to evaluate your moment equations but does not need derivatives at that time, then the derivatives() option will be empty. In our program, we check to see if that is the case, and, if so, exit without calculating derivatives. As is often the case with $[R] \mathrm{ml}$ as well, the portion of our program devoted to derivatives is longer than the code to compute the objective function.

The first part of our derivative code computes the term

$$
\begin{equation*}
\frac{1}{T} \sum_{l=1}^{l=T} x_{i l}^{(j)} \mu_{i l} \tag{29}
\end{equation*}
$$

for $x_{i t}^{(j)}=\mathrm{x} 1, \mathrm{x} 2$, and, x 3 . The 'derivatives' macro contains three variable names, corresponding to the three parameters of the 'at' matrix. We extract those names into local macros ' d 1 ', ' d 2 ', and ' d 3 ', and then fill in the variables those macros represent with the derivatives shown in (28).

With our program written, we fit our model by typing

```
. use http://www.stata-press.com/data/r13/poisson1
. gmm gmm_poideriv, nequations(1) parameters(b1 b2 b3)
> instruments(x1 x2 x3, noconstant) vce(cluster id) onestep hasderivatives
Step 1
Iteration 0: GMM criterion Q(b) = 51.99142
Iteration 1: GMM criterion Q(b) = . 04345191
Iteration 2: GMM criterion Q(b) = 8.720e-06
Iteration 3: GMM criterion Q(b) = 7.115e-13
Iteration 4: GMM criterion Q(b) = 5.129e-27
GMM estimation
```

Number of parameters $=3$
Number of moments $=3$
Initial weight matrix: Unadjusted $\quad$ Number of obs $=409$
(Std. Err. adjusted for 45 clusters in id)

|  |  | Robust |  |  |  |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: | ---: |
|  | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| /b1 | 1.94866 | .1000265 | 19.48 | 0.000 | 1.752612 | 2.144709 |
| /b2 | -2.966119 | .0923592 | -32.12 | 0.000 | -3.14714 | -2.785099 |
| /b3 | 1.008634 | .1156561 | 8.72 | 0.000 | .781952 | 1.235315 |

Instruments for equation 1: x1 x2 x3
Our results are identical to those in example 9. Another way to verify that our program calculates derivatives correctly would be to type

```
. gmm gmm_poideriv, nequations(1) parameters(b1 b2 b3)
> instruments(x1 x2 x3, noconstant) vce(cluster id) onestep
```

Without the hasderivatives or haslfderivatives option, gmm will not request derivatives from your program, even if it contains code to compute them. If you have trouble obtaining convergence with the hasderivatives or haslfderivatives option but do not have trouble without specifying one of them, then you need to recheck your derivatives.

After example 9, we remarked that the evaluator program would have to be changed to accommodate different regressors. We then showed how you can specify parameters using the syntax $<$ eqname $>:<$ varname $>$ and then use matrix score to compute linear combinations of variables. To specify derivatives when you specify parameters using this equation-name syntax, ensure that your residual equations satisfy the "linear-form restriction" analogous to the restrictions of linear-form evaluators used by ml. See [R] ml and Gould, Pitblado, and Poi (2010) for more information about linear-form evaluators.

A GMM residual equation satisfies the linear-form restriction if the equation can be written in terms of a single observation in the dataset and if the equation for observation $i$ does not depend on any observations $j \neq i$. Cross-sectional models satisfy the linear-form restriction. Time-series models satisfy the linear-form restriction only when no lags or leads are used.

Panel-data models often do not satisfy the linear-form restriction. For example, recall moment condition (13) for a panel Poisson model. That residual equation included panel-level mean terms $\bar{y}_{i}$ and $\bar{\mu}_{i}$, so the residual equation for an individual observation depends on all the observations in the same panel.

When a residual equation does not satisfy the linear-form restriction, neither will its derivatives. To apply the chain rule, we need a way to multiply the eqname-level derivative by each of the variables
in the equation to obtain parameter-level derivatives. In (28), for example, there is no way to factor out each $x_{i t}^{(j)}$ variable and obtain an eqname-level derivative that we then multiply by each of the $x_{i t}^{(j)}$ s.

Suppose we do have a model with $q=2$ moment equations, both of which do satisfy the linear-form restriction, and we specify the parameters() option like this:

```
. gmm ..., parameters(eq1:x1 eq1:x2 eq1:_cons eq2:_cons eq3:x1 eq3:x2 eq3:_cons)
```

We have specified $n=3$ eqnames in the parameters() option: eq1, eq2, and eq3. When we specify the haslfderivatives option, gmm will pass $n \times q=3 \times 2=6$ variables in the derivatives() option. The first three variables are to be filled with

$$
\frac{\partial}{\partial \mathrm{eq} 1} u_{1 i}(\boldsymbol{\beta}) \quad \frac{\partial}{\partial \mathrm{eq} 2} u_{1 i}(\boldsymbol{\beta}) \quad \text { and } \quad \frac{\partial}{\partial \mathrm{eq} 3} u_{1 i}(\boldsymbol{\beta})
$$

where $u_{1 i}(\boldsymbol{\beta})$ is the $i$ th observation for the first moment equation. Then the second three variables are to be filled with

$$
\frac{\partial}{\partial \mathrm{eq} 1} u_{2 i}(\boldsymbol{\beta}) \quad \frac{\partial}{\partial \mathrm{eq} 2} u_{2 i}(\boldsymbol{\beta}) \quad \text { and } \quad \frac{\partial}{\partial \mathrm{eq} 3} u_{2 i}(\boldsymbol{\beta})
$$

where $u_{2 i}(\boldsymbol{\beta})$ is the second moment equation. In this example, we filled in a total of six variables with derivatives. If we instead used the hasderivatives option, we would have filled $k \times q=7 \times 2=14$ variables; moreover, if we wanted to change the number of variables in our model, we would have modified our evaluator program.

## $>$ Example 20: Specifying derivatives with linear-form residual equations

In examples 7 and 8, we showed how to specify derivatives with an exponential regression model when using the interactive version of gmm. Here we show how to write a moment-evaluator program for the exponential regression model, including derivatives.

The residual equation for observation $i$ is

$$
u_{i}=y_{i}-\exp \left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}\right)
$$

where $\mathbf{x}_{i}$ may include a constant term. The derivative with respect to the linear combination $\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}$ is

$$
\frac{\partial u_{i}}{\partial \mathbf{x}_{i}^{\prime} \boldsymbol{\beta}}=-\exp \left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}\right)
$$

To verify this residual equation satisfies the linear-form restriction, we see that for the $j$ th element of $\boldsymbol{\beta}$, we have

$$
\frac{\partial u_{i}}{\partial \beta_{j}}=-x_{i j} \exp \left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}\right)=\frac{\partial u_{i}}{\partial \mathbf{x}_{i}^{\prime} \boldsymbol{\beta}} \times x_{i j}
$$

so that given $\partial u_{i} / \partial \mathbf{x}_{i}^{\prime} \boldsymbol{\beta}$, gmm can apply the chain rule to obtain the derivatives with respect to the individual parameters.

Our moment-evaluator program is

```
program gmm_poideriv2
    version 13
    syntax varlist if, at(name) [derivatives(varlist)]
    quietly {
        tempvar mu
        matrix score double 'mu' = 'at' 'if', eq(#1)
        replace 'mu' = exp('mu')
        local depvar : coleq 'at'
        local depvar : word 1 of 'depvar'
        replace 'varlist' = 'depvar' - 'mu' 'if'
        // Did -gmm- request derivatives?
        if "'derivatives'" == "" {
                exit // no, so we are done
            }
        // Calculate derivatives
        // The derivatives macro only has one variable
        // for this model.
        replace 'derivatives' = -1*'mu' 'if'
    }
```

end

To fit our model of doctor visits treating income as an endogenous regressor, we type
. use http://www.stata-press.com/data/r13/docvisits
. gmm gmm_poideriv2, nequations(1)
> instruments(private chronic female age black hispanic)
> parameters(docvis:private docvis:chronic
> docvis:female docvis:income docvis:_cons) haslfderivatives
Step 1
Iteration 0: GMM criterion $Q(b)=16.910173$
Iteration 1: GMM criterion $\mathrm{Q}(\mathrm{b})=.82270871$
Iteration 2: GMM criterion $Q(b)=.21831995$
Iteration 3: GMM criterion $Q(b)=.12685934$
Iteration 4: GMM criterion $Q(b)=.12672369$
Iteration 5: GMM criterion $Q(b)=.12672365$
Step 2
Iteration 0: GMM criterion $Q(b)=.00234641$
Iteration 1: GMM criterion $Q(b)=.00215957$
Iteration 2: GMM criterion $Q(b)=.00215911$
Iteration 3: GMM criterion $\mathrm{Q}(\mathrm{b})=.00215911$
GMM estimation
Number of parameters $=5$
Number of moments $=7$
Initial weight matrix: Unadjusted $\quad$ Number of obs $=4412$
GMM weight matrix: Robust

|  | Coef. | Robust Std. Err. | z | $P>\|z\|$ | [95\% Conf. | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| private | . 535335 | . 159904 | 3.35 | 0.001 | . 221929 | . 848741 |
| chronic | 1.090126 | . 0617659 | 17.65 | 0.000 | . 9690668 | 1.211185 |
| female | . 6636579 | . 0959885 | 6.91 | 0.000 | . 475524 | . 8517918 |
| income | . 0142855 | . 0027162 | 5.26 | 0.000 | . 0089618 | . 0196092 |
| _cons | -. 5983477 | . 138433 | -4.32 | 0.000 | -. 8696714 | -. 327024 |

Instruments for equation 1: private chronic female age black hispanic _cons
Our results match those shown in example 8.

We can change the variables in our model just by changing the parameters() and instruments () options; we do not need to make any changes to the moment-evaluator program, because we used linear-form derivatives.

Depending on your model, allowing your moment-evaluator program to accept weights may be as easy as modifying the syntax command to allow them, or it may require significantly more work. If your program uses only commands like generate and replace, then just modifying the syntax command is all you need to do; gmm takes care of applying the weights to the observation-level residuals when computing the sample moments, derivatives, and weight matrices. On the other hand, if your moment-evaluator program computes residuals using statistics that depend on multiple observations, then you must apply the weights passed to your program when computing those statistics.

In our examples of panel Poisson with strictly exogenous regressors (9 and 18), we used the statistics $\bar{\mu}_{i}$ and $\bar{y}_{i}$ when computing the residuals. If we are to allow weights with our momentevaluator program, then we must incorporate those weights when computing $\bar{\mu}_{i}$ and $\bar{y}_{i}$. Moreover, looking at the derivative in (28), the term highlighted in (29) is in fact a sample mean, so we must incorporate weights when computing it.

## > Example 21: Panel Poisson with derivatives and weights

Here we modify the program in example 19. to accept frequency weights. One complication immediately arises: we had been using egen to compute $\bar{\mu}_{i}$ and $\bar{y}_{i}$. egen does not accept weights, so we must compute $\bar{\mu}_{i}$ and $\bar{y}_{i}$ ourselves, incorporating any weights the user may specify. Our program is

```
program gmm_poiderivfw
    version 13
    syntax varlist if [fweight/], at(name) [derivatives(varlist)]
    quietly {
        if "'exp'" == "" { // no weights
        local exp 1 // weight each observation equally
    }
    // Calculate residuals as before
    tempvar mu mubar ybar sumwt
    gen double 'mu' = exp(x1*'at'[1,1] + x2*'at'[1,2] ///
        + x3*'at'[1,3]) 'if'
    bysort id: gen double 'sumwt' = sum('exp')
    by id: gen double 'mubar' = sum('mu'*'exp')
    by id: gen double 'ybar' = sum(y*'exp')
    by id: replace 'mubar' = 'mubar'[_N] / 'sumwt'[_N]
    by id: replace 'ybar' = 'ybar'[_N] / 'sumwt'[_N]
    replace 'varlist' = y - 'mu'*'ybar'/'mubar' 'if'
    // Did -gmm- request derivatives?
    if "'derivatives'" == "" {
        exit // no, so we are done
    }
    // Calculate derivatives
    // We need the panel means of x1*mu, x2*mu, and x }3*m
    tempvar work x1mubar x2mubar x3mubar
    generate double 'work' = x1*'mu' 'if'
    by id: generate double 'x1mubar' = sum('work'*'exp')
    by id: replace 'x1mubar' = 'x1mubar'[_N] / 'sumwt'[_N]
    replace 'work' = x2*'mu' 'if'
    by id: generate double 'x2mubar' = sum('work'*'exp')
    by id: replace 'x2mubar' = 'x2mubar'[_N] / 'sumwt'[_N]
    replace 'work' = x3*'mu' 'if,
    by id: generate double 'x3mubar' = sum('work'*'exp')
    by id: replace 'x3mubar' = 'x3mubar'[_N] / 'sumwt'[_N]
    local d1: word 1 of 'derivatives'
    local d2: word 2 of 'derivatives'
    local d3: word 3 of 'derivatives'
    replace 'd1' = -1*'mu'*'ybar'/'mubar'^2*(x1*'mubar' - 'x1mubar')
    replace 'd2' = -1*'mu'*'ybar'/'mubar'` 2*(x2*'mubar' - 'x2mubar')
    replace 'd3' = -1*'mu'*'ybar'/'mubar'` 2*(x3*'mubar' - 'x3mubar')
    }
```

end
Our syntax command now indicates that fweights are allowed. The first part of our code looks at the macro 'exp'. If it is empty, then the user did not specify weights in their call to gmm; and we set the macro equal to 1 , so that we weight each observation equally. After we compute $\mu_{i t}$, we calculate $\bar{\mu}_{i}$ and $\bar{y}_{i}$, taking into account weights. To compute frequency-weighted means for each panel, we just multiply each observation by its respective weight, sum over all observations in the panel, then divide by the sum of the weights for the panel. (See [U] 20.23 Weighted estimation for information on how to handle aweights and pweights.) We use the same procedure to compute the frequency-weighted variant of expression (29) in the derivative calculations.

To use our program, we type

```
. use http://www.stata-press.com/data/r13/poissonwts
. gmm gmm_poiderivfw [fw=fwt], nequations(1) parameters(b1 b2 b3)
> instruments(x1 x2 x3, noconstant) vce(cluster id) onestep hasderivatives
(sum of wgt is 819)
Step 1
Iteration 0: GMM criterion Q(b) = 49.8292
Iteration 1: GMM criterion Q(b) = . }1113673
Iteration 2: GMM criterion Q(b) = .00008519
Iteration 3: GMM criterion Q(b) = 7.110e-11
Iteration 4: GMM criterion Q(b) = 5.596e-23
GMM estimation
Number of parameters = 3
Number of moments = 3
Initial weight matrix: Unadjusted Number of obs = 819
```

(Std. Err. adjusted for 45 clusters in id)

|  |  | Robust |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
|  | /b1 | 1.967766 | .111795 | 17.60 | 0.000 | 1.748652 |
| /b2 | -3.060838 | .0935561 | -32.72 | 0.000 | -3.244205 | -2.877472 |
| /b3 | 1.037594 | .1184227 | 8.76 | 0.000 | .80549 | 1.269698 |

Instruments for equation 1: x 1 x 2 x 3
Testing whether our program works correctly with frequency weights is easy. A frequency-weighted dataset is just a compact form of a larger dataset in which identical observations are omitted and a frequency-weight variable is included to tell us how many times each observation in the smaller dataset appears in the larger dataset. Therefore, we can expand our smaller dataset by the frequency-weight variable and then refit our model without specifying frequency weights. If we obtain the same results, our program works correctly. When we type

```
. expand fw
. gmm gmm_poiderivfw, nequations(1) parameters(b1 b2 b3)
> instruments(x1 x2 x3, noconstant) vce(cluster id) onestep
```

we obtain the same results as before.

## Stored results

gmm stores the following in e():

Scalars
e(k_eq)
e(k_eq_model)
e(k_aux)
e(n_moments)
e(n_eq)
e(Q)
e(J)
e(J_df)
e(k_i)
e(has_xtinst)
e(N_clust)
e(type)
e(rank)
e(ic)
e(converged)

Macros
e(cmd)
e(cmdline)
e(title)
e(title_2)
e(clustvar)
e(inst_i)
e(eqnames)
e(winit)
e(winitname)
e(estimator)
e(rhs)
e(params_i)
e(wmatrix)
e(vce)
e(vcetype)
e(params)
e(sexp_i)
e(evalprog)
e(evalopts)
e(nocommonesample)
e(technique)
e(properties)
e(estat_cmd)
e(predict)
e(marginsnotok)
Matrices
e(b)
e(init)
e(Wuser)
e(W)
e(S)
e(N_byequation)
e(V)
e(V_modelbased)
Functions
e(sample) marks estimation sample

## Methods and formulas

Let $q$ denote the number of moment equations. For observation $i, i=1, \ldots, N$, write the $j$ th moment equation as $\mathbf{z}_{i j} u_{i j}\left(\boldsymbol{\beta}_{j}\right)$ for $j=1, \ldots, q . \mathbf{z}_{i j}$ is a $1 \times m_{j}$ vector, where $m_{j}$ is the number of instruments specified for equation $j$. Let $m=m_{1}+\cdots+m_{q}$.

Our notation can incorporate moment conditions of the form $h_{i j}\left(\mathbf{w}_{i j} ; \boldsymbol{\beta}_{j}\right)$ with instruments $\mathbf{w}_{i j}$ by defining $\mathbf{z}_{i j}=1$ and $u_{i j}\left(\boldsymbol{\beta}_{j}\right)=h_{i j}\left(\mathbf{w}_{i j} ; \beta_{j}\right)$, so except when necessary we do not distinguish between the two types of moment conditions. We could instead use notation so that all our moment conditions are of the form $h_{i j}\left(\mathbf{w}_{i j} ; \beta_{j}\right)$, or we could adopt notation that explicitly combines both forms of moment equations. However, because moment conditions of the form $\mathbf{z}_{i j}^{\prime} u_{i j}\left(\boldsymbol{\beta}_{j}\right)$ are arguably more common, we use that notation.

Let $\boldsymbol{\beta}$ denote a $k \times 1$ vector of parameters, consisting of all the unique parameters of $\boldsymbol{\beta}_{1}, \ldots, \boldsymbol{\beta}_{q}$. Then we can stack the moment conditions and write them more compactly as $\mathbf{Z}_{i}^{\prime} \mathbf{u}_{i}(\boldsymbol{\beta})$, where

$$
\mathbf{Z}_{i}=\left[\begin{array}{cccc}
\mathbf{z}_{i 1} & \mathbf{0} & \cdots & \mathbf{0} \\
\mathbf{0} & \mathbf{z}_{i 2} & \cdots & \mathbf{0} \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{0} & \mathbf{0} & \cdots & \mathbf{z}_{i q}
\end{array}\right] \quad \text { and } \quad \mathbf{u}_{i}(\boldsymbol{\beta})=\left[\begin{array}{c}
u_{i 1}\left(\boldsymbol{\beta}_{1}\right) \\
u_{i 2}\left(\boldsymbol{\beta}_{2}\right) \\
\vdots \\
u_{i q}\left(\boldsymbol{\beta}_{j}\right)
\end{array}\right]
$$

The GMM estimator $\widehat{\boldsymbol{\beta}}$ is the value of $\beta$ that minimizes

$$
\begin{equation*}
Q(\boldsymbol{\beta})=\left\{N^{-1} \sum_{i=1}^{N} \mathbf{Z}_{i}^{\prime} \mathbf{u}_{i}(\boldsymbol{\beta})\right\}^{\prime} \mathbf{W}\left\{N^{-1} \sum_{i=1}^{N} \mathbf{Z}_{i}^{\prime} \mathbf{u}_{i}(\boldsymbol{\beta})\right\} \tag{A1}
\end{equation*}
$$

for $q \times q$ weight matrix $\mathbf{W}$.
By default, gmm minimizes (A1) using the Gauss-Newton method. See Hayashi $(2000,498)$ for a derivation. This technique is typically faster than quasi-Newton methods and does not require second-order derivatives.

Methods and formulas are presented under the following headings:

```
Initial weight matrix
Weight matrix
Variance-covariance matrix
Hansen's J statistic
Panel-style instruments
```


## Initial weight matrix

If you specify winitial (identity), then we set $\mathbf{W}=\mathbf{I}_{q}$.
If you specify winitial (unadjusted), then we create matrix $\boldsymbol{\Lambda}$ with typical submatrix

$$
\boldsymbol{\Lambda}_{r s}=N^{-1} \sum_{i=1}^{N} \mathbf{z}_{i r}^{\prime} \mathbf{z}_{i s}
$$

for $r=1, \ldots, q$ and $s=1, \ldots, q$. If you include the independent suboption, then we set $\boldsymbol{\Lambda}_{r s}=\mathbf{0}$ for $r \neq s$. The weight matrix $\mathbf{W}$ equals $\boldsymbol{\Lambda}^{-1}$.

If you specify winitial (matname), then we set $\mathbf{W}$ equal to Stata matrix matname.

If you specify winitial (xt xtspec), then you must specify one or two items in xtspec, one for each equation. gmm allows you to specify at most two moment equations when you specify winitial (xt xtspec), one in first-differences and one in levels. We create the block-diagonal matrix $\mathbf{H}$ with typical block $\mathbf{H}_{j}$. If the $j$ th element of xtspec is " L ", then $\mathbf{H}_{j}$ is the identity matrix of suitable dimension. If the $j$ th element of xtspec is " D ", then

$$
\mathbf{H}_{j}=\left[\begin{array}{cccccc}
1 & -0.5 & 0 & \ldots & 0 & 0 \\
-0.5 & 1 & -0.5 & \ldots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \ldots & 1 & -0.5 \\
0 & 0 & 0 & \ldots & -0.5 & 1
\end{array}\right]
$$

Then

$$
\boldsymbol{\Lambda}_{H}=N_{g}^{-1} \sum_{g=1}^{g=N_{G}} \mathbf{Z}_{g}^{\prime} \mathbf{H} \mathbf{Z}_{g}
$$

where $g$ indexes panels in the dataset, $N_{G}$ is the number of panels, $\mathbf{Z}_{g}$ is the full instrument matrix for panel $g$, and $\mathbf{W}=\boldsymbol{\Lambda}_{H}^{-1}$. See Panel-style instruments below for a discussion of how $\mathbf{Z}_{g}$ is formed.

## Weight matrix

Specification of the weight matrix applies only to the two-step and iterative estimators. When you use the onestep option, the wmatrix() option is ignored.

We first evaluate (A1) using the initial weight matrix described above and then compute $\mathbf{u}_{i}(\widehat{\boldsymbol{\beta}})$. In all cases, $\mathbf{W}=\boldsymbol{\Lambda}^{-1}$. If you specify wmatrix (unadjusted), then we create $\boldsymbol{\Lambda}$ to have typical submatrix

$$
\boldsymbol{\Lambda}_{r s}=\sigma_{r s} N^{-1} \sum_{i=1}^{N} \mathbf{z}_{i r}^{\prime} \mathbf{z}_{i s}
$$

where

$$
\sigma_{r s}=N^{-1} \sum_{i=1}^{N} u_{i r}(\widehat{\boldsymbol{\beta}}) u_{i s}(\widehat{\boldsymbol{\beta}})
$$

and $r$ and $s$ index moment equations. For all types of weight matrices, if the independent suboption is specified, then $\boldsymbol{\Lambda}_{r s}=\mathbf{0}$ for $r \neq s$, where $\boldsymbol{\Lambda}_{r s}$ measures the covariance between moment conditions for equations $r$ and $s$.

If you specify wmatrix(robust), then

$$
\boldsymbol{\Lambda}=N^{-1} \sum_{i=1}^{N} \mathbf{Z}_{i} \mathbf{u}_{i}(\widehat{\boldsymbol{\beta}}) \mathbf{u}_{i}^{\prime}(\widehat{\boldsymbol{\beta}}) \mathbf{Z}_{i}^{\prime}
$$

If you specify wmatrix(cluster clustvar), then

$$
\boldsymbol{\Lambda}=N^{-1} \sum_{c=1}^{c=N_{C}} \mathbf{q}_{c} \mathbf{q}_{c}^{\prime}
$$

where $c$ indexes clusters, $N_{C}$ is the number of clusters, and

$$
\mathbf{q}_{c}=\sum_{i \in c_{j}} \mathbf{Z}_{i} \mathbf{u}_{i}(\widehat{\boldsymbol{\beta}})
$$

If you specify wmatrix(hac kernel [\#]), then

$$
\begin{aligned}
\boldsymbol{\Lambda}= & N^{-1} \sum_{i=1}^{N} \mathbf{Z}_{i} \mathbf{u}_{i}(\widehat{\boldsymbol{\beta}}) \mathbf{u}_{i}(\widehat{\boldsymbol{\beta}})^{\prime} \mathbf{Z}_{i}^{\prime}+ \\
& N^{-1} \sum_{l=1}^{l=n-1} \sum_{i=l+1}^{N} K(l, m)\left\{\mathbf{Z}_{i} \mathbf{u}_{i}(\widehat{\boldsymbol{\beta}}) \mathbf{u}_{i-l}^{\prime}(\widehat{\boldsymbol{\beta}}) \mathbf{Z}_{i-l}^{\prime}+\mathbf{Z}_{i-l} \mathbf{u}_{i-l}(\widehat{\boldsymbol{\beta}}) \mathbf{u}_{i}^{\prime}(\widehat{\boldsymbol{\beta}}) \mathbf{Z}_{i}^{\prime}\right\}
\end{aligned}
$$

where $m=\#$ if \# is specified and $m=N-2$ otherwise. Define $z=l /(m+1)$. If kernel is bartlett or nwest, then

$$
K(l, m)= \begin{cases}1-z & 0 \leq z \leq 1 \\ 0 & \text { otherwise }\end{cases}
$$

If kernel is parzen or gallant, then

$$
K(l, m)= \begin{cases}1-6 z^{2}+6 z^{3} & 0 \leq z \leq 0.5 \\ 2(1-z)^{3} & 0.5<z \leq 1 \\ 0 & \text { otherwise }\end{cases}
$$

If kernel is quadraticspectral or andrews, then

$$
K(l, m)= \begin{cases}1 & z=0 \\ 3\{\sin (\theta) / \theta-\cos (\theta)\} / \theta^{2} & \text { otherwise }\end{cases}
$$

where $\theta=6 \pi z / 5$.
If wmatrix (hac kernel opt) is specified, then gmm uses Newey and West's (1994) automatic lag-selection algorithm, which proceeds as follows. Define $\mathbf{h}$ to be an $m \times 1$ vector of ones. Note that this definition of $\mathbf{h}$ is slightly different than the one used by ivregress. There, the element of $\mathbf{h}$ corresponding to the constant term equals zero, effectively ignoring the effect of the constant in determining the optimal lag length. Here we include the effect of the constant term. Now define

$$
\begin{aligned}
f_{i} & =\left\{\mathbf{Z}_{i}^{\prime} \mathbf{u}_{i}(\boldsymbol{\beta})\right\}^{\prime} \mathbf{h} \\
\widehat{\sigma}_{j} & =N^{-1} \sum_{i=j+1}^{N} f_{i} f_{i-j} \quad j=0, \ldots, m^{*} \\
\widehat{s}^{(q)} & =2 \sum_{j=1}^{j=m^{*}} \widehat{\sigma}_{j} j^{q} \\
\widehat{s}^{(0)} & =\widehat{\sigma}_{0}+2 \sum_{j=1}^{j=m^{*}} \widehat{\sigma}_{j} \\
\widehat{\gamma} & =c_{\gamma}\left\{\left(\frac{\widehat{s}^{(q)}}{\widehat{s}^{(0)}}\right)^{2}\right\}^{1 /(2 q+1)} \\
m & =\widehat{\gamma} N^{1 /(2 q+1)}
\end{aligned}
$$

where $q, m^{*}$, and $c_{\gamma}$ depend on the kernel specified:

| Kernel | $q$ | $m^{*}$ | $c_{\gamma}$ |
| :---: | :---: | :---: | :---: |
| Bartlett/Newey-West | 1 | $\operatorname{int}\left\{20(T / 100)^{2 / 9}\right\}$ | 1.1447 |
| Parzen/Gallant | 2 | $\operatorname{int}\left\{20(T / 100)^{4 / 25}\right\}$ | 2.6614 |
| Quadratic spectral/Andres | 2 | int $\left\{20(T / 100)^{2 / 25}\right\}$ | 1.3221 |

where $\operatorname{int}(x)$ denotes the integer obtained by truncating $x$ toward zero. For the Bartlett and Parzen kernels, the optimal lag is $\min \left\{\operatorname{int}(m), m^{*}\right\}$. For the quadratic spectral kernel, the optimal lag is $\min \left\{m, m^{*}\right\}$.

## Variance-covariance matrix

If you specify vce(unadjusted), then the VCE matrix is computed as

$$
\begin{equation*}
\operatorname{Var}(\widehat{\boldsymbol{\beta}})=N^{-1}\left\{\overline{\mathbf{G}}(\widehat{\boldsymbol{\beta}})^{\prime} \mathbf{W} \overline{\mathbf{G}}(\widehat{\boldsymbol{\beta}})\right\}^{-1} \tag{A2}
\end{equation*}
$$

where

$$
\begin{equation*}
\overline{\mathbf{G}}(\widehat{\boldsymbol{\beta}})=\left.N^{-1} \sum_{i=1}^{N} \mathbf{Z}_{i}^{\prime} \frac{\partial \mathbf{u}_{i}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}^{\prime}}\right|_{\boldsymbol{\beta}=\widehat{\boldsymbol{\beta}}} \tag{A3}
\end{equation*}
$$

For the two-step and iterated estimators, we use the weight matrix $\mathbf{W}$ that was used to compute the final-round estimate $\widehat{\boldsymbol{\beta}}$.

When you do not specify analytic derivatives, gmm must compute the Jacobian matrix (A3) numerically. By default, gmm computes each element of the matrix individually by using the Mata deriv() function; see [M-5] deriv( ). This procedure results in accurate derivatives but can be slow if your model has many instruments or parameters.

When you specify the quickderivatives option, gmm computes all derivatives corresponding to parameter $\beta_{j}, j=1, \ldots, q$, at once, using two-sided derivatives with a step size of $\left|\beta_{j}\right| \epsilon^{1 / 3}$, where $\epsilon$ is the machine precision of a double precision number (approximately $2.22045 \times 10^{-16}$ ). This method requires just two evaluations of the model's moments to compute an entire column of (A3) and therefore has the most impact when you specify many instruments or moment equations so that (A3) has many rows.

For the one-step estimator, how the unadjusted VCE is computed depends on the type of initial weight matrix requested and the form of the moment equations. If you specify two or more moment equations of the form $h_{i j}\left(\mathbf{w}_{i j} ; \boldsymbol{\beta}_{j}\right)$, then gmm issues a warning message and computes a heteroskedasticityrobust VCE because here the matrix $\mathbf{Z}^{\prime} \mathbf{Z}$ is necessarily singular; moreover, here you must use the identity matrix as the initial weight matrix. Otherwise, if you specify winitial (identity) or winitial (unadjusted), then gmm first computes an unadjusted weight matrix based on $\widehat{\boldsymbol{\beta}}$ before evaluating (A2). If you specify winitial (matname), then (A2) is evaluated based on matname; the user is responsible for verifying that the VCE and other statistics so produced are appropriate.

All types of robust VCEs computed by gmm take the form

$$
\operatorname{Var}(\widehat{\boldsymbol{\beta}})=N^{-1}\left\{\overline{\mathbf{G}}(\widehat{\boldsymbol{\beta}})^{\prime} \mathbf{W} \overline{\mathbf{G}}(\widehat{\boldsymbol{\beta}})\right\}^{-1} \overline{\mathbf{G}}(\widehat{\boldsymbol{\beta}})^{\prime} \mathbf{W} \mathbf{S W} \overline{\mathbf{G}}(\widehat{\boldsymbol{\beta}})\left\{\overline{\mathbf{G}}(\widehat{\boldsymbol{\beta}})^{\prime} \mathbf{W} \overline{\mathbf{G}}(\widehat{\boldsymbol{\beta}})\right\}^{-1}
$$

For the one-step estimator, $\mathbf{W}$ represents the initial weight matrix requested using the winitial() option, and $\mathbf{S}$ is computed based on the specification of the vce() option. The formulas for the $\mathbf{S}$ matrix are identical to the ones that define the $\boldsymbol{\Lambda}$ matrix in Weight matrix above, except that $\mathbf{S}$ is computed after the moment equations are reevaluated using the final estimate of $\widehat{\boldsymbol{\beta}}$. For the two-step and iterated GMM estimators, computation of $\mathbf{W}$ is controlled by the wmatrix() option based on the penultimate estimate of $\widehat{\boldsymbol{\beta}}$.

For details on computation of the VCE matrix with dynamic panel-data models, see Panel-style instruments below.

## Hansen's J statistic

Hansen's (1982) $J$ test of overidentifying restrictions is $J=N \times Q(\widehat{\boldsymbol{\beta}}) . J \sim \chi^{2}(m-k)$. If $m<k$, gmm issues an error message without estimating the parameters. If $m=k$, the model is just-identified and $J$ is saved as missing ("."). For the two-step and iterated GMM estimators, the $J$ statistic is based on the last-computed weight matrix as determined by the wmatrix() option. For the one-step estimator, gmm recomputes a weight matrix as described in the second paragraph of Variance-covariance matrix above. To obtain Hansen's $J$ statistic, you use estat overid; see [R] gmm postestimation.

## Panel-style instruments

Here we discuss several issues that arise only when you specify panel-style instruments by using the xtinstruments() option. When you specify the xtinstruments() option, we can no longer consider the instruments for one observation in isolation; instead, we must consider the instrument matrix for an entire panel at once. In the following discussion, we let $T$ denote the number of time periods in a panel. To accommodate unbalanced datasets, conceptually we simply use zeros as instruments and residuals for time periods that are missing in a panel.

We consider the case where you specify both an equation in levels and an equation in differences, yielding two residual equations. Let $u_{p t}^{L}(\boldsymbol{\beta})$ denote the residual for the level equation for panel $p$ in period $t$, and let $u_{p t}^{D}(\boldsymbol{\beta})$ denote the residual for the corresponding difference equation. Now define the $2 T \times 1$ vector $\mathbf{u}_{p}(\boldsymbol{\beta})$ as

$$
\mathbf{u}_{p}(\boldsymbol{\beta})=\left[u_{p 1}^{L}(\boldsymbol{\beta}), u_{p 2}^{L}(\boldsymbol{\beta}), \ldots, u_{p T}^{L}(\boldsymbol{\beta}), u_{p 2}^{D}(\boldsymbol{\beta}), u_{p 3}^{D}(\boldsymbol{\beta}), \ldots, u_{p T}^{D}(\boldsymbol{\beta})\right]
$$

The $T+1$ element of $\mathbf{u}_{p}$ is $u_{p 2}^{D}(\boldsymbol{\beta})$ since we lose the first observation of the difference equation because of differencing.

We write the moment conditions for the $p$ th panel as $\mathbf{Z}_{p} \mathbf{u}_{p}(\boldsymbol{\beta})$. To see how $\mathbf{Z}_{p}$ is defined, let $\mathbf{w}_{p t}^{L}$ and $\mathbf{w}_{p t}^{D}$ denote the vectors of panel-style instruments for the level and difference equations, respectively, and let time be denoted by $t$; we discuss their dimensions momentarily. Also let $\mathbf{x}_{p t}^{L}$ and $\mathbf{x}_{p t}^{D}$ denote the vectors of instruments specified in instruments () for the level and difference equations at time $t$. Without loss of generality, for our discussion we assume that you specify the level equation first. Then $\mathbf{Z}_{p}$ has the form

$$
\mathbf{Z}_{p}=\left[\begin{array}{cccccccc}
\mathbf{w}_{1}^{L} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0}  \tag{A4}\\
\mathbf{0} & \mathbf{w}_{2}^{L} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\mathbf{0} & \mathbf{0} & \cdots & \mathbf{w}_{T}^{L} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\
\mathbf{x}_{1}^{L} & \mathbf{x}_{2}^{L} & \cdots & \mathbf{x}_{T}^{L} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{w}_{1}^{D} & \mathbf{0} & \cdots & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{w}_{2}^{D} & \cdots & \mathbf{0} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{w}_{T}^{D} \\
\mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{x}_{1}^{D} & \mathbf{x}_{2}^{D} & \cdots & \mathbf{x}_{T}^{D}
\end{array}\right]
$$

To see how the $\mathbf{w}$ vectors are formed, suppose you specify

```
xtinstruments(eq(1): d, lags(a/b))
```

Then $\mathbf{w}_{t}^{L}$ will be a $(b-a+1) \times 1$ vector consisting of $d_{t-a}, \ldots, d_{t-b}$. If $(t-a) \leq 0$, then instead we set $\mathbf{w}_{t}^{L}=0$. If $(t-a)>0$ but $(t-b) \leq 0$, then we create $\mathbf{w}_{t}^{L}$ to consist of $d_{t-a}, \ldots, d_{1}$. With this definition, $(b-a+1)$ defines the maximum number of lags of $d$ used, but gmm will proceed with fewer lags if all $(b-a+1)$ lags are not available. If you specify two panel-style instruments, d and e , say, then $\mathbf{w}_{t}^{L}$ will consist of $d_{t-a}, \ldots, d_{t-b}, e_{t-a}, \ldots, e_{t-b} . \mathbf{w}_{t}^{D}$ is handled analogously.

The $\mathbf{x}_{t}^{L}$ vectors are simply $j \times 1$ vectors, where $j$ is the number of regular instruments specified with the instruments() option; these vectors include a " 1 " unless you specify the noconstant suboption.

Looking carefully at (A4), you will notice that for dynamic panel-data models, moment conditions corresponding to the instruments $\mathbf{x}_{p t}^{L}$ take the form

$$
E\left[\sum_{t=1}^{t=T} \mathbf{x}_{p t}^{L} u_{p t}^{L}(\boldsymbol{\beta})\right]=\mathbf{0}
$$

and likewise for $\mathbf{x}_{p t}^{D}$. Instead of having separate moment conditions for each time period, there is one moment condition equal to the average of individual periods' moments. See Arellano and Bond (1991, 280). To include separate moment conditions for each time period, instead of specifying, say,
instruments(1: x)
you could instead first generate a variable called one equal to unity for all observations and specify

```
xtinstruments(1: x one)
```

(Creating the variable one is necessary because a constant is not automatically included in variable lists specified in xtinstruments().)

Unbalanced panels are essentially handled by including zeros rows and columns of $\mathbf{Z}_{p}$ and $\mathbf{u}_{p}(\boldsymbol{\beta})$ corresponding to missing time periods. However, the numbers of instruments and moment conditions reported by gmm do not reflect this trickery and instead reflect the numbers of instruments and moment conditions that are not manipulated in this way. Moreover, gmm includes code to work through these situations efficiently without actually having to fill in zeros.

When you specify winitial (xt ...), the one-step unadjusted VCE is computed as

$$
\operatorname{Var}(\widehat{\boldsymbol{\beta}})=\widehat{\sigma}_{1}^{2} \boldsymbol{\Lambda}_{H}
$$

where $\boldsymbol{\Lambda}_{H}$ was defined previously,

$$
\widehat{\sigma}_{1}^{2}=(N-k)^{-1} \sum_{p=1}^{p=P} \mathbf{u}_{p}^{D}(\widehat{\boldsymbol{\beta}})
$$

and $\mathbf{u}_{p}^{D}(\widehat{\boldsymbol{\beta}})=\left[u_{p 2}^{D}(\widehat{\boldsymbol{\beta}}), \ldots, u_{p T}^{D}(\widehat{\boldsymbol{\beta}})\right]$. Here we use $(N-k)^{-1}$ instead of $N^{-1}$ to match xtdpd.

## References

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## Also see

[R] gmm postestimation - Postestimation tools for gmm
$[\mathrm{R}]$ ivregress - Single-equation instrumental-variables regression
$[R]$ ml - Maximum likelihood estimation
[R] mlexp - Maximum likelihood estimation of user-specified expressions
[R] nl - Nonlinear least-squares estimation
[R] nlsur - Estimation of nonlinear systems of equations
[XT] xtabond - Arellano-Bond linear dynamic panel-data estimation
[XT] xtdpd - Linear dynamic panel-data estimation
[XT] xtdpdsys - Arellano-Bover/Blundell-Bond linear dynamic panel-data estimation
[U] 20 Estimation and postestimation commands

## Title

gmm postestimation - Postestimation tools for gmm

Description
Syntax for estat overid Reference

Syntax for predict
Menu for estat
Also see

## Description

The following postestimation command is of special interest after gmm:
Command Description
estat overid perform test of overidentifying restrictions

The following standard postestimation commands are also available:
Command Description

| estat vce | variance-covariance matrix of the estimators (VCE) <br> estimates <br> cataloging estimation results <br> point estimates, standard errors, testing, and inference for linear combinations <br> of coefficients |
| :--- | :--- |
| nlcom | point estimates, standard errors, testing, and inference for nonlinear combinations <br> of coefficients |
| predict | residuals |
| predictnl | point estimates, standard errors, testing, and inference for generalized predictions <br> test |
| testnl | Wald tests of simple and composite linear hypotheses |

## Special-interest postestimation command

estat overid reports Hansen's $J$ statistic, which is used to determine the validity of the overidentifying restrictions in a GMM model. If the model is correctly specified in the sense that $E\left\{\mathbf{z}_{i} u_{i}(\boldsymbol{\beta})\right\}=\mathbf{0}$, then the sample analog to that condition should hold at the estimated value of $\boldsymbol{\beta}$. Hansen's $J$ statistic is valid only if the weight matrix is optimal, meaning that it equals the inverse of the covariance matrix of the moment conditions. Therefore, estat overid only reports Hansen's $J$ statistic after two-step or iterated estimation, or if you specified winitial (matname) when calling gmm. In the latter case, it is your responsibility to determine the validity of the $J$ statistic.

## Syntax for predict

```
predict [type] newvar [if][in] [, equation(#eqno|eqname)]
predict [type] {stub*|newvar 
```

Residuals are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.
You specify one new variable and (optionally) equation(), or you specify stub* or $q$ new variables, where $q$ is the number of moment equations.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Option for predict

Main
equation(\#eqno $\mid$ eqname) specifies the equation for which residuals are desired. Specifying equation(\#1) indicates that the calculation is to be made for the first moment equation. Specifying equation(demand) would indicate that the calculation is to be made for the moment equation named demand, assuming there is an equation named demand in the model.
If you specify one new variable name and omit equation(), results are the same as if you had specified equation(\#1).
For more information on using predict after multiple-equation estimation commands, see $[R]$ predict.

## Syntax for estat overid

```
estat overid
```


## Menu for estat

Statistics $>$ Postestimation $>$ Reports and statistics

## Remarks and examples

As we noted in Introduction of [R] gmm, underlying generalized method of moments (GMM) estimators is a set of $l$ moment conditions, $E\left\{\mathbf{z}_{i} u_{i}(\boldsymbol{\beta})\right\}=\mathbf{0}$. When $l$ is greater than the number of parameters, $k$, any size- $k$ subset of the moment conditions would yield a consistent parameter estimate. We remarked that the parameter estimates we would obtain would in general depend on which $k$ moment conditions we used. However, if all our moment conditions are indeed valid, then the parameter estimates should not differ too much regardless of which $k$ moment conditions we used to estimate the parameters. The test of overidentifying restrictions is a model specification test based on this observation. The test of overidentifying restrictions requires that the number of moment conditions be greater than the number of parameters in the model.

Recall that the GMM criterion function is

$$
Q=\left\{\frac{1}{N} \sum_{i} \mathbf{z}_{i} u_{i}(\boldsymbol{\beta})\right\}^{\prime} \mathbf{W}\left\{\frac{1}{N} \sum_{i} \mathbf{z}_{i} u_{i}(\boldsymbol{\beta})\right\}
$$

The test of overidentifying restrictions is remarkably simple. If $\mathbf{W}$ is an optimal weight matrix, under the null hypothesis $H_{0}: E\left\{\mathbf{z}_{i} u_{i}(\boldsymbol{\beta})\right\}=\mathbf{0}$, the test statistic $J=N \times Q \sim \chi^{2}(l-k)$. A large test statistic casts doubt on the null hypothesis.

For the test to be valid, $\mathbf{W}$ must be optimal, meaning that $\mathbf{W}$ must be the inverse of the covariance matrix of the moment conditions:

$$
\mathbf{W}^{-1}=E\left\{\mathbf{z}_{i} u_{i}(\boldsymbol{\beta}) u_{i}^{\prime}(\boldsymbol{\beta}) \mathbf{z}_{i}^{\prime}\right\}
$$

Therefore, estat overid works only after the two-step and iterated estimators, or if you supplied your own initial weight matrix by using the winitial (matname) option to gmm and used the one-step estimator.

Often the overidentifying restrictions test is interpreted as a test of the validity of the instruments z. However, other forms of model misspecification can sometimes lead to a significant test statistic. See Hall (2005, sec. 5.1) for a discussion of the overidentifying restrictions test and its behavior in correctly and misspecified models.

## > Example 1

In example 6 of $[R]$ gmm, we fit an exponential regression model of the number of doctor visits based on the person's gender, income, possession of private health insurance, and presence of a chronic disease. We argued that the variable income may be endogenous; we used the person's age and race as additional instrumental variables. Here we refit the model and test the specification of the model. We type

```
. use http://www.stata-press.com/data/r13/docvisits
. gmm (docvis - exp({xb:private chronic female income} + {b0})),
> instruments(private chronic female age black hispanic)
    (output omitted)
estat overid
Test of overidentifying restriction:
Hansen's J chi2(2) = 9.52598 (p = 0.0085)
```

The $J$ statistic is significant even at the $1 \%$ significance level, so we conclude that our model is misspecified. One possibility is that age and race directly affect the number of doctor visits, so we are not justified in excluding them from the model.

A simple technique to explore whether any of the instruments is invalid is to examine the statistics

$$
r_{j}=\mathbf{W}_{j j}^{1 / 2}\left\{\frac{1}{N} \sum_{i=1}^{N} z_{i j} u_{i}(\widehat{\boldsymbol{\beta}})\right\}
$$

for $j=1, \ldots, k$, where $\mathbf{W}_{j j}$ denotes the $j$ th diagonal element of $\mathbf{W}, u_{i}(\widehat{\boldsymbol{\beta}})$ denotes the sample residuals, and $k$ is the number of instruments. If all the instruments are valid, then the scaled sample moments should at least be on the same order of magnitude. If one (or more) instrument's $r_{j}$ is large in absolute value relative to the others, then that could be an indication that instrument is not valid.

In Stata, we type

```
predict double r if e(sample) // obtain residual from the model
matrix W = e(W) // retrieve weight matrix
local i 1
// loop over each instrument and compute r_j
foreach var of varlist private chronic female age black hispanic {
2. generate double r'var' = r*'var'*sqrt(W['i', 'i'])
3. local '++i'
4. }
```

| . summarize r* |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: |
| Variable | Obs | Mean | Std. Dev. | Min | Max |
| $r$ | 4412 | .0344373 | 8.26176 | -151.1847 | 113.059 |
| rprivate | 4412 | .007988 | 3.824118 | -72.66254 | 54.33852 |
| rchronic | 4412 | .0026947 | 2.0707 | -43.7311 | 32.703 |
| rfemale | 4412 | .0028168 | 1.566397 | -12.7388 | 24.43621 |
| rage | 4412 | .0360978 | 4.752986 | -89.74112 | 55.58143 |
| rblack | 4412 | -.0379317 | 1.062027 | -24.39747 | 27.34512 |
| rhispanic | 4412 | -.017435 | 1.08567 | -5.509386 | 31.53512 |

We notice that the $r_{j}$ statistics for age, black, and hispanic are larger than those for the other instruments in our model, supporting our suspicion that age and race may have a direct impact on the number of doctor visits.

## Stored results

estat overid stores the following in $r()$ :
Scalars

$$
\begin{array}{ll}
\mathrm{r}(\mathrm{~J}) & \text { Hansen's } J \text { statistic } \\
\mathrm{r}\left(\mathrm{~J} \_ \text {df }\right) & J \text { statistic degrees of freedom } \\
\mathrm{r}\left(\mathrm{~J} \_\mathrm{p}\right) & J \text { statistic } p \text {-value }
\end{array}
$$

## Reference

Hall, A. R. 2005. Generalized Method of Moments. Oxford: Oxford University Press.

## Also see

[R] gmm - Generalized method of moments estimation
[U] 20 Estimation and postestimation commands

## Title

grmeanby - Graph means and medians by categorical variables

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | References |  |  |

## Syntax

grmeanby varlist [if] [in] [weight], summarize(varname) [, options]
options
Main

* summarize (varname)
median
Plot
cline_options
marker_options
marker_label_options
Y axis, X axis, Titles, Legend, Overall
twoway_options

Description
graph mean (or median) of varname
graph medians; default is to graph means
change the look of the lines
change look of markers (color, size, etc.)
add marker labels; change look or position
any options other than by () documented in [G-3] twoway_options
*summarize(varname) is required.
aweights and fweights are allowed; see [U] 11.1.6 weight.

## Menu

Statistics $>$ Summaries, tables, and tests $>$ Summary and descriptive statistics $>$ Graph means/medians by groups

## Description

grmeanby graphs the (optionally weighted) means or medians of varname according to the values of the variables in varlist. The variables in varlist may be string or numeric and, if numeric, may be labeled.

## Options

$\qquad$ Main
summarize (varname) is required; it specifies the name of the variable whose mean or median is to be graphed.
median specifies that the graph is to be of medians, not means.
$\qquad$ Plot
cline_options affect the rendition of the lines through the markers, including their color, pattern, and width; see [G-3] cline_options.
marker_options affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] marker_options.
marker_label_options specify if and how the markers are to be labeled; see [G-3] marker_label_options.
$\qquad$ Y axis, X axis, Titles, Legend, Overall
twoway_options are any of the options documented in [G-3] twoway_options, excluding by (). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

## Remarks and examples

The idea of graphing means of categorical variables was shown in Chambers and Hastie (1992, 3). Because this was shown in the context of an $S$ function for making such graphs, it doubtless has roots going back further than that. grmeanby is, in any case, another implementation of what we will assume is their idea.
> Example 1
Using a variation of our auto dataset, we graph the mean of mpg by foreign, rep77, rep78, and make:

```
. use http://www.stata-press.com/data/r13/auto1
(Automobile Models)
. grmeanby foreign rep77 rep78 make, sum(mpg)
```



If we had wanted a graph of medians rather than means, we could have typed

```
. grmeanby foreign rep77 rep78 make, sum(mpg) median
```


## References

Chambers, J. M., and T. J. Hastie, ed. 1992. Statistical Models in S. Pacific Grove, CA: Wadsworth and Brooks/Cole.
Gould, W. W. 1993. gr12: Graphs of means and medians by categorical variables. Stata Technical Bulletin 12: 13. Reprinted in Stata Technical Bulletin Reprints, vol. 2, pp. 44-45. College Station, TX: Stata Press.

## Title

hausman - Hausman specification test

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | Acknowledgment |
| References | Also see |  |  |

## Syntax

hausman name-consistent [name-efficient] [, options]
options
Main
constant
alleqs
skipeqs (eqlist)
equations (matchlist)
force
df (\#)
sigmamore
sigmaless

## Advanced

tconsistent (string)
tefficient (string)

Description
include estimated intercepts in comparison; default is to exclude use all equations to perform test; default is first equation only skip specified equations when performing test associate/compare the specified (by number) pairs of equations force performance of test, even though assumptions are not met use \# degrees of freedom
base both (co)variance matrices on disturbance variance estimate from efficient estimator
base both (co)variance matrices on disturbance variance estimate from consistent estimator
consistent estimator column header
efficient estimator column header
where name-consistent and name-efficient are names under which estimation results were stored via estimates store; see [R] estimates store.

A period (.) may be used to refer to the last estimation results, even if these were not already stored.
Not specifying name-efficient is equivalent to specifying the last estimation results as ".".

## Menu

Statistics $>$ Postestimation $>$ Tests $>$ Hausman specification test

## Description

hausman performs Hausman's (1978) specification test.

## Options

Main
constant specifies that the estimated intercept(s) be included in the model comparison; by default, they are excluded. The default behavior is appropriate for models in which the constant does not have a common interpretation across the two models.
alleqs specifies that all the equations in the models be used to perform the Hausman test; by default, only the first equation is used.
skipeqs (eqlist) specifies in eqlist the names of equations to be excluded from the test. Equation numbers are not allowed in this context, because the equation names, along with the variable names, are used to identify common coefficients.
equations (matchlist) specifies, by number, the pairs of equations that are to be compared.
The matchlist in equations() should follow the syntax

$$
\#_{c}: \#_{e}\left[, \#_{c}: \#_{e}[, \ldots]\right]
$$

where $\#_{c}\left(\#_{e}\right)$ is an equation number of the always-consistent (efficient under $H_{0}$ ) estimator. For instance, equations(1:1), equations ( $1: 1,2: 2$ ), or equations ( $1: 2$ ).
If equations() is not specified, then equations are matched on equation names.
equations() handles the situation in which one estimator uses equation names and the other does not. For instance, equations (1:2) means that equation 1 of the always-consistent estimator is to be tested against equation 2 of the efficient estimator. equations ( $1: 1,2: 2$ ) means that equation 1 is to be tested against equation 1 and that equation 2 is to be tested against equation 2 . If equations() is specified, the alleqs and skipeqs options are ignored.
force specifies that the Hausman test be performed, even though the assumptions of the Hausman test seem not to be met, for example, because the estimators were pweighted or the data were clustered.
$\mathrm{df}(\#)$ specifies the degrees of freedom for the Hausman test. The default is the matrix rank of the variance of the difference between the coefficients of the two estimators.
sigmamore and sigmaless specify that the two covariance matrices used in the test be based on a common estimate of disturbance variance $\left(\sigma^{2}\right)$.
sigmamore specifies that the covariance matrices be based on the estimated disturbance variance from the efficient estimator. This option provides a proper estimate of the contrast variance for so-called tests of exogeneity and overidentification in instrumental-variables regression.
sigmaless specifies that the covariance matrices be based on the estimated disturbance variance from the consistent estimator.

These options can be specified only when both estimators store e(sigma) or e(rmse), or with the xtreg command. e(sigma_e) is stored after the xtreg command with the fe or mle option. $e$ (rmse) is stored after the xtreg command with the re option.
sigmamore or sigmaless are recommended when comparing fixed-effects and random-effects linear regression because they are much less likely to produce a non-positive-definite-differenced covariance matrix (although the tests are asymptotically equivalent whether or not one of the options is specified).

[^24]
## Remarks and examples

hausman is a general implementation of Hausman's (1978) specification test, which compares an estimator $\widehat{\theta}_{1}$ that is known to be consistent with an estimator $\widehat{\theta}_{2}$ that is efficient under the assumption being tested. The null hypothesis is that the estimator $\widehat{\theta}_{2}$ is indeed an efficient (and consistent) estimator of the true parameters. If this is the case, there should be no systematic difference between the two estimators. If there exists a systematic difference in the estimates, you have reason to doubt the assumptions on which the efficient estimator is based.

The assumption of efficiency is violated if the estimator is pweighted or the data are clustered, so hausman cannot be used. The test can be forced by specifying the force option with hausman. For an alternative to using hausman in these cases, see $[R]$ suest.

To use hausman, you
. (compute the always-consistent estimator)

- estimates store name-consistent
. (compute the estimator that is efficient under H_0)
. hausman name-consistent
Alternatively, you can turn this around:
. (compute the estimator that is efficient under H_0)
- estimates store name-efficient
. (fit the less-efficient model)
. (compute the always-consistent estimator)
. hausman . name-efficient
You can, of course, also compute and store both the always-consistent and efficient-under- $H_{0}$ estimators and perform the Hausman test with
. hausman name-consistent name-efficient


## $>$ Example 1

We are studying the factors that affect the wages of young women in the United States between 1970 and 1988, and we have a panel-data sample of individual women over that time span.

```
. use http://www.stata-press.com/data/r13/nlswork4
(National Longitudinal Survey. Young Women 14-26 years of age in 1968)
. describe
Contains data from http://www.stata-press.com/data/r13/nlswork4.dta
    obs: 28,534 National Longitudinal Survey.
                                Young Women 14-26 years of age
in 1968
    vars: 6 29 Jan 2013 16:35
    size: 370,942
```

| variable name | storage <br> type | display <br> format | value <br> label | variable label |
| :--- | :--- | :--- | :--- | :--- |
| idcode | int | $\% 8.0 \mathrm{~g}$ |  | NLS ID |
| year | byte | $\% 8.0 \mathrm{~g}$ |  | interview year |
| age | byte | $\% 8.0 \mathrm{~g}$ |  | age in current year |
| msp | byte | $\% 8.0 \mathrm{~g}$ |  | 1 if married, spouse present |
| ttl_exp | float | $\% 9.0 \mathrm{~g}$ |  | total work experience |
| ln_wage | float | $\% 9.0 \mathrm{~g}$ |  | ln(wage/GNP deflator) |

Sorted by: idcode year

We believe that a random-effects specification is appropriate for individual-level effects in our model. We fit a fixed-effects model that will capture all temporally constant individual-level effects.


We assume that this model is consistent for the true parameters and store the results by using estimates store under a name, fixed:

```
. estimates store fixed
```

Now we fit a random-effects model as a fully efficient specification of the individual effects under the assumption that they are random and follow a normal distribution. We then compare these estimates with the previously stored results by using the hausman command.


```
. hausman fixed ., sigmamore
                _ Coefficients -_
\begin{tabular}{r|ccrc} 
& \begin{tabular}{c}
\((b)\) \\
fixed
\end{tabular} & \begin{tabular}{c}
\((B)\) \\
\(\cdot\)
\end{tabular} & \begin{tabular}{c}
\((b-B)\) \\
Difference
\end{tabular} & sqrt(diag(V_b-V_B)) \\
\hline age & -.005485 & -.0069749 & .0014899 & .0004803 \\
msp & .0033427 & .0046594 & -.0013167 & .0020596 \\
ttl_exp & .0383604 & .0429635 & -.0046031 & .0007181 \\
\hline
\end{tabular}
                                    b = consistent under Ho and Ha; obtained from xtreg
            B = inconsistent under Ha, efficient under Ho; obtained from xtreg
        Test: Ho: difference in coefficients not systematic
        chi2(3) = (b-B)'[(V_b-V_B)^(-1)](b-B)
                        = 260.40
            Prob>chi2 = 0.0000
```

Under the current specification, our initial hypothesis that the individual-level effects are adequately modeled by a random-effects model is resoundingly rejected. This result is based on the rest of our model specification, and random effects might be appropriate for some alternate model of wages.

Jerry Allen Hausman was born in West Virginia in 1946. He studied economics at Brown and Oxford, has been at MIT since 1972, and has made many outstanding contributions to econometrics and applied microeconomics.

## > Example 2

A stringent assumption of multinomial and conditional logit models is that outcome categories for the model have the property of independence of irrelevant alternatives (IIA). Stated simply, this assumption requires that the inclusion or exclusion of categories does not affect the relative risks associated with the regressors in the remaining categories.

One classic example of a situation in which this assumption would be violated involves the choice of transportation mode; see McFadden (1974). For simplicity, postulate a transportation model with the four possible outcomes: rides a train to work, takes a bus to work, drives the Ford to work, and drives the Chevrolet to work. Clearly, "drives the Ford" is a closer substitute to "drives the Chevrolet" than it is to "rides a train" (at least for most people). This means that excluding "drives the Ford" from the model could be expected to affect the relative risks of the remaining options and that the model would not obey the IIA assumption.

Using the data presented in [R] mlogit, we will use a simplified model to test for IIA. The choice of insurance type among indemnity, prepaid, and uninsured is modeled as a function of age and gender. The indemnity category is allowed to be the base category, and the model including all three outcomes is fit. The results are then stored under the name allcats.

| (Health insurance data) <br> . mlogit insure age male |  |  |  |
| :---: | :---: | :---: | :---: |
| Iteration 0: log likelihood $=-555.85446$ |  |  |  |
| Iteration 1: log likelihood $=-551.32973$ |  |  |  |
| Iteration 2: log likelihood $=-551.32802$ |  |  |  |
| Iteration 3: log likelihood = -551.32802 |  |  |  |
| Multinomial logistic regression | Number of obs | $=$ | 615 |
|  | LR chi2(4) | = | 9.05 |
|  | Prob > chi2 | = | 0.0598 |
| Log likelihood = -551.32802 | Pseudo R2 | = | 0.0081 |


| insure | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :--- | ---: | :--- | ---: | ---: |
| Indemnity | (base outcome) |  |  |  |  |  |
| Prepaid |  |  |  |  |  |  |
| age | -.0100251 | .0060181 | -1.67 | 0.096 | -.0218204 | .0017702 |
| male | .5095747 | .1977893 | 2.58 | 0.010 | .1219147 | .8972346 |
| _cons | .2633838 | .2787575 | 0.94 | 0.345 | -.2829708 | .8097383 |
| Uninsure |  |  |  |  |  |  |
| age | -.0051925 | .0113821 | -0.46 | 0.648 | -.0275011 | .0171161 |
| male | .4748547 | .3618462 | 1.31 | 0.189 | -.2343508 | 1.18406 |
| _cons | -1.756843 | .5309602 | -3.31 | 0.001 | -2.797506 | -.7161803 |

estimates store allcats
Under the IIA assumption, we would expect no systematic change in the coefficients if we excluded one of the outcomes from the model. (For an extensive discussion, see Hausman and McFadden [1984].) We reestimate the parameters, excluding the uninsured outcome, and perform a Hausman test against the fully efficient full model.

| Iteration 0: log likelihood = -394.8693 |  |  |  |
| :---: | :---: | :---: | :---: |
| Iteration 1: log likelihood $=-390.4871$ |  |  |  |
| Iteration 2: log likelihood $=-390.48643$ |  |  |  |
| Iteration 3: log likelihood = -390.48643 |  |  |  |
| Multinomial logistic regression | Number of obs | = | 570 |
|  | LR chi2(2) | = | 8.77 |
|  | Prob > chi2 | = | 0.0125 |
| Log likelihood $=-390.48643$ | Pseudo R2 | = | 0.0111 |


| insure | Coef. $\quad$ Std. Err. | $z$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Indemnity | (base outcome) |  |  |  |  |  |
| Prepaid |  |  |  |  |  |  |
| age | -.0101521 | .0060049 | -1.69 | 0.091 | -.0219214 | .0016173 |
| male | .5144003 | .1981735 | 2.60 | 0.009 | .1259874 | .9028133 |
| _cons | .2678043 | .2775563 | 0.96 | 0.335 | -.276196 | .8118046 |

hausman . allcats, alleqs constant
—— Coefficients __

|  | $(\mathrm{b})$ | (B) <br> allcats | $(\mathrm{b}-\mathrm{B})$ <br> Difference | sqrt(diag(V_b-V_B)) <br> S.E. |
| ---: | :---: | :---: | :---: | :---: |
| age | -.0101521 | -.0100251 | -.0001269 | . |
| male | .5144003 | .5095747 | .0048256 | .0123338 |
| _cons | .2678043 | .2633838 | .0044205 | $\cdot$ |

b = consistent under Ho and Ha; obtained from mlogit
Test: Ho: inconsistent under Ha, efficient under Ho; obtained from mlogit
difference in coefficients not systematic
chi2 3 ) $=(b-B)^{\prime}\left[\left(V_{-} b-V \_B\right)^{\wedge}(-1)\right](b-B)$
$=$

The syntax of the if condition on the mlogit command simply identified the "Uninsured" category with the insure value label; see [U] 12.6.3 Value labels. On examining the output from hausman, we see that there is no evidence that the IIA assumption has been violated.

Because the Hausman test is a standardized comparison of model coefficients, using it with mlogit requires that the base outcome be the same in both competing models. In particular, if the most-frequent category (the default base outcome) is being removed to test for IIA, you must use the baseoutcome() option in mlogit to manually set the base outcome to something else. Or you can use the equation() option of the hausman command to align the equations of the two models.

Having the missing values for the square root of the diagonal of the covariance matrix of the differences is not comforting, but it is also not surprising. This covariance matrix is guaranteed to be positive definite only asymptotically (it is a consequence of the assumption that one of the estimators is efficient), and assurances are not made about the diagonal elements. Negative values along the diagonal are possible, and the fourth column of the table is provided mainly for descriptive use.

We can also perform the Hausman IIA test against the remaining alternative in the model:

| Iteration 0: log likelihood $=-132.59913$ |  |  |  |
| :---: | :---: | :---: | :---: |
| Iteration 1: $\quad$ log likelihood $=-131.78009$ |  |  |  |
| Iteration 2: $\quad \log$ likelihood $=-131.76808$ |  |  |  |
| Iteration 3: $\quad \mathrm{log}$ likelihood $=-131.76807$ |  |  |  |
| Multinomial logistic regression | Number of obs | $=$ | 338 |
|  | LR chi2(2) | = | 1.66 |
|  | Prob > chi2 | = | 0.4356 |
| Log likelihood = -131.76807 | Pseudo R2 | = | 0.0063 |


| insure | Coef. | Std. Err. | $z$ | $\mathrm{P}>\|z\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :--- | ---: | :--- | ---: | :--- |
| Indemnity | (base outcome) |  |  |  |  |  |
| Uninsure |  |  |  |  |  |  |
| age | -.0041055 | .0115807 | -0.35 | 0.723 | -.0268033 | .0185923 |
| male | .4591074 | .3595663 | 1.28 | 0.202 | -.2456296 | 1.163844 |
| _cons | -1.801774 | .5474476 | -3.29 | 0.001 | -2.874752 | -.7287968 |

```
. hausman . allcats, alleqs constant
\begin{tabular}{|c|c|c|c|c|}
\hline & \begin{tabular}{l}
- Coeff \\
(b)
\end{tabular} & \begin{tabular}{l}
ents \(\qquad\) \\
(B) \\
allcats
\end{tabular} & \[
\begin{gathered}
\text { (b-B) } \\
\text { Difference }
\end{gathered}
\] & \[
\begin{gathered}
\operatorname{sqrt}\left(\operatorname{diag}\left(V \_b-V \_B\right)\right) \\
\text { S.E. }
\end{gathered}
\] \\
\hline age & -. 0041055 & -. 0051925 & . 001087 & . 0021355 \\
\hline male & . 4591074 & . 4748547 & -. 0157473 & \\
\hline _cons & -1.801774 & -1.756843 & -. 0449311 & . 1333421 \\
\hline
\end{tabular}
b = consistent under Ho and Ha ; obtained from mlogit
            B = inconsistent under Ha, efficient under Ho; obtained from mlogit
    Test: Ho: difference in coefficients not systematic
        chi2(3) = (b-B)'[(V_b-V_B)^(-1)](b-B)
                        = -0.18 chi2<0 ==> model fitted on these
                            data fails to meet the asymptotic
                            assumptions of the Hausman test;
                            see suest for a generalized test
```

Here the $\chi^{2}$ statistic is actually negative. We might interpret this result as strong evidence that we cannot reject the null hypothesis. Such a result is not an unusual outcome for the Hausman test, particularly when the sample is relatively small-there are only 45 uninsured individuals in this dataset.

Are we surprised by the results of the Hausman test in this example? Not really. Judging from the $z$ statistics on the original multinomial logit model, we were struggling to identify any structure in the data with the current specification. Even when we were willing to assume IIA and computed the efficient estimator under this assumption, few of the effects could be identified as statistically different from those on the base category. Trying to base a Hausman test on a contrast (difference) between two poor estimates is just asking too much of the existing data.

In example 2, we encountered a case in which the Hausman was not well defined. Unfortunately, in our experience this happens fairly often. Stata provides an alternative to the Hausman test that overcomes this problem through an alternative estimator of the variance of the difference between the two estimators. This other estimator is guaranteed to be positive semidefinite. This alternative estimator also allows a widening of the scope of problems to which Hausman-type tests can be applied by relaxing the assumption that one of the estimators is efficient. For instance, you can perform Hausman-type tests to clustered observations and survey estimators. See [R] suest for details.

## Stored results

hausman stores the following in r() :
Scalars

```
r(chi2) < < 
    r(df) degrees of freedom for the statistic
    r(p) p-value for the }\mp@subsup{\chi}{}{2
    r(rank) rank of (V_b-V_B)^(-1)
```


## Methods and formulas

The Hausman statistic is distributed as $\chi^{2}$ and is computed as

$$
H=\left(\beta_{c}-\beta_{e}\right)^{\prime}\left(V_{c}-V_{e}\right)^{-1}\left(\beta_{c}-\beta_{e}\right)
$$

where
$\beta_{c}$ is the coefficient vector from the consistent estimator
$\beta_{e}$ is the coefficient vector from the efficient estimator
$V_{c} \quad$ is the covariance matrix of the consistent estimator
$V_{e}$ is the covariance matrix of the efficient estimator
When the difference in the variance matrices is not positive definite, a Moore-Penrose generalized inverse is used. As noted in Gourieroux and Monfort (1995, 125-128), the choice of generalized inverse is not important asymptotically.

The number of degrees of freedom for the statistic is the rank of the difference in the variance matrices. When the difference is positive definite, this is the number of common coefficients in the models being compared.

## Acknowledgment

Portions of hausman are based on an early implementation by Jeroen Weesie of the Department of Sociology at Utrecht University, The Netherlands.

## References

Baltagi, B. H. 2011. Econometrics. 5th ed. Berlin: Springer.
Gourieroux, C. S., and A. Monfort. 1995. Statistics and Econometric Models, Vol 2: Testing, Confidence Regions, Model Selection, and Asymptotic Theory. Trans. Q. Vuong. Cambridge: Cambridge University Press.

Hausman, J. A. 1978. Specification tests in econometrics. Econometrica 46: 1251-1271.
Hausman, J. A., and D. L. McFadden. 1984. Specification tests for the multinomial logit model. Econometrica 52: 1219-1240.
McFadden, D. L. 1974. Measurement of urban travel demand. Journal of Public Economics 3: 303-328.

## Also see

[R] Irtest - Likelihood-ratio test after estimation
[R] suest - Seemingly unrelated estimation
[R] test - Test linear hypotheses after estimation
[XT] xtreg - Fixed-, between-, and random-effects and population-averaged linear models

## Title

heckman - Heckman selection model

Syntax
Description
Options for Heckman selection model (two-step)
Stored results
References

Menu
Options for Heckman selection model (ML)
Remarks and examples
Methods and formulas
Also see

## Syntax

Basic syntax
heckman depvar [indepvars], select(varlists) [twostep]
or
heckman depvar $[$ indepvars $]$, select (depvar ${ }_{s}=$ varlist $\left._{s}\right)$ [twostep]

Full syntax for maximum likelihood estimates only

```
heckman depvar [indepvars] [if] [in] [weight],
        select([\mp@subsup{depvar }{s}{=}]\mp@subsup{\operatorname{varlist}}{s}{}[,\underline{\mathrm{ noconstant offset (varname}}\mathrm{ ) ])}
        [heckman_ml_options]
```

Full syntax for Heckman's two-step consistent estimates only

```
heckman depvar [indepvars] [if] [in], twostep
    select([depvars = ] varlists [, noconstant]) [heckman_ts_options]
```

heckman_ml_options
Model

* select()
noconstant
offset (varname)
constraints (constraints)
collinear
SE/Robust
vce (vcetype)

Reporting
level(\#)
first
noskip
nshazard (newvar)
mills(newvar)
nocnsreport
display_options

## Maximization

maximize_options
coeflegend

Description
specify selection equation: dependent and independent variables; whether to have constant term and offset variable suppress constant term
include varname in model with coefficient constrained to 1
apply specified linear constraints
keep collinear variables
vcetype may be oim, robust, cluster clustvar, opg, bootstrap, or jackknife
set confidence level; default is level (95)
report first-step probit estimates
perform likelihood-ratio test
generate nonselection hazard variable
synonym for nshazard()
do not display constraints
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
*select () is required.
The full specification is select $\left(\left[\right.\right.$ depvar $\left._{s}=\right]$ varlist $_{s}\left[\right.$, noconstant offset $\left(\right.$ varname $\left.\left._{o}\right)\right]$ ).

## Model

* select ()
* twostep noconstant rhosigma
rhotrunc
rholimited
rhoforce
SE
vce (vcetype)
Reporting
level(\#)
first
nshazard (newvar)
mills(newvar)
display_options
coeflegend
specify selection equation: dependent and independent variables; whether to have constant term
produce two-step consistent estimate
suppress constant term
truncate $\rho$ to $[-1,1]$ with consistent $\sigma$
truncate $\rho$ to $[-1,1]$
truncate $\rho$ in limited cases
do not truncate $\rho$
vcetype may be conventional, bootstrap, or jackknife
set confidence level; default is level (95)
report first-step probit estimates
generate nonselection hazard variable
synonym for nshazard()
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
display legend instead of statistics
*select ( ) and twostep are required.
The full specification is select $\left(\left[\right.\right.$ depvar $\left._{s}=\right]$ varlist $_{s}[$, noconstant $]$ ).
indepvars and varlist ${ }_{s}$ may contain factor variables; see [U] 11.4.3 Factor variables.
depvar, indepvars, varlist $t_{s}$, and depvar ${ }_{s}$ may contain time-series operators; see [U] 11.4.4 Time-series varlists.
bootstrap, by, fp, jackknife, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the bootstrap prefix; see [R] bootstrap.
aweights are not allowed with the jackknife prefix; see [R] jackknife.
twostep, vce(), first, noskip, and weights are not allowed with the svy prefix; see [SVY] svy.
pweights, aweights, fweights, and iweights are allowed with maximum likelihood estimation; see [U] 11.1.6 weight.
No weights are allowed if twostep is specified.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.


## Menu

## heckman for maximum likelihood estimates

Statistics $>$ Sample-selection models $>$ Heckman selection model (ML)

## heckman for two-step consistent estimates

Statistics $>$ Sample-selection models $>$ Heckman selection model (two-step)

## Description

heckman fits regression models with selection by using either Heckman's two-step consistent estimator or full maximum likelihood.

## Options for Heckman selection model (ML)

```
Model
select([depvar,
    options for the selection equation. It is an integral part of specifying a Heckman model and is
    required. The selection equation should contain at least one variable that is not in the outcome
    equation.
    If depvars is specified, it should be coded as 0 or 1, with 0 indicating an observation not selected
    and }1\mathrm{ indicating a selected observation. If depvar is not specified, observations for which depvar
    is not missing are assumed selected, and those for which depvar is missing are assumed not
    selected.
    noconstant suppresses the selection constant term (intercept).
    offset (varname }\mp@subsup{O}{0}{}\mathrm{ ) specifies that selection offset varname o be included in the model with the
    coefficient constrained to be 1.
noconstant, offset(varname), constraints(constraints), collinear; see [R] estimation op-
    tions.
```

        SE/Robust
    vce (vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

Reporting
level(\#); see [R] estimation options.
first specifies that the first-step probit estimates of the selection equation be displayed before estimation.
noskip specifies that a full maximum-likelihood model with only a constant for the regression equation be fit. This model is not displayed but is used as the base model to compute a likelihood-ratio test for the model test statistic displayed in the estimation header. By default, the overall model test statistic is an asymptotically equivalent Wald test that all the parameters in the regression equation are zero (except the constant). For many models, this option can substantially increase estimation time.
nshazard(newvar) and mills (newvar) are synonyms; either will create a new variable containing the nonselection hazard - what Heckman (1979) referred to as the inverse of the Mills' ratio-from the selection equation. The nonselection hazard is computed from the estimated parameters of the selection equation.
nocnsreport; see $[\mathrm{R}]$ estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% $f m t$ ), pformat (\% fmt), sformat (\%fmt), and nolstretch; see [R] estimation options.

Maximization
maximize_options: difficult, technique (algorithm_spec), iterate (\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance(\#), nonrtolerance, and from (init_specs); see [R] maximize. These options are seldom used.
Setting the optimization type to technique (bhhh) resets the default vcetype to vce (opg).
The following option is available with heckman but is not shown in the dialog box:
coeflegend; see [R] estimation options.

## Options for Heckman selection model (two-step)

Model
select $\left(\left[\right.\right.$ depvar $\left._{s}=\right]$ varlist $_{s}[$, noconstant $\left.]\right)$ specifies the variables and options for the selection equation. It is an integral part of specifying a Heckman model and is required. The selection equation should contain at least one variable that is not in the outcome equation.

If depvar ${ }_{s}$ is specified, it should be coded as 0 or 1 , with 0 indicating an observation not selected and 1 indicating a selected observation. If depvar ${ }_{s}$ is not specified, observations for which depvar is not missing are assumed selected, and those for which depvar is missing are assumed not selected.
noconstant suppresses the selection constant term (intercept).
twostep specifies that Heckman's (1979) two-step efficient estimates of the parameters, standard errors, and covariance matrix be produced.
noconstant; see $[R]$ estimation options.
rhosigma, rhotrunc, rholimited, and rhoforce are rarely used options to specify how the two-step estimator (option twostep) handles unusual cases in which the two-step estimate of $\rho$ is outside the admissible range for a correlation, $[-1,1]$. When $\operatorname{abs}(\rho)>1$, the two-step estimate of the coefficient variance-covariance matrix may not be positive definite and thus may be unusable for testing. The default is rhosigma.
rhosigma specifies that $\rho$ be truncated, as with the rhotrunc option, and that the estimate of $\sigma$ be made consistent with $\widehat{\rho}$, the truncated estimate of $\rho$. So, $\widehat{\sigma}=\boldsymbol{\beta}_{m} \widehat{\rho}$, see Methods and formulas for the definition of $\boldsymbol{\beta}_{m}$. Both the truncated $\rho$ and the new estimate of $\widehat{\sigma}$ are used in all computations to estimate the two-step covariance matrix.
rhotrunc specifies that $\rho$ be truncated to lie in the range $[-1,1]$. If the two-step estimate is less than $-1, \rho$ is set to -1 ; if the two-step estimate is greater than $1, \rho$ is set to 1 . This truncated value of $\rho$ is used in all computations to estimate the two-step covariance matrix.
rholimited specifies that $\rho$ be truncated only in computing the diagonal matrix $\mathbf{D}$ as it enters $\mathbf{V}_{\text {twostep }}$ and $\mathbf{Q}$; see Methods and formulas. In all other computations, the untruncated estimate of $\rho$ is used.
rhoforce specifies that the two-step estimate of $\rho$ be retained, even if it is outside the admissible range for a correlation. This option may, in rare cases, lead to a non-positive-definite covariance matrix.
These options have no effect when estimation is by maximum likelihood, the default. They also have no effect when the two-step estimate of $\rho$ is in the range $[-1,1]$.
vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (conventional) and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
vce (conventional), the default, uses the two-step variance estimator derived by Heckman.

Reporting
level (\#); see [R] estimation options.
first specifies that the first-step probit estimates of the selection equation be displayed before estimation.
nshazard (newvar) and mills (newvar) are synonyms; either will create a new variable containing the nonselection hazard - what Heckman (1979) referred to as the inverse of the Mills' ratio -from the selection equation. The nonselection hazard is computed from the estimated parameters of the selection equation.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% $f m t$ ), pformat (\% fmt), sformat (\%fmt), and nolstretch; see [R] estimation options.

The following option is available with heckman but is not shown in the dialog box:
coeflegend; see $[R]$ estimation options.

## Remarks and examples

The Heckman selection model (Gronau 1974; Lewis 1974; Heckman 1976) assumes that there exists an underlying regression relationship,

$$
y_{j}=\mathbf{x}_{j} \boldsymbol{\beta}+u_{1 j} \quad \text { regression equation }
$$

The dependent variable, however, is not always observed. Rather, the dependent variable for observation $j$ is observed if

$$
\mathbf{z}_{j} \gamma+u_{2 j}>0 \quad \text { selection equation }
$$

where

$$
\begin{gathered}
u_{1} \sim N(0, \sigma) \\
u_{2} \sim N(0,1) \\
\operatorname{corr}\left(u_{1}, u_{2}\right)=\rho
\end{gathered}
$$

When $\rho \neq 0$, standard regression techniques applied to the first equation yield biased results. heckman provides consistent, asymptotically efficient estimates for all the parameters in such models.

In one classic example, the first equation describes the wages of women. Women choose whether to work, and thus, from our point of view as researchers, whether we observe their wages in our data. If women made this decision randomly, we could ignore that not all wages are observed and use ordinary regression to fit a wage model. Such an assumption of random participation, however, is unlikely to be true; women who would have low wages may be unlikely to choose to work, and thus the sample of observed wages is biased upward. In the jargon of economics, women choose not to work when their personal reservation wage is greater than the wage offered by employers. Thus women who choose not to work might have even higher offer wages than those who do work-they may have high offer wages, but they have even higher reservation wages. We could tell a story that competency is related to wages, but competency is rewarded more at home than in the labor force.

In any case, in this problem - which is the paradigm for most such problems - a solution can be found if there are some variables that strongly affect the chances for observation (the reservation wage) but not the outcome under study (the offer wage). Such a variable might be the number of children in the home. (Theoretically, we do not need such identifying variables, but without them, we depend on functional form to identify the model. It would be difficult for anyone to take such results seriously because the functional form assumptions have no firm basis in theory.)

## > Example 1

In the syntax for heckman, depvar and indepvars are the dependent variable and regressors for the underlying regression model to be fit ( $\mathbf{y}=\mathbf{X} \boldsymbol{\beta}$ ), and varlist ${ }_{s}$ are the variables $(\mathbf{Z})$ thought to determine whether depvar is observed or unobserved (selected or not selected). In our female wage example, the number of children at home would be included in the second list. By default, heckman assumes that missing values (see [U] 12.2.1 Missing values) of depvar imply that the dependent variable is unobserved (not selected). With some datasets, it is more convenient to specify a binary variable (depvar ${ }_{s}$ ) that identifies the observations for which the dependent is observed/selected ( depvar $_{s} \neq 0$ ) or not observed (depvar $=0$ ); heckman will accommodate either type of data.

We have a (fictional) dataset on 2,000 women, 1,343 of whom work:

```
. use http://www.stata-press.com/data/r13/womenwk
. summarize age educ married children wage
```

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| age | 2000 | 36.208 | 8.28656 | 20 | 59 |
| education | 2000 | 13.084 | 3.045912 | 10 | 20 |
| married | 2000 | .6705 | .4701492 | 0 | 1 |
| children | 2000 | 1.6445 | 1.398963 | 0 | 5 |
| wage | 1343 | 23.69217 | 6.305374 | 5.88497 | 45.80979 |

We will assume that the hourly wage is a function of education and age, whereas the likelihood of working (the likelihood of the wage being observed) is a function of marital status, the number of children at home, and (implicitly) the wage (via the inclusion of age and education, which we think determine the wage):

| . heckman wage educ age, select(married children educ age) |  |  |  |
| :--- | :--- | :--- | :--- |
| Iteration 0: log likelihood $=-5178.7009$ |  |  |  |
| Iteration 1: log likelihood $=-5178.3049$ |  |  |  |
| Iteration 2: log likelihood $=-5178.3045$ |  |  |  |
| Heckman selection model | Number of obs | $=$ | 2000 |
| (regression model with sample selection) | Censored obs | $=$ | 657 |
|  | Uncensored obs | $=$ | 1343 |
|  | Wald chi2(2) | $=$ | 508.44 |
| Log likelihood $=-5178.304$ |  | Prob > chi2 | $=$ |


| wage | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :--- | ---: | :--- | ---: | ---: |
| wage |  |  |  |  |  |  |
| education | .9899537 | .0532565 | 18.59 | 0.000 | .8855729 | 1.094334 |
| age | .2131294 | .0206031 | 10.34 | 0.000 | .1727481 | .2535108 |
| _cons | .4857752 | 1.077037 | 0.45 | 0.652 | -1.625179 | 2.59673 |
| select |  |  |  |  |  |  |
| married | .4451721 | .0673954 | 6.61 | 0.000 | .3130794 | .5772647 |
| children | .4387068 | .0277828 | 15.79 | 0.000 | .3842534 | .4931601 |
| education | .0557318 | .0107349 | 5.19 | 0.000 | .0346917 | .0767718 |
| age | .0365098 | .0041533 | 8.79 | 0.000 | .0283694 | .0446502 |
| _cons | -2.491015 | .1893402 | -13.16 | 0.000 | -2.862115 | -2.119915 |
| /athrho | .8742086 | .1014225 | 8.62 | 0.000 | .6754241 | 1.072993 |
| /lnsigma | 1.792559 | .027598 | 64.95 | 0.000 | 1.738468 | 1.84665 |
| rho | .7035061 | .0512264 |  |  | .5885365 | .7905862 |
| sigma | 6.004797 | .1657202 |  |  | 5.68862 | 6.338548 |
| lambda | 4.224412 | .3992265 |  |  | 3.441942 | 5.006881 |

LR test of indep. eqns. (rho $=0$ ) : chi2 (1) $=61.20 \quad$ Prob $>$ chi2 $=0.0000$
heckman assumes that wage is the dependent variable and that the first variable list (educ and age) are the determinants of wage. The variables specified in the select() option (married, children, educ, and age) are assumed to determine whether the dependent variable is observed (the selection equation). Thus we fit the model

$$
\text { wage }=\beta_{0}+\beta_{1} \text { educ }+\beta_{2} \text { age }+u_{1}
$$

and we assumed that wage is observed if

$$
\gamma_{0}+\gamma_{1} \text { married }+\gamma_{2} \text { children }+\gamma_{3} \text { educ }+\gamma_{4} \text { age }+u_{2}>0
$$

where $u_{1}$ and $u_{2}$ have correlation $\rho$.
The reported results for the wage equation are interpreted exactly as though we observed wage data for all women in the sample; the coefficients on age and education level represent the estimated marginal effects of the regressors in the underlying regression equation. The results for the two ancillary parameters require some explanation. heckman does not directly estimate $\rho$; to constrain $\rho$ within its valid limits, and for numerical stability during optimization, it estimates the inverse hyperbolic tangent of $\rho$ :

$$
\operatorname{atanh} \rho=\frac{1}{2} \ln \left(\frac{1+\rho}{1-\rho}\right)
$$

This estimate is reported as /athrho. In the bottom panel of the output, heckman undoes this transformation for you: the estimated value of $\rho$ is 0.7035061 . The standard error for $\rho$ is computed using the delta method, and its confidence intervals are the transformed intervals of /athrho.

Similarly, $\sigma$, the standard error of the residual in the wage equation, is not directly estimated; for numerical stability, heckman instead estimates $\ln \sigma$. The untransformed sigma is reported at the end of the output: 6.004797 .

Finally, some researchers - especially economists - are used to the selectivity effect summarized not by $\rho$ but by $\lambda=\rho \sigma$. heckman reports this, too, along with an estimate of the standard error and confidence interval.

## - Technical note

If each of the equations in the model had contained many regressors, the heckman command could have become long. An alternate way of specifying our wage model would be to use Stata's global macros. The following lines are an equivalent way of specifying our model:

```
. global wageeq "wage educ age"
. global seleq "married children educ age"
. heckman $wageeq, select($seleq)
(output omitted)
```


## - Technical note

The reported model $\chi^{2}$ test is a Wald test that all coefficients in the regression model (except the constant) are 0 . heckman is an estimation command, so you can use test, testnl, or lrtest to perform tests against whatever nested alternate model you choose; see [R] test, [R] testnl, and [R] lrtest.

The estimation of $\rho$ and $\sigma$ in the forms atanh $\rho$ and $\ln \sigma$ extends the range of these parameters to infinity in both directions, thus avoiding boundary problems during the maximization. Tests of $\rho$ must be made in the transformed units. However, because $\operatorname{atanh}(0)=0$, the reported test for atanh $\rho=0$ is equivalent to the test for $\rho=0$.

The likelihood-ratio test reported at the bottom of the output is an equivalent test for $\rho=0$ and is computationally the comparison of the joint likelihood of an independent probit model for the selection equation and a regression model on the observed wage data against the Heckman model likelihood. Because $\chi^{2}=61.20$, this clearly justifies the Heckman selection equation with these data.
> Example 2
heckman supports the Huber/White/sandwich estimator of variance under the vce (robust) and vce(cluster clustvar) options or when pweights are used for population-weighted data; see [U] 20.21 Obtaining robust variance estimates. We can obtain robust standard errors for our wage model by specifying clustering on county of residence (the county variable).

```
Iteration 0: log pseudolikelihood = -5178.7009
Iteration 1: log pseudolikelihood = -5178.3049
Iteration 2: log pseudolikelihood = -5178.3045
```

. heckman wage educ age, select(married children educ age) vce(cluster county)

| Heckman selection model | Number of obs | $=$ | 2000 |
| :--- | :--- | :--- | ---: |
| (regression model with sample selection) | Censored obs | $=$ | 657 |
|  | Uncensored obs | $=$ | 1343 |
|  | Wald chi2 (1) | $=$ | . |
| Log pseudolikelihood $=-5178.304$ | Prob $>$ chi2 | $=$ | . |

(Std. Err. adjusted for 10 clusters in county)

| wage | Coef. | Robust Std. Err. | z | $P>\|z\|$ | [95\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| wage |  |  |  |  |  |  |
| education | . 9899537 | . 0600061 | 16.50 | 0.000 | . 8723438 | 1.107564 |
| age | . 2131294 | . 020995 | 10.15 | 0.000 | . 17198 | . 2542789 |
| _cons | . 4857752 | 1.302103 | 0.37 | 0.709 | -2.066299 | 3.03785 |
| select |  |  |  |  |  |  |
| married | . 4451721 | . 0731472 | 6.09 | 0.000 | . 3018062 | . 5885379 |
| children | . 4387068 | . 0312386 | 14.04 | 0.000 | . 3774802 | . 4999333 |
| education | . 0557318 | . 0110039 | 5.06 | 0.000 | . 0341645 | . 0772991 |
| age | . 0365098 | . 004038 | 9.04 | 0.000 | . 0285954 | . 0444242 |
| _cons | -2.491015 | . 1153305 | -21.60 | 0.000 | -2.717059 | -2.264972 |
| /athrho | . 8742086 | . 1403337 | 6.23 | 0.000 | . 5991596 | 1.149258 |
| /lnsigma | 1.792559 | . 0258458 | 69.36 | 0.000 | 1.741902 | 1.843216 |
| rho | . 7035061 | . 0708796 |  |  | . 5364513 | . 817508 |
| sigma | 6.004797 | . 155199 |  |  | 5.708189 | 6.316818 |
| lambda | 4.224412 | . 5186709 |  |  | 3.207835 | 5.240988 |

Wald test of indep. eqns. (rho $=0$ ) : chi2(1) $=38.81$ Prob > chi2 $=0.0000$
The robust standard errors tend to be a bit larger, but we notice no systematic differences. This finding is not surprising because the data were not constructed to have any county-specific correlations or any other characteristics that would deviate from the assumptions of the Heckman model.

## Example 3

Stata also produces Heckman's (1979) two-step efficient estimator of the model with the twostep option. Maximum likelihood estimation of the parameters can be time consuming with large datasets, and the two-step estimates may provide a good alternative in such cases. Continuing with the women's wage model, we can obtain the two-step estimates with Heckman's consistent covariance estimates by typing


## - Technical note

The Heckman selection model depends strongly on the model being correct, much more so than ordinary regression. Running a separate probit or logit for sample inclusion followed by a regression, referred to in the literature as the two-part model (Manning, Duan, and Rogers 1987) - not to be confused with Heckman's two-step procedure - is an especially attractive alternative if the regression part of the model arose because of taking a logarithm of zero values. When the goal is to analyze an underlying regression model or to predict the value of the dependent variable that would be observed in the absence of selection, however, the Heckman model is more appropriate. When the goal is to predict an actual response, the two-part model is usually the better choice.

The Heckman selection model can be unstable when the model is not properly specified or if a specific dataset simply does not support the model's assumptions. For example, let's examine the solution to another simulated problem.

```
. use http://www.stata-press.com/data/r13/twopart
. heckman yt x1 x2 x3, select(z1 z2) nonrtol
Iteration 0: log likelihood = -111.94996
Iteration 1: log likelihood = -110.82258
Iteration 2: log likelihood = -110.17707
Iteration 3: log likelihood = -107.70663 (not concave)
Iteration 4: log likelihood = -107.07729 (not concave)
    (output omitted)
Iteration 33: log likelihood = -104.0825 (not concave)
Iteration 34: log likelihood = -104.0825
```

| Heckman selection model | Number of obs | $=$ | 150 |
| :--- | :--- | :--- | ---: |
| (regression model with sample selection) | Censored obs | $=$ | 87 |
|  | Uncensored obs | $=$ | 63 |
|  | Wald chi2 (3) | $=$ | $8.64 \mathrm{e}+08$ |
| Log likelihood $=-104.0825$ | Prob $>$ chi2 | $=$ | 0.0000 |


| yt | Coef . | Std. Err | z | $P>\|z\|$ | [95\% Conf | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| yt |  |  |  |  |  |  |
| x 1 | . 8974192 | . 0002247 | 3994.69 | 0.000 | . 8969789 | . 8978595 |
| x2 | -2.525303 | . 0001472 | $-1.7 e+04$ | 0.000 | -2.525591 | -2.525014 |
| x3 | 2.855786 | . 0004181 | 6829.86 | 0.000 | 2.854966 | 2.856605 |
| _cons | . 6975442 | . 0920515 | 7.58 | 0.000 | . 5171265 | . 8779619 |
| select |  |  |  |  |  |  |
| z1 | -. 6825988 | . 0900159 | -7.58 | 0.000 | -. 8590267 | -. 5061709 |
| z2 | 1.003605 | . 132347 | 7.58 | 0.000 | . 7442097 | 1.263 |
| _cons | -. 3604652 | . 1232778 | -2.92 | 0.003 | -. 6020852 | -. 1188452 |
| /athrho | 16.19193 | 280.9822 | 0.06 | 0.954 | -534.523 | 566.9069 |
| /lnsigma | -. 5396153 | . 1318714 | -4.09 | 0.000 | -. 7980786 | -. 2811521 |
| rho | 1 | $9.73 \mathrm{e}-12$ |  |  | -1 | 1 |
| sigma | . 5829725 | . 0768774 |  |  | . 4501931 | . 7549135 |
| lambda | . 5829725 | . 0768774 |  |  | . 4322955 | . 7336494 |

The model has converged to a value of $\rho$ that is 1.0 -within machine-rounding tolerances. Given the form of the likelihood for the Heckman selection model, this implies a division by zero, and it is surprising that the model solution turns out as well as it does. Reparameterizing $\rho$ has allowed the estimation to converge, but we clearly have problems with the estimates. Moreover, if this had occurred in a large dataset, waiting for convergence might take considerable time.

This dataset was not intentionally developed to cause problems. It is actually generated by a "Heckman process" and when generated starting from different random values can be easily estimated. The luck of the draw here merely led to data that, despite the source, did not support the assumptions of the Heckman model.

The two-step model is generally more stable when the data are problematic. It even tolerates estimates of $\rho$ less than -1 and greater than 1 . For these reasons, the two-step model may be preferred when exploring a large dataset. Still, if the maximum likelihood estimates cannot converge, or converge to a value of $\rho$ that is at the boundary of acceptable values, there is scant support for fitting a Heckman selection model on the data. Heckman (1979) discusses the implications of $\rho$ being exactly 1 or 0 , together with the implications of other possible covariance relationships among the model's determinants.

James Joseph Heckman was born in Chicago in 1944 and studied mathematics at Colorado College and economics at Princeton. He has taught economics at Columbia and (since 1973) at the University of Chicago. He has worked on developing a scientific basis for economic policy evaluation, with emphasis on models of individuals or disaggregated groups and the problems and possibilities created by heterogeneity, diversity, and unobserved counterfactual states. In 2000, he shared the Nobel Prize in Economics with Daniel L. McFadden.

## Stored results

heckman (maximum likelihood) stores the following in $e()$ :
Scalars
e(N)
e(N_cens)
e(k)
e(k_eq)
e(k_eq_model)
e(k_aux)
e(k_dv)
$e\left(d f \_m\right)$
e(11)
e(ll_0)
e(N_clust)
e(lambda)
e(selambda)
e(sigma)
e(chi2)
e(chi2_c)
e(p_c)
e(p)
e(rho)
e(rank)
e(rank0)
e(ic)
e(rc)
e(converged)

```
number of observations
number of censored observations
number of parameters
number of equations in e(b)
number of equations in overall model test
number of auxiliary parameters
number of dependent variables
model degrees of freedom
log likelihood
log likelihood, constant-only model
number of clusters
\lambda
standard error of }
sigma
\chi2
\chi 2 for comparison test
p-value for comparison test
significance of comparison test
\rho
rank of e(V)
rank of e(V) for constant-only model
number of iterations
return code
1 if converged, 0 otherwise
```

Macros
e(cmd)
e(cmdline)
e(depvar)
e(wtype)
e(wexp)
e(title)
e(title2)
e(clustvar)
e(offset1)
e(offset2)
e(mills)
e(chi2type)
e(chi2_ct)
e(vce)
e(vcetype)
e(opt)
e(which)
e(method)
e(ml_method)
e(user)
e(technique)
e(properties)
e(predict)
e(marginsok)
e(asbalanced)
e(asobserved)
Matrices
e(b)
e(Cns)
e(ilog)
e(gradient)
e(V)
e(V_modelbased)
Functions
e(sample)
heckman
command as typed
names of dependent variables
weight type
weight expression
title in estimation output
secondary title in estimation output
name of cluster variable
offset for regression equation
offset for selection equation
variable containing nonselection hazard (inverse of Mills')
Wald or LR; type of model $\chi^{2}$ test
Wald or LR; type of model $\chi^{2}$ test corresponding to e(chi2_c)
vcetype specified in vce()
title used to label Std. Err.
type of optimization
$\max$ or min; whether optimizer is to perform maximization or minimization ml
type of ml method
name of likelihood-evaluator program
maximization technique
b V
program used to implement predict
predictions allowed by margins
factor variables fvset as asbalanced
factor variables fvset as asobserved
coefficient vector
constraints matrix
iteration $\log$ (up to 20 iterations)
gradient vector
variance-covariance matrix of the estimators
model-based variance
marks estimation sample
heckman (two-step) stores the following in e() :
Scalars
e(N)
e(N_cens)
$e\left(d f \_m\right)$
e(lambda)
e(selambda)
e(sigma)
e(chi2)
e(p)
e(rho)
e(rank)
Macros
e(cmd)
e(cmdline)
e(depvar)
e(title)
e(title2)
e(mills)
e(chi2type)
e(vce)
e(vcetype)
$e$ (rhometh)
$e$ (method)
$e($ properties)
e(predict)
e(marginsok)
e(marginsnotok)
e(asbalanced)
e(asobserved)
Matrices
e(b)
e(Cns)
e(V)
number of observations
number of censored observations
model degrees of freedom
$\lambda$
standard error of $\lambda$
sigma
$\chi^{2}$
significance of comparison test
$\rho$
rank of $e(V)$

## heckman

command as typed
names of dependent variables
title in estimation output
secondary title in estimation output
variable containing nonselection hazard (inverse of Mills')
Wald or LR; type of model $\chi^{2}$ test
vcetype specified in vce()
title used to label Std. Err.
rhosigma, rhotrunc, rholimited, or rhoforce
twostep
b V
program used to implement predict
predictions allowed by margins
predictions disallowed by margins
factor variables fvset as asbalanced
factor variables fvset as asobserved
coefficient vector
constraints matrix
variance-covariance matrix of the estimators
marks estimation sample

## Methods and formulas

Cameron and Trivedi $(2010,556-562)$ and Greene (2012, 873-880) provide good introductions to the Heckman selection model. Adkins and Hill (2011, sec. 16.8) describe the two-step estimator with an application using Stata. Jones (2007, 35-40) illustrates Heckman estimation with an application to health economics.

Regression estimates using the nonselection hazard (Heckman 1979) provide starting values for maximum likelihood estimation.

The regression equation is

$$
y_{j}=\mathbf{x}_{j} \boldsymbol{\beta}+u_{1 j}
$$

The selection equation is

$$
\mathbf{z}_{j} \gamma+u_{2 j}>0
$$

where

$$
\begin{gathered}
u_{1} \sim N(0, \sigma) \\
u_{2} \sim N(0,1) \\
\operatorname{corr}\left(u_{1}, u_{2}\right)=\rho
\end{gathered}
$$

The $\log$ likelihood for observation $j, \ln L_{j}=l_{j}$, is
$l_{j}= \begin{cases}w_{j} \ln \Phi\left\{\frac{\mathbf{z}_{j} \gamma+\left(y_{j}-\mathbf{x}_{j} \boldsymbol{\beta}\right) \rho / \sigma}{\sqrt{1-\rho^{2}}}\right\}-\frac{w_{j}}{2}\left(\frac{y_{j}-\mathbf{x}_{j} \boldsymbol{\beta}}{\sigma}\right)^{2}-w_{j} \ln (\sqrt{2 \pi} \sigma) & y_{j} \text { observed } \\ w_{j} \ln \Phi\left(-\mathbf{z}_{j} \gamma\right) & y_{j} \text { not observed }\end{cases}$
where $\Phi(\cdot)$ is the standard cumulative normal and $w_{j}$ is an optional weight for observation $j$.
In the maximum likelihood estimation, $\sigma$ and $\rho$ are not directly estimated. Directly estimated are $\ln \sigma$ and atanh $\rho$ :

$$
\operatorname{atanh} \rho=\frac{1}{2} \ln \left(\frac{1+\rho}{1-\rho}\right)
$$

The standard error of $\lambda=\rho \sigma$ is approximated through the propagation of error (delta) method; that is,

$$
\operatorname{Var}(\lambda) \approx \mathbf{D} \operatorname{Var}\{(\operatorname{atanh} \rho \ln \sigma)\} \mathbf{D}^{\prime}
$$

where $\mathbf{D}$ is the Jacobian of $\lambda$ with respect to $\operatorname{atanh} \rho$ and $\ln \sigma$.
With maximum likelihood estimation, this command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce(robust) and vce (cluster clustvar), respectively. See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas.

The maximum likelihood version of heckman also supports estimation with survey data. For details on VCEs with survey data, see [SVY] variance estimation.

The two-step estimates are computed using Heckman's (1979) procedure.
Probit estimates of the selection equation

$$
\operatorname{Pr}\left(y_{j} \text { observed } \mid \mathbf{z}_{j}\right)=\Phi\left(\mathbf{z}_{j} \gamma\right)
$$

are obtained. From these estimates, the nonselection hazard—what Heckman (1979) referred to as the inverse of the Mills' ratio, $m_{j}$-for each observation $j$ is computed as

$$
m_{j}=\frac{\phi\left(\mathbf{z}_{j} \widehat{\gamma}\right)}{\Phi\left(\mathbf{z}_{j} \widehat{\gamma}\right)}
$$

where $\phi$ is the normal density. We also define

$$
\delta_{j}=m_{j}\left(m_{j}+\widehat{\gamma} \mathbf{z}_{j}\right)
$$

Following Heckman, the two-step parameter estimates of $\boldsymbol{\beta}$ are obtained by augmenting the regression equation with the nonselection hazard $\mathbf{m}$. Thus the regressors become [ $\mathbf{X} \mathbf{m}$ ], and we obtain the additional parameter estimate $\boldsymbol{\beta}_{m}$ on the variable containing the nonselection hazard.

A consistent estimate of the regression disturbance variance is obtained using the residuals from the augmented regression and the parameter estimate on the nonselection hazard,

$$
\widehat{\sigma}^{2}=\frac{\mathbf{e}^{\prime} \mathbf{e}+\boldsymbol{\beta}_{m}^{2} \sum_{j=1}^{N} \delta_{j}}{N}
$$

The two-step estimate of $\rho$ is then

$$
\widehat{\rho}=\frac{\boldsymbol{\beta}_{m}}{\widehat{\sigma}}
$$

Heckman derived consistent estimates of the coefficient covariance matrix on the basis of the augmented regression.

Let $\mathbf{W}=[\mathbf{X} \mathbf{~ m}]$ and $\mathbf{R}$ be a square, diagonal matrix of dimension $N$, with $\left(1-\widehat{\rho}^{2} \delta_{j}\right)$ as the diagonal elements. The conventional VCE is

$$
\mathbf{V}_{\text {twostep }}=\hat{\sigma}^{2}\left(\mathbf{W}^{\prime} \mathbf{W}\right)^{-1}\left(\mathbf{W}^{\prime} \mathbf{R} \mathbf{W}+\mathbf{Q}\right)\left(\mathbf{W}^{\prime} \mathbf{W}\right)^{-1}
$$

where

$$
\mathbf{Q}=\widehat{\rho}^{2}\left(\mathbf{W}^{\prime} \mathbf{D} \mathbf{Z}\right) \mathbf{V}_{\mathbf{p}}\left(\mathbf{Z}^{\prime} \mathbf{D W}\right)
$$

where $\mathbf{D}$ is the square, diagonal matrix of dimension $N$ with $\delta_{j}$ as the diagonal elements; $\mathbf{Z}$ is the data matrix of selection equation covariates; and $\mathbf{V}_{p}$ is the variance-covariance estimate from the probit estimation of the selection equation.

## References

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Manning, W. G., N. Duan, and W. H. Rogers. 1987. Monte Carlo evidence on the choice between sample selection and two-part models. Journal of Econometrics 35: 59-82.

## Also see

[R] heckman postestimation - Postestimation tools for heckman
$[R]$ heckoprobit - Ordered probit model with sample selection
[R] heckprobit - Probit model with sample selection
[ R ] regress - Linear regression
[R] tobit - Tobit regression
[SVY] svy estimation - Estimation commands for survey data
[TE] etregress - Linear regression with endogenous treatment effects
[U] 20 Estimation and postestimation commands

## Title

heckman postestimation - Postestimation tools for heckman

| Description | Syntax for predict | Menu for predict |
| :--- | :--- | :--- | Options for predict

## Description

The following postestimation commands are available after heckman:

| Command | Description |
| :---: | :---: |
| contrast | contrasts and ANOVA-style joint tests of estimates |
| estat ic ${ }^{1}$ | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample |
| estat vce | variance-covariance matrix of the estimators (VCE) |
| estat (svy) | postestimation statistics for survey data |
| estimates | cataloging estimation results |
| lincom | point estimates, standard errors, testing, and inference for linear combinations of coefficients |
| lrtest ${ }^{2}$ | likelihood-ratio test; not available with two-step estimator |
| margins | marginal means, predictive margins, marginal effects, and average marginal effects |
| marginsplot | graph the results from margins (profile plots, interaction plots, etc.) |
| nlcom | point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients |
| predict | predictions, residuals, influence statistics, and other diagnostic measures |
| predictnl | point estimates, standard errors, testing, and inference for generalized predictions |
| pwcompare | pairwise comparisons of estimates |
| suest ${ }^{1}$ | seemingly unrelated estimation |
| test | Wald tests of simple and composite linear hypotheses |
| testnl | Wald tests of nonlinear hypotheses |

[^25]
## Syntax for predict

After ML or twostep
predict [type] newvar [if] [in] [, statistic nooffset]
After ML
predict $[$ type $]\left\{s t u b * \mid\right.$ newvar $_{\mathrm{reg}}$ newvar $_{\text {sel }}$ newvar $_{\text {athrho }}$ newvar $\left._{\text {lnsigma }}\right\}$
[if] [in], scores

## statistic

Description

Main
xb
stdp
stdf
xbsel
stdpsel
$\overline{\operatorname{pr}(a, b)}$
$\mathrm{e}(a, b)$
ystar ( $a, b$ )
ycond
yexpected
nshazard or mills
psel
linear prediction; the default
standard error of the prediction
standard error of the forecast
linear prediction for selection equation
standard error of the linear prediction for selection equation
$\operatorname{Pr}\left(y_{j} \mid a<y_{j}<b\right)$
$E\left(y_{j} \mid a<y_{j}<b\right)$
$E\left(y_{j}^{*}\right), y_{j}^{*}=\max \left\{a, \min \left(y_{j}, b\right)\right\}$
$E\left(y_{j} \mid y_{j}\right.$ observed $)$
$E\left(y_{j}^{*}\right), y_{j}$ taken to be 0 where unobserved
nonselection hazard (also called the inverse of Mills' ratio)
$\operatorname{Pr}\left(y_{j}\right.$ observed $)$

These statistics are available both in and out of sample; type predict . . . if e (sample) ... if wanted only for the estimation sample.
stdf is not allowed with svy estimation results.
where $a$ and $b$ may be numbers or variables; $a$ missing ( $a \geq$.) means $-\infty$, and $b$ missing ( $b \geq$.) means $+\infty$; see [U] 12.2.1 Missing values.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
xb , the default, calculates the linear prediction $\mathbf{x}_{j} \mathbf{b}$.
stdp calculates the standard error of the prediction, which can be thought of as the standard error of the predicted expected value or mean for the observation's covariate pattern. The standard error of the prediction is also referred to as the standard error of the fitted value.
stdf calculates the standard error of the forecast, which is the standard error of the point prediction for 1 observation. It is commonly referred to as the standard error of the future or forecast value. By construction, the standard errors produced by stdf are always larger than those produced by stdp; see Methods and formulas in [R] regress postestimation.
xbsel calculates the linear prediction for the selection equation.
stdpsel calculates the standard error of the linear prediction for the selection equation.
$\operatorname{pr}(a, b)$ calculates $\operatorname{Pr}\left(a<\mathbf{x}_{j} \mathbf{b}+u_{1}<b\right)$, the probability that $y_{j} \mid \mathbf{x}_{j}$ would be observed in the interval $(a, b)$.
$a$ and $b$ may be specified as numbers or variable names; $l b$ and $u b$ are variable names;
$\operatorname{pr}(20,30)$ calculates $\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{1}<30\right)$; $\operatorname{pr}(l b, u b)$ calculates $\operatorname{Pr}\left(l b<\mathbf{x}_{j} \mathbf{b}+u_{1}<u b\right)$; and $\operatorname{pr}(20, u b)$ calculates $\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{1}<u b\right)$.
$a$ missing $(a \geq$.$) means -\infty ; \operatorname{pr}(., 30)$ calculates $\operatorname{Pr}\left(-\infty<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$;
$\operatorname{pr}(l b, 30)$ calculates $\operatorname{Pr}\left(-\infty<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$ in observations for which $l b \geq$.
and calculates $\operatorname{Pr}\left(l b<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$ elsewhere.
$b$ missing $\left(b \geq\right.$.) means $+\infty$; $\operatorname{pr}(20,$.$) calculates \operatorname{Pr}\left(+\infty>\mathbf{x}_{j} \mathbf{b}+u_{j}>20\right)$; $\operatorname{pr}(20, u b)$ calculates $\operatorname{Pr}\left(+\infty>\mathbf{x}_{j} \mathbf{b}+u_{j}>20\right)$ in observations for which $u b \geq$.
and calculates $\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{j}<u b\right)$ elsewhere.
$\mathrm{e}(a, b)$ calculates $E\left(\mathbf{x}_{j} \mathbf{b}+u_{1} \mid a<\mathbf{x}_{j} \mathbf{b}+u_{1}<b\right)$, the expected value of $y_{j} \mid \mathbf{x}_{j}$ conditional on $y_{j} \mid \mathbf{x}_{j}$ being in the interval $(a, b)$, meaning that $y_{j} \mid \mathbf{x}_{j}$ is truncated.
$a$ and $b$ are specified as they are for $\operatorname{pr}()$.
ystar $(a, b)$ calculates $E\left(y_{j}^{*}\right)$, where $y_{j}^{*}=a$ if $\mathbf{x}_{j} \mathbf{b}+u_{j} \leq a, y_{j}^{*}=b$ if $\mathbf{x}_{j} \mathbf{b}+u_{j} \geq b$, and $y_{j}^{*}=\mathbf{x}_{j} \mathbf{b}+u_{j}$ otherwise, meaning that $y_{j}^{*}$ is censored. $a$ and $b$ are specified as they are for pr() .
ycond calculates the expected value of the dependent variable conditional on the dependent variable being observed, that is, selected; $E\left(y_{j} \mid y_{j}\right.$ observed $)$.
yexpected calculates the expected value of the dependent variable $\left(y_{j}^{*}\right)$, where that value is taken to be 0 when it is expected to be unobserved; $y_{j}^{*}=\operatorname{Pr}\left(y_{j}\right.$ observed $) E\left(y_{j} \mid y_{j}\right.$ observed $)$.
The assumption of 0 is valid for many cases where nonselection implies nonparticipation (for example, unobserved wage levels, insurance claims from those who are uninsured) but may be inappropriate for some problems (for example, unobserved disease incidence).
nshazard and mills are synonyms; both calculate the nonselection hazard—what Heckman (1979) referred to as the inverse of the Mills' ratio-from the selection equation.
psel calculates the probability of selection (or being observed):
$\operatorname{Pr}\left(y_{j}\right.$ observed $)=\operatorname{Pr}\left(\mathbf{z}_{j} \gamma+u_{2 j}>0\right)$.
nooffset is relevant when you specify offset (varname) for heckman. It modifies the calculations made by predict so that they ignore the offset variable; the linear prediction is treated as $\mathbf{x}_{j} \mathbf{b}$ rather than as $\mathbf{x}_{j} \mathbf{b}+$ offset $_{j}$.
scores, not available with twostep, calculates equation-level score variables.
The first new variable will contain $\partial \ln L / \partial\left(\mathbf{x}_{j} \boldsymbol{\beta}\right)$.
The second new variable will contain $\partial \ln L / \partial\left(\mathbf{z}_{j} \gamma\right)$.
The third new variable will contain $\partial \ln L / \partial(\operatorname{atanh} \rho)$.
The fourth new variable will contain $\partial \ln L / \partial(\ln \sigma)$.

## Remarks and examples

## > Example 1

The default statistic produced by predict after heckman is the expected value of the dependent variable from the underlying distribution of the regression model. In the wage model of $[R]$ heckman, this is the expected wage rate among all women, regardless of whether they were observed to participate in the labor force:

```
. use http://www.stata-press.com/data/r13/womenwk
. heckman wage educ age, select(married children educ age) vce(cluster county)
(output omitted)
. predict heckwage
(option xb assumed; fitted values)
```

It is instructive to compare these predicted wage values from the Heckman model with an ordinary regression model - a model without the selection adjustment:

```
. regress wage educ age
```

| Source | SS | $d f$ | MS |
| ---: | ---: | ---: | ---: |
| Model | 13524.0337 | 2 | 6762.01687 |
| Residual | 39830.8609 | 1340 | 29.7245231 |
| Total | 53354.8946 | 1342 | 39.7577456 |


| F( 2, 1340) | $=227.49$ |
| :--- | :--- | ---: |
| Prob $>$ F | $=0.0000$ |
| R-squared | $=0.2535$ |
| Adj R-squared | $=0.2524$ |
| Root MSE | $=5.452$ |


| wage | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| education | .8965829 | .0498061 | 18.00 | 0.000 | .7988765 | .9942893 |
| age | .1465739 | .0187135 | 7.83 | 0.000 | .109863 | .1832848 |
| _cons | 6.084875 | .8896182 | 6.84 | 0.000 | 4.339679 | 7.830071 |


| Variable | Obs | Mean | Std. Dev. | Min | Max |
| :---: | :---: | :---: | :---: | :---: | :---: |
| heckwage | 2000 | 21.15532 | 3.83965 | 14.6479 | 32.85949 |
| regwage | 2000 | 23.12291 | 3.241911 | 17.98218 | 32.66439 |

Since this dataset was concocted, we know the true coefficients of the wage regression equation to be $1,0.2$, and 1 , respectively. We can compute the true mean wage for our sample.

```
. generate truewage = 1 + . 2*age + 1*educ
```

. summarize truewage

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| truewage | 2000 | 21.3256 | 3.797904 | 15 | 32.8 |

Whereas the mean of the predictions from heckman is within 18 cents of the true mean wage, ordinary regression yields predictions that are on average about $\$ 1.80$ per hour too high because of the selection effect. The regression predictions also show somewhat less variation than the true wages.

The coefficients from heckman are so close to the true values that they are not worth testing. Conversely, the regression equation is significantly off but seems to give the right sense. Would we be led far astray if we relied on the OLS coefficients? The effect of age is off by more than 5 cents per year of age, and the coefficient on education level is off by about $10 \%$. We can test the OLS coefficient on education level against the true value by using test.

```
. test educ = 1
( 1) education = 1
    F( 1, 1340) = 4.31
        Prob > F = 0.0380
```

Not only is the OLS coefficient on education substantially lower than the true parameter, but the difference from the true parameter is also statistically significant beyond the $5 \%$ level. We can perform a similar test for the OLS age coefficient:

```
. test age = . 2
( 1) age = . 2
    F( 1, 1340) = 8.15
        Prob > F = 0.0044
```

We find even stronger evidence that the OLS regression results are biased away from the true parameters.

## $\rangle$ Example 2

Several other interesting aspects of the Heckman model can be explored with predict. Continuing with our wage model, we can obtain the expected wages for women conditional on participating in the labor force with the ycond option. Let's get these predictions and compare them with actual wages for women participating in the labor force.

```
. use http://www.stata-press.com/data/r13/womenwk, clear
. heckman wage educ age, select(married children educ age)
    (output omitted)
. predict hcndwage, ycond
. summarize wage hcndwage if wage != .
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline wage & 1343 & 23.69217 & 6.305374 & 5.88497 & 45.80979 \\
hcndwage & 1343 & 23.68239 & 3.335087 & 16.18337 & 33.7567
\end{tabular}
```

We see that the average predictions from heckman are close to the observed levels but do not have the same mean. These conditional wage predictions are available for all observations in the dataset but can be directly compared only with observed wages, where individuals are participating in the labor force.

What if we were interested in making predictions about mean wages for all women? Here the expected wage is 0 for those who are not expected to participate in the labor force, with expected participation determined by the selection equation. These values can be obtained with the yexpected option of predict. For comparison, a variable can be generated where the wage is set to 0 for nonparticipants.

```
. predict hexpwage, yexpected
. generate wage0 = wage
(657 missing values generated)
. replace wage0 = 0 if wage == .
(657 real changes made)
```

| . summarize hexpwage wage0 |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: |
| Variable | Obs | Mean | Std. Dev. | Min | Max |
| hexpwage | 2000 | 15.92511 | 5.979336 | 2.492469 | 32.45858 |
| wage0 | 2000 | 15.90929 | 12.27081 | 0 | 45.80979 |

Again we note that the predictions from heckman are close to the observed mean hourly wage rate for all women. Why aren't the predictions using ycond and yexpected equal to their observed sample equivalents? For the Heckman model, unlike linear regression, the sample moments implied by the optimal solution to the model likelihood do not require that these predictions match observed data. Properly accounting for the additional variation from the selection equation requires that the model use more information than just the sample moments of the observed wages.

## Reference

Heckman, J. 1979. Sample selection bias as a specification error. Econometrica 47: 153-161.

## Also see

[R] heckman - Heckman selection model
[U] 20 Estimation and postestimation commands

## Title

heckoprobit - Ordered probit model with sample selection

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |
| Also see |  |  |  |

## Syntax

heckoprobit depvar indepvars [if] [in] [weight], select ([depvar $=]$ varlist $_{s}\left[\right.$, noconstant offset $\left(\right.$ varname $\left.\left._{o}\right)\right]$ ) [options]
options
Model

* select ()
offset (varname)
constraints (constraints)
collinear
SE/Robust
vce (vcetype)

Reporting
level(\#)
first
noheader
nofootnote
nocnsreport
display_options

Maximization
maximize_options
coeflegend
specify selection equation: dependent and independent variables; whether to have constant term and offset variable include varname in model with coefficient constrained to 1 apply specified linear constraints keep collinear variables
vcetype may be oim, robust, cluster clustvar, opg, bootstrap, or jackknife
set confidence level; default is level (95)
report first-step probit estimates
do not display header above parameter table
do not display footnotes below parameter table
do not display constraints
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
control the maximization process; seldom used
display legend instead of statistics
*select() is required.
The full specification is select ( $\left[\right.$ depvar $\left._{s}=\right] \operatorname{varlist}_{s}\left[\right.$, noconstant offset(varname ${ }_{o}$ )]). indepvars and varlist ${ }_{s}$ may contain factor variables; see [U] 11.4.3 Factor variables. depvar, indepvars, depvar ${ }_{s}$, and varlist $_{s}$ may contain time-series operators; see [U] 11.4.4 Time-series varlists. bootstrap, by, jackknife, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands. Weights are not allowed with the bootstrap prefix; see $[\mathrm{R}]$ bootstrap. vce(), first, and weights are not allowed with the svy prefix; see [SVY] svy. pweights, fweights, and iweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Sample-selection models $>$ Ordered probit model with selection

## Description

heckoprobit fits maximum-likelihood ordered probit models with sample selection.

## Options

```
Model
select \(\left(\left[\right.\right.\) depvar \(\left._{s}=\right]\) varlist \(_{s}\left[\right.\), noconstant offset \(\left(\right.\) varname \(\left.\left._{o}\right)\right]\) ). specifies the variables and options for the selection equation. It is an integral part of specifying a selection model and is required. The selection equation should contain at least one variable that is not in the outcome equation.
```



``` indicating a selected observation. If depvar \(r_{s}\) is not specified, observations for which depvar is not missing are assumed selected, and those for which depvar is missing are assumed not selected.
noconstant suppresses the selection constant term (intercept).
offset \(\left(\right.\) varname \(\left._{o}\right)\) specifies that selection offset varname \({ }_{o}\) be included in the model with the coefficient constrained to be 1 .
offset (varname), constraints (constraints), collinear; see [R] estimation options.
```


## SE/Robust

vce (vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

## Reporting

level(\#); see [R] estimation options.
first specifies that the first-step probit estimates of the selection equation be displayed before estimation.
noheader suppresses the header above the parameter table, the display that reports the final loglikelihood value, number of observations, etc.
nofootnote suppresses the footnotes displayed below the parameter table.
nocnsreport; see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% fmt), pformat (\%fmt), sformat(\%fmt), and nolstretch; see [R] estimation options.

Maximization
maximize_options: difficult, technique (algorithm_spec), iterate (\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance(\#), nonrtolerance, and from (init_specs); see [R] maximize. These options are seldom used.

Setting the optimization type to technique (bhhh) resets the default vcetype to vce (opg).
The following option is available with heckoprobit but is not shown in the dialog box: coeflegend; see $[R]$ estimation options.

## Remarks and examples

heckoprobit estimates the parameters of a regression model for an ordered categorical outcome from a nonrandom sample known as a selected sample. Selected samples suffer from "selection on unobservables" because the errors that determine whether a case is missing are correlated with the errors that determine the outcome.

For ordered categorical regression from samples that do not suffer from selection on unobservables, see $[R]$ oprobit or $[R]$ ologit. For regression of a continuous outcome variable from a selected sample, see $[R]$ heckman.

Even though we are interested in modeling a single ordinal outcome, there are two dependent variables in the ordered probit sample-selection model because we must also model the sampleselection process. First, there is the ordinal outcome $y_{j}$. Second, there is a binary variable that indicates whether each case in the sample is observed or unobserved. To handle the sample-selection problem, we model both dependent variables jointly. Both variables are categorical. Their categorical values are determined by the values of linear combinations of covariates and normally distributed error terms relative to certain cutpoints that partition the real line. The error terms used in the determination of selection and the ordinal outcome value may be correlated.

The probability that the ordinal outcome $y_{j}$ is equal to the value $v_{h}$ is given by the probability that $\mathbf{x}_{j} \boldsymbol{\beta}+u_{1 j}$ falls within the cutpoints $\kappa_{h-1}$ and $\kappa_{h}$,

$$
\operatorname{Pr}\left(y_{j}=v_{h}\right)=\operatorname{Pr}\left(\kappa_{h-1}<\mathbf{x}_{j} \boldsymbol{\beta}+u_{1 j} \leq \kappa_{h}\right)
$$

where $\mathbf{x}_{j}$ is the outcome covariates, $\boldsymbol{\beta}$ is the coefficients, and $u_{1 j}$ is a random-error term. The observed outcome values $v_{1}, \ldots, v_{H}$ are integers such that $v_{i}<v_{m}$ for $i<m . \kappa_{0}$ is taken as $-\infty$ and $\kappa_{H}$ is taken as $+\infty$.

We model the selection process for the outcome by

$$
s_{j}=\mathbf{1}\left(\mathbf{z}_{j} \gamma+u_{2 j}>0\right)
$$

where $s_{j}=1$ if we observed $y_{j}$ and 0 otherwise, $\mathbf{z}_{j}$ is the covariates used to model the selection process, $\gamma$ is the coefficients for the selection process, $\mathbf{1}(\cdot)$ denotes the indicator function, and $u_{2 j}$ is a random-error term.
$\left(u_{1 j}, u_{2 j}\right)$ have bivariate normal distribution with mean zero and variance matrix

$$
\left[\begin{array}{ll}
1 & \rho \\
\rho & 1
\end{array}\right]
$$

When $\rho \neq 0$, standard ordered probit techniques applied to the outcome equation yield inconsistent results. heckoprobit provides consistent, asymptotically efficient estimates for all the parameters in such models.

De Luca and Perotti (2011) describe the maximum likelihood estimator used in heckoprobit.

## > Example 1

We have a simulated dataset containing a sample of 5,000 women, 3,480 of whom work. The outcome of interest is a woman's job satisfaction, and we suspect that unobservables that determine job satisfaction and the unobservables that increase the likelihood of employment are correlated. Women may make a decision to work based on how satisfying their job would be. We estimate the parameters of an ordered probit sample-selection model for the outcome of job satisfaction (satisfaction) with selection on employment (work). Age (age) and years of education (education) are used as outcome covariates, and we also expect that they affect selection. Additional covariates for selection are marital status (married) and the number of children at home (children).

Here we estimate the parameters of the model with heckoprobit. We use the factorial interaction of married and children in select (). This specifies that the number of children and marital status affect selection, and it allows the effect of the number of children to differ among married and nonmarried women. The factorial interaction is specified using factor-variable notation, which is described in [U] 11.4.3 Factor variables.

```
. use http://www.stata-press.com/data/r13/womensat
(Job satisfaction, female)
. heckoprobit satisfaction education age,
> select(work=education age i.married##c.children)
Fitting oprobit model:
\begin{tabular}{ll} 
Iteration 0: & log likelihood \(=-3934.1474\) \\
Iteration 1: & log likelihood \(=\) \\
Iteration 2: & log likelihood \(=-3570.2616\) \\
Iteration 3: & log likelihood \(=-3570.2616\)
\end{tabular}
Fitting selection model:
Iteration 0: log likelihood = -3071.0775
Iteration 1: log likelihood = -2565.5092
Iteration 2: log likelihood = -2556.8369
Iteration 3: log likelihood = -2556.8237
Iteration 4: log likelihood = -2556.8237
Comparison: log likelihood = -6127.0853
Fitting full model:
Iteration 0: log likelihood = -6127.0853
Iteration 1: log likelihood = -6093.8868
Iteration 2: log likelihood = -6083.215
Iteration 3: log likelihood = -6083.0376
Iteration 4: log likelihood = -6083.0372
```

| Ordered probit model with sample selection | Number of obs | $=$ | 5000 |
| :--- | :--- | :--- | :--- |
|  | Censored obs | $=$ | 1520 |
|  | Uncensored obs | $=$ | 3480 |
|  | Wald chi2(2) | $=$ | 842.42 |
| Log likelihood $=-6083.037$ | Prob $>$ chi2 | $=$ | 0.0000 |


|  | Coef . | Std. Err | z | $P>\|z\|$ | [95\% Conf | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| satisfaction education age | $\begin{aligned} & .1536381 \\ & .0334463 \end{aligned}$ | $\begin{aligned} & .0068266 \\ & .0024049 \end{aligned}$ | $\begin{aligned} & 22.51 \\ & 13.91 \end{aligned}$ | $\begin{aligned} & 0.000 \\ & 0.000 \end{aligned}$ | $\begin{array}{r} .1402583 \\ .0287329 \end{array}$ | $\begin{array}{r} .1670179 \\ .0381598 \end{array}$ |
| work |  |  |  |  |  |  |
| education | . 0512494 | . 0068095 | 7.53 | 0.000 | . 037903 | . 0645958 |
| age | . 0288084 | . 0026528 | 10.86 | 0.000 | . 023609 | . 0340078 |
| 1. married | . 6120876 | . 0700055 | 8.74 | 0.000 | . 4748794 | . 7492958 |
| children | . 5140995 | . 0288529 | 17.82 | 0.000 | . 4575489 | . 5706501 |
| married\# <br> c.children |  |  |  |  |  |  |
| 1 | -. 1337573 | . 035126 | -3.81 | 0.000 | -. 202603 | -. 0649117 |
| _cons | -2.203036 | . 125772 | -17.52 | 0.000 | -2.449545 | -1.956528 |
| /cut1 | 1.728757 | . 1232063 | 14.03 | 0.000 | 1.487277 | 1.970237 |
| /cut2 | 2.64357 | . 116586 | 22.67 | 0.000 | 2.415066 | 2.872075 |
| /cut3 | 3.642911 | . 1178174 | 30.92 | 0.000 | 3.411993 | 3.873829 |
| /athrho | . 7430919 | . 0780998 | 9.51 | 0.000 | . 5900191 | . 8961646 |
| rho | . 6310096 | . 0470026 |  |  | . 5299093 | . 7144252 |

The output shows several iteration logs. The first iteration log corresponds to running the ordered probit model for those observations in the sample where we have observed the outcome. The second iteration $\log$ corresponds to running the selection probit model, which models whether we observe
our outcome of interest. If $\rho=0$, the sum of the log likelihoods from these two models will equal the log likelihood of the ordered probit sample-selection model; this sum is printed in the iteration $\log$ as the comparison $\log$ likelihood. The final iteration $\log$ is for fitting the full ordered probit sample-selection model.

The Wald test in the header is highly significant, indicating a good model fit. All the covariates are statistically significant. The likelihood-ratio test in the footer indicates that we can reject the null hypothesis that the errors for outcome and selection are uncorrelated. This means that we should use the ordered probit sample-selection model instead of the simple ordered probit model.

The positive estimate of 0.63 for $\rho$ indicates that unobservables that increase job satisfaction tend to occur with unobservables that increase the chance of having a job.

## Stored results

heckoprobit stores the following in e():
Scalars

| e(N) | number of observations |
| :---: | :---: |
| e(N_cens) | number of censored observations |
| e(N_cd) | number of completely determined observations |
| e(k_cat) | number of categories |
| e(k) | number of parameters |
| e(k_eq) | number of equations in e(b) |
| e(k_eq_model) | number of equations in overall model test |
| $e\left(k \_a u x\right)$ | number of auxiliary parameters |
| e(k_dv) | number of dependent variables |
| e(df_m) | model degrees of freedom |
| e(ll) | log likelihood |
| e(ll_c) | log likelihood, comparison model |
| e(N_clust) | number of clusters |
| e(chi2) | $\chi^{2}$ |
| e(chi2_c) | $\chi^{2}$ for comparison test |
| $e(p-c)$ | $p$-value for comparison test |
| $e(p)$ | significance of comparison test |
| e(rho) | $\rho$ |
| e(rank) | rank of e(V) |
| e(ic) | number of iterations |
| $e(r c)$ | return code |
| e(converged) | 1 if converged, 0 otherwise |

Macros
e(cmd)
e(cmdline)
e(depvar)
e(wtype)
e(wexp)
e(title)
e(title2)
e(clustvar)
e(offset1)
e(offset2)
e(chi2type)
e(chi2_ct)
e(vce)
e(vcetype)
e(opt)
e(which)
e(method)
e(ml_method)
e(user)
e(technique)
e(properties)
e(predict)
e(marginsok)
e(asbalanced)
e(asobserved)
Matrices
e(b)
e(Cns)
e(ilog)
e(gradient)
e(cat)
e(V)
e(V_modelbased)
Functions
e(sample) marks estimation sample

## Methods and formulas

De Luca and Perotti (2011) provide an introduction to this model.
The ordinal outcome equation is

$$
y_{j}=\sum_{h=1}^{H} v_{h} 1\left(\kappa_{h-1}<\mathbf{x}_{j} \boldsymbol{\beta}+u_{1 j} \leq \kappa_{h}\right)
$$

where $\mathbf{x}_{j}$ is the outcome covariates, $\boldsymbol{\beta}$ is the coefficients, and $u_{1 j}$ is a random-error term. The observed outcome values $v_{1}, \ldots, v_{H}$ are integers such that $v_{i}<v_{m}$ for $i<m . \kappa_{1}, \ldots, \kappa_{H-1}$ are real numbers such that $\kappa_{i}<\kappa_{m}$ for $i<m . \kappa_{0}$ is taken as $-\infty$ and $\kappa_{H}$ is taken as $+\infty$.

The selection equation is

$$
s_{j}=\mathbf{1}\left(\mathbf{z}_{j} \gamma+u_{2 j}>0\right)
$$

where $s_{j}=1$ if we observed $y_{j}$ and 0 otherwise, $\mathbf{z}_{j}$ is the covariates used to model the selection process, $\gamma$ is the coefficients for the selection process, and $u_{2 j}$ is a random-error term.
$\left(u_{1 j}, u_{2 j}\right)$ have bivariate normal distribution with mean zero and variance matrix

$$
\left[\begin{array}{ll}
1 & \rho \\
\rho & 1
\end{array}\right]
$$

Let $a_{j}=z_{j} \gamma+\operatorname{offset}_{j}^{\gamma}$ and $b_{j}=x_{j} \boldsymbol{\beta}+\operatorname{offset}_{j}^{\beta}$. This yields the log likelihood

$$
\ln L=\sum_{j \notin S} w_{j} \ln \left\{\Phi\left(-a_{j}\right)\right\}+\sum_{h=1}^{H} \sum_{\substack{j \in S \\ y_{j}=v_{h}}} w_{j} \ln \left\{\Phi_{2}\left(a_{j}, \kappa_{h}-b_{j},-\rho\right)-\Phi_{2}\left(a_{j}, \kappa_{h-1}-b_{j},-\rho\right)\right\}
$$

where $S$ is the set of observations for which $y_{j}$ is observed, $\Phi_{2}(\cdot)$ is the cumulative bivariate normal distribution function (with mean $\left.\left[\begin{array}{ll}0 & 0\end{array}\right]^{\prime}\right), \Phi(\cdot)$ is the standard cumulative normal, and $w_{j}$ is an optional weight for observation $j$.

In the maximum likelihood estimation, $\rho$ is not directly estimated. Directly estimated is atanh $\rho$ :

$$
\operatorname{atanh} \rho=\frac{1}{2} \ln \left(\frac{1+\rho}{1-\rho}\right)
$$

From the form of the likelihood, it is clear that if $\rho=0$, the log likelihood for the ordered probit sample-selection model is equal to the sum of the ordered probit model for the outcome $y$ and the selection model. We can perform a likelihood-ratio test by comparing the log likelihood of the full model with the sum of the log likelihoods for the ordered probit and selection models.

## References

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Heckman, J. 1979. Sample selection bias as a specification error. Econometrica 47: 153-161.
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## Also see

$[R]$ heckoprobit postestimation - Postestimation tools for heckoprobit
[R] heckman - Heckman selection model
[R] heckprobit - Probit model with sample selection
[R] oprobit - Ordered probit regression
[R] probit - Probit regression
[R] regress - Linear regression
[R] tobit - Tobit regression
[SVY] svy estimation - Estimation commands for survey data
[U] 20 Estimation and postestimation commands

## Title

heckoprobit postestimation - Postestimation tools for heckoprobit

| Description | Syntax for predict | Menu for predict | Options for predict |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Methods and formulas | Also see |  |

## Description

The following postestimation commands are available after heckoprobit:

| Command | Description |
| :--- | :--- |
| contrast | contrasts and ANOVA-style joint tests of estimates |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample <br> estat vce |
| variance-covariance matrix of the estimators (VCE) |  |
| estat (svy) | postestimation statistics for survey data <br> cataloging estimation results |
| lincom | point estimates, standard errors, testing, and inference for linear combinations <br> of coefficients |
| likelihood-ratio test |  |$\quad$| marginal means, predictive margins, marginal effects, and average marginal |
| :--- |
| effects |

[^26]
## Syntax for predict

```
predict [type] {stub*|newvar|newvarlist } [if] [in] [, statistic
    outcome(outcome) nooffset]
```



```
    [if][in], scores
```

    statistic Description
    Main
pmargin marginal probabilities; the default
p1 bivariate probabilities of levels with selection
p0 bivariate probabilities of levels with no selection
pcond1 probabilities of levels conditional on selection
pcond0 probabilities of levels conditional on no selection
psel selection probability
$\overline{\mathrm{xb}} \quad$ linear prediction
stdp standard error of the linear prediction
xbsel linear prediction for selection equation
stdpsel standard error of the linear prediction for selection equation

If you do not specify outcome(), pmargin (with one new variable specified) assumes outcome(\#1).
You specify one or $k$ new variables with pmargin, where $k$ is the number of outcomes.
You specify one new variable with psel, xb , stdp, xbsel, and stdpsel.
These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
pmargin, the default, calculates the predicted marginal probabilities.
You specify one or $k$ new variables, where $k$ is the number of categories of the outcome variable $y_{j}$. If you specify the outcome() option, you must specify one new variable. If you specify one new variable and do not specify outcome(), outcome(\#1) is assumed.

When outcome() is specified, the marginal probability that $y_{j}$ is equal to the level outcome() is calculated. When outcome() is not specified, the marginal probabilities for each outcome level are calculated.
p1 calculates the predicted bivariate probabilities of outcome levels with selection.
You specify one or $k$ new variables, where $k$ is the number of categories of the outcome variable $y_{j}$. If you specify the outcome() option, you must specify one new variable. If you specify one new variable and do not specify outcome(), outcome(\#1) is assumed.

When outcome() is specified, the bivariate probability that $y_{j}$ is equal to the level outcome() and that $y_{j}$ is observed is calculated. When outcome() is not specified, the bivariate probabilities for each outcome level and selection are calculated.
p0 calculates the predicted bivariate probabilities of outcome levels with no selection.
You specify one or $k$ new variables, where $k$ is the number of categories of the outcome variable $y_{j}$. If you specify the outcome () option, you must specify one new variable. If you specify one new variable and do not specify outcome(), outcome(\#1) is assumed.
When outcome() is specified, the bivariate probability that $y_{j}$ is equal to the level outcome() and that $y_{j}$ is not observed is calculated. When outcome() is not specified, the bivariate probabilities for each outcome level and no selection are calculated.
pcond1 calculates the predicted probabilities of outcome levels conditional on selection.
You specify one or $k$ new variables, where $k$ is the number of categories of the outcome variable $y_{j}$. If you specify the outcome() option, you must specify one new variable. If you specify one new variable and do not specify outcome(), outcome (\#1) is assumed.
When outcome() is specified, the probability that $y_{j}$ is equal to the level outcome() given that $y_{j}$ is observed is calculated. When outcome() is not specified, the probabilities for each outcome level conditional on selection are calculated.
pcond0 calculates the predicted probabilities of outcome levels conditional on no selection.
You specify one or $k$ new variables, where $k$ is the number of categories of the outcome variable $y_{j}$. If you specify the outcome () option, you must specify one new variable. If you specify one new variable and do not specify outcome(), outcome(\#1) is assumed.
When outcome() is specified, the probability that $y_{j}$ is equal to the level outcome() given that $y_{j}$ is not observed is calculated. When outcome() is not specified, the probabilities for each outcome level conditional on no selection are calculated.
psel calculates the predicted univariate (marginal) probability of selection.
xb calculates the linear prediction for outcome variable, which is $\mathbf{x}_{j} \boldsymbol{\beta}$ if offset () was not specified and $\mathbf{x}_{j} \boldsymbol{\beta}+\operatorname{offset}_{j}^{\beta}$ if offset() was specified.
stdp calculates the standard error of the prediction, which can be thought of as the standard error of the predicted expected value or mean for the observation's covariate pattern. The standard error of the prediction is also referred to as the standard error of the fitted value.
xbsel calculates the linear prediction for the selection equation, which is $\mathbf{z}_{j} \gamma$ if offset () was not specified in select() and $\mathbf{z}_{j} \gamma+\operatorname{offset}_{j}^{\gamma}$ if offset() was specified in select().
stdpsel calculates the standard error of the linear prediction for the selection equation.
outcome (outcome) specifies for which outcome the predicted probabilities are to be calculated. outcome() should contain either one value of the dependent variable or one of \#1, \#2, ..., with \#1 meaning the first category of the dependent variable, \#2 meaning the second category, etc.
nooffset is relevant only if you specified offset (varname) for heckoprobit. It modifies the calculations made by predict so that they ignore the offset variable; the linear prediction is treated as $\mathbf{x}_{j} \mathbf{b}$ rather than as $\mathbf{x}_{j} \mathbf{b}+$ offset $_{j}$.
scores calculates equation-level score variables.
The first new variable will contain $\partial \ln L / \partial\left(\mathbf{x}_{j} \boldsymbol{\beta}\right)$.
The second new variable will contain $\partial \ln L / \partial\left(\mathbf{z}_{j} \gamma\right)$.
When the dependent variable takes $k$ different values, new variables three through $k+1$ will contain $\partial \ln L / \partial\left(\kappa_{j-2}\right)$.
The last new variable will contain $\partial \ln L / \partial(\operatorname{atanh} \rho)$.

## Remarks and examples

## $>$ Example 1

In example 1 of $[\mathrm{R}]$ heckoprobit, we examined a simulated dataset of 5,000 women, 3,480 of whom work and can thus report job satisfaction. Using job satisfaction (satisfaction) as the outcome variable and employment (work) as the selection variable, we estimated the parameters of an ordered probit sample-selection model. Covariates age (age), years of education (education), number of children (children), and marital status (married) are expected to affect selection. The outcome, job satisfaction, is affected by age (age) and education (education).

We first reestimate the parameters of the regression, but this time we request a robust variance estimator:

```
. use http://www.stata-press.com/data/r13/womensat
(Job satisfaction, female)
. heckoprobit satisfaction education age,
> select(work=education age i.married##c.children) vce(robust)
    (output omitted)
```

We then use margins (see [R] margins) to estimate the average marginal effect of education on the probability of having low job satisfaction.

| Average marginal effects |  |  |  | Number of obs |  | = | 5000 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { Expression : Pr(satisfaction=1), predict() } \\ & \text { dy/dx w.r.t. : education } \end{aligned}$ |  |  |  |  |  |  |  |
|  | Unconditional |  |  |  |  |  |  |
|  | dy/dx | Std. Err. | z | $P>\|z\|$ | [95\% | Conf | Interval] |
| education | -. 0234776 | . 0019176 | -12.24 | 0.000 | -. 027 | 236 | -. 0197192 |

The estimated average marginal effect of education on the probability of having low job satisfaction is approximately -0.023 .

## Methods and formulas

The ordinal outcome equation is

$$
y_{j}=\sum_{h=1}^{H} v_{h} 1\left(\kappa_{h-1}<\mathbf{x}_{j} \boldsymbol{\beta}+u_{1 j} \leq \kappa_{h}\right)
$$

where $\mathbf{x}_{j}$ is the outcome covariates, $\boldsymbol{\beta}$ is the coefficients, and $u_{1 j}$ is a random-error term. The observed outcome values $v_{1}, \ldots, v_{H}$ are integers such that $v_{i}<v_{m}$ for $i<m . \kappa_{1}, \ldots, \kappa_{H-1}$ are real numbers such that $\kappa_{i}<\kappa_{m}$ for $i<m . \kappa_{0}$ is taken as $-\infty$ and $\kappa_{H}$ is taken as $+\infty$.

The selection equation is

$$
s_{j}=1\left(\mathbf{z}_{j} \gamma+u_{2 j}>0\right)
$$

where $s_{j}=1$ if we observed $y_{j}$ and 0 otherwise, $\mathbf{z}_{j}$ is the covariates used to model the selection process, $\gamma$ is the coefficients for the selection process, and $u_{2 j}$ is a random-error term.
$\left(u_{1 j}, u_{2 j}\right)$ have bivariate normal distribution with mean zero and variance matrix

$$
\left[\begin{array}{ll}
1 & \rho \\
\rho & 1
\end{array}\right]
$$

The probability of selection is

$$
\operatorname{Pr}\left(s_{j}=1\right)=\Phi\left(\mathbf{z}_{j} \gamma+\operatorname{offset}_{j}^{\gamma}\right)
$$

$\Phi(\cdot)$ is the standard cumulative normal distribution function.
The probability of selection and the outcome $y_{j}=v_{h}$ is

$$
\begin{aligned}
\operatorname{Pr}\left(y_{j}=v_{h}, s_{j}=1\right)= & \Phi_{2}\left(\mathbf{z}_{j} \boldsymbol{\gamma}+\operatorname{offset}_{j}^{\gamma}, \kappa_{h}-x_{j} \boldsymbol{\beta}-\operatorname{offset}_{j}^{\beta},-\rho\right) \\
& -\Phi_{2}\left(\mathbf{z}_{j} \boldsymbol{\gamma}+\operatorname{offset}_{j}^{\gamma}, \kappa_{h-1}-\mathbf{x}_{j} \boldsymbol{\beta}-\operatorname{offset}_{j}^{\beta},-\rho\right)
\end{aligned}
$$

$\Phi_{2}(\cdot)$ is the cumulative bivariate normal distribution function (with mean $\left[\begin{array}{ll}0 & 0\end{array}\right]^{\prime}$ ).
The probability of $y_{j}$ not being selected and the outcome $y_{j}=v_{h}$ is

$$
\begin{aligned}
\operatorname{Pr}\left(y_{j}=v_{h}, s_{j}=0\right)= & \Phi_{2}\left(-\mathbf{z}_{j} \gamma-\operatorname{offset}_{j}^{\gamma}, \kappa_{h}-x_{j} \boldsymbol{\beta}-\operatorname{offset}_{j}^{\beta}, \rho\right) \\
& -\Phi_{2}\left(-\mathbf{z}_{j} \gamma-\operatorname{offset}_{j}^{\gamma}, \kappa_{h-1}-\mathbf{x}_{j} \boldsymbol{\beta}-\operatorname{offset}_{j}^{\beta}, \rho\right)
\end{aligned}
$$

The probability of outcome $y_{j}=v_{h}$ given selection is

$$
\operatorname{Pr}\left(y_{j}=v_{h} \mid s_{j}=1\right)=\frac{\operatorname{Pr}\left(y_{j}=v_{h}, s_{j}=1\right)}{\operatorname{Pr}\left(s_{j}=1\right)}
$$

The probability of outcome $y_{j}=v_{h}$ given $y_{j}$ is not selected is

$$
\operatorname{Pr}\left(y_{j}=v_{h} \mid s_{j}=0\right)=\frac{\operatorname{Pr}\left(y_{j}=v_{h}, s_{j}=0\right)}{\operatorname{Pr}\left(s_{j}=0\right)}
$$

The marginal probabilities of the outcome $y_{j}$ are

$$
\begin{aligned}
\operatorname{Pr}\left(y_{j}=v_{1}\right) & =\Phi\left(\kappa_{1}-x_{j} \boldsymbol{\beta}-\operatorname{offset}_{j}^{\beta}\right) \\
\operatorname{Pr}\left(y_{j}=v_{H}\right) & =1-\Phi\left(\kappa_{H-1}-x_{j} \boldsymbol{\beta}-\operatorname{offset}_{j}^{\beta}\right) \\
\operatorname{Pr}\left(y_{j}=v_{h}\right) & =\Phi\left(\kappa_{h}-x_{j} \boldsymbol{\beta}-\operatorname{offset}_{j}^{\beta}\right)-\Phi\left(\kappa_{h-1}-x_{j} \boldsymbol{\beta}-\operatorname{offset}_{j}^{\beta}\right)
\end{aligned}
$$

## Also see

[R] heckoprobit - Ordered probit model with sample selection
[U] 20 Estimation and postestimation commands

## Title

heckprobit — Probit model with sample selection

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |
| Also see |  |  |  |

## Syntax

heckprobit depvar indepvars [if] [in] [weight], select ( $\left[\right.$ depvar $\left.{ }_{s}=\right]$ varlist $_{s}\left[\right.$, noconstant offset $\left(\right.$ varname $\left.\left._{o}\right)\right]$ ) [options]
options
Model

* select ()
noconstant
offset (varname)
constraints (constraints)
collinear
SE/Robust
vce (vcetype)

Reporting
level(\#)
first
noskip
nocnsreport
display_options

Maximization
maximize_options
coeflegend

Description
specify selection equation: dependent and independent variables; whether to have constant term and offset variable suppress constant term include varname in model with coefficient constrained to 1 apply specified linear constraints
keep collinear variables
vcetype may be oim, robust, cluster clustvar, opg, bootstrap, or jackknife
set confidence level; default is level (95)
report first-step probit estimates
perform likelihood-ratio test
do not display constraints
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
*select () is required.
The full specification is select ( $\left[\right.$ depvar $\left._{s}=\right] \operatorname{varlist}_{s}\left[\right.$, noconstant offset(varname ${ }_{o}$ )]). indepvars and varlist ${ }_{s}$ may contain factor variables; see [U] 11.4.3 Factor variables. depvar, indepvars, depvar ${ }_{s}$, and varlist $_{s}$ may contain time-series operators; see [U] 11.4.4 Time-series varlists. bootstrap, by, fp, jackknife, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands. Weights are not allowed with the bootstrap prefix; see [R] bootstrap.
vce(), first, noskip, and weights are not allowed with the svy prefix; see [SVY] svy.
pweights, fweights, and iweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Sample-selection models $>$ Probit model with selection

## Description

heckprobit fits maximum-likelihood probit models with sample selection. heckprob is a synonym for heckprobit.

## Options

Model
select ( $\left[\right.$ depvar $\left._{s}=\right]$ varlist $_{s}\left[\right.$, noconstant offset $\left(\right.$ varname $\left.\left._{o}\right)\right]$ ) specifies the variables and options for the selection equation. It is an integral part of specifying a selection model and is required. The selection equation should contain at least one variable that is not in the outcome equation.
If depvar ${ }_{s}$ is specified, it should be coded as 0 or 1,0 indicating an observation not selected and 1 indicating a selected observation. If depvar $r_{s}$ is not specified, observations for which depvar is not missing are assumed selected, and those for which depvar is missing are assumed not selected. noconstant suppresses the selection constant term (intercept).
offset ( varname $_{o}$ ) specifies that selection offset varname ${ }_{o}$ be included in the model with the coefficient constrained to be 1 .
noconstant, offset(varname), constraints(constraints), collinear; see [R] estimation options.

SE/Robust
vce (vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

## Reporting

level(\#); see [R] estimation options.
first specifies that the first-step probit estimates of the selection equation be displayed before estimation.
noskip specifies that a full maximum-likelihood model with only a constant for the regression equation be fit. This model is not displayed but is used as the base model to compute a likelihood-ratio test for the model test statistic displayed in the estimation header. By default, the overall model test statistic is an asymptotically equivalent Wald test that all the parameters in the regression equation are zero (except the constant). For many models, this option can substantially increase estimation time.
nocnsreport; see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% fmt), pformat (\%fmt), sformat(\%fmt), and nolstretch; see [R] estimation options.
maximize_options: difficult, technique (algorithm_spec), iterate(\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance (\#), nonrtolerance, and from(init_specs); see [R] maximize. These options are seldom used.
Setting the optimization type to technique (bhhh) resets the default vcetype to vce (opg).
The following option is available with heckprobit but is not shown in the dialog box:
coeflegend; see $[R]$ estimation options.

## Remarks and examples

The probit model with sample selection (Van de Ven and Van Pragg 1981) assumes that there exists an underlying relationship

$$
y_{j}^{*}=\mathbf{x}_{j} \boldsymbol{\beta}+u_{1 j} \quad \text { latent equation }
$$

such that we observe only the binary outcome

$$
y_{j}^{\text {probit }}=\left(y_{j}^{*}>0\right) \quad \text { probit equation }
$$

The dependent variable, however, is not always observed. Rather, the dependent variable for observation $j$ is observed if

$$
y_{j}^{\text {select }}=\left(\mathbf{z}_{j} \gamma+u_{2 j}>0\right) \quad \text { selection equation }
$$

where

$$
\begin{gathered}
u_{1} \sim N(0,1) \\
u_{2} \sim N(0,1) \\
\operatorname{corr}\left(u_{1}, u_{2}\right)=\rho
\end{gathered}
$$

When $\rho \neq 0$, standard probit techniques applied to the first equation yield biased results. heckprobit provides consistent, asymptotically efficient estimates for all the parameters in such models.

For the model to be well identified, the selection equation should have at least one variable that is not in the probit equation. Otherwise, the model is identified only by functional form, and the coefficients have no structural interpretation.

## $\lambda$ Example 1

We use the data from Pindyck and Rubinfeld (1998). In this dataset, the variables are whether children attend private school (private), number of years the family has been at the present residence (years), log of property tax (logptax), log of income (loginc), and whether one voted for an increase in property taxes (vote).

In this example, we alter the meaning of the data. Here we assume that we observe whether children attend private school only if the family votes for increasing the property taxes. This assumption is not true in the dataset, and we make it only to illustrate the use of this command.

We observe whether children attend private school only if the head of household voted for an increase in property taxes. We assume that the vote is affected by the number of years in residence, the current property taxes paid, and the household income. We wish to model whether children are sent to private school on the basis of the number of years spent in the current residence and the current property taxes paid.
. use http://www.stata-press.com/data/r13/school
. heckprob private years logptax, select(vote=years loginc logptax)
Fitting probit model:
$\begin{array}{ll}\text { Iteration 0: } & \text { log likelihood }=-17.122381 \\ \text { Iteration 1: } & \text { log likelihood }=-16.243974 \\ \begin{array}{c}\text { (output omitted })\end{array} \\ \text { Iteration 5: } & \text { log likelihood }=-15.883655\end{array}$
Fitting selection model:
Iteration 0: log likelihood $=-63.036914$
Iteration 1: log likelihood $=-58.534843$
Iteration 2: $\quad$ log likelihood $=-58.497292$
Iteration 3: log likelihood $=-58.497288$
Comparison: log likelihood = -74.380943
Fitting starting values:
Iteration 0: log likelihood $=-40.895684$
Iteration 1: log likelihood $=-16.654497$ (output omitted)
Iteration 6: log likelihood $=-15.753765$
Fitting full model:

| Iteration 0: | log likelihood $=-75.010619$ | (not concave) |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Iteration 1: | log likelihood $=-74.287786$ |  |  |  |
| Iteration 2: | log likelihood $=-74.250137$ |  |  |  |
| Iteration 3: | log likelihood $=-74.245088$ |  |  |  |
| Iteration 4: | log likelihood $=-74.244973$ |  |  |  |
| Iteration 5: | log likelihood $=-74.244973$ |  |  |  |
| Probit model with sample selection | Number of obs | $=$ | 95 |  |
|  | Censored obs | $=$ | 36 |  |
|  | Uncensored obs | $=$ | 59 |  |
|  | Wald chi2(2) | $=$ | 1.04 |  |
| Log likelihood $=-74.24497$ |  | Prob $>$ chi2 | $=$ | 0.5935 |


|  | Coef. | Std. Err. | z | $P>\|z\|$ | [95\% Conf | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| private |  |  |  |  |  |  |
| years | -. 1142597 | . 1461717 | -0.78 | 0.434 | -. 400751 | . 1722317 |
| logptax | . 3516098 | 1.016485 | 0.35 | 0.729 | -1.640665 | 2.343884 |
| _cons | -2.780665 | 6.905838 | -0.40 | 0.687 | -16.31586 | 10.75453 |
| vote |  |  |  |  |  |  |
| years | -. 0167511 | . 0147735 | -1.13 | 0.257 | -. 0457067 | . 0122045 |
| loginc | . 9923024 | . 4430009 | 2.24 | 0.025 | . 1240366 | 1.860568 |
| logptax | -1.278783 | . 5717545 | -2.24 | 0.025 | -2.399401 | -. 1581647 |
| _cons | -. 545821 | 4.070418 | -0.13 | 0.893 | -8.523694 | 7.432052 |
| /athrho | -. 8663156 | 1.450028 | -0.60 | 0.550 | -3.708318 | 1.975687 |
| rho | -. 6994973 | . 7405343 |  |  | -. 9987984 | . 962269 |
| LR test of indep. eqns. (rho = 0) : chi2 (1) = 0.27 Prob > chi2 = 0.6020 |  |  |  |  |  |  |

The output shows several iteration logs. The first iteration log corresponds to running the probit model for those observations in the sample where we have observed the outcome. The second iteration log corresponds to running the selection probit model, which models whether we observe our outcome of interest. If $\rho=0$, the sum of the log likelihoods from these two models will equal the log likelihood of the probit model with sample selection; this sum is printed in the iteration log as the comparison $\log$ likelihood. The third iteration log shows starting values for the iterations.

The final iteration $\log$ is for fitting the full probit model with sample selection. A likelihood-ratio test of the log likelihood for this model and the comparison log likelihood is presented at the end of the output. If we had specified the vce (robust) option, this test would be presented as a Wald test instead of as a likelihood-ratio test.

## > Example 2

In example 1, we could have obtained robust standard errors by specifying the vce (robust) option. We do this here and also eliminate the iteration logs by using the nolog option:

| Probit model | th sample s | lection |  | Number of obs Censored obs Uncensored obs |  | $\begin{aligned} & 95 \\ & 36 \\ & 59 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Wald ch | 2(2) | 2.55 |
| Log pseudolikelihood $=-74.24497$ |  |  |  | Prob > | hi2 | 0.2798 |
|  | Coef. | Robust <br> Std. Err. | z | $P>\|z\|$ | [95\% Con | Interval] |
| private |  |  |  |  |  |  |
| years | -. 1142597 | . 1113977 | -1.03 | 0.305 | -. 3325951 | . 1040758 |
| logptax | . 3516098 | . 7358265 | 0.48 | 0.633 | -1.090584 | 1.793803 |
| _cons | -2.780665 | 4.786678 | -0.58 | 0.561 | -12.16238 | 6.601051 |
| vote |  |  |  |  |  |  |
| years | -. 0167511 | . 0173344 | -0.97 | 0.334 | -. 0507259 | . 0172237 |
| loginc | . 9923024 | . 4228044 | 2.35 | 0.019 | . 1636209 | 1.820984 |
| logptax | -1.278783 | . 5095156 | -2.51 | 0.012 | -2.277415 | -. 2801508 |
| _cons | -. 545821 | 4.543892 | -0.12 | 0.904 | -9.451686 | 8.360044 |
| /athrho | -. 8663156 | 1.630643 | -0.53 | 0.595 | -4.062318 | 2.329687 |
| rho | -. 6994973 | . 8327753 |  |  | -. 9994079 | . 981233 |
| Wald test of indep. eqns. (rho = 0) : chi2 (1) = 0.28 Prob > chi2 = 0.5952 |  |  |  |  |  |  |

Regardless of whether we specify the vce (robust) option, the outcome is not significantly different from the outcome obtained by fitting the probit and selection models separately. This result is not surprising because the selection mechanism estimated was invented for the example rather than borne from any economic theory.

## Stored results

heckprobit stores the following in e():
Scalars

```
e(N)
    e(N_cens)
    e(k)
    e(k_eq)
    e(k_eq_model)
    e(k_aux)
    e(k_dv)
    e(df_m)
    e(ll)
    e(ll_0)
    e(ll_c)
    e(N_clust)
    e(chi2)
    e(chi2_c)
    e(p_c)
    e(p)
    e(rho)
    e(rank)
    e(rank0)
    e(ic)
    e(rc)
    e(converged)
```

Macros
e(cmd)
e(cmdline)
e(depvar)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(offset1)
e(offset2)
e(chi2type)
e(chi2_ct)
e(vce)
e(vcetype)
e(opt)
e(which)
e(ml_method)
e(user)
e(technique)
e(properties)
e(predict)
e(asbalanced)
e(asobserved)
Matrices
e(b)
e(Cns)
e(ilog)
e(gradient)
e(V)
e(V_modelbased)

Functions
e(sample) marks estimation sample

## Methods and formulas

Van de Ven and Van Pragg (1981) provide an introduction and an explanation of this model.
The probit equation is

$$
y_{j}=\left(\mathbf{x}_{j} \boldsymbol{\beta}+u_{1 j}>0\right)
$$

The selection equation is

$$
\mathbf{z}_{j} \gamma+u_{2 j}>0
$$

where

$$
\begin{gathered}
u_{1} \sim N(0,1) \\
u_{2} \sim N(0,1) \\
\operatorname{corr}\left(u_{1}, u_{2}\right)=\rho
\end{gathered}
$$

The log likelihood is

$$
\begin{aligned}
\ln L=\sum_{\substack{j \in S \\
y_{j} \neq 0}} & w_{j} \ln \left\{\Phi_{2}\left(x_{j} \beta+\operatorname{offset}_{j}^{\beta}, z_{j} \gamma+\operatorname{offset}_{j}^{\gamma}, \rho\right)\right\} \\
& +\sum_{\substack{j \in S \\
y_{j}=0}} w_{j} \ln \left\{\Phi_{2}\left(-x_{j} \beta+\operatorname{offset}_{j}^{\beta}, z_{j} \gamma+\operatorname{offset}_{j}^{\gamma},-\rho\right)\right\} \\
& +\sum_{j \notin S} w_{j} \ln \left\{1-\Phi\left(z_{j} \gamma+\operatorname{offset}_{j}^{\gamma}\right)\right\}
\end{aligned}
$$

where $S$ is the set of observations for which $y_{j}$ is observed, $\Phi_{2}(\cdot)$ is the cumulative bivariate normal distribution function (with mean $\left[\begin{array}{ll}0 & 0\end{array}\right]^{\prime}$ ), $\Phi(\cdot)$ is the standard cumulative normal, and $w_{j}$ is an optional weight for observation $j$.

In the maximum likelihood estimation, $\rho$ is not directly estimated. Directly estimated is atanh $\rho$ :

$$
\operatorname{atanh} \rho=\frac{1}{2} \ln \left(\frac{1+\rho}{1-\rho}\right)
$$

From the form of the likelihood, it is clear that if $\rho=0$, the $\log$ likelihood for the probit model with sample selection is equal to the sum of the probit model for the outcome $y$ and the selection model. We can perform a likelihood-ratio test by comparing the likelihood of the full model with the sum of the log likelihoods for the probit and selection models.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce(robust) and vce(cluster clustvar), respectively. See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas.
heckprobit also supports estimation with survey data. For details on VCEs with survey data, see [SVY] variance estimation.

## References

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Van de Ven, W. P. M. M., and B. M. S. Van Pragg. 1981. The demand for deductibles in private health insurance: A probit model with sample selection. Journal of Econometrics 17: 229-252.

## Also see

[R] heckprobit postestimation - Postestimation tools for heckprobit
[R] heckman - Heckman selection model
[R] heckoprobit - Ordered probit model with sample selection
[R] probit - Probit regression
[SVY] svy estimation - Estimation commands for survey data
[TE] etregress - Linear regression with endogenous treatment effects
[U] 20 Estimation and postestimation commands

## Title

heckprobit postestimation - Postestimation tools for heckprobit

Remarks and examples

Description Syntax for predict Menu for predict Options for predict<br>Also see

## Description

The following postestimation commands are available after heckprobit:

| Command | Description |
| :--- | :--- |
| contrast | contrasts and ANOVA-style joint tests of estimates |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample <br> estat vce |
| variance-covariance matrix of the estimators (VCE) |  |
| estat (svy) | postestimation statistics for survey data <br> cataloging estimation results |
| lincom | point estimates, standard errors, testing, and inference for linear combinations <br> of coefficients |
| likelihood-ratio test |  |$\quad$| marginal means, predictive margins, marginal effects, and average marginal |
| :--- |
| effects |

[^27]
## Syntax for predict

predict [type] newvar [if] [in] [, statistic nooffset]
predict $[$ type $]\left\{\right.$ stub $\mid$ newvar $_{\text {reg }}$ newvar $_{\text {sel }}$ newvar $\left._{\text {athrho }}\right\}[i f][i n]$, scores

## statistic Description

Main
pmargin
$\Phi\left(\mathbf{x}_{j} \mathbf{b}\right)$, success probability; the default
p11
p10
$\Phi_{2}\left(\mathbf{x}_{j} \mathbf{b}, \mathbf{z}_{j} \mathbf{g}, \rho\right)$, predicted probability $\operatorname{Pr}\left(y_{j}^{\text {probit }}=1, y_{j}^{\text {select }}=1\right)$
p01
$\Phi_{2}\left(\mathbf{x}_{j} \mathbf{b},-\mathbf{z}_{j} \mathbf{g},-\rho\right)$, predicted probability $\operatorname{Pr}\left(y_{j}^{\text {probit }}=1, y_{j}^{\text {select }}=0\right)$
$\Phi_{2}\left(-\mathbf{x}_{j} \mathbf{b}, \mathbf{z}_{j} \mathbf{g},-\rho\right)$, predicted probability $\operatorname{Pr}\left(y_{j}^{\text {probit }}=0, y_{j}^{\text {select }}=1\right)$
p00
psel
pcond
xb
stdp
xbsel
stdpsel
$\Phi_{2}\left(-\mathbf{x}_{j} \mathbf{b},-\mathbf{z}_{j} \mathbf{g}, \rho\right)$, predicted probability $\operatorname{Pr}\left(y_{j}^{\text {probit }}=0, y_{j}^{\text {select }}=0\right)$
$\Phi\left(\mathbf{z}_{j} \mathbf{g}\right)$, selection probability
$\Phi_{2}\left(\mathbf{x}_{j} \mathbf{b}, \mathbf{z}_{j} \mathbf{g}, \rho\right) / \Phi\left(\mathbf{z}_{j} \mathbf{g}\right)$, probability of success conditional on selection
linear prediction
standard error of the linear prediction
linear prediction for selection equation
standard error of the linear prediction for selection equation
where $\Phi(\cdot)$ is the standard normal distribution function and $\Phi_{2}(\cdot)$ is the bivariate normal distribution function.

These statistics are available both in and out of sample; type predict ... if e(sample)... if wanted only for the estimation sample.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
pmargin, the default, calculates the univariate (marginal) predicted probability of success $\operatorname{Pr}\left(y_{j}^{\text {probit }}=1\right)$.
p11 calculates the bivariate predicted probability $\operatorname{Pr}\left(y_{j}^{\text {probit }}=1, y_{j}^{\text {select }}=1\right)$.
p 10 calculates the bivariate predicted probability $\operatorname{Pr}\left(y_{j}^{\text {probit }}=1, y_{j}^{\text {select }}=0\right)$.
p01 calculates the bivariate predicted probability $\operatorname{Pr}\left(y_{j}^{\text {probit }}=0, y_{j}^{\text {select }}=1\right)$.
p00 calculates the bivariate predicted probability $\operatorname{Pr}\left(y_{j}^{\text {probit }}=0, y_{j}^{\text {select }}=0\right)$.
psel calculates the univariate (marginal) predicted probability of selection $\operatorname{Pr}\left(y_{j}^{\text {select }}=1\right)$.
pcond calculates the conditional (on selection) predicted probability of success

$$
\operatorname{Pr}\left(y_{j}^{\text {probit }}=1, y_{j}^{\text {select }}=1\right) / \operatorname{Pr}\left(y_{j}^{\text {select }}=1\right)
$$

xb calculates the probit linear prediction $\mathbf{x}_{j} \mathbf{b}$.
stdp calculates the standard error of the prediction, which can be thought of as the standard error of the predicted expected value or mean for the observation's covariate pattern. The standard error of the prediction is also referred to as the standard error of the fitted value.
xbsel calculates the linear prediction for the selection equation.
stdpsel calculates the standard error of the linear prediction for the selection equation.
nooffset is relevant only if you specified offset(varname) for heckprobit. It modifies the calculations made by predict so that they ignore the offset variable; the linear prediction is treated as $\mathbf{x}_{j} \mathbf{b}$ rather than as $\mathbf{x}_{j} \mathbf{b}+$ offset $_{j}$.
scores calculates equation-level score variables.
The first new variable will contain $\partial \ln L / \partial\left(\mathbf{x}_{j} \boldsymbol{\beta}\right)$.
The second new variable will contain $\partial \ln L / \partial\left(\mathbf{z}_{j} \gamma\right)$.
The third new variable will contain $\partial \ln L / \partial(\operatorname{atanh} \rho)$.

## Remarks and examples

## > Example 1

It is instructive to compare the marginal predicted probabilities with the predicted probabilities that we would obtain by ignoring the selection mechanism. To compare the two approaches, we will synthesize data so that we know the "true" predicted probabilities.

First, we need to generate correlated error terms, which we can do using a standard Cholesky decomposition approach. For our example, we will clear any data from memory and then generate errors that have a correlation of 0.5 by using the following commands. We set the seed so that interested readers can type in these same commands and obtain the same results.

```
. set seed 12309
. set obs 5000
obs was 0, now 5000
. gen c1 = rnormal()
. gen c2 = rnormal()
. matrix P = (1,.5\.5,1)
. matrix A = cholesky(P)
. local fac1 = A[2,1]
. local fac2 = A[2,2]
. gen u1 = c1
. gen u2 = 'fac1'*c1 + 'fac2'*c2
```

We can check that the errors have the correct correlation by using the correlate command. We will also normalize the errors so that they have a standard deviation of one, so we can generate a bivariate probit model with known coefficients. We do that with the following commands:

```
. correlate u1 u2
(obs=5000)
\begin{tabular}{r|rr} 
& u1 & u2 \\
\hline u1 & 1.0000 & \\
u2 & 0.5020 & 1.0000
\end{tabular}
. summarize u1
    (output omitted)
. replace u1 = u1/r(sd)
(5000 real changes made)
. summarize u2
    (output omitted)
. replace u2 = u2/r(sd)
(5000 real changes made)
. drop c1 c2
. gen x1 = runiform()-.5
. gen x2 = runiform()+1/3
. gen y1s = 0.5 + 4*x1 + u1
. gen y2s = 3 - 3*x2 + .5*x1 + u2
. gen y1 = (y1s>0)
. gen y2 = (y2s>0)
```

We have now created two dependent variables, y1 and y2, which are defined by our specified coefficients. We also included error terms for each equation, and the error terms are correlated. We run heckprobit to verify that the data have been correctly generated according to the model

$$
\begin{aligned}
& y_{1}=.5+4 x_{1}+u_{1} \\
& y_{2}=3+.5 x_{1}-3 x_{2}+u_{2}
\end{aligned}
$$

where we assume that $y_{1}$ is observed only if $y_{2}=1$.

| Probit model with sample selection | Number of obs | = | 5000 |
| :---: | :---: | :---: | :---: |
|  | Censored obs | = | 1762 |
|  | Uncensored obs | = | 3238 |
|  | Wald chi2(1) | = | 953.71 |
| Log likelihood = -3679.5 | Prob > chi2 | = | 0.0000 |


|  | Coef. | Std. Err. | z | $P>\|z\|$ | [95\% Conf | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| y1 |  |  |  |  |  |  |
| x1 | 3.784705 | . 1225532 | 30.88 | 0.000 | 3.544505 | 4.024905 |
| _cons | . 4630922 | . 0453952 | 10.20 | 0.000 | . 3741192 | . 5520653 |
| y2 |  |  |  |  |  |  |
| x 1 | . 3693052 | . 0721694 | 5.12 | 0.000 | . 2278558 | . 5107547 |
| x2 | -3.05069 | . 0832424 | -36.65 | 0.000 | -3.213842 | -2.887538 |
| _cons | 3.037696 | . 0777733 | 39.06 | 0.000 | 2.885263 | 3.190128 |
| /athrho | . 5186232 | . 083546 | 6.21 | 0.000 | . 354876 | . 6823705 |
| rho | . 4766367 | . 0645658 |  |  | . 3406927 | . 5930583 |
| LR test of indep. eqns. (rho = 0) : chi2 (1) = 40.43 Prob > chi2 $=0.0000$ |  |  |  |  |  |  |

Now that we have verified that we have generated data according to a known model, we can obtain and then compare predicted probabilities from the probit model with sample selection and a (usual) probit model.

```
. predict pmarg
(option pmargin assumed; Pr(y1=1))
. probit y1 x1 if y2==1
    (output omitted)
. predict phat
(option pr assumed; Pr(y1))
```

Using the (marginal) predicted probabilities from the probit model with sample selection (pmarg) and the predicted probabilities from the (usual) probit model (phat), we can also generate the "true" predicted probabilities from the synthesized y1s variable and then compare the predicted probabilities:

- gen ptrue $=$ normal (y1s)
. summarize pmarg ptrue phat

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| pmarg | 5000 | .6071226 | .3147861 | .0766334 | .9907113 |
| ptrue | 5000 | .5974195 | .348396 | $5.53 \mathrm{e}-06$ | .9999999 |
| phat | 5000 | .6568175 | .3025085 | .1059824 | .9954919 |

Here we see that ignoring the selection mechanism (comparing the phat variable with the true ptrue variable) results in predicted probabilities that are much higher than the true values. Looking at the marginal predicted probabilities from the model with sample selection, however, results in more accurate predictions.

## Also see

[R] heckprobit - Probit model with sample selection
[U] 20 Estimation and postestimation commands

## Title

help — Display help in Stata

| Syntax | Menu | Description |
| :--- | :--- | :--- |
| Remarks and examples | Also see |  |

## Syntax

help [command_or_topic_name] [, nonew name(viewername) marker(markername)]

## Menu

Help $>$ Stata Command...

## Description

The help command displays help information about the specified command or topic.
Stata for Mac, Stata for Unix(GUI), and Stata for Windows:
help launches a new Viewer to display help for the specified command or topic. If help is not followed by a command or a topic name, Stata launches the Viewer and displays help help_advice, advice for using the help system and documentation.

Help may be accessed either by selecting Help > Stata Command... and filling in the desired command name or by typing help followed by a command or topic name.
Stata for Unix(console):
Typing help followed by a command name or a topic name will display help on the console.
If help is not followed by a command or a topic name, a description of how to use the help system is displayed.

## Options

nonew specifies that a new Viewer window not be opened for the help topic if a Viewer window is already open. The default is for a new Viewer window to be opened each time help is typed so that multiple help files may be viewed at once. nonew causes the help file to be displayed in the topmost open Viewer.
name (viewername) specifies that help be displayed in a Viewer window named viewername. If the named window already exists, its contents will be replaced. If the named window does not exist, it will be created.
marker (markername) specifies that the help file be opened to the position of markername within the help file.

## Remarks and examples

To obtain help for any Stata command, type help command or select Help > Stata Command... and fill in command.
help is best explained by examples.

| To obtain help for ... | type |
| :--- | :--- |
| regress | help regress |
| postestimation tools for regress | help regress postestimation or |
|  | help regress post |
| graph option xlabel() | help graph xlabel() |
| Stata function strpos() | help strpos() |
| Mata function optimize() | help mata optimize() |

Tips:

- help displays advice for using the help system and documentation.
- help guide displays a table of contents for basic Stata concepts.
- help estimation commands displays an alphabetical listing of all Stata estimation commands.
- help functions displays help on Stata functions by category.
- help mata functions displays a subject table of contents for Mata's functions.
- help ts glossary displays the glossary for the time-series manual, and similarly for the other Stata specialty manuals.

If you type help topic and help for topic is not found, Stata will automatically perform a search for topic.

For instance, try typing help forecasting. A forecasting help file is not found, so Stata executes search forecasting and displays the results in the Viewer.

See [U] 4 Stata's help and search facilities for a complete description of how to use help.

## - Technical note

When you type help topic, Stata first looks along the adopath for topic.sthlp; see [U] 17.5 Where does Stata look for ado-files?.

## Video examples

Quick help in Stata

## Also see <br> $[R]$ net search - Search the Internet for installable packages <br> [R] search - Search Stata documentation and other resources <br> [GSM] 4 Getting help <br> [GSW] 4 Getting help <br> [GSU] 4 Getting help <br> [U] 4 Stata's help and search facilities

## Title

hetprobit - Heteroskedastic probit model
Syntax
Remarks and examples
Menu
Stored results

Description
Methods and formulas

Options
References

## Syntax

```
hetprobit depvar [indepvars] [if] [in] [weight],
\(\operatorname{het}\left(\right.\) varlist \(\left[\right.\), offset \(\left(\right.\) varname \(\left.\left.\left._{o}\right)\right]\right)[\) options \(]\)
```

options
Description

Model

* het (varlist [...])
noconstant
offset (varname)
asis
constraints (constraints)
collinear
SE/Robust
vce (vcetype)

Reporting
level(\#)
noskip
nolrtest
nocnsreport
display_options

## Maximization

maximize_options
coeflegend
independent variables to model the variance and possible offset variable
suppress constant term
include varname in model with coefficient constrained to 1
retain perfect predictor variables
apply specified linear constraints
keep collinear variables
vcetype may be oim, robust, cluster clustvar, opg, bootstrap, or jackknife
set confidence level; default is level (95)
perform likelihood-ratio test
perform Wald test on variance
do not display constraints
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
${ }^{*} \operatorname{het}()$ is required. The full specification is het (varlist $\left[\right.$, offset (varname ${ }_{O}$ )]). indepvars and varlist may contain factor variables; see [U] 11.4.3 Factor variables.
depvar, indepvars, and varlist may contain time-series operators; see [U] 11.4.4 Time-series varlists.
bootstrap, by, fp, jackknife, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the bootstrap prefix; see $[R]$ bootstrap.
vce(), noskip, and weights are not allowed with the svy prefix; see [SVY] svy.
fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Binary outcomes $>$ Heteroskedastic probit regression

## Description

hetprobit fits a maximum-likelihood heteroskedastic probit model.
hetprob is a synonym for hetprobit.
See [R] logistic for a list of related estimation commands.

## Options



SE/Robust
vce (vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

## Reporting

level(\#); see [R] estimation options.
noskip requests fitting of the constant-only model and calculation of the corresponding likelihood-ratio $\chi^{2}$ statistic for testing significance of the full model. By default, a Wald $\chi^{2}$ statistic is computed for testing the significance of the full model.
nolrtest specifies that a Wald test of whether lnsigma2 $=0$ be performed instead of the LR test. nocnsreport; see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% fmt), pformat (\% fmt), sformat (\% fmt), and nolstretch; see [R] estimation options.

Maximization
maximize_options: difficult, technique (algorithm_spec), iterate(\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance(\#), nonrtolerance, and from (init_specs); see [R] maximize. These options are seldom used.

Setting the optimization type to technique (bhhh) resets the default vcetype to vce (opg).

The following option is available with hetprobit but is not shown in the dialog box: coeflegend; see $[R]$ estimation options.

## Remarks and examples

Remarks are presented under the following headings:

## Introduction

Robust standard errors

## Introduction

hetprobit fits a maximum-likelihood heteroskedastic probit model, which is a generalization of the probit model. Let $y_{j}, j=1, \ldots, N$, be a binary outcome variable taking on the value 0 (failure) or 1 (success). In the probit model, the probability that $y_{j}$ takes on the value 1 is modeled as a nonlinear function of a linear combination of the $k$ independent variables $\mathbf{x}_{j}=\left(x_{1 j}, x_{2 j}, \ldots, x_{k j}\right)$,

$$
\operatorname{Pr}\left(y_{j}=1\right)=\Phi\left(\mathbf{x}_{j} \mathbf{b}\right)
$$

in which $\Phi()$ is the cumulative distribution function (CDF) of a standard normal random variable, that is, a normally distributed (Gaussian) random variable with mean 0 and variance 1. The linear combination of the independent variables, $\mathbf{x}_{j} \mathbf{b}$, is commonly called the index function, or index. Heteroskedastic probit generalizes the probit model by generalizing $\Phi()$ to a normal CDF with a variance that is no longer fixed at 1 but can vary as a function of the independent variables. hetprobit models the variance as a multiplicative function of these $m$ variables $\mathbf{z}_{j}=\left(z_{1 j}, z_{2 j}, \ldots, z_{m j}\right)$, following Harvey (1976):

$$
\sigma_{j}^{2}=\left\{\exp \left(\mathbf{z}_{j} \boldsymbol{\gamma}\right)\right\}^{2}
$$

Thus the probability of success as a function of all the independent variables is

$$
\operatorname{Pr}\left(y_{j}=1\right)=\Phi\left\{\mathbf{x}_{j} \mathbf{b} / \exp \left(\mathbf{z}_{j} \boldsymbol{\gamma}\right)\right\}
$$

From this expression, it is clear that, unlike the index $\mathbf{x}_{j} \mathbf{b}$, no constant term can be present in $\mathbf{z}_{j} \gamma$ if the model is to be identifiable.

Suppose that the binary outcomes $y_{j}$ are generated by thresholding an unobserved random variable, $w$, which is normally distributed with mean $\mathbf{x}_{j} \mathbf{b}$ and variance 1 such that

$$
y_{j}= \begin{cases}1 & \text { if } w_{j}>0 \\ 0 & \text { if } w_{j} \leq 0\end{cases}
$$

This process gives the probit model:

$$
\operatorname{Pr}\left(y_{j}=1\right)=\operatorname{Pr}\left(w_{j}>0\right)=\Phi\left(\mathbf{x}_{j} \mathbf{b}\right)
$$

Now suppose that the unobserved $w_{j}$ are heteroskedastic with variance

$$
\sigma_{j}^{2}=\left\{\exp \left(\mathbf{z}_{j} \gamma\right)\right\}^{2}
$$

Relaxing the homoskedastic assumption of the probit model in this manner yields our multiplicative heteroskedastic probit model:

$$
\operatorname{Pr}\left(y_{j}=1\right)=\Phi\left\{\mathbf{x}_{j} \mathbf{b} / \exp \left(\mathbf{z}_{j} \boldsymbol{\gamma}\right)\right\}
$$

## > Example 1

For this example, we generate simulated data for a simple heteroskedastic probit model and then estimate the coefficients with hetprobit:

```
. set obs 1000
obs was 0, now 1000
. set seed 1234567
. gen x = 1-2*runiform()
. gen xhet = runiform()
. gen sigma = exp(1.5*xhet)
. gen p = normal((0.3+2*x)/sigma)
. gen y = cond(runiform()<=p,1,0)
. hetprob y x, het(xhet)
Fitting probit model:
Iteration 0: log likelihood = -688.53208
Iteration 1: log likelihood = -591.59895
Iteration 2: log likelihood = -591.50674
Iteration 3: log likelihood = -591.50674
Fitting full model:
Iteration 0: log likelihood = -591.50674
Iteration 1: log likelihood = -572.12219
Iteration 2: log likelihood = -570.7742
Iteration 3: log likelihood = -569.48921
Iteration 4: log likelihood = -569.47828
Iteration 5: log likelihood = -569.47827
```

| Heteroskedastic probit model | Number of obs | $=$ | 1000 |
| :--- | :--- | :--- | ---: |
|  | Zero outcomes | $=$ | 452 |
|  | Nonzero outcomes | $=$ | 548 |
| Log likelihood = -569.4783 | Wald chi2 (1) | $=$ | 78.66 |
|  | Prob $>$ chi2 | $=$ | 0.0000 |


| y | Coef. | Std. Err. | z | $P>\|z\|$ | [95\% Con | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| y |  |  |  |  |  |  |
| x | 2.228031 | . 2512073 | 8.87 | 0.000 | 1.735673 | 2.720388 |
| _cons | . 2493822 | . 0862833 | 2.89 | 0.004 | . 08027 | . 4184943 |
| lnsigma2 |  |  |  |  |  |  |
| xhet | 1.602537 | . 2640131 | 6.07 | 0.000 | 1.085081 | 2.119993 |

Likelihood-ratio test of lnsigma2=0: chi2(1) $=44.06$ Prob > chi2 $=0.0000$
Above we created two variables, x and xhet, and then simulated the model

$$
\operatorname{Pr}(\mathrm{y}=1)=F\left\{\left(\beta_{0}+\beta_{1} \mathrm{x}\right) / \exp \left(\gamma_{1} \mathrm{xhet}\right)\right\}
$$

for $\beta_{0}=0.3, \beta_{1}=2$, and $\gamma_{1}=1.5$. According to hetprobit's output, all coefficients are significant, and, as we would expect, the Wald test of the full model versus the constant-only model-for example, the index consisting of $\beta_{0}+\beta_{1} \mathrm{x}$ versus that of just $\beta_{0}$-is significant with $\chi^{2}(1)=79$. Likewise, the likelihood-ratio test of heteroskedasticity, which tests the full model with heteroskedasticity against the full model without, is significant with $\chi^{2}(1)=44$. See $[R]$ maximize for more explanation of the output. For this simple model, hetprobit took five iterations to converge. As stated elsewhere (Greene 2012, 714), this is a difficult model to fit, and it is not uncommon for it to require many iterations or for the optimizer to print out warnings and informative messages during the optimization. Slow convergence is especially common for models in which one or more of the independent variables appear in both the index and variance functions.

## - Technical note

Stata interprets a value of 0 as a negative outcome (failure) and treats all other values (except missing) as positive outcomes (successes). Thus if your dependent variable takes on the values 0 and 1 , then 0 is interpreted as failure and 1 as success. If your dependent variable takes on the values 0 , 1 , and 2 , then 0 is still interpreted as failure, but both 1 and 2 are treated as successes.

## Robust standard errors

If you specify the vce(robust) option, hetprobit reports robust standard errors as described in [U] 20.21 Obtaining robust variance estimates. To illustrate the effect of this option, we will reestimate our coefficients by using the same model and data in our example, this time adding vce(robust) to our hetprobit command.
> Example 2


The vce(robust) standard errors for two of the three parameters are larger than the previously reported conventional standard errors. This is to be expected, even though (by construction) we have perfect model specification because this option trades off efficient estimation of the coefficient variance-covariance matrix for robustness against misspecification.

Specifying the vce(cluster clustvar) option relaxes the usual assumption of independence between observations to the weaker assumption of independence just between clusters; that is, hetprobit, vce(cluster clustvar) is robust with respect to within-cluster correlation. This option is less efficient than the xtgee population-averaged models because hetprobit inefficiently sums within cluster for the standard-error calculation rather than attempting to exploit what might be assumed about the within-cluster correlation.

## Stored results

hetprobit stores the following in e():
Scalars
e(N)
e(N_f)
e(N_s)
e(k)
e(k_eq)
e(k_eq_model)
e(k_dv)
e(df_m)
e(11)
e(ll_0)
e(ll_c)
e(N_clust)
e(chi2)
e(chi2_c)
e(p_c)
e(df_m_c)
$\mathrm{e}(\mathrm{p})$
e(rank)
e(rank0)
e(ic)
e(rc)
e(converged)
Macros
e(cmd)
e(cmdline)
e(depvar)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(offset1)
e(offset2)
e(chi2type)
e(chi2_ct)
e(vce)
e(vcetype)
e(opt)
e(which)
$e$ (method)
e(ml_method)
e(user)
e(technique)
e(properties)
e(predict)
e(asbalanced)
e(asobserved)
Matrices
e(b)
e(Cns)
e(ilog)
e(gradient)
e(V)
e(V_modelbased)
Functions
e(sample)
number of observations
number of zero outcomes
number of nonzero outcomes
number of parameters
number of equations in $e(b)$
number of equations in overall model test
number of dependent variables
model degrees of freedom
log likelihood
log likelihood, constant-only model
log likelihood, comparison model
number of clusters
$\chi^{2}$
$\chi^{2}$ for heteroskedasticity LR test
$p$-value for heteroskedasticity LR test
degrees of freedom for heteroskedasticity LR test
significance
rank of e(V)
rank of $e(V)$ for constant-only model
number of iterations
return code
1 if converged, 0 otherwise
hetprobit
command as typed
name of dependent variable
weight type
weight expression
title in estimation output
name of cluster variable
offset for probit equation
offset for variance equation
Wald or LR; type of model $\chi^{2}$ test
Wald or LR; type of model $\chi^{2}$ test corresponding to e(chi2_c)
vcetype specified in vce()
title used to label Std. Err.
type of optimization
max or min; whether optimizer is to perform maximization or minimization
requested estimation method
type of ml method
name of likelihood-evaluator
maximization technique
b V
program used to implement predict
factor variables fvset as asbalanced
factor variables fvset as asobserved
coefficient vector
constraints matrix
iteration $\log$ (up to 20 iterations)
gradient vector
variance-covariance matrix of the estimators
model-based variance
marks estimation sample

## Methods and formulas

The heteroskedastic probit model is a generalization of the probit model because it allows the scale of the inverse link function to vary from observation to observation as a function of the independent variables.

The log-likelihood function for the heteroskedastic probit model is

$$
\ln L=\sum_{j \in S} w_{j} \ln \Phi\left\{\mathbf{x}_{j} \boldsymbol{\beta} / \exp (\mathbf{z} \boldsymbol{\gamma})\right\}+\sum_{j \notin S} w_{j} \ln \left[1-\Phi\left\{\mathbf{x}_{j} \boldsymbol{\beta} / \exp (\mathbf{z} \boldsymbol{\gamma})\right\}\right]
$$

where $S$ is the set of all observations $j$ such that $y_{j} \neq 0$ and $w_{j}$ denotes the optional weights. $\ln L$ is maximized as described in [ R ] maximize.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce (robust) and vce (cluster clustvar), respectively. See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas.
hetprobit also supports estimation with survey data. For details on VCEs with survey data, see [SVY] variance estimation.

## References

Blevins, J. R., and S. Khan. 2013. Distribution-free estimation of heteroskedastic binary response models in Stata. Stata Journal 13: 588-602.

Greene, W. H. 2012. Econometric Analysis. 7th ed. Upper Saddle River, NJ: Prentice Hall.
Harvey, A. C. 1976. Estimating regression models with multiplicative heteroscedasticity. Econometrica 44: 461-465.

## Also see

[R] hetprobit postestimation - Postestimation tools for hetprobit
[R] logistic - Logistic regression, reporting odds ratios
[R] probit - Probit regression
[SVY] svy estimation - Estimation commands for survey data
[XT] xtprobit - Random-effects and population-averaged probit models
[U] 20 Estimation and postestimation commands

## Title

hetprobit postestimation - Postestimation tools for hetprobit

Description Syntax for predict Menu for predict Options for predict Remarks and examples Also see

## Description

The following postestimation commands are available after hetprobit:

| Command | Description |
| :--- | :--- |
| contrast | contrasts and ANOVA-style joint tests of estimates |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample |
| estat vce | variance-covariance matrix of the estimators (VCE) |
| estat (svy) | postestimation statistics for survey data <br> estaloging estimation results |
| forecast |  |$\quad$| dynamic forecasts and simulations |
| :--- |
| lincom |$\quad$| point estimates, standard errors, testing, and inference for linear combinations |
| :--- |
| of coefficients |

[^28]
## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic nooffset]
predict [type] {stub*|newvarreg newvar lnsigma2} [if] [in], scores
```

statistic Description

Main
pr probability of a positive outcome; the default
$\overline{\mathrm{xb}} \quad$ linear prediction
sigma standard deviation of the error term
These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
pr , the default, calculates the probability of a positive outcome.
xb calculates the linear prediction.
sigma calculates the standard deviation of the error term.
nooffset is relevant only if you specified offset (varname) for hetprobit. It modifies the calculations made by predict so that they ignore the offset variable; the linear prediction is treated as $\mathbf{x}_{j} \mathbf{b}$ rather than as $\mathbf{x}_{j} \mathbf{b}+$ offset $_{j}$.
scores calculates equation-level score variables.
The first new variable will contain $\partial \ln L / \partial\left(\mathbf{x}_{j} \boldsymbol{\beta}\right)$.
The second new variable will contain $\partial \ln L / \partial\left(\mathbf{z}_{j} \gamma\right)$.

## Remarks and examples

Once you have fit a model, you can use the predict command to obtain the predicted probabilities for both the estimation sample and other samples; see [U] 20 Estimation and postestimation commands and $[R]$ predict. predict without arguments calculates the predicted probability of a positive outcome. With the $\mathbf{x b}$ option, predict calculates the index function combination, $\mathbf{x}_{j} \mathbf{b}$, where $\mathbf{x}_{j}$ are the independent variables in the $j$ th observation and $\mathbf{b}$ is the estimated parameter vector. With the sigma option, predict calculates the predicted standard deviation, $\sigma_{j}=\exp \left(\mathbf{z}_{j} \boldsymbol{\gamma}\right)$.
$>$ Example 1
We use predict to compute the predicted probabilities and standard deviations based on the model in example 2 in $[\mathrm{R}]$ hetprobit to compare these with the actual values:

```
. predict phat
(option pr assumed; Pr(y))
. gen diff_p = phat - p
. summarize diff_p
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline diff_p & 1000 & -.0107081 & .0131869 & -.0466331 & .010482
\end{tabular}
. predict sigmahat, sigma
. gen diff_s = sigmahat - sigma
. summarize diff_s
\begin{tabular}{r|rrrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline diff_s & 1000 & .1558882 & .1363698 & .0000417 & .4819107
\end{tabular}
```


## Also see

[R] hetprobit - Heteroskedastic probit model
[U] 20 Estimation and postestimation commands

## Title

histogram - Histograms for continuous and categorical variables

Syntax Menu
Description
Options for use in the discrete case
Remarks and examples
Also see
Options for use in the continuous case
Options for use in the continuous and discrete cases
References

## Syntax

histogram varname $[$ if $][$ in $][$ weight $][,[$ continuous_opts $\mid$ discrete_opts $]$ options $]$
continuous_opts
Description

Main

| bin(\#) | set number of bins to \# |
| :--- | :--- |
| width(\#) | set width of bins to \# |
| start(\#) | set lower limit of first bin to \# |

discrete_opts
Description
Main
discrete
specify that data are discrete
width(\#)
set width of bins to \#
start(\#)
set theoretical minimum value to \#
options
Main
density
fraction
frequency
percent
bar_options
addlabels
addlabopts(marker_label_options)
Density plots
normal
normopts (line_options)
kdensity
kdenopts(kdensity_options)
Add plots
addplot (plot)
Y axis, X axis, Titles, Legend, Overall, By
twoway_options

Description
draw as density; the default
draw as fractions
draw as frequencies
draw as percentages
rendition of bars
add height labels to bars
affect rendition of labels
add a normal density to the graph affect rendition of normal density add a kernel density estimate to the graph affect rendition of kernel density
add other plots to the histogram
any options documented in [G-3] twoway_options
fweights are allowed; see [U] 11.1.6 weight.

## Menu

Graphics > Histogram

## Description

histogram draws histograms of varname, which is assumed to be the name of a continuous variable unless the discrete option is specified.

## Options for use in the continuous case

$\qquad$ Main
bin(\#) and width(\#) are alternatives. They specify how the data are to be aggregated into bins: bin() by specifying the number of bins (from which the width can be derived) and width() by specifying the bin width (from which the number of bins can be derived).
If neither option is specified, results are the same as if $\operatorname{bin}(k)$ had been specified, where

$$
k=\min \{\operatorname{sqrt}(N), 10 \ln (N) / \ln (10)\}
$$

and where $N$ is the (weighted) number of observations.
start(\#) specifies the theoretical minimum of varname. The default is start ( $m$ ), where $m$ is the observed minimum value of varname.

Specify start () when you are concerned about sparse data, for instance, if you know that varname can have a value of 0 , but you are concerned that 0 may not be observed.
start(\#), if specified, must be less than or equal to $m$, or else an error will be issued.

## Options for use in the discrete case

$\qquad$
discrete specifies that varname is discrete and that you want each unique value of varname to have its own bin (bar of histogram).
width (\#) is rarely specified in the discrete case; it specifies the width of the bins. The default is width $(d)$, where $d$ is the observed minimum difference between the unique values of varname.
Specify width () if you are concerned that your data are sparse. For example, in theory varname could take on the values, say, $1,2,3, \ldots, 9$, but because of the sparseness, perhaps only the values $2,4,7$, and 8 are observed. Here the default width calculation would produce width(2), and you would want to specify width(1).
start (\#) is also rarely specified in the discrete case; it specifies the theoretical minimum value of varname. The default is start $(m)$, where $m$ is the observed minimum value.
As with width(), specify start (\#) if you are concerned that your data are sparse. In the previous example, you might also want to specify start(1). start() does nothing more than add white space to the left side of the graph.
The value of \# in start () must be less than or equal to $m$, or an error will be issued.

## Options for use in the continuous and discrete cases

## Main

density, fraction, frequency, and percent specify whether you want the histogram scaled to density units, fractional units, frequencies, or percentages. density is the default.
density scales the height of the bars so that the sum of their areas equals 1 .
fraction scales the height of the bars so that the sum of their heights equals 1 .
frequency scales the height of the bars so that each bar's height is equal to the number of observations in the category. Thus the sum of the heights is equal to the total number of observations.
percent scales the height of the bars so that the sum of their heights equals 100 .
bar_options are any of the options allowed by graph twoway bar; see [G-2] graph twoway bar.
One of the most useful bar_options is barwidth(\#), which specifies the width of the bars in varname units. By default, histogram draws the bars so that adjacent bars just touch. If you want gaps between the bars, do not specify histogram's width() option-which would change how the histogram is calculated—but specify the bar_option barwidth() or the histogram option gap, both of which affect only how the bar is rendered.
The bar_option horizontal cannot be used with the addlabels option.
addlabels specifies that the top of each bar be labeled with the density, fraction, or frequency, as determined by the density, fraction, and frequency options.
addlabopts(marker_label_options) specifies how to render the labels atop the bars. See [G-3] marker_label_options. Do not specify the marker_label_option mlabel(varname), which specifies the variable to be used; this is specified for you by histogram.
addlabopts() will accept more options than those documented in [G-3] marker_label_options. All options allowed by twoway scatter are also allowed by addlabopts(); see [G-2] graph twoway scatter. One particularly useful option is yvarformat (); see [G-3] advanced_options.

Density plots
normal specifies that the histogram be overlaid with an appropriately scaled normal density. The normal will have the same mean and standard deviation as the data.
normopts(line_options) specifies details about the rendition of the normal curve, such as the color and style of line used. See [G-2] graph twoway line.
kdensity specifies that the histogram be overlaid with an appropriately scaled kernel density estimate of the density. By default, the estimate will be produced using the Epanechnikov kernel with an "optimal" half-width. This default corresponds to the default of kdensity; see [R] kdensity. How the estimate is produced can be controlled using the kdenopts () option described below.
kdenopts(kdensity_options) specifies details about how the kernel density estimate is to be produced along with details about the rendition of the resulting curve, such as the color and style of line used. The kernel density estimate is described in [G-2] graph twoway kdensity. As an example, if you wanted to produce kernel density estimates by using the Gaussian kernel with optimal half-width, you would specify kdenopts (gauss) and if you also wanted a half-width of 5, you would specify kdenopts(gauss width(5)).
$\sqrt{\text { Add plots }}$
addplot (plot) allows adding more graph twoway plots to the graph; see [G-3] addplot_option.

Y axis, X axis, Titles, Legend, Overall, By
twoway_options are any of the options documented in [G-3] twoway_options. This includes, most importantly, options for titling the graph (see [G-3] title_options), options for saving the graph to disk (see [G-3] saving_option), and the by () option, which will allow you to simultaneously graph histograms for different subsets of the data (see [G-3] by_option).

## Remarks and examples

Remarks are presented under the following headings:

```
Histograms of continuous variables
Overlaying normal and kernel density estimates
Histograms of discrete variables
Use with by()
Video example
```

For an example of editing a histogram with the Graph Editor, see Pollock (2011, 29-31).

## Histograms of continuous variables

histogram assumes that the variable is continuous, so you need type only histogram followed by the variable name:

```
. use http://www.stata-press.com/data/r13/sp500
(S&P 500)
. histogram volume
(bin=15, start=4103, width=1280.3533)
```



The small values reported for density on the $y$ axis are correct; if you added up the area of the bars, you would get 1 . Nevertheless, many people are used to seeing histograms scaled so that the bar heights sum to 1 ,

```
. histogram volume, fraction
(bin=15, start=4103, width=1280.3533)
```


and others are used to seeing histograms so that the bar height reflects the number of observations,
. histogram volume, frequency
(bin=15, start=4103, width=1280.3533)


Regardless of the scale you prefer, you can specify other options to make the graph look more impressive:
summarize volume



For an explanation of the xaxis() option-it created the upper and lower $x$ axissee [G-3] axis_choice_options. For an explanation of the ylabel() and xlabel() options, see [G-3] axis_label_options. For an explanation of the subtitle() and note() options, see [G-3] title_options.

## Overlaying normal and kernel density estimates

Specifying normal will overlay a normal density over the histogram. It would be enough to type
. histogram volume, normal
but we will add the option to our more impressive rendition:
. summarize volume

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| volume | 248 | 12320.68 | 2585.929 | 4103 | 23308.3 |

```
histogram volume, freq normal
    xaxis(1 2)
    ylabel(0(10)60, grid)
    xlabel(12321 "mean"
            9735 "-1 s.d."
            14907 "+1 s.d."
            7149 "-2 s.d."
            17493 "+2 s.d."
            20078 "+3 s.d."
            22664 "+4 s.d."
                , axis(2) grid gmax)
    xtitle("", axis(2))
    subtitle("S&P 500, January 2001 - December 2001")
    note("Source: Yahoo! Finance and Commodity Systems, Inc.")
    start=4103, width=1280.3533)
```



If we instead wanted to overlay a kernel density estimate, we could specify kdensity in place of normal.

## Histograms of discrete variables

Specify histogram's discrete option when you wish to treat the data as discrete-when you wish each unique value of the variable to be assigned its own bin. For instance, in the automobile data, mpg is a continuous variable, but the mileage ratings have been measured to integer precision. If we were to type

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. histogram mpg
(bin=8, start=12, width=3.625)
```

mpg would be treated as continuous and categorized into eight bins by the default number-of-bins calculation, which is based on the number of observations, 74.

Adding the discrete option makes a histogram with a bin for each of the 21 unique values.

```
. histogram mpg, discrete
(start=12, width=1)
```



Just as in the continuous case, the $y$ axis was reported in density, and we could specify the fraction or frequency options if we wanted it to be reported differently. Below we specify frequency, we specify addlabels to add a report of frequencies printed above the bars, we specify ylabel (, grid) to add horizontal grid lines, and we specify xlabel(12(2)42) to label the values $12,14, \ldots, 42$ on the $x$ axis:

```
. histogram mpg, discrete freq addlabels ylabel(,grid) xlabel(12(2)42)
(start=12, width=1)
```



## Use with by()

histogram may be used with graph twoway's by (); for example,
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. histogram mpg, discrete by(foreign)


Here results would be easier to compare if the graphs were presented in one column:
. histogram mpg, discrete by(foreign, col(1))

col(1) is a by () suboption-see [G-3] by_option-and there are other useful suboptions, such as total, which will add an overall total histogram. total is a suboption of by (), not an option of histogram, so you would type
. histogram mpg, discrete by(foreign, total)
and not histogram mpg, discrete by (foreign) total.

As another example, Lipset (1993) reprinted data from the New York Times (November 5, 1992) collected by the Voter Research and Surveys based on questionnaires completed by 15,490 U.S. presidential voters from 300 polling places on election day in 1992.

```
. use http://www.stata-press.com/data/r13/voter
. histogram candi [freq=pop], discrete fraction by(inc, total)
> gap(40) xlabel(2 3 4, valuelabel)
```



We specified gap(40) to reduce the width of the bars by $40 \%$. We also used xlabel()'s valuelabel suboption, which caused our bars to be labeled "Clinton", "Bush", and "Perot", rather than 2, 3, and 4; see [G-3] axis_label_options.

## Video example

Histograms in Stata

## References

Cox, N. J. 2004. Speaking Stata: Graphing distributions. Stata Journal 4: 66-88.
-_. 2005. Speaking Stata: Density probability plots. Stata Journal 5: 259-273.
Harrison, D. A. 2005. Stata tip 20: Generating histogram bin variables. Stata Journal 5: 280-281.
Lipset, S. M. 1993. The significance of the 1992 election. PS: Political Science and Politics 26: 7-16.
Pollock, P. H., III. 2011. A Stata Companion to Political Analysis. 2nd ed. Washington, DC: CQ Press.

## Also see

[ R ] kdensity - Univariate kernel density estimation
$[R]$ spikeplot - Spike plots and rootograms
[G-2] graph twoway histogram - Histogram plots

## Title

icc - Intraclass correlation coefficients

Syntax<br>Description<br>Options for two-way RE and ME models<br>Stored results<br>References

Menu
Options for one-way RE model
Remarks and examples
Methods and formulas
Also see

## Syntax

Calculate intraclass correlations for one-way random-effects model icc depvar target $[$ if $][$ in $]$ [, oneway_options $]$

Calculate intraclass correlations for two-way random-effects model icc depvar target rater [if] [in] [, twoway_re_options]

Calculate intraclass correlations for two-way mixed-effects model
icc depvar target rater $[$ if $]$ [in], mixed [twoway_me_options]
oneway_options Description
Main
absolute estimate absolute agreement; the default
testvalue(\#) test whether intraclass correlations equal \#; default is testvalue (0)

Reporting
level(\#)
set confidence level; default is level (95)
format (\% $\%$ fmt) display format for statistics and confidence intervals; default is format ( $\% 9.0 \mathrm{~g}$ )
twoway_re_options
Main
absolute
consistency
testvalue(\#)

Reporting
level(\#)
format (\% fmt)
estimate absolute agreement; the default estimate consistency of agreement
test whether intraclass correlations equal \#; default is testvalue(0)
set confidence level; default is level (95)
display format for statistics and confidence intervals; default is format (\%9.0g)

Main

* mixed
consistency
absolute
testvalue(\#)

Reporting
level (\#)
format ( $\%$ fmt)
estimate intraclass correlations for a mixed-effects model
estimate consistency of agreement; the default
estimate absolute agreement
test whether intraclass correlations equal \#; default is testvalue(0)
set confidence level; default is level (95)
display format for statistics and confidence intervals; default is format ( $\% 9.0 \mathrm{~g}$ )

* mixed is required.
bootstrap, by, jackknife, and statsby are allowed; see [U] 11.1.10 Prefix commands.


## Menu

Statistics $>$ Summaries, tables, and tests $>$ Summary and descriptive statistics $>$ Intraclass correlations

## Description

icc estimates intraclass correlations for one-way random-effects models, two-way random-effects models, or two-way mixed-effects models for both individual and average measurements. Intraclass correlations measuring consistency of agreement or absolute agreement of the measurements may be estimated.

## Options for one-way RE model

$\qquad$ Main
absolute specifies that intraclass correlations measuring absolute agreement of the measurements be estimated. This is the default for random-effects models.
testvalue(\#) tests whether intraclass correlations equal \#. The default is testvalue(0).

## Reporting

level(\#) specifies the confidence level, as a percentage, for confidence intervals. The default is level (95) or as set by set level; see [R] level.
format (\% fmt) specifies how the intraclass correlation estimates and confidence intervals are to be formatted. The default is format $(\% 9.0 \mathrm{~g})$.

## Options for two-way RE and ME models

$\qquad$ Main
mixed is required to calculate two-way mixed-effects models. mixed specifies that intraclass correlations for a mixed-effects model be estimated.
absolute specifies that intraclass correlations measuring absolute agreement of the measurements be estimated. This is the default for random-effects models. Only one of absolute or consistency may be specified.
consistency specifies that intraclass correlations measuring consistency of agreement of the measurements be estimated. This is the default for mixed-effects models. Only one of absolute or consistency may be specified.
testvalue(\#) tests whether intraclass correlations equal \#. The default is testvalue(0).

## Reporting

level(\#) specifies the confidence level, as a percentage, for confidence intervals. The default is level (95) or as set by set level; see [R] level.
format ( $\%$ fmt ) specifies how the intraclass correlation estimates and confidence intervals are to be formatted. The default is format $(\% 9.0 \mathrm{~g})$.

## Remarks and examples

Remarks are presented under the following headings:

## Introduction

One-way random effects
Two-way random effects
Two-way mixed effects
Adoption study
Relationship between ICCs
Tests against nonzero values

## Introduction

In some disciplines, such as psychology and sociology, data are often measured with error that can seriously affect statistical interpretation of the results. Thus it is important to assess the amount of measurement error by evaluating the consistency or reliability of measurements. The intraclass correlation coefficient (ICC) is often used to measure the consistency or homogeneity of measurements.

Several versions of ICCs are introduced in the literature depending on the experimental design and goals of the study (see, for example, Shrout and Fleiss [1979] and McGraw and Wong [1996a]). Following Shrout and Fleiss (1979), we describe various forms of ICCs in the context of a reliability study of ratings of different targets (or objects of measurements) by several raters.

Consider $n$ targets (for example, students, patients, athletes) that are randomly sampled from a population of interest. Each target is rated independently by a set of $k$ raters (for example, teachers, doctors, judges). One rating per target and rater is obtained. It is of interest to determine the extent of the agreement of the ratings.

As noted by Shrout and Fleiss (1979) and McGraw and Wong (1996a), you need to answer several questions to decide what version of ICC is appropriate to measure the agreement in your study:

1. Is a one-way or two-way analysis-of-variance model appropriate for your study?
2. Are differences between raters' mean ratings relevant to the reliability of interest?
3. Is the unit of analysis an individual rating or the mean rating over several raters?
4. Is the consistency of agreement or the absolute agreement of ratings of interest?

Three types of analysis-of-variance models are considered for the reliability study: one-way random effects, two-way random effects, and two-way mixed effects. Mixed models contain both fixed effects and random effects. In the one-way random-effects model, each target is rated by a different set of $k$ independent raters, who are randomly drawn from the population of raters. The target is the only random effect in this model; the effects due to raters and possibly due to rater-and-target interaction cannot be separated from random error. In the two-way random-effects model, each target is rated by the same set of $k$ independent raters, who are randomly drawn from the population of raters. The random effects in this model are target and rater and possibly their interaction, although in the absence of repeated measurements for each rater on each target, the effect of an interaction cannot be separated from random error. In the two-way mixed-effects model, each target is rated by the same set of $k$ independent raters. Because they are the only raters of interest, rater is a fixed effect. The random effects are target and possibly target-and-rater interaction, but again the interaction effect cannot be separated from random error without repeated measurements for each rater and target. The definition of ICC depends on the chosen random-effects model; see Methods and formulas for details.

In summary, use a one-way model if there are no systematic differences in measurements due to raters and use a two-way model otherwise. If you want to generalize your results to a population of raters from which the observed raters are sampled, use a two-way random-effects model, treating raters as random. If you are only interested in the effects of the observed $k$ raters, use a two-way mixed-effects model, treating raters as fixed. For example, suppose you compare judges' ratings of targets from different groups. If you use the combined data from $k$ judges to compare the groups, the random-effects model is appropriate. If you compare groups separately for each judge and then pool the differences, the mixed-effects model is appropriate.

The definition of ICC also depends on the unit of analysis in a study-whether the agreement is measured between individual ratings (individual ICC) or between the averages of ratings over several raters (average ICC). The data on individual ratings are more common. The data on average ratings are typically used when individual ratings are deemed unreliable. The average ICC can also be used when teams of raters are used to rate a target. For example, the ratings of teams of physicians may be evaluated in this manner. When the unit of analysis is an average rating, you should remember that the interpretation of ICC pertains to average ratings and not individual ratings.

Finally, depending on whether consistency of agreement or absolute agreement is of interest, two types of ICC are used: consistency-of-agreement ICC (CA-ICC) and absolute-agreement ICC (AA-ICC). Under consistency of agreement, the scores are considered consistent if the scores from any two raters differ by the same constant value for all targets. This implies that raters give the same ranking to all targets. Under absolute agreement, the scores are considered in absolute agreement if the scores from all raters match exactly.

For example, suppose we observe three targets and two raters. The ratings are $(2,4),(4,6)$, and $(6,8)$, with rater 1 giving the scores $(2,4,6)$ and rater 2 giving the scores $(4,6,8)$, two points higher than rater 1. The CA-ICC between individual ratings is 1 because the scores from rater 1 and rater 2 differ by a constant value (two points) for all targets. That rater 1 gives lower scores than rater 2 is deemed irrelevant under the consistency measure of agreement. The raters have the same difference of opinion on every target, and the variation between raters that is caused by this difference is not relevant. On the other hand, the AA-ICC between individual ratings is $8 / 12=0.67$, where 8 is the estimated between-target variance and 12 is the estimated total variance of ratings.

Either CA-ICC or AA-ICC can serve as a useful measure of agreement depending on whether rater variability is relevant for determining the degree of agreement. As McGraw and Wong (1996a) point out, CA-ICC is useful when comparative judgments are made about objects of measurement. The CA-ICC represents correlation when the rater is fixed; the AA-ICC represents correlation when the rater is random.

See Shrout and Fleiss (1979) and McGraw and Wong (1996a) for more detailed guidelines about the choice of appropriate ICC.

Shrout and Fleiss (1979) and McGraw and Wong (1996a) describe 10 versions of ICCs based on the concepts above: individual and average AA-ICCs for a one-way model (consistency of agreement is not defined for this model); individual and average AA-ICCs and CA-ICCs for a two-way random-effects model; and individual and average AA-ICCs and CA-ICCs for a two-way mixed-effects model. Although each of these ICCs has its own definition and interpretation, the estimators for some are identical, leading to the same estimates of those ICCs; see Relationship between ICCs and Methods and formulas for details.

The icc command calculates ICCs for each of the three analysis-of-variance models. You can use option absolute to compute AA-ICCs or option consistency to compute CA-ICCs. By default, icc computes ICCs corresponding to the correlation between ratings and between average ratings made on the same target: AA-ICC for a random-effects model and CA-ICC for a mixed-effects model. As pointed out by Shrout and Fleiss (1979), although the data on average ratings might be needed for reliability, the generalization of interest might be individuals. For this reason, icc reports ICCs for both units, individual and average, for each model.

In addition to estimates of ICCs, icc provides confidence intervals and one-sided $F$ tests. The $F$ test of $H_{o}: \rho=0$ versus $H_{a}: \rho>0$ is the same for the individual and average ICCs, so icc reports one test. This is not true, however, for nonzero null hypotheses (see Tests against nonzero values for details), so icc reports a separate test in this case.

The icc command requires data in long form; see [D] reshape for how to convert data in wide form to long form. The data must also be balanced and contain one observation per target and rater. For unbalanced data, icc omits all targets with fewer than $k$ ratings from computation. Under one-way models, $k$ is determined as the largest number of observed ratings for a target. Under two-way models, $k$ is the number of unique raters. If multiple observations per target and rater are detected, icc issues an error.

We demonstrate the use of icc using datasets from Shrout and Fleiss (1979) and McGraw and Wong (1996a). In the next three sections, we use an example from table 2 of Shrout and Fleiss (1979) with six targets and four judges. For instructional purposes, we analyze these data under each of the three different models: one-way random effects, two-way random effects, and two-way mixed effects.

## One-way random effects

In the one-way random-effects model, we assume that the $n$ targets being rated are randomly selected from the population of potential targets. Each is rated by a different set of $k$ raters randomly drawn from the population of potential raters. McGraw and Wong (1996a) describe an example of this setting, where behavioral genetics data are used to assess familial resemblance. Family units can be viewed as "targets", and children can be viewed as "raters". By taking a measurement on a child of the family unit, we obtain the "rating" of the family unit by the "child-rater". In this case, we can use ICC to assess similarity between children within a family or, in other words, assess if there is a family effect in these data.

As we mentioned in the introduction, only AA-ICC is defined for a one-way model. The consistency of agreement is not defined in this case, as each target is evaluated by a different set of raters. Thus there is no between-rater variability in this model.

In a one-way model, the AA-ICC corresponds to the correlation coefficient between ratings within a target. It is also a ratio of the between-target variance of ratings to the total variance of ratings, the sum of the between-target and error variances.

## > Example 1: One-way random-effects ICCs

Consider data from table 2 of Shrout and Fleiss (1979) stored in judges.dta. The data contain 24 ratings of $n=6$ targets by $k=4$ judges. We list the first eight observations:

```
. use http://www.stata-press.com/data/r13/judges
(Ratings of targets by judges)
. list in 1/8, sepby(target)
```

|  | rating | target | judge |
| :--- | ---: | ---: | ---: |
| 1. | 9 | 1 | 1 |
| 2. | 2 | 1 | 2 |
| 3. | 5 | 1 | 3 |
| 4. | 8 | 1 | 4 |
| 5. | 6 | 2 | 1 |
| 6. | 1 | 2 | 2 |
| 7. | 3 | 2 | 3 |
| 8. | 2 | 2 | 4 |

For a moment, let's ignore that targets are rated by the same set of judges. Instead, we assume that a different set of four judges is used to rate each target. In this case, the only systematic variation in the data is due to targets, so the one-way random-effects model is appropriate.

We use icc to estimate the intraclass correlations for these data. To compute ICCs for a one-way model, we specify the dependent variable rating followed by the target variable target:

```
- icc rating target
Intraclass correlations
One-way random-effects model
Absolute agreement
```

Random effects: target Number of targets $=\quad 6$
Number of raters $=4$

| rating | ICC | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: |
| Individual | .1657418 | -.1329323 | .7225601 |
| Average | .4427971 | -.8844422 | .9124154 |

F test that
ICC=0.00: F $5.0,18.0$ ) $=1.79 \quad$ Prob $>F=0.165$
Note: ICCs estimate correlations between individual measurements
and between average measurements made on the same target.
icc reports the AA-ICCs for both individual and average ratings. The individual AA-ICC corresponds to ICC(1) in McGraw and Wong (1996a) or ICC(1,1) in Shrout and Fleiss (1979). The average AA-ICC corresponds to $\operatorname{ICC}(k)$ in McGraw and Wong (1996a) or ICC $(1, k)$ in Shrout and Fleiss (1979).

The estimated correlation between individual ratings is 0.17 , indicating little similarity between ratings within a target, low reliability of individual target ratings, or no target effect. The estimated intraclass correlation between ratings averaged over $k=4$ judges is higher, 0.44 . (The average ICC will typically be higher than the individual ICC.) The estimated intraclass correlation measures the similarity or reliability of mean ratings from groups of four judges. We do not have statistical evidence that either ICC is different from zero based on reported confidence intervals and the one-sided $F$ test.

Note that although the estimates of ICCs cannot be negative in this setting, the lower bound of the computed confidence interval may be negative. A common ad-hoc way of handling this is to truncate the lower bound at zero.

The estimates of both the individual and the average AA-ICC are also computed by the loneway command (see [R] loneway), which performs a one-way analysis of variance.

## Technical note

Mean rating is commonly used when individual rating is unreliable because the reliability of a mean rating is always higher than the reliability of the individual rating when the individual reliability is positive.

In the previous example, we estimated low reliability of the individual ratings of a target, 0.17. The reliability increased to 0.44 for the ratings averaged over four judges. What if we had more judges?

We can use the Spearman-Brown formula (Spearman 1910; Brown 1910) to compute the $m$-average ICC based on the individual ICC:

$$
\operatorname{ICC}(m)=\frac{m \operatorname{ICC}(1)}{1+(m-1) \operatorname{ICC}(1)}
$$

Using this formula for the previous example, we find that the mean reliability over, say, 10 judges is $10 \times 0.17 /(1+9 \times 0.17)=0.67$.

Alternatively, we can invert the Spearman-Brown formula to determine the number of judges (or the number of ratings of a target) we need to achieve the desired reliability. Suppose we would like an average reliability of 0.9 , then

$$
m=\frac{\operatorname{ICC}(m)\{(1-\operatorname{ICC}(1))\}}{\operatorname{ICC}(1)\{1-\operatorname{ICC}(m)\}}=\frac{0.9(1-0.17)}{0.17(1-0.9)}=44
$$

See, for example, Bliese (2000) for other examples.

## Two-way random effects

As before, we assume that the targets being rated are randomly selected from the population of potential targets. We now also assume that each target is evaluated by the same set of $k$ raters, who have been randomly sampled from the population of raters. In this scenario, we want to generalize our findings to the population of raters from which the observed $k$ raters were sampled. For example, suppose we want to estimate the reliability of doctors' evaluations of patients with a certain condition. Unless the reliability at a specific hospital is of interest, the doctors may be interchanged with others in the relevant population of doctors.

As for a one-way model, the AA-ICC corresponds to the correlation between measurements on the same target and is also a ratio of the between-target variance to the total variance of measurements in a two-way random-effects model. The total variance is now the sum of the between-target, between-rater, and error variances. Unlike a one-way model, the CA-ICC can be computed for a two-way randomeffects model when the consistency of agreement is of interest rather than the absolute agreement. The CA-ICC is also the ratio of the between-target variance to the total variance, but the total variance does not include the between-rater variance because the between-rater variability is irrelevant for the consistency of agreement.

Again, the two versions, individual and average, are available for each ICC.

## > Example 2: Two-way random-effects ICCs

Continuing with example 1 , recall that we previously ignored that each target is rated by the same set of four judges and instead assumed different sets of judges. We return to the original data setting. We want to evaluate the agreement between judges' ratings of targets in a population represented by the observed set of four judges.

In a two-way model, we must specify both the target and the rater variables. In icc, we now additionally specify the rater variable judge following the target variable target; the random-effects model is assumed by default.

| Intraclass correlations Two-way random-effects model Absolute agreement |  |  |  |
| :---: | :---: | :---: | :---: |
| Random effects: target |  | f targets | 6 |
| Random effects: judge |  | f raters | 4 |
| rating | ICC | [95\% Conf | Interval] |
| Individual | . 2897638 | . 0187865 | . 7610844 |
| Average | . 6200505 | . 0711368 | . 927232 |
| $F$ test that |  |  | ICC=0.00: F $5.0,15.0)=11.03 \quad$ Prob > F $=0.000$ |
| Note: ICCs estimate correlations between individual measurement and between average measurements made on the same target. |  |  |  |

As for a one-way random-effects model, icc by default reports AA-ICCs that correspond to the correlation between ratings on a target. Notice that both individual and average ICCs are larger in the two-way random-effects model than in the previous one-way model- 0.29 versus 0.17 and 0.62 versus 0.44 , respectively. We also have statistical evidence to reject the null hypothesis that neither ICC is zero based on confidence intervals and the $F$ test. If a one-way model is used when a two-way model is appropriate, the true ICC will generally be underestimated.

The individual AA-ICC corresponds to $\operatorname{ICC}(A, 1)$ in McGraw and Wong (1996a) or ICC $(2,1)$ in Shrout and Fleiss (1979). The average AA-ICC corresponds to $\operatorname{ICC}(A, k)$ in McGraw and Wong (1996a) or $\operatorname{ICC}(2, k)$ in Shrout and Fleiss (1979).

Instead of the absolute agreement, we can also assess the consistency of agreement. The individual and average CA-ICCs are considered in McGraw and Wong (1996a) and denoted as ICC(C,1) and ICC(C, $k$ ), respectively. These ICCs are not considered in Shrout and Fleiss (1979) because they are not correlations in the strict sense. Although CA-ICCs do not estimate correlation, they can provide useful information about the reliability of the raters. McGraw and Wong (1996a) note that the practical value of the individual and average CA-ICCs in the two-way random-effects model setting is well documented in measurement theory, citing Hartmann (1982) and Suen (1988).

To estimate the individual and average CA-ICCs, we specify the consistency option:


We estimate that the consistency of agreement of ratings in the considered population of raters is high, 0.71, based on the individual CA-ICC. On the other hand, the absolute agreement of ratings is low, 0.29 , based on the individual AA-ICC from the previous output.

The measure of consistency of agreement among means, the average CA-ICC, is equivalent to Cronbach's alpha (Cronbach 1951); see [MV] alpha. The individual CA-ICC can also be equivalent to the Pearson's correlation coefficient between raters when $k=2$; see McGraw and Wong (1996a) for details.

In the next example, we will see that the actual estimates of the individual and average AA-ICCs and CA-ICCs are the same whether we examine a random-effects model or a mixed-effects model. The differences between these ICCs are in their definitions and interpretations.

## Two-way mixed effects

As in a two-way random-effects model, we assume that the targets are randomly selected from the population of potential targets and that each is evaluated by the same set of $k$ raters. In a mixed-effects model, however, we assume that these raters are the only raters of interest. So as before, the targets are random, but now the raters are fixed.

In the two-way mixed-effects model, the fixed effect of the rater does not contribute to the betweenrater random variance component to the total variance. As such, the definitions and interpretations of ICCs are different in a mixed-effects model than in a random-effects model. However, the estimates of ICCs as well as test statistics and confidence intervals are the same. The only exceptions are average AA-ICCs and CA-ICCs. These are not estimable in a two-way mixed-effects model including an interaction term between target and rater; see Relationship between ICCs and Methods and formulas for details.

In a two-way mixed-effects model, the CA-ICC corresponds to the correlation between measurements on the same target. As pointed out by Shrout and Fleiss (1979), when the rater variance is ignored, the correlation coefficient is interpreted in terms of rater consistency rather than rater absolute agreement. Formally, the CA-ICC is the ratio of the covariance between measurements on the target to the total variance of the measurements. The AA-ICC corresponds to the same ratio, but includes a variance of the fixed factor, rater, in its denominator.

## > Example 3: Two-way mixed-effects ICCs

Continuing with example 2, suppose that we are now interested in assessing the agreement of ratings from only the observed four judges. The judges are now fixed effects, and the appropriate model is a two-way mixed-effects model.

To estimate ICCs for a two-way mixed-effects model, we specify the mixed option with icc:

```
. icc rating target judge, mixed
Intraclass correlations
Two-way mixed-effects model
Consistency of agreement
Random effects: target Number of targets = 6
    Fixed effects: judge Number of raters = 4
\begin{tabular}{r|rll}
\hline rating & ICC & [95\% Conf. & Interval] \\
\hline Individual & .7148407 & .3424648 & .9458583 \\
Average & .9093155 & .6756747 & .9858917 \\
\hline
\end{tabular}
F test that
    ICC=0.00: F(5.0, 15.0) = 11.03 Prob > F = 0.000
```

Note: ICCs estimate correlations between individual measurements
and between average measurements made on the same target.

As we described in the introduction, icc by default reports ICCs corresponding to the correlations. So, for a mixed-effects model, icc reports CA-ICCs by default. The individual and average CA-ICCs are denoted as $\operatorname{ICC}(3,1)$ and $\operatorname{ICC}(3, k)$ in Shrout and Fleiss (1979) and $\operatorname{ICC}(C, 1)$ and $\operatorname{ICC}(C, k)$ in McGraw and Wong (1996a).

Our estimates of the individual and average CA-ICCs are identical to the CA-ICC estimates obtained under the two-way random-effects model in example 2, but our interpretation of the results is different. Under a mixed-effects model, 0.71 and 0.91 are the estimates, respectively, of the correlation between individual measurements and the correlation between average measurements made on the same target.

We can also estimate the AA-ICCs in this setting by specifying the absolute option:

| Intraclass correlations Two-way mixed-effects model Absolute agreement |  |  |  |
| :---: | :---: | :---: | :---: |
| ```Random effects: target Fixed effects: judge rating``` |  | targets | 6 |
|  | Num | raters | 4 |
|  | ICC | [95\% Conf | Interval] |
| Individual Average | . 2897638 | . 0187865 | . 7610844 |
|  | . 6200505 | . 0711368 | . 927232 |

The intraclass correlation estimates match the individual and average AA-ICCs obtained under the two-way random-effects model in example 2; but in a mixed-effects model, they do not represent correlations. We demonstrate the use of an individual AA-ICC in a mixed-effects setting in the next example.

The AA-ICCs under a mixed-effects model are not considered by Shrout and Fleiss (1979). They are denoted as $\operatorname{ICC}(A, 1)$ and $\operatorname{ICC}(A, k)$ in McGraw and Wong (1996a).

## Adoption study

In this section, we consider the adoption study described in McGraw and Wong (1996a). Adoption studies commonly include two effects of interest. One is the mean difference between the adopted child and its biological parents. It is used to determine if characteristics of adopted children differ on average from those of their biological parents. Another effect of interest is the correlation between genetically paired individuals and genetically unrelated individuals who live together. This effect is used to evaluate the impact of genetic differences on individual differences.

As discussed in McGraw and Wong (1996a), a consistent finding from adoption research using IQ as a trait characteristic is that while adopted children typically have higher IQs than their biological parents, their IQs correlate better with those of their biological parents than with those of their adoptive parents. Both effects are important, and there is additional need to reconcile the two findings. McGraw and Wong (1996a) propose to use the individual AA-ICC for this purpose.

## > Example 4: Absolute-agreement ICC in a mixed-effects model

The adoption.dta dataset contains the data from table 6 of McGraw and Wong (1996a) on IQ scores:

| Contains data from http://www.stata-press.com/data/r13/adoption.dta |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| obs: | 20 |  |  | Biological mother and adopted |
| vars: | 5 |  |  | 15 May 2013 13:50 |
| size: | 160 |  |  | (_dta has notes) |
| variable name | storage type | display <br> format | value <br> label | variable label |
| family | byte | \%9.0g |  | Adoptive family ID |
| mc | byte | \%9.0g | mcvalues | 1=Mother, 2=Child |
| iq3 | int | \%9.0g |  | IQ scores, mother-child difference of 3 pts |
| iq9 | int | \%9.0g |  | IQ scores, mother-child difference of 9 pts |
| iq15 | int | \%9.0g |  | IQ scores, mother-child difference of 15 pts |

Sorted by:
The family variable contains adoptive family identifiers, the mc variable records a mother or a child, and the iq3, iq9, and iq15 variables record IQ scores with differences between mother and child mean IQ scores of 3,9 , and 15 points, respectively.
. by mc, sort: summarize iq*


The variances of the mother and child IQ scores are the same.
Children are fixed effects, so the mixed-effects model is appropriate for these data. We want to compare individual CA-ICC with individual AA-ICC for each of the three IQ variables. We could issue a separate icc command for each of the three IQ variables to obtain the intraclass correlations. Instead, we use reshape to convert our data to long form with one iq variable and the new diff variable recording mean differences:

```
. reshape long iq, i(family mc) j(diff)
(note: j = 3 9 15)
\begin{tabular}{lrlr} 
Data & wide & -> & long \\
\hline Number of obs. & 20 & \(->\) & 60 \\
Number of variables & 5 & \(\rightarrow\) & 4 \\
j variable (3 values) & & -> & diff
\end{tabular}
xij variables:
iq3 iq9 iq15 -> iq
```

We can now use the by prefix with icc to estimate intraclass correlations for the three groups of interest:
. by diff, sort: icc iq family mc, mixed

```
-> diff = 3
Intraclass correlations
Two-way mixed-effects model
Consistency of agreement
Random effects: family Number of targets = 10
    Fixed effects: mc Number of raters = 2
\begin{tabular}{r|rcr}
\hline iq & ICC & [95\% Conf. & Interval] \\
\hline Individual & .7142152 & .1967504 & .920474 \\
Average & .8332853 & .3288078 & .9585904 \\
\hline
\end{tabular}
F test that
    ICC=0.00: F(9.0, 9.0) = 6.00 Prob > F = 0.007
```

Note: ICCs estimate correlations between individual measurements and between average measurements made on the same target.

```
-> diff = 9
Intraclass correlations
Two-way mixed-effects model
Consistency of agreement
Random effects: family Number of targets = 10
    Fixed effects: mc Number of raters = 2
\begin{tabular}{r|rrr}
\hline iq & ICC & [95\% Conf. Interval] \\
\hline Individual & .7142152 & .1967504 & .920474 \\
Average & .8332853 & .3288078 & .9585904 \\
\hline
\end{tabular}
F test that
    ICC=0.00: F(9.0, 9.0) = 6.00 Prob > F = 0.007
```

Note: ICCs estimate correlations between individual measurements and between average measurements made on the same target.

```
-> diff = 15
    (output omitted)
```

The estimated CA-ICCs are the same in all three groups and are equal to the corresponding estimates of the Pearson's correlation coefficients because mothers' and childrens' IQ scores have the same variability. The scores differ only in means, and mean differences are irrelevant when measuring the consistency of agreement.

The AA-ICCs, however, differ across the three groups:
. by diff, sort: icc iq family mc, mixed absolute

```
-> diff = 3
```

Intraclass correlations
Two-way mixed-effects model
Absolute agreement
Random effects: family Number of targets $=10$
Fixed effects: mc Number of raters $=$ 2

| iq | ICC | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: |
| Individual | .7204023 | .2275148 | .9217029 |
| Average | .8374812 | .3706917 | .9592564 |
| F test that |  |  |  |
| ICC=0.00: $F(9.0,9.0)=6.00$ | Prob $>F=0.007$ |  |  |

-> diff $=9$
Intraclass correlations
Two-way mixed-effects model
Absolute agreement
Random effects: family Number of targets $=10$
Fixed effects: mc Number of raters $=$ 2

| iq | ICC | [95\% Conf. Interval] |  |
| ---: | ---: | :--- | :--- |
| Individual | .6203378 | .0293932 | .8905025 |
| Average | .7656895 | .0571077 | .9420802 |

```
F test that
    ICC=0.00: F(9.0, 9.0) = 6.00 Prob > F = 0.007
```

-> diff $=15$
Intraclass correlations
Two-way mixed-effects model
Absolute agreement
Random effects: family Number of targets $=10$
Fixed effects: mc Number of raters $=\quad 2$

| iq | ICC | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | ---: |
| Individual | .4854727 | -.1194157 | .8466905 |
| Average | .6536272 | -.2712191 | .9169815 |

F test that
ICC $=0.00: F(9.0,9.0)=6.00 \quad$ Prob $>F=0.007$

As the mean differences increase, the AA-ICCs decrease. Their attenuation reflects the difference in means between biological mother and child IQs while still measuring their agreement. Notice that for small mean differences, the estimates of AA-ICCs and CA-ICCs are very similar.

Note that our estimates match those given in McGraw and Wong (1996b), who correct the original table 6 of McGraw and Wong (1996a).

## Relationship between ICCs

In examples 2 and 3, we saw that the estimates of AA-ICCs and CA-ICCs are the same for twoway random-effects and two-way mixed-effects models. In this section, we consider the relationship between various forms of ICCs in more detail; also see Methods and formulas.

There are 10 different versions of ICCs, but only six different estimators are needed to compute them. These estimators include the two estimators for the individual and average AA-ICCs in a one-way model, the two estimators for the individual and average AA-ICCs in two-way models, and the two estimators for the individual and average CA-ICCs in two-way models.

Only individual and average AA-ICCs are defined for the one-way model. The estimates of AA-ICCs based on the one-way model will typically be smaller than individual and average estimates of AA-ICCs and CA-ICCs based on two-way models. The estimates of individual and average CA-ICCs will typically be larger than the estimates of individual and average AA-ICCs.

Although AA-ICCs and CA-ICCs have the same respective estimators in two-way random-effects and mixed-effects models, their definitions and interpretations are different. The AA-ICCs based on a random-effects model contain the between-rater variance component in the denominator of the variance ratio. The AA-ICCs based on a mixed-effects model contain the variance of the fixed-factor rater instead of the random between-rater variability. The AA-ICCs in a random-effects model represent correlations between any two measurements made on a target. The AA-ICCs in a mixed-effects model measure absolute agreement of measurements treating raters as fixed. The CA-ICCs for random-effects and mixed-effects models have the same definition but different interpretations. The CA-ICCs represent correlations between any two measurements made on a target in a mixed-effects model but estimate the degree of consistency among measurements treating raters as random in a random-effects model. The difference in the definitions of AA-ICCS and CA-ICCs is that CA-ICCs do not contain the between-rater variance in the denominator of the variance ratio.

For two-way models, the definitions and interpretations (but not the estimators) of ICCs also depend on whether the model contains an interaction between target and rater. For two-way models with interaction, ICCs include an additional variance component for the target-rater interaction in the denominator of the variance ratio. This component cannot be separated from random error because there is only one observation per target and rater.

Also, under a two-way mixed-effects model including interaction, the interaction components are not mutually independent, as they are in a two-way random-effects model. The considered version of the mixed-effects model places a constraint on the interaction effects-the sum of the interaction effects over levels of the fixed factor is zero; see, for example, chapter 7 in Kuehl (2000) for an introductory discussion of mixed models. In this version of the model, there is a correlation between the interaction effects. Specifically, the two interaction effects for the same target and two different raters are negatively correlated. As a result, the estimated intraclass correlation can be negative under a two-way mixed-effects model with interaction. Also, average AA-ICC and average CA-ICC cannot be estimated in a two-way mixed-effects model including interaction; see Methods and formulas and McGraw and Wong (1996a) for details.

## Tests against nonzero values

It may be of interest to test whether the intraclass correlation is equal to a value other than zero. icc supports testing against positive values through the use of the testvalue() option. Specifying testvalue(\#) provides a one-sided hypothesis test of $H_{o}: \rho=\#$ versus $H_{a}: \rho>\#$. The test is provided separately for both individual and average ICCs.

## > Example 5: Testing ICC against a nonzero value

We return to the two-way random-effects model for the judge and target data from Shrout and Fleiss (1979). Suppose we want to test whether the individual and average AA-ICCs are each equal to 0.2 . We specify the testvalue (0.2) option with icc:

```
. use http://www.stata-press.com/data/r13/judges, clear
(Ratings of targets by judges)
. icc rating target judge, testvalue(0.2)
Intraclass correlations
Two-way random-effects model
Absolute agreement
\begin{tabular}{lll} 
Random effects: target & Number of targets \(=\) & 6 \\
Random effects: judge & Number of raters \(=\) & 4
\end{tabular}
\begin{tabular}{r|rcr}
\hline rating & ICC & [95\% Conf. Interval] \\
\hline Individual & .2897638 & .0187865 & .7610844 \\
Average & .6200505 & .0711368 & .927232 \\
\hline
\end{tabular}
F test that 
```

Note: ICCs estimate correlations between individual measurements
and between average measurements made on the same target.

We reject the null hypothesis that the average AA-ICC, labeled as $\operatorname{ICC}(k)$ in the output, is equal to 0.2 , but we do not have statistical evidence to reject the null hypothesis that the individual AA-ICC, labeled as $\operatorname{ICC}(1)$, is equal to 0.2 .

## Stored results

icc stores the following in $r()$ :
Scalars
r(N_target)
r(N_rater)
r(icc_i)
r(icc_i_F)
r(icc_i_df1)
r(icc_i_df2)
r(icc_i_p)
r(icc_i_lb)
r(icc_i_ub)
r(icc_avg)
r(icc_avg_F)
r(icc_avg_df1)
r(icc_avg_df2)
r(icc_avg_p)
r(icc_avg_lb)
r(icc_avg_ub)
$r$ (testvalue)
r(level)
Macros
$r$ (model)
r(depvar)
$r$ (target)
r(rater)
r(type)
number of targets
number of raters
intraclass correlation for individual measurements
$F$ test statistic for individual ICC
numerator degrees of freedom for r (icc_i_F)
denominator degrees of freedom for $\mathbf{r}$ (icc_i_F)
$p$-value for F test of individual ICC
lower endpoint for confidence intervals of individual ICC
upper endpoint for confidence intervals of individual ICC
intraclass correlation for average measurements
$F$ test statistic for average ICC
numerator degrees of freedom for r (icc_avg_F)
denominator degrees of freedom for r (icc_avg_F)
$p$-value for $F$ test of average ICC
lower endpoint for confidence intervals of average ICC
upper endpoint for confidence intervals of average ICC
null hypothesis value
confidence level
analysis-of-variance model
name of dependent variable
target variable
rater variable
type of ICC estimated (absolute or consistency)

## Methods and formulas

We observe $y_{i j}$, where $i=1, \ldots, n$ and $j=1, \ldots, k$. $y_{i j}$ is the $j$ th rating on the $i$ th target. Let $\alpha=1-l / 100$, where $l$ is the significance level specified by the user.

Methods and formulas are presented under the following headings:

Mean squares<br>One-way random effects<br>Two-way random effects<br>Two-way mixed effects

## Mean squares

The mean squares within targets are

$$
\mathrm{wMS}=\sum_{i} \sum_{j} \frac{\left(y_{i j}-\bar{y}_{i .}\right)^{2}}{n(k-1)}
$$

where $\bar{y}_{i} .=\sum_{j} y_{i j} / k$.
The mean squares between targets are

$$
\mathrm{BMS}=\sum_{i} \frac{\left(\bar{y}_{i .}-\bar{y}_{. .}\right)^{2}}{n-1}
$$

where $\bar{y}_{. .}=\sum_{i} \bar{y}_{i .} / n$.
These are the only mean squares needed to estimate ICC in the one-way random-effects model. For the two-way models, we need two additional mean squares.

The mean squares between raters are

$$
\mathrm{JMS}=\sum_{j} \frac{\left(\bar{y}_{\cdot j}-\bar{y}_{. .}\right)^{2}}{k-1}
$$

where $\bar{y}_{\cdot j}=\sum_{i} y_{i j} / n$ and $\bar{y}_{. .}=\sum_{j} \bar{y}_{\cdot j} / k$.
The residual or error mean square is

$$
\mathrm{EMS}=\frac{\sum_{i} \sum_{j}\left(y_{i j}-\bar{y}\right)^{2}-(k-1) \mathrm{JMS}-(n-1) \mathrm{BMS}}{(n-1)(k-1)}
$$

## One-way random effects

Under the one-way random-effects model, we observe

$$
\begin{equation*}
y_{i j}=\mu+r_{i}+\epsilon_{i j} \tag{M1}
\end{equation*}
$$

where $\mu$ is the mean rating, $r_{i}$ is the target random effect, and $\epsilon_{i j}$ is random error. The $r_{i} \mathrm{~s}$ are i.i.d. $N\left(0, \sigma_{r}^{2}\right) ; \epsilon_{i j} \mathrm{~s}$ are i.i.d. $N\left(0, \sigma_{\epsilon}^{2}\right)$ and are independent of $r_{i}$ s. There is no rater effect separate from the residual error because each target is evaluated by a different set of raters.

The individual AA-ICC is the correlation between individual measurements on the same target:

$$
\rho_{1}=\operatorname{ICC}(1)=\operatorname{Corr}\left(y_{i j}, y_{i j^{\prime}}\right)=\frac{\sigma_{r}^{2}}{\sigma_{r}^{2}+\sigma_{\epsilon}^{2}}
$$

The average AA-ICC is the correlation between average measurements of size $k$ made on the same target:

$$
\rho_{k}=\operatorname{ICC}(k)=\operatorname{Corr}\left(\bar{y}_{i .}, \bar{y}_{i .}^{\prime}\right)=\frac{\sigma_{r}^{2}}{\sigma_{r}^{2}+\sigma_{\epsilon}^{2} / k}
$$

They are estimated by

$$
\begin{aligned}
& \widehat{\rho}_{1}=\widehat{\mathrm{ICC}(1)}=\frac{\mathrm{BMS}-\mathrm{WMS}}{\mathrm{BMS}+(k-1) \mathrm{WMS}} \\
& \widehat{\rho}_{k}=\widehat{\mathrm{ICC}(k)}=\frac{\mathrm{BMS}-\mathrm{WMS}}{\mathrm{BMS}}
\end{aligned}
$$

Confidence intervals. Let $F_{\text {obs }}=$ BMS $/ \mathrm{WMS}$, let $F_{l}$ be the $(1-\alpha / 2) \times 100$ th percentile of the $F_{n-1, n(k-1)}$ distribution, and let $F_{u}$ be the $(1-\alpha / 2) \times 100$ th percentile of the $F_{n(k-1), n-1}$ distribution. Let $F_{L}=F_{\text {obs }} / F_{l}$ and $F_{U}=F_{\text {obs }} F_{u}$.

A $(1-\alpha) \times 100 \%$ confidence interval for $\rho_{1}$ is

$$
\begin{equation*}
\left(\frac{F_{L}-1}{F_{l}+k-1}, \frac{F_{U}-1}{F_{U}+k-1}\right) \tag{1}
\end{equation*}
$$

A $(1-\alpha) \times 100 \%$ confidence interval for $\rho_{k}$ is

$$
\begin{equation*}
\left(1-\frac{1}{F_{L}}, 1-\frac{1}{F_{U}}\right) \tag{2}
\end{equation*}
$$

Hypothesis tests. Consider a one-sided hypothesis test of $H_{o}$ : ICC $=\rho_{0}$ versus $H_{a}$ : ICC $>\rho_{0}$.
The test statistic for $\rho_{1}$ is

$$
\begin{equation*}
F_{\rho_{1}}=\frac{\mathrm{BMS}}{\mathrm{WMS}} \frac{1-\rho_{0}}{1+(k-1) \rho_{0}} \tag{3}
\end{equation*}
$$

The test statistic for $\rho_{k}$ is

$$
\begin{equation*}
F_{\rho_{k}}=\frac{\mathrm{BMS}}{\mathrm{WMS}}\left(1-\rho_{0}\right) \tag{4}
\end{equation*}
$$

Under the null hypothesis, both $F_{\rho_{1}}$ and $F_{\rho_{k}}$ have the $F_{n-1, n(k-1)}$ distribution. When $\rho_{0}=0$, the two test statistics coincide.

## Two-way random effects

In this setting, the target is evaluated by the same set of raters, who are randomly drawn from the population of raters. The underlying models with and without interaction are

$$
\begin{gather*}
y_{i j}=\mu+r_{i}+c_{j}+(r c)_{i j}+\epsilon_{i j}  \tag{M2}\\
y_{i j}=\mu+r_{i}+c_{j}+\epsilon_{i j} \tag{M2A}
\end{gather*}
$$

where $y_{i j}$ is the rating of the $i$ th target by the $j$ th rater, $\mu$ is the mean rating, $r_{i}$ is the target random effect, $c_{j}$ is the rater random effect, $(r c)_{i j}$ is the target-rater random effect, and $\epsilon_{i j}$ is random error. The $r_{i} \mathrm{~s}$ are i.i.d. $N\left(0, \sigma_{r}^{2}\right), c_{j} \mathrm{~s}$ are i.i.d. $N\left(0, \sigma_{c}^{2}\right),(r c)_{i j} \mathrm{~s}$ are i.i.d. $N\left(0, \sigma_{r c}^{2}\right)$, and $\epsilon_{i j} \mathrm{~s}$ are i.i.d. $N\left(0, \sigma_{\epsilon}^{2}\right)$. Each effect is mutually independent of the others.

Below we provide formulas for ICCs for model (M2). The corresponding ICCs for model (M2A) can be obtained by setting $\sigma_{r c}^{2}=0$.

The individual AA-ICC is the correlation between individual measurements on the same target:

$$
\rho_{A, 1}=\operatorname{ICC}(A, 1)=\operatorname{Corr}\left(y_{i j}, y_{i j^{\prime}}\right)=\frac{\sigma_{r}^{2}}{\sigma_{r}^{2}+\sigma_{c}^{2}+\left(\sigma_{r c}^{2}+\sigma_{\epsilon}^{2}\right)}
$$

The average AA-ICC is the correlation between average measurements of size $k$ made on the same target:

$$
\rho_{A, k}=\operatorname{ICC}(A, k)=\operatorname{Corr}\left(\bar{y}_{i .}, \bar{y}_{i .}^{\prime}\right)=\frac{\sigma_{r}^{2}}{\sigma_{r}^{2}+\left(\sigma_{c}^{2}+\sigma_{r c}^{2}+\sigma_{\epsilon}^{2}\right) / k}
$$

The consistency-of-agreement intraclass correlation for individual measurements, individual CA-ICC, is

$$
\rho_{C, 1}=\operatorname{ICC}(C, 1)=\frac{\sigma_{r}^{2}}{\sigma_{r}^{2}+\left(\sigma_{r c}^{2}+\sigma_{\epsilon}^{2}\right)}
$$

The consistency-of-agreement intraclass correlation for average measurements of size $k$, average CA-ICC, is

$$
\rho_{C, k}=\operatorname{ICC}(C, k)=\frac{\sigma_{r}^{2}}{\sigma_{r}^{2}+\left(\sigma_{r c}^{2}+\sigma_{\epsilon}^{2}\right) / k}
$$

With one observation per target and rater, $\sigma_{r c}^{2}$ and $\sigma_{\epsilon}^{2}$ cannot be estimated separately.
The estimators of intraclass correlations, confidence intervals, and test statistics are the same for models (M2) and (M2A). The estimators of ICCs are

$$
\begin{aligned}
& \left.\widehat{\rho}_{A, 1}=\operatorname{ICC(A}, 1\right)=\frac{\mathrm{BMS}-\mathrm{EMS}}{\mathrm{BMS}+(k-1) \mathrm{EMS}+\frac{k}{n}(\mathrm{JMS}-\mathrm{EMS})} \\
& \widehat{\rho}_{A, k}=\operatorname{ICC} \widehat{(A, k)}=\frac{\mathrm{BMS}-\mathrm{EMS}}{\mathrm{BMS}+\frac{1}{n}(\mathrm{JMS}-\mathrm{EMS})} \\
& \widehat{\rho}_{C, 1}=\mathrm{ICC}(C, 1)=\frac{\mathrm{BMS}-\mathrm{EMS}}{\mathrm{BMS}+(k-1) \mathrm{EMS}} \\
& \widehat{\rho}_{C, k}=\operatorname{ICC(C,\mathrm {k})}=\frac{\mathrm{BMS}-\mathrm{EMS}}{\mathrm{BMS}}
\end{aligned}
$$

Confidence intervals. Let $a=k \widehat{\rho}_{A, 1} /\left\{n\left(1-\widehat{\rho}_{A, 1}\right)\right\}, b=1+k \widehat{\rho}_{A, 1}(n-1) /\left\{n\left(1-\widehat{\rho}_{A, 1}\right)\right\}$, and

$$
\begin{equation*}
v=\frac{(a \mathrm{JMS}+b \mathrm{EMS})^{2}}{\frac{a^{2} \mathrm{JMS}}{}{ }^{2}-1}+\frac{b^{2} \mathrm{EMS}^{2}}{(n-1)(k-1)} \tag{5}
\end{equation*}
$$

Let $F_{l}$ be the $(1-\alpha / 2) \times 100$ th percentile of the $F_{n-1, v}$ distribution and $F_{u}$ be the $(1-\alpha / 2) \times 100$ th percentile of the $F_{v, n-1}$ distribution.

A $(1-\alpha) \times 100 \%$ confidence interval for $\rho_{A, 1}$ is given by $(L, U)$, where

$$
\begin{align*}
L & =\frac{n\left(\mathrm{BMS}-F_{l} \mathrm{EMS}\right)}{F_{l}\{k \mathrm{JMS}+(k n-k-n) \mathrm{EMS}\}+n \mathrm{BMS}}  \tag{6}\\
U & =\frac{n\left(F_{u} \mathrm{BMS}-\mathrm{EMS}\right)}{k \mathrm{JMS}+(k n-k-n) \mathrm{EMS}+n F_{u} \mathrm{BMS}}
\end{align*}
$$

A $(1-\alpha) \times 100 \%$ confidence intervals for $\rho_{A, k}$ is a special case of (6) with $k=1$, where $a=\widehat{\rho}_{A, k} /\left\{n\left(1-\widehat{\rho}_{A, k}\right)\right\}, b=1+\widehat{\rho}_{A, k}(n-1) /\left\{n\left(1-\widehat{\rho}_{A, k}\right)\right\}$, and $v$ is defined in (5).

To define confidence intervals for $\rho_{C, 1}$ and $\rho_{C, k}$, let $F_{\text {obs }}=$ BMS $/$ EMS, $F_{l}$ be the $(1-\alpha / 2) \times 100$ th percentile of the $F_{n-1,(n-1)(k-1)}$ distribution, and $F_{u}$ be the $(1-\alpha / 2) \times 100$ th percentile of the $F_{(n-1)(k-1), n-1}$ distribution. Let $F_{L}=F_{\text {obs }} / F_{l}$ and $F_{L}=F_{\text {obs }} F_{u}$.

A $(1-\alpha) \times 100 \%$ confidence intervals for $\rho_{C, 1}$ and $\rho_{C, k}$ are then as given by (1) and (2) for model (M1).
Hypothesis tests. Consider a one-sided hypothesis test of $H_{o}$ : ICC $=\rho_{0}$ versus $H_{a}$ : ICC $>\rho_{0}$. Let $a=k \rho_{0} /\left\{n\left(1-\rho_{0}\right)\right\}$ and $b=1+k \rho_{0}(n-1) /\left\{n\left(1-\rho_{0}\right)\right\}$.

The test statistic for $\rho_{A, 1}$ is

$$
F_{\rho_{A, 1}}=\frac{\mathrm{BMS}}{a \mathrm{JMS}+b \mathrm{EMS}}
$$

Under the null hypothesis, $F_{\rho_{A, 1}}$ has the $F_{n-1, v}$ distribution, where $v$ is defined in (5).
The test statistic for $\rho_{A, k}$ is defined similarly, except $a=\rho_{0} /\left\{n\left(1-\rho_{0}\right)\right\}$ and $b=1+$ $\rho_{0}(n-1) /\left\{n\left(1-\rho_{0}\right)\right\}$. Under the null hypothesis, $F_{\rho_{A, k}}$ has the $F_{n-1, v}$ distribution, where $v$ is defined in (5). When $\rho_{0}=0$, then $a=0, b=1$, and the two test statistics coincide.

The test statistics for $\rho_{C, 1}$ and $\rho_{C, k}$ are defined by (3) and (4), respectively, with WMS replaced by EMS. Under the null hypothesis, both $F_{\rho_{C, 1}}$ and $F_{\rho_{C, k}}$ have the $F_{n-1,(n-1)(k-1)}$ distribution. They also both have the same value when $\rho_{0}=0$.

## Two-way mixed effects

In this setting, every target is evaluated by the same set of judges, who are the only judges of interest. The underlying models with and without interaction are

$$
\begin{gather*}
y_{i j}=\mu+r_{i}+c_{j}+(r c)_{i j}+\epsilon_{i j}  \tag{M3}\\
y_{i j}=\mu+r_{i}+c_{j}+\epsilon_{i j} \tag{M3A}
\end{gather*}
$$

where $y_{i j}$ is the rating of the $i$ th target by the $j$ th rater, $\mu$ is the mean rating, $r_{i}$ is the target random effect, $c_{j}$ is the rater random effect, $(r c)_{i j}$ is an interaction effect between target and rater, and $\epsilon_{i j}$ is random error. The $r_{i}$ s are i.i.d. $N\left(0, \sigma_{r}^{2}\right),(r c)_{i j}$ s are $N\left(0, \sigma_{r c}^{2}\right)$, and $\epsilon_{i j}$ s are i.i.d. $N\left(0, \sigma_{\epsilon}^{2}\right)$. Each random effect is mutually independent of the others. The $c_{j} \mathrm{~s}$ are fixed such that $\sum_{j} c_{j}=0$. The variance of $c_{j} \mathrm{~s}$ is $\theta_{c}^{2}=\sum c_{j}^{2} /(k-1)$.

In the presence of an interaction, two versions of a mixed-effects model may be considered. One assumes that $(r c)_{i j} \mathrm{~s}$ are i.i.d. $N\left(0, \sigma_{r c}^{2}\right)$. Another assumes that $(r c)_{i j} \mathrm{~s}$ are $N\left(0, \sigma_{r c}^{2}\right)$ with an additional constraint that $\sum_{j}(r c)_{i j}=0$ (for example, Kuehl [2000]), so only interaction terms involving different targets are independent. The latter model is considered here.

We now define the intraclass correlations for individual measurements for model (M3).
The individual CA-ICC, the correlation between individual measurements on the same target, is

$$
\rho_{C, 1}=\operatorname{ICC}(C, 1)=\operatorname{Corr}\left(y_{i j}, y_{i j^{\prime}}\right)=\frac{\sigma_{r}^{2}-\sigma_{r c}^{2} /(k-1)}{\sigma_{r}^{2}+\left(\sigma_{r c}^{2}+\sigma_{\epsilon}^{2}\right)}
$$

The absolute-agreement intraclass correlation for individual measurements, individual AA-ICC, is

$$
\rho_{A, 1}=\operatorname{ICC}(A, 1)=\frac{\sigma_{r}^{2}-\sigma_{r c}^{2} /(k-1)}{\sigma_{r}^{2}+\theta_{c}^{2}+\left(\sigma_{r c}^{2}+\sigma_{\epsilon}^{2}\right)}
$$

Shrout and Fleiss (1979) show that the individual ICC could be negative in this case-a phenomenon first pointed out by Sitgreaves (1960). This can happen when the interaction term has a high variance relative to the targets and there are not many raters.

The individual intraclass correlations for model (M3A) have similar definitions with $\sigma_{r c}^{2}=0$. The individual CA-ICC is the correlation between individual measurements on the same target, $\operatorname{Corr}\left(y_{i j}, y_{i j^{\prime}}\right)$.

We now discuss the intraclass correlations that correspond to average measurements. Neither average AA-ICC, $\rho_{A, k}$, nor average CA-ICC, $\rho_{C, k}$, can be estimated under model (M3) (Shrout and Fleiss 1979; McGraw and Wong 1996a). The problem is that in this model, $\sigma_{r}^{2}$, which is the covariance between two means based on $k$ raters, cannot be estimated.

Specifically, the parameter $\sigma_{r}^{2}$ appears only in the expectation of the between-target mean squares BMS. Under the restriction $\sum_{j}(r c)_{i j}=0$,

$$
E(\mathrm{BMS})=k \sigma_{r}^{2}+\sigma_{\epsilon}^{2}
$$

Note that $\sigma_{r c}^{2}$ does not appear in the expectation of between-target mean squares. With one observation per target and rater, $\sigma_{r c}^{2}$ and $\sigma_{\epsilon}^{2}$ cannot be estimated separately (only their sum $\sigma_{r c}^{2}+\sigma_{\epsilon}^{2}$ can be estimated), so BMS alone cannot be used to estimate $\sigma_{r}^{2}$.

Under model (M3A), however, there is no interaction (and thus no interaction variance component $\sigma_{r c}^{2}$ ), so $\rho_{A, k}$ or $\rho_{C, k}$ can be estimated.

The average AA-ICC, the absolute-agreement intraclass correlation for average measurements of size $k$, is

$$
\rho_{A, k}=\operatorname{ICC}(A, k)=\frac{\sigma_{r}^{2}}{\sigma_{r}^{2}+\left(\theta_{c}^{2}+\sigma_{\epsilon}^{2}\right) / k}
$$

The average CA-ICC, the correlation between average measurements of size $k$ made on the same target, is

$$
\rho_{C, k}=\operatorname{ICC}(C, k)=\operatorname{Corr}\left(\bar{y}_{i .}, \bar{y}_{i .}^{\prime}\right)=\frac{\sigma_{r}^{2}}{\sigma_{r}^{2}+\sigma_{\epsilon}^{2} / k}
$$

The estimators of ICCs, their confidence intervals, and hypothesis tests are as described for two-way random-effects models, except $\rho_{A, k}$ and $\rho_{C, k}$ are not defined under model (M3).

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## Also see

[R] anova - Analysis of variance and covariance
$[R]$ correlate - Correlations (covariances) of variables or coefficients
[R] loneway - Large one-way ANOVA, random effects, and reliability
[MV] alpha - Compute interitem correlations (covariances) and Cronbach's alpha

## Title

inequality - Inequality measures

Remarks and examples
References

## Remarks and examples

Stata does not have commands for inequality measures, except roctab has an option to report Gini and Pietra indices; see [R] roctab. Stata users, however, have developed an excellent suite of commands, many of which have been published in the Stata Journal (SJ) and in the Stata Technical Bulletin (STB).

| Issue | Insert | Author(s) | Command | Description |
| :---: | :---: | :---: | :---: | :---: |
| SJ-12-3 | st0266 | I. Almås, T. Havnes, M. Mogstad | adgini | Adjusting for age effects in cross-sectional distributions |
| STB-48 | gr35 | N. J. Cox | psm, qsm, pdagum, qdagum | Diagnostic plots for assessing Singh-Maddala and Dagum distributions fit by MLE |
| SJ-11-3 | st0237 | A. Doris, <br> D. O'Neill, <br> O. Sweetman | gmmcovearn | GMM estimation of the covariance structure of longitudinal data |
| STB-23 | sg31 | R. Goldstein | rspread | Measures of diversity: Absolute and relative |
| STB-48 | sg104 | S. P. Jenkins | sumdist, xfrac, ineqdeco, geivars, ineqfac, povdeco | Analysis of income distributions |
| STB-48 | sg106 | S. P. Jenkins | smfit, dagumfit | Fitting Singh-Maddala and Dagum distributions by maximum likelihood |
| STB-51 | sg115 | D. Jolliffe, <br> B. Krushelnytskyy | ineqerr | Bootstrap standard errors for indices of inequality |
| STB-51 | sg117 | D. Jolliffe, <br> A. Semykina | sepov | Robust standard errors for the Foster-GreerThorbecke class of poverty indices |
| SJ-8-4 | st0100_1 | A. López-Feldman | descogini | Decomposing inequality and obtaining marginal effects |
| SJ-6-4 | snp15_7 | R. Newson | somersd | Gini coefficient is a special case of Somers' $D$ |
| SJ-7-2 | gr0001_3 | S. P. Van Kerm, P. Jenkins | glcurve | Generalized Lorenz curves and related graphs |
| STB-48 | sg 108 | P. Van Kerm | poverty | Computing poverty indices |
| STB-23 | sg30 | E. Whitehouse | lorenz, inequal, atkinson, relsgini | Measures of inequality in Stata |

More commands may be available; enter Stata and type search inequality measure, historical.

Max Otto Lorenz (1876-1959) was born in Iowa and studied at the Universities of Iowa and Wisconsin. He proposed what is now known as the Lorenz curve in 1905. Lorenz worked for the Interstate Commerce Commission between 1911 and 1944, mainly with transportation data. His hobbies included calendar reform and Interlingua, a proposed international language.

To download and install the Jenkins and Van Kerm glcurve command from the Internet, for instance, you could

1. Select Help > SJ and User-written Programs.
2. Click on Stata Journal.
3. Click on sj7-2.
4. Click on gr0001_3.
5. Click on click here to install.
or you could instead do the following:
6. Navigate to the appropriate SJ issue:
a. Type net from http://www.stata-journal.com/software Type net cd sj7-2
or
b. Type net from http://www.stata-journal.com/software/sj7-2
7. Type net describe gr0001_3
8. Type net install gr0001_3

To download and install the Jenkins sumdist command from the Internet, for instance, you could

1. Select Help > SJ and User-written Programs.
2. Click on STB.
3. Click on $s t b 48$.
4. Click on sg104.
5. Click on click here to install.
or you could instead do the following:
6. Navigate to the appropriate STB issue:
a. Type net from http://www.stata.com

Type net cd stb
Type net cd stb48
or
b. Type net from http://www.stata.com/stb/stb48
2. Type net describe sg104
3. Type net install sg104

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## Title

intreg - Interval regression

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

Also see

## Syntax

intreg depvar ${ }_{1}$ depvar 2 [indepvars] [if] [in] [weight] [, options]
options
Model
noconstant
het(varlist [, noconstant])
offset (varname)
constraints (constraints)
collinear
SE/Robust
vce (vcetype)

Reporting
level(\#)
nocnsreport
display_options

Maximization
maximize_options
coeflegend
suppress constant term
independent variables to model the variance; use noconstant to suppress constant term
include varname in model with coefficient constrained to 1
apply specified linear constraints
keep collinear variables
vcetype may be oim, robust, cluster clustvar, opg, bootstrap, or jackknife
set confidence level; default is level (95)
do not display constraints
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
control the maximization process; seldom used
display legend instead of statistics
indepvars and varlist may contain factor variables; see [U] 11.4.3 Factor variables.
depvar $_{1}$, depvar 2 , indepvars, and varlist may contain time-series operators; see [U] 11.4.4 Time-series varlists.
bootstrap, by, fp, jackknife, mfp, nestreg, rolling, statsby, stepwise, and svy are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the bootstrap prefix; see $[R]$ bootstrap.
aweights are not allowed with the jackknife prefix; see $[R]$ jackknife.
vce() and weights are not allowed with the svy prefix; see [SVY] svy.
aweights, fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Linear models and related $>$ Censored regression $>$ Interval regression

## Description

intreg fits a model of $y=\left[\right.$ depvar $_{1}$, depvar $\left.{ }_{2}\right]$ on indepvars, where $y$ for each observation is point data, interval data, left-censored data, or right-censored data.
depvar $_{1}$ and depvar ${ }_{2}$ should have the following form:

| Type of data |  | depvar $_{1}$ | depvar $_{2}$ |
| :--- | ---: | :---: | :---: |
| point data | $a=[a, a]$ | $a$ | $a$ |
| interval data | $[a, b]$ | $a$ | $b$ |
| left-censored data | $(-\infty, b]$ | $\cdot$ | $b$ |
| right-censored data | $[a,+\infty)$ | $a$ | . |

## Options

## Model

noconstant; see [R] estimation options.
het (varlist [, noconstant]) specifies that varlist be included in the specification of the conditional variance. This varlist enters the variance specification collectively as multiplicative heteroskedasticity.
offset (varname), constraints (constraints), collinear; see [R] estimation options.
vce (vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
$\qquad$ Reporting
level (\#); see [R] estimation options.
nocnsreport; see $[R]$ estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\%fmt), pformat(\%fmt), sformat(\%fmt), and nolstretch; see [R] estimation options.

Maximization
maximize_options: difficult, technique (algorithm_spec), iterate (\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance(\#), nonrtolerance, and from (init_specs); see [R] maximize. These options are seldom used.

Setting the optimization type to technique (bhhh) resets the default vcetype to vce (opg).

The following option is available with intreg but is not shown in the dialog box: coeflegend; see $[R]$ estimation options.

## Remarks and examples

intreg is a generalization of the models fit by tobit. Cameron and Trivedi (2010, 548-550) discuss the differences among censored, truncated, and interval data. If you know that the value for the $j$ th individual is somewhere in the interval [ $y_{1 j}, y_{2 j}$ ], then the likelihood contribution from this individual is simply $\operatorname{Pr}\left(y_{1 j} \leq Y_{j} \leq y_{2 j}\right)$. For censored data, their likelihoods contain terms of the form $\operatorname{Pr}\left(Y_{j} \leq y_{j}\right)$ for left-censored data and $\operatorname{Pr}\left(Y_{j} \geq y_{j}\right)$ for right-censored data, where $y_{j}$ is the observed censoring value and $Y_{j}$ denotes the random variable representing the dependent variable in the model.

Hence, intreg can fit models for data where each observation represents interval data, left-censored data, right-censored data, or point data. Regardless of the type of observation, the data should be stored in the dataset as interval data; that is, two dependent variables, depvar ${ }_{1}$ and depvar ${ }_{2}$, are used to hold the endpoints of the interval. If the data are left-censored, the lower endpoint is $-\infty$ and is represented by a missing value, '.', or an extended missing value, '. a, .b, ... . $\mathbf{z}$ ', in depvar $r_{1}$. If the data are right-censored, the upper endpoint is $+\infty$ and is represented by a missing value, '.' (or an extended missing value), in depvar. ${ }_{2}$. Point data are represented by the two endpoints being equal.

| Type of data |  | depvar $_{1}$ | depvar $_{2}$ |
| :--- | ---: | :---: | :---: |
| point data | $a=[a, a]$ | $a$ | $a$ |
| interval data | $[a, b]$ | $a$ | $b$ |
| left-censored data | $(-\infty, b]$ | $\cdot$ | $b$ |
| right-censored data | $[a,+\infty)$ | $a$ | . |

Truly missing values of the dependent variable must be represented by missing values in both depvar ${ }_{1}$ and depvar 2 .

Interval data arise naturally in many contexts, such as wage data. Often you know only that, for example, a person's salary is between $\$ 30,000$ and $\$ 40,000$. Below we give an example for wage data and show how to set up depvar ${ }_{1}$ and depvar ${ }_{2}$.
$>$ Example 1
We have a dataset that contains the yearly wages of working women. Women were asked via a questionnaire to indicate a category for their yearly income from employment. The categories were less than $5,000,5,001-10,000, \ldots, 25,001-30,000,30,001-40,000,40,001-50,000$, and more than 50,000 . The wage categories are stored in the wagecat variable.

| (Wages of women) <br> . tabulate wagecat |  |  |  |
| :---: | :---: | :---: | :---: |
| Wage category (\$1000s) | Freq. | Percent | Cum. |
| 5 | 14 | 2.87 | 2.87 |
| 10 | 83 | 17.01 | 19.88 |
| 15 | 158 | 32.38 | 52.25 |
| 20 | 107 | 21.93 | 74.18 |
| 25 | 57 | 11.68 | 85.86 |
| 30 | 30 | 6.15 | 92.01 |
| 40 | 19 | 3.89 | 95.90 |
| 50 | 14 | 2.87 | 98.77 |
| 51 | 6 | 1.23 | 100.00 |
| Total | 488 | 100.00 |  |

A value of 5 for wagecat represents the category less than 5,000, a value of 10 represents $5,001-10,000$, $\ldots$, and a value of 51 represents greater than 50,000 .

To use intreg, we must create two variables, wage1 and wage2, containing the lower and upper endpoints of the wage categories. Here is one way to do it. We first create a dataset containing the nine wage categories, lag the wage categories into wage1, and match-merge this dataset with nine observations back into the main one.

```
. by wagecat: keep if _n==1
(479 observations deleted)
. generate wage1 = wagecat[_n-1]
(1 missing value generated)
. keep wagecat wage1
. save lagwage
file lagwage.dta saved
. use http://www.stata-press.com/data/r13/womenwage
(Wages of women)
. merge m:1 wagecat using lagwage
    Result # of obs.
    not matched 0
    matched 488 (_merge==3)
```

Now we create the upper endpoint and list the new variables:

```
. generate wage2 = wagecat
. replace wage2 = . if wagecat == 51
(6 real changes made, 6 to missing)
. sort age, stable
```

. list wage1 wage2 in $1 / 10$

|  | wage1 | wage2 |
| ---: | ---: | ---: |
| 1. | . | 5 |
| 2. | 5 | 10 |
| 3. | 5 | 10 |
| 4. | 10 | 15 |
| 5. | $\cdot$ | 5 |
| 6. | . | 5 |
| 7. | . | 5 |
| 8. | 5 | 10 |
| 9. | 5 | 10 |
| 10. | 5 | 10 |

We can now run intreg:
. intreg wage1 wage2 age c.age\#c.age nev_mar rural school tenure
Fitting constant-only model:
Iteration 0: log likelihood = -967. 24956
Iteration 1: log likelihood = -967.1368
Iteration 2: log likelihood $=-967.1368$
Fitting full model:
Iteration 0: log likelihood $=-856.65324$
Iteration 1: log likelihood $=-856.33294$
Iteration 2: log likelihood $=-856.33293$

| Interval regression | Number of obs | $=$ | 488 |
| :--- | :--- | :--- | ---: |
| Log likelihood $=-856.33293$ | LR chi2 $(6)$ | $=$ | 221.61 |
|  | Prob $>$ chi2 | $=$ | 0.0000 |


|  | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| age | .7914438 | .4433604 | 1.79 | 0.074 | -.0775265 | 1.660414 |
| c.age\#c.age | -.0132624 | .0073028 | -1.82 | 0.069 | -.0275757 | .0010509 |
|  |  |  |  |  |  |  |
| nev_mar | -.2075022 | .8119581 | -0.26 | 0.798 | -1.798911 | 1.383906 |
| rural | -3.043044 | .7757324 | -3.92 | 0.000 | -4.563452 | -1.522637 |
| school | 1.334721 | .1357873 | 9.83 | 0.000 | 1.068583 | 1.600859 |
| tenure | .8000664 | .1045077 | 7.66 | 0.000 | .5952351 | 1.004898 |
| _cons | -12.70238 | 6.367117 | -1.99 | 0.046 | -25.1817 | -.2230583 |
| /lnsigma | 1.987823 | .0346543 | 57.36 | 0.000 | 1.919902 | 2.055744 |
| sigma | 7.299626 | .2529634 |  |  | 6.82029 | 7.81265 |

Observation summary: 14 left-censored observations
0 uncensored observations
6 right-censored observations
468 interval observations

We could also model these data by using an ordered probit model with oprobit (see [R] oprobit):

| Iteration 0: log likelihood $=$-881.1491 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Iteration 1: log likelihood $=-764$ |  |  |  |  |  |  |
| Iteration 2: log likelihood = -763.31191 |  |  |  |  |  |  |
| Iteration 3: log likelihood $=-763.31049$ |  |  |  |  |  |  |
| Iteration 4: log likelihood = -763.31049 |  |  |  |  |  |  |
| Ordered probit regression |  |  |  | Number of obs $=488$ |  |  |
|  |  |  |  | LR | (6) | 235.68 |
|  |  |  |  | Prob | chi2 | 0.0000 |
| Log likelihood = -763.31049 |  |  |  | Pseudo R2 |  | 0.1337 |
| wagecat | Coef. | Std. Err. | z | $P>\|z\|$ | [95\% Con | Interval] |
| age | . 1674519 | . 0620333 | 2.70 | 0.007 | . 0458689 | . 289035 |
| c.age\#c.age | -. 0027983 | . 0010214 | $-2.74$ | 0.006 | -. 0048001 | -. 0007964 |
| nev_mar | -. 0046417 | . 1126737 | -0.04 | 0.967 | -. 225478 | . 2161946 |
| rural | -. 5270036 | . 1100449 | -4.79 | 0.000 | -. 7426875 | -. 3113196 |
| school | . 2010587 | . 0201189 | 9.99 | 0.000 | . 1616263 | . 2404911 |
| tenure | . 0989916 | . 0147887 | 6.69 | 0.000 | . 0700063 | . 127977 |
| /cut1 | 2.650637 | . 8957245 |  |  | . 8950495 | 4.406225 |
| /cut2 | 3.941018 | . 8979167 |  |  | 2.181134 | 5.700903 |
| /cut3 | 5.085205 | . 9056582 |  |  | 3.310148 | 6.860263 |
| /cut4 | 5.875534 | . 9120933 |  |  | 4.087864 | 7.663204 |
| /cut5 | 6.468723 | . 918117 |  |  | 4.669247 | 8.268199 |
| /cut6 | 6.922726 | . 9215455 |  |  | 5.11653 | 8.728922 |
| /cut7 | 7.34471 | . 9237628 |  |  | 5.534168 | 9.155252 |
| /cut8 | 7.963441 | . 9338881 |  |  | 6.133054 | 9.793828 |

We can directly compare the log likelihoods for the intreg and oprobit models because both likelihoods are discrete. If we had point data in our intreg estimation, the likelihood would be a mixture of discrete and continuous terms, and we could not compare it directly with the oprobit likelihood.

Here the oprobit log likelihood is significantly larger (that is, less negative), so it fits better than the intreg model. The intreg model assumes normality, but the distribution of wages is skewed and definitely nonnormal. Normality is more closely approximated if we model the $\log$ of wages.


The log likelihood of this intreg model is close to the oprobit log likelihood, and the $z$ statistics for both models are similar.

Technical note
intreg has two parameterizations for the log-likelihood function: the transformed parameterization $(\beta / \sigma, 1 / \sigma)$ and the untransformed parameterization $(\beta, \ln (\sigma))$. By default, the log likelihood for intreg is parameterized in the transformed parameter space. This parameterization tends to be more convergent, but it requires that any starting values and constraints have the same parameterization, and it prevents the estimation with multiplicative heteroskedasticity. Therefore, when the het() option is specified, intreg switches to the untransformed $\log$ likelihood for the fit of the conditional-variance model. Similarly, specifying from () or constraints() causes the optimization in the untransformed parameter space to allow constraints on (and starting values for) the coefficients on the covariates without reference to $\sigma$.

The estimation results are all stored in the $(\beta, \ln (\sigma))$ metric.

## Stored results

intreg stores the following in e() :
Scalars
e(N)
e(N_unc)
e(N_lc)
e(N_rc)
e(N_int)
e(k)
e(k_aux)
e(k_eq)
e(k_eq_model)
e(k_dv)
e(df_m)
e(11)
e(ll_0)
e(ll_c)
e(N_clust)
e(chi2)
e (p)
e(sigma)
e(se_sigma)
e(rank)
e(rankO)
e(ic)
$e(r c)$
e(converged)
Macros
e(cmd)
e(cmdline)
e(depvar)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(offset)
e(chi2type)
e(vce)
e(vcetype)
e(het)
e(ml_score)
e(opt)
$e$ (which)
e(ml_method)
e(user)
$e$ (technique)
e(properties)
e(predict)
$e$ (footnote)
e(asbalanced)
e(asobserved)
Matrices
e(b)
e(Cns)
e(ilog)
e(gradient)
e(V)
e(V_modelbased)
number of observations
number of uncensored observations
number of left-censored observations
number of right-censored observations
number of interval observations
number of parameters
number of auxiliary parameters
number of equations in $e(b)$
number of equations in overall model test
number of dependent variables
model degrees of freedom
log likelihood
log likelihood, constant-only model
log likelihood, comparison model
number of clusters
$\chi^{2}$
$p$-value for model $\chi^{2}$ test
sigma
standard error of sigma
rank of e(V)
rank of $e(V)$ for constant-only model
number of iterations
return code
1 if converged, 0 otherwise

## intreg

command as typed
names of dependent variables
weight type
weight expression
title in estimation output
name of cluster variable
linear offset variable
Wald or LR; type of model $\chi^{2}$ test
vcetype specified in vce()
title used to label Std. Err.
heteroskedasticity, if het() specified
program used to implement scores
type of optimization
max or min; whether optimizer is to perform maximization or minimization
type of ml method
name of likelihood-evaluator program
maximization technique
b V
program used to implement predict
program and arguments to display footnote
factor variables fvset as asbalanced
factor variables fvset as asobserved
coefficient vector
constraints matrix
iteration $\log$ (up to 20 iterations)
gradient vector
variance-covariance matrix of the estimators
model-based variance

Functions
e(sample) marks estimation sample

## Methods and formulas

See Wooldridge (2013, sec. 17.4) or Davidson and MacKinnon (2004, sec. 11.6) for an introduction to censored and truncated regression models.

The likelihood for intreg subsumes that of the tobit models.
Let $\mathbf{y}=\mathbf{X} \boldsymbol{\beta}+\boldsymbol{\epsilon}$ be the model. $\mathbf{y}$ represents continuous outcomes - either observed or not observed. Our model assumes $\boldsymbol{\epsilon} \sim N\left(\mathbf{0}, \sigma^{2} \mathbf{I}\right)$.

For observations $j \in \mathcal{C}$, we observe $y_{j}$, that is, point data. Observations $j \in \mathcal{L}$ are left-censored; we know only that the unobserved $y_{j}$ is less than or equal to $y_{\mathcal{L} j}$, a censoring value that we do know. Similarly, observations $j \in \mathcal{R}$ are right-censored; we know only that the unobserved $y_{j}$ is greater than or equal to $y_{\mathcal{R} j}$. Observations $j \in \mathcal{I}$ are intervals; we know only that the unobserved $y_{j}$ is in the interval $\left[y_{1 j}, y_{2 j}\right]$.

The log likelihood is

$$
\begin{aligned}
\ln L= & -\frac{1}{2} \sum_{j \in \mathcal{C}} w_{j}\left\{\left(\frac{y_{j}-\mathbf{x} \boldsymbol{\beta}}{\sigma}\right)^{2}+\log 2 \pi \sigma^{2}\right\} \\
& +\sum_{j \in \mathcal{L}} w_{j} \log \Phi\left(\frac{y_{\mathcal{L}}-\mathbf{x} \boldsymbol{\beta}}{\sigma}\right) \\
& +\sum_{j \in \mathcal{R}} w_{j} \log \left\{1-\Phi\left(\frac{y_{\mathcal{R} j}-\mathbf{x} \boldsymbol{\beta}}{\sigma}\right)\right\} \\
& +\sum_{j \in \mathcal{I}} w_{j} \log \left\{\Phi\left(\frac{y_{2 j}-\mathbf{x} \boldsymbol{\beta}}{\sigma}\right)-\Phi\left(\frac{y_{1 j}-\mathbf{x} \boldsymbol{\beta}}{\sigma}\right)\right\}
\end{aligned}
$$

where $\Phi()$ is the standard cumulative normal and $w_{j}$ is the weight for the $j$ th observation. If no weights are specified, $w_{j}=1$. If aweights are specified, $w_{j}=1$, and $\sigma$ is replaced by $\sigma / \sqrt{a_{j}}$ in the above, where $a_{j}$ are the aweights normalized to sum to $N$.

Maximization is as described in $[\mathrm{R}]$ maximize; the estimate reported as _sigma is $\widehat{\sigma}$.
See Amemiya (1973) for a generalization of the tobit model to variable, but known, cutoffs.
This command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce (robust) and vce (cluster clustvar), respectively. See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas.
intreg also supports estimation with survey data. For details on VCEs with survey data, see [SVY] variance estimation.

## References

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## Also see

$[R]$ intreg postestimation - Postestimation tools for intreg
$[R]$ regress - Linear regression
[R] tobit - Tobit regression
[SVY] svy estimation - Estimation commands for survey data
[XT] xtintreg - Random-effects interval-data regression models
[XT] xttobit — Random-effects tobit models
[U] 20 Estimation and postestimation commands

## Title

intreg postestimation - Postestimation tools for intreg

Description Syntax for predict Menu for predict Options for predict Remarks and examples Also see

## Description

The following postestimation commands are available after intreg:

| Command | Description |
| :--- | :--- |
| contrast | contrasts and ANOVA-style joint tests of estimates |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample <br> estat vce |
| variance-covariance matrix of the estimators (VCE) |  |
| estat (svy) | postestimation statistics for survey data <br> cataloging estimation results |
| lincom | point estimates, standard errors, testing, and inference for linear combinations <br> of coefficients |
| likelihood-ratio test |  |$\quad$| marginal means, predictive margins, marginal effects, and average marginal |
| :--- |
| effects |

[^29]
## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic nooffset]
predict [type] {stub*|newvarreg newvar msigma } [if][in], scores
```

statistic Description

| Main |  |
| :--- | :--- |
| xb | linear prediction; the default |
| stdp | standard error of the prediction |
| stdf | standard error of the forecast |
| $\mathrm{pr}(a, b)$ | $\operatorname{Pr}\left(a<y_{j}<b\right)$ |
| $\mathrm{e}(a, b)$ | $E\left(y_{j} \mid a<y_{j}<b\right)$ |
| $\underline{\text { ystar }(a, b)}$ | $E\left(y_{j}^{*}\right), y_{j}^{*}=\max \left\{a, \min \left(y_{j}, b\right)\right\}$ |

These statistics are available both in and out of sample; type predict ... if e(sample)... if wanted only for the estimation sample.
stdf is not allowed with svy postestimation results.
where $a$ and $b$ may be numbers or variables; $a$ missing ( $a \geq$.) means $-\infty$, and $b$ missing ( $b \geq$.) means $+\infty$; see [U] 12.2.1 Missing values.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
xb , the default, calculates the linear prediction.
stdp calculates the standard error of the prediction, which can be thought of as the standard error of the predicted expected value or mean for the observation's covariate pattern. The standard error of the prediction is also referred to as the standard error of the fitted value.
stdf calculates the standard error of the forecast, which is the standard error of the point prediction for 1 observation. It is commonly referred to as the standard error of the future or forecast value. By construction, the standard errors produced by stdf are always larger than those produced by stdp; see Methods and formulas in [R] regress postestimation.
$\operatorname{pr}(a, b)$ calculates $\operatorname{Pr}\left(a<\mathbf{x}_{j} \mathbf{b}+u_{j}<b\right)$, the probability that $y_{j} \mid \mathbf{x}_{j}$ would be observed in the interval $(a, b)$.
$a$ and $b$ may be specified as numbers or variable names; $l b$ and $u b$ are variable names;
$\operatorname{pr}(20,30)$ calculates $\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$;
$\operatorname{pr}(l b, u b)$ calculates $\operatorname{Pr}\left(l b<\mathbf{x}_{j} \mathbf{b}+u_{j}<u b\right)$; and
$\operatorname{pr}(20, u b)$ calculates $\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{j}<u b\right)$.
$a$ missing $(a \geq$.$) means -\infty ; \operatorname{pr}(., 30)$ calculates $\operatorname{Pr}\left(-\infty<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$;
$\operatorname{pr}(l b, 30)$ calculates $\operatorname{Pr}\left(-\infty<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$ in observations for which $l b \geq$.
and calculates $\operatorname{Pr}\left(l b<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$ elsewhere.
$b$ missing $(b \geq$.$) means +\infty$; $\operatorname{pr}(20,$.$) calculates \operatorname{Pr}\left(+\infty>\mathbf{x}_{j} \mathbf{b}+u_{j}>20\right)$;
$\operatorname{pr}(20, u b)$ calculates $\operatorname{Pr}\left(+\infty>\mathbf{x}_{j} \mathbf{b}+u_{j}>20\right)$ in observations for which $u b \geq$. and calculates $\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{j}<u b\right)$ elsewhere.
$\mathrm{e}(a, b)$ calculates $E\left(\mathbf{x}_{j} \mathbf{b}+u_{j} \mid a<\mathbf{x}_{j} \mathbf{b}+u_{j}<b\right)$, the expected value of $y_{j} \mid \mathbf{x}_{j}$ conditional on $y_{j} \mid \mathbf{x}_{j}$ being in the interval $(a, b)$, meaning that $y_{j} \mid \mathbf{x}_{j}$ is truncated.
$a$ and $b$ are specified as they are for $\operatorname{pr}()$.
ystar $(a, b)$ calculates $E\left(y_{j}^{*}\right)$, where $y_{j}^{*}=a$ if $\mathbf{x}_{j} \mathbf{b}+u_{j} \leq a, y_{j}^{*}=b$ if $\mathbf{x}_{j} \mathbf{b}+u_{j} \geq b$, and $y_{j}^{*}=\mathbf{x}_{j} \mathbf{b}+u_{j}$ otherwise, meaning that $y_{j}^{*}$ is censored. $a$ and $b$ are specified as they are for pr() .
nooffset is relevant only if you specified offset (varname). It modifies the calculations made by predict so that they ignore the offset variable; the linear prediction is treated as $\mathbf{x}_{j} \mathbf{b}$ rather than as $\mathbf{x}_{j} \mathbf{b}+$ offset $_{j}$.
scores calculates equation-level score variables.
The first new variable will contain $\partial \ln L / \partial\left(\mathbf{x}_{j} \boldsymbol{\beta}\right)$.
The second new variable will contain $\partial \ln L / \partial \ln \sigma$.

## Remarks and examples

> Example 1
We continue with example 1 of $[\mathrm{R}]$ intreg.

```
. use http://www.stata-press.com/data/r13/intregxmpl
. intreg wage1 wage2 age c.age#c.age nev_mar rural school tenure
(output omitted)
```

By default, the predict command produces the linear prediction, which in this case is the expected wage for each individual.

```
. predict w1
(option xb assumed; fitted values)
```

We can use the $\mathrm{e}(a, b)$ option to compute the expected wage, conditional on it being larger than \$5,000:

```
. predict w2, e(5,.)
```

The probability of earning more than $\$ 5,000$ might vary with age. We can use margins to compute the marginal means for those probabilities for different ages.

| Predictive margins |  |  |  | Number of obs |  | 488 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Model VCE |  |  |  |  |  |  |
| Expression : $\operatorname{Pr}(\mathrm{y}>5)$, predict(pr $(5,)$. |  |  |  |  |  |  |
| 1._at | : age | = | 20 |  |  |  |
| 2._at | : age | = | 25 |  |  |  |
| 3._at | : age | = | 30 |  |  |  |
| 4._at | : age | = | 35 |  |  |  |
| 5._at | : age | = | 40 |  |  |  |
| 6._at | : age | = | 45 |  |  |  |
| 7._at | : age | = | 50 |  |  |  |
|  | Margin | elta-metho Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf | Interval] |
| _at |  |  |  |  |  |  |
| 1 | . 8912598 | . 0151773 | 58.72 | 0.000 | . 8615127 | . 9210068 |
| 2 | . 9104568 | . 0103467 | 87.99 | 0.000 | . 8901775 | . 930736 |
| 3 | . 9160005 | . 0120025 | 76.32 | 0.000 | . 892476 | . 9395251 |
| 4 | . 9096667 | . 0136693 | 66.55 | 0.000 | . 8828753 | . 9364581 |
| 5 | . 8894289 | . 0206992 | 42.97 | 0.000 | . 8488593 | . 9299985 |
| 6 | . 8491103 | . 0447429 | 18.98 | 0.000 | . 7614159 | . 9368048 |
| 7 | . 7781644 | . 0970557 | 8.02 | 0.000 | . 5879387 | . 9683902 |

We can visualize these results by using marginsplot:
. qui margins, predict(pr(5,.)) at(age=(20(5)50))
. marginsplot
Variables that uniquely identify margins: age


The probability increases until age 30, and it decreases after that age.

## Also see

[R] intreg - Interval regression
[U] 20 Estimation and postestimation commands

## Title

ivpoisson - Poisson regression with endogenous regressors

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |
| Also see |  |  |  |

## Syntax

Generalized method of moments estimator ivpoisson gmm depvar $\left[\right.$ varlist $\left._{1}\right]\left[\left(\right.\right.$ varlist $_{2}=$ varlist $\left.\left._{\mathrm{iv}}\right)\right][$ if $][$ in $][$ weight $]$ [, reg_err_opt options]

## Control-function estimator

ivpoisson cfunction depvar $\left[\right.$ varlist $\left._{1}\right] \quad\left(\right.$ varlist $_{2}=$ varlist $\left._{\mathrm{iv}}\right) \quad[$ if $][$ in $][$ weight $]$
[, options]
reg_err_opt Description
Model
additive
multiplicative
add regression errors to the conditional mean term; the default multiply regression errors by the conditional mean term

Description

```
Model
    noconstant
    exposure(varname e)
    offset(\mp@subsup{varnameo)}{o}{\prime}
* twostep
* onestep
* igmm
```

suppress constant term
include $\ln \left(\right.$ varname $\left._{e}\right)$ in model with coefficient constrained to 1 include varname ${ }_{o}$ in model with coefficient constrained to 1 use two-step GMM estimator; the default for ivpoisson gmm use one-step GMM estimator; the default for ivpoisson cfunction use iterative GMM estimator

```
Weight matrix
        wmatrix(wmtype)
        center
SE/Robust
```

winitial(iwtype[, independent])
specify initial weight matrix; iwtype may be unadjusted,
identity, or the name of a Stata matrix
(independent may not be specified with ivpoisson gmm)
vce(vcetype) vcetype may be robust, cluster clustvar, bootstrap,
Reporting
level(\#)
irr
display_options
Optimization
from(initial_values)
$\ddagger_{\text {igmmiterate (\#) }}$
$\ddagger_{\text {igmmeps }}$ (\#)
$\ddagger_{\text {igmmweps }}$ (\#)
optimization_options
set confidence level; default is level (95)
report incidence-rate ratios
control column formats, row spacing, line width, display of omitted
variables and base and empty cells, and factor-variable labeling
specify weight matrix; wmtype may be robust, cluster clustvar,
or unadjusted
center moments in weight-matrix computation
jackknife, or unadjusted
specify initial values for parameters
specify maximum number of iterations for iterated GMM estimator
specify \# for iterated GMM parameter convergence criterion; default is igmmeps ( $1 \mathrm{e}-6$ )
specify \# for iterated GMM weight-matrix convergence criterion; default is igmmweps (1e-6)
control the optimization process; seldom used

[^30]
## Menu

Statistics $>$ Endogenous covariates $>$ Poisson regression with endogenous regressors

## Description

ivpoisson estimates the parameters of a Poisson regression model in which some of the regressors are endogenous. The model is also known as an exponential conditional mean model in which some of the regressors are endogenous. The model may be specified using either additive or multiplicative error terms. The model is frequently used to model count outcomes and is also used to model nonnegative outcome variables.

## Options

## Model

noconstant, exposure ( varname $_{e}$ ), offset ( varname $_{o}$ ); see $[R]$ estimation options.
additive, the default, specifies that the regression errors be added to the conditional mean term and have mean 0 .
multiplicative specifies that the regression errors be multiplied by the conditional mean term and have mean 1.
twostep, onestep, and igmm specify which estimator is to be used.
twostep requests the two-step GMM estimator. gmm obtains parameter estimates based on the initial weight matrix, computes a new weight matrix based on those estimates, and then reestimates the parameters based on that weight matrix. twostep is the default for ivpoisson gmm.
onestep requests the one-step GMM estimator. The parameters are estimated based on an initial weight matrix, and no updating of the weight matrix is performed except when calculating the appropriate variance-covariance (VCE) matrix. onestep is the default for ivpoisson cfunction.
igmm requests the iterative GMM estimator. gmm obtains parameter estimates based on the initial weight matrix, computes a new weight matrix based on those estimates, reestimates the parameters based on that weight matrix, computes a new weight matrix, and so on, to convergence. Convergence is declared when the relative change in the parameter vector is less than igmmeps (), the relative change in the weight matrix is less than igmmweps(), or igmmiterate() iterations have been completed. Hall (2005, sec. 2.4 and 3.6) mentions that there may be gains to finite-sample efficiency from using the iterative estimator.

Weight matrix
wmatrix (wmtype) specifies the type of weight matrix to be used in conjunction with the two-step and iterated GMM estimators.

Specifying wmatrix(robust) requests a weight matrix that is appropriate when the errors are independent but not necessarily identically distributed. wmatrix (robust) is the default.

Specifying wmatrix(cluster clustvar) requests a weight matrix that accounts for arbitrary correlation among observations within clusters identified by clustvar.
Specifying wmatrix (unadjusted) requests a weight matrix that is suitable when the errors are homoskedastic.
wmatrix() cannot be specified if onestep is also specified.
center requests that the sample moments be centered (demeaned) when computing GMM weight matrices. By default, centering is not done.
winitial (wmtype [, independent]) specifies the weight matrix to use to obtain the first-step parameter estimates.
Specifying winitial(unadjusted) requests a weighting matrix that assumes the error functions are independent and identically distributed. This matrix is of the form $\left(\mathbf{Z}^{\prime} \mathbf{Z}\right)^{-1}$, where $\mathbf{Z}$ represents all the exogenous and instrumental variables.
winitial (identity) requests that the identity matrix be used.
winitial (matname) requests that Stata matrix matname be used.
Including the independent suboption creates a weight matrix that assumes error functions are independent. Elements of the weight matrix corresponding to covariances between any two error functions are set equal to zero. This suboption only applies to ivpoisson cfunction.
winitial (unadjusted) is the default for ivpoisson gmm.
winitial (unadjusted, independent) is the default for ivpoisson cfunction.

## SE/Robust

vce (vcetype) specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
vce (unadjusted) specifies that an unadjusted (nonrobust) VCE matrix be used; this, along with the twostep option, results in the "optimal two-step GMM" estimates often discussed in textbooks. vce (unadjusted) may not be set in ivpoisson cfunction.
The default vcetype is based on the wmtype specified in the wmatrix() option. If wmatrix() is specified but vce() is not, then vcetype is set equal to wmtype. To override this behavior in ivpoisson gmm and obtain an unadjusted (nonrobust) VCE matrix, specify vce(unadjusted). The default vcetype for ivpoisson cfunction is robust.
Specifying vce (bootstrap) or vce (jackknife) results in standard errors based on the bootstrap or jackknife, respectively. See $[R]$ vee_option, $[R]$ bootstrap, and $[R]$ jackknife for more information on these VCEs.

The syntax for veetypes is identical to those for wmatrix().

## Reporting

level (\#); see [R] estimation options.
irr reports estimated coefficients transformed to incidence-rate ratios, that is, $e^{\beta_{i}}$ rather than $\beta_{i}$. Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated or stored. irr may be specified at estimation or when replaying previously estimated results. irr is not allowed with additive.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% fmt), pformat (\% fmt), sformat (\% fmt), and nolstretch; see [R] estimation options.

Optimization
from(initial_values) specifies the initial values to begin the estimation. You can specify a $1 \times k$ matrix, where $k$ is the number of parameters in the model, or you can specify a parameter name,
its initial value, another parameter name, its initial value, and so on. For example, to initialize the coefficient for male to 1.23 and the constant _cons to 4.57 , you would type

```
ivpoisson ..., from(male 1.23 _cons 4.57) ...
```

Initial values declared using this option override any that are declared within substitutable expressions. If you specify a parameter that does not appear in your model, ivpoisson exits with error code 480 . If you specify a matrix, the values must be in the same order in which the parameters are declared in your model. ivpoisson ignores the row and column names of the matrix.
igmmiterate(\#), igmmeps(\#), and igmmweps(\#) control the iterative process for the iterative GMM estimator for ivpoisson. These options can be specified only if you also specify igmm.
igmmiterate(\#) specifies the maximum number of iterations to perform with the iterative GMM estimator. The default is the number set using set maxiter (set [R] maximize), which is 16,000 by default.
igmmeps (\#) specifies the convergence criterion used for successive parameter estimates when the iterative GMM estimator is used. The default is igmmeps ( $1 \mathrm{e}-6$ ). Convergence is declared when the relative difference between successive parameter estimates is less than igmmeps () and the relative difference between successive estimates of the weight matrix is less than igmmweps ().
igmmweps (\#) specifies the convergence criterion used for successive estimates of the weight matrix when the iterative GMM estimator is used. The default is igmmweps ( $1 \mathrm{e}-6$ ). Convergence is declared when the relative difference between successive parameter estimates is less than igmmeps () and the relative difference between successive estimates of the weight matrix is less than igmmweps().
optimization_options: technique(), conv_maxiter(), conv_ptol(), conv_vtol(),
conv_nrtol(), and tracelevel(). technique() specifies the optimization technique to use;
gn (the default), nr , dfp , and bfgs are allowed. conv_maxiter() specifies the maximum number of iterations; conv_ptol(), conv_vtol(), and conv_nrtol() specify the convergence criteria for the parameters, gradient, and scaled Hessian, respectively. tracelevel() allows you to obtain additional details during the iterative process. See [M-5] optimize().

## Remarks and examples

ivpoisson estimates the parameters of a Poisson regression model in which some of the regressors are endogenous. A regressor is endogenous if it is related to the unobserved error term. The model is also known as an exponential conditional mean model in which some of the regressors are endogenous. The model may be specified using either additive or multiplicative error terms.

The model is frequently used to model count outcomes and is also used to model nonnegative outcome variables. Poisson regression is a special exponential conditional mean model. See [R] poisson for more information on Poisson regression.

The exponential conditional mean model has an error form representation in which the dependent variable $y$ is a function of the exogenous regressors $\mathbf{x}$, endogenous regressors $\mathbf{y}_{2}$, and an error $\epsilon$. The regressors $\mathbf{x}$ are independent of $\epsilon$, while $\mathbf{y}_{2}$ are not.
ivpoisson allows $\epsilon$ to enter either additively,

$$
y_{i}=\exp \left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}_{1}+\mathbf{y}_{2, i}^{\prime} \boldsymbol{\beta}_{2}\right)+\epsilon_{i}
$$

or multiplicatively,

$$
y_{i}=\exp \left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}_{1}+\mathbf{y}_{2, i}^{\prime} \boldsymbol{\beta}_{2}\right) \epsilon_{i}
$$

Mullahy (1997), Cameron and Trivedi (2013), Windmeijer and Santos Silva (1997), and Wooldridge (2010) discuss the generalized method of moments (GMM) estimators implemented in ivpoisson. GMM is frequently used in modern econometrics. Many econometric and statistical models can be expressed as conditions on the population moments. The parameter estimates produced by GMM estimators make the sample-moment conditions as true as possible given the data. See $[R]$ gmm for further information on GMM estimation and how Stata performs it.

The rest of the discussion is presented under the following headings:

> GMM estimator for additive model
> GMM estimator for multiplicative model
> $C F$ estimator for multiplicative model

## GMM estimator for additive model

The GMM estimator uses additional variables, known as instruments and denoted by $\mathbf{z}_{i}$, to specify moment conditions that hold in the population. The GMM parameter estimates make the sample versions of these population-moment conditions as close to true as possible. The instrumental variables are assumed to be correlated with the endogenous regressors $\mathbf{y}_{2, i}$ but independent of the errors $\epsilon_{i}$.

Under additive errors, the dependent variable $y_{i}$ is determined by exogenous regressors $\mathbf{x}_{i}$, endogenous regressors $\mathbf{y}_{2, i}$, and zero-mean error $\epsilon_{i}$ as

$$
y_{i}=\exp \left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}_{1}+\mathbf{y}_{2, i}^{\prime} \boldsymbol{\beta}_{2}\right)+\epsilon_{i}
$$

This leads to the following error function

$$
u\left(y_{i}, \mathbf{x}_{i}, \mathbf{y}_{2, i}, \boldsymbol{\beta}_{1}, \boldsymbol{\beta}_{2}\right)=y_{i}-\exp \left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}_{1}+\mathbf{y}_{2, i}^{\prime} \boldsymbol{\beta}_{2}\right)
$$

The population-moment conditions for GMM estimation are $E\left\{\widetilde{\mathbf{z}}_{i} u\left(y_{i}, \mathbf{x}_{i}, \mathbf{y}_{2, i}, \boldsymbol{\beta}_{1}, \boldsymbol{\beta}_{2}\right)\right\}=\mathbf{0}$, where the vector $\widetilde{\mathbf{z}}_{i}$ is partitioned as $\left(\mathbf{x}_{i}^{\prime}, \mathbf{z}_{i}^{\prime}\right)$. The sample-moment conditions are formed by replacing the expectation with the corresponding sample mean. The GMM estimator solves a minimization problem to make the sample-moment conditions as close to zero as possible. Details on how estimation is performed are given in Methods and formulas.

Now we will demonstrate how ivpoisson gmm works in the additive error setting with an example.

## > Example 1: ivpoisson gmm with additive errors

This example uses simulated data based on the following story. A news website randomly samples 500 young adults in a major city. The website wants to model the number of times the sampled individuals visit its website (visits) based on their overall time spent on the Internet (time) and the number of times they receive an ad for the website through email or viewing another website (ad). The website also suspects the gender of the individual may matter, so an exogenous dummy variable, female, is included in the model.

We suspect time spent on the Internet is correlated with unobserved factors that additively affect the number of times an individual visits the website. So we treat time as an endogenous regressor. Two instruments are used for this variable. The time spent on the phone (phone) is one instrument. The other instrument is the time spent interacting with friends and family that live out of town (frfam).

We model the number of visits the website receives using an exponential conditional mean model with additive errors and use ivpoisson gmm to estimate the parameters of the regression in the output below. To allow for heteroskedasticity of the errors, we use robust standard errors, which is the default; see Obtaining standard errors in [R] gmm for a discussion of why robust standard errors is the default.

```
.use http://www.stata-press.com/data/r13/website
(Visits to website)
. ivpoisson gmm visits ad female (time = phone frfam)
Step 1
Iteration 0: GMM criterion Q(b) = . 33829416
Iteration 1: GMM criterion Q(b) = .00362656
Iteration 2: GMM criterion Q(b) = .00131886
Iteration 3: GMM criterion Q(b) = .00131876
Step 2
Iteration 0: GMM criterion Q(b) = .00027102
Iteration 1: GMM criterion Q(b) = .00025811
Iteration 2: GMM criterion Q(b) = .00025811
Exponential mean model with endogenous regressors
Number of parameters = 4 Number of obs = 500
Number of moments = 5
Initial weight matrix: Unadjusted
GMM weight matrix: Robust
```

| visits | Coef. | Robust <br> Std. Err. | z | $P>\|z\|$ | [95\% Conf. | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| time | . 0589294 | . 0107942 | 5.46 | 0.000 | . 0377732 | . 0800857 |
| ad | . 137344 | . 010157 | 13.52 | 0.000 | . 1174366 | . 1572515 |
| female | -. 0247707 | . 0376218 | -0.66 | 0.510 | -. 098508 | . 0489666 |
| _cons | 1.041505 | . 0385848 | 26.99 | 0.000 | . 9658807 | 1.11713 |

Instrumented: time
Instruments: ad female phone frfam
We find significant coefficients for all regressors but female. At fixed values of the other regressors, increased time spent on the Internet will raise the expected number of website visits. Receiving additional advertisements will also cause an increase in the expected number of website visits.

## GMM estimator for multiplicative model

Under multiplicative errors, the dependent variable $y_{i}$ is determined by exogenous regressors $\mathbf{x}_{i}$, endogenous regressors $\mathbf{y}_{2, i}$, and unit-mean errors $\epsilon_{i}$ as

$$
y_{i}=\exp \left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}_{1}+\mathbf{y}_{2, i}^{\prime} \boldsymbol{\beta}_{2}\right) \epsilon_{i}
$$

This setting yields a different error function than the additive error case. This ratio formulation is

$$
u\left(y_{i}, \mathbf{x}_{i}, \mathbf{y}_{2, i}, \boldsymbol{\beta}_{1}, \boldsymbol{\beta}_{2}\right)=y_{i} / \exp \left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}_{1}+\mathbf{y}_{2, i}^{\prime} \boldsymbol{\beta}_{2}\right)-1
$$

Given the instrumental variables $\mathbf{z}$, the population-moment conditions for GMM estimation are $E\left\{\widetilde{\mathbf{z}}_{i} u\left(y_{i}, \mathbf{x}_{i}, \mathbf{y}_{2, i}, \boldsymbol{\beta}_{1}, \boldsymbol{\beta}_{2}\right)\right\}=\mathbf{0}$. The vector $\widetilde{\mathbf{z}}_{i}$ is partitioned as $\left(\mathbf{x}_{i}^{\prime}, \mathbf{z}_{i}^{\prime}\right)$. As above, the samplemoment conditions are the sample analogs of the population-moment conditions, and the GMM estimator solves a minimization problem to make the sample-moment conditions as close to zero as possible. Details on how estimation is performed are given in Methods and formulas.

## > Example 2: ivpoisson gmm with multiplicative errors

In this example, we observe a simulated random sample of 5,000 households. We model the number of trips taken by members of the household in the 24 -hour period immediately prior to the interview time by using an exponential conditional mean model with multiplicative errors. Exogenous regressors include the distance to the central business district from the household (cbd), the distance from the household to a public transit node (ptn), whether there is a full-time worker in the household (worker), and whether the examined period is on a weekend (weekend). We suspect that the endogenous regressor, the transportation cost of the household in the prior week (tcost), is correlated with unobserved factors that affect the number of trips taken. This transportation cost includes gasoline and bus, train tickets, etc.

The ratio of the cost of a public transit day pass in the sampled area to the national average cost of such a pass ( pt ) is also observed. This is used as an instrument for transportation cost.

In the output below, we estimate the parameters of the regression with ivpoisson gmm. To allow for heteroskedasticity of the errors, we use robust standard errors, which is the default.

```
. use http://www.stata-press.com/data/r13/trip
(Household trips)
. ivpoisson gmm trips cbd ptn worker weekend (tcost = pt), multiplicative
Step 1
Iteration 0: GMM criterion Q(b) = . 04949852
Iteration 1: GMM criterion Q(b) = .00011194
Iteration 2: GMM criterion Q(b) = 1.563e-08
Iteration 3: GMM criterion Q(b) = 3.685e-16
Step 2
Iteration 0: GMM criterion Q(b) = 2.287e-16
Iteration 1: GMM criterion Q(b) = 1.413e-31
```

Exponential mean model with endogenous regressors
Number of parameters $=6 \quad$ Number of obs $=5000$
Number of moments $=6$
Initial weight matrix: Unadjusted
GMM weight matrix: Robust

|  | Robust |  |  |  |  |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| trips | Coef. | Std. Err. | $\mathbf{z}$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| tcost | .0352185 | .0098182 | 3.59 | 0.000 | .0159752 | .0544617 |
| cbd | -.008398 | .0020172 | -4.16 | 0.000 | -.0123517 | -.0044444 |
| ptn | -.0113146 | .0021819 | -5.19 | 0.000 | -.015591 | -.0070383 |
| worker | .6623018 | .0519909 | 12.74 | 0.000 | .5604015 | .764202 |
| weekend | .3009323 | .0362682 | 8.30 | 0.000 | .2298479 | .3720167 |
| _cons | .2654423 | .1550127 | 1.71 | 0.087 | -.0383769 | .5692616 |

Instrumented: tcost
Instruments: cbd ptn worker weekend pt
We find that all coefficients are significant. At fixed values of the other regressors, we see that additional mileage from the central business district and public transit nodes reduces the expected number of trips taken. Individuals who live farther away from the central business district may still be out of the house the same amount of time, but they will take fewer trips because the transit time has increased. The situation is similar for those who live farther from public transit.

To interpret the other parameters, we will look at the partial effects of their respective independent variables. The partial effects of a change in an independent variable on the modeled conditional expectation function vary over the data because the model is nonlinear. However, under the multiplicative
error model, the ratio of the new value to the old value after a discrete change in an independent variable is constant over the data.

Let $\mathbf{w}=\left(\mathbf{x}^{\prime}, \mathbf{y}_{2}^{\prime}\right)^{\prime}$. If we add 1 to the $j$ th independent variable in $\mathbf{w}$, the functional form of the model implies that

$$
\frac{E\left(y \mid \mathbf{w}, w_{j}+1, \epsilon\right)}{E\left(y \mid \mathbf{w}, w_{j}, \epsilon\right)}=\frac{E\left(y \mid w_{1}, \ldots, w_{j}+1, \ldots, w_{k}, \epsilon\right)}{E\left(y \mid w_{1}, \ldots, w_{j}, \ldots, w_{k}, \epsilon\right)}=e^{\beta_{j}}
$$

When $y$ is a count variable, this normalized effect is called the incidence-rate ratio (IRR) for a one-unit change in $w_{j}$.

More generally, the IRR for a $\Delta w_{j}$ change in $w_{j}$ is $e^{\beta_{j} \Delta w_{j}}$ under a multiplicative-error exponential conditional mean model. We can calculate incidence-rate ratios for different changes in the regressors by using lincom; see [ R ] lincom.

Here we replay the ivpoisson results by typing the command name and we specify the irr option to get the incidence-rate ratios. Each significance test for a coefficient equaling zero becomes a test for the incidence-rate ratio equaling one.

```
. ivpoisson, irr
Exponential mean model with endogenous regressors
Number of parameters = 6 Number of obs = 5000
Number of moments = 6
Initial weight matrix: Unadjusted
GMM weight matrix: Robust
```

|  | Robust <br> trips |  |  |  |  | IRR |
| ---: | ---: | :---: | :---: | :---: | :---: | ---: |
|  | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |  |
| tcost | 1.035846 | .0101701 | 3.59 | 0.000 | 1.016103 | 1.055972 |
| cbd | .9916371 | .0020003 | -4.16 | 0.000 | .9877243 | .9955655 |
| ptn | .9887491 | .0021573 | -5.19 | 0.000 | .9845299 | .9929864 |
| worker | 1.939251 | .1008234 | 12.74 | 0.000 | 1.751376 | 2.14728 |
| weekend | 1.351118 | .0490026 | 8.30 | 0.000 | 1.258409 | 1.450657 |
| _cons | 1.304008 | .2021377 | 1.71 | 0.087 | .9623501 | 1.766962 |

Instrumented: tcost
Instruments: cbd ptn worker weekend pt
Holding other regressors and the error constant, the expected number of trips made from houses with a full-time worker is nearly twice that of those houses without a full-time worker. Similarly, the expected number of trips made during a weekend day is close to $35 \%$ higher than the expected number of trips made on other days. For each additional dollar of weekly transportation cost, the expected number of household trips is increased by approximately $3.6 \%$.

## CF estimator for multiplicative model

Control-function (CF) estimators can be used to account for endogenous regressors. As Wooldridge (2010, sec. 18.5) describes, CF estimators assume a certain structural relationship between the endogenous regressors and the exogenous regressors and use functions of first-stage parameter estimates to control for the endogeneity in the second stage.

Wooldridge (2010, sec. 18.5) notes that the VCE of the second-stage estimator must be adjusted to account for estimates from the first stage. ivpoisson cfunction solves this problem by stacking the moment conditions that define each stage and applying a single GMM estimator. See Newey (1984) and Wooldridge (2010, sec. 14.2) for a description of this technique. No adjustment to the VCE is necessary because there is only one stage.

The CF estimator augments the original multiplicative model with an estimated term that controls for the endogeneity of $\mathbf{y}_{2, i}$. When $\mathbf{y}_{2, i}$ is exogenous, the coefficient on this control term is zero. Let $\mathbf{z}$ be instrumental variables, and the vector $\widetilde{\mathbf{z}}_{i}$ be $\left(\mathbf{x}_{i}^{\prime}, \mathbf{z}_{i}^{\prime}\right)$.

The augmented model is

$$
y_{i}=\exp \left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}_{1}+\mathbf{y}_{2, i}^{\prime} \boldsymbol{\beta}_{2}+\mathbf{v}_{i}^{\prime} \boldsymbol{\rho}+c_{i}\right)
$$

where

$$
\mathbf{y}_{2, i}=\mathbf{B} \widetilde{\mathbf{z}}_{i}^{\prime}+\mathbf{v}_{i}
$$

The term $\mathbf{v}_{i}^{\prime} \boldsymbol{\rho}$ controls for the endogeneity of $\mathbf{y}_{2, i}$, and we normalize $E\left\{\exp \left(c_{i}\right)\right\}=1$. The coefficient vector $\boldsymbol{\rho}$ measures the strength of the endogeneity of $\mathbf{y}_{2, i} ; \mathbf{y}_{2, i}$ is exogenous when $\boldsymbol{\rho}=\mathbf{0}$.
ivpoisson cfunction estimates $\boldsymbol{\beta}_{1}$ and $\boldsymbol{\beta}_{2}$ and the auxiliary parameters $\boldsymbol{\rho}$ and $\mathbf{B}$ by GMM; see Methods and formulas for details.

## > Example 3: Control-function estimator

We return to the previous example, where we estimated the parameters of an exponential conditional mean model for the number of trips taken by a household in a 24 -hour period. We will estimate the parameters of the regression with the CF estimator method and compare our results with those obtained with the GMM estimator in example 2.

In the output below, we estimate the parameters of the regression with the ivpoisson cfunction command.

```
. ivpoisson cfunction trips cbd ptn worker weekend (tcost = pt)
Step 1
Iteration 0: GMM criterion Q(b) = .00056156
Iteration 1: GMM criterion Q(b) = 2.366e-07
Iteration 2: GMM criterion Q(b) = 5.552e-14
Iteration 3: GMM criterion Q(b) = 9.772e-27
Exponential mean model with endogenous regressors
Number of parameters = 13 Number of obs = 5000
Number of moments = 13
Initial weight matrix: Unadjusted
GMM weight matrix: Robust
```

| trips | Coef. | Robust Std. Err. | z | $P>\|z\|$ | [95\% Conf | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| trips |  |  |  |  |  |  |
| cbd | -. 0082567 | . 0020005 | -4.13 | 0.000 | -. 0121777 | -. 0043357 |
| ptn | -. 0113719 | . 0021625 | -5.26 | 0.000 | -. 0156102 | -. 0071335 |
| worker | . 6903044 | . 0521642 | 13.23 | 0.000 | . 5880645 | . 7925444 |
| weekend | . 2978149 | . 0356474 | 8.35 | 0.000 | . 2279472 | . 3676825 |
| tcost | . 0320718 | . 0092738 | 3.46 | 0.001 | . 0138955 | . 0502481 |
| _cons | . 2145986 | . 1359327 | 1.58 | 0.114 | -. 0518246 | . 4810218 |
| tcost |  |  |  |  |  |  |
| cbd | . 0165466 | . 0043693 | 3.79 | 0.000 | . 0079829 | . 0251102 |
| ptn | -. 040652 | . 0045946 | -8.85 | 0.000 | -. 0496573 | -. 0316467 |
| worker | 1.550985 | . 0996496 | 15.56 | 0.000 | 1.355675 | 1.746294 |
| weekend | . 0423009 | . 0779101 | 0.54 | 0.587 | -. 1104002 | . 1950019 |
| pt | . 7739176 | . 0150072 | 51.57 | 0.000 | . 7445041 | . 8033312 |
| _cons | 12.13934 | . 1123471 | 108.05 | 0.000 | 11.91915 | 12.35954 |
| /c_tcost | . 1599984 | . 0111752 | 14.32 | 0.000 | . 1380954 | . 1819014 |
| Instrumented: tcost |  |  |  |  |  |  |

The output table presents results for the estimated coefficients in each of three equations. First, in the trips equation, we see the results for the estimated coefficients in the equation for the dependent variable trips. Second, in the tcost equation, we see the estimated coefficients in the regression of tcost on the instrumental and exogenous variables. Third, the /c_tcost ancillary parameter corresponds to the estimate of $\rho$, the coefficient on the residual variable included to control for the endogeneity of tcost.

We find that all coefficients are significant in the exponential conditional mean equation, trips. The coefficient estimates in the trips equation are similar to the estimates obtained by the GMM estimator in example 2. That the estimated coefficient on the tcost control variable is significantly different from zero suggests that tcost is endogenous.

## Stored results

ivpoisson stores the following in e():
Scalars

```
e(N)
e(k)
e(k_eq)
e(k_aux)
e(k_dv)
e(Q)
e(J)
e(J_df)
e(N_clust)
e(rank)
e(ic)
e(converged)
```

Macros
e(cmd)
e(cmdline)
e(depvar)
e(instd)
e(insts)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(offset1)
e(winit)
e(winitname)
e(estimator)
e(additive)
e(multiplicative)
e(gmmestimator)
e(wmatrix)
e(vce)
e(vcetype)
e(technique)
e(properties)
e(estat_cmd)
e(predict)
e(footnote)
e(marginsok)
e(asbalanced)
e(asobserved)
Matrices
e(b)
e(V)
e(init)
e(Wuser)
e(W)
e(S)
e(V_modelbased)
Functions
e(sample)
number of observations
number of parameters
number of equations
number of auxiliary parameters
number of dependent variables
criterion function
Hansen $J \chi^{2}$ statistic
$J$ statistic degrees of freedom
number of clusters
rank of $e(V)$
number of iterations used by iterative GMM estimator
1 if converged, 0 otherwise
ivpoisson
command as typed
dependent variable of regression
instrumented variable
instruments
weight type
weight expression
title in estimation output
name of cluster variable
offset variable for first equation
initial weight matrix used
name of user-supplied initial weight matrix
gmm or cfunction
additive if additive errors specified
multiplicative if multiplicative errors specified
onestep, twostep, or igmm
wmtype specified in wmatrix()
vcetype specified in vce()
title used to label Std. Err.
optimization technique
b V
program used to implement estat
program used to implement predict
program used to implement footnote display
predictions allowed by margins
factor variables fvset as asbalanced
factor variables fvset as asobserved
coefficient vector
variance-covariance matrix
initial values of the estimators
user-supplied initial weight matrix
weight matrix used for final round of estimation moment covariance matrix used in robust VCE computations model-based variance
marks estimation sample

## Methods and formulas

The estimators in ivpoisson are GMM estimators that can be expressed in terms of error functions and the instruments that are used to form the moment conditions. When offsets $o_{j}^{\beta}$ are used in the outcome variable equation, the following formulas apply with $\mathbf{x}_{j}^{\prime} \boldsymbol{\beta}_{1}$ changed to $\mathbf{x}_{j}^{\prime} \boldsymbol{\beta}_{1}+o_{j}^{\beta}$.

The error functions for the GMM estimators are given in the text.
Here we provide some details about the form of the error function used by the CF estimator.
Recall that the multiplicative model is

$$
y_{i}=\exp \left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}_{1}+\mathbf{y}_{2, i}^{\prime} \boldsymbol{\beta}_{2}\right) \epsilon_{i}
$$

We parameterize the endogenous variables in the form

$$
\mathbf{y}_{2, i}=\mathbf{B} \widetilde{\mathbf{z}}_{i}^{\prime}+\mathbf{v}_{i}
$$

This allows us to decompose $\epsilon_{i}$ as

$$
\epsilon_{i}=\exp \left(\mathbf{v}_{i}^{\prime} \boldsymbol{\rho}+c_{i}\right)
$$

Given this setup, we obtain the following conditional mean:

$$
E\left(y \mid \mathbf{x}_{i}, \mathbf{z}_{i}, \mathbf{y}_{2, i}, \mathbf{v}_{i}\right)=\exp \left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}_{1}+\mathbf{y}_{2, i}^{\prime} \boldsymbol{\beta}_{2}+\mathbf{v}_{i}^{\prime} \boldsymbol{\rho}\right)
$$

We estimate $\mathbf{v}_{i}$ as the residuals of the linear regression of $\mathbf{y}_{2, i}$ on $\widetilde{\mathbf{z}}_{i}$. The estimates of $\mathbf{v}_{i}$ are used as additional regressors in the exponential conditional mean model for $y$ to estimate $\boldsymbol{\beta}_{1}, \boldsymbol{\beta}_{2}$, and $\rho$. In essence, the estimates of $\mathbf{v}_{i}$ control for the endogeneity.

The error functions for the endogenous regressors are defined as

$$
\mathbf{u}_{e n, i}\left(\mathbf{y}_{2, i}, \widetilde{\mathbf{z}}_{i}, \mathbf{B}\right)=\mathbf{y}_{2, i}-\mathbf{B} \widetilde{\mathbf{z}}_{i}^{\prime}
$$

Now we define the error function for the dependent variable as

$$
u_{y}\left(y_{i}, \mathbf{x}_{i}, \mathbf{y}_{2, i}, \mathbf{u}_{e n, i}, \boldsymbol{\beta}_{1}, \boldsymbol{\beta}_{2}, \boldsymbol{\rho}\right)=y_{i} / \exp \left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}_{1}+\mathbf{y}_{2, i}^{\prime} \boldsymbol{\beta}_{2}+\mathbf{u}_{e n, i}^{\prime} \boldsymbol{\rho}\right)-1
$$

$\mathbf{u}_{e n, i}$ will be vector valued if we have multiple endogenous regressors $\mathbf{y}_{2, i}$. Call the dimension of $\mathbf{y}_{2, i} g . \mathbf{u}_{e n, i}$ and $u_{y, i}$ define $g+1$ separate error functions. We will use variables $\widetilde{\mathbf{z}}_{i}$ to instrument each error function in $\mathbf{u}_{e n, i}$. So for error function $j=1, \ldots, g$, we have the error function $u_{e n, i, j}$ and the population-moment conditions $E\left(\widetilde{\mathbf{z}}_{i} u_{e n, i, j}\right)=\mathbf{0}$.

We calculate $\widehat{\mathbf{v}}_{o i}$ previous to estimation as the residuals of the linear regression of $\mathbf{y}_{2, i}$ on $\widetilde{\mathbf{z}}_{i}$. We use variables $\mathbf{x}_{i}, \mathbf{y}_{2, i}$, and $\widehat{\mathbf{v}}_{o i}$ to instrument the error function $u_{y}$. This leads to the population-moment conditions $E\left\{\left(\mathbf{x}_{i}^{\prime}, \mathbf{y}_{2, i}^{\prime}, \widehat{\mathbf{v}}_{o i}^{\prime}\right) u_{y, i}\right\}=\mathbf{0}$

Details of GMM estimation can be found in Methods and formulas of $[\mathrm{R}]$ gmm. Determination of the weight matrix $\mathbf{W}_{N}$ is discussed there.

Under the GMM estimation, the GMM estimators $\widehat{\boldsymbol{\beta}}_{1}$ and $\widehat{\boldsymbol{\beta}}_{2}$ are the values of $\boldsymbol{\beta}_{1}$ and $\boldsymbol{\beta}_{2}$ that minimize

$$
\begin{equation*}
Q\left(\boldsymbol{\beta}_{1}, \boldsymbol{\beta}_{2}\right)=\left\{\frac{1}{N} \sum_{i} \widetilde{\mathbf{z}}_{i} u_{i}\left(y_{i}, \mathbf{x}_{i}, \mathbf{y}_{2, i}, \boldsymbol{\beta}_{1}, \boldsymbol{\beta}_{2}\right)\right\}^{\prime} \mathbf{W}_{N}\left\{\frac{1}{N} \sum_{i} \widetilde{\mathbf{z}}_{i} u_{i}\left(y_{i}, \mathbf{x}_{i}, \mathbf{y}_{2, i}, \boldsymbol{\beta}_{1}, \boldsymbol{\beta}_{2}\right)\right\} \tag{1}
\end{equation*}
$$

for $q \times q$ weight matrix $\mathbf{W}_{N}$, where $q$ is the dimension of $\widetilde{\mathbf{z}}_{i}$. The error functions $u_{i}$ were defined in the text.

In the CF method, we have multiple error functions as defined above. We can stack the moment conditions and write them more compactly as $\mathbf{Z}_{i}^{\prime} \mathbf{u}_{i}\left(\mathbf{B}, \boldsymbol{\beta}_{1}, \boldsymbol{\beta}_{2}, \boldsymbol{\rho}\right)$, where

$$
\mathbf{Z}_{i}=\left[\begin{array}{ccccc}
\mathbf{x}_{i}^{\prime} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\
\mathbf{0} & \mathbf{y}_{2, i}^{\prime} & \mathbf{0} & \cdots & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \widetilde{\mathbf{z}}_{i} & \cdots & \mathbf{0} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \widetilde{\mathbf{z}}_{i}
\end{array}\right]
$$

and

$$
\mathbf{u}_{i}\left(\mathbf{B}, \boldsymbol{\beta}_{1}, \boldsymbol{\beta}_{2}, \boldsymbol{\rho}\right)=\left[\begin{array}{c}
u_{y}\left(y_{i}, \mathbf{x}_{i}, \mathbf{y}_{2, i}, \mathbf{u}_{e n, i}, \boldsymbol{\beta}_{1}, \boldsymbol{\beta}_{2}, \boldsymbol{\rho}\right) \\
\mathbf{u}_{e n}\left(\mathbf{y}_{2, i}, \widetilde{\mathbf{z}}_{i}, \mathbf{B}\right)
\end{array}\right]
$$

The matrix $\mathbf{Z}_{i}$ has $g+1$ rows and $k+g z$ columns, where $k$ is the number of regressors for $y_{i}$ and $z$ is the number of exogenous regressors in $\widetilde{\mathbf{z}}_{i}$.

The GMM estimators $\widehat{\mathbf{B}}, \widehat{\boldsymbol{\beta}}_{1}, \widehat{\boldsymbol{\beta}}_{2}$, and $\widehat{\boldsymbol{\rho}}$ are the values of $\mathbf{B}, \boldsymbol{\beta}_{1}, \boldsymbol{\beta}_{2}$, and $\boldsymbol{\rho}$ that minimize

$$
\begin{align*}
Q\left(\mathbf{B}, \boldsymbol{\beta}_{1}, \boldsymbol{\beta}_{2}, \boldsymbol{\rho}\right) & = \\
& \left\{N^{-1} \sum_{i=1}^{N} \mathbf{Z}_{i}^{\prime} \mathbf{u}_{i}\left(\mathbf{B}, \boldsymbol{\beta}_{1}, \boldsymbol{\beta}_{2}, \boldsymbol{\rho}\right)\right\}^{\prime} \mathbf{W}_{N}\left\{N^{-1} \sum_{i=1}^{N} \mathbf{Z}_{i}^{\prime} \mathbf{u}_{i}\left(\mathbf{B}, \boldsymbol{\beta}_{1}, \boldsymbol{\beta}_{2}, \boldsymbol{\rho}\right)\right\} \tag{2}
\end{align*}
$$

for $(k+g z) \times(k+g z)$ weight matrix $\mathbf{W}_{N}$.
By default, ivpoisson minimizes (1) and (2) using the Gauss-Newton method. See Hayashi (2000, 498) for a derivation. This technique is typically faster than quasi-Newton methods and does not require second-order derivatives.

## References

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## Also see

$[R]$ ivpoisson postestimation - Postestimation tools for ivpoisson
$[\mathrm{R}]$ gmm - Generalized method of moments estimation
[R] ivprobit - Probit model with continuous endogenous regressors
$[\mathrm{R}]$ ivregress - Single-equation instrumental-variables regression
[R] ivtobit - Tobit model with continuous endogenous regressors
[R] nl - Nonlinear least-squares estimation
[R] nlsur - Estimation of nonlinear systems of equations
[R] poisson - Poisson regression
$[\mathrm{R}]$ regress - Linear regression
[U] 20 Estimation and postestimation commands

## Title

ivpoisson postestimation - Postestimation tools for ivpoisson

Description
Options for predict
Remarks and examples
Reference

Syntax for predict
Syntax for estat overid
Stored results
Also see

Menu for predict
Menu for estat
Methods and formulas

## Description

The following postestimation command is of special interest after ivpoisson:
Command Description
estat overid perform test of overidentifying restrictions

The following standard postestimation commands are also available:
\(\left.$$
\begin{array}{ll}\text { Command } & \text { Description } \\
\hline \text { contrast } & \text { contrasts and ANOVA-style joint tests of estimates } \\
\text { estat summarize } & \begin{array}{l}\text { summary statistics for the estimation sample } \\
\text { estat vce } \\
\text { variance-covariance matrix of the estimators (VCE) } \\
\text { estimates } \\
\text { lincom }\end{array}
$$ <br>
cataloging estimation results <br>
point estimates, standard errors, testing, and inference for linear combinations <br>

of coefficients\end{array}\right]\)| marginal means, predictive margins, marginal effects, and average marginal |
| :--- |
| effects |

## Special-interest postestimation command

estat overid reports Hansen's $J$ statistic, which is used to determine the validity of the overidentifying restrictions in a GMM model. ivpoisson gmm uses GMM estimation to obtain parameter estimates. Under additive and multiplicative errors, Hansen's $J$ statistic can be accurately reported when more instruments than endogenous regressors are specified. It is not appropriate to report the $J$ statistic after ivpoisson cfunction, because a just-identified model is fit.

If the model is correctly specified in the sense that $E\left\{\widetilde{\mathbf{z}}_{i} u\left(y_{i}, \mathbf{x}_{i}, \mathbf{y}_{2, i}, \boldsymbol{\beta}\right)\right\}=0$, then the sample analog to that condition should hold at the estimated value of $\boldsymbol{\beta}_{1}$ and $\boldsymbol{\beta}_{2}$. The $\widetilde{\mathbf{z}}_{i}$ variables are the exogenous regressors $\mathbf{x}_{i}$ and instrumental variables $\mathbf{z}_{i}$ used in ivpoisson gmm. The $\mathbf{y}_{2, i}$ are the endogenous regressors. The $u$ function is the error function, which will have a different form for multiplicative and additive errors in the regression.

Hansen's $J$ statistic is valid only if the weight matrix is optimal, meaning that it equals the inverse of the covariance matrix of the moment conditions. Therefore, estat overid only reports Hansen's $J$ statistic after two-step or iterated estimation or if you specified winitial (matname) when calling ivpoisson gmm. In the latter case, it is your responsibility to determine the validity of the $J$ statistic.

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic nooffset ]
```

statistic Description

## Main

n
xbtotal
xb
residuals
number of events; the default
linear prediction, using residual estimates for ivpoisson cfunction linear prediction residuals

These statistics are available both in and out of sample; type predict . . . if e(sample) ... if wanted only for the estimation sample.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
n , the default, calculates the predicted number of events via the exponential-form estimate. This is $\exp \left(\mathbf{x}_{j}^{\prime} \boldsymbol{\beta}_{1}+\mathbf{y}_{2, j}^{\prime} \boldsymbol{\beta}_{2}\right)$ if neither offset() nor exposure () was specified, $\exp \left(\mathbf{x}_{j}^{\prime} \boldsymbol{\beta}_{1}+\mathbf{y}_{2, j}^{\prime} \boldsymbol{\beta}_{2}+\right.$ offset $\left._{j}\right)$ if offset() was specified, or $\exp \left(\mathbf{x}_{j}^{\prime} \boldsymbol{\beta}_{1}+\mathbf{y}_{2, j}^{\prime} \boldsymbol{\beta}_{2}\right) \times$ exposure $_{j}$ if exposure() was specified.

After generalized method of moments estimation, the exponential-form estimate is not a consistent estimate of the conditional mean of $y_{j}$, because it is not corrected for $E\left(\epsilon_{j} \mid \mathbf{y}_{2, j}\right)$. More details are found in Methods and formulas.
After control-function estimation, we correct the exponential-form estimate for $E\left(\epsilon_{j} \mid \mathbf{y}_{2, j}\right)$ by using the estimated residuals of $\mathbf{y}_{2, j}$ and the $c^{\prime} *$ auxiliary parameters. This supplements the direct effect of $\mathbf{y}_{2, j}$ and $\mathbf{x}_{j}$ through $\boldsymbol{\beta}_{1}$ and $\boldsymbol{\beta}_{2}$ with the indirect effects of $\mathbf{y}_{2, j}, \mathbf{x}_{j}$, and the instruments $\mathbf{z}_{j}$ through the endogenous error $\epsilon_{j}$. Thus the exponential-form estimate consistently estimates the conditional mean of $y_{j}$.
xbtotal calculates the linear prediction, which is $\mathbf{x}_{j}^{\prime} \boldsymbol{\beta}_{1}+\mathbf{y}_{2, j}^{\prime} \boldsymbol{\beta}_{2}$ if neither offset () nor exposure() was specified, $\mathbf{x}_{j}^{\prime} \boldsymbol{\beta}_{1}+\mathbf{y}_{2, j}^{\prime} \boldsymbol{\beta}_{2}+$ offset $_{j}$ if offset () was specified, or $\mathbf{x}_{j}^{\prime} \boldsymbol{\beta}_{1}+\mathbf{y}_{2, j}^{\prime} \boldsymbol{\beta}_{2}+\ln \left(\right.$ exposure $\left._{j}\right)$ if exposure() was specified.

After control-function estimation, the estimate of the linear form $\mathbf{x}_{j}^{\prime} \boldsymbol{\beta}_{1}$ includes the estimated residuals of the endogenous regressors with coefficients from the $c_{-} *$ auxiliary parameters.
xb calculates the linear prediction, which is $\mathbf{x}_{j}^{\prime} \boldsymbol{\beta}_{1}+\mathbf{y}_{2, j}^{\prime} \boldsymbol{\beta}_{2}$ if neither offset () nor exposure () was specified, $\mathbf{x}_{j}^{\prime} \boldsymbol{\beta}_{1}+\mathbf{y}_{2, j}^{\prime} \boldsymbol{\beta}_{2}+$ offset $_{j}$ if offset() was specified, or $\mathbf{x}_{j}^{\prime} \boldsymbol{\beta}_{1}+\mathbf{y}_{2, j}^{\prime} \boldsymbol{\beta}_{2}+\ln \left(\right.$ exposure $\left._{j}\right)$ if exposure() was specified.
residuals calculates the residuals. Under additive errors, these are calculated as $y_{j}-\exp \left(\mathbf{x}_{j}^{\prime} \boldsymbol{\beta}_{1}+\right.$ $\left.\mathbf{y}_{2, j}^{\prime} \boldsymbol{\beta}_{2}\right)$. Under multiplicative errors, they are calculated as $y_{j} / \exp \left(\mathbf{x}_{j}^{\prime} \boldsymbol{\beta}_{1}+\mathbf{y}_{2, j}^{\prime} \boldsymbol{\beta}_{2}\right)-1$.
When offset () or exposure() is specified, $\mathbf{x}_{j}^{\prime} \boldsymbol{\beta}_{1}$ is not used directly in the residuals. $\mathbf{x}_{j}^{\prime} \boldsymbol{\beta}_{1}+$ offset $_{j}$ is used if offset() was specified. $\mathbf{x}_{j}^{\prime} \boldsymbol{\beta}_{1}+\ln$ (exposure $_{j}$ ) is used if exposure() was specified. See nooffset below.
After control-function estimation, the estimate of the linear form $\mathbf{x}_{j}^{\prime} \boldsymbol{\beta}_{1}$ includes the estimated residuals of the endogenous regressors with coefficients from the $c_{-} *$ auxiliary parameters.
nooffset is relevant only if you specified offset() or exposure() when you fit the model. It modifies the calculations made by predict so that they ignore the offset or exposure variable. nooffset removes the offset from calculations involving both the treat() equation and the dependent count variable.

## Syntax for estat overid

estat overid

## Menu for estat

Statistics $>$ Postestimation $>$ Reports and statistics

## Remarks and examples

estat overid reports Hansen's $J$ statistic, which is used to determine the validity of the overidentifying restrictions in a GMM model. It is not appropriate to use it after ivpoisson cfunction, because a just-identified model is fit.

Recall that the GMM criterion function is

$$
\begin{equation*}
Q(\boldsymbol{\beta})=\left\{\frac{1}{N} \sum_{i} \widetilde{\mathbf{z}}_{i} u\left(y_{i}, \mathbf{x}_{i}, \mathbf{y}_{2, i}, \boldsymbol{\beta}_{1}, \boldsymbol{\beta}_{2}\right)\right\}^{\prime} \mathbf{W}_{N}\left\{\frac{1}{N} \sum_{i} \widetilde{\mathbf{z}}_{i} u\left(y_{i}, \mathbf{x}_{i}, \mathbf{y}_{2, i}, \boldsymbol{\beta}_{1}, \boldsymbol{\beta}_{2}\right)\right\} \tag{A1}
\end{equation*}
$$

Our $u$ function within this formula will change depending on whether we use additive or multiplicative errors. The $\widetilde{\mathbf{z}}$ vector contains the exogenous regressors and instrumental variables used. ivpoisson gmm estimates regression coefficients to minimize $Q$.

Let $l$ be the dimension of $\widetilde{\mathbf{z}}$ and $k$ the number of regressors. If $\mathbf{W}_{N}$ is an optimal weight matrix, under the null hypothesis $H_{0}: E\left\{\widetilde{\mathbf{z}}_{i} u\left(y_{i}, \mathbf{x}_{i}, \mathbf{y}_{2, i}, \boldsymbol{\beta}_{1}, \boldsymbol{\beta}_{2}\right)\right\}=\mathbf{0}$, the test statistic $J=N \times Q \sim \chi^{2}(l-k)$. A large test statistic casts doubt on the null hypothesis.

Because the weight matrix $\mathbf{W}_{N}$ must be optimal, estat overid works only after the two-step and iterated estimation or if you supplied your own initial weight matrix by using the winitial (matname) option of ivpoisson gmm and used the one-step estimator.

Often the overidentifying restrictions test is interpreted as a test of the validity of the instruments z. However, other forms of model misspecification can sometimes lead to a significant test statistic. See Hall (2005, sec. 5.1) for a discussion of the overidentifying restrictions test and its behavior in correctly specified and misspecified models.

Note that ivpoisson gmm defaults to the two-step estimator when other options are not specified to override the default. Thus it is appropriate to perform the $J$ test after the regression of example 1 in $[R]$ ivpoisson.
$>$ Example 1: Specification test
Recall example 1 of $[\mathrm{R}]$ ivpoisson. We estimated the parameters of an exponential conditional mean model for the number of visits to a website. Additive errors were used. Exogenous regressors included the gender of an individual and the number of ads received from the website.

An endogenous regressor, time spent on the Internet, was also included in the model. Two instruments were used. One of the instruments measured the time spent interacting with friends and out-of-town family. The other measured the time spent on the phone.

We will reestimate the parameters of the regression here and then test the specification.

```
. use http://www.stata-press.com/data/r13/website
(Visits to website)
. ivpoisson gmm visits ad female (time = phone frfam)
    (output omitted)
    estat overid
    Test of overidentifying restriction:
    Hansen's J chi2(1) = .129055 ( }\textrm{p}=0.7194\mathrm{ )
```

We have two instruments for one endogenous variable, so the $J$ statistic has one degree of freedom. The $J$ statistic is not significant. We fail to reject the null hypothesis that the model is correctly specified.

## Stored results

estat overid stores the following in $r()$ :
Scalars

$$
\begin{array}{ll}
\mathrm{r}(\mathrm{~J}) & \text { Hansen's } J \text { statistic } \\
\mathrm{r}\left(\mathrm{~J} \_ \text {df }\right) & J \text { statistic degrees of freedom } \\
\mathrm{r}\left(\mathrm{~J} \_\mathrm{P}\right) & J \text { statistic } p \text {-value }
\end{array}
$$

## Methods and formulas

The vector $\mathbf{x}_{i}$ contains the exogenous regressors, and $\mathbf{z}_{i}$ the instruments. The vector $\widetilde{\mathbf{z}}_{i}$ is partitioned as $\left(\mathbf{x}_{i}, \mathbf{z}_{i}\right)$. The vector $\mathbf{y}_{2, i}$ contains the endogenous regressors.

Under multiplicative errors, the conditional mean of $y_{i}$ is

$$
\begin{aligned}
E\left(y_{i} \mid \mathbf{y}_{2, i}, \widetilde{\mathbf{z}}_{i}\right) & =E\left\{E\left(y_{i} \mid \mathbf{x}_{i}, \mathbf{y}_{2, i}, \epsilon_{i}\right) \mid \mathbf{y}_{2, i}, \widetilde{\mathbf{z}}_{i}\right\} \\
& =E\left\{\exp \left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}_{1}+\mathbf{y}_{2, i}^{\prime} \boldsymbol{\beta}_{2}\right) \epsilon_{i} \mid \mathbf{y}_{2, i}, \widetilde{\mathbf{z}}_{i}\right\} \\
& =\exp \left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}_{1}+\mathbf{y}_{2, i}^{\prime} \boldsymbol{\beta}_{2}\right) E\left(\epsilon_{i} \mid \mathbf{y}_{2, i}, \widetilde{\mathbf{z}}_{i}\right)
\end{aligned}
$$

Under the CF estimator,

$$
\begin{aligned}
E\left(\epsilon_{i} \mid \mathbf{y}_{2, i}, \widetilde{\mathbf{z}}_{i}\right) & =E\left\{E\left(\epsilon_{i} \mid \mathbf{v}_{i}, c_{i}\right) \mid \mathbf{y}_{2, i}, \widetilde{\mathbf{z}}_{i}\right\} \\
& =E\left\{\exp \left(\mathbf{v}_{i}^{\prime} \boldsymbol{\rho}+c_{i}\right) \mid \mathbf{y}_{2, i}, \widetilde{\mathbf{z}}_{i}\right\} \\
& =\exp \left\{\left(\mathbf{y}_{2, i}-\mathbf{B} \widetilde{\mathbf{z}}_{i}^{\prime}\right)^{\prime} \boldsymbol{\rho}\right\} E\left(c_{i} \mid \mathbf{y}_{2, i}, \widetilde{\mathbf{z}}_{i}\right) \\
& =\exp \left\{\left(\mathbf{y}_{2, i}-\mathbf{B} \widetilde{\mathbf{z}}_{i}^{\prime}\right)^{\prime} \boldsymbol{\rho}\right\}
\end{aligned}
$$

Thus under the CF estimator, we estimate the conditional mean of $y_{i}$ as

$$
E\left(y_{i} \mid \mathbf{y}_{2, i}, \widetilde{\mathbf{z}}_{i}\right)=\exp \left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}_{1}+\mathbf{y}_{2, i}^{\prime} \boldsymbol{\beta}_{2}+\left(\mathbf{y}_{2, i}-\mathbf{B} \widetilde{\mathbf{z}}_{i}^{\prime}\right)^{\prime} \boldsymbol{\rho}\right)
$$

The CF estimator explicitly models the functional form of the endogeneity of $\mathbf{y}_{2, i}$ and $\epsilon_{i}$ with the instruments and exogenous regressors $\widetilde{\mathbf{z}}_{i}$. This allows it to correct the exponential-form estimator for the $E\left(\epsilon_{i} \mid \mathbf{y}_{2, i}, \widetilde{\mathbf{z}}_{i}\right)$ term.

In contrast, the GMM estimator does not model the functional form of the endogeneity of $\mathbf{y}_{2, i}$ and $\epsilon_{i}$. Therefore, $E\left(\epsilon_{i} \mid \mathbf{y}_{2, i}, \widetilde{\mathbf{z}}_{i}\right)$ is not estimated, and the exponential-form estimator under GMM estimation simply ignores this term. Noting that because $\widetilde{\mathbf{z}}_{i}$ and $\epsilon_{i}$ are independent, $E\left(\epsilon_{i} \mid \mathbf{y}_{2, i}, \widetilde{\mathbf{z}}_{i}\right)=E\left(\epsilon_{i} \mid \mathbf{y}_{2, i}\right)$, we can obviously see that ignoring the term will lead to inconsistent estimation of the conditional mean of $y_{i} . y_{2, i}$ and $\epsilon_{i}$ are not independent, so $E\left(\epsilon_{i} \mid y_{2, i}\right)$ may vary based on $y_{2, i}$.

In the additive errors setting, a similar derivation will show that the exponential-form estimator obtained from GMM estimation is inconsistent for the conditional mean of $y_{i}$.

## Reference

Hall, A. R. 2005. Generalized Method of Moments. Oxford: Oxford University Press.

## Also see

[R] ivpoisson - Poisson regression with endogenous regressors
[U] 20 Estimation and postestimation commands

## Title

ivprobit - Probit model with continuous endogenous regressors

| Syntax | Menu | Description |
| :--- | :--- | :--- |
| Options for ML estimator | Options for two-step estimator | Remarks and examples |
| Stored results | Methods and formulas | Acknowledgments |
| References | Also see |  |

## Syntax

## Maximum likelihood estimator

ivprobit depvar $\left[\right.$ varlist $\left._{1}\right]\left(\right.$ varlist $_{2}=$ varlist $\left._{\mathrm{iv}}\right)$ [if] [in] [weight] [, mle_options $]$

Two-step estimator

```
ivprobit depvar [\mp@subsup{varlist }{1}{}](\mp@subsup{\mathrm{ varlist }}{2}{}=\mp@subsup{\mathrm{ varlist }}{\textrm{iv}}{}) [if] [in] [weight], twostep
    [tse_options]
```

mle_options
Description
Model
mle
asis
constraints (constraints)
SE/Robust
vce (vcetype)

Reporting
level(\#)
first
nocnsreport
display_options

Maximization
maximize_options
coeflegend or jackknife
use conditional maximum-likelihood estimator; the default retain perfect predictor variables
apply specified linear constraints
vcetype may be oim, robust, cluster clustvar, opg, bootstrap,
set confidence level; default is level (95)
report first-stage regression
do not display constraints
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
control the maximization process
display legend instead of statistics

## Description

## Model

*twostep
asis
SE
vce(vcetype)
Reporting
level(\#)
first
display_options
coeflegend
use Newey's two-step estimator; the default is mle retain perfect predictor variables
vcetype may be twostep, bootstrap, or jackknife
set confidence level; default is level (95)
report first-stage regression
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
display legend instead of statistics
*twostep is required.
varlist $_{1}$ and varlist $_{\mathrm{iv}}$ may contain factor variables; see [U] 11.4.3 Factor variables.
depvar, varlist ${ }_{1}$, varlist $t_{2}$, and varlist ${ }_{\mathrm{iv}}$ may contain time-series operators; see [U] 11.4.4 Time-series varlists.
bootstrap, by, jackknife, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands. fp is allowed with the maximum likelihood estimator.
Weights are not allowed with the bootstrap prefix; see [R] bootstrap.
vce(), first, twostep, and weights are not allowed with the svy prefix; see [SVY] svy.
fweights, iweights, and pweights are allowed with the maximum likelihood estimator. fweights are allowed with Newey's two-step estimator. See [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Endogenous covariates $>$ Probit model with endogenous covariates

## Description

ivprobit fits probit models where one or more of the regressors are endogenously determined. By default, ivprobit uses maximum likelihood estimation. Alternatively, Newey's (1987) minimum chi-squared estimator can be invoked with the twostep option. Both estimators assume that the endogenous regressors are continuous and are not appropriate for use with discrete endogenous regressors. See $[R]$ ivtobit for tobit estimation with endogenous regressors and $[R]$ probit for probit estimation when the model contains no endogenous regressors.

## Options for ML estimator

$\qquad$
asis requests that all specified variables and observations be retained in the maximization process. This option is typically not used and may introduce numerical instability. Normally, ivprobit drops any endogenous or exogenous variables that perfectly predict success or failure in the dependent variable. The associated observations are also dropped. For more information, see Model identification in [R] probit.
constraints (constraints); see [R] estimation options.

SE/Robust
vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

Reporting
level(\#); see [R] estimation options.
first requests that the parameters for the reduced-form equations showing the relationships between the endogenous variables and instruments be displayed. For the two-step estimator, first shows the first-stage regressions. For the maximum likelihood estimator, these parameters are estimated jointly with the parameters of the probit equation. The default is not to show these parameter estimates.
nocnsreport; see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\%fmt), pformat (\%fmt), sformat (\%fmt), and nolstretch; see [R] estimation options.
maximize_options: difficult, technique(algorithm_spec), iterate(\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance(\#), nonrtolerance, and from(init_specs); see [R] maximize. This model's likelihood function can be difficult to maximize, especially with multiple endogenous variables. The difficult and technique (bfgs) options may be helpful in achieving convergence.

Setting the optimization type to technique (bhhh) resets the default vcetype to vce (opg).
The following option is available with ivprobit but is not shown in the dialog box:
coeflegend; see $[R]$ estimation options.

## Options for two-step estimator

Model
twostep is required and requests that Newey's (1987) efficient two-step estimator be used to obtain the coefficient estimates.
asis requests that all specified variables and observations be retained in the maximization process. This option is typically not used and may introduce numerical instability. Normally, ivprobit drops any endogenous or exogenous variables that perfectly predict success or failure in the dependent variable. The associated observations are also dropped. For more information, see Model identification in [R] probit.
vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (twostep) and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

## Reporting

level (\#); see [R] estimation options.
first requests that the parameters for the reduced-form equations showing the relationships between the endogenous variables and instruments be displayed. For the two-step estimator, first shows the first-stage regressions. For the maximum likelihood estimator, these parameters are estimated jointly with the parameters of the probit equation. The default is not to show these parameter estimates.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% fmt), pformat (\% fmt), sformat (\% fmt), and nolstretch; see [R] estimation options.

The following option is available with ivprobit but is not shown in the dialog box:
coeflegend; see $[R]$ estimation options.

## Remarks and examples

Remarks are presented under the following headings:

Model setup<br>Model identification

## Model setup

ivprobit fits models with dichotomous dependent variables and endogenous regressors. You can use it to fit a probit model when you suspect that one or more of the regressors are correlated with the error term. ivprobit is to probit modeling what ivregress is to linear regression analysis; see $[R]$ ivregress for more information.

Formally, the model is

$$
\begin{aligned}
y_{1 i}^{*} & =\boldsymbol{y}_{2 i} \boldsymbol{\beta}+\boldsymbol{x}_{1 i} \boldsymbol{\gamma}+u_{i} \\
\boldsymbol{y}_{2 i} & =\boldsymbol{x}_{1 i} \boldsymbol{\Pi}_{1}+\boldsymbol{x}_{2 i} \boldsymbol{\Pi}_{2}+\boldsymbol{v}_{i}
\end{aligned}
$$

where $i=1, \ldots, N, \boldsymbol{y}_{2 i}$ is a $1 \times p$ vector of endogenous variables, $\boldsymbol{x}_{1 i}$ is a $1 \times k_{1}$ vector of exogenous variables, $\boldsymbol{x}_{2 i}$ is a $1 \times k_{2}$ vector of additional instruments, and the equation for $\boldsymbol{y}_{2 i}$ is written in reduced form. By assumption, $\left(u_{i}, \boldsymbol{v}_{i}\right) \sim \mathrm{N}(\mathbf{0}, \boldsymbol{\Sigma})$, where $\sigma_{11}$ is normalized to one to identify the model. $\boldsymbol{\beta}$ and $\gamma$ are vectors of structural parameters, and $\Pi_{1}$ and $\Pi_{2}$ are matrices of reduced-form parameters. This is a recursive model: $\mathbf{y}_{2 i}$ appears in the equation for $y_{1 i}^{*}$, but $y_{1 i}^{*}$ does not appear in the equation for $\mathbf{y}_{2 i}$. We do not observe $y_{1 i}^{*}$; instead, we observe

$$
y_{1 i}= \begin{cases}0 & y_{1 i}^{*}<0 \\ 1 & y_{1 i}^{*} \geq 0\end{cases}
$$

The order condition for identification of the structural parameters requires that $k_{2} \geq p$. Presumably, $\boldsymbol{\Sigma}$ is not block diagonal between $u_{i}$ and $\boldsymbol{v}_{i}$; otherwise, $\boldsymbol{y}_{2 i}$ would not be endogenous.

## - Technical note

This model is derived under the assumption that $\left(u_{i}, \boldsymbol{v}_{i}\right)$ is independent and identically distributed multivariate normal for all $i$. The vce (cluster clustvar) option can be used to control for a lack of independence. As with most probit models, if $u_{i}$ is heteroskedastic, point estimates will be inconsistent.

## > Example 1

We have hypothetical data on 500 two-parent households, and we wish to model whether the woman is employed. We have a variable, fem_work, that is equal to one if she has a job and zero otherwise. Her decision to work is a function of the number of children at home (kids), number of years of schooling completed (fem_educ), and other household income measured in thousands of dollars (other_inc). We suspect that unobservable shocks affecting the woman's decision to hold a job also affect the household's other income. Therefore, we treat other_inc as endogenous. As an instrument, we use the number of years of schooling completed by the man (male_educ).

The syntax for specifying the exogenous, endogenous, and instrumental variables is identical to that used in ivregress; see [R] ivregress for details.

```
. use http://www.stata-press.com/data/r13/laborsup
. ivprobit fem_work fem_educ kids (other_inc = male_educ)
Fitting exogenous probit model
Iteration 0: 
Fitting full model
```

Iteration 0: log likelihood $=-2371.4753$
Iteration 1: $\quad$ log likelihood $=-2369.3178$
Iteration 2: log likelihood $=-2368.2198$
Iteration 3: log likelihood $=-2368.2062$
Iteration 4: log likelihood $=-2368.2062$

| Probit model with endogenous regressors | Number of obs | $=$ | 500 |
| :--- | :--- | :--- | :--- |
|  | Wald chi2(3) | $=$ | 163.88 |
| Log likelihood $=-2368.2062$ |  | Prob $>$ chi2 | $=$ |


|  | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| other_inc | -.0542756 | .0060854 | -8.92 | 0.000 | -.0662027 | -.0423485 |
| fem_educ | .211111 | .0268648 | 7.86 | 0.000 | .1584569 | .2637651 |
| kids | -.1820929 | .0478267 | -3.81 | 0.000 | -.2758316 | -.0883543 |
| _cons | .3672083 | .4480724 | 0.82 | 0.412 | -.5109975 | 1.245414 |
| /athrho | .3907858 | .1509443 | 2.59 | 0.010 | .0949403 | .6866313 |
| /lnsigma | 2.813383 | .0316228 | 88.97 | 0.000 | 2.751404 | 2.875363 |
| rho | .3720374 | .1300519 |  |  | .0946561 | .5958135 |
| sigma | 16.66621 | .5270318 |  |  | 15.66461 | 17.73186 |

```
Instrumented: other_inc
Instruments: fem_educ kids male_educ
```

Wald test of exogeneity (/athrho $=0$ ) : chi2 $(1)=6.70$ Prob $>$ chi2 $=0.0096$

Because we did not specify mle or twostep, ivprobit used the maximum likelihood estimator by default. At the top of the output, we see the iteration log. ivprobit fits a probit model ignoring
endogeneity to obtain starting values for the endogenous model. The header of the output contains the sample size as well as a Wald statistic and $p$-value for the test of the hypothesis that all the slope coefficients are jointly zero. Below the table of coefficients, Stata reminds us that the endogenous variable is other_inc and that fem_educ, kids, and male_educ were used as instruments.

At the bottom of the output is a Wald test of the exogeneity of the instrumented variables. We reject the null hypothesis of no endogeneity. However, if the test statistic is not significant, there is not sufficient information in the sample to reject the null, so a regular probit regression may be appropriate. The point estimates from ivprobit are still consistent, though those from probit (see $[\mathrm{R}]$ probit) are likely to have smaller standard errors.

Various two-step estimators have also been proposed for the endogenous probit model, and Newey's (1987) minimum chi-squared estimator is available with the twostep option.
$>$ Example 2
Refitting our labor-supply model with the two-step estimator yields

| Two-step probit with endogenous regressors |  |  |  | Numb Wald Prob | of obs i2(3) chi2 | $\begin{array}{r} 500 \\ 93.97 \\ 0.0000 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Coef. | Std. Err. | z | $P>\|z\|$ | [95\% | Interval] |
| other_inc | -. 058473 | . 0093364 | -6.26 | 0.000 | -. 0767 | -. 040174 |
| fem_educ | . 227437 | . 0281628 | 8.08 | 0.000 | . 1722 | . 282635 |
| kids | -. 1961748 | . 0496323 | -3.95 | 0.000 | -. 2934 | . 0988973 |
| _cons | . 3956061 | . 4982649 | 0.79 | 0.427 | -. 5809 | 1.372187 |
| Instrumented: other_inc |  |  |  |  |  |  |
| Instruments: | fem_educ kids male_educ |  |  |  |  |  |

All the coefficients have the same signs as their counterparts in the maximum likelihood model. The Wald test at the bottom of the output confirms our earlier finding of endogeneity.

## - Technical note

In a standard probit model, the error term is assumed to have a variance of one. In the probit model with endogenous regressors, we assume that $\left(u_{i}, \boldsymbol{v}_{i}\right)$ is multivariate normal with covariance matrix

$$
\operatorname{Var}\left(u_{i}, \boldsymbol{v}_{i}\right)=\boldsymbol{\Sigma}=\left[\begin{array}{cc}
1 & \boldsymbol{\Sigma}_{21}^{\prime} \\
\boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22}
\end{array}\right]
$$

With the properties of the multivariate normal distribution, $\operatorname{Var}\left(u_{i} \mid \boldsymbol{v}_{i}\right)=1-\boldsymbol{\Sigma}_{21}^{\prime} \boldsymbol{\Sigma}_{22}^{-1} \boldsymbol{\Sigma}_{21}$. As a result, Newey's estimator and other two-step probit estimators do not yield estimates of $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ but rather $\boldsymbol{\beta} / \sigma$ and $\gamma / \sigma$, where $\sigma$ is the square root of $\operatorname{Var}\left(u_{i} \mid \boldsymbol{v}_{i}\right)$. Hence, we cannot directly compare the estimates obtained from Newey's estimator with those obtained via maximum likelihood or with those obtained from probit. See Wooldridge (2010, 585-594) for a discussion of Rivers and Vuong's (1988) two-step estimator. The issues raised pertaining to the interpretation of the coefficients of that
estimator are identical to those that arise with Newey's estimator. Wooldridge also discusses ways to obtain marginal effects from two-step estimators.

Despite the coefficients not being directly comparable to their maximum likelihood counterparts, the two-step estimator is nevertheless useful. The maximum likelihood estimator may have difficulty converging, especially with multiple endogenous variables. The two-step estimator, consisting of nothing more complicated than a probit regression, will almost certainly converge. Moreover, although the coefficients from the two models are not directly comparable, the two-step estimates can still be used to test for statistically significant relationships.

## Model identification

As in the linear simultaneous-equation model, the order condition for identification requires that the number of excluded exogenous variables (that is, the additional instruments) be at least as great as the number of included endogenous variables. ivprobit checks this for you and issues an error message if the order condition is not met.

Like probit, logit, and logistic, ivprobit checks the exogenous and endogenous variables to see if any of them predict the outcome variable perfectly. It will then drop offending variables and observations and fit the model on the remaining data. Instruments that are perfect predictors do not affect estimation, so they are not checked. See Model identification in [R] probit for more information.
ivprobit will also occasionally display messages such as

```
Note: 4 failures and O successes completely determined.
```

For an explanation of this message, see $[R]$ logit.

## Stored results

ivprobit, mle stores the following in e():
Scalars

| e(N) | number of observations |
| :--- | :--- |
| e(N_cds) | number of completely determined successes |
| e(N_cdf) | number of completely determined failures |
| e(k) | number of parameters |
| e(k_eq) | number of equations in e(b) |
| e(k_eq_model) | number of equations in overall model test |
| e(k_aux) | number of auxiliary parameters |
| e(k_dv) | number of dependent variables |
| e(df_m) | model degrees of freedom |
| e(ll) | log likelihood |
| e(N_clust) | number of clusters |
| e(endog_ct) | number of endogenous regressors |
| e(p) | model Wald $p$-value |
| e(p_exog) | exogeneity test Wald $p$-value |
| e(chi2) | model Wald $\chi^{2}$ |
| e(chi2_exog) | Wald $\chi^{2}$ test of exogeneity |
| e(rank) | rank of e(V) |
| e(ic) | number of iterations |
| e(rc) | return code |
| e(converged) | 1 if converged, 0 otherwise |

Macros

```
        e(cmd)
        e(cmdline)
```

        e(depvar)
        e(instd)
        e(insts)
        e(wtype)
        e(wexp)
        e(title)
        e(clustvar)
        e(chi2type)
        e(vce)
        e(vcetype)
        e(asis)
        e(method)
        e(opt)
        e(which)
        e(ml_method)
        e(user)
        e(technique)
        e(properties)
        e(estat_cmd)
        e(predict)
        e(footnote)
        e(marginsok)
        e(asbalanced)
        e(asobserved)
    Matrices
e(b)
e(Cns)
e(rules)
e(ilog)
e(gradient)
e(Sigma)
e(V)
e(V_modelbased)

Functions
e(sample)
ivprobit
command as typed
name of dependent variable instrumented variables
instruments
weight type
weight expression
title in estimation output
name of cluster variable
Wald; type of model $\chi^{2}$ test
vcetype specified in vce()
title used to label Std. Err.
asis, if specified
ml
type of optimization
$\max$ or min; whether optimizer is to perform maximization or minimization type of ml method
name of likelihood-evaluator program
maximization technique
b V
program used to implement estat
program used to implement predict
program used to implement the footnote display
predictions allowed by margins
factor variables fvset as asbalanced
factor variables fvset as asobserved
coefficient vector
constraints matrix
information about perfect predictors
iteration $\log$ (up to 20 iterations)
gradient vector
$\widehat{\boldsymbol{\Sigma}}$
variance-covariance matrix of the estimators
model-based variance
marks estimation sample
ivprobit, twostep stores the following in e():
Scalars

```
    e(N)
    e(N_cds)
    e(N_cdf)
    e(df_m)
    e(df_exog)
    e(p)
    e(p_exog)
    e(chi2)
    e(chi2_exog)
    e(rank)
```

Macros
e(cmd)
e(cmdline)
e(depvar)
e(instd)
e(insts)
e(wtype)
e(wexp)
e(chi2type)
e(vce)
e(vcetype)
e(asis)
$e$ (method)
e(properties)
e(estat_cmd)
e(predict)
$e$ (footnote)
e(marginsok)
e(asbalanced)
e(asobserved)
Matrices
e(b)
e(Cns)
e(rules)
e(V)
Functions
$e$ (sample) marks estimation sample

## Methods and formulas

Fitting limited-dependent variable models with endogenous regressors has received considerable attention in the econometrics literature. Building on the results of Amemiya (1978, 1979), Newey (1987) developed an efficient method of estimation that encompasses both Rivers and Vuong's (1988) simultaneous-equations probit model and Smith and Blundell's (1986) simultaneous-equations tobit model. With modern computers, maximum likelihood estimation is feasible as well. For compactness, we write the model as

$$
\begin{align*}
y_{1 i}^{*} & =\boldsymbol{z}_{i} \boldsymbol{\delta}+u_{i}  \tag{1a}\\
\boldsymbol{y}_{2 i} & =\boldsymbol{x}_{i} \boldsymbol{\Pi}+\boldsymbol{v}_{i} \tag{1b}
\end{align*}
$$

where $\boldsymbol{z}_{i}=\left(\boldsymbol{y}_{2 i}, \boldsymbol{x}_{1 i}\right), \boldsymbol{x}_{i}=\left(\boldsymbol{x}_{1 i}, \boldsymbol{x}_{2 i}\right), \boldsymbol{\delta}=\left(\boldsymbol{\beta}^{\prime}, \boldsymbol{\gamma}^{\prime}\right)^{\prime}$, and $\boldsymbol{\Pi}=\left(\boldsymbol{\Pi}_{1}^{\prime}, \boldsymbol{\Pi}_{2}^{\prime}\right)^{\prime}$.
Deriving the likelihood function is straightforward because we can write the joint density $f\left(y_{1 i}, \boldsymbol{y}_{2 i} \mid \boldsymbol{x}_{i}\right)$ as $f\left(y_{1 i} \mid \boldsymbol{y}_{2 i}, \boldsymbol{x}_{i}\right) f\left(\boldsymbol{y}_{2 i} \mid \boldsymbol{x}_{i}\right)$. When there is an endogenous regressor, the log likelihood for observation $i$ is

$$
\ln L_{i}=w_{i}\left[y_{1 i} \ln \Phi\left(m_{i}\right)+\left(1-y_{1 i}\right) \ln \left\{1-\Phi\left(m_{i}\right)\right\}+\ln \phi\left(\frac{y_{2 i}-\boldsymbol{x}_{i} \boldsymbol{\Pi}}{\sigma}\right)-\ln \sigma\right]
$$

where

$$
m_{i}=\frac{\boldsymbol{z}_{i} \boldsymbol{\delta}+\rho\left(y_{2 i}-\boldsymbol{x}_{i} \boldsymbol{\Pi}\right) / \sigma}{\left(1-\rho^{2}\right)^{\frac{1}{2}}}
$$

$\Phi(\cdot)$ and $\phi(\cdot)$ are the standard normal distribution and density functions, respectively; $\sigma$ is the standard deviation of $v_{i} ; \rho$ is the correlation coefficient between $u_{i}$ and $v_{i}$; and $w_{i}$ is the weight for observation $i$ or one if no weights were specified. Instead of estimating $\sigma$ and $\rho$, we estimate $\ln \sigma$ and atanh $\rho$, where

$$
\operatorname{atanh} \rho=\frac{1}{2} \ln \left(\frac{1+\rho}{1-\rho}\right)
$$

For multiple endogenous regressors, let

$$
\operatorname{Var}\left(u_{i}, \boldsymbol{v}_{i}\right)=\boldsymbol{\Sigma}=\left[\begin{array}{cc}
1 & \boldsymbol{\Sigma}_{21}^{\prime} \\
\boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22}
\end{array}\right]
$$

As in any probit model, we have imposed the normalization $\operatorname{Var}\left(u_{i}\right)=1$ to identify the model. The $\log$ likelihood for observation $i$ is

$$
\ln L_{i}=w_{i}\left[y_{1 i} \ln \Phi\left(m_{i}\right)+\left(1-y_{1 i}\right) \ln \left\{1-\Phi\left(m_{i}\right)\right\}+\ln f\left(\boldsymbol{y}_{2 i} \mid \boldsymbol{x}_{i}\right)\right]
$$

where

$$
\ln f\left(\boldsymbol{y}_{2 i} \mid \boldsymbol{x}_{i}\right)=-\frac{p}{2} \ln 2 \pi-\frac{1}{2} \ln \left|\boldsymbol{\Sigma}_{22}\right|-\frac{1}{2}\left(\boldsymbol{y}_{2 i}-\boldsymbol{x}_{i} \boldsymbol{\Pi}\right) \boldsymbol{\Sigma}_{22}^{-1}\left(\boldsymbol{y}_{2 i}-\boldsymbol{x}_{i} \boldsymbol{\Pi}\right)^{\prime}
$$

and

$$
m_{i}=\left(1-\boldsymbol{\Sigma}_{21}^{\prime} \boldsymbol{\Sigma}_{22}^{-1} \boldsymbol{\Sigma}_{21}\right)^{-\frac{1}{2}}\left\{\boldsymbol{z}_{i} \boldsymbol{\delta}+\left(\boldsymbol{y}_{2 i}-\boldsymbol{x}_{i} \boldsymbol{\Pi}\right) \boldsymbol{\Sigma}_{22}^{-1} \boldsymbol{\Sigma}_{21}\right\}
$$

Instead of maximizing the log-likelihood function with respect to $\boldsymbol{\Sigma}$, we maximize with respect to the Cholesky decomposition $\boldsymbol{S}$ of $\boldsymbol{\Sigma}$; that is, there exists a lower triangular matrix, $\boldsymbol{S}$, such that $\boldsymbol{S} \boldsymbol{S}^{\prime}=\boldsymbol{\Sigma}$. This maximization ensures that $\boldsymbol{\Sigma}$ is positive definite, as a covariance matrix must be. Let

$$
\boldsymbol{S}=\left[\begin{array}{ccccc}
1 & 0 & 0 & \ldots & 0 \\
s_{21} & s_{22} & 0 & \ldots & 0 \\
s_{31} & s_{32} & s_{33} & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
s_{p+1,1} & s_{p+1,2} & s_{p+1,3} & \ldots & s_{p+1, p+1}
\end{array}\right]
$$

With maximum likelihood estimation, this command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce (robust) and vce (cluster clustvar), respectively. See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas.

The maximum likelihood version of ivprobit also supports estimation with survey data. For details on VCEs with survey data, see [SVY] variance estimation.

The two-step estimates are obtained using Newey's (1987) minimum chi-squared estimator. The reduced-form equation for $y_{1 i}^{*}$ is

$$
\begin{aligned}
y_{1 i}^{*} & =\left(\boldsymbol{x}_{i} \boldsymbol{\Pi}+\boldsymbol{v}_{i}\right) \boldsymbol{\beta}+\boldsymbol{x}_{1 i} \boldsymbol{\gamma}+u_{i} \\
& =\boldsymbol{x}_{i} \boldsymbol{\alpha}+\boldsymbol{v}_{i} \boldsymbol{\beta}+u_{i} \\
& =\boldsymbol{x}_{i} \boldsymbol{\alpha}+\nu_{i}
\end{aligned}
$$

where $\nu_{i}=\boldsymbol{v}_{i} \boldsymbol{\beta}+u_{i}$. Because $u_{i}$ and $\boldsymbol{v}_{i}$ are jointly normal, $\nu_{i}$ is also normal. Note that

$$
\boldsymbol{\alpha}=\left[\begin{array}{l}
\boldsymbol{\Pi}_{1} \\
\boldsymbol{\Pi}_{2}
\end{array}\right] \boldsymbol{\beta}+\left[\begin{array}{l}
\boldsymbol{I} \\
\mathbf{0}
\end{array}\right] \boldsymbol{\gamma}=D(\boldsymbol{\Pi}) \boldsymbol{\delta}
$$

where $D(\boldsymbol{\Pi})=\left(\boldsymbol{\Pi}, \boldsymbol{I}_{1}\right)$ and $\boldsymbol{I}_{1}$ is defined such that $\boldsymbol{x}_{i} \boldsymbol{I}_{1}=\boldsymbol{x}_{1 i}$. Letting $\widehat{\boldsymbol{z}_{i}}=\left(\boldsymbol{x}_{i} \boldsymbol{\Pi}, \boldsymbol{x}_{1 i}\right), \widehat{\boldsymbol{z}_{i}} \boldsymbol{\delta}=$ $\boldsymbol{x}_{i} D(\widehat{\boldsymbol{\Pi}}) \boldsymbol{\delta}$, where $D(\widehat{\boldsymbol{\Pi}})=\left(\widehat{\boldsymbol{\Pi}}, \boldsymbol{I}_{1}\right)$. Thus one estimator of $\boldsymbol{\alpha}$ is $D(\widehat{\boldsymbol{\Pi}}) \boldsymbol{\delta}$; denote this estimator by $\widehat{\boldsymbol{D}} \boldsymbol{\delta}$. $\boldsymbol{\alpha}$ could also be estimated directly as the solution to

$$
\begin{equation*}
\max _{\boldsymbol{\alpha}, \boldsymbol{\lambda}} \sum_{i=1}^{N} l\left(y_{1 i}, \boldsymbol{x}_{i} \boldsymbol{\alpha}+\widehat{\boldsymbol{v}}_{i} \boldsymbol{\lambda}\right) \tag{2}
\end{equation*}
$$

where $l(\cdot)$ is the log likelihood for probit. Denote this estimator by $\widetilde{\boldsymbol{\alpha}}$. The inclusion of the $\widehat{\boldsymbol{v}}_{i} \boldsymbol{\lambda}$ term follows because the multivariate normality of $\left(u_{i}, \boldsymbol{v}_{i}\right)$ implies that, conditional on $\boldsymbol{y}_{2 i}$, the expected value of $u_{i}$ is nonzero. Because $\boldsymbol{v}_{i}$ is unobservable, the least-squares residuals from fitting ( $1 b$ ) are used.

Amemiya (1978) shows that the estimator of $\boldsymbol{\delta}$ defined by

$$
\max _{\boldsymbol{\delta}}(\widetilde{\boldsymbol{\alpha}}-\widehat{\boldsymbol{D}} \boldsymbol{\delta})^{\prime} \widehat{\boldsymbol{\Omega}}^{-1}(\widetilde{\boldsymbol{\alpha}}-\widehat{\boldsymbol{D}} \boldsymbol{\delta})
$$

where $\widehat{\boldsymbol{\Omega}}$ is a consistent estimator of the covariance of $\sqrt{N}(\widetilde{\boldsymbol{\alpha}}-\widehat{\boldsymbol{D}} \boldsymbol{\delta})$, is asymptotically efficient relative to all other estimators that minimize the distance between $\widetilde{\boldsymbol{\alpha}}$ and $D(\widehat{\boldsymbol{\Pi}}) \boldsymbol{\delta}$. Thus an efficient estimator of $\boldsymbol{\delta}$ is

$$
\begin{equation*}
\widehat{\boldsymbol{\delta}}=\left(\widehat{\boldsymbol{D}}^{\prime} \widehat{\boldsymbol{\Omega}}^{-1} \widehat{\boldsymbol{D}}\right)^{-1} \widehat{\boldsymbol{D}}^{\prime} \widehat{\boldsymbol{\Omega}}^{-1} \widetilde{\boldsymbol{\alpha}} \tag{3}
\end{equation*}
$$

and

$$
\begin{equation*}
\operatorname{Var}(\widehat{\boldsymbol{\delta}})=\left(\widehat{\boldsymbol{D}}^{\prime} \widehat{\boldsymbol{\Omega}}^{-1} \widehat{\boldsymbol{D}}\right)^{-1} \tag{4}
\end{equation*}
$$

To implement this estimator, we need $\widehat{\boldsymbol{\Omega}}^{-1}$.
Consider the two-step maximum likelihood estimator that results from first fitting (1b) by OLS and computing the residuals $\widehat{\boldsymbol{v}}_{i}=\boldsymbol{y}_{2 i}-\boldsymbol{x}_{i} \widehat{\boldsymbol{\Pi}}$. The estimator is then obtained by solving

$$
\max _{\boldsymbol{\delta}, \boldsymbol{\lambda}} \sum_{i=1}^{N} l\left(y_{1 i}, \boldsymbol{z}_{i} \boldsymbol{\delta}+\widehat{\boldsymbol{v}}_{i} \boldsymbol{\lambda}\right)
$$

This is the two-step instrumental variables (2SIV) estimator proposed by Rivers and Vuong (1988), and its role will become apparent shortly.

From Proposition 5 of Newey (1987), $\sqrt{N}(\widetilde{\boldsymbol{\alpha}}-\widehat{\boldsymbol{D}} \boldsymbol{\delta}) \xrightarrow{d} \mathrm{~N}(\mathbf{0}, \boldsymbol{\Omega})$, where

$$
\boldsymbol{\Omega}=\boldsymbol{J}_{\alpha \alpha}^{-1}+(\boldsymbol{\lambda}-\boldsymbol{\beta})^{\prime} \boldsymbol{\Sigma}_{22}(\boldsymbol{\lambda}-\boldsymbol{\beta}) \boldsymbol{Q}^{-1}
$$

and $\boldsymbol{\Sigma}_{22}=E\left\{\boldsymbol{v}_{i}^{\prime} \boldsymbol{v}_{i}\right\} . \boldsymbol{J}_{\alpha \alpha}^{-1}$ is simply the covariance matrix of $\widetilde{\boldsymbol{\alpha}}$, ignoring that $\widehat{\boldsymbol{\Pi}}$ is an estimated parameter matrix. Moreover, Newey shows that the covariance matrix from an olS regression of $\boldsymbol{y}_{2 i}(\widehat{\boldsymbol{\lambda}}-\widehat{\boldsymbol{\beta}})$ on $\boldsymbol{x}_{i}$ is a consistent estimator of the second term. $\widehat{\boldsymbol{\lambda}}$ can be obtained from solving (2), and the 2 SIV estimator yields a consistent estimate, $\widehat{\boldsymbol{\beta}}$.

Mechanically, estimation proceeds in several steps.

1. Each of the endogenous right-hand-side variables is regressed on all the exogenous variables, and the fitted values and residuals are calculated. The matrix $\widehat{D}=D(\widehat{\boldsymbol{\Pi}})$ is assembled from the estimated coefficients.
2. probit is used to solve (2) and obtain $\widetilde{\boldsymbol{\alpha}}$ and $\widehat{\boldsymbol{\lambda}}$. The portion of the covariance matrix corresponding to $\boldsymbol{\alpha}, \boldsymbol{J}_{\alpha \alpha}^{-1}$, is also saved.
3. The 2SIV estimator is evaluated, and the parameters $\widehat{\boldsymbol{\beta}}$ corresponding to $\boldsymbol{y}_{2 i}$ are collected.
4. $\boldsymbol{y}_{2 i}(\widehat{\boldsymbol{\lambda}}-\widehat{\boldsymbol{\beta}})$ is regressed on $\boldsymbol{x}_{i}$. The covariance matrix of the parameters from this regression is added to $\boldsymbol{J}_{\alpha \alpha}^{-1}$, yielding $\widehat{\boldsymbol{\Omega}}$.
5. Evaluating (3) and (4) yields the estimates $\widehat{\boldsymbol{\delta}}$ and $\operatorname{Var}(\widehat{\boldsymbol{\delta}})$.
6. A Wald test of the null hypothesis $H_{0}: \boldsymbol{\lambda}=\mathbf{0}$, using the 2SIV estimates, serves as our test of exogeneity.
The two-step estimates are not directly comparable to those obtained from the maximum likelihood estimator or from probit. The argument is the same for Newey's efficient estimator as for Rivers and Vuong's (1988) 2SIV estimator, so we consider the simpler 2SIV estimator. From the properties of the normal distribution,

$$
E\left(u_{i} \mid \boldsymbol{v}_{i}\right)=\boldsymbol{v}_{i} \boldsymbol{\Sigma}_{22}^{-1} \boldsymbol{\Sigma}_{21} \quad \text { and } \quad \operatorname{Var}\left(u_{i} \mid \boldsymbol{v}_{i}\right)=1-\boldsymbol{\Sigma}_{21}^{\prime} \boldsymbol{\Sigma}_{22}^{-1} \boldsymbol{\Sigma}_{21}
$$

We write $u_{i}$ as $u_{i}=\boldsymbol{v}_{i} \boldsymbol{\Sigma}_{22}^{-1} \boldsymbol{\Sigma}_{21}+e_{i}=\boldsymbol{v}_{i} \boldsymbol{\lambda}+e_{i}$, where $e_{i} \sim \mathrm{~N}\left(0,1-\rho^{2}\right), \rho^{2}=\boldsymbol{\Sigma}_{21}^{\prime} \boldsymbol{\Sigma}_{22}^{-1} \boldsymbol{\Sigma}_{21}$, and $e_{i}$ is independent of $\boldsymbol{v}_{i}$. In the second stage of 2SIV, we use a probit regression to estimate the parameters of

$$
y_{1 i}=\boldsymbol{z}_{i} \boldsymbol{\delta}+\boldsymbol{v}_{i} \boldsymbol{\lambda}+e_{i}
$$

Because $\boldsymbol{v}_{i}$ is unobservable, we use the sample residuals from the first-stage regressions.

$$
\operatorname{Pr}\left(y_{1 i}=1 \mid \boldsymbol{z}_{i}, \boldsymbol{v}_{i}\right)=\operatorname{Pr}\left(\boldsymbol{z}_{i} \boldsymbol{\delta}+\boldsymbol{v}_{i} \boldsymbol{\lambda}+e_{i}>0 \mid \boldsymbol{z}_{i}, \boldsymbol{v}_{i}\right)=\Phi\left\{\left(1-\rho^{2}\right)^{-\frac{1}{2}}\left(\boldsymbol{z}_{i} \boldsymbol{\delta}+\boldsymbol{v}_{i} \boldsymbol{\lambda}\right)\right\}
$$

Hence, as mentioned previously, 2SIV and Newey's estimator do not estimate $\boldsymbol{\delta}$ and $\boldsymbol{\lambda}$ but rather

$$
\boldsymbol{\delta}_{\rho}=\frac{1}{\left(1-\rho^{2}\right)^{\frac{1}{2}}} \boldsymbol{\delta} \quad \text { and } \quad \boldsymbol{\lambda}_{\rho}=\frac{1}{\left(1-\rho^{2}\right)^{\frac{1}{2}}} \boldsymbol{\lambda}
$$

## Acknowledgments

The two-step estimator is based on the probitiv command written by Jonah Gelbach of the Department of Economics at Yale University and the ivprob command written by Joe Harkness of the Institute of Policy Studies at Johns Hopkins University.

## References

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[^31]
## Title

ivprobit postestimation - Postestimation tools for ivprobit

| Description | Syntax for predict | Menu for predict | Options for predict |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Methods and formulas | Also see |  |

## Description

The following postestimation commands are of special interest after ivprobit:
Command Description
estat classification
lroc
lsens
report various summary statistics, including the classification table compute area under ROC curve and graph the curve graph sensitivity and specificity versus probability cutoff

These commands are not appropriate after the two-step estimator or the svy prefix.

The following standard postestimation commands are also available:

| Command | Description |
| :--- | :--- |
| contrast | contrasts and ANOVA-style joint tests of estimates |
| estat ic ${ }^{1}$ | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) <br> estat summarize <br> summary statistics for the estimation sample <br> estat vce |
| variance-covariance matrix of the estimators (VCE) <br> estimates | postestimation statistics for survey data <br> cataloging estimation results <br> dynamic forecasts and simulations |
| forecast |  |
| hausman | Hausman's specification test <br> lincom |
| point estimates, standard errors, testing, and inference for linear combinations |  |
| of coefficients |  |

[^32]
## Syntax for predict

After ML or twostep
predict [type] newvar [if][in] [, statistic rules asif]

After ML
predict [type] \{stub*|newvarlist $\}[$ if $][$ in $]$, scores
statistic Description
Main
$\mathrm{xb} \quad$ linear prediction; the default
stdp standard error of the linear prediction
pr probability of a positive outcome; not available with two-step estimator
These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
xb , the default, calculates the linear prediction.
stdp calculates the standard error of the linear prediction.
pr calculates the probability of a positive outcome. pr is not available with the two-step estimator.
rules requests that Stata use any rules that were used to identify the model when making the prediction. By default, Stata calculates missing for excluded observations. rules is not available with the two-step estimator.
asif requests that Stata ignore the rules and the exclusion criteria and calculate predictions for all observations possible using the estimated parameters from the model. asif is not available with the two-step estimator.
scores, not available with twostep, calculates equation-level score variables.
For models with one endogenous regressor, four new variables are created.
The first new variable will contain $\partial \ln L / \partial\left(\boldsymbol{z}_{i} \boldsymbol{\delta}\right)$.
The second new variable will contain $\partial \ln L / \partial\left(\boldsymbol{x}_{i} \boldsymbol{\Pi}\right)$.
The third new variable will contain $\partial \ln L / \partial$ atanh $\rho$.
The fourth new variable will contain $\partial \ln L / \partial \ln \sigma$.
For models with $p$ endogenous regressors, $p+\{(p+1)(p+2)\} / 2$ new variables are created.
The first new variable will contain $\partial \ln L / \partial\left(\boldsymbol{z}_{i} \boldsymbol{\delta}\right)$.

The second through $(p+1)$ th new variables will contain $\partial \ln L / \partial\left(\boldsymbol{x}_{i} \boldsymbol{\Pi}_{k}\right), k=1, \ldots, p$, where $\boldsymbol{\Pi}_{k}$ is the $k$ th column of $\boldsymbol{\Pi}$.
The remaining score variables will contain the partial derivatives of $\ln L$ with respect to $s_{21}$, $s_{31}, \ldots, s_{p+1,1}, s_{22}, \ldots, s_{p+1,2}, \ldots, s_{p+1, p+1}$, where $s_{m, n}$ denotes the $(m, n)$ element of the Cholesky decomposition of the error covariance matrix.

## Remarks and examples

Remarks are presented under the following headings:

Marginal effects<br>Obtaining predicted values

## Marginal effects

$>$ Example 1
We can obtain marginal effects by using the margins command after ivprobit. We will calculate average marginal effects by using the labor-supply model of example 1 in $[R]$ ivprobit.

```
. use http://www.stata-press.com/data/r13/laborsup
. ivprobit fem_work fem_educ kids (other_inc = male_educ)
    (output omitted)
. margins, dydx(*) predict(pr)
Average marginal effects Number of obs = 500
Model VCE : OIM
Expression : Probability of positive outcome, predict(pr)
dy/dx w.r.t. : other_inc fem_educ kids male_educ
```

|  | Delta-method |  |  |  |  |  |
| ---: | ---: | :---: | :---: | ---: | ---: | ---: |
|  | dy/dx | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| other_inc | -.014015 | .0009836 | -14.25 | 0.000 | -.0159428 | -.0120872 |
| fem_educ | .0545129 | .0066007 | 8.26 | 0.000 | .0415758 | .06745 |
| kids | -.0470199 | .0123397 | -3.81 | 0.000 | -.0712052 | -.0228346 |
| male_educ | 0 | (omitted) |  |  |  |  |

Here we see that a $\$ 1,000$ increase in other_inc leads to an average decrease of 0.014 in the probability that the woman has a job. male_edu has no effect because it appears only as an instrument.

## Obtaining predicted values

After fitting your model with ivprobit, you can obtain the linear prediction and its standard error for both the estimation sample and other samples by using the predict command; see [U] 20 Estimation and postestimation commands and [R] predict. If you had used the maximum likelihood estimator, you could also obtain the probability of a positive outcome.
predict's pr option calculates the probability of a positive outcome, remembering any rules used to identify the model, and calculates missing for excluded observations. predict's rules option uses the rules in predicting probabilities, whereas predict's asif option ignores both the rules and the exclusion criteria and calculates probabilities for all possible observations by using the estimated parameters from the model. See Obtaining predicted values in $[R]$ probit postestimation for an example.

## Methods and formulas

The linear prediction is calculated as $\boldsymbol{z}_{i} \widehat{\boldsymbol{\delta}}$, where $\widehat{\boldsymbol{\delta}}$ is the estimated value of $\boldsymbol{\delta}$, and $\boldsymbol{z}_{i}$ and $\boldsymbol{\delta}$ are defined in (1a) of [R] ivprobit. The probability of a positive outcome is $\Phi\left(\boldsymbol{z}_{i} \widehat{\boldsymbol{\delta}}\right)$, where $\Phi(\cdot)$ is the standard normal distribution function.

## Also see

[R] ivprobit - Probit model with continuous endogenous regressors
[R] estat classification - Classification statistics and table
$[R]$ Iroc - Compute area under ROC curve and graph the curve
[R] Isens - Graph sensitivity and specificity versus probability cutoff
[U] 20 Estimation and postestimation commands

## Title

ivregress - Single-equation instrumental-variables regression

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

ivregress estimator depvar $\left[\right.$ varlist $\left._{1}\right] \quad\left(\right.$ varlist $_{2}=$ varlist $\left._{\mathrm{iv}}\right) \quad[$ if $] \quad[$ in $] \quad[$ weight $]$ [, options]

| estimator | Description |
| :--- | :--- |
| 2 sls | two-stage least squares (2SLS) |
| liml | limited-information maximum likelihood (LIML) |
| gmm | generalized method of moments (GMM) |

## options Description

Model
noconstant
hascons
GMM ${ }^{1}$
wmatrix (wmtype)
center
igmm
eps (\#) ${ }^{2}$
weps (\#) ${ }^{2}$
optimization_options ${ }^{2}$
SE/Robust
vce (vcetype)

Reporting
level(\#)
first
small
noheader
depname (depname)
eform(string)
display_options
suppress constant term
has user-supplied constant
wmtype may be robust, cluster clustvar, hac kernel, or unadjusted center moments in weight matrix computation
use iterative instead of two-step GMM estimator
specify \# for parameter convergence criterion; default is eps (1e-6)
specify \# for weight matrix convergence criterion; default is weps ( $1 \mathrm{e}-6$ )
control the optimization process; seldom used
vcetype may be unadjusted, robust, cluster clustvar, bootstrap, jackknife, or hac kernel
set confidence level; default is level (95)
report first-stage regression
make degrees-of-freedom adjustments and report small-sample statistics
display only the coefficient table
substitute dependent variable name
report exponentiated coefficients and use string to label them control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling

## perfect do not check for collinearity between endogenous regressors and excluded instruments <br> display legend instead of statistics

${ }^{1}$ These options may be specified only when gmm is specified.
${ }^{2}$ These options may be specified only when igmm is specified.
varlist $_{1}$, varlist ${ }_{2}$, and varlist ${ }_{\mathrm{iv}}$ may contain factor variables; see [U] 11.4.3 Factor variables. depvar, varlist ${ }_{1}$, varlist $t_{2}$, and varlist ${ }_{\mathrm{iv}}$ may contain time-series operators; see [U] 11.4.4 Time-series varlists. bootstrap, by, jackknife, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands. Weights are not allowed with the bootstrap prefix; see [R] bootstrap. aweights are not allowed with the jackknife prefix; see [R] jackknife. hascons, vce(), noheader, depname(), and weights are not allowed with the svy prefix; see [SVY] svy. aweights, fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight. perfect and coeflegend do not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Endogenous covariates $>$ Single-equation instrumental-variables regression

## Description

ivregress fits a linear regression of depvar on varlist $_{1}$ and varlist $_{2}$, using varlist $_{\mathrm{iv}}$ (along with varlist $_{1}$ ) as instruments for varlist $_{2}$. ivregress supports estimation via two-stage least squares (2SLS), limited-information maximum likelihood (LIML), and generalized method of moments (GMM).

In the language of instrumental variables, varlist $_{1}$ and varlist $_{\mathrm{iv}}$ are the exogenous variables, and varlist $_{2}$ are the endogenous variables.

## Options

Model
noconstant; see [R] estimation options.
hascons indicates that a user-defined constant or its equivalent is specified among the independent variables.

## GMM

wmatrix(wmtype) specifies the type of weighting matrix to be used in conjunction with the GMM estimator.

Specifying wmatrix(robust) requests a weighting matrix that is optimal when the error term is heteroskedastic. wmatrix (robust) is the default.

Specifying wmatrix(cluster clustvar) requests a weighting matrix that accounts for arbitrary correlation among observations within clusters identified by clustvar.
Specifying wmatrix(hac kernel \#) requests a heteroskedasticity- and autocorrelation-consistent (HAC) weighting matrix using the specified kernel (see below) with \# lags. The bandwidth of a kernel is equal to $\#+1$.

Specifying wmatrix (hac kernel opt) requests an HAC weighting matrix using the specified kernel, and the lag order is selected using Newey and West's (1994) optimal lag-selection algorithm.
Specifying wmatrix (hac kernel) requests an HAC weighting matrix using the specified kernel and $N-2$ lags, where $N$ is the sample size.

There are three kernels available for HAC weighting matrices, and you may request each one by using the name used by statisticians or the name perhaps more familiar to economists:
bartlett or nwest requests the Bartlett (Newey-West) kernel;
parzen or gallant requests the Parzen (Gallant 1987) kernel; and
quadraticspectral or andrews requests the quadratic spectral (Andrews 1991) kernel.
Specifying wmatrix (unadjusted) requests a weighting matrix that is suitable when the errors are homoskedastic. The GMM estimator with this weighting matrix is equivalent to the 2SLS estimator.
center requests that the sample moments be centered (demeaned) when computing GMM weight matrices. By default, centering is not done.
igmm requests that the iterative GMM estimator be used instead of the default two-step GMM estimator. Convergence is declared when the relative change in the parameter vector from one iteration to the next is less than eps () or the relative change in the weight matrix is less than weps ().
eps (\#) specifies the convergence criterion for successive parameter estimates when the iterative GMM estimator is used. The default is eps $(1 e-6)$. Convergence is declared when the relative difference between successive parameter estimates is less than eps() and the relative difference between successive estimates of the weighting matrix is less than weps ().
weps (\#) specifies the convergence criterion for successive estimates of the weighting matrix when the iterative GMM estimator is used. The default is weps ( $1 e-6$ ). Convergence is declared when the relative difference between successive parameter estimates is less than eps() and the relative difference between successive estimates of the weighting matrix is less than weps().
optimization_options: iterate(\#), [no] log. iterate() specifies the maximum number of iterations to perform in conjunction with the iterative GMM estimator. The default is 16,000 or the number set using set maxiter (see [ R$]$ maximize). log/nolog specifies whether to show the iteration log. These options are seldom used.

## SE/Robust

vce(vcetype) specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
vce(unadjusted), the default for 2 sls and liml, specifies that an unadjusted (nonrobust) VCE matrix be used. The default for gmm is based on the wmtype specified in the wmatrix() option; see wmatrix (wmtype) above. If wmatrix() is specified with gmm but vce() is not, then vcetype is set equal to wmtype. To override this behavior and obtain an unadjusted (nonrobust) VCE matrix, specify vce(unadjusted).
ivregress also allows the following:
vce (hac kernel [\#|opt]) specifies that an HAC covariance matrix be used. The syntax used with vce(hac kernel ...) is identical to that used with wmatrix(hac kernel ...); see wmatrix(wmtype) above.
first requests that the first-stage regression results be displayed.
small requests that the degrees-of-freedom adjustment $N /(N-k)$ be made to the variance-covariance matrix of parameters and that small-sample $F$ and $t$ statistics be reported, where $N$ is the sample size and $k$ is the number of parameters estimated. By default, no degrees-of-freedom adjustment is made, and Wald and $z$ statistics are reported. Even with this option, no degrees-of-freedom adjustment is made to the weighting matrix when the GMM estimator is used.
noheader suppresses the display of the summary statistics at the top of the output, displaying only the coefficient table.
depname (depname) is used only in programs and ado-files that use ivregress to fit models other than instrumental-variables regression. depname() may be specified only at estimation time. depname is recorded as the identity of the dependent variable, even though the estimates are calculated using depvar. This method affects the labeling of the output - not the results calculated-but could affect later calculations made by predict, where the residual would be calculated as deviations from depname rather than depvar. depname() is most typically used when depvar is a temporary variable (see $[\mathrm{P}]$ macro) used as a proxy for depname.
eform(string) is used only in programs and ado-files that use ivregress to fit models other than instrumental-variables regression. eform() specifies that the coefficient table be displayed in "exponentiated form", as defined in $[\mathrm{R}]$ maximize, and that string be used to label the exponentiated coefficients in the table.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\%fmt), pformat(\%fmt), sformat(\%fmt), and nolstretch; see [R] estimation options.

The following options are available with ivregress but are not shown in the dialog box:
perfect requests that ivregress not check for collinearity between the endogenous regressors and excluded instruments, allowing one to specify "perfect" instruments. This option cannot be used with the LIML estimator. This option may be required when using ivregress to implement other estimators.
coeflegend; see $[R]$ estimation options.

## Remarks and examples

ivregress performs instrumental-variables regression and weighted instrumental-variables regression. For a general discussion of instrumental variables, see Baum (2006), Cameron and Trivedi (2005; 2010, chap. 6) Davidson and MacKinnon (1993, 2004), Greene (2012, chap. 8), and Wooldridge (2010, 2013). See Hall (2005) for a lucid presentation of GMM estimation. Angrist and Pischke (2009, chap. 4) offer a casual yet thorough introduction to instrumental-variables estimators, including their use in estimating treatment effects. Some of the earliest work on simultaneous systems can be found in Cowles Commission monographs-Koopmans and Marschak (1950) and Koopmans and Hood (1953) - with the first developments of 2SLS appearing in Theil (1953) and Basmann (1957). However, Stock and Watson (2011, 422-424) present an example of the method of instrumental variables that was first published in 1928 by Philip Wright.

The syntax for ivregress assumes that you want to fit one equation from a system of equations or an equation for which you do not want to specify the functional form for the remaining equations of the system. To fit a full system of equations, using either 2SLS equation-by-equation or three-stage least squares, see [R] reg3. An advantage of ivregress is that you can fit one equation of a multiple-equation system without specifying the functional form of the remaining equations.

Formally, the model fit by ivregress is

$$
\begin{align*}
y_{i} & =\mathbf{y}_{i} \boldsymbol{\beta}_{1}+\mathbf{x}_{1 i} \boldsymbol{\beta}_{2}+u_{i}  \tag{1}\\
\mathbf{y}_{i} & =\mathbf{x}_{1 i} \boldsymbol{\Pi}_{1}+\mathbf{x}_{2 i} \boldsymbol{\Pi}_{2}+\mathbf{v}_{i} \tag{2}
\end{align*}
$$

Here $y_{i}$ is the dependent variable for the $i$ th observation, $\mathbf{y}_{i}$ represents the endogenous regressors (varlist $t_{2}$ in the syntax diagram), $\mathbf{x}_{1 i}$ represents the included exogenous regressors (varlist $t_{1}$ in the syntax diagram), and $\mathbf{x}_{2 i}$ represents the excluded exogenous regressors (varlist ${ }_{i v}$ in the syntax diagram). $\mathbf{x}_{1 i}$ and $\mathbf{x}_{2 i}$ are collectively called the instruments. $u_{i}$ and $\mathbf{v}_{i}$ are zero-mean error terms, and the correlations between $u_{i}$ and the elements of $\mathbf{v}_{i}$ are presumably nonzero.

The rest of the discussion is presented under the following headings:
2SLS and LIML estimators
GMM estimator

## 2SLS and LIML estimators

The most common instrumental-variables estimator is 2SLS.
> Example 1: 2SLS estimator
We have state data from the 1980 census on the median dollar value of owner-occupied housing (hsngval) and the median monthly gross rent (rent). We want to model rent as a function of hsngval and the percentage of the population living in urban areas (pcturban):

$$
\operatorname{rent}_{i}=\beta_{0}+\beta_{1} \text { hsngval }_{i}+\beta_{2} \text { pcturban }_{i}+u_{i}
$$

where $i$ indexes states and $u_{i}$ is an error term.
Because random shocks that affect rental rates in a state probably also affect housing values, we treat hsngval as endogenous. We believe that the correlation between hsngval and $u$ is not equal to zero. On the other hand, we have no reason to believe that the correlation between pcturban and $u$ is nonzero, so we assume that pcturban is exogenous.

Because we are treating hsngval as an endogenous regressor, we must have one or more additional variables available that are correlated with hsngval but uncorrelated with $u$. Moreover, these excluded exogenous variables must not affect rent directly, because if they do then they should be included in the regression equation we specified above. In our dataset, we have a variable for family income (faminc) and for region of the country (region) that we believe are correlated with hsngval but not the error term. Together, pcturban, faminc, and factor variables 2.region, 3.region, and 4.region constitute our set of instruments.

To fit the equation in Stata, we specify the dependent variable and the list of included exogenous variables. In parentheses, we specify the endogenous regressors, an equal sign, and the excluded exogenous variables. Only the additional exogenous variables must be specified to the right of the equal sign; the exogenous variables that appear in the regression equation are automatically included as instruments.

Here we fit our model with the 2SLS estimator:
. use http://www.stata-press.com/data/r13/hsng
(1980 Census housing data)
. ivregress 2sls rent pcturban (hsngval = faminc i.region)
Instrumental variables (2SLS) regression

| rent | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | ---: |
| hsngval | .0022398 | .0003284 | 6.82 | 0.000 | .0015961 | .0028836 |
| pcturban | .081516 | .2987652 | 0.27 | 0.785 | -.504053 | .667085 |
| _cons | 120.7065 | 15.22839 | 7.93 | 0.000 | 90.85942 | 150.5536 |

Instrumented: hsngval
Instruments: pcturban faminc 2.region 3.region 4.region
As we would expect, states with higher housing values have higher rental rates. The proportion of a state's population that is urban does not have a significant effect on rents.

## Technical note

In a simultaneous-equations framework, we could write the model we just fit as

$$
\begin{aligned}
\text { hsngval }_{i} & =\pi_{0}+\pi_{1} \text { faminc }_{i}+\pi_{2} 2 . \text { region }_{i}+\pi_{3} 3 . \text { region }_{i}+\pi_{4} 4 . \text { region }_{i}+v_{i} \\
\text { rent }_{i} & =\beta_{0}+\beta_{1} \text { hsngval }_{i}+\beta_{2} \text { pcturban }_{i}+u_{i}
\end{aligned}
$$

which here happens to be recursive (triangular), because hsngval appears in the equation for rent but rent does not appear in the equation for hsngval. In general, however, systems of simultaneous equations are not recursive. Because this system is recursive, we could fit the two equations individually via OLS if we were willing to assume that $u$ and $v$ were independent. For a more detailed discussion of triangular systems, see Kmenta (1997, 719-720).

Historically, instrumental-variables estimation and systems of simultaneous equations were taught concurrently, and older textbooks describe instrumental-variables estimation solely in the context of simultaneous equations. However, in recent decades, the treatment of endogeneity and instrumentalvariables estimation has taken on a much broader scope, while interest in the specification of complete systems of simultaneous equations has waned. Most recent textbooks, such as Cameron and Trivedi (2005), Davidson and MacKinnon (1993, 2004), and Wooldridge (2010, 2013), treat instrumental-variables estimation as an integral part of the modern economists' toolkit and introduce it long before shorter discussions on simultaneous equations.

In addition to the 2SLS member of the $\kappa$-class estimators, ivregress implements the LIML estimator. Both theoretical and Monte Carlo exercises indicate that the LIML estimator may yield less bias and confidence intervals with better coverage rates than the 2SLS estimator. See Poi (2006) and Stock, Wright, and Yogo (2002) (and the papers cited therein) for Monte Carlo evidence.
> Example 2: LIML estimator
Here we refit our model with the LIML estimator:
. ivregress liml rent pcturban (hsngval = faminc i.region)
Instrumental variables (LIML) regression

| rent | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | :---: | ---: | ---: |
| hsngval | .0026686 | .0004173 | 6.39 | 0.000 | .0018507 | .0034865 |
| pcturban | -.1827391 | .3571132 | -0.51 | 0.609 | -.8826681 | .5171899 |
| _cons | 117.6087 | 17.22625 | 6.83 | 0.000 | 83.84587 | 151.3715 |

Instrumented: hsngval
Instruments: pcturban faminc 2.region 3.region 4.region
These results are qualitatively similar to the 2SLS results, although the coefficient on hsngval is about 19\% higher.

## GMM estimator

Since the celebrated paper of Hansen (1982), the GMM has been a popular method of estimation in economics and finance, and it lends itself well to instrumental-variables estimation. The basic principle is that we have some moment or orthogonality conditions of the form

$$
\begin{equation*}
E\left(\mathbf{z}_{i} u_{i}\right)=\mathbf{0} \tag{3}
\end{equation*}
$$

From (1), we have $u_{i}=y_{i}-\mathbf{y}_{i} \boldsymbol{\beta}_{1}-\mathbf{x}_{1 i} \boldsymbol{\beta}_{2}$. What are the elements of the instrument vector $\mathbf{z}_{i}$ ? By assumption, $\mathbf{x}_{1 i}$ is uncorrelated with $u_{i}$, as are the excluded exogenous variables $\mathbf{x}_{2 i}$, and so we use $\mathbf{z}_{i}=\left[\mathbf{x}_{1 i} \mathbf{x}_{2 i}\right]$. The moment conditions are simply the mathematical representation of the assumption that the instruments are exogenous-that is, the instruments are orthogonal to (uncorrelated with) $u_{i}$.

If the number of elements in $\mathbf{z}_{i}$ is just equal to the number of unknown parameters, then we can apply the analogy principle to (3) and solve

$$
\begin{equation*}
\frac{1}{N} \sum_{i} \mathbf{z}_{i} u_{i}=\frac{1}{N} \sum_{i} \mathbf{z}_{i}\left(y_{i}-\mathbf{y}_{i} \boldsymbol{\beta}_{1}-\mathbf{x}_{1 i} \boldsymbol{\beta}_{2}\right)=\mathbf{0} \tag{4}
\end{equation*}
$$

This equation is known as the method of moments estimator. Here where the number of instruments equals the number of parameters, the method of moments estimator coincides with the 2SLS estimator, which also coincides with what has historically been called the indirect least-squares estimator (Judge et al. 1985, 595).

The "generalized" in GMM addresses the case in which the number of instruments (columns of $\mathbf{z}_{i}$ ) exceeds the number of parameters to be estimated. Here there is no unique solution to the population moment conditions defined in (3), so we cannot use (4). Instead, we define the objective function

$$
\begin{equation*}
Q\left(\boldsymbol{\beta}_{1}, \boldsymbol{\beta}_{2}\right)=\left(\frac{1}{N} \sum_{i} \mathbf{z}_{i} u_{i}\right)^{\prime} \mathbf{W}\left(\frac{1}{N} \sum_{i} \mathbf{z}_{i} u_{i}\right) \tag{5}
\end{equation*}
$$

where $\mathbf{W}$ is a positive-definite matrix with the same number of rows and columns as the number of columns of $\mathbf{z}_{i}$. $\mathbf{W}$ is known as the weighting matrix, and we specify its structure with the wmatrix() option. The GMM estimator of $\left(\boldsymbol{\beta}_{1}, \boldsymbol{\beta}_{2}\right)$ minimizes $Q\left(\boldsymbol{\beta}_{1}, \boldsymbol{\beta}_{2}\right)$; that is, the GMM estimator chooses $\boldsymbol{\beta}_{1}$ and $\boldsymbol{\beta}_{2}$ to make the moment conditions as close to zero as possible for a given $\mathbf{W}$. For a more general GMM estimator, see $[\mathrm{R}]$ gmm. gmm does not restrict you to fitting a single linear equation, though the syntax is more complex.

A well-known result is that if we define the matrix $\mathbf{S}_{0}$ to be the covariance of $\mathbf{z}_{i} u_{i}$ and set $\mathbf{W}=\mathbf{S}_{0}^{-1}$, then we obtain the optimal two-step GMM estimator, where by optimal estimator we mean the one that results in the smallest variance given the moment conditions defined in (3).

Suppose that the errors $u_{i}$ are heteroskedastic but independent among observations. Then

$$
\mathbf{S}_{0}=E\left(\mathbf{z}_{i} u_{i} u_{i} \mathbf{z}_{i}^{\prime}\right)=E\left(u_{i}^{2} \mathbf{z}_{i} \mathbf{z}_{i}^{\prime}\right)
$$

and the sample analogue is

$$
\begin{equation*}
\widehat{\mathbf{S}}=\frac{1}{N} \sum_{i} \widehat{u}_{i}^{2} \mathbf{z}_{i} \mathbf{z}_{i}^{\prime} \tag{6}
\end{equation*}
$$

To implement this estimator, we need estimates of the sample residuals $\widehat{u}_{i}$. ivregress gmm obtains the residuals by estimating $\boldsymbol{\beta}_{1}$ and $\boldsymbol{\beta}_{2}$ by 2SLS and then evaluates (6) and sets $\mathbf{W}=\widehat{\mathbf{S}}^{-1}$. Equation (6) is the same as the center term of the "sandwich" robust covariance matrix available from most Stata estimation commands through the vce(robust) option.

## > Example 3: GMM estimator

Here we refit our model of rents by using the GMM estimator, allowing for heteroskedasticity in $u_{i}$ :
. ivregress gmm rent pcturban (hsngval = faminc i.region), wmatrix (robust)
Instrumental variables (GMM) regression

GMM weight matrix: Robust | Number of obs $=$ |
| :--- |

| rent | Coef. | Robust | Std. Err. | $z$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |
| ---: | ---: | :---: | :---: | :---: | :---: | ---: |
|  | .0014643 | .0004473 | 3.27 | 0.001 | .0005877 | .002341 |
| pcturban | .7615482 | .2895105 | 2.63 | 0.009 | .1941181 | 1.328978 |
| _cons | 112.1227 | 10.80234 | 10.38 | 0.000 | 90.95052 | 133.2949 |

Instrumented: hsngval
Instruments: pcturban faminc 2.region 3.region 4.region
Because we requested that a heteroskedasticity-consistent weighting matrix be used during estimation but did not specify the vce() option, ivregress reported standard errors that are robust to heteroskedasticity. Had we specified vce(unadjusted), we would have obtained standard errors that would be correct only if the weighting matrix $\mathbf{W}$ does in fact converge to $S_{0}^{-1}$.

- Technical note

Many software packages that implement GMM estimation use the same heteroskedasticity-consistent weighting matrix we used in the previous example to obtain the optimal two-step estimates but do not use a heteroskedasticity-consistent VCE, even though they may label the standard errors as being "robust". To replicate results obtained from other packages, you may have to use the vce (unadjusted) option. See Methods and formulas below for a discussion of robust covariance matrix estimation in the GMM framework.

By changing our definition of $\mathbf{S}_{0}$, we can obtain GMM estimators suitable for use with other types of data that violate the assumption that the errors are independent and identically distributed. For example, you may have a dataset that consists of multiple observations for each person in a sample. The observations that correspond to the same person are likely to be correlated, and the estimation technique should account for that lack of independence. Say that in your dataset, people are identified by the variable personid and you type

```
. ivregress gmm ..., wmatrix(cluster personid)
```

Here ivregress estimates $\mathbf{S}_{0}$ as

$$
\widehat{\mathbf{S}}=\frac{1}{N} \sum_{c \in C} \mathbf{q}_{c} \mathbf{q}_{c}^{\prime}
$$

where $C$ denotes the set of clusters and

$$
\mathbf{q}_{c}=\sum_{i \in c_{j}} \widehat{u}_{i} \mathbf{z}_{i}
$$

where $c_{j}$ denotes the $j$ th cluster. This weighting matrix accounts for the within-person correlation among observations, so the GMM estimator that uses this version of $\mathbf{S}_{0}$ will be more efficient than the estimator that ignores this correlation.

## > Example 4: GMM estimator with clustering

We have data from the National Longitudinal Survey on young women's wages as reported in a series of interviews from 1968 through 1988, and we want to fit a model of wages as a function of each woman's age and age squared, job tenure, birth year, and level of education. We believe that random shocks that affect a woman's wage also affect her job tenure, so we treat tenure as endogenous. As additional instruments, we use her union status, number of weeks worked in the past year, and a dummy indicating whether she lives in a metropolitan area. Because we have several observations for each woman (corresponding to interviews done over several years), we want to control for clustering on each person.

| (National Longitudinal Survey. Young Women $14-26$ years of age in 1968) <br> . ivregress gmm ln_wage age c.age\#c.age birth_yr grade <br> > (tenure $=$ union wks_work msp), wmatrix (cluster idcode) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Instrumental variables (GMM) regression |  |  |  |  | $\begin{array}{llr} \text { Number of obs } & =18625 \\ \text { Wald chi2 }(5) & =1807.17 \\ \text { Prob > chi2 } & =0.0000 \\ \text { R-squared } & = \end{array}$ |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| GMM weight matrix: Cluster (idcode) |  |  |  |  | oot MSE | $=.46951$ |
| (Std. Err. adjusted for 4110 clusters in idcode) |  |  |  |  |  |  |
| Robust |  |  |  |  |  |  |
| ln_wage | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Con | Interval] |
| tenure | . 099221 | . 0037764 | 26.27 | 0.000 | . 0918194 | . 1066227 |
| age | . 0171146 | . 0066895 | 2.56 | 0.011 | . 0040034 | . 0302259 |
| c.age\#c.age | -. 0005191 | . 000111 | -4.68 | 0.000 | -. 0007366 | -. 0003016 |
| birth_yr | -. 0085994 | . 0021932 | -3.92 | 0.000 | -. 012898 | -. 0043008 |
| grade | . 071574 | . 0029938 | 23.91 | 0.000 | . 0657062 | . 0774417 |
| _cons | . 8575071 | . 1616274 | 5.31 | 0.000 | . 5407231 | 1.174291 |

Instrumented: tenure
Instruments: age c.age\#c.age birth_yr grade union wks_work msp
Both job tenure and years of schooling have significant positive effects on wages.

Time-series data are often plagued by serial correlation. In these cases, we can construct a weighting matrix to account for the fact that the error in period $t$ is probably correlated with the errors in periods $t-1, t-2$, etc. An HAC weighting matrix can be used to account for both serial correlation and potential heteroskedasticity.

To request an HAC weighting matrix, you specify the wmatrix (hac kernel [\#|opt]) option. kernel specifies which of three kernels to use: bartlett, parzen, or quadraticspectral. kernel determines the amount of weight given to lagged values when computing the HAC matrix, and \# denotes the maximum number of lags to use. Many texts refer to the bandwidth of the kernel instead of the number of lags; the bandwidth is equal to the number of lags plus one. If neither opt nor \# is specified, then $N-2$ lags are used, where $N$ is the sample size.

If you specify wmatrix (hac kernel opt), then ivregress uses Newey and West's (1994) algorithm for automatically selecting the number of lags to use. Although the authors' Monte Carlo simulations do show that the procedure may result in size distortions of hypothesis tests, the procedure is still useful when little other information is available to help choose the number of lags.

For more on GMM estimation, see Baum (2006); Baum, Schaffer, and Stillman (2003, 2007); Cameron and Trivedi (2005); Davidson and MacKinnon (1993, 2004); Hayashi (2000); or Wooldridge (2010). See Newey and West (1987) and Wang and Wu (2012) for an introduction to HAC covariance matrix estimation.

## Stored results

ivregress stores the following in e():
Scalars
e(N)
e(mss)
e(df_m)
e(rss)
e(df_r)
e(r2)
e(r2_a)
e(F)
e(rmse)
e(N_clust)
e(chi2)
e(kappa)
e(J)
e(wlagopt)
e(vcelagopt)
$e$ (rank)
e(iterations)
Macros
e(cmd)
e(cmdline)
e(depvar)
e(instd)
$e$ (insts)
e(constant)
$e$ (wtype)
e(wexp)
e(title)
e(clustvar)
e(hac_kernel)
e(hac_lag)
e(vce)
e(vcetype)
e(estimator)
e(exogr)
$e$ (wmatrix)
e(moments)
e(small)
e(depname)
e(properties)
e(estat_cmd)
e(predict)
$e$ (footnote)
e(marginsok)
e(marginsnotok)
$e$ (asbalanced)
e(asobserved)
Matrices
e(b)
e(Cns)
e(W)
e(S)
e(V)
e(V_modelbased)
Functions
e(sample)
number of observations
model sum of squares
model degrees of freedom
residual sum of squares
residual degrees of freedom
$R^{2}$
adjusted $R^{2}$
$F$ statistic
root mean squared error
number of clusters
$\chi^{2}$
$\kappa$ used in LIML estimator
value of GMM objective function
lags used in HAC weight matrix (if Newey-West algorithm used)
lags used in HAC VCE matrix (if Newey-West algorithm used)
rank of e(V)
number of GMM iterations (0 if not applicable)

## ivregress

command as typed
name of dependent variable
instrumented variable
instruments
noconstant or hasconstant if specified
weight type
weight expression
title in estimation output
name of cluster variable
HAC kernel
HAC lag
vcetype specified in vce()
title used to label Std. Err.
2 sls , liml, or gmm
exogenous regressors
wmtype specified in wmatrix()
centered if center specified
small if small-sample statistics
depname if depname (depname) specified; otherwise same as e(depvar)
b V
program used to implement estat
program used to implement predict
program used to implement footnote display
predictions allowed by margins
predictions disallowed by margins
factor variables fvset as asbalanced
factor variables fvset as asobserved
coefficient vector
constraints matrix
weight matrix used to compute GMM estimates
moment covariance matrix used to compute GMM variance-covariance matrix
variance-covariance matrix of the estimators
model-based variance
marks estimation sample

## Methods and formulas

Methods and formulas are presented under the following headings:

Notation<br>2SLS and LIML estimators<br>GMM estimator

## Notation

Items printed in lowercase and italicized (for example, $x$ ) are scalars. Items printed in lowercase and boldfaced (for example, $\mathbf{x}$ ) are vectors. Items printed in uppercase and boldfaced (for example, $\mathbf{X )}$ are matrices.

The model is

$$
\begin{gathered}
\mathbf{y}=\mathbf{Y} \boldsymbol{\beta}_{1}+\mathbf{X}_{1} \boldsymbol{\beta}_{2}+\mathbf{u}=\mathbf{X} \boldsymbol{\beta}+\mathbf{u} \\
\mathbf{Y}=\mathbf{X}_{1} \boldsymbol{\Pi}_{1}+\mathbf{X}_{2} \boldsymbol{\Pi}_{2}+\mathbf{v}=\mathbf{Z} \boldsymbol{\Pi}+\mathbf{V}
\end{gathered}
$$

where $\mathbf{y}$ is an $N \times 1$ vector of the left-hand-side variable; $N$ is the sample size; $\mathbf{Y}$ is an $N \times p$ matrix of $p$ endogenous regressors; $\mathbf{X}_{1}$ is an $N \times k_{1}$ matrix of $k_{1}$ included exogenous regressors; $\mathbf{X}_{2}$ is an $N \times k_{2}$ matrix of $k_{2}$ excluded exogenous variables, $\mathbf{X}=\left[\mathbf{Y} \mathbf{X}_{1}\right], \mathbf{Z}=\left[\mathbf{X}_{1} \mathbf{X}_{2}\right] ; \mathbf{u}$ is an $N \times 1$ vector of errors; $\mathbf{V}$ is an $N \times p$ matrix of errors; $\boldsymbol{\beta}=\left[\boldsymbol{\beta}_{1} \boldsymbol{\beta}_{2}\right]$ is a $k=\left(p+k_{1}\right) \times 1$ vector of parameters; and $\boldsymbol{\Pi}$ is a $\left(k_{1}+k_{2}\right) \times p$ vector of parameters. If a constant term is included in the model, then one column of $\mathbf{X}_{1}$ contains all ones.

Let $\mathbf{v}$ be a column vector of weights specified by the user. If no weights are specified, $\mathbf{v}=\mathbf{1}$. Let $\mathbf{w}$ be a column vector of normalized weights. If no weights are specified or if the user specified fweights or iweights, $\mathbf{w}=\mathbf{v}$; otherwise, $\mathbf{w}=\left\{\mathbf{v} /\left(\mathbf{1}^{\prime} \mathbf{v}\right)\right\}\left(\mathbf{1}^{\prime} \mathbf{1}\right)$. Let $\mathbf{D}$ denote the $N \times N$ matrix with $\mathbf{w}$ on the main diagonal and zeros elsewhere. If no weights are specified, $\mathbf{D}$ is the identity matrix.

The weighted number of observations $n$ is defined as $\mathbf{1}^{\prime} \mathbf{w}$. For iweights, this is truncated to an integer. The sum of the weights is $\mathbf{1}^{\prime} \mathbf{v}$. Define $c=1$ if there is a constant in the regression and zero otherwise.

The order condition for identification requires that $k_{2} \geq p$ : the number of excluded exogenous variables must be at least as great as the number of endogenous regressors.

In the following formulas, if weights are specified, $\mathbf{X}_{1}^{\prime} \mathbf{X}_{1}, \mathbf{X}^{\prime} \mathbf{X}, \mathbf{X}^{\prime} \mathbf{y}, \mathbf{y}^{\prime} \mathbf{y}, \mathbf{Z}^{\prime} \mathbf{Z}, \mathbf{Z}^{\prime} \mathbf{X}$, and $\mathbf{Z}^{\prime} \mathbf{y}$ are replaced with $\mathbf{X}_{1}^{\prime} \mathbf{D} \mathbf{X}_{1}, \mathbf{X}^{\prime} \mathbf{D X}, \mathbf{X}^{\prime} \mathbf{D} \mathbf{y}, \mathbf{y}^{\prime} \mathbf{D} \mathbf{y}, \mathbf{Z}^{\prime} \mathbf{D Z}, \mathbf{Z}^{\prime} \mathbf{D X}$, and $\mathbf{Z}^{\prime} \mathbf{D} \mathbf{y}$, respectively. We suppress the $\mathbf{D}$ below to simplify the notation.

## 2SLS and LIML estimators

Define the $\kappa$-class estimator of $\boldsymbol{\beta}$ as

$$
\mathbf{b}=\left\{\mathbf{X}^{\prime}\left(\mathbf{I}-\kappa \mathbf{M}_{\mathbf{Z}}\right) \mathbf{X}\right\}^{-1} \mathbf{X}^{\prime}\left(\mathbf{I}-\kappa \mathbf{M}_{\mathbf{Z}}\right) \mathbf{y}
$$

where $\mathbf{M}_{\mathbf{Z}}=\mathbf{I}-\mathbf{Z}\left(\mathbf{Z}^{\prime} \mathbf{Z}\right)^{-1} \mathbf{Z}^{\prime}$. The 2SLS estimator results from setting $\kappa=1$. The LIML estimator results from selecting $\kappa$ to be the minimum eigenvalue of $\left(\mathbf{Y}^{\prime} \mathbf{M}_{\mathbf{Z}} \mathbf{Y}\right)^{-1 / 2} \mathbf{Y}^{\prime} \mathbf{M}_{\mathbf{X}_{1}} \mathbf{Y}\left(\mathbf{Y}^{\prime} \mathbf{M}_{\mathbf{Z}} \mathbf{Y}\right)^{-1 / 2}$, where $\mathbf{M}_{\mathbf{X}_{1}}=\mathbf{I}-\mathbf{X}_{1}\left(\mathbf{X}_{1}^{\prime} \mathbf{X}_{1}\right)^{-1} \mathbf{X}_{1}^{\prime}$.

The total sum of squares (TSS) equals $\mathbf{y}^{\prime} \mathbf{y}$ if there is no intercept and $\mathbf{y}^{\prime} \mathbf{y}-\left\{\left(\mathbf{1}^{\prime} \mathbf{y}\right)^{2} / n\right\}$ otherwise. The degrees of freedom is $n-c$. The error sum of squares (ESS) is defined as $\mathbf{y}^{\prime} \mathbf{y}-2 \mathbf{b} \mathbf{X}^{\prime} \mathbf{y}+\mathbf{b}^{\prime} \mathbf{X}^{\prime} \mathbf{X b}$. The model sum of squares (MSS) equals TSS - ESS. The degrees of freedom is $k-c$.

The mean squared error, $s^{2}$, is defined as ESS $/(n-k)$ if small is specified and ESS $/ n$ otherwise. The root mean squared error is $s$, its square root.

If $c=1$ and small is not specified, a Wald statistic, $W$, of the joint significance of the $k-1$ parameters of $\boldsymbol{\beta}$ except the constant term is calculated; $W \sim \chi^{2}(k-1)$. If $c=1$ and small is specified, then an $F$ statistic is calculated as $F=W /(k-1) ; F \sim F(k-1, n-k)$.

The $R$-squared is defined as $R^{2}=1$ - ESS/TSS.
The adjusted $R$-squared is $R_{\mathrm{a}}^{2}=1-\left(1-R^{2}\right)(n-c) /(n-k)$.
If robust is not specified, then $\operatorname{Var}(\mathbf{b})=s^{2}\left\{\mathbf{X}^{\prime}\left(\mathbf{I}-\kappa \mathbf{M}_{\mathbf{Z}}\right) \mathbf{X}\right\}^{-1}$. For a discussion of robust variance estimates in regression and regression with instrumental variables, see Methods and formulas in $[\mathrm{R}]$ regress. If small is not specified, then $k=0$ in the formulas given there.

This command also supports estimation with survey data. For details on VCEs with survey data, see [SVY] variance estimation.

## GMM estimator

We obtain an initial consistent estimate of $\boldsymbol{\beta}$ by using the 2SLS estimator; see above. Using this estimate of $\boldsymbol{\beta}$, we compute the weighting matrix $\mathbf{W}$ and calculate the GMM estimator

$$
\mathbf{b}_{\mathrm{GMM}}=\left\{\mathbf{X}^{\prime} \mathbf{Z} \mathbf{W} \mathbf{Z}^{\prime} \mathbf{X}\right\}^{-1} \mathbf{X}^{\prime} \mathbf{Z} \mathbf{W} \mathbf{Z}^{\prime} \mathbf{y}
$$

The variance of $\mathbf{b}_{\mathrm{GMM}}$ is

$$
\operatorname{Var}\left(\mathbf{b}_{\mathrm{GMM}}\right)=n\left\{\mathbf{X}^{\prime} \mathbf{Z} \mathbf{W} \mathbf{Z}^{\prime} \mathbf{X}\right\}^{-1} \mathbf{X}^{\prime} \mathbf{Z} \mathbf{W} \widehat{\mathbf{S}} \mathbf{W} \mathbf{Z}^{\prime} \mathbf{X}\left\{\mathbf{X}^{\prime} \mathbf{Z} \mathbf{W} \mathbf{Z}^{\prime} \mathbf{X}\right\}^{-1}
$$

$\operatorname{Var}\left(\mathbf{b}_{\mathrm{GMM}}\right)$ is of the sandwich form $\mathbf{D M D}$; see $[\mathrm{P}]$ _robust. If the user specifies the small option, ivregress implements a small-sample adjustment by multiplying the VCE by $N /(N-k)$.

If vce(unadjusted) is specified, then we set $\widehat{\mathbf{S}}=\mathbf{W}^{-1}$ and the VCE reduces to the "optimal" GMM variance estimator

$$
\operatorname{Var}\left(\boldsymbol{\beta}_{\mathrm{GMM}}\right)=n\left\{\mathbf{X}^{\prime} \mathbf{Z} \mathbf{W} \mathbf{Z}^{\prime} \mathbf{X}\right\}^{-1}
$$

However, if $\mathbf{W}^{-1}$ is not a good estimator of $E\left(\mathbf{z}_{i} u_{i} u_{i} \mathbf{z}_{i}^{\prime}\right)$, then the optimal GMM estimator is inefficient, and inference based on the optimal variance estimator could be misleading.
$\mathbf{W}$ is calculated using the residuals from the initial 2SLS estimates, whereas $\mathbf{S}$ is estimated using the residuals based on $\mathbf{b}_{\mathrm{GMM}}$. The wmatrix () option affects the form of $\mathbf{W}$, whereas the vce() option affects the form of $\mathbf{S}$. Except for different residuals being used, the formulas for $\mathbf{W}^{-1}$ and $\mathbf{S}$ are identical, so we focus on estimating $\mathbf{W}^{-1}$.

If wmatrix(unadjusted) is specified, then

$$
\mathbf{W}^{-1}=\frac{s^{2}}{n} \sum_{i} \mathbf{z}_{i} \mathbf{z}_{i}^{\prime}
$$

where $s^{2}=\sum_{i} u_{i}^{2} / n$. This weight matrix is appropriate if the errors are homoskedastic.
If wmatrix(robust) is specified, then

$$
\mathbf{W}^{-1}=\frac{1}{n} \sum_{i} u_{i}^{2} \mathbf{z}_{i} \mathbf{z}_{i}^{\prime}
$$

which is appropriate if the errors are heteroskedastic.

If wmatrix(cluster clustvar) is specified, then

$$
\mathbf{W}^{-1}=\frac{1}{n} \sum_{c} \mathbf{q}_{c} \mathbf{q}_{c}^{\prime}
$$

where $c$ indexes clusters,

$$
\mathbf{q}_{c}=\sum_{i \in c_{j}} u_{i} \mathbf{z}_{i}
$$

and $c_{j}$ denotes the $j$ th cluster.
If watrix (hac kernel [\#]) is specified, then

$$
\mathbf{W}^{-1}=\frac{1}{n} \sum_{i} u_{i}^{2} \mathbf{z}_{i} \mathbf{z}_{i}^{\prime}+\frac{1}{n} \sum_{l=1}^{l=n-1} \sum_{i=l+1}^{i=n} K(l, m) u_{i} u_{i-l}\left(\mathbf{z}_{i} \mathbf{z}_{i-l}^{\prime}+\mathbf{z}_{i-l} \mathbf{z}_{i}^{\prime}\right)
$$

where $m=$ \# if \# is specified and $m=n-2$ otherwise. Define $z=l /(m+1)$. If kernel is nwest, then

$$
K(l, m)= \begin{cases}1-z & 0 \leq z \leq 1 \\ 0 & \text { otherwise }\end{cases}
$$

If kernel is gallant, then

$$
K(l, m)= \begin{cases}1-6 z^{2}+6 z^{3} & 0 \leq z \leq 0.5 \\ 2(1-z)^{3} & 0.5<z \leq 1 \\ 0 & \text { otherwise }\end{cases}
$$

If kernel is quadraticspectral, then

$$
K(l, m)= \begin{cases}1 & z=0 \\ 3\{\sin (\theta) / \theta-\cos (\theta)\} / \theta^{2} & \text { otherwise }\end{cases}
$$

where $\theta=6 \pi z / 5$.
If wmatrix (hac kernel opt) is specified, then ivregress uses Newey and West's (1994) automatic lag-selection algorithm, which proceeds as follows. Define $\mathbf{h}$ to be a $\left(k_{1}+k_{2}\right) \times 1$ vector containing ones in all rows except for the row corresponding to the constant term (if present); that row contains a zero. Define

$$
\begin{aligned}
f_{i} & =\left(u_{i} \mathbf{z}_{i}\right) \mathbf{h} \\
\widehat{\sigma}_{j} & =\frac{1}{n} \sum_{i=j+1}^{n} f_{i} f_{i-j} \quad j=0, \ldots, m^{*} \\
\widehat{s}^{(q)} & =2 \sum_{j=1}^{m^{*}} \widehat{\sigma}_{j} j^{q} \\
\widehat{s}^{(0)} & =\widehat{\sigma}_{0}+2 \sum_{j=1}^{m^{*}} \widehat{\sigma}_{j} \\
\widehat{\gamma} & =c_{\gamma}\left\{\left(\frac{\widehat{s}^{(q)}}{\widehat{s}^{(0)}}\right)^{2}\right\}^{1 / 2 q+1} \\
m & =\widehat{\gamma} n^{1 /(2 q+1)}
\end{aligned}
$$

where $q, m^{*}$, and $c_{\gamma}$ depend on the kernel specified:

| Kernel | $q$ | $m^{*}$ | $c_{\gamma}$ |
| :---: | :---: | :---: | :---: |
| Bartlett | 1 | int $\left\{20(T / 100)^{2 / 9}\right\}$ | 1.1447 |
| Parzen | 2 | int $\left\{20(T / 100)^{4 / 25}\right\}$ | 2.6614 |
| Quadratic spectral | 2 | int $\left\{20(T / 100)^{2 / 25}\right\}$ | 1.3221 |

where $\operatorname{int}(x)$ denotes the integer obtained by truncating $x$ toward zero. For the Bartlett and Parzen kernels, the optimal lag is $\min \left\{\operatorname{int}(m), m^{*}\right\}$. For the quadratic spectral, the optimal lag is $\min \left\{m, m^{*}\right\}$.

If center is specified, when computing weighting matrices ivregress replaces the term $u_{i} z_{i}$ in the formulas above with $u_{i} \mathbf{z}_{i}-\overline{u \mathbf{z}}$, where $\overline{u \mathbf{z}}=\sum_{i} u_{i} \mathbf{z}_{i} / N$.

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## Also see

[R] ivregress postestimation - Postestimation tools for ivregress
[R] gmm - Generalized method of moments estimation
[R] ivprobit - Probit model with continuous endogenous regressors
[R] ivtobit - Tobit model with continuous endogenous regressors
[R] reg3 - Three-stage estimation for systems of simultaneous equations
[ R ] regress - Linear regression
[SEM] intro 5 - Tour of models
[SVY] svy estimation - Estimation commands for survey data
[TS] forecast - Econometric model forecasting
[XT] xtivreg - Instrumental variables and two-stage least squares for panel-data models
[U] 20 Estimation and postestimation commands

## Title

ivregress postestimation - Postestimation tools for ivregress

| Description | Syntax for predict | Menu for predict | Options for predict |
| :--- | :--- | :--- | :--- |
| Syntax for estat | Menu for estat | Options for estat | Remarks and examples |
| Stored results | Methods and formulas | References | Also see |

## Description

The following postestimation commands are of special interest after ivregress:
Command Description
estat endogenous perform tests of endogeneity
estat firststage report "first-stage" regression statistics
estat overid perform tests of overidentifying restrictions

These commands are not appropriate after the svy prefix.

The following postestimation commands are also available:
Command Description

| contrast | contrasts and ANOVA-style joint tests of estimates |
| :--- | :--- |
| estat summarize |  |
| estat vce | summary statistics for the estimation sample <br> variance-covariance matrix of the estimators (VCE) <br> estat (svy) <br> estimates <br> postestimation statistics for survey data <br> corecast |
| hausman | cataloging estimation results <br> dynamic forecasts and simulations |
| lincom | Hausman's specification test <br> point estimates, standard errors, testing, and inference for linear combinations <br> of coefficients |
| margins | marginal means, predictive margins, marginal effects, and average marginal <br> effects |
| marginsplot | graph the results from margins (profile plots, interaction plots, etc.) <br> point estimates, standard errors, testing, and inference for nonlinear combinations <br> of coefficients |
| nlcom | predictions, residuals, influence statistics, and other diagnostic measures |
| predict | point estimates, standard errors, testing, and inference for generalized predictions <br> predictnl <br> pwcompare |
| test | Wald tests of simple and composite linear hypotheses |
| testnl | Wald tests of nonlinear hypotheses |

[^33]
## Special-interest postestimation commands

estat endogenous performs tests to determine whether endogenous regressors in the model are in fact exogenous. After GMM estimation, the $C$ (difference-in-Sargan) statistic is reported. After 2SLS estimation with an unadjusted VCE, the Durbin (1954) and Wu-Hausman (Wu 1974; Hausman 1978) statistics are reported. After 2SLS estimation with a robust VCE, Wooldridge's (1995) robust score test and a robust regression-based test are reported. In all cases, if the test statistic is significant, then the variables being tested must be treated as endogenous. estat endogenous is not available after LIML estimation.
estat firststage reports various statistics that measure the relevance of the excluded exogenous variables. By default, whether the equation has one or more than one endogenous regressor determines what statistics are reported.
estat overid performs tests of overidentifying restrictions. If the 2SLS estimator was used, Sargan's (1958) and Basmann's (1960) $\chi^{2}$ tests are reported, as is Wooldridge's (1995) robust score test; if the LIML estimator was used, Anderson and Rubin's (1950) $\chi^{2}$ test and Basmann's $F$ test are reported; and if the GMM estimator was used, Hansen's (1982) $J$ statistic $\chi^{2}$ test is reported. A statistically significant test statistic always indicates that the instruments may not be valid.

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic]
predict [type] {stub*|newvarlist } [if] [in], scores
```

statistic Description

Main
$\mathrm{xb} \quad$ linear prediction; the default
residuals residuals
stdp standard error of the prediction
stdf standard error of the forecast
$\underline{\operatorname{pr}}(a, b) \quad \operatorname{Pr}\left(a<y_{j}<b\right)$
$\mathrm{e}(a, b) \quad E\left(y_{j} \mid a<y_{j}<b\right)$
ystar $(a, b) \quad E\left(y_{j}^{*}\right), y_{j}^{*}=\max \left\{a, \min \left(y_{j}, b\right)\right\}$
These statistics are available both in and out of sample; type predict ... if e(sample)... if wanted only for the estimation sample.
stdf is not allowed with svy estimation results.
where $a$ and $b$ may be numbers or variables; $a$ missing ( $a \geq$.) means $-\infty$, and $b$ missing ( $b \geq$. ) means $+\infty$; see [U] 12.2.1 Missing values.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

Main
xb , the default, calculates the linear prediction.
residuals calculates the residuals, that is, $y_{j}-\mathbf{x}_{j} \mathbf{b}$. These are based on the estimated equation when the observed values of the endogenous variables are used-not the projections of the instruments onto the endogenous variables.
stdp calculates the standard error of the prediction, which can be thought of as the standard error of the predicted expected value or mean for the observation's covariate pattern. This is also referred to as the standard error of the fitted value.
stdf calculates the standard error of the forecast, which is the standard error of the point prediction for 1 observation. It is commonly referred to as the standard error of the future or forecast value. By construction, the standard errors produced by stdf are always larger than those produced by stdp; see Methods and formulas in [R] regress postestimation.
$\operatorname{pr}(a, b)$ calculates $\operatorname{Pr}\left(a<\mathbf{x}_{j} \mathbf{b}+u_{j}<b\right)$, the probability that $y_{j} \mid \mathbf{x}_{j}$ would be observed in the interval $(a, b)$.
$a$ and $b$ may be specified as numbers or variable names; $l b$ and $u b$ are variable names;
$\operatorname{pr}(20,30)$ calculates $\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$;
$\operatorname{pr}(l b, u b)$ calculates $\operatorname{Pr}\left(l b<\mathbf{x}_{j} \mathbf{b}+u_{j}<u b\right)$; and
$\operatorname{pr}(20, u b)$ calculates $\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{j}<u b\right)$.
$a$ missing ( $a \geq$. ) means $-\infty$; $\operatorname{pr}(., 30)$ calculates $\operatorname{Pr}\left(-\infty<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$;
$\operatorname{pr}(l b, 30)$ calculates $\operatorname{Pr}\left(-\infty<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$ in observations for which $l b \geq$.
and calculates $\operatorname{Pr}\left(l b<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$ elsewhere.
$b$ missing ( $b \geq$.) means $+\infty$; $\operatorname{pr}(20,$.$) calculates \operatorname{Pr}\left(+\infty>\mathbf{x}_{j} \mathbf{b}+u_{j}>20\right)$;
$\operatorname{pr}(20, u b)$ calculates $\operatorname{Pr}\left(+\infty>\mathbf{x}_{j} \mathbf{b}+u_{j}>20\right)$ in observations for which $u b \geq$.
and calculates $\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{j}<u b\right)$ elsewhere.
$\mathrm{e}(a, b)$ calculates $E\left(\mathbf{x}_{j} \mathbf{b}+u_{j} \mid a<\mathbf{x}_{j} \mathbf{b}+u_{j}<b\right)$, the expected value of $y_{j} \mid \mathbf{x}_{j}$ conditional on $y_{j} \mid \mathbf{x}_{j}$ being in the interval $(a, b)$, meaning that $y_{j} \mid \mathbf{x}_{j}$ is truncated.
$a$ and $b$ are specified as they are for $\operatorname{pr}()$.
ystar $(a, b)$ calculates $E\left(y_{j}^{*}\right)$, where $y_{j}^{*}=a$ if $\mathbf{x}_{j} \mathbf{b}+u_{j} \leq a, y_{j}^{*}=b$ if $\mathbf{x}_{j} \mathbf{b}+u_{j} \geq b$, and $y_{j}^{*}=\mathbf{x}_{j} \mathbf{b}+u_{j}$ otherwise, meaning that $y_{j}^{*}$ is censored. $a$ and $b$ are specified as they are for pr() .
scores calculates the scores for the model. A new score variable is created for each endogenous regressor, as well as an equation-level score that applies to all exogenous variables and constant term (if present).

## Syntax for estat

Perform tests of endogeneity

```
estat endogenous [varlist] [, lags(#) forceweights forcenonrobust]
```

Report "first-stage" regression statistics

```
estat firststage [, all forcenonrobust]
```

Perform tests of overidentifying restrictions
estat overid [, lags(\#) forceweights forcenonrobust]

## Menu for estat

Statistics $>$ Postestimation $>$ Reports and statistics

## Options for estat

Options for estat are presented under the following headings:
Options for estat endogenous
Options for estat firststage
Options for estat overid

## Options for estat endogenous

lags (\#) specifies the number of lags to use for prewhitening when computing the heteroskedasticityand autocorrelation-consistent (HAC) version of the score test of endogeneity. Specifying lags (0) requests no prewhitening. This option is valid only when the model was fit via 2SLS and an HAC covariance matrix was requested when the model was fit. The default is lags(1).
forceweights requests that the tests of endogeneity be computed even though aweights, pweights, or iweights were used in the previous estimation. By default, these tests are conducted only after unweighted or frequency-weighted estimation. The reported critical values may be inappropriate for weighted data, so the user must determine whether the critical values are appropriate for a given application.
forcenonrobust requests that the Durbin and Wu-Hausman tests be performed after 2SLS estimation even though a robust VCE was used at estimation time. This option is available only if the model was fit by 2SLS.

## Options for estat firststage

all requests that all first-stage goodness-of-fit statistics be reported regardless of whether the model contains one or more endogenous regressors. By default, if the model contains one endogenous regressor, then the first-stage $R^{2}$, adjusted $R^{2}$, partial $R^{2}$, and $F$ statistics are reported, whereas if the model contains multiple endogenous regressors, then Shea's partial $R^{2}$ and adjusted partial $R^{2}$ are reported instead.
forcenonrobust requests that the minimum eigenvalue statistic and its critical values be reported even though a robust VCE was used at estimation time. The reported critical values assume that the errors are independent and identically distributed (i.i.d.) normal, so the user must determine whether the critical values are appropriate for a given application.

## Options for estat overid

lags (\#) specifies the number of lags to use for prewhitening when computing the heteroskedasticityand autocorrelation-consistent (HAC) version of the score test of overidentifying restrictions. Specifying lags ( 0 ) requests no prewhitening. This option is valid only when the model was fit via 2SLS and an HAC covariance matrix was requested when the model was fit. The default is lags(1).
forceweights requests that the tests of overidentifying restrictions be computed even though aweights, pweights, or iweights were used in the previous estimation. By default, these tests are conducted only after unweighted or frequency-weighted estimation. The reported critical values may be inappropriate for weighted data, so the user must determine whether the critical values are appropriate for a given application.
forcenonrobust requests that the Sargan and Basmann tests of overidentifying restrictions be performed after 2SLS or LIML estimation even though a robust VCE was used at estimation time. These tests assume that the errors are i.i.d. normal, so the user must determine whether the critical values are appropriate for a given application.

## Remarks and examples

Remarks are presented under the following headings:

```
estat endogenous
estat firststage
estat overid
```


## estat endogenous

A natural question to ask is whether a variable presumed to be endogenous in the previously fit model could instead be treated as exogenous. If the endogenous regressors are in fact exogenous, then the OLS estimator is more efficient; and depending on the strength of the instruments and other factors, the sacrifice in efficiency by using an instrumental-variables estimator can be significant. Thus, unless an instrumental-variables estimator is really needed, OLS should be used instead. estat endogenous provides several tests of endogeneity after 2SLS and GMM estimation.
$>$ Example 1
In example 1 of $[\mathrm{R}]$ ivregress, we fit a model of the average rental rate for housing in a state as a function of the percentage of the population living in urban areas and the average value of houses. We treated hsngval as endogenous because unanticipated shocks that affect rental rates probably affect house prices as well. We used family income and region dummies as additional instruments for hsngval. Here we test whether we could treat hsngval as exogenous.

```
. use http://www.stata-press.com/data/r13/hsng
(1980 Census housing data)
. ivregress 2sls rent pcturban (hsngval = faminc i.region)
(output omitted)
estat endogenous
Tests of endogeneity
Ho: variables are exogenous
Durbin (score) chi2(1) = 12.8473 (p = 0.0003)
Wu-Hausman F(1,46) = 15.9067 (p = 0.0002)
```

Because we did not specify any variable names after the estat endogenous command, Stata by default tested all the endogenous regressors (namely, hsngval) in our model. The null hypothesis of the Durbin and $\mathrm{Wu}-H a u s m a n ~ t e s t s ~ i s ~ t h a t ~ t h e ~ v a r i a b l e ~ u n d e r ~ c o n s i d e r a t i o n ~ c a n ~ b e ~ t r e a t e d ~ a s ~$ exogenous. Here both test statistics are highly significant, so we reject the null of exogeneity; we must continue to treat hsngval as endogenous.

The difference between the Durbin and Wu-Hausman tests of endogeneity is that the former uses an estimate of the error term's variance based on the model assuming the variables being tested are exogenous, while the latter uses an estimate of the error variance based on the model assuming the variables being tested are endogenous. Under the null hypothesis that the variables being tested are exogenous, both estimates of the error variance are consistent. What we label the Wu -Hausman statistic is Wu's (1974) " $T_{2}$ " statistic, which Hausman (1978) showed can be calculated very easily via linear regression. Baum, Schaffer, and Stillman $(2003,2007)$ provide a lucid discussion of these tests.

When you fit a model with multiple endogenous regressors, you can test the exogeneity of a subset of the regressors while continuing to treat the others as endogenous. For example, say you have three endogenous regressors, $\mathrm{y} 1, \mathrm{y} 2$, and y 3 , and you fit your model by typing

```
. ivregress depvar ... (y1 y2 y3 = ...)
```

Suppose you are confident that y1 must be treated as endogenous, but you are undecided about y2 and y3. To test whether y2 and y3 can be treated as exogenous, you would type

```
estat endogenous y2 y3
```

The Durbin and Wu -Hausman tests assume that the error term is i.i.d. Therefore, if you requested a robust VCE at estimation time, estat endogenous will instead report Wooldridge's (1995) score test and a regression-based test of exogeneity. Both these tests can tolerate heteroskedastic and autocorrelated errors, while only the regression-based test is amenable to clustering.

## $>$ Example 2

We refit our housing model, requesting robust standard errors, and then test the exogeneity of hsngval:

```
. use http://www.stata-press.com/data/r13/hsng
(1980 Census housing data)
. ivregress 2sls rent pcturban (hsngval = faminc i.region), vce(robust)
    (output omitted)
. estat endogenous
    Tests of endogeneity
    Ho: variables are exogenous
    Robust score chi2(1) = 2.10428 ( }\textrm{p}=0.1469\mathrm{ )
    Robust regression F(1,46) = 4.31101 (p = 0.0435)
```

Wooldridge's score test does not reject the null hypothesis that hsngval is exogenous at conventional significance levels ( $p=0.1469$ ). However, the regression-based test does reject the null hypothesis at the 5\% significance level ( $p=0.0435$ ). Typically, these two tests yield the same conclusion; the fact that our dataset has only 50 observations could be contributing to the discrepancy. Here we would be inclined to continue to treat hsngval as endogenous. Even if hsngval is exogenous, the 2SLS estimates are still consistent. On the other hand, if hsngval is in fact endogenous, the OLS estimates would not be consistent. Moreover, as we will see in our discussion of the estat overid command, our additional instruments may be invalid. To test whether an endogenous variable can be treated as exogenous, we must have a valid set of instruments to use to fit the model in the first place!

Unlike the Durbin and Wu-Hausman tests, Wooldridge's score and the regression-based tests do not allow you to test a subset of the endogenous regressors in the model; you can test only whether all the endogenous regressors are in fact exogenous.

After GMM estimation, estat endogenous calculates what Hayashi (2000, 220) calls the $C$ statistic, also known as the difference-in-Sargan statistic. The $C$ statistic can be made robust to heteroskedasticity, autocorrelation, and clustering; and the version reported by estat endogenous is determined by the weight matrix requested via the wmatrix () option used when fitting the model with ivregress. Additionally, the test can be used to determine the exogeneity of a subset of the endogenous regressors, regardless of the type of weight matrix used.

If you fit your model using the LIML estimator, you can use the hausman command to carry out a traditional Hausman (1978) test between the OLS and LIML estimates.

## estat firststage

For an excluded exogenous variable to be a valid instrument, it must be sufficiently correlated with the included endogenous regressors but uncorrelated with the error term. In recent decades, researchers have paid considerable attention to the issue of instruments that are only weakly correlated with the endogenous regressors. In such cases, the usual 2SLS, GMM, and LIML estimators are biased toward the OLS estimator, and inference based on the standard errors reported by, for example, ivregress can be severely misleading. For more information on the theory behind instrumental-variables estimation with weak instruments, see Nelson and Startz (1990); Staiger and Stock (1997); Hahn and Hausman (2003); the survey article by Stock, Wright, and Yogo (2002); and Angrist and Pischke (2009, chap. 4).

When the instruments are only weakly correlated with the endogenous regressors, some Monte Carlo evidence suggests that the LIML estimator performs better than the 2SLS and GMM estimators; see, for example, Poi (2006) and Stock, Wright, and Yogo (2002) (and the papers cited therein). On the other hand, the LIML estimator often results in confidence intervals that are somewhat larger than those from the 2SLS estimator.

Moreover, using more instruments is not a solution, because the biases of instrumental-variables estimators increase with the number of instruments. See Hahn and Hausman (2003).
estat firststage produces several statistics for judging the explanatory power of the instruments and is most easily explained with examples.

## > Example 3

Again building on the model fit in example 1 of $[\mathrm{R}]$ ivregress, we now explore the degree of correlation between the additional instruments faminc, 2.region, 3.region, and 4.region and the endogenous regressor hsngval:

```
. use http://www.stata-press.com/data/r13/hsng
(1980 Census housing data)
. ivregress 2sls rent pcturban (hsngval = faminc i.region)
(output omitted)
estat firststage
First-stage regression summary statistics
```

| Variable | R-sq. | Adjusted <br> R-sq. | Partial <br> R-sq. | $F(4,44)$ | Prob >F |
| :---: | :---: | :---: | :---: | :---: | :---: |
| hsngval | 0.6908 | 0.6557 | 0.5473 | 13.2978 | 0.0000 |


| Minimum eigenvalue statistic $=13.2978$ <br> Critical Values <br> Ho: Instruments are weak | \# of endogenous regressors: <br> \# of excluded instruments: | 1 <br> 4 |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | $5 \%$ | $10 \%$ | $20 \%$ | $30 \%$ |
| 2SLS relative bias | 16.85 | 10.27 | 6.71 | 5.34 |  |

To understand these results, recall that the first-stage regression is
hsngval $_{i}=\pi_{0}+\pi_{1}$ pcturban $_{i}+\pi_{2}$ faminc $+\pi_{3}$ 2.region $+\pi_{4}$ 3.region $+\pi_{5}$ 4.region $+v_{i}$
where $v_{i}$ is an error term. The column marked " R -sq." is the simple $R^{2}$ from fitting the first-stage regression by OLS, and the column marked "Adjusted R-sq." is the adjusted $R^{2}$ from that regression. Higher values purportedly indicate stronger instruments, and instrumental-variables estimators exhibit less bias when the instruments are strongly correlated with the endogenous variable.

Looking at just the $R^{2}$ and adjusted $R^{2}$ can be misleading, however. If hsngval were strongly correlated with the included exogenous variable pcturban but only weakly correlated with the additional instruments, then these statistics could be large even though a weak-instrument problem is present.

The partial $R^{2}$ statistic measures the correlation between hsngval and the additional instruments after partialling out the effect of pcturban. Unlike the $R^{2}$ and adjusted $R^{2}$ statistics, the partial $R^{2}$ statistic will not be inflated because of strong correlation between hsngval and pcturban. Bound, Jaeger, and Baker (1995) and others have promoted using this statistic.

The column marked " $F(4,44)$ " is an $F$ statistic for the joint significance of $\pi_{2}, \pi_{3}, \pi_{4}$, and $\pi_{5}$, the coefficients on the additional instruments. Its $p$-value is listed in the column marked "Prob $>\mathrm{F}$ ". If the $F$ statistic is not significant, then the additional instruments have no significant explanatory power for hsngval after controlling for the effect of pcturban. However, Hall, Rudebusch, and Wilcox (1996) used Monte Carlo simulation to show that simply having an $F$ statistic that is significant at the typical $5 \%$ or $10 \%$ level is not sufficient. Stock, Wright, and Yogo (2002) suggest that the $F$ statistic should exceed 10 for inference based on the 2SLS estimator to be reliable when there is one endogenous regressor.
estat firststage also presents the Cragg and Donald (1993) minimum eigenvalue statistic as a further test of weak instruments. Stock and Yogo (2005) discuss two characterizations of weak instruments: first, weak instruments cause instrumental-variables estimators to be biased; second, hypothesis tests of parameters estimated by instrumental-variables estimators may suffer from severe size distortions. The test statistic in our example is 13.30 , which is identical to the $F$ statistic just discussed because our model contains one endogenous regressor.

The null hypothesis of each of Stock and Yogo's tests is that the set of instruments is weak. To perform these tests, we must first choose either the largest relative bias of the 2SLS estimator we are willing to tolerate or the largest rejection rate of a nominal $5 \%$ Wald test we are willing to tolerate. If the test statistic exceeds the critical value, we can conclude that our instruments are not weak.

The row marked " 2 SLS relative bias" contains critical values for the test that the instruments are weak based on the bias of the 2SLS estimator relative to the bias of the OLS estimator. For example, from past experience we might know that the OLS estimate of a parameter $\beta$ may be $50 \%$ too high. Saying that we are willing to tolerate a $10 \%$ relative bias means that we are willing to tolerate a bias of the 2SLS estimator no greater than $5 \%$ (that is, $10 \%$ of $50 \%$ ). In our rental rate model, if we
are willing to tolerate a $10 \%$ relative bias, then we can conclude that our instruments are not weak because the test statistic of 13.30 exceeds the critical value of 10.22 . However, if we were willing to tolerate only a relative bias of $5 \%$, we would conclude that our instruments are weak because $13.30<16.85$.

The rows marked "2SLS Size of nominal 5\% Wald test" and "LIML Size of nominal 5\% Wald test" contain critical values pertaining to Stock and Yogo's (2005) second characterization of weak instruments. This characterization defines a set of instruments to be weak if a Wald test at the 5\% level can have an actual rejection rate of no more than $10 \%, 15 \%, 20 \%$, or $25 \%$. Using the current example, suppose that we are willing to accept a rejection rate of at most $10 \%$. Because $13.30<24.58$, we cannot reject the null hypothesis of weak instruments. On the other hand, if we use the LIML estimator instead, then we can reject the null hypothesis because $13.30>5.44$.

## Technical note

Stock and Yogo (2005) tabulated critical values for 2SLS relative biases of $5 \%, 10 \%, 20 \%$, and $30 \%$ for models with 1 , 2 , or 3 endogenous regressors and between 3 and 30 excluded exogenous variables (instruments). They also provide critical values for worst-case rejection rates of $5 \%, 10 \%$, $20 \%$, and $25 \%$ for nominal $5 \%$ Wald tests of the endogenous regressors with 1 or 2 endogenous regressors and between 1 and 30 instruments. If the model previously fit by ivregress has more instruments or endogenous regressors than these limits, the critical values are not shown. Stock and Yogo did not consider GMM estimators.

When the model being fit contains more than one endogenous regressor, the $R^{2}$ and $F$ statistics described above can overstate the relevance of the excluded instruments. Suppose that there are two endogenous regressors, $Y_{1}$ and $Y_{2}$, and that there are two additional instruments, $z_{1}$ and $z_{2}$. Say that $z_{1}$ is highly correlated with both $Y_{1}$ and $Y_{2}$ but $z_{2}$ is not correlated with either $Y_{1}$ or $Y_{2}$. Then the first-stage regression of $Y_{1}$ on $z_{1}$ and $z_{2}$ (along with the included exogenous variables) will produce large $R^{2}$ and $F$ statistics, as will the regression of $Y_{2}$ on $z_{1}, z_{2}$, and the included exogenous variables. Nevertheless, the lack of correlation between $z_{2}$ and $Y_{1}$ and $Y_{2}$ is problematic. Here, although the order condition indicates that the model is just identified (the number of excluded instruments equals the number of endogenous regressors), the irrelevance of $z_{2}$ implies that the model is in fact not identified. Even if the model is overidentified, including irrelevant instruments can adversely affect the properties of instrumental-variables estimators, because their biases increase as the number of instruments increases.

## > Example 4

estat firststage presents different statistics when the model contains multiple endogenous regressors. For illustration, we refit our model of rental rates, assuming that both hsngval and faminc are endogenously determined. We use i.region along with popden, a measure of population density, as additional instruments.

```
. ivregress 2sls rent pcturban (hsngval faminc = i.region popden)
(output omitted)
```

estat firststage
Shea's partial R-squared

| Variable | Shea's <br> Partial R-sq. | Adj. Partial R-sq. |
| ---: | :---: | :---: |
| hsngval | 0.3477 | 0.2735 |
| faminc | 0.1893 | 0.0972 |

Minimum eigenvalue statistic $=2.51666$
Critical Values \# of endogenous regressors: 2
Ho: Instruments are weak \# of excluded instruments: 4

|  |  | $5 \%$ | $10 \%$ | $20 \%$ | $30 \%$ |
| :--- | :---: | :--- | :--- | :--- | :--- |
| 2SLS relative bias | 11.04 | 7.56 | 5.57 | 4.73 |  |
|  |  | $10 \%$ | $15 \%$ | $20 \%$ | $25 \%$ |
| 2SLS Size of nominal $5 \%$ Wald test | 16.87 | 9.93 | 7.54 | 6.28 |  |
| LIML Size of nominal $5 \%$ Wald test | 4.72 | 3.39 | 2.99 | 2.79 |  |

Consider the endogenous regressor hsngval. Part of its variation is attributable to its correlation with the other regressors pcturban and faminc. The other component of hsngval's variation is peculiar to it and orthogonal to the variation in the other regressors. Similarly, we can think of the instruments as predicting the variation in hsngval in two ways, one stemming from the fact that the predicted values of hsngval are correlated with the predicted values of the other regressors and one from the variation in the predicted values of hsngval that is orthogonal to the variation in the predicted values of the other regressors.

What really matters for instrumental-variables estimation is whether the component of hsngval that is orthogonal to the other regressors can be explained by the component of the predicted value of hsngval that is orthogonal to the predicted values of the other regressors in the model. Shea's (1997) partial $R^{2}$ statistic measures this correlation. Because the bias of instrumental-variables estimators increases as more instruments are used, Shea's adjusted partial $R^{2}$ statistic is often used instead, as it makes a degrees-of-freedom adjustment for the number of instruments, analogous to the adjusted $R^{2}$ measure used in OLS regression. Although what constitutes a "low" value for Shea's partial $R^{2}$ depends on the specifics of the model being fit and the data used, these results, taken in isolation, do not strike us as being a particular cause for concern.

However, with this specification the minimum eigenvalue statistic is low. We cannot reject the null hypothesis of weak instruments for either of the characterizations we have discussed.

By default, estat firststage determines which statistics to present based on the number of endogenous regressors in the model previously fit. However, you can specify the all option to obtain all the statistics.

- Technical note

If the previous estimation was conducted using aweights, pweights, or iweights, then the first-stage regression summary statistics are computed using those weights. However, in these cases the minimum eigenvalue statistic and its critical values are not available.

If the previous estimation included a robust VCE, then the first-stage $F$ statistic is based on a robust VCE as well; for example, if you fit your model with an HAC VCE using the Bartlett kernel and four lags, then the $F$ statistic reported is based on regression results using an HAC VCE using the Bartlett kernel and four lags. By default, the minimum eigenvalue statistic and its critical values are not displayed. You can use the forcenonrobust option to obtain them in these cases; the minimum eigenvalue statistic is computed using the weights, though the critical values reported may not be appropriate.

## estat overid

In addition to the requirement that instrumental variables be correlated with the endogenous regressors, the instruments must also be uncorrelated with the structural error term. If the model is overidentified, meaning that the number of additional instruments exceeds the number of endogenous regressors, then we can test whether the instruments are uncorrelated with the error term. If the model is just identified, then we cannot perform a test of overidentifying restrictions.

The estimator you used to fit the model determines which tests of overidentifying restrictions estat overid reports. If you used the 2SLS estimator without a robust VCE, estat overid reports Sargan's (1958) and Basmann's (1960) $\chi^{2}$ tests. If you used the 2SLS estimator and requested a robust VCE, Wooldridge's robust score test of overidentifying restrictions is performed instead; without a robust VCE, Wooldridge's test statistic is identical to Sargan's test statistic. If you used the LIML estimator, estat overid reports the Anderson-Rubin (1950) likelihood-ratio test and Basmann's (1960) $F$ test. estat overid reports Hansen's (1982) $J$ statistic if you used the GMM estimator. Davidson and MacKinnon (1993, 235-236) give a particularly clear explanation of the intuition behind tests of overidentifying restrictions. Also see Judge et al. (1985, 614-616) for a summary of tests of overidentifying restrictions for the 2SLS and LIML estimators.

Tests of overidentifying restrictions actually test two different things simultaneously. One, as we have discussed, is whether the instruments are uncorrelated with the error term. The other is that the equation is misspecified and that one or more of the excluded exogenous variables should in fact be included in the structural equation. Thus a significant test statistic could represent either an invalid instrument or an incorrectly specified structural equation.
$>$ Example 5
Here we refit the model that treated just hsngval as endogenous using 2SLS, and then we perform tests of overidentifying restrictions:

```
. ivregress 2sls rent pcturban (hsngval = faminc i.region)
(output omitted)
estat overid
Tests of overidentifying restrictions:
Sargan (score) chi2(3) = 11.2877 (p = 0.0103)
Basmann chi2(3) = 12.8294 (p = 0.0050)
```

Both test statistics are significant at the $5 \%$ test level, which means that either one or more of our instruments are invalid or that our structural model is specified incorrectly.

One possibility is that the error term in our structural model is heteroskedastic. Both Sargan's and Basmann's tests assume that the errors are i.i.d.; if the errors are not i.i.d., then these tests are not valid. Here we refit the model by requesting heteroskedasticity-robust standard errors, and then we use estat overid to obtain Wooldridge's score test of overidentifying restrictions, which is robust to heteroskedasticity.

```
. ivregress 2sls rent pcturban (hsngval = faminc i.region), vce(robust)
(output omitted)
```

. estat overid
Test of overidentifying restrictions:
Score chi2 (3) $=6.8364 \quad(\mathrm{p}=0.0773)$

Here we no longer reject the null hypothesis that our instruments are valid at the $5 \%$ significance level, though we do reject the null at the $10 \%$ level. You can verify that the robust standard error on the coefficient for hsngval is more than twice as large as its nonrobust counterpart and that the robust standard error for pcturban is nearly $50 \%$ larger.

## Technical note

The test statistic for the test of overidentifying restrictions performed after GMM estimation is simply the sample size times the value of the objective function $Q\left(\boldsymbol{\beta}_{1}, \boldsymbol{\beta}_{2}\right)$ defined in (5) of [R] ivregress, evaluated at the GMM parameter estimates. If the weighting matrix $\mathbf{W}$ is optimal, meaning that $\mathbf{W}=\operatorname{Var}\left(\mathbf{z}_{i} u_{i}\right)$, then $Q\left(\boldsymbol{\beta}_{\mathbf{1}}, \boldsymbol{\beta}_{2}\right) \stackrel{A}{\sim} \chi^{2}(q)$, where $q$ is the number of overidentifying restrictions. However, if the estimated $\mathbf{W}$ is not optimal, then the test statistic will not have an asymptotic $\chi^{2}$ distribution.

Like the Sargan and Basmann tests of overidentifying restrictions for the 2SLS estimator, the Anderson-Rubin and Basmann tests after LIML estimation are predicated on the errors' being i.i.d. If the previous LIML results were reported with robust standard errors, then estat overid by default issues an error message and refuses to report the Anderson-Rubin and Basmann test statistics. You can use the forcenonrobust option to override this behavior. You can also use forcenonrobust to obtain the Sargan and Basmann test statistics after 2SLS estimation with robust standard errors.

By default, estat overid issues an error message if the previous estimation was conducted using aweights, pweights, or iweights. You can use the forceweights option to override this behavior, though the test statistics may no longer have the expected $\chi^{2}$ distributions.

## Stored results

After 2SLS estimation, estat endogenous stores the following in r() :

```
Scalars
```

    \(r\) (durbin) Durbin \(\chi^{2}\) statistic
    \(r\) (p_durbin) \(\quad p\)-value for Durbin \(\chi^{2}\) statistic
    r(wu) Wu-Hausman \(F\) statistic
    \(r\) (p_wu) \(\quad p\)-value for Wu -Hausman \(F\) statistic
    \(r(d f) \quad\) degrees of freedom
    \(r\) (wudf_r) denominator degrees of freedom for Wu -Hausman \(F\)
    \(r\) ( \(r\) _score) robust score statistic
    \(r\) (p_r_score) \(\quad p\)-value for robust score statistic
    \(r\) (hac_score) HAC score statistic
    \(r\) (p_hac_score) \(\quad p\)-value for HAC score statistic
    \(r\) (lags) lags used in prewhitening
    \(r\) (regF) regression-based \(F\) statistic
    \(r\) (p_regF) \(\quad p\)-value for regression-based \(F\) statistic
    \(r(\) regFdf_n) regression-based \(F\) numerator degrees of freedom
    \(r\) (regFdf_r) regression-based \(F\) denominator degrees of freedom
    After GMM estimation, estat endogenous stores the following in $r()$ :
Scalars
r(C)
$C \chi^{2}$ statistic
r(p_C)
$p$-value for $C \chi^{2}$ statistic
$r(d f)$ degrees of freedom
estat firststage stores the following in $r()$ :
Scalars $r$ (mineig) minimum eigenvalue statistic
Matrices

| $r$ (mineigcv) | critical values for minimum eigenvalue statistic |
| :--- | :--- |
| $r$ (multiresults) | Shea's partial $R^{2}$ statistics |
| $r$ (singleresults) | first-stage $R^{2}$ and $F$ statistics |

After 2SLS estimation, estat overid stores the following in r() :
Scalars

```
r(lags) lags used in prewhitening
r(df) \quad }\mp@subsup{\chi}{}{2}\mathrm{ degrees of freedom
r(score) score }\mp@subsup{\chi}{}{2}\mathrm{ statistic
r(p_score) }\quadp\mathrm{ -value for score }\mp@subsup{\chi}{}{2}\mathrm{ statistic
r(basmann) Basmann }\mp@subsup{\chi}{}{2}\mathrm{ statistic
    r(p_basmann) }\quadp\mathrm{ -value for Basmann }\mp@subsup{\chi}{}{2}\mathrm{ statistic
    r(sargan) Sargan }\mp@subsup{\chi}{}{2}\mathrm{ statistic
    r(p_sargan) }\quadp\mathrm{ -value for Sargan }\mp@subsup{\chi}{}{2}\mathrm{ statistic
```

After LIML estimation, estat overid stores the following in r() :
Scalars $r(a r) \quad$ Anderson-Rubin $\chi^{2}$ statistic
$r$ (p_ar) $\quad p$-value for Anderson-Rubin $\chi^{2}$ statistic
$r\left(a r \_d f\right) \quad \chi^{2}$ degrees of freedom
r (basmann) Basmann $F$ statistic
$r$ (p_basmann) $\quad p$-value for Basmann $F$ statistic
$r$ (basmann_df_n) $\quad F$ numerator degrees of freedom
r(basmann_df_d) $\quad F$ denominator degrees of freedom
After GMM estimation, estat overid stores the following in $r()$ :
Scalars
$r$ (HansenJ) Hansen's $J \chi^{2}$ statistic
$r$ (p_HansenJ) $\quad p$-value for Hansen's $J \chi^{2}$ statistic
$r\left(J \_d f\right) \quad \chi^{2}$ degrees of freedom

## Methods and formulas

Methods and formulas are presented under the following headings:
Notation
estat endogenous
estat firststage
estat overid

## Notation

Recall from [ R ] ivregress that the model is

$$
\begin{gathered}
\mathbf{y}=\mathbf{Y} \boldsymbol{\beta}_{1}+\mathbf{X}_{1} \boldsymbol{\beta}_{2}+\mathbf{u}=\mathbf{X} \boldsymbol{\beta}+\mathbf{u} \\
\mathbf{Y}=\mathbf{X}_{1} \boldsymbol{\Pi}_{1}+\mathbf{X}_{2} \boldsymbol{\Pi}_{2}+\mathbf{V}=\mathbf{Z} \boldsymbol{\Pi}+\mathbf{V}
\end{gathered}
$$

where $\mathbf{y}$ is an $N \times 1$ vector of the left-hand-side variable, $N$ is the sample size, $\mathbf{Y}$ is an $N \times p$ matrix of $p$ endogenous regressors, $\mathbf{X}_{1}$ is an $N \times k_{1}$ matrix of $k_{1}$ included exogenous regressors, $\mathbf{X}_{2}$ is an $N \times k_{2}$ matrix of $k_{2}$ excluded exogenous variables, $\mathbf{X}=\left[\mathbf{Y} \mathbf{X}_{1}\right], \mathbf{Z}=\left[\mathbf{X}_{1} \mathbf{X}_{2}\right], \mathbf{u}$ is an $N \times 1$ vector of errors, $\mathbf{V}$ is an $N \times p$ matrix of errors, $\boldsymbol{\beta}=\left[\boldsymbol{\beta}_{1} \boldsymbol{\beta}_{2}\right]$ is a $k=\left(p+k_{1}\right) \times 1$ vector of parameters, and $\boldsymbol{\Pi}$ is a $\left(k_{1}+k_{2}\right) \times p$ vector of parameters. If a constant term is included in the model, then one column of $\mathbf{X}_{1}$ contains all ones.

## estat endogenous

Partition $\mathbf{Y}$ as $\mathbf{Y}=\left[\begin{array}{ll}\mathbf{Y}_{1} & \mathbf{Y}_{2}\end{array}\right]$, where $\mathbf{Y}_{1}$ represents the $p_{1}$ endogenous regressors whose endogeneity is being tested and $\mathbf{Y}_{2}$ represents the $p_{2}$ endogenous regressors whose endogeneity is not being tested. If the endogeneity of all endogenous regressors is being tested, $\mathbf{Y}=\mathbf{Y}_{\mathbf{1}}$ and $p_{2}=0$. After GMM estimation, estat endogenous refits the model treating $\mathbf{Y}_{1}$ as exogenous using the same type of weight matrix as requested at estimation time with the wmatrix () option; denote the Sargan statistic from this model by $J_{e}$ and the estimated weight matrix by $\mathbf{W}_{e}$. Let $\mathbf{S}_{e}=\mathbf{W}_{e}^{-1}$. estat endogenous removes from $\mathbf{S}_{e}$ the rows and columns corresponding to the variables represented by $\mathbf{Y}_{1}$; denote the inverse of the resulting matrix by $\mathbf{W}_{e}^{\prime}$. Next estat endogenous fits the model treating both $\mathbf{Y}_{1}$ and $\mathbf{Y}_{2}$ as endogenous, using the weight matrix $\mathbf{W}_{e}^{\prime}$; denote the Sargan statistic from this model by $J_{c}$. Then $C=\left(J_{e}-J_{c}\right) \sim \chi^{2}\left(p_{1}\right)$. If one simply used the $J$ statistic from the original model fit by ivregress in place of $J_{c}$, then in finite samples $J_{e}-J$ might be negative. The procedure used by estat endogenous is guaranteed to yield $C \geq 0$; see Hayashi (2000, 220).

Let $\widehat{\mathbf{u}}_{c}$ denote the residuals from the model treating both $\mathbf{Y}_{1}$ and $\mathbf{Y}_{2}$ as endogenous, and let $\widehat{\mathbf{u}}_{e}$ denote the residuals from the model treating only $\mathbf{Y}_{2}$ as endogenous. Then Durbin's (1954) statistic is

$$
D=\frac{\widehat{\mathbf{u}}_{e}^{\prime} \mathbf{P}_{Z Y_{1}} \widehat{\mathbf{u}}_{e}-\widehat{\mathbf{u}}_{c}^{\prime} \mathbf{P}_{Z} \widehat{\mathbf{u}}_{c}}{\widehat{\mathbf{u}}_{e}^{\prime} \widehat{\mathbf{u}}_{e} / N}
$$

where $\mathbf{P}_{Z}=\mathbf{Z}\left(\mathbf{Z}^{\prime} \mathbf{Z}\right)^{-1} \mathbf{Z}^{\prime}$ and $\mathbf{P}_{Z Y_{1}}=\left[\mathbf{Z} \mathbf{Y}_{1}\right]\left(\left[\begin{array}{ll}\mathbf{Z} & \mathbf{Y}_{1}\end{array}\right]^{\prime}\left[\begin{array}{ll}\mathbf{Z} & \mathbf{Y}_{1}\end{array}\right]\right)^{-1}\left[\begin{array}{ll}\mathbf{Z} & \mathbf{Y}_{1}\end{array}\right]^{\prime} D \sim \chi^{2}\left(p_{1}\right)$. The Wu-Hausman (Wu 1974; Hausman 1978) statistic is

$$
W H=\frac{\left(\widehat{\mathbf{u}}_{e}^{\prime} \mathbf{P}_{Z Y_{1}} \widehat{\mathbf{u}}_{e}-\widehat{\mathbf{u}}_{c}^{\prime} \mathbf{P}_{Z} \widehat{\mathbf{u}}_{c}\right) / p_{1}}{\left\{\widehat{\mathbf{u}}_{e}^{\prime} \widehat{\mathbf{u}}_{e}-\left(\widehat{\mathbf{u}}_{e}^{\prime} \mathbf{P}_{Z Y_{1}} \widehat{\mathbf{u}}_{e}-\widehat{\mathbf{u}}_{c}^{\prime} \mathbf{P}_{Z} \widehat{\mathbf{u}}_{c}\right)\right\} /\left(N-k_{1}-p-p_{1}\right)}
$$

$W H \sim F\left(p_{1}, N-k_{1}-p-p_{1}\right)$. Baum, Schaffer, and Stillman $(2003,2007)$ discuss these tests in more detail.

Next we describe Wooldridge's (1995) score test. The nonrobust version of Wooldridge's test is identical to Durbin's test. Suppose a robust covariance matrix was used at estimation time. Let $\widehat{\mathbf{e}}$ denote the sample residuals obtained by fitting the model via OLS, treating $\mathbf{Y}$ as exogenous. We then regress each variable represented in $\mathbf{Y}$ on $\mathbf{Z}$; call the residuals for the $j$ th regression $\widehat{\mathbf{r}}_{j}, j=1, \ldots, p$. Define $\widehat{k}_{i j}=\widehat{e}_{i} \widehat{r}_{i j}, i=1, \ldots, N$. We then run the regression

$$
\mathbf{1}=\theta_{1} \widehat{\mathbf{k}}_{1}+\cdots+\theta_{p} \widehat{\mathbf{k}}_{p}+\boldsymbol{\epsilon}
$$

where 1 is an $N \times 1$ vector of ones and $\epsilon$ is a regression error term. $N-\operatorname{RSS} \sim \chi^{2}(p)$, where RSS is the residual sum of squares from the regression just described. If instead an HAC VCE was used at estimation time, then before running the final regression we prewhiten the $\widehat{\mathbf{k}}_{j}$ series by using a $\operatorname{VAR}(q)$ model, where $q$ is the number of lags specified with the lags() option.

The regression-based test proceeds as follows. Following Hausman (1978, 1259), we regress $\mathbf{Y}$ on $\mathbf{Z}$ and obtain the residuals $\widehat{\mathbf{V}}$. Next we fit the augmented regression

$$
\mathbf{y}=\mathbf{Y} \boldsymbol{\beta}_{1}+\mathbf{X}_{1} \boldsymbol{\beta}_{2}+\widehat{\mathbf{V}} \boldsymbol{\gamma}+\boldsymbol{\epsilon}
$$

by OLS regression, where $\epsilon$ is a regression error term. A test of the exogeneity of $\mathbf{Y}$ is equivalent to a test of $\gamma=\mathbf{0}$. As Cameron and Trivedi $(2005,276)$ suggest, this test can be made robust to heteroskedasticity, autocorrelation, or clustering by using the appropriate robust VCE when testing $\gamma=0$. When a nonrobust VCE is used, this test is equivalent to the Wu-Hausman test described earlier. One cannot simply fit this augmented regression via 2SLS to test the endogeneity of a subset of the endogenous regressors; Davidson and MacKinnon (1993, 229-231) discuss a test of $\gamma=\mathbf{0}$ for the homoskedastic version of the augmented regression fit by 2SLS, but an appropriate robust test is not apparent.

## estat firststage

When the structural equation includes one endogenous regressor, estat firststage fits the regression

$$
\mathbf{Y}=\mathbf{X}_{1} \boldsymbol{\pi}_{1}+\mathbf{X}_{2} \boldsymbol{\pi}_{2}+\mathbf{v}
$$

via OLS. The $R^{2}$ and adjusted $R^{2}$ from that regression are reported in the output, as well as the $F$ statistic from the Wald test of $H_{0}: \boldsymbol{\pi}_{2}=\mathbf{0}$. To obtain the partial $R^{2}$ statistic, estat firststage fits the regression

$$
\mathbf{M}_{\mathbf{X}_{1}} \mathbf{y}=\mathbf{M}_{\mathbf{X}_{1}} \mathbf{X}_{2} \boldsymbol{\xi}+\boldsymbol{\epsilon}
$$

by OLS, where $\boldsymbol{\epsilon}$ is a regression error term, $\boldsymbol{\xi}$ is a $k_{2} \times 1$ parameter vector, and $\mathbf{M}_{\mathbf{X}_{1}}=\mathbf{I}-$ $\mathbf{X}_{1}\left(\mathbf{X}_{1}^{\prime} \mathbf{X}_{1}\right)^{-1} \mathbf{X}_{1}^{\prime}$; that is, the partial $R^{2}$ is the $R^{2}$ between $\mathbf{y}$ and $\mathbf{X}_{2}$ after eliminating the effects of $\mathbf{X}_{1}$. If the model contains multiple endogenous regressors and the all option is specified, these statistics are calculated for each endogenous regressor in turn.

To calculate Shea's partial $R^{2}$, let $\mathbf{y}_{1}$ denote the endogenous regressor whose statistic is being calculated and $\mathbf{Y}_{0}$ denote the other endogenous regressors. Define $\widetilde{\mathbf{y}}_{1}$ as the residuals obtained from regressing $\mathbf{y}_{1}$ on $\mathbf{Y}_{0}$ and $\mathbf{X}_{1}$. Let $\widehat{\mathbf{y}}_{1}$ denote the fitted values obtained from regressing $\mathbf{y}_{1}$ on $\mathbf{X}_{1}$ and $\mathbf{X}_{2}$; that is, $\widehat{\mathbf{y}}_{1}$ are the fitted values from the first-stage regression for $\mathbf{y}_{1}$, and define the columns of $\widehat{\mathbf{Y}}_{0}$ analogously. Finally, let $\widetilde{\hat{\mathbf{y}}}_{1}$ denote the residuals from regressing $\widehat{\mathbf{y}}_{1}$ on $\widehat{\mathbf{Y}}_{0}$ and $\mathbf{X}_{1}$. Shea's partial $R^{2}$ is the simple $R^{2}$ from the regression of $\widetilde{\mathbf{y}}_{1}$ on $\widetilde{\widehat{\mathbf{y}}}_{1}$; denote this as $R_{S}^{2}$. Shea's adjusted partial $R^{2}$ is equal to $1-\left(1-R_{S}^{2}\right)(N-1) /\left(N-k_{Z}+1\right)$ if a constant term is included and $1-\left(1-R_{S}^{2}\right)(N-1) /\left(N-k_{Z}\right)$ if there is no constant term included in the model, where $k_{Z}=k_{1}+k_{2}$. For one endogenous regressor, one instrument, no exogenous regressors, and a constant term, $R_{S}^{2}$ equals the adjusted $R_{S}^{2}$.

The Stock and Yogo minimum eigenvalue statistic, first proposed by Cragg and Donald (1993) as a test for underidentification, is the minimum eigenvalue of the matrix

$$
\mathbf{G}=\frac{1}{k_{Z}} \widehat{\boldsymbol{\Sigma}}_{\mathbf{V V}}^{-1 / 2} \mathbf{Y}^{\prime} \mathbf{M}_{\mathbf{X}_{1}}^{\prime} \mathbf{X}_{2}\left(\mathbf{X}_{2}^{\prime} \mathbf{M}_{\mathbf{X}_{1}} \mathbf{X}_{2}\right)^{-1} \mathbf{X}_{2}^{\prime} \mathbf{M}_{\mathbf{X}_{1}} \mathbf{Y} \widehat{\boldsymbol{\Sigma}}_{\mathbf{V V}}^{-1 / 2}
$$

where

$$
\widehat{\boldsymbol{\Sigma}}_{\mathbf{V V}}=\frac{1}{N-k_{Z}} \mathbf{Y}^{\prime} \mathbf{M}_{\mathbf{Z}} \mathbf{Y}
$$

$\mathbf{M}_{\mathbf{Z}}=\mathbf{I}-\mathbf{Z}\left(\mathbf{Z}^{\prime} \mathbf{Z}\right)^{-1} \mathbf{Z}^{\prime}$, and $\mathbf{Z}=\left[\mathbf{X}_{1} \mathbf{X}_{2}\right]$. Critical values are obtained from the tables in Stock and Yogo (2005).

## estat overid

The Sargan (1958) and Basmann (1960) $\chi^{2}$ statistics are calculated by running the auxiliary regression

$$
\widehat{\mathbf{u}}=\mathbf{Z} \boldsymbol{\delta}+\mathbf{e}
$$

where $\widehat{\mathbf{u}}$ are the sample residuals from the model and $\mathbf{e}$ is an error term. Then Sargan's statistic is

$$
S=N\left(1-\frac{\hat{\mathbf{e}}^{\prime} \widehat{\mathbf{e}}}{\widehat{\mathbf{u}}^{\prime} \widehat{\mathbf{u}}}\right)
$$

where $\widehat{\mathbf{e}}$ are the residuals from that auxiliary regression. Basmann's statistic is calculated as

$$
B=S \frac{N-k_{Z}}{N-S}
$$

Both $S$ and $B$ are distributed $\chi^{2}(m)$, where $m$, the number of overidentifying restrictions, is equal to $k_{Z}-k$, where $k$ is the number of endogenous regressors.

Wooldridge's (1995) score test of overidentifying restrictions is identical to Sargan's (1958) statistic under the assumption of i.i.d. and therefore is not recomputed unless a robust VCE was used at estimation time. If a heteroskedasticity-robust VCE was used, Wooldridge's test proceeds as follows. Let $\widehat{\mathbf{Y}}$ denote the $N \times k$ matrix of fitted values obtained by regressing the endogenous regressors on $\mathbf{X}_{1}$ and $\mathbf{X}_{2}$. Let $\mathbf{Q}$ denote an $N \times m$ matrix of excluded exogenous variables; the test statistic to be calculated is invariant to whichever $m$ of the $k_{2}$ excluded exogenous variables is chosen. Define the $i$ th element of $\widehat{\mathbf{k}}_{j}, i=1, \ldots, N, j=1, \ldots, m$, as

$$
k_{i j}=\widehat{q}_{i j} u_{i}
$$

where $\widehat{q}_{i j}$ is the $i$ th element of $\widehat{\mathbf{q}}_{j}$, the fitted values from regressing the $j$ th column of $\mathbf{Q}$ on $\widehat{\mathbf{Y}}$ and $\mathbf{X}_{1}$. Finally, fit the regression

$$
\mathbf{1}=\theta_{1} \widehat{\mathbf{k}}_{1}+\cdots+\theta_{m} \widehat{\mathbf{k}}_{m}+\boldsymbol{\epsilon}
$$

where 1 is an $N \times 1$ vector of ones and $\epsilon$ is a regression error term, and calculate the residual sum of squares, RSS. Then the test statistic is $W=N-$ RSS. $W \sim \chi^{2}(m)$. If an HAC VCE was used at estimation, then the $\widehat{\mathbf{k}}_{j}$ are prewhitened using a $\operatorname{VAR}(p)$ model, where $p$ is specified using the lags() option.

The Anderson-Rubin (1950), AR, test of overidentifying restrictions for use after the LIML estimator is calculated as $\mathrm{AR}=N(\kappa-1)$, where $\kappa$ is the minimal eigenvalue of a certain matrix defined in Methods and formulas of [R] ivregress. AR $\sim \chi^{2}(m)$. (Some texts define this statistic as $N \ln (\kappa)$ because $\ln (x) \approx(x-1)$ for $x$ near one.) Basmann's $F$ statistic for use after the LIML estimator is calculated as $B_{F}=(\kappa-1)\left(N-k_{Z}\right) / m . B_{F} \sim F\left(m, N-k_{Z}\right)$.

Hansen's $J$ statistic is simply the sample size times the value of the GMM objective function defined in (5) of [R] ivregress, evaluated at the estimated parameter values. Under the null hypothesis that the overidentifying restrictions are valid, $J \sim \chi^{2}(m)$.

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## Also see

$[R]$ ivregress - Single-equation instrumental-variables regression
[U] 20 Estimation and postestimation commands

## Title

ivtobit - Tobit model with continuous endogenous regressors

| Syntax | Menu | Description |
| :--- | :--- | :--- |
| Options for ML estimator | Options for two-step estimator | Remarks and examples |
| Stored results | Methods and formulas | Acknowledgments |
| References | Also see |  |

## Syntax

Maximum likelihood estimator

```
ivtobit depvar \(\left[\right.\) varlist \(\left._{1}\right]\left(\right.\) varlist \(_{2}=\) varlist \(\left._{\mathrm{iv}}\right) \quad[\) if \(][\) in \(][\) weight \(]\),
    \(11[(\#)] \mathrm{ul}[(\#)][\) mle_options \(]\)
```

Two-step estimator
ivtobit depvar $\left[\right.$ varlist $\left._{1}\right]\left(\right.$ varlist $_{2}=$ varlist $\left._{\text {iv }}\right)[$ if $][$ in $]$ [weight $]$, twostep
$11[(\#)]$ ul [(\#)] [tse_options]
mle_options
Description
Model

| $* 11[(\#)]$ | lower limit for left censoring |
| :--- | :--- |
| $*$ ul $[(\#)]$ | upper limit for right censoring |
| $\underline{\text { mle }} \mathrm{constraints}$ (constraints) | use conditional maximum-likelihood estimator; the default |
| $\underline{\text { consply specified linear constraints }}$ |  |

```
SE/Robust
vce (vcetype)
```

vcetype may be oim, robust, cluster clustvar, opg, bootstrap, or jackknife

Reporting
level(\#)
set confidence level; default is level (95)
first report first-stage regression
nocnsreport
display_options
do not display constraints
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling

Maximization
maximize_options
control the maximization process
coeflegend
display legend instead of statistics

[^34]
## tse_options

Description

## Model

* twostep
* $11[(\#)]$
*ul $[(\#)]$
SE
vce (vcetype)
use Newey's two-step estimator; the default is mle
lower limit for left censoring
upper limit for right censoring
vcetype may be twostep, bootstrap, or jackknife
Reporting
level (\#)
first
display_options
coeflegend
set confidence level; default is level (95)
report first-stage regression
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
display legend instead of statistics
*twostep is required. You must specify at least one of $11[(\#)]$ and $u l[(\#)]$.
varlist $_{1}$ and varlist $_{\mathrm{iv}}$ may contain factor variables; see [U] 11.4.3 Factor variables.
depvar, varlist ${ }_{1}$, varlist $t_{2}$, and varlist ${ }_{\mathrm{iv}}$ may contain time-series operators; see [U] 11.4.4 Time-series varlists.
bootstrap, by, jackknife, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands. fp is allowed with the maximum likelihood estimator.
Weights are not allowed with the bootstrap prefix; see [R] bootstrap.
vce(), first, twostep, and weights are not allowed with the svy prefix; see [SVY] svy.
fweights, iweights, and pweights are allowed with the maximum likelihood estimator. fweights are allowed with Newey's two-step estimator. See [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.


## Menu

Statistics $>$ Endogenous covariates $>$ Tobit model with endogenous covariates

## Description

ivtobit fits tobit models where one or more of the regressors is endogenously determined. By default, ivtobit uses maximum likelihood estimation. Alternatively, Newey's (1987) minimum chi-squared estimator can be invoked with the twostep option. Both estimators assume that the endogenous regressors are continuous and so are not appropriate for use with discrete endogenous regressors. See $[R]$ ivprobit for probit estimation with endogenous regressors and $[R]$ tobit for tobit estimation when the model contains no endogenous regressors.

## Options for ML estimator

$\qquad$ Model
11 (\#) and ul(\#) indicate the lower and upper limits for censoring, respectively. You may specify one or both. Observations with depvar $\leq 11$ () are left-censored; observations with depvar $\geq$ ul () are right-censored; and remaining observations are not censored. You do not have to specify the censoring values at all. It is enough to type 11 , ul, or both. When you do not specify a censoring value, ivtobit assumes that the lower limit is the minimum observed in the data (if $l l$ is specified) and that the upper limit is the maximum (if $u l$ is specified).
mle requests that the conditional maximum-likelihood estimator be used. This is the default.
constraints (constraints); see [R] estimation options.

## SE/Robust

vce (vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

Reporting
level (\#); see [R] estimation options.
first requests that the parameters for the reduced-form equations showing the relationships between the endogenous variables and instruments be displayed. For the two-step estimator, first shows the first-stage regressions. For the maximum likelihood estimator, these parameters are estimated jointly with the parameters of the tobit equation. The default is not to show these parameter estimates.
nocnsreport; see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% fmt), pformat (\% fmt), sformat (\% fmt), and nolstretch; see [R] estimation options.

Maximization
maximize_options: difficult, technique (algorithm_spec), iterate (\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance(\#), nonrtolerance, and from(init_specs); see [R] maximize. This model's likelihood function can be difficult to maximize, especially with multiple endogenous variables. The difficult and technique (bfgs) options may be helpful in achieving convergence.

Setting the optimization type to technique (bhhh) resets the default vcetype to vce (opg).
The following option is available with ivtobit but is not shown in the dialog box: coeflegend; see [R] estimation options.

## Options for two-step estimator

Model
twostep is required and requests that Newey's (1987) efficient two-step estimator be used to obtain the coefficient estimates.
ll(\#) and $\mathrm{ul}(\#)$ indicate the lower and upper limits for censoring, respectively. You may specify one or both. Observations with depvar $\leq 11$ () are left-censored; observations with depvar $\geq$ ul() are right-censored; and remaining observations are not censored. You do not have to specify the censoring values at all. It is enough to type ll, ul, or both. When you do not specify a censoring value, ivtobit assumes that the lower limit is the minimum observed in the data (if 11 is specified) and that the upper limit is the maximum (if $u l$ is specified).

SE
vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (twostep) and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

Reporting
level (\#); see [R] estimation options.
first requests that the parameters for the reduced-form equations showing the relationships between the endogenous variables and instruments be displayed. For the two-step estimator, first shows the first-stage regressions. For the maximum likelihood estimator, these parameters are estimated jointly with the parameters of the tobit equation. The default is not to show these parameter estimates.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\%fmt), pformat(\%fmt), sformat(\%fmt), and nolstretch; see [R] estimation options.

The following option is available with ivtobit but is not shown in the dialog box: coeflegend; see $[R]$ estimation options.

## Remarks and examples

ivtobit fits models with censored dependent variables and endogenous regressors. You can use it to fit a tobit model when you suspect that one or more of the regressors is correlated with the error term. ivtobit is to tobit what ivregress is to linear regression analysis; see [R] ivregress for more information.

Formally, the model is

$$
\begin{aligned}
& y_{1 i}^{*}=\boldsymbol{y}_{2 i} \boldsymbol{\beta}+\boldsymbol{x}_{1 i} \boldsymbol{\gamma}+u_{i} \\
& \boldsymbol{y}_{2 i}=\boldsymbol{x}_{1 i} \boldsymbol{\Pi}_{1}+\boldsymbol{x}_{2 i} \boldsymbol{\Pi}_{2}+\boldsymbol{v}_{i}
\end{aligned}
$$

where $i=1, \ldots, N ; \boldsymbol{y}_{2 i}$ is a $1 \times p$ vector of endogenous variables; $\boldsymbol{x}_{1 i}$ is a $1 \times k_{1}$ vector of exogenous variables; $\boldsymbol{x}_{2 i}$ is a $1 \times k_{2}$ vector of additional instruments; and the equation for $\boldsymbol{y}_{2 i}$ is written in reduced form. By assumption $\left(u_{i}, \boldsymbol{v}_{i}\right) \sim \mathrm{N}(\mathbf{0}) . \boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ are vectors of structural parameters, and $\boldsymbol{\Pi}_{1}$ and $\Pi_{2}$ are matrices of reduced-form parameters. We do not observe $y_{1 i}^{*}$; instead, we observe

$$
y_{1 i}= \begin{cases}a & y_{1 i}^{*}<a \\ y_{1 i}^{*} & a \leq y_{1 i}^{*} \leq b \\ b & y_{1 i}^{*}>b\end{cases}
$$

The order condition for identification of the structural parameters is that $k_{2} \geq p$. Presumably, $\boldsymbol{\Sigma}$ is not block diagonal between $u_{i}$ and $\boldsymbol{v}_{i}$; otherwise, $\boldsymbol{y}_{2 i}$ would not be endogenous.

## - Technical note

This model is derived under the assumption that $\left(u_{i}, \boldsymbol{v}_{i}\right)$ is independent and identically distributed multivariate normal for all $i$. The vce (cluster clustvar) option can be used to control for a lack of independence. As with the standard tobit model without endogeneity, if $u_{i}$ is heteroskedastic, point estimates will be inconsistent.

## > Example 1

Using the same dataset as in [R] ivprobit, we now want to estimate women's incomes. In our hypothetical world, all women who choose not to work receive $\$ 10,000$ in welfare and child-support payments. Therefore, we never observe incomes under $\$ 10,000$ : a woman offered a job with an annual wage less than that would not accept and instead would collect the welfare payment. We model income as a function of the number of years of schooling completed, the number of children at home, and other household income. We again believe that other_inc is endogenous, so we use male_educ as an instrument.

```
. use http://www.stata-press.com/data/r13/laborsup
. ivtobit fem_inc fem_educ kids (other_inc = male_educ), ll
Fitting exogenous tobit model
Fitting full model
```

Iteration 0: log likelihood = -3228.4224
Iteration 1: $\quad$ log likelihood $=-3226.2882$
Iteration 2: log likelihood $=-3226.085$
Iteration 3: log likelihood $=-3226.0845$
Iteration 4: log likelihood $=-3226.0845$

| Tobit model with endogenous regressors | Number of obs | $=$ | 500 |
| :--- | :--- | :--- | :--- |
|  |  | Wald chi2(3) | $=$ |
| Log likelihood $=-3226.0845$ |  | Prob > chi2 | $=$ |


|  | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| other_inc | -.9045399 | .1329762 | -6.80 | 0.000 | -1.165168 | -.6439114 |
| fem_educ | 3.272391 | .3968708 | 8.25 | 0.000 | 2.494538 | 4.050243 |
| kids | -3.312357 | .7218628 | -4.59 | 0.000 | -4.727182 | -1.897532 |
| _cons | 19.24735 | 7.372391 | 2.61 | 0.009 | 4.797725 | 33.69697 |
| /alpha | .2907654 | .1379965 | 2.11 | 0.035 | .0202972 | .5612336 |
| /lns | 2.874031 | .0506672 | 56.72 | 0.000 | 2.774725 | 2.973337 |
| /lnv | 2.813383 | .0316228 | 88.97 | 0.000 | 2.751404 | 2.875363 |
| s | 17.70826 | .897228 |  |  | 16.03422 | 19.55707 |
| v | 16.66621 | .5270318 |  |  | 15.66461 | 17.73186 |

Instrumented: other_inc
Instruments: fem_educ kids male_educ
Wald test of exogeneity (/alpha $=0$ ) : chi2 $(1)=4.44$ Prob $>$ chi2 $=0.0351$
Obs. summary: 272 left-censored observations at fem_inc<=10
228 uncensored observations
0 right-censored observations

Because we did not specify mle or twostep, ivtobit used the maximum likelihood estimator by default. ivtobit fits a tobit model, ignoring endogeneity, to get starting values for the full model. The header of the output contains the maximized log likelihood, the number of observations, and a

Wald statistic and $p$-value for the test of the hypothesis that all the slope coefficients are jointly zero. At the end of the output, we see a count of the censored and uncensored observations.

Near the bottom of the output is a Wald test of the exogeneity of the instrumented variables. If the test statistic is not significant, there is not sufficient information in the sample to reject the null hypothesis of no endogeneity. Then the point estimates from ivtobit are consistent, although those from tobit are likely to have smaller standard errors.

Various two-step estimators have also been proposed for the endogenous tobit model, and Newey's (1987) minimum chi-squared estimator is available with the twostep option.

## > Example 2

Refitting our labor-supply model with the two-step estimator yields


All the coefficients have the same signs as their counterparts in the maximum likelihood model. The Wald test at the bottom of the output confirms our earlier finding of endogeneity.

## - Technical note

In the tobit model with endogenous regressors, we assume that $\left(u_{i}, \boldsymbol{v}_{i}\right)$ is multivariate normal with covariance matrix

$$
\operatorname{Var}\left(u_{i}, \boldsymbol{v}_{i}\right)=\boldsymbol{\Sigma}=\left[\begin{array}{cc}
\sigma_{u}^{2} & \boldsymbol{\Sigma}_{21}^{\prime} \\
\boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22}
\end{array}\right]
$$

Using the properties of the multivariate normal distribution, $\operatorname{Var}\left(u_{i} \mid \boldsymbol{v}_{i}\right) \equiv \sigma_{u \mid v}^{2}=\sigma_{u}^{2}-\boldsymbol{\Sigma}_{21}^{\prime} \boldsymbol{\Sigma}_{22}^{-1} \boldsymbol{\Sigma}_{21}$. Calculating the marginal effects on the conditional expected values of the observed and latent dependent variables and on the probability of censoring requires an estimate of $\sigma_{u}^{2}$. The two-step estimator identifies only $\sigma_{u \mid v}^{2}$, not $\sigma_{u}^{2}$, so only the linear prediction and its standard error are available after you have used the twostep option. However, unlike the two-step probit estimator described in [R] ivprobit, the two-step tobit estimator does identify $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$. See Wooldridge (2010, 683-684) for more information.

## Stored results

ivtobit, mle stores the following in e() :
Scalars

```
e(N)
    e(N_unc)
    e(N_lc)
    e(N_rc)
    e(llopt)
    e(ulopt)
    e(k)
    e(k_eq)
    e(k_eq_model)
    e(k_aux)
    e(k_dv)
    e(df_m)
    e(ll)
    e(N_clust)
    e(endog_ct)
    e(p)
    e(p_exog)
    e(chi2)
    e(chi2_exog)
    e(rank)
    e(ic)
    e(rc)
    e(converged)
```

Macros
number of observations
number of uncensored observations
number of left-censored observations
number of right-censored observations
contents of 11 ()
contents of ul()
number of parameters
number of equations in e(b)
number of equations in overall model test
number of auxiliary parameters
number of dependent variables
model degrees of freedom
log likelihood
number of clusters
number of endogenous regressors
model Wald $p$-value
exogeneity test Wald $p$-value
model Wald $\chi^{2}$
Wald $\chi^{2}$ test of exogeneity
rank of e(V)
number of iterations
return code
1 if converged, 0 otherwise
Macros

```
e(cmd)
e(cmdline)
e(depvar)
e(instd)
e(insts)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(chi2type)
e(vce)
e(vcetype)
e(method)
e(opt)
e(which)
e(ml_method)
e(user)
e(technique)
e(properties)
e(predict)
e(footnote)
e(marginsok)
e(asbalanced)
e(asobserved)
e(wexp)
```

ivtobit
command as typed
name of dependent variable
instrumented variables
instruments
weight type
weight expression
title in estimation output
name of cluster variable
Wald; type of model $\chi^{2}$ test
vcetype specified in vce()
title used to label Std. Err.
ml
type of optimization
max or min; whether optimizer is to perform maximization or minimization
type of ml method
name of likelihood-evaluator program
maximization technique
b V
program used to implement predict
program used to implement the footnote display
predictions allowed by margins
factor variables fvset as asbalanced
factor variables fvset as asobserved

Matrices

```
e(b)
    e(Cns)
    e(ilog)
    e(gradient)
    e(Sigma)
    e(V)
    e(V_modelbased)
```

Functions
e(sample)
coefficient vector
constraints matrix
iteration $\log$ (up to 20 iterations)
gradient vector
$\widehat{\boldsymbol{\Sigma}}$
variance-covariance matrix of the estimators model-based variance
marks estimation sample
ivtobit, twostep stores the following in e():
Scalars
e(N)
e(N_unc)
e(N_lc)
e(N_rc)
e(llopt)
e(ulopt)
e(df_m)
e(df_exog)
e(p)
e(p_exog)
e(chi2)
e(chi2_exog)
e(rank)
Macros
e(cmd)
e(cmdline)
e(depvar)
e(instd)
e(insts)
e(wtype)
e(wexp)
e(chi2type)
e(vce)
e(vcetype)
e(method)
e(properties)
e(predict)
e(footnote)
e(marginsok)
e(asbalanced)
e(asobserved)
Matrices
e(b)
e(Cns)
e(V)
e(V_modelbased)
Functions
e(sample) marks estimation sample

## Methods and formulas

The estimation procedure used by ivtobit is similar to that used by ivprobit. For compactness, we write the model as

$$
\begin{align*}
& y_{1 i}^{*}=\boldsymbol{z}_{i} \boldsymbol{\delta}+u_{i}  \tag{1a}\\
& \boldsymbol{y}_{2 i}=\boldsymbol{x}_{i} \boldsymbol{\Pi}+\boldsymbol{v}_{i} \tag{1b}
\end{align*}
$$

where $\boldsymbol{z}_{i}=\left(\boldsymbol{y}_{2 i}, \boldsymbol{x}_{1 i}\right), \boldsymbol{x}_{i}=\left(\boldsymbol{x}_{1 i}, \boldsymbol{x}_{2 i}\right), \boldsymbol{\delta}=\left(\boldsymbol{\beta}^{\prime}, \boldsymbol{\gamma}^{\prime}\right)^{\prime}$, and $\boldsymbol{\Pi}=\left(\boldsymbol{\Pi}_{1}^{\prime}, \boldsymbol{\Pi}_{2}^{\prime}\right)^{\prime}$. We do not observe $y_{1 i}^{*}$; instead, we observe

$$
y_{1 i}= \begin{cases}a & y_{1 i}^{*}<a \\ y_{1 i}^{*} & a \leq y_{1 i}^{*} \leq b \\ b & y_{1 i}^{*}>b\end{cases}
$$

$\left(u_{i}, \boldsymbol{v}_{i}\right)$ is distributed multivariate normal with mean zero and covariance matrix

$$
\boldsymbol{\Sigma}=\left[\begin{array}{cc}
\sigma_{u}^{2} & \boldsymbol{\Sigma}_{21}^{\prime} \\
\boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22}
\end{array}\right]
$$

Using the properties of the multivariate normal distribution, we can write $u_{i}=\boldsymbol{v}_{i}^{\prime} \boldsymbol{\alpha}+\epsilon_{i}$, where $\boldsymbol{\alpha}=\boldsymbol{\Sigma}_{22}^{-1} \boldsymbol{\Sigma}_{21} ; \epsilon_{i} \sim \mathrm{~N}\left(0 ; \sigma_{u \mid v}^{2}\right)$, where $\sigma_{u \mid v}^{2}=\sigma_{u}^{2}-\boldsymbol{\Sigma}_{21}^{\prime} \boldsymbol{\Sigma}_{22}^{-1} \boldsymbol{\Sigma}_{21} ;$ and $\epsilon_{i}$ is independent of $\boldsymbol{v}_{i}, \boldsymbol{z}_{i}$, and $\boldsymbol{x}_{i}$.

The likelihood function is straightforward to derive because we can write the joint density $f\left(y_{1 i}, \boldsymbol{y}_{2 i} \mid \boldsymbol{x}_{i}\right)$ as $f\left(y_{1 i} \mid \boldsymbol{y}_{2 i}, \boldsymbol{x}_{i}\right) f\left(\boldsymbol{y}_{2 i} \mid \boldsymbol{x}_{i}\right)$. With one endogenous regressor,

$$
\ln f\left(y_{2 i} \mid \boldsymbol{x}_{i}\right)=-\frac{1}{2}\left\{\ln 2 \pi+\ln \sigma_{v}^{2}+\frac{\left(y_{2 i}-\boldsymbol{x}_{i} \boldsymbol{\Pi}\right)^{2}}{\sigma_{v}^{2}}\right\}
$$

and

$$
\ln f\left(y_{1 i} \mid y_{2 i}, \boldsymbol{x}_{i}\right)= \begin{cases}\ln \left\{1-\Phi\left(\frac{m_{i}-a}{\sigma_{u \mid v}}\right)\right\} & y_{1 i}=a \\ -\frac{1}{2}\left\{\ln 2 \pi+\ln \sigma_{u \mid v}^{2}+\frac{\left(y_{1 i}-m_{i}\right)^{2}}{\sigma_{u \mid v}^{2}}\right\} & a<y_{1 i}<b \\ \ln \Phi\left(\frac{m_{i}-b}{\sigma_{u \mid v}}\right) & y_{1 i}=b\end{cases}
$$

where

$$
m_{i}=\boldsymbol{z}_{i} \boldsymbol{\delta}+\alpha\left(y_{2 i}-\boldsymbol{x}_{i} \boldsymbol{\Pi}\right)
$$

and $\Phi(\cdot)$ is the normal distribution function so that the log likelihood for observation $i$ is

$$
\ln L_{i}=w_{i}\left\{\ln f\left(y_{1 i} \mid y_{2 i}, \boldsymbol{x}_{i}\right)+\ln f\left(y_{2 i} \mid \boldsymbol{x}_{i}\right)\right\}
$$

where $w_{i}$ is the weight for observation $i$ or one if no weights were specified. Instead of estimating $\sigma_{u \mid v}$ and $\sigma_{v}$ directly, we estimate $\ln \sigma_{u \mid v}$ and $\ln \sigma_{v}$.

For multiple endogenous regressors, we have

$$
\ln f\left(\boldsymbol{y}_{2 i} \mid \boldsymbol{x}_{i}\right)=-\frac{1}{2}\left(\ln 2 \pi+\ln \left|\boldsymbol{\Sigma}_{22}\right|+\boldsymbol{v}_{i}^{\prime} \boldsymbol{\Sigma}_{22}^{-1} \boldsymbol{v}_{i}\right)
$$

and $\ln f\left(y_{1 i} \mid \boldsymbol{y}_{2 i}, \boldsymbol{x}_{i}\right)$ is the same as before, except that now

$$
m_{i}=\boldsymbol{z}_{i} \boldsymbol{\delta}+\left(\boldsymbol{y}_{2 i}-\boldsymbol{x}_{i} \boldsymbol{\Pi}\right) \boldsymbol{\Sigma}_{22}^{-1} \boldsymbol{\Sigma}_{21}
$$

Instead of maximizing the $\log$-likelihood function with respect to $\boldsymbol{\Sigma}$, we maximize with respect to the Cholesky decomposition $\boldsymbol{S}$ of $\boldsymbol{\Sigma}$; that is, there exists a lower triangular matrix $\boldsymbol{S}$ such that $\boldsymbol{S} \boldsymbol{S}^{\prime}=\boldsymbol{\Sigma}$. This maximization ensures that $\boldsymbol{\Sigma}$ is positive definite, as a covariance matrix must be. Let

$$
\boldsymbol{S}=\left[\begin{array}{ccccc}
s_{11} & 0 & 0 & \cdots & 0 \\
s_{21} & s_{22} & 0 & \cdots & 0 \\
s_{31} & s_{32} & s_{33} & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
s_{p+1,1} & s_{p+1,2} & s_{p+1,3} & \cdots & s_{p+1, p+1}
\end{array}\right]
$$

With maximum likelihood estimation, this command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce (robust) and vce (cluster clustvar), respectively. See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas.

The maximum likelihood version of ivtobit also supports estimation with survey data. For details on VCEs with survey data, see [SVY] variance estimation.

The two-step estimates are obtained using Newey's (1987) minimum chi-squared estimator. The procedure is identical to the one described in [R] ivprobit, except that tobit is used instead of probit.

## Acknowledgments

The two-step estimator is based on the tobitiv command written by Jonah Gelbach of the Department of Economics at Yale University and the ivtobit command written by Joe Harkness of the Institute of Policy Studies at Johns Hopkins University.

## References

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Miranda, A., and S. Rabe-Hesketh. 2006. Maximum likelihood estimation of endogenous switching and sample selection models for binary, ordinal, and count variables. Stata Journal 6: 285-308.

Newey, W. K. 1987. Efficient estimation of limited dependent variable models with endogenous explanatory variables. Journal of Econometrics 36: 231-250.

Wooldridge, J. M. 2010. Econometric Analysis of Cross Section and Panel Data. 2nd ed. Cambridge, MA: MIT Press.

[^35][U] 20 Estimation and postestimation commands

## Title

ivtobit postestimation - Postestimation tools for ivtobit

| Description | Syntax for predict | Menu for predict | Options for predict |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Methods and formulas | Also see |  |

## Description

The following postestimation commands are available after ivtobit:

| Command | Description |
| :--- | :--- |
| contrast | contrasts and ANOVA-style joint tests of estimates |
| estat ic ${ }^{1}$ | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample |
| estat vce | variance-covariance matrix of the estimators (VCE) |
| estat (svy) | postestimation statistics for survey data |
| estimates | cataloging estimation results |
| forecast | dynamic forecasts and simulations |
| hausman | Hausman's specification test <br> point estimates, standard errors, testing, and inference for linear combinations <br> lincom |
| of coefficients |  |

[^36]
## Syntax for predict

After ML or twostep

$$
\text { predict }[\text { type }] \text { newvar }[\text { if }][\text { in }][\text {, statistic }]
$$

After ML

$$
\text { predict }[\text { type }]\{\text { stub* } \mid \text { newvarlist }\}[\text { if }][\text { in }] \text {, scores }
$$

statistic Description

Main
xb
stdp
stdf
$\operatorname{pr}(a, b)$
$\mathrm{e}(a, b)$
ystar $(a, b)$
linear prediction; the default
standard error of the linear prediction
standard error of the forecast; not available with two-step estimator
$\operatorname{Pr}\left(a<y_{j}<b\right)$; not available with two-step estimator
$E\left(y_{j} \mid a<y_{j}<b\right)$; not available with two-step estimator
$E\left(y_{j}^{*}\right), y_{j}=\max \left\{a, \min \left(y_{j}, b\right)\right\}$; not available with two-step estimator

These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.
stdf is not allowed with svy estimation results.
where $a$ and $b$ may be numbers or variables; $a$ missing ( $a \geq$.) means $-\infty$, and $b$ missing ( $b \geq$.) means $+\infty$; see [U] 12.2.1 Missing values.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
xb , the default, calculates the linear prediction.
stdp calculates the standard error of the linear prediction. It can be thought of as the standard error of the predicted expected value or mean for the observation's covariate pattern. The standard error of the prediction is also referred to as the standard error of the fitted value.
stdf calculates the standard error of the forecast, which is the standard error of the point prediction for 1 observation. It is commonly referred to as the standard error of the future or forecast value. By construction, the standard errors produced by stdf are always larger than those produced by stdp; see Methods and formulas in [R] regress postestimation. stdf is not available with the two-step estimator.
$\operatorname{pr}(a, b)$ calculates $\operatorname{Pr}\left(a<\mathbf{x}_{j} \mathbf{b}+u_{j}<b\right)$, the probability that $y_{j} \mid \mathbf{x}_{j}$ would be observed in the interval $(a, b)$.
$a$ and $b$ may be specified as numbers or variable names; $l b$ and $u b$ are variable names;
$\operatorname{pr}(20,30)$ calculates $\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$;
$\operatorname{pr}(l b, u b)$ calculates $\operatorname{Pr}\left(l b<\mathbf{x}_{j} \mathbf{b}+u_{j}<u b\right)$; and
$\operatorname{pr}(20, u b)$ calculates $\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{j}<u b\right)$.
$a$ missing ( $a \geq$. ) means $-\infty ; \operatorname{pr}(., 30)$ calculates $\operatorname{Pr}\left(-\infty<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$;
$\operatorname{pr}(l b, 30)$ calculates $\operatorname{Pr}\left(-\infty<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$ in observations for which $l b \geq$.
and calculates $\operatorname{Pr}\left(l b<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$ elsewhere.
$b$ missing $(b \geq$.$) means +\infty$; $\operatorname{pr}(20,$.$) calculates \operatorname{Pr}\left(+\infty>\mathbf{x}_{j} \mathbf{b}+u_{j}>20\right)$; $\operatorname{pr}(20, u b)$ calculates $\operatorname{Pr}\left(+\infty>\mathbf{x}_{j} \mathbf{b}+u_{j}>20\right)$ in observations for which $u b \geq$.
and calculates $\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{j}<u b\right)$ elsewhere.
$\operatorname{pr}(a, b)$ is not available with the two-step estimator.
$\mathrm{e}(a, b)$ calculates $E\left(\boldsymbol{x}_{j} \boldsymbol{b}+u_{j} \mid a<\boldsymbol{x}_{j} \boldsymbol{b}+u_{j}<b\right)$, the expected value of $y_{j} \mid \boldsymbol{x}_{j}$ conditional on $y_{j} \mid \boldsymbol{x}_{j}$ being in the interval $(a, b)$, meaning that $y_{j} \mid \boldsymbol{x}_{j}$ is truncated. $a$ and $b$ are specified as they are for $\operatorname{pr}() . \mathrm{e}(a, b)$ is not available with the two-step estimator.
ystar $(a, b)$ calculates $E\left(y_{j}^{*}\right)$, where $y_{j}^{*}=a$ if $\boldsymbol{x}_{j} \boldsymbol{b}+u_{j} \leq a, y_{j}^{*}=b$ if $\boldsymbol{x}_{j} \boldsymbol{b}+u_{j} \geq b$, and $y_{j}^{*}=\boldsymbol{x}_{j} \boldsymbol{b}+u_{j}$ otherwise, meaning that $y_{j}^{*}$ is censored. $a$ and $b$ are specified as they are for pr() . ystar ( $a, b$ ) is not available with the two-step estimator.
scores, not available with twostep, calculates equation-level score variables.
For models with one endogenous regressor, five new variables are created.
The first new variable will contain $\partial \ln L / \partial\left(\boldsymbol{z}_{i} \boldsymbol{\delta}\right)$.
The second new variable will contain $\partial \ln L / \partial\left(\boldsymbol{x}_{i} \boldsymbol{\Pi}\right)$.
The third new variable will contain $\partial \ln L / \partial \alpha$.
The fourth new variable will contain $\partial \ln L / \partial \ln \sigma_{u \mid v}$.
The fifth new variable will contain $\partial \ln L / \partial \ln \sigma_{v}$.
For models with $p$ endogenous regressors, $p+\{(p+1)(p+2)\} / 2+1$ new variables are created.
The first new variable will contain $\partial \ln L / \partial\left(\boldsymbol{z}_{i} \boldsymbol{\delta}\right)$.
The second through $(p+1)$ th new score variables will contain $\partial \ln L / \partial\left(\boldsymbol{x}_{i} \boldsymbol{\Pi}_{k}\right), k=1, \ldots, p$, where $\Pi_{k}$ is the $k$ th column of $\boldsymbol{\Pi}$.
The remaining score variables will contain the partial derivatives of $\ln L$ with respect to $s_{11}$, $s_{21}, \ldots, s_{p+1,1}, s_{22}, \ldots, s_{p+1,2}, \ldots, s_{p+1, p+1}$, where $s_{m, n}$ denotes the $(m, n)$ element of the Cholesky decomposition of the error covariance matrix.

## Remarks and examples

Remarks are presented under the following headings:

## Marginal effects

Obtaining predicted values

## Marginal effects

> Example 1
We can obtain average marginal effects by using the margins command after ivtobit. For the labor-supply model of example 1 in [R] ivtobit, suppose that we wanted to know the average marginal effects on the woman's expected income, conditional on her income being greater than $\$ 10,000$.

```
. use http://www.stata-press.com/data/r13/laborsup
. ivtobit fem_inc fem_educ kids (other_inc = male_educ), ll
(output omitted)
. margins, dydx(*) predict(e(10, .))
Average marginal effects Number of obs = 500
Model VCE : OIM
Expression : E(fem_inc|fem_inc>10), predict(e(10, .))
dy/dx w.r.t. : other_inc fem_educ kids male_educ
```

|  | Delta-method |  |  |  |  |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
|  | dy/dx | Std. Err. | $\mathbf{z}$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| other_inc | -.3420189 | .0553591 | -6.18 | 0.000 | -.4505208 | -.233517 |
| fem_educ | 1.237336 | .1534025 | 8.07 | 0.000 | .9366723 | 1.537999 |
| kids | -1.252447 | .2725166 <br> (omitted) | -4.60 | 0.000 | -1.78657 | -.7183246 |
| male_educ | 0 |  |  |  |  |  |

In our sample, increasing the number of children in the family by one decreases the expected wage by $\$ 1,252$ on average (wages in our dataset are measured in thousands of dollars). male_edu has no effect because it appears only as an instrument.

## Obtaining predicted values

After fitting your model using ivtobit, you can obtain the linear prediction and its standard error for both the estimation sample and other samples using the predict command. If you used the maximum likelihood estimator, you can also obtain conditional expected values of the observed and latent dependent variables, the standard error of the forecast, and the probability of observing the dependent variable in a specified interval. See [U] 20 Estimation and postestimation commands and $[R]$ predict.

## Methods and formulas

The linear prediction is calculated as $\boldsymbol{z}_{i} \widehat{\boldsymbol{\delta}}$, where $\widehat{\boldsymbol{\delta}}$ is the estimated value of $\boldsymbol{\delta}$, and $\boldsymbol{z}_{i}$ and $\boldsymbol{\delta}$ are defined in $(1 a)$ of $[\mathrm{R}]$ ivtobit. Expected values and probabilities are calculated using the same formulas as those used by the standard exogenous tobit model.

## Also see

[R] ivtobit - Tobit model with continuous endogenous regressors
[U] 20 Estimation and postestimation commands

## Title

jackknife - Jackknife estimation

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

jackknife exp_list $[$, options eform_option $]$ : command
options
Description
Main
eclass
rclass
n (exp)
Options
cluster (varlist)
idcluster(newvar)
saving (filename, ...)
keep
mse
Reporting
level(\#)
notable
noheader
nolegend
verbose
nodots
noisily
trace
title (text)
display_options
eform_option
Advanced
nodrop do not drop observations
reject (exp)
coeflegend
variables identifying sample clusters
create new cluster ID variable
keep pseudovalues
use MSE formula for variance estimation
suppress table of results
suppress table header
suppress table legend
display the full table legend
suppress replication dots
display any output from command
trace command
use text as title for jackknife results
identify invalid results
display legend instead of statistics
number of observations used is stored in $\mathrm{e}(\mathrm{N})$
number of observations used is stored in $r(N)$
specify $\exp$ that evaluates to the number of observations used
save results to filename; save statistics in double precision; save results to filename every \# replications
set confidence level; default is level(95)
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling display coefficient table in exponentiated form
svy is allowed; see [SVY] svy jackknife.
All weight types supported by command are allowed except aweights; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

| exp_list contains | (name: elist) elist |
| :---: | :---: |
|  | eexp |
| elist contains | $\begin{aligned} & \text { newvar }=(\exp ) \\ & (\text { exp }) \end{aligned}$ |
| eexp is | specname |
|  | [eqno]specname |
| specname is | _b |
|  | _b [] |
|  | _se |
|  | _se[] |
| eqno is | \#\# |
|  | name |

$\exp$ is a standard Stata expression; see [U] 13 Functions and expressions.
Distinguish between [], which are to be typed, and [], which indicate optional arguments.

## Menu

Statistics $>$ Resampling $>$ Jackknife estimation

## Description

jackknife performs jackknife estimation. Typing
jackknife exp_list: command
executes command once for each observation in the dataset, leaving the associated observation out of the calculations that make up exp_list.
command defines the statistical command to be executed. Most Stata commands and user-written programs can be used with jackknife, as long as they follow standard Stata syntax and allow the if qualifier; see [U] 11 Language syntax. The by prefix may not be part of command.
exp_list specifies the statistics to be collected from the execution of command. If command changes the contents in $\mathrm{e}(\mathrm{b})$, exp_list is optional and defaults to _b.

Many estimation commands allow the vce(jackknife) option. For those commands, we recommend using vce(jackknife) over jackknife because the estimation command already handles clustering and other model-specific details for you. The jackknife prefix command is intended for use with nonestimation commands, such as summarize, user-written commands, or functions of coefficients.
jknife is a synonym for jackknife.

## Options

Main
eclass, rclass, and n (exp) specify where command stores the number of observations on which it based the calculated results. We strongly advise you to specify one of these options.
eclass specifies that command store the number of observations in $e(N)$. rclass specifies that command store the number of observations in $r(N)$.
n (exp) specifies an expression that evaluates to the number of observations used. Specifying $\mathrm{n}(\mathrm{r}(\mathrm{N})$ ) is equivalent to specifying the rclass option. Specifying $\mathrm{n}(\mathrm{e}(\mathrm{N})$ ) is equivalent to specifying the eclass option. If command stores the number of observations in $r$ (N1), specify n ( r (N1)).

If you specify no options, jackknife will assume eclass or rclass, depending on which of $e(N)$ and $r(N)$ is not missing (in that order). If both $e(N)$ and $r(N)$ are missing, jackknife assumes that all observations in the dataset contribute to the calculated result. If that assumption is incorrect, the reported standard errors will be incorrect. For instance, say that you specify

```
. jackknife coef=_b[x2]: myreg y x1 x2 x3
```

where myreg uses $e(n)$ instead of $e(N)$ to identify the number of observations used in calculations. Further assume that observation 42 in the dataset has $x 3$ equal to missing. The 42 nd observation plays no role in obtaining the estimates, but jackknife has no way of knowing that and will use the wrong $N$. If, on the other hand, you specify

```
. jackknife coef=_b[x2], n(e(n)): myreg y x1 x2 x3
```

jackknife will notice that observation 42 plays no role. The $n(e(n))$ option is specified because myreg is an estimation command but it stores the number of observations used in $e(n)$ (instead of the standard $e(N)$ ). When jackknife runs the regression omitting the 42 nd observation, jackknife will observe that $e(n)$ has the same value as when jackknife previously ran the regression using all the observations. Thus jackknife will know that myreg did not use the observation.

## Options

cluster (varlist) specifies the variables identifying sample clusters. If cluster() is specified, one cluster is left out of each call to command, instead of 1 observation.
idcluster (newvar) creates a new variable containing a unique integer identifier for each resampled cluster, starting at 1 and leading up to the number of clusters. This option may be specified only when the cluster() option is specified. idcluster() helps identify the cluster to which a pseudovalue belongs.
saving (filename [, suboptions]) creates a Stata data file (.dta file) consisting of (for each statistic in exp_list) a variable containing the replicates.
double specifies that the results for each replication be saved as doubles, meaning 8-byte reals. By default, they are saved as floats, meaning 4-byte reals. This option may be used without the saving () option to compute the variance estimates by using double precision.
every (\#) specifies that results be written to disk every \#th replication. every () should be specified only in conjunction with saving() when command takes a long time for each replication. This option will allow recovery of partial results should some other software crash your computer. See [P] postfile.
replace specifies that filename be overwritten if it exists. This option does not appear in the dialog box.
keep specifies that new variables be added to the dataset containing the pseudovalues of the requested statistics. For instance, if you typed

```
. jackknife coef=_b[x2], eclass keep: regress y x1 x2 x3
```

new variable coef would be added to the dataset containing the pseudovalues for _b[x2]. Let $b$ be the value of $\quad \mathrm{b}[\mathrm{x} 2]$ when all observations are used to fit the model, and let $b(j)$ be the value when the $j$ th observation is omitted. The pseudovalues are defined as

$$
\text { pseudovalue }_{j}=N\{b-b(j)\}+b(j)
$$

where $N$ is the number of observations used to produce $b$.
When the cluster () option is specified, each cluster is given at most one nonmissing pseudovalue. The keep option implies the nodrop option.
mse specifies that jackknife compute the variance by using deviations of the replicates from the observed value of the statistics based on the entire dataset. By default, jackknife computes the variance by using deviations of the pseudovalues from their mean.

Reporting
level (\#); see [R] estimation options.
notable suppresses the display of the table of results.
noheader suppresses the display of the table header. This option implies nolegend.
nolegend suppresses the display of the table legend. The table legend identifies the rows of the table with the expressions they represent.
verbose specifies that the full table legend be displayed. By default, coefficients and standard errors are not displayed.
nodots suppresses display of the replication dots. By default, one dot character is displayed for each successful replication. A red ' $x$ ' is displayed if command returns an error or if one of the values in exp_list is missing.
noisily specifies that any output from command be displayed. This option implies the nodots option.
trace causes a trace of the execution of command to be displayed. This option implies the noisily option.
title (text) specifies a title to be displayed above the table of jackknife results; the default title is Jackknife results or what is produced in e(title) by an estimation command.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% fint), pformat (\% fmt), sformat (\% fmt), and nolstretch; see [R] estimation options.
eform_option causes the coefficient table to be displayed in exponentiated form; see [R] eform_option. command determines which eform_option is allowed (eform(string) and eform are always allowed).
command determines which of the following are allowed (eform(string) and eform are always allowed):

| eform_option | Description |
| :--- | :--- |
| eform(string) | use string for the column title |
| eform | exponentiated coefficient, string is exp(b) |
| hr | hazard ratio, string is Haz. Ratio |
| shr | subhazard ratio, string is SHR |
| $\underline{\text { irr }}$ | incidence-rate ratio, string is IRR |
| or | odds ratio, string is Odds Ratio |
| $\underline{\text { rrr }}$ | relative-risk ratio, string is RRR |

Advanced
nodrop prevents observations outside e(sample) and the if and in qualifiers from being dropped before the data are resampled.
reject (exp) identifies an expression that indicates when results should be rejected. When exp is true, the resulting values are reset to missing values.

The following option is available with jackknife but is not shown in the dialog box: coeflegend; see [R] estimation options.

## Remarks and examples

Remarks are presented under the following headings:

Introduction<br>Jackknifed standard deviation<br>Collecting multiple statistics<br>Collecting coefficients

## Introduction

Although the jackknife-developed in the late 1940s and early 1950s-is of largely historical interest today, it is still useful in searching for overly influential observations. This feature is often forgotten. In any case, the jackknife is

- an alternative, first-order unbiased estimator for a statistic;
- a data-dependent way to calculate the standard error of the statistic and to obtain significance levels and confidence intervals; and
- a way of producing measures called pseudovalues for each observation, reflecting the observation's influence on the overall statistic.

The idea behind the simplest form of the jackknife-the one implemented here-is to repeatedly calculate the statistic in question, each time omitting just one of the dataset's observations. Assume that our statistic of interest is the sample mean. Let $y_{j}$ be the $j$ th observation of our data on some measurement $y$, where $j=1, \ldots, N$ and $N$ is the sample size. If $\bar{y}$ is the sample mean of $y$ using the entire dataset and $\bar{y}_{(j)}$ is the mean when the $j$ th observation is omitted, then

$$
\bar{y}=\frac{(N-1) \bar{y}_{(j)}+y_{j}}{N}
$$

Solving for $y_{j}$, we obtain

$$
y_{j}=N \bar{y}-(N-1) \bar{y}_{(j)}
$$

These are the pseudovalues that jackknife calculates. To move this discussion beyond the sample mean, let $\widehat{\theta}$ be the value of our statistic (not necessarily the sample mean) using the entire dataset, and let $\widehat{\theta}_{(j)}$ be the computed value of our statistic with the $j$ th observation omitted. The pseudovalue for the $j$ th observation is

$$
\widehat{\theta}_{j}^{*}=N \widehat{\theta}-(N-1) \widehat{\theta}_{(j)}
$$

The mean of the pseudovalues is the alternative, first-order unbiased estimator mentioned above, and the standard error of the mean of the pseudovalues is an estimator for the standard error of $\widehat{\theta}$ (Tukey 1958).

When the cluster () option is given, clusters are omitted instead of observations, and $N$ is the number of clusters instead of the sample size.

The jackknife estimate of variance has been largely replaced by the bootstrap estimate (see $[R]$ bootstrap), which is widely viewed as more efficient and robust. The use of jackknife pseudovalues to detect outliers is too often forgotten and is something the bootstrap does not provide. See Mosteller and Tukey (1977, 133-163) and Mooney and Duval (1993, 22-27) for more information.

## > Example 1

As our first example, we will show that the jackknife standard error of the sample mean is equivalent to the standard error of the sample mean computed using the classical formula in the ci command. We use the double option to compute the standard errors with the same precision as the ci command.


## Jackknifed standard deviation

## Example 2

Mosteller and Tukey (1977, 139-140) request a $95 \%$ confidence interval for the standard deviation of the 11 values:

$$
0.1, \quad 0.1, \quad 0.1, \quad 0.4, \quad 0.5, \quad 1.0, \quad 1.1, \quad 1.3, \quad 1.9, \quad 1.9, \quad 4.7
$$

Stata's summarize command calculates the mean and standard deviation and stores them as $r$ (mean) and $r(s d)$. To obtain the jackknifed standard deviation of the 11 values and save the pseudovalues as a new variable, sd, we would type
. clear

- input x
x

1. 0.1
2. 0.1
3. 0.1
4. 0.4
5. 0.5
6. 1.0
7. 1.1
8. 1.3
9. 1.9
10. 1.9
11. 4.7
12. end
. jackknife sd=r(sd), rclass keep: summarize x
(running summarize on estimation sample)
Jackknife replications (11)


Jackknife results $\quad$ Number of obs $\quad 11$
command: summarize x
sd: r(sd)
$n(): r(N)$

|  | Coef. | Sackknife |  |  |  |  |
| ---: | ---: | :---: | :---: | :---: | :---: | ---: |
|  | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |  |
| sd | 1.343469 | .624405 | 2.15 | 0.057 | -.047792 | 2.73473 |

Interpreting the output, the standard deviation reported by summarize mpg is 1.34 . The jackknife standard error is 0.62 . The $95 \%$ confidence interval for the standard deviation is -0.048 to 2.73 .

By specifying keep, jackknife creates in our dataset a new variable, sd, for the pseudovalues.

```
. list, sep(4)
```

|  | x | sd |
| :---: | :---: | :---: |
| 1. | . 1 | 1.139977 |
| 2. | . 1 | 1.139977 |
| 3. | . 1 | 1.139977 |
| 4. | . 4 | . 8893147 |
| 5. | . 5 | . 824267 |
| 6. | 1 | . 632489 |
| 7. | 1.1 | . 6203189 |
| 8. | 1.3 | . 6218889 |
| 9. | 1.9 | . 835419 |
| 10. | 1.9 | . 835419 |
| 11. | 4.7 | 7.703949 |

The jackknife estimate is the average of the sd variable, so sd contains the individual values of our statistic. We can see that the last observation is substantially larger than the others. The last observation is certainly an outlier, but whether that reflects the considerable information it contains or indicates that it should be excluded from analysis depends on the context of the problem. Here Mosteller and Tukey created the dataset by sampling from an exponential distribution, so the observation is informative.

## Example 3

Let's repeat the example above using the automobile dataset, obtaining the standard error of the standard deviation of mpg.


Let's look at sd more carefully:

| summarize sd, detail |  |  |  |  |
| ---: | :---: | :---: | :--- | ---: |
|  |  | pseudovalues: $r(s d)$ |  |  |
|  | Percentiles | Smallest |  |  |
| $1 \%$ | 2.870471 | 2.870471 |  | 74 |
| $5 \%$ | 2.870471 | 2.870471 |  | 74 |
| $10 \%$ | 2.906255 | 2.870471 | Obs |  |
| $25 \%$ | 3.328489 | 2.870471 | Sum of Wgt. |  |
| $50 \%$ | 3.948335 |  | Mean | 5.817374 |
|  |  |  | Largest | Std. Dev. |
| $75 \%$ | 6.844418 | 17.34316 |  | 5.22377 |
| $90 \%$ | 9.597018 | 19.7617 | Variance | 27.28777 |
| $95 \%$ | 17.34316 | 19.7617 | Skewness | 4.07202 |
| $99 \%$ | 38.60905 | 38.60905 | Kurtosis | 23.37823 |

. list make mpg sd if sd > 30

| make |  | mpg |
| :--- | ---: | ---: |
| 71. | VW Diesel | 41 |
|  | 38.60905 |  |

Here the Vw Diesel is the only diesel car in our dataset.

## Collecting multiple statistics

## > Example 4

jackknife is not limited to collecting just one statistic. For instance, we can use summarize, detail and then obtain the jackknife estimate of the standard deviation and skewness. summarize, detail stores the standard deviation in $r$ (sd) and the skewness in $r$ (skewness), so we might type


## Collecting coefficients

## Example 5

jackknife can also collect coefficients from estimation commands. For instance, using auto.dta, we might wish to obtain the jackknife standard errors of the coefficients from a regression in which we model the mileage of a car by its weight and trunk space. To do this, we could refer to the coefficients as _b[weight], _b [trunk], _se[weight], and _se[trunk] in the exp_list, or we could simply use the extended expressions _b. In fact, jackknife assumes _b by default when used with estimation commands.


If you are going to use jackknife to estimate standard errors of model coefficients, we recommend using the vce (jackknife) option when it is allowed with the estimation command; see [R] vce_option.


John Wilder Tukey (1915-2000) was born in Massachusetts. He studied chemistry at Brown and mathematics at Princeton and afterward worked at both Princeton and Bell Labs, as well as being involved in a great many government projects, consultancies, and committees. He made outstanding contributions to several areas of statistics, including time series, multiple comparisons, robust statistics, and exploratory data analysis. Tukey was extraordinarily energetic and inventive, not least in his use of terminology: he is credited with inventing the terms bit and software, in addition to ANOVA, boxplot, data analysis, hat matrix, jackknife, stem-and-leaf plot, trimming, and winsorizing, among many others. Tukey's direct and indirect impacts mark him as one of the greatest statisticians of all time.

## Stored results

jknife stores the following in e():
Scalars
e(N)
e(N_reps)
e(N_misreps)
e(N_clust)
$e\left(k \_\right.$eq)
e(k_extra)
e(k_exp)
e(k_eexp)
e(df_r)
Macros
e(cmdname)
e(cmd)
e(command)
e(cmdline)
e(prefix)
e(wtype)
e(wexp)
e(title)
e(cluster)
e(pseudo)
e(nfunction)
e(exp\#)
e(mse)
e(vce)
e(vcetype)
e(properties)
Matrices
e(b)
$e(b-j k)$
e(V)
e(V_modelbased)
sample size
number of complete replications
number of incomplete replications
number of clusters
number of equations in $e(b)$
number of extra equations
number of expressions
number of extended expressions ( $\_$b or _se)
degrees of freedom
command name from command
same as e(cmdname) or jackknife
command
command as typed
jackknife
weight type
weight expression
title in estimation output
cluster variables
new variables containing pseudovalues
$e(N), r(N), n()$ option, or empty
expression for the \#th statistic
from mse option
jackknife
title used to label Std. Err.
b V
observed statistics
jackknife estimates
jackknife variance-covariance matrix
model-based variance

When exp_list is _b, jackknife will also carry forward most of the results already in e() from command.

## Methods and formulas

Let $\widehat{\theta}$ be the observed value of the statistic, that is, the value of the statistic calculated using the original dataset. Let $\widehat{\theta}_{(j)}$ be the value of the statistic computed by leaving out the $j$ th observation (or cluster); thus $j=1,2, \ldots, N$ identifies an individual observation (or cluster), and $N$ is the total number of observations (or clusters). The $j$ th pseudovalue is given by

$$
\widehat{\theta}_{j}^{*}=\widehat{\theta}_{(j)}+N\left\{\widehat{\theta}-\widehat{\theta}_{(j)}\right\}
$$

When the mse option is specified, the standard error is estimated as

$$
\widehat{\mathrm{se}}=\left\{\frac{N-1}{N} \sum_{j=1}^{N}\left(\widehat{\theta}_{(j)}-\widehat{\theta}\right)^{2}\right\}^{1 / 2}
$$

and the jackknife estimate is

$$
\bar{\theta}_{(.)}=\frac{1}{N} \sum_{j=1}^{N} \widehat{\theta}_{(j)}
$$

Otherwise, the standard error is estimated as

$$
\widehat{\mathrm{se}}=\left\{\frac{1}{N(N-1)} \sum_{j=1}^{N}\left(\widehat{\theta}_{j}^{*}-\bar{\theta}^{*}\right)^{2}\right\}^{1 / 2} \quad \bar{\theta}^{*}=\frac{1}{N} \sum_{j=1}^{N} \widehat{\theta}_{j}^{*}
$$

where $\bar{\theta}^{*}$ is the jackknife estimate. The variance-covariance matrix is similarly computed.

## References

Brillinger, D. R. 2002. John W. Tukey: His life and professional contributions. Annals of Statistics 30: 1535-1575.
Gould, W. W. 1995. sg34: Jackknife estimation. Stata Technical Bulletin 24: 25-29. Reprinted in Stata Technical Bulletin Reprints, vol. 4, pp. 165-170. College Station, TX: Stata Press.
Mooney, C. Z., and R. D. Duval. 1993. Bootstrapping: A Nonparametric Approach to Statistical Inference. Newbury Park, CA: Sage.
Mosteller, C. F., and J. W. Tukey. 1977. Data Analysis and Regression: A Second Course in Statistics. Reading, MA: Addison-Wesley.

Tukey, J. W. 1958. Bias and confidence in not-quite large samples. Abstract in Annals of Mathematical Statistics 29: 614.

## Also see

[R] jackknife postestimation - Postestimation tools for jackknife
$[R]$ bootstrap - Bootstrap sampling and estimation
[R] permute - Monte Carlo permutation tests
[R] simulate - Monte Carlo simulations
[SVY] svy jackknife - Jackknife estimation for survey data
[U] 13.5 Accessing coefficients and standard errors
[U] 13.6 Accessing results from Stata commands
[U] 20 Estimation and postestimation commands

## Title

jackknife postestimation - Postestimation tools for jackknife

Description Syntax for predict Also see

## Description

The following postestimation commands are available after jackknife:
Command Description

| * contrast | contrasts and ANOVA-style joint tests of estimates |
| :--- | :--- |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample |
| estat vce | variance-covariance matrix of the estimators (VCE) |
| estimates | cataloging estimation results <br> point estimates, standard errors, testing, and inference for linear combinations <br> lincom |
|  | of coefficients |
| *margins | effects means, predictive margins, marginal effects, and average marginal |

*This postestimation command is allowed only if it may be used after command.

## Syntax for predict

The syntax of predict (and whether predict is even allowed) following jackknife depends on the command used with jackknife.

## Also see

[R] jackknife - Jackknife estimation
[U] 20 Estimation and postestimation commands

## Title

kappa - Interrater agreement

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

Interrater agreement, two unique raters
kap varname varname $_{2}$ [if] [in] [weight] [, options $]$

Weights for weighting disagreements

```
kapwgt wgtid [1\ # 1 [\ # # 1...]]
```

Interrater agreement, nonunique raters, variables record ratings for each rater kap varname ${ }_{1}$ varname $_{2}$ varname $_{3}[\ldots][$ if $][$ in $][$ weight $]$

Interrater agreement, nonunique raters, variables record frequency of ratings kappa varlist [if] [in]
options Description

Main

| $\underline{\text { tab }}$ | display table of assessments |
| :--- | :--- |
| $\underline{\text { wgt }}$ (wgtid) | specify how to weight disagreements; see Options for alternatives |
| absolute | treat rating categories as absolute |

fweights are allowed; see [U] 11.1.6 weight.

## Menu

## kap: two unique raters

Statistics $>$ Epidemiology and related $>$ Other $>$ Interrater agreement, two unique raters

## kapwgt

Statistics $>$ Epidemiology and related $>$ Other $>$ Define weights for the above (kap)

## kap: nonunique raters

Statistics $>$ Epidemiology and related $>$ Other $>$ Interrater agreement, nonunique raters

## kappa

Statistics $>$ Epidemiology and related $>$ Other $>$ Interrater agreement, nonunique raters with frequencies

## Description

kap (first syntax) calculates the kappa-statistic measure of interrater agreement when there are two unique raters and two or more ratings.
kapwgt defines weights for use by kap in measuring the importance of disagreements.
kap (second syntax) and kappa calculate the kappa-statistic measure when there are two or more (nonunique) raters and two outcomes, more than two outcomes when the number of raters is fixed, and more than two outcomes when the number of raters varies. kap (second syntax) and kappa produce the same results; they merely differ in how they expect the data to be organized.
kap assumes that each observation is a subject. varname $_{1}$ contains the ratings by the first rater, varname $_{2}$ by the second rater, and so on.
kappa also assumes that each observation is a subject. The variables, however, record the frequencies with which ratings were assigned. The first variable records the number of times the first rating was assigned, the second variable records the number of times the second rating was assigned, and so on.

## Options

## Main

tab displays a tabulation of the assessments by the two raters.
wgt (wgtid) specifies that wgtid be used to weight disagreements. You can define your own weights by using kapwgt; wgt () then specifies the name of the user-defined matrix. For instance, you might define

```
. kapwgt mine 1 \ . 8 1 \ 0 . 8 1 \ 0 0 . 8 1
```

and then

```
. kap rata ratb, wgt(mine)
```

Also, two prerecorded weights are available.
wgt(w) specifies weights $1-|i-j| /(k-1)$, where $i$ and $j$ index the rows and columns of the ratings by the two raters and $k$ is the maximum number of possible ratings.
wgt (w2) specifies weights $1-\{(i-j) /(k-1)\}^{2}$.
absolute is relevant only if wgt() is also specified. The absolute option modifies how $i, j$, and $k$ are defined and how corresponding entries are found in a user-defined weighting matrix. When absolute is not specified, $i$ and $j$ refer to the row and column index, not to the ratings themselves. Say that the ratings are recorded as $\{0,1,1.5,2\}$. There are four ratings; $k=4$, and $i$ and $j$ are still $1,2,3$, and 4 in the formulas above. Index 3, for instance, corresponds to rating $=1.5$. This system is convenient but can, with some data, lead to difficulties.

When absolute is specified, all ratings must be integers, and they must be coded from the set $\{1,2,3, \ldots\}$. Not all values need be used; integer values that do not occur are simply assumed to be unobserved.

## Remarks and examples

Remarks are presented under the following headings:
Two raters
More than two raters
The kappa-statistic measure of agreement is scaled to be 0 when the amount of agreement is what would be expected to be observed by chance and 1 when there is perfect agreement. For intermediate values, Landis and Koch (1977a, 165) suggest the following interpretations:

| below 0.0 | Poor |
| :--- | :--- |
| $0.00-0.20$ | Slight |
| $0.21-0.40$ | Fair |
| $0.41-0.60$ | Moderate |
| $0.61-0.80$ | Substantial |
| $0.81-1.00$ | Almost perfect |

## Two raters

## > Example 1

Consider the classification by two radiologists of 85 xeromammograms as normal, benign disease, suspicion of cancer, or cancer (a subset of the data from Boyd et al. [1982] and discussed in the context of kappa in Altman [1991, 403-405]).

```
. use http://www.stata-press.com/data/r13/rate2
(Altman p. 403)
. tabulate rada radb
\begin{tabular}{r|rrrr|r}
\begin{tabular}{r} 
Radiologist \\
A's
\end{tabular} & \multicolumn{5}{c}{\begin{tabular}{r} 
Radiologist B's assessment \\
benign \\
suspect
\end{tabular}} \\
assessment & normal & cancer & Total \\
\hline normal & 21 & 12 & 0 & 0 & 33 \\
benign & 4 & 17 & 1 & 0 & 22 \\
suspect & 3 & 9 & 15 & 2 & 29 \\
cancer & 0 & 0 & 0 & 1 & 1 \\
\hline Total & 28 & 38 & 16 & 3 & 85
\end{tabular}
```

Our dataset contains two variables: rada, radiologist A's assessment, and radb, radiologist B's assessment. Each observation is a patient.

We can obtain the kappa measure of interrater agreement by typing

```
. kap rada radb
```

                Expected
    | Agreement | Agreement | Kappa | Std. Err. | Z | Prob>Z |
| :---: | :---: | :---: | :---: | ---: | ---: |
| $63.53 \%$ | $30.82 \%$ | 0.4728 | 0.0694 | 6.81 | 0.0000 |

If each radiologist had made his determination randomly (but with probabilities equal to the overall proportions), we would expect the two radiologists to agree on $30.8 \%$ of the patients. In fact, they agreed on $63.5 \%$ of the patients, or $47.3 \%$ of the way between random agreement and perfect agreement. The amount of agreement indicates that we can reject the hypothesis that they are making their determinations randomly.

## > Example 2: Weighted kappa, prerecorded weight w

There is a difference between two radiologists disagreeing about whether a xeromammogram indicates cancer or the suspicion of cancer and disagreeing about whether it indicates cancer or is normal. The weighted kappa attempts to deal with this. kap provides two "prerecorded" weights, w and w2:


The w weights are given by $1-|i-j| /(k-1)$, where $i$ and $j$ index the rows of columns of the ratings by the two raters and $k$ is the maximum number of possible ratings. The weighting matrix is printed above the table. Here the rows and columns of the $4 \times 4$ matrix correspond to the ratings normal, benign, suspicious, and cancerous.

A weight of 1 indicates that an observation should count as perfect agreement. The matrix has 1 s down the diagonals - when both radiologists make the same assessment, they are in agreement. A weight of, say, 0.6667 means that they are in two-thirds agreement. In our matrix, they get that score if they are "one apart" - one radiologist assesses cancer and the other is merely suspicious, or one is suspicious and the other says benign, and so on. An entry of 0.3333 means that they are in one-third agreement, or, if you prefer, two-thirds disagreement. That is the score attached when they are "two apart". Finally, they are in complete disagreement when the weight is zero, which happens only when they are three apart - one says cancer and the other says normal.
$>$ Example 3: Weighted kappa, prerecorded weight w2
The other prerecorded weight is w 2 , where the weights are given by $1-\{(i-j) /(k-1)\}^{2}$ :


The w2 weight makes the categories even more alike and is probably inappropriate here.

## > Example 4: Weighted kappa, user-defined weights

In addition to using prerecorded weights, we can define our own weights with the kapwgt command. For instance, we might feel that suspicious and cancerous are reasonably similar, that benign and normal are reasonably similar, but that the suspicious/cancerous group is nothing like the benign/normal group:

```
. kapwgt xm 1 \. . 1 \ 0 0 1 \ 0 0 . 8 1
. kapwgt xm
1.0000
0.8000 1.0000
0.0000 0.0000 1.0000
0.0000 0.0000 0.8000 1.0000
```

We name the weights xm, and after the weight name, we enter the lower triangle of the weighting matrix, using \to separate rows. We have four outcomes, so we continued entering numbers until we had defined the fourth row of the weighting matrix. If we type kapwgt followed by a name and nothing else, it shows us the weights recorded under that name. Satisfied that we have entered them correctly, we now use the weights to recalculate kappa:

| Ratings weighted by: |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1.0000 | 0.8000 | 0.00000 | 000 |  |  |
| 0.8000 | 1.0000 | 0.00000 | 000 |  |  |
| 0.0000 | 0.0000 | 1.00000 | 000 |  |  |
| 0.0000 | 0.0000 | 0.80001. | 000 |  |  |
| Expected |  |  |  |  |  |
| Agreement | Agreeme | $t$ Kappa | Std. Err. | Z | Prob>Z |
| 80.47\% | 52.67 | 0.5874 | 0.0865 | 6.79 | 0.0000 |

## Technical note

In addition to using weights for weighting the differences in categories, you can specify Stata's traditional weights for weighting the data. In the examples above, we have 85 observations in our dataset - one for each patient. If we only knew the table of outcomes - that there were 21 patients rated normal by both radiologists, etc.-it would be easier to enter the table into Stata and work from it. The easiest way to enter the data is with tabi; see $[R]$ tabulate twoway.

| tabi $211200 \backslash 41710 \backslash 39152 \backslash 0001$, replace |  |  |  |  |  |  |
| ---: | ---: | :---: | :---: | :---: | :---: | ---: | ---: |
| row |  | 1 | 2 | 3 | 4 | Total |
| 1 | 21 | 12 | 0 | 0 | 33 |  |
| 2 | 4 | 17 | 1 | 0 | 22 |  |
| 3 | 3 | 9 | 15 | 2 | 29 |  |
| 4 | 0 | 0 | 0 | 1 | 1 |  |
| Total |  | 28 | 38 | 16 | 3 | 85 |
| Pearson chi2(9) $=77.8111$ | $\operatorname{Pr}=0.000$ |  |  |  |  |  |

tabi reported the Pearson $\chi^{2}$ for this table, but we do not care about it. The important thing is that, with the replace option, tabi left the table in memory:

```
. list in 1/5
```

|  | row | col | pop |
| :--- | ---: | ---: | ---: |
| 1. | 1 | 1 | 21 |
| 2. | 1 | 2 | 12 |
| 3. | 1 | 3 | 0 |
| 4. | 1 | 4 | 0 |
| 5. | 2 | 1 | 4 |
|  |  |  |  |

The variable row is radiologist A's assessment, col is radiologist B's assessment, and pop is the number so assessed by both. Thus
. kap row col [freq=pop]
Expected

| Agreement | Agreement | Kappa | Std. Err. | Z | Prob>Z |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $63.53 \%$ | $30.82 \%$ | 0.4728 | 0.0694 | 6.81 | 0.0000 |

If we are going to keep these data, the names row and col are not indicative of what the data reflect. We could type (see [U] 12.6 Dataset, variable, and value labels)

```
. rename row rada
. rename col radb
. label var rada "Radiologist A's assessment"
. label var radb "Radiologist B's assessment"
. label define assess 1 normal 2 benign 3 suspect 4 cancer
. label values rada assess
. label values radb assess
. label data "Altman p. 403"
```

kap's tab option, which can be used with or without weighted data, shows the table of assessments:

| Radiologist A's assessment | normal | iologist benign | B's assessm suspect | cancer | Total |
| :---: | :---: | :---: | :---: | :---: | :---: |
| normal | 21 | 12 | 0 | 0 | 33 |
| benign | 4 | 17 | 1 | 0 | 22 |
| suspect | 3 | 9 | 15 | 2 | 29 |
| cancer | 0 | 0 | 0 | 1 | 1 |
| Total | 28 | 38 | 16 | 3 | 85 |
| Expected |  |  |  |  |  |
| Agreement | Agreement | Kappa | Std. Err. | Z | Prob>Z |
| 63.53\% | 30.82\% | 0.4728 | 0.0694 | 6.81 | 0.0000 |

## - Technical note

You have data on individual patients. There are two raters, and the possible ratings are 1, 2, 3, and 4 , but neither rater ever used rating 3:

```
. use http://www.stata-press.com/data/r13/rate2no3, clear
. tabulate ratera raterb
\begin{tabular}{r|rrr|r} 
ratera & 1 & 2 & 4 & raterb \\
\hline 1 & 6 & 4 & 3 & 13 \\
2 & 5 & 3 & 3 & 11 \\
4 & 1 & 1 & 26 & 28 \\
\hline Total & 12 & 8 & 32 & 52
\end{tabular}
```

Here kap would determine that the ratings are from the set $\{1,2,4\}$ because those were the only values observed. kap would expect a user-defined weighting matrix to be $3 \times 3$, and if it were not, kap would issue an error message. In the formula-based weights, the calculation would be based on $i, j=1,2,3$ corresponding to the three observed ratings $\{1,2,4\}$.

Specifying the absolute option would clarify that the ratings are $1,2,3$, and 4 ; it just so happens that rating 3 was never assigned. If a user-defined weighting matrix were also specified, kap would expect it to be $4 \times 4$ or larger (larger because we can think of the ratings being $1,2,3,4,5, \ldots$ and it just so happens that ratings $5,6, \ldots$ were never observed, just as rating 3 was not observed). In the formula-based weights, the calculation would be based on $i, j=1,2,4$.

| Ratings weighted by: |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1.0000 | 0.50000. | 000 |  |  |  |
| 0.5000 | 1.0000 | 000 |  |  |  |
| 0.0000 | 0.5000 | 000 |  |  |  |
| Expected |  |  |  |  |  |
| 79.81\% | 57.17\% | 0.5285 | 0.1169 | 4.52 | 0.0000 |
| kap rat | a raterb, w | w) abso |  |  |  |
| Ratings weighted by: |  |  |  |  |  |
| 1.0000 | 0.6667 | 000 |  |  |  |
| 0.6667 | 1.0000 | 333 |  |  |  |
| 0.0000 | 0.3333 | 000 |  |  |  |
| Expected |  |  |  |  |  |
| Agreement | Agreement | Kappa | Std. Err. | Z | Prob>Z |
| 81.41\% | 55.08\% | 0.5862 | 0.1209 | 4.85 | 0.0000 |

If all conceivable ratings are observed in the data, specifying absolute makes no difference. For instance, if rater A assigns ratings $\{1,2,4\}$ and rater B assigns $\{1,2,3,4\}$, the complete set of assigned ratings is $\{1,2,3,4\}$, the same that absolute would specify. Without absolute, it makes no difference whether the ratings are coded $\{1,2,3,4\},\{0,1,2,3\},\{1,7,9,100\},\{0,1,1.5,2.0\}$, or otherwise.

## More than two raters

For more than two raters, the mathematics are such that the two raters are not considered unique. For instance, if there are three raters, there is no assumption that the three raters who rate the first subject are the same as the three raters who rate the second. Although we call this the "more than two raters" case, it can be used with two raters when the raters' identities vary.

The nonunique rater case can be usefully broken down into three subcases: 1) there are two possible ratings, which we will call positive and negative; 2) there are more than two possible ratings, but the number of raters per subject is the same for all subjects; and 3) there are more than two possible ratings, and the number of raters per subject varies. kappa handles all these cases. To emphasize that there is no assumption of constant identity of raters across subjects, the variables specified contain counts of the number of raters rating the subject into a particular category.

Jacob Cohen (1923-1998) was born in New York City. After studying psychology at City College of New York and New York University, he worked as a medical psychologist until 1959 when he became a full professor in the Department of Psychology at New York University. He made many contributions to research methods, including the kappa measure. He persistently emphasized the value of multiple regression and the importance of power and of measuring effects rather than testing significance.

## > Example 5: Two ratings

Fleiss, Levin, and Paik $(2003,612)$ offers the following hypothetical ratings by different sets of raters on 25 subjects:

| Subject | No. of <br> raters | No. of <br> pos. ratings | Subject of |  | No. of <br> raters |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 2 | 2 | 14 | 4 | Nos. ratings |
| 2 | 2 | 0 | 15 | 2 | 3 |
| 3 | 3 | 2 | 16 | 2 | 0 |
| 4 | 4 | 3 | 17 | 3 | 2 |
| 5 | 3 | 3 | 18 | 2 | 1 |
| 6 | 4 | 1 | 19 | 4 | 1 |
| 7 | 3 | 0 | 20 | 5 | 1 |
| 8 | 5 | 0 | 21 | 3 | 4 |
| 9 | 2 | 0 | 22 | 4 | 2 |
| 10 | 4 | 4 | 23 | 3 | 0 |
| 11 | 5 | 5 | 24 | 3 | 0 |
| 12 | 3 | 3 | 25 | 2 | 2 |
| 13 | 4 | 4 |  |  |  |

We have entered these data into Stata, and the variables are called subject, raters, and pos. kappa, however, requires that we specify variables containing the number of positive ratings and negative ratings, that is, pos and raters-pos:

```
. use http://www.stata-press.com/data/r13/p612
. gen neg = raters-pos
. kappa pos neg
Two-outcomes, multiple raters:
\begin{tabular}{ccc} 
Kappa & \(Z\) & Prob>Z \\
\hline 0.5415 & 5.28 & 0.0000
\end{tabular}
```

We would have obtained the same results if we had typed kappa neg pos.
> Example 6: More than two ratings, constant number of raters, kappa
Each of 10 subjects is rated into one of three categories by five raters (Fleiss, Levin, and Paik 2003, 615):

```
. use http://www.stata-press.com/data/r13/p615, clear
```

. list

|  | subject | cat1 | cat2 | cat3 |
| :---: | ---: | ---: | ---: | ---: |
| 1. | 1 | 1 | 4 | 0 |
| 2. | 2 | 2 | 0 | 3 |
| 3. | 3 | 0 | 0 | 5 |
| 4. | 4 | 4 | 0 | 1 |
| 5. | 5 | 3 | 0 | 2 |
| 6. | 6 | 1 | 4 | 0 |
| 7. | 7 | 5 | 0 | 0 |
| 8. | 8 | 0 | 4 | 1 |
| 9. | 9 | 1 | 0 | 4 |
| 10. | 10 | 3 | 0 | 2 |

We obtain the kappa statistic:

| . kappa cat1-cat3 |  |  |  |
| ---: | :---: | :---: | :---: |
| Outcome | Kappa | Z | Prob>Z |
| cat1 | 0.2917 | 2.92 | 0.0018 |
| cat2 | 0.6711 | 6.71 | 0.0000 |
| cat3 | 0.3490 | 3.49 | 0.0002 |
| combined | 0.4179 | 5.83 | 0.0000 |

The first part of the output shows the results of calculating kappa for each of the categories separately against an amalgam of the remaining categories. For instance, the cat1 line is the two-rating kappa, where positive is cat1 and negative is cat2 or cat3. The test statistic, however, is calculated differently (see Methods and formulas). The combined kappa is the appropriately weighted average of the individual kappas. There is considerably less agreement about the rating of subjects into the first category than there is for the second.

Example 7: More than two ratings, constant number of raters, kap
Now suppose that we have the same data as in the previous example but that the data are organized differently:
. use http://www.stata-press.com/data/r13/p615b
. list

|  | subject | rater1 | rater2 | rater3 | rater4 | rater5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | 1 | 1 | 2 | 2 | 2 | 2 |
| 2. | 2 | 1 | 1 | 3 | 3 | 3 |
| 3. | 3 | 3 | 3 | 3 | 3 | 3 |
| 4. | 4 | 1 | 1 | 1 | 1 | 3 |
| 5. | 5 | 1 | 1 | 1 | 3 | 3 |
| 6. | 6 | 1 | 2 | 2 | 2 | 2 |
| 7. | 7 | 1 | 1 | 1 | 1 | 1 |
| 8. | 8 | 2 | 2 | 2 | 2 | 3 |
| 9. | 9 | 1 | 3 | 3 | 3 | 3 |
| 10. | 10 | 1 | 1 | 1 | 3 | 3 |

Here we would use kap rather than kappa because the variables record ratings for each rater.

| . kap rater1 rater2 rater3 rater4 rater5 |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| There are 5 raters per subject: |  |  |  |  |
| Outcome |  | Kappa | Z | Prob>Z |
| 1 |  |  |  |  |

It does not matter which rater is which when there are more than two raters.
> Example 8: More than two ratings, varying number of raters, kappa
In this unfortunate case, kappa can be calculated, but there is no test statistic for testing against $\kappa>0$. We do nothing differently-kappa calculates the total number of raters for each subject, and, if it is not a constant, kappa suppresses the calculation of test statistics.

```
. use http://www.stata-press.com/data/r13/rvary
. list
```

|  | subject | cat1 | cat2 | cat3 |
| ---: | ---: | ---: | ---: | ---: |
| 1. | 1 | 1 | 3 | 0 |
| 2. | 2 | 2 | 0 | 3 |
| 3. | 3 | 0 | 0 | 5 |
| 4. | 4 | 4 | 0 | 1 |
| 5. | 5 | 3 | 0 | 2 |
| 6. | 6 | 1 | 4 | 0 |
| 7. | 7 | 5 | 0 | 0 |
| 8. | 8 | 0 | 4 | 1 |
| 9. | 9 | 1 | 0 | 2 |
| 10. | 10 | 3 | 0 | 2 |

. kappa cat1-cat3

|  | Outcome | Kappa | Z | Prob>Z |
| :---: | :---: | :---: | :---: | :---: |
|  | cat1 | 0.2685 | . | . |
|  | cat2 | 0.6457 | - | - |
|  | cat3 | 0.2938 | - | . |
|  | combined | 0.3816 | . | . |
| Note: number of ratings per subject vary; cannot calculate test statistics. |  |  |  |  |

## Example 9: More than two ratings, varying number of raters, kap

This case is similar to the previous example, but the data are organized differently:
. use http://www.stata-press.com/data/r13/rvary2
. list

|  | subject | rater1 | rater2 | rater3 | rater4 | rater5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | 1 | 1 | 2 | 2 | . | 2 |
| 2. | 2 | 1 | 1 | 3 | 3 | 3 |
| 3. | 3 | 3 | 3 | 3 | 3 | 3 |
| 4. | 4 | 1 | 1 | 1 | 1 | 3 |
| 5. | 5 | 1 | 1 | 1 | 3 | 3 |
| 6. | 6 | 1 | 2 | 2 | 2 | 2 |
| 7. | 7 | 1 | 1 | 1 | 1 | 1 |
| 8. | 8 | 2 | 2 | 2 | 2 | 3 |
| 9. | 9 | 1 | 3 | . | . | 3 |
| 10. | 10 | 1 | 1 | 1 | 3 | 3 |

Here we specify kap instead of kappa because the variables record ratings for each rater.


## Stored results

kap and kappa store the following in $r()$ :
Scalars
$\left.\begin{array}{llll}r(N) & \text { number of subjects (kap only) } & r(k a p p a) & \begin{array}{l}\text { kappa } \\ r(\text { prop_o })\end{array} \\ \begin{array}{c}\text { observed proportion of agreement (kap } \\ \text { only })\end{array} & r(z) & z \text { statistic }\end{array}\right]$

## Methods and formulas

The kappa statistic was first proposed by Cohen (1960). The generalization for weights reflecting the relative seriousness of each possible disagreement is due to Cohen (1968). The analysis-of-variance approach for $k=2$ and $m \geq 2$ is due to Landis and Koch (1977b). See Altman (1991, 403-409) or Dunn (2000, chap. 2) for an introductory treatment and Fleiss, Levin, and Paik (2003, chap. 18) for a more detailed treatment. All formulas below are as presented in Fleiss, Levin, and Paik (2003). Let $m$ be the number of raters, and let $k$ be the number of rating outcomes.

Methods and formulas are presented under the following headings:

```
kap: m = 2
kappa: m> 2, k=2
kappa:m>2,k>2
```


## kap: $\mathbf{m}=\mathbf{2}$

Define $w_{i j}(i=1, \ldots, k$ and $j=1, \ldots, k)$ as the weights for agreement and disagreement (wgt()), or, if the data are not weighted, define $w_{i i}=1$ and $w_{i j}=0$ for $i \neq j$. If wgt(w) is specified, $w_{i j}=1-|i-j| /(k-1)$. If wgt(w2) is specified, $w_{i j}=1-\{(i-j) /(k-1)\}^{2}$.

The observed proportion of agreement is

$$
p_{o}=\sum_{i=1}^{k} \sum_{j=1}^{k} w_{i j} p_{i j}
$$

where $p_{i j}$ is the fraction of ratings $i$ by the first rater and $j$ by the second. The expected proportion of agreement is

$$
p_{e}=\sum_{i=1}^{k} \sum_{j=1}^{k} w_{i j} p_{i \cdot p \cdot j}
$$

where $p_{i}=\sum_{j} p_{i j}$ and $p_{\cdot j}=\sum_{i} p_{i j}$.
Kappa is given by $\widehat{\kappa}=\left(p_{o}-p_{e}\right) /\left(1-p_{e}\right)$.
The standard error of $\widehat{\kappa}$ for testing against 0 is

$$
\widehat{s}_{0}=\frac{1}{\left(1-p_{e}\right) \sqrt{n}}\left(\left[\sum_{i} \sum_{j} p_{i \cdot} \cdot p_{\cdot j}\left\{w_{i j}-\left(\bar{w}_{i} \cdot+\bar{w}_{\cdot j}\right)\right\}^{2}\right]-p_{e}^{2}\right)^{1 / 2}
$$

where $n$ is the number of subjects being rated, $\bar{w}_{i \cdot}=\sum_{j} p_{\cdot j} w_{i j}$, and $\bar{w}_{\cdot j}=\sum_{i} p_{i \cdot} w_{i j}$. The test statistic $Z=\widehat{\kappa} / \widehat{s}_{0}$ is assumed to be distributed $N(0,1)$.

## kappa: $m>2, k=2$

Each subject $i, i=1, \ldots, n$, is found by $x_{i}$ of $m_{i}$ raters to be positive (the choice as to what is labeled positive is arbitrary).

The overall proportion of positive ratings is $\bar{p}=\sum_{i} x_{i} /(n \bar{m})$, where $\bar{m}=\sum_{i} m_{i} / n$. The between-subjects mean square is (approximately)

$$
B=\frac{1}{n} \sum_{i} \frac{\left(x_{i}-m_{i} \bar{p}\right)^{2}}{m_{i}}
$$

and the within-subject mean square is

$$
W=\frac{1}{n(\bar{m}-1)} \sum_{i} \frac{x_{i}\left(m_{i}-x_{i}\right)}{m_{i}}
$$

Kappa is then defined as

$$
\widehat{\kappa}=\frac{B-W}{B+(\bar{m}-1) W}
$$

The standard error for testing against 0 (Fleiss and Cuzick 1979) is approximately equal to and is calculated as

$$
\widehat{s}_{0}=\frac{1}{(\bar{m}-1) \sqrt{n \bar{m}_{H}}}\left\{2\left(\bar{m}_{H}-1\right)+\frac{\left(\bar{m}-\bar{m}_{H}\right)(1-4 \overline{p q})}{\overline{m p q}}\right\}^{1 / 2}
$$

where $\bar{m}_{H}$ is the harmonic mean of $m_{i}$ and $\bar{q}=1-\bar{p}$.
The test statistic $Z=\widehat{\kappa} / \widehat{s}_{0}$ is assumed to be distributed $N(0,1)$.

## kappa: $m>2, k>2$

Let $x_{i j}$ be the number of ratings on subject $i, i=1, \ldots, n$, into category $j, j=1, \ldots, k$. Define $\bar{p}_{j}$ as the overall proportion of ratings in category $j, \bar{q}_{j}=1-\bar{p}_{j}$, and let $\widehat{\kappa}_{j}$ be the kappa statistic given above for $k=2$ when category $j$ is compared with the amalgam of all other categories. Kappa is

$$
\bar{\kappa}=\frac{\sum_{j} \bar{p}_{j} \bar{q}_{j} \widehat{\kappa}_{j}}{\sum_{j} \bar{p}_{j} \bar{q}_{j}}
$$

(Landis and Koch 1977b). In the case where the number of raters per subject, $\sum_{j} x_{i j}$, is a constant $m$ for all $i$, Fleiss, Nee, and Landis (1979) derived the following formulas for the approximate standard errors. The standard error for testing $\widehat{\kappa}_{j}$ against 0 is

$$
\widehat{s}_{j}=\left\{\frac{2}{n m(m-1)}\right\}^{1 / 2}
$$

and the standard error for testing $\bar{\kappa}$ is

$$
\bar{s}=\frac{\sqrt{2}}{\sum_{j} \bar{p}_{j} \bar{q}_{j} \sqrt{n m(m-1)}}\left\{\left(\sum_{j} \bar{p}_{j} \bar{q}_{j}\right)^{2}-\sum_{j} \bar{p}_{j} \bar{q}_{j}\left(\bar{q}_{j}-\bar{p}_{j}\right)\right\}^{1 / 2}
$$

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## Title

kdensity - Univariate kernel density estimation

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | Acknowledgments |
| References | Also see |  |  |

## Syntax

kdensity varname $[$ if $][$ in $][$ weight $]$ [, options $]$
options
Description

Main
kernel (kernel)
bwidth(\#)
generate(newvar newvar $_{d}$ )
n(\#)
at ( $\operatorname{var}_{x}$ )
nograph
Kernel plot
cline_options
Density plots
normal
normopts (cline_options)
student (\#)
stopts (cline_options)
Add plots
addplot (plot)
Y axis, X axis, Titles, Legend, Overall
twoway_options
specify kernel function; default is kernel (epanechnikov) half-width of kernel
store the estimation points in newvar $x_{x}$ and the density estimate in newvar ${ }_{d}$
estimate density using \# points; default is $\min (N, 50)$
estimate density using the values specified by $\mathrm{var}_{x}$ suppress graph
affect rendition of the plotted kernel density estimate
add normal density to the graph
affect rendition of normal density
add Student's $t$ density with \# degrees of freedom to the graph affect rendition of the Student's $t$ density
add other plots to the generated graph
any options other than by () documented in [G-3] twoway_options
kernel
epanechnikov
epan2
biweight
cosine
gaussian
parzen
rectangle
triangle

Description
Epanechnikov kernel function; the default
alternative Epanechnikov kernel function
biweight kernel function
cosine trace kernel function
Gaussian kernel function
Parzen kernel function
rectangle kernel function
triangle kernel function
fweights, aweights, and iweights are allowed; see [U] 11.1.6 weight.

## Menu

Statistics $>$ Nonparametric analysis $>$ Kernel density estimation

## Description

kdensity produces kernel density estimates and graphs the result.

## Options

$\qquad$ Main
kernel (kernel) specifies the kernel function for use in calculating the kernel density estimate. The default kernel is the Epanechnikov kernel (epanechnikov).
bwidth(\#) specifies the half-width of the kernel, the width of the density window around each point. If bwidth() is not specified, the "optimal" width is calculated and used. The optimal width is the width that would minimize the mean integrated squared error if the data were Gaussian and a Gaussian kernel were used, so it is not optimal in any global sense. In fact, for multimodal and highly skewed densities, this width is usually too wide and oversmooths the density (Silverman 1992).
generate (newvar $x_{x}$ newvar $_{d}$ ) stores the results of the estimation. newvar ${ }_{x}$ will contain the points at which the density is estimated. newvar ${ }_{d}$ will contain the density estimate.
n (\#) specifies the number of points at which the density estimate is to be evaluated. The default is $\min (N, 50)$, where $N$ is the number of observations in memory.
at $\left(v a r_{x}\right)$ specifies a variable that contains the values at which the density should be estimated. This option allows you to more easily obtain density estimates for different variables or different subsamples of a variable and then overlay the estimated densities for comparison.
nograph suppresses the graph. This option is often used with the generate() option.
$\qquad$ Kernel plot
cline_options affect the rendition of the plotted kernel density estimate. See [G-3] cline_options.
$\qquad$ Density plots
normal requests that a normal density be overlaid on the density estimate for comparison.
normopts(cline_options) specifies details about the rendition of the normal curve, such as the color and style of line used. See [G-3] cline_options.
student (\#) specifies that a Student's $t$ density with \# degrees of freedom be overlaid on the density estimate for comparison.
stopts (cline_options) affects the rendition of the Student's $t$ density. See [G-3] cline_options.

[^37]Y axis, X axis, Titles, Legend, Overall
twoway_options are any of the options documented in [G-3] twoway_options, excluding by (). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

## Remarks and examples

Kernel density estimators approximate the density $f(x)$ from observations on $x$. Histograms do this, too, and the histogram itself is a kind of kernel density estimate. The data are divided into nonoverlapping intervals, and counts are made of the number of data points within each interval. Histograms are bar graphs that depict these frequency counts - the bar is centered at the midpoint of each interval - and its height reflects the average number of data points in the interval.

In more general kernel density estimates, the range is still divided into intervals, and estimates of the density at the center of intervals are produced. One difference is that the intervals are allowed to overlap. We can think of sliding the interval-called a window-along the range of the data and collecting the center-point density estimates. The second difference is that, rather than merely counting the number of observations in a window, a kernel density estimator assigns a weight between 0 and 1 -based on the distance from the center of the window - and sums the weighted values. The function that determines these weights is called the kernel.

Kernel density estimates have the advantages of being smooth and of being independent of the choice of origin (corresponding to the location of the bins in a histogram).

See Salgado-Ugarte, Shimizu, and Taniuchi (1993) and Fox (1990) for discussions of kernel density estimators that stress their use as exploratory data-analysis tools.

Cox (2007) gives a lucid introductory tutorial on kernel density estimation with several Stata produced examples. He provides tips and tricks for working with skewed or bounded distributions and applying the same techniques to estimate the intensity function of a point process.
> Example 1: Histogram and kernel density estimate
Goeden (1978) reports data consisting of 316 length observations of coral trout. We wish to investigate the underlying density of the lengths. To begin on familiar ground, we might draw a histogram. In $[R]$ histogram, we suggest setting the bins to $\min \left(\sqrt{n}, 10 \cdot \log _{10} n\right)$, which for $n=316$ is roughly 18 :
. use http://www.stata-press.com/data/r13/trocolen
. histogram length, bin(18)
(bin=18, start=226, width=19.777778)


The kernel density estimate, on the other hand, is smooth.
. kdensity length


Kernel density estimators are, however, sensitive to an assumption, just as are histograms. In histograms, we specify a number of bins. For kernel density estimators, we specify a width. In the graph above, we used the default width. kdensity is smarter than twoway histogram in that its default width is not a fixed constant. Even so, the default width is not necessarily best.
kdensity stores the width in the returned scalar bwidth, so typing display $r$ (bwidth) reveals it. Doing this, we discover that the width is approximately 20.

Widths are similar to the inverse of the number of bins in a histogram in that smaller widths provide more detail. The units of the width are the units of $x$, the variable being analyzed. The width is specified as a half-width, meaning that the kernel density estimator with half-width 20 corresponds to sliding a window of size 40 across the data.

We can specify half-widths for ourselves by using the bwidth() option. Smaller widths do not smooth the density as much:

```
. kdensity length, bwidth(10)
```


. kdensity length, bwidth(15)


## Example 2: Different kernels can produce different results

When widths are held constant, different kernels can produce surprisingly different results. This is really an attribute of the kernel and width combination; for a given width, some kernels are more sensitive than others at identifying peaks in the density estimate.

We can see this when using a dataset with lots of peaks. In the automobile dataset, we characterize the density of weight, the weight of the vehicles. Below we compare the Epanechnikov and Parzen kernels.
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. kdensity weight, kernel(epanechnikov) nograph generate(x epan)
. kdensity weight, kernel(parzen) nograph generate(x2 parzen)
. label var epan "Epanechnikov density estimate"
. label var parzen "Parzen density estimate"
. line epan parzen $x$, sort ytitle(Density) legend(cols(1))


We did not specify a width, so we obtained the default width. That width is not a function of the selected kernel, but of the data. See Methods and formulas for the calculation of the optimal width.

## Example 3: Density with overlaid normal density

In examining the density estimates, we may wish to overlay a normal density or a Student's $t$ density for comparison. Using automobile weights, we can get an idea of the distance from normality by using the normal option.

```
. kdensity weight, kernel(epanechnikov) normal
```



## > Example 4: Compare two densities

We also may want to compare two or more densities. In this example, we will compare the density estimates of the weights for the foreign and domestic cars.

```
. use http://www.stata-press.com/data/r13/auto, clear
(1978 Automobile Data)
. kdensity weight, nograph generate(x fx)
. kdensity weight if foreign==0, nograph generate(fx0) at(x)
. kdensity weight if foreign==1, nograph generate(fx1) at(x)
. label var fx0 "Domestic cars"
. label var fx1 "Foreign cars"
. line fx0 fx1 x, sort ytitle(Density)
```



## Technical note

Although all the examples we included had densities of less than 1 , the density may exceed 1 .
The probability density $f(x)$ of a continuous variable, $x$, has the units and dimensions of the reciprocal of $x$. If $x$ is measured in meters, $f(x)$ has units $1 /$ meter. Thus the density is not measured on a probability scale, so it is possible for $f(x)$ to exceed 1 .

To see this, think of a uniform density on the interval 0 to 1 . The area under the density curve is 1: this is the product of the density, which is constant at 1 , and the range, which is 1 . If the variable is then transformed by doubling, the area under the curve remains 1 and is the product of the density, constant at 0.5 , and the range, which is 2 . Conversely, if the variable is transformed by halving, the area under the curve also remains at 1 and is the product of the density, constant at 2 , and the range, which is 0.5 . (Strictly, the range is measured in certain units, and the density is measured in the reciprocal of those units, so the units cancel on multiplication.)

## Stored results

kdensity stores the following in $r()$ :
Scalars

```
    r(bwidth) kernel bandwidth
    r(n) number of points at which the estimate was evaluated
    r(scale) density bin width
    r(kernel) name of kernel
```

Macros

## Methods and formulas

A kernel density estimate is formed by summing the weighted values calculated with the kernel function $K$, as in

$$
\widehat{f}_{K}=\frac{1}{q h} \sum_{i=1}^{n} w_{i} K\left(\frac{x-X_{i}}{h}\right)
$$

where $q=\sum_{i} w_{i}$ if weights are frequency weights (fweight) or analytic weights (aweight), and $q=1$ if weights are importance weights (iweights). Analytic weights are rescaled so that $\sum_{i} w_{i}=n$ (see [U] 11 Language syntax). If weights are not used, then $w_{i}=1$, for $i=1, \ldots, n$. kdensity includes seven different kernel functions. The Epanechnikov is the default function if no other kernel is specified and is the most efficient in minimizing the mean integrated squared error.

| Kernel | Formula |  |
| :---: | :---: | :---: |
| Biweight | $K[z]=\left\{\begin{array}{l} \frac{15}{16}\left(1-z^{2}\right)^{2} \\ 0 \end{array}\right.$ | if $\|z\|<1$ <br> otherwise |
| Cosine | $K[z]=\left\{\begin{array}{l} 1+\cos (2 \pi z) \\ 0 \end{array}\right.$ | if $\|z\|<1 / 2$ otherwise |
| Epanechnikov | $K[z]=\left\{\begin{array}{l} \frac{3}{4}\left(1-\frac{1}{5} z^{2}\right) / \sqrt{5} \\ 0 \end{array}\right.$ | if $\|z\|<\sqrt{5}$ otherwise |
| Epan2 | $K[z]=\left\{\begin{array}{l}\frac{3}{4}\left(1-z^{2}\right) \\ 0\end{array}\right.$ | if $\|z\|<1$ <br> otherwise |
| Gaussian | $K[z]=\frac{1}{\sqrt{2 \pi}} e^{-z^{2} / 2}$ |  |
| Parzen | $K[z]=\left\{\begin{array}{l} \frac{4}{3}-8 z^{2}+8\|z\|^{3} \\ 8(1-\|z\|)^{3} / 3 \\ 0 \end{array}\right.$ | if $\|z\| \leq 1 / 2$ <br> if $1 / 2<\|z\| \leq 1$ <br> otherwise |
| Rectangular | $K[z]=\left\{\begin{array}{l} 1 / 2 \\ 0 \end{array}\right.$ | if $\|z\|<1$ otherwise |
| Triangular | $K[z]=\left\{\begin{array}{l} 1-\|z\| \\ 0 \end{array}\right.$ | if $\|z\|<1$ otherwise |

From the definitions given in the table, we can see that the choice of $h$ will drive how many values are included in estimating the density at each point. This value is called the window width or bandwidth. If the window width is not specified, it is determined as

$$
\left.\begin{array}{rl}
m & =\min \left(\sqrt{\text { variance }_{x}}, \quad \frac{\text { interquartile range }}{x}\right. \\
1.349
\end{array}\right)
$$

where $x$ is the variable for which we wish to estimate the kernel and $n$ is the number of observations.
Most researchers agree that the choice of kernel is not as important as the choice of bandwidth. There is a great deal of literature on choosing bandwidths under various conditions; see, for example, Parzen (1962) or Tapia and Thompson (1978). Also see Newton (1988) for a comparison with sample spectral density estimation in time-series applications.

## Acknowledgments

We gratefully acknowledge the previous work by Isaías H. Salgado-Ugarte of Universidad Nacional Autónoma de México, and Makoto Shimizu and Toru Taniuchi of the University of Tokyo; see Salgado-Ugarte, Shimizu, and Taniuchi (1993). Their article provides a good overview of the subject of univariate kernel density estimation and presents arguments for its use in exploratory data analysis.

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## Also see

[R] histogram - Histograms for continuous and categorical variables

## Title

ksmirnov - Kolmogorov-Smirnov equality-of-distributions test

| Syntax | Menu | Description |
| :--- | :--- | :--- |
| Options for two-sample test | Remarks and examples | Stored results |
| Methods and formulas | References | Also see |

## Syntax

One-sample Kolmogorov-Smirnov test
ksmirnov varname $=\exp [$ if $][$ in $]$

Two-sample Kolmogorov-Smirnov test
ksmirnov varname [if] [in], by (groupvar) [ exact]

## Menu

Statistics $>$ Nonparametric analysis $>$ Tests of hypotheses $>$ Kolmogorov-Smirnov test

## Description

ksmirnov performs one- and two-sample Kolmogorov-Smirnov tests of the equality of distributions. In the first syntax, varname is the variable whose distribution is being tested, and exp must evaluate to the corresponding (theoretical) cumulative. In the second syntax, groupvar must take on two distinct values. The distribution of varname for the first value of groupvar is compared with that of the second value.

When testing for normality, please see $[R]$ sktest and $[R]$ swilk.

## Options for two-sample test

$\qquad$ Main
by (groupvar) is required. It specifies a binary variable that identifies the two groups.
exact specifies that the exact $p$-value be computed. This may take a long time if $n>50$.

## Remarks and examples

$>$ Example 1: Two-sample test
Say that we have data on x that resulted from two different experiments, labeled as group==1 and group $==2$. Our data contain

```
. use http://www.stata-press.com/data/r13/ksxmpl
. list
```

|  | group | x |
| :--- | ---: | ---: |
| 1. | 2 | 2 |
| 2. | 1 | 0 |
| 3. | 2 | 3 |
| 4. | 1 | 4 |
| 5. | 1 | 5 |
| 6. | 2 | 8 |
| 7. | 2 | 10 |

We wish to use the two-sample Kolmogorov-Smirnov test to determine if there are any differences in the distribution of x for these two groups:

| Two-sample Kolmogorov-Smirnov test for equality |  |  |  |
| :---: | :---: | :---: | :---: |
| Smaller group | D | P -value | Corrected |
| 1: | 0.5000 | 0.424 |  |
| 2 : | -0.1667 | 0.909 |  |
| Combined K-S | 0.5000 | 0.785 | 0.735 |

The first line tests the hypothesis that x for group 1 contains smaller values than for group 2 . The largest difference between the distribution functions is 0.5 . The approximate $p$-value for this is 0.424 , which is not significant.

The second line tests the hypothesis that x for group 1 contains larger values than for group 2 . The largest difference between the distribution functions in this direction is 0.1667 . The approximate $p$-value for this small difference is 0.909 .

Finally, the approximate $p$-value for the combined test is 0.785 , corrected to 0.735 . The $p$-values ksmirnov calculates are based on the asymptotic distributions derived by Smirnov (1933). These approximations are not good for small samples $(n<50)$. They are too conservative-real $p$-values tend to be substantially smaller. We have also included a less conservative approximation for the nondirectional hypothesis based on an empirical continuity correction-the 0.735 reported in the third column.

That number, too, is only an approximation. An exact value can be calculated using the exact option:

| Two-sample Kolmogorov-Smirno |  | test for equality |  |
| :---: | :---: | :---: | :---: |
| Smaller group | D | P -value | Exact |
| 1: | 0.5000 | 0.424 |  |
| 2 : | -0.1667 | 0.909 |  |
| Combined K-S: | 0.5000 | 0.785 | 0.657 |

> Example 2: One-sample test
Let's now test whether x in the example above is distributed normally. Kolmogorov-Smirnov is not a particularly powerful test in testing for normality, and we do not endorse such use of it; see $[R]$ sktest and $[R]$ swilk for better tests.

In any case, we will test against a normal distribution with the same mean and standard deviation:

```
. summarize x
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline x & 7 & 4.571429 & 3.457222 & 0 & 10
\end{tabular}
. ksmirnov x = normal((x-4.571429)/3.457222)
One-sample Kolmogorov-Smirnov test against theoretical distribution
    normal((x-4.571429)/3.457222)
\begin{tabular}{lrrr} 
Smaller group & \multicolumn{1}{c}{ D } & P-value & Corrected \\
\hline x: & 0.1650 & 0.683 & \\
Cumulative: & -0.1250 & 0.803 & \\
Combined K-S: & 0.1650 & 0.991 & 0.978
\end{tabular}
```

Because Stata has no way of knowing that we based this calculation on the calculated mean and standard deviation of x , the test statistics will be slightly conservative in addition to being approximations. Nevertheless, they clearly indicate that the data cannot be distinguished from normally distributed data.

## Stored results

ksmirnov stores the following in r() :
Scalars

| $\mathrm{r}\left(\mathrm{D} \_1\right)$ | $D$ from line 1 | $\mathrm{r}(\mathrm{D})$ | combined $D$ |
| :--- | :--- | :--- | :--- |
| $\mathrm{r}(\mathrm{p}-1)$ | $p$-value from line 1 | $\mathrm{r}(\mathrm{p})$ | combined $p$-value |
| $\mathrm{r}\left(\mathrm{D} \_2\right)$ | $D$ from line 2 | $\mathrm{r}(\mathrm{p}-\mathrm{cor})$ | corrected combined $p$-value |
| $\mathrm{r}\left(\mathrm{p} \_2\right)$ | $p$-value from line 2 | $\mathrm{r}(\mathrm{p}$-exact) | exact combined $p$-value |

## Methods and formulas

In general, the Kolmogorov-Smirnov test (Kolmogorov 1933; Smirnov 1933; also see Conover [1999], 428-465) is not very powerful against differences in the tails of distributions. In return for this, it is fairly powerful for alternative hypotheses that involve lumpiness or clustering in the data.

The directional hypotheses are evaluated with the statistics

$$
\begin{aligned}
& D^{+}=\max _{x}\{F(x)-G(x)\} \\
& D^{-}=\min _{x}\{F(x)-G(x)\}
\end{aligned}
$$

where $F(x)$ and $G(x)$ are the empirical distribution functions for the sample being compared. The combined statistic is

$$
D=\max \left(\left|D^{+}\right|,\left|D^{-}\right|\right)
$$

The $p$-value for this statistic may be obtained by evaluating the asymptotic limiting distribution. Let $m$ be the sample size for the first sample, and let $n$ be the sample size for the second sample. Smirnov (1933) shows that

$$
\lim _{m, n \rightarrow \infty} \operatorname{Pr}\left\{\sqrt{m n /(m+n)} D_{m, n} \leq z\right\}=1-2 \sum_{i=1}^{\infty}(-1)^{i-1} \exp \left(-2 i^{2} z^{2}\right)
$$

The first five terms form the approximation $P_{a}$ used by Stata. The exact $p$-value is calculated by a counting algorithm; see Gibbons and Chakraborti (2011, 236-238). A corrected $p$-value was obtained by modifying the asymptotic $p$-value by using a numerical approximation technique:

$$
\begin{aligned}
Z & =\Phi^{-1}\left(P_{a}\right)+1.04 / \min (m, n)+2.09 / \max (m, n)-1.35 / \sqrt{m n /(m+n)} \\
p \text {-value } & =\Phi(Z)
\end{aligned}
$$

where $\Phi(\cdot)$ is the cumulative normal distribution.

Andrei Nikolayevich Kolmogorov (1903-1987), of Russia, was one of the great mathematicians of the twentieth century, making outstanding contributions in many different branches, including set theory, measure theory, probability and statistics, approximation theory, functional analysis, classical dynamics, and theory of turbulence. He was a faculty member at Moscow State University for more than 60 years.
Nikolai Vasilyevich Smirnov (1900-1966) was a Russian statistician whose work included contributions in nonparametric statistics, order statistics, and goodness of fit. After army service and the study of philosophy and philology, he turned to mathematics and eventually rose to be head of mathematical statistics at the Steklov Mathematical Institute in Moscow.

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## Also see <br> [R] runtest - Test for random order <br> [R] sktest - Skewness and kurtosis test for normality <br> [R] swilk - Shapiro-Wilk and Shapiro-Francia tests for normality

## Title

kwallis - Kruskal-Wallis equality-of-populations rank test

| Syntax | Menu | Description | Option |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

Also see

## Syntax

kwallis varname [if] [in], by (groupvar)

## Menu

Statistics $>$ Nonparametric analysis $>$ Tests of hypotheses $>$ Kruskal-Wallis rank test

## Description

kwallis tests the hypothesis that several samples are from the same population. In the syntax diagram above, varname refers to the variable recording the outcome, and groupvar refers to the variable denoting the population. by() is required.

## Option

by (groupvar) is required. It specifies a variable that identifies the groups.

## Remarks and examples

$>$ Example 1
We have data on the 50 states. The data contain the median age of the population, medage, and the region of the country, region, for each state. We wish to test for the equality of the median age distribution across all four regions simultaneously:

```
. use http://www.stata-press.com/data/r13/census
(1980 Census data by state)
. kwallis medage, by(region)
Kruskal-Wallis equality-of-populations rank test
\begin{tabular}{|l|r|r|}
\hline region & Obs & Rank Sum \\
\hline NE & 9 & 376.50 \\
N Cntrl & 12 & 294.00 \\
South & 16 & 398.00 \\
West & 13 & 206.50 \\
\hline
\end{tabular}
```

```
chi-squared = 17.041 with 3 d.f.
```

chi-squared = 17.041 with 3 d.f.
probability = 0.0007
probability = 0.0007
chi-squared with ties = 17.062 with 3 d.f.
chi-squared with ties = 17.062 with 3 d.f.
probability = 0.0007

```
probability = 0.0007
```

From the output, we see that we can reject the hypothesis that the populations are the same at any level below $0.07 \%$.

## Stored results

kwallis stores the following in r() :
Scalars
$r(d f) \quad$ degrees of freedom
$r($ chi2 $) \quad \chi^{2}$
$r$ (chi2_adj) $\quad \chi^{2}$ adjusted for ties

## Methods and formulas

The Kruskal-Wallis test (Kruskal and Wallis 1952, 1953; also see Altman [1991, 213-215]; Conover [1999, 288-297]; and Riffenburgh [2012, sec. 11.6]) is a multiple-sample generalization of the two-sample Wilcoxon (also called Mann-Whitney) rank sum test (Wilcoxon 1945; Mann and Whitney 1947). Samples of sizes $n_{j}, j=1, \ldots, m$, are combined and ranked in ascending order of magnitude. Tied values are assigned the average ranks. Let $n$ denote the overall sample size, and let $R_{j}=\sum_{i=1}^{n_{j}} R\left(X_{j i}\right)$ denote the sum of the ranks for the $j$ th sample. The Kruskal-Wallis one-way analysis-of-variance test, $H$, is defined as

$$
H=\frac{1}{S^{2}}\left\{\sum_{j=1}^{m} \frac{R_{j}^{2}}{n_{j}}-\frac{n(n+1)^{2}}{4}\right\}
$$

where

$$
S^{2}=\frac{1}{n-1}\left\{\sum_{\text {all ranks }} R\left(X_{j i}\right)^{2}-\frac{n(n+1)^{2}}{4}\right\}
$$

If there are no ties, this equation simplifies to

$$
H=\frac{12}{n(n+1)} \sum_{j=1}^{m} \frac{R_{j}^{2}}{n_{j}}-3(n+1)
$$

The sampling distribution of $H$ is approximately $\chi^{2}$ with $m-1$ degrees of freedom.

William Henry Kruskal (1919-2005) was born in New York City. He studied mathematics and statistics at Antioch College, Harvard, and Columbia, and joined the University of Chicago in 1951. He made many outstanding contributions to linear models, nonparametric statistics, government statistics, and the history and methodology of statistics.

Wilson Allen Wallis (1912-1998) was born in Philadelphia. He studied psychology and economics at the Universities of Minnesota and Chicago and at Columbia. He taught at Yale, Stanford, and Chicago, before moving as president (later chancellor) to the University of Rochester in 1962. He also served in several Republican administrations. Wallis served as editor of the Journal of the American Statistical Association, coauthored a popular introduction to statistics, and contributed to nonparametric statistics.

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## Also see

[R] nptrend - Test for trend across ordered groups
[R] oneway - One-way analysis of variance
[R] sdtest - Variance-comparison tests
[R] signrank - Equality tests on matched data

## Title

ladder - Ladder of powers

| Syntax | Menu | Description |
| :--- | :--- | :--- |
| Options for ladder | Options for gladder | Options for qladder |
| Remarks and examples | Stored results | Methods and formulas |
| Acknowledgment | References | Also see |

## Syntax

## Ladder of powers

ladder varname [if] [in] [, generate(newvar) noadjust]

Ladder-of-powers histograms
gladder varname [if] [in] [, histogram_options combine_options]

Ladder-of-powers quantile-normal plots qladder varname [if] [in] [, qnorm_options combine_options] by is allowed with ladder; see [D] by.

## Menu

## ladder

Statistics $>$ Summaries, tables, and tests $>$ Distributional plots and tests $>$ Ladder of powers

## gladder

Statistics $>$ Summaries, tables, and tests $>$ Distributional plots and tests $>$ Ladder-of-powers histograms

## qladder

Statistics $>$ Summaries, tables, and tests $>$ Distributional plots and tests $>$ Ladder-of-powers quantile-normal plots

## Description

ladder searches a subset of the ladder of powers (Tukey 1977) for a transform that converts varname into a normally distributed variable. sktest tests for normality; see [R] sktest. Also see [R] boxcox.
gladder displays nine histograms of transforms of varname according to the ladder of powers. gladder is useful pedagogically, but we do not advise looking at histograms for research work; ladder or qnorm (see [ R ] diagnostic plots) is preferred.
qladder displays the quantiles of transforms of varname according to the ladder of powers against the quantiles of a normal distribution.

## Options for ladder

$\qquad$ Main
generate (newvar) saves the transformed values corresponding to the minimum chi-squared value from the table. We do not recommend using generate() because it is literal in interpreting the minimum, thus ignoring nearly equal but perhaps more interpretable transforms.
noadjust is the noadjust option to sktest; see [R] sktest.

## Options for gladder

histogram_options affect the rendition of the histograms across all relevant transformations; see [R] histogram. Here the normal option is assumed, so you must supply the nonormal option to suppress the overlaid normal density. Also, gladder does not allow the width(\#) option of histogram.
combine_options are any of the options documented in [G-2] graph combine. These include options for titling the graph (see [G-3] title _options) and for saving the graph to disk (see [G-3] saving_option).

## Options for qladder

qnorm_options affect the rendition of the quantile-normal plots across all relevant transformations. See $[\mathrm{R}]$ diagnostic plots.
combine_options are any of the options documented in [G-2] graph combine. These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

## Remarks and examples

> Example 1: ladder
We have data on the mileage rating of 74 automobiles and wish to find a transform that makes the variable normally distributed:

| Transformation | formula | chi2(2) | P(chi2) |
| :---: | :---: | :---: | :---: |
| cubic | mpg^3 | 43.59 | 0.000 |
| square | $m p{ }^{\wedge} 2$ | 27.03 | 0.000 |
| identity | mpg | 10.95 | 0.004 |
| square root | sqrt (mpg) | 4.94 | 0.084 |
| log | $\log$ (mpg) | 0.87 | 0.647 |
| 1/(square root) | 1/sqrt(mpg) | 0.20 | 0.905 |
| inverse | $1 / \mathrm{mpg}$ | 2.36 | 0.307 |
| 1/square | 1/(mpg 2 ) | 11.99 | 0.002 |
| 1/cubic | 1/( $\left.\mathrm{mpg}^{\wedge} 3\right)$ | 24.30 | 0.000 |

If we had typed ladder mpg , gen (mpgx), the variable mpgx containing $1 / \sqrt{\mathrm{mpg}}$ would have been automatically generated for us. This is the perfect example of why you should not, in general, specify the generate() option. We also cannot reject the hypothesis that the inverse of mpg is normally distributed and that $1 / \mathrm{mpg}$ - gallons per mile—has a better interpretation. It is a measure of energy consumption.

## > Example 2: gladder

gladder explores the same transforms as ladder but presents results graphically:
. gladder mpg, fraction


## Technical note

gladder is useful pedagogically, but be careful when using it for research work, especially with many observations. For instance, consider the following data on the average July temperature in degrees Fahrenheit for 954 U.S. cities:

| . use http://www.stata-press.com/data/r13/citytemp <br> (City Temperature Data) |  |  |  |
| :--- | :--- | ---: | :--- |
| . ladder tempjuly <br> Transformation | formula |  |  |
| cubic | tempjuly^3 | chi2(2) | P(chi2) |
| square | tempjuly^2 | 47.49 | 0.000 |
| identity | tempjuly | 19.70 | 0.000 |
| square root | sqrt(tempjuly) | 3.83 | 0.147 |
| log | log(tempjuly) | 1.83 | 0.400 |
| 1/(square root) | 1/sqrt(tempjuly) | 5.40 | 0.067 |
| inverse | $1 /$ tempjuly | 13.72 | 0.001 |
| 1/square | $1 /$ (tempjuly^2) | 26.36 | 0.000 |
| 1/cubic | $1 /$ (tempjuly^3) | 64.43 | 0.000 |
|  |  | . | 0.000 |

The period in the last line indicates that the $\chi^{2}$ is very large; see $[\mathrm{R}]$ sktest.

From the table, we see that there is certainly a difference in normality between the square and square-root transform. If, however, you can see the difference between the transforms in the diagram below, you have better eyes than we do:

```
. gladder tempjuly, l1title("") ylabel(none) xlabel(none)
```


> Example 3: qladder
A better graph for seeing normality is the quantile-normal graph, which can be produced by qladder.
. qladder tempjuly, ylabel(none) xlabel(none)


This graph shows that for the square transform, the upper tail—and only the upper tail—diverges from what would be expected. This divergence is detected by sktest (see [R] sktest) as a problem with skewness, as we would learn from using sktest to examine tempjuly squared and square rooted.

## Stored results

ladder stores the following in $r()$ :
Scalars

```
r(N) number of observations
    r(invcube) }\mp@subsup{\chi}{}{2}\mathrm{ for inverse-cubic transformation
    r(P_invcube) significance level for inverse-cubic transformation
    r(invsq) }\quad\mp@subsup{\chi}{}{2}\mathrm{ for inverse-square transformation
    r(P_invsq) significance level for inverse-square transformation
```



```
    r(P_inv) significance level for inverse transformation
    r(invsqrt) }\quad\mp@subsup{\chi}{}{2}\mathrm{ for inverse-root transformation
    r(P_invsqrt) significance level for inverse-root transformation
```



```
    r(P_log) significance level for log transformation
```



```
    r(P_sqrt) significance level for square-root transformation
    r(ident) }\quad\mp@subsup{\chi}{}{2}\mathrm{ for untransformed data
    r(P_ident) significance level for untransformed data
    r(square) }\quad\mp@subsup{\chi}{}{2}\mathrm{ for square transformation
    r(P_square) significance level for square transformation
    r(cube) }\quad\mp@subsup{\chi}{}{2}\mathrm{ for cubic transformation
    r(P_cube) significance level for cubic transformation
```


## Methods and formulas

For ladder, results are as reported by sktest; see [R] sktest. If generate() is specified, the transform with the minimum $\chi^{2}$ value is chosen.
gladder sets the number of bins to $\min \left(\sqrt{n}, 10 \log _{10} n\right)$, rounded to the closest integer, where $n$ is the number of unique values of varname. See [R] histogram for a discussion of the optimal number of bins.

Also see Findley (1990) for a ladder-of-powers variable transformation program that produces one-way graphs with overlaid box plots, in addition to histograms with overlaid normals. Buchner and Findley (1990) discuss ladder-of-powers transformations as one aspect of preliminary data analysis. Also see Hamilton (1992, 18-23) and Hamilton (2013, 129-132).

## Acknowledgment

qladder was written by Jeroen Weesie of the Department of Sociology at Utrecht University, The Netherlands.

## References

Buchner, D. M., and T. W. Findley. 1990. Research in physical medicine and rehabilitation: VIII. Preliminary data analysis. American Journal of Physical Medicine and Rehabilitation 69: 154-169.
Cox, N. J. 2005. Speaking Stata: Density probability plots. Stata Journal 5: 259-273.
Findley, T. W. 1990. sed3: Variable transformation and evaluation. Stata Technical Bulletin 2: 15. Reprinted in Stata Technical Bulletin Reprints, vol. 1, pp. 85-86. College Station, TX: Stata Press.

Hamilton, L. C. 1992. Regression with Graphics: A Second Course in Applied Statistics. Belmont, CA: Duxbury.
——. 2013. Statistics with Stata: Updated for Version 12. 8th ed. Boston: Brooks/Cole.
Tukey, J. W. 1977. Exploratory Data Analysis. Reading, MA: Addison-Wesley.

## Also see

[R] diagnostic plots - Distributional diagnostic plots
[R] Inskew0 - Find zero-skewness log or Box-Cox transform
[R] Iv - Letter-value displays
[R] sktest - Skewness and kurtosis test for normality

## Title

level - Set default confidence level

Syntax Description Option Remarks and examples Also see

## Syntax

set level \# [, permanently]

## Description

set level specifies the default confidence level for confidence intervals for all commands that report confidence intervals. The initial value is 95 , meaning $95 \%$ confidence intervals. \# may be between 10.00 and 99.99 , and \# can have at most two digits after the decimal point.

## Option

permanently specifies that, in addition to making the change right now, the level setting be remembered and become the default setting when you invoke Stata.

## Remarks and examples

To change the level of confidence intervals reported by a particular command, you need not reset the default confidence level. All commands that report confidence intervals have a level (\#) option. When you do not specify the option, the confidence intervals are calculated for the default level set by set level, or for $95 \%$ if you have not reset set level.

## Example 1

We use the ci command to obtain the confidence interval for the mean of mpg :

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. ci mpg
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Err. & [95\% Conf. Interval] \\
\hline mpg & 74 & 21.2973 & .6725511 & 19.9569 & 22.63769
\end{tabular}
```

To obtain $90 \%$ confidence intervals, we would type
. ci mpg, level(90)

| Variable | Obs | Mean | Std. Err. | [90\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: |
| mpg | 74 | 21.2973 | .6725511 | 20.17683 | 22.41776 |

or
. set level 90
. ci mpg

| Variable | Obs | Mean | Std. Err. | [90\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: |
| mpg | 74 | 21.2973 | .6725511 | 20.17683 | 22.41776 |

If we opt for the second alternative, the next time that we fit a model (say, with regress), $90 \%$ confidence intervals will be reported. If we wanted $95 \%$ confidence intervals, we could specify level (95) on the estimation command, or we could reset the default by typing set level 95.

The current setting of level() is stored as the c-class value c(level); see [P] creturn.

## Also see

[R] query - Display system parameters
[P] creturn - Return c-class values
[U] 20 Estimation and postestimation commands
[U] 20.7 Specifying the width of confidence intervals

## Title

limits — Quick reference for limits

## Description Remarks and examples Also see

## Description

This entry provides a quick reference for the size limits in Stata. Note that most of these limits are so high that you will never encounter them.

## Remarks and examples

Remarks are presented under the following headings:

Maximum size limits<br>Matrix size<br>Determining which flavor of Stata you are running

## Maximum size limits

|  | Small | Stata/IC | Stata/MP and Stata/SE |
| :---: | :---: | :---: | :---: |
| \# of observations (1) | 1,200 | 2,147,483,647 | 2,147,483,647 |
| \# of variables | 99 | 2,047 | 32,767 |
| width of a dataset in bytes | 800 | 24,564 | 393,192 |
| value of matsize | 100 | 800 | 11,000 |
| \# of RHS variables | 98 | 798 | 10,998 |
| \# characters in a command | 13,416 | 165,216 | 1,081,527 |
| \# options for a command | 70 | 70 | 70 |
| \# of elements in a numlist | 2,500 | 2,500 | 2,500 |
| \# of interacted continuous variables | 8 | 8 | 8 |
| \# of interacted factor variables | 8 | 8 | 8 |
| \# of unique time-series operators in a command | 100 | 100 | 100 |
| \# seasonal suboperators per time-series operator | 8 | 8 | 8 |
| \# of dyadic operators in an expression | 66 | 800 | 800 |
| \# of numeric literals in an expression | 50 | 300 | 300 |
| \# of string literals in an expression | 256 | 512 | 512 |
| length of string in string expression | 2,000,000,000 | 2,000,000,000 | 2,000,000,000 |
| \# of sum functions in an expression | 5 | 5 | 5 |
| \# of pairs of nested parentheses | 249 | 249 | 249 |
| \# of characters in a macro (2) | 13,400 | 165,200 | 1,081,511 |
| \# of nested do-files | 64 | 64 | 64 |


| continued | Small | Stata/IC | Stata/MP and Stata/SE |
| :---: | :---: | :---: | :---: |
| \# of lines in a program | 3,500 | 3,500 | 3,500 |
| \# of characters in a program | 135,600 | 135,600 | 135,600 |
| length of a variable name | 32 | 32 | 32 |
| length of ado-command name | 32 | 32 | 32 |
| length of a global macro name | 32 | 32 | 32 |
| length of a local macro name | 31 | 31 | 31 |
| length of a str\# variable | 2,045 | 2,045 | 2,045 |
| length of a strL variable 2,000, | 2,000,000,000 | 2,000,000,000 | 2,000,000,000 |
| anova |  |  |  |
| \# of variables in one anova term | 8 | 8 | 8 |
| \# of terms in the repeated() option | 4 | 4 | 4 |
| char |  |  |  |
| length of one characteristic | 13,400 | 67,784 | 67,784 |
| constraint |  |  |  |
| \# of constraints | 1,999 | 1,999 | 1,999 |
| encode and decode |  |  |  |
| \# of unique values | 1,000 | 65,536 | 65,536 |
| _estimates hold |  |  |  |
| \# of stored estimation results | 300 | 300 | 300 |
| estimates store |  |  |  |
| \# of stored estimation results | 300 | 300 | 300 |
| exlogistic and expoisson |  |  |  |
| grmeanby |  |  |  |
| \# of unique values in varlist | -N/2 | _N/2 | -N/2 |
| graph twoway |  |  |  |
| \# of variables in a plot | 100 | 100 | 100 |
| \# of styles in an option's stylelist | 20 | 20 | 20 |
| infile (free format) |  |  |  |
| infile (fixed format) |  |  | 524,275 |
| infix (fixed format) record length with a dictionary | 524,275 | 524,275 | 524,275 |


| continued | Small | Stata/IC | Stata/MP and Stata/SE |
| :---: | :---: | :---: | :---: |
| label |  |  |  |
| length of dataset label | 80 | 80 | 80 |
| length of variable label | 80 | 80 | 80 |
| length of value label string | 32,000 | 32,000 | 32,000 |
| length of name of value label | 32 | 32 | 32 |
| \# of codings within one value label | 1,000 | 65,536 | 65,536 |
| label language |  |  |  |
| \# of different languages | 100 | 100 | 100 |
| macro |  |  |  |
| \# of nested macros | 20 | 20 | 20 |
| manova |  |  |  |
| \# of variables in single manova term | 8 | 8 | 8 |
| matrix (3) |  |  |  |
| dimension of single matrix | $40 \times 40$ | $800 \times 800$ | $11,000 \times 11,000$ |
| maximize options |  |  |  |
| iterate() maximum | 16,000 | 16,000 | 16,000 |
| mprobit |  |  |  |
| \# of categories in a depvar | 30 | 30 | 30 |
| net |  |  |  |
| \# of description lines in . pkg file | 100 | 100 | 100 |
| nlogit and nlogittree |  |  |  |
| \# of levels in model | 8 | 8 | 8 |
| notes |  |  |  |
| length of one note | 13,400 | 67,784 | 67,784 |
| \# of notes attached to _dta | 9,999 | 9,999 | 9,999 |
| \# of notes attached to each variable | 9,999 | 9,999 | 9,999 |
| numlist |  |  |  |
| \# of elements in the numeric list | 2,500 | 2,500 | 2,500 |
| reg3, sureg, and other system estimators |  |  |  |
| \# of equations | 40 | 800 | 11,000 |
| set adosize |  |  |  |
| memory ado-files may consume | 1000K | 1000K | 1000K |
| set scrollbufsize |  |  |  |
| memory for Results window buffer | 2000K | 2000K | 2000k |


| continued | Small | Stata/IC | Stata/MP and Stata/SE |
| :---: | :---: | :---: | :---: |
| slogit |  |  |  |
| \# of categories in a depvar | 30 | 30 | 30 |
| snapspan |  |  |  |
| length of label | 80 | 80 | 80 |
| \# of saved snapshots | 1,000 | 1,000 | 1,000 |
| stcox |  |  |  |
| \# of variables in strata() option | 5 | 5 | 5 |
| stcurve |  |  |  |
| \# of curves plotted on the same graph | 10 | 10 | 10 |
| table and tabdisp |  |  |  |
| \# of by variables | 4 | 4 | 4 |
| \# of margins, i.e., sum of rows, columns, supercolumns, and by groups | 3,000 | 3,000 | 3,000 |
| tabulate oneway |  |  |  |
| \# of rows in one-way table | 500 | 3,000 | 12,000 |
| tabulate twoway |  |  |  |
| \# of rows \& cols in two-way table | $160 \times 20$ | $300 \times 20$ | $1,200 \times 80$ |
| tabulate, summarize() <br> \# of cells (rows X cols) | 375 | 375 | 375 |
| teffects |  |  |  |
| \# of treatments | 20 | 20 | 20 |
| xt estimation commands (e.g., xtgee, xtgls, xtpoisson, xtprobit, xtreg with mle option, and xtpcse when neither option hetonly nor option independent is specified) |  |  |  |
| \# of time periods within panel | 40 | 800 | 11,000 |
| \# of integration points accepted by intpoints(\#) | 195 | 195 | 195 |

(1) $2,147,483,647$ is a theoretical maximum; memory availability will certainly impose a smaller maximum.
(2) The maximum length of the contents of a macro are fixed in Stata/IC and settable in Stata/SE and Stata/MP. The currently set maximum length is recorded in c(macrolen); type display $c$ (macrolen). The maximum length can be changed with set maxvar. If you set maxvar to a larger value, the maximum length increases; if you set maxvar to a smaller value, the maximum length decreases. The relationship between them is maximum_length $=33 \times$ maxvar +200 .
(3) In Mata, matrices are limited only by the amount of memory on your computer.

## Matrix size

See [R] matsize.

## Determining which flavor of Stata you are running

Type
. about
The response will be Stata/MP, Stata/SE, Stata/IC, or Small Stata. Other information is also shown, including your serial number. See $[R]$ about.

## Also see

[R] about - Display information about your Stata
[R] matsize - Set the maximum number of variables in a model
[D] compress - Compress data in memory
[D] data types - Quick reference for data types
[D] import - Overview of importing data into Stata
[D] infile (fixed format) - Read text data in fixed format with a dictionary
[D] infile (free format) - Read unformatted text data
[D] memory - Memory management
[D] obs - Increase the number of observations in a dataset

## Title

lincom - Linear combinations of estimators

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | References | Also see |

## Syntax

lincom exp [, options]
options Description
eform generic label; $\exp (b)$
or odds ratio
hr hazard ratio
shr subhazard ratio
irr incidence-rate ratio
rrr relative-risk ratio
level (\#) set confidence level; default is level (95)
display_options control column formats
df (\#) use $t$ distribution with \# degrees of freedom for computing $p$-values and confidence intervals
exp is any linear combination of coefficients that is a valid syntax for test; see $[\mathrm{R}]$ test. exp must not contain an equal sign.
$\mathrm{df}(\#)$ does not appear in the dialog box.

## Menu

Statistics $>$ Postestimation $>$ Linear combinations of estimates

## Description

lincom computes point estimates, standard errors, $t$ or $z$ statistics, $p$-values, and confidence intervals for linear combinations of coefficients after any estimation command. Results can optionally be displayed as odds ratios, hazard ratios, incidence-rate ratios, or relative-risk ratios.
lincom can be used with svy estimation results; see [SVY] svy postestimation.

## Options

eform, or, hr , shr, irr, and rrr all report coefficient estimates as $\exp (\widehat{\beta})$ rather than $\widehat{\beta}$. Standard errors and confidence intervals are similarly transformed. or is the default after logistic. The only difference in these options is how the output is labeled.

| Option | Label | Explanation | Example commands |
| :--- | :--- | :--- | :--- |
| eform | $\exp (b)$ | Generic label | cloglog |
| or | Odds Ratio | Odds ratio | logistic, logit |
| hr | Haz. Ratio | Hazard ratio | stcox, streg |
| $\operatorname{shr}$ | SHR | Subhazard ratio | stcrreg |
| $\operatorname{irr}$ | IRR | Incidence-rate ratio | poisson |
| $\operatorname{rrr}$ | RRR | Relative-risk ratio | mlogit |

exp may not contain any additive constants when you use the eform, or, hr, irr, or rrr option. level(\#) specifies the confidence level, as a percentage, for confidence intervals. The default is level (95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals. display_options: cformat (\%fmt), pformat (\%fmt), and sformat (\%fmt); see [R] estimation options.

The following option is available with lincom but is not shown in the dialog box:
df (\#) specifies that the $t$ distribution with \# degrees of freedom be used for computing $p$-values and confidence intervals. The default is to use e(df_r) degrees of freedom or the standard normal distribution if e(df_r) is missing.

## Remarks and examples

Remarks are presented under the following headings:
Using lincom
Odds ratios and incidence-rate ratios
Multiple-equation models

## Using lincom

After fitting a model and obtaining estimates for coefficients $\beta_{1}, \beta_{2}, \ldots, \beta_{k}$, you may want to view estimates for linear combinations of the $\beta_{i}$, such as $\beta_{1}-\beta_{2}$. lincom can display estimates for any linear combination of the form $c_{0}+c_{1} \beta_{1}+c_{2} \beta_{2}+\cdots+c_{k} \beta_{k}$.
lincom works after any estimation command for which test works. Any valid expression for test syntax 1 (see [R] test) is a valid expression for lincom.
lincom is useful for viewing odds ratios, hazard ratios, etc., for one group (that is, one set of covariates) relative to another group (that is, another set of covariates). See the examples below.

## Example 1

We perform a linear regression:
. use http://www.stata-press.com/data/r13/regress
. regress y x1 x2 x3

| Source | SS | df | MS |
| ---: | ---: | ---: | ---: |
| Model | 3259.3561 | 3 | 1086.45203 |
| Residual | 1627.56282 | 144 | 11.3025196 |
| Total | 4886.91892 | 147 | 33.2443464 |

Number of obs = 148
$F(3,144)=96.12$
Prob > F $=0.0000$
$R$-squared $=0.6670$
Adj R-squared $=0.6600$
Root MSE $=3.3619$

| y | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | ---: | ---: | ---: | ---: |
| x 1 | 1.457113 | 1.07461 | 1.36 | 0.177 | -.666934 | 3.581161 |
| x2 | 2.221682 | .8610358 | 2.58 | 0.011 | .5197797 | 3.923583 |
| x3 | -.006139 | .0005543 | -11.08 | 0.000 | -.0072345 | -.0050435 |
| _cons | 36.10135 | 4.382693 | 8.24 | 0.000 | 27.43863 | 44.76407 |

To see the difference of the coefficients of $x 2$ and $x 1$, we type
. lincom x2 - x1
(1) $-\mathrm{x} 1+\mathrm{x} 2=0$

| $y$ | Coef. | Std. Err. | $t$ | P>\|t| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | ---: |
| $(1)$ | .7645682 | .9950282 | 0.77 | 0.444 | -1.20218 | 2.731316 |

The expression can be any linear combination.

```
. lincom 3*x1 + 500*x3
```

( 1) $3 * x 1+500 * x 3=0$

| $y$ | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $(1)$ | 1.301825 | 3.396624 | 0.38 | 0.702 | -5.411858 | 8.015507 |

Nonlinear expressions are not allowed.

```
. lincom x2/x1
not possible with test
r(131);
```

For information about estimating nonlinear expressions, see $[R]$ nlcom.

## - Technical note

lincom uses the same shorthands for coefficients as does test (see [R] test). When you type x1, for instance, lincom knows that you mean the coefficient of $x 1$. The formal syntax for referencing this coefficient is actually $\_\mathrm{b}$ [x1], or alternatively, _coef [x1]. So, more formally, in the last example we could have typed

```
. lincom 3*_b[x1] + 500*_b[x3]
    (output omitted)
```


## Odds ratios and incidence-rate ratios

After logistic regression, the or option can be specified with lincom to display odds ratios for any effect. Incidence-rate ratios after commands such as poisson can be similarly obtained by specifying the irr option.

## > Example 2

Consider the low birthweight dataset from Hosmer, Lemeshow, and Sturdivant (2013, 24). We fit a logistic regression model of low birthweight (variable low) on the following variables:

| Variable | Description | Coding |
| :--- | :--- | :--- |
| age | age in years |  |
| race | race | 1 if white, 2 if black, 3 if other |
| smoke | smoking status | 1 if smoker, 0 if nonsmoker |
| ht | history of hypertension | 1 if yes, 0 if no |
| ui | materine irritability | 1 if yes, 0 if no |
| lwd | history of premature labor <br> ptd <br> c.age\#\#lwd | age main effects, lwd main effects, <br> and their interaction |
| smoke\#\#lwd | smoke main effects, lwd main effects, <br> and their interaction |  |

We first fit a model without the interaction terms by using logit.

```
. use http://www.stata-press.com/data/r13/lbw3
(Hosmer \& Lemeshow data)
. logit low age lwd i.race smoke ptd ht ui
Iteration 0: log likelihood \(=\quad-117.336\)
Iteration 1: log likelihood \(=\quad-99.3982\)
Iteration 2: log likelihood \(=-98.780418\)
Iteration 3: log likelihood \(=-98.777998\)
Iteration 4: log likelihood = -98.777998
```

| Logistic regression | Number of obs | $=$ | 189 |
| :--- | :--- | :--- | :--- |
|  | LR chi2 (8) | $=$ | 37.12 |
| Log likelihood $=-98.777998$ | Prob > chi2 | $=$ | 0.0000 |
|  | Pseudo R2 | $=$ | 0.1582 |


| low | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| age | -.0464796 | .0373888 | -1.24 | 0.214 | -.1197603 | .0268011 |
| lwd | .8420615 | .4055338 | 2.08 | 0.038 | .0472299 | 1.636893 |
| race |  |  |  |  |  |  |
| black | 1.073456 | .5150753 | 2.08 | 0.037 | .0639273 | 2.082985 |
| other | .815367 | .4452979 | 1.83 | 0.067 | -.0574008 | 1.688135 |
| smoke | .8071996 | .404446 | 2.00 | 0.046 | .0145001 | 1.599899 |
| ptd | 1.281678 | .4621157 | 2.77 | 0.006 | .3759478 | 2.187408 |
| ht | 1.435227 | .6482699 | 2.21 | 0.027 | .1646414 | 2.705813 |
| ui | .6576256 | .4666192 | 1.41 | 0.159 | -.2569313 | 1.572182 |
| _cons | -1.216781 | .9556797 | -1.27 | 0.203 | -3.089878 | .656317 |

To get the odds ratio for black smokers relative to white nonsmokers (the reference group), we type
. lincom 2.race + smoke, or
( 1) [low]2.race $+[$ low] smoke $=0$

| low | Odds Ratio | Std. Err. | $z$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | :---: |
| $(1)$ | 6.557805 | 4.744692 | 2.60 | 0.009 | 1.588176 | 27.07811 |

lincom computed $\exp \left(\beta_{2 \text {.race }}+\beta_{\text {smoke }}\right)=6.56$. To see the odds ratio for white smokers relative to black nonsmokers, we type

```
. lincom smoke - 2.race, or
( 1) - [low]2.race + [low]smoke = 0
```

| low | Odds Ratio | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | :---: |
| $(1)$ | .7662425 | .4430176 | -0.46 | 0.645 | .2467334 | 2.379603 |

Now let's add the interaction terms to the model (Hosmer and Lemeshow 1989, table 4.10). This time, we will use logistic rather than logit. By default, logistic displays odds ratios.

| . logistic low i.race ht ui ptd c.age\#\#lwd smoke\#\#lwd |  |  |  |
| :--- | :--- | :--- | :--- |
| Logistic regression | Number of obs | $=$ | 189 |
|  | LR chi2 (10) | $=$ | 42.66 |
|  |  | Prob > chi2 | $=$ |
| Log likelihood $=-96.00616$ |  | Pseudo R2 | $=$ |



Hosmer and Lemeshow (1989, table 4.13) consider the effects of smoking (smoke $=1$ ) and low maternal weight before pregnancy ( $1 \mathrm{wd}=1$ ). The effect of smoking among non-low-weight mothers $(l \mathrm{wd}=0)$ is given by the odds ratio 3.17 for smoke in the logistic output. The effect of smoking among low-weight mothers is given by
. lincom 1.smoke + 1.smoke\#1.lwd
( 1) [low]1.smoke + [low]1.smoke\#1.lwd $=0$

| low | Odds Ratio | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $(1)$ | .7755022 | .574951 | -0.34 | 0.732 | .1813465 | 3.316323 |

We did not have to specify the or option. After logistic, lincom assumes or by default.
The effect of low weight $(\mathrm{lwd}=1)$ is more complicated because we fit an age $\times$ lwd interaction. We must specify the age of mothers for the effect. The effect among 30 -year-old nonsmokers is given by
. lincom 1.lwd + 30*1.lwd\#c.age
( 1) [low]1.1wd $+30 *[$ low] 1.lwd\#c.age $=0$

| low | Odds Ratio | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | :---: |
| $(1)$ | 14.7669 | 13.5669 | 2.93 | 0.003 | 2.439264 | 89.39633 |

lincom computed $\exp \left(\beta_{\text {lwd }}+30 \beta_{\text {agelwd }}\right)=14.8$. It may seem odd that we entered it as 1.1 wd $+30 * 1.1 w d \# c . a g e$, but remember that these terms are just lincom's (and test's) shorthands for _b[1.lwd] and _b[1.lwd\#c.age]. We could have typed

```
. lincom _b[1.lwd] + 30*_b[1.lwd#c.age]
( 1) [low]1.1wd + 30*[low]1.1wd#c.age = 0
```

| low | Odds Ratio | Std. Err. | $z$ | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | ---: |
| $(1)$ | 14.7669 | 13.5669 | 2.93 | 0.003 | 2.439264 | 89.39633 |

## Multiple-equation models

lincom also works with multiple-equation models. The only difference is how you refer to the coefficients. Recall that for multiple-equation models, coefficients are referenced using the syntax

## [eqno] varname

where eqno is the equation number or equation name and varname is the corresponding variable name for the coefficient; see [U] 13.5 Accessing coefficients and standard errors and [R] test for details.

## Example 3

Let's consider example 4 from [R] mlogit (Tarlov et al. 1989; Wells et al. 1989).


| insure | Coef. | Std. Err | z | $P>\|z\|$ | [95\% Con | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Indemnity | (base outcome) |  |  |  |  |  |
| Prepaid |  |  |  |  |  |  |
| age | -. 011745 | . 0061946 | -1.90 | 0.058 | -. 0238862 | . 0003962 |
| male | . 5616934 | . 2027465 | 2.77 | 0.006 | . 1643175 | . 9590693 |
| nonwhite | . 9747768 | . 2363213 | 4.12 | 0.000 | . 5115955 | 1.437958 |
| site |  |  |  |  |  |  |
| 2 | . 1130359 | . 2101903 | 0.54 | 0.591 | -. 2989296 | . 5250013 |
| 3 | -. 5879879 | . 2279351 | -2.58 | 0.010 | -1.034733 | -. 1412433 |
| _cons | . 2697127 | . 3284422 | 0.82 | 0.412 | -. 3740222 | . 9134476 |
| Uninsure |  |  |  |  |  |  |
| age | -. 0077961 | . 0114418 | -0.68 | 0.496 | -. 0302217 | . 0146294 |
| male | . 4518496 | . 3674867 | 1.23 | 0.219 | -. 268411 | 1.17211 |
| nonwhite | . 2170589 | . 4256361 | 0.51 | 0.610 | -. 6171725 | 1.05129 |
| site |  |  |  |  |  |  |
| 2 | -1.211563 | . 4705127 | -2.57 | 0.010 | -2.133751 | -. 2893747 |
| 3 | -. 2078123 | . 3662926 | -0.57 | 0.570 | -. 9257327 | . 510108 |
| _cons | -1.286943 | . 5923219 | -2.17 | 0.030 | -2.447872 | -. 1260134 |

To see the estimate of the sum of the coefficient of male and the coefficient of nonwhite for the Prepaid outcome, we type
. lincom [Prepaid]male + [Prepaid]nonwhite
( 1) [Prepaid]male + [Prepaid]nonwhite $=0$

| insure | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | ---: |
| $(1)$ | 1.53647 | .3272489 | 4.70 | 0.000 | .8950741 | 2.177866 |

To view the estimate as a ratio of relative risks (see [R] mlogit for the definition and interpretation), we specify the rrr option.

```
. lincom [Prepaid]male + [Prepaid]nonwhite, rrr
( 1) [Prepaid]male + [Prepaid]nonwhite = 0
```

| insure | RRR | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | :---: |
| $(1)$ | 4.648154 | 1.521103 | 4.70 | 0.000 | 2.447517 | 8.827451 |

## Stored results

lincom stores the following in r() :
Scalars

```
r(estimate) point estimate
r(se) estimate of standard error
    r(df) degrees of freedom
```


## References

Hosmer, D. W., Jr., and S. A. Lemeshow. 1989. Applied Logistic Regression. New York: Wiley.
Hosmer, D. W., Jr., S. A. Lemeshow, and R. X. Sturdivant. 2013. Applied Logistic Regression. 3rd ed. Hoboken, NJ: Wiley.

Tarlov, A. R., J. E. Ware, Jr., S. Greenfield, E. C. Nelson, E. Perrin, and M. Zubkoff. 1989. The medical outcomes study. An application of methods for monitoring the results of medical care. Journal of the American Medical Association 262: 925-930.

Wells, K. B., R. D. Hays, M. A. Burnam, W. H. Rogers, S. Greenfield, and J. E. Ware, Jr. 1989. Detection of depressive disorder for patients receiving prepaid or fee-for-service care. Results from the Medical Outcomes Survey. Journal of the American Medical Association 262: 3298-3302.

## Also see

[R] nlcom - Nonlinear combinations of estimators
$[R]$ test - Test linear hypotheses after estimation
[R] testnl - Test nonlinear hypotheses after estimation
[U] 13.5 Accessing coefficients and standard errors
[U] 20 Estimation and postestimation commands

## Title

linktest - Specification link test for single-equation models

| Syntax | Menu | Description | Option |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |
| Also see |  |  |  |

## Syntax

linktest [if] [in] [, cmd_options]
When if and in are not specified, the link test is performed on the same sample as the previous estimation.

## Menu

Statistics $>$ Postestimation $>$ Tests $>$ Specification link test for single-equation models

## Description

linktest performs a link test for model specification after any single-equation estimation command, such as logistic, regress, stcox, etc.

## Option

$\qquad$ Main
cmd_options must be the same options specified with the underlying estimation command, except the display_options may differ.

## Remarks and examples

The form of the link test implemented here is based on an idea of Tukey (1949), which was further described by Pregibon (1980), elaborating on work in his unpublished thesis (Pregibon 1979). See Methods and formulas below for more details.

## > Example 1

We want to explain the mileage ratings of cars in our automobile dataset by using the weight, engine displacement, and whether the car is manufactured outside the United States:

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. regress mpg weight displ foreign
```

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | 1619.71935 | 3 | 539.906448 |
| Residual | 823.740114 | 70 | 11.7677159 |
| Total | 2443.45946 | 73 | 33.4720474 |

Number of obs $=\quad 74$ $F(3,70)=45.88$ Prob > F $=0.0000$ $R$-squared $=0.6629$ Adj R-squared $=0.6484$ Root MSE $=3.4304$

| mpg | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| weight | -.0067745 | .0011665 | -5.81 | 0.000 | -.0091011 | -.0044479 |
| displacement | .0019286 | .0100701 | 0.19 | 0.849 | -.0181556 | .0220129 |
| foreign | -1.600631 | 1.113648 | -1.44 | 0.155 | -3.821732 | .6204699 |
| _cons | 41.84795 | 2.350704 | 17.80 | 0.000 | 37.15962 | 46.53628 |

On the basis of the $R^{2}$, we are reasonably pleased with this model.
If our model really is specified correctly, then if we were to regress mpg on the prediction and the prediction squared, the prediction squared would have no explanatory power. This is what linktest does:
. linktest

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | 1670.71514 | 2 | 835.357572 |
| Residual | 772.744316 | 71 | 10.8837228 |
| Total | 2443.45946 | 73 | 33.4720474 |


| Number of obs | $=$ | 74 |
| :--- | ---: | ---: |
| F 2, | $71)$ | $=$ |
| Prob $>$ F | $=$ | 0.0000 |
| R-squared | $=$ | 0.6837 |
| Adj R-squared | $=$ | 0.6748 |
| Root MSE | $=$ | 3.299 |


| mpg | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| _hat | -.4127198 | .6577736 | -0.63 | 0.532 | -1.724283 | .8988434 |
| _hatsq | .0338198 | .015624 | 2.16 | 0.034 | .0026664 | .0649732 |
| _cons | 14.00705 | 6.713276 | 2.09 | 0.041 | .6211539 | 27.39294 |

We find that the prediction squared does have explanatory power, so our specification is not as good as we thought.

Although linktest is formally a test of the specification of the dependent variable, it is often interpreted as a test that, conditional on the specification, the independent variables are specified incorrectly. We will follow that interpretation and now include weight squared in our model:
. regress mpg weight c.weight\#c.weight displ foreign

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | 1699.02634 | 4 | 424.756584 |
| Residual | 744.433124 | 69 | 10.7888859 |
| Total | 2443.45946 | 73 | 33.4720474 |

Number of obs $=\quad 74$ $F(4,69)=39.37$ Prob $>\mathrm{F}=0.0000$ $R$-squared $=0.6953$ Adj R -squared $=0.6777$ Root MSE $=3.2846$

| mpg | Coef. | Std. Err. | t | $\mathrm{P}>\mid \mathrm{tI}$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| weight | -.0173257 | .0040488 | -4.28 | 0.000 | -.0254028 | -.0092486 |
| c.weight\# |  |  |  |  |  |  |
| c.weight | $1.87 \mathrm{e}-06$ | $6.89 \mathrm{e}-07$ | 2.71 | 0.008 | $4.93 \mathrm{e}-07$ | $3.24 \mathrm{e}-06$ |
|  |  |  |  |  |  |  |
| displacement | -.0101625 | .0106236 | -0.96 | 0.342 | -.031356 | .011031 |
| foreign | -2.560016 | 1.123506 | -2.28 | 0.026 | -4.801349 | -.3186832 |
| _cons | 58.23575 | 6.449882 | 9.03 | 0.000 | 45.36859 | 71.10291 |

Now we perform the link test on our new model:


We now pass the link test.

## Example 2

Above we followed a standard misinterpretation of the link test-when we discovered a problem, we focused on the explanatory variables of our model. We might consider varying exactly what the link test tests. The link test told us that our dependent variable was misspecified. For those with an engineering background, mpg is indeed a strange measure. It would make more sense to model energy consumption-gallons per mile-in terms of weight and displacement:
. gen $\mathrm{gpm}=1 / \mathrm{mpg}$
. regress gpm weight displ foreign

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model <br> Residual | .009157962 | 3 | .003052654 |
| Total | .002799666 | 70 | .000039995 |


| Number of obs | $=$ | 74 |
| :--- | ---: | ---: |
| $\mathrm{~F}(3, \quad 70)$ | $=$ | 76.33 |
| Prob $>$ F | $=$ | 0.0000 |
| R-squared | $=$ | 0.7659 |
| Adj R-squared | $=$ | 0.7558 |
| Root MSE | $=$ | .00632 |


| gpm | Coef. | Std. Err. | t | $\mathrm{P}>\mid \mathrm{t\mid}$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| weight | .0000144 | $2.15 \mathrm{e}-06$ | 6.72 | 0.000 | .0000102 | .0000187 |
| displacement | .0000186 | .0000186 | 1.00 | 0.319 | -.0000184 | .0000557 |
| foreign | .0066981 | .0020531 | 3.26 | 0.002 | .0026034 | .0107928 |
| _cons | .0008917 | .0043337 | 0.21 | 0.838 | -.0077515 | .009535 |

This model looks every bit as reasonable as our original model:
. linktest

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | .009175219 | 2 | .004587609 |
| Residual | .002782409 | 71 | .000039189 |
| Total | .011957628 | 73 | .000163803 |

Number of obs $=\quad 74$
$F(2,71)=117.06$
Prob > F $=0.0000$
$R$-squared $=0.7673$
Adj R-squared $=0.7608$
Root MSE $=.00626$

| gpm | Coef. | Std. Err. | t | P>\|t| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | :---: | ---: | ---: |
| _hat | .6608413 | .515275 | 1.28 | 0.204 | -.3665877 | 1.68827 |
| _hatsq | 3.275857 | 4.936655 | 0.66 | 0.509 | -6.567553 | 13.11927 |
| _cons | .008365 | .0130468 | 0.64 | 0.523 | -.0176496 | .0343795 |

Specifying the model in terms of gallons per mile also solves the specification problem and results in a more parsimonious specification.

## > Example 3

The link test can be used with any single-equation estimation procedure, not solely regression. Let's turn our problem around and attempt to explain whether a car is manufactured outside the United States by its mileage rating and weight. To save paper, we will specify logit's nolog option, which suppresses the iteration log:

| Logistic regression |  |  |  | Number of obs LR chi2(2) |  |  | 74 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | 35.72 |
|  |  |  |  |  |  |  | Prob | chi2 |  | 0.0000 |
| Log likelihood = -27.175156 |  |  |  | Pse |  |  | 0.3966 |
| foreign | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |  |
| mpg | -. 1685869 | . 0919175 | -1.83 | 0.067 | -. 3487 | 418 | . 011568 |
| weight | -. 0039067 | . 0010116 | -3.86 | 0.000 | -. 0058 | 894 | -. 001924 |
| _cons | 13.70837 | 4.518709 | 3.03 | 0.002 | 4.851 | 859 | 22.56487 |

When we run linktest after logit, the result is another logit specification:

| Logistic regression |  |  |  | Number of obs LR chi2(2) |  | $=\quad 74$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | 36.83 |
|  |  |  |  | Prob | chi2 | 0.0000 |
| Log likelihood $=-26.615714$ |  |  |  | Pseudo R2 |  | 0.4090 |
| foreign | Coef. | Std. Err. | z | $P>\|z\|$ | [95\% | Interval] |
| _hat | . 8438531 | . 2738759 | 3.08 | 0.002 | . 3070 | 1.38064 |
| _hatsq | -. 1559115 | . 1568642 | -0.99 | 0.320 | -. 46335 | . 1515366 |
| _cons | . 2630557 | . 4299598 | 0.61 | 0.541 | -. 57 | 1.105761 |

The link test reveals no problems with our specification.
If there had been a problem, we would have been virtually forced to accept the misinterpretation of the link test-we would have reconsidered our specification of the independent variables. When using logit, we have no control over the specification of the dependent variable other than to change likelihood functions.

We admit to having seen a dataset once for which the link test rejected the logit specification. We did change the likelihood function, refitting the model using probit, and satisfied the link test. Probit has thinner tails than logit. In general, however, you will not be so lucky.

## - Technical note

You should specify the same options with linktest that you do with the estimation command, although you do not have to follow this advice as literally as we did in the preceding example. logit's nolog option merely suppresses a part of the output, not what is estimated. We specified nolog both times to save space.

If you are testing a tobit model, you must specify the censoring points just as you do with the tobit command.

If you are not sure which options are important, duplicate exactly what you specified on the estimation command.

If you do not specify if exp or in range with linktest, Stata will by default perform the link test on the same sample as the previous estimation. Suppose that you omitted some data when performing your estimation, but want to calculate the link test on all the data, which you might do if you believe the model is appropriate for all the data. You would type linktest if e(sample) < . to do this.

## Stored results

linktest stores the following in r() :
Scalars
$\mathrm{r}(\mathrm{t}) \quad t$ statistic on _hatsq
$r(\mathrm{df})$ degrees of freedom
linktest is not an estimation command in the sense that it leaves previous estimation results unchanged. For instance, after running a regression and performing the link test, typing regress without arguments after the link test still replays the original regression.

For integrating an estimation command with linktest, linktest assumes that the name of the estimation command is stored in $\mathrm{e}(\mathrm{cmd})$ and that the name of the dependent variable is stored in e (depvar). After estimation, it assumes that the number of degrees of freedom for the $t$ test is given by $e\left(d f \_m\right)$ if the macro is defined.

If the estimation command reports $z$ statistics instead of $t$ statistics, linktest will also report $z$ statistics. The $z$ statistic, however, is still returned in $r(t)$, and $r(d f)$ is set to a missing value.

## Methods and formulas

The link test is based on the idea that if a regression or regression-like equation is properly specified, you should be able to find no additional independent variables that are significant except by chance. One kind of specification error is called a link error. In regression, this means that the dependent variable needs a transformation or "link" function to properly relate to the independent variables. The idea of a link test is to add an independent variable to the equation that is especially likely to be significant if there is a link error.

Let

$$
\mathbf{y}=f(\mathbf{X} \boldsymbol{\beta})
$$

be the model and $\widehat{\boldsymbol{\beta}}$ be the parameter estimates. linktest calculates

$$
\text { _hat }=\mathbf{X} \widehat{\boldsymbol{\beta}}
$$

and

$$
\text { _hatsq }=\text { hat }^{2}
$$

The model is then refit with these two variables, and the test is based on the significance of _hatsq. This is the form suggested by Pregibon (1979) based on an idea of Tukey (1949). Pregibon (1980) suggests a slightly different method that has come to be known as "Pregibon's goodness-of-link test". We prefer the older version because it is universally applicable, straightforward, and a good second-order approximation. It can be applied to any single-equation estimation technique, whereas Pregibon's more recent tests are estimation-technique specific.

## References

Pregibon, D. 1979. Data analytic methods for generalized linear models. PhD diss., University of Toronto.
-_ 1980. Goodness of link tests for generalized linear models. Applied Statistics 29: 15-24.
Tukey, J. W. 1949. One degree of freedom for non-additivity. Biometrics 5: 232-242.

## Also see <br> [R] regress postestimation - Postestimation tools for regress

## Title

Inskew0 — Find zero-skewness log or Box-Cox transform

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | Acknowledgment |
| Reference | Also see |  |  |

## Syntax

Zero-skewness log transform
Inskew0 newvar $=\exp [$ if $][$ in $][$, options $]$

Zero-skewness Box-Cox transform
bcskew0 newvar $=\exp [$ if $][$ in $][$, options $]$
options
Description
Main
delta(\#) increment for derivative of skewness function; default is delta(0.02) for lnskew0 and delta(0.01) for bcskew0
zero(\#) value for determining convergence; default is zero(0.001)
level(\#) set confidence level; default is level(95)

## Menu

## Inskew0

Data $>$ Create or change data $>$ Other variable-creation commands $>$ Zero-skewness log transform
bcskew0
Data $>$ Create or change data $>$ Other variable-creation commands $>$ Box-Cox transform

## Description

lnskew 0 creates newvar $=\ln ( \pm \exp -k)$, choosing $k$ and the sign of exp so that the skewness of newvar is zero.
bcskew0 creates newvar $=\left(\exp ^{\lambda}-1\right) / \lambda$, the Box-Cox power transformation (Box and Cox 1964), choosing $\lambda$ so that the skewness of newvar is zero. exp must be strictly positive. Also see [R] boxcox for maximum likelihood estimation of $\lambda$.

## Options

Main
delta(\#) specifies the increment used for calculating the derivative of the skewness function with respect to $k$ (lnskew 0 ) or $\lambda$ (bcskew 0 ). The default values are 0.02 for lnskew 0 and 0.01 for bcskew0.
zero(\#) specifies a value for skewness to determine convergence that is small enough to be considered zero and is, by default, 0.001 .
level (\#) specifies the confidence level for the confidence interval for $k$ (lnskew0) or $\lambda$ (bcskew0). The confidence interval is calculated only if level() is specified. \# is specified as an integer; 95 means $95 \%$ confidence intervals. The level () option is honored only if the number of observations exceeds 7.

## Remarks and examples

> Example 1: Inskew0
Using our automobile dataset (see [U] 1.2.2 Example datasets), we want to generate a new variable equal to $\ln (\mathrm{mpg}-k)$ to be approximately normally distributed. mpg records the miles per gallon for each of our cars. One feature of the normal distribution is that it has skewness 0 .

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. lnskew0 lnmpg = mpg
\begin{tabular}{c|ccc} 
Transform & k & {\([95 \%\) Conf. Interval] } & Skewness \\
\hline \(\ln (\mathrm{mpg}-\mathrm{k})\) & 5.383659 & (not calculated) & \(-7.05 \mathrm{e}-06\)
\end{tabular}
```

This created the new variable $\operatorname{lnmpg}=\ln (\mathrm{mpg}-5.384)$ :

| describe lnmpg |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| variable name | storage <br> type | display <br> format | value <br> label | variable label |
| $\operatorname{lnmpg}$ | float | $\% 9.0 \mathrm{~g}$ |  | $\ln (m p g-5.383659)$ |

Because we did not specify the level() option, no confidence interval was calculated. At the outset, we could have typed

```
. use http://www.stata-press.com/data/r13/auto, clear
(Automobile Data)
. lnskewO lnmpg = mpg, level(95)
\begin{tabular}{c|cccc} 
Transform & k & {\([95 \%\) Conf. Interval] } & Skewness \\
\hline \(\ln (\mathrm{mpg}-\mathrm{k})\) & 5.383659 & -17.12339 & 9.892416 & \(-7.05 \mathrm{e}-06\)
\end{tabular}
```

The confidence interval is calculated under the assumption that $\ln (\mathrm{mpg}-k)$ really does have a normal distribution. It would be perfectly reasonable to use lnskew0, even if we did not believe that the transformed variable would have a normal distribution-if we literally wanted the zero-skewness transform - although, then the confidence interval would be an approximation of unknown quality to the true confidence interval. If we now wanted to test the believability of the confidence interval, we could also test our new variable lnmpg by using swilk (see [R] swilk) with the lnnormal option.

## - Technical note

lnskew0 and bcskew 0 report the resulting skewness of the variable merely to reassure you of the accuracy of its results. In our example above, lnskew0 found $k$ such that the resulting skewness was $-7 \times 10^{-6}$, a number close enough to zero for all practical purposes. If we wanted to make it even smaller, we could specify the zero() option. Typing lnskew0 new=mpg, zero(1e-8) changes the estimated $k$ to 5.383552 from 5.383659 and reduces the calculated skewness to $-2 \times 10^{-11}$.

When you request a confidence interval, lnskew0 may report the lower confidence interval as '.', which should be taken as indicating the lower confidence limit $k_{L}=-\infty$. (This cannot happen with bcskew0.)

As an example, consider a sample of size $n$ on $x$ and assume that the skewness of $x$ is positive, but not significantly so, at the desired significance level-say, 5\%. Then no matter how large and negative you make $k_{L}$, there is no value extreme enough to make the skewness of $\ln \left(x-k_{L}\right)$ equal the corresponding percentile ( 97.5 for a $95 \%$ confidence interval) of the distribution of skewness in a normal distribution of the same sample size. You cannot do this because the distribution of $\ln \left(x-k_{L}\right)$ tends to that of $x$-apart from location and scale shift—as $x \rightarrow \infty$. This "problem" never applies to the upper confidence limit, $k_{U}$, because the skewness of $\ln \left(x-k_{U}\right)$ tends to $-\infty$ as $k$ tends upward to the minimum value of $x$.

## > Example 2: bcskew0

In example 1, using lnskew0 with a variable such as mpg is probably undesirable. mpg has a natural zero, and we are shifting that zero arbitrarily. On the other hand, use of lnskew 0 with a variable such as temperature measured in Fahrenheit or Celsius would be more appropriate, as the zero is indeed arbitrary.

For a variable like mpg, it makes more sense to use the Box-Cox power transform (Box and Cox 1964):

$$
y^{(\lambda)}=\frac{y^{\lambda}-1}{\lambda}
$$

$\lambda$ is free to take on any value, but $y^{(1)}=y-1, y^{(0)}=\ln (y)$, and $y^{(-1)}=1-1 / y$.
bcskew0 works like lnskew 0 :
. bcskewO bcmpg = mpg, level(95)

| Transform | L | [95\% Conf. Interval] | Skewness |  |
| ---: | ---: | ---: | ---: | ---: |
| $\left(\mathrm{mpg}^{\wedge} \mathrm{L}-1\right) / \mathrm{L}$ | -.3673283 | -1.212752 | .4339645 | .0001898 |

The $95 \%$ confidence interval includes $\lambda=-1$ ( $\lambda$ is labeled L in the output), which has a rather more pleasing interpretation-gallons per mile—than $\left(\mathrm{mpg}^{-0.3673}-1\right) /(-0.3673)$. The confidence interval, however, is calculated assuming that the power transformed variable is normally distributed. It makes perfect sense to use bcskew0, even when you do not believe that the transformed variable will be normally distributed, but then the confidence interval is an approximation of unknown quality. If you believe that the transformed data are normally distributed, you can alternatively use boxcox to estimate $\lambda$; see $[\mathrm{R}]$ boxcox.

## Stored results

lnskew 0 and bcskew0 store the following in $r()$ :
Scalars

$$
\begin{array}{ll}
\mathrm{r} \text { (gamma) } & k \text { (lnskew0) } \\
\mathrm{r}(\text { lambda }) & \lambda \text { (bcskew0) } \\
\mathrm{r}(\mathrm{lb}) & \text { lower bound of confidence interval } \\
\mathrm{r}(\text { ub }) & \text { upper bound of confidence interval } \\
\mathrm{r} \text { (skewness) } & \text { resulting skewness of transformed variable }
\end{array}
$$

## Methods and formulas

Skewness is as calculated by summarize; see [R] summarize. Newton's method with numeric, uncentered derivatives is used to estimate $k$ (lnskew0) and $\lambda$ (bcskew0). For lnskew0, the initial value is chosen so that the minimum of $x-k$ is 1 , and thus $\ln (x-k)$ is 0 . bcskew 0 starts with $\lambda=1$.

## Acknowledgment

Inskew0 and bcskew0 were written by Patrick Royston of the MRC Clinical Trials Unit, London, and coauthor of the Stata Press book Flexible Parametric Survival Analysis Using Stata: Beyond the Cox Model.

## Reference

Box, G. E. P., and D. R. Cox. 1964. An analysis of transformations. Journal of the Royal Statistical Society, Series B 26: 211-252.

## Also see

$[\mathrm{R}]$ boxcox - Box-Cox regression models
[R] ladder - Ladder of powers
[R] swilk - Shapiro-Wilk and Shapiro-Francia tests for normality

## Title

$\log$ — Echo copy of session to file

| Syntax | Menu |
| :--- | :--- |
| Description | Options for use with both log and cmdlog |
| Options for use with log | Option for use with set logtype |
| Remarks and examples | Stored results |
| Also see |  |

## Syntax

Report status of log file
$\log$
log query [logname |_all]

Open log file
log using filename $[$, append replace $[\underline{\text { text }} \mid \underline{\text { smcl }}]$ name(logname) $]$
Close log
log close [logname|_all]

Temporarily suspend logging or resume logging
$\log \{$ off $\mid$ on $\}$ [logname]
Report status of command log file
cmdlog
Open command log file
cmdlog using filename [, append replace]

Close command log, temporarily suspend logging, or resume logging cmdlog \{́lose|on|off $\}$

Set default format for logs
set logtype $\{\underline{\text { text }} \mid \underline{\text { smcl }}\}$ [, permanently]
Specify screen width
set linesize \#
In addition to using the log command, you may access the capabilities of log by selecting File $>\mathbf{L o g}$ from the menu and choosing one of the options in the list.

## Menu

File $>$ Log

## Description

log allows you to make a full record of your Stata session. A log is a file containing what you type and Stata's output. You may start multiple log files at the same time, and you may refer to them with a logname. If you do not specify a logname, Stata will use the name <unnamed>.
cmdlog allows you to make a record of what you type during your Stata session. A command log contains only what you type, so it is a subset of a full log.

You can make full logs, command logs, or both simultaneously. Neither is produced until you tell Stata to start logging.

Command logs are always text files, making them easy to convert into do-files. (In this respect, it would make more sense if the default extension of a command log file was . do because command logs are do-files. The default is .txt, not .do, however, to keep you from accidentally overwriting your important do-files.)

Full logs are recorded in one of two formats: Stata Markup and Control Language (SMCL) or plain text. The default is SMCL, but you can use set logtype to change that, or you can specify an option to state the format you wish. We recommend SMCL because it preserves fonts and colors. SMCL logs can be converted to text or to other formats by using the translate command; see [R] translate. You can also use translate to produce printable versions of SMCL logs. SMCL logs can be viewed and printed from the Viewer, as can any text file; see $[R]$ view.

When using multiple log files, you may have up to five SMCL logs and five text logs open at the same time.
log or cmdlog, typed without arguments, reports the status of logging. log query, when passed an optional logname, reports the status of that log.
log using and cmdlog using open a log file. log close and cmdlog close close the file. Between times, log off and cmdlog off, and log on and cmdlog on, can temporarily suspend and resume logging.

If filename is specified without an extension, one of the suffixes . smcl, .log, or .txt is added. The extension .smcl or. $\log$ is added by $\log$, depending on whether the file format is SMCL or text. The extension .txt is added by cmdlog. If filename contains embedded spaces, remember to enclose it in double quotes.
set logtype specifies the default format in which full logs are to be recorded. Initially, full logs are recorded in SMCL format.
set linesize specifies the maximum width, in characters, of Stata output. Most commands in Stata do not respect linesize, because it is not important for most commands. Most users never need to set linesize, because it will automatically be reset if you resize your Results window. This is also why there is no permanently option allowed with set linesize. set linesize is for use with commands such as list and display and is typically used by programmers who wish the output of those commands to be wider or narrower than the current width of the Results window.

## Options for use with both log and cmdlog

append specifies that results be appended to an existing file. If the file does not already exist, a new file is created.
replace specifies that filename, if it already exists, be overwritten. When you do not specify either replace or append, the file is assumed to be new. If the specified file already exists, an error message is issued and logging is not started.

## Options for use with log

text and smcl specify the format in which the log is to be recorded. The default is complicated to describe but is what you would expect:
If you specify the file as filename. smcl , the default is to write the $\log$ in SMCL format (regardless of the value of set logtype).
If you specify the file as filename. 1 log , the default is to write the $\log$ in text format (regardless of the value of set logtype).

If you type filename without an extension and specify neither the smcl option nor the text option, the default is to write the file according to the value of set logtype. If you have not set logtype, then the default is SMCL. Also, the filename you specified will be fixed to read filename. smcl if a SMCL $\log$ is being created or filename. 1 log if a text $\log$ is being created.

If you specify either the text or smcl option, then what you specify determines how the log is written. If filename was specified without an extension, the appropriate extension is added for you. If you open multiple log files, you may choose a different format for each file.
name (logname) specifies an optional name you may use to refer to the log while it is open. You can start multiple log files, give each a different logname, and then close, temporarily suspend, or resume them each individually.

## Option for use with set logtype

permanently specifies that, in addition to making the change right now, the logtype setting be remembered and become the default setting when you invoke Stata.

## Remarks and examples

For a detailed explanation of logs, see [U] 15 Saving and printing output-log files.
When you open a full log, the default is to show the name of the file and a time and date stamp:
. log using myfile

```
            name: <unnamed>
    log: C:\data\proj1\myfile.smcl
    log type: smcl
opened on: 12 Jan 2013, 12:28:23
```

The above information will appear in the log. If you do not want this information to appear, precede the command by quietly:

```
. quietly log using myfile
```

quietly will not suppress any error messages or anything else you need to know.

Similarly, when you close a full $\log$, the default is to show the full information,

```
. log close
    name: <unnamed>
        log: C:\data\proj1\myfile.smcl
    log type: smcl
closed on: 12 Jan 2013, 12:32:41
```

and that information will also appear in the log. If you want to suppress that, type quietly log close.

## Stored results

log and cmdlog store the following in r() :
Macros

```
r(name) logname
r(filename) name of file
r(status) on or off
r(type) smcl or text
```

log query _all stores the following in $r()$ :
Scalars
$r$ (numlogs) number of open log files
For each open $\log$ file, log query _all also stores

```
r(name#) logname
r(filename#) name of file
r(status#) on or off
r(type#) smcl or text
```

where \# varies between 1 and the value of $r$ (numlogs). Be aware that \# will not necessarily represent the order in which the $\log$ files were first opened, nor will it necessarily remain constant for a given log file upon multiple calls to log query.

## Also see

[R] query - Display system parameters
[R] translate - Print and translate logs
[GSM] 16 Saving and printing results by using logs
[GSW] 16 Saving and printing results by using logs
[GSU] 16 Saving and printing results by using logs
[U] 15 Saving and printing output-log files

## Title

logistic - Logistic regression, reporting odds ratios

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

logistic depvar indepvars [if] [in] [weight] [, options]
options Description

Model
noconstant
offset (varname)
asis
constraints (constraints)
collinear
SE/Robust
vce(vcetype)

Reporting
level (\#)
coef
nocnsreport
display_options

Maximization
maximize_options
coeflegend
suppress constant term
include varname in model with coefficient constrained to 1
retain perfect predictor variables
apply specified linear constraints
keep collinear variables
vcetype may be oim, robust, cluster clustvar, bootstrap, or jackknife
indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
depvar and indepvars may contain time-series operators; see [U] 11.4.4 Time-series varlists.
bootstrap, by, fp, jackknife, mfp, mi estimate, nestreg, rolling, statsby, stepwise, and svy are allowed; see [U] 11.1.10 Prefix commands.
vce(bootstrap) and vce(jackknife) are not allowed with the mi estimate prefix; see [MI] mi estimate.
Weights are not allowed with the bootstrap prefix; see $[R]$ bootstrap.
vce() and weights are not allowed with the svy prefix; see [SVY] svy.
fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Binary outcomes $>$ Logistic regression (reporting odds ratios)

## Description

logistic fits a logistic regression model of depvar on indepvars, where depvar is a $0 / 1$ variable (or, more precisely, a 0/non-0 variable). Without arguments, logistic redisplays the last logistic estimates. logistic displays estimates as odds ratios; to view coefficients, type logit after running logistic. To obtain odds ratios for any covariate pattern relative to another, see [R] lincom.

## Options



SE/Robust
vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

Reporting
level (\#); see [R] estimation options.
coef causes logistic to report the estimated coefficients rather than the odds ratios (exponentiated coefficients). coef may be specified when the model is fit or may be used later to redisplay results. coef affects only how results are displayed and not how they are estimated.
nocnsreport; see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% fimt), pformat (\%fmt), sformat(\%fmt), and nolstretch; see [R] estimation options.

Maximization
maximize_options: difficult, technique (algorithm_spec), iterate (\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance(\#), nonrtolerance, and from(init_specs); see [R] maximize. These options are seldom used.

The following option is available with logistic but is not shown in the dialog box:
coeflegend; see $[R]$ estimation options.

## Remarks and examples

Remarks are presented under the following headings:

logistic and logit<br>Robust estimate of variance<br>Video examples

## logistic and logit

logistic provides an alternative and preferred way to fit maximum-likelihood logit models, the other choice being logit ([R] logit).

First, let's dispose of some confusing terminology. We use the words logit and logistic to mean the same thing: maximum likelihood estimation. To some, one or the other of these words connotes transforming the dependent variable and using weighted least squares to fit the model, but that is not how we use either word here. Thus the logit and logistic commands produce the same results.

The logistic command is generally preferred to the logit command because logistic presents the estimates in terms of odds ratios rather than coefficients. To some people, this may seem disadvantageous, but you can type logit without arguments after logistic to see the underlying coefficients. You should be cautious when interpreting the odds ratio of the constant term. Usually, this odds ratio represents the baseline odds of the model when all predictor variables are set to zero. However, you must verify that a zero value for all predictor variables in the model actually makes sense before continuing with this interpretation.

Nevertheless, $[R]$ logit is still worth reading because logistic shares the same features as logit, including omitting variables due to collinearity or one-way causation.

For an introduction to logistic regression, see Lemeshow and Hosmer (2005), Pagano and Gauvreau (2000, 470-487), or Pampel (2000); for a complete but nonmathematical treatment, see Kleinbaum and Klein (2010); and for a thorough discussion, see Hosmer, Lemeshow, and Sturdivant (2013). See Gould (2000) for a discussion of the interpretation of logistic regression. See Dupont (2009) or Hilbe (2009) for a discussion of logistic regression with examples using Stata. For a discussion using Stata with an emphasis on model specification, see Vittinghoff et al. (2012).

Stata has a variety of commands for performing estimation when the dependent variable is dichotomous or polytomous. See Long and Freese (2014) for a book devoted to fitting these models with Stata. Here is a list of some estimation commands that may be of interest. See help estimation commands for a complete list of all of Stata's estimation commands.

| asclogit | [R] asclogit | Alternative-specific conditional logit (McFadden's choice) model |
| :---: | :---: | :---: |
| asmprobit | [R] asmprobit | Alternative-specific multinomial probit regression |
| asroprobit | $[R]$ asroprobit | Alternative-specific rank-ordered probit regression |
| binreg | [R] binreg | Generalized linear models for the binomial family |
| biprobit | [R] biprobit | Bivariate probit regression |
| blogit | [R] glogit | Logit regression for grouped data |
| bprobit | [R] glogit | Probit regression for grouped data |
| clogit | [R] clogit | Conditional (fixed-effects) logistic regression |
| cloglog | [R] cloglog | Complementary log-log regression |
| exlogistic | [R] exlogistic | Exact logistic regression |
| glm | [R] glm | Generalized linear models |
| glogit | [R] glogit | Weighted least-squares logistic regression for grouped data |
| gprobit | [R] glogit | Weighted least-squares probit regression for grouped data |
| heckoprobit | [R] heckoprobit | Ordered probit model with sample selection |
| heckprobit | $[\mathrm{R}]$ heckprobit | Probit model with sample selection |
| hetprobit | [R] hetprobit | Heteroskedastic probit model |
| ivprobit | [R] ivprobit | Probit model with endogenous regressors |
| logit | [R] logit | Logistic regression, reporting coefficients |
| mecloglog | [ME] mecloglog | Multilevel mixed-effects complementary log-log regression |
| meglm | [ME] meglm | Multilevel mixed-effects generalized linear model |
| melogit | [ME] melogit | Multilevel mixed-effects logistic regression |
| meprobit | [ME] meprobit | Multilevel mixed-effects probit regression |
| mlogit | [R] mlogit | Multinomial (polytomous) logistic regression |
| mprobit | [R] mprobit | Multinomial probit regression |
| nlogit | [R] nlogit | Nested logit regression (RUM-consistent and nonnormalized) |
| ologit | [R] ologit | Ordered logistic regression |
| oprobit | [R] oprobit | Ordered probit regression |
| probit | [R] probit | Probit regression |
| rologit | [R] rologit | Rank-ordered logistic regression |
| scobit | [R] scobit | Skewed logistic regression |
| slogit | [R] slogit | Stereotype logistic regression |
| svy: cmd | [SVY] svy estimation | Survey versions of many of these commands are available; see [SVY] svy estimation |
| xtcloglog | [XT] xtcloglog | Random-effects and population-averaged cloglog models |
| xtgee | [XT] xtgee | GEE population-averaged generalized linear models |
| xtlogit | [XT] xtlogit | Fixed-effects, random-effects, and population-averaged logit models |
| xtologit | [XT] xtologit | Random-effects ordered logistic models |
| xtoprobit | [XT] xtoprobit | Random-effects ordered probit models |
| xtprobit | [XT] xtprobit | Random-effects and population-averaged probit models |

## > Example 1

Consider the following dataset from a study of risk factors associated with low birthweight described in Hosmer, Lemeshow, and Sturdivant (2013, 24).

```
. use http://www.stata-press.com/data/r13/lbw
(Hosmer & Lemeshow data)
. describe
Contains data from http://www.stata-press.com/data/r13/lbw.dta
    obs: 189 Hosmer & Lemeshow data
    vars: 11 15 Jan 2013 05:01
    size: 2,646
```

| variable name | storage <br> type | display <br> format | value <br> label | variable label |
| :--- | :--- | :--- | :--- | :--- |
| id | int | $\% 8.0 \mathrm{~g}$ |  | identification code <br> low |
| age | byte | $\% 8.0 \mathrm{~g}$ |  | birthweight<2500g |
| lwt | byte | $\% 8.0 \mathrm{~g}$ |  | age of mother |
| race | byte | $\% 8.0 \mathrm{~g}$ |  | weight at last menstrual period |
| smoke | byte | $\% 9.0 \mathrm{~g}$ | race | smoke |
| ptl | byte | $\% 8.0 \mathrm{~g}$ |  | smoked during pregnancy |
| ht | byte | $\% 8.0 \mathrm{~g}$ |  | premature labor history (count) |
| ui | byte | $\% 8.0 \mathrm{~g}$ |  | has history of hypertension |
| ftv | byte | $\% 8.0 \mathrm{~g}$ |  | presence, uterine irritability <br> number of visits to physician |
| bwt | int | $\% 8.0 \mathrm{~g}$ |  | during 1st trimester |

Sorted by:
We want to investigate the causes of low birthweight. Here race is a categorical variable indicating whether a person is white (race $=1$ ), black (race $=2$ ), or some other race (race $=3$ ). We want indicator (dummy) variables for race included in the regression, so we will use factor variables.

```
. logistic low age lwt i.race smoke ptl ht ui
```

| Logistic regression | Number of obs | $=$ | 189 |
| :--- | :--- | :--- | :--- |
|  | LR chi2 (8) | $=$ | 33.22 |
| Log likelihood $=-100.724$ | Prob > chi2 | $=$ | 0.0001 |
|  |  | Pseudo R2 | $=$ | 0.1416


| low | Odds Ratio | Std. Err. | $z$ | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | :---: | :---: | ---: |
| age | .9732636 | .0354759 | -0.74 | 0.457 | .9061578 | 1.045339 |
| lwt | .9849634 | .0068217 | -2.19 | 0.029 | .9716834 | .9984249 |
| race |  |  |  |  |  |  |
| black | 3.534767 | 1.860737 | 2.40 | 0.016 | 1.259736 | 9.918406 |
| other | 2.368079 | 1.039949 | 1.96 | 0.050 | 1.001356 | 5.600207 |
|  |  |  |  |  |  |  |
| smoke | 2.517698 | 1.00916 | 2.30 | 0.021 | 1.147676 | 5.523162 |
| ptl | 1.719161 | .5952579 | 1.56 | 0.118 | .8721455 | 3.388787 |
| ht | 6.249602 | 4.322408 | 2.65 | 0.008 | 1.611152 | 24.24199 |
| ui | 2.1351 | .9808153 | 1.65 | 0.099 | .8677528 | 5.2534 |
| _cons | 1.586014 | 1.910496 | 0.38 | 0.702 | .1496092 | 16.8134 |

The odds ratios are for a one-unit change in the variable. If we wanted the odds ratio for age to be in terms of 4-year intervals, we would type
. gen age4 = age/4
. logistic low age4 lwt i.race smoke ptl ht ui (output omitted)

After logistic, we can type logit to see the model in terms of coefficients and standard errors:


If we wanted to see the logistic output again, we would type logistic without arguments.

## > Example 2

We can specify the confidence interval for the odds ratios with the level() option, and we can do this either at estimation time or when replaying the model. For instance, to see our first model in example 1 with narrower, $90 \%$ confidence intervals, we might type

| . logistic, level(90) |  |  |  |
| :--- | :--- | :--- | :--- |
| Logistic regression | Number of obs | $=$ | 189 |
|  | LR chi2 (8) | $=$ | 33.22 |
|  | Prob > chi2 | $=$ | 0.0001 |
| Log likelihood $=-100.724$ | Pseudo R2 | $=$ | 0.1416 |


| low | Odds Ratio | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [90\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | :---: | :---: | :---: |
| age4 | .8972675 | .1308231 | -0.74 | 0.457 | .7059409 | 1.140448 |
| lwt | .9849634 | .0068217 | -2.19 | 0.029 | .9738063 | .9962483 |
| race |  |  |  |  |  |  |
| black | 3.534767 | 1.860737 | 2.40 | 0.016 | 1.487028 | 8.402379 |
| other | 2.368079 | 1.039949 | 1.96 | 0.050 | 1.149971 | 4.876471 |
|  |  |  |  |  |  |  |
| smoke | 2.517698 | 1.00916 | 2.30 | 0.021 | 1.302185 | 4.867819 |
| ptl | 1.719161 | .5952579 | 1.56 | 0.118 | .9726876 | 3.038505 |
| ht | 6.249602 | 4.322408 | 2.65 | 0.008 | 2.003487 | 19.49478 |
| ui | 2.1351 | .9808153 | 1.65 | 0.099 | 1.00291 | 4.545424 |
| _cons | 1.586014 | 1.910496 | 0.38 | 0.702 | .2186791 | 11.50288 |

## Robust estimate of variance

If you specify vce (robust), Stata reports the robust estimate of variance described in [U] $\mathbf{2 0 . 2 1} \mathbf{~ O b}$ taining robust variance estimates. Here is the model previously fit with the robust estimate of variance:


Also you can specify vce(cluster clustvar) and then, within cluster, relax the assumption of independence. To illustrate this, we have made some fictional additions to the low-birthweight data.

Say that these data are not a random sample of mothers but instead are a random sample of mothers from a random sample of hospitals. In fact, that may be true-we do not know the history of these data.

Hospitals specialize, and it would not be too incorrect to say that some hospitals specialize in more difficult cases. We are going to show two extremes. In one, all hospitals are alike, but we are going to estimate under the possibility that they might differ. In the other, hospitals are strikingly different. In both cases, we assume that patients are drawn from 20 hospitals.

In both examples, we will fit the same model, and we will type the same command to fit it. Below are the same data we have been using but with a new variable, hospid, that identifies from which of the 20 hospitals each patient was drawn (and which we have made up):
. use http://www.stata-press.com/data/r13/hospid1, clear
. logistic low age lwt i.race smoke ptl ht ui, vce(cluster hospid)

| Logistic regression | Number of obs | $=$ | 189 |
| :--- | :--- | :--- | :--- |
|  | Wald chi2(8) | $=$ | 49.67 |
| Log pseudolikelihood $=$ | -100.724 | Prob > chi2 | $=$ |
|  | Pseudo R2 | $=$ | 0.0000 |
|  |  |  |  |

(Std. Err. adjusted for 20 clusters in hospid)

| low | Robust |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Odds Ratio | Std. Err . | z | $P>\|z\|$ | [95\% Conf | Interval] |
| age | . 9732636 | . 0397476 | -0.66 | 0.507 | . 898396 | 1.05437 |
| lwt | . 9849634 | . 0057101 | -2.61 | 0.009 | . 9738352 | . 9962187 |
| race |  |  |  |  |  |  |
| black | 3.534767 | 2.013285 | 2.22 | 0.027 | 1.157563 | 10.79386 |
| other | 2.368079 | . 8451325 | 2.42 | 0.016 | 1.176562 | 4.766257 |
| smoke | 2.517698 | . 8284259 | 2.81 | 0.005 | 1.321062 | 4.79826 |
| ptl | 1.719161 | . 6676221 | 1.40 | 0.163 | . 8030814 | 3.680219 |
| ht | 6.249602 | 4.066275 | 2.82 | 0.005 | 1.74591 | 22.37086 |
| ui | 2.1351 | 1.093144 | 1.48 | 0.138 | . 7827337 | 5.824014 |
| _cons | 1.586014 | 1.661913 | 0.44 | 0.660 | . 2034094 | 12.36639 |

The standard errors are similar to the standard errors we have previously obtained, whether we used the robust or conventional estimators. In this example, we invented the hospital IDs randomly.

Here are the results of the estimation with the same data but with a different set of hospital IDs:

```
. use http://www.stata-press.com/data/r13/hospid2
. logistic low age lwt i.race smoke ptl ht ui, vce(cluster hospid)
```

| Logistic regression | Number of obs | $=$ | 189 |
| :--- | :--- | :--- | ---: |
|  | Wald chi2 (8) | $=$ | 7.19 |
| Log pseudolikelihood $=$ | -100.724 | Prob > chi2 | $=$ |

(Std. Err. adjusted for 20 clusters in hospid)

|  |  | Robust |  |  |  |  |
| ---: | ---: | :---: | :---: | :---: | :---: | ---: |
| low | Odds Ratio | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| age | .9732636 | .0293064 | -0.90 | 0.368 | .9174862 | 1.032432 |
| lwt | .9849634 | .0106123 | -1.41 | 0.160 | .9643817 | 1.005984 |
| race |  |  |  |  |  |  |
| black | 3.534767 | 3.120338 | 1.43 | 0.153 | .6265521 | 19.9418 |
| other | 2.368079 | 1.297738 | 1.57 | 0.116 | .8089594 | 6.932114 |
|  |  |  |  |  |  |  |
| smoke | 2.517698 | 1.570287 | 1.48 | 0.139 | .7414969 | 8.548655 |
| ptl | 1.719161 | .6799153 | 1.37 | 0.171 | .7919045 | 3.732161 |
| ht | 6.249602 | 7.165454 | 1.60 | 0.110 | .660558 | 59.12808 |
| ui | 2.1351 | 1.411977 | 1.15 | 0.251 | .5841231 | 7.804266 |
| _cons | 1.586014 | 1.946253 | 0.38 | 0.707 | .1431423 | 17.573 |

Note the strikingly larger standard errors. What happened? In these data, women most likely to have low-birthweight babies are sent to certain hospitals, and the decision on likeliness is based not just on age, smoking history, etc., but on other things that doctors can see but that are not recorded in our data. Thus merely because a woman is at one of the centers identifies her to be more likely to have a low-birthweight baby.

## Video examples

Logistic regression, part 1: Binary predictors
Logistic regression, part 2: Continuous predictors
Logistic regression, part 3: Factor variables

## Stored results

logistic stores the following in e() :
Scalars
e(N)
e(N_cds)
e(N_cdf)
e(k)
e(k_eq)
e(k_eq_model)
e(k_dv)
e(df_m)
e(r2_p)
e(11)
e(ll_0)
e(N_clust)
e(chi2)
e(p)
e(rank)
e(ic)
$e(r c)$
e(converged)
Macros
e(cmd)
e(cmdline)
e(depvar)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(offset)
e(chi2type)
e(vce)
e(vcetype)
e(opt)
e(which)
$e(m l$ _method)
e(user)
e(technique)
e(properties)
e(estat_cmd)
e(predict)
e(marginsnotok)
e(asbalanced)
e(asobserved)
number of observations
number of completely determined successes
number of completely determined failures
number of parameters
number of equations in $e(b)$
number of equations in overall model test
number of dependent variables
model degrees of freedom
pseudo- $R$-squared
log likelihood
log likelihood, constant-only model
number of clusters
$\chi^{2}$
significance of model test
rank of $e(V)$
number of iterations
return code
1 if converged, 0 otherwise
logistic
command as typed
name of dependent variable
weight type
weight expression
title in estimation output
name of cluster variable
linear offset variable
Wald or LR; type of model $\chi^{2}$ test
vcetype specified in vce()
title used to label Std. Err.
type of optimization
max or min; whether optimizer is to perform maximization or minimization
type of ml method
name of likelihood-evaluator program
maximization technique
b V
program used to implement estat
program used to implement predict
predictions disallowed by margins
factor variables fvset as asbalanced
factor variables fvset as asobserved

```
Matrices
    e(b)
    e(Cns)
    e(ilog)
    e(gradient)
    e(mns)
    e(rules)
    e(V)
    e(V_modelbased)
Functions
```

    e(sample) marks estimation sample
    
## Methods and formulas

Define $\mathbf{x}_{j}$ as the (row) vector of independent variables, augmented by 1 , and $\mathbf{b}$ as the corresponding estimated parameter (column) vector. The logistic regression model is fit by logit; see [R] logit for details of estimation.

The odds ratio corresponding to the $i$ th coefficient is $\psi_{i}=\exp \left(b_{i}\right)$. The standard error of the odds ratio is $s_{i}^{\psi}=\psi_{i} s_{i}$, where $s_{i}$ is the standard error of $b_{i}$ estimated by logit.

Define $I_{j}=\mathbf{x}_{j} \mathbf{b}$ as the predicted index of the $j$ th observation. The predicted probability of a positive outcome is

$$
p_{j}=\frac{\exp \left(I_{j}\right)}{1+\exp \left(I_{j}\right)}
$$

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce (robust) and vce(cluster clustvar), respectively. See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas.
logistic also supports estimation with survey data. For details on VCEs with survey data, see [SVY] variance estimation.

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## Also see

[R] logistic postestimation - Postestimation tools for logistic
[R] brier - Brier score decomposition
[R] cloglog - Complementary log-log regression
[R] exlogistic - Exact logistic regression
[R] logit - Logistic regression, reporting coefficients
$[R]$ roc - Receiver operating characteristic (ROC) analysis
[MI] estimation - Estimation commands for use with mi estimate
[SVY] svy estimation - Estimation commands for survey data
[XT] xtlogit - Fixed-effects, random-effects, and population-averaged logit models
[U] 20 Estimation and postestimation commands

## Title

logistic postestimation - Postestimation tools for logistic

| Description | Syntax for predict | Menu for predict | Options for predict |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Methods and formulas | References | Also see |

## Description

The following postestimation commands are of special interest after logistic:
Command Description
estat classification report various summary statistics, including the classification table
estat gof Pearson or Hosmer-Lemeshow goodness-of-fit test
lroc compute area under ROC curve and graph the curve lsens graph sensitivity and specificity versus probability cutoff

These commands are not appropriate after the svy prefix.

The following standard postestimation commands are also available:

| Command | Description |
| :--- | :--- |
| contrast | contrasts and ANOVA-style joint tests of estimates |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample |
| estat vce | variance-covariance matrix of the estimators (VCE) |
| estat (svy) | postestimation statistics for survey data |
| estimates | cataloging estimation results |
| forecast | dynamic forecasts and simulations <br> point estimates, standard errors, testing, and inference for linear combinations <br> of coefficients |
| lincom | link test for model specification |
| linktest | likelihood-ratio test |
| marginal means, predictive margins, marginal effects, and average marginal |  |
| margins | effects |

[^38]
## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic nooffset rules asif]
```

statistic Description

Main
$\overline{\mathrm{xb}} \quad$ linear prediction
stdp

* dbeta
* deviance
* dx2
* ddeviance
* hat
* 
* residuals
*rstandard score
pr probability of a positive outcome; the default
Description
standard error of the prediction
Pregibon (1981) $\Delta \widehat{\beta}$ influence statistic
deviance residual
Hosmer, Lemeshow, and Sturdivant (2013) $\Delta \chi^{2}$ influence statistic
Hosmer, Lemeshow, and Sturdivant (2013) $\Delta D$ influence statistic
Pregibon (1981) leverage
sequential number of the covariate pattern
Pearson residuals; adjusted for number sharing covariate pattern
standardized Pearson residuals; adjusted for number sharing covariate pattern first derivative of the log likelihood with respect to $\mathbf{x}_{j} \boldsymbol{\beta}$

Unstarred statistics are available both in and out of sample; type predict . . . if e (sample) ... if wanted only for the estimation sample. Starred statistics are calculated only for the estimation sample, even when if e(sample) is not specified.
$\mathrm{pr}, \mathrm{xb}$, stdp, and score are the only options allowed with svy estimation results.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
pr , the default, calculates the probability of a positive outcome.
xb calculates the linear prediction.
stdp calculates the standard error of the linear prediction.
dbeta calculates the Pregibon (1981) $\Delta \widehat{\beta}$ influence statistic, a standardized measure of the difference in the coefficient vector that is due to deletion of the observation along with all others that share the same covariate pattern. In Hosmer, Lemeshow, and Sturdivant (2013, 154-155) jargon, this statistic is $M$-asymptotic; that is, it is adjusted for the number of observations that share the same covariate pattern.
deviance calculates the deviance residual.
dx2 calculates the Hosmer, Lemeshow, and Sturdivant $(2013,191) \Delta \chi^{2}$ influence statistic, reflecting the decrease in the Pearson $\chi^{2}$ that is due to the deletion of the observation and all others that share the same covariate pattern.
ddeviance calculates the Hosmer, Lemeshow, and Sturdivant (2013, 191) $\Delta D$ influence statistic, which is the change in the deviance residual that is due to deletion of the observation and all others that share the same covariate pattern.
hat calculates the Pregibon (1981) leverage or the diagonal elements of the hat matrix adjusted for the number of observations that share the same covariate pattern.
number numbers the covariate patterns-observations with the same covariate pattern have the same number. Observations not used in estimation have number set to missing. The first covariate pattern is numbered 1 , the second 2 , and so on.
residuals calculates the Pearson residual as given by Hosmer, Lemeshow, and Sturdivant (2013, 155) and adjusted for the number of observations that share the same covariate pattern.
rstandard calculates the standardized Pearson residual as given by Hosmer, Lemeshow, and Sturdivant $(2013,191)$ and adjusted for the number of observations that share the same covariate pattern.
score calculates the equation-level score, $\partial \ln L / \partial\left(\mathbf{x}_{j} \boldsymbol{\beta}\right)$.

Options
nooffset is relevant only if you specified offset (varname) for logistic. It modifies the calculations made by predict so that they ignore the offset variable; the linear prediction is treated as $\mathbf{x}_{j} \mathbf{b}$ rather than as $\mathbf{x}_{j} \mathbf{b}+$ offset $_{j}$.
rules requests that $S t a t a$ use any rules that were used to identify the model when making the prediction. By default, Stata calculates missing for excluded observations. See example 1 in $[R]$ logit postestimation.
asif requests that Stata ignore the rules and the exclusion criteria and calculate predictions for all observations possible by using the estimated parameter from the model. See example 1 in $[R]$ logit postestimation.

## Remarks and examples

predict is used after logistic to obtain predicted probabilities, residuals, and influence statistics for the estimation sample. The suggested diagnostic graphs below are from Hosmer, Lemeshow, and Sturdivant (2013), where they are more elaborately explained. Also see Collett (2003, 129-168) for a thorough discussion of model checking.

Remarks are presented under the following headings:

> predict without options
> predict with the xb and stdp options
> predict with the residuals option
> predict with the number option predict with the deviance option predict with the rstandard option predict with the hat option predict with the dx2 option predict with the ddeviance option predict with the dbeta option

## predict without options

Typing predict newvar after estimation calculates the predicted probability of a positive outcome.
In example 1 of [R] logistic, we ran the model logistic low age lwt i.race smoke ptl ht ui. We obtain the predicted probabilities of a positive outcome by typing
(Hosmer \& Lemeshow data)
. logistic low age lwt i.race smoke ptl ht ui
(output omitted)

- predict p
(option pr assumed; $\operatorname{Pr}($ low ))
- summarize p low

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| p | 189 | .3121693 | .1913915 | .0272559 | .8391283 |
| low | 189 | .3121693 | .4646093 | 0 | 1 |

## predict with the $\mathbf{x b}$ and stdp options

predict with the xb option calculates the linear combination $x_{j} \mathbf{b}$, where $x_{j}$ are the independent variables in the $j$ th observation and $\mathbf{b}$ is the estimated parameter vector. This is sometimes known as the index function because the cumulative distribution function indexed at this value is the probability of a positive outcome.

With the stdp option, predict calculates the standard error of the prediction, which is not adjusted for replicated covariate patterns in the data. The influence statistics described below are adjusted for replicated covariate patterns in the data.

## predict with the residuals option

predict can calculate more than predicted probabilities. The Pearson residual is defined as the square root of the contribution of the covariate pattern to the Pearson $\chi^{2}$ goodness-of-fit statistic, signed according to whether the observed number of positive responses within the covariate pattern is less than or greater than expected. For instance,

```
. predict r, residuals
. summarize r, detail
```

Pearson residual

|  | Percentiles | Smallest |  |  |
| ---: | :---: | ---: | :--- | ---: |
| $1 \%$ | -1.750923 | -2.283885 |  |  |
| $5 \%$ | -1.129907 | -1.750923 |  | 189 |
| $10 \%$ | -.9581174 | -1.636279 | Obs | 189 |
| $25 \%$ | -.6545911 | -1.636279 | Sum of Wgt. | -.0242299 |
| $50 \%$ | -.3806923 |  | Mean | .9970949 |
|  |  | Largest | Std. Dev. |  |
| $75 \%$ | .8162894 | 2.23879 |  | .9941981 |
| $90 \%$ | 1.510355 | 2.317558 | Variance | .8618271 |
| $95 \%$ | 1.747948 | 3.002206 | Skewness | 3.038448 |

We notice the prevalence of a few large positive residuals:

```
. sort r
. list id r low p age race in -5/l
```

185. 
186. 
187. 
188. 
189. 

| id | r | low | p | age | race |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 33 | 2.224501 | 1 | .1681123 | 19 | white |
| 57 | 2.23879 | 1 | .166329 | 15 | white |
| 16 | 2.317558 | 1 | .1569594 | 27 | other |
| 77 | 3.002206 | 1 | .0998678 | 26 | white |
| 36 | 3.126763 | 1 | .0927932 | 24 | white |

## predict with the number option

Covariate patterns play an important role in logistic regression. Two observations are said to share the same covariate pattern if the independent variables for the two observations are identical. Although we might think of having individual observations, the statistical information in the sample can be summarized by the covariate patterns, the number of observations with that covariate pattern, and the number of positive outcomes within the pattern. Depending on the model, the number of covariate patterns can approach or be equal to the number of observations, or it can be considerably less.

Stata calculates all the residual and diagnostic statistics in terms of covariate patterns, not observations. That is, all observations with the same covariate pattern are given the same residual and diagnostic statistics. Hosmer, Lemeshow, and Sturdivant (2013, 154-155) argue that such " $M$ asymptotic" statistics are more useful than " $N$-asymptotic" statistics.

To understand the difference, think of an observed positive outcome with predicted probability of 0.8 . Taking the observation in isolation, the residual must be positive-we expected 0.8 positive responses and observed 1 . This may indeed be the correct residual, but not necessarily. Under the $M$-asymptotic definition, we ask how many successes we observed across all observations with this covariate pattern. If that number were, say, six, and there were a total of 10 observations with this covariate pattern, then the residual is negative for the covariate pattern - we expected eight positive outcomes but observed six. predict makes this kind of calculation and then attaches the same residual to all observations in the covariate pattern.

Occasionally, you might want to find all observations sharing a covariate pattern. number allows you to do this:

```
. predict pattern, number
```

. summarize pattern

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| pattern | 189 | 89.2328 | 53.16573 | 1 | 182 |

We previously fit the model logistic low age lwt i.race smoke ptl ht ui over 189 observations. There are 182 covariate patterns in our data.

## predict with the deviance option

The deviance residual is defined as the square root of the contribution to the likelihood-ratio test statistic of a saturated model versus the fitted model. It has slightly different properties from the Pearson residual (see Hosmer, Lemeshow, and Sturdivant [2013, 155-157]):

```
. predict d, deviance
summarize d, detail
```

    deviance residual
    |  | Percentiles | Smallest |  |  |
| ---: | :---: | ---: | :--- | ---: |
| $1 \%$ | -1.843472 | -1.911621 |  |  |
| $5 \%$ | -1.33477 | -1.843472 |  | 189 |
| $10 \%$ | -1.148316 | -1.843472 | Obs | 189 |
| $25 \%$ | -.8445325 | -1.674869 | Sum of Wgt. | -.1228811 |
| $50 \%$ | -.5202702 |  | Mean | 1.049237 |
|  |  | Largest | Std. Dev. |  |
| $75 \%$ | .9129041 | 1.894089 |  | 1.100898 |
| $90 \%$ | 1.541558 | 1.924457 | Variance | .6598857 |
| $95 \%$ | 1.673338 | 2.146583 | Skewness | 2.036938 |

## predict with the rstandard option

Pearson residuals do not have a standard deviation equal to 1. rstandard generates Pearson residuals normalized to have an expected standard deviation equal to 1 .

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| :---: | :---: | :---: | :---: | :---: | :---: |
| r | 189 | -. 0242299 | . 9970949 | -2.283885 | 3.126763 |
| rs | 189 | -. 0279135 | 1.026406 | -2.4478 | 3.149081 |
| $\begin{aligned} & \text { correlate r rs } \\ & \text { (obs=189) } \end{aligned}$ |  |  |  |  |  |
| r | 1.0000 |  |  |  |  |
| rs | 0.9998 | 1.0000 |  |  |  |

Remember that we previously created $r$ containing the (unstandardized) Pearson residuals. In these data, whether we use standardized or unstandardized residuals does not matter much.

## predict with the hat option

hat calculates the leverage of a covariate pattern - a scaled measure of distance in terms of the independent variables. Large values indicate covariate patterns far from the average covariate pattern that can have a large effect on the fitted model even if the corresponding residual is small. Consider the following graph:

- predict h, hat
. scatter h r, xline(0)


The points to the left of the vertical line are observed negative outcomes; here our data contain almost as many covariate patterns as observations, so most covariate patterns are unique. In such unique patterns, we observe either 0 or 1 success and expect $p$, thus forcing the sign of the residual. If we had fewer covariate patterns-if we did not have continuous variables in our model-there would be no such interpretation, and we would not have drawn the vertical line at 0 .

Points on the left and right edges of the graph represent large residuals - covariate patterns that are not fit well by our model. Points at the top of our graph represent high leverage patterns. When analyzing the influence of observations on the model, we are most interested in patterns with high leverage and small residuals-patterns that might otherwise escape our attention.

## predict with the dx2 option

There are many ways to measure influence, and hat is one example. dx 2 measures the decrease in the Pearson $\chi^{2}$ goodness-of-fit statistic that would be caused by deleting an observation (and all others sharing the covariate pattern):

```
. predict dx2, dx2
. scatter dx2 p
```



Paraphrasing Hosmer, Lemeshow, and Sturdivant (2013, 195-197), the points going from the top left to the bottom right correspond to covariate patterns with the number of positive outcomes equal to the number in the group; the points on the other curve correspond to 0 positive outcomes. In our data, most of the covariate patterns are unique, so the points tend to lie along one or the other curves; the points that are off the curves correspond to the few repeated covariate patterns in our data in which all the outcomes are not the same.

We examine this graph for large values of dx 2 - there are two at the top left.

## predict with the ddeviance option

Another measure of influence is the change in the deviance residuals due to deletion of a covariate pattern:

```
. predict dd, ddeviance
```

As with dx2, we typically graph ddeviance against the probability of a positive outcome. We direct you to Hosmer, Lemeshow, and Sturdivant $(2013,195)$ for an example and for the interpretation of this graph.

## predict with the dbeta option

One of the more direct measures of influence of interest to model fitters is the Pregibon (1981) dbeta measure, a measure of the change in the coefficient vector that would be caused by deleting an observation (and all others sharing the covariate pattern):
. predict db, dbeta
. scatter db p


One observation has a large effect on the estimated coefficients. We can easily find this point:
. sort db
. list in 1
189.

| $\begin{array}{r} \text { id } \\ 188 \end{array}$ | $\begin{array}{r} \text { low } \\ 0 \end{array}$ | $\begin{array}{r} \text { age } \\ 25 \end{array}$ | $\begin{array}{r} \text { lwt } \\ 95 \end{array}$ | race white | smoke smoker | $\begin{array}{r} \mathrm{ptl} \\ 3 \end{array}$ | ht 0 | ui 1 | ftv 0 | $\begin{array}{r} \text { bwt } \\ 3637 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| . 839 | p | -2.28 |  | pattern $117$ | -1.91 |  | -2. |  |  | $\begin{array}{r} h \\ 4439 \end{array}$ |
| $\begin{array}{r} \mathrm{dx} 2 \\ 5.991726 \end{array}$ |  |  |  | $\begin{array}{r} \mathrm{dd} \\ 4.197658 \end{array}$ |  |  | $\begin{array}{r} \mathrm{db} \\ .8909163 \end{array}$ |  |  |  |

Hosmer, Lemeshow, and Sturdivant $(2013,196)$ suggest a graph that combines two of the influence measures:

```
. scatter dx2 p [w=db], title("Symbol size proportional to dBeta") mfcolor(none)
(analytic weights assumed)
(analytic weights assumed)
```



We can easily spot the most influential points by the dbeta and dx 2 measures.

## Methods and formulas

Let $j$ index observations. Define $M_{j}$ for each observation as the total number of observations sharing $j$ 's covariate pattern. Define $Y_{j}$ as the total number of positive responses among observations sharing $j$ 's covariate pattern.

The Pearson residual for the $j$ th observation is defined as

$$
r_{j}=\frac{Y_{j}-M_{j} p_{j}}{\sqrt{M_{j} p_{j}\left(1-p_{j}\right)}}
$$

For $M_{j}>1$, the deviance residual $d_{j}$ is defined as

$$
d_{j}= \pm\left(2\left[Y_{j} \ln \left(\frac{Y_{j}}{M_{j} p_{j}}\right)+\left(M_{j}-Y_{j}\right) \ln \left\{\frac{M_{j}-Y_{j}}{M_{j}\left(1-p_{j}\right)}\right\}\right]\right)^{1 / 2}
$$

where the sign is the same as the sign of $\left(Y_{j}-M_{j} p_{j}\right)$. In the limiting cases, the deviance residual is given by

$$
d_{j}= \begin{cases}-\sqrt{2 M_{j}\left|\ln \left(1-p_{j}\right)\right|} & \text { if } Y_{j}=0 \\ \sqrt{2 M_{j}\left|\ln p_{j}\right|} & \text { if } Y_{j}=M_{j}\end{cases}
$$

The unadjusted diagonal elements of the hat matrix $h_{\mathrm{U} j}$ are given by $h_{\mathrm{U} j}=\left(\mathbf{X V} \mathbf{X}^{\prime}\right)_{j j}$, where $V$ is the estimated covariance matrix of parameters. The adjusted diagonal elements $h_{j}$ created by hat are then $h_{j}=M_{j} p_{j}\left(1-p_{j}\right) h_{\mathrm{U} j}$.

The standardized Pearson residual $r_{\mathrm{S} j}$ is $r_{j} / \sqrt{1-h_{j}}$.
The Pregibon (1981) $\Delta \widehat{\beta}_{j}$ influence statistic is

$$
\Delta \widehat{\beta}_{j}=\frac{r_{j}^{2} h_{j}}{\left(1-h_{j}\right)^{2}}
$$

The corresponding change in the Pearson $\chi^{2}$ is $r_{\mathrm{S} j}^{2}$. The corresponding change in the deviance residual is $\Delta D_{j}=d_{j}^{2} /\left(1-h_{j}\right)$.

## References

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## Also see

[R] logistic - Logistic regression, reporting odds ratios
[R] estat classification - Classification statistics and table
[R] estat gof - Pearson or Hosmer-Lemeshow goodness-of-fit test
$[\mathrm{R}]$ Iroc - Compute area under ROC curve and graph the curve
[R] Isens - Graph sensitivity and specificity versus probability cutoff
[U] 20 Estimation and postestimation commands

## Title

logit — Logistic regression, reporting coefficients

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

logit depvar [indepvars] [if] [in] [weight] [, options]
options
Model
noconstant
offset (varname)
asis
constraints (constraints)
collinear
SE/Robust
vce (vcetype)

Reporting
level(\#)
or
nocnsreport
display_options

Maximization
maximize_options
nocoef
coeflegend

## Description

suppress constant term
include varname in model with coefficient constrained to 1
retain perfect predictor variables
apply specified linear constraints
keep collinear variables

```
vcetype may be oim, robust, cluster clustvar, bootstrap, or jackknife
```

set confidence level; default is level (95)
report odds ratios
do not display constraints
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
depvar and indepvars may contain time-series operators; see [U] 11.4.4 Time-series varlists.
bootstrap, by, fp, jackknife, mfp, mi estimate, nestreg, rolling, statsby, stepwise, and svy are allowed; see [U] 11.1.10 Prefix commands.
vce(bootstrap) and vce(jackknife) are not allowed with the mi estimate prefix; see [MI] mi estimate.
Weights are not allowed with the bootstrap prefix; see $[R]$ bootstrap.
vce(), nocoef, and weights are not allowed with the svy prefix; see [SVY] svy.
fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
nocoef and coeflegend do not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Binary outcomes $>$ Logistic regression

## Description

logit fits a logit model for a binary response by maximum likelihood; it models the probability of a positive outcome given a set of regressors. depvar equal to nonzero and nonmissing (typically depvar equal to one) indicates a positive outcome, whereas depvar equal to zero indicates a negative outcome.

Also see [R] logistic; logistic displays estimates as odds ratios. Many users prefer the logistic command to logit. Results are the same regardless of which you use-both are the maximumlikelihood estimator. Several auxiliary commands that can be run after logit, probit, or logistic estimation are described in [R] logistic postestimation. A list of related estimation commands is given in $[R]$ logistic.

If estimating on grouped data, see $[R]$ glogit.

## Options

```
Model
noconstant, offset(varname), constraints(constraints), collinear; see [R] estimation op- tions.
asis forces retention of perfect predictor variables and their associated perfectly predicted observations and may produce instabilities in maximization; see [R] probit.
```


## SE/Robust

vce (vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

Reporting
level (\#); see [R] estimation options.
or reports the estimated coefficients transformed to odds ratios, that is, $e^{b}$ rather than $b$. Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated. or may be specified at estimation or when replaying previously estimated results.
nocnsreport; see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\%fmt), pformat (\%fmt), sformat (\%fmt), and nolstretch; see [R] estimation options.

[^39]The following options are available with logit but are not shown in the dialog box: nocoef specifies that the coefficient table not be displayed. This option is sometimes used by program writers but is of no use interactively.
coeflegend; see $[R]$ estimation options.

## Remarks and examples

Remarks are presented under the following headings:
Basic usage
Model identification

## Basic usage

logit fits maximum likelihood models with dichotomous dependent (left-hand-side) variables coded as $0 / 1$ (or, more precisely, coded as 0 and not- 0 ).

## $>$ Example 1

We have data on the make, weight, and mileage rating of 22 foreign and 52 domestic automobiles. We wish to fit a logit model explaining whether a car is foreign on the basis of its weight and mileage. Here is an overview of our data:


The variable foreign takes on two unique values, 0 and 1 . The value 0 denotes a domestic car, and 1 denotes a foreign car.

The model that we wish to fit is

$$
\operatorname{Pr}(\text { foreign }=1)=F\left(\beta_{0}+\beta_{1} \text { weight }+\beta_{2} \mathrm{mpg}\right)
$$

where $F(z)=e^{z} /\left(1+e^{z}\right)$ is the cumulative logistic distribution.
To fit this model, we type

```
. logit foreign weight mpg
Iteration 0: log likelihood = -45.03321
Iteration 1: log likelihood = -29.238536
Iteration 2: log likelihood = -27.244139
Iteration 3: log likelihood = -27.175277
Iteration 4: log likelihood = -27.175156
Iteration 5: log likelihood = -27.175156
```

Logistic regression

| Number of obs | $=$ | 74 |
| :--- | :--- | ---: |
| LR chi2 (2) | $=$ | 35.72 |
| Prob > chi2 | $=$ | 0.0000 |
| Pseudo R2 | $=$ | 0.3966 |


| foreign | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | ---: | ---: | ---: |
| weight | -.0039067 | .0010116 | -3.86 | 0.000 | -.0058894 | -.001924 |
| mpg | -.1685869 | .0919175 | -1.83 | 0.067 | -.3487418 | .011568 |
| _cons | 13.70837 | 4.518709 | 3.03 | 0.002 | 4.851859 | 22.56487 |

We find that heavier cars are less likely to be foreign and that cars yielding better gas mileage are also less likely to be foreign, at least holding the weight of the car constant.

## - Technical note

Stata interprets a value of 0 as a negative outcome (failure) and treats all other values (except missing) as positive outcomes (successes). Thus if your dependent variable takes on the values 0 and 1 , then 0 is interpreted as failure and 1 as success. If your dependent variable takes on the values 0 , 1 , and 2 , then 0 is still interpreted as failure, but both 1 and 2 are treated as successes.

If you prefer a more formal mathematical statement, when you type logit $y x$, Stata fits the model

$$
\operatorname{Pr}\left(y_{j} \neq 0 \mid \mathbf{x}_{j}\right)=\frac{\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}\right)}{1+\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}\right)}
$$

## Model identification

The logit command has one more feature, and it is probably the most useful. logit automatically checks the model for identification and, if it is underidentified, drops whatever variables and observations are necessary for estimation to proceed. (logistic, probit, and ivprobit do this as well.)

## > Example 2

Have you ever fit a logit model where one or more of your independent variables perfectly predicted one or the other outcome?

For instance, consider the following data:

| Outcome $y$ | Independent variable $x$ |
| :---: | :---: |
| 0 | 1 |
| 0 | 1 |
| 0 | 0 |
| 1 | 0 |

Say that we wish to predict the outcome on the basis of the independent variable. The outcome is always zero whenever the independent variable is one. In our data, $\operatorname{Pr}(y=0 \mid x=1)=1$, which means that the logit coefficient on $x$ must be minus infinity with a corresponding infinite standard error. At this point, you may suspect that we have a problem.

Unfortunately, not all such problems are so easily detected, especially if you have a lot of independent variables in your model. If you have ever had such difficulties, you have experienced one of the more unpleasant aspects of computer optimization. The computer has no idea that it is trying to solve for an infinite coefficient as it begins its iterative process. All it knows is that at each step, making the coefficient a little bigger, or a little smaller, works wonders. It continues on its merry way until either 1) the whole thing comes crashing to the ground when a numerical overflow error occurs or 2 ) it reaches some predetermined cutoff that stops the process. In the meantime, you have been waiting. The estimates that you finally receive, if you receive any at all, may be nothing more than numerical roundoff.

Stata watches for these sorts of problems, alerts us, fixes them, and properly fits the model.
Let's return to our automobile data. Among the variables we have in the data is one called repair, which takes on three values. A value of 1 indicates that the car has a poor repair record, 2 indicates an average record, and 3 indicates a better-than-average record. Here is a tabulation of our data:

```
. use http://www.stata-press.com/data/r13/repair, clear
(1978 Automobile Data)
. tabulate foreign repair
\begin{tabular}{r|rrr|r} 
Car type & 1 & 2 & 3 & Topair \\
\hline Domestic & 10 & 27 & 9 & 46 \\
Foreign & 0 & 3 & 9 & 12 \\
\hline Total & 10 & 30 & 18 & 58
\end{tabular}
```

All the cars with poor repair records (repair $=1$ ) are domestic. If we were to attempt to predict foreign on the basis of the repair records, the predicted probability for the repair $=1$ category would have to be zero. This in turn means that the logit coefficient must be minus infinity, and that would set most computer programs buzzing.

Let's try Stata on this problem.

```
. logit foreign b3.repair
note: 1.repair != 0 predicts failure perfectly
    1.repair dropped and 10 obs not used
Iteration 0: log likelihood = -26.992087
Iteration 1: log likelihood = -22.483187
Iteration 2: log likelihood = -22.230498
Iteration 3: log likelihood = -22.229139
Iteration 4: log likelihood = -22.229138
Logistic regression
```

Log likelihood $=-22.229138$

| Number of obs | $=$ | 48 |
| :--- | :--- | ---: |
| LR chi2 (1) | $=$ | 9.53 |
| Prob > chi2 | $=$ | 0.0020 |
| Pseudo R2 | $=$ | 0.1765 |


| foreign | Coef. | Std. Err. | $z$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | :---: |
| repair | 0 | (empty) |  |  |  |  |
| 1 |  |  |  |  |  |  |
| 2 | -2.197225 | .7698003 | -2.85 | 0.004 | -3.706005 | -.6884436 |
| _cons | $-1.98 \mathrm{e}-16$ | .4714045 | -0.00 | 1.000 | -.9239359 | .9239359 |

Remember that all the cars with poor repair records (repair $=1$ ) are domestic, so the model cannot be fit, or at least it cannot be fit if we restrict ourselves to finite coefficients. Stata noted that fact "note: 1.repair !=0 predicts failure perfectly". This is Stata's mathematically precise way of saying what we said in English. When repair is 1, the car is domestic.

Stata then went on to say "1.repair dropped and 10 obs not used". This is Stata eliminating the problem. First 1.repair had to be removed from the model because it would have an infinite coefficient. Then the 10 observations that led to the problem had to be eliminated, as well, so as not to bias the remaining coefficients in the model. The 10 observations that are not used are the 10 domestic cars that have poor repair records.

Stata then fit what was left of the model, using the remaining observations. Because no observations remained for cars with poor repair records, Stata reports "(empty)" in the row for repair $=1$.

## - Technical note

Stata is pretty smart about catching problems like this. It will catch "one-way causation by a dummy variable", as we demonstrated above.

Stata also watches for "two-way causation", that is, a variable that perfectly determines the outcome, both successes and failures. Here Stata says, "so-and-so predicts outcome perfectly" and stops. Statistics dictates that no model can be fit.

Stata also checks your data for collinear variables; it will say, "so-and-so omitted because of collinearity". No observations need to be eliminated in this case, and model fitting will proceed without the offending variable.

It will also catch a subtle problem that can arise with continuous data. For instance, if we were estimating the chances of surviving the first year after an operation, and if we included in our model age, and if all the persons over 65 died within the year, Stata would say, "age $>65$ predicts failure perfectly". It would then inform us about the fix-up it takes and fit what can be fit of our model.
logit (and logistic, probit, and ivprobit) will also occasionally display messages such as Note: 4 failures and 0 successes completely determined.

There are two causes for a message like this. The first-and most unlikely-case occurs when a continuous variable (or a combination of a continuous variable with other continuous or dummy variables) is simply a great predictor of the dependent variable. Consider Stata's auto.dta dataset with 6 observations removed.

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. drop if foreign==0 & gear_ratio > 3.1
(6 observations deleted)
. logit foreign mpg weight gear_ratio, nolog
```

Logistic regression Number of obs $=\quad 68$
LR chi2(3) $=72.64$
Prob > chi2 $=0.0000$
Log likelihood = -6.4874814
Pseudo R2 $=0.8484$

| foreign | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| mpg | -.4944907 | .2655508 | -1.86 | 0.063 | -1.014961 | .0259792 |
| weight | -.0060919 | .003101 | -1.96 | 0.049 | -.0121698 | -.000014 |
| gear_ratio | 15.70509 | 8.166234 | 1.92 | 0.054 | -.300436 | 31.71061 |
| _cons | -21.39527 | 25.41486 | -0.84 | 0.400 | -71.20747 | 28.41694 |

Note: 4 failures and 0 successes completely determined.
There are no missing standard errors in the output. If you receive the "completely determined" message and have one or more missing standard errors in your output, see the second case discussed below.

Note gear_ratio's large coefficient. logit thought that the 4 observations with the smallest predicted probabilities were essentially predicted perfectly.

```
. predict p
(option pr assumed; Pr(foreign))
. sort p
. list p in 1/4
```

| $p$ |
| ---: |
| $1.34 \mathrm{e}-10$ |
| $6.26 \mathrm{e}-09$ |
| $7.84 \mathrm{e}-09$ |
| $1.49 \mathrm{e}-08$ |

If this happens to you, you do not have to do anything. Computationally, the model is sound. The second case discussed below requires careful examination.

The second case occurs when the independent terms are all dummy variables or continuous ones with repeated values (for example, age). Here one or more of the estimated coefficients will have missing standard errors. For example, consider this dataset consisting of 5 observations.

```
. use http://www.stata-press.com/data/r13/logitxmpl, clear
```

. list, separator(0)

| $y$ | $x 1$ | $x 2$ |
| :---: | :---: | :---: |
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 0 |
| 0 | 0 | 1 |
| 1 | 0 | 1 |

```
. logit y x1 x2
Iteration 0: log likelihood = -3.819085
Iteration 1: log likelihood = -2.9527336
Iteration 2: log likelihood = -2.8110282
Iteration 3: log likelihood = -2.7811973
Iteration 4: log likelihood = -2.7746107
Iteration 5: log likelihood = -2.7730128
    (output omitted)
Iteration 15996: log likelihood = -2.7725887 (not concave)
Iteration 15997: log likelihood = -2.7725887 (not concave)
Iteration 15998: log likelihood = -2.7725887 (not concave)
Iteration 15999: log likelihood = -2.7725887 (not concave)
Iteration 16000: log likelihood = -2.7725887 (not concave)
convergence not achieved
```

| Logistic regression | Number of obs | $=$ | 6 |
| :--- | :--- | :--- | :--- |
|  | LR chi2 (1) | $=$ | 2.09 |
| Log likelihood $=-2.7725887$ | Prob > chi2 | $=$ | 0.1480 |
|  | Pseudo R2 | $=$ | 0.2740 |


| y | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| x 1 | 18.3704 | 2 | 9.19 | 0.000 | 14.45047 | 22.29033 |  |
| x 2 | 18.3704 | . | . | . | . | . | . |
| _cons | -18.3704 | 1.414214 | -12.99 | 0.000 | -21.14221 | -15.5986 |  |

Note: 2 failures and 0 successes completely determined.
convergence not achieved
r(430);
Three things are happening here. First, logit iterates almost forever and then declares nonconvergence. Second, logit can fit the outcome $(y=0)$ for the covariate pattern $\mathrm{x} 1=0$ and $\mathrm{x} 2=0$ (that is, the first two observations) perfectly. This observation is the " 2 failures and 0 successes completely determined". Third, if this observation is dropped, then x 1 , x 2 , and the constant are collinear.

This is the cause of the nonconvergence, the message "completely determined", and the missing standard errors. It happens when you have a covariate pattern (or patterns) with only one outcome and there is collinearity when the observations corresponding to this covariate pattern are dropped.

If this happens to you, confirm the causes. First, identify the covariate pattern with only one outcome. (For your data, replace x 1 and x 2 with the independent variables of your model.)

```
. egen pattern = group(x1 x2)
. quietly logit y x1 x2, iterate(100)
. predict p
(option pr assumed; Pr(y))
. summarize p
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline p & 6 & .3333333 & .2581989 & \(1.05 \mathrm{e}-08\) & .5
\end{tabular}
```

If successes were completely determined, that means that there are predicted probabilities that are almost 1 . If failures were completely determined, that means that there are predicted probabilities that are almost 0 . The latter is the case here, so we locate the corresponding value of pattern:
. tabulate pattern if $p<1 e-7$

| group(x1 <br> x2) | Freq. | Percent | Cum. |
| ---: | ---: | ---: | ---: |
| 1 | 2 | 100.00 | 100.00 |
| Total | 2 | 100.00 |  |

Once we omit this covariate pattern from the estimation sample, logit can deal with the collinearity:

| Logistic regression |  |  |  | Number of obs LR chi2(1) |  |  | $=\begin{array}{ll}= & 4\end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | 0.001.0000 |  |
|  |  |  |  | Prob > chi2 |  |  |  |  |
| Log likelihood $=-2.7725887$ |  |  |  | Pseudo R2 |  |  | 0.0000 |
| y | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% | Con | Interval] |
| x 1 | 0 | 2 | 0.00 | 1.000 | -3.919 |  | 3.919928 |
| x2 | 0 | (omitted) |  |  |  |  |  |
| _cons | 0 | 1.414214 | 0.00 | 1.000 | -2.771 |  | 2.771808 |

We omit the collinear variable. Then we must decide whether to include or omit the observations with pattern $=1$. We could include them,

| . logit y x1, nolog |  |  |  |
| :--- | :--- | :--- | :--- |
| Logistic regression | Number of obs | $=$ | 6 |
|  | LR chi2 (1) | $=$ | 0.37 |
|  | Prob > chi2 | $=$ | 0.5447 |
| Log likelihood $=-3.6356349$ | Pseudo R2 | $=$ | 0.0480 |


| y | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | :---: | ---: | ---: |
| x 1 | 1.098612 | 1.825742 | 0.60 | 0.547 | -2.479776 | 4.677001 |
| _cons | -1.098612 | 1.154701 | -0.95 | 0.341 | -3.361784 | 1.164559 |

or exclude them,
. logit y x1 if pattern != 1, nolog

| Logistic regression | Number of obs | $=$ | 4 |
| :--- | :--- | :--- | :--- |
|  | LR chi2 (1) | $=$ | 0.00 |
| Log likelihood $=-2.7725887$ | Prob > chi2 | $=$ | 1.0000 |
|  | Pseudo R2 | $=$ | 0.0000 |


| y | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | ---: | ---: | ---: |
| x 1 | 0 | 2 | 0.00 | 1.000 | -3.919928 | 3.919928 |
| _cons | 0 | 1.414214 | 0.00 | 1.000 | -2.771808 | 2.771808 |

If the covariate pattern that predicts outcome perfectly is meaningful, you may want to exclude these observations from the model. Here you would report that covariate pattern such and such predicted outcome perfectly and that the best model for the rest of the data is . ... But, more likely, the perfect prediction was simply the result of having too many predictors in the model. Then you would omit the extraneous variables from further consideration and report the best model for all the data.

## Stored results

logit stores the following in e():
Scalars

| e(N) | number of observations |
| :---: | :---: |
| e(N_cds) | number of completely determined successes |
| $e($ N_cdf) | number of completely determined failures |
| $e(k)$ | number of parameters |
| $e\left(k \_\right.$eq) | number of equations in e(b) |
| e(k_eq_model) | number of equations in overall model test |
| $e\left(k \_d v\right)$ | number of dependent variables |
| $e\left(d f \_m\right)$ | model degrees of freedom |
| e(r2_p) | pseudo- $R$-squared |
| e(11) | log likelihood |
| e(11_0) | log likelihood, constant-only model |
| e(N_clust) | number of clusters |
| e(chi2) | $\chi^{2}$ |
| e (p) | significance of model test |
| $e$ (rank) | rank of e(V) |
| e(ic) | number of iterations |
| $e(r c)$ | return code |
| e(converged) | 1 if converged, 0 otherwise |

Macros
e(cmd)
e(cmdline)
e(depvar)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(offset)
e(chi2type)
e(vce)
e(vcetype)
e(opt)
e(which)
e(ml_method)
e(user)
$e$ (technique)
e(properties)
e(estat_cmd)
e(predict)
e(marginsnotok)
e(asbalanced)
e(asobserved)
Matrices
$e(b)$
e(Cns)
e(ilog)
e(gradient)
$e$ (mns)
e(rules)
e(V)
e(V_modelbased)

Functions
e(sample)
Functions
logit
command as typed
name of dependent variable
weight type
weight expression
title in estimation output
name of cluster variable
linear offset variable
Wald or LR; type of model $\chi^{2}$ test
vcetype specified in vce()
title used to label Std. Err.
type of optimization
max or min; whether optimizer is to perform maximization or minimization type of ml method
name of likelihood-evaluator program
maximization technique
b V
program used to implement estat
program used to implement predict
predictions disallowed by margins
factor variables fvset as asbalanced
factor variables fvset as asobserved
coefficient vector
constraints matrix
iteration $\log$ (up to 20 iterations)
gradient vector
vector of means of the independent variables
information about perfect predictors
variance-covariance matrix of the estimators
model-based variance
marks estimation sample

## Methods and formulas

Cramer (2003, chap. 9) surveys the prehistory and history of the logit model. The word "logit" was coined by Berkson (1944) and is analogous to the word "probit". For an introduction to probit and logit, see, for example, Aldrich and Nelson (1984), Cameron and Trivedi (2010), Greene (2012), Jones (2007), Long (1997), Long and Freese (2014), Pampel (2000), or Powers and Xie (2008).

The likelihood function for logit is

$$
\ln L=\sum_{j \in S} w_{j} \ln F\left(\mathbf{x}_{j} \mathbf{b}\right)+\sum_{j \notin S} w_{j} \ln \left\{1-F\left(\mathbf{x}_{j} \mathbf{b}\right)\right\}
$$

where $S$ is the set of all observations $j$, such that $y_{j} \neq 0, F(z)=e^{z} /\left(1+e^{z}\right)$, and $w_{j}$ denotes the optional weights. $\ln L$ is maximized as described in [R] maximize.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce (robust) and vce (cluster clustvar), respectively. See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas. The scores are calculated as $\mathbf{u}_{j}=\left\{1-F\left(\mathbf{x}_{j} \mathbf{b}\right)\right\} \mathbf{x}_{j}$ for the positive outcomes and $-F\left(\mathbf{x}_{j} \mathbf{b}\right) \mathbf{x}_{j}$ for the negative outcomes.
logit also supports estimation with survey data. For details on VCEs with survey data, see [SVY] variance estimation.

Joseph Berkson (1899-1982) was born in New York City and studied at the College of the City of New York, Columbia, and Johns Hopkins, earning both an MD and a doctorate in statistics. He then worked at Johns Hopkins before moving to the Mayo Clinic in 1931 as a biostatistician. Among many other contributions, his most influential one drew upon a long-sustained interest in the logistic function, especially his 1944 paper on bioassay, in which he introduced the term "logit". Berkson was a frequent participant in controversy-sometimes humorous, sometimes bitter-on subjects such as the evidence for links between smoking and various diseases and the relative merits of probit and logit methods and of different calculation methods.

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[^40]
## Title

logit postestimation - Postestimation tools for logit

| Description | Syntax for predict | Menu for predict | Options for predict |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Methods and formulas | References | Also see |

## Description

The following postestimation commands are of special interest after logit:
Command Description
estat classification
estat gof
lroc
lsens
report various summary statistics, including the classification table Pearson or Hosmer-Lemeshow goodness-of-fit test compute area under ROC curve and graph the curve graph sensitivity and specificity versus probability cutoff

These commands are not appropriate after the svy prefix.

The following standard postestimation commands are also available:
Command Description

| contrast | contrasts and ANOVA-style joint tests of estimates |
| :--- | :--- |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample |
| estat vce | variance-covariance matrix of the estimators (VCE) |
| estat (svy) | postestimation statistics for survey data <br> estimates <br> cataloging estimation results |
| forecast | dynamic forecasts and simulations <br> lincom |
|  | point estimates, standard errors, testing, and inference for linear combinations <br> of coefficients |
| linktest | link test for model specification <br> liktest |
| margins | marginal means, predictive margins, marginal effects, and average marginal <br> effects |
| marginsplot | graph the results from margins (profile plots, interaction plots, etc.) <br> nlcom |
| preint estimates, standard errors, testing, and inference for nonlinear combinations |  |
| of coefficients |  |

[^41]
## Syntax for predict

predict [type] newvar [if] [in] [, statistic nooffset rules asif]

## statistic Description

Main
$\mathrm{pr} \quad$ probability of a positive outcome; the default
$\mathrm{xb} \quad$ linear prediction
stdp

* dbeta
standard error of the prediction
* deviance
* dx 2

Pregibon (1981) $\Delta \widehat{\beta}$ influence statistic
deviance residual

* ddeviance

Hosmer, Lemeshow, and Sturdivant (2013) $\Delta \chi^{2}$ influence statistic

* hat
* number

Hosmer, Lemeshow, and Sturdivant (2013) $\Delta D$ influence statistic
*residuals
*rstandard
score
sequential number of the covariate pattern
Pearson residuals; adjusted for number sharing covariate pattern
standardized Pearson residuals; adjusted for number sharing covariate pattern first derivative of the $\log$ likelihood with respect to $\mathbf{x}_{j} \boldsymbol{\beta}$

Unstarred statistics are available both in and out of sample; type predict . . . if e (sample) ... if wanted only for the estimation sample. Starred statistics are calculated only for the estimation sample, even when if e(sample) is not specified.
$\mathrm{pr}, \mathrm{xb}$, stdp, and score are the only options allowed with svy estimation results.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
pr , the default, calculates the probability of a positive outcome.
xb calculates the linear prediction.
stdp calculates the standard error of the linear prediction.
dbeta calculates the Pregibon (1981) $\Delta \widehat{\beta}$ influence statistic, a standardized measure of the difference in the coefficient vector that is due to deletion of the observation along with all others that share the same covariate pattern. In Hosmer, Lemeshow, and Sturdivant (2013, 154-155) jargon, this statistic is $M$-asymptotic; that is, it is adjusted for the number of observations that share the same covariate pattern.
deviance calculates the deviance residual.
dx2 calculates the Hosmer, Lemeshow, and Sturdivant $(2013,191) \Delta \chi^{2}$ influence statistic, reflecting the decrease in the Pearson $\chi^{2}$ that is due to deletion of the observation and all others that share the same covariate pattern.
ddeviance calculates the Hosmer, Lemeshow, and Sturdivant (2013, 191) $\Delta D$ influence statistic, which is the change in the deviance residual that is due to deletion of the observation and all others that share the same covariate pattern.
hat calculates the Pregibon (1981) leverage or the diagonal elements of the hat matrix adjusted for the number of observations that share the same covariate pattern.
number numbers the covariate patterns-observations with the same covariate pattern have the same number. Observations not used in estimation have number set to missing. The first covariate pattern is numbered 1 , the second 2 , and so on.
residuals calculates the Pearson residual as given by Hosmer, Lemeshow, and Sturdivant (2013, 155) and adjusted for the number of observations that share the same covariate pattern.
rstandard calculates the standardized Pearson residual as given by Hosmer, Lemeshow, and Sturdivant $(2013,191)$ and adjusted for the number of observations that share the same covariate pattern.
score calculates the equation-level score, $\partial \ln L / \partial\left(\mathbf{x}_{j} \boldsymbol{\beta}\right)$.

## Options

nooffset is relevant only if you specified offset (varname) for logit. It modifies the calculations made by predict so that they ignore the offset variable; the linear prediction is treated as $\mathbf{x}_{j} \mathbf{b}$ rather than as $\mathbf{x}_{j} \mathbf{b}+$ offset $_{j}$.
rules requests that Stata use any rules that were used to identify the model when making the prediction. By default, Stata calculates missing for excluded observations.
asif requests that Stata ignore the rules and exclusion criteria and calculate predictions for all observations possible by using the estimated parameter from the model.

## Remarks and examples

Once you have fit a logit model, you can obtain the predicted probabilities by using the predict command for both the estimation sample and other samples; see [U] 20 Estimation and postestimation commands and $[R]$ predict. Here we will make only a few more comments.
predict without arguments calculates the predicted probability of a positive outcome, that is, $\operatorname{Pr}\left(y_{j}=1\right)=F\left(\mathbf{x}_{j} \mathbf{b}\right)$. With the xb option, predict calculates the linear combination $\mathbf{x}_{j} \mathbf{b}$, where $\mathbf{x}_{j}$ are the independent variables in the $j$ th observation and $\mathbf{b}$ is the estimated parameter vector. This is sometimes known as the index function because the cumulative distribution function indexed at this value is the probability of a positive outcome.

In both cases, Stata remembers any rules used to identify the model and calculates missing for excluded observations, unless rules or asif is specified. For information about the other statistics available after predict, see [R] logistic postestimation.

## > Example 1: Predicted probabilities

In example 2 of $[R]$ logit, we fit the logit model logit foreign b3.repair. To obtain predicted probabilities, type

```
. use http://www.stata-press.com/data/r13/repair
(1978 Automobile Data)
. logit foreign b3.repair
note: 1.repair != 0 predicts failure perfectly
        1.repair dropped and 10 obs not used
    (output omitted)
. predict p
(option pr assumed; Pr(foreign))
(10 missing values generated)
```

| . summarize foreign p |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Variable | Obs | Mean | Std. Dev. | Min | Max |
| foreign | 58 | .2068966 | .4086186 | 0 | 1 |
| p | 48 | .25 | .1956984 | .1 | .5 |

Stata remembers any rules used to identify the model and sets predictions to missing for any excluded observations. logit dropped the variable 1.repair from our model and excluded 10 observations. Thus when we typed predict p, those same 10 observations were again excluded, and their predictions were set to missing.
predict's rules option uses the rules in the prediction. During estimation, we were told "1.repair
 probability of success or a positive outcome:

```
. predict p2, rules
(option pr assumed; Pr(foreign))
. summarize foreign p p2
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline foreign & 58 & .2068966 & .4086186 & 0 & 1 \\
p & 48 & .25 & .1956984 & .1 & .5 \\
p2 & 58 & .2068966 & .2016268 & 0 & .5
\end{tabular}
```

predict's asif option ignores the rules and exclusion criteria and calculates predictions for all observations possible by using the estimated parameters from the model:

```
. predict p3, asif
(option pr assumed; Pr(foreign))
. summarize foreign p p2 p3
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline foreign & 58 & .2068966 & .4086186 & 0 & 1 \\
p & 48 & .25 & .1956984 & .1 & .5 \\
p2 & 58 & .2068966 & .2016268 & 0 & .5 \\
p3 & 58 & .2931035 & .2016268 & .1 & .5
\end{tabular}
```

Which is right? What predict does by default is the most conservative approach. If many observations had been excluded because of a simple rule, we could be reasonably certain that the rules prediction is correct. The asif prediction is correct only if the exclusion is a fluke, and we would be willing to exclude the variable from the analysis anyway. Then, however, we would refit the model to include the excluded observations.

## > Example 2: Predictive margins

We can use the command margins, contrast after logit to make comparisons on the probability scale. Let's fit a model predicting low birthweight from characteristics of the mother:

[^42]```
. logit low age i.race i.smoke ptl i.ht i.ui
Iteration 0: log likelihood = -117.336
Iteration 1: log likelihood = -103.81846
Iteration 2: log likelihood = -103.40486
Iteration 3: log likelihood = -103.40384
Iteration 4: log likelihood = -103.40384
```

| Logistic regression | Number of obs | $=$ | 189 |
| :--- | :--- | :--- | ---: |
|  | LR chi2(7) | $=$ | 27.86 |
| Log likelihood $=-103.40384$ | Prob > chi2 | $=$ | 0.0002 |
|  | Pseudo R2 | $=$ | 0.1187 |


| low | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| age | -.0403293 | .0357127 | -1.13 | 0.259 | -.1103249 | .0296663 |
| race |  |  |  |  |  |  |
| black | 1.009436 | .5025122 | 2.01 | 0.045 | .0245302 | 1.994342 |
| other | 1.001908 | .4248342 | 2.36 | 0.018 | .1692485 | 1.834568 |
| smoke |  |  |  |  |  |  |
| smoker | .9631876 | .3904357 | 2.47 | 0.014 | .1979477 | 1.728427 |
| ptl | .6288678 | .3399067 | 1.85 | 0.064 | -.0373371 | 1.295073 |
| 1.ht | 1.358142 | .6289555 | 2.16 | 0.031 | .125412 | 2.590872 |
| 1.ui | .8001832 | .4572306 | 1.75 | 0.080 | -.0959724 | 1.696339 |
| _cons | -1.184127 | .9187461 | -1.29 | 0.197 | -2.984837 | .6165818 |

The coefficients are log odds-ratios: conditional on the other predictors, smoking during pregnancy is associated with an increase of 0.96 in the log odds-ratios of low birthweight. The model is linear in the $\log$ odds-scale, so the estimate of 0.96 has the same interpretation, whatever the values of the other predictors might be. We could convert 0.96 to an odds ratio by replaying the results with logit, or.

But what if we want to talk about the probability of low birthweight, and not the odds? Then we will need the command margins, contrast. We will use the $r$. contrast operator to compare each level of smoke with a reference level. (smoke has only two levels, so there will be only one comparison: a comparison of smokers with nonsmokers.)

```
. margins r.smoke, contrast
Contrasts of predictive margins
Model VCE : OIM
Expression : Pr(low), predict()
```

|  | df | chi2 | P>chi2 |
| ---: | :---: | :---: | :---: |
| smoke | 1 | 6.32 | 0.0119 |


|  | Delta-method <br> Std. Err. |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| Contrast | [95\% Conf. Interval] |  |  |  |
| (smoker vs nonsmoker) | .1832779 | .0728814 | .0404329 | .3261229 |

We see that maternal smoking is associated with an $18.3 \%$ increase in the probability of low birthweight. (We received a contrast in the probability scale because predicted probabilities are the default when margins is used after logit.)

The contrast of $18.3 \%$ is a difference of margins that are computed by averaging over the predictions for observations in the estimation sample. If the values of the other predictors were different, the contrast for smoke would be different, too. Let's estimate the contrast for 25 -year-old mothers:


Specifying a maternal age of 25 changed the contrast to $18.1 \%$. Our contrast of probabilities changed because the logit model is nonlinear in the probability scale. A contrast of log odds-ratios would not have changed.

## Methods and formulas

See Methods and formulas of the individual postestimation commands for details.

## References

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[^43]
## Title

Ioneway - Large one-way ANOVA, random effects, and reliability

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | Acknowledgment |
| References | Also see |  |  |

## Syntax

loneway response_var group_var [if] [in] [weight] [, options]

## options Description

Main
mean
median
exact
level(\#)
expected value of $F$ distribution; default is 1
median of $F$ distribution; default is 1
exact confidence intervals (groups must be equal with no weights)
set confidence level; default is level (95)
by is allowed; see [D] by.
aweights are allowed; see [U] 11.1.6 weight.

## Menu

Statistics $>$ Linear models and related $>$ ANOVA/MANOVA $>$ Large one-way ANOVA

## Description

loneway fits one-way analysis-of-variance (ANOVA) models on datasets with many levels of group_var and presents different ancillary statistics from oneway (see [R] oneway):

| Feature | oneway | loneway |
| :--- | :---: | :---: |
| Fit one-way model | x | x |
| $\quad$ on fewer than 376 levels | x | x |
| $\quad$ on more than 376 levels |  | x |
| Bartlett's test for equal variance | x |  |
| Multiple-comparison tests | x |  |
| Intragroup correlation and SE |  | x |
| Intragroup correlation confidence interval |  | x |
| Est. reliability of group-averaged score |  | x |
| Est. SD of group effect | x |  |
| Est. SD within group | x |  |

## Options

$\qquad$ Main
mean specifies that the expected value of the $F_{k-1, N-k}$ distribution be used as the reference point $F_{m}$ in the estimation of $\rho$ instead of the default value of 1 .
median specifies that the median of the $F_{k-1, N-k}$ distribution be used as the reference point $F_{m}$ in the estimation of $\rho$ instead of the default value of 1 .
exact requests that exact confidence intervals be computed, as opposed to the default asymptotic confidence intervals. This option is allowed only if the groups are equal in size and weights are not used.
level(\#) specifies the confidence level, as a percentage, for confidence intervals of the coefficients. The default is level (95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals.

## Remarks and examples

Remarks are presented under the following headings:
The one-way ANOVA model
$R$-squared
The random-effects ANOVA model
Intraclass correlation
Estimated reliability of the group-averaged score

## The one-way ANOVA model

## > Example 1

loneway's output looks like that of oneway, except that loneway presents more information at the end. Using our automobile dataset, we have created a (numeric) variable called manufacturer_grp identifying the manufacturer of each car, and within each manufacturer we have retained a maximum of four models, selecting those with the lowest mpg. We can compute the intraclass correlation of mpg for all manufacturers with at least four models as follows:


In addition to the standard one-way ANOVA output, loneway produces the $R$-squared, the estimated standard deviation of the group effect, the estimated standard deviation within group, the intragroup correlation, the estimated reliability of the group-averaged mean, and, for unweighted data, the asymptotic standard error and confidence interval for the intragroup correlation.

## R-squared

The $R$-squared is, of course, simply the underlying $R^{2}$ for a regression of response_var on the levels of group_var, or mpg on the various manufacturers here.

## The random-effects ANOVA model

loneway assumes that we observe a variable, $y_{i j}$, measured for $n_{i}$ elements within $k$ groups or classes such that

$$
y_{i j}=\mu+\alpha_{i}+\epsilon_{i j}, \quad i=1,2, \ldots, k, \quad j=1,2, \ldots, n_{i}
$$

and $\alpha_{i}$ and $\epsilon_{i j}$ are independent zero-mean random variables with variance $\sigma_{\alpha}^{2}$ and $\sigma_{\epsilon}^{2}$, respectively. This is the random-effects ANOVA model, also known as the components-of-variance model, in which it is typically assumed that the $y_{i j}$ are normally distributed.

The interpretation with respect to our example is that the observed value of our response variable, mpg , is created in two steps. First, the $i$ th manufacturer is chosen, and a value, $\alpha_{i}$, is determined-the typical mpg for that manufacturer less the overall $\mathrm{mpg} \mu$. Then a deviation, $\epsilon_{i j}$, is chosen for the $j$ th model within this manufacturer. This is how much that particular automobile differs from the typical mpg value for models from this manufacturer.

For our sample of 36 car models, the estimated standard deviations are $\sigma_{\alpha}=3.8$ and $\sigma_{\epsilon}=4.6$. Thus a little more than half of the variation in mpg between cars is attributable to the car model, with the rest attributable to differences between manufacturers. These standard deviations differ from those that would be produced by a (standard) fixed-effects regression in that the regression would require the sum within each manufacturer of the $\epsilon_{i j}, \epsilon_{i}$. for the $i$ th manufacturer, to be zero, whereas these estimates merely impose the constraint that the sum is expected to be zero.

## Intraclass correlation

There are various estimators of the intraclass correlation, such as the pairwise estimator, which is defined as the Pearson product-moment correlation computed over all possible pairs of observations that can be constructed within groups. For a discussion of various estimators, see Donner (1986). loneway computes what is termed the analysis of variance, or ANOVA, estimator. This intraclass correlation is the theoretical upper bound on the variation in response_var that is explainable by group_var, of which $R$-squared is an overestimate because of the serendipity of fitting. This correlation is comparable to an $R$-squared-you do not have to square it.

In our example, the intra-manu correlation, the correlation of mpg within manufacturer, is 0.40 . Because aweights were not used and the default correlation was computed (that is, the mean and median options were not specified), loneway also provided the asymptotic confidence interval and standard error of the intraclass correlation estimate.

## Estimated reliability of the group-averaged score

The estimated reliability of the group-averaged score or mean has an interpretation similar to that of the intragroup correlation; it is a comparable number if we average response_var by group_var, or mpg by manu in our example. It is the theoretical upper bound of a regression of manufactureraveraged mpg on characteristics of manufacturers. Why would we want to collapse our 36-observation dataset into a 9 -observation dataset of manufacturer averages? Because the 36 observations might be a mirage. When General Motors builds cars, do they sometimes put a Pontiac label and sometimes a Chevrolet label on them, so that it appears in our data as if we have two cars when we really have
only one, replicated? If that were the case, and if it were the case for many other manufacturers, then we would be forced to admit that we do not have data on 36 cars; we instead have data on nine manufacturer-averaged characteristics.

## Stored results

loneway stores the following in $r()$ :

| Scalars |  |  |  |
| :--- | :--- | :--- | :--- |
| $r(N)$ | number of observations | $r\left(r h o \_t\right)$ | estimated reliability |
| $r(r h o)$ | intraclass correlation | $r(s e)$ | asymp. SE of intraclass correlation |
| $r(l b)$ | lower bound of $95 \%$ CI for rho | $r(s d-w)$ | estimated SD within group |
| $r(u b)$ | upper bound of $95 \%$ CI for rho | $r\left(s d \_b\right)$ | estimated SD of group effect |

## Methods and formulas

The mean squares in the loneway's ANOVA table are computed as

$$
\mathrm{MS}_{\alpha}=\sum_{i} w_{i \cdot} \cdot\left(\bar{y}_{i} .-\bar{y}_{. .}\right)^{2} /(k-1)
$$

and

$$
\mathrm{MS}_{\epsilon}=\sum_{i} \sum_{j} w_{i j}\left(y_{i j}-\bar{y}_{i} .\right)^{2} /(N-k)
$$

in which

$$
w_{i .}=\sum_{j} w_{i j} \quad w_{. .}=\sum_{i} w_{i .} \quad \bar{y}_{i .}=\sum_{j} w_{i j} y_{i j} / w_{i .} \quad \text { and } \quad \bar{y} . .=\sum_{i} w_{i} \cdot \bar{y}_{i \cdot} / w .
$$

The corresponding expected values of these mean squares are

$$
E\left(\mathrm{MS}_{\alpha}\right)=\sigma_{\epsilon}^{2}+g \sigma_{\alpha}^{2} \quad \text { and } \quad E\left(\mathrm{MS}_{\epsilon}\right)=\sigma_{\epsilon}^{2}
$$

in which

$$
g=\frac{w_{. .}-\sum_{i} w_{i \cdot}^{2} / w_{. .}}{k-1}
$$

In the unweighted case, we get

$$
g=\frac{N-\sum_{i} n_{i}^{2} / N}{k-1}
$$

As expected, $g=m$ for the case of no weights and equal group sizes in the data, that is, $n_{i}=m$ for all $i$. Replacing the expected values with the observed values and solving yields the ANOVA estimates of $\sigma_{\alpha}^{2}$ and $\sigma_{\epsilon}^{2}$. Substituting these into the definition of the intraclass correlation

$$
\rho=\frac{\sigma_{\alpha}^{2}}{\sigma_{\alpha}^{2}+\sigma_{\epsilon}^{2}}
$$

yields the ANOVA estimator of the intraclass correlation:

$$
\rho_{A}=\frac{F_{\mathrm{obs}}-1}{F_{\mathrm{obs}}-1+g}
$$

$F_{\text {obs }}$ is the observed value of the $F$ statistic from the ANOVA table. For no weights and equal $n_{i}, \rho_{A}$ $=$ roh, which is the intragroup correlation defined by Kish (1965). Two slightly different estimators are available through the mean and median options (Gleason 1997). If either of these options is specified, the estimate of $\rho$ becomes

$$
\rho=\frac{F_{\text {obs }}-F_{m}}{F_{\text {obs }}+(g-1) F_{m}}
$$

For the mean option, $F_{m}=E\left(F_{k-1, N-K}\right)=(N-k) /(N-k-2)$, that is, the expected value of the anOva table's $F$ statistic. For the median option, $F_{m}$ is simply the median of the $F$ statistic. Setting $F_{m}$ to 1 gives $\rho_{A}$, so for large samples, these different point estimators are essentially the same. Also, because the intraclass correlation of the random-effects model is by definition nonnegative, for any of the three possible point estimators, $\rho$ is truncated to zero if $F_{\text {obs }}$ is less than $F_{m}$.

For no weighting, interval estimators for $\rho_{A}$ are computed. If the groups are equal sized (all $n_{i}$ equal) and the exact option is specified, the following exact (assuming that the $y_{i j}$ are normally distributed) $100(1-\alpha) \%$ confidence interval is computed:

$$
\left\{\frac{F_{\mathrm{obs}}-F_{m} F_{u}}{F_{\mathrm{obs}}+(g-1) F_{m} F_{u}}, \frac{F_{\mathrm{obs}}-F_{m} F_{l}}{F_{\mathrm{obs}}+(g-1) F_{m} F_{l}}\right\}
$$

with $F_{m}=1, F_{l}=F_{\alpha / 2, k-1, N-k}$, and $F_{u}=F_{1-\alpha / 2, k-1, N-k}, F_{\cdot, k-1, N-k}$ being the cumulative distribution function for the $F$ distribution with $k-1$ and $N-k$ degrees of freedom. If mean or median is specified, $F_{m}$ is defined as above. If the groups are equal sized and exact is not specified, the following asymptotic $100(1-\alpha) \%$ confidence interval for $\rho_{A}$ is computed,

$$
\left[\rho_{A}-z_{\alpha / 2} \sqrt{V\left(\rho_{A}\right)}, \rho_{A}+z_{\alpha / 2} \sqrt{V\left(\rho_{A}\right)}\right]
$$

where $z_{\alpha / 2}$ is the $100(1-\alpha / 2)$ percentile of the standard normal distribution and $\sqrt{V\left(\rho_{A}\right)}$ is the asymptotic standard error of $\rho$ defined below. This confidence interval is also available for unequal groups. It is not applicable and, therefore, not computed for the estimates of $\rho$ provided by the mean and median options. Again, because the intraclass coefficient is nonnegative, if the lower bound is negative for either confidence interval, it is truncated to zero. As might be expected, the coverage probability of a truncated interval is higher than its nominal value.

The asymptotic standard error of $\rho_{A}$, assuming that the $y_{i j}$ are normally distributed, is also computed when appropriate, namely, for unweighted data and when $\rho_{A}$ is computed (neither the mean option nor the median option is specified):

$$
V\left(\rho_{A}\right)=\frac{2(1-\rho)^{2}}{g^{2}}(A+B+C)
$$

with

$$
\begin{gathered}
A=\frac{\{1+\rho(g-1)\}^{2}}{N-k} \\
B=\frac{(1-\rho)\{1+\rho(2 g-1)\}}{k-1} \\
C=\frac{\rho^{2}\left\{\sum n_{i}^{2}-2 N^{-1} \sum n_{i}^{3}+N^{-2}\left(\sum n_{i}^{2}\right)^{2}\right\}}{(k-1)^{2}}
\end{gathered}
$$

and $\rho_{A}$ is substituted for $\rho$ (Donner 1986).

The estimated reliability of the group-averaged score, known as the Spearman-Brown prediction formula in the psychometric literature (Winer, Brown, and Michels 1991, 1014), is

$$
\rho_{t}=\frac{t \rho}{1+(t-1) \rho}
$$

for group size $t$. loneway computes $\rho_{t}$ for $t=g$.
The estimated standard deviation of the group effect is $\sigma_{\alpha}=\sqrt{\left(\mathrm{MS}_{\alpha}-\mathrm{MS}_{\epsilon}\right) / g}$. This deviation comes from the assumption that an observation is derived by adding a group effect to a within-group effect.

The estimated standard deviation within group is the square root of the mean square due to error, or $\sqrt{\mathrm{MS}_{\epsilon}}$.

## Acknowledgment

We thank John Gleason of Syracuse University (retired) for his contributions to improving loneway.

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## Also see

[R] anova - Analysis of variance and covariance
[R] icc - Intraclass correlation coefficients
[R] oneway - One-way analysis of variance

## Title

lowess - Lowess smoothing

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Methods and formulas | Acknowledgment | References |

## Syntax

lowess yvar xvar [if] [in] [, options]

## options

## Description

Main
mean
noweight
bwidth(\#)
logit
adjust
nograph
generate (newvar)
Plot
marker_options
marker_label_options
Smoothed line
lineopts (cline_options)
Add plots
addplot (plot) add other plots to generated graph
Y axis, X axis, Titles, Legend, Overall, By
twoway_options any of the options documented in [G-3] twoway_options
yvar and xvar may contain time-series operators; see [U] 11.4.4 Time-series varlists.

## Menu

Statistics $>$ Nonparametric analysis $>$ Lowess smoothing

## Description

lowess carries out a locally weighted regression of yvar on xvar, displays the graph, and optionally saves the smoothed variable.

Warning: lowess is computationally intensive and may therefore take a long time to run on a slow computer. Lowess calculations on 1,000 observations, for instance, require performing 1,000 regressions.

## Options

$\qquad$ Main
mean specifies running-mean smoothing; the default is running-line least-squares smoothing.
noweight prevents the use of Cleveland's (1979) tricube weighting function; the default is to use the weighting function.
bwidth(\#) specifies the bandwidth. Centered subsets of bwidth() $\times N$ observations are used for calculating smoothed values for each point in the data except for the end points, where smaller, uncentered subsets are used. The greater the bwidth(), the greater the smoothing. The default is 0.8 .
logit transforms the smoothed yvar into logits. Predicted values less than 0.0001 or greater than 0.9999 are set to $1 / N$ and $1-1 / N$, respectively, before taking logits.
adjust adjusts the mean of the smoothed yvar to equal the mean of $y v a r$ by multiplying by an appropriate factor. This option is useful when smoothing binary ( $0 / 1$ ) data.
nograph suppresses displaying the graph.
generate (newvar) creates newvar containing the smoothed values of yvar.

Plot
marker_options affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] marker_options.
marker_label_options specify if and how the markers are to be labeled; see [G-3] marker_label_options.

Smoothed line
lineopts (cline_options) affects the rendition of the lowess-smoothed line; see [G-3] cline_options.

Add plots
addplot (plot) provides a way to add other plots to the generated graph; see [G-3] addplot_option.

Y axis, X axis, Titles, Legend, Overall, By
twoway_options are any of the options documented in [G-3] twoway_options. These include options for titling the graph (see [G-3] title _options), options for saving the graph to disk (see [G-3] saving_option), and the by () option (see [G-3] by_option).

## Remarks and examples

By default, lowess provides locally weighted scatterplot smoothing. The basic idea is to create a new variable (newvar) that, for each yvar $y_{i}$, contains the corresponding smoothed value. The smoothed values are obtained by running a regression of $y v a r$ on $x v a r$ by using only the data $\left(x_{i}, y_{i}\right)$ and a few of the data near this point. In lowess, the regression is weighted so that the central point $\left(x_{i}, y_{i}\right)$ gets the highest weight and points that are farther away (based on the distance $\left.\left|x_{j}-x_{i}\right|\right)$ receive less weight. The estimated regression line is then used to predict the smoothed value $\widehat{y}_{i}$ for $y_{i}$ only. The procedure is repeated to obtain the remaining smoothed values, which means that a separate weighted regression is performed for every point in the data.

Lowess is a desirable smoother because of its locality -it tends to follow the data. Polynomial smoothing methods, for instance, are global in that what happens on the extreme left of a scatterplot can affect the fitted values on the extreme right.

## > Example 1

The amount of smoothing is affected by bwidth(\#). You are warned to experiment with different values. For instance,

```
. use http://www.stata-press.com/data/r13/lowess1
(example data for lowess)
. lowess h1 depth
```



Now compare that with
. lowess h1 depth, bwidth(.4)


In the first case, the default bandwidth of 0.8 is used, meaning that $80 \%$ of the data are used in smoothing each point. In the second case, we explicitly specified a bandwidth of 0.4 . Smaller bandwidths follow the original data more closely.

## > Example 2

Two lowess options are especially useful with binary (0/1) data: adjust and logit. adjust adjusts the resulting curve (by multiplication) so that the mean of the smoothed values is equal to the mean of the unsmoothed values. logit specifies that the smoothed curve be in terms of the log of the odds ratio:

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. lowess foreign mpg, ylabel(0 "Domestic" 1 "Foreign") jitter(5) adjust
```


. lowess foreign mpg, logit yline(0)


With binary data, if you do not use the logit option, it is a good idea to specify graph's jitter () option; see [G-2] graph twoway scatter. Because the underlying data (whether the car was manufactured outside the United States here) take on only two values, raw data points are more likely to be on top of each other, thus making it impossible to tell how many points there are. graph's jitter () option adds some noise to the data to shift the points around. This noise affects only the location of points on the graph, not the lowess curve.

When you specify the logit option, the display of the raw data is suppressed.

## - Technical note

lowess can be used for more than just lowess smoothing. Lowess can be usefully thought of as a combination of two smoothing concepts: the use of predicted values from regression (rather than means) for imputing a smoothed value and the use of the tricube weighting function (rather than a constant weighting function). lowess allows you to combine these concepts freely. You can use line smoothing without weighting (specify noweight), mean smoothing with tricube weighting (specify mean), or mean smoothing without weighting (specify mean and noweight).

## Methods and formulas

Let $y_{i}$ and $x_{i}$ be the two variables, and assume that the data are ordered so that $x_{i} \leq x_{i+1}$ for $i=1, \ldots, N-1$. For each $y_{i}$, a smoothed value $y_{i}^{s}$ is calculated.

The subset used in calculating $y_{i}^{s}$ is indices $i_{-}=\max (1, i-k)$ through $i_{+}=\min (i+k, N)$, where $k=\lfloor(N \times$ bwidth -0.5$) / 2\rfloor$. The weights for each of the observations between $j=i_{-}, \ldots, i_{+}$ are either 1 (noweight) or the tricube (default),

$$
w_{j}=\left\{1-\left(\frac{\left|x_{j}-x_{i}\right|}{\Delta}\right)^{3}\right\}^{3}
$$

where $\Delta=1.0001 \max \left(x_{i_{+}}-x_{i}, x_{i}-x_{i_{-}}\right)$. The smoothed value $y_{i}^{s}$ is then the (weighted) mean or the (weighted) regression prediction at $x_{i}$.

William Swain Cleveland (1943- ) studied mathematics and statistics at Princeton and Yale. He worked for several years at Bell Labs in New Jersey and now teaches statistics and computer science at Purdue. He has made key contributions in many areas of statistics, including graphics and data visualization, time series, environmental applications, and analysis of Internet traffic data.

## Acknowledgment

lowess is a modified version of a command originally written by Patrick Royston of the MRC Clinical Trials Unit, London, and coauthor of the Stata Press book Flexible Parametric Survival Analysis Using Stata: Beyond the Cox Model.

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## Also see

[R] lpoly - Kernel-weighted local polynomial smoothing
[R] smooth - Robust nonlinear smoother
[D] ipolate - Linearly interpolate (extrapolate) values

## Title

Ipoly - Kernel-weighted local polynomial smoothing

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

$$
\text { lpoly yvar xvar }[\text { if }][\text { in }][\text { weight }][\text {, options }]
$$

options
Main
kernel (kernel)
bwidth (\#|varname)
degree(\#)
generate ([ newvar $\left._{x}\right]$ newvar $_{s}$ )
n(\#)
at (varname)
nograph
noscatter
SE/CI
ci
level(\#)
se(newvar)
pwidth (\#)
var(\#|varname)
Scatterplot
marker_options
marker_label_options
Smoothed line
lineopts (cline_options)
Cl plot
ciopts(cline_options)
Add plots addplot (plot)

Y axis, X axis, Titles, Legend, Overall twoway_options

Description
specify kernel function; default is kernel (epanechnikov) specify kernel bandwidth
specify degree of the polynomial smooth; default is degree (0) store smoothing grid in newvar $r_{x}$ and smoothed points in newvars
obtain the smooth at \# points; default is $\min (N, 50)$
obtain the smooth at the values specified by varname suppress graph
suppress scatterplot only
plot confidence bands
set confidence level; default is level(95)
store standard errors in newvar
specify pilot bandwidth for standard error calculation
specify estimates of residual variance
change look of markers (color, size, etc.)
add marker labels; change look or position
affect rendition of the smoothed line
affect rendition of the confidence bands
add other plots to the generated graph
any options other than by () documented in [G-3] twoway_options
kernel
Description
epanechnikov
epan2
biweight
cosine
gaussian
parzen
rectangle
triangle

Epanechnikov kernel function; the default alternative Epanechnikov kernel function biweight kernel function cosine trace kernel function Gaussian kernel function Parzen kernel function rectangle kernel function triangle kernel function
fweights and aweights are allowed; see [U] 11.1.6 weight.

## Menu

Statistics $>$ Nonparametric analysis $>$ Local polynomial smoothing

## Description

lpoly performs a kernel-weighted local polynomial regression of yvar on xvar and displays a graph of the smoothed values with (optional) confidence bands.

## Options

kernel (kernel) specifies the kernel function for use in calculating the weighted local polynomial estimate. The default is kernel (epanechnikov).
bwidth (\#|varname) specifies the half-width of the kernel-the width of the smoothing window around each point. If bwidth() is not specified, a rule-of-thumb (ROT) bandwidth estimator is calculated and used. A local variable bandwidth may be specified in varname, in conjunction with an explicit smoothing grid using the at () option.
degree(\#) specifies the degree of the polynomial to be used in the smoothing. The default is degree ( 0 ), meaning local-mean smoothing.
generate ( $\left[\right.$ newvar $\left._{x}\right]$ newvar $_{s}$ ) stores the smoothing grid in newvar ${ }_{x}$ and the smoothed values in newvar ${ }_{s}$. If at () is not specified, then both newvar ${ }_{x}$ and newvar ${ }_{s}$ must be specified. Otherwise, only newvar ${ }_{s}$ is to be specified.
n (\#) specifies the number of points at which the smooth is to be calculated. The default is $\min (N, 50)$, where $N$ is the number of observations.
at (varname) specifies a variable that contains the values at which the smooth should be calculated. By default, the smoothing is done on an equally spaced grid, but you can use at () to instead perform the smoothing at the observed $x$ 's, for example. This option also allows you to more easily obtain smooths for different variables or different subsamples of a variable and then overlay the estimates for comparison.
nograph suppresses drawing the graph of the estimated smooth. This option is often used with the generate() option.
noscatter suppresses superimposing a scatterplot of the observed data over the smooth. This option is useful when the number of resulting points would be so large as to clutter the graph.

```
SE/CI
```

ci plots confidence bands, using the confidence level specified in level().
level(\#) specifies the confidence level, as a percentage, for confidence intervals. The default is level (95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals.
se(newvar) stores the estimates of the standard errors in newvar. This option requires specifying generate() or at ().
pwidth(\#) specifies the pilot bandwidth to be used for standard-error computations. The default is chosen to be 1.5 times the value of the ROT bandwidth selector. If you specify pwidth() without specifying se() or ci, then the ci option is assumed.
$\operatorname{var}(\# \mid$ varname $)$ specifies an estimate of a constant residual variance or a variable containing estimates of the residual variances at each grid point required for standard-error computation. By default, the residual variance at each smoothing point is estimated by the normalized weighted residual sum of squares obtained from locally fitting a polynomial of order $p+2$, where $p$ is the degree specified in degree(). var (varname) is allowed only if at () is specified. If you specify var () without specifying se() or ci, then the ci option is assumed.

Scatterplot
marker_options affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] marker_options.
marker_label_options specify if and how the markers are to be labeled; see [G-3] marker_label_options.
$\qquad$ Smoothed line
lineopts (cline_options) affects the rendition of the smoothed line; see [G-3] cline_options.
$\qquad$ $\sqrt{\mathrm{Cl} \text { plot }}$
ciopts (cline_options) affects the rendition of the confidence bands; see [G-3] cline_options.

Add plots
addplot (plot) provides a way to add other plots to the generated graph; see [G-3] addplot_option.
$\longleftarrow \sqrt{\mathrm{Y}}$ axis, X axis, Titles, Legend, Overall
twoway_options are any of the options documented in [G-3] twoway_options, excluding by (). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

## Remarks and examples

Remarks are presented under the following headings:

[^44]
## Introduction

The last 25 years or so has seen a significant outgrowth in the literature on scatterplot smoothing, otherwise known as univariate nonparametric regression. Of most appeal is the idea of making no assumptions about the functional form for the expected value of a response given a regressor, but instead allowing the data to "speak for themselves". Various methods and estimators fall into the category of nonparametric regression, including local mean smoothing as described independently by Nadaraya (1964) and Watson (1964), the Gasser and Müller (1979) estimator, locally weighted scatterplot smoothing (LOWESS) as described by Cleveland (1979), wavelets (for example, Donoho [1995]), and splines (Eubank 1999), to name a few. Much of the vast literature focuses on automating the amount of smoothing to be performed and dealing with the bias/variance tradeoff inherent to this type of estimation. For example, for Nadaraya-Watson the amount of smoothing is controlled by choosing a bandwidth.

Smoothing via local polynomials is by no means a new idea but instead one that has been rediscovered in recent years in articles such as Fan (1992). A natural extension of the local mean smoothing of Nadaraya-Watson, local polynomial regression involves fitting the response to a polynomial form of the regressor via locally weighted least squares. Higher-order polynomials have better bias properties than the zero-degree local polynomials of the Nadaraya-Watson estimator; in general, higher-order polynomials do not require bias adjustment at the boundary of the regression space. For a definitive reference on local polynomial smoothing, see Fan and Gijbels (1996).

## Local polynomial smoothing

Consider a set of scatterplot data $\left\{\left(x_{1}, y_{1}\right), \ldots,\left(x_{n}, y_{n}\right)\right\}$ from the model

$$
\begin{equation*}
y_{i}=m\left(x_{i}\right)+\sigma\left(x_{i}\right) \epsilon_{i} \tag{1}
\end{equation*}
$$

for some unknown mean and variance functions $m(\cdot)$ and $\sigma^{2}(\cdot)$, and symmetric errors $\epsilon_{i}$ with $E\left(\epsilon_{i}\right)=0$ and $\operatorname{Var}\left(\epsilon_{i}\right)=1$. The goal is to estimate $m\left(x_{0}\right)=E\left[Y \mid X=x_{0}\right]$, making no assumption about the functional form of $m(\cdot)$.
lpoly estimates $m\left(x_{0}\right)$ as the constant term (intercept) of a regression, weighted by the kernel function specified in kernel (), of yvar on the polynomial terms (xvar-x $x_{0}$ ), (xvar- $\left.x_{0}\right)^{2}, \ldots,(x v a r-$ $\left.x_{0}\right)^{p}$ for each smoothing point $x_{0}$. The degree of the polynomial, $p$, is specified in degree (), the amount of smoothing is controlled by the bandwidth specified in bwidth(), and the chosen kernel function is specified in kernel().

## > Example 1

Consider the motorcycle data as examined (among other places) in Fan and Gijbels (1996). The data consist of 133 observations and measure the acceleration (accel measured in grams [g]) of a dummy's head during impact over time (time measured in milliseconds). For these data, we use lpoly to fit a local cubic polynomial with the default bandwidth (obtained using the ROT method) and the default Epanechnikov kernel.

```
. use http://www.stata-press.com/data/r13/motorcycle
(Motorcycle data from Fan & Gijbels (1996))
. lpoly accel time, degree(3)
```



The default bandwidth and kernel settings do not provide a satisfactory fit in this example. To improve the fit, we can either supply a different bandwidth by using the bwidth() option or specify a different kernel by using the kernel() option. For example, using the alternative Epanechnikov kernel, kernel (epan2), below provides a better fit for these data.

```
. lpoly accel time, degree(3) kernel(epan2)
```


lpoly allows specifying in degree() both odd and even orders of the polynomial to be used for the smoothing. However, the odd-order, $2 k+1$, polynomial approximations are preferable. They have
an extra parameter compared with the even-order, $2 k$, approximations, which leads to a significant bias reduction and there is no increase of variability associated with adding this extra parameter. Using an odd order when estimating the regression function is therefore usually sufficient. For a more thorough discussion, see Fan and Gijbels (1996).

## Choice of a bandwidth

The choice of a bandwidth is crucial for many smoothing techniques, including local polynomial smoothing. In general, using a large bandwidth gives smooths with a large bias, whereas a small bandwidth may result in highly variable smoothed values. Various techniques exist for optimal bandwidth selection. By default, lpoly uses the ROT method to estimate the bandwidth used for the smoothing; see Methods and formulas for details.
$>$ Example 2
Using the motorcycle data, we demonstrate how a local linear polynomial fit changes using different bandwidths.

```
. lpoly accel time, degree(1) kernel(epan2) bwidth(1) generate(at smooth1)
> nograph
. lpoly accel time, degree(1) kernel(epan2) bwidth(7) at(at) generate(smooth2)
> nograph
. label variable smooth1 "smooth: width = 1"
. label variable smooth2 "smooth: width = 7"
. lpoly accel time, degree(1) kernel(epan2) at(at) addplot(line smooth* at)
> legend(label(2 "smooth: width = 3.42 (ROT)")) note("kernel = epan2, degree = 1")
```



From this graph, we can see that the local linear polynomial fit with larger bandwidth (width $=$ 7) corresponds to a smoother line but fails to fit the curvature of the scatterplot data. The smooth obtained using the width equal to one seems to fit most data points, but the corresponding line has several spikes indicating larger variability. The smooth obtained using the ROT bandwidth estimator seems to have a good tradeoff between the fit and variability in this example.

In the above, we also demonstrated how the generate() and addplot () options may be used to produce overlaid plots obtained from lpoly with different options. The nograph option saves time when you need to save only results with generate().

However, to avoid generating variables manually, one can use twoway lpoly instead; see [G-2] graph twoway lpoly for more details.

```
. twoway scatter accel time ||
> lpoly accel time, degree(1) kernel(epan2) lpattern(solid) ||
> lpoly accel time, degree(1) kernel(epan2) bwidth(1) ||
> lpoly accel time, degree(1) kernel(epan2) bwidth(7) ||
> , legend(label(2 "smooth: width = 3.42 (ROT)") label(3 "smooth: width = 1")
            label(4 "smooth: width = 7"))
        title("Local polynomial smooth") note("kernel = epan2, degree = 1")
        xtitle("time (msec)") ytitle("acceleration (g)")
```



The ROT estimate is commonly used as an initial guess for the amount of smoothing; this approach may be sufficient when the choice of a bandwidth is less important. In other cases, you can pick your own bandwidth.

When the shape of the regression function has a combination of peaked and flat regions, a variable bandwidth may be preferable over the constant bandwidth to allow for different degrees of smoothness in different regions. The bwidth() option allows you to specify the values of the local variable bandwidths as those stored in a variable in your data.

Similar issues with bias and variability arise when choosing a pilot bandwidth (the pwidth() option) used to compute standard errors of the local polynomial smoother. The default value is chosen to be $1.5 \times$ ROT. For a review of methods for pilot bandwidth selection, see Fan and Gijbels (1996).

## Confidence bands

The established asymptotic normality of the local polynomial estimators under certain conditions allows the construction of approximate confidence bands. lpoly offers the ci option to plot these bands.

## > Example 3

Let us plot the confidence bands for the local polynomial fit from example 1.

```
. lpoly accel time, degree(3) kernel(epan2) ci
```



You can obtain graphs with overlaid confidence bands by using twoway lpolyci; see [G-2] graph twoway lpolyci for examples.

Constructing the confidence intervals involves computing standard errors obtained by taking a square root of the estimate of the conditional variance of the local polynomial estimator at each grid point $x_{0}$. Estimating the conditional variance requires fitting a polynomial of a higher order locally by using a different bandwidth, the pilot bandwidth. The value of the pilot bandwidth may be supplied by using pwidth(). By default, the value of $1.5 \times$ ROT is used. Also, estimates of the residual variance $\sigma^{2}\left(x_{0}\right)$ at each grid point, $x_{0}$, are required to obtain the estimates of the conditional variances. These estimates may be supplied by using the var () option. By default, they are computed using the normalized weighted residual sum of squares from a local polynomial fit of a higher order. See Methods and formulas for details. The standard errors may be saved by using se().

## Stored results

lpoly stores the following in $r()$ :

| Scalars |  |  |  |
| :--- | :--- | :--- | :--- |
| $\quad r$ (degree) | smoothing polynomial degree | $r($ bwidth $)$ | bandwidth of the smooth <br> $r($ ngrid $)$ |
| $r(N)$ | number of successful regressions | $r($ pwidth $)$ | pilot bandwidth |
| Macros | sample size |  |  |
| $r($ kernel $)$ | name of kernel |  |  |

## Methods and formulas

Consider model (1), written in matrix notation,

$$
\mathbf{y}=m(\mathbf{x})+\boldsymbol{\epsilon}
$$

where $\mathbf{y}$ and $\mathbf{x}$ are the $n \times 1$ vectors of scatterplot values, $\boldsymbol{\epsilon}$ is the $n \times 1$ vector of errors with zero mean and covariance matrix $\boldsymbol{\Sigma}=\operatorname{diag}\left\{\sigma\left(x_{i}\right)\right\} \mathbf{I}_{n}$, and $m()$ and $\sigma()$ are some unknown functions. Define $m\left(x_{0}\right)=E\left[Y \mid X=x_{0}\right]$ and $\sigma^{2}\left(x_{0}\right)=\operatorname{Var}\left[Y \mid X=x_{0}\right]$ to be the conditional mean and conditional variance of random variable $Y$ (residual variance), respectively, for some realization $x_{0}$ of random variable $X$.

The method of local polynomial smoothing is based on the approximation of $m(x)$ locally by a $p$ th order polynomial in $\left(x-x_{0}\right)$ for some $x$ in the neighborhood of $x_{0}$. For the scatterplot data $\left\{\left(x_{1}, y_{1}\right), \ldots,\left(x_{n}, y_{n}\right)\right\}$, the $p$ th-order local polynomial smooth $\widehat{m}\left(x_{0}\right)$ is equal to $\widehat{\beta}_{0}$, an estimate of the intercept of the weighted linear regression,

$$
\begin{equation*}
\widehat{\beta}=\left(\mathbf{X}^{T} \mathbf{W} \mathbf{X}\right)^{-1} \mathbf{X}^{T} \mathbf{W} \mathbf{y} \tag{2}
\end{equation*}
$$

where $\widehat{\beta}=\left(\widehat{\beta}_{0}, \widehat{\beta}_{1}, \ldots, \widehat{\beta}_{p}\right)^{T}$ is the vector of estimated regression coefficients (with $\left\{\widehat{\beta}_{j}=\right.$ $\left.\left.(j!)^{-1} \widehat{m}^{(j)}(x)\right|_{x=x_{0}}, j=0, \ldots, p\right\}$ also representing estimated coefficients from a corresponding Taylor expansion); $\mathbf{X}=\left\{\left(x_{i}-x_{0}\right)^{j}\right\}_{i, j=1,0}^{n, p}$ is a design matrix; and $\mathbf{W}=\operatorname{diag}\left\{K_{h}\left(x_{i}-x_{0}\right)\right\}_{n \times n}$ is a weighting matrix with weights $K_{h}(\cdot)$ defined as $K_{h}(x)=h^{-1} K(x / h)$, with $K(\cdot)$ being a kernel function and $h$ defining a bandwidth. The kernels are defined in Methods and formulas of [ R ] kdensity.

The default bandwidth is obtained using the ROT method of bandwidth selection. The ROT bandwidth is the plugin estimator of the asymptotically optimal constant bandwidth. This is the bandwidth that minimizes the conditional weighted mean integrated squared error. The ROT plugin bandwidth selector for the smoothing bandwidth $h$ is defined as follows; assuming constant residual variance $\sigma^{2}\left(x_{0}\right)=\sigma^{2}$ and odd degree $p$ :

$$
\begin{equation*}
\widehat{h}=C_{0, p}(K)\left[\frac{\widehat{\sigma}^{2} \int w_{0}(x) d x}{n \int\left\{\widehat{m}^{(p+1)}(x)\right\}^{2} w_{0}(x) f(x) d x}\right]^{1 /(2 p+3)} \tag{3}
\end{equation*}
$$

where $C_{0, p}(K)$ is a constant, as defined in Fan and Gijbels (1996), that depends on the kernel function $K(\cdot)$, and the degree of a polynomial $p$ and $w_{0}$ is chosen to be an indicator function on the interval $\left[\min _{\mathbf{x}}+0.05 \times\right.$ range $_{\mathbf{x}}, \max _{\mathbf{x}}-0.05 \times$ range $\left._{\mathbf{x}}\right]$ with $\min _{\mathbf{x}}, \max _{\mathbf{x}}$, and range ${ }_{\mathbf{x}}$ being, respectively, the minimum, maximum, and the range of $\mathbf{x}$. To obtain the estimates of a constant residual variance, $\widehat{\sigma}^{2}$, and $(p+1)$ th order derivative of $m(x)$, denoted as $\widehat{m}^{(p+1)}(x)$, a polynomial in $\mathbf{x}$ of order $(p+3)$ is fit globally to $\mathbf{y} . \widehat{\sigma}^{2}$ is estimated as a standardized residual sum of squares from this fit.

The expression for the asymptotically optimal constant bandwidth used in constructing the ROT bandwidth estimator is derived for the odd-order polynomial approximations. For even-order polynomial fits the expression would depend not only on $m^{(p+1)}(x)$ but also on $m^{(p+2)}(x)$ and the design density and its derivative, $f(x)$ and $f^{\prime}(x)$. Therefore, the ROT bandwidth selector would require estimation of these additional quantities. Instead, for an even-degree $p$ of the local polynomial, lpoly uses the value of the ROT estimator (3) computed using degree $p+1$. As such, for even degrees this is not a plugin estimator of the asymptotically optimal constant bandwidth.

The estimates of the conditional variance of local polynomial estimators are obtained using

$$
\begin{equation*}
\widehat{\operatorname{Var}}\left\{\widehat{m}\left(x_{0}\right) \mid X=x_{0}\right\}=\widehat{\sigma}_{m}^{2}\left(x_{0}\right)=\left(\mathbf{X}^{T} \mathbf{W} \mathbf{X}\right)^{-1}\left(\mathbf{X}^{T} \mathbf{W}^{\mathbf{2}} \mathbf{X}\right)\left(\mathbf{X}^{T} \mathbf{W} \mathbf{X}\right)^{-1} \widehat{\sigma}^{2}\left(x_{0}\right) \tag{4}
\end{equation*}
$$

where $\widehat{\sigma}^{2}\left(x_{0}\right)$ is estimated by the normalized weighted residual sum of squares from the $(p+2)$ th order polynomial fit using pilot bandwidth $h^{\star}$.

When the bias is negligible the normal-approximation method yields a $(1-\alpha) \times 100 \%$ confidence interval for $m\left(x_{0}\right)$,

$$
\left\{\widehat{m}\left(x_{0}\right)-z_{(1-\alpha / 2)} \widehat{\sigma}_{m}\left(x_{0}\right), \widehat{m}\left(x_{0}\right)+z_{(1-\alpha / 2)} \widehat{\sigma}_{m}\left(x_{0}\right)\right\}
$$

where $z_{(1-\alpha / 2)}$ is the $(1-\alpha / 2)$ th quantile of the standard Gaussian distribution, and $\widehat{m}\left(x_{0}\right)$ and $\widehat{\sigma}_{m}\left(x_{0}\right)$ are as defined in (2) and (4), respectively.

## References

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## Also see

[R] kdensity - Univariate kernel density estimation
[ R$]$ lowess - Lowess smoothing
[R] smooth - Robust nonlinear smoother
[G-2] graph twoway lpoly - Local polynomial smooth plots
[G-2] graph twoway lpolyci - Local polynomial smooth plots with CIs

## Title

Iroc - Compute area under ROC curve and graph the curve

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |
| Also see |  |  |  |

## Syntax

```
lroc \([\) depvar \(][\) if \(][\) in \(][\) weight \(][\), options \(]\)
```

options Description

Main
all nograph

Advanced
beta (matname) row vector containing model coefficients
Plot
cline_options change the look of the line
marker_options
marker_label_options
Reference line rlopts(cline_options)

Add plots
addplot (plot) add other plots to the generated graph

Y axis, X axis, Titles, Legend, Overall
twoway_options
fweights are allowed; see [U] 11.1.6 weight.
lroc is not appropriate after the svy prefix.

## Menu

Statistics $>$ Binary outcomes $>$ Postestimation $>$ ROC curve after logistic/logit/probit/ivprobit

## Description

lroc graphs the ROC curve and calculates the area under the curve.
lroc requires that the current estimation results be from logistic, logit, probit, or ivprobit; see $[R]$ logistic, $[R]$ logit, $[R]$ probit, or $[R]$ ivprobit.

## Options

$\qquad$ Main
all requests that the statistic be computed for all observations in the data, ignoring any if or in restrictions specified by the estimation command.
nograph suppresses graphical output.

Advanced
beta (matname) specifies a row vector containing model coefficients. The columns of the row vector must be labeled with the corresponding names of the independent variables in the data. The dependent variable depvar must be specified immediately after the command name. See Models other than the last fitted model later in this entry.
$\qquad$ Plot
cline_options, marker_options, and marker_label_options affect the rendition of the ROC curve-the plotted points connected by lines. These options affect the size and color of markers, whether and how the markers are labeled, and whether and how the points are connected; see [G-3] cline _options, [G-3] marker_options, and [G-3] marker_label_options.

Reference line
rlopts(cline_options) affects the rendition of the reference line; see [G-3] cline_options.

Add plots
addplot (plot) provides a way to add other plots to the generated graph; see [G-3] addplot_option.
$\qquad$ Y axis, X axis, Titles, Legend, Overall
twoway_options are any of the options documented in [G-3] twoway_options, excluding by (). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

## Remarks and examples

Remarks are presented under the following headings:
Introduction
Samples other than the estimation sample
Models other than the last fitted model

## Introduction

Stata also has a suite of commands for performing both parametric and nonparametric receiver operating characteristic (ROC) analysis. See [R] roc for an overview of these commands.
lroc graphs the ROC curve-a graph of sensitivity versus one minus specificity as the cutoff $c$ is varied-and calculates the area under it. Sensitivity is the fraction of observed positive-outcome cases that are correctly classified; specificity is the fraction of observed negative-outcome cases that are correctly classified. When the purpose of the analysis is classification, you must choose a cutoff.

The curve starts at $(0,0)$, corresponding to $c=1$, and continues to $(1,1)$, corresponding to $c=0$. A model with no predictive power would be a $45^{\circ}$ line. The greater the predictive power, the more bowed the curve, and hence the area beneath the curve is often used as a measure of the predictive power. A model with no predictive power has area 0.5 ; a perfect model has area 1.

The ROC curve was first discussed in signal detection theory (Peterson, Birdsall, and Fox 1954) and then was quickly introduced into psychology (Tanner and Swets 1954). It has since been applied in other fields, particularly medicine (for instance, Metz [1978]). For a classic text on ROC techniques, see Green and Swets (1966).
lsens also plots sensitivity and specificity; see [R] Isens.

## > Example 1

Hardin and Hilbe (2012) examine data from the National Canadian Registry of Cardiovascular Disease (FASTRAK), sponsored by Hoffman-La Roche Canada. They model death within 48 hours based on whether a patient suffers an anterior infarct (heart attack) rather than an inferior infarct using a logistic regression and evaluate the model using an ROC curve. We replicate their analysis here.

Both anterior and inferior refer to sites on the heart where damage occurs. The model is also adjusted for hcabg, whether the subject has had a cardiac bypass surgery (CABG); age, a four-category age-group indicator; and killip, a four-level risk indicator.

We load the data and then estimate the parameters of the logistic regression with logistic. Factor-variable notation is used for each predictor, because they are categorical; see [U] 11.4.3 Factor variables.


The odds ratios for a unit change in each covariate are reported by logistic. At fixed values of the other covariates, patients who enter Canadian hospitals with an anterior infarct have nearly twice the odds of death within 48 hours than those with an inferior infarct. Those who have had a previous CABG have approximately twice the risk of death of those who have not. Those with higher Killip risks and those who are older are also at greater risk of death.

We use lroc to draw the ROC curve for the model. The area under the curve of approximately 0.8 indicates acceptable discrimination for the model.

```
. lroc
Logistic model for death
number of observations = 4483
area under ROC curve = 0.7965
```



## Samples other than the estimation sample

lroc can be used with samples other than the estimation sample. By default, lroc remembers the estimation sample used with the last logistic, logit, probit, or ivprobit command. To override this, simply use an if or in restriction to select another set of observations, or specify the all option to force the command to use all the observations in the dataset.

See example 3 in $[R]$ estat gof for an example of using lroc with a sample other than the estimation sample.

## Models other than the last fitted model

By default, lroc uses the last model fit by logistic, logit, probit, or ivprobit. You may also directly specify the model to lroc by inputting a vector of coefficients with the beta() option and passing the name of the dependent variable depvar to lroc.
$\lambda$ Example 2
Suppose that someone publishes the following logistic model of low birthweight:
$\operatorname{Pr}($ low $=1)=F(-0.02$ age -0.01 lwt +1.3 black +1.1 smoke $+0.5 \mathrm{ptl}+1.8 \mathrm{ht}+0.8 \mathrm{ui}+0.5)$
where $F$ is the cumulative logistic distribution. These coefficients are not odds ratios; they are the equivalent of what logit produces.

We can see whether this model fits our data. First we enter the coefficients as a row vector and label its columns with the names of the independent variables plus _cons for the constant (see $[\mathrm{P}]$ matrix define and $[\mathrm{P}]$ matrix rownames).

```
. use http://www.stata-press.com/data/r13/lbw3, clear
(Hosmer & Lemeshow data)
. matrix input b = (-.02, -.01, 1.3, 1.1, .5, 1.8, .8, .5)
. matrix colnames b = age lwt black smoke ptl ht ui _cons
```

Here we use lroc to examine the predictive ability of the model:

```
. lroc low, beta(b) nograph
Logistic model for low
number of observations = 189
area under ROC curve = 0.7275
```

The area under the curve indicates that this model does have some predictive power. We can obtain a graph of sensitivity and specificity as a function of the cutoff probability by typing . Isens low, beta(b)


See [R] Isens.

## Stored results

lroc stores the following in $r()$ :
Scalars
$\begin{array}{ll}r(N) & \text { number of observations } \\ r(\text { area }) & \text { area under the ROC curve }\end{array}$

## Methods and formulas

The ROC curve is a graph of specificity against ( 1 - sensitivity). This is guaranteed to be a monotone nondecreasing function because the number of correctly predicted successes increases and the number of correctly predicted failures decreases as the classification cutoff $c$ decreases.

The area under the ROC curve is the area on the bottom of this graph and is determined by integrating the curve. The vertices of the curve are determined by sorting the data according to the predicted index, and the integral is computed using the trapezoidal rule.

## References

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Peterson, W. W., T. G. Birdsall, and W. C. Fox. 1954. The theory of signal detectability. Transactions IRE Professional Group on Information Theory PGIT-4: 171-212.

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Tilford, J. M., P. K. Roberson, and D. H. Fiser. 1995. sbe12: Using lfit and lroc to evaluate mortality prediction models. Stata Technical Bulletin 28: 14-18. Reprinted in Stata Technical Bulletin Reprints, vol. 5, pp. 77-81. College Station, TX: Stata Press.

## Also see

[ R$]$ logistic - Logistic regression, reporting odds ratios
[R] logit - Logistic regression, reporting coefficients
[R] probit - Probit regression
[R] ivprobit - Probit model with continuous endogenous regressors
[R] Isens - Graph sensitivity and specificity versus probability cutoff
[R] estat classification - Classification statistics and table
[R] estat gof - Pearson or Hosmer-Lemeshow goodness-of-fit test
$[\mathrm{R}]$ roc - Receiver operating characteristic (ROC) analysis
[U] 20 Estimation and postestimation commands

## Title

Irtest — Likelihood-ratio test after estimation

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

lrtest modelspec ${ }_{1}$ [ modelspec $_{2}$ ] [, options $]$
where modelspec is

```
name|.|(namelist)
```

where name is the name under which estimation results were stored using estimates store (see $[R]$ estimates store), and "." refers to the last estimation results, whether or not these were already stored.
options Description
stats display statistical information about the two models
dir display descriptive information about the two models
df (\#) override the automatic degrees-of-freedom calculation; seldom used
force force testing even when apparently invalid

## Menu

Statistics $>$ Postestimation $>$ Tests $>$ Likelihood-ratio test

## Description

lrtest performs a likelihood-ratio test of the null hypothesis that the parameter vector of a statistical model satisfies some smooth constraint. To conduct the test, both the unrestricted and the restricted models must be fit using the maximum likelihood method (or some equivalent method), and the results of at least one must be stored using estimates store; see $[R]$ estimates store.
modelspec $_{1}$ and modelspec $_{2}$ specify the restricted and unrestricted model in any order. modelspec ${ }_{1}$ and modelspec $_{2}$ cannot have names in common; for example, lrtest (A B C) (C D E) is not allowed because both model specifications include C. If modelspec $_{2}$ is not specified, the last estimation result is used; this is equivalent to specifying modelspec $_{2}$ as a period (.).
lrtest supports composite models specified by a parenthesized list of model names. In a composite model, we assume that the log likelihood and dimension (number of free parameters) of the full model are obtained as the sum of the log-likelihood values and dimensions of the constituting models.
lrtest provides an important alternative to test (see [R] test) for models fit via maximum likelihood or equivalent methods.

## Options

stats displays statistical information about the unrestricted and restricted models, including the information indices of Akaike and Schwarz.
dir displays descriptive information about the unrestricted and restricted models; see estimates dir in [R] estimates store.
$\mathrm{df}(\#)$ is seldom specified; it overrides the automatic degrees-of-freedom calculation.
force forces the likelihood-ratio test calculations to take place in situations where lrtest would normally refuse to do so and issue an error. Such situations arise when one or more assumptions of the test are violated, for example, if the models were fit with vce (robust), vce(cluster clustvar), or pweights; when the dependent variables in the two models differ; when the null log likelihoods differ; when the samples differ; or when the estimation commands differ. If you use the force option, there is no guarantee as to the validity or interpretability of the resulting test.

## Remarks and examples

The standard way to use lrtest is to do the following:

1. Fit either the restricted model or the unrestricted model by using one of Stata's estimation commands and then store the results using estimates store name.
2. Fit the alternative model (the unrestricted or restricted model) and then type 'lrtest name .'. lrtest determines for itself which of the two models is the restricted model by comparing the degrees of freedom.

Often you may want to store the alternative model with estimates store name ${ }_{2}$, for instance, if you plan additional tests against models yet to be fit. The likelihood-ratio test is then obtained as lrtest name name ${ }_{2}$.

Remarks are presented under the following headings:
Nested models
Composite models

## Nested models

lrtest may be used with any estimation command that reports a log likelihood, including heckman, logit, poisson, stcox, and streg. You must check that one of the model specifications implies a statistical model that is nested within the model implied by the other specification. Usually, this means that both models are fit with the same estimation command (for example, both are fit by logit, with the same dependent variables) and that the set of covariates of one model is a subset of the covariates of the other model. Second, lrtest is valid only for models that are fit by maximum likelihood or by some equivalent method, so it does not apply to models that were fit with probability weights or clusters. Specifying the vce (robust) option similarly would indicate that you are worried about the valid specification of the model, so you would not use lrtest. Third, lrtest assumes that under the null hypothesis, the test statistic is (approximately) distributed as chi-squared. This assumption is not true for likelihood-ratio tests of "boundary conditions", such as tests for the presence of overdispersion or random effects (Gutierrez, Carter, and Drukker 2001).
$>$ Example 1
We have data on infants born with low birthweights along with the characteristics of the mother (Hosmer, Lemeshow, and Sturdivant 2013; see also [R] logistic). We fit the following model:


| low | Odds Ratio | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| age | .9732636 | .0354759 | -0.74 | 0.457 | .9061578 | 1.045339 |
| lwt | .9849634 | .0068217 | -2.19 | 0.029 | .9716834 | .9984249 |
| race |  |  |  |  |  |  |
| black | 3.534767 | 1.860737 | 2.40 | 0.016 | 1.259736 | 9.918406 |
| other | 2.368079 | 1.039949 | 1.96 | 0.050 | 1.001356 | 5.600207 |
|  |  |  |  |  |  |  |
| smoke | 2.517698 | 1.00916 | 2.30 | 0.021 | 1.147676 | 5.523162 |
| ptl | 1.719161 | .5952579 | 1.56 | 0.118 | .8721455 | 3.388787 |
| ht | 6.249602 | 4.322408 | 2.65 | 0.008 | 1.611152 | 24.24199 |
| ui | 2.1351 | .9808153 | 1.65 | 0.099 | .8677528 | 5.2534 |
| _cons | 1.586014 | 1.910496 | 0.38 | 0.702 | .1496092 | 16.8134 |

We now wish to test the constraint that the coefficients on age, lwt, ptl, and ht are all zero or, equivalently here, that the odds ratios are all 1 . One solution is to type

```
. test age lwt ptl ht
(1) [low] age = 0
( 2) [low]lwt = 0
( 3) [low]ptl = 0
(4) [low]ht = 0
chi2( 4) = 12.38
Prob > chi2 = 0.0147
```

This test is based on the inverse of the information matrix and is therefore based on a quadratic approximation to the likelihood function; see $[R]$ test. A more precise test would be to refit the model, applying the proposed constraints, and then calculate the likelihood-ratio test.

We first save the current model:

```
. estimates store full
```

We then fit the constrained model, which here is the model omitting age, lwt, ptl, and ht:

| - logistic low i.race smoke ui |  |  |  |
| :--- | :--- | :--- | :--- |
| Logistic regression | Number of obs | $=$ | 189 |
|  | LR chi2 (4) | $=$ | 18.80 |
|  | Prob > chi2 | $=$ | 0.0009 |
| Log likelihood $=-107.93404$ | Pseudo R2 | $=$ | 0.0801 |


| low | Odds Ratio | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :--- | :--- | :--- | :--- | :--- |
| race |  |  |  |  |  |  |
| black | 3.052746 | 1.498087 | 2.27 | 0.023 | 1.166747 | 7.987382 |
| other | 2.922593 | 1.189229 | 2.64 | 0.008 | 1.316457 | 6.488285 |
|  |  |  |  |  |  |  |
| smoke | 2.945742 | 1.101838 | 2.89 | 0.004 | 1.415167 | 6.131715 |
| ui | 2.419131 | 1.047359 | 2.04 | 0.041 | 1.035459 | 5.651788 |
| _cons | .1402209 | .0512295 | -5.38 | 0.000 | .0685216 | .2869447 |

That done, lrtest compares this model with the model we previously stored:

```
. lrtest full .
Likelihood-ratio test LR chi2(4) = 14.42
(Assumption: . nested in full) Prob > chi2 = 0.0061
```

Let's compare results. test reported that age, lwt, ptl, and ht were jointly significant at the $1.5 \%$ level; lrtest reports that they are significant at the $0.6 \%$ level. Given the quadratic approximation made by test, we could argue that lrtest's results are more accurate.
lrtest explicates the assumption that, from a comparison of the degrees of freedom, it has assessed that the last fit model (.) is nested within the model stored as full. In other words, full is the unconstrained model and . is the constrained model.

The names in "(Assumption: . nested in full)" are actually links. Click on a name, and the results for that model are replayed.

Aside: The nestreg command provides a simple syntax for performing likelihood-ratio tests for nested model specifications; see [R] nestreg. In the previous example, we fit a full logistic model, used estimates store to store the full model, fit a constrained logistic model, and used lrtest to report a likelihood-ratio test between two models. To do this with one call to nestreg, use the lrtable option.

## Technical note

lrtest determines the degrees of freedom of a model as the rank of the (co)variance matrix $e(V)$. There are two issues here. First, the numerical determination of the rank of a matrix is a subtle problem that can, for instance, be affected by the scaling of the variables in the model. The rank of a matrix depends on the number of (independent) linear combinations of coefficients that sum exactly to zero. In the world of numerical mathematics, it is hard to tell whether a very small number is really nonzero or is a real zero that happens to be slightly off because of roundoff error from the finite precision with which computers make floating-point calculations. Whether a small number is being classified as one or the other, typically on the basis of a threshold, affects the determined degrees of freedom. Although Stata generally makes sensible choices, it is bound to make mistakes occasionally. The moral of this story is to make sure that the calculated degrees of freedom is as you expect before interpreting the results.

## Technical note

A second issue involves regress and related commands such as anova. Mainly for historical reasons, regress does not treat the residual variance, $\sigma^{2}$, the same way that it treats the regression coefficients. Type estat vce after regress, and you will see the regression coefficients, not $\widehat{\sigma}^{2}$. Most estimation commands for models with ancillary parameters (for example, streg and heckman) treat all parameters as equals. There is nothing technically wrong with regress here; we are usually focused on the regression coefficients, and their estimators are uncorrelated with $\widehat{\sigma}^{2}$. But, formally, $\sigma^{2}$ adds a degree of freedom to the model, which does not matter if you are comparing two regression models by a likelihood-ratio test. This test depends on the difference in the degrees of freedom, and hence being "off by 1 " in each does not matter. But, if you are comparing a regression model with a larger model-for example, a heteroskedastic regression model fit by arch-the automatic determination of the degrees of freedom is incorrect, and you must specify the $\mathrm{df}(\#)$ option.

## > Example 2

Returning to the low-birthweight data in the example 1, we now wish to test that the coefficient on 2.race (black) is equal to that on 3.race (other). The base model is still stored under the name full, so we need only fit the constrained model and perform the test. With $z$ as the index of the logit model, the base model is

$$
z=\beta_{0}+\beta_{1} \text { age }+\beta_{2} \text { lwt }+\beta_{3} 2 . \text { race }+\beta_{4} 3 . \text { race }+\cdots
$$

If $\beta_{3}=\beta_{4}$, this can be written as

$$
z=\beta_{0}+\beta_{1} \text { age }+\beta_{2} \text { lwt }+\beta_{3}(2 . \text { race }+3 . \text { race })+\cdots
$$

We can fit the constrained model as follows:

```
. constraint 1 2.race = 3.race
. logistic low age lwt i.race smoke ptl ht ui, constraints(1)
```

| Logistic regression | Number of obs | $=$ | 189 |
| :--- | :--- | :--- | :--- |
|  | Wald chi2(7) | $=$ | 25.17 |
| Log likelihood $=-100.9997$ |  | Prob > chi2 | $=$ |

( 1) [low]2.race - [low]3.race $=0$

| low | Odds Ratio | Std. Err. | $z$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :--- | :---: | :---: | :---: | :---: |
| age | .9716799 | .0352638 | -0.79 | 0.429 | .9049649 | 1.043313 |
| lwt | .9864971 | .0064627 | -2.08 | 0.038 | .9739114 | .9992453 |
| race |  |  |  |  |  |  |
| black | 2.728186 | 1.080207 | 2.53 | 0.011 | 1.255586 | 5.927907 |
| other | 2.728186 | 1.080207 | 2.53 | 0.011 | 1.255586 | 5.927907 |
|  |  |  |  |  |  |  |
| smoke | 2.664498 | 1.052379 | 2.48 | 0.013 | 1.228633 | 5.778414 |
| ptl | 1.709129 | .5924776 | 1.55 | 0.122 | .8663666 | 3.371691 |
| ht | 6.116391 | 4.215585 | 2.63 | 0.009 | 1.58425 | 23.61385 |
| ui | 2.09936 | .9699702 | 1.61 | 0.108 | .8487997 | 5.192407 |
| _cons | 1.309371 | 1.527398 | 0.23 | 0.817 | .1330839 | 12.8825 |

Comparing this model with our original model, we obtain

```
. lrtest full .
Likelihood-ratio test
(Assumption: . nested in full)
```

| LR chi2 (1) $=$ | 0.55 |
| :--- | ---: |
| Prob $>$ chi2 $=$ | 0.4577 |

By comparison, typing test 2. race $=3$.race after fitting our base model results in a significance level of 0.4572 . Alternatively, we can first store the restricted model, here using the name equal. Next lrtest is invoked specifying the names of the restricted and unrestricted models (we do not care about the order). This time, we also add the option stats requesting a table of model statistics, including the model selection indices AIC and BIC.
. estimates store equal
. lrtest equal full, stats
Likelihood-ratio test LR chi2(1) = 0.55
(Assumption: equal nested in full) Prob > chi2 = 0.4577

| Model | Obs | ll(null) | ll(model) | df | AIC | BIC |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| equal | 189 |  | (10. | -100.9997 | 8 | 217.9994 |
| full | 189 | -117.336 | -100.724 | 9 | 219.448 | 248.9334 |

Note: $N=O b s$ used in calculating BIC; see [R] BIC note

## Composite models

lrtest supports composite models; that is, models that can be fit by fitting a series of simpler models or by fitting models on subsets of the data. Theoretically, a composite model is one in which the likelihood function, $L(\theta)$, of the parameter vector, $\theta$, can be written as the product

$$
L(\theta)=L_{1}\left(\theta_{1}\right) \times L_{2}\left(\theta_{2}\right) \times \cdots \times L_{k}\left(\theta_{k}\right)
$$

of likelihood terms with $\theta=\left(\theta_{1}, \ldots, \theta_{k}\right)$ a partitioning of the full parameter vector. In such a case, the full-model likelihood $L(\theta)$ is maximized by maximizing the likelihood terms $L_{j}\left(\theta_{j}\right)$ in turn. Obviously, $\log L(\widehat{\theta})=\sum_{j=1}^{k} \log L_{j}\left(\widehat{\theta}_{j}\right)$. The degrees of freedom for the composite model is obtained as the sum of the degrees of freedom of the constituting models.

## > Example 3

As an example of the application of composite models, we consider a test of the hypothesis that the coefficients of a statistical model do not differ between different portions ("regimes") of the covariate space. Economists call a test for such a hypothesis a Chow test.

We continue the analysis of the data on children of low birthweight by using logistic regression modeling and study whether the regression coefficients are the same among the three races: white, black, and other. A likelihood-ratio Chow test can be obtained by fitting the logistic regression model for each of the races and then comparing the combined results with those of the model previously stored as full. Because the full model included dummies for the three races, this version of the Chow test allows the intercept of the logistic regression model to vary between the regimes (races).

| . logistic low age lwt smoke ptl ht ui if 1.race, nolog |  |  |  |
| :--- | :--- | :--- | :--- |
| Logistic regression | Number of obs | $=$ | 96 |
|  | LR chi2 $(6)$ | $=$ | 13.86 |
|  |  | Prob $>$ chi2 | $=$ |
| Log likelihood $=-45.927061$ |  | Pseudo R2 | $=$ |


| low | Odds Ratio | Std. Err. | $\mathbf{z}$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | :---: | ---: | ---: |
| age | .9869674 | .0527757 | -0.25 | 0.806 | .8887649 | 1.096021 |
| lwt | .9900874 | .0106101 | -0.93 | 0.353 | .9695089 | 1.011103 |
| smoke | 4.208697 | 2.680133 | 2.26 | 0.024 | 1.20808 | 14.66222 |
| ptl | 1.592145 | .7474264 | 0.99 | 0.322 | .6344379 | 3.995544 |
| ht | 2.900166 | 3.193537 | 0.97 | 0.334 | .3350554 | 25.1032 |
| ui | 1.229523 | .9474768 | 0.27 | 0.789 | .2715165 | 5.567715 |
| _cons | .4891008 | .993785 | -0.35 | 0.725 | .0091175 | 26.23746 |

```
. estimates store white
```

. logistic low age lwt smoke ptl ht ui if 2.race, nolog

| Logistic regression | Number of obs | $=$ | 26 |
| :--- | :--- | :--- | ---: |
|  | LR chi2 $(6)$ | $=$ | 10.12 |
|  | Prob $>$ chi2 | $=$ | 0.1198 |
| Log likelihood $=-12.654157$ | Pseudo R2 | $=$ | 0.2856 |


| low | Odds Ratio | Std. Err. | $\mathbf{z}$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | :---: | :---: | :---: |
| age | .8735313 | .1377846 | -0.86 | 0.391 | .6412332 | 1.189983 |
| lwt | .9747736 | .016689 | -1.49 | 0.136 | .9426065 | 1.008038 |
| smoke | 16.50373 | 24.37044 | 1.90 | 0.058 | .9133647 | 298.2083 |
| ptl | 4.866916 | 9.33151 | 0.83 | 0.409 | .1135573 | 208.5895 |
| ht | 85.05605 | 214.6382 | 1.76 | 0.078 | .6049308 | 11959.27 |
| ui | 67.61338 | 133.3313 | 2.14 | 0.033 | 1.417399 | 3225.322 |
| _cons | 48.7249 | 169.9216 | 1.11 | 0.265 | .0523961 | 45310.94 |


| Logistic regression | Number of obs | = | 67 |
| :---: | :---: | :---: | :---: |
|  | LR chi2(6) | = | 14.06 |
|  | Prob > chi2 | = | 0.0289 |
| Log likelihood = -37.228444 | Pseudo R2 | = | 0.1589 |


| low | Odds Ratio | Std. Err. | $z$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | :---: | :---: | ---: |
| age | .9263905 | .0665386 | -1.06 | 0.287 | .8047407 | 1.06643 |
| lwt | .9724499 | .015762 | -1.72 | 0.085 | .9420424 | 1.003839 |
| smoke | .7979034 | .6340585 | -0.28 | 0.776 | .1680885 | 3.787586 |
| ptl | 2.845675 | 1.777944 | 1.67 | 0.094 | .8363053 | 9.682908 |
| ht | 7.767503 | 10.00537 | 1.59 | 0.112 | .6220764 | 96.98826 |
| ui | 2.925006 | 2.046473 | 1.53 | 0.125 | .7423107 | 11.52571 |
| _cons | 49.09444 | 113.9165 | 1.68 | 0.093 | .5199275 | 4635.769 |

. estimates store other
We are now ready to perform the likelihood-ratio Chow test:

| . Irtest (full) (white black other), stats |  |
| :--- | :--- |
| Likelihood-ratio test | LR chi2(12) $=$ |
|  | Prob $>$ chi2 $=$ |
|  | 0.8310 |

Assumption: (full) nested in (white, black, other)

| Model | Obs | ll(null) | ll(model) | df | AIC | BIC |
| ---: | ---: | ---: | :---: | :---: | ---: | ---: |
| full | 189 | -117.336 | -100.724 | 9 | 219.448 | 248.6237 |
| white | 96 | -52.85752 | -45.92706 | 7 | 105.8541 | 123.8046 |
| black | 26 | -17.71291 | -12.65416 | 7 | 39.30831 | 48.11499 |
| other | 67 | -44.26039 | -37.22844 | 7 | 88.45689 | 103.8897 |

We cannot reject the hypothesis that the logistic regression model applies to each of the races at any reasonable significance level. By specifying the stats option, we can verify the degrees of freedom of the test: $12=7+7+7-9$. We can obtain the same test by fitting an expanded model with interactions between all covariates and race.
. logistic low race\#\#c. (age lwt smoke ptl ht ui)

| Logistic regression | Number of obs | $=$ | 189 |
| :--- | :--- | :--- | ---: |
|  | LR chi2 20$)$ | $=$ | 43.05 |
|  | Prob > chi2 | $=$ | 0.0020 |
| Log likelihood $=-95.809661$ | Pseudo R2 | $=$ | 0.1835 |


| low | Odds Ratio | Std. Err. | z | $P>\|z\|$ | [95\% Con | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| race |  |  |  |  |  |  |
| black | 99.62137 | 402.0829 | 1.14 | 0.254 | . 0365434 | 271578.9 |
| other | 100.3769 | 309.586 | 1.49 | 0.135 | . 2378638 | 42358.38 |
| age | . 9869674 | . 0527757 | -0.25 | 0.806 | . 8887649 | 1.096021 |
| lwt | . 9900874 | . 0106101 | -0.93 | 0.353 | . 9695089 | 1.011103 |
| smoke | 4.208697 | 2.680133 | 2.26 | 0.024 | 1.20808 | 14.66222 |
| ptl | 1.592145 | . 7474264 | 0.99 | 0.322 | . 6344379 | 3.995544 |
| ht | 2.900166 | 3.193537 | 0.97 | 0.334 | . 3350554 | 25.1032 |
| ui | 1.229523 | . 9474768 | 0.27 | 0.789 | . 2715165 | 5.567715 |
| race\#c.age |  |  |  |  |  |  |
| black | . 885066 | . 1474079 | -0.73 | 0.464 | . 638569 | 1.226714 |
| other | . 9386232 | . 0840486 | -0.71 | 0.479 | . 7875366 | 1.118695 |
| race\#c.lwt |  |  |  |  |  |  |
| black | . 9845329 | . 0198857 | -0.77 | 0.440 | . 9463191 | 1.02429 |
| other | . 9821859 | . 0190847 | -0.93 | 0.355 | . 9454839 | 1.020313 |
| race\#c.smoke |  |  |  |  |  |  |
| black | 3.921338 | 6.305992 | 0.85 | 0.395 | . 167725 | 91.67917 |
| other | . 1895844 | . 1930601 | -1.63 | 0.102 | . 025763 | 1.395113 |
| race\#c.ptl |  |  |  |  |  |  |
| black | 3.05683 | 6.034089 | 0.57 | 0.571 | . 0638301 | 146.3918 |
| other | 1.787322 | 1.396789 | 0.74 | 0.457 | . 3863582 | 8.268285 |
| race\#c.ht |  |  |  |  |  |  |
| black | 29.328 | 80.7482 | 1.23 | 0.220 | . 1329492 | 6469.623 |
| other | 2.678295 | 4.538712 | 0.58 | 0.561 | . 0966916 | 74.18702 |
| race\#c.ui |  |  |  |  |  |  |
| black | 54.99155 | 116.4274 | 1.89 | 0.058 | . 8672471 | 3486.977 |
| other | 2.378976 | 2.476124 | 0.83 | 0.405 | . 309335 | 18.29579 |
| _cons | . 4891008 | . 993785 | -0.35 | 0.725 | . 0091175 | 26.23746 |

```
. lrtest full .
Likelihood-ratio test LR chi2(12) = 9.83
(Assumption: full nested in .) Prob > chi2 = 0.6310
```

Applying lrtest for the full model against the model with all interactions yields the same test statistic and $p$-value as for the full model against the composite model for the three regimes. Here the specification of the model with interactions was convenient, and logistic had no problem computing the estimates for the expanded model. In models with more complicated likelihoods, such as Heckman's selection model (see [R] heckman) or complicated survival-time models (see [ST] streg), fitting the models with all interactions may be numerically demanding and may be much more time consuming than fitting a series of models separately for each regime.

Given the model with all interactions, we could also test the hypothesis of no differences among the regions (races) by a Wald version of the Chow test by using the testparm command; see $[R]$ test.

```
. testparm race#c.(age lwt smoke ptl ht ui)
( 1) [low]2.race#c.age = 0
( 2) [low]3.race#c.age = 0
( 3) [low]2.race#c.lwt = 0
(4) [low]3.race#c.lwt = 0
( 5) [low]2.race#c.smoke = 0
( 6) [low]3.race#c.smoke = 0
( 7) [low]2.race#c.ptl = 0
( 8) [low]3.race#c.ptl = 0
( 9) [low]2.race#c.ht = 0
(10) [low]3.race#c.ht = 0
(11) [low]2.race#c.ui = 0
(12) [low]3.race#c.ui = 0
    chi2( 12) = 8.24
        Prob > chi2 = 0.7663
```

We conclude that, here, the Wald version of the Chow test is similar to the likelihood-ratio version of the Chow test.

## Stored results

lrtest stores the following in r() :
Scalars

$$
\begin{array}{ll}
\mathrm{r}(\mathrm{p}) & \text { level of significance } \\
\mathrm{r}(\mathrm{df}) & \text { degrees of freedom } \\
\mathrm{r}(\mathrm{chi2}) & \text { LR test statistic }
\end{array}
$$

Programmers wishing their estimation commands to be compatible with lrtest should note that lrtest requires that the following results be returned:

```
e(cmd) name of estimation command
e(11) log likelihood
e(V) variance-covariance matrix of the estimators
e(N) number of observations
```

lrtest also verifies that e(N), e(ll_0), and e(depvar) are consistent between two noncomposite models.

## Methods and formulas

Let $L_{0}$ and $L_{1}$ be the log-likelihood values associated with the full and constrained models, respectively. The test statistic of the likelihood-ratio test is $\mathrm{LR}=-2\left(L_{1}-L_{0}\right)$. If the constrained model is true, LR is approximately $\chi^{2}$ distributed with $d_{0}-d_{1}$ degrees of freedom, where $d_{0}$ and $d_{1}$ are the model degrees of freedom associated with the full and constrained models, respectively (Greene 2012, 526-527).
lrtest determines the degrees of freedom of a model as the rank of $e(V)$, computed as the number of nonzero diagonal elements of invsym(e(V)).

## References

Greene, W. H. 2012. Econometric Analysis. 7th ed. Upper Saddle River, NJ: Prentice Hall.
Gutierrez, R. G., S. L. Carter, and D. M. Drukker. 2001. sg160: On boundary-value likelihood-ratio tests. Stata Technical Bulletin 60: 15-18. Reprinted in Stata Technical Bulletin Reprints, vol. 10, pp. 269-273. College Station, TX: Stata Press.

Hosmer, D. W., Jr., S. A. Lemeshow, and R. X. Sturdivant. 2013. Applied Logistic Regression. 3rd ed. Hoboken, NJ: Wiley.

Kleinbaum, D. G., and M. Klein. 2010. Logistic Regression: A Self-Learning Text. 3rd ed. New York: Springer.
Pérez-Hoyos, S., and A. Tobías. 1999. sg111: A modified likelihood-ratio test command. Stata Technical Bulletin 49: 24-25. Reprinted in Stata Technical Bulletin Reprints, vol. 9, pp. 171-173. College Station, TX: Stata Press.

Wang, Z. 2000. sg133: Sequential and drop one term likelihood-ratio tests. Stata Technical Bulletin 54: 46-47. Reprinted in Stata Technical Bulletin Reprints, vol. 9, pp. 332-334. College Station, TX: Stata Press.

## Also see

[R] test - Test linear hypotheses after estimation
[R] testnl - Test nonlinear hypotheses after estimation
[R] nestreg - Nested model statistics

## Title

Isens - Graph sensitivity and specificity versus probability cutoff

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | Reference |
| Also see |  |  |  |

## Syntax

lsens [depvar] [if] [in] [weight] [, options]
options Description
Main
all
graph all observations in the data
genprob (varname) create variable containing probability cutoffs
gensens (varname)
genspec (varname)
replace
nograph
Advanced
beta (matname) row vector containing model coefficients
Plot
connect_options
Add plots
addplot (plot) add other plots to the generated graph
Y axis, X axis, Titles, Legend, Overall
twoway_options
any options other than by () documented in [G-3] twoway_options
fweights are allowed; see [U] 11.1.6 weight.
lsens is not appropriate after the svy prefix.

## Menu

Statistics $>$ Binary outcomes $>$ Postestimation $>$ Sensitivity/specificity plot

## Description

lsens graphs sensitivity and specificity versus probability cutoff and optionally creates new variables containing these data.
lsens requires that the current estimation results be from logistic, logit, probit, or ivprobit; see $[R]$ logistic, $[R]$ logit, $[R]$ probit, or $[R]$ ivprobit.

## Options

$\qquad$ Main
all requests that the statistic be computed for all observations in the data, ignoring any if or in restrictions specified by the estimation command.
genprob (varname), gensens (varname), and genspec (varname) specify the names of new variables created to contain, respectively, the probability cutoffs and the corresponding sensitivity and specificity.
replace requests that existing variables specified for genprob(), gensens(), or genspec() be overwritten.
nograph suppresses graphical output.

Advanced
beta (matname) specifies a row vector containing model coefficients. The columns of the row vector must be labeled with the corresponding names of the independent variables in the data. The dependent variable depvar must be specified immediately after the command name. See Models other than the last fitted model later in this entry.

Plot
connect_options affect the rendition of the plotted points connected by lines; see connect_options in [G-2] graph twoway scatter.
$\qquad$ Add plots
addplot (plot) provides a way to add other plots to the generated graph. See [G-3] addplot_option.
$\qquad$ Y axis, X axis, Titles, Legend, Overall
twoway_options are any of the options documented in [G-3] twoway_options, excluding by (). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

## Remarks and examples

Remarks are presented under the following headings:

```
Introduction
Models other than the last fitted model
```


## Introduction

lsens plots sensitivity and specificity; it plots both sensitivity and specificity versus probability cutoff $c$. The graph is equivalent to what you would get from estat classification (see $[\mathrm{R}]$ estat classification) if you varied the cutoff probability $c$ from 0 to 1 .

## > Example 1

We illustrate lsens after logistic; see [R] logistic.

```
. use http://www.stata-press.com/data/r13/lbw
(Hosmer & Lemeshow data)
. logistic low age i.race smoke ui
    (output omitted)
. lsens
```


lsens optionally creates new variables containing the probability cutoff, sensitivity, and specificity.

```
. lsens, genprob(p) gensens(sens) genspec(spec) nograph
```

The variables created will have $M+2$ distinct nonmissing values: one for each of the $M$ covariate patterns, one for $c=0$, and another for $c=1$. Values are recorded for $p=0$, for each of the observed predicted probabilities, and for $p=1$. The total number of observations required to do this can be fewer than $\_\mathrm{N}$, the same as $\_\mathrm{N}$, or $\_\mathrm{N}+1$, or $\_\mathrm{N}+2$. If more observations are added, they are added at the end of the dataset and the values of the original variables are set to missing in the added observations. How the values added align with existing observations is irrelevant.

## - Technical note

logistic, logit, probit, or ivprobit and lsens keep track of the estimation sample. If you type, for instance, logistic ... if $x==1$, then when you type lsens, the statistics will be calculated on the $x==1$ subsample of the data automatically.

You should specify if or in with lsens only when you wish to produce graphs and calculate statistics for a set of observations other than the estimation sample.

If the logistic model was fit with fweights, lsens properly accounts for the weights in its calculations. You do not have to specify the weights when you run lsens. Weights should be specified with lsens only when you wish to use a different set of weights.

## Models other than the last fitted model

By default, lsens uses the last model fit. You may also directly specify the model to lsens by inputting a vector of coefficients with the beta() option and passing the name of the dependent variable depvar to lsens.

## > Example 2

Suppose that someone publishes the following logistic model of low birthweight:
$\operatorname{Pr}($ low $=1)=F(-0.02$ age -0.01 lwt $+1.3 \mathrm{black}+1.1$ smoke $+0.5 \mathrm{ptl}+1.8 \mathrm{ht}+0.8 \mathrm{ui}+0.5)$
where $F$ is the cumulative logistic distribution. These coefficients are not odds ratios; they are the equivalent of what logit produces.

We can see whether this model fits our data. First we enter the coefficients as a row vector and label its columns with the names of the independent variables plus _cons for the constant (see $[\mathrm{P}]$ matrix define and $[\mathrm{P}]$ matrix rownames).

```
. use http://www.stata-press.com/data/r13/lbw3, clear
(Hosmer & Lemeshow data)
. matrix input b = (-0.02, -.01, 1.3, 1.1, .5, 1.8, .8, .5)
. matrix colnames b = age lwt black smoke ptl ht ui _cons
```

We can use lroc (see [R] lroc) to examine the predictive ability of the model:

```
. lroc low, beta(b) nograph
Logistic model for low
number of observations = 189
area under ROC curve = 0.7275
```

The area under the curve indicates that this model does have some predictive power. We can obtain a graph of sensitivity and specificity as a function of the cutoff probability by typing

```
. lsens low, beta(b)
```



## Stored results

lsens stores the following in $r()$ :
Scalars

$$
r(N) \quad \text { number of observations }
$$

## Methods and formulas

Let $j$ index observations and $c$ be the cutoff probability. Let $p_{j}$ be the predicted probability of a positive outcome and $y_{j}$ be the actual outcome, which we will treat as 0 or 1 , although Stata treats it as 0 and non- 0 , excluding missing observations.

A prediction is classified as positive if $p_{j} \geq c$ and otherwise is classified as negative. The classification is correct if it is positive and $y_{j}=1$ or if it is negative and $y_{j}=0$.

Sensitivity is the fraction of $y_{j}=1$ observations that are correctly classified. Specificity is the percentage of $y_{j}=0$ observations that are correctly classified.

## Reference

Hosmer, D. W., Jr., S. A. Lemeshow, and R. X. Sturdivant. 2013. Applied Logistic Regression. 3rd ed. Hoboken, NJ: Wiley.

[^45]
## Title

Iv - Letter-value displays

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |
| Also see |  |  |  |

## Syntax

lv [varlist] [if] [in] [, generate tail(\#)]
by is allowed; see [D] by.

## Menu

Statistics $>$ Summaries, tables, and tests $>$ Distributional plots and tests $>$ Letter-value display

## Description

lv shows a letter-value display (Tukey 1977, 44-49; Hoaglin 1983) for each variable in varlist. If no variables are specified, letter-value displays are shown for each numeric variable in the data.

## Options

Main
generate adds four new variables to the data: _mid, containing the midsummaries; _spread, containing the spreads; _psigma, containing the pseudosigmas; and _z2, containing the squared values from a standard normal distribution corresponding to the particular letter value. If the variables _mid, _spread, _psigma, and _z2 already exist, their contents are replaced. At most, only the first 11 observations of each variable are used; the remaining observations contain missing. If varlist specifies more than one variable, the newly created variables contain results for the last variable specified. The generate option may not be used with the by prefix.
tail(\#) indicates the inverse of the tail density through which letter values are to be displayed: 2 corresponds to the median (meaning half in each tail), 4 to the fourths (roughly the 25th and 75th percentiles), 8 to the eighths, and so on. \# may be specified as $4,8,16,32,64,128,256,512$, or 1,024 and defaults to a value of \# that has corresponding depth just greater than 1 . The default is taken as 1,024 if the calculation results in a number larger than 1,024 . Given the intelligent default, this option is rarely specified.

## Remarks and examples

Letter-value displays are a collection of observations drawn systematically from the data, focusing especially on the tails rather than the middle of the distribution. The displays are called letter-value displays because letters have been (almost arbitrarily) assigned to tail densities:

| Letter | Tail area | Letter | Tail area |
| :--- | :--- | :--- | :--- |
| M | $1 / 2$ | B | $1 / 64$ |
| F | $1 / 4$ | A | $1 / 128$ |
| E | $1 / 8$ | Z | $1 / 256$ |
| D | $1 / 16$ | Y | $1 / 512$ |
| C | $1 / 32$ | X | $1 / 1024$ |

We have data on the mileage ratings of 74 automobiles. To obtain a letter-value display, we type


The decimal points can be made to line up and thus the output made more readable by specifying a display format for the variable; see [U] 12.5 Formats: Controlling how data are displayed.

| \# 74 | Mileage (mpg) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| M $\quad 37.5$ |  | 20.00 |  | spread | pseudosigma |
| F 19 | 18.00 | 21.50 | 25.00 | 7.00 | 5.22 |
| E 10 | 15.00 | 21.50 | 28.00 | 13.00 | 5.77 |
| D 5.5 | 14.00 | 22.25 | 30.50 | 16.50 | 5.58 |
| C 3 | 14.00 | 24.50 | 35.00 | 21.00 | 5.83 |
| B 2 | 12.00 | 23.50 | 35.00 | 23.00 | 5.73 |
| A 1.5 | 12.00 | 25.00 | 38.00 | 26.00 | 6.04 |
| 1 | 12.00 | 26.50 | 41.00 | 29.00 | 6.17 |
|  |  |  |  | \# below | \# above |
| inner fence | 7.50 |  | 35.50 | 0 | 1 |
| outer fence | -3.00 |  | 46.00 | 0 | 0 |

At the top, the number of observations is indicated as 74 . The first line shows the statistics associated with M, the letter value that puts half the density in each tail, or the median. The median has depth 37.5 (that is, in the ordered data, M is 37.5 observations in from the extremes) and has value 20 . The next line shows the statistics associated with F or the fourths. The fourths have depth 19 (that is, in the ordered data, the lower fourth is observation 19, and the upper fourth is observation $74-19+1$ ), and the values of the lower and upper fourths are 18 and 25 . The number in the middle is the point halfway between the fourths-called a midsummary. If the distribution were perfectly symmetric, the midsummary would equal the median. The spread is the difference between the lower and upper summaries $(25-18=7)$. For fourths, half the data lie within a $7-\mathrm{mpg}$ band. The pseudosigma is a calculation of the standard deviation using only the lower and upper summaries and assuming that the variable is normally distributed. If the data really were normally distributed, all the pseudosigmas would be roughly equal.

After the letter values, the line labeled with depth 1 reports the minimum and maximum values. Here the halfway point between the extremes is 26.5 , which is greater than the median, indicating that 41 is more extreme than 12, at least relative to the median. And with each letter value, the midsummaries are increasing - our data are skewed. The pseudosigmas are also increasing, indicating
that the data are spreading out relative to a normal distribution, although, given the evident skewness, this elongation may be an artifact of the skewness.

At the end is an attempt to identify outliers, although the points so identified are merely outside some predetermined cutoff. Points outside the inner fence are called outside values or mild outliers. Points outside the outer fence are called severe outliers. The inner fence is defined as $(3 / 2) \mathrm{IQR}$ and the outer fence as 3 IQR above and below the F summaries, where the interquartile range ( IQR ) is the spread of the fourths.

## Technical note

The form of the letter-value display has varied slightly with different authors. Iv displays appear as described by Hoaglin (1983) but as modified by Emerson and Stoto (1983), where they included the midpoint of each of the spreads. This format was later adopted by Hoaglin (1985). If the distribution is symmetric, the midpoints will all be roughly equal. On the other hand, if the midpoints vary systematically, the distribution is skewed.

The pseudosigmas are obtained from the lower and upper summaries for each letter value. For each letter value, they are the standard deviation a normal distribution would have if its spread for the given letter value were to equal the observed spread. If the pseudosigmas are all roughly equal, the data are said to have neutral elongation. If the pseudosigmas increase systematically, the data are said to be more elongated than a normal, that is, have thicker tails. If the pseudosigmas decrease systematically, the data are said to be less elongated than a normal, that is, have thinner tails.

Interpretation of the number of mild and severe outliers is more problematic. The following discussion is drawn from Hamilton (1991):

Obviously, the presence of any such outliers does not rule out that the data have been drawn from a normal distribution; in large datasets, there will most certainly be observations outside (3/2)IQR and 3IQR. Severe outliers, however, make up about two per million ( $0.0002 \%$ ) of a normal population. In samples, they lie far enough out to have substantial effects on means, standard deviations, and other classical statistics. The $0.0002 \%$, however, should be interpreted carefully; outliers appear more often in small samples than one might expect from population proportions because of sampling variation in estimated quartiles. Monte Carlo simulation by Hoaglin, Iglewicz, and Tukey (1986) obtained these results on the percentages and numbers of outliers in random samples from a normal population:

|  | percentage |  | number |  |
| :---: | :---: | :---: | :---: | :---: |
| n | any outliers | severe | any outliers | severe |
| 10 | 2.83 | .362 | .283 | .0362 |
| 20 | 1.66 | .074 | .332 | .0148 |
| 50 | 1.15 | .011 | .575 | .0055 |
| 100 | .95 | .002 | .95 | .002 |
| 200 | .79 | .001 | 1.58 | .002 |
| 300 | .75 | .001 | 2.25 | .003 |
| $\infty$ | .70 | .0002 | $\infty$ | $\infty$ |

Thus the presence of any severe outliers in samples of less than 300 is sufficient to reject normality. Hoaglin, Iglewicz, and Tukey (1981) suggested the approximation $0.00698+0.4 / n$ for the fraction of mild outliers in a sample of size $n$ or, equivalently, $0.00698 n+0.4$ for the number of outliers.

## > Example 2

The generate option adds the _mid, _spread, _psigma, and _z2 variables to our data, making possible many of the diagnostic graphs suggested by Hoaglin (1985).

```
. lv mpg, generate
    (output omitted)
. list _mid _spread _psigma _z2 in 1/12
```

|  | _mid | _spread | _psigma | _z2 |
| :---: | :---: | :---: | :---: | :---: |
| 1. | 20 | . | . |  |
| 2. | 21.5 | 7 | 5.216359 | . 4501955 |
| 3. | 21.5 | 13 | 5.771728 | 1.26828 |
| 4. | 22.25 | 16.5 | 5.576303 | 2.188846 |
| 5. | 24.5 | 21 | 5.831039 | 3.24255 |
| 6. | 23.5 | 23 | 5.732448 | 4.024532 |
| 7. | 25 | 26 | 6.040635 | 4.631499 |
| 8. | . | . | . |  |
| 9. | - | - | - |  |
| 10. | . | . | . |  |
| 11. | 26.5 | 29 | 6.16562 | 5.53073 |
| 12. |  |  | . |  |

Observations 12 through the end are missing for these new variables. The definition of the observations is always the same. The first observation contains the M summary; the second, the F; the third, the E; and so on. Observation 11 always contains the summary for depth 1. Observations $8-10$-corresponding to letter values $\mathrm{Z}, \mathrm{Y}$, and X - contain missing because these statistics were not calculated. We have only 74 observations, and their depth would be 1.

Hoaglin (1985) suggests graphing the midsummary against $z^{2}$. If the distribution is not skewed, the points in the resulting graph will be along a horizontal line:
. scatter _mid _z2


The graph clearly indicates the skewness of the distribution. We might also graph _psigma against _z2 to examine elongation.

## Stored results

lv stores the following in $r()$ :
Scalars

| r(N) | number of observations | r(u_C) | upper 32nd |
| :---: | :---: | :---: | :---: |
| $r(\min )$ | minimum | r(l_B) | lower 64th |
| $r$ (max) | maximum | r(u_B) | upper 64th |
| r (median) | median | r(l_A) | lower 128th |
| r(l_F) | lower 4th | r(u_A) | upper 128th |
| r (u_F) | upper 4th | r(1_Z) | lower 256th |
| r(l_E) | lower 8th | r(u_Z) | upper 256th |
| r(u_E) | upper 8th | r(l_Y) | lower 512th |
| r(1_D) | lower 16th | r(u_Y) | upper 512th |
| r (u_D) | upper 16th | r(1_X) | lower 1024th |
| r(l_C) | lower 32nd | r(u_X) | upper 1024th |

The lower/upper 8ths, 16ths, ..., 1024ths will be defined only if there are sufficient data.

## Methods and formulas

Let $N$ be the number of (nonmissing) observations on $x$, and let $x_{(i)}$ refer to the ordered data when $i$ is an integer. Define $x_{(i+0.5)}=\left(x_{(i)}+x_{(i+1)}\right) / 2$; the median is defined as $x_{\{(N+1) / 2\}}$.

Define $x_{[d]}$ as the pair of numbers $x_{(d)}$ and $x_{(N+1-d)}$, where $d$ is called the depth. Thus $x_{[1]}$ refers to the minimum and maximum of the data. Define $m=(N+1) / 2$ as the depth of the median, $f=(\lfloor m\rfloor+1) / 2$ as the depth of the fourths, $e=(\lfloor f\rfloor+1) / 2$ as the depth of the eighths, and so on. Depths are reported on the far left of the letter-value display. The corresponding fourths of the data are $x_{[f]}$, the eighths are $x_{[e]}$, and so on. These values are reported inside the display. The middle value is defined as the corresponding midpoint of $x_{[\cdot]}$. The spreads are defined as the difference in $x_{[\cdot]}$.

The corresponding point $z_{i}$ on a standard normal distribution is obtained as (Hoaglin 1985, 456-457)

$$
z_{i}= \begin{cases}F^{-1}\left\{\left(d_{i}-1 / 3\right) /(N+1 / 3)\right\} & \text { if } d_{i}>1 \\ F^{-1}\{0.695 /(N+0.390)\} & \text { otherwise }\end{cases}
$$

where $d_{i}$ is the depth of the letter value. The corresponding pseudosigma is obtained as the ratio of the spread to $-2 z_{i}$ (Hoaglin 1985, 431).

Define $\left(F_{l}, F_{u}\right)=x_{[f]}$. The inner fence has cutoffs $F_{l}-\frac{3}{2}\left(F_{u}-F_{l}\right)$ and $F_{u}+\frac{3}{2}\left(F_{u}-F_{l}\right)$. The outer fence has cutoffs $F_{l}-3\left(F_{u}-F_{l}\right)$ and $F_{u}+3\left(F_{u}-F_{l}\right)$.

The inner-fence values reported by lv are almost equal to those used by graph, box to identify outside points. The only difference is that graph uses a slightly different definition of fourths, namely, the 25 th and 75 th percentiles as defined by summarize; see $[R]$ summarize.

## References

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1986. Performance of some resistant rules for outlier labeling. Journal of the American Statistical Association 81: 991-999.

Tukey, J. W. 1977. Exploratory Data Analysis. Reading, MA: Addison-Wesley.

## Also see

[R] diagnostic plots - Distributional diagnostic plots
[R] stem - Stem-and-leaf displays
[R] summarize - Summary statistics

## Title

margins - Marginal means, predictive margins, and marginal effects

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

Also see

## Syntax

margins [marginlist] [if] [in] [weight] [, response_options options]
where marginlist is a list of factor variables or interactions that appear in the current estimation results. The variables may be typed with or without the i. prefix, and you may use any factor-variable syntax:
. margins i.sex i.group i.sex\#i.group
. margins sex group sex\#i.group
. margins sex\#\#group
response_options Description
Main
predict (pred_opt) estimate margins for predict, pred_opt expression(pnl_exp) dydx (varlist)
eyex (varlist)
dyex (varlist)
eydx (varlist)
continuous
estimate margins for pnl_exp
estimate marginal effect of variables in varlist
estimate elasticities of variables in varlist
estimate semielasticity- $d(y) / d(\ln x)$
estimate semielasticity— $d(\ln y) / d(x)$
treat factor-level indicators as continuous
options
Main
grand
At
at (atspec)
atmeans
asbalanced
if/in/over
over (varlist)
subpop (subspec)
Within
within(varlist)
Contrast
contrast_options
Pairwise comparisons
pwcompare_options
add the overall margin; default if no marginlist
estimate margins at specified values of covariates
estimate margins at the means of covariates
treat all factor variables as balanced
estimate margins at unique values of varlist
estimate margins for subpopulation
estimate margins at unique values of the nesting factors in varlist
any options documented in [R] margins, contrast
any options documented in [R] margins, pwcompare

SE
vce(delta) estimate SEs using delta method; the default
vce(unconditional)
nose
Advanced
noweights
noesample
emptycells (empspec)
estimtolerance (tol)
noestimcheck
force
chainrule
nochainrule
Reporting
level(\#)
mcompare (method)
noatlegend
post
display_options
df (\#) estimate SEs allowing for sampling of covariates do not estimate SEs
ignore weights specified in estimation
do not restrict margins to the estimation sample treatment of empty cells for balanced factors specify numerical tolerance used to determine estimable functions; default is estimtolerance (1e-5)
suppress estimability checks
estimate margins despite potential problems use the chain rule when computing derivatives do not use the chain rule
set confidence level; default is level (95)
adjust for multiple comparisons; default is mcompare (noadjust) suppress legend of fixed covariate values
post margins and their VCE as estimation results control columns and column formats, row spacing, line width and factor-variable labeling
use $t$ distribution with \# degrees of freedom for computing $p$-values and confidence intervals
method
noadjust
bonferroni [adjustall] sidak [adjustall]
scheffe

## Description

do not adjust for multiple comparisons; the default
Bonferroni's method; adjust across all terms
Šidák's method; adjust across all terms
Scheffé's method

Time-series operators are allowed if they were used in the estimation.
See at () under Options for a description of atspec.
fweights, aweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
$\mathrm{df}(\#)$ does not appear in the dialog box.

## Menu

Statistics $>$ Postestimation $>$ Marginal means and predictive margins
Statistics $>$ Postestimation $>$ Marginal effects

## Description

Margins are statistics calculated from predictions of a previously fit model at fixed values of some covariates and averaging or otherwise integrating over the remaining covariates.

The margins command estimates margins of responses for specified values of covariates and presents the results as a table.

Capabilities include estimated marginal means, least-squares means, average and conditional marginal and partial effects (which may be reported as derivatives or as elasticities), average and conditional adjusted predictions, and predictive margins.

## Options

Warning: The option descriptions are brief and use jargon. Skip to Remarks and examples if you are reading about margins for the first time.
predict (pred_opt) and expression(pnl_exp) are mutually exclusive; they specify the response. If neither is specified, the response will be the default prediction that would be produced by predict after the underlying estimation command.
predict (pred_opt) specifies the option(s) to be specified with the predict command to produce the variable that will be used as the response. After estimation by logistic, you could specify predict(xb) to obtain linear predictions rather than the predict command's default, the probabilities.
expression(pnl_exp) specifies the response as an expression. See [R] predictnl for a full description of pnl_exp. After estimation by logistic, you might specify expression $(\exp ($ predict $(x b)))$ to use relative odds rather than probabilities as the response. For examples, see Example 12: Margins of a specified expression.
dydx (varlist), eyex (varlist), dyex (varlist), and eydx (varlist) request that margins report derivatives of the response with respect to varlist rather than on the response itself. eyex(), dyex(), and eydx() report derivatives as elasticities; see Expressing derivatives as elasticities.
continuous is relevant only when one of dydx() or eydx() is also specified. It specifies that the levels of factor variables be treated as continuous; see Derivatives versus discrete differences. This option is implied if there is a single-level factor variable specified in dydx() or eydx().
grand specifies that the overall margin be reported. grand is assumed when marginlist is empty.
at (atspec) specifies values for covariates to be treated as fixed.
at (age=20) fixes covariate age to the value specified. at () may be used to fix continuous or factor covariates.
at (age $=20$ sex=1) simultaneously fixes covariates age and sex at the values specified.
at (age $=(20304050)$ ) fixes age first at 20 , then at $30, \ldots$ margins produces separate results for each specified value.
at $($ age $=(20(10) 50))$ does the same as at (age $=(20304050)$ ); that is, you may specify a numlist.
at ((mean) age (median) distance) fixes the covariates at the summary statistics specified. at ( p 25 ) _all) fixes all covariates at their 25 th percentile values. See Syntax of at() for the full list of summary-statistic modifiers.
at ((mean) _all (median) $\times \times 2=1.2 \mathrm{z}=(123)$ ) is read from left to right, with latter specifiers overriding former ones. Thus all covariates are fixed at their means except for x (fixed at its median), $x 2$ (fixed at 1.2 ), and $z$ (fixed first at 1 , then at 2 , and finally at 3 ).
at ((means) _all (asobserved) x2) is a convenient way to set all covariates except $x 2$ to the mean.

Multiple at () options can be specified, and each will produce a different set of margins.
See Syntax of at() for more information.
atmeans specifies that covariates be fixed at their means and is shorthand for at ((mean) _all). atmeans differs from at ((mean) _all) in that atmeans will affect subsequent at () options. For instance,

```
. margins ..., atmeans at((p25) x) at((p75) x)
```

produces two sets of margins with both sets evaluated at the means of all covariates except x .
asbalanced is shorthand for at ((asbalanced) _factor) and specifies that factor covariates be evaluated as though there were an equal number of observations in each level; see Obtaining margins as though the data were balanced. asbalanced differs from at ((asbalanced) _factor) in that asbalanced will affect subsequent at() options in the same way as atmeans does.

[^46]Within
within (varlist) allows for nested designs. varlist contains the nesting variable(s) over which margins are to be estimated. See Obtaining margins with nested designs. As with over (varlist), when within(varlist) is combined with vce(unconditional), each level of the variables in varlist is treated as a subpopulation.
$\sqrt{\text { Contrast }}$
contrast_options are any of the options documented in [R] margins, contrast.

Pairwise comparisons
pwcompare_options are any of the options documented in [R] margins, pwcompare.
$\square$ SE
vce(delta) and vce(unconditional) specify how the VCE and, correspondingly, standard errors are calculated.
vce (delta) is the default. The delta method is applied to the formula for the response and the VCE of the estimation command. This method assumes that values of the covariates used to calculate the response are given or, if all covariates are not fixed using at (), that the data are given.
vce(unconditional) specifies that the covariates that are not fixed be treated in a way that accounts for their having been sampled. The VCE is estimated using the linearization method. This method allows for heteroskedasticity or other violations of distributional assumptions and allows for correlation among the observations in the same manner as vce (robust) and vce (cluster ...), which may have been specified with the estimation command. This method also accounts for complex survey designs if the data are svyset. See Obtaining margins with survey data and representative samples. When you use complex survey data, this method requires that the linearized variance estimation method be used for the model. See [SVY] svy postestimation for an example of margins with replication-based methods.
nose suppresses calculation of the VCE and standard errors. See Requirements for model specification for an example of the use of this option.

## Advanced

noweights specifies that any weights specified on the previous estimation command be ignored by margins. By default, margins uses the weights specified on the estimator to average responses and to compute summary statistics. If weights are specified on the margins command, they override previously specified weights, making it unnecessary to specify noweights. The noweights option is not allowed after svy: estimation when the vce (unconditional) option is specified.
noesample specifies that margins not restrict its computations to the estimation sample used by the previous estimation command. See Example 15: Margins evaluated out of sample.
With the default delta-method VCE, noesample margins may be estimated on samples other than the estimation sample; such results are valid under the assumption that the data used are treated as being given.

You can specify noesample and vce(unconditional) together, but if you do, you should be sure that the data in memory correspond to the original e (sample). To show that you understand that, you must also specify the force option. Be aware that making the vce (unconditional) calculation on a sample different from the estimation sample would be equivalent to estimating the coefficients on one set of data and computing the scores used by the linearization on another set; see [P] _robust.
emptycells(strict) and emptycells (reweight) are relevant only when the asbalanced option is also specified. emptycells() specifies how empty cells are handled in interactions involving factor variables that are being treated as balanced; see Obtaining margins as though the data were balanced.
emptycells(strict) is the default; it specifies that margins involving empty cells be treated as not estimable.
emptycells(reweight) specifies that the effects of the observed cells be increased to accommodate any missing cells. This makes the margin estimable but changes its interpretation. emptycells (reweight) is implied when the within() option is specified.
estimtolerance (tol) specifies the numerical tolerance used to determine estimable functions. The default is estimtolerance ( $1 \mathrm{e}-5$ ).

A linear combination of the model coefficients $z$ is found to be not estimable if

$$
\text { mreldif }(z, z \times H)>t o l
$$

where $H$ is defined in Methods and formulas.
noestimcheck specifies that margins not check for estimability. By default, the requested margins are checked and those found not estimable are reported as such. Nonestimability is usually caused by empty cells. If noestimcheck is specified, estimates are computed in the usual way and reported even though the resulting estimates are manipulable, which is to say they can differ across equivalent models having different parameterizations. See Estimability of margins.
force instructs margins to proceed in some situations where it would otherwise issue an error message because of apparent violations of assumptions. Do not be casual about specifying force. You need to understand and fully evaluate the statistical issues. For an example of the use of force, see Using margins after the estimates use command.
chainrule and nochainrule specify whether margins uses the chain rule when numerically computing derivatives. You need not specify these options when using margins after any official Stata estimator, margins will choose the appropriate method automatically.
Specify nochainrule after estimation by a user-written command. We recommend using nochainrule, even though chainrule is usually safe and is always faster. nochainrule is safer because it makes no assumptions about how the parameters and covariates join to form the response.
nochainrule is implied when the expression() option is specified.

Reporting
level(\#) specifies the confidence level, as a percentage, for confidence intervals. The default is level(95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals. mcompare (method) specifies the method for computing $p$-values and confidence intervals that account for multiple comparisons within a factor-variable term.
Most methods adjust the comparisonwise error rate, $\alpha_{c}$, to achieve a prespecified experimentwise error rate, $\alpha_{e}$.
mcompare(noadjust) is the default; it specifies no adjustment.

$$
\alpha_{c}=\alpha_{e}
$$

mcompare (bonferroni) adjusts the comparisonwise error rate based on the upper limit of the Bonferroni inequality

$$
\alpha_{e} \leq m \alpha_{c}
$$

where $m$ is the number of comparisons within the term.
The adjusted comparisonwise error rate is

$$
\alpha_{c}=\alpha_{e} / m
$$

mcompare (sidak) adjusts the comparisonwise error rate based on the upper limit of the probability inequality

$$
\alpha_{e} \leq 1-\left(1-\alpha_{c}\right)^{m}
$$

where $m$ is the number of comparisons within the term.
The adjusted comparisonwise error rate is

$$
\alpha_{c}=1-\left(1-\alpha_{e}\right)^{1 / m}
$$

This adjustment is exact when the $m$ comparisons are independent.
mcompare (scheffe) controls the experimentwise error rate using the $F$ or $\chi^{2}$ distribution with degrees of freedom equal to the rank of the term.
mcompare(method adjustall) specifies that the multiple-comparison adjustments count all comparisons across all terms rather than performing multiple comparisons term by term. This leads to more conservative adjustments when multiple variables or terms are specified in marginslist. This option is compatible only with the bonferroni and sidak methods.
noatlegend specifies that the legend showing the fixed values of covariates be suppressed.
post causes margins to behave like a Stata estimation (e-class) command. margins posts the vector of estimated margins along with the estimated variance-covariance matrix to e(), so you can treat the estimated margins just as you would results from any other estimation command. For example, you could use test to perform simultaneous tests of hypotheses on the margins, or you could use lincom to create linear combinations. See Example 10: Testing margins-contrasts of margins.
display_options: noci, nopvalues, vsquish, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% fmt), pformat (\%fmt), sformat (\%fmt), and nolstretch.
noci suppresses confidence intervals from being reported in the coefficient table.
nopvalues suppresses $p$-values and their test statistics from being reported in the coefficient table.
vsquish specifies that the blank space separating factor-variable terms or time-series-operated variables from other variables in the model be suppressed.
nofvlabel displays factor-variable level values rather than attached value labels. This option overrides the fvlabel setting; see [R] set showbaselevels.
fvwrap(\#) allows long value labels to wrap the first \# lines in the coefficient table. This option overrides the fvwrap setting; see [R] set showbaselevels.
fvwrapon(style) specifies whether value labels that wrap will break at word boundaries or break based on available space.
fvwrapon(word), the default, specifies that value labels break at word boundaries.
fvwrapon(width) specifies that value labels break based on available space.
This option overrides the fvwrapon setting; see [R] set showbaselevels.
cformat (\% fint) specifies how to format margins, standard errors, and confidence limits in the table of estimated margins.
pformat (\% fint) specifies how to format $p$-values in the table of estimated margins.
sformat (\%fmt) specifies how to format test statistics in the table of estimated margins.
nolstretch specifies that the width of the table of estimated margins not be automatically widened to accommodate longer variable names. The default, lstretch, is to automatically widen the table of estimated margins up to the width of the Results window. To change the default, use set lstretch off. nolstretch is not shown in the dialog box.

The following option is available with margins but is not shown in the dialog box:
df (\#) specifies that the $t$ distribution with \# degrees of freedom be used for computing $p$-values and confidence intervals. The default typically is to use the standard normal distribution. However, if the estimation command computes the residual degrees of freedom (e(df_r)) and predict (xb) is specified with margins, the default is to use the $t$ distribution with e(df_r) degrees of freedom.

## Remarks and examples

Remarks are presented under the following headings:

```
Introduction
Obtaining margins of responses
    Example 1: A simple case after regress
    Example 2: A simple case after logistic
    Example 3: Average response versus response at average
    Example 4: Multiple margins from one command
    Example 5: Margins with interaction terms
    Example 6: Margins with continuous variables
    Example 7: Margins of continuous variables
    Example 8: Margins of interactions
    Example 9: Decomposing margins
    Example 10: Testing margins - contrasts of margins
    Example 11: Margins of a specified prediction
    Example 12: Margins of a specified expression
    Example 13: Margins with multiple outcomes (responses)
    Example 14: Margins with multiple equations
    Example 15: Margins evaluated out of sample
Obtaining margins of derivatives of responses (a.k.a. marginal effects)
    Do not specify marginlist when you mean over()
    Use at() freely, especially with continuous variables
    Expressing derivatives as elasticities
    Derivatives versus discrete differences
    Example 16: Average marginal effect (partial effects)
    Example 17: Average marginal effect of all covariates
    Example 18: Evaluating marginal effects over the response surface
Obtaining margins with survey data and representative samples
    Example 19: Inferences for populations, margins of response
    Example 20: Inferences for populations, marginal effects
    Example 21: Inferences for populations with svyset data
Standardizing margins
Obtaining margins as though the data were balanced
    Balancing using asbalanced
    Balancing by standardization
    Balancing nonlinear responses
    Treating a subset of covariates as balanced
    Using fvset design
    Balancing in the presence of empty cells
Obtaining margins with nested designs
    Introduction
    Margins with nested designs as though the data were balanced
    Coding of nested designs
Special topics
    Requirements for model specification
    Estimability of margins
    Manipulability of tests
    Using margins after the estimates use command
    Syntax of at()
    Estimation commands that may be used with margins
Video examples
Glossary
```


## Introduction

margins is a postestimation command, a command for use after you have fit a model using an estimation command such as regress or logistic, or using almost any other estimation command.
margins estimates and reports margins of responses and margins of derivatives of responses, also known as marginal effects. A margin is a statistic based on a fitted model in which some of or all the
covariates are fixed. Marginal effects are changes in the response for change in a covariate, which can be reported as a derivative, elasticity, or semielasticity.

For a brief overview of margins, see Williams (2012).

## Obtaining margins of responses

What we call margins of responses are also known as predictive margins, adjusted predictions, and recycled predictions. When applied to balanced data, margins of responses are also called estimated marginal means and least-squares means.

A margin is a statistic based on a fitted model calculated over a dataset in which some of or all the covariates are fixed at values different from what they really are. For instance, after a linear regression fit on males and females, the marginal mean (margin of mean) for males is the predicted mean of the dependent variable, where every observation is treated as if it represents a male; thus those observations that in fact do represent males are included, as well as those observations that represent females. The marginal mean for female would be similarly obtained by treating all observations as if they represented females.

In making the calculation, sex is treated as male or female everywhere it appears in the model. The model might be

```
. regress y age bp i.sex sex#c.age sex#c.bp
```

and then, in making the marginal calculation of the mean for males and females, margins not only accounts for the direct effect of i.sex but also for the indirect effects of sex\#c. age and sex\#c.bp.

The response being margined can be any statistic produced by $[\mathrm{R}]$ predict, or any expression of those statistics.

Standard errors are obtained by the delta method, at least by default. The delta method assumes that the values at which the covariates are evaluated to obtain the marginal responses are fixed. When your sample represents a population, whether you are using svy or not (see [SVY] svy), you can specify margins' vce (unconditional) option and margins will produce standard errors that account for the sampling variability of the covariates. Some researchers reserve the term predictive margins to describe this.

The best way to understand margins is to see some examples. You can run the following examples yourself if you type

```
. use http://www.stata-press.com/data/r13/margex
(Artificial data for margins)
```


## Example 1: A simple case after regress



The numbers reported in the "Margin" column are average values of $y$. Based on a linear regression of $y$ on sex and group, 60.6 would be the average value of $y$ if everyone in the data were treated as if they were male, and 78.9 would be the average value if everyone were treated as if they were female.

## Example 2: A simple case after logistic

margins may be used after almost any estimation command.


The numbers reported in the "Margin" column are average predicted probabilities. Based on a logistic regression of outcome on sex and group, 0.13 would be the average probability of outcome if everyone in the data were treated as if they were male, and 0.19 would be the average probability if everyone were treated as if they were female.
margins reports average values after regress and average probabilities after logistic. By default, margins makes tables of whatever it is that predict (see [R] predict) predicts by default. Alternatively, margins can make tables of anything that predict can produce if you use margins' predict() option; see Example 11: Margins of a specified prediction.

## Example 3: Average response versus response at average

In example 2, margins reported average probabilities of outcome for sex $=0$ and $\operatorname{sex}=1$. If we instead wanted the predicted probabilities evaluated at the mean of the covariates, we would specify margins' atmeans option. We previously typed

```
. logistic outcome i.sex i.group
    (output omitted)
```

. margins sex
(output omitted)
and now we type


|  | Delta-method |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Margin | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| sex |  |  |  |  |  |  |
| male | .0966105 | .0089561 | 10.79 | 0.000 | .0790569 | .1141641 |
| female | .1508362 | .0118064 | 12.78 | 0.000 | .127696 | .1739764 |

The prediction at the average of the covariates is different from the average of the predictions. The first is the expected probability of a person with average characteristics, a person who, in another problem, might be $3 / 4$ married and have 1.2 children. The second is the average of the probability among actual persons in the data.

When you specify atmeans or any other at option, margins reports the values used for the covariates in the legend above the table margins lists the values for all the covariates, including values it may not use, in the results that follow. In this example, margins reported means for sex even though those means were not used. They were not used because we asked for the margins of sex, so sex was fixed first at 0 and then at 1 .

If you wish to suppress this legend, specify the nolegend option.

## Example 4: Multiple margins from one command

More than one margin can be reported by just one margins command. You can type

```
. margins sex group
```

and doing that is equivalent in terms of the output to typing

```
. margins sex
```

. margins group

When multiple margins are requested on the same command, each is estimated separately. There is, however, a difference when you also specify margins' post option. Then the variance-covariance matrix for all margins requested is posted, and that is what allows you to test equality of margins, etc. Testing equality of margins is covered in Example 10: Testing margins-contrasts of margins.

In any case, below we request margins for sex and for group.

```
. margins sex group
Predictive margins \(\quad\) Number of obs \(=3000\)
Model VCE : OIM
Expression : Pr(outcome), predict()
```

|  | Delta-method <br> Margin <br> Std. Err. |  |  |  | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| sex |  |  |  |  |  |  |  |
| male | .1286796 | .0111424 | 11.55 | 0.000 | .106841 | .1505182 |  |
| female | .1905087 | .0089719 | 21.23 | 0.000 | .1729241 | .2080933 |  |
| group |  |  |  |  |  |  |  |
| 1 | .2826207 | .0146234 | 19.33 | 0.000 | .2539593 | .311282 |  |
| 2 | .1074814 | .0094901 | 11.33 | 0.000 | .0888812 | .1260817 |  |
| 3 | .0291065 | .0073417 | 3.96 | 0.000 | .0147169 | .043496 |  |

## Example 5: Margins with interaction terms

The estimation command on which margins bases its calculations may contain interaction terms, such as an interaction of sex and group:

```
. logistic outcome i.sex i.group sex#group
    (output omitted)
. margins sex group
Predictive margins Number of obs = 3000
Model VCE : OIM
Expression : Pr(outcome), predict()
```

|  | Delta-method |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Margin | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Con | Interval] |
| sex |  |  |  |  |  |  |
| male | . 1561738 | . 0132774 | 11.76 | 0.000 | . 1301506 | . 182197 |
| female | . 1983749 | . 0101546 | 19.54 | 0.000 | . 1784723 | . 2182776 |
| group |  |  |  |  |  |  |
| 1 | . 3211001 | . 0176403 | 18.20 | 0.000 | . 2865257 | . 3556744 |
| 2 | . 1152127 | . 0099854 | 11.54 | 0.000 | . 0956417 | . 1347838 |
| 3 | . 0265018 | . 0109802 | 2.41 | 0.016 | . 0049811 | . 0480226 |

We fit the model by typing logistic outcome i.sex i.group sex\#group, but the meaning would have been the same had we typed logistic outcome sex\#\#group.

As mentioned in example 4, the results for sex and the results for group are calculated independently, and we would have obtained the same results had we typed margins sex followed by margins group.

The margin for male $(\operatorname{sex}=0)$ is 0.16 . The probability 0.16 is the average probability if everyone in the data were treated as if sex $=0$, including sex $=0$ in the main effect and sex $=0$ in the interaction of sex with group.

Had we specified margins sex, atmeans, we would have obtained not average probabilities but the probabilities evaluated at the average. Rather than obtaining 0.16 , we would have obtained 0.10
for sex $=0$. The 0.10 is calculated by taking the fitted model, plugging in sex $=0$ everywhere, and plugging in the average value of the group indicator variables everywhere they are used. That is, rather than treating the group indicators as being $(1,0,0),(0,1,0)$, or $(0,0,1)$ depending on observation, the group indicators are treated as being $(0.40,0.37,0.23)$, which are the average values of group $=1$, group $=2$, and group $=3$.

## Example 6: Margins with continuous variables

To the above example, we will add the continuous covariate age to the model and then rerun margins sex group.

| Predictive margins |  |  |  | Number of obs |  | 3000 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Expression : Pr(outcome), predict() |  |  |  |  |  |  |
| Delta-method |  |  |  |  |  |  |
|  | Margin | Std. Err | z | $P>\|z\|$ | [95\% Conf | Interval] |
| sex |  |  |  |  |  |  |
| male | . 1600644 | . 0125653 | 12.74 | 0.000 | . 1354368 | . 184692 |
| female | . 1966902 | . 0100043 | 19.66 | 0.000 | . 1770821 | . 2162983 |
| group |  |  |  |  |  |  |
| 1 | . 2251302 | . 0123233 | 18.27 | 0.000 | . 200977 | . 2492834 |
| 2 | . 150603 | . 0116505 | 12.93 | 0.000 | . 1277685 | . 1734376 |
| 3 | . 0736157 | . 0337256 | 2.18 | 0.029 | . 0075147 | . 1397167 |

Compared with the results presented in example 5, results for sex change little, but results for groups 1 and 3 change markedly. The tables differ because now we are adjusting for the continuous covariate age, as well as for sex and group.

We will continue examining interactions in example 8. Because we have added a continuous variable, let's take a detour to explain how to obtain margins for continuous variables and to explain their interpretation.

## Example 7: Margins of continuous variables

Continuing with our example of
. logistic outcome i.sex i.group sex\#group age
let's examine the continuous covariate age.

You are not allowed to type margins age; doing that will produce an error:

```
. margins age
'age' not found in list of covariates
r(322);
```

The message "'age' not found in list of covariates" is margins' way of saying, "Yes, age might be in the model, but if it is, it is not included as a factor variable; it is in as a continuous variable." Sometimes, Stata is overly terse. margins might also say that because age is continuous there are an infinite number of values at which it could evaluate the margins. At what value(s) should age be fixed? margins requires more guidance with continuous covariates. We can provide that guidance by using the at () option and typing

```
. margins, at(age=40)
```

To understand why that yields the desired result, let us tell you that if you were to type

```
. margins
```

margins would report the overall margin-the margin that holds nothing constant. Because our model is logistic, the average value of the predicted probabilities would be reported. The at () option fixes one or more covariates to the value(s) specified and can be used with both factor and continuous variables. Thus, if you typed margins, at (age=40), then margins would average over the data the responses for everybody, setting age=40. Here is what happens when you type that:


Reported is the margin for age $=40$, adjusted for the other covariates in our model.
If we wanted to obtain the margins for age $30,35,40,45$, and 50 , we could type

```
. margins, at(age=(30 35 40 45 50))
```

or, equivalently,

```
. margins, at(age=(30(5)50))
```


## Example 8: Margins of interactions

Our model is
. logistic outcome i.sex i.group sex\#group age

We can obtain the margins of all possible combinations of the levels of sex and the levels of group by typing

```
. margins sex#group 
```

|  | Delta-method |  |  |  |  |  |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | MarginStd. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |  |
| sex\#group |  |  |  |  |  |  |
| male\#1 | .2379605 | .0237178 | 10.03 | 0.000 | .1914745 | .2844465 |
| male\#2 | .0658294 | .0105278 | 6.25 | 0.000 | .0451953 | .0864636 |
| male\#3 | .0538001 | .0136561 | 3.94 | 0.000 | .0270347 | .0805656 |
| female\#1 | .2158632 | .0112968 | 19.11 | 0.000 | .1937218 | .2380045 |
| female\#2 | .2054406 | .0183486 | 11.20 | 0.000 | .1694781 | .2414032 |
| female\#3 | .085448 | .0533914 | 1.60 | 0.110 | -.0191973 | .1900932 |

The first line in the table reports the marginal probability for sex $=0$ (male) and group $=1$. That is, it reports the estimated probability if everyone in the data were treated as if they were sex $=0$ and group $=1$.

Also reported are all the other combinations of sex and group.
By the way, we could have typed margins sex\#group even if our fitted model did not include sex\#group. Estimation is one thing, and asking questions about the nature of the estimates is another. margins does, however, require that i.sex and i.group appear somewhere in the model, because fixing a value outside the model would just produce the grand margin, and you can separately ask for that if you want it by typing margins without arguments.

## Example 9: Decomposing margins

We have the model

- logistic outcome i.sex i.group sex\#group age

In example 6, we typed margins sex and obtained 0.160 for males and 0.197 for females. We are going to decompose each of those numbers. Let us explain:

1. The margin for males, 0.160 , treats everyone as if they were male, and that amounts to simultaneously

1a. treating males as males and
1 b . treating females as males.
2. The margin for females, 0.197 , treats everyone as if they were female, and that amounts to simultaneously

2 a . treating males as females and
2 b . treating females as females.
The margins 1 a and 1 b are the decomposition of 1 , and the margins 2 a and 2 b are the decomposition of 2 .

We could obtain 1a and 2a by typing

```
. margins if sex==0, at(sex=(0 1))
```

because the qualifier if sex=$=0$ would restrict margins to running on only the males. Similarly, we could obtain 1 b and 2 b by typing

```
. margins if sex==1, at(sex=(0 1))
```

We run these examples below:

| Predictive margins |  |  | Number of obs | $=$ | 1498 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Model VCE | : OIM |  |  |  |  |
| Expression | : $\operatorname{Pr}($ | di |  |  |  |
| 1._at | : sex | = |  |  |  |
| 2._at | : sex | $=$ |  |  |  |


|  | Delta-method |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Margin | Std. Err. | $\mathbf{z}$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| -at |  |  |  |  |  |  |
| 1 | .0794393 | .0062147 | 12.78 | 0.000 | .0672586 | .0916199 |
| 2 | .1335584 | .0127351 | 10.49 | 0.000 | .1085981 | .1585187 |

. margins if sex==1, at (sex=(0 1))
Predictive margins $\quad$ Number of obs $=1502$

Model VCE : OIM
Expression : Pr(outcome), predict()
1._at : sex $=\quad 0$
2._at : sex $=\quad 1$

|  | Delta-method |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Margin | Std. Err. | $\mathbf{z}$ | P>\|z| | [95\% Conf. Interval] |  |
| at |  |  |  |  |  |  |
| 1 | .2404749 | .0199709 | 12.04 | 0.000 | .2013326 | .2796171 |
| 2 | .2596538 | .0104756 | 24.79 | 0.000 | .2391219 | .2801857 |

Putting together the results from example 6 and the results above, we have

| Margin treating everybody as themself | 0.170 |
| :---: | :--- |
| Margin treating everybody as male | 0.160 |
| Margin treating male as male | 0.079 |
| Margin treating female as male | 0.240 |
| Margin treating everybody as female | 0.197 |
| Margin treating male as female | 0.134 |
| Margin treating female as female | 0.260 |

## Example 10: Testing margins-contrasts of margins

Continuing with the previous example, it would be interesting to test the equality of 2 b and 1 b , to test whether the average probability of a positive outcome for females treated as females is equal to that for females treated as males. That test would be different from testing the overall significance of sex in our model. The test performed on our model would be a test of whether the probability of a positive outcome differs between males and females when they have equal values of the other covariates. The test of equality of margins is a test of whether the average probabilities differ given the different pattern of values of the other covariates that the two sexes have in our data.

We can also perform such tests by treating the results from margins as estimation results. There are three steps required to perform tests on margins. First, you must arrange it so that all the margins of interest are reported by just one margins command. Second, you must specify margins' post option. Third, you perform the test with the test command.

Such tests and comparisons can be readily performed by contrasting margins; see [R] margins, contrast. Also see Contrasts of margins—effects (discrete marginal effects) in [R] marginsplot.

In the previous example, we used two commands to obtain our results, namely,
. margins if sex==0, at (sex=(01))
. margins if sex==1, at (sex=(0 1) )
We could, however, have obtained the same results by typing just one command:

```
. margins, over(sex) at(sex=(0 1))
```

Performing margins, over (sex) first restricts the sample to sex==0 and then restricts it to $\operatorname{sex}==1$, and that is equivalent to the two different if conditions that we specified before.

To test whether females treated as females is equal to females treated as males, we will need to type

```
. margins, over(sex) at(sex=(0 1)) post
. test _b[2._at#1.sex] = _b[1._at#1.sex]
```

We admit that the second command may seem to have come out of nowhere. When we specify post on the margins command, margins behaves as if it were an estimation command, which means that 1) it posts its estimates and full VCE to e(), 2) it gains the ability to replay results just as any estimation command can, and 3) it gains access to the standard postestimation commands. Item 3 explains why we could use test. We learned that we wanted to test _b [2._at\#1.sex] and _b [1._at\#1.sex] by replaying the estimation results, but this time with the standard estimation command coeflegend option. So what we typed was
. margins, over(sex) at(sex=(0 1)) post
. margins, coeflegend
. test _b[2._at\#1.sex] = _b[1._at\#1.sex]

We will let you try margins, coeflegend for yourself. The results of running the other two commands are

| Predictive margins |  |  |  | Number of obs |  | 3000 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Model VCE | OIM |  |  |  |  |  |
| $\begin{array}{ll} \text { Expression } & : \text { Pr(outcome), predict() } \\ \text { over } & : \text { sex } \end{array}$ |  |  |  |  |  |  |
| 1._at | 0. sex |  |  |  |  |  |
|  | sex | = |  | O |  |  |
|  | 1.sex |  |  |  |  |  |
|  | sex | = |  |  |  |  |
| 2._at | 0. sex |  |  |  |  |  |
|  | sex | = |  | 1 |  |  |
|  | 1.sex |  |  |  |  |  |
|  | sex |  |  | 1 |  |  |
|  | Delta-method |  |  |  |  |  |
|  | Margin | Std. Err. | z | $P>\|z\|$ | [95\% Conf. | Interval] |
| _at\#sex |  |  |  |  |  |  |
| 1\#male | . 0794393 | . 0062147 | 12.78 | 0.000 | . 0672586 | . 0916199 |
| 1\#female | . 2404749 | . 0199709 | 12.04 | 0.000 | . 2013326 | . 2796171 |
| 2\#male | . 1335584 | . 0127351 | 10.49 | 0.000 | . 1085981 | . 1585187 |
| 2\#female | . 2596538 | . 0104756 | 24.79 | 0.000 | . 2391219 | . 2801857 |
| . test _b[2._at\#1.sex] = _b[1._at\#1.sex] |  |  |  |  |  |  |
| ( 1) - 1bn._at\#1.sex + 2._at\#1.sex $=0$ |  |  |  |  |  |  |
| $\operatorname{chi2}(1)=0.72$ |  |  |  |  |  |  |
| Prob > chi2 $=0.3951$ |  |  |  |  |  |  |

We can perform the same test in one command using contrasts of margins:

| . logistic outcome i.sex i.group sex\#group age (output omitted) |  |  |  |
| :---: | :---: | :---: | :---: |
| Contrasts of predictive margins Model VCE : OIM |  |  |  |
| $\begin{array}{ll} \text { Expression } & \text { : Pr } \\ \text { over } & \text { : s } \end{array}$ | ```Pr(outcome), predict() sex``` |  |  |
| 1._at : 0 | 0. sex |  |  |
|  |  | $=$ | 0 |
|  | 1.sex |  |  |
|  |  | = | 0 |
| 2._at $\begin{array}{r}\text { a } \\ \\ 1\end{array}$ | 0. sex |  |  |
|  |  | $=$ | 1 |
|  | 1.sex |  |  |
|  |  | = | 1 |
|  | df | chi2 | P>chi2 |
| _at@sex |  |  |  |
| (2 vs 1) male | 1 | 14.59 | 0.0001 |
| (2 vs 1) female | 1 | 0.72 | 0.3951 |
| Joint | 2 | 16.13 | 0.0003 |


|  | Delta-method |  |  |  |
| :---: | ---: | :--- | ---: | ---: |
| Contrast | Err. | [95\% Conf. Interval] |  |  |
| (2 vs 1) male | .0541192 | .0141706 | .0263453 | .081893 |
| $(2$ vs 1) female | .0191789 | .0225516 | -.0250215 | .0633793 |

We refitted our logistic model because its estimation results were replaced when we posted our margins. The syntax to perform the contrast we want is admittedly not obvious. Contrasting (testing) across at () groups is more difficult than contrasting across the margins themselves or across over () groups, because we have no natural place for the contrast operators (r., in our case). We also explicitly requested Wald tests of the contrasts, which are not provided by default. Nevertheless, the chi-squared statistic and its $p$-value for ( 2 vs 1 ) for male matches the results of our test command. We also obtain the test of whether the response of males treated as males is equal to the response of males treated as females.

For a gentler introduction to contrasts of margins, see $[\mathrm{R}]$ margins, contrast.

## Example 11: Margins of a specified prediction

We will fit the model
. use http://www.stata-press.com/data/r13/margex
. tobit ycn i.sex i.group sex\#group age, ul(90)
and we will tell the following story about the variables: We run a peach orchard where we allow people to pick their own peaches. A person receives one empty basket in exchange for $\$ 20$, along with the right to enter the orchard. There is no official limit on how many peaches a person can pick, but only 90 peaches will fit into a basket. The dependent variable in the above tobit model, ycn, is the number of peaches picked. We use tobit, a special case of censored-normal regression, because ycn is censored at 90 .

After fitting this model, if we typed

```
. margins sex
```

we would obtain the margins for males and for females of the uncensored number of peaches picked. We would obtain that because predict after tobit produces the uncensored number by default. To obtain the censored prediction, we would have to specify predict's ystar(.,90) option. If we want the margins based on that response, we type

```
. margins sex, predict(ystar(.,90))
```

The results of typing that are

```
. tobit ycn i.sex i.group sex#group age, ul(90)
    (output omitted)
. margins sex, predict(ystar(.,90))
Predictive margins Number of obs = 3000
Model VCE : OIM
Expression : E(ycn*|ycn<90), predict(ystar(.,90))
```

|  | MarginDelta-method <br> Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |  |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| sex |  |  |  |  |  |  |
| male | 62.21804 | .5996928 | 103.75 | 0.000 | 61.04266 | 63.39342 |
| female | 78.34272 | .455526 | 171.98 | 0.000 | 77.4499 | 79.23553 |

In our previous examples, sex $=1$ has designated females, so evidently the females visiting our orchard are better at filling baskets than the men.

## Example 12: Margins of a specified expression

Continuing with our peach orchard example and the previously fit model
. use http://www.stata-press.com/data/r13/margex
. tobit ycn i.sex i.group sex\#group age, ul(90)
let's examine how well our baskets are working for us. What is the proportion of the number of peaches actually picked to the number that would have been picked were the baskets larger? As mentioned in example 11, predict, ystar (.,90) produces the expected number picked given the limit of basket size. predict, xb would predict the expected number without a limit. We want the ratio of those two predictions. That ratio will measure as a proportion how well the baskets work. Thus we could type

```
. margins sex, expression(predict(ystar(.,90))/predict(xb))
```

That would give us the proportion for everyone treated as male and everyone treated as female, but what we want to know is how well baskets work for true males and true females, so we will type

```
. margins, over(sex) expression(predict(ystar(.,90))/predict(xb))
. margins, over(sex) expression(predict(ystar(0,90))/predict(xb))
Predictive margins Number of obs = 3000
Model VCE : OIM
Expression : predict(ystar(0,90))/predict(xb)
over : sex
```

|  | Delta-method |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | MarginStd. Err. | $\mathbf{z}$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |  |
| sex |  |  |  |  |  |  |
| male |  |  |  |  |  |  |
| female | .9811785 | .0013037 | 752.60 | 0.000 | .9786233 | .9837337 |

By the way, we could count the number of peaches saved by the limited basket size during the period of data collection by typing

```
. count
    3000
. margins, expression(3000*(predict(xb)-predict(ystar(.,90))))
(output omitted)
```

The number of peaches saved turns outs to be 9,183 .

## Example 13: Margins with multiple outcomes (responses)

Estimation commands such as mlogit and mprobit (see [R] mlogit and $[R]$ mprobit) calculate multiple responses, and those multiple responses are reflected in the options available with predict after estimation. Obtaining margins for such estimators is thus the same as obtaining margins of a specified prediction, which was demonstrated in example 11. The solution is to include the predict_opt that selects the desired response in margins' predict (predict_opt) option.

If we fit the multinomial logistic model
. mlogit group i.sex age
then to obtain the margins for the probability that group $=1$, we would type
. margins sex, predict(outcome(1))
and to obtain the margins for the probability that group $=3$, we would type
. margins sex, predict(outcome(3))
We learned about the outcome (1) and outcome (3) options by looking in [R] mlogit postestimation. For an example using margins with a multiple-outcome estimator, see example 4 in $[R]$ mlogit postestimation.

## Example 14: Margins with multiple equations

Estimation commands such as mvreg, manova, sureg, and reg3 (see [MV] mvreg, [MV] manova, $[R]$ sureg, and $[R]$ reg3) fit multiple equations. Obtaining margins for such estimators is the same as obtaining margins with multiple outcomes (see example 13), which in turn is the same as obtaining margins of a specified prediction (see example 11). You place the relevant option from the estimator's predict command into margins' predict (predict_opt) option.

If we fit the seemingly unrelated regression model

```
. sureg (y = i.sex age) (distance = i.sex i.group)
```

we can obtain the marginal means of $y$ for males and females by typing

```
. margins sex, predict(equation(y))
```

and we can obtain the marginal means of distance by typing

```
. margins sex, predict(equation(distance))
```

We could obtain the difference between the margins of y and distance by typing

```
. margins sex, expression(predict(equation(y)) -
```

> predict (equation(distance)))

More examples can be found in [MV] manova and [MV] manova postestimation.

## Example 15: Margins evaluated out of sample

You can fit your model on one dataset and use margins on another if you specify margins' noesample option. Remember that margins reports estimated average responses, and, unless you lock all the covariates at fixed values by using the at () option, the remaining variables are allowed to vary as they are observed to vary in the data. That is indeed the point of using margins. The fitted model provides the basis for adjusting for the remaining variables, and the data provide their values. The predictions produced by margins are of interest assuming the data used by margins are in some sense interesting or representative. In some cases, you might need to fit your model on one set of data and perform margins on another.

In example 11, we fit the model

```
. tobit ycn i.sex i.group sex#group age, ul(90)
```

and we told a story about our peach orchard in which we charged people $\$ 20$ to collect a basket of peaches, where baskets could hold at most 90 peaches. Let us now tell you that we believe the data on which we estimated those margins were unrepresentative, or at least, we have a more representative sample stored in another .dta file. That dataset includes the demographics of our customers but does not include counts of peaches picked. It is a lot of work counting those peaches.

Thus we will fit our model just as we did previously using the detailed data, but we will bring the other, more representative dataset into memory before issuing the margins sex, predict (ystar (.,90)) command, and we will add noesample to it.

| (Artificial data for margins) <br> tobit ycn i.sex i.group sex\#group age, ul(90) (output omitted) <br> . use http://www.stata-press.com/data/r13/peach <br> . margins sex, predict(ystar(.,90)) noesample |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |
| Predictive margins <br> Model VCE : OIM |  |  |  | Numb | of obs | 2727 |
| Expression : E $\mathrm{ycn} * \mid \mathrm{ycn}<90$ ), predict (ystar (.,90)) |  |  |  |  |  |  |
|  | Delta-method |  |  | $P>\|z\|$ | [95\% | Interval] |
| sex |  |  |  |  |  |  |
| 0 | 56.79774 | 1.003727 | 56.59 | 0.000 | 54.83 | 58.76501 |
| 1 | 75.02146 | . 643742 | 116.54 | 0.000 | 73.75 | 76.28317 |

In example 12, we produced an estimate of the number of peaches saved by the limited-size baskets. We can update that estimate using the new demographic data by typing

```
. count
    2727
. margins, exp(2727*(predict(xb)-predict(ystar(.,90)))) noesample
    (output omitted)
```

By running the above, we find that the updated number of peaches saved is 6,408 .

## Obtaining margins of derivatives of responses (a.k.a. marginal effects)

Derivatives of responses are themselves responses, so everything said above in Obtaining margins of responses is equally true of derivatives of responses, and every example above could be repeated here substituting the derivative of the response for the response.

Derivatives are of interest because they are an informative way of summarizing fitted results. The change in a response for a change in the covariate is easy to understand and to explain. In simple models, one hardly needs margins to assist in obtaining such margins. Consider the simple linear regression

$$
y=\beta_{0}+\beta_{1} \times \operatorname{sex}+\beta_{2} \times \text { age }+\epsilon
$$

The derivatives of the responses are

$$
\begin{aligned}
& d y / d(\text { sex })=\beta_{1} \\
& d y / d(\text { age })=\beta_{2}
\end{aligned}
$$

The derivatives are the fitted coefficients. How does $y$ change between males and females? It changes by $\beta_{1}$. How does $y$ change with age? It changes by $\beta_{2}$ per year.

If you make the model a little more complicated, however, the need for margins arises. Consider the model

$$
y=\beta_{0}+\beta_{1} \times \operatorname{sex}+\beta_{2} \times \text { age }+\beta_{3} \times \text { age }^{2}+\epsilon
$$

Now the derivative with respect to age is

$$
d y / d(\text { age })=\beta_{2}+2 \times \beta_{3} \times \text { age }
$$

The change in $y$ for a change in age itself changes with age, and so to better understand the fitted results, you might want to make a table of the change in $y$ for a change in age for age $=30$, age $=40$, and age $=50$. margins can do that.

Consider an even more complicated model, such as

$$
\begin{align*}
y=\beta_{0} & +\beta_{1} \times \mathrm{sex}+\beta_{2} \times \text { age }+\beta_{3} \times \mathrm{age}^{2}+\beta_{4} \times \mathrm{bp}+\beta_{5} \times \mathrm{sex} \times \mathrm{bp}+\beta_{6} \times \mathrm{tmt} \\
& +\beta_{7} \times \mathrm{tmt} \times \mathrm{age}+\beta_{8} \times \mathrm{tmt} \times \mathrm{age}^{2}+\epsilon \tag{1}
\end{align*}
$$

The derivatives are

$$
\begin{aligned}
d y / d(\mathrm{sex}) & =\beta_{1}+\beta_{5} \times \mathrm{bp} \\
d y / d(\mathrm{age}) & =\beta_{2}+2 \times \beta_{3} \times \text { age }+\beta_{7} \times \mathrm{tmt}+2 \times \beta_{8} \times \mathrm{tmt} \times \text { age } \\
d y / d(\mathrm{bp}) & =\beta_{4}+\beta_{5} \times \mathrm{sex} \\
d y / d(\mathrm{tmt}) & =\beta_{6}+\beta_{7} \times \text { age }+\beta_{8} \times \operatorname{age}^{2}
\end{aligned}
$$

At this point, margins becomes indispensable.

## Do not specify marginlist when you mean over()

margins has the same syntax when used with derivatives of responses as when used with responses. To obtain derivatives, one specifies the $d y d x$ () option. If we wanted to examine the response variable $d y / d(\operatorname{tmt})$, we would specify margins' dydx (tmt) option. The rest of the margins command has the same syntax as ordinarily, although one tends to specify different syntactical elements. For instance, one usually does not specify a marginlist. If we typed

```
. margins sex, dydx(tmt)
```

we would obtain $d y / d(\mathrm{tmt})$ calculated first as if everyone were male and then as if everyone were female. At the least, we would probably want to specify
. margins sex, dydx(tmt) grand
so as also to obtain $d y / d(\mathrm{tmt})$, the overall margin, the margin with everyone having their own value of sex. Usually, however, all we want is the overall margin, and because grand is the default when the marginlist is not specified, we would just type

```
. margins, dydx(tmt)
```

Alternatively, if we were interested in the decomposition by sex, then rather than type margins sex, dydx (tmt), we probably want to type

```
. margins, over(sex) dydx(tmt)
```

This command gives us the average effect of tmt for males and again for females rather than the average effect with everyone treated as male and then again with everyone treated as female.

## Use at() freely, especially with continuous variables

Another option one tends to use more often with derivatives of responses than one does with responses is at (). Such use is often to better understand or to communicate how the response varies, or, in technical jargon, to explore the nature of the response surface.

For instance, the effect $d y / d(\mathrm{tmt})$ in (1) is equal to $\beta_{6}+\beta_{7} \times$ age $+\beta_{8} \times \mathrm{age}^{2}$, and so simply to understand how treatment varies with age, we may want to fix age at various values. We might type

```
. margins, dydx(tmt) over(sex) at(age=(30 40 50))
```


## Expressing derivatives as elasticities

You specify the dydx (varname) option on the margins command to use $d y / d($ varname) as the response variable. If you want that derivative expressed as an elasticity, you can specify eyex (varname), eydx (varname), or dyex (varname). You substitute e for d where you want an elasticity. The formulas are

$$
\begin{aligned}
& \operatorname{dydx}()=d y / d x \\
& \operatorname{eyex}()=d y / d x \times(x / y) \\
& \operatorname{eydx}()=d y / d x \times(1 / y) \\
& \operatorname{dyex}()=d y / d x \times(x)
\end{aligned}
$$

and the interpretations are

| $\operatorname{dydx}():$ |  | change in $y$ for a |  | change in $x$ |
| :--- | :--- | :--- | :--- | :--- |
| eyex (): | proportional | change in $y$ for a | proportional | change in $x$ |
| eydx(): | proportional | change in $y$ for a |  | change in $x$ |
| dyex(): |  | change in $y$ for a | proportional | change in $x$ |

As margins always does with response functions, calculations are made at the observational level and are then averaged. Let's assume that in observation $5, d y / d x=0.5, y=15$, and $x=30$; then

$$
\begin{array}{r}
\operatorname{dydx}()=0.5 \\
\operatorname{eyex}()=1.0 \\
\operatorname{eydx}()=0.03 \\
\operatorname{dyex}()=15.0
\end{array}
$$

Many social scientists would informally explain the meaning of eyex () = 1 as " $y$ increases $100 \%$ when $x$ increases $100 \%$ " or as " $y$ doubles when $x$ doubles", although neither statement is literally true. eyex (), eydx (), and dyex() are rates evaluated at a point, just as dydx() is a rate, and all such interpretations are valid only for small (infinitesimal) changes in $x$. It is true that eyex ()$=1$ means $y$ increases with $x$ at a rate such that, if the rate were constant, $y$ would double if $x$ doubled. This issue of casual interpretation is no different from casually interpreting dydx () as if it represents the response to a unit change. It is not necessarily true that $\operatorname{dydx}()=0.5$ means that " $y$ increases by 0.5 if $x$ increases by 1 ". It is true that " $y$ increases with $x$ at a rate such that, if the rate were constant, $y$ would increase by 0.5 if $x$ increased by 1 ".
dydx (), eyex(), eydx(), and dyex() may be used with continuous $x$ variables. dydx() and eydx() may also be used with factor variables.

## Derivatives versus discrete differences

In (1),

$$
\begin{aligned}
y=\beta_{0} & +\beta_{1} \times \mathrm{sex}+\beta_{2} \times \text { age }+\beta_{3} \times \mathrm{age}^{2}+\beta_{4} \times \mathrm{bp}+\beta_{5} \times \mathrm{sex} \times \mathrm{bp}+\beta_{6} \times \mathrm{tmt} \\
& +\beta_{7} \times \mathrm{tmt} \times \text { age }+\beta_{8} \times \mathrm{tmt} \times \mathrm{age}^{2}+\epsilon
\end{aligned}
$$

Let us call your attention to the derivatives of $y$ with respect to age and sex:

$$
\begin{gather*}
d y / d(\text { age })=\beta_{2}+2 \times \beta_{3} \times \text { age }+\beta_{7} \times \mathrm{tmt}+2 \times \beta_{8} \times \mathrm{tmt} \times \text { age }  \tag{2}\\
d y / d(\operatorname{sex})=\beta_{1}+\beta_{5} \times \mathrm{bp} \tag{3}
\end{gather*}
$$

age is presumably a continuous variable and (2) is precisely how margins calculates its derivatives when you type margins, dydx (age). sex, however, is presumably a factor variable, and margins does not necessarily make the calculation using (3) were you to type margins, dydx (sex). We will explain, but let us first clarify what we mean by a continuous and a factor variable. Say that you fit (1) by typing

```
. regress y i.sex age c.age#c.age i.bp bp#sex
> i.tmt tmt#c.age tmt#c.age#c.age
```

It is important that sex entered the model as a factor variable. It would not do to type regress y sex... because then sex would be a continuous variable, or at least it would be a continuous variable from Stata's point of view. The model estimates would be the same, but margins' understanding of those estimates would be a little different. With the model estimated using i.sex, margins understands that either sex is 0 or sex is 1 . With the model estimated using sex, margins thinks sex is continuous and, for instance, sex $=1.5$ is a possibility.
margins calculates $d y d x()$ differently for continuous and for factor variables. For continuous variables, margins calculates $d y / d x$. For factor variables, margins calculates the discrete firstdifference from the base category. To obtain that for sex, write down the model and then subtract from it the model evaluated at the base category for sex, which is sex $=0$. If you do that, you will get the same formula as we obtained for the derivative, namely,

$$
\text { discrete difference }\{(\operatorname{sex}=1)-(\operatorname{sex}=0)\}=\beta_{1}+\beta_{5} \times \mathrm{bp}
$$

We obtain the same formula because our model is linear regression. Outside of linear regression, and outside of linear response functions generally, the discrete difference is not equal to the derivative. The discrete difference is not equal to the derivative for logistic regression, probit, etc. The discrete difference calculation is generally viewed as better for factor variables than the derivative calculation because the discrete difference is what would actually be observed.

If you want the derivative calculation for your factor variables, specify the continuous option on the margins command.

## Example 16: Average marginal effect (partial effects)

Concerning the title of this example, the way we use the term marginal effect, the effects of factor variables are calculated using discrete first-differences. If you wanted the continuous calculation, you would specify margins' continuous option in what follows.


Note: dy/dx for factor levels is the discrete change from the base level.
The average marginal effect of treatment on the probability of a positive outcome is 0.039 .

## Example 17: Average marginal effect of all covariates

We will continue with the model

```
. logistic outcome treatment##group age c.age#c.age treatment#c.age
```

if we wanted the average marginal effects for all covariates, we would type margins, dydx (*) or margins, dydx (_all); they mean the same thing. This is probably the most common way margins, dydx() is used.

```
. margins, dydx(*)
Average marginal effects Number of obs = 3000
Model VCE : OIM
Expression : Pr(outcome), predict()
dy/dx w.r.t. : 1.treatment 2.group 3.group age
```

|  | Delta-method |  |  |  |  |  |
| ---: | ---: | ---: | :---: | :---: | :---: | :---: | :---: |
|  | dy/dx | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| 1. treatment | .0385625 | .0162848 | 2.37 | 0.018 | .0066449 | .0704801 |
| group |  |  |  |  |  |  |
| 2 | -.0776906 | .0181584 | -4.28 | 0.000 | -.1132805 | -.0421007 |
| 3 | -.1505652 | .0400882 | -3.76 | 0.000 | -.2291366 | -.0719937 |
| age | .0095868 | .0007796 | 12.30 | 0.000 | .0080589 | .0111148 |

Note: dy/dx for factor levels is the discrete change from the base level.

## Example 18: Evaluating marginal effects over the response surface

Continuing with the model
. logistic outcome treatment\#\#group age c.age\#c.age treatment\#c.age
What follows maps out the entire response surface of our fitted model. We report the marginal effect of treatment evaluated at age $=20,30, \ldots, 60$, by each level of group.

| Conditional marginal effects <br> Model VCE : OIM |  |  |  | Numb | of obs | 3000 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Expression : Pr(outcome), predict() <br> dy/dx w.r.t. : 1.treatment |  |  |  |  |  |  |
| 1._at |  | = | 20 |  |  |  |
| 2._at | age | = | 30 |  |  |  |
| 3._at | age | = | 40 |  |  |  |
| 4._at |  | = | 50 |  |  |  |
| 5._at | age | = | 60 |  |  |  |
|  | $d y / d x$ | lta-metho Std. Err. | z | $P>\|z\|$ | [95\% Conf. | Interval] |
| 1.treatment _at\#group |  |  |  |  |  |  |
| 11 | -. 0208409 | . 0152862 | -1.36 | 0.173 | -. 0508013 | . 0091196 |
| 12 | . 009324 | . 0059896 | 1.56 | 0.120 | -. 0024155 | . 0210635 |
| 13 | . 0006558 | . 0048682 | 0.13 | 0.893 | -. 0088856 | . 0101972 |
| 21 | -. 0436964 | . 0279271 | -1.56 | 0.118 | -. 0984325 | . 0110397 |
| 22 | . 0382959 | . 0120405 | 3.18 | 0.001 | . 014697 | . 0618949 |
| 23 | . 0064564 | . 0166581 | 0.39 | 0.698 | -. 0261929 | . 0391057 |
| 31 | -. 055676 | . 0363191 | -1.53 | 0.125 | -. 1268601 | . 015508 |
| 32 | . 1152235 | . 0209858 | 5.49 | 0.000 | . 074092 | . 156355 |
| 33 | . 0284808 | . 0471293 | 0.60 | 0.546 | -. 0638908 | . 1208524 |
| 41 | -. 027101 | . 0395501 | -0.69 | 0.493 | -. 1046177 | . 0504158 |
| 42 | . 2447682 | . 0362623 | 6.75 | 0.000 | . 1736954 | . 315841 |
| 43 | . 0824401 | . 1025028 | 0.80 | 0.421 | -. 1184616 | . 2833418 |
| 51 | . 0292732 | . 0587751 | 0.50 | 0.618 | -. 0859239 | . 1444703 |
| 52 | . 3757777 | . 0578106 | 6.50 | 0.000 | . 2624709 | . 4890844 |
| 53 | . 1688268 | . 1642191 | 1.03 | 0.304 | -. 1530368 | . 4906904 |

Note: dy/dx for factor levels is the discrete change from the base level.

## Obtaining margins with survey data and representative samples

The standard errors and confidence intervals produced by margins are based by default on the delta method applied to the VCE of the current estimates. Delta-method standard errors treat the covariates at which the response is evaluated as given or fixed. Such standard errors are appropriate if you specify at () to fix the covariates, and they are appropriate when you are making inferences about groups exactly like your sample whether you specify at () or not.

On the other hand, if you have a representative sample of the population or if you have complex survey data and if you want to make inferences about the underlying population, you need to account for the variation in the covariates that would arise in repeated sampling. You do that using vce (unconditional), which invokes a different standard-error calculation based on Korn and Graubard (1999). Syntactically, there are three cases. They all involve specifying the vce (unconditional) option on the margins command:

1. You have a representative random sample, and you have not svyset your data. When you fit the model, you need to specify the vce (robust) or vce (cluster clustvar) option. When you issue the margins command, you need to specify the vce (unconditional) option.
2. You have a weighted sample, and you have not svyset your data.

You need to specify [pw=weight] when you fit the model and, of course, specify the vce (unconditional) option on the margins command. You do not need to specify the weights on the margins command because margins will obtain them from the estimation results.
3. You have svyset your data, whether it be a simple random sample or something more complex including weights, strata, sampling units, or poststratification, and you are using the linearized variance estimator.
You need to use the svy prefix when you fit the model. You need to specify vce (unconditional) when you issue the margins command. You do not need to respecify the weights.
Even though the data are svyset, and even though the estimation was svy estimation, margins does not default to vce(unconditional). It does not default to vce (unconditional) because there are valid reasons to want the data-specific, vce (delta) standard-error estimates. Whether you specify vce (unconditional) or not, margins uses the weights, so you do not need to respecify them even if you are using vce (unconditional).
vce(unconditional) is allowed only after estimation with vce(robust), vce(cluster ...), or the svy prefix with the linearized variance estimator. If the VCE of the current estimates was specified as clustered, so will be the VCE estimates of margins. If the estimates were from a survey estimation, the survey settings in the dataset will be used by margins.

When you use vce(unconditional), never specify if exp or in range on the margins command; instead, specify the subpop(if exp) option. You do that for the usual reasons; see [SVY] subpopulation estimation. If you specify over (varlist) to examine subgroups, the subgroups will automatically be treated as subpopulations.

If you are using a replication-based variance estimator, you may want to use this method to estimate the variance of your margins; see [SVY] svy postestimation.

## Example 19: Inferences for populations, margins of response

In example 6, we fit the model
. logistic outcome i.sex i.group sex\#group age
and we obtained margins by sex and margins by group,

```
. margins sex group
```

If our data were randomly drawn from the population of interest and we wanted to account for this, we would have typed

```
. logistic outcome i.sex i.group sex#group age, vce(robust)
```

. margins sex group, vce(unconditional)

We do that below:

| Predictive margins |  |  |  | Number of obs |  | 3000 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Expression : Pr(outcome), predict() |  |  |  |  |  |  |
|  | Unconditional |  |  |  |  |  |
|  | Margin | Std. Err. | z | $P>\|z\|$ | [95\% Conf | Interval] |
| sex |  |  |  |  |  |  |
| male | . 1600644 | . 0131685 | 12.16 | 0.000 | . 1342546 | . 1858743 |
| female | . 1966902 | . 0104563 | 18.81 | 0.000 | . 1761963 | . 2171841 |
| group |  |  |  |  |  |  |
| 1 | . 2251302 | . 0127069 | 17.72 | 0.000 | . 200225 | . 2500354 |
| 2 | . 150603 | . 0118399 | 12.72 | 0.000 | . 1273972 | . 1738088 |
| 3 | . 0736157 | . 0343188 | 2.15 | 0.032 | . 0063522 | . 1408793 |

The estimated margins are the same as they were in example 6, but the standard errors and confidence intervals differ, although not by much. Given that we have 3,000 observations in our randomly drawn sample, we should expect this.

## Example 20: Inferences for populations, marginal effects

In example 17, we fit a logistic model and then obtained the average marginal effects for all covariates by typing
. logistic outcome treatment\#\#group age c.age\#c.age treatment\#c.age
. margins, dydx(*)
To repeat that and also obtain standard errors for our population, we would type

```
. logistic outcome treatment##group age c.age#c.age treatment#c.age,
> vce(robust)
. margins, dydx(*) vce(unconditional)
```

The results are

```
. logistic outcome treatment##group age c.age#c.age treatment#c.age, vce(robust)
    (output omitted)
. margins, dydx(*) vce(unconditional)
Average marginal effects Number of obs = 3000
Expression : Pr(outcome), predict()
dy/dx w.r.t. : 1.treatment 2.group 3.group age
```

|  | Unconditional |  |  |  |  |  |
| ---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | dy/dx | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| 1.treatment | .0385625 | .0163872 | 2.35 | 0.019 | .0064442 | .0706808 |
| group |  |  |  |  |  |  |
| 2 | -.0776906 | .0179573 | -4.33 | 0.000 | -.1128863 | -.0424949 |
| 3 | -.1505652 | .0411842 | -3.66 | 0.000 | -.2312848 | -.0698456 |
| age | .0095868 | .0007814 | 12.27 | 0.000 | .0080553 | .0111183 |

Note: dy/dx for factor levels is the discrete change from the base level.

## Example 21: Inferences for populations with svyset data

See example 3 in [SVY] svy postestimation.

## Standardizing margins

A standardized margin is the margin calculated on data different from the data used to fit the model. Typically, the word standardized is reserved for situations in which the alternate population is a reference population, which may be real or artificial, and which is treated as fixed.

Say that you work for a hospital and have fit a model of mortality on the demographic characteristics of the hospital's patients. At this stage, were you to type
. margins
you would obtain the mortality rate for your hospital. You have another dataset, hstandard.dta, that contains demographic characteristics of patients across all hospitals along with the population of each hospital recorded in the pop variable. You could obtain the expected mortality rate at your hospital if your patients matched the characteristics of the standard population by typing
. use http://www.stata-press.com/data/r13/hstandard, clear
. margins [fw=pop], noesample
You specified noesample because the margin is being calculated on data other than the data used to estimate the model. You specified [fw=pop] because the reference dataset you are using included population counts, as many reference datasets do.

## Obtaining margins as though the data were balanced

Here we discuss what are commonly called estimated marginal means or least-squares means. These are margins assuming that all levels of factor variables are equally likely or, equivalently, that the design is balanced. The seminal reference on these margins is Searle, Speed, and Milliken (1980).

In designed experiments, observations are often allocated in a balanced way so that the variances can be easily compared and decomposed. At the Acme Portable Widget Company, they are experimenting with a new machine. The machine has three temperature settings and two pressure settings; a combination of settings will be optimal on any particular day, determined by the weather. At start-up, one runs a quick test and chooses the optimal setting for the day. Across different days, each setting will be used about equally, says the manufacturer.

In experiments with the machine, 10 widgets were collected for stress testing at each of the settings over a six-week period. We wish to know the average stress-test value that can be expected from these machines over a long period.

## Balancing using asbalanced

The data were intended to be balanced, but unfortunately, the stress test sometimes destroys samples before the stress can be measured. Thus even though the experiment was designed to be balanced, the data are not balanced. You specify the asbalanced option to estimate the margins as if the data were balanced. We will type
. use http://www.stata-press.com/data/r13/acmemanuf
. regress y pressure\#\#temp
. margins, asbalanced

So that you can compare the asbalanced results with the observed results, we will also include margins without the asbalanced option in what follows:


## Technical note

Concerning how asbalanced calculations are performed, if a factor variable has $l$ levels, then each level's coefficient contributes to the response weighted by $1 / l$. If two factors, $a$ and $b$, interact, then each coefficient associated with their interaction is weighted by $1 /\left(l_{a} \times l_{b}\right)$.

If a balanced factor interacts with a continuous variable, then each coefficient in the interaction is applied to the value of the continuous variable, and the results are weighted equally. So, if the factor being interacted has $l_{a}$ levels, the effect of each coefficient on the value of the continuous covariate is weighted by $1 / l_{a}$.

## Balancing by standardization

To better understand the balanced results, we can perform the balancing ourselves by using the standardizing method shown in Standardizing margins. To do that, we will input a balanced dataset and then type margins, noesample.

```
. use http://www.stata-press.com/data/r13/acmemanuf
. regress y pressure##temp
    (output omitted)
. drop _all
. input pressure temp
        pressure temp
    1. 1 1
    2. 1 2
    3. 1 3
    4. 2 1
    5. 22
    6. 2 3
    7. end
. margins, noesample
Predictive margins Number of obs = 6
Model VCE : OLS
Expression : Linear prediction, predict()
\begin{tabular}{r|rccccc}
\hline & \multicolumn{4}{|c}{ Delta-method } \\
& Margin & Std. Err. & t & \(\mathrm{P}>|\mathrm{t}|\) & [95\% Conf. Interval] \\
\hline _cons & 115.3758 & 1.530199 & 75.40 & 0.000 & 112.2899 & 118.4618 \\
\hline
\end{tabular}
```

We obtain the same results as previously.

## Balancing nonlinear responses

If our testing had produced a binary outcome, say, acceptable/unacceptable, rather than a continuous variable, we would type
. use http://www.stata-press.com/data/r13/acmemanuf, clear

- logistic acceptable pressure\#\#temp
. margins, asbalanced
The result of doing that would be 0.680 . If we omitted the asbalanced option, the result would have been 0.667 . The two results are so similar because acmemanuf.dta is nearly balanced.

Even though the asbalanced option can be used on both linear and nonlinear responses, such as probabilities, there is an issue of which you should be aware. The most widely used formulas for balancing responses apply the balancing to the linear prediction, average that as if it were balanced, and then apply the nonlinear transform. That is the calculation that produced 0.680 .

An alternative would be to apply the standardization method. That amounts to making the linear predictions observation by observation, applying the nonlinear transform to each, and then averaging the nonlinear result as if it were balanced. You could do that by typing
. use http://www.stata-press.com/data/r13/acmemanuf, clear
. logistic acceptable pressure\#\#temp
. clear
. input pressure temp (see above for entered data)
. margins, noesample

The result from the standardization procedure would be 0.672 . These two ways of averaging nonlinear responses are discussed in detail in Lane and Nelder (1982) within the context of general linear models.

Concerning the method used by the asbalanced option, if your data start balanced and you have a nonlinear response, you will get different results with and without the asbalanced option!

## Treating a subset of covariates as balanced

So far, we have treated all the covariates as if they were balanced. margins will allow you to treat a subset of the covariates as balanced, too. For instance, you might be performing an experiment in which you are randomly allocating patients to a treatment arm and so want to balance on arm, but you do not want to balance the other characteristics because you want mean effects for the experiment's population.

In this example, we will imagine that the outcome of the experiment is continuous. We type
. use http://www.stata-press.com/data/r13/margex, clear
. regress y arm\#\#sex sex\#\#agegroup
. margins, at((asbalanced) arm)
If we wanted results balanced on agegroup as well, we could type

```
. margins, at((asbalanced) arm agegroup)
```

If we wanted results balanced on all three covariates, we could type
. margins, at((asbalanced) arm agegroup sex)
or we could type

```
. margins, at((asbalanced) _factor)
```

or we could type

```
. margins, asbalanced
```


## Using fvset design

As a convenience feature, equivalent to

- regress y arm\#\#sex sex\#\#agegroup
. margins, at((asbalanced) arm sex)
is
. fvset design asbalanced arm sex
. regress y arm\#\#sex sex\#\#agegroup
. margins
The advantage of the latter is that you have to set the variables as balanced only once. This is useful when balancing is a design characteristic of certain variables and you wish to avoid accidentally treating them as unbalanced.

If you save your data after fvsetting, the settings will be remembered in future sessions. If you want to clear the setting(s), type
. fvset clear varlist
See [R] fvset.

## Balancing in the presence of empty cells

The issue of empty cells is not exclusively an issue of balancing, but there are special considerations when balancing. Empty cells are discussed generally in Estimability of margins.

An empty cell is an interaction of levels of two or more factor variables for which you have no data. Usually, margins involving empty cells cannot be estimated. When balancing, there is an alternate definition of the margin that allows the margin to be estimated. margins makes the alternate calculation when you specify the emptycells(reweight) option. By default, margins uses the emptycells(strict) option.

If you have empty cells in your data and you request margins involving the empty cells, those margins will be marked as not estimable even if you specify the asbalanced option.


This example is discussed in Estimability of margins, although without the asbalanced option. What is said there is equally relevant to the asbalanced case. For reasons explained there, the margin for sex $=1$ (female) cannot be estimated.

The margin for sex $=1$ can be estimated in the asbalanced case if you are willing to make an assumption. Remember that margins makes the balanced calculation by summing the responses associated with the levels and then dividing by the number of levels. If you specify emptycells(reweight), margins sums what is available and divides by the number available. Thus you are assuming that, whatever the responses in the empty cells, those responses are such that they would not change the overall mean of what is observed.

The results of specifying emptycells(reweight) are

| Adjusted predictionsModel VCE : OLS |  |  |  | Number of obs |  | 69 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |
| Expression : Linear prediction, predict()  <br> Empty cells : reweight  <br> at sex (asbalanced) <br>  group (asbalanced) |  |  |  |  |  |  |
|  | Delta-method |  |  | $p>\|t\|$ | [95\% Conf. Interval] |  |
|  | Margin | Std. Err. | t |  |  |  |  |
| sex |  |  |  |  |  |  |
| male | 21.91389 | 1.119295 | 19.58 | 0.000 | 19.67 | 24.15206 |
| female | 24.85185 | 1.232304 | 20.17 | 0.000 | 22.38 | 27.316 |

## Obtaining margins with nested designs

## Introduction

Factors whose meaning depends on other factors are called nested factors, and the factors on which their meaning depends are called the nesting factors. For instance, assume that we have a sample of patients and each patient is assigned to one doctor. Then patient is nested within doctor. Let the identifiers of the first 5 observations of our data be

| Doctor | Patient | Name |
| :--- | :--- | :--- |
| 1 | 1 | Fred |
| 1 | 2 | Mary |
| 1 | 3 | Bob |
| 2 | 1 | Karen |
| 2 | 2 | Hank |

The first patient on one doctor's list has nothing whatsoever to do with the first patient on another doctor's list. The meaning of patient $=1$ is defined only when the value of doctor is supplied.

Nested factors enter into models as interactions of nesting and nested; the nested factor does not appear by itself. We might estimate a model such as

```
regress y ... i.doctor doctor#patient ...
```

You do not include i.patient because the coding for patient has no meaning except within doctor. Patient 1 is Fred for doctor 1 and Karen for doctor 2, etc.
margins provides an option to help account for the structure of nested models. The within (varlist) option specifies that margins estimate and report a set of margins for the value combinations of varlist. We might type

```
. margins, within(doctor)
```

Margin calculations are performed first for doctor $=1$, then for doctor $=2$, and so on.
Sometimes you need to specify within(), and other times you do not. Let's consider the particular model

The guidelines are the following:

1. You may compute overall margins by typing margins.
2. You may compute overall margins within levels of a nesting factor by typing margins, within(doctor).
3. You may compute margins of a nested factor within levels of its nesting factor by typing margins patient, within(doctor).
4. You may compute margins of factors in your model, as long as the factor does not nest other factors and is not nested within other factors, by typing margins sex.
5. You may not compute margins of a nesting factor, such as margins doctor, because they are not estimable.

For examples using within(), see [R] anova.

## Margins with nested designs as though the data were balanced

To obtain margins with nested designs as though the data were balanced, the guidelines are the same as above except that 1) you add the asbalanced option and 2) whenever you do not specify within(), you specify emptycells(reweight). The updated guidelines are

1. You may compute overall margins by typing margins, asbalanced emptycells(reweight).
2. You may compute overall margins within levels of a nesting factor by typing margins, asbalanced within(doctor).
3. You may compute margins of a nested factor within levels of its nesting factor by typing margins patient, asbalanced within(doctor).
4. You may compute margins of factors in your model, as long as the factor does not nest other factors and is not nested within other factors, by typing margins sex, asbalanced emptycells(reweight).
5. You may not compute margins of a nesting factor, such as margins doctor, because they are not estimable.

Just as explained in Using fvset design, rather than specifying the asbalanced option, you may set the balancing characteristic on the factor variables once and for all by using the command fvset design asbalanced varlist.

## - Technical note

Specifying either emptycells(reweight) or within(varlist) causes margins to rebalance over all empty cells in your model. If you have interactions in your model that are not involved in the nesting, margins will lose its ability to detect estimability.

## - Technical note

Careful readers will note that the description of within(varlist) matches closely the description of over (varlist). The concept of nesting is similar to the concept of subpopulations. within() differs from over () in that it gracefully handles the missing cells when margins are computed as balanced.

## Coding of nested designs

In the Introduction to this section, we showed a coding of the nested variable patient, where the coding started over with each doctor:

| Doctor | Patient | Name |
| :--- | :--- | :--- |
| 1 | 1 | Fred |
| 1 | 2 | Mary |
| 1 | 3 | Bob |
| 2 | 1 | Karen |
| 2 | 2 | Hank |

That coding style is not required. The data could just as well have been coded

| Doctor | Patient | Name |
| :--- | :--- | :--- |
| 1 | 1 | Fred |
| 1 | 2 | Mary |
| 1 | 3 | Bob |
| 2 | 4 | Karen |
| 2 | 5 | Hank |

or even

| Doctor | Patient | Name |
| :--- | :--- | :--- |
| 1 | 1037239 | Fred |
| 1 | 2223942 | Mary |
| 1 | 0611393 | Bob |
| 2 | 4433329 | Karen |
| 2 | 6110271 | Hank |

Actually, either of the above two alternatives are better than the first one because margins will be better able to give you feedback about estimability should you make a mistake following the guidelines. On the other hand, both of these two alternatives require more memory at the estimation step. If you run short of memory, you will need to recode your patient ID to the first coding style, which you could do by typing
. sort doctor patient
. by doctor: gen newpatient = _n
Alternatively, you can set emptycells drop and continue to use your patient ID variable just as it is coded. If you do this, we recommend that you remember to type set emptycells keep when you are finished; margins is better able to determine estimability that way. If you regularly work with large nested models, you can set emptycells keep, permanently so that the setting persists across sessions. See $[R]$ set emptycells.

## Special topics

## Requirements for model specification

The results that margins reports are based on the most recently fit model or, in Stata jargon, the most recently issued estimation command. Here we discuss 1) mechanical requirements for how you specify that estimation command, 2) work-arounds to use when those restrictions prove impossible, and 3) requirements for margins' predict (pred_opt) option to work.

Concerning 1, when you specify the estimation command, covariates that are logically factor variables must be Stata factor variables, and that includes indicator variables, binary variables, and dummies. It will not do to type
. regress y ... female ...
even if female is a $0 / 1$ variable. You must type
. regress y ... i.female ...
If you violate this rule, you will not get incorrect results, but you will discover that you will be unable to obtain margins on female:

```
. margins female
factor female not found in e(b)
r(111);
```

It is also important that if the same continuous variable appears in your model more than once, differently transformed, those transforms be performed via Stata's factor-variable notation. It will not do to type

- generate age2 = age~2
. regress y ... age age2 ...


## You must type

. regress y ... age c.age\#c.age ...
You must do that because margins needs to know everywhere that variable appears in the model if it is to be able to set covariates to fixed values.

Concerning 2, sometimes the transformations you desire may not be achievable using the factorvariable notation; in those situations, there is a work-around. Let's assume you wish to estimate
. generate age1_5 = age^1.5
. regress y ... age age1_5 ...
There is no factor-variable notation for including age and age ${ }^{1.5}$ in a model, so obviously you are going to obtain the estimates by typing just what we have shown. In what follows, it would be okay if there are interactions of age and age1_5 with other variables specified by the factor-variable notation, so the model could just as well be

```
. regress y ... age age1_5 sex#c.age sex#c.age1_5 ...
```

Let's assume you have fit one of these two models. On any subsequent margins command where you leave age free to vary, there will be no issue. You can type
and results will be correct. Issues arise when you attempt to fix age at predetermined values. The following would produce incorrect results:

```
. margins female, at(age=20)
```

The results would be incorrect because they leave age1_5 free to vary, and, logically, fixing age implies that age1_5 should also be fixed. Because we were unable to state the relationship between age and age1_5 using the factor-variable notation, margins does not know to fix age1_5 at $20^{1.5}$ when it fixes age at 20 . To get the correct results, you must fix the value of age1_5 yourself:

```
. margins female, at(age=20 age1_5=89.442719)
```

That command produces correct results. In the command, 89.442719 is $20^{1.5}$.
In summary, when there is a functional relationship between covariates of your model and that functional relationship is not communicated to margins via the factor-variable notation, then it becomes your responsibility to ensure that all variables that are functionally related are set to the appropriate fixed values when any one of them is set to a fixed value.

Concerning 3, we wish to amend our claim that you can calculate margins for anything that predict will produce. We need to add a qualifier. Let us show you an example where the statement is not true. After regress, predict will predict something it calls $\operatorname{pr}(a, b)$, which is the probability $a \leq y \leq b$. Yet if we attempted to use $\operatorname{pr}$ () with margins after estimation by regress, we would obtain

```
. margins sex, predict(pr (10,20))
prediction is a function of possibly stochastic quantities other than e(b)
r(498);
```

What we should have stated was that you can calculate margins for anything that predict will produce for which all the estimated quantities used in its calculation appear in e(V), the estimated VCE. $\operatorname{pr}()$ is a function of $\boldsymbol{\beta}$, the estimated coefficients, and of $s^{2}$, the estimated variance of the residual. regress does not post the variance of the residual variance (sic) in $e(V)$, or even estimate it, and therefore, predict $(\operatorname{pr}(10,20)$ ) cannot be specified with margins after estimation by regress.

It is unlikely that you will ever encounter these kinds of problems because there are so few predictions where the components are not posted to $e(V)$. If you do encounter the problem, the solution may be to specify nose to suppress the standard-error calculation. If the problem is not with computing the margin, but with computing its standard error, margins will report the result:

```
. margins sex, predict(pr (10,20)) nose
    (output appears with SEs, tests, and CIs left blank)
```


## - Technical note

Programmers: If you run into this after running an estimation command that you have written, be aware that as of Stata 11, you are supposed to set in e(marginsok) the list of options allowed with predict that are okay to use with margins. When that list is not set, margins looks for violations of its assumptions and, if it finds any, refuses to proceed.

## Estimability of margins

Sometimes margins will report that a margin cannot be estimated:

```
. use http://www.stata-press.com/data/r13/estimability, clear
(margins estimability)
. regress y sex##group
    (output omitted)
. margins sex
Predictive margins Number of obs = 69
Model VCE : OLS
Expression : Linear prediction, predict()
```

|  | MarginDelta-method <br> Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| sex <br> male <br> female | 21.8500245 | 24.71 | 0.000 | 19.30027 | 22.69973 |

In the above output, the margin for sex $=0$ (male) is estimated, but the margin for sex $=1$ (female) is not estimable. This occurs because of empty cells. An empty cell is an interaction of levels of two or more factor variables for which you have no data. In the example, the lack of estimability arises because we have two empty cells:

```
. table sex group
```

|  | group |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: |
| sex | 1 | 2 | 3 | 4 | 5 |
| male | 2 | 9 | 27 | 8 | 2 |
| female | 9 | 9 | 3 |  |  |

To calculate the marginal mean response for sex $=1$, we have no responses to average over for group $=4$ and group $=5$. We obviously could calculate that mean for the observations that really are sex $=1$, but remember, the marginal calculation for sex $=1$ treats everyone as if female, and we will thus have 8 and 2 observations for which we have no basis for estimating the response.

There is no solution for this problem unless you are willing to treat the data as if it were balanced and adjust your definition of a margin; see Balancing in the presence of empty cells.

## Manipulability of tests

Manipulability is a problem that arises with some tests, and in particular, arises with Wald tests. Tests of margins are based on Wald tests, hence our interest. This is a generic issue and not specific to the margins command.

Let's understand the problem. Consider performing a test of whether some statistic $\phi$ is 0 . Whatever the outcome of that test, it would be desirable if the outcome were the same were we to test whether the $\operatorname{sqrt}(\phi)$ were 0 , or whether $\phi^{2}$ were 0 , or whether any other monotonic transform of $\phi$ were 0 (for $\phi^{2}$, we were considering only the positive half of the number line). If a test does not have that property, it is manipulable.

Wald tests are manipulable, and that means the tests produced by margins are manipulable. You can see this for yourself by typing
. use http://www.stata-press.com/data/r13/margex, clear
. replace y = y - 65
. regress y sex\#\#group
. margins, df(.)
. margins, expression(predict(xb)^2)
To compare the results from the two margins commands, we added the $\mathrm{df}($.$) option to the$ first one, forcing it to report a $z$ statistic even though a $t$ statistic would have been appropriate in this case. We would prefer if the test against zero produced by margins, $\mathrm{df}($.$) was equal to the$ test produced by margins, expression(predict (xb) $\wedge^{2}$ ). But alas, they produce different results. The first produces $z=12.93$, and the second produces $z=12.57$.

The difference is not much in our example, but behind the scenes, we worked to make it small. We subtracted 65 from y so that the experiment would be for a case where it might be reasonable that you would be testing against 0 . One does not typically test whether the mean income in the United States is zero or whether the mean blood pressure of live patients is zero. Had we left y as it was originally, we would have obtained $z=190$ and $z=96$. We did not want to show that comparison to you first because the mean of $y$ is so far from 0 that you probably would never be testing it. The corresponding difference in $\phi$ is tiny.

Regardless of the example, it is important that you base your tests in the metric where the likelihood surface is most quadratic. For further discussion on manipulability, see Manipulability in [R] predictnl.

This manipulability is not limited to Wald tests after estimation; you can also see the manipulability of results produced by linear regression just by applying nonlinear transforms to a covariate (Phillips and Park 1988; Gould 1996).

## Using margins after the estimates use command

Assume you fit and used estimates save (see [R] estimates save) to save the estimation results:

```
. regress y sex##group age c.age*c.age if site==1
. estimates save mymodel
(file mymodel.ster saved)
```

Later, perhaps in a different Stata session, you reload the estimation results by typing

```
. estimates use mymodel
```

You plan to use margins with the reloaded results. You must remember that margins bases its results not only on the current estimation results but also on the current data in memory. Before you can use margins, you must reload the dataset on which you fit the model or, if you wish to produce standardized margins, some other dataset.

```
. use mydata, clear
(data for fitting models)
```

If the dataset you loaded contained the data for standardization, you can stop reading; you know that to produce standardized margins, you need to specify the noesample option.

We reloaded the original data and want to produce margins for the estimation sample. In addition to the data, margins requires that e(sample) be set, as margins will remind us:

```
. margins sex
e(sample) does not identify the estimation sample
r(322);
```

The best solution is to use estimates esample to rebuild e(sample):

```
. estimates esample: y sex group age if site==1
```

If we knew we had no missing values in y and the covariates, we could type

```
. estimates esample: if site==1
```

Either way, margins would now work:

```
. margins sex
(usual output appears)
```

There is an alternative. We do not recommend it, but we admit that we have used it. Rather than rebuilding e(sample), you can use margins' noesample option to tell margins to skip using e (sample). You could then specify the appropriate if statement (if necessary) to identify the estimation sample:

```
. estimates use mymodel
. use mydata, clear
(data for fitting models)
. margins sex if !missing(y, sex, group age) & site==1, noesample
(usual output appears)
```

In the above, we are not really running on a sample different from the estimation sample; we are merely using noesample to fool margins, and then we are specifying on the margins command the conditions equivalent to re-create e(sample).

If we wish to obtain vce (unconditional) results, however, noesample will be insufficient. We must also specify the force option,

```
. margins sex if !missing(y, sex, group age) & site==1,
> vce(unconditional) noesample force
(usual output appears)
```

Regardless of the approach you choose-resetting e(sample) or specifying noesample and possibly force-make sure you are right. In the vce (delta) case, you want to be right to ensure that you obtain the results you want. In the vce (unconditional) case, you need to be right because otherwise results will be statistically invalid.

## Syntax of at()

In at (atspec), atspec may contain one or more of the following specifications:
varlist
(stat) varlist
varname = \#
varname $=$ (numlist $)$
varname $=$ generate $(\exp )$
where

1. varnames must be covariates in the previously fit model (estimation command).
2. Variable names (whether in varname or varlist) may be continuous variables, factor variables, or specific level variables, such as age, group, or 3.group.
3. varlist may also be one of three standard lists:
a. _all (all covariates),
b. - factor (all factor-variable covariates), or
c. -continuous (all continuous covariates).
4. Specifications are processed from left to right with latter specifications overriding previous ones.
5. stat can be any of the following:

| stat | Description | Variables <br> allowed |
| :--- | :--- | :--- |
| asobserved | at observed values in the sample (default) <br> means (default for varlist) | all <br> all |
| median | medians | continuous <br> p1 |
| p2 | 1st percentile | continuous |
| $\ldots$ | 2nd percentile | continuous |
| p50 | 3rd-49th percentiles | continuous |
| $\ldots$ | 50th percentile (same as median) | continuous |
| p98 | 51st-97th percentiles | continuous |
| p99 | 98th percentile | continuous |
| min | 99th percentile | continuous |
| max | minimums | continuous |
| zero | maximums | continuous |
| base | fixed at zero | base level |
| asbalanced | all levels equally probable and sum to 1 | factors |

Any stat except zero, base, and asbalanced may be prefixed with an o to get the overall statistic-the sample over all over() groups. For example, omean, omedian, and op25. Overall statistics differ from their correspondingly named statistics only when the over() or within() option is specified. When no stat is specified, mean is assumed.

## Estimation commands that may be used with margins

margins may be used after most estimation commands.
margins cannot be used after estimation commands that do not produce full variance matrices, such as exlogistic and expoisson (see [ $R$ ] exlogistic and $[R]$ expoisson).
margins is all about covariates and cannot be used after estimation commands that do not post the covariates, which eliminates gmm (see $[\mathrm{R}]$ gmm).
margins cannot be used after estimation commands that have an odd data organization, and that excludes asclogit, asmprobit, asroprobit, and nlogit (see [R] asclogit, [R] asmprobit, [R] asroprobit, and [R] nlogit).

## Video examples

Introduction to margins, part 1: Categorical variables
Introduction to margins, part 2: Continuous variables
Introduction to margins, part 3: Interactions

## Glossary

adjusted mean. A margin when the response is the linear predictor from linear regression, ANOVA, etc. For some authors, adjusting also implies adjusting for unbalanced data. See Obtaining margins of responses and see Obtaining margins as though the data were balanced.
average marginal effect. See marginal effect and average marginal effect.
average partial effect. See partial effect and average partial effect.
conditional margin. A margin when the response is evaluated at fixed values of all the covariates. If any covariates are left to vary, the margin is called a predictive margin.
effect. The effect of $x$ is the derivative of the response with respect to covariate $x$, or it is the difference in responses caused by a discrete change in $x$. Also see marginal effect.
The effect of $x$ measures the change in the response for a change in $x$. Derivatives or differences might be reported as elasticities. If $x$ is continuous, the effect is measured continuously. If $x$ is a factor, the effect is measured with respect to each level of the factor and may be calculated as a discrete difference or as a continuous change, as measured by the derivative. margins calculates the discrete difference by default and calculates the derivative if the continuous option is specified.
elasticity and semielasticity. The elasticity of $y$ with respect to $x$ is $d(\ln y) / d(\ln x)=(x / y) \times(d y / d x)$, which is approximately equal to the proportional change in $y$ for a proportional change in $x$.

The semielasticity of $y$ with respect to $x$ is either 1$) d y / d(\ln x)=x \times(d y / d x)$ or 2$) d(\ln y) / d x=$ $(1 / y) \times(d y / d x)$, which is approximately 1) the change in $y$ for a proportional change in $x$ or 2) the proportional change in $y$ for a change in $x$.
empty cell. An interaction of levels of two or more factor variables for which you have no data. For instance, you have sex interacted with group in your model, and in your data there are no females in group 1. Empty cells affect which margins can be estimated; see Estimability of margins.
estimability. Estimability concerns whether a margin can be uniquely estimated (identified); see Estimability of margins.
estimated marginal mean. This is one of the few terms that has the same meaning across authors. An estimated marginal mean is a margin assuming the levels of each factor covariate are equally likely (balanced), including interaction terms. This is obtained using margins' asbalanced option. In addition, there is an alternate definition of estimated marginal mean in which margins involving empty cells are redefined so that they become estimable. This is invoked by margins' emptycells(reweight) option. See Balancing in the presence of empty cells.
least-squares mean. Synonym for estimated marginal mean.
margin. A statistic calculated from predictions or other statistics of a previously fit model at fixed values of some covariates and averaging or otherwise integrating over the remaining covariates. The prediction or other statistic on which the margin is based is called the response.

If all the covariates are fixed, then the margin is called a conditional margin. If any covariates are left to vary, the margin is called a predictive margin.

In this documentation, we divide margins on the basis of whether the statistic is a response or a derivative of a response; see Obtaining margins of responses and Obtaining margins of derivatives of responses.
marginal effect and average marginal effect. The marginal effect of $x$ is the margin of the effect of $x$. The term is popular with social scientists, and because of that, you might think the word marginal in marginal effect means derivative because of terms like marginal cost and marginal revenue. Marginal used in that way, however, refers to the derivative of revenue and the derivative of cost; it refers to the numerator, whereas marginal effect refers to the denominator. Moreover, effect is already a derivative or difference.
Some researchers interpret marginal in marginal effect to mean instantaneous, and thus a marginal effect is the instantaneous derivative rather than the discrete first-difference, corresponding to margins' continuous option. Researchers who use marginal in this way refer to the discrete difference calculation of an effect as a partial effect.
Other researchers define marginal effect to be the margin when all covariates are held fixed and the average marginal effect when some covariates are not fixed.
out-of-sample prediction. Predictions made in one dataset using the results from a model fit on another. Sample here refers to the sample on which the model was fit, and out-of-sample refers to the dataset on which the predictions are made.
partial effect and average partial effect. Some authors restrict the term marginal effect to mean derivatives and use the term partial effect to denote discrete differences; see marginal effect and average marginal effect.
population marginal mean. The theoretical (true) value that is estimated by estimated marginal mean. We avoid this term because it can be confused with the concept of a population in survey statistics, with which the population marginal mean has no connection.
posting results, posting margins. A Stata concept having to do with storing the results from the margins command in e() so that those results can be used as if they were estimation results, thus allowing the subsequent use of postestimation commands, such as test, testnl, lincom, and nlcom (see $[R]$ test, $[R]$ testnl, $[R]$ lincom, and $[R]$ nlcom). This is achieved by specifying margins' post option. See Example 10: Testing margins-contrasts of margins.
predictive margin. A margin in which all the covariates are not fixed. When all covariates are fixed, it is called a conditional margin.
recycled prediction. A synonym for predictive margin.
response. A prediction or other statistic derived from combining the parameter estimates of a fitted model with data or specified values on covariates. Derivatives of responses are themselves responses. Responses are what we take margins of.
standardized margin. The margin calculated on data different from the data used to fit the model. The term standardized is usually reserved for situations in which the alternate population is a reference population, which may be real or artificial, and which is treated as fixed.
subpopulation. A subset of your sample that represents a subset of the population, such as the males in a sample of people. In survey contexts when it is desired to account for sampling of the covariates, standard errors for marginal statistics and effects need to account for both the population and the subpopulation. This is accomplished by specifying the vce (unconditional) option and one of the subpop() or over() options. In fact, the above is allowed even when your data are not svyset because vce(unconditional) implies that the sample represents a population.

## Stored results

margins stores the following in $r()$ :
Scalars

```
r(N)
    r(N_sub)
    r(N_clust)
    r(N_psu)
    r(N_strata)
    r(df_r)
    r(N_poststrata)
    r(k_margins)
    r(k_by)
    r(k_at)
    r(level)
```

Macros
r(cmd)
r(cmdline)
r (est_cmd)
r(est_cmdline)
$r$ (title)
r(subpop)
$r$ (model_vce)
$r$ (model_vcetype)
r(vce)
r(vcetype)
r(clustvar)
$r$ (margins)
r(predict_label)
$r$ (expression)
$r$ (xvars)
r(derivatives)
r(over)
$r$ (within)
r(by)
r(by\#)
r(atstats\#)
r(emptycells)
$r$ (mcmethod)
r(mcadjustall)

Matrices
$r(b)$
$r$ (V)
r(Jacobian)
r(_N)
$r$ (at)
$r$ (chainrule)
$r$ (error)
$r$ (table)
number of observations
subpopulation observations
number of clusters
number of sampled PSUs, survey data only
number of strata, survey data only
variance degrees of freedom, survey data only
number of post strata, survey data only
number of terms in marginlist
number of subpopulations
number of at() options
confidence level of confidence intervals
margins
command as typed
$e(\mathrm{cmd})$ from original estimation results
$e$ (cmdline) from original estimation results
title in output
subspec from subpop()
vcetype from estimation command
Std. Err. title from estimation command
vcetype specified in vce()
title used to label Std. Err.
name of cluster variable
marginlist
label from predict()
response expression
varlist from dydx(), dyex(), eydx(), or eyex()
"", "dy/dx", "dy/ex", "ey/dx", "ey/ex"
varlist from over()
varlist from within()
union of $r$ (over) and $r$ (within) lists
interaction notation identifying the \#th subpopulation
the \#th at() specification
empspec from emptycells()
method from mcompare()
adjustall or empty
estimates
variance-covariance matrix of the estimates
Jacobian matrix
sample size corresponding to each margin estimate
matrix of values from the at () options
chainrule information from the fitted model
margin estimability codes;
0 means estimable,
8 means not estimable
matrix containing the margins with their standard errors, test statistics, $p$-values, and confidence intervals
margins with the post option also stores the following in e():
Scalars
e(N)
e(N_sub)
e(N_clust)
e(N_psu)
e(N_strata)
e(df_r)
e(N_poststrata)
e(k_margins)
e(k_by)
e(k_at)
Macros
e(cmd)
e(cmdline)
e(est_cmd)
e(est_cmdline)
e(title)
e(subpop)
e(model_vce)
e(model_vcetype)
e(vce)
e(vcetype)
e(clustvar)
e(margins)
e(predict_label)
e(expression)
e(xvars)
e(derivatives)
e(over)
e(within)
e(by)
e(by\#)
e(atstats\#)
e(emptycells)
e(mcmethod)
e(mcadjustall)
Matrices
e(b)
e(V)
e(Jacobian)
e (_N)
e(at)
e(chainrule)
Functions
e(sample)
number of observations subpopulation observations number of clusters number of sampled PSUs, survey data only
number of strata, survey data only
variance degrees of freedom, survey data only
number of post strata, survey data only
number of terms in marginlist
number of subpopulations
number of at () options

## margins

command as typed
$e(c m d)$ from original estimation results
$e$ (cmdline) from original estimation results
title in estimation output
subspec from subpop()
vcetype from estimation command
Std. Err. title from estimation command
vcetype specified in vce()
title used to label Std. Err.
name of cluster variable
marginlist
label from predict()
prediction expression
varlist from dydx(), dyex(), eydx(), or eyex()
"", "dy/dx", "dy/ex", "ey/dx", "ey/ex"
varlist from over()
varlist from within()
union of $r$ (over) and $r$ (within) lists
interaction notation identifying the \#th subpopulation
the \#th at () specification
empspec from emptycells()
method from mcompare()
adjustall or empty
estimates
variance-covariance matrix of the estimates
Jacobian matrix
sample size corresponding to each margin estimate
matrix of values from the at () options
chainrule information from the fitted model
marks estimation sample

## Methods and formulas

Margins are statistics calculated from predictions of a previously fit model at fixed values of some covariates and averaging or otherwise integrating over the remaining covariates. There are many names for the different statistics that margins can compute: estimates marginal means (see Searle, Speed, and Milliken [1980]), predictive margins (see Graubard and Korn [2004]), marginal effects (see Greene [2012]), and average marginal/partial effects (see Wooldridge [2010] and Bartus [2005]).

Methods and formulas are presented under the following headings:
Notation
Marginal effects
Fixing covariates and balancing factors
Estimable functions
Standard errors conditional on the covariates
Unconditional standard errors

## Notation

Let $\boldsymbol{\theta}$ be the vector of parameters in the current model fit, let $\mathbf{z}$ be a vector of covariate values, and let $f(\mathbf{z}, \boldsymbol{\theta})$ be a scalar-valued function returning the value of the predictions of interest. The following table illustrates the parameters and default prediction for several of Stata's estimation commands.

| Command | $\boldsymbol{\theta}$ | $\mathbf{z}$ | $f(\mathbf{z}, \boldsymbol{\theta})$ |
| :--- | :--- | :--- | :--- |
| regress | $\boldsymbol{\beta}$ | $\mathbf{x}$ | $\mathbf{x} \boldsymbol{\beta}$ |
| cloglog | $\boldsymbol{\beta}$ | $\mathbf{x}$ | $1-e^{-e^{x \beta}}$ |
| logit | $\boldsymbol{\beta}$ | $\mathbf{x}$ | $1 /\left(1+e^{-x \beta}\right)$ |
| poisson | $\boldsymbol{\beta}$ | $\mathbf{x}$ | $e^{x \beta}$ |
| probit | $\boldsymbol{\beta}$ | $\mathbf{x}$ | $\Phi(\mathbf{x} \boldsymbol{\beta})$ |
| biprobit | $\boldsymbol{\beta}_{1}, \boldsymbol{\beta}_{2}, \rho$ | $\mathbf{x}_{1}, \mathbf{x}_{2}$ | $\Phi_{2}\left(\mathbf{x}_{1} \boldsymbol{\beta}_{1}, \mathbf{x}_{2} \boldsymbol{\beta}_{2}, \rho\right)$ |
| mlogit | $\boldsymbol{\beta}_{1}, \boldsymbol{\beta}_{2}, \ldots, \boldsymbol{\beta}_{k}$ | $\mathbf{x}$ | $e^{-x \beta_{1}} /\left(\sum_{i} e^{-x \beta_{i}}\right)$ |
| nbreg | $\boldsymbol{\beta}, \ln \alpha$ | $\mathbf{x}$ | $e^{x \boldsymbol{\beta}}$ |

$\Phi()$ and $\Phi_{2}()$ are cumulative distribution functions: $\Phi()$ for the standard normal distribution and $\Phi_{2}()$ for the standard bivariate normal distribution.
margins computes estimates of

$$
p(\boldsymbol{\theta})=\frac{1}{M_{S_{\mathrm{p}}}} \sum_{j=1}^{M} \delta_{j}\left(S_{\mathrm{p}}\right) f\left(\mathbf{z}_{j}, \boldsymbol{\theta}\right)
$$

where $\delta_{j}\left(S_{\mathrm{p}}\right)$ identifies elements within the subpopulation $S_{\mathrm{p}}$ (for the prediction of interest),

$$
\delta_{j}\left(S_{\mathrm{p}}\right)= \begin{cases}1, & j \in S_{\mathrm{p}} \\ 0, & j \notin S_{\mathrm{p}}\end{cases}
$$

$M_{S_{\mathrm{p}}}$ is the subpopulation size,

$$
M_{S_{\mathrm{p}}}=\sum_{j=1}^{M} \delta_{j}\left(S_{\mathrm{p}}\right)
$$

and $M$ is the population size.
Let $\widehat{\boldsymbol{\theta}}$ be the vector of parameter estimates. Then margins estimates $p(\boldsymbol{\theta})$ via

$$
\widehat{p}=\frac{1}{w .} \sum_{j=1}^{N} \delta_{j}\left(S_{\mathrm{p}}\right) w_{j} f\left(\mathbf{z}_{j}, \widehat{\boldsymbol{\theta}}\right)
$$

where

$$
w .=\sum_{j=1}^{N} \delta_{j}\left(S_{\mathrm{p}}\right) w_{j}
$$

$\delta_{j}\left(S_{\mathrm{p}}\right)$ indicates whether observation $j$ is in subpopulation $S_{\mathrm{p}}, w_{j}$ is the weight for the $j$ th observation, and $N$ is the sample size.

## Marginal effects

margins also computes marginal/partial effects. For the marginal effect of continuous covariate $x$, margins computes

$$
\widehat{p}=\frac{1}{w} \sum_{j=1}^{N} \delta_{j}\left(S_{\mathrm{p}}\right) w_{j} h\left(\mathbf{z}_{j}, \widehat{\boldsymbol{\theta}}\right)
$$

where

$$
h(\mathbf{z}, \boldsymbol{\theta})=\frac{\partial f(\mathbf{z}, \boldsymbol{\theta})}{\partial x}
$$

The marginal effect for level $k$ of factor variable $A$ is the simple contrast (a.k.a. difference) comparing its margin with the margin at the base level.

$$
h(\mathbf{z}, \boldsymbol{\theta})=f(\mathbf{z}, \boldsymbol{\theta} \mid A=k)-f(\mathbf{z}, \boldsymbol{\theta} \mid A=\text { base })
$$

## Fixing covariates and balancing factors

margins controls the values in each $\mathbf{z}$ vector through the marginlist, the at () option, the atmeans option, and the asbalanced and emptycells() options. Suppose $\mathbf{z}$ is composed of the elements from the equation specification

## A\#\#B x

where A is a factor variable with $a$ levels, B is a factor variable with $b$ levels, and x is a continuous covariate. To simplify the notation for this discussion, assume the levels of A and B start with 1 and are contiguous. Then

$$
\mathbf{z}=\left(A_{1}, \ldots, A_{a}, B_{1}, \ldots, B_{b}, A_{1} B_{1}, A_{1} B_{2}, \ldots, A_{a} B_{b}, x, 1\right)
$$

where $A_{i}, B_{j}$, and $A_{i} B_{j}$ represent the indicator values for the factor variables A and B and the interaction A\#B.

When factor A is in the marginlist, margins replaces A with $i$ and then computes the mean of the subsequent prediction, for $i=1, \ldots, a$. When the interaction term A\#B is in the marginlist, margins replaces A with $i$ and B with $j$, and then computes the mean of the subsequent prediction, for all combinations of $i=1, \ldots, a$ and $j=1, \ldots, b$.

The at () option sets model covariates to fixed values. For example, at ( $x=15$ ) causes margins to temporarily set $x$ to 15 for each observation in the dataset before computing any predictions. Similarly, at ((median) x ) causes margins to temporarily set $x$ to the median of x using the current dataset.

When factor variable A is specified as asbalanced, margins sets each $A_{i}$ to $1 / a$. Thus each $\mathbf{z}$ vector will look like

$$
\mathbf{z}=\left(1 / a, \ldots, 1 / a, B_{1}, \ldots, B_{b}, B_{1} / a, B_{2} / a, \ldots, B_{b} / a, x, 1\right)
$$

If B is also specified as asbalanced, then each $B_{j}$ is set to $1 / b$, and each $\mathbf{z}$ vector will look like

$$
\mathbf{z}=(1 / a, \ldots, 1 / a, 1 / b, \ldots, 1 / b, 1 / a b, 1 / a b, \ldots, 1 / a b, x, 1)
$$

If emptycells (reweight) is also specified, then margins uses a different balancing weight for each element of $\mathbf{z}$, depending on how many empty cells the element is associated with. Let $\delta_{i j}$ indicate that the $i j$ th cell of A\#B was observed in the estimation sample.

$$
\delta_{i j}= \begin{cases}0, & \mathrm{~A}=i \text { and } \mathrm{B}=j \text { was an empty cell } \\ 1, & \text { otherwise }\end{cases}
$$

For the grand margin, the affected elements of $\mathbf{z}$ and their corresponding balancing weights are

$$
\begin{aligned}
A_{i} & =\frac{\sum_{j} \delta_{i j}}{\sum_{k} \sum_{j} \delta_{k j}} \\
B_{j} & =\frac{\sum_{i} \delta_{i j}}{\sum_{i} \sum_{k} \delta_{i k}} \\
A_{i} B_{j} & =\frac{\delta_{i j}}{\sum_{k} \sum_{l} \delta_{k l}}
\end{aligned}
$$

For the $j$ th margin of B , the affected elements of $\mathbf{z}$ and their corresponding balancing weights are

$$
\begin{gathered}
A_{i}=\frac{\delta_{i j}}{\sum_{k} \delta_{k j}} \\
B_{l}= \begin{cases}1, & \text { if } l=j \text { and not all } \delta_{i j} \text { are zero } \\
0, & \text { otherwise }\end{cases} \\
A_{i} B_{l}=\frac{\delta_{i l}}{\sum_{k} \delta_{k l}} B_{l}
\end{gathered}
$$

## Estimable functions

The fundamental idea behind estimable functions is clearly defined in the statistical literature for linear models; see Searle (1971). Assume that we are working with the following linear model:

$$
\mathbf{y}=\mathbf{X b}+\mathbf{e}
$$

where $\mathbf{y}$ is an $N \times 1$ vector of responses, $\mathbf{X}$ is an $N \times p$ matrix of covariate values, $\mathbf{b}$ is a $p \times 1$ vector of coefficients, and $\mathbf{e}$ is a vector of random errors. Assuming a constant variance for the random errors, the normal equations for the least-squares estimator, $\widehat{\mathbf{b}}$, are

$$
\mathbf{X}^{\prime} \mathbf{X} \widehat{b}=\mathbf{X}^{\prime} \mathbf{y}
$$

When $\mathbf{X}$ is not of full column rank, we will need a generalized inverse (g-inverse) of $\mathbf{X}^{\prime} \mathbf{X}$ to solve for $\widehat{\mathbf{b}}$. Let $\mathbf{G}$ be a g-inverse of $\mathbf{X}^{\prime} \mathbf{X}$.

Searle (1971) defines a linear function of the parameters as estimable if it is identically equal to some linear function of the expected values of the $\mathbf{y}$ vector. Let $\mathbf{H}=\mathbf{G X}^{\prime} \mathbf{X}$. Then this definition simplifies to the following rule:
$\mathbf{z b}$ is estimable if $\mathbf{z}=\mathbf{z H}$
margins generalizes this to nonlinear functions by assuming the prediction function $f(\mathbf{z}, \boldsymbol{\theta})$ is a function of one or more of the linear predictions from the equations in the model that $\boldsymbol{\theta}$ represents.

$$
f(\mathbf{z}, \boldsymbol{\theta})=h\left(\mathbf{z}_{1} \boldsymbol{\beta}_{1}, \mathbf{z}_{2} \boldsymbol{\beta}_{2}, \ldots, \mathbf{z}_{k} \boldsymbol{\beta}_{k}\right)
$$

$\mathbf{z}_{i} \boldsymbol{\beta}_{i}$ is considered estimable if $\mathbf{z}_{i}=\mathbf{z}_{i} \mathbf{H}_{i}$, where $\mathbf{H}_{i}=\mathbf{G}_{i} \mathbf{X}_{i}^{\prime} \mathbf{X}_{i}, \mathbf{G}_{i}$ is a $g$-inverse for $\mathbf{X}_{i}^{\prime} \mathbf{X}_{i}$, and $\mathbf{X}_{i}$ is the matrix of covariates from the $i$ th equation of the fitted model. margins considers $p(\boldsymbol{\theta})$ to be estimable if every $\mathbf{z}_{i} \boldsymbol{\beta}_{i}$ is estimable.

## Standard errors conditional on the covariates

By default, margins uses the delta method to estimate the variance of $\widehat{p}$.

$$
\widehat{\operatorname{Var}}(\widehat{p} \mid \mathbf{z})=\mathbf{v}^{\prime} \mathbf{V} \mathbf{v}
$$

where $\mathbf{V}$ is a variance estimate for $\widehat{\boldsymbol{\theta}}$ and

$$
\mathbf{v}=\left.\frac{\partial \widehat{p}}{\partial \boldsymbol{\theta}}\right|_{\theta=\widehat{\boldsymbol{\theta}}}
$$

This variance estimate is conditional on the $\mathbf{z}$ vectors used to compute the marginalized predictions.

## Unconditional standard errors

margins with the vce(unconditional) option uses linearization to estimate the unconditional variance of $\widehat{\boldsymbol{\theta}}$. Linearization uses the variance estimator for the total of a score variable for $\widehat{p}$ as an approximate estimator for $\operatorname{Var}(\widehat{p})$; see [SVY] variance estimation. margins requires that the model was fit using some form of linearized variance estimator and that predict, scores computes the appropriate score values for the linearized variance estimator.

The score for $\widehat{p}$ from the $j$ th observation is given by

$$
s_{j}=\frac{\partial \widehat{p}}{\partial w_{j}}=-\frac{\delta_{j}\left(S_{\mathrm{p}}\right)}{w .} \widehat{p}+\frac{\delta_{j}\left(S_{\mathrm{p}}\right)}{w .} f\left(\mathbf{z}_{j}, \widehat{\boldsymbol{\theta}}\right)+\frac{1}{w} \sum_{i=1}^{N} \delta_{i}\left(S_{\mathrm{p}}\right) w_{i} \frac{\partial f\left(\mathbf{z}_{i}, \widehat{\boldsymbol{\theta}}\right)}{\partial w_{j}}
$$

The remaining partial derivative can be decomposed using the chain rule.

$$
\frac{\partial f\left(\mathbf{z}_{i}, \widehat{\boldsymbol{\theta}}\right)}{\partial w_{j}}=\left(\left.\frac{\partial f\left(\mathbf{z}_{i}, \boldsymbol{\theta}\right)}{\partial \boldsymbol{\theta}}\right|_{\theta=\widehat{\theta}}\right)\left(\frac{\partial \widehat{\boldsymbol{\theta}}}{\partial w_{j}}\right)^{\prime}
$$

This is the inner product of two vectors, the second of which is not a function of the $i$ index. Thus the score is

$$
s_{j}=-\frac{\delta_{j}\left(S_{\mathrm{p}}\right)}{w .} \widehat{p}+\frac{\delta_{j}\left(S_{\mathrm{p}}\right)}{w .} f\left(\mathbf{z}_{j}, \widehat{\boldsymbol{\theta}}\right)+\left(\left.\frac{\partial \widehat{p}}{\partial \boldsymbol{\theta}}\right|_{\theta=\widehat{\theta}}\right)\left(\frac{\partial \widehat{\boldsymbol{\theta}}}{\partial w_{j}}\right)^{\prime}
$$

If $\widehat{\boldsymbol{\theta}}$ was derived from a system of equations (such as in linear regression or maximum likelihood estimation), then $\widehat{\boldsymbol{\theta}}$ is the solution to

$$
\mathbf{G}(\boldsymbol{\theta})=\sum_{j=1}^{N} \delta_{j}\left(S_{\mathrm{m}}\right) w_{j} \mathbf{g}\left(\boldsymbol{\theta}, \mathbf{y}_{j}, \mathbf{x}_{j}\right)=\mathbf{0}
$$

where $S_{\mathrm{m}}$ identifies the subpopulation used to fit the model, $\mathbf{g}()$ is the model's gradient function, and $\mathbf{y}_{j}$ and $\mathbf{x}_{j}$ are the values of the dependent and independent variables for the $j$ th observation. We can use linearization to derive a first-order approximation for $\partial \widehat{\boldsymbol{\theta}} / \partial w_{j}$.

$$
\mathbf{G}(\widehat{\boldsymbol{\theta}}) \approx \mathbf{G}\left(\boldsymbol{\theta}_{0}\right)+\left.\frac{\partial \mathbf{G}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right|_{\theta=\theta_{0}}\left(\widehat{\boldsymbol{\theta}}-\boldsymbol{\theta}_{0}\right)
$$

Let $\mathbf{H}$ be the Hessian matrix

$$
\mathbf{H}=\left.\frac{\partial \mathbf{G}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right|_{\theta=\theta_{0}}
$$

Then

$$
\widehat{\boldsymbol{\theta}} \approx \boldsymbol{\theta}_{0}+(-\mathbf{H})^{-1} \mathbf{G}\left(\boldsymbol{\theta}_{0}\right)
$$

and

$$
\left.\frac{\partial \widehat{\boldsymbol{\theta}}}{\partial w_{j}} \approx(-\mathbf{H})^{-1} \frac{\partial \mathbf{G}(\boldsymbol{\theta})}{\partial w_{j}}\right|_{\theta=\widehat{\boldsymbol{\theta}}}=(-\mathbf{H})^{-1} \delta_{j}\left(S_{\mathrm{m}}\right) \mathbf{g}\left(\widehat{\boldsymbol{\theta}}, \mathbf{y}_{j}, \mathbf{x}_{j}\right)
$$

The computed value of the score for $\widehat{p}$ for the $j$ th observation is

$$
s_{j}=\mathbf{v}^{\prime} \mathbf{u}_{j}
$$

where

$$
\mathbf{v}=\left[\begin{array}{c}
-\frac{\widehat{p}}{w .} \\
\frac{1}{w .} \\
\frac{\partial \widehat{p}}{\partial \widehat{\boldsymbol{\theta}}}(-\mathbf{H})^{-1}
\end{array}\right]
$$

and

$$
\mathbf{u}_{j}=\left[\begin{array}{c}
\delta_{j}\left(S_{\mathrm{p}}\right) \\
\delta_{j}\left(S_{\mathrm{p}}\right) f\left(\mathbf{z}_{j}, \widehat{\boldsymbol{\theta}}\right) \\
\delta_{j}\left(S_{\mathrm{m}}\right) \mathbf{g}\left(\widehat{\boldsymbol{\theta}}, \mathbf{y}_{j}, \mathbf{x}_{j}\right)
\end{array}\right]
$$

Thus the variance estimate for $\widehat{p}$ is

$$
\widehat{\operatorname{Var}}(\widehat{p})=\mathbf{v}^{\prime} \widehat{\operatorname{Var}}(\widehat{\mathbf{U}}) \mathbf{v}
$$

where

$$
\widehat{\mathbf{U}}=\sum_{j=1}^{N} w_{j} \mathbf{u}_{j}
$$

margins uses the model-based variance estimates for $(-\mathbf{H})^{-1}$ and the scores from predict for $\mathbf{g}\left(\widehat{\boldsymbol{\theta}}, \mathbf{y}_{j}, \mathbf{x}_{j}\right)$.

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## Also see

[R] contrast - Contrasts and linear hypothesis tests after estimation
[R] margins, contrast - Contrasts of margins
[R] margins, pwcompare - Pairwise comparisons of margins
[R] margins postestimation - Postestimation tools for margins
$[R]$ marginsplot - Graph results from margins (profile plots, etc.)
[R] lincom - Linear combinations of estimators
[R] nlcom - Nonlinear combinations of estimators
[R] predict - Obtain predictions, residuals, etc., after estimation
[R] predictnl - Obtain nonlinear predictions, standard errors, etc., after estimation
[U] 20 Estimation and postestimation commands

## Title

margins postestimation - Postestimation tools for margins

## Description Remarks and examples Also see

## Description

The following standard postestimation command is available after margins:

| Command | Description |
| :--- | :--- |
| marginsplot | graph the results from margins-profile plots, interaction plots, etc. |

For information on marginsplot, see [R] marginsplot.

The following standard postestimation commands are available after margins, post:

| Command | Description |
| :--- | :--- |
| contrast | contrasts and ANOVA-style joint tests of estimates |
| estat summarize | summary statistics for the estimation sample <br> estimates <br> cataloging estimation results <br> point estimates, standard errors, testing, and inference for linear combinations <br> of coefficients |
| nlcom | point estimates, standard errors, testing, and inference for nonlinear combinations <br> of coefficients |
| pwcompare | pairwise comparisons of estimates |
| test | Wald tests of simple and composite linear hypotheses |
| testnl | Wald tests of nonlinear hypotheses |

## Remarks and examples

Continuing with the example from Example 8: Margins of interactions in [R] margins, we use the dataset and reestimate the logistic model of outcome:

```
. use http://www.stata-press.com/data/r13/margex
(Artificial data for margins)
. logistic outcome sex##group age
    (output omitted)
```

We then estimate the margins for males and females and post the margins as estimation results with a full VCE.

```
. margins sex, post
Predictive margins Number of obs = 3000
Model VCE : OIM
Expression : Pr(outcome), predict()
```

|  | Delta-method |  |  |  |  |  |
| :---: | ---: | :---: | :---: | :---: | :---: | :---: | ---: |
|  | Margin | Std. Err. | $\mathbf{z}$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| sex |  |  |  |  |  |  |
| male | .1600644 | .0125653 | 12.74 | 0.000 | .1354368 | .184692 |
| female | .1966902 | .0100043 | 19.66 | 0.000 | .1770821 | .2162983 |

We can now use nlcom (see [R] nlcom) to estimate a risk ratio of females to males using the average probabilities for females and males posted by margins:

```
. nlcom (risk_ratio: _b[1.sex] / _b[0.sex])
    risk_ratio: _b[1.sex] / _b[0.sex]
```

|  | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | :---: |
| risk_ratio | 1.228819 | .1149538 | 10.69 | 0.000 | 1.003514 | 1.454124 |

We could similarly estimate the average risk difference between females and males:

```
. nlcom (risk_diff: _b[1.sex] - _b[0.sex])
    risk_diff: _b[1.sex] - _b[0.sex]
```

|  | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | ---: |
| risk_diff | .0366258 | .0160632 | 2.28 | 0.023 | .0051425 | .068109 |

## Also see

[R] margins - Marginal means, predictive margins, and marginal effects
[R] marginsplot - Graph results from margins (profile plots, etc.)
[U] 20 Estimation and postestimation commands

## Title

margins, contrast - Contrasts of margins
Syntax
Remarks and examples
Also see
Menu
Stored results

Description
Methods and formulas

Suboptions
Reference

## Syntax

```
margins [marginlist] [if] [in] [weight] [, contrast margins_options]
margins [marginlist] [if] [in] [weight] [, contrast(suboptions) margins_options]
```

where marginlist is a list of factor variables or interactions that appear in the current estimation results. The variables may be typed with or without contrast operators, and you may use any factor-variable syntax:
. margins sex\#\#group, contrast
. margins sex\#\#g.group, contrast
. margins sex@group, contrast

See the operators (op.) table in [R] contrast for the list of contrast operators. Contrast operators may also be specified on the variables in margins' over() and within() options to perform contrasts across the levels of those variables.

See [R] margins for the available margins_options.
suboptions Description
Contrast
overall
lincom
atcontrast (op[.-at])
atjoint
overjoint
withinjoint
marginswithin
cieffects
pveffects
effects
nowald
noatlevels
nosvyadjust
add a joint hypothesis test for all specified contrasts
treat user-defined contrasts as linear combinations
apply the $o p$. contrast operator to the groups defined by at () test jointly across all groups defined by at ()
test jointly across all levels of the unoperated over() variables test jointly across all levels of the unoperated within() variables perform contrasts within the levels of the unoperated terms in marginlist
show effects table with confidence intervals
show effects table with $p$-values
show effects table with confidence intervals and $p$-values
suppress table of Wald tests
report only the overall Wald test for terms that use the within © or nested | operator
compute unadjusted Wald tests for survey results

## Menu

Statistics $>$ Postestimation $>$ Contrasts of margins

## Description

margins with the contrast option or with contrast operators performs contrasts of margins. This extends the capabilities of contrast to any of the nonlinear responses, predictive margins, or other margins that can be estimated by margins.

## Suboptions

## Contrast <br> overall specifies that a joint hypothesis test over all terms be performed.

lincom specifies that user-defined contrasts be treated as linear combinations. The default is to require that all user-defined contrasts sum to zero. (Summing to zero is part of the definition of a contrast.)
atcontrast ( $o p\left[. \_a t\right]$ ) specifies that the $o p$. contrast operator be applied to the groups defined by the at () option(s). The default behavior, by comparison, is to perform tests and contrasts within the groups defined by the at () option(s).

See example 6 in Remarks and examples.
atjoint specifies that joint tests be performed across all groups defined by the at () option. The default behavior, by comparison, is to perform contrasts and tests within each group.
See example 5 in Remarks and examples.
overjoint specifies how unoperated variables in the over() option are treated.
Each variable in the over() option may be specified either with or without a contrast operator. For contrast-operated variables, the specified contrast comparisons are always performed.
overjoint specifies that joint tests be performed across all levels of the unoperated variables. The default behavior, by comparison, is to perform contrasts and tests within each combination of levels of the unoperated variables.
See example 3 in Remarks and examples.
withinjoint specifies how unoperated variables in the within() option are treated.
Each variable in the within() option may be specified either with or without a contrast operator. For contrast-operated variables, the specified contrast comparisons are always performed.
withinjoint specifies that joint tests be performed across all levels of the unoperated variables. The default behavior, by comparison, is to perform contrasts and tests within each combination of levels of the unoperated variables.
marginswithin specifies how unoperated variables in marginlist are treated.
Each variable in marginlist may be specified either with or without a contrast operator. For contrast-operated variables, the specified contrast comparisons are always performed.
marginswithin specifies that contrasts and tests be performed within each combination of levels of the unoperated variables. The default behavior, by comparison, is to perform joint tests across all levels of the unoperated variables.

See example 4 in Remarks and examples.
cieffects specifies that a table containing a confidence interval for each individual contrast be reported.
pveffects specifies that a table containing a $p$-value for each individual contrast be reported.
effects specifies that a single table containing a confidence interval and $p$-value for each individual contrast be reported.
nowald suppresses the table of Wald tests.
noatlevels indicates that only the overall Wald test be reported for each term containing within or nested (@ or I) operators.
nosvyadjust is for use with svy estimation commands. It specifies that the Wald test be carried out without the default adjustment for the design degrees of freedom. That is to say the test is carried out as $W / k \sim F(k, d)$ rather than as $(d-k+1) W /(k d) \sim F(k, d-k+1)$, where $k$ is the dimension of the test and $d$ is the total number of sampled PSUs minus the total number of strata.

## Remarks and examples

Remarks are presented under the following headings:
Contrasts of margins
Contrasts and the over() option
The overjoint suboption
The marginswithin suboption
Contrasts and the at() option
Estimating treatment effects with margins
Conclusion

## Contrasts of margins

## > Example 1

Estimating contrasts of margins is as easy as adding a contrast operator to the variable name. Let's review Example 2: A simple case after logistic of [R] margins. Variable sex is coded 0 for males and 1 for females.


The first margin, 0.13 , is the average probability of a positive outcome, treating everyone as if they were male. The second margin, 0.19 , is the average probability of a positive outcome, treating everyone as if they were female. We can compare females with males by rerunning margins and adding a contrast operator:


The r. prefix for sex is the reference-category contrast operator-see [R] contrast. (The default reference category is zero, the lowest value of sex.) Contrast operators in a marginlist work just as they do in the termlist of a contrast command.

The contrast estimate of 0.06 says that unconditional on group, females on average are about $6 \%$ more likely than males to have a positive outcome. The chi-squared statistic of 16.61 shows that the contrast is significantly different from zero.

You may be surprised that we did not need to include the contrast option to estimate our contrast. If we had included the option, our output would not have changed:

```
. margins r.sex, contrast
Contrasts of predictive margins
Model VCE : OIM
Expression : Pr(outcome), predict()
```

|  | df | chi2 | P>chi2 |
| ---: | :---: | :---: | :---: |
| sex | 1 | 16.61 | 0.0000 |


|  | Delta-method <br> Std. Err. |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Contrast | [95\% Conf. Interval] |  |  |  |
| sex | .0618291 | .0151719 | .0320927 | .0915656 |

The contrast option is useful mostly for its suboptions, which control the output and how contrasts are estimated in more complicated situations. But contrast may be specified on its own (without contrast operators or suboptions) if we do not need estimates or confidence intervals:

```
. margins sex group, contrast
Contrasts of predictive margins
Model VCE : OIM
Expression : Pr(outcome), predict()
```

|  | df | chi2 | P>chi2 |
| ---: | :---: | :---: | :---: |
| sex | 1 | 16.61 | 0.0000 |
| group | 2 | 225.76 | 0.0000 |

Each chi-squared statistic is a joint test of constituent contrasts. The test for group has two degrees of freedom because group has three levels.

## Contrasts and the over() option

## > Example 2

It is common to estimate margins at combinations of factor levels, and margins, contrast includes several suboptions for contrasting such margins. Let's fit a model with two categorical predictors and their interaction:

| . logistic outcome agegroup\#\#group |  |  |  |
| :--- | :--- | :--- | :--- |
| Logistic regression | Number of obs | $=$ | 3000 |
|  | LR chi2 (8) | $=$ | 520.64 |
|  | Prob $>$ chi2 | $=$ | 0.0000 |
| Log likelihood $=-1105.7504$ |  | Pseudo R2 | $=$ | 0.1906


| outcome | Odds Ratio | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| agegroup |  |  |  |  |  |  |
| $30-39$ | 3.54191 | 2.226951 | 2.01 | 0.044 | 1.032882 | 12.14576 |
| $40+$ | 16.23351 | 9.61188 | 4.71 | 0.000 | 5.086452 | 51.80955 |
|  |  |  |  |  |  |  |
| group |  |  |  |  |  |  |
| 2 | .834507 | .5663738 | -0.27 | 0.790 | .2206611 | 3.15598 |
| 3 | .2146729 | .1772897 | -1.86 | 0.062 | .0425407 | 1.083303 |
|  |  |  |  |  |  |  |
| agegroup\# |  |  |  |  |  |  |
| group |  |  |  |  |  |  |
| 30-39\#2 | .4426927 | .3358505 | -1.07 | 0.283 | .1000772 | 1.958257 |
| 30-39\#3 | 1.160885 | 1.103527 | 0.16 | 0.875 | .1801543 | 7.480553 |
| $40+\# 2$ | .440672 | .3049393 | -1.18 | 0.236 | .1135259 | 1.71055 |
| $40+\# 3$ | .4407912 | .4034688 | -0.89 | 0.371 | .0733 | 2.650709 |
|  |  |  |  |  |  |  |
| _cons | .0379747 | .0223371 | -5.56 | 0.000 | .0119897 | .1202762 |

Each of agegroup and group has three levels. To compare each age group with the reference category on the probability scale, we can again use margins with the $r$. contrast operator.

```
. margins r.agegroup
Contrasts of predictive margins
Model VCE : OIM
Expression : Pr(outcome), predict()
```

|  | df | chi2 | P>chi2 |
| :---: | :---: | :---: | :---: |
| agegroup |  |  |  |
| $(30-39$ vs 20-29) | 1 | 10.04 | 0.0015 |
| $(40+$ vs 20-29) | 1 | 224.44 | 0.0000 |
| Joint | 2 | 238.21 | 0.0000 |


|  | Delta-method |  |  |  |
| :---: | ---: | :--- | :--- | :--- |
|  | Contrast | Std. Err. | [95\% Conf. Interval] |  |
| agegroup <br> $(30-39$ vs 20-29) <br> $(40+$ vs 20-29) | .2059281 | .0137455 | .1789874 | .2328688 |

Our model includes an interaction, though, so it would be nice to estimate the contrasts separately for each value of group. We need the over () option:

```
. margins r.agegroup, over(group)
Contrasts of predictive margins
Model VCE : OIM
Expression : Pr(outcome), predict()
over : group
```

|  | df | chi2 | P>chi2 |
| ---: | ---: | ---: | ---: |
| agegroup@group |  |  |  |
| (30-39 vs 20-29) 1 | 1 | 6.94 | 0.0084 |
| (30-39 vs 20-29) 2 | 1 | 1.18 | 0.2783 |
| (30-39 vs 20-29) 3 | 1 | 3.10 | 0.0783 |
| (40+ vs 20-29) 1 | 1 | 173.42 | 0.0000 |
| (40+ vs 20-29) 2 | 1 | 57.77 | 0.0000 |
| (40+ vs 20-29) 3 | 1 | 5.12 | 0.0236 |
| Joint | 6 | 266.84 | 0.0000 |


|  | Delta-method <br> Std. Err. |  |  |  |
| ---: | ---: | ---: | ---: | ---: |
|  | Contrast | [95\% Conf. Interval] |  |  |
| agegroup@group |  |  |  |  |
| (30-39 vs 20-29) 1 | .0819713 | .0311208 | .0209757 | .142967 |
| $(30-39$ vs 20-29) 2 | .0166206 | .0153309 | -.0134275 | .0466686 |
| $(30-39$ vs 20-29) 3 | .0243462 | .0138291 | -.0027583 | .0514508 |
| $(40+$ vs 20-29) 1 | .3447797 | .0261811 | .2934658 | .3960937 |
| $(40+$ vs 20-29) 2 | .1540882 | .0202722 | .1143554 | .193821 |
| $(40+$ vs $20-29)$ | 3 | .0470319 | .0207774 | .006309 |

The effect of agegroup appears to be greatest for the first level of group.
Including a variable in the over () option is not equivalent to including the variable in the main marginlist. The variables in the marginlist are manipulated in the analysis, so that we can measure, for example, the effect of being in age group 3 and not age group 1. (The manipulation could be mimicked by running replace and then predict, but the manipulations actually performed by margins do not
change the data in memory.) The variables in the over() option are not so manipulated-the values of the over () variables are left as they were observed, and the marginlist variables are manipulated separately for each observed over() group. For more information, see Do not specify marginlist when you mean over() in [R] margins.

## The overjoint suboption

## Example 3

Each variable in an over () option may be specified with or without contrast operators. Our option over (group) did not include a contrast operator, so margins estimated the contrasts separately for each level of group. If we had instead specified over (r.group), we would have received differences of the contrasts:

| . margins $r$ r.agegroup, over(r.group) |  |
| :--- | :--- |
| Contrasts of | predictive margins |
| Model VCE | $:$ OIM |
| Expression | $: \operatorname{Pr}$ (outcome), predict() |
| over | $:$ group |


|  | df | chi2 | P>chi2 |
| ---: | ---: | ---: | ---: |
| (2 vs 1) (30-39 vs 20-29) |  |  |  |
| (2 vs 1) (40+ vs 20-29) | 1 | 3.55 | 0.0596 |
| $(3$ vs 1) (30-39 vs 20-29) | 1 | 33.17 | 0.0000 |
| $(3$ vs 1) (40+ vs 20-29) | 1 | 2.86 | 0.0906 |
|  |  | 1 | 79.36 |


|  | Delta-method |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| group\#agegroup |  |  |  |  |
| (2 vs 1) (30-39 vs 20-29) | -. 0653508 | . 0346921 | -. 133346 | . 0026445 |
| (2 vs 1) (40+ vs 20-29) | -. 1906915 | . 0331121 | -. 25559 | -. 1257931 |
| (3 vs 1) (30-39 vs 20-29) | -. 0576251 | . 0340551 | -. 1243719 | . 0091216 |
| (3 vs 1) (40+ vs 20-29) | -. 2977479 | . 0334237 | -. 3632572 | -. 2322385 |

The contrasts are double differences: the estimate of -0.19 , for example, says that the difference in the probability of success between age group 3 and age group 1 is smaller in group 2 than in group 1 . We can jointly test pairs of the double differences with the overjoint suboption:

```
. margins r.agegroup, over(group) contrast(overjoint)
Contrasts of predictive margins
Model VCE : OIM
Expression : Pr(outcome), predict()
over : group
```

|  | df | chi2 | P>chi2 |
| ---: | ---: | ---: | ---: |
| group\#agegroup |  |  |  |
| (joint) (30-39 vs 20-29) | 2 | 3.62 | 0.1641 |
| (joint) (40+ vs 20-29) | 2 | 79.45 | 0.0000 |
|  | Joint | 4 | 83.88 |

The contrast (overjoint) option overrides the default behavior of over() and requests joint tests over the levels of the unoperated variable group. The chi-squared statistic of 3.62 tests that the first and third contrasts from the previous table are jointly zero. The chi-squared statistic of 79.45 jointly tests the other pair of contrasts.

## The marginswithin suboption

## > Example 4

Another suboption that may usefully be combined with over() is marginswithin marginswithin requests that contrasts be performed within the levels of unoperated variables in the main marginlist, instead of performing them jointly across the levels. marginswithin affects only unoperated variables because contrast operators take precedence over suboptions.

Let's first look at the default behavior, which occurs when marginswithin is not specified:

| Contrasts of predictive margins Model VCE : OIM |  |  |  |
| :---: | :---: | :---: | :---: |
| $\begin{array}{ll} \text { Expression } & \text { : Pr(outcome), predict() } \\ \text { over } & \text { : group } \end{array}$ |  |  |  |
|  | df | chi2 | $\mathrm{P}>\mathrm{chi2}$ |
| group\#agegroup |  |  |  |
| (2 vs 1) (joint) | 2 | 33.94 | 0.0000 |
| (3 vs 1) (joint) | 2 | 83.38 | 0.0000 |
| Joint | 4 | 83.88 | 0.0000 |



Here agegroup in the main marginlist is an unoperated variable, so margins by default performs joint tests across the levels of agegroup: the chi-squared statistic of 33.94 , for example, jointly tests whether the first two contrast estimates in the lower table differ significantly from zero.

When we specify marginswithin, the contrasts will instead be performed within the levels of agegroup:

```
. margins agegroup, over(r.group) contrast(marginswithin effects)
Contrasts of predictive margins
Model VCE : OIM
Expression : Pr(outcome), predict()
over : group
```

|  | df | chi2 | $\mathrm{P}>\mathrm{chi} 2$ |
| :---: | :---: | :---: | :---: |
| group@agegroup |  |  |  |
| (2 vs 1) 20-29 | 1 | 0.06 | 0.7991 |
| (2 vs 1) 30-39 | 1 | 7.55 | 0.0060 |
| (2 vs 1) 40+ | 1 | 68.39 | 0.0000 |
| (3 vs 1) $20-29$ | 1 | 1.80 | 0.1798 |
| (3 vs 1) 30-39 | 1 | 10.47 | 0.0012 |
| (3 vs 1) 40+ | 1 | 159.89 | 0.0000 |
| Joint | 6 | 186.87 | 0.0000 |


|  | Delta-method |  |  |  | [95\% Conf. | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Contrast | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ |  |  |
| group@ agegroup |  |  |  |  |  |  |
| (2 vs 1) |  |  |  |  |  |  |
| 20-29 | -. 0058686 | . 0230533 | -0.25 | 0.799 | -. 0510523 | . 039315 |
| (2 vs 1) |  |  |  |  |  |  |
| 30-39 | -. 0712194 | . 0259246 | -2.75 | 0.006 | -. 1220308 | -. 0204081 |
| ( 2 vs 1 ) |  |  |  |  |  |  |
| 40+ | -. 1965602 | . 0237688 | -8.27 | 0.000 | -. 2431461 | -. 1499742 |
| (3 vs 1) |  |  |  |  |  |  |
| 20-29 | -. 0284991 | . 0212476 | -1.34 | 0.180 | -. 0701436 | . 0131453 |
| (3 vs 1) |  |  |  |  |  |  |
| 30-39 | -. 0861243 | . 0266137 | -3.24 | 0.001 | -. 1382862 | -. 0339624 |
| (3 vs 1) |  |  |  |  |  |  |
| 40+ | -. 326247 | . 0258009 | -12.64 | 0.000 | -. 3768159 | -. 2756781 |

The joint tests in the top table have been replaced by one-degree-of-freedom tests, one for each combination of the two reference comparisons and three levels of agegroup. The reference-category contrasts for group have been performed within levels of agegroup.

## Contrasts and the at() option

## Example 5

The at () option of margins is used to set predictors to particular values. When at() is used, contrasts are by default performed within each at () level:

| . margins r.agegroup, at (group=(1/3)) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Contrasts of adjusted predictionsModel VCE : OIM |  |  |  |  |
| Expression : Pr (outcome), predict() |  |  |  |  |
| 1._at : group | = | 1 |  |  |
| 2._at : group | = | 2 |  |  |
| 3._at : group | = | 3 |  |  |
|  | df | chi2 | P>chi2 |  |
| agegroup@_at |  |  |  |  |
|  |  |  |  |  |
| $\begin{array}{lllll}(30-39\end{array}$ vs $\left.\left.20-29\right)^{2} \quad 10.18\right)$ |  |  |  |  |
| $\begin{array}{lllll}(30-39 & \text { vs 20-29) } 3 & 1 & 3.10 & 0.0783\end{array}$ |  |  |  |  |
| $\begin{array}{lllll}(40+\text { vs 20-29) } & 1 & 1 & 173.42 & 0.0000\end{array}$ |  |  |  |  |
| $\begin{array}{lllll}(40+\text { vs 20-29) } 2 & 1 & 57.77 & 0.0000\end{array}$ |  |  |  |  |
| $\begin{array}{lllll}(40+\text { vs 20-29) } 3 & 1 & 5.12 & 0.0236\end{array}$ |  |  |  |  |
| Joint | 266.84 |  | 0.0000 |  |
|  | Delta-method |  |  |  |
|  | Contrast | Std. Err. | [95\% Con | Interval] |
| agegroup@_at |  |  |  |  |
| (30-39 vs 20-29) 1 | . 0819713 | . 0311208 | . 0209757 | . 142967 |
| (30-39 vs 20-29) 2 | . 0166206 | . 0153309 | -. 0134275 | . 0466686 |
| (30-39 vs 20-29) 3 | . 0243462 | . 0138291 | -. 0027583 | . 0514508 |
| (40+ vs 20-29) 1 | . 3447797 | . 0261811 | . 2934658 | . 3960937 |
| (40+ vs 20-29) 2 | . 1540882 | . 0202722 | . 1143554 | . 193821 |
| (40+ vs 20-29) 3 | . 0470319 | . 0207774 | . 006309 | . 0877548 |

Our option at (group=(1/3)) manipulates the values of group and is therefore not equivalent to over (group). We see that the reference-category contrasts for agegroup have been performed within each at () level. For a similar example that uses the ._at operator instead of the at () option, see Contrasts of at() groups-discrete effects in [R] marginsplot.

The default within behavior of at () may be changed to joint behavior with the atjoint suboption:

| . margins r.agegroup, at(group=(1/3)) contrast(atjoint) |  |  |  |
| :--- | :--- | :--- | :--- |
| Contrasts of adjusted predictions |  |  |  |
| Model VCE | $:$ OIM |  |  |
| Expression | $: \operatorname{Pr}$ (outcome), predict() |  |  |
| 1._at | : group | $=$ | 1 |
| 2._at | : group | $=$ | 2 |
| 3._at | : group | $=$ | 3 |


|  | df | chi2 | P>chi2 |
| ---: | ---: | ---: | ---: |
| (joint) (30-39 vs 20-29) |  |  |  |
| (joint) (40+ vs 20-29) | 2 | 3.62 | 0.1641 |
|  | Joint | 2 | 79.45 |

Now the tests are performed jointly over the levels of group, the at() variable. The atjoint suboption is the analogue for at() of the overjoint suboption from example 3.
> Example 6
What if we would like to apply a contrast operator, like r., to the at () levels? It is not possible to specify the operator inside the at () option. Instead, we need a new suboption, atcontrast():

| Contrasts of adjusted predictions Model VCE : OIM |  |  |  |
| :---: | :---: | :---: | :---: |
| Expression : Pr(outcome), predict() |  |  |  |
| 1._at : group | = |  |  |
| 2._at : group | = |  |  |
| 3._at : group = 3 |  |  |  |
|  | df | chi2 | $\mathrm{P}>\mathrm{chi} 2$ |
| -at\#agegroup |  |  |  |
| (2 vs 1) (30-39 vs 20-29) | 1 | 3.55 | 0.0596 |
| (2 vs 1) (40+ vs 20-29) | 1 | 33.17 | 0.0000 |
| (3 vs 1) (30-39 vs 20-29) | 1 | 2.86 | 0.0906 |
| (3 vs 1) (40+ vs 20-29) | 1 | 79.36 | 0.0000 |
| Joint | 4 | 83.88 | 0.0000 |


|  | Delta-method <br> Std. Err. |  |  |  |
| :--- | :--- | :--- | :--- | ---: |
|  | Contrast | [95\% Conf. Interval] |  |  |

When we specify contrast(atcontrast(r)), margins will apply the r. reference-category operator to the levels of group, the variable specified inside at (). The default reference category is 1, the lowest level of group.

## Estimating treatment effects with margins

margins with the contrast option can also be used to estimate treatment effects in certain cases. A treatment effect represents the change in an outcome variable that is attributable to a particular event, controlling for all other factors that could affect the outcome. For example, we might want to know how a person's wage changes as a result of being in a union. Here the outcome variable is the person's wage, and the "event" is membership in a union. The treatment effect measures the difference in a person's wage as a result of being or not being in a union once we control for the person's educational background, level of experience, industry, and other factors.

In fact, Stata has an entire manual dedicated to estimators designed specifically for estimating treatment effects; see the Stata Treatment-Effects Reference Manual. Here we show how margins can be used to estimate treatment effects using the regression-adjustment estimator when the conditional independence assumption is met; see [TE] teffects intro. Regression adjustment simply means that we are going to use a regression model to predict the outcome variable, controlling for treatment status and other characteristics. The conditional independence assumption implies that we have enough variables in our dataset so that once we control for them in our regression model, the outcomes one would obtain with and without treatment are independent of how treatment status is determined.

## > Example 7: Regression adjustment with a binary treatment variable

nlsw88.dta contains women's wages (wage) in dollars per hour, a binary variable indicating their union status (union), years of experience (ttl_exp), and a variable, grade, indicating the number of years of schooling completed. We want to know how being in a union (the treatment) affects women's wages. Traditionally, a wage equation of the form

$$
\ln \text { wage }_{i}=\beta_{0}+\beta_{1} \text { union }_{i}+\beta_{2} \text { grade }_{i}+\beta_{3} \text { ttl_exp }+\beta_{4} \text { ttl_exp }^{2}+\epsilon_{i}
$$

would be fit. However, there are two shortcomings that we will improve upon. First, to avoid the problem of predicting the level of a log-transformed dependent variable, we will use poisson with the vce (robust) option to fit an exponential regression model; see Wooldridge (2010, sec. 18.2) for background on this approach. Second, the previous equation implies that factors other than union status have the same impact on wages for both union and nonunion workers. Regression-adjustment estimators allow all the variables to have different impacts depending on the level of the treatment variable, and we can accomplish that here using factor-variable notation. In Stata, we fit our model by typing

```
. use http://www.stata-press.com/data/r13/nlsw88
(NLSW, 1988 extract)
```

. poisson wage i.union\#\#(c.grade c.ttl_exp\#\#c.ttl_exp), vce(robust)
note: you are responsible for interpretation of noncount dep. variable
Iteration 0: $\quad$ log pseudolikelihood $=-4770.7957$
Iteration 1: $\quad$ log pseudolikelihood $=-4770.7693$
Iteration 2: $\quad$ log pseudolikelihood $=-4770.7693$

| Poisson regression | Number of obs | $=$ | 1876 |
| :--- | :--- | :--- | :--- |
|  | Wald chi2(7) | $=$ | 1047.11 |
|  | Prob $>$ chi2 | $=$ | 0.0000 |
| Log pseudolikelihood $=-4770.7693$ | Pseudo R2 | $=$ | 0.1195 |


| wage | Coef. | Robust <br> Std. Err. | z | $P>\|z\|$ | [95\% Conf. | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| union |  |  |  |  |  |  |
| union | . 8638376 | . 168233 | 5.13 | 0.000 | . 534107 | 1.193568 |
| grade | . 0895252 | . 0056874 | 15.74 | 0.000 | . 0783782 | . 1006722 |
| ttl_exp | . 0805737 | . 0114534 | 7.03 | 0.000 | . 0581255 | . 103022 |
| $\begin{aligned} & \text { c.ttl_exp\# } \\ & \text { c.ttl_exp } \end{aligned}$ | -. 0015502 | . 0004612 | -3.36 | 0.001 | -. 0024541 | -. 0006463 |
| union\# <br> c.grade union | -. 0310298 | . 0088259 | -3.52 | 0.000 | -. 0483282 | -. 0137314 |
| union\# <br> c.ttl_exp union | -. 0404226 | . 0230113 | -1.76 | 0.079 | -. 085524 | . 0046788 |
| ```union# c.ttl_exp# c.ttl_exp``` |  |  |  |  |  |  |
| union | . 0011808 | . 0008428 | 1.40 | 0.161 | -. 0004711 | . 0028327 |
| _cons | . 017488 | . 0893602 | 0.20 | 0.845 | -. 1576547 | . 1926308 |

To see how union status affects wages, we can use margins:

```
. margins r.union, vce(unconditional)
Contrasts of predictive margins
Expression : Predicted number of events, predict()
```

|  | df | chi2 | P>chi2 |
| :---: | :---: | :---: | :---: |
| union | 1 | 26.22 | 0.0000 |


|  | Unconditional |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Contrast | Std. Err. | [95\% Conf. Interval] |  |
| union | 1.004119 | .1960944 | .6197815 | 1.388457 |

The estimated contrast 1.004 indicates that on average, belonging to a union causes a woman's wage to be slightly more than a dollar higher than if she were not in the union. This estimated contrast is called the average treatment effect (ATE). Conceptually, we predicted the wage of each woman in the estimation sample assuming she was in a union and obtained the sample mean. We then predicted each woman's wage assuming she was not in a union and obtained that sample mean. The difference between these two sample means represents the ATE.

We obtain essentially the same results by using teffects ra:

```
. teffects ra (wage c.grade c.ttl_exp##c.ttl_exp, poisson) (union)
Iteration 0: EE criterion = 2.611e-13
Iteration 1: EE criterion = 1.098e-26
Treatment-effects estimation Number of obs = 1876
Estimator : regression adjustment
Outcome model : Poisson
Treatment model: none
```

| wage | Coef. | Robust |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Std. Err. | $z$ | $\mathrm{P}>\|z\|$ | [95\% Conf. Interval] |  |  |  |
| ATEunion <br> (union <br> vs <br> nonunion) | 1.004119 | .1960421 | 5.12 | 0.000 | .619884 | 1.388355 |
| POmeanunion <br> nonunion <br> 7.346493 | .1096182 | 67.02 | 0.000 | 7.131645 | 7.561341 |  |

The point estimates of the ATE are identical to those we obtained using margins, though the standard errors differ slightly from those reported by margins. The standard errors from the two estimators are, however, asymptotically equivalent, meaning they would coincide with a sufficiently large dataset. The last statistic in this output table indicates the untreated potential-outcome mean (untreated POM), which is the mean predicted wage assuming each woman did not belong to a union.

If we specify the pomeans option with teffects ra, we can obtain both the treated and the untreated POMs, which represent the predicted mean wages assuming all women were or were not in the union:


Notice that the difference between these two POMs equals 1.004119 , which is the ATE we obtained earlier.

In some applications, the average treatment effect of the treated (ATET) is more germane than the ATE. For example, if the untreated subjects in the sample could not possibly receive treatment (perhaps because a medical condition precludes their taking an experimental drug), then considering the counterfactual outcome had those subjects taken the drug may not be relevant. In these cases, the ATET is a better statistic because it measures the effect of the treatment only for those subjects who actually did receive treatment. Like the ATE, the ATET involves computing predicted outcomes for each treatment level, obtaining the sample means, and computing the difference between those two means. Unlike the ATE, however, we only use observations corresponding to treated subjects.

## > Example 8: Regression adjustment with a binary treatment variable (continued)

Here we calculate the ATET of union membership, first using margins. Because teffects ra overwrote our estimation results, we first quietly refit our poisson model. We then call margins to obtain the ATET:

```
. quietly poisson wage i.union##(c.grade c.ttl_exp##c.ttl_exp), vce(robust)
. margins r.union, subpop(union) vce(unconditional)
Contrasts of predictive margins
Expression : Predicted number of events, predict()
```

|  | df | chi2 | P>chi2 |
| ---: | :---: | :---: | :---: |
| union | 1 | 18.86 | 0.0000 |


|  | Unconditional |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Contrast | Std. Err. | [95\% Conf. Interval] |  |
| union | .901419 | .2075863 | .4945574 | 1.308281 |

The key here was specifying the subpop(union) option to restrict margin's computations to those women who are union members. The results indicate that being in the union causes the union members' wages to be about 90 cents higher than they would otherwise be.

To replicate these results using teffects ra, we include the atet option to obtain ATETs:

```
. teffects ra (wage c.grade c.ttl_exp##c.ttl_exp, poisson) (union), atet
Iteration 0: EE criterion = 2.611e-13
Iteration 1: EE criterion = 9.324e-27
Treatment-effects estimation Number of obs = 1876
Estimator : regression adjustment
Outcome model : Poisson
Treatment model: none
```

| wage | Coef. | Robust |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Std. Err. | $z$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |  |  |
| ATETunion <br> (union <br> vs <br> nonunion) | .901419 | .2075309 | 4.34 | 0.000 | .4946658 | 1.308172 |
| POmeanunion <br> nonunion | 7.776417 | .162121 | 47.97 | 0.000 | 7.458665 | 8.094168 |

We obtain the same point estimate of the effect of union status as with margins. As before, the standard errors differ slightly between the two estimators, but they are asymptotically equivalent. The output also indicates that among the women who are in a union, their average wage would be $\$ 7.78$ if they were not in a union.

## - Technical note

One advantage of the ATET over the ATE is that the ATET can be consistently estimated with slightly weaker assumptions than are required to consistently estimate the ATE. See Comparing the ATE and ATET in Remarks and examples of [TE] teffects intro advanced.

Both margins and teffects can estimate treatment effects using regression adjustment, so which should you use? In addition to regression adjustment, the teffects command implements other estimators of treatment effects; some of these estimators possess desirable robustness properties that we cannot replicate using margins. Moreover, all the teffects estimators use a common syntax and automatically present the estimated treatment effects, whereas we must first fit our own regression model and then call margins to obtain the treatment effects.

On the other hand, particularly with the at() option, margins gives us more flexibility in specifying our scenarios. The teffects commands allow us to measure the effect of a single binary or multinomial treatment, but we can have margins compute the effects of arbitrary interventions, as we illustrate in the next example.

## > Example 9: Interventions involving multiple variables

Suppose we want to see how women's wages would be affected if we could increase each woman's education level by one year. That is, we want to measure the treatment effect of an additional year of schooling. We assume that if a woman attains another year of schooling, she cannot simultaneously work. Thus an additional year of education implies her total work experience must decrease by a year. The flexible at () option of margins allows us to manipulate both variables at once:

```
. quietly poisson wage i.union##(c.grade c.ttl_exp##c.ttl_exp), vce(robust)
. margins, at((asobserved) _all)
> at(grade=generate(grade+1) ttl_exp=generate(ttl_exp-1))
> contrast(atcontrast(r._at))
Contrasts of predictive margins
Model VCE : Robust
Expression : Predicted number of events, predict()
1._at : (asobserved)
2._at : grade = grade+1
                                ttl_exp = ttl_exp-1
\begin{tabular}{r|ccc}
\hline & df & chi2 & P>chi2 \\
\hline at & 1 & 58.53 & 0.0000 \\
\hline
\end{tabular}
\begin{tabular}{c|cccc}
\hline & \multicolumn{4}{|c}{\begin{tabular}{c} 
Delta-method \\
Std. Err.
\end{tabular}} \\
\hline [95\% Conf. Interval] \\
\hline (2 vs 1) ºntrast \(^{\text {[ }}\) & .3390392 & .0443161 & .2521813 & .4258971 \\
\hline
\end{tabular}
```

The first at () option instructs margins to obtain predicted wages for all women in the sample using their existing values for grade and $t t l_{\text {_ exp }}$ and to record the mean of those predictions. The second at () option instructs margins to obtain the mean predicted wage under the counterfactual scenario where each woman's education level is increased by one year and total work experience is simultaneously decreased by one year. The contrast () option instructs margins to compute the difference between the two means. The output indicates that increasing education by one year, which will necessarily decrease work experience by the same amount, will cause the average wage to increase by about 34 cents per hour, a statistically significant amount.

## Conclusion

margins, contrast is a powerful command, and its abundance of suboptions may seem daunting. The suboptions are in the service of only three goals, however. There are three things that margins, contrast can do with a factor variable or a set of at () definitions:

1. Perform contrasts across the levels of the factor or set (as in example 1).
2. Perform a joint test across the levels of the factor or set (as in example 5).
3. Perform other tests and contrasts within each level of the factor or set (as in example 4).

The default behavior for variables specified inside at(), over(), and within() is to perform contrasts within groups; the default behavior for variables in the marginlist is to perform joint tests across groups.

## Stored results

margins, contrast stores the following additional results in $r()$ :
Scalars $r$ (k_terms) number of terms participating in contrasts
Macros

```
        r(cmd)
        r(cmd2)
        r(overall) overall or empty
contrast
        margins
        r(L)
        r(chi2)
        r(p)
        r(df)
                                matrix of contrasts applied to the margins
                                vector of }\mp@subsup{\chi}{}{2}\mathrm{ statistics
                                vector of p-values corresponding to r(chi2)
                                vector of degrees of freedom corresponding to r(p)
```

Matrices
margins, contrast with the post option also stores the following additional results in e():
Scalars
e(k_terms) number of terms participating in contrasts
Macros
e(cmd)

```
contrast
margins
```

    e(cmd2)
    e(overall) overall or empty
    Matrices
e(L) matrix of contrasts applied to the margins
e(chi2)
vector of $\chi^{2}$ statistics
e (p)
vector of $p$-values corresponding to e (chi2)
$e(d f)$
vector of degrees of freedom corresponding to e(p)

## Methods and formulas

See Methods and formulas in $[R]$ margins and Methods and formulas in $[R]$ contrast.

## Reference

Wooldridge, J. M. 2010. Econometric Analysis of Cross Section and Panel Data. 2nd ed. Cambridge, MA: MIT Press.

## Also see

[R] contrast - Contrasts and linear hypothesis tests after estimation
[R] lincom - Linear combinations of estimators
[R] margins - Marginal means, predictive margins, and marginal effects
[R] margins postestimation - Postestimation tools for margins
[R] margins, pwcompare - Pairwise comparisons of margins
[R] pwcompare - Pairwise comparisons

## Title

margins, pwcompare - Pairwise comparisons of margins

| Syntax | Menu | Description | Suboptions |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | Also see |

## Syntax

```
margins [marginlist] [if] [in] [weight] [, pwcompare margins_options]
margins [marginlist] [if] [in] [weight] [, pwcompare(suboptions) margins_options]
```

where marginlist is a list of factor variables or interactions that appear in the current estimation results. The variables may be typed with or without the i. prefix, and you may use any factor-variable syntax:
. margins i.sex i.group i.sex\#i.group, pwcompare
. margins sex group sex\#i.group, pwcompare
. margins sex\#\#group, pwcompare

See [R] margins for the available margins_options.
suboptions Description

Pairwise comparisons
cieffects
pveffects
effects
cimargins
groups
sort
show effects table with confidence intervals; the default show effects table with $p$-values
show effects table with confidence intervals and $p$-values
show table of margins and confidence intervals
show table of margins and group codes
sort the margins or contrasts in each term
fweights, aweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.

## Menu

Statistics $>$ Postestimation $>$ Pairwise comparisons of margins

## Description

margins with the pwcompare option performs pairwise comparisons of margins. margins, pwcompare extends the capabilities of pwcompare to any of the nonlinear responses, predictive margins, or other margins that can be estimated by margins.

## Suboptions

Pairwise comparisons
cieffects specifies that a table of the pairwise comparisons with their standard errors and confidence intervals be reported. This is the default.
pveffects specifies that a table of the pairwise comparisons with their standard errors, test statistics, and $p$-values be reported.
effects specifies that a table of the pairwise comparisons with their standard errors, test statistics, $p$-values, and confidence intervals be reported.
cimargins specifies that a table of the margins with their standard errors and confidence intervals be reported.
groups specifies that a table of the margins with their standard errors and group codes be reported. Margins with the same letter in the group code are not significantly different at the specified significance level.
sort specifies that the reported tables be sorted on the margins or contrasts in each term.

## Remarks and examples

You should be familiar with the concepts and syntax of both margins and pwcompare before using the pwcompare option of margins. These remarks build on those in $[R]$ margins and $[R]$ pwcompare.
margins can perform pairwise comparisons of any of the margins that it estimates.
We begin by fitting a logistic regression model using the NHANES II dataset, ignoring the complex survey nature of the data. Our dependent variable is highbp, an indicator for whether a person has high blood pressure. We fit an interacted model including two factor variables representing the region of the country as well as the continuous covariate bmi.
. use http://www.stata-press.com/data/r13/nhanes2
. logistic highbp region\#\#c.bmi
(output omitted)
By default, margins will compute the predictive margins of the probability of a positive outcome for each of the terms in marginlist after logistic regression. We will margin on region so that margins will estimate the average predicted probabilities of having high blood pressure conditional on being in each of the four regions and unconditional on BMI. We can specify the pwcompare option to obtain all possible pairwise comparisons of these predictive margins:

```
. margins region, pwcompare
Pairwise comparisons of predictive margins
Model VCE : OIM
Expression : Pr(highbp), predict()
```

|  | Delta-method <br> Std. Err. |  |  | Unadjusted <br> [95\% Conf. Interval] |  |
| :---: | ---: | ---: | ---: | ---: | :---: |
|  | Contrast |  |  |  |  |
| region |  |  | -.0638987 | -.01154 |  |
| MW vs NE | -.0377194 | .0133571 | -.041945 | .0105764 |  |
| S vs NE | -.0156843 | .0133986 | -.0336451 | .019899 |  |
| W vs NE | -.006873 | .0136595 | -.0023789 | .0464492 |  |
| S vs MW | .0220351 | .0124564 | .0058831 | .0558096 |  |
| W vs MW | .0308463 | .0127366 | -.0162373 | .0338598 |  |
| W vs S | .0088112 | .0127801 |  |  |  |

This table gives each of the pairwise differences with confidence intervals. We can see that the confidence interval in the row labeled MW vs NE does not include 0 . At the $5 \%$ level, the predictive margins for the first and second regions, the Northeast and the Midwest, are significantly different. The same is true of the second and fourth regions, the Midwest and the West. With many pairwise comparisons, output in this format can be difficult to sort through. We can organize it by adding the group suboption:
. margins region, pwcompare(group)
Pairwise comparisons of predictive margins
Model VCE $\quad:$ OIM
Expression $\quad: \operatorname{Pr}($ highbp ), predict()

|  | Delta-method Unadjusted <br> Gtd. Err. |  |  |
| :---: | :---: | :---: | :---: |
| Margin | Groups |  |  |
| region |  |  |  |
| NE | .4388358 | .010069 | B |
| MW | .4011164 | .0087764 | AB |
| S | .4231516 | .0088395 | B |
| W | .4319628 | .0092301 |  |

Note: Margins sharing a letter in the group label are not significantly different at the $5 \%$ level.

The group output includes the predictive margins for each region and letters denoting margins that are not significantly different from one another. In this case, the Northeast (NE), South (S), and West (W) regions have the letter B in the "Unadjusted Groups" column. The letter B indicates that the average predicted probability for the Northeast region is not significantly different from the average predicted probabilities for the South and West regions at the $5 \%$ significance level. The Midwest (MW) region does not share a letter with the Northeast region nor the West region, which indicates that the average predicted probability for the Midwest region is significantly different for each of the other two regions at our 5\% level.

We can also include the mcompare(bonferroni) option to perform tests using Bonferroni's method to account for making multiple comparisons.
. margins region, pwcompare(group) mcompare(bonferroni)
Pairwise comparisons of predictive margins
Model VCE : OIM
Expression : Pr(highbp), predict()

|  | Number of <br> Comparisons |
| ---: | ---: |
| region | 6 |


|  | Delta-method Bonferroni <br> Std. Err. |  |  |
| :---: | :---: | :---: | :---: |
| Margin | Groups |  |  |
| region |  |  |  |
| NE | .4388358 | .010069 | B |
| MW | .4011164 | .0087764 | A |
| S | .4231516 | .0088395 | AB |
| W | .4319628 | .0092301 | AB |

```
Note: Margins sharing a letter in the group label
    are not significantly different at the 5%
    level.
```

We now see the letter A on the row corresponding to the West region. At the $5 \%$ level and with Bonferroni's adjustment, the predictive margins for the probability in the Midwest and West regions are not significantly different.

## Stored results

margins, pwcompare stores the following additional results in $r()$ :
Scalars
$r$ (k_terms) number of terms participating in pairwise comparisons
Macros

```
    r(cmd)
    r(cmd2)
    r(group#)
```

    r (mcmethod_vs) method from mcompare()
    \(r\) (mctitle_vs) title for method from mcompare()
    \(r\) (mcadjustall_vs) adjustall or empty
        pwcompare
        margins
        group code for the \#th margin in r (b)
    Matrices
r(b)
r(V)
r(b_vs)
r(V-vs)
r(error_vs)
r(table_vs)
r(L)
margin estimates variance-covariance matrix of the margin estimates margin difference estimates variance-covariance margin difference of the margin estimates margin difference estimability codes;
0 means estimable,
8 means not estimable
matrix containing the margin differences with their standard errors, test statistics, $p$-values, and confidence intervals
matrix that produces the margin differences
margins, pwcompare with the post option also stores the following additional results in e() :
Scalars
e(k_terms) number of terms participating in pairwise comparisons
Macros
e(cmd)
pwcompare
e(cmd2)
margins
Matrices
e(b)
e(V)
e(b_vs)
margin estimates
variance-covariance matrix of the margin estimates margin difference estimates
variance-covariance margin difference of the margin estimates margin difference estimability codes;

0 means estimable,
8 means not estimable
e(L) matrix that produces the margin differences

## Methods and formulas

See Methods and formulas in $[R]$ margins and Methods and formulas in $[R]$ pwcompare.

## Also see

[R] contrast - Contrasts and linear hypothesis tests after estimation
[R] margins - Marginal means, predictive margins, and marginal effects
[R] margins, contrast - Contrasts of margins
[R] margins postestimation - Postestimation tools for margins
[R] pwcompare - Pairwise comparisons

## Title

marginsplot - Graph results from margins (profile plots, etc.)

| Syntax | Menu | Description |
| :--- | :--- | :--- |
| Options | Remarks and examples | Addendum: Advanced uses of dimlist |
| Acknowledgments | References | Also see |

## Syntax

```
marginsplot [, options]
```

options
Description

Main
$\underline{x} d i m e n s i o n(d i m l i s t ~[, ~ d i m o p t s]) \quad u s e ~ d i m l i s t ~ t o ~ d e f i n e ~ x ~ a x i s ~$ plotdimension(dimlist [, dimopts]) bydimension(dimlist [, dimopts])
graphdimension(dimlist [, dimopts]) horizontal
noci
name (name $\mid$ stub [, replace])
Labels
allxlabels
nolabels
allsimplelabels
nosimplelabels
separator (string)
noseparator
Plot plotopts(plot_options)
plot\#opts(plot_options)
recast (plottype)
CI plot
ciopts(rcap_options)
ci\#opts (rcap_options)
recastci(plottype)
mcompare (method)
level(\#)

Pairwise
unique
csort
Add plots addplot (plot)
create plots for groups in dimlist create subgraphs for groups in dimlist create graphs for groups in dimlist swap $x$ and $y$ axes
do not plot confidence intervals name of graph, or stub if multiple graphs
place ticks and labels on the $x$ axis for each value label groups with their values, not their labels forgo variable name and equal signs in all labels include variable name and equal signs in all labels separator for labels when multiple variables are specified in a dimension
do not use a separator
affect rendition of all margin plots affect rendition of \#th margin plot plot margins using plottype
affect rendition of all confidence interval plots affect rendition of \#th confidence interval plot plot confidence intervals using plottype adjust for multiple comparisons set confidence level
plot only unique pairwise comparisons
sort comparison categories first
add other plots to the graph
twoway_options
byopts (byopts)
any options documented in [G-3] twoway_options how subgraphs are combined, labeled, etc.
where dimlist may be any of the dimensions across which margins were computed in the immediately preceding margins command; see [R] margins. That is to say, dimlist may be any variable used in the margins command, including variables specified in the at(), over(), and within() options. More advanced specifications of dimlist are covered in Addendum: Advanced uses of dimlist.
dimopts
labels(lablist)
elabels(elablist)
nolabels
allsimplelabels
nosimplelabels
separator (string)
noseparator

Description
list of quoted strings to label each level of the dimension list of enumerated labels
label groups with their values, not their labels forgo variable name and equal signs in all labels include variable name and equal signs in all labels separator for labels when multiple variables are specified in the dimension
do not use a separator
where lablist is defined as
"label" ["label" [...]]
elablist is defined as
\# "label" [\# "label" [...]]
and the \#s are the indices of the levels of the dimension-1 is the first level, 2 is the second level, and so on.
plot_options Description
marker_options
marker_label_options
cline_options
change look of markers (color, size, etc.) add marker labels; change look or position change look of the line
method
noadjust
bonferroni [adjustall]
sidak [adjustall]
scheffe

Description
do not adjust for multiple comparisons
Bonferroni's method; adjust across all terms
Šidák's method; adjust across all terms
Scheffé's method

## Menu

Statistics $>$ Postestimation $>$ Margins plots and profile plots

## Description

marginsplot graphs the results of the immediately preceding margins command; see [R] margins. Common names for some of the graphs that marginsplot can produce are profile plots and interaction plots.

## Options

Main
xdimension(), plotdimension(), bydimension(), and graphdimension() specify the variables from the preceding margins command whose group levels will be used for the graph's $x$ axis, plots, by () subgraphs, and graphs.
marginsplot chooses default dimensions based on the margins command. In most cases, the first variable appearing in an at () option and evaluated over more than one value is used for the $x$ axis. If no at () variable meets this condition, the first variable in the marginlist is usually used for the $x$ axis and the remaining variables determine the plotted lines or markers. Pairwise comparisons and graphs of marginal effects (derivatives) have different defaults. In all cases, you may override the defaults and explicitly control which variables are used on each dimension of the graph by using these dimension options.
Each of these options supports suboptions that control the labeling of the dimension-axis labels for xdimension(), plot labels for plotdimension(), subgraph titles for bydimension(), and graph titles for graphdimension() titles.
For examples using the dimension options, see Controlling the graph's dimensions.
xdimension(dimlist [, dimopts]) specifies the variables for the $x$ axis in dimlist and controls the content of those labels with dimopts.
plotdimension(dimlist [, dimopts]) specifies in dimlist the variables whose group levels determine the plots and optionally specifies in dimopts the content of the plots' labels.
bydimension(dimlist [, dimopts]) specifies in dimlist the variables whose group levels determine the by () subgraphs and optionally specifies in dimopts the content of the subgraphs' titles. For an example using by (), see Three-way interactions.
graphdimension(dimlist [, dimopts]) specifies in dimlist the variables whose group levels determine the graphs and optionally specifies in dimopts the content of the graphs' titles.
horizontal reverses the default $x$ and $y$ axes. By default, the $y$ axis represents the estimates of the margins and the $x$ axis represents one or more factors or continuous covariates. Specifying horizontal swaps the axes so that the $x$ axis represents the estimates of the margins. This option can be useful if the labels on the factor or continuous covariates are long.

The horizontal option is discussed in Horizontal is sometimes better.
noci removes plots of the pointwise confidence intervals. The default is to plot the confidence intervals.
name (name $\mid$ stub [, replace]) specifies the name of the graph or graphs. If the graphdimension() option is specified, or if the default action is to produce multiple graphs, then the argument of name() is taken to be stub and graphs named stub1, stub2, ... are created.

The replace suboption causes existing graphs with the specified name or names to be replaced.
If name() is not specified, default names are used and the graphs may be replaced by subsequent marginsplot or other graphing commands.

With the exception of allxlabels, all of these options may be specified either directly as options or as dimopts within options xdimension(), plotdimension(), bydimension(), and graphdimension(). When specified in one of the dimension options, only the labels for that dimension are affected. When specified outside the dimension options, all labels on all dimensions are affected. Specifications within the dimension options take precedence.
allxlabels specifies that tick marks and labels be placed on the $x$ axis for each value of the $x$-dimension variables. By default, if there are more than 25 ticks, default graph axis labeling rules are applied. Labeling may also be specified using the standard graph twoway $x$-axis label rules and options-xlabel(); see [G-3] axis_label_options.
nolabels specifies that value labels not be used to construct graph labels and titles for the group levels in the dimension. By default, if a variable in a dimension has value labels, those labels are used to construct labels and titles for axis ticks, plots, subgraphs, and graphs.

Graphs of contrasts and pairwise comparisons are an exception to this rule and are always labeled with values rather than value labels.
allsimplelabels and nosimplelabels control whether graphs' labels and titles include just the values of the variables or include variable names and equal signs. The default is to use just the value label for variables that have value labels and to use variable names and equal signs for variables that do not have value labels. An example of the former is "Female" and the latter is "country=2".
Sometimes value labels are universally descriptive, and sometimes they have meaning only when considered in relation to their variable. For example, "Male" and "Female" are typically universal, regardless of the variable from which they are taken. "High" and "Low" may not have meaning unless you know they are in relation to a specific measure, say, blood-pressure level. The allsimplelabels and nosimplelabels options let you override the default labeling.
allsimplelabels specifies that all titles and labels use just the value or value label of the variable.
nosimplelabels specifies that all titles and labels include varname $=$ before the value or value label of the variable.
separator (string) and noseparator control the separator between label sections when more than one variable is used to specify a dimension. The default separator is a comma followed by a space, but no separator may be requested with noseparator or the default may be changed to any string with separator ().
For example, if plotdimension( a b) is specified, the plot labels in our graph legend might be " $a=1, b=1 ", " a=1, b=2 ", \ldots$ Specifying separator (: ) would create labels " $a=1: b=1 "$, $" a=1: b=2 ", \ldots$

## Plot

plotopts(plot_options) affects the rendition of all margin plots. The plot_options can affect the size and color of markers, whether and how the markers are labeled, and whether and how the points are connected; see [G-3] marker_options, [G-3] marker_label_options, and [G-3] cline_options.

These settings may be overridden for specific plots by using the plot\#opts() option.
plot\#opts (plot_options) affects the rendition of the \#th margin plot. The plot_options can affect the size and color of markers, whether and how the markers are labeled, and whether and how the points are connected; see [G-3] marker_options, [G-3] marker_label_options, and [G-3] cline_options.
recast (plottype) specifies that margins be plotted using plottype. plottype may be scatter, line, connected, bar, area, spike, dropline, or dot; see [G-2] graph twoway. When recast() is specified, the plot-rendition options appropriate to the specified plottype may be used in lieu of plot_options. For details on those options, follow the appropriate link from [G-2] graph twoway. For an example using recast(), see Continuous covariates.
You may specify recast() within a plotopts() or plot\#opts() option. It is better, however, to specify it as documented here, outside those options. When specified outside those options, you have greater access to the plot-specific rendition options of your specified plottype.

```
Cl plot
```

ciopts(rcap_options) affects the rendition of all confidence interval plots; see [G-3] rcap_options.
These settings may be overridden for specific confidence interval plots with the ci\#opts() option. ci\#opts(rcap_options) affects the rendition of the \#th confidence interval; see [G-3] rcap_options. recastci(plottype) specifies that confidence intervals be plotted using plottype. plottype may be rarea, rbar, rspike, rcap, rcapsym, rline, rconnected, or rscatter; see [G-2] graph twoway. When recastci() is specified, the plot-rendition options appropriate to the specified plottype may be used in lieu of rcap_options. For details on those options, follow the appropriate link from [G-2] graph twoway.
For an example using recastci(), see Continuous covariates.
You may specify recastci() within a ciopts() or ci\#opts() option. It is better, however, to specify it as documented here, outside those options. When specified outside those options, you have greater access to the plot-specific rendition options of your specified plottype.
mcompare (method) specifies the method for confidence intervals that account for multiple comparisons within a factor-variable term. The default is determined by the margins results stored in $r()$. If marginsplot is working from margins results stored in e(), the default is mcompare (noadjust).
level(\#) specifies the confidence level, as a percentage, for confidence intervals. The default is determined by the margins results stored in $r()$. If marginsplot is working from margins results stored in e(), the default is level(95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals.

## Pairwise

These options have an effect only when the pwcompare option was specified on the preceding margins command.
unique specifies that only unique pairwise comparisons be plotted. The default is to plot all pairwise comparisons, including those that are mirror images of each other-"male" versus "female" and "female" versus "male". margins reports only the unique pairwise comparisons. unique also changes the default xdimension() for graphs of pairwise comparisons from the reference categories (_pw0) to the comparisons of each pairwise category (_pw).
Unique comparisons are often preferred with horizontal graphs that put all pairwise comparisons on the $x$ axis, whereas including the full matrix of comparisons is preferred for charts showing the reference groups on an axis and the comparison groups as plots; see Pairwise comparisons and Horizontal is sometimes better.
csort specifies that comparison categories are sorted first, and then reference categories are sorted within comparison category. The default is to sort reference categories first, and then sort comparison categories within reference categories. This option has an observable effect only when _pw is also
specified in one of the dimension options. It then determines the order of the labeling in the dimension where _pw is specified.

## Add plots

addplot (plot) provides a way to add other plots to the generated graph; see [G-3] addplot_option.
For an example using addplot(), see Adding scatterplots of the data.
If multiple graphs are drawn by a single marginsplot command or if plot specifies plots with multiple $y$ variables, for example, scatter y1 y2 x , then the graph's legend will not clearly identify all the plots and will require customization using the legend() option; see [G-3] legend_options.

Y axis, X axis, Titles, Legend, Overall, By
twoway_options are any of the options documented in [G-3] twoway_options. These include options for titling the graph (see [G-3] title_options); for saving the graph to disk (see [G-3] saving_option); for controlling the labeling and look of the axes (see [G-3] axis_options); for controlling the look, contents, position, and organization of the legend (see [G-3] legend_options); for adding lines (see [G-3] added_line_options) and text (see [G-3] added_text_options); and for controlling other aspects of the graph's appearance (see [G-3] twoway_options).
The label () suboption of the legend() option has no effect on marginsplot. Use the order () suboption instead.
byopts (byopts) affects the appearance of the combined graph when bydimension() is specified or when the default graph has subgraphs, including the overall graph title, the position of the legend, and the organization of subgraphs. See [G-3] by_option.

## Remarks and examples

Remarks are presented under the following headings:

```
Introduction
Dataset
Profile plots
Interaction plots
Contrasts of margins—effects (discrete marginal effects)
Three-way interactions
Continuous covariates
Plots at every value of a continuous covariate
Contrasts of at() groups-discrete effects
Controlling the graph's dimensions
Pairwise comparisons
Horizontal is sometimes better
Marginal effects
Plotting a subset of the results from margins
Advanced usage
    Plots with multiple terms
    Plots with multiple at() options
    Adding scatterplots of the data
Video examples
```


## Introduction

marginsplot is a post-margins command. It graphs the results of the margins command, whether those results are marginal means, predictive margins, marginal effects, contrasts, pairwise comparisons, or other statistics; see [R] margins.

By default, the margins are plotted on the $y$ axis, and all continuous and factor covariates specified in the margins command will usually be placed on the $x$ axis or used to identify plots. Exceptions are discussed in the following sections and in Addendum: Advanced uses of dimlist below.
marginsplot produces classic plots, such as profile plots and interaction plots. Beyond that, anything that margins can compute, marginsplot can graph.

We will be using some relatively complicated margins commands with little explanation of the syntax. We will also avoid lengthy interpretations of the results of margins. See [R] margins for the complete syntax of margins and discussions of its results.

All graphs in this entry were drawn using the s2gcolor scheme; see [G-4] scheme $\mathbf{s} \mathbf{2}$.
Mitchell (2012) shows in many examples how to use marginsplot to understand a fitted model.

## Dataset

For continuity, we will use one dataset for most examples-the Second National Health and Nutrition Examination Survey (NHANES II) (McDowell et al. 1981). NHANES II is part of a study to assess the health and nutritional status of adults and children in the United States. It is designed to be a nationally representative sample of the U.S. population. This particular sample is from 1976 to 1980.

The survey nature of the dataset-weights, strata, and sampling units-will be ignored in our analyses. We are discussing graphing, not survey statistics. If you would like to see the results with the appropriate adjustments for the survey design, just add svy : before each estimation command, and if you wish, add vce (unconditional) as an option to each margins command. See [R] margins, particularly the discussion and examples under Obtaining margins with survey data and representative samples, for reasons why you probably would want to add vce(unconditional) when analyzing survey data. For the most part, adjusting for survey design produces moderately larger confidence intervals and relatively small changes in point estimates.

## Profile plots

What does my estimation say about how my response varies as one (or more) of my covariates changes? That is the question that is answered by profile plots. Profile plots are also referred to as plots of estimated (or expected, or least-squares) means, though that is unnecessarily restrictive when considering models of binary, count, and ordered outcomes. In the latter cases, we might prefer to say they plot conditional expectations of responses, where a response might be a probability.

What we do with the other covariates depends on the questions we wish to answer. Sometimes we wish to hold other covariates at fixed values, and sometimes we wish to average the response over their values. margins can do either, so you can graph either.

We can fit a fully factorial two-way ANOVA of systolic blood pressure on age group and sex using the NHANES II data.
. use http://www.stata-press.com/data/r13/nhanes2
. anova bpsystol agegrp\#\#sex

|  | $\begin{array}{rlr} \text { Number of obs } & =10351 \\ \text { Root MSE } & =20.2209 \end{array}$ |  |  | R-squared <br> Adj R-squared | $\begin{aligned} & =0.2497 \\ & =0.2489 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Source | Partial SS | df | MS | F | Prob > F |
| Model | 1407229.28 | 11 | 127929.935 | $5 \quad 312.88$ | 0.0000 |
| agegrp | 1243037.82 | 5 | 248607.565 | $5 \quad 608.02$ | 0.0000 |
| sex | 27728.3794 | 1 | 27728.3794 | $4 \quad 67.81$ | 0.0000 |
| agegrp\#sex | 88675.043 | 5 | 17735.0086 | $6 \quad 43.37$ | 0.0000 |
| Residual | 4227440.75 | 10339 | 408.882943 |  |  |
| Total | 5634670.03 | 10350 | 544.412563 |  |  |

If you are more comfortable with regression than ANOVA, then type

```
. regress bpsystol agegrp##sex
```

The anova and regress commands fit identical models. The output from anova displays all the terms in the model and thus tends to be more conducive to exploration with margins and marginsplot.

We estimate the predictive margins of systolic blood pressure for each age group using margins.

```
. margins agegrp
Predictive margins Number of obs = 10351
Expression : Linear prediction, predict()
```

|  | Delta-method <br> Std. Err. |  |  |  |  |  |  | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | :---: | :---: | :---: | :---: |
| agegrp |  |  |  |  |  |  |  |  |  |  |
| $20-29$ | 117.2684 | .419845 | 279.31 | 0.000 | 116.4454 | 118.0914 |  |  |  |  |
| $30-39$ | 120.2383 | .5020813 | 239.48 | 0.000 | 119.2541 | 121.2225 |  |  |  |  |
| $40-49$ | 126.9255 | .56699 | 223.86 | 0.000 | 125.8141 | 128.0369 |  |  |  |  |
| $50-59$ | 135.682 | .5628593 | 241.06 | 0.000 | 134.5787 | 136.7853 |  |  |  |  |
| $60-69$ | 141.5285 | .3781197 | 374.30 | 0.000 | 140.7873 | 142.2696 |  |  |  |  |
| $70+$ | 148.1096 | .6445073 | 229.80 | 0.000 | 146.8463 | 149.373 |  |  |  |  |

The six predictive margins are just the averages of the predictions over the estimation sample, holding agegrp to each of its six levels. If this were a designed experiment rather than survey data, we might wish to assume the cells are balanced-that they have the same number of observations-and thus estimate what are often called expected means or least-squares means. To do that, we would simply add the asbalanced option to the margins command. The NHANES II data are decidedly unbalanced over sex\#agegrp cells. So much so that it is unreasonable to assume the cells are balanced.

We graph the results:

```
marginsplot
Variables that uniquely identify margins: agegrp
```



Profile plots are often drawn without confidence intervals (CIs). The CIs may be removed by adding the noci option. We prefer to see the CIs.

Disciplines vary widely in their use of the term profile plot. Some disciplines consider any connected plot of a response over values of other variables to be a profile plot. By that definition, most graphs in this entry are profile plots.

## Interaction plots

Interaction plots are often used to explore the form of an interaction. The interaction term in our ANOVA results is highly significant. Are the interaction effects also large enough to matter? What form do they take? We can answer these questions by fixing agegrp and sex to each possible combination of the two covariates and estimating the margins for those cells.

Then we can graph the results:

```
. marginsplot
    Variables that uniquely identify margins: agegrp sex
```



It is clear that the effect of age differs by sex-there is an interaction. If there were no interaction, then the two lines would be parallel.

While males start out with higher systolic blood pressure, females catch up to the males as age increases and may even surpass males in the upper age groups. We say "may" because we cannot tell if the differences are statistically significant. The CIs overlap for the top three age groups. It is tempting to conclude from this overlap that the differences are not statistically significant. Do not fall into this trap. Likewise, do not fall into the trap that the first three age groups are different because their CIs do not overlap. The CIs are for the point estimates, not the differences. There is a covariance between the differences that we must consider if we are to make statements about those differences.

## Contrasts of margins-effects (discrete marginal effects)

To assess the differences, all we need do is ask margins to contrast the sets of effects that we just estimated; see $[\mathrm{R}]$ margins, contrast. With only two groups in sex, it does not matter much which contrast operator we choose. We will use the reference contrast. It will compare the difference between males and females, with males (the first category) as the reference category.

```
. margins r.sex@agegrp
Contrasts of adjusted predictions
Expression : Linear prediction, predict()
```

|  | df | F | $\mathrm{P}>\mathrm{F}$ |
| ---: | ---: | ---: | ---: |
| sex@agegrp |  |  |  |
| (Female vs Male) 20-29 | 1 | 224.92 | 0.0000 |
| (Female vs Male) 30-39 | 1 | 70.82 | 0.0000 |
| (Female vs Male) 40-49 | 1 | 12.15 | 0.0005 |
| (Female vs Male) 50-59 | 1 | 0.47 | 0.4949 |
| (Female vs Male) 60-69 | 1 | 3.88 | 0.0488 |
| (Female vs Male) 70+ | 1 | 6.37 | 0.0116 |
| Joint | 6 | 53.10 | 0.0000 |
| Denominator | 10339 |  |  |


|  | Delta-method <br> Ctd. Err. |  |  |  |
| :---: | ---: | ---: | ---: | ---: |
| [95\% Contrast. Interval] |  |  |  |  |
| sex@agegrp |  |  |  |  |
| (Female vs Male) 20-29 | -12.60132 | .8402299 | -14.24833 | -10.9543 |
| (Female vs Male) 30-39 | -8.461161 | 1.005448 | -10.43203 | -6.490288 |
| (Female vs Male) 40-49 | -3.956451 | 1.134878 | -6.181031 | -1.731871 |
| (Female vs Male) 50-59 | -.7699782 | 1.128119 | -2.981309 | 1.441353 |
| (Female vs Male) 60-69 | 1.491684 | .756906 | .0080022 | 2.975367 |
| (Female vs Male) 70+ | 3.264762 | 1.293325 | .729594 | 5.79993 |

Because we are looking for effects that are different from 0 , we will add a reference line at 0 to our graph.

```
marginsplot, yline(0)
Variables that uniquely identify margins: agegrp
```



We can now say that females' systolic blood pressure is substantially and significantly lower than males' in the first three age groups but is significantly higher in the last two age groups. Despite the overlapping CIs for the last two age groups in the interaction graph, the effect of sex is significant in these age groups.

The terminology for what we just estimated and graphed varies widely across disciplines. Those versed in design of experiments refer to these values as contrasts or effects. Economists and some other social scientists call them marginal or partial effects. The latter groups might be more comfortable if we avoided the whole concept of contrasts and instead estimated the effects by typing

```
. margins agegrp, dydx(sex)
```

This will produce estimates that are identical to those shown above, and we can graph them by typing marginsplot.

The advantage of using the contrast notation and thinking in contrasts is most evident when we take marginal effects with respect to a categorical covariate with more than two levels. Marginal effects for each level of the covariate will be taken with respect to a specified base level. Contrasts are much more flexible. Using the r. operator, we can reproduce the marginal-effects results by taking derivatives with respect to a reference level (as we saw above.) We can also estimate the marginal effect of first moving from level 1 to level 2, then from level 2 to level 3, then from level 3 to level $4, \ldots$ using the ar. or "reverse adjacent" operator. Adjacent effects (marginal effects) can be valuable when evaluating an ordinal covariate, such as agegrp in our current model. For a discussion of contrasts, see $[R]$ contrast and $[R]$ margins, contrast.

## Three-way interactions

marginsplot can handle any number of covariates in your margins command. Consider the three-way ANOVA model that results from adding an indicator for whether an individual has been diagnosed with diabetes. We will fully interact the new covariate with the others in the model.
. anova bpsystol agegrp\#\#sex\#\#diabetes


The three-way interaction is significant, as is the main effect of diabetes and its interaction with agegrp.

Again, if you are more comfortable with regression than ANOVA, you may type

```
. regress bpsystol agegrp\#\#sex\#\#diabetes
```

The margins and marginsplot results will be the same.

We estimate the expected cell means for each combination of agegrp, sex, and diabetes, and then graph the results by typing

```
. margins agegrp#sex#diabetes
(output omitted)
. marginsplot
Variables that uniquely identify margins: agegrp sex diabetes
```



The graph is busy and difficult to interpret.
We can make it better by putting those with diabetes on one subgraph and those without on another:

```
. marginsplot, by(diabetes)
    Variables that uniquely identify margins: agegrp sex diabetes
```

Adjusted Predictions of agegrp\#sex\#diabetes with 95\% CIs


We notice much larger CIs for diabetics. That is not surprising because our sample contains only 499 diabetics compared with 9,850 nondiabetics.

A more interesting way to arrange the plots is by grouping the subgraphs on sex:

```
marginsplot, by(sex)
Variables that uniquely identify margins: agegrp sex diabetes
```

Adjusted Predictions of agegrp\#sex\#diabetes with 95\% Cls


Aside from increased systolic blood pressure in the upper-age groups, which we saw earlier, it appears that those with diabetes are at greater risk of higher systolic blood pressure for many upper-age groups. We can check that by having margins estimate the differences between diabetics and nondiabetics, and graphing the results.

```
. margins r.diabetes@agegrp#sex
(output omitted)
marginsplot, by(sex) yline(0)
Variables that uniquely identify margins: agegrp sex
```

Contrasts of Adjusted Predictions of diabetes@agegrp\#sex with 95\% CIs


With CIs above 0 for six of eight age groups over 40, this graph provides evidence that diabetes is related to higher blood pressure in those over 40.

## Continuous covariates

margins and marginsplot are just as useful with continuous covariates as they are with factor variables. As a variation on our ANOVA/regression models, let's move to a logistic regression, using as our dependent variable an indicator for whether a person has high blood pressure. We introduce a continuous covariate-body mass index (BMI), a measure of weight relative to height. High BMI is often associated with high blood pressure. We will allow the effect of BMI to vary across sexes, age groups, and sex/age combinations by fully interacting the covariates.

```
. logistic highbp sex##agegrp##c.bmi
```

If we wished, we could perform all the analyses above on this model. Instead of estimating margins, contrasts, and marginal effects on the level of systolic blood pressure, we would be estimating margins, contrasts, and marginal effects on the probability of having high blood pressure. You can see those results by repeating any of the prior commands that involve sex and agegrp. In this section, we will focus on the continuous covariate bmi.

With continuous covariates, rather than specify them in the marginlist of margins, we specify the specific values at which we want the covariate evaluated in an at () option. at () options are very flexible, and there are many ways to specify values; see Syntax of at() in [R] margins.

BMI in our sample ranges from 12.4 to 61.1. Let's estimate the predictive margins for males and females at levels of BMI from 10 through 65 at intervals of 5 and graph the results:

```
. margins sex, at(bmi=(10(5)65))
    (output omitted)
. marginsplot, xlabel(10(10)60)
    Variables that uniquely identify margins: bmi sex
```



We added the xlabel(10(10)60) option to improve the labeling of the $x$ axis. You may add any twoway_options (see [G-3] twoway_options) to the marginsplot command.

For a given BMI, males are generally more susceptible to high blood pressure, though the effect is attenuated by the logistic response when the probabilities approach 0 or 1 .

Because bmi is continuous, we might prefer to see the response graphed using a line. We might also prefer that the CIs be plotted as areas. We change the plottype of the response by using the recast() option and the plottype of the CI by using the recastci() option:

```
. marginsplot, xlabel(10(10)60) recast(line) recastci(rarea)
Variables that uniquely identify margins: bmi sex
```



The CIs are a little dark for our tastes. You can dim them a bit by reducing the intensity of their color. Adding ciopts (color (*.8)) to our marginsplot command will do that. Any plot option accepted by twoway rarea (see [G-2] graph twoway rarea) may be specified in a ciopts() option.

Given their confidence regions, the male and female profiles appear to be statistically different over most of the range of BMI. As with the profiles of categorical covariates, we can check that assertion by contrasting the two profiles on sex and graphing the results. Let's improve the smoothness of the response by specifying intervals of 1 instead of 5 .

```
. margins r.sex, at(bmi=(10(1)65))
(output omitted)
marginsplot, xlabel(10(10)60) recast(line) recastci(rarea)
    Variables that uniquely identify margins: bmi
```



We see that the difference between the sexes is largest at a BMI of about 35 and that the sexes respond more similarly with very high and very low BMI. This shape is largely determined by the
response of the logistic function, which is attenuated near probabilities 0 and 1 , combined with the fact that the lowest measured BMIs are associated with low probabilities of high blood pressure and the highest measured BMIs are associated with high probabilities of high blood pressure.

As when we contrasted profiles of categorical variables, different disciplines will think of this graph differently. Those familiar with designed experiments will be comfortable with the terms used above-this is a contrast of profiles, or a profile of effects, or a profile of a contrast. Many social scientists will prefer to think of this as a graph of marginal or partial effects. For them, this is a plot of the discrete marginal effect of being female for various levels of BMI. They can obtain an identical graph, with labeling more appropriate for the marginal effect's interpretation, by typing

```
. margins, at(bmi=(10(1)65)) dydx(sex)
. marginsplot, xlabel(10(10)60) recast(line) recastci(rarea)
```

We can also plot profiles of the response of BMI by levels of another continuous covariate (rather than by the categorical variable sex). To do so, we will need another continuous variable in our model. We have been using age groups as a covariate to emphasize the treatment of categorical variables and to allow the effect of age to be flexible. Our dataset also has age recorded in integer years. We replace agegrp with continuous age in our logistic regression.
. logistic highbp sex\#\#c.age\#\#c.bmi
We can now obtain profiles of BMI for different ages by specifying ranges for both bmi and age in a single at () option on the margins command:

```
. margins sex, at(bmi=(10(5)60) age=(20(10)70))
```

With six ages specified, we have many profiles, so we will dispense with the CIs by adding the noci option and also tidy up the graph by asking for three columns in the legend:

```
. marginsplot, noci by(sex) legend(cols(3))
Variables that uniquely identify margins: bmi age sex
```



Our model seems to indicate that males have a sharper reaction to body mass indices than do females. Likewise, younger subjects display a sharper response, while older subjects have a more gradual response with earlier onset. That interpretation might be a result of our parametric treatment of age. As it turns out, the interpretation holds if we allow age to take more flexible forms or return to our use of age groups, which allows each of seven age groups to have unique BMI profiles. Here are the commands to perform that analysis:
. logistic highbp sex\#\#agegrp\#\#c.bmi
(output omitted)
. margins sex\#agegrp, at (bmi=(10(5)60)) (output omitted)
. marginsplot, noci by(sex) legend(cols(3))
Variables that uniquely identify margins: bmi sex agegrp


## Plots at every value of a continuous covariate

In some cases, the specific values of a continuous covariate are important, and we want to plot the response at those specific values. Return to our logistic example with age treated as a continuous covariate.
. logistic highbp sex\#\#c.age\#\#c.bmi
We can use a programming trick to extract all the values of age and then supply them in an at () option, just as we would any list of values.
. levelsof age
. margins sex, at(age=('r(levels)'))
See [ P$]$ levelsof for a discussion of the levelsof command. levelsof returns in $r$ (levels) the sorted list of unique values of the specified varlist, in our case, age.

We can then plot the results using marginsplot.
This is not a very interesting trick when using our age variable, which is recorded as integers from 20 to 74, but the approach will work with almost any continuous variable. In our model, bmi might seem more interesting, but there are 9,941 unique values of bmi in our dataset. A graph cannot resolve so many different values. For that reason, we usually recommend against plotting at every value of a covariate. Instead, graph at reasonable values over the range of the covariate by using the at () option, as we did earlier. This trick is best reserved for variables with a few, or at most a few dozen, unique values.

## Contrasts of at() groups-discrete effects

We have previously contrasted across the values of factor variables in our model. Put another way, we have estimated the discrete marginal effects of factor variables. We can do the same for the levels of variables in at () specifications and across separate at () specifications.

Returning to one of our logistic models and its margins, we earlier estimated the predictive margins of BMI at 5 -unit intervals for both sexes. These are the commands we typed:

```
. logistic highbp sex##agegrp##c.bmi
. margins sex, at(bmi=(10(5)65))
. marginsplot, xlabel(10(10)60)
```

We can estimate the discrete effects by sex of bmi moving from 10 to 15 , then from 15 to $20, \ldots$, and then from 60 to 65 by contrasting the levels of the at () groups using the reverse-adjacent contrast operator (ar.). We specify the operator within the atcontrast() suboption of the contrast() option. We need to specify one other option. By default, margins, contrast will apply a contrast to all variables in its marginlist when a contrast has been requested. In this case, we do not want to contrast across sexes but rather to contrast across the levels of BMI within each sex. To prevent margins from contrasting across the sexes, we specify the marginswithin option. Our margins command is

```
. margins sex, at(bmi=(10(5)65)) contrast(atcontrast(ar._at) marginswithin)
```

And we graph the results using marginsplot:

```
. marginsplot
Variables that uniquely identify margins: bmi sex
```



The graph shows the contrasts (or if you prefer, discrete changes) in the probability of high blood pressure by sex as one increases BMI in 5-unit increments.

We can even estimate contrasts (discrete effects) across at () options. To start, let's compare the age-group profiles of the probability of high blood pressure for those in the 25 th and 75 th percentile of BMI.

```
. margins agegrp, at((p25) bmi) at((p75) bmi)
(output omitted)
. marginsplot
    Variables that uniquely identify margins: agegrp _atopt
    Multiple at() options specified:
    _atoption=1: (p25) bmi
    _atoption=2: (p75) bmi
```



For each age group, people whose BMI is at the 75th percentile have a much higher probability of high blood pressure than those at the 25th percentile. What is that difference in probability and its CI? To contrast across the percentiles of BMI within age groups, we again specify a contrast operator on the at() groups using atcontrast (), and we also tell margins to perform that contrast within the levels of the marginlist by using the marginswithin option.

```
. margins agegrp, at((p25) bmi) at((p75) bmi)
> contrast(atcontrast(r._at) marginswithin)
    (output omitted)
. marginsplot
    Variables that uniquely identify margins: agegrp _atopt
    Multiple at() options specified:
    _atoption=1: (p25) bmi
    _atoption=2: (p75) bmi
```



The differences in probability between 25 th and 75 th BMI percentiles are clearly significantly greater than 0 . The differences appear to be smallest for those in the youngest and oldest age groups.

## Controlling the graph's dimensions

Thus far, marginsplot has miraculously done almost exactly what we want in most cases. The things we want on the $x$ axis have been there, the choice of plots has made sense, etc. Some of that luck sprang from the relatively simple analyses we were performing, and some was from careful specification of our margins command. Sometimes, we will not be so lucky.

Consider the following regress, margins, and marginsplot commands:

```
. regress bpsystol agegrp##sex##c.bmi
    (output omitted)
. margins agegrp, over(sex) at(bmi=(10(10)60))
    (output omitted)
    marginsplot
    Variables that uniquely identify margins: agegrp _atopt
    Multiple at() options specified:
        _atoption=1: (p25) bmi
        _atoption=2: (p75) bmi
```



By default, marginsplot places the levels of the first multilevel at() specification on the $x$ axis, and then usually plots the levels of all remaining variables as connected lines. That is what we see in the graph above-bmi, the at () variable, is on the $x$ axis, and each combination of agegrp and sex is plotted as a separate connected line. If there is no multilevel at () specification, then the first variable in marginlist becomes the $x$ axis. There are many more rules, but it is usually best to simply type marginsplot and see what happens. If you do not like marginsplot's choices, change them.

What if we wanted agegrp on the $x$ axis instead of BMI? We tell marginsplot to make that change by specifying agegrp in the xdimension() option:

```
. marginsplot, xdimension(agegrp)
Variables that uniquely identify margins: bmi agegrp sex
```



We have been suppressing the Results window output for marginsplot, but that output is helpful if we want to change how things are plotted. You may specify any variable used in your margins command in any of the dimension options-xdimension(), plotdimension(), bydimension(), and graphdimension(). (In fact, there are some pseudovariables that you may also specify in some cases; see Addendum: Advanced uses of dimlist for details.) marginsplot tries to help you narrow your choices by listing a set of variables that uniquely identify all your margins. You are not restricted to this list.

We have a different $x$ axis and a different set of plots, but our graph is still busy and difficult to read. We can make it better by creating separate graph panels for each sex. We do that by adding a bydimension() option with sex as the argument.

```
. marginsplot, xdimension(agegrp) bydimension(sex)
    Variables that uniquely identify margins: bmi agegrp sex
```



The patterns and the differences between males and females are now easier to see.

If our interest is in comparing males and females, we might even choose to create a separate panel for each level of BMI:

```
. marginsplot, xdimension(agegrp) bydimension(bmi) xlabel(, angle(45))
Variables that uniquely identify margins: bmi agegrp sex
```



The $x$-axis labels did not fit, so we angled them.
We leave you to explore the use of the graphdimension() option. It is much like bydimension() but creates separate graphs rather than separate panels. Operationally, the plotdimension() option is rarely used. All variables not in the $x$ dimension and not specified elsewhere become the plotted connected lines.

You will likely use the dimension options frequently. This is one of the rare cases where we recommend using the minimal abbreviations of the options- x () for xdimension(), plot() for plotdimension(), by() for bydimension(), and graph() for graphdimension(). The abbreviations are easy to read and just as meaningful as the full option names. The full names exist to reinforce the relationship between the dimension options.

## Pairwise comparisons

marginsplot can graph the results of margins, pwcompare; see [R] margins, pwcompare. We return to one of our ANOVA examples. Here we request pairwise comparisons with the pwcompare option of margins, and we request Bonferroni-adjusted CIs with the mcompare() option:

```
. anova bpsystol agegrp##sex
(output omitted)
. margins agegrp, pwcompare mcompare(bonferroni)
(output omitted)
marginsplot
Variables that uniquely identify margins: _pw1 _pw0
_pw enumerates all pairwise comparisons; _pwO enumerates the reference
categories; _pw1 enumerates the comparison categories.
```



Each connected line plot in the graph represents a reference age-group category for the pairwise comparison. The ticks on the $x$ axis represent comparison age-group categories. So, each plot is a profile for a reference category showing its comparison to each other category.

## Horizontal is sometimes better

Another interesting way to graph pairwise comparisons is to simply plot each comparison and label the two categories being compared. This type of graph works better if it is oriented horizontally rather than vertically.

Continuing with the example above, we will switch the graph to horizontal. We will also make several changes to display the graph better. We specify that only unique comparisons be plotted. The graph above plotted both 1 versus 2 and 2 versus 1 , which are the same comparison with opposite signs. We add a reference line at 0 because we are interested in comparisons that differ from 0 . This graph looks better without the connecting lines, so we add the option recast(scatter). We also reverse the $y$ scale so that the smallest levels of age group appear at the top of the axis.

```
marginsplot, horizontal unique xline(0) recast(scatter) yscale(reverse)
Variables that uniquely identify margins: _pw1 _pw0
    _pw enumerates all pairwise comparisons; _pw0 enumerates the reference
    categories; _pw1 enumerates the comparison categories.
```



All the comparisons differ from 0 , so all our age groups are statistically different from each other.
The horizontal option can be useful outside of pairwise comparisons. Profile plots are usually oriented vertically. However, when your covariates have long labels or there are many levels at which the margins are being evaluated, the graph may be easier to read when rendered horizontally.

## Marginal effects

We have seen how to graph discrete effects for factor variables and continuous variables by using contrasts, and optionally by using the dydx() option of margins: Contrasts of margins-effects (discrete marginal effects) and Continuous covariates. Let's now consider graphing instantaneous marginal effects for continuous covariates. Begin by refitting our logistic model of high blood pressure as a function of sex, age, and BMI:

```
. logistic highbp sex##agegrp##c.bmi
```

We estimate the average marginal effect of BMI on the probability of high blood pressure for each age group and then graph the results by typing

```
. margins agegrp, dydx(bmi)
(output omitted)
. marginsplot
    Variables that uniquely identify margins: agegrp
```



These are the conditional expectations of the marginal effects treating everyone in the sample as though they were in each age group. We can estimate fully conditional marginal effects that do not depend on averaging over the sample by also margining on our one remaining covariate-sex.

```
. margins agegrp#sex, dydx(bmi)
    (output omitted)
    marginsplot
    Variables that uniquely identify margins: agegrp sex
```



The effect of BMI on the probability of high blood pressure looks to increase with age for females. The marginal effect is higher for males than females in the younger age groups but then decreases with age for males after the 40-49 age group.

You may want to test for differences in the marginal effect of BMI for males and females by contrasting across sexes within agegrp:

```
. margins r.sex@agegrp, dydx(bmi)
```


## Plotting a subset of the results from margins

marginsplot plots all the margins produced by the preceding margins command. If you want a graph that does not include all the margins, then enter a margins command that produces a reduced set of margins. Obvious ways to reduce the number of margins include not specifying some factors or interactions in the marginlist of margins, not specifying some at () or over () options, or reducing the values specified in an at () option. A less obvious technique uses selection lists in factor operators to select specific sets of levels from factor variables specified in the marginlist.

Instead of typing
. margins agegrp
which will give you margins for all six age groups in our sample, type
. margins i(2/4).agegrp
which will give you only three margins—those for groups 2, 3, and 4. See [U] 11.4.3.4 Selecting levels.

## Advanced usage

margins is incredibly flexible in the statistics it can estimate and in the grouping of those estimates. Many of the estimates that margins can produce do not make convincing graphs. marginsplot plots the results of any margins command, regardless of whether the resulting graph is easily interpreted. Here we demonstrate some options that can make complicated margins into graphs that are somewhat more useful than those produced by marginsplot's defaults. Others may find truly useful applications for these approaches.

## Plots with multiple terms

Margins plots are rarely interesting when you specify multiple terms on your margins command, for example, margins a b. Such plots often compare things that are not comparable. The defaults for marginsplot rarely produce useful plots with multiple terms. Perhaps the most interesting graph in such cases puts all the levels of all the terms together on the vertical axis and plots their margins on the horizontal axis. We do that by including the marginlist from margins in an xdimension() option on marginsplot. The long labels on such graphs look better with a horizontal orientation, and there is no need to connect the margin estimates, so we specify the recast (scatter) option.

Using one of our ANOVA examples from earlier,

```
. anova bpsystol agegrp##sex
    (output omitted)
- margins agegrp sex
    (output omitted)
marginsplot, xdimension(agegrp sex) horizontal recast(scatter)
    Variables that uniquely identify margins: agegrp sex
```



The "asobserved" notations in the $y$-axis labels are informing us that, for example, when the margin for females is evaluated, the values of age group are taken as they are observed in the dataset. The margin is computed as an average over those values.

## Plots with multiple at() options

Some disciplines like to compute margins at the means of other covariates in their model and others like to compute the response for each observation and then take the means of the response. These correspond to the margins options at ((mean) _all) and at ((asobserved) _all). For responses that are linear functions of the coefficients, such as predict after regress, the two computations yield identical results. For responses that are nonlinear functions of the coefficients, the two computations estimate different things.

Using one of our logistic models of high blood pressure,

```
. logistic highbp sex##agegrp##c.bmi
```

and computing both sets of margins for each age group,

```
. margins agegrp, at((mean) _all) at((asobserved) _all)
```

we can use marginsplot to compare the approaches:

```
. marginsplot
    Variables that uniquely identify margins: agegrp _atopt
    Multiple at() options specified:
    _atoption=1: (mean)_all
    _atoption=2: (asobserved) _all
```



For the first three age groups, the probabilities of high blood pressure are lower at the means of sex and bpi than are the mean probabilities of high blood pressure averaged over the observed values of sex and bpi. The reverse is true for the last three age groups, although the values are very similar in these older age groups.

Such comparisons come up even more frequently when evaluating marginal effects. We can estimate the marginal effects of sex at each age group and graph the results by adding dydx (sex) to our margins command:

```
. margins agegrp, at((mean) _all) at((asobserved) _all) dydx(sex)
(output omitted)
. marginsplot
    Variables that uniquely identify margins: agegrp _atopt
    Multiple at() options specified:
    _atoption=1: (mean)_all
    _atoption=2: (asobserved) _all
```



The average marginal effect is smaller for most age groups, but the CIs for both sets of estimates are wide. Can we tell the difference between the estimates? To answer that, we use the now-familiar tactic of taking the contrast of our estimated marginal-effects profiles. That means adding contrast (atjoint marginswithin) to our margins command. We will also add mcompare(bonferroni) to account for the fact that we will be comparing six contrasts.

```
. margins agegrp, at((mean) _all) at((asobserved) _all) dydx(sex)
> contrast(atjoint marginswithin) mcompare(bonferroni)
```

We will also add the familiar reference line at 0 to our graph of the contrasts.

```
marginsplot, yline(0)
Variables that uniquely identify margins: agegrp _atopt
Multiple at() options specified:
    _atoption=1: (mean)_all
    _atoption=2: (asobserved) _all
```



While the difference in the estimates of marginal effects is not large, we can distinguish the estimates for the 30-39 and 70+ age groups.

The at () option of margins provides far more flexibility than demonstrated above. It can be used to evaluate a response or marginal effect at almost any point of interest or combinations of such points. See Syntax of at() in [R] margins.

## Adding scatterplots of the data

We can add scatterplots of the observed data to our plots of the margins. The NHANES II dataset is too large for this to be interesting, so for this example, we will use auto.dta. We fit mileage on whether the care is foreign and on a quadratic in the weight of the car. We convert the weight into tons (U.S. definition) to improve the scaling, and we format the new tons variable to improve its labels on the graph. For our graph, we create separate variables for mileage of domestic and of foreign cars. We fit a fully interacted model so that the effect of weight on mileage can be different for foreign and for domestic cars.

```
. use http://www.stata-press.com/data/r13/auto
. generate tons = weight/2000
. format tons %6.2f
. separate mpg, by(foreign)
. regress mpg foreign##c.tons##c.tons
```

We then estimate the margins over the range of tons, using the option over (foreign) to obtain separate estimates for foreign and domestic cars.

```
. margins, at(tons=(.8(.05)2.4)) over(foreign)
```

Adding scatterplots of mileage for domestic and foreign cars is easy. We insert into an addplot() option of marginsplot the same scatterplot syntax for twoway that we would type to produce a scatterplot of the data:

```
marginsplot, addplot(scatter mpg0 tons || scatter mpg1 tons) recast(line) noci
Variables that uniquely identify margins: tons foreign
```



Many will be surprised that the mileage profile is higher in 1978 for domestic (U.S. built) cars. Is the difference significant?

```
. margins, at(tons=(.8(.05)2.4)) over(r.for)
(output omitted)
. marginsplot, yline(0)
Variables that uniquely identify margins: tons
```



As we did earlier, we contrast the two profiles. We can discern some difference between the two profiles for midweight vehicles, but otherwise there is insufficient information to believe mileage differs across domestic and foreign cars.

## Video examples

Profile plots and interaction plots, part 1: A single categorical variable
Profile plots and interaction plots, part 2: A single continuous variable
Profile plots and interaction plots, part 3: Interactions between categorical variables
Profile plots and interaction plots, part 4: Interactions of continuous and categorical variables
Profile plots and interaction plots, part 5: Interactions of two continuous variables

## Addendum: Advanced uses of dimlist

dimlist specifies the dimensions from the immediately preceding margins command that are to be used for the marginsplot's $x$ axis, plots, subgraphs, and graphs. dimlist may contain:

> dim

Description
varname
at (varname) If a variable is specified in both the marginlist or the over() option and in the Any variable referenced in the preceding margins command. at () option of margins, then the two uses can be distinguished in marginsplot by typing the at () variables as at (varname) in dimlist.
_deriv If the preceding margins command included a dydx(), eyex(), dyex(), or eydx() option, dimlist may also contain _deriv to specify all the variables over which derivatives were taken.
_term If the preceding margins command included multiple terms (for example, margins a b), then dimlist may contain _term to enumerate those terms.
_atopt If the preceding margins command included multiple at () options, then dimist may contain _atopt to enumerate those at () options.

When the pairwise option is specified on margins, you may specify dimensions that enumerate the pairwise comparisons.

| _pw | enumerates all the pairwise comparisons |
| :--- | :--- |
| -pw0 | enumerates the reference categories of the comparisons |
| -pw1 | enumerates the comparison categories of the comparisons |

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## References

McDowell, A., A. Engel, J. T. Massey, and K. Maurer. 1981. Plan and operation of the Second National Health and Nutrition Examination Survey, 1976-1980. Vital and Health Statistics 1(15): 1-144.

Mitchell, M. N. 2012. Interpreting and Visualizing Regression Models Using Stata. College Station, TX: Stata Press. Royston, P. 2013. marginscontplot: Plotting the marginal effects of continuous predictors. Stata Journal 13: 510-527.
Williams, R. 2012. Using the margins command to estimate and interpret adjusted predictions and marginal effects. Stata Journal 12: 308-331.

## Also see

[R] margins - Marginal means, predictive margins, and marginal effects
[R] margins, contrast - Contrasts of margins
[R] margins, pwcompare - Pairwise comparisons of margins
[R] margins postestimation - Postestimation tools for margins

## Title

matsize - Set the maximum number of variables in a model
Syntax Description Option Remarks and examples Also see

## Syntax

```
set matsize # [, permanently ]
```

where $10 \leq \# \leq 11000$ for Stata/MP and Stata/SE and where $10 \leq \# \leq 800$ for Stata/IC.

## Description

set matsize sets the maximum number of variables that can be included in any of Stata's estimation commands.

For Stata/MP and Stata/SE, the default value is 400, but it may be changed upward or downward. The upper limit is 11,000 .

For Stata/IC, the initial value is 400 , but it may be changed upward or downward. The upper limit is 800 .

This command may not be used with Small Stata; matsize is permanently frozen at 100 .
Changing matsize has no effect on Mata.

## Option

permanently specifies that, in addition to making the change right now, the matsize setting be remembered and become the default setting when you invoke Stata.

## Remarks and examples

set matsize controls the internal size of matrices that Stata uses. The default of 400 for Stata/IC, for instance, means that linear regression models are limited to 198 independent variables - 198 because the constant uses one position and the dependent variable another, making a total of 200.

You may change matsize with data in memory, but increasing matsize increases the amount of memory consumed by Stata, increasing the probability of page faults and thus of making Stata run more slowly.
> Example 1
We wish to fit a model of $y$ on the variables $x 1$ through $x 400$. Without thinking, we type

```
. regress y x1-x400
matsize too small
    You have attempted to create a matrix with more than 400 rows or columns
    or to fit a model with more than 400 variables plus ancillary parameters.
    You need to increase matsize by using the set matsize command; see help
    matsize.
r(908);
```

We realize that we need to increase matsize, so we type
. set matsize 450

- regress y x1-x400 (output omitted)

Programmers should note that the current setting of matsize is stored as the c-class value $c$ (matsize); see [P] creturn.

## Also see

[R] query - Display system parameters
[D] memory - Memory management
[U] 6 Managing memory

## Title

maximize - Details of iterative maximization

| Syntax | Description | Maximization options |
| :--- | :--- | :--- |
| Option for set maxiter | Remarks and examples | Stored results |
| Methods and formulas | References | Also see |

## Syntax

Maximum likelihood optimization

$$
\text { mle_cmd } \ldots[\text {, options }]
$$

Set default maximum iterations

```
set maxiter # [, 隹manently ]
```

options
difficult
technique (algorithm_spec)
iterate(\#)
[no] log
trace
gradient
showstep
hessian
showtolerance
tolerance (\#)
ltolerance(\#)
nrtolerance(\#)
qtolerance(\#)
nonrtolerance
from(init_specs)

Description
use a different stepping algorithm in nonconcave regions maximization technique perform maximum of \# iterations; default is iterate (16000) display an iteration $\log$ of the log likelihood; typically, the default display current parameter vector in iteration log display current gradient vector in iteration log report steps within an iteration in iteration log
display current negative Hessian matrix in iteration log report the calculated result that is compared to the effective convergence criterion
tolerance for the coefficient vector; see Options for the defaults tolerance for the log likelihood; see Options for the defaults tolerance for the scaled gradient; see Options for the defaults when specified with algorithms bhhh, dfp , or bfgs , the $\mathbf{q}-\mathbf{H}$ matrix is used as the final check for convergence rather than nrtolerance() and the $\mathbf{H}$ matrix; seldom used
ignore the nrtolerance() option
initial values for the coefficients
where algorithm_spec is
algorithm $[\#[$ algorithm [\#] $] \ldots]$
algorithm is $\{\mathrm{nr} \mid$ bhhh $|\mathrm{dfp}| \mathrm{bfgs}\}$
and init_specs is one of

$$
\begin{aligned}
& \text { matname }[, \text { skip copy }] \\
& \{[\text { eqname }:] \text { name }=\# \mid \text { /eqname }=\#\}[\ldots] \\
& \#[\# \ldots], \text { copy }
\end{aligned}
$$

## Description

All Stata commands maximize likelihood functions using moptimize() and optimize(); see Methods and formulas below. Commands use the Newton-Raphson method with step halving and special fixups when they encounter nonconcave regions of the likelihood. For details, see [M-5] moptimize( ) and [M-5] optimize( ). For more information about programming maximum likelihood estimators in ado-files and Mata, see [R] ml and the fourth edition of Maximum Likelihood Estimation with Stata (Gould, Pitblado, and Poi 2010).
set maxiter specifies the default maximum number of iterations for estimation commands that iterate. The initial value is 16000 , and \# can be 0 to 16000 . To change the maximum number of iterations performed by a particular estimation command, you need not reset maxiter; you can specify the iterate(\#) option. When iterate(\#) is not specified, the maxiter value is used.

## Maximization options

difficult specifies that the likelihood function is likely to be difficult to maximize because of nonconcave regions. When the message "not concave" appears repeatedly, ml's standard stepping algorithm may not be working well. difficult specifies that a different stepping algorithm be used in nonconcave regions. There is no guarantee that difficult will work better than the default; sometimes it is better and sometimes it is worse. You should use the difficult option only when the default stepper declares convergence and the last iteration is "not concave" or when the default stepper is repeatedly issuing "not concave" messages and producing only tiny improvements in the log likelihood.
technique (algorithm_spec) specifies how the likelihood function is to be maximized. The following algorithms are allowed. For details, see Gould, Pitblado, and Poi (2010).
technique ( nr ) specifies Stata's modified Newton-Raphson (NR) algorithm.
technique (bhhh) specifies the Berndt-Hall-Hall-Hausman (BHHH) algorithm.
technique (dfp) specifies the Davidon-Fletcher-Powell (DFP) algorithm.
technique (bfgs) specifies the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm.
The default is technique ( nr ).
You can switch between algorithms by specifying more than one in the technique() option. By default, an algorithm is used for five iterations before switching to the next algorithm. To specify a different number of iterations, include the number after the technique in the option. For example, specifying technique (bhhh 10 nr 1000 ) requests that ml perform 10 iterations with the BHHH algorithm followed by 1000 iterations with the NR algorithm, and then switch back to BHHH for 10 iterations, and so on. The process continues until convergence or until the maximum number of iterations is reached.
iterate(\#) specifies the maximum number of iterations. When the number of iterations equals iterate(), the optimizer stops and presents the current results. If convergence is declared before this threshold is reached, it will stop when convergence is declared. Specifying iterate(0) is useful for viewing results evaluated at the initial value of the coefficient vector. Specifying iterate ( 0 ) and from() together allows you to view results evaluated at a specified coefficient vector; however, not all commands allow the from() option. The default value of iterate(\#) for both estimators programmed internally and estimators programmed with ml is the current value of set maxiter, which is iterate (16000) by default.
log and nolog specify whether an iteration $\log$ showing the progress of the log likelihood is to be displayed. For most commands, the log is displayed by default, and nolog suppresses it. For a few commands (such as the svy maximum likelihood estimators), you must specify log to see the $\log$.
trace adds to the iteration $\log$ a display of the current parameter vector.
gradient adds to the iteration $\log$ a display of the current gradient vector.
showstep adds to the iteration log a report on the steps within an iteration. This option was added so that developers at StataCorp could view the stepping when they were improving the ml optimizer code. At this point, it mainly provides entertainment.
hessian adds to the iteration $\log$ a display of the current negative Hessian matrix.
showtolerance adds to the iteration $\log$ the calculated value that is compared with the effective convergence criterion at the end of each iteration. Until convergence is achieved, the smallest calculated value is reported.
shownrtolerance is a synonym of showtolerance.

Below we describe the three convergence tolerances. Convergence is declared when the nrtolerance() criterion is met and either the tolerance() or the ltolerance() criterion is also met.
tolerance (\#) specifies the tolerance for the coefficient vector. When the relative change in the coefficient vector from one iteration to the next is less than or equal to tolerance(), the tolerance () convergence criterion is satisfied.
tolerance ( $1 \mathrm{e}-4$ ) is the default for estimators programmed with ml .
tolerance ( $1 \mathrm{e}-6$ ) is the default.
ltolerance(\#) specifies the tolerance for the log likelihood. When the relative change in the log likelihood from one iteration to the next is less than or equal to ltolerance(), the ltolerance() convergence is satisfied.
ltolerance ( 0 ) is the default for estimators programmed with ml .
ltolerance ( $1 \mathrm{e}-7$ ) is the default.
nrtolerance(\#) specifies the tolerance for the scaled gradient. Convergence is declared when $\mathbf{g} \mathbf{H}^{-1} \mathbf{g}^{\prime}<$ nrtolerance (). The default is nrtolerance (1e-5).
qtolerance(\#) when specified with algorithms bhhh, dfp, or bfgs uses the $\mathbf{q}-\mathbf{H}$ matrix as the final check for convergence rather than nrtolerance() and the $\mathbf{H}$ matrix.
Beginning with Stata 12, by default, Stata now computes the $\mathbf{H}$ matrix when the $\mathbf{q}-\mathbf{H}$ matrix passes the convergence tolerance, and Stata requires that $\mathbf{H}$ be concave and pass the nrtolerance() criterion before concluding convergence has occurred.
qtolerance() provides a way for the user to obtain Stata's earlier behavior.
nonrtolerance specifies that the default nrtolerance() criterion be turned off.
from() specifies initial values for the coefficients. Not all estimators in Stata support this option. You can specify the initial values in one of three ways: by specifying the name of a vector containing the initial values (for example, from (b0), where b0 is a properly labeled vector); by specifying coefficient names with the values (for example, from (age=2.1/sigma=7.4) ; or by specifying a list of values (for example, from ( 2.17 .4 , copy)). from() is intended for use when doing bootstraps (see [R] bootstrap) and in other special situations (for example, with iterate( 0 ) ). Even when the values specified in from() are close to the values that maximize the likelihood, only a few iterations may be saved. Poor values in from() may lead to convergence problems.
skip specifies that any parameters found in the specified initialization vector that are not also found in the model be ignored. The default action is to issue an error message.
copy specifies that the list of values or the initialization vector be copied into the initial-value vector by position rather than by name.

## Option for set maxiter

permanently specifies that, in addition to making the change right now, the maxiter setting be remembered and become the default setting when you invoke Stata.

## Remarks and examples

Only in rare circumstances would you ever need to specify any of these options, except nolog. The nolog option is useful for reducing the amount of output appearing in $\log$ files.

The following is an example of an iteration log:

| Iteration 0: | log likelihood = -3791.0251 |  |
| :---: | :---: | :---: |
| Iteration 1: | log likelihood = -3761.738 |  |
| Iteration 2: | log likelihood = -3758.0632 | (not concave) |
| Iteration 3: | log likelihood = -3758.0447 |  |
| Iteration 4: | log likelihood = -3757.5861 |  |
| Iteration 5: | log likelihood = -3757.474 |  |
| Iteration 6: | log likelihood = -3757.4613 |  |
| Iteration 7: | log likelihood = -3757.4606 |  |
| Iteration 8: <br> (table of resu | $\begin{aligned} & \text { log likelihood }=-3757.4606 \\ & \text { omitted ) } \end{aligned}$ |  |

At iteration 8, the model converged. The message "not concave" at the second iteration is notable. This example was produced using the heckman command; its likelihood is not globally concave, so it is not surprising that this message sometimes appears. The other message that is occasionally seen is "backed up". Neither of these messages should be of any concern unless they appear at the final iteration.

If a "not concave" message appears at the last step, there are two possibilities. One is that the result is valid, but there is collinearity in the model that the command did not otherwise catch. Stata checks for obvious collinearity among the independent variables before performing the maximization, but strange collinearities or near collinearities can sometimes arise between coefficients and ancillary parameters. The second, more likely cause for a "not concave" message at the final step is that the optimizer entered a flat region of the likelihood and prematurely declared convergence.

If a "backed up" message appears at the last step, there are also two possibilities. One is that Stata found a perfect maximum and could not step to a better point; if this is the case, all is fine, but this is a highly unlikely occurrence. The second is that the optimizer worked itself into a bad concave spot where the computed gradient and Hessian gave a bad direction for stepping.

If either of these messages appears at the last step, perform the maximization again with the gradient option. If the gradient goes to zero, the optimizer has found a maximum that may not be unique but is a maximum. From the standpoint of maximum likelihood estimation, this is a valid result. If the gradient is not zero, it is not a valid result, and you should try tightening up the convergence criterion, or try ltol ( 0 ) tol ( $1 \mathrm{e}-7$ ) to see if the optimizer can work its way out of the bad region.

If you get repeated "not concave" steps with little progress being made at each step, try specifying the difficult option. Sometimes difficult works wonderfully, reducing the number of iterations and producing convergence at a good (that is, concave) point. Other times, difficult works poorly, taking much longer to converge than the default stepper.

## Stored results

Maximum likelihood estimators store the following in e():

| Scalars |  |  |
| :---: | :---: | :---: |
| e(N) | number of observations | always stored |
| e(k) | number of parameters | always stored |
| e(k_eq) | number of equations in e(b) | usually stored |
| e(k_eq_model) | number of equations in overall model test | usually stored |
| e (k_dv) | number of dependent variables | usually stored |
| $e\left(d f \_m\right)$ | model degrees of freedom | always stored |
| e (r2_p) | pseudo- $R$-squared | sometimes stored |
| e(11) | log likelihood | always stored |
| e(ll_0) | log likelihood, constant-only model | stored when constant-only model is fit |
| e(N_clust) | number of clusters | stored when vce(cluster clustvar) is specified; see [U] 20.21 Obtaining robust variance estimates |
| e(chi2) | $\chi^{2}$ | usually stored |
| e(p) | significance of model of test | usually stored |
| e (rank) | rank of e(V) | always stored |
| e(rank0) | rank of e(V) for constant-only model | stored when constant-only model is fit |
| e(ic) | number of iterations | usually stored |
| e(rc) | return code | usually stored |
| e(converged) | 1 if converged, 0 otherwise | usually stored |

Macros

```
e(cmd)
e(cmdline)
e(depvar)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(chi2type)
e(vce)
e(vcetype)
e(opt)
e(which)
name of command
command as typed
names of dependent variables
weight type
weight expression
title in estimation output
name of cluster variable
Wald or LR; type of model \(\chi^{2}\) test vcetype specified in vce() title used to label Std. Err. type of optimization
\(\max\) or min; whether optimizer is to perform maximization or
```

e(ml_method)
e(user)
e(technique)
e(singularHmethod) m-marquardt or hybrid; method
e(crittype) optimization criterion
e(properties)
e(predict)
Matrices

Functions
e(sample)
used when Hessian is singular minimization
type of ml method
name of likelihood-evaluator program always stored
from technique() option
optimization criterion
estimator properties
program used to implement predict usually stored

Matrices

```
e(b)
e(Cns)
e(ilog)
e(gradient)
e(V)
e(V_modelbased)
```

    coefficient vector
    constraints matrix
    iteration \(\log\) (up to 20 iterations)
    gradient vector
    variance-covariance matrix of
        the estimators
    model-based variance
    Functions
always stored
sometimes stored
usually stored
usually stored
always stored
only stored when $e(V)$ is neither the OIM nor
OPG variance

1. Type ereturn list, all to view these results; see [P] return.

See Stored results in the manual entry for any maximum likelihood estimator for a list of returned results.

## Methods and formulas

Optimization is currently performed by moptimize() and optimize(), with the former implemented in terms of the latter; see [M-5] moptimize() and [M-5] optimize(). Some estimators use moptimize() and optimize() directly, and others use the ml ado-file interface to moptimize().

Prior to Stata 11, Stata had three separate optimization engines: an internal one used by estimation commands implemented in C code; ml implemented in ado-code separately from moptimize() and used by most estimators; and moptimize() and optimize() used by a few recently written
estimators. These days, the internal optimizer and the old version of ml are used only under version control. In addition, arch and arima (see [TS] arch and [TS] arima) are currently implemented using the old ml .

Let $L_{1}$ be the log likelihood of the full model (that is, the log-likelihood value shown on the output), and let $L_{0}$ be the log likelihood of the "constant-only" model. The likelihood-ratio $\chi^{2}$ model test is defined as $2\left(L_{1}-L_{0}\right)$. The pseudo- $R^{2}$ (McFadden 1974) is defined as $1-L_{1} / L_{0}$. This is simply the log likelihood on a scale where 0 corresponds to the "constant-only" model and 1 corresponds to perfect prediction for a discrete model (in which case the overall log likelihood is 0 ).

Some maximum likelihood routines can report coefficients in an exponentiated form, for example, odds ratios in logistic. Let $b$ be the unexponentiated coefficient, $s$ its standard error, and $b_{0}$ and $b_{1}$ the reported confidence interval for $b$. In exponentiated form, the point estimate is $e^{b}$, the standard error $e^{b} s$, and the confidence interval $e^{b_{0}}$ and $e^{b_{1}}$. The displayed $Z$ (or $t$ ) statistics and $p$-values are the same as those for the unexponentiated results. This is justified because $e^{b}=1$ and $b=0$ are equivalent hypotheses, and normality is more likely to hold in the $b$ metric.

## References

Gould, W. W., J. S. Pitblado, and B. P. Poi. 2010. Maximum Likelihood Estimation with Stata. 4th ed. College Station, TX: Stata Press.
McFadden, D. L. 1974. Conditional logit analysis of qualitative choice behavior. In Frontiers in Econometrics, ed. P. Zarembka, 105-142. New York: Academic Press.

## Also see

[R] ml - Maximum likelihood estimation
[SVY] ml for svy - Maximum pseudolikelihood estimation for survey data
[M-5] moptimize( ) - Model optimization
[M-5] optimize( ) - Function optimization

## Title

mean - Estimate means

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

mean varlist [if] [in][weight] [, options]

## options Description

## Model <br> stdize (varname) <br> stdweight (varname) <br> nostdrescale

if/in/over
$\operatorname{over}(\operatorname{varlist}[, \underline{\text { nolabel }])}$

SE/Cluster
vce (vcetype)

Reporting
level(\#)
noheader
nolegend
display_options
coeflegend
variable identifying strata for standardization weight variable for standardization do not rescale the standard weight variable
group over subpopulations defined by varlist; optionally, suppress group labels
vcetype may be analytic, cluster clustvar, bootstrap, or jackknife
bootstrap, jackknife, mi estimate, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands. vce (bootstrap) and vce (jackknife) are not allowed with the mi estimate prefix; see [MI] mi estimate.
Weights are not allowed with the bootstrap prefix; see $[R]$ bootstrap.
aweights are not allowed with the jackknife prefix; see [R] jackknife.
vce() and weights are not allowed with the svy prefix; see [SVY] svy.
fweights, aweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Summaries, tables, and tests $>$ Summary and descriptive statistics $>$ Means

## Description

mean produces estimates of means, along with standard errors.

## Options

Model
stdize(varname) specifies that the point estimates be adjusted by direct standardization across the strata identified by varname. This option requires the stdweight() option.
stdweight (varname) specifies the weight variable associated with the standard strata identified in the stdize() option. The standardization weights must be constant within the standard strata.
nostdrescale prevents the standardization weights from being rescaled within the over() groups. This option requires stdize() but is ignored if the over() option is not specified.
if/in/over
over (varlist [, nolabel]) specifies that estimates be computed for multiple subpopulations, which are identified by the different values of the variables in varlist.

When this option is supplied with one variable name, such as over (varname), the value labels of varname are used to identify the subpopulations. If varname does not have labeled values (or there are unlabeled values), the values themselves are used, provided that they are nonnegative integers. Noninteger values, negative values, and labels that are not valid Stata names are substituted with a default identifier.

When over() is supplied with multiple variable names, each subpopulation is assigned a unique default identifier.
nolabel requests that value labels attached to the variables identifying the subpopulations be ignored.

[^47]
## Reporting

level (\#); see [R] estimation options.
noheader prevents the table header from being displayed. This option implies nolegend. nolegend prevents the table legend identifying the subpopulations from being displayed. display_options: cformat (\%fmt) and nolstretch; see [R] estimation options.

The following option is available with mean but is not shown in the dialog box:
coeflegend; see [R] estimation options.

## Remarks and examples

> Example 1
Using the fuel data from example 3 of [R] ttest, we estimate the average mileage of the cars without the fuel treatment (mpg1) and those with the fuel treatment (mpg2).
. use http://www.stata-press.com/data/r13/fuel
. mean mpg1 mpg2
Mean estimation Number of obs $=12$

|  | Mean | Std. Err. | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: |
| mpg1 | 21 | .7881701 | 19.26525 | 22.73475 |
| mpg2 | 22.75 | .9384465 | 20.68449 | 24.81551 |

Using these results, we can test the equality of the mileage between the two groups of cars.

```
. test mpg1 = mpg2
(1) mpg1 - mpg2 = 0
    F(1, 11) = 5.04
        Prob > F = 0.0463
```


## - Example 2

In example 1, the joint observations of mpg1 and mpg2 were used to estimate a covariance between their means.

```
. matrix list e(V)
symmetric e(V)[2,2]
    mpg1 mpg2
mpg1 . }6212121
mpg2 . }4469697\mathrm{ . }8806818
```

If the data were organized this way out of convenience but the two variables represent independent samples of cars (coincidentally of the same sample size), we should reshape the data and use the over () option to ensure that the covariance between the means is zero.

```
. use http://www.stata-press.com/data/r13/fuel
. stack mpg1 mpg2, into(mpg) clear
. mean mpg, over(_stack)
Mean estimation Number of obs = 24
    1: _stack = 1
    2: _stack = 2
\begin{tabular}{rr|rrrr}
\hline & Over & Mean & Std. Err. & [95\% Conf. Interval] \\
\hline mpg & & & & & \\
& 1 & 21 & .7881701 & 19.36955 & 22.63045 \\
& 2 & 22.75 & .9384465 & 20.80868 & 24.69132 \\
\hline
\end{tabular}
```

```
. matrix list e(V)
symmetric e(V)[2,2]
    mpg: mpg:
        1 2
mpg:1 . }6212121
mpg:2 0 . 88068182
```

Now we can test the equality of the mileage between the two independent groups of cars.

```
. test [mpg]1 = [mpg]2
( 1) [mpg]1 - [mpg]2 = 0
    F( 1, 23) = 2.04
        Prob > F = 0.1667
```


## Example 3: standardized means

Suppose that we collected the blood pressure data from example 2 of $[R]$ dstdize, and we wish to obtain standardized high blood pressure rates for each city in 1990 and 1992, using, as the standard, the age, sex, and race distribution of the four cities and two years combined. Our rate is really the mean of a variable that indicates whether a sampled individual has high blood pressure. First, we generate the strata and weight variables from our standard distribution, and then use mean to compute the rates.

```
. use http://www.stata-press.com/data/r13/hbp, clear
. egen strata = group(age race sex) if inlist(year, 1990, 1992)
(675 missing values generated)
. by strata, sort: gen stdw = _N
. mean hbp, over(city year) stdize(strata) stdweight(stdw)
Mean estimation
N. of std strata = Number of obs = 44 N
            Over: city year
        _subpop_1: 1 1990
        _subpop_2: 1 1992
        _subpop_3: 2 1990
        _subpop_4: 2 1992
        _subpop_5: 3 1990
        _subpop_6: 3 1992
        _subpop_7: 5 1990
        _subpop_8: 5 1992
\begin{tabular}{r|rrrr}
\hline Over & Mean & Std. Err. & [95\% Conf. Interval] \\
\hline hbp & & & & \\
_subpop_1 & .058642 & .0296273 & .0004182 & .1168657 \\
_subpop_2 & .0117647 & .0113187 & -.0104789 & .0340083 \\
_subpop_3 & .0488722 & .0238958 & .0019121 & .0958322 \\
_subpop_4 & .014574 & .007342 & .0001455 & .0290025 \\
_subpop_5 & .1011211 & .0268566 & .0483425 & .1538998 \\
_subpop_6 & .0810577 & .0227021 & .0364435 & .1256719 \\
_subpop_7 & .0277778 & .0155121 & -.0027066 & .0582622 \\
_subpop_8 & .0548926 & 0 &. & \\
\hline
\end{tabular}
```

The standard error of the high blood pressure rate estimate is missing for city 5 in 1992 because there was only one individual with high blood pressure; that individual was the only person observed in the stratum of white males 30-35 years old.

By default, mean rescales the standard weights within the over() groups. In the following, we use the nostdrescale option to prevent this, thus reproducing the results in $[R]$ dstdize.

```
. mean hbp, over(city year) nolegend stdize(strata) stdweight(stdw)
> nostdrescale
Mean estimation
N. of std strata = Number of obs = 44 455
```

| Over | Mean | Std. Err. | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: |
| hbp |  |  |  |  |
| _subpop_1 | .0073302 | .0037034 | .0000523 | .0146082 |
| _subpop_2 | .0015432 | .0014847 | -.0013745 | .004461 |
| _subpop_3 | .0078814 | .0038536 | .0003084 | .0154544 |
| _subpop_4 | .0025077 | .0012633 | .000025 | .0049904 |
| _subpop_5 | .0155271 | .0041238 | .007423 | .0236312 |
| _subpop_6 | .0081308 | .0022772 | .0036556 | .012606 |
| _subpop_7 | .0039223 | .0021904 | -.0003822 | .0082268 |
| _subpop_8 | .0088735 | 0 | . |  |

## Video example

Descriptive statistics in Stata

## Stored results

mean stores the following in e() :
Scalars
$e(N) \quad$ number of observations
e(N_over) number of subpopulations
e(N_stdize) number of standard strata
$e$ (N_clust) number of clusters
$e\left(k \_\right.$eq) number of equations in $e(b)$
$e\left(d f \_r\right) \quad$ sample degrees of freedom
$e$ (rank) rank of e(V)
Macros
e(cmd)
mean
e(cmdline)
e(varlist)
command as typed
varlist
$e$ (stdize)
e(stdweight)
varname from stdize()
varname from stdweight()
e(wtype)
weight type
e(wexp)
e(title)
e(cluster)
weight expression
title in estimation output
name of cluster variable
e(over)
varlist from over()
e(over_labels) labels from over() variables
e(over_namelist) names from e(over_labels)
e(vce)
e(vcetype)
$e($ properties)
vcetype specified in vce()
title used to label Std. Err.
b V
e(estat_cmd)
program used to implement estat
e(marginsnotok)
predictions disallowed by margins

Matrices
e(b) vector of mean estimates
e(V) (co)variance estimates
$e\left(\_N\right) \quad$ vector of numbers of nonmissing observations
e(_N_stdsum) number of nonmissing observations within the standard strata
e(_p_stdize)
standardizing proportions
e(error)
error code corresponding to $e(b)$
Functions
e(sample) marks estimation sample

## Methods and formulas

Methods and formulas are presented under the following headings:
The mean estimator
Survey data
The survey mean estimator
The standardized mean estimator
The poststratified mean estimator
The standardized poststratified mean estimator
Subpopulation estimation

## The mean estimator

Let $y$ be the variable on which we want to calculate the mean and $y_{j}$ an individual observation on $y$, where $j=1, \ldots, n$ and $n$ is the sample size. Let $w_{j}$ be the weight, and if no weight is specified, define $w_{j}=1$ for all $j$. For aweights, the $w_{j}$ are normalized to sum to $n$. See The survey mean estimator for pweighted data.

Let $W$ be the sum of the weights

$$
W=\sum_{j=1}^{n} w_{j}
$$

The mean is defined as

$$
\bar{y}=\frac{1}{W} \sum_{j=1}^{n} w_{j} y_{j}
$$

The default variance estimator for the mean is

$$
\widehat{V}(\bar{y})=\frac{1}{W(W-1)} \sum_{j=1}^{n} w_{j}\left(y_{j}-\bar{y}\right)^{2}
$$

The standard error of the mean is the square root of the variance.
If $x, x_{j}$, and $\bar{x}$ are similarly defined for another variable (observed jointly with $y$ ), the covariance estimator between $\bar{x}$ and $\bar{y}$ is

$$
\widehat{\operatorname{Cov}}(\bar{x}, \bar{y})=\frac{1}{W(W-1)} \sum_{j=1}^{n} w_{j}\left(x_{j}-\bar{x}\right)\left(y_{j}-\bar{y}\right)
$$

## Survey data

See [SVY] variance estimation, [SVY] direct standardization, and [SVY] poststratification for discussions that provide background information for the following formulas. The following formulas are derived from the fact that the mean is a special case of the ratio estimator where the denominator variable is one, $x_{j}=1$; see [ R$]$ ratio.

## The survey mean estimator

Let $Y_{j}$ be a survey item for the $j$ th individual in the population, where $j=1, \ldots, M$ and $M$ is the size of the population. The associated population mean for the item of interest is $\bar{Y}=Y / M$ where

$$
Y=\sum_{j=1}^{M} Y_{j}
$$

Let $y_{j}$ be the survey item for the $j$ th sampled individual from the population, where $j=1, \ldots, m$ and $m$ is the number of observations in the sample.

The estimator for the mean is $\bar{y}=\widehat{Y} / \widehat{M}$, where

$$
\widehat{Y}=\sum_{j=1}^{m} w_{j} y_{j} \quad \text { and } \quad \widehat{M}=\sum_{j=1}^{m} w_{j}
$$

and $w_{j}$ is a sampling weight. The score variable for the mean estimator is

$$
z_{j}(\bar{y})=\frac{y_{j}-\bar{y}}{\widehat{M}}=\frac{\widehat{M} y_{j}-\widehat{Y}}{\widehat{M}^{2}}
$$

## The standardized mean estimator

Let $D_{g}$ denote the set of sampled observations that belong to the $g$ th standard stratum and define $I_{D_{g}}(j)$ to indicate if the $j$ th observation is a member of the $g$ th standard stratum; where $g=1, \ldots$, $L_{D}$ and $L_{D}$ is the number of standard strata. Also, let $\pi_{g}$ denote the fraction of the population that belongs to the $g$ th standard stratum, thus $\pi_{1}+\cdots+\pi_{L_{D}}=1 . \pi_{g}$ is derived from the stdweight () option.

The estimator for the standardized mean is

$$
\bar{y}^{D}=\sum_{g=1}^{L_{D}} \pi_{g} \frac{\widehat{Y}_{g}}{\widehat{M}_{g}}
$$

where

$$
\widehat{Y}_{g}=\sum_{j=1}^{m} I_{D_{g}}(j) w_{j} y_{j} \quad \text { and } \quad \widehat{M}_{g}=\sum_{j=1}^{m} I_{D_{g}}(j) w_{j}
$$

The score variable for the standardized mean is

$$
z_{j}\left(\bar{y}^{D}\right)=\sum_{g=1}^{L_{D}} \pi_{g} I_{D_{g}}(j) \frac{\widehat{M}_{g} y_{j}-\widehat{Y}_{g}}{\widehat{M}_{g}^{2}}
$$

## The poststratified mean estimator

Let $P_{k}$ denote the set of sampled observations that belong to poststratum $k$ and define $I_{P_{k}}(j)$ to indicate if the $j$ th observation is a member of poststratum $k$; where $k=1, \ldots, L_{P}$ and $L_{P}$ is the number of poststrata. Also let $M_{k}$ denote the population size for poststratum $k . P_{k}$ and $M_{k}$ are identified by specifying the poststrata() and postweight() options on svyset; see [SVY] svyset.

The estimator for the poststratified mean is

$$
\bar{y}^{P}=\frac{\widehat{Y}^{P}}{\widehat{M}^{P}}=\frac{\widehat{Y}^{P}}{M}
$$

where

$$
\widehat{Y}^{P}=\sum_{k=1}^{L_{P}} \frac{M_{k}}{\widehat{M}_{k}} \widehat{Y}_{k}=\sum_{k=1}^{L_{P}} \frac{M_{k}}{\widehat{M}_{k}} \sum_{j=1}^{m} I_{P_{k}}(j) w_{j} y_{j}
$$

and

$$
\widehat{M}^{P}=\sum_{k=1}^{L_{P}} \frac{M_{k}}{\widehat{M}_{k}} \widehat{M}_{k}=\sum_{k=1}^{L_{P}} M_{k}=M
$$

The score variable for the poststratified mean is

$$
z_{j}\left(\bar{y}^{P}\right)=\frac{z_{j}\left(\widehat{Y}^{P}\right)}{M}=\frac{1}{M} \sum_{k=1}^{L_{P}} I_{P_{k}}(j) \frac{M_{k}}{\widehat{M}_{k}}\left(y_{j}-\frac{\widehat{Y}_{k}}{\widehat{M}_{k}}\right)
$$

## The standardized poststratified mean estimator

The estimator for the standardized poststratified mean is

$$
\bar{y}^{D P}=\sum_{g=1}^{L_{D}} \pi_{g} \frac{\widehat{Y}_{g}^{P}}{\widehat{M}_{g}^{P}}
$$

where

$$
\widehat{Y}_{g}^{P}=\sum_{k=1}^{L_{p}} \frac{M_{k}}{\widehat{M}_{k}} \widehat{Y}_{g, k}=\sum_{k=1}^{L_{p}} \frac{M_{k}}{\widehat{M}_{k}} \sum_{j=1}^{m} I_{D_{g}}(j) I_{P_{k}}(j) w_{j} y_{j}
$$

and

$$
\widehat{M}_{g}^{P}=\sum_{k=1}^{L_{p}} \frac{M_{k}}{\widehat{M}_{k}} \widehat{M}_{g, k}=\sum_{k=1}^{L_{p}} \frac{M_{k}}{\widehat{M}_{k}} \sum_{j=1}^{m} I_{D_{g}}(j) I_{P_{k}}(j) w_{j}
$$

The score variable for the standardized poststratified mean is

$$
z_{j}\left(\bar{y}^{D P}\right)=\sum_{g=1}^{L_{D}} \pi_{g} \frac{\widehat{M}_{g}^{P} z_{j}\left(\widehat{Y}_{g}^{P}\right)-\widehat{Y}_{g}^{P} z_{j}\left(\widehat{M}_{g}^{P}\right)}{\left(\widehat{M}_{g}^{P}\right)^{2}}
$$

where

$$
z_{j}\left(\widehat{Y}_{g}^{P}\right)=\sum_{k=1}^{L_{P}} I_{P_{k}}(j) \frac{M_{k}}{\widehat{M}_{k}}\left\{I_{D_{g}}(j) y_{j}-\frac{\widehat{Y}_{g, k}}{\widehat{M}_{k}}\right\}
$$

and

$$
z_{j}\left(\widehat{M}_{g}^{P}\right)=\sum_{k=1}^{L_{P}} I_{P_{k}}(j) \frac{M_{k}}{\widehat{M}_{k}}\left\{I_{D_{g}}(j)-\frac{\widehat{M}_{g, k}}{\widehat{M}_{k}}\right\}
$$

## Subpopulation estimation

Let $S$ denote the set of sampled observations that belong to the subpopulation of interest, and define $I_{S}(j)$ to indicate if the $j$ th observation falls within the subpopulation.

The estimator for the subpopulation mean is $\bar{y}^{S}=\widehat{Y}^{S} / \widehat{M}^{S}$, where

$$
\widehat{Y}^{S}=\sum_{j=1}^{m} I_{S}(j) w_{j} y_{j} \quad \text { and } \quad \widehat{M}^{S}=\sum_{j=1}^{m} I_{S}(j) w_{j}
$$

Its score variable is

$$
z_{j}\left(\bar{y}^{S}\right)=I_{S}(j) \frac{y_{j}-\bar{y}^{S}}{\widehat{M}^{S}}=I_{S}(j) \frac{\widehat{M}^{S} y_{j}-\widehat{Y}^{S}}{\left(\widehat{M}^{S}\right)^{2}}
$$

The estimator for the standardized subpopulation mean is

$$
\bar{y}^{D S}=\sum_{g=1}^{L_{D}} \pi_{g} \frac{\widehat{Y}_{g}^{S}}{\widehat{M}_{g}^{S}}
$$

where

$$
\widehat{Y}_{g}^{S}=\sum_{j=1}^{m} I_{D_{g}}(j) I_{S}(j) w_{j} y_{j} \quad \text { and } \quad \widehat{M}_{g}^{S}=\sum_{j=1}^{m} I_{D_{g}}(j) I_{S}(j) w_{j}
$$

Its score variable is

$$
z_{j}\left(\bar{y}^{D S}\right)=\sum_{g=1}^{L_{D}} \pi_{g} I_{D_{g}}(j) I_{S}(j) \frac{\widehat{M}_{g}^{S} y_{j}-\widehat{Y}_{g}^{S}}{\left(\widehat{M}_{g}^{S}\right)^{2}}
$$

The estimator for the poststratified subpopulation mean is

$$
\bar{y}^{P S}=\frac{\widehat{Y}^{P S}}{\widehat{M}^{P S}}
$$

where

$$
\widehat{Y}^{P S}=\sum_{k=1}^{L_{P}} \frac{M_{k}}{\widehat{M}_{k}} \widehat{Y}_{k}^{S}=\sum_{k=1}^{L_{P}} \frac{M_{k}}{\widehat{M}_{k}} \sum_{j=1}^{m} I_{P_{k}}(j) I_{S}(j) w_{j} y_{j}
$$

and

$$
\widehat{M}^{P S}=\sum_{k=1}^{L_{P}} \frac{M_{k}}{\widehat{M}_{k}} \widehat{M}_{k}^{S}=\sum_{k=1}^{L_{P}} \frac{M_{k}}{\widehat{M}_{k}} \sum_{j=1}^{m} I_{P_{k}}(j) I_{S}(j) w_{j}
$$

Its score variable is

$$
z_{j}\left(\bar{y}^{P S}\right)=\frac{\widehat{M}^{P S} z_{j}\left(\widehat{Y}^{P S}\right)-\widehat{Y}^{P S} z_{j}\left(\widehat{M}^{P S}\right)}{\left(\widehat{M}^{P S}\right)^{2}}
$$

where

$$
z_{j}\left(\widehat{Y}^{P S}\right)=\sum_{k=1}^{L_{P}} I_{P_{k}}(j) \frac{M_{k}}{\widehat{M}_{k}}\left\{I_{S}(j) y_{j}-\frac{\widehat{Y}_{k}^{S}}{\widehat{M}_{k}}\right\}
$$

and

$$
z_{j}\left(\widehat{M}^{P S}\right)=\sum_{k=1}^{L_{P}} I_{P_{k}}(j) \frac{M_{k}}{\widehat{M}_{k}}\left\{I_{S}(j)-\frac{\widehat{M}_{k}^{S}}{\widehat{M}_{k}}\right\}
$$

The estimator for the standardized poststratified subpopulation mean is

$$
\bar{y}^{D P S}=\sum_{g=1}^{L_{D}} \pi_{g} \frac{\widehat{Y}_{g}^{P S}}{\widehat{M}_{g}^{P S}}
$$

where

$$
\widehat{Y}_{g}^{P S}=\sum_{k=1}^{L_{p}} \frac{M_{k}}{\widehat{M}_{k}} \widehat{Y}_{g, k}^{S}=\sum_{k=1}^{L_{p}} \frac{M_{k}}{\widehat{M}_{k}} \sum_{j=1}^{m} I_{D_{g}}(j) I_{P_{k}}(j) I_{S}(j) w_{j} y_{j}
$$

and

$$
\widehat{M}_{g}^{P S}=\sum_{k=1}^{L_{p}} \frac{M_{k}}{\widehat{M}_{k}} \widehat{M}_{g, k}^{S}=\sum_{k=1}^{L_{p}} \frac{M_{k}}{\widehat{M}_{k}} \sum_{j=1}^{m} I_{D_{g}}(j) I_{P_{k}}(j) I_{S}(j) w_{j}
$$

Its score variable is

$$
z_{j}\left(\bar{y}^{D P S}\right)=\sum_{g=1}^{L_{D}} \pi_{g} \frac{\widehat{M}_{g}^{P S} z_{j}\left(\widehat{Y}_{g}^{P S}\right)-\widehat{Y}_{g}^{P S} z_{j}\left(\widehat{M}_{g}^{P S}\right)}{\left(\widehat{M}_{g}^{P S}\right)^{2}}
$$

where

$$
z_{j}\left(\widehat{Y}_{g}^{P S}\right)=\sum_{k=1}^{L_{P}} I_{P_{k}}(j) \frac{M_{k}}{\widehat{M}_{k}}\left\{I_{D_{g}}(j) I_{S}(j) y_{j}-\frac{\widehat{Y}_{g, k}^{S}}{\widehat{M}_{k}}\right\}
$$

and

$$
z_{j}\left(\widehat{M}_{g}^{P S}\right)=\sum_{k=1}^{L_{P}} I_{P_{k}}(j) \frac{M_{k}}{\widehat{M}_{k}}\left\{I_{D_{g}}(j) I_{S}(j)-\frac{\widehat{M}_{g, k}^{S}}{\widehat{M}_{k}}\right\}
$$

## References

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## Also see

[R] mean postestimation - Postestimation tools for mean
[R] ameans - Arithmetic, geometric, and harmonic means
$[\mathrm{R}]$ proportion - Estimate proportions
[ R ] ratio - Estimate ratios
[R] summarize - Summary statistics
$[\mathrm{R}]$ total - Estimate totals
[MI] estimation - Estimation commands for use with mi estimate
[SVY] direct standardization - Direct standardization of means, proportions, and ratios
[SVY] poststratification - Poststratification for survey data
[SVY] subpopulation estimation - Subpopulation estimation for survey data
[SVY] svy estimation - Estimation commands for survey data
[SVY] variance estimation - Variance estimation for survey data
[U] 20 Estimation and postestimation commands

## Title

mean postestimation - Postestimation tools for mean

## Description Remarks and examples Also see

## Description

The following postestimation commands are available after mean:
Command Description
estat vce variance-covariance matrix of the estimators (VCE)
estat (svy) postestimation statistics for survey data
estimates cataloging estimation results
lincom point estimates, standard errors, testing, and inference for linear combinations of coefficients
nlcom point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients
test Wald tests of simple and composite linear hypotheses
testnl Wald tests of nonlinear hypotheses

## Remarks and examples

## > Example 1

We have a dataset with monthly rates of returns on the Dow and NASDAQ stock indices. We can use mean to compute the average quarterly rates of return for the two indices separately;
. use http://www.stata-press.com/data/r13/rates
. mean dow nasdaq
Mean estimation Number of obs = 357

|  | Mean | Std. Err. | [95\% Conf. | Interval] |
| ---: | ---: | ---: | ---: | ---: |
| dow | .2489137 | 6.524386 | -12.58227 | 13.0801 |
| nasdaq | 10.78477 | 4.160821 | 2.601887 | 18.96765 |

If you chose just one of the indices for your portfolio, you either did rather well or rather poorly, depending on which one you picked. However, as we now show with the postestimation command lincom, if you diversified your portfolio, you would have earned a respectable $5.5 \%$ rate of return without having to guess which index would be the better performer.

| Mean | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Conf. | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (1) | 5.51684 | 4.262673 | 1.29 | 0.196 | -2.866347 | 13.90003 |

## Also see

[R] mean - Estimate means
[U] 20 Estimation and postestimation commands

## Title

meta - Meta-analysis

Remarks and examples References

## Remarks and examples

Stata does not have a meta-analysis command. Stata users, however, have developed an excellent suite of commands for performing meta-analysis, including commands for performing standard and cumulative meta-analysis, commands for producing forest plots and contour-enhanced funnel plots, and commands for nonparametric analysis of publication bias.

Many articles describing these commands have been published in the Stata Technical Bulletin and the Stata Journal. These articles were updated and published in a cohesive collection: Meta-Analysis in Stata: An Updated Collection from the Stata Journal.

In this collection, editor Jonathan Sterne discusses how these articles relate to each other and how they fit in the overall literature of meta-analysis. Sterne has organized the collection into four areas: classic meta-analysis; meta-regression; graphical and analytic tools for detecting bias; and recent advances such as meta-analysis for dose-response curves, diagnostic accuracy, multivariate analysis, and studies containing missing values.

All meta-analysis commands discussed in this collection may be downloaded by visiting http://www.stata-press.com/books/mais.html.

We highly recommend that Stata users interested in meta-analysis read this book. Since the publication of the meta-analysis collection, Kontopantelis and Reeves (2010) published an article in the Stata Journal describing a new command metaan that performs fixed- or random-effects meta-analysis.

Please also see the following FAQ on the Stata website:

# What meta-analysis features are available in Stata? http://www.stata.com/support/faqs/stat/meta.html 

## References

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White, I. R. 2011. Multivariate random-effects meta-regression: Updates to mvmeta. Stata Journal 11: 240-254.

## Title

mfp - Multivariable fractional polynomial models

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Acknowledgments | References |
| Also see |  |  |  |

## Syntax

$\operatorname{mfp}[$, options $]:$ regression_cmd $\left[\right.$ yvar $_{1}\left[\right.$ yvar $\left.\left._{2}\right]\right]$ xvarlist $[$ if $][$ in $][$ weight $]$ [, regression_cmd_options]
options
Description
Model 2
sequential
cycles(\#)
dfdefault(\#)
center (cent_list)
alpha(alpha_list)
df (df_list)
powers (numlist)
use the Royston and Altman model-selection algorithm; default uses closed-test procedure
maximum number of iteration cycles; default is cycles(5)
default maximum degrees of freedom; default is dfdefault (4)
specification of centering for the independent variables
$p$-values for testing between FP models; default is alpha(0.05)
degrees of freedom for each predictor
list of FP powers to use; default is

```
powers(-2 -1(.5)1 2 3)
```

Adv. model
xorder ( $+|-| n$ )
select (select_list)
xpowers (xp_list)
zero (varlist)
catzero (varlist)
all
Reporting
level(\#)
display_options
order of entry into model-selection algorithm; default is xorder (+)
nominal $p$-values for selection on each predictor
FP powers for each predictor
treat nonpositive values of specified predictors as zero when FP transformed
add indicator variable for specified predictors
include out-of-sample observations in generated variables
set confidence level; default is level (95)
control column formats and line width
regression_cmd_options Description
Adv. model
regression_cmd_options
options appropriate to the regression command in use

All weight types supported by regression_cmd are allowed; see [U] 11.1.6 weight.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands. fp generate may be used to create new variables containing fractional polynomial powers. See $[\mathrm{R}] \mathrm{fp}$.
where
regression_cmd may be clogit, glm, intreg, logistic, logit, mlogit, nbreg, ologit, oprobit, poisson, probit, qreg, regress, rreg, stcox, stcrreg, streg, or xtgee.
$y v a r_{1}$ is not allowed for streg, stcrreg, and stcox. For these commands, you must first stset your data.
$y v a r_{1}$ and $y^{v a r_{2}}$ must both be specified when regression_cmd is intreg.
xvarlist has elements of type varlist and/or (varlist), for example, x 1 x 2 ( $\mathrm{x} 3 \mathrm{x} 4 \times 5$ )
Elements enclosed in parentheses are tested jointly for inclusion in the model and are not eligible for fractional polynomial transformation.

## Menu

Statistics $>$ Linear models and related $>$ Fractional polynomials $>$ Multivariable fractional polynomial models

## Description

mfp selects the multivariable fractional polynomial (MFP) model that best predicts the outcome variable from the right-hand-side variables in xvarlist.

For univariate fractional polynomials, $f p$ can be used to fit a wider range of models than $\mathrm{mf} p$. See $[R]$ fp for more details.

## Options

$\qquad$ Model 2
sequential chooses the sequential fractional polynomial (FP) selection algorithm (see Methods of FP model selection).
cycles(\#) sets the maximum number of iteration cycles permitted. cycles(5) is the default.
dfdefault(\#) determines the default maximum degrees of freedom (df) for a predictor. The default is dfdefault (4) (second-degree FP).
center (cent_list) defines the centering of the covariates xvar $_{1}$, xvar $_{2}, \ldots$ of xvarlist. The default is center(mean), except for binary covariates, where it is center(\#), with \# being the lower of the two distinct values of the covariate. A typical item in cent_list is varlist: \{mean|\#|no\}. Items are separated by commas. The first item is special in that varlist is optional, and if it is omitted, the default is reset to the specified value (mean, \#, or no). For example, center (no, age:mean) sets the default to no (that is, no centering) and the centering of age to mean.
alpha(alpha_list) sets the significance levels for testing between FP models of different degrees.
The rules for alpha_list are the same as those for $d f \_l i s t$ in the df () option (see below). The default nominal $p$-value (significance level, selection level) is 0.05 for all variables.

Example: alpha (0.01) specifies that all variables have an FP selection level of $1 \%$.
Example: alpha( 0.05 , weight: 0.1 ) specifies that all variables except weight have an FP selection level of $5 \%$; weight has a level of $10 \%$.
df ( $d f$ _list) sets the df for each predictor. The df (not counting the regression constant, _cons) is twice the degree of the FP, so, for example, an xvar fit as a second-degree FP (FP2) has 4 df . The first item in $d f \_l i s t$ may be either \# or varlist:\#. Subsequent items must be varlist:\#. Items are separated by commas, and varlist is specified in the usual way for variables. With the first type of item, the df for all predictors is taken to be \#. With the second type of item, all members of varlist (which must be a subset of xvarlist) have \# df.
The default number of degrees of freedom for a predictor of type varlist specified in xvarlist but not in $d f$ _list is assigned according to the number of distinct (unique) values of the predictor, as follows:

| \# of distinct values | Default df |
| :---: | :--- |
| 1 | (invalid predictor) |
| $2-3$ | 1 |
| $4-5$ | $\min (2, \operatorname{dfdefault}())$ |
| $\geq 6$ | dfdefault() |

Example: df (4)
All variables have 4 df .
Example: df (2, weight displ:4)
weight and displ have 4 df ; all other variables have 2 df .
Example: df (weight displ:4, mpg:2)
weight and displ have 4 df , mpg has 2 df ; all other variables have default df.
powers (numlist) is the set of FP powers to be used. The default set is $-2,-1,-0.5,0,0.5,1,2$, 3 (0 means log).

Adv. model
xorder $(+|-| n)$ determines the order of entry of the covariates into the model-selection algorithm. The default is xorder $(+)$, which enters them in decreasing order of significance in a multiple linear regression (most significant first). xorder ( - ) places them in reverse significance order, whereas xorder ( $n$ ) respects the original order in xvarlist.
select (select_list) sets the nominal $p$-values (significance levels) for variable selection by backward elimination. A variable is dropped if its removal causes a nonsignificant increase in deviance. The rules for select_list are the same as those for $d f$ _list in the df() option (see above). Using the default selection level of 1 for all variables forces them all into the model. Setting the nominal $p$-value to be 1 for a given variable forces it into the model, leaving others to be selected or not. The nominal $p$-value for elements of xvarlist bound by parentheses is specified by including (varlist) in select_list.

Example: select (0.05)
All variables have a nominal $p$-value of $5 \%$.
Example: select ( 0.05 , weight: 1 )
All variables except weight have a nominal $p$-value of $5 \%$; weight is forced into the model.
Example: select (a (b c):0.05)
All variables except $a, b$, and $c$ are forced into the model. $b$ and $c$ are tested jointly with 2 df at the $5 \%$ level, and a is tested singly at the $5 \%$ level.
xpowers ( $x p \_l i s t$ ) sets the permitted FP powers for covariates individually. The rules for $x p \_l i s t$ are the same as for $d f \_l i s t$ in the df() option. The default selection is the same as that for the powers() option.
Example: xpowers (-1 01 )
All variables have powers $-1,0,1$.
Example: xpowers (x5:-1 0 1)
All variables except x 5 have default powers; x 5 has powers $-1,0,1$.
zero(varlist) treats negative and zero values of members of varlist as zero when FP transformations are applied. By default, such variables are subjected to a preliminary linear transformation to avoid negative and zero values, as described in the scale option of [R] fp. varlist must be part of xvarlist.
catzero(varlist) is a variation on zero(); see Zeros and zero categories below. varlist must be part of xvarlist.
regression_cmd_options may be any of the options appropriate to regression_cmd.
all includes out-of-sample observations when generating the FP variables. By default, the generated FP variables contain missing values outside the estimation sample.

Reporting
level(\#) specifies the confidence level, as a percentage, for confidence intervals. The default is level(95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals. display_options: cformat (\%fmt), pformat (\%fmt), sformat (\%fmt), and nolstretch; see [R] estimation options.

## Remarks and examples

Remarks are presented under the following headings:
Iteration report
Estimation algorithm
Methods of FP model selection
Zeros and zero categories
For elements in xvarlist not enclosed in parentheses, mfp leaves variables in the data named Ixvar__1, Ixvar__2, ..., where xvar represents the first four letters of the name of $x^{2} a r_{1}$, and so on, for $x v a r_{2}$, xvar $_{3}$, etc. The new variables contain the best-fitting FP powers of $x^{2 v a r} r_{1}, x v a r_{2}, \ldots$

## Iteration report

By default, for each continuous predictor, $x$, mfp compares null, linear, and FP1 models for $x$ with an FP2 model. The deviance for each of these nested submodels is given in the column labeled "Deviance". The line labeled "Final" gives the deviance for the selected model and its powers. All the other predictors currently selected are included, with their transformations (if any). For models specified as having 1 df , the only choice is whether the variable enters the model.

## Estimation algorithm

The estimation algorithm in mfp processes the xvars in turn. Initially, mfp silently arranges xvarlist in order of increasing $p$-value (that is, of decreasing statistical significance) for omitting each predictor from the model comprising xvarlist, with each term linear. The aim is to model relatively important variables before unimportant ones. This approach may help to reduce potential model-fitting difficulties caused by collinearity or, more generally, "concurvity" among the predictors. See the xorder() option above for details on how to change the ordering.

At the initial cycle, the best-fitting FP function for xvar $_{1}$ (the first of $x$ varlist) is determined, with all the other variables assumed to be linear. Either the default or the alternative procedure is used (see Methods of FP model selection below). The functional form (but not the estimated regression coefficients) for $x^{2 v a r} r_{1}$ is kept, and the process is repeated for $x v a r_{2}, x^{2} a r_{3}$, etc. The first iteration concludes when all the variables have been processed in this way. The next cycle is similar, except that the functional forms from the initial cycle are retained for all variables except the one currently being processed.

A variable whose functional form is prespecified to be linear (that is, to have 1 df ) is tested for exclusion within the above procedure when its nominal $p$-value (selection level) according to select () is less than 1 ; otherwise, it is included.

Updating of FP functions and candidate variables continues until the functions and variables included in the overall model do not change (convergence). Convergence is usually achieved within 1-4 cycles.

## Methods of FP model selection

mfp includes two algorithms for FP model selection, both of which combine backward elimination with the selection of an FP function. For each continuous variable in turn, they start from a mostcomplex permitted FP model and attempt to simplify the model by reducing the degree. The default algorithm resembles a closed-test procedure, a sequence of tests maintaining the overall type I error rate at a prespecified nominal level, such as $5 \%$. All significance tests are approximate; therefore, the algorithm is not precisely a closed-test procedure (Royston and Sauerbrei 2008, chap. 6).

The closed-test algorithm for choosing an FP model with maximum permitted degree $m=2$ (that is, an FP2 model with 4 df ) for one continuous predictor, $x$, is as follows:

1. Inclusion: Test FP2 against the null model for $x$ on 4 df at the significance level determined by select(). If $x$ is significant, continue; otherwise, drop $x$ from the model.
2. Nonlinearity: Test FP2 against a straight line in $x$ on 3 df at the significance level determined by alpha(). If significant, continue; otherwise, stop, with the chosen model for $x$ being a straight line.
3. Simplification: Test FP2 against FP1 on 2 df at the significance level determined by alpha(). If significant, the final model is FP2; otherwise, it is FP1.

The first step is omitted if $x$ is to be retained in the model, that is, if its nominal $p$-value, according to the select () option, is 1 .

An alternative algorithm is available with the sequential option, as originally suggested by Royston and Altman (1994):

1. Test FP2 against FP1 on 2 df at the alpha() significance level. If significant, the final model is FP2; otherwise, continue.
2. Test FP1 against a straight line on 1 df at the alpha() level. If significant, the final model is FP1; otherwise, continue.
3. Test a straight line against omitting $x$ on 1 df at the select () level. If significant, the final model is a straight line; otherwise, drop $x$.
The final step is omitted if $x$ is to be retained in the model, that is, if its nominal $p$-value, according to the select () option, is 1 .

If $x$ is uninfluential, the overall type I error rate of this procedure is about double that of the closed-test procedure, for which the rate is close to the nominal value. This inflated type I error rate confers increased apparent power to detect nonlinear relationships.

## Zeros and zero categories

The zero() option permits fitting an FP model to the positive values of a covariate, taking nonpositive values as zero. An application is the assessment of the effect of cigarette smoking as a risk factor in an epidemiological study. Nonsmokers may be qualitatively different from smokers, so the effect of smoking (regarded as a continuous variable) may not be continuous between one and zero cigarettes. To allow for this, the risk may be modeled as constant for the nonsmokers and as an FP function of the number of cigarettes for the smokers:

```
. generate byte nonsmokr = cond(n_cigs==0, 1, 0) if n_cigs != .
. mfp, zero(n_cigs) df(4, nonsmokr:1): logit case n_cigs nonsmokr age
```

Omission of zero(n_cigs) would cause n_cigs to be transformed before analysis by the addition of a suitable constant, probably 1.

A closely related approach involves the catzero() option. The command

```
. mfp, catzero(n_cigs): logit case n_cigs age
```

would achieve a similar result to the previous command but with important differences. First, mfp would create the equivalent of the binary variable nonsmokr automatically and include it in the model. Second, the two smoking variables would be treated as one predictor in the model. With the select() option active, the two variables would be tested jointly for inclusion in the model. A modified version is described in Royston and Sauerbrei (2008, sec. 4.15).
$>$ Example 1
We illustrate two of the analyses performed by Sauerbrei and Royston (1999). We use brcancer.dta, which contains prognostic factors data from the German Breast Cancer Study Group of patients with node-positive breast cancer. The response variable is recurrence-free survival time (rectime), and the censoring variable is censrec. There are 686 patients with 299 events. We use Cox regression to predict the log hazard of recurrence from prognostic factors of which five are continuous ( $\mathrm{x} 1, \mathrm{x} 3, \mathrm{x} 5, \mathrm{x} 6, \mathrm{x} 7$ ) and three are binary ( $\mathrm{x} 2, \mathrm{x} 4 \mathrm{a}, \mathrm{x} 4 \mathrm{~b}$ ). Hormonal therapy (hormon) is known to reduce recurrence rates and is forced into the model. We use mfp to build a model from the initial set of eight predictors by using the backfitting model-selection algorithm. We set the nominal $p$-value for variable and FP selection to 0.05 for all variables except hormon, which it is set to 1 :

```
. use http://www.stata-press.com/data/r13/brcancer
(German breast cancer data)
. stset rectime, fail(censrec)
    (output omitted)
```

. mfp, alpha(.05) select(.05, hormon:1) : stcox x 1 x 2 x 3 x 4 a x4b x5 x6 x7 hormon, > nohr

Deviance for model with all terms untransformed $=3471.637$, 686 observations

| Variable | Model (vs.) | Deviance | Dev diff. | P | Powers | (vs.) |  |
| :--- | :--- | :--- | :--- | ---: | :--- | :--- | :--- | :--- |
| x5 | null | FP2 | 3503.610 | 61.366 | $0.000 *$ | . | .53 |
|  | lin. |  | 3471.637 | 29.393 | $0.000+$ | 1 |  |
|  | FP1 |  | 3449.203 | 6.959 | $0.031+$ | 0 |  |
| x6 | Final |  | 3442.244 |  |  | .53 |  |
|  | null | FP2 | 3464.113 | 29.917 | $0.000 *$ | . | -2.5 |
|  | lin. |  | 3442.244 | 8.048 | $0.045+$ | 1 |  |
|  | FP1 |  | 3435.550 | 1.354 | 0.508 | .5 |  |
|  | Final | 3435.550 |  |  | .5 |  |  |

[hormon included with 1 df in model]

| x4a | null <br> Final | lin. | $\begin{aligned} & 3440.749 \\ & 3435.550 \end{aligned}$ | 5.199 | 0.023* | $1$ | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| x3 | null <br> Final | FP2 | $\begin{aligned} & 3436.832 \\ & 3436.832 \end{aligned}$ | 3.560 | 0.469 | $\cdot$ | $-23$ |
| x 2 | null <br> Final | lin. | $\begin{aligned} & 3437.589 \\ & 3437.589 \end{aligned}$ | 0.756 | 0.384 | $\cdot$ | 1 |
| x4b | null <br> Final | lin. | $\begin{aligned} & 3437.848 \\ & 3437.848 \end{aligned}$ | 0.259 | 0.611 | . | 1 |
| x1 | null <br> lin. <br> FP1 <br> Final | FP2 | $\begin{aligned} & 3437.893 \\ & 3437.848 \\ & 3433.628 \\ & 3419.808 \end{aligned}$ | 18.085 18.040 <br> 13.820 | $\begin{aligned} & 0.001 * \\ & 0.000+ \\ & 0.001+ \end{aligned}$ | $\begin{aligned} & 1 \\ & -2 \\ & -2 \end{aligned}$ | -2-. 5 |
| x7 | null <br> Final | FP2 | $\begin{aligned} & 3420.805 \\ & 3420.805 \end{aligned}$ | 3.715 | 0.446 | $\cdot$ | -. 53 |


| End of Cycle 1: deviance $=$ |  | 3420.805 |  |  |  |  |  |
| :--- | :--- | ---: | ---: | ---: | :--- | :--- | :--- |
| x5 | null | FP2 | 3494.867 | 74.143 | $0.000 *$ | . | -2 |
|  | lin. |  | 3451.795 | 31.071 | $0.000+$ | 1 |  |
|  | FP1 | 3428.023 | 7.299 | $0.026+$ | 0 |  |  |
|  | Final |  | 3420.724 |  |  | -2 | -1 |
| x6 | null | FP2 | 3452.093 | 32.704 | $0.000 *$ | . | 0 |
|  | lin. |  | 3427.703 | 8.313 | $0.040+$ | 1 | 0 |
|  | FP1 | 3420.724 | 1.334 | 0.513 | .5 |  |  |
|  | Final | 3420.724 |  |  | .5 |  |  |

[hormon included with 1 df in model]

| x4a | null | lin. | 3425.310 | 4.586 | $0.032 *$ | . | 1 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| x3 | Final |  | 3420.724 |  |  | 1 |  |  |
|  | null | FP2 | 3420.724 | 5.305 | 0.257 | . | -.5 | 0 |
| x2 | Final |  | 3420.724 |  |  | . |  |  |
|  | null | lin. | 3420.724 | 0.214 | 0.644 | . | 1 |  |
| x4b | Final |  | 3420.724 |  |  | . |  |  |
|  | null | lin. | 3420.724 | 0.145 | 0.703 | . | 1 |  |
| x1 | Final |  | 3420.724 |  |  | . |  |  |
|  | null | FP2 | 3440.057 | 19.333 | $0.001 *$ | . | $-2-5$ |  |
|  | lin. |  | 3440.038 | 19.314 | $0.000+$ | 1 |  |  |
|  | FP1 |  | 3436.949 | 16.225 | $0.000+$ | -2 |  |  |
| x7 | Final |  | 3420.724 |  |  | $-2-.5$ |  |  |
|  | null | FP2 | 3420.724 | 2.152 | 0.708 | . | -13 |  |

Fractional polynomial fitting algorithm converged after 2 cycles.

Transformations of covariates:
-> gen double Ix1_-1 = $X^{\wedge}-2-.0355294635$ if e(sample)
-> gen double $\operatorname{Ix} 1_{--2}=X^{\wedge}-.5-.4341573547$ if e(sample)
(where: $\mathrm{X}=\mathrm{x} 1 / 10$ )
-> gen double Ix5__1 = $X^{\wedge}-2-3.983723313$ if e(sample)
-> gen double Ix5__2 = X^-1-1.99592668 if e(sample)
(where: $X=x 5 / 10$ )
-> gen double Ix6_-1 = $X^{\wedge} .5-.3331600619$ if e(sample)
(where: $X=(x 6+1) / 1000)$
Final multivariable fractional polynomial model for _t

| Variable | -_Initial__ |  |  | _-Final |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | df | Select | Alpha | Status | df | Powers |
| x 1 | 4 | 0.0500 | 0.0500 | in | 4 | -2-. 5 |
| x2 | 1 | 0.0500 | 0.0500 | out | 0 |  |
| x3 | 4 | 0.0500 | 0.0500 | out | 0 |  |
| x 4 a | 1 | 0.0500 | 0.0500 | in | 1 | 1 |
| x4b | 1 | 0.0500 | 0.0500 | out | 0 |  |
| x5 | 4 | 0.0500 | 0.0500 | in | 4 | -2 -1 |
| x 6 | 4 | 0.0500 | 0.0500 | in | 2 | . 5 |
| x7 | 4 | 0.0500 | 0.0500 | out | 0 |  |
| hormon | 1 | 1.0000 | 0.0500 | in | 1 | 1 |

Cox regression -- Breslow method for ties

| Entry time _t0 | Number of obs | $=$ | 686 |
| :--- | :--- | :--- | :--- |
|  | LR chi2 $(7)$ | $=$ | 155.62 |
|  | Prob > chi2 | $=$ | 0.0000 |
| Log likelihood $=-1710.3619$ |  | Pseudo R2 | $=$ | 0.0435


| _t | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Ix1__1 | 44.73377 | 8.256682 | 5.42 | 0.000 | 28.55097 | 60.91657 |
| Ix1__2 | -17.92302 | 3.909611 | -4.58 | 0.000 | -25.58571 | -10.26032 |
| x4a | .5006982 | .2496324 | 2.01 | 0.045 | .0114276 | .9899687 |
| Ix5__1 | .0387904 | .0076972 | 5.04 | 0.000 | .0237041 | .0538767 |
| Ix5__2 | -.5490645 | .0864255 | -6.35 | 0.000 | -.7184554 | -.3796736 |
| Ix6__1 | -1.806966 | .3506314 | -5.15 | 0.000 | -2.494191 | -1.119741 |
| hormon | -.4024169 | .1280843 | -3.14 | 0.002 | -.6534575 | -.1513763 |

Deviance: 3420.724.
Some explanation of the output from the model-selection algorithm is desirable. Consider the first few lines of output in the iteration log:

| 1. Deviance for model with all terms untransformed $=3471.637,686$ | observations |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Variable | Model (vs.) | Deviance | Dev diff. | P | Powers | (vs.) |  |  |
| 2. x5 | null | FP2 | 3503.610 | 61.366 | $0.000 *$ |  | .5 | .5 |
| 3. | lin. |  | 3471.637 | 29.393 | $0.000+$ | 1 |  |  |
| 4. | FP1 |  | 3449.203 | 6.959 | $0.031+$ | 0 |  |  |
| 5. | Final | 3442.244 |  |  | .5 | 3 |  |  |

Line 1 gives the deviance ( $-2 \times \log$ partial likelihood) for the Cox model with all terms linear, the place where the algorithm starts. The model is modified variable by variable in subsequent steps. The most significant linear term turns out to be x 5 , which is therefore processed first. Line 2 compares the best-fitting FP2 for $x 5$ with a model omitting $x 5$. The FP has powers $(0.5,3)$, and the test for inclusion of $x 5$ is highly significant. The reported deviance of $3,503.610$ is of the null model, not for the FP2 model. The deviance for the FP2 model may be calculated by subtracting the deviance
difference (Dev diff.) from the reported deviance, giving 3,503.610-61.366 $=3,442.244$. Line 3 shows that the FP2 model is also a significantly better fit than a straight line (lin.) and line 4 that FP2 is also somewhat better than FP1 $(p=0.031)$. Thus at this stage in the model-selection procedure, the final model for x 5 (line 5) is FP2 with powers $(0.5,3)$. The overall model with an FP2 for x 5 and all other terms linear has a deviance of $3,442.244$.

After all the variables have been processed (cycle 1) and reprocessed (cycle 2) in this way, convergence is achieved because the functional forms (FP powers and variables included) after cycle 2 are the same as they were after cycle 1 . The model finally chosen is Model II as given in tables 3 and 4 of Sauerbrei and Royston (1999). Because of scaling of variables, the regression coefficients reported there are different, but the model and its deviance are identical. The model includes x 1 with powers $(-2,-0.5), \mathrm{x} 4 \mathrm{a}, \mathrm{x} 5$ with powers $(-2,-1)$, and x 6 with power 0.5 . There is strong evidence of nonlinearity for x 1 and for x 5 , the deviance differences for comparison with a straight-line model (FP2 vs lin.) being, respectively, 19.3 and 31.1 at convergence (cycle 2). Predictors x 2 , x 3 , x 4 b , and $x 7$ are dropped, as may be seen from their status out in the table Final multivariable fractional polynomial model for _t (the assumed depvar when using stcox).

All predictors except x 4 a and hormon, which are binary, have been centered on the mean of the original variable. For example, the mean of x 1 (age) is 53.05 years. The first FP-transformed variable for x 1 is $\mathrm{x} 1^{\wedge}-2$ and is created by the expression gen double Ix1__1 = $\mathrm{X}^{\wedge}-2-.0355$ if e(sample). The value 0.0355 is obtained from $(53.05 / 10)^{-2}$. The division by 10 is applied automatically to improve the scaling of the regression coefficient for Ix1__1.

According to Sauerbrei and Royston (1999), medical knowledge dictates that the estimated risk function for x5 (number of positive nodes), which was based on the above FP with powers $(-2,-1)$, should be monotonic, but it was not. They improved Model II by estimating a preliminary exponential transformation, $\mathrm{x} 5 \mathrm{e}=\exp (-0.12 \cdot \mathrm{x} 5)$, for x 5 and fitting a degree 1 FP for x 5 e , thus obtaining a monotonic risk function. The value of -0.12 was estimated univariately using nonlinear Cox regression with the ado-file boxtid (Royston and Ambler 1999b, 1999d). To ensure a negative exponent, Sauerbrei and Royston (1999) restricted the powers for x5e to be positive. Their Model III may be fit by using the following command:

```
. mfp, alpha(.05) select(.05, hormon:1) df(x5e:2) xpowers(x5e:0.5 1 2 3):
> stcox x1 x2 x3 x4a x4b x5e x6 x7 hormon
```

Other than the customization for x 5 e , the command is the same as it was before. The resulting model is as reported in table 4 of Sauerbrei and Royston (1999):
. use http://www.stata-press.com/data/r13/brcancer, clear
(German breast cancer data)
. stset rectime, fail(censrec)
(output omitted)
. mfp, alpha(.05) select(.05, hormon:1) df(x5e:2) xpowers(x5e:0.5 123 ):
> stcox x1 x2 x3 x4a x4b x5e x6 x7 hormon, nohr
(output omitted)
Final multivariable fractional polynomial model for _t

| Variable | _-Initial__ |  |  | Status | $\underset{\text { Final }}{ }$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | df | Select | Alpha |  |  |  |
| x 1 | 4 | 0.0500 | 0.0500 | in | 4 | -2-. 5 |
| x2 | 1 | 0.0500 | 0.0500 | out | 0 |  |
| x3 | 4 | 0.0500 | 0.0500 | out | 0 |  |
| x4a | 1 | 0.0500 | 0.0500 | in | 1 | 1 |
| x4b | 1 | 0.0500 | 0.0500 | out | 0 |  |
| x5e | 2 | 0.0500 | 0.0500 | in | 1 | 1 |
| x 6 | 4 | 0.0500 | 0.0500 | in | 2 | . 5 |
| x7 | 4 | 0.0500 | 0.0500 | out | 0 |  |
| hormon | 1 | 1.0000 | 0.0500 | in | 1 | 1 |

Cox regression -- Breslow method for ties
Entry time _t0

| Number of obs | $=$ | 686 |
| :--- | :--- | ---: |
| LR chi2 $(6)$ | $=$ | 153.11 |
| Prob > chi2 | $=$ | 0.0000 |
| Pseudo R2 | $=$ | 0.0428 |


| _t | Coef. | Std. Err. | $z$ | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| Ix1__1 | 43.55382 | 8.253433 | 5.28 | 0.000 | 27.37738 | 59.73025 |
| Ix1__2 | -17.48136 | 3.911882 | -4.47 | 0.000 | -25.14851 | -9.814212 |
| x4a | .5174351 | .2493739 | 2.07 | 0.038 | .0286713 | 1.006199 |
| Ix5e__1 | -1.981213 | .2268903 | -8.73 | 0.000 | -2.425909 | -1.536516 |
| Ix6__1 | -1.84008 | .3508432 | -5.24 | 0.000 | -2.52772 | -1.15244 |
| hormon | -.3944998 | .128097 | -3.08 | 0.002 | -.6455654 | -.1434342 |

Deviance: 3423.237.

## Stored results

In addition to what regression_cmd stores, mfp stores the following in e() :
Scalars
$\mathrm{e}\left(\mathrm{f} p \_\mathrm{nx}\right) \quad$ number of predictors in xvarlist
e(fp_dev) deviance of final model fit
e(Fp_id\#) initial degrees of freedom for the \#th element of xvarlist
e(Fp_fd\#) final degrees of freedom for the \#th element of xvarlist
e(Fp_al\#) FP selection level for the \#th element of xvarlist
e(Fp_se\#) backward elimination selection level for the \#th element of xvarlist
Macros
e(fp_cmd) fracpoly
$e(f p-c m d 2)$ mfp
e(cmdline) command as typed
e(fracpoly) command used to fit the selected model using fracpoly
$e\left(f p \_f v l\right)$ variables in final model
e(fp_depv) yvar ( $^{\left(\mathrm{yvar}_{2}\right)}$
e(fp_opts) estimation command options
e(fp_x1) first variable in xvarlist
$e\left(f p \_x 2\right)$ second variable in xvarlist
e $\left(f p \_x N\right) \quad$ last variable in xvarlist, $N=e\left(f p \_n x\right)$
e(fp_k1) power for first variable in xvarlist (*)
$e\left(f p \_k 2\right) \quad$ power for second variable in xvarlist (*)
$\mathrm{e}\left(\mathrm{f} p \_\mathrm{k} N\right) \quad$ power for last var. in $\operatorname{xvarlist}(*), N=\mathrm{e}\left(\mathrm{fp} \_\mathrm{nx}\right)$
Note: (*) contains '.' if the variable is not selected in the final model.

## Acknowledgments

mfp is an update of mfracpol by Royston and Ambler (1998).

## References

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Sauerbrei, W., and P. Royston. 1999. Building multivariable prognostic and diagnostic models: Transformation of the predictors by using fractional polynomials. Journal of the Royal Statistical Society, Series A 162: 71-94.
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## Also see

[R] mfp postestimation - Postestimation tools for mfp
$[\mathrm{R}] \mathbf{f p}$ - Fractional polynomial regression
[U] 20 Estimation and postestimation commands

## Title

mfp postestimation - Postestimation tools for mfp

| Description | Syntax for fracplot and fracpred | Menu for fracplot and fracpred |
| :--- | :--- | :--- |
| Options for fracplot | Options for fracpred | Remarks and examples |

## Description

The following postestimation commands are of special interest after mfp:

| Command | Description |
| :--- | :--- |
| fracplot | plot data and fit from most recently fit fractional polynomial model <br> fracpred |

The following standard postestimation commands are also available if available after regression_cmd:
Command Description

| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| :--- | :--- |
| estat summarize | summary statistics for the estimation sample |
| estat vce | variance-covariance matrix of the estimators (VCE) |
| estimates | cataloging estimation results |
| lincom | point estimates, standard errors, testing, and inference for linear combinations <br> of coefficients |
| linktest | link test for model specification |
| lrtest | likelihood-ratio test <br> nlcom |
| point estimates, standard errors, testing, and inference for nonlinear combinations |  |
| of coefficients |  |

## Special-interest postestimation commands

fracplot plots the data and fit, with $95 \%$ confidence limits, from the most recently fit fractional polynomial (FP) model. The data and fit are plotted against varname, which may be $x v a r_{1}$ or another of the covariates (xvar $2, \ldots$, or a variable from xvarlist). If varname is not specified, xvar ${ }_{1}$ is assumed.
fracpred creates newvar containing the fitted index or deviance residuals for the whole model, or the fitted index or its standard error for varname, which may be $x_{v a r_{1}}$ or another covariate.

## Syntax for fracplot and fracpred

Plot data and fit from most recently fit fractional polynomial model

```
fracplot [varname] [if] [in] [, fracplot_options]
```

Create variable containing the prediction, deviance residuals, or SEs of fitted values fracpred newvar [, fracpred_options]
fracplot_options
Description
Plot
marker_options change look of markers (color, size, etc.)
marker_label_options add marker labels; change look or position
Fitted line
lineopts(cline_options) affect rendition of the fitted line
Cl plot
ciopts (area_options) affect rendition of the confidence bands
Add plots
addplot (plot) add other plots to the generated graph
Y axis, X axis, Titles, Legend, Overall
twoway_options any options other than by() documented in [G-3] twoway_options
fracpred_options
Description
for (varname) compute prediction for varname
dresid
compute deviance residuals
stdp
compute standard errors of the fitted values varname
fracplot is not allowed after mfp with clogit, mlogit, or stcrreg. fracpred, dresid is not allowed after mfp with clogit, mlogit, or stcrreg.

## Menu for fracplot and fracpred

## fracplot

Statistics $>$ Linear models and related $>$ Fractional polynomials $>$ Multivariable fractional polynomial plot

## fracpred

Statistics $>$ Linear models and related $>$ Fractional polynomials $>$ Multivariable fractional polynomial prediction

## Options for fracplot

$\qquad$ Plot
marker_options affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] marker_options.
marker_label_options specify if and how the markers are to be labeled; see [G-3] marker_label_options.
$\qquad$ Fitted line
lineopts(cline_options) affect the rendition of the fitted line; see [G-3] cline_options.

## $\curvearrowright$ Cl plot

ciopts(area_options) affect the rendition of the confidence bands; see [G-3] area_options.

Add plots
addplot (plot) provides a way to add other plots to the generated graph. See [G-3] addplot_option.
$\qquad$ Y axis, X axis, Titles, Legend, Overall
twoway_options are any of the options documented in [G-3] twoway_options, excluding by (). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

## Options for fracpred

for(varname) specifies (partial) prediction for variable varname. The fitted values are adjusted to the value specified by the center () option in mfp.
dresid specifies that deviance residuals be calculated.
stdp specifies calculation of the standard errors of the fitted values varname, adjusted for all the other predictors at the values specified by center ().

## Remarks and examples

fracplot actually produces a component-plus-residual plot. For normal-error models with constant weights and one covariate, this amounts to a plot of the observations with the fitted line inscribed. For other normal-error models, weighted residuals are calculated and added to the fitted values.

For models with additional covariates, the line is the partial linear predictor for the variable in question (xvar ${ }_{1}$ or a covariate) and includes the intercept $\beta_{0}$.

For generalized linear and Cox models, the fitted values are plotted on the scale of the "index" (linear predictor). Deviance residuals are added to the (partial) linear predictor to give component-plus-residual values. These values are plotted as small circles.

## > Example 1

In example 1 of $[\mathrm{R}] \mathbf{m f p}$, we used Cox regression to predict the $\log$ hazard of breast cancer recurrence from prognostic factors of which five are continuous ( $x 1, x 3, x 5, x 6, x 7$ ) and three are binary ( $\mathrm{x} 2, \mathrm{x} 4 \mathrm{a}, \mathrm{x} 4 \mathrm{~b}$ ). We also controlled for hormonal therapy (hormon). We used mfp to build a model from the initial set of eight predictors by using the backfitting model-selection algorithm. The nominal $p$-value for variable and FP selection was set to 0.05 for all variables except hormon, which is set to 1 .

```
. use http://www.stata-press.com/data/r13/brcancer
(German breast cancer data)
. stset rectime, fail(censrec)
    (output omitted)
. mfp, alpha(.05) select(.05, hormon:1): stcox x1 x2 x3 x4a x4b x5 x6 x7 hormon,
> nohr
    (output omitted)
```

We can use fracplot to produce component-plus-residual plots of the continuous variables. We produce the component-plus-residual plot for x 1 with fracplot by specifying x 1 after the command name.

```
. fracplot x1
```



We use fracpred with the stdp option to predict the standard error of the fractional polynomial prediction for x 1 . The standard error prediction will be stored in variable sepx1. We specify that prediction is made for x 1 with the for () option. After prediction, we use summarize to show how the standard error estimate varies over different values of $x 1$.

```
. fracpred sepx1, stdp for(x1)
```

. summarize sepx1

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| sepx1 | 686 | .0542654 | .0471993 | .0003304 | .6862065 |

## Methods and formulas

The general definition of an FP, accommodating possible repeated powers, may be written for functions $H_{1}(x), \ldots, H_{m}(x)$ as

$$
\beta_{0}+\sum_{j=1}^{m} \beta_{j} H_{j}(x)
$$

where $H_{1}(x)=x^{\left(p_{1}\right)}$ and for $j=2, \ldots, m$,

$$
H_{j}(x)= \begin{cases}x^{\left(p_{j}\right)} & \text { if } p_{j} \neq p_{j-1} \\ H_{j-1}(x) \log x & \text { if } p_{j}=p_{j-1}\end{cases}
$$

For example, an FP of degree 3 with powers $(1,3,3)$ has $H_{1}(x)=x, H_{2}(x)=x^{3}$, and $H_{3}(x)=$ $x^{3} \log x$ and equals $\beta_{0}+\beta_{1} x+\beta_{2} x^{3}+\beta_{3} x^{3} \log x$.

The component-plus-residual values graphed by fracplot are calculated as follows: Let the data consist of triplets $\left(y_{i}, x_{i}, \mathbf{z}_{i}\right), i=1, \ldots, n$, where $\mathbf{z}_{i}$ is the vector of covariates for the $i$ th observation, after applying possible fractional polynomial transformation and adjustment as described earlier. Let $\widehat{\eta}_{i}=\widehat{\beta}_{0}+\left\{\mathbf{H}\left(x_{i}\right)-\mathbf{H}\left(x_{0}\right)\right\}^{\prime} \widehat{\boldsymbol{\beta}}+\mathbf{z}_{i}^{\prime} \widehat{\boldsymbol{\gamma}}$ be the linear predictor from the FP model, as given by the fracpred command or, equivalently, by the predict command with the xb option, following mfp. Here $\mathbf{H}\left(x_{i}\right)=\left\{H_{1}\left(x_{i}\right), \ldots, H_{m}\left(x_{i}\right)\right\}^{\prime}$ is the vector of FP functions described above, $\mathbf{H}\left(x_{0}\right)=\left\{H_{1}\left(x_{0}\right), \ldots, H_{m}\left(x_{0}\right)\right\}^{\prime}$ is the vector of adjustments to $x_{0}$ (often, $x_{0}$ is chosen to be the mean of the $\left.x_{i}\right), \widehat{\boldsymbol{\beta}}$ is the estimated parameter vector, and $\widehat{\gamma}$ is the estimated parameter vector for the covariates. The values $\widehat{\eta}_{i}^{*}=\widehat{\beta}_{0}+\left\{\mathbf{H}\left(x_{i}\right)-\mathbf{H}\left(x_{0}\right)\right\}^{\prime} \widehat{\boldsymbol{\beta}}$ represent the behavior of the FP model for $x$ at fixed values $\mathbf{z}=\mathbf{0}$ of the (adjusted) covariates. The $i$ th component-plus-residual is defined as $\widehat{\eta}_{i}^{*}+d_{i}$, where $d_{i}$ is the deviance residual for the $i$ th observation. For normal-errors models, $d_{i}=\sqrt{w_{i}}\left(y_{i}-\widehat{\eta}_{i}\right)$, where $w_{i}$ is the case weight (or 1 , if weight is not specified). For logistic, Cox, and generalized linear regression models, see $[R]$ logistic, $[R]$ probit, $[S T]$ stcox, and $[R]$ glm for the formula for $d_{i}$. The formula for poisson models is the same as that for glm with family (poisson). For stcox, $d_{i}$ is the partial martingale residual (see [ST] stcox postestimation).
fracplot plots the values of $d_{i}$ and the curve represented by $\widehat{\eta}_{i}^{*}$ against $x_{i}$. The confidence interval for $\widehat{\eta}_{i}^{*}$ is obtained from the variance-covariance matrix of the entire model and takes into account the uncertainty in estimating $\beta_{0}, \boldsymbol{\beta}$, and $\gamma$ (but not in estimating the FP powers for $x$ ).
fracpred with the for (varname) option calculates the predicted index at $x_{i}=x_{0}$ and $\mathbf{z}_{i}=\mathbf{0}$; that is, $\widehat{\eta}_{i}=\widehat{\beta}_{0}+\left\{\mathbf{H}\left(x_{i}\right)-\mathbf{H}\left(x_{0}\right)\right\}^{\prime} \widehat{\boldsymbol{\beta}}$. The standard error is calculated from the variance-covariance matrix of $\left(\widehat{\beta}_{0}, \widehat{\boldsymbol{\beta}}\right)$, again ignoring estimation of the powers.

## Also see

[R] mfp - Multivariable fractional polynomial models
[U] 20 Estimation and postestimation commands

## Title

misstable - Tabulate missing values

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Also see |  |

## Syntax

Report counts of missing values

```
misstable summarize [varlist] [if] [in] [, summarize_options]
```

Report pattern of missing values

```
misstable patterns [varlist] [if] [in] [, patterns_options]
```

Present a tree view of the pattern of missing values

```
misstable tree [varlist] [if] [in] [, tree_options]
```

List the nesting rules that describe the missing-value pattern misstable nested [varlist] [if] [in] [, nested_options]
summarize_options Description
all show all variables
showzeros show zeros in table
generate (stub [, exok]) generate missing-value indicators
patterns_options Description

| asis | use variables in order given <br> report frequencies instead of percentages <br> freat $. a, . b, \ldots, . z$ as nonmissing |
| :--- | :--- |
| exok | replace data in memory with dataset of patterns <br> okay to replace even if original unsaved |
| replace | list by patterns rather than by frequency |
| clear | bypatterns |

tree_options Description

| asis | use variables in order given |
| :--- | :--- |
| frequency | report frequencies instead of percentages |

exok treat .a, .b,..., .z as nonmissing
nested_options
Description
exok treat .a,.b,..., .z as nonmissing

In addition, programmer's option nopreserve is allowed with all syntaxes; see [P] nopreserve option.

## Menu

Statistics $>$ Summaries, tables, and tests $>$ Other tables $>$ Tabulate missing values

## Description

misstable makes tables that help you understand the pattern of missing values in your data.

## Options

Options are presented under the following headings:
Options for misstable summarize
Options for misstable patterns
Options for misstable tree
Option for misstable nested
Common options

## Options for misstable summarize

all specifies that the table should include all the variables specified or all the variables in the dataset.
The default is to include only numeric variables that contain missing values.
showzeros specifies that zeros in the table should display as 0 rather than being omitted.
generate (stub [, exok]) requests that a missing-value indicator newvar, a new binary variable containing 0 for complete observations and 1 for incomplete observations, be generated for every numeric variable in varlist containing missing values. If the all option is specified, missing-value indicators are created for all the numeric variables specified or for all the numeric variables in the dataset. If exok is specified within generate(), the extended missing values .a, b, ..., .z are treated as if they do not designate missing.
For each variable in varlist, newvar is the corresponding variable name varname prefixed with stub. If the total length of stub and varname exceeds 32 characters, newvar is abbreviated so that its name does not exceed 32 characters.

## Options for misstable patterns

asis, frequency, and exok - see Common options below.
replace specifies that the data in memory be replaced with a dataset corresponding to the table just displayed; see misstable patterns under Remarks and examples below.
clear is for use with replace; it specifies that it is okay to change the data in memory even if they have not been saved to disk.
bypatterns specifies the table be ordered by pattern rather than by frequency. That is, bypatterns specifies that patterns containing one incomplete variable be listed first, followed by those for two incomplete variables, and so on. The default is to list the most frequent pattern first, followed by the next most frequent pattern, etc.

## Options for misstable tree

asis, frequency, and exok - see Common options below.

## Option for misstable nested

exok - see Common options below.

## Common options

asis specifies that the order of the variables in the table be the same as the order in which they are specified on the misstable command. The default is to order the variables by the number of missing values, and within that, by the amount of overlap of missing values.
frequency specifies that the table should report frequencies instead of percentages.
exok specifies that the extended missing values .a, .b, ..., .z should be treated as if they do not designate missing. Some users use extended missing values to designate values that are missing for a known and valid reason.
nopreserve is a programmer's option allowed with all misstable commands; see [P] nopreserve option.

## Remarks and examples

Remarks are presented under the following headings:

```
misstable summarize
misstable patterns
misstable tree
misstable nested
Execution time of misstable nested
```

In what follows, we will use data from a 125-observation, fictional, student-satisfaction survey:
. use http://www.stata-press.com/data/r13/studentsurvey
(Student Survey)
. summarize
Variable

The m 1 and m 2 variables record the student's satisfaction with teaching and with academics. comment is a string variable recording any comments the student might have had.

## misstable summarize

## > Example 1

misstable summarize reports counts of missing values:
. misstable summarize
Obs<.

| Variable | Obs=. | Obs>. | Obs<. | Unique <br> values | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| age | 3 | 122 | 5 | 17 | 21 |  |
| female | 3 | 122 | 2 | 0 | 1 |  |
| dept | 9 | 116 | 4 | 1 | 4 |  |

Stata provides 27 different missing values, namely, ., .a, .b, ...,.z. The first of those, ., is often called system missing. The remaining missing values are called extended missings. The nonmissing and missing values are ordered nonmissing $<.<. \mathrm{a}<. \mathrm{b}<\cdots<. \mathrm{z}$. Thus reported in the column "Obs=." are counts of system missing values; in the column "Obs $>$.", extended missing values; and in the column "Obs<.", nonmissing values.

The rightmost portion of the table is included to remind you how the variables are encoded.
Our data contain seven variables and yet misstable reported only three of them. The omitted variables contain no missing values or are string variables. Even if we specified the varlist explicitly, those variables would not appear in the table unless we specified the all option.

We can also create missing-value indicators for each of the variables above using the generate() option:

```
. quietly misstable summarize, generate(miss_)
. describe miss_*
\begin{tabular}{lllll} 
variable name & \begin{tabular}{c} 
storage \\
type
\end{tabular} & \begin{tabular}{l} 
display \\
format
\end{tabular} & \begin{tabular}{l} 
value \\
label
\end{tabular} & variable label \\
\hline miss_age & byte & \(\% 8.0 \mathrm{~g}\) & & (age>=.) \\
miss_female & byte & \(\% 8.0 \mathrm{~g}\) & & (female>=.) \\
miss_dept & byte & \(\% 8.0 \mathrm{~g}\) & & (dept>=.)
\end{tabular}
```

For each variable containing missing values, the generate() option creates a new binary variable containing 0 for complete observations and 1 for incomplete observations. In our example, three new missing-value indicators are generated, one for each of the incomplete variables age, female, and dept. The naming convention of generate() is to prefix the corresponding variable names with the specified stub, which is miss_ in this example.

Missing-value indicators are useful, for example, for checking whether data are missing completely at random. They are also often used within the multiple-imputation context to identify the observed and imputed data; see [MI] intro substantive for a general introduction to multiple imputation. Within Stata's multiple-imputation commands, an incomplete value is identified by the system missing value, a dot. By default, misstable summarize, generate() marks the extended missing values as incomplete values, as well. You can use exok within generate() to treat extended missing values as complete when creating missing-value identifiers.

## misstable patterns

## > Example 2

misstable patterns reports the pattern of missing values:


There are three patterns in these data: $(1,1,1),(1,1,0)$, and $(0,0,0)$. By default, the rows of the table are ordered by frequency. In larger tables that have more patterns, it is sometimes useful to order the rows by pattern. We could have obtained that by typing mi misstable patterns, bypatterns.

In a pattern, 1 indicates that all values of the variable are nonmissing and 0 indicates that all values are missing. Thus pattern $(1,1,1)$ means no missing values, and $93 \%$ of our data have that pattern. There are two patterns in which variables are missing, $(1,1,0)$ and $(0,0,0)$. Pattern $(1,1,0)$ means that age is nonmissing, female is nonmissing, and dept is missing. The order of the variables in the patterns appears in the key at the bottom of the table. Five percent of the observations have pattern $(1,1,0)$. The remaining $2 \%$ have pattern $(0,0,0)$, meaning that all three variables contain missing.

As with misstable summarize, only numeric variables that contain missing are listed, so had we typed misstable patterns comments age female offcampus dept, we still would have obtained the same table. Variables that are automatically omitted contain no missing values or are string variables.

The variables in the table are ordered from lowest to highest frequency of missing values, although you cannot see that from the information presented in the table. The variables are ordered this way even if you explicitly specify the varlist with a different ordering. Typing misstable patterns dept female age would produce the same table as above. Specify the asis option if you want the variables in the order in which you specify them.

You can obtain a dataset of the patterns by specifying the replace option:

. list

| _freq | age | female | dept |
| ---: | ---: | ---: | ---: |
| 3 | 0 | 0 | 0 |
| 6 | 1 | 1 | 0 |
| 116 | 1 | 1 | 1 |

The differences between the dataset and the printed table are that 1 ) the dataset always records frequency and 2) the rows are reversed.

## misstable tree

> Example 3
misstable tree presents a tree view of the pattern of missing values:

```
. use http://www.stata-press.com/data/r13/studentsurvey, clear
(Student Survey)
. misstable tree, frequency
    Nested pattern of missing values
        dept age female
            9 3
            6 0
                        0 6
            116 0 0
                            116 0
                        116
```

(number missing listed first)

In this example, we specified the frequency option to see the table in frequency rather than percentage terms. In the table, each column sums to the total number of observations in the data, 125. Variables are ordered from those with the most missing values to those with the least. Start with the first column. The dept variable is missing in 9 observations and, farther down, the table reports that it is not missing in 116 observations.

Go back to the first row and read across, but only to the second column. The dept variable is missing in 9 observations. Within those 9, age is missing in 3 of them and is not missing in the remaining 6 . Reading down the second column, within the 116 observations that dept is not missing, age is missing in 0 and not missing in 116.

Reading straight across the first row again, dept is missing in 9 observations, and within the 9 , age is missing in 3 , and within the 3 , female is also missing in 3 . Skipping down just a little, within the 6 observations for which dept is missing and age is not missing, female is not missing, too.

## misstable nested

> Example 4
misstable nested lists the nesting rules that describe the missing-value pattern,

```
. misstable nested
    1. female(3) <-> age(3) -> dept(9)
```

This line says that in observations in which female is missing, so is age missing, and vice versa, and in observations in which age (or female) is missing, so is dept. The numbers in parentheses are counts of the missing values. The female variable happens to be missing in 3 observations, and the same is true for age; the dept variable is missing in 9 observations. Thus dept is missing in the 3 observations for which age and female are missing, and in 6 more observations, too.

In these data, it turns out that the missing-value pattern can be summarized in one statement. In a larger dataset, you might see something like this:

```
. misstable nested
```

1. female(50) <-> age(50) -> dept (120)
2. female(50) $\rightarrow$ m1 (58)
3. offcampus(11)
misstable nested accounts for every missing value. In the above, in addition to female <-> age $->$ dept, we have that female $->m 1$, and we have offcampus, the last all by itself. The last line says that the 11 missing values in offcampus are not themselves nested in the missing value of any other variable, nor do they imply the missing values in another variable. In some datasets, all the statements will be of this last form.

In our data, however, we have one statement:

```
. misstable nested
```

1. female(3) <-> age (3) $\rightarrow$ dept (9)

When the missing-value pattern can be summarized in one misstable nested statement, the pattern of missing values in the data is said to be monotone.

## Execution time of misstable nested

The execution time of misstable nested is affected little by the number of observations but can grow quickly with the number of variables, depending on the fraction of missing values within variable. The execution time of the example above, which has 3 variables containing missing, is instant. In worst-case scenarios, with 500 variables, the time might be 25 seconds; with 1,000 variables, the execution time might be closer to an hour.

In situations where misstable nested takes a long time to complete, it will produce thousands of rules that will defy interpretation. A 523-variable dataset we have seen ran in 20 seconds and produced 8,040 rules. Although we spotted a few rules in the output that did not surprise us, such as the year of the date being missing implied that the month and the day were also missing, mostly the output was not helpful.

If you have such a dataset, we recommend you run misstable on groups of variables that you have reason to believe the pattern of missing values might be related.

## Stored results

misstable summarize stores the following values of the last variable summarized in $r()$ :
Scalars

```
    r(N_eq_dot) number of observations containing .
    r(N_gt_dot) number of observations containing .a, .b, ..., .z
    r(N_lt_dot) number of observations containing nonmissing
    r(K_uniq) number of unique, nonmissing values
    r(min) variable's minimum value
    r(max) variable's maximum value
    r(vartype) numeric, string, or none
```

Macros
$r$ (K_uniq) contains . if the number of unique, nonmissing values is greater than 500. $r$ (vartype) contains none if no variables are summarized, and in that case, the value of the scalars are all set to missing (.). Programmers intending to access results after misstable summarize should specify the all option.
misstable patterns stores the following in r() :
Scalars
$r$ (N_complete) number of complete observations
$r$ (N_incomplete) number of incomplete observations
$r(K) \quad$ number of patterns
Macros
$r$ (vars) variables used in order presented
$r$ (N_complete) and $r$ ( $N$ _incomplete) are defined with respect to the variables specified if variables were specified and otherwise, defined with respect to all the numeric variables in the dataset. $r$ (N_complete) is the number of observations that contain no missing values.
misstable tree stores the following in $r()$ :
Macros
$r$ (vars) variables used in order presented
misstable nested stores the following in $r()$ :
Scalars
$r(K) \quad$ number of statements
Macros
$r$ (stmt1) first statement
$r$ (stmt2) second statement

-     - 

$r($ stmt ' $r(K)$ ') last statement
$r$ (stmt1wc) $\quad r(s t m t 1)$ with missing-value counts
$r$ (vars) variables considered
A statement is encoded "varname", "varname op varname", or "varname op varname op varname", and so on; $o p$ is either "->" or "<->".

## Also see <br> [MI] mi misstable - Tabulate pattern of missing values <br> [R] summarize - Summary statistics <br> [R] tabulate oneway - One-way table of frequencies <br> [R] tabulate twoway - Two-way table of frequencies

## Title

mkspline - Linear and restricted cubic spline construction

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Methods and formulas | Acknowledgment | References |
| Also see |  |  |  |

## Syntax

Linear spline with knots at specified points

```
mkspline newvar. # # [newvar }\mp@subsup{|}{2}{*}[\ldots]]\mp@subsup{\mathrm{ newvar }}{k}{}=\mathrm{ oldvar [if ] [in] [, marginal
    displayknots]
```

Linear spline with knots equally spaced or at percentiles of data

```
mkspline stubname # = oldvar [if] [in] [weight] [, marginal pctile
    displayknots]
```

Restricted cubic spline

```
mkspline stubname = oldvar [if] [in] [weight], cubic [\underline{nknots(#) knots(numlist)}
displayknots]
```

fweights are allowed with the second and third syntax; see [U] 11.1.6 weight.

## Menu

Data $>$ Create or change data $>$ Other variable-creation commands $>$ Linear and cubic spline construction

## Description

mkspline creates variables containing a linear spline or a restricted cubic spline of oldvar.
In the first syntax, mkspline creates newvar,$\ldots$, newvar $_{k}$ containing a linear spline of oldvar with knots at the specified $\#_{1}, \ldots, \#_{k-1}$.

In the second syntax, mkspline creates \# variables named stubname $1, \ldots$, stubname\# containing a linear spline of oldvar. The knots are equally spaced over the range of oldvar or are placed at the percentiles of oldvar.

In the third syntax, mkspline creates variables containing a restricted cubic spline of oldvar. This is also known as a natural spline. The location and spacing of the knots is determined by the specification of the nknots() and knots() options.

## Options

$\qquad$ Options
marginal is allowed with the first or second syntax. It specifies that the new variables be constructed so that, when used in estimation, the coefficients represent the change in the slope from the preceding interval. The default is to construct the variables so that, when used in estimation, the coefficients measure the slopes for the interval.
displayknots displays the values of the knots that were used in creating the linear or restricted cubic spline.
pctile is allowed only with the second syntax. It specifies that the knots be placed at percentiles of the data rather than being equally spaced over the range.
nknots (\#) is allowed only with the third syntax. It specifies the number of knots that are to be used for a restricted cubic spline. This number must be between 3 and 7 unless the knot locations are specified using knots (). The default number of knots is 5 .
knots (numlist) is allowed only with the third syntax. It specifies the exact location of the knots to be used for a restricted cubic spline. The values of these knots must be given in increasing order. When this option is omitted, the default knot values are based on Harrell's recommended percentiles with the additional restriction that the smallest knot may not be less than the fifth-smallest value of oldvar and the largest knot may not be greater than the fifth-largest value of oldvar. If both nknots () and knots () are given, they must specify the same number of knots.

## Remarks and examples

Remarks are presented under the following headings:
Linear splines
Restricted cubic splines

## Linear splines

Linear splines allow estimating the relationship between $y$ and $x$ as a piecewise linear function, which is a function composed of linear segments - straight lines. One linear segment represents the function for values of $x$ below $x_{0}$, another linear segment handles values between $x_{0}$ and $x_{1}$, and so on. The linear segments are arranged so that they join at $x_{0}, x_{1}, \ldots$, which are called the knots. An example of a piecewise linear function is shown below.


## > Example 1

We wish to fit a model of log income on education and age by using a piecewise linear function for age:

$$
\operatorname{lninc}=b_{0}+b_{1} \text { educ }+f(\text { age })+u
$$

The knots are to be placed at 10-year intervals: 20, 30, 40, 50, and 60 .

```
. use http://www.stata-press.com/data/r13/mksp1
. mkspline age1 20 age2 30 age3 40 age4 50 age5 60 age6 = age, marginal
. regress lninc educ age1-age6
    (output omitted)
```

Because we specified the marginal option, we could test whether the age effect is the same in the $30-40$ and $40-50$ intervals by asking whether the age 4 coefficient is zero. With the marginal option, coefficients measure the change in slope from the preceding group. Specifying marginal changes only the interpretation of the coefficients; the same model is fit in either case. Without the marginal option, the interpretation of the coefficients would have been

$$
\frac{d y}{d \text { age }}= \begin{cases}a_{1} & \text { if age }<20 \\ a_{2} & \text { if } 20 \leq \text { age }<30 \\ a_{3} & \text { if } 30 \leq \text { age }<40 \\ a_{4} & \text { if } 40 \leq \text { age }<50 \\ a_{5} & \text { if } 50 \leq \text { age }<60 \\ a_{6} & \text { otherwise }\end{cases}
$$

With the marginal option, the interpretation is

$$
\frac{d y}{d \text { age }}= \begin{cases}a_{1} & \text { if age }<20 \\ a_{1}+a_{2} & \text { if } 20 \leq \text { age }<30 \\ a_{1}+a_{2}+a_{3} & \text { if } 30 \leq \text { age }<40 \\ a_{1}+a_{2}+a_{3}+a_{4} & \text { if } 40 \leq \text { age }<50 \\ a_{1}+a_{2}+a_{3}+a_{4}+a_{5} & \text { if } 50 \leq \text { age }<60 \\ a_{1}+a_{2}+a_{3}+a_{4}+a_{5}+a_{6} & \text { otherwise }\end{cases}
$$

## Example 2

Say that we have a binary outcome variable called outcome. We are beginning an analysis and wish to parameterize the effect of dosage on outcome. We wish to divide the data into five equal-width groups of dosage for the piecewise linear function.

```
. use http://www.stata-press.com/data/r13/mksp2, clear
. mkspline dose 5 = dosage, displayknots
```

|  | $\operatorname{knot} 1$ | $\operatorname{knot} 2$ | $\operatorname{knot} 3$ | $\operatorname{knot} 4$ |
| ---: | ---: | ---: | ---: | ---: |
| dosage | 20 | 40 | 60 | 80 |

. logistic outcome dose1-dose5
(output omitted)
mkspline dose 5 = dosage creates five variables-dose1, dose2, ... dose5—equally spacing the knots over the range of dosage. Because dosage varied between 0 and 100, the mkspline command above has the same effect as typing

```
. mkspline dose1 20 dose2 40 dose3 60 dose4 80 dose5 = dosage
```

The pctile option sets the knots to divide the data into five equal sample-size groups rather than five equal-width ranges. Typing

| . mkspline pctdose $5=$ dosage, pctile displayknots |  |  |  |  |
| ---: | :---: | :---: | :---: | :---: |
|  | knot1 | knot2 | knot3 | knot4 |
| dosage | 16 | 36.4 | 55.6 | 82 |

places the knots at the 20th, 40th, 60th, and 80th percentiles of the data.

## Restricted cubic splines

A linear spline can be used to fit many functions well. However, a restricted cubic spline may be a better choice than a linear spline when working with a very curved function. When using a restricted cubic spline, one obtains a continuous smooth function that is linear before the first knot, a piecewise cubic polynomial between adjacent knots, and linear again after the last knot.

## > Example 3

Returning to the data from example 1, we may feel that a curved function is a better fit. First, we will use the knots() option to specify the five knots that we used previously.

```
. use http://www.stata-press.com/data/r13/mksp1, clear
. mkspline agesp = age, cubic knots(20 30 40 50 60)
. regress lninc educ agesp*
    (output omitted)
```

Harrell $(2001,23)$ recommends placing knots at equally spaced percentiles of the original variable's marginal distribution. If we do not specify the knots () option, variables will be created containing a restricted cubic spline with five knots determined by Harrell's default percentiles.

```
. use http://www.stata-press.com/data/r13/mksp1, clear
. mkspline agesp = age, cubic displayknots
. regress lninc educ agesp*
(output omitted)
```


## Methods and formulas

Methods and formulas are presented under the following headings:
Linear splines Restricted cubic splines

## Linear splines

Let $V_{i}, i=1, \ldots, n$, be the variables to be created; $k_{i}, i=1, \ldots, n-1$, be the corresponding knots; and $\mathcal{V}$ be the original variable (the command is mkspline $V_{1} k_{1} V_{2} k_{2} \ldots V_{n}=\mathcal{V}$ ). Then

$$
\begin{aligned}
V_{1} & =\min \left(\mathcal{V}, k_{1}\right) \\
V_{i} & =\max \left\{\min \left(\mathcal{V}, k_{i}\right), k_{i-1}\right\}-k_{i-1} \quad i=2, \ldots, n-1 \\
V_{n} & =\max \left(\mathcal{V}, k_{n-1}\right)-k_{n-1}
\end{aligned}
$$

If the marginal option is specified, the definitions are

$$
\begin{aligned}
& V_{1}=\mathcal{V} \\
& V_{i}=\max \left(0, \mathcal{V}-k_{i-1}\right) \quad i=2, \ldots, n
\end{aligned}
$$

In the second syntax, mkspline stubname $\#=\mathcal{V}$, so let $m$ and $M$ be the minimum and maximum of $\mathcal{V}$. Without the pctile option, knots are set at $m+(M-m)(i / n)$ for $i=1, \ldots, n-1$. If pctile is specified, knots are set at the $100(i / n)$ percentiles, for $i=1, \ldots, n-1$. Percentiles are calculated by centile; see [R] centile.

## Restricted cubic splines

Let $k_{i}, i=1, \ldots, n$, be the knot values; $V_{i}, i=1, \ldots, n-1$, be the variables to be created; and $\mathcal{V}$ be the original variable. Then

$$
\begin{aligned}
& V_{1}= \mathcal{V} \\
& V_{i+1}= \frac{\left(\mathcal{V}-k_{i}\right)_{+}^{3}-\left(k_{n}-k_{n-1}\right)^{-1}\left\{\left(\mathcal{V}-k_{n-1}\right)_{+}^{3}\left(k_{n}-k_{i}\right)-\left(\mathcal{V}-k_{n}\right)_{+}^{3}\left(k_{n-1}-k_{i}\right)\right\}}{\left(k_{n}-k_{1}\right)^{2}} \\
& \quad i=1, \ldots, n-2
\end{aligned}
$$

where

$$
(u)_{+}=\left\{\begin{array}{l}
u, \text { if } \quad u>0 \\
0, \text { if } \quad u \leq 0
\end{array}\right.
$$

Without the knots() option, the locations of the knots are determined by the percentiles recommended in Harrell (2001, 23). These percentiles are based on the chosen number of knots as follows:

| $\begin{array}{c}\text { No. } \\ \text { of knots }\end{array}$ | Percentiles |  |  |  |  |  |  |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- |$]$|  |  |  |  |  |  |  |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| 3 | 10 | 50 | 90 |  |  |  |
| 4 | 5 | 35 | 65 | 95 |  |  |
| 5 | 5 | 27.5 | 50 | 72.5 | 95 |  |
| 6 | 5 | 23 | 41 | 59 | 77 | 95 |
| 7 | 2.5 | 18.33 | 34.17 | 50 | 65.83 | 81.67 |
| 7 |  |  | 97.5 |  |  |  |

Harrell provides default percentiles when the number of knots is between 3 and 7. When using a number of knots outside this range, the location of the knots must be specified in knots().

## Acknowledgment

The restricted cubic spline portion of mkspline is based on the rc_spline command by William Dupont of the Department of Biostatistics at Vanderbilt University.

## References

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## Also see

[R] fp - Fractional polynomial regression

## Title

ml — Maximum likelihood estimation

| Syntax | Description | Options | Remarks and examples |
| :--- | :--- | :--- | :--- |
| Stored results | Methods and formulas | References | Also see |

## Syntax

ml model in interactive mode
ml model method progname eq $[$ eq...] [if][in] [weight]
[, model_options svy diparm_options]
ml model method funcname() eq [eq...] [if] [in] [weight] [, model_options svy diparm_options]
ml model in noninteractive mode
ml model method progname eq [eq...] [if] [in] [weight], maximize [model_options svy diparm_options noninteractive_options]
ml model method funcname() eq [eq...] [if] [in] [weight], maximize [model_options svy diparm_options noninteractive_options]

Noninteractive mode is invoked by specifying the maximize option. Use maximize when ml will be used as a subroutine of another ado-file or program and you want to carry forth the problem, from definition to posting of results, in one command.
ml clear
ml query
ml check
ml search $\left[[/]\right.$ eqname $\left.[:] \#_{l b} \#_{u b}\right][\ldots][$, search_options $]$
ml plot [eqname: $]$ name $[\#[\#[\#]]][$, saving(filename[, replace])]
ml init $\{[$ eqname $:]$ name $=\# \mid$ eqname $=\#\}[. .$.
ml init \# [\#...], copy
ml init matname [, copy skip]
ml report
ml trace $\quad\{$ on $\mid$ off $\}$

```
ml count }\quad[clear|on|off
ml maximize [, ml_maximize_options display_options eform_option]
ml graph [#][, saving(filename[, replace ])]
ml display [, display_options eform_option]
ml footnote
ml score newvar [if] [in] [, equation(eqname) missing]
ml score newvarlist [if ][in] [, missing}
ml score [type] stub* [if] [in] [, missing]
```

where method is one of

| lf | d0 | lf0 | gf0 |
| :--- | :--- | :--- | :--- |
|  | d1 | lf1 |  |
|  | d1debug | lf1debug |  |
|  | d2 | lf2 |  |
|  | d2debug | lf2debug |  |

or method can be specified using one of the longer, more descriptive names

| method | Longer name |
| :--- | :--- |
| lf | linearform |
| d0 | derivative0 |
| d1 | derivative1 |
| d1debug | derivative1debug |
| d2 | derivative2 |
| d2debug | derivative2debug |
| lf0 | linearform0 |
| lf1 | linearform1 |
| lf1debug | linearform1debug |
| lf2 | linearform2 |
| lf2debug | linearform2debug |
| gf0 | generalform0 |

$e q$ is the equation to be estimated, enclosed in parentheses, and optionally with a name to be given to the equation, preceded by a colon,

$$
\left([\text { eqname }:]\left[\text { varlist }_{y}=\right]\left[\text { varlist }_{x}\right][\text {, eq_options }]\right)
$$

or eq is the name of a parameter, such as sigma, with a slash in front
/eqname which is equivalent to (eqname:)
and diparm_options is one or more diparm(diparm_args) options where diparm_args is either __sep__ or anything accepted by the "undocumented"_diparm command; see help _diparm.
noconstant
offset (varname ${ }_{o}$ ) exposure (varname ${ }_{e}$ )
do not include an intercept in the equation include varname $_{o}$ in model with coefficient constrained to 1 include $\ln \left(\right.$ varname $\left._{e}\right)$ in model with coefficient constrained to 1
model_options

| group(varname) <br> vce(vcetype) <br> constraints (numlist) |
| :--- |
| constraints (matname) |
| nocnsnotes |
| title(string) |
| nopreserve |
| collinear |
| missing |
| lf0 $\left(\#_{k} \#_{l l}\right)$ |

continue
waldtest(\#)
obs(\#)
crittype (string)
subpop(varname)
nosvyadjust
technique (nr)
technique (bhhh)
technique (dfp)
technique (bfgs)

Description
use varname to identify groups
vcetype may be robust, cluster clustvar, oim, or opg
constraints by number to be applied
matrix that contains the constraints to be applied
do not display notes when constraints are dropped
place a title on the estimation output
do not preserve the estimation subsample in memory
keep collinear variables within equations
keep observations containing variables with missing values
number of parameters and log-likelihood value of the constant-only model
specifies that a model has been fit and sets the initial values $\mathbf{b}_{0}$ for the model to be fit based on those results
perform a Wald test; see Options for use with ml model in interactive or noninteractive mode below
number of observations
describe the criterion optimized by ml
compute estimates for the single subpopulation
carry out Wald test as $W / k \sim F(k, d)$
Stata's modified Newton-Raphson (NR) algorithm
Berndt-Hall-Hall-Hausman (BHHH) algorithm
Davidon-Fletcher-Powell (DFP) algorithm
Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm
noninteractive_options

Description

| init (ml_init_args) | set the initial values $\mathbf{b}_{0}$ |
| :---: | :---: |
| search(on) | equivalent to ml search, repeat (0) ; the default |
| search(norescale) | equivalent to ml search, repeat (0) norescale |
| search(quietly) | same as search(on), except that output is suppressed |
| search (off) | prevents calling ml search |
| repeat (\#) | ml search's repeat () option; see below |
| bounds (ml_search_bounds) | specify bounds for ml search |
| nowarning | suppress "convergence not achieved" message of iterate(0) |
| novce | substitute the zero matrix for the variance matrix |
| negh | indicates that the evaluator returns the negative Hessian matrix |
| score(newvars) | new variables containing the contribution to the score |
| maximize_options | control the maximization process; seldom used |

search_options
Description
repeat (\#)
restart use random actions to find starting values; not recommended norescale
maximize_options
number of random attempts to find better initial-value vector; default is repeat (10) in interactive mode and repeat ( 0 ) in noninteractive mode do not rescale to improve parameter vector; not recommended control the maximization process; seldom used
ml_maximize_options Description
nowarning
novce
negh
score (newvars $\mid$ stub*)
nooutput
noclear
maximize_options

Description
suppress "convergence not achieved" message of iterate (0) substitute the zero matrix for the variance matrix indicates that the evaluator returns the negative Hessian matrix new variables containing the contribution to the score suppress display of final results
do not clear ml problem definition after model has converged control the maximization process; seldom used
display_options
noheader
nofootnote
level(\#)
first
neq(\#)
showeqns
plus
nocnsreport
noomitted
vsquish
noemptycells
baselevels
allbaselevels
cformat (\% fint)
pformat (\% $\%$ fmt)
sformat (\% fint)
nolstretch
coeflegend

## Description

suppress header display above the coefficient table suppress footnote display below the coefficient table set confidence level; default is level (95)
display coefficient table reporting results for first equation only display coefficient table reporting first \# equations display equation names in the coefficient table display coefficient table ending in dashes-plus-sign-dashes suppress constraints display above the coefficient table suppress display of omitted variables
suppress blank space separating factor-variable terms or time-series-operated variables from other variables
suppress empty cells for interactions of factor variables
report base levels of factor variables and interactions
display all base levels of factor variables and interactions
format the coefficients, standard errors, and confidence limits in the coefficient table
format the $p$-values in the coefficient table
format the test statistics in the coefficient table
do not automatically widen the coefficient table to accommodate longer variable names
display legend instead of statistics
eform_option
Description
eform (string)
eform
hr
shr
irr
or
rrr
display exponentiated coefficients; column title is "string"
display exponentiated coefficients; column title is " $\exp (\mathrm{b})$ "
report hazard ratios
report subhazard ratios
report incidence-rate ratios
report odds ratios
report relative-risk ratios
fweights, aweights, iweights, and pweights are allowed; see [U] 11.1.6 weight. With all but method lf, you must write your likelihood-evaluation program carefully if pweights are to be specified, and pweights may not be specified with method d0, d1, d1debug, d2, or d2debug. See Gould, Pitblado, and Poi (2010, chap. 6) for details.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.
To redisplay results, type ml display.

## Syntax of subroutines for use by evaluator programs

```
mleval newvar \(=\) vecname \([\), eq(\#) \(]\)
mleval scalarname \(=\) vecname, \(\operatorname{scalar}[\mathrm{eq}(\#)]\)
mlsum \(\quad\) scalarname \(_{\operatorname{lnf}}=\exp [\) if \(][\), noweight \(]\)
mlvecsum \(\quad\) scalarname \(_{\operatorname{lnf}}\) rowvecname \(=\exp [\) if \(][\), eq(\#) \(]\)
mlmatsum \(\quad\) scalarname \(_{\operatorname{lnf}}\) matrixname \(=\exp [\) if \(][\), eq \((\#[, \#])]\)
mlmatbysum \(^{\text {scalarname }_{\operatorname{lnf}}}\) matrixname varname \({ }_{a}\) varname \(_{b}\left[\right.\) varname \(\left._{c}\right][\) if \(]\),
by (varname) [ eq(\#[,\#])]
```


## Syntax of user-written evaluator

## Summary of notation

The log-likelihood function is $\ln L\left(\theta_{1 j}, \theta_{2 j}, \ldots, \theta_{E j}\right)$, where $\theta_{i j}=\mathbf{x}_{i j} \mathbf{b}_{i}, j=1, \ldots, N$ indexes observations, and $i=1, \ldots, E$ indexes the linear equations defined by ml model. If the likelihood satisfies the linear-form restrictions, it can be decomposed as $\ln L=\sum_{j=1}^{N} \ln \ell\left(\theta_{1 j}, \theta_{2 j}, \ldots, \theta_{E j}\right)$.

## Method-lf evaluators

program progname
version 13
args lnfj theta1 theta2 ...
// if you need to create any intermediate results:
tempvar tmp1 tmp2...
quietly gen double 'tmp1' = ...
quietly replace ' $\operatorname{lnf} \mathrm{j}$ ' = ...
end
where
' $\operatorname{lnf} \mathrm{j}$, variable to be filled in with observation-by-observation values of $\ln \ell_{j}$
'theta1' variable containing evaluation of first equation $\theta_{1 j}=\mathbf{x}_{1 j} \mathbf{b}_{1}$
'theta2' variable containing evaluation of second equation $\theta_{2 j}=\mathbf{x}_{2 j} \mathbf{b}_{2}$

## Method-d0 evaluators

```
program progname
        version 13
        args todo b lnf
        tempvar thetal theta2 ...
        mleval 'theta1' = 'b', eq(1)
        mleval 'theta2' = ' b', eq(2) // if there is a }\mp@subsup{0}{2}{
        // if you need to create any intermediate results:
        tempvar tmp1 tmp2...
        gen double 'tmpl' = ...
        mlsum 'lnf' = ...
end
```

where

| 'todo' | always contains 0 (may be ignored) |
| :--- | :--- |
| ' $\mathbf{b}$ ' | full parameter row vector $\mathbf{b}=\left(\mathbf{b}_{1}, \mathbf{b}_{2}, \ldots, \mathbf{b}_{E}\right)$ |
| ' $\operatorname{lnf}$, | scalar to be filled in with overall $\ln L$ |

## Method-d1 evaluators

```
program progname
    version 13
    args todo b lnf g
    tempvar thetal theta2 ...
    mleval 'theta1' = 'b', eq(1)
    mleval 'theta2' = 'b', eq(2) // if there is a }\mp@subsup{0}{2}{
    // if you need to create any intermediate results:
    tempvar tmp1 tmp2...
    gen double 'tmpl' = ...
    mlsum 'lnf' = ...
    if ('todo'==0 | 'lnf'>=.) exit
    tempname d1 d2 ...
    mlvecsum 'lnf' 'd1' = formula for }\partial\operatorname{ln}\mp@subsup{\ell}{j}{}/\partial\mp@subsup{0}{1j}{}\mathrm{ , eq(1)
    mlvecsum 'lnf' 'd2' = formula for }\partial\operatorname{ln}\mp@subsup{\ell}{j}{}/\partial\mp@subsup{0}{2j}{}\mathrm{ , eq(2)
    matrix 'g' = ('d1','d2', ... )
end
```

where

```
'todo' contains 0 or 1
                        0=>'lnf'to be filled in;
                        1=>'lnf' and 'g' to be filled in
'b' full parameter row vector b}\mathbf{b}=(\mp@subsup{\mathbf{b}}{1}{},\mp@subsup{\mathbf{b}}{2}{},\ldots,\mp@subsup{\mathbf{b}}{E}{}
'lnf' scalar to be filled in with overall }\operatorname{ln}
' g' row vector to be filled in with overall g=\partial ln}L/\partial\mathbf{b
```


## Method-d2 evaluators

```
program progname
    version 13
    args todo b lnf g H
    tempvar thetal theta2...
    mleval 'theta1' = 'b', eq(1)
    mleval 'theta2' = 'b', eq(2) // if there is a }\mp@subsup{0}{2}{
```

    // if you need to create any intermediate results:
    tempvar tmp1 tmp2...
    gen double 'tmpl' = ...
    mlsum 'lnf' = ...
    if ('todo'==0 | 'lnf'>=.) exit
    tempname d1 d2 ...
    mlvecsum 'lnf' 'd1' = formula for \(\partial \ln \ell_{j} / \partial \theta_{1 j}\), eq(1)
    mlvecsum 'lnf' ' d 2 ' \(=\) formula for \(\partial \ln \ell_{j} / \partial \theta_{2 j}\), eq(2)
    matrix 'g' \(=(' d 1\) ',' \(d 2\) ', ... )
    if ('todo'==1 | 'lnf'>=.) exit
    tempname d11 d12 d22 ...
    mlmatsum 'lnf' 'd11' \(=\) formula for \(\partial^{2} \ln \ell_{j} / \partial \theta_{1 j}^{2}\), eq(1)
    mlmatsum 'lnf' 'd12' = formula for \(\partial^{2} \ln \ell_{j} / \partial \theta_{1 j} \partial \theta_{2 j}\), eq(1,2)
    mlmatsum ' \(\operatorname{lnf}\) ' 'd22' \(=\) formula for \(\partial^{2} \ln \ell_{j} / \partial \theta_{2 j}^{2}\), eq(2)
    
end
where

```
'todo' contains 0, 1, or 2
                        0=>'lnf' to be filled in;
                        1=> 'lnf' and 'g' to be filled in;
                        2=>'lnf', 'g', and 'H' to be filled in
'b' full parameter row vector b}\mathbf{b}=(\mp@subsup{\mathbf{b}}{1}{},\mp@subsup{\mathbf{b}}{2}{},\ldots,\mp@subsup{\mathbf{b}}{E}{}
'lnf' scalar to be filled in with overall }\operatorname{ln}
'g' row vector to be filled in with overall g=\partial ln L/\partialb
''H' matrix to be filled in with overall Hessian H=\partial' ln L/\partial\mathbf{b}\partial\mp@subsup{\mathbf{b}}{}{\prime}
```


## Method-lf0 evaluators

```
program progname
    version 13
    args todo b lnfj
    tempvar thetal theta2...
    mleval 'theta1' = 'b', eq(1)
    mleval 'theta2' = 'b', eq(2) // if there is a }\mp@subsup{0}{2}{
    // if you need to create any intermediate results:
    tempvar tmp1 tmp2 ...
    gen double 'tmp1' = ...
    quietly replace 'lnfj' = ...
end
```

where

| 'todo' | always contains 0 (may be ignored) |
| :--- | :--- |
| ' $b$ ' | full parameter row vector $\mathbf{b}=\left(\mathbf{b}_{1}, \mathbf{b}_{2}, \ldots, \mathbf{b}_{E}\right)$ |
| ' $\operatorname{lnf} \mathrm{j}$, | variable to be filled in with observation-by-observation values of $\ln \ell_{j}$ |

## Method-lf1 evaluators

```
program progname
    version 13
    args todo b lnfj g1 g2 ...
    tempvar thetal theta2 ...
    mleval 'theta1' = 'b', eq(1)
    mleval 'theta2' = 'b', eq(2) // if there is a }\mp@subsup{0}{2}{
```

    // if you need to create any intermediate results:
    tempvar tmp1 tmp2...
    gen double 'tmpl' = ...
    quietly replace ' \(\operatorname{lnf} \mathrm{j}\) ' = ...
    if ('todo'==0) exit
    quietly replace 'g1' = formula for \(\partial \ln \ell_{j} / \partial \theta_{1 j}\)
    quietly replace ' g 2 ' \(=\) formula for \(\partial \ln \ell_{j} / \partial \theta_{2 j}\)
    end
where

```
'todo' contains 0 or 1
                        0=>'lnfj'to be filled in;
                        1=>'lnfj',' 'g1', 'g2',..., to be filled in
' b' full parameter row vector }\mathbf{b}=(\mp@subsup{\mathbf{b}}{1}{},\mp@subsup{\mathbf{b}}{2}{},\ldots,\mp@subsup{\mathbf{b}}{E}{}
'lnfj' variable to be filled in with observation-by-observation values of }\operatorname{ln}\mp@subsup{\ell}{j}{
'g1' variable to be filled in with }\partial\operatorname{ln}\mp@subsup{\ell}{j}{}/\partial\mp@subsup{0}{1j}{
'g2' variable to be filled in with }\partial\operatorname{ln}\mp@subsup{\ell}{j}{}/\partial\mp@subsup{0}{2j}{
```


## Method-lf2 evaluators

```
program progname
    version 13
    args todo b lnfj g1 g2 ... H
    tempvar thetal theta2 ...
    mleval 'theta1' = 'b', eq(1)
    mleval 'theta2' = 'b', eq(2) // if there is a }\mp@subsup{0}{2}{
    // if you need to create any intermediate results:
    tempvar tmp1 tmp2...
    gen double 'tmpl' = ...
    quietly replace 'lnfj' = ...
    if ('todo'==0) exit
    quietly replace 'g1' = formula for }\partial\operatorname{ln}\mp@subsup{\ell}{j}{}/\partial\mp@subsup{0}{1j}{
    quietly replace 'g2' = formula for }\partial\operatorname{ln}\mp@subsup{\ell}{j}{}/\partial\mp@subsup{0}{2j}{
    if ('todo'==1) exit
    tempname d11 d12 d22 lnf ...
    mlmatsum 'lnf' 'd11' = formula for }\mp@subsup{\partial}{}{2}\operatorname{ln}\mp@subsup{\ell}{j}{}/\partial\mp@subsup{0}{1j}{2}\mathrm{ , eq(1)
    mlmatsum 'lnf' 'd12' = formula for }\mp@subsup{\partial}{}{2}\operatorname{ln}\mp@subsup{\ell}{j}{}/\partial\mp@subsup{0}{1j}{}\partial\mp@subsup{0}{2j}{}\mathrm{ , eq (1,2)
    mlmatsum 'lnf' 'd22' = formula for }\mp@subsup{\partial}{}{2}\operatorname{ln}\mp@subsup{\ell}{j}{}/\partial\mp@subsup{0}{2j}{2}\mathrm{ , eq(2)
    matrix 'H' = ('d11','d12', ... \ 'd12'','d22', ... )
end
```

where

```
'todo' contains 0 or 1
    0=>'lnfj'to be filled in;
    1=>'lnfj',' 'g1', 'g2',..., to be filled in
    2=>'lnfj', 'g1', 'g2', ..., and 'H' to be filled in
''b' full parameter row vector b}=(\mp@subsup{\mathbf{b}}{1}{},\mp@subsup{\mathbf{b}}{2}{},\ldots,\mp@subsup{\mathbf{b}}{E}{}
'Infj' scalar to be filled in with observation-by-observation }\operatorname{ln}
'g1' variable to be filled in with }\partial\operatorname{ln}\mp@subsup{\ell}{j}{}/\partial\mp@subsup{0}{1j}{
'g2' variable to be filled in with }\partial\operatorname{ln}\mp@subsup{\ell}{j}{}/\partial\mp@subsup{0}{2j}{
''H' matrix to be filled in with overall Hessian H=\partial ' ln L/\partial\mathbf{b}\partial\mp@subsup{\mathbf{b}}{}{\prime}
```


## Method-gf0 evaluators

```
program progname
    version 13
    args todo b lnfj
    tempvar thetal theta2 ...
    mleval 'theta1' = 'b', eq(1)
    mleval 'theta2' = ' b', eq(2) // if there is a }\mp@subsup{0}{2}{
    // if you need to create any intermediate results:
    tempvar tmp1 tmp2...
    gen double 'tmp1' = . . .
    quietly replace 'lnfj' = ...
end
where
'todo' always contains 0 (may be ignored)
'b' full parameter row vector b=(b
'lnfj' variable to be filled in with the values of the log-likelihood }\operatorname{ln}\mp@subsup{\ell}{j}{
```


## Global macros for use by all evaluators

```
$ML_y1 name of first dependent variable
$ML_y2 name of second dependent variable, if any
...
$ML_samp variable containing 1 if observation to be used; 0 otherwise
$ML_w variable containing weight associated with observation or 1 if no weights specified
```

Method-lf evaluators can ignore \$ML_samp, but restricting calculations to the \$ML_samp==1 subsample will speed execution. Method-lf evaluators must ignore \$ML_w; application of weights is handled by the method itself.

Methods d0, d1, d2, lf0, lf1, lf2, and gf0 can ignore \$ML_samp as long as ml model's nopreserve option is not specified. These methods will run more quickly if nopreserve is specified. These evaluators can ignore \$ML_w only if they use mlsum, mlvecsum, mlmatsum, and mlmatbysum to produce all final results.

## Description

ml model defines the current problem.
ml clear clears the current problem definition. This command is rarely used because when you type ml model, any previous problem is automatically cleared.
ml query displays a description of the current problem.
ml check verifies that the log-likelihood evaluator you have written works. We strongly recommend using this command.
ml search searches for (better) initial values. We recommend using this command.
ml plot provides a graphical way of searching for (better) initial values.
ml init provides a way to specify initial values.
ml report reports $\ln L$ 's values, gradient, and Hessian at the initial values or current parameter estimates, $\mathbf{b}_{0}$.
ml trace traces the execution of the user-defined log-likelihood evaluation program.
ml count counts the number of times the user-defined log-likelihood evaluation program is called; this command is seldom used. ml count clear clears the counter. ml count on turns on the counter. ml count without arguments reports the current values of the counter. ml count off stops counting calls.
ml maximize maximizes the likelihood function and reports results. Once ml maximize has successfully completed, the previously mentioned ml commands may no longer be used unless noclear is specified. ml graph and ml display may be used whether or not noclear is specified.
ml graph graphs the log-likelihood values against the iteration number.
ml display redisplays results.
ml footnote displays a warning message when the model did not converge within the specified number of iterations.
ml score creates new variables containing the equation-level scores. The variables generated by ml score are equivalent to those generated by specifying the score() option of ml maximize (and ml model ..., ... maximize).
progname is the name of a Stata program you write to evaluate the log-likelihood function.
funcname () is the name of a Mata function you write to evaluate the log-likelihood function.
In this documentation, progname and funcname() are referred to as the user-written evaluator, the likelihood evaluator, or sometimes simply as the evaluator. The program you write is written in the style required by the method you choose. The methods are lf, d0, d1, d2, lf0, lf1, lf2, and gf0. Thus, if you choose to use method lf, your program is called a method-lf evaluator.
Method-lf evaluators are required to evaluate the observation-by-observation $\log$ likelihood $\ln \ell_{j}$, $j=1, \ldots, N$.
Method-d0 evaluators are required to evaluate the overall $\log$ likelihood $\ln L$. Method-d1 evaluators are required to evaluate the overall $\log$ likelihood and its gradient vector $\mathbf{g}=\partial \ln L / \partial \mathbf{b}$. Method-d2 evaluators are required to evaluate the overall log likelihood, its gradient, and its Hessian matrix $H=\partial^{2} \ln L / \partial \mathbf{b} \partial \mathbf{b}^{\prime}$.

Method-lf0 evaluators are required to evaluate the observation-by-observation $\log$ likelihood $\ln \ell_{j}$, $j=1, \ldots, N$. Method-lf1 evaluators are required to evaluate the observation-by-observation log likelihood and its equation-level scores $g_{j i}=\partial \ln \ell / \partial \mathbf{x}_{j i} \mathbf{b}_{i}$. Method-lf2 evaluators are required to evaluate the observation-by-observation log likelihood, its equation-level scores, and its Hessian matrix $H=\partial^{2} \ln \ell / \partial \mathbf{b} \partial \mathbf{b}^{\prime}$.

Method-gf0 evaluators are required to evaluate the summable pieces of the $\log$ likelihood $\ln \ell_{k}$, $k=1, \ldots, K$.
mleval is a subroutine used by evaluators of methods d0, d1, d2, lf0, lf1, lf2, and gf0 to evaluate the coefficient vector, $\mathbf{b}$, that they are passed.
mlsum is a subroutine used by evaluators of methods $\mathrm{d} 0, \mathrm{~d} 1$, and d 2 to define the value, $\ln L$, that is to be returned.
ml vecsum is a subroutine used by evaluators of methods d 1 and d 2 to define the gradient vector, $\mathbf{g}$, that is to be returned. It is suitable for use only when the likelihood function meets the linear-form restrictions.
mlmatsum is a subroutine used by evaluators of methods d2 and lf2 to define the Hessian matrix, $\mathbf{H}$, that is to be returned. It is suitable for use only when the likelihood function meets the linear-form restrictions.
mlmatbysum is a subroutine used by evaluator of method d2 to help define the Hessian matrix, $\mathbf{H}$, that is to be returned. It is suitable for use when the likelihood function contains terms made up of grouped sums, such as in panel-data models. For such models, use mlmatsum to compute the observation-level outer products and mlmatbysum to compute the group-level outer products. mlmatbysum requires that the data be sorted by the variable identified in the by () option.

## Options

Options are presented under the following headings:

```
Options for use with ml model in interactive or noninteractive mode
Options for use with ml model in noninteractive mode
Options for use when specifying equations
Options for use with ml search
Option for use with ml plot
Options for use with ml init
Options for use with ml maximize
Option for use with ml graph
Options for use with ml display
Options for use with mleval
Option for use with mlsum
Option for use with mlvecsum
Option for use with mlmatsum
Options for use with mlmatbysum
Options for use with ml score
```


## Options for use with $\mathbf{m l}$ model in interactive or noninteractive mode

group (varname) specifies the numeric variable that identifies groups. This option is typically used to identify panels for panel-data models.
vce(vcetype) specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that are derived from asymptotic theory (oim, opg); see [R] vce_option.
vce(robust), vce(cluster clustvar), pweight, and svy will work with evaluators of methods lf, lf0, lf1, lf2, and gf0; all you need do is specify them.
These options will not work with evaluators of methods d 0 , d 1 , or d 2 , and specifying these options will produce an error message.
constraints (numlist $\mid$ matname) specifies the linear constraints to be applied during estimation. constraints (numlist) specifies the constraints by number. Constraints are defined by using the constraint command; see $[\mathrm{R}]$ constraint. constraint (matname) specifies a matrix that contains the constraints.
nocnsnotes prevents notes from being displayed when constraints are dropped. A constraint will be dropped if it is inconsistent, contradicts other constraints, or causes some other error when the constraint matrix is being built. Constraints are checked in the order in which they are specified.
title(string) specifies the title for the estimation output when results are complete.
nopreserve specifies that ml need not ensure that only the estimation subsample is in memory when the user-written likelihood evaluator is called. nopreserve is irrelevant when you use method lf.
For the other methods, if nopreserve is not specified, ml saves the data in a file (preserves the original dataset) and drops the irrelevant observations before calling the user-written evaluator. This way, even if the evaluator does not restrict its attentions to the \$ML_samp==1 subsample, results will still be correct. Later, ml automatically restores the original dataset.
ml need not go through these machinations for method lf because the user-written evaluator calculates observation-by-observation values, and ml itself sums the components.
ml goes through these machinations if and only if the estimation sample is a subsample of the data in memory. If the estimation sample includes every observation in memory, ml does not preserve the original dataset. Thus programmers must not alter the original dataset unless they preserve the data themselves.
We recommend that interactive users of ml not specify nopreserve; the speed gain is not worth the possibility of getting incorrect results.
We recommend that programmers specify nopreserve, but only after verifying that their evaluator really does restrict its attentions solely to the $\$$ ML_samp $==1$ subsample.
collinear specifies that ml not remove the collinear variables within equations. There is no reason to leave collinear variables in place, but this option is of interest to programmers who, in their code, have already removed collinear variables and do not want ml to waste computer time checking again.
missing specifies that observations containing variables with missing values not be eliminated from the estimation sample. There are two reasons you might want to specify missing:
Programmers may wish to specify missing because, in other parts of their code, they have already eliminated observations with missing values and do not want ml to waste computer time looking again.
You may wish to specify missing if your model explicitly deals with missing values. Stata's heckman command is a good example of this. In such cases, there will be observations where missing values are allowed and other observations where they are not-where their presence should cause the observation to be eliminated. If you specify missing, it is your responsibility to specify an if exp that eliminates the irrelevant observations.
$\operatorname{lf0}\left(\#_{k} \#_{l l}\right)$ is typically used by programmers. It specifies the number of parameters and log-likelihood value of the constant-only model so that ml can report a likelihood-ratio test rather than a Wald test. These values may have been analytically determined, or they may have been determined by a previous fitting of the constant-only model on the estimation sample.
Also see the continue option directly below.
If you specify lfO() , it must be safe for you to specify the missing option, too, else how did you calculate the log likelihood for the constant-only model on the same sample? You must have identified the estimation sample, and done so correctly, so there is no reason for ml to waste time rechecking your results. All of which is to say, do not specify lfO() unless you are certain your code identifies the estimation sample correctly.
lf0(), even if specified, is ignored if vce(robust), vce(cluster clustvar), pweight, or svy is specified because, in that case, a likelihood-ratio test would be inappropriate.
continue is typically specified by programmers and does two things:
First, it specifies that a model has just been fit by either ml or some other estimation command, such as logit, and that the likelihood value stored in e(ll) and the number of parameters stored in $e(b)$ as of that instant are the relevant values of the constant-only model. The current value of the log likelihood is used to present a likelihood-ratio test unless vce(robust), vce(cluster clustvar), pweight, svy, or constraints() is specified. A likelihood-ratio test is inappropriate when vce(robust), vce(cluster clustvar), pweight, or svy is specified. We suggest using lrtest when constraints() is specified; see [R] Irtest.
Second, continue sets the initial values, $\mathbf{b}_{0}$, for the model about to be fit according to the $\mathrm{e}(\mathrm{b})$ currently stored.

The comments made about specifying missing with 1 f 0() apply equally well here.
waldtest (\#) is typically specified by programmers. By default, ml presents a Wald test, but that is overridden if the $\operatorname{lfO} 0$ () or continue option is specified. A Wald test is performed if vce (robust), vce(cluster clustvar), or pweight is specified.
waldtest (0) prevents even the Wald test from being reported.
waldtest $(-1)$ is the default. It specifies that a Wald test be performed by constraining all coefficients except the intercept to 0 in the first equation. Remaining equations are to be unconstrained. A Wald test is performed if neither $\operatorname{lf0} 0()$ nor continue was specified, and a Wald test is forced if vce(robust), vce(cluster clustvar), or pweight was specified.
waldtest ( $k$ ) for $k \leq-1$ specifies that a Wald test be performed by constraining all coefficients except intercepts to 0 in the first $|k|$ equations; remaining equations are to be unconstrained. A Wald test is performed if neither $\operatorname{lfO} 0()$ nor continue was specified, and a Wald test is forced if vce(robust), vce(cluster clustvar), or pweight was specified.
waldtest ( $k$ ) for $k \geq 1$ works like the options above, except that it forces a Wald test to be reported even if the information to perform the likelihood-ratio test is available and even if none of vce(robust), vce(cluster clustvar), or pweight was specified. waldtest ( $k$ ), $k \geq 1$, may not be specified with lf0().
obs(\#) is used mostly by programmers. It specifies that the number of observations reported and ultimately stored in $e(N)$ be \#. Ordinarily, ml works that out for itself. Programmers may want to specify this option when, for the likelihood evaluator to work for $N$ observations, they first had to modify the dataset so that it contained a different number of observations.
crittype (string) is used mostly by programmers. It allows programmers to supply a string (up to 32 characters long) that describes the criterion that is being optimized by ml. The default is "log likelihood" for nonrobust and "log pseudolikelihood" for robust estimation.
svy indicates that ml is to pick up the svy settings set by svyset and use the robust variance estimator. This option requires the data to be svyset; see [SVY] svyset. svy may not be specified with vce() or weights.
subpop (varname) specifies that estimates be computed for the single subpopulation defined by the observations for which varname $\neq 0$. Typically, varname $=1$ defines the subpopulation, and varname $=0$ indicates observations not belonging to the subpopulation. For observations whose subpopulation status is uncertain, varname should be set to missing ('.'). This option requires the svy option.
nosvyadjust specifies that the model Wald test be carried out as $W / k \sim F(k, d)$, where $W$ is the Wald test statistic, $k$ is the number of terms in the model excluding the constant term, $d$ is the total number of sampled PSUs minus the total number of strata, and $F(k, d)$ is an $F$ distribution with $k$ numerator degrees of freedom and $d$ denominator degrees of freedom. By default, an adjusted

Wald test is conducted: $(d-k+1) W /(k d) \sim F(k, d-k+1)$. See Korn and Graubard (1990) for a discussion of the Wald test and the adjustments thereof. This option requires the svy option.
technique (algorithm_spec) specifies how the likelihood function is to be maximized. The following algorithms are currently implemented in ml. For details, see Gould, Pitblado, and Poi (2010).
technique ( nr ) specifies Stata's modified Newton-Raphson (NR) algorithm.
technique (bhhh) specifies the Berndt-Hall-Hall-Hausman (BHHH) algorithm.
technique (dfp) specifies the Davidon-Fletcher-Powell (DFP) algorithm.
technique (bfgs) specifies the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm.
The default is technique ( nr ).
You can switch between algorithms by specifying more than one in the technique() option. By default, ml will use an algorithm for five iterations before switching to the next algorithm. To specify a different number of iterations, include the number after the technique in the option. For example, technique (bhhh 10 nr 1000 ) requests that ml perform 10 iterations using the BHHH algorithm, followed by 1,000 iterations using the NR algorithm, and then switch back to BHHH for 10 iterations, and so on. The process continues until convergence or until reaching the maximum number of iterations.

## Options for use with ml model in noninteractive mode

The following extra options are for use with ml model in noninteractive mode. Noninteractive mode is for programmers who use ml as a subroutine and want to issue one command that will carry forth the estimation from start to finish.
maximize is required. It specifies noninteractive mode.
init(ml_init_args) sets the initial values, $\mathbf{b}_{0}$. ml_init_args are whatever you would type after the ml init command.
search (on|norescale|quietly|off) specifies whether ml search is to be used to improve the initial values. search (on) is the default and is equivalent to separately running ml search, repeat (0). search (norescale) is equivalent to separately running ml search, repeat (0) norescale. search (quietly) is equivalent to search(on), except that it suppresses ml search's output. search (off) prevents calling ml search.
repeat (\#) is ml search's repeat() option. repeat ( 0 ) is the default.
bounds ( $m l$ _search_bounds) specifies the search bounds. ml_search_bounds is specified as
[eqn_name] lower_bound upper_bound . . [eqn_name] lower_bound upper_bound for instance, bounds (100 100 lnsigma 010 ). The ml model command issues ml search ml_search_bounds, repeat(\#). Specifying search bounds is optional.
nowarning, novce, negh, and score() are ml maximize's equivalent options.
maximize_options: difficult, technique (algorithm_spec), iterate (\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance(\#), nonrtolerance, and from (init_specs); see [R] maximize. These options are seldom used.

## Options for use when specifying equations

noconstant specifies that the equation not include an intercept.
offset (varname ${ }_{o}$ ) specifies that the equation be $\mathbf{x b}+$ varname $_{o}$ - that it include varname ${ }_{o}$ with coefficient constrained to be 1 .
exposure (varname $e_{e}$ ) is an alternative to offset (varname ${ }_{o}$ ); it specifies that the equation be $\mathbf{x b}+\ln \left(\right.$ varname $\left._{e}\right)$. The equation is to include $\ln \left(\right.$ varname $\left._{e}\right)$ with coefficient constrained to be 1.

## Options for use with ml search

repeat (\#) specifies the number of random attempts that are to be made to find a better initial-value vector. The default is repeat (10).
repeat ( 0 ) specifies that no random attempts be made. More precisely, repeat (0) specifies that no random attempts be made if the first initial-value vector is a feasible starting point. If it is not, ml search will make random attempts, even if you specify repeat (0), because it has no alternative. The repeat () option refers to the number of random attempts to be made to improve the initial values. When the initial starting value vector is not feasible, ml search will make up to 1,000 random attempts to find starting values. It stops when it finds one set of values that works and then moves into its improve-initial-values logic.
repeat $(k), k>0$, specifies the number of random attempts to be made to improve the initial values.
restart specifies that random actions be taken to obtain starting values and that the resulting starting values not be a deterministic function of the current values. Generally, you should not specify this option because, with restart, ml search intentionally does not produce as good a set of starting values as it could. restart is included for use by the optimizer when it gets into serious trouble. The random actions ensure that the optimizer and ml search, working together, do not cause an endless loop.
restart implies norescale, which is why we recommend that you do not specify restart. In testing, sometimes rescale worked so well that, even after randomization, the rescaler would bring the starting values right back to where they had been the first time and thus defeat the intended randomization.
norescale specifies that ml search not engage in its rescaling actions to improve the parameter vector. We do not recommend specifying this option because rescaling tends to work so well.
maximize_options: [ㅡㅡ] log and trace; see [R] maximize. These options are seldom used.

## Option for use with ml plot

saving (filename[, replace]) specifies that the graph be saved in filename.gph.
See [G-3] saving_option.

## Options for use with ml init

copy specifies that the list of numbers or the initialization vector be copied into the initial-value vector by position rather than by name.
skip specifies that any parameters found in the specified initialization vector that are not also found in the model be ignored. The default action is to issue an error message.

## Options for use with ml maximize

nowarning is allowed only with iterate(0). nowarning suppresses the "convergence not achieved" message. Programmers might specify iterate ( 0 ) nowarning when they have a vector $\mathbf{b}$ already containing the final estimates and want ml to calculate the variance matrix and postestimation results. Then specify init(b) search(off) iterate(0) nowarning nolog.
novce is allowed only with iterate (0). novce substitutes the zero matrix for the variance matrix, which in effect posts estimation results as fixed constants.
negh indicates that the evaluator returns the negative Hessian matrix. By default, ml assumes d2 and lf2 evaluators return the Hessian matrix.
score (newvars $\mid$ stub*) creates new variables containing the contributions to the score for each equation and ancillary parameter in the model; see [U] 20.22 Obtaining scores.
If score(newvars) is specified, the newvars must contain $k$ new variables. For evaluators of methods lf, lf0, lf1, and lf2, $k$ is the number of equations. For evaluators of method gf0, $k$ is the number of parameters. If score (stub*) is specified, variables named stub1, stub2, ..., stubk are created.

For evaluators of methods lf, lf0, lf1, and lf2, the first variable contains $\partial \ln \ell_{j} / \partial\left(\mathbf{x}_{1 j} \mathbf{b}_{1}\right)$, the second variable contains $\partial \ln \ell_{j} / \partial\left(\mathbf{x}_{2 j} \mathbf{b}_{2}\right)$, and so on.
For evaluators of method gf0, the first variable contains $\partial \ln \ell_{j} / \partial \mathbf{b}_{1}$, the second variable contains $\partial \ln \ell_{j} / \partial \mathbf{b}_{2}$, and so on.
nooutput suppresses display of results. This option is different from prefixing ml maximize with quietly in that the iteration $\log$ is still displayed (assuming that nolog is not specified).
noclear specifies that the ml problem definition not be cleared after the model has converged. Perhaps you are having convergence problems and intend to run the model to convergence. If so, use ml search to see if those values can be improved, and then restart the estimation.
maximize_options: difficult, iterate(\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance(\#), nonrtolerance; see [R] maximize. These options are seldom used.
display_options; see Options for use with ml display below.
eform_option; see Options for use with ml display below.

## Option for use with ml graph

saving (filename[, replace]) specifies that the graph be saved in filename.gph. See [G-3] saving_option.

## Options for use with ml display

noheader suppresses the header display above the coefficient table that displays the final log-likelihood value, the number of observations, and the model significance test.
nofootnote suppresses the footnote display below the coefficient table, which displays a warning if the model fit did not converge within the specified number of iterations. Use ml footnote to display the warning if 1) you add to the coefficient table using the plus option or 2) you have your own footnotes and want the warning to be last.
level(\#) is the standard confidence-level option. It specifies the confidence level, as a percentage, for confidence intervals of the coefficients. The default is level (95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals.
first displays a coefficient table reporting results for the first equation only, and the report makes it appear that the first equation is the only equation. This option is used by programmers who estimate ancillary parameters in the second and subsequent equations and who wish to report the values of such parameters themselves.
neq(\#) is an alternative to first. neq(\#) displays a coefficient table reporting results for the first \# equations. This option is used by programmers who estimate ancillary parameters in the \# + 1 and subsequent equations and who wish to report the values of such parameters themselves.
showeqns is a seldom-used option that displays the equation names in the coefficient table. ml display uses the numbers stored in $e\left(k_{-} e q\right)$ and $e\left(k \_a u x\right)$ to determine how to display the coefficient table. e(k_eq) identifies the number of equations, and e( $k$ _aux) identifies how many of these are for ancillary parameters. The first option is implied when showeqns is not specified and all but the first equation are for ancillary parameters.
plus displays the coefficient table, but rather than ending the table in a line of dashes, ends it in dashes-plus-sign-dashes. This is so that programmers can write additional display code to add more results to the table and make it appear as if the combined result is one table. Programmers typically specify plus with the first or neq() options. This option implies nofootnote.
nocnsreport suppresses the display of constraints above the coefficient table. This option is ignored if constraints were not used to fit the model.
noomitted specifies that variables that were omitted because of collinearity not be displayed. The default is to include in the table any variables omitted because of collinearity and to label them as "(omitted)".
vsquish specifies that the blank space separating factor-variable terms or time-series-operated variables from other variables in the model be suppressed.
noemptycells specifies that empty cells for interactions of factor variables not be displayed. The default is to include in the table interaction cells that do not occur in the estimation sample and to label them as "(empty)".
baselevels and allbaselevels control whether the base levels of factor variables and interactions are displayed. The default is to exclude from the table all base categories.
baselevels specifies that base levels be reported for factor variables and for interactions whose bases cannot be inferred from their component factor variables.
allbaselevels specifies that all base levels of factor variables and interactions be reported.
cformat ( $\%$ fmt) specifies how to format coefficients, standard errors, and confidence limits in the coefficient table.
pformat (\%fmt) specifies how to format $p$-values in the coefficient table.
sformat ( $\%$ fmt) specifies how to format test statistics in the coefficient table.
nolstretch specifies that the width of the coefficient table not be automatically widened to accommodate longer variable names. The default, lstretch, is to automatically widen the coefficient table up to the width of the Results window. To change the default, use set lstretch off. nolstretch is not shown in the dialog box.
coeflegend specifies that the legend of the coefficients and how to specify them in an expression be displayed rather than displaying the statistics for the coefficients.
eform_option: eform(string), eform, hr, shr, irr, or, and rrr display the coefficient table in exponentiated form: for each coefficient, $\exp (b)$ rather than $b$ is displayed, and standard errors and confidence intervals are transformed. string is the table header that will be displayed above the transformed coefficients and must be 11 characters or shorter in length-for example, eform("Odds ratio"). The options eform, hr, shr, irr, or, and rrr provide a default string equivalent to "exp(b)", "Haz. Ratio", "SHR", "IRR", "Odds Ratio", and "RRR", respectively. These options may not be combined.
ml display looks at e (k_eform) to determine how many equations are affected by an eform_option; by default, only the first equation is affected. Type ereturn list, all to view e(k_eform); see [P] ereturn.

## Options for use with mleval

eq(\#) specifies the equation number, $i$, for which $\theta_{i j}=\mathbf{x}_{i j} \mathbf{b}_{i}$ is to be evaluated. eq(1) is assumed if eq() is not specified.
scalar asserts that the $i$ th equation is known to evaluate to a constant, meaning that the equation was specified as (), (name:), or /name on the ml model statement. If you specify this option, the new variable created is created as a scalar. If the $i$ th equation does not evaluate to a scalar, an error message is issued.

## Option for use with mlsum

noweight specifies that weights (\$ML_w) be ignored when summing the likelihood function.

## Option for use with mlvecsum

eq(\#) specifies the equation for which a gradient vector $\partial \ln L / \partial \mathbf{b}_{i}$ is to be constructed. The default is eq(1).

## Option for use with mlmatsum

eq(\#[,\#]) specifies the equations for which the Hessian matrix is to be constructed. The default is eq (1), which is the same as eq $(1,1)$, which means $\partial^{2} \ln L / \partial \mathbf{b}_{1} \partial \mathbf{b}_{1}^{\prime}$. Specifying eq $(i, j)$ results in $\partial^{2} \ln L / \partial \mathbf{b}_{i} \partial \mathbf{b}_{j}^{\prime}$.

## Options for use with mimatbysum

by (varname) is required and specifies the group variable.
eq (\#[,\#]) specifies the equations for which the Hessian matrix is to be constructed. The default is eq (1), which is the same as eq $(1,1)$, which means $\partial^{2} \ln L / \partial \mathbf{b}_{1} \partial \mathbf{b}_{1}^{\prime}$. Specifying eq $(i, j)$ results in $\partial^{2} \ln L / \partial \mathbf{b}_{i} \partial \mathbf{b}_{j}^{\prime}$.

## Options for use with ml score

equation(eqname) identifies from which equation the observation scores are to come. This option may be used only when generating one variable.
missing specifies that observations containing variables with missing values not be eliminated from the estimation sample.

## Remarks and examples

For a thorough discussion of ml, see the fourth edition of Maximum Likelihood Estimation with Stata (Gould, Pitblado, and Poi 2010). The book provides a tutorial introduction to ml , notes on advanced programming issues, and a discourse on maximum likelihood estimation from both theoretical and practical standpoints. See Survey options and ml at the end of Remarks and examples for examples of the new svy options. For more information about survey estimation, see [SVY] survey, [SVY] svy estimation, and [SVY] variance estimation.
ml requires that you write a program that evaluates the log-likelihood function and, possibly, its first and second derivatives. The style of the program you write depends upon the method you choose. Methods lf, lf0, d0, and gf0 require that your program evaluate the log likelihood only. Methods d1 and lf1 require that your program evaluate the log likelihood and its first derivatives. Methods d2 and lf2 requires that your program evaluate the log likelihood and its first and second derivatives. Methods lf, lf0, d0, and gf0 differ from each other in that, with methods if and lf0, your program is required to produce observation-by-observation $\log$-likelihood values $\ln \ell_{j}$ and it is assumed that $\ln L=\sum_{j} \ln \ell_{j}$; with method d0, your program is required to produce only the overall value $\ln L$; and with method gf0, your program is required to produce the summable pieces of the log likelihood, such as those in panel-data models.

Once you have written the program-called an evaluator-you define a model to be fit using ml model and obtain estimates using ml maximize. You might type

```
. ml model ...
. ml maximize
```

but we recommend that you type

```
. ml model ...
. ml check
. ml search
. ml maximize
```

ml check verifies your evaluator has no obvious errors, and ml search finds better initial values.
You fill in the ml model statement with 1) the method you are using, 2) the name of your program, and 3) the "equations". You write your evaluator in terms of $\theta_{1}, \theta_{2}, \ldots$, each of which has a linear equation associated with it. That linear equation might be as simple as $\theta_{i}=b_{0}$, it might be $\theta_{i}=b_{1} \mathrm{mpg}+b_{2}$ weight $+b_{3}$, or it might omit the intercept $b_{3}$. The equations are specified in parentheses on the ml model line.

Suppose that you are using method lf and the name of your evaluator program is myprog. The statement

```
. ml model lf myprog (mpg weight)
```

would specify one equation with $\theta_{i}=b_{1} \mathrm{mpg}+b_{2}$ weight $+b_{3}$. If you wanted to omit $b_{3}$, you would type
. ml model lf myprog (mpg weight, nocons)
and if all you wanted was $\theta_{i}=b_{0}$, you would type
. ml model lf myprog ()
With multiple equations, you list the equations one after the other; so, if you typed

```
. ml model lf myprog (mpg weight) ()
```

you would be specifying $\theta_{1}=b_{1} \mathrm{mpg}+b_{2}$ weight $+b_{3}$ and $\theta_{2}=b_{4}$. You would write your likelihood in terms of $\theta_{1}$ and $\theta_{2}$. If the model was linear regression, $\theta_{1}$ might be the $\mathbf{x b}$ part and $\theta_{2}$ the variance of the residuals.

When you specify the equations, you also specify any dependent variables. If you typed

```
. ml model lf myprog (price = mpg weight) ()
```

price would be the one and only dependent variable, and that would be passed to your program in $\$ M L-y 1$. If your model had two dependent variables, you could type

```
. ml model lf myprog (price displ = mpg weight) ()
```

Then $\$$ ML_y 1 would be price and $\$$ ML_y 2 would be displ. You can specify however many dependent variables are necessary and specify them on any equation. It does not matter on which equation you specify them; the first one specified is placed in ${ }^{(M L}-y 1$, the second in \$ML_y2, and so on.

## $>$ Example 1: Method If

Using method lf, we want to produce observation-by-observation values of the log likelihood. The probit log-likelihood function is

$$
\begin{aligned}
\ln \ell_{j} & = \begin{cases}\ln \Phi\left(\theta_{1 j}\right) & \text { if } y_{j}=1 \\
\ln \Phi\left(-\theta_{1 j}\right) & \text { if } y_{j}=0\end{cases} \\
\theta_{1 j} & =\mathbf{x}_{j} \mathbf{b}_{1}
\end{aligned}
$$

The following is the method-lf evaluator for this likelihood function:

```
program myprobit
    version 13
    args lnf theta1
    quietly replace 'lnf' = ln(normal('theta1')) if $ML_y1==1
    quietly replace 'lnf' = ln(normal(-'theta1')) if $ML_y1==0
end
```

If we wanted to fit a model of foreign on mpg and weight, we would type the following commands. The 'foreign =' part specifies that $y$ is foreign. The 'mpg weight' part specifies that $\theta_{1 j}=b_{1} \mathrm{mpg}_{j}+b_{2}$ weight $_{j}+b_{3}$.

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. ml model lf myprobit (foreign = mpg weight)
. ml maximize
initial: log likelihood = -51.292891
alternative: log likelihood = -45.055272
rescale: log likelihood = -45.055272
Iteration 0: log likelihood = -45.055272
Iteration 1: log likelihood = -27.905385
Iteration 2: log likelihood = -26.858058
Iteration 3: log likelihood = -26.844198
Iteration 4: log likelihood = -26.844189
Iteration 5: log likelihood = -26.844189
```

| Number of obs | $=$ | 74 |
| :--- | :--- | ---: |
| Wald chi2(2) | $=$ | 20.75 |
| Prob > chi2 | $=$ | 0.0000 |


| foreign | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| mpg | -.1039503 | .0515689 | -2.02 | 0.044 | -.2050235 | -.0028772 |
| weight | -.0023355 | .0005661 | -4.13 | 0.000 | -.003445 | -.0012261 |
| _cons | 8.275464 | 2.554142 | 3.24 | 0.001 | 3.269438 | 13.28149 |

## Example 2: Method If for two-equation, two-dependent-variable model

A two-equation, two-dependent-variable model is a little different. Rather than receiving one $\theta$, our program will receive two. Rather than there being one dependent variable in \$ML_y1, there will be dependent variables in \$ML_y1 and \$ML_y2. For instance, the Weibull regression log-likelihood function is

$$
\begin{aligned}
\ln \ell_{j} & =-\left(t_{j} e^{-\theta_{1 j}}\right)^{\exp \left(\theta_{2 j}\right)}+d_{j}\left\{\theta_{2 j}-\theta_{1 j}+\left(e^{\theta_{2 j}}-1\right)\left(\ln t_{j}-\theta_{1 j}\right)\right\} \\
\theta_{1 j} & =\mathbf{x}_{j} \mathbf{b}_{1} \\
\theta_{2 j} & =s
\end{aligned}
$$

where $t_{j}$ is the time of failure or censoring and $d_{j}=1$ if failure and 0 if censored. We can make the log likelihood a little easier to program by introducing some extra variables:

$$
\begin{aligned}
p_{j} & =\exp \left(\theta_{2 j}\right) \\
M_{j} & =\left\{t_{j} \exp \left(-\theta_{1 j}\right)\right\}^{p_{j}} \\
R_{j} & =\ln t_{j}-\theta_{1 j} \\
\ln \ell_{j} & =-M_{j}+d_{j}\left\{\theta_{2 j}-\theta_{1 j}+\left(p_{j}-1\right) R_{j}\right\}
\end{aligned}
$$

The method-lf evaluator for this is

```
program myweib
    version 13
    args lnf theta1 theta2
    tempvar p M R
    quietly gen double 'p' = exp('theta2')
    quietly gen double 'M' = ($ML_y1*exp(-'theta1')) ^'p'
    quietly gen double 'R' = ln($ML_y1)-'theta1'
    quietly replace 'lnf' = -'M' + $ML_y2*('theta2'-'theta1' + ('p'-1)*'R')
end
```

We can fit a model by typing

```
. ml model lf myweib (studytime died = i.drug age) ()
. ml maximize
```

Note that we specified '()' for the second equation. The second equation corresponds to the Weibull shape parameter $s$, and the linear combination we want for $s$ contains just an intercept. Alternatively, we could type

```
. ml model lf myweib (studytime died = i.drug age) /s
```

Typing /s means the same thing as typing ( $\mathrm{s}:$ ), and both really mean the same thing as (). The $s$, either after a slash or in parentheses before a colon, labels the equation. It makes the output look prettier, and that is all:

```
. use http://www.stata-press.com/data/r13/cancer, clear
(Patient Survival in Drug Trial)
. ml model lf myweib (studytime died = i.drug age) /s
. ml maximize
initial: log likelihood = -744
alternative: log likelihood = -356.14276
rescale: log likelihood = -200.80201
rescale eq: log likelihood = -136.69232
Iteration 0: log likelihood = -136.69232 (not concave)
Iteration 1: log likelihood = -124.11726
Iteration 2: log likelihood = -113.91566
Iteration 3: log likelihood = -110.30559
Iteration 4: log likelihood = -110.26747
Iteration 5: log likelihood = -110.26736
Iteration 6: log likelihood = -110.26736
```

| Number of obs | $=$ | 48 |
| :--- | :--- | ---: |
| Wald chi2 $(3)$ | $=$ | 35.25 |
| Prob $>$ chi2 | $=$ | 0.0000 |


|  |  | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| eq1 |  |  |  |  |  |  |  |
|  | drug |  |  |  |  |  |  |
|  | 2 | 1.012966 | .2903917 | 3.49 | 0.000 | .4438086 | 1.582123 |
|  | 1.45917 | .2821195 | 5.17 | 0.000 | .9062261 | 2.012114 |  |
|  | age | -.0671728 | .0205688 | -3.27 | 0.001 | -.1074868 | -.0268587 |
|  | _cons | 6.060723 | 1.152845 | 5.26 | 0.000 | 3.801188 | 8.320259 |
| s |  |  |  |  |  |  |  |
|  | _cons | .5573333 | .1402154 | 3.97 | 0.000 | .2825162 | .8321504 |

## Example 3: Method d0

Method-d0 evaluators receive $\mathbf{b}=\left(\mathbf{b}_{1}, \mathbf{b}_{\mathbf{2}}, \ldots, \mathbf{b}_{E}\right)$, the coefficient vector, rather than the already evaluated $\theta_{1}, \theta_{2}, \ldots, \theta_{E}$, and they are required to evaluate the overall $\log$-likelihood $\ln L$ rather than $\ln \ell_{j}, j=1, \ldots, N$.

Use mleval to produce the thetas from the coefficient vector.
Use mlsum to sum the components that enter into $\ln L$.

In the case of Weibull, $\ln L=\sum \ln \ell_{j}$, and our method-d0 evaluator is

```
program weib0
    version 13
    args todo b lnf
    tempvar theta1 theta2
    mleval 'theta1' = 'b', eq(1)
    mleval 'theta2' = 'b', eq(2)
    local t "$ML_y1" // this is just for readability
    local d "$ML_y2"
    tempvar p M R
    quietly gen double 'p' = exp('theta2')
    quietly gen double 'M' = ('t'*exp(-'theta1')) ' 'p'
    quietly gen double 'R' = ln('t')-'theta1'
    mlsum 'lnf' = -'M' + 'd'*('theta2'-'theta1' + ('p'-1)*'R')
end
```

To fit our model using this evaluator, we would type

```
. ml model dO weibO (studytime died = i.drug age) /s
. ml maximize
```


## - Technical note

Method d0 does not require $\ln L=\sum_{j} \ln \ell_{j}, j=1, \ldots, N$, as method if does. Your likelihood function might have independent components only for groups of observations. Panel-data estimators have a $\log$-likelihood value $\ln L=\sum_{i} \ln L_{i}$, where $i$ indexes the panels, each of which contains multiple observations. Conditional logistic regression has $\ln L=\sum_{k} \ln L_{k}$, where $k$ indexes the risk pools. Cox regression has $\ln L=\sum_{(t)} \ln L_{(t)}$, where $(t)$ denotes the ordered failure times.

To evaluate such likelihood functions, first calculate the within-group log-likelihood contributions. This usually involves generate and replace statements prefixed with by, as in

```
tempvar sumd
by group: gen double 'sumd' = sum($ML_y1)
```

Structure your code so that the log-likelihood contributions are recorded in the last observation of each group. Say that a variable is named 'cont'. To sum the contributions, code

```
tempvar last
quietly by group: gen byte 'last' = (_n==_N)
mlsum 'lnf' = 'cont' if 'last'
```

You must inform mlsum which observations contain log-likelihood values to be summed. First, you do not want to include intermediate results in the sum. Second, mlsum does not skip missing values. Rather, if mlsum sees a missing value among the contributions, it sets the overall result, 'lnf', to missing. That is how ml maximize is informed that the likelihood function could not be evaluated at the particular value of $\mathbf{b} . \mathrm{ml}$ maximize will then take action to escape from what it thinks is an infeasible area of the likelihood function.

When the likelihood function violates the linear-form restriction $\ln L=\sum_{j} \ln \ell_{j}, j=1, \ldots, N$, with $\ln \ell_{j}$ being a function solely of values within the $j$ th observation, use method d 0 . In the following examples, we will demonstrate methods d 1 and d 2 with likelihood functions that meet this linear-form restriction. The d 1 and d 2 methods themselves do not require the linear-form restriction, but the utility routines mlvecsum and mlmatsum do. Using method d1 or d2 when the restriction is violated is difficult; however, mlmatbysum may be of some help for method-d2 evaluators.

## > Example 4: Method d1

Method-d1 evaluators are required to produce the gradient vector $\mathbf{g}=\partial \ln L / \partial \mathbf{b}$, as well as the overall log-likelihood value. Using mlvecsum, we can obtain $\partial \ln L / \partial \mathbf{b}$ from $\partial \ln L / \partial \theta_{i}, i=$ $1, \ldots, E$. The derivatives of the Weibull log-likelihood function are

$$
\begin{aligned}
\frac{\partial \ln \ell_{j}}{\partial \theta_{1 j}} & =p_{j}\left(M_{j}-d_{j}\right) \\
\frac{\partial \ln \ell_{j}}{\partial \theta_{2 j}} & =d_{j}-R_{j} p_{j}\left(M_{j}-d_{j}\right)
\end{aligned}
$$

The method-d1 evaluator for this is

```
program weib1
    version 13
    args todo b lnf g // g is new
    tempvar t1 t2
    mleval 't1' = 'b', eq(1)
    mleval 't2' = 'b', eq(2)
    local t "$ML_y1"
    local d "$ML_y2"
    tempvar p M R
    quietly gen double 'p' = exp('t2')
    quietly gen double 'M' = ('t'*exp(-'t1'))^'p'
    quietly gen double 'R' = ln('t')-'t1'
    mlsum 'lnf' = -'M' + 'd'*('t2'-'t1' + ('p'-1)*'R')
    if ('todo'==0 | 'lnf'>=.) exit /* <-- new */
    tempname d1 d2 /* <-- new */
    mlvecsum 'lnf' 'd1' = 'p'*('M'-'d'), eq(1) /* <-- new */
    mlvecsum 'lnf' 'd2' = 'd' - 'R'*'p'*('M'-'d'), eq(2) /* <-- new */
    matrix 'g' = ('d1','d2') /* <-- new */
end
```

We obtained this code by starting with our method-d0 evaluator and then adding the extra lines that method d1 requires. To fit our model using this evaluator, we could type

```
. ml model d1 weib1 (studytime died = drug2 drug3 age) /s
```

. ml maximize
but we recommend substituting method d1debug for method d1 and typing

```
. ml model d1debug weib1 (studytime died = drug2 drug3 age) /s
. ml maximize
```

Method d1debug will compare the derivatives we calculate with numerical derivatives and thus verify that our program is correct. Once we are certain the program is correct, then we would switch from method d1debug to method d1.

## Example 5: Method d2

Method-d2 evaluators are required to produce $\mathbf{H}=\partial^{2} \ln L / \partial \mathbf{b} \partial \mathbf{b}^{\prime}$, the Hessian matrix, as well as the gradient and log-likelihood value. mlmatsum will help calculate $\partial^{2} \ln L / \partial \mathbf{b} \partial \mathbf{b}^{\prime}$ from the second derivatives with respect to $\theta$. For the Weibull model, these second derivatives are

$$
\begin{aligned}
\frac{\partial^{2} \ln \ell_{j}}{\partial \theta_{1 j}^{2}} & =-p_{j}^{2} M_{j} \\
\frac{\partial^{2} \ln \ell_{j}}{\partial \theta_{1 j} \partial \theta_{2 j}} & =p_{j}\left(M_{j}-d_{j}+R_{j} p_{j} M_{j}\right) \\
\frac{\partial^{2} \ln \ell_{j}}{\partial \theta_{2 j}^{2}} & =-p_{j} R_{j}\left(R_{j} p_{j} M_{j}+M_{j}-d_{j}\right)
\end{aligned}
$$

The method-d2 evaluator is

```
program weib2
    version 13
    args todo b lnf g H // H added
    tempvar t1 t2
    mleval 't1' = 'b', eq(1)
    mleval 't2' = 'b', eq(2)
    local t "$ML_y1"
    local d "$ML_y2"
    tempvar p M R
    quietly gen double 'p' = exp('t2')
    quietly gen double 'M' = ('t'*exp(-'t1'))^'p'
    quietly gen double 'R' = ln('t')-'t1'
    mlsum 'lnf' = -'M' + 'd'*('t2'-'t1' + ('p'-1)*'R')
    if ('todo'==0 | 'lnf'>=.) exit
    tempname d1 d2
    mlvecsum 'lnf' 'd1' = 'p'*('M'-'d'), eq(1)
    mlvecsum 'lnf' 'd2' = 'd' - 'R'*'p'*('M'-'d'), eq(2)
    matrix 'g' = ('d1','d2')
    if ('todo'==1 | 'lnf'>=.) exit // new from here down
    tempname d11 d12 d22
    mlmatsum 'lnf' 'd11' = -'p'^2 * 'M', eq(1)
    mlmatsum 'lnf' 'd12' = 'p'*('M'-'d' + 'R'*'p'*'M'), eq(1,2)
    mlmatsum 'lnf, 'd22' = -'p'*'R'*('R'*'p'*'M' + 'M' - 'd') , eq(2)
    matrix 'H' = ('d11','d12' \ 'd12'','d22')
end
```

We started with our previous method-d1 evaluator and added the lines that method d2 requires. We could now fit a model by typing
. ml model d2 weib2 (studytime died = drug2 drug3 age) /s
. ml maximize
but we would recommend substituting method d2debug for method d2 and typing
. ml model d2debug weib2 (studytime died = drug2 drug3 age) /s
. ml maximize
Method d2debug will compare the first and second derivatives we calculate with numerical derivatives and thus verify that our program is correct. Once we are certain the program is correct, then we would switch from method d2debug to method d2.

As we stated earlier, to produce the robust variance estimator with method lf, there is nothing to do except specify vce(robust), vce(cluster clustvar), or pweight. For methods d0, d1, and d2, these options do not work. If your likelihood function meets the linear-form restrictions, you can use methods lf0, lf1, and lf2, then these options will work. The equation scores are defined as

$$
\frac{\partial \ln \ell_{j}}{\partial \theta_{1 j}}, \frac{\partial \ln \ell_{j}}{\partial \theta_{2 j}}, \ldots
$$

Your evaluator will be passed variables, one for each equation, which you fill in with the equation scores. For both method lf1 and lf2, these variables are passed in the fourth and subsequent positions of the argument list. That is, you must process the arguments as

```
args todo b lnf g1 g2 ... H
```

Note that for method lf1, the ' H ' argument is not used and can be ignored.

## > Example 6: Robust variance estimates

If you have used mlvecsum in your evaluator of method d1 or d2, it is easy to turn it into evaluator of method lf1 or lf2 that allows the computation of the robust variance estimator. The expression that you specified on the right-hand side of mlvecsum is the equation score.

Here we turn the program that we gave earlier in the method-d1 example into a method-lf1 evaluator that allows vce(robust), vce(cluster clustvar), or pweight.

```
program weib1
    version 13
    args todo b lnfj g1 g2 // g1 and g2 are new
    tempvar t1 t2
    mleval 't1' = 'b', eq(1)
    mleval 't2' = 'b', eq(2)
    local t "$ML_y1"
    local d "$ML_y2"
    tempvar p M R
    quietly gen double 'p' = exp('t2')
    quietly gen double 'M' = ('t'*exp(-'t1')) '(p'
    quietly gen double 'R' = ln('t')-'t1'
    quietly replace 'lnfj' = -'M' + 'd'*('t2'-'t1' + ('p'-1)*'R')
    if ('todo'==0) exit
    quietly replace 'g1' = 'p'*('M'-'d') /* <-- new */
    quietly replace 'g2' = 'd' - 'R'*'p'*('M'-'d') /* <-- new */
end
```

To fit our model and get the robust variance estimates, we type

```
. ml model lf1 weib1 (studytime died = drug2 drug3 age) /s, vce(robust)
. ml maximize
```


## Survey options and ml

ml can handle stratification, poststratification, multiple stages of clustering, and finite population corrections. Specifying the svy option implies that the data come from a survey design and also implies that the survey linearized variance estimator is to be used; see [SVY] variance estimation.

## $>$ Example 7

Suppose that we are interested in a probit analysis of data from a survey in which q1 is the answer to a yes/no question and x 1 , x 2 , x 3 are demographic responses. The following is a lf 2 evaluator for the probit model that meets the requirements for vce(robust) (linear form and computes the scores).

```
program mylf2probit
    version 13
    args todo b lnfj g1 H
    tempvar z Fz lnf
    mleval 'z' = 'b'
    quietly gen double 'Fz' = normal( 'z') if $ML_y1 == 1
    quietly replace 'Fz' = normal(-'z') if $ML_y1 == 0
    quietly replace ''lnfj' = log('Fz')
    if ('todo'==0) exit
    quietly replace 'g1' = normalden('z')/'Fz' if $ML_y1 == 1
    quietly replace 'g1' = -normalden('z')/'Fz' if $ML_y1 == 0
    if ('todo'==1) exit
    mlmatsum 'lnf' 'H' = -'g1'*('g1'+'z'), eq(1,1)
end
```

To fit a model, we svyset the data, then use svy with ml .

```
. svyset psuid [pw=w], strata(strid)
. ml model lf2 mylf2probit (q1 = x1 x2 x3), svy
. ml maximize
```

We could also use the subpop() option to make inferences about the subpopulation identified by the variable sub:

```
. svyset psuid [pw=w], strata(strid)
```

. ml model lf2 mylf2probit (q1 = x1 x2 x3), svy subpop(sub)
. ml maximize

## Stored results

For results stored by ml without the svy option, see [R] maximize.
For results stored by ml with the svy option, see [SVY] svy.

## Methods and formulas

ml is implemented using moptimize(); see [M-5] moptimize().

## References

Gould, W. W., J. S. Pitblado, and B. P. Poi. 2010. Maximum Likelihood Estimation with Stata. 4th ed. College Station, TX: Stata Press.
Korn, E. L., and B. I. Graubard. 1990. Simultaneous testing of regression coefficients with complex survey data: Use of Bonferroni $t$ statistics. American Statistician 44: 270-276.

Royston, P. 2007. Profile likelihood for estimation and confidence intervals. Stata Journal 7: 376-387.

## Also see

[ R ] maximize - Details of iterative maximization
[R] mlexp - Maximum likelihood estimation of user-specified expressions
[R] nl - Nonlinear least-squares estimation
[M-5] moptimize( ) - Model optimization
[M-5] optimize( ) - Function optimization

## Title

mlexp - Maximum likelihood estimation of user-specified expressions

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

Also see

## Syntax

$$
\operatorname{mlexp}(<\text { lexp }>) \ldots[\text { if }][\text { in }][\text { weight }][\text {, options }]
$$

where
$<$ lexp $>$ is a substitutable expression representing the log-likelihood function.

## options

Description
Model
variables (varlist)
from(initial_values)

## Derivatives

derivative(/name $=<$ dexp $>$ )

SE/Robust
vce (vcetype)

Reporting
level(\#)
title(string)
title2(string)
display_options
Maximization
maximize_options
coeflegend
specify variables in model
specify initial values for parameters
specify derivative of $<$ lexp $>$ with respect to parameter name; can be specified more than once
vcetype may be oim, opg, robust, cluster clustvar, bootstrap, or jackknife
<lexp> may contain time-series operators; see [U] 13.9 Time-series operators.
bootstrap, by, jackknife, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the bootstrap prefix; see $[R]$ bootstrap.
aweights are not allowed with the jackknife prefix; see [R] jackknife.
aweights, fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.
$<$ lexp $>$ and $<\operatorname{dexp}>$ are extensions of valid Stata expressions that also contain parameters to be estimated. The parameters are enclosed in curly braces and must otherwise satisfy the naming requirements for variables; \{beta\} is an example of a parameter. Also allowed is a notation of the
form $\{<$ eqname $>$ : varlist \} for linear combinations of multiple covariates and their parameters. For example, $\{x b$ : mpg price turn\} defines a linear combination of the variables mpg, price, and turn. See Substitutable expressions under Remarks and examples below.

## Menu

Statistics $>$ Other $>$ Maximum likelihood estimation of expression

## Description

mlexp performs maximum likelihood estimation of models that satisfy the linear-form restrictions, which is to say models for which you can write down the log likelihood for an individual observation and for which the overall log likelihood is simply the sum of the individual observations' log likelihoods.

You express the observation-level log-likelihood function by using a substitutable expression. Unlike models fit using ml , you do not need to do any programming. However, ml can fit classes of models that cannot be fit by mlexp.

## Options

Model
variables (varlist) specifies the variables in the model. mlexp ignores observations for which any of these variables has missing values. If you do not specify variables(), then mlexp assumes all the observations are valid. If the log likelihood cannot be calculated at the initial values for any observation, mlexp will exit with an error message.
from(initial_values) specifies the initial values to begin the estimation. You can specify a $1 \times k$ matrix, where $k$ is the number of parameters in the model, or you can specify parameter names and values. For example, to initialize alpha to 1.23 and delta to 4.57 , you would type

```
mlexp ..., from(alpha=1.23 delta=4.57) ...
```

Initial values declared using this option override any that are declared within substitutable expressions. If you specify a parameter that does not appear in your model, mlexp exits with an error. If you specify a matrix, the values must be in the same order in which the parameters are declared in your model. mlexp ignores the row and column names of the matrix.

Derivatives
derivative(/name $=<$ dexp $>$ ) specifies the derivative of the observation-level log-likelihood function with respect to parameter name.
$<d \exp >$ uses the same substitutable expression syntax as is used to specify the log-likelihood function. If you declare a linear combination in the log-likelihood function, you provide the derivative for the linear combination; mlexp then applies the chain rule for you. See Specifying derivatives under Remarks and examples below for examples.

If you do not specify the derivative() option, mlexp calculates derivatives numerically. You must either specify no derivatives or specify all the derivatives; you cannot specify some analytic derivatives and have mlexp compute the rest numerically.
If you are estimating multiple parameters, you supply derivatives using multiple derivative() specifications.
vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
$\qquad$ Reporting
level (\#); see [R] estimation options.
title (string) specifies an optional title that will be displayed just above the table of parameter estimates.
title2 (string) specifies an optional subtitle that will be displayed between the title specified in title() and the table of parameter estimates. If title2() is specified but title() is not, then title2() has the same effect as title().
display_options: cformat ( $\%$ fmt) , pformat ( $\%$ fmt) , and sformat ( $\%$ fmt) ; see $[\mathrm{R}]$ estimation options.

Maximization
maximize_options: difficult, technique (algorithm_spec), iterate(\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance(\#), and nonrtolerance; see [R] maximize. These options are seldom used.

The following option is available with mlexp but is not shown in the dialog box: coeflegend; see $[R]$ estimation options.

## Remarks and examples

Remarks are presented under the following headings:

Introduction<br>Substitutable expressions<br>Parameter constraints<br>Specifying derivatives

## Introduction

mlexp performs maximum likelihood estimation of models that satisfy the linear-form restrictions, which is to say models for which you can write down the log likelihood for a single observation and for which the overall log likelihood is simply the sum of the individual observations' log likelihoods. Models designed for use with cross-sectional data usually meet the linear-form restrictions, including linear regression, many discrete-choice models, limited-dependent-variable models, and selection models. Examples of models that do not satisfy the linear-form restrictions are random-effects paneldata models (because the likelihood function is defined at the panel level) and Cox proportional hazards models (because the likelihood function is defined for risk sets).

Because of its straightforward syntax and accessibility from the menu system, mlexp is particularly suited to users who are new to Stata and to those using Stata for pedagogical purposes. You express the observation-level log-likelihood function by using a substitutable expression, which we explain below. Unlike models fit using ml , you do not need to do any programming. However, ml can fit classes of models that cannot be fit by mlexp, including those that do not meet the linear-form restrictions.

## Substitutable expressions

You specify the log-likelihood function that mlexp is to maximize by using substitutable expressions that are similar to those used by nl, nlsur, and gmm. You specify substitutable expressions just as you would specify any other mathematical expression involving scalars and variables, such as those expressions you would use with Stata's generate command, except that the parameters to be estimated are bound in braces. See [U] 13.2 Operators and [U] 13.3 Functions for more information on expressions. Parameter names must follow the same conventions as variable names. See [U] 11.3 Naming conventions.

For example, say that you have observations on variable $x$, and the $\log$ likelihood for the $i$ th observation is

$$
\ln \ell_{i}=\ln \lambda-\lambda x_{i}
$$

where $\lambda$ is a parameter to be estimated. Then you would type

```
. mlexp (ln({lambda}) - {lambda}*x)
```

Because $\lambda$ is a parameter, we enclosed it in braces. To specify initial values for a parameter, you can include an equal sign and the initial value after the parameter; for example,
. $\operatorname{mlexp}(\ln (\{\operatorname{lambda}=0.75\})-\{\operatorname{lambda}\} * x)$
would initialize $\lambda$ to be 0.75 . If you do not initialize a parameter, mlexp initializes it to zero.
Frequently, even nonlinear functions contain linear combinations of variables. Continuing the previous example, say that we want to parameterize $\lambda$ as

$$
\lambda_{i}=\alpha_{1} u_{i}+\alpha_{2} v_{i}
$$

where $u$ and $v$ are variables in the dataset. Instead of typing
. $\operatorname{mlexp}(\ln (\{a l p h a 1\} * u+\{a l p h a 2\} * v)-(\{a l p h a 1\} * u+\{a l p h a 2\} * v) * x)$
you can instead type
. $\operatorname{mlexp}(\ln (\{l a m b d a: ~ u v\})-\{l a m b d a:\} * x)$
The notation \{lambda: $\mathrm{u} v\}$ indicates to mlexp that you want a linear combination of the variables $u$ and $v$. We named the linear combination lambda, so mlexp will name the parameters for the two variables lambda_u and lambda_v, respectively. Once you have declared a linear combination, you can subsequently refer to the linear combination by specifying its name and a colon inside braces, as we did with this example. You cannot use the same name for both an individual parameter and a linear combination. However, after a linear combination has been declared, you can refer to the parameter of an individual variable within that linear combination by using the notation $\left\{l c_{-} z\right\}$, where $l c$ is the name of the linear combination and $z$ is the variable whose parameter you want to reference. Linear combinations do not include a constant term.

There are three rules to follow when defining substitutable expressions:

1. Parameters of the model are bound in braces: $\{b 0\}$, \{param\}, etc.
2. Initial values for parameters are given by including an equal sign and the initial value inside the braces: $\{b 0=1\}$, $\{$ param $=3.571\}$, etc.
3. Linear combinations of variables can be included using the notation \{eqname:varlist\}: \{xb: mpg price weight\}, \{score: w x z\}, etc. Parameters of linear combinations are initialized to zero.

If you specify initial values by using the from() option, they override whatever initial values are given within the substitutable expression. Substitutable expressions are so named because once values are assigned to the parameters, the resulting expressions can be handled by generate and replace.

Regardless of whether you specify initial values, mlexp performs a search procedure for better starting values before commencing the first iteration of the maximization routine. If you specify initial values, the search procedure tries to improve upon those values. Otherwise, the search procedure begins with all parameters set to zero.
> Example 1: The gamma density function
The two-parameter gamma density function for $y \geq 0$ is

$$
f(y)=\frac{\lambda^{P}}{\Gamma(P)} \exp (-\lambda y) y^{P-1} \quad \lambda>0, P>0
$$

so that the log likelihood for the $i$ th observation is

$$
\ln \ell_{i}=P \ln \lambda-\ln \Gamma(P)-\lambda y_{i}+(P-1) \ln y_{i}
$$

The dataset greenegamma.dta, based on Greene (2012, 460-461), contains 20 observations drawn randomly from the two-parameter gamma distribution. We want to estimate the parameters of that distribution. We type

```
. use http://www.stata-press.com/data/r13/greenegamma
. mlexp ({P}*ln({lambda}) - lngamma({P}) - {lambda}*y + ({P}-1)*ln(y))
initial: log likelihood = -<inf> (could not be evaluated)
feasible: log likelihood = -363.37264
rescale: log likelihood = -153.09898
rescale eq: log likelihood = -88.863468
Iteration 0: log likelihood = -88.863468
Iteration 1: log likelihood = -85.405011
Iteration 2: log likelihood = -85.375857
Iteration 3: log likelihood = -85.375669
Iteration 4: log likelihood = -85.375669
Maximum likelihood estimation
Log likelihood = -85.375669 Number of obs = 20
\begin{tabular}{r|rrcccr}
\hline & Coef. & Std. Err. & z & P>|z| & [95\% Conf. Interval] \\
\hline /P & 2.410602 & .7158452 & 3.37 & 0.001 & 1.007571 & 3.813633 \\
/lambda & .0770702 & .0254361 & 3.03 & 0.002 & .0272164 & .1269241 \\
\hline
\end{tabular}
```

In our substitutable expression for the log-likelihood function, we enclosed the two parameters of our model, $P$ and $\lambda$, in curly braces. We used the lngamma() function to compute $\ln \Gamma(P)$ because Stata (unlike Mata) does not have a built-in function to compute $\Gamma(P)$, and numerical algorithms for computing $\ln \Gamma(P)$ directly are more accurate than taking the natural logarithm of $\Gamma(P)$, anyway.

Because we did not specify initial values, mlexp initialized $P$ and $\lambda$ to be zero. When both parameters are zero, the log-likelihood function cannot be evaluated because $\ln (0)$ is undefined. Therefore, in the iteration $\log$ above the coefficient table, we see that mlexp reported the initial $\log$ likelihood to be -<inf> (could not be evaluated). mlexp uses a search routine to find alternative initial values that do allow the log-likelihood function to be calculated.

## > Example 2: Obtaining alternative VCEs

In example 1, by default mlexp reported standard errors based on the observed information matrix of the log-likelihood function. See [R] vce_option for an overview or Gould, Pitblado, and Poi (2010) for an in-depth discussion of different ways of obtaining the VCE in maximum likelihood estimation. With mlexp, we can use the vce() option to obtain standard errors based on alternative VCEs. For example, to obtain the outer product of gradients (OPG) standard errors, we type

```
. mlexp ({P}*ln({lambda}) - lngamma({P}) - {lambda}*y + ({P}-1)*ln(y)), vce(opg)
initial: log likelihood = -<inf> (could not be evaluated)
feasible: log likelihood = -363.37264
rescale: log likelihood = -153.09898
rescale eq: log likelihood = -88.863468
Iteration 0: log likelihood = -88.863468
Iteration 1: log likelihood = -85.405011
Iteration 2: log likelihood = -85.375857
Iteration 3: log likelihood = -85.375669
Iteration 4: log likelihood = -85.375669
Maximum likelihood estimation
Log likelihood \(=-85.375669 \quad\) Number of obs \(=\quad 20\)
```

|  | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | :---: |
| /P | 2.410602 | .8768255 | 2.75 | 0.006 | .6920557 | 4.129149 |
|  | .0770702 | .0270771 | 2.85 | 0.004 | .0240001 | .1301404 |

## Parameter constraints

In examples 1 and 2, we were lucky. The two-parameter gamma density function is defined only when both $\lambda$ and $P$ are positive. However, mlexp does not know this; when maximizing the log-likelihood function, it will consider all real values for the parameters. Rather than relying on luck, we should instead reparameterize our model so that we avoid having to directly estimate parameters that are restricted.

For example, consider the parameter $\lambda>0$, and suppose we define the new parameter $\theta=\ln (\lambda)$ so that $\lambda=\exp (\theta)$. With this parameterization, for any real value of $\theta$ that mlexp might try to use when evaluating the log-likelihood function, $\lambda$ is guaranteed to be positive.
> Example 3: Classical normal regression
In the classical normal variant of linear regression (Goldberger 1991, chap. 19), we assume not only that $y_{i}=\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}+\epsilon_{i}$ but also that $\mathbf{x}_{i}$ is nonstochastic and that $\epsilon_{i}$ is distributed independently and identically normal with mean zero and variance $\sigma^{2}$. This is equivalent to assuming that

$$
y_{i} \mid \mathbf{x}_{i} \sim N\left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}, \sigma^{2}\right)
$$

Using the properties of the normal distribution, we could write the $\log$ likelihood for the $i$ th observation as

$$
\ln \ell_{i}=\ln \left\{\frac{1}{\sigma} \phi\left(\frac{y_{i}-\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}}{\sigma}\right)\right\}
$$

where $\phi(\cdot)$ is the standard normal density function. In Stata, we use the three-argument version of the normalden() function and directly specify the conditional mean ( $\mathrm{x}_{i}^{\prime} \boldsymbol{\beta}$ ) and standard deviation $(\sigma)$ as the additional arguments.

The normal density function is defined only for $\sigma>0$, so instead of estimating $\sigma$ directly, we will instead estimate the unconstrained parameter $\theta$ and let $\sigma=\exp (\theta)$. For any real value of $\theta$, this transformation ensures that $\sigma>0$.

Using auto.dta, say that we want to fit the classical normal regression

$$
\operatorname{mpg}_{i}=\beta_{0}+\beta_{1} \text { weight }_{i}+\epsilon_{i}
$$

We type

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. mlexp (ln(normalden(mpg, {b0} + {b1}*weight, exp({theta}))))
initial: log likelihood = -<inf> (could not be evaluated)
feasible: log likelihood = -882.02886
rescale: log likelihood = -882.02886
rescale eq: log likelihood = -274.09391
Iteration 0: log likelihood = -274.09391 (not concave)
    (output omitted)
Iteration 13: log likelihood = -195.38869
```

Maximum likelihood estimation
Log likelihood $=-195.38869 \quad$ Number of obs $=\quad 74$

|  | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | :---: | ---: | ---: |
| /b0 | 39.44028 | 1.592043 | 24.77 | 0.000 | 36.31993 | 42.56063 |
| /b1 | -.0060087 | .0005108 | -11.76 | 0.000 | -.0070099 | -.0050075 |
| /theta | 1.221449 | .0821995 | 14.86 | 0.000 | 1.060341 | 1.382557 |

To recover our estimate of $\sigma$, we can use nlcom:

```
. nlcom (sigma: exp(_b[/theta]))
    sigma: exp(_b[/theta])
```

|  | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | ---: |
| sigma | 3.392099 | .2788288 | 12.17 | 0.000 | 2.845605 | 3.938594 |

In the previous example, we named the unconstrained parameter $\theta$ theta. In actual practice, however, we would generally name that parameter lnsigma to indicate that it is $\ln (\sigma)$.

Two other parameter restrictions often appear in maximum likelihood estimation. Consider the correlation coefficient $\rho$. In general, it must be true that $-1<\rho<1$. Define the parameter $\eta=\tanh ^{-1}(\rho)$, where $\tanh ^{-1}(\cdot)$ is the hyperbolic arctangent function. Then $\rho=\tanh (\eta)$, and by the properties of the hyperbolic tangent function, for any real value of $\eta$, we will have $-1<\rho<1$. Stata has the built-in function $\tanh ()$, so recovering $\rho$ from $\eta$ is easy. In practice, instead of naming the unconstrained parameter eta in our likelihood expression, we would name it atanhrho to remind us that it is the hyperbolic arctangent of $\rho$.

Other parameters, such as those that represent probabilities or ratios of variances, are often restricted to be between 0 and 1. Say that we have the restriction $0<\kappa<1$. Consider the parameter $\psi=\ln \{\kappa /(1-\kappa)\}$. Then $\kappa=e^{\psi} /\left(1+e^{\psi}\right)$ and for any real value of $\psi$, we have $0<\kappa<1$. The formula for $\kappa$ in terms of $\psi$ is known as the inverse logit transformation and is available as invlogit() in Stata. Thus in our likelihood expression, we would code invlogit(\{logitk\}) to map the unconstrained parameter logitk into the $(0,1)$ interval.

## Specifying derivatives

By default, mlexp calculates derivatives of the log-likelihood function numerically using a sophisticated algorithm that produces accurate results. However, mlexp will fit your model more quickly (and even more accurately) if you specify analytic derivatives.

You specify derivatives by using substitutable expressions in much the same way as you specify the log-likelihood function. If you specify a linear combination in your log-likelihood function, then you supply a derivative with respect to that linear combination; mlexp then uses the chain rule to obtain the derivatives with respect to the individual parameters.

We will illustrate how to specify derivatives using the probit model for dichotomous outcomes. The $\log$ likelihood for the probit model is often written as

$$
\ln \ell_{i}= \begin{cases}\ln \Phi\left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}\right) & y_{i}=1 \\ \ln \Phi\left(-\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}\right) & y_{i}=0\end{cases}
$$

using the fact that $1-\Phi\left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}\right)=\Phi\left(-\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}\right)$, where $\Phi(\cdot)$ is the cumulative standard normal distribution function. If we use the trick suggested by Greene (2012, 691, fn. 7), we can simplify the log-likelihood function, making the derivative calculation easier. Let $q_{i}=2 y_{i}-1$. Then we can write the loglikelihood function as

$$
\begin{equation*}
\ln \ell_{i}=\ln \Phi\left(q_{i} \mathbf{x}_{i}^{\prime} \boldsymbol{\beta}\right) \tag{1}
\end{equation*}
$$

and the first derivative as

$$
\begin{equation*}
\frac{\partial \ln \ell_{i}}{\partial \boldsymbol{\beta}}=\frac{q_{i} \phi\left(q_{i} \mathbf{x}_{i}^{\prime} \boldsymbol{\beta}\right)}{\Phi\left(q_{i} \mathbf{x}_{i}^{\prime} \boldsymbol{\beta}\right)} \mathbf{x}_{i} \tag{2}
\end{equation*}
$$

$>$ Example 4: Probit with one regressor
Say that we want to fit a probit model of foreign on mpg and a constant term. We have two parameters, so we will need to specify two derivatives; $\mathbf{x}_{i}$ consists of the $i$ th observation on mpg and a 1 as the constant term. Because the term $q_{i} \mathbf{x}_{i}^{\prime} \boldsymbol{\beta}$ will appear several times in our command, we create a macro to store it. We type

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. generate q = 2*foreign - 1
. global qxb "q*({b1}*mpg + {b0})"
. mlexp (ln(normal($qxb))), derivative(/b1 = q*normalden($qxb)/normal($qxb)*mpg)
> deriv(/b0 = q*normalden($qxb)/normal($qxb))
initial: log likelihood = -51.292891
alternative: log likelihood = -2017.3105
rescale: log likelihood = -47.888213
rescale eq: log likelihood = -46.343247
Iteration 0: log likelihood = -46.343247
Iteration 1: log likelihood = -39.268764
Iteration 2: log likelihood = -39.258972
Iteration 3: log likelihood = -39.258972
Maximum likelihood estimation
Log likelihood = -39.258972 Number of obs = 74
```

|  | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| /b1 | .0960601 | .0301523 | 3.19 | 0.001 | .0369627 | .1551575 |
| /b0 | -2.635268 | .6841462 | -3.85 | 0.000 | -3.97617 | -1.294366 |

When you specify a linear combination of variables, you specify the derivative with respect to the linear combination. That way, if you change the variables that comprise the linear combination, you do not need to change the derivative at all. To see why this is the case, consider the function $f\left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}\right)$, where $\mathbf{x}_{i}^{\prime} \beta$ is a linear combination. Then, using the chain rule,

$$
\frac{\partial f\left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}\right)}{\partial \beta_{j}}=\frac{\partial f\left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}\right)}{\partial \mathbf{x}_{i}^{\prime} \boldsymbol{\beta}} \times \frac{\partial \mathbf{x}_{i}^{\prime} \boldsymbol{\beta}}{\partial \beta_{j}}=\frac{\partial f\left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}\right)}{\partial \mathbf{x}_{i}^{\prime} \boldsymbol{\beta}} \times x_{i j}
$$

Once the derivative with respect to the linear combination is known, mlexp can then multiply it by each of the variables in the linear combination to get the full set of derivatives with respect to the parameters needed to maximize the likelihood function. Moreover, the derivative with respect to the linear combination does not depend on the variables within the linear combination, so even if you change the variables in it, you will not need to modify the specification of the corresponding derivative() option.

## > Example 5: Probit with a linear combination

Now let's fit a probit model of foreign on mpg and gear_ratio. We could specify the parameters and independent variables individually, but we will use a linear combination instead. First, note that

$$
\frac{\partial \ln \ell_{i}}{\partial \mathbf{x}_{i}^{\prime} \boldsymbol{\beta}}=\frac{q_{i} \phi\left(q_{i} \mathbf{x}_{i}^{\prime} \boldsymbol{\beta}\right)}{\Phi\left(q_{i} \mathbf{x}_{i}^{\prime} \boldsymbol{\beta}\right)}
$$

We type

```
. use http://www.stata-press.com/data/r13/auto, clear
(1978 Automobile Data)
. generate q = 2*foreign - 1
. global qxb "q*({xb:} + {bO})"
. mlexp (ln(normal(q*({xb:mpg gear_ratio}+{b0})))),
> deriv(/xb = q*normalden($qxb)/normal($qxb))
> deriv(/b0 = q*normalden($qxb)/normal($qxb))
initial: log likelihood = -51.292891
alternative: log likelihood = -2556.2172
rescale: log likelihood = -47.865271
rescale eq: log likelihood = -46.658776
Iteration 0: log likelihood = -46.658776
Iteration 1: log likelihood = -22.541058
Iteration 2: log likelihood = -21.467371
Iteration 3: log likelihood = -21.454446
Iteration 4: log likelihood = -21.454436
Iteration 5: log likelihood = -21.454436
Maximum likelihood estimation
Log likelihood = -21.454436 Number of obs = 74
\begin{tabular}{r|rcccrr}
\hline & Coef. & Std. Err. & z & P>|z| & [95\% Conf. Interval] \\
\hline /xb_mpg & -.0282433 & .0464514 & -0.61 & 0.543 & -.1192864 & .0627998 \\
/xb_gear_r~o & 3.699635 & .8368276 & 4.42 & 0.000 & 2.059483 & 5.339787 \\
/b0 & -11.57588 & 2.337239 & -4.95 & 0.000 & -16.15678 & -6.994972 \\
\hline
\end{tabular}
```

We first redefined our global macro \$qxb to contain the linear combination xb and a constant term b0. More importantly, we did not specify the variables in the linear combination just yet. Instead, we will use \$qxb after we explicitly declare the variables in $x b$ when we specify our model. To avoid making mistakes, you can declare the variables in a linear combination only once when you set up your model. If we had declared the variables when we defined $\$ q x b$, we would have received an error because, upon substituting for $\$ q x b$, we would have declared the variables multiple times in our call to mlexp.

## Stored results

mlexp stores the following in e():
Scalars

```
e(N)
    e(k)
    e(k_aux)
    e(k_eq)
    e(k_eq_model)
    e(df_m)
    e(ll)
    e(N_clust)
    e(rank)
    e(ic)
    e(rc)
    e(converged)
```

Macros
e(cmd)
e(cmdline)
e(lexp)
e(wtype)
e(wexp)
e(usrtitle)
e(usrtitle2)
e(vce)
e(vcetype)
e(params)
e(hasderiv)
e(d_j)
e(rhs)
e(opt)
e(ml_method)
$e$ (technique)
e(singularHmethod)
e(crittype)
e(properties)
e(estat_cmd)
e(predict)
e(marginsnotok)
e(marginsprop)
Matrices
e(b)
e(ilog)
e(init)
e(gradient)
e(V)
e(V_modelbased)
Functions
e(sample)
number of observations
number of parameters
number of ancillary parameters
number of equations in $e(b)$
number of equations in overall model test
model degrees of freedom
log likelihood
number of clusters
rank of $e(V)$
number of iterations
return code
1 if converged, 0 otherwise
mlexp
command as typed
likelihood expression
weight type
weight expression
user-specified title
user-specified secondary title
vcetype specified in vce()
title used to label Std. Err.
names of parameters
yes, if derivative() is specified
derivative expression for parameter $j$
contents of variables()
type of optimization
type of ml method
maximization technique
$m$-marquardt or hybrid; method used when Hessian is singular ${ }^{1}$
optimization criterion ${ }^{1}$
b V
program used to implement estat
program used to implement predict
predictions disallowed by margins
signals to the margins command
coefficient vector
iteration $\log$ (up to 20 iterations)
initial values
gradient vector
variance-covariance matrix of the estimators
model-based variance
marks estimation sample

1. Type ereturn list, all to view these results; see $[P]$ return.

## Methods and formulas

Optimization is carried out using moptimize(); see [M-5] moptimize().

## References

Goldberger, A. S. 1991. A Course in Econometrics. Cambridge, MA: Harvard University Press.
Gould, W. W., J. S. Pitblado, and B. P. Poi. 2010. Maximum Likelihood Estimation with Stata. 4th ed. College Station, TX: Stata Press.

Greene, W. H. 2012. Econometric Analysis. 7th ed. Upper Saddle River, NJ: Prentice Hall.

## Also see

[R] mlexp postestimation - Postestimation tools for mlexp
[R] gmm - Generalized method of moments estimation
$[\mathrm{R}]$ maximize - Details of iterative maximization
[R] ml - Maximum likelihood estimation
[R] nl - Nonlinear least-squares estimation
[R] nlsur - Estimation of nonlinear systems of equations

## Title

mlexp postestimation - Postestimation tools for mlexp

Description Syntax for predict Menu for predict Option for predict Also see

## Description

The following postestimation commands are available after mlexp:
Command Description
estat ic
estat summarize
estat vce
estimates
lincom
lrtest ${ }^{1}$
nlcom
predict
predictnl
suest
test
testnl

Akaike's and Schwarz's Bayesian information criteria (AIC and BIC)
summary statistics for the estimation sample
variance-covariance matrix of the estimators (VCE)
cataloging estimation results
point estimates, standard errors, testing, and inference for linear combinations of coefficients
likelihood-ratio test
point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients
scores
point estimates, standard errors, testing, and inference for generalized predictions
seemingly unrelated estimation
Wald tests of simple and composite linear hypotheses
Wald tests of nonlinear hypotheses

1 lrtest is not appropriate with svy estimation results.

## Syntax for predict

predict [type] \{stub*|newvar ${ }_{1} \ldots$ newvar $\left._{k}\right\}[$ if ] [in] [, scores ]
This statistic is only available for observations within the estimation sample. $k$ represents the number of parameters in the model.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Option for predict

scores, the default, calculates the equation-level score variables. The $j$ th new variable will contain the scores for the $j$ th parameter of the model. Linear combinations are expanded prior to computing scores, so each variable's parameter will have its own score variable.

## Also see

[R] mlexp - Maximum likelihood estimation of user-specified expressions
[U] 20 Estimation and postestimation commands

## Title

mlogit - Multinomial (polytomous) logistic regression

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

Also see

## Syntax

$$
\text { mlogit depvar [indepvars] [if][in][weight] [, options }]
$$

options
Description
Model
noconstant
baseoutcome(\#)
constraints (clist)
collinear
SE/Robust
vce (vcetype)

Reporting
level(\#)
rrr
nocnsreport
display_options
suppress constant term
value of depvar that will be the base outcome
apply specified linear constraints; clist has the form \#[-\#] [, \#[-\#]...] keep collinear variables
vcetype may be oim, robust, cluster clustvar, bootstrap, or jackknife
set confidence level; default is level (95)
report relative-risk ratios
do not display constraints
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling

Maximization
maximize_options
coeflegend
control the maximization process; seldom used
display legend instead of statistics
indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
indepvars may contain time-series operators; see [U] 11.4.4 Time-series varlists.
bootstrap, by, fp, jackknife, mfp, mi estimate, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands.
vce(bootstrap) and vce(jackknife) are not allowed with the mi estimate prefix; see [MI] mi estimate.
Weights are not allowed with the bootstrap prefix; see [R] bootstrap.
vce() and weights are not allowed with the svy prefix; see [SVY] svy.
fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Categorical outcomes $>$ Multinomial logistic regression

## Description

mlogit fits maximum-likelihood multinomial logit models, also known as polytomous logistic regression. You can define constraints to perform constrained estimation. Some people refer to conditional logistic regression as multinomial logit. If you are one of them, see $[R]$ clogit.

See [R] logistic for a list of related estimation commands.

## Options

```
        Model
    noconstant; see [R] estimation options.
    baseoutcome(#) specifies the value of depvar to be treated as the base outcome. The default is to
    choose the most frequent outcome.
constraints(clist), collinear; see [R] estimation options.
```

SE/Robust
vce (vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
If specifying vce(bootstrap) or vce(jackknife), you must also specify baseoutcome().
$\qquad$ Reporting
level (\#); see [R] estimation options.
rrr reports the estimated coefficients transformed to relative-risk ratios, that is, $e^{b}$ rather than $b$; see Description of the model below for an explanation of this concept. Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated. rrr may be specified at estimation or when replaying previously estimated results.
nocnsreport; see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\%fmt), pformat(\% fmt), sformat (\%fmt), and nolstretch; see [R] estimation options.

Maximization
maximize_options: difficult, technique (algorithm_spec), iterate (\#), [no $]$ log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance (\#), nonrtolerance, and from(init_specs); see [R] maximize. These options are seldom used.

The following option is available with mlogit but is not shown in the dialog box:
coeflegend; see [R] estimation options.

## Remarks and examples

Remarks are presented under the following headings:
Description of the model
Fitting unconstrained models
Fitting constrained models
mlogit fits maximum likelihood models with discrete dependent (left-hand-side) variables when the dependent variable takes on more than two outcomes and the outcomes have no natural ordering. If the dependent variable takes on only two outcomes, estimates are identical to those produced by logistic or logit; see [R] logistic or [R] logit. If the outcomes are ordered, see [R] ologit.

## Description of the model

For an introduction to multinomial logit models, see Greene (2012, 763-766), Hosmer, Lemeshow, and Sturdivant (2013, 269-289), Long (1997, chap. 6), Long and Freese (2014, chap. 8), and Treiman (2009, 336-341). For a description emphasizing the difference in assumptions and data requirements for conditional and multinomial logit, see Davidson and MacKinnon (1993).

Consider the outcomes $1,2,3, \ldots, m$ recorded in $y$, and the explanatory variables $X$. Assume that there are $m=3$ outcomes: "buy an American car", "buy a Japanese car", and "buy a European car". The values of $y$ are then said to be "unordered". Even though the outcomes are coded 1, 2, and 3, the numerical values are arbitrary because $1<2<3$ does not imply that outcome 1 (buy American) is less than outcome 2 (buy Japanese) is less than outcome 3 (buy European). This unordered categorical property of $y$ distinguishes the use of mlogit from regress (which is appropriate for a continuous dependent variable), from ologit (which is appropriate for ordered categorical data), and from logit (which is appropriate for two outcomes, which can be thought of as ordered).

In the multinomial logit model, you estimate a set of coefficients, $\beta^{(1)}, \beta^{(2)}$, and $\beta^{(3)}$, corresponding to each outcome:

$$
\begin{aligned}
& \operatorname{Pr}(y=1)=\frac{e^{X \beta^{(1)}}}{e^{X \beta^{(1)}}+e^{X \beta^{(2)}}+e^{X \beta^{(3)}}} \\
& \operatorname{Pr}(y=2)=\frac{e^{X \beta^{(2)}}}{e^{X \beta^{(1)}}+e^{X \beta^{(2)}}+e^{X \beta^{(3)}}} \\
& \operatorname{Pr}(y=3)=\frac{e^{X \beta^{(3)}}}{e^{X \beta^{(1)}}+e^{X \beta^{(2)}}+e^{X \beta^{(3)}}}
\end{aligned}
$$

The model, however, is unidentified in the sense that there is more than one solution to $\beta^{(1)}, \beta^{(2)}$, and $\beta^{(3)}$ that leads to the same probabilities for $y=1, y=2$, and $y=3$. To identify the model, you arbitrarily set one of $\beta^{(1)}, \beta^{(2)}$, or $\beta^{(3)}$ to 0 -it does not matter which. That is, if you arbitrarily set $\beta^{(1)}=0$, the remaining coefficients $\beta^{(2)}$ and $\beta^{(3)}$ will measure the change relative to the $y=1$ group. If you instead set $\beta^{(2)}=0$, the remaining coefficients $\beta^{(1)}$ and $\beta^{(3)}$ will measure the change relative to the $y=2$ group. The coefficients will differ because they have different interpretations, but the predicted probabilities for $y=1,2$, and 3 will still be the same. Thus either parameterization will be a solution to the same underlying model.

Setting $\beta^{(1)}=0$, the equations become

$$
\begin{aligned}
& \operatorname{Pr}(y=1)=\frac{1}{1+e^{X \beta^{(2)}}+e^{X \beta^{(3)}}} \\
& \operatorname{Pr}(y=2)=\frac{e^{X \beta^{(2)}}}{1+e^{X \beta^{(2)}}+e^{X \beta^{(3)}}} \\
& \operatorname{Pr}(y=3)=\frac{e^{X \beta^{(3)}}}{1+e^{X \beta^{(2)}}+e^{X \beta^{(3)}}}
\end{aligned}
$$

The relative probability of $y=2$ to the base outcome is

$$
\frac{\operatorname{Pr}(y=2)}{\operatorname{Pr}(y=1)}=e^{X \beta^{(2)}}
$$

Let's call this ratio the relative risk, and let's further assume that $X$ and $\beta_{k}^{(2)}$ are vectors equal to $\left(x_{1}, x_{2}, \ldots, x_{k}\right)$ and $\left(\beta_{1}^{(2)}, \beta_{2}^{(2)}, \ldots, \beta_{k}^{(2)}\right)^{\prime}$, respectively. The ratio of the relative risk for a one-unit change in $x_{i}$ is then

$$
\frac{e^{\beta_{1}^{(2)} x_{1}+\cdots+\beta_{i}^{(2)}\left(x_{i}+1\right)+\cdots+\beta_{k}^{(2)} x_{k}}}{e^{\beta_{1}^{(2)} x_{1}+\cdots+\beta_{i}^{(2)} x_{i}+\cdots+\beta_{k}^{(2)} x_{k}}}=e^{\beta_{i}^{(2)}}
$$

Thus the exponentiated value of a coefficient is the relative-risk ratio for a one-unit change in the corresponding variable (risk is measured as the risk of the outcome relative to the base outcome).

## Fitting unconstrained models

## > Example 1: A first example

We have data on the type of health insurance available to 616 psychologically depressed subjects in the United States (Tarlov et al. 1989; Wells et al. 1989). The insurance is categorized as either an indemnity plan (that is, regular fee-for-service insurance, which may have a deductible or coinsurance rate) or a prepaid plan (a fixed up-front payment allowing subsequent unlimited use as provided, for instance, by an HMO). The third possibility is that the subject has no insurance whatsoever. We wish to explore the demographic factors associated with each subject's insurance choice. One of the demographic factors in our data is the race of the participant, coded as white or nonwhite:

```
. use http://www.stata-press.com/data/r13/sysdsn1
(Health insurance data)
. tabulate insure nonwhite, chi2 col
```

| Key |
| :--- |
| frequency |
| column percentage |


| insure | nonwhite |  |  |
| ---: | ---: | ---: | ---: |
|  | 0 | 1 | Total |
| Indemnity | 251 | 43 | 294 |
|  | 50.71 | 35.54 | 47.73 |
| Prepaid | 208 | 69 | 277 |
|  | 42.02 | 57.02 | 44.97 |
| Uninsure | 36 | 9 | 45 |
|  | 7.27 | 7.44 | 7.31 |
| Total | 495 | 121 | 616 |
|  | 100.00 | 100.00 | 100.00 |
| Pearson chi2(2) $=$ | 9.5599 | $\operatorname{Pr}=0.008$ |  |

Although insure appears to take on the values Indemnity, Prepaid, and Uninsure, it actually takes on the values 1,2 , and 3 . The words appear because we have associated a value label with the numeric variable insure; see [U] 12.6.3 Value labels.

When we fit a multinomial logit model, we can tell mlogit which outcome to use as the base outcome, or we can let mlogit choose. To fit a model of insure on nonwhite, letting mlogit choose the base outcome, we type

```
. mlogit insure nonwhite
Iteration 0: log likelihood = -556.59502
Iteration 1: log likelihood = -551.78935
Iteration 2: log likelihood = -551.78348
Iteration 3: log likelihood = -551.78348
```

$\begin{array}{lllr}\text { Multinomial logistic regression } & \text { Number of obs } & = & 616 \\ & \text { LR chi2 }(2) & = & 9.62 \\ & \text { Prob }>\operatorname{chi2} & = & 0.0081\end{array}$
Log likelihood = -551.78348

| Number of obs | $=$ | 616 |
| :--- | :--- | ---: |
| LR chi2 $(2)$ | $=$ | 9.62 |
| Prob > chi2 | $=$ | 0.0081 |
| Pseudo R2 | $=$ | 0.0086 |


| insure | Coef. | Std. Err. | $z$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Indemnity | (base outcome) |  |  |  |  |  |
| Prepaid <br> nonwhite <br> _cons | -.6608212 | .2157321 | 3.06 | 0.002 | .2379942 | 1.083648 |
| Uninsure <br> nonwhite <br> _cons | -1879149 | .0937644 | -2.00 | 0.045 | -.3716896 | -.0041401 |

mlogit chose the indemnity outcome as the base outcome and presented coefficients for the outcomes prepaid and uninsured. According to the model, the probability of prepaid for whites (nonwhite $=0$ ) is

$$
\operatorname{Pr}(\text { insure }=\text { Prepaid })=\frac{e^{-.188}}{1+e^{-.188}+e^{-1.942}}=0.420
$$

Similarly, for nonwhites, the probability of prepaid is

$$
\operatorname{Pr}(\text { insure }=\text { Prepaid })=\frac{e^{-.188+.661}}{1+e^{-.188+.661}+e^{-1.942+.378}}=0.570
$$

These results agree with the column percentages presented by tabulate because the mlogit model is fully saturated. That is, there are enough terms in the model to fully explain the column percentage in each cell. The model chi-squared and the tabulate chi-squared are in almost perfect agreement; both test that the column percentages of insure are the same for both values of nonwhite.

## > Example 2: Specifying the base outcome

By specifying the baseoutcome () option, we can control which outcome of the dependent variable is treated as the base. Left to its own, mlogit chose to make outcome 1, indemnity, the base outcome. To make outcome 2, prepaid, the base, we would type


The baseoutcome() option requires that we specify the numeric value of the outcome, so we could not type base(Prepaid).

Although the coefficients now appear to be different, the summary statistics reported at the top are identical. With this parameterization, the probability of prepaid insurance for whites is

$$
\operatorname{Pr}(\text { insure }=\operatorname{Prepaid})=\frac{1}{1+e^{.188}+e^{-1.754}}=0.420
$$

This is the same answer we obtained previously.

## D Example 3: Displaying relative-risk ratios

By specifying rrr, which we can do at estimation time or when we redisplay results, we see the model in terms of relative-risk ratios:

| . mlogit, rrr |  |  |  |
| :--- | :--- | :--- | :--- |
| Multinomial logistic regression | Number of obs | $=$ | 616 |
|  | LR chi2 2 ) | $=$ | 9.62 |
|  | Prob $>$ chi2 | $=$ | 0.0081 |
| Log likelihood $=-551.78348$ | Pseudo R2 | $=$ | 0.0086 |


| insure | RRR | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| :---: | ---: | :---: | :---: | :---: | :---: | :---: |
| Indemnity <br> nonwhite <br> _cons | 1.206731 | .1131483 | 2.00 | 0.045 | 1.004149 | 1.450183 |
| Prepaid | (base outcome) |  |  |  |  |  |
| Uninsure <br> nonwhite <br> _cons | .7536233 | .2997387 | -0.71 | 0.477 | .3456255 | 1.643247 |

Looked at this way, the relative risk of choosing an indemnity over a prepaid plan is 0.516 for nonwhites relative to whites.

To illustrate, from the output and discussions of examples 1 and 2 we find that

$$
\operatorname{Pr}(\text { insure }=\text { Indemnity } \mid \text { white })=\frac{1}{1+e^{-.188}+e^{-1.942}}=0.507
$$

and thus the relative risk of choosing indemnity over prepaid (for whites) is

$$
\frac{\operatorname{Pr}(\text { insure }=\text { Indemnity } \mid \text { white })}{\operatorname{Pr}(\text { insure }=\text { Prepaid } \mid \text { white })}=\frac{0.507}{0.420}=1.207
$$

For nonwhites,

$$
\operatorname{Pr}(\text { insure }=\text { Indemnity } \mid \text { not white })=\frac{1}{1+e^{-.188+.661}+e^{-1.942+.378}}=0.355
$$

and thus the relative risk of choosing indemnity over prepaid (for nonwhites) is

$$
\frac{\operatorname{Pr}(\text { insure }=\text { Indemnity } \mid \text { not white })}{\operatorname{Pr}(\text { insure }=\text { Prepaid } \mid \text { not white })}=\frac{0.355}{0.570}=0.623
$$

The ratio of these two relative risks, hence the name "relative-risk ratio", is $0.623 / 1.207=0.516$, as given in the output under the heading "RRR".

## - Technical note

In models where only two categories are considered, the mlogit model reduces to standard logit. Consequently the exponentiated regression coefficients, labeled as RRR within mlogit, are equal to the odds ratios as given when the or option is specified under logit; see [R] logit.

As such, always referring to mlogit's exponentiated coefficients as odds ratios may be tempting. However, the discussion in example 3 demonstrates that doing so would be incorrect. In general mlogit models, the exponentiated coefficients are ratios of relative risks, not ratios of odds.

## Example 4: Model with continuous and multiple categorical variables

One of the advantages of mlogit over tabulate is that we can include continuous variables and multiple categorical variables in the model. In examining the data on insurance choice, we decide that we want to control for age, gender, and site of study (the study was conducted in three sites):

| Iteration 0: log likelihood $=-555.85446$ |  |  |  |
| :---: | :---: | :---: | :---: |
| Iteration 1: log likelihood $=-534.67443$ |  |  |  |
| Iteration 2: log likelihood $=-534.36284$ |  |  |  |
| Iteration 3: log likelihood $=-534.36165$ |  |  |  |
| Iteration 4: log likelihood = -534.36165 |  |  |  |
| Multinomial logistic regression | Number of obs | = | 615 |
|  | LR chi2(10) |  | 42.99 |
|  | Prob > chi2 | = | 0.0000 |
| Log likelihood = -534.36165 | Pseudo R2 | = | 0.0387 |


| insure | Coef. | Std. Err. | z | $P>\|z\|$ | [95\% Con | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Indemnity | (base outcome) |  |  |  |  |  |
| Prepaid |  |  |  |  |  |  |
| age | -. 011745 | . 0061946 | -1.90 | 0.058 | -. 0238862 | . 0003962 |
| male | . 5616934 | . 2027465 | 2.77 | 0.006 | . 1643175 | . 9590693 |
| nonwhite | . 9747768 | . 2363213 | 4.12 | 0.000 | . 5115955 | 1.437958 |
| site |  |  |  |  |  |  |
| 2 | . 1130359 | . 2101903 | 0.54 | 0.591 | -. 2989296 | . 5250013 |
| 3 | -. 5879879 | . 2279351 | -2.58 | 0.010 | -1.034733 | -. 1412433 |
| _cons | . 2697127 | . 3284422 | 0.82 | 0.412 | -. 3740222 | . 9134476 |
| Uninsure |  |  |  |  |  |  |
| age | -. 0077961 | . 0114418 | -0.68 | 0.496 | -. 0302217 | . 0146294 |
| male | . 4518496 | . 3674867 | 1.23 | 0.219 | -. 268411 | 1.17211 |
| nonwhite | . 2170589 | . 4256361 | 0.51 | 0.610 | -. 6171725 | 1.05129 |
| site |  |  |  |  |  |  |
| 2 | -1.211563 | . 4705127 | -2.57 | 0.010 | -2.133751 | -. 2893747 |
| 3 | -. 2078123 | . 3662926 | -0.57 | 0.570 | -. 9257327 | . 510108 |
| _cons | -1.286943 | . 5923219 | -2.17 | 0.030 | -2.447872 | -. 1260134 |

These results suggest that the inclination of nonwhites to choose prepaid care is even stronger than it was without controlling. We also see that subjects in site 2 are less likely to be uninsured.

## Fitting constrained models

mlogit can fit models with subsets of coefficients constrained to be zero, with subsets of coefficients constrained to be equal both within and across equations, and with subsets of coefficients arbitrarily constrained to equal linear combinations of other estimated coefficients.

Before fitting a constrained model, you define the constraints with the constraint command; see $[R]$ constraint. Once the constraints are defined, you estimate using mlogit, specifying the constraint() option. Typing constraint (4) would use the constraint you previously saved as 4. Typing constraint $(1,4,6)$ would use the previously stored constraints 1,4 , and 6 . Typing constraint $(1-4,6)$ would use the previously stored constraints $1,2,3,4$, and 6 .

Sometimes you will not be able to specify the constraints without knowing the omitted outcome. In such cases, assume that the omitted outcome is whatever outcome is convenient for you, and include the baseoutcome() option when you specify the mlogit command.
> Example 5: Specifying constraints to test hypotheses
We can use constraints to test hypotheses, among other things. In our insurance-choice model, let's test the hypothesis that there is no distinction between having indemnity insurance and being uninsured. Indemnity-style insurance was the omitted outcome, so we type

```
. test [Uninsure]
( 1) [Uninsure]age = 0
( 2) [Uninsure]male = 0
( 3) [Uninsure]nonwhite = 0
(4) [Uninsure] 1b.site = 0
( 5) [Uninsure]2.site = 0
( 6) [Uninsure]3.site = 0
    Constraint 4 dropped
        chi2( 5) = 9.31
    Prob > chi2 = 0.0973
```

If indemnity had not been the omitted outcome, we would have typed test [Uninsure=Indemnity].
The results produced by test are an approximation based on the estimated covariance matrix of the coefficients. Because the probability of being uninsured is low, the log likelihood may be nonlinear for the uninsured. Conventional statistical wisdom is not to trust the asymptotic answer under these circumstances but to perform a likelihood-ratio test instead.

To use Stata's lrtest (likelihood-ratio test) command, we must fit both the unconstrained and constrained models. The unconstrained model is the one we have previously fit. Following the instruction in [R] lrtest, we first store the unconstrained model results:

```
. estimates store unconstrained
```

To fit the constrained model, we must refit our model with all the coefficients except the constant set to 0 in the Uninsure equation. We define the constraint and then refit:
. constraint 1 [Uninsure]
. mlogit insure age male nonwhite i.site, constraints(1)
Iteration 0: log likelihood $=-555.85446$
Iteration 1: log likelihood $=-539.80523$
Iteration 2: log likelihood $=-539.75644$
Iteration 3: log likelihood = -539.75643

| Multinomial logistic regression | Number of obs | $=$ |
| :--- | :--- | :--- |
|  | Wald chi2(5) | $=$ |
| Log likelihood $=-539.75643$ | Prob > chi2 | $=$ |

( 1) [Uninsure]o.age $=0$
( 2) [Uninsure]o.male $=0$
( 3) [Uninsure]o.nonwhite $=0$
( 4) [Uninsure]2o.site $=0$
( 5) [Uninsure] 30. site $=0$

| insure | Coef. | Std. Err. | z | $P>\|z\|$ | [95\% Con | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Indemnity | (base outcome) |  |  |  |  |  |
| Prepaid |  |  |  |  |  |  |
| age | -. 0107025 | . 0060039 | -1.78 | 0.075 | -. 0224699 | . 0010649 |
| male | . 4963616 | . 1939683 | 2.56 | 0.010 | . 1161907 | . 8765324 |
| nonwhite | . 9421369 | . 2252094 | 4.18 | 0.000 | . 5007346 | 1.383539 |
| site |  |  |  |  |  |  |
| 2 | . 2530912 | . 2029465 | 1.25 | 0.212 | -. 1446767 | . 6508591 |
| 3 | -. 5521773 | . 2187237 | -2.52 | 0.012 | -. 9808678 | -. 1234869 |
| _cons | . 1792752 | . 3171372 | 0.57 | 0.572 | -. 4423023 | . 8008527 |
| Uninsure |  |  |  |  |  |  |
| age | 0 | (omitted) |  |  |  |  |
| male | 0 | (omitted) |  |  |  |  |
| nonwhite | 0 | (omitted) |  |  |  |  |
| site |  |  |  |  |  |  |
| 2 | 0 | (omitted) |  |  |  |  |
| 3 | 0 | (omitted) |  |  |  |  |
| _cons | -1.87351 | . 1601099 | -11.70 | 0.000 | -2.18732 | -1.5597 |

We can now perform the likelihood-ratio test:

```
. lrtest unconstrained .
Likelihood-ratio test LR chi2(5) = 10.79
(Assumption: . nested in unconstrained) Prob > chi2 = 0.0557
```

The likelihood-ratio chi-squared is 10.79 with 5 degrees of freedom-just slightly greater than the magic $p=0.05$ level-so we should not call this difference significant.

## - Technical note

In certain circumstances, you should fit a multinomial logit model with conditional logit; see $[R]$ clogit. With substantial data manipulation, clogit can handle the same class of models with some interesting additions. For example, if we had available the price and deductible of the most competitive insurance plan of each type, mlogit could not use this information, but clogit could.

## Stored results

mlogit stores the following in e():
Scalars

```
e(N)
e(N_cd)
e(k_out)
e(k)
e(k_eq)
e(k_eq_model)
e(k_dv)
e(df_m)
e(r2_p)
e(ll)
e(ll_0)
e(N_clust)
e(chi2)
e(p)
e(k_eq_base)
e(baseout)
e(ibaseout)
e(rank)
e(ic)
e(rc)
e(converged)
```

Macros
e(cmd)
e(cmdline)
e(depvar)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(chi2type)
e(vce)
e(vcetype)
e(eqnames)
e(baselab)
e(opt)
e(which)
e(ml_method)
e(user)
e(technique)
e(properties)
e(predict)
e(marginsnotok)
e(asbalanced)
e(asobserved)
number of observations
number of completely determined observations
number of outcomes
number of parameters
number of equations in $e(b)$
number of equations in overall model test
number of dependent variables
model degrees of freedom
pseudo- $R$-squared
log likelihood
log likelihood, constant-only model
number of clusters
$\chi^{2}$
significance
equation number of the base outcome
the value of depvar to be treated as the base outcome
index of the base outcome
rank of $e(V)$
number of iterations
return code
1 if converged, 0 otherwise
mlogit
command as typed
name of dependent variable
weight type
weight expression
title in estimation output
name of cluster variable
Wald or LR; type of model $\chi^{2}$ test
vcetype specified in vce()
title used to label Std. Err.
names of equations
value label corresponding to base outcome
type of optimization
max or min; whether optimizer is to perform maximization or minimization
type of ml method
name of likelihood-evaluator program
maximization technique
b V
program used to implement predict
predictions disallowed by margins
factor variables fvset as asbalanced
factor variables fvset as asobserved

Matrices
e(b)
e(out)
e(Cns)
e(ilog)
e(gradient)
e(V)
e(V_modelbased)
Functions
e(sample) marks estimation sample

## Methods and formulas

The multinomial logit model is described in Greene (2012, 763-766).
Suppose that there are $k$ categorical outcomes and-without loss of generality-let the base outcome be 1 . The probability that the response for the $j$ th observation is equal to the $i$ th outcome is

$$
p_{i j}=\operatorname{Pr}\left(y_{j}=i\right)= \begin{cases}\frac{1}{1+\sum_{m=2}^{k} \exp \left(\mathbf{x}_{j} \boldsymbol{\beta}_{m}\right)}, & \text { if } \\ \frac{\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}_{i}\right)}{1+\sum_{m=2}^{k} \exp \left(\mathbf{x}_{j} \boldsymbol{\beta}_{m}\right)}, & \text { if } \\ i>1\end{cases}
$$

where $\mathbf{x}_{j}$ is the row vector of observed values of the independent variables for the $j$ th observation and $\boldsymbol{\beta}_{m}$ is the coefficient vector for outcome $m$. The $\log$ pseudolikelihood is

$$
\ln L=\sum_{j} w_{j} \sum_{i=1}^{k} I_{i}\left(y_{j}\right) \ln p_{i k}
$$

where $w_{j}$ is an optional weight and

$$
I_{i}\left(y_{j}\right)=\left\{\begin{array}{l}
1, \text { if } y_{j}=i \\
0, \text { otherwise }
\end{array}\right.
$$

Newton-Raphson maximum likelihood is used; see [R] maximize.
For constrained equations, the set of constraints is orthogonalized, and a subset of maximizable parameters is selected. For example, a parameter that is constrained to zero is not a maximizable parameter. If two parameters are constrained to be equal to each other, only one is a maximizable parameter.

Let $\mathbf{r}$ be the vector of maximizable parameters. $\mathbf{r}$ is physically a subset of the solution parameters, b. A matrix, $\mathbf{T}$, and a vector, $\mathbf{m}$, are defined as

$$
\mathbf{b}=\operatorname{Tr}+\mathbf{m}
$$

so that

$$
\begin{aligned}
\frac{\partial f}{\partial \mathbf{b}} & =\frac{\partial f}{\partial \mathbf{r}} \mathbf{T}^{\prime} \\
\frac{\partial^{2} f}{\partial \mathbf{b}^{2}} & =\mathbf{T} \frac{\partial^{2} f}{\partial \mathbf{r}^{2}} \mathbf{T}^{\prime}
\end{aligned}
$$

$\mathbf{T}$ consists of a block form in which one part is a permutation of the identity matrix and the other part describes how to calculate the constrained parameters from the maximizable parameters.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce(robust) and vce(cluster clustvar), respectively. See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas.
mlogit also supports estimation with survey data. For details on VCEs with survey data, see [SVY] variance estimation.

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## Also see

[R] mlogit postestimation - Postestimation tools for mlogit
[R] clogit - Conditional (fixed-effects) logistic regression
$[\mathrm{R}]$ logistic - Logistic regression, reporting odds ratios
$[\mathrm{R}]$ logit — Logistic regression, reporting coefficients
[R] mprobit - Multinomial probit regression
[R] nlogit - Nested logit regression
[R] ologit - Ordered logistic regression
[R] rologit - Rank-ordered logistic regression
[R] slogit - Stereotype logistic regression
[MI] estimation - Estimation commands for use with mi estimate
[SVY] svy estimation - Estimation commands for survey data
[U] 20 Estimation and postestimation commands

## Title

mlogit postestimation - Postestimation tools for mlogit

| Description | Syntax for predict | Menu for predict | Options for predict |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Reference | Also see |  |

## Description

The following postestimation commands are available after mlogit:
\(\left.$$
\begin{array}{ll}\text { Command } & \text { Description } \\
\hline \text { contrast } & \text { contrasts and ANOVA-style joint tests of estimates } \\
\text { estat ic } & \text { Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) } \\
\text { estat summarize } & \text { summary statistics for the estimation sample } \\
\text { estat vce } & \text { variance-covariance matrix of the estimators (VCE) } \\
\text { estat (svy) } & \text { postestimation statistics for survey data } \\
\text { estimates } & \text { cataloging estimation results } \\
\text { forecast }\end{array}
$$ \quad $$
\begin{array}{l}\text { dynamic forecasts and simulations } \\
\text { hausman }\end{array}
$$ \quad $$
\begin{array}{l}\text { Hausman's specification test } \\
\text { lincom }\end{array}
$$ \quad \begin{array}{l}point estimates, standard errors, testing, and inference for linear combinations <br>

of coefficients\end{array}\right]\)| likelihood-ratio test |
| :--- |
| lrtest |

[^48]
## Syntax for predict

```
    predict [type] {stub*|newvar|newvarlist } [if][in] [, statistic outcome(outcome)]
    predict [type] {stub*|newvarlist} [if] [in], scores
```

statistic Description

| Main |  |
| :--- | :--- |
| $\underline{\mathrm{pr}}$ | probability of a positive outcome; the default |
| xb | linear prediction |
| stdp | standard error of the linear prediction |
| stddp | standard error of the difference in two linear predictions |

If you do not specify outcome(), pr (with one new variable specified), xb, and stdp assume outcome(\#1). You must specify outcome() with the stddp option.
You specify one or $k$ new variables with pr , where $k$ is the number of outcomes.
You specify one new variable with xb , stdp, and stddp.
These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
pr , the default, calculates the probability of each of the categories of the dependent variable or the probability of the level specified in outcome(outcome). If you specify the outcome (outcome) option, you need to specify only one new variable; otherwise, you must specify a new variable for each category of the dependent variable.
xb calculates the linear prediction. You must also specify the outcome (outcome) option.
stdp calculates the standard error of the linear prediction. You must also specify the outcome (outcome) option.
stddp calculates the standard error of the difference in two linear predictions. You must specify the outcome (outcome) option, and here you specify the two particular outcomes of interest inside the parentheses, for example, predict sed, stddp outcome $(1,3)$.
outcome (outcome) specifies the outcome for which the statistic is to be calculated. equation() is a synonym for outcome(): it does not matter which you use. outcome() or equation() can be specified using
\#1, \#2, ..., where \#1 means the first category of the dependent variable, \#2 means the second category, etc.;
the values of the dependent variable; or
the value labels of the dependent variable if they exist.
scores calculates equation-level score variables. The number of score variables created will be one less than the number of outcomes in the model. If the number of outcomes in the model were $k$, then
the first new variable will contain $\partial \ln L / \partial\left(\mathbf{x}_{j} \boldsymbol{\beta}_{1}\right)$;
the second new variable will contain $\partial \ln L / \partial\left(\mathbf{x}_{j} \boldsymbol{\beta}_{2}\right)$;
the $(k-1)$ th new variable will contain $\partial \ln L / \partial\left(\mathbf{x}_{j} \boldsymbol{\beta}_{k-1}\right)$.

## Remarks and examples

Remarks are presented under the following headings:
Obtaining predicted values
Calculating marginal effects
Testing hypotheses about coefficients

## Obtaining predicted values

> Example 1: Obtaining predicted probabilities
After estimation, we can use predict to obtain predicted probabilities, index values, and standard errors of the index, or differences in the index. For instance, in example 4 of [R] mlogit, we fit a model of insurance choice on various characteristics. We can obtain the predicted probabilities for outcome 1 by typing

```
. use http://www.stata-press.com/data/r13/sysdsn1
(Health insurance data)
. mlogit insure age i.male i.nonwhite i.site
    (output omitted)
. predict p1 if e(sample), outcome(1)
(option pr assumed; predicted probability)
(29 missing values generated)
```

. summarize p1

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| p1 | 615 | .4764228 | .1032279 | .1698142 | .71939 |

We added the i. prefix to the male, nonwhite, and site variables to explicitly identify them as factor variables. That makes no difference in the estimated results, but we will take advantage of it in later examples. We also included if e(sample) to restrict the calculation to the estimation sample. In example 4 of $[R]$ mlogit, the multinomial logit model was fit on 615 observations, so there must be missing values in our dataset.

Although we typed outcome(1), specifying 1 for the indemnity outcome, we could have typed outcome (Indemnity). For instance, to obtain the probabilities for prepaid, we could type

```
. predict p2 if e(sample), outcome(Prepaid)
(option pr assumed; predicted probability)
(29 missing values generated)
. summarize p2
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline p2 & 615 & .4504065 & .1125962 & .1964103 & .7885724
\end{tabular}
```

We must specify the label exactly as it appears in the underlying value label (or how it appears in the mlogit output), including capitalization.

Here we have used predict to obtain probabilities for the same sample on which we estimated. That is not necessary. We could use another dataset that had the independent variables defined (in our example, age, male, nonwhite, and site) and use predict to obtain predicted probabilities; here, we would not specify if e(sample).

## > Example 2: Obtaining index values

predict can also be used to obtain the index values-the $\sum x_{i} \widehat{\beta}_{i}^{(k)}$ - as well as the probabilities:

```
. predict idx1, outcome(Indemnity) xb
(1 missing value generated)
. summarize idx1
```

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | :---: | :---: | :---: | :---: | ---: |
| idx1 | 643 | 0 | 0 | 0 | 0 |

The indemnity outcome was our base outcome - the outcome for which all the coefficients were set to 0 -so the index is always 0 . For the prepaid and uninsured outcomes, we type

```
. predict idx2, outcome(Prepaid) xb
(1 missing value generated)
. predict idx3, outcome(Uninsure) xb
(1 missing value generated)
. summarize idx2 idx3
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline idx2 & 643 & -.0566113 & .4962973 & -1.298198 & 1.700719 \\
idx3 & 643 & -1.980747 & .6018139 & -3.112741 & -.8258458
\end{tabular}
```

We can obtain the standard error of the index by specifying the stdp option:

```
. predict se2, outcome(Prepaid) stdp
(1 missing value generated)
. list p2 idx2 se2 in 1/5
```

| p 2 | idx2 | $\mathrm{se2}$ |
| ---: | ---: | ---: |
| .3709022 | -.4831167 | .2437772 |
| .4977667 | .055111 | .1694686 |
| .4113073 | -.1712106 | .1793498 |
| .5424927 | .3788345 | .2513701 |
| . | -.0925817 | .1452616 |

We obtained the probability, p 2 , in the previous example.

Finally, predict can calculate the standard error of the difference in the index values between two outcomes with the stddp option:

```
. predict se_2_3, outcome(Prepaid,Uninsure) stddp
(1 missing value generated)
. list idx2 idx3 se_2_3 in 1/5
```

| idx2 | idx3 | se_2_3 |
| ---: | ---: | ---: |
| -.4831167 | -3.073253 | .5469354 |
| .055111 | -2.715986 | .4331918 |
| -.1712106 | -1.579621 | .3053815 |
| .3788345 | -1.462007 | .4492552 |
| -.0925817 | -2.814022 | .4024784 |

In the first observation, the difference in the indexes is $-0.483-(-3.073)=2.59$. The standard error of that difference is 0.547 .

## $>$ Example 3: Interpreting results using predictive margins

It is more difficult to interpret the results from mlogit than those from clogit or logit because there are multiple equations. For example, suppose that one of the independent variables in our model takes on the values 0 and 1 , and we are attempting to understand the effect of this variable. Assume that the coefficient on this variable for the second outcome, $\beta^{(2)}$, is positive. We might then be tempted to reason that the probability of the second outcome is higher if the variable is 1 rather than 0 . Most of the time, that will be true, but occasionally we will be surprised. The probability of some other outcome could increase even more (say, $\beta^{(3)}>\beta^{(2)}$ ), and thus the probability of outcome 2 would actually fall relative to that outcome. We can use predict to help interpret such results.

Continuing with our previously fit insurance-choice model, we wish to describe the model's predictions by race. For this purpose, we can use the method of predictive margins (also known as recycled predictions), in which we vary characteristics of interest across the whole dataset and average the predictions. That is, we have data on both whites and nonwhites, and our individuals have other characteristics as well. We will first pretend that all the people in our data are white but hold their other characteristics constant. We then calculate the probabilities of each outcome. Next we will pretend that all the people in our data are nonwhite, still holding their other characteristics constant. Again we calculate the probabilities of each outcome. The difference in those two sets of calculated probabilities, then, is the difference due to race, holding other characteristics constant.

```
. gen byte nonwhold=nonwhite // save real race
. replace nonwhite=0 // make everyone white
(126 real changes made)
. predict wpind, outcome(Indemnity) // predict probabilities
(option pr assumed; predicted probability)
(1 missing value generated)
. predict wpp, outcome(Prepaid)
(option pr assumed; predicted probability)
(1 missing value generated)
. predict wpnoi, outcome(Uninsure)
(option pr assumed; predicted probability)
(1 missing value generated)
. replace nonwhite=1 // make everyone nonwhite
(644 real changes made)
```

```
. predict nwpind, outcome(Indemnity)
(option pr assumed; predicted probability)
(1 missing value generated)
. predict nwpp, outcome(Prepaid)
(option pr assumed; predicted probability)
(1 missing value generated)
. predict nwpnoi, outcome(Uninsure)
(option pr assumed; predicted probability)
(1 missing value generated)
. replace nonwhite=nonwhold // restore real race
(518 real changes made)
```

. summarize wp* nwp*, sep(3)

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| wpind | 643 | .5141673 | .0872679 | .3092903 | .71939 |
| wpp | 643 | .4082052 | .0993286 | .1964103 | .6502247 |
| wpnoi | 643 | .0776275 | .0360283 | .0273596 | .1302816 |
| nwpind | 643 | .3112809 | .0817693 | .1511329 | .535021 |
| nwpp | 643 | .630078 | .0979976 | .3871782 | .8278881 |
| nwpnoi | 643 | .0586411 | .0287185 | .0209648 | .0933874 |

In example 1 of $[\mathrm{R}]$ mlogit, we presented a cross-tabulation of insurance type and race. Those values were unadjusted. The means reported above are the values adjusted for age, sex, and site. Combining the results gives

|  | Unadjusted |  | Adjusted |  |
| :--- | :---: | :---: | :---: | :---: |
|  | white | nonwhite | white | nonwhite |
| Indemnity | 0.51 | 0.36 | 0.51 | 0.31 |
| Prepaid | 0.42 | 0.57 | 0.41 | 0.63 |
| Uninsured | 0.07 | 0.07 | 0.08 | 0.06 |

We find, for instance, after adjusting for age, sex, and site, that although $57 \%$ of nonwhites in our data had prepaid plans, $63 \%$ of nonwhites chose prepaid plans.

Computing predictive margins by hand was instructive, but we can compute these values more easily using the margins command (see [R] margins). The two margins for the indemnity outcome can be estimated by typing

margins also estimates the standard errors and confidence intervals of the margins. By default, margins uses only the estimation sample. We added the noesample option so that margins would use the entire sample and produce results comparable to our earlier analysis.

We can use marginsplot to graph the results from margins:

```
. marginsplot
Variables that uniquely identify margins: nonwhite
```



The margins for the other two outcomes can be computed by typing

```
. margins nonwhite, predict(outcome(Prepaid)) noesample
    (output omitted)
. margins nonwhite, predict(outcome(Uninsure)) noesample
    (output omitted)
```


## - Technical note

You can use predict to classify predicted values and compare them with the observed outcomes to interpret a multinomial logit model. This is a variation on the notions of sensitivity and specificity for logistic regression. Here we will classify indemnity and prepaid as definitely predicting indemnity, definitely predicting prepaid, and ambiguous.

```
. predict indem, outcome(Indemnity) index // obtain indexes
(1 missing value generated)
. predict prepaid, outcome(Prepaid) index
(1 missing value generated)
. gen diff = prepaid-indem // obtain difference
(1 missing value generated)
. predict sediff, outcome(Indemnity,Prepaid) stddp // & its standard error
(1 missing value generated)
. gen type = 1 if diff/sediff < -1.96 // definitely indemnity
(504 missing values generated)
. replace type = 3 if diff/sediff > 1.96 // definitely prepaid
(100 real changes made)
. replace type = 2 if type>=. & diff/sediff < . // ambiguous
(404 real changes made)
. label def type 1 "Def Ind" 2 "Ambiguous" 3 "Def Prep"
. label values type type // label results
```

| • tabulate insure type |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: |
| insure | Def | Ind | type <br> Ambiguous | Def Prep |$\quad$ Total

We can see that the predictive power of this model is modest. There are many misclassifications in both directions, though there are more correctly classified observations than misclassified observations.

Also the uninsured look overwhelmingly as though they might have come from the indemnity system rather than from the prepaid system.

## Calculating marginal effects

## $\lambda$ Example 4

We have already noted that the coefficients from multinomial logit can be difficult to interpret because they are relative to the base outcome. Another way to evaluate the effect of covariates is to examine the marginal effect of changing their values on the probability of observing an outcome.

The margins command can be used for this too. We can estimate the marginal effect of each covariate on the probability of observing the first outcome-indemnity insurance-by typing

```
. margins, dydx(*) predict(outcome(Indemnity))
Average marginal effects Number of obs = 615
Model VCE : OIM
Expression : Pr(insure==Indemnity), predict(outcome(Indemnity))
dy/dx w.r.t. : age 1.male 1.nonwhite 2.site 3.site
```

|  | Delta-method |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | dy/dx | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| age | .0026655 | .001399 | 1.91 | 0.057 | -.0000765 | .0054074 |
| 1.male | -.1295734 | .0450945 | -2.87 | 0.004 | -.2179571 | -.0411898 |
| 1.nonwhite | -.2032404 | .0482554 | -4.21 | 0.000 | -.2978192 | -.1086616 |
|  |  |  |  |  |  |  |
| site |  |  |  |  |  |  |
| 2 | .0070995 | .0479993 | 0.15 | 0.882 | -.0869775 | .1011765 |
| 3 | .1216165 | .0505833 | 2.40 | 0.016 | .022475 | .220758 |

Note: dy/dx for factor levels is the discrete change from the base level.
By default, margins estimates the average marginal effect over the estimation sample, and that is what we see above. Being male decreases the average probability of having indemnity insurance by 0.130 . We also see, from the note at the bottom of the table, that the marginal effect was computed as a discrete change in the probability of being male rather than female. That is why we made male a factor variable when fitting the model.

The dydx (*) option requested that margins estimate the marginal effect for each regressor, dydx (age) would have produced estimates only for the effect of age. margins has many options for controlling how the marginal effect is computed, including the ability to average over subgroups or to compute estimates for specified values of the regressors; see $[R]$ margins.

We could evaluate the marginal effects on the other two outcomes by typing

```
. margins, dydx(*) predict(outcome(Prepaid))
    (output omitted)
. margins, dydx(*) predict(outcome(Uninsure))
(output omitted)
```


## Testing hypotheses about coefficients

## > Example 5

test tests hypotheses about the coefficients just as after any estimation command; see $[R]$ test. Note, however, test's syntax for dealing with multiple-equation models. Because test bases its results on the estimated covariance matrix, we might prefer a likelihood-ratio test; see example 5 in [ R ] mlogit for an example of lrtest.

If we simply list variables after the test command, we are testing that the corresponding coefficients are zero across all equations:

```
. test 2.site 3.site
( 1) [Indemnity]2o.site = 0
( 2) [Prepaid]2.site = 0
( 3) [Uninsure]2.site = 0
( 4) [Indemnity]3o.site = 0
( 5) [Prepaid]3.site = 0
( 6) [Uninsure]3.site = 0
    Constraint 1 dropped
    Constraint 4 dropped
    chi2( 4) = 19.74
        Prob > chi2 = 0.0006
```

We can test that all the coefficients (except the constant) in an equation are zero by simply typing the outcome in square brackets:

```
. test [Uninsure]
( 1) [Uninsure] age = 0
( 2) [Uninsure]0b.male = 0
( 3) [Uninsure]1.male = 0
(4) [Uninsure]0b.nonwhite = 0
( 5) [Uninsure]1.nonwhite = 0
( 6) [Uninsure]1b.site = 0
( 7) [Uninsure]2.site = 0
( 8) [Uninsure]3.site = 0
    Constraint 2 dropped
    Constraint 4 dropped
    Constraint 6 dropped
    chi2( 5) = 9.31
        Prob > chi2 = 0.0973
```

We specify the outcome just as we do with predict; we can specify the label if the outcome variable is labeled, or we can specify the numeric value of the outcome. We would have obtained the same test as above if we had typed test [3] because 3 is the value of insure for the outcome uninsured.

We can combine the two syntaxes. To test that the coefficients on the site variables are 0 in the equation corresponding to the outcome prepaid, we can type

```
. test [Prepaid]: 2.site 3.site
( 1) [Prepaid]2.site = 0
( 2) [Prepaid]3.site = 0
    chi2( 2) = 10.78
    Prob > chi2 = 0.0046
```

We specified the outcome and then followed that with a colon and the variables we wanted to test.
We can also test that coefficients are equal across equations. To test that all coefficients except the constant are equal for the prepaid and uninsured outcomes, we can type

```
. test [Prepaid=Uninsure]
( 1) [Prepaid]age - [Uninsure] age = 0
( 2) [Prepaid]0b.male - [Uninsure]Ob.male = 0
( 3) [Prepaid]1.male - [Uninsure]1.male = 0
( 4) [Prepaid]0b.nonwhite - [Uninsure]0b.nonwhite = 0
( 5) [Prepaid]1.nonwhite - [Uninsure]1.nonwhite = 0
( 6) [Prepaid]1b.site - [Uninsure]1b.site = 0
( 7) [Prepaid]2.site - [Uninsure]2.site = 0
( 8) [Prepaid]3.site - [Uninsure]3.site = 0
    Constraint 2 dropped
    Constraint 4 dropped
        Constraint 6 dropped
    chi2( 5) = 13.80
        Prob > chi2 = 0.0169
```

To test that only the site variables are equal, we can type

```
. test [Prepaid=Uninsure]: 2.site 3.site
( 1) [Prepaid]2.site - [Uninsure]2.site = 0
( 2) [Prepaid]3.site - [Uninsure]3.site = 0
    chi2( 2) = 12.68
    Prob > chi2 = 0.0018
```

Finally, we can test any arbitrary constraint by simply entering the equation and specifying the coefficients as described in [U] 13.5 Accessing coefficients and standard errors. The following hypothesis is senseless but illustrates the point:

```
. test ([Prepaid]age+[Uninsure]2.site)/2 = 2-[Uninsure]1.nonwhite
( 1) . 5*[Prepaid]age + [Uninsure]1.nonwhite + . 5*[Uninsure]2.site = 2
    chi2( 1) = 22.45
    Prob > chi2 = 0.0000
```

See [R] test for more information about test. The information there about combining hypotheses across test commands (the accumulate option) also applies after mlogit.

## Reference

Fagerland, M. W., and D. W. Hosmer, Jr. 2012. A generalized HosmerLemeshow goodness-of-fit test for multinomial logistic regression models. Stata Journal 12: 447-453.

## Also see

[R] mlogit - Multinomial (polytomous) logistic regression
[U] 20 Estimation and postestimation commands

## Title

more - The -more- message

Syntax Description Option Remarks and examples Also see

## Syntax

Tell Stata to pause or not pause for -more- messages
set more $\{$ on $\mid$ off $\}[$, permanently $]$

Set number of lines between -more- messages
set pagesize \#

## Description

set more on, which is the default, tells Stata to wait until you press a key before continuing when a -more- message is displayed.
set more off tells Stata not to pause or display the -more - message.
set pagesize \# sets the number of lines between -more - messages. The permanently option is not allowed with set pagesize.

## Option

permanently specifies that, in addition to making the change right now, the more setting be remembered and become the default setting when you invoke Stata.

## Remarks and examples

When you see -more- at the bottom of the screen,

| Press $\ldots$ | and Stata ... |
| :--- | :--- |
| letter $l$ or Enter | displays the next line |
| letter $q$ | acts as if you pressed Break |
| Spacebar or any other key | displays the next screen |

You can also click on the More button or click on -more- to display the next screen.
-more- is Stata's way of telling you that it has something more to show you but that showing it to you will cause the information on the screen to scroll off.

If you type set more off, -more- conditions will never arise, and Stata's output will scroll by at full speed.

If you type set more on, -more- conditions will be restored at the appropriate places.
Programmers should see [P] more for information on the more programming command.

## Also see

[R] query - Display system parameters
[P] creturn - Return c-class values
[ P ] more - Pause until key is pressed
[P] sleep - Pause for a specified time
[U] 7 -more- conditions

## Title

mprobit - Multinomial probit regression

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

mprobit depvar [indepvars] [if] [in] [weight] [, options]
options
Description

Model
noconstant
baseoutcome (\#|lbl)
probitparam
constraints (constraints)
collinear
SE/Robust
vce (vcetype)

Reporting
level(\#)
nocnsreport
display_options

Integration
intpoints(\#)

Maximization
maximize_options
coeflegend
suppress constant terms
outcome used to normalize location
use the probit variance parameterization
apply specified linear constraints
keep collinear variables
vcetype may be oim, robust, cluster clustvar, opg, bootstrap, or jackknife
set confidence level; default is level (95)
do not display constraints
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
bootstrap, by, fp, jackknife, mi estimate, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands.
vce (bootstrap) and vce (jackknife) are not allowed with the mi estimate prefix; see [MI] mi estimate.
Weights are not allowed with the bootstrap prefix; see $[R]$ bootstrap.
vce() and weights are not allowed with the svy prefix; see [SVY] svy.
fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Categorical outcomes $>$ Independent multinomial probit

## Description

mprobit fits multinomial probit (MNP) models via maximum likelihood. depvar contains the outcome for each observation, and indepvars are the associated covariates. The error terms are assumed to be independent, standard normal, random variables. See $[R]$ asmprobit for the case where the latent-variable errors are correlated or heteroskedastic and you have alternative-specific variables.

## Options

## Model

noconstant suppresses the $J-1$ constant terms.
baseoutcome (\#|lbl) specifies the outcome used to normalize the location of the latent variable. The base outcome may be specified as a number or a label. The default is to use the most frequent outcome. The coefficients associated with the base outcome are zero.
probitparam specifies to use the probit variance parameterization by fixing the variance of the differenced latent errors between the scale and the base alternatives to be one. The default is to make the variance of the base and scale latent errors one, thereby making the variance of the difference to be two.
constraints (constraints), collinear; see [R] estimation options.

SE/Robust
vce (vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
If specifying vce(bootstrap) or vce(jackknife), you must also specify baseoutcome().
$\qquad$ Reporting
level(\#); see [R] estimation options.
nocnsreport; see $[\mathrm{R}]$ estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% fmt), pformat (\% fmt), sformat (\% fmt), and nolstretch; see [R] estimation options.
$\sqrt{\text { Integration }}$
intpoints(\#) specifies the number of Gaussian quadrature points to use in approximating the likelihood. The default is 15 .
maximize_options: difficult, technique (algorithm_spec), iterate (\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance (\#), nonrtolerance, and from(init_specs); see [R] maximize. These options are seldom used.
Setting the optimization type to technique (bhhh) resets the default vcetype to vce (opg).
The following option is available with mprobit but is not shown in the dialog box:
coeflegend; see $[R]$ estimation options.

## Remarks and examples

The MNP model is used with discrete dependent variables that take on more than two outcomes that do not have a natural ordering. The stochastic error terms for this implementation of the model are assumed to have independent, standard normal distributions. To use mprobit, you must have one observation for each decision maker in the sample. See [R] asmprobit for another implementation of the MNP model that permits correlated and heteroskedastic errors and is suitable when you have data for each alternative that a decision maker faced.

The MNP model is frequently motivated using a latent-variable framework. The latent variable for the $j$ th alternative, $j=1, \ldots, J$, is

$$
\eta_{i j}=\mathbf{z}_{i} \boldsymbol{\alpha}_{j}+\xi_{i j}
$$

where the $1 \times q$ row vector $\mathbf{z}_{i}$ contains the observed independent variables for the $i$ th decision maker. Associated with $\mathbf{z}_{i}$ are the $J$ vectors of regression coefficients $\boldsymbol{\alpha}_{j}$. The $\xi_{i, 1}, \ldots, \xi_{i, J}$ are distributed independently and identically standard normal. The decision maker chooses the alternative $k$ such that $\eta_{i k} \geq \eta_{i m}$ for $m \neq k$.

Suppose that case $i$ chooses alternative $k$, and take the difference between latent variable $\eta_{i k}$ and the $J-1$ others:

$$
\begin{align*}
v_{i j k} & =\eta_{i j}-\eta_{i k} \\
& =\mathbf{z}_{i}\left(\boldsymbol{\alpha}_{j}-\boldsymbol{\alpha}_{k}\right)+\xi_{i j}-\xi_{i k}  \tag{1}\\
& =\mathbf{z}_{i} \gamma_{j^{\prime}}+\epsilon_{i j^{\prime}}
\end{align*}
$$

where $j^{\prime}=j$ if $j<k$ and $j^{\prime}=j-1$ if $j>k$ so that $j^{\prime}=1, \ldots, J-1 . \operatorname{Var}\left(\epsilon_{i j^{\prime}}\right)=\operatorname{Var}\left(\xi_{i j}-\xi_{i k}\right)=2$ and $\operatorname{Cov}\left(\epsilon_{i j^{\prime}}, \epsilon_{i l^{\prime}}\right)=1$ for $j^{\prime} \neq l^{\prime}$. The probability that alternative $k$ is chosen is

$$
\begin{aligned}
\operatorname{Pr}(i \text { chooses } k) & =\operatorname{Pr}\left(v_{i 1 k} \leq 0, \ldots, v_{i, J-1, k} \leq 0\right) \\
& =\operatorname{Pr}\left(\epsilon_{i 1} \leq-\mathbf{z}_{i} \boldsymbol{\gamma}_{1}, \ldots, \epsilon_{i, J-1} \leq-\mathbf{z}_{i} \boldsymbol{\gamma}_{J-1}\right)
\end{aligned}
$$

Hence, evaluating the likelihood function involves computing probabilities from the multivariate normal distribution. That all the covariances are equal simplifies the problem somewhat; see Methods and formulas for details.

In (1), not all $J$ of the $\boldsymbol{\alpha}_{j}$ are identifiable. To remove the indeterminacy, $\boldsymbol{\alpha}_{l}$ is set to the zero vector, where $l$ is the base outcome as specified in the baseoutcome() option. That fixes the $l$ th latent variable to zero so that the remaining variables measure the attractiveness of the other alternatives relative to the base.

## > Example 1

As discussed in example 1 of $[\mathrm{R}]$ mlogit, we have data on the type of health insurance available to 616 psychologically depressed subjects in the United States (Tarlov et al. 1989; Wells et al. 1989). Patients may have either an indemnity (fee-for-service) plan or a prepaid plan such as an HMO, or the patient may be uninsured. Demographic variables include age, gender, race, and site. Indemnity insurance is the most popular alternative, so mprobit will choose it as the base outcome by default.
. use http://www.stata-press.com/data/r13/sy
(Health insurance data)
. mprobit insure age male nonwhite i.site
Iteration 0: log likelihood $=-535.89424$
Iteration 1: $\quad$ log likelihood $=-534.56173$
Iteration 2: $\quad$ log likelihood $=-534.52835$
Iteration 3: $\quad$ log likelihood $=-534.52833$

| Multinomial probit regression | Number of obs | = | 615 |
| :---: | :---: | :---: | :---: |
|  | Wald chi2(10) |  | 40.18 |
| Log likelihood = -534.52833 | Prob > chi2 |  | 0.0000 |


| insure | Coef. | Std. Err. | z | $P>\|z\|$ | [95\% Con | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Indemnity | (base outcome) |  |  |  |  |  |
| Prepaid |  |  |  |  |  |  |
| age | -. 0098536 | . 0052688 | -1.87 | 0.061 | -. 0201802 | . 000473 |
| male | . 4774678 | . 1718316 | 2.78 | 0.005 | . 1406841 | . 8142515 |
| nonwhite | . 8245003 | . 1977582 | 4.17 | 0.000 | . 4369013 | 1.212099 |
| site |  |  |  |  |  |  |
| 2 | . 0973956 | . 1794546 | 0.54 | 0.587 | -. 2543289 | . 4491201 |
| 3 | -. 495892 | . 1904984 | -2.60 | 0.009 | -. 869262 | -. 1225221 |
| _cons | . 22315 | . 2792424 | 0.80 | 0.424 | -. 324155 | . 7704549 |
| Uninsure |  |  |  |  |  |  |
| age | -. 0050814 | . 0075327 | -0.67 | 0.500 | -. 0198452 | . 0096823 |
| male | . 3332637 | . 2432986 | 1.37 | 0.171 | -. 1435929 | . 8101203 |
| nonwhite | . 2485859 | . 2767734 | 0.90 | 0.369 | -. 29388 | . 7910518 |
| site |  |  |  |  |  |  |
| 2 | -. 6899485 | . 2804497 | -2.46 | 0.014 | -1.23962 | -. 1402771 |
| 3 | -. 1788447 | . 2479898 | -0.72 | 0.471 | -. 6648957 | . 3072063 |
| _cons | -. 9855917 | . 3891873 | -2.53 | 0.011 | -1.748385 | -. 2227986 |

The likelihood function for mprobit is derived under the assumption that all decision-making units face the same choice set, which is the union of all outcomes observed in the dataset. If that is not true for your model, then an alternative is to use the asmprobit command, which does not require this assumption. To do that, you will need to expand the dataset so that each decision maker has $k_{i}$ observations, where $k_{i}$ is the number of alternatives in the choice set faced by decision maker $i$. You will also need to create a binary variable to indicate the choice made by each decision maker. Moreover, you will need to use the correlation(independent) and stddev(homoskedastic) options with asmprobit unless you have alternative-specific variables.

## Stored results

mprobit stores the following in e():
Scalars

```
e(N)
e(k_out)
e(k_points)
e(k)
e(k_eq)
e(k_eq_model)
e(k_indvars)
e(k_dv)
e(df_m)
e(ll)
e(N_clust)
e(chi2)
e(p)
e(i_base)
e(const)
e(probitparam)
e(rank)
e(ic)
e(rc)
e(converged)
```

Macros
e(cmd)
e(cmdline)
e(depvar)
e(indvars)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(chi2type)
e(vce)
e(vcetype)
e(outeqs)
e(out\#)
e(opt)
e(which)
e(ml_method)
e(user)
e(technique)
e(properties)
e(predict)
e(marginsnotok)
e(asbalanced)
e(asobserved)
number of observations
number of outcomes
number of quadrature points
number of parameters
number of equations in $e(b)$
number of equations in overall model test
number of independent variables
number of dependent variables
model degrees of freedom
log simulated-likelihood
number of clusters
$\chi^{2}$
significance
base outcome index
0 if noconstant is specified, 1 otherwise
1 if probitparam is specified, 0 otherwise
rank of e(V)
number of iterations
return code
1 if converged, 0 otherwise
mprobit
command as typed
name of dependent variable
independent variables
weight type
weight expression
title in estimation output
name of cluster variable
Wald, type of model $\chi^{2}$ test
vcetype specified in vce()
title used to label Std. Err.
outcome equations
outcome labels, $\#=1, \ldots, e$ (k_out)
type of optimization
$\max$ or min; whether optimizer is to perform maximization or minimization
type of ml method
name of likelihood-evaluator program
maximization technique
b V
program used to implement predict
predictions disallowed by margins
factor variables fvset as asbalanced
factor variables fvset as asobserved

Matrices

```
    e(b)
    e(outcomes)
    e(Cns)
    e(ilog)
    e(gradient)
    e(V)
    e(V_modelbased)
```

Functions e(sample)
coefficient vector
outcome values
constraints matrix
iteration $\log$ (up to 20 iterations)
gradient vector
variance-covariance matrix of the estimators
model-based variance
marks estimation sample

## Methods and formulas

See Cameron and Trivedi (2005, chap. 15) for a discussion of multinomial models, including multinomial probit. Long and Freese (2014, chap. 8) discuss the multinomial logistic, multinomial probit, and stereotype logistic regression models, with examples using Stata.

As discussed in Remarks and examples, the latent variables for a $J$-alternative model are $\eta_{i j}=$ $\mathbf{z}_{i} \boldsymbol{\alpha}_{j}+\xi_{i j}$, for $j=1, \ldots, J, i=1, \ldots, n$, and $\left\{\xi_{i, 1}, \ldots, \xi_{i, \mathrm{~J}}\right\} \sim$ i.i.d. $N(0,1)$. The experimenter observes alternative $k$ for the $i$ th observation if $\eta_{i k}>\eta_{i l}$ for $l \neq k$. For $j \neq k$, let

$$
\begin{aligned}
v_{i j^{\prime}} & =\eta_{i j}-\eta_{i k} \\
& =\mathbf{z}_{i}\left(\boldsymbol{\alpha}_{j}-\boldsymbol{\alpha}_{k}\right)+\xi_{i j}-\xi_{i k} \\
& =\mathbf{z}_{i} \boldsymbol{\gamma}_{j^{\prime}}+\epsilon_{i j^{\prime}}
\end{aligned}
$$

where $j^{\prime}=j$ if $j<k$ and $j^{\prime}=j-1$ if $j>k$ so that $j^{\prime}=1, \ldots, J-1 . \boldsymbol{\epsilon}_{i}=\left(\epsilon_{i 1}, \ldots, \epsilon_{i, J-1}\right) \sim$ $\operatorname{MVN}(\mathbf{0}, \mathbf{\Sigma})$, where

$$
\boldsymbol{\Sigma}=\left(\begin{array}{ccccc}
2 & 1 & 1 & \ldots & 1 \\
1 & 2 & 1 & \ldots & 1 \\
1 & 1 & 2 & \ldots & 1 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & 1 & 1 & \ldots & 2
\end{array}\right)
$$

Denote the deterministic part of the model as $\lambda_{i j^{\prime}}=\mathbf{z}_{i} \gamma_{j^{\prime}}$; the probability that subject $i$ chooses outcome $k$ is

$$
\begin{aligned}
\operatorname{Pr}\left(y_{i}\right. & =k)=\operatorname{Pr}\left(v_{i 1} \leq 0, \ldots, v_{i, J-1} \leq 0\right) \\
& =\operatorname{Pr}\left(\epsilon_{i 1} \leq-\lambda_{i 1}, \ldots, \epsilon_{i, J-1} \leq-\lambda_{i, J-1}\right) \\
& =\frac{1}{(2 \pi)^{(J-1) / 2}|\mathbf{\Sigma}|^{1 / 2}} \int_{-\infty}^{-\lambda_{i 1}} \cdots \int_{-\infty}^{-\lambda_{i, J-1}} \exp \left(-\frac{1}{2} \mathbf{z}^{\prime} \mathbf{\Sigma}^{-1} \mathbf{z}\right) d \mathbf{z}
\end{aligned}
$$

Because of the exchangeable correlation structure of $\boldsymbol{\Sigma}\left(\rho_{i j}=1 / 2\right.$ for all $\left.i \neq j\right)$, we can use Dunnett's (1989) result to reduce the multidimensional integral to one dimension:

$$
\operatorname{Pr}\left(y_{i}=k\right)=\frac{1}{\sqrt{\pi}} \int_{0}^{\infty}\left\{\prod_{j=1}^{J-1} \Phi\left(-z \sqrt{2}-\lambda_{i j}\right)+\prod_{j=1}^{J-1} \Phi\left(z \sqrt{2}-\lambda_{i j}\right)\right\} e^{-z^{2}} d z
$$

Gaussian quadrature is used to approximate this integral, resulting in the $K$-point quadrature formula

$$
\operatorname{Pr}\left(y_{i}=k\right) \approx \frac{1}{2} \sum_{k=1}^{K} w_{k}\left\{\prod_{j=1}^{J-1} \Phi\left(-\sqrt{2 x_{k}}-\lambda_{i j}\right)+\prod_{j=1}^{J-1} \Phi\left(\sqrt{2 x_{k}}-\lambda_{i j}\right)\right\}
$$

where $w_{k}$ and $x_{k}$ are the weights and roots of the Laguerre polynomial of order $K$. In mprobit, $K$ is specified by the intpoints() option.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce (robust) and vce(cluster clustvar), respectively. See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas.
mprobit also supports estimation with survey data. For details on VCEs with survey data, see [SVY] variance estimation.

## References

Cameron, A. C., and P. K. Trivedi. 2005. Microeconometrics: Methods and Applications. New York: Cambridge University Press.
Dunnett, C. W. 1989. Algorithm AS 251: Multivariate normal probability integrals with product correlation structure. Journal of the Royal Statistical Society, Series C 38: 564-579.

Haan, P., and A. Uhlendorff. 2006. Estimation of multinomial logit models with unobserved heterogeneity using maximum simulated likelihood. Stata Journal 6: 229-245.
Hole, A. R. 2007. Fitting mixed logit models by using maximum simulated likelihood. Stata Journal 7: 388-401.
Long, J. S., and J. Freese. 2014. Regression Models for Categorical Dependent Variables Using Stata. 3rd ed. College Station, TX: Stata Press.
Tarlov, A. R., J. E. Ware, Jr., S. Greenfield, E. C. Nelson, E. Perrin, and M. Zubkoff. 1989. The medical outcomes study. An application of methods for monitoring the results of medical care. Journal of the American Medical Association 262: 925-930.

Wells, K. B., R. D. Hays, M. A. Burnam, W. H. Rogers, S. Greenfield, and J. E. Ware, Jr. 1989. Detection of depressive disorder for patients receiving prepaid or fee-for-service care. Results from the Medical Outcomes Survey. Journal of the American Medical Association 262: 3298-3302.

## Also see

[R] mprobit postestimation - Postestimation tools for mprobit
[R] asmprobit - Alternative-specific multinomial probit regression
$[\mathrm{R}]$ clogit - Conditional (fixed-effects) logistic regression
[R] mlogit - Multinomial (polytomous) logistic regression
[R] nlogit - Nested logit regression
$[\mathrm{R}]$ ologit - Ordered logistic regression
[R] oprobit - Ordered probit regression
[MI] estimation - Estimation commands for use with mi estimate
[SVY] svy estimation - Estimation commands for survey data
[U] 20 Estimation and postestimation commands

## Title

mprobit postestimation - Postestimation tools for mprobit

| Description | Syntax for predict | Menu for predict |
| :--- | :--- | :--- | Options for predict

## Description

The following postestimation commands are available after mprobit:
\(\left.$$
\begin{array}{ll}\text { Command } & \text { Description } \\
\hline \text { contrast } & \text { contrasts and ANOVA-style joint tests of estimates } \\
\text { estat ic } & \text { Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) } \\
\text { estat summarize } & \text { summary statistics for the estimation sample } \\
\text { estat vce } & \text { variance-covariance matrix of the estimators (VCE) } \\
\text { estat (svy) } & \text { postestimation statistics for survey data } \\
\text { estimates } & \text { cataloging estimation results } \\
\text { forecast }\end{array}
$$ \quad $$
\begin{array}{l}\text { dynamic forecasts and simulations } \\
\text { hausman } \\
\text { lincom }\end{array}
$$ \quad \begin{array}{l}Hausman's specification test <br>
point estimates, standard errors, testing, and inference for linear combinations <br>

of coefficients\end{array}\right]\)| likelihood-ratio test |
| :--- |
| lrtest |

[^49]
## Syntax for predict

```
predict [type]{stub*|newvar|newvarlist } [if][in] [, statistic outcome(outcome)]
predict [type] {stub*|newvarlist } [if] [in], scores
```

statistic
Description
Main
$\mathrm{pr} \quad$ probability of a positive outcome; the default
$\overline{\mathrm{xb}} \quad$ linear prediction
stdp standard error of the linear prediction
If you do not specify outcome(), pr (with one new variable specified), xb, and stdp assume outcome(\#1).
You specify one or $k$ new variables with pr, where $k$ is the number of outcomes.
You specify one new variable with xb and stdp.
These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
pr , the default, calculates the probability of each of the categories of the dependent variable or the probability of the level specified in outcome(outcome). If you specify the outcome(outcome) option, you need to specify only one new variable; otherwise, you must specify a new variable for each category of the dependent variable.
xb calculates the linear prediction, $\mathbf{x}_{i} \boldsymbol{\alpha}_{j}$, for alternative $j$ and individual $i$. The index, $j$, corresponds to the outcome specified in outcome().
stdp calculates the standard error of the linear prediction.
outcome (outcome) specifies the outcome for which the statistic is to be calculated. equation() is a synonym for outcome(): it does not matter which you use. outcome() or equation() can be specified using
\#1, \#2, ..., where \#1 means the first category of the dependent variable, \#2 means the second category, etc.;
the values of the dependent variable; or
the value labels of the dependent variable if they exist.
scores calculates the equation-level score variables. The $j$ th new variable will contain the scores for the $j$ th fitted equation.

## Remarks and examples

Once you have fit a multinomial probit model, you can use predict to obtain probabilities that an individual will choose each of the alternatives for the estimation sample, as well as other samples; see [U] 20 Estimation and postestimation commands and [R] predict.

## > Example 1

In example 1 of [R] mprobit, we fit the multinomial probit model to a dataset containing the type of health insurance available to 616 psychologically depressed subjects in the United States (Tarlov et al. 1989; Wells et al. 1989). We can obtain the predicted probabilities by typing

```
. use http://www.stata-press.com/data/r13/sysdsn1
(Health insurance data)
. mprobit insure age male nonwhite i.site
    (output omitted)
- predict p1-p3
(option pr assumed; predicted probabilities)
. list p1-p3 insure in 1/10
```

| p1 | p2 | p3 | insure |
| ---: | ---: | ---: | ---: |
| .5961306 | .3741824 | .029687 | Indemnity |
| .4719296 | .4972289 | .0308415 | Prepaid <br> .4896086 <br> .4121961 |
| .3730529 | .5416623 | .0852848 | Prepnity |
| .5063069 | .4629773 | .0307158 | . |
| .4768125 | .4923548 | .0308327 | Prepaid |
| .5035672 | .4657016 | .0307312 | Prepaid |
| .3326361 | .5580404 | .1093235 | . |
| .4758165 | .4384811 | .0857024 | Uninsure |
| .5734057 | .3316601 | .0949342 | Prepaid |

insure contains a missing value for observations 5 and 8 . Because of that, those two observations were not used in the estimation. However, because none of the independent variables is missing, predict can still calculate the probabilities. Had we typed

```
. predict p1-p3 if e(sample)
```

predict would have filled in missing values for $\mathrm{p} 1, \mathrm{p} 2$, and p 3 for those observations because they were not used in the estimation.

## References

Tarlov, A. R., J. E. Ware, Jr., S. Greenfield, E. C. Nelson, E. Perrin, and M. Zubkoff. 1989. The medical outcomes study. An application of methods for monitoring the results of medical care. Journal of the American Medical Association 262: 925-930.
Wells, K. B., R. D. Hays, M. A. Burnam, W. H. Rogers, S. Greenfield, and J. E. Ware, Jr. 1989. Detection of depressive disorder for patients receiving prepaid or fee-for-service care. Results from the Medical Outcomes Survey. Journal of the American Medical Association 262: 3298-3302.

## Also see

[R] mprobit - Multinomial probit regression
[U] 20 Estimation and postestimation commands

## Title

nbreg - Negative binomial regression

| Syntax | Menu | Description | Options for nbreg |
| :--- | :--- | :--- | :--- |
| Options for gnbreg | Remarks and examples | Stored results | Methods and formulas |
| References | Also see |  |  |

## Syntax

Negative binomial regression model

```
nbreg depvar [indepvars] [if] [in] [weight] [, nbreg_options]
```

Generalized negative binomial model

$$
\text { gnbreg depvar }[\text { indepvars }][\text { if }][\text { in }][\text { weight }][\text {, gnbreg_options }]
$$

nbreg_options
Model
noconstant
dispersion(mean)
dispersion(constant)
exposure ( varname $_{e}$ )
offset ( varname $_{o}$ )
constraints (constraints)
collinear
SE/Robust
vce (vcetype)

Reporting
level(\#)
nolrtest
irr
nocnsreport
display_options

Maximization
maximize_options
coeflegend

Description
suppress constant term
parameterization of dispersion; the default
constant dispersion for all observations
include $\ln \left(\right.$ varname $\left._{e}\right)$ in model with coefficient constrained to 1 include varname ${ }_{o}$ in model with coefficient constrained to 1 apply specified linear constraints keep collinear variables
vcetype may be oim, robust, cluster clustvar, opg, bootstrap, or jackknife
set confidence level; default is level (95)
suppress likelihood-ratio test
report incidence-rate ratios
do not display constraints
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
control the maximization process; seldom used
display legend instead of statistics

## Model

noconstant
lnalpha (varlist) exposure (varname ${ }_{e}$ )
offset (varname ${ }_{o}$ )
constraints (constraints)
collinear
SE/Robust
vce (vcetype)

Reporting
level(\#)
irr
nocnsreport
display_options

## Maximization

maximize_options
coeflegend
suppress constant term
dispersion model variables
include $\ln \left(\right.$ varname $\left._{e}\right)$ in model with coefficient constrained to 1 include varname ${ }_{o}$ in model with coefficient constrained to 1
apply specified linear constraints
keep collinear variables
vcetype may be oim, robust, cluster clustvar, opg, bootstrap, or jackknife
set confidence level; default is level (95)
report incidence-rate ratios
do not display constraints
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
control the maximization process; seldom used
display legend instead of statistics
indepvars and varlist may contain factor variables; see [U] 11.4.3 Factor variables.
depvar, indepvars, varname $e_{e}$, and varname ${ }_{O}$ may contain time-series operators (nbreg only); see [U] 11.4.4 Time-series varlists.
bootstrap, by (nbreg only), fp (nbreg only), jackknife, mfp (nbreg only), mi estimate, nestreg (nbreg only), rolling, statsby, stepwise, and svy are allowed; see [U] 11.1.10 Prefix commands.
vce (bootstrap) and vce (jackknife) are not allowed with the mi estimate prefix; see [MI] mi estimate.
Weights are not allowed with the bootstrap prefix; see $[R]$ bootstrap.
vce() and weights are not allowed with the svy prefix; see [SVY] svy.
fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

nbreg
Statistics $>$ Count outcomes $>$ Negative binomial regression

## gnbreg

Statistics $>$ Count outcomes $>$ Generalized negative binomial regression

## Description

nbreg fits a negative binomial regression model of depvar on indepvars, where depvar is a nonnegative count variable. In this model, the count variable is believed to be generated by a Poissonlike process, except that the variation is greater than that of a true Poisson. This extra variation is referred to as overdispersion. See $[R]$ poisson before reading this entry.
gnbreg fits a generalization of the negative binomial mean-dispersion model; the shape parameter $\alpha$ may also be parameterized.

If you have panel data, see [XT] xtnbreg and [ME] menbreg.

## Options for nbreg

Model
noconstant; see [R] estimation options.
dispersion(mean $\mid$ constant) specifies the parameterization of the model. dispersion(mean), the default, yields a model with dispersion equal to $1+\alpha \exp \left(\mathbf{x}_{j} \boldsymbol{\beta}+\right.$ offset $\left._{j}\right)$; that is, the dispersion is a function of the expected mean: $\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}+\right.$ offset $\left._{j}\right)$. dispersion (constant) has dispersion equal to $1+\delta$; that is, it is a constant for all observations.
exposure( varname $_{e}$ ), offset ( varname $_{o}$ ), constraints (constraints), collinear; see [R] estimation options.

SE/Robust
vce (vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

## Reporting

level (\#); see [R] estimation options.
nolrtest suppresses fitting the Poisson model. Without this option, a comparison Poisson model is fit, and the likelihood is used in a likelihood-ratio test of the null hypothesis that the dispersion parameter is zero.
irr reports estimated coefficients transformed to incidence-rate ratios, that is, $e^{\beta_{i}}$ rather than $\beta_{i}$. Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated or stored. irr may be specified at estimation or when replaying previously estimated results.
nocnsreport; see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\%fmt), pformat(\%fmt), sformat(\%fmt), and nolstretch; see [R] estimation options.

Maximization
maximize_options: difficult, technique (algorithm_spec), iterate(\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance(\#), nonrtolerance, and from(init_specs); see [R] maximize. These options are seldom used.

Setting the optimization type to technique (bhhh) resets the default vcetype to vce (opg).
The following option is available with nbreg but is not shown in the dialog box:
coeflegend; see $[R]$ estimation options.

## Options for gnbreg


#### Abstract

Model noconstant; see $[R]$ estimation options. lnalpha(varlist) allows you to specify a linear equation for $\ln \alpha$. Specifying lnalpha(male old) means that $\ln \alpha=\gamma_{0}+\gamma_{1}$ male $+\gamma_{2}$ old, where $\gamma_{0}, \gamma_{1}$, and $\gamma_{2}$ are parameters to be estimated along with the other model coefficients. If this option is not specified, gnbreg and nbreg will produce the same results because the shape parameter will be parameterized as a constant. exposure ( varname $_{e}$ ), offset (varname ${ }_{o}$ ), constraints (constraints), collinear; see [R] estimation options.


SE/Robust
vce (vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
$\qquad$ Reporting
level(\#); see [R] estimation options.
irr reports estimated coefficients transformed to incidence-rate ratios, that is, $e^{\beta_{i}}$ rather than $\beta_{i}$. Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated or stored. irr may be specified at estimation or when replaying previously estimated results.
nocnsreport; see $[\mathrm{R}]$ estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\%fmt), pformat(\%fmt), sformat(\%fmt), and nolstretch; see [R] estimation options.

Maximization
maximize_options: difficult, technique (algorithm_spec), iterate(\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance (\#), nonrtolerance, and from(init_specs); see [R] maximize. These options are seldom used.
Setting the optimization type to technique (bhhh) resets the default vcetype to vce (opg).
The following option is available with gnbreg but is not shown in the dialog box:
coeflegend; see [R] estimation options.

## Remarks and examples

Remarks are presented under the following headings:

## Introduction to negative binomial regression

Negative binomial regression models the number of occurrences (counts) of an event when the event has extra-Poisson variation, that is, when it has overdispersion. The Poisson regression model is

$$
y_{j} \sim \operatorname{Poisson}\left(\mu_{j}\right)
$$

where

$$
\mu_{j}=\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}+\text { offset }_{j}\right)
$$

for observed counts $y_{j}$ with covariates $\mathbf{x}_{j}$ for the $j$ th observation. One derivation of the negative binomial mean-dispersion model is that individual units follow a Poisson regression model, but there is an omitted variable $\nu_{j}$, such that $e^{\nu_{j}}$ follows a gamma distribution with mean 1 and variance $\alpha$ :

$$
y_{j} \sim \operatorname{Poisson}\left(\mu_{j}^{*}\right)
$$

where

$$
\mu_{j}^{*}=\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}+\operatorname{offset}_{j}+\nu_{j}\right)
$$

and

$$
e^{\nu_{j}} \sim \operatorname{Gamma}(1 / \alpha, \alpha)
$$

With this parameterization, a $\operatorname{Gamma}(a, b)$ distribution will have expectation $a b$ and variance $a b^{2}$.
We refer to $\alpha$ as the overdispersion parameter. The larger $\alpha$ is, the greater the overdispersion. The Poisson model corresponds to $\alpha=0$. nbreg parameterizes $\alpha$ as $\ln \alpha$. gnbreg allows $\ln \alpha$ to be modeled as $\ln \alpha_{j}=\mathbf{z}_{j} \gamma$, a linear combination of covariates $\mathbf{z}_{j}$.
nbreg will fit two different parameterizations of the negative binomial model. The default, described above and also given by the dispersion(mean) option, has dispersion for the $j$ th observation equal to $1+\alpha \exp \left(\mathbf{x}_{j} \boldsymbol{\beta}+\right.$ offset $\left._{j}\right)$. This is seen by noting that the above implies that

$$
\mu_{j}^{*} \sim \operatorname{Gamma}\left(1 / \alpha, \alpha \mu_{j}\right)
$$

and thus

$$
\begin{aligned}
\operatorname{Var}\left(y_{j}\right) & =E\left\{\operatorname{Var}\left(y_{j} \mid \mu_{j}^{*}\right)\right\}+\operatorname{Var}\left\{E\left(y_{j} \mid \mu_{j}^{*}\right)\right\} \\
& =E\left(\mu_{j}^{*}\right)+\operatorname{Var}\left(\mu_{j}^{*}\right) \\
& =\mu_{j}\left(1+\alpha \mu_{j}\right)
\end{aligned}
$$

The alternative parameterization, given by the dispersion(constant) option, has dispersion equal to $1+\delta$; that is, it is constant for all observations. This is so because the constant-dispersion model assumes instead that

$$
\mu_{j}^{*} \sim \operatorname{Gamma}\left(\mu_{j} / \delta, \delta\right)
$$

and thus $\operatorname{Var}\left(y_{j}\right)=\mu_{j}(1+\delta)$. The Poisson model corresponds to $\delta=0$.
For detailed derivations of both models, see Cameron and Trivedi (2013, 80-89). In particular, note that the mean-dispersion model is known as the NB2 model in their terminology, whereas the constant-dispersion model is referred to as the NB1 model.

See Long and Freese (2014) and Cameron and Trivedi (2010, chap. 17) for a discussion of the negative binomial regression model with Stata examples and for a discussion of other regression models for count data.

Hilbe (2011) provides an extensive review of the negative binomial model and its variations, using Stata examples.

## nbreg

It is not uncommon to posit a Poisson regression model and observe a lack of model fit. The following data appeared in Rodríguez (1993):

```
. use http://www.stata-press.com/data/r13/rod93
```

. list, sepby(cohort)

|  | cohort | age_mos | deaths | exposure |
| :---: | :---: | :---: | :---: | :---: |
| 1. | 1941-1949 | 0.5 | 168 | 278.4 |
| 2. | 1941-1949 | 2.0 | 48 | 538.8 |
| 3. | 1941-1949 | 4.5 | 63 | 794.4 |
| 4. | 1941-1949 | 9.0 | 89 | 1,550.8 |
| 5. | 1941-1949 | 18.0 | 102 | 3,006.0 |
| 6. | 1941-1949 | 42.0 | 81 | 8,743.5 |
| 7. | 1941-1949 | 90.0 | 40 | 14,270.0 |
| 8. | 1960-1967 | 0.5 | 197 | 403.2 |
| 9. | 1960-1967 | 2.0 | 48 | 786.0 |
| 10. | 1960-1967 | 4.5 | 62 | 1,165.3 |
| 11. | 1960-1967 | 9.0 | 81 | 2,294.8 |
| 12. | 1960-1967 | 18.0 | 97 | 4,500.5 |
| 13. | 1960-1967 | 42.0 | 103 | 13,201.5 |
| 14. | 1960-1967 | 90.0 | 39 | 19,525.0 |
| 15. | 1968-1976 | 0.5 | 195 | 495.3 |
| 16. | 1968-1976 | 2.0 | 55 | 956.7 |
| 17. | 1968-1976 | 4.5 | 58 | 1,381.4 |
| 18. | 1968-1976 | 9.0 | 85 | 2,604.5 |
| 19. | 1968-1976 | 18.0 | 87 | 4,618.5 |
| 20. | 1968-1976 | 42.0 | 70 | 9,814.5 |
| 21. | 1968-1976 | 90.0 | 10 | 5,802.5 |

. generate logexp = ln(exposure)
. poisson deaths i.cohort, offset(logexp)
Iteration 0: log likelihood $=-2160.0544$
Iteration 1: $\quad$ log likelihood $=-2159.5162$
Iteration 2: log likelihood $=-2159.5159$
Iteration 3: log likelihood $=-2159.5159$


```
. estat gof
Deviance goodness-of-fit = 4190.689
Prob > chi2(18) = 0.0000
Pearson goodness-of-fit = 15387.67
Prob > chi2(18) = 0.0000
```

The extreme significance of the goodness-of-fit $\chi^{2}$ indicates that the Poisson regression model is inappropriate, suggesting to us that we should try a negative binomial model:

| Negative binomial regression |  |  |  | Number of obs LR chi2(2) |  |  | $\begin{array}{r} 21 \\ 0.40 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Dispersion | = mean |  |  | Prob | chi2 | = | 0.8171 |
| Log likelihood $=-131.3799$ |  |  |  | Pseudo R2 |  |  | 0.0015 |
| deaths | Coef. | Std. Err . | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |  |
|  |  |  |  |  |  |  |  |
| $\begin{aligned} & 1960-1967 \\ & 1968-1976 \end{aligned}$ | -. 2676187 | . 7237203 | -0.37 | 0.712 | -1.686 |  | 1.150847 |
|  | -. 4573957 | . 7236651 | -0.63 | 0.527 | -1.875 |  | . 9609618 |
| _cons | -2.086731 | . 511856 | -4.08 | 0.000 | -3.08 |  | -1.083511 |
| logexp |  | (offset) |  |  |  |  |  |
| /lnalpha | . 5939963 | . 2583615 |  |  | . 0876 |  | 1.100376 |
| alpha | 1.811212 | . 4679475 |  |  | 1.09 |  | 3.005295 |

Likelihood-ratio test of alpha=0: chibar2(01) $=4056.27$ Prob>=chibar2 $=0.000$
Our original Poisson model is a special case of the negative binomial-it corresponds to $\alpha=0$. nbreg, however, estimates $\alpha$ indirectly, estimating instead $\ln \alpha$. In our model, $\ln \alpha=0.594$, meaning that $\alpha=1.81$ (nbreg undoes the transformation for us at the bottom of the output).

To test $\alpha=0$ (equivalent to $\ln \alpha=-\infty$ ), nbreg performs a likelihood-ratio test. The staggering $\chi^{2}$ value of 4,056 asserts that the probability that we would observe these data conditional on $\alpha=0$ is virtually zero, that is, conditional on the process being Poisson. The data are not Poisson. It is not accidental that this $\chi^{2}$ value is close to the goodness-of-fit statistic from the Poisson regression itself.

## - Technical note

The usual Gaussian test of $\alpha=0$ is omitted because this test occurs on the boundary, invalidating the usual theory associated with such tests. However, the likelihood-ratio test of $\alpha=0$ has been modified to be valid on the boundary. In particular, the null distribution of the likelihood-ratio test statistic is not the usual $\chi_{1}^{2}$, but rather a $50: 50$ mixture of a $\chi_{0}^{2}$ (point mass at zero) and a $\chi_{1}^{2}$, denoted as $\bar{\chi}_{01}^{2}$. See Gutierrez, Carter, and Drukker (2001) for more details.

## - Technical note

The negative binomial model deals with cases in which there is more variation than would be expected if the process were Poisson. The negative binomial model is not helpful if there is less than Poisson variation-if the variance of the count variable is less than its mean. However, underdispersion is uncommon. Poisson models arise because of independently generated events. Overdispersion comes about if some of the parameters (causes) of the Poisson processes are unknown. To obtain underdispersion, the sequence of events somehow would have to be regulated; that is, events would not be independent but controlled based on past occurrences.

## gnbreg

gnbreg is a generalization of nbreg, dispersion(mean). Whereas in nbreg, one $\ln \alpha$ is estimated, gnbreg allows $\ln \alpha$ to vary, observation by observation, as a linear combination of another set of covariates: $\ln \alpha_{j}=\mathbf{z}_{j} \gamma$.

We will assume that the number of deaths is a function of age, whereas the $\ln \alpha$ parameter is a function of cohort. To fit the model, we type

```
. gnbreg deaths age_mos, lnalpha(i.cohort) offset(logexp)
Fitting constant-only model:
Iteration 0: log likelihood = -187.067 (not concave)
Iteration 1: log likelihood = -137.4064
Iteration 2: log likelihood = -134.07766
Iteration 3: log likelihood = -131.60668
Iteration 4: log likelihood = -131.57951
Iteration 5: log likelihood = -131.57948
Iteration 6: log likelihood = -131.57948
Fitting full model:
Iteration 0: log likelihood = -124.34327
Iteration 1: log likelihood = -117.70256
Iteration 2: log likelihood = -117.56373
Iteration 3: log likelihood = -117.56164
Iteration 4: log likelihood = -117.56164
Generalized negative binomial regression
Log likelihood = -117.56164
\begin{tabular}{llr} 
Number of obs & \(=\) & 21 \\
LR chi2 (1) & \(=\) & 28.04 \\
Prob > chi2 & \(=\) & 0.0000 \\
Pseudo R2 & \(=\) & 0.1065
\end{tabular}
```

| deaths | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| :---: | ---: | :--- | :--- | :--- | :--- | :--- |
| deaths |  |  |  |  |  |  |
| age_mos | -.0516657 | .0051747 | -9.98 | 0.000 | -.061808 | -.0415233 |
| _cons | -1.867225 | .2227944 <br> logexp | 1 | -8.38 | 0.000 | -2.303894 |
| (offset) |  |  |  |  |  |  |
| lnalpha |  |  |  |  |  |  |
| cohort <br> 1960-1967 <br> $1968-1976$ | .0939546 | .7187747 | 0.13 | 0.896 | -1.314818 | 1.502727 |
| _cons | -.4759581 | .5156502 | -0.92 | 0.356 | -1.486614 | .5346978 |

We find that age is a significant determinant of the number of deaths. The standard errors for the variables in the $\ln \alpha$ equation suggest that the overdispersion parameter does not vary across cohorts. We can test this assertion by typing

```
. test 2.cohort 3.cohort
( 1) [lnalpha]2.cohort = 0
( 2) [lnalpha]3.cohort = 0
```



There is no evidence of variation by cohort in these data.

## - Technical note

Note the intentional absence of a likelihood-ratio test for $\alpha=0$ in gnbreg. The test is affected by the same boundary condition that affects the comparison test in nbreg; however, when $\alpha$ is parameterized by more than a constant term, the null distribution becomes intractable. For this reason, we recommend using nbreg to test for overdispersion and, if you have reason to believe that overdispersion exists, only then modeling the overdispersion using gnbreg.

## Stored results

nbreg and gnbreg store the following in e():
Scalars
e(N)
e(k)
e(k_aux)
e(k_eq)
e(k_eq_model)
e(k_dv)
e(df_m)
e(r2_p)
e(11)
e(11_0)
e(ll_c)
e(alpha)
$e(d e l t a)$
e(N_clust)
e(chi2)
e(chi2_c)
$e(p)$
e(rank)
e(rank0)
e(ic)
e(rc)
e(converged)
Macros
e(cmd)
e(cmdline)
e(depvar)
$e$ (wtype)
e(wexp)
e(title)
e(clustvar)
e(offset)
e(offset1)
e(chi2type)
e(chi2_ct)
e(dispers)
e(vce)
e(vcetype)
e(opt)
$e$ (which)
e(ml_method)
e(user)
e(technique)
e(properties)
e(predict)
e(asbalanced)
e(asobserved)
number of observations
number of parameters
number of auxiliary parameters
number of equations in $e(b)$
number of equations in overall model test
number of dependent variables
model degrees of freedom
pseudo- $R$-squared
log likelihood
log likelihood, constant-only mode
log likelihood, comparison model
value of alpha
value of delta
number of clusters
$\chi^{2}$
$\chi^{2}$ for comparison test
significance
rank of e(V)
rank of $e(V)$ for constant-only model
number of iterations
return code
1 if converged, 0 otherwise
nbreg or gnbreg
command as typed
name of dependent variable
weight type
weight expression
title in estimation output
name of cluster variable
linear offset variable (nbreg)
linear offset variable (gnbreg)
Wald or LR; type of model $\chi^{2}$ test
Wald or LR; type of model $\chi^{2}$ test corresponding to e(chi2_c)
mean or constant
vcetype specified in vce()
title used to label Std. Err.
type of optimization
max or min; whether optimizer is to perform maximization or minimization
type of ml method
name of likelihood-evaluator program
maximization technique
b V
program used to implement predict
factor variables fvset as asbalanced
factor variables fvset as asobserved

Matrices

```
    e(b)
    e(Cns)
    e(ilog)
    e(gradient)
    e(V)
    e(V_modelbased)
```

Functions
e(sample)
coefficient vector
constraints matrix
iteration $\log$ (up to 20 iterations)
gradient vector variance-covariance matrix of the estimators model-based variance
marks estimation sample

## Methods and formulas

See [R] poisson and Johnson, Kemp, and Kotz (2005, chap. 4) for an introduction to the Poisson distribution.

Methods and formulas are presented under the following headings:
Mean-dispersion model
Constant-dispersion model

## Mean-dispersion model

A negative binomial distribution can be regarded as a gamma mixture of Poisson random variables. The number of times something occurs, $y_{j}$, is distributed as Poisson $\left(\nu_{j} \mu_{j}\right)$. That is, its conditional likelihood is

$$
f\left(y_{j} \mid \nu_{j}\right)=\frac{\left(\nu_{j} \mu_{j}\right)^{y_{j}} e^{-\nu_{j} \mu_{j}}}{\Gamma\left(y_{j}+1\right)}
$$

where $\mu_{j}=\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}+\right.$ offset $\left._{j}\right)$ and $\nu_{j}$ is an unobserved parameter with a $\operatorname{Gamma}(1 / \alpha, \alpha)$ density:

$$
g(\nu)=\frac{\nu^{(1-\alpha) / \alpha} e^{-\nu / \alpha}}{\alpha^{1 / \alpha} \Gamma(1 / \alpha)}
$$

This gamma distribution has mean 1 and variance $\alpha$, where $\alpha$ is our ancillary parameter.
The unconditional likelihood for the $j$ th observation is therefore

$$
f\left(y_{j}\right)=\int_{0}^{\infty} f\left(y_{j} \mid \nu\right) g(\nu) d \nu=\frac{\Gamma\left(m+y_{j}\right)}{\Gamma\left(y_{j}+1\right) \Gamma(m)} p_{j}^{m}\left(1-p_{j}\right)^{y_{j}}
$$

where $p_{j}=1 /\left(1+\alpha \mu_{j}\right)$ and $m=1 / \alpha$. Solutions for $\alpha$ are handled by searching for $\ln \alpha$ because $\alpha$ must be greater than zero.

The log likelihood (with weights $w_{j}$ and offsets) is given by

$$
\begin{aligned}
& m=1 / \alpha \quad p_{j}=1 /\left(1+\alpha \mu_{j}\right) \quad \mu_{j}=\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}+\text { offset }_{j}\right) \\
& \ln L=\sum_{j=1}^{n} w_{j}\left[\ln \left\{\Gamma\left(m+y_{j}\right)\right\}-\ln \left\{\Gamma\left(y_{j}+1\right)\right\}\right. \\
& \\
& \left.\quad-\ln \{\Gamma(m)\}+m \ln \left(p_{j}\right)+y_{j} \ln \left(1-p_{j}\right)\right]
\end{aligned}
$$

For gnbreg, $\alpha$ can vary across the observations according to the parameterization $\ln \alpha_{j}=\mathbf{z}_{j} \gamma$.

## Constant-dispersion model

The constant-dispersion model assumes that $y_{j}$ is conditionally distributed as Poisson $\left(\mu_{j}^{*}\right)$, where $\mu_{j}^{*} \sim \operatorname{Gamma}\left(\mu_{j} / \delta, \delta\right)$ for some dispersion parameter $\delta$ (by contrast, the mean-dispersion model assumes that $\mu_{j}^{*} \sim \operatorname{Gamma}\left(1 / \alpha, \alpha \mu_{j}\right)$ ). The log likelihood is given by

$$
\begin{gathered}
m_{j}=\mu_{j} / \delta \quad p=1 /(1+\delta) \\
\ln L=\sum_{j=1}^{n} w_{j}\left[\ln \left\{\Gamma\left(m_{j}+y_{j}\right)\right\}-\ln \left\{\Gamma\left(y_{j}+1\right)\right\}\right. \\
\left.-\ln \left\{\Gamma\left(m_{j}\right)\right\}+m_{j} \ln (p)+y_{j} \ln (1-p)\right]
\end{gathered}
$$

with everything else defined as before in the calculations for the mean-dispersion model.
nbreg and gnbreg support the Huber/White/sandwich estimator of the variance and its clustered version using vce (robust) and vce (cluster clustvar), respectively. See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas.

These commands also support estimation with survey data. For details on VCEs with survey data, see [SVY] variance estimation.

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## Also see

[R] nbreg postestimation - Postestimation tools for nbreg and gnbreg
[ R ] glm - Generalized linear models
[R] poisson - Poisson regression
[R] tnbreg - Truncated negative binomial regression
[R] zinb - Zero-inflated negative binomial regression
[ME] menbreg - Multilevel mixed-effects negative binomial regression
[MI] estimation - Estimation commands for use with mi estimate
[SVY] svy estimation - Estimation commands for survey data
[XT] xtnbreg - Fixed-effects, random-effects, \& population-averaged negative binomial models
[U] 20 Estimation and postestimation commands

## Title

nbreg postestimation - Postestimation tools for nbreg and gnbreg

| Description | Syntax for predict | Menu for predict | Options for predict |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Methods and formulas | Also see |  |

## Description

The following postestimation commands are available after nbreg and gnbreg:

| Command | Description |
| :--- | :--- |
| contrast | contrasts and ANOVA-style joint tests of estimates |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample |
| estat vce | variance-covariance matrix of the estimators (VCE) |
| estat (svy) | postestimation statistics for survey data <br> estaloging estimation results |
| forecast |  |$\quad$| dynamic forecasts and simulations |
| :--- |
| lincom |$\quad$| point estimates, standard errors, testing, and inference for linear combinations |
| :--- |
| of coefficients |

[^50]
## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic nooffset]
predict [type] {stub*|newvar reg newvar risp } [if] [in], scores
```

statistic Description

| Main |  |
| :--- | :--- |
| n | number of events; the default |
| ir | incidence rate (equivalent to predict $\ldots, \mathrm{n}$ nooffset $)$ |
| $\operatorname{pr}(n)$ | probability $\operatorname{Pr}\left(y_{j}=n\right)$ |
| $\operatorname{pr}(a, b)$ | probability $\operatorname{Pr}\left(a \leq y_{j} \leq b\right)$ |
| xb | linear prediction |
| stdp | standard error of the linear prediction |

In addition, relevant only after gnbreg are the following:
statistic Description
Main

| alpha | predicted values of $\alpha_{j}$ |
| :--- | :--- |
| lnalpha | predicted values of $\ln \alpha_{j}$ |
| stdplna | standard error of predicted $\ln \alpha_{j}$ |

These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
n , the default, calculates the predicted number of events, which is $\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}\right)$ if neither offset ( varname $_{o}$ ) nor exposure (varname $e_{e}$ ) was specified when the model was fit; $\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}+\right.$ offset $\left._{j}\right)$ if offset () was specified; or $\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}\right) \times$ exposure $_{j}$ if exposure() was specified.
ir calculates the incidence rate $\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}\right)$, which is the predicted number of events when exposure is 1 . This is equivalent to specifying both the n and the nooffset options.
$\operatorname{pr}(n)$ calculates the probability $\operatorname{Pr}\left(y_{j}=n\right)$, where $n$ is a nonnegative integer that may be specified as a number or a variable.
$\operatorname{pr}(a, b)$ calculates the probability $\operatorname{Pr}\left(a \leq y_{j} \leq b\right)$, where $a$ and $b$ are nonnegative integers that may be specified as numbers or variables;
$b$ missing ( $b \geq$.) means $+\infty$;
$\operatorname{pr}(20,$.$) calculates \operatorname{Pr}\left(y_{j} \geq 20\right)$;
$\operatorname{pr}(20, b)$ calculates $\operatorname{Pr}\left(y_{j} \geq 20\right)$ in observations for which $b \geq$. and calculates
$\operatorname{Pr}\left(20 \leq y_{j} \leq b\right)$ elsewhere.
$\operatorname{pr}(., b)$ produces a syntax error. A missing value in an observation of the variable $a$ causes a missing value in that observation for $\operatorname{pr}(a, b)$.
xb calculates the linear prediction, which is $\mathbf{x}_{j} \boldsymbol{\beta}$ if neither offset () nor exposure() was specified; $\mathbf{x}_{j} \boldsymbol{\beta}+$ offset $_{j}$ if offset () was specified; or $\mathbf{x}_{j} \boldsymbol{\beta}+\ln$ (exposure $_{j}$ ) if exposure() was specified; see nooffset below.
stdp calculates the standard error of the linear prediction.
alpha, lnalpha, and stdplna are relevant after gnbreg estimation only; they produce the predicted values of $\alpha_{j}, \ln \alpha_{j}$, and the standard error of the predicted $\ln \alpha_{j}$, respectively.
nooffset is relevant only if you specified offset() or exposure() when you fit the model. It modifies the calculations made by predict so that they ignore the offset or exposure variable; the linear prediction is treated as $\mathbf{x}_{j} \boldsymbol{\beta}$ rather than as $\mathbf{x}_{j} \boldsymbol{\beta}+$ offset $_{j}$ or $\mathbf{x}_{j} \boldsymbol{\beta}+\ln \left(\right.$ exposure $\left._{j}\right)$. Specifying predict..., nooffset is equivalent to specifying predict..., ir.
scores calculates equation-level score variables.
The first new variable will contain $\partial \ln L / \partial\left(\mathbf{x}_{j} \boldsymbol{\beta}\right)$.
The second new variable will contain $\partial \ln L / \partial\left(\ln \alpha_{j}\right)$ for dispersion(mean) and gnbreg.
The second new variable will contain $\partial \ln L / \partial(\ln \delta)$ for dispersion(constant).

## Remarks and examples

After nbreg and gnbreg, predict returns the expected number of deaths per cohort and the probability of observing the number of deaths recorded or fewer.


Likelihood-ratio test of alpha=0: chibar2 $(01)=434.62$ Prob>=chibar2 $=0.000$

- predict count
(option n assumed; predicted number of events)
. predict p, pr(0, deaths)
. summarize deaths count $p$

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| deaths | 21 | 84.66667 | 48.84192 | 10 | 197 |
| count | 21 | 84.66667 | 4.00773 | 80 | 89.57143 |
| p | 21 | .4991542 | .2743702 | .0070255 | .9801285 |

The expected number of deaths ranges from 80 to 90 . The probability $\operatorname{Pr}\left(y_{i} \leq\right.$ deaths $)$ ranges from 0.007 to 0.98 .

## Methods and formulas

In the following, we use the same notation as in [R] nbreg.
Methods and formulas are presented under the following headings:
Mean-dispersion model
Constant-dispersion model

## Mean-dispersion model

The equation-level scores are given by

$$
\begin{aligned}
\operatorname{score}(\mathbf{x} \boldsymbol{\beta})_{j} & =p_{j}\left(y_{j}-\mu_{j}\right) \\
\operatorname{score}(\tau)_{j} & =-m\left\{\frac{\alpha_{j}\left(\mu_{j}-y_{j}\right)}{1+\alpha_{j} \mu_{j}}-\ln \left(1+\alpha_{j} \mu_{j}\right)+\psi\left(y_{j}+m\right)-\psi(m)\right\}
\end{aligned}
$$

where $\tau_{j}=\ln \alpha_{j}$ and $\psi(z)$ is the digamma function.

## Constant-dispersion model

The equation-level scores are given by

$$
\begin{aligned}
\operatorname{score}(\mathbf{x} \boldsymbol{\beta})_{j} & =m_{j}\left\{\psi\left(y_{j}+m_{j}\right)-\psi\left(m_{j}\right)+\ln (p)\right\} \\
\operatorname{score}(\tau)_{j} & =y_{j}-\left(y_{j}+m_{j}\right)(1-p)-\operatorname{score}(\mathbf{x} \boldsymbol{\beta})_{j}
\end{aligned}
$$

where $\tau_{j}=\ln \delta_{j}$.

## Also see

$[R]$ nbreg - Negative binomial regression
[U] 20 Estimation and postestimation commands

## Title

nestreg - Nested model statistics

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Acknowledgment | Reference |

## Syntax

Standard estimation command syntax

```
nestreg [, options]: command_name depvar (varlist) [(varlist) ...]
    [if] [in] [weight] [command_options]
```

Survey estimation command syntax

```
nestreg [, options]: svy [vcetype] [, svy_options]: command_name depvar
    (varlist) [(varlist) ...] [if] [in] [, command_options]
```

options
Description

Reporting

| waldtable | report Wald test results; the default <br> report likelihood-ratio test results |
| :--- | :--- |
| lrtable | suppress any output from command_name |
| quietly | store nested estimation results in _est_stub\# |

by is allowed; see [U] 11.1.10 Prefix commands.
Weights are allowed if command_name allows them; see [U] 11.1 .6 weight.
A varlist in parentheses indicates that this list of variables is to be considered as a block. Each variable in a varlist not bound in parentheses will be treated as its own block.
All postestimation commands behave as they would after command_name without the nestreg prefix; see the postestimation manual entry for command_name.

## Menu

Statistics $>$ Other $>$ Nested model statistics

## Description

nestreg fits nested models by sequentially adding blocks of variables and then reports comparison tests between the nested models.

## Options

$\qquad$ Reporting
waldtable specifies that the table of Wald test results be reported. waldtable is the default.
lrtable specifies that the table of likelihood-ratio tests be reported. This option is not allowed if pweights, the vce(robust) option, or the vce(cluster clustvar) option is specified. lrtable is also not allowed with the svy prefix.
quietly suppresses the display of any output from command_name.
store (stub) specifies that each model fit by nestreg be stored under the name _est_stub\#, where \# is the nesting order from first to last.

## Remarks and examples

Remarks are presented under the following headings:

Estimation commands<br>Wald tests<br>Likelihood-ratio tests<br>Programming for nestreg

## Estimation commands

nestreg removes collinear predictors and observations with missing values from the estimation sample before calling command_name.

The following Stata commands are supported by nestreg:

| clogit | nbreg | regress |
| :--- | :--- | :--- |
| cloglog | ologit | scobit |
| glm | oprobit | stcox |
| intreg | poisson | stcrreg |
| logistic | probit | streg |
| logit | qreg | tobit |

You do not supply a depvar for stcox, stcrreg, or streg; otherwise, depvar is required. You must supply two depvars for intreg.

## Wald tests

Use nestreg to test the significance of blocks of predictors, building the regression model one block at a time. Using the data from example 1 of $[R]$ test, we wish to test the significance of the following predictors of birth rate: medage, medagesq, and region (already partitioned into four indicator variables: reg1, reg2, reg3, and reg4).
. use http://www.stata-press.com/data/r13/census4
(birth rate, median age)
. nestreg: regress brate (medage) (medagesq) (reg2-reg4)
Block 1: medage

| Source | SS | df | MS |
| ---: | ---: | ---: | :---: |
| Model | 32675.1044 | 1 | 32675.1044 |
| Residual | 9521.71561 | 48 | 198.369075 |
| Total | 42196.82 | 49 | 861.159592 |

Number of obs $=50$
F ( 1,48 ) $=164.72$
Prob $>$ F $=0.0000$
R-squared $=0.7743$
Adj R-squared $=0.7696$
Root MSE = 14.084

| brate | Coef. | Std. Err. | t | P>\|t| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| medage | -15.24893 | 1.188141 | -12.83 | 0.000 | -17.63785 | -12.86002 |
| _cons | 618.3935 | 35.15416 | 17.59 | 0.000 | 547.7113 | 689.0756 |

Block 2: medagesq

| Source | SS | df | MS |
| ---: | ---: | ---: | :---: |
| Model | 36755.8524 | 2 | 18377.9262 |
| Residual | 5440.96755 | 47 | 115.765267 |
| Total | 42196.82 | 49 | 861.159592 |

Number of obs $=\quad 50$
F( 2, 47) = 158.75
Prob > F $=0.0000$
R-squared $=0.8711$
Adj R-squared $=0.8656$
Root MSE $=10.759$

| brate | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| medage | -109.8925 | 15.96663 | -6.88 | 0.000 | -142.0132 | -77.7718 |
| medagesq | 1.607332 | .2707228 | 5.94 | 0.000 | 1.062708 | 2.151956 |
| _cons | 2007.071 | 235.4316 | 8.53 | 0.000 | 1533.444 | 2480.698 |

Block 3: reg2 reg3 reg4

| Source | SS | df | MS |
| ---: | ---: | ---: | :---: |
| Model | 38803.419 | 5 | 7760.68381 |
| Residual | 3393.40095 | 44 | 77.1227489 |
| Total | 42196.82 | 49 | 861.159592 |

of obs $=\quad 50$ F (5, 44) $=100.63$ Prob $>$ F $=0.0000$ R-squared $=0.9196$ Adj R-squared $=0.9104$ Root MSE $=8.782$

| brate | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| medage | -109.0957 | 13.52452 | -8.07 | 0.000 | -136.3526 | -81.83886 |
| medagesq | 1.635208 | .2290536 | 7.14 | 0.000 | 1.173581 | 2.096835 |
| reg2 | 15.00284 | 4.252068 | 3.53 | 0.001 | 6.433365 | 23.57233 |
| reg3 | 7.366435 | 3.953336 | 1.86 | 0.069 | -.6009898 | 15.33386 |
| reg4 | 21.39679 | 4.650602 | 4.60 | 0.000 | 12.02412 | 30.76946 |
| _cons | 1947.61 | 199.8405 | 9.75 | 0.000 | 1544.858 | 2350.362 |


| Block | F | Block <br> df | Residual <br> df | $\operatorname{Pr}>\mathrm{F}$ | R 2 | Change <br> in R2 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 164.72 | 1 | 48 | 0.0000 | 0.7743 |  |
| 2 | 35.25 | 1 | 47 | 0.0000 | 0.8711 | 0.0967 |
| 3 | 8.85 | 3 | 44 | 0.0001 | 0.9196 | 0.0485 |

This single call to nestreg ran regress three times, adding a block of predictors to the model for each run as in
. regress brate medage

| Source | SS | df MS |  |  | Number of obs $=50$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | F ( 1, 48) | $=164.72$ |
| Model | 32675.1044 | 1326 | 5.1044 |  | Prob > F | $=0.0000$ |
| Residual | 9521.71561 | 48198 | 369075 |  | R -squared | 0.7743 |
|  |  |  |  |  | Adj R-squared | $=0.7696$ |
| Total | 42196.82 | 49861 | 159592 |  | Root MSE | $=14.084$ |
| brate | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Conf. | Interval] |
| medage | -15.24893 | 1.188141 | -12.83 | 0.000 | -17.63785 | -12.86002 |
| _cons | 618.3935 | 35.15416 | 17.59 | 0.000 | 547.7113 | 689.0756 |

. regress brate medage medagesq

| Source | SS | df | MS |
| ---: | ---: | ---: | :---: |
| Model | 36755.8524 | 2 | 18377.9262 |
| Residual | 5440.96755 | 47 | 115.765267 |
| Total | 42196.82 | 49 | 861.159592 |

Number of obs = 50
$F(2,47)=158.75$
Prob $>\mathrm{F}=0.0000$
$R$-squared $=0.8711$
Adj R-squared $=0.8656$
Root MSE $=10.759$

| brate | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| medage | -109.8925 | 15.96663 | -6.88 | 0.000 | -142.0132 | -77.7718 |
| medagesq | 1.607332 | .2707228 | 5.94 | 0.000 | 1.062708 | 2.151956 |
| _cons | 2007.071 | 235.4316 | 8.53 | 0.000 | 1533.444 | 2480.698 |

. regress brate medage medagesq reg2-reg4

| Source | SS | df | MS |
| ---: | ---: | ---: | :---: |
| Model | 38803.419 | 5 | 7760.68381 |
| Residual | 3393.40095 | 44 | 77.1227489 |
| Total | 42196.82 | 49 | 861.159592 |


| Number of obs | $=$ | 50 |
| :--- | ---: | ---: |
| F $(5$, | $44)$ | $=100.63$ |
| Prob $>$ F | $=$ | 0.0000 |
| R-squared | $=0.9196$ |  |
| Adj R-squared | $=$ | 0.9104 |
| Root MSE | $=8.782$ |  |


| brate | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| medage | -109.0957 | 13.52452 | -8.07 | 0.000 | -136.3526 | -81.83886 |
| medagesq | 1.635208 | .2290536 | 7.14 | 0.000 | 1.173581 | 2.096835 |
| reg2 | 15.00284 | 4.252068 | 3.53 | 0.001 | 6.433365 | 23.57233 |
| reg3 | 7.366435 | 3.953336 | 1.86 | 0.069 | -.6009898 | 15.33386 |
| reg4 | 21.39679 | 4.650602 | 4.60 | 0.000 | 12.02412 | 30.76946 |
| _cons | 1947.61 | 199.8405 | 9.75 | 0.000 | 1544.858 | 2350.362 |

nestreg collected the $F$ statistic for the corresponding block of predictors and the model $R^{2}$ statistic from each model fit.

The $F$ statistic for the first block, 164.72, is for a test of the joint significance of the first block of variables; it is simply the $F$ statistic from the regression of brate on medage. The $F$ statistic for the second block, 35.25, is for a test of the joint significance of the second block of variables in a regression of both the first and second blocks of variables. In our example, it is an $F$ test of medagesq in the regression of brate on medage and medagesq. Similarly, the third block's $F$ statistic of 8.85 corresponds to a joint test of reg2, reg3, and reg4 in the final regression.

## Likelihood-ratio tests

The nestreg command provides a simple syntax for performing likelihood-ratio tests for nested model specifications; also see lrtest. Using the data from example 1 of [R] Irtest, we wish to jointly test the significance of the following predictors of low birthweight: age, lwt, ptl, and ht.


| low | Odds Ratio | Std. Err. | $\mathbf{z}$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | :---: | :---: | ---: |
| _Irace_2 | 3.534767 | 1.860737 | 2.40 | 0.016 | 1.259736 | 9.918406 |
| _Irace_3 | 2.368079 | 1.039949 | 1.96 | 0.050 | 1.001356 | 5.600207 |
| smoke | 2.517698 | 1.00916 | 2.30 | 0.021 | 1.147676 | 5.523162 |
| ui | 2.1351 | .9808153 | 1.65 | 0.099 | .8677528 | 5.2534 |
| age | .9732636 | .0354759 | -0.74 | 0.457 | .9061578 | 1.045339 |
| lwt | .9849634 | .0068217 | -2.19 | 0.029 | .9716834 | .9984249 |
| ptl | 1.719161 | .5952579 | 1.56 | 0.118 | .8721455 | 3.388787 |
| ht | 6.249602 | 4.322408 | 2.65 | 0.008 | 1.611152 | 24.24199 |
| _cons | 1.586014 | 1.910496 | 0.38 | 0.702 | .1496092 | 16.8134 |


| Block | LL | LR | df | Pr $>$ LR | AIC | BIC |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | -107.934 | 18.80 | 4 | 0.0009 | 225.8681 | 242.0768 |
| 2 | -100.724 | 14.42 | 4 | 0.0061 | 219.448 | 248.6237 |

The estimation results from the full model are left in e(), so we can later use estat and other postestimation commands.

```
. estat gof
Logistic model for low, goodness-of-fit test
    number of observations = 189
    number of covariate patterns = 182
        Pearson chi2(173) = 179.24
            Prob > chi2 = 0.3567
```


## Programming for nestreg

If you want your user-written command (command_name) to work with nestreg, it must follow standard Stata syntax and allow the if qualifier. Furthermore, command_name must have sw or swml as a program property; see [P] program properties. If command_name has swml as a property, command_name must store the log-likelihood value in $\mathrm{e}(11)$ and the model degrees of freedom in e(df_m).

## Stored results

nestreg stores the following in r() :
Matrices
$r$ (wald) matrix corresponding to the Wald table
$r(l r) \quad$ matrix corresponding to the likelihood-ratio table

## Acknowledgment

We thank Paul H. Bern of Syracuse University for developing the hierarchical regression command that inspired nestreg.

## Reference

Acock, A. C. 2014. A Gentle Introduction to Stata. 4th ed. College Station, TX: Stata Press.

## Also see

[P] program properties - Properties of user-defined programs

## Title

net - Install and manage user-written additions from the Internet

| Syntax | Description | Options | Remarks and examples |
| :--- | :--- | :--- | :--- |
| References | Also see |  |  |

## Syntax

Set current location for net net from directory_or_url

Change to a different net directory net cd path_or_url

Change to a different net site net link linkname

Search for installed packages
net search (see $[R]$ net search)

Report current net location
net

Describe a package
net describe pkgname $[$, $\underline{\text { from( }}$ (directory_or_url) $]$

Set location where packages will be installed net set ado dirname

Set location where ancillary files will be installed net set other dirname

Report net 'from', 'ado', and 'other' settings net query

Install ado-files and help files from a package net install pkgname [, all replace force $\underline{\text { from }}$ (directory_or_url)]

Install ancillary files from a package

```
net get pkgname [, all replace force from(directory_or_url)]
```

Shortcut to access Stata Journal (SJ) net site

```
net sj vol-issue [insert]
```

Shortcut to access Stata Technical Bulletin (STB) net site

```
net stb issue [insert]
```

List installed packages

```
ado [,\underline{find(string) from(dirname)]}
ado dir [pkgid] [, \underline{find(string) from(dirname)]}
```

Describe installed packages

```
ado describe [pkgid] [, find(string) from(dirname)]
```

Uninstall an installed package

```
ado uninstall pkgid [, \underline{from(dirname)]}
```

where
pkgname is name of a package
pkgid is name of a package
or a number in square brackets: [\#]
dirname is a directory name
or PLUS (default)
or PERSONAL
or SITE

## Description

net downloads and installs additions to Stata. The additions can be obtained from the Internet or from physical media. The additions can be ado-files (new commands), help files, or even datasets. Collections of files are bound together into packages. For instance, the package named zz49 might add the xyz command to Stata. At a minimum, such a package would contain xyz.ado, the code to implement the new command, and xyz.sthlp, the system help to describe it. That the package contains two files is a detail: you use net to download the package zz49, regardless of the number of files.
ado manages the packages you have installed by using net. The ado command lets you list and uninstall previously installed packages.

You can also access the net and ado features by selecting Help > SJ and User-written Programs; this is the recommended method to find and install additions to Stata.

## Options

all is used with net install and net get. Typing it with either one makes the command equivalent to typing net install followed by net get.
replace is for use with net install and net get. It specifies that the downloaded files replace existing files if any of the files already exists.
force specifies that the downloaded files replace existing files if any of the files already exists, even if Stata thinks all the files are the same. force implies replace.
find (string) is for use with ado, ado dir, and ado describe. It specifies that the descriptions of the packages installed on your computer be searched, and that the package descriptions containing string be listed.
from(dirname), when used with ado, specifies where the packages are installed. The default is from(PLUS). PLUS is a code word that Stata understands to correspond to a particular directory on your computer that was set at installation time. On Windows computers, PLUS probably means the directory $\mathrm{c}: \backslash \mathrm{ado} \backslash \mathrm{plus}$, but it might mean something else. You can find out what it means by typing sysdir, but doing so is irrelevant if you use the defaults.
from (directory_or_url), when used with net, specifies the directory or URL where installable packages may be found. The directory or URL is the same as the one that would have been specified with net from.

## Remarks and examples

For an introduction to using net and ado, see [U] 28 Using the Internet to keep up to date. The purpose of this documentation is

- to briefly, but accurately, describe net and ado and all their features and
- to provide documentation to those who wish to set up their own sites to distribute additions to Stata.

Remarks are presented under the following headings:

```
Definition of a package
The purpose of the net and ado commands
Content pages
Package-description pages
Where packages are installed
A summary of the net command
A summary of the ado command
Relationship of net and ado to the point-and-click interface
Creating your own site
Format of content and package-description files
Example 1
Example 2
Additional package directives
SMCL in content and package-description files
Error-free file delivery
```


## Definition of a package

A package is a collection of files-typically, . ado and .sthlp files-that together provide a new feature in Stata. Packages contain additions that you wish had been part of Stata at the outset. We write such additions, and so do other users.

One source of these additions is the Stata Journal, a printed and electronic journal with corresponding software. If you want the journal, you must subscribe, but the software is available for free from our website.

## The purpose of the net and ado commands

The net command makes it easy to distribute and install packages. The goal is to get you quickly to a package-description page that summarizes the addition, for example,

```
. net describe rte_stat, from(http://www.wemakeitupaswego.edu/faculty/sgazer/)
package rte_stat from http://www.wemakeitupaswego.edu/faculty/sgazer/
TITLE
    rte_stat. The robust-to-everything statistic; update.
DESCRIPTION/AOTHOR(S)
    S. Gazer, Dept. of Applied Theoretical Mathematics, WMIUAWG Univ.
    Aleph-0 100% confidence intervals proved too conservative for some
    applications; Aleph-1 confidence intervals have been substituted.
    The new robust-to-everything supplants the previous robust-to-
    everything-conceivable statistic. See "Inference in the absence
    of data" (forthcoming). After installation, see help rte.
INSTALLATION FILES (type net install rte_stat)
    rte.ado
    rte.sthlp
    nullset.ado
    random.ado
```

If you decide that the addition might prove useful, net makes the installation easy:

```
. net install rte_stat
checking rte_stat consistency and verifying not already installed...
installing into c:\ado\plus\ ...
installation complete.
```

The ado command helps you manage packages installed with net. Perhaps you remember that you installed a package that calculates the robust-to-everything statistic, but you cannot remember the command's name. You could use ado to search what you have previously installed for the rte command,

- ado
[1] package sg145 from http://www.stata.com/stb/stb56
STB-56 sg145. Scalar measures of fit for regression models. (output omitted)
[15] package rte_stat from http://www.wemakeitupaswego.edu/faculty/sgazer
rte_stat. The robust-to-everything statistic; update.
(output omitted)
[21] package st0119 from http://www.stata-journal.com/software/sj7-1
SJ7-1 st0119. Rasch analysis
or you might type

```
. ado, find("robust-to-everything")
[15] package rte_stat from http://www.wemakeitupaswego.edu/faculty/sgazer
    rte_stat. The robust-to-everything statistic; update.
```

Perhaps you decide that rte, despite the author's claims, is not worth the disk space it occupies. You can use ado to erase it:

```
. ado uninstall rte_stat
package rte_stat from http://www.wemakeitupaswego.edu/faculty/sgazer
    rte_stat. The robust-to-everything statistic; update.
(package uninstalled)
```

ado uninstall is easier than erasing the files by hand because ado uninstall erases every file associated with the package, and, moreover, ado knows where on your computer rte_stat is installed; you would have to hunt for these files.

## Content pages

There are two types of pages displayed by net: content pages and package-description pages. When you type net from, net cd, net link, or net without arguments, Stata goes to the specified place and displays the content page:

```
. net from http://www.stata.com
```

http://www.stata.com/
StataCorp

```
Welcome to StataCorp.
Below we provide links to sites providing additions to Stata, including
the Stata Journal, STB, and Statalist. These are NOT THE OFFICIAL UPDATES;
you fetch and install the official updates by typing -update-.
PLACES you could -net link- to:
    sj The Stata Journal
DIRECTORIES you could -net cd- to:
    stb materials published in the Stata Technical Bulletin
    users materials written by various people, including StataCorp
    employees
    meetings software packages from Stata Users Group meetings
    links links to other locations providing additions to Stata
```

A content page tells you about other content pages and package-description pages. The example above lists other content pages only. Below we follow one of the links for the Stata Journal:

```
. net link sj
```

http://www.stata-journal.com/
The Stata Journal

```
The Stata Journal is a refereed, quarterly journal containing articles
of interest to Stata users. For more details and subscription information,
visit the Stata Journal website at http://www.stata-journal.com.
PLACES you could -net link- to:
    stata StataCorp website
DIRECTORIES you could -net cd- to:
    production Files for authors of the Stata Journal
    software Software associated with Stata Journal articles
```

. net cd software
http://www.stata-journal.com/software/
The Stata Journal

```
PLACES you could -net link- to:
    stata StataCorp website
    stb Stata Technical Bulletin (STB) software archive
DIRECTORIES you could -net cd- to:
    (output omitted)
        sj7-1 volume 7, issue 1
    (output omitted)
        sj1-1 volume 1, issue 1
```

. net cd sj7-1
http://www.stata-journal.com/software/sj7-1/
Stata Journal volume 7, issue 1
DIRECTORIES you could -net cd- to:
.. Other Stata Journals
PACKAGES you could -net describe-:
dm0027 File filtering in Stata: handling complex data
formats and navigating log files efficiently
st0119 Rasch analysis
st0120 Multivariable regression spline models
st0121 mhbounds - Sensitivity Analysis for Average
Treatment Effects
dm0027, st0119, ..., st0121 are links to package-description pages.

1. When you type net from, you follow that with a location to display the location's content page.
a. The location could be a URL, such as http://www.stata.com. The content page at that location would then be listed.
b. The location could be e: on a Windows computer or a mounted volume on a Mac computer. The content page on that source would be listed. That would work if you had special media obtained from StataCorp or special media prepared by another user.
c. The location could even be a directory on your computer, but that would work only if that directory contained the right kind of files.
2. Once you have specified a location, typing net cd will take you into subdirectories of that location, if there are any. Typing
. net from http://www.stata-journal.com
. net cd software
is equivalent to typing
. net from http://www.stata-journal.com/software
Typing net cd displays the content page from that location.
3. Typing net without arguments redisplays the current content page, which is the content page last displayed.
4. net link is similar to net cd in that the result is to change the location, but rather than changing to subdirectories of the current location, net link jumps to another location:
```
. net from http://www.stata-journal.com
```

http://www.stata-journal.com/
The Stata Journal

```
The Stata Journal is a refereed, quarterly journal containing articles
of interest to Stata users. For more details and subscription information,
visit the Stata Journal website at
http://www.stata-journal.com.
PLACES you could -net link- to:
    stata StataCorp website
DIRECTORIES you could -net cd- to:
    production Files for authors of the Stata Journal
    software Software associated with Stata Journal articles
```

Typing net link stata would jump to http://www.stata.com:

```
. net link stata
```

http://www.stata.com/
StataCorp
Welcome to StataCorp.
(output omitted)

## Package-description pages

Package-description pages describe what could be installed:
. net from http://www.stata-journal.com/software/sj7-1

```
http://www.stata-journal.com/software/sj7-1/
    (output omitted)
. net describe st0119
```

package st0119 from http://www.stata-journal.com/software/sj7-1
TITLE
SJ7-1 st0119. Rasch analysis
DESCRIPTION/AUTHOR (S)
Rasch analysis
by Jean-Benoit Hardouin, University of Nantes, France
Support: jean-benoit.hardouin@univ-nantes.fr
After installation, type help gammasym, gausshermite,
geekel2d, raschtest, and raschtestv7
INSTALLATION FILES (type net install st0119)
st0119/raschtest.ado
st0119/raschtest.hlp
st0119/raschtestv7.ado
st0119/raschtestv7.hlp
st0119/gammasym.ado
st0119/gammasym.hlp
st0119/gausshermite.ado
st0119/gausshermite.hlp
st0119/geekel2d.ado
st0119/geekel2d.hlp
ANCILLARY FILES
(type net get st0119)
st0119/data.dta
st0119/outrasch.do

A package-description page describes the package and tells you how to install the component files. Package-description pages potentially describe two types of files:

1. Installation files: files that you type net install to install and that are required to make the addition work.
2. Ancillary files: additional files that you might want to install—you type net get to install thembut that you can ignore. Ancillary files are typically datasets that are useful for demonstration purposes. Ancillary files are not really installed in the sense of being copied to an official place for use by Stata itself. They are merely copied into the current directory so that you may use them if you wish.
You install the official files by typing net install followed by the package name. For example, to install st0119, you would type
```
. net install st0119
checking st0119 consistency and verifying not already installed...
installing into c:\ado\plus\ ...
installation complete.
```

You get the ancillary files-if there are any and if you want them—by typing net get followed by the package name:

```
. net get st0119
checking st0119 consistency and verifying not already installed...
copying into current directory...
    copying data.dta
    copying outrasch.do
ancillary files successfully copied.
```

Most users ignore the ancillary files.
Once you have installed a package-by typing net install-use ado to redisplay the packagedescription page whenever you wish:
. ado describe st0119
[1] package st0119 from http://www.stata-journal.com/software/sj7-1

TITLE
SJ7-1 st0119. Rasch analysis
DESCRIPTION/AUTHOR (S)
Rasch analysis
by Jean-Benoit Hardouin, University of Nantes, France
Support: jean-benoit.hardouin@univ-nantes.fr
After installation, type help gammasym, gausshermite,
geekel2d, raschtest, and raschtestv7
INSTALLATION FILES
r/raschtest.ado
r/raschtest.hlp
r/raschtestv7.ado
r/raschtestv7.hlp
g/gammasym.ado
g/gammasym.hlp
g/gausshermite.ado
g/gausshermite.hlp
g/geekel2d.ado
g/geekel2d.hlp
INSTALLED ONT
24 Apr 2013

The package-description page shown by ado includes the location from which we got the package and when we installed it. It does not mention the ancillary files that were originally part of this package because they are not tracked by ado.

## Where packages are installed

Packages should be installed in PLUS or SITE, which are code words that Stata understands and that correspond to some real directories on your computer. Typing sysdir will tell you where these are, if you care.

```
. sysdir
    STATA: C:\Program Files\Stata13\
        BASE: C:\Program Files\Stata13\ado\base\
        SITE: C:\Program Files\Stata13\ado\site\
        PLUS: c:\ado\plus\
PERSONAL: c:\ado\personal\
OLDPLACE: c:\ado\
```

If you type sysdir, you may obtain different results.

By default, net installs in the PLUS directory, and ado tells you about what is installed there. If you are on a multiple-user system, you may wish to install some packages in the SITE directory. This way, they will be available to other Stata users. To do that, before using net install, type

```
. net set ado SITE
```

and when reviewing what is installed or removing packages, redirect ado to that directory:

```
. ado ..., from(SITE)
```

In both cases, you type SITE because Stata will understand that SITE means the site ado-directory as defined by sysdir. To install into SITE, you must have write access to that directory.

If you reset where net installs and then, in the same session, wish to install into your private ado-directory, type

```
. net set ado PLUS
```

That is how things were originally. If you are confused as to where you are, type net query.

## A summary of the net command

The net command displays content pages and package-description pages. Such pages are provided over the Internet, and most users get them there. We recommend that you start at http://www.stata.com and work out from there. We also recommend using net search to find packages of interest to you; see $[R]$ net search.
net from moves you to a location and displays the content page.
net cd and net link change from your current location to other locations. net cd enters subdirectories of the original location. net link jumps from one location to another, depending on the code on the content page.
net describe lists a package-description page. Packages are named, and you type net describe pkgname.
net install installs a package into your copy of Stata. net get copies any additional files (ancillary files) to your current directory.
net sj and net stb simplify loading files from the Stata Journal and its predecessor, the Stata Technical Bulletin.

```
net sj vol-issue
```

is a synonym for typing

```
net from http://www.stata-journal.com/software/sjvol-issue
```

whereas
net sj vol-issue insert
is a synonym for typing

```
net from http://www.stata-journal.com/software/sjvol-issue
net describe insert
```

net set controls where net installs files. By default, net installs in the PLUS directory; see [P] sysdir. net set ado SITE would cause subsequent net commands to install in the SITE directory. net set other sets where ancillary files, such as .dta files, are installed. The default is the current directory.

```
net query displays the current net from, net set ado, and net set other settings.
```


## A summary of the ado command

The ado command lists the package descriptions of previously installed packages.
Typing ado without arguments is the same as typing ado dir. Both list the names and titles of the packages you have installed.
ado describe lists full package-description pages.
ado uninstall removes packages from your computer.
Because you can install packages from a variety of sources, the package names may not always be unique. Thus the packages installed on your computer are numbered sequentially, and you may refer to them by name or by number. For instance, say that you wanted to get rid of the robust-to-everything statistic command you installed. Type

```
. ado, find("robust-to-everything")
[15] package rte_stat from http://www.wemakeitupaswego.edu/faculty/sgazer
    rte_stat. The robust-to-everything statistic; update.
```

You could then type

```
. ado uninstall rte_stat
```

or

```
. ado uninstall [15]
```

Typing ado uninstall rte_stat would work only if the name rte_stat were unique; otherwise, ado would refuse, and you would have to type the number.

The find() option is allowed with ado dir and ado describe. It searches the package description for the word or phrase you specify, ignoring case (alpha matches Alpha). The complete package description is searched, including the author's name and the name of the files. Thus if rte was the name of a command that you wanted to eliminate, but you could not remember the name of the package, you could type

```
. ado, find(rte)
[15] package rte_stat from http://www.wemakeitupaswego.edu/faculty/sgazer
    rte_stat. The robust-to-everything statistic; update.
```


## Relationship of net and ado to the point-and-click interface

Users may instead select Help > SJ and User-written Programs. There are advantages and disadvantages:

1. Flipping through content and package-description pages is easier; it is much like a browser. See [GS] 19 Updating and extending Stata-Internet functionality (GSM, GSU, or GSW).
2. When browsing a product-description page, note that the .sthlp files are highlighted. You may click on .sthlp files to review them before installing the package.
3. You may not redirect from where ado searches for files.

## Creating your own site

The rest of this entry concerns how to create your own site to distribute additions to Stata. The idea is that you have written additions for use with Stata—say, xyz.ado and xyz.sthlp—and you wish to put them out so that coworkers or researchers at other institutions can easily install them. Or, perhaps you just have a dataset that you and others want to share.

In any case, all you need is a webpage. You place the files that you want to distribute on your webpage (or in a subdirectory), and you add two more files-a content file and a package-description file-and you are done.

## Format of content and package-description files

The content file describes the content page. It must be named stata.toc:

| OFF <br> * lines starting with $*$ are comments; they are ignored <br> * blank lines are ignored, too <br> * v indicates version-specify v 3 , which is the current ver v 3 <br> * d lines display description text <br> * the first d line is the title, and the remaining ones are text <br> * blank d lines display a blank line <br> d title <br> d text <br> d text <br> d <br> * 1 lines display links <br> 1 word-to-show path-or-url [description] <br> 1 word-to-show path-or-url [description] <br> ... <br> * $t$ lines display other directories within the site <br> t path [description] <br> t path [description] <br> ... <br> * p lines display packages <br> p pkgname [description] <br> p pkgname [description] <br> ... |  |
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Package files describe packages and are named pkgname. pkg:

```
* lines starting with * are comments; they are ignored
* blank lines are ignored, too
* v indicates version-specify v 3, which is the current version of .toc files
v 3
* d lines display package description text
* the first d line is the title, and the remaining ones are text
* blank d lines display a blank line
d title
d text
d Distribution-Date: date
d text
d
* f identifies the component files
f [path/]filename [description]
f [path/]filename [description]
* e line is optional; it means stop reading
e
```

Note the Distribution-Date description line. This line is optional but recommended. Stata can look for updates to user-written programs with the adoupdate command if the package files from which those programs were installed contain a Distribution-Date description line.

## Example 1

Say that we want the user to see the following:

```
. net from http://www.university.edu/~me
http://www.university.edu/~me
Chris Farrar, Uni Oniversity
PACKAGES you could -net describe-:
    xyz interval-truncated survival
. net describe xyz
package xyz from http://www.university.edu/~me
TITLE
    xyz. interval-truncated survival.
DESCRIPTION/AOTHOR(S)
    C. Farrar, Uni University.
INSTALLATION FILES (type net install xyz)
    xyz.ado
    xyz.sthlp
ANCILLARY FILES (type net get xyz)
    sample.dta
```

The files needed to do this would be

```
v 3
d Chris Farrar, Uni University
p xyz interval-truncated survival
```

```
v 3
    d xyz. interval-truncated survival.
    d C. Farrar, Uni University.
    f xyz.ado
    f xyz.sthlp
    f sample.dta
```

On his homepage, Chris would place the following files:

| stata.toc | (shown above) |
| :--- | :--- |
| xyz.pkg | (shown above) |
| xyz.ado | file to be delivered (for use by net install) |
| xyz.sthlp | file to be delivered (for use by net install) |
| sample.dta | file to be delivered (for use by net get) |

Chris does nothing to distinguish ancillary files from installation files.

## Example 2

S. Gazer wants to create a more complex site:
. net from http://www.wemakeitupaswego.edu/faculty/sgazer
http://www. wemakeitupaswego.edu/faculty/sgazer
Data-free inference materials
S. Gazer, Department of Applied Theoretical Mathematics

Also see my homepage for the preprint of "Irrefutable inference".
PLACES you could -net link- to:
stata StataCorp website
DIRECTORIES you could -net cd- to:
ir irrefutable inference programs (work in progress)
PACKAGES you could -net describe-:
rtec Robust-to-everything-conceivable statistic
rte Robust-to-everything statistic

```
. net describe rte
```

package rte from http://www.wemakeitupaswego.edu/faculty/sgazer/

## TITLE

rte. The robust-to-everything statistic; update.
DESCRIPTION/AUTHOR (S)
S. Gazer, Dept. of Applied Theoretical Mathematics, WMIUAWG Univ. Aleph-0 100\% confidence intervals proved too conservative for some applications; Aleph-1 confidence intervals have been substituted. The new robust-to-everything supplants the previous robust-to-everything-conceivable statistic. See "Inference in the absence of data" (forthcoming). After installation, see help rte.

Distribution-Date: 20130420
Support: email sgazer@wemakeitupaswego.edu
INSTALLATION FILES (type net install rte_stat)
rte.ado
rte.sthlp
nullset.ado
random.ado
ANCILLARY FILES (type net get rte_stat)
empty.dta

The files needed to do this would be

```
v 3
d Data-free inference materials
d S. Gazer, Department of Applied Theoretical Mathematics
d
d Also see my homepage for the preprint of "Irrefutable inference".
l stata http://www.stata.com
t ir irrefutable inference programs (work in progress)
p rtec Robust-to-everything-conceivable statistic
p rte Robust-to-everything statistic
```

```
v 3
d rte. The robust-to-everything statistic; update.
d {bf:S. Gazer, Dept. of Applied Theoretical Mathematics, WMIUAWG Univ.}
d Aleph-0 100% confidence intervals proved too conservative for some
d applications; Aleph-1 confidence intervals have been substituted.
d The new robust-to-everything supplants the previous robust-to-
d everything-conceivable statistic. See "Inference in the absence
d of data" (forthcoming). After installation, see help {bf:rte}.
d
d Distribution-Date: 20130420
d
d Support: email sgazer@wemakeitupaswego.edu
f rte.ado
f rte.sthlp
f nullset.ado
f random.ado
f empty.dta
```

On his homepage, Mr. Gazer would place the following files:

```
stata.toc (shown above)
rte.pkg (shown above)
rte.ado (file to be delivered)
rte.sthlp (file to be delivered)
nullset.ado (file to be delivered)
random.ado (file to be delivered)
empty.dta (file to be delivered)
rtec.pkg the other package referred to in stata.toc
rtec.ado the corresponding files to be delivered
rtec.sthlp
ir/stata.toc the contents file for when the user types net cd ir
ir/... whatever other .pkg files are referred to
ir/... whatever other files are to be delivered
```

If Mr. Gazer later updated the rte package, he could change the Distribution-Date description line in his package. Then, if someone who had previously installed the rte packaged wanted to obtain the latest version, that person could use the adoupdate command; see [R] adoupdate.

For complex sites, a different structure may prove more convenient:

```
stata.toc (shown above)
rte.pkg (shown above)
rtec.pkg
rte/
rte/rte.ado
rte/rte.sthlp
rte/nullset.ado
rte/random.ado
rte/empty.dta
rtec/
rtec/...
ir/stata.toc
ir/*.pkg
ir/*/...
```

(shown above)
(shown above)
the other package referred to in stata.toc
directory containing rte files to be delivered:
(file to be delivered)
(file to be delivered)
(file to be delivered)
(file to be delivered)
(file to be delivered)
directory containing rtec files to be delivered:
(files to be delivered)
the contents file for when the user types net cd ir
whatever other package files are referred to
whatever other files are to be delivered

If you prefer this structure, it is simply a matter of changing the bottom of the rte.pkg from

```
f rte.ado
f rte.sthlp
f nullset.ado
f random.ado
f empty.dta
```

to

```
f rte/rte.ado
f rte/rte.sthlp
f rte/nullset.ado
f rte/random.ado
f rte/empty.dta
```

In writing paths and files, the directory separator forward slash (/) is used, regardless of operating system, because this is what the Internet uses.

It does not matter whether the files you put out are in Windows, Mac, or Unix format (how lines end is recorded differently). When Stata reads the files over the Internet, it will figure out the file format on its own and will automatically translate the files to what is appropriate for the receiver.

## Additional package directives

F filename is similar to $f$ filename, except that, when the file is installed, it will always be copied to the system directories (and not the current directory).
With $f$ filename, the file is installed into a directory according to the file's suffix. For instance, xyz.ado would be installed in the system directories, whereas xyz.dta would be installed in the current directory.

Coding F xyz.ado would have the same result as coding f xyz.ado.
Coding F xyz.dta, however, would state that xyz.dta is to be installed in the system directories.
g platformname filename is also a variation on $f$ filename. It specifies that the file be installed only if the user's operating system is of type platformname; otherwise, the file is ignored. The platform names are WIN64A (64-bit x86-64) and WIN (32-bit x86) for Windows; MACINTEL64 (64-bit Intel, GUI), OSX . X8664 (64-bit Intel, console), MACINTEL (32-bit Intel, GUI), and OSX. X86 (32-bit Intel, console) for Mac; and LINUX64 (64-bit x86-64), LINUX (32-bit x86), SOLX8664 (64-bit x86-64), and SOL64 for Unix.

G platformname filename is a variation on F filename. The file, if not ignored, is to be installed in the system directories.
g platformname filename1 filename2 is a more detailed version of g platformname filename. In this case, filename 1 is the name of the file on the server (the file to be copied), and filename 2 is to be the name of the file on the user's system; for example, you might code

```
g WIN mydll.forwin mydll.plugin
g LINUX mydll.forlinux mydll.plugin
```

When you specify one filename, the result is the same as specifying two identical filenames.
G platformname filename1 filename2 is the install-in-system-directories version of g platformname filename1 filename2.
h filename asserts that filename must be loaded, or this package is not to be installed; for example, you might code

```
g WIN mydll.forwin mydll.plugin
g LINUX mydll.forlinux mydll.plugin
h mydll.plugin
```

if you were offering the plugin mydll. plugin for Windows and Linux only.

## SMCL in content and package-description files

The text listed on the second and subsequent d lines in both stata.toc and pkgname. pkg may contain SMCL as long as you include v 3; see [P] smcl.

Thus, in rte.pkg, S. Gazer coded the third line as

```
d {bf:S. Gazer, Dept. of Applied Theoretical Mathematics, WMIUAWG Univ.}
```


## Error-free file delivery

Most people transport files over the Internet and never worry about the file being corrupted in the process because corruption rarely occurs. If, however, the files must be delivered perfectly or not at all, you can include checksum files in the directory.

For instance, say that big.dta is included in your package and that it must be sent perfectly. First, use Stata to make the checksum file for big.dta
. checksum big.dta, save
That command creates a small file called big.sum; see [D] checksum. Then copy both big.dta and big.sum to your homepage. If set checksum is on (the default is off), whenever Stata reads filename.whatever over the net, it also looks for filename. sum. If it finds such a file, it uses the information recorded in it to verify that what was copied was error free.

If you do this, be cautious. If you put big.dta and big.sum on your homepage and then later change big.dta without changing big.sum, people will think that there are transmission errors when they try to download big.dta.

## References

Baum, C. F., and N. J. Cox. 1999. ip29: Metadata for user-written contributions to the Stata programming language. Stata Technical Bulletin 52: 10-12. Reprinted in Stata Technical Bulletin Reprints, vol. 9, pp. 121-124. College Station, TX: Stata Press.

Cox, N. J., and C. F. Baum. 2000. ip29.1: Metadata for user-written contributions to the Stata programming language. Stata Technical Bulletin 54: 21-22. Reprinted in Stata Technical Bulletin Reprints, vol. 9, pp. 124-126. College Station, TX: Stata Press.

## Also see

[R] adoupdate - Update user-written ado-files
[R] net search - Search the Internet for installable packages
[R] netio - Control Internet connections
[R] search - Search Stata documentation and other resources
[R] sj - Stata Journal and STB installation instructions
[R] ssc - Install and uninstall packages from SSC
[R] update - Check for official updates
[D] checksum - Calculate checksum of file
[P] smcl - Stata Markup and Control Language
[GSM] 19 Updating and extending Stata-Internet functionality
[GSU] 19 Updating and extending Stata-Internet functionality
[GSW] 19 Updating and extending Stata-Internet functionality
[U] 28 Using the Internet to keep up to date

## Title

net search - Search the Internet for installable packages
Syntax Description Options Remarks and examples

References Also see

## Syntax

net search word [word ...] [, options]
options Description
or list packages that contain any of the keywords; default is all
nosj search non-SJ and non-STB sources
tocpkg search both tables of contents and packages; the default
toc search tables of contents only
pkg search packages only
everywhere search packages for match
$\underline{f} i l e n a m e s \quad$ search filenames associated with package for match
errnone make return code 111 instead of 0 when no matches found

## Description

net search searches the Internet for user-written additions to Stata, including, but not limited to, user-written additions published in the Stata Journal (SJ) and in the Stata Technical Bulletin (STB). net search lists the available additions that contain the specified keywords.

The user-written materials found are available for immediate download by using the net command or by clicking on the link.

In addition to typing net search, you may select Help > Search... and choose Search net resources. This is the recommended way to search for user-written additions to Stata.

## Options

or is relevant only when multiple keywords are specified. By default, net search lists only packages that include all the keywords. or changes the command to list packages that contain any of the keywords.
nosj specifies that net search not list matches that were published in the SJ or in the STB.
tocpkg, toc, and pkg determine what is searched. tocpkg is the default, meaning that both tables of contents (tocs) and packages (pkgs) are searched. toc restricts the search to tables of contents. pkg restricts the search to packages.
everywhere and filenames determine where in packages net search looks for keywords. The default is everywhere. filenames restricts net search to search for matches only in the filenames associated with a package. Specifying everywhere implies pkg.
errnone is a programmer's option that causes the return code to be 111 instead of 0 when no matches are found.

## Remarks and examples

net search searches the Internet for user-written additions to Stata. If you want to search the Stata documentation for a particular topic, command, or author, see [R] search. net search word [word ...] (without options) is equivalent to typing search word [word ...], net.

Remarks are presented under the following headings:

```
Topic searches
Author searches
Command searches
Where does net search look?
How does net search work?
```


## Topic searches

## Example: Find what is available about random effects

```
. net search random effect
```

Comments:

- It is best to search using the singular form of a word. net search random effect will find both "random effect" and "random effects".
- net search random effect will also find "random-effect" because net search performs a string search and not a word search.
- net search random effect lists all packages containing the words "random" and "effect", not necessarily used together.
- If you wanted all packages containing the word "random" or the word "effect", you would type net search random effect, or.


## Author searches

## Example: Find what is available by author Jeroen Weesie

```
. net search weesie
```

Comments:

- You could type net search jeroen weesie, but that might list fewer results because sometimes the last name is used without the first.
- You could type net search Weesie, but it would not matter. Capitalization is ignored in the search.


## Example: Find what is available by Jeroen Weesie, excluding SJ and STB materials

. net search weesie, nosj

- The SJ and the STB tend to dominate search results because so much has been published in them. If you know that what you are looking for is not in the SJ or in the STB, specifying the nosj option will narrow the search.
- net search weesie lists everything that net search weesie, nosj lists, and more. If you just type net search weesie, look down the list. SJ and STB materials are listed first, and non-SJ and non-STB materials are listed last.


## Command searches

## Example: Find the user-written command kursus

```
. net search kursus, file
```

- You could just type net search kursus, and that will list everything net search kursus, file lists, and more. Because you know kursus is a command, however, there must be a kursus.ado file associated with the package. Typing net search kursus, file narrows the search.
- You could also type net search kursus.ado, file to narrow the search even more.


## Where does net search look?

net search looks everywhere, not just at http://www.stata.com.
net search begins by looking at http://www.stata.com, but then follows every link, which takes it to other places, and then follows every link again, which takes it to even more places, and so on.

Authors: Please let us know if you have a site that we should include in our search by sending an email to webmaster@stata.com. We will then link to your site from ours to ensure that net search finds your materials. That is not strictly necessary, however, as long as your site is directly or indirectly linked from some site that is linked to ours.

## How does net search work?



Our website maintains a database of Stata resources. When you use net search, it contacts http://www.stata.com with your request, http://www.stata.com searches its database, and Stata returns the results to you.

Another part of the system is called the crawler, which searches the web for new Stata resources to add to the net search database and verifies that the resources already found are still available. When a new resource becomes available, the crawler takes about 2 days to add it to the database, and, similarly, if a resource disappears, the crawler takes roughly 2 days to remove it from the database.

## References

Baum, C. F., and N. J. Cox. 1999. ip29: Metadata for user-written contributions to the Stata programming language. Stata Technical Bulletin 52: 10-12. Reprinted in Stata Technical Bulletin Reprints, vol. 9, pp. 121-124. College Station, TX: Stata Press.

Cox, N. J., and C. F. Baum. 2000. ip29.1: Metadata for user-written contributions to the Stata programming language. Stata Technical Bulletin 54: 21-22. Reprinted in Stata Technical Bulletin Reprints, vol. 9, pp. 124-126. College Station, TX: Stata Press.

Gould, W. W., and A. R. Riley. 2000. stata55: Search web for installable packages. Stata Technical Bulletin 54: 4-6. Reprinted in Stata Technical Bulletin Reprints, vol. 9, pp. 10-13. College Station, TX: Stata Press.

## Also see

[ R$]$ adoupdate - Update user-written ado-files
[R] net - Install and manage user-written additions from the Internet
$[R]$ search - Search Stata documentation and other resources
[R] sj - Stata Journal and STB installation instructions
[R] ssc - Install and uninstall packages from SSC
[R] update - Check for official updates

## Title

netio - Control Internet connections

Syntax Description Options Remarks and examples Also see

## Syntax

Turn on or off the use of a proxy server set httpproxy $\left\{\begin{array}{l}\text { on } \mid \text { off }\}[, \text { init] }\end{array}\right.$

Set proxy host name
set httpproxyhost ["]name["]
Set the proxy port number
set httpproxyport \#

Turn on or off proxy authorization

```
set httpproxyauth {on | off}
```

Set proxy authorization user ID
set httpproxyuser ["]name["]
Set proxy authorization password set httpproxypw ["] password ["]

Set time limit for establishing initial connection
set timeout1 \#seconds [, permanently]
Set time limit for data transfer

```
set timeout2 #seconds [, permanently]
```


## Description

Several commands (for example, net, news, and update) are designed specifically for use over the Internet. Many other Stata commands that read a file (for example, copy, type, and use) can also read directly from a URL. All of these commands will usually work without your ever needing to concern yourself with the set commands discussed here. These set commands provide control over network system parameters.

If you experience problems when using Stata's network features, ask your system administrator if your site uses a proxy. A proxy is a server between your computer and the rest of the Internet, and your computer may need to communicate with other computers on the Internet through this proxy. If your site uses a proxy, your system administrator can provide you with its host name and the port your computer can use to communicate with it. If your site's proxy requires you to $\log$ in to it before it will respond, your system administrator will provide you with a user ID and password.
set httpproxyhost sets the name of the host to be used as a proxy server. set httpproxyport sets the port number. set httpproxy turns on or off the use of a proxy server, leaving the proxy host name and port intact, even when not in use.

Under the Mac and Windows operating systems, when you set httpproxy on, Stata will attempt to obtain the values of httpproxyhost and httpproxyport from the operating system if they have not been previously set. set httpproxy on, init attempts to obtain these values from the operating system, even if they have been previously set.

If the proxy requires authorization (user ID and password), set authorization on via set httpproxyauth on. The proxy user and proxy password must also be set to the appropriate user ID and password by using set httpproxyuser and set httpproxypw.

Stata remembers the various proxy settings between sessions and does not need a permanently option.
set timeout1 changes the time limit in seconds that Stata imposes for establishing the initial connection with a remote host. The default value is 30 . set timeout2 changes the time limit in seconds that Stata imposes for subsequent data transfer with the host. The default value is 180. If these time limits are exceeded, a "connection timed out" message and error code 2 are produced. You should seldom need to change these settings.

## Options

init specifies that set httpproxy on attempts to initialize httpproxyhost and httpproxyport from the operating system (Mac and Windows only).
permanently specifies that, in addition to making the change right now, the timeout1 and timeout2 settings be remembered and become the default setting when you invoke Stata.
The various httpproxy settings do not have a permanently option because permanently is implied.

## Remarks and examples

If you receive an error message, see http://www.stata.com/support/faqs/web/ for the latest information.

## 1. remote connection failed $r(677)$;

If you see
remote connection failed
r(677) ;
then you asked for something to be done over the web, and Stata tried but could not contact the specified host. Stata was able to talk over the network and look up the host but was not able to establish a connection to that host. Perhaps the host is down; try again later.

If all your web accesses result in this message, then perhaps your network connection is through a proxy server. If it is, then you must tell Stata.

Contact your system administrator. Ask for the name and port of the "HTTP proxy server". Say that you are told

HTTP proxy server: jupiter.myuni.edu
port number: 8080
In Stata, type
. set httpproxyhost jupiter.myuni.edu
. set httpproxyport 8080
. set httpproxy on
Your web accesses should then work.

## 2. connection timed out $\mathbf{r}(2)$;

If you see
connection timed out
$r(2)$;
then an Internet connection has timed out. This can happen when
a. the connection between you and the host is slow, or
b. the connection between you and the host has disappeared, and so it eventually "timed out".

For (b), wait a while (say, 5 minutes) and try again (sometimes pieces of the Internet can break for up to a day, but that is rare). For (a), you can reset the limits for what constitutes "timed out". There are two numbers to set.

The time to establish the initial connection is timeout1. By default, Stata waits 30 seconds before declaring a timeout. You can change the limit:

```
. set timeout1 #seconds
```

You might try doubling the usual limit and specify 60 ; \#seconds must be between 1 and 32,000 .
The time to retrieve data from an open connection is timeout2. By default, Stata waits 180 seconds ( 3 minutes) before declaring a timeout. To change the limit, type

```
. set timeout2 #seconds
```

You might try doubling the usual limit and specify 360 ; \#seconds must be between 1 and 32,000 .

## Also see

[R] query - Display system parameters
[P] creturn - Return c-class values
[U] 28 Using the Internet to keep up to date

## Title

news - Report Stata news
Syntax Menu Description Remarks and examples Also see

## Syntax

news

## Menu

Help $>$ News

## Description

news displays a brief listing of recent Stata news and information, which it obtains from Stata's website. news requires that your computer be connected to the Internet.

You may also execute news by selecting Help > News.

## Remarks and examples

news provides an easy way of displaying a brief list of the latest Stata news:
. news
/_-/ / / /_---/ / / /_---/ News The latest from http://www.stata.com

24 June 2013. Stata 13 available
Stata 13 is now available. Visit http://www.stata.com/stata13/ for more information.

Quick summary: 1) 2-billion character long string variables.
2) Treatment-effects estimators. 3) More multilevel
mixed-effects models. 4) Multivariate mixed-effects and generalized linear SEM. 5) Forecasts. 6) Power and sample size. 7) New and extended random-effects panel-data estimators.
8) Effect sizes. 9) Project Manager. 10) Java plugins.
11) ...

12 June 2013. New public training dates announced.
Visit http://www.stata.com/public-training/ for course offerings and dates.

## 20 May 2013. Official update available for download

Click here (equivalent to pulling down Help and selecting
Check for Updates) or type update from http://www.stata.com.
25 March 2013. NetCourse schedule updated
See http://www.stata.com/netcourse/ for more information.
(output omitted)

## Also see

[U] 28 Using the Internet to keep up to date

## Title

nl - Nonlinear least-squares estimation

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | Acknowledgments |
| References | Also see |  |  |

## Syntax

Interactive version

$$
\mathrm{nl}(\text { depvar }=\langle\operatorname{sexp}>)[\text { if }][\text { in }][\text { weight }][\text {, options }]
$$

Programmed substitutable expression version
nl sexp_prog : depvar $[$ varlist $][$ if $][$ in $][$ weight $][$, options $]$

Function evaluator program version
nl func_prog @ depvar [varlist] [if] [in][weight],

where
depvar is the dependent variable;
$<\operatorname{sexp}>$ is a substitutable expression;
sexp_prog is a substitutable expression program; and func_prog is a function evaluator program.
options
Model
variables (varlist)
initial (initial_values)

* parameters(namelist)
* nparameters (\#)
sexp_options
func_options
Model 2


## noconstant

hasconstant (name)
SE/Robust
vce (vcetype)

Reporting
level (\#)
leave
title (string)
title2 (string)
display_options
Optimization
optimization_options
eps (\#)
delta(\#)
coeflegend

Description
variables in model
initial values for parameters
parameters in model (function evaluator program version only)
number of parameters in model
(function evaluator program version only)
options for substitutable expression program
options for function evaluator program
use $\log$ least-squares where $\ln ($ depvar $-\#)$ is assumed to be normally distributed
the model has no constant term; seldom used use name as constant term; seldom used
vcetype may be gnr, robust, cluster clustvar, bootstrap, jacknife, hac kernel, hc2, or hc3
set confidence level; default is level (95)
create variables containing derivative of $E(y)$
display string as title above the table of parameter estimates
display string as subtitle
control column formats and line width
control the optimization process; seldom used
specify \# for convergence criterion; default is eps (1e-5)
specify \# for computing derivatives; default is delta (4e-7)
display legend instead of statistics

* For function evaluator program version, you must specify parameters (namelist) or nparameters (\#), or both. bootstrap, by, jackknife, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the bootstrap prefix; see [R] bootstrap. aweights are not allowed with the jackknife prefix; see [R] jackknife. vce(), leave, and weights are not allowed with the svy prefix; see [SVY] svy.
aweights, fweights, and iweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.


## Menu

Statistics $>$ Linear models and related $>$ Nonlinear least squares

## Description

nl fits an arbitrary nonlinear regression function by least squares. With the interactive version of the command, you enter the function directly on the command line or in the dialog box by using a substitutable expression. If you have a function that you use regularly, you can write a substitutable expression program and use the second syntax to avoid having to reenter the function every time. The function evaluator program version gives you the most flexibility in exchange for increased complexity; with this version, your program is given a vector of parameters and a variable list, and your program computes the regression function.

When you write a substitutable expression program or function evaluator program, the first two letters of the name must be nl. sexp_prog and func_prog refer to the name of the program without the first two letters. For example, if you wrote a function evaluator program named nlregss, you would type $n l$ regss © ... to estimate the parameters.

## Options


#### Abstract

Model variables (varlist) specifies the variables in the model. nl ignores observations for which any of these variables have missing values. If you do not specify variables(), then $n l$ issues an error message with return code 480 if the estimation sample contains any missing values. initial(initial_values) specifies the initial values to begin the estimation. You can specify a $1 \times k$ matrix, where $k$ is the number of parameters in the model, or you can specify a parameter name, its initial value, another parameter name, its initial value, and so on. For example, to initialize alpha to 1.23 and delta to 4.57 , you would type


nl ... , initial(alpha 1.23 delta 4.57) ...
Initial values declared using this option override any that are declared within substitutable expressions. If you specify a parameter that does not appear in your model, nl exits with error code 480. If you specify a matrix, the values must be in the same order that the parameters are declared in your model. nl ignores the row and column names of the matrix.
parameters (namelist) specifies the names of the parameters in the model. The names of the parameters must adhere to the naming conventions of Stata's variables; see [U] 11.3 Naming conventions. If you specify both parameters() and nparameters(), the number of names in the former must match the number specified in the latter; if not, nl issues an error message with return code 198.
nparameters (\#) specifies the number of parameters in the model. If you do not specify names with the parameters() option, nl names them $\mathrm{b} 1, \mathrm{~b} 2, \ldots, \mathrm{~b} \#$. If you specify both parameters() and nparameters (), the number of names in the former must match the number specified in the latter; if not, nl issues an error message with return code 198.
sexp_options refer to any options allowed by your sexp_prog.
func_options refer to any options allowed by your func_prog.

[^51]noconstant indicates that the function does not include a constant term. This option is generally not needed, even if there is no constant term in the model, unless the coefficient of variation (over observations) of the partial derivative of the function with respect to a parameter is less than eps() and that parameter is not a constant term.
hasconstant (name) indicates that parameter name be treated as the constant term in the model and that $n l$ should not use its default algorithm to find a constant term. As with noconstant, this option is seldom used.

SE/Robust
vce (vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (gnr), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
vce (gnr), the default, uses the conventionally derived variance estimator for nonlinear models fit using Gauss-Newton regression.
nl also allows the following:
vce(hac kernel [\#]) specifies that a heteroskedasticity- and autocorrelation-consistent (HAC) variance estimate be used. HAC refers to the general form for combining weighted matrices to form the variance estimate. There are three kernels available for nl :

## nwest | gallant | anderson

\# specifies the number of lags. If \# is not specified, $N-2$ is assumed.
vce (hac kernel $[\#]$ ) is not allowed if weights are specified.
vce (hc2) and vce (hc3) specify alternative bias corrections for the robust variance calculation. vce (hc2) and vce (hc3) may not be specified with the svy prefix. By default, vce (robust) uses $\widehat{\sigma}_{j}^{2}=\{n /(n-k)\} u_{j}^{2}$ as an estimate of the variance of the $j$ th observation, where $u_{j}$ is the calculated residual and $n /(n-k)$ is included to improve the overall estimate's small-sample properties.
vce (hc2) instead uses $u_{j}^{2} /\left(1-h_{j j}\right)$ as the observation's variance estimate, where $h_{j j}$ is the $j$ th diagonal element of the hat (projection) matrix. This produces an unbiased estimate of the covariance matrix if the model is homoskedastic. vce (hc2) tends to produce slightly more conservative confidence intervals than vce (robust).
vce (hc3) uses $u_{j}^{2} /\left(1-h_{j j}\right)^{2}$ as suggested by Davidson and MacKinnon (1993 and 2004), who report that this often produces better results when the model is heteroskedastic. vce (hc3) produces confidence intervals that tend to be even more conservative.
See, in particular, Davidson and MacKinnon (2004, 239), who advocate the use of vce(hc2) or vce (hc3) instead of the plain robust estimator for nonlinear least squares.

Reporting
level (\#); see [R] estimation options.
leave leaves behind after estimation a set of new variables with the same names as the estimated parameters containing the derivatives of $E(y)$ with respect to the parameters. If the dataset contains an existing variable with the same name as a parameter, then using leave causes nl to issue an error message with return code 110 .
leave may not be specified with vce(cluster clustvar) or the svy prefix.
title (string) specifies an optional title that will be displayed just above the table of parameter estimates.
title2 (string) specifies an optional subtitle that will be displayed between the title specified in title() and the table of parameter estimates. If title2() is specified but title() is not, title2() has the same effect as title().
display_options: cformat (\%fmt), pformat (\%fmt), sformat(\%fmt), and nolstretch; see [R] estimation options.

Optimization
optimization_options: iterate(\#), [no] log, trace. iterate() specifies the maximum number of iterations, $l o g / n o l o g$ specifies whether to show the iteration $\log$, and trace specifies that the iteration log should include the current parameter vector. These options are seldom used.
eps (\#) specifies the convergence criterion for successive parameter estimates and for the residual sum of squares. The default is eps $(1 e-5)$.
delta(\#) specifies the relative change in a parameter to be used in computing the numeric derivatives. The derivative for parameter $\beta_{i}$ is computed as $\left\{f\left(X, \beta_{1}, \beta_{2}, \ldots, \beta_{i}+d, \beta_{i+1}, \ldots\right)-\right.$ $\left.f\left(X, \beta_{1}, \beta_{2}, \ldots, \beta_{i}, \beta_{i+1}, \ldots\right)\right\} / d$, where $d$ is $\delta\left(\beta_{i}+\delta\right)$. The default is delta(4e-7).

The following options are available with nl but are not shown in the dialog box:
coeflegend; see $[R]$ estimation options.

## Remarks and examples

Remarks are presented under the following headings:

```
Substitutable expressions
Substitutable expression programs
Built-in functions
Lognormal errors
Other uses
Weights
Potential errors
General comments on fitting nonlinear models
Function evaluator programs
```

nl fits an arbitrary nonlinear function by least squares. The interactive version allows you to enter the function directly on the command line or dialog box using substitutable expressions. You can write a substitutable expression program for functions that you fit frequently to save yourself time. Finally, function evaluator programs give you the most flexibility in defining your nonlinear function, though they are more complicated to use.

The next section explains the substitutable expressions that are used to define the regression function, and the section thereafter explains how to write substitutable expression program files so that you do not need to type in commonly used functions over and over. Later sections highlight other features of nl .

The final section discusses function evaluator programs. If you find substitutable expressions adequate to define your nonlinear function, then you can skip that section entirely. Function evaluator programs are generally needed only for complicated problems, such as multistep estimators. The program receives a vector of parameters at which it is to compute the function and a variable into which the results are to be placed.

## Substitutable expressions

You define the nonlinear function to be fit by nl by using a substitutable expression. Substitutable expressions are just like any other mathematical expressions involving scalars and variables, such as those you would use with Stata's generate command, except that the parameters to be estimated are bound in braces. See [U] 13.2 Operators and [U] 13.3 Functions for more information on expressions.

For example, suppose that you wish to fit the function

$$
y_{i}=\beta_{0}\left(1-e^{-\beta_{1} x_{i}}\right)+\epsilon_{i}
$$

where $\beta_{0}$ and $\beta_{1}$ are the parameters to be estimated and $\epsilon_{i}$ is an error term. You would simply type

$$
\text { . } \mathrm{nl}(\mathrm{y}=\{\mathrm{b} 0\} *(1-\exp (-1 *\{\mathrm{~b} 1\} * x)))
$$

You must enclose the entire equation in parentheses. Because b0 and b1 are enclosed in braces, nl knows that they are parameters in the model. nl will initialize b0 and b1 to zero by default. To request that nl initialize b 0 to 1 and b 1 to 0.25 , you would type

$$
\text { . } \mathrm{nl}(\mathrm{y}=\{\mathrm{b} 0=1\} *(1-\exp (-1 *\{\mathrm{~b} 1=0.25\} * x)))
$$

That is, inside the braces denoting a parameter, you put the parameter name followed by an equal sign and the initial value. If a parameter appears in your function multiple times, you need only specify an initial value only once (or never, if you wish to set the initial value to zero). If you do specify more than one initial value for the same parameter, nl will use the last value given. Parameter names must follow the same conventions as variable names. See [U] 11.3 Naming conventions.

Frequently, even nonlinear functions contain linear combinations of variables. As an example, suppose that you wish to fit the function

$$
y_{i}=\beta_{0}\left\{1-e^{-\left(\beta_{1} x_{1 i}+\beta_{2} x_{2 i}+\beta_{3} x_{3 i}\right)}\right\}+\epsilon_{i}
$$

nl allows you to declare a linear combination of variables by using the shorthand notation

```
. nl (y = {b0=1}*(1 - exp(-1*{xb: x1 x2 x3})))
```

In the syntax $\{x b: x 1 \times 2 x 3\}$, you are telling $n l$ that you are declaring a linear combination named xb that is a function of three variables, $\mathrm{x} 1, \mathrm{x} 2$, and $\mathrm{x} 3 . \mathrm{nl}$ will create three parameters, named $\mathrm{xb} \_\mathrm{x} 1$, $\mathrm{xb} \_\mathrm{x} 2$, and $\mathrm{xb} \_\mathrm{x} 3$, and initialize them to zero. Instead of typing the previous command, you could have typed

$$
\text { . } \mathrm{nl}\left(\mathrm{y}=\{\mathrm{b} 0=1\} *\left(1-\exp \left(-1 *\left(\left\{\mathrm{xb} \_\mathrm{x} 1\right\} * \mathrm{x} 1+\left\{\mathrm{xb} \_\mathrm{x} 2\right\} * \mathrm{x} 2+\left\{\mathrm{xb} \_\mathrm{x} 3\right\} * \mathrm{x} 3\right)\right)\right)\right)
$$

and yielded the same result. You can refer to the parameters created by nl in the linear combination later in the function, though you must declare the linear combination first if you intend to do that. When creating linear combinations, nl ensures that the parameter names it chooses are unique and have not yet been used in the function.

In general, there are three rules to follow when defining substitutable expressions:

1. Parameters of the model are bound in braces: $\{b 0\}$, \{param\}, etc.
2. Initial values for parameters are given by including an equal sign and the initial value inside the braces: $\{b 0=1\}$, $\{$ param $=3.571\}$, etc.
3. Linear combinations of variables can be included using the notation \{eqname:varlist\}, for example, $\{\mathrm{xb}$ : mpg price weight\}, \{score: wx z$\}$, etc. Parameters of linear combinations are initialized to zero.

If you specify initial values by using the initial() option, they override whatever initial values are given within the substitutable expression. Substitutable expressions are so named because, once values are assigned to the parameters, the resulting expression can be handled by generate and replace.

## Example 1

We wish to fit the CES production function

$$
\begin{equation*}
\ln Q_{i}=\beta_{0}-\frac{1}{\rho} \ln \left\{\delta K_{i}^{-\rho}+(1-\delta) L_{i}^{-\rho}\right\}+\epsilon_{i} \tag{1}
\end{equation*}
$$

where $\ln Q_{i}$ is the $\log$ of output for firm $i ; K_{i}$ and $L_{i}$ are firm $i$ 's capital and labor usage, respectively; and $\epsilon_{i}$ is a regression error term. Because $\rho$ appears in the denominator of a fraction, zero is not a feasible initial value; for a CES production function, $\rho=1$ is a reasonable choice. Setting $\delta=0.5$ implies that labor and capital have equal impacts on output, which is also a reasonable choice for an initial value. We type


Parameter b0 taken as constant term in model \& ANOVA table
nl will attempt to find a constant term in the model and, if one is found, mention it at the bottom of the output. nl found b 0 to be a constant because the partial derivative $\partial \ln Q_{i} / \partial \mathrm{b} 0$ has a coefficient of variation less than eps () in the estimation sample.

The elasticity of substitution for the CES production function is $\sigma=1 /(1+\rho)$; and, having fit the model, we can use nlcom to estimate it:
. nlcom (1/(1 + _b[/rho]))
_nl_1: 1/(1 + _b[/rho])

| lnoutput | Coef. | Std. Err. | $z$ | $P>\|z\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | ---: |
| _nl_1 | .4189372 | .0829424 | 5.05 | 0.000 | .256373 | .5815014 |

See $[R]$ nlcom and $[U]$ 13.5 Accessing coefficients and standard errors for more information.
nl 's output closely mimics that of regress; see [R] regress for more information. The $R^{2}$, sums of squares, and similar statistics are calculated in the same way that regress calculates them. If no "constant" term is specified, the usual caveats apply to the interpretation of the $R^{2}$ statistic; see the comments and references in Goldstein (1992). Unlike regress, nl does not report a model $F$ statistic, because a test of the joint significance of all the parameters except the constant term may not be relevant in a nonlinear model.

## Substitutable expression programs

If you fit the same model often or if you want to write an estimator that will operate on whatever variables you specify, then you will want to write a substitutable expression program. That program will return a macro containing a substitutable expression that nl can then evaluate, and it may optionally calculate initial values as well. The name of the program must begin with the letters nl.

To illustrate, suppose that you use the CES production function often in your work. Instead of typing in the formula each time, you can write a program like this:

```
program nlces, rclass
    version 13
    syntax varlist(min=3 max=3) [if]
    local logout : word 1 of 'varlist'
    local capital : word 2 of 'varlist'
    local labor : word 3 of 'varlist'
    // Initial value for b0 given delta=0.5 and rho=1
    tempvar y
    generate double 'y' = 'logout' + ln(0.5*'capital'^-1 + 0.5*'labor'^-1)
    summarize 'y' 'if', meanonly
    local bOval = r(mean)
    // Terms for substitutable expression
    local capterm "{delta=0.5}*'capital'^(-1*{rho})"
    local labterm "(1-{delta})*'labor'^(-1*{rho})"
    local term2 "1/{rho=1}*ln('capterm' + 'labterm')"
    // Return substitutable expression and title
    return local eq "'logout' = {b0='b0val'} - 'term2'"
    return local title "CES ftn., ln Q=`logout', K='capital', L=`labor'"
end
```

The program accepts three variables for log output, capital, and labor, and it accepts an if exp qualifier to restrict the estimation sample. All programs that you write to use with nl must accept an if $\exp$ qualifier because, when nl calls the program, it passes a binary variable that marks the estimation sample (the variable equals one if the observation is in the sample and zero otherwise). When calculating initial values, you will want to restrict your computations to the estimation sample, and you can do so by using if with any commands that accept if exp qualifiers. Even if your program does not calculate initial values or otherwise use the if qualifier, the syntax statement must still allow it. See [P] syntax for more information on the syntax command and the use of if.

As in the previous example, reasonable initial values for $\delta$ and $\rho$ are 0.5 and 1 , respectively. Conditional on those values, (1) can be rewritten as

$$
\begin{equation*}
\beta_{0}=\ln Q_{i}+\ln \left(0.5 K_{i}^{-1}+0.5 L_{i}^{-1}\right)-\epsilon_{i} \tag{2}
\end{equation*}
$$

so a good initial value for $\beta_{0}$ is the mean of the right-hand side of (2) ignoring $\epsilon_{i}$. Lines 7-10 of the function evaluator program calculate that mean and store it in a local macro. Notice the use of if in the summarize statement so that the mean is calculated only for the estimation sample.

The final part of the program returns two macros. The macro title is optional and defines a short description of the model that will be displayed in the output immediately above the table of parameter estimates. The macro eq is required and defines the substitutable expression that nl will use. If the expression is short, you can define it all at once. However, because the expression used here is somewhat lengthy, defining local macros and then building up the final expression from them is easier.

To verify that there are no errors in your program, you can call it directly and then use return list:

```
. use http://www.stata-press.com/data/r13/production
. nlces lnoutput capital labor
    (output omitted)
. return list
macros:
    r(title) : "CES ftn., ln Q=lnoutput, K=capital, L=labor"
    r(eq) : "lnoutput = {b0=3.711606264663641} - 1/{rho=1}*ln({delt
> a=0.5}*capital^(-1*{rho}) + (1-{delta})*labor^(-1*{rho}))"
```

The macro $\mathrm{r}(\mathrm{eq})$ contains the same substitutable expression that we specified at the command line in the preceding example, except for the initial value for b0. In short, an nl substitutable expression program should return in $r$ (eq) the same substitutable expression you would type at the command line. The only difference is that when writing a substitutable expression program, you do not bind the entire expression inside parentheses.

Having written the program, you can use it by typing

```
. nl ces: lnoutput capital labor
```

(There is a space between nl and ces.) The output is identical to that shown in example 1, save for the title defined in the function evaluator program that appears immediately above the table of parameter estimates.

## - Technical note

You will want to store nlces as an ado-file called nlces.ado. The alternative is to type the code into Stata interactively or to place the code in a do-file. While those alternatives are adequate for occasional use, if you save the program as an ado-file, you can use the function anytime you use Stata without having to redefine the program. When nl attempts to execute nlces, if the program is not in Stata's memory, Stata will search the disk(s) for an ado-file of the same name and, if found, automatically load it. All you have to do is name the file with the .ado suffix and then place it in a directory where Stata will find it. You should put the file in the directory Stata reserves for user-written ado-files, which, depending on your operating system, is $c: \backslash a d o \backslash p e r s o n a l$ (Windows), ~/ado/personal (Unix), or ~:ado:personal (Mac). See [U] 17 Ado-files.

Sometimes you may want to pass additional options to the substitutable expression program. You can modify the syntax statement of your program to accept whatever options you wish. Then when you call nl with the syntax

```
. nl func_prog: varlist, options
```

any options that are not recognized by nl (see the table of options at the beginning of this entry) are passed on to your function evaluator program. The only other restriction is that your program cannot accept an option named at because nl uses that option with function evaluator programs.

## Built-in functions

Some functions are used so often that nl has them built in so that you do not need to write them yourself. nl automatically chooses initial values for the parameters, though you can use the initial(...) option to override them.

Three alternatives are provided for exponential regression with one asymptote:

$$
\begin{array}{ll}
\exp 3 & y_{i}=\beta_{0}+\beta_{1} \beta_{2}^{x_{i}}+\epsilon_{i} \\
\exp 2 & y_{i}=\beta_{1} \beta_{2}^{x_{i}}+\epsilon_{i} \\
\exp 2 \mathrm{a} & y_{i}=\beta_{1}\left(1-\beta_{2}^{x_{i}}\right)+\epsilon_{i}
\end{array}
$$

For instance, typing nl exp3: ras dvl fits the three-parameter exponential model (parameters $\beta_{0}$, $\beta_{1}$, and $\beta_{2}$ ) using $y_{i}=$ ras and $x_{i}=\mathrm{dvl}$.

Two alternatives are provided for the logistic function (symmetric sigmoid shape; not to be confused with logistic regression):

$$
\begin{array}{ll}
\log 4 & y_{i}=\beta_{0}+\beta_{1} /\left[1+\exp \left\{-\beta_{2}\left(x_{i}-\beta_{3}\right)\right\}\right]+\epsilon_{i} \\
\log 3 & y_{i}=\beta_{1} /\left[1+\exp \left\{-\beta_{2}\left(x_{i}-\beta_{3}\right)\right\}\right]+\epsilon_{i}
\end{array}
$$

Finally, two alternatives are provided for the Gompertz function (asymmetric sigmoid shape):

$$
\begin{array}{ll}
\text { gom4 } & y_{i}=\beta_{0}+\beta_{1} \exp \left[-\exp \left\{-\beta_{2}\left(x_{i}-\beta_{3}\right)\right\}\right]+\epsilon_{i} \\
\text { gom3 } & y_{i}=\beta_{1} \exp \left[-\exp \left\{-\beta_{2}\left(x_{i}-\beta_{3}\right)\right\}\right]+\epsilon_{i}
\end{array}
$$

## Lognormal errors

A nonlinear model with errors that are independent and identically distributed normal may be written as

$$
\begin{equation*}
y_{i}=f\left(\mathbf{x}_{i}, \boldsymbol{\beta}\right)+u_{i}, \quad u_{i} \sim N\left(0, \sigma^{2}\right) \tag{3}
\end{equation*}
$$

for $i=1, \ldots, n$. If the $y_{i}$ are thought to have a $k$-shifted lognormal instead of a normal distributionthat is, $\ln \left(y_{i}-k\right) \sim N\left(\zeta_{i}, \tau^{2}\right)$, and the systematic part $f\left(\mathbf{x}_{i}, \boldsymbol{\beta}\right)$ of the original model is still thought appropriate for $y_{i}$-the model becomes

$$
\begin{equation*}
\ln \left(y_{i}-k\right)=\zeta_{i}+v_{i}=\ln \left\{f\left(\mathbf{x}_{i}, \boldsymbol{\beta}\right)-k\right\}+v_{i}, \quad v_{i} \sim N\left(0, \tau^{2}\right) \tag{4}
\end{equation*}
$$

This model is fit if lnlsq $(k)$ is specified.
If model (4) is correct, the variance of $\left(y_{i}-k\right)$ is proportional to $\left\{f\left(\mathbf{x}_{i}, \boldsymbol{\beta}\right)-k\right\}^{2}$. Probably the most common case is $k=0$, sometimes called "proportional errors" because the standard error of $y_{i}$ is proportional to its expectation, $f\left(\mathbf{x}_{i}, \boldsymbol{\beta}\right)$. Assuming that the value of $k$ is known, (4) is just another nonlinear model in $\boldsymbol{\beta}$, and it may be fit as usual. However, we may wish to compare the fit of (3) with that of (4) using the residual sum of squares (RSS) or the deviance $D, D=-2 \times \log$-likelihood, from each model. To do so, we must allow for the change in scale introduced by the log transformation.

Assuming, then, the $y_{i}$ to be normally distributed, Atkinson (1985, 85-87, 184), by considering the Jacobian $\prod\left|\partial \ln \left(y_{i}-k\right) / \partial y_{i}\right|$, showed that multiplying both sides of (4) by the geometric mean of $y_{i}-k$, $\dot{y}$, gives residuals on the same scale as those of $y_{i}$. The geometric mean is given by

$$
\dot{y}=e^{n^{-1}} \sum \ln \left(y_{i}-k\right)
$$

which is a constant for a given dataset. The residual deviance for (3) and for (4) may be expressed as

$$
\begin{equation*}
D(\widehat{\boldsymbol{\beta}})=\left\{1+\ln \left(2 \pi \widehat{\sigma}^{2}\right)\right\} n \tag{5}
\end{equation*}
$$

where $\widehat{\boldsymbol{\beta}}$ is the maximum likelihood estimate (MLE) of $\boldsymbol{\beta}$ for each model and $n \widehat{\sigma}^{2}$ is the RSS from (3), or that from (4) multiplied by $\dot{y}^{2}$.

Because (3) and (4) are models with different error structures but the same functional form, the arithmetic difference in their RSS or deviances is not easily tested for statistical significance. However, if the deviance difference is large ( $>4$, say), we would naturally prefer the model with the smaller deviance. Of course, the residuals for each model should be examined for departures from assumptions (nonconstant variance, nonnormality, serial correlations, etc.) in the usual way.

Alternatively, consider modeling

$$
\begin{gather*}
E\left(y_{i}\right)=1 /\left(C+A e^{B x_{i}}\right)  \tag{6}\\
E\left(1 / y_{i}\right)=E\left(y_{i}^{\prime}\right)=C+A e^{B x_{i}} \tag{7}
\end{gather*}
$$

where $C, A$, and $B$ are parameters to be estimated. Using the data $(y, x)=(0.04,5),(0.06,12)$, $(0.08,25),(0.1,35),(0.15,42),(0.2,48),(0.25,60),(0.3,75)$, and $(0.5,120)$ (Danuso 1991), fitting the models yields

| Model | $C$ | $A$ | $B$ | RSS | Deviance |
| :--- | ---: | ---: | ---: | ---: | ---: |
| (6) | 1.781 | 25.74 | -0.03926 | -0.001640 | -51.95 |
| (6) with lnlsq(0) | 1.799 | 25.45 | -0.04051 | -0.001431 | -53.18 |
| (7) | 1.781 | 25.74 | -0.03926 | 8.197 | 24.70 |
| (7) with lnlsq(0) | 1.799 | 27.45 | -0.04051 | 3.651 | 17.42 |

There is little to choose between the two versions of the logistic model (6), whereas for the exponential model (7), the fit using lnlsq(0) is much better (a deviance difference of 7.28). The reciprocal transformation has introduced heteroskedasticity into $y_{i}^{\prime}$, which is countered by the proportional errors property of the lognormal distribution implicit in lnlsq(0). The deviances are not comparable between the logistic and exponential models because the change of scale has not been allowed for, although in principle it could be.

## Other uses

Even if you are fitting linear regression models, you may find that nl can save you some typing. Because you specify the parameters of your model explicitly, you can impose constraints on them directly.
$>$ Example 2
In example 2 of $[\mathrm{R}]$ ensreg, we showed how to fit the model

$$
\begin{aligned}
\operatorname{mpg}= & \beta_{0}+\beta_{1} \text { price }+\beta_{2} \text { weight }+\beta_{3} \text { displ }+\beta_{4} \text { gear_ratio }+\beta_{5} \text { foreign }+ \\
& \beta_{6} \text { length }+u
\end{aligned}
$$

subject to the constraints

$$
\begin{aligned}
& \beta_{1}=\beta_{2}=\beta_{3}=\beta_{6} \\
& \beta_{4}=-\beta_{5}=\beta_{0} / 20
\end{aligned}
$$

An alternative way is to use nl :


The point estimates and standard errors for $\beta_{0}$ and $\beta_{1}$ are identical to those reported in example 2 of $[\mathrm{R}]$ ensreg. To get the estimate for $\beta_{4}$, we can use nlcom:

```
. nlcom _b[/b0]/20
    _nl_1: _b[/b0]/20
```

| mpg | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| _nl_1 | 1.326114 | .0687589 | 19.29 | 0.000 | 1.191349 | 1.460879 |

The advantage to using nl is that we do not need to use the constraint command six times.
nl is also a useful tool when doing exploratory data analysis. For example, you may want to run a regression of $y$ on a function of $x$, though you have not decided whether to use $\operatorname{sqrt}(\mathrm{x})$ or $\ln (\mathrm{x})$. You can use $n l$ to run both regressions without having first to generate two new variables:

```
. nl (y = {b0} + {b1}*ln(x))
. nl (y = {b0} + {b1}*sqrt(x))
```

Poi (2008) shows the advantages of using nl when marginal effects of transformed variables are desired as well.

## Weights

Weights are specified in the usual way-analytic and frequency weights as well as iweights are supported; see [U] 20.23 Weighted estimation. Use of analytic weights implies that the $y_{i}$ have different variances. Therefore, model (3) may be rewritten as

$$
\begin{equation*}
y_{i}=f\left(\mathbf{x}_{i}, \boldsymbol{\beta}\right)+u_{i}, \quad u_{i} \sim N\left(0, \sigma^{2} / w_{i}\right) \tag{3a}
\end{equation*}
$$

where $w_{i}$ are (positive) weights, assumed to be known and normalized such that their sum equals the number of observations. The residual deviance for ( $3 a$ ) is

$$
\begin{equation*}
D(\widehat{\boldsymbol{\beta}})=\left\{1+\ln \left(2 \pi \widehat{\sigma}^{2}\right)\right\} n-\sum \ln \left(w_{i}\right) \tag{5a}
\end{equation*}
$$

[compare with (5)], where

$$
n \widehat{\sigma}^{2}=\operatorname{RSS}=\sum w_{i}\left\{y_{i}-f\left(\mathbf{x}_{i}, \widehat{\boldsymbol{\beta}}\right)\right\}^{2}
$$

Defining and fitting a model equivalent to (4) when weights have been specified as in (3a) is not straightforward and has not been attempted. Thus deviances using and not using the lnlsq() option may not be strictly comparable when analytic weights (other than 0 and 1) are used.

You do not need to modify your substitutable expression in any way to use weights. If, however, you write a substitutable expression program, then you should account for weights when obtaining initial values. When $n l$ calls your program, it passes whatever weight expression (if any) was specified by the user. Here is an outline of a substitutable expression program that accepts weights:

```
program nl name, rclass
    version 13
    syntax varlist [aw fw iw] if
    // Obtain initial values allowing weights
    // Use the syntax ['weight''exp']. For example,
    summarize varname ['weight''exp'] 'if'
    regress depvar varlist ['weight''exp'] 'if'
    // Return substitutable expression
    return local eq "substitutable expression"
    return local title "description of estimator"
end
```

For details on how the syntax command processes weight expressions, see [P] syntax.

## Potential errors

nl is reasonably robust to the inability of your nonlinear function to be evaluated at some parameter values. nl does assume that your function can be evaluated at the initial values of the parameters. If your function cannot be evaluated at the initial values, an error message is issued with return code 480. Recall that if you do not specify an initial value for a parameter, then nl initializes it to zero. Many nonlinear functions cannot be evaluated when some parameters are zero, so in those cases specifying alternative initial values is crucial.

Thereafter, as nl changes the parameter values, it monitors your function for unexpected missing values. If these are detected, nl backs up. That is, nl finds a point between the previous, known-to-be-good parameter vector and the new, known-to-be-bad vector at which the function can be evaluated and continues its iterations from that point.
nl requires that once a parameter vector is found where the predictions can be calculated, small changes to the parameter vector be made to calculate numeric derivatives. If a boundary is encountered at this point, an error message is issued with return code 481.

When specifying lnlsq(), an attempt to take logarithms of $y_{i}-k$ when $y_{i} \leq k$ results in an error message with return code 482.

If iterate() iterations are performed and estimates still have not converged, results are presented with a warning, and the return code is set to 430 .

If you use the programmed substitutable expression version of $n l$ with a function evaluator program, or vice versa, Stata issues an error message. Verify that you are using the syntax appropriate for the program you have.

## General comments on fitting nonlinear models

Achieving convergence is often problematic. For example, a unique minimum of the sum-ofsquares function may not exist. Much literature exists on different algorithms that have been used, on strategies for obtaining good initial parameter values, and on tricks for parameterizing the model to make its behavior as linear-like as possible. Selected references are Kennedy and Gentle (1980, chap. 10) for computational matters and Ross (1990) and Ratkowsky (1983) for all three aspects. Ratkowsky's book is particularly clear and approachable, with useful discussion on the meaning and practical implications of intrinsic and parameter-effects nonlinearity. An excellent text on nonlinear estimation is Gallant (1987). Also see Davidson and MacKinnon (1993 and 2004).

To enhance the success of nl , pay attention to the form of the model fit, along the lines of Ratkowsky and Ross. For example, Ratkowsky (1983, 49-59) analyzes three possible three-parameter yield-density models for plant growth:

$$
E\left(y_{i}\right)=\left\{\begin{array}{l}
\left(\alpha+\beta x_{i}\right)^{-1 / \theta} \\
\left(\alpha+\beta x_{i}+\gamma x_{i}^{2}\right)^{-1} \\
\left(\alpha+\beta x_{i}^{\phi}\right)^{-1}
\end{array}\right.
$$

All three models give similar fits. However, he shows that the second formulation is dramatically more linear-like than the other two and therefore has better convergence properties. In addition, the parameter estimates are virtually unbiased and normally distributed, and the asymptotic approximation to the standard errors, correlations, and confidence intervals is much more accurate than for the other models. Even within a given model, the way the parameters are expressed (for example, $\phi^{x_{i}}$ or $e^{\theta x_{i}}$ ) affects the degree of linearity and convergence behavior.

## Function evaluator programs

Occasionally, a nonlinear function may be so complex that writing a substitutable expression for it is impractical. For example, there could be many parameters in the model. Alternatively, if you are implementing a two-step estimator, writing a substitutable expression may be altogether impossible. Function evaluator programs can be used in these situations.
nl will pass to your function evaluator program a list of variables, a weight expression, a variable marking the estimation sample, and a vector of parameters. Your program is to replace the dependent variable, which is the first variable in the variables list, with the values of the nonlinear function evaluated at those parameters. As with substitutable expression programs, the first two letters of the name must be nl.

To focus on the mechanics of the function evaluator program, again let's compare the CES production function to the previous examples. The function evaluator program is

```
program nlces2
    version 13
    syntax varlist(min=3 max=3) if, at(name)
    local logout : word 1 of 'varlist'
    local capital : word 2 of 'varlist'
    local labor : word 3 of 'varlist'
    // Retrieve parameters out of at matrix
    tempname b0 rho delta
    scalar 'b0' = 'at'[1, 1]
    scalar 'rho' = 'at'[1, 2]
    scalar 'delta' = 'at'[1, 3]
    tempvar kterm lterm
    generate double 'kterm' = 'delta'*'capital'^(-1*'rho') 'if,
    generate double 'lterm' = (1-'delta')*'labor'`(-1*'rho') 'if'
    // Fill in dependent variable
    replace 'logout' = 'b0' - 1/'rho'*ln('kterm' + 'lterm') 'if'
end
```

Unlike the previous nlces program, this one is not declared to be r-class. The syntax statement again accepts three variables: one for log output, one for capital, and one for labor. An if exp is again required because $n l$ will pass a binary variable marking the estimation sample. All function evaluator programs must accept an option named at () that takes a name as an argument-that is how nl passes the parameter vector to your program.

The next part of the program retrieves the output, labor, and capital variables from the variables list. It then breaks up the temporary matrix at and retrieves the parameters b0, rho, and delta. Pay careful attention to the order in which the parameters refer to the columns of the at matrix because that will affect the syntax you use with nl . The temporary names you use inside this program are immaterial, however.

The rest of the program computes the nonlinear function, using some temporary variables to hold intermediate results. The final line of the program then replaces the dependent variable with the values of the function. Notice the use of 'if' to restrict attention to the estimation sample. nl makes a copy of your dependent variable so that when the command is finished your data are left unchanged.

To use the program and fit your model, you type

```
. use http://www.stata-press.com/data/r13/production, clear
. nl ces2 @ lnoutput capital labor, parameters(b0 rho delta)
> initial(b0 0 rho 1 delta 0.5)
```

The output is again identical to that shown in example 1. The order in which the parameters were specified in the parameters () option is the same in which they are retrieved from the at matrix in the program. To initialize them, you simply list the parameter name, a space, the initial value, and so on.

If you use the nparameters() option instead of the parameters() option, the parameters are named $\mathrm{b} 1, \mathrm{~b} 2, \ldots, \mathrm{~b} k$, where $k$ is the number of parameters. Thus you could have typed

```
. nl ces2 @ lnoutput capital labor, nparameters(3) initial(b1 0 b2 1 b3 0.5)
```

With that syntax, the parameters called b0, rho, and delta in the program will be labeled b1, b2, and b3, respectively. In programming situations or if there are many parameters, instead of listing the parameter names and initial values in the initial() option, you may find it more convenient to pass a column vector. In those cases, you could type

```
. matrix myvals = (0, 1, 0.5)
. nl ces2 @ lnoutput capital labor, nparameters(3) initial(myvals)
```

In summary, a function evaluator program receives a list of variables, the first of which is the dependent variable that you are to replace with the values of your nonlinear function. Additionally, it must accept an if exp, as well as an option named at that will contain the vector of parameters at which nl wants the function evaluated. You are then free to do whatever is necessary to evaluate your function and replace the dependent variable.

If you wish to use weights, your function evaluator program's syntax statement must accept them. If your program consists only of, for example, generate statements, you need not do anything with the weights passed to your program. However, if in calculating the nonlinear function you use commands such as summarize or regress, then you will want to use the weights with those commands.

As with substitutable expression programs, nl will pass to it any options specified that nl does not accept, providing you with a way to pass more information to your function.

## - Technical note

Before version 9 of Stata, the nl command used a different syntax, which required you to write an nlfcn program, and it did not have a syntax for interactive use other than the seven functions that were built-in. The old syntax of nl still works, and you can still use those nlfcn programs. If nl does not see a colon, an at sign, or a set of parentheses surrounding the equation in your command, it assumes that the old syntax is being used.

The current version of nl uses scalars and matrices to store intermediate calculations instead of local and global macros as the old version did, so the current version produces more accurate results. In practice, however, any discrepancies are likely to be small.

## Stored results

nl stores the following in e() :
Scalars
e(N)
e(k)
e(k_eq_model)
$e\left(d f \_m\right)$
e(df_r)
e(df_t)
e(mss)
e(rss)
e(tss)
e(mms)
$e(\mathrm{msr})$
e(11)
e(r2)
e(r2_a)
e(rmse)
e(dev)
e(N_clust)
$e($ lnlsq)
e(log_t)
e(gm_2)
$e(c j)$
$e$ (delta)
e(rank)
$e(i c)$
e(converge)
Macros
e(cmd)
e(cmdline)
e(depvar)
e(wtype)
e(wexp)
e(title)
e(title_2)
e(clustvar)
e(hac_kernel)
e(hac_lag)
e(vce)
e(vcetype)
e(type)
e(sexp)
e(params)
$e$ (funcprog)
e(rhs)
e(properties)
e(predict)
$e$ (marginsnotok)
Matrices
e(b)
$e$ (init)
e(V)
Functions

$$
\text { e(sample) } \quad \text { marks estimation sample }
$$

## Methods and formulas

The derivation here is based on Davidson and MacKinnon (2004, chap. 6). Let $\boldsymbol{\beta}$ denote the $k \times 1$ vector of parameters, and write the regression function using matrix notation as $\mathbf{y}=\mathbf{f}(\mathbf{x}, \boldsymbol{\beta})+\mathbf{u}$ so that the objective function can be written as

$$
\operatorname{SSR}(\boldsymbol{\beta})=\{\mathbf{y}-\mathbf{f}(\mathbf{x}, \boldsymbol{\beta})\}^{\prime} \mathbf{D}\{\mathbf{y}-\mathbf{f}(\mathbf{x}, \boldsymbol{\beta})\}
$$

The $\mathbf{D}$ matrix contains the weights and is defined in [R] regress; if no weights are specified, then $\mathbf{D}$ is the $N \times N$ identity matrix. Taking a second-order Taylor series expansion centered at $\boldsymbol{\beta}_{0}$ yields

$$
\begin{equation*}
\operatorname{SSR}(\boldsymbol{\beta}) \approx \operatorname{sSR}\left(\boldsymbol{\beta}_{0}\right)+\mathbf{g}^{\prime}\left(\boldsymbol{\beta}_{0}\right)\left(\boldsymbol{\beta}-\boldsymbol{\beta}_{0}\right)+\frac{1}{2}\left(\boldsymbol{\beta}-\boldsymbol{\beta}_{0}\right)^{\prime} \mathbf{H}\left(\boldsymbol{\beta}_{0}\right)\left(\boldsymbol{\beta}-\boldsymbol{\beta}_{0}\right) \tag{8}
\end{equation*}
$$

where $\mathbf{g}\left(\boldsymbol{\beta}_{0}\right)$ denotes the $k \times 1$ gradient of $\operatorname{SSR}(\boldsymbol{\beta})$ evaluated at $\boldsymbol{\beta}_{0}$ and $\mathbf{H}\left(\boldsymbol{\beta}_{0}\right)$ denotes the $k \times k$ Hessian of $\operatorname{SSR}(\boldsymbol{\beta})$ evaluated at $\boldsymbol{\beta}_{0}$. Letting $\mathbf{X}$ denote the $N \times k$ matrix of derivatives of $\mathbf{f}(\mathbf{x}, \boldsymbol{\beta})$ with respect to $\boldsymbol{\beta}$, the gradient $\mathbf{g}(\boldsymbol{\beta})$ is

$$
\begin{equation*}
\mathbf{g}(\boldsymbol{\beta})=-2 \mathbf{X}^{\prime} \mathbf{D u} \tag{9}
\end{equation*}
$$

$\mathbf{X}$ and $\mathbf{u}$ are obviously functions of $\boldsymbol{\beta}$, though for notational simplicity that dependence is not shown explicitly. The $(m, n)$ element of the Hessian can be written as

$$
\begin{equation*}
H_{m n}(\boldsymbol{\beta})=-2 \sum_{i=1}^{i=N} d_{i i}\left[\frac{\partial^{2} f_{i}}{\partial \beta_{m} \partial \beta_{n}} u_{i}-X_{i m} X_{i n}\right] \tag{10}
\end{equation*}
$$

where $d_{i i}$ is the $i$ th diagonal element of $\mathbf{D}$. As discussed in Davidson and MacKinnon (2004, chap. 6), the first term inside the brackets of (10) has expectation zero, so the Hessian can be approximated as

$$
\begin{equation*}
\mathbf{H}(\boldsymbol{\beta})=2 \mathbf{X}^{\prime} \mathbf{D} \mathbf{X} \tag{11}
\end{equation*}
$$

Differentiating the Taylor series expansion of $\operatorname{SSR}(\boldsymbol{\beta})$ shown in (8) yields the first-order condition for a minimum

$$
\mathbf{g}\left(\boldsymbol{\beta}_{0}\right)+\mathbf{H}\left(\boldsymbol{\beta}_{0}\right)\left(\boldsymbol{\beta}-\boldsymbol{\beta}_{0}\right)=\mathbf{0}
$$

which suggests the iterative procedure

$$
\begin{equation*}
\boldsymbol{\beta}_{j+1}=\boldsymbol{\beta}_{j}-\alpha \mathbf{H}^{-1}\left(\boldsymbol{\beta}_{j}\right) \mathbf{g}\left(\boldsymbol{\beta}_{j}\right) \tag{12}
\end{equation*}
$$

where $\alpha$ is a "step size" parameter chosen at each iteration to improve convergence. Using (9) and (11), we can write (12) as

$$
\begin{equation*}
\boldsymbol{\beta}_{j+1}=\boldsymbol{\beta}_{j}+\alpha\left(\mathbf{X}^{\prime} \mathbf{D} \mathbf{X}\right)^{-1} \mathbf{X}^{\prime} \mathbf{D} \mathbf{u} \tag{13}
\end{equation*}
$$

where $\mathbf{X}$ and $\mathbf{u}$ are evaluated at $\boldsymbol{\beta}_{j}$. Apart from the scalar $\alpha$, the second term on the right-hand side of (13) can be computed via a (weighted) regression of the columns of $\mathbf{X}$ on the errors. nl computes the derivatives numerically and then calls regress. At each iteration, $\alpha$ is set to one, and a candidate value $\boldsymbol{\beta}_{j+1}^{*}$ is computed by (13). If $\operatorname{SSR}\left(\boldsymbol{\beta}_{j+1}^{*}\right)<\operatorname{SSR}\left(\boldsymbol{\beta}_{j}\right)$, then $\boldsymbol{\beta}_{j+1}=\boldsymbol{\beta}_{j+1}^{*}$ and the iteration is complete. Otherwise, $\alpha$ is halved, a new $\boldsymbol{\beta}_{j+1}^{*}$ is calculated, and the process is repeated. Convergence is declared when $\alpha\left|\beta_{j+1, m}\right| \leq \epsilon\left(\left|\beta_{j m}\right|+\tau\right)$ for all $m=1, \ldots, k$.nl uses $\tau=10^{-3}$ and, by default, $\epsilon=10^{-5}$, though you can specify an alternative value of $\epsilon$ with the eps() option.

As derived, for example, in Davidson and MacKinnon (2004, chap. 6), an expedient way to obtain the covariance matrix is to compute $\mathbf{u}$ and the columns of $\mathbf{X}$ at the final estimate $\widehat{\boldsymbol{\beta}}$ and then regress that $\mathbf{u}$ on $\mathbf{X}$. The covariance matrix of the estimated parameters of that regression serves as an estimate of $\operatorname{Var}(\widehat{\boldsymbol{\beta}})$. If that regression employs a robust covariance matrix estimator, then the covariance matrix for the parameters of the nonlinear regression will also be robust.

All other statistics are calculated analogously to those in linear regression, except that the nonlinear function $f\left(\mathbf{x}_{i}, \boldsymbol{\beta}\right)$ plays the role of the linear function $\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}$. See $[\mathrm{R}]$ regress.

This command supports estimation with survey data. For details on VCEs with survey data, see [SVY] variance estimation.

## Acknowledgments

The original version of nl was written by Patrick Royston (1992) of the MRC Clinical Trials Unit, London, and coauthor of the Stata Press book Flexible Parametric Survival Analysis Using Stata: Beyond the Cox Model. Francesco Danuso's menu-driven nonlinear regression program (1991) provided the inspiration.

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## Also see

[R] nl postestimation - Postestimation tools for nl
[R] gmm - Generalized method of moments estimation
[R] ml - Maximum likelihood estimation
[R] mlexp - Maximum likelihood estimation of user-specified expressions
[R] nlcom - Nonlinear combinations of estimators
[R] nlsur - Estimation of nonlinear systems of equations
$[\mathrm{R}]$ regress - Linear regression
[SVY] svy estimation - Estimation commands for survey data
[U] 20 Estimation and postestimation commands

## Title

nl postestimation - Postestimation tools for nl

| Description | Syntax for predict $\quad$ Menu for predict $\quad$ Options for predict |
| :--- | :--- |
| Remarks and examples | Also see |

## Description

The following postestimation commands are available after nl:

| Command | Description |
| :---: | :---: |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample |
| estat vce | variance-covariance matrix of the estimators (VCE) |
| estat (svy) | postestimation statistics for survey data |
| estimates | cataloging estimation results |
| forecast ${ }^{1}$ | dynamic forecasts and simulations |
| lincom | point estimates, standard errors, testing, and inference for linear combinations of coefficients |
| lrtest ${ }^{2}$ | likelihood-ratio test |
| margins ${ }^{3}$ | marginal means, predictive margins, marginal effects, and average marginal effects |
| marginsplot | graph the results from margins (profile plots, interaction plots, etc.) |
| nlcom | point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients |
| predict | predictions and residuals |
| predictnl | point estimates, standard errors, testing, and inference for generalized predictions |
| test | Wald tests of simple and composite linear hypotheses |
| testnl | Wald tests of nonlinear hypotheses |

[^52]
## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic]
predict [type] {stub*|newvar 
```

where $k$ is the number of parameters in the model.
statistic Description
Main
yhat fitted values; the default
residuals residuals
$\underline{\operatorname{pr}}(a, b) \quad \operatorname{Pr}\left(y_{j} \mid a<y_{j}<b\right)$
$\mathrm{e}(a, b) \quad E\left(y_{j} \mid a<y_{j}<b\right)$
ystar $(a, b) \quad E\left(y_{j}^{*}\right), y_{j}^{*}=\max \left\{a, \min \left(y_{j}, b\right)\right\}$
These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
yhat, the default, calculates the fitted values.
residuals calculates the residuals.
$\operatorname{pr}(a, b)$ calculates $\operatorname{Pr}\left(a<\mathbf{x}_{j} \mathbf{b}+u_{j}<b\right)$, the probability that $y_{j} \mid \mathbf{x}_{j}$ would be observed in the interval $(a, b)$.
$a$ and $b$ may be specified as numbers or variable names; $l b$ and $u b$ are variable names;
$\operatorname{pr}(20,30)$ calculates $\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$;
$\operatorname{pr}(l b, u b)$ calculates $\operatorname{Pr}\left(l b<\mathbf{x}_{j} \mathbf{b}+u_{j}<u b\right)$; and
$\operatorname{pr}(20, u b)$ calculates $\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{j}<u b\right)$.
$a$ missing ( $a \geq$. ) means $-\infty$; $\operatorname{pr}(., 30)$ calculates $\operatorname{Pr}\left(-\infty<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$;
$\operatorname{pr}(l b, 30)$ calculates $\operatorname{Pr}\left(-\infty<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$ in observations for which $l b \geq$.
and calculates $\operatorname{Pr}\left(l b<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$ elsewhere.
$b$ missing ( $b \geq$. ) means $+\infty$; $\operatorname{pr}(20,$.$) calculates \operatorname{Pr}\left(+\infty>\mathbf{x}_{j} \mathbf{b}+u_{j}>20\right)$;
$\operatorname{pr}(20, u b)$ calculates $\operatorname{Pr}\left(+\infty>\mathbf{x}_{j} \mathbf{b}+u_{j}>20\right)$ in observations for which $u b \geq$.
and calculates $\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{j}<u b\right)$ elsewhere.
$\mathrm{e}(a, b)$ calculates $E\left(\mathbf{x}_{j} \mathbf{b}+u_{j} \mid a<\mathbf{x}_{j} \mathbf{b}+u_{j}<b\right)$, the expected value of $y_{j} \mid \mathbf{x}_{j}$ conditional on $y_{j} \mid \mathbf{x}_{j}$ being in the interval $(a, b)$, meaning that $y_{j} \mid \mathbf{x}_{j}$ is truncated. $a$ and $b$ are specified as they are for pr() .
ystar $(a, b)$ calculates $E\left(y_{j}^{*}\right)$, where $y_{j}^{*}=a$ if $\mathbf{x}_{j} \mathbf{b}+u_{j} \leq a, y_{j}^{*}=b$ if $\mathbf{x}_{j} \mathbf{b}+u_{j} \geq b$, and $y_{j}^{*}=\mathbf{x}_{j} \mathbf{b}+u_{j}$ otherwise, meaning that $y_{j}^{*}$ is censored. $a$ and $b$ are specified as they are for $\operatorname{pr}()$.
scores calculates the scores. The $j$ th new variable created will contain the score for the $j$ th parameter in $e(b)$.

## Remarks and examples

## Example 1

Obtaining predictions after fitting a nonlinear regression model with nl is no more difficult than obtaining predictions after fitting a linear regression model with regress. Here we fit a model of mpg on weight, allowing for a nonlinear relationship:

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. nl (mpg = {b0} + {b1}*weight`{gamma=-.5}), variables(weight) nolog
(obs = 74)
```

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | 1646.43761 | 2 | 823.218806 |
| Residual | 797.021847 | 71 | 11.2256598 |
| Total | 2443.45946 | 73 | 33.4720474 |

                                    Number of obs = 74
                                    R -squared \(=0.6738\)
                                    Adj \(R\)-squared \(=0.6646\)
                                    Root MSE \(=3.350472\)
                                    Res. dev. \(=385.8874\)
    | mpg | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :--- | :---: | :---: | ---: | ---: |
| /b0 | -18.17583 | 60.61713 | -0.30 | 0.765 | -139.0429 | 102.6913 |
| /b1 | 1377.267 | 5292.443 | 0.26 | 0.795 | -9175.564 | 11930.1 |
| /gamma | -.4460916 | .6763643 | -0.66 | 0.512 | -1.794724 | .9025405 |

Parameter b0 taken as constant term in model \& ANOVA table
Now we obtain the predicted values of mpg and plot them in a graph along with the observed values:

```
. predict mpghat
(option yhat assumed; fitted values)
. scatter mpg weight || line mpghat weight, sort
```



Suppose we wanted to know how sensitive mpg is to changes in weight for cars that weigh 3,000 pounds. We can use margins to find out:


With the eyex () option, margins reports elasticities. These results show that if we increase weight by $1 \%$, then mpg decreases by about $0.84 \%$.

## - Technical note

Observant readers will notice that margins issued a warning message stating that it could not perform its usual check for estimable functions. In the case of nl , as long as you do not specify the predict() option of margins or specify the default predict (yhat), you can safely ignore that message. The predicted values that nl produces are suitable for use with margins. However, if you specify any predict() options other than yhat, then the output from margins after using nl will not be correct.

## Also see

[R] nl - Nonlinear least-squares estimation
[U] 20 Estimation and postestimation commands

## Title

nlcom - Nonlinear combinations of estimators
Syntax
Remarks and examples

Menu
Stored results

Description
Methods and formulas

Options
References

## Syntax

Nonlinear combination of estimators-one expression
nlcom [name: $] \exp [$, options]

Nonlinear combinations of estimators-more than one expression
nlcom ([name: $] \exp$ ) [([name: $] \exp [$, options $]$
options
level(\#)
iterate(\#)
post
display_options
noheader
df(\#)

Description
set confidence level; default is level (95)
maximum number of iterations
post estimation results
control column formats and line width
suppress output header
use $t$ distribution with \# degrees of freedom for computing $p$-values and confidence intervals
noheader and $\mathrm{df}(\#)$ do not appear in the dialog box.

The second syntax means that if more than one expression is specified, each must be surrounded by parentheses. The optional name is any valid Stata name and labels the transformations.
$\exp$ is a possibly nonlinear expression containing

$$
\begin{aligned}
& \mathrm{-b}[\text { coef }] \\
& \mathrm{-b}[e q n o: \text { coef }] \\
& {[\text { eqno }] \text { coef }} \\
& {[e q n o] \_\mathrm{b}[\text { coef }]}
\end{aligned}
$$

eqno is

$$
\begin{aligned}
& \text { \#\# } \\
& \text { name }
\end{aligned}
$$

coef identifies a coefficient in the model. coef is typically a variable name, a level indicator, an interaction indicator, or an interaction involving continuous variables. Level indicators identify one level of a factor variable and interaction indicators identify one combination of levels of an interaction; see [U] 11.4.3 Factor variables. coef may contain time-series operators; see [U] 11.4.4 Time-series varlists.

Distinguish between [], which are to be typed, and [], which indicate optional arguments.

## Menu

Statistics $>$ Postestimation $>$ Nonlinear combinations of estimates

## Description

nlcom computes point estimates, standard errors, test statistics, significance levels, and confidence intervals for (possibly) nonlinear combinations of parameter estimates after any Stata estimation command. Results are displayed in the usual table format used for displaying estimation results. Calculations are based on the "delta method", an approximation appropriate in large samples.
nlcom can be used with svy estimation results; see [SVY] svy postestimation.

## Options

level(\#) specifies the confidence level, as a percentage, for confidence intervals. The default is level (95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals.
iterate(\#) specifies the maximum number of iterations used to find the optimal step size in calculating numerical derivatives of the transformation(s) with respect to the original parameters. By default, the maximum number of iterations is 100 , but convergence is usually achieved after only a few iterations. You should rarely have to use this option.
post causes nlcom to behave like a Stata estimation (eclass) command. When post is specified, nlcom will post the vector of transformed estimators and its estimated variance-covariance matrix to e(). This option, in essence, makes the transformation permanent. Thus you could, after posting, treat the transformed estimation results in the same way as you would treat results from other Stata estimation commands. For example, after posting, you could redisplay the results by typing nlcom without any arguments, or use test to perform simultaneous tests of hypotheses on linear combinations of the transformed estimators; see [R] test.

Specifying post clears out the previous estimation results, which can be recovered only by refitting the original model or by storing the estimation results before running nlcom and then restoring them; see $[R]$ estimates store.
display_options: cformat (\%fmt), pformat (\%fmt), sformat (\% fmt), and nolstretch; see [R] estimation options.

The following options are available with nlcom but are not shown in the dialog box:
noheader suppresses the output header.
$\mathrm{df}(\#)$ specifies that the $t$ distribution with \# degrees of freedom be used for computing $p$-values and confidence intervals.

## Remarks and examples

Remarks are presented under the following headings:

[^53]
## Introduction

nlcom and predictnl both use the delta method. They take nonlinear transformations of the estimated parameter vector from some fitted model and apply the delta method to calculate the variance, standard error, Wald test statistic, etc., of the transformations. nlcom is designed for functions of the parameters, and predictnl is designed for functions of the parameters and of the data, that is, for predictions.
nlcom generalizes lincom (see [R] lincom) in two ways. First, nlcom allows the transformations to be nonlinear. Second, nlcom can be used to simultaneously estimate many transformations (whether linear or nonlinear) and to obtain the estimated variance-covariance matrix of these transformations.

## Basics

In [R] lincom, the following regression was performed:
. use http://www.stata-press.com/data/r13/regress
. regress y x1 x2 x3

| Source | SS | df | MS |
| ---: | ---: | ---: | :---: |
| Model | 3259.3561 | 3 | 1086.45203 |
| Residual | 1627.56282 | 144 | 11.3025196 |
| Total | 4886.91892 | 147 | 33.2443464 |


| Number of obs | $=$ | 148 |
| :--- | ---: | ---: |
| $\mathrm{~F}(3,144)$ | $=$ | 96.12 |
| Prob $>$ F | $=$ | 0.0000 |
| R-squared | $=$ | 0.6670 |
| Adj R-squared | $=$ | 0.6600 |
| Root MSE | $=3.3619$ |  |


| y | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| x 1 | 1.457113 | 1.07461 | 1.36 | 0.177 | -.666934 | 3.581161 |
| x 2 | 2.221682 | .8610358 | 2.58 | 0.011 | .5197797 | 3.923583 |
| x3 | -.006139 | .0005543 | -11.08 | 0.000 | -.0072345 | -.0050435 |
| _cons | 36.10135 | 4.382693 | 8.24 | 0.000 | 27.43863 | 44.76407 |

Then lincom was used to estimate the difference between the coefficients of x 1 and x 2 :
. lincom _b[x2] - _b[x1]
( 1) $-\mathrm{x} 1+\mathrm{x} 2=0$

| y | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $(1)$ | .7645682 | .9950282 | 0.77 | 0.444 | -1.20218 | 2.731316 |

It was noted, however, that nonlinear expressions are not allowed with lincom:

```
. lincom _b[x2]/_b[x1]
not possible with test
r(131);
```

Nonlinear transformations are instead estimated using nlcom:

```
. nlcom _b[x2]/_b[x1]
```

    _nl_1: _b[x2]/_b[x1]
    | $y$ | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| _nl_1 | 1.524714 | .9812848 | 1.55 | 0.120 | -.3985686 | 3.447997 |

- Technical note

The notation _b [name] is the standard way in Stata to refer to regression coefficients; see [U] 13.5 Accessing coefficients and standard errors. Some commands, such as lincom and test, allow you to drop the $\_\mathrm{b}[]$ and just refer to the coefficients by name. nlcom, however, requires the full specification _b [name].

Returning to our linear regression example, nlcom also allows simultaneous estimation of more than one combination:

```
. nlcom (_b[x2]/_b[x1]) (_b[x3]/_b[x1]) (_b[x3]/_b[x2])
        _nl_1: _b[x2]/_b[x1]
        _nl_2: _b[x3]/_b[x1]
        _nl_3: _b[x3]/_b[x2]
```

| y | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| _nl_1 | 1.524714 | .9812848 | 1.55 | 0.120 | -.3985686 | 3.447997 |
| _nl_2 | -.0042131 | .0033483 | -1.26 | 0.208 | -.0107756 | .0023494 |
| _nl_3 | -.0027632 | .0010695 | -2.58 | 0.010 | -.0048594 | -.000667 |

We can also label the transformations to produce more informative names in the estimation table:

```
. nlcom (ratio21:_b[x2]/_b[x1]) (ratio31:_b[x3]/_b[x1]) (ratio32:_b[x3]/_b[x2])
    ratio21: _b[x2]/_b[x1]
    ratio31: _b[x3]/_b[x1]
    ratio32: _b[x3]/_b[x2]
```

| $y$ | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| ratio21 | 1.524714 | .9812848 | 1.55 | 0.120 | -.3985686 | 3.447997 |
| ratio31 | -.0042131 | .0033483 | -1.26 | 0.208 | -.0107756 | .0023494 |
| ratio32 | -.0027632 | .0010695 | -2.58 | 0.010 | -.0048594 | -.000667 |

nlcom stores the vector of estimated combinations and its estimated variance-covariance matrix in $r()$.

```
. matrix list r(b)
r(b)[1,3]
    ratio21 ratio31 ratio32
c1 1.5247143 -.00421315 -. 00276324
. matrix list r(V)
symmetric r(V)[3,3]
                ratio21 ratio31 ratio32
ratio21 . }9629198
ratio31 -.00287781 . 00001121
ratio32 -.00014234 2.137e-06 1.144e-06
```


## Using the post option

When used with the post option, nlcom stores the estimation vector and variance-covariance matrix in e() , making the transformation permanent:

```
. quietly nlcom (ratio21:_b[x2]/_b[x1]) (ratio31:_b[x3]/_b[x1])
> (ratio32:_b[x3]/_b[x2]), post
. matrix list e(b)
```

$e(b)[1,3]$
ratio21 ratio31 ratio32
y1 $1.5247143-.00421315-.00276324$
. matrix list e(V)
symmetric e(V) $[3,3]$
ratio21 ratio31 ratio32
ratio21 . 96291982
ratio31 -. 00287781 . 00001121
ratio32 -. $00014234 \quad 2.137 \mathrm{e}-06 \quad 1.144 \mathrm{e}-06$

After posting, we can proceed as if we had just run a Stata estimation (eclass) command. For instance, we can replay the results,
. nlcom

| $y$ | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| ratio21 | 1.524714 | .9812848 | 1.55 | 0.120 | -.3985686 | 3.447997 |
| ratio31 | -.0042131 | .0033483 | -1.26 | 0.208 | -.0107756 | .0023494 |
| ratio32 | -.0027632 | .0010695 | -2.58 | 0.010 | -.0048594 | -.000667 |

or perform other postestimation tasks in the transformed metric, this time making reference to the new "coefficients":

```
. display _b[ratio31]
-.00421315
. estat vce, correlation
Correlation matrix of coefficients of nlcom model
\begin{tabular}{r|rrr}
\(e(V)\) & ratio21 & ratio31 & ratio32 \\
\hline ratio21 & 1.0000 & & \\
ratio31 & -0.8759 & 1.0000 & \\
ratio32 & -0.1356 & 0.5969 & 1.0000
\end{tabular}
. test _b[ratio21] = 1
( 1) ratio21 = 1
                chi2( 1) = 0.29
            Prob > chi2 = 0.5928
```

We see that testing $\_\mathrm{b}[$ ratio21] $=1$ in the transformed metric is equivalent to testing using testnl _b $[\mathrm{x} 2] / / \mathrm{b}[\mathrm{x} 1]=1$ in the original metric:

```
. quietly regress y x1 x2 x3
. testnl _b[x2]/_b[x1] = 1
    (1) _bb[x2]/_b[x1] = 1
                chi2(1) = 0.29
            Prob > chi2 = 0.5928
```

We needed to refit the regression model to recover the original parameter estimates.

- Technical note

In a previous technical note, we mentioned that commands such as lincom and test permit reference to name instead of $\_\mathrm{b}[$ name $]$. This is not the case when lincom and test are used after nlcom, post. In the above, we used

```
. test _b[ratio21] = 1
```

rather than

```
. test ratio21 = 1
```

which would have returned an error. Consider this a limitation of Stata. For the shorthand notation to work, you need a variable named name in the data. In nlcom, however, name is just a coefficient label that does not necessarily correspond to any variable in the data.

## Reparameterizing ML estimators for univariate data

When run using only a response and no covariates, Stata's maximum likelihood (ML) estimation commands will produce ML estimates of the parameters of some assumed univariate distribution for the response. The parameterization, however, is usually not one we are used to dealing with in a nonregression setting. In such cases, nlcom can be used to transform the estimation results from a regression model to those from a maximum likelihood estimation of the parameters of a univariate probability distribution in a more familiar metric.
> Example 1
Consider the following univariate data on $Y=\#$ of traffic accidents at a certain intersection in a given year:

```
. use http://www.stata-press.com/data/r13/trafint
```

. summarize accidents

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| accidents | 12 | 13.83333 | 14.47778 | 0 | 41 |

A quick glance of the output from summarize leads us to quickly reject the assumption that $Y$ is distributed as Poisson because the estimated variance of $Y$ is much greater than the estimated mean of $Y$.

Instead, we choose to model the data as univariate negative binomial, of which a common parameterization is

$$
\operatorname{Pr}(Y=y)=\frac{\Gamma(r+y)}{\Gamma(r) \Gamma(y+1)} p^{r}(1-p)^{y} \quad 0 \leq p \leq 1, \quad r>0, \quad y=0,1, \ldots
$$

with

$$
E(Y)=\frac{r(1-p)}{p} \quad \operatorname{Var}(Y)=\frac{r(1-p)}{p^{2}}
$$

There exist no closed-form solutions for the maximum likelihood estimates of $p$ and $r$, yet they may be estimated by the iterative method of Newton-Raphson. One way to get these estimates would be to write our own Newton-Raphson program for the negative binomial. Another way would be to write our own ML evaluator; see [R] ml.

The easiest solution, however, would be to use Stata's existing negative binomial ML regression command, nbreg. The only problem with this solution is that nbreg estimates a different parameterization of the negative binomial, but we can worry about that later.
. nbreg accidents
Fitting Poisson model:
Iteration 0: log likelihood $=-105.05361$
Iteration 1: log likelihood $=-105.05361$
Fitting constant-only model:
Iteration 0: log likelihood $=-43.948619$
Iteration 1: log likelihood $=-43.891483$
Iteration 2: log likelihood $=$
Iteration 3: log likelihood $=-43.89144$
Fitting full model:
Iteration 0: log likelihood $=-43.89144$
Iteration 1: log likelihood $=-43.89144$
Negative binomial regression
Dispersion $\quad=$ mean
Log likelihood $=-43.89144$

| Number of obs | $=$ | 12 |
| :--- | :--- | ---: |
| LR chi2 (0) | $=$ | 0.00 |
| Prob > chi2 | $=$ | . |
| Pseudo R2 | $=$ | 0.0000 |


| accidents | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | ---: |
| _cons | 2.627081 | .3192233 | 8.23 | 0.000 | 2.001415 | 3.252747 |
| /lnalpha | .1402425 | .4187147 |  |  | -.6804233 | .9609083 |
| alpha | 1.150553 | .4817534 |  |  | .5064026 | 2.61407 |

Likelihood-ratio test of alpha=0: chibar2 (01) $=122.32$ Prob>=chibar2 $=0.000$
. nbreg, coeflegend

| Negative binomial regression | Number of obs | $=$ | 12 |
| :--- | :--- | :--- | ---: |
| Dispersion $=$ mean | LR chi2 $(0)$ | $=$ | 0.00 |
| Log likelihood $=-43.89144$ |  | Prob > chi2 | $=$ |


| accidents | Coef. Legend |  |
| ---: | ---: | :--- |
| _cons | 2.627081 | _b [accidents:_cons] |
| /lnalpha | .1402425 | _b[lnalpha:_cons] |
| alpha | 1.150553 |  |

Likelihood-ratio test of alpha=0: chibar2(01) $=122.32$ Prob>=chibar2 $=0.000$
From this output, we see that, when used with univariate data, nbreg estimates a regression intercept, $\beta_{0}$, and the logarithm of some parameter $\alpha$. This parameterization is useful in regression models: $\beta_{0}$ is the intercept meant to be augmented with other terms of the linear predictor, and $\alpha$ is an overdispersion parameter used for comparison with the Poisson regression model.

However, we need to transform $\left(\beta_{0}, \ln \alpha\right)$ to $(p, r)$. Examining Methods and formulas of $[\mathrm{R}]$ nbreg reveals the transformation as

$$
p=\left\{1+\alpha \exp \left(\beta_{0}\right)\right\}^{-1} \quad r=\alpha^{-1}
$$

which we apply using nlcom:

```
. nlcom (p:1/(1 + exp([lnalpha]_b[_cons] + _b[_cons])))
> (r:exp(-[lnalpha]_b[_cons]))
    p: 1/(1 + exp([lnalpha]_b[_cons] + _b[_cons]))
    r: exp(-[lnalpha]_b[_cons])
\begin{tabular}{r|rccccr}
\hline accidents & Coef. & Std. Err. & z & P>|z| & [95\% Conf. Interval] \\
\hline p & .0591157 & .0292857 & 2.02 & 0.044 & .0017168 & .1165146 \\
r & .8691474 & .3639248 & 2.39 & 0.017 & .1558679 & 1.582427 \\
\hline
\end{tabular}
```

Given the invariance of maximum likelihood estimators and the properties of the delta method, the above parameter estimates, standard errors, etc., are precisely those we would have obtained had we instead performed the Newton-Raphson optimization in the $(p, r)$ metric.

## Technical note

Note how we referred to the estimate of $\ln \alpha$ above as [lnalpha]_b[_cons]. This is not entirely evident from the output of nbreg, which is why we redisplayed the results using the coeflegend option so that we would know how to refer to the coefficients; [U] 13.5 Accessing coefficients and standard errors.

## nlcom versus eform

Many Stata estimation commands allow you to display exponentiated regression coefficients, some by default, some optionally. Known as "eform" in Stata terminology, this reparameterization serves many uses: it gives odds ratios for logistic models, hazard ratios in survival models, incidence-rate ratios in Poisson models, and relative-risk ratios in multinomial logit models, to name a few.

For example, consider the following estimation taken directly from the technical note in [R] poisson:

```
. use http://www.stata-press.com/data/r13/airline
. generate lnN = ln(n)
. poisson injuries XYZowned lnN
Iteration 0: log likelihood = -22.333875
Iteration 1: log likelihood = -22.332276
Iteration 2: log likelihood = -22.332276
```

| Poisson regression | Number of obs | $=$ | 9 |
| :--- | :--- | :--- | :--- |
|  | LR chi2(2) | $=$ | 19.15 |
| Log likelihood $=-22.332276$ | Prob > chi2 | $=$ | 0.0001 |
|  | Pseudo R2 | $=$ | 0.3001 |


| injuries | Coef. | Std. Err. | $z$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | ---: |
| XYZowned | .6840667 | .3895877 | 1.76 | 0.079 | -.0795111 | 1.447645 |
| lnN | 1.424169 | .3725155 | 3.82 | 0.000 | .6940517 | 2.154285 |
| _cons | 4.863891 | .7090501 | 6.86 | 0.000 | 3.474178 | 6.253603 |

When we replay results and specify the irr (incidence-rate ratios) option,

we obtain the exponentiated regression coefficients and their estimated standard errors.
Contrast this with what we obtain if we exponentiate the coefficients manually by using nlcom:

```
. nlcom (E_XYZowned:exp(_b[XYZowned])) (E_lnN:exp(_b[lnN]))
    E_XYZowned: exp(_b[XYZowned])
            E_lnN: exp(_b[lnN])
```

| injuries | Coef. | Std. Err. | $z$ | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | ---: |
| E_XYZowned | 1.981921 | .7721322 | 2.57 | 0.010 | .4685701 | 3.495273 |
| E_InN | 4.154402 | 1.547579 | 2.68 | 0.007 | 1.121203 | 7.187602 |

There are three things to note when comparing poisson, irr (and eform in general) with nlcom:

1. The exponentiated coefficients and standard errors are identical. This is certainly good news.
2. The Wald test statistic ( $\mathbf{z}$ ) and level of significance are different. When using poisson, irr and other related eform options, the Wald test does not change from what you would have obtained without the eform option, and you can see this by comparing both versions of the poisson output given previously.

When you use eform, Stata knows that what is usually desired is a test of

$$
H_{0}: \exp (\beta)=1
$$

and not the uninformative-by-comparison

$$
H_{0}: \exp (\beta)=0
$$

The test of $H_{0}: \exp (\beta)=1$ is asymptotically equivalent to a test of $H_{0}: \beta=0$, the Wald test in the original metric, but the latter has better small-sample properties. Thus if you specify eform, you get a test of $H_{0}: \beta=0$.
nlcom, however, is general. It does not attempt to infer the test of greatest interest for a given transformation, and so a test of

$$
H_{0}: \text { transformed coefficient }=0
$$

is always given, regardless of the transformation.
3. You may be surprised to see that, even though the coefficients and standard errors are identical, the confidence intervals (both 95\%) are different.
eform confidence intervals are standard confidence intervals with the endpoints transformed. For example, the confidence interval for the coefficient on $\operatorname{lnN}$ is $[0.694,2.154]$, whereas the confidence interval for the incidence-rate ratio due to $\operatorname{lnN}$ is $[\exp (0.694), \exp (2.154)]=[2.002,8.619]$, which, except for some roundoff error, is what we see from the output of poisson, irr. For exponentiated coefficients, confidence intervals based on transform-the-endpoints methodology generally have better small-sample properties than their asymptotically equivalent counterparts.
The transform-the-endpoints method, however, gives valid coverage only when the transformation is monotonic. nlcom uses a more general and asymptotically equivalent method for calculating confidence intervals, as described in Methods and formulas.

## Stored results

nlcom stores the following in $r()$ :
Scalars

```
        r(N)
        number of observations
        r(df_r) residual degrees of freedom
        r(b) vector of transformed coefficients
        r(V) estimated variance-covariance matrix of the transformed coefficients
```

    Matrices
    If post is specified, nlcom also stores the following in e() :
Scalars
$e(N) \quad$ number of observations
e(df_r) residual degrees of freedom
e(N_strata) number of strata $L$, if used after svy
e(N_psu) number of sampled PSUs $n$, if used after svy
$e$ (rank) rank of $e(V)$
Macros
e(cmd) nlcom
$e$ (predict) program used to implement predict
e(properties) b V
Matrices
e(b) vector of transformed coefficients
$e(V) \quad$ estimated variance-covariance matrix of the transformed coefficients
e(V_srs) simple-random-sampling-without-replacement (co)variance $\widehat{V}_{\text {srswor }}$, if svy
e(V_srswr) simple-random-sampling-with-replacement (co)variance $\widehat{V}_{\text {srswr }}$, if svy and fpc()
$e\left(V \_m s p\right) \quad$ misspecification (co)variance $\widehat{V}_{\text {msp }}$, if svy and available
Functions
e(sample) marks estimation sample

## Methods and formulas

Given a $1 \times k$ vector of parameter estimates, $\widehat{\boldsymbol{\theta}}=\left(\widehat{\theta}_{1}, \ldots, \widehat{\theta}_{k}\right)$, consider the estimated $p$-dimensional transformation

$$
g(\widehat{\boldsymbol{\theta}})=\left[g_{1}(\widehat{\boldsymbol{\theta}}), g_{2}(\widehat{\boldsymbol{\theta}}), \ldots, g_{p}(\widehat{\boldsymbol{\theta}})\right]
$$

The estimated variance-covariance of $g(\widehat{\boldsymbol{\theta}})$ is given by

$$
\widehat{\operatorname{Var}}\{g(\widehat{\boldsymbol{\theta}})\}=\mathbf{G V G}^{\prime}
$$

where $\mathbf{G}$ is the $p \times k$ matrix of derivatives for which

$$
\mathbf{G}_{i j}=\left.\frac{\partial g_{i}(\boldsymbol{\theta})}{\partial \theta_{j}}\right|_{\boldsymbol{\theta}=\widehat{\boldsymbol{\theta}}} \quad i=1, \ldots, p \quad j=1, \ldots, k
$$

and $\mathbf{V}$ is the estimated variance-covariance matrix of $\widehat{\boldsymbol{\theta}}$. Standard errors are obtained as the square roots of the variances.

The Wald test statistic for testing

$$
H_{0}: g_{i}(\boldsymbol{\theta})=0
$$

versus the two-sided alternative is given by

$$
Z_{i}=\frac{g_{i}(\widehat{\boldsymbol{\theta}})}{\left[\widehat{\operatorname{Var}}_{i i}\{g(\widehat{\boldsymbol{\theta}})\}\right]^{1 / 2}}
$$

When the variance-covariance matrix of $\widehat{\boldsymbol{\theta}}$ is an asymptotic covariance matrix, $Z_{i}$ is approximately distributed as Gaussian. For linear regression, $Z_{i}$ is taken to be approximately distributed as $t_{1, r}$ where $r$ is the residual degrees of freedom from the original fitted model.

A $(1-\alpha) \times 100 \%$ confidence interval for $g_{i}(\boldsymbol{\theta})$ is given by

$$
g_{i}(\widehat{\boldsymbol{\theta}}) \pm z_{\alpha / 2}\left[\widehat{\operatorname{Var}}_{i i}\{g(\widehat{\boldsymbol{\theta}})\}\right]^{1 / 2}
$$

for those cases where $Z_{i}$ is Gaussian and

$$
g_{i}(\widehat{\boldsymbol{\theta}}) \pm t_{\alpha / 2, r}\left[\widehat{\operatorname{Var}}_{i i}\{g(\widehat{\boldsymbol{\theta}})\}\right]^{1 / 2}
$$

for those cases where $Z_{i}$ is $t$ distributed. $z_{p}$ is the $1-p$ quantile of the standard normal distribution, and $t_{p, r}$ is the $1-p$ quantile of the $t$ distribution with $r$ degrees of freedom.

## References

Feiveson, A. H. 1999. FAQ: What is the delta method and how is it used to estimate the standard error of a transformed parameter? http://www.stata.com/support/faqs/stat/deltam.html.
Gould, W. W. 1996. cre43: Wald test of nonlinear hypotheses after model estimation. Stata Technical Bulletin 29: 2-4. Reprinted in Stata Technical Bulletin Reprints, vol. 5, pp. 15-18. College Station, TX: Stata Press.

Oehlert, G. W. 1992. A note on the delta method. American Statistician 46: 27-29.
Phillips, P. C. B., and J. Y. Park. 1988. On the formulation of Wald tests of nonlinear restrictions. Econometrica 56: 1065-1083.

## Also see

[R] lincom - Linear combinations of estimators
$[\mathrm{R}]$ predictnl - Obtain nonlinear predictions, standard errors, etc., after estimation
[R] test - Test linear hypotheses after estimation
[R] testnl - Test nonlinear hypotheses after estimation
[U] 20 Estimation and postestimation commands

## Title

nlogit - Nested logit regression

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |
| Also see |  |  |  |

## Syntax

Nested logit regression

$$
\begin{aligned}
& \text { nlogit depvar }[\text { indepvars }][\text { if }][\text { in }][\text { weight }][\text { I। lev1_equation } \\
& \qquad[\text { I। lev2_equation...]] || altvar: }[\text { byaltvarlist }] \text {, case(varname) } \text { [options }]
\end{aligned}
$$

where the syntax of lev\#_equation is
altvar: [byaltvarlist ] [, base(\#|lbl) estconst]

Create variable based on specification of branches

$$
\text { nlogitgen newaltvar }=\text { altvar }(\text { branchlist })[\text {, nolog }]
$$

where branchlist is

$$
\text { branch, branch }[\text {, branch ... }]
$$

and branch is

$$
[\text { label }:] \text { alternative }[\text { I alternative }[\text { I alternative } . . .]]
$$

Display tree structure

$$
\text { nlogittree altvarlist }[\text { if }][\text { in }][\text { weight }][\text {, choice(depvar) nolabel nobranches }]
$$

## options

Description

## Model

* case (varname)
base (\#|lbl)
noconstant
nonnormalized
altwise
constraints (constraints)
collinear
SE/Robust
vce (vcetype)

Reporting
level(\#)
notree
nocnsreport
display_options
Maximization
maximize_options
use varname to identify cases
use the specified level or label of altvar as the base alternative for the bottom level
suppress the constant terms for the bottom-level alternatives
use the nonnormalized parameterization
use alternativewise deletion instead of casewise deletion
apply specified linear constraints
keep collinear variables
vcetype may be oim, robust, cluster clustvar, bootstrap, or jackknife

* case (varname) is required.
bootstrap, by, fp, jackknife, statsby, and xi are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the bootstrap prefix; see [R] bootstrap.
fweights, iweights, and pweights are allowed with nlogit, and fweights are allowed with nlogittree; see
[U] 11.1.6 weight. Weights for nlogit must be constant within case.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.


## Menu

nlogit
Statistics $>$ Categorical outcomes $>$ Nested logit regression

## nlogitgen

Statistics $>$ Categorical outcomes $>$ Setup for nested logit regression

## nlogittree

Statistics $>$ Categorical outcomes $>$ Display nested logit tree structure

## Description

nlogit performs full information maximum-likelihood estimation for nested logit models. These models relax the assumption of independently distributed errors and the independence of irrelevant alternatives inherent in conditional and multinomial logit models by clustering similar alternatives into nests.

By default, nlogit uses a parameterization that is consistent with random utility maximization (RUM). Before version 10 of Stata, a nonnormalized version of the nested logit model was fit, which you can request by specifying the nonnormalized option.

You must use nlogitgen to generate a new categorical variable to specify the branches of the decision tree before calling nlogit.

## Options

## Specification and options for lev\#_equation

altvar is a variable identifying alternatives at this level of the hierarchy.
byaltvarlist specifies the variables to be used to compute the by-alternative regression coefficients for that level. For each variable specified in the variable list, there will be one regression coefficient for each alternative of that level of the hierarchy. If the variable is constant across each alternative (a case-specific variable), the regression coefficient associated with the base alternative is not identifiable. These regression coefficients are labeled as (base) in the regression table. If the variable varies among the alternatives, a regression coefficient is estimated for each alternative.
base $(\# \mid l b l)$ can be specified in each level equation where it identifies the base alternative to be used at that level. The default is the alternative that has the highest frequency.
If vce(bootstrap) or vce (jackknife) is specified, you must specify the base alternative for each level that has a byaltvarlist or if the constants will be estimated. Doing so ensures that the same model is fit with each call to nlogit.
estconst applies to all the level equations except the bottom-level equation. Specifying estconst requests that constants for each alternative (except the base alternative) be estimated. By default, no constant is estimated at these levels. Constants can be estimated in only one level of the tree hierarchy. If you specify estconst for one of the level equations, you must specify noconstant for the bottom-level equation.

## Options for nlogit

 Modelcase (varname) specifies the variable that identifies each case. case() is required.
base (\#|lbl) can be specified in each level equation where it identifies the base alternative to be used at that level. The default is the alternative that has the highest frequency.
If vce(bootstrap) or vce(jackknife) is specified, you must specify the base alternative for each level that has a byaltvarlist or if the constants will be estimated. Doing so ensures that the same model is fit with each call to nlogit.
noconstant applies only to the equation defining the bottom level of the hierarchy. By default, constants are estimated for each alternative of altvar, less the base alternative. To suppress the constant terms for this level, specify noconstant. If you do not specify noconstant, you cannot specify estconst for the higher-level equations.
nonnormalized requests a nonnormalized parameterization of the model that does not scale the inclusive values by the degree of dissimilarity of the alternatives within each nest. Use this option to replicate results from older versions of Stata. The default is to use the RUM-consistent parameterization.
altwise specifies that alternativewise deletion be used when marking out observations because of missing values in your variables. The default is to use casewise deletion. This option does not apply to observations that are marked out by the if or in qualifier or the by prefix.
constraints (constraints); see [R] estimation options.
The inclusive-valued/dissimilarity parameters are parameterized as ml ancillary parameters. They are labeled as [alternative_tau]_const, where alternative is one of the alternatives defining a branch in the tree. To constrain the inclusive-valued/dissimilarity parameter for alternative a1 to be, say, equal to alternative a2, you would use the following syntax:
. constraint 1 [a1_tau]_cons = [a2_tau]_cons
. nlogit ..., constraints (1)
collinear prevents collinear variables from being dropped. Use this option when you know that you have collinear variables and you are applying constraints() to handle the rank reduction. See [R] estimation options for details on using collinear with constraints().
nlogit will not allow you to specify an independent variable in more than one level equation. Specifying the collinear option will allow execution to proceed in this case, but it is your responsibility to ensure that the parameters are identified.

SE/Robust
vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
If vce(robust) or vce(cluster clustvar) is specified, the likelihood-ratio test for the independence of irrelevant alternatives (IIA) is not computed.

Reporting
level (\#); see [R] estimation options.
notree specifies that the tree structure of the nested logit model not be displayed. See also nolabel and nobranches below for when notree is not specified.
nocnsreport; see [R] estimation options.
display_options: cformat ( $\% f m t$ ), pformat ( $\% f m t$ ), sformat ( $\% f m t$ ), and nolstretch; see [R] estimation options.

Maximization
maximize_options: difficult, technique (algorithm_spec), iterate (\#), [no $] \underline{\text { log, trace, }}$ gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance(\#), nonrtolerance, and from(init_specs); see [R] maximize. These options are seldom used.

The technique (bhhh) option is not allowed.

## Specification and options for nlogitgen

newaltvar and altvar are variables identifying alternatives at each level of the hierarchy.
label defines a label to associate with the branch. If no label is given, a numeric value is used.
alternative specifies an alternative, of altvar specified in the syntax, to be included in the branch. It is either a numeric value or the label associated with that value. An example of nlogitgen is

```
. nlogitgen type = restaurant(fast: 1 | 2,
> family: CafeEccell | LosNortenos | WingsNmore, fancy: 6 | 7)
```

nolog suppresses the display of the iteration log.

## Specification and options for nlogittree

$\qquad$ Main
altvarlist is a list of alternative variables that define the tree hierarchy. The first variable must define bottom-level alternatives, and the order continues to the variable defining the top-level alternatives. choice (depvar) defines the choice indicator variable and forces nlogittree to compute and display choice frequencies for each bottom-level alternative.
nolabel forces nlogittree to suppress value labels in tree-structure output.
nobranches forces nlogittree to suppress drawing branches in the tree-structure output.

## Remarks and examples

Remarks are presented under the following headings:

Introduction<br>Data setup and the tree structure<br>Estimation<br>Testing for the IIA<br>Nonnormalized model

## Introduction

nlogit performs full information maximum-likelihood estimation for nested logit models. These models relax the assumption of independently distributed errors and the IIA inherent in conditional and multinomial logit models by clustering similar alternatives into nests. Because the nested logit model is a direct generalization of the alternative-specific conditional logit model (also known as McFadden's choice model), you may want to read [R] asclogit before continuing.

By default, nlogit uses a parameterization that is consistent with RUM. Before version 10 of Stata, a nonnormalized version of the nested logit model was fit, which you can request by specifying the nonnormalized option. We recommend using the RUM-consistent version of the model for new projects because it is based on a sound model of consumer behavior.

McFadden $(1977,1981)$ showed how this model can be derived from a rational choice framework. Amemiya (1985, chap. 9) contains a nice discussion of how this model can be derived under the assumption of utility maximization. Hensher, Rose, and Greene (2005) provide a lucid introduction to choice models including nested logit.

Throughout this entry, we consider a model of restaurant choice. We begin by introducing the data.

## Example 1

We have fictional data on 300 families and their choice of seven local restaurants. Freebirds and Mama's Pizza are fast food restaurants; Café Eccell, Los Norteños, and Wings 'N More are family restaurants; and Christopher's and Mad Cows are fancy restaurants. We want to model the decision of where to eat as a function of household income (income, in thousands of dollars), the number of children in the household (kids), the rating of the restaurant according to a local restaurant guide (rating, coded $0-5$ ), the average meal cost per person (cost), and the distance between the household and the restaurant (distance, in miles). income and kids are attributes of the family, rating is an attribute of the alternative (the restaurant), and cost and distance are attributes of the alternative as perceived by the families-that is, each family has its own cost and distance for each restaurant.

We begin by loading the data and listing some of the variables for the first three families:

```
. use http://www.stata-press.com/data/r13/restaurant
. describe
Contains data from http://www.stata-press.com/data/r13/restaurant.dta
    obs: 2,100
    vars: 8 10 Mar 2013 01:17
size: 67,200
```

| variable name | storage type | display <br> format | value <br> label | variable label |
| :---: | :---: | :---: | :---: | :---: |
| family_id | float | \%9.0g |  | family ID |
| restaurant | float | \%12.0g | names | choices of restaurants |
| income | float | \%9.0g |  | household income |
| cost | float | $\% 9.0 \mathrm{~g}$ |  | average meal cost per person |
| kids | float | $\% 9.0 \mathrm{~g}$ |  | number of kids in the household |
| rating | float | \%9.0g |  | ratings in local restaurant guide |
| distance | float | \%9.0g |  | distance between home and restaurant |
| chosen | float | \%9.0g |  | 0 no 1 yes |

Sorted by: family_id

```
. list family_id restaurant chosen kids rating distance in 1/21, sepby(fam)
> abbrev(10)
```

|  | family_id | restaurant | chosen | kids | rating | distance |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | 1 | Freebirds | 1 | 1 | 0 | 1.245553 |
| 2. | 1 | MamasPizza | 0 | 1 | 1 | 2.82493 |
| 3. | 1 | CafeEccell | 0 | 1 | 2 | 4.21293 |
| 4. | 1 | LosNortenos | 0 | 1 | 3 | 4.167634 |
| 5. | 1 | WingsNmore | 0 | 1 | 2 | 6.330531 |
| 6. | 1 | Christophers | 0 | 1 | 4 | 10.19829 |
| 7. | 1 | MadCows | 0 | 1 | 5 | 5.601388 |
| 8. | 2 | Freebirds | 0 | 3 | 0 | 4.162657 |
| 9. | 2 | MamasPizza | 0 | 3 | 1 | 2.865081 |
| 10. | 2 | CafeEccell | 0 | 3 | 2 | 5.337799 |
| 11. | 2 | LosNortenos | 1 | 3 | 3 | 4.282864 |
| 12. | 2 | WingsNmore | 0 | 3 | 2 | 8.133914 |
| 13. | 2 | Christophers | 0 | 3 | 4 | 8.664631 |
| 14. | 2 | MadCows | 0 | 3 | 5 | 9.119597 |
| 15. | 3 | Freebirds | 1 | 3 | 0 | 2.112586 |
| 16. | 3 | MamasPizza | 0 | 3 | 1 | 2.215329 |
| 17. | 3 | CafeEccell | 0 | 3 | 2 | 6.978715 |
| 18. | 3 | LosNortenos | 0 | 3 | 3 | 5.117877 |
| 19. | 3 | WingsNmore | 0 | 3 | 2 | 5.312941 |
| 20. | 3 | Christophers | 0 | 3 | 4 | 9.551273 |
| 21. | 3 | MadCows | 0 | 3 | 5 | 5.539806 |

Because each family chose among seven restaurants, there are 7 observations in the dataset for each family. The variable chosen is coded $0 / 1$, with 1 indicating the chosen restaurant and 0 otherwise.

We could fit a conditional logit model to our data. Because income and kids are constant within each family, we would use the asclogit command instead of clogit. However, the conditional logit may be inappropriate. That model assumes that the random errors are independent, and as a result it forces the odds ratio of any two alternatives to be independent of the other alternatives, a property known as the IIA. We will discuss the IIA assumption in more detail later.

Assuming that unobserved shocks influencing a decision maker's attitude toward one alternative have no effect on his attitudes toward the other alternatives may seem innocuous, but often this assumption is too restrictive. Suppose that when a family was deciding which restaurant to visit, they were pressed for time because of plans to attend a movie later. The unobserved shock (being in a hurry) would raise the likelihood that the family goes to either fast food restaurant (Freebirds or Mama's Pizza). Similarly, another family might be choosing a restaurant to celebrate a birthday and therefore be inclined to attend a fancy restaurant (Christopher's or Mad Cows).

Nested logit models relax the independence assumption and allow us to group alternatives for which unobserved shocks may have concomitant effects. Here we suspect that restaurants should be grouped by type (fast, family, or fancy). The tree structure of a family's decision about where to eat might look like this:


At the bottom of the tree are the individual restaurants, indicating that there are some random shocks that affect a family's decision to eat at each restaurant independently. Above the restaurants are the three types of restaurants, indicating that other random shocks affect the type of restaurant chosen. As is customary when drawing decision trees, at the top level is one box, representing the family making the decision.

We use the following terms to describe nested logit models.
level, or decision level, is the level or stage at which a decision is made. The example above has only two levels. In the first level, a type of restaurant is chosen-fast food, family, or fancy-and in the second level, a specific restaurant is chosen.
bottom level is the level where the final decision is made. In our example, this is when we choose a specific restaurant.
alternative set is the set of all possible alternatives at any given decision level.
bottom alternative set is the set of all possible alternatives at the bottom level. This concept is often referred to as the choice set in the economics-choice literature. In our example, the bottom alternative set is all seven of the specific restaurants.
alternative is a specific alternative within an alternative set. In the first level of our example, "fast food" is an alternative. In the second or bottom level, "Mad Cows" is an alternative. Not all alternatives within an alternative set are available to someone making a choice at a specific stage, only those that are nested within all higher-level decisions.
chosen alternative is the alternative from an alternative set that we observe someone having chosen.

## Technical note

Although decision trees in nested logit analysis are often interpreted as implying that the highestlevel decisions are made first, followed by decisions at lower levels, and finally the decision among alternatives at the bottom level, no such temporal ordering is implied. See Hensher, Rose, and Greene (2005, chap. 13). In our example, we are not assuming that families first choose whether to attend a fast, family, or fancy restaurant and then choose the particular restaurant; we assume merely that they choose one of the seven restaurants.

## Data setup and the tree structure

To fit a nested logit model, you must first create a variable that defines the structure of your decision tree.

## > Example 2

To run nlogit, we need to generate a categorical variable that identifies the first-level set of alternatives: fast food, family restaurants, or fancy restaurants. We can do so easily by using nlogitgen.

```
. nlogitgen type = restaurant(fast: Freebirds | MamasPizza,
> family: CafeEccell | LosNortenos| WingsNmore, fancy: Christophers | MadCows)
new variable type is generated with 3 groups
label list lb_type
lb_type:
    1 fast
    2 family
    fancy
. nlogittree restaurant type, choice(chosen)
tree structure specified for the nested logit model
```



```
k = number of times alternative is chosen
N = number of observations at each level
```

The new categorical variable is type, which takes on value 1 (fast) if restaurant is Freebirds or Mama's Pizza; value 2 (family) if restaurant is Café Eccell, Los Norteños, or Wings 'N More; and value 3 (fancy) otherwise. nlogittree displays the tree structure.

## - Technical note

We could also use values instead of value labels of restaurant in nlogitgen. Value labels are optional, and the default value labels for type are type1, type2, and type3. The vertical bar is also optional.

```
. use http://www.stata-press.com/data/r13/restaurant, clear
. nlogitgen type = restaurant(1 2, 3 4 5, 6 7)
new variable type is generated with 3 groups
label list lb_type
lb_type:
    1 type1
    2 type2
    3 type3
```

```
. nlogittree restaurant type
tree structure specified for the nested logit model
type N restaurant N
type1 600 T Freebirds }\begin{array}{ll}{300}\\{\mathrm{ MamasPizza }}&{300}
type2 900 T CafeEccell 300
        - LosNortenos 300
    type3 600 T Christophers 300
                        total 2100
N = number of observations at each level
```

In our dataset, every family was able to choose among all seven restaurants. However, in other applications some decision makers may not have been able to choose among all possible alternatives. For example, two cases may have choice hierarchies of

| case 1 |  | case 2 |  |
| :---: | :---: | :---: | :---: |
| type | restaurant | type | restaurant |
| fast | Freebirds <br> MamasPizza | fast | Freebirds <br> MamasPizza |
| family | - CafeEccell | family | LosNortenos <br> WingsNmore |
| fancy | Christophers <br> - MadCows | fancy | hristophers |

where the second case does not have the restaurant alternatives Café Eccell or Mad Cows available to them. The only restriction is that the relationships between higher- and lower-level alternative sets be the same for all decision makers. In this two-level example, Freebirds and Mama's Pizza are classified as fast food restaurants for both cases; Café Eccell, Los Norteños, and Wings 'N More are family restaurants; and Christopher's and Mad Cows are fancy restaurants. nlogit requires only that hierarchy be maintained for all cases.

## Estimation

## > Example 3

With our type variable created that defines the three types of restaurants, we can now examine how the alternative-specific attributes (cost, rating, and distance) apply to the bottom alternative set (the seven restaurants) and how family-specific attributes (income and kid) apply to the alternative set at the first decision level (the three types of restaurants).
. use http://www.stata-press.com/data/r13/restaurant, clear
. qui nlogitgen type = restaurant (fast: Freebirds | MamasPizza,
> family: CafeEccell | LosNortenos| WingsNmore, fancy: Christophers | MadCows)
. nlogit chosen cost rating distance \| type: income kids, base(family) \|
> restaurant:, noconstant case(family_id)
tree structure specified for the nested logit model

| type | N | restaurant | N | k |
| :---: | :---: | :---: | :---: | :---: |
| fast |  | - Freebirds | 300 | 12 |
|  |  | - MamasPizza | 300 | 15 |
| family | 900 | - CafeEccell | 300 | 78 |
|  |  | - LosNortenos | 300 | 75 |
|  |  | - WingsNmore | 300 | 69 |
| fancy | 600 | [ Christophers | 300 | 27 |
|  |  | $\boxed{\text { MadCows }}$ | 300 | 24 |

$\mathrm{k}=$ number of times alternative is chosen
$\mathrm{N}=$ number of observations at each level
Iteration 0: log likelihood $=-541.93581$
(output omitted)
Iteration 17: log likelihood $=-485.47331$
RUM-consistent nested logit regression Number of obs = 2100
Case variable: family_id

| Number of obs | $=$ | 2100 |
| :--- | :--- | ---: |
| Number of cases | $=$ | 300 |

Alternative variable: restaurant

Log likelihood $=-485.47331$
Number of cases $=300$

Alts per case: min $=\quad 7$
$\operatorname{avg}=\quad 7.0$
$\max =\quad 7$
Wald chi2(7) $=46.71$

| chosen | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | :--- | :--- | :--- | :--- | :--- |
| restaurant |  |  |  |  |  |  |
| cost | -.1843847 | .0933975 | -1.97 | 0.048 | -.3674404 | -.0013289 |
| rating | .463694 | .3264935 | 1.42 | 0.156 | -.1762215 | 1.10361 |
| distance | -.3797474 | .1003828 | -3.78 | 0.000 | -.5764941 | -.1830007 |

type equations

| fast |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| income | -. 0266038 | . 0117306 | -2.27 | 0.023 | -. 0495952 | -. 0036123 |
| kids | -. 0872584 | . 1385026 | -0.63 | 0.529 | -. 3587184 | . 1842016 |
| family |  |  |  |  |  |  |
| income | 0 | (base) |  |  |  |  |
| kids | 0 | (base) |  |  |  |  |
| fancy |  |  |  |  |  |  |
| income | . 0461827 | . 0090936 | 5.08 | 0.000 | . 0283595 | . 0640059 |
| kids | -. 3959413 | . 1220356 | -3.24 | 0.001 | -. 6351267 | -. 1567559 |

dissimilarity parameters

| type |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| /fast_tau | 1.712878 | 1.48685 | -1.201295 | 4.627051 |
| /family_tau | 2.505113 | .9646351 | .614463 | 4.395763 |
| /fancy_tau | 4.099844 | 2.810123 | -1.407896 | 9.607583 |
| LR test for IIA (tau $=1):$ |  | chi2 (3) $=$ | 6.87 | Prob $>$ chi2 $=0.0762$ |

First, let's examine how we called nlogit. The delimiters (||) separate equations. The first equation specifies the dependent variable, chosen, and three alternative-specific variables, cost, rating, and distance. We refer to these variables as alternative-specific because they vary among the bottom-level alternatives, the restaurants. We obtain one parameter estimate for each variable. These estimates are listed in the equation subtable labeled restaurant.

For the second equation, we specify the type variable. It identifies the first-level alternatives, the restaurant types. Following the colon after type, we specify two case-specific variables, income and kids. Here we obtain a parameter estimate for each variable for each alternative at this level. That is why we call these variable lists by-alternative variables. Because income and kids do not vary within each case, to identify the model one alternative's set of parameters must be set to zero. We specified the base (family) option with this equation to restrict the parameters for the family alternative.

The variable identifying the bottom-level alternatives, restaurant, is specified after the second equation delimiter. We do not specify any variables after the colon delimiter at this level. Had we specified variables here, we would have obtained an estimate for each variable in each equation. As we will see below, these variables parameterize the constant term in the utility equation for each bottom-level alternative. The noconstant option suppresses bottom-level alternative-specific constant terms.

Near the bottom of the output are the dissimilarity parameters, which measure the degree of correlation of random shocks within each of the three types of restaurants. Dissimilarity parameters greater than one imply that the model is inconsistent with RUM; Hensher, Rose, and Greene (2005, sec. 13.6) discuss this in detail. We will ignore the fact that all our dissimilarity parameters exceed one.

The conditional logit model is a special case of nested logit in which all the dissimilarity parameters are equal to one. At the bottom of the output, we find a likelihood-ratio test of this hypothesis. Here we have mixed evidence of the null hypothesis that all the parameters are one. Equivalently, the property known as the IIA imposed by the conditional logit model holds if and only if all dissimilarity parameters are equal to one. We discuss the IIA in more detail now.

## Testing for the IIA

The IIA is a property of the multinomial and conditional logit models that forces the odds of choosing one alternative over another to be independent of the other alternatives. For simplicity, suppose that a family was choosing only between Freebirds and Mama's Pizza, and the family was equally likely to choose either of the restaurants. The probability of going to each restaurant is $50 \%$. Now suppose that Bill's Burritos opens up next door to Freebirds, which is also a burrito restaurant. If the IIA holds, then the probability of going to each restaurant must now be $33.33 \%$ so that the family remains equally likely to go to Mama's Pizza or Freebirds.

The IIA may sometimes be a plausible assumption. However, a more likely scenario would be for the probability of going to Mama's Pizza to remain at $50 \%$ and the probabilities of going to Freebirds and Bill's Burritos to be $25 \%$ each, because the two restaurants are next door to each other and serve the same food. Nested logit analysis would allow us to relax the IIA assumption of conditional logit. We could group Bill's Burritos and Freebirds into one nest that encompasses all burrito restaurants and create a second nest for pizzerias.

The IIA is a consequence of assuming that the errors are independent and identically distributed (i.i.d.). Because the errors are i.i.d., they cannot contain any alternative-specific unobserved information, and therefore adding a new alternative cannot affect the relationship between a pair of existing alternatives.

In the previous example, we saw that a joint test that the dissimilarity parameters were equal to one is one way to test for IIA. However, that test required us to specify a decision tree for the nested logit model, and different specifications could lead to conflicting results of the test. Hausman and McFadden (1984) suggest that if part of the choice set truly is irrelevant with respect to the other alternatives, omitting that subset from the conditional logit model will not lead to inconsistent estimates. Therefore, Hausman's (1978) specification test can be used to test for IIA, and this test will not be sensitive to the tree structure we specify for a nested logit model.

## > Example 4

We want to test the IIA for the subset of family restaurants against the alternatives of fast food and fancy restaurants. To do so, we need to use Stata's hausman command; see [R] hausman.

We first run the estimation on the full bottom alternative set, store the results by using estimates store, and then run the estimation on the bottom alternative set, excluding the alternatives of family restaurants. We then run the hausman test.

```
. generate incFast = (type == 1) * income
. generate incFancy = (type == 3) * income
. generate kidFast = (type == 1) * kids
. generate kidFancy = (type == 3) * kids
. clogit chosen cost rating distance incFast incFancy kidFast kidFancy,
> group(family_id) nolog
\begin{tabular}{llll} 
Conditional (fixed-effects) logistic regression & Number of obs & \(=\) & 2100 \\
& LR chi2 (7) & \(=\) & 189.73 \\
& Prob > chi2 & \(=\) & 0.0000 \\
Log likelihood \(=-488.90834\) & & Pseudo R2 & \(=\)
\end{tabular}
```

| chosen | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| cost | -.1367799 | .0358479 | -3.82 | 0.000 | -.2070404 | -.0665193 |
| rating | .3066622 | .1418291 | 2.16 | 0.031 | .0286823 | .584642 |
| distance | -.1977505 | .0471653 | -4.19 | 0.000 | -.2901927 | -.1053082 |
| incFast | -.0390183 | .0094018 | -4.15 | 0.000 | -.0574455 | -.0205911 |
| incFancy | .0407053 | .0080405 | 5.06 | 0.000 | .0249462 | .0564644 |
| kidFast | -.2398757 | .1063674 | -2.26 | 0.024 | -.448352 | -.0313994 |
| kidFancy | -.3893862 | .1143797 | -3.40 | 0.001 | -.6135662 | -.1652061 |

```
. estimates store fullset
. clogit chosen cost rating distance incFast kidFast if type != 2,
> group(family_id) nolog
note: 222 groups (888 obs) dropped because of all positive or
    all negative outcomes.
\begin{tabular}{|c|c|c|c|c|}
\hline \multirow[t]{3}{*}{Conditional} & logistic regression & Number of obs & = & 312 \\
\hline & & LR chi2(5) & = & 44.35 \\
\hline & & Prob > chi2 & = & 0.0000 \\
\hline Log likelihood = -85.955324 & & Pseudo R2 & & 0.2051 \\
\hline
\end{tabular}
```

| chosen | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| cost | -.0616621 | .067852 | -0.91 | 0.363 | -.1946496 | .0713254 |
| rating | .1659001 | .2832041 | 0.59 | 0.558 | -.3891698 | .72097 |
| distance | -.244396 | .0995056 | -2.46 | 0.014 | -.4394234 | -.0493687 |
| incFast | -.0737506 | .0177444 | -4.16 | 0.000 | -.108529 | -.0389721 |
| kidFast | .4105386 | .2137051 | 1.92 | 0.055 | -.0083157 | .8293928 |

```
. hausman . fullset
                    _Coefficients -
\begin{tabular}{r|ccrr} 
& \((\mathrm{b})\) & \begin{tabular}{c} 
(B) \\
fullset
\end{tabular} & \begin{tabular}{c}
\((\mathrm{b}-\mathrm{B})\) \\
Difference
\end{tabular} & \begin{tabular}{c} 
sqrt(diag(V_b-V_B)) \\
\\
\end{tabular}\(\quad \cdot\)
\end{tabular}
                        b = consistent under Ho and Ha; obtained from clogit
            B = inconsistent under Ha, efficient under Ho; obtained from clogit
Test: Ho: difference in coefficients not systematic
                        chi2(5) = (b-B)'[(V_b-V_B)^(-1)](b-B)
                        = 10.70
                Prob>chi2 = 0.0577
                (V_b-V_B is not positive definite)
```

Similar to our findings in example 3, the results of the test of the IIA are mixed. We cannot reject the IIA at the commonly used $5 \%$ significance level, but we could at the $10 \%$ level. Substantively, a significant test result suggests that the odds of going to one of the fancy restaurants versus going to one of the fast food restaurants changes if we include the family restaurants in the alternative set and that a nested logit specification may be warranted.

## Nonnormalized model

Previous versions of Stata fit a nonnormalized nested logit model that is available via the nonnormalized option. The nonnormalized version is presented in, for example, Greene (2012, 768-770). Here we outline the differences between the RUM-consistent and nonnormalized models. Our discussion follows Heiss (2002) and assumes the decision tree has two levels, with $M$ alternatives at the upper level and a total of $J$ alternatives at the bottom level.

In a RUM framework, by consuming alternative $j$, decision maker $i$ obtains utility

$$
U_{i j}=V_{i j}+\epsilon_{i j}=\alpha_{j}+\mathbf{x}_{i j} \boldsymbol{\beta}_{j}+\mathbf{z}_{i} \gamma_{j}+\epsilon_{i j}
$$

where $V_{i j}$ is the deterministic part of utility and $\epsilon_{i j}$ is the random part. $\mathbf{x}_{i j}$ are alternative-specific variables and $\mathbf{z}_{i}$ are case-specific variables. The set of errors $\epsilon_{i 1}, \ldots, \epsilon_{i J}$ are assumed to follow the generalized extreme-value (GEV) distribution, which is a generalization of the type 1 extreme-value distribution that allows for alternatives within nests of the tree structure to be correlated. Let $\rho_{m}$ denote the correlation in nest $m$, and define the dissimilarity parameter $\tau_{m}=\sqrt{1-\rho_{m}} . \tau_{m}=0$ implies that the alternatives in nest $m$ are perfectly correlated, whereas $\tau_{m}=1$ implies independence.

The inclusive value for the $m$ th nest corresponds to the expected value of the utility that decision maker $i$ obtains by consuming an alternative in nest $m$. Denote this value by $\mathrm{IV}_{m}$ :

$$
\begin{equation*}
\mathrm{IV}_{m}=\ln \sum_{j \in B_{m}} \exp \left(V_{k} / \tau_{m}\right) \tag{1}
\end{equation*}
$$

where $B_{m}$ denotes the set of alternatives in nest $m$. Given the inclusive values, we can show that the probability that random-utility-maximizing decision maker $i$ chooses alternative $j$ is

$$
\operatorname{Pr}_{j}=\frac{\exp \left\{V_{j} / \tau(j)\right\}}{\exp \{\operatorname{IV}(j)\}} \frac{\exp \{\tau(j) \operatorname{IV}(j)\}}{\sum_{m} \exp \left(\tau_{m} \mathrm{IV}_{m}\right)}
$$

where $\tau(j)$ and $\operatorname{IV}(j)$ are the dissimilarity parameter and inclusive value for the nest in which alternative $j$ lies.

In contrast, for the nonnormalized model, we have a latent variable

$$
\widetilde{V}_{i, j}=\widetilde{\alpha}_{j}+\mathbf{x}_{i, j} \widetilde{\boldsymbol{\beta}}_{j}+\mathbf{z}_{i} \widetilde{\boldsymbol{\gamma}}_{j}
$$

and corresponding inclusive values

$$
\begin{equation*}
\widetilde{\mathrm{IV}}_{m}=\ln \sum_{j \in B_{m}} \exp \left(\widetilde{V}_{k}\right) \tag{2}
\end{equation*}
$$

The probability of choosing alternative $j$ is

$$
\operatorname{Pr}_{j}=\frac{\exp \left(\widetilde{V}_{j}\right)}{\exp \{\widetilde{\mathrm{IV}}(j)\}} \frac{\exp \{\tau(j) \widetilde{\mathrm{IV}}(j)\}}{\sum_{m} \exp \left(\tau_{m} \widetilde{\mathrm{IV}}_{m}\right)}
$$

Equations (1) and (2) represent the key difference between the RUM-consistent and nonnormalized models. By scaling the $V_{i j}$ within each nest, the RUM-consistent model allows utilities to be compared across nests. Without the rescaling, utilities can be compared only for goods within the same nest. Moreover, adding a constant to each $V_{i j}$ for consumer $i$ will not affect the probabilities of the RUMconsistent model, but adding a constant to each $\widetilde{V}_{i j}$ will affect the probabilities from the nonnormalized model. Decisions based on utility maximization can depend only on utility differences and not the scale or zero point of the utility function because utility is an ordinal concept, so the nonnormalized model cannot be consistent with utility maximization.

Heiss (2002) showed that the nonnormalized model can be RUM consistent in the special case where all the variables are specified in the bottom-level equation. Then multiplying the nonnormalized coefficients by the respective dissimilarity parameters results in the RUM-consistent coefficients.

## - Technical note

Degenerate nests occur when there is only one alternative in a branch of the tree hierarchy. The associated dissimilarity parameter of the RUM model is not defined. The inclusive-valued parameter of the nonnormalized model will be identifiable if there are alternative-specific variables specified in (1) of the model specification (the indepvars in the model syntax). Numerically, you can skirt the issue of nonidentifiable/undefined parameters by setting constraints on them. For the RUM model constraint, set the dissimilarity parameter to 1 . See the description of constraints () in Options for details on setting constraints on the dissimilarity parameters.

## Stored results

nlogit stores the following in e():

| Scalars |  |
| :---: | :---: |
| e(N) | number of observations |
| e(N_case) | number of cases |
| e(k_eq) | number of equations in e(b) |
| e(k_eq_model) | number of equations in overall model test |
| e(k_alt) | number of alternatives for bottom level |
| e(k_altj) | number of alternatives for $j$ th level |
| e(k_indvars) | number of independent variables |
| e(k_ind2vars) | number of by-alternative variables for bottom level |
| e(k_ind2varsj) | number of by-alternative variables for $j$ th level |
| e (df_m) | model degrees of freedom |
| e (df_c) | clogit model degrees of freedom |
| e(ll) | log likelihood |
| e(ll_c) | clogit model log likelihood |
| e(N_clust) | number of clusters |
| e(chi2) | $\chi^{2}$ |
| e(chi2_c) | likelihood-ratio test for IIA |
| e(p) | $p$-value for model Wald test |
| e(p_c) | $p$-value for IIA test |
| e(i_base) | base index for bottom level |
| e(i_basej) | base index for $j$ th level |
| e(levels) | number of levels |
| e(alt_min) | minimum number of alternatives |
| e(alt_avg) | average number of alternatives |
| e(alt_max) | maximum number of alternatives |
| e(const) | constant indicator for bottom level |
| e(constj) | constant indicator for $j$ th level |
| e(rum) | 1 if RUM model, 0 otherwise |
| e(rank) | rank of e(V) |
| e(ic) | number of iterations |
| e(rc) | return code |
| e(converged) | 1 if converged, 0 otherwise |

```
Macros
    e(cmd) nlogit
    e(cmdline) command as typed
    e(depvar)
    e(indvars)
    e(ind2vars)
    e(ind2varsj)
    e(case)
    e(altvar)
    e(altvarj)
    e(alteqs)
    e(alteqsj)
    e(alti)
    e(altj_i)
    e(wtype)
    e(wexp)
    e(title)
    e(clustvar)
    e(chi2type)
    e(vce)
    e(vcetype)
    e(opt)
    e(which)
    e(ml_method)
    e(user)
    e(technique)
    e(datasignature)
    e(datasignaturevars) variables used in calculation of checksum
    e(properties) b V
    e(estat_cmd) program used to implement estat
    e(predict) program used to implement predict
    e(marginsnotok)
    predictions disallowed by margins
Matrices
    e(b)
    e(Cns)
e(k_altern)
e(k_branchj)
e(stats)
e(statsj)
e(altidxj)
e(alt_ind2vars)
e(alt_ind2varsj)
e(ilog)
e(gradient)
e(V)
e(V_modelbased)
coefficient vector
constraints matrix
number of alternatives at each level
number of branches at each alternative of jth level
alternative statistics for bottom level
alternative statistics for jth level
alternative indices for jth level
indicators for bottom level estimated by-alternative
    variable-e(k_alt)}\timese(k_ind2vars
indicators for jth level estimated by-alternative variable-e(k_altj) }\timese(k_ind2varsj
iteration log (up to 20 iterations)
gradient vector
variance-covariance matrix of the estimators
model-based variance
Functions
e(sample) marks estimation sample
```


## Methods and formulas

Methods and formulas are presented under the following headings:

[^54]
## Two-level nested logit model

Consider our two-level nested logit model for restaurant choice. We define $T=\{1,2,3\}$ to be the set of indices denoting the three restaurant types and $R_{1}=\{1,2\}, R_{2}=\{3,4,5\}$, and $R_{3}=\{6,7\}$ to be the set of indices representing each restaurant within type $t \in T$. Let $C_{1}$ and $C_{2}$ be the random variables that represent the choices made for the first level, restaurant type, and second level, restaurant, of the hierarchy, where we observe the choices $C_{1}=t, t \in T$, and $C_{2}=j, j \in R_{t}$. Let $\mathbf{z}_{t}$ and $\mathbf{x}_{t j}$, for $t \in T$ and $j \in R_{t}$, refer to the row vectors of explanatory variables for the first-level alternatives and bottom-level alternatives for one case, respectively. We write the utilities (latent variables) as $U_{t j}=\mathbf{z}_{t} \boldsymbol{\alpha}_{t}+\mathbf{x}_{t j} \boldsymbol{\beta}_{j}+\epsilon_{t j}=\boldsymbol{\eta}_{t j}+\epsilon_{t j}$, where $\boldsymbol{\alpha}_{t}$ and $\boldsymbol{\beta}_{j}$ are column vectors and the $\epsilon_{t j}$ are random disturbances. When the $\mathbf{x}_{t j}$ are alternative specific, we can drop the indices from $\boldsymbol{\beta}$, where we estimate one coefficient for each alternative in $R_{t}, t \in T$. These variables are specified in the first equation of the nlogit syntax (see example 3).

When the random-utility framework is used to describe the choice behavior, the alternative that is chosen is the alternative that has the highest utility. Assume for our restaurant example that we choose restaurant type $t \in T$. For the RUM parameterization of nlogit, the conditional distribution of $\epsilon_{t j}$ given choice of restaurant type $t$ is a multivariate version of Gumbel's extreme-value distribution,

$$
\begin{equation*}
F_{R \mid T}(\boldsymbol{\epsilon} \mid t)=\exp \left[-\left\{\sum_{m \in R_{t}} \exp \left(\epsilon_{t m} / \tau_{t}\right)\right\}^{\tau_{t}}\right] \tag{3}
\end{equation*}
$$

where it has been shown that the $\epsilon_{t j}, j \in R_{t}$, are exchangeable with correlation $1-\tau_{t}^{2}$, for $\tau_{t} \in(0,1]$ (Kotz and Nadarajah 2000). For example, the probability of choosing Christopher's, $j=6$ given type $t=3$, is

$$
\begin{aligned}
\operatorname{Pr}\left(C_{2}=6 \mid C_{1}=3\right) & =\operatorname{Pr}\left(U_{36}-U_{37}>0\right) \\
& =\operatorname{Pr}\left(\epsilon_{37} \leq \epsilon_{36}+\eta_{36}-\eta_{37}\right) \\
& =\int_{-\infty}^{\infty}\left\{\int_{-\infty}^{\epsilon_{36}+\eta_{36}-\eta_{37}} f_{R \mid T}\left(\epsilon_{36}, \epsilon_{37}\right) d \epsilon_{37}\right\} d \epsilon_{36}
\end{aligned}
$$

where $f=\frac{\partial F}{\partial \epsilon_{36} \partial \epsilon_{37}}$ is the joint density function of $\epsilon$ given $t . U_{37}$ is the utility of eating at Mad Cows, the other fancy $(t=3)$ restaurant. Amemiya (1985) demonstrates that this integral evaluates to the logistic function

$$
\begin{aligned}
\operatorname{Pr}\left(C_{2}=6 \mid C_{1}=3\right) & =\frac{\exp \left(\boldsymbol{\eta}_{36} / \tau_{3}\right)}{\exp \left(\boldsymbol{\eta}_{36} / \tau_{3}\right)+\exp \left(\boldsymbol{\eta}_{37} / \tau_{3}\right)} \\
& =\frac{\exp \left(\mathbf{x}_{36} \boldsymbol{\beta}_{6} / \tau_{3}\right)}{\exp \left(\mathbf{x}_{36} \boldsymbol{\beta}_{6} / \tau_{3}\right)+\exp \left(\mathbf{x}_{37} \boldsymbol{\beta}_{7} / \tau_{3}\right)}
\end{aligned}
$$

and in general

$$
\begin{equation*}
\operatorname{Pr}\left(C_{2}=j \mid C_{1}=t\right)=\frac{\exp \left(\mathbf{x}_{t j} \boldsymbol{\beta}_{j} / \tau_{t}\right)}{\sum_{m \in R_{t}} \exp \left(\mathbf{x}_{t m} \boldsymbol{\beta}_{m} / \tau_{t}\right)} \tag{4}
\end{equation*}
$$

Letting $\tau_{t}=1$ in (3) reduces to the product of independent extreme-value distributions, and (4) reduces to the multinomial logistic function.

For the logistic function in (4), we scale the linear predictors by the dissimilarity parameters. Another formulation of the conditional probability of choosing alternative $j \in R_{t}$ given choice $t \in T$ is the logistic function without this normalization:

$$
\operatorname{Pr}\left(C_{2}=j \mid C_{1}=t\right)=\frac{\exp \left(\mathbf{x}_{t j} \boldsymbol{\beta}_{j}\right)}{\sum_{m \in R_{t}} \exp \left(\mathbf{x}_{t m} \boldsymbol{\beta}_{m}\right)}
$$

and this is what is used in nlogit's nonnormalized parameterization.
Amemiya (1985) defines the general form for the joint distribution of the $\epsilon$ 's as

$$
F_{T, R}(\boldsymbol{\epsilon})=\exp \left\{-\sum_{k \in T} \theta_{k}\left(\sum_{m \in R_{k}} \exp \left(-\epsilon_{k m} / \tau_{k}\right)\right)^{\tau_{k}}\right\}
$$

from which the probability of choice $t, t \in T$ can be derived as

$$
\begin{equation*}
\operatorname{Pr}\left(C_{1}=t\right)=\frac{\theta_{t}\left\{\sum_{m \in R_{t}} \exp \left(\boldsymbol{\eta}_{t m} / \tau_{t}\right)\right\}^{\tau_{t}}}{\sum_{k \in T} \theta_{k}\left\{\sum_{m \in R_{k}} \exp \left(\boldsymbol{\eta}_{k m} / \tau_{k}\right)\right\}^{\tau_{k}}} \tag{5}
\end{equation*}
$$

nlogit sets $\theta_{t}=1$. Noting that

$$
\begin{aligned}
\left\{\sum_{m \in R_{t}} \exp \left(\boldsymbol{\eta}_{t m} / \tau_{t}\right)\right\}^{\tau_{t}} & =\left\{\sum_{m \in R_{t}} \exp \left(\frac{\mathbf{z}_{t} \boldsymbol{\alpha}_{t}+\mathbf{x}_{t m} \boldsymbol{\beta}_{m}}{\tau_{t}}\right)\right\}^{\tau_{t}} \\
& =\exp \left(\mathbf{z}_{t} \boldsymbol{\alpha}_{t}\right)\left\{\sum_{m \in R_{t}} \exp \left(\mathbf{x}_{t m} \boldsymbol{\beta}_{m} / \tau_{t}\right)\right\}^{\tau_{t}} \\
& =\exp \left(\mathbf{z}_{t} \boldsymbol{\alpha}_{t}+\tau_{t} I_{t}\right)
\end{aligned}
$$

we define the inclusive values $I_{t}$ as

$$
I_{t}=\ln \left\{\sum_{m \in R_{t}} \exp \left(\mathbf{x}_{t m} \boldsymbol{\beta}_{m} / \tau_{t}\right)\right\}
$$

and we can view

$$
\exp \left(\tau_{t} I_{t}\right)=\left\{\sum_{m \in R_{t}} \exp \left(x_{t m} \boldsymbol{\beta}_{m}\right)^{1 / \tau_{t}}\right\}^{\tau_{t}}
$$

as a weighted average of the $\exp \left(x_{t m} \boldsymbol{\beta}_{m}\right)$, for $m \in R_{t}$. For the nlogit RUM parameterization, we can express (5) as

$$
\operatorname{Pr}\left(C_{1}=t\right)=\frac{\exp \left(\mathbf{z}_{t} \boldsymbol{\alpha}_{t}+\tau_{t} I_{t}\right)}{\sum_{k \in T} \exp \left(\mathbf{z}_{k} \boldsymbol{\alpha}_{k}+\tau_{k} I_{k}\right)}
$$

Next we define inclusive values for the nonnormalized model to be

$$
\widetilde{I}_{t}=\ln \left\{\sum_{m \in R_{t}} \exp \left(\mathbf{x}_{t m} \boldsymbol{\beta}_{m}\right)\right\}
$$

and we express $\operatorname{Pr}\left(C_{1}=t\right)$ as

$$
\begin{equation*}
\operatorname{Pr}\left(C_{1}=t\right)=\frac{\exp \left(\mathbf{z}_{t} \boldsymbol{\alpha}_{t}+\tau_{t} \widetilde{I}_{t}\right)}{\sum_{k \in T} \exp \left(\mathbf{z}_{k} \boldsymbol{\alpha}_{k}+\tau_{k} \widetilde{I}_{k}\right)} \tag{6}
\end{equation*}
$$

Equation (5) is consistent with (6) only when $\boldsymbol{\eta}_{i j}=\mathbf{x}_{i j} \boldsymbol{\beta}_{j}$, so in general the nlogit nonnormalized model is not consistent with the RUM model.

Now assume that we have $N$ cases where we add a third subscript, $i$, to denote case $i, i=1, \ldots, N$. Denote $y_{i t j}$ to be a binary variable indicating the choice made by case $i$ so that for each $i$ only one $y_{i t j}$ is 1 and the rest are 0 for all $t \in T$ and $j \in R_{t}$. The $\log$ likelihood for the two-level RUM-consistent model is

$$
\begin{aligned}
& \log \ell= \sum_{i=1}^{N} \sum_{k \in T} \sum_{m \in R_{k}} y_{i t j} \log \left\{\operatorname{Pr}\left(C_{i 1}=k\right) \operatorname{Pr}\left(C_{i 2}=m \mid C_{i 1}=k\right)\right\} \\
&= \sum_{i=1}^{N} \sum_{k \in T} \sum_{m \in R_{k}} y_{i t j}\left[\mathbf{z}_{i k} \boldsymbol{\alpha}_{k}+\tau_{k} I_{i k}-\log \left\{\sum_{l \in T} \exp \left(\mathbf{z}_{i l} \boldsymbol{\alpha}_{l}+\tau_{l} I_{i l}\right)\right\}+\right. \\
&\left.\quad \mathbf{x}_{i t j} \boldsymbol{\beta}_{m} / \tau_{k}-\log \left\{\sum_{l \in R_{k}} \exp \left(\mathbf{x}_{i k l} \boldsymbol{\beta}_{l} / \tau_{k}\right)\right\}\right]
\end{aligned}
$$

The likelihood for the nonnormalized model has a similar form, replacing $I$ with $\widetilde{I}$ and by not scaling $\mathbf{x}_{i k j} \boldsymbol{\beta}_{j}$ by $\tau_{k}$.

## Three-level nested logit model

Here we define a three-level nested logit model that can be generalized to the four-level and higher models. As before, let the integer set $T$ be the indices for the first level of choices. Let sets $S_{t}$, $t \in T$, be mutually exclusive sets of integers representing the choices of the second level of the hierarchy. Finally, let $R_{j}, j \in S_{t}$, be the bottom-level choices. Let $U_{t j k}=\eta_{t j k}+\epsilon_{t j k}, k \in R_{j}$, and the distribution of $\epsilon_{t j k}$ be Gumbel's multivariate extreme value of the form

$$
F(\boldsymbol{\epsilon})=\exp \left(-\sum_{t \in T}\left[\sum_{j \in S_{t}}\left\{\sum_{k \in R_{j}} \exp \left(-\eta_{t j k} / \tau_{j}\right)\right\}^{\tau_{j} / v_{t}}\right]^{v_{j}}\right)
$$

Let $C_{1}, C_{2}$, and $C_{3}$ represent the choice random variables for levels 1,2 , and the bottom, respectively. Then the set of conditional probabilities is

$$
\begin{aligned}
& \operatorname{Pr}\left(C_{3}=k \mid C_{1}=t, C_{2}=j\right)=\frac{\exp \left(\eta_{t j k} / \tau_{j}\right)}{\sum_{l \in R_{j}} \exp \left(\eta_{t j l} / \tau_{j}\right)} \\
& \operatorname{Pr}\left(C_{2}=j \mid C_{1}=t\right)=\frac{\left\{\sum_{k \in R_{j}} \exp \left(\eta_{t j k} / \tau_{j}\right)\right\}^{\tau_{j} / v_{t}}}{\sum_{l \in S_{t}}\left\{\sum_{k \in R_{l}} \exp \left(\eta_{t l k} / \tau_{l}\right)\right\}^{\tau_{l} / v_{t}}} \\
& \operatorname{Pr}\left(C_{1}=t\right)=\frac{\left[\sum_{j \in S_{t}}\left\{\sum_{k \in R_{j}} \exp \left(\eta_{t j k} / \tau_{j}\right)\right\}^{\tau_{j} / v_{t}}\right]^{v_{t}}}{\sum_{l \in T}\left[\sum_{j \in S_{l}}\left\{\sum_{k \in R_{j}} \exp \left(\eta_{l j k} / \tau_{j}\right)\right\}^{\tau_{j} / v_{l}}\right]^{v_{l}}}
\end{aligned}
$$

Assume that we can decompose the linear predictor as $\boldsymbol{\eta}_{t j k}=\mathbf{z}_{t} \boldsymbol{\alpha}_{t}+\mathbf{u}_{t j} \boldsymbol{\gamma}_{j}+\mathbf{x}_{t j k} \boldsymbol{\beta}_{k}$. Here $\mathbf{z}_{t}$, $\mathbf{u}_{t j}$, and $\mathbf{x}_{t j k}$ are the row vectors of explanatory variables for the first, second, and bottom levels of the hierarchy, respectively, and $\boldsymbol{\alpha}_{t}, \gamma_{j}$, and $\boldsymbol{\beta}_{k}$ are the corresponding column vectors of regression coefficients for $t \in T, j \in S_{t}$, and $k \in R_{j}$. We then can define the inclusive values for the first and second levels as

$$
\begin{aligned}
I_{t j} & =\log \sum_{k \in R_{j}} \exp \left(\mathbf{x}_{t j k} \boldsymbol{\beta}_{k} / \tau_{j}\right) \\
J_{t} & =\log \sum_{j \in S_{t}} \exp \left(\mathbf{u}_{t j} \boldsymbol{\gamma}_{j} / v_{t}+\frac{\tau_{j}}{v_{t}} I_{t j}\right)
\end{aligned}
$$

and rewrite the probabilities

$$
\begin{aligned}
& \operatorname{Pr}\left(C_{3}=k \mid C_{1}=t, C_{2}=j\right)=\frac{\exp \left(\mathbf{x}_{t j k} \boldsymbol{\beta}_{k} / \tau_{j}\right)}{\sum_{l \in R_{j}} \exp \left(\mathbf{x}_{t j l} \boldsymbol{\beta}_{l} / \tau_{j}\right)} \\
& \operatorname{Pr}\left(C_{2}=j \mid C_{1}=t\right)=\frac{\exp \left(\mathbf{u}_{t j} \gamma_{j} / v_{t}+\frac{\tau_{j}}{v_{t}} I_{t j}\right)}{\sum_{l \in S_{t}} \exp \left(\mathbf{u}_{t l} \boldsymbol{\gamma}_{l} / v_{t}+\frac{\tau_{l}}{v_{t}} I_{t l}\right)} \\
& \operatorname{Pr}\left(C_{1}=t\right)=\frac{\exp \left(\mathbf{z}_{t} \boldsymbol{\alpha}_{t}+v_{t} J_{t}\right)}{\sum_{l \in T} \exp \left(\mathbf{z}_{l} \boldsymbol{\alpha}_{l}+v_{l} J_{l}\right)}
\end{aligned}
$$

We add a fourth index, $i$, for case and define the indicator variable $y_{i t j k}, i=1, \ldots, N$, to indicate the choice made by case $i, t \in T, j \in S_{t}$, and $k \in R_{j}$. The log likelihood for the nlogit RUM-consistent model is

$$
\begin{gathered}
\ell=\sum_{i=1}^{N} \sum_{t \in T} \sum_{j \in S_{t}} \sum_{k \in R_{j}} y_{i t j k}\left\{\mathbf{z}_{i t} \boldsymbol{\alpha}_{t}+v_{t} J_{i t}-\log \left(\sum_{m \in T} \mathbf{z}_{i m} \boldsymbol{\alpha}_{m}+v_{m} J_{i m}\right)+\right. \\
\mathbf{u}_{i t j} \gamma_{j} / v_{t}+\frac{\tau_{j}}{v_{t}} I_{i t j}-\log \left(\sum_{m \in S_{t}} \mathbf{u}_{i t m} \boldsymbol{\gamma}_{m} / v_{t}+\frac{\tau_{m}}{v_{t}} I_{i t m}\right)+ \\
\left.\mathbf{x}_{i t j k} \boldsymbol{\beta}_{k} / \tau_{k}-\sum_{m \in R_{t}} \exp \left(\mathbf{x}_{i t j m} \boldsymbol{\beta}_{m} / \tau_{k}\right)\right\}
\end{gathered}
$$

and for the nonnormalized nlogit model the log likelihood is

$$
\begin{gathered}
\ell=\sum_{i=1}^{N} \sum_{t \in T} \sum_{j \in S_{t}} \sum_{k \in R_{j}} y_{i t j k}\left\{\mathbf{z}_{i t} \boldsymbol{\alpha}_{t}+v_{t} J_{i t}-\log \left(\sum_{m \in T} \mathbf{z}_{i m} \boldsymbol{\alpha}_{m}+v_{m} J_{i m}\right)+\right. \\
\mathbf{u}_{i t j} \gamma_{j}+\tau_{j} I_{i t j}-\log \left(\sum_{m \in S_{t}} \mathbf{u}_{i t m} \boldsymbol{\gamma}_{m}+\tau_{m} I_{i t m}\right)+ \\
\left.\mathbf{x}_{i t j k} \boldsymbol{\beta}_{k}-\sum_{m \in R_{t}} \exp \left(\mathbf{x}_{i t j m} \boldsymbol{\beta}_{m}\right)\right\}
\end{gathered}
$$

Extending the model to more than three levels is straightforward, albeit notationally cumbersome.
This command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce (robust) and vce (cluster clustvar), respectively. See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas.

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## Also see

[R] nlogit postestimation - Postestimation tools for nlogit
$[\mathrm{R}]$ asclogit - Alternative-specific conditional logit (McFadden's choice) model
[R] clogit - Conditional (fixed-effects) logistic regression
[R] mlogit - Multinomial (polytomous) logistic regression
[R] ologit - Ordered logistic regression
[R] rologit - Rank-ordered logistic regression
[R] slogit - Stereotype logistic regression
[U] 20 Estimation and postestimation commands

## Title

nlogit postestimation - Postestimation tools for nlogit

| Description | Syntax for predict | Menu for predict |
| :--- | :--- | :--- |
| Options for predict | Syntax for estat alternatives | Menu for estat |
| Remarks and examples | Also see |  |

## Description

The following postestimation command is of special interest after nlogit:
Command Description
estat alternatives
alternative summary statistics

The following standard postestimation commands are also available:

| Command | Description |
| :--- | :--- |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample <br> estat vce <br> variance-covariance matrix of the estimators (VCE) <br> estimates <br> hausman <br> lincom |
| cataloging estimation results |  |
| Hausman's specification test |  |
| point estimates, standard errors, testing, and inference for linear combinations |  |
| of coefficients |  |$\quad$| likelihood-ratio test |
| :--- |
| nlcom |$\quad$| point estimates, standard errors, testing, and inference for nonlinear combinations |
| :--- |
| of coefficients |

## Special-interest postestimation command

estat alternatives displays summary statistics about the alternatives in the estimation sample for each level of the tree structure.

## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic hlevel(#) altwise]
predict [type] {stub*|newvarlist } [if] [in], scores
```

statistic Description
Main
$\underline{\mathrm{pr}} \quad$ predicted probabilities of choosing the alternatives at all levels of the hierarchy or at level \#, where \# is specified by hlevel(\#); the default
xb linear predictors for all levels of the hierarchy or at level \#, where \# is specified by hlevel(\#)
condp predicted conditional probabilities at all levels of the hierarchy or at level \#, where \# is specified by hlevel (\#)
iv inclusive values for levels 2,..., e(levels) or for hlevel(\#)
The inclusive value for the first-level alternatives is not used in estimation; therefore, it is not calculated.
These statistics are available both in and out sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
pr calculates the probability of choosing each alternative at each level of the hierarchy. Use the hlevel(\#) option to compute the alternative probabilities at level \#. When hlevel(\#) is not specified, $j$ new variables must be given, where $j$ is the number of levels, or use the stub* option to have predict generate $j$ variables with the prefix stub and numbered from 1 to $j$. The pr option is the default and if one new variable is given, the probability of the bottom-level alternatives are computed. Otherwise, probabilities for all levels are computed and the stub* option is still valid.
xb calculates the linear prediction for each alternative at each level. Use the hlevel (\#) option to compute the linear predictor at level \#. When hlevel(\#) is not specified, $j$ new variables must be given, where $j$ is the number of levels, or use the stub* option to have predict generate $j$ variables with the prefix stub and numbered from 1 to $j$.
condp calculates the conditional probabilities for each alternative at each level. Use the hlevel (\#) option to compute the conditional probabilities of the alternatives at level \#. When hlevel (\#) is not specified, $j$ new variables must be given, where $j$ is the number of levels, or use the stub* option to have predict generate $j$ variables with the prefix stub and numbered from 1 to $j$.
iv calculates the inclusive value for each alternative at each level. Use the hlevel (\#) option to compute the inclusive value at level \#. There is no inclusive value at level 1. If hlevel(\#) is not used, $j-1$ new variables are required, where $j$ is the number of levels, or use stub* to have predict generate $j-1$ variables with the prefix stub and numbered from 2 to $j$. See Methods and formulas in $[R]$ nlogit for a definition of the inclusive values.
hlevel(\#) calculates the prediction only for hierarchy level \#.
altwise specifies that alternativewise deletion be used when marking out observations due to missing values in your variables. The default is to use casewise deletion. The xb option always uses alternativewise deletion.
scores calculates the scores for each coefficient in e(b). This option requires a new-variable list of length equal to the number of columns in e(b). Otherwise, use the stub* option to have predict generate enumerated variables with prefix stub.

## Syntax for estat alternatives

estat alternatives

## Menu for estat

Statistics $>$ Postestimation $>$ Reports and statistics

## Remarks and examples

predict may be used after nlogit to obtain the predicted values of the probabilities, the conditional probabilities, the linear predictions, and the inclusive values for each level of the nested logit model. Predicted probabilities for nlogit must be interpreted carefully. Probabilities are estimated for each case as a whole and not for individual observations.

## > Example 1

Continuing with our model in example 3 of $[\mathrm{R}]$ nlogit, we refit the model and then examine a summary of the alternatives and their frequencies in the estimation sample.

```
. use http://www.stata-press.com/data/r13/restaurant
. nlogitgen type = restaurant(fast: Freebirds | MamasPizza,
> family: CafeEccell | LosNortenos | WingsNmore, fancy: Christophers | MadCows)
    (output omitted)
. nlogit chosen cost rating distance || type: income kids, base(family) ||
> restaurant:, noconst case(family_id)
    (output omitted)
```

. estat alternatives
Alternatives summary for type

| index | Alternative <br> value | label | Cases <br> present | Frequency <br> selected | Percent <br> selected |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 1 | fast | 600 | 27 | 9.00 |
| 2 | 2 | family | 900 | 222 | 74.00 |
| 3 | 3 | fancy | 600 | 51 | 17.00 |

Alternatives summary for restaurant

| index | Alternative <br> value | label | Cases <br> present | Frequency <br> selected | Percent <br> selected |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 1 | Freebirds | 300 | 12 | 4.00 |
| 2 | 2 | MamasPizza | 300 | 15 | 5.00 |
| 3 | 3 | CafeEccell | 300 | 78 | 26.00 |
| 4 | 4 | LosNortenos | 300 | 75 | 25.00 |
| 5 | 5 | WingsNmore | 300 | 69 | 23.00 |
| 6 | 6 | Christophers | 300 | 27 | 9.00 |
| 7 | 7 | MadCows | 300 | 24 | 8.00 |

Next we predict $\mathrm{p} 2=\operatorname{Pr}($ restaurant $) ; \mathrm{p} 1=\operatorname{Pr}($ type $) ;$ condp $=\operatorname{Pr}($ restaurant $\mid$ type $) ;$ xb2, the linear prediction for the bottom-level alternatives; xb1, the linear prediction for the first-level alternatives; and iv, the inclusive values for the bottom-level alternatives.

```
. predict p*
(option pr assumed)
. predict condp, condp hlevel(2)
. sort family_id type restaurant
. list restaurant type chosen p2 p1 condp in 1/14, sepby(family_id) divider
```

|  | restaurant | type | chosen | p2 | p1 | condp |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | Freebirds | fast | 1 | . 0642332 | . 1189609 | . 5399519 |
| 2. | MamasPizza | fast | 0 | . 0547278 | . 1189609 | . 4600481 |
| 3. | CafeEccell | family | 0 | . 284409 | . 7738761 | . 3675124 |
| 4. | LosNortenos | family | 0 | . 3045242 | . 7738761 | . 3935051 |
| 5. | WingsNmore | family | 0 | . 1849429 | . 7738761 | . 2389825 |
| 6. | Christophers | fancy | 0 | . 0429508 | . 107163 | . 4007991 |
| 7. | MadCows | fancy | 0 | . 0642122 | . 107163 | . 5992009 |
| 8. | Freebirds | fast | 0 | . 0183578 | . 0488948 | . 3754559 |
| 9. | MamasPizza | fast | 0 | . 030537 | . 0488948 | . 6245441 |
| 10. | CafeEccell | family | 0 | . 2832149 | . 756065 | . 3745907 |
| 11. | LosNortenos | family | 1 | . 3038883 | . 756065 | . 4019341 |
| 12. | WingsNmore | family | 0 | . 1689618 | . 756065 | . 2234752 |
| 13. | Christophers | fancy | 0 | . 1041277 | . 1950402 | . 533878 |
| 14. | MadCows | fancy | 0 | . 0909125 | . 1950402 | . 466122 |

predict $x b *$, $x b$

- predict iv, iv
. list restaurant type chosen $x b *$ iv in $1 / 14$, sepby (family_id) divider

|  | restaurant | type | chosen | xb1 | xb2 | iv |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | Freebirds | fast | 1 | -1.124805 | -1.476914 | -. 2459659 |
| 2. | MamasPizza | fast | 0 | -1.124805 | -1.751229 | -. 2459659 |
| 3. | CafeEccell | family | 0 | 0 | -2.181112 | . 1303341 |
| 4. | LosNortenos | family | 0 | 0 | -2.00992 | . 1303341 |
| 5. | WingsNmore | family | 0 | 0 | -3.259229 | . 1303341 |
| 6. | Christophers | fancy | 0 | 1.405185 | -6.804211 | -. 745332 |
| 7. | MadCows | fancy | 0 | 1.405185 | -5.155514 | -. 745332 |
| 8. | Freebirds | fast | 0 | -1.804794 | -2.552233 | -. 5104123 |
| 9. | MamasPizza | fast | 0 | -1.804794 | -1.680583 | -. 5104123 |
| 10. | CafeEccell | family | 0 | 0 | -2.400434 | . 0237072 |
| 11. | LosNortenos | family | 1 | 0 | -2. 223939 | . 0237072 |
| 12. | WingsNmore | family | 0 | 0 | -3.694409 | . 0237072 |
| 13. | Christophers | fancy | 0 | 1.490775 | -5.35932 | -. 6796131 |
| 14. | MadCows | fancy | 0 | 1.490775 | -5.915751 | -. 6796131 |

## Also see

[R] nlogit — Nested logit regression
[U] 20 Estimation and postestimation commands

## Title

nlsur - Estimation of nonlinear systems of equations

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |
| Also see |  |  |  |

## Syntax

Interactive version

$$
\text { nlsur }\left(\text { depvar_1 }=\left\langle\operatorname{sexp} \beta_{-} 1>\right)\left(\text { depvar_2 }=\left\langle\operatorname{sexp} \_2>\right) \ldots[\text { if }][\text { in }][\text { weight }][\text {, options }]\right.\right.
$$

Programmed substitutable expression version

```
nlsur sexp_prog : depvar_1 depvar_2 ... [varlist] [if ] [in] [weight] [,options]
```

Function evaluator program version
nlsur func_prog @ depvar_1 depvar_2 ... [varlist] [if][in] [weight],

```
nequations(#) { parameters(namelist)| nparameters(#)}[options]
```

where
depvar $j$ is the dependent variable for equation $j$;
$<\operatorname{sexp}>{ }_{-} j$ is the substitutable expression for equation $j$;
sexp_prog is a substitutable expression program; and func_prog is a function evaluator program.
options
Model
fgnls
ifgnls
nls
variables (varlist)
initial(initial_values)
nequations(\#)

* parameters (namelist)
* nparameters (\#)
sexp_options
func_options
SE/Robust
vce (vcetype)

Reporting
level(\#)
title(string)
title2(string)
display_options
Optimization
optimization_options
eps(\#)
ifgnlsiterate(\#)
ifgnlseps(\#)
delta(\#)
noconstants
hasconstants (namelist)
coeflegend

Description
use two-step FGNLS estimator; the default
use iterative FGNLS estimator
use NLS estimator
variables in model
initial values for parameters
number of equations in model (function evaluator program version only)
parameters in model (function evaluator program version only)
number of parameters in model
(function evaluator program version only)
options for substitutable expression program
options for function evaluator program
vcetype may be gnr, robust, cluster clustvar, bootstrap, or jackknife
set confidence level; default is level (95)
display string as title above the table of parameter estimates display string as subtitle
control column formats and line width
control the optimization process; seldom used
specify \# for convergence criteria; default is eps ( $1 \mathrm{e}-5$ )
set maximum number of FGNLS iterations
specify \# for FGNLS convergence criterion; default is ifgnlseps (1e-10)
specify stepsize \# for computing derivatives; default is delta (4e-7)
no equations have constant terms
use namelist as constant terms
display legend instead of statistics

* You must specify parameters (namelist), nparameters(\#), or both. bootstrap, by, jackknife, rolling, and statsby are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the bootstrap prefix; see $[R]$ bootstrap.
aweights are not allowed with the jackknife prefix; see [R] jackknife.
aweights, fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.


## Menu

Statistics $>$ Linear models and related $>$ Multiple-equation models $>$ Nonlinear seemingly unrelated regression

## Description

nlsur fits a system of nonlinear equations by feasible generalized nonlinear least squares (FGNLS). With the interactive version of the command, you enter the system of equations on the command line or in the dialog box by using substitutable expressions. If you have a system that you use regularly, you can write a substitutable expression program and use the second syntax to avoid having to reenter the system every time. The function evaluator program version gives you the most flexibility in exchange for increased complexity; with this version, your program is given a vector of parameters and a variable list, and your program computes the system of equations.

When you write a substitutable expression program or a function evaluator program, the first five letters of the name must be nlsur. sexp_prog and func_prog refer to the name of the program without the first five letters. For example, if you wrote a function evaluator program named nlsurregss, you would type nlsur regss © ... to estimate the parameters.

## Options

## Model

fgnls requests the two-step FGNLS estimator; this is the default.
ifgnls requests the iterative FGNLS estimator. For the nonlinear systems estimator, this is equivalent to maximum likelihood estimation.
nls requests the nonlinear least-squares (NLS) estimator.
variables (varlist) specifies the variables in the system. nlsur ignores observations for which any of these variables has missing values. If you do not specify variables(), nlsur issues an error message if the estimation sample contains any missing values.
initial (initial_values) specifies the initial values to begin the estimation. You can specify a $1 \times k$ matrix, where $k$ is the total number of parameters in the system, or you can specify a parameter name, its initial value, another parameter name, its initial value, and so on. For example, to initialize alpha to 1.23 and delta to 4.57, you would type

```
. nlsur ..., initial(alpha 1.23 delta 4.57) ...
```

Initial values declared using this option override any that are declared within substitutable expressions. If you specify a matrix, the values must be in the same order in which the parameters are declared in your model. nlsur ignores the row and column names of the matrix.
nequations (\#) specifies the number of equations in the system.
parameters (namelist) specifies the names of the parameters in the system. The names of the parameters must adhere to the naming conventions of Stata's variables; see [U] 11.3 Naming conventions. If you specify both parameters() and nparameters(), the number of names in the former must match the number specified in the latter.
nparameters (\#) specifies the number of parameters in the system. If you do not specify names with the parameters () options, nlsur names them b1, b2, .., b\#. If you specify both parameters() and nparameters(), the number of names in the former must match the number specified in the latter.
sexp_options refer to any options allowed by your sexp_prog.
func_options refer to any options allowed by your func_prog.
vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (gnr), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
vce (gnr), the default, uses the conventionally derived variance estimator for nonlinear models fit using Gauss-Newton regression.

Reporting
level(\#); see [R] estimation options.
title (string) specifies an optional title that will be displayed just above the table of parameter estimates.
title2(string) specifies an optional subtitle that will be displayed between the title specified in title() and the table of parameter estimates. If title2() is specified but title() is not, title2() has the same effect as title().
display_options: cformat (\%fmt), pformat (\%fmt), sformat (\%fmt), and nolstretch; see [R] estimation options.

Optimization
optimization_options: iterate(\#), [no]log, trace. iterate() specifies the maximum number of iterations to use for NLS at each round of FGNLS estimation. This option is different from ifgnlsiterate(), which controls the maximum rounds of FGNLS estimation to use when the ifgnls option is specified. $\log / n o l o g$ specifies whether to show the iteration $\log$, and trace specifies that the iteration $\log$ should include the current parameter vector.
eps (\#) specifies the convergence criterion for successive parameter estimates and for the residual sum of squares (RSS). The default is eps ( $1 \mathrm{e}-5$ ) ( 0.00001 ). eps () also specifies the convergence criterion for successive parameter estimates between rounds of iterative FGNLS estimation when ifgnls is specified.
ifgnlsiterate(\#) specifies the maximum number of FGNLS iterations to perform. The default is the number set using set maxiter (see [R] maximize), which is 16,000 by default. To use this option, you must also specify the ifgnls option.
ifgnlseps (\#) specifies the convergence criterion for successive estimates of the error covariance matrix during iterative FGNLS estimation. The default is ifgnlseps ( $1 \mathrm{e}-10$ ). To use this option, you must also specify the ifgnls option.
delta(\#) specifies the relative change in a parameter, $\delta$, to be used in computing the numeric derivatives. The derivative for parameter $\beta_{i}$ is computed as

$$
\left\{f_{i}\left(\mathbf{x}_{i}, \beta_{1}, \beta_{2}, \ldots, \beta_{i}+d, \beta_{i+1}, \ldots\right)-f_{i}\left(\mathbf{x}_{i}, \beta_{1}, \beta_{2}, \ldots, \beta_{i}, \beta_{i+1}, \ldots\right)\right\} / d
$$

where $d=\delta\left(\left|\beta_{i}\right|+\delta\right)$. The default is delta(4e-7).
noconstants indicates that none of the equations in the system includes constant terms. This option is generally not needed, even if there are no constant terms in the system; though in rare cases without this option, nlsur may claim that there is one or more constant terms even if there are none.
hasconstants (namelist) indicates the parameters that are to be treated as constant terms in the system of equations. The number of elements of namelist must equal the number of equations in the system. The $i$ th entry of namelist specifies the constant term in the $i$ th equation. If an equation does not include a constant term, specify a period (.) instead of a parameter name. This option is seldom needed with the interactive and programmed substitutable expression versions, because in those cases nlsur can almost always find the constant terms automatically.

The following options are available with nlsur but are not shown in the dialog box:
coeflegend; see $[R]$ estimation options.

## Remarks and examples

Remarks are presented under the following headings:
Introduction
Substitutable expression programs
Function evaluator programs

## Introduction

nlsur fits a system of nonlinear equations by FGNLS. It can be viewed as a nonlinear variant of Zellner's seemingly unrelated regression model (Zellner 1962; Zellner and Huang 1962; Zellner 1963) and is therefore commonly called nonlinear SUR or nonlinear SURE. The model is also discussed in textbooks such as Davidson and MacKinnon (1993, 2004) and Greene (2012, 305-306). Formally, the model fit by nlsur is

$$
\begin{aligned}
y_{i 1} & =f_{1}\left(\mathbf{x}_{i}, \boldsymbol{\beta}\right)+u_{i 1} \\
y_{i 2} & =f_{2}\left(\mathbf{x}_{i}, \boldsymbol{\beta}\right)+u_{i 2} \\
\vdots & =\quad \vdots \\
y_{i M} & =f_{M}\left(\mathbf{x}_{i}, \boldsymbol{\beta}\right)+u_{i M}
\end{aligned}
$$

for $i=1, \ldots, N$ observations and $m=1, \ldots, M$ equations. The errors for the $i$ th observation, $u_{i 1}, u_{i 2}, \ldots, u_{i M}$, may be correlated, so fitting the $m$ equations jointly may lead to more efficient estimates. Moreover, fitting the equations jointly allows us to impose cross-equation restrictions on the parameters. Not all elements of the parameter vector $\boldsymbol{\beta}$ and data vector $\mathbf{x}_{i}$ must appear in all the equations, though each element of $\boldsymbol{\beta}$ must appear in at least one equation for $\boldsymbol{\beta}$ to be identified. For this model, iterative FGNLS estimation is equivalent to maximum likelihood estimation with multivariate normal disturbances.

The syntax you use with nlsur closely mirrors that used with nl. In particular, you use substitutable expressions with the interactive and programmed substitutable expression versions to define the functions in your system. See $[R] \mathbf{n l}$ for more information on substitutable expressions. Here we reiterate the three rules that you must follow:

1. Parameters of the model are bound in braces: $\{b 0\}$, \{param\}, etc.
2. Initial values for parameters are given by including an equal sign and the initial value inside the braces: $\{\mathrm{b} 0=1\}$, \{param=3.571\}, etc. If you do not specify an initial value, that parameter is initialized to zero. The initial() option overrides initial values in substitutable expressions.
3. Linear combinations of variables can be included using the notation \{eqname:varlist\}, for example, $\{\mathrm{xb}$ : mpg price weight\}, \{score: wx z\}, etc. Parameters of linear combinations are initialized to zero.

## Example 1: Interactive version using two-step FGNLS estimator

We have data from an experiment in which two closely related types of bacteria were placed in a Petri dish, and the number of each type of bacteria were recorded every hour. We suspect a two-parameter exponential growth model can be used to model each type of bacteria, but because they shared the same dish, we want to allow for correlation in the error terms. We want to fit the system of equations

$$
\begin{aligned}
& p_{1}=\beta_{1} \beta_{2}^{t}+u_{1} \\
& p_{2}=\gamma_{1} \gamma_{2}^{t}+u_{2}
\end{aligned}
$$

where $p_{1}$ and $p_{2}$ are the two populations and $t$ is time, and we want to allow for nonzero correlation between $u_{1}$ and $u_{2}$. We type

```
. use http://www.stata-press.com/data/r13/petridish
. nlsur (p1 = {b1}*{b2}^t) (p2 = {g1}*{g2}^t)
(obs = 25)
Calculating NLS estimates...
Iteration 0: Residual SS = 335.5286
Iteration 1: Residual SS = 333.8583
Iteration 2: Residual SS = 219.9233
Iteration 3: Residual SS = 127.9355
Iteration 4: Residual SS = 14.86765
Iteration 5: Residual SS = 8.628459
Iteration 6: Residual SS = 8.281268
Iteration 7: Residual SS = 8.28098
Iteration 8: Residual SS = 8.280979
Iteration 9: Residual SS = 8.280979
Calculating FGNLS estimates...
Iteration 0: Scaled RSS = 49.99892
Iteration 1: Scaled RSS = 49.99892
Iteration 2: Scaled RSS = 49.99892
FGNLS regression
```

|  | Equation | Obs | Parms | RMSE | R-sq | Constant |
| :--- | ---: | ---: | ---: | ---: | :---: | ---: |
| 1 | p1 | 25 | 2 | .4337019 | $0.9734 *$ | (none) |
| 2 | p2 | 25 | 2 | .3783479 | $0.9776 *$ | (none) |


| * Uncentered R-sq |
| :--- |

The header of the output contains a summary of each equation, including the number of observations and parameters and the root mean squared error of the residuals. nlsur checks to see whether each equation contains a constant term, and if an equation does contain a constant term, an $R^{2}$ statistic is
presented. If an equation does not have a constant term, an uncentered $R^{2}$ is instead reported. The $R^{2}$ statistic for each equation measures the percentage of variance explained by the nonlinear function and may be useful for descriptive purposes, though it does not have the same formal interpretation in the context of FGNLS as it does with NLS estimation. As we would expect, $\beta_{2}$ and $\gamma_{2}$ are both greater than one, indicating the two bacterial populations increased in size over time.

The model we fit in the next three examples is in fact linear in the parameters, so it could be fit using the sureg command. However, we will fit the model using nlsur so that we can focus on the mechanics of using the command. Moreover, using nlsur will obviate the need to generate several variables as well as the need to use the constraint command to impose parameter restrictions.

## Example 2: Interactive version using iterative FGNLS estimator-the translog production function

Greene (1997, sec. 15.6) discusses the transcendental logarithmic (translog) cost function and provides cost and input price data for capital, labor, energy, and materials for the U.S. economy. One way to fit the translog production function to these data is to fit the system of three equations

$$
\begin{aligned}
s_{k} & =\beta_{k}+\delta_{k k} \ln \left(\frac{p_{k}}{p_{m}}\right)+\delta_{k l} \ln \left(\frac{p_{l}}{p_{m}}\right)+\delta_{k e} \ln \left(\frac{p_{e}}{p_{m}}\right)+u_{1} \\
s_{l} & =\beta_{l}+\delta_{k l} \ln \left(\frac{p_{k}}{p_{m}}\right)+\delta_{l l} \ln \left(\frac{p_{l}}{p_{m}}\right)+\delta_{l e} \ln \left(\frac{p_{e}}{p_{m}}\right)+u_{2} \\
s_{e} & =\beta_{e}+\delta_{k e} \ln \left(\frac{p_{k}}{p_{m}}\right)+\delta_{l e} \ln \left(\frac{p_{l}}{p_{m}}\right)+\delta_{e e} \ln \left(\frac{p_{e}}{p_{m}}\right)+u_{3}
\end{aligned}
$$

where $s_{k}$ is capital's cost share, $s_{l}$ is labor's cost share, and $s_{e}$ is energy's cost share; $p_{k}, p_{l}, p_{e}$, and $p_{m}$ are the prices of capital, labor, energy, and materials, respectively; the $u$ 's are regression error terms; and the $\beta \mathrm{s}$ and $\delta \mathrm{s}$ are parameters to be estimated. There are three cross-equation restrictions on the parameters: $\delta_{k l}, \delta_{k e}$, and $\delta_{l e}$ each appear in two equations. To fit this model by using the iterative FGNLS estimator, we type

```
. use http://www.stata-press.com/data/r13/mfgcost
. nlsur (s_k = {bk} + {dkk}*ln(pk/pm) + {dkl}*ln(pl/pm) + {dke}*ln(pe/pm))
> (s_l = {bl} + {dkl}*ln(pk/pm) + {dll}*ln(pl/pm) + {dle}*ln(pe/pm))
> (s_e = {be} + {dke}*ln(pk/pm) + {dle}*ln(pl/pm) + {dee}*ln(pe/pm)),
> ifgnls
(obs = 25)
Calculating NLS estimates...
Iteration 0: Residual SS = .0009989
Iteration 1: Residual SS = .0009989
Calculating FGNLS estimates...
Iteration 0: Scaled RSS = 65.45197
Iteration 1: Scaled RSS = 65.45197
    (output omitted)
FGNLS iteration 10...
Iteration 0: Scaled RSS = 75
Iteration 1: Scaled RSS = 75
Parameter change = 4.074e-06
Covariance matrix change = 6.265e-10
FGNLS regression
```

|  | Equation | Obs | Parms | RMSE | R-sq | Constant |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | s_k | 25 | 4 | .0031722 | 0.4776 | bk |
| 2 | s_1 | 25 | 4 | .0053963 | 0.8171 | bl |
| 3 | s_e | 25 | 4 | .00177 | 0.6615 | be |


|  | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| /bk | .0568925 | .0013454 | 42.29 | 0.000 | .0542556 | .0595294 |
| /dkk | .0294833 | .0057956 | 5.09 | 0.000 | .0181241 | .0408425 |
| /dkl | -.0000471 | .0038478 | -0.01 | 0.990 | -.0075887 | .0074945 |
| /dke | -.0106749 | .0033882 | -3.15 | 0.002 | -.0173157 | -.0040341 |
| /bl | .253438 | .0020945 | 121.00 | 0.000 | .2493329 | .2575432 |
| /dll | .0754327 | .0067572 | 11.16 | 0.000 | .0621889 | .0886766 |
| /dle | -.004756 | .002344 | -2.03 | 0.042 | -.0093501 | -.0001619 |
| /be | .0444099 | .0008533 | 52.04 | 0.000 | .0427374 | .0460823 |
| /dee | .0183415 | .0049858 | 3.68 | 0.000 | .0085694 | .0281135 |

We draw your attention to the iteration log at the top of the output. When iterative FGNLS estimation is used, the final scaled RSS will equal the product of the number of observations in the estimation sample and the number of equations; see Methods and formulas for details. Because the RSS is scaled by the error covariance matrix during each round of FGNLS estimation, the scaled RSS is not comparable from one FGNLS iteration to the next.

## $\square$ Technical note

You may have noticed that we mentioned having data for four factors of production, yet we fit only three share equations. Because the four shares sum to one, we must drop one of the equations to avoid having a singular error covariance matrix. The iterative FGNLS estimator is equivalent to maximum likelihood estimation, and thus it is invariant to which one of the four equations we choose to drop. The (linearly restricted) parameters of the fourth equation can be obtained using the lincom command. Nonlinear functions of the parameters, such as the elasticities of substitution, can be computed using nlcom.

## Substitutable expression programs

If you fit the same model repeatedly or you want to share code with colleagues, you can write a substitutable expression program to define your system of equations and avoid having to retype the system every time. The first five letters of the program's name must be nlsur, and the program must set the r-class macro $r$ ( $n \_e q$ ) to the number of equations in your system. The first equation's substitutable expression must be returned in $r$ (eq_1), the second equation's in $r$ (eq_2), and so on. You may optionally set $r$ (title) to label your output; that has the same effect as specifying the title() option.
> Example 3: Programmed substitutable expression version
We return to our translog cost function, for which a substitutable expression program is

```
program nlsurtranslog, rclass
    version 13
    syntax varlist(min=7 max=7) [if]
    tokenize 'varlist'
    args sk sl se pk pl pe pm
    local pkpm ln('pk'/'pm')
    local plpm ln('pl'/'pm')
    local pepm ln('pe'/'pm')
    return scalar n_eq = 3
    return local eq_1 "'sk'= {bk} + {dkk}*'pkpm' + {dkl}*'plpm' + {dke}*'pepm'"
    return local eq_2 "'sl'= {bl} + {dkl}*'pkpm' + {dll}*'plpm' + {dle}*'pepm'"
    return local eq_3 "'se'= {be} + {dke}*'pkpm' + {dle}*'plpm' + {dee}*'pepm'"
    return local title "4-factor translog cost function"
end
```

We made our program accept seven variables, for the three dependent variables $s_{k}, s_{l}$, and $s_{e}$, and the four factor prices $p_{k}, p_{l}, p_{m}$, and $p_{e}$. The tokenize command assigns to macros ' 1 ', ' 2 ', $\ldots$, ' 7 ' the seven variables stored in 'varlist', and the args command transfers those numbered macros to macros 'sk', 'sl', ..., 'pm'. Because we knew our substitutable expressions were going to be somewhat long, we created local macros to hold the log price ratios. These are simply macros that hold strings such as $\ln (\mathrm{pk} / \mathrm{pm})$, not variables, and they will save us some repetitious typing when we define our substitutable expressions. Our program returns the number of equations in $r$ ( $n \_e q$ ), and we defined our substitutable expressions in eq_1, eq_2, and eq_3. We do not bind the expressions in parentheses as we do with the interactive version of nlsur. Finally, we put a title in $r$ (title) to label our output.

Our syntax command also accepts an if clause, and that is how nlsur indicates the estimation sample to our program. In this application, we can safely ignore it, because our program does not compute initial values. However, had we used commands such as summarize or regress to obtain initial values, then we would need to restrict those commands to analyze only the estimation sample. In those cases, typically, you simply need to include 'if' with the commands you are using. For example, instead of the command

```
summarize 'depvar', meanonly
```

you would use

```
summarize 'depvar' 'if', meanonly
```

We can check our program by typing

```
. nlsurtranslog s_k s_l s_e pk pl pe pm
. return list
scalars:
    r(n_eq) = 3
macros:
```

```
r(title) : "4-factor translog cost function"
```

r(title) : "4-factor translog cost function"
r(eq_3) : "s_e= {be} + {dke}*ln(pk/pm) + {dle}*ln(pl/pm) + {.."
r(eq_3) : "s_e= {be} + {dke}*ln(pk/pm) + {dle}*ln(pl/pm) + {.."
r(eq_2) : "s_l= {bl} + {dkl}*ln(pk/pm) + {dll}*ln(pl/pm) + {.."
r(eq_2) : "s_l= {bl} + {dkl}*ln(pk/pm) + {dll}*ln(pl/pm) + {.."
r(eq_1) : "s_k= {bk} + {dkk}*ln(pk/pm) + {dkl}*ln(pl/pm) + {.."

```
    r(eq_1) : "s_k= {bk} + {dkk}*ln(pk/pm) + {dkl}*ln(pl/pm) + {.."
```

Now that we know that our program works, we fit our model by typing

```
. nlsur translog: s_k s_l s_e pk pl pe pm, ifgnls
(obs = 25)
Calculating NLS estimates...
Iteration 0: Residual SS = .0009989
Iteration 1: Residual SS = . }000998
Calculating FGNLS estimates...
Iteration 0: Scaled RSS = 65.45197
Iteration 1: Scaled RSS = 65.45197
FGNLS iteration 2...
Iteration 0: Scaled RSS = 73.28311
Iteration 1: Scaled RSS = 73.28311
Iteration 2: Scaled RSS = 73.28311
Parameter change = 6.537e-03
Covariance matrix change = 1.002e-06
    (output omitted)
FGNLS iteration 10...
Iteration 0: Scaled RSS = 75
Iteration 1: Scaled RSS = 75
Parameter change = 4.074e-06
Covariance matrix change = 6.265e-10
FGNLS regression
```

|  | Equation | Obs | Parms | RMSE | R-sq | Constant |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | s_k | 25 | 4 | .0031722 | 0.4776 | bk |
| 2 | s_l | 25 | 4 | .0053963 | 0.8171 | bl |
| 3 | s_e | 25 | 4 | .00177 | 0.6615 | be |

4-factor translog cost function

|  | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| /bk | .0568925 | .0013454 | 42.29 | 0.000 | .0542556 | .0595294 |
| /dkk | .0294833 | .0057956 | 5.09 | 0.000 | .0181241 | .0408425 |
| /dkl | -.0000471 | .0038478 | -0.01 | 0.990 | -.0075887 | .0074945 |
| /dke | -.0106749 | .0033882 | -3.15 | 0.002 | -.0173157 | -.0040341 |
| /bl | .253438 | .0020945 | 121.00 | 0.000 | .2493329 | .2575432 |
| /dll | .0754327 | .0067572 | 11.16 | 0.000 | .0621889 | .0886766 |
| /dle | -.004756 | .002344 | -2.03 | 0.042 | -.0093501 | -.0001619 |
| /be | .0444099 | .0008533 | 52.04 | 0.000 | .0427374 | .0460823 |
| /dee | .0183415 | .0049858 | 3.68 | 0.000 | .0085694 | .0281135 |

Because we set r (title) in our substitutable expression program, the coefficient table has a title attached to it. The estimates are identical to those we obtained in example 2.

- Technical note
nlsur accepts frequency and analytic weights as well as pweights (sampling weights) and iweights (importance weights). You do not need to modify your substitutable expressions in any way to perform weighted estimation, though you must make two changes to your substitutable expression program. The general outline of a sexp_prog program is

```
program nlsur name, rclass
    version 13
    syntax varlist [fw aw pw iw] [if]
    // Obtain initial values incorporating weights. For example,
    summarize varname ['weight''exp'] 'if'
    // Return n_eqn and substitutable expressions
    return scalar n_eq = #
    return local eq_1 = ...
end
```

First, we wrote the syntax statement to accept a weight expression. Here we allow all four types of weights, but if you know that your estimator is valid, say, for only frequency weights, then you should modify the syntax line to accept only fweights. Second, if your program computes starting values, then any commands you use must incorporate the weights passed to the program; you do that by including ['weight' 'exp'] when calling those commands.

## Function evaluator programs

Although substitutable expressions are extremely flexible, there are some problems for which the nonlinear system cannot be defined using them. You can use the function evaluator program version of nlsur in these cases. We present two examples, a simple one to illustrate the mechanics of function evaluator programs and a more complicated one to illustrate the power of nlsur.
> Example 4: Function evaluator program version
Here we write a function evaluator program to fit the translog cost function used in examples 2 and 3. The function evaluator program is

```
program nlsurtranslog2
    version 13
    syntax varlist(min=7 max=7) [if], at(name)
    tokenize 'varlist'
    args sk sl se pk pl pe pm
    tempname bk dkk dkl dke bl dll dle be dee
    scalar 'bk' = 'at'[1,1]
    scalar 'dkk' = 'at'[1,2]
    scalar 'dkl' = 'at'[1,3]
    scalar 'dke' = 'at'[1,4]
    scalar 'bl' = 'at'[1,5]
    scalar 'dll' = 'at'[1,6]
    scalar 'dle' = 'at'[1,7]
    scalar 'be' = 'at'[1,8]
    scalar 'dee' = 'at'[1,9]
    local pkpm ln('pk'/'pm')
    local plpm ln('pl'/'pm')
    local pepm ln('pe'/'pm')
    quietly {
        replace 'sk' = 'bk' + 'dkk'*'pkpm' + 'dkl'*'plpm' + ///
        'dke'*'pepm' 'if'
        replace 'sl' = 'bl' + 'dkl'*'pkpm' + 'dll'*'plpm' + ///
        'dle'*'pepm' 'if'
        replace 'se' = 'be' + 'dke'*'pkpm' + 'dle'*'plpm' + ///
        'dee'*'pepm' 'if'
    }
end
```

Unlike the substitutable expression program we wrote in example 3, nlsurtranslog2 is not declared as r-class because we will not be returning any stored results. We are again expecting seven variables: three shares and four factor prices, and nlsur will again mark the estimation sample with an if expression.

Our function evaluator program also accepts an option named at (), which will receive a parameter vector at which we are to evaluate the system of equations. All function evaluator programs must accept this option. Our model has nine parameters to estimate, and we created nine temporary scalars to hold the elements of the 'at' matrix.

Because our model has three equations, the first three variables passed to our program are the dependent variables that we are to fill in with the function values. We replaced only the observations in our estimation sample by including the 'if' qualifier in the replace statements. Here we could have ignored the 'if' qualifier because nlsur will skip over observations not in the estimation sample and we did not perform any computations requiring knowledge of the estimation sample. However, including the 'if' is good practice and may result in a slight speed improvement if the functions of your model are complicated and the estimation sample is much smaller than the dataset in memory.

We could have avoided creating temporary scalars to hold our individual parameters by writing the replace statements as, for example,

```
replace 'sk' = 'at'[1,1] + 'at'[1,2]*'pkpm' + 'at'[1,3]*'plpm' + 'at'[1,4]*'pepm' 'if'
```

You can use whichever method you find more appealing, though giving the parameters descriptive names reduces the chance for mistakes and makes debugging easier.

To fit our model by using the function evaluator program version of nlsur, we type

```
. nlsur translog2 @ s_k s_l s_e pk pl pe pm, ifgnls nequations(3)
> parameters(bk dkk dkl dke bl dll dle be dee)
> hasconstants(bk bl be)
(obs = 25)
Calculating NLS estimates...
Iteration 0: Residual SS = . 0009989
Iteration 1: Residual SS = .0009989
Calculating FGNLS estimates...
Iteration 0: Scaled RSS = 65.45197
Iteration 1: Scaled RSS = 65.45197
FGNLS iteration 2...
Iteration 0: Scaled RSS = 73.28311
Iteration 1: Scaled RSS = 73.28311
Iteration 2: Scaled RSS = 73.28311
Parameter change = 6.537e-03
Covariance matrix change = 1.002e-06
FGNLS iteration 3...
Iteration 0: Scaled RSS = 74.7113
Iteration 1: Scaled RSS = 74.7113
Parameter change = 2.577e-03
Covariance matrix change = 3.956e-07
FGNLS iteration 4...
Iteration 0: Scaled RSS = 74.95356
Iteration 1: Scaled RSS = 74.95356
Iteration 2: Scaled RSS = 74.95356
Parameter change = 1.023e-03
Covariance matrix change = 1.571e-07
FGNLS iteration 5...
Iteration 0: Scaled RSS = 74.99261
Iteration 1: Scaled RSS = 74.99261
Parameter change = 4.067e-04
Covariance matrix change = 6.250e-08
FGNLS iteration 6...
Iteration 0: Scaled RSS = 74.99883
Iteration 1: Scaled RSS = 74.99883
Iteration 2: Scaled RSS = 74.99883
Parameter change = 1.619e-04
Covariance matrix change = 2.489e-08
FGNLS iteration 7...
Iteration 0: Scaled RSS = 74.99981
Iteration 1: Scaled RSS = 74.99981
Iteration 2: Scaled RSS = 74.99981
Parameter change = 6.449e-05
Covariance matrix change = 9.912e-09
FGNLS iteration 8...
Iteration 0: Scaled RSS = 74.99997
Iteration 1: Scaled RSS = 74.99997
Iteration 2: Scaled RSS = 74.99997
Parameter change = 2.569e-05
Covariance matrix change = 3.948e-09
FGNLS iteration 9...
Iteration 0: Scaled RSS = 75
Iteration 1: Scaled RSS = 75
Parameter change = 1.023e-05
Covariance matrix change = 1.573e-09
FGNLS iteration 10...
Iteration 0: Scaled RSS = 75
Iteration 1: Scaled RSS = 75
Parameter change = = 4.074e-06
Covariance matrix change = 6.265e-10
```


## FGNLS regression

|  | Equation | Obs | Parms | RMSE | R-sq | Constant |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | s_k | 25 | . | .0031722 | 0.4776 | bk |
| 2 | s_l | 25 | . | .0053963 | 0.8171 | bl |
| 3 | s_e | 25 | . | .00177 | 0.6615 | be |


|  | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| /bk | .0568925 | .0013454 | 42.29 | 0.000 | .0542556 | .0595294 |
| /dkk | .0294833 | .0057956 | 5.09 | 0.000 | .0181241 | .0408425 |
| /dkl | -.0000471 | .0038478 | -0.01 | 0.990 | -.0075887 | .0074945 |
| /dke | -.0106749 | .0033882 | -3.15 | 0.002 | -.0173157 | -.0040341 |
| /bl | .253438 | .0020945 | 121.00 | 0.000 | .2493329 | .2575432 |
| /dll | .0754327 | .0067572 | 11.16 | 0.000 | .0621889 | .0886766 |
| /dle | -.004756 | .002344 | -2.03 | 0.042 | -.0093501 | -.0001619 |
| /be | .0444099 | .0008533 | 52.04 | 0.000 | .0427374 | .0460823 |
| /dee | .0183415 | .0049858 | 3.68 | 0.000 | .0085694 | .0281135 |

When we use the function evaluator program version, nlsur requires us to specify the number of equations in nequations(), and it requires us to either specify names for each of our parameters or the number of parameters in the model. Here we used the parameters () option to name our parameters; the order in which we specified them in this option is the same as the order in which we extracted them from the 'at' matrix in our program. Had we instead specified nparameters (9), our parameters would have been labeled /b1, /b2, ..., /b9 in the output.
nlsur has no way of telling how many parameters appear in each equation, so the Parms column in the header contains missing values. Moreover, the function evaluator program version of nlsur does not attempt to identify constant terms, so we used the hasconstant option to tell nlsur which parameter in each equation is a constant term.

The estimates are identical to those we obtained in examples 2 and 3 .

## - Technical note

As with substitutable expression programs, if you intend to do weighted estimation with a function evaluator program, you must modify your func_prog program's syntax statement to accept weights. Moreover, if you use any statistical commands when computing your nonlinear functions, then you must include the weight expression with those commands.

## > Example 5: Fitting the basic AIDS model using nlsur

Deaton and Muellbauer (1980) introduce the almost ideal demand system (AIDS), and Poi (2012) presents a set of commands and several extensions for fitting the AIDS automatically. Here we show how to fit the basic AIDS model, which is a common example of a nonlinear system of equations, by manually using nlsur. The dataset food.dta contains household expenditures, expenditure shares, and $\log$ prices for four broad food groups. For a four-good demand system, we need to fit the following system of three equations:

$$
\begin{aligned}
& w_{1}=\alpha_{1}+\gamma_{11} \ln p_{1}+\gamma_{12} \ln p_{2}+\gamma_{13} \ln p_{3}+\beta_{1} \ln \left\{\frac{m}{P(\mathbf{p})}\right\}+u_{1} \\
& w_{2}=\alpha_{2}+\gamma_{12} \ln p_{1}+\gamma_{22} \ln p_{2}+\gamma_{23} \ln p_{3}+\beta_{2} \ln \left\{\frac{m}{P(\mathbf{p})}\right\}+u_{2} \\
& w_{3}=\alpha_{3}+\gamma_{13} \ln p_{1}+\gamma_{23} \ln p_{2}+\gamma_{33} \ln p_{3}+\beta_{3} \ln \left\{\frac{m}{P(\mathbf{p})}\right\}+u_{3}
\end{aligned}
$$

where $w_{k}$ denotes a household's fraction of expenditures on good $k, \ln p_{k}$ denotes the logarithm of the price paid for good $k, m$ denotes a household's total expenditure on all four goods, the $u$ 's are regression error terms, and

$$
\ln P(\mathbf{p})=\alpha_{0}+\sum_{i=1}^{4} \alpha_{i} \ln p_{i}+\frac{1}{2} \sum_{i=1}^{4} \sum_{j=1}^{4} \gamma_{i j} \ln p_{i} \ln p_{j}
$$

The parameters for the fourth good's share equation can be recovered from the following constraints that are imposed by economic theory:

$$
\sum_{i=1}^{4} \alpha_{i}=1 \quad \sum_{i=1}^{4} \beta_{i}=0 \quad \gamma_{i j}=\gamma_{j i} \quad \text { and } \quad \sum_{i=1}^{4} \gamma_{i j}=0 \text { for all } j
$$

Our model has a total of 12 unrestricted parameters. We will not estimate $\alpha_{0}$ directly. Instead, we will set it equal to 5; see Deaton and Muellbauer (1980) for a discussion of why treating $\alpha_{0}$ as fixed is acceptable.

Our function evaluator program is

```
program nlsuraids
    version 13
    syntax varlist(min=8 max=8) if, at(name)
    tokenize 'varlist'
    args w1 w2 w3 lnp1 lnp2 lnp3 lnp4 lnm
    tempname a1 a2 a3 a4
    scalar 'a1' = 'at'[1,1]
    scalar 'a2' = 'at'[1,2]
    scalar 'a3' = 'at'[1,3]
    scalar 'a4' = 1 - 'a1' - 'a2' - 'a3'
    tempname b1 b2 b3
    scalar 'b1' = 'at'[1,4]
    scalar 'b2' = 'at'[1,5]
    scalar 'b3' = 'at'[1,6]
    tempname g11 g12 g13 g14
    tempname g21 g22 g23 g24
    tempname g31 g32 g33 g34
    tempname g41 g42 g43 g44
    scalar 'g11' = 'at'[1,7]
    scalar 'g12' = 'at'[1,8]
    scalar 'g13' = 'at'[1,9]
    scalar 'g14' = -'g11'-'g12'-'g13'
    scalar 'g21' = 'g12'
    scalar 'g22' = 'at'[1,10]
    scalar 'g23' = 'at'[1,11]
    scalar 'g24' = -'g21'-'g22'-'g23'
    scalar 'g31' = 'g13'
    scalar 'g32' = 'g23'
    scalar 'g33' = 'at'[1,12]
    scalar 'g34' = -'g31'-'g32'-'g33'
    scalar 'g41' = 'g14'
    scalar 'g42' = 'g24'
    scalar 'g43' = 'g34'
    scalar 'g44' = -'g41'-'g42'-'g43'
    quietly {
        tempvar lnpindex
        gen double 'lnpindex' = 5 + 'a1'*'lnp1' + 'a2'*'lnp2' + ///
                                    'a3'*'lnp3' + 'a4'*'lnp4'
        forvalues i = 1/4 {
            forvalues j = 1/4 {
                    replace 'lnpindex' = 'lnpindex' + ///
                        0.5*'g'i''j''*'lnp'i''*'lnp'j'',
            }
        }
        replace 'w1' = 'a1' + 'g11'*'lnp1' + 'g12'*'lnp2' + ///
                        'g13'*'lnp3' + 'g14'*'lnp4' + ///
                            'b1'*('lnm' - 'lnpindex')
        replace 'w2' = 'a2' + 'g21'*'lnp1' + 'g22'*'lnp2' + ///
        'g23'*'lnp3' + 'g24'*'lnp4' + ///
        'b2'*('lnm' - 'lnpindex')
        replace 'w3' = 'a3' + 'g31'*'lnp1' + 'g32'*'`lnp2' + ///
        'g33'*'lnp3' + 'g34'*'lnp4' + ///
                        'b3'*('lnm' - 'lnpindex')
    }
end
```

The syntax statement accepts eight variables: three expenditure share variables, all four log-price variables, and a variable for $\log$ expenditures $(\ln m)$. Most of the code simply extracts the parameters
from the 'at' matrix. Although we are estimating only 12 parameters, to calculate the price index term and the expenditure share equations, we need the restricted parameters as well. Notice how we impose the constraints on the parameters. We then created a temporary variable to hold $\ln P(\mathbf{p})$, and we filled the three dependent variables with the predicted expenditure shares.

To fit our model, we type

```
. use http://www.stata-press.com/data/r13/food
. nlsur aids @ w1 w2 w3 lnp1 lnp2 lnp3 lnp4 lnexp,
> parameters(a1 a2 a3 b1 b2 b3
> g11 g12 g13 g22 g32 g33)
> neq(3) ifgnls
(obs = 4048)
Calculating NLS estimates...
Iteration 0: Residual SS = 126.9713
Iteration 1: Residual SS = 125.669
Iteration 2: Residual SS = 125.669
Iteration 3: Residual SS = 125.669
Iteration 4: Residual SS = 125.669
Calculating FGNLS estimates...
Iteration 0: Scaled RSS = 12080.14
Iteration 1: Scaled RSS = 12080.14
Iteration 2: Scaled RSS = 12080.14
Iteration 3: Scaled RSS = 12080.14
FGNLS iteration 2...
Iteration 0: Scaled RSS = 12143.99
Iteration 1: Scaled RSS = 12143.99
Iteration 2: Scaled RSS = 12143.99
Parameter change = 1.972e-04
Covariance matrix change = 2.936e-06
FGNLS iteration 3...
Iteration 0: Scaled RSS = 12144
Iteration 1: Scaled RSS = 12144
Parameter change = 2.178e-06
Covariance matrix change = 3.469e-08
FGNLS regression
```

|  | Equation | Obs | Parms | RMSE | R-sq | Constant |
| :--- | ---: | ---: | :---: | :---: | :---: | :---: |
| 1 | w1 | 4048 | . | .1333175 | $0.9017 *$ | (none) |
| 2 | w2 | 4048 | . | .1024166 | $0.8480 *$ | (none) |
| 3 | w3 | 4048 | . | .053777 | $0.7906 *$ | (none) |

* Uncentered R-sq

|  | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| /a1 | .3163959 | .0073871 | 42.83 | 0.000 | .3019175 | .3308742 |
| /a2 | .2712501 | .0056938 | 47.64 | 0.000 | .2600904 | .2824097 |
| /a3 | .1039898 | .0029004 | 35.85 | 0.000 | .0983051 | .1096746 |
| /b1 | .0161044 | .0034153 | 4.72 | 0.000 | .0094105 | .0227983 |
| /b2 | -.0260771 | .002623 | -9.94 | 0.000 | -.0312181 | -.0209361 |
| /b3 | .0014538 | .0013776 | 1.06 | 0.291 | -.0012463 | .0041539 |
| /g11 | .1215838 | .0057186 | 21.26 | 0.000 | .1103756 | .1327921 |
| /g12 | -.0522943 | .0039305 | -13.30 | 0.000 | -.0599979 | -.0445908 |
| /g13 | -.0351292 | .0021788 | -16.12 | 0.000 | -.0393996 | -.0308588 |
| /g22 | .0644298 | .0044587 | 14.45 | 0.000 | .0556909 | .0731687 |
| /g32 | -.0011786 | .0019767 | -0.60 | 0.551 | -.0050528 | .0026957 |
| /g33 | .0424381 | .0017589 | 24.13 | 0.000 | .0389909 | .0458854 |

To get the restricted parameters for the fourth share equation, we can use lincom. For example, to obtain $\alpha_{4}$, we type

|  | Coef. | Std. Err. | z | $P>\|z\|$ | [95\% Conf | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (1) | . 3083643 | . 0052611 | 58.61 | 0.000 | . 2980528 | . 3186758 |

For more information on lincom, see [ R ] lincom.

## Stored results

nlsur stores the following in e():
Scalars
e(N)
e(k)
e(k_\#)
e(k_eq)
e(k_eq_model)
$e\left(n \_e q\right)$
e(mss_\#)
e(rss_\#)
e(rmse_\#)
e(r2_\#)
e(11)
e(N_clust)
e(rank)
$e$ (converge)
Macros
e(cmd)
e(cmdline)
$e$ (method)
e(depvar)
e(depvar_\#)
e(wtype)
e(wexp)
e(title)
e(title_2)
e(clustvar)
e(vce)
e(vcetype)
e(type)
e(sexpprog)
e(sexp_\#)
e(params)
e(params_\#)
e(funcprog)
e(rhs)
e(constants)
e(properties)
e(predict)
number of observations
number of parameters
number of parameters for equation \#
number of equation names in $e(b)$
number of equations in overall model test
number of equations
model sum of squares for equation \#
RSS for equation \#
root mean squared error for equation \#
$R^{2}$ for equation \#
Gaussian log likelihood (iflgs version only)
number of clusters
rank of e(V)
1 if converged, 0 otherwise
nlsur
command as typed
fgnls, ifgnls, or nls
names of dependent variables
dependent variable for equation \#
weight type
weight expression
title in estimation output
secondary title in estimation output
name of cluster variable
vcetype specified in vce()
title used in label Std. Err.
$1=$ interactively entered expression
$2=$ substitutable expression program
$3=$ function evaluator program
substitutable expression program
substitutable expression for equation \#
names of all parameters
parameters in equation \#
function evaluator program
contents of variables()
identifies constant terms
b V
program used to implement predict

Matrices
e(b) coefficient vector
$e$ (init) initial values vector
e(Sigma) error covariance matrix ( $\widehat{\boldsymbol{\Sigma}}$ )
e(V) variance-covariance matrix of the estimators
Functions
$e$ (sample) marks estimation sample

## Methods and formulas

Write the system of equations for the $i$ th observation as

$$
\begin{equation*}
\mathbf{y}_{i}=\mathbf{f}\left(\mathbf{x}_{i}, \boldsymbol{\beta}\right)+\mathbf{u}_{i} \tag{1}
\end{equation*}
$$

where $\mathbf{y}_{i}$ and $\mathbf{u}_{i}$ are $1 \times M$ vectors, for $i=1, \ldots, N ; \mathbf{f}$ is a function that returns a $1 \times M$ vector; $\mathbf{x}_{i}$ represents all the exogenous variables in the system; and $\boldsymbol{\beta}$ is a $1 \times k$ vector of parameters. The generalized nonlinear least-squares system estimator is defined as

$$
\widehat{\boldsymbol{\beta}} \equiv \operatorname{argmin}_{\boldsymbol{\beta}} \sum_{i=1}^{N}\left\{\mathbf{y}_{i}-\mathbf{f}\left(\mathbf{x}_{i}, \boldsymbol{\beta}\right)\right\} \boldsymbol{\Sigma}^{-1}\left\{\mathbf{y}_{i}-\mathbf{f}\left(\mathbf{x}_{i}, \boldsymbol{\beta}\right)\right\}^{\prime}
$$

where $\boldsymbol{\Sigma}=E\left(\mathbf{u}_{i}^{\prime} \mathbf{u}_{i}\right)$ is an $M \times M$ positive-definite weight matrix. Let $\mathbf{T}$ be the Cholesky decomposition of $\boldsymbol{\Sigma}^{-1}$; that is, $\mathbf{T T}^{\prime}=\boldsymbol{\Sigma}^{-1}$. Postmultiply (1) by $\mathbf{T}$ :

$$
\begin{equation*}
\mathbf{y}_{i} \mathbf{T}=\mathbf{f}\left(\mathbf{x}_{i}, \boldsymbol{\beta}\right) \mathbf{T}+\mathbf{u}_{i} \mathbf{T} \tag{2}
\end{equation*}
$$

Because $E\left(\mathbf{T}^{\prime} \mathbf{u}_{i}^{\prime} \mathbf{u}_{i} \mathbf{T}\right)=\mathbf{I}$, we can "stack" the columns of (2) and write

$$
\begin{align*}
\mathbf{y}_{1} \mathbf{T}_{1} & =\mathbf{f}\left(\mathbf{x}_{1}, \boldsymbol{\beta}\right) \mathbf{T}_{1}+\widetilde{u}_{11} \\
\mathbf{y}_{1} \mathbf{T}_{2} & =\mathbf{f}\left(\mathbf{x}_{1}, \boldsymbol{\beta}\right) \mathbf{T}_{2}+\widetilde{u}_{12} \\
\vdots & =\vdots \\
\mathbf{y}_{1} \mathbf{T}_{M} & =\mathbf{f}\left(\mathbf{x}_{1}, \boldsymbol{\beta}\right) \mathbf{T}_{M}+\widetilde{u}_{1 M} \\
\vdots & =\vdots  \tag{3}\\
\mathbf{y}_{N} \mathbf{T}_{1} & =\mathbf{f}\left(\mathbf{x}_{N}, \boldsymbol{\beta}\right) \mathbf{T}_{1}+\widetilde{u}_{N 1} \\
\mathbf{y}_{N} \mathbf{T}_{2} & =\mathbf{f}\left(\mathbf{x}_{N}, \boldsymbol{\beta}\right) \mathbf{T}_{2}+\widetilde{u}_{N 2} \\
\vdots & =\vdots \\
\mathbf{y}_{N} \mathbf{T}_{M} & =\mathbf{f}\left(\mathbf{x}_{N}, \boldsymbol{\beta}\right) \mathbf{T}_{M}+\widetilde{u}_{N M}
\end{align*}
$$

where $\mathbf{T}_{j}$ denotes the $j$ th column of $\mathbf{T}$. By construction, all $\widetilde{u}_{i j}$ are independently distributed with unit variance. As a result, by transforming the model in (1) to that shown in (3), we have reduced the multivariate generalized nonlinear least-squares system estimator to a univariate nonlinear least-squares problem; and the same parameter estimation technique used by nl can be used here. See $[\mathrm{R}] \mathrm{nl}$ for the details. Moreover, because the $\widetilde{u}_{i j}$ all have variance 1 , the final scaled RSS reported by nlsur is equal to $N M$.

To make the estimator feasible, we require an estimate $\widehat{\boldsymbol{\Sigma}}$ of $\boldsymbol{\Sigma}$. nlsur first sets $\widehat{\boldsymbol{\Sigma}}=\mathbf{I}$. Although not efficient, the resulting estimate, $\widehat{\boldsymbol{\beta}}_{\mathrm{NLS}}$, is consistent. If the nls option is specified, estimation is complete. Otherwise, the residuals

$$
\widehat{\mathbf{u}}_{i}=\mathbf{y}_{i}-\mathbf{f}\left(\mathbf{x}_{i}, \widehat{\boldsymbol{\beta}}_{\mathrm{NLS}}\right)
$$

are calculated and used to compute

$$
\widehat{\boldsymbol{\Sigma}}=\frac{1}{N} \sum_{i=1}^{N} \widehat{\mathbf{u}}_{i}^{\prime} \widehat{\mathbf{u}}_{i}
$$

With $\widehat{\boldsymbol{\Sigma}}$ in hand, a new estimate $\widehat{\boldsymbol{\beta}}$ is then obtained.
If the ifgnls option is specified, the new $\widehat{\boldsymbol{\beta}}$ is used to recompute the residuals and obtain a new estimate of $\widehat{\boldsymbol{\Sigma}}$, from which $\widehat{\boldsymbol{\beta}}$ can then be reestimated. Iterations stop when the relative change in $\widehat{\boldsymbol{\beta}}$ is less than eps(), the relative change in $\widehat{\boldsymbol{\Sigma}}$ is less than ifgnlseps(), or if ifgnlsiterate() iterations have been performed.

If the vce(robust) and vce(cluster clustvar) options were not specified, then

$$
V(\widehat{\boldsymbol{\beta}})=\left(\sum_{i=1}^{N} \mathbf{X}_{i}^{\prime} \widehat{\boldsymbol{\Sigma}}^{-1} \mathbf{X}_{i}\right)^{-1}
$$

where the $M \times k$ matrix $\mathbf{X}_{i}$ has typical element $X_{i s t}$, the derivative of the $s$ th element of $\mathbf{f}$ with respect to the $t$ th element of $\boldsymbol{\beta}$, evaluated at $\mathbf{x}_{i}$ and $\widehat{\boldsymbol{\beta}}$. As a practical matter, once the model is written in the form of (3), the variance-covariance matrix can be calculated via a Gauss-Newton regression; see Davidson and MacKinnon (1993, chap. 6).

If robust is specified, then

$$
V_{R}(\widehat{\boldsymbol{\beta}})=\left(\sum_{i=1}^{N} \mathbf{X}_{i}^{\prime} \hat{\boldsymbol{\Sigma}}^{-1} \mathbf{X}_{i}\right)^{-1} \sum_{i=1}^{N} \mathbf{X}_{i}^{\prime} \widehat{\boldsymbol{\Sigma}}^{-1} \widehat{\mathbf{u}}_{i}^{\prime} \widehat{\mathbf{u}}_{i} \hat{\boldsymbol{\Sigma}}^{-1} \mathbf{X}_{i}\left(\sum_{i=1}^{N} \mathbf{X}_{i}^{\prime} \widehat{\boldsymbol{\Sigma}}^{-1} \mathbf{X}_{i}\right)^{-1}
$$

The cluster-robust variance matrix is

$$
V_{C}(\widehat{\boldsymbol{\beta}})=\left(\sum_{i=1}^{N} \mathbf{X}_{i}^{\prime} \widehat{\boldsymbol{\Sigma}}^{-1} \mathbf{X}_{i}\right)^{-1} \sum_{c=1}^{N_{C}} \mathbf{w}_{c}^{\prime} \mathbf{w}_{c}\left(\sum_{i=1}^{N} \mathbf{X}_{i}^{\prime} \widehat{\boldsymbol{\Sigma}}^{-1} \mathbf{X}_{i}\right)^{-1}
$$

where $N_{C}$ is the number of clusters and

$$
\mathbf{w}_{c}=\sum_{j \in C_{k}} \mathbf{X}_{j}^{\prime} \widehat{\boldsymbol{\Sigma}}^{-1} \widehat{\mathbf{u}}_{j}^{\prime}
$$

with $C_{k}$ denoting the set of observations in the $k$ th cluster. In evaluating these formulas, we use the value of $\widehat{\boldsymbol{\Sigma}}$ used in calculating the final estimate of $\widehat{\boldsymbol{\beta}}$. That is, we do not recalculate $\widehat{\boldsymbol{\Sigma}}$ after we obtain the final value of $\widehat{\boldsymbol{\beta}}$.

The RSS for the $j$ th equation, $\operatorname{RSS}_{j}$, is

$$
\operatorname{RSS}_{j}=\sum_{i=1}^{N}\left(\widehat{y}_{i j}-y_{i j}\right)^{2}
$$

where $\widehat{y}_{i j}$ is the predicted value of the $i$ th observation on the $j$ th dependent variable; the total sum of squares (TSS) for the $j$ th equation, $\operatorname{TSS}_{j}$, is

$$
\operatorname{TSS}_{j}=\sum_{i=1}^{N}\left(y_{i j}-\bar{y}_{j}\right)^{2}
$$

if there is a constant term in the $j$ th equation, where $\bar{y}_{j}$ is the sample mean of the $j$ th dependent variable, and

$$
\operatorname{TSS}_{j}=\sum_{i=1}^{N} y_{i j}^{2}
$$

if there is no constant term in the $j$ th equation; and the model sum of squares (MSS) for the $j$ th equation, $\mathrm{MSS}_{j}$, is $\mathrm{TSS}_{j}-\mathrm{RSS}_{j}$.

The $R^{2}$ for the $j$ th equation is $\operatorname{MSS}_{j} / \mathrm{TSS}_{j}$. If an equation does not have a constant term, then the reported $R^{2}$ for that equation is "uncentered" and based on the latter definition of $\operatorname{TSS}_{j}$.

Under the assumption that the $\mathbf{u}_{i}$ are independent and identically distributed $N(\mathbf{0}, \widehat{\boldsymbol{\Sigma}})$, the $\log$ likelihood for the model is

$$
\ln L=-\frac{M N}{2}\{1+\ln (2 \pi)\}-\frac{N}{2} \ln |\widehat{\boldsymbol{\Sigma}}|
$$

The log likelihood is reported only when the ifgnls option is specified.

## References

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## Also see

$[\mathrm{R}]$ nlsur postestimation - Postestimation tools for nlsur
[R] nl — Nonlinear least-squares estimation
[R] gmm - Generalized method of moments estimation
[R] ml — Maximum likelihood estimation
[R] mlexp - Maximum likelihood estimation of user-specified expressions
[R] reg3 - Three-stage estimation for systems of simultaneous equations
[R] sureg - Zellner's seemingly unrelated regression
[U] 20 Estimation and postestimation commands

## Title

nlsur postestimation - Postestimation tools for nlsur

Description Syntax for predict Menu for predict Options for predict Remarks and examples Also see

## Description

The following postestimation commands are available after nlsur:

| Command | Description |
| :--- | :--- |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize |  |
| estat vce | summary statistics for the estimation sample <br> variance-covariance matrix of the estimators (VCE) <br> estimates <br> forecast <br> cataloging estimation results |
| lincom | dynamic forecasts and simulations <br> point estimates, standard errors, testing, and inference for linear combinations <br> of coefficients |
| lrtest | likelihood-ratio test <br> marginal means, predictive margins, marginal effects, and average marginal <br> effects |
| marginsplot | graph the results from margins (profile plots, interaction plots, etc.) <br> point estimates, standard errors, testing, and inference for nonlinear combinations <br> of coefficients |
| nlcom | predictions, residuals, influence statistics, and other diagnostic measures <br> point estimates, standard errors, testing, and inference for generalized predictions |
| predict | Wald tests of simple and composite linear hypotheses |
| predictnl | Wald tests of nonlinear hypotheses |
| test | testnl |

${ }^{1}$ You must specify the variables() option with nlsur.

## Syntax for predict

predict [type] newvar [if] [in] [, equation(\#eqno) yhat residuals]
These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict


#### Abstract

Main equation(\#eqno) specifies to which equation you are referring. equation(\#1) would mean that the calculation is to be made for the first equation, equation(\#2) would mean the second, and so on. If you do not specify equation(), results are the same as if you had specified equation(\#1). yhat, the default, calculates the fitted values for the specified equation. residuals calculates the residuals for the specified equation.


## Remarks and examples

## $>$ Example 1

In example 2 of [R] nlsur, we fit a four-factor translog cost function to data for the U.S. economy. The own-price elasticity for a factor measures the percentage change in its usage as a result of a $1 \%$ increase in the factor's price, assuming that output is held constant. For the translog production function, the own-price factor elasticities are

$$
\eta_{i}=\frac{\delta_{i i}+s_{i}\left(s_{i}-1\right)}{s_{i}}
$$

Here we compute the elasticity for capital at the sample mean of capital's factor share. First, we use summarize to get the mean of $\mathrm{s} \_\mathrm{k}$ and store that value in a scalar:

| summarize s_k |  |  |  |  |  |  |
| ---: | :---: | ---: | :---: | ---: | :---: | :---: |
| Variable | Obs | Mean | Std. Dev. | Min | Max |  |
| s_k | 25 | .053488 | .0044795 | .04602 | .06185 |  |

scalar kmean $=r($ mean $)$
Now we can use nlcom to calculate the elasticity:

```
. nlcom (([dkk]_cons + kmean*(kmean-1)) / kmean)
```

    _nl_1: ([dkk]_cons + kmean*(kmean-1)) / kmean
    |  | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $-n l \_1$ | -.3952986 | .1083535 | -3.65 | 0.000 | -.6076676 | -.1829295 |

If the price of capital increases by $1 \%$, its usage will decrease by about $0.4 \%$. To maintain its current level of output, a firm would increase its usage of other inputs to compensate for the lower capital usage. The standard error reported by nlcom reflects the sampling variance of the estimated parameter $\widehat{\delta_{k k}}$, but nlcom treats the sample mean of $\mathrm{s} \_\mathrm{k}$ as a fixed parameter that does not contribute to the sampling variance of the estimated elasticity.

## Also see

[R] nlsur - Estimation of nonlinear systems of equations
[U] 20 Estimation and postestimation commands

## Title

nptrend - Test for trend across ordered groups

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | Acknowledgments |
| References | Also see |  |  |

## Syntax

nptrend varname [if] [in], by (groupvar) [nodetail score(scorevar)]

## Menu

Statistics $>$ Nonparametric analysis $>$ Tests of hypotheses $>$ Trend test across ordered groups

## Description

nptrend performs a nonparametric test for trend across ordered groups.

## Options

$\qquad$ Main
by (groupvar) is required; it specifies the group on which the data are to be ordered.
nodetail suppresses the listing of group rank sums.
score(scorevar) defines scores for groups. When it is not specified, the values of groupvar are used for the scores.

## Remarks and examples

nptrend performs the nonparametric test for trend across ordered groups developed by Cuzick (1985), which is an extension of the Wilcoxon rank-sum test (see [R] ranksum). A correction for ties is incorporated into the test. nptrend is a useful adjunct to the Kruskal-Wallis test; see [ R ] kwallis.

If your data are not grouped, you can test for trend with the signtest and spearman commands; see $[R]$ signrank and $[R]$ spearman. With signtest, you can perform the Cox and Stuart test, a sign test applied to differences between equally spaced observations of varname. With spearman, you can perform the Daniels test, a test of zero Spearman correlation between varname and a time index. See Conover (1999, 169-175, 323) for a discussion of these tests and their asymptotic relative efficiency.

## > Example 1

The following data (Altman 1991, 217) show ocular exposure to ultraviolet radiation for 32 pairs of sunglasses classified into three groups according to the amount of visible light transmitted.


Entering these data into Stata, we have

```
. use http://www.stata-press.com/data/r13/sg
```

. list, $\operatorname{sep}(6)$

|  | group | exposure |
| :---: | :---: | :---: |
| 1. | 1 | 1.4 |
| 2. | 1 | 1.4 |
| 3. | 1 | 1.4 |
| 4. | 1 | 1.6 |
| 5. | 1 | 2.3 |
| 6. | 1 | 2.3 |
| 7. | $\begin{gathered} 2 \\ \text { (outpu } \end{gathered}$ | $\text { omitted ) }{ }^{.}$ |
| 31. | 3 | 8.9 |
| 32. | 3 | 13.5 |

We use nptrend to test for a trend of (increasing) exposure across the three groups by typing

- nptrend exposure, by (group)
group
1

When the groups are given any equally spaced scores (such as $-1,0,1$ ), we will obtain the same answer as above. To illustrate the effect of changing scores, an analysis of these data with scores 1 , 2 , and 5 (admittedly not sensible here) produces

```
. gen mysc = cond(group==3,5,group)
. nptrend exposure, by(group) score(mysc)
\begin{tabular}{rrrc} 
group & score & obs & sum of ranks \\
1 & 1 & 6 & 76 \\
2 & 2 & 18 & 290 \\
3 & 5 & 8 & 162 \\
\(z=\) & 1.46 & & \\
Prob \(\rangle|z|=0.143\) & &
\end{tabular}
```

This example suggests that the analysis is not all that sensitive to the scores chosen.

- Technical note

The grouping variable may be either a string variable or a numeric variable. If it is a string variable and no score variable is specified, the natural numbers $1,2,3, \ldots$ are assigned to the groups in the sort order of the string variable. This may not always be what you expect. For example, the sort order of the strings "one", "two", "three" is "one", "three", "two".

## Stored results

nptrend stores the following in $r()$ :
Scalars
$r(N)$ number of observations $\quad r(z) \quad z$ statistic
$r(\mathrm{p})$ two-sided $p$-value $\quad \mathrm{r}(\mathrm{T})$ test statistic

## Methods and formulas

nptrend is based on a method in Cuzick (1985). The following description of the statistic is from Altman (1991, 215-217). We have $k$ groups of sample sizes $n_{i}(i=1, \ldots, k)$. The groups are given scores, $l_{i}$, which reflect their ordering, such as 1,2 , and 3 . The scores do not have to be equally spaced, but they usually are. $N=\sum n_{i}$ observations are ranked from 1 to $N$, and the sums of the ranks in each group, $R_{i}$, are obtained. $L$, the weighted sum of all the group scores, is

$$
L=\sum_{i=1}^{k} l_{i} n_{i}
$$

The statistic $T$ is calculated as

$$
T=\sum_{i=1}^{k} l_{i} R_{i}
$$

Under the null hypothesis, the expected value of $T$ is $E(T)=0.5(N+1) L$, and its standard error is

$$
\operatorname{se}(T)=\sqrt{\frac{N+1}{12}\left(N \sum_{i=1}^{k} l_{i}^{2} n_{i}-L^{2}\right)}
$$

so that the test statistic, $z$, is given by $z=\{T-E(T)\} / \operatorname{se}(T)$, which has an approximately standard normal distribution when the null hypothesis of no trend is true.

The correction for ties affects the standard error of $T$. Let $\tilde{N}$ be the number of unique values of the variable being tested $(\widetilde{N} \leq N)$, and let $t_{j}$ be the number of times the $j$ th unique value of the variable appears in the data. Define

$$
a=\frac{\sum_{j=1}^{\widetilde{N}} t_{j}\left(t_{j}^{2}-1\right)}{N\left(N^{2}-1\right)}
$$

The corrected standard error of $T$ is $\widetilde{\mathrm{se}}(T)=\sqrt{1-a} \operatorname{se}(T)$.

## Acknowledgments

nptrend was written by K. A. Stepniewska and D. G. Altman (1992) of the Cancer Research UK.

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## Also see

[R] kwallis - Kruskal-Wallis equality-of-populations rank test
[ R$]$ signrank - Equality tests on matched data
[R] spearman - Spearman's and Kendall's correlations
$[\mathrm{R}]$ symmetry - Symmetry and marginal homogeneity tests
[ST] epitab - Tables for epidemiologists
[ST] strate - Tabulate failure rates and rate ratios

## Title

ologit - Ordered logistic regression

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

```
ologit depvar [indepvars] [if] [in] [weight] [, options]
```

options
Description

Model
offset (varname)
constraints (constraints)
collinear
SE/Robust

Reporting
level(\#)
or
nocnsreport
display_options

## Maximization

maximize_options
coeflegend
include varname in model with coefficient constrained to 1
apply specified linear constraints
keep collinear variables

## vce(vcetype) vcetype may be oim, robust, cluster clustvar, bootstrap, or jackknife

set confidence level; default is level (95)
report odds ratios
do not display constraints
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
depvar and indepvars may contain time-series operators; see [U] 11.4.4 Time-series varlists.
bootstrap, by, fp, jackknife, mfp, mi estimate, nestreg, rolling, statsby, stepwise, and svy are allowed; see [U] 11.1.10 Prefix commands.
vce (bootstrap) and vce (jackknife) are not allowed with the mi estimate prefix; see [MI] mi estimate.
Weights are not allowed with the bootstrap prefix; see $[R]$ bootstrap.
vce() and weights are not allowed with the svy prefix; see [SVY] svy.
fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Ordinal outcomes $>$ Ordered logistic regression

## Description

ologit fits ordered logit models of ordinal variable depvar on the independent variables indepvars. The actual values taken on by the dependent variable are irrelevant, except that larger values are assumed to correspond to "higher" outcomes.

See [R] logistic for a list of related estimation commands.

## Options

```
Model
offset (varname), constraints (constraints), collinear; see [R] estimation options.
SE/Robust
vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
```

Reporting
level (\#); see [R] estimation options.
or reports the estimated coefficients transformed to odds ratios, that is, $e^{b}$ rather than $b$. Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated or may be specified at estimation or when replaying previously estimated results.
nocnsreport; see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% fmt), pformat (\% fmt), sformat (\% fmt), and nolstretch; see [R] estimation options.

Maximization
maximize_options: difficult, technique (algorithm_spec), iterate (\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance (\#), nonrtolerance, and from(init_specs); see [ R$]$ maximize. These options are seldom used.

The following option is available with ologit but is not shown in the dialog box:
coeflegend; see [R] estimation options.

## Remarks and examples

Ordered logit models are used to estimate relationships between an ordinal dependent variable and a set of independent variables. An ordinal variable is a variable that is categorical and ordered, for instance, "poor", "good", and "excellent", which might indicate a person's current health status or the repair record of a car. If there are only two outcomes, see $[R]$ logistic, $[R]$ logit, and $[R]$ probit. This entry is concerned only with more than two outcomes. If the outcomes cannot be ordered (for example, residency in the north, east, south, or west), see [R] mlogit. This entry is concerned only with models in which the outcomes can be ordered.

In ordered logit, an underlying score is estimated as a linear function of the independent variables and a set of cutpoints. The probability of observing outcome $i$ corresponds to the probability that the estimated linear function, plus random error, is within the range of the cutpoints estimated for the outcome:

$$
\operatorname{Pr}\left(\text { outcome }_{j}=i\right)=\operatorname{Pr}\left(\kappa_{i-1}<\beta_{1} x_{1 j}+\beta_{2} x_{2 j}+\cdots+\beta_{k} x_{k j}+u_{j} \leq \kappa_{i}\right)
$$

$u_{j}$ is assumed to be logistically distributed in ordered logit. In either case, we estimate the coefficients $\beta_{1}, \beta_{2}, \ldots, \beta_{k}$ together with the cutpoints $\kappa_{1}, \kappa_{2}, \ldots, \kappa_{k-1}$, where $k$ is the number of possible outcomes. $\kappa_{0}$ is taken as $-\infty$, and $\kappa_{k}$ is taken as $+\infty$. All of this is a direct generalization of the ordinary two-outcome logit model.

## > Example 1

We wish to analyze the 1977 repair records of 66 foreign and domestic cars. The data are a variation of the automobile dataset described in [U] 1.2.2 Example datasets. The 1977 repair records, like those in 1978, take on values "Poor", "Fair", "Average", "Good", and "Excellent". Here is a cross-tabulation of the data:


Although it appears that foreign takes on the values Domestic and Foreign, it is actually a numeric variable taking on the values 0 and 1 . Similarly, rep77 takes on the values $1,2,3,4$, and 5, corresponding to Poor, Fair, and so on. The more meaningful words appear because we have attached value labels to the data; see [U] 12.6.3 Value labels.

Because the chi-squared value is significant, we could claim that there is a relationship between foreign and rep77. Literally, however, we can only claim that the distributions are different; the chi-squared test is not directional. One way to model these data is to model the categorization that took place when the data were created. Cars have a true frequency of repair, which we will assume is given by $S_{j}=\beta$ foreign $_{j}+u_{j}$, and a car is categorized as "poor" if $S_{j} \leq \kappa_{0}$, as "fair" if $\kappa_{0}<S_{j} \leq \kappa_{1}$, and so on:
$\left.\begin{array}{llllll}\text {. ologit rep77 foreign } \\ \text { Iteration 0: } & \text { log likelihood }=-89.895098\end{array}\right)$

Our model is $S_{j}=1.46$ foreign $_{j}+u_{j}$; the expected value for foreign cars is 1.46 and, for domestic cars, 0 ; foreign cars have better repair records.

The estimated cutpoints tell us how to interpret the score. For a foreign car, the probability of a poor record is the probability that $1.46+u_{j} \leq-2.77$, or equivalently, $u_{j} \leq-4.23$. Making this calculation requires familiarity with the logistic distribution: the probability is $1 /\left(1+e^{4.23}\right)=0.014$. On the other hand, for domestic cars, the probability of a poor record is the probability $u_{j} \leq-2.77$, which is 0.059 .

This, it seems to us, is a far more reasonable prediction than we would have made based on the table alone. The table showed that 2 of 45 domestic cars had poor records, whereas 1 of 21 foreign cars had poor records-corresponding to probabilities $2 / 45=0.044$ and $1 / 21=0.048$. The predictions from our model imposed a smoothness assumption-foreign cars should not, overall, have better repair records without the difference revealing itself in each category. In our data, the fractions of foreign and domestic cars in the poor category are virtually identical only because of the randomness associated with small samples.

Thus if we were asked to predict the true fractions of foreign and domestic cars that would be classified in the various categories, we would choose the numbers implied by the ordered logit model:

|  | tabulate |  | logit |  |
| :--- | :---: | :---: | :---: | ---: |
|  | Domestic | Foreign | Domestic | Foreign |
| Poor | 0.044 | 0.048 | 0.059 | 0.014 |
| Fair | 0.222 | 0.048 | 0.210 | 0.065 |
| Average | 0.444 | 0.333 | 0.450 | 0.295 |
| Good | 0.289 | 0.333 | 0.238 | 0.467 |
| Excellent | 0.000 | 0.238 | 0.043 | 0.159 |

See $[\mathrm{R}]$ ologit postestimation for a more complete explanation of how to generate predictions from an ordered logit model.

## - Technical note

Here ordered logit provides an alternative to ordinary two-outcome logistic models with an arbitrary dichotomization, which might otherwise have been tempting. We could, for instance, have summarized these data by converting the five-outcome rep77 variable to a two-outcome variable, combining cars in the average, fair, and poor categories to make one outcome and combining cars in the good and excellent categories to make the second.

Another even less appealing alternative would have been to use ordinary regression, arbitrarily labeling "excellent" as 5, "good" as 4 , and so on. The problem is that with different but equally valid labelings (say, 10 for "excellent"), we would obtain different estimates. We would have no way of choosing one metric over another. That assertion is not, however, true of ologit. The actual values used to label the categories make no difference other than through the order they imply.

In fact, our labeling was 5 for "excellent", 4 for "good", and so on. The words "excellent" and "good" appear in our output because we attached a value label to the variables; see [U] 12.6.3 Value labels. If we were to now go back and type replace rep $77=10$ if rep $77==5$, changing all the 5 s to 10 s , we would still obtain the same results when we refit our model.

## > Example 2

In the example above, we used ordered logit as a way to model a table. We are not, however, limited to including only one explanatory variable or to including only categorical variables. We can explore the relationship of rep77 with any of the variables in our data. We might, for instance, model rep77 not only in terms of the origin of manufacture, but also including length (a proxy for size) and mpg:

```
. ologit rep77 foreign length mpg
Iteration 0: log likelihood = -89.895098
Iteration 1: log likelihood = -78.775147
Iteration 2: log likelihood = -78.254294
Iteration 3: log likelihood = -78.250719
Iteration 4: log likelihood = -78.250719
```

| Ordered logistic regression | Number of obs | $=$ | 66 |
| :--- | :--- | :--- | :--- |
|  | LR chi2 (3) | $=$ | 23.29 |
|  | Prob > chi2 | $=$ | 0.0000 |
| Log likelihood $=-78.250719$ |  | Pseudo R2 | $=$ | 0.1295


| rep77 | Coef. | Std. Err. | $z$ | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | :---: | ---: | ---: |
| foreign | 2.896807 | .7906411 | 3.66 | 0.000 | 1.347179 | 4.446435 |
| length | .0828275 | .02272 | 3.65 | 0.000 | .0382972 | .1273579 |
| mpg | .2307677 | .0704548 | 3.28 | 0.001 | .0926788 | .3688566 |
| /cut1 | 17.92748 | 5.551191 |  |  | 7.047344 | 28.80761 |
| /cut2 | 19.86506 | 5.59648 |  |  | 8.896161 | 30.83396 |
| /cut3 | 22.10331 | 5.708936 |  |  | 10.914 | 33.29262 |
| /cut4 | 24.69213 | 5.890754 |  |  | 13.14647 | 36.2378 |

foreign still plays a role-and an even larger role than previously. We find that larger cars tend to have better repair records, as do cars with better mileage ratings.

## Stored results

ologit stores the following in e():
Scalars
e(N)
e(N_cd)
e(k_cat)
e(k)
e(k_aux)
e(k_eq)
e(k_eq_model)
e(k_dv)
e(df_m)
e(r2_p)
e(11)
e(ll_0)
e(N_clust)
e(chi2)
e (p)
e(rank)
e(ic)
$e(r c)$
$e$ (converged)
Macros
e(cmd)
e(cmdline)
e(depvar)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(offset)
e(chi2type)
e(vce)
e(vcetype)
e(opt)
$e$ (which)
e(ml_method)
e(user)
e(technique)
e(properties)
e(predict)
e(asbalanced)
$e$ (asobserved)
Matrices
e(b)
e(Cns)
e(ilog)
e(gradient)
e(cat)
e(V)
e(V_modelbased)
Functions
e(sample) marks estimation sample

## Methods and formulas

See Long and Freese (2014, chap. 7) for a discussion of models for ordinal outcomes and examples that use Stata. Cameron and Trivedi (2005, chap. 15) describe multinomial models, including the model fit by ologit. When you have a qualitative dependent variable, several estimation procedures are available. A popular choice is multinomial logistic regression (see [R] mlogit), but if you use this procedure when the response variable is ordinal, you are discarding information because multinomial logit ignores the ordered aspect of the outcome. Ordered logit and probit models provide a means to exploit the ordering information.

There is more than one "ordered logit" model. The model fit by ologit, which we will call the ordered logit model, is also known as the proportional odds model. Another popular choice, not fit by ologit, is known as the stereotype model; see [R] slogit. All ordered logit models have been derived by starting with a binary logit/probit model and generalizing it to allow for more than two outcomes.

The proportional-odds ordered logit model is so called because, if we consider the odds odds $(k)=$ $P(Y \leq k) / P(Y>k)$, then odds $\left(k_{1}\right)$ and odds $\left(k_{2}\right)$ have the same ratio for all independent variable combinations. The model is based on the principle that the only effect of combining adjoining categories in ordered categorical regression problems should be a loss of efficiency in estimating the regression parameters (McCullagh 1980). This model was also described by McKelvey and Zavoina (1975) and, previously, by Aitchison and Silvey (1957) in a different algebraic form. Brant (1990) offers a set of diagnostics for the model.

Peterson and Harrell (1990) suggest a model that allows nonproportional odds for a subset of the explanatory variables. ologit does not allow this, but a model similar to this was implemented by Fu (1998).

The stereotype model rejects the principle on which the ordered logit model is based. Anderson (1984) argues that there are two distinct types of ordered categorical variables: "grouped continuous", such as income, where the "type a" model applies; and "assessed", such as extent of pain relief, where the stereotype model applies. Greenland (1985) independently developed the same model. The stereotype model starts with a multinomial logistic regression model and imposes constraints on this model.

Goodness of fit for ologit can be evaluated by comparing the likelihood value with that obtained by fitting the model with mlogit. Let $\ln L_{1}$ be the log-likelihood value reported by ologit, and let $\ln L_{0}$ be the log-likelihood value reported by mlogit. If there are $p$ independent variables (excluding the constant) and $k$ categories, mlogit will estimate $p(k-1)$ additional parameters. We can then perform a "likelihood-ratio test", that is, calculate $-2\left(\ln L_{1}-\ln L_{0}\right)$, and compare it with $\chi^{2}\{p(k-2)\}$. This test is suggestive only because the ordered logit model is not nested within the multinomial logit model. A large value of $-2\left(\ln L_{1}-\ln L_{0}\right)$ should, however, be taken as evidence of poorness of fit. Marginally large values, on the other hand, should not be taken too seriously.

The coefficients and cutpoints are estimated using maximum likelihood as described in [R] maximize. In our parameterization, no constant appears, because the effect is absorbed into the cutpoints.
ologit and oprobit begin by tabulating the dependent variable. Category $i=1$ is defined as the minimum value of the variable, $i=2$ as the next ordered value, and so on, for the empirically determined $k$ categories.

The probability of a given observation for ordered logit is

$$
\begin{aligned}
p_{i j}=\operatorname{Pr}\left(y_{j}=i\right) & =\operatorname{Pr}\left(\kappa_{i-1}<\mathbf{x}_{j} \boldsymbol{\beta}+u \leq \kappa_{i}\right) \\
& =\frac{1}{1+\exp \left(-\kappa_{i}+\mathbf{x}_{j} \boldsymbol{\beta}\right)}-\frac{1}{1+\exp \left(-\kappa_{i-1}+\mathbf{x}_{j} \boldsymbol{\beta}\right)}
\end{aligned}
$$

$\kappa_{0}$ is defined as $-\infty$ and $\kappa_{k}$ as $+\infty$.
For ordered probit, the probability of a given observation is

$$
\begin{aligned}
p_{i j}=\operatorname{Pr}\left(y_{j}=i\right) & =\operatorname{Pr}\left(\kappa_{i-1}<\mathbf{x}_{j} \boldsymbol{\beta}+u \leq \kappa_{i}\right) \\
& =\Phi\left(\kappa_{i}-\mathbf{x}_{j} \boldsymbol{\beta}\right)-\Phi\left(\kappa_{i-1}-\mathbf{x}_{j} \boldsymbol{\beta}\right)
\end{aligned}
$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function.
The log likelihood is

$$
\ln L=\sum_{j=1}^{N} w_{j} \sum_{i=1}^{k} I_{i}\left(y_{j}\right) \ln p_{i j}
$$

where $w_{j}$ is an optional weight and

$$
I_{i}\left(y_{j}\right)=\left\{\begin{array}{l}
1, \text { if } y_{j}=i \\
0, \text { otherwise }
\end{array}\right.
$$

ologit and oprobit support the Huber/White/sandwich estimator of the variance and its clustered version using vce (robust) and vce (cluster clustvar), respectively. See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas.

These commands also support estimation with survey data. For details on VCEs with survey data, see [SVY] variance estimation.

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## Also see

$[\mathrm{R}]$ ologit postestimation - Postestimation tools for ologit
$[\mathrm{R}]$ clogit - Conditional (fixed-effects) logistic regression
$[\mathrm{R}]$ logistic - Logistic regression, reporting odds ratios
[R] logit - Logistic regression, reporting coefficients
[R] mlogit - Multinomial (polytomous) logistic regression
$[\mathrm{R}]$ oprobit - Ordered probit regression
[R] rologit - Rank-ordered logistic regression
[R] slogit - Stereotype logistic regression
[ME] meologit - Multilevel mixed-effects ordered logistic regression
[MI] estimation - Estimation commands for use with mi estimate
[SVY] svy estimation - Estimation commands for survey data
[XT] xtologit - Random-effects ordered logistic models
[U] 20 Estimation and postestimation commands

## Title

ologit postestimation - Postestimation tools for ologit

| Description | Syntax for predict |
| :--- | :--- |
| Remarks and examples | Also see for predict |$\quad$ Options for predict

## Description

The following postestimation commands are available after ologit:

| Command | Description |
| :--- | :--- |
| contrast | contrasts and ANOVA-style joint tests of estimates |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample |
| estat vce | variance-covariance matrix of the estimators (VCE) |
| estat (svy) | postestimation statistics for survey data <br> estaloging estimation results |
| forecast |  |$\quad$| dynamic forecasts and simulations |
| :--- |
| lincom |$\quad$| point estimates, standard errors, testing, and inference for linear combinations |
| :--- |
| of coefficients |

[^55]
## Syntax for predict

```
predict [type] {stub*|newvar|newvarlist } [if] [in] [, statistic
    outcome(outcome) nooffset]
predict [type] {stub*|newvarlist } [if] [in], scores
```

statistic Description

Main
$\underline{\mathrm{pr}} \quad$ predicted probabilities; the default
xb linear prediction
stdp standard error of the linear prediction
If you do not specify outcome(), pr (with one new variable specified) assumes outcome(\#1).
You specify one or $k$ new variables with pr , where $k$ is the number of outcomes.
You specify one new variable with xb and stdp.
These statistics are available both in and out of sample; type predict ... if e(sample)... if wanted only for the estimation sample.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

[^56]Main
scores calculates equation-level score variables. The number of score variables created will equal the number of outcomes in the model. If the number of outcomes in the model was $k$, then the first new variable will contain $\partial \ln L / \partial\left(\mathbf{x}_{j} \mathbf{b}\right)$;
the second new variable will contain $\partial \ln L / \partial \kappa_{1}$;
the third new variable will contain $\partial \ln L / \partial \kappa_{2}$;
and the $k$ th new variable will contain $\partial \ln L / \partial \kappa_{k-1}$, where $\kappa_{i}$ refers to the $i$ th cutpoint.

## Remarks and examples

See [U] 20 Estimation and postestimation commands for instructions on obtaining the variancecovariance matrix of the estimators, predicted values, and hypothesis tests. Also see [R] lrtest for performing likelihood-ratio tests.

## > Example 1

In example 2 of [R] ologit, we fit the model ologit rep77 foreign length mpg. The predict command can be used to obtain the predicted probabilities.

We type predict followed by the names of the new variables to hold the predicted probabilities, ordering the names from low to high. In our data, the lowest outcome is "poor", and the highest is "excellent". We have five categories, so we must type five names following predict; the choice of names is up to us:

```
. use http://www.stata-press.com/data/r13/fullauto
(Automobile Models)
. ologit rep77 foreign length mpg
(output omitted)
. predict poor fair avg good exc
(option pr assumed; predicted probabilities)
. list exc good make model rep78 if rep77>=., sep(4) divider
```

| exc | good | make | model | rep78 |
| ---: | ---: | ---: | ---: | ---: |
| .0033341 | .0393056 | AMC <br> .0098392 | Spirit <br> Opel | . |
| .0023406 | .0279497 | Buick <br> Ford <br> Merc. | Fiesta <br> Monarch | Good <br> Average |
| .015697 | .1594413 | .065272 | .4165188 | Peugeot <br> Plym. |
| .005187 | .059727 | Horizon | Average |  |
| .0261461 | .2371826 | Plym. |  |  |
| .0294961 | .2585825 | Sapporo |  |  |
| Pont. | Phoenix | . |  |  |

The eight cars listed were introduced after 1977, so they do not have 1977 repair records in our data. We predicted what their 1977 repair records might have been using the fitted model. We see that, based on its characteristics, the Peugeot 604 had about a $41.65+6.53 \approx 48.2 \%$ chance of a good or excellent repair record. The Ford Fiesta, which had only a $3 \%$ chance of a good or excellent repair record, in fact, had a good record when it was introduced in the following year.

- Technical note

For ordered logit, predict, xb produces $S_{j}=x_{1 j} \beta_{1}+x_{2 j} \beta_{2}+\cdots+x_{k j} \beta_{k}$. The ordered-logit predictions are then the probability that $S_{j}+u_{j}$ lies between a pair of cutpoints, $\kappa_{i-1}$ and $\kappa_{i}$. Some handy formulas are

$$
\begin{aligned}
\operatorname{Pr}\left(S_{j}+u_{j}<\kappa\right) & =1 /\left(1+e^{S_{j}-\kappa}\right) \\
\operatorname{Pr}\left(S_{j}+u_{j}>\kappa\right) & =1-1 /\left(1+e^{S_{j}-\kappa}\right) \\
\operatorname{Pr}\left(\kappa_{1}<S_{j}+u_{j}<\kappa_{2}\right) & =1 /\left(1+e^{S_{j}-\kappa_{2}}\right)-1 /\left(1+e^{S_{j}-\kappa_{1}}\right)
\end{aligned}
$$

Rather than using predict directly, we could calculate the predicted probabilities by hand. If we wished to obtain the predicted probability that the repair record is excellent and the probability that it is good, we look back at ologit's output to obtain the cutpoints. We find that "good" corresponds to the interval /cut3 $<S_{j}+u</$ cut4 and "excellent" to the interval $S_{j}+u>/$ cut4:

- predict score, xb
. generate probgood $=1 /(1+\exp ($ score-_b[/cut4] $))-1 /(1+\exp ($ score-_b[/cut3]) $)$
. generate probexc = $1-1 /(1+\exp ($ score-_b[/cut4] $))$
The results of our calculation will be the same as those produced in the previous example. We refer to the estimated cutpoints just as we would any coefficient, so _b [/cut3] refers to the value of the /cut3 coefficient; see [U] 13.5 Accessing coefficients and standard errors.


## Also see

$[\mathrm{R}]$ ologit - Ordered logistic regression
[U] 20 Estimation and postestimation commands

## Title

oneway - One-way analysis of variance

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |
| Also see |  |  |  |

## Syntax

oneway response_var factor_var $[$ if $][$ in $][$ weight $][$, options $]$
options Description
Main
bonferroni
scheffe
sidak
tabulate
[no] means
[no] standard
[no] freq
[no] obs
noanova
nolabel
wrap
missing

Bonferroni multiple-comparison test
Scheffé multiple-comparison test
Šidák multiple-comparison test produce summary table
include or suppress means; default is means
include or suppress standard deviations; default is standard
include or suppress frequencies; default is freq
include or suppress number of obs; default is obs if data are weighted
suppress the ANOVA table
show numeric codes, not labels
do not break wide tables
treat missing values as categories
by is allowed; see [D] by.
aweights and fweights are allowed; see [U] 11.1.6 weight.

## Menu

Statistics $>$ Linear models and related $>$ ANOVA/MANOVA $>$ One-way ANOVA

## Description

The oneway command reports one-way analysis-of-variance (ANOVA) models and performs multiplecomparison tests.

If you wish to fit more complicated ANOVA layouts or wish to fit analysis-of-covariance (ANCOVA) models, see [R] anova.

See [D] encode for examples of fitting ANOVA models on string variables.
See [R] loneway for an alternative oneway command with slightly different features.

## Options

## Main

bonferroni reports the results of a Bonferroni multiple-comparison test.
scheffe reports the results of a Scheffé multiple-comparison test.
sidak reports the results of a Šidák multiple-comparison test.
tabulate produces a table of summary statistics of the response_var by levels of the factor_var. The table includes the mean, standard deviation, frequency, and, if the data are weighted, the number of observations. Individual elements of the table may be included or suppressed by using the [no]means, [no]standard, [no]freq, and [no]obs options. For example, typing
oneway response factor, tabulate means standard
produces a summary table that contains only the means and standard deviations. You could achieve the same result by typing
oneway response factor, tabulate nofreq
[no]means includes or suppresses only the means from the table produced by the tabulate option. See tabulate above.
[no]standard includes or suppresses only the standard deviations from the table produced by the tabulate option. See tabulate above.
[no]freq includes or suppresses only the frequencies from the table produced by the tabulate option. See tabulate above.
[no]obs includes or suppresses only the reported number of observations from the table produced by the tabulate option. If the data are not weighted, only the frequency is reported. If the data are weighted, the frequency refers to the sum of the weights. See tabulate above.
noanova suppresses the display of the ANOVA table.
nolabel causes the numeric codes to be displayed rather than the value labels in the ANOVA and multiple-comparison test tables.
wrap requests that Stata not break up wide tables to make them more readable.
missing requests that missing values of factor_var be treated as a category rather than as observations to be omitted from the analysis.

## Remarks and examples

Remarks are presented under the following headings:

Introduction<br>Obtaining observed means<br>Multiple-comparison tests<br>Weighted data<br>Video example

## Introduction

The oneway command reports one-way ANOVA models. To perform a one-way layout of a variable called endog on exog, type oneway endog exog.

## Example 1

We run an experiment varying the amount of fertilizer used in growing apple trees. We test four concentrations, using each concentration in three groves of 12 trees each. Later in the year, we measure the average weight of the fruit.

If all had gone well, we would have had 3 observations on the average weight for each of the four concentrations. Instead, two of the groves were mistakenly leveled by a confused man on a large bulldozer. We are left with the following dataset:

| . use http://www.stata-press.com/data/r13/apple <br> (Apple trees) |
| :--- |
| describe <br> Contains <br> obs: <br> vars: <br> size: |

Sorted by:
. list, abbreviate(10)

|  | treatment | weight |
| ---: | ---: | ---: |
| 1. | 1 | 117.5 |
| 2. | 1 | 113.8 |
| 3. | 1 | 104.4 |
| 4. | 2 | 48.9 |
| 5. | 2 | 50.4 |
| 6. | 2 | 58.9 |
| 7. | 3 | 70.4 |
| 8. | 3 | 86.9 |
| 9. | 4 | 87.7 |
| 10. | 4 | 67.3 |

To obtain the one-way ANOVA results, we type
. oneway weight treatment
Analysis of Variance

| Source | SS | df | MS | F | Prob $>\mathrm{F}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Between groups <br> Within groups | 5295.54433 | 3 | 1765.18144 | 21.46 | 0.0013 |
| Total | 493.591667 | 6 | 82.2652778 |  |  |

We find significant (at better than the $1 \%$ level) differences among the four concentrations.

## - Technical note

Rather than using the oneway command, we could have performed this analysis by using anova. Example 1 in $[R]$ anova repeats this same analysis. You may wish to compare the output.

You will find the oneway command quicker than the anova command, and, as you will learn, oneway allows you to perform multiple-comparison tests. On the other hand, anova will let you generate predictions, examine the covariance matrix of the estimators, and perform more general hypothesis tests.

## - Technical note

Although the output is a usual ANOVA table, let's run through it anyway. The between-group sum of squares for the model is 5295.5 with 3 degrees of freedom, resulting in a mean square of $5295.5 / 3 \approx 1765.2$. The corresponding $F$ statistic is 21.46 and has a significance level of 0.0013 . Thus the model appears to be significant at the $0.13 \%$ level.

The second line summarizes the within-group (residual) variation. The within-group sum of squares is 493.59 with 6 degrees of freedom, resulting in a mean squared error of 82.27.

The between- and residual-group variations sum to the total sum of squares (TSS), which is reported as 5789.1 in the last line of the table. This is the TSS of weight after removal of the mean. Similarly, the between plus residual degrees of freedom sum to the total degrees of freedom, 9. Remember that there are 10 observations. Subtracting 1 for the mean, we are left with 9 total degrees of freedom.

At the bottom of the table, Bartlett's test for equal variances is reported. The value of the statistic is 1.39 . The corresponding significance level ( $\chi^{2}$ with 3 degrees of freedom) is 0.708 , so we cannot reject the assumption that the variances are homogeneous.

## Obtaining observed means

> Example 2
We typed oneway weight treatment to obtain an ANOVA table of weight of fruit by fertilizer concentration. Although we obtained the table, we obtained no information on which fertilizer seems to work the best. If we add the tabulate option, we obtain that additional information:


We find that the average weight was largest when we used fertilizer concentration 1 .

## Multiple-comparison tests

> Example 3: Bonferroni multiple-comparison test
oneway can also perform multiple-comparison tests using either Bonferroni, Scheffé, or Šidák normalizations. For instance, to obtain the Bonferroni multiple-comparison test, we specify the bonferroni option:

```
. oneway weight treatment, bonferroni
```

| Analysis of Variance |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Between groups | 5295.54433 | 3 | 1765.18144 | 21.46 | 0.0013 |
| Within groups | 493.591667 | 6 | 82.2652778 |  |  |
| Total | 5789.136 | 9 | 643.237333 |  |  |
| Bartlett's test Con | equal vari ison of Ave |  | $\operatorname{ii2}(3)=$ <br> t in grams oni) | $\begin{aligned} & \text { Foro } \\ & \text { Fertili } \end{aligned}$ | $>\operatorname{chi2}=$ |


| Row Mean- <br> Col Mean | 1 | 2 | 3 |
| ---: | ---: | ---: | ---: |
| 2 | -59.1667 |  |  |
|  | 0.001 |  |  |
| 3 | -33.25 | 25.9167 |  |
|  | 0.042 | 0.122 |  |
| 4 | -34.4 | 24.7667 | -1.15 |
|  | 0.036 | 0.146 | 1.000 |

The results of the Bonferroni test are presented as a matrix. The first entry, -59.17 , represents the difference between fertilizer concentrations 2 and 1 (labeled "Row Mean - Col Mean" in the upper stub of the table). Remember that in the previous example we requested the tabulate option. Looking back, we find that the means of concentrations 1 and 2 are 111.90 and 52.73 , respectively. Thus $52.73-111.90=-59.17$.

Underneath that number is reported " 0.001 ". This is the Bonferroni-adjusted significance of the difference. The difference is significant at the $0.1 \%$ level. Looking down the column, we see that concentration 3 is also worse than concentration 1 ( $4.2 \%$ level), as is concentration 4 ( $3.6 \%$ level).

On the basis of this evidence, we would use concentration 1 if we grew apple trees.

## > Example 4: Scheffé multiple-comparison test

We can just as easily obtain the Scheffé-adjusted significance levels. Rather than specifying the bonferroni option, we specify the scheffe option.

We will also add the noanova option to prevent Stata from redisplaying the ANOVA table:

```
. oneway weight treatment, noanova scheffe
            Comparison of Average weight in grams by Fertilizer
                            (Scheffe)
\begin{tabular}{r|rrr}
\begin{tabular}{l} 
Row Mean- \\
Col Mean
\end{tabular} & 1 & 2 & 3 \\
\hline 2 & -59.1667 & & \\
& 0.001 & & \\
3 & -33.25 & 25.9167 & \\
& 0.039 & 0.101 & \\
4 & -34.4 & 24.7667 & -1.15 \\
& 0.034 & 0.118 & 0.999
\end{tabular}
```

The differences are the same as those we obtained in the Bonferroni output, but the significance levels are not. According to the Bonferroni-adjusted numbers, the significance of the difference between fertilizer concentrations 1 and 3 is $4.2 \%$. The Scheffé-adjusted significance level is $3.9 \%$.

We will leave it to you to decide which results are more accurate.

## Example 5: Šidák multiple-comparison test

Let's conclude this example by obtaining the Šidák-adjusted multiple-comparison tests. We do this to illustrate Stata's capabilities to calculate these results, because searching across adjustment methods until you find the results you want is not a valid technique for obtaining significance levels.

```
. oneway weight treatment, noanova sidak
    Comparison of Average weight in grams by Fertilizer
        (Sidak)
\begin{tabular}{r|rrr}
\begin{tabular}{l} 
Row Mean- \\
Col Mean
\end{tabular} & 1 & 2 & 3 \\
\hline 2 & -59.1667 & & \\
& 0.001 & & \\
3 & -33.25 & 25.9167 & \\
& 0.041 & 0.116 & \\
4 & & & \\
& -34.4 & 24.7667 & -1.15 \\
& 0.035 & 0.137 & 1.000
\end{tabular}
```

We find results that are similar to the Bonferroni-adjusted numbers.

Henry Scheffé (1907-1977) was born in New York. He studied mathematics at the University of Wisconsin, gaining a doctorate with a dissertation on differential equations. He taught mathematics at Wisconsin, Oregon State University, and Reed College, but his interests changed to statistics and he joined Wilks at Princeton. After periods at Syracuse, UCLA, and Columbia, Scheffé settled in Berkeley from 1953. His research increasingly focused on linear models and particularly ANOVA, on which he produced a celebrated monograph. His death was the result of a bicycle accident.

## Weighted data

## $>$ Example 6

oneway can work with both weighted and unweighted data. Let's assume that we wish to perform a one-way layout of the death rate on the four census regions of the United States using state data. Our data contain three variables, drate (the death rate), region (the region), and pop (the population of the state).

To fit the model, we type oneway drate region [weight=pop], although we typically abbreviate weight as w. We will also add the tabulate option to demonstrate how the table of summary statistics differs for weighted data:

```
. use http://www.stata-press.com/data/r13/census8
(1980 Census data by state)
. oneway drate region [w=pop], tabulate
(analytic weights assumed)
```



```
\begin{tabular}{cccccc} 
Source & SS & df & MS & F & Prob > F \\
\hline \begin{tabular}{c} 
Between groups \\
Within groups
\end{tabular} & 2360.92281 & 3 & 786.974272 & 12.17 & 0.0000 \\
\hline Total & 5335.01916 & 49 & 108.877942 & & \\
\hline
\end{tabular}
```

When the data are weighted, the summary table has four columns rather than three. The column labeled "Freq." reports the sum of the weights. The overall frequency is $2.259 \times 10^{8}$, meaning that there are approximately 226 million people in the United States.

The ANOVA table is appropriately weighted. Also see [U] 11.1.6 weight.

## Video example

One-way ANOVA in Stata

## Stored results

oneway stores the following in $r()$ :
Scalars

| $r(N)$ | number of observations | $r\left(\mathrm{df} \_\mathrm{m}\right)$ | between-group degrees of freedom |
| :--- | :--- | :--- | :--- |
| $r(\mathrm{~F})$ | $F$ statistic | $\mathrm{r}(\mathrm{rss})$ | within-group sum of squares |
| $r\left(\mathrm{df} \_r\right)$ | within-group degrees of freedom | $\mathrm{r}(\mathrm{chi2bart})$ | Bartlett's $\chi^{2}$ |
| $r(\mathrm{mss})$ | between-group sum of squares | $r\left(\mathrm{df} \_\mathrm{bart}\right)$ | Bartlett's degrees of freedom |

## Methods and formulas

Methods and formulas are presented under the following headings:
One-way analysis of variance
Bartlett's test
Multiple-comparison tests

## One-way analysis of variance

The model of one-way ANOVA is

$$
y_{i j}=\mu+\alpha_{i}+\epsilon_{i j}
$$

for levels $i=1, \ldots, k$ and observations $j=1, \ldots, n_{i}$. Define $\bar{y}_{i}$ as the (weighted) mean of $y_{i j}$ over $j$ and $\bar{y}$ as the overall (weighted) mean of $y_{i j}$. Define $w_{i j}$ as the weight associated with $y_{i j}$, which is 1 if the data are unweighted. $w_{i j}$ is normalized to sum to $n=\sum_{i} n_{i}$ if aweights are used and is otherwise not normalized. $w_{i}$ refers to $\sum_{j} w_{i j}$, and $w$ refers to $\sum_{i} w_{i}$.

The between-group sum of squares is then

$$
S_{1}=\sum_{i} w_{i}\left(\bar{y}_{i}-\bar{y}\right)^{2}
$$

The TSS is

$$
S=\sum_{i} \sum_{j} w_{i j}\left(y_{i j}-\bar{y}\right)^{2}
$$

The within-group sum of squares is given by $S_{e}=S-S_{1}$.
The between-group mean square is $s_{1}^{2}=S_{1} /(k-1)$, and the within-group mean square is $s_{e}^{2}=S_{e} /(w-k)$. The test statistic is $F=s_{1}^{2} / s_{e}^{2}$. See, for instance, Snedecor and Cochran (1989).

## Bartlett's test

Bartlett's test assumes that you have $m$ independent, normal, random samples and tests the hypothesis $\sigma_{1}^{2}=\sigma_{2}^{2}=\cdots=\sigma_{m}^{2}$. The test statistic, $M$, is defined as

$$
M=\frac{(T-m) \ln \widehat{\sigma}^{2}-\sum\left(T_{i}-1\right) \ln \widehat{\sigma}_{i}^{2}}{1+\frac{1}{3(m-1)}\left\{\left(\sum \frac{1}{T_{i}-1}\right)-\frac{1}{T-m}\right\}}
$$

where there are $T$ overall observations, $T_{i}$ observations in the $i$ th group, and

$$
\begin{aligned}
& \left(T_{i}-1\right) \widehat{\sigma}_{i}^{2}=\sum_{j=1}^{T_{i}}\left(y_{i j}-\bar{y}_{i}\right)^{2} \\
& (T-m) \widehat{\sigma}^{2}=\sum_{i=1}^{m}\left(T_{i}-1\right) \widehat{\sigma}_{i}^{2}
\end{aligned}
$$

An approximate test of the homogeneity of variance is based on the statistic $M$ with critical values obtained from the $\chi^{2}$ distribution of $m-1$ degrees of freedom. See Bartlett (1937) or Draper and Smith (1998, 56-57).

## Multiple-comparison tests

Let's begin by reviewing the logic behind these adjustments. The "standard" $t$ statistic for the comparison of two means is

$$
t=\frac{\bar{y}_{i}-\bar{y}_{j}}{s \sqrt{\frac{1}{n_{i}}+\frac{1}{n_{j}}}}
$$

where $s$ is the overall standard deviation, $\bar{y}_{i}$ is the measured average of $y$ in group $i$, and $n_{i}$ is the number of observations in the group. We perform hypothesis tests by calculating this $t$ statistic. We simultaneously choose a critical level, $\alpha$, and look up the $t$ statistic corresponding to that level in a table. We reject the hypothesis if our calculated $t$ exceeds the value we looked up. Alternatively, because we have a computer at our disposal, we calculate the significance level $e$ corresponding to our calculated $t$ statistic, and if $e<\alpha$, we reject the hypothesis.

This logic works well when we are performing one test. Now consider what happens when we perform several separate tests, say, $n$ of them. Let's assume, just for discussion, that we set $\alpha$ equal to 0.05 and that we will perform six tests. For each test, we have a 0.05 probability of falsely rejecting the equality-of-means hypothesis. Overall, then, our chances of falsely rejecting at least one of the hypotheses is $1-(1-0.05)^{6} \approx 0.26$ if the tests are independent.

The idea behind multiple-comparison tests is to control for the fact that we will perform multiple tests and to reduce our overall chances of falsely rejecting each hypothesis to $\alpha$ rather than letting our chances increase with each additional test. (See Miller [1981] and Hochberg and Tamhane [1987] for rather advanced texts on multiple-comparison procedures.)

The Bonferroni adjustment (see Miller [1981]; also see van Belle et al. [2004, 534-537]) does this by (falsely but approximately) asserting that the critical level we should use, $a$, is the true critical level, $\alpha$, divided by the number of tests, $n$; that is, $a=\alpha / n$. For instance, if we are going to perform six tests, each at the 0.05 significance level, we want to adopt a critical level of $0.05 / 6 \approx 0.00833$.

We can just as easily apply this logic to $e$, the significance level associated with our $t$ statistic, as to our critical level $\alpha$. If a comparison has a calculated significance of $e$, then its "real" significance, adjusted for the fact of $n$ comparisons, is $n \times e$. If a comparison has a significance level of, say, 0.012 , and we perform six tests, then its "real" significance is 0.072 . If we adopt a critical level of 0.05 , we cannot reject the hypothesis. If we adopt a critical level of 0.10 , we can reject it.

Of course, this calculation can go above 1 , but that just means that there is no $\alpha<1$ for which we could reject the hypothesis. (This situation arises because of the crude nature of the Bonferroni adjustment.) Stata handles this case by simply calling the significance level 1. Thus the formula for the Bonferroni significance level is

$$
e_{b}=\min (1, e n)
$$

where $n=k(k-1) / 2$ is the number of comparisons.

The Šidák adjustment (Šidák [1967]; also see Winer, Brown, and Michels [1991, 165-166]) is slightly different and provides a tighter bound. It starts with the assertion that

$$
a=1-(1-\alpha)^{1 / n}
$$

Turning this formula around and substituting calculated significance levels, we obtain

$$
e_{s}=\min \left\{1,1-(1-e)^{n}\right\}
$$

For example, if the calculated significance is 0.012 and we perform six tests, the "real" significance is approximately 0.07 .

The Scheffé test (Scheffé [1953, 1959]; also see Kuehl [2000, 97-98]) differs in derivation, but it attacks the same problem. Let there be $k$ means for which we want to make all the pairwise tests. Two means are declared significantly different if

$$
t \geq \sqrt{(k-1) F(\alpha ; k-1, \nu)}
$$

where $F(\alpha ; k-1, \nu)$ is the $\alpha$-critical value of the $F$ distribution with $k-1$ numerator and $\nu$ denominator degrees of freedom. Scheffé's test has the nicety that it never declares a contrast significant if the overall $F$ test is not significant.

Turning the test around, Stata calculates a significance level

$$
\widehat{e}=F\left(\frac{t^{2}}{k-1}, k-1, \nu\right)
$$

For instance, you have a calculated $t$ statistic of 4.0 with 50 degrees of freedom. The simple $t$ test says that the significance level is 0.00021 . The $F$ test equivalent, 16 with 1 and 50 degrees of freedom, says the same. If you are comparing three means, however, you calculate an $F$ test of 8.0 with 2 and 50 degrees of freedom, which says that the significance level is 0.0010 .

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## Also see

[R] anova - Analysis of variance and covariance
[R] loneway - Large one-way ANOVA, random effects, and reliability [PSS] power oneway - Power analysis for one-way analysis of variance

## Title

oprobit - Ordered probit regression

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

$$
\text { oprobit depvar }[\text { indepvars }][\text { if }][\text { in }][\text { weight }][\text {, options }]
$$

options
Description

Model
offset (varname)
constraints (constraints)
collinear
SE/Robust
vce (vcetype)

Reporting
level(\#)
nocnsreport
display_options

Maximization
maximize_options
coeflegend
include varname in model with coefficient constrained to 1
apply specified linear constraints
keep collinear variables
vcetype may be oim, robust, cluster clustvar, bootstrap, or jackknife
indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
depvar and indepvars may contain time-series operators; see [U] 11.4.4 Time-series varlists.
bootstrap, by, fp, jackknife, mfp, mi estimate, nestreg, rolling, statsby, stepwise, and svy are allowed; see [U] 11.1.10 Prefix commands.
vce(bootstrap) and vce(jackknife) are not allowed with the mi estimate prefix; see [MI] mi estimate.
Weights are not allowed with the bootstrap prefix; see $[R]$ bootstrap.
vce() and weights are not allowed with the svy prefix; see [SVY] svy.
fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Ordinal outcomes $>$ Ordered probit regression

## Description

oprobit fits ordered probit models of ordinal variable depvar on the independent variables indepvars. The actual values taken on by the dependent variable are irrelevant, except that larger values are assumed to correspond to "higher" outcomes.

See [R] logistic for a list of related estimation commands.

## Options

$\qquad$ Model
offset (varname), constraints (constraints), collinear; see [R] estimation options.


#### Abstract

SE/Robust vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.


$\sqrt{\text { Reporting }}$
level (\#); see [R] estimation options.
nocnsreport; see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% $f m t$ ), pformat (\% fmt), sformat (\%fmt), and nolstretch; see [R] estimation options.

Maximization
maximize_options: difficult, technique (algorithm_spec), iterate(\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance(\#), nonrtolerance, and from (init_specs); see [R] maximize. These options are seldom used.

The following option is available with oprobit but is not shown in the dialog box: coeflegend; see $[R]$ estimation options.

## Remarks and examples

An ordered probit model is used to estimate relationships between an ordinal dependent variable and a set of independent variables. An ordinal variable is a variable that is categorical and ordered, for instance, "poor", "good", and "excellent", which might indicate a person's current health status or the repair record of a car. If there are only two outcomes, see $[R]$ logistic, $[R]$ logit, and $[R]$ probit. This entry is concerned only with more than two outcomes. If the outcomes cannot be ordered (for example, residency in the north, east, south, or west), see [R] mlogit. This entry is concerned only with models in which the outcomes can be ordered.

In ordered probit, an underlying score is estimated as a linear function of the independent variables and a set of cutpoints. The probability of observing outcome $i$ corresponds to the probability that the estimated linear function, plus random error, is within the range of the cutpoints estimated for the outcome:

$$
\operatorname{Pr}\left(\text { outcome }_{j}=i\right)=\operatorname{Pr}\left(\kappa_{i-1}<\beta_{1} x_{1 j}+\beta_{2} x_{2 j}+\cdots+\beta_{k} x_{k j}+u_{j} \leq \kappa_{i}\right)
$$

$u_{j}$ is assumed to be normally distributed. In either case, we estimate the coefficients $\beta_{1}, \beta_{2}, \ldots$, $\beta_{k}$ together with the cutpoints $\kappa_{1}, \kappa_{2}, \ldots, \kappa_{I-1}$, where $I$ is the number of possible outcomes. $\kappa_{0}$ is taken as $-\infty$, and $\kappa_{I}$ is taken as $+\infty$. All of this is a direct generalization of the ordinary two-outcome probit model.

## $>$ Example 1

In example 2 of $[R]$ ologit, we use a variation of the automobile dataset (see [U] 1.2.2 Example datasets) to analyze the 1977 repair records of 66 foreign and domestic cars. We use ordered logit to explore the relationship of rep77 in terms of foreign (origin of manufacture), length (a proxy for size), and mpg. Here we fit the same model using ordered probit rather than ordered logit:

| (Automobile Models) <br> . oprobit rep77 foreign length mpg |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Iteration 0: log likelihood $=-89.895098$ |  |  |  |  |  |  |
| Iteration 1: log likelihood $=-78.106316$ |  |  |  |  |  |  |
| Iteration 2: log likelihood $=-78.020086$ |  |  |  |  |  |  |
| Iteration 3: log likelihood $=-78.020025$ |  |  |  |  |  |  |
| Iteration 4: log likelihood $=-78.020025$ |  |  |  |  |  |  |
| Ordered probit regression |  |  |  | Number of obs $=66$ |  |  |
|  |  |  |  | LR | (3) | 23.75 |
|  |  |  |  | Prob | chi2 | 0.0000 |
| Log likelihood $=-78.020025$ |  |  |  | Pseudo R2 $=0.1321$ |  |  |
| rep77 | Coef. | Std. Err. | z | $\mathrm{P}>\|z\|$ | [95\% | Interval] |
| foreign | 1.704861 | . 4246796 | 4.01 | 0.000 | . 8725 | 2.537217 |
| length | . 0468675 | . 012648 | 3.71 | 0.000 | . 022 | . 0716571 |
| mpg | . 1304559 | . 0378628 | 3.45 | 0.001 | . 0562 | . 2046656 |
| /cut1 | 10.1589 | 3.076754 |  |  | 4.128 | 16.18923 |
| /cut2 | 11.21003 | 3.107527 |  |  | 5.119 | 17.30067 |
| /cut3 | 12.54561 | 3.155233 |  |  | 6.361 | 18.72975 |
| /cut4 | 13.98059 | 3.218793 |  |  | 7.671 | 20.28931 |

We find that foreign cars have better repair records, as do larger cars and cars with better mileage ratings.

## Stored results

oprobit stores the following in e():
Scalars

```
e(N)
e(N_cd)
e(k_cat)
e(k)
e(k_aux)
e(k_eq)
e(k_eq_model)
e(k_dv)
e(df_m)
e(r2_p)
e(ll)
e(ll_0)
e(N_clust)
e(chi2)
e(p)
e(rank)
e(ic)
e(rc)
e(converged)
```

Macros
e(cmd)
e(cmdline)
e(depvar)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(offset)
e(chi2type)
e(vce)
e(vcetype)
e(opt)
e(which)
e(ml_method)
e(user)
e(technique)
e(properties)
e(predict)
e(asbalanced)
e(asobserved)
Matrices
e(b)
e(Cns)
e(ilog)
e(gradient)
e(cat)
e(V)
e(V_modelbased)
Functions
e(sample)
number of observations
number of completely determined observations
number of categories
number of parameters
number of auxiliary parameters
number of equations in e(b)
number of equations in overall model test
number of dependent variables
model degrees of freedom
pseudo- $R$-squared
log likelihood
log likelihood, constant-only model
number of clusters
$\chi^{2}$
significance of model test
rank of e(V)
number of iterations
return code
1 if converged, 0 otherwise

## oprobit

command as typed
name of dependent variable
weight type
weight expression
title in estimation output
name of cluster variable
linear offset variable
Wald or LR; type of model $\chi^{2}$ test
vcetype specified in vce()
title used to label Std. Err.
type of optimization
max or min; whether optimizer is to perform maximization or minimization
type of ml method
name of likelihood-evaluator program
maximization technique
b V
program used to implement predict
factor variables fvset as asbalanced
factor variables fvset as asobserved
coefficient vector
constraints matrix
iteration $\log$ (up to 20 iterations)
gradient vector
category values
variance-covariance matrix of the estimators
model-based variance
marks estimation sample

## Methods and formulas

See Methods and formulas of [R] ologit.

## References

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## Also see

[R] oprobit postestimation - Postestimation tools for oprobit
$[R]$ heckoprobit - Ordered probit model with sample selection
[R] logistic - Logistic regression, reporting odds ratios
[R] mlogit - Multinomial (polytomous) logistic regression
[R] mprobit - Multinomial probit regression
[R] ologit - Ordered logistic regression
[R] probit - Probit regression
[ME] meoprobit - Multilevel mixed-effects ordered probit regression
[MI] estimation - Estimation commands for use with mi estimate
[SVY] svy estimation - Estimation commands for survey data
[XT] xtoprobit — Random-effects ordered probit models
[U] 20 Estimation and postestimation commands

## Title

oprobit postestimation - Postestimation tools for oprobit

| Description | Syntax for predict $\quad$ Menu for predict $\quad$ Options for predict |
| :--- | :--- |
| Remarks and examples | Also see |

## Description

The following postestimation commands are available after oprobit:

| Command | Description |
| :--- | :--- |
| contrast | contrasts and ANOVA-style joint tests of estimates |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample |
| estat vce | variance-covariance matrix of the estimators (VCE) |
| estat (svy) | postestimation statistics for survey data <br> estaloging estimation results |
| forecast |  |$\quad$| dynamic forecasts and simulations |
| :--- |
| lincom |$\quad$| point estimates, standard errors, testing, and inference for linear combinations |
| :--- |
| of coefficients |

[^57]
## Syntax for predict

```
predict [type] {stub*|newvar|newvarlist } [if] [in] [, statistic
    outcome(outcome) nooffset]
predict [type] {stub*|newvarlist } [if] [in], scores
```

statistic Description
Main
$\underline{\mathrm{pr}} \quad$ predicted probabilities; the default
$\mathrm{xb} \quad$ linear prediction
stdp standard error of the linear prediction

If you do not specify outcome(), pr (with one new variable specified) assumes outcome(\#1).
You specify one or $k$ new variables with pr , where $k$ is the number of outcomes.
You specify one new variable with xb and stdp.
These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
pr , the default, calculates the predicted probabilities. If you do not also specify the outcome() option, you specify $k$ new variables, where $k$ is the number of categories of the dependent variable. Say that you fit a model by typing oprobit result x1 x2, and result takes on three values. Then you could type predict p1 p2 p3 to obtain all three predicted probabilities. If you specify the outcome() option, you must specify one new variable. Say that result takes on values 1 , 2 , and 3 . Typing predict p1, outcome(1) would produce the same p1.
xb calculates the linear prediction. You specify one new variable, for example, predict linear, xb . The linear prediction is defined, ignoring the contribution of the estimated cutpoints.
stdp calculates the standard error of the linear prediction. You specify one new variable, for example, predict se, stdp.
outcome (outcome) specifies for which outcome the predicted probabilities are to be calculated. outcome() should contain either one value of the dependent variable or one of $\# 1, \# 2, \ldots$, with \#1 meaning the first category of the dependent variable, \#2 meaning the second category, etc.
nooffset is relevant only if you specified offset (varname) for oprobit. It modifies the calculations made by predict so that they ignore the offset variable; the linear prediction is treated as $\mathbf{x}_{j} \mathbf{b}$ rather than as $\mathbf{x}_{j} \mathbf{b}+$ offset $_{j}$.
scores calculates equation-level score variables. The number of score variables created will equal the number of outcomes in the model. If the number of outcomes in the model was $k$, then the first new variable will contain $\partial \ln L / \partial\left(\mathbf{x}_{j} \mathbf{b}\right)$;
the second new variable will contain $\partial \ln L / \partial \kappa_{1}$;
the third new variable will contain $\partial \ln L / \partial \kappa_{2}$;
and the $k$ th new variable will contain $\partial \ln L / \partial \kappa_{k-1}$, where $\kappa_{i}$ refers to the $i$ th cutpoint.

## Remarks and examples

See [U] 20 Estimation and postestimation commands for instructions on obtaining the variancecovariance matrix of the estimators, predicted values, and hypothesis tests. Also see [R] lrtest for performing likelihood-ratio tests.

## > Example 1

In example 1 of [R] oprobit, we fit the model oprobit rep77 foreign length mpg. The predict command can be used to obtain the predicted probabilities. We type predict followed by the names of the new variables to hold the predicted probabilities, ordering the names from low to high. In our data, the lowest outcome is "poor" and the highest is "excellent". We have five categories, so we must type five names following predict; the choice of names is up to us:

```
. use http://www.stata-press.com/data/r13/fullauto
(Automobile Models)
. oprobit rep77 foreign length mpg
    (output omitted)
. predict poor fair avg good exc
(option pr assumed; predicted probabilities)
. list make model exc good if rep77>=., sep(4) divider
```

|  | make | model | exc | good |
| ---: | ---: | ---: | ---: | ---: |
|  | AMC | Spirit | .0006044 | .0351813 |
| 10. | Buick | Opel | .0043803 | .1133763 |
| 32. | Ford | Fiesta | .0002927 | .0222789 |
| 44. | Merc. | Monarch | .0093209 | .1700846 |
| 53. | Peugeot | 604 | .0734199 | .4202766 |
| 56. | Plym. | Horizon | .001413 | .0590294 |
| 57. | Plym. | Sapporo | .0197543 | .2466034 |
| 63. | Pont. | Phoenix | .0234156 | .266771 |

## - Technical note

For ordered probit, predict, xb produces $S_{j}=x_{1 j} \beta_{1}+x_{2 j} \beta_{2}+\cdots+x_{k j} \beta_{k}$. Ordered probit is identical to ordered logit, except that we use different distribution functions for calculating probabilities. The ordered-probit predictions are then the probability that $S_{j}+u_{j}$ lies between a pair of cutpoints $\kappa_{i-1}$ and $\kappa_{i}$. The formulas for ordered probit are

$$
\begin{aligned}
\operatorname{Pr}\left(S_{j}+u<\kappa\right) & =\Phi\left(\kappa-S_{j}\right) \\
\operatorname{Pr}\left(S_{j}+u>\kappa\right) & =1-\Phi\left(\kappa-S_{j}\right)=\Phi\left(S_{j}-\kappa\right) \\
\operatorname{Pr}\left(\kappa_{1}<S_{j}+u<\kappa_{2}\right) & =\Phi\left(\kappa_{2}-S_{j}\right)-\Phi\left(\kappa_{1}-S_{j}\right)
\end{aligned}
$$

Rather than using predict directly, we could calculate the predicted probabilities by hand.
. predict pscore, xb
. generate probexc = normal (pscore-_b[/cut4])
. generate probgood = normal(_b[/cut4]-pscore) - normal(_b[/cut3]-pscore)

## Also see

[R] oprobit - Ordered probit regression
[U] 20 Estimation and postestimation commands

## Title

orthog - Orthogonalize variables and compute orthogonal polynomials

Syntax
Options for orthog Methods and formulas

## Menu <br> Options for orthpoly References

## Description

Remarks and examples
Also see

## Syntax

Orthogonalize variables

```
orthog [varlist] [if] [in] [weight], generate(newvarlist) [白者rix(matname)]
```

Compute orthogonal polynomial
orthpoly varname [if] [in] [weight],
$\{$ generate(newvarlist) $\mid \underline{p o l y}($ matname $)\}[\underline{\text { degree (\#) }}]$
orthpoly requires that generate(newvarlist) or poly(matname), or both, be specified.
varlist may contain time-series operators; see [U] 11.4.4 Time-series varlists.
iweights, aweights, fweights, and pweights are allowed, see [U] 11.1.6 weight.

## Menu

## orthog

Data $>$ Create or change data $>$ Other variable-creation commands $>$ Orthogonalize variables
orthpoly
Data $>$ Create or change data $>$ Other variable-creation commands $>$ Orthogonal polynomials

## Description

orthog orthogonalizes a set of variables, creating a new set of orthogonal variables (all of type double), using a modified Gram-Schmidt procedure (Golub and Van Loan 1996). The order of the variables determines the orthogonalization; hence, the "most important" variables should be listed first.

Execution time is proportional to the square of the number of variables. With many ( $>10$ ) variables, orthog will be fairly slow.
orthpoly computes orthogonal polynomials for one variable.

## Options for orthog

$\qquad$ Main
generate (newvarlist) is required. generate() creates new orthogonal variables of type double. For orthog, newvarlist will contain the orthogonalized varlist. If varlist contains $d$ variables, then so will newvarlist. newvarlist can be specified by giving a list of exactly $d$ new variable names, or it can be abbreviated using the styles newvar1-newvard or newvar*. For these two styles of abbreviation, new variables newvar1, newvar2, ..., newvard are generated.
matrix (matname) creates a $(d+1) \times(d+1)$ matrix containing the matrix $R$ defined by $X=Q R$, where $X$ is the $N \times(d+1)$ matrix representation of varlist plus a column of ones and $Q$ is the $N \times(d+1)$ matrix representation of newvarlist plus a column of ones ( $d=$ number of variables in varlist, and $N=$ number of observations).

## Options for orthpoly

$\qquad$ Main
generate (newvarlist) or poly(), or both, must be specified. generate() creates new orthogonal variables of type double. newvarlist will contain orthogonal polynomials of degree 1, 2, ..., $d$ evaluated at varname, where $d$ is as specified by degree $(d)$. newvarlist can be specified by giving a list of exactly $d$ new variable names, or it can be abbreviated using the styles newvar1newvard or newvar*. For these two styles of abbreviation, new variables newvar1, newvar2, ..., newvard are generated.
poly (matname) creates a $(d+1) \times(d+1)$ matrix called matname containing the coefficients of the orthogonal polynomials. The orthogonal polynomial of degree $i \leq d$ is

```
matname \([i, d+1]+\) matname \([i, 1] * v a r n a m e+m a t n a m e[i, 2] * v a r n a m e ~ 2 ~\)
    \(+\cdots+\) matname \([i, i] *\) varname \({ }^{i}\)
```

The coefficients corresponding to the constant term are placed in the last column of the matrix. The last row of the matrix is all zeros, except for the last column, which corresponds to the constant term.
degree(\#) specifies the highest-degree polynomial to include. Orthogonal polynomials of degree 1 , $2, \ldots, d=\#$ are computed. The default is $d=1$.

## Remarks and examples

Orthogonal variables are useful for two reasons. The first is numerical accuracy for highly collinear variables. Stata's regress and other estimation commands can face much collinearity and still produce accurate results. But, at some point, these commands will drop variables because of collinearity. If you know with certainty that the variables are not perfectly collinear, you may want to retain all their effects in the model. If you use orthog or orthpoly to produce a set of orthogonal variables, all variables will be present in the estimation results.

Users are more likely to find orthogonal variables useful for the second reason: ease of interpreting results. orthog and orthpoly create a set of variables such that the "effects" of all the preceding variables have been removed from each variable. For example, if we issue the command

```
. orthog x1 x2 x3, generate(q1 q2 q3)
```

the effect of the constant is removed from x 1 to produce q 1 ; the constant and x 1 are removed from x 2 to produce q 2 ; and finally the constant, x 1 , and x 2 are removed from x 3 to produce q 3 . Hence,

$$
\begin{aligned}
& \mathrm{q} 1=r_{01}+r_{11} \mathrm{x} 1 \\
& \mathrm{q} 2=r_{02}+r_{12} \mathrm{x} 1+r_{22} \mathrm{x} 2 \\
& \mathrm{q} 3=r_{03}+r_{13} \mathrm{x} 1+r_{23} \mathrm{x} 2+r_{33} \times 3
\end{aligned}
$$

This effect can be generalized and written in matrix notation as

$$
X=Q R
$$

where $X$ is the $N \times(d+1)$ matrix representation of varlist plus a column of ones, and $Q$ is the $N \times(d+1)$ matrix representation of newvarlist plus a column of ones ( $d=$ number of variables in varlist and $N=$ number of observations). The $(d+1) \times(d+1)$ matrix $R$ is a permuted uppertriangular matrix, that is, $R$ would be upper triangular if the constant were first, but the constant is last, so the first row/column has been permuted with the last row/column. Because Stata's estimation commands list the constant term last, this allows $R$, obtained via the matrix () option, to be used to transform estimation results.

## > Example 1: orthog

Consider Stata's auto.dta dataset. Suppose that we postulate a model in which price depends on the car's length, weight, headroom, and trunk size (trunk). These predictors are collinear, but not extremely so-the correlations are not that close to 1 :

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. correlate length weight headroom trunk
(obs=74)
\begin{tabular}{r|rrrr} 
& length & weight headroom & trunk \\
\hline length & 1.0000 & & & \\
weight & 0.9460 & 1.0000 & & \\
headroom & 0.5163 & 0.4835 & 1.0000 & \\
trunk & 0.7266 & 0.6722 & 0.6620 & 1.0000
\end{tabular}
```

regress certainly has no trouble fitting this model:


| Source | SS | df | MS |
| ---: | ---: | ---: | ---: |
| Model | 236016580 | 4 | 59004145 |
| Residual | 399048816 | 69 | 5783316.17 |
| Total | 635065396 | 73 | 8699525.97 |


| Number of obs | $=$ | 74 |
| :--- | ---: | ---: |
| F 4, | $69)$ | $=10.20$ |
| Prob $>$ F | $=$ | 0.0000 |
| R-squared | $=$ | 0.3716 |
| Adj R-squared | $=$ | 0.3352 |
| Root MSE | $=$ | 2404.9 |


| price | Coef. | Std. Err. | t | P>\|t| | [95\% Conf. Interval] |  |
| ---: | ---: | :--- | :---: | :---: | ---: | ---: |
| length | -101.7092 | 42.12534 | -2.41 | 0.018 | -185.747 | -17.67147 |
| weight | 4.753066 | 1.120054 | 4.24 | 0.000 | 2.518619 | 6.987512 |
| headroom | -711.5679 | 445.0204 | -1.60 | 0.114 | -1599.359 | 176.2236 |
| trunk | 114.0859 | 109.9488 | 1.04 | 0.303 | -105.2559 | 333.4277 |
| _cons | 11488.47 | 4543.902 | 2.53 | 0.014 | 2423.638 | 20553.31 |

However, we may believe a priori that length is the most important predictor, followed by weight, headroom, and trunk. We would like to remove the "effect" of length from all the other predictors, remove weight from headroom and trunk, and remove headroom from trunk. We can do this by running orthog, and then we fit the model again using the orthogonal variables:

```
. orthog length weight headroom trunk, gen(olength oweight oheadroom otrunk)
> matrix(R)
```

. regress price olength oweight oheadroom otrunk

| Source | SS | df | MS |
| ---: | ---: | ---: | ---: |
| Model | 236016580 | 4 | 59004145 |
| Residual | 399048816 | 69 | 5783316.17 |
| Total | 635065396 | 73 | 8699525.97 |


| Number of obs | $=$ | 74 |
| :--- | ---: | ---: |
| F $(4$, | $69)$ | $=$ |
| Prob $>$ F | $=0.20$ |  |
| R-squared | $=$ | 0.3716 |
| Adj R-squared | $=$ | 0.3352 |
| Root MSE | $=$ | 2404.9 |


| price | Coef. | Std. Err. | t | P>\|t| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| olength | 1265.049 | 279.5584 | 4.53 | 0.000 | 707.3454 | 1822.753 |
| oweight | 1175.765 | 279.5584 | 4.21 | 0.000 | 618.0617 | 1733.469 |
| oheadroom | -349.9916 | 279.5584 | -1.25 | 0.215 | -907.6955 | 207.7122 |
| otrunk | 290.0776 | 279.5584 | 1.04 | 0.303 | -267.6262 | 847.7815 |
| _cons | 6165.257 | 279.5584 | 22.05 | 0.000 | 5607.553 | 6722.961 |

Using the matrix R , we can transform the results obtained using the orthogonal predictors back to the metric of original predictors:

```
. matrix b = e(b)*inv(R)'
. matrix list b
b [1,5]
```



## - Technical note

The matrix $R$ obtained using the matrix() option with orthog can also be used to recover $X$ (the original varlist) from $Q$ (the orthogonalized newvarlist), one variable at a time. Continuing with the previous example, we illustrate how to recover the trunk variable:

```
. matrix C = R[1...,"trunk"]'
. matrix score double rtrunk = C
. compare rtrunk trunk
```

|  |  | count | minimum | difference <br> average |
| :--- | ---: | ---: | ---: | ---: |
| rtrunk>trunk | 74 | $1.42 \mathrm{e}-14$ | $2.27 \mathrm{e}-14$ | $3.55 \mathrm{e}-14$ |
|  |  |  |  |  |
| jointly defined | 74 | $1.42 \mathrm{e}-14$ | $2.27 \mathrm{e}-14$ | $3.55 \mathrm{e}-14$ |
| total | 74 |  |  |  |

Here the recovered variable rtrunk is almost exactly the same as the original trunk variable. When you are orthogonalizing many variables, this procedure can be performed to check the numerical soundness of the orthogonalization. Because of the ordering of the orthogonalization procedure, the last variable and the variables near the end of the varlist are the most important ones to check.

The orthpoly command effectively does for polynomial terms what the orthog command does for an arbitrary set of variables.

## Example 2: orthpoly

Again consider the auto.dta dataset. Suppose that we wish to fit the model

$$
\operatorname{mpg}=\beta_{0}+\beta_{1} \text { weight }+\beta_{2} \text { weight }^{2}+\beta_{3} \text { weight }^{3}+\beta_{4} \text { weight }^{4}+\epsilon
$$

We will first compute the regression with natural polynomials:

```
. gen double w1 = weight
. gen double w2 = w1*w1
. gen double w3 = w2*w1
. gen double w4 = w3*w1
. correlate w1-w4
(obs=74)
```

|  | w1 | w2 | w3 | w4 |
| ---: | ---: | ---: | ---: | ---: |
| w1 | 1.0000 |  |  |  |
| w2 | 0.9915 | 1.0000 |  |  |
| w3 | 0.9665 | 0.9916 | 1.0000 |  |
| w4 | 0.9279 | 0.9679 | 0.9922 | 1.0000 |

. regress mpg w1-w4

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | 1652.73666 | 4 | 413.184164 |
| Residual | 790.722803 | 69 | 11.4597508 |
| Total | 2443.45946 | 73 | 33.4720474 |


| Number of obs | $=$ | 74 |
| :--- | ---: | ---: |
| F ( 4, 69) | $=36.06$ |  |
| Prob $>$ F | $=0.0000$ |  |
| R-squared | $=$ | 0.6764 |
| Adj R-squared | $=$ | 0.6576 |
| Root MSE | $=3.3852$ |  |


| mpg | Coef. | Std. Err. | t | P>\|t| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | :---: |
| w1 | .0289302 | .1161939 | 0.25 | 0.804 | -.2028704 | .2607307 |
| w2 | -.0000229 | .0000566 | -0.40 | 0.687 | -.0001359 | .0000901 |
| w3 | $5.74 \mathrm{e}-09$ | $1.19 \mathrm{e}-08$ | 0.48 | 0.631 | $-1.80 \mathrm{e}-08$ | $2.95 \mathrm{e}-08$ |
| w4 | $-4.86 \mathrm{e}-13$ | $9.14 \mathrm{e}-13$ | -0.53 | 0.596 | $-2.31 \mathrm{e}-12$ | $1.34 \mathrm{e}-12$ |
| _cons | 23.94421 | 86.60667 | 0.28 | 0.783 | -148.8314 | 196.7198 |

Some of the correlations among the powers of weight are very large, but this does not create any problems for regress. However, we may wish to look at the quadratic trend with the constant removed, the cubic trend with the quadratic and constant removed, etc. orthpoly will generate polynomial terms with this property:
. orthpoly weight, generate(pw*) deg(4) poly(P)
. regress mpg pw1-pw4

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | 1652.73666 | 4 | 413.184164 |
| Residual | 790.722803 | 69 | 11.4597508 |
| Total | 2443.45946 | 73 | 33.4720474 |


| Number of obs | $=$ | 74 |
| :--- | ---: | ---: |
| F 4, | $69)$ | $=36.06$ |
| Prob $>$ F | $=$ | 0.0000 |
| R-squared | $=$ | 0.6764 |
| Adj R-squared | $=$ | 0.6576 |
| Root MSE | $=3.3852$ |  |


| mpg | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| pw1 | -4.638252 | .3935245 | -11.79 | 0.000 | -5.423312 | -3.853192 |
| pw2 | .8263545 | .3935245 | 2.10 | 0.039 | .0412947 | 1.611414 |
| pw3 | -.3068616 | .3935245 | -0.78 | 0.438 | -1.091921 | .4781982 |
| pw4 | -.209457 | .3935245 | -0.53 | 0.596 | -.9945168 | .5756028 |
| _cons | 21.2973 | .3935245 | 54.12 | 0.000 | 20.51224 | 22.08236 |

Compare the $p$-values of the terms in the natural polynomial regression with those in the orthogonal polynomial regression. With orthogonal polynomials, it is easy to see that the pure cubic and quartic trends are not significant and that the constant, linear, and quadratic terms each have $p<0.05$.

The matrix P obtained with the poly () option can be used to transform coefficients for orthogonal polynomials to coefficients for natural polynomials:

```
. orthpoly weight, poly(P) deg(4)
. matrix b = e(b)*P
. matrix list b
b[1,5]
\begin{tabular}{rrrrrr} 
& deg 1 & deg 2 & deg 3 & deg 4 & -cons \\
y 1 & .02893016 & -.00002291 & \(5.745 \mathrm{e}-09\) & \(-4.862 \mathrm{e}-13\) & 23.944212
\end{tabular}
```


## Methods and formulas

orthog's orthogonalization can be written in matrix notation as

$$
X=Q R
$$

where $X$ is the $N \times(d+1)$ matrix representation of varlist plus a column of ones and $Q$ is the $N \times(d+1)$ matrix representation of newvarlist plus a column of ones ( $d=$ number of variables in varlist, and $N=$ number of observations). The $(d+1) \times(d+1)$ matrix $R$ is a permuted uppertriangular matrix; that is, $R$ would be upper triangular if the constant were first, but the constant is last, so the first row/column has been permuted with the last row/column.
$Q$ and $R$ are obtained using a modified Gram-Schmidt procedure; see Golub and Van Loan (1996, 218-219) for details. The traditional Gram-Schmidt procedure is notoriously unsound, but the modified procedure is good. orthog performs two passes of this procedure.
orthpoly uses the Christoffel-Darboux recurrence formula (Abramowitz and Stegun 1972).
Both orthog and orthpoly normalize the orthogonal variables such that

$$
Q^{\prime} W Q=M I
$$

where $W=\operatorname{diag}\left(w_{1}, w_{2}, \ldots, w_{N}\right)$ with weights $w_{1}, w_{2}, \ldots, w_{N}$ (all 1 if weights are not specified), and $M$ is the sum of the weights (the number of observations if weights are not specified).

## References

Abramowitz, M., and I. A. Stegun, ed. 1972. Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables. 10th ed. Washington, DC: National Bureau of Standards.
Golub, G. H., and C. F. Van Loan. 1996. Matrix Computations. 3rd ed. Baltimore: Johns Hopkins University Press. Sribney, W. M. 1995. sg37: Orthogonal polynomials. Stata Technical Bulletin 25: 17-18. Reprinted in Stata Technical Bulletin Reprints, vol. 5, pp. 96-98. College Station, TX: Stata Press.

## Also see

$[\mathrm{R}]$ regress - Linear regression

## Title

pcorr - Partial and semipartial correlation coefficients

| Syntax | Menu | Description | Remarks and examples |
| :--- | :--- | :--- | :--- |
| Stored results | Methods and formulas | Acknowledgment | References |

## Syntax

pcorr varname ${ }_{1}$ varlist [if] [in] [weight]
varname $_{1}$ and varlist may contain time-series operators; see [U] 11.4.4 Time-series varlists.
by is allowed; see [D] by.
aweights and fweights are allowed; see [U] 11.1.6 weight.

## Menu

Statistics $>$ Summaries, tables, and tests $>$ Summary and descriptive statistics $>$ Partial correlations

## Description

pcorr displays the partial and semipartial correlation coefficients of varname ${ }_{1}$ with each variable in varlist after removing the effects of all other variables in varlist. The squared correlations and corresponding significance are also reported.

## Remarks and examples

Assume that $y$ is determined by $x_{1}, x_{2}, \ldots, x_{k}$. The partial correlation between $y$ and $x_{1}$ is an attempt to estimate the correlation that would be observed between $y$ and $x_{1}$ if the other $x$ 's did not vary. The semipartial correlation, also called part correlation, between $y$ and $x_{1}$ is an attempt to estimate the correlation that would be observed between $y$ and $x_{1}$ after the effects of all other $x$ 's are removed from $x_{1}$ but not from $y$.

Both squared correlations estimate the proportion of the variance of $y$ that is explained by each predictor. The squared semipartial correlation between $y$ and $x_{1}$ represents the proportion of variance in $y$ that is explained by $x_{1}$ only. This squared correlation can also be interpreted as the decrease in the model's $R^{2}$ value that results from removing $x_{1}$ from the full model. Thus one could use the squared semipartial correlations as criteria for model selection. The squared partial correlation between $y$ and $x_{1}$ represents the proportion of variance in $y$ not associated with any other $x$ 's that is explained by $x_{1}$. Thus the squared partial correlation gives an estimate of how much of the variance of $y$ not explained by the other $x$ 's is explained by $x_{1}$.
> Example 1
Using our automobile dataset (described in [U] 1.2.2 Example datasets), we can obtain the simple correlations between price, mpg, weight, and foreign from correlate (see [R] correlate):

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. correlate price mpg weight foreign
(obs=74)
\begin{tabular}{r|rrrr} 
& price & mpg & weight & foreign \\
\hline price & 1.0000 & & & \\
mpg & -0.4686 & 1.0000 & & \\
weight & 0.5386 & -0.8072 & 1.0000 & \\
foreign & 0.0487 & 0.3934 & -0.5928 & 1.0000
\end{tabular}
```

Although correlate gave us the full correlation matrix, our interest is in just the first column. We find, for instance, that the higher the mpg, the lower the price. We obtain the partial and semipartial correlation coefficients by using pcorr:

| Partial and semipartial correlations of price with |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Variable | Partial Corr. | Semipartial Corr. | Partial <br> Corr. ${ }^{\text {² }}$ | Semipartial Corr. ${ }^{\text {² }}$ | Significance <br> Value |
| mpg | 0.0352 | 0.0249 | 0.0012 | 0.0006 | 0.7693 |
| weight | 0.5488 | 0.4644 | 0.3012 | 0.2157 | 0.0000 |
| foreign | 0.5402 | 0.4541 | 0.2918 | 0.2062 | 0.0000 |

We now find that the partial and semipartial correlations of price with mpg are near 0 . In the simple correlations, we found that price and foreign were virtually uncorrelated. In the partial and semipartial correlations, we find that price and foreign are positively correlated. The nonsignificance of mpg tells us that the amount in which $R^{2}$ decreases by removing mpg from the model is not significant. We find that removing either weight or foreign results in a significant drop in the $R^{2}$ of the model.

## - Technical note

Use caution when interpreting the above results. As we said at the outset, the partial and semipartial correlation coefficients are an attempt to estimate the correlation that would be observed if the effects of all other variables were taken out of both $y$ and $x$ or only $x$. pcorr makes it too easy to ignore the fact that we are fitting a model. In the example above, the model is

$$
\text { price }=\beta_{0}+\beta_{1} \mathrm{mpg}+\beta_{2} \text { weight }+\beta_{3} \text { foreign }+\epsilon
$$

which is, in all honesty, a rather silly model. Even if we accept the implied economic assumptions of the model-that consumers value mpg, weight, and foreign-do we really believe that consumers place equal value on every extra 1,000 pounds of weight? That is, have we correctly parameterized the model? If we have not, then the estimated partial and semipartial correlation coefficients may not represent what they claim to represent. Partial and semipartial correlation coefficients are a reasonable way to summarize data if we are convinced that the underlying model is reasonable. We should not, however, pretend that there is no underlying model and that these correlation coefficients are unaffected by the assumptions and parameterization.

## Stored results

pcorr stores the following in $r()$ :
Scalars
$r(N) \quad$ number of observations
$r(d f) \quad$ degrees of freedom
Matrices
$r$ (p_corr) partial correlation coefficient vector
$r$ (sp_corr) semipartial correlation coefficient vector

## Methods and formulas

Results are obtained by fitting a linear regression of varname $1_{1}$ on varlist; see [R] regress. The partial correlation coefficient between varname $_{1}$ and each variable in varlist is then calculated as

$$
\frac{t}{\sqrt{t^{2}+n-k}}
$$

(Greene 2012, 37), where $t$ is the $t$ statistic, $n$ is the number of observations, and $k$ is the number of independent variables, including the constant but excluding any dropped variables.

The semipartial correlation coefficient between varname $e_{1}$ and each variable in varlist is calculated as

$$
\operatorname{sign}(t) \sqrt{\frac{t^{2}\left(1-R^{2}\right)}{n-k}}
$$

(Cohen et al. 2003, 89), where $R^{2}$ is the model $R^{2}$ value, and $t, n$, and $k$ are as described above.
The significance is given by $2 \operatorname{Pr}\left(t_{n-k}>|t|\right)$, where $t_{n-k}$ follows a Student's $t$ distribution with $n-k$ degrees of freedom.

## Acknowledgment

The addition of semipartial correlation coefficients to pcorr is based on the pcorr2 command by Richard Williams of the Department of Sociology at the University of Notre Dame.

## References

Cohen, J., P. Cohen, S. G. West, and L. S. Aiken. 2003. Applied Multiple Regression/Correlation Analysis for the Behavioral Sciences. 3rd ed. Hillsdale, NJ: Erlbaum.

Greene, W. H. 2012. Econometric Analysis. 7th ed. Upper Saddle River, NJ: Prentice Hall.

## Also see

$[R]$ correlate - Correlations (covariances) of variables or coefficients
[R] spearman - Spearman's and Kendall's correlations

## Title

permute - Monte Carlo permutation tests

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | References | Also see |

## Syntax

Compute permutation test

```
permute permvar exp_list [, options] : command
```

Report saved results

```
permute [varlist] [using filename] [, display_options]
```

options
Description
Main
reps (\#) perform \# random permutations; default is reps(100)
left|right
compute one-sided $p$-values; default is two-sided
Options
strata (varlist)
saving (filename, ...)
permute within strata save results to filename; save statistics in double precision;
save results to filename every \# replications
Reporting
level (\#)
noheader
nolegend
verbose
nodrop
nodots
noisily
trace
title (text)
Advanced
eps (\#) numerical tolerance; seldom used
nowarn do not warn when e (sample) is not set
force do not check for weights or svy commands; seldom used
reject (exp)
seed (\#)
set confidence level; default is level (95)
suppress table header
suppress table legend
display full table legend
do not drop observations
suppress replication dots
display any output from command
trace command
use text as title for permutation results
identify invalid results
set random-number seed to \#
weights are not allowed in command.
display_options
left|right
level(\#)
noheader
nolegend
verbose
title(text)
eps(\#)

Description
compute one-sided $p$-values; default is two-sided
set confidence level; default is level(95)
suppress table header
suppress table legend
display full table legend
use text as title for results
numerical tolerance; seldom used

| exp_list contains | (name: elist) <br> elist |
| :---: | :---: |
|  | eexp |
| elist contains | $\begin{aligned} & \text { newvar }=(\text { exp }) \\ & (\text { exp }) \end{aligned}$ |
| eexp is | specname |
|  | [eqno]specname |
| specname is | _b |
|  | _b [] |
|  | _se |
|  | _se[] |
| eqno is | \#\# |
|  | name |

exp is a standard Stata expression; see [U] 13 Functions and expressions.
Distinguish between [], which are to be typed, and [], which indicate optional arguments.

## Menu

Statistics $>$ Resampling $>$ Permutation tests

## Description

permute estimates $p$-values for permutation tests on the basis of Monte Carlo simulations. Typing - permute permvar exp_list, reps(\#): command
randomly permutes the values in permvar \# times, each time executing command and collecting the associated values from the expression in exp_list.

These $p$-value estimates can be one-sided: $\operatorname{Pr}\left(T^{*} \leq T\right)$ or $\operatorname{Pr}\left(T^{*} \geq T\right)$. The default is two-sided: $\operatorname{Pr}\left(\left|T^{*}\right| \geq|T|\right)$. Here $T^{*}$ denotes the value of the statistic from a randomly permuted dataset, and $T$ denotes the statistic as computed on the original data. permvar identifies the variable whose observed values will be randomly permuted.
command defines the statistical command to be executed. Most Stata commands and user-written programs can be used with permute, as long as they follow standard Stata syntax; see [U] 11 Language syntax. The by prefix may not be part of command.
exp_list specifies the statistics to be collected from the execution of command.
permute may be used for replaying results, but this feature is appropriate only when a dataset generated by permute is currently in memory or is identified by the using option. The variables specified in varlist in this context must be present in the respective dataset.

## Options

$\qquad$ Main
reps (\#) specifies the number of random permutations to perform. The default is 100 .
left or right requests that one-sided $p$-values be computed. If left is specified, an estimate of $\operatorname{Pr}\left(T^{*} \leq T\right)$ is produced, where $T^{*}$ is the test statistic and $T$ is its observed value. If right is specified, an estimate of $\operatorname{Pr}\left(T^{*} \geq T\right)$ is produced. By default, two-sided $p$-values are computed; that is, $\operatorname{Pr}\left(\left|T^{*}\right| \geq|T|\right)$ is estimated.

## Options

strata(varlist) specifies that the permutations be performed within each stratum defined by the values of varlist.
saving (filename [, suboptions]) creates a Stata data file (.dta file) consisting of (for each statistic in exp_list) a variable containing the replicates.
double specifies that the results for each replication be saved as doubles, meaning 8-byte reals. By default, they are saved as floats, meaning 4-byte reals.
every (\#) specifies that results are to be written to disk every \#th replication. every() should be specified only in conjunction with saving() when command takes a long time for each replication. This will allow recovery of partial results should some other software crash your computer. See [P] postfile.
replace specifies that filename be overwritten if it exists. This option does not appear in the dialog box.
$\qquad$ Reporting
level(\#) specifies the confidence level, as a percentage, for confidence intervals. The default is level (95) or as set by set level; see [R] level.
noheader suppresses display of the table header. This option implies the nolegend option.
nolegend suppresses display of the table legend. The table legend identifies the rows of the table with the expressions they represent.
verbose requests that the full table legend be displayed. By default, coefficients and standard errors are not displayed.
nodrop prevents permute from dropping observations outside the if and in qualifiers. nodrop will also cause permute to ignore the contents of e(sample) if it exists as a result of running command. By default, permute temporarily drops out-of-sample observations.
nodots suppresses display of the replication dots. By default, one dot character is displayed for each successful replication. A red ' $x$ ' is displayed if command returns an error or if one of the values in exp_list is missing.
noisily requests that any output from command be displayed. This option implies the nodots option.
trace causes a trace of the execution of command to be displayed. This option implies the noisily option.
title (text) specifies a title to be displayed above the table of permutation results; the default title is Monte Carlo permutation results.

Advanced
eps(\#) specifies the numerical tolerance for testing $\left|T^{*}\right| \geq|T|, T^{*} \leq T$, or $T^{*} \geq T$. These are considered true if, respectively, $\left|T^{*}\right| \geq|T|-\#, T^{*} \leq T+\#$, or $T^{*} \geq T-\#$. The default is $1 \mathrm{e}-7$. You will not have to specify eps () under normal circumstances.
nowarn suppresses the printing of a warning message when command does not set e(sample).
force suppresses the restriction that command may not specify weights or be a svy command. permute is not suited for weighted estimation, thus permute should not be used with weights or svy. permute reports an error when it encounters weights or svy in command if the force option is not specified. This is a seldom used option, so use it only if you know what you are doing!
reject (exp) identifies an expression that indicates when results should be rejected. When exp is true, the resulting values are reset to missing values.
seed(\#) sets the random-number seed. Specifying this option is equivalent to typing the following command prior to calling permute:
. set seed \#

## Remarks and examples

Permutation tests determine the significance of the observed value of a test statistic in light of rearranging the order (permuting) of the observed values of a variable.

## > Example 1: A simple two-sample test

Suppose that we conducted an experiment to determine the effect of a treatment on the development of cells. Further suppose that we are restricted to six experimental units because of the extreme cost of the experiment. Thus three units are to be given a placebo, and three units are given the treatment. The measurement is the number of newly developed healthy cells. The following listing gives the hypothetical data, along with some summary statistics.

```
. input y treatment
    y treatment
1. 70
2. 90
3. 11 0
4. 10 1
5. 12 1
6. 14 1
7. end
sort treatment
```

| summarize y <br> Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | :---: | :---: | :---: | :---: | ---: |
| $y$ | 6 | 10.5 | 2.428992 | 7 | 14 |

. by treatment: summarize y

```
-> treatment = 0
```

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| $y$ | 3 | 9 | 2 | 7 | 11 |

```
-> treatment = 1
```

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $y$ | 3 | 12 | 2 | 10 | 14 |

Clearly, there are more cells in the treatment group than in the placebo group, but a statistical test is needed to conclude that the treatment does affect the development of cells. If the sum of the treatment measures is our test statistic, we can use permute to determine the probability of observing 36 or more cells, given the observed data and assuming that there is no effect due to the treatment.

```
. set seed 1234
. permute y sum=r(sum), saving(permdish) right nodrop nowarn: sum y if treatment
(running summarize on estimation sample)
Permutation replications (100)
```



```
..................................................... . . . . 50
........................................................... . . . . }10
Monte Carlo permutation results
                                    Number of obs = 6
        command: summarize y if treatment
        sum: r(sum)
    permute var: y
```

| $T$ | T (obs) | c | n | $\mathrm{p}=\mathrm{c} / \mathrm{n}$ | $\mathrm{SE}(\mathrm{p})$ | $[95 \%$ Conf. Interval] |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| sum | 36 | 10 | 100 | 0.1000 | 0.0300 | .0490047 | .1762226 |

```
Note: confidence interval is with respect to p=c/n.
Note: c = #{T >= T(obs)}
```

We see that 10 of the 100 randomly permuted datasets yielded sums from the treatment group larger than or equal to the observed sum of 36 . Thus the evidence is not strong enough, at the $5 \%$ level, to reject the null hypothesis that there is no effect of the treatment.

Because of the small size of this experiment, we could have calculated the exact permutation $p$-value from all possible permutations. There are six units, but we want the sum of the treatment units. Thus there are $\binom{6}{3}=20$ permutation sums from the possible unique permutations.

$$
\begin{array}{rrrr}
7+9+10=26 & 7+10+12=29 & 9+10+11=30 & 9+12+14=35 \\
7+9+11=27 & 7+10+14=31 & 9+10+12=31 & 10+11+12=33 \\
7+9+12=28 & 7+11+12=30 & 9+10+14=33 & 10+11+14=35 \\
7+9+14=30 & 7+11+14=32 & 9+11+12=32 & 10+12+14=36 \\
7+10+11=28 & 7+12+14=33 & 9+11+14=34 & 11+12+14=37
\end{array}
$$

Two of the 20 permutation sums are greater than or equal to 36 . Thus the exact $p$-value for this permutation test is 0.1 . Tied values will decrease the number of unique permutations.

When the saving() option is supplied, permute saves the values of the permutation statistic to the indicated file, in our case, permdish.dta. This file can be used to replay the result of permute. The level() option controls the confidence level of the confidence interval for the permutation $p$-value. This confidence interval is calculated using cii with the reported n (number of nonmissing replications) and $c$ (the counter for events of significance).

```
. permute using permdish, level(80)
    command: summarize y if treatment
        sum: r(sum)
    permute var: y
```

Monte Carlo permutation results $\quad$ Number of obs $=\quad 6$

| $T$ | $T$ (obs) | $c$ | $n$ | $p=c / n$ | SE (p) | [80\% Conf. Interval] |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| sum | 36 | 10 | 100 | 0.1000 | 0.0300 | .0631113 | .1498826 |

Note: confidence interval is with respect to $\mathrm{p}=\mathrm{c} / \mathrm{n}$.
Note: $c=\#\{|T|>=|T(o b s)|\}$

## > Example 2: Permutation tests with ANOVA

Consider some fictional data from a randomized complete-block design in which we wish to determine the significance of five treatments.

```
. use http://www.stata-press.com/data/r13/permute1, clear
. list y treatment in 1/10, abbrev(10)
```

| $y$ | treatment |
| ---: | ---: |
| 4.407557 | 1 |
| 5.693386 | 1 |
| 7.099699 | 1 |
| 3.12132 | 1 |
| 5.242648 | 1 |
| 4.280349 | 2 |
| 4.508785 | 2 |
| 4.079967 | 2 |
| 5.904368 | 2 |
| 3.010556 | 2 |

These data may be analyzed using anova.
. anova y treatment subject

|  | Number of obs Root MSE | 50 R-squared |  |  | $\begin{aligned} & =0.3544 \\ & =0.1213 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Source | Partial SS | df | MS | F | Prob > F |
| Model | 16.5182188 | 13 | 1.27063221 | 1.52 | 0.1574 |
| treatment | 13.0226706 | 9 | 1.44696341 | 1.73 | 0.1174 |
| subject | 3.49554813 | 4 | . 873887032 | 1.05 | 0.3973 |
| Residual | 30.0847503 | 36 | . 835687509 |  |  |
| Total | 46.6029691 | 49 | . 951081002 |  |  |

Suppose that we want to compute the significance of the $F$ statistic for treatment by using permute. All we need to do is write a short program that will save the result of this statistic for permute to use. For example,

```
program panova, rclass
    version 13
    args response fac_intrst fac_other
    anova 'response' 'fac_intrst' 'fac_other'
    return scalar Fmodel = e(F)
    test 'fac_intrst'
    return scalar F = r(F)
end
```

Now in panova, test saves the $F$ statistic for the factor of interest in $r(F)$. This is different from e(F), which is the overall model $F$ statistic for the model fit by anova that panova saves in $r$ (Fmodel). In the following example, we use the strata() option so that the treatments are randomly rearranged within each subject. It should not be too surprising that the estimated $p$-values are equal for this example, because the two $F$ statistics are equivalent when controlling for differences between subjects. However, we would not expect to always get the same $p$-values every time we reran permute.

```
. set seed 1234
. permute treatment treatmentF=r(F) modelF=e(F), reps(1000) strata(subject)
> saving(permanova) nodots: panova y treatment subject
Monte Carlo permutation results
```

Number of strata $=5 \quad$ Number of obs $=50$
command: panova y treatment subject
treatmentF: $r(F)$
modelF: $e(F)$
permute var: treatment

| T | T (obs) | c | n | $\mathrm{p}=\mathrm{c} / \mathrm{n}$ | $\mathrm{SE}(\mathrm{p})$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| treatmentF | 1.731465 | 118 | 1000 | 0.1180 | 0.0102 | .0986525 | .1396277 |
| modelF | 1.520463 | 118 | 1000 | 0.1180 | 0.0102 | .0986525 | .1396277 |

```
Note: confidence intervals are with respect to p=c/n.
Note: c = #{|T| >= |T(obs)|}
```


## > Example 3: Wilcoxon rank-sum test

As a final example, let's consider estimating the $p$-value of the $Z$ statistic returned by ranksum. Suppose that we collected data from some experiment: y is some measure we took on 17 individuals, and group identifies the group that an individual belongs to.

```
. use http://www.stata-press.com/data/r13/permute2
```

. list

|  | group | y |
| :---: | :---: | :---: |
| 1. | 1 | 6 |
| 2. | 1 | 11 |
| 3. | 1 | 20 |
| 4. | 1 | 2 |
| 5. | 1 | 9 |
| 6. | 1 | 5 |
| 7. | 0 | 2 |
| 8. | 0 | 1 |
| 9. | 0 | 6 |
| 10. | 0 | 0 |
| 11. | 0 | 2 |
| 12. | 0 | 3 |
| 13. | 0 | 3 |
| 14. | 0 | 12 |
| 15. | 0 | 4 |
| 16. | 0 | 1 |
| 17. | 0 | 5 |

Next we analyze the data using ranksum and notice that the observed value of the test statistic (stored as $r(z)$ ) is -2.02 with an approximate $p$-value of 0.0434 .
. ranksum y, by (group)
Two-sample Wilcoxon rank-sum (Mann-Whitney) test
group

The observed value of the rank-sum statistic is 79 , with an expected value (under the null hypothesis of no group effect) of 99 . There are 17 observations, so the permutation distribution contains $\binom{17}{6}=12,376$ possible values of the rank-sum statistic if we ignore ties. With ties, we have fewer possible values but still too many to want to count them. Thus we use permute with 10,000 replications and see that the Monte Carlo permutation test agrees with the result of the test based on the normal approximation.

```
. set seed 18385766
. permute y z=r(z), reps(10000) nowarn nodots: ranksum y, by(group)
Monte Carlo permutation results Number of obs = 17
    command: ranksum y, by(group)
        z: r(z)
    permute var: y
```

| T | T (obs) | c | n | $\mathrm{p}=\mathrm{c} / \mathrm{n}$ | $\mathrm{SE}(\mathrm{p})$ | [95\% Conf. Interval] |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| z | -2.020002 | 468 | 10000 | 0.0468 | 0.0021 | .0427429 | .0511236 |

Note: confidence interval is with respect to $\mathrm{p}=\mathrm{c} / \mathrm{n}$.
Note: $c=\#\{|T|>=|T(o b s)|\}$

For an application of a permutation test to a problem in epidemiology, see Hayes and Moulton (2009, 190-193).

## Technical note

permute reports confidence intervals for $p$ to emphasize that it is based on the binomial estimator for proportions. When the variability implied by the confidence interval makes conclusions difficult, you may increase the number of replications to determine more precisely the significance of the test statistic of interest. In other words, the value of $p$ from permute will converge to the true permutation $p$-value as the number of replications gets arbitrarily large.

## Stored results

permute stores the following in $r()$ :

| Scalars |  |  |  |
| :---: | :---: | :---: | :---: |
| $r$ ( $)^{\text {) }}$ | sample size | r (k_exp) | number of standard expressions |
| $\begin{aligned} & \text { r(N_reps) } \\ & \text { r(level) } \end{aligned}$ | number of requested replications confidence level | r(k_eexp) | number of _b/_se expressions |
| Macros |  |  |  |
| r (cmd) | permute | r (left) | left or empty |
| r(command) | command following colon | r (right) | right or empty |
| r(permvar) | permutation variable | r (seed) | initial random-number seed |
| r(title) | title in output | $r$ (event) | $\mathrm{T}\langle=\mathrm{T}$ (obs), $\mathrm{T}>$ > T (obs), |
| $r$ (exp\#) | \#th expression |  | or $\|T\|<=\|T(o b s)\|$ |
| Matrices |  |  |  |
| r (b) | observed statistics | $\mathrm{r}(\mathrm{p})$ | observed proportions |
| r (c) | count when $r$ (event) is true | r (se) | standard errors of observed proportions |
| r(reps) | number of nonmissing results | r(ci) | confidence intervals of observed proportions |

## References

Ängquist, L. 2010. Stata tip 92: Manual implementation of permutations and bootstraps. Stata Journal 10: 686-688.
Good, P. I. 2006. Resampling Methods: A Practical Guide to Data Analysis. 3rd ed. Boston: Birkhäuser.
Hayes, R. J., and L. H. Moulton. 2009. Cluster Randomised Trials. Boca Raton, FL: Chapman \& Hall/CRC.
Kaiser, J. 2007. An exact and a Monte Carlo proposal to the Fisher-Pitman permutation tests for paired replicates and for independent samples. Stata Journal 7: 402-412.

Kaiser, J., and M. G. Lacy. 2009. A general-purpose method for two-group randomization tests. Stata Journal 9: 70-85.

## Also see

[R] bootstrap - Bootstrap sampling and estimation
[R] jackknife - Jackknife estimation
[R] simulate - Monte Carlo simulations

Description Remarks and examples References

## Description

The term pk refers to pharmacokinetic data and the Stata commands, all of which begin with the letters pk , designed to do some of the analyses commonly performed in the pharmaceutical industry. The system is intended for the analysis of pharmacokinetic data, although some of the commands are for general use.

The pk commands are

| pkexamine | $[\mathrm{R}]$ pkexamine | Calculate pharmacokinetic measures |
| :--- | :--- | :--- |
| pksumm | $[\mathrm{R}]$ pksumm | Summarize pharmacokinetic data |
| pkshape | $[\mathrm{R}]$ pkshape | Reshape (pharmacokinetic) Latin-square data |
| pkcross | $[\mathrm{R}]$ pkcross | Analyze crossover experiments |
| pkequiv | $[\mathrm{R}]$ pkequiv | Perform bioequivalence tests |
| pkcollapse | $[\mathrm{R}]$ pkcollapse | Generate pharmacokinetic measurement dataset |

## Remarks and examples

Several types of clinical trials are commonly performed in the pharmaceutical industry. Examples include combination trials, multicenter trials, equivalence trials, and active control trials. For each type of trial, there is an optimal study design for estimating the effects of interest. Currently, the pk system can be used to analyze equivalence trials, which are usually conducted using a crossover design; however, it is possible to use a parallel design and still draw conclusions about equivalence.

Equivalence trials assess bioequivalence between two drugs. Although proving that two drugs behave the same is impossible, the United States Food and Drug Administration believes that if the absorption properties of two drugs are similar, the two drugs will produce similar effects and have similar safety profiles. Generally, the goal of an equivalence trial is to assess the equivalence of a generic drug to an existing drug. This goal is commonly accomplished by comparing a confidence interval about the difference between a pharmacokinetic measurement of two drugs with a confidence limit constructed from U.S. federal regulations. If the confidence interval is entirely within the confidence limit, the drugs are declared bioequivalent. Another approach to assessing bioequivalence is to use the method of interval hypotheses testing. pkequiv is used to conduct these tests of bioequivalence.

Several pharmacokinetic measures can be used to ascertain how available a drug is for cellular absorption. The most common measure is the area under the time-versus-concentration curve (AUC). Another common measure of drug availability is the maximum concentration ( $C_{\max }$ ) achieved by the drug during the follow-up period. Stata reports these and other less common measures of drug availability, including the time at which the maximum drug concentration was observed and the duration of the period during which the subject was being measured. Stata also reports the elimination rate, that is, the rate at which the drug is metabolized, and the drug's half-life, that is, the time it takes for the drug concentration to fall to one-half of its maximum concentration.
pkexamine computes and reports all the pharmacokinetic measures that Stata produces, including four calculations of the area under the time-versus-concentration curve. The standard area under the curve from 0 to the maximum observed time $\left(\mathrm{AUC}_{0, t_{\text {max }}}\right)$ is computed using cubic splines or the trapezoidal rule. Additionally, pkexamine also computes the area under the curve from 0 to infinity by extending the standard time-versus-concentration curve from the maximum observed time by using three different methods. The first method simply extends the standard curve by using a least-squares linear fit through the last few data points. The second method extends the standard curve by fitting a decreasing exponential curve through the last few data points. Finally, the third method extends the curve by fitting a least-squares linear regression line on the log concentration. The mathematical details of these extensions are described in Methods and formulas of [R] pkexamine.

Data from an equivalence trial may also be analyzed using methods appropriate to the particular study design. When you have a crossover design, pkcross can be used to fit an appropriate ANOVA model. As an aside, a crossover design is simply a restricted Latin square; therefore, pkcross can also be used to analyze any Latin-square design.

There are some practical concerns when dealing with data from equivalence trials. Primarily, the data must be organized in a manner that Stata can use. The pk commands include pkcollapse and pkshape, which are designed to help transform data from a common format to one that is suitable for analysis with Stata.

In the following example, we illustrate several different data formats that are often encountered in pharmaceutical research and describe how these formats can be transformed to formats that can be analyzed with Stata.

## > Example 1

Assume that we have one subject and are interested in determining the drug profile for that subject. A reasonable experiment would be to give the subject the drug and then measure the concentration of the drug in the subject's blood over a given period. For example, here is a part of a dataset from Chow and Liu (2009, 13):

```
. use http://www.stata-press.com/data/r13/auc
. list, abbrev(14)
```

|  | id | time | concentration |
| :---: | :---: | :---: | :---: |
| 1. | 1 | 0 | 0 |
| 2. | 1 | . 5 | 0 |
| 3. | 1 | 1 | 2.8 |
| 4. | 1 | 1.5 | 4.4 |
| 5. | 1 | 2 | 4.4 |
| 6. | 1 | 3 | 4.7 |
| 7. | 1 | 4 | 4.1 |
| 8. | 1 | 6 | 4 |
| 9. | 1 | 8 | 3.6 |
| 10. | 1 | 12 | 3 |
| 11. | 1 | 16 | 2.5 |
| 12. | 1 | 24 | 2 |
| 13. | 1 | 32 | 1.6 |

Examining these data, we notice that the concentration quickly increases, plateaus for a short period, and then slowly decreases over time. pkexamine is used to calculate the pharmacokinetic measures of interest. pkexamine is explained in detail in [R] pkexamine. The output is

```
pkexamine time conc
```

| Maximum concentration | $=$ | 4.7 |
| ---: | :--- | ---: |
| Time of maximum concentration | $=$ | 3 |
| Time of last observation (Tmax) | $=$ | 32 |
| Elimination rate | $=$ | 0.0279 |
| Half life | $=$ | 24.8503 |

Area under the curve

| AUC [0, Tmax] | AUC [0, inf.) <br> Linear of log conc. | AUC [0, inf.) <br> Linear fit | AUC [0, inf.) <br> Exponential fit |
| :---: | :---: | :---: | :---: |
| 85.24 | 142.603 | 107.759 | 142.603 |

Fit based on last 3 points.
Clinical trials, however, require that data be collected on more than one subject. There are several ways to enter raw measured data collected on several subjects. It would be reasonable to enter for each subject the drug concentration value at specific points in time. Such data could be

| id | conc1 | conc2 | conc3 | conc4 | conc5 | conc6 | conc7 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 0 | 1 | 4 | 7 | 5 | 3 | 1 |
| 2 | 0 | 2 | 6 | 5 | 4 | 3 | 2 |
| 3 | 0 | 1 | 2 | 3 | 5 | 4 | 1 |

where conc1 is the concentration at the first measured time, conc2 is the concentration at the second measured time, etc. This format requires that each drug concentration measurement be made at the same time on each subject. Another more flexible way to enter the data is to have an observation with three variables for each time measurement on a subject. Each observation would have a subject ID, the time at which the measurement was made, and the corresponding drug concentration at that time. The data would be
. use http://www.stata-press.com/data/r13/pkdata
. list id concA time, sepby(id)

| id | concA | time |
| ---: | ---: | ---: |
| 1 | 0 | 0 |
| 1 | 3.073403 | .5 |
| 1 | 5.188444 | 1 |
| 1 | 5.898577 | 1.5 |
| 1 | 5.096378 | 2 |
| 1 | 6.094085 | 3 |
| 1 | 5.158772 | 4 |
| 1 | 5.7065 | 6 |
| 1 | 5.272467 | 8 |
| 1 | 4.4576 | 12 |
| 1 | 5.146423 | 16 |
| 1 | 4.947427 | 24 |
| 1 | 1.920421 | 32 |
| 2 |  | 0 |
| 2 | 2.48462 | 0 |
| 2 | 4.883569 | 1 |
| 2 | 7.253442 | 1.5 |
| 2 | 5.849345 | 2 |
| 2 | 6.761085 | 3 |
| 2 | 4.33839 | 4 |
| 2 | 5.04199 | 6 |
| 2 | 4.25128 | 8 |
| 2 | 6.205004 | 12 |
| 2 | 5.566165 | 16 |
| 2 | 3.689007 | 24 |
| 2 | 3.644063 | 32 |
| 3 | (output omitted) |  |
| 20 | 4.673281 | 24 |
| 20 | 3.487347 | 32 |
|  |  |  |

Stata expects the data to be organized in the second form. If your data are organized as described in the first dataset, you will need to use reshape to change the data to the second form; see [D] reshape. Because the data in the second (or long) format contain information for one drug on several subjects, pksumm can be used to produce summary statistics of the pharmacokinetic measurements. The output is

| Summary statistics for the pharmacokinetic measures |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Number of | ervation | 6 |
| Measure | Mean | Median | Variance | Skewness | Kurtosis | p-value |
| auc | 151.63 | 152.18 | 127.58 | -0.34 | 2.07 | 0.55 |
| aucline | 397.09 | 219.83 | 178276.59 | 2.69 | 9.61 | 0.00 |
| aucexp | 668.60 | 302.96 | 720356.98 | 2.67 | 9.54 | 0.00 |
| auclog | 665.95 | 298.03 | 752573.34 | 2.71 | 9.70 | 0.00 |
| half | 90.68 | 29.12 | 17750.70 | 2.36 | 7.92 | 0.00 |
| ke | 0.02 | 0.02 | 0.00 | 0.88 | 3.87 | 0.08 |
| cmax | 7.37 | 7.42 | 0.40 | -0.64 | 2.75 | 0.36 |
| tomc | 3.38 | 3.00 | 7.25 | 2.27 | 7.70 | 0.00 |
| tmax | 32.00 | 32.00 | 0.00 | . | . |  |

Until now, we have been concerned with the profile of only one drug. We have characterized the profile of that drug by individual subjects by using pkexamine and by a group of subjects by using pksumm. The goal of an equivalence trial, however, is to compare two drugs, which we will do in the rest of this example.

For equivalence trials, the study design most often used is the crossover design. For a complete discussion of crossover designs, see Ratkowsky, Evans, and Alldredge (1993).

In brief, crossover designs require that each subject be given both treatments at two different times. The order in which the treatments are applied changes between groups. For example, if we had 20 subjects numbered $1-20$, the first 10 would receive treatment A during the first period of the study, and then they would be given treatment B . The second 10 subjects would be given treatment B during the first period of the study, and then they would be given treatment A. Each subject in the study will have four variables that describe the observation: a subject identifier, a sequence identifier that indicates the order of treatment, and two outcome variables, one for each treatment. The outcome variables for each subject are the pharmacokinetic measures. The data must be transformed from a series of measurements on individual subjects to data containing the pharmacokinetic measures for each subject. In Stata parlance, this is referred to as a collapse, which can be done with pkcollapse; see [R] pkcollapse.

Here is a part of our data:

|  | id | seq | time | concA | concB |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | 1 | 1 | 0 | 0 | 0 |
| 2. | 1 | 1 | . 5 | 3.073403 | 3.712592 |
| 3. | 1 | 1 | 1 | 5.188444 | 6.230602 |
| 4. | 1 | 1 | 1.5 | 5.898577 | 7.885944 |
| 5. | 1 | 1 | 2 | 5.096378 | 9.241735 |
| 6. | 1 | 1 | 3 | 6.094085 | 13.10507 |
| 7. | 1 | 1 | 4 | 5.158772 | . 169429 |
| 8. | 1 | 1 | 6 | 5.7065 | 8.759894 |
| 9. | 1 | 1 | 8 | 5.272467 | 7.985409 |
| 10. | 1 | 1 | 12 | 4.4576 | 7.740126 |
| 11. | 1 | 1 | 16 | 5.146423 | 7.607208 |
| 12. | 1 | 1 | 24 | 4.947427 | 7.588428 |
| 13. | 1 | 1 | 32 | 1.920421 | 2.791115 |
| 14. | 2 | 1 | 0 | 0 | 0 |
| 15. | 2 | 1 | . 5 | 2.48462 | . 9209593 |
| 16. | 2 | 1 | 1 | 4.883569 | 5.925818 |
| 17. | 2 | 1 | 1.5 | 7.253442 | 8.710549 |
| 18. | 2 | 1 | 2 | 5.849345 | 10.90552 |
| 19. | 2 | 1 | 3 | 6.761085 | 8.429898 |
| 20. | 2 | 1 | 4 | 4.33839 | 5.573152 |
| 21. | 2 | 1 | 6 | 5.04199 | 6.32341 |
| 22. | 2 | 1 | 8 | 4.25128 | . 5251224 |
| 23. | 2 | 1 | 12 | 6.205004 | 7.415988 |
| 24. | 2 | 1 | 16 | 5.566165 | 6.323938 |
| 25. | 2 | 1 | 24 | 3.689007 | 1.133553 |
| 26. | 2 | 1 | 32 | 3.644063 | 5.759489 |
| 27. | 3 | 1 | 0 <br> (outpu | $\text { omitted ) } 0$ | 0 |
| 207. | 20 | 2 | 24 | 4.673281 | 6.059818 |
| 208. | 20 | 2 | 32 | 3.487347 | 5.213639 |

This format is similar to the second format described above, except that now we have measurements for two drugs at each time for each subject. We transform these data with pkcollapse:

```
. pkcollapse time concA concB, id(id) keep(seq) stat(auc)
. list, sep(8) abbrev(10)
```

| id | seq | auc_concA | auc_concB |
| ---: | ---: | ---: | ---: |
| 1 | 1 | 150.9643 | 218.5551 |
| 2 | 1 | 146.7606 | 133.3201 |
| 3 | 1 | 160.6548 | 126.0635 |
| 4 | 1 | 157.8622 | 96.17461 |
| 5 | 1 | 133.6957 | 188.9038 |
| 7 | 1 | 160.639 | 223.6922 |
| 8 | 1 | 131.2604 | 104.0139 |
| 9 | 1 | 168.5186 | 237.8962 |
| 10 | 2 | 137.0627 | 139.7382 |
| 12 | 2 | 153.4038 | 202.3942 |
| 13 | 2 | 163.4593 | 136.7848 |
| 14 | 2 | 146.0462 | 104.5191 |
| 15 | 2 | 158.1457 | 165.8654 |
| 18 | 2 | 147.1977 | 139.235 |
| 19 | 2 | 164.9988 | 166.2391 |
| 20 | 2 | 145.3823 | 158.5146 |

For this example, we chose to use the AUC for two drugs as our pharmacokinetic measure. We could have used any of the measures computed by pkexamine. In addition to the AUCs, the dataset also contains a sequence variable for each subject indicating when each treatment was administered.

The data produced by pkcollapse are in what Stata calls wide format; that is, there is one observation per subject containing two or more outcomes. To use pkcross and pkequiv, we need to transform these data to long format. This goal can be accomplished using pkshape; see $[R]$ pkshape.

Consider the first subject in the dataset. This subject is in sequence one, which means that treatment A was applied during the first period of the study and treatment B was applied in the second period of the study. We need to split the first observation into two observations so that the outcome measure is only in one variable. Also we need two new variables, one indicating the treatment the subject received and another recording the period of the study when the subject received that treatment. We might expect the expansion of the first subject to be

| id | sequence | auc | treat | period |
| ---: | ---: | ---: | ---: | ---: |
| 1 | 1 | 150.9643 | $A$ | 1 |
| 1 | 1 | 218.5551 | $B$ | 2 |

We see that subject number 1 was in sequence 1 , had an AUC of 150.9643 when treatment A was applied in the first period of the study, and had an AUC of 218.5551 when treatment B was applied.

Similarly, the expansion of subject 10 (the first subject in sequence 2 ) would be

| id | sequence | auc | treat | period |
| ---: | ---: | ---: | ---: | ---: |
| 10 | 2 | 137.0627 | $B$ | 1 |
| 10 | 2 | 139.7382 | $A$ | 2 |

Here treatment B was applied to the subject during the first period of the study, and treatment A was applied to the subject during the second period of the study.

An additional complication is common in crossover study designs. The treatment applied in the first period of the study might still have some effect on the outcome in the second period. In this example,
each subject was given one treatment followed by another treatment. To get accurate estimates of treatment effects, it is necessary to account for the effect that the first treatment has in the second period of the study. This is called the carryover effect. We must, therefore, have a variable that indicates which treatment was applied in the first treatment period. pkshape creates a variable that indicates the carryover effect. For treatments applied during the first treatment period, there will never be a carryover effect. Thus the expanded data created by pkshape for subject 1 will be

| id | sequence | outcome | treat | period | carry |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 1 | 150.9643 | A | 1 | 0 |
| 1 | 1 | 218.5551 | B | 2 | A |

and the data for subject 10 will be

| id | sequence | outcome | treat | period | carry |
| :--- | ---: | ---: | ---: | ---: | ---: |
| 10 | 2 | 137.0627 | B | 1 | 0 |
| 10 | 2 | 139.7382 | $A$ | 2 | B |

We pkshape the data:

```
. pkshape id seq auc*, order(ab ba)
. sort id sequence period
. list, sep(16)
```

|  | id | sequence | outcome | treat | carry | period |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | 1 | 1 | 150.9643 | 1 | 0 | 1 |
| 2. | 1 | 1 | 218.5551 | 2 | 1 | 2 |
| 3. | 2 | 1 | 146.7606 | 1 | 0 | 1 |
| 4. | 2 | 1 | 133.3201 | 2 | 1 | 2 |
| 5. | 3 | 1 | 160.6548 | 1 | 0 | 1 |
| 6. | 3 | 1 | 126.0635 | 2 | 1 | 2 |
| 7. | 4 | 1 | 157.8622 | 1 | 0 | 1 |
| 8. | 4 | 1 | 96.17461 | 2 | 1 | 2 |
| 9. | 5 | 1 | 133.6957 | 1 | 0 | 1 |
| 10. | 5 | 1 | 188.9038 | 2 | 1 | 2 |
| 11. | 7 | 1 | 160.639 | 1 | 0 | 1 |
| 12. | 7 | 1 | 223.6922 | 2 | 1 | 2 |
| 13. | 8 | 1 | 131.2604 | 1 | 0 | 1 |
| 14. | 8 | 1 | 104.0139 | 2 | 1 | 2 |
| 15. | 9 | 1 | 168.5186 | 1 | 0 | 1 |
| 16. | 9 | 1 | 237.8962 | 2 | 1 | 2 |
| 17. | 10 | 2 | 137.0627 | 2 | 0 | 1 |
| 18. | 10 | 2 | 139.7382 | 1 | 2 | 2 |
| 19. | 12 | 2 | 153.4038 | 2 | 0 | 1 |
| 20. | 12 | 2 | 202.3942 | 1 | 2 | 2 |
| 21. | 13 | 2 | 163.4593 | 2 | 0 | 1 |
| 22. | 13 | 2 | 136.7848 | 1 | 2 | 2 |
| 23. | 14 | 2 | 146.0462 | 2 | 0 | 1 |
| 24. | 14 | 2 | 104.5191 | 1 | 2 | 2 |
| 25. | 15 | 2 | 158.1457 | 2 | 0 | 1 |
| 26. | 15 | 2 | 165.8654 | 1 | 2 | 2 |
| 27. | 18 | 2 | 147.1977 | 2 | 0 | 1 |
| 28. | 18 | 2 | 139.235 | 1 | 2 | 2 |
| 29. | 19 | 2 | 164.9988 | 2 | 0 | 1 |
| 30. | 19 | 2 | 166.2391 | 1 | 2 | 2 |
| 31. | 20 | 2 | 145.3823 | 2 | 0 | 1 |
| 32. | 20 | 2 | 158.5146 | 1 | 2 | 2 |

As an aside, crossover designs do not require that each subject receive each treatment, but if they do, the crossover design is referred to as a complete crossover design.

The last dataset is organized in a manner that can be analyzed with Stata. To fit an ANOVA model to these data, we can use anova or pkcross. To conduct equivalence tests, we can use pkequiv. This example is further analyzed in $[R]$ pkcross and $[R]$ pkequiv.

## References

Chow, S.-C., and J.-P. Liu. 2009. Design and Analysis of Bioavailability and Bioequivalence Studies. 3rd ed. Boca Raton, FL: Chapman \& Hall/CRC.
Ratkowsky, D. A., M. A. Evans, and J. R. Alldredge. 1993. Cross-over Experiments: Design, Analysis, and Application. New York: Dekker.

## Title

pkcollapse - Generate pharmacokinetic measurement dataset

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Methods and formulas | Also see |  |

## Syntax

pkcollapse time concentration [if], id(id_var) [options]
options Description

Main

| *id(id_var) | subject ID variable |
| :--- | :--- |
| stat(measures) | create specified measures; default is all |
| trapezoid | use trapezoidal rule; default is cubic splines |
| fit(\#) | use \# points to estimate $\mathrm{AUC}_{0, \infty}$; default is fit(3) |
| keep(varlist) | keep variables in varlist |
| force | force collapse |
| nodots | suppress dots during calculation |

*id(id_var) is required.

| measures | Description |
| :--- | :--- |
| auc | area under the concentration-time curve $\left(\mathrm{AUC}_{0}, \infty\right)$ |
| aucline | area under the concentration-time curve from 0 to $\infty$ using a linear extension <br> area under the concentration-time curve from 0 to $\infty$ using an exponential <br> extension |
| auclog | area under the log-concentration-time curve extended with a linear fit <br> half |
| half-life of the drug |  |
| ke | elimination rate |
| cmax | maximum concentration |
| tmax | time at last concentration |
| tomc | time of maximum concentration |

## Menu

Statistics $>$ Epidemiology and related $>$ Other $>$ Generate pharmacokinetic measurement dataset

## Description

pkcollapse generates new variables with the pharmacokinetic summary measures of interest. pkcollapse is one of the pk commands. Please read $[\mathrm{R}] \mathbf{p k}$ before reading this entry.

## Options

$\qquad$ Main
id (id_var) is required and specifies the variable that contains the subject ID over which pkcollapse is to operate.
stat (measures) specifies the measures to be generated. The default is to generate all the measures.
trapezoid tells Stata to use the trapezoidal rule when calculating the AUC. The default is to use cubic splines, which give better results for most functions. When the curve is irregular, trapezoid may give better results.
fit(\#) specifies the number of points to use in estimating the $\mathrm{AUC}_{0, \infty}$. The default is fit(3), the last three points. This number should be viewed as a minimum; the appropriate number of points will depend on your data.
keep (varlist) specifies the variables to be kept during the collapse. Variables not specified with the keep () option will be dropped. When keep() is specified, the keep variables are checked to ensure that all values of the variables are the same within id_var.
force forces the collapse, even when the values of the keep() variables are different within the id_var.
nodots suppresses the display of dots during calculation.

## Remarks and examples

pkcollapse generates all the summary pharmacokinetic measures.

## > Example 1

We demonstrate the use of pkcollapse with the data described in [R] pk. We have drug concentration data on 15 subjects. Each subject is measured at 13 time points over a 32 -hour period. Some of the records are

```
. use http://www.stata-press.com/data/r13/pkdata
. list, sep(0)
```

|  | id | seq | time | concA | concB |
| :--- | ---: | ---: | ---: | ---: | ---: |
|  | 1. | 1 | 1 | 0 | 0 |
| 2. | 1 | 1 | .5 | 3.073403 | 3.712592 |
| 3. | 1 | 1 | 1 | 5.188444 | 6.230602 |
| 4. | 1 | 1 | 1.5 | 5.898577 | 7.885944 |
| 5. | 1 | 1 | 2 | 5.096378 | 9.241735 |
| 6. | 1 | 1 | 3 | 6.094085 | 13.10507 |
|  |  |  | (output omitted) |  |  |
| 14. | 2 | 1 | 0 | 0 | 0 |
| 15. | 2 | 1 | .5 | 2.48462 | .9209593 |
| 16. | 2 | 1 | 1 | 4.883569 | 5.925818 |
| 17. | 2 | 1 | 1.5 | 7.253442 | 8.710549 |
| 18. | 2 | 1 | 2 | 5.849345 | 10.90552 |
| 19. | 2 | 1 | 3 | 6.761085 | 8.429898 |

Although pksumm allows us to view all the pharmacokinetic measures, we can create a dataset with the measures by using pkcollapse.

```
. pkcollapse time concA concB, id(id) stat(auc) keep(seq)
. list, sep(8) abbrev(10)
```

|  | id | seq | auc_concA | auc_concB |
| :--- | ---: | ---: | ---: | ---: |
| 1. | 1 | 1 | 150.9643 | 218.5551 |
| 2. | 2 | 1 | 146.7606 | 133.3201 |
| 3. | 3 | 1 | 160.6548 | 126.0635 |
| 4. | 4 | 1 | 157.8622 | 96.17461 |
| 5. | 5 | 1 | 133.6957 | 188.9038 |
| 6. | 7 | 1 | 160.639 | 223.6922 |
| 7. | 8 | 1 | 131.2604 | 104.0139 |
| 8. | 9 | 1 | 168.5186 | 237.8962 |
|  |  |  |  |  |
| 9. | 10 | 2 | 137.0627 | 139.7382 |
| 10. | 12 | 2 | 153.4038 | 202.3942 |
| 11. | 13 | 2 | 163.4593 | 136.7848 |
| 12. | 14 | 2 | 146.0462 | 104.5191 |
| 13. | 15 | 2 | 158.1457 | 165.8654 |
| 14. | 18 | 2 | 147.1977 | 139.235 |
| 15. | 19 | 2 | 164.9988 | 166.2391 |
| 16. | 20 | 2 | 145.3823 | 158.5146 |

The resulting dataset, which we will call pkdata2, contains 1 observation per subject. This dataset is in wide format. If we want to use pkcross or pkequiv, we must transform these data to long format, which we do in the last example of $[R]$ pkshape.

## Methods and formulas

The statistics generated by pkcollapse are described in [R] pkexamine.

## Also see

[R] pk - Pharmacokinetic (biopharmaceutical) data

## Title

pkcross - Analyze crossover experiments

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Methods and formulas | References | Also see |

## Syntax

pkcross outcome [if] [in] [, options]
options Description

Model
sequence (varname) sequence variable; default is sequence (sequence) treatment (varname) treatment variable; default is treatment (treat) period(varname) period variable; default is period(period) id(varname) ID variable carryover (varname) name of carryover variable; default is carryover(carry) carryover (none) model (string)
sequential
Parameterization
param (3)
param(1)
param (4)
$\underline{p} \operatorname{aram}(2) \quad$ estimate mean, period and treatment effects, and period-by-treatment interaction; assume no sequence or carryover effects exist
estimate mean and the period, treatment, and sequence effects; assume no carryover effects exist; the default
estimate mean and the period, treatment, and carryover effects; assume no sequence effects exist
estimate mean, period and treatment effects, and period-by-treatment interaction; assume no period or crossover effects exist

## Menu

Statistics $>$ Epidemiology and related $>$ Other $>$ Analyze crossover experiments

## Description

pkcross analyzes data from a crossover design experiment. When analyzing pharmaceutical trial data, if the treatment, carryover, and sequence variables are known, the omnibus test for separability of the treatment and carryover effects is calculated.
pkcross is one of the pk commands. Please read [R] pk before reading this entry.

## Options

sequence (varname) specifies the variable that contains the sequence in which the treatment was administered. If this option is not specified, sequence (sequence) is assumed.
treatment (varname) specifies the variable that contains the treatment information. If this option is not specified, treatment (treat) is assumed.
period(varname) specifies the variable that contains the period information. If this option is not specified, period(period) is assumed.
id (varname) specifies the variable that contains the subject identifiers. If this option is not specified, id(id) is assumed.
carryover (varname ${ }^{\text {none) }}$ specifies the variable that contains the carryover information. If carry (none) is specified, the carryover effects are omitted from the model. If this option is not specified, carryover (carry) is assumed.
model (string) specifies the model to be fit. For higher-order crossover designs, this option can be useful if you want to fit a model other than the default. However, anova (see $[R]$ anova) can also be used to fit a crossover model. The default model for higher-order crossover designs is outcome predicted by sequence, period, treatment, and carryover effects. By default, the model statement is model (sequence period treat carry).
sequential specifies that sequential sums of squares be estimated.

## Parameterization

param(\#) specifies which of the four parameterizations to use for the analysis of a $2 \times 2$ crossover experiment. This option is ignored with higher-order crossover designs. The default is param (3). See the technical note for $2 \times 2$ crossover designs for more details.
param(3) estimates the overall mean, the period effects, the treatment effects, and the sequence effects, assuming that no carryover effects exist. This is the default parameterization.
param(1) estimates the overall mean, the period effects, the treatment effects, and the carryover effects, assuming that no sequence effects exist.
param(2) estimates the overall mean, the period effects, the treatment effects, and the period-bytreatment interaction, assuming that no sequence or carryover effects exist.
param (4) estimates the overall mean, the sequence effects, the treatment effects, and the sequence-by-treatment interaction, assuming that no period or crossover effects exist. When the sequence by treatment is equivalent to the period effect, this reduces to the third parameterization.

## Remarks and examples

pkcross is designed to analyze crossover experiments. Use pkshape first to reshape your data; see [R] pkshape. pkcross assumes that the data were reshaped by pkshape or are organized in the same manner as produced with pkshape. Washout periods are indicated by the number 0 . See the technical note in this entry for more information on analyzing $2 \times 2$ crossover experiments.

## - Technical note

The $2 \times 2$ crossover design cannot be used to estimate more than four parameters because there are only four pieces of information (the four cell means) collected. pkcross uses ANOVA models to analyze the data, so one of the four parameters must be the overall mean of the model, leaving just 3 degrees of freedom to estimate the remaining effects (period, sequence, treatment, and carryover). Thus the model is overparameterized. Estimation of treatment and carryover effects requires the assumption of either no period effects or no sequence effects. Some researchers maintain that it estimating carryover effects at the expense of other effects is a bad idea. This is a limitation of this design. pkcross implements four parameterizations for this model. They are numbered sequentially from one to four and are described in Options.

## > Example 1

Consider the example data published in Chow and Liu $(2009,71)$ and described in [R] pkshape. We have entered and reshaped the data with pkshape and have variables that identify the subjects, periods, treatments, sequence, and carryover treatment. To compute the ANOVA table, use pkcross:
. use http://www.stata-press.com/data/r13/chowliu

- pkshape id seq period1 period2, order(ab ba)
- pkcross outcome

There is evidence of intersubject variability, but there are no other significant effects. The omnibus test for separability is a measure reflecting the degree to which the study design allows the treatment effects to be estimated independently of the carryover effects. The measure of separability of the treatment and carryover effects indicates approximately $29 \%$ separability, which can be interpreted as the degree to which the treatment and carryover effects are orthogonal. This is a characteristic of the design of the study. For a complete discussion, see Ratkowsky, Evans, and Alldredge (1993). Compared to the output in Chow and Liu (2009), the sequence effect is mislabeled as a carryover effect. See Ratkowsky, Evans, and Alldredge (1993, sec. 3.2) for a complete discussion of the mislabeling.

By specifying param(1), we obtain parameterization 1 for this model.

```
pkcross outcome, param(1)
```

```
        sequence variable = sequence
    period variable = period
treatment variable = treat
carryover variable = carry
    id variable = id
```

| Analysis of variance (ANOVA) for a $2 \times 2$ crossover study |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Source of Variation | Partial SS | df | MS | F | Prob > F |
| Treatment effect | 301.04 | 1 | 301.04 | 0.67 | 0.4189 |
| Period effect | 255.62 | 1 | 255.62 | 0.57 | 0.4561 |
| Carryover effect | 276.00 | 1 | 276.00 | 0.61 | 0.4388 |
| Residuals | 19890.92 | 44 | 452.07 |  |  |
| Total | 20265.68 | 47 |  |  |  |
| Omnibus measure of separability of treatment and carryover $=29.2893 \%$ |  |  |  |  |  |

## > Example 2

Consider the case of a two-treatment, four-sequence, two-period crossover design. This design is commonly referred to as Balaam's design (Balaam 1968). Ratkowsky, Evans, and Alldredge (1993, 140) published the following data from an amantadine trial, originally published by Taka and Armitage (1983):

```
. use http://www.stata-press.com/data/r13/balaam, clear
. list, sep(0)
```

| id | seq | period1 | period2 | period3 |
| ---: | ---: | ---: | ---: | ---: |
| 1 | $-a b$ | 9 | 8.75 | 8.75 |
| 2 | $-a b$ | 12 | 10.5 | 9.75 |
| 3 | $-a b$ | 17 | 15 | 18.5 |
| 4 | $-a b$ | 21 | 21 | 21.5 |
| 1 | $-b a$ | 23 | 22 | 18 |
| 2 | $-b a$ | 15 | 15 | 13 |
| 3 | $-b a$ | 13 | 14 | 13.75 |
| 4 | $-b a$ | 24 | 22.75 | 21.5 |
| 5 | $-b a$ | 18 | 17.75 | 16.75 |
| 1 | $-a a$ | 14 | 12.5 | 14 |
| 2 | - aa | 27 | 24.25 | 22.5 |
| 3 | $-a a$ | 19 | 17.25 | 16.25 |
| 4 | - aa | 30 | 28.25 | 29.75 |
| 1 | $-b b$ | 21 | 20 | 19.51 |
| 2 | $-b b$ | 11 | 10.5 | 10 |
| 3 | $-b b$ | 20 | 19.5 | 20.75 |
| 4 | $-b b$ | 25 | 22.5 | 23.5 |

The sequence identifier must be a string with zeros to indicate washout or baseline periods, or a number. If the sequence identifier is numeric, the order option must be specified with pkshape. If the sequence identifier is a string, pkshape will create sequence, period, and treatment identifiers without the order option. In this example, the dash is used to indicate a baseline period, which is an invalid code for this purpose. As a result, the data must be encoded; see [D] encode.

```
. encode seq, gen(num_seq)
. pkshape id num_seq period1 period2 period3, order(Oaa Oab Oba Obb)
. pkcross outcome, se
    sequence variable = sequence
    period variable = period
treatment variable = treat
carryover variable = carry
                                    id variable = id
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multicolumn{6}{|c|}{Analysis of variance (ANOVA) for a crossover study} \\
\hline Intersubjects & & & & & \\
\hline Sequence effect & 285.82 & 3 & 95.27 & 1.01 & 0.4180 \\
\hline Residuals & 1221.49 & 13 & 93.96 & 59.96 & 0.0000 \\
\hline Intrasubjects & & & & & \\
\hline Period effect & 15.13 & 2 & 7.56 & 6.34 & 0.0048 \\
\hline Treatment effect & 8.48 & 1 & 8.48 & 8.86 & 0.0056 \\
\hline Carryover effect & 0.11 & 1 & 0.11 & 0.12 & 0.7366 \\
\hline Residuals & 29.56 & 30 & 0.99 & & \\
\hline Total & 1560.59 & 50 & & & \\
\hline
\end{tabular}
```

In this example, the sequence specifier used dashes instead of zeros to indicate a baseline period during which no treatment was given. For pkcross to work, we need to encode the string sequence variable and then use the order option with pkshape. A word of caution: encode does not necessarily choose the first sequence to be sequence 1, as in this example. Always double-check the sequence numbering when using encode.

## Example 3

Continuing with the example from $[\mathrm{R}]$ pkshape, we fit an ANOVA model.
. use http://www.stata-press.com/data/r13/pkdata3, clear . list, sep(8)

|  | id | sequence | outcome | treat | carry | period |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | 1 | 1 | 150.9643 | A | 0 | 1 |
| 2. | 2 | 1 | 146.7606 | A | 0 | 1 |
| 3. | 3 | 1 | 160.6548 | A | 0 | 1 |
| 4. | 4 | 1 | 157.8622 | A | 0 | 1 |
| 5. | 5 | 1 | 133.6957 | A | 0 | 1 |
| 6. | 7 | 1 | 160.639 | A | 0 | 1 |
| 7. | 8 | 1 | 131.2604 | A | 0 | 1 |
| 8. | 9 | 1 | 168.5186 | A | 0 | 1 |
| 9. | 10 | 2 | 137.0627 | B | 0 | 1 |
| 10. | 12 | 2 | 153.4038 | B | 0 | 1 |
| 11. | 13 | 2 | 163.4593 | B | 0 | 1 |
| 12. | 14 | 2 | 146.0462 | B | 0 | 1 |
| 13. | 15 | 2 | 158.1457 | B | 0 | 1 |
| 14. | 18 | 2 | 147.1977 | B | 0 | 1 |
| 15. | 19 | 2 | 164.9988 | B | 0 | 1 |
| 16. | 20 | 2 | 145.3823 | B | 0 | 1 |
| 17. | 1 | 1 | 218.5551 | B | A | 2 |
| 18. | 2 | 1 | 133.3201 | B | A | 2 |
| 19. | 3 | 1 | 126.0635 | B | A | 2 |
| 20. | 4 | 1 | 96.17461 | B | A | 2 |
| 21. | 5 | 1 | 188.9038 | B | A | 2 |
| 22. | 7 | 1 | 223.6922 | B | A | 2 |
| 23. | 8 | 1 | 104.0139 | B | A | 2 |
| 24. | 9 | 1 | 237.8962 | B | A | 2 |
| 25. | 10 | 2 | 139.7382 | A | B | 2 |
| 26. | 12 | 2 | 202.3942 | A | B | 2 |
| 27. | 13 | 2 | 136.7848 | A | B | 2 |
| 28. | 14 | 2 | 104.5191 | A | B | 2 |
| 29. | 15 | 2 | 165.8654 | A | B | 2 |
| 30. | 18 | 2 | 139.235 | A | B | 2 |
| 31. | 19 | 2 | 166.2391 | A | B | 2 |
| 32. | 20 | 2 | 158.5146 | A | B | 2 |

The ANOVA model is fit using pkcross:
. pkcross outcome

$$
\begin{aligned}
\text { sequence variable } & =\text { sequence } \\
\text { period variable } & =\text { period } \\
\text { treatment variable } & =\text { treat } \\
\text { carryover variable } & =\text { carry } \\
\text { id variable } & =\text { id }
\end{aligned}
$$

| Analysis of variance (ANOVA) for a $2 \times 2$ crossover study |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Source of Variation | Partial SS | df | MS | F | Prob > F |
| Intersubjects |  |  |  |  |  |
| Sequence effect | 378.04 | 1 | 378.04 | 0.29 | 0.5961 |
| Residuals | 17991.26 | 14 | 1285.09 | 1.40 | 0.2691 |
| Intrasubjects |  |  |  |  |  |
| Treatment effect | 455.04 | 1 | 455.04 | 0.50 | 0.4931 |
| Period effect | 419.47 | 1 | 419.47 | 0.46 | 0.5102 |
| Residuals | 12860.78 | 14 | 918.63 |  |  |
| Total | 32104.59 | 31 |  |  |  |

Omnibus measure of separability of treatment and carryover $=29.2893 \%$

## Example 4

Consider the case of a six-treatment crossover trial in which the squares are not variance balanced. The following dataset is from a partially balanced crossover trial published by Patterson and Lucas (1962) and reproduced in Ratkowsky, Evans, and Alldredge (1993, 231):
. use http://www.stata-press.com/data/r13/nobalance
. list, $\operatorname{sep}(4)$

|  | cow | seq | period1 | period2 | period3 | period4 | block |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | 1 | adbe | 38.7 | 37.4 | 34.3 | 31.3 | 1 |
| 2. | 2 | baed | 48.9 | 46.9 | 42 | 39.6 | 1 |
| 3. | 3 | ebda | 34.6 | 32.3 | 28.5 | 27.1 | 1 |
| 4. | 4 | deab | 35.2 | 33.5 | 28.4 | 25.1 | 1 |
| 5. | 1 | dafc | 32.9 | 33.1 | 27.5 | 25.1 | 2 |
| 6. | 2 | fdca | 30.4 | 29.5 | 26.7 | 23.1 | 2 |
| 7. | 3 | cfad | 30.8 | 29.3 | 26.4 | 23.2 | 2 |
| 8. | 4 | acdf | 25.7 | 26.1 | 23.4 | 18.7 | 2 |
| 9. | 1 | efbc | 25.4 | 26 | 23.9 | 19.9 | 3 |
| 10. | 2 | becf | 21.8 | 23.9 | 21.7 | 17.6 | 3 |
| 11. | 3 | fceb | 21.4 | 22 | 19.4 | 16.6 | 3 |
| 12. | 4 | cbfe | 22.8 | 21 | 18.6 | 16.1 | 3 |

When there is no variance balance in the design, a square or blocking variable is needed to indicate in which treatment cell a sequence was observed, but the mechanical steps are the same.

```
. pkshape cow seq period1 period2 period3 period4
. pkcross outcome, model(block cow|block period|block treat carry) se
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multirow[b]{2}{*}{Source} & \multicolumn{3}{|l|}{\[
\begin{array}{rlr}
\text { Number of obs } & = & 48 \\
\text { Root MSE } & =.740408
\end{array}
\]} & R-squared & \[
\begin{aligned}
& =0.9965 \\
& =0.9903
\end{aligned}
\] \\
\hline & Seq. SS & df & MS & F & Prob > F \\
\hline Model & 2650.1331 & 30 & 88.3377701 & 161.14 & 0.0000 \\
\hline block & 1607.01128 & 2 & 803.505642 & 1465.71 & 0.0000 \\
\hline cow|block & 628.706274 & 9 & 69.8562527 & 127.43 & 0.0000 \\
\hline period|block & 408.031253 & 9 & 45.3368059 & 82.70 & 0.0000 \\
\hline treat & 2.50000057 & 5 & . 500000114 & 0.91 & 0.4964 \\
\hline carry & 3.88428906 & 5 & . 776857812 & 1.42 & 0.2680 \\
\hline Residual & 9.31945887 & 17 & . 548203463 & & \\
\hline Total & 2659.45256 & 47 & 56.584097 & & \\
\hline
\end{tabular}
```

When the model statement is used and the omnibus measure of separability is desired, specify the variables in the treatment(), carryover(), and sequence() options to pkcross.

## Methods and formulas

pkcross uses ANOVA to fit models for crossover experiments; see $[R]$ anova.
The omnibus measure of separability is

$$
S=100(1-V) \%
$$

where $V$ is Cramér's $V$ and is defined as

$$
V=\left\{\frac{\frac{\chi^{2}}{N}}{\min (r-1, c-1)}\right\}^{\frac{1}{2}}
$$

The $\chi^{2}$ is calculated as

$$
\chi^{2}=\sum_{i} \sum_{j}\left\{\frac{\left(O_{i j}-E_{i j}\right)^{2}}{E_{i j}}\right\}
$$

where $O$ and $E$ are the observed and expected counts in a table of the number of times each treatment is followed by the other treatments.

## References

Balaam, L. N. 1968. A two-period design with $t^{2}$ experimental units. Biometrics 24: 61-73.
Chow, S.-C., and J.-P. Liu. 2009. Design and Analysis of Bioavailability and Bioequivalence Studies. 3rd ed. Boca Raton, FL: Chapman \& Hall/CRC.
Kutner, M. H., C. J. Nachtsheim, J. Neter, and W. Li. 2005. Applied Linear Statistical Models. 5th ed. New York: McGraw-Hill/Irwin.

Patterson, H. D., and H. L. Lucas. 1962. Change-over designs. Technical Bulletin 147, North Carolina Agricultural Experiment Station and the USDA.

Ratkowsky, D. A., M. A. Evans, and J. R. Alldredge. 1993. Cross-over Experiments: Design, Analysis, and Application. New York: Dekker.
Taka, M. T., and P. Armitage. 1983. Autoregressive models in clinical trials. Communications in Statistics—Theory and Methods 12: 865-876.

## Also see

[R] pk - Pharmacokinetic (biopharmaceutical) data

## Title

pkequiv - Perform bioequivalence tests

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

pkequiv outcome treatment period sequence id [if] [in] [, options] options Description

Options
compare (string) compare the two specified values of the treatment variable
limit(\#) equivalence limit (between 0.10 and 0.99 ); default is 0.2
level (\#) set confidence level; default is level (90)
fieller calculate confidence interval by Fieller's theorem
symmetric
anderson
tost
noboot
calculate symmetric equivalence interval

Anderson and Hauck hypothesis test for bioequivalence two one-sided hypothesis tests for bioequivalence do not estimate probability that CI lies within confidence limits

## Menu

Statistics $>$ Epidemiology and related $>$ Other $>$ Bioequivalence tests

## Description

pkequiv performs bioequivalence testing for two treatments. By default, pkequiv calculates a standard confidence interval symmetric about the difference between the two treatment means. pkequiv also calculates confidence intervals symmetric about zero and intervals based on Fieller's theorem. Also, pkequiv can perform interval hypothesis tests for bioequivalence.
pkequiv is one of the pk commands. Please read [R] pk before reading this entry.

## Options

[^58]fieller specifies that an equivalence interval based on Fieller's theorem be calculated.
symmetric specifies that a symmetric equivalence interval be calculated.
anderson specifies that the Anderson and Hauck (1983) hypothesis test for bioequivalence be computed. This option is ignored when calculating equivalence intervals based on Fieller's theorem or when calculating a confidence interval that is symmetric about zero.
tost specifies that the two one-sided hypothesis tests for bioequivalence be computed. This option is ignored when calculating equivalence intervals based on Fieller's theorem or when calculating a confidence interval that is symmetric about zero.
noboot prevents the estimation of the probability that the confidence interval lies within the confidence limits. If this option is not specified, this probability is estimated by resampling the data.

## Remarks and examples

pkequiv is designed to conduct tests for bioequivalence based on data from a crossover experiment. pkequiv requires that the user specify the outcome, treatment, period, sequence, and id variables. The data must be in the same format as that produced by pkshape; see [R] pkshape.

## Example 1

We have the following data on which we want to conduct a bioequivalence test between treat $=A$ and treat $=B$.

```
. use http://www.stata-press.com/data/r13/pkdata3
. list, sep(4)
```

|  | id | sequence | outcome | treat | carry | period |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | 1 | 1 | 150.9643 | A | 0 | 1 |
| 2. | 2 | 1 | 146.7606 | A | 0 | 1 |
| 3. | 3 | 1 | 160.6548 | A | 0 | 1 |
| 4. | 4 | 1 | 157.8622 | A | 0 | 1 |
| 5. | 5 | 1 | 133.6957 | A | 0 | 1 |
| 6. | 7 | 1 | 160.639 | A | 0 | 1 |
| 7. | 8 | 1 | 131.2604 | A | 0 | 1 |
| 8. | 9 | 1 | 168.5186 | A | 0 | 1 |
| 9. | 10 | 2 | 137.0627 | B | 0 | 1 |
| 10. | 12 | 2 | 153.4038 | B | 0 | 1 |
| 11. | 13 | 2 | 163.4593 | B | 0 | 1 |
| 12. | 14 | 2 | 146.0462 | B | 0 | 1 |
| 13. | 15 | 2 | 158.1457 | B | 0 | 1 |
| 14. | 18 | 2 | 147.1977 | B | 0 | 1 |
| 15. | 19 | 2 | 164.9988 | B | 0 | 1 |
| 16. | 20 | 2 | 145.3823 | B | 0 | 1 |
| 17. | 1 | 1 | 218.5551 | B | A | 2 |
| 18. | 2 | 1 | 133.3201 | B | A | 2 |
| 19. | 3 | 1 | 126.0635 | B | A | 2 |
| 20. | 4 | 1 | 96.17461 | B | A | 2 |
| 21. | 5 | 1 | 188.9038 | B | A | 2 |
| 22. | 7 | 1 | 223.6922 | B | A | 2 |
| 23. | 8 | 1 | 104.0139 | B | A | 2 |
| 24. | 9 | 1 | 237.8962 | B | A | 2 |
| 25. | 10 | 2 | 139.7382 | A | B | 2 |
| 26. | 12 | 2 | 202.3942 | A | B | 2 |
| 27. | 13 | 2 | 136.7848 | A | B | 2 |
| 28. | 14 | 2 | 104.5191 | A | B | 2 |
| 29. | 15 | 2 | 165.8654 | A | B | 2 |
| 30. | 18 | 2 | 139.235 | A | B | 2 |
| 31. | 19 | 2 | 166.2391 | A | B | 2 |
| 32. | 20 | 2 | 158.5146 | A | B | 2 |

set seed 1
pkequiv outcome treat period seq id
Classic confidence interval for bioequivalence

|  | [equivalence limits] | [ | test limits | ] |
| ---: | ---: | ---: | ---: | :---: |
| difference: | -30.296 | 30.296 | -11.332 | 26.416 |
| ratio: | $80 \%$ | $120 \%$ | $92.519 \%$ | $117.439 \%$ |
| probability test limits are within equivalence limits |  | 0.6410 |  |  |
| note: reference treatment $=1$ |  |  |  |  |

The default output for pkequiv shows a confidence interval for the difference of the means (test limits), the ratio of the means, and the federal equivalence limits. The classic confidence interval can be constructed around the difference between the average measure of effect for the two drugs or around the ratio of the average measure of effect for the two drugs. pkequiv reports both the difference measure and the ratio measure. For these data, U.S. federal government regulations state that the confidence interval for the difference must be entirely contained within the range [ $-30.296,30.296$ ] and between $80 \%$ and $120 \%$ for the ratio. Here the test limits are within the equivalence limits. Although the test limits are inside the equivalence limits, there is only a $64 \%$ assurance that the observed confidence interval will be within the equivalence limits in the long run. This is an interesting case because, although this sample shows bioequivalence, the evaluation of the long-run performance indicates possible problems. These fictitious data were generated with high intersubject variability, which causes poor long-run performance.

If we conduct a bioequivalence test with the data published in Chow and Liu $(2009,71)$, which we introduced in $[R] \mathbf{p k}$ and fully described in $[R]$ pkshape, we observe that the probability that the test limits are within the equivalence limits is high.

```
. use http://www.stata-press.com/data/r13/chowliu2
. set seed 1
. pkequiv outcome treat period seq id
Classic confidence interval for bioequivalence
\begin{tabular}{rrrrr}
\hline & [equivalence limits] & [ & test limits & ] \\
difference: & -16.512 & 16.512 & -8.698 & 4.123 \\
ratio: & \(80 \%\) & \(120 \%\) & \(89.464 \%\) & \(104.994 \%\) \\
\hline probability test limits are within equivalence & limits \(=\) & 0.9980 \\
note: reference treatment \(=1\)
\end{tabular}
```

For these data, the test limits are well within the equivalence limits, and the probability that the test limits are within the equivalence limits is $99.8 \%$.

## > Example 2

We compute a confidence interval that is symmetric about zero:
. pkequiv outcome treat period seq id, symmetric
Westlake's symmetric confidence interval for bioequivalence

|  | [Equivalence limits] | [ | Test mean ] |
| :--- | :---: | :---: | :---: |
| Test formulation: | 75.145 | 89.974 | 80.272 |
| note: reference treatment $=1$ |  |  |  |

The reported equivalence limit is constructed symmetrically about the reference mean, which is equivalent to constructing a confidence interval symmetric about zero for the difference in the two drugs. In the output above, we see that the test formulation mean of 80.272 is within the equivalence limits, indicating that the test drug is bioequivalent to the reference drug.
pkequiv displays interval hypothesis tests of bioequivalence if you specify the tost or the anderson option, or both. For example,

```
. pkequiv outcome treat period seq id, tost anderson
    Classic confidence interval for bioequivalence
\begin{tabular}{rrrlc}
\hline & [equivalence limits] & [ & \multicolumn{1}{c}{ test limits } & ] \\
\hline difference: & -16.512 & 16.512 & -8.698 & 4.123 \\
ratio: & \(80 \%\) & \(120 \%\) & \(89.464 \%\) & \(104.994 \%\) \\
\hline
\end{tabular}
probability test limits are within equivalence limits = 0.9990
Schuirmann's two one-sided tests
lrer test statistic = 
p-value = 0.001
Anderson and Hauck's test
```



```
note: reference treatment = 1
```

Both of Schuirmann's one-sided tests are highly significant, suggesting that the two drugs are bioequivalent. A similar conclusion is drawn from the Anderson and Hauck test of bioequivalence.

## Stored results

pkequiv stores the following in $r()$ :

```
Scalars
    r(stddev) pooled-sample standard deviation of period differences from both sequences
    r(uci) upper confidence interval for a classic interval
    r(lci) lower confidence interval for a classic interval
    r(delta) delta value used in calculating a symmetric confidence interval
    r(u3) upper confidence interval for Fieller's confidence interval
    r(13) lower confidence interval for Fieller's confidence interval
```


## Methods and formulas

The lower confidence interval for the difference in the two treatments for the classic shortest confidence interval is

$$
L_{1}=\left(\bar{Y}_{T}-\bar{Y}_{R}\right)-t_{\left(\alpha, n_{1}+n_{2}-2\right)} \widehat{\sigma}_{d} \sqrt{\frac{1}{n_{1}}+\frac{1}{n_{2}}}
$$

The upper limit is

$$
U_{1}=\left(\bar{Y}_{T}-\bar{Y}_{R}\right)+t_{\left(\alpha, n_{1}+n_{2}-2\right)} \widehat{\sigma}_{d} \sqrt{\frac{1}{n_{1}}+\frac{1}{n_{2}}}
$$

The limits for the ratio measure are

$$
L_{2}=\left(\frac{L_{1}}{\bar{Y}_{R}}+1\right) 100 \%
$$

and

$$
U_{2}=\left(\frac{U_{1}}{\bar{Y}_{R}}+1\right) 100 \%
$$

where $\bar{Y}_{T}$ is the mean of the test formulation of the drug, $\bar{Y}_{R}$ is the mean of the reference formulation of the drug, and $t_{\left(\alpha, n_{1}+n_{2}-2\right)}$ is the $t$ distribution with $n_{1}+n_{2}-2$ degrees of freedom. $\widehat{\sigma}_{d}$ is the pooled sample variance of the period differences from both sequences, defined as

$$
\widehat{\sigma}_{d}=\frac{1}{n_{1}+n_{2}-2} \sum_{k=1}^{2} \sum_{i=1}^{n_{k}}\left(d_{i k}-\bar{d}_{. k}\right)^{2}
$$

The upper and lower limits for the symmetric confidence interval are $\bar{Y}_{R}+\Delta$ and $\bar{Y}_{R}-\Delta$, where

$$
\Delta=k_{1} \widehat{\sigma}_{d} \sqrt{\frac{1}{n_{1}}+\frac{1}{n_{2}}}-\left(\bar{Y}_{T}-\bar{Y}_{R}\right)
$$

and (simultaneously)

$$
\Delta=-k_{2} \widehat{\sigma}_{d} \sqrt{\frac{1}{n_{1}}+\frac{1}{n_{2}}}+2\left(\bar{Y}_{T}-\bar{Y}_{R}\right)
$$

and $k_{1}$ and $k_{2}$ are computed iteratively to satisfy the above equalities and the condition

$$
\int_{k_{1}}^{k_{2}} f(t) d t=1-2 \alpha
$$

where $f(t)$ is the probability density function of the $t$ distribution with $n_{1}+n_{2}-2$ degrees of freedom.

See Chow and Liu (2009, 88-92) for details about calculating the confidence interval based on Fieller's theorem.

The two test statistics for the two one-sided tests of equivalence are

$$
T_{L}=\frac{\left(\bar{Y}_{T}-\bar{Y}_{R}\right)-\theta_{L}}{\widehat{\sigma}_{d} \sqrt{\frac{1}{n_{1}}+\frac{1}{n_{2}}}}
$$

and

$$
T_{U}=\frac{\left(\bar{Y}_{T}-\bar{Y}_{R}\right)-\theta_{U}}{\widehat{\sigma}_{d} \sqrt{\frac{1}{n_{1}}+\frac{1}{n_{2}}}}
$$

where $-\theta_{L}=\theta_{U}$ and are the regulated confidence limits.

The logic of the Anderson and Hauck test is tricky; see Chow and Liu (2009) for a complete explanation. However, the test statistic is

$$
T_{A H}=\frac{\left(\bar{Y}_{T}-\bar{Y}_{R}\right)-\left(\frac{\theta_{L}+\theta_{U}}{2}\right)}{\widehat{\sigma}_{d} \sqrt{\frac{1}{n_{1}}+\frac{1}{n_{2}}}}
$$

and the noncentrality parameter is estimated by

$$
\widehat{\delta}=\frac{\theta_{U}-\theta_{L}}{2 \widehat{\sigma}_{d} \sqrt{\frac{1}{n_{1}}+\frac{1}{n_{2}}}}
$$

The empirical $p$-value is calculated as

$$
p=F_{t}\left(\left|T_{A H}\right|-\widehat{\delta}\right)-F_{t}\left(-\left|T_{A H}\right|-\widehat{\delta}\right)
$$

where $F_{t}$ is the cumulative distribution function of the $t$ distribution with $n_{1}+n_{2}-2$ degrees of freedom.

## References

Anderson, S., and W. W. Hauck. 1983. A new procedure for testing equivalence in comparative bioavailability and other clinical trials. Communications in Statistics—Theory and Methods 12: 2663-2692.

Chow, S.-C., and J.-P. Liu. 2009. Design and Analysis of Bioavailability and Bioequivalence Studies. 3rd ed. Boca Raton, FL: Chapman \& Hall/CRC.
Fieller, E. C. 1954. Some problems in interval estimation. Journal of the Royal Statistical Society, Series B 16: 175-185.

Kutner, M. H., C. J. Nachtsheim, J. Neter, and W. Li. 2005. Applied Linear Statistical Models. 5th ed. New York: McGraw-Hill/Irwin.
Locke, C. S. 1984. An exact confidence interval from untransformed data for the ratio of two formulation means. Journal of Pharmacokinetics and Biopharmaceutics 12: 649-655.

Schuirmann, D. J. 1989. Confidence intervals for the ratio of two means from a cross-over study. In Proceedings of the Biopharmaceutical Section, 121-126. Washington, DC: American Statistical Association.
Westlake, W. J. 1976. Symmetrical confidence intervals for bioequivalence trials. Biometrics 32: 741-744.

## Also see

[R] pk - Pharmacokinetic (biopharmaceutical) data

## Title

pkexamine - Calculate pharmacokinetic measures

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | Reference |
| Also see |  |  |  |

## Syntax

pkexamine time concentration [if] [in] [, options]
options
Main
fit(\#) use \# points to estimate $\mathrm{AUC}_{0, \infty}$; default is fit(3)
trapezoid use trapezoidal rule; default is cubic splines
graph
line
log
exp(\#)
AUC plot
cline_options
marker_options
marker_label_options
Add plots
addplot (plot) add other plots to the generated graph
Y axis, X axis, Titles, Legend, Overall
twoway_options any options other than by() documented in [G-3] twoway_options
by is allowed; see [D] by.

## Menu

Statistics $>$ Epidemiology and related $>$ Other $>$ Pharmacokinetic measures

## Description

pkexamine calculates pharmacokinetic measures from time-and-concentration subject-level data. pkexamine computes and displays the maximum measured concentration, the time at the maximum measured concentration, the time of the last measurement, the elimination time, the half-life, and the area under the concentration-time curve (AUC). Three estimates of the area under the concentration-time curve from 0 to infinity $\left(\mathrm{AUC}_{0, \infty}\right)$ are also calculated.
pkexamine is one of the pk commands. Please read [R] pk before reading this entry.

## Options

## Main

fit(\#) specifies the number of points, counting back from the last measurement, to use in fitting the extension to estimate the $\mathrm{AUC}_{0, \infty}$. The default is fit(3), or the last three points. This value should be viewed as a minimum; the appropriate number of points will depend on your data.
trapezoid specifies that the trapezoidal rule be used to calculate the AUC. The default is cubic splines, which give better results for most functions. When the curve is irregular, trapezoid may give better results.
graph tells pkexamine to graph the concentration-time curve.
line and log specify the estimates of the $\mathrm{AUC}_{0, \infty}$ to display when graphing the $\mathrm{AUC}_{0, \infty}$. These options are ignored, unless they are specified with the graph option.
$\exp (\#)$ specifies that the exponential fit for the $\mathrm{AUC}_{0, \infty}$ be plotted. You must specify the maximum time value to which you want to plot the curve, and this time value must be greater than the maximum time measurement in the data. If you specify 0 , the curve will be plotted to the point at which the linear extension would cross the $x$ axis. This option is not valid with the line or log option and is ignored, unless the graph option is also specified.

AUC plot
cline_options affect the rendition of the plotted points connected by lines; see [G-3] cline_options. marker_options specify the look of markers. This look includes the marker symbol, the marker size, and its color and outline; see [G-3] marker_options.
marker_label_options specify if and how the markers are to be labeled; see [G-3] marker_label_options.

Add plots
addplot (plot) provides a way to add other plots to the generated graph. See [G-3] addplot_option.

[^59]twoway_options are any of the options documented in [G-3] twoway_options, excluding by (). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

## Remarks and examples

pkexamine computes summary statistics for a given patient in a pharmacokinetic trial. If by idvar: is specified, statistics will be displayed for each subject in the data.
$>$ Example 1
Chow and Liu $(2009,13)$ present data on a study examining primidone concentrations versus time for a subject over a 32 -hour period after dosing.
. use http://www.stata-press.com/data/r13/auc
. list, abbrev(14)

|  | id | time | concentration |
| ---: | ---: | ---: | ---: |
| 1. | 1 | 0 | 0 |
| 2. | 1 | .5 | 0 |
| 3. | 1 | 1 | 2.8 |
| 4. | 1 | 1.5 | 4.4 |
| 5. | 1 | 2 | 4.4 |
| 6. | 1 | 3 | 4.7 |
| 7. | 1 | 4 | 4.1 |
| 8. | 1 | 6 | 4 |
| 9. | 1 | 8 | 3.6 |
| 10. | 1 | 12 | 3 |
|  |  | 2.5 |  |
| 11. | 1 | 16 | 2 |
| 12. | 1 | 24 | 1.6 |
| 13. | 1 | 32 |  |
|  |  |  |  |

We use pkexamine to produce the summary statistics:

- pkexamine time conc, graph

$$
\begin{array}{rlr}
\text { Maximum concentration } & = & 4.7 \\
\text { Time of maximum concentration } & = & 3 \\
\text { Time of last observation (Tmax) } & = & 32 \\
\text { Elimination rate } & = & 0.0279 \\
\text { Half life } & =24.8503
\end{array}
$$

Area under the curve

| AUC [0, Tmax] | AUC [0, inf.) <br> Linear of log conc. | AUC [0, inf.) <br> Linear fit | AUC [0, inf.) <br> Exponential fit |
| :---: | :---: | :---: | :---: |
| 85.24 | 142.603 | 107.759 | 142.603 |

Fit based on last 3 points.


The maximum concentration of 4.7 occurs at time 3, and the time of the last observation (Tmax) is 32. In addition to the AUC, which is calculated from 0 to the maximum value of time, pkexamine also reports the area under the curve, computed by extending the curve with each of three methods: a linear fit to the $\log$ of the concentration, a linear regression line, and a decreasing exponential regression line. See Methods and formulas for details on these three methods.

By default, all extensions to the AUC are based on the last three points. Looking at the graph for these data, it seems more appropriate to use the last seven points to estimate the $\operatorname{AUC}_{0, \infty}$ :

```
. pkexamine time conc, fit(7)
```

| Maximum concentration | $=$ | 4.7 |
| ---: | :--- | ---: |
| Time of maximum concentration | $=$ | 3 |
| Time of last observation (Tmax) | $=$ | 32 |
| Elimination rate | $=$ | 0.0349 |
| Half life | $=$ | 19.8354 |

Area under the curve

| AUC [0, Tmax] | AUC [0, inf.) <br> Linear of log conc. | AUC [0, inf.) <br> Linear fit | AUC [0, inf.) <br> Exponential fit |
| :---: | :---: | :---: | :---: |
| 85.24 | 131.027 | 96.805 | 129.181 |

Fit based on last 7 points.
This approach decreased the estimate of the $\mathrm{AUC}_{0, \infty}$ for all extensions. To see a graph of the $\mathrm{AUC}_{0, \infty}$ using a linear extension, specify the graph and line options.


Fit based on last 7 points.


## Stored results

pkexamine stores the following in r() :
Scalars
$r$ (auc) area under the concentration curve
$r$ (half) half-life of the drug
$r(k e) \quad$ elimination rate
$r(\operatorname{tmax}) \quad$ time at last concentration measurement
$r$ (cmax) maximum concentration
$r$ (tomc) time of maximum concentration
$r$ (auc_line) $\quad \mathrm{AUC}_{0, \infty}$ estimated with a linear fit
$r$ (auc_exp) $\quad \mathrm{AUC}_{0, \infty}$ estimated with an exponential fit
$r$ (auc_ln) $\quad \mathrm{AUC}_{0, \infty}$ estimated with a linear fit of the natural $\log$

## Methods and formulas

Let $i$ index the observations sorted by time, let $k$ be the number of observations, and let $f$ be the number of points specified in the fit(\#) option.

The $\mathrm{AUC}_{0, t_{\text {max }}}$ is defined as

$$
\mathrm{AUC}_{0, t_{\max }}=\int_{0}^{t_{\max }} C_{t} d t
$$

where $C_{t}$ is the concentration at time $t$. By default, the integral is calculated numerically using cubic splines. However, if the trapezoidal rule is used, the $\mathrm{AUC}_{0, t_{\text {max }}}$ is given as

$$
\mathrm{AUC}_{0, t_{\max }}=\sum_{i=2}^{k} \frac{C_{i-1}+C_{i}}{2}\left(t_{i}-t_{i-1}\right)
$$

The $\mathrm{AUC}_{0, \infty}$ is the $\mathrm{AUC}_{0, t_{\text {max }}}+\mathrm{AUC}_{t_{\text {max }}, \infty}$, or

$$
\mathrm{AUC}_{0, \infty}=\int_{0}^{t_{\max }} C_{t} d t+\int_{t_{\max }}^{\infty} C_{t} d t
$$

When using the linear extension to the $\mathrm{AUC}_{0, t_{\max }}$, the integration is cut off when the line crosses the $x$ axis. The $\log$ extension is a linear extension on the $\log$ concentration scale. The area for the exponential extension is

$$
\operatorname{AUC}_{0, \infty}=\int_{t_{\max }}^{\infty} e^{-\left(\beta_{0}+t \beta_{1}\right)} d t=-\frac{e^{-\left(\beta_{0}+t_{\max } \beta_{1}\right)}}{\beta_{1}}
$$

The elimination rate $K_{\text {eq }}$ is the negative of the slope from a linear regression of log concentration on time fit to the number of points specified in the fit(\#) option:

$$
K_{\mathrm{eq}}=-\frac{\sum_{i=k-f+1}^{k}\left(t_{i}-\bar{t}\right)\left(\ln C_{i}-\overline{\ln C}\right)}{\sum_{i=k-f+1}^{k}\left(t_{i}-\bar{t}\right)^{2}}
$$

The half-life is

$$
t_{\mathrm{half}}=\frac{\ln 2}{K_{\mathrm{eq}}}
$$

## Reference

Chow, S.-C., and J.-P. Liu. 2009. Design and Analysis of Bioavailability and Bioequivalence Studies. 3rd ed. Boca Raton, FL: Chapman \& Hall/CRC.

## Also see

[R] pk - Pharmacokinetic (biopharmaceutical) data

## Title

pkshape - Reshape (pharmacokinetic) Latin-square data

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | References | Also see |  |

## Syntax

pkshape id sequence periodl period 2 [period list] [, options $]$
options Description
order (string) apply treatments in specified order
outcome (newvar) name for outcome variable; default is outcome (outcome)
treatment (newvar) name for treatment variable; default is treatment (treat)
carryover(newvar) name for carryover variable; default is carryover (carry)
sequence(newvar) name for sequence variable; default is sequence (sequence)
period(newvar) name for period variable; default is period(period)

## Menu

Statistics $>$ Epidemiology and related $>$ Other $>$ Reshape pharmacokinetic latin-square data

## Description

pkshape reshapes the data for use with anova, pkcross, and pkequiv; see $[R]$ anova, $[R]$ pkcross, and $[R]$ pkequiv. Latin-square and crossover data are often organized in a manner that cannot be analyzed easily with Stata. pkshape reorganizes the data in memory for use in Stata.
pkshape is one of the pk commands. Please read [R] pk before reading this entry.

## Options

order (string) specifies the order in which treatments were applied. If the sequence() specifier is a string variable that specifies the order, this option is not necessary. Otherwise, order () specifies how to generate the treatment and carryover variables. Any string variable can be used to specify the order. For crossover designs, any washout periods can be indicated with the number 0.
outcome (newvar) specifies the name for the outcome variable in the reorganized data. By default, outcome (outcome) is used.
treatment (newvar) specifies the name for the treatment variable in the reorganized data. By default, treatment (treat) is used.
carryover (newvar) specifies the name for the carryover variable in the reorganized data. By default, carryover (carry) is used.
sequence (newvar) specifies the name for the sequence variable in the reorganized data. By default, sequence (sequence) is used.
period(newvar) specifies the name for the period variable in the reorganized data. By default, period(period) is used.

## Remarks and examples

Often data from a Latin-square experiment are naturally organized in a manner that Stata cannot manage easily. pkshape reorganizes Latin-square data so that they can be used with anova (see $[R]$ anova) or any pk command. This includes the classic $2 \times 2$ crossover design commonly used in pharmaceutical research, as well as many other Latin-square designs.

## > Example 1

Consider the example data published in Chow and Liu (2009, 71). There are 24 patients, 12 in each sequence. Sequence 1 consists of the reference formulation followed by the test formulation; sequence 2 is the test formulation followed by the reference formulation. The measurements reported are the $\mathrm{AUC}_{0-t_{\text {max }}}$ for each patient and for each period.

```
. use http://www.stata-press.com/data/r13/chowliu
. list, sep(4)
```

|  | id | seq | period1 | period2 |
| :---: | :---: | :---: | :---: | :---: |
| 1. | 1 | 1 | 74.675 | 73.675 |
| 2. | 4 | 1 | 96.4 | 93.25 |
| 3. | 5 | 1 | 101.95 | 102.125 |
| 4. | 6 | 1 | 79.05 | 69.45 |
| 5. | 11 | 1 | 79.05 | 69.025 |
| 6. | 12 | 1 | 85.95 | 68.7 |
| 7. | 15 | 1 | 69.725 | 59.425 |
| 8. | 16 | 1 | 86.275 | 76.125 |
| 9. | 19 | 1 | 112.675 | 114.875 |
| 10. | 20 | 1 | 99.525 | 116.25 |
| 11. | 23 | 1 | 89.425 | 64.175 |
| 12. | 24 | 1 | 55.175 | 74.575 |
| 13. | 2 | 2 | 74.825 | 37.35 |
| 14. | 3 | 2 | 86.875 | 51.925 |
| 15. | 7 | 2 | 81.675 | 72.175 |
| 16. | 8 | 2 | 92.7 | 77.5 |
| 17. | 9 | 2 | 50.45 | 71.875 |
| 18. | 10 | 2 | 66.125 | 94.025 |
| 19. | 13 | 2 | 122.45 | 124.975 |
| 20. | 14 | 2 | 99.075 | 85.225 |
| 21. | 17 | 2 | 86.35 | 95.925 |
| 22. | 18 | 2 | 49.925 | 67.1 |
| 23. | 21 | 2 | 42.7 | 59.425 |
| 24. | 22 | 2 | 91.725 | 114.05 |

Because the outcome for one person is in two different variables, the treatment that was applied to an individual is a function of the period and the sequence. To analyze this treatment using anova, all the outcomes must be in one variable, and each covariate must be in its own variable. To reorganize these data, use pkshape:

```
pkshape id seq period1 period2, order(ab ba)
. sort seq id treat
```

```
list, sep(8)
```

|  | id | sequence | outcome | treat | carry | period |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | 1 | 1 | 74.675 | 1 | 0 | 1 |
| 2. | 1 | 1 | 73.675 | 2 | 1 | 2 |
| 3. | 4 | 1 | 96.4 | 1 | 0 | 1 |
| 4. | 4 | 1 | 93.25 | 2 | 1 | 2 |
| 5. | 5 | 1 | 101.95 | 1 | 0 | 1 |
| 6. | 5 | 1 | 102.125 | 2 | 1 | 2 |
| 7. | 6 | 1 | 79.05 | 1 | 0 | 1 |
| 8. | 6 | 1 | 69.45 | 2 | 1 | 2 |
| 9. | 11 | 1 | 79.05 | 1 | 0 | 1 |
| 10. | 11 | 1 | 69.025 | 2 | 1 | 2 |
| 11. | 12 | 1 | 85.95 | 1 | 0 | 1 |
| 12. | 12 | 1 | 68.7 | 2 | 1 | 2 |
| 13. | 15 | 1 | 69.725 | 1 | 0 | 1 |
| 14. | 15 | 1 | 59.425 | 2 | 1 | 2 |
| 15. | 16 | 1 | 86.275 | 1 | 0 | 1 |
| 16. | 16 | 1 | 76.125 | 2 | 1 | 2 |
| 17. | 19 | 1 | 112.675 | 1 | 0 | 1 |
| 18. | 19 | 1 | 114.875 | 2 | 1 | 2 |
| 19. | 20 | 1 | 99.525 | 1 | 0 | 1 |
| 20. | 20 | 1 | 116.25 | 2 | 1 | 2 |
| 21. | 23 | 1 | 89.425 | 1 | 0 | 1 |
| 22. | 23 | 1 | 64.175 | 2 | 1 | 2 |
| 23. | 24 | 1 | 55.175 | 1 | 0 | 1 |
| 24. | 24 | 1 | 74.575 | 2 | 1 | 2 |
| 25. | 2 | 2 | 37.35 | 1 | 2 | 2 |
| 26. | 2 | 2 | 74.825 | 2 | 0 | 1 |
| 27. | 3 | 2 | 51.925 | 1 | 2 | 2 |
| 28. | 3 | 2 | 86.875 | 2 | 0 | 1 |
| 29. | 7 | 2 | 72.175 | 1 | 2 | 2 |
| 30. | 7 | 2 | 81.675 | 2 | 0 | 1 |
| 31. | 8 | 2 | 77.5 | 1 | 2 | 2 |
| 32. | 8 | 2 | 92.7 | 2 | 0 | 1 |
| 33. | 9 | 2 | 71.875 | 1 | 2 | 2 |
| 34. | 9 | 2 | 50.45 | 2 | 0 | 1 |
| 35. | 10 | 2 | 94.025 | 1 | 2 | 2 |
| 36. | 10 | 2 | 66.125 | 2 | 0 | 1 |
| 37. | 13 | 2 | 124.975 | 1 | 2 | 2 |
| 38. | 13 | 2 | 122.45 | 2 | 0 | 1 |
| 39. | 14 | 2 | 85.225 | 1 | 2 | 2 |
| 40. | 14 | 2 | 99.075 | 2 | 0 | 1 |
| 41. | 17 | 2 | 95.925 | 1 | 2 | 2 |
| 42. | 17 | 2 | 86.35 | 2 | 0 | 1 |
| 43. | 18 | 2 | 67.1 | 1 | 2 | 2 |
| 44. | 18 | 2 | 49.925 | 2 | 0 | 1 |
| 45. | 21 | 2 | 59.425 | 1 | 2 | 2 |
| 46. | 21 | 2 | 42.7 | 2 | 0 | 1 |
| 47. | 22 | 2 | 114.05 | 1 | 2 | 2 |
| 48. | 22 | 2 | 91.725 | 2 | 0 | 1 |

Now the data are organized into separate variables that indicate each factor level for each of the covariates, so the data may be used with anova or pkcross; see [R] anova and [R] pkcross.

## > Example 2

Consider the study of background music on bank teller productivity published in Kutner et al. (2005). The data are

| Week | Monday | Tuesday | Wednesday | Thursday | Friday |
| ---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $18(\mathrm{D})$ | $17(\mathrm{C})$ | $14(\mathrm{~A})$ | $21(\mathrm{~B})$ | $17(\mathrm{E})$ |
| 2 | $13(\mathrm{C})$ | $34(\mathrm{~B})$ | $21(\mathrm{E})$ | $16(\mathrm{~A})$ | $15(\mathrm{D})$ |
| 3 | $7(\mathrm{~A})$ | $29(\mathrm{D})$ | $32(\mathrm{~B})$ | $27(\mathrm{E})$ | $13(\mathrm{C})$ |
| 4 | $17(\mathrm{E})$ | $13(\mathrm{~A})$ | $24(\mathrm{C})$ | $31(\mathrm{D})$ | $25(\mathrm{~B})$ |
| 5 | $21(\mathrm{~B})$ | $26(\mathrm{E})$ | $26(\mathrm{D})$ | $31(\mathrm{C})$ | $7(\mathrm{~A})$ |

The numbers are the productivity scores, and the letters represent the treatment. We entered the data into Stata:

```
. use http://www.stata-press.com/data/r13/music, clear
. list
```

|  | id | seq | day1 | day2 | day3 | day4 | day5 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | 1. | 1 | dcabe | 18 | 17 | 14 | 21 |
| 2. | 2 | cbead | 13 | 34 | 21 | 16 | 15 |
| 3. | 3 | adbec | 7 | 29 | 32 | 27 | 13 |
| 4. | 4 | eacdb | 17 | 13 | 24 | 31 | 25 |
| 5. | 5 | bedca | 21 | 26 | 26 | 31 | 7 |

We reshape these data with pkshape:

```
. pkshape id seq day1 day2 day3 day4 day5
. list, sep(0)
```

|  | id | sequence | outcome | treat | carry | period |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | 3 | 1 | 7 | 1 | 0 | 1 |
| 2. | 5 | 2 | 21 | 3 | 0 | 1 |
| 3. | 2 | 3 | 13 | 5 | 0 | 1 |
| 4. | 1 | 4 | 18 | 2 | 0 | 1 |
| 5. | 4 | 5 | 17 | 4 | 0 | 1 |
| 6. | 3 | 1 | 29 | 2 | 1 | 2 |
| 7. | 5 | 2 | 26 | 4 | 3 | 2 |
| 8. | 2 | 3 | 34 | 3 | 5 | 2 |
| 9. | 1 | 4 | 17 | 5 | 2 | 2 |
| 10. | 4 | 5 | 13 | 1 | 4 | 2 |
| 11. | 3 | 1 | 32 | 3 | 2 | 3 |
| 12. | 5 | 2 | 26 | 2 | 4 | 3 |
| 13. | 2 | 3 | 21 | 4 | 3 | 3 |
| 14. | 1 | 4 | 14 | 1 | 5 | 3 |
| 15. | 4 | 5 | 24 | 5 | 1 | 3 |
| 16. | 3 | 1 | 27 | 4 | 3 | 4 |
| 17. | 5 | 2 | 31 | 5 | 2 | 4 |
| 18. | 2 | 3 | 16 | 1 | 4 | 4 |
| 19. | 1 | 4 | 21 | 3 | 1 | 4 |
| 20. | 4 | 5 | 31 | 2 | 5 | 4 |
| 21. | 3 | 1 | 13 | 5 | 4 | 5 |
| 22. | 5 | 2 | 7 | 1 | 5 | 5 |
| 23. | 2 | 3 | 15 | 2 | 1 | 5 |
| 24. | 1 | 4 | 17 | 4 | 3 | 5 |
| 25. | 4 | 5 | 25 | 3 | 2 | 5 |

Here the sequence variable is a string variable that specifies how the treatments were applied, so the order option is not used. When the sequence variable is a string and the order is specified, the arguments from the order option are used. We could now produce an ANOVA table:


## Example 3

Consider the Latin-square crossover example published in Kutner et al. (2005). The example is about apple sales given different methods for displaying apples.

| Pattern | Store | Week 1 | Week 2 | Week 3 |
| ---: | :---: | :---: | :---: | :---: |
| 1 | 1 | $9(\mathrm{~B})$ | $12(\mathrm{C})$ | $15(\mathrm{~A})$ |
|  | 2 | 4(B) | $12(\mathrm{C})$ | $9(\mathrm{~A})$ |
| 2 | 1 | $12(\mathrm{~A})$ | $14(\mathrm{~B})$ | $3(\mathrm{C})$ |
|  | 2 | $13(\mathrm{~A})$ | $14(\mathrm{~B})$ | $3(\mathrm{C})$ |
| 3 | 1 | $7(\mathrm{C})$ | $18(\mathrm{~A})$ | $6(\mathrm{~B})$ |
|  | 2 | $5(\mathrm{C})$ | $20(\mathrm{~A})$ | $4(\mathrm{~B})$ |

We entered the data into Stata:

```
. use http://www.stata-press.com/data/r13/applesales, clear
. list, sep(2)
```

|  | id | seq | p1 | p2 | p3 | square |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
|  | 2. | 1 | 1 | 9 | 12 | 15 |

Now the data can be reorganized using descriptive names for the outcome variables.

```
. pkshape id seq p1 p2 p3, order(bca abc cab) seq(pattern) period(order)
```

> treat(displays)
. anova outcome pattern order display id|pattern


These are the same results reported by Kutner et al. (2005).

## Example 4

We continue with example 1 from [R] pkcollapse; the data are . use http://www.stata-press.com/data/r13/pkdata2, clear . list, sep(4) abbrev(10)

|  | id | seq | auc_concA | auc_concB |
| :--- | ---: | ---: | ---: | ---: |
| 1. | 1 | 1 | 150.9643 | 218.5551 |
| 2. | 2 | 1 | 146.7606 | 133.3201 |
| 3. | 3 | 1 | 160.6548 | 126.0635 |
| 4. | 4 | 1 | 157.8622 | 96.17461 |
| 5. | 5 | 1 | 133.6957 | 188.9038 |
| 6. | 7 | 1 | 160.639 | 223.6922 |
| 7. | 8 | 1 | 131.2604 | 104.0139 |
| 8. | 9 | 1 | 168.5186 | 237.8962 |
| 9. | 10 | 2 | 137.0627 | 139.7382 |
| 10. | 12 | 2 | 153.4038 | 202.3942 |
| 11. | 13 | 2 | 163.4593 | 136.7848 |
| 12. | 14 | 2 | 146.0462 | 104.5191 |
| 13. | 15 | 2 | 158.1457 | 165.8654 |
| 14. | 18 | 2 | 147.1977 | 139.235 |
| 15. | 19 | 2 | 164.9988 | 166.2391 |
| 16. | 20 | 2 | 145.3823 | 158.5146 |
|  |  |  |  |  |

pkshape id seq auc_concA auc_concB, order (ab ba)
sort period id
. list, sep(4)

|  | id | sequence | outcome | treat | carry | period |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | 1 | 1 | 150.9643 | 1 | 0 | 1 |
| 2. | 2 | 1 | 146.7606 | 1 | 0 | 1 |
| 3. | 3 | 1 | 160.6548 | 1 | 0 | 1 |
| 4. | 4 | 1 | 157.8622 | 1 | 0 | 1 |
| 5. | 5 | 1 | 133.6957 | 1 | 0 | 1 |
| 6. | 7 | 1 | 160.639 | 1 | 0 | 1 |
| 7. | 8 | 1 | 131.2604 | 1 | 0 | 1 |
| 8. | 9 | 1 | 168.5186 | 1 | 0 | 1 |
| 9. | 10 | 2 | 137.0627 | 2 | 0 | 1 |
| 10. | 12 | 2 | 153.4038 | 2 | 0 | 1 |
| 11. | 13 | 2 | 163.4593 | 2 | 0 | 1 |
| 12. | 14 | 2 | 146.0462 | 2 | 0 | 1 |
| 13. | 15 | 2 | 158.1457 | 2 | 0 | 1 |
| 14. | 18 | 2 | 147.1977 | 2 | 0 | 1 |
| 15. | 19 | 2 | 164.9988 | 2 | 0 | 1 |
| 16. | 20 | 2 | 145.3823 | 2 | 0 | 1 |
| 17. | 1 | 1 | 218.5551 | 2 | 1 | 2 |
| 18. | 2 | 1 | 133.3201 | 2 | 1 | 2 |
| 19. | 3 | 1 | 126.0635 | 2 | 1 | 2 |
| 20. | 4 | 1 | 96.17461 | 2 | 1 | 2 |
| 21. | 5 | 1 | 188.9038 | 2 | 1 | 2 |
| 22. | 7 | 1 | 223.6922 | 2 | 1 | 2 |
| 23. | 8 | 1 | 104.0139 | 2 | 1 | 2 |
| 24. | 9 | 1 | 237.8962 | 2 | 1 | 2 |
| 25. | 10 | 2 | 139.7382 | 1 | 2 | 2 |
| 26. | 12 | 2 | 202.3942 | 1 | 2 | 2 |
| 27. | 13 | 2 | 136.7848 | 1 | 2 | 2 |
| 28. | 14 | 2 | 104.5191 | 1 | 2 | 2 |
| 29. | 15 | 2 | 165.8654 | 1 | 2 | 2 |
| 30. | 18 | 2 | 139.235 | 1 | 2 | 2 |
| 31. | 19 | 2 | 166.2391 | 1 | 2 | 2 |
| 32. | 20 | 2 | 158.5146 | 1 | 2 | 2 |

We call the resulting dataset pkdata3. We conduct equivalence testing on the data in $[\mathrm{R}]$ pkequiv, and we fit an ANOVA model to these data in the third example of $[R]$ pkcross.

## References

Chow, S.-C., and J.-P. Liu. 2009. Design and Analysis of Bioavailability and Bioequivalence Studies. 3rd ed. Boca Raton, FL: Chapman \& Hall/CRC.

Kutner, M. H., C. J. Nachtsheim, J. Neter, and W. Li. 2005. Applied Linear Statistical Models. 5th ed. New York: McGraw-Hill/Irwin.

## Also see

[R] pk - Pharmacokinetic (biopharmaceutical) data

## Title

pksumm - Summarize pharmacokinetic data

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Methods and formulas | Also see |  |

## Syntax

pksumm id time concentration [if] [in] [, options]
options Description

Main
trapezoid use trapezoidal rule to calculate AUC; default is cubic splines
fit(\#) use \# points to estimate AUC; default is fit (3)
notimechk do not check whether follow-up time for all subjects is the same
nodots suppress the dots during calculation
graph graph the distribution of statistic
stat (statistic) graph the specified statistic; default is stat (auc)
Histogram, Density plots, Y axis, X axis, Titles, Legend, Overall
histogram_options any option other than by () documented in [R] histogram
statistic Description
auc area under the concentration-time curve $\left(\mathrm{AUC}_{0}, \infty\right)$; the default
aucline area under the concentration-time curve from 0 to $\infty$ using a linear extension
aucexp area under the concentration-time curve from 0 to $\infty$ using an exponential extension
auclog area under the log-concentration-time curve extended with a linear fit
half
ke elimination rate
cmax maximum concentration
tmax time at last concentration
tomc time of maximum concentration

## Menu

Statistics $>$ Epidemiology and related $>$ Other $>$ Summarize pharmacokinetic data

## Description

pksumm obtains summary measures based on the first four moments from the empirical distribution of each pharmacokinetic measurement and tests the null hypothesis that the distribution of that measurement is normally distributed.
pksumm is one of the pk commands. Please read $[\mathrm{R}] \mathrm{pk}$ before reading this entry.

## Options

$\qquad$ Main
trapezoid specifies that the trapezoidal rule be used to calculate the AUC. The default is cubic splines, which give better results for most situations. When the curve is irregular, the trapezoidal rule may give better results.
fit(\#) specifies the number of points, counting back from the last time measurement, to use in fitting the extension to estimate the $\mathrm{AUC}_{0, \infty}$. The default is fit(3), the last three points. This default should be viewed as a minimum; the appropriate number of points will depend on the data.
notimechk suppresses the check that the follow-up time for all subjects is the same. By default, pksumm expects the maximum follow-up time to be equal for all subjects.
nodots suppresses the progress dots during calculation. By default, a period is displayed for every call to calculate the pharmacokinetic measures.
graph requests a graph of the distribution of the statistic specified with stat().
stat (statistic) specifies the statistic that pksumm should graph. The default is stat(auc). If the graph option is not specified, this option is ignored.

Histogram, Density plots, Y axis, X axis, Titles, Legend, Overall
histogram_options are any of the options documented in $[\mathrm{R}]$ histogram, excluding by (). For pksumm, fraction is the default, not density.

## Remarks and examples

pksumm produces summary statistics for the distribution of nine common pharmacokinetic measurements. If there are more than eight subjects, pksumm also computes a test for normality on each measurement. The nine measurements summarized by pksumm are listed above and are described in Methods and formulas of [R] pkexamine.
> Example 1
We demonstrate the use of pksumm on a variation of the data described in [R] pk. We have drug concentration data on 15 subjects, each measured at 13 time points over a 32 -hour period. A few of the records are

```
. use http://www.stata-press.com/data/r13/pksumm
```

. list, sep(0)

| id | time | conc |
| :---: | :---: | ---: |
| 1 | 0 | 0 |
| 1 | .5 | 3.073403 |
| 1 | 1 | 5.188444 |
| 1 | 1.5 | 5.898577 |
| 1 | 2 | 5.096378 |
| 1 | 3 | 6.094085 |


| 183. | 15 | 0 | 0 |
| :--- | ---: | ---: | ---: |
| 184. | 15 | .5 | 3.86493 |
| 185. | 15 | 1 | 6.432444 |
| 186. | 15 | 1.5 | 6.969195 |
| 187. | 15 | 2 | 6.307024 |
| 188. | 15 | 3 | 6.509584 |
| 189. | 15 | 4 | 6.555091 |
| 190. | 15 | 6 | 7.318319 |
| 191. | 15 | 8 | 5.329813 |
| 192. | 15 | 12 | 5.411624 |
| 193. | 15 | 16 | 3.891397 |
| 194. | 15 | 24 | 5.167516 |
| 195. | 15 | 32 | 2.649686 |

We can use pksumm to view the summary statistics for all the pharmacokinetic parameters.

| Summary statistics for the pharmacokinetic measures |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Number of | servations | $=15$ |
| Measure | Mean | Median | Variance | Skewness | Kurtosis | p -value |
| auc | 150.74 | 150.96 | 123.07 | -0.26 | 2.10 | 0.69 |
| aucline | 408.30 | 214.17 | 188856.87 | 2.57 | 8.93 | 0.00 |
| aucexp | 691.68 | 297.08 | 762679.94 | 2.56 | 8.87 | 0.00 |
| auclog | 688.98 | 297.67 | 797237.24 | 2.59 | 9.02 | 0.00 |
| half | 94.84 | 29.39 | 18722.13 | 2.26 | 7.37 | 0.00 |
| ke | 0.02 | 0.02 | 0.00 | 0.89 | 3.70 | 0.09 |
| cmax | 7.36 | 7.42 | 0.42 | -0.60 | 2.56 | 0.44 |
| tomc | 3.47 | 3.00 | 7.62 | 2.17 | 7.18 | 0.00 |
| tmax | 32.00 | 32.00 | 0.00 | . | . | . |

For the 15 subjects, the mean $\mathrm{AUC}_{0, t_{\text {max }}}$ is 150.74 , and $\sigma^{2}=123.07$. The skewness of -0.26 indicates that the distribution is slightly skewed left. The $p$-value of 0.69 for the $\chi^{2}$ test of normality indicates that we cannot reject the null hypothesis that the distribution is normal.
If we were to consider any of the three variants of the $\mathrm{AUC}_{0, \infty}$, we would see that there is huge variability and that the distribution is heavily skewed. A skewness different from 0 and a kurtosis different from 3 are expected because the distribution of the $\mathrm{AUC}_{0, \infty}$ is not normal.

We now graph the distribution of $\mathrm{AUC}_{0, t_{\text {max }}}$ by specifying the graph option.
. pksumm id time conc, graph bin(20)

Summary statistics for the pharmacokinetic measures

|  |  | Number of observations $=$ |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Measure | Mean | Median | Variance | Skewness | Kurtosis | p-value |
| auc | 150.74 | 150.96 | 123.07 | -0.26 | 2.10 | 0.69 |
| aucline | 408.30 | 214.17 | 188856.87 | 2.57 | 8.93 | 0.00 |
| aucexp | 691.68 | 297.08 | 762679.94 | 2.56 | 8.87 | 0.00 |
| auclog | 688.98 | 297.67 | 797237.24 | 2.59 | 9.02 | 0.00 |
| half | 94.84 | 29.39 | 18722.13 | 2.26 | 7.37 | 0.00 |
| ke | 0.02 | 0.02 | 0.00 | 0.89 | 3.70 | 0.09 |
| cmax | 7.36 | 7.42 | 0.42 | -0.60 | 2.56 | 0.44 |
| tomc | 3.47 | 3.00 | 7.62 | 2.17 | 7.18 | 0.00 |
| tmax | 32.00 | 32.00 | 0.00 | . | . | . |


graph, by default, plots $\mathrm{AUC}_{0, t_{\max }}$. To plot a graph of one of the other pharmacokinetic measurements, we need to specify the stat () option. For example, we can ask Stata to produce a plot of the $\mathrm{AUC}_{0, \infty}$ using the $\log$ extension:



## Methods and formulas

The $\chi^{2}$ test for normality is conducted with sktest; see $[\mathrm{R}]$ sktest for more information on the test of normality.

The statistics reported by pksumm are identical to those reported by summarize and sktest; see $[R]$ summarize and $[R]$ sktest.

## Also see

[R] pk - Pharmacokinetic (biopharmaceutical) data

## Title

poisson - Poisson regression

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

$$
\text { poisson depvar }[\text { indepvars }][\text { if }][\text { in }][\text { weight }][\text {, options }]
$$

options Description

Model

## noconstant

 exposure( varname $_{e}$ )offset ( varname $_{o}$ ) constraints (constraints)
collinear

## SE/Robust <br> vce (vcetype)

Reporting
level(\#)
irr
nocnsreport
display_options

Maximization
maximize_options
coeflegend
suppress constant term
include $\ln \left(\right.$ varname $\left._{e}\right)$ in model with coefficient constrained to 1 include varname $_{o}$ in model with coefficient constrained to 1 apply specified linear constraints
keep collinear variables
vcetype may be oim, robust, cluster clustvar, opg, bootstrap, or jackknife
indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
depvar, indepvars, varname $e_{e}$, and varname $_{o}$ may contain time-series operators; see [U] 11.4.4 Time-series varlists.
bootstrap, by, fp, jackknife, mfp, mi estimate, nestreg, rolling, statsby, stepwise, and svy are allowed; see [U] 11.1.10 Prefix commands.
vce(bootstrap) and vce(jackknife) are not allowed with the mi estimate prefix; see [MI] mi estimate.
Weights are not allowed with the bootstrap prefix; see $[R]$ bootstrap.
vce() and weights are not allowed with the svy prefix; see [SVY] svy.
fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Count outcomes $>$ Poisson regression

## Description

poisson fits a Poisson regression of depvar on indepvars, where depvar is a nonnegative count variable.

If you have panel data, see [XT] xtpoisson.

## Options

> Model
> noconstant, exposure (varname $e$ ), offset ( varname $_{o}$ ), constraints (constraints), collinear; see $[R]$ estimation options.

SE/Robust
vce (vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

Reporting
level(\#); see [R] estimation options.
irr reports estimated coefficients transformed to incidence-rate ratios, that is, $e^{\beta_{i}}$ rather than $\beta_{i}$. Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated or stored. irr may be specified at estimation or when replaying previously estimated results.
nocnsreport; see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% fmt), pformat (\%fmt), sformat (\%fmt), and nolstretch; see [R] estimation options.

Maximization
maximize_options: difficult, technique (algorithm_spec), iterate(\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance(\#), nonrtolerance, and from (init_specs); see [R] maximize. These options are seldom used.

Setting the optimization type to technique (bhhh) resets the default vcetype to vce (opg).
The following option is available with poisson but is not shown in the dialog box:
coeflegend; see $[R]$ estimation options.

## Remarks and examples

The basic idea of Poisson regression was outlined by Coleman (1964, 378-379). See Cameron and Trivedi (2013; 2010, chap. 17) and Johnson, Kemp, and Kotz (2005, chap. 4) for information about the Poisson distribution. See Cameron and Trivedi (2013), Long (1997, chap. 8), Long and Freese (2014, chap. 9), McNeil (1996, chap. 6), and Selvin (2011, chap. 6) for an introduction to Poisson regression. Also see Selvin (2004, chap. 5) for a discussion of the analysis of spatial distributions, which includes a discussion of the Poisson distribution. An early example of Poisson regression was Cochran (1940).

Poisson regression fits models of the number of occurrences (counts) of an event. The Poisson distribution has been applied to diverse events, such as the number of soldiers kicked to death by horses in the Prussian army (von Bortkiewicz 1898); the pattern of hits by buzz bombs launched against London during World War II (Clarke 1946); telephone connections to a wrong number (Thorndike 1926); and disease incidence, typically with respect to time, but occasionally with respect to space. The basic assumptions are as follows:

1. There is a quantity called the incidence rate that is the rate at which events occur. Examples are 5 per second, 20 per 1,000 person-years, 17 per square meter, and 38 per cubic centimeter.
2. The incidence rate can be multiplied by exposure to obtain the expected number of observed events. For example, a rate of 5 per second multiplied by 30 seconds means that 150 events are expected; a rate of 20 per 1,000 person-years multiplied by 2,000 person-years means that 40 events are expected; and so on.
3. Over very small exposures $\epsilon$, the probability of finding more than one event is small compared with $\epsilon$.
4. Nonoverlapping exposures are mutually independent.

With these assumptions, to find the probability of $k$ events in an exposure of size $E$, you divide $E$ into $n$ subintervals $E_{1}, E_{2}, \ldots, E_{n}$, and approximate the answer as the binomial probability of observing $k$ successes in $n$ trials. If you let $n \rightarrow \infty$, you obtain the Poisson distribution.

In the Poisson regression model, the incidence rate for the $j$ th observation is assumed to be given by

$$
r_{j}=e^{\beta_{0}+\beta_{1} x_{1, j}+\cdots+\beta_{k} x_{k, j}}
$$

If $E_{j}$ is the exposure, the expected number of events, $C_{j}$, will be

$$
\begin{aligned}
C_{j} & =E_{j} e^{\beta_{0}+\beta_{1} x_{1, j}+\cdots+\beta_{k} x_{k, j}} \\
& =e^{\ln \left(E_{j}\right)+\beta_{0}+\beta_{1} x_{1, j}+\cdots+\beta_{k} x_{k, j}}
\end{aligned}
$$

This model is fit by poisson. Without the exposure() or offset() options, $E_{j}$ is assumed to be 1 (equivalent to assuming that exposure is unknown), and controlling for exposure, if necessary, is your responsibility.

Comparing rates is most easily done by calculating incidence-rate ratios (IRRs). For instance, what is the relative incidence rate of chromosome interchanges in cells as the intensity of radiation increases; the relative incidence rate of telephone connections to a wrong number as load increases; or the relative incidence rate of deaths due to cancer for females relative to males? That is, you want to hold all the $x$ 's in the model constant except one, say, the $i$ th. The IRR for a one-unit change in $x_{i}$ is

$$
\frac{e^{\ln (E)+\beta_{1} x_{1}+\cdots+\beta_{i}\left(x_{i}+1\right)+\cdots+\beta_{k} x_{k}}}{e^{\ln (E)+\beta_{1} x_{1}+\cdots+\beta_{i} x_{i}+\cdots+\beta_{k} x_{k}}}=e^{\beta_{i}}
$$

More generally, the IRR for a $\Delta x_{i}$ change in $x_{i}$ is $e^{\beta_{i} \Delta x_{i}}$. The lincom command can be used after poisson to display incidence-rate ratios for any group relative to another; see $[\mathrm{R}]$ lincom.

## > Example 1

Chatterjee and Hadi $(2012,174)$ give the number of injury incidents and the proportion of flights for each airline out of the total number of flights from New York for nine major U.S. airlines in one year:

```
. use http://www.stata-press.com/data/r13/airline
```

. list

|  | airline | injuries | n | XYZowned |
| :---: | :---: | :---: | :---: | :---: |
| 1. | 1 | 11 | 0.0950 | 1 |
| 2. | 2 | 7 | 0.1920 | 0 |
| 3. | 3 | 7 | 0.0750 | 0 |
| 4. | 4 | 19 | 0.2078 | 0 |
| 5. | 5 | 9 | 0.1382 | 0 |
| 6. | 6 | 4 | 0.0540 | 1 |
| 7. | 7 | 3 | 0.1292 | 0 |
| 8. | 8 | 1 | 0.0503 | 0 |
| 9. | 9 | 3 | 0.0629 | 1 |

To their data, we have added a fictional variable, XYZowned. We will imagine that an accusation is made that the airlines owned by XYZ Company have a higher injury rate.

```
. poisson injuries XYZowned, exposure(n) irr
Iteration 0: log likelihood = -23.027197
Iteration 1: log likelihood = -23.027177
Iteration 2: log likelihood = -23.027177
\begin{tabular}{llll} 
& Number of obs & \(=\) & 9 \\
Poisson regression & LR chi2 \((1)\) & \(=\) & 1.77 \\
& Prob \(>\) chi2 & \(=\) & 0.1836 \\
Log likelihood \(=-23.027177\) & & Pseudo R2 & \(=\)
\end{tabular} 0.0370
```

| injuries | IRR | Std. Err. | $z$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| XYZowned | 1.463467 | .406872 | 1.37 | 0.171 | .8486578 | 2.523675 |
| _cons | 58.04416 | 8.558145 | 27.54 | 0.000 | 43.47662 | 77.49281 |
| ln(n) | 1 | (exposure) |  |  |  |  |

We specified irr to see the IRRs rather than the underlying coefficients. We estimate that XYZ Airlines' injury rate is 1.46 times larger than that for other airlines, but the $95 \%$ confidence interval is 0.85 to 2.52; we cannot even reject the hypothesis that XYZ Airlines has a lower injury rate.

## Technical note

In example 1, we assumed that each airline's exposure was proportional to its fraction of flights out of New York. What if "large" airlines, however, also used larger planes, and so had even more passengers than would be expected, given this measure of exposure? A better measure would be each airline's fraction of passengers on flights out of New York, a number that we do not have. Even so, we suppose that n represents this number to some extent, so a better estimate of the effect might be


Here rather than specifying the exposure() option, we explicitly included the variable that would normalize for exposure in the model. We did not specify the irr option, so we see coefficients rather than IRRs. We started with the model

$$
\text { rate }=e^{\beta_{0}+\beta_{1} \text { XYZowned }}
$$

The observed counts are therefore

$$
\text { count }=n e^{\beta_{0}+\beta_{1} \text { XYZowned }}=e^{\ln (n)+\beta_{0}+\beta_{1} \mathrm{XYZowned}}
$$

which amounts to constraining the coefficient on $\ln (\mathrm{n})$ to 1 . This is what was estimated when we specified the exposure ( $n$ ) option. In the above model, we included the normalizing exposure ourselves and, rather than constraining the coefficient to be 1 , estimated the coefficient.

The estimated coefficient is 1.42 , a respectable distance away from 1 , and is consistent with our speculation that larger airlines also use larger airplanes. With this small amount of data, however, we also have a wide confidence interval that includes 1.

Our estimated coefficient on XYZowned is now 0.684 , and the implied IRR is $e^{0.684} \approx 1.98$ (which we could also see by typing poisson, irr). The $95 \%$ confidence interval for the coefficient still includes 0 (the interval for the IRR includes 1 ), so although the point estimate is now larger, we still cannot be certain of our results.

Our expert opinion would be that, although there is not enough evidence to support the charge, there is enough evidence to justify collecting more data.

## > Example 2

In a famous age-specific study of coronary disease deaths among male British doctors, Doll and Hill (1966) reported the following data (reprinted in Rothman, Greenland, and Lash [2008, 264]):

|  | Smokers |  | Nonsmokers |  |
| :--- | ---: | ---: | ---: | ---: |
| Age | Deaths | Person-years | Deaths | Person-years |
| $35-44$ | 32 | 52,407 | 2 | 18,790 |
| $45-54$ | 104 | 43,248 | 12 | 10,673 |
| $55-64$ | 206 | 28,612 | 28 | 5,710 |
| $65-74$ | 186 | 12,663 | 28 | 2,585 |
| $75-84$ | 102 | 5,317 | 31 | 1,462 |

The first step is to enter these data into Stata, which we have done:
. use http://www.stata-press.com/data/r13/dollhill3, clear
. list

|  | agecat | smokes | deaths | pyears |
| ---: | ---: | ---: | ---: | ---: |
| 1. | $35-44$ | 1 | 32 | 52,407 |
| 2. | $45-54$ | 1 | 104 | 43,248 |
| 3. | $55-64$ | 1 | 206 | 28,612 |
| 4. | $65-74$ | 1 | 186 | 12,663 |
| 5. | $75-84$ | 1 | 102 | 5,317 |
| 6. | $35-44$ | 0 | 2 | 18,790 |
| 7. | $45-54$ | 0 | 12 | 10,673 |
| 8. | $55-64$ | 0 | 28 | 5,710 |
| 9. | $65-74$ | 0 | 28 | 2,585 |
| 10. | $75-84$ | 0 | 31 | 1,462 |
|  |  |  |  |  |

The most "natural" analysis of these data would begin by introducing indicator variables for each age category and one indicator for smoking:


In the above, we specified irr to obtain IRRs. We estimate that smokers have 1.43 times the mortality rate of nonsmokers. See, however, example 1 in [R] poisson postestimation.

## Stored results

poisson stores the following in e():
Scalars

```
e(N)
e(k)
e(k_eq)
e(k_eq_model)
e(k_dv)
e(df_m)
e(r2_p)
e(ll)
e(ll_0)
e(N_clust)
e(chi2)
e(p)
e(rank)
e(ic)
e(rc)
e(converged)
```

Macros

```
e(cmd)
    e(cmdline)
    e(depvar)
    e(wtype)
    e(wexp)
    e(title)
    e(clustvar)
    e(offset)
    e(chi2type)
    e(vce)
    e(vcetype)
    e(opt)
    e(which)
    e(ml_method)
    e(user)
    e(technique)
    e(properties)
    e(estat_cmd)
    e(predict)
    e(asbalanced)
    e(asobserved)
```

Matrices

```
e(b)
e(Cns)
e(ilog)
e(gradient)
e(V)
e(V_modelbased)
```

Functions
e(sample)
number of observations
number of parameters
number of equations in e(b)
number of equations in overall model test
number of dependent variables
model degrees of freedom
pseudo- $R$-squared
log likelihood
log likelihood, constant-only model
number of clusters
$\chi^{2}$
significance
rank of $e(V)$
number of iterations
return code
1 if converged, 0 otherwise
poisson
command as typed
name of dependent variable
weight type
weight expression
title in estimation output
name of cluster variable
linear offset variable
Wald or LR; type of model $\chi^{2}$ test
vcetype specified in vce()
title used to label Std. Err.
type of optimization
max or min; whether optimizer is to perform maximization or minimization type of ml method
name of likelihood-evaluator program
maximization technique
b V
program used to implement estat
program used to implement predict
factor variables fvset as asbalanced
factor variables fvset as asobserved
coefficient vector
constraints matrix
iteration $\log$ (up to 20 iterations)
gradient vector
variance-covariance matrix of the estimators
model-based variance
marks estimation sample

## Methods and formulas

The $\log$ likelihood (with weights $w_{j}$ and offsets) is given by

$$
\begin{aligned}
\operatorname{Pr}(Y=y) & =\frac{e^{-\lambda} \lambda^{y}}{y!} \\
\xi_{j} & =\mathbf{x}_{j} \boldsymbol{\beta}+\operatorname{offset}_{j} \\
f\left(y_{j}\right) & =\frac{e^{-\exp \left(\xi_{j}\right)} e^{\xi_{j} y_{j}}}{y_{j}!} \\
\ln L & =\sum_{j=1}^{n} w_{j}\left\{-e^{\xi_{j}}+\xi_{j} y_{j}-\ln \left(y_{j}!\right)\right\}
\end{aligned}
$$

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce (robust) and vce(cluster clustvar), respectively. See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas.
poisson also supports estimation with survey data. For details on VCEs with survey data, see [SVY] variance estimation.

Siméon-Denis Poisson (1781-1840) was a French mathematician and physicist who contributed to several fields: his name is perpetuated in Poisson brackets, Poisson's constant, Poisson's differential equation, Poisson's integral, and Poisson's ratio. Among many other results, he produced a version of the law of large numbers. His rather misleadingly titled Recherches sur la probabilité des jugements embraces a complete treatise on probability, as the subtitle indicates, including what is now known as the Poisson distribution. That, however, was discovered earlier by the Huguenot-British mathematician Abraham de Moivre (1667-1754).

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## Also see

$[R]$ poisson postestimation - Postestimation tools for poisson
[ R ] glm - Generalized linear models
[R] nbreg - Negative binomial regression
[R] tpoisson - Truncated Poisson regression
[R] zip — Zero-inflated Poisson regression
[ME] mepoisson - Multilevel mixed-effects Poisson regression
[MI] estimation - Estimation commands for use with mi estimate
[SVY] svy estimation - Estimation commands for survey data
[XT] xtpoisson - Fixed-effects, random-effects, and population-averaged Poisson models
[U] 20 Estimation and postestimation commands

## Title

poisson postestimation - Postestimation tools for poisson

| Description | Syntax for predict | Menu for predict |
| :--- | :--- | :--- |
| Options for predict | Syntax for estat gof | Menu for estat |
| Remarks and examples | Methods and formulas | Also see |

## Description

The following postestimation command is of special interest after poisson:
Command Description
estat gof goodness-of-fit test
estat gof is not appropriate after the svy prefix.

The following standard postestimation commands are also available:

| Command | Description |
| :--- | :--- |
| contrast | contrasts and ANOVA-style joint tests of estimates |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample |
| estat vce | variance-covariance matrix of the estimators (VCE) |
| estat (svy) | postestimation statistics for survey data |
| estimates | cataloging estimation results |
| forecast | dynamic forecasts and simulations <br> point estimates, standard errors, testing, and inference for linear combinations <br> of coefficients |
| lincom | link test for model specification |
| linktest | likelihood-ratio test |
| marginal means, predictive margins, marginal effects, and average marginal |  |
| margins | effects |

[^60]
## Special-interest postestimation command

estat gof performs a goodness-of-fit test of the model. Both the deviance statistic and the Pearson statistic are reported. If the tests are significant, the Poisson regression model is inappropriate. Then you could try a negative binomial model; see [R] nbreg.

## Syntax for predict

predict [type] newvar [if][in] [, statistic nooffset]
statistic Description
Main
n number of events; the default
ir incidence rate
$\operatorname{pr}(n) \quad$ probability $\operatorname{Pr}\left(y_{j}=n\right)$
$\operatorname{pr}(a, b) \quad$ probability $\operatorname{Pr}\left(a \leq y_{j} \leq b\right)$
$\mathrm{xb} \quad$ linear prediction
stdp standard error of the linear prediction
score $\quad$ first derivative of the $\log$ likelihood with respect to $\mathbf{x}_{j} \boldsymbol{\beta}$
These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
n , the default, calculates the predicted number of events, which is $\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}\right)$ if neither offset () nor exposure() was specified when the model was fit; $\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}+\right.$ offset $\left._{j}\right)$ if offset() was specified; or $\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}\right) \times$ exposure $_{j}$ if exposure() was specified.
ir calculates the incidence rate $\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}\right)$, which is the predicted number of events when exposure is 1 . Specifying ir is equivalent to specifying $n$ when neither offset() nor exposure() was specified when the model was fit.
$\operatorname{pr}(n)$ calculates the probability $\operatorname{Pr}\left(y_{j}=n\right)$, where $n$ is a nonnegative integer that may be specified as a number or a variable.
$\operatorname{pr}(a, b)$ calculates the probability $\operatorname{Pr}\left(a \leq y_{j} \leq b\right)$, where $a$ and $b$ are nonnegative integers that may be specified as numbers or variables;
$b$ missing ( $b \geq$.) means $+\infty$;
$\operatorname{pr}(20,$.$) calculates \operatorname{Pr}\left(y_{j} \geq 20\right)$;
$\operatorname{pr}(20, b)$ calculates $\operatorname{Pr}\left(y_{j} \geq 20\right)$ in observations for which $b \geq$. and calculates $\operatorname{Pr}\left(20 \leq y_{j} \leq b\right)$ elsewhere.
$\operatorname{pr}(., b)$ produces a syntax error. A missing value in an observation of the variable $a$ causes a missing value in that observation for $\mathrm{pr}(a, b)$.
xb calculates the linear prediction, which is $\mathbf{x}_{j} \boldsymbol{\beta}$ if neither offset () nor exposure() was specified; $\mathbf{x}_{j} \boldsymbol{\beta}+$ offset $_{j}$ if offset () was specified; or $\mathbf{x}_{j} \boldsymbol{\beta}+\ln$ (exposure $_{j}$ ) if exposure() was specified; see nooffset below.
stdp calculates the standard error of the linear prediction.
score calculates the equation-level score, $\partial \ln L / \partial\left(\mathbf{x}_{j} \boldsymbol{\beta}\right)$.
nooffset is relevant only if you specified offset() or exposure() when you fit the model. It modifies the calculations made by predict so that they ignore the offset or exposure variable; the linear prediction is treated as $\mathbf{x}_{j} \boldsymbol{\beta}$ rather than as $\mathbf{x}_{j} \boldsymbol{\beta}+$ offset $_{j}$ or $\mathbf{x}_{j} \boldsymbol{\beta}+\ln$ (exposure $\left.{ }_{j}\right)$. Specifying predict ..., nooffset is equivalent to specifying predict..., ir.

## Syntax for estat gof

estat gof

## Menu for estat

Statistics $>$ Postestimation $>$ Reports and statistics

## Remarks and examples

## $>$ Example 1

Continuing with example 2 of $[R]$ poisson, we use estat gof to determine whether the model fits the data well.

```
. use http://www.stata-press.com/data/r13/dollhill3
. poisson deaths smokes i.agecat, exp(pyears) irr
(output omitted)
. estat gof
    Deviance goodness-of-fit = 12.13244
    Prob > chi2(4) = 0.0164
    Pearson goodness-of-fit = 11.15533
    Prob > chi2(4) = 0.0249
```

The deviance goodness-of-fit test tells us that, given the model, we can reject the hypothesis that these data are Poisson distributed at the $1.64 \%$ significance level. The Pearson goodness-of-fit test tells us that we can reject the hypothesis at the $2.49 \%$ significance level.

So let us now back up and be more careful. We can most easily obtain the incidence-rate ratios within age categories by using ir; see [ST] epitab:
. ir deaths smokes pyears, by (agecat) nohet

| age category | IRR | [95\% Conf. Interval] | M-H Weight |  |
| ---: | ---: | ---: | ---: | ---: |
| $35-44$ | 5.736638 | 1.463557 | 49.40468 | 1.472169 (exact) |
| $45-54$ | 2.138812 | 1.173714 | 4.272545 | 9.624747 (exact) |
| $55-64$ | 1.46824 | .9863624 | 2.264107 | 23.34176 (exact) |
| $65-74$ | 1.35606 | .9081925 | 2.096412 | 23.25315 (exact) |
| $75-84$ | .9047304 | .6000757 | 1.399687 | 24.31435 (exact) |
|  |  |  |  | (exact) |

We find that the mortality incidence ratios are greatly different within age category, being highest for the youngest categories and actually dropping below 1 for the oldest. (In the last case, we might argue that those who smoke and who have not died by age 75 are self-selected to be particularly robust.)

Seeing this, we will now parameterize the smoking effects separately for each category, although we will begin by constraining the smoking effects on third and fourth age categories to be equivalent:

| . poisson deaths c.smokes\#agecat i.agecat, exposure(pyears) irr constraints (1) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Iteration 0: log likelihood = -31.95424 |  |  |  |  |  |  |
| Iteration 1: log likelihood = -27.796801 |  |  |  |  |  |  |
| Iteration 2: log likelihood $=-27.574177$ |  |  |  |  |  |  |
| Iteration 3: log likelihood = -27.572645 |  |  |  |  |  |  |
| Iteration 4: log likelihood = -27.572645 |  |  |  |  |  |  |
| Poisson regression |  |  |  | Number of obs = 10 |  |  |
|  |  |  |  | Wald | i2(8) | 632.14 |
| ( 1) [deaths]3.agecat\#c.smokes - [deaths]4.agecat\#c.smokes $=0$ |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| deaths | IRR | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf | Interval] |
| agecat\# <br> c.smokes |  |  |  |  |  |  |
| 35-44 | 5.736637 | 4.181256 | 2.40 | 0.017 | 1.374811 | 23.93711 |
| 45-54 | 2.138812 | . 6520701 | 2.49 | 0.013 | 1.176691 | 3.887609 |
| 55-64 | 1.412229 | . 2017485 | 2.42 | 0.016 | 1.067343 | 1.868557 |
| 65-74 | 1.412229 | . 2017485 | 2.42 | 0.016 | 1.067343 | 1.868557 |
| 75-84 | . 9047304 | . 1855513 | -0.49 | 0.625 | . 6052658 | 1.35236 |
| agecat |  |  |  |  |  |  |
| 45-54 | 10.5631 | 8.067701 | 3.09 | 0.002 | 2.364153 | 47.19623 |
| 55-64 | 47.671 | 34.37409 | 5.36 | 0.000 | 11.60056 | 195.8978 |
| 65-74 | 98.22765 | 70.85012 | 6.36 | 0.000 | 23.89324 | 403.8244 |
| 75-84 | 199.2099 | 145.3356 | 7.26 | 0.000 | 47.67693 | 832.3648 |
| _cons | . 0001064 | . 0000753 | -12.94 | 0.000 | . 0000266 | . 0004256 |
| ln(pyears) | 1 | (exposure) |  |  |  |  |

```
. estat gof
    Deviance goodness-of-fit = . 0774185
    Prob > chi2(1) = 0.7808
    Pearson goodness-of-fit = . 0773882
    Prob > chi2(1) = 0.7809
```

The goodness-of-fit is now small; we are no longer running roughshod over the data. Let us now consider simplifying the model. The point estimate of the incidence-rate ratio for smoking in age category 1 is much larger than that for smoking in age category 2 , but the confidence interval for smokes\#1.agecat is similarly wide. Is the difference real?

```
. test smokes#1.agecat = smokes#2.agecat
    ( 1) [deaths]1b.agecat#c.smokes - [deaths]2.agecat#c.smokes = 0
    chi2( 1) = 1.56
    Prob > chi2 = 0.2117
```

The point estimates of the incidence-rate ratio for smoking in the 35-44 age category is much larger than that for smoking in the $45-54$ age category, but there is insufficient data, and we may be observing random differences. With that success, might we also combine the smokers in the third and fourth categories with those in the first and second categories?
. test smokes\#2.agecat $=$ smokes\#3.agecat, accum
( 1) [deaths]1b.agecat\#c.smokes - [deaths]2.agecat\#c.smokes = 0
( 2) [deaths]2.agecat\#c.smokes - [deaths]3.agecat\#c.smokes = 0

$$
\begin{array}{rll}
\text { chi2 }(2) & = & 4.73 \\
\text { Prob }>\text { chi2 } & =0.0938
\end{array}
$$

Combining the first four categories may be overdoing it - the $9.38 \%$ significance level is enough to stop us, although others may disagree.

Thus we now fit our final model:
. constraint 2 smokes\#1.agecat = smokes\#2.agecat
. poisson deaths c.smokes\#agecat i.agecat, exposure (pyears) irr constraints(1/2)
Iteration 0: log likelihood $=-31.550722$
Iteration 1: log likelihood $=-28.525057$
Iteration 2: log likelihood $=-28.514535$
Iteration 3: log likelihood $=-28.514535$

| Poisson regression |  |  |  | Number of obs Wald chi2(7) |  | $\begin{array}{r} 10 \\ 642.25 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Log likelihood $=-28.514535$ |  |  |  | Prob > chi2 |  | 0.0000 |
| [deaths]3.agecat\#c.smokes - [deaths]4.agecat\#c.smokes = 0 |  |  |  |  |  |  |
| ( 2) [deaths]1b.agecat\#c.smokes - [deaths]2.agecat\#c.smokes |  |  |  |  |  |  |
| deaths | IRR | Std. Err. | z | $P>\|z\|$ | [95\% Conf | Interval] |
| agecat\# c.smokes |  |  |  |  |  |  |
| 35-44 | 2.636259 | . 7408403 | 3.45 | 0.001 | 1.519791 | 4.572907 |
| 45-54 | 2.636259 | . 7408403 | 3.45 | 0.001 | 1.519791 | 4.572907 |
| 55-64 | 1.412229 | . 2017485 | 2.42 | 0.016 | 1.067343 | 1.868557 |
| 65-74 | 1.412229 | . 2017485 | 2.42 | 0.016 | 1.067343 | 1.868557 |
| 75-84 | . 9047304 | . 1855513 | -0.49 | 0.625 | . 6052658 | 1.35236 |
| agecat |  |  |  |  |  |  |
| 45-54 | 4.294559 | . 8385329 | 7.46 | 0.000 | 2.928987 | 6.296797 |
| 55-64 | 23.42263 | 7.787716 | 9.49 | 0.000 | 12.20738 | 44.94164 |
| 65-74 | 48.26309 | 16.06939 | 11.64 | 0.000 | 25.13068 | 92.68856 |
| 75-84 | 97.87965 | 34.30881 | 13.08 | 0.000 | 49.24123 | 194.561 |
| _cons | . 0002166 | . 0000652 | -28.03 | 0.000 | . 0001201 | . 0003908 |
| ln(pyears) | 1 | (exposure) |  |  |  |  |

The above strikes us as a fair representation of the data. The probabilities of observing the deaths seen in these data are estimated using the following predict command:

```
. predict p, pr(0, deaths)
```

. list deaths p

|  | deaths | p |
| ---: | ---: | ---: |
| 1. | 32 | .6891766 |
| 2. | 104 | .4456625 |
| 3. | 206 | .5455328 |
| 4. | 186 | .4910622 |
| 5. | 102 | .5263011 |
| 6. | 2 | .227953 |
| 7. | 12 | .7981917 |
| 8. | 28 | .4772961 |
| 9. | 28 | .6227565 |
| 10. | 31 | .5475718 |

The probability $\operatorname{Pr}(y \leq$ deaths $)$ ranges from 0.23 to 0.80 .

## Methods and formulas

In the following, we use the same notation as in $[\mathrm{R}]$ poisson.
The equation-level scores are given by

$$
\operatorname{score}(\mathbf{x} \boldsymbol{\beta})_{j}=y_{j}-e^{\xi_{j}}
$$

The deviance (D) and Pearson (P) goodness-of-fit statistics are given by

$$
\begin{aligned}
\ln L_{\max } & =\sum_{j=1}^{n} w_{j}\left[-y_{j}\left\{\ln \left(y_{j}\right)-1\right\}-\ln \left(y_{j}!\right)\right] \\
\chi_{D}^{2} & =-2\left\{\ln L-\ln L_{\max }\right\} \\
\chi_{P}^{2} & =\sum_{j=1}^{n} \frac{w_{j}\left(y_{j}-e^{\xi_{j}}\right)^{2}}{e^{\xi_{j}}}
\end{aligned}
$$

## Also see

[R] poisson - Poisson regression
[U] 20 Estimation and postestimation commands

## Title

predict - Obtain predictions, residuals, etc., after estimation

| Syntax | Menu for predict | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Methods and formulas | Also see |  |

## Syntax

After single-equation (SE) models

```
predict [type] newvar [if] [in] [, single_options]
```

After multiple-equation (ME) models

```
predict [type] newvar [if] [in] [, multiple_options]
predict [type] {stub*|newvar 
```

single_options
Description
Main
xb calculate linear prediction
stdp calculate standard error of the prediction
score $\quad$ calculate first derivative of the $\log$ likelihood with respect to $\mathbf{x}_{j} \boldsymbol{\beta}$
Options
nooffset ignore any offset() or exposure() variable
other_options
command-specific options
multiple_options
Description
Main
equation (eqno [, eqno]) specify equations
xb
stdp
stddp
Options
nooffset
other_options
ignore any offset () or exposure() variable command-specific options

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Description

predict calculates predictions, residuals, influence statistics, and the like after estimation. Exactly what predict can do is determined by the previous estimation command; command-specific options are documented with each estimation command. Regardless of command-specific options, the actions of predict share certain similarities across estimation commands:

1. predict newvar creates newvar containing "predicted values"-numbers related to the $E\left(y_{j} \mid \mathbf{x}_{j}\right)$. For instance, after linear regression, predict newvar creates $\mathbf{x}_{j} \mathbf{b}$ and, after probit, creates the probability $\Phi\left(\mathbf{x}_{j} \mathbf{b}\right)$.
2. predict newvar, xb creates newvar containing $\mathbf{x}_{j} \mathbf{b}$. This may be the same result as option 1 (for example, linear regression) or different (for example, probit), but regardless, option xb is allowed.
3. predict newvar, stdp creates newvar containing the standard error of the linear prediction $\mathbf{x}_{j} \mathbf{b}$.
4. predict newvar, other_options may create newvar containing other useful quantities; see help or the reference manual entry for the particular estimation command to find out about other available options.
5. nooffset added to any of the above commands requests that the calculation ignore any offset or exposure variable specified by including the offset (varname ${ }_{o}$ ) or exposure (varname ${ }_{e}$ ) option when you fit the model.
predict can be used to make in-sample or out-of-sample predictions:
6. predict calculates the requested statistic for all possible observations, whether they were used in fitting the model or not. predict does this for standard options $1-3$ and generally does this for estimator-specific options 4.
7. predict newvar if e(sample), ... restricts the prediction to the estimation subsample.
8. Some statistics make sense only with respect to the estimation subsample. In such cases, the calculation is automatically restricted to the estimation subsample, and the documentation for the specific option states this. Even so, you can still specify if e(sample) if you are uncertain.
9. predict can make out-of-sample predictions even using other datasets. In particular, you can
. use ds1

- (fit a model)
. use two /* another dataset */
. predict yhat, ... /* fill in the predictions */


## Options

Main
xb calculates the linear prediction from the fitted model. That is, all models can be thought of as estimating a set of parameters $b_{1}, b_{2}, \ldots, b_{k}$, and the linear prediction is $\widehat{y}_{j}=b_{1} x_{1 j}+b_{2} x_{2 j}+$ $\cdots+b_{k} x_{k j}$, often written in matrix notation as $\widehat{\mathbf{y}}_{j}=\mathbf{x}_{j} \mathbf{b}$. For linear regression, the values $\widehat{y}_{j}$ are called the predicted values or, for out-of-sample predictions, the forecast. For logit and probit, for example, $\widehat{y}_{j}$ is called the logit or probit index.
$x_{1 j}, x_{2 j}, \ldots, x_{k j}$ are obtained from the data currently in memory and do not necessarily correspond to the data on the independent variables used to fit the model (obtaining $b_{1}, b_{2}, \ldots, b_{k}$ ).
stdp calculates the standard error of the linear prediction. Here the prediction means the same thing as the "index", namely, $\mathbf{x}_{j} \mathbf{b}$. The statistic produced by stdp can be thought of as the standard error of the predicted expected value, or mean index, for the observation's covariate pattern. The standard error of the prediction is also commonly referred to as the standard error of the fitted value. The calculation can be made in or out of sample.
stddp is allowed only after you have previously fit a multiple-equation model. The standard error of the difference in linear predictions ( $\mathbf{x}_{1 j} \mathbf{b}-\mathbf{x}_{2 j} \mathbf{b}$ ) between equations 1 and 2 is calculated. This option requires that equation $\left(\right.$ eqno $_{1}, e^{2}$ no $\left._{2}\right)$ be specified.
score calculates the equation-level score, $\partial \ln L / \partial\left(\mathbf{x}_{j} \boldsymbol{\beta}\right)$. Here $\ln L$ refers to the log-likelihood function.
scores is the ME model equivalent of the score option, resulting in multiple equation-level score variables. An equation-level score variable is created for each equation in the model; ancillary parameters-such as $\ln \sigma$ and $\operatorname{atanh} \rho$-make up separate equations.
equation (eqno[, eqno]) - synonym outcome() - is relevant only when you have previously fit a multiple-equation model. It specifies the equation to which you are referring.
equation() is typically filled in with one eqno-it would be filled in that way with options xb and stdp, for instance. equation(\#1) would mean the calculation is to be made for the first equation, equation(\#2) would mean the second, and so on. You could also refer to the equations by their names. equation(income) would refer to the equation named income and equation(hours) to the equation named hours.
If you do not specify equation(), results are the same as if you specified equation(\#1).
Other statistics, such as stddp, refer to between-equation concepts. In those cases, you might specify equation(\#1,\#2) or equation(income, hours). When two equations must be specified, equation() is required.

Options
nooffset may be combined with most statistics and specifies that the calculation should be made, ignoring any offset or exposure variable specified when the model was fit.
This option is available, even if it is not documented for predict after a specific command. If neither the offset $\left(\right.$ varname $_{o}$ ) option nor the exposure (varname $e_{e}$ ) option was specified when the model was fit, specifying nooffset does nothing.
other_options refers to command-specific options that are documented with each command.

## Remarks and examples

Remarks are presented under the following headings:

```
Estimation-sample predictions
Out-of-sample predictions
Residuals
Single-equation (SE) models
SE model scores
Multiple-equation (ME) models
ME model scores
```

Most of the examples are presented using linear regression, but the general syntax is applicable to all estimators.

You can think of any estimation command as estimating a set of coefficients $b_{1}, b_{2}, \ldots, b_{k}$ corresponding to the variables $x_{1}, x_{2}, \ldots, x_{k}$, along with a (possibly empty) set of ancillary statistics $\gamma_{1}, \gamma_{2}, \ldots, \gamma_{m}$. All estimation commands store the $b_{i} \mathrm{~s}$ and $\gamma_{i} \mathrm{~s}$. predict accesses that stored information and combines it with the data currently in memory to make various calculations. For instance, predict can calculate the linear prediction, $\widehat{y}_{j}=b_{1} x_{1 j}+b_{2} x_{2 j}+\cdots+b_{k} x_{k j}$. The data on which predict makes the calculation can be the same data used to fit the model or a different dataset-it does not matter. predict uses the stored parameter estimates from the model, obtains the corresponding values of $x$ for each observation in the data, and then combines them to produce the desired result.

## Estimation-sample predictions

> Example 1
We have a 74-observation dataset on automobiles, including the mileage rating (mpg), the car's weight (weight), and whether the car is foreign (foreign). We fit the model

| Source | SS | df | MS |  | Number of obs | $=\quad 22$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | F( 1, 20) | $=\quad 17.47$ |
| Model | 427.990298 | 1427 | 990298 |  | Prob > F | $=0.0005$ |
| Residual | 489.873338 | 2024. | 936669 |  | R-squared | $=0.4663$ |
|  |  |  |  |  | Adj R-squared | $=0.4396$ |
| Total | 917.863636 | 2143. | 077922 |  | Root MSE | $=4.9491$ |
| mpg | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Conf. | Interval] |
| weight | -. 010426 | . 0024942 | -4.18 | 0.000 | -. 0156287 | -. 0052232 |
| _cons | 48.9183 | 5.871851 | 8.33 | 0.000 | 36.66983 | 61.16676 |

If we were to type predict pmpg now, we would obtain the linear predictions for all 74 observations. To obtain the predictions just for the sample on which we fit the model, we could type

```
. predict pmpg if e(sample)
(option xb assumed; fitted values)
(52 missing values generated)
```

Here e (sample) is true only for foreign cars because we typed if foreign when we fit the model and because there are no missing values among the relevant variables. If there had been missing values, e(sample) would also account for those.

By the way, the if e(sample) restriction can be used with any Stata command, so we could obtain summary statistics on the estimation sample by typing

```
. summarize if e(sample)
    (output omitted)
```


## Out-of-sample predictions

By out-of-sample predictions, we mean predictions extending beyond the estimation sample. In the example above, typing predict pmpg would generate linear predictions using all 74 observations.
predict will work on other datasets, too. You can use a new dataset and type predict to obtain results for that sample.

## $>$ Example 2

Using the same auto dataset, assume that we wish to fit the model

$$
\mathrm{mpg}=\beta_{1} \text { weight }+\beta_{2} \ln (\text { weight })+\beta_{3} \text { foreign }+\beta_{4}
$$

We first create the $\ln$ (weight) variable, and then type the regress command:

```
. use http://www.stata-press.com/data/r13/auto, clear
(1978 Automobile Data)
. generate lnweight = ln(weight)
. regress mpg weight lnweight foreign
```

| Source | SS | df | MS | Number of obs | 74 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | F ( 3, 70) | 52.36 |
| Model | 1690.27997 | 3 | 563.426657 | Prob > F | 0.0000 |
| Residual | 753.179489 | 70 | 10.759707 | R -squared | 0.6918 |
|  |  |  |  | Adj R-squared | 0.6785 |
| Total | 2443.45946 | 73 | 33.4720474 | Root MSE | 3.2802 |


| mpg | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| weight | .003304 | .0038995 | 0.85 | 0.400 | -.0044734 | .0110813 |
| lnweight | -29.59133 | 11.52018 | -2.57 | 0.012 | -52.5676 | -6.615061 |
| foreign | -2.125299 | 1.052324 | -2.02 | 0.047 | -4.224093 | -.0265044 |
| _cons | 248.0548 | 80.37079 | 3.09 | 0.003 | 87.76035 | 408.3493 |

If we typed predict pmpg now, we would obtain predictions for all 74 cars in the current data. Instead, we are going to use a new dataset.

The dataset newautos.dta contains the make, weight, and place of manufacture of two cars, the Pontiac Sunbird and the Volvo 260. Let's use the dataset and create the predictions:

```
. use http://www.stata-press.com/data/r13/newautos, clear
(New Automobile Models)
. list
\begin{tabular}{|rrr|}
\hline make & weight & foreign \\
\hline Pont. Sunbird & 2690 & Domestic \\
Volvo 260 & 3170 & Foreign \\
\hline
\end{tabular}
. predict mpg
(option xb assumed; fitted values)
variable lnweight not found
r(111);
```

Things did not work. We typed predict mpg, and Stata responded with the message "variable lnweight not found". predict can calculate predicted values on a different dataset only if that dataset contains the variables that went into the model. Here our dataset does not contain a variable called lnweight. lnweight is just the $\log$ of weight, so we can create it and try again:

```
. generate lnweight = ln(weight)
. predict mpg
(option xb assumed; fitted values)
. list
```

| make | weight | foreign | lnweight | mpg |
| ---: | ---: | ---: | ---: | ---: |
| Pont. Sunbird | 2690 | Domestic | 7.897296 | 23.25097 |
| Volvo 260 | 3170 | Foreign | 8.061487 | 17.85295 |

We obtained our predicted values. The Pontiac Sunbird has a predicted mileage rating of 23.3 mpg , whereas the Volvo 260 has a predicted rating of 17.9 mpg .

## Residuals

## > Example 3

With many estimators, predict can calculate more than predicted values. With most regressiontype estimators, we can, for instance, obtain residuals. Using our regression example, we return to our original data and obtain residuals by typing

```
. use http://www.stata-press.com/data/r13/auto, clear
(1978 Automobile Data)
. generate lnweight = ln(weight)
. regress mpg weight lnweight foreign
(output omitted)
. predict double resid, residuals
. summarize resid
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline resid & 74 & \(-1.51 \mathrm{e}-15\) & 3.212091 & -5.453078 & 13.83719
\end{tabular}
```

We could do this without refitting the model. Stata always remembers the last set of estimates, even as we use new datasets.

It was not necessary to type the double in predict double resid, residuals, but we wanted to remind you that you can specify the type of a variable in front of the variable's name; see [U] 11.4.2 Lists of new variables. We made the new variable resid a double rather than the default float.

If you want your residuals to have a mean as close to zero as possible, remember to request the extra precision of double. If we had not specified double, the mean of resid would have been roughly $10^{-9}$ rather than $10^{-14}$. Although $10^{-14}$ sounds more precise than $10^{-9}$, the difference really does not matter.

For linear regression, predict can also calculate standardized residuals and Studentized residuals with the options rstandard and rstudent; for examples, see [R] regress postestimation.

## Single-equation (SE) models

If you have not read the discussion above on using predict after linear regression, please do so. And predict's default calculation almost always produces a statistic in the same metric as the dependent variable of the fitted model-for example, predicted counts for Poisson regression. In any case, xb can always be specified to obtain the linear prediction.
predict can calculate the standard error of the prediction, which is obtained by using the covariance matrix of the estimators.
$>$ Example 4
After most binary outcome models (for example, logistic, logit, probit, cloglog, scobit), predict calculates the probability of a positive outcome if we do not tell it otherwise. We can specify the xb option if we want the linear prediction (also known as the logit or probit index). The odd abbreviation xb is meant to suggest $\mathbf{x} \boldsymbol{\beta}$. In logit and probit models, for example, the predicted probability is $p=F(\mathbf{x} \boldsymbol{\beta})$, where $F()$ is the logistic or normal cumulative distribution function, respectively.

```
. logistic foreign mpg weight
    (output omitted)
. predict phat
(option pr assumed; Pr(foreign))
. predict idxhat, xb
. summarize foreign phat idxhat
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline foreign & 74 & .2972973 & .4601885 & 0 & 1 \\
phat & 74 & .2972973 & .3052979 & .000729 & .8980594 \\
idxhat & 74 & -1.678202 & 2.321509 & -7.223107 & 2.175845
\end{tabular}
```

Because this is a logit model, we could obtain the predicted probabilities ourselves from the predicted index

```
. generate phat2 = exp(idxhat)/(1+exp(idxhat))
```

but using predict without options is easier.

## > Example 5

For all models, predict attempts to produce a predicted value in the same metric as the dependent variable of the model. We have seen that for dichotomous outcome models, the default statistic produced by predict is the probability of a success. Similarly, for Poisson regression, the default statistic produced by predict is the predicted count for the dependent variable. You can always specify the xb option to obtain the linear combination of the coefficients with an observation's $x$ values (the inner product of the coefficients and $x$ values). For poisson (without an explicit exposure), this is the natural log of the count.

```
. use http://www.stata-press.com/data/r13/airline, clear
. poisson injuries XYZowned
(output omitted)
```



We note that our "hand-computed" prediction of the count (exp_idx) matches what was produced by the default operation of predict.

If our model has an exposure-time variable, we can use predict to obtain the linear prediction with or without the exposure. Let's verify what we are getting by obtaining the linear prediction with and without exposure, transforming these predictions to count predictions and comparing them with the default count prediction from predict. We must remember to multiply by the exposure time when using predict..., nooffset.

```
. use http://www.stata-press.com/data/r13/airline, clear
. poisson injuries XYZowned, exposure(n)
(output omitted)
. predict double injhat
(option n assumed; predicted number of events)
. predict double idx, xb
. gen double exp_idx = exp(idx)
. predict double idxn, xb nooffset
. gen double exp_idxn = exp(idxn)*n
. summarize injuries injhat exp_idx exp_idxn idx idxn
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline injuries & 9 & 7.111111 & 5.487359 & 1 & 19 \\
injhat & 9 & 7.111111 & 3.10936 & 2.919621 & 12.06158 \\
exp_idx & 9 & 7.111111 & 3.10936 & 2.919621 & 12.06158 \\
exp_idxn & 9 & 7.111111 & 3.10936 & 2.919621 & 12.06158 \\
idx & 9 & 1.869722 & .4671044 & 1.071454 & 2.490025 \\
\hline idxn & 9 & 4.18814 & .1904042 & 4.061204 & 4.442013
\end{tabular}
```

Looking at the identical means and standard deviations for injhat, exp_idx, and exp_idxn, we see that we can reproduce the default computations of predict for poisson estimations. We have also demonstrated the relationship between the count predictions and the linear predictions with and without exposure.

## SE model scores

## $>$ Example 6

With most maximum likelihood estimators, predict can calculate equation-level scores. The first derivative of the $\log$ likelihood with respect to $\mathbf{x}_{j} \boldsymbol{\beta}$ is the equation-level score.

```
. use http://www.stata-press.com/data/r13/auto, clear
(1978 Automobile Data)
. logistic foreign mpg weight
    (output omitted)
. predict double sc, score
. summarize sc
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline sc & 74 & \(-1.37 e-12\) & .3533133 & -.8760856 & .8821309
\end{tabular}
```

See $[\mathrm{P}]$ _robust and [SVY] variance estimation for details regarding the role equation-level scores play in linearization-based variance estimators.

## - Technical note

predict after some estimation commands, such as regress and cnsreg, allows the score option as a synonym for the residuals option.

## Multiple-equation (ME) models

If you have not read the above discussion on using predict after SE models, please do so. With the exception of the ability to select specific equations to predict from, the use of predict after ME models follows almost the same form that it does for SE models.

## > Example 7

The details of prediction statistics that are specific to particular ME models are documented with the estimation command. If you are using ME commands that do not have separate discussions on obtaining predictions, read Obtaining predicted values in [R] mlogit postestimation, even if your interest is not in multinomial logistic regression. As a general introduction to the ME models, we will demonstrate predict after sureg:

```
. use http://www.stata-press.com/data/r13/auto, clear
(1978 Automobile Data)
. sureg (price foreign displ) (weight foreign length)
Seemingly unrelated regression
```

| Equation | Obs | Parms | RMSE | "R-sq" | chi2 | $P$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| price | 74 | 2 | 2202.447 | 0.4348 | 45.21 | 0.0000 |
| weight | 74 | 2 | 245.5238 | 0.8988 | 658.85 | 0.0000 |


|  | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| price |  |  |  |  |  |  |
| foreign | 3137.894 | 697.3805 | 4.50 | 0.000 | 1771.054 | 4504.735 |
| displacement | 23.06938 | 3.443212 | 6.70 | 0.000 | 16.32081 | 29.81795 |
| _cons | 680.8438 | 859.8142 | 0.79 | 0.428 | -1004.361 | 2366.049 |
| weight |  |  |  |  |  |  |
| foreign | -154.883 | 75.3204 | -2.06 | 0.040 | -302.5082 | -7.257674 |
| length | 30.67594 | 1.531981 | 20.02 | 0.000 | 27.67331 | 33.67856 |
| _cons | -2699.498 | 302.3912 | -8.93 | 0.000 | -3292.173 | -2106.822 |

sureg estimated two equations, one called price and the other weight; see $[R]$ sureg.

```
. predict pred_p, equation(price)
(option xb assumed; fitted values)
. predict pred_w, equation(weight)
(option xb assumed; fitted values)
```

. summarize price pred_p weight pred_w

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| price | 74 | 6165.257 | 2949.496 | 3291 | 15906 |
| pred_p | 74 | 6165.257 | 1678.805 | 2664.81 | 10485.33 |
| weight | 74 | 3019.459 | 777.1936 | 1760 | 4840 |
| pred_w | 74 | 3019.459 | 726.0468 | 1501.602 | 4447.996 |

You may specify the equation by name, as we did above, or by number: equation(\#1) means the same thing as equation(price) in this case.

## ME model scores

## > Example 8

For ME models, predict allows you to specify a stub when generating equation-level score variables. predict generates new variables using this stub by appending an equation index. Depending upon the command, the index will start with 0 or 1 . Here is an example where predict starts indexing the score variables with 0 .

```
. ologit rep78 mpg weight
(output omitted)
. predict double sc*, scores
. summarize sc*
```

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| sc0 | 69 | $-1.33 e-11$ | .5337363 | -.9854088 | .921433 |
| sc1 | 69 | $-7.69 e-13$ | .186919 | -.2738537 | .9854088 |
| sc2 | 69 | $-2.87 e-11$ | .4061637 | -.5188487 | 1.130178 |
| sc3 | 69 | $-1.04 e-10$ | .5315368 | -1.067351 | .8194842 |
| sc4 | 69 | $1.47 e-10$ | .360525 | -.921433 | .6140182 |

Although it involves much more typing, we could also specify the new variable names individually.
. predict double (sc_xb sc_1 sc_2 sc_3 sc_4), scores
• summarize sc_*
Variable

## Methods and formulas

Denote the previously estimated coefficient vector as $\mathbf{b}$ and its estimated variance matrix as $\mathbf{V}$. predict works by recalling various aspects of the model, such as $\mathbf{b}$, and combining that information with the data currently in memory. Let's write $\mathbf{x}_{j}$ for the $j$ th observation currently in memory.

The predicted value ( xb option) is defined as $\widehat{y}_{j}=\mathbf{x}_{j} \mathbf{b}+$ offset $_{j}$
The standard error of the prediction (the stdp option) is defined as $s_{p_{j}}=\sqrt{\mathbf{x}_{j} \mathbf{V \mathbf { x } _ { j } ^ { \prime }}}$
The standard error of the difference in linear predictions between equations 1 and 2 is defined as

$$
s_{d p_{j}}=\left\{\left(\mathbf{x}_{1 j},-\mathbf{x}_{2 j}, \mathbf{0}, \ldots, \mathbf{0}\right) \mathbf{V}\left(\mathbf{x}_{1 j},-\mathbf{x}_{2 j}, \mathbf{0}, \ldots, \mathbf{0}\right)^{\prime}\right\}^{\frac{1}{2}}
$$

See the individual estimation commands for information about calculating command-specific predict statistics.

## Also see

$[R]$ predictnl - Obtain nonlinear predictions, standard errors, etc., after estimation
[P] _predict - Obtain predictions, residuals, etc., after estimation programming command
[U] 20 Estimation and postestimation commands

## Title

predictnl - Obtain nonlinear predictions, standard errors, etc., after estimation

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Methods and formulas | References | Also see |

## Syntax

$$
\text { predictnl }[\text { type }] \text { newvar }=\text { pnl_exp }[\text { if }][\text { in }][\text {, options }]
$$

options
Main
se(newvar)
variance (newvar)
wald(newvar)
p(newvar)
ci(newvars)
level(\#)
g (stub)

Advanced
iterate(\#)
force
df(\#)

Description
create newvar containing standard errors
create newvar containing variances
create newvar containing the Wald test statistic
create newvar containing the significance level ( $p$-value) of the Wald test
create newvars containing lower and upper confidence intervals
set confidence level; default is level (95)
create $s t u b 1$, stub2, ..., stubk variables containing observationspecific derivatives
maximum iterations for finding optimal step size; default is 100 calculate standard errors, etc., even when possibly inappropriate use $F$ distribution with \# denominator degrees of freedom for the reference distribution of the test statistic
$\mathrm{df}(\#)$ does not appear in the dialog box.

## Menu

Statistics $>$ Postestimation $>$ Nonlinear predictions

## Description

predictnl calculates (possibly) nonlinear predictions after any Stata estimation command and optionally calculates the variances, standard errors, Wald test statistics, significance levels, and confidence limits for these predictions. Unlike its companion nonlinear postestimation commands testnl and nlcom, predictnl generates functions of the data (that is, predictions), not scalars. The quantities generated by predictnl are thus vectorized over the observations in the data.

Consider some general prediction, $g\left(\boldsymbol{\theta}, \mathbf{x}_{i}\right)$, for $i=1, \ldots, n$, where $\boldsymbol{\theta}$ are the model parameters and $\mathbf{x}_{i}$ are some data for the $i$ th observation; $\mathbf{x}_{i}$ is assumed fixed. Typically, $g\left(\boldsymbol{\theta}, \mathbf{x}_{i}\right)$ is estimated by $g\left(\widehat{\boldsymbol{\theta}}, \mathbf{x}_{i}\right)$, where $\widehat{\boldsymbol{\theta}}$ are the estimated model parameters, which are stored in e(b) following any Stata estimation command.

In its most common use, predictnl generates two variables: one containing the estimated prediction, $g\left(\widehat{\boldsymbol{\theta}}, \mathbf{x}_{i}\right)$, the other containing the estimated standard error of $g\left(\widehat{\boldsymbol{\theta}}, \mathbf{x}_{i}\right)$. The calculation of standard errors (and other obtainable quantities that are based on the standard errors, such as test statistics) is based on the delta method, an approximation appropriate in large samples; see Methods and formulas.
predictnl can be used with svy estimation results (assuming that predict is also allowed), see [SVY] svy postestimation.

The specification of $g\left(\widehat{\boldsymbol{\theta}}, \mathbf{x}_{i}\right)$ is handled by specifying pnl_exp, and the values of $g\left(\widehat{\boldsymbol{\theta}}, \mathbf{x}_{i}\right)$ are stored in the new variable newvar of storage type type. pnl_exp is any valid Stata expression and may also contain calls to two special functions unique to predictnl:

1. predict ([predict_options]): When you are evaluating pnl_exp, predict() is a convenience function that replicates the calculation performed by the command
```
predict ..., predict_options
```

As such, the predict () function may be used either as a shorthand for the formula used to make this prediction or when the formula is not readily available. When used without arguments, predict() replicates the default prediction for that particular estimation command.
2. $\mathrm{xb}([$ eqno $])$ : The xb() function replicates the calculation of the linear predictor $\mathbf{x}_{i} \mathbf{b}$ for equation eqno. If xb() is specified without eqno, the linear predictor for the first equation (or the only equation in single-equation estimation) is obtained.
For example, $\mathrm{xb}(\# 1)$ (or equivalently, xb() with no arguments) translates to the linear predictor for the first equation, $\mathrm{xb}(\# 2)$ for the second, and so on. You could also refer to the equations by their names, such as xb (income).

When specifying pnl_exp, both of these functions may be used repeatedly, in combination, and in combination with other Stata functions and expressions. See Remarks and examples for examples that use both of these functions.

## Options

$\qquad$ Main
se (newvar) adds newvar of storage type type, where for each i in the prediction sample, newvar [i] contains the estimated standard error of $g\left(\widehat{\boldsymbol{\theta}}, \mathbf{x}_{i}\right)$.
variance (newvar) adds newvar of storage type type, where for each i in the prediction sample, newvar[i] contains the estimated variance of $g\left(\widehat{\boldsymbol{\theta}}, \mathbf{x}_{i}\right)$.
wald (newvar) adds newvar of storage type type, where for each i in the prediction sample, newvar [i] contains the Wald test statistic for the test of the hypothesis $H_{0}: g\left(\boldsymbol{\theta}, \mathbf{x}_{i}\right)=0$.
p (newvar) adds newvar of storage type type, where newvar [i] contains the significance level ( $p$-value) of the Wald test of $H_{0}: g\left(\boldsymbol{\theta}, \mathbf{x}_{i}\right)=0$ versus the two-sided alternative.
ci (newvars) requires the specification of two newvars, such that the $i$ th observation of each will contain the left and right endpoints (respectively) of a confidence interval for $g\left(\boldsymbol{\theta}, \mathbf{x}_{i}\right)$. The level of the confidence intervals is determined by level(\#).
level(\#) specifies the confidence level, as a percentage, for confidence intervals. The default is level (95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals.
$\mathrm{g}(s t u b)$ specifies that new variables, stub1, stub2, ..., stubk be created, where k is the dimension of $\boldsymbol{\theta}$. stub1 will contain the observation-specific derivatives of $g\left(\boldsymbol{\theta}, \mathbf{x}_{i}\right)$ with respect to the first element, $\theta_{1}$, of $\boldsymbol{\theta}$; stub2 will contain the derivatives of $g\left(\boldsymbol{\theta}, \mathbf{x}_{i}\right)$ with respect to $\theta_{2}$, etc.; If the derivative of $g\left(\boldsymbol{\theta}, \mathbf{x}_{i}\right)$ with respect to a particular coefficient in $\boldsymbol{\theta}$ equals zero for all observations in the prediction sample, the stub variable for that coefficient is not created. The ordering of the parameters in $\boldsymbol{\theta}$ is precisely that of the stored vector of parameter estimates e(b).
iterate(\#) specifies the maximum number of iterations used to find the optimal step size in the calculation of numerical derivatives of $g\left(\boldsymbol{\theta}, \mathbf{x}_{i}\right)$ with respect to $\boldsymbol{\theta}$. By default, the maximum number of iterations is 100 , but convergence is usually achieved after only a few iterations. You should rarely have to use this option.
force forces the calculation of standard errors and other inference-related quantities in situations where predictnl would otherwise refuse to do so. The calculation of standard errors takes place by evaluating (at $\widehat{\boldsymbol{\theta}})$ the numerical derivative of $g\left(\boldsymbol{\theta}, \mathbf{x}_{i}\right)$ with respect to $\boldsymbol{\theta}$. If predictnl detects that $g()$ is possibly a function of random quantities other than $\widehat{\boldsymbol{\theta}}$, it will refuse to calculate standard errors or any other quantity derived from them. The force option forces the calculation to take place anyway. If you use the force option, there is no guarantee that any inference quantities (for example, standard errors) will be correct or that the values obtained can be interpreted.

The following option is available with predictnl but is not shown in the dialog box:
$\mathrm{df}(\#)$ specifies that the $F$ distribution with \# denominator degrees of freedom be used for the reference distribution of the test statistic.

## Remarks and examples

Remarks are presented under the following headings:

Introduction<br>Nonlinear transformations and standard errors<br>Using $x b()$ and predict()<br>Multiple-equation (ME) estimators<br>Test statistics and significance levels Manipulability<br>Confidence intervals

## Introduction

predictnl and nlcom both use the delta method. They take a nonlinear transformation of the estimated parameter vector from some fitted model and apply the delta method to calculate the variance, standard error, Wald test statistic, etc., of this transformation. nlcom is designed for scalar functions of the parameters, and predictnl is designed for functions of the parameters and of the data, that is, for predictions.

## Nonlinear transformations and standard errors

We begin by fitting a probit model to the low-birthweight data of Hosmer, Lemeshow, and Sturdivant (2013, 24). The data are described in detail in example 1 of $[R]$ logistic.
(Hosmer \& Lemeshow data)
. probit low lwt smoke ptl ht


| low | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | ---: | ---: | ---: |
| lwt | -.0095164 | .0036875 | -2.58 | 0.010 | -.0167438 | -.0022891 |
| smoke | .3487004 | .2041772 | 1.71 | 0.088 | -.0514794 | .7488803 |
| ptl | .365667 | .1921201 | 1.90 | 0.057 | -.0108815 | .7422154 |
| ht | 1.082355 | .410673 | 2.64 | 0.008 | .2774503 | 1.887259 |
| _cons | .4238985 | .4823224 | 0.88 | 0.379 | -.5214361 | 1.369233 |

After we fit such a model, we first would want to generate the predicted probabilities of a low birthweight, given the covariate values in the estimation sample. This is easily done using predict after probit, but it doesn't answer the question, "What are the standard errors of those predictions?"

For the time being, we will consider ourselves ignorant of any automated way to obtain the predicted probabilities after probit. The formula for the prediction is

$$
\operatorname{Pr}\left(y \neq 0 \mid \mathbf{x}_{i}\right)=\Phi\left(\mathbf{x}_{i} \boldsymbol{\beta}\right)
$$

where $\Phi$ is the standard cumulative normal. Thus for this example, $g\left(\boldsymbol{\theta}, \mathbf{x}_{i}\right)=\Phi\left(\mathbf{x}_{i} \boldsymbol{\beta}\right)$. Armed with the formula, we can use predictnl to generate the predictions and their standard errors:

```
. predictnl phat = normal(_b[_cons] + _b[ht]*ht + _b[ptl]*ptl +
> _b[smoke]*smoke + _b[lwt]*lwt), se(phat_se)
. list phat phat_se lwt smoke ptl ht in -10/l
```

|  | phat | phat_se | lwt | smoke | ptl | ht |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| 180. | .2363556 | .042707 | 120 | 0 | 0 | 0 |
| 181. | .6577712 | .1580714 | 154 | 0 | 1 | 1 |
| 182. | .2793261 | .0519958 | 106 | 0 | 0 | 0 |
| 183. | .1502118 | .0676339 | 190 | 1 | 0 | 0 |
| 184. | .5702871 | .0819911 | 101 | 1 | 1 | 0 |
| 185. | .4477045 | .079889 | 95 | 1 | 0 | 0 |
| 186. | .2988379 | .0576306 | 100 | 0 | 0 | 0 |
| 187. | .4514706 | .080815 | 94 | 1 | 0 | 0 |
| 188. | .5615571 | .1551051 | 142 | 0 | 0 | 1 |
| 189. | .7316517 | .1361469 | 130 | 1 | 0 | 1 |

Thus subject 180 in our data has an estimated probability of low birthweight of $23.6 \%$ with standard error $4.3 \%$.

Used without options, predictnl is not much different from generate. By specifying the se(phat_se) option, we were able to obtain a variable containing the standard errors of the predictions; therein lies the utility of predictnl.

## Using $\mathbf{x b}()$ and predict()

As was the case above, a prediction is often not a function of a few isolated parameters and their corresponding variables but instead is some (possibly elaborate) function of the entire linear predictor. For models with many predictors, the brute-force expression for the linear predictor can be cumbersome to type. An alternative is to use the inline function xb()$. \mathrm{xb}()$ is a shortcut for having to type _b[_cons] + _b[ht]*ht + _b[ptl]*ptl + ...,

```
. drop phat phat_se
. predictnl phat = norm(xb()), se(phat_se)
. list phat phat_se lwt smoke ptl ht in -10/l
```

|  | phat | phat_se | lwt | smoke | ptl | ht |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| 180. | .2363556 | .042707 | 120 | 0 | 0 | 0 |
| 181. | .6577712 | .1580714 | 154 | 0 | 1 | 1 |
| 182. | .2793261 | .0519958 | 106 | 0 | 0 | 0 |
| 183. | .1502118 | .0676339 | 190 | 1 | 0 | 0 |
| 184. | .5702871 | .0819911 | 101 | 1 | 1 | 0 |
| 185. | .4477045 | .079889 | 95 | 1 | 0 | 0 |
| 186. | .2988379 | .0576306 | 100 | 0 | 0 | 0 |
| 187. | .4514706 | .080815 | 94 | 1 | 0 | 0 |
| 188. | .5615571 | .1551051 | 142 | 0 | 0 | 1 |
| 189. | .7316517 | .1361469 | 130 | 1 | 0 | 1 |

which yields the same results. This approach is easier, produces more readable code, and is less prone to error, such as forgetting to include a term in the sum.

Here we used xb() without arguments because we have only one equation in our model. In multiple-equation (ME) settings, xb() (or equivalently $\mathrm{xb}(\# 1)$ ) yields the linear predictor from the first equation, $\mathrm{xb}(\# 2)$ from the second, etc. You can also refer to equations by their names, for example, xb(income).

## Technical note

Most estimation commands in Stata allow the postestimation calculation of linear predictors and their standard errors via predict. For example, to obtain these for the first (or only) equation in the model, you could type

```
predict xbvar, xb
predict stdpvar, stdp
```

Equivalently, you could type

```
predictnl xbvar = xb(), se(stdpvar)
```

but we recommend the first method, as it is faster. As we demonstrated above, however, predictnl is more general.

Returning to our probit example, we can further simplify the calculation by using the inline function predict(). predict(pred_options) works by substituting, within our predictnl expression, the calculation performed by

```
predict ..., pred_options
```

In our example, we are interested in the predicted probabilities after a probit regression, normally obtained via

```
predict ..., p
```

We can obtain these predictions (and standard errors) by using

```
. drop phat phat_se
. predictnl phat = predict(p), se(phat_se)
. list phat phat_se lwt smoke ptl ht in -10/l
```

|  | phat | phat_se | lwt | smoke | ptl | ht |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| 180. | .2363556 | .042707 | 120 | 0 | 0 | 0 |
| 181. | .6577712 | .1580714 | 154 | 0 | 1 | 1 |
| 182. | .2793261 | .0519958 | 106 | 0 | 0 | 0 |
| 183. | .1502118 | .0676339 | 190 | 1 | 0 | 0 |
| 184. | .5702871 | .0819911 | 101 | 1 | 1 | 0 |
| 185. | .4477045 | .079889 | 95 | 1 | 0 | 0 |
| 186. | .2988379 | .0576306 | 100 | 0 | 0 | 0 |
| 187. | .4514706 | .080815 | 94 | 1 | 0 | 0 |
| 188. | .5615571 | .1551051 | 142 | 0 | 0 | 1 |
| 189. | .7316517 | .1361469 | 130 | 1 | 0 | 1 |

which again replicates what we have already done by other means. However, this version did not require knowledge of the formula for the predicted probabilities after a probit regression—predict (p) took care of that for us.

Because the predicted probability is the default prediction after probit, we could have just used predict() without arguments, namely,

```
. predictnl phat = predict(), se(phat_se)
```

Also, the expression pnl_exp can be inordinately complicated, with multiple calls to predict () and xb() . For example,

```
. predictnl phat = normal(invnormal(predict()) + predict(xb)/xb() - 1),
> se(phat_se)
```

is perfectly valid and will give the same result as before, albeit a bit inefficiently.

## Technical note

When using predict() and xb() , the formula for the calculation is substituted within pnl_exp, not the values that result from the application of that formula. To see this, note the subtle difference between

```
. predict xbeta, xb
. predictnl phat = normal(xbeta), se(phat_se)
```

and

```
. predictnl phat = normal(xb()), se(phat_se)
```

Both sequences will yield the same phat, yet for the first sequence, phat_se will equal zero for all observations. The reason is that, once evaluated, xbeta will contain the values of the linear predictor, yet these values are treated as fixed and nonstochastic as far as predictnl is concerned. By contrast, because xb() is shorthand for the formula used to calculate the linear predictor, it contains not values, but references to the estimated regression coefficients and corresponding variables. Thus the second method produces the desired result.

## Multiple-equation (ME) estimators

In [R] mlogit, data on insurance choice (Tarlov et al. 1989; Wells et al. 1989) were examined, and a multinomial logit was used to assess the effects of age, gender, race, and site of study (one of three sites) on the type of insurance:

| . use http://www.stata-press.com/data/r13/sysdsn1, clear |  |  |
| :--- | :--- | :--- |
| (Health insurance data) |  |  |
| . mlogit insure age male nonwhite i.site, nolog |  |  |
| Multinomial logistic regression | Number of obs | $=$ |
|  | LR chi2 (10) | $=$ |


| insure | Coef . | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Indemnity | (base outcome) |  |  |  |  |  |
| Prepaid |  |  |  |  |  |  |
| age | -. 011745 | . 0061946 | -1.90 | 0.058 | -. 0238862 | . 0003962 |
| male | . 5616934 | . 2027465 | 2.77 | 0.006 | . 1643175 | . 9590693 |
| nonwhite | . 9747768 | . 2363213 | 4.12 | 0.000 | . 5115955 | 1.437958 |
| site |  |  |  |  |  |  |
| 2 | . 1130359 | . 2101903 | 0.54 | 0.591 | -. 2989296 | . 5250013 |
| 3 | -. 5879879 | . 2279351 | -2.58 | 0.010 | -1.034733 | -. 1412433 |
| _cons | . 2697127 | . 3284422 | 0.82 | 0.412 | -. 3740222 | . 9134476 |
| Uninsure |  |  |  |  |  |  |
| age | -. 0077961 | . 0114418 | -0.68 | 0.496 | -. 0302217 | . 0146294 |
| male | . 4518496 | . 3674867 | 1.23 | 0.219 | -. 268411 | 1.17211 |
| nonwhite | . 2170589 | . 4256361 | 0.51 | 0.610 | -. 6171725 | 1.05129 |
| site |  |  |  |  |  |  |
| 2 | -1.211563 | . 4705127 | -2.57 | 0.010 | -2.133751 | -. 2893747 |
| 3 | -. 2078123 | . 3662926 | -0.57 | 0.570 | -. 9257327 | . 510108 |
| _cons | -1.286943 | . 5923219 | -2.17 | 0.030 | -2.447872 | -. 1260134 |

Of particular interest is the estimation of the relative risk, which, for a given selection, is the ratio of the probability of making that selection to the probability of selecting the base category (Indemnity here), given a set of covariate values. In a multinomial logit model, the relative risk (when comparing to the base category) simplifies to the exponentiated linear predictor for that selection.

Using this example, we can estimate the observation-specific relative risks of selecting a prepaid plan over the base category (with standard errors) by either referring to the Prepaid equation by name or number,

```
. predictnl RRppaid = exp(xb(Prepaid)), se(SERRppaid)
```

or

```
. predictnl RRppaid = exp(xb(#1)), se(SERRppaid)
```

because Prepaid is the first equation in the model.
Those of us for whom the simplified formula for the relative risk does not immediately come to mind may prefer to calculate the relative risk directly from its definition, that is, as a ratio of two predicted probabilities. After mlogit, the predicted probability for a category may be obtained using predict, but we must specify the category as the outcome:

```
. predictnl RRppaid = predict(outcome(Prepaid))/predict(outcome(Indemnity)),
> se(SERRppaid)
(1 missing value generated)
. list RRppaid SERRppaid age male nonwhite site in 1/10
```

| RRppaid | SERRpp~d | age | male | nonwhite | site |
| ---: | ---: | ---: | ---: | ---: | ---: |
| .6168578 | .1503759 | 73.722107 | 0 | 0 | 2 |
| 1.056658 | .1790703 | 27.89595 | 0 | 0 | 2 |
| .8426442 | .1511281 | 37.541397 | 0 | 0 | 1 |
| 1.460581 | .3671465 | 23.641327 | 0 | 1 | 3 |
| .9115747 | .1324168 | 40.470901 | 0 | 0 | 2 |
| 1.034701 | .1696923 | 29.683777 | 0 | 0 | 2 |
| .9223664 | .1344981 | 39.468857 | 0 | 0 | 2 |
| 1.678312 | .4216626 | 26.702255 | 1 | 0 | 1 |
| .9188519 | .2256017 | 63.101974 | 0 | 1 | 3 |
| .5766296 | .1334877 | 69.839828 | 0 | 0 | 1 |

The "(1 missing value generated)" message is not an error; further examination of the data would reveal that age is missing in one observation and that the offending observation (among others) is not in the estimation sample. Just as with predict, predictnl can generate predictions in or out of the estimation sample.

Thus we estimate (among other things) that a white, female, 73-year-old from site 2 is less likely to choose a prepaid plan over an indemnity plan-her relative risk is about $62 \%$ with standard error $15 \%$.

## Test statistics and significance levels

Often a standard error calculation is just a means to an end, and what is really desired is a test of the hypothesis,

$$
H_{0}: g\left(\boldsymbol{\theta}, \mathbf{x}_{i}\right)=0
$$

versus the two-sided alternative.
We can use predictnl to obtain the Wald test statistics or significance levels (or both) for the above tests, whether or not we want standard errors. To obtain the Wald test statistics, we use the wald() option; for significance levels, we use $p()$.

Returning to our mlogit example, suppose that we wanted for each observation a test of whether the relative risk of choosing a prepaid plan over an indemnity plan is different from one. One way to do this would be to define $g()$ to be the relative risk minus one and then test whether $g()$ is different from zero.

```
. predictnl RRm1 = exp(xb(Prepaid)) - 1, wald(W_RRm1) p(sig_RRm1)
(1 missing value generated)
```

```
note: significance levels are with respect to the chi-squared(1) distribution . list RRm1 W_RRm1 sig_RRm1 age male nonwhite in \(1 / 10\)
```

| RRm1 | W_RRm1 | sig_RRm1 | age | male | nonwhite |
| ---: | ---: | ---: | ---: | ---: | ---: |
| -.3831422 | 6.491778 | .0108375 | 73.722107 | 0 | 0 |
| .0566578 | .100109 | .7516989 | 27.89595 | 0 | 0 |
| -.1573559 | 1.084116 | .2977787 | 37.541397 | 0 | 0 |
| .4605812 | 1.573743 | .2096643 | 23.641327 | 0 | 1 |
| -.0884253 | .4459299 | .5042742 | 40.470901 | 0 | 0 |
| .0347015 | .0418188 | .8379655 | 29.683777 | 0 | 0 |
| -.0776336 | .3331707 | .563798 | 39.468857 | 0 | 0 |
| .6783119 | 2.587788 | .1076906 | 26.702255 | 1 | 0 |
| -.0811482 | .1293816 | .719074 | 63.101974 | 0 | 1 |
| -.4233705 | 10.05909 | .001516 | 69.839828 | 0 | 0 |

The newly created variable W_RRm1 contains the Wald test statistic for each observation, and sig_RRm1 contains the level of significance. Thus our 73-year-old white female represented by the first observation would have a relative risk of choosing prepaid over indemnity that is significantly different from 1, at least at the $5 \%$ level. For this test, it was not necessary to generate a variable containing the standard error of the relative risk minus 1 , but we could have done so had we wanted. We could have also omitted specifying wald (W_RRm1) if all we cared about were, say, the significance levels of the tests.

In this regard, predictnl acts as an observation-specific version of testnl, with the test results vectorized over the observations in the data. The significance levels are pointwise-they are not adjusted to reflect any simultaneous testing over the observations in the data.

## Manipulability

There are many ways to specify $g\left(\boldsymbol{\theta}, \mathbf{x}_{i}\right)$ to yield tests such that, for multiple specifications of $g()$, the theoretical conditions for which

$$
H_{0}: g\left(\boldsymbol{\theta}, \mathbf{x}_{i}\right)=0
$$

is true will be equivalent. However, this does not mean that the tests themselves will be equivalent. This is known as the manipulability of the Wald test for nonlinear hypotheses; also see $[R]$ boxcox.

As an example, consider the previous section where we defined $g()$ to be the relative risk between choosing a prepaid plan over an indemnity plan, minus 1 . We could also have defined $g()$ to be the risk difference-the probability of choosing a prepaid plan minus the probability of choosing an indemnity plan. Either specification of $g()$ yields a mathematically equivalent specification of $H_{0}: g()=0$; that is, the risk difference will equal zero when the relative risk equals one. However, the tests themselves do not give the same results:

```
. predictnl RD = predict(outcome(Prepaid)) - predict(outcome(Indemnity)),
> wald(W_RD) p(sig_RD)
(1 missing value generated)
note: significance levels are with respect to the chi-squared(1) distribution
list RD W_RD sig_RD RRm1 W_RRm1 sig_RRm1 in 1/10
```

|  | RD | W_RD | sig_RD | RRm1 | W_RRm1 | sig_RRm1 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | 1. | -.2303744 | 4.230243 | .0397097 | -.3831422 | 6.491778 |
| 2. | .0266902 | .1058542 | .7449144 | .0566578 | .100109 | .7516975 |
| 3. | -.0768078 | .9187646 | .3377995 | -.1573559 | 1.084116 | .2977787 |
| 4. | .1710702 | 2.366535 | .1239619 | .4605812 | 1.573743 | .2096643 |
| 5. | -.0448509 | .4072922 | .5233471 | -.0884253 | .4459299 | .5042742 |
| 6. | .0165251 | .0432816 | .835196 | .0347015 | .0418188 | .8379655 |
| 7. | -.0391535 | .3077611 | .5790573 | -.0776336 | .3331707 | .563798 |
| 8. | .22382 | 4.539085 | .0331293 | .6783119 | 2.587788 | .1076906 |
| 9. | -.0388409 | .1190183 | .7301016 | -.0811482 | .1293816 | .719074 |
| 10. | -.2437626 | 6.151558 | .0131296 | -.4233705 | 10.05909 | .001516 |

In certain cases (such as subject 8 ), the difference can be severe enough to potentially change the conclusion. The reason for this inconsistency is that the nonlinear Wald test is actually a standard Wald test of a first-order Taylor approximation of $g()$, and this approximation can differ according to how $g()$ is specified.

As such, keep in mind the manipulability of nonlinear Wald tests when drawing scientific conclusions.

## Confidence intervals

We can also use predictnl to obtain confidence intervals for the observation-specific $g\left(\boldsymbol{\theta}, \mathbf{x}_{i}\right)$ by using the ci() option to specify two new variables to contain the left and right endpoints of the confidence interval, respectively. For example, we could generate confidence intervals for the risk differences calculated previously:

```
. drop RD
. predictnl RD = predict(outcome(Prepaid)) - predict(outcome(Indemnity)),
> ci(RD_lcl RD_rcl)
(1 missing value generated)
note: Confidence intervals calculated using Z critical values
. list RD RD_lcl RD_rcl age male nonwhite in 1/10
```

| RD | RD_lcl | RD_rcl | age | male | nonwhite |
| ---: | ---: | ---: | ---: | ---: | ---: |
| -.2303744 | -.4499073 | -.0108415 | 73.722107 | 0 | 0 |
| .0266902 | -.1340948 | .1874752 | 27.89595 | 0 | 0 |
| -.0768078 | -.2338625 | .080247 | 37.541397 | 0 | 0 |
| .1710702 | -.0468844 | .3890248 | 23.641327 | 0 | 1 |
| -.0448509 | -.1825929 | .092891 | 40.470901 | 0 | 0 |
| .0165251 | -.1391577 | .1722078 | 29.683777 | 0 | 0 |
| -.0391535 | -.177482 | .099175 | 39.468857 | 0 | 0 |
| .22382 | .0179169 | .4297231 | 26.702255 | 1 | 0 |
| -.0388409 | -.2595044 | .1818226 | 63.101974 | 0 | 1 |
| -.2437626 | -.4363919 | -.0511332 | 69.839828 | 0 | 0 |

The confidence level, here, $95 \%$, is either set using the level() option or obtained from the current default level, c (level); see [U] 20.7 Specifying the width of confidence intervals.

From the above output, we can see that, for subjects 1,8 , and 10 , a $95 \%$ confidence interval for the risk difference does not contain zero, meaning that, for these subjects, there is some evidence of a significant difference in risks.

The confidence intervals calculated by predictnl are pointwise; there is no adjustment (such as a Bonferroni correction) made so that these confidence intervals may be considered jointly at the specified level.

## Methods and formulas

For the $i$ th observation, consider the transformation $g\left(\boldsymbol{\theta}, \mathbf{x}_{i}\right)$, estimated by $g\left(\widehat{\boldsymbol{\theta}}, \mathbf{x}_{i}\right)$, for the $1 \times k$ parameter vector $\boldsymbol{\theta}$ and data $\mathbf{x}_{i}$ ( $\mathbf{x}_{i}$ is assumed fixed). The variance of $g\left(\widehat{\boldsymbol{\theta}}, \mathbf{x}_{i}\right)$ is estimated by

$$
\widehat{\operatorname{Var}}\left\{g\left(\widehat{\boldsymbol{\theta}}, \mathbf{x}_{i}\right)\right\}=\mathbf{G V G}^{\prime}
$$

where $\mathbf{G}$ is the vector of derivatives

$$
\mathbf{G}=\left\{\left.\frac{\partial g\left(\boldsymbol{\theta}, \mathbf{x}_{i}\right)}{\partial \boldsymbol{\theta}}\right|_{\boldsymbol{\theta}=\widehat{\boldsymbol{\theta}}}\right\}_{(1 \times k)}
$$

and $\mathbf{V}$ is the estimated variance-covariance matrix of $\widehat{\boldsymbol{\theta}}$. Standard errors, $\widehat{\operatorname{se}}\left\{g\left(\widehat{\boldsymbol{\theta}}, \mathbf{x}_{i}\right)\right\}$, are obtained as the square roots of the variances.

The Wald test statistic for testing

$$
H_{0}: g\left(\boldsymbol{\theta}, \mathbf{x}_{i}\right)=0
$$

versus the two-sided alternative is given by

$$
W_{i}=\frac{\left\{g\left(\widehat{\boldsymbol{\theta}}, \mathbf{x}_{i}\right)\right\}^{2}}{\widehat{\operatorname{Var}}\left\{g\left(\widehat{\boldsymbol{\theta}}, \mathbf{x}_{i}\right)\right\}}
$$

When the variance-covariance matrix of $\widehat{\boldsymbol{\theta}}$ is an asymptotic covariance matrix, $W_{i}$ is approximately distributed as $\chi^{2}$ with 1 degree of freedom. For linear regression, $W_{i}$ is taken to be approximately distributed as $F_{1, r}$, where $r$ is the residual degrees of freedom from the original model fit. The levels of significance of the observation-by-observation tests of $H_{0}$ versus the two-sided alternative are given by

$$
p_{i}=\operatorname{Pr}\left(T>W_{i}\right)
$$

where $T$ is either a $\chi^{2}$ - or $F$-distributed random variable, as described above.
A $(1-\alpha) \times 100 \%$ confidence interval for $g\left(\boldsymbol{\theta}, \mathbf{x}_{i}\right)$ is given by

$$
g\left(\widehat{\boldsymbol{\theta}}, \mathbf{x}_{i}\right) \pm z_{\alpha / 2}\left[\widehat{\operatorname{se}}\left\{g\left(\widehat{\boldsymbol{\theta}}, \mathbf{x}_{i}\right)\right\}\right]
$$

when $W_{i}$ is $\chi^{2}$-distributed, and

$$
g\left(\widehat{\boldsymbol{\theta}}, \mathbf{x}_{i}\right) \pm t_{\alpha / 2, r}\left[\widehat{\operatorname{se}}\left\{g\left(\widehat{\boldsymbol{\theta}}, \mathbf{x}_{i}\right)\right\}\right]
$$

when $W_{i}$ is $F$-distributed. $z_{p}$ is the $1-p$ quantile of the standard normal distribution, and $t_{p, r}$ is the $1-p$ quantile of the $t$ distribution with $r$ degrees of freedom.

## References

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Phillips, P. C. B., and J. Y. Park. 1988. On the formulation of Wald tests of nonlinear restrictions. Econometrica 56: 1065-1083.

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Wells, K. B., R. D. Hays, M. A. Burnam, W. H. Rogers, S. Greenfield, and J. E. Ware, Jr. 1989. Detection of depressive disorder for patients receiving prepaid or fee-for-service care. Results from the Medical Outcomes Survey. Journal of the American Medical Association 262: 3298-3302.

## Also see

[R] lincom - Linear combinations of estimators
[R] nlcom - Nonlinear combinations of estimators
$[R]$ predict - Obtain predictions, residuals, etc., after estimation
$[R]$ test - Test linear hypotheses after estimation
[R] testnl - Test nonlinear hypotheses after estimation
[U] 20 Estimation and postestimation commands

## Title

probit - Probit regression

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

$$
\text { probit depvar }[\text { indepvars }][\text { if }][\text { in }][\text { weight }][\text {, options }]
$$

options
Model
noconstant
offset (varname)
asis
constraints (constraints)
collinear
SE/Robust
vce (vcetype)

Reporting
level(\#)
nocnsreport
display_options

Maximization
maximize_options
nocoef
coeflegend

Description
suppress constant term
include varname in model with coefficient constrained to 1
retain perfect predictor variables
apply specified linear constraints
keep collinear variables
vcetype may be oim, robust, cluster clustvar, bootstrap, or jackknife
indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
depvar and indepvars may contain time-series operators; see [U] 11.4.4 Time-series varlists.
bootstrap, by, fp, jackknife, mfp, mi estimate, nestreg, rolling, statsby, stepwise, and svy are allowed; see [U] 11.1.10 Prefix commands.
vce(bootstrap) and vce(jackknife) are not allowed with the mi estimate prefix; see [MI] mi estimate.
Weights are not allowed with the bootstrap prefix; see $[\mathrm{R}]$ bootstrap.
vce(), nocoef, and weights are not allowed with the svy prefix; see [SVY] svy.
fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
nocoef and coeflegend do not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Binary outcomes $>$ Probit regression

## Description

probit fits a maximum-likelihood probit model.
If estimating on grouped data, see the bprobit command described in [R] glogit.
Several auxiliary commands may be run after probit, logit, or logistic; see [R] logistic postestimation for a description of these commands.

See [R] logistic for a list of related estimation commands.

## Options

$\qquad$ Model
noconstant, offset(varname), constraints(constraints), collinear; see [R] estimation options.
asis specifies that all specified variables and observations be retained in the maximization process. This option is typically not specified and may introduce numerical instability. Normally probit drops variables that perfectly predict success or failure in the dependent variable along with their associated observations. In those cases, the effective coefficient on the dropped variables is infinity (negative infinity) for variables that completely determine a success (failure). Dropping the variable and perfectly predicted observations has no effect on the likelihood or estimates of the remaining coefficients and increases the numerical stability of the optimization process. Specifying this option forces retention of perfect predictor variables and their associated observations.
vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

## Reporting

level(\#); see [R] estimation options.
nocnsreport; see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% fmt), pformat (\% fmt), sformat (\% fmt), and nolstretch; see [R] estimation options.

Maximization
maximize_options: difficult, technique (algorithm_spec), $\underline{\text { iterate (\#), [no] log, trace, }}$ gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance(\#), nonrtolerance, and from(init_specs); see [R] maximize. These options are seldom used.

The following options are available with probit but are not shown in the dialog box:
nocoef specifies that the coefficient table not be displayed. This option is sometimes used by programmers but is of no use interactively.
coeflegend; see $[R]$ estimation options.

## Remarks and examples

Remarks are presented under the following headings:
Robust standard errors
Model identification
probit fits maximum likelihood models with dichotomous dependent (left-hand-side) variables coded as $0 / 1$ (more precisely, coded as 0 and not 0 ).

## > Example 1

We have data on the make, weight, and mileage rating of 22 foreign and 52 domestic automobiles. We wish to fit a probit model explaining whether a car is foreign based on its weight and mileage. Here is an overview of our data:

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. keep make mpg weight foreign
. describe
Contains data from http://www.stata-press.com/data/r13/auto.dta
    obs: 74 1978 Automobile Data
    vars: 4 13 Apr 2013 17:45
    size: 1,702 (_dta has notes)
```

| variable name | storage <br> type | display <br> format | value <br> label | variable label |
| :--- | :--- | :--- | :--- | :--- |
| make | str18 | $\%-18 \mathrm{~s}$ |  | Make and Model |
| mpg | int | $\% 8.0 \mathrm{~g}$ |  | Mileage (mpg) |
| weight | int | $\% 8.0 \mathrm{gc}$ |  | Weight (lbs.) |
| foreign | byte | $\% 8.0 \mathrm{~g}$ | origin | Car type |

Sorted by: foreign
Note: dataset has changed since last saved
. inspect foreign
foreign: Car type

|  | Number of Observations |  |  |
| :---: | :---: | :---: | :---: |
|  | Total | Integers | Nonintegers |
| Negative | - | - | - |
| Zero | 52 | 52 | - |
| Positive | 22 | 22 | - |
| Total | 74 | 74 | - |
| Missing | - |  |  |
|  | 74 |  |  |



The foreign variable takes on two unique values, 0 and 1 . The value 0 denotes a domestic car, and 1 denotes a foreign car.

The model that we wish to fit is

$$
\operatorname{Pr}(\text { foreign }=1)=\Phi\left(\beta_{0}+\beta_{1} \text { weight }+\beta_{2} \mathrm{mpg}\right)
$$

where $\Phi$ is the cumulative normal distribution.
To fit this model, we type

```
. probit foreign weight mpg
Iteration 0: log likelihood = -45.03321
Iteration 1: log likelihood = -27.914626
    (output omitted)
Iteration 5: log likelihood = -26.844189
Probit regression Number of obs = 74
    LR chi2(2) = 36.38
    Prob > chi2 = 0.0000
    Pseudo R2 = 0.4039
```

| foreign | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| weight | -.0023355 | .0005661 | -4.13 | 0.000 | -.003445 | -.0012261 |
| mpg | -.1039503 | .0515689 | -2.02 | 0.044 | -.2050235 | -.0028772 |
| _cons | 8.275464 | 2.554142 | 3.24 | 0.001 | 3.269437 | 13.28149 |

We find that heavier cars are less likely to be foreign and that cars yielding better gas mileage are also less likely to be foreign, at least holding the weight of the car constant.

See [R] maximize for an explanation of the output.

## Technical note

Stata interprets a value of 0 as a negative outcome (failure) and treats all other values (except missing) as positive outcomes (successes). Thus if your dependent variable takes on the values 0 and 1 , then 0 is interpreted as failure and 1 as success. If your dependent variable takes on the values 0 , 1 , and 2, then 0 is still interpreted as failure, but both 1 and 2 are treated as successes.

If you prefer a more formal mathematical statement, when you type probit $y x$, Stata fits the model

$$
\operatorname{Pr}\left(y_{j} \neq 0 \mid \mathbf{x}_{j}\right)=\Phi\left(\mathbf{x}_{j} \boldsymbol{\beta}\right)
$$

where $\Phi$ is the standard cumulative normal.

## Robust standard errors

If you specify the vce(robust) option, probit reports robust standard errors; see [U] $\mathbf{2 0 . 2 1} \mathbf{O b}$ taining robust variance estimates.

## > Example 2

For the model from example 1, the robust calculation increases the standard error of the coefficient on mpg by almost $15 \%$ :

| Probit regression <br> Log pseudolikelihood $=-26.844189$ |  |  |  | Number of obs Wald chi2(2) Prob > chi2 Pseudo R2 |  | = | $\begin{array}{r} 74 \\ 30.26 \\ 0.0000 \\ 0.4039 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| foreign | Coef. | Robust Std. Err. | z | $P>\|z\|$ | [95\% | onf | Interval] |
| weight | -. 0023355 | . 0004934 | -4.73 | 0.000 | -. 0033 |  | -. 0013686 |
| mpg | -. 1039503 | . 0593548 | -1.75 | 0.080 | -. 2202 |  | . 0123829 |
| _cons | 8.275464 | 2.539177 | 3.26 | 0.001 | 3.298 |  | 13.25216 |

Without vce(robust), the standard error for the coefficient on mpg was reported to be 0.052 with a resulting confidence interval of $[-0.21,-0.00]$.

## > Example 3

The vce(cluster clustvar) option can relax the independence assumption required by the probit estimator to independence between clusters. To demonstrate, we will switch to a different dataset.

We are studying unionization of women in the United States and have a dataset with 26,200 observations on 4,434 women between 1970 and 1988 . We will use the variables age (the women were 14-26 in 1968, and our data span the age range of 16-46), grade (years of schooling completed, ranging from 0 to 18), not_smsa ( $28 \%$ of the person-time was spent living outside an SMSA—standard metropolitan statistical area), south ( $41 \%$ of the person-time was in the South), and year. Each of these variables is included in the regression as a covariate along with the interaction between south and year. This interaction, along with the south and year variables, is specified in the probit command using factor-variables notation, south\#\#c.year. We also have variable union, indicating union membership. Overall, $22 \%$ of the person-time is marked as time under union membership, and $44 \%$ of these women have belonged to a union.

We fit the following model, ignoring that the women are observed an average of 5.9 times each in these data:


| union | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| age | .0118481 | .0029072 | 4.08 | 0.000 | .0061502 | .017546 |
| grade | .0267365 | .0036689 | 7.29 | 0.000 | .0195457 | .0339273 |
| not_smsa | -.1293525 | .0202595 | -6.38 | 0.000 | -.1690604 | -.0896445 |
| 1.south | -.8281077 | .2472219 | -3.35 | 0.001 | -1.312654 | -.3435618 |
| year | -.0080931 | .0033469 | -2.42 | 0.016 | -.0146529 | -.0015333 |
| south\#c.year |  |  |  |  |  |  |
| 1 | .0057369 | .0030917 | 1.86 | 0.064 | -.0003226 | .0117965 |
| _cons | -.6542487 | .2007777 | -3.26 | 0.001 | -1.047766 | -.2607316 |

The reported standard errors in this model are probably meaningless. Women are observed repeatedly, and so the observations are not independent. Looking at the coefficients, we find a large southern effect against unionization and a time trend for the south that is almost significantly different from the overall downward trend. The vce (cluster clustvar) option provides a way to fit this model and obtains correct standard errors:

| Iteration 0: | log pseudolikelihood $=-13864.23$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Iteration 1: | log pseudolikelihood $=-13545.541$ |  |  |  |  |  |
| Iteration 2: | log pseudolikelihood $=-13544.385$ |  |  |  |  |  |
| Iteration 3: | log pseudolikelihood $=-13544.385$ |  |  |  |  |  |
|  |  |  |  | Number of obs $=26200$ |  |  |
| Probit regression |  |  |  | Wald chi2(6) = 166.53 |  |  |
|  |  |  |  | Prob > chi2 $=0.0000$ |  |  |
| Log pseudolikelihood $=-13544.385$ |  |  |  | Pseudo R2 $=0.0231$ |  |  |
| (Std. Err. adjusted for 4434 clusters in idcode) |  |  |  |  |  |  |
|  |  | Robust |  |  |  |  |
| union | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf | Interval] |
| age | . 0118481 | . 0056625 | 2.09 | 0.036 | . 0007499 | . 0229463 |
| grade | . 0267365 | . 0078124 | 3.42 | 0.001 | . 0114244 | . 0420486 |
| not_smsa | -. 1293525 | . 0403885 | -3.20 | 0.001 | -. 2085125 | -. 0501925 |
| 1. south | -. 8281077 | . 3201584 | -2.59 | 0.010 | -1.455607 | -. 2006089 |
| year | -. 0080931 | . 0060829 | -1.33 | 0.183 | -. 0200153 | . 0038292 |
| south\#c.year |  |  |  |  |  |  |
| 1 | . 0057369 | . 0040133 | 1.43 | 0.153 | -. 002129 | . 0136029 |
| _cons | -. 6542487 | . 3485976 | -1.88 | 0.061 | -1.337487 | . 02899 |

These standard errors are larger than those reported by the inappropriate conventional calculation. By comparison, another model we could fit is an equal-correlation population-averaged probit model:

| Iteration 1: tolerance $=.12544249$ |  |  |  |
| :---: | :---: | :---: | :---: |
| Iteration 2: tolerance $=$. 0034686 |  |  |  |
| Iteration 3: tolerance $=.00017448$ |  |  |  |
| Iteration 4: tolerance $=8.382 \mathrm{e}-06$ |  |  |  |
| Iteration 5: tolerance $=3.997 \mathrm{e}-07$ |  |  |  |
| GEE population-averaged model | Number of obs | = | 26200 |
| Group variable: idcode | Number of groups | = | 4434 |
| Link: probit | Obs per group: min | = | 1 |
| Family: binomial | avg | $=$ | 5.9 |
| Correlation: exchangeable | max | = | 12 |
|  | Wald chi2(6) | = | 242.57 |
| Scale parameter: 1 | Prob > chi2 | = | 0.0000 |


| union | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| age | .0089699 | .0053208 | 1.69 | 0.092 | -.0014586 | .0193985 |
| grade | .0333174 | .0062352 | 5.34 | 0.000 | .0210966 | .0455382 |
| not_smsa | -.0715717 | .027543 | -2.60 | 0.009 | -.1255551 | -.0175884 |
| 1.south | -1.017368 | .207931 | -4.89 | 0.000 | -1.424905 | -.6098308 |
| year | -.0062708 | .0055314 | -1.13 | 0.257 | -.0171122 | .0045706 |
| south\#c.year |  |  |  |  |  |  |
| 1 | .0086294 | .00258 | 3.34 | 0.001 | .0035727 | .013686 |
| _cons | -.8670997 | .294771 | -2.94 | 0.003 | -1.44484 | -.2893592 |

The coefficient estimates are similar, but these standard errors are smaller than those produced by probit, vce(cluster clustvar), as we would expect. If the equal-correlation assumption is valid, the population-averaged probit estimator above should be more efficient.

Is the assumption valid? That is a difficult question to answer. The default population-averaged estimates correspond to an assumption of exchangeable correlation within person. It would not be unreasonable to assume an $\operatorname{AR}(1)$ correlation within person or to assume that the observations are correlated but that we do not wish to impose any structure. See [XT] xtprobit and [XT] xtgee for full details.
probit, vce(cluster clustvar) is robust to assumptions about within-cluster correlation. That is, it inefficiently sums within cluster for the standard error calculation rather than attempting to exploit what might be assumed about the within-cluster correlation.

## Model identification

The probit command has one more feature that is probably the most useful. It will automatically check the model for identification and, if the model is underidentified, drop whatever variables and observations are necessary for estimation to proceed.
$>$ Example 4
Have you ever fit a probit model where one or more of your independent variables perfectly predicted one or the other outcome?

For instance, consider the following data:

| Outcome $y$ | Independent variable $x$ |
| :---: | :---: |
| 0 | 1 |
| 0 | 1 |
| 0 | 0 |
| 1 | 0 |

Say that we wish to predict the outcome on the basis of the independent variable. The outcome is always zero when the independent variable is one. In our data, $\operatorname{Pr}(y=0 \mid x=1)=1$, which means that the probit coefficient on $x$ must be minus infinity with a corresponding infinite standard error. At this point, you may suspect that we have a problem.

Unfortunately, not all such problems are so easily detected, especially if you have many independent variables in your model. If you have ever had such difficulties, then you have experienced one of the more unpleasant aspects of computer optimization. The computer has no idea that it is trying to solve for an infinite coefficient as it begins its iterative process. All it knows is that, at each step, making the coefficient a little bigger, or a little smaller, works wonders. It continues on its merry way until either 1) the whole thing comes crashing to the ground when a numerical overflow error occurs or 2) it reaches some predetermined cutoff that stops the process. Meanwhile, you have been waiting. And the estimates that you finally receive, if any, may be nothing more than numerical roundoff.

Stata watches for these sorts of problems, alerts you, fixes them, and then properly fits the model.
Let's return to our automobile data. Among the variables we have in the data is one called repair that takes on three values. A value of 1 indicates that the car has a poor repair record, 2 indicates an average record, and 3 indicates a better-than-average record. Here is a tabulation of our data:

```
. use http://www.stata-press.com/data/r13/repair
(1978 Automobile Data)
. tabulate foreign repair
\begin{tabular}{|c|c|c|c|c|}
\hline & \multicolumn{3}{|c|}{repair} & \multirow[b]{2}{*}{Total} \\
\hline Car type & 1 & 2 & 3 & \\
\hline Domestic & 10 & 27 & 9 & 46 \\
\hline Foreign & 0 & 3 & 9 & 12 \\
\hline Total & 10 & 30 & 18 & 58 \\
\hline
\end{tabular}
```

All the cars with poor repair records (repair $=1$ ) are domestic. If we were to attempt to predict foreign on the basis of the repair records, the predicted probability for the repair $=1$ category would have to be zero. This in turn means that the probit coefficient must be minus infinity, and that would set most computer programs buzzing.

Let's try using Stata on this problem.

```
. probit foreign b3.repair
note: 1.repair != 0 predicts failure perfectly
    1.repair dropped and 10 obs not used
Iteration 0: log likelihood = -26.992087
Iteration 1: log likelihood = -22.276479
Iteration 2: log likelihood = -22.229184
Iteration 3: log likelihood = -22.229138
Iteration 4: log likelihood = -22.229138
```

Probit regression $\quad$ Number of obs $=\quad 48$
LR chi2 (1) $=9.53$
Prob > chi2 $=0.0020$
Pseudo R2 $=0.1765$

| foreign | Coef. | Std. Err. | $z$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | :---: |
| repair |  | 0 | (empty) |  |  |  |
| 1 |  |  |  |  |  |  |
| 2 | -1.281552 | .4297326 | -2.98 | 0.003 | -2.123812 | -.4392911 |
| _cons | $9.89 \mathrm{e}-17$ | .295409 | 0.00 | 1.000 | -.578991 | .578991 |

Remember that all the cars with poor repair records (repair $=1$ ) are domestic, so the model cannot be fit, or at least it cannot be fit if we restrict ourselves to finite coefficients. Stata noted that fact "note: 1.repair != 0 predicts failure perfectly". This is Stata's mathematically precise way of saying what we said in English. When repair is 1 , the car is domestic.

Stata then went on to say, "1.repair dropped and 10 obs not used". This is Stata eliminating the problem. First, 1.repair had to be removed from the model because it would have an infinite coefficient. Then the 10 observations that led to the problem had to be eliminated, as well, so as not to bias the remaining coefficients in the model. The 10 observations that are not used are the 10 domestic cars that have poor repair records.

Stata then fit what was left of the model, using the remaining observations. Because no observations remained for cars with poor repair records, Stata reports "(empty)" in the row for repair $=1$.

- Technical note

Stata is pretty smart about catching these problems. It will catch "one-way causation by a dummy variable", as we demonstrated above.

Stata also watches for "two-way causation", that is, a variable that perfectly determines the outcome, both successes and failures. Here Stata says that the variable "predicts outcome perfectly" and stops. Statistics dictate that no model can be fit.

Stata also checks your data for collinear variables; it will say "so-and-so omitted because of collinearity". No observations need to be eliminated here and model fitting will proceed without the offending variable.

It will also catch a subtle problem that can arise with continuous data. For instance, if we were estimating the chances of surviving the first year after an operation, and if we included in our model age, and if all the persons over 65 died within the year, Stata will say, "age $>65$ predicts failure perfectly". It will then inform us about how it resolves the issue and fit what can be fit of our model.
probit (and logit, logistic, and ivprobit) will also occasionally fail to converge and then display messages such as

Note: 4 failures and 0 successes completely determined.
The cause of this message and what to do if you see it are described in [R] logit.

## Stored results

probit stores the following in e():
Scalars

| e(N) | number of observations |
| :---: | :---: |
| e(N_cds) | number of completely determined successes |
| e(N_cdf) | number of completely determined failures |
| e(k) | number of parameters |
| e(k_eq) | number of equations in $\mathrm{e}(\mathrm{b})$ |
| e(k_eq_model) | number of equations in overall model test |
| e(k_dv) | number of dependent variables |
| $e\left(d f \_m\right)$ | model degrees of freedom |
| e(r2_p) | pseudo- $R$-squared |
| e(11) | log likelihood |
| e(11_0) | log likelihood, constant-only model |
| e(N_clust) | number of clusters |
| e(chi2) | $\chi^{2}$ |
| e (p) | significance of model test |
| $e(r a n k)$ | rank of e(V) |
| e(ic) | number of iterations |
| e(rc) | return code |
| e(converged) | 1 if converged, 0 otherwise |

Macros
e(cmd)
$e$ (cmdline)
e(depvar)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(offset)
e(chi2type)
e(vce)
e(vcetype)
e(opt)
e(which)
e(ml_method)
e(user)
$e$ (technique)
e(properties)
e(estat_cmd)
e(predict)
e(asbalanced)
e(asobserved)
Matrices
e(b)
e(Cns)
e(ilog)
e(gradient)
$e$ (mns)
e(rules)
e(V)
e(V_modelbased)
Functions
e(sample)
probit
command as typed
name of dependent variable
weight type
weight expression
title in estimation output
name of cluster variable
linear offset variable
Wald or LR; type of model $\chi^{2}$ test
vcetype specified in vce()
title used to label Std. Err.
type of optimization
max or min; whether optimizer is to perform maximization or minimization type of ml method
name of likelihood-evaluator program
maximization technique
b V
program used to implement estat
program used to implement predict
factor variables fvset as asbalanced
factor variables fvset as asobserved
coefficient vector
constraints matrix
iteration $\log$ (up to 20 iterations)
gradient vector
vector of means of the independent variables information about perfect predictors
variance-covariance matrix of the estimators
model-based variance
marks estimation sample

## Methods and formulas

Probit analysis originated in connection with bioassay, and the word probit, a contraction of "probability unit", was suggested by Bliss (1934a, 1934b). For an introduction to probit and logit, see, for example, Aldrich and Nelson (1984), Cameron and Trivedi (2010), Greene (2012), Long (1997), Pampel (2000), or Powers and Xie (2008). Long and Freese (2014, chap. 5 and 6) and Jones (2007, chap. 3) provide introductions to probit and logit, along with Stata examples.

The log-likelihood function for probit is

$$
\ln L=\sum_{j \in S} w_{j} \ln \Phi\left(\mathbf{x}_{j} \boldsymbol{\beta}\right)+\sum_{j \notin S} w_{j} \ln \left\{1-\Phi\left(\mathbf{x}_{j} \boldsymbol{\beta}\right)\right\}
$$

where $\Phi$ is the cumulative normal and $w_{j}$ denotes the optional weights. $\ln L$ is maximized, as described in [R] maximize.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce (robust) and vce (cluster clustvar), respectively. See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas. The scores are calculated as $\mathbf{u}_{j}=$ $\left\{\phi\left(\mathbf{x}_{j} \mathbf{b}\right) / \Phi\left(\mathbf{x}_{j} \mathbf{b}\right)\right\} \mathbf{x}_{j}$ for the positive outcomes and $-\left[\phi\left(\mathbf{x}_{j} \mathbf{b}\right) /\left\{1-\Phi\left(\mathbf{x}_{j} \mathbf{b}\right)\right\}\right] \mathbf{x}_{j}$ for the negative outcomes, where $\phi$ is the normal density.
probit also supports estimation with survey data. For details on VCEs with survey data, see [SVY] variance estimation.

Chester Ittner Bliss (1899-1979) was born in Ohio. He was educated as an entomologist, earning degrees from Ohio State and Columbia, and was employed by the United States Department of Agriculture until 1933. When he lost his job because of the Depression, Bliss then worked with R. A. Fisher in London and at the Institute of Plant Protection in Leningrad before returning to a post at the Connecticut Agricultural Experiment Station in 1938. He was also a lecturer at Yale for 25 years. Among many contributions to biostatistics, his development and application of probit methods to biological problems are outstanding.

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## Also see

[R] probit postestimation - Postestimation tools for probit
[R] asmprobit - Alternative-specific multinomial probit regression
[R] biprobit - Bivariate probit regression
[R] brier - Brier score decomposition
[R] glm - Generalized linear models
$[R]$ heckoprobit - Ordered probit model with sample selection
[R] hetprobit - Heteroskedastic probit model
[R] ivprobit - Probit model with continuous endogenous regressors
[R] logistic - Logistic regression, reporting odds ratios
[R] logit - Logistic regression, reporting coefficients
[R] mprobit - Multinomial probit regression
$[\mathrm{R}]$ roc - Receiver operating characteristic (ROC) analysis
[R] scobit - Skewed logistic regression
[ME] meprobit - Multilevel mixed-effects probit regression
[MI] estimation - Estimation commands for use with mi estimate
[SVY] svy estimation - Estimation commands for survey data
[XT] xtprobit - Random-effects and population-averaged probit models
[U] 20 Estimation and postestimation commands

## Title

probit postestimation - Postestimation tools for probit

| Description | Syntax for predict | Menu for predict |
| :--- | :--- | :--- | Options for predict

## Description

The following postestimation commands are of special interest after probit:
Command Description

```
estat classification report various summary statistics, including the classification table
estat gof Pearson or Hosmer-Lemeshow goodness-of-fit test
lroc compute area under ROC curve and graph the curve
lsens graph sensitivity and specificity versus probability cutoff
```

These commands are not appropriate after the svy prefix.

The following standard postestimation commands are also available:

| Command | Description |
| :--- | :--- |
| contrast | contrasts and ANOVA-style joint tests of estimates |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample <br> estat vce |
| variance-covariance matrix of the estimators (VCE) |  |
| estat (svy) | postestimation statistics for survey data <br> estimates <br> forecaloging estimation results <br> hausman |
| dynamic forecasts and simulations |  |
| lincom | Hausman's specification test |
| point estimates, standard errors, testing, and inference for linear |  |
| linktest | combinations of coefficients |
| lrtest | link test for model specification <br> margins |
| likelihood-ratio test |  |

[^61]
## Syntax for predict

predict [type] newvar [if] [in] [, statistic nooffset rules asif]
statistic Description
Main
pr probability of a positive outcome; the default
$\mathrm{xb} \quad$ linear prediction
stdp standard error of the linear prediction

* deviance
deviance residual
score
first derivative of the $\log$ likelihood with respect to $\mathbf{x}_{j} \boldsymbol{\beta}$
Unstarred statistics are available both in and out of sample; type predict . . . if e (sample) ... if wanted only for the estimation sample. Starred statistics are calculated only for the estimation sample, even when if e(sample) is not specified.


## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
pr , the default, calculates the probability of a positive outcome.
xb calculates the linear prediction.
stdp calculates the standard error of the linear prediction.
deviance calculates the deviance residual.
score calculates the equation-level score, $\partial \ln L / \partial\left(\mathbf{x}_{j} \boldsymbol{\beta}\right)$.
nooffset is relevant only if you specified offset (varname) for probit. It modifies the calculations made by predict so that they ignore the offset variable; the linear prediction is treated as $\mathbf{x}_{j} \mathbf{b}$ rather than as $\mathbf{x}_{j} \mathbf{b}+$ offset $_{j}$.
rules requests that Stata use any rules that were used to identify the model when making the prediction. By default, Stata calculates missing for excluded observations.
asif requests that Stata ignore the rules and exclusion criteria and calculate predictions for all observations possible using the estimated parameter from the model.

## Remarks and examples

Remarks are presented under the following headings:
Obtaining predicted values
Performing hypothesis tests

## Obtaining predicted values

Once you have fit a probit model, you can obtain the predicted probabilities by using the predict command for both the estimation sample and other samples; see [U] 20 Estimation and postestimation commands and $[\mathrm{R}]$ predict. Here we will make only a few additional comments.
predict without arguments calculates the predicted probability of a positive outcome. With the xb option, predict calculates the linear combination $\mathbf{x}_{j} \mathbf{b}$, where $\mathbf{x}_{j}$ are the independent variables in the $j$ th observation and $\mathbf{b}$ is the estimated parameter vector. This is known as the index function because the cumulative density indexed at this value is the probability of a positive outcome.

In both cases, Stata remembers any rules used to identify the model and calculates missing for excluded observations unless rules or asif is specified. This is covered in the following example.

With the stdp option, predict calculates the standard error of the prediction, which is not adjusted for replicated covariate patterns in the data.

You can calculate the unadjusted-for-replicated-covariate-patterns diagonal elements of the hat matrix, or leverage, by typing
. predict pred
. predict stdp, stdp
. generate hat $=$ stdp^ $2 *$ pred $*(1-$ pred $)$

## > Example 1

In example 4 of [R] probit, we fit the probit model probit foreign b3.repair. To obtain predicted probabilities, we type

```
. predict p
(option pr assumed; Pr(foreign))
(10 missing values generated)
```

. summarize foreign p

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| foreign | 58 | .2068966 | .4086186 | 0 | 1 |
| $p$ | 48 | .25 | .1956984 | .1 | .5 |

Stata remembers any rules used to identify the model and sets predictions to missing for any excluded observations. In example 4 of $[\mathrm{R}]$ probit, probit dropped the variable 1.repair from our model and excluded 10 observations. When we typed predict $p$, those same 10 observations were again excluded and their predictions set to missing.
predict's rules option uses the rules in the prediction. During estimation, we were told, "1.repair
 probability of success or a positive outcome:

```
. predict p2, rules
(option pr assumed; Pr(foreign))
. summarize foreign p p2
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline foreign & 58 & .2068966 & .4086186 & 0 & 1 \\
p & 48 & .25 & .1956984 & .1 & .5 \\
p2 & 58 & .2068966 & .2016268 & 0 & .5
\end{tabular}
```

predict's asif option ignores the rules and the exclusion criteria and calculates predictions for all observations possible using the estimated parameters from the model:

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| :---: | :---: | :---: | :---: | :---: | :---: |
| foreign | 58 | . 2068966 | . 4086186 | 0 | 1 |
| p | 48 | . 25 | . 1956984 | . 1 | . 5 |
| p2 | 58 | . 2068966 | . 2016268 | 0 | . 5 |
| p3 | 58 | . 2931034 | . 2016268 | . 1 | . 5 |

Which is right? By default, predict uses the most conservative approach. If many observations had been excluded due to a simple rule, we could be reasonably certain that the rules prediction is correct. The asif prediction is correct only if the exclusion is a fluke and we would be willing to exclude the variable from the analysis, anyway. Then, however, we should refit the model to include the excluded observations.

## Performing hypothesis tests

After estimation with probit, you can perform hypothesis tests by using the test or testnl command; see [U] 20 Estimation and postestimation commands.

## Methods and formulas

Let index $j$ be used to index observations. Define $M_{j}$ for each observation as the total number of observations sharing $j$ 's covariate pattern. Define $Y_{j}$ as the total number of positive responses among observations sharing $j$ 's covariate pattern. Define $p_{j}$ as the predicted probability of a positive outcome for observation $j$.

For $M_{j}>1$, the deviance residual $d_{j}$ is defined as

$$
d_{j}= \pm\left(2\left[Y_{j} \ln \left(\frac{Y_{j}}{M_{j} p_{j}}\right)+\left(M_{j}-Y_{j}\right) \ln \left\{\frac{M_{j}-Y_{j}}{M_{j}\left(1-p_{j}\right)}\right\}\right]\right)^{1 / 2}
$$

where the sign is the same as the sign of $\left(Y_{j}-M_{j} p_{j}\right)$. In the limiting cases, the deviance residual is given by

$$
d_{j}= \begin{cases}-\sqrt{2 M_{j}\left|\ln \left(1-p_{j}\right)\right|} & \text { if } Y_{j}=0 \\ \sqrt{2 M_{j}\left|\ln p_{j}\right|} & \text { if } Y_{j}=M_{j}\end{cases}
$$

## Also see

[R] probit - Probit regression
[R] estat classification - Classification statistics and table
[R] estat gof - Pearson or Hosmer-Lemeshow goodness-of-fit test
$[R]$ lroc - Compute area under ROC curve and graph the curve
$[R]$ Isens - Graph sensitivity and specificity versus probability cutoff
[U] 20 Estimation and postestimation commands

## Title

proportion - Estimate proportions

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

$$
\text { proportion varlist }[\text { if }][\text { in }][\text { weight }][\text {, options }]
$$

options

```
Model
    stdize(varname)
    stdweight(varname)
    nostdrescale
    nolabel
    missing
```

iffin/over
over (varlist [, nolabel])
SE/Cluster
vce(vcetype)
Reporting
level(\#)
citype(logit|normal)
noheader
nolegend
display_options
coeflegend

Description
variable identifying strata for standardization weight variable for standardization do not rescale the standard weight variable suppress value labels from varlist treat missing values like other values
group over subpopulations defined by varlist; optionally, suppress group labels
vcetype may be analytic, cluster clustvar, bootstrap, or jackknife
set confidence level; default is level (95)
method to compute limits of confidence intervals; default is citype(logit)
suppress table header
suppress table legend
control column formats and line width
display legend instead of statistics
bootstrap, jackknife, mi estimate, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands.
vce (bootstrap) and vce (jackknife) are not allowed with the mi estimate prefix; see [MI] mi estimate.
Weights are not allowed with the bootstrap prefix; see $[R]$ bootstrap.
vce() and weights are not allowed with the svy prefix; see [SVY] svy.
fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Summaries, tables, and tests $>$ Summary and descriptive statistics $>$ Proportions

## Description

proportion produces estimates of proportions, along with standard errors, for the categories identified by the values in each variable of varlist.

## Options

$\qquad$ Model
stdize (varname) specifies that the point estimates be adjusted by direct standardization across the strata identified by varname. This option requires the stdweight() option.
stdweight (varname) specifies the weight variable associated with the standard strata identified in the stdize() option. The standardization weights must be constant within the standard strata.
nostdrescale prevents the standardization weights from being rescaled within the over() groups.
This option requires stdize() but is ignored if the over() option is not specified.
nolabel specifies that value labels attached to the variables in varlist be ignored.
missing specifies that missing values in varlist be treated as valid categories, rather than omitted from the analysis (the default).


When this option is supplied with one variable name, such as over (varname), the value labels of varname are used to identify the subpopulations. If varname does not have labeled values (or there are unlabeled values), the values themselves are used, provided that they are nonnegative integers. Noninteger values, negative values, and labels that are not valid Stata names are substituted with a default identifier.
When over() is supplied with multiple variable names, each subpopulation is assigned a unique default identifier.
nolabel requests that value labels attached to the variables identifying the subpopulations be ignored.

[^62]citype (logit |normal) specifies how to compute the limits of confidence intervals.
citype(logit), the default, uses the logit transformation to compute the limits of confidence intervals.
citype (normal) uses the normal approximation to compute the limits of confidence intervals.
noheader prevents the table header from being displayed. This option implies nolegend.
nolegend prevents the table legend identifying the subpopulations from being displayed.
display_options: cformat (\%fmt) and nolstretch; see [R] estimation options.
The following option is available with proportion but is not shown in the dialog box: coeflegend; see $[R]$ estimation options.

## Remarks and examples

## > Example 1

We can estimate the proportion of each repair rating in auto2.dta:

| Proportion estimation |  | Number of obs |  | $=69$ |
| :---: | :---: | :---: | :---: | :---: |
|  | Proportion | Std. Err. | [95\% Conf | Interval] |
| rep78 |  |  |  |  |
| Poor | . 0289855 | . 0203446 | . 0070061 | . 1121326 |
| Fair | . 115942 | . 0388245 | . 0580159 | . 2183014 |
| Average | . 4347826 | . 0601159 | . 3207109 | . 556206 |
| Good | . 2608696 | . 0532498 | . 1690271 | . 3798066 |
| Excellent | . 1594203 | . 0443922 | . 089188 | . 2686455 |

Here we use the missing option to include missing values as a category of rep78:

```
. proportion rep78, missing
Proportion estimation Number of obs = 74
    _prop_6: rep78 = .
```

|  | Proportion | Std. Err. | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: |
| rep78 |  |  |  |  |
| Poor | .027027 | .0189796 | .0065484 | .1047932 |
| Fair | .1081081 | .0363433 | .054094 | .204402 |
| Average | .4054054 | .0574637 | .2977369 | .523012 |
| Good | .2432432 | .0502154 | .1572724 | .3563376 |
| Excellent | .1486486 | .0416364 | .0831005 | .2517065 |
| _prop_6 | .0675676 | .0293776 | .0278144 | .1550743 |

## Example 2

We can also estimate proportions over groups:

```
. proportion rep78, over(foreign)
    Poor: rep78 = Poor
        Fair: rep78 = Fair
        Average: rep78 = Average
        Good: rep78 = Good
    Excellent: rep78 = Excellent
        Domestic: foreign = Domestic
        Foreign: foreign = Foreign
```

Proportion estimation Number of obs $=69$

| Over | Proportion | Std. Err. | [95\% Con | Interval] |
| :---: | :---: | :---: | :---: | :---: |
| Poor |  |  |  |  |
| Domestic | . 0416667 | . 0291477 | . 0100299 | . 1572433 |
| Foreign | (no observations) |  |  |  |
| Fair |  |  |  |  |
| Domestic | . 1666667 | . 0543607 | . 0839032 | . 3039797 |
| Foreign |  | (no observa |  |  |
| Average |  |  |  |  |
| Domestic | . 5625 | . 0723605 | . 4169211 | . 6980553 |
| Foreign | . 1428571 | . 0782461 | . 0444941 | . 3736393 |
| Good |  |  |  |  |
| Domestic | . 1875 | . 0569329 | . 0986718 | . 3272601 |
| Foreign | . 4285714 | . 1106567 | . 2333786 | . 6488451 |
| Excellent |  |  |  |  |
| Domestic | . 0416667 | . 0291477 | . 0100299 | . 1572433 |
| Foreign | . 4285714 | . 1106567 | . 2333786 | . 6488451 |

## Stored results

proportion stores the following in e():
Scalars
e(N)
e(N_over)
e(N_stdize)
e(N_clust)
e(k_eq)
e(df_r)
e(rank)
Macros
e(cmd
$e$ (cmdline)
e(varlist)
e(stdize)
e(stdweight)
e(wtype)
e(wexp)
e(title)
e(cluster)
e(over)
e(over_labels)
e(over_namelist)
e(namelist)
e(label\#)
e(vce)
e(vcetype)
e(properties)
e(estat_cmd)
e(marginsnotok)
Matrices
e(b)
e(V)
e(_N)
e(_N_stdsum)
e(_p_stdize)
e(error)
Functions
e(sample) marks estimation sample

## Methods and formulas

Proportions are means of indicator variables; see $[\mathrm{R}]$ mean.

## Confidence intervals

Confidence intervals for proportions are calculated using a logit transform so that the endpoints lie between 0 and 1 . Let $\widehat{p}$ be an estimated proportion and $\widehat{s}$ be an estimate of its standard error. Let

$$
f(\widehat{p})=\ln \left(\frac{\widehat{p}}{1-\widehat{p}}\right)
$$

be the logit transform of the proportion. In this metric, an estimate of the standard error is

$$
\widehat{\mathrm{SE}}\{f(\widehat{p})\}=f^{\prime}(\widehat{p}) \widehat{s}=\frac{\widehat{s}}{\widehat{p}(1-\widehat{p})}
$$

Thus a $100(1-\alpha) \%$ confidence interval in this metric is

$$
\ln \left(\frac{\widehat{p}}{1-\widehat{p}}\right) \pm \frac{t_{1-\alpha / 2, \nu} \widehat{s}}{\widehat{p}(1-\widehat{p})}
$$

where $t_{1-\alpha / 2, \nu}$ is the $(1-\alpha / 2)$ th quantile of Student's $t$ distribution with $\nu$ degrees of freedom. The endpoints of this confidence interval are transformed back to the proportion metric by using the inverse of the logit transform

$$
f^{-1}(y)=\frac{e^{y}}{1+e^{y}}
$$

Hence, the displayed confidence intervals for proportions are

$$
f^{-1}\left\{\ln \left(\frac{\widehat{p}}{1-\widehat{p}}\right) \pm \frac{t_{1-\alpha / 2, \nu} \widehat{s}}{\widehat{p}(1-\widehat{p})}\right\}
$$

## References

Cochran, W. G. 1977. Sampling Techniques. 3rd ed. New York: Wiley.
Stuart, A., and J. K. Ord. 1994. Kendall's Advanced Theory of Statistics: Distribution Theory, Vol I. 6th ed. London: Arnold.

## Also see

$[\mathrm{R}]$ proportion postestimation - Postestimation tools for proportion
[R] mean - Estimate means
[ R$]$ ratio - Estimate ratios
[R] total - Estimate totals
[MI] estimation - Estimation commands for use with mi estimate
[SVY] direct standardization - Direct standardization of means, proportions, and ratios
[SVY] poststratification - Poststratification for survey data
[SVY] subpopulation estimation - Subpopulation estimation for survey data
[SVY] svy estimation - Estimation commands for survey data
[SVY] variance estimation - Variance estimation for survey data
[U] 20 Estimation and postestimation commands

## Title

proportion postestimation - Postestimation tools for proportion

## Description Remarks and examples Also see

## Description

The following postestimation commands are available after proportion:

| Command | Description |
| :--- | :--- |
| estat vce | variance-covariance matrix of the estimators (VCE) <br> estat (svy) <br> estimates <br> lincom |
| cataloging estimation results <br> point estimates, standard errors, testing, and inference for linear <br> combinations of coefficients |  |
| nlcom | point estimates, standard errors, testing, and inference for nonlinear <br> combinations of coefficients |
| test | Wald tests of simple and composite linear hypotheses |
| testnl | Wald tests of nonlinear hypotheses |

## Remarks and examples

## > Example 1

In example 2 of $[\mathrm{R}]$ proportion, we computed the proportions of cars with different repair records for each group, foreign or domestic. We use test to test whether the proportion of cars with repair record equal to 4 is the same for domestic and foreign cars.

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. proportion rep78, over(foreign)
    (output omitted)
. test [_prop_4]:Domestic=[_prop_4]:Foreign
( 1) [_prop_4]Domestic - [_prop_4]Foreign = 0
    F( 1, 68) = 3.75
            Prob > F = 0.0569
```

There is not a significant difference between those proportions at the $5 \%$ level.

## $>$ Example 2

Continuing with auto.dta from example 1, we generate a new variable, highprice, that indicates if the price is larger than $\$ 5,000$ and then use proportion to see the proportion of cars with high price among domestic and foreign cars separately.

| Proportion es _prop_1 _prop_2 <br> Domestic Foreign | imation <br> highprice <br> highprice <br> foreign = <br> foreign = | estic <br> eign | of obs | 74 |
| :---: | :---: | :---: | :---: | :---: |
| Over | Proportion | Std. Err. | [95\% Con | Interval] |
| _prop_1 |  |  |  |  |
| Domestic | . 5576923 | . 0695464 | . 4182157 | . 6886264 |
| Foreign | . 3636364 | . 1049728 | . 1879015 | . 5852765 |
| _prop_2 |  |  |  |  |
| Domestic | . 4423077 | . 0695464 | . 3113736 | . 5817843 |
| Foreign | . 6363636 | . 1049728 | . 4147235 | . 8120985 |

We will compute the odds ratio of having a high price in group Foreign to having a high price in group Domestic. Usually, odds ratios are computed by using the logistic command, but here we will perform the computation by using nlcom after proportion.

```
. nlcom OR: ([_prop_2]_b[Foreign]/[_prop_1]_b[Foreign])/([_prop_2]_b[Domestic]/
> [_prop_1]_b[Domestic])
```

    OR: ([_prop_2]_b[Foreign]/[_prop_1]_b[Foreign])/([_prop_2]_b[Domesti
    > c]/[_prop_1]_b[Domestic])

| Proportion | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| OR | 2.206522 | 1.178522 | 1.87 | 0.061 | -.1033393 | 4.516383 |

This is the same odds ratio that we would obtain from

```
. logistic highprice foreign
```

The odds ratio is slightly larger than 2 , which means that the odds of having a high price among foreign cars are more than twice that of having a high price among domestic cars.

## Also see

[R] proportion - Estimate proportions
[SVY] svy postestimation - Postestimation tools for svy
[U] 20 Estimation and postestimation commands

## Title

prtest - Tests of proportions

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

One-sample test of proportion

$$
\text { prtest varname }==\#_{p}[\text { if }][\text { in }][, \underline{l e v e l}(\#)]
$$

Two-sample test of proportions using groups

```
prtest varname [if] [in], by(groupvar) [level(#)]
```

Two-sample test of proportions using variables

$$
\text { prtest } \text { varname }_{1}==\text { varname }_{2}[\text { if }][\text { in }][\text {, level }(\#)]
$$

Immediate form of one-sample test of proportion prtesti $\#_{\text {obs1 }} \#_{p 1} \#_{p 2}[$, level (\#) count $]$

Immediate form of two-sample test of proportions prtesti $\#_{\mathrm{obs} 1} \#_{p 1} \#_{\mathrm{obs} 2} \#_{p 2}[$, level(\#) count $]$ by is allowed with prtest; see [D] by.

## Menu

## prtest

Statistics $>$ Summaries, tables, and tests $>$ Classical tests of hypotheses $>$ Proportion test

## prtesti

Statistics $>$ Summaries, tables, and tests $>$ Classical tests of hypotheses $>$ Proportion test calculator

## Description

prtest performs tests on the equality of proportions using large-sample statistics.
In the first form, prtest tests that varname has a proportion of $\#_{p}$. In the second form, prtest tests that varname has the same proportion within the two groups defined by groupvar. In the third form, prtest tests that varname $1_{1}$ and varname $_{2}$ have the same proportion.
prtesti is the immediate form of prtest; see [U] 19 Immediate commands.

The bitest command is a better version of the first form of prtest in that it gives exact $p$-values. Researchers should use bitest when possible, especially for small samples; see $[R]$ bitest.

## Options

$\qquad$ Main
by (groupvar) specifies a numeric variable that contains the group information for a given observation. This variable must have only two values. Do not confuse the by () option with the by prefix; both may be specified.
level(\#) specifies the confidence level, as a percentage, for confidence intervals. The default is level (95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals.
count specifies that integer counts instead of proportions be used in the immediate forms of prtest.
In the first syntax, prtesti expects that $\#_{\mathrm{obs} 1}$ and $\#_{p 1}$ are counts- $\#_{p 1} \leq \#_{\mathrm{obs} 1}$-and $\#_{p 2}$ is a proportion. In the second syntax, prtesti expects that all four numbers are integer counts, that $\#_{\mathrm{obs} 1} \geq \#_{p 1}$, and that $\#_{\mathrm{obs} 2} \geq \#_{p 2}$.

## Remarks and examples

The prtest output follows the output of ttest in providing a lot of information. Each proportion is presented along with a confidence interval. The appropriate one- or two-sample test is performed, and the two-sided and both one-sided results are included at the bottom of the output. For a two-sample test, the calculated difference is also presented with its confidence interval. This command may be used for both large-sample testing and large-sample interval estimation.

## > Example 1: One-sample test of proportion

In the first form, prtest tests whether the mean of the sample is equal to a known constant. Assume that we have a sample of 74 automobiles. We wish to test whether the proportion of automobiles that are foreign is different from $40 \%$.


The test indicates that we cannot reject the hypothesis that the proportion of foreign automobiles is 0.40 at the $5 \%$ significance level.

## > Example 2: Two-sample test of proportions

We have two headache remedies that we give to patients. Each remedy's effect is recorded as 0 for failing to relieve the headache and 1 for relieving the headache. We wish to test the equality of the proportion of people relieved by the two treatments.

```
. use http://www.stata-press.com/data/r13/cure
```

- prtest cure1 == cure2
Two-sample test of proportions cure1: Number of obs $=\quad 50$
cure2: Number of obs = 59

| Variable | Mean | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | :---: |
| cure1 | .52 | .0706541 |  |  | .3815205 | .6584795 |
| cure2 | .7118644 | .0589618 |  |  | .5963013 | .8274275 |
| diff | -.1918644 <br> under Ho: | .0920245 |  |  | -.372229 | -.0114998 |
|  |  | -2.06 | 0.039 |  |  |  |


| diff $=$ prop(cure1) | $-\operatorname{prop}($ cure2 $)$ | $z=-2.0605$ |
| ---: | ---: | ---: |
| Ho: diff $=0$ |  |  |
| Ha: diff $<0$ | $H a: \operatorname{diff}!=0$ | Ha: diff $>0$ |
| $\operatorname{Pr}(Z<z)=0.0197$ | $\operatorname{Pr}(\|Z\|<\|z\|)=0.0394$ | $\operatorname{Pr}(Z>z)=0.9803$ |

We find that the proportions are statistically different from each other at any level greater than $3.9 \%$.

## $>$ Example 3: Immediate form of one-sample test of proportion

prtesti is like prtest, except that you specify summary statistics rather than variables as arguments. For instance, we are reading an article that reports the proportion of registered voters among 50 randomly selected eligible voters as 0.52 . We wish to test whether the proportion is 0.7 :

| One-sample test of proportion |  |  | x : Number of obs $=50$ |
| :---: | :---: | :---: | :---: |
| Variable | Mean | Std. Err. | [95\% Conf. Interval] |
| x | . 52 | . 0706541 | . 3815205.6584795 |
| $\mathrm{p}=$ proportion( x ) |  |  | $z=-2.7775$ |
| Ho: $p=0.7$ |  |  |  |
| На: $\mathrm{p}<0.7$ |  |  | $\begin{aligned} H a: p & >0.7 \\ \operatorname{Pr}(Z>z) & =0.9973 \end{aligned}$ |
| $\operatorname{Pr}(\mathrm{Z}<\mathrm{z})=0.0027$ |  | $\operatorname{Pr}(\|z\|>\|z\|)=0.0055$ |  |

## > Example 4: Immediate form of two-sample test of proportions

To judge teacher effectiveness, we wish to test whether the same proportion of people from two classes will answer an advanced question correctly. In the first classroom of 30 students, $40 \%$ answered the question correctly, whereas in the second classroom of 45 students, $67 \%$ answered the question correctly.
. prtesti 30.445 . 67
Two-sample test of proportions $x:$ Number of obs $=\quad 30$
$y$ : Number of obs $=\quad 45$

| Variable | Mean | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| x | .4 | .0894427 |  |  | .2246955 | .5753045 |
| y | .67 | .0700952 |  |  | .532616 | .807384 |
| diff | -.27 | .1136368 |  |  | -.4927241 | -.0472759 |
|  | under Ho: | .1169416 | -2.31 | 0.021 |  |  |


| diff $=$ prop $(x)-\operatorname{prop}(y)$ | $z=-2.3088$ |  |
| :---: | ---: | ---: |
| Ho: diff $=0$ |  |  |
| Ha: diff $<0$ | $H a: \operatorname{diff}!=0$ | Ha: diff $>0$ |
| $\operatorname{Pr}(Z<z)=0.0105$ | $\operatorname{Pr}(\|Z\|<\|z\|)=0.0210$ | $\operatorname{Pr}(Z>z)=0.9895$ |

## Stored results

prtest and prtesti store the following in $r()$ :
Scalars
$\mathrm{r}(\mathrm{z}) \quad z$ statistic
r(P_\#) proportion for variable \#
$r\left(N \_\#\right)$ number of observations for variable \#

## Methods and formulas

See Acock $(2014,155-161)$ for additional examples of tests of proportions using Stata.
A large-sample $100(1-\alpha) \%$ confidence interval for a proportion $p$ is

$$
\widehat{p} \pm z_{1-\alpha / 2} \sqrt{\frac{\widehat{p} \widehat{q}}{n}}
$$

and a $100(1-\alpha) \%$ confidence interval for the difference of two proportions is given by

$$
\left(\widehat{p}_{1}-\widehat{p}_{2}\right) \pm z_{1-\alpha / 2} \sqrt{\frac{\widehat{p}_{1} \widehat{q}_{1}}{n_{1}}+\frac{\widehat{p}_{2} \widehat{q}_{2}}{n_{2}}}
$$

where $\widehat{q}=1-\widehat{p}$ and $z$ is calculated from the inverse cumulative standard normal distribution.
The one-tailed and two-tailed tests of a population proportion use a normally distributed test statistic calculated as

$$
z=\frac{\widehat{p}-p_{0}}{\sqrt{p_{0} q_{0} / n}}
$$

where $p_{0}$ is the hypothesized proportion. A test of the difference of two proportions also uses a normally distributed test statistic calculated as

$$
z=\frac{\widehat{p}_{1}-\widehat{p}_{2}}{\sqrt{\widehat{p}_{p} \widehat{q}_{p}\left(1 / n_{1}+1 / n_{2}\right)}}
$$

where

$$
\widehat{p}_{p}=\frac{x_{1}+x_{2}}{n_{1}+n_{2}}
$$

and $x_{1}$ and $x_{2}$ are the total number of successes in the two populations.

## References

Acock, A. C. 2014. A Gentle Introduction to Stata. 4th ed. College Station, TX: Stata Press.
Wang, D. 2000. sg154: Confidence intervals for the ratio of two binomial proportions by Koopman's method. Stata Technical Bulletin 58: 16-19. Reprinted in Stata Technical Bulletin Reprints, vol. 10, pp. 244-247. College Station, TX: Stata Press.

## Also see

[R] bitest - Binomial probability test
[R] proportion - Estimate proportions
[R] ttest - $t$ tests (mean-comparison tests)
[MV] hotelling - Hotelling's T-squared generalized means test

## Title

pwcompare - Pairwise comparisons

Syntax Menu
Remarks and examples
Stored results
Description
Options
Also see

## Syntax

pwcompare marginlist [, options]
where marginlist is a list of factor variables or interactions that appear in the current estimation results or _eqns to reference equations. The variables may be typed with or without the i. prefix, and you may use any factor-variable syntax:
. pwcompare i.sex i.group i.sex\#i.group

- pwcompare sex group sex\#group
. pwcompare sex\#\#group
options
Main
mcompare (method)
asobserved
Equations
equation(eqspec)
atequations
Advanced
emptycells (empspec) treatment of empty cells for balanced factors
noestimcheck suppress estimability checks
Reporting
level(\#)
cieffects
pveffects
effects
cimargins
groups
sort
post
display_options
eform_option
df (\#) use $t$ distribution with \# degrees of freedom for computing $p$-values and confidence intervals
$\mathrm{df}(\#)$ does not appear in the dialog box.
method
Description
noadjust
bonferroni [adjustall]
sidak[adjustall]
scheffe
*tukey
*snk
* duncan
* dunnett
do not adjust for multiple comparisons; the default
Bonferroni's method; adjust across all terms
Šidák's method; adjust across all terms
Scheffé's method
Tukey's method
Student-Newman-Keuls’ method
Duncan's method
Dunnett's method

[^63]Time-series operators are allowed if they were used in the estimation.

## Menu

Statistics $>$ Postestimation $>$ Pairwise comparisons

## Description

pwcompare performs pairwise comparisons across the levels of factor variables from the most recently fit model. pwcompare can compare estimated cell means, marginal means, intercepts, marginal intercepts, slopes, or marginal slopes-collectively called margins. pwcompare reports the comparisons as contrasts (differences) of margins along with significance tests or confidence intervals for the contrasts. The tests and confidence intervals can be adjusted for multiple comparisons.
pwcompare can be used with svy estimation results; see [SVY] svy postestimation.
See [R] margins, pwcompare for performing pairwise comparisons of margins of linear and nonlinear predictions.

## Options

## Main

mcompare (method) specifies the method for computing $p$-values and confidence intervals that account for multiple comparisons within a factor-variable term.
Most methods adjust the comparisonwise error rate, $\alpha_{c}$, to achieve a prespecified experimentwise error rate, $\alpha_{e}$.
mcompare(noadjust) is the default; it specifies no adjustment.

$$
\alpha_{c}=\alpha_{e}
$$

mcompare (bonferroni) adjusts the comparisonwise error rate based on the upper limit of the
Bonferroni inequality:

$$
\alpha_{e} \leq m \alpha_{c}
$$

where $m$ is the number of comparisons within the term.
The adjusted comparisonwise error rate is

$$
\alpha_{c}=\alpha_{e} / m
$$

mcompare(sidak) adjusts the comparisonwise error rate based on the upper limit of the probability inequality

$$
\alpha_{e} \leq 1-\left(1-\alpha_{c}\right)^{m}
$$

where $m$ is the number of comparisons within the term.
The adjusted comparisonwise error rate is

$$
\alpha_{c}=1-\left(1-\alpha_{e}\right)^{1 / m}
$$

This adjustment is exact when the $m$ comparisons are independent.
mcompare (scheffe) controls the experimentwise error rate using the $F$ (or $\chi^{2}$ ) distribution with degrees of freedom equal to the rank of the term.

For results from anova, regress, manova, and mvreg (see [R] anova, [R] regress, [MV] manova, and [MV] mvreg), pwcompare allows the following additional methods. These methods are not allowed with results that used vce(robust) or vce(cluster clustvar).
mcompare (tukey) uses what is commonly referred to as Tukey's honestly significant difference. This method uses the Studentized range distribution instead of the $t$ distribution.
mcompare (snk) is a variation on mcompare (tukey) that counts only the number of margins in the range for a given comparison instead of the full number of margins.
mcompare (duncan) is a variation on mcompare (snk) with additional adjustment to the significance probabilities.
mcompare (dunnett) uses Dunnett's method for making comparisons with a reference category. mcompare (method adjustall) specifies that the multiple-comparison adjustments count all comparisons across all terms rather than performing multiple comparisons term by term. This leads to more conservative adjustments when multiple variables or terms are specified in marginlist. This option is compatible only with the bonferroni and sidak methods.
asobserved specifies that factor covariates be evaluated using the cell frequencies observed when the model was fit. The default is to treat all factor covariates as though there were an equal number of observations at each level.

Equations
equation(eqspec) specifies the equation from which margins are to be computed. The default is to compute margins from the first equation.
atequations specifies that the margins be computed within each equation.

## Advanced

emptycells (empspec) specifies how empty cells are handled in interactions involving factor variables that are being treated as balanced.
emptycells(strict) is the default; it specifies that margins involving empty cells be treated as not estimable.
emptycells (reweight) specifies that the effects of the observed cells be increased to accommodate any missing cells. This makes the margins estimable but changes their interpretation.
noestimcheck specifies that pwcompare not check for estimability. By default, the requested margins are checked and those found not estimable are reported as such. Nonestimability is usually caused by empty cells. If noestimcheck is specified, estimates are computed in the usual way and reported even though the resulting estimates are manipulable, which is to say they can differ across equivalent models having different parameterizations.
level(\#) specifies the confidence level, as a percentage, for confidence intervals. The default is level (95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals. The significance level used by the groups option is $100-\#$, expressed as a percentage.
cieffects specifies that a table of the pairwise comparisons with their standard errors and confidence intervals be reported. This is the default.
pveffects specifies that a table of the pairwise comparisons with their standard errors, test statistics, and $p$-values be reported.
effects specifies that a table of the pairwise comparisons with their standard errors, test statistics, $p$-values, and confidence intervals be reported.
cimargins specifies that a table of the margins with their standard errors and confidence intervals be reported.
groups specifies that a table of the margins with their standard errors and group codes be reported. Margins with the same letter in the group code are not significantly different at the specified significance level.
sort specifies that the reported tables be sorted on the margins or differences in each term.
post causes pwcompare to behave like a Stata estimation (e-class) command. pwcompare posts the vector of estimated margins along with the estimated variance-covariance matrix to e(), so you can treat the estimated margins just as you would results from any other estimation command. For example, you could use test to perform simultaneous tests of hypotheses on the margins, or you could use lincom to create linear combinations.
display_options: vsquish, nofvlabel, fvwrap(\#), fvwrapon(style), cformat(\%fmt), pformat (\% $\% m t$ ), sformat ( $\% f m t$ ), and nolstretch.
vsquish specifies that the blank space separating factor-variable terms or time-series-operated variables from other variables in the model be suppressed.
nofvlabel displays factor-variable level values rather than attached value labels. This option overrides the fvlabel setting; see [R] set showbaselevels.
fvwrap(\#) specifies how many lines to allow when long value labels must be wrapped. Labels requiring more than \# lines are truncated. This option overrides the fvwrap setting; see [R] set showbaselevels.
fvwrapon(style) specifies whether value labels that wrap will break at word boundaries or break based on available space.
fvwrapon(word), the default, specifies that value labels break at word boundaries. fvwrapon(width) specifies that value labels break based on available space.
This option overrides the fvwrapon setting; see [R] set showbaselevels.
cformat (\% firt) specifies how to format contrasts or margins, standard errors, and confidence limits in the table of pairwise comparisons.
pformat (\%fmt) specifies how to format $p$-values in the table of pairwise comparisons.
sformat (\%fmt) specifies how to format test statistics in the table of pairwise comparisons.
nolstretch specifies that the width of the table of pairwise comparisons not be automatically widened to accommodate longer variable names. The default, lstretch, is to automatically widen the table of pairwise comparisons up to the width of the Results window. To change the default, use set lstretch off. nolstretch is not shown in the dialog box.
eform_option specifies that the contrasts table be displayed in exponentiated form. $e^{\text {contrast }}$ is displayed rather than contrast. Standard errors and confidence intervals are also transformed. See [R] eform_option for the list of available options.

The following option is available with pwcompare but is not shown in the dialog box:
df (\#) specifies that the $t$ distribution with \# degrees of freedom be used for computing $p$-values and confidence intervals. The default is to use e(df_r) degrees of freedom or the standard normal distribution if $e\left(d f \_r\right)$ is missing.

## Remarks and examples

pwcompare performs pairwise comparisons of margins across the levels of factor variables from the most recently fit model. The margins can be estimated cell means, marginal means, intercepts, marginal intercepts, slopes, or marginal slopes. With the exception of slopes, we can also consider these margins to be marginal linear predictions.

The margins are calculated as linear combinations of the coefficients. Let $k$ be the number of levels for a factor term in our model; then there are $k$ margins for that term, and

$$
m=\binom{k}{2}=\frac{k(k-1)}{2}
$$

unique pairwise comparisons of those margins.
The confidence intervals and $p$-values for these pairwise comparisons can be adjusted to account for multiple comparisons. Bonferroni's, Šidák's, and Scheffé's adjustments can be made for multiple comparisons after fitting any type of model. In addition, Tukey's, Student-Newman-Keuls', Duncan's, and Dunnett's adjustments are available when fitting ANOVA, linear regression, MANOVA, or multivariate regression models.

Remarks are presented under the following headings:
Pairwise comparisons of means
Marginal means
All pairwise comparisons
Overview of multiple-comparison methods
Fisher's protected least-significant difference (LSD)
Bonferroni's adjustment
Šidák's adjustment
Scheffé's adjustment
Tukey's HSD adjustment
Student-Newman-Keuls' adjustment
Duncan's adjustment
Dunnett's adjustment
Example adjustments using one-way models
Fisher's protected LSD
Tukey's HSD
Dunnett's method for comparisons to a control
Two-way models
Pairwise comparisons of slopes
Nonlinear models
Multiple-equation models
Unbalanced data
Empty cells

## Pairwise comparisons of means

Suppose we are interested in the effects of five different fertilizers on wheat yield. We could estimate the following linear regression model to determine the effect of each type of fertilizer on the yield.

| . use http://www.stata-press.com/data/r13/yield |
| :--- |
| (Artificial wheat yield dataset) |
| . regress yield i.fertilizer |
| Source |
| Model SS df MS <br> Residual 9859.55334 195 50.561812 <br> Total 10938.3954 199 54.9668111 |


| Number of obs | $=$ | 200 |
| :--- | ---: | ---: |
| F 4, 195) | $=$ | 5.33 |
| Prob $>$ F | $=$ | 0.0004 |
| R-squared | $=$ | 0.0986 |
| Adj R-squared | $=$ | 0.0801 |
| Root MSE | $=$ | 7.1107 |


| yield | Coef. | Std. Err. | t | P>\|t| | [95\% Conf. Interval] |  |
| :---: | ---: | :---: | ---: | ---: | ---: | ---: |
| fertilizer |  |  |  |  |  |  |
| $10-08-22$ | 3.62272 | 1.589997 | 2.28 | 0.024 | .4869212 | 6.758518 |
| $16-04-08$ | .4906299 | 1.589997 | 0.31 | 0.758 | -2.645169 | 3.626428 |
| $18-24-06$ | 4.922803 | 1.589997 | 3.10 | 0.002 | 1.787005 | 8.058602 |
| $29-03-04$ | -1.238328 | 1.589997 | -0.78 | 0.437 | -4.374127 | 1.89747 |
|  |  |  |  |  |  |  |
| _cons | 41.36243 | 1.124298 | 36.79 | 0.000 | 39.14509 | 43.57977 |

In this simple case, the coefficients for fertilizers 10-08-22, 16-04-08, 18-24-06, and 29-03-04 indicate the difference in the mean yield for that fertilizer versus the mean yield for fertilizer 10-10-10. That the standard errors of all four coefficients are identical results from having perfectly balanced data.

## Marginal means

We can use pwcompare with the cimargins option to compute the mean yield for each of the fertilizers.

```
. pwcompare fertilizer, cimargins
Pairwise comparisons of marginal linear predictions
Margins : asbalanced
```

|  |  |  |  | Unadjusted <br> [95\% Conf. Interval] |  |
| :---: | ---: | :---: | ---: | ---: | :---: |
|  | Margin | Std. Err. |  |  |  |
| fertilizer |  |  | 39.14509 | 43.57977 |  |
| $10-10-10$ | 41.36243 | 1.124298 | 42.7678 | 47.20249 |  |
| $10-08-22$ | 44.98515 | 1.124298 | 39.63571 | 44.0704 |  |
| $16-04-08$ | 41.85306 | 1.124298 | 44.06789 | 48.50258 |  |
| $18-24-06$ | 46.28523 | 1.124298 | 37.90676 | 42.34145 |  |
| $29-03-04$ | 40.1241 | 1.124298 |  |  |  |

Looking at the confidence intervals for fertilizers 10-10-10 and 10-08-22 in the table above, we might be tempted to conclude that these means are not significantly different because the intervals overlap. However, as discussed in Interaction plots of [R] marginsplot, we cannot draw conclusions about the differences in means by looking at confidence intervals for the means themselves. Instead, we would need to look at confidence intervals for the difference in means.

## All pairwise comparisons

By default, pwcompare calculates all pairwise differences of the margins, in this case pairwise differences of the mean yields.

| Pairwise comparisons of marginal linear predictions Margins : asbalanced |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Contrast | Std. Err. | Unad [95\% Conf | usted <br> Interval] |
| fertilizer |  |  |  |  |
| $10-08-22$ vs $10-10-10$ | 3.62272 | 1.589997 | . 4869212 | 6.758518 |
| $16-04-08$ vs 10-10-10 | . 4906299 | 1.589997 | -2.645169 | 3.626428 |
| $18-24-06$ vs $10-10-10$ | 4.922803 | 1.589997 | 1.787005 | 8.058602 |
| $29-03-04$ vs $10-10-10$ | -1.238328 | 1.589997 | -4.374127 | 1.89747 |
| $16-04-08$ vs $10-08-22$ | -3.13209 | 1.589997 | -6.267889 | . 0037086 |
| $18-24-06$ vs $10-08-22$ | 1.300083 | 1.589997 | -1.835715 | 4.435882 |
| $29-03-04$ vs $10-08-22$ | -4.861048 | 1.589997 | -7.996847 | -1.725249 |
| $18-24-06$ vs 16-04-08 | 4.432173 | 1.589997 | 1.296375 | 7.567972 |
| 29-03-04 vs 16-04-08 | -1.728958 | 1.589997 | -4.864757 | 1.406841 |
| $29-03-04$ vs 18-24-06 | -6.161132 | 1.589997 | -9.29693 | -3.025333 |

If a confidence interval does not include zero, the means for the compared fertilizers are significantly different. Therefore, at the $5 \%$ significance level, we would reject the hypothesis that the means for fertilizers 10-10-10 and 10-08-22 are equivalent-as we would do for 18-24-06 vs 10-10-10, $29-03-04$ vs $10-08-22,18-24-06$ vs $16-04-08$, and $29-03-04$ vs $18-24-06$.

We may prefer to see the $p$-values instead of looking at confidence intervals to determine whether the pairwise differences are significantly different from zero. We could use the pveffects option to see the differences with standard errors and $p$-values, or we could use the effects option to see both $p$-values and confidence intervals in the same table. Here we specify effects as well as the sort option so that the differences are sorted from smallest to largest.

```
. pwcompare fertilizer, effects sort
Pairwise comparisons of marginal linear predictions
Margins : asbalanced
```

|  | Contrast | Std. Err. | Unadjusted |  | Unadjusted |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | t | $P>\|t\|$ | [95\% Conf | Interval] |
| fertilizer |  |  |  |  |  |  |
| 29-03-04 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 18-24-06 | -6.161132 | 1.589997 | -3.87 | 0.000 | -9.29693 | -3.025333 |
| 29-03-04 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-08-22 | -4.861048 | 1.589997 | -3.06 | 0.003 | -7.996847 | -1.725249 |
| 16-04-08 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| $\begin{array}{llllllllll}10-08-22 \\ 29-03-04 & -3.13209 & 1.589997 & -1.97 & 0.050 & -6.267889\end{array}$ |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| $16-04-08$ <br> $29-03-04$ |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-10-10 | -1.238328 | 1.589997 | -0.78 | 0.437 | -4.374127 | 1.89747 |
| 16-04-08 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
|  | . 4906299 | 1.589997 | 0.31 | 0.758 | -2.645169 | 3.626428 |
| $18-24-06$ |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-08-22 | 1.300083 | 1.589997 | 0.82 | 0.415 | -1.835715 | 4.435882 |
| 10-08-22 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-10-10 | 3.62272 | 1.589997 | 2.28 | 0.024 | . 4869212 | 6.758518 |
| 18-24-06 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 16-04-08 | 4.432173 | 1.589997 | 2.79 | 0.006 | 1.296375 | 7.567972 |
| 18-24-06 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-10-10 | 4.922803 | 1.589997 | 3.10 | 0.002 | 1.787005 | 8.058602 |

We find that 5 of the 10 pairs of means are significantly different at the $5 \%$ significance level.
We can use the groups option to obtain a table that identifies groups whose means are not significantly different by assigning them the same letter.

```
. pwcompare fertilizer, groups sort
Pairwise comparisons of marginal linear predictions
Margins : asbalanced
\begin{tabular}{c|rcr}
\hline & Margin & Std. Err. & \begin{tabular}{r} 
Unadjusted \\
Groups
\end{tabular} \\
\hline fertilizer & & & \\
\(29-03-04\) & 40.1241 & 1.124298 & A \\
\(10-10-10\) & 41.36243 & 1.124298 & A \\
\(16-04-08\) & 41.85306 & 1.124298 & AB \\
\(10-08-22\) & 44.98515 & 1.124298 & BC \\
\(18-24-06\) & 46.28523 & 1.124298 & C \\
\hline
\end{tabular}
Note: Margins sharing a letter in the group label are not significantly different at the \(5 \%\) level.
```

The letter A that is assigned to fertilizers 29-03-04, 10-10-10, and 16-04-08 designates that the mean yields for these fertilizers are not different at the $5 \%$ level.

## Overview of multiple-comparison methods

For a single test, if we choose a 5\% significance level, we would have a 5\% chance of concluding that two margins are different when the population values are actually equal. This is known as making a type I error. When we perform $m=k(k-1) / 2$ pairwise comparisons of the $k$ margins, we have $m$ opportunities to make a type I error.
pwcompare with the mcompare() option allows us to adjust the confidence intervals and $p$-values for each comparison to account for the increased probability of making a type I error when making multiple comparisons. Bonferroni's adjustment, Šidák's adjustment, and Scheffé's adjustment can be used when making pairwise comparisons of the margins after any estimation command. Tukey's honestly significant difference, Student-Newman-Keuls' method, Duncan's method, and Dunnett's method are only available when fitting linear models after anova, manova, regress, or mvreg.

## Fisher's protected least-significant difference (LSD)

pwcompare does not offer an mcompare() option specifically for Fisher's protected least-significant difference (LSD). In this methodology, no adjustment is made to the confidence intervals or $p$-values. However, it is protected in the sense that no pairwise comparisons are tested unless the joint test for the corresponding term in the model is significant. Therefore, the default mcompare (noadjust) corresponds to Fisher's protected LSD assuming that the corresponding joint test was performed before using pwcompare.

Milliken and Johnson (2009) recommend using this methodology for planned comparisons, assuming the corresponding joint test is significant.

## Bonferroni's adjustment

mcompare(bonferroni) adjusts significance levels based on the Bonferroni inequality, which, in the case of multiple testing, tells us that the maximum error rate for all comparisons is the sum of the error rates for the individual comparisons. Assuming that we are using the same significance level for all tests, the experimentwise error rate is the error rate for a single test multiplied by the
number of comparisons. Therefore, a $p$-value for each comparison can be computed by multiplying the unadjusted $p$-value by the total number of comparisons. If the adjusted $p$-value is greater than 1 , then pwcompare will report a $p$-value of 1 .

Bonferroni's adjustment is popular because it is easy to compute manually and because it can be applied to any set of tests, not only the pairwise comparisons available in pwcompare. In addition, this method does not require equal sample sizes.

Because Bonferroni's adjustment is so general, it is more conservative than many of the other adjustments. It is especially conservative when a large number of tests is being performed.

## Šidák's adjustment

mcompare (sidak) performs an adjustment using Šidák’s method. This adjustment, like Bonferroni's adjustment, is derived from an inequality. However, in this case, the inequality is based on the probability of not making a type I error. For a single test, the probability that we do not make a type I error is $1-\alpha$. For two independent tests, both using $\alpha$ as a significance level, the probability is $(1-\alpha)(1-\alpha)$. Likewise, for $m$ independent tests, the probability of not making a type I error is $(1-\alpha)^{m}$. Therefore, the probability of making one or more type I errors is $1-(1-\alpha)^{m}$. When tests are not independent, the probability of making at least one error is less than $1-(1-\alpha)^{m}$. Therefore, we can compute an adjusted $p$-value as $1-\left(1-{ }_{u} p\right)^{m}$, where ${ }_{u} p$ is the unadjusted $p$-value for a single comparison.

Šidák's method is also conservative although slightly less so than Bonferroni's method. Like Bonferroni's method, this method does not require equal sample sizes.

## Scheffé's adjustment

Scheffe's adjustment is used when mcompare (scheffe) is specified. This adjustment is derived from the joint $F$ test and its correspondence to the maximum normalized comparison. To adjust for multiple comparisons, the absolute value of the $t$ statistic for a particular comparison can be compared with a critical value of $\sqrt{(k-1) F_{k-1, \nu}}$, where $\nu$ is the residual degrees of freedom. $F_{k-1, \nu}$ is the distribution of the joint $F$ test for the corresponding term in a one-way ANOVA model. Winer, Brown, and Michels (1991, 191-195) discuss this in detail. For estimation commands that report $z$ statistics instead of $t$ statistics for the tests on coefficients, a $\chi^{2}$ distribution is used instead of an $F$ distribution.

Scheffés method allows for making all possible comparisons of the $k$ margins, not just the pairwise comparisons. Unlike the methods described above, it does not take into account the number of comparisons that are currently being made. Therefore, this method is even more conservative than the others. Because this method adjusts for all possible comparisons of the levels of the term, Milliken and Johnson (2009) recommend using this procedure when making unplanned contrasts that are suggested by the data. As Winer, Brown, and Michels $(1991,191)$ put it, this method is often used to adjust for "unfettered data snooping". When using this adjustment, a contrast will never be significant if the joint $F$ or $\chi^{2}$ test for the term is not also significant.

This is another method that does not require equal sample sizes.

## Tukey's HSD adjustment

Tukey's adjustment is also referred to as Tukey's honestly significant difference (HSD) and is used when mcompare (tukey) is specified. It is often applied to all pairwise comparisons of means. Tukey's HSD is commonly used as a post hoc test although this is not a requirement.

To adjust for multiple comparisons, Tukey's method compares the absolute value of the $t$ statistic from the individual comparison with a critical value based on a Studentized range distribution with parameter equal to the number of levels in the term. When applied to pairwise comparisons of means,

$$
q=\frac{\operatorname{mean}_{\max }-\operatorname{mean}_{\min }}{s}
$$

follows a Studentized range distribution with parameter $k$ and $\nu$ degrees of freedom. Here mean $\max$ and mean $\min$ are the largest and smallest marginal means, and $s$ is an estimate of the standard error of the means.

Now for the comparison of the smallest and largest means, we can say that the probability of not making a type I error is

$$
\operatorname{Pr}\left(\frac{\operatorname{mean}_{\max }-\operatorname{mean}_{\min }}{s} \leq q_{k, \nu}\right)=1-\alpha
$$

Then the following inquality holds for all pairs of means simultaneously:

$$
\operatorname{Pr}\left(\frac{\left|\operatorname{mean}_{i}-\operatorname{mean}_{j}\right|}{s} \leq q_{k, \nu}\right) \geq 1-\alpha
$$

Based on this procedure, Tukey's HSD computes the $p$-value for each of the individual comparisons using the Studentized range distribution. However, because the equality holds only for the difference in the largest and smallest means, this procedure produces conservative tests for the remaining comparisons. Winer, Brown, and Michels (1991, 172-182) discuss this in further detail.

With unequal sample sizes, mcompare(tukey) produces the Tukey-Kramer adjustment (Tukey 1953; Kramer 1956).

## Student-Newman-Keuls' adjustment

The Student-Newman-Keuls (SNK) method is used when mcompare (snk) is specified. It is a modification to Tukey's method and is less conservative. In this procedure, we first order the means. We then test the difference in the smallest and largest means using a critical value from the Studentized range distribution with parameter $k$, where $k$ is the number of levels in the term. This step uses the same methodology as in Tukey's procedure. However, in the next step, we will then test for differences in the two sets of means that are the endpoints of the two ranges including $k-1$ means. Specifically, we test the difference in the smallest mean and the second-largest mean using a critical value from the Studentized range distribution with parameter $k-1$. We would also test the difference in the second-smallest mean and the largest mean using this critical value. Likewise, the means that are the endpoints of ranges including $k-2$ means when ordered are tested using the Studentized range distribution with parameter $k-2$, and so on.

Equal sample sizes are required for this method.

## Duncan's adjustment

When mcompare (duncan) is specified, tests are adjusted for multiple comparisons using Duncan's method, which is sometimes referred to as Duncan's new multiple range method. This adjustment produces tests that are less conservative than both Tukey's HSD and SNK. This procedure is performed in the same manner as SNK except that the $p$-values for the individual comparisons are adjusted as $1-\left(1-{ }_{\mathrm{snk}} p_{i}\right)^{1 /(r+1)}$, where ${ }_{\mathrm{snk}} p$ is the $p$-value computed using the SNK method and $r$ represents the number of means that, when ordered, fall between the two that are being compared.

Again equal sample sizes are required for this adjustment.

## Dunnett's adjustment

Dunnett's adjustment is obtained by specifying mcompare(dunnett). It is used when one of the levels of a factor can be considered a control or reference level with which each of the other levels is being compared. When Dunnett's adjustment is requested, $k-1$ instead of $k(k-1) / 2$ pairwise comparisons are made. Dunnett $(1955,1964)$ developed tables of critical values for what Miller (1981, 76) refers to as the "many-one $t$ statistic". The $t$ statistics for individual comparisons are compared with these critical values when making many comparisons to a single reference level.

This method also requires equal sample sizes.

## Example adjustments using one-way models

## Fisher's protected LSD

Fisher's protected LSD requires that we first verify that the joint test for a term in our model is significant before proceeding with pairwise comparisons. Using our previous example, we could have first used the contrast command to obtain a joint test for the effects of fertilizer.

| . contrast fertilizer <br> Contrasts of marginal <br> Margins | asbalanced |  |  |
| :--- | :---: | :---: | :---: |
|  | df |  |  |
| fertilizer | 4 | 5.33 | 0.0004 |
| Denominator | 195 |  |  |

This test for the effects of fertilizer is highly significant. Now we can say we are using Fisher's protected LSD when looking at the unadjusted $p$-values that were obtained from our previous command,

```
. pwcompare fertilizer, effects sort
```


## Tukey's HSD

Because we fit a linear regression model and are interested in all pairwise comparisons of the marginal means, we may instead choose to use Tukey's HSD.

```
. pwcompare fertilizer, effects sort mcompare(tukey)
Pairwise comparisons of marginal linear predictions
Margins : asbalanced
```

|  | Number of <br> Comparisons |
| ---: | ---: |
| fertilizer | 10 |


|  | Contrast | Std. Err. | Tukey |  | Tukey |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | t | $P>\|t\|$ | [95\% Conf | Interval] |
| $\begin{gathered} \text { fertilizer } \\ 29-03-04 \end{gathered}$ |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 18-24-06 | -6.161132 | 1.589997 | -3.87 | 0.001 | -10.53914 | -1.78312 |
| 29-03-04 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-08-22 | -4.861048 | 1.589997 | -3.06 | 0.021 | -9.239059 | -. 4830368 |
| 16-04-08 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-08-22 | -3.13209 | 1.589997 | -1.97 | 0.285 | -7.510101 | 1.245921 |
| 29-03-04 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 16-04-08 | -1.728958 | 1.589997 | -1.09 | 0.813 | -6.106969 | 2.649053 |
| 29-03-04 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-10-10 | -1.238328 | 1.589997 | -0.78 | 0.936 | -5.616339 | 3.139683 |
| 16-04-08 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
|  | . 4906299 | 1.589997 | 0.31 | 0.998 | -3.887381 | 4.868641 |
| $18-24-06$ |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-08-22 | 1.300083 | 1.589997 | 0.82 | 0.925 | -3.077928 | 5.678095 |
| 10-08-22 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-10-10 | 3.62272 | 1.589997 | 2.28 | 0.156 | -. 7552913 | 8.000731 |
| 18-24-06 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 16-04-08 | 4.432173 | 1.589997 | 2.79 | 0.046 | . 0541623 | 8.810185 |
| 18-24-06 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-10-10 | 4.922803 | 1.589997 | 3.10 | 0.019 | . 5447922 | 9.300815 |

This time, our $p$-values have been modified, and we find that only four of the pairwise differences are considered significantly different from zero at the $5 \%$ level.

If we only are interested in performing pairwise comparisons of a subset of our means, we can use factor-variable operators to select the levels of the factor that we want to compare. Here we exclude all comparisons involving fertilizer 10-10-10.

| Pairwise comparisons of marginal linear predictions <br> Margins : asbalanced |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of Comparisons |  |  |  |  |  |  |
| fertilizer | 6 |  |  |  |  |  |
|  |  |  | Tukey |  | Tukey |  |
|  | Contrast | Std. Err. | t | $P>\|t\|$ | [95\% Conf | Interval] |
| $\begin{gathered} \text { fertilizer } \\ 29-03-04 \end{gathered}$ |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 18-24-06 | -6.161132 | 1.589997 | -3.87 | 0.001 | -10.28133 | -2.040937 |
| 29-03-04 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-08-22 | -4.861048 | 1.589997 | -3.06 | 0.013 | -8.981242 | -. 7408538 |
| 16-04-08 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-08-22 | -3.13209 | 1.589997 | -1.97 | 0.203 | -7.252284 | . 9881042 |
| 29-03-04 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 16-04-08 | -1.728958 | 1.589997 | -1.09 | 0.698 | -5.849152 | 2.391236 |
| 18-24-06 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-08-22 | 1.300083 | 1.589997 | 0.82 | 0.846 | -2.820111 | 5.420278 |
| 18-24-06 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 16-04-08 | 4.432173 | 1.589997 | 2.79 | 0.030 | . 3119792 | 8.552368 |

The adjusted $p$-values and confidence intervals differ from those in the previous output because Tukey's adjustment takes into account the total number of comparisons being made when determining the appropriate degrees of freedom to use for the Studentized range distribution.

## Dunnett's method for comparisons to a control

If one of our five fertilizer groups represents fields where no fertilizer was applied, we may want to use Dunnett's method to compare each of the four fertilizers with the control group. In this case, we make only $k-1$ comparisons for $k$ groups.

| Pairwise comparisons of marginal linear predictions <br> Margins : asbalanced |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of Comparisons |  |  |  |  |  |  |
| fertilizer | 4 |  |  |  |  |  |
|  |  |  | Dunnett |  | Dunnett |  |
|  | Contrast | Std. Err. | t | $P>\|t\|$ | [95\% Conf | Interval] |
| $\begin{gathered} \text { fertilizer } \\ 10-08-22 \end{gathered}$ |  |  |  |  |  |  |
| $\begin{array}{r} \text { vs } \\ 10-10-10 \end{array}$ | 3.62272 | 1.589997 | 2.28 | 0.079 | -. 2918331 | 7.537273 |
| 16-04-08 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-10-10 | . 4906299 | 1.589997 | 0.31 | 0.994 | -3.423923 | 4.405183 |
| 18-24-06 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-10-10 | 4.922803 | 1.589997 | 3.10 | 0.008 | 1.00825 | 8.837356 |
| 29-03-04 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-10-10 | -1.238328 | 1.589997 | -0.78 | 0.852 | -5.152881 | 2.676225 |

In our previous regress command, fertilizer 10-10-10 was treated as the base. Therefore, by default, it was treated as the control when using Dunnett's adjustment, and the pairwise comparisons are equivalent to the coefficients reported by regress. Based on our regress output, we would conclude that fertilizers 10-08-22 and 18-24-06 are different from fertilizer 10-10-10 at the 5\% level. However, using Dunnett's adjustment, we find only fertilizer 18-24-06 to be different from fertilizer 10-10-10 at this same significance level.

If the model is fit without a base level for a factor variable, then pwcompare will choose the first level as the reference level. If we want to make comparisons with a different level than the one mcompare (dunnett) chooses by default, we can use the b. operator to override the default. Here we use fertilizer 5 (29-03-04) as the reference level.
. pwcompare b5.fertilizer, effects sort mcompare(dunnett)
Pairwise comparisons of marginal linear predictions
Margins : asbalanced

|  | Number of <br> Comparisons |
| :--- | ---: |
| fertilizer | 4 |


|  | Contrast | Std. Err. | Dunnett |  | Dunnett |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | t | $P>\|t\|$ | [95\% Conf | Interval] |
| $\begin{gathered} \text { fertilizer } \\ 10-10-10 \end{gathered}$ |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 29-03-04 | 1.238328 | 1.589997 | 0.78 | 0.852 | -2.676225 | 5.152881 |
| 16-04-08 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 29-03-04 | 1.728958 | 1.589997 | 1.09 | 0.649 | -2.185595 | 5.643511 |
| 10-08-22 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 29-03-04 | 4.861048 | 1.589997 | 3.06 | 0.009 | . 9464951 | 8.775601 |
| 18-24-06 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 29-03-04 | 6.161132 | 1.589997 | 3.87 | 0.001 | 2.246579 | 10.07568 |

## Two-way models

In the previous examples, we have performed pairwise comparisons after fitting a model with a single factor. Now we include two factors and their interaction in our model.

| Source | SS | df MS |  |  | $\begin{aligned} & \text { Number of obs } \\ & \text { F( } 9,190) \end{aligned}$ | $=200$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | $=27.63$ |
| Model | 6200.81605 | 968 | 688.979561 |  | Prob > F | $=0.0000$ |
| Residual | 4737.57936 | 19024 | 24.9346282 |  | R-squared <br> Adj R-squared <br> Root MSE | 0.5669 |
|  |  |  |  |  |  | $=0.5464$ |
| Total | 10938.3954 | 19954. | 68111 |  |  | $=4.9935$ |
| yield | Coef. | Std. Err | t | $P>\|t\|$ | [95\% Conf. Interval] |  |
| fertilizer |  |  |  |  |  |  |
| 10-08-22 | 1.882256 | 1.57907 | 1.19 | 0.235 | -1.232505 | 4.997016 |
| 16-04-08 | -. 5687418 | 1.57907 | -0.36 | 0.719 | -3.683502 | 2.546019 |
| 18-24-06 | 4.904999 | 1.57907 | 3.11 | 0.002 | 1.790239 | 8.01976 |
| 29-03-04 | -1.217496 | 1.57907 | -0.77 | 0.442 | -4.332257 | 1.897264 |
| 1.irrigation | 8.899721 | 1.57907 | 5.64 | 0.000 | 5.784961 | 12.01448 |
| fertilizer\# irrigation |  |  |  |  |  |  |
| 10-08-22\#1 | 3.480928 | 2.233143 | 1.56 | 0.121 | -. 9240084 | 7.885865 |
| 16-04-08\#1 | 2.118743 | 2.233143 | 0.95 | 0.344 | -2.286193 | 6.52368 |
| 18-24-06\#1 | . 0356082 | 2.233143 | 0.02 | 0.987 | -4.369328 | 4.440545 |
| 29-03-04\#1 | -. 0416636 | 2.233143 | -0.02 | 0.985 | -4.4466 | 4.363273 |
| _cons | 36.91257 | 1.116571 | 33.06 | 0.000 | 34.7101 | 39.11504 |

We can perform pairwise comparisons of the cell means defined by the fertilizer and irrigation interaction.
. pwcompare fertilizer\#irrigation, sort groups mcompare(tukey)
Pairwise comparisons of marginal linear predictions
Margins $\quad:$ asbalanced

|  | Number of <br> Comparisons |
| :--- | ---: |
| fertilizer\#irrigation | 45 |


|  | Margin | Std. Err. | Tukey <br> Groups |
| ---: | ---: | ---: | :---: |
| fertilizer\#irrigation |  |  |  |
| $29-03-04 \# 0$ | 35.69507 | 1.116571 | A |
| $16-04-08 \# 0$ | 36.34383 | 1.116571 | A |
| $10-10-10 \# 0$ | 36.91257 | 1.116571 | AB |
| $10-08-22 \# 0$ | 38.79482 | 1.116571 | AB |
| $18-24-06 \# 0$ | 41.81757 | 1.116571 | BC |
| $29-03-04 \# 1$ | 44.55313 | 1.116571 | CD |
| $10-10-10 \# 1$ | 45.81229 | 1.116571 | CDE |
| $16-04-08 \# 1$ | 47.36229 | 1.116571 | DEF |
| $18-24-06 \# 1$ | 50.7529 | 1.116571 | EF |
| $10-08-22 \# 1$ | 51.17547 | 1.116571 | F |

```
Note: Margins sharing a letter in the group label are not significantly different at the 5\% level.
```

Based on Tukey's HSD and a 5\% significance level, we would conclude that the mean yield for fertilizer 29-03-04 without irrigation is not significantly different from the mean yields for fertilizers $10-10-10,10-08-22$, and 16-04-08 when used without irrigation but is significantly different from the remaining means.

Up to this point, most of the pairwise comparisons that we have performed could have also been obtained with pwmean (see [R] pwmean) if we had not been interested in examining the results from the estimation command before making pairwise comparisons of the means. For instance, we could reproduce the results from the above pwcompare command by typing

```
. pwmean yield, over(fertilizer irrigation) sort group mcompare(tukey)
```

However, pwcompare extends the capabilities of pwmean in many ways. For instance, pwmean only allows for pairwise comparisons of the cell means determined by the highest level interaction of the variables specified in the over () option. However, pwcompare allows us to fit a single model, such as the two-way model that we fit above,

```
. regress yield fertilizer##irrigation
```

and compute pairwise comparisons of the marginal means for only one of the variables in the model:

```
. pwcompare fertilizer, sort effects mcompare(tukey)
Pairwise comparisons of marginal linear predictions
Margins : asbalanced
```

|  | Number of <br> Comparisons |
| ---: | ---: |
| fertilizer | 10 |


|  | Contrast | Std. Err. | Tukey |  | Tukey |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf | Interval] |
| $\begin{gathered} \text { fertilizer } \\ 29-03-04 \end{gathered}$ |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 18-24-06 | -6.161132 | 1.116571 | -5.52 | 0.000 | -9.236338 | -3.085925 |
| 29-03-04 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-08-22 | -4.861048 | 1.116571 | -4.35 | 0.000 | -7.936255 | -1.785841 |
| 16-04-08 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-08-22 | -3.13209 | 1.116571 | -2.81 | 0.044 | -6. 207297 | -. 0568832 |
| 29-03-04 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 16-04-08 | -1.728958 | 1.116571 | -1.55 | 0.532 | -4.804165 | 1.346249 |
| 29-03-04 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-10-10 | -1.238328 | 1.116571 | -1.11 | 0.802 | -4.313535 | 1.836879 |
| 16-04-08 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-10-10 | . 4906299 | 1.116571 | 0.44 | 0.992 | -2.584577 | 3.565837 |
| 18-24-06 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-08-22 | 1.300083 | 1.116571 | 1.16 | 0.772 | -1.775123 | 4.37529 |
| 10-08-22 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-10-10 | 3.62272 | 1.116571 | 3.24 | 0.012 | . 5475131 | 6.697927 |
| 18-24-06 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 16-04-08 | 4.432173 | 1.116571 | 3.97 | 0.001 | 1.356967 | 7.50738 |
| 18-24-06 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-10-10 | 4.922803 | 1.116571 | 4.41 | 0.000 | 1.847597 | 7.99801 |

Here the standard errors for the differences in marginal means and the residual degrees of freedom are based on the full model. Therefore, the results will differ from those obtained from pwcompare after fitting the one-way model with only fertilizer (or equivalently using pwmean).

## Pairwise comparisons of slopes

If we fit a model with a factor variable that is interacted with a continuous variable, pwcompare will even allow us to make pairwise comparisons of the slopes of the continuous variable for the levels of the factor variable.

In this case, we have a continuous variable, NO3_N, indicating the amount of nitrate nitrogen already existing in the soil, based on a sample taken from each field.
. regress yield fertilizer\#\#c.NO3_N

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | 7005.69932 | 9 | 778.411035 |
| Residual | 3932.69609 | 190 | 20.6984005 |
| Total | 10938.3954 | 199 | 54.9668111 |

Number of obs $=200$
$F(9,190)=37.61$
Prob $>\mathrm{F}=0.0000$
$R$-squared $=0.6405$
Adj R-squared $=0.6234$
Root MSE $=4.5495$

| yield | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Con | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| fertilizer |  |  |  |  |  |  |
| 10-08-22 | 18.65019 | 8.452061 | 2.21 | 0.029 | 1.97826 | 35.32212 |
| 16-04-08 | -13.34076 | 10.07595 | -1.32 | 0.187 | -33.21585 | 6.534327 |
| 18-24-06 | 24.35061 | 9.911463 | 2.46 | 0.015 | 4.799973 | 43.90125 |
| 29-03-04 | 17.58529 | 8.446736 | 2.08 | 0.039 | . 9238646 | 34.24671 |
| N03_N | 4.915653 | . 7983509 | 6.16 | 0.000 | 3.340884 | 6.490423 |
| $\begin{gathered} \text { fertilizer\# } \\ \text { c.NO3_N } \end{gathered}$ |  |  |  |  |  |  |
| 10-08-22 | -1.282039 | . 8953419 | -1.43 | 0.154 | -3.048126 | . 4840487 |
| 16-04-08 | -1.00571 | . 9025862 | -1.11 | 0.267 | -2.786087 | . 7746662 |
| 18-24-06 | -2.97627 | . 9136338 | -3.26 | 0.001 | -4.778438 | -1.174102 |
| 29-03-04 | -3.275947 | . 8247385 | -3.97 | 0.000 | -4.902767 | -1.649127 |
| _cons | -5.459168 | 7.638241 | -0.71 | 0.476 | -20.52581 | 9.607477 |

These are the pairwise differences of the slopes of NO3_N for each pair of fertilizers:
. pwcompare fertilizer\#c.N03_N, pveffects sort mcompare(scheffe)
Pairwise comparisons of marginal linear predictions
Margins : asbalanced

|  | Number of <br> Comparisons |
| :--- | ---: |
| fertilizer\#c.NO3_N | 10 |


|  | Contrast | Std. Err. | Scheffe |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | t | $P>\|t\|$ |
| fertilizer\#c.N03_N |  |  |  |  |
| 29-03-04 vs 10-10-10 | -3.275947 | . 8247385 | -3.97 | 0.004 |
| $18-24-06$ vs $10-10-10$ | -2.97627 | . 9136338 | -3.26 | 0.034 |
| 29-03-04 vs 16-04-08 | -2.270237 | . 4691771 | -4.84 | 0.000 |
| $29-03-04$ vs 10-08-22 | -1.993909 | . 4550851 | -4.38 | 0.001 |
| $18-24-06$ vs 16-04-08 | -1.97056 | . 612095 | -3.22 | 0.038 |
| $18-24-06$ vs $10-08-22$ | -1.694232 | . 6013615 | -2.82 | 0.099 |
| $10-08-22$ vs $10-10-10$ | -1.282039 | . 8953419 | -1.43 | 0.727 |
| $16-04-08$ vs $10-10-10$ | -1.00571 | . 9025862 | -1.11 | 0.871 |
| 29-03-04 vs 18-24-06 | -. 2996772 | . 4900939 | -0.61 | 0.984 |
| $16-04-08$ vs $10-08-22$ | . 276328 | . 5844405 | 0.47 | 0.994 |

Using Scheffés adjustment, we find that five of the pairs have significantly different slopes at the 5\% level.

## Nonlinear models

pwcompare can also perform pairwise comparisons of the marginal linear predictions after fitting a nonlinear model. For instance, we can use the dataset from Beyond linear models in $[R]$ contrast and fit the following logistic regression model of patient satisfaction on hospital:

| (Artificial hospital satisfac . logit satisfied i.hospital |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Iteration 0: log likelihood $=-393.72216$ |  |  |  |  |  |  |
| Iteration 1: log likelihood $=-387.55736$ |  |  |  |  |  |  |
| Iteration 2: log likelihood $=-387.4768$ |  |  |  |  |  |  |
| Iteration 3: log likelihood $=-387.47679$ |  |  |  |  |  |  |
| Logistic regression |  |  |  | Numb | of obs | 802 |
|  |  |  |  | LR | (2) | 12.49 |
|  |  |  |  | Prob | chi2 | 0.0019 |
| Log likelihood $=-387.47679$ |  |  |  | Pse |  | 0.0159 |
| satisfied | Coef. | Std. Err. | z | $P>\|z\|$ | [95\% | Interval] |
| hospital |  |  |  |  |  |  |
| 2 | . 5348129 | . 2136021 | 2.50 | 0.012 | . 1161 | . 9534654 |
| 3 | . 7354519 | . 2221929 | 3.31 | 0.001 | . 2999 | 1.170942 |
| _cons | 1.034708 | . 1391469 | 7.44 | 0.000 | . 7619 | 1.307431 |

For this model, the marginal linear predictions are the predicted log odds for each hospital and can be obtained with the cimargins option:

| . pwcompare hospital, cimargins <br> Pairwise comparisons of marginal linear predictions <br> Margins <br> : asbalanced |
| :--- |

The pairwise comparisons are, therefore, differences in the log odds. We can specify mcompare(bonferroni) and effects to request Bonferroni-adjusted $p$-values and confidence intervals.

| . pwcompare hospital, effect <br> Pairwise comparisons of marg <br> Margins <br> : asbalanced |
| :--- |
| Number of <br> Comparisons |
| satisfied <br> hospital |


|  |  |  | Bonferroni |  | Bonferroni |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Contrast | Std. Err. | z | $\mathrm{P}>\|z\|$ | [95\% Con | Interval] |
| satisfied |  |  |  |  |  |  |
| hospital |  |  |  |  |  |  |
| 2 vs 1 | . 5348129 | . 2136021 | 2.50 | 0.037 | . 0234537 | 1.046172 |
| 3 vs 1 | . 7354519 | . 2221929 | 3.31 | 0.003 | . 2035265 | 1.267377 |
| 3 vs 2 | . 200639 | . 2372169 | 0.85 | 1.000 | -. 3672535 | . 7685314 |

For nonlinear models, only Bonferroni's adjustment, Šidák's adjustment, and Scheffé's adjustment are available.

If we want pairwise comparisons reported as odds ratios, we can specify the or option.

| . pwcompare hospital, effects mcompare(bonferroni) or |
| :--- |
| Pairwise comparisons of marginal linear predictions <br> Margins <br> : asbalanced |
| Number of <br> Comparisons |
| satisfied <br> hospital |


|  | Odds Ratio | Std. Err. | Bonferroni |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| z | P>\|z| | [95\% Conf. Interval] |  |

Notice that these tests are still performed on the marginal linear predictions. The odds ratios reported here are the exponentiated versions of the pairwise differences of $\log$ odds in the previous output. For further discussion, see $[R]$ contrast.

## Multiple-equation models

pwcompare works with models containing multiple equations. Commands such as intreg and gnbreg allow their ancillary parameters to be modeled as a function of independent variables, and pwcompare can compare the margins within these equations. The equation() option can be used to specify the equation for which pairwise comparisons of the margins should be made. The atequations option specifies that pairwise comparisons be computed for each equation. In addition, pwcompare allows a special pseudofactor for equation-called _eqns-when working with results from manova, mvreg, mlogit, and mprobit.

Here we use the jaw fracture dataset described in example 4 of [MV] manova. We fit a multivariate regression model including one independent factor variable, fracture.
. use http://www.stata-press.com/data/r13/jaw
(Table 4.6 Two-Way Unbalanced Data for Fractures of the Jaw -- Rencher

| . mvreg y1 y2 y3 = i.fracture |
| :--- |
| Equation |


|  |  |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| y1 | Obs | Parms | RMSE | "R-sq" | F | P |
| y2 | 27 | 3 | 10.42366 | 0.2966 | 5.060804 | 0.0147 |
| y3 | 27 | 3 | 6.325398 | 0.1341 | 1.858342 | 0.1777 |


|  | Coef. | Std. Err | t | $P>\|t\|$ | [95\% Conf. | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| y1 |  |  |  |  |  |  |
| fracture two compo.. | -8.833333 | 4.957441 | -1.78 | 0.087 | -19.06499 | 1.398322 |
| one simpl.. | 6 | 5.394759 | 1.11 | 0.277 | -5.134235 | 17.13423 |
| _cons | 37 | 3.939775 | 9.39 | 0.000 | 28.8687 | 45.1313 |
| y2 |  |  |  |  |  |  |
| fracture <br> two compo.. | -5.761905 | 3.008327 | -1.92 | 0.067 | -11.97079 | . 446977 |
| one simpl.. | -3.053571 | 3.273705 | -0.93 | 0.360 | -9.810166 | 3.703023 |
| _cons | 38.42857 | 2.390776 | 16.07 | 0.000 | 33.49425 | 43.36289 |
| y3 |  |  |  |  |  |  |
| fracture |  | 2.842618 | 1.50 |  |  |  |
| one simpl.. | $.9285714$ | 3.093377 | 0.30 | 0.767 | -5.455846 | $7.312989$ |
| _cons | 58.57143 | 2.259083 | 25.93 | 0.000 | 53.90891 | 63.23395 |

pwcompare performs pairwise comparisons of the margins using the coefficients from the first equation by default:


We can use the equation() option to get pwcompare to perform comparisons in the y 2 equation:


Because we are working with mvreg results, we can use the _eqns pseudofactor to compare the margins between the three dependent variables. The levels of _eqns index the equations: 1 for the first equation, 2 for the second, and 3 for the third.

```
. pwcompare _eqns, mcompare(bonferroni)
Pairwise comparisons of marginal linear predictions
Margins : asbalanced
```

|  | Number of <br> Comparisons |
| ---: | ---: |
| _eqns | 3 |


|  |  |  | Bonferroni <br> [95\% Conf. Interval] |  |
| :--- | ---: | ---: | ---: | ---: |
|  | Contrast | Std. Err. |  |  |
| _eqns |  |  |  |  |
| 2 vs 1 | -.5654762 | 2.545923 | -7.117768 | 5.986815 |
| 3 vs 1 | 24.24603 | 2.320677 | 18.27344 | 30.21862 |
| 3 vs 2 | 24.81151 | 2.368188 | 18.71664 | 30.90637 |

For the previous command, the only methods available are mcompare(bonferroni), mcompare(sidak), or mcompare(scheffe). Methods that use the Studentized range are not appropriate for making comparisons across equations.

## Unbalanced data

pwcompare treats all factors as balanced when it computes the marginal means. By "balanced", we mean that the number of observations in each combination of factor levels (in each cell mean) is equal. We can alternatively specify the asobserved option when we have unbalanced data to obtain marginal means that are based on the observed cell frequencies from the model fit. For more details on the difference in these two types of marginal means and a discussion of when each may be appropriate, see $[R]$ margins and $[R]$ contrast.

In addition, when our data are not balanced, some of the multiple-comparison adjustments are no longer appropriate. Student-Newman-Keuls' method, Duncan's method, and Dunnett's method assume equal numbers of observations per group.

Here we use an unbalanced dataset and fit a two-way ANOVA model for cholesterol levels on race and age group. Then we perform pairwise comparisons of the mean cholesterol levels for each race, requesting Šidák's adjustment as well as marginal means that are computed using the observed cell frequencies.

```
. use http://www.stata-press.com/data/r13/cholesterol3
(Artificial cholesterol data, unbalanced)
```

. anova chol race\#\#agegrp

| Source | $\begin{array}{llr} \text { Number of obs } & = & 67 \\ \text { Root MSE } & =8.37496 \end{array}$ |  |  | R -squared | $\begin{aligned} & =0.8179 \\ & =0.7689 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Partial SS | df | MS | F | Prob > F |
| Model | 16379.9926 | 14 | 1169.99947 | 16.68 | 0.0000 |
| race | 230.754396 | 2 | 115.377198 | 1.64 | 0.2029 |
| agegrp | 13857.9877 | 4 | 3464.49693 | 49.39 | 0.0000 |
| race\#agegrp | 857.815209 | 8 | 107.226901 | 1.53 | 0.1701 |
| Residual | 3647.2774 | 52 | 70.13995 |  |  |
| Total | 20027.27 | 66 | 303.443485 |  |  |

- pwcompare race, asobserved mcompare(sidak)

Pairwise comparisons of marginal linear predictions
Margins : asobserved

|  | Number of <br> Comparisons |
| ---: | ---: |
| race | 3 |


|  |  |  | Sidak <br> [95\% Conf. Interval] |  |
| :--- | ---: | ---: | ---: | ---: |
|  | Contrast | Std. Err. |  |  |
| race |  |  |  |  |
| white vs black | -7.232433 | 2.686089 | -13.85924 | -.6056277 |
| other vs black | -5.231198 | 2.651203 | -11.77194 | 1.309541 |
| other vs white | 2.001235 | 2.414964 | -3.956682 | 7.959152 |

## Empty cells

An empty cell is a combination of the levels of factor variables that is not observed in the estimation sample. When we have empty cells in our data, the marginal means involving those empty cells are not estimable as described in [R] margins. In addition, all pairwise comparisons involving a marginal mean that is not estimable are themselves not estimable. Here we use a dataset where we do not have any observations for white individuals in the $20-29$ age group. We can use the emptycells (reweight) option to reweight the nonempty cells so that we can estimate the marginal mean for whites and compute pairwise comparisons involving that marginal mean.

```
. use http://www.stata-press.com/data/r13/cholesterol2
(Artificial cholesterol data, empty cells)
```

. tabulate race agegrp

| race | $10-19$ | $20-29$ | agegrp <br> $30-39$ | $40-59$ | $60-79$ | Total |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| black | 5 | 5 | 5 | 5 | 5 | 25 |
| white | 5 | 0 | 5 | 5 | 5 | 20 |
| other | 5 | 5 | 5 | 5 | 5 | 25 |
| Total | 15 | 10 | 15 | 15 | 15 | 70 |

. anova chol race\#\#agegrp

|  | Number of obs $=r 70$ <br> Root MSE $=9.47055$ |  |  | R-squared | $\begin{aligned} & =0.7582 \\ & =0.7021 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Source | Partial SS | df | MS | F | Prob > F |
| Model | 15751.6113 | 13 | 1211.66241 | 13.51 | 0.0000 |
| race | 305.49046 | 2 | 152.74523 | 1.70 | 0.1914 |
| agegrp | 14387.8559 | 4 | 3596.96397 | 40.10 | 0.0000 |
| race\#agegrp | 795.807574 | 7 | 113.686796 | 1.27 | 0.2831 |
| Residual | 5022.71559 | 56 | 89.6913498 |  |  |
| Total | 20774.3269 | 69 | 301.077201 |  |  |

. pwcompare race, emptycells(reweight)
Pairwise comparisons of marginal linear predictions
Margins : asbalanced
Empty cells : reweight

|  |  |  | Unadjusted |  |
| :--- | ---: | :---: | :---: | :---: |
|  | Contrast | Std. Err. | [95\% Conf. Interval] |  |
| race |  |  |  |  |
| white vs black | 2.922769 | 2.841166 | -2.768769 | 8.614308 |
| other vs black | -4.12621 | 2.678677 | -9.492244 | 1.239824 |
| other vs white | -7.048979 | 2.841166 | -12.74052 | -1.35744 |

For further details on the emptycells(reweight) option, see $[R]$ margins and [R] contrast.

## Stored results

pwcompare stores the following in $r()$ :
Scalars

```
    r(df_r)
    r(k_terms)
    r(level)
    r(balanced)
```

Macros

```
    r(cmd)
    r(cmdline)
    r(est_cmd)
    r(est_cmdline)
    r(title)
    r(emptycells)
    r(groups#)
    r(mcmethod_vs)
    r(mctitle_vs)
    r(mcadjustall_vs)
    r(margin_method)
    r(vce)
```

Matrices
r(b)
r(V)
$r$ (error)
$r$ (table)
r(M)
r(b_vs)
r(V_vs)
r(error_vs)
r(table_vs)
$r(\mathrm{~L})$
r(k_groups)
variance degrees of freedom
number of terms in marginlist
confidence level of confidence intervals
1 if fully balanced data; 0 otherwise
pwcompare
command as typed
$e(\mathrm{cmd})$ from original estimation results
$e$ (cmdline) from original estimation results
title in output
empspec from emptycells()
group codes for the \#th margin in $r(b)$
method from mcompare()
title for method from mcompare()
adjustall or empty
asbalanced or asobserved
vcetype specified in vce() in original estimation command
margin estimates
variance-covariance matrix of the margin estimates
margin estimability codes;
0 means estimable,
8 means not estimable
matrix containing the margins with their standard errors, test statistics, $p$-values,
and confidence intervals
matrix that produces the margins from the model coefficients
margin difference estimates
variance-covariance matrix of the margin difference estimates
margin difference estimability codes;
0 means estimable,
8 means not estimable
matrix containing the margin differences with their standard errors, test statistics,
$p$-values, and confidence intervals
matrix that produces the margin differences from the model coefficients
number of significance groups for each term
pwcompare with the post option also stores the following in e():
Scalars
$e\left(d f \_r\right) \quad$ variance degrees of freedom
e(k_terms)
e(balanced)
Macros
e(cmd)
e(cmdline)
e(est_cmd)
e(est_cmdline)
e(title)
e(emptycells)
e(margin_method)
number of terms in marginlist
1 if fully balanced data; 0 otherwise
e(vce)
e(properties)
pwcompare
command as typed
$e(c m d)$ from original estimation results
$e$ (cmdline) from original estimation results
title in output
empspec from emptycells()
asbalanced or asobserved
vcetype specified in vce() in original estimation command
b V
Matrices
e(b)
margin estimates
variance-covariance matrix of the margin estimates
margin estimability codes;
0 means estimable,
8 means not estimable
matrix that produces the margins from the model coefficients
margin difference estimates
variance-covariance matrix of the margin difference estimates
margin difference estimability codes;
0 means estimable,
8 means not estimable
matrix that produces the margin differences from the model coefficients number of significance groups for each term

## Methods and formulas

Methods and formulas are presented under the following headings:
Notation
Unadjusted comparisons
Bonferroni's method
Šidák's method
Scheffé's method
Tukey's method
Student-Newman-Keuls' method
Duncan's method
Dunnett's method

## Notation

pwcompare performs comparisons of margins; see Methods and formulas in $[\mathrm{R}]$ contrast.
If there are $k$ margins for a given factor term, then there are

$$
m=\binom{k}{2}=\frac{k(k-1)}{2}
$$

unique pairwise comparisons. Let the $i$ th pairwise comparison be denoted by

$$
\widehat{\delta}_{i}=l_{i}^{\prime} \mathbf{b}
$$

where $\mathbf{b}$ is a column vector of coefficients from the fitted model and $l_{i}$ is a column vector that forms the corresponding linear combination. If $\widehat{\mathbf{V}}$ denotes the estimated variance matrix for $\mathbf{b}$, then the standard error for $\widehat{\delta}_{i}$ is given by

$$
\widehat{\operatorname{se}}\left(\widehat{\delta}_{i}\right)=\sqrt{l_{i}^{\prime} \widehat{\mathbf{V}} l_{i}}
$$

The corresponding test statistic is then

$$
t_{i}=\frac{\widehat{\delta}_{i}}{\widehat{\operatorname{se}}\left(\widehat{\delta}_{i}\right)}
$$

and the limits for a $100(1-\alpha) \%$ confidence interval for the expected value of $\widehat{\delta}_{i}$ are

$$
\widehat{\delta}_{i} \pm c_{i}(\alpha) \widehat{\operatorname{se}}\left(\widehat{\delta}_{i}\right)
$$

where $c_{i}(\alpha)$ is the critical value corresponding to the chosen multiple-comparison method.

## Unadjusted comparisons

pwcompare computes unadjusted $p$-values and confidence intervals by default. pwcompare uses the $t$ distribution with $\nu=\mathrm{e}\left(\mathrm{df} \_\mathrm{r}\right)$ degrees of freedom when $\mathrm{e}\left(\mathrm{df} \_\mathrm{r}\right)$ is posted by the estimation command. The unadjusted two-sided $p$-value is

$$
{ }_{u} p_{i}=2 \operatorname{Pr}\left(t_{\nu}>\left|t_{i}\right|\right)
$$

and the unadjusted critical value ${ }_{u} c_{i}(\alpha)$ satisfies the following probability statement:

$$
\alpha=2 \operatorname{Pr}\left\{t_{\nu}>{ }_{u} c_{i}(\alpha)\right\}
$$

pwcompare uses the standard normal distribution when $e\left(d f \_r\right)$ is not posted.

## Bonferroni's method

For mcompare(bonferroni), the adjusted $p$-value is

$$
{ }_{b} p_{i}=\min \left(1, m_{u} p_{i}\right)
$$

and the adjusted critical value is

$$
{ }_{b} c_{i}(\alpha)={ }_{u} c_{i}(\alpha / m)
$$

## Šidák's method

For mcompare(sidak), the adjusted $p$-value is

$$
{ }_{\mathrm{si}} p_{i}=1-\left(1-{ }_{u} p_{i}\right)^{m}
$$

and the adjusted critical value is

$$
{ }_{\mathrm{si}} c_{i}(\alpha)={ }_{u} c_{i}\left\{1-(1-\alpha)^{1 / m}\right\}
$$

## Scheffés method

For mcompare(scheffe), the adjusted $p$-value is

$$
{ }_{\mathrm{sc}} p_{i}=\operatorname{Pr}\left(F_{d, \nu}>t_{i}^{2} / d\right)
$$

where $F_{d, \nu}$ is distributed as an $F$ with $d$ numerator and $\nu$ denominator degrees of freedom and $d$ is the rank of the VCE for the term. The adjusted critical value satisfies the following probability statement:

$$
\alpha=\operatorname{Pr}\left[F_{d, \nu}>\left\{_{\mathrm{sc}} c_{i}(\alpha)\right\}^{2} / d\right]
$$

pwcompare uses the $\chi^{2}$ distribution when $e\left(d f \_r\right)$ is not posted.

## Tukey's method

For mcompare(tukey), the adjusted $p$-value is

$$
{ }_{t} p_{i}=\operatorname{Pr}\left(q_{k, \nu}>\left|t_{i}\right| \sqrt{2}\right)
$$

where $q_{k, \nu}$ is distributed as the Studentized range statistic for $k$ means and $\nu$ residual degrees of freedom (Miller 1981). The adjusted critical value satisfies the following probability statement:

$$
\alpha=\operatorname{Pr}\left\{q_{k, \nu}>{ }_{t} c_{i}(\alpha) \sqrt{2}\right\}
$$

## Student-Newman-Keuls' method

For mcompare (snk), suppose $t_{i}$ is comparing two margins that have $r$ other margins between them. Then the adjusted $p$-value is

$$
{ }_{\mathrm{snk}} p_{i}=\operatorname{Pr}\left(q_{r+2, \nu}>\left|t_{i}\right| \sqrt{2}\right)
$$

where $r$ ranges from 0 to $k-2$. The adjusted critical value ${ }_{\mathrm{snk}} c_{i}(\alpha)$ satisfies the following probability statement:

$$
\alpha=\operatorname{Pr}\left\{q_{r+2, \nu}>_{\mathrm{snk}} c_{i}(\alpha) \sqrt{2}\right\}
$$

## Duncan's method

For mcompare(duncan), the adjusted $p$-value is

$$
\operatorname{dunc} p_{i}=1-\left(1-\mathrm{snk} p_{i}\right)^{1 /(r+1)}
$$

and the adjusted critical value is

$$
\operatorname{dunc} c_{i}(\alpha)={ }_{\operatorname{snk}} c_{i}\left\{1-(1-\alpha)^{r+1}\right\}
$$

## Dunnett's method

For mcompare (dunnett), the margins are compared with a reference category, resulting in only $k-1$ pairwise comparisons. The adjusted $p$-value is

$$
{ }_{\mathrm{dunn}} p_{i}=\operatorname{Pr}\left(d_{k-1, \nu}>\left|t_{i}\right|\right)
$$

where $d_{k-1, \nu}$ is distributed as the many-one $t$ statistic (Miller 1981, 76). The adjusted critical value dunn $c_{i}(\alpha)$ satisfies the following probability statement:

$$
\alpha=\operatorname{Pr}\left\{d_{k-1, \nu}>_{\operatorname{dunn}} c_{i}(\alpha)\right\}
$$

The multiple-comparison methods for mcompare(tukey), mcompare(snk), mcompare(duncan), and mcompare (dunnett) assume the normal distribution with equal variance; thus these methods are allowed only with results from anova, regress, manova, and mvreg. mcompare (snk), mcompare (duncan), and mcompare (dunnett) assume equal sample size for each marginal mean. These options will cause pwcompare to report a footnote if unbalanced factors are detected.

## References

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## Also see

[R] pwcompare postestimation - Postestimation tools for pwcompare
[R] contrast - Contrasts and linear hypothesis tests after estimation
[R] lincom - Linear combinations of estimators
$[\mathrm{R}]$ margins - Marginal means, predictive margins, and marginal effects
[R] margins, pwcompare - Pairwise comparisons of margins
[R] pwmean - Pairwise comparisons of means
[R] test - Test linear hypotheses after estimation
[U] 20 Estimation and postestimation commands

## Title

pwcompare postestimation - Postestimation tools for pwcompare

## Description Remarks and examples Also see

## Description

The following postestimation commands are available after pwcompare, post:

Command Description
estat vce variance-covariance matrix of the estimators (VCE)
estat (svy) postestimation statistics for survey data
estimates cataloging estimation results
lincom point estimates, standard errors, testing, and inference for linear combinations of coefficients
nlcom point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients
test Wald tests of simple and composite linear hypotheses
testnl Wald tests of nonlinear hypotheses

## Remarks and examples

When we use the post option with pwcompare, the marginal linear predictions are posted as estimation results, and we can use postestimation commands to perform further analysis on them.

In Pairwise comparisons of means of [R] pwcompare, we fit a regression of wheat yield on types of fertilizers.

```
. use http://www.stata-press.com/data/r13/yield
(Artificial wheat yield dataset)
. regress yield i.fertilizer
    (output omitted)
```

We also used pwcompare with the cimargins option to obtain the marginal mean yield for each fertilizer. We can add the post option to this command to post these marginal means and their VCEs as estimation results.


Now we can use nlcom to compute a percentage improvement in the mean yield for fertilizer 2 when compared with fertilizer 1.

```
. nlcom (pct_chg: 100*(_b[2.fertilizer] - _b[1.fertilizer])/_b[1.fertilizer])
```

    pct_chg: 100*(_b[2.fertilizer] - _b[1.fertilizer])/_b[1.fertilizer]
    |  | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | ---: |
| pct_chg | 8.758479 | 4.015932 | 2.18 | 0.029 | .8873982 | 16.62956 |

The mean yield for fertilizer 2 is about $9 \%$ higher than that of fertilizer 1, with a standard error of $4 \%$.

## Also see

[R] pwcompare - Pairwise comparisons
[U] 20 Estimation and postestimation commands

## Title

pwmean - Pairwise comparisons of means

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | Reference |
| Also see |  |  |  |

## Syntax

pwmean varname, over(varlist) [options]
options Description
Main

* over (varlist) compare means across each combination of the levels in varlist mcompare (method) adjust for multiple comparisons; default is mcompare(noadjust)

Reporting
level(\#) confidence level; default is level(95)
cieffects display a table of mean differences and confidence intervals; the default
pveffects display a table of mean differences and $p$-values
effects display a table of mean differences with $p$-values and confidence intervals
cimeans display a table of means and confidence intervals
groups display a table of means with codes that group them with other means that are not significantly different
sort sort results tables by displayed mean or difference
display_options control column formats, line width, and factor-variable labeling

* over (varlist) is required.
method
Description
noadjust do not adjust for multiple comparisons; the default
bonferroni Bonferroni's method
sidak
scheffe
tukey
snk
duncan
Šidák's method
Scheffé's method
Tukey's method
Student-Newman-Keuls' method
Duncan's method
dunnett


## Menu

Statistics $>$ Summaries, tables, and tests $>$ Summary and descriptive statistics $>$ Pairwise comparisons of means

## Description

pwmean performs pairwise comparisons of means. It computes all pairwise differences of the means of varname over the combination of the levels of the variables in varlist. The tests and confidence intervals for the pairwise comparisons assume equal variances across groups. pwmean also allows for adjusting the confidence intervals and $p$-values to account for multiple comparisons using Bonferroni's method, Scheffe's method, Tukey's method, Dunnett's method, and others.

See [ R ] pwcompare for performing pairwise comparisons of means, estimated marginal means, and other types of marginal linear predictions after anova, regress, and most other estimation commands.

See $[\mathrm{R}]$ margins, pwcompare for performing pairwise comparisons of marginal probabilities and other linear and nonlinear predictions after estimation commands.

## Options

Main
over (varlist) is required and specifies that means are computed for each combination of the levels of the variables in varlist.
mcompare (method) specifies the method for computing $p$-values and confidence intervals that account for multiple comparisons.

Most methods adjust the comparisonwise error rate, $\alpha_{c}$, to achieve a prespecified experimentwise error rate, $\alpha_{e}$.
mcompare(noadjust) is the default; it specifies no adjustment.

$$
\alpha_{c}=\alpha_{e}
$$

mcompare (bonferroni) adjusts the comparisonwise error rate based on the upper limit of the Bonferroni inequality:

$$
\alpha_{e} \leq m \alpha_{c}
$$

where $m$ is the number of comparisons within the term.
The adjusted comparisonwise error rate is

$$
\alpha_{c}=\alpha_{e} / m
$$

mcompare (sidak) adjusts the comparisonwise error rate based on the upper limit of the probability inequality

$$
\alpha_{e} \leq 1-\left(1-\alpha_{c}\right)^{m}
$$

where $m$ is the number of comparisons within the term.
The adjusted comparisonwise error rate is

$$
\alpha_{c}=1-\left(1-\alpha_{e}\right)^{1 / m}
$$

This adjustment is exact when the $m$ comparisons are independent.
mcompare (scheffe) controls the experimentwise error rate using the $F$ (or $\chi^{2}$ ) distribution with degrees of freedom equal to $k-1$ where $k$ is the number of means being compared.
mcompare(tukey) uses what is commonly referred to as Tukey's honestly significant difference. This method uses the Studentized range distribution instead of the $t$ distribution.
mcompare (snk) is a variation on mcompare(tukey) that counts only the number of means participating in the range for a given comparison instead of the full number of means.
mcompare (duncan) is a variation on mcompare (snk) with additional adjustment to the significance probabilities.
mcompare (dunnett) uses Dunnett's method for making comparisons with a reference category.

Reporting
level(\#) specifies the confidence level, as a percentage, for confidence intervals. The default is level (95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals. The significance level used by the groups option is $100-$ \#, expressed as a percentage.
cieffects specifies that a table of the pairwise comparisons of means with their standard errors and confidence intervals be reported. This is the default.
pveffects specifies that a table of the pairwise comparisons of means with their standard errors, test statistics, and $p$-values be reported.
effects specifies that a table of the pairwise comparisons of means with their standard errors, test statistics, $p$-values, and confidence intervals be reported.
cimeans specifies that a table of the means with their standard errors and confidence intervals be reported.
groups specifies that a table of the means with their standard errors and group codes be reported. Means with the same letter in the group code are not significantly different at the specified significance level.
sort specifies that the reported tables be sorted by the mean or difference that is displayed in the table.
display_options: nofvlabel, fvwrap(\#), fvwrapon(style), cformat(\%fmt), pformat(\%fmt), sformat (\%fmt), and nolstretch.
nofvlabel displays factor-variable level values rather than attached value labels. This option overrides the fvlabel setting; see [R] set showbaselevels.
fvwrap(\#) specifies how many lines to allow when long value labels must be wrapped. Labels requiring more than \# lines are truncated. This option overrides the fvwrap setting; see [R] set showbaselevels.
fvwrapon (style) specifies whether value labels that wrap will break at word boundaries or break based on available space.
fvwrapon(word), the default, specifies that value labels break at word boundaries. fvwrapon(width) specifies that value labels break based on available space.
This option overrides the fvwrapon setting; see [R] set showbaselevels.
cformat (\% fint) specifies how to format means, standard errors, and confidence limits in the table of pairwise comparison of means.
pformat ( $\% f m t$ ) specifies how to format $p$-values in the table of pairwise comparison of means.
sformat (\%fmt) specifies how to format test statistics in the table of pairwise comparison of means.
nolstretch specifies that the width of the table of pairwise comparisons not be automatically widened to accommodate longer variable names. The default, lstretch, is to automatically
widen the table of pairwise comparisons up to the width of the Results window. To change the default, use set lstretch off. nolstretch is not shown in the dialog box.

## Remarks and examples

pwmean performs pairwise comparisons (differences) of means, assuming a common variance among groups. It can easily adjust the $p$-values and confidence intervals for the differences to account for the elevated type I error rate due to multiple comparisons. Adjustments for multiple comparisons can be made using Bonferroni's method, Scheffé's method, Tukey's method, Dunnett's method, and others.

Remarks are presented under the following headings:

Group means<br>Pairwise differences of means<br>Group output<br>Adjusting for multiple comparisons<br>Tukey's method<br>Dunnett's method<br>Multiple over() variables<br>Equal variance assumption

## Group means

Suppose we have data on the wheat yield of fields that were each randomly assigned an application of one of five types of fertilizers. Let's first look at the mean yield for each type of fertilizer.
. use http://www.stata-press.com/data/r13/yield
(Artificial wheat yield dataset)
. pwmean yield, over(fertilizer) cimeans
Pairwise comparisons of means with equal variances
over $\quad:$ fertilizer

|  |  |  | Unadjusted |  |
| :---: | ---: | :---: | ---: | ---: |
| yield | Mean | Std. Err. | [95\% Conf. Interval] |  |

## Pairwise differences of means

We can compute all pairwise differences in mean wheat yields for the types of fertilizers.

```
. pwmean yield, over(fertilizer) effects
Pairwise comparisons of means with equal variances
over : fertilizer
```

| yield | Contrast | Std. Err. | Unadjusted |  | Unadjusted |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf | Interval] |
| fertilizer |  |  |  |  |  |  |
| 10-08-22 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-10-10 | 3.62272 | 1.589997 | 2.28 | 0.024 | . 4869212 | 6.758518 |
| 16-04-08 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-10-10 | . 4906299 | 1.589997 | 0.31 | 0.758 | -2.645169 | 3.626428 |
| 18-24-06 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-10-10 | 4.922803 | 1.589997 | 3.10 | 0.002 | 1.787005 | 8.058602 |
| 29-03-04 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-10-10 | -1.238328 | 1.589997 | -0.78 | 0.437 | $-4.374127$ | 1.89747 |
| $16-04-08$ |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
|  | -3.13209 | 1.589997 | -1.97 | 0.050 | -6.267889 | . 0037086 |
| $18-24-06$ |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
|  | 1.300083 | 1.589997 | 0.82 | 0.415 | $-1.835715$ | 4.435882 |
| 29-03-04 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-08-22 | -4.861048 | 1.589997 | -3.06 | 0.003 | -7.996847 | -1.725249 |
| 18-24-06 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 16-04-08 | 4.432173 | 1.589997 | 2.79 | 0.006 | 1.296375 | 7.567972 |
| 29-03-04 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 16-04-08 | -1.728958 | 1.589997 | -1.09 | 0.278 | $-4.864757$ | 1.406841 |
| 29-03-04 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 18-24-06 | -6.161132 | 1.589997 | -3.87 | 0.000 | -9.29693 | -3.025333 |

The contrast in the row labeled ( $10-08-22$ vs $10-10-10$ ) is the difference in the mean wheat yield for fertilizer 10-08-22 and fertilizer 10-10-10. At a $5 \%$ significance level, we conclude that there is a difference in the means for these two fertilizers. Likewise, the rows labeled (18-24-06 vs $10-10-10$ ), ( $29-03-04$ vs $10-08-22$ ), ( $18-24-06$ vs $16-04-08$ ) and ( $29-03-04$ vs $18-24-06$ ) show differences in these pairs of means. In all, we find that 5 of the 10 mean differences are significantly different from zero at a 5\% significance level.

We can specify the sort option to order the differences from smallest to largest in the table.

```
. pwmean yield, over(fertilizer) effects sort
Pairwise comparisons of means with equal variances
over : fertilizer
```

| yield | Contrast | Std. Err. | Unadjusted |  | Unadjusted |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | t | $p>\|t\|$ | [95\% Conf | Interval] |
| $\begin{gathered} \text { fertilizer } \\ 29-03-04 \end{gathered}$ |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 18-24-06 | -6.161132 | 1.589997 | -3.87 | 0.000 | -9.29693 | -3.025333 |
| 29-03-04 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-08-22 | -4.861048 | 1.589997 | -3.06 | 0.003 | -7.996847 | -1.725249 |
| 16-04-08 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-08-22 | -3.13209 | 1.589997 | -1.97 | 0.050 | -6.267889 | . 0037086 |
| 29-03-04 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 16-04-08 | -1.728958 | 1.589997 | -1.09 | 0.278 | $-4.864757$ | 1.406841 |
| 29-03-04 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
|  | -1.238328 | 1.589997 | -0.78 | 0.437 | $-4.374127$ | 1.89747 |
| $16-04-08$ |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-10-10 | . 4906299 | 1.589997 | 0.31 | 0.758 | $-2.645169$ | 3.626428 |
| 18-24-06 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-08-22 | 1.300083 | 1.589997 | 0.82 | 0.415 | $-1.835715$ | 4.435882 |
| 10-08-22 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
|  | 3.62272 | 1.589997 | 2.28 | 0.024 | . 4869212 | 6.758518 |
| $10-10-10$$18-24-06$$\quad \begin{array}{llllll} \\ 18\end{array}$ |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| $16-04-08$$18-24-06$ |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-10-10 | 4.922803 | 1.589997 | 3.10 | 0.002 | 1.787005 | 8.058602 |

Ordering the pairwise differences is particularly convenient when we are comparing means for a large number of groups.

## Group output

We can use the group option to see the mean of each group and a visual representation of the tests for differences.

| . pwmean yield, over(fertilizer) group sort |
| :--- |
| Pairwise comparisons of means with equal variances |
| over |
| : fertilizer |


| yield | Mean | Std. Err. | Unadjusted <br> Groups |
| ---: | ---: | ---: | ---: |
| fertilizer |  |  |  |
| $29-03-04$ | 40.1241 | 1.124298 | A |
| $10-10-10$ | 41.36243 | 1.124298 | A |
| $16-04-08$ | 41.85306 | 1.124298 | AB |
| $10-08-22$ | 44.98515 | 1.124298 | BC |
| $18-24-06$ | 46.28523 | 1.124298 | C |

Note: Means sharing a letter in the group label are not significantly different at the $5 \%$ level.

Fertilizers 29-03-04, 10-10-10, and 16-04-08 are all in group A. This means that at our 5\% level of significance, we have insufficient information to distinguish their means. Likewise, fertilizers 16-0408 and 10-08-22 are in group B and cannot be distinguished at the $5 \%$ level. The same is true for fertilizers 10-08-22 and 18-24-06 in group C.

Fertilizer 29-03-04 and fertilizer 10-08-22 have no letters in common, indicating that the mean yields of these two groups are significantly different at the $5 \%$ level. We can conclude that any other fertilizers without a letter in common have significantly different means as well.

## Adjusting for multiple comparisons

The statistics in the examples above take no account that we are performing 10 comparisons. With our $5 \%$ significance level and assuming the comparisons are independent, we expect 1 in 20 tests of comparisons to be significant, even if all the population means are truly the same. If we are performing many comparisons, then we should account for the fact that some tests will be found significant by chance alone. More formally, the test for each pairwise comparison is made without adjusting for the elevated type I experimentwise error rate that is introduced when performing multiple tests. We can use the mcompare () option to adjust the confidence intervals and $p$-values for multiple comparisons.

## Tukey's method

Of the available adjustments for multiple comparisons, Tukey's honestly significant difference, Student-Newman-Keuls' method, and Duncan's method are most often used when performing all pairwise comparisons of means. Of these, Tukey's method is the most conservative and Duncan's method is the least conservative. For further discussion of each of the multiple-comparison adjustments, see [R] pwcompare.

Here we use Tukey's adjustment to compute $p$-values and confidence intervals for the pairwise differences.

| Pairwise comparisons of means with equal variances over : fertilizer |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Number of Comparisons |  |  |  |  |  |
| fertilizer | 10 |  |  |  |  |  |
| yield | Contrast | Std. Err. | Tukey |  | Tukey |  |
|  |  |  | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Con | Interval] |
| $\begin{gathered} \text { fertilizer } \\ 29-03-04 \end{gathered}$ |  |  |  |  |  |  |
| vs |  |  |  |  |  | -1.78312 |
| 18-24-06 | -6.161132 | 1.589997 | -3.87 | 0.001 | -10.53914 |  |
| 29-03-04 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-08-22 | -4.861048 | 1.589997 | -3.06 | 0.021 | -9.239059 | -. 4830368 |
| 16-04-08 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-08-22 | -3.13209 | 1.589997 | -1.97 | 0.285 | -7.510101 | 1.245921 |
| 29-03-04 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 16-04-08 | -1.728958 | 1.589997 | -1.09 | 0.813 | -6.106969 | 2.649053 |
| 29-03-04 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-10-10 | -1.238328 | 1.589997 | -0.78 | 0.936 | -5.616339 | 3.139683 |
| 16-04-08 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-10-10 | . 4906299 | 1.589997 | 0.31 | 0.998 | -3.887381 | 4.868641 |
| 18-24-06 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-08-22 | 1.300083 | 1.589997 | 0.82 | 0.925 | -3.077928 | 5.678095 |
| 10-08-22 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-10-10 | 3.62272 | 1.589997 | 2.28 | 0.156 | -. 7552913 | 8.000731 |
| 18-24-06 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 16-04-08 | 4.432173 | 1.589997 | 2.79 | 0.046 | . 0541623 | 8.810185 |
| 18-24-06 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-10-10 | 4.922803 | 1.589997 | 3.10 | 0.019 | . 5447922 | 9.300815 |

When using a 5\% significance level, Tukey's adjustment indicates that four pairs of means are different. With the adjustment, we no longer conclude that the difference in the mean yields for fertilizers $10-08-22$ and $10-10-10$ is significantly different from zero.

## Dunnett's method

Now let's suppose that fertilizer 10-10-10 actually represents fields on which no fertilizer was applied. In this case, we can use Dunnett's method for comparing each of the fertilizers to the control.

```
. pwmean yield, over(fertilizer) effects mcompare(dunnett)
Pairwise comparisons of means with equal variances
over : fertilizer
\begin{tabular}{l|r}
\hline & \begin{tabular}{r} 
Number of \\
Comparisons
\end{tabular} \\
\hline fertilizer & 4 \\
\hline
\end{tabular}
```

| yield | Contrast | Std. Err. | Dunnett |  | Dunnett |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf | Interval] |
| fertilizer |  |  |  |  |  |  |
| 10-08-22 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-10-10 | 3.62272 | 1.589997 | 2.28 | 0.079 | -. 2918331 | 7.537273 |
| 16-04-08 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
|  | . 4906299 | 1.589997 | 0.31 | 0.994 | -3.423923 | 4.405183 |
| $18-24-06$ |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
|  | 4.922803 | 1.589997 | 3.10 | 0.008 | 1.00825 | 8.837356 |
| 29-03-04 |  |  |  |  |  |  |
| vs |  |  |  |  |  |  |
| 10-10-10 | -1.238328 | 1.589997 | -0.78 | 0.852 | -5.152881 | 2.676225 |

Using Dunnett's adjustment, we conclude that only fertilizer 4 (18-24-06) produces a mean yield that is significantly different from the mean yield of the field with no fertilizer applied.

By default, pwmean treats the lowest level of the group variable as the control. If, for instance, fertilizer 3 (16-04-08) was our control group, we could type

```
. pwmean yield, over(b3.fertilizer) effects mcompare(dunnett)
```

using the b3. factor-variable operator to specify this level as the reference level.

## Multiple over() variables

When we specify more than one variable in the over () option, pairwise comparisons are performed for the means defined by each combination of levels of these variables.

```
. pwmean yield, over(fertilizer irrigation) group
Pairwise comparisons of means with equal variances
over : fertilizer irrigation
```

| yield | Mean | Std. Err. | Unadjusted |
| :---: | :---: | :---: | :---: |
|  |  |  | Groups |
| fertilizer\#irrigation |  |  |  |
| 10-10-10\#0 | 36.91257 | 1.116571 | A |
| 10-10-10\#1 | 45.81229 | 1.116571 | B |
| 10-08-22\#0 | 38.79482 | 1.116571 | A C |
| 10-08-22\#1 | 51.17547 | 1.116571 | E |
| 16-04-08\#0 | 36.34383 | 1.116571 | A |
| 16-04-08\#1 | 47.36229 | 1.116571 | B |
| 18-24-06\#0 | 41.81757 | 1.116571 | CD |
| 18-24-06\#1 | 50.7529 | 1.116571 | E |
| 29-03-04\#0 | 35.69507 | 1.116571 | A |
| 29-03-04\#1 | 44.55313 | 1.116571 | B D |

Note: Means sharing a letter in the group label are not significantly different at the 5\% level.

Here the row labeled $10-10-10 \# 0$ is the mean for the fields treated with fertilizer 10-10-10 and without irrigation. This mean is significantly different from the mean of all fertilizer/irrigation pairings that do not have an A in the "Unadjusted Groups" column. These include all pairings where the fields were irrigated as well as the fields treated with fertilizer 18-24-06 but without irrigation.

## Equal variance assumption

pwmean performs multiple comparisons assuming that there is a common variance for all groups. In the case of two groups, this is equivalent to performing the familiar two-sample $t$ test when equal variances are assumed.
. ttest yield, by(irrigation)
Two-sample t test with equal variances

| Group | Obs | Mean | Std. Err. | Std. Dev. | [95\% Conf | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 100 | 37.91277 | . 5300607 | 5.300607 | 36.86102 | 38.96453 |
| 1 | 100 | 47.93122 | . 5630353 | 5.630353 | 46.81403 | 49.0484 |
| combined | 200 | 42.92199 | . 5242462 | 7.413961 | 41.8882 | 43.95579 |
| diff |  | -10.01844 | . 7732872 |  | -11.54338 | -8.493509 |
| $\begin{aligned} \text { diff } & =\text { mean }(0)-\operatorname{mean}(1) \\ \text { Ho: } \operatorname{diff} & =0\end{aligned}$ |  |  |  | degrees of freedom |  | - 12.9557 |
|  |  |  |  |  |  | 198 |
| Ha: |  | Ha: diff != 0 |  |  | Ha: diff > 0 <br> $\operatorname{Pr}(T>t)=1.0000$ |  |
| $\operatorname{Pr}(\mathrm{T}<$ | 0000 | $\operatorname{Pr}(\|T\|>\|t\|)=0.0000$ |  |  |  |  |

. pwmean yield, over(irrigation) effects
Pairwise comparisons of means with equal variances
over : irrigation

|  |  |  | Unadjusted |  | Unadjusted |  |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: |
| yield | Contrast | Std. Err. | t | $\mathrm{P}>\|\mathrm{t\mid}\|$ | [95\% Conf. Interval] |  |

The signs for the difference, the test statistic, and the confidence intervals are reversed because the difference is taken in the opposite direction. The $p$-value from pwmean is equivalent to the one for the two-sided test in the ttest output.
pwmean extends the capabilities of ttest to allow for simultaneously comparing all pairs of means and to allow for using one common variance estimate for all the tests instead of computing a separate pooled variance for each pair of means when using multiple ttest commands. In addition, pwmean allows adjustments for multiple comparisons, many of which rely on an assumption of equal variances among groups.

## Stored results

pwmean stores the following in $e()$ :
Scalars
e(df_r)
$e$ (balanced)
Macros
e(cmd)
e(cmdline)
e(title)
e(depvar)
e(over)
e(properties)
Matrices
e(b)
e(V)
e(error)
e(b_vs)
e(V_vs)
e(error_vs)
e(k_groups)
variance degrees of freedom
1 if fully balanced data; 0 otherwise
pwmean
command as typed
title in output
name of variable from which the means are computed
varlist from over()
b V
mean estimates
variance-covariance matrix of the mean estimates
mean estimability codes;
0 means estimable,
8 means not estimable
mean difference estimates
variance-covariance matrix of the mean difference estimates
mean difference estimability codes;
0 means estimable,
8 means not estimable
number of significance groups for each term

## Methods and formulas

pwmean is a convenience command that uses pwcompare after fitting a fully factorial linear model.
See Methods and formulas described in [ R$]$ pwcompare.

## Reference

Searle, S. R. 1997. Linear Models for Unbalanced Data. New York: Wiley.

## Also see

[R] pwmean postestimation - Postestimation tools for pwmean
[R] contrast - Contrasts and linear hypothesis tests after estimation
[R] margins - Marginal means, predictive margins, and marginal effects
[R] margins, pwcompare - Pairwise comparisons of margins
[R] pwcompare - Pairwise comparisons
$[R]$ ttest $-t$ tests (mean-comparison tests)
[U] 20 Estimation and postestimation commands

## Title

pwmean postestimation - Postestimation tools for pwmean

## Description Remarks and examples Also see

## Description

The following postestimation commands are available after pwmean:

| Command | Description |
| :--- | :--- |
| estat vce | variance-covariance matrix of the estimators (VCE) <br> estimates <br> cataloging estimation results |
| lincom | point estimates, standard errors, testing, and inference for linear combinations <br> of coefficients <br> point estimates, standard errors, testing, and inference for nonlinear combinations <br> of coefficients |
| nlcom | Wald tests of simple and composite linear hypotheses |
| test | Wald tests of nonlinear hypotheses |

## Remarks and examples

In Pairwise differences of means of [R] pwmean, we computed all pairwise differences in mean wheat yields for five fertilizers.

```
. use http://www.stata-press.com/data/r13/yield
(Artificial wheat yield dataset)
. pwmean yield, over(fertilizer)
Pairwise comparisons of means with equal variances
over : fertilizer
```

|  |  |  | Unadjusted <br> [95\% Conf. Interval] |  |
| :--- | ---: | ---: | ---: | ---: |
|  | yield | Contrast | Std. Err. |  |
| fertilizer |  |  |  |  |
| $10-08-22$ vs $10-10-10$ | 3.62272 | 1.589997 | -4869212 | 6.758518 |
| $16-04-08$ | vs $10-10-10$ | .4906299 | 1.589997 | -2.645169 |
| $18-24-06$ vs $10-10-10$ | 4.922803 | 1.589997 | 1.787005 | 8.0586428 |
| $29-03-04$ vs $10-10-10$ | -1.238328 | 1.589997 | -4.374127 | 1.89747 |
| $16-04-08$ vs $10-08-22$ | -3.13209 | 1.589997 | -6.267889 | .0037086 |
| $18-24-06$ vs $10-08-22$ | 1.300083 | 1.589997 | -1.835715 | 4.435882 |
| $29-03-04$ vs $10-08-22$ | -4.861048 | 1.589997 | -7.996847 | -1.725249 |
| $18-24-06$ vs $16-04-08$ | 4.432173 | 1.589997 | 1.296375 | 7.567972 |
| $29-03-04$ vs $16-04-08$ | -1.728958 | 1.589997 | -4.864757 | 1.406841 |
| $29-03-04$ vs $18-24-06$ | -6.161132 | 1.589997 | -9.29693 | -3.025333 |

After pwmean, we can use testnl to test whether the improvement in mean wheat yield when using fertilizer 18-24-06 instead of fertilizer 29-03-04 is significantly different from $10 \%$.
. testnl (_b[4.fertilizer] - _b[5.fertilizer])/_b[5.fertilizer] = 0.1
(1) (_b[4.fertilizer] - _b[5.fertilizer])/_b[5.fertilizer] = 0.1 $\operatorname{chi2}(1)=1.57$ Prob $>$ chi2 $=0.2106$

The improvement is not significantly different from $10 \%$.

## Also see

[R] pwmean - Pairwise comparisons of means
[U] 20 Estimation and postestimation commands

## Title

qc - Quality control charts

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |
| Also see |  |  |  |

## Syntax

Draw a c chart
cchart defect_var unit_var [, cchart_options]

Draw a p (fraction-defective) chart
pchart reject_var unit_var ssize_var [, pchart_options]

Draw an $R$ (range or dispersion) chart rchart varlist [if] [in] [, rchart_options]

Draw an $\overline{\mathrm{X}}$ (control line) chart
xchart varlist [if] [in] [, xchart_options $]$

Draw vertically aligned $\overline{\mathrm{X}}$ and R charts
shewhart varlist [if] [in] [, shewhart_options]
cchart_options
Description
Main

## nograph

Plot
connect_options
marker_options
marker_label_options
Control limits
clopts(cline_options)
Add plots
addplot (plot)
Y axis, X axis, Titles, Legend, Overall
twoway_options
suppress graph
affect rendition of the plotted points change look of markers (color, size, etc.) add marker labels; change look or position
affect rendition of the control limits
add other plots to the generated graph
any options other than by () documented in [G-3] twoway_options

## pchart_options

Description

## Main

## stabilized

nograph
generate(newvar newvar $_{\text {lcl }}$ newvar $_{\text {ucl }}$ )

Plot
connect_options
marker_options
marker_label_options
Control limits
clopts(cline_options)
Add plots
addplot (plot)
Y axis, X axis, Titles, Legend, Overall
twoway_options
stabilize the p chart when sample sizes are unequal suppress graph
store the fractions of defective elements and the lower and upper control limits
affect rendition of the plotted points
change look of markers (color, size, etc.)
add marker labels; change look or position
affect rendition of the control limits
add other plots to the generated graph
any options other than by () documented in [G-3] twoway_options
rchart_options
Main
std (\#)
nograph
Plot
connect_options
marker_options
marker_label_options
Control limits
clopts (cline_options)
Add plots
addplot (plot)
Y axis, X axis, Titles, Legend, Overall
twoway_options

Description
user-specified standard deviation
suppress graph
affect rendition of the plotted points change look of markers (color, size, etc.) add marker labels; change look or position
affect rendition of the control limits
add other plots to the generated graph
any options other than by () documented in [G-3] twoway_options
xchart_options
Main

```
std(#)
```

    mean (\#)
    lower(\#) upper (\#)
    nograph
    Plot
connect_options
marker_options
marker_label_options
Control limits
clopts (cline_options)
Add plots
addplot (plot)
Y axis, X axis, Titles, Legend, Overall
twoway_options

Description
user-specified standard deviation user-specified mean
lower and upper limits of the X-bar limits suppress graph
affect rendition of the plotted points change look of markers (color, size, etc.) add marker labels; change look or position
affect rendition of the control limits
add other plots to the generated graph
any options other than by () documented in [G-3] twoway_options
shewhart_options
Main

| $\underline{\text { std (\#) }}$ | user-specified standard deviation <br> mean(\#) |
| :--- | :--- |
| $\underline{\text { nograph }}$ | user-specified mean <br> suppress graph |
| Plot |  |
| connect_options | affect rendition of the plotted points <br> change look of markers (color, size, etc.) |
| marker_options | add marker labels; change look or positions |

Control limits
clopts (cline_options)
Y axis, X axis, Titles, Legend, Overall
combine_options

Description
affect rendition of the plotted points change look of markers (color, size, etc.) add marker labels; change look or position
affect rendition of the control limits
any options documented in [G-2] graph combine

## Menu

cchart
Statistics > Other > Quality control > C chart
pchart
Statistics $>$ Other $>$ Quality control $>\mathrm{P}$ chart
rchart
Statistics $>$ Other $>$ Quality control $>\mathrm{R}$ chart

## xchart

Statistics $>$ Other $>$ Quality control $>$ X-bar chart

## shewhart

Statistics $>$ Other $>$ Quality control $>$ Vertically aligned X-bar and R chart

## Description

These commands provide standard quality-control charts. cchart draws a c chart; pchart, a p (fraction-defective) chart; rchart, an $R$ (range or dispersion) chart; xchart, an $\overline{\mathrm{X}}$ (control line) chart; and shewhart, vertically aligned $\bar{X}$ and $R$ charts.

## Options

Main
stabilized stabilizes the p chart when sample sizes are unequal.
std (\#) specifies the standard deviation of the process. The R chart is calculated (based on the range) if this option is not specified.
mean(\#) specifies the grand mean, which is calculated if not specified.
lower(\#) and upper(\#) must be specified together or not at all. They specify the lower and upper limits of the X chart. Calculations based on the mean and standard deviation (whether specified by option or calculated) are used otherwise.
nograph suppresses the graph.
generate (newvar ${ }_{f}$ newvar $_{\text {lcl }}$ newvar $_{\mathrm{ucl}}$ ) stores the plotted values in the p chart. newvar ${ }_{f}$ will contain the fractions of defective elements; newvar $r_{\mathrm{lcl}}$ and newvar ucl will contain the lower and upper control limits, respectively.
$\qquad$ Plot
connect_options affect whether lines connect the plotted points and the rendition of those lines; see [G-3] connect_options.
marker_options affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] marker_options.
marker_label_options specify if and how the markers are to be labeled; see [G-3] marker_label_options.
$\qquad$ Control limits
clopts (cline_options) affects the rendition of the control limits; see [G-3] cline_options.

Add plots
addplot (plot) provides a way to add other plots to the generated graph. See [G-3] addplot_option.

Y axis, X axis, Titles, Legend, Overall
twoway_options are any of the options documented in [G-3] twoway_options, excluding by (). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).
combine_options (shewhart only) are any of the options documented in [G-2] graph combine. These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

## Remarks and examples

Control charts may be used to define the goal of a repetitive process, to control that process, and to determine if the goal has been achieved. Walter A. Shewhart of Bell Telephone Laboratories devised the first control chart in 1924. In 1931, Shewhart published Economic Control of Quality of Manufactured Product. According to Burr, "Few fields of knowledge have ever been so completely explored and charted in the first exposition" $(1976,29)$. Shewhart states that "a phenomenon will be said to be controlled when, through the use of past experience, we can predict, at least within limits, how the phenomenon may be expected to vary in the future. Here it is understood that prediction within limits means that we can state, at least approximately, the probability that the observed phenomenon will fall within given limits" $(1931,6)$.

For more information on quality-control charts, see Burr (1976), Duncan (1986), Harris (1999), or Ryan (2000).

## > Example 1: cchart

cchart graphs a c chart showing the number of nonconformities in a unit, where defect_var records the number of defects in each inspection unit and unit_var records the unit number. The unit numbers need not be in order. For instance, consider the following example dataset from Ryan (2000, 156):

```
. use http://www.stata-press.com/data/r13/ncu
. describe
Contains data from http://www.stata-press.com/data/r13/ncu.dta
    obs: 30
    vars: 2 31 Mar 2013 03:56
    size: 240
```

| variable name | storage <br> type | display <br> format | value <br> label | variable label |
| :--- | :--- | :--- | :--- | :--- |
| day | float | $\% 9.0 \mathrm{~g}$ |  | Days in April <br> defects |
| float | $\% 9.0 \mathrm{~g}$ |  |  |  |

Sorted by:
. list in $1 / 5$

| day | defects |
| ---: | ---: |
| 1 | 7 |
| 2 | 5 |
| 3 | 11 |
| 4 | 13 |
| 5 | 9 |

```
. cchart defects day, title(c Chart for Nonconforming Transistors)
```



The expected number of defects is 10.6 , with lower and upper control limits of 0.8327 and 20.37, respectively. No units are out of control.

## Example 2: pchart

pchart graphs a p chart, which shows the fraction of nonconforming items in a subgroup, where reject_var records the number rejected in each inspection unit, unit_var records the inspection unit number, and ssize_var records the number inspected in each unit.

Consider the example dataset from Ryan $(2000,156)$ of the number of nonconforming transistors out of 1,000 inspected each day during the month of April:

```
. use http://www.stata-press.com/data/r13/ncu2
. describe
Contains data from http://www.stata-press.com/data/r13/ncu2.dta
    obs: 30
    vars: 3 31 Mar 2013 14:13
    size: 360
```

| variable name | storage type | display <br> format | value <br> label | variable label |
| :---: | :---: | :---: | :---: | :---: |
| day | float | \%9.0g |  | Days in April |
| rejects | float | \%9.0g |  | Numbers of Nonconforming Units |
| ssize | float | \%9.0g |  | Sample size |

[^64]. list in $1 / 5$

|  | day | rejects | ssize |
| :--- | ---: | ---: | ---: |
| 1. | 1 | 7 | 1000 |
| 2. | 2 | 5 | 1000 |
| 3. | 3 | 11 | 1000 |
| 4. | 4 | 13 | 1000 |
| 5. | 5 | 9 | 1000 |
|  |  |  |  |

pchart rejects day ssize


All the points are within the control limits, which are 0.0009 for the lower limit and 0.0203 for the upper limit.

Here the sample sizes are fixed at 1,000 , so the ssize variable contains 1,000 for each observation. Sample sizes need not be fixed, however. Say that our data were slightly different:

```
. use http://www.stata-press.com/data/r13/ncu3
```

. list in $1 / 5$

|  | day | rejects | ssize |
| :--- | ---: | ---: | ---: |
| 1. | 1 | 7 | 920 |
| 2. | 2 | 5 | 920 |
| 3. | 3 | 11 | 920 |
| 4. | 4 | 13 | 950 |
| 5. | 5 | 9 | 950 |
|  |  |  |  |

. pchart rejects day ssize


Here the control limits are, like the sample size, no longer constant. The stabilize option will stabilize the control chart:
. pchart rejects day ssize, stabilize

> Example 3: rchart
rchart displays an R chart showing the range for repeated measurements at various times. Variables within observations record measurements. Observations represent different samples.

For instance, say that we take five samples of 5 observations each. In our first sample, our measurements are $10,11,10,11$, and 12 . The data are
. list

|  | m 1 | m 2 | m 3 | m 4 | m 5 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| 1. | 10 | 11 | 10 | 11 | 12 |
| 2. | 12 | 10 | 9 | 10 | 9 |
| 3. | 10 | 11 | 10 | 12 | 10 |
| 4. | 9 | 9 | 9 | 10 | 11 |
| 5. | 12 | 12 | 12 | 12 | 13 |

. rchart m1-m5, connect(1)


The expected range in each sample is 2 with lower and upper control limits of 0 and 4.23 , respectively. If we know that the process standard deviation is 0.3 , we could specify

```
. rchart m1-m5, connect(1) std(.3)
```



## > Example 4: xchart

xchart graphs an $\bar{X}$ chart for repeated measurements at various times. Variables within observations record measurements, and observations represent different samples. Using the same data as in the previous example, we type
. xchart m1-m5, connect(1)


The average measurement in the sample is 10.64 , and the lower and upper control limits are 9.486 and 11.794 , respectively. Suppose that we knew from prior information that the mean of the process is 11 . Then we would type
. xchart m1-m5, connect(1) mean(11)


If we also know that the standard deviation of the process is 0.3 , we could type . xchart m1-m5, connect(1) mean(11) std(.3)


Finally, xchart allows us to specify our own control limits:

```
. xchart m1-m5, connect(1) mean(11) lower(10) upper(12)
```



Walter Andrew Shewhart (1891-1967) was born in Illinois and educated as a physicist, with degrees from the Universities of Illinois and California. After a brief period teaching physics, he worked for the Western Electric Company and (from 1925) the Bell Telephone Laboratories. His name is most associated with control charts used in quality controls, but his many other interests ranged generally from quality assurance to the philosophy of science.
$>$ Example 5: shewhart
shewhart displays a vertically aligned $\bar{X}$ and $R$ chart in the same image. To produce the bestlooking combined image possible, you will want to use the xchart and rchart commands separately and then combine the graphs. shewhart, however, is more convenient.

Using the same data as previously, but realizing that the standard deviation should have been 0.4 , we type


## Stored results

cchart stores the following in $r()$ :
Scalars

```
r(cbar) expected number of nonconformities
r(lcl_c) lower control limit
r(ucl_c) upper control limit
r(N) number of observations
r(out_c) number of units out of control
r(below_c) number of units below the lower limit
r(above_c) number of units above the upper limit
```

pchart stores the following in $r()$ :
Scalars

| $r(p b a r)$ | average fraction of nonconformities |
| :--- | :--- |
| $r\left(\operatorname{lcl} \_p\right)$ | lower control limit |
| $r\left(u c l \_p\right)$ | upper control limit |
| $r(N)$ | number of observations |
| $r($ out_p | number of units out of control |
| $r($ below_p | number of units below the lower limit |
| $r($ above_p | number of units above the upper limit |

rchart stores the following in $r()$ :
Scalars
$r$ (central_line) ordinate of the central line
$r\left(1 c l \_r\right) \quad$ lower control limit
$r$ (ucl_r) upper control limit
$r(N) \quad$ number of observations
$r$ (out_r) number of units out of control
$r$ (below_r) number of units below the lower limit
$r$ (above_r) number of units above the upper limit
xchart stores the following in $r()$ :
Scalars

| $r(x b a r)$ | grand mean |
| :--- | :--- |
| $r\left(l c l \_x\right)$ | lower control limit |
| $r\left(u c l \_x\right)$ | upper control limit |
| $r(N)$ | number of observations |
| $r($ out_x $)$ | number of units out of control |
| $r($ below_x $)$ | number of units below the lower limit |
| $r($ above_x $)$ | number of units above the upper limit |

shewhart stores in $r()$ the combination of stored results from xchart and rchart.

## Methods and formulas

For the c chart, the number of defects per unit, $C$, is taken to be a value of a random variable having a Poisson distribution. If $k$ is the number of units available for estimating $\lambda$, the parameter of the Poisson distribution, and if $C_{i}$ is the number of defects in the $i$ th unit, then $\lambda$ is estimated by $\bar{C}=\sum_{i} C_{i} / k$. Then

$$
\begin{aligned}
\text { central line } & =\bar{C} \\
\mathrm{UCL} & =\bar{C}+3 \sqrt{\bar{C}} \\
\mathrm{LCL} & =\bar{C}-3 \sqrt{\bar{C}}
\end{aligned}
$$

Control limits for the p chart are based on the sampling theory for proportions, using the normal approximation to the binomial. If $k$ samples are taken, the estimator of $p$ is given by $\bar{p}=\sum_{i} \widehat{p_{i}} / k$, where $\widehat{p}_{i}=x_{i} / n_{i}$, and $x_{i}$ is the number of defects in the $i$ th sample of size $n_{i}$. The central line and the control limits are given by

$$
\begin{aligned}
\text { central line } & =\bar{p} \\
\mathrm{UCL} & =\bar{p}+3 \sqrt{\bar{p}(1-\bar{p}) / n_{i}} \\
\mathrm{LCL} & =\bar{p}-3 \sqrt{\bar{p}(1-\bar{p}) / n_{i}}
\end{aligned}
$$

Control limits for the R chart are based on the distribution of the range of samples of size $n$ from a normal population. If the standard deviation of the process, $\sigma$, is known,

$$
\begin{aligned}
\text { central line } & =d_{2} \sigma \\
\mathrm{UCL} & =D_{2} \sigma \\
\mathrm{LCL} & =D_{1} \sigma
\end{aligned}
$$

where $d_{2}, D_{1}$, and $D_{2}$ are functions of the number of observations in the sample and are obtained from the table published in Beyer (1976).

When $\sigma$ is unknown,

$$
\begin{aligned}
\text { central line } & =\bar{R} \\
\mathrm{UCL} & =\left(D_{2} / d_{2}\right) \bar{R} \\
\mathrm{LCL} & =\left(D_{1} / d_{2}\right) \bar{R}
\end{aligned}
$$

where $\bar{R}=\sum_{i} R_{i} / k$ is the range of the $k$ sample ranges $R_{i}$.

Control limits for the $\overline{\mathrm{X}}$ chart are given by

$$
\begin{aligned}
\text { central line } & =\bar{x} \\
\mathrm{UCL} & =\bar{x}+(3 / \sqrt{n}) \sigma \\
\mathrm{LCL} & =\bar{x}-(3 / \sqrt{n}) \sigma
\end{aligned}
$$

if $\sigma$ is known. If $\sigma$ is unknown,

$$
\begin{aligned}
\text { central line } & =\bar{x} \\
\mathrm{UCL} & =\bar{x}+A_{2} \bar{R} \\
\mathrm{LCL} & =\bar{x}-A_{2} \bar{R}
\end{aligned}
$$

where $\bar{R}$ is the average range as defined above and $A_{2}$ is a function (op. cit.) of the number of observations in the sample.

## References

Bayart, D. 2001. Walter Andrew Shewhart. In Statisticians of the Centuries, ed. C. C. Heyde and E. Seneta, 398-401. New York: Springer.
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## Also see

[R] serrbar - Graph standard error bar chart

## Title

qreg - Quantile regression

| Syntax | Menu | Description | Options for qreg |
| :--- | :--- | :--- | :--- |
| Options for iqreg | Options for sqreg | Options for bsqreg | Remarks and examples |
| Stored results | Methods and formulas | References | Also see |

## Syntax

Quantile regression
qreg depvar [indepvars] [if] [in] [weight] [, qreg_options]

Interquantile range regression
iqreg depvar [indepvars] [if] [in] [, iqreg_options]

Simultaneous-quantile regression
sqreg depvar [indepvars] [if] [in] [, sqreg_options]

Bootstrapped quantile regression
bsqreg depvar [indepvars] [if] [in] [, bsqreg_options]
qreg_options Description
Model
quantile(\#) estimate \# quantile; default is quantile(.5)
SE/Robust
$\operatorname{vce}([$ vcetype $],[$ vceopts $]$ ) technique used to estimate standard errors
Reporting
level(\#)
display_options
set confidence level; default is level (95)
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling

Optimization
optimization_options
wlsiter(\#)
control the optimization process; seldom used attempt \# weighted least-squares iterations before doing linear programming iterations
vcetype
iid
robust

Description
compute the VCE assuming the residuals are i.i.d. compute the robust VCE
denmethod
bwidth
nonparametric density estimation technique bandwidth method used by the density estimator
denmethod Description
fitted use the empirical quantile function using fitted values; the default
residual use the empirical residual quantile function
kernel [(kernel)] use a nonparametric kernel density estimator; default is epanechnikov
bwidth Description

| hsheather | Hall-Sheather's bandwidth; the default |
| :--- | :--- |
| bofinger | Bofinger's bandwidth |
| chamberlain | Chamberlain's bandwidth |

kernel
Description
epanechnikov
epan2
biweight
cosine
gaussian
parzen
rectangle
triangle

Epanechnikov kernel function; the default
alternative Epanechnikov kernel function
biweight kernel function
cosine trace kernel function
Gaussian kernel function
Parzen kernel function
rectangle kernel function
triangle kernel function
iqreg_options
Description
Model
quantiles(\# \#)
interquantile range; default is quantiles(.25 .75)
reps(\#)
perform \# bootstrap replications; default is reps (20)
Reporting
level(\#)
nodots
display_options
set confidence level; default is level (95)
suppress display of the replication dots
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling

## Model

quantiles(\# [\# [\# ...]]) reps (\#)

Reporting
level(\#)
nodots
display_options
estimate \# quantiles; default is quantiles(.5) perform \# bootstrap replications; default is reps (20)
set confidence level; default is level (95)
suppress display of the replication dots
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
bsqreg_options
Model
quantile(\#)
reps(\#)
Reporting
level(\#)
display_options

## Description

estimate \# quantile; default is quantile(.5)
perform \# bootstrap replications; default is reps (20)
set confidence level; default is level (95)
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
by, mi estimate, rolling, and statsby, are allowed by qreg, iqreg, sqreg, and bsqreg; mfp, nestreg, and stepwise are allowed only with qreg; see [U] 11.1.10 Prefix commands.
qreg allows fweights, iweights, and pweights; see [U] 11.1.6 weight.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

## qreg

Statistics $>$ Nonparametric analysis $>$ Quantile regression

## iqreg

Statistics $>$ Nonparametric analysis $>$ Interquantile regression

## sqreg

Statistics $>$ Nonparametric analysis $>$ Simultaneous-quantile regression

## bsqreg

Statistics $>$ Nonparametric analysis $>$ Bootstrapped quantile regression

## Description

qreg fits quantile (including median) regression models, also known as least-absolute-value models (LAV or MAD) and minimum L1-norm models. The quantile regression models fit by qreg express the quantiles of the conditional distribution as linear functions of the independent variables.
iqreg estimates interquantile range regressions, regressions of the difference in quantiles. The estimated variance-covariance matrix of the estimators (VCE) is obtained via bootstrapping.
sqreg estimates simultaneous-quantile regression. It produces the same coefficients as qreg for each quantile. Reported standard errors will be similar, but sqreg obtains an estimate of the VCE via bootstrapping, and the VCE includes between-quantile blocks. Thus you can test and construct confidence intervals comparing coefficients describing different quantiles.
bsqreg is equivalent to sqreg with one quantile.

## Options for qreg

$\qquad$ Model
quantile(\#) specifies the quantile to be estimated and should be a number between 0 and 1 , exclusive. Numbers larger than 1 are interpreted as percentages. The default value of 0.5 corresponds to the median.

vcetype specifies the type of VCE to compute. Available types are iid and robust.
vce (iid), the default, computes the VCE under the assumption that the residuals are independent and identically distributed (i.i.d.).
vce (robust) computes the robust VCE under the assumption that the residual density is continuous and bounded away from 0 and infinity at the specified quantile(); see Koenker (2005, sec. 4.2).
vceopts consists of available denmethod and bwidth options.
denmethod specifies the method to use for the nonparametric density estimator. Available methods are fitted, residual, or kernel [(kernel)], where the optional kernel must be one of the kernel choices listed below.
fitted and residual specify that the nonparametric density estimator use some of the structure imposed by quantile regression. The default fitted uses a function of the fitted values and residual uses a function of the residuals. vce(robust, residual) is not allowed.
kernel() specifies that the nonparametric density estimator use a kernel method. The available kernel functions are epanechnikov, epan2, biweight, cosine, gaussian, parzen, rectangle, and triangle. The default is epanechnikov. See [R] kdensity for the kernel function forms.
bwidth specifies the bandwidth method to use by the nonparametric density estimator. Available methods are hsheather for the Hall-Sheather bandwidth, bofinger for the Bofinger bandwidth, and chamberlain for the Chamberlain bandwidth.

See Koenker (2005, sec. 3.4 and 4.10 ) for a description of the sparsity estimation techniques and the Hall-Sheather and Bofinger bandwidth formulas. See Chamberlain (1994, eq. 2.2) for the Chamberlain bandwidth.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\%fmt), pformat(\%fmt), sformat(\%fmt), and nolstretch; see [R] estimation options.

Optimization
optimization_options: iterate(\#), [no] log, trace. iterate() specifies the maximum number of iterations; log/nolog specifies whether to show the iteration log; and trace specifies that the iteration $\log$ should include the current parameter vector. These options are seldom used.
wlsiter (\#) specifies the number of weighted least-squares iterations that will be attempted before the linear programming iterations are started. The default value is 1 . If there are convergence problems, increasing this number should help.

## Options for iqreg

Model
quantiles(\# \#) specifies the quantiles to be compared. The first number must be less than the second, and both should be between 0 and 1, exclusive. Numbers larger than 1 are interpreted as percentages. Not specifying this option is equivalent to specifying quantiles(.25 .75), meaning the interquantile range.
reps(\#) specifies the number of bootstrap replications to be used to obtain an estimate of the variance-covariance matrix of the estimators (standard errors). reps (20) is the default and is arguably too small. reps (100) would perform 100 bootstrap replications. reps(1000) would perform 1,000 replications.

Reporting
level(\#); see [R] estimation options.
nodots suppresses display of the replication dots.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\%fmt), pformat(\%fmt), sformat(\%fmt), and nolstretch; see [R] estimation options.

## Options for sqreg

Model
quantiles (\# [\#[\#...]]) specifies the quantiles to be estimated and should contain numbers between 0 and 1 , exclusive. Numbers larger than 1 are interpreted as percentages. The default value of 0.5 corresponds to the median.
reps(\#) specifies the number of bootstrap replications to be used to obtain an estimate of the variance-covariance matrix of the estimators (standard errors). reps (20) is the default and is arguably too small. reps (100) would perform 100 bootstrap replications. reps(1000) would perform 1,000 replications.
level (\#); see [R] estimation options.
nodots suppresses display of the replication dots.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% fimt), pformat (\%fmt), sformat (\% fint), and nolstretch; see [R] estimation options.

## Options for bsqreg

$\qquad$ Model
quantile(\#) specifies the quantile to be estimated and should be a number between 0 and 1 , exclusive. Numbers larger than 1 are interpreted as percentages. The default value of 0.5 corresponds to the median.
reps(\#) specifies the number of bootstrap replications to be used to obtain an estimate of the variance-covariance matrix of the estimators (standard errors). reps (20) is the default and is arguably too small. reps (100) would perform 100 bootstrap replications. reps (1000) would perform 1,000 replications.

Reporting
level (\#); see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\%fmt), pformat(\%fmt), sformat(\%fmt), and nolstretch; see [R] estimation options.

## Remarks and examples

Remarks are presented under the following headings:
Median regression
Quantile regression
Estimated standard errors
Interquantile and simultaneous-quantile regression What are the parameters?

## Median regression

qreg fits quantile regression models. The default form is median regression, where the objective is to estimate the median of the dependent variable, conditional on the values of the independent variables. This method is similar to ordinary regression, where the objective is to estimate the conditional mean of the dependent variable. Simply put, median regression finds a line through the data that minimizes the sum of the absolute residuals rather than the sum of the squares of the residuals, as in ordinary regression. Equivalently, median regression expresses the median of the conditional distribution of the dependent variable as a linear function of the conditioning (independent) variables. Cameron and Trivedi (2010, chap. 7) provide a nice introduction to quantile regression using Stata.

## > Example 1: Estimating the conditional median

Consider a two-group experimental design with 5 observations per group:

```
. use http://www.stata-press.com/data/r13/twogrp
. list
```

| $x$ | $y$ |
| :---: | ---: |
| 0 | 0 |
| 0 | 1 |
| 0 | 3 |
| 0 | 4 |
| 0 | 95 |
| 1 | 14 |
| 1 | 19 |
| 1 | 20 |
| 1 | 22 |
| 1 | 23 |


| Iteration 1: WLS sum of weighted deviations $=60.941342$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Iteration 1: sum of abs. weighted deviations <br> Iteration 2: sum of abs. weighted deviations |  |  |  | $\begin{array}{r} 55.5 \\ 55 \end{array}$ |  |  |
|  |  |  |  |  |  |  |
| Median regression |  |  |  | Number of obs $=10$ |  |  |
| Raw sum of deviations |  | 78.5 (abo |  |  |  |  |
| Min sum of | ons | 55 |  |  | do R2 | 0.2994 |
| y | Coef. | Std. Err | t | $P>\|t\|$ | [95\% Con | Interval] |
| x | 17 | 18.23213 | 0.93 | 0.378 | -25.04338 | 59.04338 |
| _cons | 3 | 12.89207 | 0.23 | 0.822 | -26.72916 | 32.72916 |

We have estimated the equation

$$
\mathrm{y}_{\text {median }}=3+17 \mathrm{x}
$$

We look back at our data. x takes on the values 0 and 1 , so the median for the $\mathrm{x}=0$ group is 3 , whereas for $\mathrm{x}=1$ it is $3+17=20$. The output reports that the raw sum of absolute deviations about 14 is 78.5 ; that is, the sum of $|y-14|$ is 78.5 . Fourteen is the unconditional median of $y$, although in these data, any value between 14 and 19 could also be considered an unconditional median (we have an even number of observations, so the median is bracketed by those two values). In any case, the raw sum of deviations of $y$ about the median would be the same no matter what number we choose between 14 and 19. (With a "median" of 14 , the raw sum of deviations is 78.5 . Now think of choosing a slightly larger number for the median and recalculating the sum. Half the observations will have larger negative residuals, but the other half will have smaller positive residuals, resulting in no net change.)

We turn now to the actual estimated equation. The sum of the absolute deviations about the solution $\mathrm{y}_{\text {median }}=3+17 \mathrm{x}$ is 55 . The pseudo- $R^{2}$ is calculated as $1-55 / 78.5 \approx 0.2994$. This result is based on the idea that the median regression is the maximum likelihood estimate for the double-exponential distribution.

## - Technical note

qreg is an alternative to regular regression or robust regression-see $[R]$ regress and $[R]$ rreg. Let's compare the results:

| Source | SS | df | MS |  | Number of obs $=10$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | F 1, 8) | 0.00 |
| Model | 2.5 | 1 | 2.5 |  | Prob > F | $=0.9586$ |
| Residual | 6978.4 | 8 | 872.3 |  | R -squared | $=0.0004$ |
|  |  |  |  |  | Adj R-squared | $=-0.1246$ |
| Total | 6980.9 | 9775 | 655556 |  | Root MSE | $=29.535$ |
| y | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Conf. | Interval] |
| x | -1 | 18.6794 | -0.05 | 0.959 | -44.07477 | 42.07477 |
| _cons | 20.6 | 13.20833 | 1.56 | 0.157 | -9.858465 | 51.05847 |

Unlike qreg, regress fits ordinary linear regression and is concerned with predicting the mean rather than the median, so both results are, in a technical sense, correct. Putting aside those technicalities, however, we tend to use either regression to describe the central tendency of the data, of which the mean is one measure and the median another. Thus we can ask, "which method better describes the central tendency of these data?"

Means-and therefore ordinary linear regression-are sensitive to outliers, and our data were purposely designed to contain two such outliers: 95 for $\mathrm{x}=0$ and 14 for $\mathrm{x}=1$. These two outliers dominated the ordinary regression and produced results that do not reflect the central tendency well-you are invited to enter the data and graph y against x .

Robust regression attempts to correct the outlier-sensitivity deficiency in ordinary regression:


Here rreg discarded the first outlier completely. (We know this because we included the genwt () option on rreg and, after fitting the robust regression, examined the weights.) For the other "outlier", rreg produced a weight of 0.47 .

In any case, the answers produced by qreg and rreg to describe the central tendency are similar, but the standard errors are different. In general, robust regression will have smaller standard errors because it is not as sensitive to the exact placement of observations near the median. You are welcome to try removing the first outlier in the qreg estimation to observe an improvement in the standard errors by typing

```
. qreg y x if _n!=5
```

Also, some authors (Rousseeuw and Leroy 1987, 11) have noted that quantile regression, unlike the unconditional median, may be sensitive to even one outlier if its leverage is high enough. Rousseeuw and Leroy (1987) discuss estimators that are more robust to perturbations to the data than either mean regression or quantile regression.

In the end, quantile regression may be more useful for the interpretation of the parameters that it estimates than for its robustness to perturbations to the data.

## > Example 2: Median regression

Let's now consider a less artificial example using the automobile data described in [U] 1.2.2 Example datasets. Using median regression, we will regress each car's price on its weight and length and whether it is of foreign manufacture:


The estimated equation is

$$
\text { price }_{\text {median }}=3.93 \text { weight }-41.25 \text { length }+3377.8 \text { foreign }+344.65
$$

The output may be interpreted in the same way as linear regression output; see [ R ] regress. The variables weight and foreign are significant, but length is not significant. The median price of the cars in these data is $\$ 4,934$. This value is a median (one of the two center observations), not the median, which would typically be defined as the midpoint of the two center observations.

## Quantile regression

Quantile regression is similar to median regression in that it estimates an equation expressing a quantile of the conditional distribution, albeit one that generally differs from the 0.5 quantile that is the median. For example, specifying quantile(.25) estimates the parameters that describe the 25 th percentile (first quartile) of the conditional distribution.

Quantile regression allows for effects of the independent variables to differ over the quantiles. For example, Chamberlain (1994) finds that union membership has a larger effect on the lower quantiles than on the higher quantiles of the conditional distribution of U.S. wages. That the effects of the independent variables may vary over quantiles of the conditional distribution is an important advantage of quantile regression over mean regression.

## $>$ Example 3: Estimating quantiles other than the median

Returning to real data, the equation for the 25 th percentile of price conditional on weight, length, and foreign in our automobile data is

| . use http://www.stata-press.com/data/r13/auto (1978 Automobile Data) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Iteration 1: WLS sum of weighted deviations $=49469.235$ |  |  |  |  |  |  |
| Iteration 1: sum of abs. weighted deviations $=49728.883$ |  |  |  |  |  |  |
| Iteration 2: sum of abs. weighted deviations $=45669.89$ |  |  |  |  |  |  |
| Iteration 3: sum of abs. weighted deviations $=43416.646$ |  |  |  |  |  |  |
| Iteration 4: sum of abs. weighted deviations $=41947.221$ |  |  |  |  |  |  |
| Iteration 5: sum of abs. weighted deviations $=41093.025$ |  |  |  |  |  |  |
| Iteration 6: sum of abs. weighted deviations $=37623.424$ |  |  |  |  |  |  |
| Iteration 7: sum of abs. weighted deviations $=35721.453$ |  |  |  |  |  |  |
| Iteration 8: sum of abs. weighted deviations $=35226.308$ |  |  |  |  |  |  |
| Iteration 9: sum of abs. weighted deviations $=34823.319$ |  |  |  |  |  |  |
| Iteration 10: sum of abs. weighted deviations $=34801.777$ |  |  |  |  |  |  |
| .25 Quantile regression Number of obs $=$ 74 <br> Raw sum of deviations 41912.75 (about 4187)   <br> Min sum of deviations 34801.78 Pseudo R2 $=\quad 0.1697$ |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| price | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Con | Interval] |
| weight | 1.831789 | . 6328903 | 2.89 | 0.005 | . 5695289 | 3.094049 |
| length | 2.84556 | 21.65558 | 0.13 | 0.896 | -40.34514 | 46.03626 |
| foreign | 2209.925 | 421.7401 | 5.24 | 0.000 | 1368.791 | 3051.059 |
| _cons | -1879.775 | 2468.46 | -0.76 | 0.449 | -6802.963 | 3043.413 |

Compared with our previous median regression, the coefficient on length now has a positive sign, and the coefficients on foreign and weight are reduced. The actual lower quantile is $\$ 4,187$, substantially less than the median $\$ 4,934$.

We can also estimate the upper quartile as a function of the same three variables:
qreg price weight length foreign, quantile(.75)
Iteration 1: WLS sum of weighted deviations $=55465.741$
Iteration $1:$ sum of abs. weighted deviations $=55652.957$
Iteration $2:$ sum of abs. weighted deviations $=52994.785$
Iteration $3:$ sum of abs. weighted deviations $=50189.446$
Iteration $4:$ sum of abs. weighted deviations $=49898.245$
Iteration $5:$ sum of abs. weighted deviations $=49398.106$
Iteration $6:$ sum of abs. weighted deviations $=49241.835$
Iteration $7:$ sum of abs. weighted deviations $=49197.967$
. 75 Quantile regression
Number of obs $=$
74
Raw sum of deviations 79860.75 (about 6342)
Min sum of deviations 49197.97 Pseudo R2 $=0.3840$

| price | Coef. | Std. Err. | t | P>\|t| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| weight | 9.22291 | 1.785767 | 5.16 | 0.000 | 5.66131 | 12.78451 |
| length | -220.7833 | 61.10352 | -3.61 | 0.001 | -342.6504 | -98.91616 |
| foreign | 3595.133 | 1189.984 | 3.02 | 0.004 | 1221.785 | 5968.482 |
| _cons | 20242.9 | 6965.02 | 2.91 | 0.005 | 6351.61 | 34134.2 |

This result tells a different story: weight is much more important, and length is now significant - with a negative coefficient! The prices of high-priced cars seem to be determined by factors different from those affecting the prices of low-priced cars.

## Technical note

One explanation for having substantially different regression functions for different quantiles is that the data are heteroskedastic, as we will demonstrate below. The following statements create a sharply heteroskedastic set of data:

```
. drop _all
. set obs 10000
obs was 0, now 10000
. set seed 50550
. gen x = . 1 + .9 * runiform()
. gen y = x * runiform()^2
```

Let's now fit the regressions for the 5th and 95th quantiles:

| qreg y x, quantile(.05) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Iteration 1: sum of abs. weighted deviations $=539.15959$ |  |  |  |  |  |  |
| Iteration 2: sum of abs. weighted deviations $=141.36772$ |  |  |  |  |  |  |
| Iteration 3: sum of abs. weighted deviations $=91.234609$ |  |  |  |  |  |  |
| Iteration 4: sum of abs. weighted deviations $=91.127281$ |  |  |  |  |  |  |
| Iteration 5: sum of abs. weighted deviations $=91.126351$ |  |  |  |  |  |  |
| Iteration 6: sum of abs. weighted deviations $=91.126236$ |  |  |  |  |  |  |
| Iteration 7: sum of abs. weighted deviations $=91.126229$ |  |  |  |  |  |  |
| Iteration 8: sum of abs. weighted deviations $=91.126224$ |  |  |  |  |  |  |
| Iteration 9: sum of abs. weighted deviations $=91.126221$ |  |  |  |  |  |  |
| .05 Quantile regression Number of obs $=$ 10000 <br> Raw sum of deviations 91.17849 (about .0009234)   <br> Min sum of deviations 91.12622 Pseudo R2 $=0.0006$ |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| y | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Conf | terval] |
| x | . 002601 | . 0004576 | 5.68 | 0.000 | . 001704 | . 003498 |
| _cons | -. 0001393 | . 0002782 | -0.50 | 0.617 | -. 0006846 | . 000406 |



| y | Coef. | Std. Err. | t | P>\|t| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| x | .8898259 | .0090984 | 97.80 | 0.000 | .8719912 | .9076605 |
| _cons | .0021514 | .0055307 | 0.39 | 0.697 | -.00869 | .0129927 |

The coefficient on x , in particular, differs markedly between the two estimates. For the mathematically inclined, it is not too difficult to show that the theoretical lines are $\mathrm{y}=0.0025 \mathrm{x}$ for the 5 th percentile and $\mathrm{y}=0.9025 \mathrm{x}$ for the 95 th, numbers in close agreement with our numerical results.

The estimator for the standard errors computed by qreg assumes that the sample is independent and identically distributed (i.i.d.); see Estimated standard errors and Methods and formulas for details. Because the data are conditionally heteroskedastic, we should have used bsqreg to consistently estimate the standard errors using a bootstrap method.

## Estimated standard errors

The variance-covariance matrix of the estimator (VCE) depends on the reciprocal of the density of the dependent variable evaluated at the quantile of interest. This function, known as the "sparsity function", is hard to estimate.

The default method, which uses the fitted values for the predicted quantiles, generally performs well, but other methods may be preferred in larger samples. The vce() suboptions denmethod and bwidth provide other estimators of the sparsity function, the details of which are described in Methods and formulas.

For models with heteroskedastic errors, option vce(robust) computes a Huber (1967) form of sandwich estimate (Koenker 2005). Alternatively, Gould (1992, 1997b) introduced generalized versions of qreg that obtain estimates of the standard errors by using bootstrap resampling (see Efron and Tibshirani [1993] or Wu [1986] for an introduction to bootstrap standard errors). The iqreg, sqreg, and bsqreg commands provide a bootstrapped estimate of the entire variance-covariance matrix of the estimators.

## > Example 4: Obtaining robust standard errors

Example 2 of qreg on real data above was a median regression of price on weight, length, and foreign using auto.dta. Suppose, after investigation, we are convinced that car price observations are not independent. We decide that standard errors robust to non-i.i.d. errors would be appropriate and use the option vce (robust).
. use http://www.stata-press.com/data/r13/auto, clear
(1978 Automobile Data)
. qreg price weight length foreign, vce(robust)
Iteration 1: WLS sum of weighted deviations $=$
Iteration 1: sum of abs. weighted deviations $=$
Iteration $2:$ sum of abs. weighted deviations $=$
Iteration 3: sum of abs. weighted deviations $=$
Iteration $4:$ sum of abs. weighted deviations $=$
Iteration 5: sum of abs. weighted deviations $=$
note: alternate solutions exist
Iteration 6: sum of abs. weighted deviations $=$
Iteration $7:$ sum of abs. weighted deviations $=$
Iteration $8:$ sum of abs. weighted deviations $=$

| Median regression |  | Number of obs | $=$ |
| :--- | ---: | :--- | :--- |
| Raw sum of deviations | 71102.5 (about 4934) |  | 74 |
| Min sum of deviations 54411.29 | Pseudo R2 | $=$ | 0.2347 |


|  | Robust |  |  |  |  |  |
| ---: | ---: | :---: | ---: | :---: | ---: | ---: |
| price | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| weight | 3.933588 | 1.694477 | 2.32 | 0.023 | .55406 | 7.313116 |
| length | -41.25191 | 51.73571 | -0.80 | 0.428 | -144.4355 | 61.93171 |
| foreign | 3377.771 | 728.5115 | 4.64 | 0.000 | 1924.801 | 4830.741 |
| _cons | 344.6489 | 5096.528 | 0.07 | 0.946 | -9820.055 | 10509.35 |

We see that the robust standard error for weight increases making it less significant in modifying the median automobile price. The standard error for length also increases, but the standard error for the foreign indicator decreases.

For comparison, we repeat the estimation using bootstrap standard errors:

| . use http://www.stata-press.com/data (1978 Automobile Data) <br> . set seed 1001 <br> . bsqreg price weight length foreign (fitting base model) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |
| Median regres <br> Raw sum of <br> Min sum of | n, bootst viations iations 5 | $\begin{array}{ll} p(20) & \text { SEs } \\ 102.5 & \text { (abo } \\ 11.29 & \end{array}$ | 4934) |  | er of ob do R2 | $\begin{array}{r} 74 \\ 0.2347 \end{array}$ |
| price | Coef. | Std. Err | t | $P>\|t\|$ | [95\% Con | Interval] |
| weight | 3.933588 | 3.12446 | 1.26 | 0.212 | -2.297951 | 10.16513 |
| length | -41.25191 | 83.71267 | -0.49 | 0.624 | -208.2116 | 125.7077 |
| foreign | 3377.771 | 1057.281 | 3.19 | 0.002 | 1269.09 | 5486.452 |
| _cons | 344.6489 | 7053.301 | 0.05 | 0.961 | -13722.72 | 14412.01 |

The coefficient estimates are the same-indeed, they are obtained using the same technique. Only the standard errors differ. Therefore, the $t$ statistics, significance levels, and confidence intervals also differ.

Because bsqreg (as well as sqreg and iqreg) obtains standard errors by randomly resampling the data, the standard errors it produces will not be the same from run to run unless we first set the random-number seed to the same number; see $[R]$ set seed.

By default, bsqreg, sqreg, and iqreg use 20 replications. We can control the number of replications by specifying the reps () option:


| Median regression, bootstrap(1000) SEs | Number of obs |  | 74 |
| ---: | :--- | :--- | :--- |
| Raw sum of deviations 71102.5 (about 4934) |  |  |  |
| Min sum of deviations 54411.29 | Pseudo R2 | $=0.2347$ |  |


| price | Coef. | Std. Err. | t | P>\|t| | [95\% Conf. Interval] |  |
| :---: | ---: | :---: | :---: | :---: | ---: | ---: |
| weight | 3.933588 | 2.659381 | 1.48 | 0.144 | -1.370379 | 9.237555 |
| length | -41.25191 | 69.29771 | -0.60 | 0.554 | -179.4618 | 96.95802 |
| foreign |  |  |  |  |  |  |
| Foreign | 3377.771 | 1094.264 | 3.09 | 0.003 | 1195.331 | 5560.211 |
| _cons | 344.6489 | 5916.906 | 0.06 | 0.954 | -11456.25 | 12145.55 |

A comparison of the standard errors is informative.

| Variable | qreg | qreg <br> vce(robust) | bsqreg <br> reps(20) | bsqreg <br> reps (1000) |
| :--- | :---: | :---: | :---: | :---: |
| weight | 1.329 | 1.694 | 3.124 | 2.660 |
| length | 45.46 | 51.74 | 83.71 | 69.30 |
| 1.foreign | 885.4 | 728.5 | 1057. | 1094. |
| _cons | 5182. | 5096. | 7053. | 5917. |

The results shown above are typical for models with heteroskedastic errors. (Our dependent variable is price; if our model had been in terms of $\ln$ (price), the standard errors estimated by qreg and bsqreg would have been nearly identical.) Also, even for heteroskedastic errors, 20 replications is generally sufficient for hypothesis tests against 0 .

## Interquantile and simultaneous-quantile regression

Consider a quantile regression model where the $q$ th quantile is given by

$$
Q_{q}(y)=a_{q}+b_{q, 1} x_{1}+b_{q, 2} x_{2}
$$

For instance, the 75 th and 25 th quantiles are given by

$$
\begin{aligned}
& Q_{0.75}(y)=a_{0.75}+b_{0.75,1} x_{1}+b_{0.75,2} x_{2} \\
& Q_{0.25}(y)=a_{0.25}+b_{0.25,1} x_{1}+b_{0.25,2} x_{2}
\end{aligned}
$$

The difference in the quantiles is then

$$
Q_{0.75}(y)-Q_{0.25}(y)=\left(a_{0.75}-a_{0.25}\right)+\left(b_{0.75,1}-b_{0.25,1}\right) x_{1}+\left(b_{0.75,2}-b_{0.25,2}\right) x_{2}
$$

qreg fits models such as $Q_{0.75}(y)$ and $Q_{0.25}(y)$. iqreg fits interquantile models, such as $Q_{0.75}(y)-$ $Q_{0.25}(y)$. The relationships of the coefficients estimated by qreg and iqreg are exactly as shown: iqreg reports coefficients that are the difference in coefficients of two qreg models, and, of course, iqreg reports the appropriate standard errors, which it obtains by bootstrapping.
sqreg is like qreg in that it estimates the equations for the quantiles

$$
\begin{aligned}
& Q_{0.75}(y)=a_{0.75}+b_{0.75,1} x_{1}+b_{0.75,2} x_{2} \\
& Q_{0.25}(y)=a_{0.25}+b_{0.25,1} x_{1}+b_{0.25,2} x_{2}
\end{aligned}
$$

The coefficients it obtains are the same that would be obtained by estimating each equation separately using qreg. sqreg differs from qreg in that it estimates the equations simultaneously and obtains an estimate of the entire variance-covariance matrix of the estimators by bootstrapping. Thus you can perform hypothesis tests concerning coefficients both within and across equations.

For example, to fit the above model, you could type

```
. qreg y x1 x2, quantile(.25)
. qreg y x1 x2, quantile(.75)
```

By doing this, you would obtain estimates of the parameters, but you could not test whether $b_{0.25,1}=b_{0.75,1}$ or, equivalently, $b_{0.75,1}-b_{0.25,1}=0$. If your interest really is in the difference of coefficients, you could type

```
. iqreg y x1 x2, quantiles(.25 .75)
```

The "coefficients" reported would be the difference in quantile coefficients. You could also estimate both quantiles simultaneously and then test the equality of the coefficients:

```
. sqreg y x1 x2, quantiles(.25 .75)
. test [q25]x1 = [q75]x1
```

Whether you use iqreg or sqreg makes no difference for this test. sqreg, however, because it estimates the quantiles simultaneously, allows you to test other hypotheses. iqreg, by focusing on quantile differences, presents results in a way that is easier to read.

Finally, sqreg can estimate quantiles singly,

```
. sqreg y x1 x2, quantiles(.5)
```

and can thereby be used as a substitute for the slower bsqreg. (Gould [1997b] presents timings demonstrating that sqreg is faster than bsqreg.) sqreg can also estimate more than two quantiles simultaneously:

## > Example 5: Simultaneous quantile estimation

In demonstrating qreg, we performed quantile regressions using auto.dta. We discovered that the regression of price on weight, length, and foreign produced vastly different coefficients for the $0.25,0.5$, and 0.75 quantile regressions. Here are the coefficients that we obtained:

| Variable | 25th <br> percentile | 50th <br> percentile | 75th <br> percentile |
| :--- | :---: | :---: | :---: |
| weight | 1.83 | 3.93 | 9.22 |
| length | 2.85 | -41.25 | -220.8 |
| foreign | 2209.9 | 3377.8 | 3595.1 |
| -cons | -1879.8 | 344.6 | 20242.9 |

All we can say, having estimated these equations separately, is that price seems to depend differently on the weight, length, and foreign variables depending on the portion of the price distribution we examine. We cannot be more precise because the estimates have been made separately. With sqreg, however, we can estimate all the effects simultaneously:

```
. use http://www.stata-press.com/data/r13/auto, clear
(1978 Automobile Data)
. set seed 1001
. sqreg price weight length foreign, q(.25 .5 .75) reps(100)
(fitting base model)
Bootstrap replications (100)
```



```
. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . }5
. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . }10
Simultaneous quantile regression
Number of obs = 74
    bootstrap(100) SEs
.25 Pseudo R2 = 0.1697
.50 Pseudo R2 = 0.2347
.75 Pseudo R2 = 0.3840
```

| price | Coef. | Bootstrap <br> Std. Err. | t | $P>\|t\|$ | [95\% Conf | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| q25 |  |  |  |  |  |  |
| weight | 1.831789 | 1.574777 | 1.16 | 0.249 | -1.309005 | 4.972583 |
| length | 2.84556 | 38.63523 | 0.07 | 0.941 | -74.20998 | 79.9011 |
| foreign | 2209.925 | 1008.521 | 2.19 | 0.032 | 198.494 | 4221.357 |
| _cons | -1879.775 | 3665.184 | -0.51 | 0.610 | -9189.753 | 5430.204 |
| q50 |  |  |  |  |  |  |
| weight | 3.933588 | 2.529541 | 1.56 | 0.124 | -1.111423 | 8.978599 |
| length | -41.25191 | 68.62258 | -0.60 | 0.550 | -178.1153 | 95.61151 |
| foreign | 3377.771 | 1025.882 | 3.29 | 0.002 | 1331.715 | 5423.827 |
| _cons | 344.6489 | 6199.257 | 0.06 | 0.956 | -12019.38 | 12708.68 |
| q75 |  |  |  |  |  |  |
| weight | 9.22291 | 2.483676 | 3.71 | 0.000 | 4.269374 | 14.17645 |
| length | -220.7833 | 86.17422 | -2.56 | 0.013 | -392.6524 | -48.91421 |
| foreign | 3595.133 | 1145.124 | 3.14 | 0.002 | 1311.255 | 5879.011 |
| _cons | 20242.9 | 9414.242 | 2.15 | 0.035 | 1466.79 | 39019.02 |

The coefficient estimates above are the same as those previously estimated, although the standard error estimates are a little different. sqreg obtains estimates of variance by bootstrapping. The important thing here, however, is that the full covariance matrix of the estimators has been estimated and stored, and thus it is now possible to perform hypothesis tests. Are the effects of weight the same at the 25th and 75 th percentiles?

```
. test [q25]weight = [q75]weight
( 1) [q25]weight - [q75]weight = 0
    F( 1, 70) = 8.97
        Prob > F = 0.0038
```

It appears that they are not. We can obtain a confidence interval for the difference by using lincom:

| price | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Conf | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (1) | 7.391121 | 2.467548 | 3.00 | 0.004 | 2.469752 | 12.31249 |

Indeed, we could test whether the weight and length sets of coefficients are equal at the three quantiles estimated:

```
. quietly test [q25]weight = [q50]weight
. quietly test [q25]weight = [q75]weight, accumulate
. quietly test [q25]length = [q50]length, accumulate
. test [q25]length = [q75]length, accumulate
( 1) [q25]weight - [q50]weight = 0
( 2) [q25]weight - [q75]weight = 0
( 3) [q25]length - [q50]length = 0
(4) [q25]length - [q75]length = 0
    F( 4, 70) = 2.43
        Prob > F = 0.0553
```

iqreg focuses on one quantile comparison but presents results that are more easily interpreted:

```
. set seed 1001
. iqreg price weight length foreign, q(.25 . 75) reps(100) nolog
M5-.25 Interquantile regression 
```

|  | Coef. | Bootstrap <br> Std. Err. | t | P>\|t| | [95\% Conf. Interval] |  |  |
| ---: | ---: | :--- | :---: | :---: | ---: | ---: | ---: |
| price | weight | 7.391121 | 2.467548 | 3.00 | 0.004 | 2.469752 | 12.31249 |
| length | -223.6288 | 83.09868 | -2.69 | 0.009 | -389.3639 | -57.89376 |  |
| foreign | 1385.208 | 1191.018 | 1.16 | 0.249 | -990.2036 | 3760.619 |  |
| _cons | 22122.68 | 9009.159 | 2.46 | 0.017 | 4154.478 | 40090.88 |  |

Looking only at the 0.25 and 0.75 quantiles (the interquartile range), the iqreg command output is easily interpreted. Increases in weight correspond significantly to increases in price dispersion. Increases in length correspond to decreases in price dispersion. The foreign variable does not significantly change price dispersion.

Do not make too much of these results; the purpose of this example is simply to illustrate the sqreg and iqreg commands and to do so in a context that suggests why analyzing dispersion might be of interest.
lincom after sqreg produced the same $t$ statistic for the interquartile range of weight, as did the iqreg command above. In general, they will not agree exactly because of the randomness of bootstrapping, unless the random-number seed is set to the same value before estimation (as was done here).

Gould (1997a) presents simulation results showing that the coverage-the actual percentage of confidence intervals containing the true value-for iqreg is appropriate.

## What are the parameters?

In this section, we use a specific data-generating process (DGP) to illustrate the interpretation of the parameters estimated by qreg. If simulation experiments are not intuitive to you, skip this section.

In general, quantile regression parameterizes the quantiles of the distribution of $y$ conditional on the independent variables $\mathbf{x}$ as $\mathbf{x} \boldsymbol{\beta}$, where $\boldsymbol{\beta}$ is a vector of estimated parameters. In our example, we include a constant term and a single independent variable, and we express quantiles of the distribution of $y$ conditional on $x$ as $\beta_{0}+\beta_{1} x$.

We use simulated data to illustrate what we mean by a conditional distribution and how to interpret the parameters $\boldsymbol{\beta}$ estimated by qreg. We also note how we could change our example to illustrate a DGP for which the estimator in qreg would be misspecified.

We suppose that the distribution of $y$ conditional on $x$ has a Weibull form. If $y$ has a Weibull distribution, the distribution function is $F(y)=1-\exp \left\{-(y / \lambda)^{k}\right\}$, where the scale parameter $\lambda>0$ and the shape parameter $k>0$. We can make $y$ have a Weibull distribution function conditional on $x$ by making the scale parameter or the shape parameter functions of $x$. In our example, we specify a particular DGP by supposing that $\lambda=(1+\alpha x), \alpha=1.5, x=1+\sqrt{\nu}$, and that $\nu$ has a $\chi^{2}(1)$ distribution. For the moment, we leave the parameter $k$ as is so that we can discuss how this decision relates to model specification.

Plugging in for $\lambda$ yields the functional form for the distribution of $y$ conditional on $x$, which is known as the conditional distribution function and is denoted $F(y \mid x) . F(y \mid x)$ is the distribution for $y$ for each given value of $x$.

Some algebra yields that $F(y \mid x)=1-\exp \left[-\{y /(1+\alpha x)\}^{k}\right]$. Letting $\tau=F(y \mid x)$ implies that $0 \leq \tau \leq 1$, because probabilities must be between 0 and 1 .

To obtain the $\tau$ quantile of the distribution of $y$ conditional on $x$, we solve

$$
\tau=1-\exp \left[-\{y /(1+\alpha x)\}^{k}\right]
$$

for $y$ as a function of $\tau, x, \alpha$, and $k$. The solution is

$$
\begin{equation*}
y=(1+\alpha x)\{-\ln (1-\tau)\}^{(1 / k)} \tag{1}
\end{equation*}
$$

For any value of $\tau \in(0,1)$, expression (1) gives the $\tau$ quantile of the distribution of $y$ conditional on $x$. To use qreg, we must rewrite (1) as a function of $x, \beta_{0}$, and $\beta_{1}$. Some algebra yields that (1) can be rewritten as

$$
y=\beta_{0}+\beta_{1} * x
$$

where $\beta_{0}=\{-\ln (1-\tau)\}^{(1 / k)}$ and $\beta_{1}=\alpha\{-\ln (1-\tau)\}^{(1 / k)}$. We can express the conditional quantiles as linear combinations of $x$, which is a property of the estimator implemented in qreg.

If we parameterize $k$ as a nontrivial function of $x$, the conditional quantiles will not be linear in $x$. If the conditional quantiles cannot be represented as linear functions of $x$, we cannot estimate the true parameters of the DGP. This restriction illustrates the limits of the estimator implemented in qreg.

We set $k=2$ for our example.
Conditional quantile regression allows the coefficients to change with the specified quantile. For our DGP, the coefficients $\beta_{0}$ and $\beta_{1}$ increase as $\tau$ gets larger. Substituting in for $\alpha$ and $k$ yields that $\beta_{0}=\sqrt{-\ln (1-\tau)}$ and $\beta_{1}=1.5 \sqrt{-\ln (1-\tau)}$. Table 1 presents the true values for $\beta_{0}$ and $\beta_{1}$ implied by our DGP when $\tau \in\{0.25,0.5,0.8\}$.

Table 1: True values for $\beta_{0}$ and $\beta_{1}$

| $\tau$ | $\beta_{0}$ | $\beta_{1}$ |
| :--- | :--- | :--- |
| 0.25 | 0.53636 | 0.80454 |
| 0.5 | 0.8325546 | 1.248832 |
| 0.8 | 1.268636 | 1.902954 |

We can also use (1) to generate data from the specified distribution of $y$ conditional on $x$ by plugging in random uniform numbers for $\tau$. Each random uniform number substituted in for $\tau$ in (1) yields a draw from the conditional distribution of $y$ given $x$.

## > Example 6

In this example, we generate 100,000 observations from our specified DGP by substituting random uniform numbers for $\tau$ in (1), with $\alpha=1.5, k=2, x=1+\sqrt{\nu}$, and $\nu$ coming from a $\chi^{2}(1)$ distribution.

We begin by executing the code that implements this method; below we discuss each line of the output produced.

```
. clear // drop existing variables
. set seed 1234571 // set random-number seed
. set obs 100000 // set number of observations
obs was 0, now 100000
. generate double tau = runiform() // generate uniform variate
. generate double x = 1 + sqrt(rchi2(1)) // generate values for x
. generate double lambda = 1 + 1.5*x // lambda is 1 + alpha*x
. generate double k = 2
    // fix value of k
    // generate random values for y
    // given x
. generate double y = lambda*((-ln(1-tau))^(1/k))
```

Although the comments at the end of each line briefly describe what each line is doing, we provide a more careful description. The first line drops any variables in memory. The second sets the seed of the random-number generator so that we will always get the same sequence of random uniform numbers. The third line sets the sample size to 100,000 observations, and the fourth line reports the change in sample size.

The fifth line substitutes random uniform numbers for $\tau$. This line is the key to the algorithm. This standard method, known as inverse-probability transforms, for computing random numbers is discussed by Cameron and Trivedi (2010, 126-127), among others.

Lines 6-8 generate $x, \lambda$, and $k$ per our specified DGP. Lines $9-11$ implement (1) using the previously generated $\lambda, x$, and $k$.

At the end, we have 100,000 observations on $y$ and $x$, with $y$ coming from the conditional distribution that we specified above.

## > Example 7

In the example below, we use qreg to estimate $\beta_{1}$ and $\beta_{0}$, the parameters from the conditional quantile function, for the 0.5 quantile from our simulated data.
qreg $y$ x, quantile (.5)
Iteration 1: WLS sum of weighted deviations $=68975.517$
Iteration 1: sum of abs. weighted deviations $=68975.325$
Iteration $2:$ sum of abs. weighted deviations $=68843.958$
Iteration 3: sum of abs. weighted deviations $=$
Iteration $4:$ sum of abs. weighted deviations $=68629.64$
Iteration $5:$ sum of abs. weighted deviations $=68625.382$
Iteration $6:$ sum of abs. weighted deviations $=68625.657$
Iteration $7:$ sum of abs. weighted deviations $=68625.657$

Iteration 7: sum of abs. weighted deviations $=68625.657$
Median regression $\quad$ Number of obs $=10000$
Raw sum of deviations 73840.51 (about 2.944248)
Min sum of deviations 68625.66 Pseudo R2 $=0.0706$

| y | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| x | 1.228536 | .0118791 | 103.42 | 0.000 | 1.205253 | 1.251819 |
| _cons | .8693355 | .0225288 | 38.59 | 0.000 | .8251793 | .9134917 |

In the qreg output, the results for x correspond to the estimate of $\beta_{1}$, and the results for $\quad$ cons correspond to the estimate of $\beta_{0}$. The reported estimates are close to their true values of 1.248832 and 0.8325546 , which are given in table 1 .

The intuition in this example comes from the ability of qreg to recover the true parameters of our specified DGP. As we increase the number of observations in our sample size, the qreg estimates will get closer to the true values.

## > Example 8

In the example below, we estimate the parameters of the conditional quantile function for the 0.25 quantile and compare them with the true values.

```
. qreg y x, quantile(.25)
Iteration 1: WLS sum of weighted deviations = 65497.284
Iteration 1: sum of abs. weighted deviations = 65492.359
Iteration 2: sum of abs. weighted deviations = 60139.477
Iteration 3: sum of abs. weighted deviations = 49999.793
Iteration 4: sum of abs. weighted deviations = 49999.479
Iteration 5: sum of abs. weighted deviations = 49999.465
Iteration 6: sum of abs. weighted deviations = 49999.465
```

| .25 Quantile regression | Number of obs $=$ | 100000 |
| :--- | :--- | :--- |
| Raw sum of deviations 52014.79 (about 1.857329) |  |  |
| Min sum of deviations 49999.47 | Pseudo R2 | $=0.0387$ |


| y | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | ---: |
| x | .7844305 | .0107092 | 73.25 | 0.000 | .7634405 | .8054204 |
| _cons | .5633285 | .0203102 | 27.74 | 0.000 | .5235209 | .6031362 |

As above, qreg reports the estimates of $\beta_{1}$ and $\beta_{0}$ in the output table for x and _cons, respectively. The reported estimates are close to their true values of 0.80454 and 0.53636 , which are given in table 1. As expected, the estimates are close to their true values. Also as expected, the estimates for the 0.25 quantile are smaller than the estimates for the 0.5 quantile.

## - Example 9

We finish this section by estimating the parameters of the conditional quantile function for the 0.8 quantile and comparing them with the true values.


As above, qreg reports the estimates of $\beta_{1}$ and $\beta_{0}$ in the output table for x and _cons, respectively. The reported estimates are close to their true values of 1.902954 and 1.268636 , which are given in table 1. As expected, the estimates are close to their true values. Also as expected, the estimates for the 0.8 quantile are larger than the estimates for the 0.5 quantile.

## Stored results

qreg stores the following in e() :
Scalars

```
e(N)
e(df_m)
e(df_r)
e(q)
e(q-v)
e(sum_adev)
e(sum_rdev)
e(sum_w)
e(f_r)
e(sparsity)
e(bwidth)
e(kbwidth)
e(rank)
e(convcode)
```

number of observations
model degrees of freedom
number of observations model degrees of freedom
residual degrees of freedom
quantile requested
value of the quantile
sum of absolute deviations
sum of raw deviations
sum of weights
density estimate
sparsity estimate
bandwidth
kernel bandwidth
rank of $e(V)$
0 if converged; otherwise, return code for why nonconvergence
Macros

```
e(cmd)
e(cmdline)
e(depvar)
e(bwmethod)
e(denmethod)
e(kernel)
e(wtype)
e(wexp)
e(vce)
e(vcetype)
e(properties) b V
e(marginsnotok)
```

qreg
command as typed
name of dependent variable
bandwidth method; hsheather, bofinger, or chamberlain
density estimation method; fitted, residual, or kernel
kernel function
weight type
weight expression
vcetype specified in vce()
e (predict) program used to implement predict
predictions disallowed by margins

Matrices
e(b)
coefficient vector
$e(V) \quad$ variance-covariance matrix of the estimators

Functions

```
e(sample) marks estimation sample
```

iqreg stores the following in e():
Scalars
e(N)
e(df_r)
e(q0)
e(q1)
e(reps)
e(sumrdev0)
e(sumrdev1)
e(sumadev0)
e(sumadev1)
e(rank)
e(convcode)
Macros
e(cmd)
e(cmdline)
e(depvar)
e(vcetype)
e(properties)
e(predict)
e(marginsnotok)
Matrices
e(b)
e(V)
Functions

```
e(sample) marks estimation sample
```

sqreg stores the following in e():
Scalars
e(N)
e(df_r)
e(n_q)
e(q\#)
e(reps)
e(sumrdv\#)
e(sumadv\#)
e(rank)
e(convcode)
Macros

$$
\mathrm{e}(\mathrm{cmd})
$$

e(cmdline)
e(depvar)
$e$ (eqnames)
e(vcetype)
e(properties)
e(predict)
e(marginsnotok)
Matrices
e(b)
e(V)
Functions
e(sample) marks estimation sample
bsqreg stores the following in e():
Scalars
e(N)
e(df_r)
$e(q)$
e(q-v)
e(reps)
e(sum_adev)
e(sum_rdev)
$e$ (rank)
e(convcode)
Macros
e(cmd)
e(cmdline)
e(depvar)
e(properties)
e(predict)
e(marginsnotok)
Matrices
e(b)
e(V)
Functions
$e($ sample) marks estimation sample

## Methods and formulas

Methods and formulas are presented under the following headings:

## Introduction

Linear programming formulation of quantile regression
Standard errors when residuals are i.i.d.
Pseudo- $R^{2}$

## Introduction

According to Stuart and Ord (1991, 1084), the method of minimum absolute deviations was first proposed by Boscovich in 1757 and was later developed by Laplace; Stigler (1986, 39-55) and Hald (1998, 97-103, 112-116) provide historical details. According to Bloomfield and Steiger (1980), Harris (1950) later observed that the problem of minimum absolute deviations could be turned into the linear programming problem that was first implemented by Wagner (1959). Interest has grown in this method because robust methods and extreme value modeling have become more popular. Statistical and computational properties of minimum absolute deviation estimators are surveyed by Narula and Wellington (1982). Cameron and Trivedi (2005), Hao and Naiman (2007), and Wooldridge (2010) provide excellent introductions to quantile regression methods, while Koenker (2005) gives an in-depth review of the topic.

## Linear programming formulation of quantile regression

Define $\tau$ as the quantile to be estimated; the median is $\tau=0.5$. For each observation $i$, let $\varepsilon_{i}$ be the residual

$$
\varepsilon_{i}=y_{i}-\mathbf{x}_{i}^{\prime} \widehat{\boldsymbol{\beta}}_{\tau}
$$

The objective function to be minimized is

$$
\begin{align*}
c_{\tau}\left(\varepsilon_{i}\right) & =\left(\tau \mathbf{1}\left\{\varepsilon_{i} \geq 0\right\}+(1-\tau) \mathbf{1}\left\{\varepsilon_{i}<0\right\}\right)\left|\varepsilon_{i}\right| \\
& =\left(\tau \mathbf{1}\left\{\varepsilon_{i} \geq 0\right\}-(1-\tau) \mathbf{1}\left\{\varepsilon_{i}<0\right\}\right) \varepsilon_{i}  \tag{2}\\
& =\left(\tau-\mathbf{1}\left\{\varepsilon_{i}<0\right\}\right) \varepsilon_{i}
\end{align*}
$$

where $1\{\cdot\}$ is the indicator function. This function is sometimes referred to as the check function because it resembles a check mark (Wooldridge 2010, 450); the slope of $c_{\tau}\left(\varepsilon_{i}\right)$ is $\tau$ when $\varepsilon_{i}>0$ and is $\tau-1$ when $\varepsilon_{i}<0$, but is undefined for $\varepsilon_{i}=0$. Choosing the $\widehat{\boldsymbol{\beta}}_{\tau}$ that minimize $c_{\tau}\left(\varepsilon_{i}\right)$ is equivalent to finding the $\widehat{\boldsymbol{\beta}}_{\tau}$ that make $\mathbf{x} \widehat{\boldsymbol{\beta}}_{\tau}$ best fit the quantiles of the distribution of $y$ conditional on $\mathbf{x}$.

This minimization problem is set up as a linear programming problem and is solved with linear programming techniques, as suggested by Armstrong, Frome, and Kung (1979) and described in detail by Koenker (2005). Here $2 n$ slack variables, $\mathbf{u}_{n \times 1}$ and $\mathbf{v}_{n \times 1}$, are introduced, where $u_{i} \geq 0, v_{i} \geq 0$, and $u_{i} \times v_{i}=0$, reformulating the problem as

$$
\min _{\boldsymbol{\beta}_{\tau}, \mathbf{u}, \mathbf{v}}\left\{\tau \mathbf{1}_{n}^{\prime} \mathbf{u}+(1-\tau) \mathbf{1}_{n}^{\prime} \mathbf{v} \mid \mathbf{y}-\mathbf{X} \boldsymbol{\beta}_{\tau}=\mathbf{u}-\mathbf{v}\right\}
$$

where $\mathbf{1}_{n}$ is a vector of 1 s . This is a linear objective function on a polyhedral constraint set with $\binom{n}{k}$ vertices, and our goal is to find the vertex that minimizes (2). Each step in the search is described by a set of $k$ observations through which the regression plane passes, called the basis. A step is taken by replacing a point in the basis if the linear objective function can be improved. If this occurs, a line is printed in the iteration log. The definition of convergence is exact in the sense that no amount of added iterations could improve the objective function.

A series of weighted least-squares (WLS) regressions is used to identify a set of observations as a starting basis. The WLS algorithm for $\tau=0.5$ is taken from Schlossmacher (1973) with a generalization for $0<\tau<1$ implied from Hunter and Lange (2000).

## Standard errors when residuals are i.i.d.

The estimator for the VCE implemented in qreg assumes that the errors of the model are independent and identically distributed (i.i.d.). When the errors are i.i.d., the large-sample VCE is

$$
\begin{equation*}
\operatorname{cov}\left(\boldsymbol{\beta}_{\tau}\right)=\frac{\tau(1-\tau)}{f_{Y}^{2}\left(\xi_{\tau}\right)}\left\{E\left(\mathbf{x}_{i} \mathbf{x}_{i}^{\prime}\right)\right\}^{-1} \tag{3}
\end{equation*}
$$

where $\xi_{\tau}=F_{Y}^{-1}(\tau)$ and $F_{Y}(y)$ is the distribution function of $Y$ with density $f_{Y}(y)$. See Koenker (2005, 73) for this result. From (3), we see that the regression precision depends on the inverse of the density function, termed the sparsity function, $s_{\tau}=1 / f_{Y}\left(\xi_{\tau}\right)$.

While $1 / n \sum_{i=1}^{n} \mathbf{x}_{i} \mathbf{x}_{i}^{\prime}$ estimates $E\left(\mathbf{x}_{i} \mathbf{x}_{i}^{\prime}\right)$, estimating the sparsity function is more difficult. qreg provides several methods to estimate the sparsity function. The different estimators are specified through the suboptions of vce(iid, denmethod bwidth). The suboption denmethod specifies the functional form for the sparsity estimator. The default is fitted.

Here we outline the logic underlying the fitted estimator. Because $F_{Y}(y)$ is the distribution function for $Y$, we have $f_{Y}(y)=\left\{d F_{y}(y)\right\} / d y, \tau=F_{Y}\left(\xi_{\tau}\right)$, and $\xi_{\tau}=F_{Y}^{-1}(\tau)$. When differentiating the identity $F_{Y}\left\{F_{Y}^{-1}(\tau)\right\}=\tau$, the sparsity function can be written as $s_{\tau}=\left\{F_{Y}^{-1}(\tau)\right\} / d t$. Numerically, we can approximate the derivative using the centered difference,

$$
\begin{equation*}
\frac{F_{Y}^{-1}(\tau)}{d t} \approx \frac{F_{Y}^{-1}(\tau+h)-F_{Y}^{-1}(\tau-h)}{2 h}=\frac{\xi_{\tau+h}-\xi_{\tau-h}}{2 h}=\widehat{s}_{\tau} \tag{4}
\end{equation*}
$$

where $h$ is the bandwidth.
The empirical quantile function is computed by first estimating $\boldsymbol{\beta}_{\tau+h}$ and $\boldsymbol{\beta}_{\tau-h}$, and then computing $\widehat{F}_{Y}^{-1}(\tau+h)=\overline{\mathbf{x}}^{\prime} \widehat{\boldsymbol{\beta}}_{\tau+h}$ and $\widehat{F}_{Y}^{-1}(\tau-h)=\overline{\mathbf{x}}^{\prime} \widehat{\boldsymbol{\beta}}_{\tau-h}$, where $\overline{\mathbf{x}}$ is the sample mean of the independent variables $\mathbf{x}$. These quantities are then substituted into (4).

Alternatively, as the option suggests, vce(iid, residual) specifies that qreg use the empirical quantile function of the residuals to estimate the sparsity. Here we substitute $F_{\epsilon}$, the distribution of the residuals, for $F_{Y}$, which only differ by their first moments.

The $k$ residuals associated with the linear programming basis will be zero, where $k$ is the number of regression coefficients. These zero residuals are removed before computing the $\tau+h$ and $\tau-h$ quantiles, $\varepsilon_{(\tau+h)}=\widehat{F}_{\epsilon}^{-1}(\tau+h)$ and $\varepsilon_{(\tau-h)}=\widehat{F}_{\epsilon}^{-1}(\tau-h)$. The $\widehat{F}_{\epsilon}^{-1}$ estimates are then substituted for $F_{Y}^{-1}$ in (4).

Each of the estimators for the sparsity function depends on a bandwidth. The vce () suboption bwidth specifies the bandwidth method to use. The three bandwidth options and their citations are hsheather (Hall and Sheather 1988), bofinger (Bofinger 1975), and chamberlain (Chamberlain 1994).

Their formulas are

$$
\begin{aligned}
& h_{s}=n^{-1 / 3} \Phi^{-1}\left(1-\frac{\alpha}{2}\right)^{2 / 3}\left[\frac{3}{2} \times \frac{\phi\left\{\Phi^{-1}(\tau)\right\}^{4}}{2 \Phi^{-1}(\tau)^{2}+1}\right]^{1 / 3} \\
& h_{b}=n^{-1 / 5}\left[\frac{\frac{9}{2} \phi\left\{2 \Phi^{-1}(\tau)\right\}^{4}}{\left\{2 \Phi^{-1}(\tau)^{2}+1\right\}^{2}}\right]^{1 / 5} \\
& h_{c}=\Phi^{-1}\left(1-\frac{\alpha}{2}\right) \sqrt{\frac{\tau(1-\tau)}{n}}
\end{aligned}
$$

where $h_{s}$ is the Hall-Sheather bandwidth, $h_{b}$ is the Bofinger bandwidth, $h_{c}$ is the Chamberlain bandwidth, $\Phi()$ and $\phi()$ are the standard normal distribution and density functions, $n$ is the sample size, and $100(1-\alpha)$ is the confidence level set by the level () option. Koenker (2005) discusses the derivation of the Hall-Sheather and the Bofinger bandwidth formulas. You should avoid modifying the confidence level when replaying estimates that use the Hall-Sheather or Chamberlain bandwidths because these methods use the confidence level to estimate the coefficient standard errors.

Finally, the vce() suboption kernel (kernel) specifies that qreg use one of several kernel-density estimators to estimate the sparsity function. kernel allows you to choose which kernel function to use, where the default is the Epanechnikov kernel. See [R] kdensity for the functional form of the eight kernels.

The kernel bandwidth is computed using an adaptive estimate of scale

$$
h_{k}=\min \left(\widehat{\sigma}, \frac{r_{q}}{1.34}\right) \times\left\{\Phi^{-1}(\tau+h)-\Phi^{-1}(\tau-h)\right\}
$$

where $h$ is one of $h_{s}, h_{b}$, or $h_{c} ; r_{q}$ is the interquartile range; and $\widehat{\sigma}$ is the standard deviation of $\mathbf{y}$; see Silverman $(1992,47)$ and Koenker $(2005,81)$ for discussions. Let $\widehat{f}_{\epsilon}\left(\varepsilon_{i}\right)$ be the kernel density estimate for the $i$ th residual, and then the kernel estimator for the sparsity function is

$$
\widehat{s}_{\tau}=\frac{n h_{k}}{\sum_{i=1}^{n} \widehat{f}_{\epsilon}\left(\varepsilon_{i}\right)}
$$

Finally, substituting your choice of sparsity estimate into (3) results in the i.i.d. variance-covariance matrix

$$
\mathbf{V}_{n}=\widehat{s}_{\tau}^{2} \tau(1-\tau)\left(\sum_{i=1}^{n} \mathbf{x}_{i} \mathbf{x}_{i}^{\prime}\right)^{-1}
$$

## Pseudo-R ${ }^{2}$

The pseudo- $R^{2}$ is calculated as

$$
1-\frac{\text { sum of weighted deviations about estimated quantile }}{\text { sum of weighted deviations about raw quantile }}
$$

This is based on the likelihood for a double-exponential distribution $e^{v_{i}\left|\varepsilon_{i}\right|}$, where $v_{i}$ are multipliers

$$
v_{i}= \begin{cases}2 \tau & \text { if } \varepsilon_{i}>0 \\ 2(1-\tau) & \text { otherwise }\end{cases}
$$

Minimizing the objective function (2) with respect to $\boldsymbol{\beta}_{\tau}$ also minimizes $\sum_{i}\left|\varepsilon_{i}\right| v_{i}$, the sum of weighted least absolute deviations. For example, for the 50 th percentile $v_{i}=1$, for all $i$, and we have median regression. If we want to estimate the 75th percentile, we weight the negative residuals by 0.50 and the positive residuals by 1.50 . It can be shown that the criterion is minimized when $75 \%$ of the residuals are negative.

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## Also see

[R] qreg postestimation - Postestimation tools for qreg, iqreg, sqreg, and bsqreg
$[R]$ bootstrap - Bootstrap sampling and estimation
$[R]$ regress - Linear regression
[R] rreg - Robust regression
[MI] estimation - Estimation commands for use with mi estimate
[U] 20 Estimation and postestimation commands

## Title

qreg postestimation - Postestimation tools for qreg, iqreg, sqreg, and bsqreg

| Description | Syntax for predict |
| :--- | :--- |
| Remarks and examples | Also see for predict |$\quad$ Options for predict

## Description

The following postestimation commands are available after qreg, iqreg, bsqreg, and sqreg:

Command Description
estat summarize summary statistics for the estimation sample
estat vce variance-covariance matrix of the estimators (VCE)
estimates cataloging estimation results
forecast ${ }^{1}$ dynamic forecasts and simulations
lincom point estimates, standard errors, testing, and inference for linear combinations of coefficients
linktest link test for model specification
margins marginal means, predictive margins, marginal effects, and average marginal effects
marginsplot graph the results from margins (profile plots, interaction plots, etc.)
nlcom
predict predictions, residuals, influence statistics, and other diagnostic measures
predictnl point estimates, standard errors, testing, and inference for generalized predictions
test Wald tests of simple and composite linear hypotheses
testnl Wald tests of nonlinear hypotheses

[^65]
## Syntax for predict

For qreg, iqreg, and bsqreg
predict [type] newvar [if] [in] [, [xb|stdp|residuals]]

For sqreg
predict [type] newvar [if] [in] [, equation(eqno[,eqno]) statistic]
statistic Description

Main
$\mathrm{xb} \quad$ linear prediction; the default
stdp standard error of the linear prediction
stddp standard error of the difference in linear predictions
residuals residuals

These statistics are available both in and out of sample; type predict ... if e(sample)... if wanted only for the estimation sample.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
xb , the default, calculates the linear prediction.
stdp calculates the standard error of the linear prediction.
stddp is allowed only after you have fit a model using sqreg. The standard error of the difference in linear predictions ( $\mathbf{x}_{1 j} \mathbf{b}-\mathbf{x}_{2 j} \mathbf{b}$ ) between equations 1 and 2 is calculated.
residuals calculates the residuals, that is, $y_{j}-\mathbf{x}_{j} \mathbf{b}$. equation (eqno [, eqno $]$ ) specifies the equation to which you are making the calculation.
equation() is filled in with one eqno for the xb , $s t d p$, and residuals options. equation(\#1) would mean that the calculation is to be made for the first equation, equation(\#2) would mean the second, and so on. You could also refer to the equations by their names. equation(income) would refer to the equation named income and equation(hours) to the equation named hours.
If you do not specify equation(), results are the same as if you had specified equation(\#1).
To use stddp, you must specify two equations. You might specify equation(\#1, \#2) or equation (q80, q20) to indicate the 80th and 20th quantiles.

## Remarks and examples

## > Example 1

In example 4 of $[\mathrm{R}]$ qreg, we fit regressions for the lower and the upper quartile of the price variable. The predict command can be used to obtain the linear prediction after each regression.

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. qreg price weight length foreign, quantile(.25)
    (output omitted)
. predict q25
(option xb assumed; fitted values)
. qreg price weight length foreign, quantile(.75)
    (output omitted)
- predict q75
(option xb assumed; fitted values)
```

We can use the variables generated by predict to compute the predicted interquartile range, that is,

```
. generate iqr1 = q75 - q25
```

If we directly perform the interquartile range regression with the iqreg command, we can predict the interquartile range and also the standard error for the prediction.

```
. iqreg price weight length foreign, quantile(.25 .75)
    (output omitted)
. predict iqr2
(option xb assumed; fitted values)
. predict stdp, stdp
```

We now plot the predicted interquartile range versus variable length:

```
. scatter iqr2 length
```



As stated in example 5 of [R] qreg, the negative coefficient for the length variable means that increases in length imply decreases in the interquartile range and therefore in price dispersion. Consequently, we could have expected a downward trend in the plot, but there is not. This is because
the regression output indicates that when we hold the rest of the variables constant, an increase in length leads to a decrease in iqr2. However, there is a high correlation between weight and length, which could be masking the effect of length on iqr2. We can achieve a better visualization by using a contour plot.

```
. twoway contour iqr2 weight length, level(10)
```



We can see the effect by setting a fixed value of length on the vertical axis, say, 3,000 lbs. When we move from left to right on the horizontal axis, we see that for small values of length, iqr2 values are shown in red, meaning high values, and when we move toward the right, the graph indicates transition into increasingly smaller values.

## Also see

[R] qreg - Quantile regression
[U] 20 Estimation and postestimation commands

## Title

query - Display system parameters

Syntax Description Remarks and examples Also see

## Syntax

query [ memory $\mid$ output $\mid$ interface $\mid$ graphics $\mid$ efficiency $\mid$ network $\mid$
$\quad$ update $\mid$ trace $\mid$ mata $\mid$ other $]$

## Description

query displays the settings of various Stata parameters.

## Remarks and examples

query provides more system information than you will ever want to know. You do not need to understand every line of output that query produces if all you need is one piece of information. Here is what happens when you type query:

- query

| Memory settings |  |  |
| :---: | :---: | :---: |
| set maxvar | 5000 | 2048-32767; max. vars allowed |
| set matsize | 400 | 10-11000; max. \# vars in models |
| set niceness | 5 | 0-10 |
| set min_memory | 0 | 0-1600g |
| set max_memory |  | $32 \mathrm{~m}-1600 \mathrm{~g}$ or |
| set segmentsize | 32m | $1 \mathrm{~m}-32 \mathrm{~g}$ |
| Output settings |  |  |
| set more | on |  |
| set rmsg | off |  |
| set dp | period | may be period or comma |
| set linesize | 80 | characters |
| set pagesize | 28 | lines |
| set level | 95 | percent confidence intervals |
| set showbaselevels |  | may be empty, off, on, or all |
| set showemptycells |  | may be empty, off, or on |
| set showomitted |  | may be empty, off, or on |
| set fvlabel | on |  |
| set fvwrap | 1 |  |
| set fvwrapon | word | may be word or width |
| set lstretch |  | may be empty, off, or on |
| set cformat |  | may be empty or a numerical format |
| set pformat |  | may be empty or a numerical format |
| set sformat |  | may be empty or a numerical format |

set coeftabresults on
set logtype smcl may be smcl or text

Interface settings
set dockable on
set dockingguides on
set floatwindows off
set locksplitters off
set pinnable on
set doublebuffer on

| set linegap <br> set scrollbufsize <br> set fastscroll <br> set varlabelpos <br> set reventries | 204800 <br> on | pixels <br> characters |
| :--- | :--- | :--- |
| set maxdb | 5000 | (not relevant) <br> lines |

Graphics settings
set graphics on
set autotabgraphs off
set scheme s2color
set printcolor automatic may be automatic, asis, gs1, gs2, gs3
set copycolor automatic may be automatic, asis, gs1, gs2, gs3

| Efficiency settings <br> set adosize | 1000 | kilobytes |
| :---: | :--- | :--- |
| Network settings |  |  |
| set checksum | off | seconds |
| set timeout1 | 30 | seconds |
| set timeout2 | 180 | off |
| set httpproxy <br> set httpproxyhost <br> set httpproxyport | 80 |  |
| set httpproxyauth <br> set httpproxyuser <br> set httpproxypw | off |  |

Update settings
set update_query on
set update_interval 7
set update_prompt on
Trace (programming debugging) settings
set trace off
set tracedepth 32000
set traceexpand on
set tracesep on
set traceindent on
set tracenumber off
set tracehilite

```
    Mata settings
    set matastrict off
    set matalnum off
    set mataoptimize on
    set matafavor space may be space or speed
    set matacache kilobytes
    set matalibs lmatabase;lmataado;lmatafc;lmatagsem;lmataopt;
> lmatapath;lmatapss;lmatasem
    set matamofirst off
Other settings
    set type float may be float or double
    set maxiter 16000 max iterations for estimation commands
    set searchdefault all may be local, net, or all
    set seed
    set varabbrev on
    set emptycells keep may be keep or drop
    set processors 2 1-2
    set haverdir
```

The output is broken into several divisions: memory, output, interface, graphics, efficiency, network, update, trace, mata, and other settings. We will discuss each one in turn.

We generated the output above using Stata/MP for Windows. Here is what happens when we type query and we are running Stata/SE for Mac:

| Memory settings |  |  |
| :---: | :---: | :---: |
| set maxvar | 5000 | 2048-32767; max. vars allowed |
| set matsize | 400 | 10-11000; max. \# vars in models |
| set niceness | 5 | 0-10 |
| set min_memory | 0 | 0-1600g |
| set max_memory |  | $32 \mathrm{~m}-1600 \mathrm{~g}$ or |
| set segmentsize | 32m | $1 \mathrm{~m}-32 \mathrm{~g}$ |
| Output settings |  |  |
| set more | off |  |
| set rmsg | off |  |
| set dp | period | may be period or comma |
| set linesize | 80 | characters |
| set pagesize | 23 | lines |
| set level | 95 | percent confidence intervals |
| set showbaselevels |  | may be empty, off, on, or all |
| set showemptycells |  | may be empty, off, or on |
| set showomitted |  | may be empty, off, or on |
| set fvlabel | on |  |
| set fvwrap | 1 |  |
| set fvwrapon | word | may be word or width |
| set lstretch |  | may be empty, off, or on |
| set cformat |  | may be empty or a numerical format |
| set pformat |  | may be empty or a numerical format |
| set sformat |  | may be empty or a numerical format |
| set coeftabresults | on |  |
| set logtype | smcl | may be smcl or text |


| set charset | mac | may be mac or latin1 |
| :--- | :--- | :--- |
| set eolchar | unix | may be mac or unix |
| set notifyuser | on |  |
| set playsnd | off |  |
| set include_bitmap on |  |  |



```
    Mata settings
    set matastrict off
    set matalnum off
    set mataoptimize on
    set matafavor space may be space or speed
    set matacache kilobytes
    set matalibs lmatabase;lmataado;lmatafc;lmatagsem;lmataopt;
> lmatapath;lmatapss;lmatasem
    set matamofirst off
Other settings
    set type float may be float or double
    set maxiter max iterations for estimation commands
    set searchdefault local may be local, net, or all
    set seed X075bcd151f123bb5159a55e50022865700043e55
    set varabbrev on
    set emptycells keep may be keep or drop
    set processors 1
```


## Memory settings

Memory settings indicate how memory is allocated, the maximum number of variables, and the maximum size of a matrix.

For more information, see

| maxvar | $[\mathrm{D}]$ memory |
| ---: | :--- |
| matsize | $[\mathrm{R}]$ matsize |
| niceness | $[\mathrm{D}]$ memory |
| min_memory | $[\mathrm{D}]$ memory |
| max_memory | $[\mathrm{D}]$ memory |
| segmentsize | $[\mathrm{D}]$ memory |

## Output settings

Output settings show how Stata displays output on the screen and in $\log$ files.
For more information, see

| more | $[R]$ more |
| ---: | :--- |
| rmsg | $[P]$ rmsg |
| $d p$ | $[D]$ format |
| linesize | $[R]$ log |
| pagesize | $[R]$ more |
| level | $[R]$ level |
| showbaselevels | $[R]$ set showbaselevels |
| showemptycells | $[R]$ set showbaselevels |
| showomitted | $[R]$ set showbaselevels |
| fvlabel | $[R]$ set showbaselevels |
| fvwrap | $[R]$ set showbaselevels |
| fvwrapon | $[R]$ set showbaselevels |
| cformat | $[R]$ set cformat |
| pformat | $[R]$ set cformat |
| sformat | $[R]$ set cformat |

```
coeftabresults [R] set
    lstretch [R] set
        logtype [R] log
        charset [R] set
        eolchar [R] set
    notifyuser [R] set
        playsnd [R] set
include_bitmap [R] set
```


## Interface settings

Interface settings control how Stata's interface works.
For more information, see

| dockable | $[R]$ set |
| ---: | :--- |
| dockingguides | $[R]$ set |
| floatwindows | $[R]$ set |
| locksplitters | $[R]$ set |
| pinnable | $[R]$ set |
| doublebuffer | $[R]$ set |
| revkeyboard | $[R]$ set |
| varkeyboard | $[R]$ set |
| smoothfonts | $[R]$ set |
| linegap | $[R]$ set |
| scrollbufsize | $[R]$ set |
| fastscroll | $[R]$ set |
| reventries | $[R]$ set |
| maxdb | $[R]$ db |

## Graphics settings

Graphics settings indicate how Stata's graphics are displayed.
For more information, see

```
            graphics [G-2] set graphics
autotabgraphs [R] set
            scheme [G-2] set scheme
    printcolor [G-2] set printcolor
    copycolor [G-2] set printcolor
```


## Efficiency settings

The efficiency settings set the maximum amount of memory allocated to automatically loaded do-files, the maximum number of remembered-contents dialog boxes, and the use of virtual memory.

For more information, see

```
adosize [P] sysdir
```


## Network settings

Network settings determine how Stata interacts with the Internet.
For more information, see $[R]$ netio.

## Update settings

Update settings determine how Stata performs updates.
For more information, see [R] update.

## Trace settings

Trace settings adjust Stata's behavior and are particularly useful in debugging code.
For more information, see [P] trace.

## Mata settings

Mata settings affect Mata's system parameters.
For more information, see [M-3] mata set.

## Other settings

The other settings are a miscellaneous collection.
For more information, see

```
            type [D] generate
            maxiter [R] maximize
searchdefault [R] search
            seed [R] set seed
        varabbrev [R] set
emptycells [R] set
processors [R] set
            odbcmgr [D] odbc
        haverdir [D] import haver
```

In general, the parameters displayed by query can be changed by set; see [R] set.

## Also see

$[R]$ set - Overview of system parameters
[P] creturn - Return c-class values
[M-3] mata set - Set and display Mata system parameters

## Title

ranksum - Equality tests on unmatched data

| Syntax | Menu | Description | Options for ranksum |
| :--- | :--- | :--- | :--- |
| Options for median | Remarks and examples | Stored results | Methods and formulas |
| References | Also see |  |  |

## Syntax

Wilcoxon rank-sum test
ranksum varname [if] [in], by(groupvar) [porder]

Nonparametric equality-of-medians test
median varname $[$ if $][$ in $]$ [weight], by (groupvar) [median_options]
ranksum_options Description

Main

* by (groupvar)
porder
grouping variable
probability that variable for first group is larger than variable for second group
median_options
Description


## Main

* by (groupvar)
exact
medianties (below)
medianties(above)
medianties (drop) medianties(split)
grouping variable
perform Fisher's exact test
assign values equal to the median to below group
assign values equal to the median to above group
drop values equal to the median from the analysis
split values equal to the median equally between the two groups
*by (groupvar) is required.
by is allowed with ranksum and median; see [D] by.
fweights are allowed with median; see [U] 11.1.6 weight.


## Menu

## ranksum

Statistics $>$ Nonparametric analysis $>$ Tests of hypotheses $>$ Wilcoxon rank-sum test

## median

Statistics $>$ Nonparametric analysis $>$ Tests of hypotheses $>$ K-sample equality-of-medians test

## Description

ranksum tests the hypothesis that two independent samples (that is, unmatched data) are from populations with the same distribution by using the Wilcoxon rank-sum test, which is also known as the Mann-Whitney two-sample statistic (Wilcoxon 1945; Mann and Whitney 1947).
median performs a nonparametric $k$-sample test on the equality of medians. It tests the null hypothesis that the $k$ samples were drawn from populations with the same median. For two samples, the chi-squared test statistic is computed both with and without a continuity correction.
ranksum and median are for use with unmatched data. For equality tests on matched data, see [R] signrank.

## Options for ranksum

$\qquad$ Main
by (groupvar) is required. It specifies the name of the grouping variable.
porder displays an estimate of the probability that a random draw from the first population is larger than a random draw from the second population.

## Options for median

Main
by (groupvar) is required. It specifies the name of the grouping variable.
exact displays the significance calculated by Fisher's exact test. For two samples, both one- and two-sided probabilities are displayed.
medianties (below $\mid$ above $\mid$ drop $\mid$ split) specifies how values equal to the overall median are to be handled. The median test computes the median for varname by using all observations and then divides the observations into those falling above the median and those falling below the median. When values for an observation are equal to the sample median, they can be dropped from the analysis by specifying medianties (drop); added to the group above or below the median by specifying medianties(above) or medianties (below), respectively; or if there is more than 1 observation with values equal to the median, they can be equally divided into the two groups by specifying medianties (split). If this option is not specified, medianties (below) is assumed.

## Remarks and examples

> Example 1
We are testing the effectiveness of a new fuel additive. We run an experiment with 24 cars: 12 cars with the fuel treatment and 12 cars without. We input these data by creating a dataset with 24 observations. mpg records the mileage rating, and treat records 0 if the mileage corresponds to untreated fuel and 1 if it corresponds to treated fuel.
. use http://www.stata-press.com/data/r13/fuel2
. ranksum mpg, by(treat)
Two-sample Wilcoxon rank-sum (Mann-Whitney) test

| treat | obs | rank sum | expected |
| :---: | :---: | :---: | :---: |
| untreated | 12 | 128 | 150 |
| treated | 12 | 172 | 150 |
| combined | 24 | 300 | 300 |
| unadjusted variance adjustment for ties |  | 300.00 |  |
|  |  | -4.04 |  |
| adjusted variance | 295.96 |  |  |
| Ho: mpg(treat==untreated) = mpg (treat==treated) |  |  |  |
|  | . 279 |  |  |
| Prob > \|z | . 201 |  |  |

These results indicate that the medians are not statistically different at any level smaller than $20.1 \%$. Similarly, the median test,

| . median mpg, by (treat) exact |
| :--- |
| Median test |
| Greater <br> than the <br> median |
| whether car received <br> fuel additive <br> untreated <br> treated |
| no |
| yes |

fails to reject the null hypothesis that there is no difference between the fuel with the additive and the fuel without the additive.

Compare these results from these two tests with those obtained from the signrank and signtest where we found significant differences; see [R] signrank. An experiment run on 24 different cars is not as powerful as a before-and-after comparison using the same 12 cars.

## Stored results

ranksum stores the following in $r()$ :
Scalars

```
r(N_1) sample size n
    r(N_2) sample size n2
    r(z) z statistic
    r(Var_a) adjusted variance
    r(group1) value of variable for first group
    r(sum_obs) actual sum of ranks for first group
    r(sum_exp) expected sum of ranks for first group
    r(porder) probability that draw from first population is larger than draw from second population
```

median stores the following in $r()$ :
Scalars

```
    r(N) sample size
    r(chi2) Pearson's }\mp@subsup{\chi}{}{2
    r(p) significance of Pearson's }\mp@subsup{\chi}{}{2
    r(p_exact) Fisher's exact p
    r(groups) number of groups compared
    r(chi2_cc) continuity-corrected Pearson's }\mp@subsup{\chi}{}{2
    r(p_cc) continuity-corrected significance
    r(p1_exact) one-sided Fisher's exact p
```


## Methods and formulas

For a practical introduction to these techniques with an emphasis on examples rather than theory, see Acock (2014), Bland (2000), or Sprent and Smeeton (2007). For a summary of these tests, see Snedecor and Cochran (1989).

Methods and formulas are presented under the following headings:

```
ranksum
median
```


## ranksum

For the Wilcoxon rank-sum test, there are two independent random variables, $X_{1}$ and $X_{2}$, and we test the null hypothesis that $X_{1} \sim X_{2}$. We have a sample of size $n_{1}$ from $X_{1}$ and another of size $n_{2}$ from $X_{2}$.

The data are then ranked without regard to the sample to which they belong. If the data are tied, averaged ranks are used. Wilcoxon's test statistic (1945) is the sum of the ranks for the observations in the first sample:

$$
T=\sum_{i=1}^{n_{1}} R_{1 i}
$$

Mann and Whitney's $U$ statistic (1947) is the number of pairs $\left(X_{1 i}, X_{2 j}\right)$ such that $X_{1 i}>X_{2 j}$. These statistics differ only by a constant:

$$
U=T-\frac{n_{1}\left(n_{1}+1\right)}{2}
$$

Again Fisher's principle of randomization provides a method for calculating the distribution of the test statistic, ties or not. The randomization distribution consists of the $\binom{n}{n_{1}}$ ways to choose $n_{1}$ ranks from the set of all $n=n_{1}+n_{2}$ ranks and assign them to the first sample.

It is a straightforward exercise to verify that

$$
E(T)=\frac{n_{1}(n+1)}{2} \quad \text { and } \quad \operatorname{Var}(T)=\frac{n_{1} n_{2} s^{2}}{n}
$$

where $s$ is the standard deviation of the combined ranks, $r_{i}$, for both groups:

$$
s^{2}=\frac{1}{n-1} \sum_{i=1}^{n}\left(r_{i}-\bar{r}\right)^{2}
$$

This formula for the variance is exact and holds both when there are no ties and when there are ties and we use averaged ranks. (Indeed, the variance formula holds for the randomization distribution of choosing $n_{1}$ numbers from any set of $n$ numbers.)

Using a normal approximation, we calculate

$$
z=\frac{T-E(T)}{\sqrt{\operatorname{Var}(T)}}
$$

When the porder option is specified, the probability

$$
p=\frac{U}{n_{1} n_{2}}
$$

is computed.

## - Technical note

We follow the great majority of the literature in naming these tests for Wilcoxon, Mann, and Whitney. However, they were independently developed by several other researchers in the late 1940s and early 1950s. In addition to Wilcoxon, Mann, and Whitney, credit is due to Festinger (1946), Whitfield (1947), Haldane and Smith (1947), and Van der Reyden (1952). Leon Festinger (1919-1989), John Burdon Sanderson Haldane (1892-1964), and Cedric Austen Bardell Smith (1917-2002) are well known for other work, but little seems to be known about Whitfield or van der Reyden. For a detailed study, including information on these researchers, see Berry, Mielke, and Johnston (2012).

## median

The median test examines whether it is likely that two or more samples came from populations with the same median. The null hypothesis is that the samples were drawn from populations with the same median. The alternative hypothesis is that at least one sample was drawn from a population with a different median. The test should be used only with ordinal or interval data.

Assume that there are score values for $k$ independent samples to be compared. The median test is performed by first computing the median score for all observations combined, regardless of the sample group. Each score is compared with this computed grand median and is classified as being above the grand median, below the grand median, or equal to the grand median. Observations with scores equal to the grand median can be dropped, added to the "above" group, added to the "below" group, or split between the two groups.

Once all observations are classified, the data are cast into a $2 \times k$ contingency table, and a Pearson's chi-squared test or Fisher's exact test is performed.

Henry Berthold Mann (1905-2000) was born in Vienna, Austria, where he completed a doctorate in algebraic number theory. He moved to the United States in 1938 and for several years made his livelihood by tutoring in New York. During this time, he proved a celebrated conjecture in number theory and studied statistics at Columbia with Abraham Wald, with whom he wrote three papers. After the war, he taught at Ohio State and the Universities of Wisconsin and Arizona. In addition to his work in number theory and statistics, he made major contributions to algebra and combinatorics.

Donald Ransom Whitney (1915-2007) studied at Oberlin, Princeton, and Ohio State Universities and worked at the latter throughout his career. His PhD thesis under Henry Mann was on nonparametric statistics. It was this work that produced the test that bears their names.

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Wilcoxon, F. 1945. Individual comparisons by ranking methods. Biometrics 1: 80-83.

## Also see

[R] signrank - Equality tests on matched data
$[\mathrm{R}]$ ttest $-t$ tests (mean-comparison tests)

## Title

ratio - Estimate ratios

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

## Basic syntax

```
ratio [name:] varname [/] varname
```

Full syntax

```
ratio ([name:] varname [/] varname)
    \([([\) name \(:]\) varname [/] varname \() \ldots][\) if \(][\) in \(][\) weight \(][\), options \(]\)
```

options
Model
stdize (varname)
stdweight (varname)
nostdrescale
iffin/over
over (varlist [, nolabel])

## SE/Cluster

vce (vcetype)

Reporting
level(\#)
noheader
nolegend
display_options
coeflegend
variable identifying strata for standardization
weight variable for standardization
do not rescale the standard weight variable
group over subpopulations defined by varlist; optionally, suppress group labels
vcetype may be linearized, cluster clustvar, bootstrap, or jackknife
set confidence level; default is level (95)
suppress table header
suppress table legend
control column formats and line width
display legend instead of statistics
bootstrap, jackknife, mi estimate, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands. vce(bootstrap) and vce(jackknife) are not allowed with the mi estimate prefix; see [MI] mi estimate.

Weights are not allowed with the bootstrap prefix; see [R] bootstrap.
vce() and weights are not allowed with the svy prefix; see [SVY] svy.
fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Summaries, tables, and tests $>$ Summary and descriptive statistics $>$ Ratios

## Description

ratio produces estimates of ratios, along with standard errors.

## Options

$\qquad$
stdize (varname) specifies that the point estimates be adjusted by direct standardization across the strata identified by varname. This option requires the stdweight() option.
stdweight (varname) specifies the weight variable associated with the standard strata identified in the stdize() option. The standardization weights must be constant within the standard strata.
nostdrescale prevents the standardization weights from being rescaled within the over() groups.
This option requires stdize() but is ignored if the over() option is not specified.
if/in/over
over (varlist [, nolabel]) specifies that estimates be computed for multiple subpopulations, which are identified by the different values of the variables in varlist.

When this option is supplied with one variable name, such as over (varname), the value labels of varname are used to identify the subpopulations. If varname does not have labeled values (or there are unlabeled values), the values themselves are used, provided that they are nonnegative integers. Noninteger values, negative values, and labels that are not valid Stata names are substituted with a default identifier.

When over () is supplied with multiple variable names, each subpopulation is assigned a unique default identifier.
nolabel requests that value labels attached to the variables identifying the subpopulations be ignored.

## SE/Cluster

vce (vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (linearized), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
vce(linearized), the default, uses the linearized or sandwich estimator of variance.

## Reporting

level (\#); see [R] estimation options.
noheader prevents the table header from being displayed. This option implies nolegend. nolegend prevents the table legend identifying the subpopulations from being displayed. display_options: cformat (\%fmt) and nolstretch; see [R] estimation options.

The following option is available with ratio but is not shown in the dialog box:
coeflegend; see [R] estimation options.

## Remarks and examples

## > Example 1

Using the fuel data from example 3 of $[\mathrm{R}]$ ttest, we estimate the ratio of mileage for the cars without the fuel treatment (mpg1) to those with the fuel treatment (mpg2).

```
. use http://www.stata-press.com/data/r13/fuel
. ratio myratio: mpg1/mpg2
Ratio estimation Number of obs = 12
    myratio: mpg1/mpg2
\begin{tabular}{l|rccc}
\hline & Ratio & \begin{tabular}{c} 
Linearized \\
Std. Err.
\end{tabular} & [95\% Conf. Interval] \\
\hline myratio & .9230769 & .032493 & .8515603 & .9945936
\end{tabular}
```

Using these results, we can test to see if this ratio is significantly different from one.

```
. test _b[myratio] = 1
( 1) myratio = 1
    F( 1, 11) = 5.60
        Prob > F = 0.0373
```

We find that the ratio is different from one at the $5 \%$ significance level but not at the $1 \%$ significance level.

## > Example 2

Using state-level census data, we want to test whether the marriage rate is equal to the death rate.

```
. use http://www.stata-press.com/data/r13/census2
(1980 Census data by state)
. ratio (deathrate: death/pop) (marrate: marriage/pop)
Ratio estimation Number of obs = 50
    deathrate: death/pop
        marrate: marriage/pop
\begin{tabular}{r|rcrr}
\hline & Ratio & \begin{tabular}{c} 
Linearized \\
Std. Err.
\end{tabular} & [95\% Conf. Interval] \\
\hline deathrate & .0087368 & .0002052 & .0083244 & .0091492 \\
marrate & .0105577 & .0006184 & .009315 & .0118005 \\
\hline
\end{tabular}
```

```
. test _b[deathrate] = _b[marrate]
```

. test _b[deathrate] = _b[marrate]
( 1) deathrate - marrate = 0
( 1) deathrate - marrate = 0
F( 1, 49) = 6.93
F( 1, 49) = 6.93
Prob > F = 0.0113

```
                Prob > F = 0.0113
```


## Stored results

ratio stores the following in e():
Scalars

| $e(N)$ | number of observations |
| :--- | :--- |
| $e\left(N_{\text {_over }}\right)$ | number of subpopulations |
| $e\left(N_{\text {stdize }}\right)$ | number of standard strata |
| $e\left(N_{c l u}\right)$ | number of clusters |
| $e\left(k \_e q\right)$ | number of equations in $e(b)$ |
| $e(d f-r)$ | sample degrees of freedom |
| $e($ rank $)$ | rank of $e(V)$ |

Macros
e(cmd)
e(cmdline)
e(varlist)
e(stdize)
e(stdweight)
e(wtype)
e(wexp)
e(title)
e(cluster)
e(over)
e(over_labels)
e(over_namelist)
$e$ (namelist)
e(vce)
e(vcetype)
e(properties)
e(estat_cmd)
e(marginsnotok)
Matrices
e(b)
e(V)
e(_N)
e(_N_stdsum)
e(_p_stdize)
e(error)
Functions
e(sample) marks estimation sample

## Methods and formulas

Methods and formulas are presented under the following headings:

[^66]
## The ratio estimator

Let $R=Y / X$ be the ratio to be estimated, where $Y$ and $X$ are totals; see [ R$]$ total. The estimate for $R$ is $\widehat{R}=\widehat{Y} / \widehat{X}$ (the ratio of the sample totals). From the delta method (that is, a first-order Taylor expansion), the approximate variance of the sampling distribution of the linearized $\widehat{R}$ is

$$
V(\widehat{R}) \approx \frac{1}{X^{2}}\left\{V(\widehat{Y})-2 R \operatorname{Cov}(\widehat{Y}, \widehat{X})+R^{2} V(\widehat{X})\right\}
$$

Direct substitution of $\widehat{X}, \widehat{R}$, and the estimated variances and covariance of $\widehat{X}$ and $\widehat{Y}$ leads to the following variance estimator:

$$
\begin{equation*}
\widehat{V}(\widehat{R})=\frac{1}{\widehat{X}^{2}}\left\{\widehat{V}(\widehat{Y})-2 \widehat{R} \widehat{\operatorname{Cov}}(\widehat{Y}, \widehat{X})+\widehat{R}^{2} \widehat{V}(\widehat{X})\right\} \tag{1}
\end{equation*}
$$

## Survey data

See [SVY] variance estimation, [SVY] direct standardization, and [SVY] poststratification for discussions that provide background information for the following formulas.

## The survey ratio estimator

Let $Y_{j}$ and $X_{j}$ be survey items for the $j$ th individual in the population, where $j=1, \ldots, M$ and $M$ is the size of the population. The associated population ratio for the items of interest is $R=Y / X$ where

$$
Y=\sum_{j=1}^{M} Y_{j} \quad \text { and } \quad X=\sum_{j=1}^{M} X_{j}
$$

Let $y_{j}$ and $x_{j}$ be the corresponding survey items for the $j$ th sampled individual from the population, where $j=1, \ldots, m$ and $m$ is the number of observations in the sample.

The estimator $\widehat{R}$ for the population ratio $R$ is $\widehat{R}=\widehat{Y} / \widehat{X}$, where

$$
\widehat{Y}=\sum_{j=1}^{m} w_{j} y_{j} \quad \text { and } \quad \widehat{X}=\sum_{j=1}^{m} w_{j} x_{j}
$$

and $w_{j}$ is a sampling weight. The score variable for the ratio estimator is

$$
z_{j}(\widehat{R})=\frac{y_{j}-\widehat{R} x_{j}}{\widehat{X}}=\frac{\widehat{X} y_{j}-\widehat{Y} x_{j}}{\widehat{X}^{2}}
$$

## The standardized ratio estimator

Let $D_{g}$ denote the set of sampled observations that belong to the $g$ th standard stratum and define $I_{D_{g}}(j)$ to indicate if the $j$ th observation is a member of the $g$ th standard stratum; where $g=1$, $\ldots, L_{D}$ and $L_{D}$ is the number of standard strata. Also, let $\pi_{g}$ denote the fraction of the population that belongs to the $g$ th standard stratum, thus $\pi_{1}+\cdots+\pi_{L_{D}}=1$. Note that $\pi_{g}$ is derived from the stdweight() option.

The estimator for the standardized ratio is

$$
\widehat{R}^{D}=\sum_{g=1}^{L_{D}} \pi_{g} \frac{\widehat{Y}_{g}}{\widehat{X}_{g}}
$$

where

$$
\widehat{Y}_{g}=\sum_{j=1}^{m} I_{D_{g}}(j) w_{j} y_{j}
$$

and $\widehat{X}_{g}$ is similarly defined. The score variable for the standardized ratio is

$$
z_{j}\left(\widehat{R}^{D}\right)=\sum_{g=1}^{L_{D}} \pi_{g} I_{D_{g}}(j) \frac{\widehat{X}_{g} y_{j}-\widehat{Y}_{g} x_{j}}{\widehat{X}_{g}^{2}}
$$

## The poststratified ratio estimator

Let $P_{k}$ denote the set of sampled observations that belong to poststratum $k$, and define $I_{P_{k}}(j)$ to indicate if the $j$ th observation is a member of poststratum $k$, where $k=1, \ldots, L_{P}$ and $L_{P}$ is the number of poststrata. Also, let $M_{k}$ denote the population size for poststratum $k . P_{k}$ and $M_{k}$ are identified by specifying the poststrata() and postweight () options on svyset; see [SVY] svyset.

The estimator for the poststratified ratio is

$$
\widehat{R}^{P}=\frac{\widehat{Y}^{P}}{\widehat{X}^{P}}
$$

where

$$
\widehat{Y}^{P}=\sum_{k=1}^{L_{P}} \frac{M_{k}}{\widehat{M}_{k}} \widehat{Y}_{k}=\sum_{k=1}^{L_{P}} \frac{M_{k}}{\widehat{M}_{k}} \sum_{j=1}^{m} I_{P_{k}}(j) w_{j} y_{j}
$$

and $\widehat{X}^{P}$ is similarly defined. The score variable for the poststratified ratio is

$$
z_{j}\left(\widehat{R}^{P}\right)=\frac{z_{j}\left(\widehat{Y}^{P}\right)-\widehat{R}^{P} z_{j}\left(\widehat{X}^{P}\right)}{\widehat{X}^{P}}=\frac{\widehat{X}^{P} z_{j}\left(\widehat{Y}^{P}\right)-\widehat{Y}^{P} z_{j}\left(\widehat{X}^{P}\right)}{\left(\widehat{X}^{P}\right)^{2}}
$$

where

$$
z_{j}\left(\widehat{Y}^{P}\right)=\sum_{k=1}^{L_{P}} I_{P_{k}}(j) \frac{M_{k}}{\widehat{M}_{k}}\left(y_{j}-\frac{\widehat{Y}_{k}}{\widehat{M}_{k}}\right)
$$

and $z_{j}\left(\widehat{X}^{P}\right)$ is similarly defined.

## The standardized poststratified ratio estimator

The estimator for the standardized poststratified ratio is

$$
\widehat{R}^{D P}=\sum_{g=1}^{L_{D}} \pi_{g} \frac{\widehat{Y}_{g}^{P}}{\widehat{X}_{g}^{P}}
$$

where

$$
\widehat{Y}_{g}^{P}=\sum_{k=1}^{L_{p}} \frac{M_{k}}{\widehat{M}_{k}} \widehat{Y}_{g, k}=\sum_{k=1}^{L_{p}} \frac{M_{k}}{\widehat{M}_{k}} \sum_{j=1}^{m} I_{D_{g}}(j) I_{P_{k}}(j) w_{j} y_{j}
$$

and $\widehat{X}_{g}^{P}$ is similarly defined. The score variable for the standardized poststratified ratio is

$$
z_{j}\left(\widehat{R}^{D P}\right)=\sum_{g=1}^{L_{D}} \pi_{g} \frac{\widehat{X}_{g}^{P} z_{j}\left(\widehat{Y}_{g}^{P}\right)-\widehat{Y}_{g}^{P} z_{j}\left(\widehat{X}_{g}^{P}\right)}{\left(\widehat{X}_{g}^{P}\right)^{2}}
$$

where

$$
z_{j}\left(\widehat{Y}_{g}^{P}\right)=\sum_{k=1}^{L_{P}} I_{P_{k}}(j) \frac{M_{k}}{\widehat{M}_{k}}\left\{I_{D_{g}}(j) y_{j}-\frac{\widehat{Y}_{g, k}}{\widehat{M}_{k}}\right\}
$$

and $z_{j}\left(\widehat{X}_{g}^{P}\right)$ is similarly defined.

## Subpopulation estimation

Let $S$ denote the set of sampled observations that belong to the subpopulation of interest, and define $I_{S}(j)$ to indicate if the $j$ th observation falls within the subpopulation.

The estimator for the subpopulation ratio is $\widehat{R}^{S}=\widehat{Y}^{S} / \widehat{X}^{S}$, where

$$
\widehat{Y}^{S}=\sum_{j=1}^{m} I_{S}(j) w_{j} y_{j} \quad \text { and } \quad \widehat{X}^{S}=\sum_{j=1}^{m} I_{S}(j) w_{j} x_{j}
$$

Its score variable is

$$
z_{j}\left(\widehat{R}^{S}\right)=I_{S}(j) \frac{y_{j}-\widehat{R}^{S} x_{j}}{\widehat{X}^{S}}=I_{S}(j) \frac{\widehat{X}^{S} y_{j}-\widehat{Y}^{S} x_{j}}{\left(\widehat{X}^{S}\right)^{2}}
$$

The estimator for the standardized subpopulation ratio is

$$
\widehat{R}^{D S}=\sum_{g=1}^{L_{D}} \pi_{g} \frac{\widehat{Y}_{g}^{S}}{\widehat{X}_{g}^{S}}
$$

where

$$
\widehat{Y}_{g}^{S}=\sum_{j=1}^{m} I_{D_{g}}(j) I_{S}(j) w_{j} y_{j}
$$

and $\widehat{X}_{g}^{S}$ is similarly defined. Its score variable is

$$
z_{j}\left(\widehat{R}^{D S}\right)=\sum_{g=1}^{L_{D}} \pi_{g} I_{D_{g}}(j) I_{S}(j) \frac{\widehat{X}_{g}^{S} y_{j}-\widehat{Y}_{g}^{S} x_{j}}{\left(\widehat{X}_{g}^{S}\right)^{2}}
$$

The estimator for the poststratified subpopulation ratio is

$$
\widehat{R}^{P S}=\frac{\widehat{Y}^{P S}}{\widehat{X}^{P S}}
$$

where

$$
\widehat{Y}^{P S}=\sum_{k=1}^{L_{P}} \frac{M_{k}}{\widehat{M}_{k}} \widehat{Y}_{k}^{S}=\sum_{k=1}^{L_{P}} \frac{M_{k}}{\widehat{M}_{k}} \sum_{j=1}^{m} I_{P_{k}}(j) I_{S}(j) w_{j} y_{j}
$$

and $\widehat{X}^{P S}$ is similarly defined. Its score variable is

$$
z_{j}\left(\widehat{R}^{P S}\right)=\frac{\widehat{X}^{P S} z_{j}\left(\widehat{Y}^{P S}\right)-\widehat{Y}^{P S} z_{j}\left(\widehat{X}^{P S}\right)}{\left(\widehat{X}^{P S}\right)^{2}}
$$

where

$$
z_{j}\left(\widehat{Y}^{P S}\right)=\sum_{k=1}^{L_{P}} I_{P_{k}}(j) \frac{M_{k}}{\widehat{M}_{k}}\left\{I_{S}(j) y_{j}-\frac{\widehat{Y}_{k}^{S}}{\widehat{M}_{k}}\right\}
$$

and $z_{j}\left(\widehat{X}^{P S}\right)$ is similarly defined.
The estimator for the standardized poststratified subpopulation ratio is

$$
\widehat{R}^{D P S}=\sum_{g=1}^{L_{D}} \pi_{g} \frac{\widehat{Y}_{g}^{P S}}{\widehat{X}_{g}^{P S}}
$$

where

$$
\widehat{Y}_{g}^{P S}=\sum_{k=1}^{L_{p}} \frac{M_{k}}{\widehat{M}_{k}} \widehat{Y}_{g, k}^{S}=\sum_{k=1}^{L_{p}} \frac{M_{k}}{\widehat{M}_{k}} \sum_{j=1}^{m} I_{D_{g}}(j) I_{P_{k}}(j) I_{S}(j) w_{j} y_{j}
$$

and $\widehat{X}_{g}^{P S}$ is similarly defined. Its score variable is

$$
z_{j}\left(\widehat{R}^{D P S}\right)=\sum_{g=1}^{L_{D}} \pi_{g} \frac{\widehat{X}_{g}^{P S} z_{j}\left(\widehat{Y}_{g}^{P S}\right)-\widehat{Y}_{g}^{P S} z_{j}\left(\widehat{X}_{g}^{P S}\right)}{\left(\widehat{X}_{g}^{P S}\right)^{2}}
$$

where

$$
z_{j}\left(\widehat{Y}_{g}^{P S}\right)=\sum_{k=1}^{L_{P}} I_{P_{k}}(j) \frac{M_{k}}{\widehat{M}_{k}}\left\{I_{D_{g}}(j) I_{S}(j) y_{j}-\frac{\widehat{Y}_{g, k}^{S}}{\widehat{M}_{k}}\right\}
$$

and $z_{j}\left(\widehat{X}_{g}^{P S}\right)$ is similarly defined.

## References

Cochran, W. G. 1977. Sampling Techniques. 3rd ed. New York: Wiley.
Stuart, A., and J. K. Ord. 1994. Kendall's Advanced Theory of Statistics: Distribution Theory, Vol I. 6th ed. London: Arnold.

## Also see

$[R]$ ratio postestimation - Postestimation tools for ratio
[R] mean - Estimate means
[R] proportion - Estimate proportions
[R] total - Estimate totals
[MI] estimation - Estimation commands for use with mi estimate
[SVY] direct standardization - Direct standardization of means, proportions, and ratios
[SVY] poststratification - Poststratification for survey data
[SVY] subpopulation estimation - Subpopulation estimation for survey data
[SVY] svy estimation - Estimation commands for survey data
[SVY] variance estimation - Variance estimation for survey data
[U] 20 Estimation and postestimation commands

## Title

ratio postestimation - Postestimation tools for ratio

## Description Remarks and examples Also see

## Description

The following postestimation commands are available after ratio:

| Command | Description |
| :--- | :--- |
| estat vce | variance-covariance matrix of the estimators (VCE) <br> estat (svy) <br> postestimation statistics for survey data <br> estimates <br> lincom |
| cataloging estimation results <br> point estimates, standard errors, testing, and inference for linear combinations <br> of coefficients |  |
| nlcom | point estimates, standard errors, testing, and inference for nonlinear combinations <br> of coefficients |
| test | Wald tests of simple and composite linear hypotheses <br> testnl |
|  | Wald tests of nonlinear hypotheses |

## Remarks and examples

For examples of the use of test after ratio, see [R] ratio.

## Also see

[ R ] ratio - Estimate ratios
[U] 20 Estimation and postestimation commands

## Title

reg3 - Three-stage estimation for systems of simultaneous equations

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |
| Also see |  |  |  |

## Syntax

## Basic syntax

reg3 (depvar ${ }_{1}$ varlist $_{1}$ ) (depvar varlist $_{2}$ ) $\ldots\left(\right.$ depvar $_{N}$ varlist $\left._{N}\right)[$ if $][$ in] [weight $]$
Full syntax


```
    ([\mp@subsup{eqname }{2}{}:]\mp@subsup{\mathrm{ depvar }}{2a}{}[\mp@subsup{\mathrm{ depvar }}{2b}{}\ldots=]\mp@subsup{\mathrm{ varlist }}{2}{[}[\mathrm{ , noconstant ])}
    ([\mp@subsup{eqname N}{N}{}:]\mp@subsup{\mathrm{ depvar }}{Na}{}[\mp@subsup{\mathrm{ depvar }}{Nb}{}\ldots=]\mp@subsup{v}{\mathrm{ varlist }}{N}}[\mp@code{, noconstant])
    [if] [in] [weight] [, options]
```

options
Description
Model

$$
\begin{array}{ll}
\underline{\text { ireg3 }} & \text { iterate until estimates converge } \\
\underline{\text { constraints (constraints) }} & \text { apply specified linear constraints }
\end{array}
$$

Model 2
exog (varlist)
endog (varlist)
inst (varlist)
allexog
noconstant
Est. method
3sls
2sls
ols
sure
mvreg
corr (correlation)
df adj.
small
dfk
dfk2
exogenous variables not specified in system equations
additional right-hand-side endogenous variables
full list of exogenous variables
all right-hand-side variables are exogenous
suppress constant from instrument list
three-stage least squares; the default
two-stage least squares
ordinary least squares (OLS)
seemingly unrelated regression estimation (SURE)
sure with OLS degrees-of-freedom adjustment
unstructured or independent correlation structure; default is unstructured
report small-sample statistics
use small-sample adjustment
use alternate adjustment

Reporting
level(\#)
first
nocnsreport
display_options
set confidence level; default is level (95)
report first-stage regression
do not display constraints
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling

Optimization
optimization_options
noheader
notable
nofooter
coeflegend
control the optimization process; seldom used
suppress display of header
suppress display of coefficient table
suppress display of footer
display legend instead of statistics
varlist $_{1}, \ldots$, varlist $_{N}$ and the exog() and the inst() varlist may contain factor variables; see [U] 11.4.3 Factor variables. You must have the same levels of factor variables in all equations that have factor variables.
depvar and varlist may contain time-series operators; see [U] 11.4.4 Time-series varlists.
bootstrap, by, fp, jackknife, rolling, and statsby are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the bootstrap prefix; see $[R]$ bootstrap.
aweights are not allowed with the jackknife prefix; see $[R]$ jackknife.
aweights and fweights are allowed; see [U] 11.1.6 weight.
noheader, notable, nofooter, and coeflegend do not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.
Explicit equation naming (eqname:) cannot be combined with multiple dependent variables in an equation specification.

## Menu

Statistics $>$ Endogenous covariates $>$ Three-stage least squares

## Description

reg3 estimates a system of structural equations, where some equations contain endogenous variables among the explanatory variables. Estimation is via three-stage least squares (3SLS); see Zellner and Theil (1962). Typically, the endogenous explanatory variables are dependent variables from other equations in the system. reg3 supports iterated GLS estimation and linear constraints.
reg3 can also estimate systems of equations by seemingly unrelated regression estimation (SURE), multivariate regression (MVREG), and equation-by-equation ordinary least squares (OLS) or two-stage least squares (2SLS).

## Nomenclature

Under 3SLS or 2SLS estimation, a structural equation is defined as one of the equations specified in the system. A dependent variable will have its usual interpretation as the left-hand-side variable in an equation with an associated disturbance term. All dependent variables are explicitly taken to be endogenous to the system and are treated as correlated with the disturbances in the system's equations. Unless specified in an endog() option, all other variables in the system are treated as exogenous to the system and uncorrelated with the disturbances. The exogenous variables are taken to be instruments for the endogenous variables.

## Options


#### Abstract

Model ireg3 causes reg3 to iterate over the estimated disturbance covariance matrix and parameter estimates until the parameter estimates converge. Although the iteration is usually successful, there is no guarantee that it will converge to a stable point. Under SURE, this iteration converges to the maximum likelihood estimates.


constraints (constraints); see [R] estimation options.
$\qquad$ Model 2
exog (varlist) specifies additional exogenous variables that are included in none of the system equations. This can occur when the system contains identities that are not estimated. If implicitly exogenous variables from the equations are listed here, reg3 will just ignore the additional information. Specified variables will be added to the exogenous variables in the system and used in the first stage as instruments for the endogenous variables. By specifying dependent variables from the structural equations, you can use exog() to override their endogeneity.
endog (varlist) identifies variables in the system that are not dependent variables but are endogenous to the system. These variables must appear in the variable list of at least one equation in the system. Again the need for this identification often occurs when the system contains identities. For example, a variable that is the sum of an exogenous variable and a dependent variable may appear as an explanatory variable in some equations.
inst (varlist) specifies a full list of all exogenous variables and may not be used with the endog() or exog() options. It must contain a full list of variables to be used as instruments for the endogenous regressors. Like exog(), the list may contain variables not specified in the system of equations. This option can be used to achieve the same results as the endog() and exog() options, and the choice is a matter of convenience. Any variable not specified in the varlist of the inst () option is assumed to be endogenous to the system. As with exog(), including the dependent variables from the structural equations will override their endogeneity.
allexog indicates that all right-hand-side variables are to be treated as exogenous-even if they appear as the dependent variable of another equation in the system. This option can be used to enforce a SURE or MVREG estimation even when some dependent variables appear as regressors.
noconstant; see $[R]$ estimation options.
$\sqrt{\text { Est. method }}$
3sls specifies the full 3SLS estimation of the system and is the default for reg3.
2sls causes reg3 to perform equation-by-equation 2SLS on the full system of equations. This option implies dfk, small, and corr(independent).
Cross-equation testing should not be performed after estimation with this option. With 2s1s, no covariance is estimated between the parameters of the equations. For cross-equation testing, use 3sls.
ols causes reg3 to perform equation-by-equation OLS on the system-even if dependent variables appear as regressors or the regressors differ for each equation; see [MV] mvreg. ols implies allexog, dfk, small, and corr(independent); nodfk and nosmall may be specified to override dfk and small.
The covariance of the coefficients between equations is not estimated under this option, and cross-equation tests should not be performed after estimation with ols. For cross-equation testing, use sure or 3sls (the default).
sure causes reg3 to perform a SURE of the system-even if dependent variables from some equations appear as regressors in other equations; see $[R]$ sureg. sure is a synonym for allexog.
mvreg is identical to sure, except that the disturbance covariance matrix is estimated with an olS degrees-of-freedom adjustment-the dfk option. If the regressors are identical for all equations, the parameter point estimates will be the standard MVREG results. If any of the regressors differ, the point estimates are those for SURE with an OLS degrees-of-freedom adjustment in computing the covariance matrix. nodfk and nosmall may be specified to override dfk and small.
corr (correlation) specifies the assumed form of the correlation structure of the equation disturbances and is rarely requested explicitly. For the family of models fit by reg3, the only two allowable correlation structures are unstructured and independent. The default is unstructured.

This option is used almost exclusively to estimate a system of equations by 2SLS or to perform OLS regression with reg3 on multiple equations. In these cases, the correlation is set to independent, forcing reg3 to treat the covariance matrix of equation disturbances as diagonal in estimating model parameters. Thus a set of two-stage coefficient estimates can be obtained if the system contains endogenous right-hand-side variables, or OLS regression can be imposed, even if the regressors differ across equations. Without imposing independent disturbances, reg3 would estimate the former by 3SLS and the latter by SURE.
Any tests performed after estimation with the independent option will treat coefficients in different equations as having no covariance; cross-equation tests should not be used after specifying corr (independent).
df adj.
small specifies that small-sample statistics be computed. It shifts the test statistics from $\chi^{2}$ and $z$ statistics to $F$ statistics and $t$ statistics. This option is intended primarily to support MVREG. Although the standard errors from each equation are computed using the degrees of freedom for the equation, the degrees of freedom for the $t$ statistics are all taken to be those for the first equation. This approach poses no problem under MVREG because the regressors are the same across equations.
dfk specifies the use of an alternative divisor in computing the covariance matrix for the equation residuals. As an asymptotically justified estimator, reg3 by default uses the number of sample observations $n$ as a divisor. When the dfk option is set, a small-sample adjustment is made, and the divisor is taken to be $\sqrt{\left(n-k_{i}\right)\left(n-k_{j}\right)}$, where $k_{i}$ and $k_{j}$ are the numbers of parameters in equations $i$ and $j$, respectively.
dfk2 specifies the use of an alternative divisor in computing the covariance matrix for the equation errors. When the dfk2 option is set, the divisor is taken to be the mean of the residual degrees of freedom from the individual equations.

Reporting
level (\#); see [R] estimation options.
first requests that the first-stage regression results be displayed during estimation.
nocnsreport; see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\%fmt), pformat(\%fmt), sformat(\%fmt), and nolstretch; see [R] estimation options.
optimization_options control the iterative process that minimizes the sum of squared errors when ireg3 is specified. These options are seldom used.
iterate(\#) specifies the maximum number of iterations. When the number of iterations equals \#, the optimizer stops and presents the current results, even if the convergence tolerance has not been reached. The default value of iterate() is the current value of set maxiter (see [ R$]$ maximize), which is iterate (16000) if maxiter has not been changed.
trace adds to the iteration $\log$ a display of the current parameter vector.
nolog suppresses the display of the iteration log.
tolerance (\#) specifies the tolerance for the coefficient vector. When the relative change in the coefficient vector from one iteration to the next is less than or equal to \#, the optimization process is stopped. tolerance $(1 e-6)$ is the default.

The following options are available with reg3 but are not shown in the dialog box:
noheader suppresses display of the header reporting the estimation method and the table of equation summary statistics.
notable suppresses display of the coefficient table.
nofooter suppresses display of the footer reporting the list of endogenous and exogenous variables in the model.
coeflegend; see $[R]$ estimation options.

## Remarks and examples

reg3 estimates systems of structural equations where some equations contain endogenous variables among the explanatory variables. Generally, these endogenous variables are the dependent variables of other equations in the system, though not always. The disturbance is correlated with the endogenous variables-violating the assumptions of OLS. Further, because some of the explanatory variables are the dependent variables of other equations in the system, the error terms among the equations are expected to be correlated. reg3 uses an instrumental-variables approach to produce consistent estimates and generalized least squares (GLS) to account for the correlation structure in the disturbances across the equations. Good general references on three-stage estimation include Davidson and MacKinnon (1993, 651-661) and Greene (2012, 331-334).

Three-stage least squares can be thought of as producing estimates from a three-step process.
Step 1. Develop instrumented values for all endogenous variables. These instrumented values can simply be considered as the predicted values resulting from a regression of each endogenous variable on all exogenous variables in the system. This stage is identical to the first step in 2SLS and is critical for the consistency of the parameter estimates.
Step 2. Obtain a consistent estimate for the covariance matrix of the equation disturbances. These estimates are based on the residuals from a 2SLS estimation of each structural equation.
Step 3. Perform a GLS-type estimation using the covariance matrix estimated in the second stage and with the instrumented values in place of the right-hand-side endogenous variables.

## - Technical note

The estimation and use of the covariance matrix of disturbances in three-stage estimation is almost identical to the SURE method-sureg. As with SURE, using this covariance matrix improves the efficiency of the three-stage estimator. Even without the covariance matrix, the estimates would be consistent. (They would be 2SLS estimates.) This improvement in efficiency comes with a caveat. All the parameter estimates now depend on the consistency of the covariance matrix estimates. If one equation in the system is misspecified, the disturbance covariance estimates will be inconsistent, and the resulting coefficients will be biased and inconsistent. Alternatively, if each equation is estimated separately by 2 SLS ( $[R]$ regress), only the coefficients in the misspecified equation are affected.

## - Technical note

If an equation is just identified, the 3SLS point estimates for that equation are identical to the 2SLS estimates. However, as with sureg, even if all equations are just identified, fitting the model via reg3 has at least one advantage over fitting each equation separately via ivregress; by using reg3, tests involving coefficients in different equations can be performed easily using test or testnl.

## > Example 1

A simple macroeconomic model relates consumption (consump) to private and government wages paid (wagepriv and wagegovt). Simultaneously, private wages depend on consumption, total government expenditures (govt), and the lagged stock of capital in the economy (capital1). Although this is not a plausible model, it does meet the criterion of being simple. This model could be written as

$$
\begin{aligned}
\text { consump } & =\beta_{0}+\beta_{1} \text { wagepriv }+\beta_{2} \text { wagegovt }+\epsilon_{1} \\
\text { wagepriv } & =\beta_{3}+\beta_{4} \text { consump }+\beta_{5} \text { govt }+\beta_{6} \text { capital1 }+\epsilon_{2}
\end{aligned}
$$

If we assume that this is the full system, consump and wagepriv will be endogenous variables, with wagegovt, govt, and capitall exogenous. Data for the U.S. economy on these variables are taken from Klein (1950). This model can be fit with reg3 by typing
. use http://www.stata-press.com/data/r13/klein
. reg3 (consump wagepriv wagegovt) (wagepriv consump govt capital1)
Three-stage least-squares regression

| Equation | Obs | Parms | RMSE | "R-sq" | chi2 | P |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| consump | 22 | 2 | 1.776297 | 0.9388 | 208.02 | 0.0000 |
| wagepriv | 22 | 3 | 2.372443 | 0.8542 | 80.04 | 0.0000 |


|  | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| :---: | ---: | :---: | :---: | :---: | :---: | :---: |
| consump |  |  |  |  |  |  |
| wagepriv | .8012754 | .1279329 | 6.26 | 0.000 | .5505314 | 1.052019 |
| wagegovt | 1.029531 | .3048424 | 3.38 | 0.001 | .432051 | 1.627011 |
| _cons | 19.3559 | 3.583772 | 5.40 | 0.000 | 12.33184 | 26.37996 |
| wagepriv |  |  |  |  |  |  |
| consump | .4026076 | .2567312 | 1.57 | 0.117 | -.1005764 | .9057916 |
| govt | 1.177792 | .5421253 | 2.17 | 0.030 | .1152461 | 2.240338 |
| capital1 | -.0281145 | .0572111 | -0.49 | 0.623 | -.1402462 | .0840173 |
| _cons | 14.63026 | 10.26693 | 1.42 | 0.154 | -5.492552 | 34.75306 |

Endogenous variables: consump wagepriv
Exogenous variables: wagegovt govt capital1

Without showing the 2SLS results, we note that the consumption function in this system falls under the conditions noted earlier. That is, the 2SLS and 3SLS coefficients for the equation are identical.

## $>$ Example 2

Some of the most common simultaneous systems encountered are supply-and-demand models. A simple system could be specified as

$$
\begin{aligned}
& \text { qDemand }=\beta_{0}+\beta_{1} \text { price }+\beta_{2} \text { pcompete }+\beta_{3} \text { income }+\epsilon_{1} \\
& \text { qSupply }=\beta_{4}+\beta_{5} \text { price }+\beta_{6} \text { praw }+\epsilon_{2} \\
& \text { Equilibrium condition: quantity }=\text { qDemand }=\text { qSupply }
\end{aligned}
$$

where
quantity is the quantity of a product produced and sold, price is the price of the product, pcompete is the price of a competing product, income is the average income level of consumers, and praw is the price of raw materials used to produce the product.

In this system, price is assumed to be determined simultaneously with demand. The important statistical implications are that price is not a predetermined variable and that it is correlated with the disturbances of both equations. The system is somewhat unusual: quantity is associated with two disturbances. This fact really poses no problem because the disturbances are specified on the behavioral demand and supply equations-two separate entities. Often one of the two equations is rewritten to place price on the left-hand side, making this endogeneity explicit in the specification.

To provide a concrete illustration of the effects of simultaneous equations, we can simulate data for the above system by using known coefficients and disturbance properties. Specifically, we will simulate the data as

$$
\begin{aligned}
& \text { qDemand }=40-1.0 \text { price }+0.25 \text { pcompete }+0.5 \text { income }+\epsilon_{1} \\
& \text { qSupply }=0.5 \text { price }-0.75 \text { praw }+\epsilon_{2}
\end{aligned}
$$

where

$$
\begin{aligned}
& \epsilon_{1} \sim N(0,3.8) \\
& \epsilon_{2} \sim N(0,2.4)
\end{aligned}
$$

For comparison, we can estimate the supply and demand equations separately by OLS. The estimates for the demand equation are
. use http://www.stata-press.com/data/r13/supDem
. regress quantity price pcompete income

| Source | SS | df | MS | Number of obs | 49 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | F ( 3, 45) | 1.00 |
| Model | 23.1579302 | 3 | 7.71931008 | Prob > F | 0.4004 |
| Residual | 346.459313 | 45 | 7.69909584 | R -squared | 0.0627 |
|  |  |  |  | Adj R-squared | 0.0002 |
| Total | 369.617243 | 48 | 7.70035923 | Root MSE | 2.7747 |


| quantity | Coef. | Std. Err. | t | $\mathrm{P}>\mid \mathrm{tI}$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | ---: |
| price | .1186265 | .1716014 | 0.69 | 0.493 | -.2269965 | .4642496 |
| pcompete | .0946416 | .1200815 | 0.79 | 0.435 | -.1472149 | .3364981 |
| income | .0785339 | .1159867 | 0.68 | 0.502 | -.1550754 | .3121432 |
| _cons | 7.563261 | 5.019479 | 1.51 | 0.139 | -2.54649 | 17.67301 |

The OLS estimates for the supply equation are

| Source | SS | df | MS |  | Number of obs $=\quad 49$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Model | 224.819549 | 2112 | 09774 |  | Prob > F | $=0.0000$ |
| Residual | 144.797694 | 463.1 | 77596 |  | R -squared | $=0.6082$ |
| Total | 369.617243 | $48 \quad 7.7$ | 35923 |  | Adj R-squared <br> Root MSE | $\begin{aligned} & =0.5912 \\ & =1.7742 \end{aligned}$ |
| quantity | Coef. | Std. Err . | t | $P>\|t\|$ | [95\% Conf. | Interval] |
| price | . 724675 | . 1095657 | 6.61 | 0.000 | . 5041307 | . 9452192 |
| praw | -. 8674796 | . 1066114 | -8.14 | 0.000 | -1.082077 | -. 652882 |
| _cons | -6.97291 | 3.323105 | -2.10 | 0.041 | -13.66197 | -. 283847 |

Examining the coefficients from these regressions, we note that they are not close to the known parameters used to generate the simulated data. In particular, the positive coefficient on price in the demand equation stands out. We constructed our simulated data to be consistent with economic theory-people demand less of a product if its price rises and more if their personal income rises. Although the price coefficient is statistically insignificant, the positive value contrasts starkly with what is predicted from economic price theory and the -1.0 value that we used in the simulation. Likewise, we are disappointed with the insignificance and level of the coefficient on average income. The supply equation has correct signs on the two main parameters, but their levels are different from the known values. In fact, the coefficient on price (0.724675) is different from the simulated parameter (0.5) at the $5 \%$ level of significance.

All of these problems are to be expected. We explicitly constructed a simultaneous system of equations that violated one of the assumptions of least squares. Specifically, the disturbances were correlated with one of the regressors-price.

Two-stage least squares can be used to address the correlation between regressors and disturbances. Using instruments for the endogenous variable, price, 2SLS will produce consistent estimates of the parameters in the system. Let's use ivregress (see [R] ivregress) to see how our simulated system behaves when fit using 2SLS.


| quantity | Coef. | Std. Err. | $z$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | :---: | ---: | ---: |
| price | -1.015817 | .374209 | -2.71 | 0.007 | -1.749253 | -.282381 |
| pcompete | .3319504 | .172912 | 1.92 | 0.055 | -.0069508 | .6708517 |
| income | .5090607 | .1919482 | 2.65 | 0.008 | .1328491 | .8852723 |
| _cons | 39.89988 | 10.77378 | 3.70 | 0.000 | 18.78366 | 61.01611 |

Instrumented: price
Instruments: pcompete income praw
. ivregress 2sls quantity (price = pcompete income) praw
Instrumental variables (2SLS) regression $\quad$ Number of obs $=\quad 49$ Wald chi2(2) $=39.25$ Prob > chi2 $=0.0000$
R-squared $=0.5928$
Root MSE $=1.7525$

| quantity | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | :---: | ---: | ---: | ---: |
| price | .5773133 | .1749974 | 3.30 | 0.001 | .2343247 | .9203019 |
| praw | -.7835496 | .1312414 | -5.97 | 0.000 | -1.040778 | -.5263213 |
| _cons | -2.550694 | 5.273067 | -0.48 | 0.629 | -12.88571 | 7.784327 |

Instrumented: price
Instruments: praw pcompete income
We are now much happier with the estimation results. All the coefficients from both equations are close to the true parameter values for the system. In particular, the coefficients are all well within $95 \%$ confidence intervals for the parameters. The missing $R$-squared in the demand equation seems unusual; we will discuss that more later.

Finally, this system could be estimated using 3SLS. To demonstrate how large systems might be handled and to avoid multiline commands, we will use global macros (see [P] macro) to hold the specifications for our equations.
. global demand "(qDemand: quantity price pcompete income)"
. global supply "(qSupply: quantity price praw)"
. reg3 \$demand \$supply, endog(price)
We must specify price as endogenous because it does not appear as a dependent variable in either equation. Without this option, reg3 would assume that there are no endogenous variables in the system and produce seemingly unrelated regression (sureg) estimates. The reg3 output from our series of commands is

Three-stage least-squares regression

| Equation | Obs Parms | RMSE | "R-sq" |  | chi2 |  | P |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| qDemand | 49 | 3.739686 | -0.8540 |  | 8.680 .0338 |  |  |
| qSupply | 49 | 1.752501 | 0.5928 |  | 39.250 .0000 |  |  |
|  | Coef. | Std. Err. | z | $P>\|z\|$ | [95\% | Conf | Interval] |
| qDemand |  |  |  |  |  |  |  |
| price | -1.014345 | . 3742036 | -2.71 | 0.007 | -1.747 | 777 | -. 2809194 |
| pcompete | . 2647206 | . 1464194 | 1.81 | 0.071 | -. 0222 | 561 | . 5516973 |
| income | . 5299146 | . 1898161 | 2.79 | 0.005 | . 1578 | 819 | . 9019472 |
| _cons | 40.08749 | 10.77072 | 3.72 | 0.000 | 18.97 | 726 | 61.19772 |
| qSupply |  |  |  |  |  |  |  |
| price | . 5773133 | . 1749974 | 3.30 | 0.001 | . 2343 | 247 | . 9203019 |
| praw | -. 7835496 | . 1312414 | -5.97 | 0.000 | -1.040 | 778 | -. 5263213 |
| _cons | -2.550694 | 5.273067 | -0.48 | 0.629 | -12.88 | 571 | 7.784327 |

Endogenous variables: quantity price
Exogenous variables: pcompete income praw

The use of 3SLS over 2SLS is essentially an efficiency issue. The coefficients of the demand equation from 3SLS are close to the coefficients from two-stage least squares, and those of the supply equation are identical. The latter case was mentioned earlier for systems with some exactly identified equations. However, even for the demand equation, we do not expect the coefficients to change systematically. What we do expect from three-stage least squares are more precise estimates of the parameters given the validity of our specification and reg3's use of the covariances among the disturbances.

Let's summarize the results. With OLS, we got obviously biased estimates of the parameters. No amount of data would have improved the OLS estimates-they are inconsistent in the face of the violated OLS assumptions. With 2SLS, we obtained consistent estimates of the parameters, and these would have improved with more data. With 3SLS, we obtained consistent estimates of the parameters that are more efficient than those obtained by 2SLS.

## Technical note

We noted earlier that the $R$-squared was missing from the two-stage estimates of the demand equation. Now we see that the $R$-squared is negative for the three-stage estimates of the same equation. How can we have a negative $R$-squared?

In most estimators, other than least squares, the $R$-squared is no more than a summary measure of the overall in-sample predictive power of the estimator. The computational formula for $R$-squared is $R$-squared $=1-$ RSS/TSS, where RSS is the residual sum of squares (sum of squared residuals) and TSS is the total sum of squared deviations about the mean of the dependent variable. In a standard linear model with a constant, the model from which the TSS is computed is nested within the full model from which RSS is computed-they both have a constant term based on the same data. Thus it must be that TSS $\geq$ RSS and $R$-squared is constrained between 0 and 1 .

For 2SLS and 3SLS, some of the regressors enter the model as instruments when the parameters are estimated. However, because our goal is to fit the structural model, the actual values, not the instruments for the endogenous right-hand-side variables, are used to determine $R$-squared. The model residuals are computed over a different set of regressors from those used to fit the model. The two-
or three-stage estimates are no longer nested within a constant-only model of the dependent variable, and the residual sum of squares is no longer constrained to be smaller than the total sum of squares.

A negative $R$-squared in 3SLS should be taken for exactly what it is-an indication that the structural model predicts the dependent variable worse than a constant-only model. Is this a problem? It depends on the application. Three-stage least squares applied to our contrived supply-and-demand example produced good estimates of the known true parameters. Still, the demand equation produced an $R$-squared of -0.854 . How do we feel about our parameter estimates? This should be determined by the estimates themselves, their associated standard errors, and the overall model significance. On this basis, negative $R$-squared and all, we feel pretty good about all the parameter estimates for both the supply and demand equations. Would we want to make predictions about equilibrium quantity by using the demand equation alone? Probably not. Would we want to make these quantity predictions by using the supply equation? Possibly, because based on in-sample predictions, they seem better than those from the demand equations. However, both the supply and demand estimates are based on limited information. If we are interested in predicting quantity, a reduced-form equation containing all our independent variables would usually be preferred.

## Technical note

As a matter of syntax, we could have specified the supply-and-demand model on one line without using global macros.
. reg3 (quantity price pcompete income) (quantity price praw), endog(price)
Three-stage least-squares regression

| Equation | Obs | Parms | RMSE | "R-sq" | chi2 | P |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| quantity | 49 | 3 | 3.739686 | -0.8540 | 8.68 | 0.0338 |
| 2quantity | 49 | 2 | 1.752501 | 0.5928 | 39.25 | 0.0000 |


|  | Coef. | Std. Err. | $z$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| :---: | ---: | :---: | :---: | :---: | ---: | ---: |
| quantity |  |  |  |  |  |  |
| price | -1.014345 | .3742036 | -2.71 | 0.007 | -1.74777 | -.2809194 |
| pcompete | .2647206 | .1464194 | 1.81 | 0.071 | -.0222561 | .5516973 |
| income | .5299146 | .1898161 | 2.79 | 0.005 | .1578819 | .9019472 |
| _cons | 40.08749 | 10.77072 | 3.72 | 0.000 | 18.97726 | 61.19772 |
| 2quantity |  |  |  |  |  |  |
| price | .5773133 | .1749974 | 3.30 | 0.001 | .2343247 | .9203019 |
| praw | -.7835496 | .1312414 | -5.97 | 0.000 | -1.040778 | -.5263213 |
| _cons | -2.550694 | 5.273067 | -0.48 | 0.629 | -12.88571 | 7.784327 |

Endogenous variables: quantity price
Exogenous variables: pcompete income praw

However, here reg3 has been forced to create a unique equation name for the supply equation2quantity. Both the supply and demand equations could not be designated as quantity, so a number was prefixed to the name for the supply equation.

We could have specified

```
. reg3 (qDemand: quantity price pcompete income) (qSupply: quantity price praw),
> endog(price)
```

and obtained the same results and equation labeling as when we used global macros to hold the equation specifications.

Without explicit equation names, reg3 always assumes that the dependent variable should be used to name equations. When each equation has a different dependent variable, this rule causes no problems and produces easily interpreted result tables. If the same dependent variable appears in more than one equation, however, reg3 will create a unique equation name based on the dependent variable name. Because equation names must be used for cross-equation tests, you have more control in this situation if explicit names are placed on the equations.
> Example 3: Using the full syntax of reg3
Klein's (1950) model of the U.S. economy is often used to demonstrate system estimators. It contains several common features that will serve to demonstrate the full syntax of reg3. The Klein model is defined by the following seven relationships:

$$
\begin{align*}
\mathrm{c} & =\beta_{0}+\beta_{1} \mathrm{p}+\beta_{2} \mathrm{~L} \cdot \mathrm{p}+\beta_{3} \mathrm{w}+\epsilon_{1}  \tag{1}\\
\mathrm{i} & =\beta_{4}+\beta_{5} \mathrm{p}+\beta_{6} \mathrm{~L} \cdot \mathrm{p}+\beta_{7} \mathrm{~L} \cdot \mathrm{k}+\epsilon_{2}  \tag{2}\\
\mathrm{wp} & =\beta_{8}+\beta_{9} \mathrm{y}+\beta_{10} \mathrm{~L} \cdot \mathrm{y}+\beta_{11} \mathrm{yr}+\epsilon_{3}  \tag{3}\\
\mathrm{y} & =\mathrm{c}+\mathrm{i}+\mathrm{g}  \tag{4}\\
\mathrm{p} & =\mathrm{y}-\mathrm{t}-\mathrm{wp}  \tag{5}\\
\mathrm{k} & =\mathrm{L} \cdot \mathrm{k}+\mathrm{i}  \tag{6}\\
\mathrm{w} & =\mathrm{wg}+\mathrm{wp} \tag{7}
\end{align*}
$$

Here we have used Stata's lag operator L. to represent variables that appear with a one-period lag in our model; see [U] 13.9 Time-series operators.

The variables in the model are listed below. Two sets of variable names are shown. The concise first name uses traditional economics mnemonics, whereas the second name provides more guidance for everyone else. The concise names serve to keep the specification of the model small (and quite understandable to economists).

| Short name | Long name | Variable definition | Type |
| :--- | :--- | :--- | :--- |
| c | consump | Consumption | endogenous |
| p | profits | Private industry profits | endogenous |
| wp | wagepriv | Private wage bill | endogenous |
| wg | wagegovt | Government wage bill | exogenous |
| w | wagetot | Total wage bill | endogenous |
| i | invest | Investment | endogenous |
| k | capital | Capital stock | endogenous |
| y | totinc | Total income/demand | endogenous |
| g | govt | Government spending | exogenous |
| t | taxnetx | Indirect bus. taxes + net exports | exogenous |
| yr | year | Year-1931 | exogenous |

Equations (1)-(3) are behavioral and contain explicit disturbances ( $\epsilon_{1}, \epsilon_{2}$, and $\epsilon_{3}$ ). The remaining equations are identities that specify additional variables in the system and their accounting relationships with the variables in the behavioral equations. Some variables are explicitly endogenous by appearing as dependent variables in (1)-(3). Others are implicitly endogenous as linear combinations that contain other endogenous variables (for example, w and p). Still other variables are implicitly exogenous by appearing in the identities but not in the behavioral equations (for example, wg and g ).

Using the concise names, we can fit Klein's model with the following command:

```
. use http://www.stata-press.com/data/r13/klein2
. reg3 (c p L.p w) (i p L.p L.k) (wp y L.y yr), endog(w p y) exog(t wg g)
```

Three-stage least-squares regression

| Equation | Obs | Parms | RMSE | "R-sq" | chi2 | P |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| c | 21 | 3 | .9443305 | 0.9801 | 864.59 | 0.0000 |
| i | 21 | 3 | 1.446736 | 0.8258 | 162.98 | 0.0000 |
| wp | 21 | 3 | .7211282 | 0.9863 | 1594.75 | 0.0000 |


|  | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Con | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| c |  |  |  |  |  |  |
| p |  |  |  |  |  |  |
| --. | . 1248904 | . 1081291 | 1.16 | 0.248 | -. 0870387 | . 3368194 |
| L1. | . 1631439 | . 1004382 | 1.62 | 0.104 | -. 0337113 | . 3599992 |
| W | . 790081 | . 0379379 | 20.83 | 0.000 | . 715724 | . 8644379 |
| _cons | 16.44079 | 1.304549 | 12.60 | 0.000 | 13.88392 | 18.99766 |
| i |  |  |  |  |  |  |
| p |  |  |  |  |  |  |
| -- | -. 0130791 | . 1618962 | -0.08 | 0.936 | -. 3303898 | . 3042316 |
| L1. | . 7557238 | . 1529331 | 4.94 | 0.000 | . 4559805 | 1.055467 |
| k |  |  |  |  |  |  |
| L1. | -. 1948482 | . 0325307 | -5.99 | 0.000 | -. 2586072 | -. 1310893 |
| _cons | 28.17785 | 6.793768 | 4.15 | 0.000 | 14.86231 | 41.49339 |
| wp |  |  |  |  |  |  |
| y |  |  |  |  |  |  |
| -- | . 4004919 | . 0318134 | 12.59 | 0.000 | . 3381388 | . 462845 |
| L1. | . 181291 | . 0341588 | 5.31 | 0.000 | . 1143411 | . 2482409 |
| yr | . 149674 | . 0279352 | 5.36 | 0.000 | . 094922 | . 2044261 |
| _cons | 1.797216 | 1.115854 | 1.61 | 0.107 | -. 3898181 | 3.984251 |

Endogenous variables: c i wp w p y
Exogenous variables: L.p L.k L.y yr $t$ wg g

We used the $\operatorname{exog}()$ option to identify $t$, $w g$, and $g$ as exogenous variables in the system. These variables must be identified because they are part of the system but appear directly in none of the behavioral equations. Without this option, reg3 would not know they were part of the system and could be used as instrumental variables. The endog() option specifying w, p, and y is also required. Without this information, reg3 would be unaware that these variables are linear combinations that include endogenous variables. We did not include k in the endog() option because only its lagged value appears in the behavioral equations.

- Technical note

Rather than listing additional endogenous and exogenous variables, we could specify the full list of exogenous variables in an inst() option,

```
. reg3 (c p L.p w) (i p L.p L.k) (wp y L.y yr), inst(g t wg yr L.p L.k L.y)
```

or equivalently,

```
. global conseqn "(c p L.p w)"
. global inveqn "(i p L.p L.k)"
. global wageqn "(wp y L.y yr)"
. global inlist "g t wg yr L.p L.k L.y"
. reg3 $conseqn $inveqn $wageqn, inst($inlist)
```

Macros and explicit equations can also be mixed in the specification

```
. reg3 $conseqn (i p L.p L.k) $wageqn, endog(w p y) exog(t wg g)
```

or

```
. reg3 (c p L.p w) $inveqn (wp y L.y yr), endog(w p y) exog(t wg g)
```

Placing the equation-binding parentheses in the global macros was also arbitrary. We could have used

```
. global consump "c p L.p w"
. global invest "i p L.p L.k"
. global wagepriv "wp y L.y yr"
. reg3 ($consump) ($invest) ($wagepriv), endog(w p y) exog(t wg g)
```

reg3 is tolerant of all combinations, and these commands will produce identical output.

Switching to the full variable names, we can fit Klein's model with the commands below. We will use global macros to store the lists of endogenous and exogenous variables. Again this is not necessary: these lists could have been typed directly on the command line. However, assigning the lists to local macros makes additional processing easier if alternative models are to be fit. We will also use the ireg3 option to produce the iterated estimates.

```
. use http://www.stata-press.com/data/r13/kleinfull
. global conseqn "(consump profits L.profits wagetot)"
. global inveqn "(invest profits L.profits L.capital)"
. global wageqn "(wagepriv totinc L.totinc year)"
. global enlist "wagetot profits totinc"
. global exlist "taxnetx wagegovt govt"
. reg3 $conseqn $inveqn $wageqn, endog($enlist) exog($exlist) ireg3
Iteration 1: tolerance = . }371254
Iteration 2: tolerance = . }189471
Iteration 3: tolerance = . }107640
    (output omitted)
Iteration 24: tolerance = 7.049e-07
Three-stage least-squares regression, iterated
\begin{tabular}{lrrrrrr}
\hline Equation & Obs & Parms & RMSE & "R-sq" & chi2 & P \\
\hline consump & 21 & 3 & .9565088 & 0.9796 & 970.31 & 0.0000 \\
invest & 21 & 3 & 2.134327 & 0.6209 & 56.78 & 0.0000 \\
wagepriv & 21 & 3 & .7782334 & 0.9840 & 1312.19 & 0.0000 \\
\hline
\end{tabular}
```

|  | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Con | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| consump profits |  |  |  |  |  |  |
| -- | . 1645096 | . 0961979 | 1.71 | 0.087 | -. 0240348 | . 3530539 |
| L1. | . 1765639 | . 0901001 | 1.96 | 0.050 | -. 0000291 | . 3531569 |
| wagetot | . 7658011 | . 0347599 | 22.03 | 0.000 | . 6976729 | . 8339294 |
| _cons | 16.55899 | 1.224401 | 13.52 | 0.000 | 14.15921 | 18.95877 |
| invest |  |  |  |  |  |  |
| profits |  |  |  |  |  |  |
| - | -. 3565316 | . 2601568 | -1.37 | 0.171 | -. 8664296 | . 1533664 |
| L1. | 1.011299 | . 2487745 | 4.07 | 0.000 | . 5237098 | 1.498888 |
| capital |  |  |  |  |  |  |
| L1. | -. 2602 | . 0508694 | -5.12 | 0.000 | -. 3599022 | -. 1604978 |
| _cons | 42.89629 | 10.59386 | 4.05 | 0.000 | 22.13271 | 63.65987 |
| wagepriv |  |  |  |  |  |  |
| - | . 3747792 | . 0311027 | 12.05 | 0.000 | . 3138191 | . 4357394 |
| L1. | . 1936506 | . 0324018 | 5.98 | 0.000 | . 1301443 | . 257157 |
| year | . 1679262 | . 0289291 | 5.80 | 0.000 | . 1112263 | . 2246261 |
| _cons | 2.624766 | 1.195559 | 2.20 | 0.028 | . 2815124 | 4.968019 |

Endogenous variables: consump invest wagepriv wagetot profits totinc
Exogenous variables: L.profits L.capital L.totinc year taxnetx wagegovt govt

## > Example 4: Constraints with reg3

As a simple example of constraints, (1) above may be rewritten with both wages explicitly appearing (rather than as a variable containing the sum). Using the longer variable names, we have

$$
\text { consump }=\beta_{0}+\beta_{1} \text { profits }+\beta_{2} \text { L.profits }+\beta_{3} \text { wagepriv }+\beta_{12} \text { wagegovt }+\epsilon_{1}
$$

To retain the effect of the identity in (7), we need $\beta_{3}=\beta_{12}$ as a constraint on the system. We obtain this result by defining the constraint in the usual way and then specifying its use in reg3. Because reg3 is a system estimator, we will need to use the full equation syntax of constraint. The assumption that the following commands are entered after the model above has been estimated. We are simply changing the definition of the consumption equation (consump) and adding a constraint on two of its parameters. The rest of the model definition is carried forward.

```
. global conseqn "(consump profits L.profits wagepriv wagegovt)"
. constraint define 1 [consump]wagepriv = [consump]wagegovt
. reg3 $conseqn $inveqn $wageqn, endog($enlist) exog($exlist) constr(1) ireg3
note: additional endogenous variables not in the system have no effect
    and are ignored: wagetot
Iteration 1: tolerance = . }371254
Iteration 2: tolerance = . }18947
Iteration 3: tolerance = . }1076
    (output omitted)
Iteration 24: tolerance = 7.049e-07
```

Three-stage least-squares regression, iterated

| Equation | Obs Pa | RMSE | "R-sq" |  | chi2 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| consump | 21 | . 9565086 | 0.9796 |  | 970.310 .0000 |  |  |
| invest | 21 | 2.134326 | 0.6209 |  | $56.78 \quad 0.0000$ |  |  |
| wagepriv | 21 | . 7782334 | 0.9840 |  | 1312.190 .0000 |  |  |
| ( 1) [consump]wagepriv - [consump]wagegovt $=0$ |  |  |  |  |  |  |  |
|  | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% | Con | Interval] |
| consump profits |  |  |  |  |  |  |  |
| -- | . 1645097 | . 0961978 | 1.71 | 0.087 | -. 0240 | 346 | . 353054 |
| L1. | . 1765639 | . 0901001 | 1.96 | 0.050 | -. 0000 | 291 | . 3531568 |
| wagepriv | . 7658012 | . 0347599 | 22.03 | 0.000 | . 6976 | 729 | . 8339294 |
| wagegovt | . 7658012 | . 0347599 | 22.03 | 0.000 | . 6976 | 729 | . 8339294 |
| _cons | 16.55899 | 1.224401 | 13.52 | 0.000 | 14.1 | 592 | 18.95877 |
| invest |  |  |  |  |  |  |  |
| profirs | -. 3565311 | . 2601567 | -1.37 | 0.171 | -. 8664 | 288 | 1533666 |
| L1. | 1.011298 | . 2487744 | 4.07 | 0.000 | . 5237 | 096 | 1.498887 |
| capital |  |  |  |  |  |  |  |
| L1. | -. 2601999 | . 0508694 | -5.12 | 0.000 | -. 359 | 992 | -. 1604977 |
| _cons | 42.89626 | 10.59386 | 4.05 | 0.000 | 22.13 | 269 | 63.65984 |
| wagepriv |  |  |  |  |  |  |  |
| --. | . 3747792 | . 0311027 | 12.05 | 0.000 | . 313 | 3819 | . 4357394 |
| L1. | . 1936506 | . 0324018 | 5.98 | 0.000 | . 1301 | 443 | . 257157 |
| year | . 1679262 | . 0289291 | 5.80 | 0.000 | . 1112 | 263 | . 2246261 |
| _cons | 2.624766 | 1.195559 | 2.20 | 0.028 | . 281 | 512 | 4.968019 |

Endogenous variables: consump invest wagepriv wagetot profits totinc
Exogenous variables: L.profits wagegovt L.capital L.totinc year taxnetx govt

As expected, none of the parameter or standard error estimates has changed from the previous estimates (before the seventh significant digit). We have simply decomposed the total wage variable into its two parts and constrained the coefficients on these parts. The warning about additional endogenous variables was just reg3's way of letting us know that we had specified some information that was irrelevant to the estimation of the system. We had left the wagetot variable in our endog macro. It does not mean anything to the system to specify wagetot as endogenous because it is no longer in the system. That's fine with reg3 and fine for our current purposes.

We can also impose constraints across the equations. For example, the admittedly meaningless constraint of requiring profits to have the same effect in both the consumption and investment equations could be imposed. Retaining the constraint on the wage coefficients, we would estimate this constrained system.
. constraint define 2 [consump]profits = [invest]profits
. reg3 \$conseqn \$inveqn \$wageqn, endog(\$enlist) exog(\$exlist) constr(1 2) ireg3 note: additional endogenous variables not in the system have no effect
and are ignored: wagetot
Iteration 1: tolerance = . 1427927
Iteration 2: tolerance $=$. 032539
Iteration 3: tolerance $=.00307811$
Iteration 4: tolerance $=.00016903$
Iteration 5: tolerance $=.00003409$
Iteration 6: tolerance $=7.763 \mathrm{e}-06$
Iteration 7: tolerance $=9.240 \mathrm{e}-07$
Three-stage least-squares regression, iterated

| Equation | Obs | Parms | RMSE | "R-sq" | chi2 | P |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| consump | 21 | 3 | .9504669 | 0.9798 | 1019.54 | 0.0000 |
| invest | 21 | 3 | 1.247066 | 0.8706 | 144.57 | 0.0000 |
| wagepriv | 21 | 3 | .7225276 | 0.9862 | 1537.45 | 0.0000 |

( 1) [consump]wagepriv - [consump] wagegovt $=0$
( 2) [consump] profits - [invest]profits $=0$

|  | Coef . | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| consump profits |  |  |  |  |  |  |
| -- | . 1075413 | . 0957767 | 1.12 | 0.262 | -. 0801777 | . 2952602 |
| L1. | . 1712756 | . 0912613 | 1.88 | 0.061 | -. 0075932 | . 3501444 |
| wagepriv | . 798484 | . 0340876 | 23.42 | 0.000 | . 7316734 | . 8652946 |
| wagegovt | . 798484 | . 0340876 | 23.42 | 0.000 | . 7316734 | . 8652946 |
| _cons | 16.2521 | 1.212157 | 13.41 | 0.000 | 13.87631 | 18.62788 |
| invest |  |  |  |  |  |  |
| --. | . 1075413 | . 0957767 | 1.12 | 0.262 | -. 0801777 | . 2952602 |
| L1. | . 6443378 | . 1058682 | 6.09 | 0.000 | . 43684 | . 8518356 |
| capital |  |  |  |  |  |  |
| L1. | -. 1766669 | . 0261889 | -6.75 | 0.000 | -. 2279962 | -. 1253375 |
| _cons | 24.31931 | 5.284325 | 4.60 | 0.000 | 13.96222 | 34.6764 |
| wagepriv <br> totinc |  |  |  |  |  |  |
| -- | . 4014106 | . 0300552 | 13.36 | 0.000 | . 3425035 | . 4603177 |
| L1. | . 1775359 | . 0321583 | 5.52 | 0.000 | . 1145068 | . 240565 |
| year | . 1549211 | . 0282291 | 5.49 | 0.000 | . 099593 | . 2102492 |
| _cons | 1.959788 | 1.14467 | 1.71 | 0.087 | -. 2837242 | 4.203299 |

Endogenous variables: consump invest wagepriv wagetot profits totinc
Exogenous variables: L.profits wagegovt L.capital L.totinc year taxnetx govt

## - Technical note

Identification in a system of simultaneous equations involves the notion that there is enough information to estimate the parameters of the model given the specified functional form. Underidentification usually manifests itself as one matrix in the 3SLS computations. The most commonly violated order condition for 2SLS or 3SLS involves the number of endogenous and exogenous variables. There must be at least as many noncollinear exogenous variables in the remaining system as there are endogenous right-hand-side variables in an equation. This condition must hold for each structural equation in the system.

Put as a set of rules the following:

1. Count the number of right-hand-side endogenous variables in an equation and call this $m_{i}$.
2. Count the number of exogenous variables in the same equation and call this $k_{i}$.
3. Count the total number of exogenous variables in all the structural equations plus any additional variables specified in an $\operatorname{exog}()$ or inst() option and call this $K$.
4. If $m_{i}>\left(K-k_{i}\right)$ for any structural equation (i), then the system is underidentified and cannot be estimated by 3SLS.
We are also possibly in trouble if any of the exogenous variables are linearly dependent. We must have $m_{i}$ linearly independent variables among the exogenous variables represented by ( $K-k_{i}$ ).

The complete conditions for identification involve rank-order conditions on several matrices. For a full treatment, see Theil (1971) or Greene (2012, 331-334).

Henri Theil (1924-2000) was born in Amsterdam and awarded a PhD in 1951 by the University of Amsterdam. He researched and taught econometric theory, statistics, microeconomics, macroeconomic modeling, and economic forecasting, and policy at (now) Erasmus University Rotterdam, the University of Chicago, and the University of Florida. Theil's many specific contributions include work on 2SLS and 3SLS, inequality and concentration, and consumer demand.

## Stored results

reg3 stores the following in e() :
Scalars

| e(N) | number of observations |
| :---: | :---: |
| e(k) | number of parameters |
| e(k_eq) | number of equations in e (b) |
| e(mss_\#) | model sum of squares for equation \# |
| $e\left(d f \_m \#\right)$ | model degrees of freedom for equation \# |
| e(rss_\#) | residual sum of squares for equation \# |
| $e\left(d f \_r\right)$ | residual degrees of freedom (small) |
| e(r2_\#) | $R$-squared for equation \# |
| e(F_\#) | $F$ statistic for equation \# (small) |
| e(rmse_\#) | root mean squared error for equation \# |
| e(dfk2_adj) | divisor used with VCE when dfk2 specified |
| e(11) | log likelihood |
| e(chi2_\#) | $\chi^{2}$ for equation \# |
| e(p_\#) | significance for equation \# |
| e(cons_\#) | 1 when equation \# has a constant, 0 otherwise |
| $e(r a n k)$ | rank of e(V) |
| e(ic) | number of iterations |

Macros
e(cmd)
e(cmdline)
e(depvar)
e(exog)
e(endog)
e(eqnames)
e(corr)
e(wtype)
e(wexp)
e(method)
e(small)
e(dfk)
e(properties)
e(predict)
e(marginsok)
e(marginsnotok)
e(asbalanced)
e(asobserved)
Matrices
e(b)
e(Cns)
e(Sigma)
e(V)
Functions
e(sample) marks estimation sample

## Methods and formulas

The most concise way to represent a system of equations for 3SLS requires thinking of the individual equations and their associated data as being stacked. reg3 does not expect the data in this format, but it is a convenient shorthand. The system could then be formulated as

$$
\left[\begin{array}{c}
\mathbf{y}_{1} \\
\mathbf{y}_{2} \\
\vdots \\
\mathbf{y}_{M}
\end{array}\right]=\left[\begin{array}{cccc}
\mathbf{Z}_{1} & 0 & \ldots & 0 \\
0 & \mathbf{Z}_{2} & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & \mathbf{Z}_{M}
\end{array}\right]\left[\begin{array}{c}
\boldsymbol{\beta}_{1} \\
\boldsymbol{\beta}_{2} \\
\vdots \\
\boldsymbol{\beta}_{M}
\end{array}\right]+\left[\begin{array}{c}
\boldsymbol{\epsilon}_{1} \\
\boldsymbol{\epsilon}_{2} \\
\vdots \\
\boldsymbol{\epsilon}_{M}
\end{array}\right]
$$

In full matrix notation, this is just

$$
\mathbf{y}=\mathbf{Z} \mathbf{B}+\boldsymbol{\epsilon}
$$

The $\mathbf{Z}$ elements in these matrices represent both the endogenous and the exogenous right-hand-side variables in the equations.

Also assume that there will be correlation between the disturbances of the equations so that

$$
E\left(\boldsymbol{\epsilon} \boldsymbol{\epsilon}^{\prime}\right)=\boldsymbol{\Sigma}
$$

where the disturbances are further assumed to have an expected value of $0 ; E(\boldsymbol{\epsilon})=0$.
The first stage of 3SLS regression requires developing instrumented values for the endogenous variables in the system. These values can be derived as the predictions from a linear regression of each endogenous regressor on all exogenous variables in the system or, more succinctly, as the projection of each regressor through the projection matrix of all exogenous variables onto the regressors. Designating the set of all exogenous variables as $\mathbf{X}$ results in

$$
\widehat{\mathbf{z}}_{i}=\mathbf{X}\left(\mathbf{X}^{\prime} \mathbf{X}\right)^{-1} \mathbf{X}^{\prime} \mathbf{z}_{i} \quad \text { for each } i
$$

Taken collectively, these $\widehat{\mathbf{Z}}$ contain the instrumented values for all the regressors. They take on the actual values for the exogenous variables and first-stage predictions for the endogenous variables. Given these instrumented variables, a generalized least squares (GLS) or Aitken (1935) estimator can be formed for the parameters of the system

$$
\widehat{\mathbf{B}}=\left\{\widehat{\mathbf{Z}}^{\prime}\left(\boldsymbol{\Sigma}^{-1} \otimes \mathbf{I}\right) \widehat{\mathbf{Z}}\right\}^{-1} \widehat{\mathbf{Z}}^{\prime}\left(\boldsymbol{\Sigma}^{-1} \otimes \mathbf{I}\right) \mathbf{y}
$$

All that remains is to obtain a consistent estimator for $\boldsymbol{\Sigma}$. This estimate can be formed from the residuals of 2SLS estimates of each equation in the system. Alternately, and identically, the residuals can be computed from the estimates formed by taking $\boldsymbol{\Sigma}$ to be an identity matrix. This maintains the full system of coefficients and allows constraints to be applied when the residuals are computed.

If we take $\mathbf{E}$ to be the matrix of residuals from these estimates, a consistent estimate of $\boldsymbol{\Sigma}$ is

$$
\widehat{\boldsymbol{\Sigma}}=\frac{\mathbf{E}^{\prime} \mathbf{E}}{n}
$$

where $n$ is the number of observations in the sample. An alternative divisor for this estimate can be obtained with the dfk option as outlined under options.

With the estimate of $\widehat{\boldsymbol{\Sigma}}$ placed into the GLS estimating equation,

$$
\widehat{\mathbf{B}}=\left\{\widehat{\mathbf{Z}}^{\prime}\left(\widehat{\boldsymbol{\Sigma}}^{-1} \otimes \mathbf{I}\right) \widehat{\mathbf{Z}}\right\}^{-1} \widehat{\mathbf{Z}}^{\prime}\left(\widehat{\boldsymbol{\Sigma}}^{-1} \otimes \mathbf{I}\right) \mathbf{y}
$$

is the 3SLS estimates of the system parameters.
The asymptotic variance-covariance matrix of the estimator is just the standard formulation for a GLS estimator

$$
\mathbf{V}_{\widehat{\mathbf{B}}}=\left\{\widehat{\mathbf{Z}}^{\prime}\left(\widehat{\boldsymbol{\Sigma}}^{-1} \otimes \mathbf{I}\right) \widehat{\mathbf{Z}}\right\}^{-1}
$$

Iterated 3SLS estimates can be obtained by computing the residuals from the three-stage parameter estimates, using these to formulate a new $\widehat{\boldsymbol{\Sigma}}$, and recomputing the parameter estimates. This process is repeated until the estimates $\widehat{\mathbf{B}}$ converge-if they converge. Convergence is not guaranteed. When estimating a system by SURE, these iterated estimates will be the maximum likelihood estimates for the system. The iterated solution can also be used to produce estimates that are invariant to choice of system and restriction parameterization for many linear systems under full 3SLS.

The exposition above follows the parallel developments in Greene (2012) and Davidson and MacKinnon (1993).

## References

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## Also see

[R] reg3 postestimation - Postestimation tools for reg3
$[R]$ ivregress - Single-equation instrumental-variables regression
[R] nlsur - Estimation of nonlinear systems of equations
[ R ] regress - Linear regression
[R] sureg - Zellner's seemingly unrelated regression
[MV] mvreg - Multivariate regression
[SEM] example 7 - Nonrecursive structural model
[SEM] intro 5 - Tour of models
[TS] forecast — Econometric model forecasting
[U] 20 Estimation and postestimation commands

## Title

reg3 postestimation - Postestimation tools for reg3

Description<br>Remarks and examples<br>Syntax for predict<br>Methods and formulas<br>Menu for predict<br>Reference<br>Options for predict<br>Also see

## Description

The following postestimation commands are available after reg3:

| Command | Description |
| :--- | :--- |
| contrast | contrasts and ANOVA-style joint tests of estimates |
| * estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample <br> estat vce |
| estimates | variance-covariance matrix of the estimators (VCE) |
| forecast | dynamic forecasts and simulations |
| hausman | Hausman's specification test <br> point estimates, standard errors, testing, and inference for linear combinations <br> of coefficients |
| margins | marginal means, predictive margins, marginal effects, and average marginal <br> effects |
| marginsplot | graph the results from margins (profile plots, interaction plots, etc.) |
| nlcom | point estimates, standard errors, testing, and inference for nonlinear combinations |
| of coefficients |  |

*estat ic is not appropriate after reg3, 2sls.

## Syntax for predict

predict [type] newvar [if] [in] [, equation(eqno[,eqno]) statistic]
statistic Description

## Main

xb
stdp
residuals
difference stddp
linear prediction; the default
standard error of the linear prediction
residuals
difference between the linear predictions of two equations
standard error of the difference in linear predictions

These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
equation (eqno [,eqno]) specifies to which equation you are referring.
equation() is filled in with one eqno for the xb , stdp, and residuals options. equation(\#1) would mean the calculation is to be made for the first equation, equation(\#2) would mean the second, and so on. You could also refer to the equations by their names. equation(income) would refer to the equation named income and equation(hours) to the equation named hours. If you do not specify equation(), results are the same as if you specified equation(\#1).
difference and stddp refer to between-equation concepts. To use these options, you must specify two equations, for example, equation(\#1,\#2) or equation(income,hours). When two equations must be specified, equation() is required.
xb , the default, calculates the linear prediction (fitted values) - the prediction of $\mathbf{x}_{j} \mathbf{b}$ for the specified equation.
stdp calculates the standard error of the prediction for the specified equation. It can be thought of as the standard error of the predicted expected value or mean for the observation's covariate pattern. The standard error of the prediction is also referred to as the standard error of the fitted value.
residuals calculates the residuals.
difference calculates the difference between the linear predictions of two equations in the system. With equation(\#1,\#2), difference computes the prediction of equation(\#1) minus the prediction of equation(\#2).
stddp is allowed only after you have previously fit a multiple-equation model. The standard error of the difference in linear predictions ( $\mathbf{x}_{1 j} \mathbf{b}-\mathbf{x}_{2 j} \mathbf{b}$ ) between equations 1 and 2 is calculated.

For more information on using predict after multiple-equation estimation commands, see $[R]$ predict.

## Remarks and examples

## > Example 1: Using predict

In example 2 of $[R]$ reg3, we fit a simple supply-and-demand model. Here we obtain the fitted supply and demand curves assuming that the exogenous regressors equal their sample means. We first replace each of the three exogenous regressors with their sample means, then we call predict to obtain the predictions.

```
. use http://www.stata-press.com/data/r13/supDem
global demand "(qDemand: quantity price pcompete income)"
global supply "(qSupply: quantity price praw)"
reg3 $demand $supply, endog(price)
(output omitted)
summarize pcompete, meanonly
. replace pcompete = r(mean)
(49 real changes made)
```

```
. summarize income, meanonly
. replace income = r(mean)
(49 real changes made)
. summarize praw, meanonly
. replace praw = r(mean)
(49 real changes made)
. predict demand, equation(qDemand)
(option xb assumed; fitted values)
. predict supply, equation(qSupply)
(option xb assumed; fitted values)
. graph twoway line demand price, sort || line supply price, ytitle(" ")
> legend(label(1 "Fitted values: qDemand") label(2 "Fitted values: qSupply"))
```



As we would expect based on economic theory, the demand curve slopes downward while the supply curve slopes upward. With the exogenous variables at their mean levels, the equilibrium price and quantity are slightly less than 33 and 13 , respectively.

## > Example 2: Obtaining forecasts

In example 3 of [R] reg3, we fit Klein's (1950) model of the U.S. economy. That model includes three stochastic equations we fit using reg3 as well as four identities. Here we briefly illustrate how the forecast command can be used to obtain forecasts for all the endogenous variables in the model. For a more detailed discussion of how to forecast with this model, see [TS] forecast.

In Stata, we type

```
. use http://www.stata-press.com/data/r13/klein2, clear
. reg3 (c p L.p w) (i p L.p L.k) (wp y L.y yr), endog(w p y) exog(t wg g)
(output omitted)
. estimates store kleineqs
. forecast create kleinmodel
Forecast model kleinmodel started.
. forecast estimates kleineqs
Added estimation results from reg3.
Forecast model kleinmodel now contains 3 endogenous variables.
```

```
    forecast identity y = c + i + g
    Forecast model kleinmodel now contains 4 endogenous variables.
    forecast identity p = y - t - wp
    Forecast model kleinmodel now contains 5 endogenous variables.
    forecast identity k = L.k + i
    Forecast model kleinmodel now contains 6 endogenous variables.
    forecast identity w = wg + wp
    Forecast model kleinmodel now contains 7 endogenous variables.
. forecast solve, begin(1937)
Computing dynamic forecasts for model kleinmodel.
Starting period: 1937
Ending period: 1941
Forecast prefix: f_
1937:
1938: .........................................................
1939: ......................................................
1940: ......................................................
1941: ...........................................................
Forecast }7\mathrm{ variables spanning 5 periods.
```

Here we have obtained dynamic forecasts for our 7 endogenous variables beginning in 1937. By default, the variables containing the forecasts begin with the prefix $f_{\_}$. Next we plot the forecast and actual values of consumption:

```
. tsline c f_c
```



For more information about producing forecasts, see [TS] forecast.

## Methods and formulas

The computational formulas for the statistics produced by predict can be found in $[R]$ predict and $[R]$ regress postestimation.

## Reference

Klein, L. R. 1950. Economic Fluctuations in the United States 1921-1941. New York: Wiley.

## Also see

[R] reg3 - Three-stage estimation for systems of simultaneous equations
[U] 20 Estimation and postestimation commands

## Title

regress - Linear regression

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | Acknowledgments |
| References | Also see |  |  |

## Syntax

$$
\underline{\text { regress depvar }[\text { indepvars }][\text { if }][\text { in }][\text { weight }][\text {,options }]}
$$

options
Description
Model
noconstant
hascons
tsscons
suppress constant term
has user-supplied constant compute total sum of squares with constant; seldom used
vcetype may be ols, robust, cluster clustvar, bootstrap, jackknife, hc2, or hc3

Reporting
level(\#)
beta
eform(string)
depname (varname)
display_options
noheader
notable
plus
mse1
coeflegend
set confidence level; default is level (95)
report standardized beta coefficients
report exponentiated coefficients and label as string
substitute dependent variable name; programmer's option
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
suppress output header
suppress coefficient table
make table extendable
force mean squared error to 1
display legend instead of statistics
indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
depvar and indepvars may contain time-series operators; see [U] 11.4.4 Time-series varlists.
bootstrap, by, fp, jackknife, mfp, mi estimate, nestreg, rolling, statsby, stepwise, and svy are allowed; see [U] 11.1.10 Prefix commands.
vce(bootstrap) and vce(jackknife) are not allowed with the mi estimate prefix; see [MI] mi estimate.
Weights are not allowed with the bootstrap prefix; see $[\mathrm{R}]$ bootstrap.
aweights are not allowed with the jackknife prefix; see [R] jackknife.
hascons, tsscons, vce(), beta, noheader, notable, plus, depname(), mse1, and weights are not allowed with the svy prefix; see [SVY] svy.
aweights, fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
noheader, notable, plus, mse1, and coeflegend do not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Linear models and related $>$ Linear regression

## Description

regress fits a model of depvar on indepvars using linear regression.
Here is a short list of other regression commands that may be of interest. See help estimation commands for a complete list.

| Command | Entry | Description |
| :---: | :---: | :---: |
| areg | [R] areg | an easier way to fit regressions with many dummy variables |
| arch | [TS] arch | regression models with ARCH errors |
| arima | [TS] arima | ARIMA models |
| boxcox | [R] boxcox | Box-Cox regression models |
| cnsreg | [R] cnsreg | constrained linear regression |
| eivreg | [R] eivreg | errors-in-variables regression |
| etregress | [TE] etregress | Linear regression with endogenous treatment effects |
| frontier | [R] frontier | stochastic frontier models |
| gmm | [R] gmm | generalized method of moments estimation |
| heckman | [R] heckman | Heckman selection model |
| intreg | $[R]$ intreg | interval regression |
| ivregress | [R] ivregress | single-equation instrumental-variables regression |
| ivtobit | [R] ivtobit | tobit regression with endogenous variables |
| newey | [TS] newey | regression with Newey-West standard errors |
| nl | [R] nl | nonlinear least-squares estimation |
| nlsur | [R] nlsur | estimation of nonlinear systems of equations |
| qreg | [R] qreg | quantile (including median) regression |
| reg3 | [R] reg3 | three-stage least-squares (3SLS) regression |
| rreg | [R] rreg | a type of robust regression |
| gsem | [SEM] intro 5 | generalized structural equation models |
| sem | [SEM] intro 5 | linear structural equation models |
| sureg | [R] sureg | seemingly unrelated regression |
| tobit | [R] tobit | tobit regression |
| truncreg | [R] truncreg | truncated regression |
| xtabond | [XT] xtabond | Arellano-Bond linear dynamic panel-data estimation |
| xtdpd | [XT] xtdpd | linear dynamic panel-data estimation |
| xtfrontier | [XT] xtfrontier | panel-data stochastic frontier models |
| $x t g 1 s$ | [XT] xtgls | panel-data GLS models |
| xthtaylor | [XT] xthtaylor | Hausman-Taylor estimator for error-components models |
| xtintreg | [XT] xtintreg | panel-data interval regression models |
| xtivreg | [XT] xtivreg | panel-data instrumental-variables (2SLS) regression |
| xtpcse | [XT] xtpese | linear regression with panel-corrected standard errors |
| xtreg | [XT] xtreg | fixed- and random-effects linear models |
| xtregar | [XT] xtregar | fixed- and random-effects linear models with an AR(1) disturbance |
| xttobit | [XT] xttobit | panel-data tobit models |

## Options


hascons indicates that a user-defined constant or its equivalent is specified among the independent variables in indepvars. Some caution is recommended when specifying this option, as resulting estimates may not be as accurate as they otherwise would be. Use of this option requires "sweeping" the constant last, so the moment matrix must be accumulated in absolute rather than deviation form. This option may be safely specified when the means of the dependent and independent variables are all reasonable and there is not much collinearity between the independent variables. The best procedure is to view hascons as a reporting option-estimate with and without hascons and verify that the coefficients and standard errors of the variables not affected by the identity of the constant are unchanged.
tsscons forces the total sum of squares to be computed as though the model has a constant, that is, as deviations from the mean of the dependent variable. This is a rarely used option that has an effect only when specified with noconstant. It affects the total sum of squares and all results derived from the total sum of squares.

SE/Robust
vce (vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (ols), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
vce(ols), the default, uses the standard variance estimator for ordinary least-squares regression. regress also allows the following:
vce (hc2) and vce (hc3) specify an alternative bias correction for the robust variance calculation. vce(hc2) and vce(hc3) may not be specified with svy prefix. In the unclustered case, vce (robust) uses $\widehat{\sigma}_{j}^{2}=\{n /(n-k)\} u_{j}^{2}$ as an estimate of the variance of the $j$ th observation, where $u_{j}$ is the calculated residual and $n /(n-k)$ is included to improve the overall estimate's small-sample properties.
vce (hc2) instead uses $u_{j}^{2} /\left(1-h_{j j}\right)$ as the observation's variance estimate, where $h_{j j}$ is the diagonal element of the hat (projection) matrix. This estimate is unbiased if the model really is homoskedastic. vce (hc2) tends to produce slightly more conservative confidence intervals. vce (hc3) uses $u_{j}^{2} /\left(1-h_{j j}\right)^{2}$ as suggested by Davidson and MacKinnon (1993), who report that this method tends to produce better results when the model really is heteroskedastic. vce (hc3) produces confidence intervals that tend to be even more conservative.
See Davidson and MacKinnon (1993, 554-556) and Angrist and Pischke (2009, 294-308) for more discussion on these two bias corrections.

## Reporting

level (\#); see [R] estimation options.
beta asks that standardized beta coefficients be reported instead of confidence intervals. The beta coefficients are the regression coefficients obtained by first standardizing all variables to have a mean of 0 and a standard deviation of 1 . beta may not be specified with vce (cluster clustvar) or the svy prefix.
eform (string) is used only in programs and ado-files that use regress to fit models other than linear regression. eform() specifies that the coefficient table be displayed in exponentiated form as defined in $[R]$ maximize and that string be used to label the exponentiated coefficients in the table.
depname (varname) is used only in programs and ado-files that use regress to fit models other than linear regression. depname() may be specified only at estimation time. varname is recorded as the identity of the dependent variable, even though the estimates are calculated using depvar. This method affects the labeling of the output - not the results calculated-but could affect subsequent calculations made by predict, where the residual would be calculated as deviations from varname rather than depvar. depname() is most typically used when depvar is a temporary variable (see [P] macro) used as a proxy for varname.
depname() is not allowed with the svy prefix.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\%fmt), pformat(\%fmt), sformat(\%fmt), and nolstretch; see [R] estimation options.

The following options are available with regress but are not shown in the dialog box:
noheader suppresses the display of the ANOVA table and summary statistics at the top of the output; only the coefficient table is displayed. This option is often used in programs and ado-files.
notable suppresses display of the coefficient table.
plus specifies that the output table be made extendable. This option is often used in programs and ado-files.
mse1 is used only in programs and ado-files that use regress to fit models other than linear regression and is not allowed with the svy prefix. mse1 sets the mean squared error to 1 , forcing the variance-covariance matrix of the estimators to be $\left(\mathbf{X}^{\prime} \mathbf{D X}\right)^{-1}$ (see Methods and formulas below) and affecting calculated standard errors. Degrees of freedom for $t$ statistics is calculated as $n$ rather than $n-k$.
coeflegend; see [R] estimation options.

## Remarks and examples

Remarks are presented under the following headings:
Ordinary least squares
Treatment of the constant
Robust standard errors
Weighted regression
Instrumental variables and two-stage least-squares regression Video example
regress performs linear regression, including ordinary least squares and weighted least squares. For a general discussion of linear regression, see Draper and Smith (1998), Greene (2012), or Kmenta (1997).

See Wooldridge (2013) for an excellent treatment of estimation, inference, interpretation, and specification testing in linear regression models. This presentation stands out for its clarification of the statistical issues, as opposed to the algebraic issues. See Wooldridge (2010, chap. 4) for a more advanced discussion along the same lines.

See Hamilton (2013, chap. 7) and Cameron and Trivedi (2010, chap. 3) for an introduction to linear regression using Stata. Dohoo, Martin, and Stryhn (2012, 2010) discuss linear regression using examples from epidemiology, and Stata datasets and do-files used in the text are available. Cameron and Trivedi (2010) discuss linear regression using econometric examples with Stata. Mitchell (2012) shows how to use graphics and postestimation commands to understand a fitted regression model.

Chatterjee and Hadi (2012) explain regression analysis by using examples containing typical problems that you might encounter when performing exploratory data analysis. We also recommend Weisberg (2005), who emphasizes the importance of the assumptions of linear regression and problems resulting from these assumptions. Becketti (2013) discusses regression analysis with an emphasis on time-series data. Angrist and Pischke (2009) approach regression as a tool for exploring relationships, estimating treatment effects, and providing answers to public policy questions. For a discussion of model-selection techniques and exploratory data analysis, see Mosteller and Tukey (1977). For a mathematically rigorous treatment, see Peracchi (2001, chap. 6). Finally, see Plackett (1972) if you are interested in the history of regression. Least squares, which dates back to the 1790s, was discovered independently by Legendre and Gauss.

## Ordinary least squares

> Example 1: Basic linear regression
Suppose that we have data on the mileage rating and weight of 74 automobiles. The variables in our data are mpg , weight, and foreign. The last variable assumes the value 1 for foreign and 0 for domestic automobiles. We wish to fit the model

$$
\mathrm{mpg}=\beta_{0}+\beta_{1} \text { weight }+\beta_{2} \text { foreign }+\epsilon
$$

This model can be fit with regress by typing
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. regress mpg weight foreign
Source

| mpg | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{tl}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| weight | -.0065879 | .0006371 | -10.34 | 0.000 | -.0078583 | -.0053175 |
| foreign | -1.650029 | 1.075994 | -1.53 | 0.130 | -3.7955 | .4954422 |
| _cons | 41.6797 | 2.165547 | 19.25 | 0.000 | 37.36172 | 45.99768 |

regress produces a variety of summary statistics along with the table of regression coefficients. At the upper left, regress reports an analysis-of-variance (ANOVA) table. The column headings SS, df , and MS stand for "sum of squares", "degrees of freedom", and "mean square", respectively. In this example, the total sum of squares is $2,443.5$ : $1,619.3$ accounted for by the model and 824.2 left unexplained. Because the regression included a constant, the total sum reflects the sum after removal of means, as does the sum of squares due to the model. The table also reveals that there are 73 total degrees of freedom (counted as 74 observations less 1 for the mean removal), of which 2 are consumed by the model, leaving 71 for the residual.

To the right of the ANOVA table are presented other summary statistics. The $F$ statistic associated with the ANOVA table is 69.75 . The statistic has 2 numerator and 71 denominator degrees of freedom. The $F$ statistic tests the hypothesis that all coefficients excluding the constant are zero. The chance of observing an $F$ statistic that large or larger is reported as 0.0000 , which is Stata's way of indicating a number smaller than 0.00005 . The $R$-squared $\left(R^{2}\right)$ for the regression is 0.6627 , and the $R$-squared adjusted for degrees of freedom $\left(R_{a}^{2}\right)$ is 0.6532 . The root mean squared error, labeled Root MSE, is 3.4071. It is the square root of the mean squared error reported for the residual in the ANOVA table.

Finally, Stata produces a table of the estimated coefficients. The first line of the table indicates that the left-hand-side variable is mpg. Thereafter follow the estimated coefficients. Our fitted model is

$$
\text { mpg_hat }=41.68-0.0066 \text { weight }-1.65 \text { foreign }
$$

Reported to the right of the coefficients in the output are the standard errors. For instance, the standard error for the coefficient on weight is 0.0006371 . The corresponding $t$ statistic is -10.34 , which has a two-sided significance level of 0.000 . This number indicates that the significance is less than 0.0005 . The $95 \%$ confidence interval for the coefficient is $[-0.0079,-0.0053]$.

## Example 2: Transforming the dependent variable

If we had a graph comparing mpg with weight, we would notice that the relationship is distinctly nonlinear. This is to be expected because energy usage per distance should increase linearly with weight, but mpg is measuring distance per energy used. We could obtain a better model by generating a new variable measuring the number of gallons used per 100 miles (gp100m) and then using this new variable in our model:

$$
\operatorname{gp} 100 \mathrm{~m}=\beta_{0}+\beta_{1} \text { weight }+\beta_{2} \text { foreign }+\epsilon
$$

We can now fit this model:

```
. generate gp100m = 100/mpg
. regress gp100m weight foreign
```

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model <br> Residual | 91.1761694 | 28 | 45.5880847 |
| Total | 119.576261 | 73 | 1.63803097 |

                                    Number of obs \(=\quad 74\)
                                    \(F(2, \quad 71)=113.97\)
                                    Prob \(>\mathrm{F}=0.0000\)
                                    \(R\)-squared \(=0.7625\)
                                    Adj R-squared \(=0.7558\)
                                    Root MSE \(=.63246\)
    | gp100m | Coef. | Std. Err. | t | P>\|t| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| weight | .0016254 | .0001183 | 13.74 | 0.000 | .0013896 | .0018612 |
| foreign | .6220535 | .1997381 | 3.11 | 0.003 | .2237871 | 1.02032 |
| _cons | -.0734839 | .4019932 | -0.18 | 0.855 | -.8750354 | .7280677 |

Fitting the physically reasonable model increases our $R$-squared to 0.7625 .

## > Example 3: Obtaining beta coefficients

regress shares the features of all estimation commands. Among other things, this means that after running a regression, we can use test to test hypotheses about the coefficients, estat vce to examine the covariance matrix of the estimators, and predict to obtain predicted values, residuals, and influence statistics. See [U] 20 Estimation and postestimation commands. Options that affect how estimates are displayed, such as beta or level(), can be used when replaying results.

Suppose that we meant to specify the beta option to obtain beta coefficients (regression coefficients normalized by the ratio of the standard deviation of the regressor to the standard deviation of the dependent variable). Even though we forgot, we can specify the option now:

| . regress, beta |  |  |  |
| ---: | :---: | ---: | :---: |
| Source | SS | df | MS |
| Model <br> Residual | 91.1761694 | 2 | 45.5880847 |
| Total | 119.576261 | 73 | 1.63803097 |


| Number of obs | $=$ | 74 |
| :--- | ---: | ---: |
| F $(2$, | $71)$ | $=$ |
| Prob $>$ F | $=0.0000$ |  |
| R-squared | $=$ | 0.7625 |
| Adj R-squared | $=$ | 0.7558 |
| Root MSE | $=$ | .63246 |


| gp100m | Coef. | Std. Err. | t | $\mathrm{P}>\mid \mathrm{tI}$ | Beta |
| ---: | ---: | :---: | ---: | :---: | ---: |
| weight | .0016254 | .0001183 | 13.74 | 0.000 | .9870255 |
| foreign | .6220535 | .1997381 | 3.11 | 0.003 | .2236673 |
| _cons | -.0734839 | .4019932 | -0.18 | 0.855 | . |

## Treatment of the constant

By default, regress includes an intercept (constant) term in the model. The noconstant option suppresses it, and the hascons option tells regress that the model already has one.
> Example 4: Suppressing the constant term
We wish to fit a regression of the weight of an automobile against its length, and we wish to impose the constraint that the weight is zero when the length is zero.

If we simply type regress weight length, we are fitting the model

$$
\text { weight }=\beta_{0}+\beta_{1} \text { length }+\epsilon
$$

Here a length of zero corresponds to a weight of $\beta_{0}$. We want to force $\beta_{0}$ to be zero or, equivalently, estimate an equation that does not include an intercept:

$$
\text { weight }=\beta_{1} \text { length }+\epsilon
$$

We do this by specifying the noconstant option:
. regress weight length, noconstant

| Source | SS | df | MS |
| ---: | ---: | ---: | ---: |
| Model | 703869302 | 1 | 703869302 |
| Residual | 14892897.8 | 73 | 204012.299 |
| Total | 718762200 | 74 | 9713002.7 |

Number of obs $=\quad 74$
$F(1, \quad 73)=3450.13$
Prob $>$ F $=0.0000$
$R$-squared $=0.9793$
Adj R-squared $=0.9790$
Root MSE $=451.68$

| weight | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | ---: |
| length | 16.29829 | .2774752 | 58.74 | 0.000 | 15.74528 | 16.8513 |

In our data, length is measured in inches and weight in pounds. We discover that each inch of length adds 16 pounds to the weight.

Sometimes there is no need for Stata to include a constant term in the model. Most commonly, this occurs when the model contains a set of mutually exclusive indicator variables. hascons is a variation of the noconstant option-it tells Stata not to add a constant to the regression because the regression specification already has one, either directly or indirectly.

For instance, we now refit our model of weight as a function of length and include separate constants for foreign and domestic cars by specifying bn.foreign. bn.foreign is factor-variable notation for "no base for foreign" or "include all levels of variable foreign in the model"; see [U] 11.4.3 Factor variables.
. regress weight length bn.foreign, hascons

| Source | SS | df | MS |
| ---: | ---: | ---: | :---: |
| Model | 39647744.7 | 2 | 19823872.3 |
| Residual | 4446433.7 | 71 | 62625.8268 |
| Total | 44094178.4 | 73 | 604029.841 |


| Number of obs | $=$ | 74 |
| :--- | ---: | ---: |
| F $(2$, | $71)$ | $=$ |
| Prob $>$ F | $=$ | 0.0000 |
| R-squared | $=$ | 0.8992 |
| Adj R-squared | $=$ | 0.8963 |
| Root MSE | $=$ | 250.25 |


| weight | Coef. | Std. Err. | t | $\mathrm{P}>\mid \mathrm{tI}$ | [95\% Conf. Interval] |  |
| :---: | ---: | :---: | :---: | :---: | :---: | :---: |
| length | 31.44455 | 1.601234 | 19.64 | 0.000 | 28.25178 | 34.63732 |
| foreign |  |  |  |  |  |  |
| Domestic <br> Foreign | -2850.25 | 315.9691 | -9.02 | 0.000 | -3480.274 | -2220.225 |

## - Technical note

There is a subtle distinction between the hascons and noconstant options. We can most easily reveal it by refitting the last regression, specifying noconstant rather than hascons:

| regress weight length bn.foreign, noconstant |  |  |  |
| ---: | ---: | ---: | ---: |
| Source | SS | df | MS |
| Model | 714315766 | 3 | 238105255 |
| Residual | 4446433.7 | 71 | 62625.8268 |
| Total | 718762200 | 74 | 9713002.7 |


| ber of obs |  |
| :---: | :---: |
| 3, | 3802.03 |
| cob > F | 0.000 |
| -squared | 0.993 |
| Adj R-squared |  |
| Root MSE | 250 |


| weight | Coef. | Std. Err. | t | $\mathrm{P}>\mid \mathrm{tl}$ | [95\% Conf. Interval] |  |
| :---: | ---: | :---: | :---: | :---: | :---: | :---: |
| length | 31.44455 | 1.601234 | 19.64 | 0.000 | 28.25178 | 34.63732 |
| foreign |  |  |  |  |  |  |
| Domestic <br> Foreign | -2850.25 | 315.9691 | -9.02 | 0.000 | -3480.274 | -2220.225 |

Comparing this output with that produced by the previous regress command, we see that they are almost, but not quite, identical. The parameter estimates and their associated statistics-the second half of the output - are identical. The overall summary statistics and the ANOVA table-the first half of the output-are different, however.

In the first case, the $R^{2}$ is shown as 0.8992 ; here it is shown as 0.9938 . In the first case, the $F$ statistic is 316.54 ; now it is $3,802.03$. The numerator degrees of freedom is different as well. In the first case, the numerator degrees of freedom is 2 ; now the degrees of freedom is 3 . Which is correct?

Both are. Specifying the hascons option causes regress to adjust the ANOVA table and its associated statistics for the explanatory power of the constant. The regression in effect has a constant; it is just written in such a way that a separate constant is unnecessary. No such adjustment is made with the noconstant option.

## Technical note

When the hascons option is specified, regress checks to make sure that the model does in fact have a constant term. If regress cannot find a constant term, it automatically adds one. Fitting a model of weight on length and specifying the hascons option, we obtain


Even though we specified hascons, regress included a constant, anyway. It also added a note to our output: "note: hascons false".

## - Technical note

Even if the model specification effectively includes a constant term, we need not specify the hascons option. regress is always on the lookout for collinear variables and omits them from the model. For instance,
regress weight length bn.foreign
note: 1.foreign omitted because of collinearity
Source

| Number of obs | $=$ | 74 |
| :--- | ---: | ---: |
| F $(2$, | $71)$ | $=316.54$ |
| Prob $>$ F | $=$ | 0.0000 |
| R-squared | $=$ | 0.8992 |
| Adj R-squared | $=$ | 0.8963 |
| Root MSE | $=$ | 250.25 |


| weight | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| length | 31.44455 | 1.601234 | 19.64 | 0.000 | 28.25178 | 34.63732 |
| foreign <br> Domestic <br> Foreign | 133.6775 | 77.47615 <br> (omitted) | 1.73 | 0.089 | -20.80555 | 288.1605 |
| _cons | -2983.927 | 275.1041 | -10.85 | 0.000 | -3532.469 | -2435.385 |

## Robust standard errors

regress with the vce(robust) option substitutes a robust variance matrix calculation for the conventional calculation, or if vce (cluster clustvar) is specified, allows relaxing the assumption of independence within groups. How this method works is explained in [U] 20.21 Obtaining robust variance estimates. Below we show how well this approach works.
> Example 5: Heteroskedasticity and robust standard errors
Specifying the vce (robust) option is equivalent to requesting White-corrected standard errors in the presence of heteroskedasticity. We use the automobile data and, in the process of looking at the energy efficiency of cars, analyze a variable with considerable heteroskedasticity.

We will examine the amount of energy - measured in gallons of gasoline-that the cars in the data need to move 1,000 pounds of their weight 100 miles. We are going to examine the relative efficiency of foreign and domestic cars.

```
. gen gpmw = ((1/mpg)/weight)*100*1000
. summarize gpmw
\begin{tabular}{r|rrrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline gpmw & 74 & 1.682184 & .2426311 & 1.09553 & 2.30521
\end{tabular}
```

In these data, the engines consume between 1.10 and 2.31 gallons of gas to move 1,000 pounds of the car's weight 100 miles. If we ran a regression with conventional standard errors of gpmw on foreign, we would obtain
. regress gpmw foreign

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | .936705572 | 1 | .936705572 |
| Residual | 3.36079459 | 72 | .046677703 |
| Total | 4.29750017 | 73 | .058869865 |


| Number of obs | $=$ | 74 |
| :--- | ---: | ---: |
| $\mathrm{~F}(1$, | $72)$ | $=20.07$ |
| Prob $>$ F | $=$ | 0.0000 |
| R-squared | $=$ | 0.2180 |
| Adj R-squared | $=$ | 0.2071 |
| Root MSE | $=$ | .21605 |


| gpmw | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| foreign | .2461526 | .0549487 | 4.48 | 0.000 | .1366143 | .3556909 |
| _cons | 1.609004 | .0299608 | 53.70 | 0.000 | 1.549278 | 1.66873 |

regress with the vce(robust) option, on the other hand, reports

```
. regress gpmw foreign, vce(robust)
```

Linear regression

| Number of obs | $=$ | 74 |
| :--- | ---: | ---: |
| F ( 1, 72) | $=13.13$ |  |
| Prob $>$ F | $=0.0005$ |  |
| R-squared | $=$ | 0.2180 |
| Root MSE | $=.21605$ |  |


|  |  | Robust |  |  |  |  |
| ---: | ---: | :---: | ---: | :---: | ---: | ---: |
| gpmw | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| foreign | .2461526 | .0679238 | 3.62 | 0.001 | .1107489 | .3815563 |
| _cons | 1.609004 | .0234535 | 68.60 | 0.000 | 1.56225 | 1.655758 |

The point estimates are the same (foreign cars need one-quarter gallon more gas), but the standard errors differ by roughly $20 \%$. Conventional regression reports the $95 \%$ confidence interval as [0.14, 0.36], whereas the robust standard errors make the interval [ $0.11,0.38$ ].

Which is right? Notice that gpmw is a variable with considerable heteroskedasticity:

| . tabulate foreign, summarize (gpmw) |  |  |  |
| ---: | ---: | :--- | ---: |
| Car type | Summary of gpmw <br> Mean |  | Std. Dev. | Freq.

Thus here we favor the robust standard errors. In [U] 20.21 Obtaining robust variance estimates, we show another example using linear regression where it makes little difference whether we specify vce (robust). The linear-regression assumptions were true, and we obtained nearly linear-regression results. The advantage of the robust estimate is that in neither case did we have to check assumptions.

## - Technical note

regress purposefully suppresses displaying the ANOVA table when vce (robust) is specified, as it is no longer appropriate in a statistical sense, even though, mechanically, the numbers would be unchanged. That is, sums of squares remain unchanged, but the meaning of those sums is no longer relevant. The $F$ statistic, for instance, is no longer based on sums of squares; it becomes a Wald test based on the robustly estimated variance matrix. Nevertheless, regress continues to report the $R^{2}$
and the root MSE even though both numbers are based on sums of squares and are, strictly speaking, irrelevant. In this, the root MSE is more in violation of the spirit of the robust estimator than is $R^{2}$. As a goodness-of-fit statistic, $R^{2}$ is still fine; just do not use it in formulas to obtain $F$ statistics because those formulas no longer apply. The root MSE is valid in a literal sense-it is the square root of the mean squared error, but it is no longer an estimate of $\sigma$ because there is no single $\sigma$; the variance of the residual varies observation by observation.

## > Example 6: Alternative robust standard errors

The vce (hc2) and vce (hc3) options modify the robust variance calculation. In the context of linear regression without clustering, the idea behind the robust calculation is somehow to measure $\sigma_{j}^{2}$, the variance of the residual associated with the $j$ th observation, and then to use that estimate to improve the estimated variance of $\widehat{\boldsymbol{\beta}}$. Because residuals have (theoretically and practically) mean 0 , one estimate of $\sigma_{j}^{2}$ is the observation's squared residual itself- $u_{j}^{2}$. A finite-sample correction could improve that by multiplying $u_{j}^{2}$ by $n /(n-k)$, and, as a matter of fact, vce (robust) uses $\{n /(n-k)\} u_{j}^{2}$ as its estimate of the residual's variance.
vce(hc2) and vce(hc3) use alternative estimators of the observation-specific variances. For instance, if the residuals are homoskedastic, we can show that the expected value of $u_{j}^{2}$ is $\sigma^{2}\left(1-h_{j j}\right)$, where $h_{j j}$ is the $j$ th diagonal element of the projection (hat) matrix. $h_{j j}$ has average value $k / n$, so $1-h_{j j}$ has average value $1-k / n=(n-k) / n$. Thus the default robust estimator $\widehat{\sigma}_{j}=\{n /(n-k)\} u_{j}^{2}$ amounts to dividing $u_{j}^{2}$ by the average of the expectation.
vce (hc2) divides $u_{j}^{2}$ by $1-h_{j j}$ itself, so it should yield better estimates if the residuals really are homoskedastic. vce (hc3) divides $u_{j}^{2}$ by $\left(1-h_{j j}\right)^{2}$ and has no such clean interpretation. Davidson and MacKinnon (1993) show that $u_{j}^{2} /\left(1-h_{j j}\right)^{2}$ approximates a more complicated estimator that they obtain by jackknifing (MacKinnon and White 1985). Angrist and Pischke (2009) also illustrate the relative merits of these adjustments.

Here are the results of refitting our efficiency model using vce(hc2) and vce(hc3):

```
. regress gpmw foreign, vce(hc2)
```

| Number of obs | $=$ | 74 |
| :--- | ---: | ---: |
| F $(1$, | $72)$ | $=12.93$ |
| Prob $>$ F | $=0.0006$ |  |
| R-squared | $=0.2180$ |  |
| Root MSE | $=.21605$ |  |


|  | Robust HC2 |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| gpmw | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| foreign | .2461526 | .0684669 | 3.60 | 0.001 | .1096662 | .3826389 |
| _cons | 1.609004 | .0233601 | 68.88 | 0.000 | 1.562437 | 1.655571 |

. regress gpmw foreign, vce(hc3)

| Linear regres |  |  |  |  | $\begin{aligned} & \text { Number of obs } \\ & \text { F( } 1, \quad 72) \\ & \text { Prob > F } \\ & \text { R-squared } \\ & \text { Root MSE } \end{aligned}$ | $\begin{array}{lr} = & 74 \\ = & 12.38 \\ = & 0.0008 \\ = & 0.2180 \\ = & .21605 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| gpmw | Coef. | Robust HC3 <br> Std. Err. | t | $P>\|t\|$ | [95\% Conf. | Interval] |
| foreign | . 2461526 | . 069969 | 3.52 | 0.001 | . 1066719 | . 3856332 |
| _cons | 1.609004 | . 023588 | 68.21 | 0.000 | 1.561982 | 1.656026 |

## > Example 7: Standard errors for clustered data

The vce (cluster clustvar) option relaxes the assumption of independence. Below we have 28,534 observations on 4,711 women aged 14-46 years. Data were collected on these women between 1968 and 1988. We are going to fit a classic earnings model, and we begin by ignoring that each woman appears an average of 6.057 times in the data.

```
. use http://www.stata-press.com/data/r13/regsmpl, clear
(NLS Women 14-26 in 1968)
```

. regress $l_{n}$ _wage age c.age\#c.age tenure

| Source | SS | df | MS | Number of obs | 28101 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | F ( 3, 28097) | 1842.45 |
| Model | 1054.52501 | 3 | 351.508335 | Prob > F | 0.0000 |
| Residual | 5360.43962 | 28097 | . 190783344 | R-squared | 0.1644 |
|  |  |  |  | Adj R-squared | 0.1643 |
| Total | 6414.96462 | 28100 | . 228290556 | Root MSE | . 43679 |


| ln_wage | Coef. | Std. Err. | t | $\mathrm{P}>\mid \mathrm{tI}$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | :---: | :---: | :---: |
| age | .0752172 | .0034736 | 21.65 | 0.000 | .0684088 | .0820257 |
| c.age\#c.age | -.0010851 | .0000575 | -18.86 | 0.000 | -.0011979 | -.0009724 |
|  |  |  |  |  |  |  |
| tenure | .0390877 | .0007743 | 50.48 | 0.000 | .0375699 | .0406054 |
| _cons | .3339821 | .0504413 | 6.62 | 0.000 | .2351148 | .4328495 |

The number of observations in our model is 28,101 because Stata drops observations that have a missing value for one or more of the variables in the model. We can be reasonably certain that the standard errors reported above are meaningless. Without a doubt, a woman with higher-than-average wages in one year typically has higher-than-average wages in other years, and so the residuals are not independent. One way to deal with this would be to fit a random-effects model-and we are going to do that—but first we fit the model using regress specifying vce(cluster id), which treats only observations with different person ids as truly independent:
. regress ln_wage age c.age\#c.age tenure, vce(cluster id)
Linear regression

| Number of obs | $=28101$ |
| :--- | ---: | ---: |
| $\mathrm{~F}(3,4698)$ | $=748.82$ |
| Prob $>\mathrm{F}$ | $=0.0000$ |
| R-squared | $=0.1644$ |
| Root MSE | $=.43679$ |

(Std. Err. adjusted for 4699 clusters in idcode)

|  | Robust |  |  |  |  |  |
| ---: | ---: | ---: | :---: | ---: | ---: | ---: |
| ln_wage | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| age | .0752172 | .0045711 | 16.45 | 0.000 | .0662557 | .0841788 |
| c.age\#c.age | -.0010851 | .0000778 | -13.94 | 0.000 | -.0012377 | -.0009325 |
|  |  |  |  |  |  |  |
| tenure | .0390877 | .0014425 | 27.10 | 0.000 | .0362596 | .0419157 |
| _cons | .3339821 | .0641918 | 5.20 | 0.000 | .208136 | .4598282 |

For comparison, we focus on the tenure coefficient, which in economics jargon can be interpreted as the rate of return for keeping your job. The $95 \%$ confidence interval we previously estimated - an interval we do not believe-is $[0.038,0.041]$. The robust interval is twice as wide, being [ $0.036,0.042$ ].

As we said, one correct way to fit this model is by random-effects regression. Here is the random-effects result:


Robust regression estimated the $95 \%$ interval [ $0.036,0.042$ ], and xtreg (see [XT] xtreg) estimates [ $0.025,0.027]$. Which is better? The random-effects regression estimator assumes a lot. We can check some of these assumptions by performing a Hausman test. Using estimates (see [R] estimates store), we store the random-effects estimation results, and then we run the required fixed-effects regression to perform the test.

```
. estimates store random
. xtreg ln_wage age c.age#c.age tenure, fe
Fixed-effects (within) regression Number of obs = 28101
Group variable: idcode Number of groups = 4699
```




```
F(3,23399) = 1243.00
corr(u_i, Xb) = 0.1380
Prob > F = 0.0000
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline ln_wage & Coef. & Std. Err. & t & \(P>|t|\) & [95\% Con & Interval] \\
\hline age & . 0522751 & . 002783 & 18.78 & 0.000 & . 0468202 & . 05773 \\
\hline c.age\#c.age & -. 0006717 & . 0000461 & -14.56 & 0.000 & -. 0007621 & -. 0005813 \\
\hline tenure & . 021738 & . 000799 & 27.21 & 0.000 & . 020172 & . 023304 \\
\hline _cons & . 687178 & . 0405944 & 16.93 & 0.000 & . 6076103 & . 7667456 \\
\hline sigma_u & . 38743138 & \multicolumn{5}{|l|}{\multirow[b]{3}{*}{(fraction of variance due to u_i)}} \\
\hline sigma_e & . 29674679 & & & & & \\
\hline rho & . 6302569 & & & & & \\
\hline \multicolumn{2}{|l|}{\(F\) test that all \(u_{\text {_ }} \mathrm{i}=0\) :} & \multicolumn{2}{|l|}{\(F(4698,23399)=\)} & 7.98 & \multicolumn{2}{|l|}{Prob > F \(=0.0000\)} \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|}
\hline \multirow[t]{2}{*}{} & \multicolumn{2}{|r|}{Coefficients} & \multirow[b]{2}{*}{\[
\begin{gathered}
\text { (b-B) } \\
\text { Difference }
\end{gathered}
\]} & \multirow[b]{2}{*}{\[
\begin{gathered}
\operatorname{sqrt}\left(\operatorname{diag}\left(V_{-} b-V \_B\right)\right) \\
\text { S.E. }
\end{gathered}
\]} \\
\hline & (b) & \begin{tabular}{l}
(B) \\
random
\end{tabular} & & \\
\hline age & . 0522751 & . 0568296 & -. 0045545 & . 0006913 \\
\hline c.age\#c.age & -. 0006717 & -. 0007566 & . 0000849 & . 0000115 \\
\hline tenure & . 021738 & . 0260135 & -. 0042756 & . 0002816 \\
\hline
\end{tabular}
```

```
                            b = consistent under Ho and Ha; obtained from xtreg
```

                            b = consistent under Ho and Ha; obtained from xtreg
                B = inconsistent under Ha, efficient under Ho; obtained from xtreg
                B = inconsistent under Ha, efficient under Ho; obtained from xtreg
    Test: Ho: difference in coefficients not systematic

```
Test: Ho: difference in coefficients not systematic
```




The Hausman test casts grave suspicions on the random-effects model we just fit, so we should be careful in interpreting those results.

Meanwhile, our robust regression results still stand, as long as we are careful about the interpretation. The correct interpretation is that, if the data collection were repeated (on women sampled the same way as in the original sample), and if we were to refit the model, $95 \%$ of the time we would expect the estimated coefficient on tenure to be in the range [0.036, 0.042].

Even with robust regression, we must be careful about going beyond that statement. Here the Hausman test is probably picking up something that differs within and between person, which would cast doubt on our robust regression model in terms of interpreting [ $0.036,0.042$ ] to contain the rate of return for keeping a job, economywide, for all women, without exception.

## Weighted regression

regress can perform weighted and unweighted regression. We indicate the weight by specifying the [weight] qualifier. By default, regress assumes analytic weights; see the technical note below.

## > Example 8: Using means as regression variables

We have census data recording the death rate (drate) and median age (medage) for each state. The data also record the region of the country in which each state is located and the overall population of the state:

```
. use http://www.stata-press.com/data/r13/census9
(1980 Census data by state)
. describe
Contains data from http://www.stata-press.com/data/r13/census9.dta
    obs: 50 1980 Census data by state
    vars: 6 6 Apr 2013 15:43
    size: 1,450
```

| variable name | storage <br> type | display <br> format | value <br> label | variable label |
| :--- | :--- | :--- | :--- | :--- |
| state | str14 | $\%-14 \mathrm{~s}$ |  | State |
| state2 | str2 | $\%-2 \mathrm{~s}$ |  | Two-letter state abbreviation |
| drate | float | $\% 9.0 \mathrm{~g}$ |  | Death Rate |
| pop | long | $\% 12.0 \mathrm{gc}$ |  | Population |
| medage | float | $\% 9.2 \mathrm{f}$ |  | Median age |
| region | byte | $\%-8.0 \mathrm{~g}$ | cenreg | Census region |

Sorted by:
We can use factor variables to include dummy variables for region. Because the variables in the regression reflect means rather than individual observations, the appropriate method of estimation is analytically weighted least squares (Davidson and MacKinnon 2004, 261-262), where the weight is total population:

| Source | SS | df | MS |  | Number of obs | $=50$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Model | 4096.6093 | 4102 | 15232 |  | Prob > F | $=0.0000$ |
| Residual | 1238.40987 | 4527. | 202192 |  | R-squared | $=0.7679$ |
| Total | 5335.01916 | 49108 | 877942 |  | Root MSE | $=5.246$ |
| drate | Coef. | Std. Err | t | $P>\|t\|$ | [95\% Conf. | Interval] |
| medage | 4.283183 | . 5393329 | 7.94 | 0.000 | 3.196911 | 5.369455 |
| region | 3138738 | 2.456431 | 0. 13 | 0. 899 | -4. 633632 | 5.26138 |
| South | -1.438452 | 2.320244 | -0.62 | 0.538 | -6.111663 | 3.234758 |
| West | -10.90629 | 2.681349 | -4.07 | 0.000 | -16.30681 | -5.505777 |
| _cons | -39.14727 | 17.23613 | -2.27 | 0.028 | -73.86262 | -4.431915 |

To weight the regression by population, we added the qualifier [ $\mathrm{w}=\mathrm{pop}$ ] to the end of the regress command. Our qualifier was vague (we did not say [aweight=pop]), but unless told otherwise, Stata
assumes analytic weights for regress. Stata informed us that the sum of the weight is $2.2591 \times 10^{8}$; there were approximately 226 million people residing in the United States according to our 1980 data.

## - Technical note

Once we fit a weighted regression, we can obtain the appropriately weighted variance-covariance matrix of the estimators using estat vce and perform appropriately weighted hypothesis tests using test.

In the weighted regression in example 8 , we see that 4 .region is statistically significant but that 2.region and 3.region are not. We use test to test the joint significance of the region variables:

```
. test 2.region 3.region 4.region
(1) 2.region = 0
( 2) 3.region = 0
( 3) 4.region = 0
    F( 3, 45) = 9.84
        Prob > F = 0.0000
```

The results indicate that the region variables are jointly significant.
regress also accepts frequency weights (fweights). Frequency weights are appropriate when the data do not reflect cell means, but instead represent replicated observations. Specifying aweights or fweights will not change the parameter estimates, but it will change the corresponding significance levels.

For instance, if we specified [fweight=pop] in the weighted regression example above-which would be statistically incorrect-Stata would treat the data as if the data represented 226 million independent observations on death rates and median age. The data most certainly do not represent that - they represent 50 observations on state averages.

With aweights, Stata treats the number of observations on the process as the number of observations in the data. When we specify fweights, Stata treats the number of observations as if it were equal to the sum of the weights; see Methods and formulas below.

## Technical note

A popular request on the help line is to describe the effect of specifying [aweight=exp] with regress in terms of transformation of the dependent and independent variables. The mechanical answer is that typing

```
. regress y x1 x2 [aweight=n]
```

is equivalent to fitting the model

$$
y_{j} \sqrt{n_{j}}=\beta_{0} \sqrt{n_{j}}+\beta_{1} x_{1 j} \sqrt{n_{j}}+\beta_{2} x_{2 j} \sqrt{n_{j}}+u_{j} \sqrt{n_{j}}
$$

This regression will reproduce the coefficients and covariance matrix produced by the aweighted regression. The mean squared errors (estimates of the variance of the residuals) will, however, be different. The transformed regression reports $s_{t}^{2}$, an estimate of $\operatorname{Var}\left(u_{j} \sqrt{n_{j}}\right)$. The aweighted regression reports $s_{a}^{2}$, an estimate of $\operatorname{Var}\left(u_{j} \sqrt{n_{j}} \sqrt{N / \sum_{k} n_{k}}\right)$, where $N$ is the number of observations. Thus

$$
\begin{equation*}
s_{a}^{2}=\frac{N}{\sum_{k} n_{k}} s_{t}^{2}=\frac{s_{t}^{2}}{\bar{n}} \tag{1}
\end{equation*}
$$

The logic for this adjustment is as follows: Consider the model

$$
y=\beta_{0}+\beta_{1} x_{1}+\beta_{2} x_{2}+u
$$

Assume that, were this model fit on individuals, $\operatorname{Var}(u)=\sigma_{u}^{2}$, a constant. Assume that individual data are not available; what is available are averages $\left(\bar{y}_{j}, \bar{x}_{1 j}, \bar{x}_{2 j}\right)$ for $j=1, \ldots, N$, and each average is calculated over $n_{j}$ observations. Then it is still true that

$$
\bar{y}_{j}=\beta_{0}+\beta_{1} \bar{x}_{1 j}+\beta_{2} \bar{x}_{2 j}+\bar{u}_{j}
$$

where $\bar{u}_{j}$ is the average of $n_{j}$ mean 0 , variance $\sigma_{u}^{2}$ deviates and has variance $\sigma_{\bar{u}}^{2}=\sigma_{u}^{2} / n_{j}$. Thus multiplying through by $\sqrt{n_{j}}$ produces

$$
\bar{y}_{j} \sqrt{n_{j}}=\beta_{0} \sqrt{n_{j}}+\beta_{1} \bar{x}_{1 j} \sqrt{n_{j}}+\beta_{2} \bar{x}_{2 j} \sqrt{n_{j}}+\bar{u}_{j} \sqrt{n_{j}}
$$

and $\operatorname{Var}\left(\bar{u}_{j} \sqrt{n_{j}}\right)=\sigma_{u}^{2}$. The mean squared error, $s_{t}^{2}$, reported by fitting this transformed regression is an estimate of $\sigma_{u}^{2}$. The coefficients and covariance matrix could also be obtained by aweighted regress. The only difference would be in the reported mean squared error, which from (1) is $\sigma_{u}^{2} / \bar{n}$. On average, each observation in the data reflects the averages calculated over $\bar{n}=\sum_{k} n_{k} / N$ individuals, and thus this reported mean squared error is the average variance of an observation in the dataset. We can retrieve the estimate of $\sigma_{u}^{2}$ by multiplying the reported mean squared error by $\bar{n}$.

More generally, aweights are used to solve general heteroskedasticity problems. In these cases, we have the model

$$
y_{j}=\beta_{0}+\beta_{1} x_{1 j}+\beta_{2} x_{2 j}+u_{j}
$$

and the variance of $u_{j}$ is thought to be proportional to $a_{j}$. If the variance is proportional to $a_{j}$, it is also proportional to $\alpha a_{j}$, where $\alpha$ is any positive constant. Not quite arbitrarily, but with no loss of generality, we could choose $\alpha=\sum_{k}\left(1 / a_{k}\right) / N$, the average value of the inverse of $a_{j}$. We can then write $\operatorname{Var}\left(u_{j}\right)=k \alpha a_{j} \sigma^{2}$, where $k$ is the constant of proportionality that is no longer a function of the scale of the weights.

Dividing this regression through by the $\sqrt{a_{j}}$,

$$
y_{j} / \sqrt{a_{j}}=\beta_{0} / \sqrt{a_{j}}+\beta_{1} x_{1 j} / \sqrt{a_{j}}+\beta_{2} x_{2 j} / \sqrt{a_{j}}+u_{j} / \sqrt{a_{j}}
$$

produces a model with $\operatorname{Var}\left(u_{j} / \sqrt{a_{j}}\right)=k \alpha \sigma^{2}$, which is the constant part of $\operatorname{Var}\left(u_{j}\right)$. This variance is a function of $\alpha$, the average of the reciprocal weights; if the weights are scaled arbitrarily, then so is this variance.

We can also fit this model by typing

```
. regress y x1 x2 [aweight=1/a]
```

This input will produce the same estimates of the coefficients and covariance matrix; the reported mean squared error is, from (1), $\left\{N / \sum_{k}\left(1 / a_{k}\right)\right\} k \alpha \sigma^{2}=k \sigma^{2}$. This variance is independent of the scale of $a_{j}$.

## Instrumental variables and two-stage least-squares regression

An alternate syntax for regress can be used to produce instrumental-variables (two-stage least squares) estimates.
regress depvar $\left[\right.$ varlist $_{1}\left[\left(\right.\right.$ varlist $\left.\left.\left._{2}\right)\right]\right][$ if $][\mathrm{in}][$ weight $][$, regress_options $]$
This syntax is used mainly by programmers developing estimators using the instrumental-variables estimates as intermediate results. ivregress is normally used to directly fit these models; see [ R$]$ ivregress.

With this syntax, regress fits a structural equation of depvar on varlist $t_{1}$ using instrumental variables regression; ( varlist $_{2}$ ) indicates the list of instrumental variables. With the exception of vce(hc2) and vce(hc3), all standard regress options are allowed.

## Video example

Simple linear regression in Stata

## Stored results

regress stores the following in e():
Scalars
e(N)
e(mss)
e(df_m)
e(rss)
e(df_r)
e(r2)
e(r2_a)
e(F)
e(rmse)
e(11)
e(11_0)
e(N_clust)
$e$ (rank)
Macros
e(cmd)
e(cmdline)
e(depvar)
e(model)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(vce)
e(vcetype)
e(properties)
e(estat_cmd)
$e$ (predict)
$e$ (marginsok)
e(asbalanced)
e(asobserved)
Matrices
e(b)
e(v)
e(V_modelbased)
Functions
$e$ (sample) marks estimation sample

## Methods and formulas

Methods and formulas are presented under the following headings:
Coefficient estimation and ANOVA table
A general notation for the robust variance calculation
Robust calculation for regress

## Coefficient estimation and ANOVA table

Variables printed in lowercase and not boldfaced (for example, $x$ ) are scalars. Variables printed in lowercase and boldfaced (for example, $\mathbf{x}$ ) are column vectors. Variables printed in uppercase and boldfaced (for example, $\mathbf{X}$ ) are matrices.

Let $\mathbf{v}$ be a column vector of weights specified by the user. If no weights are specified, $\mathbf{v}=\mathbf{1}$. Let $\mathbf{w}$ be a column vector of normalized weights. If no weights are specified or if the user specified fweights or iweights, $\mathbf{w}=\mathbf{v}$. Otherwise, $\mathbf{w}=\left\{\mathbf{v} /\left(\mathbf{1}^{\prime} \mathbf{v}\right)\right\}\left(\mathbf{1}^{\prime} \mathbf{1}\right)$.

The number of observations, $n$, is defined as $\mathbf{1}^{\prime} \mathbf{w}$. For iweights, this is truncated to an integer. The sum of the weights is $\mathbf{1}^{\prime} \mathbf{v}$. Define $c=1$ if there is a constant in the regression and zero otherwise. Define $k$ as the number of right-hand-side variables (including the constant).

Let $\mathbf{X}$ denote the matrix of observations on the right-hand-side variables, $\mathbf{y}$ the vector of observations on the left-hand-side variable, and $\mathbf{Z}$ the matrix of observations on the instruments. If the user specifies no instruments, then $\mathbf{Z}=\mathbf{X}$. In the following formulas, if the user specifies weights, then $\mathbf{X}^{\prime} \mathbf{X}$, $\mathbf{X}^{\prime} \mathbf{y}, \mathbf{y}^{\prime} \mathbf{y}, \mathbf{Z}^{\prime} \mathbf{Z}, \mathbf{Z}^{\prime} \mathbf{X}$, and $\mathbf{Z}^{\prime} \mathbf{y}$ are replaced by $\mathbf{X}^{\prime} \mathbf{D X}, \mathbf{X}^{\prime} \mathbf{D} \mathbf{y}, \mathbf{y}^{\prime} \mathbf{D} \mathbf{y}, \mathbf{Z}^{\prime} \mathbf{D Z}, \mathbf{Z}^{\prime} \mathbf{D X}$, and $\mathbf{Z}^{\prime} \mathbf{D} \mathbf{y}$, respectively, where $\mathbf{D}$ is a diagonal matrix whose diagonal elements are the elements of $\mathbf{w}$. We suppress the $\mathbf{D}$ below to simplify the notation.

If no instruments are specified, define $\mathbf{A}$ as $\mathbf{X}^{\prime} \mathbf{X}$ and $\mathbf{a}$ as $\mathbf{X}^{\prime} \mathbf{y}$. Otherwise, define $\mathbf{A}$ as $\mathbf{X}^{\prime} \mathbf{Z}\left(\mathbf{Z}^{\prime} \mathbf{Z}\right)^{-1}\left(\mathbf{X}^{\prime} \mathbf{Z}\right)^{\prime}$ and $\mathbf{a}$ as $\mathbf{X}^{\prime} \mathbf{Z}\left(\mathbf{Z}^{\prime} \mathbf{Z}\right)^{-1} \mathbf{Z}^{\prime} \mathbf{y}$.

The coefficient vector $\mathbf{b}$ is defined as $\mathbf{A}^{-1} \mathbf{a}$. Although not shown in the notation, unless hascons is specified, $\mathbf{A}$ and a are accumulated in deviation form and the constant is calculated separately. This comment applies to all statistics listed below.

The total sum of squares, TSS, equals $\mathbf{y}^{\prime} \mathbf{y}$ if there is no intercept and $\mathbf{y}^{\prime} \mathbf{y}-\left\{\left(\mathbf{1}^{\prime} \mathbf{y}\right)^{2} / n\right\}$ otherwise. The degrees of freedom is $n-c$.

The error sum of squares, ESS, is defined as $\mathbf{y}^{\prime} \mathbf{y}-2 \mathbf{b} \mathbf{X}^{\prime} \mathbf{y}+\mathbf{b}^{\prime} \mathbf{X}^{\prime} \mathbf{X b}$ if there are instruments and as $\mathbf{y}^{\prime} \mathbf{y}-\mathbf{b}^{\prime} \mathbf{X}^{\prime} \mathbf{y}$ otherwise. The degrees of freedom is $n-k$.

The model sum of squares, MSS, equals TSS - ESS. The degrees of freedom is $k-c$.
The mean squared error, $s^{2}$, is defined as ESS $/(n-k)$. The root mean squared error is $s$, its square root.

The $F$ statistic with $k-c$ and $n-k$ degrees of freedom is defined as

$$
F=\frac{\mathrm{MSS}}{(k-c) s^{2}}
$$

if no instruments are specified. If instruments are specified and $c=1$, then $F$ is defined as

$$
F=\frac{(\mathbf{b}-\mathbf{c})^{\prime} \mathbf{A}(\mathbf{b}-\mathbf{c})}{(k-1) s^{2}}
$$

where $\mathbf{c}$ is a vector of $k-1$ zeros and $k$ th element $\mathbf{1}^{\prime} \mathbf{y} / n$. Otherwise, $F$ is defined as "missing". (Here you may use the test command to construct any $F$ test that you wish.)

The R-squared, $R^{2}$, is defined as $R^{2}=1-\mathrm{ESS} / \mathrm{TSS}$.
The adjusted R-squared, $R_{a}^{2}$, is $1-\left(1-R^{2}\right)(n-c) /(n-k)$.
If vce (robust) is not specified, the conventional estimate of variance is $s^{2} \mathbf{A}^{-1}$. The handling of vce(robust) is described below.

## A general notation for the robust variance calculation

Put aside all context of linear regression and the notation that goes with it - we will return to it. First, we are going to establish a notation for describing robust variance calculations.

The calculation formula for the robust variance calculation is

$$
\widehat{\mathcal{V}}=q_{c} \widehat{\mathbf{V}}\left(\sum_{k=1}^{M} \mathbf{u}_{k}^{(G) \prime} \mathbf{u}_{k}^{(G)}\right) \widehat{\mathbf{V}}
$$

where

$$
\mathbf{u}_{k}^{(G)}=\sum_{j \in G_{k}} w_{j} \mathbf{u}_{j}
$$

$G_{1}, G_{2}, \ldots, G_{M}$ are the clusters specified by vce (cluster clustvar), and $w_{j}$ are the user-specified weights, normalized if aweights or pweights are specified and equal to 1 if no weights are specified.

For fweights without clusters, the variance formula is

$$
\widehat{\mathcal{V}}=q_{c} \widehat{\mathbf{V}}\left(\sum_{j=1}^{N} w_{j} \mathbf{u}_{j}^{\prime} \mathbf{u}_{j}\right) \widehat{\mathbf{V}}
$$

which is the same as expanding the dataset and making the calculation on the unweighted data.
If vce(cluster clustvar) is not specified, $M=N$, and each cluster contains 1 observation. The inputs into this calculation are

- $\widehat{\mathbf{V}}$, which is typically a conventionally calculated variance matrix;
- $\mathbf{u}_{j}, j=1, \ldots, N$, a row vector of scores; and
- $q_{\mathrm{c}}$, a constant finite-sample adjustment.

Thus we can now describe how estimators apply the robust calculation formula by defining $\widehat{\mathbf{V}}, \mathbf{u}_{j}$, and $q_{\mathrm{c}}$.

Two definitions are popular enough for $q_{\mathrm{c}}$ to deserve a name. The regression-like formula for $q_{\mathrm{c}}$ (Fuller et al. 1986) is

$$
q_{\mathrm{c}}=\frac{N-1}{N-k} \frac{M}{M-1}
$$

where $M$ is the number of clusters and $N$ is the number of observations. For weights, $N$ refers to the sum of the weights if weights are frequency weights and the number of observations in the dataset (ignoring weights) in all other cases. Also note that, weighted or not, $M=N$ when vce(cluster clustvar) is not specified, and then $q_{\mathrm{c}}=N /(N-k)$.

The asymptotic-like formula for $q_{c}$ is

$$
q_{\mathrm{c}}=\frac{M}{M-1}
$$

where $M=N$ if vce(cluster clustvar) is not specified.

See [U] 20.21 Obtaining robust variance estimates and [P] _robust for a discussion of the robust variance estimator and a development of these formulas.

## Robust calculation for regress

For regress, $\widehat{\mathbf{V}}=\mathbf{A}^{-1}$. The other terms are
No instruments, vce(robust), but not vce(hc2) or vce(hc3),

$$
\mathbf{u}_{j}=\left(y_{j}-\mathbf{x}_{j} \mathbf{b}\right) \mathbf{x}_{j}
$$

and $q_{\mathrm{c}}$ is given by its regression-like definition.
No instruments, vce(hc2),

$$
\mathbf{u}_{j}=\frac{1}{\sqrt{1-h_{j j}}}\left(y_{j}-\mathbf{x}_{j} \mathbf{b}\right) \mathbf{x}_{j}
$$

where $q_{\mathrm{c}}=1$ and $h_{j j}=\mathbf{x}_{j}\left(\mathbf{X}^{\prime} \mathbf{X}\right)^{-1} \mathbf{x}_{j}{ }^{\prime}$.
No instruments, vce(hc3),

$$
\mathbf{u}_{j}=\frac{1}{1-h_{j j}}\left(y_{j}-\mathbf{x}_{j} \mathbf{b}\right) \mathbf{x}_{j}
$$

where $q_{\mathrm{c}}=1$ and $h_{j j}=\mathbf{x}_{j}\left(\mathbf{X}^{\prime} \mathbf{X}\right)^{-1} \mathbf{x}_{j}{ }^{\prime}$.
Instrumental variables,

$$
\mathbf{u}_{j}=\left(y_{j}-\mathbf{x}_{j} \mathbf{b}\right) \widehat{\mathbf{x}}_{j}
$$

where $q_{\mathrm{c}}$ is given by its regression-like definition, and

$$
\widehat{\mathbf{x}}_{j}^{\prime}=\mathbf{P} \mathbf{z}_{j}{ }^{\prime}
$$

where $\mathbf{P}=\left(\mathbf{X}^{\prime} \mathbf{Z}\right)\left(\mathbf{Z}^{\prime} \mathbf{Z}\right)^{-1}$.

## Acknowledgments

The robust estimate of variance was first implemented in Stata by Mead Over of the Center for Global Development, Dean Jolliffe of the World Bank, and Andrew Foster of the Department of Economics at Brown University (Over, Jolliffe, and Foster 1996).

The history of regression is long and complicated: the books by Stigler (1986) and Hald (1998) are devoted largely to the story. Legendre published first on least squares in 1805. Gauss published later in 1809, but he had the idea earlier. Gauss, and especially Laplace, tied least squares to a normal errors assumption. The idea of the normal distribution can itself be traced back to De Moivre in 1733. Laplace discussed a variety of other estimation methods and error assumptions over his long career, while linear models long predate either innovation. Most of this work was linked to problems in astronomy and geodesy.

A second wave of ideas started when Galton used graphical and descriptive methods on data bearing on heredity to develop what he called regression. His term reflects the common phenomenon that characteristics of offspring are positively correlated with those of parents but with regression slope such that offspring "regress toward the mean". Galton's work was rather intuitive: contributions from Pearson, Edgeworth, Yule, and others introduced more formal machinery, developed related ideas on correlation, and extended application into the biological and social sciences. So most of the elements of regression as we know it were in place by 1900.

Pierre-Simon Laplace (1749-1827) was born in Normandy and was early recognized as a remarkable mathematician. He weathered a changing political climate well enough to rise to Minister of the Interior under Napoleon in 1799 (although only for 6 weeks) and to be made a Marquis by Louis XVIII in 1817. He made many contributions to mathematics and physics, his two main interests being theoretical astronomy and probability theory (including statistics). Laplace transforms are named for him.

Adrien-Marie Legendre (1752-1833) was born in Paris (or possibly in Toulouse) and educated in mathematics and physics. He worked in number theory, geometry, differential equations, calculus, function theory, applied mathematics, and geodesy. The Legendre polynomials are named for him. His main contribution to statistics is as one of the discoverers of least squares. He died in poverty, having refused to bow to political pressures.

Johann Carl Friedrich Gauss (1777-1855) was born in Braunschweig (Brunswick), now in Germany. He studied there and at Göttingen. His doctoral dissertation at the University of Helmstedt was a discussion of the fundamental theorem of algebra. He made many fundamental contributions to geometry, number theory, algebra, real analysis, differential equations, numerical analysis, statistics, astronomy, optics, geodesy, mechanics, and magnetism. An outstanding genius, Gauss worked mostly in isolation in Göttingen.

Francis Galton (1822-1911) was born in Birmingham, England, into a well-to-do family with many connections: he and Charles Darwin were first cousins. After an unsuccessful foray into medicine, he became independently wealthy at the death of his father. Galton traveled widely in Europe, the Middle East, and Africa, and became celebrated as an explorer and geographer. His pioneering work on weather maps helped in the identification of anticyclones, which he named. From about 1865, most of his work was centered on quantitative problems in biology, anthropology, and psychology. In a sense, Galton (re)invented regression, and he certainly named it. Galton also promoted the normal distribution, correlation approaches, and the use of median and selected quantiles as descriptive statistics. He was knighted in 1909.

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## Also see

[R] regress postestimation - Postestimation tools for regress
$[R]$ regress postestimation diagnostic plots - Postestimation plots for regress
$[\mathrm{R}]$ regress postestimation time series - Postestimation tools for regress with time series
[R] anova - Analysis of variance and covariance
[R] contrast - Contrasts and linear hypothesis tests after estimation
[MI] estimation - Estimation commands for use with mi estimate
[SEM] example 6 - Linear regression
[SEM] intro 5 - Tour of models
[SVY] svy estimation - Estimation commands for survey data
[TS] forecast — Econometric model forecasting
[U] 20 Estimation and postestimation commands

## Title

regress postestimation - Postestimation tools for regress

Description
Tests for violation of assumptions
Methods and formulas
Also see

Predictions
Variance inflation factors
Acknowledgments

DFBETA influence statistics
Measures of effect size References

## Description

The following postestimation commands are of special interest after regress:

| Command | Description |
| :--- | :--- |
| dfbeta | DFBETA influence statistics |
| estat hettest | tests for heteroskedasticity |
| estat imtest | information matrix test |
| estat ovtest | Ramsey regression specification-error test for omitted variables |
| estat szroeter | Szroeter's rank test for heteroskedasticity |
| estat vif | variance inflation factors for the independent variables |
| estat esize | $\eta^{2}$ and $\omega^{2}$ effect sizes |

These commands are not appropriate after the svy prefix.

The following standard postestimation commands are also available:

| Command | Description |
| :---: | :---: |
| contrast | contrasts and ANOVA-style joint tests of estimates |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample |
| estat vce | variance-covariance matrix of the estimators (VCE) |
| estat (svy) | postestimation statistics for survey data |
| estimates | cataloging estimation results |
| forecast ${ }^{1}$ | dynamic forecasts and simulations |
| hausman | Hausman's specification test |
| lincom | point estimates, standard errors, testing, and inference for linear combinations of coefficients |
| linktest | link test for model specification |
| lrtest ${ }^{2}$ | likelihood-ratio test |
| margins | marginal means, predictive margins, marginal effects, and average marginal effects |
| marginsplot | graph the results from margins (profile plots, interaction plots, etc.) |
| nlcom | point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients |
| predict | predictions, residuals, influence statistics, and other diagnostic measures |
| predictnl | point estimates, standard errors, testing, and inference for generalized predictions |
| pwcompare | pairwise comparisons of estimates |
| suest | seemingly unrelated estimation |
| test | Wald tests of simple and composite linear hypotheses |
| testnl | Wald tests of nonlinear hypotheses |

[^67]
## Predictions

## Syntax for predict

$$
\text { predict }[\text { type }] \text { newvar }[\text { if }][\text { in }][\text {, statistic }]
$$

statistic

## Main

xb
residuals
score
rstandard
rstudent
cooksd
leverage | hat
$\underline{\operatorname{pr}}(a, b)$
$\mathrm{e}(a, b)$
ystar ( $a, b$ )

* dfbeta (varname)
stdp
stdf
stdr
* covratio
* dfits
* ${ }^{\text {wel }}$ sch
linear prediction; the default residuals
score; equivalent to residuals
standardized residuals
Studentized (jackknifed) residuals
Cook's distance
leverage (diagonal elements of hat matrix)
$\operatorname{Pr}\left(y_{j} \mid a<y_{j}<b\right)$
$E\left(y_{j} \mid a<y_{j}<b\right)$
$E\left(y_{j}^{*}\right), y_{j}^{*}=\max \left\{a, \min \left(y_{j}, b\right)\right\}$
DFBETA for varname
standard error of the linear prediction
standard error of the forecast
standard error of the residual
COVRATIO
DFITS
Welsch distance

Unstarred statistics are available both in and out of sample; type predict . . . if e (sample) ... if wanted only for the estimation sample. Starred statistics are calculated only for the estimation sample, even when if e(sample) is not specified.
rstandard, rstudent, cooksd, leverage, dfbeta(), stdf, stdr, covratio, dfits, and welsch are not available if any vce() other than vce(ols) was specified with regress.
xb , residuals, score, and stdp are the only options allowed with svy estimation results.
where $a$ and $b$ may be numbers or variables; $a$ missing ( $a \geq$.) means $-\infty$, and $b$ missing ( $b \geq$.) means $+\infty$; see [U] 12.2.1 Missing values.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

Main
xb , the default, calculates the linear prediction.
residuals calculates the residuals.
score is equivalent to residuals in linear regression.
rstandard calculates the standardized residuals. rstudent calculates the Studentized (jackknifed) residuals. cooksd calculates the Cook's $D$ influence statistic (Cook 1977).
leverage or hat calculates the diagonal elements of the projection ("hat") matrix.
$\operatorname{pr}(a, b)$ calculates $\operatorname{Pr}\left(a<\mathbf{x}_{j} \mathbf{b}+u_{j}<b\right)$, the probability that $y_{j} \mid \mathbf{x}_{j}$ would be observed in the interval $(a, b)$.
$a$ and $b$ may be specified as numbers or variable names; $l b$ and $u b$ are variable names;
$\operatorname{pr}(20,30)$ calculates $\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$;
$\operatorname{pr}(l b, u b)$ calculates $\operatorname{Pr}\left(l b<\mathbf{x}_{j} \mathbf{b}+u_{j}<u b\right)$; and
$\operatorname{pr}(20, u b)$ calculates $\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{j}<u b\right)$.
$a$ missing $\left(a \geq\right.$.) means $-\infty ; \operatorname{pr}(., 30)$ calculates $\operatorname{Pr}\left(-\infty<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$;
$\operatorname{pr}(l b, 30)$ calculates $\operatorname{Pr}\left(-\infty<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$ in observations for which $l b \geq$.
and calculates $\operatorname{Pr}\left(l b<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)$ elsewhere.
$b$ missing $(b \geq$.$) means +\infty$; $\operatorname{pr}(20,$.$) calculates \operatorname{Pr}\left(+\infty>\mathbf{x}_{j} \mathbf{b}+u_{j}>20\right)$;
$\operatorname{pr}(20, u b)$ calculates $\operatorname{Pr}\left(+\infty>\mathbf{x}_{j} \mathbf{b}+u_{j}>20\right)$ in observations for which $u b \geq$.
and calculates $\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{j}<u b\right)$ elsewhere.
$\mathrm{e}(a, b)$ calculates $E\left(\mathbf{x}_{j} \mathbf{b}+u_{j} \mid a<\mathbf{x}_{j} \mathbf{b}+u_{j}<b\right)$, the expected value of $y_{j} \mid \mathbf{x}_{j}$ conditional on $y_{j} \mid \mathbf{x}_{j}$ being in the interval $(a, b)$, meaning that $y_{j} \mid \mathbf{x}_{j}$ is truncated. $a$ and $b$ are specified as they are for pr() .
ystar $(a, b)$ calculates $E\left(y_{j}^{*}\right)$, where $y_{j}^{*}=a$ if $\mathbf{x}_{j} \mathbf{b}+u_{j} \leq a, y_{j}^{*}=b$ if $\mathbf{x}_{j} \mathbf{b}+u_{j} \geq b$, and $y_{j}^{*}=\mathbf{x}_{j} \mathbf{b}+u_{j}$ otherwise, meaning that $y_{j}^{*}$ is censored. $a$ and $b$ are specified as they are for pr() .
dfbeta (varname) calculates the DFBETA for varname, the difference between the regression coefficient when the $j$ th observation is included and excluded, said difference being scaled by the estimated standard error of the coefficient. varname must have been included among the regressors in the previously fitted model. The calculation is automatically restricted to the estimation subsample.
stdp calculates the standard error of the prediction, which can be thought of as the standard error of the predicted expected value or mean for the observation's covariate pattern. The standard error of the prediction is also referred to as the standard error of the fitted value.
stdf calculates the standard error of the forecast, which is the standard error of the point prediction for 1 observation. It is commonly referred to as the standard error of the future or forecast value. By construction, the standard errors produced by stdf are always larger than those produced by stdp; see Methods and formulas.
stdr calculates the standard error of the residuals.
covratio calculates COVRATIO (Belsley, Kuh, and Welsch 1980), a measure of the influence of the $j$ th observation based on considering the effect on the variance-covariance matrix of the estimates.
The calculation is automatically restricted to the estimation subsample.
dfits calculates DFITS (Welsch and Kuh 1977) and attempts to summarize the information in the leverage versus residual-squared plot into one statistic. The calculation is automatically restricted to the estimation subsample.
welsch calculates Welsch distance (Welsch 1982) and is a variation on dfits. The calculation is automatically restricted to the estimation subsample.

## Remarks and examples for predict

Remarks are presented under the following headings:

Terminology<br>Fitted values and residuals<br>Prediction standard errors<br>Prediction with weighted data<br>Leverage statistics<br>Standardized and Studentized residuals DFITS, Cook's Distance, and Welsch Distance COVRATIO

## Terminology

Many of these commands concern identifying influential data in linear regression. This is, unfortunately, a field that is dominated by jargon, codified and partially begun by Belsley, Kuh, and Welsch (1980). In the words of Chatterjee and Hadi (1986, 416), "Belsley, Kuh, and Welsch's book, Regression Diagnostics, was a very valuable contribution to the statistical literature, but it unleashed on an unsuspecting statistical community a computer speak (à la Orwell), the likes of which we have never seen." Things have only gotten worse since then. Chatterjee and Hadi's $(1986$, 1988) own attempts to clean up the jargon did not improve matters (see Hoaglin and Kempthorne [1986], Velleman [1986], and Welsch [1986]). We apologize for the jargon, and for our contribution to the jargon in the form of inelegant command names, we apologize most of all.

Model sensitivity refers to how estimates are affected by subsets of our data. Imagine data on $y$ and $x$, and assume that the data are to be fit by the regression $y_{i}=\alpha+\beta x_{i}+\epsilon_{i}$. The regression estimates of $\alpha$ and $\beta$ are $a$ and $b$, respectively. Now imagine that the estimated $a$ and $b$ would be different if a small portion of the dataset, perhaps even one observation, were deleted. As a data analyst, you would like to think that you are summarizing tendencies that apply to all the data, but you have just been told that the model you fit is unduly influenced by one point or just a few points and that, as a matter of fact, there is another model that applies to the rest of the data-a model that you have ignored. The search for subsets of the data that, if deleted, would change the results markedly is a predominant theme of this entry.

There are three key issues in identifying model sensitivity to individual observations, which go by the names residuals, leverage, and influence. In our $y_{i}=a+b x_{i}+e_{i}$ regression, the residuals are, of course, $e_{i}$-they reveal how much our fitted value $\widehat{y}_{i}=a+b x_{i}$ differs from the observed $y_{i}$. A point $\left(x_{i}, y_{i}\right)$ with a corresponding large residual is called an outlier. Say that you are interested in outliers because you somehow think that such points will exert undue influence on your estimates. Your feelings are generally right, but there are exceptions. A point might have a huge residual and yet not affect the estimated $b$ at all. Nevertheless, studying observations with large residuals almost always pays off.
$\left(x_{i}, y_{i}\right)$ can be an outlier in another way-just as $y_{i}$ can be far from $\widehat{y}_{i}, x_{i}$ can be far from the center of mass of the other $x$ 's. Such an "outlier" should interest you just as much as the more traditional outliers. Picture a scatterplot of $y$ against $x$ with thousands of points in some sort of mass at the lower left of the graph and one point at the upper right of the graph. Now run a regression line through the points-the regression line will come close to the point at the upper right of the graph and may in fact, go through it. That is, this isolated point will not appear as an outlier as measured by residuals because its residual will be small. Yet this point might have a dramatic effect on our resulting estimates in the sense that, were you to delete the point, the estimates would change markedly. Such a point is said to have high leverage. Just as with traditional outliers, a high leverage point does not necessarily have an undue effect on regression estimates, but if it does not, it is more the exception than the rule.

Now all this is a most unsatisfactory state of affairs. Points with large residuals may, but need not, have a large effect on our results, and points with small residuals may still have a large effect. Points with high leverage may, but need not, have a large effect on our results, and points with low leverage may still have a large effect. Can you not identify the influential points and simply have the computer list them for you? You can, but you will have to define what you mean by "influential".
"Influential" is defined with respect to some statistic. For instance, you might ask which points in your data have a large effect on your estimated $a$, which points have a large effect on your estimated $b$, which points have a large effect on your estimated standard error of $b$, and so on, but do not be surprised when the answers to these questions are different. In any case, obtaining such measures is not difficult-all you have to do is fit the regression excluding each observation one at a time and record the statistic of interest which, in the day of the modern computer, is not too onerous. Moreover, you can save considerable computer time by doing algebra ahead of time and working out formulas that will calculate the same answers as if you ran each of the regressions. (Ignore the question of pairs of observations that, together, exert undue influence, and triples, and so on, which remains largely unsolved and for which the brute force fit-every-possible-regression procedure is not a viable alternative.)

## Fitted values and residuals

Typing predict newvar with no options creates newvar containing the fitted values. Typing predict newvar, resid creates newvar containing the residuals.

## > Example 1

Continuing with example 1 from $[\mathrm{R}]$ regress, we wish to fit the following model:

$$
\mathrm{mpg}=\beta_{0}+\beta_{1} \text { weight }+\beta_{2} \text { foreign }+\epsilon
$$

. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. regress mpg weight foreign

| Source | SS | df | MS |
| ---: | ---: | ---: | ---: |
| Model | 1619.2877 | 2 | 809.643849 |
| Residual | 824.171761 | 71 | 11.608053 |
| Total | 2443.45946 | 73 | 33.4720474 |


| Number of obs | $=$ | 74 |
| :--- | ---: | ---: |
| $\mathrm{~F}(2$, | $71)$ | $=69.75$ |
| Prob $>$ F | $=$ | 0.0000 |
| R-squared | $=$ | 0.6627 |
| Adj R-squared | $=$ | 0.6532 |
| Root MSE | $=3.4071$ |  |


| mpg | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| weight | -.0065879 | .0006371 | -10.34 | 0.000 | -.0078583 | -.0053175 |
| foreign | -1.650029 | 1.075994 | -1.53 | 0.130 | -3.7955 | .4954422 |
| _cons | 41.6797 | 2.165547 | 19.25 | 0.000 | 37.36172 | 45.99768 |

That done, we can now obtain the predicted values from the regression. We will store them in a new variable called pmpg by typing predict pmpg. Because predict produces no output, we will follow that by summarizing our predicted and observed values.
. predict pmpg
(option xb assumed; fitted values)
. summarize pmpg mpg

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| pmpg | 74 | 21.2973 | 4.709779 | 9.794333 | 29.82151 |
| mpg | 74 | 21.2973 | 5.785503 | 12 | 41 |

## > Example 2: Out-of-sample predictions

We can just as easily obtain predicted values from the model by using a wholly different dataset from the one on which the model was fit. The only requirement is that the data have the necessary variables, which here are weight and foreign.

Using the data on two new cars (the Pontiac Sunbird and the Volvo 260) from the newautos.dta dataset, we can obtain out-of-sample predictions (or forecasts) by typing

```
. use http://www.stata-press.com/data/r13/newautos, clear
(New Automobile Models)
. predict pmpg
(option xb assumed; fitted values)
. list, divider
```

| make | weight | foreign | pmpg |
| ---: | ---: | ---: | ---: |
| Pont. Sunbird | 2690 | Domestic | 23.95829 |
| Volvo 260 | 3170 | Foreign | 19.14607 |

The Pontiac Sunbird has a predicted mileage rating of 23.96 mpg , whereas the Volvo 260 has a predicted rating of 19.15 mpg . In comparison, the actual mileage ratings are 24 for the Pontiac and 17 for the Volvo.

## Prediction standard errors

predict can calculate the standard error of the forecast (stdf option), the standard error of the prediction (stdp option), and the standard error of the residual (stdr option). It is easy to confuse stdf and stdp because both are often called the prediction error. Consider the prediction $\widehat{y}_{j}=\mathbf{x}_{j} \mathbf{b}$, where $\mathbf{b}$ is the estimated coefficient (column) vector and $\mathbf{x}_{j}$ is a (row) vector of independent variables for which you want the prediction. First, $\widehat{y}_{j}$ has a variance due to the variance of the estimated coefficient vector $\mathbf{b}$,

$$
\operatorname{Var}\left(\widehat{y}_{j}\right)=\operatorname{Var}\left(\mathbf{x}_{j} \mathbf{b}\right)=s^{2} h_{j}
$$

where $h_{j}=\mathbf{x}_{j}\left(\mathbf{X}^{\prime} \mathbf{X}\right)^{-1} \mathbf{x}_{j}^{\prime}$ and $s^{2}$ is the mean squared error of the regression. Do not panic over the algebra-just remember that $\operatorname{Var}\left(\widehat{y}_{j}\right)=s^{2} h_{j}$, whatever $s^{2}$ and $h_{j}$ are. stdp calculates this quantity. This is the error in the prediction due to the uncertainty about $\mathbf{b}$.

If you are about to hand this number out as your forecast, however, there is another error. According to your model, the true value of $y_{j}$ is given by

$$
y_{j}=\mathbf{x}_{j} \mathbf{b}+\epsilon_{j}=\widehat{y}_{j}+\epsilon_{j}
$$

and thus the $\operatorname{Var}\left(y_{j}\right)=\operatorname{Var}\left(\widehat{y}_{j}\right)+\operatorname{Var}\left(\epsilon_{j}\right)=s^{2} h_{j}+s^{2}$, which is the square of stdf. stdf, then, is the sum of the error in the prediction plus the residual error.
stdr has to do with an analysis-of-variance decomposition of $s^{2}$, the estimated variance of $y$. The standard error of the prediction is $s^{2} h_{j}$, and therefore $s^{2} h_{j}+s^{2}\left(1-h_{j}\right)=s^{2}$ decomposes $s^{2}$ into the prediction and residual variances.
> Example 3: standard error of the forecast
Returning to our model of mpg on weight and foreign, we previously predicted the mileage rating for the Pontiac Sunbird and Volvo 260 as 23.96 and 19.15 mpg , respectively. We now want to put a standard error around our forecast. Remember, the data for these two cars were in newautos.dta:

```
. use http://www.stata-press.com/data/r13/newautos, clear
(New Automobile Models)
. predict pmpg
(option xb assumed; fitted values)
. predict se_pmpg, stdf
. list, divider
```

| make | weight | foreign | pmpg | se_pmpg |
| ---: | ---: | ---: | ---: | ---: |
| Pont. Sunbird | 2690 | Domestic | 23.95829 | 3.462791 |
| Volvo 260 | 3170 | Foreign | 19.14607 | 3.525875 |

Thus an approximate $95 \%$ confidence interval for the mileage rating of the Volvo 260 is $19.15 \pm 2 \cdot 3.53=$ [12.09, 26.21].

## Prediction with weighted data

predict can be used after frequency-weighted (fweight) estimation, just as it is used after unweighted estimation. The technical note below concerns the use of predict after analytically weighted (aweight) estimation.

- Technical note

After analytically weighted estimation, predict is willing to calculate only the prediction (no options), residual (residual option), standard error of the prediction (stdp option), and diagonal elements of the projection matrix (hat option). Moreover, the results produced by hat need to be adjusted, as will be described. For analytically weighted estimation, the standard error of the forecast and residuals, the standardized and Studentized residuals, and Cook's $D$ are not statistically well-defined concepts.

## Leverage statistics

In addition to providing fitted values and the associated standard errors, the predict command can also be used to generate various statistics used to detect the influence of individual observations. This section provides a brief introduction to leverage (hat) statistics, and some of the following subsections discuss other influence statistics produced by predict.

## > Example 4: diagonal elements of projection matrix

The diagonal elements of the projection matrix, obtained by the hat option, are a measure of distance in explanatory variable space. leverage is a synonym for hat.

```
. use http://www.stata-press.com/data/r13/auto, clear
(1978 Automobile Data)
. regress mpg weight foreign
    (output omitted)
    predict xdist, hat
. summarize xdist, detail
```

                                    Leverage
    |  | Percentiles | Smallest |  |  |
| ---: | :---: | :---: | :--- | ---: |
| $1 \%$ | .0192325 | .0192325 |  | 74 |
| $5 \%$ | .0192686 | .0192366 |  | 74 |
| $10 \%$ | .0193448 | .019241 | Obs |  |
| $25 \%$ | .0220291 | .0192686 | Sum of Wgt. | .0405405 |
| $50 \%$ | .0383797 |  | Mean | .0207624 |
|  |  | Largest | Std. Dev. |  |
| $75 \%$ | .0494002 | .0880814 |  | .0004311 |
| $90 \%$ | .0693432 | .099715 | Variance | 1.159745 |
| $95 \%$ | .0880814 | .099715 | Skewness | 4.083313 |

Some $5 \%$ of our sample has an xdist measure in excess of 0.08 . Let's force them to reveal their identities:
. list foreign make mpg if xdist>.08, divider

|  | foreign | make | mpg |
| :--- | :---: | :--- | :---: |
| 24. | Domestic | Ford Fiesta | 28 |
| 26. | Domestic | Linc. Continental | 12 |
| 27. | Domestic | Linc. Mark V | 12 |
| 43. | Domestic | Plym. Champ | 34 |
| 64. | Foreign | Peugeot 604 | 14 |

To understand why these cars are on this list, we must remember that the explanatory variables in our model are weight and foreign and that xdist measures distance in this metric. The Ford Fiesta and the Plymouth Champ are the two lightest domestic cars in our data. The Lincolns are the two heaviest domestic cars, and the Peugeot is the heaviest foreign car.

See lvr2plot in [R] regress postestimation diagnostic plots for information on a leverage-versus-squared-residual plot.

## Standardized and Studentized residuals

The terms standardized and Studentized residuals have meant different things to different authors. In Stata, predict defines the standardized residual as $\widehat{e}_{i}=e_{i} /\left(s \sqrt{1-h_{i}}\right)$ and the Studentized residual as $r_{i}=e_{i} /\left(s_{(i)} \sqrt{1-h_{i}}\right)$, where $s_{(i)}$ is the root mean squared error of a regression with the $i$ th observation removed. Stata's definition of the Studentized residual is the same as the one given in Bollen and Jackman $(1990,264)$ and is what Chatterjee and Hadi $(1988,74)$ call the "externally Studentized" residual. Stata's "standardized" residual is the same as what Chatterjee and Hadi (1988, 74) call the "internally Studentized" residual.

Standardized and Studentized residuals are attempts to adjust residuals for their standard errors. Although the $\epsilon_{i}$ theoretical residuals are homoskedastic by assumption (that is, they all have the same variance), the calculated $e_{i}$ are not. In fact,

$$
\operatorname{Var}\left(e_{i}\right)=\sigma^{2}\left(1-h_{i}\right)
$$

where $h_{i}$ are the leverage measures obtained from the diagonal elements of hat matrix. Thus observations with the greatest leverage have corresponding residuals with the smallest variance.

Standardized residuals use the root mean squared error of the regression for $\sigma$. Studentized residuals use the root mean squared error of a regression omitting the observation in question for $\sigma$. In general, Studentized residuals are preferable to standardized residuals for purposes of outlier identification. Studentized residuals can be interpreted as the $t$ statistic for testing the significance of a dummy variable equal to 1 in the observation in question and 0 elsewhere (Belsley, Kuh, and Welsch 1980). Such a dummy variable would effectively absorb the observation and so remove its influence in determining the other coefficients in the model. Caution must be exercised here, however, because of the simultaneous testing problem. You cannot simply list the residuals that would be individually significant at the $5 \%$ level-their joint significance would be far less (their joint significance level would be far greater).

## > Example 5: standardized and Studentized residuals

In the Terminology section of Remarks and examples for predict, we distinguished residuals from leverage and speculated on the impact of an observation with a small residual but large leverage. If we adjust the residuals for their standard errors, however, the adjusted residual would be (relatively) larger and perhaps large enough so that we could simply examine the adjusted residuals. Taking our price on weight and foreign\#\#c.mpg model from example 1 of [R] regress postestimation diagnostic plots, we can obtain the in-sample standardized and Studentized residuals by typing

```
. use http://www.stata-press.com/data/r13/auto, clear
(1978 Automobile Data)
. regress price weight foreign##c.mpg
(output omitted)
. predict esta if e(sample), rstandard
. predict estu if e(sample), rstudent
```

In the lvr2plot section of $[\mathrm{R}]$ regress postestimation diagnostic plots, we discovered that the VW Diesel has the highest leverage in our data, but a corresponding small residual. The standardized and Studentized residuals for the VW Diesel are
. list make price esta estu if make=="VW Diesel"
71.

| make | price | esta | estu |
| :--- | ---: | ---: | ---: |
| VW Diesel | 5,397 | .6142691 | .6114758 |

The Studentized residual of 0.611 can be interpreted as the $t$ statistic for including a dummy variable for VW Diesel in our regression. Such a variable would not be significant.

## DFITS, Cook's Distance, and Welsch Distance

DFITS (Welsch and Kuh 1977), Cook's Distance (Cook 1977), and Welsch Distance (Welsch 1982) are three attempts to summarize the information in the leverage versus residual-squared plot into one statistic. That is, the goal is to create an index that is affected by the size of the residuals - outliers - and the size of $h_{i}$-leverage. Viewed mechanically, one way to write DFITS (Bollen and Jackman 1990, 265) is

$$
\operatorname{DFITS}_{i}=r_{i} \sqrt{\frac{h_{i}}{1-h_{i}}}
$$

where $r_{i}$ are the Studentized residuals. Thus large residuals increase the value of DFITS, as do large values of $h_{i}$. Viewed more traditionally, DFITS is a scaled difference between predicted values for the $i$ th case when the regression is fit with and without the $i$ th observation, hence the name.

The mechanical relationship between DFITS and Cook's Distance, $D_{i}$ (Bollen and Jackman 1990, 266), is

$$
D_{i}=\frac{1}{k} \frac{s_{(i)}^{2}}{s^{2}} \operatorname{DFITS}_{i}^{2}
$$

where $k$ is the number of variables (including the constant) in the regression, $s$ is the root mean squared error of the regression, and $s_{(i)}$ is the root mean squared error when the $i$ th observation is omitted. Viewed more traditionally, $D_{i}$ is a scaled measure of the distance between the coefficient vectors when the $i$ th observation is omitted.

The mechanical relationship between DFITS and Welsch's Distance, $W_{i}$ (Chatterjee and Hadi 1988, 123), is

$$
W_{i}=\operatorname{DFITS}_{i} \sqrt{\frac{n-1}{1-h_{i}}}
$$

The interpretation of $W_{i}$ is more difficult, as it is based on the empirical influence curve. Although DFITS and Cook's distance are similar, the Welsch distance measure includes another normalization by leverage.

Belsley, Kuh, and Welsch $(1980,28)$ suggest that DFITS values greater than $2 \sqrt{k / n}$ deserve more investigation, and so values of Cook's distance greater than $4 / n$ should also be examined (Bollen and Jackman 1990, 265-266). Through similar logic, the cutoff for Welsch distance is approximately $3 \sqrt{k}$ (Chatterjee and Hadi 1988, 124).

## > Example 6: DFITS influence measure

Continuing with our model of price on weight and foreign\#\#c.mpg, we can obtain the DFITS influence measure:

```
. predict e if e(sample), resid
. predict dfits, dfits
```

We did not specify if e (sample) in computing the DFITS statistic. DFITS is available only over the estimation sample, so specifying if e(sample) would have been redundant. It would have done no harm, but it would not have changed the results.

Our model has $k=5$ independent variables ( $k$ includes the constant) and $n=74$ observations; following the $2 \sqrt{k / n}$ cutoff advice, we type

```
. list make price e dfits if abs(dfits) > 2*sqrt(5/74), divider
```

|  | make | price | e | dfits |
| :--- | :--- | ---: | ---: | ---: |
| 12. | Cad. Eldorado | 14,500 | 7271.96 | .9564455 |
| 13. | Cad. Seville | 15,906 | 5036.348 | 1.356619 |
| 24. | Ford Fiesta | 4,389 | 3164.872 | .5724172 |
| 27. | Linc. Mark V | 13,594 | 3109.193 | .5200413 |
| 28. | Linc. Versailles | 13,466 | 6560.912 | .8760136 |
| 42. | Plym. Arrow | 4,647 | -3312.968 | -.9384231 |

We calculate Cook's distance and list the observations greater than the suggested $4 / n$ cutoff:

```
. predict cooksd if e(sample), cooksd
. list make price e cooksd if cooksd > 4/74, divider
```

12. 

| make | price | e | cooksd |
| :--- | ---: | ---: | ---: |
| Cad. Eldorado | 14,500 | 7271.96 | .1492676 |
| Cad. Seville | 15,906 | 5036.348 | .3328515 |
| Ford Fiesta | 4,389 | 3164.872 | .0638815 |
| Linc. Versailles | 13,466 | 6560.912 | .1308004 |
| Plym. Arrow | 4,647 | -3312.968 | .1700736 |

Here we used if e(sample) because Cook's distance is not restricted to the estimation sample by default. It is worth comparing this list with the preceding one.

Finally, we use Welsch distance and the suggested $3 \sqrt{k}$ cutoff:

```
. predict wd, welsch
. list make price e wd if abs(wd) > 3*sqrt(5), divider
```

12. 
13. 
14. 
15. 

| make | price | e | wd |
| :--- | ---: | ---: | ---: |
| Cad. Eldorado | 14,500 | 7271.96 | 8.394372 |
| Cad. Seville | 15,906 | 5036.348 | 12.81125 |
| Linc. Versailles | 13,466 | 6560.912 | 7.703005 |
| Plym. Arrow | 4,647 | -3312.968 | -8.981481 |

Here we did not need to specify if e(sample) because welsch automatically restricts the prediction to the estimation sample.

## COVRATIO

covratio (Belsley, Kuh, and Welsch 1980) measures the influence of the $i$ th observation by considering the effect on the variance-covariance matrix of the estimates. The measure is the ratio of the determinants of the covariances matrix, with and without the $i$ th observation. The resulting formula is

$$
\operatorname{COVRATIO}_{i}=\frac{1}{1-h_{i}}\left(\frac{n-k-\widehat{e}_{i}^{2}}{n-k-1}\right)^{k}
$$

where $\widehat{e}_{i}$ is the standardized residual.
For noninfluential observations, the value of COVRATIO is approximately 1. Large values of the residuals or large values of leverage will cause deviations from 1, although if both are large, COVRATIO may tend back toward 1 and therefore not identify such observations (Chatterjee and Hadi 1988, 139).

Belsley, Kuh, and Welsch (1980) suggest that observations for which

$$
\left|\operatorname{COVRATIO}_{i}-1\right| \geq \frac{3 k}{n}
$$

are worthy of further examination.

## > Example 7: COVRATIO influence measure

Using our model of price on weight and foreign\#\#c.mpg, we can obtain the COVRATIO measure and list the observations outside the suggested cutoff by typing

```
. predict covr, covratio
. list make price e covr if abs(covr-1) >= 3*5/74, divider
```

| make | price | e | covr |
| :--- | ---: | ---: | ---: |
| Cad. Eldorado | 14,500 | 7271.96 | .3814242 |
| Cad. Seville | 15,906 | 5036.348 | .7386969 |
| Linc. Versailles | 13,466 | 6560.912 | .4761695 |
| Plym. Champ | 4,425 | 1621.747 | 1.27782 |
| Audi 5000 | 9,690 | 591.2883 | 1.206842 |
| Datsun 210 | 4,589 | 19.81829 | 1.284801 |
| Peugeot 604 | 12,990 | 1037.184 | 1.348219 |
| Subaru | 3,798 | -909.5894 | 1.264677 |
| VW Diesel | 5,397 | 999.7209 | 1.630653 |
| Volvo 260 | 11,995 | 1327.668 | 1.211888 |

The covratio option automatically restricts the prediction to the estimation sample.

## DFBETA influence statistics

## Syntax for dfbeta

dfbeta [indepvar [indepvar [...]]] [, stub(name)]

## Menu for dfbeta

Statistics $>$ Linear models and related $>$ Regression diagnostics $>$ DFBETAs

## Description for dfbeta

dfbeta will calculate one, more than one, or all the DFBETAs after regress. Although predict will also calculate DFBETAs, predict can do this for only one variable at a time. dfbeta is a convenience tool for those who want to calculate DFBETAs for multiple variables. The names for the new variables created are chosen automatically and begin with the letters _dfbeta_.

## Option for dfbeta

stub (name) specifies the leading characters dfbeta uses to name the new variables to be generated. The default is stub(_dfbeta_).

## Remarks and examples for dfbeta

DFBETAs are perhaps the most direct influence measure of interest to model builders. DFBETAs focus on one coefficient and measure the difference between the regression coefficient when the $i$ th observation is included and excluded, the difference being scaled by the estimated standard error of the coefficient. Belsley, Kuh, and Welsch $(1980,28)$ suggest observations with $\mid$ DFBETA $_{i} \mid>2 / \sqrt{n}$ as deserving special attention, but it is also common practice to use 1 (Bollen and Jackman 1990, 267), meaning that the observation shifted the estimate at least one standard error.
> Example 8: DFBETAs influence measure; the dfbeta() option
Using our model of price on weight and foreign\#\#c.mpg, let's first ask which observations have the greatest impact on the determination of the coefficient on 1.foreign. We will use the suggested $2 / \sqrt{n}$ cutoff:

```
. use http://www.stata-press.com/data/r13/auto, clear
(1978 Automobile Data)
. regress price weight foreign##c.mpg
    (output omitted)
```

```
. sort foreign make
. predict dfor, dfbeta(1.foreign)
. list make price foreign dfor if abs(dfor) > 2/sqrt(74), divider
```

12. 
13. 
14. 
15. 
16. 
17. 
18. 

| make | price | foreign | dfor |
| :--- | ---: | ---: | ---: |
| Cad. Eldorado | 14,500 | Domestic | -.5290519 |
| Cad. Seville | 15,906 | Domestic | .8243419 |
| Linc. Versailles | 13,466 | Domestic | -.5283729 |
| Plym. Arrow | 4,647 | Domestic | -.6622424 |
| Plym. Champ | 4,425 | Domestic | .2371104 |
| Peugeot 604 | 12,990 | Foreign | .2552032 |
| Toyota Corona | 5,719 | Foreign | -.256431 |

The Cadillac Seville shifted the coefficient on 1.foreign 0.82 standard deviations!
Now let us ask which observations have the greatest effect on the mpg coefficient:

```
. predict dmpg, dfbeta(mpg)
```

. list make price mpg dmpg if abs(dmpg) > 2/sqrt(74), divider

| make | price | mpg | dmpg |
| :--- | ---: | ---: | ---: |
| Cad. Eldorado | 14,500 | 14 | -.5970351 |
| Cad. Seville | 15,906 | 21 | 1.134269 |
| Linc. Versailles | 13,466 | 14 | -.6069287 |
| Plym. Arrow | 4,647 | 28 | -.8925859 |
| Plym. Champ | 4,425 | 34 | .3186909 |

Once again we see the Cadillac Seville heading the list, indicating that our regression results may be dominated by this one car.

## > Example 9: DFBETAs influence measure; the dfbeta command

We can use predict, dfbeta() or the dfbeta command to generate the DFBETAs. dfbeta makes up names for the new variables automatically and, without arguments, generates the DFBETAS for all the variables in the regression:

```
. dfbeta
```

```
_dfbeta_1: dfbeta(weight)
_dfbeta_2: dfbeta(1.foreign)
_dfbeta_3: dfbeta(mpg)
_dfbeta_4: dfbeta(1.foreign#c.mpg)
```

dfbeta created four new variables in our dataset: _dfbeta_1, containing the DFBETAS for weight; _dfbeta_2, containing the DFBETAs for mpg; and so on. Had we wanted only the DFBETAs for mpg and weight, we might have typed

```
. dfbeta mpg weight
```

```
_dfbeta_5: dfbeta(weight)
_dfbeta_6: dfbeta(mpg)
```

In the example above, we typed dfbeta mpg weight instead of dfbeta; if we had typed dfbeta followed by dfbeta mpg weight, here is what would have happened:
. dfbeta

```
    _dfbeta_7: dfbeta(weight)
    _dfbeta_8: dfbeta(1.foreign)
    _dfbeta_9: dfbeta(mpg)
_dfbeta_10: dfbeta(1.foreign#c.mpg)
_dfbeta_11: dfbeta(weight)
_dfbeta_12: dfbeta(mpg)
```

. dfbeta mpg weight
dfbeta would have made up different names for the new variables. dfbeta never replaces existing variables - it instead makes up a different name, so we need to pay attention to dfbeta's output.

## Tests for violation of assumptions

## Syntax for estat hettest

estat hettest $[$ varlist] [, $\underline{\underline{\text { rhs }}}$ [normal $|\underline{\text { iid }}| \underline{\text { fstat }] ~ \underline{\underline{m} t e s t ~}[(s p e c)]]}$

## Menu for estat

Statistics $>$ Postestimation $>$ Reports and statistics

## Description for estat hettest

estat hettest performs three versions of the Breusch-Pagan (1979) and Cook-Weisberg (1983) test for heteroskedasticity. All three versions of this test present evidence against the null hypothesis that $\mathbf{t}=\mathbf{0}$ in $\operatorname{Var}(e)=\sigma^{2} \exp (\mathbf{z t})$. In the normal version, performed by default, the null hypothesis also includes the assumption that the regression disturbances are independent-normal draws with variance $\sigma^{2}$. The normality assumption is dropped from the null hypothesis in the iid and fstat versions, which respectively produce the score and $F$ tests discussed in Methods and formulas. If varlist is not specified, the fitted values are used for $\mathbf{z}$. If varlist or the rhs option is specified, the variables specified are used for $\mathbf{z}$.

## Options for estat hettest

rhs specifies that tests for heteroskedasticity be performed for the right-hand-side (explanatory) variables of the fitted regression model. The rhs option may be combined with a varlist.
normal, the default, causes estat hettest to compute the original Breusch-Pagan/Cook-Weisberg test, which assumes that the regression disturbances are normally distributed.
iid causes estat hettest to compute the $N * R^{2}$ version of the score test that drops the normality assumption.
fstat causes estat hettest to compute the $F$-statistic version that drops the normality assumption.
mtest $[$ (spec $)]$ specifies that multiple testing be performed. The argument specifies how $p$-values are adjusted. The following specifications, spec, are supported:

| bonferroni | Bonferroni's multiple testing adjustment |
| :---: | :--- |
| holm | Holm's multiple testing adjustment |
| sidak | Šidák's multiple testing adjustment |
| noadjust | no adjustment is made for multiple testing |

mtest may be specified without an argument. This is equivalent to specifying mtest (noadjust); that is, tests for the individual variables should be performed with unadjusted $p$-values. By default, estat hettest does not perform multiple testing. mtest may not be specified with iid or fstat.

## Syntax for estat imtest

estat imtest [, preserve white]

## Menu for estat

Statistics > Postestimation > Reports and statistics

## Description for estat imtest

estat imtest performs an information matrix test for the regression model and an orthogonal decomposition into tests for heteroskedasticity, skewness, and kurtosis due to Cameron and Trivedi (1990); White's test for homoskedasticity against unrestricted forms of heteroskedasticity (1980) is available as an option. White's test is usually similar to the first term of the Cameron-Trivedi decomposition.

## Options for estat imtest

preserve specifies that the data in memory be preserved, all variables and cases that are not needed in the calculations be dropped, and at the conclusion the original data be restored. This option is costly for large datasets. However, because estat imtest has to perform an auxiliary regression on $k(k+1) / 2$ temporary variables, where $k$ is the number of regressors, it may not be able to perform the test otherwise.
white specifies that White's original heteroskedasticity test also be performed.

## Syntax for estat ovtest

estat ovtest [, rhs]

## Menu for estat

Statistics $>$ Postestimation $>$ Reports and statistics

## Description for estat ovtest

estat ovtest performs two versions of the Ramsey (1969) regression specification-error test (RESET) for omitted variables. This test amounts to fitting $y=\mathbf{x b}+\mathbf{z t}+u$ and then testing $\mathbf{t}=\mathbf{0}$. If the rhs option is not specified, powers of the fitted values are used for $\mathbf{z}$. If $\mathbf{r h s}$ is specified, powers of the individual elements of $\mathbf{x}$ are used.

## Option for estat ovtest

rhs specifies that powers of the right-hand-side (explanatory) variables be used in the test rather than powers of the fitted values.

## Syntax for estat szroeter

estat szroeter [varlist] [, rhs mtest(spec)]
Either varlist or rhs must be specified.

## Menu for estat

Statistics $>$ Postestimation $>$ Reports and statistics

## Description for estat szroeter

estat szroeter performs Szroeter's rank test for heteroskedasticity for each of the variables in varlist or for the explanatory variables of the regression if rhs is specified.

## Options for estat szroeter

rhs specifies that tests for heteroskedasticity be performed for the right-hand-side (explanatory) variables of the fitted regression model. Option rhs may be combined with a varlist.
mtest (spec) specifies that multiple testing be performed. The argument specifies how $p$-values are adjusted. The following specifications, spec, are supported:

| bonferroni | Bonferroni's multiple testing adjustment |
| :---: | :--- |
| holm | Holm's multiple testing adjustment |
| sidak | Sidák's multiple testing adjustment |
| noadjust | no adjustment is made for multiple testing |

estat szroeter always performs multiple testing. By default, it does not adjust the $p$-values.

## Remarks and examples for estat hettest, estat imtest, estat ovtest, and estat szroeter

We introduce some regression diagnostic commands that are designed to test for certain violations that rvfplot (see $[R]$ regress postestimation diagnostic plots) less formally attempts to detect. estat ovtest provides Ramsey's test for omitted variables-a pattern in the residuals. estat hettest provides a test for heteroskedasticity - the increasing or decreasing variation in the residuals with fitted values, with respect to the explanatory variables, or with respect to yet other variables. The score test implemented in estat hettest (Breusch and Pagan 1979; Cook and Weisberg 1983) performs a score test of the null hypothesis that $b=0$ against the alternative hypothesis of multiplicative heteroskedasticity. estat szroeter provides a rank test for heteroskedasticity, which is an alternative to the score test computed by estat hettest. Finally, estat imtest computes an information matrix test, including an orthogonal decomposition into tests for heteroskedasticity, skewness, and kurtosis (Cameron and Trivedi 1990). The heteroskedasticity test computed by estat imtest is similar to the general test for heteroskedasticity that was proposed by White (1980). Cameron and Trivedi (2010, chap. 3) discuss most of these tests and provides more examples.
> Example 10: estat ovtest, estat hettest, estat szroeter, and estat imtest
We use our model of price on weight and foreign\#\#c.mpg.

```
. use http://www.stata-press.com/data/r13/auto, clear
(1978 Automobile Data)
. regress price weight foreign##c.mpg
    (output omitted)
. estat ovtest
Ramsey RESET test using powers of the fitted values of price
        Ho: model has no omitted variables
                        F(3,66) = 7.77
                        Prob > F = 0.0002
. estat hettest
Breusch-Pagan / Cook-Weisberg test for heteroskedasticity
    Ho: Constant variance
    Variables: fitted values of price
    chi2(1) = 6.50
    Prob > chi2 = 0.0108
```

Testing for heteroskedasticity in the right-hand-side variables is requested by specifying the rhs option. By specifying the mtest (bonferroni) option, we request that tests be conducted for each of the variables, with a Bonferroni adjustment for the $p$-values to accommodate our testing multiple hypotheses.

```
. estat hettest, rhs mtest(bonf)
Breusch-Pagan / Cook-Weisberg test for heteroskedasticity
    Ho: Constant variance
```

| Variable | chi2 | df | p |
| :---: | :---: | :---: | :---: |
| weight <br> foreign <br> Foreign <br> mpg <br> foreign\# <br> c.mpg | 15.24 | 1 | 0.0004 \# |
| Foreign |  |  |  |

\# Bonferroni-adjusted p-values
. estat szroeter, rhs mtest(holm)
Szroeter's test for homoskedasticity
Ho: variance constant
Ha: variance monotonic in variable

| Variable | chi2 | df | p |
| :---: | ---: | :---: | :---: |
| weight <br> foreign | 17.07 | 1 | 0.0001 \# |
| Foreign | 6.15 | 1 | 0.0131 \# |
| mpg <br> foreign\# <br> c.mpg <br> Foreign | 11.45 | 1 | 0.0021 \# |

\# Holm-adjusted p-values
Finally, we request the information matrix test, which is a conditional moments test with second-, third-, and fourth-order moment conditions.

```
. estat imtest
Cameron & Trivedi's decomposition of IM-test
```

| Source | chi2 | df | p |
| ---: | ---: | ---: | :---: |
| Heteroskedasticity | 18.86 | 10 | 0.0420 |
| Skewness | 11.69 | 4 | 0.0198 |
| Kurtosis | 2.33 | 1 | 0.1273 |
| Total | 32.87 | 15 | 0.0049 |

We find evidence for omitted variables, heteroskedasticity, and nonnormal skewness.
So, why bother with the various graphical commands when the tests seem so much easier to interpret? In part, it is a matter of taste: both are designed to uncover the same problem, and both are, in fact, going about it in similar ways. One is based on a formal calculation, whereas the other is based on personal judgment in evaluating a graph. On the other hand, the tests are seeking evidence of specific problems, whereas judgment is more general. The careful analyst will use both.

We performed the omitted-variable test first. Omitted variables are a more serious problem than heteroskedasticity or the violations of higher moment conditions tested by estat imtest. If this
were not a manual, having found evidence of omitted variables, we would never have run the estat hettest, estat szroeter, and estat imtest commands, at least not until we solved the omitted-variable problem.

## - Technical note

estat ovtest and estat hettest both perform two flavors of their respective tests. By default, estat ovtest looks for evidence of omitted variables by fitting the original model augmented by $\widehat{y}^{2}, \widehat{y}^{3}$, and $\widehat{y}^{4}$, which are the fitted values from the original model. Under the assumption of no misspecification, the coefficients on the powers of the fitted values will be zero. With the rhs option, estat ovtest instead augments the original model with powers (second through fourth) of the explanatory variables (except for dummy variables).
estat hettest, by default, looks for heteroskedasticity by modeling the variance as a function of the fitted values. If, however, we specify a variable or variables, the variance will be modeled as a function of the specified variables. In our example, if we had, a priori, some reason to suspect heteroskedasticity and that the heteroskedasticity is a function of a car's weight, then using a test that focuses on weight would be more powerful than the more general tests such as White's test or the first term in the Cameron-Trivedi decomposition test.
estat hettest, by default, computes the original Breusch-Pagan/Cook-Weisberg test, which includes the assumption of normally distributed errors. Koenker (1981) derived an $N * R^{2}$ version of this test that drops the normality assumption. Wooldridge (2013) gives an $F$-statistic version that does not require the normality assumption.

## Stored results for estat hettest, estat imtest, and estat ovtest

estat hettest stores the following results for the (multivariate) score test in $\mathrm{r}($ ):
Scalars
$r($ chi2 $) \quad \chi^{2}$ test statistic
$r(\mathrm{df}) \quad$ \#df for the asymptotic $\chi^{2}$ distribution under $H_{0}$
$r(p)$ $p$-value
estat hettest, fstat stores results for the (multivariate) score test in $r()$ :
Scalars
$r(F) \quad$ test statistic
$\mathrm{r}\left(\mathrm{df} \_\mathrm{m}\right) \quad$ \#df of the test for the $F$ distribution under $H_{0}$
r(df_r) \#df of the residuals for the $F$ distribution under $H_{0}$
$r(p) \quad p$-value
estat hettest (if mtest is specified) and estat szroeter store the following in $r()$ :
Matrices
$r$ (mtest) a matrix of test results, with rows corresponding to the univariate tests
mtest [., 1] $\chi^{2}$ test statistic
mtest[.,2] \#df
mtest [., 3] unadjusted $p$-value
mtest [.,4] adjusted $p$-value (if an mtest() adjustment method is specified)
Macros
$r$ (mtmethod) adjustment method for $p$-values
estat imtest stores the following in r() :
Scalars

$$
\begin{array}{ll}
r\left(c h i 2 \_t\right) & \text { IM-test statistic }\left(=r\left(c h i 2 \_h\right)+r\left(c h i 2 \_s\right)+r\left(c h i 2 \_k\right)\right) \\
r\left(d f \_t\right) & \text { df for limiting } \chi^{2} \text { distribution under } H_{0}\left(=r\left(d f \_h\right)+r\left(d f \_s\right)+r\left(d f \_k\right)\right) \\
r\left(c h i 2 \_h\right) & \text { heteroskedasticity test statistic } \\
r(d f-h) & \text { df for limiting } \chi^{2} \text { distribution under } H_{0} \\
r\left(c h i 2 \_s\right) & \text { skewness test statistic } \\
r\left(d f \_s\right) & \text { df for limiting } \chi^{2} \text { distribution under } H_{0} \\
r\left(c h i 2 \_k\right) & \text { kurtosis test statistic } \\
r\left(d f \_k\right) & \text { df for limiting } \chi^{2} \text { distribution under } H_{0} \\
r\left(c h 2 \_w\right) & \text { White's heteroskedasticity test (if white specified }) \\
r\left(d f \_w\right) & \text { df for limiting } \chi^{2} \text { distribution under } H_{0}
\end{array}
$$

estat ovtest stores the following in $r()$ :
Scalars

| $r(p)$ | two-sided $p$-value |
| :--- | :--- |
| $r(\mathrm{~F})$ | $F$ statistic |
| $r(d f)$ | degrees of freedom |
| $r\left(d f \_r\right)$ | residual degrees of freedom |

## Variance inflation factors

## Syntax for estat vif

estat vif [, uncentered]

## Menu for estat

Statistics $>$ Postestimation $>$ Reports and statistics

## Description for estat vif

estat vif calculates the centered or uncentered variance inflation factors (VIFs) for the independent variables specified in a linear regression model.

## Option for estat vif

uncentered requests that the computation of the uncentered variance inflation factors. This option is often used to detect the collinearity of the regressors with the constant. estat vif, uncentered may be used after regression models fit without the constant term.

## Remarks and examples for estat vif

Problems arise in regression when the predictors are highly correlated. In this situation, there may be a significant change in the regression coefficients if you add or delete an independent variable. The estimated standard errors of the fitted coefficients are inflated, or the estimated coefficients may not be statistically significant even though a statistical relation exists between the dependent and independent variables.

Data analysts rely on these facts to check informally for the presence of multicollinearity. estat vif, another command for use after regress, calculates the variance inflation factors and tolerances for each of the independent variables.

The output shows the variance inflation factors together with their reciprocals. Some analysts compare the reciprocals with a predetermined tolerance. In the comparison, if the reciprocal of the VIF is smaller than the tolerance, the associated predictor variable is removed from the regression model. However, most analysts rely on informal rules of thumb applied to the VIF; see Chatterjee and Hadi (2012). According to these rules, there is evidence of multicollinearity if

1. The largest VIF is greater than 10 (some choose a more conservative threshold value of 30 ).
2. The mean of all the VIFs is considerably larger than 1.

## > Example 11: estat vif

We examine a regression model fit using the ubiquitous automobile dataset:

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
```

. regress price mpg rep78 trunk headroom length turn displ gear_ratio

| Source | SS | df | MS | Number of obs | 69 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | F( 8, 60) | 6.33 |
| Model | 264102049 | 8 | 33012756.2 | Prob > F | 0.0000 |
| Residual | 312694909 | 60 | 5211581.82 | R-squared | 0.4579 |
|  |  |  |  | Adj R-squared | 0.3856 |
| Total | 576796959 | 68 | 8482308.22 | Root MSE | 2282.9 |


| price | Coef. | Std. Err. | t | P>\|t| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | ---: | ---: | ---: |
| mpg | -144.84 | 82.12751 | -1.76 | 0.083 | -309.1195 | 19.43948 |
| rep78 | 727.5783 | 337.6107 | 2.16 | 0.035 | 52.25638 | 1402.9 |
| trunk | 44.02061 | 108.141 | 0.41 | 0.685 | -172.2935 | 260.3347 |
| headroom | -807.0996 | 435.5802 | -1.85 | 0.069 | -1678.39 | 64.19062 |
| length | -8.688914 | 34.89848 | -0.25 | 0.804 | -78.49626 | 61.11843 |
| turn | -177.9064 | 137.3455 | -1.30 | 0.200 | -452.6383 | 96.82551 |
| displacement | 30.73146 | 7.576952 | 4.06 | 0.000 | 15.5753 | 45.88762 |
| gear_ratio | 1500.119 | 1110.959 | 1.35 | 0.182 | -722.1303 | 3722.368 |
| _cons | 6691.976 | 7457.906 | 0.90 | 0.373 | -8226.058 | 21610.01 |


| . estat vif |  |  |
| ---: | ---: | ---: |
| Variable | VIF | $1 / \mathrm{VIF}$ |
| length | 8.22 | 0.121614 |
| displacement | 6.50 | 0.153860 |
| turn | 4.85 | 0.205997 |
| gear_ratio | 3.45 | 0.290068 |
| mpg | 3.03 | 0.330171 |
| trunk | 2.88 | 0.347444 |
| headroom | 1.80 | 0.554917 |
| rep78 | 1.46 | 0.686147 |
| Mean VIF | 4.02 |  |

The results are mixed. Although we have no VIFs greater than 10 , the mean VIF is greater than 1 , though not considerably so. We could continue the investigation of collinearity, but given that other authors advise that collinearity is a problem only when VIFs exist that are greater than 30 (contradicting our rule above), we will not do so here.

## Example 12: estat vif, with strong evidence of multicollinearity

This example comes from a dataset described in Kutner, Nachtsheim, and Neter $(2004,257)$ that examines body fat as modeled by caliper measurements on the triceps, midarm, and thigh.

| . use http://www.stata-press.com/data/r13/bodyfat |
| :--- |
| (Body Fat) |
| . regress bodyfat tricep thigh midarm |
| Source |
| Model |
| Residual |


| Number of obs | $=$ | 20 |
| :--- | ---: | ---: |
| F $(3$, | $16)$ | $=$ |
| Prob $>$ F | $=$ | 0.0000 |
| R-squared | $=$ | 0.8014 |
| Adj R-squared | $=$ | 0.7641 |
| Root MSE | $=$ | 2.48 |


| bodyfat | Coef. | Std. Err. | t | $\mathrm{P}>\mid \mathrm{tI}$ | [95\% Conf. Interval] |  |
| ---: | ---: | :--- | :---: | :---: | ---: | ---: |
| triceps | 4.334085 | 3.015511 | 1.44 | 0.170 | -2.058512 | 10.72668 |
| thigh | -2.856842 | 2.582015 | -1.11 | 0.285 | -8.330468 | 2.616785 |
| midarm | -2.186056 | 1.595499 | -1.37 | 0.190 | -5.568362 | 1.19625 |
| _cons | 117.0844 | 99.78238 | 1.17 | 0.258 | -94.44474 | 328.6136 |

. estat vif

| Variable | VIF | $1 /$ VIF |
| ---: | ---: | ---: |
| triceps | 708.84 | 0.001411 |
| thigh | 564.34 | 0.001772 |
| midarm | 104.61 | 0.009560 |
| Mean VIF | 459.26 |  |

Here we see strong evidence of multicollinearity in our model. More investigation reveals that the measurements on the thigh and the triceps are highly correlated:

```
. correlate triceps thigh midarm
(obs=20)
```

|  | triceps | thigh | midarm |
| ---: | :---: | :---: | :---: |
| triceps | 1.0000 |  |  |
| thigh | 0.9238 | 1.0000 |  |
| midarm | 0.4578 | 0.0847 | 1.0000 |

If we remove the predictor tricep from the model (because it had the highest VIF), we get
. regress bodyfat thigh midarm

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | 384.279748 | 2 | 192.139874 |
| Residual | 111.109765 | 17 | 6.53586854 |
| Total | 495.389513 | 19 | 26.0731323 |


| Number of obs | $=$ | 20 |
| :--- | ---: | ---: |
| F 2, | $17)$ | $=29.40$ |
| Prob $>$ F | $=$ | 0.0000 |
| R-squared | $=0.7757$ |  |
| Adj R-squared | $=$ | 0.7493 |
| Root MSE | $=2.5565$ |  |


| bodyfat | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| thigh | .8508818 | .1124482 | 7.57 | 0.000 | .6136367 | 1.088127 |
| midarm | .0960295 | .1613927 | 0.60 | 0.560 | -.2444792 | .4365383 |
| _cons | -25.99696 | 6.99732 | -3.72 | 0.002 | -40.76001 | -11.2339 |


| estat vif <br> Variable | VIF | $1 /$ VIF |
| ---: | ---: | ---: |
| midarm | 1.01 | 0.992831 |
| thigh | 1.01 | 0.992831 |
| Mean VIF | 1.01 |  |

Note how the coefficients change and how the estimated standard errors for each of the regression coefficients become much smaller. The calculated value of $R^{2}$ for the overall regression for the subset model does not appreciably decline when we remove the correlated predictor. Removing an independent variable from the model is one way to deal with multicollinearity. Other methods include ridge regression, weighted least squares, and restricting the use of the fitted model to data that follow the same pattern of multicollinearity. In economic studies, it is sometimes possible to estimate the regression coefficients from different subsets of the data by using cross-section and time series.

All examples above demonstrated the use of centered VIFs. As pointed out by Belsley (1991), the centered VIFs may fail to discover collinearity involving the constant term. One solution is to use the uncentered VIFs instead. According to the definition of the uncentered VIFs, the constant is viewed as a legitimate explanatory variable in a regression model, which allows one to obtain the VIF value for the constant term.

## $>$ Example 13: estat vif, with strong evidence of collinearity with the constant term

Consider the extreme example in which one of the regressors is highly correlated with the constant. We simulate the data and examine both centered and uncentered VIF diagnostics after fitted regression model as follows.
. use http://www.stata-press.com/data/r13/extreme_collin

- regress y one x z

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | 223801.985 | 3 | 74600.6617 |
| Residual | 2642.42124 | 96 | 27.5252213 |
| Total | 226444.406 | 99 | 2287.31723 |

Number of obs $=100$
F ( 3, 96) $=2710.27$
Prob > F $=0.0000$
$R$-squared $=0.9883$
Adj R-squared $=0.9880$
Root MSE $=5.2464$

| y | Coef. | Std. Err. | t | $\mathrm{P}>\mid \mathrm{tI}$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | :---: | ---: | ---: |
| one | -3.278582 | 10.5621 | -0.31 | 0.757 | -24.24419 | 17.68702 |
| x | 2.038696 | .0242673 | 84.01 | 0.000 | 1.990526 | 2.086866 |
| z | 4.863137 | .2681036 | 18.14 | 0.000 | 4.330956 | 5.395319 |
| _cons | 9.760075 | 10.50935 | 0.93 | 0.355 | -11.10082 | 30.62097 |


| estat vif <br> Variable | VIF | $1 / \mathrm{VIF}$ |
| ---: | ---: | ---: |
| z | 1.03 | 0.968488 |
| x | 1.03 | 0.971307 |
| one | 1.00 | 0.995425 |
| Mean VIF | 1.02 |  |


| . estat vif, uncentered |  |  |
| ---: | ---: | ---: |
| Variable | VIF | $1 /$ VIF |
| one | 402.94 | 0.002482 |
| intercept | 401.26 | 0.002492 |
| z | 2.93 | 0.341609 |
| x | 1.13 | 0.888705 |
| Mean VIF | 202.06 |  |

According to the values of the centered VIFs (1.03, 1.03, 1.00), no harmful collinearity is detected in the model. However, by the construction of these simulated data, we know that one is highly collinear with the constant term. As such, the large values of uncentered VIFs for one (402.94) and intercept (401.26) reveal high collinearity of the variable one with the constant term.

## Measures of effect size

## Syntax for estat esize

estat esize $[$, omega level (\#) $]$

## Menu for estat

Statistics $>$ Postestimation $>$ Reports and statistics

## Description for estat esize

estat esize calculates effect sizes for linear models after regress or anova. By default, estat esize reports $\eta^{2}$ (eta-squared) estimates (Kerlinger 1964), which are equivalent to $R^{2}$ estimates. If the option omega is specified, estat esize reports $\omega^{2}$ estimates (Hays 1963), which are equivalent to adjusted $R^{2}$ estimates. Confidence intervals for $\eta^{2}$ and $\omega^{2}$ estimates are estimated by using the noncentral $F$ distribution (Smithson 2001). See Kline (2013) or Thompson (2006) for further information.

## Options for estat esize

omega specifies that the $\omega^{2}$ estimates of effect size be reported. The default is $\eta^{2}$ estimates.
level(\#) specifies the confidence level, as a percentage, for confidence intervals. The default is level(95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals.

## Remarks and examples for estat esize

Whereas $p$-values are used to assess the statistical significance of a result, measures of effect size are used to assess the practical significance of a result. Effect sizes can be broadly categorized as "measures of group differences" (the $d$ family) and "measures of association" (the $r$ family); see Ellis (2010, table 1.1). The $d$ family includes estimators such as Cohen's $D$, Hedges's $G$, and Glass's $\Delta$ (also see [R] esize). The $r$ family includes estimators such as the point-biserial correlation coefficient, $\omega^{2}$, and $\eta^{2}$. For an introduction to the concepts and calculation of effect sizes, see Kline (2013) or Thompson (2006). For a more detailed discussion, see Kirk (1996), Ellis (2010), Cumming (2012), Grissom and Kim (2012), and Kelley and Preacher (2012).
> Example 14: Calculating effect sizes for a linear regression model
Suppose we fit a linear regression model for low-birthweight infants.

| Source | SS | df | MS |  | Number of obs | $=189$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Model | 12346897.6 | 3411 | 632.54 |  | Prob > F | $=0.0000$ |
| Residual | 87568400.9 | 185473 | 42.708 |  | R -squared | $=0.1236$ |
|  |  |  |  |  | Adj R-squared | $=0.1094$ |
| Total | 99915298.6 | 188531 | 64.354 |  | Root MSE | $=688$ |
| bwt | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Conf. | Interval] |
| smoke | -428.0254 | 109.0033 | -3.93 | 0.000 | -643.0746 | -212.9761 |
| race |  |  |  |  |  |  |
| black | -450.54 | 153.066 | -2.94 | 0.004 | -752.5194 | -148.5607 |
| other | -454.1813 | 116.436 | -3.90 | 0.000 | -683.8944 | -224.4683 |
| _cons | 3334.858 | 91.74301 | 36.35 | 0.000 | 3153.86 | 3515.855 |

We can use the estat esize command to calculate $\eta^{2}$ for the entire model and a partial $\eta^{2}$ for each term in the model.
. estat esize
Effect sizes for linear models

| Source | Eta-Squared | df | [95\% Conf. Interval] |  |
| ---: | :---: | :---: | :---: | :---: |
| Model | .1235736 | 3 | .0399862 | .2041365 |
| smoke | .0769345 | 1 | .0193577 | .1579213 |
| race | .0908394 | 2 | .0233037 | .1700334 |

The omega option causes estat esize to report $\omega^{2}$ and partial $\omega^{2}$.
. estat esize, omega
Effect sizes for linear models

| Source | Omega-Squared | df | [95\% Conf. Interval] |  |
| ---: | :---: | :---: | :---: | :---: |
| Model | .1093613 | 3 | .0244184 | .1912306 |
|  |  |  |  |  |
| smoke | .0719449 | 1 | .0140569 | .1533695 |
| race | .0810106 | 2 | .0127448 | .1610608 |

## Example 15: Calculating effect size for an ANOVA model

We can use estat esize after ANOVA models as well.
. anova bwt smoke race

|  | $\begin{array}{llr} \text { Number of obs } & =189 \\ \text { Root MSE } & =687.999 \end{array}$ |  |  | R -squared | $\begin{aligned} & =0.1236 \\ & =0.1094 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Source | Partial SS | df | MS | F | Prob > F |
| Model | 12346897.6 | 3 | 4115632.54 | 8.69 | 0.0000 |
| smoke | 7298536.57 | 1 | 7298536.57 | 15.42 | 0.0001 |
| race | 8749453.3 | 2 | 4374726.65 | 9.24 | 0.0001 |
| Residual | 87568400.9 | 185 | 473342.708 |  |  |
| Total | 99915298.6 | 188 | 531464.354 |  |  |

. estat esize
Effect sizes for linear models

| Source | Eta-Squared | df | [95\% Conf. Interval] |  |
| ---: | :---: | :---: | :---: | :---: |
| Model | .1235736 | 3 | .0399862 | .2041365 |
|  |  |  |  |  |
| smoke | .0769345 | 1 | .0193577 | .1579213 |
| race | .0908394 | 2 | .0233037 | .1700334 |

- Technical note
$\eta^{2}$ and $\omega^{2}$ were developed in the context of analysis of variance. Thus, the published research on the calculation of their confidence intervals focuses on cases where the numerator degrees of freedom are relatively small (for example, $\mathrm{df}<20$ ).

Some combinations of the $F$ statistic, numerator degrees of freedom, and denominator degrees of freedom yield confidence limits that do not contain the corresponding estimated value for an $\eta^{2}$ or $\omega^{2}$. This problem is most commonly observed for larger numerator degrees of freedom.

Nothing in the literature suggests alternative methods for constructing confidence intervals in such cases; therefore, we recommend cautious interpretation of confidence intervals for $\eta^{2}$ and $\omega^{2}$ when the numerator degrees of freedom are greater than 20.

## Stored results for estat esize

estat esize stores the following results in $r()$ :
Scalars
$r$ (level) confidence level
Matrices
r(esize) a matrix of effect sizes, confidence intervals, degrees of freedom, and $F$ statistics with rows corresponding to each term in the model

$$
\text { esize }[., 1] \quad \eta^{2}
$$

esize $[., 2]$ lower confidence bound for $\eta^{2}$
esize $[., 3]$ upper confidence bound for $\eta^{2}$
esize[.,4] $\omega^{2}$
esize[.,5] lower confidence bound for $\omega^{2}$
esize[.,6] upper confidence bound for $\omega^{2}$
esize[.,7] numerator degrees of freedom
esize $[., 8]$ denominator degrees of freedom
esize[.,9] $F$ statistic

## Methods and formulas

See Hamilton (2013, chap. 7), Kohler and Kreuter (2012, sec. 9.3), or Baum (2006, chap. 5) for an overview of using Stata to perform regression diagnostics. See Peracchi (2001, chap. 8) for a mathematically rigorous discussion of diagnostics.

Methods and formulas are presented under the following headings:

## predict

Special-interest postestimation commands

## predict

Assume that you have already fit the regression model

$$
\mathbf{y}=\mathbf{X b}+\mathbf{e}
$$

where $\mathbf{X}$ is $n \times k$.

Denote the previously estimated coefficient vector by $\mathbf{b}$ and its estimated variance matrix by $\mathbf{V}$. predict works by recalling various aspects of the model, such as $\mathbf{b}$, and combining that information with the data currently in memory. Let $\mathbf{x}_{j}$ be the $j$ th observation currently in memory, and let $s^{2}$ be the mean squared error of the regression.

If the user specified weights in regress, then $\mathbf{X}^{\prime} \mathbf{X}$ in the following formulas is replaced by $\mathbf{X}^{\prime} \mathbf{D X}$, where $\mathbf{D}$ is defined in Coefficient estimation and ANOVA table under Methods and formulas in $[R]$ regress.

Let $\mathbf{V}=s^{2}\left(\mathbf{X}^{\prime} \mathbf{X}\right)^{-1}$. Let $k$ be the number of independent variables including the intercept, if any, and let $y_{j}$ be the observed value of the dependent variable.

The predicted value ( xb option) is defined as $\widehat{y}_{j}=\mathbf{x}_{j} \mathbf{b}$.
Let $\ell_{j}$ represent a lower bound for an observation $j$ and $u_{j}$ represent an upper bound. The probability that $y_{j} \mid \mathbf{x}_{j}$ would be observed in the interval $\left(\ell_{j}, u_{j}\right)$-the $\operatorname{pr}(\ell, u)$ option-is

$$
P\left(\ell_{j}, u_{j}\right)=\operatorname{Pr}\left(\ell_{j}<\mathbf{x}_{j} \mathbf{b}+e_{j}<u_{j}\right)=\Phi\left(\frac{u_{j}-\widehat{y}_{j}}{s}\right)-\Phi\left(\frac{\ell_{j}-\widehat{y}_{j}}{s}\right)
$$

where for the $\operatorname{pr}(\ell, u), \mathrm{e}(\ell, u)$, and $\operatorname{ystar}(\ell, u)$ options, $\ell_{j}$ and $u_{j}$ can be anywhere in the range $(-\infty,+\infty)$.

The option e $(\ell, u)$ computes the expected value of $y_{j} \mid \mathbf{x}_{j}$ conditional on $y_{j} \mid \mathbf{x}_{j}$ being in the interval $\left(\ell_{j}, u_{j}\right)$, that is, when $y_{j} \mid \mathbf{x}_{j}$ is truncated. It can be expressed as

$$
E\left(\ell_{j}, u_{j}\right)=E\left(\mathbf{x}_{j} \mathbf{b}+e_{j} \mid \ell_{j}<\mathbf{x}_{j} \mathbf{b}+e_{j}<u_{j}\right)=\widehat{y}_{j}-s \frac{\phi\left(\frac{u_{j}-\widehat{y}_{j}}{s}\right)-\phi\left(\frac{\ell_{j}-\widehat{y}_{j}}{s}\right)}{\Phi\left(\frac{u_{j}-\widehat{y}_{j}}{s}\right)-\Phi\left(\frac{\ell_{j}-\widehat{y}_{j}}{s}\right)}
$$

where $\phi$ is the normal density and $\Phi$ is the cumulative normal.
You can also compute ystar $(\ell, u)$ - the expected value of $y_{j} \mid \mathbf{x}_{j}$, where $y_{j}$ is assumed censored at $\ell_{j}$ and $u_{j}$ :

$$
y_{j}^{*}= \begin{cases}\ell_{j} & \text { if } \mathbf{x}_{j} \mathbf{b}+e_{j} \leq \ell_{j} \\ \mathbf{x}_{j} \mathbf{b}+u & \text { if } \ell_{j}<\mathbf{x}_{j} \mathbf{b}+e_{j}<u_{j} \\ u_{j} & \text { if } \mathbf{x}_{j} \mathbf{b}+e_{j} \geq u_{j}\end{cases}
$$

This computation can be expressed in several ways, but the most intuitive formulation involves a combination of the two statistics just defined:

$$
y_{j}^{*}=P\left(-\infty, \ell_{j}\right) \ell_{j}+P\left(\ell_{j}, u_{j}\right) E\left(\ell_{j}, u_{j}\right)+P\left(u_{j},+\infty\right) u_{j}
$$

A diagonal element of the projection matrix (hat) or (leverage) is given by

$$
h_{j}=\mathbf{x}_{j}\left(\mathbf{X}^{\prime} \mathbf{X}\right)^{-1} \mathbf{x}_{j}^{\prime}
$$

The standard error of the prediction (the stdp option) is defined as $s_{p_{j}}=\sqrt{\mathbf{x}_{j} \mathbf{V} \mathbf{x}_{j}^{\prime}}$ and can also be written as $s_{p_{j}}=s \sqrt{h_{j}}$.

The standard error of the forecast (stdf) is defined as $s_{f_{j}}=s \sqrt{1+h_{j}}$.

The standard error of the residual (stdr) is defined as $s_{r_{j}}=s \sqrt{1-h_{j}}$.
The residuals (residuals) are defined as $\widehat{e}_{j}=y_{j}-\widehat{y}_{j}$.
The standardized residuals (rstandard) are defined as $\widehat{e}_{s_{j}}=\widehat{e}_{j} / s_{r_{j}}$.
The Studentized residuals (rstudent) are defined as

$$
r_{j}=\frac{\widehat{e}_{j}}{s_{(j)} \sqrt{1-h_{j}}}
$$

where $s_{(j)}$ represents the root mean squared error with the $j$ th observation removed, which is given by

$$
s_{(j)}^{2}=\frac{s^{2}(T-k)}{T-k-1}-\frac{\widehat{e}_{j}^{2}}{(T-k-1)\left(1-h_{j}\right)}
$$

Cook's $D$ (cooksd) is given by

$$
D_{j}=\frac{\widehat{e}_{s_{j}}^{2}\left(s_{p_{j}} / s_{r_{j}}\right)^{2}}{k}=\frac{h_{j} \widehat{e}_{j}^{2}}{k s^{2}\left(1-h_{j}\right)^{2}}
$$

DFITS (dfits) is given by

$$
\operatorname{DFITS}_{j}=r_{j} \sqrt{\frac{h_{j}}{1-h_{j}}}
$$

Welsch distance (welsch) is given by

$$
W_{j}=\frac{r_{j} \sqrt{h_{j}(n-1)}}{1-h_{j}}
$$

COVRATIO (covratio) is given by

$$
\operatorname{COVRATIO}_{j}=\frac{1}{1-h_{j}}\left(\frac{n-k-\widehat{e}_{j}^{2}}{n-k-1}\right)^{k}
$$

The DFBETAS (dfbeta) for a particular regressor $x_{i}$ are given by

$$
\mathrm{DFBETA}_{j}=\frac{r_{j} u_{j}}{\sqrt{U^{2}\left(1-h_{j}\right)}}
$$

where $u_{j}$ are the residuals obtained from a regression of $x_{i}$ on the remaining $x$ 's and $U^{2}=\sum_{j} u_{j}^{2}$.

## Special-interest postestimation commands

The omitted-variable test (Ramsey 1969) reported by estat ovtest fits the regression $y_{i}=$ $\mathbf{x}_{i} \mathbf{b}+\mathbf{z}_{i} \mathbf{t}+u_{i}$ and then performs a standard $F$ test of $\mathbf{t}=\mathbf{0}$. The default test uses $\mathbf{z}_{i}=\left(\widehat{y}_{i}^{2}, \widehat{y}_{i}^{3}, \widehat{y}_{i}^{4}\right)$. If $\mathbf{r h s}$ is specified, $\mathbf{z}_{i}=\left(x_{1 i}^{2}, x_{1 i}^{3}, x_{1 i}^{4}, x_{2 i}^{2}, \ldots, x_{m i}^{4}\right)$. In either case, the variables are normalized to have minimum 0 and maximum 1 before powers are calculated.

The test for heteroskedasticity (Breusch and Pagan 1979; Cook and Weisberg 1983) models $\operatorname{Var}\left(e_{i}\right)=\sigma^{2} \exp (\mathbf{z t})$, where $\mathbf{z}$ is a variable list specified by the user, the list of right-hand-side variables, or the fitted values $\mathbf{x} \widehat{\boldsymbol{\beta}}$. The test is of $\mathbf{t}=\mathbf{0}$. Mechanically, estat hettest fits the augmented regression $\widehat{e}_{i}^{2} / \widehat{\sigma}^{2}=a+\mathbf{z}_{i} \mathbf{t}+v_{i}$.

The original Breusch-Pagan/Cook-Weisberg version of the test assumes that the $e_{i}$ are normally distributed under the null hypothesis which implies that the score test statistic $S$ is equal to the model sum of squares from the augmented regression divided by 2 . Under the null hypothesis, $S$ has the $\chi^{2}$ distribution with $m$ degrees of freedom, where $m$ is the number of columns of $\mathbf{z}$.

Koenker (1981) derived a score test of the null hypothesis that $\mathbf{t}=\mathbf{0}$ under the assumption that the $e_{i}$ are independent and identically distributed (i.i.d.). Koenker showed that $S=N * R^{2}$ has a large-sample $\chi^{2}$ distribution with $m$ degrees of freedom, where $N$ is the number of observations and $R^{2}$ is the $R$-squared in the augmented regression and $m$ is the number of columns of $\mathbf{z}$. estat hettest, iid produces this version of the test.

Wooldridge (2013) showed that an $F$ test of $\mathbf{t}=\mathbf{0}$ in the augmented regression can also be used under the assumption that the $e_{i}$ are i.i.d. estat hettest, fstat produces this version of the test.

Szroeter's class of tests for homoskedasticity against the alternative that the residual variance increases in some variable $x$ is defined in terms of

$$
H=\frac{\sum_{i=1}^{n} h\left(x_{i}\right) e_{i}^{2}}{\sum_{i=1}^{n} e_{i}^{2}}
$$

where $h(x)$ is some weight function that increases in $x$ (Szroeter 1978). $H$ is a weighted average of the $h(x)$, with the squared residuals serving as weights. Under homoskedasticity, $H$ should be approximately equal to the unweighted average of $h(x)$. Large values of $H$ suggest that $e_{i}^{2}$ tends to be large where $h(x)$ is large; that is, the variance indeed increases in $x$, whereas small values of $H$ suggest that the variance actually decreases in $x$. estat szroeter uses $h\left(x_{i}\right)=\operatorname{rank}\left(x_{i}\right.$ in $\left.x_{1} \ldots x_{n}\right)$; see Judge et al. [1985, 452] for details. estat szroeter displays a normalized version of $H$,

$$
Q=\sqrt{\frac{6 n}{n^{2}-1}} H
$$

which is approximately $N(0,1)$ distributed under the null (homoskedasticity).
estat hettest and estat szroeter provide adjustments of $p$-values for multiple testing. The supported methods are described in [R] test.
estat imtest performs the information matrix test for the regression model, as well as an orthogonal decomposition into tests for heteroskedasticity $\delta_{1}$, nonnormal skewness $\delta_{2}$, and nonnormal kurtosis $\delta_{3}$ (Cameron and Trivedi 1990; Long and Trivedi 1993). The decomposition is obtained via three auxiliary regressions. Let $e$ be the regression residuals, $\widehat{\sigma}^{2}$ be the maximum likelihood estimate of $\sigma^{2}$ in the regression, $n$ be the number of observations, $X$ be the set of $k$ variables specified with estat imtest, and $R_{\text {un }}^{2}$ be the uncentered $R^{2}$ from a regression. $\delta_{1}$ is obtained as $n R_{\text {un }}^{2}$ from a regression of $e^{2}-\widehat{\sigma}^{2}$ on the cross products of the variables in $X . \delta_{2}$ is computed as $n R_{\text {un }}^{2}$ from a regression of $e^{3}-3 \widehat{\sigma}^{2} e$ on $X$. Finally, $\delta_{3}$ is obtained as $n R_{\text {un }}^{2}$ from a regression of $e^{4}-6 \widehat{\sigma}^{2} e^{2}-3 \widehat{\sigma}^{4}$
on $X . \delta_{1}, \delta_{2}$, and $\delta_{3}$ are asymptotically $\chi^{2}$ distributed with $1 / 2 k(k+1), K$, and 1 degree of freedom. The information test statistic $\delta=\delta_{1}+\delta_{2}+\delta_{3}$ is asymptotically $\chi^{2}$ distributed with $1 / 2 k(k+3)$ degrees of freedom. White's test for heteroskedasticity is computed as $n R^{2}$ from a regression of $\widehat{u}^{2}$ on $X$ and the cross products of the variables in $X$. This test statistic is usually close to $\delta_{1}$.
estat vif calculates the centered variance inflation factor $\left(\mathrm{VIF}_{c}\right)$ (Chatterjee and Hadi 2012, 248-251) for $x_{j}$, given by

$$
\operatorname{VIF}_{c}\left(x_{j}\right)=\frac{1}{1-\widehat{R}_{j}^{2}}
$$

where $\widehat{R}_{j}^{2}$ is the square of the centered multiple correlation coefficient that results when $x_{j}$ is regressed with intercept against all the other explanatory variables.

The uncentered variance inflation factor (VIF ${ }_{u c}$ ) (Belsley 1991, 28-29) for $x_{j}$ is given by

$$
\operatorname{VIF}_{u c}\left(x_{j}\right)=\frac{1}{1-\widetilde{R}_{j}^{2}}
$$

where $\widetilde{R}_{j}^{2}$ is the square of the uncentered multiple correlation coefficient that results when $x_{j}$ is regressed without intercept against all the other explanatory variables including the constant term.

The methods and formulas for estat esize are described in Methods and formulas of [R] esize.

## Acknowledgments

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## Also see

[ R ] regress - Linear regression
[R] regress postestimation diagnostic plots - Postestimation plots for regress
$[\mathrm{R}]$ regress postestimation time series - Postestimation tools for regress with time series
[U] 20 Estimation and postestimation commands

## Title

regress postestimation diagnostic plots - Postestimation plots for regress

| Description | rvfplot | avplot | avplots | cprplot | acprplot |
| :--- | :--- | :--- | :--- | :--- | :--- |
| rvpplot | Ivr2plot | Methods and formulas | References | Also see |  |

## Description

The following postestimation commands are of special interest after regress:

## Command Description

| rvfplot | residual-versus-fitted plot |
| :--- | :--- |
| avplot | added-variable plot |
| avplots | all added-variables plots in one image |
| cprplot | component-plus-residual plot |
| acprplot | augmented component-plus-residual plot |
| rvpplot | residual-versus-predictor plot |
| lvr2plot | leverage-versus-squared-residual plot |

These commands are not appropriate after the svy prefix.
For a discussion of the terminology used in this entry, see the Terminology section of
Remarks and examples for predict in [R] regress postestimation.

## rvfplot

## Syntax for rvfplot

> rvfplot [, rvfplot_options]
rvfplot_options
Description

Plot
marker_options change look of markers (color, size, etc.)
marker_label_options
add marker labels; change look or position
Add plots
addplot (plot) add plots to the generated graph
Y axis, X axis, Titles, Legend, Overall
twoway_options
any options other than by () documented in [G-3] twoway_options

## Menu for rvfplot

Statistics $>$ Linear models and related $>$ Regression diagnostics $>$ Residual-versus-fitted plot

## Description for rvfplot

rvfplot graphs a residual-versus-fitted plot, a graph of the residuals against the fitted values.

## Options for rvfplot

$\qquad$ Plot
marker_options affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] marker_options.
marker_label_options specify if and how the markers are to be labeled; see [G-3] marker_label_options.
$\qquad$
Add plots
addplot (plot) provides a way to add plots to the generated graph. See [G-3] addplot_option.
$\qquad$ Y axis, X axis, Titles, Legend, Overall
twoway_options are any of the options documented in [G-3] twoway_options, excluding by (). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

## Remarks and examples for rvfplot

rvfplot graphs the residuals against the fitted values.
$>$ Example 1
Using auto.dta described in [U] 1.2.2 Example datasets, we will use regress to fit a model of price on weight, mpg, foreign, and the interaction of foreign with mpg. We specify foreign\#\#c.mpg to obtain the interaction of foreign with mpg ; see [U] 11.4.3 Factor variables.

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. regress price weight foreign##c.mpg
```

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | 350319665 | 4 | 87579916.3 |
| Residual | 284745731 | 69 | 4126749.72 |
| Total | 635065396 | 73 | 8699525.97 |


| Number of obs | $=$ | 74 |
| :--- | ---: | ---: |
| $\mathrm{~F}(4$, | $69)$ | $=21.22$ |
| Prob $>$ F | $=0.0000$ |  |
| R-squared | $=0.5516$ |  |
| Adj R-squared | $=0.5256$ |  |
| Root MSE | $=2031.4$ |  |


| price | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | :---: |
| weight | 4.613589 | .7254961 | 6.36 | 0.000 | 3.166263 | 6.060914 |
| foreign <br> Foreign <br> mpg | 11240.33 | 2751.681 | 4.08 | 0.000 | 5750.878 | 16729.78 |
| 263.1875 | 110.7961 | 2.38 | 0.020 | 42.15527 | 484.2197 |  |
| foreign\#c.mpg |  |  |  |  |  |  |
| Foreign | -307.2166 | 108.5307 | -2.83 | 0.006 | -523.7294 | -90.70368 |
| _cons | -14449.58 | 4425.72 | -3.26 | 0.002 | -23278.65 | -5620.51 |

Once we have fit a model, we may use any of the regression diagnostics commands. rvfplot (read residual-versus-fitted plot) graphs the residuals against the fitted values:

```
. rvfplot, yline(0)
```



All the diagnostic plot commands allow the graph twoway and graph twoway scatter options; we specified a yline (0) to draw a line across the graph at $y=0$; see [G-2] graph twoway scatter.

In a well-fitted model, there should be no pattern to the residuals plotted against the fitted values - something not true of our model. Ignoring the two outliers at the top center of the graph, we see curvature in the pattern of the residuals, suggesting a violation of the assumption that price is linear in our independent variables. We might also have seen increasing or decreasing variation in the residuals-heteroskedasticity. Any pattern whatsoever indicates a violation of the least-squares assumptions.

## avplot

## Syntax for avplot

```
avplot indepvar [, avplot_options \(]\)
```

avplot_options Description
Plot
marker_options change look of markers (color, size, etc.)
marker_label_options
add marker labels; change look or position
Reference line
rlopts (cline_options) affect rendition of the reference line
Add plots
addplot (plot) add other plots to the generated graph
Y axis, X axis, Titles, Legend, Overall
twoway_options any options other than by () documented in [G-3] twoway_options

## Menu for avplot

Statistics $>$ Linear models and related $>$ Regression diagnostics $>$ Added-variable plot

## Description for avplot

avplot graphs an added-variable plot (a.k.a. partial-regression leverage plot, partial regression plot, or adjusted partial residual plot) after regress. indepvar may be an independent variable (a.k.a. predictor, carrier, or covariate) that is currently in the model or not.

## Options for avplot

$\qquad$ Plot
marker_options affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] marker_options. marker_label_options specify if and how the markers are to be labeled; see [G-3] marker_label_options.
$\qquad$ Reference line
rlopts (cline_options) affects the rendition of the reference line. See [G-3] cline_options.
$\qquad$ Add plots
addplot (plot) provides a way to add other plots to the generated graph. See [G-3] addplot_option.
$\qquad$ Y axis, X axis, Titles, Legend, Overall
twoway_options are any of the options documented in [G-3] twoway_options, excluding by (). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

## Remarks and examples for avplot

avplot graphs an added-variable plot, also known as the partial-regression leverage plot.
One of the wonderful features of one-regressor regressions (regressions of $y$ on one $x$ ) is that we can graph the data and the regression line. There is no easier way to understand the regression than to examine such a graph. Unfortunately, we cannot do this when we have more than one regressor. With two regressors, it is still theoretically possible-the graph must be drawn in three dimensions, but with three or more regressors no graph is possible.

The added-variable plot is an attempt to project multidimensional data back to the two-dimensional world for each of the original regressors. This is, of course, impossible without making some concessions. Call the coordinates on an added-variable plot $y$ and $x$. The added-variable plot has the following properties:

- There is a one-to-one correspondence between $\left(x_{i}, y_{i}\right)$ and the $i$ th observation used in the original regression.
- A regression of $y$ on $x$ has the same coefficient and standard error (up to a degree-of-freedom adjustment) as the estimated coefficient and standard error for the regressor in the original regression.
- The "outlierness" of each observation in determining the slope is in some sense preserved.

It is equally important to note the properties that are not listed. The $y$ and $x$ coordinates of the added-variable plot cannot be used to identify functional form, or, at least, not well (see Mallows [1986]). In the construction of the added-variable plot, the relationship between $y$ and $x$ is forced to be linear.

## $>$ Example 2

Let's use the same model as we used in example 1.

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. regress price weight foreign##c.mpg
    (output omitted)
```

We can now examine the added-variable plot for mpg.
. avplot mpg


This graph suggests a problem in determining the coefficient on mpg. Were this a one-regressor regression, the two points at the top-left corner and the one at the top right would cause us concern, and so it does in our more complicated multiple-regressor case. To identify the problem points, we retyped our command, modifying it to read avplot mpg, mlabel (make), and discovered that the two cars at the top left are the Cadillac Eldorado and the Lincoln Versailles; the point at the top right is the Cadillac Seville. These three cars account for $100 \%$ of the luxury cars in our data, suggesting that our model is misspecified. By the way, the point at the lower right of the graph, also cause for concern, is the Plymouth Arrow, our data entry error.

## - Technical note

Stata's avplot command can be used with regressors already in the model, as we just did, or with potential regressors not yet in the model. In either case, avplot will produce the correct graph. The name "added-variable plot" is unfortunate in the case when the variable is already among the list of regressors but is, we think, still preferable to the name "partial-regression leverage plot" assigned by Belsley, Kuh, and Welsch $(1980,30)$ and more in the spirit of the original use of such plots by Mosteller and Tukey (1977, 271-279). Welsch (1986, 403), however, disagrees: "I am sorry to see that Chatterjee and Hadi [1986] endorse the term 'added-variable plot' when $X_{j}$ is part of the original model" and goes on to suggest the name "adjusted partial residual plot".

## avplots

## Syntax for avplots

```
    avplots [, avplots_options]
    avplots_options
        Description
```

Plot
marker_options
marker_label_options
combine_options
Reference line
rlopts (cline_options)
$Y$ axis, $X$ axis, Titles, Legend, Overall
twoway_options
change look of markers (color, size, etc.)
add marker labels; change look or position
any of the options documented in [G-2] graph combine
affect rendition of the reference line
any options other than by () documented in [G-3] twoway_options

## Menu for avplots

Statistics $>$ Linear models and related $>$ Regression diagnostics $>$ Added-variable plot

## Description for avplots

avplots graphs all the added-variable plots in one image.

## Options for avplots

$\qquad$ Plot
marker_options affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] marker_options.
marker_label_options specify if and how the markers are to be labeled; see [G-3] marker_label_options. combine_options are any of the options documented in [G-2] graph combine. These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).
$\qquad$ Reference line
rlopts (cline_options) affects the rendition of the reference line. See [G-3] cline_options.
$\qquad$ Y axis, X axis, Titles, Legend, Overall
twoway_options are any of the options documented in [G-3] twoway_options, excluding by (). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

## Remarks and examples for avplots

$\lambda$ Example 3
In example 2, we used avplot to examine the added-variable plot for mpg in our regression of price on weight and foreign\#\#c.mpg. Now let's use avplots to graph an added-variable plot for every regressor in the data.
. avplots


## cprplot

## Syntax for cprplot

cprplot indepvar [, cprplot_options]
cprplot_options Description
Plot
marker_options
marker_label_options
Reference line
rlopts(cline_options)
Options
lowess
lsopts(lowess_options)
mspline
msopts (mspline_options)
Add plots
addplot (plot) add other plots to the generated graph
Y axis, X axis, Titles, Legend, Overall twoway_options any options other than by () documented in [G-3] twoway_options

## Menu for cprplot

Statistics $>$ Linear models and related $>$ Regression diagnostics $>$ Component-plus-residual plot

## Description for cprplot

cprplot graphs a component-plus-residual plot (a.k.a. partial residual plot) after regress. indepvar must be an independent variable that is currently in the model.

## Options for cprplot

$\qquad$ Plot
marker_options affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] marker_options. marker_label_options specify if and how the markers are to be labeled; see [G-3] marker_label_options.

Options
lowess adds a lowess smooth of the plotted points to assist in detecting nonlinearities.
lsopts(lowess_options) affects the rendition of the lowess smooth. For an explanation of these options, especially the bwidth() option, see [R] lowess. Specifying lsopts() implies the lowess option.
mspline adds a median spline of the plotted points to assist in detecting nonlinearities.
msopts (mspline_options) affects the rendition of the spline. For an explanation of these options, especially the bands() option, see [G-2] graph twoway mspline. Specifying msopts() implies the mspline option.

Add plots
addplot (plot) provides a way to add other plots to the generated graph. See [G-3] addplot_option.

Y axis, X axis, Titles, Legend, Overall
twoway_options are any of the options documented in [G-3] twoway_options, excluding by (). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

## Remarks and examples for cprplot

Added-variable plots are successful at identifying outliers, but they cannot be used to identify functional form. The component-plus-residual plot (Ezekiel 1924; Larsen and McCleary 1972) is another attempt at projecting multidimensional data into a two-dimensional form, but with different properties. Although the added-variable plot can identify outliers, the component-plus-residual plot cannot. It can, however, be used to examine the functional form assumptions of the model. Both plots have the property that a regression line through the coordinates has a slope equal to the estimated coefficient in the regression model.
> Example 4
We illustrate component-plus-residual plots using a variation of auto.dta.

```
. use http://www.stata-press.com/data/r13/auto1
(Automobile Models)
```

. regress price mpg weight

| Source | SS | df | MS |
| ---: | ---: | ---: | ---: |
| Model | 187716578 | 2 | 93858289 |
| Residual | 447348818 | 71 | 6300687.58 |
| Total | 635065396 | 73 | 8699525.97 |


| Number of obs | $=$ | 74 |
| :--- | ---: | ---: |
| $\mathrm{~F}(2$, | $71)$ | $=$ |
| Prob $>\mathrm{F}$ | 14.90 |  |
| R-squared | $=$ | 0.0000 |
| Adj R-squared | $=$ | 0.2956 |
| Root MSE | $=$ | 2510.1 |


| price | Coef. | Std. Err. | $t$ | P>\|t| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| mpg | -55.9393 | 75.24136 | -0.74 | 0.460 | -205.9663 | 94.08771 |
| weight | 1.710992 | .5861682 | 2.92 | 0.005 | .5422063 | 2.879779 |
| _cons | 2197.9 | 3190.768 | 0.69 | 0.493 | -4164.311 | 8560.11 |

In fact, we know that the effects of mpg in this model are nonlinear-if we added mpg squared to the model, its coefficient would have a $t$ statistic of 2.38 , the $t$ statistic on mpg would become -2.48 , and weight's effect would become about one-third of its current value and become statistically insignificant. Pretend that we do not know this.

The component-plus-residual plot for mpg is

```
. cprplot mpg, mspline msopts(bands(13))
```



We are supposed to examine the above graph for nonlinearities or, equivalently, ask if the regression line, which has slope equal to the estimated effect of mpg in the original model, fits the data adequately. To assist our eyes, we added a median spline. Perhaps some people may detect nonlinearity from this graph, but we assert that if we had not previously revealed the nonlinearity of mpg and if we had not added the median spline, the graph would not overly bother us.

## acprplot

## Syntax for acprplot

acprplot indepvar [, acprplot_options]

| acprplot_options | Description |
| :---: | :---: |
| Plot |  |
| marker_options | change look of markers (color, size, etc.) |
| marker_label_options | add marker labels; change look or position |
| Reference line |  |
| rlopts (cline_options) | affect rendition of the reference line |
| Options |  |
| lowess | add a lowess smooth of the plotted points |
| lsopts(lowess_options) | affect rendition of the lowess smooth |
| mspline | add median spline of the plotted points |
| msopts (mspline_options) | affect rendition of the spline |
| Add plots |  |
| addplot (plot) | add other plots to the generated graph |
| Y axis, X axis, Titles, Legend, Overall |  |
| twoway_options | any options other than by () documented in [G-3] twoway_options |

## Menu for acprplot

Statistics $>$ Linear models and related $>$ Regression diagnostics $>$ Augmented component-plus-residual plot

## Description for acprplot

acprplot graphs an augmented component-plus-residual plot (a.k.a. augmented partial residual plot) as described by Mallows (1986). This seems to work better than the component-plus-residual plot for identifying nonlinearities in the data.

## Options for acprplot

marker_options affect the rendition of markers drawn at the plotted points, including their shape,
size, color, and outline; see [G-3] marker_options.
marker_label_options specify if and how the markers are to be labeled; see [G-3] marker_label_options.

[^68]lowess adds a lowess smooth of the plotted points to assist in detecting nonlinearities.
lsopts(lowess_options) affects the rendition of the lowess smooth. For an explanation of these options, especially the bwidth() option, see [R] lowess. Specifying lsopts () implies the lowess option.
mspline adds a median spline of the plotted points to assist in detecting nonlinearities.
msopts (mspline_options) affects the rendition of the spline. For an explanation of these options, especially the bands() option, see [G-2] graph twoway mspline. Specifying msopts() implies the mspline option.

Add plots
addplot (plot) provides a way to add other plots to the generated graph. See [G-3] addplot_option.

Y axis, X axis, Titles, Legend, Overall
twoway_options are any of the options documented in [G-3] twoway_options, excluding by (). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

## Remarks and examples for acprplot

In the cprplot section above, we discussed the component-plus-residual plot. Mallows (1986) proposed an augmented component-plus-residual plot that is often more sensitive to detecting nonlinearity.

## > Example 5

Let's compare the augmented component-plus-residual plot with the component-plus-residual plot of example 4.

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. regress price weight foreign##c.mpg
    (output omitted)
```

```
. acprplot mpg, mspline msopts(bands(13))
```



It does do somewhat better.

## rvpplot

## Syntax for rvpplot

$$
\text { rvpplot indepvar }[\text {, rvpplot_options }]
$$

rvpplot_options
Plot
marker_options
marker_label_options
Add plots
addplot (plot) add other plots to the generated graph
Y axis, X axis, Titles, Legend, Overall
twoway_options
any options other than by () documented in [G-3] twoway_options

## Menu for rvpplot

[^69]
## Description for rvpplot

rvpplot graphs a residual-versus-predictor plot (a.k.a. independent variable plot or carrier plot), a graph of the residuals against the specified predictor.

## Options for rvpplot

$\qquad$ Plot
marker_options affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] marker_options.
marker_label_options specify if and how the markers are to be labeled; see [G-3] marker_label_options.
$\qquad$ Add plots
addplot (plot) provides a way to add other plots to the generated graph; see [G-3] addplot_option.
$\qquad$ Y axis, X axis, Titles, Legend, Overall
twoway_options are any of the options documented in [G-3] twoway_options, excluding by (). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

## Remarks and examples for rvpplot

The residual-versus-predictor plot is a simple way to look for violations of the regression assumptions. If the assumptions are correct, there should be no pattern on the graph.
> Example 6
Let's use our model of price on mpg and weight.

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. regress price weight foreign##c.mpg
    (output omitted)
. rvpplot mpg, yline(0)
```



Remember, any pattern counts as a problem, and in this graph, we see that the variation in the residuals decreases as mpg increases.

## Ivr2plot

## Syntax for Ivr2plot



## Menu for Ivr2plot

Statistics $>$ Linear models and related $>$ Regression diagnostics $>$ Leverage-versus-squared-residual plot

## Description for Ivr2plot

lvr2plot graphs a leverage-versus-squared-residual plot (a.k.a. L-R plot).

## Options for Ivr2plot

$\qquad$ Plot
marker_options affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] marker_options.
marker_label_options specify if and how the markers are to be labeled; see [G-3] marker_label_options.
$\qquad$ Add plots
addplot (plot) provides a way to add other plots to the generated graph. See [G-3] addplot_option.
$\qquad$ Y axis, X axis, Titles, Legend, Overall
twoway_options are any of the options documented in [G-3] twoway_options, excluding by (). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

## Remarks and examples for Ivr2plot

One of the most useful diagnostic graphs is provided by lvr2plot (leverage-versus-residual-squared plot), a graph of leverage against the (normalized) residuals squared.
$>$ Example 7
We illustrate lvr2plot using our model in example 1.

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. regress price weight foreign##c.mpg
    (output omitted)
. lvr2plot
```



The lines on the chart show the average values of leverage and the (normalized) residuals squared. Points above the horizontal line have higher-than-average leverage; points to the right of the vertical line have larger-than-average residuals.

One point immediately catches our eye, and four more make us pause. The point at the top of the graph has high leverage and a smaller-than-average residual. The other points that bother us all have higher-than-average leverage, two with smaller-than-average residuals and two with larger-than-average residuals.

A less pretty but more useful version of the above graph specifies that make be used as the symbol (see [G-3] marker_label_options):

```
. lvr2plot, mlabel(make) mlabp(0) m(none) mlabsize(small)
```



The Vw Diesel, Plymouth Champ, Plymouth Arrow, and Peugeot 604 are the points that cause us the most concern. When we further examine our data, we discover that the vw Diesel is the only diesel in our data and that the data for the Plymouth Arrow were entered incorrectly into the computer. No such simple explanations were found for the Plymouth Champ and Peugeot 604.

## Methods and formulas

See Hamilton (2013, 209-214) and Kohler and Kreuter (2012, sec. 9.3) for a discussion of these diagnostic graphs.

The lvr2plot command plots leverage against the squares of the normalized residuals. The normalized residuals are defined as $\widehat{e}_{n_{j}}=\widehat{e}_{j} /\left(\sum_{i} \widehat{e}_{i}^{2}\right)^{1 / 2}$.

## References

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Welsch, R. E. 1986. Comment [on Chatterjee and Hadi 1986]. Statistical Science 1: 403-405.

## Also see

$[R]$ regress - Linear regression
$[R]$ regress postestimation - Postestimation tools for regress
$[R]$ regress postestimation time series - Postestimation tools for regress with time series
[U] 20 Estimation and postestimation commands

## Title

regress postestimation time series - Postestimation tools for regress with time series

Description
Syntax for estat bgodfrey
Options for estat durbinalt
Remarks and examples
Acknowledgment

Syntax for estat archlm
Options for estat bgodfrey
Syntax for estat dwatson
Stored results
References

Options for estat archlm
Syntax for estat durbinalt
Menu for estat
Methods and formulas
Also see

## Description

The following postestimation commands for time series are available for regress:
Command Description
estat archlm test for ARCH effects in the residuals
estat bgodfrey Breusch-Godfrey test for higher-order serial correlation
estat durbinalt Durbin's alternative test for serial correlation
estat dwatson
Durbin-Watson $d$ statistic to test for first-order serial correlation

These commands provide regression diagnostic tools specific to time series. You must tsset your data before using these commands; see [TS] tsset.
estat archlm tests for time-dependent volatility. estat bgodfrey, estat durbinalt, and estat dwatson test for serial correlation in the residuals of a linear regression. For non-time-series regression diagnostic tools, see $[R]$ regress postestimation.
estat archlm performs Engle's Lagrange multiplier (LM) test for the presence of autoregressive conditional heteroskedasticity.
estat bgodfrey performs the Breusch-Godfrey test for higher-order serial correlation in the disturbance. This test does not require that all the regressors be strictly exogenous.
estat durbinalt performs Durbin's alternative test for serial correlation in the disturbance. This test does not require that all the regressors be strictly exogenous.
estat dwatson computes the Durbin-Watson $d$ statistic (Durbin and Watson 1950) to test for first-order serial correlation in the disturbance when all the regressors are strictly exogenous.

## Syntax for estat archlm

estat archlm [, archlm_options]
archlm_options Description

| lags (numlist) | test numlist lag orders |
| :--- | :--- |
| force | allow test after regress, vce(robust) |

## Options for estat archlm

lags (numlist) specifies a list of numbers, indicating the lag orders to be tested. The test will be performed separately for each order. The default is order one.
force allows the test to be run after regress, vce(robust). The command will not work if the vce(cluster clustvar) option is specified with regress; see [ R$]$ regress.

## Syntax for estat bgodfrey

estat bgodfrey [, bgodfrey_options]
bgodfrey_options Description

| lags(numlist) | test numlist lag orders |
| :--- | :--- |
| nomiss0 | do not use Davidson and MacKinnon's approach |
| small | obtain $p$-values using the $F$ or $t$ distribution |

## Options for estat bgodfrey

lags(numlist) specifies a list of numbers, indicating the lag orders to be tested. The test will be performed separately for each order. The default is order one.
nomiss0 specifies that Davidson and MacKinnon's approach (1993, 358), which replaces the missing values in the initial observations on the lagged residuals in the auxiliary regression with zeros, not be used.
small specifies that the $p$-values of the test statistics be obtained using the $F$ or $t$ distribution instead of the default chi-squared or normal distribution.

## Syntax for estat durbinalt

```
estat durbinalt [, durbinalt_options]
```

durbinalt_options Description

| lags(numlist) | test numlist lag orders |
| :--- | :--- |
| $\underline{\text { nomiss0 }}$ | do not use Davidson and MacKinnon's approach |
| robust | compute standard errors using the robust/sandwich estimator |
| small | obtain $p$-values using the $F$ or $t$ distribution |
| force | allow test after regress, vce(robust) or after newey |

## Options for estat durbinalt

lags (numlist) specifies a list of numbers, indicating the lag orders to be tested. The test will be performed separately for each order. The default is order one.
nomiss0 specifies that Davidson and MacKinnon's approach (1993, 358), which replaces the missing values in the initial observations on the lagged residuals in the auxiliary regression with zeros, not be used.
robust specifies that the Huber/White/sandwich robust estimator of the variance-covariance matrix be used in Durbin's alternative test.
small specifies that the $p$-values of the test statistics be obtained using the $F$ or $t$ distribution instead of the default chi-squared or normal distribution. This option may not be specified with robust, which always uses an $F$ or $t$ distribution.
force allows the test to be run after regress, vce (robust) and after newey (see [R] regress and [TS] newey). The command will not work if the vce(cluster clustvar) option is specified with regress.

## Syntax for estat dwatson

estat dwatson

## Menu for estat

Statistics $>$ Postestimation $>$ Reports and statistics

## Remarks and examples

The Durbin-Watson test is used to determine whether the error term in a linear regression model follows an $\operatorname{AR}(1)$ process. For the linear model

$$
y_{t}=\mathbf{x}_{t} \boldsymbol{\beta}+u_{t}
$$

the $\operatorname{AR}(1)$ process can be written as

$$
u_{t}=\rho u_{t-1}+\epsilon_{t}
$$

In general, an $\operatorname{AR}(1)$ process requires only that $\epsilon_{t}$ be independent and identically distributed (i.i.d.). The Durbin-Watson test, however, requires $\epsilon_{t}$ to be distributed $N\left(0, \sigma^{2}\right)$ for the statistic to have an exact distribution. Also, the Durbin-Watson test can be applied only when the regressors are strictly exogenous. A regressor $x$ is strictly exogenous if $\operatorname{Corr}\left(x_{s}, u_{t}\right)=0$ for all $s$ and $t$, which precludes the use of the Durbin-Watson statistic with models where lagged values of the dependent variable are included as regressors.

The null hypothesis of the test is that there is no first-order autocorrelation. The Durbin-Watson $d$ statistic can take on values between 0 and 4 and under the null $d$ is equal to 2 . Values of $d$ less than 2 suggest positive autocorrelation $(\rho>0)$, whereas values of $d$ greater than 2 suggest negative autocorrelation $(\rho<0)$. Calculating the exact distribution of the $d$ statistic is difficult, but empirical upper and lower bounds have been established based on the sample size and the number of regressors. Extended tables for the $d$ statistic have been published by Savin and White (1977). For example, suppose you have a model with 30 observations and three regressors (including the constant term). For a test of the null hypothesis of no autocorrelation versus the alternative of positive autocorrelation, the lower bound of the $d$ statistic is 1.284 , and the upper bound is 1.567 at the $5 \%$ significance level. You would reject the null if $d<1.284$, and you would fail to reject if $d>1.567$. A value falling within the range $(1.284,1.567)$ leads to no conclusion about whether or not to reject the null hypothesis.

When lagged dependent variables are included among the regressors, the past values of the error term are correlated with those lagged variables at time $t$, implying that they are not strictly exogenous regressors. The inclusion of covariates that are not strictly exogenous causes the $d$ statistic to be biased toward the acceptance of the null hypothesis. Durbin (1970) suggested an alternative test for models with lagged dependent variables and extended that test to the more general $\operatorname{AR}(p)$ serial correlation process

$$
u_{t}=\rho_{1} u_{t-1}+\cdots+\rho_{p} u_{t-p}+\epsilon_{t}
$$

where $\epsilon_{t}$ is i.i.d. with variance $\sigma^{2}$ but is not assumed or required to be normal for the test.
The null hypothesis of Durbin's alternative test is

$$
H_{0}: \rho_{1}=0, \ldots, \rho_{p}=0
$$

and the alternative is that at least one of the $\rho$ 's is nonzero. Although the null hypothesis was originally derived for an $\operatorname{AR}(p)$ process, this test turns out to have power against MA $(p)$ processes as well. Hence, the actual null of this test is that there is no serial correlation up to order $p$ because the MA $(p)$ and the $\operatorname{AR}(p)$ models are locally equivalent alternatives under the null. See Godfrey $(1988,113-115)$ for a discussion of this result.

Durbin's alternative test is in fact a LM test, but it is most easily computed with a Wald test on the coefficients of the lagged residuals in an auxiliary OLS regression of the residuals on their lags and all the covariates in the original regression. Consider the linear regression model

$$
\begin{equation*}
y_{t}=\beta_{1} x_{1 t}+\cdots+\beta_{k} x_{k t}+u_{t} \tag{1}
\end{equation*}
$$

in which the covariates $x_{1}$ through $x_{k}$ are not assumed to be strictly exogenous and $u_{t}$ is assumed to be i.i.d. and to have finite variance. The process is also assumed to be stationary. (See Wooldridge [2013] for a discussion of stationarity.) Estimating the parameters in (1) by OLS obtains the residuals $\widehat{u}_{t}$. Next another OLS regression is performed of $\widehat{u}_{t}$ on $\widehat{u}_{t-1}, \ldots, \widehat{u}_{t-p}$ and the other regressors,

$$
\begin{equation*}
\widehat{u}_{t}=\gamma_{1} \widehat{u}_{t-1}+\cdots+\gamma_{p} \widehat{u}_{t-p}+\beta_{1} x_{1 t}+\cdots+\beta_{k} x_{k t}+\epsilon_{t} \tag{2}
\end{equation*}
$$

where $\epsilon_{t}$ stands for the random-error term in this auxiliary OLS regression. Durbin's alternative test is then obtained by performing a Wald test that $\gamma_{1}, \ldots, \gamma_{p}$ are jointly zero. The test can be made robust to an unknown form of heteroskedasticity by using a robust VCE estimator when estimating the regression in (2). When there are only strictly exogenous regressors and $p=1$, this test is asymptotically equivalent to the Durbin-Watson test.

The Breusch-Godfrey test is also an LM test of the null hypothesis of no autocorrelation versus the alternative that $u_{t}$ follows an $\operatorname{AR}(p)$ or $\mathrm{MA}(p)$ process. Like Durbin's alternative test, it is based on the auxiliary regression (2), and it is computed as $N R^{2}$, where $N$ is the number of observations and $R^{2}$ is the simple $R^{2}$ from the regression. This test and Durbin's alternative test are asymptotically equivalent. The test statistic $N R^{2}$ has an asymptotic $\chi^{2}$ distribution with $p$ degrees of freedom. It is valid with or without the strict exogeneity assumption but is not robust to conditional heteroskedasticity, even if a robust VCE is used when fitting (2).

In fitting (2), the values of the lagged residuals will be missing in the initial periods. As noted by Davidson and MacKinnon (1993), the residuals will not be orthogonal to the other covariates in the model in this restricted sample, which implies that the $R^{2}$ from the auxiliary regression will not be zero when the lagged residuals are left out. Hence, Breusch and Godfrey's $N R^{2}$ version of the test may overreject in small samples. To correct this problem, Davidson and MacKinnon (1993) recommend setting the missing values of the lagged residuals to zero and running the auxiliary regression in (2) over the full sample used in (1). This small-sample correction has become conventional for both the Breusch-Godfrey and Durbin's alternative test, and it is the default for both commands. Specifying the nomiss0 option overrides this default behavior and treats the initial missing values generated by regressing on the lagged residuals as missing. Hence, nomiss0 causes these initial observations to be dropped from the sample of the auxiliary regression.

Durbin's alternative test and the Breusch-Godfrey test were originally derived for the case covered by regress without the vce(robust) option. However, after regress, vce(robust) and newey, Durbin's alternative test is still valid and can be invoked if the robust and force options are specified.

## > Example 1: tests for serial correlation

Using data from Klein (1950), we first fit an OLS regression of consumption on the government wage bill:

```
. use http://www.stata-press.com/data/r13/klein
. tsset yr
    time variable: yr, 1920 to 1941
    delta: 1 unit
```

. regress consump wagegovt

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | 532.567711 | 1 | 532.567711 |
| Residual | 601.207167 | 20 | 30.0603584 |
| Total | 1133.77488 | 21 | 53.9892799 |


| Number of obs | $=$ | 22 |
| :--- | ---: | ---: |
| $\mathrm{~F}(1$, 20) | $=$ | 17.72 |
| Prob $>$ F | $=$ | 0.0004 |
| R-squared | $=$ | 0.4697 |
| Adj R-squared | $=$ | 0.4432 |
| Root MSE | $=$ | 5.4827 |


| consump | Coef. | Std. Err. | t | P>\|t| | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | :---: | ---: |
| wagegovt | 2.50744 | .5957173 | 4.21 | 0.000 | 1.264796 | 3.750085 |
| _cons | 40.84699 | 3.192183 | 12.80 | 0.000 | 34.18821 | 47.50577 |

If we assume that wagegov is a strictly exogenous variable, we can use the Durbin-Watson test to check for first-order serial correlation in the errors.

```
. estat dwatson
Durbin-Watson d-statistic( 2, 22) = . 3217998
```

The Durbin-Watson $d$ statistic, 0.32 , is far from the center of its distribution $(d=2.0)$. Given 22 observations and two regressors (including the constant term) in the model, the lower 5\% bound is about 0.997 , much greater than the computed $d$ statistic. Assuming that wagegov is strictly exogenous, we can reject the null of no first-order serial correlation. Rejecting the null hypothesis does not necessarily mean an AR process; other forms of misspecification may also lead to a significant test statistic. If we are willing to assume that the errors follow an $\operatorname{AR}(1)$ process and that wagegov is strictly exogenous, we could refit the model using arima or prais and model the error process explicitly; see [TS] arima and [TS] prais.

If we are not willing to assume that wagegov is strictly exogenous, we could instead use Durbin's alternative test or the Breusch-Godfrey to test for first-order serial correlation. Because we have only 22 observations, we will use the small option.

| . estat durbinalt, small |
| :--- |
| Durbin's alternative test for autocorrelation |
| lags $(p)$ |

HO: no serial correlation
. estat bgodfrey, small
Breusch-Godfrey LM test for autocorrelation

| $\operatorname{lags}(p)$ | F | df | Prob $>\mathrm{F}$ |
| :---: | :---: | :---: | :---: |
| 1 | 14.264 | $(1, \quad 19)$ | 0.0013 |

HO: no serial correlation
Both tests strongly reject the null of no first-order serial correlation, so we decide to refit the model with two lags of consump included as regressors and then rerun estat durbinalt and estat bgodfrey. Because the revised model includes lagged values of the dependent variable, the Durbin-Watson test is not applicable.
. regress consump wagegovt L.consump L2.consump

| Source | SS | df MS |  |  | Number of obs $=20$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | F ( 3, 16) | $=44.01$ |
| Model | 702.660311 | 323 | 234.220104 |  | Prob > F | $=0.0000$ |
| Residual | 85.1596011 | 165. | . 32247507 |  | R-squared | $=0.8919$ |
|  |  |  |  |  | Adj R-squared | $=0.8716$ |
| Total | 787.819912 | 1941 | 41.4642059 |  | Root MSE | $=2.307$ |
| consump | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Conf. Interval] |  |
| wagegovt | . 6904282 | . 3295485 | 2.10 | 0.052 | -. 0081835 | 1.38904 |
| consump |  |  |  |  |  |  |
| L1. | 1.420536 | . 197024 | 7.21 | 0.000 | 1.002864 | 1.838208 |
| L2. | -. 650888 | . 1933351 | -3.37 | 0.004 | -1.06074 | -. 241036 |
| _cons | 9.209073 | 5.006701 | 1.84 | 0.084 | -1.404659 | 19.82281 |

. estat durbinalt, small lags(1/2)
Durbin's alternative test for autocorrelation

| lags $(p)$ | F | df |  |
| :---: | :---: | :---: | :---: |
| 1 | 0.080 | $\left(\begin{array}{cc}1, & 15 \\ 2 & 0.260\end{array}\right.$ | Prob $>\mathrm{F}$ |

HO: no serial correlation
. estat bgodfrey, small lags(1/2)
Breusch-Godfrey LM test for autocorrelation

| lags $(p)$ | F | df | Prob $>\mathrm{F}$ |
| :---: | :---: | :---: | :---: |
| 1 | 0.107 | $\left(\begin{array}{cc}1, & 15\end{array}\right)$ | 0.7484 |
| 2 | 0.358 | $(2, \quad 14)$ | 0.7056 |

HO: no serial correlation
Although wagegov and the constant term are no longer statistically different from zero at the $5 \%$ level, the output from estat durbinalt and estat bgodfrey indicates that including the two lags of consump has removed any serial correlation from the errors.

Engle (1982) suggests an LM test for checking for autoregressive conditional heteroskedasticity (ARCH) in the errors. The $p$ th-order ARCH model can be written as

$$
\begin{aligned}
\sigma_{t}^{2} & =E\left(u_{t}^{2} \mid u_{t-1}, \ldots, u_{t-p}\right) \\
& =\gamma_{0}+\gamma_{1} u_{t-1}^{2}+\cdots+\gamma_{p} u_{t-p}^{2}
\end{aligned}
$$

To test the null hypothesis of no autoregressive conditional heteroskedasticity (that is, $\gamma_{1}=\cdots=$ $\gamma_{p}=0$ ), we first fit the OLS model (1), obtain the residuals $\widehat{u}_{t}$, and run another OLS regression on the lagged residuals:

$$
\begin{equation*}
\widehat{u}_{t}^{2}=\gamma_{0}+\gamma_{1} \widehat{u}_{t-1}^{2}+\cdots+\gamma_{p} \widehat{u}_{t-p}^{2}+\epsilon \tag{3}
\end{equation*}
$$

The test statistic is $N R^{2}$, where $N$ is the number of observations in the sample and $R^{2}$ is the $R^{2}$ from the regression in (3). Under the null hypothesis, the test statistic follows a $\chi_{p}^{2}$ distribution.

## Example 2: estat archlm

We refit the original model that does not include the two lags of consump and then use estat archlm to see if there is any evidence that the errors are autoregressive conditional heteroskedastic.

| Source | SS | df | MS |  | Number of obs $=\quad 22$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Model | 532.567711 | 1532 | 67711 |  | Prob > F | 0.0004 |
| Residual | 601.207167 | 2030. | 03584 |  | R -squared | $=0.4697$ |
| Total | 1133.77488 | 2153. | 92799 |  | Adj R-squared <br> Root MSE | $\begin{aligned} & =0.4432 \\ & =5.4827 \end{aligned}$ |
| consump | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Conf. | Interval] |
| wagegovt | 2.50744 | . 5957173 | 4.21 | 0.000 | 1.264796 | 3.750085 |
| _cons | 40.84699 | 3.192183 | 12.80 | 0.000 | 34.18821 | 47.50577 |

. estat archlm, lags(1 223 )
LM test for autoregressive conditional heteroskedasticity (ARCH)

| $\operatorname{lags}(p)$ | chi2 | df | Prob >chi2 |
| :---: | :---: | :---: | :---: |
| 1 | 5.543 | 1 | 0.0186 |
| 2 | 9.431 | 2 | 0.0090 |
| 3 | 9.039 | 3 | 0.0288 |
| H0: no ARCH effects |  |  | vs. |
| H1: $\operatorname{ARCH}(p)$ disturbance |  |  |  |

estat archlm shows the results for tests of ARCH(1), ARCH(2), and ARCH(3) effects, respectively. At the $5 \%$ significance level, all three tests reject the null hypothesis that the errors are not autoregressive conditional heteroskedastic. See [TS] arch for information on fitting ARCH models.

## Stored results

estat archlm stores the following in $r()$ :
Scalars
$r(N) \quad$ number of observations $\quad r\left(N \_\right.$gaps $) \quad$ number of gaps
$r(k) \quad$ number of regressors
Macros
r(lags) lag order
Matrices

| $r(\operatorname{arch})$ | test statistic for each lag order <br> degrees of freedom | $r(p)$ | two-sided $p$-values |
| :--- | :--- | :--- | :--- |

estat bgodfrey stores the following in $r()$ :
Scalars
$r(N) \quad$ number of observation
$r$ (N_gaps) number of gaps
$r(k) \quad$ number of regressors
Macros
r(lags) lag order
Matrices

```
        r(chi2)
    r(F) F
    r(df_r) residual degrees of freedom (small
    only)
```

$r(\mathrm{p}) \quad$ two-sided $p$-values
r(df)
degrees of freedom
estat durbinalt stores the following in $r()$ :
Scalars
$r(N) \quad$ number of observation
$r$ (N_gaps) number of gaps
$r(k) \quad$ number of regressors
Macros
r(lags) lag order
Matrices
$r($ chi2 $) \quad \chi^{2}$ statistic for each lag order $\quad r(p) \quad$ two-sided $p$-values
$r\left(d f \_r\right)$ residual degrees of freedom (small
only)
estat dwatson stores the following in $r()$ :
Scalars

| $r(N)$ | number of observations | $r\left(N \_\right.$gaps $)$ | number of gaps |
| :--- | :--- | :--- | :--- |
| $r(k)$ | number of regressors | $r(d w)$ | Durbin-Watson statistic |

## Methods and formulas

Consider the regression

$$
\begin{equation*}
y_{t}=\beta_{1} x_{1 t}+\cdots+\beta_{k} x_{k t}+u_{t} \tag{4}
\end{equation*}
$$

in which some of the covariates are not strictly exogenous. In particular, some of the $x_{i t}$ may be lags of the dependent variable. We are interested in whether the $u_{t}$ are serially correlated.

The Durbin-Watson $d$ statistic reported by estat dwatson is

$$
d=\frac{\sum_{t=1}^{n-1}\left(\widehat{u}_{t+1}-\widehat{u}_{t}\right)^{2}}{\sum_{t=1}^{n} \widehat{u}_{t}^{2}}
$$

where $\widehat{u}_{t}$ represents the residual of the $t$ th observation.
To compute Durbin's alternative test and the Breusch-Godfrey test against the null hypothesis that there is no $p$ th order serial correlation, we fit the regression in (4), compute the residuals, and then fit the following auxiliary regression of the residuals $\widehat{u}_{t}$ on $p$ lags of $\widehat{u}_{t}$ and on all the covariates in the original regression in (4):

$$
\begin{equation*}
\widehat{u}_{t}=\gamma_{1} \widehat{u}_{t-1}+\cdots+\gamma_{p} \widehat{u}_{t-p}+\beta_{1} x_{1 t}+\cdots+\beta_{k} x_{k t}+\epsilon \tag{5}
\end{equation*}
$$

Durbin's alternative test is computed by performing a Wald test to determine whether the coefficients of $\widehat{u}_{t-1}, \ldots, \widehat{u}_{t-p}$ are jointly different from zero. By default, the statistic is assumed to be distributed $\chi^{2}(\mathrm{p})$. When small is specified, the statistic is assumed to follow an $F(p, N-p-k)$ distribution. The reported $p$-value is a two-sided $p$-value. When robust is specified, the Wald test is performed using the Huber/White/sandwich estimator of the variance-covariance matrix, and the test is robust to an unspecified form of heteroskedasticity.

The Breusch-Godfrey test is computed as $N R^{2}$, where $N$ is the number of observations in the auxiliary regression (5) and $R^{2}$ is the $R^{2}$ from the same regression (5). Like Durbin's alternative test, the Breusch-Godfrey test is asymptotically distributed $\chi^{2}(p)$, but specifying small causes the $p$-value to be computed using an $F(p, N-p-k)$.

By default, the initial missing values of the lagged residuals are replaced with zeros, and the auxiliary regression is run over the full sample used in the original regression of (4). Specifying the nomiss0 option causes these missing values to be treated as missing values, and the observations are dropped from the sample.

Engle's LM test for $\operatorname{ARCH}(p)$ effects fits an OLS regression of $\widehat{u}_{t}^{2}$ on $\widehat{u}_{t-1}^{2}, \ldots, \widehat{u}_{t-p}^{2}$ :

$$
\widehat{u}_{t}^{2}=\gamma_{0}+\gamma_{1} \widehat{u}_{t-1}^{2}+\cdots+\gamma_{p} \widehat{u}_{t-p}^{2}+\epsilon
$$

The test statistic is $n R^{2}$ and is asymptotically distributed $\chi^{2}(p)$.

## Acknowledgment

The original versions of estat archlm, estat bgodfrey, and estat durbinalt were written by Christopher F. Baum of the Department of Economics at Boston College and author of the Stata Press books An Introduction to Modern Econometrics Using Stata and An Introduction to Stata Programming.

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James Durbin (1923-2012) was a British statistician who was born in Wigan, near Manchester. He studied mathematics at Cambridge and after military service and various research posts joined the London School of Economics in 1950. Later in life, he was also affiliated with University College London. His many contributions to statistics centered on serial correlation, time series (including major contributions to structural or unobserved components models), sample survey methodology, goodness-of-fit tests, and sample distribution functions, with emphasis on applications in the social sciences. He served terms as president of the Royal Statistical Society and the International Statistical Institute.

Geoffrey Stuart Watson (1921-1998) was born in Victoria, Australia, and earned degrees at Melbourne University and North Carolina State University. After a visit to the University of Cambridge, he returned to Australia, working at Melbourne and then the Australian National University. Following periods at Toronto and Johns Hopkins, he settled at Princeton. Throughout his wide-ranging career, he made many notable accomplishments and important contributions, including the Durbin-Watson test for serial correlation, the Nadaraya-Watson estimator in nonparametric regression, and methods for analyzing directional data.

Leslie G. Godfrey (1946- ) was born in London and earned degrees at the Universities of Exeter and London. He is now a professor of econometrics at the University of York. His interests center on implementation and interpretation of tests of econometric models, including nonnested models.

Trevor Stanley Breusch (1949- ) was born in Queensland and earned degrees at the University of Queensland and Australian National University (ANU). After a post at the University of Southampton, he returned to work at ANU. His background is in econometric methods and his recent interests include political values and social attitudes, earnings and income, and measurement of underground economic activity.

## Also see

[ R ] regress - Linear regression
[TS] tsset - Declare data to be time-series data
$[R]$ regress postestimation - Postestimation tools for regress
$[R]$ regress postestimation diagnostic plots - Postestimation plots for regress

## Title

\#review - Review previous commands

## Syntax Description Remarks and examples

## Syntax

```
#review [## [#2]]
```


## Description

The \#review command displays the last few lines typed at the terminal.

## Remarks and examples

\#review (pronounced pound-review) is a Stata preprocessor command. \#commands do not generate a return code or generate ordinary Stata errors. The only error message associated with \#commands is "unrecognized \#command".

The \#review command displays the last few lines typed at the terminal. If no arguments follow \#review, the last five lines typed at the terminal are displayed. The first argument specifies the number of lines to be reviewed, so \#review 10 displays the last 10 lines typed. The second argument specifies the number of lines to be displayed, so \#review 105 displays five lines, starting at the 10th previous line.

Stata reserves a buffer for \#review lines and stores as many previous lines in the buffer as will fit, rolling out the oldest line to make room for the newest. Requests to \#review lines no longer stored will be ignored. Only lines typed at the terminal are placed in the \#review buffer. See [U] 10.5 Editing previous lines in Stata.
$>$ Example 1
Typing \#review by itself will show the last five lines you typed at the terminal:

```
. #review
5 use mydata
4 * comments go into the #review buffer, too
3 describe
2 tabulate marriage educ [freq=number]
1 tabulate marriage educ [freq=number], chi2
```

Typing \#review 152 shows the 15th and 14th previous lines:

```
. #review 15 2
15 replace x=. if x<200
14 summarize x
```


## Title

roc - Receiver operating characteristic (ROC) analysis

## Description Reference

## Description

ROC analysis quantifies the accuracy of diagnostic tests or other evaluation modalities used to discriminate between two states or conditions, which are here referred to as normal and abnormal or control and case. The discriminatory accuracy of a diagnostic test is measured by its ability to correctly classify known normal and abnormal subjects. For this reason, we often refer to the diagnostic test as a classifier. The analysis uses the ROC curve, a graph of the sensitivity versus 1 - specificity of the diagnostic test. The sensitivity is the fraction of positive cases that are correctly classified by the diagnostic test, whereas the specificity is the fraction of negative cases that are correctly classified. Thus the sensitivity is the true-positive rate, and the specificity is the true-negative rate.

There are six ROC commands:

| Command | Entry | Description |
| :--- | :--- | :--- |
| roccomp | $[R]$ roccomp | Tests of equality of ROC areas |
| rocgold | $[R]$ roccomp | Tests of equality of ROC areas against a standard ROC curve |
| rocfit | $[R]$ rocfit | Parametric ROC models |
| rocreg | $[R]$ rocreg | Nonparametric and parametric ROC regression models |
| rocregplot | $[R]$ rocregplot | Plot marginal and covariate-specific ROC curves |
| roctab | $[R]$ roctab | Nonparametric ROC analysis |

Postestimation commands are available after rocfit and rocreg; see [R] rocfit postestimation and [R] rocreg postestimation.

Both nonparametric and parametric (semiparametric) methods have been suggested for generating the ROC curve. The roctab command performs nonparametric ROC analysis for a single classifier. roccomp extends the nonparametric ROC analysis function of roctab to situations where we have multiple diagnostic tests of interest to be compared and tested. The rocgold command also provides ROC analysis for multiple classifiers. rocgold compares each classifier's ROC curve to a "gold standard" ROC curve and makes adjustments for multiple comparisons in the analysis. Both rocgold and roccomp also allow parametric estimation of the ROC curve through a binormal fit. In a binormal fit, both the control and the case populations are normal.

The rocfit command also estimates the ROC curve of a classifier through a binormal fit. Unlike roctab, roccomp, and rocgold, rocfit is an estimation command. In postestimation, graphs of the ROC curve and confidence bands can be produced. Additional tests on the parameters can also be conducted.

ROC analysis can be interpreted as a two-stage process. First, the control distribution of the classifier is estimated, assuming a normal model or using a distribution-free estimation technique. The classifier is standardized using the control distribution to 1 - percentile value, the false-positive rate. Second, the ROC curve is estimated as the case distribution of the standardized classifier values.

Covariates may affect both stages of ROC analysis. The first stage may be affected, yielding a covariate-adjusted ROC curve. The second stage may also be affected, producing multiple covariatespecific ROC curves.

The rocreg command performs ROC analysis under both types of covariate effects. Both parametric (semiparametric) and nonparametric methods may be used by rocreg. Like rocfit, rocreg is an estimation command and provides many postestimation capabilities.

The global performance of a diagnostic test is commonly summarized by the area under the ROC curve (AUC). This area can be interpreted as the probability that the result of a diagnostic test of a randomly selected abnormal subject will be greater than the result of the same diagnostic test from a randomly selected normal subject. The greater the AUC, the better the global performance of the diagnostic test. Each of the ROC commands provides computation of the AUC.

Citing a lack of clinical relevance for the AUC, other ROC summary measures have been suggested. These include the partial area under the ROC curve for a given false-positive rate $t$ [pAUC $(t)$ ]. This is the area under the ROC curve from the false-positive rate of 0 to $t$. The ROC value at a particular false-positive rate and the false-positive rate for a particular ROC value are also useful summary measures for the ROC curve. These three measures are directly estimated by rocreg during the model fit or postestimation stages. Point estimates of ROC value are computed by the other ROC commands, but no standard errors are reported.

See Pepe (2003) for a discussion of ROC analysis. Pepe has posted Stata datasets and programs used to reproduce results presented in the book (http://www.stata.com/bookstore/pepe.html).

## Reference

Pepe, M. S. 2003. The Statistical Evaluation of Medical Tests for Classification and Prediction. New York: Oxford University Press.

## Title

roccomp - Tests of equality of ROC areas

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |
| Also see |  |  |  |

## Syntax

Test equality of ROC areas roccomp refvar classvar [classvars] [if] [in] [weight] [, roccomp_options]

Test equality of ROC area against a standard ROC curve
rocgold refvar goldvar classvar [classvars] [if] [in] [weight] [, rocgold_options]
roccomp_options
Main
by (varname)
test (matname)
graph
norefline
separate
summary
binormal
line\#opts(cline_options)
level(\#)
Plot
plot\#opts(plot_options)
Reference line
rlopts (cline_options)
Y axis, X axis, Titles, Legend, Overall twoway_options

Description
split into groups by variable
use contrast matrix for comparing ROC areas graph the ROC curve
suppress plotting the 45 -degree reference line
place each ROC curve on its own graph
report the area under the ROC curve
estimate areas by using binormal distribution assumption
affect rendition of the \#th binormal fit line
set confidence level; default is level (95)
affect rendition of the \#th ROC curve
affect rendition of the reference line
any options other than by () documented in [G-3] twoway_options
rocgold_options

Main
sidak
test (matname)
graph
norefline
separate
summary
binormal
line\#opts (cline_options)
level(\#)

## Plot

plot\#opts (plot_options)
Reference line
rlopts(cline_options)
Y axis, X axis, Titles, Legend, Overall twoway_options

Description
adjust the significance probability by using Šidák's method
use contrast matrix for comparing ROC areas
graph the ROC curve
suppress plotting the 45 -degree reference line
place each ROC curve on its own graph
report the area under the ROC curve
estimate areas by using binormal distribution assumption
affect rendition of the \#th binormal fit line
set confidence level; default is level (95)
affect rendition of the \#th ROC curve; plot 1 is the "gold standard" affect rendition of the reference line
any options other than by () documented in [G-3] twoway_options
plot_options Description
marker_options
marker_label_options
cline_options
change look of markers (color, size, etc.)
add marker labels; change look or position
change the look of the line
fweights are allowed; see [U] 11.1.6 weight.

## Menu

## roccomp

Statistics $>$ Epidemiology and related $>$ ROC analysis $>$ Test equality of two or more ROC areas
rocgold
Statistics $>$ Epidemiology and related $>$ ROC analysis $>$ Test equality of ROC area against gold standard

## Description

The above commands are used to perform receiver operating characteristic (ROC) analyses with rating and discrete classification data.

The two variables refvar and classvar must be numeric. The reference variable indicates the true state of the observation, such as diseased and nondiseased or normal and abnormal, and must be coded as 0 and 1 . The rating or outcome of the diagnostic test or test modality is recorded in classvar, which must be at least ordinal, with higher values indicating higher risk.
roccomp tests the equality of two or more ROC areas obtained from applying two or more test modalities to the same sample or to independent samples. roccomp expects the data to be in wide form when comparing areas estimated from the same sample and in long form for areas estimated from independent samples.
rocgold independently tests the equality of the ROC area of each of several test modalities, specified by classvar, against a "gold standard" ROC curve, goldvar. For each comparison, rocgold reports the raw and the Bonferroni-adjusted significance probability. Optionally, Šidák's adjustment for multiple comparisons can be obtained.

See $[R]$ rocfit and $[R]$ rocreg for commands that fit maximum-likelihood ROC models.

## Options

$\qquad$
Main
by (varname) (roccomp only) is required when comparing independent ROC areas. The by () variable identifies the groups to be compared.
sidak (rocgold only) requests that the significance probability be adjusted for the effect of multiple comparisons by using Šidák's method. Bonferroni's adjustment is reported by default.
test (matname) specifies the contrast matrix to be used when comparing ROC areas. By default, the null hypothesis that all areas are equal is tested.
graph produces graphical output of the ROC curve.
norefline suppresses plotting the 45 -degree reference line from the graphical output of the ROC curve.
separate is meaningful only with roccomp and specifies that each ROC curve be placed on its own graph rather than one curve on top of the other.
summary reports the area under the ROC curve, its standard error, and its confidence interval. This option is needed only when also specifying graph.
binormal specifies that the areas under the ROC curves to be compared should be estimated using the binormal distribution assumption. By default, areas to be compared are computed using the trapezoidal rule.
line\#opts(cline_options) affects the rendition of the line representing the \#th ROC curve drawn using the binormal distribution assumption; see [G-3] cline_options. These lines are drawn only if the binormal option is specified.
level(\#) specifies the confidence level, as a percentage, for the confidence intervals. The default is level (95) or as set by set level; see [R] level.

Plot
plot\#opts(plot_options) affects the rendition of the \#th ROC curve-the curve's plotted points connected by lines. The plot_options can affect the size and color of markers, whether and how the markers are labeled, and whether and how the points are connected; see [G-3] marker_options, [G-3] marker_label_options, and [G-3] cline_options.
For rocgold, plot1opts() are applied to the ROC for the gold standard.

Reference line
rlopts(cline_options) affects the rendition of the reference line; see [G-3] cline_options.
twoway_options are any of the options documented in [G-3] twoway_options. These include options for titling the graph (see [G-3] title_options), options for saving the graph to disk (see [G-3] saving_option), and the by () option (see [G-3] by_option).

## Remarks and examples

Remarks are presented under the following headings:

Introduction<br>Comparing areas under the ROC curve<br>Correlated data<br>Independent data<br>Comparing areas with a gold standard

## Introduction

roccomp provides comparison of the ROC curves of multiple classifiers. rocgold compares the ROC curves of multiple classifiers with a single "gold standard" classifier. Adjustment of inference for multiple comparisons is also provided by rocgold.

See Pepe (2003) for a discussion of ROC analysis. Pepe has posted Stata datasets and programs used to reproduce results presented in the book (http://www.stata.com/bookstore/pepe.html).

## Comparing areas under the ROC curve

The area under multiple ROC curves can be compared by using roccomp. The command syntax is slightly different if the ROC curves are correlated (that is, different diagnostic tests are applied to the same sample) or independent (that is, diagnostic tests are applied to different samples).

## Correlated data

$>$ Example 1
Hanley and McNeil (1983) presented data from an evaluation of two computer algorithms designed to reconstruct CT images from phantoms. We will call these two algorithms' modalities 1 and 2. A sample of 112 phantoms was selected; 58 phantoms were considered normal, and the remaining 54 were abnormal. Each of the two modalities was applied to each phantom, and the resulting images were rated by a reviewer using a six-point scale: $1=$ definitely normal, $2=$ probably normal, 3 $=$ possibly normal, $4=$ possibly abnormal, $5=$ probably abnormal, and $6=$ definitely abnormal. Because each modality was applied to the same sample of phantoms, the two sets of outcomes are correlated.

We list the first 7 observations:

```
. use http://www.stata-press.com/data/r13/ct
. list in 1/7, sep(0)
```

|  | $\bmod 1$ | $\bmod 2$ | status |
| :--- | ---: | ---: | ---: |
| 1. | 2 | 1 | 0 |
| 2. | 5 | 5 | 1 |
| 3. | 2 | 1 | 0 |
| 4. | 2 | 3 | 0 |
| 5. | 5 | 6 | 1 |
| 6. | 2 | 2 | 0 |
| 7. | 3 | 2 | 0 |

The data are in wide form, which is required when dealing with correlated data. Each observation corresponds to one phantom. The variable mod1 identifies the rating assigned for the first modality, and mod2 identifies the rating assigned for the second modality. The true status of the phantoms is given by status=0 if they are normal and status=1 if they are abnormal. The observations with at least one missing rating were dropped from the analysis.

We plot the two ROC curves and compare their areas.
. roccomp status mod1 mod2, graph summary

|  | ROC |  | Std. Err. | -Asymptotic Normal[95\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| mod1 | 112 | 0.8828 | 0.0317 | 0.82067 | 0.94498 |
| mod2 | 112 | 0.9302 | 0.0256 | 0.88005 | 0.98042 |

```
Ho: area(mod1) = area(mod2)
    chi2(1) = 2.31 Prob>chi2 = 0.1282
```



By default, roccomp, with the graph option specified, plots the ROC curves on the same graph. Optionally the curves can be plotted side by side, each on its own graph, by also specifying separate.

For each curve, roccomp reports summary statistics and provides a test for the equality of the area under the curves, using an algorithm suggested by DeLong, DeLong, and Clarke-Pearson (1988).

Although the area under the ROC curve for modality 2 is larger than that of modality 1 , the chi-squared test yielded a significance probability of 0.1282 , suggesting that there is no significant difference between these two areas.

The roccomp command can also be used to compare more than two ROC areas. To illustrate this, we modified the previous dataset by including a fictitious third modality.

|  | Obs | $\begin{array}{r} \text { ROC } \\ \text { Area } \end{array}$ | Std. Err. | -Asymptot [95\% Conf | $\begin{gathered} \text { c Normal- } \\ \text { Interval] } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| mod1 | 112 | 0.8828 | 0.0317 | 0.82067 | 0.94498 |
| mod2 | 112 | 0.9302 | 0.0256 | 0.88005 | 0.98042 |
| mod3 | 112 | 0.9240 | 0.0241 | 0.87670 | 0.97132 |
| ```Ho: area(mod1) = area(mod2) = area(mod3) chi2(2) = 6.54 Prob>chi2 = 0.0381``` |  |  |  |  |  |



By default, roccomp tests whether the areas under the ROC curves are all equal. Other comparisons can be tested by creating a contrast matrix and specifying test (matname), where matname is the name of the contrast matrix.

For example, assume that we are interested in testing whether the area under the ROC for mod1 is equal to that of mod3. To do this, we can first create an appropriate contrast matrix and then specify its name with the test() option.

Of course, this is a trivial example because we could have just specified

```
. roccomp status mod1 mod3
```

without including mod2 to obtain the same test results. However, for illustration, we will continue with this example.

The contrast matrix must have its number of columns equal to the number of classvars (that is, the total number of ROC curves) and a number of rows less than or equal to the number of classvars, and the elements of each row must add to zero.

```
. matrix C=(1,0,-1)
```

. roccomp status mod1 mod2 mod3, test(C)

|  | ROC |  |  | -Asymptotic Normal- |  |
| :--- | :---: | ---: | :---: | ---: | ---: |
|  | Obs | Area | Std. Err. | [95\% Conf. Interval] |  |
| mod1 | 112 | 0.8828 | 0.0317 | 0.82067 | 0.94498 |
| mod2 | 112 | 0.9302 | 0.0256 | 0.88005 | 0.98042 |
| mod3 | 112 | 0.9240 | 0.0241 | 0.87670 | 0.97132 |

Ho: Comparison as defined by contrast matrix: C
$\operatorname{chi2}(1)=5.25 \quad$ Prob>chi2 $=0.0220$
Although all three areas are reported, the comparison is made using the specified contrast matrix.
Perhaps more interesting would be a comparison of the area from mod1 and the average area of $\bmod 2$ and mod3.

```
. matrix C=(1,-. 5,-.5)
. roccomp status mod1 mod2 mod3, test(C)
```

|  | ROC |  |  | -Asymptotic Normal- <br> [95\% Conf. |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | Obs | Area | Std. Err. | 0.94498 |  |
| mod1 | 112 | 0.8828 | 0.0317 | 0.82067 | 0.940 |
| mod2 | 112 | 0.9302 | 0.0256 | 0.88005 | 0.98042 |
| mod3 | 112 | 0.9240 | 0.0241 | 0.87670 | 0.97132 |

```
Ho: Comparison as defined by contrast matrix: C
    chi2(1) = 3.43 Prob>chi2 = 0.0642
```

Other contrasts could be made. For example, we could test if mod3 is different from at least one of the other two by first creating the following contrast matrix:

```
. matrix C=(-1,0,1 \ 0,-1,1)
. mat list C
C[2,3]
    c1 c2 c3
r1 -1 0
r2 0
```


## Independent data

## > Example 2

In example 1, we noted that because each test modality was applied to the same sample of phantoms, the classification outcomes were correlated. Now assume that we have collected the same data presented by Hanley and McNeil (1983), except that we applied the first test modality to one sample of phantoms and the second test modality to a different sample of phantoms. The resulting measurements are now considered independent.

Here are a few of the observations.

```
. use http://www.stata-press.com/data/r13/ct3
. list in 1/7, sep(0)
```

| pop | status | rating | mod |
| ---: | ---: | ---: | ---: |
| 12 | 0 | 1 | 1 |
| 31 | 0 | 1 | 2 |
| 1 | 1 | 1 | 1 |
| 3 | 1 | 1 | 2 |
| 28 | 0 | 2 | 1 |
| 19 | 0 | 2 | 2 |
| 3 | 1 | 2 | 1 |

The data are in long form, which is required when dealing with independent data. The data consist of 24 observations: 6 observations corresponding to abnormal phantoms and 6 to normal phantoms evaluated using the first modality, and similarly 6 observations corresponding to abnormal phantoms and 6 to normal phantoms evaluated using the second modality. The number of phantoms corresponding to each observation is given by the pop variable. Once again we have frequency-weighted data. The variable mod identifies the modality, and rating is the assigned classification.

We can better view our data by using the table command.
. table status rating [fw=pop], by(mod) row col

| mod and <br> status | rating |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | Total |
| 1 |  |  |  |  |  |  |  |
| 0 | 12 | 28 | 8 | 6 | 4 |  | 58 |
| 1 | 1 | 3 | 6 | 13 | 22 | 9 | 54 |
| Total | 13 | 31 | 14 | 19 | 26 | 9 | 112 |
| 2 |  |  |  |  |  |  |  |
| 0 | 31 | 19 | 5 | 3 |  |  | 58 |
| 1 | 3 | 2 | 5 | 19 | 15 | 10 | 54 |
| Total | 34 | 21 | 10 | 22 | 15 | 10 | 112 |

The status variable indicates the true status of the phantoms: status $=0$ if they are normal and status $=1$ if they are abnormal.

We now compare the areas under the two ROC curves.

```
. roccomp status rating [fw=pop], by(mod) graph summary
```

|  |  |  |  |  | ROC |
| :--- | :---: | ---: | :---: | ---: | ---: |
| mod | Obs | Area | Std. Err. | -Asymptotic Normal- <br> [95\% Conf. Interval] |  |
| 1 | 112 | 0.8828 | 0.0317 | 0.82067 | 0.94498 |
| 2 | 112 | 0.9302 | 0.0256 | 0.88005 | 0.98042 |

```
Ho: area(1) = area(2)
    chi2(1) = 1.35 Prob>chi2 = 0.2447
```



## Comparing areas with a gold standard

The area under multiple ROC curves can be compared with a gold standard using rocgold. The command syntax is similar to that of roccomp. The tests are corrected for the effect of multiple comparisons.

## > Example 3

We will use the same data (presented by Hanley and McNeil [1983]) as in the roccomp examples. Let's assume that the first modality is considered to be the standard against which both the second and third modalities are compared.

We want to plot and compare both the areas of the ROC curves of mod2 and mod3 with mod1. Because we consider mod1 to be the gold standard, it is listed first after the reference variable in the rocgold command line.
. use http://www.stata-press.com/data/r13/ct2
. rocgold status mod1 mod2 mod3, graph summary

|  | ROC <br> Area | Std. Err. | chi2 | df | Pr>chi2 | Bonferroni <br> Pr>chi2 |
| :--- | ---: | :---: | :---: | :---: | :---: | ---: |
| mod1 (standard) | 0.8828 | 0.0317 |  |  |  |  |
| mod2 | 0.9302 | 0.0256 | 2.3146 | 1 | 0.1282 | 0.2563 |
| mod3 | 0.9240 | 0.0241 | 5.2480 | 1 | 0.0220 | 0.0439 |



Equivalently, we could have done this in two steps by using the roccomp command.
. roccomp status mod1 mod2, graph summary
. roccomp status mod1 mod3, graph summary

## Stored results

roccomp stores the following in $r()$ :
Scalars

$$
\begin{array}{llll}
r\left(N \_g\right) & \text { number of groups } & r(d f) & \chi^{2} \text { degrees of freedom } \\
r(p) & \text { significance probability } & r(\text { chi2 }) & \chi^{2}
\end{array}
$$

Matrices

$$
r(V) \quad \text { variance-covariance matrix }
$$

rocgold stores the following in $r()$ :
Scalars
$r\left(N \_g\right)$ number of groups
Matrices

| $r(V)$ | variance-covariance matrix | $r(p)$ | significance-probability vector <br> $r(c h i 2)$$\chi^{2}$ vector |
| :--- | :--- | :--- | :--- |
| $r(d f)$ | $\chi^{2}$ degrees-of-freedom vector | $r(p-a d j)$ | adjusted significance-probability vector |

## Methods and formulas

Assume that we applied a diagnostic test to each of $N_{n}$ normal and $N_{a}$ abnormal subjects. Further assume that the higher the outcome value of the diagnostic test, the higher the risk of the subject being abnormal. Let $\widehat{\theta}$ be the estimated area under the curve, and let $X_{i}, i=1,2, \ldots, N_{a}$ and $Y_{j}, j=1,2, \ldots, N_{n}$ be the values of the diagnostic test for the abnormal and normal subjects, respectively.

Areas under ROC curves are compared using an algorithm suggested by DeLong, DeLong, and Clarke-Pearson (1988). Let $\widehat{\boldsymbol{\theta}}=\left(\widehat{\theta^{1}}, \widehat{\theta^{2}}, \ldots, \widehat{\theta^{k}}\right)$ be a vector representing the areas under $k$ ROC curves. See Methods and formulas in [R] roctab for the definition of these area estimates.

For the $r$ th area, define

$$
V_{10}^{r}\left(X_{i}\right)=\frac{1}{N_{n}} \sum_{j=1}^{N_{n}} \psi\left(X_{i}^{r}, Y_{j}^{r}\right)
$$

and for each normal subject, $j$, define

$$
V_{01}^{r}\left(Y_{j}\right)=\frac{1}{N_{a}} \sum_{i=1}^{N_{a}} \psi\left(X_{i}^{r}, Y_{j}^{r}\right)
$$

where

$$
\psi\left(X^{r}, Y^{r}\right)= \begin{cases}1 & Y^{r}<X^{r} \\ \frac{1}{2} & Y^{r}=X^{r} \\ 0 & Y^{r}>X^{r}\end{cases}
$$

Define the $k \times k$ matrix $\mathbf{S}_{\mathbf{1 0}}$ such that the $(r, s)$ th element is

$$
S_{10}^{r, s}=\frac{1}{N_{a}-1} \sum_{i=1}^{N_{a}}\left\{V_{10}^{r}\left(X_{i}\right)-\widehat{\theta^{r}}\right\}\left\{V_{10}^{s}\left(X_{i}\right)-\widehat{\theta^{s}}\right\}
$$

and $\mathbf{S}_{\mathbf{0 1}}$ such that the $(r, s)$ th element is

$$
S_{01}^{r, s}=\frac{1}{N_{n}-1} \sum_{j=1}^{N_{n}}\left\{V_{01}^{r}\left(Y_{i}\right)-\widehat{\theta^{r}}\right\}\left\{V_{01}^{s}\left(Y_{i}\right)-\widehat{\theta^{s}}\right\}
$$

Then the covariance matrix is

$$
S=\frac{1}{N_{a}} S_{10}+\frac{1}{N_{n}} S_{01}
$$

Let $\mathbf{L}$ be a contrast matrix defining the comparison, so that

$$
\left.(\widehat{\theta}-\theta)^{\prime} \mathbf{L}^{\prime}(\mathbf{L} \mathbf{S L})^{\prime}\right)^{-\mathbf{1}} \mathbf{L}(\widehat{\theta}-\theta)
$$

has a chi-squared distribution with degrees of freedom equal to the rank of $\mathbf{L S L}$.

## References

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## Also see <br> [R] logistic postestimation - Postestimation tools for logistic <br> $[R]$ roc - Receiver operating characteristic (ROC) analysis <br> [R] rocfit - Parametric ROC models <br> $[R]$ rocreg - Receiver operating characteristic (ROC) regression <br> [R] roctab - Nonparametric ROC analysis

## Title

rocfit — Parametric ROC models

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |
| Also see |  |  |  |

## Syntax

$$
\text { rocfit refvar classvar }[\text { if }][\text { in }][\text { weight }][\text {, rocfit_options }]
$$

rocfit_options Description

Model
continuous (\#)
generate(newvar)
SE
vce (vcetype) vcetype may be oim or opg
Reporting
level(\#)
Maximization
maximize_options
divide classvar into \# groups of approximately equal length create newvar containing classification groups
set confidence level; default is level(95)
control the maximization process; seldom used
fp, is allowed; see [U] 11.1.10 Prefix commands.
fweights are allowed; see [U] 11.1.6 weight.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Epidemiology and related $>$ ROC analysis $>$ Parametric ROC analysis without covariates

## Description

rocfit fits maximum-likelihood ROC models assuming a binormal distribution of the latent variable.
The two variables refvar and classvar must be numeric. The reference variable indicates the true state of the observation, such as diseased and nondiseased or normal and abnormal, and must be coded as 0 and 1 . The rating or outcome of the diagnostic test or test modality is recorded in classvar, which must be at least ordinal, with higher values indicating higher risk.

See $[R]$ roc for other commands designed to perform receiver operating characteristic (ROC) analyses with rating and discrete classification data.

## Options

> Model
> continuous(\#) specifies that the continuous classvar be divided into \# groups of approximately equal length. This option is required when classvar takes on more than 20 distinct values.
> continuous(.) may be specified to indicate that classvar be used as it is, even though it could have more than 20 distinct values.
> generate (newvar) specifies the new variable that is to contain the values indicating the groups produced by continuous(\#). generate() may be specified only with continuous().

Reporting
level(\#); see [R] estimation options.
maximize_options: difficult, technique (algorithm_spec), iterate (\#), [no log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance(\#), nonrtolerance, and from(init_specs); see [R] maximize. These options are seldom used.

Setting the optimization type to technique (bhhh) resets the default vcetype to vce (opg).

## Remarks and examples

Dorfman and Alf (1969) developed a generalized approach for obtaining maximum likelihood estimates of the parameters for a smooth fitting ROC curve. The most commonly used method for ordinal data, and the one implemented here, is based upon the binormal model; see Pepe (2003), Pepe, Longton, and Janes (2009), and Janes, Longton, and Pepe (2009) for methods of ROC analysis for continuous data, including methods for adjusting for covariates.

The model assumes the existence of an unobserved, continuous, latent variable that is normally distributed (perhaps after a monotonic transformation) in both the normal and abnormal populations with means $\mu_{n}$ and $\mu_{a}$ and variances $\sigma_{n}^{2}$ and $\sigma_{a}^{2}$, respectively. The model further assumes that the $K$ categories of the rating variable result from partitioning the unobserved latent variable by $K-1$ fixed boundaries. The method fits a straight line to the empirical ROC points plotted using normal probability scales on both axes. Maximum likelihood estimates of the line's slope and intercept and the $K-1$ boundaries are obtained simultaneously. See Methods and formulas for details.

The intercept from the fitted line is a measurement of $\left(\mu_{a}-\mu_{n}\right) / \sigma_{a}$, and the slope measures $\sigma_{n} / \sigma_{a}$.

Thus the intercept is the standardized difference between the two latent population means, and the slope is the ratio of the two standard deviations. The null hypothesis that there is no difference between the two population means is evaluated by testing that the intercept $=0$, and the null hypothesis that the variances in the two populations are equal is evaluated by testing that the slope $=1$.

## Example 1

We use Hanley and McNeil's (1982) dataset, described in example 1 of [R] roctab, to fit a smooth ROC curve assuming a binormal model.

```
. use http://www.stata-press.com/data/r13/hanley
. rocfit disease rating
Fitting binormal model:
Iteration 0: log likelihood = -123.68069
Iteration 1: log likelihood = -123.64867
Iteration 2: log likelihood = -123.64855
Iteration 3: log likelihood = -123.64855
Binormal model of disease on rating Number of obs = 109
Goodness-of-fit chi2(2) = 0.21
Prob > chi2 = 0.9006
Log likelihood = -123.64855
```

|  | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| intercept | 1.656782 | 0.310456 | 5.34 | 0.000 | 1.048300 | 2.265265 |
| slope (*) | 0.713002 | 0.215882 | -1.33 | 0.184 | 0.289881 | 1.136123 |
| /cut1 | 0.169768 | 0.165307 | 1.03 | 0.304 | -0.154227 | 0.493764 |
| /cut2 | 0.463215 | 0.167235 | 2.77 | 0.006 | 0.135441 | 0.790990 |
| /cut3 | 0.766860 | 0.174808 | 4.39 | 0.000 | 0.424243 | 1.109477 |
| /cut4 | 1.797938 | 0.299581 | 6.00 | 0.000 | 1.210770 | 2.385106 |
| Index | Indices from binormal fit |  |  |  |  |  |
| ROC area | 0.911331 | 0.029506 |  |  | 0.853501 | 0.969161 |
| delta(m) | 2.323671 | 0.502370 |  |  | 1.339044 | 3.308298 |
| d(e) | 1.934361 | 0.257187 |  |  | 1.430284 | 2.438438 |
| d(a) | 1.907771 | 0.259822 |  |  | 1.398530 | 2.417012 |

(*) z test for slope==1
rocfit outputs the MLE for the intercept and slope of the fitted regression line along with, here, four boundaries (because there are five ratings) labeled /cut1 through /cut4. Also rocfit computes and reports four indices based on the fitted ROC curve: the area under the curve (labeled ROC area), $\delta(m)$ (labeled delta $(\mathrm{m})$ ), $d_{e}$ (labeled $\mathrm{d}(\mathrm{e})$ ), and $d_{a}$ (labeled $\mathrm{d}(\mathrm{a})$ ). More information about these indices can be found in Methods and formulas and in Erdreich and Lee (1981).

## Stored results

rocfit stores the following in e():
Scalars

```
e(N)
e(k)
e(k_eq)
e(k_eq_model)
e(k_dv)
e(df_m)
e(ll)
e(chi2_gf)
e(df_gf)
e(p_gf)
e(area)
e(se_area)
e(deltam)
e(se_delm)
e(de)
e(se_de)
e(da)
e(se_da)
e(rank)
e(ic)
e(rc)
e(converged)
```

Macros
e(cmd)
e(cmdline)
e(depvar)
e(wtype)
e(wexp)
e(title)
e(chi2type)
e(vce)
e(vcetype)
e(opt)
e(which)
e(ml_method)
e(user)
e(technique)
e(properties)
Matrices
e(b)
e(ilog)
e(gradient)
e(V)

Functions

```
e(sample)
marks estimation sample
```

number of observations
number of parameters
number of equations in e(b)
number of equations in overall model test
number of dependent variables
model degrees of freedom
log likelihood
goodness-of-fit $\chi^{2}$
goodness-of-fit degrees of freedom
$\chi^{2}$ goodness-of-fit significance probability
area under the ROC curve
standard error for the area under the ROC curve
delta (m)
standard area for delta (m)
d(e) index
standard error for $d(e)$ index
d(a) index
standard error for $d(a)$ index
rank of e(V)
number of iterations
return code
1 if converged, 0 otherwise
rocfit
command as typed
refvar and classvar
weight type
weight expression
title in estimation output
GOF; type of model $\chi^{2}$ test
vcetype specified in vce()
title used to label Std. Err.
type of optimization
max or min; whether optimizer is to perform maximization or minimization
type of ml method
name of likelihood-evaluator program
maximization technique
b V
coefficient vector
iteration $\log$ (up to 20 iterations)
gradient vector
variance-covariance matrix of the estimators

## Methods and formulas

Dorfman and Alf (1969) developed a general procedure for obtaining maximum likelihood estimates of the parameters of a smooth-fitting ROC curve. The most common method, and the one implemented in Stata, is based upon the binormal model.

The model assumes that there is an unobserved continuous latent variable that is normally distributed in both the normal and abnormal populations. The idea is better explained with the following illustration:


The latent variable is assumed to be normally distributed for both the normal and abnormal subjects, perhaps after a monotonic transformation, with means $\mu_{n}$ and $\mu_{a}$ and variances $\sigma_{n}^{2}$ and $\sigma_{a}^{2}$, respectively.

This latent variable is assumed to be partitioned into the $k$ categories of the rating variable by $k-1$ fixed boundaries. In the above figure, the $k=5$ categories of the rating variable identified on the bottom result from the partition of the four boundaries $Z_{1}$ through $Z_{4}$.

Let $R_{j}$ for $j=1,2, \ldots, k$ indicate the categories of the rating variable, let $i=1$ if the subject belongs to the normal group, and let $i=2$ if the subject belongs to the abnormal group.

Then

$$
p\left(R_{j} \mid i=1\right)=F\left(Z_{j}\right)-F\left(Z_{j-1}\right)
$$

where $Z_{k}=\left(x_{k}-\mu_{n}\right) / \sigma_{n}, F$ is the cumulative normal distribution, $F\left(Z_{0}\right)=0$, and $F\left(Z_{k}\right)=1$. Also,

$$
p\left(R_{j} \mid i=2\right)=F\left(b Z_{j}-a\right)-F\left(b Z_{j-1}-a\right)
$$

where $b=\sigma_{n} / \sigma_{a}$ and $a=\left(\mu_{a}-\mu_{n}\right) / \sigma_{a}$.
The parameters $a, b$ and the $k-1$ fixed boundaries $Z_{j}$ are simultaneously estimated by maximizing the log-likelihood function

$$
\log L=\sum_{i=1}^{2} \sum_{j=1}^{k} r_{i j} \log \left\{p\left(R_{j} \mid i\right)\right\}
$$

where $r_{i j}$ is the number of $R_{j} \mathrm{~s}$ in group $i$.

The area under the fitted ROC curve is computed as

$$
\Phi\left(\frac{a}{\sqrt{1+b^{2}}}\right)
$$

where $\Phi$ is the standard normal cumulative distribution function.
Point estimates for the ROC curve indices are as follows:

$$
\delta(m)=\frac{a}{b} \quad d_{e}=\frac{2 a}{b+1} \quad d_{a}=\frac{a \sqrt{2}}{\sqrt{1+b^{2}}}
$$

Variances for these indices are computed using the delta method.
The $\delta(m)$ estimates $\left(\mu_{a}-\mu_{n}\right) / \sigma_{n}, d_{e}$ estimates $2\left(\mu_{a}-\mu_{n}\right) /\left(\sigma_{a}-\sigma_{n}\right)$, and $d_{a}$ estimates $\sqrt{2}\left(\mu_{a}-\mu_{n}\right) /\left(\sigma_{a}^{2}-\sigma_{n}^{2}\right)^{2}$.

Simultaneous confidence bands for the entire curve are obtained, as suggested by Ma and Hall (1993), by first obtaining Working-Hotelling (1929) confidence bands for the fitted straight line in normal probability coordinates and then transforming them back to ROC coordinates.

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## Also see

[R] rocfit postestimation - Postestimation tools for rocfit
$[R]$ roc - Receiver operating characteristic (ROC) analysis
$[R]$ rocreg - Receiver operating characteristic (ROC) regression
[U] 20 Estimation and postestimation commands

## Title

rocfit postestimation - Postestimation tools for rocfit

| Description | Syntax for rocplot Menu | Options for rocplot |
| :--- | :--- | :--- |
| Remarks and examples | Also see |  |

## Description

The following command is of special interest after rocfit:

| Command | Description |
| :--- | :--- |
| rocplot | plot the fitted ROC curve and simultaneous confidence bands |

The following standard postestimation commands are also available:

| Command | Description |
| :--- | :--- |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample <br> estat vce |
| variance-covariance matrix of the estimators (VCE) |  |
| estimates | cataloging estimation results <br> point estimates, standard errors, testing, and inference for linear combinations <br> of coefficients |
| *test | Wald tests of simple and composite linear hypotheses |

*See Using lincom and test below.

## Special-interest postestimation command

rocplot plots the fitted ROC curve and simultaneous confidence bands.

## Syntax for rocplot


plot_options
marker_options
marker_label_options cline_options

Description
change look of markers (color, size, etc.) add marker labels; change look or position change the look of the line

## Menu

Statistics $>$ Epidemiology and related $>$ ROC analysis $>$ ROC curves after rocfit

## Options for rocplot

[^70]plotopts(plot_options) affects the rendition of the plotted ROC points, including the size and color of markers, whether and how the markers are labeled, and whether and how the points are connected. For the full list of available plot_options, see [G-3] marker_options, [G-3] marker_label_options, and [G-3] cline_options.
$\qquad$ Fit line
lineopts(cline_options) affects the rendition of the fitted ROC line; see [G-3] cline_options.
$\qquad$
ciopts (area_options) affects the rendition of the confidence bands; see [G-3] area_options.

```
\mathrm{ rlopts (cline_options) affects the rendition of the reference line; see [G-3] cline_options.}
```

$\stackrel{\text { Add plots }}{ }$
addplot (plot) provides a way to add other plots to the generated graph. See [G-3] addplot_option.
$\qquad$ Y axis, X axis, Titles, Legend, Overall
twoway_options are any of the options documented in [G-3] twoway_options, excluding by (). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

## Remarks and examples

Remarks are presented under the following headings:
Using lincom and test
Using rocplot

## Using lincom and test

intercept, slope, and cut\#, shown in example 1 of $[R]$ rocfit, are equation names and not variable names, so they need to be referenced as described in Special syntaxes after multiple-equation estimation of $[\mathrm{R}]$ test. For example, instead of typing

```
. test intercept
intercept not found
r(111);
```

you should type

```
. test [intercept]_cons
( 1) [intercept]_cons = 0
    chi2( 1) = 28.48
    Prob > chi2 = 0.0000
```


## Using rocplot

$>$ Example 1
In example 1 of [ R$]$ rocfit, we fit a ROC curve by typing rocfit disease rating.
In the output table for our model, we are testing whether the variances of the two latent populations are equal by testing that the slope $=1$.

We plot the fitted ROC curve.

```
. rocplot, confband
```



## Also see

[R] rocfit - Parametric ROC models
[U] 20 Estimation and postestimation commands

## Title

rocreg - Receiver operating characteristic (ROC) regression

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | Acknowledgments |
| References | Also see |  |  |

## Syntax

Perform nonparametric analysis of ROC curve under covariates, using bootstrap rocreg refvar classvar [classvars] [if] [in] [, np_options]

Perform parametric analysis of ROC curve under covariates, using bootstrap rocreg refvar classvar [classvars] [if] [in], probit [probit_options]

Perform parametric analysis of ROC curve under covariates, using maximum likelihood

```
rocreg refvar classvar [classvars] [if] [in] [weight], probit ml
    [probit_ml_options]
```

$n p \_o p t i o n s$
Model
auc
roc(numlist)
invroc (numlist)
pauc(numlist)
cluster (varname)
ctrlcov(varlist)
ctrlmodel(strata|linear)
pvc(empirical|normal)
tiecorrected
Bootstrap
nobootstrap
bseed (\#)
breps(\#)
bootcc
nobstrata
nodots
Reporting
level(\#)
estimate total area under the ROC curve; the default estimate ROC for given false-positive rates estimate false-positive rates for given ROC values
estimate partial area under the ROC curve (pAUC) up to each false-positive rate
variable identifying resampling clusters
adjust control distribution for covariates in varlist
stratify or regress on covariates; default is ctrlmodel (strata) use empirical or normal distribution percentile value estimates; default is pvc(empirical)
adjust for tied observations; not allowed with pvc (normal)
do not perform bootstrap, just output point estimates random-number seed for bootstrap
number of bootstrap replications; default is breps (1000)
perform case-control (stratified on refvar) sampling rather than cohort sampling in bootstrap
ignore covariate stratification in bootstrap sampling
suppress bootstrap replication dots
set confidence level; default is level(95)
probit_options
Description

Model

* probit
roccov(varlist)
fprpts(\#)
ctrlfprall
cluster (varname)
ctrlcov(varlist)
ctrlmodel (strata|linear)
pvc(empirical|normal)
tiecorrected
Bootstrap
nobootstrap
bseed (\#)
breps (\#)
bootcc
nobstrata
nodots
bsave (filename, . . . )
bfile(filename)
Reporting
level(\#)
fit the probit model
covariates affecting ROC curve
number of false-positive rate points to use in fitting ROC curve; default is fprpts (10)
fit ROC curve at each false-positive rate in control population variable identifying resampling clusters
adjust control distribution for covariates in varlist stratify or regress on covariates; default is ctrlmodel (strata)
use empirical or normal distribution percentile value estimates; default is pvc(empirical)
adjust for tied observations; not allowed with pvc(normal)
do not perform bootstrap, just output point estimates random-number seed for bootstrap
number of bootstrap replications; default is breps (1000)
perform case-control (stratified on refvar) sampling rather than cohort sampling in bootstrap
ignore covariate stratification in bootstrap sampling
suppress bootstrap replication dots
save bootstrap replicates from parametric estimation
use bootstrap replicates dataset for estimation replay
set confidence level; default is level (95)

[^71]Description

## Model

* probit
* ml roccov(varlist) cluster (varname) ctrlcov(varlist)

Reporting level(\#) display_options

Maximization maximize_options
fit the probit model
fit the probit model by maximum likelihood estimation covariates affecting ROC curve
variable identifying clusters
adjust control distribution for covariates in varlist
set confidence level; default is level (95)
control column formats, line width, and display of omitted variables
control the maximization process; seldom used

[^72]fweights, iweights, and pweights are allowed with maximum likelihood estimation; see [U] 11.1.6 weight.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Epidemiology and related $>$ ROC analysis $>$ ROC regression models

## Description

The rocreg command is used to perform receiver operating characteristic (ROC) analyses with rating and discrete classification data under the presence of covariates.

The two variables refvar and classvar must be numeric. The reference variable indicates the true state of the observation-such as diseased and nondiseased or normal and abnormal-and must be coded as 0 and 1. The refvar coded as 0 can also be called the control population, while the refvar coded as 1 comprises the case population. The rating or outcome of the diagnostic test or test modality is recorded in classvar, which must be ordinal, with higher values indicating higher risk.
rocreg can fit three models: a nonparametric model, a parametric probit model that uses the bootstrap for inference, and a parametric probit model fit using maximum likelihood.

## Options

Options are presented under the following headings:
Options for nonparametric ROC estimation, using bootstrap
Options for parametric ROC estimation, using bootstrap
Options for parametric ROC estimation, using maximum likelihood

## Options for nonparametric ROC estimation, using bootstrap

$\qquad$ Model
auc estimates the total area under the ROC curve. This is the default summary statistic.
roc (numlist) estimates the ROC corresponding to each of the false-positive rates in numlist. The
values of numlist must be in the range $(0,1)$.
invroc (numlist) estimates the false-positive rates corresponding to each of the ROC values in numlist. The values of numlist must be in the range $(0,1)$.
pauc (numlist) estimates the partial area under the ROC curve up to each false-positive rate in numlist.
The values of numlist must in the range $(0,1]$.
cluster (varname) specifies the variable identifying resampling clusters.
ctrlcov(varlist) specifies the covariates to be used to adjust the control population.
ctrlmodel (strata|linear) specifies how to model the control population of classifiers on ctrlcov(). When ctrlmodel(linear) is specified, linear regression is used. The default is ctrlmodel(strata); that is, the control population of classifiers is stratified on the control variables.
pvc(empirical $\mid$ normal) determines how the percentile values of the control population will be calculated. When pvc (normal) is specified, the standard normal cumulative distribution function (CDF) is used for calculation. Specifying pvc (empirical) will use the empirical CDFs of the control population classifiers for calculation. The default is pvc(empirical).
tiecorrected adjusts the percentile values for ties. For each value of the classifier, one half the probability that the classifier equals that value under the control population is added to the percentile value. tiecorrected is not allowed with pvc(normal).

## Bootstrap

nobootstrap specifies that bootstrap standard errors not be calculated.
bseed (\#) specifies the random-number seed to be used in the bootstrap.
breps (\#) sets the number of bootstrap replications. The default is breps (1000).
bootcc performs case-control (stratified on refvar) sampling rather than cohort bootstrap sampling. nobstrata ignores covariate stratification in bootstrap sampling.
nodots suppresses bootstrap replicate dots.

Reporting
level(\#); see [R] estimation options.

## Options for parametric ROC estimation, using bootstrap

Model
probit fits the probit model. This option is required and implies parametric estimation.
roccov(varlist) specifies the covariates that will affect the ROC curve.
fprpts(\#) sets the number of false-positive rate points to use in modeling the ROC curve. These points form an equispaced grid on ( 0,1 ). The default is fprpts(10).
ctrlfprall models the ROC curve at each false-positive rate in the control population. cluster (varname) specifies the variable identifying resampling clusters.
ctrlcov(varlist) specifies the covariates to be used to adjust the control population.
ctrlmodel (strata|linear) specifies how to model the control population of classifiers on ctrlcov(). When ctrlmodel(linear) is specified, linear regression is used. The default is ctrlmodel(strata); that is, the control population of classifiers is stratified on the control variables.
pvc(empirical $\mid$ normal) determines how the percentile values of the control population will be calculated. When pvc (normal) is specified, the standard normal CDF is used for calculation. Specifying pvc (empirical) will use the empirical CDFs of the control population classifiers for calculation. The default is pvc(empirical).
tiecorrected adjusts the percentile values for ties. For each value of the classifier, one half the probability that the classifier equals that value under the control population is added to the percentile value. tiecorrected is not allowed with pvc(normal).

## Bootstrap

nobootstrap specifies that bootstrap standard errors not be calculated.
bseed (\#) specifies the random-number seed to be used in the bootstrap.
breps(\#) sets the number of bootstrap replications. The default is breps (1000).
bootcc performs case-control (stratified on refvar) sampling rather than cohort bootstrap sampling. nobstrata ignores covariate stratification in bootstrap sampling.
nodots suppresses bootstrap replicate dots.
bsave (filename, ...) saves bootstrap replicates from parametric estimation in the given filename with specified options (that is, replace). bsave() is only allowed with parametric analysis using bootstrap.
bfile(filename) specifies to use the bootstrap replicates dataset for estimation replay. bfile() is only allowed with parametric analysis using bootstrap.
$\qquad$ Reporting
level(\#); see [R] estimation options.

## Options for parametric ROC estimation, using maximum likelihood

Model
probit fits the probit model. This option is required and implies parametric estimation.
ml fits the probit model by maximum likelihood estimation. This option is required and must be specified with probit.
roccov(varlist) specifies the covariates that will affect the ROC curve.
cluster (varname) specifies the variable used for clustering.
ctrlcov(varlist) specifies the covariates to be used to adjust the control population.
level(\#); see [R] estimation options.
display_options: noomitted, cformat (\%fmt), pformat (\%fmt), sformat (\%fmt), and nolstretch; see [R] estimation options.

Maximization
maximize_options: difficult, technique (algorithm_spec), iterate (\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance(\#), nonrtolerance, and from (init_specs); see [R] maximize. These options are seldom used. The technique(bhhh) option is not allowed.

## Remarks and examples

Remarks are presented under the following headings:

Introduction<br>ROC statistics<br>Covariate-adjusted ROC curves<br>Parametric ROC curves: Estimating equations<br>Parametric ROC curves: Maximum likelihood

## Introduction

Receiver operating characteristic (ROC) analysis provides a quantitative measure of the accuracy of diagnostic tests to discriminate between two states or conditions. These conditions may be referred to as normal and abnormal, nondiseased and diseased, or control and case. We will use these terms interchangeably. The discriminatory accuracy of a diagnostic test is measured by its ability to correctly classify known control and case subjects.

The analysis uses the ROC curve, a graph of the sensitivity versus 1 - specificity of the diagnostic test. The sensitivity is the fraction of positive cases that are correctly classified by the diagnostic test, whereas the specificity is the fraction of negative cases that are correctly classified. Thus the sensitivity is the true-positive rate, and the specificity is the true-negative rate. We also call 1 - specificity the false-positive rate.

These rates are functions of the possible outcomes of the diagnostic test. At each outcome, a decision will be made by the user of the diagnostic test to classify the tested subject as either normal or abnormal. The true-positive and false-positive rates measure the probability of correct classification or incorrect classification of the subject as abnormal. Given the classification role of the diagnostic test, we will refer to it as the classifier.

Using this basic definition of the ROC curve, Pepe (2000) and Pepe (2003) describe how ROC analysis can be performed as a two-stage process. In the first stage, the control distribution of the classifier is estimated. The specificity is then determined as the percentiles of the classifier values calculated based on the control population. The false-positive rates are calculated as 1 - specificity. In the second stage, the ROC curve is estimated as the cumulative distribution of the case population's "false-positive" rates, also known as the survival function under the case population of the previously calculated percentiles. We use the terms ROC value and true-positive value interchangeably.

This formulation of ROC curve analysis provides simple, nonparametric estimates of several ROC curve summary parameters: area under the ROC curve, partial area under the ROC curve, ROC value for a given false-positive rate, and false-positive rate (also known as invROC) for a given ROC value. In the next section, we will show how to use rocreg to compute these estimates with bootstrap inference. There we will also show how rocreg complements the other nonparametric Stata ROC commands roctab and roccomp.

Other factors beyond condition status and the diagnostic test may affect both stages of ROC analysis. For example, a test center may affect the control distribution of the diagnostic test. Disease severity may affect the distribution of the standardized diagnostic test under the case population. Our analysis of the ROC curve in these situations will be more accurate if we take these covariates into account.

In a nonparametric ROC analysis, covariates may only affect the first stage of estimation; that is, they may be used to adjust the control distribution of the classifier. In a parametric ROC analysis, it is assumed that ROC follows a normal distribution, and thus covariates may enter the model at both stages; they may be used to adjust the control distribution and to model ROC as a function of these covariates and the false-positive rate. In parametric models, both sets of covariates need not be distinct but, in fact, they are often the same.

To model covariate effects on the first stage of ROC analysis, Janes and Pepe (2009) propose a covariate-adjusted ROC curve. We will demonstrate the covariate adjustment capabilities of rocreg in Covariate-adjusted ROC curves.

To account for covariate effects at the second stage, we assume a parametric model. Particularly, the ROC curve is a generalized linear model of the covariates. We will thus have a separate ROC curve for each combination of the relevant covariates. In Parametric ROC curves: Estimating equations, we show how to fit the model with estimating equations and bootstrap inference using rocreg. This method, documented as the "pdf" approach in Alonzo and Pepe (2002), works well with weak assumptions about the control distribution.

Also in Parametric ROC curves: Estimating equations, we show how to fit a constant-only parametric model (involving no covariates) of the ROC curve with weak assumptions about the control distribution. The constant-only model capabilities of rocreg in this context will be compared with those of rocfit. roccomp has the binormal option, which will allow it to compute area under the ROC curve according to a normal ROC curve, equivalent to that obtained by rocfit. We will compare this functionality with that of rocreg.

In Parametric ROC curves: Maximum likelihood, we demonstrate maximum likelihood estimation of the ROC curve model with rocreg. There we assume a normal linear model for the classifier on the covariates and case-control status. This method is documented in Pepe (2003). We will also demonstrate how to use this method with no covariates, and we will compare rocreg under the constant-only model with rocfit and roccomp.

The rocregplot command is used repeatedly in this entry. This command provides graphical output for rocreg and is documented in [ R$]$ rocregplot.

## ROC statistics

roctab computes the ROC curve by calculating the false-positive rate and true-positive rate empirically at every value of the input classifier. It makes no distributional assumptions about the case or control distributions. We can get identical behavior from rocreg by using the default option settings.

## > Example 1: Nonparametric ROC, AUC

Hanley and McNeil (1982) presented data from a study in which a reviewer was asked to classify, using a five-point scale, a random sample of 109 tomographic images from patients with neurological problems. The rating scale was as follows: 1 is definitely normal, 2 is probably normal, 3 is questionable, 4 is probably abnormal, and 5 is definitely abnormal. The true disease status was normal for 58 of the patients and abnormal for the remaining 51 patients.

Here we list 9 of the 109 observations:

```
. use http://www.stata-press.com/data/r13/hanley
```

. list disease rating in $1 / 9$

|  | disease | rating |
| :--- | ---: | ---: |
| 1. | 1 | 5 |
| 2. | 0 | 1 |
| 3. | 1 | 5 |
| 4. | 0 | 4 |
| 5. | 0 | 1 |
| 6. | 0 | 3 |
| 7. | 1 | 5 |
| 8. | 0 | 5 |
| 9. | 0 | 1 |

For each observation, disease identifies the true disease status of the subject ( 0 is normal, 1 is abnormal), and rating contains the classification value assigned by the reviewer.

We run roctab on these data, specifying the graph option so that the ROC curve is rendered. We then calculate the false-positive and true-positive rates of the ROC curve by using rocreg. We graph the rates with rocregplot. Because we focus on rocreg output later, for now we use the quietly prefix to omit the output of rocreg. Both graphs are combined using graph combine (see [G-2] graph combine) for comparison. To ease the comparison, we specify the aspectratio(1) option in roctab; this is the default aspect ratio in rocregplot.

```
. roctab disease rating, graph aspectratio(1) name(a) nodraw title("roctab")
. quietly rocreg disease rating
. rocregplot, name(b) nodraw legend(off) title("rocreg")
. graph combine a b
```



Both roctab and rocreg compute the same false-positive rate and ROC values. The stairstep line connection style of the graph on the right emphasizes the empirical nature of its estimates. The control distribution of the classifier is estimated using the empirical CDF estimate. Similarly, the ROC curve, the distribution of the resulting case observation false-positive rate values, is estimated using the empirical CDF. Note the footnote in the roctab plot. By default, roctab will estimate the area
under the ROC curve (AUC) using a trapezoidal approximation to the estimated false-positive rate and true-positive rate points.

The AUC can be interpreted as the probability that a randomly selected member of the case population will have a larger classifier value than a randomly selected member of the control population. It can also be viewed as the average ROC value, averaged uniformly over the $(0,1)$ false-positive rate domain (Pepe 2003).

The nonparametric estimator of the AUC (DeLong, DeLong, and Clarke-Pearson 1988; Hanley and Hajian-Tilaki 1997) used by rocreg is equivalent to the sample mean of the percentile values of the case observations. Thus to calculate the nonparametric AUC estimate, we only need to calculate the percentile values of the case observations with respect to the control distribution.

This estimate can differ from the trapezoidal approximation estimate. Under discrete classification data, like we have here, there may be ties between classifier values from case to control. The trapezoidal approximation uses linear interpolation between the classifier values to correct for ties. Correcting the nonparametric estimator involves adding a correction term to each observation's percentile value, which measures the probability that the classifier is equal to (instead of less than) the observation's classifier value.

The tie-corrected nonparametric estimate (trapezoidal approximation) is used when we think the true ROC curve is smooth. This means that the classifier we measure is a discretized approximation of a true latent and a continuous classifier.

We now recompute the ROC curve of rating for classifying disease and calculate the AUC. Specifying the tiecorrected option allows tie correction to be used in the rocreg calculation. Under nonparametric estimation, rocreg bootstraps to obtain standard errors and confidence intervals for requested statistics. We use the default 1,000 bootstrap replications to obtain confidence intervals for our parameters. This is a reasonable lower bound to the number of replications (Mooney and Duval 1993) required for estimating percentile confidence intervals. By specifying the summary option in roctab, we will obtain output showing the trapezoidal approximation of the AUC estimate, along with standard error and confidence interval estimates for the trapezoidal approximation suggested by DeLong, DeLong, and Clarke-Pearson (1988).

| Obs | $\begin{array}{r} \text { ROC } \\ \text { Area } \end{array}$ | Std. Err. | -Asymptotic Normal[95\% Conf. Interval] |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 109 | 0.8932 | 0.0307 | 0.83 | 0.95 |  |  |
| . rocreg disease rating, tiecorrected bseed(29092) (running rocregstat on estimation sample) |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| ...................................................... 50 |  |  |  |  |  |  |
| (output omitted) ${ }^{\text {( }}$. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 100 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| 950 |  |  |  |  |  |  |
| 1000 |  |  |  |  |  |  |
| Bootstrap results |  |  | Number of obs Replications |  | = | 109 |
|  |  |  | = | 1000 |
| Nonparametric ROC estimation |  |  |  |  |  |  |
| Control standardization: empirical, corrected for tiesROC method $:$ empirical |  |  |  |  |  |  |
| Area under the ROC curve |  |  |  |  |  |  |
| Status : disease |  |  |  |  |  |  |
| Classifier: rating |  |  |  |  |  |  |
| AUC | Observed Coef. | Bootstrap |  |  |  |  |
|  | . 8931711 | . 000108 |  |  | . 0292028 | . 8359347 | . 9504075 | (N) |
|  |  |  | . 8290958 | . 9457951 |  | (P) |
|  |  |  | . 8280714 | . 9450642 |  | (BC) |

The estimates of AUC match well. The standard error from roctab is close to the bootstrap standard error calculated by rocreg. The bootstrap standard error generalizes to the more complex models that we consider later, whereas the roctab standard-error calculation does not.

The AUC can be used to compare different classifiers. It is the most popular summary statistic for comparisons (Pepe, Longton, and Janes 2009). roccomp will compute the trapezoidal approximation of the AUC and graph the ROC curves of multiple classifiers. Using the DeLong, DeLong, and ClarkePearson (1988) covariance estimates for the AUC estimate, roccomp performs a Wald test of the null hypothesis that all classifier AUC values are equal. rocreg has similar capabilities.

## > Example 2: Nonparametric ROC, AUC, multiple classifiers

Hanley and McNeil (1983) presented data from an evaluation of two computer algorithms designed to reconstruct CT images from phantoms. We will call these two algorithms modalities 1 and 2 . A sample of 112 phantoms was selected; 58 phantoms were considered normal, and the remaining 54 were abnormal. Each of the two modalities was applied to each phantom, and the resulting images were rated by a reviewer using a six-point scale: 1 is definitely normal, 2 is probably normal, 3 is possibly normal, 4 is possibly abnormal, 5 is probably abnormal, and 6 is definitely abnormal. Because each modality was applied to the same sample of phantoms, the two sets of outcomes are correlated.

We list the first seven observations:

```
. use http://www.stata-press.com/data/r13/ct, clear
. list in 1/7, sep(0)
```

|  | $\bmod 1$ | $\bmod 2$ | status |
| :--- | ---: | ---: | ---: |
| 1. | 2 | 1 | 0 |
| 2. | 5 | 5 | 1 |
| 3. | 2 | 1 | 0 |
| 4. | 2 | 3 | 0 |
| 5. | 5 | 6 | 1 |
| 6. | 2 | 2 | 0 |
| 7. | 3 | 2 | 0 |
|  |  |  |  |

Each observation corresponds to one phantom. The mod1 variable identifies the rating assigned for the first modality, and the mod2 variable identifies the rating assigned for the second modality. The true status of the phantoms is given by status $==0$ if they are normal and status $==1$ if they are abnormal. The observations with at least one missing rating were dropped from the analysis.

A fictitious dataset was created from this true dataset, adding a third test modality. We will use roccomp to compute the AUC statistic for each modality in these data and compare the AUC of the three modalities. We obtain the same behavior from rocreg. As before, the tiecorrected option is specified so that the AUC is calculated with the trapezoidal approximation.
. use http://www.stata-press.com/data/r13/ct2
. roccomp status mod1 mod2 mod3, summary

|  | ROC |  |  |  |  |  | -Asymptotic Normal- <br> [95\% Conf. Interval] |  |
| :--- | :---: | ---: | :---: | ---: | ---: | :---: | :---: | :---: |
|  | Obs | Area | Std. Err. | 0.82067 | 0.94498 |  |  |  |
| mod1 | 112 | 0.8828 | 0.0317 | 0.88005 | 0.98042 |  |  |  |
| mod2 | 112 | 0.9302 | 0.0256 | 0.87670 | 0.97132 |  |  |  |

```
Ho: area(mod1) = area(mod2) = area(mod3)
    chi2(2) = 6.54 Prob>chi2 = 0.0381
```



Ho: All classifiers have equal AUC values.
Ha: At least one classifier has a different AUC value.
P-value: . 0389797 Test based on bootstrap (N) assumptions.
We see that the AUC estimates are equivalent, and the standard errors are quite close as well. The $p$-value for the tests of equal AUC under rocreg leads to similar inference as the $p$-value from roccomp. The Wald test performed by rocreg uses the joint bootstrap estimate variance matrix of the three AUC estimators rather than the DeLong, DeLong, and Clarke-Pearson (1988) variance estimate used by roccomp.
roccomp is used here on potentially correlated classifiers that are recorded in wide-format data. It can also be used on long-format data to compare independent classifiers. Further details can be found in [R] roccomp.

Citing the AUC's lack of clinical relevance, there is argument against using it as a key summary statistic of the ROC curve (Pepe 2003; Cook 2007). Pepe, Longton, and Janes (2009) suggest using the estimate of the ROC curve itself at a particular point, or the estimate of the false-positive rate at a given ROC value, also known as invROC.

Recall from example 1 how nonparametric rocreg graphs look, with the stairstep pattern in the ROC curve. In an ideal world, the graph would be a smooth one-to-one function, and it would be trivial to map a false-positive rate to its corresponding true-positive rate and vice versa.

However, smooth ROC curves can only be obtained by assuming a parametric model that uses linear interpolation between observed false-positive rates and between observed true-positive rates, and rocreg is certainly capable of that; see example 1 of $[R]$ rocregplot. However, under nonparametric estimation, the mapping between false-positive rates and true-positive rates is not one to one, and estimates tend to be less reliable the further you are from an observed data point. This is somewhat mitigated by using tie-corrected rates (the tiecorrected option).

When we examine continuous data, the difference between the tie-corrected estimates and the standard estimates becomes negligible, and the empirical estimate of the ROC curve becomes close to the smooth ROC curve obtained by linear interpolation. So the nonparametric ROC and invROC estimates work well.

Fixing one rate value of interest can be difficult and subjective (Pepe 2003). A compromise measure is the partial area under the ROC curve (pAUC) (McClish 1989; Thompson and Zucchini 1989). This is the integral of the ROC curve from 0 and above to a given false-positive rate (perhaps the largest clinically acceptable value). Like the AUC estimate, the nonparametric estimate of the paUC can be written as a sample average of the case observation percentiles, but with an adjustment based on the prescribed maximum false-positive rate (Dodd and Pepe 2003). A tie correction may also be applied so that it reflects the trapezoidal approximation.

We cannot compare rocreg with roctab or roccomp on the estimation of pAUC, because pAUC is not computed by the latter two.

## > Example 3: Nonparametric ROC, other statistics

To see how rocreg estimates ROC, invROC, and pAUC, we will examine a new study. Wieand et al. (1989) examined a pancreatic cancer study with two continuous classifiers, here called y1 (CA 19-9) and y2 (CA 125). This study was also examined in Pepe, Longton, and Janes (2009). The indicator of cancer in a subject is recorded as d. The study was a case-control study, stratifying participants on disease status.

We list the first five observations:

```
. use http://labs.fhcrc.org/pepe/book/data/wiedat2b, clear
(S. Wieand - Pancreatic cancer diagnostic marker data)
```

. list in $1 / 5$

| y 1 | y 2 | d |
| ---: | ---: | ---: |
| 28 | 13.3 | no |
| 15.5 | 11.1 | no |
| 8.2 | 16.7 | no |
| 3.4 | 12.6 | no |
| 17.3 | 7.4 | no |

We will estimate the ROC curves at a large value (0.7) and a small value (0.2) of the false-positive rate. These values are specified in $\operatorname{roc}()$. The false-positive rate for ROC or sensitivity value of 0.6 will also be estimated by specifying invroc(). Percentile confidence intervals for these parameters are displayed in the graph obtained by rocregplot after rocreg. The pAUC statistic will be calculated for the false-positive rate of 0.5 , which is specified as an argument to the pauc () option. Following Pepe, Longton, and Janes (2009), we use a stratified bootstrap, sampling separately from the case
and control populations by specifying the bootcc option. This reflects the case-control nature of the study.

All four statistics can be estimated simultaneously by rocreg. For clarity, however, we will estimate each statistic with a separate call to rocreg. rocregplot is used after estimation to graph the ROC and false-positive rate estimates. The display of the individual, observation-specific false-positive rate and ROC values will be omitted in the plot. This is accomplished by specifying msymbol(i) in our plot1opts() and plot2opts() options to rocregplot.

| . rocreg d y1 y2, roc(.7) bseed(8378923) bootcc nodots |  |  |  |
| :--- | :--- | :--- | :--- |
| Bootstrap results |  |  |  |
| Number of strata $=$ | 2 | Number of obs | $=$ |
|  |  | Replications | $=$ | | 141 |  |
| ---: | :--- |
|  |  |

Nonparametric ROC estimation
Control standardization: empirical
ROC method : empirical
ROC curve
Status : d
Classifier: y1

| ROC | Observed <br> Coef. | Bias | Bootstrap <br> Std. Err. | [95\% Conf. Interval] |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | :--- |
|  | .9222222 | -.0021889 | .0323879 | .8587432 | .9857013 | $(\mathrm{~N})$ |
|  |  |  |  | .8444445 | .9777778 (P) |  |
|  |  |  |  | .8555555 | .9777778 (BC) |  |

Status : d
Classifier: y2

| ROC | Observed <br> Coef. | Bias | Bootstrap <br> Std. Err. | [95\% Conf. Interval] |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| .7 | .8888889 | -.0035556 | .0414215 | .8077043 | .9700735 | $(\mathrm{~N})$ |
|  |  |  |  | .8 | .9611111 | (P) |
|  |  |  |  | .7888889 | .9555556 (BC) |  |

Ho: All classifiers have equal ROC values.
Ha: At least one classifier has a different ROC value.
Test based on bootstrap (N) assumptions.

| ROC | P-value |
| ---: | ---: |
|  | .7 |
|  | .5423044 |

. rocregplot, plot1opts(msymbol(i)) plot2opts(msymbol(i))


In this study, we see that classifier y1 (CA 19-9) is a uniformly better test than is classifier y2 (CA 125) until high levels of false-positive rate and sensitivity or ROC value are reached. At the high level of false-positive rate, 0.7 , the ROC value does not significantly differ between the two classifiers. This can be seen in the plot by the overlapping confidence intervals.
. rocreg d y1 y2, roc(.2) bseed(8378923) bootcc nodots
Bootstrap results

| Number of strata $=$ | 2 | Number of obs <br> Replications | $=$ |
| :--- | :--- | :--- | :--- |

Nonparametric ROC estimation
Control standardization: empirical
ROC method : empirical
ROC curve
Status : d
Classifier: y1

| ROC | Observed <br> Coef. | Bias | Bootstrap <br> Std. Err. | [95\% Conf. Interval] |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | :--- |
|  | .7777778 | .0011778 | .0483655 | .6829831 | .8725725 | (N) |
|  |  |  |  | .6888889 | .8777778 (P) |  |
|  |  |  |  | .6777778 | .8666667 (BC) |  |

Status : d
Classifier: y2

| ROC | Observed <br> Coef. | Bias | Bootstrap <br> Std. Err. | [95\% Conf. Interval] |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | .4888889 | -.0091667 | .1339863 | .2262806 | .7514971 | $(\mathrm{~N})$ |
|  |  |  |  | .2222222 | .7 | $(\mathrm{P})$ |
|  |  |  |  | .2111111 | .7 (BC) |  |

Ho: All classifiers have equal ROC values.
Ha: At least one classifier has a different ROC value.
Test based on bootstrap (N) assumptions.

| ROC | P-value |
| :--- | ---: |
| .2 | .043234 |

. rocregplot, plot1opts(msymbol(i)) plot2opts(msymbol(i))


The sensitivity for the false-positive rate of 0.2 is found to be higher under y1 than under y 2 , and this difference is significant at the 0.05 level. In the plot, this is shown by the vertical confidence intervals.


Ho: All classifiers have equal invROC values.
Ha: At least one classifier has a different invROC value.
Test based on bootstrap (N) assumptions.

| invROC | P-value |
| ---: | ---: |
| .6 | .0016562 |

. rocregplot, plot1opts(msymbol(i)) plot2opts(msymbol(i))


We find significant evidence that false-positive rates corresponding to a sensitivity of 0.6 are different from y1 to y2. This is visually indicated by the horizontal confidence intervals, which are separated from each other.

```
. rocreg d y1 y2, pauc(.5) bseed(8378923) bootcc nodots
Bootstrap results
\begin{tabular}{llll} 
Number of strata \(=\) & 2 & \begin{tabular}{ll} 
Number of obs & \(=\)
\end{tabular} & 141 \\
& Replications & \(=\) & 1000
\end{tabular}
Nonparametric ROC estimation
Control standardization: empirical
ROC method : empirical
Partial area under the ROC curve
    Status : d
    Classifier: y1
```

| pAUC | Observed Coef. | Bias | Bootstrap <br> Std. Err. | [95\% Conf. | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| . 5 | . 3932462 | -. 0000769 | . 021332 | . 3514362 | . 4350562 (N) |
|  |  |  |  | . 3492375 | . 435512 (P) |
|  |  |  |  | . 3492375 | . 435403 (BC) |
| Status : d <br> Classifier: y2 |  |  |  |  |  |
|  |  |  |  |  |  |
| pAUC | Observed Coef. | Bias | Bootstrap <br> Std. Err. | [95\% Conf. Interval] |  |
| . 5 | . 2496732 | . 0019168 | . 0374973 | . 1761798 | . 3231666 (N) |
|  |  |  |  | . 177451 | . 3253268 (P) |
|  |  |  |  | . 1738562 | . 3233115 (BC) |

Ho: All classifiers have equal pAUC values.
Ha: At least one classifier has a different pAUC value.
Test based on bootstrap (N) assumptions.

| pAUC | P-value |
| :--- | ---: |
| .5 | .0011201 |

We also find significant evidence supporting the hypothesis that the pAUC for y 1 up to a false-positive rate of 0.5 differs from the area of the same region under the ROC curve of y 2 .

## Covariate-adjusted ROC curves

When covariates affect the control distribution of the diagnostic test, thresholds for the test being classified as abnormal may be chosen that vary with the covariate values. These conditional thresholds will be more accurate than the marginal thresholds that would normally be used, because they take into account the specific distribution of the diagnostic test under the given covariate values as opposed to the marginal distribution over all covariate values.

By using these covariate-specific thresholds, we are essentially creating new classifiers for each covariate-value combination, and thus we are creating multiple ROC curves. As explained in Pepe (2003), when the case and control distributions of the covariates are the same, the marginal ROC curve will always be bound above by these covariate-specific ROC curves. So using conditional thresholds will never provide a less powerful test diagnostic in this case.

In the marginal ROC curve calculation, the classifiers are standardized to percentiles according to the control distribution, marginalized over the covariates. Thus the ROC curve is the CDF of the standardized case observations. The covariate-adjusted ROC curve is the CDF of one minus the conditional control percentiles for the case observations, and the marginal ROC curve is the CDF of one minus the marginal control percentiles for the case observations (Pepe and Cai 2004). Thus the standardization of classifier to false-positive rate value is conditioned on the specific covariate values under the covariate-adjusted ROC curve.

The covariate-adjusted ROC curve (Janes and Pepe 2009) at a given false-positive rate $t$ is equivalent to the expected value of the covariate-specific ROC at $t$ over all covariate combinations. When the covariates in question do not affect the case distribution of the classifier, the covariate-specific ROC will have the same value at each covariate combination. So here the covariate-adjusted ROC is equivalent to the covariate-specific ROC, regardless of covariate values.

When covariates do affect the case distribution of the classifier, users of the diagnostic test would likely want to model the covariate-specific ROC curves separately. Tools to do this can be found in the parametric modeling discussion in the following two sections. Regardless, the covariate-adjusted ROC curve can serve as a meaningful summary of covariate-adjusted accuracy.

Also note that the ROC summary statistics defined in the previous section have covariate-adjusted analogs. These analogs are estimated in a similar manner as under the marginal ROC curve (Janes, Longton, and Pepe 2009). The options for their calculation in rocreg are identical to those given in the previous section. Further details can be found in Methods and formulas.
> Example 4: Nonparametric ROC, linear covariate adjustment
Norton et al. (2000) studied data from a neonatal audiology study on three tests to identify hearing impairment in newborns. These data were also studied in Janes, Longton, and Pepe (2009). Here we list 5 of the 5,058 observations.

```
. use http://www.stata-press.com/data/r13/nnhs, clear
(Norton - neonatal audiology data)
. list in 1/5
```

| id | ear | male | currage | d | y1 | y2 | y3 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| B0157 | R | M | 42.42 | 0 | -3.1 | -9 | -1.5 |
| B0157 | L | M | 42.42 | 0 | -4.5 | -8.7 | -2.71 |
| B0158 | R | M | 40.14 | 1 | -3.2 | -13.2 | -2.64 |
| B0161 | L | F | 38.14 | 0 | -22.1 | -7.8 | -2.59 |
| B0167 | R | F | 37 | 0 | -10.9 | -6.6 | -1.42 |

The classifiers y1 (DPOAE 65 at 2 kHz ), y2 (TEOAE 80 at 2 kHz ), and y3 (ABR) and the hearing impairment indicator $d$ are recorded along with some relevant covariates. The infant's age is recorded in months as currage, and the infant's gender is indicated by male. Over $90 \%$ of the newborns were tested in each ear (ear), so we will cluster on infant ID (id).

Following the strategy of Janes, Longton, and Pepe (2009), we will first perform ROC analysis for the classifiers while adjusting for the covariate effects of the infant's gender and age. This is done by specifying these variables in the ctrlcov() option. We adjust using a linear regression rule, by specifying ctrlmodel (linear). This means that when a user of the diagnostic test chooses a threshold conditional on the age and gender covariates, they assume that the diagnostic test classifier has some linear dependence on age and gender and equal variance as their levels vary. Our cluster adjustment is made by specifying the cluster() option.

We will focus on the first classifier. The percentile, or specificity, values are calculated empirically by default, and thus so are the false-positive rates, ( $1-$ specificity). Also by default, the ROC curve values are empirically defined by the false-positive rates. To draw the ROC curve, we again use rocregplot.

The AUC is calculated by default. For brevity, we specify the nobootstrap option so that bootstrap sampling is not performed. The AUC point estimate will be sufficient for our purposes.


Area under the ROC curve
Status : d
Classifier: y1

| AUC | Observed <br> Coef. | Bias | Bootstrap <br> Std. Err. | [95\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | .6293994 | . | . | . | . |
|  |  |  | . | . | $(\mathrm{~N})$ |
|  |  |  |  | . | $(\mathrm{PC})$ |



Our covariate control adjustment model shows that currage has a negative effect on y1 (DPOAE 65 at 2 kHz ) under the control population. At the 0.001 significance level, we reject that its contribution to y 1 is zero, and the point estimate has a negative sign. This result does not directly tell us about the effect of currage on the ROC curve of y1 as a classifier of d. None of the case observations are used in the linear regression, so information on currage for abnormal cases is not used in the model. This result does show us how to calculate false-positive rates for tests that use thresholds conditional on a child's sex and current age. We will see how currage affects the ROC curve when y1 is used as a classifier and conditional thresholds are used based on male and currage in the following section, Parametric ROC curves: Estimating equations.

## Technical note

Under this nonparametric estimation, rocreg saved the false-positive rate for each observation's y1 values in the utility variable $-f p r-y 1$. The true-positive rates are stored in the utility variable _roc_y1. For other models, say with classifier yname, these variables would be named _fpr_yname and _roc_yname. They will also be overwritten with each call of rocreg. The variables _roc_* and _fpr_* are usually for internal rocreg use only and are overwritten with each call of rocreg. They are only created for nonparametric models or parametric models that do not involve ROC covariates. In these models, covariates may only affect the first stage of estimation, the control distribution, and not the ROC curve itself. In parametric models that allow ROC covariates, different covariate values would lead to different ROC curves.

To see how the covariate-adjusted ROC curve estimate differs from the standard marginal estimate, we will reestimate the ROC curve for classifier y1 without covariate adjustment. We rename these variables before the new estimation and then draw an overlaid twoway line (see [G-2] graph twoway line) plot to compare the two.

```
. rename _fpr_y1 o_fpr_y1
. rename _roc_y1 o_roc_y1
. label variable o_roc_y1 "covariate_adjusted"
. rocreg d y1, cluster(id) nobootstrap
Nonparametric ROC estimation
Control standardization: empirical
ROC method : empirical
Area under the ROC curve
    Status : d
    Classifier: y1
```

| AUC | Observed <br> Coef. | Bias | Bootstrap <br> Std. Err. | [95\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | .6279645 | . | . | . | . |
|  |  |  | . | . | $(\mathrm{~N})$ |
|  |  |  |  | . | $(\mathrm{PC})$ |

```
. label variable _roc_y1 "marginal"
. twoway line _roc_y1 _fpr_y1, sort(_fpr_y1 _roc_y1) connect(J) ||
        line o_roc_y1 o_fpr_y1, sort(o_fpr_y1 o_roc_y1)
        connect(J) lpattern(dash) aspectratio(1) legend(cols(1))
```



Though they are close, particularly in AUC, there are clearly some points of difference between the estimates. So the covariate-adjusted ROC curve may be useful here.

In our examples thus far, we have used the empirical CDF estimator to estimate the control distribution. rocreg allows some flexibility here. The pvc (normal) option may be specified to calculate the percentile values according to a Gaussian distribution of the control.

Covariate adjustment in rocreg may also be performed with stratification instead of linear regression. Under the stratification method, the unique values of the stratified covariates each define separate parameters for the control distribution of the classifier. A user of the diagnostic test chooses a threshold based on the control distribution conditioned on the unique covariate value parameters.

We will demonstrate the use of normal percentile values and covariate stratification in our next example.

## > Example 5: Nonparametric ROC, covariate stratification

The hearing test study of Stover et al. (1996) examined the effectiveness of negative signal-to-noise ratio, nsnr, as a classifier of hearing loss. The test was administered under nine different settings, corresponding to different frequency, xf, and intensity, $x l$, combinations. Here we list 10 of the 1,848 observations.

```
. use http://www.stata-press.com/data/r13/dp, clear
(Stover - DPOAE test data)
. list in 1/10
```

| id | d | nsnr | xf | xl | xd |
| :---: | :---: | ---: | ---: | ---: | ---: |
| 101 | 1 | 18 | 10.01 | 5.5 | 3.5 |
| 101 | 1 | 19 | 20.02 | 5.5 | 3 |
| 101 | 1 | 7.6 | 10.01 | 6 | 3.5 |
| 101 | 1 | 15 | 20.02 | 6 | 3 |
| 101 | 1 | 16 | 10.01 | 6.5 | 3.5 |
| 101 | 1 | 5.8 | 20.02 | 6.5 | 3 |
| 102 | 0 | -2.6 | 10.01 | 5.5 | . |
| 102 | 0 | -3 | 14.16 | 5.5 | . |
| 102 | 1 | 10 | 20.02 | 5.5 | 1 |
| 102 | 0 | -5.8 | 10.01 | 6 | . |

Hearing loss is represented by d . The covariate xd is a measure of the degree of hearing loss. We will use this covariate in later analysis, because it only affects the case distribution of the classifier. Multiple measurements are taken for each individual, id, so we will cluster by individual.

We evaluate the effectiveness of nsnr using xf and $x l$ as stratification covariates with rocreg; the default method of covariate adjustment.

As mentioned before, the default false-positive rate calculation method in rocreg estimates the conditional control distribution of the classifiers empirically. For comparison, we will also estimate a separate ROC curve using false-positive rates assuming the conditional control distribution is normal. This behavior is requested by specifying the pvc (normal) option. Using the rocregplot option name() to store the ROC plots and using the graph combine command, we are able to compare the Gaussian and empirical ROC curves side by side. As before, for brevity we specify the nobootstrap option to suppress bootstrap sampling.

| Nonparametric ROC estimation |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Covariate control $\quad:$ stratificationControl variables $:$ xf xlControl standardization: empiricalROC method $\quad$ : empiricalArea under the ROC curveStatus : d <br> Classifier: nsnr |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| AUC | Observed Coef. | Bias | Bootstrap Std. Err. | [95\% | Conf | Interval] |
|  | . 9264192 | . |  |  | $\cdot$ |  |

```
. rocreg d nsnr, pvc(normal) ctrlcov(xf xl) cluster(id) nobootstrap
Nonparametric ROC estimation
Covariate control : stratification
Control variables : xf xl
Control standardization: normal
ROC method : empirical
Area under the ROC curve
    Status : d
    Classifier: nsnr
```

| AUC | Observed Coef. | Bias | Bootstrap <br> Std. Err. | [95\% Conf. | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | . 9309901 | . | . |  |  |

. rocregplot, title(Normal FPR) name(b) nodraw
. graph combine a b, xsize(5)


On cursory visual inspection, we see little difference between the two curves. The AUC values are close as well. So it is sensible to assume that we have Gaussian percentile values for control standardization.

## Parametric ROC curves: Estimating equations

We now assume a parametric model for covariate effects on the second stage of ROC analysis. Particularly, the ROC curve is a probit model of the covariates. We will thus have a separate ROC curve for each combination of the relevant covariates.

Under weak assumptions about the control distribution of the classifier, we can fit this model by using estimating equations as described in Alonzo and Pepe (2002). This method can be also be used without covariate effects in the second stage, assuming a parametric model for the single (constant only) ROC curve. Covariates may still affect the first stage of estimation, so we parametrically model the single covariate-adjusted ROC curve (from the previous section). The marginal ROC curve, involving no covariates in either stage of estimation, can be fit parametrically as well.

In addition to the Alonzo and Pepe (2002) explanation, further details are given in Pepe, Longton, and Janes (2009); Janes, Longton, and Pepe (2009); Pepe (2003); and Janes and Pepe (2009).

The parametric models that we consider assume that the ROC curve is a cumulative distribution function $g$ invoked with input of a linear polynomial in the corresponding quantile function invoked on the false-positive rate $u$. In this context, we assume that $g$ corresponds to a standard normal cumulative distribution function, $\Phi$. So the corresponding quantile function is $\Phi^{-1}$. The constant intercept of the polynomial may depend on covariates, but the slope term $\alpha$ (the quantile coefficient) may not.

$$
\operatorname{ROC}(u)=g\left\{\mathbf{x}^{\prime} \boldsymbol{\beta}+\alpha g^{-1}(u)\right\}
$$

The first step of the algorithm involves the choice of false-positive rates to use in the parametric fit. These are typically a set of equispaced points spanning the interval ( 0,1 ). Alonzo and Pepe (2002) examined the effect of fitting large and small sets of points, finding that relatively small sets could be used with little loss of efficiency. Alternatively, the set can be formed by using the observed false-positive rates in the data (Pepe 2003). Further details on the algorithm are provided in Methods and formulas.

Under parametric estimation, all the summary measures we defined earlier, except the AUC, are not calculated until postestimation. In models with covariates, each covariate combination would yield a different ROC curve and thus different summary parameters, so no summary parameters are initially estimated. In marginal parametric models (where there are no ROC covariates, but there are potentially control covariates), we will calculate the AUC and leave the other measures for postestimation; see $[R]$ rocreg postestimation. As with the other parameters, we bootstrap for standard errors and inference.

We will now demonstrate how rocreg performs the Alonzo and Pepe (2002) algorithm using the previous section's examples and others.

## > Example 6: Parametric ROC, linear covariate adjustment

We return to the neonatal audiology study with gender and age covariates (Norton et al. 2000), which we discussed in example 4. Janes, Longton, and Pepe (2009) suspected the current age of the infant would play a role in the case distribution of the classifier y1 (DPOAE 65 at 2 kHz ). They postulated a probit link between the ROC curve and the covariate-adjusted false-positive rates. We follow their investigation and reach similar results.

In example 4, we saw the results of adjusting for the currage and male variables in the control population for classifier y1. Now we see how currage affects the ROC curve when y1 is used with thresholds conditioned on male and currage.

We specify the covariates that should affect the ROC curve in the roccov() option. By default, rocreg will choose 10 equally spaced false-positive rates in the $(0,1)$ interval as fitting points. The fprpts() option allows the user to specify more or fewer points. We specify the bsave() option with the nnhs 2 y 1 dataset so that we can use the bootstrap resamples in postestimation.

```
. use http://www.stata-press.com/data/r13/nnhs, clear
(Norton - neonatal audiology data)
. rocreg d y1, probit ctrlcov(currage male) ctrlmodel(linear) roccov(currage)
> cluster(id) bseed(56930) bsave(nnhs2y1) nodots
```

Bootstrap results

| Number of obs | $=$ | 5056 |
| :--- | :--- | :--- |
| Replications | $=$ | 1000 |

Parametric ROC estimation
Covariate control : linear regression
Control variables : currage male
Control standardization: empirical
ROC method : parametric Link: probit
Status : d
Classifier: y1
Covariate control adjustment model:
Linear regression $\quad$ Number of obs $=\quad 4907$
$F(2,2685)=13.80$
Prob > F $=0.0000$
R-squared $=0.0081$
Root MSE $=7.7515$
(Std. Err. adjusted for 2686 clusters in id)

|  |  | Robust |  |  |  |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: | ---: |
| y1 | Coef. | Std. Err. | t | $\mathrm{P}\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| currage | -.2032456 | .0389032 | -5.22 | 0.000 | -.2795288 | -.1269624 |
| male | .2471744 | .2603598 | 0.95 | 0.343 | -.2633516 | .7577005 |
| _cons | -1.239484 | 1.487855 | -0.83 | 0.405 | -4.156942 | 1.677973 |

    Status : d
    Classifier: y1
    ROC Model :
    (Replications based on 2741 clusters in id)

| y1 | Observed Coef. | Bias | Bootstrap <br> Std. Err. | [95\% Conf. Interval] |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| _cons | -1.272505 | -. 0566737 | 1.076706 | -3.38281 | . 8377993 | ( N ) |
|  |  |  |  | -3.509356 | . 7178385 | (P) |
|  |  |  |  | -3.487457 | . 7813575 | (BC) |
| currage | . 0448228 | . 0015878 | . 0280384 | -. 0101316 | . 0997771 | (N) |
|  |  |  |  | -. 007932 | . 1033131 | (P) |
|  |  |  |  | -. 0102905 | . 101021 | (BC) |
| probit |  |  |  |  |  |  |
| _cons | . 9372393 | . 0128376 | . 0747228 | . 7907853 | 1.083693 | (N) |
|  |  |  |  | . 8079087 | 1.101941 | (P) |
|  |  |  |  | . 7928988 | 1.083399 | (BC) |

Note how the number of clusters-here infants-changes from the covariate control adjustment model fit to the ROC model. The control fit is limited to control cases and thus fewer infants. The ROC is fit on all the data, so the variance is adjusted for all clustering on all infants.

With a 0.05 level of statistical significance, we cannot reject the null hypothesis that currage has no effect on the ROC curve at a given false-positive rate. This is because each of our $95 \%$ bootstrap confidence intervals contains 0. This corresponds with the finding in Janes, Longton, and Pepe (2009) where the reported $95 \%$ intervals each contained 0 . We cannot reject that the intercept parameter $\beta_{0}$, reported as _cons in the main table, is 0 at the 0.05 level either. The slope parameter $\alpha$, reported
as _cons in the probit table, is close to 1 and cannot be rejected as being 1 at the 0.05 level. Under the assumption that the ROC coefficients except $\alpha$ are 0 and that $\alpha=1$, the ROC curve at false-positive rate $u$ is equal to $u$. In other words, we cannot reject that the false-positive rate is equal to the true-positive rate, and so the test is noninformative. Further investigation of the results requires postestimation; see $[R]$ rocreg postestimation.

The fitting point set can be formed by using the observed false-positive rates (Pepe 2003). Our next example will illustrate this.

## > Example 7: Parametric ROC, covariate stratification

We return to the hearing test study of Stover et al. (1996), which we discussed in example 5. Pepe (2003) suspected that intensity, xd, would play a role in the case distribution of the negative signal-to-noise ratio (nsnr) classifier. A ROC regression was fit with covariate adjustment for $x f$ and xl with stratification, and for ROC covariates $\mathrm{xf}, \mathrm{xl}$, and xd . There is no prohibition against the same covariate being used in the first and second stages of ROC calculation. The false-positive rate fitting point set was composed of all observed false-positive rates in the control data.

We fit the model with rocreg here. Using observed false-positive rates as the fitting point set can make the dataset very large, so fitting the model is computationally intensive. We demonstrate the fitting algorithm without precise confidence intervals, focusing instead on the coefficient estimates and standard errors. We will thus perform only 50 bootstrap replications, a reasonable number to obtain accurate standard error estimates (Mooney and Duval 1993). The number of replications is specified in the breps () option.

The ROC covariates are specified in roccov(). We specify that all observed false-positive rates in the control observations be used as fitting points with the ctrlfprall option. The nobstrata option specifies that the bootstrap is not stratified. The covariate stratification in the first stage of estimation does not affect the resampling. We will return to this example in postestimation, so we save the bootstrap results in the nsnrf dataset with the bsave() option.

| . rocreg d nsnr, probit ctrlcov(xf xl) roccov(xf xl xd) ctrlfprall cluster(id) > nobstrata bseed(156385) breps(50) bsave(nsnrf) <br> (running rocregstat on estimation sample) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |
| Bootstrap results |  |  |  | er of obs ications | $\begin{aligned} & = \\ & = \end{aligned}$ | $\begin{array}{r} 1848 \\ 50 \end{array}$ |
| Parametric ROC estimation |  |  |  |  |  |  |
| Covariate control : stratification  <br> Control variables : xf xl  <br> Control standardization: empirical   <br> ROC method : parametric Link: probit |  |  |  |  |  |  |
| Status : dClassifier: nsnrROC Model : |  |  |  |  |  |  |
| nsnr | Observed Coef. | Bias | Bootstrap <br> Std. Err. | [95\% Con | . Interva |  |
| _cons | 3.247872 | -. 0846178 | . 8490006 | 1.583862 | 4.911883 |  |
|  |  |  |  | 1.598022 | 4.690076 | (P) |
|  |  |  |  | 1.346904 | 4.690076 | (BC) |
| xf | . 0502557 | . 014478 | . 0329044 | -. 0142357 | . 1147471 | (N) |
|  |  |  |  | -. 0031814 | . 1186107 | (P) |
|  |  |  |  | -. 0053095 | . 1132185 | (BC) |
| xl | -. 4327223 | -. 0194846 | . 1116309 | -. 6515149 | -. 2139298 |  |
|  |  |  |  | -. 6570321 | -. 2499706 | (P) |
|  |  |  |  | -. 6570321 | -. 231854 | (BC) |
| xd | . 4431764 | . 0086147 | . 0936319 | . 2596612 | . 6266916 |  |
|  |  |  |  | . 330258 | . 6672749 |  |
|  |  |  |  | . 3487118 | . 7674865 | (BC) |
| probit |  |  |  |  |  |  |
| _cons | 1.032657 | -. 0188887 | . 1224993 | . 7925628 | 1.272751 |  |
|  |  |  |  | . 7815666 | 1.236179 | (P) |
|  |  |  |  | . 7815666 | 1.237131 |  |

We obtain results similar to those reported in Pepe (2003, 159). We find that the coefficients for xl and xd differ from 0 at the 0.05 level of significance. So over certain covariate combinations, we can have a variety of informative tests using nsnr as a classifier.

As mentioned before, when there are no covariates, rocreg can still fit a parametric model for the ROC curve of a classifier by using the Alonzo and Pepe (2002) method. roccomp and rocfit can fit marginal probit models as well. We will compare the behavior of rocreg with that of roccomp and rocfit for probit models without covariates.

When the binormal option is specified, roccomp calculates the AUC for input classifiers according to the maximum likelihood algorithm of rocfit. The rocfit algorithm expects discrete classifiers but can slice continuous classifiers into discrete partitions. Further, the case and control distributions are both assumed normal. Actually, the observed classification values are taken as discrete indicators
of the latent normally distributed classification values. This method is documented in Dorfman and Alf (1969).

Alonzo and Pepe (2002) compared their estimating equations probability density function method (with empirical estimation of the false-positive rates) to the maximum likelihood approach of Dorfman and Alf (1969) and found that they had similar efficiency and mean squared error. So we should expect rocfit and rocreg to give similar results when fitting a simple probit model.

## > Example 8: Parametric ROC, marginal model

We return to the Hanley and McNeil (1982) data. We will fit a probit model to the ROC curve, assuming that the rating variable is a discrete indicator of an underlying latent normal random variable in both the case and control populations of disease. We invoke rocfit with the default options. rocreg is invoked with the probit option. The percentile values are calculated empirically. Because there are fewer categories than 10, there will be fewer than 10 false-positive rates that trigger a different true-positive rate value. So for efficiency, we invoke rocreg with the ctrlfprall option.

```
. use http://www.stata-press.com/data/r13/hanley
. rocfit disease rating, nolog
Binormal model of disease on rating Number of obs = 109
Goodness-of-fit chi2(2) = 0.21
Prob > chi2 = 0.9006
Log likelihood = -123.64855
```

|  | Coef. | Std. Err. | z | $P>\|z\|$ | [95\% Conf | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| intercept | 1.656782 | 0.310456 | 5.34 | 0.000 | 1.048300 | 2.265265 |
| slope (*) | 0.713002 | 0.215882 | -1.33 | 0.184 | 0.289881 | 1.136123 |
| /cut1 | 0.169768 | 0.165307 | 1.03 | 0.304 | -0.154227 | 0.493764 |
| /cut2 | 0.463215 | 0.167235 | 2.77 | 0.006 | 0.135441 | 0.790990 |
| /cut3 | 0.766860 | 0.174808 | 4.39 | 0.000 | 0.424243 | 1.109477 |
| /cut4 | 1.797938 | 0.299581 | 6.00 | 0.000 | 1.210770 | 2.385106 |
|  | Indices from binormal fit |  |  |  |  |  |
| Index | Estimate | Std. Err. |  |  | [95\% Conf. Interval] |  |
| ROC area | 0.911331 | 0.029506 |  |  | 0.853501 | 0.969161 |
| delta(m) | 2.323671 | 0.502370 |  |  | 1.339044 | 3.308298 |
| d(e) | 1.934361 | 0.257187 |  |  | 1.430284 | 2.438438 |
| d(a) | 1.907771 | 0.259822 |  |  | 1.398530 | 2.417012 |

(*) $z$ test for slope==1


We see that the intercept and slope parameter estimates are close. The intercept (_cons in the main table) is clearly nonzero. Under rocreg, the slope (_cons in the probit table) and its percentile and bias-corrected confidence intervals are close to those of rocfit. The area under the ROC curve for each of the rocreg and rocfit estimators also matches closely.

Now we will compare the parametric fit of rocreg under the constant probit model with roccomp.

## > Example 9: Parametric ROC, marginal model, multiple classifiers

We now use the fictitious dataset generated from Hanley and McNeil (1983). To fit a probit model using roccomp, we specify the binormal option. Our specification of rocreg remains the same as before.
rocregplot is used to render the model produced by rocreg. We specify several graph options to both roccomp and rocregplot to ease comparison. When the binormal option is specified along with graph, roccomp will draw the binormal fitted lines in addition to connected line plots of the empirical false-positive and true-positive rates.

In this plot, we overlay scatterplots of the empirical false-positive rates (because percentile value calculation defaulted to pvc(empirical)) and the parametric true-positive rates.
. use http://www.stata-press.com/data/r13/ct2, clear
. roccomp status mod1 mod2 mod3, summary binormal graph aspectratio(1)
$>\quad$ plot1opts(connect(i) msymbol(o))
$>\quad$ plot2opts (connect(i) msymbol(s))
$>\quad$ plot3opts (connect(i) msymbol(t))
> legend(label(1 "mod1") label(3 "mod2") label(5 "mod3")
label(2 "mod1 fit") label(4 "mod2 fit")
label(6 "mod3 fit") order(1 35246 ) cols(1))
> title(roccomp) name(a) nodraw
Fitting binormal model for: mod1
Fitting binormal model for: mod2
Fitting binormal model for: mod3
ROC

|  | Obs | Area | Std. Err. | [95\% Conf. Interval] |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| mod1 | 112 | 0.8945 | 0.0305 | 0.83482 | 0.95422 |
| mod2 | 112 | 0.9382 | 0.0264 | 0.88647 | 0.99001 |
| mod3 | 112 | 0.9376 | 0.0223 | 0.89382 | 0.98139 |

Ho: | $\operatorname{area}(\bmod 1)=\operatorname{area}(\bmod 2)=\operatorname{area}(\bmod 3)$ |
| :--- |
| $\operatorname{chi2}(2)=8.27 \quad$ Prob>chi2 $=0.0160$ |

. rocreg status mod1 mod2 mod3, probit ctrlfprall bseed(867340912) nodots
Bootstrap results Number of obs = 112
Replications $=1000$
Parametric ROC estimation
Control standardization: empirical
ROC method : parametric Link: probit
Status : status
Classifier: mod1
ROC Model :

| mod1 | Observed Coef. | Bias | Bootstrap Std. Err. | [95\% Conf. Interval] |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| _cons | 1.726034 | . 1363112 | . 5636358 | . 6213277 | 2.83074 |  |
|  |  |  |  | 1.162477 | 3.277376 |  |
|  |  |  |  | 1.152112 | 3.187595 | (BC) |
| probit |  |  |  |  |  |  |
| _cons | . 9666323 | . 0872018 | . 4469166 | . 0906919 | 1.842573 | (N) |
|  |  |  |  | . 518082 | 2.219548 | (P) |
|  |  |  |  | . 5568404 | 2.394036 | (BC) |


| AUC | Observed <br> Coef. | Bias | Bootstrap <br> Std. Err. | [95\% Conf. Interval] |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
|  | .8927007 | -.0011794 | .0313951 | .8311675 | .954234 | $(\mathrm{~N})$ |
|  |  |  |  | .8245637 | .9466904 | (P) |
|  |  |  |  | .8210562 | .9432855 (BC) |  |

tatus : status
Classifier: mod2
ROC Model :

| mod2 | Observed Coef. | Bias | Bootstrap <br> Std. Err. | [95\% Conf. Interval] |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| _cons | 1.696811 | . 0918364 | . 5133386 | . 6906858 | 2.702936 | (N) |
|  |  |  |  | 1.21812 | 2.973929 | (P) |
|  |  |  |  | 1.22064 | 3.068454 | (BC) |
| probit |  |  |  |  |  |  |
| _cons | . 4553828 | . 047228 | . 3345303 | -. 2002845 | 1.11105 | (N) |
|  |  |  |  | . 1054933 | 1.18013 | (P) |
|  |  |  |  | . 1267796 | 1.272523 | (BC) |


| AUC | Observed <br> Coef. | Bias | Bootstrap <br> Std. Err. | [95\% Conf. Interval] |  |  |
| :---: | ---: | :---: | :---: | :---: | :---: | :---: |
|  | .938734 | -.0037989 | .0261066 | .8875659 | .9899021 | $(\mathrm{~N})$ |
|  |  |  |  | .8777664 | .9778214 | $(\mathrm{P})$ |
|  |  |  |  | .8823555 | .9792451 | $(\mathrm{BC})$ |

Status : status
Classifier: mod3
ROC Model :

| mod3 | Observed Coef. | Bias | Bootstrap Std. Err. | [95\% Con | . Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| _cons | 2.281359 | . 1062846 | . 6615031 | . 9848363 | 3.577881 (N) |
|  |  |  |  | 1.637764 | 4.157873 (P) |
|  |  |  |  | 1.666076 | 4.474779 (BC) |
| probit |  |  |  |  |  |
| _cons | 1.107736 | . 0514693 | . 4554427 | . 2150843 | 2.000387 (N) |
|  |  |  |  | . 58586 | 2.28547 (P) |
|  |  |  |  | . 6385949 | 2.671192 (BC) |
| AUC | Observed Coef. |  | Bootstrap <br> Std. Err. | [95\% Conf. Interval] |  |
|  |  | Bias |  |  |  |
|  | . 9368321 | -. 0023853 | . 0231363 | . 8914859 | . 9821784 (N) |
|  |  |  |  | . 8844096 | . 9722485 (P) |
|  |  |  |  | . 8836259 | . 9718463 (BC) |

Ho: All classifiers have equal AUC values.
Ha: At least one classifier has a different AUC value.
P-value: . 0778556 Test based on bootstrap (N) assumptions.
. rocregplot, title(rocreg) nodraw name(b)
> plot1opts(msymbol(o)) plot2opts(msymbol(s)) plot3opts(msymbol(t))

- graph combine a b, xsize(5)



We see differing true-positive rate values in the scattered points, which is expected because roccomp gives the empirical estimate and rocreg gives the parametric estimate. However, the estimated curves and areas under the ROC curve look similar. Using the Wald test based on the bootstrap covariance, rocreg rejects the null hypothesis that each test has the same AUC at the 0.1 significance level. roccomp formulates the asymptotic covariance using the rocfit estimates of AUC. Examination of its output leads to rejection of the null hypothesis that the AUCs are equal across each test at the 0.05 significance level.

## Parametric ROC curves: Maximum likelihood

The Alonzo and Pepe (2002) method of fitting a parametric model to the ROC curve is powerful because it can be generally applied, but that can be a limitation as well. Whenever we invoke the method and want anything other than point estimates of the parameters, we must perform bootstrap resampling.

An alternative is to use maximum likelihood inference to fit the ROC curve. This method can save computational time by avoiding the bootstrap.
rocreg implements maximum likelihood estimation for ROC curve analysis when both the case and control populations are normal. Particularly, the classifier is a normal linear model on certain covariates, and the covariate effect and variance of the classifier may change between the case and control populations. This model is defined in Pepe (2003, 145).

$$
y=\mathbf{z}^{\prime} \boldsymbol{\beta}_{\mathbf{0}}+D \mathbf{x}^{\prime} \boldsymbol{\beta}_{\mathbf{1}}+\sigma(D) \epsilon
$$

Our error term, $\epsilon$, is a standard normal random variable. The variable $D$ is our true status variable, being 1 for the case population observations and 0 for the control population observations. The variance function $\sigma$ is defined as

$$
\sigma(D)=\sigma_{0}(D=0)+\sigma_{1}(D=1)
$$

This provides two variance parameters in the model and does not depend on covariate values.

Suppose a covariate $x_{i}$ is present in $\mathbf{z}$ and $\mathbf{x}$. The coefficient $\beta_{1 i}$ represents the interaction effect of the $x_{i}$ and $D$. It is the extra effect that $x_{i}$ has on classifier $y$ under the case population, $D=1$, beyond the main effect $\beta_{0 i}$. These $\beta_{1}$ coefficients are directly related to the ROC curve of $y$.

Under this model, the ROC curve is derived to be

$$
\operatorname{ROC}(u)=\Phi\left[\frac{1}{\sigma_{1}}\left\{\mathbf{x}^{\prime} \boldsymbol{\beta}_{\mathbf{1}}+\sigma_{0} \Phi^{-1}(u)\right\}\right]
$$

For convenience, we reparameterize the model at this point, creating the parameters $\beta_{i}=\sigma_{1}^{-1} \beta_{1 i}$ and $\alpha=\sigma_{1}^{-1} \sigma_{0}$. We refer to $\beta_{0}$ as the constant intercept, i_cons. The parameter $\alpha$ is referred to as the constant slope, s_cons.

$$
\operatorname{ROC}(u)=\Phi\left\{\mathbf{x}^{\prime} \boldsymbol{\beta}+\alpha \Phi^{-1}(u)\right\}
$$

We may interpret the final coefficients as the standardized linear effect of the ROC covariate on the classifier under the case population. The marginal effect of the covariate on the classifier in the control population is removed, and it is rescaled by the case population standard deviation of the classifier when all ROC covariate effects are removed. An appreciable effect on the classifier by a ROC covariate in this measure leads to an appreciable effect on the classifier's ROC curve by the ROC covariate.

The advantage of estimating the control coefficients $\boldsymbol{\beta}_{0}$ is similar to the gains of estimating the covariate control models in the estimating equations ROC method and nonparametric ROC estimation. This model would similarly apply when evaluating a test that is conditioned on control covariates.

Again we note that under parametric estimation, all the summary measures we defined earlier except the AUC are not calculated until postestimation. In models with covariates, each covariate combination would yield a different ROC curve and thus different summary parameters, so no summary parameters are estimated initially. In marginal parametric models, we will calculate the AUC and leave the other measures for postestimation. There is a simple closed-form formula for the AUC under the probit model. Using this formula, the delta method can be invoked for inference on the AUC. Details on AUC estimation for probit marginal models are found in Methods and formulas.

We will demonstrate the maximum likelihood method of rocreg by revisiting the models of the previous section.

## > Example 10: Maximum likelihood ROC, single classifier

Returning to the hearing test study of Stover et al. (1996), we use a similar covariate grouping as before. The frequency $x f$ and intensity xl are control covariates ( $\mathbf{z}$ ), while all three covariates $\mathrm{xf}, \mathrm{xl}$, and hearing loss degree xd are case covariates ( $\mathbf{x}$ ). In example 7, we fit this model using the Alonzo and Pepe (2002) method. Earlier we stratified on the control covariates and estimated the conditioned control distribution of nsnr empirically. Now we assume a normal linear model for nsnr on $x f$ and $x l$ under the control population.

We fit the model by specifying the control covariates in the ctrlcov() option and the case covariates in the roccov() option. The ml option tells rocreg to perform maximum likelihood estimation.

```
. use http://www.stata-press.com/data/r13/dp, clear
(Stover - DPOAE test data)
. rocreg d nsnr, ctrlcov(xf xl) roccov(xf xl xd) probit ml cluster(id) nolog
Parametric ROC estimation
Covariate control : linear regression
Control variables : xf xl
Control standardization: normal
ROC method : parametric Link: probit
Status : d
Classifiers: nsnr
Classifier : nsnr
Covariate control adjustment model:
```

(Std. Err. adjusted for 208 clusters in id)

|  | Coef. | Robust Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| casecov |  |  |  |  |  |  |
| xf | . 4690907 | . 1408683 | 3.33 | 0.001 | . 192994 | . 7451874 |
| xl | -3.187785 | . 8976521 | -3.55 | 0.000 | -4.947151 | -1.42842 |
| xd | 3.042998 | . 3569756 | 8.52 | 0.000 | 2.343339 | 3.742657 |
| _cons | 23.48064 | 5.692069 | 4.13 | 0.000 | 12.32439 | 34.63689 |
| casesd |  |  |  |  |  |  |
| _cons | 7.979708 | . 354936 | 22.48 | 0.000 | 7.284047 | 8.67537 |
| ctrlcov |  |  |  |  |  |  |
| xf | -. 1447499 | . 0615286 | -2.35 | 0.019 | -. 2653438 | -. 0241561 |
| xl | -. 8631348 | . 2871976 | -3.01 | 0.003 | -1.426032 | -. 3002378 |
| _cons | 1.109477 | 1.964004 | 0.56 | 0.572 | -2.7399 | 4.958854 |
| ctrlsd |  |  |  |  |  |  |
| _cons | 7.731203 | . 3406654 | 22.69 | 0.000 | 7.063511 | 8.398894 |

Status : d
ROC Model :
(Std. Err. adjusted for 208 clusters in id)

|  |  |  | Robust |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  |  | Coef. | Std. Err. | $z$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| nsnr |  |  |  |  |  |  |  |
|  | i_cons | 2.942543 | .7569821 | 3.89 | 0.000 | 1.458885 | 4.426201 |
|  | xf | .0587854 | .0175654 | 3.35 | 0.001 | .024358 | .0932129 |
|  | xl | -.3994865 | .1171914 | -3.41 | 0.001 | -.6291775 | -.1697955 |
|  | xd | .381342 | .0449319 | 8.49 | 0.000 | .2932771 | .4694068 |
|  | s_cons | .9688578 | .0623476 | 15.54 | 0.000 | .8466587 | 1.091057 |

We find the results are similar to those of example 7. Frequency (xf) and intensity (xl) have a negative effect on the classifier nsnr in the control population.

The negative control effect is mitigated for xf in the case population, but the effect for xl is even more negative there. Hearing loss severity, xd, has a positive effect on nsnr in the case population, and it is undefined in the control population.

The ROC coefficients are shown in the ROC Model table. Each are different from 0 at the 0.05 level. At this level, we also cannot conclude that the variances differ from case to control populations, because 1 is in the $95 \%$ confidence interval for s_cons, the ratio of the case to control standard deviation parameters.

Both frequency ( xf ) and hearing loss severity ( xd ) make a positive contribution to the ROC curve and thus make the test more powerful. Intensity ( $x l$ ) has a negative effect on the ROC curve and weakens the test. We previously saw in example 5 that the control distribution appears to be normal, so using maximum likelihood to fit this model is a reasonable approach.

This model was also fit in Pepe (2003, 147). Pepe used separate least-squares estimates for the case and control samples. We obtain similar results for the coefficients, but the maximum likelihood fitting yields slightly different standard deviations by considering both case and control observations concurrently. In addition, a misprint in Pepe (2003, 147) reports a coefficient of -4.91 for $x l$ in the case population instead of -3.19 as reported by Stata.

Inference on multiple classifiers using the Alonzo and Pepe (2002) estimating equation method is performed by fitting each model separately and bootstrapping to determine the dependence of the estimates. Using the maximum likelihood method, we also fit each model separately. We use suest (see [R] suest) to estimate the joint variance-covariance of our parameter estimates.

For our models, we can view the score equation for each model as an estimating equation. The estimate that solves the estimating equation (that makes the score 0 ) is asymptotically normal with a variance matrix that can be estimated using the inverse of the squared scores. By stacking the score equations of the separate models, we can estimate the variance matrix for all the parameter estimates by using this rule. This is an informal explanation; further details can be found in $[R]$ suest and in the references Rogers (1993); White (1982 and 1996).

Now we will examine a case with multiple classification variables.

## > Example 11: Maximum likelihood ROC, multiple classifiers

We return to the neonatal audiology study with gender and age covariates (Norton et al. 2000). In example 6, we fit a model with male and currage as control covariates, and currage as a ROC covariate for the classifier y1 (DPOAE 65 at 2 kHz ). We will refit this model, extending it to include the classifier y2 (TEOAE 80 at 2 kHz ).

```
. use http://www.stata-press.com/data/r13/nnhs
(Norton - neonatal audiology data)
. rocreg d y1 y2, probit ml ctrlcov(currage male) roccov(currage) cluster(id)
> nolog
```

Parametric ROC estimation
Covariate control : linear regression
Control variables : currage male
Control standardization: normal
ROC method : parametric Link: probit

```
Status : d
Classifiers: y1 y2
Classifier : y1
```

Covariate control adjustment model:

|  | Coef. | Std. Err. | z | $P>\|z\|$ | [95\% Con | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| casecov |  |  |  |  |  |  |
| currage | . 494211 | . 2126672 | 2.32 | 0.020 | . 077391 | . 9110311 |
| _cons | -15.00403 | 8.238094 | -1.82 | 0.069 | -31.1504 | 1.142338 |
| casesd |  |  |  |  |  |  |
| _cons | 8.49794 | . 4922792 | 17.26 | 0.000 | 7.533091 | 9.46279 |
| ctrlcov |  |  |  |  |  |  |
| currage | -. 2032048 | . 0323803 | -6.28 | 0.000 | -. 266669 | -. 1397406 |
| male | . 2369359 | . 2201391 | 1.08 | 0.282 | -. 1945288 | . 6684006 |
| _cons | -1.23534 | 1.252775 | -0.99 | 0.324 | -3.690734 | 1.220055 |
| ctrlsd |  |  |  |  |  |  |
| _cons | 7.749156 | . 0782225 | 99.07 | 0.000 | 7.595843 | 7.902469 |

Classifier : y2
Covariate control adjustment model:

|  | Coef. | Std. Err. | z | $P>\|z\|$ | [95\% Con | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| casecov |  |  |  |  |  |  |
| currage | . 5729861 | . 2422662 | 2.37 | 0.018 | . 0981532 | 1.047819 |
| _cons | -18.2597 | 9.384968 | -1.95 | 0.052 | -36.6539 | . 1344949 |
| casesd |  |  |  |  |  |  |
| _cons | 9.723858 | . 5632985 | 17.26 | 0.000 | 8.619813 | 10.8279 |
| ctrlcov |  |  |  |  |  |  |
| currage | -. 1694575 | . 0291922 | -5.80 | 0.000 | -. 2266732 | -. 1122419 |
| male | . 7122587 | . 1993805 | 3.57 | 0.000 | . 3214802 | 1.103037 |
| _cons | -5.651728 | 1.129452 | -5.00 | 0.000 | -7.865415 | -3.438042 |
| ctrlsd |  |  |  |  |  |  |
| _cons | 6.986167 | . 0705206 | 99.07 | 0.000 | 6.84795 | 7.124385 |

```
Status : d
ROC Model :
```

(Std. Err. adjusted for 2741 clusters in id)

|  |  |  | Robust |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  |  | Coef. | Std. Err. | $\mathbf{z}$ | P>\|z| | [95\% Conf. Interval] |  |
| y1 |  |  |  |  |  |  |  |
|  | i_cons | -1.765608 | 1.105393 | -1.60 | 0.110 | -3.932138 | .4009225 |
|  | currage | .0581566 | .0290177 | 2.00 | 0.045 | .0012828 | .1150303 |
|  | s_cons | .9118864 | .0586884 | 15.54 | 0.000 | .7968593 | 1.026913 |
| y2 |  |  |  |  |  |  |  |
|  | i_cons | -1.877825 | .905174 | -2.07 | 0.038 | -3.651933 | -.1037167 |
|  | currage | .0589258 | .0235849 | 2.50 | 0.012 | .0127002 | .1051514 |
|  | s_cons | .7184563 | .0565517 | 12.70 | 0.000 | .607617 | .8292957 |

Both classifiers have similar results. The results for y1 show the same direction as the estimating equation results in example 6 . However, we can now reject the null hypothesis that the ROC currage coefficient is 0 at the 0.05 level.

In example 6, we could not reject that the slope parameter s_cons was 1 and that the constant intercept or ROC coefficient for current age was 0 . The resulting ROC curve implied a noninformative test using y1 as a classifier. This is not the case with our current results. As currage increases, we expect a steeper ROC curve and thus a more powerful test, for both classifiers y1 (DPOAE 65 at 2 kHz ) and y2 (TEOAE 80 at 2 kHz ).

In example 10, the clustering of observations within infant id was adjusted in the individual fit of nsnr. In our current example, the adjustment for the clustering of observations within id is performed during concurrent estimation, as opposed to during the individual classifier fits (as in example 10). This adjustment, performed by suest, is still accurate.

Now we will fit constant probit models and compare rocreg with rocfit and roccomp with the binormal option. Our first applications of rocfit and roccomp are taken directly from examples 8 and 9. The Dorfman and Alf (1969) algorithm that rocfit works with uses discrete classifiers or uses slicing to make a classifier discrete. So we are applying the maximum likelihood method of rocreg on discrete classification data here, where it expects continuous data. We expect to see some discrepancies, but we do not find great divergence in the estimates. After revisiting examples 8 and 9 , we will fit a probit model with a continuous classifier and no covariates using rocreg, and we will compare the results with those from rocfit.

## > Example 12: Maximum likelihood ROC, marginal model

Using the Hanley and McNeil (1982) data, discussed in example 1 and in example 8, we fit a constant probit model of the classifier rating with true status disease. rocreg is invoked with the ml option and compared with rocfit.
. use http://www.stata-press.com/data/r13/hanley, clear
. rocfit disease rating, nolog
Binormal model of disease on rating Number of obs $=109$
Goodness-of-fit chi2(2) = 0.21
Prob > chi2 $=0.9006$
Log likelihood $=-123.64855$

|  | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| intercept | 1.656782 | 0.310456 | 5.34 | 0.000 | 1.048300 | 2.265265 |
| slope (*) | 0.713002 | 0.215882 | -1.33 | 0.184 | 0.289881 | 1.136123 |
| /cut1 | 0.169768 | 0.165307 | 1.03 | 0.304 | -0.154227 | 0.493764 |
| /cut2 | 0.463215 | 0.167235 | 2.77 | 0.006 | 0.135441 | 0.790990 |
| /cut3 | 0.766860 | 0.174808 | 4.39 | 0.000 | 0.424243 | 1.109477 |
| /cut4 | 1.797938 | 0.299581 | 6.00 | 0.000 | 1.210770 | 2.385106 |


|  |  | Indices from binormal fit |  |  |  |  |
| ---: | ---: | :--- | ---: | ---: | :---: | :---: |
| Index | Estimate | Std. Err. | [95\% Conf. Interval] |  |  |  |
| ROC area | 0.911331 | 0.029506 | 0.853501 | 0.969161 |  |  |
| delta(m) | 2.323671 | 0.502370 | 1.339044 | 3.308298 |  |  |
| d(e) | 1.934361 | 0.257187 | 1.430284 | 2.438438 |  |  |
| d(a) | 1.907771 | 0.259822 | 1.398530 | 2.417012 |  |  |

(*) z test for slope==1
. rocreg disease rating, probit ml nolog
Parametric ROC estimation
Control standardization: normal
ROC method : parametric Link: probit

> Status : disease
> Classifiers: rating

Classifier : rating
Covariate control adjustment model:

|  | Coef. | Std. Err. | $z$ | P>\|z| | [95\% Conf. Interval] |  |
| :--- | ---: | :--- | :--- | :--- | :--- | :--- | :--- |
| casecov _cons | 2.3357 | .2334285 | 10.01 | 0.000 | 1.878188 | 2.793211 |
| casesd _cons | 1.117131 | .1106124 | 10.10 | 0.000 | .9003344 | 1.333927 |
| ctrlcov_cons | 2.017241 | .1732589 | 11.64 | 0.000 | 1.67766 | 2.356823 |
| ctrlsd _cons | 1.319501 | .1225125 | 10.77 | 0.000 | 1.07938 | 1.559621 |


| Status <br> ROC Model : disease |  | Coef. | Std. Err. | $z$ | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
|  |  |  |  |  |  |  |
| rating |  |  |  |  |  |  |
| i_cons | 2.090802 | .2941411 | 7.11 | 0.000 | 1.514297 | 2.667308 |
| S_cons | 1.181151 | .1603263 | 7.37 | 0.000 | .8669177 | 1.495385 |
| auc | .9116494 | .0261658 | 34.84 | 0.000 | .8603654 | .9629333 |

We compare the estimates for these models:

|  | rocfit | rocreg, ml |
| :--- | :--- | :--- |
| slope | 0.7130 | 1.1812 |
| SE of slope | 0.2159 | 0.1603 |
| intercept | 1.6568 | 2.0908 |
| SE of intercept | 0.3105 | 0.2941 |
| AUC | 0.9113 | 0.9116 |
| SE of AUC | 0.0295 | 0.0262 |

We find that both the intercept and the slope are estimated as higher with the maximum likelihood method under rocreg than with rocfit. The AUC (ROC area in rocfit) is close for both commands. We find that the standard errors of each of these estimates is slightly lower under rocreg than rocfit as well.

Both rocfit and rocreg suggest that the slope parameter of the ROC curve (slope in rocfit and s_cons in rocreg) is not significantly different from 1 . Thus we cannot reject that the classifier has the same variance in both case and control populations. There is, however, significant evidence that the intercepts (i_cons in rocreg and intercept in rocfit) differ from 0 . Because of the positive direction of the intercept estimates, the ROC curve for rating as a classifier of disease suggests that rating provides an informative test. This is also suggested by the high AUC, which is significantly different from 0.5 , that is, a flip of a coin.

## > Example 13: Maximum likelihood ROC, marginal model, multiple classifiers

We use the fictitious dataset generated from Hanley and McNeil (1983), which we previously used in example 2 and in example 9. To fit a probit model using roccomp, we specify the binormal option. We perform parametric, maximum likelihood ROC analysis using rocreg. We use rocregplot to plot the ROC curves created by rocreg.

```
. use http://www.stata-press.com/data/r13/ct2, clear
. roccomp status mod1 mod2 mod3, summary binormal graph aspectratio(1)
> plot1opts(connect(i) msymbol(o))
> plot2opts(connect(i) msymbol(s))
> plot3opts(connect(i) msymbol(t))
> legend(label(1 "mod1") label(3 "mod2") label(5 "mod3")
> label(2 "mod1 fit") label(4 "mod2 fit") label(6 "mod3 fit")
> order(1 3 5 2 4 6) cols(1)) title(roccomp) name(a) nodraw
Fitting binormal model for: mod1
Fitting binormal model for: mod2
Fitting binormal model for: mod3
                                    ROC
\begin{tabular}{lccccc} 
& Obs & Area & Std. Err. & {\([95 \%\) Conf. Interval] } \\
\hline mod1 & 112 & 0.8945 & 0.0305 & 0.83482 & 0.95422 \\
mod2 & 112 & 0.9382 & 0.0264 & 0.88647 & 0.99001 \\
mod3 & 112 & 0.9376 & 0.0223 & 0.89382 & 0.98139
\end{tabular}
```

```
Ho: area(mod1) = area(mod2) = area(mod3)
    chi2(2) = 8.27 Prob>chi2 = 0.0160
```

. rocreg status mod1 mod2 mod3, probit ml nolog
Parametric ROC estimation
Control standardization: normal

| ROC method $\quad$ : parametric |  |
| :--- | :--- |
| Status $:$ status |  |
| Classifiers: mod1 mod2 mod3 |  |
| Classifier : mod1 |  |
| Covariate control adjustment model: |  |


|  | Coef. | Std. Err. | $z$ | P>\|z| | [95\% Conf. Interval] |  |
| :--- | ---: | :--- | :--- | :--- | :--- | :--- | :--- |
| casecov _cons | 2.118135 | .2165905 | 9.78 | 0.000 | 1.693626 | 2.542645 |
| casesd _cons | 1.166078 | .1122059 | 10.39 | 0.000 | .9461589 | 1.385998 |
| ctrlcov _cons | 2.344828 | .1474147 | 15.91 | 0.000 | 2.0559 | 2.633755 |
| ctrlsd _cons | 1.122677 | .1042379 | 10.77 | 0.000 | .9183746 | 1.32698 |

Classifier : mod2
Covariate control adjustment model:

|  | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| :--- | ---: | :--- | ---: | :--- | ---: | :--- |
| casecov _cons | 2.659642 | .2072731 | 12.83 | 0.000 | 2.253395 | 3.06589 |
| casesd _cons | 1.288468 | .1239829 | 10.39 | 0.000 | 1.045466 | 1.53147 |
| ctrlcov _cons | 1.655172 | .1105379 | 14.97 | 0.000 | 1.438522 | 1.871823 |
| ctrlsd _cons | .8418313 | .0781621 | 10.77 | 0.000 | .6886365 | .9950262 |

Classifier : mod3
Covariate control adjustment model:

|  | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| :--- | ---: | :--- | :--- | :--- | :--- | :--- | :--- |
| casecov _cons | 2.353768 | .1973549 | 11.93 | 0.000 | 1.966959 | 2.740576 |
| casesd _cons | 1.143359 | .1100198 | 10.39 | 0.000 | .9277243 | 1.358994 |
| ctrlcov _cons | 2.275862 | .1214094 | 18.75 | 0.000 | 2.037904 | 2.51382 |
| ctrlsd _cons | .9246267 | .0858494 | 10.77 | 0.000 | .7563649 | 1.092888 |


| Status <br> ROC Model : status |  |  |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  |  | Coef. | Robust <br> Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
|  |  |  |  |  |  |  |  |
| mod1 |  |  |  |  |  |  |  |
|  | i_cons | 1.81646 | .3144804 | 5.78 | 0.000 | 1.20009 | 2.432831 |
|  | s_cons | .9627801 | .1364084 | 7.06 | 0.000 | .6954245 | 1.230136 |
|  | auc | .904657 | .0343518 | 26.34 | 0.000 | .8373287 | .9719853 |
| mod2 |  |  |  |  |  |  |  |
|  | i_cons | 2.064189 | .3267274 | 6.32 | 0.000 | 1.423815 | 2.704563 |
|  | s_cons | .6533582 | .1015043 | 6.44 | 0.000 | .4544135 | .8523029 |
|  | auc | .9580104 | .0219713 | 43.60 | 0.000 | .9149473 | 1.001073 |
| mod3 |  |  |  |  |  |  |  |
|  | i_cons | 2.058643 | .2890211 | 7.12 | 0.000 | 1.492172 | 2.625113 |
|  | s_cons | .8086932 | .1163628 | 6.95 | 0.000 | .5806262 | 1.03676 |
|  | auc | .9452805 | .0236266 | 40.01 | 0.000 | .8989732 | .9915877 |

Ho: All classifiers have equal AUC values.
Ha: At least one classifier has a different AUC value.
P-value: . 0808808
. rocregplot, title(rocreg) nodraw name(b)
> plot1opts(msymbol(o)) plot2opts(msymbol(s)) plot3opts(msymbol(t))
. graph combine a b, xsize(5)


We compare the AUC estimates for these models:

|  | roccomp | rocreg, ml |
| :--- | :--- | :--- |
| $\bmod 1$ | 0.8945 | 0.9047 |
| $\bmod 2$ | 0.9382 | 0.9580 |
| $\bmod 3$ | 0.9376 | 0.9453 |

Each classifier has a higher estimated AUC under rocreg than roccomp. Each curve appears to be raised and smoothed in the rocreg fit as compared with roccomp. They are different, but not drastically different. The inference on whether the curve areas are the same is similar to example 9 . We reject equality at the 0.10 level under rocreg and at the 0.05 level under roccomp.

Each intercept is significantly different from 0 at the 0.05 level and is estimated in a positive direction. Though all but classifier mod2 has 1 in their slope confidence intervals, the high intercepts suggest steep ROC curves and powerful tests.

Also note that the false-positive and true-positive rate points are calculated empirically in the roccomp graph and parametrically in rocreg. In example 9, the false-positive rates calculated by rocreg were calculated empirically, similar to roccomp. But in this example, the rates are calculated based on normal percentiles.

Now we will generate an example to compare rocfit and rocreg under maximum likelihood estimation of a continuous classifier.

## > Example 14: Maximum likelihood ROC, graphical comparison with rocfit

We generate 500 realizations of a population under threat of disease. One quarter of the population has the disease. A classifier x is measured, which has a control distribution of $N(1,3)$ and a case distribution of $N(1+5,2)$. We will invoke rocreg with the ml option on this generated data. We specify the continuous () option for rocfit and invoke it on the data as well. The continuous() option tells rocfit how many discrete slices to partition the data into before fitting.

For comparison of the two curves, we will use the rocfit postestimation command, rocplot; see [R] rocfit postestimation. This command graphs the empirical false-positive and true-positive rates with an overlaid fit of the binormal curve estimated by rocfit. rocplot also supports an addplot() option. We use the saved variables from rocreg in this option to overlay a line plot of the rocreg fit.

```
. clear
. set seed 8675309
. set obs 500
obs was 0, now 500
. generate d = runiform() < . 25
. quietly generate double epsilon = 3*invnormal(runiform()) if d == 0
. quietly replace epsilon = 2*invnormal(runiform()) if d == 1
. quietly generate double x = 1 + d*5 + epsilon
```

. rocreg $\mathrm{d} x$, probit ml nolog
Parametric ROC estimation
Control standardization: normal
ROC method : parametric Link: probit
Status : d
Classifiers: x
Classifier : x
Covariate control adjustment model:

|  | Coef. | Std. Err. | $z$ | P>\|z| | [95\% Conf. Interval] |  |
| :--- | ---: | :--- | :--- | :--- | :--- | :--- | :--- |
| casecov _cons | 4.905612 | .2411624 | 20.34 | 0.000 | 4.432943 | 5.378282 |
| casesd _cons | 2.038278 | .1299559 | 15.68 | 0.000 | 1.783569 | 2.292987 |
| ctrlcov _cons | 1.010382 | .1561482 | 6.47 | 0.000 | .7043377 | 1.316427 |
| ctrlsd _cons | 3.031849 | .1104134 | 27.46 | 0.000 | 2.815443 | 3.248255 |


|  | Coef. | Std. Err. | z | $P>\|z\|$ | [95\% Conf | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| i_cons | 2.406743 | . 193766 | 12.42 | 0.000 | 2.026969 | 2.786518 |
| s_cons | 1.487456 | . 1092172 | 13.62 | 0.000 | 1.273394 | 1.701518 |
| auc | . 9103292 | . 012754 | 71.38 | 0.000 | . 8853318 | . 9353266 |

. rocfit d x, continuous(10) nolog
Binormal model of $d$ on $x \quad$ Number of obs $=500$
Goodness-of-fit chi2(7) = 1.69

Prob > chi2 $=0.9751$
Log likelihood = -911.91338

|  | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :--- | ---: | :--- | ---: | ---: |
| intercept | 2.207250 | 0.232983 | 9.47 | 0.000 | 1.750611 | 2.663888 |
| slope (*) | 1.281443 | 0.158767 | 1.77 | 0.076 | 0.970265 | 1.592620 |
| /cut1 | -1.895707 | 0.130255 | -14.55 | 0.000 | -2.151001 | -1.640412 |
| /cut2 | -1.326900 | 0.089856 | -14.77 | 0.000 | -1.503015 | -1.150784 |
| /cut3 | -0.723677 | 0.070929 | -10.20 | 0.000 | -0.862695 | -0.584660 |
| /cut4 | -0.116960 | 0.064666 | -1.81 | 0.070 | -0.243702 | 0.009782 |
| /cut5 | 0.442769 | 0.066505 | 6.66 | 0.000 | 0.312422 | 0.573116 |
| /cut6 | 1.065183 | 0.075744 | 14.06 | 0.000 | 0.916728 | 1.213637 |
| /cut7 | 1.689570 | 0.102495 | 16.48 | 0.000 | 1.488683 | 1.890457 |
| /cut8 | 2.495841 | 0.185197 | 13.48 | 0.000 | 2.132861 | 2.858821 |
| /cut9 | 3.417994 | 0.348485 | 9.81 | 0.000 | 2.734976 | 4.101012 |


|  |  | Indices from binormal fit |  |  |
| ---: | ---: | :--- | ---: | ---: |
| Index | Estimate | Std. Err. | [95\% Conf. Interval] |  |
| ROC area | 0.912757 | 0.013666 | 0.885972 | 0.939542 |
| delta(m) | 1.722473 | 0.127716 | 1.472153 | 1.972792 |
| d(e) | 1.934960 | 0.125285 | 1.689405 | 2.180515 |
| d(a) | 1.920402 | 0.121804 | 1.681670 | 2.159135 |

(*) z test for slope==1
. rocplot, plotopts(msymbol(i)) lineopts(lpattern(dash))
$>\quad$ norefline addplot(line _roc_x _fpr_x, sort(_fpr_x _roc_x)
> lpattern(solid)) aspectratio(1) legend(off)


We find that the curves are close. As before, the rocfit estimates are lower for the slope and intercept than under rocreg. The AUC estimates are close. Though the slope confidence interval contains 1, a high ROC intercept suggests a steep ROC curve and thus a powerful test.

## Stored results

Nonparametric rocreg stores the following in e():
Scalars

| $e(N)$ | number of observations |
| :--- | :--- |
| $e\left(N_{\text {_strata }}\right)$ | number of covariate strata |
| $e\left(N_{\text {_clust }}\right)$ | number of clusters |
| $e($ rank $)$ | rank of $e(V)$ |

Macros

```
    e(cmd)
    e(cmdline)
    e(classvars)
```

    e(refvar)
    e(ctrlmodel)
    e(ctrlcov)
    e(pvc)
    e(title)
    e(tiecorrected)
    e(nobootstrap)
    e(bseed)
    e(breps)
    e(cc)
    e(nobstrata)
    e(clustvar)
    e(roc)
    e(invroc)
    e(pauc)
    e(auc)
    e(vce)
    e(properties) b V (or b if bootstrap not performed)
    Matrices
e(b)
e(V)
e(b_bs)
e(bias)
e(se)
e(z0)
e(ci_normal)
e(ci_percentile)
e(ci_bc)

Functions

```
    e(sample) marks estimation sample
```

Parametric, bootstrap rocreg stores the following in e():
Scalars
e(N)
e(N_strata)
e(N_clust)
e(rank)
Macros
e(cmd)
e(cmdline)
e(title)
e(classvars)
e(refvar)
e(ctrlmodel)
e(ctrlcov)
e(pvc)
e(title)
e(tiecorrected)
e(probit)
e(roccov)
e(fprpts)
e(ctrlfprall)
e(nobootstrap)
e(bseed)
e(breps)
e(cc)
e(nobstrata)
e(clustvar)
e(vce)
e(properties)
e(predict)
Matrices
e(b)
e(V)
e(b_bs)
e(reps)
e(bias)
e(se)
e(z0)
e(ci_normal)
e(ci_percentile)
e(ci_bc)
Functions
e(sample) marks estimation sample
number of observations
number of covariate strata
number of clusters
rank of e(V)

## rocreg

command as typed
title in estimation output
classification variable list status variable, reference variable covariate-adjustment specification covariate-adjustment variables percentile value calculation method title in estimation output probit
ROC covariates indicates that bootstrap was performed
seed used in bootstrap
number of bootstrap resamples
name of cluster variable
bootstrap
b V (or b if nobootstrap is specified)
program used to implement predict
coefficient vector
bootstrap estimates
number of nonmissing results
estimated biases
estimated standard errors
median biases
percentile confidence intervals
bias-corrected confidence intervals
indicates whether tie correction was used
number of points used as false-positive rate fit points
indicates whether all observed false-positive rates were used as fit points
indicates whether case-control groups were used as resampling strata
indicates whether resampling should stratify based on control covariates
variance-covariance matrix of the estimators
normal-approximation confidence intervals

Parametric, maximum likelihood rocreg stores the following in e():
Scalars
e(N)
e(N_clust)
e(rank)
Macros
e(cmd)
e(cmdline)
e(classvars)
e(refvar)
e(ctrlmodel)
e(ctrlcov)
e(roccov)
e(probit)
e(pvc)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(vce)
e(vcetype)
e(ml)
e(predict)
Matrices
e(b)
e(V)
Functions
e(sample) marks estimation sample

## Methods and formulas

Assume that we applied a diagnostic test to each of $N_{0}$ control and $N_{1}$ case subjects. Further assume that the higher the outcome value of the diagnostic test, the higher the risk of the subject being abnormal. Let $y_{1 i}, i=1,2, \ldots, N_{1}$, and $y_{0 j}, j=1,2, \ldots, N_{0}$, be the values of the diagnostic test for the case and control subjects, respectively. The true status variable $D$ identifies an observation as case $D=1$ or control $D=0$. The CDF of the classifier $Y$ is $F$. Conditional on $D$, we write the CDF as $F_{D}$.

Methods and formulas are presented under the following headings:

ROC statistics<br>Covariate-adjusted ROC curves<br>Parametric ROC curves: Estimating equations<br>Parametric ROC curves: Maximum likelihood

## ROC statistics

We obtain these definitions and their estimates from Pepe (2003) and Pepe, Longton, and Janes (2009). The false-positive and true-positive rates at cutoff $c$ are defined as

$$
\begin{aligned}
& \operatorname{FPR}(y)=P(Y \geq y \mid D=0) \\
& \operatorname{TPR}(y)=P(Y \geq y \mid D=1)
\end{aligned}
$$

The true-positive rate, or ROC value at false-positive rate $u$, is given by

$$
\operatorname{ROC}(u)=P\left(1-F_{0}(Y) \leq u \mid D=1\right)
$$

When $Y$ is continuous, the false-positive rate can be written as

$$
\operatorname{FPR}(y)=1-F_{0}(y)
$$

The empirical CDF for the sample $z_{1}, \ldots, z_{n}$ is given by

$$
\widehat{F}(z)=\sum_{i=1}^{n} \frac{I\left(z<z_{i}\right)}{n}
$$

The empirical estimates $\widehat{\text { PPR }}$ and $\widehat{\text { ROC }}$ both use this empirical CDF estimator.
The area under the ROC curve is defined as

$$
\mathrm{AUC}=\int_{0}^{1} \operatorname{ROC}(u) d_{u}
$$

The partial area under the ROC curve for false-positive rate $a$ is defined as

$$
\operatorname{pAUC}(a)=\int_{0}^{a} \operatorname{ROC}(u) d_{u}
$$

The nonparametric estimate for the AUC is given by

$$
\widehat{\mathrm{AUC}}=\sum_{i=1}^{N_{1}} \frac{1-\widehat{\mathrm{FPR}}\left(y_{1 i}\right)}{N_{1}}
$$

The nonparametric estimate of pAUC is given by

$$
\widehat{\operatorname{pAUC}}(a)=\sum_{i=1}^{N_{1}} \frac{\max \left\{1-\widehat{\mathrm{FPR}}\left(y_{1 i}\right)-(1-a), 0\right\}}{N_{1}}
$$

For discrete classifiers, a correction term is subtracted from the false-positive rate estimate so that the $\widehat{A U C}$ and $\widehat{p A U C}$ estimates correspond with a trapezoidal approximation to the area of the ROC curve.

$$
\operatorname{FPR}^{c}(y)=1-\widehat{F}_{0}(y)-\frac{1}{2} \sum_{j=1}^{N_{0}} \frac{I\left(y=y_{0 j}\right)}{N_{0}}
$$

In the nonparametric estimation of the ROC curve, all inference is performed using the bootstrap command (see [R] bootstrap). rocreg also allows users to calculate the ROC curve and related statistics by assuming a normal control distribution. So these formulas are updated by replacing $F_{0}$ by $\Phi$ (with adjustment of the marginal mean and variance of the control distribution).

## Covariate-adjusted ROC curves

Suppose we observe covariate vector $Z$ in addition to the classifier $Y$. Let $Z_{1 i}, i=1,2, \ldots, N_{1}$, and $Z_{0 j}, j=1,2, \ldots, N_{0}$, be the values of the covariates for the case and control subjects, respectively.

The covariate-adjusted ROC curve is defined by Janes and Pepe (2009) as

$$
\operatorname{AROC}(t)=E\left\{\operatorname{ROC}\left(t \mid Z_{0}\right)\right\}
$$

It is calculated by replacing the marginal control CDF estimate, $\widehat{F}_{0}$, with the conditional control CDF estimate, $\widehat{F}_{0 Z}$. If we used a normal control CDF, then we would replace the marginal control mean and variance with the conditional control mean and variance. The formulas of the previous section can be updated for covariate-adjustment by making this substitution of the conditional CDF for the marginal CDF in the false-positive rate calculation.

Because the calculation of the ROC value is now performed based on the conditionally calculated false-positive rate, no further conditioning is made in its calculation under nonparametric estimation.
rocreg supports covariate adjustment with stratification and linear regression. Under stratification, separate parameters are estimated for the control distribution at each level of the covariates. Under linear regression, the classifier is regressed on the covariates over the control distribution, and the resulting coefficients serve as parameters for $\widehat{F}_{0 Z}$.

## Parametric ROC curves: Estimating equations

Under nonparametric estimation of the ROC curve with covariate adjustment, no further conditioning occurs in the ROC curve calculation beyond the use of covariate-adjusted false-positive rates as inputs.

Under parametric estimation of the ROC curve, we can relax this restriction. We model the ROC curve as a cumulative distribution function $g$ (standard normal $\Phi$ ) invoked with input of a linear polynomial in the corresponding quantile function (here $\Phi^{-1}$ ) invoked on the false-positive rate $u$. The constant intercept of the polynomial may depend on covariates; the slope term $\alpha$ (quantile coefficient) may not.

$$
\operatorname{ROC}(u)=g\left\{\mathbf{x}^{\prime} \boldsymbol{\beta}+\alpha g^{-1}(u)\right\}
$$

Pepe (2003) notes that having a binormal ROC $(g=\Phi)$ is equivalent to specifying that some monotone transformation of the data exists to make the case and control classifiers normally distributed. This specification applies to the marginal case and control.

Under weak assumptions about the control distribution of the classifier, we can fit this model by using estimating equations (Alonzo and Pepe 2002). The method can be used without covariate effects in the second stage, assuming a parametric model for the single ROC curve. Using the Alonzo and Pepe (2002) method, the covariate-adjusted ROC curve may be fit parametrically. The marginal ROC curve, involving no covariates in either stage of estimation, can be fit parametrically as well. In addition to the Alonzo and Pepe (2002) explanation, further details are given in Pepe, Longton, and Janes (2009); Janes, Longton, and Pepe (2009); Pepe (2003); and Janes and Pepe (2009).

The algorithm can be described as follows:

1. Estimate the false-positive rates of the classifier fpr. These may be computed in any fashion outlined so far: covariate-adjusted, empirically, etc.
2. Determine a set of $n_{p}$ false-positive rates to use as fitting points $f_{1}, \ldots, f_{n_{p}}$. These may be an equispaced grid on $(0,1)$ or the set of observed false-positive rates from part 1.
3. Expand the case observation portion of the data to include a subobservation for each fitting point. So there are now $N_{1}\left(n_{p}-1\right)$ additional observations in the data.
4. Generate a new dummy variable u . For subobservation $j, \mathrm{u}=I\left(\mathrm{fpr} \leq f_{j}\right)$.
5. Generate a new variable quant containing the quantiles of the false-positive rate fitting points. For subobservation $j$, quant $=g^{-1}\left(f_{j}\right)$.
6. Perform a binary regression (probit, $g=\Phi$ ) of fpr on the covariates x and quantile variable quant.
The coefficients of part 6 are the coefficients of the ROC model. The coefficients of the covariates coincide naturally with estimates of $\boldsymbol{\beta}$, and the $\alpha$ parameter is estimated by the coefficient on quant. Because the method is so general and makes few distributional assumptions, bootstrapping must be performed for inference. If multiple classifiers are to be fit, the algorithm is performed separately for each in each bootstrap, and the bootstrap is used to estimate covariances.

We mentioned earlier that in parametric estimation, the AUC was the only summary parameter that could be estimated initially. This is true when we fit the marginal probit model because there are no covariates in part 6 of the algorithm.

To calculate the AUC statistic under a marginal probit model, we use the formula

$$
\mathrm{AUC}=\Phi\left(\frac{\beta_{0}}{\sqrt{1+\alpha^{2}}}\right)
$$

Alternatively, the AUC for the probit model can be calculated as pAUC(1) in postestimation. Under both models, bootstrapping is performed for inference on the AUC.

## Parametric ROC curves: Maximum likelihood

rocreg supports another form of parametric ROC estimation: maximum likelihood with a normally distributed classifier. This method assumes that the classifier is a normal linear model on certain covariates, and the covariate effect and variance of the classifier may change between the case and control populations. The model is defined in Pepe (2003, 145).

$$
y=\mathbf{z}^{\prime} \boldsymbol{\beta}_{\mathbf{0}}+D \mathbf{x}^{\prime} \boldsymbol{\beta}_{\mathbf{1}}+\sigma(D) \epsilon
$$

Our error term, $\epsilon$, is a standard normal random variable. The variable $D$ is our true status variable, being 1 for the case population observations and 0 for the control population observations. The variance function $\sigma$ is defined as

$$
\sigma(D)=\sigma_{0}(D=0)+\sigma_{1}(D=1)
$$

This provides two variance parameters in the model and does not depend on covariate values.
Under this model, the ROC curve is easily derived to be

$$
\operatorname{ROC}(u)=\Phi\left[\frac{1}{\sigma_{1}}\left\{\mathbf{x}^{\prime} \boldsymbol{\beta}_{\mathbf{1}}+\sigma_{0} \Phi^{-1}(u)\right\}\right]
$$

We reparameterize the model, creating the parameters $\beta_{i}=\sigma_{1}^{-1} \beta_{1 i}$ and $\alpha=\sigma_{1}^{-1} \sigma_{0}$. We refer to $\beta_{0}$ as the constant intercept, i_cons. The parameter $\alpha$ is referred to as the constant slope, s_cons.

$$
\operatorname{ROC}(u)=\Phi\left\{\mathbf{x}^{\prime} \boldsymbol{\beta}+\alpha \Phi^{-1}(u)\right\}
$$

The original model defining the classifier $y$ leads to the following single observation likelihoods for $D=0$ and $D=1$ :

$$
\begin{gathered}
L\left(\boldsymbol{\beta}_{\mathbf{0}}, \boldsymbol{\beta}_{\mathbf{1}}, \sigma_{1}, \sigma_{0}, \mid D=0, y, \mathbf{z}, \mathbf{x}\right)=\frac{1}{\sqrt{2 \pi} \sigma_{0}} \exp \frac{-\left(y-\mathbf{z}^{\prime} \boldsymbol{\beta}_{\mathbf{0}}\right)^{2}}{2 \sigma_{0}^{2}} \\
L\left(\boldsymbol{\beta}_{\mathbf{0}}, \boldsymbol{\beta}_{\mathbf{1}}, \sigma_{1}, \sigma_{0}, \mid D=1, y, \mathbf{z}, \mathbf{x}\right)=\frac{1}{\sqrt{2 \pi} \sigma_{1}} \exp \frac{-\left(y-\mathbf{z}^{\prime} \boldsymbol{\beta}_{\mathbf{0}}-\mathbf{x}^{\prime} \boldsymbol{\beta}_{\mathbf{1}}\right)^{2}}{2 \sigma_{1}^{2}}
\end{gathered}
$$

These can be combined to yield the observation-level log likelihood:

$$
\begin{aligned}
\ln L\left(\boldsymbol{\beta}_{\mathbf{0}}, \boldsymbol{\beta}_{\mathbf{1}}, \sigma_{1}, \sigma_{0}, \mid D, y, \mathbf{z}, \mathbf{x}\right)= & -\frac{\ln 2 \pi}{2} \\
& -I(D=0)\left\{\ln \sigma_{0}+\frac{\left(y-\mathbf{z}^{\prime} \boldsymbol{\beta}_{\mathbf{0}}\right)^{2}}{2 \sigma_{0}^{2}}\right\} \\
& -I(D=1)\left\{\ln \sigma_{1}+\frac{\left(y-\mathbf{z}^{\prime} \boldsymbol{\beta}_{\mathbf{0}}-\mathbf{x}^{\prime} \boldsymbol{\beta}_{\mathbf{1}}\right)^{2}}{2 \sigma_{1}^{2}}\right\}
\end{aligned}
$$

When there are multiple classifiers, each classifier is fit separately with maximum likelihood. Then the results are combined by stacking the scores and using the sandwich variance estimator. For more information, see [R] suest and the references White (1982); Rogers (1993); and White (1996).

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## Also see

[R] rocreg postestimation - Postestimation tools for rocreg
[R] rocregplot - Plot marginal and covariate-specific ROC curves after rocreg
[R] rocfit - Parametric ROC models
$[R]$ roc - Receiver operating characteristic (ROC) analysis

## Title

rocreg postestimation - Postestimation tools for rocreg

| Description | Syntax for predict | Menu for predict |
| :--- | :--- | :--- |
| Options for predict | Syntax for estat nproc | Menu for estat |
| Options for estat nproc | Remarks and examples | Stored results |
| Methods and formulas | References | Also see |

## Description

The following commands are of special interest after rocreg:
Command Description
estat nproc nonparametric ROC curve estimation, keeping fit information from rocreg rocregplot plot marginal and covariate-specific ROC curves

The following standard postestimation commands are also available:

Command Description

| estimates | cataloging estimation results <br> lincom |
| :--- | :--- |
| point estimates, standard errors, testing, and inference for linear combinations |  |
| nlcom | point estimates, standard errors, testing, and inference for nonlinear combinations <br> of coefficients |
| predict | predictions for parametric ROC curve estimation |
| test | Wald tests of simple and composite linear hypotheses |
| testnl | Wald tests of nonlinear hypotheses |

## Special-interest postestimation command

The estat nproc command allows calculation of all the ROC curve summary statistics for covariate-specific ROC curves, as well as for a nonparametric ROC estimation. Under nonparametric estimation, a single ROC curve is estimated by rocreg. Covariates can affect this estimation, but there are no separate covariate-specific ROC curves. Thus the input arguments for estat nproc are taken in the command line rather than from the data as variable values.

## Syntax for predict

predict [type] newvar [if] [in] [, statistic options]
statistic
Description
Main
at (varname)
auc
roc
invroc
pauc
classvar (varname)
input variable for statistic
total area under the ROC curve; the default
ROC values for given false-positive rates in at ()
false-positive rate for given ROC values in at ()
partial area under the ROC curve up to each false-positive rate in at ()
statistic for given classifier
options
Options
intpts(\#)
se(newvar)
ci (stubname)
level(\#) set confidence level; default is level (95)
*bfile(filename, ...) load dataset containing bootstrap replicates from rocreg
*btype ( $\mathrm{n}|\mathrm{p}| \mathrm{bc}$ ) produce normal-based ( n ), percentile ( p ), or bias-corrected (bc) confidence intervals; default is btype ( n )

* bfile() and btype() are only allowed with parametric analysis using bootstrap inference.


## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
at (varname) records the variable to be used as input for the above predictions.
auc predicts the total area under the ROC curve defined by the covariate values in the data. This is the default statistic.
roc predicts the ROC values for false-positive rates stored in varname specified in at ().
invroc predicts the false-positive rates for given ROC values stored in varname specified in at (). pauc predicts the partial area under the ROC curve up to each false-positive rate stored in varname specified in at ().
classvar (varname) performs the prediction for the specified classifier.
intpts(\#) specifies that \# points be used in the pAUC calculation.
se(newvar) specifies that standard errors be produced and stored in newvar.
ci(stubname) requests that confidence intervals be produced and the lower and upper bounds be stored in stubname_l and stubname_u, respectively.
level(\#) specifies the confidence level, as a percentage, for confidence intervals. The default is level (95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals.
bfile (filename, ...) uses bootstrap replicates of parameters from rocreg stored in filename to estimate standard errors and confidence intervals of predictions.
btype ( $n|p| b c$ ) specifies whether to produce normal-based (n), percentile (p), or bias-corrected (bc) confidence intervals. The default is btype ( $n$ ).

## Syntax for estat nproc

estat nproc [, estat_nproc_options]
estat_nproc_options Description

Main
auc
roc(numlist)
invroc(numlist)
pauc (numlist)
estimate total area under the ROC curve
estimate ROC values for given false-positive rates
estimate false-positive rate for given ROC values
estimate partial area under the ROC curve up to each false-positive rate

At least one option must be specified.

## Menu for estat

Statistics $>$ Postestimation $>$ Reports and statistics

## Options for estat nproc

$\qquad$ Main
auc estimates the total area under the ROC curve.
roc (numlist) estimates the ROC for each of the false-positive rates in numlist. The values in numlist must be in the range $(0,1)$.
invroc (numlist) estimates the false-positive rate for each of the ROC values in numlist. The values in numlist must be in the range $(0,1)$.
pauc (numlist) estimates the partial area under the ROC curve up to each false-positive rate in numlist.
The values in numlist must be in the range $(0,1]$.

## Remarks and examples

Remarks are presented under the following headings:
Using predict after rocreg
Using estat nproc

## Using predict after rocreg

predict, after parametric rocreg, predicts the AUC, the ROC value, the false-positive rate (invROC), or the paUC value. The default is auc.

We begin by estimating the area under the ROC curve for each of the three age-specific ROC curves in example 1 of [ R$]$ rocregplot: 30,40 , and 50 months.

## > Example 1: Parametric ROC, AUC

In example 6 of $[\mathrm{R}]$ rocreg, a probit ROC model was fit to audiology test data from Norton et al. (2000). The estimating equations method of Alonzo and Pepe (2002) was used to fit the model. Gender and age were covariates that affected the control distribution of the classifier y1 (DPOAE 65 at 2 kHz ). Age was a ROC covariate for the model, so we fit separate ROC curves at each age.

Following Janes, Longton, and Pepe (2009), we drew the ROC curves for ages 30, 40, and 50 months in example 1 of $[R]$ rocregplot. Now we use predict to estimate the AUC for the ROC curve at each of those ages.

The bootstrap dataset saved by rocreg in example 6 of [R] rocreg, nnhs $2 y 1 . d t a$, is used in the bfile() option.

We will store the AUC prediction in the new variable predAUC. We specify the se() option with the new variable name seAUC to produce an estimate of the prediction's standard error. By specifying the stubname cin in ci(), we tell predict to create normal-based confidence intervals (the default) as new variables cin_l and cin_u.

```
. use http://www.stata-press.com/data/r13/nnhs
(Norton - neonatal audiology data)
. rocreg d y1, probit ctrlcov(currage male) ctrlmodel(linear) roccov(currage)
> cluster(id) bseed(56930) bsave(nnhs2y1)
    (output omitted)
. set obs 5061
obs was 5058, now 5061
. quietly replace currage = 30 in 5059
. quietly replace currage = 40 in 5060
. quietly replace currage = 50 in 5061
. predict predAUC in 5059/5061, auc se(seAUC) ci(cin) bfile(nnhs2y1)
. list currage predAUC seAUC cin* in 5059/5061
```

5059. 
5060. 
5061. 

| currage | predAUC | seAUC | cin_l | cin_u |
| ---: | ---: | ---: | ---: | ---: |
| 30 | .5209999 | .0712928 | .3812686 | .6607312 |
| 40 | .6479176 | .0286078 | .5918474 | .7039879 |
| 50 | .7601378 | .0746157 | .6138937 | .9063819 |

As expected, we find the AUC to increase with age.
Essentially, we have a stored bootstrap sample of ROC covariate coefficient estimates in nnhs $2 \mathrm{y} 1 . \mathrm{dta}$. We calculate the AUC using each set of coefficient estimates, resulting in a sample of AUC estimates. Then the bootstrap standard error and confidence intervals are calculated based on this AUC sample. Further details of the computation of the standard error and percentile confidence intervals can be found in Methods and formulas and in [R] bootstrap.

We can also produce percentile or bias-corrected confidence intervals by specifying btype (p) or btype (bc), which we now demonstrate.

```
. drop *AUC*
. predict predAUC in 5059/5061, auc se(seAUC) ci(cip) bfile(nnhs2y1) btype(p)
. list currage predAUC cip* in 5059/5061
```

|  | currage | predAUC | cip_l | cip_u |
| :--- | ---: | ---: | ---: | ---: |
| 5059. | 30 | .5209999 | .3760555 | .6513149 |
| 5060. | 40 | .6479176 | .5893397 | .7032645 |
| 5061. | 50 | .7601378 | .5881404 | .8836223 |

. drop *AUC*
. predict predAUC in 5059/5061, auc se(seAUC) ci(cibc) bfile(nnhs2y1) btype(bc)
. list currage predAUC cibc* in 5059/5061

|  | currage | predAUC | cibc_l | cibc_u |
| :--- | ---: | ---: | ---: | ---: |
| 5059. | 30 | .5209999 | .3736968 | .6500064 |
| 5060. | 40 | .6479176 | .588947 | .7010052 |
| 5061. | 50 | .7601378 | .5812373 | .8807758 |

predict can also estimate the ROC value and the false-positive rate (invROC).

## $>$ Example 2: Parametric ROC, invROC, and ROC value

In example 7 of [R] rocreg, we fit the ROC curve for status variable hearing loss (d) and classifier negative signal-to-noise ratio $n s n r$ with ROC covariates frequency ( $x f$ ), intensity ( xl ), and hearing loss severity (xd). The data were obtained from Stover et al. (1996). The model fit was probit with bootstrap resampling. We saved 50 bootstrap replications in the dataset nsnrf.dta.

The covariate value combinations $\mathrm{xf}=10.01$, $\mathrm{xl}=5.5$, and $\mathrm{xd}=.5$, and $\mathrm{xf}=10.01$, $\mathrm{xl}=$ 6.5 , and $x d=4$ are of interest. In example 3 of $[R]$ rocregplot, we estimated the ROC values for false-positive rates 0.2 and 0.7 and the false-positive rate for a ROC value of 0.5 by using rocregplot. We will use predict to replicate the estimation.

We begin by appending observations with our desired covariate combinations to the data. We also create two new variables: rocinp, which contains the ROC values for which we wish to predict the corresponding invROC values, and invrocinp, which contains the invROC values corresponding to the ROC values we wish to predict.

```
. clear
. input xf xl xd rocinp invrocinp
    xf xl xd rocinp invrocinp
    1. 10.01 5.5 .5 .2 .
    2. 10.01 6.5 4 .2 .
    3. 10.01 5.5 .5 .7 .5
    4. 10.01 6.5 4 .7 . 5
    5. end
. save newdata
file newdata.dta saved
. use http://www.stata-press.com/data/r13/dp
(Stover - DPOAE test data)
. quietly rocreg d nsnr, ctrlcov(xf xl) roccov(xf xl xd) probit cluster(id)
> nobstrata ctrlfprall bseed(156385) breps(50) ctrlmodel(strata) bsave(nsnrf)
. append using newdata
. list xf xl xd invrocinp rocinp in 1849/1852
```

|  | xf | xl | xd | invroc $\sim \mathrm{p}$ | rocinp |
| :--- | ---: | ---: | ---: | ---: | ---: |
|  | 1849. | 10.01 | 5.5 | .5 | . |
| 1850. | 10.01 | 6.5 | 4 | .2 |  |
| 1851. | 10.01 | 5.5 | .5 | .5 | .2 |
| 1852. | 10.01 | 6.5 | 4 | .5 | .7 |

Now we will use predict to estimate the ROC value for the false-positive rates stored in rocinp. We specify the roc option, and we specify rocinp in the at () option. The other options, se() and ci(), are used to obtain standard errors and confidence intervals, respectively. The dataset of bootstrap samples, nsnrf.dta, is specified in bfile(). After prediction, we list the point estimates and standard errors.

```
. predict rocit in 1849/1852, roc at(rocinp) se(seroc) ci(cin) bfile(nsnrf)
. list xf xl xd rocinp rocit seroc if !missing(rocit)
```

|  | xf | xl | xd | rocinp | rocit | seroc |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
|  | 1849. | 10.01 | 5.5 | .5 | .2 | .7652956 |
| 1850. | 10.01 | 6.5 | 4 | .2 | .9672505 | .0227977 |
| 1851. | 10.01 | 5.5 | .5 | .7 | .9835816 | .0204353 |
| 1852. | 10.01 | 6.5 | 4 | .7 | .999428 | .0011309 |

These results match example 3 of $[\mathrm{R}]$ rocregplot. We list the confidence intervals next. These also conform to the rocregplot results from example 3 in $[R]$ rocregplot. We begin with the confidence intervals for ROC under the covariate values $\mathrm{xf}=10.01, \mathrm{xl}=5.5$, and $\mathrm{xd}=.5$.
. list xf xl xd rocinp rocit cin* if inlist(_n, 1849, 1851)

|  | xf | xl | xd | rocinp | rocit | cin_l | cin_u |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | 1849. | 10.01 | 5.5 | .5 | .2 | .7652956 | .6211391 |
| 1851. | 10.01 | 5.5 | .5 | .7 | .9835816 | .9435292 | 1.023634 |

Now we list the ROC confidence intervals under the covariate values $\mathrm{xf}=10.01, \mathrm{xl}=6.5$, and $\mathrm{xd}=4$.
. list xf xl xd rocinp rocit cin* if inlist(_n, 1850, 1852)

|  | xf | xl | xd | rocinp | rocit | cin_l | cin_u |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | 1850. | 10.01 | 6.5 | 4 | .2 | .9672505 | .9225678 |
| 1852. | 10.01 | 6.5 | 4 | .7 | .999428 | .9972115 | 1.001644 |

Now we will predict the false-positive rate for a ROC value by specifying the invroc option. We pass the invrocinp variable as an argument to the at () option. Again we list the point estimates and standard errors first.

```
. drop ci*
. predict invrocit in 1849/1852, invroc at(invrocinp) se(serocinv) ci(cin)
> bfile(nsnrf)
. list xf xl xd invrocinp invrocit serocinv if !missing(invrocit)
```

|  | xf | xl | xd | invroc~p | invrocit | serocinv |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
|  | 10.01 | 5.5 | .5 | .5 | .0615144 | .0254042 |
| 1852. | 10.01 | 6.5 | 4 | .5 | .0043298 | .0045938 |

These also match those of example 3 of $[R]$ rocregplot. Listing the confidence intervals shows identical results as well. First we list the confidence intervals under the covariate values $\mathrm{xf}=10.01$, $\mathrm{xl}=5.5$, and $\mathrm{xd}=.5$.
. list xf xl xd invrocinp invrocit cin* in 1851
1851.

| xf | xl | xd | invroc $\sim p$ | invrocit | cin_l | cin_u |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 10.01 | 5.5 | .5 | .5 | .0615144 | .0117231 | .1113057 |

Now we list the confidence intervals for false-positive rate under the covariate values $\mathrm{xf}=10.01$, $\mathrm{xl}=6.5$, and $\mathrm{xd}=4$.
. list xf xl xd invrocinp invrocit cin* in 1852
1852.

| xf | xl | xd | invroc $\sim p$ | invrocit | cin_l | cin_u |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 10.01 | 6.5 | 4 | .5 | .0043298 | -.004674 | .0133335 |

The predict command can also be used after a maximum-likelihood ROC model is fit.
> Example 3: Maximum likelihood ROC, invROC, and ROC value
In the previous example, we revisited the estimating equations fit of a probit model with ROC covariates frequency (xf), intensity (xl), and hearing loss severity (xd) to the Stover et al. (1996) audiology study data. A maximum likelihood fit of the same model was performed in example 10 of $[R]$ rocreg. In example 2 of $[R]$ rocregplot, we used rocregplot to estimate ROC values and false-positive rates for this model under two covariate configurations. We will use predict to obtain the same estimates. We will also estimate the partial area under the ROC curve.

We append the data as in the previous example. This leads to the following four final observations in the data.

```
. use http://www.stata-press.com/data/r13/dp, clear
(Stover - DPOAE test data)
. rocreg d nsnr, probit ctrlcov(xf xl) roccov(xf xl xd) ml cluster(id)
    (output omitted)
. append using newdata
. list xf xl xd invrocinp rocinp in 1849/1852
```

|  | xf | xl | xd | invroc $\sim p$ | rocinp |
| :--- | ---: | ---: | ---: | ---: | ---: |
| 1849. | 10.01 | 5.5 | .5 | . | .2 |
| 1850. | 10.01 | 6.5 | 4 | . | .2 |
| 1851. | 10.01 | 5.5 | .5 | .5 | .7 |
| 1852. | 10.01 | 6.5 | 4 | .5 | .7 |

Now we predict the ROC value for false-positive rates of 0.2 and 0.7 . Under maximum likelihood prediction, only Wald-type confidence intervals are produced. We specify a new variable name for the standard error in the se() option and a stubname for the confidence interval variables in the ci() option.

- predict rocit in 1849/1852, roc at(rocinp) se(seroc) ci(ci)
. list xf xl xd rocinp rocit seroc ci_l ci_u if !missing(rocit), noobs

| xf | xl | xd | rocinp | rocit | seroc | ci_l | ci_u |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 10.01 | 5.5 | .5 | .2 | .7608593 | .0510501 | .660803 | .8609157 |
| 10.01 | 6.5 | 4 | .2 | .9499408 | .0179824 | .914696 | .9851856 |
| 10.01 | 5.5 | .5 | .7 | .978951 | .0097382 | .9598644 | .9980376 |
| 10.01 | 6.5 | 4 | .7 | .9985001 | .0009657 | .9966073 | 1.000393 |

These results match our estimates in example 2 of $[R]$ rocregplot. We also match example 2 of $[R]$ rocregplot when we estimate the false-positive rate for a ROC value of 0.5 .

```
. drop ci*
. predict invrocit in 1851/1852, invroc at(invrocinp) se(serocinv) ci(ci)
. list xf xl xd invrocinp invrocit serocinv ci_l ci_u if !missing(invrocit),
> noobs
```

| xf | xl | xd | invroc $\sim p$ | invrocit | serocinv | ci_l | ci_u |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 10.01 | 5.5 | .5 | .5 | .0578036 | .0198626 | .0188736 | .0967336 |
| 10.01 | 6.5 | 4 | .5 | .0055624 | .0032645 | -.0008359 | .0119607 |

## > Example 4: Maximum likelihood ROC, pAUC, and ROC value

In example 13 of [R] rocreg, we fit a maximum-likelihood marginal probit model to each classifier of the fictitious dataset generated from Hanley and McNeil (1983). In example 5 of [R] rocregplot, rocregplot was used to draw the ROC for the mod1 and mod3 classifiers. Estimates of the ROC value and false-positive rate were also obtained with Wald-type confidence intervals.

We return to this example, this time using predict to estimate the ROC value and false-positive rate. We will also estimate the pAUC for the false-positive rates of 0.3 and 0.8 .

First, we add the input variables to the data. The variable paucinp will hold the 0.3 and 0.8 false-positive rates that we will input to pAUC. The variable invrocinp holds the ROC value of 0.8 for which we will estimate the false-positive rate. Finally, the variable rocinp holds the false-positive rates of 0.15 and 0.75 for which we will estimate the ROC value.

```
. use http://www.stata-press.com/data/r13/ct2, clear
. rocreg status mod1 mod2 mod3, probit ml
(output omitted)
. quietly generate paucinp = . 3 in 111
. quietly replace paucinp = . }8\mathrm{ in }11
. quietly generate invrocinp = . }8\mathrm{ in }11
. quietly generate rocinp = . 15 in 111
. quietly replace rocinp = . }75\mathrm{ in }11
```

Then, we estimate the ROC value for false-positive rates 0.15 and 0.75 under classifier mod1. The point estimate is stored in roc1. Wald confidence intervals and standard errors are also estimated. We find that these results match those of example 5 of [R] rocregplot.

```
. predict roc1 in 111/112, classvar(mod1) roc at(rocinp) se(sr1) ci(cir1)
. list rocinp roc1 sr1 cir1* in 111/112
```

|  | rocinp | roc1 | sr1 | cir1_1 | cir1_u |
| :--- | ---: | ---: | ---: | ---: | ---: |
| 111. | .15 | .7934935 | .0801363 | .6364293 | .9505578 |
| 112. | .75 | .9931655 | .0069689 | .9795067 | 1.006824 |

Now we perform the same estimation under the classifier mod3.

```
. predict roc3 in 111/112, classvar(mod3) roc at(roci) se(sr3) ci(cir3)
. list rocinp roc3 sr3 cir3* in 111/112
```

|  | rocinp | roc3 | sr3 | cir3_1 | cir3_u |
| :--- | ---: | ---: | ---: | ---: | ---: |
|  | 111. | .15 | .8888596 | .0520118 | .7869184 |
| 112. | .75 | .9953942 | .0043435 | .9868811 | 1.003907 |

Next we estimate the false-positive rate for the ROC value of 0.8 . These results also match example 5 of $[R]$ rocregplot.

```
. predict invroc1 in 112, classvar(mod1) invroc at(invrocinp) se(sir1) ci(ciir1)
. list invrocinp invroc1 sir1 ciir1* in 112
```

112. 

| invroc~p | invroc1 | sir1 | ciir1_1 | ciir1_u |
| ---: | ---: | ---: | ---: | ---: |
| .8 | .1556435 | .069699 | .0190361 | .292251 |

. predict invroc3 in 112, classvar(mod3) invroc at(invrocinp) se(sir3) ci(ciir3)
. list invrocinp invroc3 sir3 ciir3* in 112
112.

| invroc $\sim$ p | invroc3 | sir3 | ciir3_1 | ciir3_u |
| ---: | ---: | ---: | ---: | ---: |
| .8 | .0661719 | .045316 | -.0226458 | .1549896 |

Finally, we estimate the pAUC for false-positive rates of 0.3 and 0.8 . The point estimate is calculated by numeric integration. Wald confidence intervals are obtained with the delta method. Further details are presented in Methods and formulas.

```
. predict pauc1 in 111/112, classvar(mod1) pauc at(paucinp) se(sp1) ci(cip1)
. list paucinp pauc1 sp1 cip1* in 111/112
```

|  | paucinp | pauc1 | sp1 | cip1_1 | cip1_u |
| :--- | ---: | ---: | ---: | ---: | ---: |
| 111. | .3 | .221409 | .0240351 | .174301 | .268517 |
| 112. | .8 | .7033338 | .0334766 | .6377209 | .7689466 |

. predict pauc3 in $111 / 112$, classvar(mod3) pauc at (paucinp) se(sp3) ci(cip3)
. list paucinp pauc3 sp3 cip3* in 111/112

|  | paucinp | pauc3 | sp3 | cip3_1 | cip3_u |
| :--- | ---: | ---: | ---: | ---: | ---: |
| 111. | .3 | .2540215 | .0173474 | .2200213 | .2880217 |
| 112. | .8 | .7420408 | .0225192 | .6979041 | .7861776 |

## Using estat nproc

When you initially use rocreg to fit a nonparametric ROC curve, you can obtain bootstrap estimates of a ROC value, false-positive rate, area under the ROC curve, and partial area under the ROC curve. The estat nproc command allows the user to estimate these parameters after rocreg has originally been used.

The seed and resampling settings used by rocreg are used by estat nproc. So the results for these new statistics are identical to what they would be if they had been initially estimated in the rocreg command. These new statistics, together with those previously estimated in rocreg, are returned in $r()$.

We demonstrate with an example.

## > Example 5: Nonparametric ROC, invROC, and pAUC

In example 3 of $[\mathrm{R}]$ rocreg, we examined data from a pancreatic cancer study (Wieand et al. 1989). Two continuous classifiers, y1 (CA 19-9) and y2 (CA 125), were used for the true status variable d. In that example, we estimated various quantities including the false-positive rate for a ROC value of 0.6 and the pAUC for a false-positive rate of 0.5 . Here we replicate that estimation with a call to rocreg to estimate the former and follow that with a call to estat nproc to estimate the latter. For simplicity, we restrict estimation to classifier y1 (CA 19-9).

We start by executing rocreg, estimating the false-positive rate for a ROC value of 0.6. This value is specified in invroc(). Case-control resampling is used by specifying the bootcc option.
. use http://labs.fhcrc.org/pepe/book/data/wiedat2b, clear
(S. Wieand - Pancreatic cancer diagnostic marker data)
. rocreg d y1, invroc(.6) bseed(8378923) bootcc nodots
Bootstrap results

| Number of strata $=$ | 2 | Number of obs $=$ | 141 |
| :--- | :--- | :--- | :--- |
|  | Replications | $=$ | 1000 |

Nonparametric ROC estimation
Control standardization: empirical
ROC method : empirical
False-positive rate
Status : d
Classifier: y1

| invROC | Observed <br> Coef. | Bias | Bootstrap <br> Std. Err. | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: |
|  | 0 | .0158039 | .0267288 | -.0523874 | .0523874 |
|  |  |  |  | 0 | .0784314 |
|  |  |  | 0 | .1372549 | (BC) |
|  |  |  |  |  |  |

Now we will estimate the pAUC for the false-positive rate of 0.5 using estat nproc and the pauc() option.

```
. matrix list e(b)
symmetric e(b)[1,1]
            y1:
    invroc_1
y1 0
. estat nproc, pauc(.5)
Bootstrap results
Nonparametric ROC estimation
Control standardization: empirical
ROC method : empirical
False-positive rate
    Status : d
    Classifier: y1
```

| Number of strata $=$ | 2 | Number of obs  <br> Replications $=$ | 141 |
| :--- | :--- | :--- | :--- |
|  |  | 1000 |  |


| invROC | Observed <br> Coef. | Bias | Bootstrap <br> Std. Err. | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: |
|  | 0 | .0158039 | .0267288 | -.0523874 | .0523874 |
|  |  |  |  | 0 | .0784314 |
|  |  |  |  | (P) |  |
|  |  |  |  | .1372549 | (BC) |

Partial area under the ROC curve
Status : d
Classifier: y1

|  | Observed <br> Coef. | Bias | Bootstrap <br> Std. Err. | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | :--- |
| .5 | .3932462 | -.0000769 | .021332 | .3514362 | .4350562 (N) |
|  |  |  |  | .3492375 | .435512 (P) |
|  |  |  |  | .3492375 | .435403 (BC) |

```
. matrix list r(b)
r(b)[1,2]
    invroc_1 pauc_1
y1 0 . 39324619
. matrix list e(b)
symmetric e(b)[1,1]
            y1:
        invroc_1
y1 0
. matrix list r(V)
symmetric r(V)[2,2]
            y1: y1:
y1:invroc_1 .00071443
    y1:pauc_1 -.000326 .00045506
. matrix list e(V)
symmetric e(V)[1,1]
            y1:
        invroc_1
y1:invroc_1 . 00071443
```

The advantages of using estat nproc are twofold. First, you can estimate additional parameters of interest without having to respecify the bootstrap settings you did with rocreg; instead estat nproc uses the bootstrap settings that were stored by rocreg. Second, parameters estimated with estat nproc are added to those parameters estimated by rocreg and returned in the matrices $r$ (b) (parameter estimates) and $r(V)$ (variance-covariance matrix). Thus you can also obtain correlations between any quantities you wish to estimate.

## Stored results

estat nproc stores the following in $r()$ :
$r(b) \quad$ coefficient vector
$r(V) \quad$ variance-covariance matrix of the estimators
$r$ (ci_normal) normal-approximation confidence intervals
$r(c i-p e r c e n t i l e)$ percentile confidence intervals
$r$ (ci_bc) bias-corrected confidence intervals

## Methods and formulas

Details on computation of the nonparametric ROC curve and the estimation of the parametric ROC curve model coefficients can be found in $[\mathrm{R}]$ rocreg. Here we describe how to estimate the ROC curve summary statistics for a parametric model. The cumulative distribution function, $g$, can be the standard normal cumulative distribution function, $\Phi$.

Methods and formulas are presented under the following headings:

[^73]
## Parametric model: Summary parameter definition

Conditioning on covariates $\mathbf{x}$, we have the following ROC curve model:

$$
\operatorname{ROC}(u)=g\left\{\mathbf{x}^{\prime} \boldsymbol{\beta}+\alpha g^{-1}(u)\right\}
$$

$\mathbf{x}$ can be constant, and $\boldsymbol{\beta}=\beta_{0}$, the constant intercept.
We can solve this equation to obtain the false-positive rate value $u$ for a ROC value of $r$ :

$$
u=g\left[\left\{g^{-1}(r)-\mathbf{x}^{\prime} \boldsymbol{\beta}\right\} \alpha^{-1}\right]
$$

The partial area under the ROC curve for the false-positive rate $u$ is defined by

$$
\operatorname{pAUC}(u)=\int_{o}^{u} g\left\{\mathbf{x}^{\prime} \boldsymbol{\beta}+\alpha g^{-1}(t)\right\} d t
$$

The area under the ROC curve is defined by

$$
\mathrm{AUC}=\int_{o}^{1} g\left\{\mathbf{x}^{\prime} \boldsymbol{\beta}+\alpha g^{-1}(t)\right\} d t
$$

When $g$ is the standard normal cumulative distribution function $\Phi$, we can express the AUC as

$$
\mathrm{AUC}=\Phi\left(\frac{\mathbf{x}^{\prime} \boldsymbol{\beta}}{\sqrt{1+\alpha^{2}}}\right)
$$

## Maximum likelihood estimation

We allow maximum likelihood estimation under probit parametric models, so $g=\Phi$. The ROC value, false-positive rate, and AUC parameters all have closed-form expressions in terms of the covariate values $\mathbf{x}$, coefficient vector $\boldsymbol{\beta}$, and slope parameter $\alpha$. So to estimate these three types of summary parameters, we use the delta method (Oehlert 1992; Phillips and Park 1988). Particularly, we use the nl com command (see [R] nlcom) to implement the delta method.

To estimate the partial area under the ROC curve for false-positive rate $u$, we use numeric integration. A trapezoidal approximation is used in calculating the integrals. A numeric integral of the $\operatorname{ROC}(t)$ function conditioned on the covariate values $\mathbf{x}$, coefficient vector estimate $\widehat{\boldsymbol{\beta}}$, and slope parameter estimate $\widehat{\alpha}$ is computed over the range $t=[0, u]$. This gives us the point estimate of pAUC $(u)$.

To calculate the standard error and confidence intervals for the point estimate of pAUC $(u)$, we again use the delta method. Details on the delta method algorithm can be found in Methods and formulas of $[\mathrm{R}]$ nlcom and the earlier mentioned references.

Under maximum likelihood estimation, the coefficient estimates $\widehat{\boldsymbol{\beta}}$ and slope estimate $\widehat{\alpha}$ are asymptotically normal with variance matrix V. For convenience, we rename the parameter vector $\left[\boldsymbol{\beta}^{\prime}, \alpha\right]$ to the $k$-parameter vector $\boldsymbol{\theta}=\left[\theta_{1}, \ldots, \theta_{k}\right]$. We will also explicitly refer to the conditioning of the ROC curve by $\boldsymbol{\theta}$ in its mention as $\operatorname{ROC}(t, \boldsymbol{\theta})$.

Under the delta method, the continuous scalar function of the estimate $\widehat{\boldsymbol{\theta}}, f(\widehat{\boldsymbol{\theta}})$ has asymptotic mean $f(\boldsymbol{\theta})$ and asymptotic covariance

$$
\widehat{\operatorname{Var}}\{f(\widehat{\boldsymbol{\theta}})\}=\mathbf{f V f}^{\prime}
$$

where $\mathbf{f}$ is the $1 \times k$ matrix of derivatives for which

$$
\mathbf{f}_{1 j}=\frac{\partial f(\boldsymbol{\theta})}{\partial \theta_{j}} \quad j=1, \ldots, k
$$

The asymptotic covariance of $f(\widehat{\boldsymbol{\theta}})$ is estimated and then used in conjunction with $f(\widehat{\boldsymbol{\theta}})$ for further inference, including Wald confidence intervals, standard errors, and hypothesis testing.

In the case of $\operatorname{pAUC}(u)$ estimation, our $f(\widehat{\boldsymbol{\theta}})$ is the aforementioned numeric integral of the ROC curve. It estimates $f(\boldsymbol{\theta})$, the true integral of the ROC curve on the $[0, u]$ range. The $\mathbf{V}$ variance matrix is estimated using the likelihood information that rocreg calculated, and the estimation is performed by rocreg itself.

The partial derivatives of $f(\boldsymbol{\theta})$ can be determined by using Leibnitz's rule (Weisstein 2011):

$$
\mathbf{f}_{1 j}=\frac{\partial}{\partial \theta_{j}} \int_{0}^{u} \operatorname{ROC}(t, \boldsymbol{\theta}) d t=\int_{0}^{u} \frac{\partial}{\partial \theta_{j}} \operatorname{ROC}(t, \boldsymbol{\theta}) d t \quad j=1, \ldots, k
$$

When $\theta_{j}$ corresponds with the slope parameter $\alpha$, we obtain the following partial derivative:

$$
\frac{\partial}{\partial \alpha} \operatorname{pAUC}(u)=\int_{0}^{u} \phi\left\{\mathbf{x}^{\prime} \boldsymbol{\beta}+\alpha \Phi^{-1}(t)\right\} \Phi^{-1}(t) d t
$$

The partial derivative of $f(\boldsymbol{\theta})$ [pAUC( $\mathbf{u})$ ] for $\beta_{0}$ is the following:

$$
\frac{\partial}{\partial \beta_{0}} \operatorname{pAUC}(u)=\int_{0}^{u} \phi\left\{\mathbf{x}^{\prime} \boldsymbol{\beta}+\alpha \Phi^{-1}(t)\right\} d t
$$

For a nonintercept coefficient, we obtain the following:

$$
\frac{\partial}{\partial \beta_{i}} \operatorname{pAUC}(u)=\int_{0}^{u} x_{i} \phi\left\{\mathbf{x}^{\prime} \boldsymbol{\beta}+\alpha \Phi^{-1}(t)\right\} d t
$$

We can estimate each of these integrals by numeric integration, plugging in the estimates $\widehat{\boldsymbol{\beta}}$ and $\widehat{\alpha}$ for the parameters. This, together with the previously calculated estimate $\widehat{\mathbf{V}}$, provides an estimate of the asymptotic covariance of $f(\widehat{\boldsymbol{\theta}})=\widehat{\mathrm{pAUC}}(u)$, which allows us to perform further statistical inference on $\operatorname{padc}(u)$.

## Estimating equations estimation

When we fit a model using the Alonzo and Pepe (2002) estimating equations method, we use the bootstrap to perform inference on the ROC curve summary parameters. Each bootstrap sample provides a sample of the coefficient estimates $\boldsymbol{\beta}$ and the slope estimates $\alpha$. Using the formulas in Parametric model: Summary parameter definition under Methods and formulas, we can obtain an estimate of the ROC, false-positive rate, or AUC for each resample. Using numeric integration (with the trapezoidal approximation), we can also estimate the pAUC of the resample.

By making these calculations, we obtain a bootstrap sample of our summary parameter estimate. We then obtain bootstrap standard errors, normal approximation confidence intervals, percentile confidence intervals, and bias-corrected confidence intervals using this bootstrap sample. Further details can be found in [R] bootstrap.

## References

Alonzo, T. A., and M. S. Pepe. 2002. Distribution-free ROC analysis using binary regression techniques. Biostatistics 3: 421-432.
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## Also see

[R] rocreg - Receiver operating characteristic (ROC) regression
[R] rocregplot - Plot marginal and covariate-specific ROC curves after rocreg
[U] 20 Estimation and postestimation commands

## Title

rocregplot - Plot marginal and covariate-specific ROC curves after rocreg

| Syntax | Menu | Description | probit_options |
| :--- | :--- | :--- | :--- |
| common_options | boot_options | Remarks and examples | Methods and formulas |
| References | Also see |  |  |

## Syntax

Plot ROC curve after nonparametric analysis
rocregplot [, common_options boot_options]

Plot ROC curve after parametric analysis using bootstrap rocregplot [, probit_options common_options boot_options]

Plot ROC curve after parametric analysis using maximum likelihood rocregplot [, probit_options common_options]
probit_options
Description
Main
at (varname $=\#[$ varname=\#...] ) value of specified covariates/mean of unspecified covariates
[at1 (varname=\# [varname=\#...])
[at2 $($ varname $=\#[$ varname $=\# \ldots])$
[...]]]
*roc(numlist)
*invroc (numlist)
level(\#)
show estimated ROC values for given false-positive rates show estimated false-positive rates for given ROC values set confidence level; default is level (95)

Curve line\#opts(cline_options)
affect rendition of ROC curve \#

[^74]common_options
Description
Main
classvars (varlist)
norefline
Scatter
plot\#opts (scatter_options)

Reference line
rlopts(cline_options)
Y axis, X axis, Titles, Legend, Overall
twoway_options
restrict plotting of ROC curves to specified classifiers suppress plotting the reference line
affect rendition of classifier \#s false-positive rate and ROC scatter points; not allowed with at ()
affect rendition of the reference line
any options other than by () documented in [G-3] twoway_options
boot_options

## Bootstrap

$\dagger$ bfile (filename) btype ( $\mathrm{n}|\mathrm{p}| \mathrm{bc}$ )

Description
load dataset containing bootstrap replicates from rocreg plot normal-based (n), percentile (p), or bias-corrected (bc) confidence intervals; default is btype ( $n$ )
$\dagger$ bfile() is only allowed with parametric analysis using bootstrap inference; in which case this option is required with roc() or invroc().

## Menu

Statistics $>$ Epidemiology and related $>$ ROC analysis $>$ ROC curves after rocreg

## Description

Under parametric estimation, rocregplot plots the fitted ROC curves for specified covariate values and classifiers. If rocreg, probit or rocreg, probit ml were previously used, the false-positive rates (for specified ROC values) and ROC values (for specified false-positive rates) for each curve may also be plotted, along with confidence intervals.

Under nonparametric estimation, rocregplot will plot the fitted ROC curves using the _fpr_* and _roc_* variables produced by rocreg. Point estimates and confidence intervals for false-positive rates and ROC values that were computed in rocreg may be plotted as well.

## probit_options

## Main

at (varname=\# . . ) requests that the covariates specified by varname be set to \#. By default, rocreg evaluates the function by setting each covariate to its mean value. This option causes the ROC curve to be evaluated at the value of the covariates listed in at () and at the mean of all unlisted covariates.
at1 (varname=\# ...), at2 (varname=\# ...), ..., at10 (varname=\# ...) specify that ROC curves (up to 10) be plotted on the same graph. at1(), at2(), ..., at10() work like the at () option. They request that the function be evaluated at the value of the covariates specified and at the mean of all unlisted covariates. at1() specifies the values of the covariates for the first curve, at2() specifies the values of the covariates for the second curve, and so on.
roc (numlist) specifies that estimated ROC values for given false-positive rates be graphed. invroc (numlist) specifies that estimated false-positive rates for given ROC values be graphed.
level(\#) specifies the confidence level, as a percentage, for confidence intervals. The default is level (95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals. level() may be specified with either roc() or invroc().

Curve
line\#opts(cline_options) affects the rendition of ROC curve \#. See [G-3] cline_options.

## common_options

```
Main
classvars (varlist) restricts plotting ROC curves to specified classification variables. norefline suppresses plotting the reference line.
```


## Scatter <br> plot\#opts (scatter_options) affects the rendition of classifier \#'s false-positive rate and ROC scatter points. This option applies only to non-ROC covariate estimation graphing. See [G-2] graph twoway scatter.

$\curvearrowright \sqrt{\text { Reference line }}$
rlopts(cline_options) affects rendition of the reference line. See [G-3] cline_options.
$\longleftarrow \sqrt{\mathrm{Y}}$ axis, X axis, Titles, Legend, Overall
twoway_options are any of the options documented in [G-3] twoway_options, excluding by (). These include options for titling the graph (see [G-3] title_options) and options for saving the graph to disk (see [G-3] saving_option).

## boot_options

Bootstrap
bfile(filename) uses bootstrap replicates of parameters from rocreg stored in filename to estimate standard errors and confidence intervals of predictions. bfile() must be specified with either roc() or invroc() if parametric estimation with bootstrapping was used.
btype ( $n|p| b c$ ) indicates the desired type of confidence interval rendering. $n$ draws normal-based, $p$ draws percentile, and bc draws bias-corrected confidence intervals for specified false-positive rates and ROC values in roc () and invroc (). The default is btype(n).

## Remarks and examples

Remarks are presented under the following headings:
Plotting covariate-specific ROC curves
Plotting marginal ROC curves

## Plotting covariate-specific ROC curves

The rocregplot command is also demonstrated in [ R$]$ rocreg. We will further demonstrate its use with several examples. Particularly, we will show how rocregplot can draw the ROC curves of covariate models that have been fit using rocreg.

## > Example 1: Parametric ROC

In example 6 of $[\mathrm{R}]$ rocreg, we fit a probit ROC model to audiology test data from Norton et al. (2000). The estimating equation method of Alonzo and Pepe (2002) was used to the fit the model. Gender and age were covariates that affected the control distribution of the classifier y1 (DPOAE 65 at 2 kHz ). Age was a ROC covariate for the model, so we fit separate ROC curves at each age.

Following Janes, Longton, and Pepe (2009), we draw the ROC curves for ages 30, 40, and 50 months. The at1(), at2(), and at3() options are used to specify the age covariates.

```
. use http://www.stata-press.com/data/r13/nnhs
(Norton - neonatal audiology data)
. rocreg d y1, probit ctrlcov(currage male) ctrlmodel(linear) roccov(currage)
> cluster(id) bseed(56930) bsave(nnhs2y1, replace)
    (output omitted)
. rocregplot, at1(currage=30) at2(currage=40) at3(currage=50)
```



Here we use the default entries of the legend, which indicate the "at \#" within the specified at* options and the classifier to which the curve corresponds. ROC curve one corresponds with currage $=30$, two with currage=40, and three with currage=50. The positive effect of age on the ROC curve is evident. At an age of 30 months (currage=30), the ROC curve of y1 (DPOAE 65 at 2 kHz ) is nearly equivalent to that of a noninformative test that gives equal probability to hearing loss. At age 50 months (currage=50), corresponding to some of the oldest children in the study, the ROC curve shows that test y1 (DPOAE 65 at 2 kHz ) is considerably more powerful than the noninformative test.

You may create your own legend by specifying the legend() option. The default legend is designed for the possibility of multiple covariates. Here we could change the legend entries to currage values and gain some extra clarity. However, this may not be feasible when there are many covariates present.

We can also use rocregplot after maximum likelihood estimation.

## > Example 2: Maximum likelihood ROC

We return to the audiology study with frequency (xf), intensity (xl), and hearing loss severity (xd) covariates from Stover et al. (1996) that we examined in example 10 of [R] rocreg. Negative signal-to-noise ratio is again used as a classifier. Using maximum likelihood, we fit a probit model to these data with the indicated ROC covariates.

After fitting the model, we wish to compare the ROC curves of two covariate combinations. The first has an intensity value of 5.5 (the lowest intensity, corresponding to 55 decibels) and a frequency of 10.01 (the lowest frequency, corresponding to 1001 hertz). We give the first combination a hearing loss severity value of 0.5 (the lowest). The second covariate combination has the same frequency, but the highest intensity value of 6.5 ( 65 decibels). We give this second covariate set a higher severity value of 4 . We will visually compare the two ROC curves resulting from these two covariate value combinations.

We specify false-positive rates of 0.7 first followed by 0.2 in the roc () option to visually compare the size of the ROC curve at large and small false-positive rates. Because maximum likelihood estimation was used to fit the model, a Wald confidence interval is produced for the estimated ROC value and false-positive rate parameters. Further details are found in Methods and formulas.

```
. use http://www.stata-press.com/data/r13/dp
(Stover - DPOAE test data)
. rocreg d nsnr, probit ctrlcov(xf xl) roccov(xf xl xd) ml cluster(id)
    (output omitted)
```




At the higher false-positive rate value of 0.7 , we see little difference in the ROC values and note that the confidence intervals nearly overlap. Now we view the same curves with the lower false-positive rate compared.
. rocregplot, at $1(x f=10.01, x l=5.5, x d=.5)$ at2 $(x f=10.01, x l=6.5, x d=4) \operatorname{roc}(.2)$
ROC curve
Status : d
Classifier: nsnr
Under covariates:

|  | at1 |
| ---: | ---: |
| xf | 10.01 |
| xl | 5.5 |
| xd | .5 |


| ROC | Coef. | Std. Err. | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :--- |
| .2 | .7608593 | .0510501 | .660803 | .8609157 |

Under covariates:

|  | at2 |
| ---: | ---: |
| $x f$ | 10.01 |
| $x l$ | 6.5 |
| $x d$ | 4 |


| ROC | Coef. | Std. Err. | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | :--- |
| .2 | .9499408 | .0179824 | .914696 | .9851856 |



The lower false-positive rate of 0.2 shows clearly distinguishable ROC values. Now we specify option invroc (.5) to view how the false-positive rates vary at a ROC value of 0.5 .

```
. rocregplot, at1(xf=10.01, xl=5.5, xd=.5) at2(xf=10.01, xl=6.5, xd=4) invroc(.5)
False-positive rate
    Status : d
    Classifier: nsnr
Under covariates:
\begin{tabular}{r|r} 
& at1 \\
\hline\(x f\) & 10.01 \\
xl & 5.5 \\
xd & .5
\end{tabular}
\begin{tabular}{r|rcrl}
\hline invROC & Coef. & Std. Err. & [95\% Conf. Interval] \\
\hline .5 & .0578036 & .0198626 & .0188736 & .0967336 \\
\hline
\end{tabular}
```

Under covariates:

|  | at2 |
| ---: | ---: |
| $x f$ | 10.01 |
| $x l$ | 6.5 |
| xd | 4 |


| invROC | Coef. | Std. Err. | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | :--- |
| .5 | .0055624 | .0032645 | -.0008359 | .0119607 |



At a ROC value of 0.5 , the false-positive rates for both curves are small and close to one another.

## Technical note

We can use the testnl command to support our visual observations with statistical inference. We use it to perform a Wald test of the null hypothesis that the two ROC curves just rendered are equal at a false-positive rate of 0.7.

```
. testnl normal(_b[i_cons]+10.01*_b[xf]+5.5*_b[xl]
> +.5*_b[xd]+_b[s_cons]*invnormal (.7)) =
> normal(_b[i_cons]+10.01*_b[xf]+6.5*_b[xl]
> + 4*_b[xd]+_b[s_cons]*invnormal(.7))
(1) normal(_b[i_cons]+10.01*_b[xf]+5.5*_b[xl] +.5*_b[xd]+_b[s_cons]*invnormal(.7))=
    normal(_b[i_cons]+10.01*_b[xf]+6.5*_b[xl] + 4*_b[xd]+_b[s_cons]*invnormal(.7))
        chi2(1) = 
```

The test is significant at the 0.05 level, and thus we find that the two curves are significantly different. Now we will use testnl again to test equality of the false-positive rates for each curve with a ROC value of 0.5 . The inverse ROC formula used is derived in Methods and formulas.

```
. testnl normal((invnormal(.5)-(_b[i_cons]+10.01*_b[xf]+5.5*_b[xl]+.5*_b[xd]))
                        /_b[s_cons]) =
        normal((invnormal(.5)-(_b[i_cons]+10.01*_b[xf]+6.5*_b[xl]+4*_b[xd]))
            /_b[s_cons])
(1) normal((invnormal(.5)-(_b[i_cons]+10.01*_b[xf]+5.5*_b[xl]+.5*_b[xd]))
        /_b[s_cons]) =
    normal((invnormal(.5)-(_b[i_cons]+10.01*_b[xf]+6.5*_b[xl]+4*_b[xd]))
        /_b[s_cons])
                chi2(1) = 8.01
    Prob > chi2 = 0.0046
```

We again reject the null hypothesis that the two curves are equal at the 0.05 level.

The model of our last example was also fit using the estimating equations method in example 7 of $[R]$ rocreg. We will demonstrate rocregplot after that model fit as well.

## > Example 3: Parametric ROC, invROC, and ROC value

In example 2, we used rocregplot after a maximum likelihood model fit of the ROC curve for classifier nsnr and covariates frequency (xf), intensity (xl), and hearing loss severity (xd). The data were obtained from the audiology study described in Stover et al. (1996). In example 7 of $[R]$ rocreg, we fit the model using the estimating equations method of Alonzo and Pepe (2002). Under this method, bootstrap resampling is used to make inferences. We saved 50 bootstrap replications in nsnrf.dta, which we re-create below.

We use rocregplot to draw the ROC curves for $n s n r$ under the covariate values $\mathrm{xf}=10.01$, $x l=5.5$, and $x d=.5$, and $x f=10.01, x l=6.5$, and $x d=4$. The at\#() options are used to specify the covariate values. The previous bootstrap results are made available to rocregplot with the bfile() option. As before, we will specify 0.2 and 0.7 as false-positive rates in the roc() option and 0.5 as a ROC value in the invroc () option. We do not specify btype() and thus our graph will contain normal-based bootstrap confidence bands, the default.
. use http://www.stata-press.com/data/r13/dp
(Stover - DPOAE test data)
. rocreg d nsnr, probit ctrlcov(xf xl) roccov(xf xl xd) cluster(id)
> nobstrata ctrlfprall bseed(156385) breps(50) bsave(nsnrf, replace) (output omitted)
. rocregplot, at1 $(x f=10.01, x l=5.5, x d=.5)$ at2 $(x f=10.01, x l=6.5, x d=4)$
$>\operatorname{roc}(.7)$ bfile(nsnrf)
ROC curve
Status : d
Classifier: nsnr
Under covariates:

|  | at1 |
| ---: | ---: |
| xf | 10.01 |
| xl | 5.5 |
| xd | .5 |

(Replications based on 208 clusters in id)

| ROC | Observed <br> Coef. | Bias | Bootstrap <br> Std. Err. | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | .9835816 | .0087339 | .0204353 | .9435292 | $1.023634(\mathrm{~N})$ |
|  |  |  |  | .9155462 | $.9974037(\mathrm{P})$ |
|  |  |  |  | .9392258 | .9976629 (BC) |

Under covariates:

|  | at2 |
| ---: | ---: |
| xf | 10.01 |
| $x l$ | 6.5 |
| xd | 4 |

(Replications based on 208 clusters in id)

| ROC | Observed <br> Coef. | Bias | Bootstrap <br> Std. Err. | [95\% Conf. Interval] |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | .999428 | .0006059 | .0011309 | .9972115 | 1.001644 | $(\mathrm{~N})$ |
|  |  |  |  | .9958003 | .9999675 | (P) |
|  |  |  |  | .9968304 | .9999901 | (BC) |



As shown in the graph, we find that the ROC values at a false-positive rate of 0.7 are close together, as they were in the maximum likelihood estimation in example 2 . We now repeat this process for the lower false-positive rate of 0.2 by using the roc (.2) option.

| ```. rocregplot, at1(xf=10.01, xl=5.5, xd=.5) at2(xf=10.01, xl=6.5, xd=4) > roc(.2) bfile(nsnrf)``` |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ROC curve |  |  |  |  |  |  |
| Status : d <br> Classifier: nsnr |  |  |  |  |  |  |
| Under covariates: |  |  |  |  |  |  |
| at1 |  |  |  |  |  |  |
| xf | 10.01 |  |  |  |  |  |
| xl | 5.5 |  |  |  |  |  |
| xd | . 5 |  |  |  |  |  |
|  |  | (Replications based on 208 clusters in id) |  |  |  |  |
| ROC | Observed Coef. | Bootstrap |  |  | Interva |  |
| . 2 | . 7652956 | . 0145111 | . 0735506 | . 6211391 | . 9094522 | (N) |
|  |  |  |  | . 6054495 | . 878052 | (P) |
|  |  |  |  | . 6394838 | . 9033081 | (BC) |

Under covariates:

|  | at2 |
| ---: | ---: |
| xf | 10.01 |
| xl | 6.5 |
| xd | 4 |

(Replications based on 208 clusters in id)

| ROC | Observed <br> Coef. | Bias | Bootstrap <br> Std. Err. | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | .9672505 | .0072429 | .0227977 | .9225679 | 1.011933 (N) |
|  |  |  |  | .9025254 | $.9931714(\mathrm{P})$ |
|  |  |  |  | .9235289 | .9979637 (BC) |



The ROC values are slightly higher at the false-positive rate of 0.2 than they were in the maximum likelihood estimation in example 2. To see if the false-positive rates differ at a ROC value of 0.5 , we specify the invroc (.5) option.

```
. rocregplot, at1(xf=10.01, xl=5.5, xd=.5) at2(xf=10.01, xl=6.5, xd=4)
> invroc(.5) bfile(nsnrf)
False-positive rate
    Status : d
    Classifier: nsnr
```

Under covariates:

|  | at1 |
| ---: | ---: |
| xf | 10.01 |
| $x l$ | 5.5 |
| xd | .5 |

(Replications based on 208 clusters in id)

| invROC | Observed <br> Coef. | Bias | Bootstrap <br> Std. Err. | [95\% Conf. Interval] |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | :--- |
|  | .0615144 | -.0063531 | .0254042 | .0117231 | .1113057 | (N) |
|  |  |  |  | .0225159 | .1265046 (P) |  |
|  |  |  |  | .0224352 | .1265046 (BC) |  |

Under covariates:

|  | at2 |
| ---: | ---: |
| $x f$ | 10.01 |
| $x l$ | 6.5 |
| xd | 4 |

(Replications based on 208 clusters in id)

| invROC | Observed <br> Coef. | Bias | Bootstrap <br> Std. Err. | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | .0043298 | -.0012579 | .0045938 | -.004674 | $.0133335(\mathrm{~N})$ |
|  |  |  |  | .0002773 | $.0189199(\mathrm{P})$ |
|  |  |  |  | .0001292 | .0134801 (BC) |



The point estimates of the ROC value and false-positive rate are both computed directly using the point estimates of the ROC coefficients. Calculation of the standard errors and confidence intervals is slightly more complicated. Essentially, we have stored a sample of our ROC covariate coefficient estimates in nsnrf.dta. We then calculate the ROC value or false-positive rate estimates using each set of coefficient estimates, resulting in a sample of point estimates. Then the bootstrap standard error and confidence intervals are calculated based on these bootstrap samples. Details of the computation of the standard error and percentile confidence intervals can be found in Methods and formulas and in [R] bootstrap.

As mentioned in [R] rocreg, 50 resamples is a reasonable lower bound for obtaining bootstrap standard errors (Mooney and Duval 1993). However, it may be too low for obtaining percentile and bias-corrected confidence intervals. Normal-based confidence intervals are valid when the bootstrap distribution exhibits normality. See $[R]$ bootstrap postestimation for more details.

We can assess the normality of the bootstrap distribution by using a normal probability plot. Stata provides this in the pnorm command (see [R] diagnostic plots). We will use nsnrf.dta to draw a normal probability plot for the ROC estimate corresponding to a false-positive rate of 0.2 . We use the covariate values $\mathrm{xf}=10.01, \mathrm{xl}=6.5$, and $\mathrm{xd}=4$.

```
. use nsnrf
(bootstrap: rocregstat)
. generate double rocp2 = nsnr_b_i_cons + 10.01*nsnr_b_xf + 6.5*nsnr_b_xl +
> 4*nsnr_b_xd+nsnr_b_s_cons*invnormal(.2)
. replace rocp2 = normal(rocp2)
(50 real changes made)
```

- pnorm rocp2


The closeness of the points to the horizontal line on the normal probability plot shows us that the bootstrap distribution is approximately normal. So it is reasonable to use the normal-based confidence intervals for ROC at a false-positive rate of 0.2 under covariate values $\mathrm{xf}=10.01$, $\mathrm{xl}=6.5$, and $x d=4$.

## Plotting marginal ROC curves

The rocregplot command can also be used after fitting models with no covariates. We will demonstrate this with an empirical ROC model fit in $[R]$ rocreg.

## > Example 4: Nonparametric ROC

We run rocregplot after fitting the single-classifier, empirical ROC model shown in example 1 of $[R]$ rocreg. There we empirically predicted the ROC curve of the classifier rating for the true status variable disease from the Hanley and McNeil (1982) data. The rocreg command saves variables _roc_rating and _fpr_rating, which give the ROC values and false-positive rates, respectively, for every value of rating. These variables are used by rocregplot to render the ROC curve.

```
. use http://www.stata-press.com/data/r13/hanley, clear
. rocreg disease rating, noboot
Nonparametric ROC estimation
Control standardization: empirical
ROC method : empirical
Area under the ROC curve
    Status : disease
    Classifier: rating
```


. rocregplot


We end our discussion of rocregplot by showing its use after a marginal probit model.

Example 5: Maximum likelihood ROC, invROC, and ROC value
In example 13 of [ R$]$ rocreg, we fit a maximum-likelihood probit model to each classifier of the fictitious dataset generated from Hanley and McNeil (1983).

We use rocregplot after the original rocreg command to draw the ROC curves for classifiers mod1 and mod3. This is accomplished by specifying the two variables in the classvars() option. We will use the roc() option to obtain confidence intervals for ROC values at false-positive rates of 0.15 and 0.75 . We will specify the invroc () option to obtain false-positive rate confidence intervals for a ROC value of 0.8 . As mentioned previously, these are Wald confidence intervals.

First, we will view results for a false-positive rate of 0.75 .
. use http://www.stata-press.com/data/r13/ct2, clear
. rocreg status mod1 mod2 mod3, probit ml (output omitted)
. rocregplot, classvars(mod1 mod3) roc(.75)
ROC curve
Status : status
Classifier: mod1

| ROC | Coef. | Std. Err. | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :--- | :--- |
| .75 | .9931655 | .0069689 | .9795067 | 1.006824 |

Status : status
Classifier: mod3

| ROC | Coef. | Std. Err. | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :--- | :--- |
| .75 | .9953942 | .0043435 | .9868811 | 1.003907 |



We see that the estimates for each of the two ROC curves are close. Because this is a marginal model, the actual false-positive rate and the true-positive rate for each observation are plotted in the graph. The added point estimates of the ROC value at false-positive rate 0.75 are shown as diamond (mod3) and circle (mod1) symbols in the upper-right-hand corner of the graph at FPR $=0.75$. Confidence bands are also plotted at $\operatorname{FPR}=0.75$ but are so narrow that they are barely noticeable. Under both classifiers, the ROC value at 0.75 is very high. Now we will compare these results to those with a lower false-positive rate of 0.15 .

```
. rocregplot, classvars(mod1 mod3) roc(.15)
ROC curve
    Status : status
    Classifier: mod1
```

| ROC | Coef. | Std. Err. | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :--- | :--- |
| .15 | .7934935 | .0801363 | .6364292 | .9505578 |

Status : status
Classifier: mod3

| ROC | Coef. | Std. Err. | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :--- | :--- |
| .15 | .8888596 | .0520118 | .7869184 | .9908008 |



The ROC value for the false-positive rate of 0.15 is more separated in the two classifiers. Here we see that mod3 has a larger ROC value than mod1 for this false-positive rate, but the confidence intervals of the estimates overlap.

By specifying invroc (.8), we obtain invROC confidence intervals corresponding to a ROC value of 0.8 .

```
. rocregplot, classvars(mod1 mod3) invroc(.8)
False-positive rate
    Status : status
    Classifier: mod1
```

| invROC | Coef. | Std. Err. | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | :--- |
| .8 | .1556435 | .069699 | .019036 | .2922509 |

    Status : status
    Classifier: mod3
    | invROC | Coef. | Std. Err. | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :--- |
| .8 | .0661719 | .045316 | -.0226458 | .1549896 |



For estimation of the false-positive rate at a ROC value of 0.8 , the confidence intervals overlap. Both classifiers require only a small false-positive rate to achieve a ROC value of 0.8 .

## Methods and formulas

Details on computation of the nonparametric ROC curve and the estimation of the parametric ROC curve model coefficients can be found in $[\mathrm{R}]$ rocreg. Here we describe how to estimate the ROC values and false-positive rates of a parametric model. The cumulative distribution function $g$ can be the standard normal cumulative distribution function.

Methods and formulas are presented under the following headings:
Parametric model: Summary parameter definition
Maximum likelihood estimation
Estimating equations estimation

## Parametric model: Summary parameter definition

Conditioning on covariates $\mathbf{x}$, we have the following ROC curve model:

$$
\operatorname{ROC}(u)=g\left\{\mathbf{x}^{\prime} \boldsymbol{\beta}+\alpha g^{-1}(u)\right\}
$$

$\mathbf{x}$ can be constant, and $\boldsymbol{\beta}=\beta_{0}$, the constant intercept.
With simple algebra, we can solve this equation to obtain the false-positive rate value $u$ for a ROC value of $r$ :

$$
u=g\left[\left\{g^{-1}(r)-\mathbf{x}^{\prime} \boldsymbol{\beta}\right\} \alpha^{-1}\right]
$$

## Maximum likelihood estimation

We allow maximum likelihood estimation under probit parametric models, so $g=\Phi$. The ROC value and false-positive rate parameters all have closed-form expressions in terms of the covariate values $\mathbf{x}$, coefficient vector $\boldsymbol{\beta}$, and slope parameter $\alpha$. Thus to estimate these two types of summary parameters, we use the delta method (Oehlert 1992; Phillips and Park 1988). Particularly, we use the nlcom command (see [R] nlcom) to implement the delta method.

Under maximum likelihood estimation, the coefficient estimates $\widehat{\boldsymbol{\beta}}$ and slope estimate $\widehat{\alpha}$ are asymptotically normal with variance matrix V. For convenience, we rename the parameter vector [ $\left.\boldsymbol{\beta}^{\prime}, \alpha\right]$ to the $k$-parameter vector $\boldsymbol{\theta}=\left[\theta_{1}, \ldots, \theta_{k}\right]$. We will also explicitly refer to the conditioning of the ROC curve by $\boldsymbol{\theta}$ in its mention as $\operatorname{ROC}(t, \boldsymbol{\theta})$.

Under the delta method, the continuous scalar function of the estimate $\widehat{\boldsymbol{\theta}}, f(\widehat{\boldsymbol{\theta}})$ has asymptotic mean $f(\boldsymbol{\theta})$ and asymptotic covariance

$$
\widehat{\operatorname{Var}}\{f(\widehat{\boldsymbol{\theta}})\}=\mathbf{f V f}^{\prime}
$$

where $\mathbf{f}$ is the $1 \times k$ matrix of derivatives for which

$$
\mathbf{f}_{1 j}=\frac{\partial f(\boldsymbol{\theta})}{\partial \theta_{j}} \quad j=1, \ldots, k
$$

The asymptotic covariance of $f(\widehat{\boldsymbol{\theta}})$ is estimated and then used in conjunction with $f(\widehat{\boldsymbol{\theta}})$ for further inference, including Wald confidence intervals, standard errors, and hypothesis testing.

## Estimating equations estimation

When we fit a model using the Alonzo and Pepe (2002) estimating equations method, we use the bootstrap to perform inference on the ROC curve summary parameters. Each bootstrap sample provides a sample of the coefficient estimates $\boldsymbol{\beta}$ and the slope estimates $\alpha$. Using the formulas above, we can obtain an estimate of the ROC value or false-positive rate for each resample.

By making these calculations, we obtain a bootstrap sample of our summary parameter estimate. We then obtain bootstrap standard errors, normal approximation confidence intervals, percentile confidence intervals, and bias-corrected confidence intervals using this bootstrap sample. Further details can be found in $[R]$ bootstrap.

## References

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## Also see

[R] rocreg - Receiver operating characteristic (ROC) regression
$[\mathrm{R}]$ rocreg postestimation - Postestimation tools for rocreg
[U] 20 Estimation and postestimation commands

## Title

roctab - Nonparametric ROC analysis

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

roctab refvar classvar [if] [in] [weight] [, options]
roctab_options
Main
lorenz
binomial
nolabel
detail
table
bamber
hanley
graph
norefline
summary
specificity
level(\#)
Plot
plotopts(plot_options)
Reference line
rlopts (cline_options)
Add plots addplot (plot)

Y axis, X axis, Titles, Legend, Overall twoway_options

Description
report Gini and Pietra indices
calculate exact binomial confidence intervals display numeric codes rather than value labels show details on sensitivity/specificity for each cutpoint display the raw data in a $2 \times k$ contingency table calculate standard errors by using the Bamber method calculate standard errors by using the Hanley method graph the ROC curve suppress plotting the 45 -degree reference line report the area under the ROC curve graph sensitivity versus specificity set confidence level; default is level (95)
affect rendition of the ROC curve
affect rendition of the reference line
add other plots to the generated graph
any options other than by () documented in [G-3] twoway_options
fweights are allowed; see [U] 11.1.6 weight.
plot_options
Description
marker_options
marker_label_options
cline_options
change look of markers (color, size, etc.) add marker labels; change look or position change the look of the line

## Menu

Statistics $>$ Epidemiology and related $>$ ROC analysis $>$ Nonparametric ROC analysis without covariates

## Description

The above command is used to perform receiver operating characteristic (ROC) analyses with rating and discrete classification data.

The two variables refvar and classvar must be numeric. The reference variable indicates the true state of the observation, such as diseased and nondiseased or normal and abnormal, and must be coded as 0 and 1 . The rating or outcome of the diagnostic test or test modality is recorded in classvar, which must be at least ordinal, with higher values indicating higher risk.
roctab performs nonparametric ROC analyses. By default, roctab calculates the area under the ROC curve. Optionally, roctab can plot the ROC curve, display the data in tabular form, and produce Lorenz-like plots.

See [R] rocfit for a command that fits maximum-likelihood ROC models.

## Options

## Main

lorenz specifies that Gini and Pietra indices be reported. Optionally, graph will plot the Lorenz-like curve.
binomial specifies that exact binomial confidence intervals be calculated.
nolabel specifies that numeric codes be displayed rather than value labels.
detail outputs a table displaying the sensitivity, specificity, the percentage of subjects correctly classified, and two likelihood ratios for each possible cutpoint of classvar.
table outputs a $2 \times k$ contingency table displaying the raw data.
bamber specifies that the standard error for the area under the ROC curve be calculated using the method suggested by Bamber (1975). Otherwise, standard errors are obtained as suggested by DeLong, DeLong, and Clarke-Pearson (1988).
hanley specifies that the standard error for the area under the ROC curve be calculated using the method suggested by Hanley and McNeil (1982). Otherwise, standard errors are obtained as suggested by DeLong, DeLong, and Clarke-Pearson (1988).
graph produces graphical output of the ROC curve. If lorenz is specified, graphical output of a Lorenz-like curve will be produced.
norefline suppresses plotting the 45-degree reference line from the graphical output of the ROC curve.
summary reports the area under the ROC curve, its standard error, and its confidence interval. If lorenz is specified, Lorenz indices are reported. This option is needed only when also specifying graph.
specificity produces a graph of sensitivity versus specificity instead of sensitivity versus (1 - specificity). specificity implies graph.
level(\#) specifies the confidence level, as a percentage, for the confidence intervals. The default is level (95) or as set by set level; see [R] level.
plotopts(plot_options) affects the rendition of the plotted ROC curve-the curve's plotted points connected by lines. The plot_options can affect the size and color of markers, whether and how the markers are labeled, and whether and how the points are connected; see [G-3] marker_options, [G-3] marker_label_options, and [G-3] cline_options.
$\sqrt{\text { Add plots }}$
addplot (plot) provides a way to add other plots to the generated graph; see [G-3] addplot_option.

twoway_options are any of the options documented in [G-3] twoway_options, excluding by (). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

## Remarks and examples

Remarks are presented under the following headings:
Introduction
Nonparametric ROC curves
Lorenz-like curves

## Introduction

The roctab command provides nonparametric estimation of the ROC for a given classifier and true-status reference variable. The Lorenz curve functionality of roctab, which provides an alternative to standard ROC analysis, is discussed in Lorenz-like curves.

See Pepe (2003) for a discussion of ROC analysis. Pepe has posted Stata datasets and programs used to reproduce results presented in the book (http://www.stata.com/bookstore/pepe.html).

## Nonparametric ROC curves

The points on the nonparametric ROC curve are generated using each possible outcome of the diagnostic test as a classification cutpoint and computing the corresponding sensitivity and $1-$ specificity. These points are then connected by straight lines, and the area under the resulting ROC curve is computed using the trapezoidal rule.

## $>$ Example 1

Hanley and McNeil (1982) presented data from a study in which a reviewer was asked to classify, using a five-point scale, a random sample of 109 tomographic images from patients with neurological problems. The rating scale was as follows: $1=$ definitely normal, $2=$ probably normal, $3=$ questionable, $4=$ probably abnormal, and $5=$ definitely abnormal. The true disease status was normal for 58 of the patients and abnormal for the remaining 51 patients.

Here we list 9 of the 109 observations:

```
. use http://www.stata-press.com/data/r13/hanley
. list disease rating in 1/9
```

|  | disease | rating |
| :--- | ---: | ---: |
| 1. | 1 | 5 |
| 2. | 0 | 1 |
| 3. | 1 | 5 |
| 4. | 0 | 4 |
| 5. | 0 | 1 |
| 6. | 0 | 3 |
| 7. | 1 | 5 |
| 8. | 0 | 5 |
| 9. | 0 | 1 |

For each observation, disease identifies the true disease status of the subject $(0=$ normal, $1=$ abnormal), and rating contains the classification value assigned by the reviewer.

We can use roctab to calculate and plot the nonparametric ROC curve by specifying both the summary and graph options. By also specifying the table option, we obtain a contingency table summarizing our dataset.
. roctab disease rating, table graph summary
disease


By default, roctab reports the area under the curve, its standard error, and its confidence interval. The graph option can be used to plot the ROC curve.

The ROC curve is plotted by computing the sensitivity and specificity using each value of the rating variable as a possible cutpoint. A point is plotted on the graph for each of the cutpoints. These plotted points are joined by straight lines to form the ROC curve, and the area under the ROC curve is computed using the trapezoidal rule.

We can tabulate the computed sensitivities and specificities for each of the possible cutpoints by specifying detail.
. roctab disease rating, detail
Detailed report of sensitivity and specificity

| Cutpoint |  | Sensitivity | Specificity | Correctly <br> Classified | LR+ | LR- |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ( >= 1 ) |  | 100.00\% | 0.00\% | 46.79\% | 1.0000 |  |
| ( >= 2 ) |  | 94.12\% | 56.90\% | 74.31\% | 2.1835 | 0.1034 |
| ( >= 3 ) |  | 90.20\% | 67.24\% | 77.98\% | 2.7534 | 0.1458 |
| ( >= 4 ) |  | 86.27\% | 77.59\% | 81.65\% | 3.8492 | 0.1769 |
| ( >= 5 ) |  | 64.71\% | 96.55\% | 81.65\% | 18.7647 | 0.3655 |
| ( > 5 ) |  | 0.00\% | 100.00\% | 53.21\% |  | 1.0000 |
|  | ROC |  |  | -Asymptotic Normal[95\% Conf. Interval] |  |  |
|  | 109 | 0.8932 | 0.0307 | 0.83295 | 0.95339 |  |

Each cutpoint in the table indicates the ratings used to classify tomographs as being from an abnormal subject. For example, the first cutpoint ( $>=1$ ) indicates that all tomographs rated as 1 or greater are classified as coming from abnormal subjects. Because all tomographs have a rating of 1 or greater, all are considered abnormal. Consequently, all abnormal cases are correctly classified (sensitivity $=100 \%$ ), but none of the normal patients is classified correctly (specificity $=0 \%$ ). For the second cutpoint ( $>=2$ ), tomographs with ratings of 1 are classified as normal, and those with ratings of 2 or greater are classified as abnormal. The resulting sensitivity and specificity are $94.12 \%$ and $56.90 \%$, respectively. Using this cutpoint, we correctly classified $74.31 \%$ of the 109 tomographs. Similar interpretations can be used on the remaining cutpoints. As mentioned, each cutpoint corresponds to a point on the nonparametric ROC curve. The first cutpoint $(>=1)$ corresponds to the point at $(1,1)$, and the last cutpoint $(>5)$ corresponds to the point at $(0,0)$.
detail also reports two likelihood ratios suggested by Choi (1998): the likelihood ratio for a positive test result ( $\mathrm{LR}+$ ) and the likelihood ratio for a negative test result ( $\mathrm{LR}-$ ). The $\mathrm{LR}+$ is the ratio of the probability of a positive test among the truly positive subjects to the probability of a positive test among the truly negative subjects. The LR- is the ratio of the probability of a negative test among the truly positive subjects to the probability of a negative test among the truly negative subjects. Choi points out that LR+ corresponds to the slope of the line from the origin to the point on the ROC curve determined by the cutpoint. Similarly, LR- corresponds to the slope from the point $(1,1)$ to the point on the ROC curve determined by the cutpoint.

By default, roctab calculates the standard error for the area under the curve by using an algorithm suggested by DeLong, DeLong, and Clarke-Pearson (1988) and asymptotic normal confidence intervals. Optionally, standard errors based on methods suggested by Bamber (1975) or Hanley and McNeil (1982) can be computed by specifying bamber or hanley, respectively, and an exact binomial confidence interval can be obtained by specifying binomial.

```
. roctab disease rating, bamber
```



## Lorenz-like curves

For applications where it is known that the risk status increases or decreases monotonically with increasing values of the diagnostic test, the ROC curve and associated indices are useful in assessing the overall performance of a diagnostic test. When the risk status does not vary monotonically with increasing values of the diagnostic test, however, the resulting ROC curve can be nonconvex and its indices can be unreliable. For these situations, Lee (1999) proposed an alternative to the ROC analysis based on Lorenz-like curves and the associated Pietra and Gini indices.

Lee (1999) mentions at least three specific situations where results from Lorenz curves are superior to those obtained from ROC curves: 1) a diagnostic test with similar means but very different standard deviations in the abnormal and normal populations, 2) a diagnostic test with bimodal distributions in either the normal or abnormal population, and 3) a diagnostic test distributed symmetrically in the normal population and skewed in the abnormal.

When the risk status increases or decreases monotonically with increasing values of the diagnostic test, the ROC and Lorenz curves yield interchangeable results.

## > Example 2

To illustrate the use of the lorenz option, we constructed a fictitious dataset that yields results similar to those presented in Table III of Lee (1999). The data assume that a 12-point rating scale was used to classify 442 diseased and 442 healthy subjects. We list a few of the observations.

```
. use http://www.stata-press.com/data/r13/lorenz, clear
. list in 1/7, noobs sep(0)
```

| disease | class | pop |
| ---: | ---: | ---: |
| 0 | 5 | 66 |
| 1 | 11 | 17 |
| 0 | 6 | 85 |
| 0 | 3 | 19 |
| 0 | 10 | 19 |
| 0 | 2 | 7 |
| 1 | 4 | 16 |

The data consist of 24 observations: 12 observations from diseased individuals and 12 from nondiseased individuals. Each observation corresponds to one of the 12 classification values of the rating-scale variable, class. The number of subjects represented by each observation is given by the pop variable, making this a frequency-weighted dataset. The data were generated assuming a binormal distribution of the latent variable with similar means for the normal and abnormal populations but with the standard deviation for the abnormal population five times greater than that of the normal population.
. roctab disease class [fweight=pop], graph summary

| Obs | $\begin{array}{r} \text { ROC } \\ \text { Area } \end{array}$ | Std. Err. | -Asymptotic Normal[95\% Conf. Interval] |
| :---: | :---: | :---: | :---: |
| 884 | 0.5774 | 0.0215 | $0.53517 \quad 0.61959$ |



The resulting ROC curve is nonconvex or, as termed by Lee, "wiggly". Lee argues that for this and similar situations, the Lorenz curve and indices are preferred.

```
. roctab disease class [fweight=pop], lorenz summary graph
Lorenz curve
Pietra index = 0.6493
Gini index = 0.7441
```

Lorenz curve


Like ROC curves, a more bowed Lorenz curve suggests a better diagnostic test. This bowedness is quantified by the Pietra index, which is geometrically equivalent to twice the largest triangle that can be inscribed in the area between the curve and the diagonal line, and the Gini index, which is
equivalent to twice the area between the Lorenz curve and the diagonal. Lee (1999) provides several additional interpretations for the Pietra and Gini indices.

## Stored results

roctab stores the following in $r()$ :
Scalars

| $r(N)$ | number of observations |
| :--- | :--- |
| $r$ (se) | standard error for the area under the ROC curve |
| $r(1 \mathrm{~b})$ | lower bound of CI for the area under the ROC curve |
| $r$ (ub) | upper bound of CI for the area under the ROC curve |
| $r$ (area) | area under the ROC curve |
| $r$ (pietra) | Pietra index |
| $r$ (gini) | Gini index |

## Methods and formulas

Assume that we applied a diagnostic test to each of $N_{n}$ normal and $N_{a}$ abnormal subjects. Further assume that the higher the outcome value of the diagnostic test, the higher the risk of the subject being abnormal. Let $\widehat{\theta}$ be the estimated area under the curve, and let $X_{i}, i=1,2, \ldots, N_{a}$ and $Y_{j}, j=1,2, \ldots, N_{n}$ be the values of the diagnostic test for the abnormal and normal subjects, respectively.

The points on the nonparametric ROC curve are generated using each possible outcome of the diagnostic test as a classification cutpoint and computing the corresponding sensitivity and $1-$ specificity. These points are then connected by straight lines, and the area under the resulting ROC curve is computed using the trapezoidal rule.

The default standard error for the area under the ROC curve is computed using the algorithm described by DeLong, DeLong, and Clarke-Pearson (1988). For each abnormal subject, $i$, define

$$
V_{10}\left(X_{i}\right)=\frac{1}{N_{n}} \sum_{j=1}^{N_{n}} \psi\left(X_{i}, Y_{j}\right)
$$

and for each normal subject, $j$, define

$$
V_{01}\left(Y_{j}\right)=\frac{1}{N_{a}} \sum_{i=1}^{N_{a}} \psi\left(X_{i}, Y_{j}\right)
$$

where

$$
\psi(X, Y)= \begin{cases}1 & Y<X \\ \frac{1}{2} & Y=X \\ 0 & Y>X\end{cases}
$$

Define

$$
S_{10}=\frac{1}{N_{a}-1} \sum_{i=1}^{N_{a}}\left\{V_{10}\left(X_{i}\right)-\widehat{\theta}\right\}^{2}
$$

and

$$
S_{01}=\frac{1}{N_{n}-1} \sum_{j=1}^{N_{n}}\left\{V_{01}\left(Y_{j}\right)-\widehat{\theta}\right\}^{2}
$$

The variance of the estimated area under the ROC curve is given by

$$
\operatorname{var}(\widehat{\theta})=\frac{1}{N_{a}} S_{10}+\frac{1}{N_{n}} S_{01}
$$

The hanley standard error for the area under the ROC curve is computed using the algorithm described by Hanley and McNeil (1982). It requires the calculation of two quantities: $Q_{1}$ is $\operatorname{Pr}$ (two randomly selected abnormal subjects will both have a higher score than a randomly selected normal subject), and $Q_{2}$ is $\operatorname{Pr}$ (one randomly selected abnormal subject will have a higher score than any two randomly selected normal subjects). The Hanley and McNeil variance of the estimated area under the ROC curve is

$$
\operatorname{var}(\widehat{\theta})=\frac{\widehat{\theta}(1-\widehat{\theta})+\left(N_{a}-1\right)\left(Q_{1}-\widehat{\theta}^{2}\right)+\left(N_{n}-1\right)\left(Q_{2}-\widehat{\theta}^{2}\right)}{N_{a} N_{n}}
$$

The bamber standard error for the area under the ROC curve is computed using the algorithm described by Bamber (1975). For any two $Y$ values, $Y_{j}$ and $Y_{k}$, and any $X_{i}$ value, define

$$
b_{y y x}=p\left(Y_{j}, Y_{k}<X_{i}\right)+p\left(X_{i}<Y_{j}, Y_{k}\right)-2 p\left(Y_{j}<X_{i}<Y_{k}\right)
$$

and similarly, for any two $X$ values, $X_{i}$ and $X_{l}$, and any $Y_{j}$ value, define

$$
b_{x x y}=p\left(X_{i}, X_{l}<Y_{j}\right)+p\left(Y_{j}<X_{i}, X_{l}\right)-2 p\left(X_{i}<Y_{j}<X_{l}\right)
$$

Bamber's unbiased estimate of the variance for the area under the ROC curve is
$\operatorname{var}(\widehat{\theta})=\frac{1}{4}\left(N_{a}-1\right)\left(N_{n}-1\right)\left\{p(X \neq Y)+\left(N_{a}-1\right) b_{x x y}+\left(N_{n}-1\right) b_{y y x}-4\left(N_{a}+N_{n}-1\right)(\widehat{\theta}-0.5)^{2}\right\}$
Asymptotic confidence intervals are constructed and reported by default, assuming a normal distribution for the area under the ROC curve.

Exact binomial confidence intervals are calculated as described in $[\mathrm{R}] \mathbf{c i}$, with $p$ equal to the area under the ROC curve.

## References

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## Also see

[R] logistic postestimation - Postestimation tools for logistic
$[R]$ roc - Receiver operating characteristic (ROC) analysis
[ R$]$ roccomp - Tests of equality of ROC areas
[R] rocfit - Parametric ROC models
$[\mathrm{R}]$ rocreg - Receiver operating characteristic (ROC) regression

## Title

rologit — Rank-ordered logistic regression

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | Acknowledgment |
| References | Also see |  |  |

## Syntax

rologit depvar indepvars [if] [in] [weight], group(varname) [options]

## options Description

Model

* group (varname) offset (varname)
incomplete(\#) reverse notestrhs ties (spec)

SE/Robust vce (vcetype)

Reporting
level(\#)
display_options
identifier variable that links the alternatives include varname in model with coefficient constrained to 1 use \# to code unranked alternatives; default is incomplete (0) reverse the preference order
keep right-hand-side variables that do not vary within group method to handle ties: exactm, breslow, efron, or none
vcetype may be oim, robust, cluster clustvar, bootstrap, or jackknife
set confidence level; default is level (95)
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling

Maximization
maximize_options
coeflegend
control the maximization process; seldom used
display legend instead of statistics

* group (varname) is required.
indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
bootstrap, by, fp, jackknife, rolling, and statsby are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the bootstrap prefix; see [R] bootstrap.
fweights, iweights, and pweights are allowed, except with ties(efron); see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.


## Menu

Statistics $>$ Ordinal outcomes $>$ Rank-ordered logistic regression

## Description

rologit fits the rank-ordered logistic regression model by maximum likelihood (Beggs, Cardell, and Hausman 1981). This model is also known as the Plackett-Luce model (Marden 1995), as the exploded logit model (Punj and Staelin 1978), and as the choice-based method of conjoint analysis (Hair et al. 2010).
rologit expects the data to be in long form, similar to clogit (see [R] clogit), in which each of the ranked alternatives forms an observation; all observations related to an individual are linked together by the variable that you specify in the group () option. The distinction from clogit is that depvar in rologit records the rankings of the alternatives, whereas for clogit, depvar marks only the best alternative by a value not equal to zero. rologit interprets equal scores of depvar as ties. The ranking information may be incomplete "at the bottom" (least preferred alternatives). That is, unranked alternatives may be coded as 0 or as a common value that may be specified with the incomplete() option.

If your data record only the unique best alternative, rologit fits the same model as clogit.

## Options

## Model

group (varname) is required, and it specifies the identifier variable (numeric or string) that links the alternatives for an individual, which have been compared and rank ordered with respect to one another.
offset (varname); see [R] estimation options.
incomplete(\#) specifies the numeric value used to code alternatives that are not ranked. It is assumed that unranked alternatives are less preferred than the ranked alternatives (that is, the data record the ranking of the most preferred alternatives). It is not assumed that subjects are indifferent between the unranked alternatives. \# defaults to 0 .
reverse specifies that in the preference order, a higher number means a less attractive alternative. The default is that higher values indicate more attractive alternatives. The rank-ordered logit model is not symmetric in the sense that reversing the ordering simply leads to a change in the signs of the coefficients.
notestrhs suppresses the test that the independent variables vary within (at least some of) the groups. Effects of variables that are always constant are not identified. For instance, a rater's gender cannot directly affect his or her rankings; it could affect the rankings only via an interaction with a variable that does vary over alternatives.
ties (spec) specifies the method for handling ties (indifference between alternatives) (see [ST] stcox for details):

| exactm | exact marginal likelihood (default) <br> Breslow's method (default if pweights specified) |
| :--- | :--- |
| breslow | Efron's method (default if robust VCE) |
| none | no ties allowed |

SE/Robust
vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
If ties (exactm) is specified, vcetype may be only oim, bootstrap, or jackknife.

Reporting
level (\#); see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\%fmt), pformat(\%fint), sformat(\%fmt), and nolstretch; see [R] estimation options.

Maximization
maximize_options: iterate(\#), trace, [no] log, tolerance(\#), ltolerance(\#), nrtolerance(\#), and nonrtolerance; see [R] maximize. These options are seldom used.

The following option is available with rologit but is not shown in the dialog box:
coeflegend; see [R] estimation options.

## Remarks and examples

The rank-ordered logit model can be applied to analyze how decision makers combine attributes of alternatives into overall evaluations of the attractiveness of these alternatives. The model generalizes a version of McFadden's choice model without alternative-specific covariates, as fit by the clogit command. It uses richer information about the comparison of alternatives, namely, how decision-makers rank the alternatives rather than just specifying the alternative that they like best.

Remarks are presented under the following headings:

Examples<br>Comparing respondents<br>Incomplete rankings and ties<br>Clustered choice data<br>Comparison of rologit and clogit<br>On reversals of rankings

## Examples

A popular way to study employer preferences for characteristics of employees is the quasiexperimental "vignette method". As an example, we consider the research by de Wolf on the labor market position of social science graduates (de Wolf 2000). This study addresses how the educational portfolio (for example, general skills versus specific knowledge) affects short-term and long-term labor-market opportunities. De Wolf asked 22 human resource managers (the respondents) to rank order the six most suitable candidates of 20 fictitious applicants and to rank order these six candidates for three jobs, namely, 1) researcher, 2) management trainee, and 3) policy adviser. Applicants were described by 10 attributes, including their age, gender, details of their portfolio, and work experience. In this example, we analyze a subset of the data. Also, to simplify the output, we drop, at random, 10 nonselected applicants per case. The resulting dataset includes 29 cases, consisting of 10 applicants each. The data are in long form: observations correspond to alternatives (the applications), and alternatives that figured in one decision task are identified by the variable caseid. We list the observations for caseid==7, in which the respondent considered applicants for a social-science research position.
. use http://www.stata-press.com/data/r13/evignet
(Vignet study employer prefs (Inge de Wolf 2000))
. list pref female age grades edufit workexp boardexp if caseid==7, noobs

| pref | female | age | grades | edufit | workexp | boardexp |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0 | yes | 28 | A/B | no | none | no |
| 0 | no | 25 | C/D | yes | one year | no |
| 0 | no | 25 | C/D | yes | none | yes |
| 0 | yes | 25 | C/D | no | internship | yes |
| 1 | no | 25 | C/D | yes | one year | yes |
| 2 | no | 25 | A/B | yes | none | no |
| 3 | yes | 25 | A/B | yes | one year | no |
| 4 | yes | 25 | A/B | yes | none | yes |
| 5 | no | 25 | A/B | yes | internship | no |
| 6 | yes | 28 | A/B | yes | one year | yes |

Here six applicants were selected. The rankings are stored in the variable pref, where a value of 6 corresponds to "best among the candidates", a value of 5 corresponds to "second-best among the candidates", etc. The applicants with a ranking of 0 were not among the best six candidates for the job. The respondent was not asked to express his preferences among these four applicants, but by the elicitation procedure, it is known that he ranks these four applicants below the six selected applicants. The best candidate was a female, 28 years old, with education fitting the job, with good grades (A/B), with 1 year of work experience, and with experience being a board member of a fraternity, a sports club, etc. The profiles of the other candidates read similarly. Here the respondent completed the task; that is, he selected and rank ordered the six most suitable applicants. Sometimes the respondent performed only part of the task.

```
- list pref female age grades edufit workexp boardexp if caseid==18, noobs
```

| pref | female | age | grades | edufit | workexp | boardexp |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0 | no | 25 | C/D | yes | none | yes |
| 0 | no | 25 | C/D | no | internship | yes |
| 0 | no | 28 | C/D | no | internship | yes |
| 0 | yes | 25 | A/B | no | one year | no |
| 2 | yes | 25 | A/B | no | none | yes |
| 2 | no | 25 | A/B | no | none | yes |
| 2 | no | 25 | A/B | no | one year | yes |
| 5 | no | 25 | A/B | no | none | yes |
| 5 | no | 25 | A/B | no | none | yes |
| 5 | yes | 25 | A/B | no | none | no |

The respondent selected the six best candidates and segmented these six candidates into two groups: one group with the three best candidates, and a second group of three candidates that were "still acceptable". The numbers 2 and 5, indicating these two groups, are arbitrary apart from the implied ranking of the groups. The ties between the candidates in a group indicate that the respondent was not able to rank the candidates within the group.

The purpose of the vignette experiment was to explore and test hypotheses about which of the employees' attributes are valued by employers, how these attributes are weighted depending on the type of job (described by variable job in these data), etc. In the psychometric tradition of Thurstone (1927), value is assumed to be linear in the attributes, with the coefficients expressing the direction and weight of the attributes. In addition, it is assumed that valuation is to some extent a random procedure,
captured by an additive random term. For instance, if value depends only on an applicant's age and gender, we would have

$$
\operatorname{value}\left(\mathrm{female}_{i}, \text { age }_{i}\right)=\beta_{1} \mathrm{female}_{i}+\beta_{2} \text { age }_{i}+\epsilon_{i}
$$

where the random residual, $\epsilon_{i}$, captures all omitted attributes. Thus $\beta_{1}>0$ means that the employer assigns higher value to a woman than to a man. Given this conceptualization of value, it is straightforward to model the decision (selection) among alternatives or the ranking of alternatives: the alternative with the highest value is selected (chosen), or the alternatives are ranked according to their value. To complete the specification of a model of choice and of ranking, we assume that the random residual $\epsilon_{i}$ follows an "extreme value distribution of type I", introduced in this context by Luce (1959). This specific assumption is made mostly for computational convenience.

This model is known by many names. Among others, it is known as the rank-ordered logit model in economics (Beggs, Cardell, and Hausman 1981), as the exploded logit model in marketing research (Punj and Staelin 1978), as the choice-based conjoint analysis model (Hair et al. 2010), and as the Plackett-Luce model (Marden 1995). The model coefficients are estimated using the method of maximum likelihood. The implementation in rologit uses an analogy between the rank-ordered logit model and the Cox regression model observed by Allison and Christakis (1994); see Methods and formulas. The rologit command implements this method for rankings, whereas clogit deals with the variant of choices, that is, only the most highly valued alternative is recorded. In the latter case, the model is also known as the Luce-McFadden choice model. In fact, when the data record the most preferred (unique) alternative and no additional ranking information about preferences is available, rologit and clogit return the same information, though formatted somewhat differently.

| Iteration 0: log likelihood = -95.41087 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Iteration 1: | log likelihood = -71.180903 |  |  |  |  |  |
| Iteration 2: | log likelihood = -68.47734 |  |  |  |  |  |
| Iteration 3: | log likelihood = -68.345918 |  |  |  |  |  |
| Iteration 4: | log likelihood $=-68.345389$ |  |  |  |  |  |
| Refining estimates: |  |  |  |  |  |  |
| Iteration 0: | log likelihood = -68.345389 |  |  |  |  |  |
| Rank-ordered logistic regression Group variable: caseid |  |  |  | Number | $f$ obs | 80 |
|  |  |  |  | Number | $f$ groups | 8 |
| No ties in data |  |  |  | Obs per group: min $=10$ |  |  |
|  |  |  |  |  | av | 10.00 |
|  |  |  |  |  | max | 10 |
|  |  |  |  | LR chi |  | 54.13 |
| Log likelihood = -68.34539 |  |  |  | Prob > chi2 |  | 0.0000 |
| pref | Coef. | Std. Err. | z | $P>\|z\|$ | [95\% Con | Interval] |
| female | -. 4487287 | . 3671307 | -1.22 | 0.222 | -1.168292 | . 2708343 |
| age | -. 0984926 | . 0820473 | -1.20 | 0.230 | -. 2593024 | . 0623172 |
| grades | 3.064534 | . 6148245 | 4.98 | 0.000 | 1.8595 | 4.269568 |
| edufit | . 7658064 | . 3602366 | 2.13 | 0.034 | . 0597556 | 1.471857 |
| workexp | 1.386427 | . 292553 | 4.74 | 0.000 | . 8130341 | 1.959821 |
| boardexp | . 6944377 | . 3762596 | 1.85 | 0.065 | -. 0430176 | 1.431893 |

Focusing only on the variables whose coefficients are significant at the $10 \%$ level (we are analyzing 8 respondents only!), the estimated value of an applicant for a job of type 1 (research positions) can be written as

```
value = 3.06*grades + 0.77*edufit + 1.39*workexp + 0.69*boardexp
```

Thus employers prefer applicants for a research position (job==1) whose educational portfolio fits the job, who have better grades, who have more relevant work experience, and who have (extracurricular) board experience. They do not seem to care much about the sex and age of applicants, which is comforting.

Given these estimates of the valuation by employers, we consider the probabilities that each of the applications is ranked first. Under the assumption that the $\epsilon_{i}$ are independent and follow an extreme value type I distribution, Luce (1959) showed that the probability, $\pi_{i}$, that alternative $i$ is valued higher than alternatives $2, \ldots, k$ can be written in the multinomial logit form

$$
\pi_{i}=\operatorname{Pr}\left\{\text { value }_{1}>\max \left(\text { value }_{2}, \ldots, \text { value }_{m}\right)\right\}=\frac{\exp \left(\text { value }_{i}\right)}{\sum_{j=1}^{k} \exp \left(\text { value }_{i}\right)}
$$

The probability of observing a specific ranking can be written as the product of such terms, representing a sequential decision interpretation in which the rater first chooses the most preferred alternative, and then the most preferred alternative among the rest, etc.

The probabilities for alternatives to be ranked first are conveniently computed by predict.

```
. predict p if e(sample)
(option pr assumed; conditional probability that alternative is ranked first)
( 210 missing values generated)
. sort caseid pref p
. list pref p grades edufit workexp boardexp if caseid==7, noobs
```

| pref | p | grades | edufit | workexp | boardexp |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 0 | .0027178 | C/D | yes | none | yes |
| 0 | .0032275 | C/D | no | internship | yes |
| 0 | .0064231 | A/B | no | none | no |
| 0 | .0217202 | C/D | yes | one year | no |
| 1 | .0434964 | C/D | yes | one year | yes |
| 2 | .0290762 | A/B | yes | none | no |
| 3 | .2970933 | A/B | yes | one year | no |
| 4 | .0371747 | A/B | yes | none | yes |
| 5 | .1163203 | A/B | yes | internship | no |
| 6 | .4427504 | A/B | yes | one year | yes |

There clearly is a positive relation between the stated ranking and the predicted probabilities for alternatives to be ranked first, but the association is not perfect. In fact, we would not have expected a perfect association, as the model specifies a (nondegenerate) probability distribution over the possible rankings of the alternatives. These predictions for sets of 10 candidates can also be used to make predictions for subsets of the alternatives. For instance, suppose that only the last three candidates listed in this table would be available. According to parameter estimates of the rank-ordered logit model, the probability that the last of these candidates is selected equals $0.443 /(0.037+0.116+0.443)=0.743$.

## Comparing respondents

The rologit model assumes that all respondents, HR managers in large public-sector organizations in The Netherlands, use the same valuation function; that is, they apply the same decision weights. This is the substantive interpretation of the assumption that the $\beta$ 's are constant between the respondents. To probe this assumption, we could test whether the coefficients vary between different groups of respondents. For a metric characteristic of the HR manager, such as firmsize, we can consider a trend-model in the valuation weights,

$$
\beta_{i j}=\alpha_{i 0}+\alpha_{i 1} \text { firmsize }_{j}
$$

and we can test that the slopes $\alpha_{i 1}$ of firmsize are zero.


| pref | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| :---: | ---: | :---: | :---: | :---: | :---: | :---: |
| edufit | 1.29122 | 1.13764 | 1.13 | 0.256 | -.9385127 | 3.520953 |
| grades <br> workexp | 6.439776 | 2.288056 | 2.81 | 0.005 | 1.955267 | 10.92428 |
| c.firmsize\# <br> c.edufit | -.0173333 | .0711942 | -0.24 | 0.808 | -.1568714 | .1222048 |
| c.firmsize\# <br> c.grades | -.2099279 | .1218251 | -1.72 | 0.085 | -.4487008 | .028845 |
| c.firmsize\# <br> c.workexp | .0097508 | .0525081 | 0.19 | 0.853 | -.0931632 | .1126649 |
| c.firmsize\# |  |  |  |  |  |  |

```
( 1) c.firmsize#c.edufit = 0
( 2) c.firmsize#c.grades = 0
( 3) c.firmsize#c.workexp = 0
(4) c.firmsize#c.boardexp = 0
    chi2( 4) = 7.14
        Prob > chi2 = 0.1288
```

. testparm c.firmsize\#c. (edufit grades workexp boardexp)

The Wald test that the slopes of the interacted firmsize variables are jointly zero provides no evidence upon which we would reject the null hypothesis; that is, we do not find evidence against the assumption of constant valuation weights of the attributes by firms of different size. We did not enter firmsize as a predictor variable. Characteristics of the decision-making agent do not vary between alternatives. Thus an additive effect of these characteristics on the valuation of alternatives does not affect the agent's ranking of alternatives and his choice. Consequently the coefficient of firmsize is not identified. rologit would in fact have diagnosed the problem and dropped firmsize from the analysis. Diagnosing this problem can slow the estimation considerably; the test may be suppressed by specifying the notestrhs option.

## Incomplete rankings and ties

rologit allows incomplete rankings and ties in the rankings as proposed by Allison and Christakis (1994). rologit permits rankings to be incomplete only "at the bottom"; namely, that the ranking of the least attractive alternatives for subjects may not be known-do not confuse this with the situation that a subject is indifferent between these alternatives. This form of incompleteness occurred in the example discussed here, because the respondents were instructed to select and rank only the top six alternatives. It may also be that respondents refused to rank the alternatives that are very unattractive. rologit does not allow other forms of incompleteness, for instance, data in which respondents indicate which of four cars they like best, and which one they like least, but not how they rank the two intermediate cars. Another example of incompleteness that cannot be analyzed with rologit is data in which respondents select the three alternatives they like best but are not requested to express their preferences among the three selected alternatives.
rologit also permits ties in rankings. rologit assumes that if a subject expresses a tie between two or more alternatives, he or she actually holds one particular strict preference ordering, but with all possibilities of a strict ordering consistent with the expressed weak ordering being equally probable. For instance, suppose that a respondent ranks alternative 1 highest. He prefers alternatives 2 and 3 over alternative 4, and he is indifferent between alternatives 2 and 3 . We assume that this respondent either has the strict preference ordering $1>2>3>4$ or $1>3>2>4$, with both possibilities being equally likely. From a psychometric perspective, it may actually be more appropriate to also assume that the alternatives 2 and 3 are close; for instance, the difference between the associated valuations (utilities) is less than some threshold or minimally discernible difference. Computationally, however, this is a more demanding model.

## Clustered choice data

We have seen that applicants with work experience are in a relatively favorable position. To test whether the effects of work experience vary between the jobs, we can include interactions between the type of job and the attributes of applicants. Such interactions can be obtained using factor variables.

Because some HR managers contributed data for more than one job, we cannot assume that their selection decisions for different jobs are independent. We can account for this by specifying the vce(cluster clustvar) option. By treating choice data as incomplete ranking data with only the most preferred alternative marked, rologit may be used to estimate the model parameters for clustered choice data.


The parameter estimates for the first job type are very similar to those that would have been obtained from an analysis isolated to these data. Differences are due only to an implied change in the method of handling ties. With clustered observations, rologit uses Efron's method. If we had specified the ties (efron) option with the separate analyses, then the parameter estimates would have been identical to the simultaneous results. Another difference is that rologit now reports robust standard errors, adjusted for clustering within respondents. These could have been obtained for the separate analyses, as well by specifying the vce (robust) option. In fact, this option would also have forced rologit to switch to Efron's method as well.

Given the combined results for the three types of jobs, we can test easily whether the weights for the attributes of applicants vary between the jobs, in other words, whether employers are looking for different qualifications in applicants for different jobs. A Wald test for the equality hypothesis of no difference can be obtained with the testparm command:

```
. testparm job#c.(female grades edufit workexp)
( 1) 2.job#c.female = 0
( 2) 3.job#c.female = 0
( 3) 2.job#c.grades = 0
(4) 3.job#c.grades = 0
( 5) 2.job#c.edufit = 0
( 6) 3.job#c.edufit = 0
( 7) 2.job#c.workexp = 0
( 8) 3.job#c.workexp = 0
        chi2( 8) = 14.96
        Prob > chi2 = 0.0599
```

We find only mild evidence that employers look for different qualities in candidates according to the job for which they are being considered.

- Technical note

Allison (1999) stressed that the comparison between groups of the coefficients of logistic regression is problematic, especially in its latent-variable interpretation. In many common latent-variable models, only the regression coefficients divided by the scale of the latent variable are identified. Thus a comparison of logit regression coefficients between, say, men and women is meaningful only if one is willing to argue that the standard deviation of the latent residual does not differ between the sexes. The rank-ordered logit model is also affected by this problem. While we formulated the model with a scale-free residual, we can actually think of the model for the value of an alternative as being scaled by the standard deviation of the random term, representing other relevant attributes of alternatives. Again comparing attribute weights between jobs is meaningful to the extent that we are willing to defend the proposition that "all omitted attributes" are equally important for different kinds of jobs.

## Comparison of rologit and clogit

The rank-ordered logit model also has a sequential interpretation. A subject first chooses the best among the alternatives. Next he or she selects the best alternative among the remaining alternatives, etc. The decisions at each of the subsequent stages are described by a conditional logit model, and a subject is assumed to apply the same decision weights at each stage. Some authors have expressed concern that later choices may well be made more randomly than the first few decisions. A formalization of this idea is a heteroskedastic version of the rank-ordered logit model in which the scale of the random term increases with the number of decisions made (for example, Hausman and Ruud [1987]). This extended model is currently not supported by rologit. However, the hypothesis that the same decision weights are applied at the first stage and at later stages can be tested by applying a Hausman test.

First, we fit the rank-ordered logit model on the full ranking data for the first type of job,

| Rank-ordered logistic regression | Number of obs |  | 80 |
| :---: | :---: | :---: | :---: |
| Group variable: caseid | Number of groups |  | 8 |
| No ties in data | Obs per group: min |  | 10 |
|  | avg |  | 10.00 |
|  | max |  | 10 |
|  | LR chi2(6) |  | 54.13 |
| Log likelihood = -68.34539 | Prob > chi2 | = | 0.0000 |


| pref | Coef. | Std. Err. | z | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | ---: | ---: | ---: |
| age | -.0984926 | .0820473 | -1.20 | 0.230 | -.2593024 | .0623172 |
| female | -.4487287 | .3671307 | -1.22 | 0.222 | -1.168292 | .2708343 |
| edufit | .7658064 | .3602366 | 2.13 | 0.034 | .0597556 | 1.471857 |
| grades | 3.064534 | .6148245 | 4.98 | 0.000 | 1.8595 | 4.269568 |
| workexp | 1.386427 | .292553 | 4.74 | 0.000 | .8130341 | 1.959821 |
| boardexp | .6944377 | .3762596 | 1.85 | 0.065 | -.0430176 | 1.431893 |

and we save the estimates for later use with the estimates command.
. estimates store Ranking
To estimate the decision weights on the basis of the most preferred alternatives only, we create a variable, best, that is 1 for the best alternatives, and 0 otherwise. The by prefix is useful here.

```
. by caseid (pref), sort: gen best = pref == pref[_N] if job==1
(210 missing values generated)
```

By specifying (pref) with by caseid, we ensured that the data were sorted in increasing order on pref within caseid. Hence, the most preferred alternatives are last in the sort order. The expression pref $==\operatorname{pref}\left[\_\mathrm{N}\right]$ is true (1) for the most preferred alternatives, even if the alternative is not unique, and false (0) otherwise. If the most preferred alternatives were sometimes tied, we could still fit the model for the based-alternatives-only data via rologit, but clogit would yield different results because it deals with ties in a less appropriate way for continuous valuations. To ascertain whether there are ties in the selected data regarding applicants for research positions, we can combine by with assert:

```
. by caseid (pref), sort: assert pref[_N-1] != pref[_N] if job==1
```

There are no ties. We can now fit the model on the choice data by using either clogit or rologit.

| . rologit best age edufit grades workexp boardexp if job==1, group (caseid) nolog |  |  |
| :--- | :--- | :--- |
| Rank-ordered logistic regression | Number of obs | $=$ |
| Group variable: caseid | Number of groups | $=$ |
| No ties in data | Obs per group: $\min =$ | 80 |
|  |  | avg |


| best | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| age | -.1048959 | .2017068 | -0.52 | 0.603 | -.5002339 | .2904421 |
| edufit | .4558387 | .9336775 | 0.49 | 0.625 | -1.374136 | 2.285813 |
| grades | 3.443851 | 1.969002 | 1.75 | 0.080 | -.4153223 | 7.303025 |
| workexp | 2.545648 | 1.099513 | 2.32 | 0.021 | .3906422 | 4.700655 |
| boardexp | 1.765176 | 1.112763 | 1.59 | 0.113 | -.4157988 | 3.946152 |

```
. estimates store Choice
```

The same results, though with a slightly different formatted header, would have been obtained by using clogit on these data.


The parameters of the ranking and choice models look different, but the standard errors based on the choice data are much larger. Are we estimating parameters with the ranking data that are different from those with the choice data? A Hausman test compares two estimators of a parameter. One of the estimators should be efficient under the null hypothesis, namely, that choosing the second-best alternative is determined with the same decision weights as the best, etc. In our case, the efficient estimator of the decision weights uses the ranking information. The other estimator should be consistent, even if the null hypothesis is false. In our application, this is the estimator that uses the first-choice data only.


We do not find evidence for misspecification. We have to be cautious, though, because Hausmantype tests are often not powerful, and the number of observations in our example is very small, which makes the quality of the method of the null distribution by a chi-squared test rather uncertain.

## On reversals of rankings

The rank-ordered logit model has a property that you may find unexpected and even unfortunate. Compare two analyses with the rank-ordered logit model, one in which alternatives are ranked from "most attractive" to "least attractive", the other a reversed analysis in which these alternatives are ranked from "most unattractive" to "least unattractive". By unattractiveness, you probably mean just the opposite of attractiveness, and you expect that the weights of the attributes in predicting "attractiveness" to be minus the weights in predicting "unattractiveness". This is, however, not true for the rank-ordered logit model. The assumed distribution of the random residual takes the form $F(\epsilon)=1-\exp \{\exp (-\epsilon)\}$. This distribution is right-skewed. Therefore, slightly different models result from adding and subtracting the random residual, corresponding with high-to-low and low-to-high rankings. Thus the estimated coefficients will differ between the two specifications, though usually not in an important way. You may observe the difference by specifying the reverse option of rologit. Reversing the rank order makes rankings that are incomplete at the bottom become incomplete at the top. Only the first kind of incompleteness is supported by rologit. Thus, for this comparison, we exclude the alternatives that are not ranked, omitting the information that ranked alternatives are preferred over excluded ones.

```
. rologit pref grades edufit workexp boardexp if job==1 & pref!=0, group(caseid)
(output omitted)
. estimates store Original
. rologit pref grades edufit workexp boardexp if job==1 & pref!=0, group(caseid)
> reverse
(output omitted)
estimates store Reversed
```

. estimates table Original Reversed, stats(aic bic)

| Variable | Original | Reversed |
| ---: | ---: | :--- |
| grades | 2.0032332 | -1.0955335 |
| edufit | -.13111006 | -.05710681 |
| workexp | 1.2805373 | -1.2096383 |
| boardexp | .46213212 | -.27200317 |
| aic | 96.750452 | 99.665642 |
| bic | 104.23526 | 107.15045 |

Thus, although the weights of the attributes for reversed rankings are indeed mostly of opposite signs, the magnitudes of the weights and their standard errors differ. Which one is more appropriate? We have no advice to offer here. The specific science of the problem will determine what is appropriate, though we would be surprised indeed if this helps here. Formal testing does not help much either, as the models for the original and reversed rankings are not nested. The model-selection indices, such as the AIC and BIC, however, suggest that you stick to the rank-ordered logit model applied to the original ranking rather than to the reversed ranking.

## Stored results

rologit stores the following in e():

Scalars
e(N)
e(ll_0)
e(ll)
e(df_m)
e(chi2)
e(p)
e(r2_p)
e(N_g)
e(g_min)
e(g_avg)
e(g_max)
e(code_inc)
e(N_clust)
e(rank)
Macros
e(cmd)
e(cmdline)
e(depvar)
e(group)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(offset)
e(chi2type)
e(reverse)
e(ties)
e(vce)
e(vcetype)
e(properties)
e(predict)
e(marginsok)
e(marginsnotok)
e(asbalanced)
e(asobserved)
Matrices
e(b)
e(V)
e(V_modelbased)
Functions
e(sample) marks estimation sample

## Methods and formulas

Allison and Christakis (1994) demonstrate that maximum likelihood estimates for the rank-ordered logit model can be obtained as the maximum partial-likelihood estimates of an appropriately specified Cox regression model for waiting time ([ST] stcox). In this analogy, a higher value for an alternative is formally equivalent to a higher hazard rate of failure. rologit uses stcox to fit the rank-ordered logit model based on such a specification of the data in Cox terms. A higher stated preference is represented by a shorter waiting time until failure. Incomplete rankings are dealt with via censoring. Moreover, decision situations (subjects) are to be treated as strata. Finally, as proposed by Allison and Christakis, ties in rankings are handled by the marginal-likelihood method, specifying that all strict preference orderings consistent with the stated weak preference ordering are equally likely.

The marginal-likelihood estimator is available in stcox via the exactm option. The methods of the marginal likelihood due to Breslow and Efron are also appropriate for the analysis of rank-ordered logit models. Because in most applications the number of ranked alternatives by one subject will be fairly small (at most, say, 20), the number of ties is small as well, and so you rarely will need to turn to methods to restrict computer time. Because the marginal-likelihood estimator in stcox does not support the cluster adjustment or pweights, you should use the Efron method in such cases.

This command supports the clustered version of the Huber/White/sandwich estimator of the variance using vce (robust) and vce (cluster clustvar). See [ P ] _robust, particularly Maximum likelihood estimators and Methods and formulas. Specifying vce (robust) is equivalent to specifying vce(cluster groupvar), where groupvar is the identifier variable that links the alternatives.

## Acknowledgment

The rologit command was written by Jeroen Weesie of the Department of Sociology at Utrecht University, The Netherlands.

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## Also see

[R] rologit postestimation - Postestimation tools for rologit
$[\mathrm{R}]$ clogit - Conditional (fixed-effects) logistic regression
[R] logistic - Logistic regression, reporting odds ratios
[R] mlogit - Multinomial (polytomous) logistic regression
$[R]$ nlogit — Nested logit regression
[R] slogit - Stereotype logistic regression
[U] 20 Estimation and postestimation commands

## Title

rologit postestimation - Postestimation tools for rologit

Description Syntax for predict Menu for predict Options for predict
Remarks and examples

## Description

The following postestimation commands are available after rologit:
Command Description

| contrast | contrasts and ANOVA-style joint tests of estimates |
| :--- | :--- |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample |
| estat vce | variance-covariance matrix of the estimators (VCE) |
| estimates | cataloging estimation results |
| hausman | Hausman's specification test <br> point estimates, standard errors, testing, and inference for linear combinations <br> of coefficients |
| lincom | link test for model specification |
| linktest | likelihood-ratio test <br> marginal means, predictive margins, marginal effects, and average marginal <br> lrtest |
| margins ${ }^{1}$ | graph the results from margins (profile plots, interaction plots, etc.) <br> point estimates, standard errors, testing, and inference for nonlinear combinations <br> of coefficients |
| marginsplot | predictions, residuals, influence statistics, and other diagnostic measures |
| nlcom | point estimates, standard errors, testing, and inference for generalized predictions <br> pairwise comparisons of estimates |
| predict | Wald tests of simple and composite linear hypotheses <br> predictnl <br> pwcompare <br> test |
| testnl | Wald tests of nonlinear hypotheses |

1 The default prediction statistic pr cannot be correctly handled by margins; however, margins can be used after
rologit with the predict (xb) option.

## Syntax for predict

predict [type] newvar [if] [in] [, statistic nooffset]
statistic
Description
Main

| pr | probability that alternatives are ranked first; the default <br> xb |
| :--- | :--- |
| linear prediction |  |

These statistics are available both in and out of sample; type predict ... if esample() ... if wanted only for the estimation sample.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
pr , the default, calculates the probability that alternatives are ranked first.
xb calculates the linear prediction.
stdp calculates the standard error of the linear prediction.
nooffset is relevant only if you specified offset (varname) for rologit. It modifies the calculations made by predict so that they ignore the offset variable; the linear prediction is treated as $\mathbf{x}_{j} \mathbf{b}$ rather than as $\mathbf{x}_{j} \mathbf{b}+$ offset $_{j}$.

## Remarks and examples

See Comparing respondents and Clustered choice data in [R] rologit for examples of the use of testparm, an alternative to the test command.

See Comparison of rologit and clogit and On reversals of rankings in [R] rologit for examples of the use of estimates.

See Comparison of rologit and clogit in [R] rologit for an example of the use of hausman.

## Also see

[R] rologit - Rank-ordered logistic regression
[U] 20 Estimation and postestimation commands

## Title

rreg - Robust regression

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | Acknowledgment |
| References | Also see |  |  |

## Syntax

$$
\text { rreg depvar [indepvars }][\text { if }][\text { in }][\text {, options }]
$$

options
Model
tune (\#)
Reporting
level (\#)
genwt (newvar)
display_options

Optimization
optimization_options
graph
coeflegend
use \# as the biweight tuning constant; default is tune(7)
set confidence level; default is level (95)
create newvar containing the weights assigned to each observation control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
depvar and indepvars may contain time-series operators; see [U] 11.4.4 Time-series varlists.
by, mfp, mi estimate, rolling, and statsby are allowed; see [U] 11.1.10 Prefix commands.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Linear models and related $>$ Other $>$ Robust regression

## Description

rreg performs one version of robust regression of depvar on indepvars.
Also see Robust standard errors in [R] regress for standard regression with robust variance estimates and $[R]$ qreg for quantile (including median or least-absolute-residual) regression.

## Options

Model
tune(\#) is the biweight tuning constant. The default is 7, meaning seven times the median absolute deviation (MAD) from the median residual; see Methods and formulas. Lower tuning constants downweight outliers rapidly but may lead to unstable estimates (less than 6 is not recommended). Higher tuning constants produce milder downweighting.
$\qquad$ Reporting
level(\#); see [R] estimation options.
genwt (newvar) creates the new variable newvar containing the weights assigned to each observation.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% fmt), pformat (\% fmt), sformat (\% fmt), and nolstretch; see [R] estimation options.

## Optimization

optimization_options: iterate(\#), tolerance(\#), [no] log. iterate() specifies the maximum number of iterations; iterations stop when the maximum change in weights drops below tolerance(); and log/nolog specifies whether to show the iteration log. These options are seldom used.
graph allows you to graphically watch the convergence of the iterative technique. The weights obtained from the most recent round of estimation are graphed against the weights obtained from the previous round.

The following option is available with rreg but is not shown in the dialog box:
coeflegend; see [R] estimation options.

## Remarks and examples

rreg first performs an initial screening based on Cook's distance $>1$ to eliminate gross outliers before calculating starting values and then performs Huber iterations followed by biweight iterations, as suggested by Li (1985).
$>$ Example 1
We wish to examine the relationship between mileage rating, weight, and location of manufacture for the 74 cars in our automobile data. As a point of comparison, we begin by fitting an ordinary regression:

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. regress mpg weight foreign
```

| Source | SS | df | MS |
| ---: | ---: | ---: | ---: |
| Model | 1619.2877 | 2 | 809.643849 |
| Residual | 824.171761 | 71 | 11.608053 |
| Total | 2443.45946 | 73 | 33.4720474 |

Number of obs $=\quad 74$ $F(2,71)=69.75$ Prob > F $=0.0000$ R -squared $=0.6627$ Adj R-squared $=0.6532$ Root MSE $=3.4071$

| mpg | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| weight | -.0065879 | .0006371 | -10.34 | 0.000 | -.0078583 | -.0053175 |
| foreign | -1.650029 | 1.075994 | -1.53 | 0.130 | -3.7955 | .4954422 |
| _cons | 41.6797 | 2.165547 | 19.25 | 0.000 | 37.36172 | 45.99768 |

We now compare this with the results from rreg:
. rreg mpg weight foreign
Huber iteration 1: maximum difference in weights $=.80280176$
Huber iteration 2: maximum difference in weights $=.2915438$
Huber iteration 3: maximum difference in weights $=.08911171$
Huber iteration 4: maximum difference in weights $=.02697328$
Biweight iteration 5: maximum difference in weights $=.29186818$
Biweight iteration 6: maximum difference in weights $=.11988101$
Biweight iteration 7: maximum difference in weights $=.03315872$
Biweight iteration 8: maximum difference in weights $=.00721325$

Robust regression $\quad$ Number of obs $=\quad 74$
F ( 2, 71) $=168.32$
Prob $>\mathrm{F}=0.0000$

| mpg | Coef. | Std. Err. | t | P>\|t| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| weight | -.0063976 | .0003718 | -17.21 | 0.000 | -.007139 | -.0056562 |
| foreign | -3.182639 | .627964 | -5.07 | 0.000 | -4.434763 | -1.930514 |
| _cons | 40.64022 | 1.263841 | 32.16 | 0.000 | 38.1202 | 43.16025 |

Note the large change in the foreign coefficient.

## Technical note

It would have been better if we had fit the previous robust regression by typing rreg mpg weight foreign, genwt(w). The new variable, w, would then contain the estimated weights. Let's pretend that we did this:
. rreg mpg weight foreign, genwt(w)
(output omitted)
. summarize w, detail

Robust Regression Weight

|  | Percentiles | Smallest |  |  |
| ---: | ---: | ---: | :--- | ---: |
| $1 \%$ | 0 | 0 |  | 74 |
| $5 \%$ | .0442957 | 0 | 0 | Obs |
| $10 \%$ | .4674935 |  | Sum of Wgt. | 74 |
| $25 \%$ | .8894815 | .0442957 |  | Mean |
| $50 \%$ | .9690193 |  | Largest | Std. Dev. |
|  |  | .9996715 |  | .8509966 |
| $75 \%$ | .9949395 | .9996953 | Variance | .07542951 |
| $90 \%$ | .9989245 | .9997343 | Skewness | -2.287952 |
| $95 \%$ | .9996715 | .9998585 | Kurtosis | 6.874605 |

We discover that 3 observations in our data were dropped altogether (they have weight 0 ). We could further explore our data:

```
. sort w
. list make mpg weight w if w <.467, sep(0)
```

| make | mpg | weight | w |
| :--- | ---: | ---: | ---: |
| VW Diesel | 41 | 2,040 | 0 |
| Subaru | 35 | 2,050 | 0 |
| Datsun 210 | 35 | 2,020 | 0 |
| Plym. Arrow | 28 | 3,260 | .04429567 |
| Cad. Seville | 21 | 4,290 | .08241943 |
| Toyota Corolla | 31 | 2,200 | .10443129 |
| Olds 98 | 21 | 4,060 | .28141296 |

Being familiar with the automobile data, we immediately spotted two things: the VW is the only diesel car in our data, and the weight recorded for the Plymouth Arrow is incorrect.

## > Example 2

If we specify no explanatory variables, rreg produces a robust estimate of the mean:

```
. rreg mpg
    Huber iteration 1: maximum difference in weights = . }6447187
    Huber iteration 2: maximum difference in weights = . }0509833
    Huber iteration 3: maximum difference in weights = . 0099887
Biweight iteration 4: maximum difference in weights = . 25197391
Biweight iteration 5: maximum difference in weights = . 00358606
```

Robust regression $\quad \begin{aligned} & \text { Number of obs }= \\ & 74\end{aligned}$
$F(0,73)=0.00$
Prob > F =

| mpg | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| _cons | 20.68825 | .641813 | 32.23 | 0.000 | 19.40912 | 21.96738 |

The estimate is given by the coefficient on _cons. The mean is 20.69 with an estimated standard error of 0.6418 . The $95 \%$ confidence interval is [19.4,22.0]. By comparison, ci (see [R] ci) gives us the standard calculation:
. ci mpg

| Variable | Obs | Mean | Std. Err. | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: |
| mpg | 74 | 21.2973 | .6725511 | 19.9569 | 22.63769 |

## Stored results

rreg stores the following in e() :
Scalars
e(N)
$e$ (mss)
e(df_m)
e(rss)
e(df_r)
$e(r 2)$
e(r2_a)
e(F)
e(rmse)
$e$ (rank)
Macros
e(cmd)
e(cmdline)
e(depvar)
$e$ (genwt)
e(title)
e(model)
e(vce)
e(properties)
e(predict)
e(marginsok)
e(asbalanced)
e(asobserved)
Matrices
e(b)
$e(V) \quad$ variance-covariance matrix of the estimators
Functions
e(sample) marks estimation sample

## Methods and formulas

See Berk (1990), Goodall (1983), and Rousseeuw and Leroy (1987) for a general description of the issues and methods. Hamilton (1991a, 1992) provides a more detailed description of rreg and some Monte Carlo evaluations.
rreg begins by fitting the regression (see [R] regress), calculating Cook's $D$ (see [R] predict and $[\mathrm{R}]$ regress postestimation), and excluding any observation for which $D>1$.

Thereafter rreg works iteratively: it performs a regression, calculates case weights from absolute residuals, and regresses again using those weights. Iterations stop when the maximum change in weights drops below tolerance(). Weights derive from one of two weight functions, Huber weights
and biweights. Huber weights (Huber 1964) are used until convergence, and then, from that result, biweights are used until convergence. The biweight was proposed by Beaton and Tukey (1974, 151152) after the Princeton robustness study (Andrews et al. 1972) had compared various estimators. Both weighting functions are used because Huber weights have problems dealing with severe outliers, whereas biweights sometimes fail to converge or have multiple solutions. The initial Huber weighting should improve the behavior of the biweight estimator.

In Huber weighting, cases with small residuals receive weights of 1 ; cases with larger residuals receive gradually smaller weights. Let $e_{i}=y_{i}-\mathbf{X}_{i} \mathbf{b}$ represent the $i$ th-case residual. The $i$ th scaled residual $u_{i}=e_{i} / s$ is calculated, where $s=M / 0.6745$ is the residual scale estimate and $M=\operatorname{med}\left(\left|e_{i}-\operatorname{med}\left(e_{i}\right)\right|\right)$ is the median absolute deviation from the median residual. Huber estimation obtains case weights:

$$
w_{i}= \begin{cases}1 & \text { if }\left|u_{i}\right| \leq c_{h} \\ c_{h} /\left|u_{i}\right| & \text { otherwise }\end{cases}
$$

rreg defines $c_{h}=1.345$, so downweighting begins with cases whose absolute residual exceeds (1.345/0.6745) $M \approx 2 M$.

With biweights, all cases with nonzero residuals receive some downweighting, according to the smoothly decreasing biweight function

$$
w_{i}= \begin{cases}\left\{1-\left(u_{i} / c_{b}\right)^{2}\right\}^{2} & \text { if }\left|u_{i}\right| \leq c_{b} \\ 0 & \text { otherwise }\end{cases}
$$

where $c_{b}=4.685 \times$ tune () $/ 7$. Thus when tune ( $)=7$, cases with absolute residuals of $(4.685 / 0.6745) M \approx 7 M$ or more are assigned 0 weight and thus are effectively dropped. Goodall (1983, 377) suggests using a value between 6 and 9, inclusive, for tune() in the biweight case and states that performance is good between 6 and 12, inclusive.

The tuning constants $c_{h}=1.345$ and $c_{b}=4.685$ (assuming tune() is set at the default 7) give rreg about $95 \%$ of the efficiency of OLS when applied to data with normally distributed errors (Hamilton 1991b). Lower tuning constants downweight outliers more drastically (but give up Gaussian efficiency); higher tuning constants make the estimator more like OLS.

Standard errors are calculated using the pseudovalues approach described in Street, Carroll, and Ruppert (1988).

## Acknowledgment

The current version of rreg is due to the work of Lawrence Hamilton of the Department of Sociology at the University of New Hampshire.

## References

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## Also see

[R] rreg postestimation - Postestimation tools for rreg
[R] qreg - Quantile regression
$[\mathrm{R}]$ regress - Linear regression
[MI] estimation - Estimation commands for use with mi estimate
[U] 20 Estimation and postestimation commands

## Title

rreg postestimation - Postestimation tools for rreg
Description Syntax for predict Menu for predict Options for predict Also see

## Description

The following postestimation commands are available after rreg:
Command Description

| contrast | contrasts and ANOVA-style joint tests of estimates |
| :--- | :--- |
| estat summarize |  |
| estat vce | summary statistics for the estimation sample <br> variance-covariance matrix of the estimators (VCE) <br> estimates $^{\text {cataloging estimation results }}$ <br> forecast |
| lincom | dynamic forecasts and simulations <br> point estimates, standard errors, testing, and inference for linear combinations <br> of coefficients |
| margins | marginal means, predictive margins, marginal effects, and average marginal <br> effects |
| marginsplot | graph the results from margins (profile plots, interaction plots, etc.) <br> point estimates, standard errors, testing, and inference for nonlinear combinations <br> of coefficients |
| nlcom | predictions, residuals, influence statistics, and other diagnostic measures <br> point estimates, standard errors, testing, and inference for generalized predictions |
| predict | pairwise comparisons of estimates |
| predictnl | Wald tests of simple and composite linear hypotheses |
| pwampare | Wald tests of nonlinear hypotheses |

1 forecast is not appropriate with mi estimation results.

## Syntax for predict

predict [type] newvar $[$ if $][$ in $][$, statistic $]$
statistic Description
Main
xb
stdp
residuals
hat
linear prediction; the default standard error of the linear prediction residuals
diagonal elements of the hat matrix
These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

Main
xb , the default, calculates the linear prediction.
stdp calculates the standard error of the linear prediction.
residuals calculates the residuals.
hat calculates the diagonal elements of the hat matrix. You must have run the rreg command with the genwt () option.

## Also see

[R] rreg - Robust regression
[U] 20 Estimation and postestimation commands

## Title

runtest - Test for random order

Syntax
Remarks and examples References

Menu
Stored results

Description
Methods and formulas

Options
Acknowledgment

## Syntax

runtest varname [in] [, options]
options Description

| continuity | continuity correction <br> ignore values equal to the threshold <br> drop |
| :--- | :--- |
| split | threshold; default is to count as below |
| mean | use mean as threshold; default is median <br> assign arbitrary threshold; default is median |
| threshold(\#) |  |

## Menu

Statistics $>$ Nonparametric analysis $>$ Tests of hypotheses $>$ Test for random order

## Description

runtest tests whether the observations of varname are serially independent-that is, whether they occur in a random order - by counting how many runs there are above and below a threshold. By default, the median is used as the threshold. A small number of runs indicates positive serial correlation; a large number indicates negative serial correlation.

## Options

continuity specifies a continuity correction that may be helpful in small samples. If there are fewer than 10 observations either above or below the threshold, however, the tables in Swed and Eisenhart (1943) provide more reliable critical values. By default, no continuity correction is used.
drop directs runtest to ignore any values of varname that are equal to the threshold value when counting runs and tabulating observations. By default, runtest counts a value as being above the threshold when it is strictly above the threshold and as being below the threshold when it is less than or equal to the threshold.
split directs runtest to randomly split values of varname that are equal to the threshold. In other words, when varname is equal to threshold, a "coin" is flipped. If it comes up heads, the value is counted as above the threshold. If it comes up tails, the value is counted as below the threshold. mean directs runtest to tabulate runs above and below the mean rather than the median.
threshold(\#) specifies an arbitrary threshold to use in counting runs. For example, if varname has already been coded as a $0 / 1$ variable, the median generally will not be a meaningful separating value.

## Remarks and examples

runtest performs a nonparametric test of the hypothesis that the observations of varname occur in a random order by counting how many runs there are above and below a threshold. If varname is positively serially correlated, it will tend to remain above or below its median for several observations in a row; that is, there will be relatively few runs. If, on the other hand, varname is negatively serially correlated, observations above the median will tend to be followed by observations below the median and vice versa; that is, there will be relatively many runs.

By default, runtest uses the median for the threshold, and this is not necessarily the best choice. If mean is specified, the mean is used instead of the median. If threshold(\#) is specified, \# is used. Because runtest divides the data into two states-above and below the threshold-it is appropriate for data that are already binary; for example, win or lose, live or die, rich or poor, etc. Such variables are often coded as 0 for one state and 1 for the other. Here you should specify threshold (0) because, by default, runtest separates the observations into those that are greater than the threshold and those that are less than or equal to the threshold.

As with most nonparametric procedures, the treatment of ties complicates the test. Observations equal to the threshold value are ties and can be treated in one of three ways. By default, they are treated as if they were below the threshold. If drop is specified, they are omitted from the calculation and the total number of observations is adjusted. If split is specified, each is randomly assigned to the above- and below-threshold groups. The random assignment is different each time the procedure is run unless you specify the random-number seed; see [R] set seed.

## > Example 1

We can use runtest to check regression residuals for serial correlation.

```
. use http://www.stata-press.com/data/r13/run1
. scatter resid year, connect(1) yline(0) title(Regression residuals)
```



The graph gives the impression that these residuals are positively correlated. Excursions above or below zero-the natural threshold for regression residuals - tend to last for several observations. runtest can evaluate the statistical significance of this impression.

```
. runtest resid, thresh(0)
N(resid <= 0) = 8
N(resid > 0) = 8
            obs = 16
        N(runs) = 5
            z = -2.07
    Prob>|z| = .04
```

There are five runs in these 16 observations. Using the normal approximation to the true distribution of the number of runs, the five runs in this series are fewer than would be expected if the residuals were serially independent. The $p$-value is 0.04 , indicating a two-sided significant result at the $5 \%$ level. If the alternative hypothesis is positive serial correlation, rather than any deviation from randomness, then the one-sided $p$-value is $0.04 / 2=0.015$. With so few observations, however, the normal approximation may be inaccurate. (Tables compiled by Swed and Eisenhart list five runs as the $5 \%$ critical value for a one-sided test.)
runtest is a nonparametric test. It ignores the magnitudes of the observations and notes only whether the values are above or below the threshold. We can demonstrate this feature by reducing the information about the regression residuals in this example to a $0 / 1$ variable that indicates only whether a residual is positive or negative.

```
. generate byte sign = resid>0
. runtest sign, thresh(0)
N(sign <= 0) = 8
N(sign > 0) = 8
            obs = 16
    N(runs) = 5
            z = -2.07
    Prob>|z| = . 04
```

As expected, runtest produces the same answer as before.

## - Technical note

The run test can also be used to test the null hypothesis that two samples are drawn from the same underlying distribution. The run test is sensitive to differences in the shapes, as well as the locations, of the empirical distributions.

Suppose, for example, that two different additives are added to the oil in 10 different cars during an oil change. The cars are run until a viscosity test determines that another oil change is needed, and the number of miles traveled between oil changes is recorded. The data are

```
. use http://www.stata-press.com/data/r13/additive, clear
```

. list

|  | additive | miles |
| ---: | ---: | ---: |
| 1. | 1 | 4024 |
| 2. | 1 | 4756 |
| 3. | 1 | 7993 |
| 4. | 1 | 5025 |
| 5. | 1 | 4188 |
| 6. | 2 | 3007 |
| 7. | 2 | 1988 |
| 8. | 2 | 1051 |
| 9. | 2 | 4478 |
| 10. | 2 | 4232 |

To test whether the additives generate different distributions of miles between oil changes, we sort the data by miles and then use runtest to see whether the marker for each additive occurs in random order:

```
sort miles
. runtest additive, thresh(1)
N(additive <= 1) = 5
N(additive > 1) = 5
            obs = 10
        N(runs) = 4
            z = -1.34
    Prob>|z| = . 18
```

Here the additives do not produce statistically different results.

## Technical note

A test that is related to the run test is the runs up-and-down test. In the latter test, the data are classified not by whether they lie above or below a threshold but by whether they are steadily increasing or decreasing. Thus an unbroken string of increases in the variable of interest is counted as one run, as is an unbroken string of decreases. According to Madansky (1988), the run test is superior to the runs up-and-down test for detecting trends in the data, but the runs up-and-down test is superior for detecting autocorrelation.
runtest can be used to perform a runs up-and-down test. Using the regression residuals from the example above, we can perform a runtest on their first differences:

```
. use http://www.stata-press.com/data/r13/run1
. generate resid_D = resid - resid[_n-1]
(1 missing value generated)
. runtest resid_D, thresh(0)
N(resid_D <= 0) = 7
N(resid_D > 0) = 8
            obs = 15
        N(runs) = 6
            z = -1.33
    Prob>|z| = . 18
```

Edgington (1961) has compiled a table of the small sample distribution of the runs up-and-down statistic, and this table is reprinted in Madansky (1988). For large samples, the $z$ statistic reported by runtest is incorrect for the runs up-and-down test. Let $N$ be the number of observations ( 15 here), and let $r$ be the number of runs (6). The expected number of runs in the runs up-and-down test is

$$
\mu_{r}=\frac{2 N-1}{3}
$$

the variance is

$$
\sigma_{r}^{2}=\frac{16 N-29}{90}
$$

and the correct $z$ statistic is

$$
\widehat{z}=\frac{r-\mu_{r}}{\sigma_{r}}
$$

## - Technical note

runtest will tolerate missing values at the beginning or end of a series, as occurred in the technical note above (generating first differences resulted in a missing value for the first observation). runtest, however, will issue an error message if there are any missing observations in the interior of the series (in the portion covered by the in range modifier). To perform the test anyway, simply drop the missing observations before using runtest.

## Stored results

runtest stores the following in $r()$ :
Scalars
$\mathrm{r}(\mathrm{N}) \quad$ number of observations $\quad \mathrm{r}(\mathrm{p}) \quad p$-value of $z$
r(N_below) number below the threshold
$r$ (N_above) number above the threshold
$r$ (mean)
expected number of runs

```
r(z) z statistic
r(n_runs) number of runs
r(Var) variance of the number of runs
```


## Methods and formulas

runtest begins by calculating the number of observations below the threshold, $n_{0}$; the number of observations above the threshold, $n_{1}$; the total number of observations, $N=n_{0}+n_{1}$; and the number of runs, $r$. These statistics are always reported, so the exact tables of critical values in Swed and Eisenhart (1943) may be consulted if necessary.

The expected number of runs under the null is

$$
\mu_{r}=\frac{2 n_{0} n_{1}}{N}+1
$$

the variance is

$$
\sigma_{r}^{2}=\frac{2 n_{0} n_{1}\left(2 n_{0} n_{1}-N\right)}{N^{2}(N-1)}
$$

and the normal approximation test statistic is

$$
\widehat{z}=\frac{r-\mu_{r}}{\sigma_{r}}
$$

## Acknowledgment

runtest was written by Sean Becketti, a past editor of the Stata Technical Bulletin and author of the Stata Press book Introduction to Time Series Using Stata.

## References

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## Title

scobit - Skewed logistic regression

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |

## Syntax

$$
\text { scobit depvar }[\text { indepvars }][\text { if }][\text { in }][\text { weight }][\text {, options }]
$$

## options

## Description

Model
noconstant
offset (varname)
asis
constraints (constraints)
collinear
SE/Robust
vce (vcetype)

Reporting
level(\#)
or
nocnsreport
display_options

Maximization
maximize_options
coeflegend
suppress constant term
include varname in model with coefficient constrained to 1
retain perfect predictor variables
apply specified linear constraints
keep collinear variables
vcetype may be oim, robust, cluster clustvar, opg, bootstrap, or jackknife
indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
bootstrap, by, fp, jackknife, nestreg, rolling, statsby, stepwise, and svy are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the bootstrap prefix; see $[R]$ bootstrap.
vce() and weights are not allowed with the svy prefix; see [SVY] svy.
fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Menu

Statistics $>$ Binary outcomes $>$ Skewed logit regression

## Description

scobit fits a maximum-likelihood skewed logit model.
See [R] logistic for a list of related estimation commands.

## Options

$\qquad$ Model
noconstant, offset(varname), constraints(constraints), collinear; see [R] estimation options.
asis forces retention of perfect predictor variables and their associated perfectly predicted observations and may produce instabilities in maximization; see [R] probit.

## SE/Robust

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

Reporting
level(\#); see [R] estimation options.
or reports the estimated coefficients transformed to odds ratios, that is, $e^{b}$ rather than $b$. Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated. or may be specified at estimation or when replaying previously estimated results.
nocnsreport; see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\%fmt), pformat(\%fmt), sformat(\%fmt), and nolstretch; see [R] estimation options.

Maximization
maximize_options: difficult, technique (algorithm_spec), iterate(\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance(\#), nonrtolerance, and from (init_specs); see [R] maximize. These options are seldom used.

Setting the optimization type to technique (bhhh) resets the default vcetype to vce (opg).
The following option is available with scobit but is not shown in the dialog box:
coeflegend; see [R] estimation options.

## Remarks and examples

Remarks are presented under the following headings:

## Skewed logistic model

scobit fits maximum likelihood models with dichotomous dependent variables coded as $0 / 1$ (or, more precisely, coded as 0 and not 0 ).

## > Example 1

We have data on the make, weight, and mileage rating of 22 foreign and 52 domestic automobiles. We wish to fit a model explaining whether a car is foreign based on its mileage. Here is an overview of our data:

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. keep make mpg weight foreign
. describe
Contains data from http://www.stata-press.com/data/r13/auto.dta
    obs: 74 1978 Automobile Data
    vars: 4 13 Apr 2013 17:45
    size: 1,702 (_dta has notes)
\begin{tabular}{lllll} 
variable name & \begin{tabular}{c} 
storage \\
type
\end{tabular} & \begin{tabular}{l} 
display \\
format
\end{tabular} & \begin{tabular}{l} 
value \\
label
\end{tabular} & variable label \\
\hline make & str18 & \(\%-18 \mathrm{~s}\) & & Make and Model \\
mpg & int & \(\% 8.0 \mathrm{~g}\) & & Mileage (mpg) \\
weight & int & \(\% 8.0 \mathrm{gc}\) & & Weight (lbs.) \\
foreign & byte & \(\% 8.0 \mathrm{~g}\) & origin & Car type \\
\hline
\end{tabular}
Sorted by: foreign
        Note: dataset has changed since last saved
. inspect foreign
foreign: Car type
\begin{tabular}{lrcc} 
& \multicolumn{3}{c}{ Number of Observations } \\
\cline { 2 - 4 } & Total & Integers & Nonintegers \\
Negative & - & - & - \\
Zero & 52 & 52 & - \\
Positive & 22 & 22 & - \\
\cline { 2 - 3 } & 74 & 74 & - \\
Total & - & & \\
Missing & 74 & &
\end{tabular}
    (2 unique values)
        foreign is labeled and all values are documented in the label.
```

The variable foreign takes on two unique values, 0 and 1 . The value 0 denotes a domestic car, and 1 denotes a foreign car.

The model that we wish to fit is

$$
\operatorname{Pr}(\text { foreign }=1)=F\left(\beta_{0}+\beta_{1} \mathrm{mpg}\right)
$$

where $F(z)=1-1 /\{1+\exp (z)\}^{\alpha}$.
To fit this model, we type

| Fitting logistic model: |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Iteration 0: log likelihood $=-45.03321$ |  |  |  |  |  |  |
| Iteration 1: $\quad$ log likelihood $=-39.380959$ |  |  |  |  |  |  |
| Iteration 2: log likelihood $=-39.288802$ |  |  |  |  |  |  |
| Iteration 3: log likelihood $=-39.28864$ |  |  |  |  |  |  |
| Iteration 4: log likelihood $=-39.28864$ |  |  |  |  |  |  |
| Fitting full model: |  |  |  |  |  |  |
| Iteration 0: log likelihood $=-39.28864$ |  |  |  |  |  |  |
| Iteration 1: log likelihood $=-39.286393$ |  |  |  |  |  |  |
| Iteration 2: log likelihood $=-39.284415$ |  |  |  |  |  |  |
| Iteration 3: log likelihood $=-39.284234$ |  |  |  |  |  |  |
| Iteration 4: $\quad \log$ likelihood $=-39.284197$ |  |  |  |  |  |  |
| Iteration 5: $\quad \mathrm{log}$ likelihood $=-39.284196$ |  |  |  |  |  |  |
| Skewed logistic regression |  |  |  | Numbe | $f$ obs | 74 |
|  |  |  |  | Zero | comes | 52 |
| Log likelihood $=-39.2842$ |  |  |  | Nonze | outcomes | 22 |
| foreign | Coef. | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Con | Interval] |
| mpg | . 1813879 | . 2407362 | 0.75 | 0.451 | -. 2904463 | . 6532222 |
| _cons | -4.274883 | 1.399305 | -3.06 | 0.002 | -7.017471 | -1.532295 |
| /lnalpha | -. 4450405 | 3.879885 | -0.11 | 0.909 | -8.049476 | 7.159395 |
| alpha | . 6407983 | 2.486224 |  |  | . 0003193 | 1286.133 |
| Likelihood-ratio test of alpha=1: chi2 (1) = 0.01 Prob > chi2 = 0.9249 |  |  |  |  |  |  |
| Note: likelihood-ratio tests are recommended for inference with scobit models. |  |  |  |  |  |  |

We find that cars yielding better gas mileage are less likely to be foreign. The likelihood-ratio test at the bottom of the output indicates that the model is not significantly different from a logit model. Therefore, we should use the more parsimonious model.

## Technical note

Stata interprets a value of 0 as a negative outcome (failure) and treats all other values (except missing) as positive outcomes (successes). Thus if the dependent variable takes on the values 0 and 1 , then 0 is interpreted as failure and 1 as success. If the dependent variable takes on the values 0 , 1 , and 2, then 0 is still interpreted as failure, but both 1 and 2 are treated as successes.

Formally, when we type scobit $y x$, Stata fits the model

$$
\operatorname{Pr}\left(y_{j} \neq 0 \mid \mathbf{x}_{j}\right)=1-1 /\left\{1+\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}\right)\right\}^{\alpha}
$$

## Robust standard errors

If you specify the vce (robust) option, scobit reports robust standard errors as described in [U] 20.21 Obtaining robust variance estimates. For the model of foreign on mpg, the robust calculation increases the standard error of the coefficient on mpg by around $25 \%$ :
. scobit foreign mpg, vce (robust) nolog
Skewed logistic regression
Log pseudolikelihood $=-39.2842$

Without vce(robust), the standard error for the coefficient on mpg was reported to be 0.241 , with a resulting confidence interval of $[-0.29,0.65]$.

Specifying the vce (cluster clustvar) option relaxes the independence assumption required by the skewed logit estimator to being just independence between clusters. To demonstrate this, we will switch to a different dataset.

## Example 2

We are studying the unionization of women in the United States and have a dataset with 26,200 observations on 4,434 women between 1970 and 1988. For our purposes, we will use the variables age (the women were $14-26$ in 1968 and the data thus span the age range of 16-46), grade (years of schooling completed, ranging from 0 to 18), not_smsa ( $28 \%$ of the person-time was spent living outside an SMSA-standard metropolitan statistical area), south ( $41 \%$ of the person-time was in the South), and year. Each of these variables is included in the regression as a covariate along with the interaction between south and year. This interaction, along with the south and year variables, is specified in the scobit command using factor-variables notation, south\#\#c.year. We also have variable union. Overall, $22 \%$ of the person-time is marked as time under union membership and $44 \%$ of these women have belonged to a union.

We fit the following model, ignoring that women are observed an average of 5.9 times each in these data:


The reported standard errors in this model are probably meaningless. Women are observed repeatedly, so the observations are not independent. Looking at the coefficients, we find a large southern effect against unionization and a different time trend for the south. The vce (cluster clustvar) option provides a way to fit this model and obtains correct standard errors:

| . scobit union age grade not_smsa south\#\#c.year, vce(cluster id) nrtol(1e-3) |  |  |  |
| :--- | :--- | :--- | :--- |
| (output omitted) |  |  |  |
| Skewed logistic regression | Number of obs | $=$ | 26200 |
|  | Zero outcomes | $=$ | 20389 |
| Log pseudolikelihood $=-13540.61$ | Nonzero outcomes | $=$ | 5811 |

(Std. Err. adjusted for 4434 clusters in idcode)

| union | Coef. | Robust Std. Err. | z | $P>\|z\|$ | [95\% Conf. | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| age | . 0185365 | . 0084867 | 2.18 | 0.029 | . 0019029 | . 0351701 |
| grade | . 0452803 | . 0125764 | 3.60 | 0.000 | . 0206311 | . 0699296 |
| not_smsa | -. 1886849 | . 0642035 | -2.94 | 0.003 | -. 3145214 | -. 0628484 |
| 1. south | -1.422381 | . 5064916 | -2.81 | 0.005 | -2.415086 | -. 4296756 |
| year | -. 0133017 | . 0090621 | -1.47 | 0.142 | -. 0310632 | . 0044597 |
| south\#c.year <br> 1 | . 0105663 | . 0063172 | 1.67 | 0.094 | -. 0018152 | . 0229478 |
| _cons | -10.19247 | . 945772 | -10.78 | 0.000 | -12.04615 | -8.33879 |
| /lnalpha | 8.972796 | . 7482517 | 11.99 | 0.000 | 7.506249 | 10.43934 |
| alpha | 7885.616 | 5900.426 |  |  | 1819.377 | 34178.16 |

scobit, vce(cluster clustvar) is robust to assumptions about within-cluster correlation. That is, it inefficiently sums within cluster for the standard error calculation rather than attempting to exploit what might be assumed about the within-cluster correlation (as do the xtgee population-averaged models; see [XT] xtgee).

## - Technical note

The scobit model can be difficult to fit because of the functional form. Often it requires many iterations, or the optimizer prints out warning and informative messages during the optimization. For example, without the nrtol ( $1 \mathrm{e}-3$ ) option, the model using the union dataset will not converge. See $[R]$ maximize for details about the optimizer.

## - Technical note

The main reason for using scobit rather that logit is that the effects of the regressors on the probability of success are not constrained to be the largest when the probability is 0.5 . Rather, the independent variables might show their largest impact when the probability of success is 0.3 or 0.6 . This added flexibility results because the scobit function, unlike the logit function, can be skewed and is not constrained to be mirror symmetric about the 0.5 probability of success.

As Nagler (1994) pointed out, the point of maximum impact is constrained under the scobit model to fall within the interval $\left(0,1-e^{(-1)}\right)$ or approximately $(0,0.63)$. Achen (2002) notes that if we believe the maximum impact to be outside that range, we can instead estimate the "power logit" model by simply reversing the 0 s and 1 s of our outcome variable and estimating a scobit model on failure, rather than success. We would need to reverse the signs of the coefficients if we wanted to interpret them in terms of impact on success, or we could leave them as they are and interpret them in terms of impact on failure. The important thing to remember is that the scobit model, unlike the logit model, is not invariant to the choice of which result is assigned to success.

## Stored results

scobit stores the following in e():
Scalars
e(N)
e(k)
e(k_eq)
e(k_aux)
e(k_dv)
e(ll)
e(ll_c)
e(N_f)
e(N_s)
e(alpha)
e(N_clust)
e(chi2)
e(chi2_c)
e(p)
e(rank)
e(ic)
e(rc)
e(converged)
Macros
e(cmd)
e(cmdline)
e(depvar)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(offset)
e(chi2type)
e(chi2_ct)
e(vce)
e(vcetype)
e(opt)
e(which)
e(ml_method)
e(user)
e(technique)
e(properties)
e(predict)
e(footnote)
e(asbalanced)
e(asobserved)
Matrices
e(b)
e(Cns)
e(ilog)
e(gradient)
e(V)
e(V_modelbased)
Functions
e(sample)
number of observations
number of parameters
number of equations in e(b)
number of auxiliary parameters
number of dependent variables
log likelihood
log likelihood, comparison model
number of failures (zero outcomes)
number of successes (nonzero outcomes)
alpha
number of clusters
$\chi^{2}$
$\chi^{2}$ for comparison test
significance
rank of e(V)
number of iterations
return code
1 if converged, 0 otherwise
scobit
command as typed
name of dependent variable
weight type
weight expression
title in estimation output
name of cluster variable
linear offset variable
Wald or LR; type of model $\chi^{2}$ test
Wald or LR; type of model $\chi^{2}$ test corresponding to e(chi2_c)
vcetype specified in vce()
title used to label Std. Err.
type of optimization
max or min; whether optimizer is to perform maximization or minimization
type of ml method
name of likelihood-evaluator program
maximization technique
b V
program used to implement predict
program used to implement the footnote display
factor variables fvset as asbalanced
factor variables fvset as asobserved
coefficient vector
constraints matrix
iteration $\log$ (up to 20 iterations)
gradient vector
variance-covariance matrix of the estimators
model-based variance
marks estimation sample

## Methods and formulas

Skewed logit analysis is an alternative to logit that relaxes the assumption that individuals with initial probability of 0.5 are most sensitive to changes in independent variables.

The log-likelihood function for skewed logit is

$$
\ln L=\sum_{j \in S} w_{j} \ln F\left(\mathbf{x}_{j} \mathbf{b}\right)+\sum_{j \notin S} w_{j} \ln \left\{1-F\left(\mathbf{x}_{j} \mathbf{b}\right)\right\}
$$

where $S$ is the set of all observations $j$ such that $y_{j} \neq 0, F(z)=1-1 /\{1+\exp (z)\}^{\alpha}$, and $w_{j}$ denotes the optional weights. $\ln L$ is maximized as described in [ R$]$ maximize.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce (robust) and vce(cluster clustvar), respectively. See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas.
scobit also supports estimation with survey data. For details on VCEs with survey data, see [SVY] variance estimation.

## References

Achen, C. H. 2002. Toward a new political methodology: Microfoundations and ART. Annual Review of Political Science 5: 423-450.

Nagler, J. 1994. Scobit: An alternative estimator to logit and probit. American Journal of Political Science 38: 230-255.

## Also see

[R] scobit postestimation - Postestimation tools for scobit
[R] cloglog - Complementary log-log regression
[R] glm - Generalized linear models
$[\mathrm{R}]$ logistic - Logistic regression, reporting odds ratios
[SVY] svy estimation - Estimation commands for survey data
[U] 20 Estimation and postestimation commands

## Title

scobit postestimation - Postestimation tools for scobit

| Description | Syntax for predict |
| :--- | :--- |
| Remarks and examples | Also see for predict |$\quad$ Options for predict

## Description

The following postestimation commands are available after scobit:

| Command | Description |
| :--- | :--- |
| contrast | contrasts and ANOVA-style joint tests of estimates |
| estat ic | Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) |
| estat summarize | summary statistics for the estimation sample <br> variance-covariance matrix of the estimators (VCE) |
| estat vce | verat (svy) <br> postestimation statistics for survey data |
| estimates | cataloging estimation results <br> dynamic forecasts and simulations |
| forecast ${ }^{1}$ | point estimates, standard errors, testing, and inference for linear combinations <br> of coefficients |
| lincom | likelihood-ratio test |
| marginal means, predictive margins, marginal effects, and average marginal |  |
| margins | effects |
| marginsplot | graph the results from margins (profile plots, interaction plots, etc.) <br> point estimates, standard errors, testing, and inference for nonlinear combinations <br> of coefficients |
| nlcom | predictions, residuals, influence statistics, and other diagnostic measures <br> point estimates, standard errors, testing, and inference for generalized predictions <br> predict |
| predictnl | peemingly unrelated estimation |
| pwcompare | Wald tests of simple and composite linear hypotheses |
| suest | Wald tests of nonlinear hypotheses |
| test |  |

[^76]
## Syntax for predict

```
predict [type] newvar [if] [in] [, statistic nooffset]
predict [type] {stub*|newvarreg newvar rnalpha } [if][in], scores
```

statistic Description

Main
pr probability of a positive outcome; the default
$\overline{\mathrm{xb}} \quad \mathbf{x}_{j} \mathbf{b}$, linear prediction
stdp standard error of the linear prediction

These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

## Menu for predict

Statistics $>$ Postestimation $>$ Predictions, residuals, etc.

## Options for predict

$\qquad$ Main
pr , the default, calculates the probability of a positive outcome.
xb calculates the linear prediction.
stdp calculates the standard error of the linear prediction.
nooffset is relevant only if you specified offset (varname) for scobit. It modifies the calculations made by predict so that they ignore the offset variable; the linear prediction is treated as $\mathbf{x}_{j} \mathbf{b}$ rather than as $\mathbf{x}_{j} \mathbf{b}+$ offset $_{j}$.
scores calculates equation-level score variables.
The first new variable will contain $\partial \ln L / \partial\left(\mathbf{x}_{j} \boldsymbol{\beta}\right)$.
The second new variable will contain $\partial \ln L / \partial \ln \alpha$.

## Remarks and examples

Once you have fit a model, you can obtain the predicted probabilities by using the predict command for both the estimation sample and other samples; see [U] 20 Estimation and postestimation commands and $[R]$ predict. Here we will make only a few additional comments.
predict without arguments calculates the predicted probability of a positive outcome. With the xb option, it calculates the linear combination $\mathbf{x}_{j} \mathbf{b}$, where $\mathbf{x}_{j}$ are the independent variables in the $j$ th observation and $\mathbf{b}$ is the estimated parameter vector.

With the stdp option, predict calculates the standard error of the prediction, which is not adjusted for replicated covariate patterns in the data.
> Example 1
In example 1 of $[R]$ scobit, we fit the model scobit foreign mpg. To obtain predicted probabilities, we type

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. keep make mpg weight foreign
. scobit foreign mpg
    (output omitted)
. predict p
(option pr assumed; Pr(foreign))
. summarize foreign p
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline foreign & 74 & .2972973 & .4601885 & 0 & 1 \\
\(p\) & 74 & .2974049 & .182352 & .0714664 & .871624
\end{tabular}
```


## Also see

[R] scobit - Skewed logistic regression
[U] 20 Estimation and postestimation commands

## Title

sdtest - Variance-comparison tests

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Stored results | Methods and formulas | References |
| Also see |  |  |  |

## Syntax

One-sample variance-comparison test

$$
\text { sdtest varname }==\#[\text { if }][\text { in }][, \text { level }(\#)]
$$

Two-sample variance-comparison test using groups sdtest varname [if] [in], by (groupvar) [level(\#)]

Two-sample variance-comparison test using variables sdtest varname $_{1}=$ varname $_{2}[$ if $][$ in $][$, level $(\#)]$

Immediate form of one-sample variance-comparison test
sdtesti $\#_{\text {obs }}\left\{\#_{\text {mean }} \mid \cdot\right\} \#_{\text {sd }} \#_{\text {val }}[$, level (\#) $]$

Immediate form of two-sample variance-comparison test

```
sdtesti # #obs,1 { #mean,1 |.} # (sd,1 # #obs,2 { #mean,2}|.} ##sd,2 [, level(#)
```

Robust tests for equality of variances robvar varname [if] [in], by (groupvar)
by is allowed with sdtest and robvar; see [D] by.

## Menu

## sdtest

Statistics $>$ Summaries, tables, and tests $>$ Classical tests of hypotheses $>$ Variance-comparison test

## sdtesti

Statistics $>$ Summaries, tables, and tests $>$ Classical tests of hypotheses $>$ Variance-comparison test calculator robvar

Statistics $>$ Summaries, tables, and tests $>$ Classical tests of hypotheses $>$ Robust equal-variance test

## Description

sdtest performs tests on the equality of standard deviations (variances). In the first form, sdtest tests that the standard deviation of varname is \#. In the second form, sdtest performs the same test, using the standard deviations of the two groups defined by groupvar. In the third form, sdtest tests that varname $_{1}$ and varname ${ }_{2}$ have the same standard deviation.
sdtesti is the immediate form of sdtest; see [U] 19 Immediate commands.
Both the traditional $F$ test for the homogeneity of variances and Bartlett's generalization of this test to $K$ samples are sensitive to the assumption that the data are drawn from an underlying Gaussian distribution. See, for example, the cautionary results discussed by Markowski and Markowski (1990). Levene (1960) proposed a test statistic for equality of variance that was found to be robust under nonnormality. Then Brown and Forsythe (1974) proposed alternative formulations of Levene's test statistic that use more robust estimators of central tendency in place of the mean. These reformulations were demonstrated to be more robust than Levene's test when dealing with skewed populations.
robvar reports Levene's robust test statistic $\left(W_{0}\right)$ for the equality of variances between the groups defined by groupvar and the two statistics proposed by Brown and Forsythe that replace the mean in Levene's formula with alternative location estimators. The first alternative ( $W_{50}$ ) replaces the mean with the median. The second alternative replaces the mean with the $10 \%$ trimmed mean $\left(W_{10}\right)$.

## Options

level(\#) specifies the confidence level, as a percentage, for confidence intervals of the means. The default is level (95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals.
by (groupvar) specifies the groupvar that defines the groups to be compared. For sdtest, there should be two groups, but for robvar there may be more than two groups. Do not confuse the by () option with the by prefix; both may be specified.

## Remarks and examples

Remarks are presented under the following headings:
Basic form
Immediate form
Robust test

## Basic form

sdtest performs two different statistical tests: one testing equality of variances and the other testing that the standard deviation is equal to a known constant. Which test it performs is determined by whether you type a variable name or a number to the right of the equal sign.
> Example 1: One-sample test of variance
We have a sample of 74 automobiles. For each automobile, we know the mileage rating. We wish to test whether the overall standard deviation is 5 mpg :


## Example 2: Variance ratio test

We are testing the effectiveness of a new fuel additive. We run an experiment on 12 cars, running each without and with the additive. The data can be found in $[R]$ ttest. The results for each car are stored in the variables mpg1 and mpg2:

| Variable | Obs | Mean | Std. Err | Std. Dev. | [95\% Conf | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| mpg1 | 12 | 21 | . 7881701 | 2.730301 | 19.26525 | 22.73475 |
| mpg2 | 12 | 22.75 | . 9384465 | 3.250874 | 20.68449 | 24.81551 |
| combined | 24 | 21.875 | . 6264476 | 3.068954 | 20.57909 | 23.17091 |
| ```ratio = sd(mpg1) / sd(mpg2) Ho: ratio = 1``` |  |  |  | degrees of freedom |  | $=0.7054$ |
|  |  |  |  |  |  | -11, 11 |
| Ha: ratio < 1 |  | Ha: ratio != 1 |  |  | $\begin{gathered} \text { Ha: ratio > } 1 \\ \operatorname{Pr}(F>f)=0.7138 \end{gathered}$ |  |
| $\operatorname{Pr}(\mathrm{F}<$ | . 2862 | $2 * \operatorname{Pr}(\mathrm{~F}<\mathrm{f})=0.5725$ |  |  |  |  |

We cannot reject the hypothesis that the standard deviations are the same.
In $[\mathrm{R}]$ ttest, we draw an important distinction between paired and unpaired data, which, in this example, means whether there are 12 cars in a before-and-after experiment or 24 different cars. For sdtest, on the other hand, there is no distinction. If the data had been unpaired and stored as described in [R] ttest, we could have typed sdtest mpg, by (treated), and the results would have been the same.

## Immediate form

$>$ Example 3: sdtesti
Immediate commands are used not with data, but with reported summary statistics. For instance, to test whether a variable on which we have 75 observations and a reported standard deviation of 6.5 comes from a population with underlying standard deviation 6 , we would type
. sdtesti 75 . 6.56
One-sample test of variance

|  | Obs | Mean | Std. Err. | Std. Dev. | [95\% Conf. Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| x | 75 | . | . 7505553 | 6.5 | . . |
| sd $=$ sd (x) $\quad c=$ chi2 $=86.8472$ |  |  |  |  |  |
| Ho: sd = 6 degrees of freedom |  |  |  |  |  |
| $\begin{gathered} \mathrm{Ha}: \text { sd }<6 \\ \mathrm{r}(\mathrm{C}<\mathrm{c})=0.8542 \end{gathered}$ |  | $\begin{array}{cc} \text { Ha: sd }!=6 & \text { Ha: sd > }> \\ 2 * \operatorname{Pr}(\mathrm{C}>\mathrm{c})=0.2916 & \operatorname{Pr}(\mathrm{C}>\mathrm{c})=0.1458 \end{array}$ |  |  |  |
|  |  |  |  |  |  |

The mean plays no role in the calculation, so it may be omitted.
To test whether the variable comes from a population with the same standard deviation as another for which we have a calculated standard deviation of 7.5 over 65 observations, we would type
. sdtesti 75 . 6.565 . 7.5
Variance ratio test

|  | Obs | Mean | Std. Err. | Std. Dev. | [95\% Conf | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| x | 75 | . | . 7505553 | 6.5 | . |  |
| y | 65 | . | . 9302605 | 7.5 | . | . |
| combined | 140 | . | . | . | . | . |
| ratio $=s d(x) / s d(y)$ <br> Ho: ratio$=1 \quad$$f$ $=0.7511$ |  |  |  |  |  |  |

Ha: ratio < 1 Ha: ratio != $1 \quad$ Ha: ratio > 1
$\operatorname{Pr}(F<f)=0.1172 \quad 2 * \operatorname{Pr}(F<f)=0.2344 \quad \operatorname{Pr}(F>f)=0.8828$

## Robust test

## Example 4: robvar

We wish to test whether the standard deviation of the length of stay for patients hospitalized for a given medical procedure differs by gender. Our data consist of observations on the length of hospital stay for 1778 patients: 884 males and 894 females. Length of stay, lengthstay, is highly skewed (skewness coefficient $=4.912591$ ) and thus violates Bartlett's normality assumption. Therefore, we use robvar to compare the variances.


For these data, we cannot reject the null hypothesis that the variances are equal. However, Bartlett's test yields a significance probability of 0.0319 because of the pronounced skewness of the data.

## - Technical note

robvar implements both the conventional Levene's test centered at the mean and a median-centered test. In a simulation study, Conover, Johnson, and Johnson (1981) compare the properties of the two tests and recommend using the median test for asymmetric data, although for small sample sizes the test is somewhat conservative. See Carroll and Schneider (1985) for an explanation of why both mean- and median-centered tests have approximately the same level for symmetric distributions, but for asymmetric distributions the median test is closer to the correct level.

## Stored results

sdtest and sdtesti store the following in $r()$ :
Scalars

| $r(N)$ | number of observations |
| :--- | :--- |
| $r(p-l)$ | lower one-sided $p$-value |
| $r(p-u)$ | upper one-sided $p$-value |
| $r(p)$ | two-sided $p$-value |
| $r(F)$ | $F$ statistic |
| $r(s d)$ | standard deviation |
| $r(s d-1)$ | standard deviation for first variable |
| $r\left(s d \_2\right)$ | standard deviation for second variable |
| $r(d f)$ | degrees of freedom |
| $r(d f-1)$ | numerator degrees of freedom |
| $r(d f-2)$ | denominator degrees of freedom |
| $r(c h i 2)$ | $\chi^{2}$ |

robvar stores the following in $r()$ :
Scalars
$r(N) \quad$ number of observations
r(w50) Brown and Forsythe's $F$ statistic (median)
$r$ (p_w50) Brown and Forsythe's $p$-value
$r$ (w0) Levene's $F$ statistic
$r$ (p_w0) Levene's $p$-value
$r$ (w10) Brown and Forsythe's $F$ statistic (trimmed mean)
$r$ (p_w10) Brown and Forsythe's $p$-value (trimmed mean)
$r\left(d f \_1\right) \quad$ numerator degrees of freedom
$r\left(d f \_2\right)$ denominator degrees of freedom

## Methods and formulas

See Armitage et al. (2002, 149-153) or Bland (2000, 171-172) for an introduction and explanation of the calculation of these tests.

The test for $\sigma=\sigma_{0}$ is given by

$$
\chi^{2}=\frac{(n-1) s^{2}}{\sigma_{0}^{2}}
$$

which is distributed as $\chi^{2}$ with $n-1$ degrees of freedom.

The test for $\sigma_{x}^{2}=\sigma_{y}^{2}$ is given by

$$
F=\frac{s_{x}^{2}}{s_{y}^{2}}
$$

which is distributed as $F$ with $n_{x}-1$ and $n_{y}-1$ degrees of freedom.
Let $X_{i j}$ be the $j$ th observation of $X$ for the $i$ th group. Let $Z_{i j}=\left|X_{i j}-\bar{X}_{i}\right|$, where $\bar{X}_{i}$ is the mean of $X$ in the $i$ th group. Levene's test statistic is

$$
W_{0}=\frac{\sum_{i} n_{i}\left(\bar{Z}_{i}-\bar{Z}\right)^{2} /(g-1)}{\sum_{i} \sum_{j}\left(Z_{i j}-\bar{Z}_{i}\right)^{2} / \sum_{i}\left(n_{i}-1\right)}
$$

where $n_{i}$ is the number of observations in group $i$ and $g$ is the number of groups. $W_{50}$ is obtained by replacing $\bar{X}_{i}$ with the $i$ th group median of $X_{i j}$, whereas $W_{10}$ is obtained by replacing $\bar{X}_{i}$ with the $10 \%$ trimmed mean for group $i$.

## References

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## Also see

$[\mathrm{R}]$ ttest $-t$ tests (mean-comparison tests)

## Title

search - Search Stata documentation and other resources

| Syntax | Menu | Description |
| :--- | :--- | :--- |
| Options for search | Option for set searchdefault | Remarks and examples |
| Acknowledgment | Also see |  |

## Syntax

```
search word [word ...] [, search_options]
set searchdefault {all|local|net } [, permanently]
```

search_options Description
all search across both the local keyword database and the net material; the default
local search using Stata's keyword database
net search across materials available via Stata's net command
author search by author's name
entry search by entry ID
exact search across both the local keyword database and the net materials; prevents matching on abbreviations
faq search the FAQs posted to the Stata website
historical search entries that are of historical interest only
or
manual search the entries in the Stata Documentation
sj search the entries in the Stata Journal and the STB

## Menu

Help $>$ Search...

## Description

search searches a keyword database and the Internet for Stata materials related to your query.
Capitalization of the words following search is irrelevant, as is the inclusion or exclusion of special characters such as commas and hyphens.
set searchdefault affects the default behavior of the search command. all is the default. search, all is the best way to search for information on a topic across all sources, including the system help, the FAQs at the Stata website, the Stata Journal, and all Stata-related Internet sources including user-written additions. From the results, you can click to go to a source or to install additions.

## Options for search

all, the default (unless changed by set searchdefault), specifies that the search be performed across both the local keyword database and the net materials. The results of a search performed with all and no other options will be displayed in the Viewer window.
local specifies that the search be performed using only Stata's keyword database. The results of a search performed with local and no other options will be displayed in the Viewer window.
net specifies that the search be performed across the materials available via Stata's net command. Using search word [word ...], net is equivalent to typing net search word [word ...] (without options); see [R] net search. The results of a search performed with net and no other options will be displayed in the Viewer window.
author specifies that the search be performed on the basis of author's name rather than keywords. A search with the author option is performed on the local keyword database only, and the results are displayed in the Results window.
entry specifies that the search be performed on the basis of entry IDs rather than keywords. A search with the entry option is performed on the local keyword database only, and the results are displayed in the Results window.
exact prevents matching on abbreviations. A search with the exact option is performed across both the local keyword database and the net materials, and the results are displayed in the Results window.
faq limits the search to the FAQs posted on the Stata website: http://www.stata.com. A search with the faq option is performed on the local keyword database only, and the results are displayed in the Results window.
historical adds to the search entries that are of historical interest only. By default, such entries are not listed. Past entries are classified as historical if they discuss a feature that later became an official part of Stata. Updates to historical entries will always be found, even if historical is not specified. A search with the historical option is performed on the local keyword database only, and the results are displayed in the Results window.
or specifies that an entry be listed if any of the words typed after search are associated with the entry. The default is to list the entry only if all the words specified are associated with the entry. A search with the or option is performed on the local keyword database only, and the results are displayed in the Results window.
manual limits the search to entries in the Stata Documentation; that is, the search is limited to the User's Guide and all the reference manuals. A search with the manual option is performed on the local keyword database only, and the results are displayed in the Results window.
sj limits the search to entries in the Stata Journal and its predecessor, the Stata Technical Bulletin; see $[\mathrm{R}]$ sj. A search with the sj option is performed on the local keyword database only, and the results are displayed in the Results window.

## Option for set searchdefault

permanently specifies that, in addition to making the change right now, the searchdefault setting be remembered and become the default setting when you invoke Stata.

## Remarks and examples

Remarks are presented under the following headings:

Introduction<br>Internet searches<br>Author searches<br>Entry ID searches<br>Return codes

## Introduction

See [U] 4 Stata's help and search facilities for a tutorial introduction to search. search is one of Stata's most useful commands. To understand the advanced features of search, you need to know how it works.
search has a database-files-containing the titles, etc., of every entry in the User's Guide, the Base Reference Manual, the Data Management Reference Manual, the Graphics Reference Manual, the Longitudinal-Data/Panel-Data Reference Manual, the Multilevel Mixed-Effects Reference Manual, the Multiple-Imputation Reference Manual, the Multivariate Statistics Reference Manual, the Power and Sample-Size Reference Manual, the Programming Reference Manual, the Structural Equation Modeling Reference Manual, the Survey Data Reference Manual, the Survival Analysis and Epidemiological Tables Reference Manual, the Treatment-Effects Reference Manual, the Time-Series Reference Manual, the Mata Reference Manual, undocumented help files, NetCourses, Stata Press books, FAQs posted on the Stata website, videos posted on the Stata YouTube channel, selected articles on StataCorp's official blog, selected user-written FAQs and examples, and the articles in the Stata Journal and the Stata Technical Bulletin. In these files is a list of words, called keywords, associated with each entry.

When you type search $x y z$, search reads the database and compares the list of keywords with $x y z$. If it finds $x y z$ in the list or a keyword that allows an abbreviation of $x y z$, it displays the entry.

When you type search $x y z a b c$, search does the same thing but displays an entry only if it contains both keywords. The order does not matter, so you can search linear regression or search regression linear.

Obviously, how many entries search finds depends on how the search database was constructed. We have included a plethora of keywords under the theory that, for a given request, it is better to list too much rather than risk listing nothing at all. Still, you are in the position of guessing the keywords. Do you look up normality test, normality tests, or tests of normality? Well, normality test would be best, but all would work. In general, use the singular, and strike the unnecessary words. For guidelines for specifying keywords, see [U] 4.6 More on search.
set searchdefault allows you to specify where search searches. set searchdefault all, the default, indicates that both the keyword database and the Internet are to be searched. set searchdefault local restricts search to using only Stata's keyword database. set searchdefault net restricts search to searching only the Internet.

## Internet searches

search with the net option searches the Internet for user-written additions to Stata, including, but not limited to, user-written additions published in the Stata Journal (SJ) and the Stata Technical Bulletin (STB). search keywords, net performs the same search as the command net search (with no options); see [R] net search.

```
. search random effect, net
```


## Web resources from Stata and other users

```
(contacting http://www.stata.com)
```

70 packages found (Stata Journal and STB listed first)
st0156_1 from http://www.stata-journal.com/software/sj11-2
SJ11-2 st0156_1. Update: Multivariate random-effects... / Update:
Multivariate random-effects meta-regression / by Ian White / Support:
ian.white@mrc-bsu.cam.ac.uk / After installation, type help mvmeta and
mvmeta_make
st0201 from http://www.stata-journal.com/software/sj10-3
SJ10-3 st0201. metaan: Random-effects meta-analysis / metaan:
Random-effects meta-analysis / by Evangelos Kontopantelis, / National
Primary Care Research and Development Centre (NPCRDC), / University of
Manchester, Manchester, UK / David Reeves, / National Primary Care
sbe24_3 from http://www.stata-journal.com/software/sj9-2
SJ9-2 sbe24_3. Update: metan: fixed- and random-effects... / Update:
metan: fixed- and random-effects meta-analysis / by Ross J. Harris, Roger
M. Harbord, and Jonathan A. C. Sterne, / Department of Social Medicine,
University of Bristol / Jonathan J. Deeks, Department of Primary Care
(output omitted)
(end of search)

## Author searches

search ordinarily compares the words following search with the keywords for the entry. If you specify the author option, however, it compares the words with the author's name. In the search database, we have filled in author names for all SJ and STB inserts.

For instance, in [R] kdensity in this manual you will discover that Isaías H. Salgado-Ugarte wrote the first version of Stata's kdensity command and published it in the STB. Assume that you read his original insert and found the discussion useful. You might now wonder what else he has written in the SJ or STB. To find out, you type

```
. search Salgado-Ugarte, author
(output omitted)
```

Names like Salgado-Ugarte are confusing to many people. search does not require you to specify the entire name; what you type is compared with each "word" of the name and, if any part matches, the entry is listed. The hyphen is a special character, and you can omit it. Thus you can obtain the same list by looking up Salgado, Ugarte, or Salgado Ugarte without the hyphen.

Actually, to find all entries written by Salgado-Ugarte, you need to type

```
. search Salgado-Ugarte, author historical
    (output omitted)
```

Prior inserts in the SJ or STB that provide a feature that later was superseded by a built-in feature of Stata are marked as historical in the search database and, by default, are not listed. The historical option ensures that all entries are listed.

## Entry ID searches

If you specify the entry option, search compares what you have typed with the entry ID. The entry ID is not the title-it is the reference listed to the left of the title that tells you where to look. For instance, in
[R]

```
regress . . . . . . . . . . . . . . . . . . . . . . Linear regression
(help regress)
```

[R] regress is the entry ID. This is a reference, of course, to this manual. In
FAQ

```
. . . . . . . . . . . Analysis of multiple failure-time survival data
. . . . . . . . . . . . . . . . . . . . . . . M. Cleves and I. Canette
07/09 How do I analyze multiple failure-time data using Stata?
    http://www.stata.com/support/faqs/statistics/multiple-failure-
    type-data/
```

"FAQ" is the entry ID. In

discusses survey issues in analyzing complex survey
data and describes some of Stata's capabilities for
such analyses
"SJ-7-1" is the entry ID.
search with the entry option searches these entry IDs.
Thus you could generate a table of contents for the User's Guide by typing

```
. search [U], entry
(output omitted)
```

You could generate a table of contents for Stata Journal, Volume 1, Issue 1, by typing

```
. search sj-1-1, entry
```

    (output omitted)
    To generate a table of contents for the 26th issue of the STB, you would type

```
. search STB-26, entry historical
    (output omitted)
```

The historical option here is possibly important. STB-26 was published in July 1995, and perhaps some of its inserts have already been marked historical.

You could obtain a list of all inserts associated with sg53 by typing

```
. search sg53, entry historical
    (output omitted)
```

Again we include the historical option in case any of the relevant inserts have been marked historical.

## Return codes

In addition to indexing the entries in the User's Guide and all the Reference manuals, search also can be used to search return codes.

To see information on return code 131, type

```
. search rc 131
```

[P] error . . . . . . . . . . . . . . . . . . . . . . . . Return code 131
not possible with test;
You requested a test of a hypothesis that is nonlinear in the variables. test tests only linear hypotheses. Use testnl.

If you want a list of all Stata return codes, type

- search error, entry
(output omitted)


## Acknowledgment

We thank Nicholas J. Cox of the Department of Geography at Durham University, UK, and coeditor of the Stata Journal for his contributions to the search command.

## Also see

[R] help - Display help in Stata
[R] net search - Search the Internet for installable packages
[U] 4 Stata's help and search facilities

## Title

serrbar - Graph standard error bar chart

| Syntax | Menu | Description | Options |
| :--- | :--- | :--- | :--- |
| Remarks and examples | Acknowledgment | Also see |  |

## Syntax

serrbar mvar svar xvar [if] [in] [, options]
options Description
Main
scale(\#) scale length of graph bars; default is scale(1)
Error bars
rcap_options affect rendition of capped spikes
Plotted points
mvopts (scatter_options) affect rendition of plotted points
Add plots
addplot (plot) add other plots to generated graph
Y axis, X axis, Titles, Legend, Overall
twoway_options any options other than by () documented in [G-3] twoway_options

## Menu

Statistics $>$ Other $>$ Quality control $>$ Standard error bar chart

## Description

serrbar graphs mvar $\pm$ scale() $\times$ svar against xvar. Usually, but not necessarily, mvar and svar will contain means and standard errors or standard deviations of some variable so that a standard error bar chart is produced.

## Options

$\qquad$ Main
scale(\#) controls the length of the bars. The upper and lower limits of the bars will be mvar + scale () $\times$ svar and mvar - scale ( $) \times$ svar. The default is scale(1).

## Error bars

rcap_options affect the rendition of the plotted error bars (the capped spikes). See [G-2] graph twoway rcap.

Plotted points
mvopts (scatter_options) affects the rendition of the plotted points (mvar versus xvar). See [G-2] graph twoway scatter.

Add plots
addplot (plot) provides a way to add other plots to the generated graph; see [G-3] addplot_option.

Y axis, X axis, Titles, Legend, Overall
twoway_options are any of the options documented in [G-3] twoway_options, excluding by (). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

## Remarks and examples

## > Example 1

In quality-control applications, the three most commonly used variables with this command are the process mean, process standard deviation, and time. For instance, we have data on the average weights and standard deviations from an assembly line in San Francisco for the period January 8 to January 16. Our data are

```
. use http://www.stata-press.com/data/r13/assembly
. list, sep(0) divider
```

| date | mean | std |
| :---: | ---: | ---: |
| 108 | 192.22 | 3.94 |
| 109 | 192.64 | 2.83 |
| 110 | 192.37 | 4.58 |
| 113 | 194.76 | 3.25 |
| 114 | 192.69 | 2.89 |
| 115 | 195.02 | 1.73 |
| 116 | 193.40 | 2.62 |

We type serrbar mean std date, scale(2) but, after seeing the result, decide to make it fancier:

```
. serrbar mean std date, scale(2) title("Observed Weight Variation")
> sub("San Francisco plant, 1/8 to 1/16") yline(195) yaxis(1 2)
> ylab(195, axis(2)) ytitle("", axis(2))
```



## Acknowledgment

serrbar was written by Nicholas J. Cox of the Department of Geography at Durham University, UK, and coeditor of the Stata Journal.

## Also see

[R] qc - Quality control charts

## Title

set - Overview of system parameters

## Syntax Description Remarks and examples Also see

## Syntax

set $[$ setcommand ...]
set typed without arguments is equivalent to query typed without arguments.

## Description

This entry provides a reference to Stata's set commands. For many entries, more thorough information is provided elsewhere; see the Reference field in each entry below for the location of this information.

To reset system parameters to factory defaults, see $[R]$ set_defaults.

## Remarks and examples

set adosize
Syntax: set adosize \# [, permanently]
Default: 1,000
Description: sets the maximum amount of memory that automatically loaded do-files may consume. $10 \leq \# \leq 10000$.
Reference: [P] sysdir
set autotabgraphs (Windows only)
Syntax: set autotabgraphs \{on |off $\} \quad[$, permanently $]$
Default: off
Description: determines whether graphs are created as tabs within one window or as separate windows.
set cformat
Syntax: set cformat [fint] [, permanently]
Description: specifies the output format of coefficients, standard errors, and confidence limits in coefficient tables. fint is a numerical format; see [D] format.
Reference: [R] set cformat
set charset (Mac only)
Syntax: set charset $\{\operatorname{mac} \mid$ latin1 $\} \quad[$, permanently $]$
Default: mac
Description: sets the character set used by Stata for Mac for rendering of ASCII text.
set checksum
Syntax: set checksum $\{$ on $\mid$ off $\} \quad[$, permanently $]$
Default: off
Description: determines whether files should be prevented from being downloaded from the Internet if checksums do not match.
Reference: [D] checksum
set coeftabresults
Syntax: set coeftabresults \{on|off $\}$
Default: on
Description: determines whether coefficient table results are stored in $r()$.
There is no permanently option because permanently is implied.
set conren (Unix console only)
Syntax 1: set conren
Syntax 2: set conren clear
Syntax 3: set conren $[\mathrm{sf}|\mathrm{bf}| \mathrm{it}]$
$\{\underline{\text { result } \mid[\text { txt } \mid \text { text }] \mid \text { input }|\underline{\text { error }}| \underline{\text { link }} \mid \underline{\text { hilite }}\}}$
[char[char...]]
Syntax 4: set conren $\{$ ulon | uloff $\}$ [char [char ...]]
Syntax 5: set conren reset [char [char...]]
Description: can possibly make the output on your screen appear prettier.
set conren displays a list of the currently defined display codes.
set conren clear clears all codes.
set conren followed by a font type (bf, sf, or it) and display context (result, error, link, or hilite) and then followed by a series of space-separated characters sets the code for the specified font type and display context. If the font type is omitted, the code is set to the same specified code for all three font types. set conren ulon and set conren uloff set the codes for turning on and off underlining.
set conren reset sets the code that will turn off all display and underlining codes.
Reference: [GSU] conren
set copycolor (Mac and Windows only)
Syntax: set copycolor \{automatic|asis|gs1|gs2|gs3\} [, permanently]
Default: automatic
Description: determines how colors are handled when graphs are copied to the Clipboard.
Reference: [G-2] set printcolor
set dockable (Windows only)
Syntax: set dockable $\{$ on $\mid$ off $\} \quad$ [, permanently]
Default: on
Description: determines whether to enable the use of dockable window characteristics, including the ability to dock or tab a window into another window.

## set dockingguides (Windows only)

Syntax: set dockingguides $\{$ on $\mid$ off $\} \quad[$, permanently $]$
Default: on
Description: determines whether to enable the use of dockable guides when repositioning a dockable window.
set doublebuffer (Windows only)
Syntax: set doublebuffer $\{$ on $\mid$ off $\}$ [, permanently]
Default: on
Description: enables or disables double buffering of the Results, Viewer, and Data Editor windows. Double buffering prevents the windows from flickering when redrawn or resized. Users who encounter performance problems such as the Results window outputting very slowly should disable double buffering.
set $d p$
Syntax: set dp \{comma|period\} [, permanently]
Default: period
Description: determines whether a period or a comma is to be used as the decimal point.
Reference: [D] format
set emptycells
Syntax: set emptycells $\{$ keep $\mid$ drop $\}$ [, permanently $]$
Default: keep
Description: sets what to do with empty cells in interactions.
Reference: [R] set emptycells
set eolchar (Mac only)
Syntax: set eolchar $\{$ mac $\mid$ unix $\}$ [, permanently]
Default: unix
Description: sets the default end-of-line delimiter for text files created in Stata.
set fastscroll (Unix and Windows only)
Syntax: set fastscroll $\{$ on $\mid$ off $\}$ [, permanently]
Default: on
Description: sets the scrolling method for new output in the Results window. Setting fastscroll to on is faster but can be jumpy. Setting fastscroll to off is slower but smoother.
set floatwindows (Windows only)
Syntax: set floatwindows $\{$ on $\mid$ off $\}$
Default: off
Description: determines whether to enable floating window behavior for dialog boxes and dockable window. The term "float" in this context means that a window will always float over the main Stata window; these windows cannot be placed behind the main Stata window. There is no permanently option because permanently is implied.

```
set fvlabel
```

Syntax:
Description:
Reference: [R] set showbaselevels
set fvwrap
Syntax:
Description:
Reference:
set fvwrap \# [, permanently]
specifies that long value labels wrap \# lines in coefficient tables.
[ $R$ ] set showbaselevels
set fvwrapon
Syntax: set fvwrapon $\{$ word|width $\}$ [, permanently]
Description: specifies whether value labels that wrap will break at word bounderies or break based on available space.
Reference: [R] set showbaselevels
set graphics
Syntax: set graphics $\{$ on $\mid$ off $\}$
Default: on; default is off for console Stata
Description: determines whether graphs are displayed on your monitor.
Reference: [G-2] set graphics
set haverdir
Syntax: set haverdir "path" [, permanently]
Description: specifies the directory where the Haver databases are stored.
Reference: [D] import haver
set httpproxy
Syntax: set httpproxy $\{$ on $\mid$ off $\}[$, init $]$
Default: off
Description: turns on/off the use of a proxy server. There is no permanently option because permanently is implied.
Reference: [R] netio
set httpproxyauth
Syntax: set httpproxyauth $\{$ on $\mid$ off $\}$
Default: off
Description: determines whether authorization is required for the proxy server.
There is no permanently option because permanently is implied.
Reference: [R] netio
set httpproxyhost
Syntax: set httpproxyhost ["] name["]
Description: sets the name of a host to be used as a proxy server. There is no permanently option because permanently is implied.
Reference: [R] netio

```
set httpproxyport
    Syntax: set httpproxyport #
    Default: 8080 if Stata cannot autodetect the proper setting for your computer.
    Description: sets the port number for a proxy server. There is no permanently option
                because permanently is implied.
    Reference: [R] netio
set httpproxypw
    Syntax: set httpproxypw ["] password ["]
    Description: sets the appropriate password. There is no permanently option because
    permanently is implied.
    Reference: [R] netio
```

set httpproxyuser
Syntax: set httpproxyuser ["]name["]
Description: sets the appropriate user ID. There is no permanently option because
permanently is implied.
Reference: [R] netio
set include_bitmap (Mac only)
Syntax: set include_bitmap $\{$ on $\mid$ off $\}$ [, permanently]
Default: on
Description: sets the output behavior when copying an image to the Clipboard.
set level
Syntax: set level \# [, permanently $]$
Default: 95
Description: sets the default confidence level for confidence intervals for all commands
that report confidence intervals. $10.00 \leq \# \leq 99.99$, and \# can have at
most two digits after the decimal point.
Reference: [R] level
set linegap
Syntax: set linegap \#
Default: 1
Description: sets the space between lines, in pixels, in the Results window. There is no
permanently option because permanently is implied.
set linesize
Syntax: set linesize \#
Default: 1 less than the full width of the screen
Description: sets the line width, in characters, for both the screen and the $\log$ file.
Reference: [R] log

```
set locksplitters (Windows only)
    Syntax: set locksplitters {on|off} [, permanently]
    Default: off
    Description: determines whether splitters should be locked so that docked windows
    cannot be resized.
```

set logtype
Syntax: set logtype $\{\underline{\text { text }} \mid \underline{\text { smcl }}\}$ [, permanently]
Default: smcl
Description: sets the default log filetype.
Reference: $[R] \log$
set lstretch
Syntax: set lstretch $\{$ on $\mid$ off $\} \quad$ [, permanently]
Description: specifies whether to automatically widen the coefficient table up to the width of
the Results window to accommodate longer variable names.
set matacache, set matafavor, set matalibs, set matalnum, set matamofirst,
set mataoptimize, and set matastrict; see [M-3] mata set.
set matsize
Syntax: set matsize \# [, permanently]
Default: $\quad 400$ for Stata/MP, Stata/SE, and Stata/IC; 40 for Small Stata
Description: sets the maximum number of variables that can be included in any estimation
command. This setting cannot be changed in Small Stata.
$10 \leq \# \leq 11000$ for Stata/MP and Stata/SE; $10 \leq \# \leq 800$ for Stata/IC.
Reference: [R] matsize
set max_memory
Syntax: set max_memory \#[b|k|m|g][, permanently]
Default: . (all the memory the operating system will supply)
Description: specifies the maximum amount of memory Stata can use to store your data.
$2 \times$ segmentsize $\leq \# \leq$.
Reference: [D] memory
set maxdb
Syntax: set maxdb \# [, permanently]
Default: 50
Description: sets the maximum number of dialog boxes whose contents are remembered
from one invocation to the next during a session. $5 \leq \# \leq 1000$
Reference: $[\mathrm{R}] \mathbf{d b}$
set maxiter
Syntax: set maxiter \# [, permanently]
Default: 16000
Description: sets the default maximum number of iterations for estimation commands.
$0 \leq \# \leq 16000$
Reference: [R] maximize
set maxvar
Syntax: set maxvar \# [, permanently ]
Default: 5000 for Stata/MP and Stata/SE, 2048 for Stata/IC, and 99 for Small Stata
Description: sets the maximum number of variables. This can be changed only in Stata/MP and Stata/SE. $2048 \leq \# \leq 32767$
Reference: [D] memory
set min_memory
Syntax: set min_memory $\#[b|k| m \mid g][$, permanently $]$
Default: 0
Description: specifies an amount of memory Stata will not fall below. This setting affects efficiency, not the size of datasets you can analyze. $0 \leq \# \leq$ max_memory
Reference: [D] memory
set more
Syntax: set more $\{$ on $\mid$ off $\}$ [, permanently]
Default: on
Description: pauses when -more- is displayed, continuing only when the user presses a key.
Reference: [R] more
set niceness
Syntax: set niceness \# [, permanently]
Default: 5
Description: affects how soon Stata gives back unused segments to the operating system.
$0 \leq \# \leq 10$
Reference: [D] memory
set notifyuser (Mac only)
Syntax: set notifyuser $\{$ on $\mid$ off $\}$ [, permanently $]$
Default: on
Description: sets the default Notification Manager behavior in Stata.
set obs
Syntax: set obs \#
Default: current number of observations
Description: changes the number of observations in the current dataset. \# must be at least as large as the current number of observations. If there are variables in memory, the values of all new observations are set to missing.
Reference: [D] obs
set odbcmgr (Unix only)
Syntax: set odbcmgr \{iodbc|unixodbc $\}$ [, permanently]
Default: iodbc
Description: determines whether iODBC or unixODBC is your ODBC driver manager.
Reference: [D] odbc
set output
Syntax:
Default:
set output $\{$ proc $\mid$ inform $\mid$ error $\}$
proc
Description: specifies the output to be displayed. proc means display all output; inform suppresses procedure output but displays informative messages and error messages; error suppresses all output except error messages. set output is seldom used.
Reference: [P] quietly
set pagesize
Syntax: set pagesize \#
Default: 2 less than the physical number of lines on the screen
Description: sets the number of lines between -more- messages.
Reference: [R] more
set pformat
Syntax: set pformat [fmt] [, permanently]
Description: specifies the output format of $p$-values in coefficient tables.
fint is a numerical format; see [D] format.
Reference: [R] set cformat
set pinnable (Windows only)
Syntax: set pinnable $\{$ on $\mid$ off $\} \quad$ [, permanently]
Default: on
Description: determines whether to enable the use of pinnable window characteristics for certain windows in Stata.
set playsnd (Mac only)
Syntax: set playsnd $\{$ on $\mid$ off $\}$ [, permanently]
Default: on
Description: sets the sound behavior for the Notification Manager behavior in Stata.
set printcolor
Syntax: set printcolor \{automatic|asis $\mid$ gs1 $\mid$ gs2 $\mid$ gs3 $\}$ [, permanently]
Default: automatic
Description: determines how colors are handled when graphs are printed.
Reference: [G-2] set printcolor
set processors
Syntax: set processors \#
Description: sets the number of processors or cores that Stata/MP will use. The default is the number of processors available on the computer, or the number of processors allowed by Stata/MP's license, whichever is less.
set reventries
Syntax: set reventries \# [, permanently]
Default: 5000
Description: sets the number of scrollback lines available in the Review window. $5 \leq \# \leq 32000$.

```
set revkeyboard (Mac only)
    Syntax: set revkeyboard {on|off } [, permanently]
    Default: on
    Description: sets the keyboard navigation behavior for the Review window. on indicates
        that you can use the keyboard to navigate and enter items from the Review
        window into the Command window. off indicates that all keyboard input be
        directed at the Command window; items can be entered from the Review
        window only by using the mouse.
```

set rmsg
Syntax: set rmsg $\{$ on $\mid$ off $\}$ [, permanently]
Default: off
Description: indicates whether a return message telling the execution time is to be displayed at
the completion of each command.
Reference: [P] rmsg
set scheme
Syntax: set scheme schemename [, permanently]
Default: s2color
Description: determines the overall look for graphs.
Reference: [G-2] set scheme
set scrollbufsize
Syntax: set scrollbufsize \#
Default: 200000
Description: sets the scrollback buffer size, in bytes, for the Results window;
may be set between 10,000 and $2,000,000$.
set searchdefault
Syntax: set searchdefault \{local|net|all\}[, permanently]
Default: local
Description: sets the default behavior of the search command. set searchdefault local
restricts search to use only Stata's keyword database. set searchdefault net
restricts search to searching only the Internet. set searchdefault all
indicates that both the keyword database and the Internet are to be searched.
Reference: [R] search
set seed
Syntax: set seed $\{\# \mid$ code $\}$
Default: 123456789
Description: specifies initial value of the random-number seed used by the runiform() function.
Reference: [R] set seed
set segmentsize
Syntax: set segmentsize \#[b|k|m|g][, permanently]
Default: $\quad 32 \mathrm{~m}$ for 64 -bit machines; 16 m for 32 -bit machines
Description: Stata allocates memory for data in units of segmentsize. This setting changes the amount of memory in a single segment.
$1 \mathrm{~m} \leq \# \leq 32 \mathrm{~g}$ for 64 -bit machines; $1 \mathrm{~m} \leq \# \leq 1 \mathrm{~g}$ for 32-bit machines

Reference: [D] memory

## set sformat

Syntax: set sformat $[f m t]$ [, permanently]
Description: specifies the output format of test statistics in coefficient tables. fint is a numerical format; see [D] format.
Reference: [R] set cformat
set showbaselevels
Syntax: set showbaselevels $\{$ on $\mid$ off $\mid$ all $\}$ [, permanently]
Description: specifies whether to display base levels of factor variables and their interactions in coefficient tables.
Reference: [R] set showbaselevels
set showemptycells
Syntax: set showemptycells $\{$ on $\mid$ off $\}$ [, permanently]
Description: specifies whether to display empty cells in coefficient tables.
Reference: [R] set showbaselevels
set showomitted
Syntax: set showomitted $\{$ on |off $\}$ [, permanently]
Description: specifies whether to display omitted coefficients in coefficient tables.
Reference: [R] set showbaselevels
set smoothfonts (Mac only)
Syntax: set smoothfonts $\{$ on $\mid$ off $\}$
Default: on
Description: determines whether to use font smoothing (antialiased text) in the Results, Viewer, and Data Editor windows.
set timeout1
Syntax: set timeout1 \#seconds [, permanently]
Default: 30
Description: sets the number of seconds Stata will wait for a remote host to respond to an initial contact before giving up. In general, users should not modify this value unless instructed to do so by Stata Technical Services.
Reference: [R] netio
set timeout2
Syntax: set timeout2 \#seconds [, permanently]
Default: 180
Description: sets the number of seconds Stata will keep trying to get information from a remote host after initial contact before giving up. In general, users should not modify this value unless instructed to do so by Stata Technical Services.
Reference: [R] netio
set trace
Syntax: set trace $\{$ on $\mid$ off $\}$
Default: off
Description: determines whether to trace the execution of programs for debugging.
Reference: [P] trace

```
set tracedepth
    Syntax: set tracedepth #
    Default: }\quad32000\mathrm{ (equivalent to }\infty\mathrm{ )
    Description: if trace is set on, traces execution of programs and nested programs up to
                tracedepth. For example, if tracedepth is 2, the current program and any
                subroutine called would be traced, but subroutines of subroutines would not
                be traced.
    Reference: [P] trace
set traceexpand
    Syntax: set traceexpand {on|off } [, permanently]
    Default: on
    Description: if trace is set on, shows lines both before and after macro expansion. If
        traceexpand is set off, only the line before macro expansion is shown.
    Reference: [P] trace
set tracehilite
    Syntax: set tracehilite "pattern" [, word]
    Default: ""
    Description: highlights pattern in the trace output.
    Reference: [P] trace
```

set traceindent
Syntax: set traceindent $\{$ on $\mid$ off $\}$ [, permanently $]$
Default: on
Description: if trace is set on, indents displayed lines according to their nesting level. The
lines of the main program are not indented. Two spaces of indentation are used for
each level of nested subroutine.
Reference: [P] trace
set tracenumber
Syntax: set tracenumber $\{$ on $\mid$ off $\}$ [, permanently $]$
Default: off
Description: if trace is set on, shows the nesting level numerically in front of the line.
Lines of the main program are preceded by 01 , lines of subroutines called by the
main program are preceded by 02 , etc.
Reference: [P] trace
set tracesep
Syntax: set tracesep $\{$ on $\mid$ off $\}$ [, permanently]
Default: on
Description: if trace is set on, displays a horizontal separator line that displays the name
of the subroutine whenever a subroutine is called or exits.
Reference: [P] trace
set type
Syntax: set type $\{$ float $\mid$ double $\}[$, permanently]
Default: float
Description: specifies the default storage type assigned to new variables.
Reference: [D] generate
set update_interval (Mac and Windows only)
Syntax: set update_interval \#
Default: 7
Description: sets the number of days to elapse before performing the next automatic update query.
Reference: [R] update
set update_prompt (Mac and Windows only)
Syntax: set update_prompt $\{$ on $\mid$ off $\}$
Default: on
Description: determines wheter a dialog is to be displayed before performing an automatic update query. There is no permanently option because permanently is implied.
Reference: [R] update
set update_query (Mac and Windows only)
Syntax: set update_query $\{$ on $\mid$ off $\}$
Default: on
Description: determines whether update query is to be automatically performed when Stata is launched. There is no permanently option because permanently is implied.
Reference: [R] update
set varabbrev
Syntax: set varabbrev $\{$ on $\mid$ off $\}$ [, permanently]
Default: on
Description: indicates whether Stata should allow variable abbreviations.
Reference: [P] varabbrev
set varkeyboard (Mac only)
Syntax: set varkeyboard $\{$ on $\mid$ off $\}$ [, permanently]
Default: on
Description: sets the keyboard navigation behavior for the Variables window. on indicates that you can use the keyboard to navigate and enter items from the Variables window into the Command window. off indicates that all keyboard input be directed at the Command window; items can be entered from the Variables window only by using the mouse.

## Also see

[R] query - Display system parameters
[R] set_defaults - Reset system parameters to original Stata defaults
[P] creturn - Return c-class values
[M-3] mata set - Set and display Mata system parameters

## Title

set cformat - Format settings for coefficient tables

Syntax Description Option Remarks and examples Also see

## Syntax

```
set cformat [fmt] [, permanently]
set pformat [fmt] [, permanently]
set sformat [fmt] [, permanently]
```

where frit is a numerical format.

## Description

set cformat specifies the output format of coefficients, standard errors, and confidence limits in coefficient tables.
set pformat specifies the output format of $p$-values in coefficient tables.
set sformat specifies the output format of test statistics in coefficient tables.

## Option

permanently specifies that, in addition to making the change right now, the setting be remembered and become the default setting when you invoke Stata.

## Remarks and examples

The formatting of the numbers in the coefficient table can be controlled by using the set cformat, set pformat, and set sformat commands or by using the cformat (\%fmt), pformat (\%fmt), and sformat (\%fmt) options at the time of estimation or on replay of the estimation command. See $[\mathrm{R}]$ estimation options.

The maximum format widths for set cformat, set pformat, and set sformat in coefficient tables are 9,5 , and 8 , respectively.

## Example 1

We use auto.dta to illustrate.

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. regress mpg weight displacement
```

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | 1595.40969 | 2 | 797.704846 |
| Residual | 848.049768 | 71 | 11.9443629 |
| Total | 2443.45946 | 73 | 33.4720474 |


| Number of obs | $=$ |
| :--- | ---: |
| F 2, | 74 |
| Prob $>$ F | $=66.79$ |
| R-squared | $=$ |
| Adj R-squared | $=0.0000$ |
| Root MSE | $=0.6432$ |
|  | $=3.4561$ |


| mpg | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | :---: | ---: | ---: |
| weight | -.0065671 | .0011662 | -5.63 | 0.000 | -.0088925 | -.0042417 |
| displacement | .0052808 | .0098696 | 0.54 | 0.594 | -.0143986 | .0249602 |
| _cons | 40.08452 | 2.02011 | 19.84 | 0.000 | 36.05654 | 44.11251 |

. set cformat \%9.2f
. regress mpg weight displacement

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | 1595.40969 | 2 | 797.704846 |
| Residual | 848.049768 | 71 | 11.9443629 |
| Total | 2443.45946 | 73 | 33.4720474 |


| Number of obs | $=$ | 74 |
| :--- | ---: | ---: |
| F 2, | $71)$ | $=$ |
| Prob $>$ F | $=$ | 0.0000 |
| R-squared | $=$ | 0.6529 |
| Adj R-squared | $=$ | 0.6432 |
| Root MSE | $=$ | 3.4561 |


| mpg | Coef. | Std. Err. | $t$ | P>\|t| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| weight | -0.01 | 0.00 | -5.63 | 0.000 | -0.01 | -0.00 |
| displacement | 0.01 | 0.01 | 0.54 | 0.594 | -0.01 | 0.02 |
| _cons | 40.08 | 2.02 | 19.84 | 0.000 | 36.06 | 44.11 |

. regress mpg weight displacement, cformat(\%9.3f)

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | 1595.40969 | 2 | 797.704846 |
| Residual | 848.049768 | 71 | 11.9443629 |
| Total | 2443.45946 | 73 | 33.4720474 |


| Number of obs | $=$ | 74 |
| :--- | ---: | ---: |
| $\mathrm{~F}(2$, | $71)$ | $=$ |
| Prob $>$ F | $=$ | 0.0000 |
| R-squared | $=$ | 0.6529 |
| Adj R-squared | $=$ | 0.6432 |
| Root MSE | $=3.4561$ |  |


| mpg | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | :---: | :---: | ---: | ---: |
| weight | -0.007 | 0.001 | -5.63 | 0.000 | -0.009 | -0.004 |
| displacement | 0.005 | 0.010 | 0.54 | 0.594 | -0.014 | 0.025 |
| _cons | 40.085 | 2.020 | 19.84 | 0.000 | 36.057 | 44.113 |

To reset the cformat setting to its command-specific default, type
. set cformat
. regress mpg weight displacement

| Source | SS | df | MS |
| ---: | :---: | ---: | :---: |
| Model | 1595.40969 | 2 | 797.704846 |
| Residual | 848.049768 | 71 | 11.9443629 |
| Total | 2443.45946 | 73 | 33.4720474 |


| Number of obs | $=$ | 74 |
| :--- | ---: | ---: |
| F $(2$, | $71)$ | $=$ |
| Prob $>$ F | $=$ | 0.0000 |
| R-squared | $=$ | 0.6529 |
| Adj R-squared | $=$ | 0.6432 |
| Root MSE | $=$ | 3.4561 |


| mpg | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| weight | -.0065671 | .0011662 | -5.63 | 0.000 | -.0088925 | -.0042417 |
| displacement | .0052808 | .0098696 | 0.54 | 0.594 | -.0143986 | .0249602 |
| _cons | 40.08452 | 2.02011 | 19.84 | 0.000 | 36.05654 | 44.11251 |

## Also see

[R] estimation options - Estimation options
[R] query - Display system parameters
[R] set - Overview of system parameters
[U] 20.8 Formatting the coefficient table

## Title

set_defaults - Reset system parameters to original Stata defaults
Syntax Description Option Remarks and examples Also see

## Syntax

```
set_defaults { category| _all} [, permanently]
```

where category is one of memory | output | interface \| graphics | efficiency network | update | trace | mata | other

## Description

set_defaults resets settings made by set to the original default settings that were shipped with Stata.
set_defaults _all resets all the categories, whereas set defaults category resets only the settings for the specified category.

## Option

permanently specifies that, in addition to making the change right now, the settings be remembered and become the default settings when you invoke Stata.

## Remarks and examples

$>$ Example 1
To assist us in debugging a new command, we modified some of the trace settings. To return them to their original values, we type

```
. set_defaults trace
-> set trace off
-> set tracedepth 32000
-> set traceexpand on
-> set tracesep on
-> set traceindent on
-> set tracenumber off
-> set tracehilite ""
(preferences reset)
```


## Also see

[R] query - Display system parameters
[R] set - Overview of system parameters
[M-3] mata set - Set and display Mata system parameters

## Title

set emptycells - Set what to do with empty cells in interactions

## Syntax Description Option Remarks and examples Also see

## Syntax

```
set emptycells { keep|drop} [, permanently]
```


## Description

set emptycells allows you to control how Stata handles interaction terms with empty cells. Stata can keep empty cells or drop them. The default is to keep empty cells.

## Option

permanently specifies that, in addition to making the change right now, the setting be remembered and become the default setting when you invoke Stata.

## Remarks and examples

By default, Stata keeps empty cells so they can be reported in the coefficient table. For example, type
. use http://www.stata-press.com/data/r13/auto
. regress mpg rep78\#foreign, baselevels
and you will see a regression of mpg on 10 indicator variables because rep78 takes on 5 values and foreign takes on 2 values in the auto dataset. Two of those cells will be reported as empty because the data contain no observations of foreign cars with a rep78 value of 1 or 2.

Many real datasets contain a large number of empty cells, and this could cause the "matsize too small" error, r(908). In that case, type
. set emptycells drop
to get Stata to drop empty cells from the list of coefficients. If you commonly fit models with empty cells, you can permanently set Stata to drop empty cells by typing the following:

```
. set emptycells drop, permanently
```


## Also see

[R] set - Overview of system parameters

## Title

set seed - Specify initial value of random-number seed

## Syntax Description Remarks and examples Also see

## Syntax

```
set seed #
```

set seed statecode
where
\# is any number between 0 and $2^{31}-1(2,147,483,647)$, and statecode is a random-number state previously obtained from creturn value c(seed).

## Description

set seed \# specifies the initial value of the random-number seed used by the random-number functions, such as runiform() and rnormal().
set seed statecode resets the state of the random-number functions to the value specified, which is a state previously obtained from creturn value $c$ (seed).

## Remarks and examples

Remarks are presented under the following headings:

```
Examples
Setting the seed
How to choose a seed
Do not set the seed too often
Preserving and restoring the random-number generator state
```


## Examples

1. Specify initial value of random-number seed
. set seed 339487731
2. Create variable $u$ containing uniformly distributed pseudorandom numbers on the interval $[0,1)$
```
. generate u = runiform()
```

3. Create variable $z$ containing normally distributed random numbers with mean 0 and standard deviation 1
```
. generate z = rnormal()
```

4. Obtain state of pseudorandom-number generator and store it in a local macro named state
. local state $=c($ seed $)$
5. Restore pseudorandom-number generator state to that previously stored in local macro named state
. set seed 'state'

## Setting the seed

Stata's random-number generation functions, such as runiform() and rnormal(), do not really produce random numbers. These functions are deterministic algorithms that produce numbers that can pass for random. runiform() produces numbers that can pass for independent draws from a rectangular distribution over $[0,1)$; rnormal () produces numbers that can pass for independent draws from $N(0,1)$. Stata's random-number functions are formally called pseudorandom-number functions.

The sequences these functions produce are determined by the seed, which is just a number and which is set to 123456789 every time Stata is launched. This means that runiform() produces the same sequence each time you start Stata. The first time you use runiform() after Stata is launched, runiform() returns 0.136984078446403146 . The second time you use it, runiform() returns 0.643220667960122228 . The third time you use it, ....

To obtain different sequences, you must specify different seeds using the set seed command. You might specify the seed 472195 :

```
. set seed 472195
```

If you were now to use runiform (), the first call would return 0.247166610788553953 , the second call would return 0.593119932804256678 , and so on. Whenever you set seed 472195 , runiform () will return those numbers the first two times you use it.

Thus you set the seed to obtain different pseudorandom sequences from the pseudorandom-number functions.

If you record the seed you set, pseudorandom results such as results from a simulation or imputed values from mi impute can be reproduced later. Whatever you do after setting the seed, if you set the seed to the same value and repeat what you did, you will obtain the same results.

## How to choose a seed

Your best choice for the seed is an element chosen randomly from the set $\{0,1, \ldots, 2,147,483,647\}$. We recommend that, but that is difficult to achieve because finding easy-to-access, truly random sources is difficult.

One person we know uses digits from the serial numbers from dollar bills he finds in his wallet. Of course, the numbers he obtains are not really random, but they are good enough, and they are probably a good deal more random than the seeds most people choose. Some people use dates and times, although we recommend against that because, over the day, it just gets later and later, and that is a pattern. Others try to make up a random number, figuring if they include enough digits, the result just has to be random. This is a variation on the five-second rule for dropped food, and we admit to using both of these rules.

It does not really matter how you set the seed, as long as there is no obvious pattern in the seeds that you set and as long as you do not set the seed too often during a session.

Nonetheless, here are two methods that we have seen used but you should not use:

1. The first time you set the seed, you set the number 1 . The next time, you set 2 , and then 3 , and so on. Variations on this included setting 1001, 1002, 1003, ..., or setting 1001, 2001, 3001, and so on.

Do not follow any of these procedures. The seeds you set must not exhibit a pattern.
2. To set the seed, you obtain a pseudorandom number from runiform() and then use the digits from that to form the seed.
This is a bad idea because the pseudorandom-number generator can converge to a cycle. If you obtained the pseudorandom-number generator unrelated to those in Stata, this would work well, but then you would have to find a rule to set the first generator's seed. In any case, the pseudorandom-number generators in Stata are all closely related, and so you must not follow this procedure.
Choosing seeds that do not exhibit a pattern is of great importance. That the seeds satisfy the other properties of randomness is minor by comparison.

## Do not set the seed too often

We cannot emphasize this enough: Do not set the seed too often.
To see why this is such a bad idea, consider the limiting case: You set the seed, draw one pseudorandom number, reset the seed, draw again, and so continue. The pseudorandom numbers you obtain will be nothing more than the seeds you run through a mathematical function. The results you obtain will not pass for random unless the seeds you choose pass for random. If you already had such numbers, why are you even bothering to use the pseudorandom-number generator?

The definition of too often is more than once per problem.
If you are running a simulation of 10,000 replications, set the seed at the start of the simulation and do not reset it until the 10,000 th replication is finished. The pseudorandom-number generators provided by Stata have long periods. The longer you go between setting the seed, the more random-like are the numbers produced.

It is sometimes useful later to be able to reproduce in isolation any one of the replications, and so you might be tempted to set the seed to a known value for each of the replications. We negatively mentioned setting the seed to $1,2, \ldots$, and it is in exactly such situations that we have seen this done. The advantage, however, is that you could reproduce the fifth replication merely by setting the seed to 5 and then repeating whatever it is that is to be replicated. If this is your goal, you do not need to reset the seed. You can record the state of the random-number generator, save the state with your replication results, and then use the recorded states later to reproduce whichever of the replications that you wish. This will be discussed in Preserving and restoring the random-number generator state.

There is another reason you might be tempted to set the seed more than once per problem. It sometimes happens that you run a simulation, let's say for 5,000 replications, and then you decide you should have run it for 10,000 replications. Instead of running all 10,000 replications afresh, you decide to save time by running another 5,000 replications and then combining those results with your previous 5,000 results. That is okay. We at StataCorp do this kind of thing. If you do this, it is important that you set the seed especially well, particularly if you repeat this process to add yet another 5,000 replications. It is also important that in each run there be a large enough number of replications, which is say thousands of them.

Even so, do not do this: You want 500,000 replications. To obtain them, you run in batches of 1,000 , setting the seed 500 times. Unless you have a truly random source for the seeds, it is unlikely you can produce a patternless sequence of 500 seeds. The fact that you ran 1,000 replications in between choosing the seeds does not mitigate the requirement that there be no pattern to the seeds you set.

In all cases, the best solution is to set the seed only once and then use the method we suggest in the next section.

## Preserving and restoring the random-number generator state

In the previous section, we discussed the case in which you might be tempted to set the seed more frequently than otherwise necessary, either to save time or to be able to rerun any one of the replications. In such cases, there is an alternative to setting a new seed: recording the state of the pseudorandom-number generator and then restoring the state later should the need arise.

The state of the random-number generator is a string that looks like this:
Xb5804563c43f462544a474abacbdd93d00021fb3
You can obtain the state from c (seed):

```
. display c(seed)
Xb5804563c43f462544a474abacbdd93d00021fb3
```

The name c (seed) is unfortunate because it suggests that Xb5804563c43f462544a474abacbdd93d00021fb3 is nothing more than a seed such as 1073741823 in a different guise. It is not. A better name for c (seed) would have been c(rng_state). The state string specifies an entry point into the sequence produced by the pseudorandom-number generator. Let us explain.

The best way to use a pseudorandom-number generator would be to choose a seed once, draw random numbers until you use up the generator, and then get a new generator and choose a new key. Pseudorandom-number generators have a period, after which they repeat the original sequence. That is what we mean by using up a generator. The period of the pseudorandom-number generator that Stata is currently using is over $2^{123}$. Stata uses the KISS generator. It is difficult to imagine that you could ever use up KISS.

The string reported by $c$ (seed) reports an encoded form of the information necessary for Stata to reestablish exactly where it is located in the pseudorandom-number generator's sequence.

We are not seriously suggesting you choose only one seed over your entire lifetime, but let's look at how you might do that. Sometime after birth, when you needed your first random number, you would set your seed,

```
. set seed 1073741823
```

On that day, you would draw, say, 10,000 pseudorandom numbers, perhaps to impute some missing values. Being done for the day, you type
. display c(seed)
X15b512f3b2143ab434f1c92f4e7058e400023bc3
The next day, after launching Stata, you type
. set seed X15b512f3b2143ab434f1c92f4e7058e400023bc3
When you type set seed followed by a state string rather than a number, instead of setting the seed, Stata reestablishes the previous state. Thus the next time you draw a pseudorandom number, Stata will produce the 10,001 st result after setting seed 1073741823 . Let's assume that you draw 100,000 numbers this day. Done for the day, you display c (seed).
. display c(seed)
X5d13d693a72ad0602b093cc4f61e07a500020381
On the third day, after setting the seed to the string above, you will be in a position to draw the 110,001 st pseudorandom number.

In this way, you would eat your way though the $2^{123}$ random numbers, but you would be unlikely ever to make it to the end. Assuming you did this every day for 100 years, to arrive at the end of the sequence you would need to consume $2.9 \mathrm{e}+32$ pseudorandom numbers per day.

We do not expect you to set the seed just once in your life, but using the state string makes it easy to set the seed just once for a problem.

When we do simulations at StataCorp, we record c (seed) for each replication. Just like everybody else, we record results from replications as observations in datasets; we just happen to have an extra variable in the dataset, namely, a string variable named state. That string is filled in observation by observation from the then-current values of $c$ (seed), which is a function and so can be used in any context that a function can be used in Stata.

Anytime we want to reproduce a particular replication, we thus have the information we need to reset the pseudorandom-number generator, and having it in the dataset is convenient because we had to go there anyway to determine which replication we wanted to reproduce.

In addition to recording each of the state strings for each replication, we record the closing value of $c$ (seed) as a note, which is easy enough to do:
. note: closing state 'c(seed)'
If we want to add more replications later, we have a state string that we can use to continue from where we left off.

## Also see

$[R]$ set - Overview of system parameters
[D] functions - Functions

## Title

set showbaselevels - Display settings for coefficient tables

Syntax Description Option Remarks and examples Also see

## Syntax

```
set showbaselevels {on|off|all} [, permanently]
set showemptycells {on|off } [, permanently]
set showomitted {on|off } [, permanently]
set fvlabel {on|off} [, permanently]
set fvwrap # [, permanently]
set fvwrapon {word|width} [, permanently]
```


## Description

set showbaselevels specifies whether to display base levels of factor variables and their interactions in coefficient tables. set showbaselevels on specifies that base levels be reported for factor variables and for interactions whose bases cannot be inferred from their component factor variables. set showbaselevels all specifies that all base levels of factor variables and interactions be reported.
set showemptycells specifies whether to display empty cells in coefficient tables.
set showomitted specifies whether to display omitted coefficients in coefficient tables.
set fvlabel specifies whether to display factor-variable value labels in coefficient tables. set fvlabel on, the default, specifies that the labels be displayed. set fvlabel off specifies that the levels of factor variables rather than the labels be displayed.
set fvwrap \# specifies that long value labels wrap \# lines in the coefficient table. The default is set fvwrap 1, which means that long value labels will be abbreviated to fit on one line.
set fvwrapon specifies whether value labels that wrap will break at word boundaries or break based on available space. set fvwrapon word, the default, specifies that value labels break at word boundaries. set fvwrapon width specifies that value labels break based on available space.

## Option

permanently specifies that, in addition to making the change right now, the setting be remembered and become the default setting when you invoke Stata.

## Remarks and examples

## Example 1

We illustrate the first three set commands using cholesterol2.dta.

| (Artificial cholesterol data, empty cells) <br> - generate $\mathrm{x}=$ race <br> . regress chol race\#\#agegrp x <br> note: 2.race\#2.agegrp identifies no observations in the sample <br> note: $x$ omitted because of collinearity |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Source | SS | df | MS | Number of obs | 70 |
|  |  |  |  | F ( 13, 56) | 13.51 |
| Model | 15751.6113 | 13 | 1211.66241 | Prob > F | 0.0000 |
| Residual | 5022.71559 | 56 | 89.6913498 | R -squared | 0.7582 |
|  |  |  |  | Adj R-squared | 0.7021 |
| Total | 20774.3269 | 69 | 301.077201 | Root MSE | 9.4706 |


| chol | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Conf | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| race |  |  |  |  |  |  |
| white | 12.84185 | 5.989703 | 2.14 | 0.036 | . 8430383 | 24.84067 |
| other | -. 167627 | 5.989703 | -0.03 | 0.978 | -12.16644 | 11.83119 |
| agegrp |  |  |  |  |  |  |
| 20-29 | 17.24681 | 5.989703 | 2.88 | 0.006 | 5.247991 | 29.24562 |
| 30-39 | 31.43847 | 5.989703 | 5.25 | 0.000 | 19.43966 | 43.43729 |
| 40-59 | 34.86613 | 5.989703 | 5.82 | 0.000 | 22.86732 | 46.86495 |
| 60-79 | 44.43374 | 5.989703 | 7.42 | 0.000 | 32.43492 | 56.43256 |
| race\#agegrp |  |  |  |  |  |  |
| white 20-29 | 0 | (empty) |  |  |  |  |
| white 30-39 | -22.83983 | 8.470719 | -2.70 | 0.009 | -39.80872 | -5.870939 |
| white 40-59 | -14.67558 | 8.470719 | -1.73 | 0.089 | -31.64447 | 2.293306 |
| white 60-79 | -10.51115 | 8.470719 | -1.24 | 0.220 | -27.48004 | 6.457735 |
| other 20-29 | -6.054425 | 8.470719 | -0.71 | 0.478 | -23.02331 | 10.91446 |
| other 30-39 | -11.48083 | 8.470719 | -1.36 | 0.181 | -28.44971 | 5.488063 |
| other 40-59 | -. 6796112 | 8.470719 | -0.08 | 0.936 | -17.6485 | 16.28928 |
| other 60-79 | -1.578052 | 8.470719 | -0.19 | 0.853 | -18.54694 | 15.39084 |
| x | 0 | (omitted) |  |  |  |  |
| _cons | 175.2309 | 4.235359 | 41.37 | 0.000 | 166.7464 | 183.7153 |

[^77]. set showomitted off
. set showbaselevels all
. regress chol race\#\#agegrp x
note: 2.race\#2.agegrp identifies no observations in the sample
note: $x$ omitted because of collinearity

| Source | SS | $d f$ | MS |
| ---: | :---: | :---: | :---: |
| Model | 15751.6113 | 13 | 1211.66241 |
| Residual | 5022.71559 | 56 | 89.6913498 |
| Total | 20774.3269 | 69 | 301.077201 |


| Number of obs | $=$ | 70 |
| :--- | ---: | ---: |
| $\mathrm{~F}(13$, | $56)$ | $=13.51$ |
| Prob $>\mathrm{F}$ | $=$ | 0.0000 |
| R-squared | $=$ | 0.7582 |
| Adj R-squared | $=$ | 0.7021 |
| Root MSE | $=$ | 9.4706 |


| chol | Coef. | Std. Err. | t | $\mathrm{P}>\|\mathrm{t}\|$ | [95\% Conf | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| race |  |  |  |  |  |  |
| black | 0 | (base) |  |  |  |  |
| white | 12.84185 | 5.989703 | 2.14 | 0.036 | . 8430383 | 24.84067 |
| other | -. 167627 | 5.989703 | -0.03 | 0.978 | -12.16644 | 11.83119 |
| agegrp |  |  |  |  |  |  |
| 10-19 | 0 | (base) |  |  |  |  |
| 20-29 | 17.24681 | 5.989703 | 2.88 | 0.006 | 5.247991 | 29.24562 |
| 30-39 | 31.43847 | 5.989703 | 5.25 | 0.000 | 19.43966 | 43.43729 |
| 40-59 | 34.86613 | 5.989703 | 5.82 | 0.000 | 22.86732 | 46.86495 |
| 60-79 | 44.43374 | 5.989703 | 7.42 | 0.000 | 32.43492 | 56.43256 |
| race\#agegrp |  |  |  |  |  |  |
| black 10-19 | 0 | (base) |  |  |  |  |
| black 20-29 | 0 | (base) |  |  |  |  |
| black 30-39 | 0 | (base) |  |  |  |  |
| black 40-59 | 0 | (base) |  |  |  |  |
| black 60-79 | 0 | (base) |  |  |  |  |
| white 10-19 | 0 | (base) |  |  |  |  |
| white 30-39 | -22.83983 | 8.470719 | -2.70 | 0.009 | -39.80872 | -5.870939 |
| white 40-59 | -14.67558 | 8.470719 | -1.73 | 0.089 | -31.64447 | 2.293306 |
| white 60-79 | -10.51115 | 8.470719 | -1.24 | 0.220 | -27.48004 | 6.457735 |
| other 10-19 | 0 | (base) |  |  |  |  |
| other 20-29 | -6.054425 | 8.470719 | -0.71 | 0.478 | -23.02331 | 10.91446 |
| other 30-39 | -11.48083 | 8.470719 | -1.36 | 0.181 | -28.44971 | 5.488063 |
| other 40-59 | -. 6796112 | 8.470719 | -0.08 | 0.936 | -17.6485 | 16.28928 |
| other 60-79 | -1.578052 | 8.470719 | -0.19 | 0.853 | -18.54694 | 15.39084 |
| _cons | 175.2309 | 4.235359 | 41.37 | 0.000 | 166.7464 | 183.7153 |

To restore the display of empty cells, omitted predictors, and baselevels to their command-specific default behavior, type
. set showemptycells
. set showomitted
. set showbaselevels
. regress chol race\#\#agegrp x
note: 2.race\#2.agegrp identifies no observations in the sample
note: $x$ omitted because of collinearity

| Source | SS | df | MS | Number of obs | 70 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | F ( 13, 56) | 13.51 |
| Model | 15751.6113 | 13 | 1211.66241 | Prob > F | 0.0000 |
| Residual | 5022.71559 | 56 | 89.6913498 | R -squared | 0.7582 |
|  |  |  |  | Adj R-squared | 0.7021 |
| Total | 20774.3269 | 69 | 301.077201 | Root MSE | 9.4706 |


| chol | Coef. | Std. Err | t | $P>\|t\|$ | [95\% Conf | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| race |  |  |  |  |  |  |
| white | 12.84185 | 5.989703 | 2.14 | 0.036 | . 8430383 | 24.84067 |
| other | -. 167627 | 5.989703 | -0.03 | 0.978 | -12.16644 | 11.83119 |
| agegrp |  |  |  |  |  |  |
| 20-29 | 17.24681 | 5.989703 | 2.88 | 0.006 | 5.247991 | 29.24562 |
| 30-39 | 31.43847 | 5.989703 | 5.25 | 0.000 | 19.43966 | 43.43729 |
| 40-59 | 34.86613 | 5.989703 | 5.82 | 0.000 | 22.86732 | 46.86495 |
| 60-79 | 44.43374 | 5.989703 | 7.42 | 0.000 | 32.43492 | 56.43256 |
| race\#agegrp |  |  |  |  |  |  |
| white 30-39 | -22.83983 | 8.470719 | -2.70 | 0.009 | -39.80872 | -5.870939 |
| white 40-59 | -14.67558 | 8.470719 | -1.73 | 0.089 | -31.64447 | 2.293306 |
| white 60-79 | -10.51115 | 8.470719 | -1.24 | 0.220 | -27.48004 | 6.457735 |
| other 20-29 | -6.054425 | 8.470719 | -0.71 | 0.478 | -23.02331 | 10.91446 |
| other 30-39 | -11.48083 | 8.470719 | -1.36 | 0.181 | -28.44971 | 5.488063 |
| other 40-59 | -. 6796112 | 8.470719 | -0.08 | 0.936 | -17.6485 | 16.28928 |
| other 60-79 | -1.578052 | 8.470719 | -0.19 | 0.853 | -18.54694 | 15.39084 |
| x | 0 | (omitted) |  |  |  |  |
| _cons | 175.2309 | 4.235359 | 41.37 | 0.000 | 166.7464 | 183.7153 |

## Example 2

We illustrate the last three set commands using jaw.dta.
. use http://www.stata-press.com/data/r13/jaw, clear
(Table 4.6 Two-Way Unbalanced Data for Fractures of the Jaw -- Rencher (1998))
. mvreg y1 y2 y3 = i.fracture

| Equation | Obs | Parms | RMSE | "R-sq" | F | $P$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| y1 | 27 | 3 | 10.42366 | 0.2966 | 5.060804 | 0.0147 |
| y2 | 27 | 3 | 6.325398 | 0.1341 | 1.858342 | 0.1777 |
| y3 | 27 | 3 | 5.976973 | 0.1024 | 1.368879 | 0.2735 |


|  | Coef . | Std. Err. | t | $p>\|t\|$ | [95\% Conf | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| y1 |  |  |  |  |  |  |
| fracture two compou.. | -8.833333 | 4.957441 | -1.78 | 0.087 | -19.06499 | 1.398322 |
| one simple.. | 6 | 5.394759 | 1.11 | 0.277 | -5.134235 | 17.13423 |
| _cons | 37 | 3.939775 | 9.39 | 0.000 | 28.8687 | 45.1313 |
| y2 |  |  |  |  |  |  |
| fracture two compou.. | -5.761905 | 3.008327 | -1.92 | 0.067 | -11.97079 | . 446977 |
| one simple.. | -3.053571 | 3.273705 | -0.93 | 0.360 | -9.810166 | 3.703023 |
| _cons | 38.42857 | 2.390776 | 16.07 | 0.000 | 33.49425 | 43.36289 |
| y3 |  |  |  |  |  |  |
| fracture two compou.. | 4.261905 | 2.842618 | 1.50 | 0.147 | -1.60497 | 10.12878 |
| one simple.. | . 9285714 | 3.093377 | 0.30 | 0.767 | -5.455846 | 7.312989 |
| _cons | 58.57143 | 2.259083 | 25.93 | 0.000 | 53.90891 | 63.23395 |

## . set fvwrap 2

. mvreg y1 y2 y3 = i.fracture

| Equation | Obs | Parms | RMSE | "R-sq" | F | P |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| y1 | 27 | 3 | 10.42366 | 0.2966 | 5.060804 | 0.0147 |
| y2 | 27 | 3 | 6.325398 | 0.1341 | 1.858342 | 0.1777 |
| y3 | 27 | 3 | 5.976973 | 0.1024 | 1.368879 | 0.2735 |


|  | Coef . | Std. Err. | t | $P>\|t\|$ | [95\% Conf. | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| y1 |  |  |  |  |  |  |
| fracture two compound |  |  |  |  |  |  |
| fractures | -8.833333 | 4.957441 | -1.78 | 0.087 | -19.06499 | 1.398322 |
| fracture | 6 | 5.394759 | 1.11 | 0.277 | -5.134235 | 17.13423 |
| _cons | 37 | 3.939775 | 9.39 | 0.000 | 28.8687 | 45.1313 |
| y2 |  |  |  |  |  |  |
| fracture <br> two compound |  |  |  |  |  |  |
| fractures | -5.761905 | 3.008327 | -1.92 | 0.067 | -11.97079 | . 446977 |
| one simple fracture | -3.053571 | 3.273705 | -0.93 | 0.360 | -9.810166 | 3.703023 |
| _cons | 38.42857 | 2.390776 | 16.07 | 0.000 | 33.49425 | 43.36289 |
| y3 |  |  |  |  |  |  |
| fracture <br> two compound |  |  |  |  |  |  |
| fractures | 4.261905 | 2.842618 | 1.50 | 0.147 | -1.60497 | 10.12878 |
| one simple |  |  |  |  |  |  |
| fracture | . 9285714 | 3.093377 | 0.30 | 0.767 | -5.455846 | 7.312989 |
| _cons | 58.57143 | 2.259083 | 25.93 | 0.000 | 53.90891 | 63.23395 |

. set fvwrapon width
. mvreg y1 y2 y3 = i.fracture

| Equation | Obs | Parms | RMSE | "R-sq" | F | P |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| y1 | 27 | 3 | 10.42366 | 0.2966 | 5.060804 | 0.0147 |
| y2 | 27 | 3 | 6.325398 | 0.1341 | 1.858342 | 0.1777 |
| y3 | 27 | 3 | 5.976973 | 0.1024 | 1.368879 | 0.2735 |


|  | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Conf | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| y1 |  |  |  |  |  |  |
| fracture <br> two compound fractures | -8.833333 | 4.957441 | -1.78 | 0.087 | -19.06499 | 1.398322 |
| racture | 6 | 5.394759 | 1.11 | 0.277 | -5.134235 | 17.13423 |
| _cons | 37 | 3.939775 | 9.39 | 0.000 | 28.8687 | 45.1313 |
| y2 |  |  |  |  |  |  |
| fracture <br> two compound |  |  |  |  |  |  |
| fractures | -5.761905 | 3.008327 | -1.92 | 0.067 | -11.97079 | . 446977 |
| racture | -3.053571 | 3.273705 | -0.93 | 0.360 | -9.810166 | 3.703023 |
| _cons | 38.42857 | 2.390776 | 16.07 | 0.000 | 33.49425 | 43.36289 |
| y3 |  |  |  |  |  |  |
| fracture two compound |  |  |  |  |  |  |
| fractures | 4.261905 | 2.842618 | 1.50 | 0.147 | -1.60497 | 10.12878 |
| one simple f racture | . 9285714 | 3.093377 | 0.30 | 0.767 | -5.455846 | 7.312989 |
| _cons | 58.57143 | 2.259083 | 25.93 | 0.000 | 53.90891 | 63.23395 |

. set showfvlabel off
. mvreg y1 y2 y3 = i.fracture

| Equation | Obs | Parms | RMSE | "R-sq" | F | P |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| y1 | 27 | 3 | 10.42366 | 0.2966 | 5.060804 | 0.0147 |
| y2 | 27 | 3 | 6.325398 | 0.1341 | 1.858342 | 0.1777 |
| y3 | 27 | 3 | 5.976973 | 0.1024 | 1.368879 | 0.2735 |


|  | Coef . | Std. Err | t | $P>\|t\|$ | [95\% Conf. | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| y1 |  |  |  |  |  |  |
| fracture |  |  |  |  |  |  |
| 2 | -8.833333 | 4.957441 | -1.78 | 0.087 | -19.06499 | 1.398322 |
| 3 | 6 | 5.394759 | 1.11 | 0.277 | -5.134235 | 17.13423 |
| _cons | 37 | 3.939775 | 9.39 | 0.000 | 28.8687 | 45.1313 |
| y2 |  |  |  |  |  |  |
| fracture |  |  |  |  |  |  |
| 2 | -5.761905 | 3.008327 | -1.92 | 0.067 | -11.97079 | . 446977 |
| 3 | -3.053571 | 3.273705 | -0.93 | 0.360 | -9.810166 | 3.703023 |
| _cons | 38.42857 | 2.390776 | 16.07 | 0.000 | 33.49425 | 43.36289 |
| y3 |  |  |  |  |  |  |
| fracture |  |  |  |  |  |  |
| 2 | 4.261905 | 2.842618 | 1.50 | 0.147 | -1.60497 | 10.12878 |
| 3 | . 9285714 | 3.093377 | 0.30 | 0.767 | -5.455846 | 7.312989 |
| _cons | 58.57143 | 2.259083 | 25.93 | 0.000 | 53.90891 | 63.23395 |

To restore these last three set commands to their defaults, type

| Equation | Obs | Parms | RMSE | "R-sq" | F | P |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| y1 | 27 | 3 | 10.42366 | 0.2966 | 5.060804 | 0.0147 |
| y2 | 27 | 3 | 6.325398 | 0.1341 | 1.858342 | 0.1777 |
| y3 | 27 | 3 | 5.976973 | 0.1024 | 1.368879 | 0.2735 |


|  | Coef. | Std. Err . | t | $P>\|t\|$ | [95\% Con | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| y1 |  |  |  |  |  |  |
| fracture <br> two compou.. | -8.833333 | 4.957441 | -1.78 | 0.087 | -19.06499 | 1.398322 |
| one simple.. | 6 | 5.394759 | 1.11 | 0.277 | -5.134235 | 17.13423 |
| _cons | 37 | 3.939775 | 9.39 | 0.000 | 28.8687 | 45.1313 |
| y2 |  |  |  |  |  |  |
| fracture two compou.. | -5.761905 | 3.008327 | -1.92 | 0.067 | -11.97079 | . 446977 |
| one simple.. | -3.053571 | 3.273705 | -0.93 | 0.360 | -9.810166 | 3.703023 |
| _cons | 38.42857 | 2.390776 | 16.07 | 0.000 | 33.49425 | 43.36289 |
| y3 |  |  |  |  |  |  |
| fracture two compou.. | 4.261905 | 2.842618 | 1.50 | 0.147 | -1.60497 | 10.12878 |
| one simple.. | . 9285714 | 3.093377 | 0.30 | 0.767 | -5.455846 | 7.312989 |
| _cons | 58.57143 | 2.259083 | 25.93 | 0.000 | 53.90891 | 63.23395 |

## Also see

[R] set - Overview of system parameters
[R] query - Display system parameters

## Title

signrank - Equality tests on matched data

| Syntax | Menu | Description | Remarks and examples |
| :--- | :--- | :--- | :--- |
| Stored results | Methods and formulas | References | Also see |

## Syntax

Wilcoxon matched-pairs signed-ranks test
signrank varname $=\exp [$ if $][$ in $]$

Sign test of matched pairs
signtest varname $=\exp [$ if $][$ in $]$
by is allowed with signrank and signtest; see [D] by.

## Menu

## signrank

Statistics $>$ Nonparametric analysis $>$ Tests of hypotheses $>$ Wilcoxon matched-pairs signed-rank test

## signtest

Statistics $>$ Nonparametric analysis $>$ Tests of hypotheses $>$ Test equality of matched pairs

## Description

signrank tests the equality of matched pairs of observations by using the Wilcoxon matched-pairs signed-ranks test (Wilcoxon 1945). The null hypothesis is that both distributions are the same.
signtest also tests the equality of matched pairs of observations (Arbuthnott [1710], but better explained by Snedecor and Cochran [1989]) by calculating the differences between varname and the expression. The null hypothesis is that the median of the differences is zero; no further assumptions are made about the distributions. This, in turn, is equivalent to the hypothesis that the true proportion of positive (negative) signs is one-half.

For equality tests on unmatched data, see [R] ranksum.

## Remarks and examples

> Example 1: signrank
We are testing the effectiveness of a new fuel additive. We run an experiment with 12 cars. We first run each car without the fuel treatment and measure the mileage. We then add the fuel treatment and repeat the experiment. The results of the experiment are

| Without <br> treatment | With <br> treatment | Without <br> treatment | With <br> treatment |
| :---: | :---: | :---: | :---: |
| 20 | 24 | 18 | 17 |
| 23 | 25 | 24 | 28 |
| 21 | 21 | 20 | 24 |
| 25 | 22 | 24 | 27 |
| 18 | 23 | 23 | 21 |
| 17 | 18 | 19 | 23 |

We create two variables called mpg1 and mpg2, representing mileage without and with the treatment, respectively. We can test the null hypothesis that the treatment had no effect by typing

```
. use http://www.stata-press.com/data/r13/fuel
. signrank mpg1=mpg2
Wilcoxon signed-rank test
\begin{tabular}{r|rrr} 
sign & obs & sum ranks & expected \\
\hline positive & 3 & 13.5 & 38.5 \\
negative & 8 & 63.5 & 38.5 \\
zero & 1 & 1 & 1 \\
\hline all & 12 & 78 & 78
\end{tabular}
unadjusted variance
    162.50
adjustment for ties -1.62
adjustment for zeros -0.25
adjusted variance
    160.62
Ho: mpg1 = mpg2
    z = -1.973
    Prob > |z| = 0.0485
```

The output indicates that we can reject the null hypothesis at any level above 4.85\%.

## > Example 2: signtest

signtest tests that the median of the differences is zero, making no further assumptions, whereas signrank assumed that the distributions are equal as well. Using the data above, we type


```
Two-sided test:
    Ho: median of mpg1 - mpg2 = 0 vs.
    Ha: median of mpg1 - mpg2 != 0
        Pr(#positive >= 8 or #negative >= 8) =
            min(1, 2*Binomial(n = 11, x >= 8, p = 0.5)) = 0.2266
```

The summary table indicates that there were three comparisons for which mpg1 exceeded mpg2, eight comparisons for which mpg2 exceeded mpg1, and one comparison for which they were the same.

The output below the summary table is based on the binomial distribution. The significance of the one-sided test, where the alternative hypothesis is that the median of mpg2-mpg1 is greater than zero, is 0.1133 . The significance of the two-sided test, where the alternative hypothesis is simply that the median of the differences is different from zero, is $0.2266=2 \times 0.1133$.

## Stored results

signrank stores the following in r() :
Scalars

| $r$ (N_neg) | number of negative comparisons | $r($ sum_neg $)$ | sum of the negative ranks |
| :--- | :--- | :--- | :--- |
| $r$ (N_pos) | number of positive comparisons | $r(z)$ | $z$ statistic |
| $r$ (N_tie) | number of tied comparisons | $r\left(V a r \_a\right)$ | adjusted variance |
| $r$ (sum_pos) | sum of the positive ranks |  |  |

signtest stores the following in $r()$ :
Scalars

| $r\left(N \_n e g\right)$ | number of negative comparisons | $r\left(p \_2\right)$ | two-sided probability <br> one-sided probability of negative |
| :--- | :--- | :--- | :--- |
| $r$ (N_pos) | number of positive comparisons | $r\left(p \_n e g\right)$ | comparison <br> $r$ (N_tie) |
| number of tied comparisons | $r\left(p \_p o s\right)$ | one-sided probability of positive <br> comparison |  |

## Methods and formulas

For a practical introduction to these techniques with an emphasis on examples rather than theory, see Bland (2000) or Sprent and Smeeton (2007). For a summary of these tests, see Snedecor and Cochran (1989).

Methods and formulas are presented under the following headings:
signrank
signtest

## signrank

Both the sign test and Wilcoxon signed-rank tests test the null hypothesis that the distribution of a random variable $D=$ varname - exp has median zero. The sign test makes no additional assumptions, but the Wilcoxon signed-rank test makes the additional assumption that the distribution of $D$ is symmetric. If $D=X_{1}-X_{2}$, where $X_{1}$ and $X_{2}$ have the same distribution, then it follows that the distribution of $D$ is symmetric about zero. Thus the Wilcoxon signed-rank test is often described as a test of the hypothesis that two distributions are the same, that is, $X_{1} \sim X_{2}$.

Let $d_{j}$ denote the difference for any matched pair of observations,

$$
d_{j}=x_{1 j}-x_{2 j}=\text { varname }-\exp
$$

for $j=1,2, \ldots, n$.
Rank the absolute values of the differences, $\left|d_{j}\right|$, and assign any tied values the average rank. Consider the signs of $d_{j}$, and let

$$
r_{j}=\operatorname{sign}\left(d_{j}\right) \operatorname{rank}\left(\left|d_{j}\right|\right)
$$

be the signed ranks. The test statistic is

$$
T_{\mathrm{obs}}=\sum_{j=1}^{n} r_{j}=(\text { sum of ranks for }+ \text { signs })-(\text { sum of ranks for }- \text { signs })
$$

The null hypothesis is that the distribution of $d_{j}$ is symmetric about 0 . Hence the likelihood is unchanged if we flip signs on the $d_{j}$, and thus the randomization datasets are the $2^{n}$ possible sign changes for the $d_{j}$. Thus the randomization distribution of our test statistic $T$ can be computed by considering all the $2^{n}$ possible values of

$$
T=\sum_{j=1}^{n} S_{j} r_{j}
$$

where the $r_{j}$ are the observed signed ranks (considered fixed) and $S_{j}$ is either +1 or -1 .
With this distribution, the mean and variance of $T$ are given by

$$
E(T)=0 \quad \text { and } \quad \operatorname{Var}_{\text {adj }}(T)=\sum_{j=1}^{n} r_{j}^{2}
$$

The test statistic for the Wilcoxon signed-rank test is often expressed (equivalently) as the sum of the positive signed-ranks, $T_{+}$, where

$$
E\left(T_{+}\right)=\frac{n(n+1)}{4} \quad \text { and } \quad \operatorname{Var}_{\mathrm{adj}}\left(T_{+}\right)=\frac{1}{4} \sum_{j=1}^{n} r_{j}^{2}
$$

Zeros and ties do not affect the theory above, and the exact variance is still given by the above formula for $\operatorname{Var}_{\mathrm{adj}}\left(T_{+}\right)$. When $d_{j}=0$ is observed, $d_{j}$ will always be zero in each of the randomization datasets (using $\operatorname{sign}(0)=0$ ). When there are ties, you can assign averaged ranks for each group of ties and then treat them the same as the other ranks.

The "unadjusted variance" reported by signrank is the variance that the randomization distribution would have had if there had been no ties or zeros:

$$
\operatorname{Var}_{\mathrm{unadj}}\left(T_{+}\right)=\frac{1}{4} \sum_{j=1}^{n} j^{2}=\frac{n(n+1)(2 n+1)}{24}
$$

The adjustment for zeros is the change in the variance when the ranks for the zeros are signed to make $r_{j}=0$,

$$
\Delta \operatorname{Var}_{\text {zero adj }}\left(T_{+}\right)=-\frac{1}{4} \sum_{j=1}^{n_{0}} j^{2}=-\frac{n_{0}\left(n_{0}+1\right)\left(2 n_{0}+1\right)}{24}
$$

where $n_{0}$ is the number of zeros. The adjustment for ties is the change in the variance when the ranks (for nonzero observations) are replaced by averaged ranks:

$$
\Delta \operatorname{Var}_{\text {ties adj }}\left(T_{+}\right)=\operatorname{Var}_{\text {adj }}\left(T_{+}\right)-\operatorname{Var}_{\text {unadj }}\left(T_{+}\right)-\Delta \operatorname{Var}_{\text {zero adj }}\left(T_{+}\right)
$$

A normal approximation is used to calculate

$$
z=\frac{T_{+}-E\left(T_{+}\right)}{\sqrt{\operatorname{Var}_{\mathrm{adj}}\left(T_{+}\right)}}
$$

## signtest

The test statistic for the sign test is the number $n_{+}$of differences

$$
d_{j}=x_{1 j}-x_{2 j}=\text { varname }-\exp
$$

greater than zero. Assuming that the probability of a difference being equal to zero is exactly zero, then, under the null hypothesis, $n_{+} \sim \operatorname{binomial}(n, p=1 / 2)$, where $n$ is the total number of observations.

But what if some differences are zero? This question has a ready answer if you view the test from the perspective of Fisher's Principle of Randomization (Fisher 1935). Fisher's idea (stated in a modern way) was to look at a family of transformations of the observed data such that the a priori likelihood (under the null hypothesis) of the transformed data is the same as the likelihood of the observed data. The distribution of the test statistic is then produced by calculating its value for each of the transformed "randomization" datasets, assuming that each dataset is equally likely.

For the sign test, the "data" are simply the set of signs of the differences. Under the null hypothesis of the sign test, the probability that $d_{j}$ is less than zero is equal to the probability that $d_{j}$ is greater than zero. Thus you can transform the observed signs by flipping any number of them, and the set of signs will have the same likelihood. The $2^{n}$ possible sign changes form the family of randomization datasets. If you have no zeros, this procedure again leads to $n_{+} \sim \operatorname{binomial}(n, p=1 / 2)$.

If you do have zeros, changing their signs leaves them as zeros. So, if you observe $n_{0}$ zeros, each of the $2^{n}$ sign-change datasets will also have $n_{0}$ zeros. Hence, the values of $n_{+}$calculated over the sign-change datasets range from 0 to $n-n_{0}$, and the "randomization" distribution of $n_{+}$is $\operatorname{binomial}\left(n-n_{0}, p=1 / 2\right)$.

The work of Arbuthnott (1710) and later eighteenth-century contributions is discussed by Hald (2003, chap. 17).

Frank Wilcoxon (1892-1965) was born in Ireland to American parents. After working in various occupations (including merchant seaman, oil-well pump attendant, and tree surgeon), he settled in chemistry, gaining degrees from Rutgers and Cornell and employment from various companies. Working mainly on the development of fungicides and insecticides, Wilcoxon became interested in statistics in 1925 and made several key contributions to nonparametric methods. After retiring from industry, he taught statistics at Florida State until his death.

## References

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## Also see

[R] ranksum - Equality tests on unmatched data
$[\mathrm{R}]$ ttest $-t$ tests (mean-comparison tests)

## Title

simulate - Monte Carlo simulations

| Syntax | Description | Options | Remarks and examples |
| :--- | :--- | :--- | :--- |
| References | Also see |  |  |

## Syntax

simulate $[$ exp_list $], \underline{r e p s}(\#)[$ options $]:$ command
options
Description
nodots
noisily
trace
saving (filename, ...)
nolegend
verbose
seed (\#)
suppress replication dots
display any output from command
trace command
save results to filename
suppress table legend
display the full table legend
set random-number seed to \#

All weight types supported by command are allowed; see [U] 11.1.6 weight.

| exp_list contains | (name: elist) |
| :---: | :---: |
|  | elist |
|  | eexp |
| elist contains | newvar $=(\exp )$ |
|  | (exp) |
| eexp is | specname |
|  | [eqno]specname |
| specname is | _b |
|  | _b [] |
|  | _se |
|  | _se [] |
| eqno is | \#\# |
|  | name |

$\exp$ is a standard Stata expression; see [U] 13 Functions and expressions.
Distinguish between [], which are to be typed, and [], which indicate optional arguments.

## Description

simulate eases the programming task of performing Monte Carlo-type simulations. Typing . simulate exp_list, reps(\#): command
runs command for \# replications and collects the results in exp_list.
command defines the command that performs one simulation. Most Stata commands and userwritten programs can be used with simulate, as long as they follow standard Stata syntax; see [U] 11 Language syntax. The by prefix may not be part of command.
exp_list specifies the expression to be calculated from the execution of command. If no expressions are given, exp_list assumes a default, depending upon whether command changes results in e() or $r()$. If command changes results in $e()$, the default is _b. If command changes results in $r()$ (but not e() ), the default is all the scalars posted to $r()$. It is an error not to specify an expression in exp_list otherwise.

## Options

reps (\#) is required-it specifies the number of replications to be performed.
nodots suppresses display of the replication dots. By default, one dot character is displayed for each successful replication. A red ' $x$ ' is displayed if command returns an error or if one of the values in exp_list is missing.
noisily requests that any output from command be displayed. This option implies the nodots option.
trace causes a trace of the execution of command to be displayed. This option implies the noisily option.
saving (filename [, suboptions]) creates a Stata data file (.dta file) consisting of (for each statistic in exp_list) a variable containing the simulated values.
double specifies that the results for each replication be saved as doubles, meaning 8-byte reals. By default, they are saved as floats, meaning 4-byte reals.
every (\#) specifies that results be written to disk every \#th replication. every () should be specified only in conjunction with saving() when command takes a long time for each replication. This will allow recovery of partial results should some other software crash your computer. See [P] postfile. replace specifies that filename be overwritten if it exists.
nolegend suppresses display of the table legend. The table legend identifies the rows of the table with the expressions they represent.
verbose requests that the full table legend be displayed. By default, coefficients and standard errors are not displayed.
seed (\#) sets the random-number seed. Specifying this option is equivalent to typing the following command before calling simulate:
. set seed \#

## Remarks and examples

For an introduction to Monte Carlo methods, see Cameron and Trivedi (2010, chap. 4). White (2010) provides a command for analyzing results of simulation studies.

## > Example 1: Simulating basic summary statistics

We have a dataset containing means and variances of 100 -observation samples from a lognormal distribution (as a first step in evaluating, say, the coverage of a $95 \%$, $t$-based confidence interval). Then we perform the experiment 1,000 times.

The following command definition will generate 100 independent observations from a lognormal distribution and compute the summary statistics for this sample.

```
program lnsim, rclass
    version 13
    drop _all
    set obs 100
    gen z = exp(rnormal())
    summarize z
    return scalar mean = r(mean)
    return scalar Var = r(Var)
end
```

We can save 1,000 simulated means and variances from lnsim by typing

```
. set seed 1234
. simulate mean=r(mean) var=r(Var), reps(1000) nodots: lnsim
        command: lnsim
            mean: r(mean)
            var: r(Var)
. describe *
\begin{tabular}{lllll} 
variable name & \begin{tabular}{c} 
storage \\
type
\end{tabular} & \begin{tabular}{l} 
display \\
format
\end{tabular} & \begin{tabular}{l} 
value \\
label
\end{tabular} & variable label \\
\hline mean & float & \(\% 9.0 \mathrm{~g}\) & & \(r\) (mean) \\
\(\operatorname{var}\) & float & \(\% 9.0 \mathrm{~g}\) & & \(r(\) Var \()\)
\end{tabular}
. summarize
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline mean & 1000 & 1.638466 & .214371 & 1.095099 & 2.887392 \\
var & 1000 & 4.63856 & 6.428406 & .8626 & 175.3746
\end{tabular}
```


## - Technical note

Before executing our lnsim simulator, we can verify that it works by executing it interactively.

```
. set seed 1234
. lnsim
obs was 0, now 100
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline z & 100 & 1.597757 & 1.734328 & .0625807 & 12.71548
\end{tabular}
. return list
scalars:
```

$$
\begin{aligned}
r(\text { Var }) & =3.007893773683719 \\
r(\text { mean }) & =1.59775722913444
\end{aligned}
$$

## > Example 2: Simulating a regression model

Consider a more complicated problem. Let's experiment with fitting $y_{j}=a+b x_{j}+u_{j}$ when the true model has $a=1, b=2, u_{j}=z_{j}+c x_{j}$, and when $z_{j}$ is $N(0,1)$. We will save the parameter estimates and standard errors and experiment with varying $c . x_{j}$ will be fixed across experiments but will originally be generated as $N(0,1)$. We begin by interactively making the true data:

```
. drop _all
. set obs 100
obs was 0, now 100
. set seed 54321
. gen x = rnormal()
. gen true_y = 1+2*x
. save truth
file truth.dta saved
```

Our program is

```
program hetero1
    version 13
    args c
    use truth, clear
    gen y = true_y + (rnormal() + 'c'*x)
    regress y x
end
```

Note the use of ' $c$ ' in our statement for generating $y$. $c$ is a local macro generated from args $c$ and thus refers to the first argument supplied to hetero1. If we want $c=3$ for our experiment, we type

```
. simulate _b _se, reps(10000): hetero1 3
```

(output omitted)
Our program hetero1 could, however, be more efficient because it rereads the file truth once every replication. It would be better if we could read the data just once. In fact, if we read in the data right before running simulate, we really should not have to reread for each subsequent replication. A faster version reads

```
program hetero2
        version 13
        args c
        capture drop y
        gen y = true_y + (rnormal() + 'c'*x)
        regress y x
end
```

Requiring that the current dataset has the variables true_y and x may become inconvenient. Another improvement would be to require that the user supply variable names, such as in

```
program hetero3
        version 13
        args truey x c
        capture drop y
        gen y = 'truey' + (rnormal() + 'c'*'x')
        regress y x
end
```

Thus we can type

```
. simulate _b _se, reps(10000): hetero3 true_y x 3
    (output omitted)
```


## > Example 3: Simulating a ratio of statistics

Now let's consider the problem of simulating the ratio of two medians. Suppose that each sample of size $n_{i}$ comes from a normal population with a mean $\mu_{i}$ and standard deviation $\sigma_{i}$, where $i=1,2$. We write the program below and save it as a text file called myratio.ado (see [U] $\mathbf{1 7}$ Ado-files). Our program is an rclass command that requires six arguments as input, identified by the local macros n 1 , mu1, sigma1, n 2 , mu2, and sigma2, which correspond to $n_{1}, \mu_{1}, \sigma_{1}, n_{2}, \mu_{2}$, and $\sigma_{2}$, respectively. With these arguments, myratio will generate the data for the two samples, use summarize to compute the two medians and store the ratio of the medians in $r$ (ratio).

```
program myratio, rclass
    version 13
    args n1 mu1 sigma1 n2 mu2 sigma2
    // generate the data
    drop _all
    local N = 'n1'+'n2'
    set obs 'N'
    tempvar y
    generate 'y' = rnormal()
    replace 'y' = cond(_n<='n1','mu1'+'y'*'sigma1','mu2'+'y'*'sigma2')
    // calculate the medians
    tempname m1
    summarize 'y' if _n<='n1', detail
    scalar 'm1' = r(p50)
    summarize 'y' if _n>'n1', detail
    // store the results
    return scalar ratio = 'm1' / r(p50)
end
```

The result of running our simulation is

```
. set seed 19192
```

. simulate ratio=r(ratio), reps(1000) nodots: myratio 5311032
command: myratio 5311032
ratio: r(ratio)
. summarize

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| ratio | 1000 | 1.08571 | .4427828 | .3834799 | 6.742217 |

## - Technical note

Stata lets us do simulations of simulations and simulations of bootstraps. Stata's bootstrap command (see [R] bootstrap) works much like simulate, except that it feeds the user-written program a bootstrap sample. Say that we want to evaluate the bootstrap estimator of the standard error of the median when applied to lognormally distributed data. We want to perform a simulation, resulting in a dataset of medians and bootstrap estimated standard errors.

As background, summarize (see [R] summarize) calculates summary statistics, leaving the mean in $r$ (mean) and the standard deviation in $r(s d)$. summarize with the detail option also calculates summary statistics, but more of them, and leaves the median in $r(p 50)$.

Thus our plan is to perform simulations by randomly drawing a dataset: we calculate the median of our random sample, we use bootstrap to obtain a dataset of medians calculated from bootstrap samples of our random sample, the standard deviation of those medians is our estimate of the standard error, and the summary statistics are stored in the results of summarize.

```
Our simulator is
program define bsse, rclass
    version 13
    drop _all
    set obs 100
    gen x = rnormal()
    tempfile bsfile
    bootstrap midp=r(p50), rep(100) saving('bsfile'): summarize x, detail
    use 'bsfile', clear
    summarize midp
    return scalar mean = r(mean)
    return scalar sd = r(sd)
end
```

We can obtain final results, running our simulation 1,000 times, by typing

```
set seed 48901
simulate med=r(mean) bs_se=r(sd), reps(1000): bsse
    command: bsse
        med: r(mean)
        bs_se: r(sd)
Simulations (1000)
```



```
50 100 150 200250
. summarize
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline med & 1000 & -.0008696 & .1210451 & -.3132536 & .4058724 \\
bs_se & 1000 & .126236 & .029646 & .0326791 & .2596813
\end{tabular}

This is a case where the simulation dots (drawn by default, unless the nodots option is specified) will give us an idea of how long this simulation will take to finish as it runs.

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\section*{Also see}
[R] bootstrap - Bootstrap sampling and estimation
[R] jackknife - Jackknife estimation
[R] permute - Monte Carlo permutation tests

\section*{Description}

The Stata Journal (SJ) is a quarterly journal containing articles about statistics, data analysis, teaching methods, and effective use of Stata's language. The SJ publishes reviewed papers together with shorter notes and comments, regular columns, tips, book reviews, and other material of interest to researchers applying statistics in a variety of disciplines. You can read all about the Stata Journal at http://www.stata-journal.com.

The Stata Journal is a printed and electronic journal with corresponding software. If you want the journal, you must subscribe, but the software is available for no charge from our website at http://www.stata-journal.com. PDF copies of SJ articles that are older than three years are available for download for no charge at http://www.stata-journal.com/archives.html. More recent articles may be individually purchased.

The predecessor to the Stata Journal was the Stata Technical Bulletin (STB). The STB was also a printed and electronic journal with corresponding software. PDF copies of all STB journals are available for download for no charge at http://www.stata-press.com/journals/stbj.html. The STB software is available for no charge from our website at http://www.stata.com.

Below are instructions for installing the Stata Journal and the Stata Technical Bulletin software from our website.

\section*{Remarks and examples}

Remarks are presented under the following headings:
Installing the Stata Journal software
Obtaining from the Internet by pointing and clicking Obtaining from the Internet via command mode
Installing the STB software
Obtaining from the Internet by pointing and clicking
Obtaining from the Internet via command mode

\section*{Installing the Stata Journal software}

Each issue of the Stata Journal is labeled Volume \#, Number \#. Volume 1 refers to the first year of publication, Volume 2 to the second, and so on. Issues are numbered 1, 2, 3, and 4 within each year. The first issue of the Journal was published in the fourth quarter of 2001, and that issue is numbered Volume 1, Number 1. For installation purposes, we refer to this issue as sj1-1.

The articles, columns, notes, and comments that make up the Stata Journal are assigned a letter-and-number code, called an insert tag, such as st0001, an0034, or ds0011. The letters represent a category: st is the statistics category, an is the announcements category, etc. The numbers are assigned sequentially, so st0001 is the first article in the statistics category.

Sometimes inserts are subsequently updated, either to fix bugs or to add new features. A number such as st0001_1 indicates that this article, column, note, or comment is an update to the original st0001 article. Updates are complete; that is, installing st \(0001 \_1\) provides all the features of the original article and more.

The Stata Journal software may be obtained by pointing and clicking or by using command mode.
The sections below detail how to install an insert. In all cases, pretend that you wish to install insert st0274 from sj12-4.

\section*{Obtaining from the Internet by pointing and clicking}
1. Select Help > SJ and User-written Programs.
2. Click on Stata Journal.
3. Click on sj12-4.
4. Click on st0274.
5. Click on (click here to install).

\section*{Obtaining from the Internet via command mode}

Type the following:
```

. net from http://www.stata-journal.com/software
. net cd sj12-4
. net describe st0274
. net install st0274

```

The above could be shortened to
```

. net from http://www.stata-journal.com/software/sj12-4
. net describe st0274
. net install st0274

```

Alternatively, you could type
```

. net sj 12-4
. net describe st0274
. net install st0274

```
but going about it the long way is more entertaining, at least the first time.

\section*{Installing the STB software}

Each issue of the STB is numbered. STB-1 refers to the first issue (published May 1991), STB-2 refers to the second (published July 1991), and so on.

An issue of the STB consists of inserts-articles-and these are assigned letter-and-number combinations, such as \(\operatorname{sg} 84\), dm80, sbe26.1, etc. The letters represent a category; for example, sg is the general statistics category and dm the data management category. The numbers are assigned sequentially, so sbe39 is the 39th insert in the biostatistics and epidemiology series.

Insert sbe39, it turns out, provides a method of accounting for publication bias in meta-analysis; it adds a new command called metatrim to Stata. If you installed sbe39, you would have that command and its help file. Insert sbe39 was published in STB-57 (September 2000). Obtaining metatrim simply requires going to STB-57 and getting sbe 39 .

Sometimes inserts were subsequently updated, either to fix bugs or to add new features. sbe 39 was updated: the first update is sbe39.1 and the second is sbe39.2. You could install insert sbe39.2, and it would not matter whether you had previously installed sbe39.1. Updates are complete: installing sbe 39.2 provides all the features of the original insert and more.

For computer naming purposes, insert sbe39.2 is referred to as sbe39_2. When referred to in normal text, however, the insert is still called sbe 39.2 because that looks nicer.

Inserts are easily available from the Internet. Inserts may be obtained by pointing and clicking or by using command mode.

The sections below detail how to install an insert. In all cases, pretend that you wish to install insert sbe39.2 from STB-61.

\section*{Obtaining from the Internet by pointing and clicking}
1. Select Help > SJ and User-written Programs.
2. Click on STB.
3. Click on stb61.
4. Click on sbe39_2.
5. Click on (click here to install).

\section*{Obtaining from the Internet via command mode}

Type the following:
```

. net from http://www.stata.com
. net cd stb
. net cd stb61
. net describe sbe39_2
. net install sbe39_2

```

The above could be shortened to
. net from http://www.stata.com/stb/stb61
. net describe sbe39_2
. net install sbe39_2
but going about it the long way is more entertaining, at least the first time.

\section*{Also see}
[R] search - Search Stata documentation and other resources
\([R]\) net - Install and manage user-written additions from the Internet
\([R]\) net search - Search the Internet for installable packages
[R] update - Check for official updates
[U] 3.5 The Stata Journal
[U] 28 Using the Internet to keep up to date
[GSM] 19 Updating and extending Stata-Internet functionality
[GSU] 19 Updating and extending Stata-Internet functionality
[GSW] 19 Updating and extending Stata-Internet functionality

\section*{Title}
sktest - Skewness and kurtosis test for normality
\begin{tabular}{llll} 
Syntax & Menu & Description & Option \\
Remarks and examples & Stored results & Methods and formulas & Acknowledgments \\
References & Also see & &
\end{tabular}

\section*{Syntax}
sktest varlist [if] [in][weight] [, noadjust]
aweights and fweights are allowed; see [U] 11.1.6 weight.

\section*{Menu}

Statistics \(>\) Summaries, tables, and tests \(>\) Distributional plots and tests \(>\) Skewness and kurtosis normality test

\section*{Description}

For each variable in varlist, sktest presents a test for normality based on skewness and another based on kurtosis and then combines the two tests into an overall test statistic. sktest requires a minimum of 8 observations to make its calculations. See [MV] mvtest normality for multivariate tests of normality.

\section*{Option}

\section*{Main}
noadjust suppresses the empirical adjustment made by Royston (1991c) to the overall \(\chi^{2}\) and its significance level and presents the unaltered test as described by D'Agostino, Belanger, and D'Agostino (1990).

\section*{Remarks and examples}

Also see [R] swilk for the Shapiro-Wilk and Shapiro-Francia tests for normality. Those tests are, in general, preferred for nonaggregated data (Gould and Rogers 1991; Gould 1992; Royston 1991c). Moreover, a normal quantile plot should be used with any test for normality; see [R] diagnostic plots for more information.

\section*{> Example 1}

Using our automobile dataset, we will test whether the variables mpg and trunk are normally distributed:
```

. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. sktest mpg trunk

```

Skewness/Kurtosis tests for Normality
\begin{tabular}{r|ccccc} 
Variable & Obs & \(\operatorname{Pr}(\) Skewness \()\) & \(\operatorname{Pr}(\) Kurtosis) & adj chi2(2) & joint \(\overline{\text { Prob>chi2 }}\) \\
\hline mpg & 74 & 0.0015 & 0.0804 & 10.95 & 0.0042 \\
trunk & 74 & 0.9115 & 0.0445 & 4.19 & 0.1228
\end{tabular}

We can reject the hypothesis that mpg is normally distributed, but we cannot reject the hypothesis that trunk is normally distributed, at least at the \(12 \%\) level. The kurtosis for trunk is 2.19 , as can be verified by issuing the command
. summarize trunk, detail
(output omitted)
and the \(p\)-value of 0.0445 shown in the table above indicates that it is significantly different from the kurtosis of a normal distribution at the \(5 \%\) significance level. However, on the basis of skewness alone, we cannot reject the hypothesis that trunk is normally distributed.

\section*{- Technical note}
sktest implements the test as described by D'Agostino, Belanger, and D'Agostino (1990) but with the adjustment made by Royston (1991c). In the above example, if we had specified the noadjust option, the \(\chi^{2}\) values would have been 13.13 for mpg and 4.05 for trunk. With the adjustment, the \(\chi^{2}\) value might show as ' \(\quad\) '. This result should be interpreted as an absurdly large number; the data are most certainly not normal.

\section*{Stored results}
sktest stores the following in r() :
Scalars
\(r\) (chi2) \(\quad \chi^{2}\)
r(P_skew) Pr(skewness)
\(r\) (P_kurt) \(\quad\) Pr(kurtosis)
r(P_chi2) Prob > chi2
Matrices
\(r(N) \quad\) matrix of observations
\(r\) (Utest) matrix of test results, one row per variable

\section*{Methods and formulas}
sktest implements the test described by D'Agostino, Belanger, and D'Agostino (1990) with the empirical correction developed by Royston (1991c).

Let \(g_{1}\) denote the coefficient of skewness and \(b_{2}\) denote the coefficient of kurtosis as calculated by summarize, and let \(n\) denote the sample size. If weights are specified, then \(g_{1}, b_{2}\), and \(n\) denote the weighted coefficients of skewness and kurtosis and weighted sample size, respectively. See \([R]\) summarize for the formulas for skewness and kurtosis.

To perform the test of skewness, we compute
and
\[
\begin{aligned}
Y & =g_{1}\left\{\frac{(n+1)(n+3)}{6(n-2)}\right\}^{1 / 2} \\
\beta_{2}\left(g_{1}\right) & =\frac{3\left(n^{2}+27 n-70\right)(n+1)(n+3)}{(n-2)(n+5)(n+7)(n+9)} \\
W^{2} & =-1+\left[2\left\{\beta_{2}\left(g_{1}\right)-1\right\}\right]^{1 / 2} \\
\alpha & =\left\{2 /\left(W^{2}-1\right)\right\}^{1 / 2}
\end{aligned}
\]

Then the distribution of the test statistic
\[
Z_{1}=\frac{1}{\sqrt{\ln W}} \ln \left[Y / \alpha+\left\{(Y / \alpha)^{2}+1\right\}^{1 / 2}\right]
\]
is approximately standard normal under the null hypothesis that the data are distributed normally.
To perform the test of kurtosis, we compute
and
\[
\begin{aligned}
E\left(b_{2}\right) & =\frac{3(n-1)}{n+1} \\
\operatorname{var}\left(b_{2}\right) & =\frac{24 n(n-2)(n-3)}{(n+1)^{2}(n+3)(n+5)} \\
X & =\left\{b_{2}-E\left(b_{2}\right)\right\} / \sqrt{\operatorname{var}\left(b_{2}\right)} \\
\sqrt{\beta_{1}\left(b_{2}\right)} & =\frac{6\left(n^{2}-5 n+2\right)}{(n+7)(n+9)}\left\{\frac{6(n+3)(n+5)}{n(n-2)(n-3)}\right\}^{1 / 2} \\
A & =6+\frac{8}{\sqrt{\beta_{1}\left(b_{2}\right)}}\left[\frac{2}{\sqrt{\beta_{1}\left(b_{2}\right)}}+\left\{1+\frac{4}{\beta_{1}\left(b_{2}\right)}\right\}^{1 / 2}\right]
\end{aligned}
\]

Then the distribution of the test statistic
\[
Z_{2}=\frac{1}{\sqrt{2 /(9 A)}}\left[\left(1-\frac{2}{9 A}\right)-\left\{\frac{1-2 / A}{1+X \sqrt{2 /(A-4)}}\right\}^{1 / 3}\right]
\]
is approximately standard normal under the null hypothesis that the data are distributed normally.
D'Agostino, Balanger, and D'Agostino Jr.'s omnibus test of normality uses the statistic
\[
K^{2}=Z_{1}^{2}+Z_{2}^{2}
\]
which has approximately a \(\chi^{2}\) distribution with 2 degrees of freedom under the null of normality.

Royston (1991c) proposed the following adjustment to the test of normality, which sktest uses by default. Let \(\Phi(x)\) denote the cumulative standard normal distribution function for \(x\), and let \(\Phi^{-1}(p)\) denote the inverse cumulative standard normal function [that is, \(x=\Phi^{-1}\{\Phi(x)\}\) ]. Define the following terms:
and
\[
\begin{aligned}
Z_{c} & =-\Phi^{-1}\left\{\exp \left(-\frac{1}{2} K^{2}\right)\right\} \\
Z_{t} & =0.55 n^{0.2}-0.21 \\
a_{1} & =(-5+3.46 \ln n) \exp (-1.37 \ln n) \\
b_{1} & =1+(0.854-0.148 \ln n) \exp (-0.55 \ln n) \\
a_{2} & =a_{1}-\{2.13 /(1-2.37 \ln n)\} Z_{t} \\
b_{2} & =2.13 /(1-2.37 \ln n)+b 1
\end{aligned}
\]

If \(Z_{c}<-1\) set \(Z=Z_{c}\); else if \(Z_{c}<Z_{t}\) set \(Z=a_{1}+b_{1} Z_{c}\); else set \(Z=a_{2}+b_{2} Z_{c}\). Define \(P=1-\Phi(Z)\). Then \(K^{2}=-2 \ln P\) is approximately distributed \(\chi^{2}\) with 2 degrees of freedom.

The relative merits of the skewness and kurtosis test versus the Shapiro-Wilk and Shapiro-Francia tests have been a subject of debate. The interested reader is directed to the articles in the Stata Technical Bulletin. Our recommendation is to use the Shapiro-Francia test whenever possible, that is, whenever dealing with nonaggregated or ungrouped data (Gould and Rogers 1991; Gould 1992); see [R] swilk. If normality is rejected, use sktest to determine the source of the problem.

As both D'Agostino, Belanger, and D'Agostino (1990) and Royston (1991d) mention, researchers should also examine the normal quantile plot to determine normality rather than blindly relying on a few test statistics. See the qnorm command documented in \([R]\) diagnostic plots for more information on normal quantile plots.
sktest is similar in spirit to the Jarque-Bera (1987) test of normality. The Jarque-Bera test statistic is also calculated from the sample skewness and kurtosis, though it is based on asymptotic standard errors with no corrections for sample size. In effect, sktest offers two adjustments for sample size, that of Royston (1991c) and that of D'Agostino, Belanger, and D'Agostino (1990).

\section*{Acknowledgments}
sktest has benefited greatly by the comments and work of Patrick Royston of the MRC Clinical Trials Unit, London, and coauthor of the Stata Press book Flexible Parametric Survival Analysis Using Stata: Beyond the Cox Model. At this point, the program should be viewed as due as much to Royston as to us, except, of course, for any errors. We are also indebted to Nicholas J. Cox of the Department of Geography at Durham University, UK, and coeditor of the Stata Journal for his helpful comments.

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\section*{Also see}
[R] diagnostic plots - Distributional diagnostic plots
[R] ladder - Ladder of powers
[R] lv - Letter-value displays
[R] swilk - Shapiro-Wilk and Shapiro-Francia tests for normality
[MV] mvtest normality - Multivariate normality tests

\section*{Title}
slogit - Stereotype logistic regression
\begin{tabular}{llll} 
Syntax & Menu & Description & Options \\
Remarks and examples & Stored results & Methods and formulas & References
\end{tabular}

\section*{Syntax}
\[
\text { slogit depvar [indepvars }][\text { if }][\text { in }][\text { weight }][\text {, options }]
\]

\section*{options}

Description

Model
dimension(\#)
baseoutcome (\#|lbl)
constraints (numlist)
collinear
nocorner
SE/Robust
vce (vcetype)

Reporting
level (\#)
nocnsreport
display_options
dimension of the model; default is dimension(1)
set the base outcome to \# or \(l b l\); default is the last outcome apply specified linear constraints keep collinear variables do not generate the corner constraints
vcetype may be oim, robust, cluster clustvar, opg, bootstrap, or jackknife
set confidence level; default is level (95)
do not display constraints
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling

Maximization
maximize_options
initialize (initype)
nonormalize
coeflegend
control the maximization process; seldom used
method of initializing scale parameters; initype can be constant, random, or svd; see Options for details
do not normalize the numeric variables
display legend instead of statistics
indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
bootstrap, by, fp, jackknife, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the bootstrap prefix; see \([R]\) bootstrap.
vce() and weights are not allowed with the svy prefix; see [SVY] svy.
fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

\section*{Menu}

Statistics \(>\) Categorical outcomes \(>\) Stereotype logistic regression

\section*{Description}
slogit fits maximum-likelihood stereotype logistic regression models as developed by Anderson (1984). Like multinomial logistic and ordered logistic models, stereotype logistic models are for use with categorical dependent variables. In a multinomial logistic model, the categories cannot be ranked, whereas in an ordered logistic model the categories follow a natural ranking scheme. You can view stereotype logistic models as a compromise between those two models. You can use them when you are unsure of the relevance of the ordering, as is often the case when subjects are asked to assess or judge something. You can also use them in place of multinomial logistic models when you suspect that some of the alternatives are similar. Unlike ordered logistic models, stereotype logistic models do not impose the proportional-odds assumption.

\section*{Options}

Model
dimension(\#) specifies the dimension of the model, which is the number of equations required to describe the relationship between the dependent variable and the independent variables. The maximum dimension is \(\min (m-1, p)\), where \(m\) is the number of categories of the dependent variable and \(p\) is the number of independent variables in the model. The stereotype model with maximum dimension is a reparameterization of the multinomial logistic model.
baseoutcome (\#|lbl) specifies the outcome level whose scale parameters and intercept are constrained to be zero. The base outcome may be specified as a number of a label. By default, slogit assumes that the outcome levels are ordered and uses the largest level of the dependent variable as the base outcome.
constraints(numlist), collinear; see [R] estimation options.
By default, the linear equality constraints suggested by Anderson (1984), termed the corner constraints, are generated for you. You can add constraints to these as needed, or you can turn off the corner constraints by specifying nocorner. These constraints are in addition to the constraints placed on the \(\phi\) parameters corresponding to baseoutcome(\#).
nocorner specifies that slogit not generate the corner constraints. If you specify nocorner, you must specify at least dimension() \(\times\) dimension() constraints for the model to be identified.

SE/Robust
vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

If specifying vce(bootstrap) or vce(jackknife), you must also specify baseoutcome().
level(\#); see [R] estimation options.
nocnsreport; see \([R]\) estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% fint), pformat (\% fmt), sformat (\% fint), and nolstretch; see [R] estimation options.

Maximization
maximize_options: difficult, technique (algorithm_spec), iterate(\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#),
nrtolerance(\#), nonrtolerance, and from(init_specs); see [ R\(]\) maximize. These options are seldom used.

Setting the optimization type to technique (bhhh) resets the default vcetype to vce (opg).
 initialize (constant), is to set the scale parameters to the constant \(\min (1 / 2,1 / d)\), where \(d\) is the dimension specified in dimension().
initialize(random) requests that uniformly distributed random numbers between 0 and 1 be used as initial values for the scale parameters. If you specify this option, you should also use set seed to ensure that you can replicate your results; see \([R]\) set seed.
initialize(svd) requests that a singular value decomposition (SVD) be performed on the matrix of regression estimates from mlogit to reduce its rank to the dimension specified in dimension(). slogit uses the reduced-rank components of the SVD as initial estimates for the scale and regression coefficients. For details, see Methods and formulas.
nonormalize specifies that the numeric variables not be normalized. Normalization of the numeric variables improves numerical stability but consumes more memory in generating temporary doubleprecision variables. Variables that are of type byte are not normalized, and if initial estimates are specified using the from() option, normalization of variables is not performed. See Methods and formulas for more information.

The following option is available with slogit but is not shown in the dialog box:
coeflegend; see \([R]\) estimation options.

\section*{Remarks and examples}

Remarks are presented under the following headings:

\section*{Introduction}

One-dimensional model
Higher-dimension models

\section*{Introduction}

Stereotype logistic models are often used when subjects are requested to assess or judge something. For example, consider a survey in which consumers may be asked to rate the quality of a product on a scale from 1 to 5 , with 1 indicating poor quality and 5 indicating excellent quality. If the categories are monotonically related to an underlying latent variable, the ordered logistic model is appropriate. However, suppose that consumers assess quality not just along one dimension, but rather weigh two or three latent factors. Stereotype logistic regression allows you to specify multiple equations to capture the effects of those latent variables, which you then parameterize in terms of observable characteristics. Unlike with multinomial logit, the number of equations you specify could be less than \(m-1\), where \(m\) is the number of categories of the dependent variable.

Stereotype logistic models are also used when categories may be indistinguishable. Suppose that a consumer must choose among \(\mathrm{A}, \mathrm{B}, \mathrm{C}\), or D . Multinomial logistic modeling assumes that the four choices are distinct in the sense that a consumer choosing one of the goods can distinguish its characteristics from the others. If goods A and B are in fact similar, consumers may be randomly picking between the two. One alternative is to combine the two categories and fit a three-category multinomial logistic model. A more flexible alternative is to use a stereotype logistic model.

In the multinomial logistic model, you estimate \(m-1\) parameter vectors \(\widetilde{\boldsymbol{\beta}}_{k}, k=1, \ldots, m-1\), where \(m\) is the number of categories of the dependent variable. The stereotype logistic model is a restriction on the multinomial model in the sense that there are \(d\) parameter vectors, where \(d\) is between one and \(\min (m-1, p)\), and \(p\) is the number of regressors. The relationship between the stereotype model's coefficients \(\boldsymbol{\beta}_{j}, j=1, \ldots, d\), and the multinomial model's coefficients is \(\widetilde{\boldsymbol{\beta}}_{k}=-\sum_{j=1}^{d} \phi_{j k} \boldsymbol{\beta}_{j}\). The \(\phi \mathrm{s}\) are scale parameters to be estimated along with the \(\boldsymbol{\beta}_{j} \mathrm{~s}\).

Given a row vector of covariates \(\mathbf{x}\), let \(\eta_{k}=\theta_{k}-\sum_{j=1}^{d} \phi_{j k} \mathbf{x} \boldsymbol{\beta}_{j}\). The probability of observing outcome \(k\) is
\[
\operatorname{Pr}\left(Y_{i}=k\right)= \begin{cases}\frac{\exp \left(\eta_{k}\right)}{1+\sum_{l=1}^{m-1} \exp \left(\eta_{l}\right)} & k<m \\ \frac{1}{1+\sum_{l=1}^{m-1} \exp \left(\eta_{l}\right)} & k=m\end{cases}
\]

This model includes a set of \(\theta\) parameters so that each equation has an unrestricted constant term. If \(d=m-1\), the stereotype model is just a reparameterization of the multinomial logistic model. To identify the \(\phi \mathrm{s}\) and the \(\boldsymbol{\beta}\) s, you must place at least \(d^{2}\) restrictions on the parameters. By default, slogit uses the "corner constraints" \(\phi_{j j}=1\) and \(\phi_{j k}=0\) for \(j \neq k, k \leq d\), and \(j \leq d\).

For a discussion of the stereotype logistic model, see Lunt (2005).

\section*{One-dimensional model}
> Example 1
We have 2 years of repair rating data on the make, price, mileage rating, and gear ratio of 104 foreign and 44 domestic automobiles (with 13 missing values on repair rating). We wish to fit a stereotype logistic model to discriminate between the levels of repair rating using mileage, price, gear ratio, and origin of the manufacturer. Here is an overview of our data:
\begin{tabular}{|c|c|c|c|}
\hline \multicolumn{4}{|l|}{\begin{tabular}{l}
. use http://www.s (Automobile Models) \\
. tabulate repair
\end{tabular}} \\
\hline Repair rating & Freq. & Percent & Cum. \\
\hline Poor & 5 & 3.70 & 3.70 \\
\hline Fair & 19 & 14.07 & 17.78 \\
\hline Average & 57 & 42.22 & 60.00 \\
\hline Good & 38 & 28.15 & 88.15 \\
\hline Excellent & 16 & 11.85 & 100.00 \\
\hline Total & 135 & 100.00 & \\
\hline
\end{tabular}

The variable repair can take five values, \(1, \ldots, 5\), which represent the subjective rating of the car model's repair record as Poor, Fair, Average, Good, and Excellent.

We wish to fit the one-dimensional stereotype logistic model
\[
\eta_{k}=\theta_{k}-\phi_{k}\left(\beta_{1} \text { foreign }+\beta_{2} \mathrm{mpg}+\beta_{3} \text { price }+\beta_{4} \text { gratio }\right)
\]
for \(k<5\) and \(\eta_{5}=0\). To fit this model, we type

(repair=Excellent is the base outcome)
The coefficient associated with the first scale parameter, \(\phi_{11}\), is 1 , and its standard error and other statistics are missing. This is the corner constraint applied to the one-dimensional model; in the header, this constraint is listed as [phi1_1]_cons \(=1\). Also, the \(\phi\) and \(\theta\) parameters that are associated with the base outcome are identified. Keep in mind, though, that there are no coefficient estimates for [phi1_5]_cons or [theta5]_cons in the ereturn matrix e(b). The Wald statistic is for a test of the joint significance of the regression coefficients on foreign, mpg, price, and gratio.

The one-dimensional stereotype model restricts the multinomial logistic regression coefficients \(\widetilde{\boldsymbol{\beta}}_{k}, k=1, \ldots, m-1\) to be parallel; that is, \(\widetilde{\boldsymbol{\beta}}_{k}=-\phi_{k} \boldsymbol{\beta}\). As Lunt (2001) discusses, in the one-dimensional stereotype model, one linear combination \(\mathbf{x}_{i} \boldsymbol{\beta}\) best discriminates the outcomes of the dependent variable, and the scale parameters \(\phi_{k}\) measure the distance between the outcome levels and the linear predictor. If \(\phi_{1} \geq \phi_{2} \geq \cdots \phi_{m-1} \geq \phi_{m} \equiv 0\), the model suggests that the subjective assessment of the dependent variable is indeed ordered. Here the maximum likelihood estimates of the \(\phi \mathrm{s}\) are not monotonic, as would be assumed in an ordered logit model.

We test that \(\phi_{1}=\phi_{2}\) by typing
```

. test [phi1_2]_cons = [phi1_1]_cons
( 1) - [phi1_1]_cons + [phi1_2]_cons = 0
chi2( 1) = 0.55
Prob > chi2 = 0.4576

```

Because the two parameters are not statistically different, we decide to add a constraint to force \(\phi_{1}=\phi_{2}\) :

(repair=Excellent is the base outcome)
The \(\phi\) estimates are now monotonically decreasing and the standard errors of the \(\phi\) s are small relative to the size of the estimates, so we conclude that, with the exception of outcomes Poor and Fair, the groups are distinguishable for the one-dimensional model and that the quality assessment can be ordered.

\section*{Higher-dimension models}

The stereotype logistic model is not limited to ordered categorical dependent variables; you can use it on nominal data to reduce the dimension of the regressions. Recall that a multinomial model fit to a categorical dependent variable with \(m\) levels will have \(m-1\) sets of regression coefficients. However, a model with fewer dimensions may fit the data equally well, suggesting that some of the categories are indistinguishable.

\section*{- Example 2}

As discussed in [R] mlogit, we have data on the type of health insurance available to 616 psychologically depressed subjects in the United States (Tarlov et al. 1989; Wells et al. 1989). Patients may have either an indemnity (fee-for-service) plan or a prepaid plan, such as an HMO, or may be uninsured. Demographic variables include age, gender, race, and site.

First, we fit the saturated, two-dimensional model that is equivalent to a multinomial logistic model. We choose the base outcome to be 1 (indemnity insurance) because that is the default for mlogit.
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \multicolumn{7}{|l|}{\begin{tabular}{l}
(Health insurance data) \\
. slogit insure age male nonwhite i.site, dim(2) base(1)
\end{tabular}} \\
\hline \multicolumn{7}{|l|}{\(\begin{array}{ll}\text { Iteration 0: } & \text { log likelihood }=-534.36165 \\ \text { Iteration 1: } & \text { log likelihood }=-534.36165\end{array}\)} \\
\hline \multicolumn{4}{|l|}{Stereotype logistic regression} & Numb & of obs
chi2(10) & \[
\begin{array}{r}
615 \\
38.17
\end{array}
\] \\
\hline \multicolumn{4}{|l|}{Log likelihood \(=-534.36165\)} & Prob & chi2 & 0.0000 \\
\hline \multicolumn{7}{|l|}{( 1) [phi1_2]_cons \(=1\)} \\
\hline \multicolumn{7}{|l|}{( 2) [phi1_3]_cons \(=0\)} \\
\hline \multicolumn{7}{|l|}{( 3) [phi2_2]_cons \(=0\)} \\
\hline \multicolumn{7}{|l|}{( 4) [phi2_3]_cons} \\
\hline insure & Coef. & Std. Err. & z & \(P>|z|\) & [95\% Con & Interval] \\
\hline \multicolumn{7}{|l|}{dim1} \\
\hline age & . 011745 & . 0061946 & 1.90 & 0.058 & -. 0003962 & . 0238862 \\
\hline male & -. 5616934 & . 2027465 & -2.77 & 0.006 & -. 9590693 & -. 1643175 \\
\hline nonwhite & -. 9747768 & . 2363213 & -4.12 & 0.000 & -1.437958 & -. 5115955 \\
\hline \multicolumn{7}{|l|}{site} \\
\hline 2 & -. 1130359 & . 2101903 & -0.54 & 0.591 & -. 5250013 & . 2989296 \\
\hline 3 & . 5879879 & . 2279351 & 2.58 & 0.010 & . 1412433 & 1.034733 \\
\hline \multicolumn{7}{|l|}{dim2} \\
\hline age & . 0077961 & . 0114418 & 0.68 & 0.496 & -. 0146294 & . 0302217 \\
\hline male & -. 4518496 & . 3674867 & -1.23 & 0.219 & -1.17211 & . 268411 \\
\hline nonwhite & -. 2170589 & . 4256361 & -0.51 & 0.610 & -1.05129 & . 6171725 \\
\hline \multicolumn{7}{|l|}{site} \\
\hline 2 & 1.211563 & . 4705127 & 2.57 & 0.010 & . 2893747 & 2.133751 \\
\hline 3 & . 2078123 & . 3662926 & 0.57 & 0.570 & -. 510108 & . 9257327 \\
\hline /phi1_1 & & (base outc & & & & \\
\hline /phi1_2 & 1 & (constrain & & & & \\
\hline /phi1_3 & 0 & (omitted) & & & & \\
\hline /phi2_1 & 0 & (base outc & & & & \\
\hline /phi2_2 & 0 & (omitted) & & & & \\
\hline /phi2_3 & 1 & (constrain & & & & \\
\hline /theta1 & 0 & (base outc & & & & \\
\hline /theta2 & . 2697127 & . 3284422 & 0.82 & 0.412 & -. 3740222 & . 9134476 \\
\hline /theta3 & -1.286943 & . 5923219 & -2.17 & 0.030 & -2.447872 & -. 1260134 \\
\hline
\end{tabular}
(insure=Indemnity is the base outcome)

For comparison, we also fit the model by using mlogit:
\begin{tabular}{llll}
. mlogit insure age male nonwhite i.site, nolog & & \\
Multinomial logistic regression & Number of obs & \(=\) & 615 \\
& LR chi2 (10) & \(=\) & 42.99 \\
& Prob > chi2 & \(=\) & 0.0000 \\
Log likelihood \(=-534.36165\) & Pseudo R2 & \(=\) & 0.0387
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline insure & Coef. & Std. Err. & z & \(P>|z|\) & [95\% Con & Interval] \\
\hline Indemnity & \multicolumn{6}{|l|}{(base outcome)} \\
\hline \multicolumn{7}{|l|}{Prepaid} \\
\hline age & -. 011745 & . 0061946 & -1.90 & 0.058 & -. 0238862 & . 0003962 \\
\hline male & . 5616934 & . 2027465 & 2.77 & 0.006 & . 1643175 & . 9590693 \\
\hline nonwhite & . 9747768 & . 2363213 & 4.12 & 0.000 & . 5115955 & 1.437958 \\
\hline site & & & & & & \\
\hline 2 & . 1130359 & . 2101903 & 0.54 & 0.591 & -. 2989296 & . 5250013 \\
\hline 3 & -. 5879879 & . 2279351 & -2.58 & 0.010 & -1.034733 & -. 1412433 \\
\hline _cons & . 2697127 & . 3284422 & 0.82 & 0.412 & -. 3740222 & . 9134476 \\
\hline \multicolumn{7}{|l|}{Uninsure} \\
\hline age & -. 0077961 & . 0114418 & -0.68 & 0.496 & -. 0302217 & . 0146294 \\
\hline male & . 4518496 & . 3674867 & 1.23 & 0.219 & -. 268411 & 1.17211 \\
\hline nonwhite & . 2170589 & . 4256361 & 0.51 & 0.610 & -. 6171725 & 1.05129 \\
\hline site & & & & & & \\
\hline 2 & -1.211563 & . 4705127 & -2.57 & 0.010 & -2.133751 & -. 2893747 \\
\hline 3 & -. 2078123 & . 3662926 & -0.57 & 0.570 & -. 9257327 & . 510108 \\
\hline _cons & -1.286943 & . 5923219 & -2.17 & 0.030 & -2.447872 & -. 1260134 \\
\hline
\end{tabular}

Apart from having opposite signs, the coefficients from the stereotype logistic model are identical to those from the multinomial logit model. Recall the definition of \(\eta_{k}\) given in the Remarks and examples, particularly the minus sign in front of the summation. One other difference in the output is that the constant estimates labeled /theta in the slogit output are the constants labeled _cons in the mlogit output.

Next we examine the one-dimensional model.
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline \multicolumn{4}{|l|}{Stereotype logistic regression} & \multicolumn{3}{|l|}{\multirow[t]{2}{*}{Number of obs Wald chi2(5) Prob > chi2}} & \[
\begin{array}{r}
615 \\
28.20
\end{array}
\] \\
\hline \multicolumn{4}{|l|}{Log likelihood \(=-539.75205\)} & & & & \multirow[t]{2}{*}{0.0000} \\
\hline \multicolumn{6}{|l|}{( 1) [phi1_2]_cons = 1} & = & \\
\hline insure & Coef. & Std. Err & z & \(P>|z|\) & [95\% & Conf & Interval] \\
\hline age & . 0108366 & . 0061918 & 1.75 & 0.080 & -. 00129 & 992 & . 0229723 \\
\hline male & -. 5032537 & . 2078171 & -2.42 & 0.015 & -. 910567 & 678 & -. 0959396 \\
\hline nonwhite & -. 9480351 & . 2340604 & -4.05 & 0.000 & -1.4067 & 785 & -. 489285 \\
\hline \multicolumn{8}{|l|}{site} \\
\hline 2 & -. 2444316 & . 2246366 & -1.09 & 0.277 & -. 68471 & 113 & . 1958481 \\
\hline 3 & . 556665 & . 2243799 & 2.48 & 0.013 & . 11688 & 886 & . 9964415 \\
\hline /phi1_1 & \multicolumn{7}{|c|}{0 (base outcome)} \\
\hline \multicolumn{8}{|l|}{/phi1_2 1 (constrained)} \\
\hline /phi1_3 & . 0383539 & . 4079705 & 0.09 & 0.925 & -. 76125 & 535 & . 8379613 \\
\hline /theta1 & 0 & (base outc & & & & & \\
\hline /theta2 & . 187542 & . 3303847 & 0.57 & 0.570 & -. 46000 & 001 & . 835084 \\
\hline /theta3 & -1.860134 & . 2158898 & -8.62 & 0.000 & -2.283 & 327 & -1.436997 \\
\hline
\end{tabular}
(insure=Indemnity is the base outcome)
We have reduced a two-dimensional multinomial model to one dimension, reducing the number of estimated parameters by four and decreasing the model likelihood by \(\approx 5.4\).
slogit does not report a model likelihood-ratio test. The test of \(d=1\) (a one-dimensional model) versus \(d=0\) (the null model) does not have an asymptotic \(\chi^{2}\) distribution because the unconstrained \(\phi\) parameters (/phi1_3 in this example) cannot be identified if \(\boldsymbol{\beta}=\mathbf{0}\). More generally, this problem precludes testing any hierarchical model of dimension \(d\) versus \(d-1\). Of course, the likelihood-ratio test of a full-dimension model versus \(d=0\) is valid because the full model is just multinomial logistic, and all the \(\phi\) parameters are fixed at 0 or 1 .

\section*{- Technical note}

The stereotype model is a special case of the reduced-rank vector generalized linear model discussed by Yee and Hastie (2003). If we define \(\eta_{i k}=\theta_{k}-\sum_{j=1}^{d} \phi_{j k} \mathbf{x}_{i} \boldsymbol{\beta}_{j}\), for \(k=1, \ldots, m-1\), we can write the expression in matrix notation as
\[
\boldsymbol{\eta}_{i}=\boldsymbol{\theta}+\boldsymbol{\Phi}\left(\mathbf{x}_{i} \mathbf{B}\right)^{\prime}
\]
where \(\boldsymbol{\Phi}\) is a \((m-1) \times d\) matrix containing the \(\phi_{j k}\) parameters and \(\mathbf{B}\) is a \(p \times d\) matrix with columns containing the \(\boldsymbol{\beta}_{j}\) parameters, \(j=1, \ldots, d\). The factorization \(\boldsymbol{\Phi} \mathbf{B}^{\prime}\) is not unique because \(\boldsymbol{\Phi} \mathbf{B}^{\prime}=\boldsymbol{\Phi} \mathbf{M} \mathbf{M}^{-1} \mathbf{B}^{\prime}\) for any nonsingular \(d \times d\) matrix \(\mathbf{M}\). To avoid this identifiability problem, we choose \(\mathbf{M}=\boldsymbol{\Phi}_{1}^{-1}\), where
\[
\boldsymbol{\Phi}=\binom{\boldsymbol{\Phi}_{1}}{\boldsymbol{\Phi}_{2}}
\]
and \(\boldsymbol{\Phi}_{1}\) is \(d \times d\) of rank \(d\) so that
\[
\boldsymbol{\Phi M}=\binom{\mathbf{I}_{d}}{\boldsymbol{\Phi}_{2} \boldsymbol{\Phi}_{1}^{-1}}
\]
and \(\mathbf{I}_{d}\) is a \(d \times d\) identity matrix. Thus the corner constraints used by slogit are \(\phi_{j j} \equiv 1\) and \(\phi_{j k} \equiv 0\) for \(j \neq k\) and \(k, j \leq d\).

\section*{Stored results}
slogit stores the following in e():
Scalars
```

e(N)
e(k)
e(k_indvars)
e(k_out)
e(k_eq)
e(k_eq_model)
e(df_m)
e(df_0)
e(k_dim)
e(i_base)
e(11)
e(ll_0)
e(N_clust)
e(chi2)
e(p)
e(ic)
e(rank)
e(rc)
e(converged)

```

Macros
    e(cmd)
    e(cmdline)
    e(depvar)
    e(indvars)
    e(wtype)
    e(wexp)
    e(title)
    e(clustvar)
    e(out\#)
    e(chi2type)
    e(labels)
    e(vce)
    e(vcetype)
    e(opt)
    e(which)
    e(ml_method)
    e(user)
    e(technique)
    e(properties)
    e(predict)
    e(marginsnotok)
    e(footnote)
    e(asbalanced)
    e(asobserved)
Matrices
    e(b)
    e(outcomes)
    e(Cns)
    e(ilog)
    e(gradient)
    e(V)
    e(V_modelbased)

Functions
e(sample) marks estimation sample

\section*{Methods and formulas}
slogit obtains the maximum likelihood estimates for the stereotype logistic model by using ml; see \([\mathrm{R}] \mathrm{ml}\). Each set of regression estimates, one set of \(\boldsymbol{\beta}_{j} \mathrm{~s}\) for each dimension, constitutes one ml model equation. The \(d \times(m-1) \phi \mathrm{s}\) and the \((m-1) \theta \mathrm{s}\) are ml ancillary parameters.

Without loss of generality, let the base outcome level be the \(m\) th level of the dependent variable. Define the row vector \(\phi_{k}=\left(\phi_{1 k}, \ldots, \phi_{d k}\right)\) for \(k=1, \ldots, m-1\), and define the \(p \times d\) matrix \(\mathbf{B}=\left(\boldsymbol{\beta}_{1}, \ldots, \boldsymbol{\beta}_{d}\right)\). For observation \(i\), the \(\log\) odds of outcome level \(k\) relative to level \(m, k=1\), \(\ldots, m-1\) is the index
\[
\begin{aligned}
\ln \left\{\frac{\operatorname{Pr}\left(Y_{i}=k\right)}{\operatorname{Pr}\left(Y_{i}=m\right)}\right\}=\eta_{i k} & =\theta_{k}-\boldsymbol{\phi}_{k}\left(\mathbf{x}_{i} \mathbf{B}\right)^{\prime} \\
& =\theta_{k}-\boldsymbol{\phi}_{k} \boldsymbol{\nu}_{i}^{\prime}
\end{aligned}
\]

The row vector \(\nu_{i}\) can be interpreted as a latent variable reducing the \(p\)-dimensional vector of covariates to a more interpretable \(d<p\) dimension.

The probability of the \(i\) th observation having outcome level \(k\) is then
\[
\operatorname{Pr}\left(Y_{i}=k\right)=p_{i k}= \begin{cases}\frac{e^{\eta_{i k}}}{1+\sum_{j=1}^{m-1} e^{\eta_{i j}}}, \text { if } & k<m \\ \frac{1}{1+\sum_{j=1}^{m-1} e^{\eta_{i j}}}, \text { if } & k=m\end{cases}
\]
from which the log-likelihood function is computed as
\[
\begin{equation*}
L=\sum_{i=1}^{n} w_{i} \sum_{k=1}^{m} I_{k}\left(y_{i}\right) \ln \left(p_{i k}\right) \tag{1}
\end{equation*}
\]

Here \(w_{i}\) is the weight for observation \(i\) and
\[
I_{k}\left(y_{i}\right)=\left\{\begin{array}{l}
1, \text { if observation } y_{i} \text { has outcome } k \\
0, \text { otherwise }
\end{array}\right.
\]

Numeric variables are normalized for numerical stability during optimization where a new doubleprecision variable \(\widetilde{x}_{j}\) is created from variable \(x_{j}, j=1, \ldots, p\), such that \(\widetilde{x}_{j}=\left(x_{j}-\bar{x}_{j}\right) / s_{j}\). This feature is turned off if you specify nonormalize, or if you use the from() option for initial estimates. Normalization is not performed on byte variables, including the indicator variables generated by \([\mathrm{R}]\) xi. The linear equality constraints for regression parameters, if specified, must be scaled also. Assume that a constraint is applied to the regression parameter associated with variable \(j\) and dimension \(i\), \(\beta_{j i}\), and the corresponding element of the constraint matrix (see [ P\(]\) makecns) is divided by \(s_{j}\).

After convergence, the parameter estimates for variable \(j\) and dimension \(i-\widetilde{\beta}_{j i}\), say-are transformed back to their original scale, \(\beta_{j i}=\widetilde{\beta}_{j i} / s_{j}\). For the intercepts, you compute
\[
\theta_{k}=\widetilde{\theta}_{k}+\sum_{i=1}^{d} \phi_{i k} \sum_{j=1}^{p} \frac{\widetilde{\beta}_{j i} \bar{x}_{j}}{s_{j}}
\]

Initial values are computed using estimates obtained using mlogit to fit a multinomial logistic model. Let the \(p \times(m-1)\) matrix \(\widetilde{\mathbf{B}}\) contain the multinomial logistic regression parameters less the \(m-1\) intercepts. Each \(\phi\) is initialized with constant values \(\min (1 / 2,1 / d)\), the initialize (constant) option (the default), or, with uniform random numbers, the initialize (random) option. Constraints are then applied to the starting values so that the structure of the \((m-1) \times d\) matrix \(\boldsymbol{\Phi}\) is
\[
\Phi=\left(\begin{array}{c}
\phi_{1} \\
\phi_{2} \\
\vdots \\
\phi_{m-1}
\end{array}\right)=\binom{\mathbf{I}_{d}}{\boldsymbol{\Phi}}
\]
where \(\mathbf{I}_{d}\) is a \(d \times d\) identity matrix. Assume that only the corner constraints are used, but any constraints you place on the scale parameters are also applied to the initial scale estimates, so the structure of \(\Phi\) will change accordingly. The \(\phi\) parameters are invariant to the scale of the covariates, so initial estimates in \([0,1]\) are reasonable. The constraints guarantee that the rank of \(\Phi\) is at least \(d\), so the initial estimates for the stereotype regression parameters are obtained from \(\mathbf{B}=\widetilde{\mathbf{B}} \boldsymbol{\Phi}\left(\Phi^{\prime} \boldsymbol{\Phi}\right)^{-1}\).

One other approach for initial estimates is provided: initialize(svd). It starts with the mlogit estimates and computes \(\widetilde{\mathbf{B}}^{\prime}=\mathbf{U D V}{ }^{\prime}\), where \(\mathbf{U}_{m-1 \times p}\) and \(\mathbf{V}_{p \times p}\) are orthonormal matrices and \(\mathbf{D}_{p \times p}\) is a diagonal matrix containing the singular values of \(\widetilde{\mathbf{B}}\). The estimates for \(\boldsymbol{\Phi}\) and \(\mathbf{B}\) are the first \(d\) columns of \(\mathbf{U}\) and \(\mathbf{V D}\), respectively (Yee and Hastie 2003).

The score for regression coefficients is
\[
\mathbf{u}_{i}\left(\boldsymbol{\beta}_{j}\right)=\frac{\partial L_{i k}}{\partial \boldsymbol{\beta}_{j}}=\mathbf{x}_{i}\left(\sum_{l=1}^{m-1} \phi_{j l} p_{i l}-\phi_{j k}\right)
\]
the score for the scale parameters is
\[
u_{i}\left(\phi_{j l}\right)=\frac{\partial L_{i k}}{\partial \phi_{j l}}= \begin{cases}\mathbf{x}_{i} \boldsymbol{\beta}_{j}\left(p_{i k}-1\right), & \text { if } l=k \\ \mathbf{x}_{i} \boldsymbol{\beta}_{j} p_{i l}, & \text { if } l \neq k\end{cases}
\]
for \(l=1, \ldots, m-1\); and the score for the intercepts is
\[
u_{i}\left(\theta_{l}\right)=\frac{\partial L_{i k}}{\partial \theta_{l}}= \begin{cases}1-p_{i k}, & \text { if } l=k \\ -p_{i l}, & \text { if } l \neq k\end{cases}
\]

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce (robust) and vce (cluster clustvar), respectively. See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas.
slogit also supports estimation with survey data. For details on VCEs with survey data, see [SVY] variance estimation.

\section*{References}

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Yee, T. W., and T. J. Hastie. 2003. Reduced-rank vector generalized linear models. Statistical Modelling 3: 15-41.

\section*{Also see}
[R] slogit postestimation - Postestimation tools for slogit
[R] logistic - Logistic regression, reporting odds ratios
[R] mlogit - Multinomial (polytomous) logistic regression
\([\mathrm{R}]\) ologit — Ordered logistic regression
\([\mathrm{R}]\) oprobit - Ordered probit regression
\([R]\) roc - Receiver operating characteristic (ROC) analysis
[SVY] svy estimation - Estimation commands for survey data
[U] 20 Estimation and postestimation commands

\section*{Title}
slogit postestimation - Postestimation tools for slogit
\begin{tabular}{llll} 
Description & Syntax for predict & Menu for predict & Options for predict \\
Remarks and examples & Methods and formulas & Also see &
\end{tabular}

\section*{Description}

The following postestimation commands are available after slogit:
\begin{tabular}{|c|c|}
\hline Command & Description \\
\hline contrast & contrasts and ANOVA-style joint tests of estimates \\
\hline estat ic & Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) \\
\hline estat summarize & summary statistics for the estimation sample \\
\hline estat vce & variance-covariance matrix of the estimators (VCE) \\
\hline estat (svy) & postestimation statistics for survey data \\
\hline estimates & cataloging estimation results \\
\hline lincom & point estimates, standard errors, testing, and inference for linear combinations of coefficients \\
\hline lrtest \({ }^{1}\) & likelihood-ratio test \\
\hline margins & marginal means, predictive margins, marginal effects, and average marginal effects \\
\hline marginsplot & graph the results from margins (profile plots, interaction plots, etc.) \\
\hline nlcom & point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients \\
\hline predict & predicted probabilities, estimated index and its approximate standard error \\
\hline predictnl & point estimates, standard errors, testing, and inference for generalized predictions \\
\hline pwcompare & pairwise comparisons of estimates \\
\hline suest & seemingly unrelated estimation \\
\hline test & Wald tests of simple and composite linear hypotheses \\
\hline testnl & Wald tests of nonlinear hypotheses \\
\hline
\end{tabular}

\footnotetext{
1 lrtest is not appropriate with svy estimation results.
}

\section*{Syntax for predict}
```

predict [type] {stub*| newvar|newvarlist } [if][in] [, statistic outcome(outcome)]
predict [type] {stub*|newvarlist } [if] [in], scores

```
statistic
Main
xb
stdp
\(\underline{\mathrm{pr}} \quad\) probability of one or all of the dependent variable outcomes; the default
Description index for the \(k\) th outcome
standard error of the index for the \(k\) th outcome

If you do not specify outcome(), pr (with one new variable specified), xb , and stdp assume outcome(\#1).
You specify one or \(k\) new variables with pr , where \(k\) is the number of outcomes.
You specify one new variable with xb and stdp.
These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

\section*{Menu for predict}

Statistics \(>\) Postestimation \(>\) Predictions, residuals, etc.

\section*{Options for predict}
\(\qquad\) Main
pr , the default, calculates the probability of each of the categories of the dependent variable or the probability of the level specified in outcome(outcome). If you specify the outcome (outcome) option, you need to specify only one new variable; otherwise, you must specify a new variable for each category of the dependent variable.
xb calculates the index, \(\theta_{k}-\sum_{j=1}^{d} \phi_{\mathrm{jk}} \mathbf{x}_{i} \boldsymbol{\beta}_{j}\), for outcome level \(k \neq \mathrm{e}\) (i_base) and dimension \(d=\mathrm{e}\) (k_dim). It returns a vector of zeros if \(k=\mathrm{e}\) ( \(\mathrm{i} \_\mathrm{b} a \mathrm{se}\) ). A synonym for xb is index. If outcome() is not specified, outcome(\#1) is assumed.
stdp calculates the standard error of the index. A synonym for stdp is seindex. If outcome () is not specified, outcome(\#1) is assumed.
outcome (outcome) specifies the outcome for which the statistic is to be calculated. equation() is a synonym for outcome(): it does not matter which you use outcome() or equation() can be specified using
\#1, \#2, ..., where \#1 means the first category of the dependent variable, \#2 means the second category, etc.;
the values of the dependent variable; or
the value labels of the dependent variable if they exist.
scores calculates the equation-level score variables. For models with \(d\) dimensions and \(m\) levels, \(d+(d+1)(m-1)\) new variables are created. Assume \(j=1, \ldots, d\) and \(k=1, \ldots, m\) in the following.

The first \(d\) new variables will contain \(\partial \ln L / \partial\left(\mathbf{x} \boldsymbol{\beta}_{j}\right)\).

The next \(d(m-1)\) new variables will contain \(\partial \ln L / \partial \phi_{j k}\).
The last \(m-1\) new variables will contain \(\partial \ln L / \partial \theta_{k}\).

\section*{Remarks and examples}

Once you have fit a stereotype logistic model, you can obtain the predicted probabilities by using the predict command for both the estimation sample and other samples; see [U] 20 Estimation and postestimation commands and \([R]\) predict.
predict without arguments (or with the pr option) calculates the predicted probability of each outcome of the dependent variable. You must therefore give a new variable name for each of the outcomes. To compute the estimated probability of one outcome, you use the outcome (outcome) option where outcome is the level encoding the outcome. If the dependent variable's levels are labeled, the outcomes can also be identified by the label values (see [D] label).

The xb option in conjunction with outcome (outcome) specifies that the index be computed for the outcome encoded by level outcome. Its approximate standard error is computed if the stdp option is specified. Only one of the \(\mathrm{pr}, \mathrm{xb}\), or stdp options can be specified with a call to predict.

\section*{\(>\) Example 1}

In example 2 of [R] slogit, we fit the one-dimensional stereotype model, where the depvar is insure with levels \(k=1\) for outcome Indemnity, \(k=2\) for Prepaid, and \(k=3\) for Uninsure. The base outcome for the model is Indemnity, so for \(k \neq 1\) the vector of indices for the \(k\) th level is
\[
\boldsymbol{\eta}_{k}=\theta_{k}-\phi_{k}\left(\beta_{1} \text { age }+\beta_{2} \text { male }+\beta_{3} \text { nonwhite }+\beta_{4} 2 . \text { site }+\beta_{5} 3 . \text { site }\right)
\]

We estimate the group probabilities by calling predict after slogit.
```

. use http://www.stata-press.com/data/r13/sysdsn1
(Health insurance data)
. slogit insure age male nonwhite i.site, dim(1) base(1) nolog
(output omitted)
. predict pIndemnity pPrepaid pUninsure, p
. list pIndemnity pPrepaid pUninsure insure in 1/10

```
\begin{tabular}{|crrr|}
\hline pIndem~y & pPrepaid & pUnins~e & insure \\
\hline .5419344 & .3754875 & .0825782 & Indemnity \\
.4359638 & .496328 & .0677081 & \begin{tabular}{r} 
Prepaid \\
.5111583 \\
.3941132
\end{tabular} \\
.4105107 & .0783309 & Indemnity \\
.4655651 & .4625064 & .0616633 & Prepaid \\
\hline .4401779 & .4915102 & .0683118 & Prepaid \\
.4632122 & .4651931 & .0715948 & Prepaid \\
.3772302 & .5635696 & .0592002 &. \\
.4867758 & .4383018 & .0749225 & Uninsure \\
.5823668 & .3295802 & .0880531 & Prepaid \\
\hline
\end{tabular}

Observations 5 and 8 are not used to fit the model because insure is missing at these points, but predict estimates the probabilities for these observations since none of the independent variables is missing. You can use if e(sample) in the call to predict to use only those observations that are used to fit the model.

\section*{Methods and formulas}

\section*{predict}

Let level \(b\) be the base outcome that is used to fit the stereotype logistic regression model of dimension \(d\). The index for observation \(i\) and level \(k \neq b\) is \(\eta_{i k}=\theta_{k}-\sum_{j=1}^{d} \phi_{j k} \mathbf{x}_{i} \boldsymbol{\beta}_{j}\). This is the \(\log\) odds of outcome encoded as level \(k\) relative to that of \(b\) so that we define \(\eta_{i b} \equiv 0\). The outcome probabilities for this model are defined as \(\operatorname{Pr}\left(Y_{i}=k\right)=e^{\eta_{i k}} / \sum_{j=1}^{m} e^{\eta_{i j}}\). Unlike in mlogit, ologit, and oprobit, the index is no longer a linear function of the parameters. The standard error of index \(\eta_{i k}\) is thus computed using the delta method (see also [R] predictnl).

The equation-level score for regression coefficients is
\[
\frac{\partial \ln L_{i k}}{\partial \mathbf{x}_{i} \boldsymbol{\beta}_{j}}=\left(\sum_{l=1}^{m-1} \phi_{j l} p_{i l}-\phi_{j k}\right)
\]
the equation-level score for the scale parameters is
\[
\frac{\partial \ln L_{i k}}{\partial \phi_{j l}}= \begin{cases}\mathbf{x}_{i} \boldsymbol{\beta}_{j}\left(p_{i k}-1\right), & \text { if } l=k \\ \mathbf{x}_{i} \boldsymbol{\beta}_{j} p_{i l}, & \text { if } l \neq k\end{cases}
\]
for \(l=1, \ldots, m-1\); and the equation-level score for the intercepts is
\[
\frac{\partial \ln L_{i k}}{\partial \theta_{l}}= \begin{cases}1-p_{i k}, & \text { if } l=k \\ -p_{i l}, & \text { if } l \neq k\end{cases}
\]

\section*{Also see}
[R] slogit - Stereotype logistic regression
[U] 20 Estimation and postestimation commands

\section*{Title}
smooth — Robust nonlinear smoother
\begin{tabular}{llll} 
Syntax & Menu & Description & Option \\
Remarks and examples & Methods and formulas & Acknowledgments & References
\end{tabular}

\section*{Syntax}
smooth smoother [, twice] varname [if] [in], generate(newvar)
where smoother is specified as \(\operatorname{Sm}[\operatorname{Sm}[\ldots]]\) and \(S m\) is one of
\[
\begin{aligned}
& \{1|2| 3|4| 5|6| 7|8| 9\}[R] \\
& 3[R] S[S \mid R][S \mid R] \ldots \\
& \mathrm{E} \\
& \mathrm{H}
\end{aligned}
\]

Letters may be specified in lowercase if preferred. Examples of smoother [,twice] include
\begin{tabular}{lllll} 
3RSSH & 3RSSH,twice & 4253H & 4253H,twice & 43RSR2H,twice \\
3rssh & 3rssh,twice & 4253 h & 4253 h, twice & 43rsr2h,twice
\end{tabular}

\section*{Menu}

Statistics \(>\) Nonparametric analysis \(>\) Robust nonlinear smoother

\section*{Description}
smooth applies the specified resistant, nonlinear smoother to varname and stores the smoothed series in newvar.

\section*{Option}
generate(newvar) is required; it specifies the name of the new variable that will contain the smoothed values.

\section*{Remarks and examples}

Smoothing is an exploratory data-analysis technique for making the general shape of a series apparent. In this approach (Tukey 1977), the observed data series is assumed to be the sum of an underlying process that evolves smoothly (the smooth) and of an unsystematic noise component (the rough); that is,
\[
\text { data }=\text { smooth }+ \text { rough }
\]

Smoothed values \(z_{t}\) are obtained by taking medians (or some other location estimate) of each point in the original data \(y_{t}\) and a few of the points around it. The number of points used is called the span of the smoother. Thus a span- 3 smoother produces \(z_{t}\) by taking the median of \(y_{t-1}, y_{t}\), and \(y_{t+1}\). smooth provides running median smoothers of spans 1 to 9 -indicated by the digit that specifies their span. Median smoothers are resistant to isolated outliers, so they provide robustness to spikes in the data. Because the median is also a nonlinear operator, such smoothers are known as robust (or resistant) nonlinear smoothers.
smooth also provides the Hanning linear, nonrobust smoother, indicated by the letter H. Hanning is a span-3 smoother with binomial weights. Repeated applications of \(\mathrm{H}-\mathrm{HH}\), HHH, etc.- provide binomial smoothers of span 5, 7, etc. See Cox \((1997,2004)\) for a graphical application of this fact.

Because one smoother usually cannot adequately separate the smooth from the rough, compound smoothers - multiple smoothers applied in sequence-are used. The smoother 35 H , for instance, then smooths the data with a span-3 median smoother, smooths the result with a span- 5 median smoother, and finally smooths that result with the Hanning smoother. smooth allows you to specify any number of smoothers in any sequence.

Three refinements can be combined with the running median and Hanning smoothers. First, the endpoints of a smooth can be given special treatment. This is specified by the E operator. Second, smoothing by 3 , the span- 3 running median, tends to produce flat-topped hills and valleys. The splitting operator, S, "splits" these repeated values, applies the endpoint operator to them, and then "rejoins" the series. Finally, it is sometimes useful to repeat an odd-span median smoother or the splitting operator until the smooth no longer changes. Following a digit or an \(S\) with an \(R\) specifies this type of repetition.

Even the best smoother may fail to separate the smooth from the rough adequately. To guard against losing any systematic components of the data series, after smoothing, the smoother can be reapplied to the resulting rough, and any recovered signal can be added back to the original smooth. The twice operator specifies this procedure. More generally, an arbitrary smoother can be applied to the rough (using a second smooth command), and the recovered signal can be added back to the smooth. This more general procedure is called reroughing (Tukey 1977).

The details of each of the smoothers and operators are explained in Methods and formulas below.

\section*{> Example 1}
smooth is designed to recover the general features of a series that has been contaminated with noise. To demonstrate this, we construct a series, add noise to it, and then smooth the noisy version to recover an estimate of the original data. First, we construct and display the data:
```

. drop _all

```
. set obs 10
. set seed 123456789
. generate time = _n
. label variable time "Time"
- generate \(\mathrm{x}=\) _n^3-10*_n^2 + 5*_n
. label variable x "Signal"
. generate \(\mathrm{z}=\mathrm{x}+50\) *rnormal()
. label variable z "Observed series"
```

. scatter x z time, c(l .) m(i o) ytitle("")

```


Now we smooth the noisy series, \(z\), assumed to be the only data we would observe:
```

. smooth 4253eh,twice z, gen(sz)
. label variable sz "Smoothed series"
. scatter x z sz time, c(l . l) m(i o i) ytitle("") || scatter sz time,
> c(l . l) m(i o i) ytitle("") clpattern(dash_dot)

```


\section*{Example 2}

Salgado-Ugarte and Curts-García (1993) provide data on the frequencies of observed fish lengths. In this example, the series to be smoothed - the frequencies - is ordered by fish length rather than by time.
```

. scatter sfreq freq length, c(l .) m(i o)
> title("Smoothed frequencies of fish lengths") ytitle("") xlabel(\#4)

```

- Technical note
smooth allows missing values at the beginning and end of the series, but missing values in the middle are not allowed. Leading and trailing missing values are ignored. If you wish to ignore missing values in the middle of the series, you must drop the missing observations before using smooth. Doing so, of course, would violate smooth's assumption that observations are equally spaced-each observation represents a year, a quarter, or a month (or a 1-year birth-rate category). In practice, smooth produces good results as long as the spaces between adjacent observations do not vary too much.

Smoothing is usually applied to time series, but any variable with a natural order can be smoothed. For example, a smoother might be applied to the birth rate recorded by the age of the mothers (birth rate for 17 -year-olds, birth rate for 18 -year-olds, and so on).

\section*{Methods and formulas}

Methods and formulas are presented under the following headings:
Running median smoothers of odd span
Running median smoothers of even span
Repeat operator
Endpoint rule
Splitting operator
Hanning smoother
Twicing

\section*{Running median smoothers of odd span}

The smoother 3 defines
\[
z_{t}=\operatorname{median}\left(y_{t-1}, y_{t}, y_{t+1}\right)
\]

The smoother 5 defines
\[
z_{t}=\operatorname{median}\left(y_{t-2}, y_{t-1}, y_{t}, y_{t+1}, y_{t+2}\right)
\]
and so on. The smoother 1 defines \(z_{t}=\operatorname{median}\left(y_{t}\right)\), so it does nothing.
Endpoints are handled by using smoothers of shorter, odd span. Thus for 3,
\[
\begin{aligned}
z_{1} & =y_{1} \\
z_{2} & =\operatorname{median}\left(y_{1}, y_{2}, y_{3}\right) \\
\vdots & \\
z_{N-1} & =\operatorname{median}\left(y_{N-2}, y_{N-1}, y_{N}\right) \\
Z_{N} & =y_{N}
\end{aligned}
\]

For 5,
\[
\begin{aligned}
z_{1} & =y_{1} \\
z_{2} & =\operatorname{median}\left(y_{1}, y_{2}, y_{3}\right) \\
z_{3} & =\operatorname{median}\left(y_{1}, y_{2}, y_{3}, y_{4}, y_{5}\right) \\
z_{4} & =\operatorname{median}\left(y_{2}, y_{3}, y_{4}, y_{5}, y_{6}\right) \\
\vdots & \\
z_{N-2} & =\operatorname{median}\left(y_{N-4}, y_{N-3}, y_{N-2}, y_{N-1}, y_{N}\right) \\
z_{N-1} & =\operatorname{median}\left(y_{N-2}, y_{N-1}, y_{N}\right) \\
Z_{N} & =y_{N}
\end{aligned}
\]
and so on.

\section*{Running median smoothers of even span}

Define the median() function as returning the linearly interpolated value when given an even number of arguments. Thus the smoother 2 defines
\[
z_{t+0.5}=\left(y_{t}+y_{t+1}\right) / 2
\]

The smoother 4 defines \(z_{t+0.5}\) as the linearly interpolated median of \(\left(y_{t-1}, y_{t}, y_{t+1}, y_{t+2}\right)\), and so on. Endpoints are always handled using smoothers of shorter, even span. Thus for 4,
\[
\begin{aligned}
z_{0.5} & =y_{1} \\
z_{1.5} & =\operatorname{median}\left(y_{1}, y_{2}\right)=\left(y_{1}+y_{2}\right) / 2 \\
z_{2.5} & =\operatorname{median}\left(y_{1}, y_{2}, y_{3}, y_{4}\right) \\
\vdots & \\
z_{N-2.5} & =\operatorname{median}\left(y_{N-4}, y_{N-3}, y_{N-2}, y_{N}\right) \\
z_{N-1.5} & =\operatorname{median}\left(y_{N-2}, y_{N-1}\right) \\
z_{N-0.5} & =\operatorname{median}\left(y_{N-1}, y_{N}\right) \\
z_{N+0.5} & =y_{N}
\end{aligned}
\]

As defined above, an even-span smoother increases the length of the series by 1 observation. However, the series can be recentered on the original observation numbers, and the "extra" observation can be eliminated by smoothing the series again with another even-span smoother. For instance, the smooth of 4 illustrated above could be followed by a smooth of 2 to obtain
\[
\begin{aligned}
z_{1}^{*} & =\left(z_{0.5}+z_{1.5}\right) / 2 \\
z_{2}^{*} & =\left(z_{1.5}+z_{2.5}\right) / 2 \\
z_{3}^{*} & =\left(z_{2.5}+z_{3.5}\right) / 2 \\
\vdots & \\
z_{N-2}^{*} & =\left(z_{N-2.5}+z_{N-1.5}\right) / 2 \\
z_{N-1}^{*} & =\left(z_{N-1.5}+z_{N}-0.5\right) / 2 \\
z_{N}^{*} & =\left(z_{N-0.5}+z_{N}+0.5\right) / 2
\end{aligned}
\]
smooth keeps track of the number of even smoothers applied to the data and expands and shrinks the length of the series accordingly. To ensure that the final smooth has the same number of observations as varname, smooth requires you to specify an even number of even-span smoothers. However, the pairs of even-span smoothers need not be contiguous; for instance, 4253 and 4523 are both allowed.

\section*{Repeat operator}
\(R\) indicates that a smoother is to be repeated until convergence, that is, until repeated applications of the smoother produce the same series. Thus 3 applies the smoother of running medians of span 3. 33 applies the smoother twice. \(3 R\) produces the result of repeating 3 an infinite number of times. \(R\) should be used only with odd-span smoothers because even-span smoothers are not guaranteed to converge.

The smoother 453R2 applies a span-4 smoother, followed by a span-5 smoother, followed by repeated applications of a span- 3 smoother, followed by a span- 2 smoother.

\section*{Endpoint rule}

The endpoint rule E modifies the values \(z_{1}\) and \(z_{N}\) according to the following formulas:
\[
\begin{aligned}
z_{1} & =\operatorname{median}\left(3 z_{2}-2 z_{3}, z_{1}, z_{2}\right) \\
z_{N} & =\operatorname{median}\left(3 z_{N-2}-2 z_{N-1}, z_{N}, z_{N-1}\right)
\end{aligned}
\]

When the endpoint rule is not applied, endpoints are typically "copied in"; that is, \(z_{1}=y_{1}\) and \(z_{N}=y_{N}\).

\section*{Splitting operator}

The smoothers 3 and 3R can produce flat-topped hills and valleys. The split operator attempts to eliminate such hills and valleys by splitting the sequence, applying the endpoint rule E , rejoining the series, and then resmoothing by 3R.

The \(S\) operator may be applied only after \(3,3 R\), or \(S\).
We recommend that the \(S\) operator be repeated once (SS) or until no further changes take place (SR).

\section*{Hanning smoother}

H is the Hanning linear smoother:
\[
z_{t}=\left(y_{t-1}+2 y_{t}+y_{t+1}\right) / 4
\]

Endpoints are copied in: \(z_{1}=y_{1}\) and \(z_{N}=y_{N} . \mathrm{H}\) should be applied only after all nonlinear smoothers.

\section*{Twicing}

A smoother divides the data into a smooth and a rough:
\[
\text { data }=\text { smooth }+ \text { rough }
\]

If the smoothing is successful, the rough should exhibit no pattern. Twicing refers to applying the smoother to the observed, calculating the rough, and then applying the smoother to the rough. The resulting "smoothed rough" is then added back to the smooth from the first step.

\section*{Acknowledgments}
smooth was originally written by William Gould (1992) - at which time it was named nlsm—and was inspired by Salgado-Ugarte and Curts-García (1992). Salgado-Ugarte and Curts-García (1993) subsequently reported anomalies in nlsm's treatment of even-span median smoothers. smooth corrects these problems and incorporates other improvements but otherwise is essentially the same as originally published.

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\section*{Also see}
[ R ] lowess - Lowess smoothing
[R] Ipoly - Kernel-weighted local polynomial smoothing
[TS] tssmooth - Smooth and forecast univariate time-series data

\section*{Title}
spearman - Spearman's and Kendall's correlations
\begin{tabular}{llll} 
Syntax & Menu & Description & Options for spearman \\
Options for ktau & Remarks and examples & Stored results & Methods and formulas \\
Acknowledgment & References & Also see &
\end{tabular}

\section*{Syntax}

Spearman's rank correlation coefficients
spearman [varlist] [if][in] [, spearman_options]

Kendall's rank correlation coefficients
ktau [varlist] [if] [in] [, ktau_options]
spearman_options Description

Main
stats (spearman_list) list of statistics; select up to three statistics; default is stats (rho)
print(\#)
star(\#)
bonferroni
sidak
pw
matrix
significance level for displaying coefficients significance level for displaying with a star use Bonferroni-adjusted significance level use Šidák-adjusted significance level calculate all pairwise correlation coefficients by using all available data display output in matrix form
ktau_options
Main
print(\#)
star (\#)
bonferroni
sidak
pw
matrix
stats (ktau_list) list of statistics; select up to six statistics; default is stats (taua)
Description significance level for displaying coefficients
significance level for displaying with a star
use Bonferroni-adjusted significance level
use Šidák-adjusted significance level
calculate all pairwise correlation coefficients by using all available data display output in matrix form
by is allowed with spearman and ktau; see [D] by.
where the elements of spearman_list may be
\begin{tabular}{ll} 
rho & correlation coefficient \\
obs & number of observations \\
p & significance level
\end{tabular}
and the elements of ktau_list may be
\begin{tabular}{ll} 
taua & correlation coefficient \(\tau_{a}\) \\
taub & correlation coefficient \(\tau_{b}\) \\
score & score \\
se & standard error of score \\
obs & number of observations \\
p & significance level
\end{tabular}

\section*{Menu}

\section*{spearman}

Statistics \(>\) Nonparametric analysis \(>\) Tests of hypotheses \(>\) Spearman's rank correlation
ktau
Statistics \(>\) Nonparametric analysis \(>\) Tests of hypotheses \(>\) Kendall's rank correlation

\section*{Description}
spearman displays Spearman's rank correlation coefficients for all pairs of variables in varlist or, if varlist is not specified, for all the variables in the dataset.
ktau displays Kendall's rank correlation coefficients between the variables in varlist or, if varlist is not specified, for all the variables in the dataset. ktau is intended for use on small- and moderate-sized datasets; it requires considerable computation time for larger datasets.

\section*{Options for spearman}
\(\qquad\) Main
stats (spearman_list) specifies the statistics to be displayed in the matrix of output. stats (rho) is the default. Up to three statistics may be specified; stats (rho obs p) would display the correlation coefficient, number of observations, and significance level. If varlist contains only two variables, all statistics are shown in tabular form, and stats(), print(), and star() have no effect unless the matrix option is specified.
print (\#) specifies the significance level of correlation coefficients to be printed. Correlation coefficients with larger significance levels are left blank in the matrix. Typing spearman, print (.10) would list only those correlation coefficients that are significant at the \(10 \%\) level or lower.
star(\#) specifies the significance level of correlation coefficients to be marked with a star. Typing spearman, star(.05) would "star" all correlation coefficients significant at the \(5 \%\) level or lower.
bonferroni makes the Bonferroni adjustment to calculated significance levels. This adjustment affects printed significance levels and the print() and star() options. Thus spearman, print (.05) bonferroni prints coefficients with Bonferroni-adjusted significance levels of 0.05 or less.
sidak makes the Šidák adjustment to calculated significance levels. This adjustment affects printed significance levels and the print() and star() options. Thus spearman, print(.05) sidak prints coefficients with Šidák-adjusted significance levels of 0.05 or less.
pw specifies that correlations be calculated using pairwise deletion of observations with missing values. By default, spearman uses casewise deletion, where observations are ignored if any of the variables in varlist are missing.
matrix forces spearman to display the statistics as a matrix, even if varlist contains only two variables. matrix is implied if more than two variables are specified.

\section*{Options for ktau}
\(\qquad\) Main
stats (ktau_list) specifies the statistics to be displayed in the matrix of output. stats(taua) is the default. Up to six statistics may be specified; stats (taua taub score se obs p) would display the correlation coefficients \(\tau_{a}, \tau_{b}\), score, standard error of score, number of observations, and significance level. If varlist contains only two variables, all statistics are shown in tabular form and stats(), print(), and star() have no effect unless the matrix option is specified. print(\#) specifies the significance level of correlation coefficients to be printed. Correlation coefficients with larger significance levels are left blank in the matrix. Typing ktau, print (.10) would list only those correlation coefficients that are significant at the \(10 \%\) level or lower.
star(\#) specifies the significance level of correlation coefficients to be marked with a star. Typing ktau, star(.05) would "star" all correlation coefficients significant at the \(5 \%\) level or lower.
bonferroni makes the Bonferroni adjustment to calculated significance levels. This adjustment affects printed significance levels and the print() and star() options. Thus ktau, print (.05) bonferroni prints coefficients with Bonferroni-adjusted significance levels of 0.05 or less.
sidak makes the Šidák adjustment to calculated significance levels. This adjustment affects printed significance levels and the print() and star() options. Thus ktau, print(.05) sidak prints coefficients with Šidák-adjusted significance levels of 0.05 or less.
pw specifies that correlations be calculated using pairwise deletion of observations with missing values. By default, ktau uses casewise deletion, where observations are ignored if any of the variables in varlist are missing.
matrix forces ktau to display the statistics as a matrix, even if varlist contains only two variables. matrix is implied if more than two variables are specified.

\section*{Remarks and examples}
\(>\) Example 1
We wish to calculate the correlation coefficients among marriage rate (mrgrate), divorce rate (divorce_rate), and median age (medage) in state data. We can calculate the standard Pearson correlation coefficients and significance by typing
```

.use http://www.stata-press.com/data/r13/states2
(State data)

```
. pwcorr mrgrate divorce_rate medage, sig
\begin{tabular}{r|rrr} 
& mrgrate divorc~e & medage \\
\hline mrgrate & 1.0000 & & \\
& & & \\
divorce_rate & 0.7895 & 1.0000 & \\
& 0.0000 & & \\
medage & 0.0011 & -0.1526 & 1.0000 \\
& 0.9941 & 0.2900 & \\
& & &
\end{tabular}

We can calculate Spearman's rank correlation coefficients by typing
```

. spearman mrgrate divorce_rate medage, stats(rho p)
(obs=50)

```
\begin{tabular}{|l|}
\hline Key \\
\hline rho \\
Sig. level \\
\hline
\end{tabular}
\begin{tabular}{r|rrr} 
& mrgrate divorc~e & medage \\
\hline mrgrate & 1.0000 & & \\
divorce_rate & 0.6933 & 1.0000 & \\
& 0.0000 & & \\
medage & -0.4869 & -0.2455 & 1.0000 \\
& 0.0003 & 0.0857 & \\
& & &
\end{tabular}

The large difference in the results is caused by one observation. Nevada's marriage rate is almost 10 times higher than the state with the next-highest marriage rate. An important feature of the Spearman rank correlation coefficient is its reduced sensitivity to extreme values compared with the Pearson correlation coefficient.

We can calculate Kendall's rank correlations by typing
. Ktau mrgrate divorce_rate medage, stats (taua taub p)
(obs=50)

\begin{tabular}{r|rrr} 
& mrgrate & divorc~e & medage \\
\hline mrgrate & 0.9829 & & \\
& 1.0000 & & \\
divorce_rate & & & \\
& 0.5110 & 0.9804 & \\
& 0.5206 & 1.0000 & \\
0.0000 & & 0.9845 \\
& -0.3486 & -0.1698 & 1.0000 \\
& -0.3544 & -0.1728 & \\
& 0.0004 & 0.0828 &
\end{tabular}

There are tied values for variables mrgrate, divorce_rate, and medage, so tied ranks are used. As a result, \(\tau_{a}<1\) on the diagonal (see Methods and formulas for the definition of \(\tau_{a}\) ).

\section*{- Technical note}

According to Conover \((1999,323)\), "Spearman's \(\rho\) tends to be larger than Kendall's \(\tau\) in absolute value. However, as a test of significance, there is no strong reason to prefer one over the other because both will produce nearly identical results in most cases."

\section*{> Example 2}

We illustrate spearman and ktau with the auto data, which contains some missing values.
```

.use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. spearman mpg rep78
Number of obs = 69
Spearman's rho = 0.3098
Test of Ho: mpg and rep78 are independent
Prob > |t| = 0.0096

```

Because we specified two variables, spearman displayed the sample size, correlation, and \(p\)-value in tabular form. To obtain just the correlation coefficient displayed in matrix form, we type
```

. spearman mpg rep78, stats(rho) matrix
(obs=69)

|  | mpg | rep78 |
| ---: | ---: | ---: |
| mpg | 1.0000 |  |
| rep78 | 0.3098 | 1.0000 |

```

The pw option instructs spearman and ktau to use all nonmissing observations between a pair of variables when calculating their correlation coefficient. In the output below, some correlations are based on 74 observations, whereas others are based on 69 because 5 observations contain a missing value for rep78.
```

. spearman mpg price rep78, pw stats(rho obs p) star(0.01)

```
\begin{tabular}{|l|}
\hline Key \\
\hline rho \\
Number of obs \\
Sig. level \\
\hline
\end{tabular}
\begin{tabular}{r|rrr} 
& mpg & price & rep78 \\
\hline mpg & 1.0000 & & \\
& 74 & & \\
& & & \\
price & \(-0.5419 *\) & 1.0000 & \\
& 0.0000 & 74 & \\
& \(0.3098 *\) & 0.1028 & 1.0000 \\
69 & 69 & 69
\end{tabular}

Finally, the bonferroni and sidak options provide adjusted significance levels:
```

. ktau mpg price rep78, stats(taua taub score se p) bonferroni
(obs=69)

```
\begin{tabular}{|l|}
\hline Key \\
\hline tau_a \\
tau_b \\
score \\
se of score \\
Sig. level \\
\hline
\end{tabular}
\begin{tabular}{r|rrr} 
& mpg & price & rep78 \\
\hline \multirow{4}{*}{ mpg } & 0.9471 & & \\
& 1.0000 & & \\
& 2222.0000 & & \\
& 191.8600 & & \\
& & & \\
& -0.3973 & 1.0000 & \\
& -0.4082 & 1.0000 & \\
& -932.0000 & 2346.0000 & \\
& 192.4561 & 193.0682 & \\
& 0.0000 & & 0.7136 \\
& 0.2076 & 0.0648 & 1.0000 \\
& 0.2525 & 0.0767 & 162.0000 \\
& 181.7024 & 182.0000 & 1674.0000 \\
& 0.0224 & 1.0000 & 172.2161
\end{tabular}

Charles Edward Spearman (1863-1945) was a British psychologist who made contributions to correlation, factor analysis, test reliability, and psychometrics. After several years' military service, he obtained a PhD in experimental psychology at Leipzig and became a professor at University College London, where he sustained a long program of work on the interpretation of intelligence tests. Ironically, the rank correlation version bearing his name is not the formula he advocated.

Maurice George Kendall (1907-1983) was a British statistician who contributed to rank correlation, time series, multivariate analysis, among other topics, and wrote many statistical texts. Most notably, perhaps, his advanced survey of the theory of statistics went through several editions, later ones with Alan Stuart; the baton has since passed to others. Kendall was employed in turn as a government and business statistician, as a professor at the London School of Economics, as a consultant, and as director of the World Fertility Survey. He was knighted in 1974.

\section*{Stored results}
spearman stores the following in \(r()\) :
Scalars \(r(N) \quad\) number of observations (last variable pair) r(rho)
\(r(p)\)
\[
\rho \text { (last variable pair) }
\]
two-sided \(p\)-value (last variable pair)
Matrices
\(r\) (Nobs) number of observations
r(Rho)
\(r(\mathrm{P}) \quad\) two-sided \(p\)-value
ktau stores the following in r() :
Scalars
\begin{tabular}{|c|c|}
\hline r (N) & number of observations (last variable pair) \\
\hline r(tau_a) & \(\tau_{\mathrm{a}}\) (last variable pair) \\
\hline r(tau_b) & \(\tau_{\mathrm{b}}\) (last variable pair) \\
\hline r(score) & Kendall's score (last variable pair) \\
\hline r(se_score) & se of score (last variable pair) \\
\hline r (p) & two-sided \(p\)-value (last variable pair) \\
\hline \multicolumn{2}{|l|}{trices} \\
\hline r(Nobs) & number of observations \\
\hline r(Tau_a) & \(\tau_{\mathrm{a}}\) \\
\hline r(Tau_b) & \(\tau_{\text {b }}\) \\
\hline r(Score) & Kendall's score \\
\hline r(Se_Score) & standard error of score \\
\hline \(r\) (P) & two-sided \(p\)-value \\
\hline
\end{tabular}

\section*{Methods and formulas}

Spearman's (1904) rank correlation is calculated as Pearson's correlation computed on the ranks and average ranks (Conover 1999, 314-315). Ranks are as calculated by egen; see [D] egen. The significance is calculated using the approximation
\[
p=2 \times \operatorname{ttail}\left(n-2,|\widehat{\rho}| \sqrt{n-2} / \sqrt{1-\widehat{\rho}^{2}}\right)
\]

For any two pairs of ranks \(\left(x_{i}, y_{i}\right)\) and \(\left(x_{j}, y_{j}\right)\) of one variable pair (varname \({ }_{1}\), varname \({ }_{2}\) ), \(1 \leq i, j \leq n\), where \(n\) is the number of observations, define them as concordant if
\[
\left(x_{i}-x_{j}\right)\left(y_{i}-y_{j}\right)>0
\]
and discordant if this product is less than zero.
Kendall's (1938; also see Kendall and Gibbons [1990] or Bland [2000], 222-225) score \(S\) is defined as \(C-D\), where \(C(D)\) is the number of concordant (discordant) pairs. Let \(N=n(n-1) / 2\) be the total number of pairs, so \(\tau_{\mathrm{a}}\) is given by
\[
\tau_{\mathrm{a}}=S / N
\]
and \(\tau_{\mathrm{b}}\) is given by
\[
\tau_{\mathrm{b}}=\frac{S}{\sqrt{N-U} \sqrt{N-V}}
\]
where
\[
\begin{aligned}
U & =\sum_{i=1}^{N_{1}} u_{i}\left(u_{i}-1\right) / 2 \\
V & =\sum_{j=1}^{N_{2}} v_{j}\left(v_{j}-1\right) / 2
\end{aligned}
\]
and where \(N_{1}\) is the number of sets of tied \(x\) values, \(u_{i}\) is the number of tied \(x\) values in the \(i\) th set, \(N_{2}\) is the number of sets of tied \(y\) values, and \(v_{j}\) is the number of tied \(y\) values in the \(j\) th set. Under the null hypothesis of independence between varname \(1_{1}\) and varname \({ }_{2}\), the variance of \(S\) is exactly (Kendall and Gibbons 1990, 66)
\[
\begin{aligned}
\operatorname{Var}(S)= & \frac{1}{18}\left\{n(n-1)(2 n+5)-\sum_{i=1}^{N_{1}} u_{i}\left(u_{i}-1\right)\left(2 u_{i}+5\right)-\sum_{j=1}^{N_{2}} v_{j}\left(v_{j}-1\right)\left(2 v_{j}+5\right)\right\} \\
& +\frac{1}{9 n(n-1)(n-2)}\left\{\sum_{i=1}^{N_{1}} u_{i}\left(u_{i}-1\right)\left(u_{i}-2\right)\right\}\left\{\sum_{j=1}^{N_{2}} v_{j}\left(v_{j}-1\right)\left(v_{j}-2\right)\right\} \\
& +\frac{1}{2 n(n-1)}\left\{\sum_{i=1}^{N_{1}} u_{i}\left(u_{i}-1\right)\right\}\left\{\sum_{j=1}^{N_{2}} v_{j}\left(v_{j}-1\right)\right\}
\end{aligned}
\]

Using a normal approximation with a continuity correction,
\[
z=\frac{|S|-1}{\sqrt{\operatorname{Var}(S)}}
\]

For the hypothesis of independence, the statistics \(S, \tau_{\mathrm{a}}\), and \(\tau_{\mathrm{b}}\) produce equivalent tests and give the same significance.

For Kendall's \(\tau\), the normal approximation is surprisingly accurate for sample sizes as small as 8 , at least for calculating \(p\)-values under the null hypothesis for continuous variables. (See Kendall and Gibbons [1990, chap. 4], who also present some tables for calculating exact \(p\)-values for \(n<10\).) For Spearman's \(\rho\), the normal approximation requires larger samples to be valid.

Let \(v\) be the number of variables specified so that \(k=v(v-1) / 2\) correlation coefficients are to be estimated. If bonferroni is specified, the adjusted significance level is \(p^{\prime}=\min (1, k p)\). If sidak is specified, \(p^{\prime}=\min \left\{1,1-(1-p)^{n}\right\}\). See Methods and formulas in \([\mathrm{R}]\) oneway for a more complete description of the logic behind these adjustments.

Early work on rank correlation is surveyed by Kruskal (1958).

\section*{Acknowledgment}

The original version of ktau was written by Sean Becketti, a past editor of the Stata Technical Bulletin and author of the Stata Press book Introduction to Time Series Using Stata.

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\section*{Also see}
[R] correlate - Correlations (covariances) of variables or coefficients
\([R]\) nptrend - Test for trend across ordered groups

\section*{Title}
spikeplot - Spike plots and rootograms
\begin{tabular}{llll} 
Syntax & Menu & Description & Options \\
Remarks and examples & Acknowledgments & References & Also see
\end{tabular}

\section*{Syntax}
spikeplot varname \([\) if \(][\) in \(][\) weight \(][\), options \(]\)

\section*{options}

Description
Main
round (\#) round varname to nearest multiple of \# (bin width)
fraction make vertical scale the proportion of total values; default is frequencies
root make vertical scale show square roots of frequencies
Plot
spike_options affect rendition of plotted spikes
Add plots
addplot (plot) add other plots to generated graph
Y axis, X axis, Titles, Legend, Overall, By
twoway_options any options documented in [G-3] twoway_options
fweights, aweights, and iweights are allowed; see [U] 11.1.6 weight.

\section*{Menu}

Graphics \(>\) Distributional graphs \(>\) Spike plot and rootogram

\section*{Description}
spikeplot produces a frequency plot for a variable in which the frequencies are depicted as vertical lines from zero. The frequency may be a count, a fraction, or the square root of the count (Tukey's rootogram, circa 1965). The vertical lines may also originate from a baseline other than zero at the user's option.

\section*{Options}

\footnotetext{
round (\#) rounds the values of varname to the nearest multiple of \#. This action effectively specifies the bin width.
fraction specifies that the vertical scale be the proportion of total values (percentage) rather than the count.
root specifies that the vertical scale show square roots. This option may not be specified if fraction is specified.
}
\(\qquad\) Plot
spike_options affect the rendition of the plotted spikes; see [G-2] graph twoway spike.
\(\qquad\) Add plots
addplot (plot) provides a way to add other plots to the generated graph. See [G-3] addplot_option.
\(\qquad\) \(\sqrt{\mathrm{Y}}\) axis, X axis, Titles, Legend, Overall, By
twoway_options are any of the options documented in [G-3] twoway_options. These include options for titling the graph (see [G-3] title_options), options for saving the graph to disk (see [G-3] saving_option), and the by () option (see [G-3] by_option).

\section*{Remarks and examples}
> Example 1
Cox and Brady (1997a) present an illustrative example using the age structure of the population of Ghana from the 1960 census (rounded to the nearest 1,000 ). The dataset has ages from 0 (less than 1 year) to 90 . To view the distribution of ages, we would like to use each integer from 0 to 90 as the bins for the dataset.
```

. use http://www.stata-press.com/data/r13/ghanaage
. spikeplot age [fw=pop], ytitle("Population in 1000s") xlab(0(10)90)
> xmtick(5(10)85)

```


The resulting graph shows a "heaping" of ages at the multiples of 5. Also, ages ending in even numbers are more frequent than ages ending in odd numbers (except for 5). This preference for reporting ages is well known in demography and other social sciences.

Note also that we used the ytitle() option to override the default title of "Frequency" and that we used the xlab() and xmtick() options with numlists to further customize the resulting graph. See [U] 11.1.8 numlist for details on specifying numlists.

\section*{> Example 2}

The rootogram is a plot of the square-root transformation of the frequency counts. The square root of a normal distribution is a multiple of another normal distribution.
. clear
. set seed 1234567
. set obs 5000
obs was 0, now 5000
. generate normal = rnormal()
. label variable normal "Gaussian(0,1) random numbers"
. spikeplot normal, round(.10) xlab(-4(1)4)

. spikeplot normal, round(.10) xlab(-4(1)4) root


Interpreting a histogram in terms of normality is thus similar to interpreting the rootogram for normality.

This example also shows how the round() option is used to bin the values for a spike plot of a continuous variable.

\section*{> Example 3}
spikeplot can also be used to produce time-series plots. varname should be the time variable, and weights should be specified as the values for those times. To get a plot of daily rainfalls, we type
```

. spikeplot day [w=rain] if rain, ytitle("Daily rainfall in mm")

```

The base() option of graph twoway spike may be used to set a different baseline, such as when we want to show variations relative to an average or to some other measure of level.

\section*{Acknowledgments}

The original version of spikeplot was written by Nicholas J. Cox of the Department of Geography at Durham University, UK, and coeditor of the Stata Journal and Anthony R. Brady of the Imperial College School of Medicine (1997a, 1997b).

\section*{References}

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\section*{Also see}
[R] histogram - Histograms for continuous and categorical variables

\section*{Title}
sSc - Install and uninstall packages from SSC
\begin{tabular}{llll} 
Syntax & Description & Options & Remarks and examples \\
Acknowledgments & References & Also see &
\end{tabular}

\section*{Syntax}

Summary of packages most recently added or updated at SSC ssc new [, saving(filename[, replace]) type]

Summary of most popular packages at SSC
```

ssc hot [, n(\#) author(name)]

```

Describe a specified package at SSC
```

ssc describe {pkgname | letter} [, saving(filename[, replace])]

```

Install a specified package from SSC
```

ssc install pkgname [, all replace]

```

Uninstall from your computer a previously installed package from SSC ssc uninstall pkgname

Type a specific file stored at SSC
```

ssc type filename [, asis]

```

Copy a specific file from SSC to your computer
ssc copy filename [, plus personal replace public binary]
where letter in ssc describe is a-z or _.

\section*{Description}
ssc works with packages (and files) from the Statistical Software Components (SSC) archive, which is often called the Boston College Archive and is provided by http://repec.org.

The SSC has become the premier Stata download site for user-written software on the web. ssc provides a convenient interface to the resources available there. For example, on Statalist (see http://www.statalist.org/), users will often write

The program can be found by typing ssc install newprogramname.

Typing that would load everything associated with newprogramname, including the help files.
If you are searching for what is available, type ssc new and ssc hot, and see \([R]\) search. search searches the SSC and other places, too. search provides a GUI interface from which programs can be installed, including the programs at the SSC archive.

You can uninstall particular packages by using ssc uninstall. For the packages that you keep, see [R] adoupdate for an automated way of keeping those packages up to date.

\section*{Command overview}
ssc new summarizes the packages made available or updated recently. Output is presented in the Stata Viewer, and from there you may click to find out more about individual packages or to install them.
ssc hot lists the most popular packages-popular based on a moving average of the number of downloads in the past three months. By default, 10 packages are listed.
ssc describe pkgname describes, but does not install, the specified package. Use search to find packages; see \([R]\) search. If you know the package name but do not know the exact spelling, type ssc describe followed by one letter, a-z or _ (underscore), to list all the packages starting with that letter.
ssc install pkgname installs the specified package. You do not have to describe a package before installing it. (You may also install a package by using net install; see [R] net.)
ssc uninstall pkgname removes the previously installed package from your computer. It does not matter how the package was installed. (ssc uninstall is a synonym for ado uninstall, so either may be used to installed any package.)
ssc type filename types a specific file stored at SSC. ssc cat is a synonym for ssc type, which may appeal to those familiar with Unix.
ssc copy filename copies a specific file stored at SSC to your computer. By default, the file is copied to the current directory, but you can use options to change this. ssc copy is a rarely used alternative to ssc install ..., all. ssc cp is a synonym for ssc copy.

\section*{Options}

Options are presented under the following headings:
Options for use with ssc new
Options for use with ssc hot
Option for use with ssc describe
Options for use with ssc install
Option for use with ssc type Options for use with ssc copy

\section*{Options for use with ssc new}
saving(filename[, replace]) specifies that the "what's new" summary be saved in filename. If filename is specified without a suffix, filename. \(\operatorname{smcl}\) is assumed. If \(\operatorname{saving}()\) is not specified, saving(ssc_results.smcl) is assumed.
type specifies that the "what's new" results be displayed in the Results window rather than in the Viewer.

\section*{Options for use with ssc hot}
n (\#) specifies the number of packages to list; \(\mathrm{n}(10)\) is the default. Specify \(\mathrm{n}(\).\() to list all packages\) in order of popularity.
author (name) lists the 10 most popular packages by the specified author. If \(n(\#)\) is also specified, the top \# packages are listed.

\section*{Option for use with ssc describe}
saving (filename [, replace]) specifies that, in addition to the descriptions being displayed on your screen, it be saved in the specified file.
If filename is specified without an extension, .smcl will be assumed, and the file will be saved as a SMCL file.
If filename is specified with an extension, no default extension is added. If the extension is .log, the file will be stored as a text file.
If replace is specified, filename is replaced if it already exists.

\section*{Options for use with ssc install}
all specifies that any ancillary files associated with the package be downloaded to your current directory, in addition to the program and help files being installed. Ancillary files are files that do not end in .ado or .sthlp and typically contain datasets or examples of the use of the new command.
You can find out which files are associated with the package by typing ssc describe pkgname before or after installing. If you install without using the all option and then want the ancillary files, you can ssc install again.
replace specifies that any files being downloaded that already exist on your computer be replaced by the downloaded files. If replace is not specified and any files already exist, none of the files from the package is downloaded or installed.
It is better not to specify the replace option and wait to see if there is a problem. If there is a problem, it is usually better to uninstall the old package by using ssc uninstall or ado uninstall (which are, in fact, the same command).

\section*{Option for use with ssc type}
asis affects how files with the suffixes .smcl and .sthlp are displayed. The default is to interpret SMCL directives the file might contain. asis specifies that the file be displayed in raw, uninterpreted form.

\section*{Options for use with ssc copy}
plus specifies that the file be copied to the PLUS directory, the directory where user-written additions are installed. Typing sysdir will display the identity of the PLUS directory on your computer; see [P] sysdir.
personal specifies that the file be copied to your PERSONAL directory as reported by sysdir; see [P] sysdir.
If neither plus nor personal is specified, the default is to copy the file to the current directory.
replace specifies that, if the file already exists on your computer, the new file replace it.
public specifies that the new file be made readable by everyone; otherwise, the file will be created according to the default permission you have set with your operating system.
binary specifies that the file being copied is a binary file and that it is to be copied as is. The default is to assume that the file is a text file and change the end-of-line characters to those appropriate for your computer/operating system.

\section*{Remarks and examples}

Users can add new features to Stata, and some users choose to make new features that they have written available to others via the web. The files that comprise a new feature are called a package, and a package usually consists of one or more ado-files and help files. The net command (see [R] net) makes it reasonably easy to install and uninstall packages regardless of where they are on the web. One site, the SSC, has become particularly popular as a repository for additions to Stata. Command ssc is an easier to use version of net designed especially for the SSC.

Many packages are available at the SSC. Packages have names, such as oaxaca, estout, or egenmore. At SSC, capitalization is not significant, so Oaxaca, ESTOUT, and EGENmore are ways of writing the same package names.

When you type
```

. ssc install oaxaca

```
the files associated with the package are downloaded and installed on your computer. Package names usually correspond to the names of the command being added to Stata, so one would expect that installing the package oaxaca will add command oaxaca to Stata on your computer, and expect that typing help oaxaca will provide the documentation. That is the situation here, but that is not always so. Before or after installing a package, type ssc describe pkgname to obtain the details.

\section*{\(>\) Example 1}
ssc new summarizes the packages most recently made available or updated. Output is presented in the Viewer, from which you may click on a package name to find out more or install it. For example,
```

. ssc new
(contacting http://repec.org)
(output omitted)
GEOCODE3
module to retrieve coordinates or addresses from Google Geocoding API Version3
Authors: Stefan Bernhard Req: Stata version 12, insheetjson and libjson
> from SSC (q.v.)
Created: 2013-05-19
GGTAX
module to identify the most suitable GG family model
Authors: Andres L Gonzalez Rangel Req: Stata version 11
Created: 2013-05-19
ASL_NORM
module computing bootstrap Gaussianity tests
Authors: Maarten L. Buis Req: Stata version 11
Created: 2013-05-16
(output omitted)

```
        End of recent additions and updates
ssc hot provides a list of the most popular packages at SSC.
. ssc hot
Top 10 packages at SSC
\begin{tabular}{rrll} 
Rank & \begin{tabular}{c} 
Apr 2013 \\
\# hits
\end{tabular} & Package & Author(s) \\
\hline 1 & 12621.4 & estout & Ben Jann \\
2 & 12606.8 & outreg2 & Roy Wada \\
3 & 8508.8 & ranktest & Mark E Schaffer, Frank Kleibergen \\
4 & 8061.9 & ivreg2 & Mark E Schaffer, Steven Stillman, \\
& & & Christopher F Baum \\
5 & 3595.9 & psmatch2 & Edwin Leuven, Barbara Sianesi \\
6 & 2862.6 & tabout & Ian Watson \\
7 & 2358.2 & outreg & John Luke Gallup \\
8 & 2300.3 & winsor & Nicholas J. Cox \\
9 & 1743.0 & xtabond2 & David Roodman \\
10 & 1482.2 & xtivreg2 & Mark E Schaffer \\
\hline
\end{tabular}
(Click on package name for description)
Use the \(n(\#)\) option to change the number of packages listed:
. ssc hot, \(\mathrm{n}(20)\)
Top 20 packages at SSC
\begin{tabular}{rrll} 
Rank & \begin{tabular}{l} 
Apr 2013 \\
\# hits
\end{tabular} & Package & Author(s) \\
\hline 1 & 12621.4 & estout & Ben Jann \\
2 & 12606.8 & outreg2 & Roy Wada \\
3 & 8508.8 & ranktest & Mark E Schaffer, Frank Kleibergen \\
4 & 8061.9 & ivreg2 & \begin{tabular}{l} 
Mark E Schaffer, Steven Stillman, \\
\\
\\
5
\end{tabular} \\
& 3595.9 & psmatch2 & Edwin Leuven, Barbara Sianesi \\
6 & 2862.6 & tabout & Ian Watson \\
7 & 2358.2 & outreg & John Luke Gallup \\
8 & 2300.3 & winsor & Nicholas J. Cox \\
9 & 1743.0 & xtabond2 & David Roodman \\
10 & 1482.2 & xtivreg2 & Mark E Schaffer \\
11 & 1481.7 & fre & Ben Jann \\
12 & 1361.7 & cem & Stefano Iacus, Gary King, Matthew \\
& & & Blackwell, Giuseppe Porro \\
13 & 1279.3 & xttest3 & Christopher F Baum \\
14 & 1205.7 & mdesc & Rose Anne Medeiros, Dan Blanchette \\
15 & 1188.7 & bcuse & Christopher F Baum \\
16 & 1154.4 & usespss & Sergiy Radyakin \\
17 & 1113.3 & distinct & Gary Longton, Nicholas J. Cox \\
18 & 1111.1 & egenmore & Nicholas J. Cox \\
19 & 950.6 & gllamm & Sophia Rabe-Hesketh \\
20 & 948.3 & hprescott & Christopher F Baum
\end{tabular}
(Click on package name for description)

The author (name) option allows you to list the most popular packages by a specific person:
. ssc hot, author(baum)
\begin{tabular}{|c|c|c|c|}
\hline Rank & \[
\begin{gathered}
\text { Apr } 2013 \\
\text { \# hits }
\end{gathered}
\] & Package & Author (s) \\
\hline 4 & 8061.9 & ivreg2 & Mark E Schaffer, Steven Stillman, Christopher F Baum \\
\hline 13 & 1279.3 & xttest3 & Christopher F Baum \\
\hline 15 & 1188.7 & bcuse & Christopher F Baum \\
\hline 20 & 948.3 & hprescott & Christopher F Baum \\
\hline 27 & 806.8 & tscollap & Christopher F Baum \\
\hline 31 & 742.7 & whitetst & Christopher F Baum, Nicholas J. Cox \\
\hline 34 & 696.3 & xttest2 & Christopher F Baum \\
\hline 42 & 574.0 & overid & Christopher F Baum, Mark E Schaffer, Vince Wiggins, Steven Stillman \\
\hline 46 & 495.3 & kpss & Christopher F Baum \\
\hline 58 & 437.0 & ivendog & Christopher F Baum, Mark E Schaffer, Steven Stillman \\
\hline
\end{tabular}
(Click on package name for description)
ssc describe pkgname describes, but does not install, the specified package. You must already know the name of the package. See [R] search for assistance in searching for packages. Sometimes you know the package name, but you do not know the exact spelling. Then you can type ssc describe followed by one letter, a-z or _, to list all the packages starting with that letter; even so, using search is better.
```

. ssc describe bidensity

```
package bidensity from http://fmwww.bc.edu/repec/bocode/b

\section*{TITLE}
'BIDENSITY': module to produce and graph bivariate density estimates

\section*{DESCRIPTION/AUTHOR (S)}
bidensity produces bivariate kernel density estimates and graphs the result using a twoway contourline plot, optionally overlaying a scatterplot. The default kernel is Epanechnikov; all of the kernels provided by -kdensity- are also available. Compared to Baum's -kdens2- (SSC), which was recently enhanced to produce contourline plots, -bidensity- computes the bivariate kernel densities much more efficiently through use of Mata, and provides a choice of kernel estimators. The estimated densities can be saved in a Stata dataset or accessed as Mata matrices.
KW: density estimation
KW: bivariate density
KW: contourline plots
Requires: Stata version 12.1 and moremata from SSC (q.v.) Distribution-Date: 20130119
Author: John Luke Gallup, Portland State University
Support: email jlgallup@pdx.edu
Author: Christopher F Baum, Boston College
Support: email baum@bc.edu

\section*{INSTALLATION FILES}
bidensity.ado
bidensity.sthlp
(type -ssc install bidensity- to install)

The default setting for the saving() option is for the output to be saved with the . smcl extension. You could also save the file with a log extension, and in this case, the file would be stored as a text file.
```

. ssc describe b, saving(b.index)
(output omitted)
. ssc describe bidensity, saving(bidensity.log)
(output omitted)

```
ssc install pkgname installs the specified package. You do not have to describe a package before installing it. There are ways of installing packages other than ssc install, such as net; see \([R]\) net. It does not matter how a package is installed. For instance, a package can be installed using net and still be uninstalled using ssc.
```

. ssc install bidensity
checking bidensity consistency and verifying not already installed...
installing into C:\ado\plus\...
installation complete.

```
ssc uninstall pkgname removes the specified, previously installed package from your computer. You can uninstall immediately after installation or at any time in the future. (Technical note: ssc uninstall is a synonym for ado uninstall, so it can uninstall any installed package, not just packages obtained from the SSC.)
```

. ssc uninstall bidensity
package bidensity from http://fmwww.bc.edu/repec/bocode/b
'BIDENSITY': module to produce and graph bivariate density estimates
(package uninstalled)

```
ssc type filename types a specific file stored at the SSC. Although not shown in the syntax diagram, ssc cat is a synonym for ssc type, which may appeal to those familiar with Unix. To view only the bidensity help file for the bidensity package, you would type
```

. ssc type bidensity.sthlp
help for bidensity
Bivariate kernel density estimation
bidensity varnameY varnameX [if exp] [in range] [, n(\#)
kernel(kernelname) xwidth(\#) ywidth(\#) saving(name) replace
nograph scatter[(scatter_options)] contourline_options
mname(name)
(output omitted)

```
ssc copy filename copies a specific file stored at the SSC to your computer. By default, the file is copied to the current directory, but you can use options to change this. ssc copy is a rarely used alternative to ssc install ..., all. ssc cp is a synonym for ssc copy.
```

. ssc copy bidensity.ado
(file bidensity.ado copied to current directory)

```

For more details on the SSC archive and for information on how to submit your own programs to the SSC, see http://repec.org/bocode/s/sscsubmit.html.

\section*{Acknowledgments}
ssc is based on archutil by Nicholas J. Cox of the Department of Geography at Durham University, UK, and coeditor of the Stata Journal and Christopher F. Baum of the Department of Economics at Boston College and author of the Stata Press books An Introduction to Modern Econometrics Using Stata and An Introduction to Stata Programming. The reworking of the original was done with their blessing and their participation.

Christopher Baum maintains the Stata-related files stored at the SSC archive. We thank him for this contribution to the Stata community.

\section*{References}

Baum, C. F., and N. J. Cox. 1999. ip29: Metadata for user-written contributions to the Stata programming language. Stata Technical Bulletin 52: 10-12. Reprinted in Stata Technical Bulletin Reprints, vol. 9, pp. 121-124. College Station, TX: Stata Press.
Cox, N. J., and C. F. Baum. 2000. ip29.1: Metadata for user-written contributions to the Stata programming language. Stata Technical Bulletin 54: 21-22. Reprinted in Stata Technical Bulletin Reprints, vol. 9, pp. 124-126. College Station, TX: Stata Press.

\footnotetext{
Also see
[ R\(]\) adoupdate - Update user-written ado-files
\([\mathrm{R}]\) net - Install and manage user-written additions from the Internet
[R] search - Search Stata documentation and other resources
[R] sj - Stata Journal and STB installation instructions
[P] sysdir - Query and set system directories
}

\section*{Title}
stem - Stem-and-leaf displays
\begin{tabular}{llll} 
Syntax & Menu & Description & Options \\
Remarks and examples & Stored results & References & Also see
\end{tabular}

\section*{Syntax}
stem varname [if] [in][, options]
options
Description
Main
prune do not print stems that have no leaves
round (\#) round data to this value; default is round(1)
truncate (\#) truncate data to this value
digits(\#) digits per leaf; default is digits(1)
lines(\#) number of stems per interval of \(10^{\text {digits }}\)
width(\#) stem width; equal to \(10^{\text {digits }} /\) width
by is allowed; see [D] by.

\section*{Menu}

Statistics \(>\) Summaries, tables, and tests \(>\) Distributional plots and tests \(>\) Stem-and-leaf display

\section*{Description}
stem displays stem-and-leaf plots.

\section*{Options}

\section*{Main}
prune prevents printing any stems that have no leaves.
round (\#) rounds the data to this value and displays the plot in these units. If round () is not specified, noninteger data will be rounded automatically.
truncate(\#) truncates the data to this value and displays the plot in these units.
digits(\#) sets the number of digits per leaf. The default is 1 .
lines (\#) sets the number of stems per every data interval of \(10^{\text {digits }}\). The value of lines () must divide \(10^{\text {digits }}\); that is, if digits (1) is specified, then lines () must divide 10. If digits (2) is specified, then lines() must divide 100, etc. Only one of lines() or width() may be specified. If neither is specified, an appropriate value will be set automatically.
width(\#) sets the width of a stem. lines () is equal to \(10^{\text {digits }} /\) width, and this option is merely an alternative way of setting lines(). The value of width() must divide \(10^{\text {digits }}\). Only one of width() or lines() may be specified. If neither is specified, an appropriate value will be set automatically.

Note: If lines() or width() is not specified, digits() may be decreased in some circumstances to make a better-looking plot. If lines() or width() is set, the user-specified value of digits() will not be altered.

\section*{Remarks and examples}
> Example 1
Stem-and-leaf displays are a compact way to present considerable information about a batch of data. For instance, using our automobile data (described in [U] 1.2.2 Example datasets):
```

. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. stem mpg
Stem-and-leaf plot for mpg (Mileage (mpg))
1t | 22
1f 44444455
1s 66667777
1. }88888888899999999
2* 00011111
2t 22222333
2f 444455555
2s 666
2. }888
3* 001
3t
3f 45
3s
3.
4* 1

```

The stem-and-leaf display provides a way to list our data. The expression to the left of the vertical bar is called the stem; the digits to the right are called the leaves. All the stems that begin with the same digit and the corresponding leaves, written beside each other, reconstruct an observation of the data. Thus, if we look at the four stems that begin with the digit 1 and their corresponding leaves, we see that we have two cars rated at \(12 \mathrm{mpg}, 6\) cars at 14,2 at 15 , and so on. The car with the highest mileage rating in our data is rated at 41 mpg .

The above plot is a five-line plot with lines() equal to 5 (five lines per interval of 10) and width() equal to 2 (two leaves per stem).

Instead, we could specify lines(2):
```

. stem mpg, lines(2)
Stem-and-leaf plot for mpg (Mileage (mpg))
1* | 22444444
1. 556666777788888888899999999
2* 00011111222223334444
2. 555556668889
3* 0014
3. 55
4* 1

```
stem mpg, width(5) would produce the same plot as above.

The stem-and-leaf display provides a crude histogram of our data, one not so pretty as that produced by histogram (see \([R]\) histogram), but one that is nonetheless informative.

\section*{> Example 2}

The miles per gallon rating fits easily into a stem-and-leaf display because, in our data, it has two digits. However, stem does not require two digits.
```

. stem price, lines(1) digits(3)
Stem-and-leaf plot for price (Price)
3***
4*** 010,060,082,099,172,181,187,195,296,389,424,425,453,482,499, ... (26)
5*** 079,104,172,189,222,379,397,705,719,788,798,799,886,899
6*** 165,229,295,303,342,486,850
7*** 140,827
8*** 129,814
9*** 690,735
10*** 371,372
11*** 385,497,995
12*** 990
13*** 466,594
14*** 500
15*** 906

```

The (26) at the right of the second stem shows that there were 26 leaves on this stem - too many to display on one line.

We can make a more compact stem-and-leaf plot by rounding. To display stem in units of 100 , we could type
```

. stem price, round(100)
Stem-and-leaf plot for price (Price)
price rounded to nearest multiple of 100
plot in units of 100
3* 33778889
4* 00001112222344455555667777899
5* 11222447788899
6* 2233359
7* 18
8* 18
9* 77
10* 44
11* 45
12* 0
13* 056
14* 5
15* 9

```
price, in our data, has four or five digits. stem presented the display in terms of units of 100 , so a car that cost \(\$ 3,291\) was treated for display purposes as \(\$ 3,300\).

\section*{- Technical note}

Stem-and-leaf diagrams have been used in Japanese railway timetables, as shown in Tufte (1990, 46-47).

\section*{Stored results}
stem stores the following in r() :
Scalars
r(width) width of a stem
\(r\) (digits) number of digits per leaf; default is 1
Macros
\(r\) (round) number specified in round ()
\(r\) (truncate) number specified in truncate()

\section*{References}

Cox, N. J. 2007. Speaking Stata: Turning over a new leaf. Stata Journal 7: 413-433.
Emerson, J. D., and D. C. Hoaglin. 1983. Stem-and-leaf displays. In Understanding Robust and Exploratory Data Analysis, ed. D. C. Hoaglin, C. F. Mosteller, and J. W. Tukey, 7-32. New York: Wiley.
Tufte, E. R. 1990. Envisioning Information. Cheshire, CT: Graphics Press.
Tukey, J. W. 1972. Some graphic and semigraphic displays. In Statistical Papers in Honor of George W. Snedecor, ed. T. A. Bancroft and S. A. Brown, 293-316. Ames, IA: Iowa State University Press.
__. 1977. Exploratory Data Analysis. Reading, MA: Addison-Wesley.

\section*{Also see}
[R] histogram - Histograms for continuous and categorical variables
[R] lv - Letter-value displays

\section*{Title}
stepwise - Stepwise estimation

Syntax
Remarks and examples
Also see

Menu
Stored results

Description
Methods and formulas

Options
References

\section*{Syntax}
stepwise [, options] : command
options Description

Model
*pr (\#)
* pe(\#)

Model2
forward perform forward-stepwise selection
hierarchical perform hierarchical selection
lockterm1 keep the first term
lr
Reporting
display_options
control column formats and line width
* At least one of \(\mathrm{pr}(\#)\) or pe(\#) must be specified.
by and xi are allowed; see [U] 11.1.10 Prefix commands.
Weights are allowed if command allows them; see [U] 11.1.6 weight.
All postestimation commands behave as they would after command without the stepwise prefix; see the postestimation manual entry for command.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

\section*{Menu}

Statistics \(>\) Other \(>\) Stepwise estimation

\section*{Description}
stepwise performs stepwise estimation. Typing
. stepwise, \(\operatorname{pr}(\#):\) command
performs backward-selection estimation for command. The stepwise selection method is determined by the following option combinations:
\begin{tabular}{ll} 
options & Description \\
\hline pr(\#) & backward selection \\
pr(\#) hierarchical & backward hierarchical selection \\
pr(\#) pe(\#) & backward stepwise \\
pe(\#) & forward selection \\
pe(\#) hierarchical & forward hierarchical selection \\
pr(\#) pe(\#) forward & forward stepwise \\
\hline
\end{tabular}
command defines the estimation command to be executed. The following Stata commands are supported by stepwise:
\begin{tabular}{lll} 
clogit & nbreg & regress \\
cloglog & ologit & scobit \\
glm & oprobit & stcox \\
intreg & poisson & stcrreg \\
logistic & probit & streg \\
logit & qreg & tobit
\end{tabular}
stepwise expects command to have the following form:
command_name [depvar] term [term...] [if] [in] [weight] [, command_options]
where term is either varname or (varlist) (a varlist in parentheses indicates that this group of variables is to be included or excluded together). depvar is not present when command_name is stcox, stcrreg, or streg; otherwise, depvar is assumed to be present. For intreg, depvar is actually two dependent variable names (depvar \({ }_{1}\) and depvar \({ }_{2}\) ).
sw is a synonym for stepwise.

\section*{Options}

Model
\(\operatorname{pr}(\#)\) specifies the significance level for removal from the model; terms with \(p \geq \operatorname{pr}()\) are eligible for removal.
\(\mathrm{pe}(\#)\) specifies the significance level for addition to the model; terms with \(p<\mathrm{pe}\) () are eligible for addition.

Model 2
forward specifies the forward-stepwise method and may be specified only when both pr() and pe() are also specified. Specifying both pr() and pe() without forward results in backward-stepwise selection. Specifying only \(\operatorname{pr}()\) results in backward selection, and specifying only pe() results in forward selection.
hierarchical specifies hierarchical selection.
lockterm1 specifies that the first term be included in the model and not be subjected to the selection criteria.
lr specifies that the test of term significance be the likelihood-ratio test. The default is the less computationally expensive Wald test; that is, the test is based on the estimated variance-covariance matrix of the estimators.

\footnotetext{
Reporting
display_options: cformat (\%fmt), pformat (\%fmt), sformat (\%fmt), and nolstretch; see [R] estimation options.
}

\section*{Remarks and examples}

Remarks are presented under the following headings:

\author{
Introduction \\ Search logic for a step \\ Full search logic \\ Examples \\ Estimation sample considerations \\ Messages \\ Programming for stepwise
}

\section*{Introduction}

Typing
```

. stepwise, pr(.10): regress y1 x1 x2 d1 d2 d3 x4 x5

```
performs a backward-selection search for the regression model \(y 1\) on \(x 1, x 2, d 1, d 2, d 3, x 4\), and \(x 5\). In this search, each explanatory variable is said to be a term. Typing
```

. stepwise, pr(.10): regress y1 x1 x2 (d1 d2 d3) (x4 x5)

```
performs a similar backward-selection search, but the variables d 1 , d 2 , and d 3 are treated as one term, as are x 4 and x 5 . That is, \(\mathrm{d} 1, \mathrm{~d} 2\), and d 3 may or may not appear in the final model, but they appear or do not appear together.

\section*{> Example 1}

Using the automobile dataset, we fit a backward-selection model of mpg:
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multicolumn{6}{|l|}{\begin{tabular}{l}
. use http://www.stata-press.com/data/r13/auto \\
. generate weight2 = weight*weight
\end{tabular}} \\
\hline \multicolumn{6}{|l|}{begin with full model} \\
\hline \multicolumn{6}{|l|}{\(\mathrm{p}=0.7116>=0.2000\) removing headroom} \\
\hline \multicolumn{6}{|l|}{\(p=0.6138>=0.2000\) removing displacement} \\
\hline \multicolumn{6}{|l|}{\(\mathrm{p}=0.3278>=0.2000\) removing price} \\
\hline Source & SS & df & MS & Number of obs & 74 \\
\hline & & & & F ( 5, 68) & 33.39 \\
\hline Model & 1736.31455 & 5 & 347.262911 & Prob > F & 0.0000 \\
\hline Residual & 707.144906 & 68 & 10.3991898 & R-squared & 0.7106 \\
\hline & & & & Adj R-squared & 0.6893 \\
\hline Total & 2443.45946 & 73 & 33.4720474 & Root MSE & 3.2248 \\
\hline
\end{tabular}
\begin{tabular}{r|rcccrr}
\hline mpg & Coef. & Std. Err. & \multicolumn{1}{c}{t} & \(\mathrm{P}>|\mathrm{t}|\) & [95\% Conf. Interval] \\
\hline weight & -.0158002 & .0039169 & -4.03 & 0.000 & -.0236162 & -.0079842 \\
weight2 & \(1.77 \mathrm{e}-06\) & \(6.20 \mathrm{e}-07\) & 2.86 & 0.006 & \(5.37 \mathrm{e}-07\) & \(3.01 \mathrm{e}-06\) \\
foreign & -3.615107 & 1.260844 & -2.87 & 0.006 & -6.131082 & -1.099131 \\
gear_ratio & 2.011674 & 1.468831 & 1.37 & 0.175 & -.9193321 & 4.94268 \\
turn & -.3087038 & .1763099 & -1.75 & 0.084 & -.6605248 & .0431172 \\
_cons & 59.02133 & 9.3903 & 6.29 & 0.000 & 40.28327 & 77.75938 \\
\hline
\end{tabular}

This estimation treated each variable as its own term and thus considered each one separately. The engine displacement and gear ratio should really be considered together:
```

. stepwise, pr(.2): regress mpg weight weight2 (displ gear) turn headroom
> foreign price
p = 0.7116 >= 0.2000 removing headroom
p = 0.3944 >= 0.2000 removing displacement gear_ratio
p = 0.2798 >= 0.2000 removing price

```
\begin{tabular}{|c|c|c|c|c|c|}
\hline Source & SS & df & MS & Number of obs & 74 \\
\hline & & & & F ( 4, 69) & 40.76 \\
\hline Model & 1716.80842 & 4 & 429.202105 & Prob > F & 0.0000 \\
\hline Residual & 726.651041 & 69 & 10.5311745 & R -squared & 0.7026 \\
\hline & & & & Adj R-squared & 0.6854 \\
\hline Total & 2443.45946 & 73 & 33.4720474 & Root MSE & 3.2452 \\
\hline
\end{tabular}
\begin{tabular}{r|rcccrr}
\hline mpg & Coef. & Std. Err. & t & \(\mathrm{P}>|\mathrm{t}|\) & [95\% Conf. Interval] \\
\hline weight & -.0160341 & .0039379 & -4.07 & 0.000 & -.0238901 & -.0081782 \\
weight2 & \(1.70 \mathrm{e}-06\) & \(6.21 \mathrm{e}-07\) & 2.73 & 0.008 & \(4.58 \mathrm{e}-07\) & \(2.94 \mathrm{e}-06\) \\
foreign & -2.758668 & 1.101772 & -2.50 & 0.015 & -4.956643 & -.5606925 \\
turn & -.2862724 & .176658 & -1.62 & 0.110 & -.6386955 & .0661508 \\
_cons & 65.39216 & 8.208778 & 7.97 & 0.000 & 49.0161 & 81.76823 \\
\hline
\end{tabular}

\section*{Search logic for a step}

Before discussing the complete search logic, consider the logic for a step-the first step-in detail. The other steps follow the same logic. If you type
```

. stepwise, pr(.20): regress y1 x1 x2 (d1 d2 d3) (x4 x5)

```
the logic is
1. Fit the model y on x 1 x 2 d 1 d 2 d 3 x 4 x 5 .
2. Consider dropping \(\times 1\).
3. Consider dropping x 2 .
4. Consider dropping d1 d2 d3.
5. Consider dropping \(\mathrm{x} 4 \times 5\).
6. Find the term above that is least significant. If its significance level is \(\geq 0.20\), remove that term.

If you type
```

. stepwise, pr(.20) hierarchical: regress y1 x1 x2 (d1 d2 d3) (x4 x5)

```
the logic would be different because the hierarchical option states that the terms are ordered. The initial logic would become
1. Fit the model y on x 1 x 2 d 1 d 2 d 3 x 4 x 5 .
2. Consider dropping \(x 4 \times 5\)-the last term.
3. If the significance of this last term is \(\geq 0.20\), remove the term.

The process would then stop or continue. It would stop if x 4 x 5 were not dropped, and otherwise, stepwise would continue to consider the significance of the next-to-last term, d1 d2 d3.

Specifying pe() rather than pr() switches to forward estimation. If you type
```

. stepwise, pe(.20): regress y1 x1 x2 (d1 d2 d3) (x4 x5)

```
stepwise performs forward-selection search. The logic for the first step is
1. Fit a model of \(y\) on nothing (meaning a constant).
2. Consider adding \(\times 1\).
3. Consider adding x2.
4. Consider adding d1 d2 d3.
5. Consider adding \(\mathrm{x} 4 \times 5\).
6. Find the term above that is most significant. If its significance level is \(<0.20\), add that term.

As with backward estimation, if you specify hierarchical,
```

stepwise, pe(.20) hierarchical: regress y1 x1 x2 (d1 d2 d3) (x4 x5)

```
the search for the most significant term is restricted to the next term:
1. Fit a model of y on nothing (meaning a constant).
2. Consider adding x 1 -the first term.
3. If the significance is \(<0.20\), add the term.

If x 1 were added, stepwise would next consider x 2 ; otherwise, the search process would stop.
stepwise can also use a stepwise selection logic that alternates between adding and removing terms. The full logic for all the possibilities is given below.

\section*{Full search logic}
\begin{tabular}{|c|c|}
\hline Option & Logic \\
\hline \begin{tabular}{l}
\[
\mathrm{pr}()
\] \\
(backward selection)
\end{tabular} & \begin{tabular}{l}
Fit the full model on all explanatory variables. \\
While the least-significant term is "insignificant", remove it and reestimate.
\end{tabular} \\
\hline pr() hierarchical (backward hierarchical selection) & Fit full model on all explanatory variables. While the last term is "insignificant", remove it and reestimate. \\
\hline \begin{tabular}{l}
pr() pe() \\
(backward stepwise)
\end{tabular} & \begin{tabular}{l}
Fit full model on all explanatory variables. \\
If the least-significant term is "insignificant", remove it and reestimate; otherwise, stop. \\
Do that again: if the least-significant term is "insignificant", remove it and reestimate; otherwise, stop. \\
Repeatedly, \\
if the most-significant excluded term is "significant", add it and reestimate; \\
if the least-significant included term is "insignificant", remove it and reestimate; \\
until neither is possible.
\end{tabular} \\
\hline \begin{tabular}{l}
pe() \\
(forward selection)
\end{tabular} & \begin{tabular}{l}
Fit "empty" model. \\
While the most-significant excluded term is "significant", add it and reestimate.
\end{tabular} \\
\hline pe() hierarchical (forward hierarchical selection) & \begin{tabular}{l}
Fit "empty" model. \\
While the next term is "significant", add it and reestimate.
\end{tabular} \\
\hline pr() pe() forward (forward stepwise) & \begin{tabular}{l}
Fit "empty" model. \\
If the most-significant excluded term is "significant", add it and reestimate; otherwise, stop. \\
Do that again: if the most-significant excluded term is "significant", add it and reestimate; otherwise, stop. Repeatedly, \\
if the least-significant included term is "insignificant", remove it and reestimate; \\
if the most-significant excluded term is "significant", add it and reestimate; \\
until neither is possible.
\end{tabular} \\
\hline
\end{tabular}

\section*{Examples}

The following two statements are equivalent; both include solely single-variable terms:
. stepwise, \(\operatorname{pr}(.2)\) : regress price mpg weight displ
. stepwise, \(\operatorname{pr}(.2)\) : regress price (mpg) (weight) (displ)
The following two statements are equivalent; the last term in each is \(r 1, \ldots, r 4\) :
```

. stepwise, pr(.2) hierarchical: regress price mpg weight displ (r1-r4)
. stepwise, pr(.2) hierarchical: regress price (mpg) (weight) (displ) (r1-r4)

```

To group variables weight and displ into one term, type
```

. stepwise, pr(.2) hierarchical: regress price mpg (weight displ) (r1-r4)

```
stepwise can be used with commands other than regress; for instance,
. stepwise, \(\operatorname{pr}(.2)\) : logit outcome (sex weight) treated1 treated2
. stepwise, \(\operatorname{pr}(.2)\) : logistic outcome (sex weight) treated1 treated2
Either statement would fit the same model because logistic and logit both perform logistic regression; they differ only in how they report results; see [R] logit and [R] logistic.

We use the lockterm1 option to force the first term to be included in the model. To keep treated1 and treated2 in the model no matter what, we type
```

. stepwise, pr(.2) lockterm1: logistic outcome (treated1 treated2) ...

```

After stepwise estimation, we can type stepwise without arguments to redisplay results,
. stepwise
(output from logistic appears)
or type the underlying estimation command:
. logistic
(output from logistic appears )
At estimation time, we can specify options unique to the command being stepped:
```

. stepwise, pr(.2): logit outcome (sex weight) treated1 treated2, or

```
or is logit's option to report odds ratios rather than coefficients; see \([\mathrm{R}] \operatorname{logit}\).

\section*{Estimation sample considerations}

Whether you use backward or forward estimation, stepwise forms an estimation sample by taking observations with nonmissing values of all the variables specified (except for depvar \(r_{1}\) and depvar \({ }_{2}\) for intreg). The estimation sample is held constant throughout the stepping. Thus if you type
```

. stepwise, pr(.2) hierarchical: regress amount sk edul sval

```
and variable sval is missing in half the data, that half of the data will not be used in the reported model, even if sval is not included in the final model.

The function e (sample) identifies the sample that was used. e(sample) contains 1 for observations used and 0 otherwise. For instance, if you type
and the final model is outcome on \(\mathrm{x} 1, \mathrm{x} 5\), x 6 , and x 7 , you could re-create the final regression by typing
```

logistic outcome x1 x5 x6 x7 if e(sample)

```

You could obtain summary statistics within the estimation sample of the independent variables by typing
```

. summarize x1 x5 x6 x7 if e(sample)

```

If you fit another model, e(sample) will automatically be redefined. Typing
```

. stepwise, lock pr(.2): logistic outcome (x1 x2) (x3 x4) (x5 x6 x7)

```
would automatically drop e(sample) and re-create it.

\section*{Messages}
note: \(\qquad\) dropped because of collinearity
Each term is checked for collinearity, and variables within the term are dropped if collinearity is found. For instance, say that you type
```

stepwise, pr(.2): regress y x1 x2 (r1-r4) (x3 x4)

```
and assume that variables \(r 1\) through \(r 4\) are mutually exclusive and exhaustive dummy variables - perhaps \(r 1, \ldots, r 4\) indicate in which of four regions the subject resides. One of the \(r 1\), \(\ldots, r 4\) variables will be automatically dropped to identify the model.

This message should cause you no concern.

\section*{Error message: between-term collinearity, variable}

After removing any within-term collinearity, if stepwise still finds collinearity between terms, it refuses to continue. For instance, assume that you type
```

. stepwise, pr(.2): regress y1 x1 x2 (d1-d8) (r1-r4)

```

Assume that \(\mathrm{r} 1, \ldots, \mathrm{r} 4\) identify in which of four regions the subject resides, and that \(\mathrm{d} 1, \ldots, \mathrm{~d} 8\) identify the same sort of information, but more finely. r1, say, amounts to d1 and d2; r2 to d3, d4, and \(d 5\); r3 to d 6 and d 7 ; and r 4 to d 8 . You can estimate the \(\mathrm{d} *\) variables or the \(r *\) variables, but not both.

It is your responsibility to specify noncollinear terms.
note: \(\qquad\) dropped because of estimability
note: obs. dropped because of estimability

You probably received this message in fitting a logistic or probit model. Regardless of estimation strategy, stepwise checks that the full model can be fit. The indicated variable had a 0 or infinite standard error.

For logistic, logit, and probit, this message is typically caused by one-way causation. Assume that you type
and assume that variable d 1 is an indicator (dummy) variable. Further assume that whenever \(\mathrm{d} 1=1\), outcome \(=1\) in the data. Then the coefficient on d 1 is infinite. One (conservative) solution to this problem is to drop the d 1 variable and the \(\mathrm{d} 1==1\) observations. The underlying estimation commands probit, logit, and logistic report the details of the difficulty and solution; stepwise simply accumulates such problems and reports the above summary messages. Thus if you see this message, you could type
```

. logistic outcome x1 x2 x3 d1

```
to see the details. Although you should think carefully about such situations, Stata's solution of dropping the offending variables and observations is, in general, appropriate.

\section*{Programming for stepwise}
stepwise requires that command_name follow standard Stata syntax and allow the if qualifier; see [U] 11 Language syntax. Furthermore, command_name must have sw or swml as a program property; see \([\mathrm{P}]\) program properties. If command_name has swml as a property, command_name must store the log-likelihood value in \(e(l l)\) and model degrees of freedom in \(e\left(d f \_m\right)\).

\section*{Stored results}
stepwise stores whatever is stored by the underlying estimation command.
Also, stepwise stores stepwise in e(stepwise).

\section*{Methods and formulas}

Some statisticians do not recommend stepwise procedures; see Sribney (1998) for a summary.

\section*{References}

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\section*{Also see}
\([\mathrm{R}]\) nestreg - Nested model statistics

\section*{Title}
stored results - Stored results

Syntax Description Option Remarks and examples References Also see

\section*{Syntax}

List results from general commands, stored in \(r()\)
```

return list [, all]

```

List results from estimation commands, stored in e()
```

ereturn list [, all]

```

List results from parsing commands, stored in \(s()\)
```

sreturn list

```

\section*{Description}

Results of calculations are stored by many Stata commands so that they can be easily accessed and substituted into later commands. return list lists results stored in \(r()\). ereturn list lists results stored in e(). sreturn list lists results stored in s() .

This entry discusses using stored results. Programmers wishing to store results should see \([\mathrm{P}]\) return and \([\mathrm{P}]\) ereturn.

\section*{Option}
all is for use with return list and ereturn list. all specifies that hidden and historical stored results be listed along with the usual stored results. This option is seldom used. See Using hidden and historical stored results and Programming hidden and historical stored results under Remarks and examples of \([\mathrm{P}]\) return for more information. These sections are written in terms of return list, but everything said there applies equally to ereturn list.
all is not allowed with sreturn list because \(s()\) does not allow hidden or historical results.

\section*{Remarks and examples}

Stata commands are classified as being
\[
\begin{array}{ll}
\mathrm{r} \text {-class } & \text { general commands that store results in } r() \\
\text { e-class } & \text { estimation commands that store results in } e() \\
\text { s-class } & \text { parsing commands that store results in } s() \\
\mathrm{n} \text {-class } & \text { commands that do not store in } r(), \mathrm{e}(), \text { or } s()
\end{array}
\]

There is also a c-class, c() , containing the values of system parameters and settings, along with certain constants, such as the value of pi; see [ P\(]\) creturn. A program, however, cannot be c-class.

You can look at the Stored results section of the manual entry of a command to determine whether it is \(\mathrm{r}-\), e-, s -, or n -class, but it is easy enough to guess.

Commands producing statistical results are either r-class or e-class. They are e-class if they present estimation results and r-class otherwise. s-class is a class used by programmers and is primarily used in subprograms performing parsing. n-class commands explicitly state where the result is to go. For instance, generate and replace are n-class because their syntax is generate varname \(=\ldots\) and replace varname =....

After executing a command, you can type return list, ereturn list, or sreturn list to see what has been stored.
\(>\) Example 1
```

. use http://www.stata-press.com/data/r13/auto4
(1978 Automobile Data)
. describe
Contains data from http://www.stata-press.com/data/r13/auto4.dta
obs: 74 1978 Automobile Data
vars: 6 6 Apr 2013 00:20
size: 2,072

```
\begin{tabular}{lllll}
\hline variable name & \begin{tabular}{c} 
storage \\
type
\end{tabular} & \begin{tabular}{l} 
display \\
format
\end{tabular} & \begin{tabular}{l} 
value \\
label
\end{tabular} & variable label \\
\hline price & int & \(\% 8.0 \mathrm{gc}\) & & Price \\
weight & int & \(\% 8.0 \mathrm{gc}\) & & Weight (lbs.) \\
mpg & int & \(\% 8.0 \mathrm{~g}\) & & Mileage (mpg) \\
make & str18 & \(\%-18 \mathrm{~s}\) & & Make and Model \\
length & int & \(\% 8.0 \mathrm{~g}\) & & Length (in.) \\
rep78 & int & \(\% 8.0 \mathrm{~g}\) & & Repair Record 1978 \\
\hline
\end{tabular}

Sorted by:
. return list
scalars:
\[
\begin{aligned}
r(\text { changed }) & =0 \\
r(\text { width }) & =28 \\
r(k) & =6 \\
r(N) & =74
\end{aligned}
\]

To view all stored results, including those that are historical or hidden, specify the all option.
```

. return list, all
scalars:
r(changed) = 0
r(width) = 28
r(k) = 6
r(N) = 74

```

Historical; used before Stata 12, may exist only under version control
scalars:
\[
\begin{aligned}
r(w i d t h m a x) & =1048576 \\
r\left(k \_\max \right) & =2048 \\
r\left(N \_\max \right) & =2147483646
\end{aligned}
\]
\(r(w i d t h m a x), r\left(k \_m a x\right)\), and \(r\left(N \_m a x\right)\) are historical stored results. They are no longer relevant because Stata dynamically adjusts memory beginning with Stata 12.

\section*{- Technical note}

In the above example, we stated that \(r\) (widthmax) and \(r\left(N \_m a x\right)\) are no longer relevant. In fact, they are not useful. Stata no longer has a fixed memory size, so the methods used to calculate \(r\) (widthmax) and \(r\) ( \(N \_\)max) are no longer appropriate.
> Example 2
You can use stored results in expressions.
\begin{tabular}{r|rrrrr}
\begin{tabular}{c} 
summarize mpg \\
Variable
\end{tabular} & Obs & Mean & Std. Dev. & Min & Max \\
\hline mpg & 74 & 21.2973 & 5.785503 & 12 & 41
\end{tabular}
- return list
scalars:
\[
\begin{aligned}
r(N) & =74 \\
r(\text { sum_w }) & =74 \\
r(\text { mean }) & =21.2972972972973 \\
r(\text { Var }) & =33.47204738985561 \\
r(\text { sd }) & =5.785503209735141 \\
r(\min ) & =12 \\
r(\max ) & =41 \\
r(\text { sum }) & =1576
\end{aligned}
\]
. generate double mpgstd \(=(m p g-r(\) mean \()) / r(s d)\)
. summarize mpgstd
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline mpgstd & 74 & \(-1.64 \mathrm{e}-16\) & 1 & -1.606999 & 3.40553
\end{tabular}

Be careful to use results stored in \(r()\) soon because they will be replaced the next time you execute another r-class command. For instance, although r(mean) was 21.3 (approximately) after summarize mpg , it is \(-1.64 \mathrm{e}-16\) now because you just ran summarize with mpgstd.

\section*{> Example 3}
e-class is really no different from r-class, except for where results are stored and that, when an estimation command stores results, it tends to store a lot of them:
```

. regress mpg weight length
(output omitted)
. ereturn list
scalars:

```
```

            e(N) = 74
    ```
            e(N) = 74
                e(df_m) = 2
                e(df_m) = 2
                e(df_r) = 71
                e(df_r) = 71
            e(F) = 69.34050004300228
            e(F) = 69.34050004300228
                            e(r2) = .6613903979336324
                            e(r2) = .6613903979336324
                                    e(rmse) = 3.413681741382589
                                    e(rmse) = 3.413681741382589
                                    e(mss) = 1616.08062422659
                                    e(mss) = 1616.08062422659
                                    e(rss) = 827.3788352328694
                                    e(rss) = 827.3788352328694
                                    e(r2_a) = .6518520992838756
                                    e(r2_a) = .6518520992838756
                                    e(ll) = -194.3267619410807
                                    e(ll) = -194.3267619410807
                                    e(ll_0) = -234.3943376482347
                                    e(ll_0) = -234.3943376482347
                                    e(rank) = 3
```

                                    e(rank) = 3
    ```
```

macros:
e(cmdline) : "regress mpg weight length"
e(title) : "Linear regression"
e(marginsok) : "XB default"
e(vce) : "ols"
e(depvar) : "mpg"
e(cmd) : "regress"
e(properties) : "b V"
e(predict) : "regres_p"
e(model) : "ols"
e(estat_cmd) : "regress_estat"
matrices:
e(b) : 1 x 3
functions:
e(sample)

```

These e-class results will stick around until you run another estimation command. Typing return list and ereturn list is the easy way to find out what a command stores.

Both r- and e-class results come in four flavors: scalars, macros, matrices, and functions. (s-class results come in only one flavor-macros-and as earlier noted, s-class is used solely by programmers, so ignore it.)

Scalars are just that—numbers by any other name. You can subsequently refer to \(r\) (mean) or \(e\) (rmse) in numeric expressions and obtain the result to full precision.

Macros are strings. For instance, e (depvar) contains "mpg". You can refer to it, too, in subsequent expressions, but really that would be of most use to programmers, who will refer to it using constructs like "'e(depvar)'". In any case, macros are macros, and you obtain their contents just as you would a local macro, by enclosing their name in single quotes. The name here is the full name, so 'e(depvar)' is mpg.

Matrices are matrices, and all estimation commands store e(b) and e(V) containing the coefficient vector and variance-covariance matrix of the estimates (VCE).

Functions are stored by e-class commands only, and the only function existing is e(sample). \(e\) (sample) evaluates to 1 (meaning true) if the observation was used in the previous estimation and to 0 (meaning false) otherwise.

\section*{- Technical note}

Say that some command set \(r\) (scalar) and \(r\) (macro), the first being stored as a scalar and the second as a macro. In theory, in subsequent use you are supposed to refer to \(r\) (scalar) and ' \(r\) (macro)'. In fact, however, you can refer to either one with or without quotes, so you could refer to 'r(scalar)' and \(r\) (macro). Programmers sometimes do this.

When you refer to \(r\) (scalar), you are referring to the full double-precision stored result. Think of \(r\) (scalar) without quotes as a function returning the value of the stored result scalar. When you refer to \(r\) (scalar) in quotes, Stata understands ' \(r\) (scalar)' to mean "substitute the printed result of evaluating r(scalar)". Pretend that r(scalar) equals the number 23. Then 'r(scalar)' is 23, the character 2 followed by 3.

Referring to \(r\) (scalar) in quotes is sometimes useful. Say that you want to use the immediate command ci with r(scalar). The immediate command ci requires its arguments to be numbersnumeric literals in programmer's jargon-and it will not take an expression. Thus you could not type
'cir(scalar) ...'. You could, however, type 'ci 'r(scalar)' ...' because 'r(scalar)' is just a numeric literal.

For \(r\) (macro), you are supposed to refer to it in quotes: ' \(r\) (macro)'. If, however, you omit the quotes in an expression context, Stata evaluates the macro and then pretends that it is the result of function-returning-string. There are side effects of this, the most important being that the result is trimmed to 80 characters.

Referring to \(r\) (macro) without quotes is never a good idea; the feature was included merely for completeness.

You can even refer to \(r\) (matrix) in quotes (assume that \(r\) (matrix) is a matrix). ' \(r\) (matrix) ' does not result in the matrix being substituted; it returns the word matrix. Programmers sometimes find that useful.

\section*{References}

Jann, B. 2005. Making regression tables from stored estimates. Stata Journal 5: 288-308. ——. 2007. Making regression tables simplified. Stata Journal 7: 227-244.

\section*{Also see}
[P] ereturn - Post the estimation results
[P] return - Return stored results
[U] 18.8 Accessing results calculated by other programs
[U] 18.9 Accessing results calculated by estimation commands

\section*{Title}
suest - Seemingly unrelated estimation
\begin{tabular}{llll} 
Syntax & Menu & Description & Options \\
Remarks and examples & Stored results & Methods and formulas & Acknowledgment \\
References & Also see & &
\end{tabular}

\section*{Syntax}
suest namelist [, options]
where namelist is a list of one or more names under which estimation results were stored via estimates store; see [R] estimates store. Wildcards may be used. * and _all refer to all stored results. A period (.) may be used to refer to the last estimation results, even if they have not (yet) been stored.
\begin{tabular}{|c|c|}
\hline options & Description \\
\hline \multicolumn{2}{|l|}{SE/Robust} \\
\hline svy & survey data estimation \\
\hline vce(vcetype) & vcetype may be robust or cluster clustvar \\
\hline \multicolumn{2}{|l|}{Reporting} \\
\hline level(\#) & set confidence level; default is level (95) \\
\hline dir & display a table describing the models \\
\hline eform(string) & report exponentiated coefficients and label as string \\
\hline display_options & control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling \\
\hline coeflegend & display legend instead of statistics \\
\hline
\end{tabular}
coeflegend does not appear in the dialog box.

\section*{Menu}

Statistics \(>\) Postestimation \(>\) Tests \(>\) Seemingly unrelated estimation

\section*{Description}
suest is a postestimation command; see [U] 20 Estimation and postestimation commands.
suest combines the estimation results-parameter estimates and associated (co)variance matricesstored under namelist into one parameter vector and simultaneous (co)variance matrix of the sandwich/robust type. This (co)variance matrix is appropriate even if the estimates were obtained on the same or on overlapping data.

Typical applications of suest are tests for intramodel and cross-model hypotheses using test or testnl, for example, a generalized Hausman specification test. lincom and nlcom may be used after suest to estimate linear combinations and nonlinear functions of coefficients. suest may also be used to adjust a standard VCE for clustering or survey design effects.

Different estimators are allowed, for example, a regress model and a probit model; the only requirement is that predict produce equation-level scores with the score option after an estimation command. The models may be estimated on different samples, due either to explicit if or in selection or to missing values. If weights are applied, the same weights (type and values) should be applied to all models in namelist. The estimators should be estimated without vce (robust) or vce (cluster clustvar) options. suest returns the robust VCE, allows the vce(cluster clustvar) option, and automatically works with results from the svy prefix command (only for vce(linearized)). See example 7 in [SVY] svy postestimation for an example using suest with svy: ologit.

Because suest posts its results like a proper estimation command, its results can be stored via estimates store. Moreover, like other estimation commands, suest typed without arguments replays the results.

\section*{Options}
\(\qquad\) SE/Robust
svy specifies that estimation results should be modified to reflect the survey design effects according to the svyset specifications, see [SVY] svyset.
The svy option is implied when suest encounters survey estimation results from the svy prefix; see [SVY] svy. Poststratification is allowed only with survey estimation results from the svy prefix.
vce(vcetype) specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (robust) and that allow for intragroup correlation (cluster clustvar; see [R] vce_option.
The vce() option may not be combined with the svy option or estimation results from the svy prefix.

\section*{Reporting}
level(\#) specifies the confidence level, as a percentage, for confidence intervals of the coefficients; see [ R\(]\) level.
dir displays a table describing the models in namelist just like estimates dir namelist.
eform (string) displays the coefficient table in exponentiated form: for each coefficient, \(\exp (b)\) rather than \(b\) is displayed, and standard errors and confidence intervals are transformed. string is the table header that will be displayed above the transformed coefficients and must be 11 characters or fewer, for example, eform("Odds ratio").
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\%fmt), pformat(\%fmt), sformat (\%fmt), and nolstretch; see [R] estimation options.

The following option is available with suest but is not shown in the dialog box:
coeflegend; see [R] estimation options.

\section*{Remarks and examples}

Remarks are presented under the following headings:

\section*{Using suest}

Remarks on regress
Testing the assumption of the independence of irrelevant alternatives
Testing proportionality
Testing cross-model hypotheses

\section*{Using suest}

If you plan to use suest, you must take precautions when fitting the original models. These restrictions are relaxed when using svy commands; see [SVY] svy postestimation.
1. suest works with estimation commands that allow predict to generate equation-level score variables when supplied with the score (or scores) option. For example, equation-level score variables are generated after running mlogit by typing
```

. predict sc*, scores

```
2. Estimation should take place without the vce (robust) or vce (cluster clustvar) option. suest always computes the robust estimator of the (co)variance, and suest has a vce(cluster clustvar) option.

The within-model covariance matrices computed by suest are identical to those obtained by specifying a vce(robust) or vce(cluster clustvar) option during estimation. suest, however, also estimates the between-model covariances of parameter estimates.
3. Finally, the estimation results to be combined should be stored by estimates store; see \([R]\) estimates store.

After estimating and storing a series of estimation results, you are ready to combine the estimation results with suest,
```

. suest namel [name2 ...] [, vce(cluster clustvar)]

```
and you can subsequently use postestimation commands, such as test, to test hypotheses. Here an important issue is how suest assigns names to the equations. If you specify one model name, the original equation names are left unchanged; otherwise, suest constructs new equation names. The coefficients of a single-equation model (such as logit and poisson) that was estimate stored under name \(X\) are collected under equation \(X\). With a multiequation model stored under name \(X\), suest prefixes \(X_{-}\)to an original equation name eq, forming equation name, \(X_{-}\)eq.

\section*{- Technical note}

Earlier we said that standard errors from suest are identical to those obtained by specifying the vce (robust) option with each command individually. Thus if you fit a logistic model using logit with the vce(robust) option, you will get the same standard errors when you type
```

. suest .

```
directly after logit using the same data without the vce (robust) option.
This is not true for multiple estimation results when the estimation samples are not all the same. The standard errors from suest will be slightly smaller than those from individual model fits using the vce (robust) option because suest uses a larger number of observations to estimate the simultaneous (co)variance matrix.

\section*{Technical note}

In rare circumstances, suest may have to truncate equation names to 32 characters. When equation names are not unique because of truncation, suest numbers the equations within models, using equations named \(X \_\#\).

\section*{Remarks on regress}
regress (see [R] regress) does not include its ancillary parameter, the residual variance, in its coefficient vector and (co)variance matrix. Moreover, while the score option is allowed with predict after regress, a score variable is generated for the mean but not for the variance parameter. suest contains special code that assigns the equation name mean to the coefficients for the mean, adds the equation Invar for the log variance, and computes the appropriate two score variables itself.

\section*{Testing the assumption of the independence of irrelevant alternatives}

The multinomial logit model and the closely related conditional logit model satisfy a probabilistic version of the assumption of the independence of irrelevant alternatives (IIA), implying that the ratio of the probabilities for two alternatives does not depend on what other alternatives are available. Hausman and McFadden (1984) proposed a test for this assumption that is implemented in the hausman command. The standard Hausman test has several limitations. First, the test statistic may be undefined because the estimated VCE does not satisfy the required asymptotic properties of the test. Second, the classic Hausman test applies only to the test of the equality of two estimators. Third, the test requires access to a fully efficient estimator; such an estimator may not be available, for example, if you are analyzing complex survey data. Using suest can overcome these three limitations.
\(>\) Example 1
In our first example, we follow the analysis of the type of health insurance reported in [R] mlogit and demonstrate the hausman command with the suest/test combination. We fit the full multinomial logit model for all three alternatives and two restricted multinomial models in which one alternative is excluded. After fitting each of these models, we store the results by using the store subcommand of estimates. title() simply documents the models.
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \multicolumn{7}{|l|}{\begin{tabular}{l}
(Health insurance data) \\
. mlogit insure age male
\end{tabular}} \\
\hline \multicolumn{7}{|l|}{Iteration 0: log likelihood \(=-555.85446\)} \\
\hline \multicolumn{7}{|l|}{Iteration 1: log likelihood \(=-551.32973\)} \\
\hline \multicolumn{7}{|l|}{Iteration 2: log likelihood \(=-551.32802\)} \\
\hline \multicolumn{7}{|l|}{Iteration 3: log likelihood = -551.32802} \\
\hline \multicolumn{4}{|l|}{\multirow[t]{3}{*}{Multinomial logistic regression}} & Numb & of obs & 615 \\
\hline & & & & & 2(4) & 9.05 \\
\hline & & & & Prob & chi2 & 0.0598 \\
\hline \multicolumn{4}{|l|}{Log likelihood \(=-551.32802\)} & Pseu & & 0.0081 \\
\hline insure & Coef. & Std. Err. & z & \(P>|z|\) & \multicolumn{2}{|l|}{[95\% Conf. Interval]} \\
\hline Indemnity & \multicolumn{6}{|l|}{(base outcome)} \\
\hline \multicolumn{7}{|l|}{Prepaid} \\
\hline age & -. 0100251 & . 0060181 & -1.67 & 0.096 & -. 0218 & . 0017702 \\
\hline male & . 5095747 & . 1977893 & 2.58 & 0.010 & . 1219 & . 8972346 \\
\hline _cons & . 2633838 & . 2787575 & 0.94 & 0.345 & -. 2829 & . 8097383 \\
\hline \multicolumn{7}{|l|}{Uninsure} \\
\hline age & -. 0051925 & . 0113821 & -0.46 & 0.648 & -. 0275 & . 0171161 \\
\hline male & . 4748547 & . 3618462 & 1.31 & 0.189 & -. 2343 & 1.18406 \\
\hline _cons & -1.756843 & . 5309602 & -3.31 & 0.001 & -2.797 & -. 7161803 \\
\hline
\end{tabular}

\footnotetext{
. estimates store m 1 , title(all three insurance forms)
}
. quietly mlogit insure age male if insure != "Uninsure":insure
. estimates store m2, title(insure != "Uninsure":insure)
. quietly mlogit insure age male if insure != "Prepaid":insure
. estimates store m3, title(insure != "Prepaid":insure)
Having performed the three estimations, we inspect the results. estimates dir provides short descriptions of the models that were stored using estimates store. Typing estimates table lists the coefficients, displaying blanks for a coefficient not contained in a model.
. estimates dir
\begin{tabular}{r|llrl}
\hline name & command & depvar & npar & title \\
\hline m1 & mlogit & insure & 9 & all three insurance forms \\
m2 & mlogit & insure & 6 & insure \(!=\) Uninsure :insure \\
m3 & mlogit & insure & 6 & insure ! \(=\) Prepaid :insure \\
\hline
\end{tabular}
. estimates table m1 m2 m3, star stats(N ll) keep(Prepaid: Uninsure:)
\begin{tabular}{|c|c|c|c|}
\hline Variable & m1 & m2 & m3 \\
\hline \multicolumn{4}{|l|}{Prepaid} \\
\hline age & -. 01002511 & -. 01015205 & \\
\hline male & .50957468** & .51440033** & \\
\hline _cons & . 26338378 & . 26780432 & \\
\hline \multicolumn{4}{|l|}{Uninsure} \\
\hline age & -. 00519249 & & -. 00410547 \\
\hline male & . 47485472 & & . 45910738 \\
\hline _cons & -1.7568431*** & & -1.8017743*** \\
\hline \multicolumn{4}{|l|}{Statistics} \\
\hline N & 615 & 570 & 338 \\
\hline 11 & -551.32802 & -390.48643 & -131.76807 \\
\hline
\end{tabular}

Comparing the coefficients between models does not suggest substantial differences. We can formally test that coefficients are the same for the full model m 1 and the restricted models m 2 and m 3 by using the hausman command. hausman expects the models to be specified in the order "always consistent" first and "efficient under \(H_{0}\) " second.
\begin{tabular}{|c|c|c|c|c|}
\hline & \begin{tabular}{l}
- Coef \\
(b) \\
m2
\end{tabular} & \begin{tabular}{l}
ients \\
(B) \\
m1
\end{tabular} & \[
\begin{gathered}
\text { (b-B) } \\
\text { Difference }
\end{gathered}
\] & \[
\begin{gathered}
\operatorname{sqrt}\left(\operatorname{diag}\left(V \_b-V \_B\right)\right) \\
\text { S.E. }
\end{gathered}
\] \\
\hline age & -. 0101521 & -. 0100251 & -. 0001269 & \\
\hline male & . 5144003 & . 5095747 & . 0048256 & . 0123338 \\
\hline _cons & . 2678043 & . 2633838 & . 0044205 & \\
\hline \multicolumn{5}{|l|}{\multirow[t]{6}{*}{```
            b = consistent under Ho and Ha; obtained from mlogit
B = inconsistent under Ha, efficient under Ho; obtained from mlogit
Test: Ho: difference in coefficients not systematic
        chi2(3) = (b-B)'[(V_b-V_B)^(-1)](b-B)
                = 0.08
    Prob>chi2 = 0.9944
    (V_b-V_B is not positive definite)
```}} \\
\hline & & & & \\
\hline & & & & \\
\hline & & & & \\
\hline & & & & \\
\hline & & & & \\
\hline
\end{tabular}
hausman m3 m1, alleqs constant
—_ Coefficients -_
\begin{tabular}{r|ccrc} 
& \begin{tabular}{c} 
(b) \\
m 3
\end{tabular} & \multicolumn{1}{c}{\begin{tabular}{c} 
(B) \\
m 1
\end{tabular}} & \begin{tabular}{c} 
(b-B) \\
Difference
\end{tabular} & \begin{tabular}{c} 
sqrt (diag(V_b-V_B)) \\
S.E.
\end{tabular} \\
\hline age & -.0041055 & -.0051925 & .001087 & .0021355 \\
male & .4591074 & .4748547 & -.0157473 & .1333421 \\
\hline cons & -1.801774 & -1.756843 & -.0449311 &. \\
\hline
\end{tabular}

> b = consistent under Ho and Ha ; obtained from mlogit
> \(B=\) inconsistent under Ha, efficient under Ho; obtained from mlogit
> Test: Ho: difference in coefficients not systematic \(\operatorname{chi2}(3)=(b-B) \cdot\left[\left(V \_b-V \_B\right)^{\wedge}(-1)\right](b-B)\)
> \(=-0.18 \quad\) chi2 \(<0==>\) model fitted on these data fails to meet the asymptotic assumptions of the Hausman test; see suest for a generalized test

According to the test of m 1 against m 2 , we cannot reject the hypothesis that the coefficients of m 1 and m 2 are the same. The second Hausman test is not well defined-something that happens fairly often. The problem is due to the estimator of the variance \(V(b-B)\) as \(V(b)-V(B)\), which is a feasible estimator only asymptotically. Here it simply is not a proper variance matrix, and the Hausman test becomes undefined.
suest m 1 m 2 estimates the simultaneous (co)variance of the coefficients of models m 1 and m 2 . Although suest is technically a postestimation command, it acts like an estimation command in that it stores the simultaneous coefficients in e(b) and the full (co)variance matrix in e(V). We could have used the estat vce command to display the full (co)variance matrix to show that the cross-model covariances were indeed estimated. Typically, we would not have a direct interest in e(V).

suest created equation names by combining the name under which we stored the results using estimates store with the original equation names. Thus, in the simultaneous estimation result, equation Prepaid originating in model m1 is named m1_Prepaid. According to the McFaddenHausman specification of a test for IIA, the coefficients of the equations m1_PrePaid and m2_PrePaid should be equal. This equality can be tested easily with the test command. The cons option specifies that the intercept _cons be included in the test.
```

. test [m1_Prepaid = m2_Prepaid], cons
( 1) [m1_Prepaid]age - [m2_Prepaid]age = 0
( 2) [m1_Prepaid]male - [m2_Prepaid]male = 0
( 3) [m1_Prepaid]_cons - [m2_Prepaid]_cons = 0
chi2( 3) = 0.89
Prob > chi2 = 0.8266

```

The Hausman test via suest is comparable to that computed by hausman, but they use different estimators of the variance of the difference of the estimates. The hausman command estimates \(V(b-B)\) by \(V(b)-V(B)\), whereas suest estimates \(V(b-B)\) by \(V(b)-\operatorname{cov}(b, B)-\operatorname{cov}(B, b)+V(B)\). One advantage of the second estimator is that it is always admissible, so the resulting test is always well defined. This quality is illustrated in the Hausman-type test of IIA comparing models m1 and m3.
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \multicolumn{4}{|l|}{Simultaneous results for m1, m3} & \multicolumn{2}{|l|}{Number of obs} & 615 \\
\hline & Coef. & \begin{tabular}{l}
Robust \\
Std. Err.
\end{tabular} & z & \(P>|z|\) & [95\% Conf. & Interval] \\
\hline \multicolumn{7}{|l|}{m1_Indemnity} \\
\hline \multicolumn{7}{|l|}{m1_Prepaid} \\
\hline age & -. 0100251 & . 0059403 & -1.69 & 0.091 & -. 0216679 & . 0016176 \\
\hline male & . 5095747 & . 1988159 & 2.56 & 0.010 & . 1199027 & . 8992467 \\
\hline _cons & . 2633838 & . 277307 & 0.95 & 0.342 & -. 280128 & . 8068956 \\
\hline \multicolumn{7}{|l|}{m1_Uninsure} \\
\hline age & -. 0051925 & . 0109005 & -0.48 & 0.634 & -. 0265571 & . 0161721 \\
\hline male & . 4748547 & . 3677326 & 1.29 & 0.197 & -. 2458879 & 1.195597 \\
\hline _cons & -1.756843 & . 4971383 & -3.53 & 0.000 & -2.731216 & -. 78247 \\
\hline \multicolumn{7}{|l|}{m3_Indemnity} \\
\hline \multicolumn{7}{|l|}{m3_Uninsure} \\
\hline age & -. 0041055 & . 0111185 & -0.37 & 0.712 & -. 0258974 & . 0176865 \\
\hline male & . 4591074 & . 3601307 & 1.27 & 0.202 & -. 2467357 & 1.164951 \\
\hline _cons & -1.801774 & . 5226351 & -3.45 & 0.001 & -2.82612 & -. 7774283 \\
\hline
\end{tabular}
```

. test [m1_Uninsure = m3_Uninsure], cons
( 1) [m1_Uninsure]age - [m3_Uninsure]age = 0
( 2) [m1_Uninsure]male - [m3_Uninsure]male = 0
( 3) [m1_Uninsure]_cons - [m3_Uninsure]_cons = 0
chi2( 3) = 1.49
Prob > chi2 = 0.6845

```

Although the classic Hausman test computed by hausman is not defined here, the suest-based test is just fine. We cannot reject the equality of the common coefficients across m 1 and m 3 .

A second advantage of the suest approach is that we can estimate the (co)variance matrix of the multivariate normal distribution of the estimators of the three models m1, m2, and m3 and test that the common coefficients are equal.
. suest m*, noomitted
Simultaneous results for m1, m2, m3
Number of obs \(=615\)
\begin{tabular}{r|rrrrrr}
\hline & Coef. & Robust \\
& Std. Err. & z & P>|z| & [95\% Conf. Interval] \\
\hline m1_Indemnity & & & & & & \\
\hline m1_Prepaid \\
age & -.0100251 & .0059403 & -1.69 & 0.091 & -.0216679 & .0016176 \\
male & .5095747 & .1988159 & 2.56 & 0.010 & .1199027 & .8992467 \\
_cons & .2633838 & .277307 & 0.95 & 0.342 & -.280128 & .8068956 \\
\hline m1_Uninsure & & & & & & \\
age & -.0051925 & .0109005 & -0.48 & 0.634 & -.0265571 & .0161721 \\
male & .4748547 & .3677326 & 1.29 & 0.197 & -.2458879 & 1.195597 \\
_cons & -1.756843 & .4971383 & -3.53 & 0.000 & -2.731216 & -.78247 \\
\hline m2_Indemnity & & & & & & \\
\hline m2_Prepaid & & & & & & \\
age & -.0101521 & .0058988 & -1.72 & 0.085 & -.0217135 & .0014094 \\
male & .5144003 & .1996133 & 2.58 & 0.010 & .1231654 & .9056352 \\
_cons & .2678043 & .2744019 & 0.98 & 0.329 & -.2700134 & .8056221 \\
\hline m3_Indemnity & & & & & & \\
\hline m3_Uninsure & & & & & & \\
age & -.0041055 & .0111185 & -0.37 & 0.712 & -.0258974 & .0176865 \\
male & .4591074 & .3601307 & 1.27 & 0.202 & -.2467357 & 1.164951 \\
_cons & -1.801774 & .5226351 & -3.45 & 0.001 & -2.82612 & -.7774283 \\
\hline
\end{tabular}
```

. test [m1_Prepaid = m2_Prepaid] , cons notest
( 1) [m1_Prepaid]age - [m2_Prepaid]age = 0
( 2) [m1_Prepaid]male - [m2_Prepaid]male = 0
( 3) [m1_Prepaid]_cons - [m2_Prepaid]_cons = 0
. test [m1_Uninsure = m3_Uninsure], cons acc
( 1) [m1_Prepaid]age - [m2_Prepaid]age = 0
( 2) [m1_Prepaid]male - [m2_Prepaid]male = 0
( 3) [m1_Prepaid]_cons - [m2_Prepaid]_cons = 0
( 4) [m1_Uninsure]age - [m3_Uninsure]age = 0
( 5) [m1_Uninsure]male - [m3_Uninsure]male = 0
( 6) [m1_Uninsure]_cons - [m3_Uninsure]_cons = 0
chi2( 6) = 1.95
Prob > chi2 = 0.9240

```

Again we do not find evidence against the correct specification of the multinomial logit for type of insurance. The classic Hausman test assumes that one of the estimators (named B in hausman) is efficient, that is, it has minimal (asymptotic) variance. This assumption ensures that \(V(b)-V(B)\) is an admissible, viable estimator for \(V(b-B)\). The assumption that we have an efficient estimator is a restrictive one. It is violated, for instance, if our data are clustered. We want to adjust for clustering via a vce (cluster clustvar) option by requesting the cluster-adjusted sandwich estimator of variance. Consequently, in such a case, hausman cannot be used. This problem does not exist with the suest version of the Hausman test. To illustrate this feature, we suppose that the data are clustered by city-we constructed an imaginary variable cityid for this illustration. If we plan to apply suest, we would not specify the vce(cluster clustvar) option the time of estimation.
suest has a vce(cluster clustvar) option. Thus we do not need to refit the models; we can call suest and test right away.
```

. suest m1 m2, vce(cluster cityid) noomitted
Simultaneous results for m1, m2

```

Number of obs \(=615\)
(Std. Err. adjusted for 260 clusters in cityid)
\begin{tabular}{c|rcrrrr}
\hline & Coef. & Robust & Std. Err. & z & P>|z| & [95\% Conf. Interval] \\
\hline m1_Indemnity & & & & & & \\
\hline m1_Prepaid & & & & & & \\
age & -.0100251 & .005729 & -1.75 & 0.080 & -.0212538 & .0012035 \\
male & .5095747 & .1910496 & 2.67 & 0.008 & .1351244 & .884025 \\
_cons & .2633838 & .2698797 & 0.98 & 0.329 & -.2655708 & .7923384 \\
\hline m1_Uninsure & & & & & & \\
age & -.0051925 & .0104374 & -0.50 & 0.619 & -.0256495 & .0152645 \\
male & .4748547 & .3774021 & 1.26 & 0.208 & -.2648399 & 1.214549 \\
_cons & -1.756843 & .4916613 & -3.57 & 0.000 & -2.720481 & -.7932048 \\
\hline m2_Indemnity & & & & & & \\
\hline m2_Prepaid & & & & & & \\
age & -.0101521 & .0057164 & -1.78 & 0.076 & -.0213559 & .0010518 \\
male & .5144003 & .1921385 & 2.68 & 0.007 & .1378158 & .8909848 \\
_cons & .2678043 & .2682193 & 1.00 & 0.318 & -.2578959 & .7935045 \\
\hline
\end{tabular}
```

. test [m1_Prepaid = m2_Prepaid], cons
( 1) [m1_Prepaid]age - [m2_Prepaid]age = 0
( 2) [m1_Prepaid]male - [m2_Prepaid]male = 0
( 3) [m1_Prepaid]_cons - [m2_Prepaid]_cons = 0
chi2( 3) = 0.79
Prob > chi2 = 0.8529

```
suest provides some descriptive information about the clustering on cityid. Like any other estimation command, suest informs us that the standard errors are adjusted for clustering. The Hausman-type test obtained from the test command uses a simultaneous (co)variance of m1 and m2 appropriately adjusted for clustering. In this example, we still do not have reason to conclude that the multinomial logit model in this application is misspecified, that is, that IIA is violated.

The multinomial logistic regression model is a special case of the conditional logistic regression model; see \([\mathrm{R}]\) clogit. Like the multinomial logistic regression model, the conditional logistic regression model also makes the IIA assumption. Consider an example, introduced in [R] asclogit, in which the demand for American, Japanese, and European cars is modeled in terms of the number of local dealers of the respective brands and of some individual attributes incorporated in interaction with the nationality of cars. We want to perform a Hausman-type test for IIA comparing the decision between all nationalities with the decision between non-American cars. The following code fragment demonstrates how to conduct a Hausman test for IIA via suest in this case.
```

. clogit choice japan europe maleJap maleEur incJap incEur dealer, group(id)
. estimates store allcars
. clogit choice japan maleJap incJap dealer if car!=1 , group(id)
. estimates store foreign

```
```

. suest allcars foreign
. test [allcars_choice=foreign_choice], common

```

\section*{Testing proportionality}

The applications of suest that we have discussed so far concern Hausman-type tests for misspecification. To test such a hypothesis, we compared two estimators that have the same probability limit if the hypothesis holds true, but otherwise have different limits. We may also want to compare the coefficients of models (estimators) for other substantive reasons. Although we most often want to test whether coefficients differ between models or estimators, we may occasionally want to test other constraints (see Hausman and Ruud [1987]).

\section*{\(>\) Example 2}

In this example, using simulated labor market data for siblings, we consider two dependent variables, income (inc) and whether a person was promoted in the last year (promo). We apply familiar economic arguments regarding human capital, according to which employees have a higher income and a higher probability of being promoted, by having more human capital. Human capital is acquired through formal education (edu) and on-the-job training experience (exp). We study whether income and promotion are "two sides of the same coin", that is, whether they reflect a common latent variable, "human capital". Accordingly, we want to compare the effects of different aspects of human capital on different outcome variables.

We estimate fairly simple labor market equations. The income model is estimated with regress, and the estimation results are stored under the name Inc.
. use http://www.stata-press.com/data/r13/income
. regress inc edu exp male
\begin{tabular}{r|rrc} 
Source & SS & df & MS \\
\hline \begin{tabular}{r} 
Model \\
Residual
\end{tabular} & 2058.44672 & 3 & 686.148908 \\
\hline Total & 6482.49855 & 276 & 23.4873136
\end{tabular}
\begin{tabular}{lrr} 
Number of obs & \(=\) & 277 \\
F 3, 273) & \(=42.34\) \\
Prob \(>\) F & \(=0.0000\) \\
R-squared & \(=0.3175\) \\
Adj R-squared & \(=0.3100\) \\
Root MSE & \(=4.0256\)
\end{tabular}
\begin{tabular}{r|rccccc}
\hline inc & Coef. & Std. Err. & t & \(\mathrm{P}>|\mathrm{t}|\) & [95\% Conf. Interval] \\
\hline edu & 2.213707 & .243247 & 9.10 & 0.000 & 1.734828 & 2.692585 \\
exp & 1.47293 & .231044 & 6.38 & 0.000 & 1.018076 & 1.927785 \\
male & .5381153 & .4949466 & 1.09 & 0.278 & -.436282 & 1.512513 \\
_cons & 1.255497 & .3115808 & 4.03 & 0.000 & .642091 & 1.868904 \\
\hline
\end{tabular}
. est store Inc
Being sibling data, the observations are clustered on family of origin, famid. In the estimation of the regression parameters, we did not specify a vce (cluster famid) option to adjust standard errors for clustering on family (famid). Thus the standard errors reported by regress are potentially flawed. This problem will, however, be corrected by specifying a vce(cluster clustvar) option with suest.

Next we estimate the promotion equation with probit and again store the results under an appropriate name.


The coefficients in the income and promotion equations definitely seem to be different. However, because the scales of the two variables are different, we would not expect the coefficients to be equal. The correct hypothesis here is that the proportionality of the coefficients of the two models, apart from the constant, are equal. This formulation would still reflect that the relative effects of the different aspects of human capital do not differ between the dependent variables. We can obtain a nonlinear Wald test for the hypothesis of proportionality by using the testnl command on the combined estimation results of the two estimators. Thus we first have to form the combined estimation results. At this point, we specify the vce(cluster famid) option to adjust for the clustering of observations on famid.
```

. suest Inc Promo, vce(cluster famid)
Simultaneous results for Inc, Promo

```

Number of obs = 277
(Std. Err. adjusted for 135 clusters in famid)
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline & Coef. & Robust Std. Err. & z & \(P>|z|\) & [95\% Conf. & Interval] \\
\hline \multicolumn{7}{|l|}{Inc_mean} \\
\hline edu & 2.213707 & . 2483907 & 8.91 & 0.000 & 1.72687 & 2.700543 \\
\hline exp & 1.47293 & . 1890583 & 7.79 & 0.000 & 1.102383 & 1.843478 \\
\hline male & . 5381153 & . 4979227 & 1.08 & 0.280 & -. 4377952 & 1.514026 \\
\hline _cons & 1.255497 & . 3374977 & 3.72 & 0.000 & . 594014 & 1.916981 \\
\hline \multicolumn{7}{|l|}{Inc_lnvar} \\
\hline _cons & 2.785339 & . 079597 & 34.99 & 0.000 & 2.629332 & 2.941347 \\
\hline \multicolumn{7}{|l|}{Promo_promo} \\
\hline edu & . 4593002 & . 0886982 & 5.18 & 0.000 & . 2854549 & . 6331454 \\
\hline exp & . 3593023 & . 079772 & 4.50 & 0.000 & . 2029522 & . 5156525 \\
\hline male & . 2079983 & . 1691053 & 1.23 & 0.219 & -. 1234419 & . 5394386 \\
\hline _cons & -. 464622 & . 1042169 & -4.46 & 0.000 & -. 6688833 & -. 2603607 \\
\hline
\end{tabular}

The standard errors reported by suest are identical to those reported by the respective estimation commands when invoked with the vce(cluster famid) option. We are now ready to test for proportionality:
\[
H_{0}: \frac{\beta_{\text {edu }}^{\text {Income }}}{\beta_{\text {edu }}^{\text {Promotion }}}=\frac{\beta_{\text {exp }}^{\text {Income }}}{\beta_{\text {exp }}^{\text {Promotion }}}=\frac{\beta_{\text {male }}^{\text {Income }}}{\beta_{\text {male }}^{\text {Protion }}}
\]

It is straightforward to translate this into syntax suitable for testnl, recalling that the coefficient of variable \(v\) in equation \(e q\) is denoted by [eq] \(v\).
```

. testnl [Inc_mean]edu/[Promo_promo]edu =
[Inc_mean] exp/[Promo_promo] exp =
[Inc_mean]male/[Promo_promo]male
(1) [Inc_mean] edu/[Promo_promo]edu = [Inc_mean] exp/[Promo_promo] exp
(2) [Inc_mean] edu/[Promo_promo]edu = [Inc_mean]male/[Promo_promo]male
chi2(2) = 0.61
Prob > chi2 = 0.7385

```

From the evidence, we fail to reject the hypotheses that the coefficients of the income and promotion equations are proportional. Thus it is not unreasonable to assume that income and promotion can be explained by the same latent variable, "labor market success".

A disadvantage of the nonlinear Wald test is that it is not invariant with respect to representation: a Wald test for a mathematically equivalent formulation of the nonlinear constraint usually leads to a different test result. An equivalent formulation of the proportionality hypothesis is
\[
\begin{aligned}
H_{0}: & \beta_{\mathrm{edu}}^{\text {Income }} \beta_{\mathrm{exp}}^{\text {Promotion }}=\beta_{\mathrm{edu}}^{\text {Promotion }} \beta_{\mathrm{exp}}^{\text {Income }} \quad \text { and } \\
& \beta_{\mathrm{edu}}^{\text {Income }} \beta_{\text {male }}^{\text {Promotion }}=\beta_{\mathrm{edu}}^{\text {Promotion }} \beta_{\text {male }}^{\text {Income }}
\end{aligned}
\]

This formulation is "more linear" in the coefficients. The asymptotic \(\chi^{2}\) distribution of the nonlinear Wald statistic can be expected to be more accurate for this representation.
```

. testnl ([Inc_mean] edu*[Promo_promo] exp = [Inc_mean] exp*[Promo_promo]edu)
> ([Inc_mean] edu*[Promo_promo]male = [Inc_mean]male*[Promo_promo]edu)
(1) [Inc_mean] edu*[Promo_promo] exp = [Inc_mean] exp*[Promo_promo]edu
(2) [Inc_mean]edu*[Promo_promo]male = [Inc_mean]male*[Promo_promo]edu
chi2(2) = 0.46
Prob > chi2 = 0.7936

```

Here the two representations lead to similar test statistics and \(p\)-values. As before, we fail to reject the hypothesis of proportionality of the coefficients of the two models.

\section*{Testing cross-model hypotheses}
> Example 3
In this example, we demonstrate how some cross-model hypotheses can be tested using the facilities already available in most estimation commands. This demonstration will explain the intricate relationship between the cluster adjustment of the robust estimator of variance and the suest command. It will also be made clear that a new facility is required to perform more general cross-model testing.

We want to test whether the effect of \(x_{1}\) on the binary variable \(y_{1}\) is the same as the effect of \(x_{2}\) on the binary \(y_{2}\); see Clogg, Petkova, and Haritou (1995). In this setting, \(x_{1}\) may equal \(x_{2}\), and \(y_{1}\) may equal \(y_{2}\). We assume that logistic regression models can be used to model the responses, and for simplicity, we ignore further predictor variables in these models. If the two logit models are fit on independent samples so that the estimators are (stochastically) independent, a Wald test for \(\_\mathrm{b}\) [x1] \(=\_b[x 2]\) rejects the null hypothesis if
\[
\frac{\widehat{b}\left(x_{1}\right)-\widehat{b}\left(x_{2}\right)}{\left[\widehat{\sigma}^{2}\left\{\widehat{b}\left(x_{1}\right)\right\}+\widehat{\sigma}^{2}\left\{\widehat{b}\left(x_{2}\right)\right\}\right]^{1 / 2}}
\]
is larger than the appropriate \(\chi_{1}^{2}\) threshold. If the models are fit on the same sample (or on dependent samples), so that the estimators are stochastically dependent, the above test that ignores the covariance between the estimators is not appropriate.

It is instructive to see how this problem can be tackled by "stacking" data. In the stacked format, we doubled the number of observations. The dependent variable is \(y_{1}\) in the first half of the data and is \(y_{2}\) in the second half of the data. The predictor variable \(z_{1}\) is set to \(x_{1}\) in the first half of the expanded data and to 0 in the rest. Similarly, \(z_{2}\) is 0 in the first half and \(x_{2}\) in the second half. The following diagram illustrates the transformation, in the terminology of the reshape command, from wide to long format.
\[
\left(\begin{array}{ccccc}
\text { id } & y_{1} & y_{2} & x_{1} & x_{2} \\
\hline 1 & y_{11} & y_{21} & x_{11} & x_{21} \\
2 & y_{12} & y_{22} & x_{12} & x_{22} \\
3 & y_{13} & y_{23} & x_{13} & x_{23}
\end{array}\right) \Longrightarrow\left(\begin{array}{ccccc}
\hline 1 & y_{11} & x_{11} & 0 & 1 \\
2 & y_{12} & x_{12} & 0 & 1 \\
3 & y_{13} & x_{13} & 0 & 1 \\
1 & y_{21} & 0 & x_{21} & 2 \\
2 & y_{22} & 0 & x_{22} & 2 \\
3 & y_{23} & 0 & x_{23} & 2
\end{array}\right)
\]

The observations in the long format data organization are clustered on the original subjects and are identified with the identifier id. The clustering on id has to be accounted for when fitting a simultaneous model. The simplest way to deal with clustering is to use the cluster adjustment of the robust or sandwich estimator; see \([\mathrm{P}]\) _robust. The data manipulation can be accomplished easily with the stack command; see [D] stack. Subsequently, we fit a simultaneous logit model and perform a Wald test for the hypothesis that the coefficients of \(z 1\) and \(z 2\) are the same. A full setup to obtain the cross-model Wald test could then be as follows:
```

. generate zero = 0 // a variable that is always 0
. generate one = 1 // a variable that is always 1
. generate two = 2 // a variable that is always 2
. stack id y1 x1 zero one id y2 zero x2 two, into(id y z1 z2 model)
. generate model2 = (model==2)
. logit y model2 z1 z2, vce(cluster id)
. test _b[z1] = _b[z2]

```

The coefficient of \(z 1\) represents the effect of \(x 1\) on \(y 1\), and similarly, \(z 2\) for the effect of \(x 2\) on y2. The variable model2 is a dummy for the "second model", which is included to allow the intercept in the second model to differ from that in the first model. The estimates of the coefficient of \(z 1\) and its standard error in the combined model are the same as the estimates of the coefficient of z 1 and its standard error if we fit the model on the unstacked data.
```

. logit y1 x1, vce(robust)

```

The vce (cluster clustvar) option specified with the logit command for the stacked data ensures that the covariances of \(\_b[z 1]\) and \(\_b[z 2]\) are indeed estimated. This estimation ensures that the Wald test for the equality of the coefficients is correct. If we had not specified the vce (cluster clustvar) option, the (co)variance matrix of the coefficients would have been block-diagonal; that is, the covariances of \(\_\mathrm{b}[\mathrm{z} 1]\) and \(\_\mathrm{b}[\mathrm{z} 2]\) would have been 0 . Then test would have effectively used the invalid formula for the Wald test for two independent samples.

In this example, the two logit models were fit on the same data. The same setup would apply, without modification, when the two logit models were fit on overlapping data that resulted, for instance, if the \(y\) or \(x\) variables were missing in some observations.

The suest command allows us to obtain the above Wald test more efficiently by avoiding the data manipulation, obviating the need to fit a model with twice the number of coefficients. The test statistic produced by the above code fragment is identical to that obtained via suest on the original (unstacked) data:
. logit y1 x1
. estimates store M1
. \(\operatorname{logit~y2~x2~}\)
. estimates store M2
. suest M1 M2
. test \([\mathrm{M} 1] \times 1=[\mathrm{M} 2] \mathrm{x} 2\)
The stacking method can be applied not only to the testing of cross-model hypotheses for logit models but also to any estimation command that supports the vce (cluster clustvar) option. The stacking approach clearly generalizes to stacking more than two logit or other models, testing more general linear hypotheses, and testing nonlinear cross-model hypotheses (see [R] testnl). In all of these cases, suest would yield identical statistical results but at smaller costs in terms of data management, computer storage, and computer time.

Is suest nothing but a convenience command? No, there are two disadvantages to the stacking method, both of which are resolved via suest. First, if the models include ancillary parameters (in a regression model, the residual variance; in an ordinal response model, the cutpoints; and in lognormal survival-time regression, the time scale parameter), these parameters are constrained to be equal between the stacked models. In suest, this constraint is relaxed. Second, the stacking method does not generalize to compare different statistical models, such as a probit model and a regression model. As demonstrated in the previous section, suest can deal with this situation.

\section*{Stored results}
suest stores the following in e():
Scalars
\(e(N) \quad\) number of observations
e(N_clust) number of clusters
\(e\) (rank) rank of \(e(V)\)
Macros
e(cmd)
e(eqnames\#)
suest
\(e\) (names) list of model names
\(e\) (wtype) weight type
e(wexp)
e(clustvar)
eight expression
e(vce)
e(vcetype)
e(properties)
Matrices
e(b) stacked coefficient vector of the models
e(V)
variance-covariance matrix of the estimators
Functions
e(sample) marks estimation sample

\section*{Methods and formulas}

The estimation of the simultaneous (co)variance of a series of \(k\) estimators is a nonstandard application of the sandwich estimator, as implemented by the command [P] _robust. You may want to read this manual entry before reading further.

The starting point is that we have fit \(k\) different models on the same data-partially overlapping or nonoverlapping data are special cases. We want to derive the simultaneous distribution of these \(k\) estimators, for instance, to test a cross-estimator hypothesis \(H_{0}\). As in the framework of Hausman testing, \(H_{0}\) will often be of the form that different estimators have the same probability limit under some hypothesis, while the estimators have different limits if the hypothesis is violated.

We consider (vector) estimators \(\widehat{\boldsymbol{\beta}}_{i}\) to be defined as "the" solution of the estimation equations \(\mathbf{G}_{i}\),
\[
\mathbf{G}_{i}\left(\mathbf{b}_{i}\right)=\sum_{j} w_{i j} \mathbf{u}_{i j}\left(\mathbf{b}_{i}\right)=\mathbf{0}, \quad i=1, \ldots, k
\]

We refer to the \(\mathbf{u}_{i j}\) as the "scores". Specifying some weights \(w_{i j}=0\) trivially accommodates for partially overlapping or even disjointed data. Under "suitable regularity conditions" (see White [1982; 1996] for details), the \(\widehat{\boldsymbol{\beta}}_{i}\) are asymptotically normally distributed, with the variance estimated consistently by the sandwich estimator
\[
V_{i}=\operatorname{Var}\left(\widehat{\boldsymbol{\beta}}_{i}\right)=\mathbf{D}_{i}^{-1} \sum_{j} w_{i j} \mathbf{u}_{i j} \mathbf{u}_{i j}^{\prime} \mathbf{D}_{i}^{-1}
\]
where \(\mathbf{D}_{i}\) is the Jacobian of \(\mathbf{G}_{i}\) evaluated at \(\widehat{\boldsymbol{\beta}}_{i}\). In the context of maximum likelihood estimation, \(\mathbf{D}_{i}\) can be estimated consistently by (minus) the Hessian of the log likelihood or by the Fisher information matrix. If the model is also well specified, the sandwiched term \(\left(\sum_{j} w_{i j} \mathbf{u}_{i j} \mathbf{u}_{i j}^{\prime}\right)\) converges in probability to \(\mathbf{D}_{i}\), so \(V_{i}\) may be consistently estimated by \(\mathbf{D}_{i}^{-1}\).

To derive the simultaneous distribution of the estimators, we consider the "stacked" estimation equation,
\[
\mathbf{G}(\widehat{\boldsymbol{\beta}})=\left\{\begin{array}{llll}
\mathbf{G}_{1}\left(\widehat{\boldsymbol{\beta}}_{1}\right)^{\prime} & \mathbf{G}_{1}\left(\widehat{\boldsymbol{\beta}}_{2}\right)^{\prime} & \ldots & \mathbf{G}_{k}\left(\widehat{\boldsymbol{\beta}}_{k}\right)^{\prime}
\end{array}\right\}^{\prime}=\mathbf{0}
\]

Under "suitable regularity conditions" (see White [1996] for details), \(\widehat{\boldsymbol{\beta}}\) is asymptotically jointly normally distributed. The Jacobian and scores of the simultaneous equation are easily expressed in the Jacobian and scores of the separate equations. The Jacobian of \(\mathbf{G}\),
\[
\mathbf{D}(\widehat{\boldsymbol{\beta}})=\left.\frac{d \mathbf{G}(\boldsymbol{\beta})}{d \boldsymbol{\beta}}\right|_{\boldsymbol{\beta}=\widehat{\boldsymbol{\beta}}}
\]
is block diagonal with blocks \(\mathbf{D}_{1}, \ldots, \mathbf{D}_{k}\). The inverse of \(\mathbf{D}(\widehat{\boldsymbol{\beta}})\) is again block diagonal, with the inverses of \(\mathbf{D}_{i}\) on the diagonal. The scores \(\mathbf{u}\) of \(\mathbf{G}\) are simply obtained as the concatenated scores of the separate equations:
\[
\mathbf{u}_{j}=\left(\begin{array}{llll}
\mathbf{u}_{1 j}^{\prime} & \mathbf{u}_{2 j}^{\prime} & \ldots & \mathbf{u}_{k j}^{\prime}
\end{array}\right)^{\prime}
\]

Out-of-sample (that is, where \(w_{i j}=0\) ) values of the score variables are defined as 0 (thus we drop the \(i\) subscript from the common weight variable). The sandwich estimator for the asymptotic variance of \(\widehat{\boldsymbol{\beta}}\) reads
\[
V=\operatorname{Var}(\widehat{\boldsymbol{\beta}})=\mathbf{D}(\widehat{\boldsymbol{\beta}})^{-1}\left(\sum_{j} w_{j} \mathbf{u}_{j} \mathbf{u}_{j}^{\prime}\right) \mathbf{D}(\widehat{\boldsymbol{\beta}})^{-1}
\]

Taking a "partitioned" look at this expression, we see that \(V\left(\widehat{\boldsymbol{\beta}}_{i}\right)\) is estimated by
\[
\mathbf{D}_{i}^{-1}\left(\sum_{j} w_{j} \mathbf{u}_{i j} \mathbf{u}_{i j}^{\prime}\right) \mathbf{D}_{i}^{-1}
\]
which is, yet again, the familiar sandwich estimator for \(\widehat{\boldsymbol{\beta}}_{i}\) based on the separate estimation equation \(\mathbf{G}_{i}\). Thus considering several estimators simultaneously in this way does not affect the estimators of the asymptotic variances of these estimators. However, as a bonus of stacking, we obtained a sandwich-type estimate of the covariance \(V_{i h}\) of estimators \(\widehat{\boldsymbol{\beta}}_{i}\) and \(\widehat{\boldsymbol{\beta}}_{h}\),
\[
V_{i h}=\operatorname{Cov}\left(\widehat{\boldsymbol{\beta}}_{i}, \widehat{\boldsymbol{\beta}}_{h}\right)=\mathbf{D}_{i}^{-1}\left(\sum_{j} w_{j} \mathbf{u}_{i j} \mathbf{u}_{i h}^{\prime}\right) \mathbf{D}_{h}^{-1}
\]
which is also obtained by White (1982).
This estimator for the covariance of estimators is an application of the cluster modification of the sandwich estimator proposed by Rogers (1993). Consider the stacked data format as discussed in the logit example, and assume that Stata would be able to estimate a "stacked model" in which different models apply to different observations, for example, a probit model for the first half, a regression model for the second half, and a one-to-one cluster relation between the first and second half. If there are no common parameters to both models, the score statistics of parameters for the stacked models are zero in the half of the data in which they do not occur. In Rogers' method, we have to sum the score statistics over the observations within a cluster. This step boils down to concatenating the score statistics at the level of the cluster.

We compare the sandwich estimator of the (co)variance \(V_{12}\) of two estimators with the estimator of variance \(\widetilde{V}_{12}\) applied in the classic Hausman test. Hausman (1978) showed that if \(\widehat{\boldsymbol{\beta}}_{1}\) is consistent under \(H_{0}\) and \(\widehat{\boldsymbol{\beta}}_{2}\) is efficient under \(H_{0}\), then asymptotically
\[
\operatorname{Cov}\left(\widehat{\boldsymbol{\beta}}_{1}, \widehat{\boldsymbol{\beta}}_{2}\right)=\operatorname{Var}\left(\widehat{\boldsymbol{\beta}}_{2}\right)
\]
and so \(\operatorname{var}\left(\widehat{\boldsymbol{\beta}}_{1}-\widehat{\boldsymbol{\beta}}_{2}\right)\) is consistently estimated by \(V_{1}-V_{2}\).

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An earlier version of suest was published in the Stata Technical Bulletin (1999). The current version of suest is not backward compatible with the STB version because of the introduction of new ways to manage estimation results via the estimates command.

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\section*{Also see}
[R] estimates - Save and manipulate estimation results
[R] hausman - Hausman specification test
[R] lincom - Linear combinations of estimators
[R] nlcom - Nonlinear combinations of estimators
[R] test - Test linear hypotheses after estimation
[R] testnl - Test nonlinear hypotheses after estimation
[P] _robust - Robust variance estimates

\section*{Title}
summarize - Summary statistics
\begin{tabular}{llll} 
Syntax & Menu & Description & Options \\
Remarks and examples & Stored results & Methods and formulas & References
\end{tabular}

Also see

\section*{Syntax}
summarize \([\) varlist \(][\) if \(][\) in \(][\) weight \(]\) [, options]
options Description
Main
detail display additional statistics
meanonly
format
suppress the display; calculate only the mean; programmer's option use variable's display format
separator (\#) draw separator line after every \# variables; default is separator (5)
display_options control spacing, line width, and base and empty cells
varlist may contain factor variables; see [U] 11.4.3 Factor variables.
varlist may contain time-series operators; see [U] 11.4.4 Time-series varlists.
by, rolling, and statsby are allowed; see [U] 11.1.10 Prefix commands.
aweights, fweights, and iweights are allowed. However, iweights may not be used with the detail option; see
[U] 11.1.6 weight.

\section*{Menu}

Statistics \(>\) Summaries, tables, and tests \(>\) Summary and descriptive statistics \(>\) Summary statistics

\section*{Description}
summarize calculates and displays a variety of univariate summary statistics. If no varlist is specified, summary statistics are calculated for all the variables in the dataset.

Also see \([R]\) ci for calculating the standard error and confidence intervals of the mean.

\section*{Options}
\(\xrightarrow{\text { Main }}\)
detail produces additional statistics, including skewness, kurtosis, the four smallest and largest values, and various percentiles.
meanonly, which is allowed only when detail is not specified, suppresses the display of results and calculation of the variance. Ado-file writers will find this useful for fast calls.
format requests that the summary statistics be displayed using the display formats associated with the variables rather than the default \(g\) display format; see [U] 12.5 Formats: Controlling how data are displayed.
separator(\#) specifies how often to insert separation lines into the output. The default is separator (5), meaning that a line is drawn after every five variables. separator (10) would draw a line after every 10 variables. separator ( 0 ) suppresses the separation line.
display_options: vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap (\#), and fvwrapon(style); see [R] estimation options.

\section*{Remarks and examples}
summarize can produce two different sets of summary statistics. Without the detail option, the number of nonmissing observations, the mean and standard deviation, and the minimum and maximum values are presented. With detail, the same information is presented along with the variance, skewness, and kurtosis; the four smallest and four largest values; and the 1st, 5th, 10th, 25th, 50th (median), 75th, 90th, 95th, and 99th percentiles.
\(>\) Example 1: summarize with the separator() option
We have data containing information on various automobiles, among which is the variable mpg, the mileage rating. We can obtain a quick summary of the mpg variable by typing
```

. use http://www.stata-press.com/data/r13/auto2
(1978 Automobile Data)
. summarize mpg

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | :---: | :---: | :---: | :---: | ---: |
| mpg | 74 | 21.2973 | 5.785503 | 12 | 41 |

```

We see that we have 74 observations. The mean of mpg is 21.3 miles per gallon, and the standard deviation is 5.79 . The minimum is 12 , and the maximum is 41 .

If we had not specified the variable (or variables) we wanted to summarize, we would have obtained summary statistics on all the variables in the dataset:
\begin{tabular}{r|rrrrr}
. summarize, separator(4) \\
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline make & 0 & & & & \\
price & 74 & 6165.257 & 2949.496 & 3291 & 15906 \\
mpg & 74 & 21.2973 & 5.785503 & 12 & 41 \\
rep78 & 69 & 3.405797 & .9899323 & 1 & 5 \\
\hline headroom & 74 & 2.993243 & .8459948 & 1.5 & 5 \\
trunk & 74 & 13.75676 & 4.277404 & 5 & 23 \\
weight & 74 & 3019.459 & 777.1936 & 1760 & 4840 \\
length & 74 & 187.9324 & 22.26634 & 142 & 233 \\
\hline turn & 74 & 39.64865 & 4.399354 & 31 & 51 \\
displacement & 74 & 197.2973 & 91.83722 & 79 & 425 \\
gear_ratio & 74 & 3.014865 & .4562871 & 2.19 & 3.89 \\
foreign & 74 & .2972973 & .4601885 & 0 & 1
\end{tabular}

There are only 69 observations on rep78, so some of the observations are missing. There are no observations on make because it is a string variable.

The idea of the mean is quite old (Plackett 1958), but its extension to a scheme of moment-based measures was not done until the end of the 19th century. Between 1893 and 1905, Pearson discussed and named the standard deviation, skewness, and kurtosis, but he was not the first to use any of these. Thiele (1889), in contrast, had earlier firmly grasped the notion that the \(m_{r}\) provide a systematic basis for discussing distributions. However, even earlier anticipations can also be found. For example, Euler in 1778 used \(m_{2}\) and \(m_{3}\) in passing in a treatment of estimation (Hald 1998, 87), but seemingly did not build on that.

Similarly, the idea of the median is quite old. The history of the interquartile range is tangled up with that of the probable error, a long-popular measure. Extending this in various ways to a more general approach based on quantiles (to use a later term) occurred to several people in the nineteenth century. Galton (1875) is a nice example, particularly because he seems so close to the key idea of the quantiles as a function, which took another century to reemerge strongly.

Thorvald Nicolai Thiele (1838-1910) was a Danish scientist who worked in astronomy, mathematics, actuarial science, and statistics. He made many pioneering contributions to statistics, several of which were overlooked until recently. Thiele advocated graphical analysis of residuals checking for trends, symmetry of distributions, and changes of sign, and he even warned against overinterpreting such graphs.

\section*{> Example 2: summarize with the detail option}

The detail option provides all the information of a normal summarize and more. The format of the output also differs, as shown here:
\begin{tabular}{|c|c|c|c|c|}
\hline \multicolumn{5}{|c|}{Mileage (mpg)} \\
\hline & Percentiles & Smallest & & \\
\hline 1\% & 12 & 12 & & \\
\hline 5\% & 14 & 12 & & \\
\hline 10\% & 14 & 14 & Obs & 74 \\
\hline 25\% & 18 & 14 & Sum of Wgt. & 74 \\
\hline 50\% & 20 & & Mean & 21.2973 \\
\hline & & Largest & Std. Dev. & 5.785503 \\
\hline 75\% & 25 & 34 & & \\
\hline 90\% & 29 & 35 & Variance & 33.47205 \\
\hline 95\% & 34 & 35 & Skewness & . 9487176 \\
\hline 99\% & 41 & 41 & Kurtosis & 3.975005 \\
\hline
\end{tabular}

As in the previous example, we see that the mean of mpg is 21.3 miles per gallon and that the standard deviation is 5.79 . We also see the various percentiles. The median of mpg (the 50 th percentile) is 20 miles per gallon. The 25 th percentile is 18 , and the 75 th percentile is 25 .

When we performed summarize, we learned that the minimum and maximum were 12 and 41, respectively. We now see that the four smallest values in our dataset are \(12,12,14\), and 14 . The four largest values are \(34,35,35\), and 41 . The skewness of the distribution is 0.95 , and the kurtosis is 3.98. (A normal distribution would have a skewness of 0 and a kurtosis of 3.)

Skewness is a measure of the lack of symmetry of a distribution. If the distribution is symmetric, the coefficient of skewness is 0 . If the coefficient is negative, the median is usually greater than the mean and the distribution is said to be skewed left. If the coefficient is positive, the median is usually less than the mean and the distribution is said to be skewed right. Kurtosis (from the Greek kyrtosis, meaning curvature) is a measure of peakedness of a distribution. The smaller the coefficient of kurtosis, the flatter the distribution. The normal distribution has a coefficient of kurtosis of 3 and provides a convenient benchmark.

\section*{- Technical note}

The convention of calculating the median of an even number of values by averaging the central two order statistics is of long standing. (That is, given 8 values, average the 4 th and 5 th smallest values, or given 42, average the 21 st and 22 nd smallest.) Stigler (1977) filled a much-needed gap in the literature by naming such paired central order statistics as "comedians", although it remains unclear how far he was joking.

\section*{Example 3: summarize with the by prefix}
summarize can usefully be combined with the by varlist: prefix. In our dataset, we have a variable, foreign, that distinguishes foreign and domestic cars. We can obtain summaries of mpg and weight within each subgroup by typing
. by foreign: summarize mpg weight
\begin{tabular}{|c|c|c|c|c|c|}
\hline Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline mpg & 52 & 19.82692 & 4.743297 & 12 & 34 \\
\hline weight & 52 & 3317.115 & 695.3637 & 1800 & 4840 \\
\hline \multicolumn{6}{|l|}{-> foreign = Foreign} \\
\hline Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline mpg & 22 & 24.77273 & 6.611187 & 14 & 41 \\
\hline weight & 22 & 2315.909 & 433.0035 & 1760 & 3420 \\
\hline
\end{tabular}

Domestic cars in our dataset average 19.8 miles per gallon, whereas foreign cars average 24.8 .
Because by varlist: can be combined with summarize, it can also be combined with summarize, detail:
by foreign: summarize mpg, detail
```

-> foreign = Domestic

```
\begin{tabular}{rrrlr}
\multicolumn{4}{c}{ Mileage (mpg) } \\
& Percentiles & Smallest & \\
\(1 \%\) & 12 & 12 & & 52 \\
\(5 \%\) & 14 & 12 & Obs & 52 \\
\(10 \%\) & 14 & 14 & Sum of Wgt. & \\
\(25 \%\) & 16.5 & 14 & Mean & 19.82692 \\
\(50 \%\) & 19 & & Std. Dev. & 4.743297 \\
& & Largest & & \\
\(75 \%\) & 22 & 28 & Variance & 22.49887 \\
\(90 \%\) & 26 & 39 & Skewness & .7712432 \\
\(95 \%\) & 29 & 34 & Kurtosis & 3.441459 \\
\(99 \%\) & 34 & & &
\end{tabular}
-> foreign = Foreign
Mileage (mpg)
\begin{tabular}{rrrlr}
\hline & Percentiles & Smallest & & \\
\(1 \%\) & 14 & 14 & & \\
\(5 \%\) & 17 & 17 & & \\
\(10 \%\) & 17 & 17 & Obs & 22 \\
\(25 \%\) & 21 & 18 & Sum of Wgt. & \\
\(50 \%\) & 24.5 & & Mean & 24.77273 \\
& & Largest & Std. Dev. & 6.611187 \\
\(75 \%\) & 28 & 31 & & \\
\(90 \%\) & 35 & 35 & Variance & 43.70779 \\
\(95 \%\) & 35 & 35 & Skewness & .657329 \\
\(99 \%\) & 41 & 41 & Kurtosis & 3.10734
\end{tabular}

\section*{- Technical note}
summarize respects display formats if we specify the format option. When we type summarize price weight, we obtain
. summarize price weight
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline price & 74 & 6165.257 & 2949.496 & 3291 & 15906 \\
weight & 74 & 3019.459 & 777.1936 & 1760 & 4840
\end{tabular}

The display is accurate but is not as aesthetically pleasing as we may wish, particularly if we plan to use the output directly in published work. By placing formats on the variables, we can control how the table appears:
. format price weight \%9.2fc
. summarize price weight, format
\begin{tabular}{r|rrrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline price & 74 & \(6,165.26\) & \(2,949.50\) & \(3,291.00\) & \(15,906.00\) \\
weight & 74 & \(3,019.46\) & 777.19 & \(1,760.00\) & \(4,840.00\)
\end{tabular}

If you specify a weight (see [U] 11.1.6 weight), each observation is multiplied by the value of the weighting expression before the summary statistics are calculated so that the weighting expression is interpreted as the discrete density of each observation.
> Example 4: summarize with factor variables
You can also use summarize to obtain summary statistics for factor variables. For example, if you type
```

. summarize i.rep78

```
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline rep78 & & & & & \\
Fair & 69 & .115942 & .3225009 & 0 & 1 \\
Average & 69 & .4347826 & .4993602 & 0 & 1 \\
Good & 69 & .2608696 & .4423259 & 0 & 1 \\
Excellent & 69 & .1594203 & .3687494 & 0 & 1
\end{tabular}
you obtain the sample proportions for four of the five levels of the rep78 variable. For example, \(11.6 \%\) of the 69 cars with nonmissing values of rep78 have a fair repair record. When you use factor-variable notation, the base category is suppressed by default. If you type
. summarize bn.rep78
\begin{tabular}{c|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline rep78 & & & & & \\
Poor & 69 & .0289855 & .1689948 & 0 & 1 \\
Fair & 69 & .115942 & .3225009 & 0 & 1 \\
Average & 69 & .4347826 & .4993602 & 0 & 1 \\
Good & 69 & .2608696 & .4423259 & 0 & 1 \\
Excellent & 69 & .1594203 & .3687494 & 0 & 1
\end{tabular}
the notation bn.rep78 indicates that Stata should not suppress the base category so that we see the proportions for all five levels.

We could have used tabulate oneway rep78 to obtain the sample proportions along with the cumulative proportions. Alternatively, we could have used proportions rep78 to obtain the sample proportions along with the standard errors of the proportions instead of the standard deviations of the proportions.

\section*{> Example 5: summarize with weights}

We have 1980 census data on each of the 50 states. Included in our variables is medage, the median age of the population of each state. If we type summarize medage, we obtain unweighted statistics:
```

. use http://www.stata-press.com/data/r13/census
(1980 Census data by state)
. summarize medage

| Variable | Obs | Mean | Std. Dev. | Min | Max |
| ---: | :---: | :---: | :---: | :---: | :---: |
| medage | 50 | 29.54 | 1.693445 | 24.2 | 34.7 |

```

Also among our variables is pop, the population in each state. Typing summarize medage [ \(\mathrm{w}=\mathrm{pop}\) ] produces population-weighted statistics:
```

. summarize medage [w=pop]
(analytic weights assumed)

| Variable | Obs | Weight | Mean | Std. Dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| medage | 50 | 225907472 | 30.11047 | 1.66933 | 24.2 | 34.7 |

```

The number listed under Weight is the sum of the weighting variable, pop, indicating that there are roughly 226 million people in the United States. The pop-weighted mean of medage is 30.11 (compared with 29.54 for the unweighted statistic), and the weighted standard deviation is 1.67 (compared with 1.69).

\section*{> Example 6: summarize with weights and the detail option}

We can obtain detailed summaries of weighted data as well. When we do this, all the statistics are weighted, including the percentiles.
```

. summarize medage [w=pop], detail
(analytic weights assumed)

```

Median age
\begin{tabular}{rcrlr}
\hline & Percentiles & Smallest & & \\
\(1 \%\) & 27.1 & 24.2 & & \\
\(5 \%\) & 27.7 & 26.1 & & 50 \\
\(10 \%\) & 28.2 & 27.1 & Obs & \\
\(25 \%\) & 29.2 & 27.4 & Sum of Wgt. & 225907472 \\
\(50 \%\) & 29.9 & & Mean & 30.11047 \\
& & Largest & Std. Dev. & 1.66933 \\
\(75 \%\) & 30.9 & 32 & & \\
\(90 \%\) & 32.1 & 32.1 & Variance & 2.786661 \\
\(95 \%\) & 32.2 & 32.2 & Skewness & .5281972 \\
\(99 \%\) & 34.7 & 34.7 & Kurtosis & 4.494223
\end{tabular}

\section*{Technical note}

If you are writing a program and need to access the mean of a variable, the meanonly option provides for fast calls. For example, suppose that your program reads as follows:
```

program mean
summarize '1', meanonly
display " mean = " r(mean)
end

```

The result of executing this is
```

. use http://www.stata-press.com/data/r13/auto2
(1978 Automobile Data)
. mean price
mean = 6165.2568

```

\section*{Video example}

Descriptive statistics in Stata

\section*{Stored results}
summarize stores the following in \(r()\) :
Scalars
\begin{tabular}{|c|c|c|c|}
\hline r(N) & number of observations & r(p50) & 50th percentile (detail only) \\
\hline \(r\) (mean) & mean & r(p75) & 75th percentile (detail only) \\
\hline r(skewness) & skewness (detail only) & r(p90) & 90th percentile (detail only) \\
\hline \(r(\min )\) & minimum & r(p95) & 95th percentile (detail only) \\
\hline \(r\) (max) & maximum & r (p99) & 99th percentile (detail only) \\
\hline r(sum_w) & sum of the weights & r (Var) & variance \\
\hline \(\mathrm{r}(\mathrm{p} 1)\) & 1st percentile (detail only) & r(kurtosis) & kurtosis (detail only) \\
\hline r \({ }^{\text {p5 }}\) ) & 5 th percentile (detail only) & r(sum) & sum of variable \\
\hline r(p10) & 10th percentile (detail only) & r(sd) & standard deviation \\
\hline r(p25) & 25 th percentile (detail only) & & \\
\hline
\end{tabular}

\section*{Methods and formulas}

Let \(x\) denote the variable on which we want to calculate summary statistics, and let \(x_{i}, i=1, \ldots, n\), denote an individual observation on \(x\). Let \(v_{i}\) be the weight, and if no weight is specified, define \(v_{i}=1\) for all \(i\).

Define \(V\) as the sum of the weight:
\[
V=\sum_{i=1}^{n} v_{i}
\]

Define \(w_{i}\) to be \(v_{i}\) normalized to sum to \(n, w_{i}=v_{i}(n / V)\).
The mean, \(\bar{x}\), is defined as
\[
\bar{x}=\frac{1}{n} \sum_{i=1}^{n} w_{i} x_{i}
\]

The variance, \(s^{2}\), is defined as
\[
s^{2}=\frac{1}{n-1} \sum_{i=1}^{n} w_{i}\left(x_{i}-\bar{x}\right)^{2}
\]

The standard deviation, \(s\), is defined as \(\sqrt{s^{2}}\).
Define \(m_{r}\) as the \(r\) th moment about the mean \(\bar{x}\) :
\[
m_{r}=\frac{1}{n} \sum_{i=1}^{n} w_{i}\left(x_{i}-\bar{x}\right)^{r}
\]

The coefficient of skewness is then defined as \(m_{3} m_{2}^{-3 / 2}\). The coefficient of kurtosis is defined as \(m_{4} m_{2}^{-2}\).

Let \(x_{(i)}\) refer to the \(x\) in ascending order, and let \(w_{(i)}\) refer to the corresponding weights of \(x_{(i)}\). The four smallest values are \(x_{(1)}, x_{(2)}, x_{(3)}\), and \(x_{(4)}\). The four largest values are \(x_{(n)}, x_{(n-1)}\), \(x_{(n-2)}\), and \(x_{(n-3)}\).

To obtain the \(p\) th percentile, which we will denote as \(x_{[p]}\), let \(P=n p / 100\). Let
\[
W_{(i)}=\sum_{j=1}^{i} w_{(j)}
\]

Find the first index \(i\) such that \(W_{(i)}>P\). The \(p\) th percentile is then
\[
x_{[p]}= \begin{cases}\frac{x_{(i-1)}+x_{(i)}}{2} & \text { if } W_{(i-1)}=P \\ x_{(i)} & \text { otherwise }\end{cases}
\]

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\section*{Also see}
[R] ameans - Arithmetic, geometric, and harmonic means
\([R]\) centile - Report centile and confidence interval
[R] mean - Estimate means
[R] proportion - Estimate proportions
[R] ratio - Estimate ratios
[R] table - Flexible table of summary statistics
[R] tabstat - Compact table of summary statistics
\([R]\) tabulate, summarize() - One- and two-way tables of summary statistics
[R] total - Estimate totals
[D] codebook - Describe data contents
[D] describe - Describe data in memory or in file
[D] inspect - Display simple summary of data's attributes
[ST] stsum - Summarize survival-time data
[SVY] svy estimation - Estimation commands for survey data
[XT] xtsum - Summarize xt data

\section*{Title}
sunflower - Density-distribution sunflower plots
\begin{tabular}{llll} 
Syntax & Menu & Description & Options \\
Remarks and examples & Acknowledgments & References &
\end{tabular}

\section*{Syntax}
```

sunflower yvar xvar [if] [in] [weight] [, options]

```
options
Description
Main
\begin{tabular}{ll} 
nograph & do not show graph \\
notable & do not show summary table; implied when by () is specified
\end{tabular}
marker_options
Bins/Petals
binwidth(\#) width of the hexagonal bins
binar(\#) aspect ratio of the hexagonal bins
bin_options
light(\#)
affect rendition of hexagonal bins
dark(\#)
xcenter(\#)
ycenter(\#)
petalweight(\#)
petallength(\#)
petal_options
flowersonly
nosinglepetal
minimum observations for a light sunflower; default is light(3)
minimum observations for a dark sunflower; default is dark(13)
\(x\)-coordinate of the reference bin
\(y\)-coordinate of the reference bin
observations in a dark sunflower petal
length of sunflower petal as a percentage
affect rendition of sunflower petals
show petals only; do not render bins
suppress single petals
Add plots
addplot (plot) add other plots to generated graph
Y axis, X axis, Titles, Legend, Overall, By
twoway_options
any options documented in [G-3] twoway_options
bin_options Description
\([\underline{l} \mid \underline{d}]\) bstyle (areastyle) overall look of hexagonal bins
[1] \(]\) bcolor (colorstyle) outline and fill color
\([\underline{1} \mid \underline{d}]\) bfcolor (colorstyle) fill color
[1] d] blstyle(linestyle) overall look of outline
[1] \(\underline{d}]\) blcolor (colorstyle) outline color
[l|d]blwidth(linewidthstyle) thickness of outline
petal_options Description
\([\underline{1} \mid \underline{\mathrm{d}}] \underline{\mathrm{fl}} \mathrm{tyle}\) (linestyle) overall style of sunflower petals
\([\underline{1} \mid \underline{d}] \underline{f l c o l o r}\) (colorstyle) color of sunflower petals
[l|d]flwidth(linewidthstyle) thickness of sunflower petals

All options are rightmost; see [G-4] concept: repeated options.
fweights are allowed; see [U] 11.1.6 weight.

\section*{Menu}

Graphics \(>\) Smoothing and densities \(>\) Density-distribution sunflower plot

\section*{Description}
sunflower draws density-distribution sunflower plots (Plummer and Dupont 2003). These plots are useful for displaying bivariate data whose density is too great for conventional scatterplots to be effective.

A sunflower is several line segments of equal length, called petals, that radiate from a central point. There are two varieties of sunflowers: light and dark. Each petal of a light sunflower represents 1 observation. Each petal of a dark sunflower represents several observations. Dark and light sunflowers represent high- and medium-density regions of the data, and marker symbols represent individual observations in low-density regions.

The plane defined by the variables yvar and xvar is divided into contiguous hexagonal bins. The number of observations contained within a bin determines how the bin will be represented.
- When there are fewer than light(\#) observations in a bin, each point is plotted using the usual marker symbols in a scatterplot.
- Bins with at least light (\#) but fewer than dark(\#) observations are represented by a light sunflower.
- Bins with at least dark(\#) observations are represented by a dark sunflower.

\section*{Options}

Main
nograph prevents the graph from being generated.
notable prevents the summary table from being displayed. This option is implied when the by () option is specified.
marker_options affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] marker_options.

Bins/Petals
binwidth(\#) specifies the horizontal width of the hexagonal bins in the same units as xvar. By default,
\[
\text { binwidth }=\max (\mathrm{rbw}, \mathrm{nbw})
\]
where
\[
\begin{gathered}
\text { rbw }=\text { range of } x v a r / 40 \\
\mathrm{nbw}=\text { range of } x \mathrm{var} / \max (1, \mathrm{nb})
\end{gathered}
\]
and
\[
\mathrm{nb}=\operatorname{int}(\min (\operatorname{sqrt}(n), 10 * \log 10(n)))
\]
where
\(n=\) the number of observations in the dataset
binar(\#) specifies the aspect ratio for the hexagonal bins. The height of the bins is given by
\[
\text { binheight }=\text { binwidth } \times \# \times 2 / \sqrt{3}
\]
where binheight and binwidth are specified in the units of yvar and xvar, respectively. The default is binar ( \(r\) ), where \(r\) results in the rendering of regular hexagons.
bin_options affect how the hexagonal bins are rendered.
lbstyle(areastyle) and dbstyle(areastyle) specify the look of the light and dark hexagonal bins, respectively. The options listed below allow you to change each attribute, but lbstyle() and dbstyle() provide the starting points. See [G-4] areastyle for a list of available area styles.
lbcolor (colorstyle) and dbcolor (colorstyle) specify one color to be used both to outline the shape and to fill the interior of the light and dark hexagonal bins, respectively. See [G-4] colorstyle for a list of color choices.
lbfcolor (colorstyle) and dbfcolor (colorstyle) specify the color to be used to fill the interior of the light and dark hexagonal bins, respectively. See [G-4] colorstyle for a list of color choices.
lblstyle(linestyle) and dblstyle (linestyle) specify the overall style of the line used to outline the area, which includes its pattern (solid, dashed, etc.), thickness, and color. The other options listed below allow you to change the line's attributes, but lblstyle() and dblstyle() are the starting points. See [G-4] linestyle for a list of choices.
lblcolor (colorstyle) and dblcolor(colorstyle) specify the color to be used to outline the light and dark hexagonal bins, respectively. See [G-4] colorstyle for a list of color choices.
lblwidth (linewidthstyle) and dblwidth(linewidthstyle) specify the thickness of the line to be used to outline the light and dark hexagonal bins, respectively. See [G-4] linewidthstyle for a list of choices.
light(\#) specifies the minimum number of observations needed for a bin to be represented by a light sunflower. The default is light(3).
dark(\#) specifies the minimum number of observations needed for a bin to be represented by a dark sunflower. The default is dark(13).
xcenter(\#) and ycenter(\#) specify the center of the reference bin. The default values are the median values of xvar and yvar, respectively. The centers of the other bins are implicitly defined by the location of the reference bin together with the common bin width and height.
petalweight (\#) specifies the number of observations represented by each petal of a dark sunflower. The default value is chosen so that the maximum number of petals on a dark sunflower is 14 .
petallength(\#) specifies the length of petals in the sunflowers. The value specified is interpreted as a percentage of half the bin width. The default is \(100 \%\).
petal_options affect how the sunflower petals are rendered.
lflstyle(linestyle) and dflstyle(linestyle) specify the overall style of the light and dark sunflower petals, respectively.
lflcolor (colorstyle) and dflcolor (colorstyle) specify the color of the light and dark sunflower petals, respectively.
lflwidth(linewidthstyle) and dflwidth(linewidthstyle) specify the width of the light and dark sunflower petals, respectively.
flowersonly suppresses rendering of the bins. This option is equivalent to specifying lbcolor (none) and dbcolor(none).
nosinglepetal suppresses flowers from being drawn in light bins that contain only 1 observation and dark bins that contain as many observations as the petal weight (see the petalweight() option).
addplot (plot) provides a way to add other plots to the generated graph; see [G-3] addplot_option.

Y axis, X axis, Titles, Legend, Overall, By
twoway_options are any of the options documented in [G-3] twoway_options. These include options for titling the graph (see [G-3] title_options), options for saving the graph to disk (see [G-3] saving_option), and the by () option (see [G-3] by_option).

\section*{Remarks and examples}

See Dupont (2009, 87-92) for a discussion of sunflower plots and how to create them using Stata.

\section*{> Example 1}

Using the auto dataset, we want to examine the relationship between weight and mpg. To do that, we type



The three darkly shaded sunflowers immediately catch our eyes, indicating a group of eight cars that are heavy (nearly 4,000 pounds) and fuel inefficient and two groups of cars that get about 20 miles per gallon and weight in the neighborhood of 3,000 pounds, one with 10 cars and one with 8 cars. The lighter sunflowers with seven petals each indicate groups of seven cars that share similar weight and fuel economy characteristics. To obtain the number of cars in each group, we counted the number of petals in each flower and consulted the graph legend to see how many observations each petal represents.

\section*{Acknowledgments}

We thank William D. Dupont and W. Dale Plummer Jr., both of the Department of Biostatistics at Vanderbilt University, who are the authors of the original sunflower command, for their assistance in producing this version.

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\section*{Title}
sureg - Zellner's seemingly unrelated regression
\begin{tabular}{llll} 
Syntax & Menu & Description & Options \\
Remarks and examples & Stored results & Methods and formulas & References \\
Also see & &
\end{tabular}

\section*{Syntax}

Basic syntax
\[
\begin{aligned}
& \text { sureg (depvar } \left.\text { varlist }_{1}\right)\left(\text { depvar }_{2} \text { varlist }_{2}\right) \ldots\left(\text { depvar }_{N} \text { varlist }_{N}\right) \\
& \quad[\text { if }][\text { in }][\text { weight }]
\end{aligned}
\]

Full syntax
sureg ([eqname \(\left.{ }_{1}:\right]\) depvar \(_{1 a}\left[\right.\) depvar \(\left._{1 b} \ldots=\right]\) varlist \(_{1}[\), noconstant \(\left.]\right)\)
\[
\begin{aligned}
& \left(\left[\text { eqname }_{2}:\right] \text { depvar }_{2 a}\left[\text { depvar }_{2 b} \ldots=\right] \text { varlist }_{2}[\text {, noconstant }]\right) \\
& \ldots \\
& \left(\left[\text { eqname }_{N}:\right] \text { depvar }_{N a}\left[\text { depvar }_{N b} \ldots=\right] \text { varlist }_{N}[\text {, noconstant }]\right) \\
& {[\text { if }][\text { in }][\text { weight }][\text {, options }]}
\end{aligned}
\]

Explicit equation naming (eqname:) cannot be combined with multiple dependent variables in an equation specification.
options

\section*{Model}
isure
constraints (constraints)
df adj.
small
dfk
dfk2
Reporting
level(\#)
corr
nocnsreport
display_options

Optimization
optimization_options
noheader
notable
coeflegend

Description
iterate until estimates converge apply specified linear constraints
report small-sample statistics
use small-sample adjustment use alternate adjustment
set confidence level; default is level (95)
perform Breusch-Pagan test
do not display constraints
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
control the optimization process; seldom used
suppress header table from above coefficient table suppress coefficient table display legend instead of statistics
varlist \(_{1}, \ldots\), varlist \(_{N}\) may contain factor variables; see [U] 11.4.3 Factor variables. You must have the same levels of factor variables in all equations that have factor variables.
depvars and the varlists may contain time-series operators; see [U] 11.4.4 Time-series varlists.
bootstrap, by, fp, jackknife, rolling, and statsby are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the bootstrap prefix; see [R] bootstrap.
aweights are not allowed with the jackknife prefix; see \([R]\) jackknife.
aweights and fweights are allowed; see [U] 11.1.6 weight.
noheader, notable, and coeflegend do not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

\section*{Menu}

Statistics \(>\) Linear models and related \(>\) Multiple-equation models \(>\) Seemingly unrelated regression

\section*{Description}
sureg fits seemingly unrelated regression models (Zellner 1962; Zellner and Huang 1962; Zellner 1963). The acronyms SURE and SUR are often used for the estimator.

\section*{Options}

\begin{abstract}
Model
isure specifies that sureg iterate over the estimated disturbance covariance matrix and parameter estimates until the parameter estimates converge. Under seemingly unrelated regression, this iteration converges to the maximum likelihood results. If this option is not specified, sureg produces two-step estimates.
constraints(constraints); see [R] estimation options.
\end{abstract}
df adj.
small specifies that small-sample statistics be computed. It shifts the test statistics from chi-squared and \(z\) statistics to \(F\) statistics and \(t\) statistics. Although the standard errors from each equation are computed using the degrees of freedom for the equation, the degrees of freedom for the \(t\) statistics are all taken to be those for the first equation.
dfk specifies the use of an alternate divisor in computing the covariance matrix for the equation residuals. As an asymptotically justified estimator, sureg by default uses the number of sample observations \((n)\) as a divisor. When the dfk option is set, a small-sample adjustment is made and the divisor is taken to be \(\sqrt{\left(n-k_{i}\right)\left(n-k_{j}\right)}\), where \(k_{i}\) and \(k_{j}\) are the numbers of parameters in equations \(i\) and \(j\), respectively.
dfk 2 specifies the use of an alternate divisor in computing the covariance matrix for the equation residuals. When the dfk2 option is set, the divisor is taken to be the mean of the residual degrees of freedom from the individual equations.

Reporting
level(\#); see [R] estimation options.
corr displays the correlation matrix of the residuals between equations and performs a Breusch-Pagan test for independent equations; that is, the disturbance covariance matrix is diagonal.
nocnsreport; see \([\mathrm{R}]\) estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\%fmt), pformat (\% fint), sformat (\% fint), and nolstretch; see \([\mathrm{R}]\) estimation options.

\section*{Optimization}
optimization_options control the iterative process that minimizes the sum of squared errors when isure is specified. These options are seldom used.
iterate (\#) specifies the maximum number of iterations. When the number of iterations equals \#, the optimizer stops and presents the current results, even if the convergence tolerance has not been reached. The default value of iterate() is the current value of set maxiter (see \([\mathrm{R}]\) maximize), which is iterate (16000) if maxiter has not been changed.
trace adds to the iteration \(\log\) a display of the current parameter vector nolog suppresses the display of the iteration log.
tolerance(\#) specifies the tolerance for the coefficient vector. When the relative change in the coefficient vector from one iteration to the next is less than or equal to \#, the optimization process is stopped. tolerance (1e-6) is the default.

The following options are available with sureg but are not shown in the dialog box:
noheader suppresses display of the table reporting \(F\) statistics, \(R\)-squared, and root mean squared error above the coefficient table.
notable suppresses display of the coefficient table.
coeflegend; see [R] estimation options.

\section*{Remarks and examples}

Seemingly unrelated regression models are so called because they appear to be joint estimates from several regression models, each with its own error term. The regressions are related because the (contemporaneous) errors associated with the dependent variables may be correlated. Chapter 5 of Cameron and Trivedi (2010) contains a discussion of the seemingly unrelated regression model and the feasible generalized least-squares estimator underlying it.

\section*{\(>\) Example 1}

When we fit models with the same set of right-hand-side variables, the seemingly unrelated regression results (in terms of coefficients and standard errors) are the same as fitting the models separately (using, say, regress). The same is true when the models are nested. Even in such cases, sureg is useful when we want to perform joint tests. For instance, let us assume that we think
\[
\begin{aligned}
\text { price } & =\beta_{0}+\beta_{1} \text { foreign }+\beta_{2} \text { length }+u_{1} \\
\text { weight } & =\gamma_{0}+\gamma_{1} \text { foreign }+\gamma_{2} \text { length }+u_{2}
\end{aligned}
\]

Because the models have the same set of explanatory variables, we could estimate the two equations separately. Yet, we might still choose to estimate them with sureg because we want to perform the joint test \(\beta_{1}=\gamma_{1}=0\).

We use the small and dfk options to obtain small-sample statistics comparable with regress or mvreg.
```

. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. sureg (price foreign length) (weight foreign length), small dfk
Seemingly unrelated regression

```
\begin{tabular}{lrrrrrr}
\hline Equation & Obs & Parms & RMSE & "R-sq" & F-Stat & \(P\) \\
\hline price & 74 & 2 & 2474.593 & 0.3154 & 16.35 & 0.0000 \\
weight & 74 & 2 & 250.2515 & 0.8992 & 316.54 & 0.0000 \\
\hline
\end{tabular}
\begin{tabular}{r|rrrrrr}
\hline & Coef. & Std. Err. & t & \(\mathrm{P}>\mid \mathrm{tI}\) & [95\% Conf. Interval] \\
\hline price & & & & & & \\
foreign & 2801.143 & 766.117 & 3.66 & 0.000 & 1286.674 & 4315.611 \\
length & 90.21239 & 15.83368 & 5.70 & 0.000 & 58.91219 & 121.5126 \\
_cons & -11621.35 & 3124.436 & -3.72 & 0.000 & -17797.77 & -5444.93 \\
\hline weight & & & & & & \\
foreign & -133.6775 & 77.47615 & -1.73 & 0.087 & -286.8332 & 19.4782 \\
length & 31.44455 & 1.601234 & 19.64 & 0.000 & 28.27921 & 34.60989 \\
_cons & -2850.25 & 315.9691 & -9.02 & 0.000 & -3474.861 & -2225.639 \\
\hline
\end{tabular}

These two equations have a common set of regressors, and we could have used a shorthand syntax to specify the equations:
```

. sureg (price weight = foreign length), small dfk

```

Here the results presented by sureg are the same as if we had estimated the equations separately:
```

. regress price foreign length
(output omitted)
. regress weight foreign length
(output omitted)

```

There is, however, a difference. We have allowed \(u_{1}\) and \(u_{2}\) to be correlated and have estimated the full variance-covariance matrix of the coefficients. sureg has estimated the correlations, but it does not report them unless we specify the corr option. We did not remember to specify corr when we fit the model, but we can redisplay the results:
```

. sureg, notable noheader corr
Correlation matrix of residuals:
price weight
price 1.0000
weight 0.5840 1.0000
Breusch-Pagan test of independence: chi2(1) = 25.237, Pr = 0.0000

```

The notable and noheader options prevented sureg from redisplaying the header and coefficient tables. We find that, for the same cars, the correlation of the residuals in the price and weight equations is 0.5840 and that we can reject the hypothesis that this correlation is zero.

We can test that the coefficients on foreign are jointly zero in both equations - as we set out to do-by typing test foreign; see \([R]\) test. When we type a variable without specifying the equation, that variable is tested for zero in all equations in which it appears:
```

. test foreign
( 1) [price]foreign = 0
( 2) [weight]foreign = 0
F( 2, 142) = 17.99
Prob > F = 0.0000

```

\section*{> Example 2}

When the models do not have the same set of explanatory variables and are not nested, sureg may lead to more efficient estimates than running the models separately as well as allowing joint tests. This time, let us assume that we believe
\[
\begin{aligned}
\text { price } & =\beta_{0}+\beta_{1} \text { foreign }+\beta_{2} \text { mpg }+\beta_{3} \text { displ }+u_{1} \\
\text { weight } & =\gamma_{0}+\gamma_{1} \text { foreign }+\gamma_{2} \text { length }+u_{2}
\end{aligned}
\]

To fit this model, we type
. sureg (price foreign mpg displ) (weight foreign length), corr
Seemingly unrelated regression
\begin{tabular}{lrrrrrr}
\hline Equation & Obs & Parms & RMSE & "R-sq" & chi2 & P \\
\hline price & 74 & 3 & 2165.321 & 0.4537 & 49.64 & 0.0000 \\
weight & 74 & 2 & 245.2916 & 0.8990 & 661.84 & 0.0000 \\
\hline
\end{tabular}
\begin{tabular}{r|rrrrrr}
\hline & Coef. & Std. Err. & \(z\) & P>|z| & [95\% Conf. Interval] \\
\hline price & & & & & & \\
foreign & 3058.25 & 685.7357 & 4.46 & 0.000 & 1714.233 & 4402.267 \\
mpg & -104.9591 & 58.47209 & -1.80 & 0.073 & -219.5623 & 9.644042 \\
displacement & 18.18098 & 4.286372 & 4.24 & 0.000 & 9.779842 & 26.58211 \\
_cons & 3904.336 & 1966.521 & 1.99 & 0.047 & 50.0263 & 7758.645 \\
\hline weight & & & & & & \\
foreign & -147.3481 & 75.44314 & -1.95 & 0.051 & -295.2139 & .517755 \\
length & 30.94905 & 1.539895 & 20.10 & 0.000 & 27.93091 & 33.96718 \\
_cons & -2753.064 & 303.9336 & -9.06 & 0.000 & -3348.763 & -2157.365 \\
\hline
\end{tabular}

Correlation matrix of residuals:
```

                price weight
    price 1.0000
    weight 0.3285 1.0000
Breusch-Pagan test of independence: chi2(1) $=7.984, \operatorname{Pr}=0.0047$

```

In comparison, if we had fit the price model separately,
. regress price foreign mpg displ
\begin{tabular}{r|crc} 
Source & SS & df & MS \\
\hline Model & 294104790 & 3 & 98034929.9 \\
Residual & 340960606 & 70 & 4870865.81 \\
\hline Total & 635065396 & 73 & 8699525.97
\end{tabular}
\begin{tabular}{lrr} 
Number of obs & \(=\) & 74 \\
F 3, & \(70)\) & \(=\) \\
Prob \(>\) F & \(=\) & 20.13 \\
R-squared & \(=0000\) \\
Adj R-squared & \(=\) & 0.4631 \\
Root MSE & \(=2201\)
\end{tabular}
\begin{tabular}{r|rlccrr}
\hline price & Coef. & Std. Err. & t & P>|t| & [95\% Conf. Interval] \\
\hline foreign & 3545.484 & 712.7763 & 4.97 & 0.000 & 2123.897 & 4967.072 \\
mpg & -98.88559 & 63.17063 & -1.57 & 0.122 & -224.8754 & 27.10426 \\
displacement & 22.40416 & 4.634239 & 4.83 & 0.000 & 13.16146 & 31.64686 \\
_cons & 2796.91 & 2137.873 & 1.31 & 0.195 & -1466.943 & 7060.763 \\
\hline
\end{tabular}

The coefficients are slightly different, but the standard errors are uniformly larger. This would still be true if we specified the dfk option to make a small-sample adjustment to the estimated covariance of the disturbances.

\section*{- Technical note}

Constraints can be applied to SURE models using Stata's standard syntax for constraints. For a general discussion of constraints, see \([R]\) constraint; for examples similar to seemingly unrelated regression models, see [R] reg3.

\section*{Stored results}
sureg stores the following in e():
Scalars
e(N)
e(k)
e(k_eq)
e(mss_\#)
e(df_m\#)
e(rss_\#)
e(df_r)
e(r2_\#)
e(F_\#)
e(rmse_\#)
e(dfk2_adj)
e(ll)
e(chi2_\#)
e(p_\#)
e(cons_\#)
e(chi2_bp)
e(df_bp)
e(cons_\#)
e(rank)
e(ic)
Macros
e(cmd)
\(e\) (cmdline)
e(method)
e(depvar)
e(exog)
e(eqnames)
e(wtype)
e(wexp)
e(corr)
e(small)
e(dfk)
e(properties)
e(predict)
e(marginsok)
e(marginsnotok)
e(asbalanced)
e(asobserved)
Matrices
e(b)
e(Cns)
e(Sigma)
e(V)
Functions
e(sample) marks estimation sample

\section*{Methods and formulas}
sureg uses the asymptotically efficient, feasible, generalized least-squares algorithm described in Greene (2012, 292-304). The computing formulas are given on page 293-294.

The \(R\)-squared reported is the percent of variance explained by the predictors. It may be used for descriptive purposes, but \(R\)-squared is not a well-defined concept when GLS is used.
sureg will refuse to compute the estimators if the same equation is named more than once or the covariance matrix of the residuals is singular.

The Breusch and Pagan (1980) \(\chi^{2}\) statistic—a Lagrange multiplier statistic—is given by
\[
\lambda=T \sum_{m=1}^{M} \sum_{n=1}^{m-1} r_{m n}^{2}
\]
where \(r_{m n}\) is the estimated correlation between the residuals of the \(M\) equations and \(T\) is the number of observations. It is distributed as \(\chi^{2}\) with \(M(M-1) / 2\) degrees of freedom.

Arnold Zellner (1927-2010) was born in New York. He studied physics at Harvard and economics at Berkeley, and then he taught economics at the Universities of Washington and Wisconsin before settling in Chicago in 1966. Among his many major contributions to econometrics and statistics are his work on seemingly unrelated regression, three-stage least squares, and Bayesian econometrics.

\section*{References}

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\section*{Also see}
[R] sureg postestimation - Postestimation tools for sureg
[R] nlsur - Estimation of nonlinear systems of equations
[R] reg3 - Three-stage estimation for systems of simultaneous equations
\([R]\) regress - Linear regression
[MV] mvreg - Multivariate regression
[SEM] example 12 - Seemingly unrelated regression
[SEM] intro 5 - Tour of models
[TS] dfactor - Dynamic-factor models
[U] 20 Estimation and postestimation commands

\section*{Title}
sureg postestimation - Postestimation tools for sureg
Description Syntax for predict Menu for predict Options for predict Remarks and examples

\section*{Description}

The following postestimation commands are available after sureg:
\begin{tabular}{ll} 
Command & Description \\
\hline contrast & contrasts and ANOVA-style joint tests of estimates \\
estat ic & Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) \\
estat summarize & \begin{tabular}{l} 
summary statistics for the estimation sample \\
estat vce
\end{tabular} \\
\begin{tabular}{l} 
variance-covariance matrix of the estimators (VCE) \\
estimates \\
forecast \\
cataging estimation results
\end{tabular} \\
dynamic forecasts and simulations \\
margins & \begin{tabular}{l} 
point estimates, standard errors, testing, and inference for linear \\
combinations of coefficients
\end{tabular} \\
marginal means, predictive margins, marginal effects, and average marginal \\
mlcomsplot & \begin{tabular}{l} 
graph the results from margins (profile plots, interaction plots, etc.) \\
point estimates, standard errors, testing, and inference for nonlinear \\
combinations of coefficients
\end{tabular} \\
predict & \begin{tabular}{l} 
predictions, residuals, influence statistics, and other diagnostic measures \\
predictnl
\end{tabular} \\
point estimates, standard errors, testing, and inference for generalized predictions \\
pwcompare & pairwise comparisons of estimates \\
test & Wald tests of simple and composite linear hypotheses \\
testnl & Wald tests of nonlinear hypotheses
\end{tabular}

\section*{Syntax for predict}
predict [type] newvar [if] [in] [, equation(eqno[,eqno]) statistic]
statistic Description

\section*{Main}
\(\mathrm{xb} \quad\) linear prediction; the default
stdp standard error of the linear prediction
residuals residuals
difference difference between the linear predictions of two equations
stddp standard error of the difference in linear predictions
These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

\section*{Menu for predict}

Statistics \(>\) Postestimation \(>\) Predictions, residuals, etc.

\section*{Options for predict}
\(\qquad\) Main
equation (eqno [,eqno]) specifies to which equation(s) you are referring.
equation() is filled in with one eqno for the xb , stdp, and residuals options. equation(\#1) would mean that the calculation is to be made for the first equation, equation(\#2) would mean the second, and so on. You could also refer to the equations by their names. equation(income) would refer to the equation named income and equation(hours) to the equation named hours. If you do not specify equation(), the results are the same as if you specified equation(\#1).
difference and stddp refer to between-equation concepts. To use these options, you must specify two equations, for example, equation(\#1,\#2) or equation(income,hours). When two equations must be specified, equation() is required.
xb , the default, calculates the linear prediction (fitted values) - the prediction of \(\mathbf{x}_{j} \mathbf{b}\) for the specified equation.
stdp calculates the standard error of the prediction for the specified equation. It can be thought of as the standard error of the predicted expected value or mean for the observation's covariate pattern. The standard error of the prediction is also referred to as the standard error of the fitted value.
residuals calculates the residuals.
difference calculates the difference between the linear predictions of two equations in the system. With equation(\#1,\#2), difference computes the prediction of equation(\#1) minus the prediction of equation(\#2).
stddp is allowed only after you have previously fit a multiple-equation model. The standard error of the difference in linear predictions ( \(\mathbf{x}_{1 j} \mathbf{b}-\mathbf{x}_{2 j} \mathbf{b}\) ) between equations 1 and 2 is calculated.
For more information on using predict after multiple-equation estimation commands, see \([\mathrm{R}]\) predict.

\section*{Remarks and examples}

For an example of cross-equation testing of parameters using the test command, see example 1 in \([R]\) sureg.

\section*{\(>\) Example 1}

In example 1 of \([R]\) sureg, we fit a seemingly unrelated regressions model of price and weight. Here we obtain the fitted values.
```

. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. sureg (price foreign length) (weight foreign length), small dfk
(output omitted)
. predict phat, equation(price)
(option xb assumed; fitted values)
. predict what, equation(weight)
(option xb assumed; fitted values)

```
. summarize price phat weight what
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline price & 74 & 6165.257 & 2949.496 & 3291 & 15906 \\
phat & 74 & 6165.257 & 1656.407 & 1639.872 & 9398.138 \\
weight & 74 & 3019.459 & 777.1936 & 1760 & 4840 \\
what & 74 & 3019.459 & 736.9666 & 1481.199 & 4476.331
\end{tabular}

Just as in single-equation OLS regression, in a SURE model the sample mean of the fitted values for an equation equals the sample mean of the dependent variable.

\section*{> Example 2}

Suppose that for whatever reason we were interested in the difference between the predicted values of price and weight. predict has an option to compute this difference in one step:
```

. predict diff, equation(price, weight) difference

```
diff is the same as phat - what:
- generate mydiff = phat - what
. summarize diff mydiff
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline diff & 74 & 3145.797 & 1233.26 & -132.2275 & 5505.914 \\
mydiff & 74 & 3145.797 & 1233.26 & -132.2275 & 5505.914
\end{tabular}

\section*{Also see}
[R] sureg - Zellner's seemingly unrelated regression
[U] 20 Estimation and postestimation commands

\section*{Title}
swilk - Shapiro-Wilk and Shapiro-Francia tests for normality
\begin{tabular}{llll} 
Syntax & Menu & Description & Options for swilk \\
Options for sfrancia & Remarks and examples & Stored results & Methods and formulas \\
Acknowledgment & References & Also see &
\end{tabular}

\section*{Syntax}

Shapiro-Wilk normality test
swilk varlist [if] [in] [, swilk_options]

Shapiro-Francia normality test sfrancia varlist [if] [in][, sfrancia_options]
swilk_options
Description
Main
generate (newvar) create newvar containing \(W\) test coefficients
Innormal test for three-parameter lognormality
noties do not use average ranks for tied values
sfrancia_options
Description
Main
boxcox use the Box-Cox transformation for \(W^{\prime}\); the default is to use the log transformation
noties
do not use average ranks for tied values
by is allowed with swilk and sfrancia; see [D] by.

\section*{Menu}

\section*{swilk}

Statistics \(>\) Summaries, tables, and tests \(>\) Distributional plots and tests \(>\) Shapiro-Wilk normality test
sfrancia
Statistics \(>\) Summaries, tables, and tests \(>\) Distributional plots and tests \(>\) Shapiro-Francia normality test

\section*{Description}
swilk performs the Shapiro-Wilk \(W\) test for normality, and sfrancia performs the Shapiro-Francia \(W^{\prime}\) test for normality. swilk can be used with \(4 \leq n \leq 2000\) observations, and sfrancia can be used with \(5 \leq n \leq 5000\) observations; see \([\mathrm{R}]\) sktest for a test allowing more observations. See [MV] mvtest normality for multivariate tests of normality.

\section*{Options for swilk}
\(\qquad\) Main
generate (newvar) creates new variable newvar containing the \(W\) test coefficients.
Innormal specifies that the test be for three-parameter lognormality, meaning that \(\ln (X-k)\) is tested for normality, where \(k\) is calculated from the data as the value that makes the skewness coefficient zero. When simply testing \(\ln (X)\) for normality, do not specify this option. See [R] Inskew0 for estimation of \(k\).
noties suppresses use of averaged ranks for tied values when calculating the \(W\) test coefficients.

\section*{Options for sfrancia}

Main
boxcox specifies that the Box-Cox transformation of Royston (1983) for calculating \(W^{\prime}\) test coefficients be used instead of the default log transformation (Royston 1993a). Under the Box-Cox transformation, the normal approximation to the sampling distribution of \(W^{\prime}\), used by sfrancia, is valid for \(5 \leq n \leq 1000\). Under the log transformation, it is valid for \(10 \leq n \leq 5000\).
noties suppresses use of averaged ranks for tied values when calculating the \(W^{\prime}\) test coefficients.

\section*{Remarks and examples}
> Example 1
Using our automobile dataset, we will test whether the variables mpg and trunk are normally distributed:
```

. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. swilk mpg trunk

```
            Shapiro-Wilk W test for normal data
\begin{tabular}{r|ccccc} 
Variable & Obs & W & V & z & Prob>z \\
\hline mpg & 74 & 0.94821 & 3.335 & 2.627 & 0.00430 \\
trunk & 74 & 0.97921 & 1.339 & 0.637 & 0.26215
\end{tabular}
. sfrancia mpg trunk
                            Shapiro-Francia \(W\) ' test for normal data
\begin{tabular}{r|rcccc} 
Variable & Obs & W, & V' & z & Prob>z \\
\hline mpg & 74 & 0.94872 & 3.650 & 2.510 & 0.00604 \\
trunk & 74 & 0.98446 & 1.106 & 0.195 & 0.42271
\end{tabular}

We can reject the hypothesis that mpg is normally distributed, but we cannot reject that trunk is normally distributed.

The values reported under \(W\) and \(W^{\prime}\) are the Shapiro-Wilk and Shapiro-Francia test statistics. The tests also report \(V\) and \(V^{\prime}\), which are more appealing indexes for departure from normality. The median values of \(V\) and \(V^{\prime}\) are 1 for samples from normal populations. Large values indicate nonnormality. The \(95 \%\) critical values of \(V\left(V^{\prime}\right)\), which depend on the sample size, are between 1.2 and 2.4 (2.0 and 2.8); see Royston (1991b). There is no more information in \(V\left(V^{\prime}\right)\) than in \(W\) ( \(W^{\prime}\) )—one is just the transform of the other.

\section*{> Example 2}

We have data on a variable called studytime, which we suspect is distributed lognormally:
```

. use http://www.stata-press.com/data/r13/cancer
(Patient Survival in Drug Trial)
. generate lnstudytime = ln(studytime)
. swilk lnstudytime
Shapiro-Wilk W test for normal data

| Variable | Obs | W | V | z | Prob>z |
| :---: | :---: | :---: | :---: | :---: | :---: |
| lnstudytime | 48 | 0.92731 | 3.311 | 2.547 | 0.00543 |

```

We can reject the lognormal assumption. We do not specify the lnnormal option when testing for lognormality. The lnnormal option is for three-parameter lognormality.

\section*{> Example 3}

Having discovered that \(\ln\) (studytime) is not distributed normally, we now test that \(\ln (\) studytime \(-k)\) is normally distributed, where \(k\) is chosen so that the resulting skewness is zero. We obtain the estimate for \(k\) from lnskew0; see [R] Inskew0:
. InskewO lnstudytimek = studytime, level(95)
\begin{tabular}{r|rcrr} 
Transform & k & [95\% Conf. Interval] & Skewness \\
\hline \(\ln\) (studytim-k) & -11.01181 & -infinity & -.9477328 & -.0000173
\end{tabular}
. swilk lnstudytimek, lnnormal
Shapiro-Wilk W test for 3-parameter lognormal data
\begin{tabular}{c|ccccc} 
Variable & Obs & W & V & z & Prob>z \\
\hline lnstudytimek & 48 & 0.97064 & 1.337 & 1.261 & 0.10363
\end{tabular}

We cannot reject the hypothesis that \(\ln (\) studytime +11.01181\()\) is distributed normally. We do specify the lnnormal option when using an estimated value of \(k\).

\section*{Stored results}
swilk and sfrancia store the following in \(r()\) :
Scalars
\begin{tabular}{lll}
\(\mathrm{r}(\mathrm{N})\) & number of observations & \(\mathrm{r}(\mathrm{W})\) \\
\(\mathrm{r}(\mathrm{p})\) & significance & \(\mathrm{r}(\mathrm{V})\)
\end{tabular}

\section*{Methods and formulas}

The Shapiro-Wilk test is based on Shapiro and Wilk (1965) with a new approximation accurate for \(4 \leq n \leq 2000\) (Royston 1992). The calculations made by swilk are based on Royston (1982, 1992, 1993b).

The Shapiro-Francia test (Shapiro and Francia 1972; Royston 1983; Royston 1993a) is an approximate test that is similar to the Shapiro-Wilk test for very large samples.

Samuel Sanford Shapiro (1930- ) earned degrees in statistics and engineering from City College of New York, Columbia, and Rutgers. After employment in the U.S. Army and industry, he joined the faculty at Florida International University in 1972. Shapiro has coauthored various texts in statistics and published several papers on distributional testing and other statistical topics.

\section*{Acknowledgment}
swilk and sfrancia were written by Patrick Royston of the MRC Clinical Trials Unit, London and coauthor of the Stata Press book Flexible Parametric Survival Analysis Using Stata: Beyond the Cox Model.

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Shapiro, S. S., and R. S. Francia. 1972. An approximate analysis of variance test for normality. Journal of the American Statistical Association 67: 215-216.

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\section*{Also see}
[R] Inskew0 - Find zero-skewness log or Box-Cox transform
[R] lv - Letter-value displays
[R] sktest - Skewness and kurtosis test for normality
[MV] mvtest normality - Multivariate normality tests

\section*{Title}
symmetry - Symmetry and marginal homogeneity tests
\begin{tabular}{llll} 
Syntax & Menu & Description & Options \\
Remarks and examples & Stored results & Methods and formulas & References \\
Also see & &
\end{tabular}

\section*{Syntax}

Symmetry and marginal homogeneity tests symmetry casevar controlvar [if] [in] [weight] [, options]

Immediate form of symmetry and marginal homogeneity tests
\[
\text { symmi } \#_{11} \#_{12}[\ldots] \backslash \#_{21} \#_{22}[\ldots][\backslash \ldots][\text { if }][\text { in }][\text {, options }]
\]
options

\section*{Main}
notable
contrib
exact
mh
trend
cc

Description
suppress output of contingency table report contribution of each off-diagonal cell pair perform exact test of table symmetry perform two marginal homogeneity tests perform a test for linear trend in the (log) relative risk (RR) use continuity correction when calculating test for linear trend
fweights are allowed; see [U] 11.1.6 weight.

\section*{Menu}

\section*{symmetry}

Statistics \(>\) Epidemiology and related \(>\) Other \(>\) Symmetry and marginal homogeneity test

\section*{symmi}

Statistics \(>\) Epidemiology and related \(>\) Other \(>\) Symmetry and marginal homogeneity test calculator

\section*{Description}
symmetry performs asymptotic symmetry and marginal homogeneity tests, as well as an exact symmetry test on \(K \times K\) tables where there is a 1 -to-1 matching of cases and controls (nonindependence). This testing is used to analyze matched-pair case-control data with multiple discrete levels of the exposure (outcome) variable. In genetics, the test is known as the transmission/disequilibrium test (TDT) and is used to test the association between transmitted and nontransmitted parental marker alleles to an affected child (Spieldman, McGinnis, and Ewens 1993). For \(2 \times 2\) tables, the asymptotic test statistics reduce to the McNemar test statistic, and the exact symmetry test produces an exact McNemar test; see [ST] epitab. For many exposure variables, symmetry can optionally perform a test for linear trend in the \(\log\) relative risk.
symmetry expects the data to be in the wide format; that is, each observation contains the matched case and control values in variables casevar and controlvar. Variables can be numeric or string.
symmi is the immediate form of symmetry. The symmi command uses the values specified on the command line; rows are separated by ' \(\backslash\) ', and options are the same as for symmetry. See [U] 19 Immediate commands for a general introduction to immediate commands.

\section*{Options}
\(\qquad\)
notable suppresses the output of the contingency table. By default, symmetry displays the \(n \times n\) contingency table at the top of the output.
contrib reports the contribution of each off-diagonal cell pair to the overall symmetry \(\chi^{2}\).
exact performs an exact test of table symmetry. This option is recommended for sparse tables. CAUTION: The exact test requires substantial amounts of time and memory for large tables.
mh performs two marginal homogeneity tests that do not require the inversion of the variance-covariance matrix.

By default, symmetry produces the Stuart-Maxwell test statistic, which requires the inversion of the nondiagonal variance-covariance matrix, \(\mathbf{V}\). When the table is sparse, the matrix may not be of full rank, and then the command substitutes a generalized inverse \(\mathbf{V}^{*}\) for \(\mathbf{V}^{-1}\). mh calculates optional marginal homogeneity statistics that do not require the inversion of the variance-covariance matrix. These tests may be preferred in certain situations. See Methods and formulas and Bickeböller and Clerget-Darpoux (1995) for details on these test statistics.
trend performs a test for linear trend in the (log) relative risk (RR). This option is allowed only for numeric exposure (outcome) variables, and its use should be restricted to measurements on the ordinal or the interval scales.
cc specifies that the continuity correction be used when calculating the test for linear trend. This correction should be specified only when the levels of the exposure variable are equally spaced.

\section*{Remarks and examples}
symmetry and symmi may be used to analyze 1 -to- 1 matched case-control data with multiple discrete levels of the exposure (outcome) variable.
\(>\) Example 1
Consider a survey of 344 individuals (BMDP 1990, 267-270) who were asked in October 1986 whether they agreed with President Reagan's handling of foreign affairs. In January 1987, after the Iran-Contra affair became public, these same individuals were surveyed again and asked the same question. We would like to know if public opinion changed over this period.

We first describe the dataset and list a few observations.
\begin{tabular}{|c|c|c|c|c|}
\hline ```
Contains data
    obs:
    vars:
    size:
``` & \multicolumn{4}{|l|}{```
from http://www.stata-press.com/data/r13/iran.dta
    344
    2 688 29 Jan 2013 02:37
```} \\
\hline variable name & storage type & \begin{tabular}{l}
display \\
format
\end{tabular} & \begin{tabular}{l}
value \\
label
\end{tabular} & variable label \\
\hline before & byte & \(\% 8.0 \mathrm{~g}\) & vlab & Public Opinion before IC \\
\hline after & byte & \%8.0g & vlab & Public Opinion after IC \\
\hline
\end{tabular}

Sorted by:
. list in \(1 / 5\)
\begin{tabular}{|rr|}
\hline before & after \\
\hline agree & agree \\
agree & disagree \\
agree & unsure \\
disagree & agree \\
disagree & disagree \\
\hline
\end{tabular}

Each observation corresponds to one of the 344 individuals. The data are in wide form so that each observation has a before and an after measurement. We now perform the test without options.
. symmetry before after
\begin{tabular}{|c|c|c|c|c|}
\hline Public Opinion before IC & \multicolumn{3}{|l|}{Public Opinion after IC
agree disagree unsure} & Total \\
\hline agree & 47 & 56 & 38 & 141 \\
\hline disagree & 28 & 61 & 31 & 120 \\
\hline unsure & 26 & 47 & 10 & 83 \\
\hline Total & 101 & 164 & 79 & 344 \\
\hline
\end{tabular}
\begin{tabular}{l|ccr} 
& chi2 & df & Prob>chi2 \\
\hline Symmetry (asymptotic) (Stuart-Maxwell) & 14.87 & 3 & 0.0019 \\
Marginal homogeneity (Stua \\
\hline
\end{tabular}

The test first tabulates the data in a \(K \times K\) table and then performs Bowker's (1948) test for table symmetry and the Stuart-Maxwell (Stuart 1955; Maxwell 1970) test for marginal homogeneity.

Both the symmetry test and the marginal homogeneity test are highly significant, thus indicating a shift in public opinion.

An exact test of symmetry is provided for use on sparse tables. This test is computationally intensive, so it should not be used on large tables. Because we are working on a fast computer, we will run the symmetry test again and this time include the exact option. We will suppress the output of the contingency table by specifying notable and include the contrib option so that we may further examine the cells responsible for the significant result.
. symmetry before after, contrib exact mh notable
\begin{tabular}{c} 
Contribution \\
to symmetry \\
chi-squared
\end{tabular}
Cells

The largest contribution to the symmetry \(\chi^{2}\) is due to cells \(n_{12}\) and \(n_{21}\). These correspond to changes between the agree and disagree categories. Of the 344 individuals, 56 ( \(16.3 \%\) ) changed from the agree to the disagree response, whereas only \(28(8.1 \%)\) changed in the opposite direction.

For these data, the results from the exact test are similar to those from the asymptotic test.

\section*{Example 2}

Breslow and Day (1980, 163) reprinted data from Mack et al. (1976) from a case-control study of the effect of exogenous estrogen on the risk of endometrial cancer. The data consist of 59 elderly women diagnosed with endometrial cancer and 59 disease-free control subjects living in the same community as the cases. Cases and controls were matched on age, marital status, and time living in the community. The data collected included information on the daily dose of conjugated estrogen therapy. Breslow and Day analyzed these data by creating four levels of the dose variable. Here are the data as entered into a Stata dataset:
. use http://www.stata-press.com/da
. list, noobs divider
\begin{tabular}{|r|r|r|}
\hline case & control & count \\
\hline 0 & 0 & 6 \\
0 & \(0.1-0.299\) & 2 \\
0 & \(0.3-0.625\) & 3 \\
0 & \(0.626+\) & 1 \\
\(0.1-0.299\) & 0 & 9 \\
\hline \(0.1-0.299\) & \(0.1-0.299\) & 4 \\
\(0.1-0.299\) & \(0.3-0.625\) & 2 \\
\(0.1-0.299\) & \(0.626+\) & 1 \\
\(0.3-0.625\) & 0 & 9 \\
\(0.3-0.625\) & \(0.1-0.299\) & 2 \\
\hline \(0.3-0.625\) & \(0.3-0.625\) & 3 \\
\(0.3-0.625\) & \(0.626+\) & 1 \\
\(0.626+\) & 0 & 12 \\
\(0.626+\) & \(0.1-0.299\) & 1 \\
\(0.626+\) & \(0.3-0.625\) & 2 \\
\hline \(0.626+\) & \(0.626+\) & 1 \\
\hline
\end{tabular}

This dataset is in a different format from that of the previous example. Instead of each observation representing one matched pair, each observation represents possibly multiple pairs indicated by the count variable. For instance, the first observation corresponds to six matched pairs where neither the case nor the control was on estrogen, the second observation corresponds to two matched pairs where the case was not on estrogen and the control was on 0.1 to \(0.299 \mathrm{mg} /\) day, etc.

To use symmetry to analyze this dataset, we must specify fweight to indicate that in our data there are observations corresponding to more than one matched pair.
. symmetry case control [fweight=count]
\begin{tabular}{r|ccccc}
\hline case & 0 & \(0.1-0.299\) & \begin{tabular}{c} 
control \\
\(0.3-0.625\)
\end{tabular} & \(0.626+\) & Total \\
\hline 0 & 6 & 2 & 3 & 1 & 12 \\
\(0.1-0.299\) & 9 & 4 & 2 & 1 & 16 \\
\(0.3-0.625\) & 9 & 2 & 3 & 1 & 15 \\
\(0.626+\) & 12 & 1 & 2 & 1 & 16 \\
Total & 36 & 9 & 10 & 4 & 59 \\
\hline
\end{tabular}
\begin{tabular}{l|rrr} 
& chi2 & df & Prob>chi2 \\
\hline Symmetry (asymptotic) (Stuart-Maxwell) & 17.10 & 6 & 0.0089 \\
Marginal homogeneity (Sta6 & 16.96 & 3 & 0.0007 \\
\hline
\end{tabular}

Both the test of symmetry and the test of marginal homogeneity are highly significant, thus leading us to reject the null hypothesis that there is no effect of exposure to estrogen on the risk of endometrial cancer.

Breslow and Day perform a test for trend assuming that the estrogen exposure levels were equally spaced by recoding the exposure levels as \(1,2,3\), and 4.

We can easily reproduce their results by recoding our data in this way and by specifying the trend option. Two new numeric variables were created, ca and co, corresponding to the variables case and control, respectively. Below we list some of the data and our results from symmetry:
```

. encode case, gen(ca)
. encode control, gen(co)
. label values ca
. label values co
. list in 1/4

```
\begin{tabular}{l|rrrrr|}
\cline { 2 - 6 } & case & control & count & ca & co \\
\hline & 0 & 0 & 6 & 1 & 1 \\
1. & 0 & \(0.1-0.299\) & 2 & 1 & 2 \\
3. & 0 & \(0.3-0.625\) & 3 & 1 & 3 \\
4. & 0 & \(0.626+\) & 1 & 1 & 4 \\
\hline
\end{tabular}
. symmetry ca co [fw=count], notable trend cc
chi2 df Prob>chi2
\begin{tabular}{l|lll}
\hline Symmetry (asymptotic) & 17.10 & 6 & 0.0089 \\
Marginal homogeneity (Stuart-Maxwell) & 16.96 & 3 & 0.0007 \\
\hline Linear trend in the (log) RR & 14.43 & 1 & 0.0001 \\
\hline
\end{tabular}

We requested the continuity correction by specifying cc. Doing so is appropriate because our coded exposure levels are equally spaced.

The test for trend was highly significant, indicating an increased risk of endometrial cancer with increased dosage of conjugated estrogen.

You must be cautious: the way in which you code the exposure variable affects the linear trend statistic. If instead of coding the levels as \(1,2,3\), and 4 , we had instead used \(0,0.2,0.46\), and 0.7 (roughly the midpoint in the range of each level), we would have obtained a \(\chi^{2}\) statistic of 11.19 for these data.

\section*{Stored results}
symmetry stores the following in r() :
\begin{tabular}{ll} 
Scalars & \\
\(r(\) N_pair \()\) & number of matched pairs \\
\(r(c h i 2)\) & asymptotic symmetry \(\chi^{2}\) \\
\(r(d f)\) & asymptotic symmetry degrees of freedom \\
\(r(p)\) & asymptotic symmetry \(p\)-value \\
\(r(\) chi2_sm \()\) & MH (Stuart-Maxwell) \(\chi^{2}\) \\
\(r\left(d f \_s m\right)\) & MH (Stuart-Maxwell) degrees of freedom \\
\(r\left(p \_s m\right)\) & MH (Stuart-Maxwell) \(p\)-value \\
\(r\left(c h i 2 \_b\right)\) & MH (Bickenböller) \(\chi^{2}\) \\
\(r\left(d f \_b\right)\) & MH (Bickenböller) degrees of freedom \\
\(r\left(p \_b\right)\) & MH (Bickenböller) \(p\)-value \\
\(r\left(c h i 2 \_n d\right)\) & MH (no diagonals) \(\chi^{2}\) \\
\(r\left(d f \_n d\right)\) & MH (no diagonals) degrees of freedom \\
\(r\left(p \_n d\right)\) & MH (no diagonals) \(p\)-value \\
\(r(c h i 2-t)\) & \(\chi^{2}\) for linear trend \\
\(r\left(p \_t r e n d\right)\) & \(p\)-value for linear trend \\
\(r\left(p \_e x a c t\right)\) & exact symmetry \(p\)-value
\end{tabular}

\section*{Methods and formulas}

Methods and formulas are presented under the following headings:
Asymptotic tests
Exact symmetry test

\section*{Asymptotic tests}

Consider a square table with \(K\) exposure categories, that is, \(K\) rows and \(K\) columns. Let \(n_{i j}\) be the count corresponding to row \(i\) and column \(j\) of the table, \(N_{i j}=n_{i j}+n_{j i}\), for \(i, j=1,2, \ldots, K\), and \(n_{i .}\), and let \(n_{. j}\) be the marginal totals for row \(i\) and column \(j\), respectively. Asymptotic tests for symmetry and marginal homogeneity for this \(K \times K\) table are calculated as follows:

The null hypothesis of complete symmetry \(p_{i j}=p_{j i}, i \neq j\), is tested by calculating the test statistic (Bowker 1948)
\[
T_{\mathrm{cs}}=\sum_{i<j} \frac{\left(n_{i j}-n_{j i}\right)^{2}}{n_{i j}+n_{j i}}
\]
which is asymptotically distributed as \(\chi^{2}\) with \(K(K-1) / 2-R\) degrees of freedom, where \(R\) is the number of off-diagonal cells with \(N_{i j}=0\).

The null hypothesis of marginal homogeneity, \(p_{i .}=p_{. i}\), is tested by calculating the Stuart-Maxwell test statistic (Stuart 1955; Maxwell 1970),
\[
T_{\mathrm{sm}}=\mathbf{d}^{\prime} \mathbf{V}^{-1} \mathbf{d}
\]
where \(\mathbf{d}\) is a column vector with elements equal to the differences \(d_{i}=n_{i .}-n_{. i}\) for \(i=1,2, \ldots, K\), and \(\mathbf{V}\) is the variance-covariance matrix with elements
\[
\begin{aligned}
v_{i i} & =n_{i .}+n_{. i}-2 n_{i i} \\
v_{i j} & =-\left(n_{i j}+n_{j i}\right), \quad i \neq j
\end{aligned}
\]
\(T_{\mathrm{sm}}\) is asymptotically \(\chi^{2}\) with \(K-1\) degrees of freedom.
This test statistic properly accounts for the dependence between the table's rows and columns. When the matrix \(\mathbf{V}\) is not of full rank, a generalized inverse \(\mathbf{V}^{*}\) is substituted for \(\mathbf{V}^{-1}\).

The Bickeböller and Clerget-Darpoux (1995) marginal homogeneity test statistic is calculated by
\[
T_{\mathrm{mh}}=\sum_{i} \frac{\left(n_{i .}-n_{. i}\right)^{2}}{n_{i .}+n_{. i}}
\]

This statistic is asymptotically distributed, under the assumption of marginal independence, as \(\chi^{2}\) with \(K-1\) degrees of freedom.

The marginal homogeneity (no diagonals) test statistic \(T_{\mathrm{mh}}^{0}\) is calculated in the same way as \(T_{\mathrm{mh}}\), except that the diagonal elements do not enter into the calculation of the marginal totals. Unlike the previous test statistic, \(T_{\mathrm{mh}}^{0}\) reduces to a McNemar test statistic for \(2 \times 2\) tables. The test statistic \(\{(K-1) / 2\} T_{\mathrm{mh}}^{0}\) is asymptotically distributed as \(\chi^{2}\) with \(K-1\) degrees of freedom (Cleves, Olson, and Jacobs 1997; Spieldman and Ewens 1996).

Breslow and Day's test statistic for linear trend in the (log) of RR is
\[
\frac{\left\{\sum_{i<j}\left(n_{i j}-n_{j i}\right)\left(X_{j}-X_{i}\right)-c c\right\}^{2}}{\sum_{i<j}\left(n_{i j}+n_{j i}\right)\left(X_{j}-X_{i}\right)^{2}}
\]
where the \(X_{j}\) are the doses associated with the various levels of exposure and \(c c\) is the continuity correction; it is asymptotically distributed as \(\chi^{2}\) with 1 degree of freedom.

The continuity correction option is applicable only when the levels of the exposure variable are equally spaced.

\section*{Exact symmetry test}

The exact test is based on a permutation algorithm applied to the null distribution. The distribution of the off-diagonal elements \(n_{i j}, i \neq j\), conditional on the sum of the complementary off-diagonal cells, \(N_{i j}=n_{i j}+n_{j i}\), can be written as the product of \(K(K-1) / 2\) binomial random variables,
\[
P(\mathbf{n})=\prod_{i<j}\binom{N_{i j}}{n_{i j}} \pi_{i j}^{n_{i j}}\left(1-\pi_{i j}\right)^{n_{i j}}
\]
where \(\mathbf{n}\) is a vector with elements \(n_{i j}\) and \(\pi_{i j}=E\left(n_{i j} / N_{i j} \mid N_{i j}\right)\). Under the null hypothesis of complete symmetry, \(\pi_{i j}=\pi_{j i}=1 / 2\), and thus the permutation distribution is given by
\[
P_{0}(\mathbf{n})=\prod_{i<j}\binom{N_{i j}}{n_{i j}}\left(\frac{1}{2}\right)^{N_{i j}}
\]

The exact significance test is performed by evaluating
\[
P_{\mathrm{cs}}=\sum_{n \in p} P_{0}(\mathbf{n})
\]
where \(p=\left\{n: P_{0}(\mathbf{n})<P_{0}\left(\mathbf{n}^{*}\right)\right\}\) and \(\mathbf{n}^{*}\) is the observed contingency table data vector. The algorithm evaluates \(p_{\text {cs }}\) exactly. For information about permutation tests, see Good (2005, 2006).

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\section*{Also see}
[ST] epitab - Tables for epidemiologists

\section*{Title}
table - Flexible table of summary statistics
\begin{tabular}{llll} 
Syntax & Menu & Description & Options \\
Remarks and examples & Methods and formulas & Also see &
\end{tabular}

\section*{Syntax}
table rowvar \([\) colvar \([\) supercolvar \(]][\) if \(][\) in \(][\) weight \(][\), options \(]\)
options
Main
contents(clist)
by (superrowvarlist)
Options
cellwidth(\#)
csepwidth(\#)
stubwidth(\#)
scsepwidth (\#)
center
left
cw
row
column
scolumn
concise
missing
replace
name (string)
format ( \(\%\) fmt)

Description
contents of table cells; select up to five statistics; default is contents (freq)
superrow variables
cell width
column-separation width
stub width
supercolumn-separation width
center-align table cells; default is right-align
left-align table cells; default is right-align
perform casewise deletion
add row totals
add column totals
add supercolumn totals
suppress rows with all missing entries
show missing statistics with period
replace current data with table statistics
name new variables with prefix string
display format for numbers in cells; default is format ( \(\% 9.0 \mathrm{~g}\) )
by is allowed; see [D] by.
fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight. pweights may not be used with sd, semean, sebinomial, or sepoisson. iweights may not be used with semean, sebinomial, or sepoisson.
where the elements of clist may be
\begin{tabular}{|c|c|c|c|}
\hline freq & frequency & n varname & same as count \\
\hline mean varname & mean of varname & ax varname & maximum \\
\hline sd varname & standard deviation & min varname & minimum \\
\hline semean varname & standard error of the mean & median varname & median \\
\hline & (sd/sqrt(n)) & p1 varname & 1st percentile \\
\hline sebinomial varname & standard error of the mean, binomial distribution \((\operatorname{sqrt}(p(1-p) / n))\) & p2 varname & 2nd percentile 3rd-49th percentiles \\
\hline sepoisson varname & standard error of the mean, Poisson distribution (sqrt (mean)) & p50 varname & 50th percentile (median) 51st-97th percentiles \\
\hline sum varname & m & p98 varname & 98th percentile \\
\hline rawsum varname & sums ignoring optionally specified weight & p99 varname & 99th percentile \\
\hline count varname & count of nonmissing observations & iqr varname & interquartile range \\
\hline
\end{tabular}

Rows, columns, supercolumns, and superrows are thus defined as

\begin{tabular}{c|cccc} 
& \multicolumn{2}{|c}{ supercol 1 } & \multicolumn{2}{c}{ supercol 2 } \\
& col 1 & col 2 & col 1 & col 2 \\
\hline row 1 &. &. &. &. \\
row 2 &. &. &. &. \\
\hline
\end{tabular}
\begin{tabular}{c|cc} 
& \(\operatorname{col} 1\) & \(\operatorname{col} 2\) \\
\hline row 1 &. &. \\
row 2 &. &. \\
\hline
\end{tabular}
\begin{tabular}{c|cccc} 
& \multicolumn{2}{|c}{ supercol 1 } & \multicolumn{2}{c}{ supercol 2 } \\
& col 1 & col 2 & col 1 & col 2 \\
\hline superrow 1: & & & & \\
row 1 & \(\cdot\) & \(\cdot\) & \(\cdot\) & \(\cdot\) \\
row 2 & \(\cdot\) & \(\cdot\) & \(\cdot\) & \(\cdot\) \\
superrow 2: & & & & \\
row 1 & \(\cdot\) & \(\cdot\) & \(\cdot\) & \(\cdot\) \\
row 2 & \(\cdot\) & \(\cdot\) & \(\cdot\) & \(\cdot\) \\
\hline
\end{tabular}

\section*{Menu}

Statistics \(>\) Summaries, tables, and tests \(>\) Other tables \(>\) Flexible table of summary statistics

\section*{Description}
table calculates and displays tables of statistics.

\section*{Options}
\(\qquad\) Main
contents (clist) specifies the contents of the table's cells; if not specified, contents (freq) is used by default. contents (freq) produces a table of frequencies. contents (mean mpg) produces a table of the means of variable mpg . contents (freq mean mpg sd mpg ) produces a table of frequencies together with the mean and standard deviation of variable mpg . Up to five statistics may be specified.
by (superrowvarlist) specifies that numeric or string variables be treated as superrows. Up to four variables may be specified in superrowvarlist. The by() option may be specified with the by prefix.
cellwidth (\#) specifies the width of the cell in units of digit widths; 10 means the space occupied by 10 digits, which is 0123456789 . The default cellwidth() is not a fixed number, but a number chosen by table to spread the table out while presenting a reasonable number of columns across the page.
csepwidth(\#) specifies the separation between columns in units of digit widths. The default is not a fixed number, but a number chosen by table according to what it thinks looks best.
stubwidth(\#) specifies the width, in units of digit widths, to be allocated to the left stub of the table. The default is not a fixed number, but a number chosen by table according to what it thinks looks best.
scsepwidth(\#) specifies the separation between supercolumns in units of digit widths. The default is not a fixed number, but a number chosen by table to present the results best.
center specifies that results be centered in the table's cells. The default is to right-align results. For centering to work well, you typically need to specify a display format as well. center format ( \(\% 9.2 \mathrm{f}\) ) is popular.
left specifies that column labels be left-aligned. The default is to right-align column labels to distinguish them from supercolumn labels, which are left-aligned.
cw specifies casewise deletion. If cw is not specified, all observations possible are used to calculate each of the specified statistics. cw is relevant only when you request a table containing statistics on multiple variables. For instance, contents (mean mpg mean weight) would produce a table reporting the means of variables mpg and weight. Consider an observation in which mpg is known but weight is missing. By default, that observation will be used in the calculation of the mean of mpg . If you specify cw , the observation will be excluded in the calculation of the means of both mpg and weight.
row specifies that a row be added to the table reflecting the total across the rows.
column specifies that a column be added to the table reflecting the total across columns.
scolumn specifies that a supercolumn be added to the table reflecting the total across supercolumns.
concise specifies that rows with all missing entries not be displayed.
missing specifies that missing statistics be shown in the table as periods (Stata's missing-value indicator). The default is that missing entries be left blank.
replace specifies that the data in memory be replaced with data containing 1 observation per cell (row, column, supercolumn, and superrow) and with variables containing the statistics designated in contents().

This option is rarely specified. If you do not specify this option, the data in memory remain unchanged.

If you do specify this option, the first statistic will be named table1, the second table2, and so on. For instance, if contents (mean mpg sd mpg ) was specified, the means of mpg would be in variable table1 and the standard deviations in table2.
name (string) is relevant only if you specify replace name() allows changing the default stub name that replace uses to name the new variables associated with the statistics. If you specify name (stat), the first statistic will be placed in variable stat1, the second in stat2, and so on.
format ( \(\% \mathrm{fm} t\) ) specifies the display format for presenting numbers in the table's cells. format ( \(\% 9.0 \mathrm{~g}\) ) is the default; format ( \(\% 9.2 \mathrm{f}\) ) and format ( \(\% 9.2 \mathrm{fc}\) ) are popular alternatives. The width of the format you specify does not matter, except that \%fmt must be valid. The width of the cells is chosen by table to present the results best. The cellwidth() option allows you to override table's choice.

\section*{Limits}

Up to four variables may be specified in the by (), so with the three row, column, and supercolumn variables, seven-way tables may be displayed.

Up to five statistics may be displayed in each cell of the table.
The sum of the number of rows, columns, supercolumns, and superrows is called the number of margins. A table may contain up to 3,000 margins. Thus a one-way table may contain 3,000 rows. A two-way table could contain 2,998 rows and two columns, 2,997 rows and three columns, \(\ldots, 1,500\) rows and 1,500 columns, \(\ldots\), two rows and 2,998 columns. A three-way table is similarly limited by the sum of the number of rows, columns, and supercolumns. A \(r \times c \times d\) table is feasible if \(r+c+d \leq 3,000\). The limit is set in terms of the sum of the rows, columns, supercolumns, and superrows, and not, as you might expect, in terms of their product.

\section*{Remarks and examples}

Remarks are presented under the following headings:

\footnotetext{
One-way tables
Two-way tables
Three-way tables
Four-way and higher-dimensional tables
Video example
}

\section*{One-way tables}
> Example 1
From the automobile dataset, here is a simple one-way table:
```

. use http://www.stata-press.com/data/r13/auto2
(1978 Automobile Data)
. table rep78, contents(mean mpg)

| Repair <br> Record <br> 1978 | mean(mpg) |
| ---: | ---: |
| Poor | 21 |
| Fair | 19.125 |
| Average | 19.4333 |
| Good | 21.6667 |
| Excellent | 27.3636 |

```

We are not limited to including only one statistic:
. table rep78, c(n mpg mean mpg sd mpg median mpg )
\begin{tabular}{r|rrrr}
\hline \begin{tabular}{l} 
Repair \\
Record \\
1978
\end{tabular} & \(\mathrm{~N}(\mathrm{mpg})\) & mean(mpg) & sd(mpg) & med (mpg) \\
\hline Poor & 2 & 21 & 4.24264 & 21 \\
Fair & 8 & 19.125 & 3.758324 & 18 \\
Average & 30 & 19.4333 & 4.141325 & 19 \\
Good & 18 & 21.6667 & 4.93487 & 22.5 \\
Excellent & 11 & 27.3636 & 8.732385 & 30 \\
\hline
\end{tabular}

We abbreviated contents () as c(). The format () option will allow us to better format the numbers in the table:
. table rep78, \(\mathrm{c}(\mathrm{n} \mathrm{mpg}\) mean mpg sd mpg median mpg ) format (\%9.2f)
\begin{tabular}{l|rrrr}
\hline \begin{tabular}{l} 
Repair \\
Record \\
1978
\end{tabular} & \(\mathrm{~N}(\mathrm{mpg})\) & mean(mpg) & sd(mpg) & med(mpg) \\
\hline Poor & 2 & 21.00 & 4.24 & 21.00 \\
Fair & 8 & 19.12 & 3.76 & 18.00 \\
Average & 30 & 19.43 & 4.14 & 19.00 \\
Good & 18 & 21.67 & 4.93 & 22.50 \\
Excellent & 11 & 27.36 & 8.73 & 30.00 \\
\hline
\end{tabular}

The center option will center the results under the headings:
. table rep78, \(c(n \mathrm{mpg}\) mean mpg sd mpg median mpg ) format ( \(\% 9.2 \mathrm{f}\) ) center
\begin{tabular}{l|rccc}
\hline \begin{tabular}{l} 
Repair \\
Record \\
1978
\end{tabular} & & & & \\
\hline Poor & \(\mathrm{N}(\mathrm{mpg})\) & mean(mpg) & sd(mpg) & med (mpg) \\
Fair & 2 & 21.00 & 4.24 & 21.00 \\
Average & 8 & 19.12 & 3.76 & 18.00 \\
Good & 30 & 19.43 & 4.14 & 19.00 \\
Excellent & 18 & 21.67 & 4.93 & 22.50 \\
\hline
\end{tabular}

\section*{Two-way tables}

\section*{Example 2}

In example 1, when we typed 'table rep78, ...', we obtained a one-way table. If we were to type 'table rep78 foreign, ...', we would obtain a two-way table:
. table rep78 foreign, c(mean mpg)
\begin{tabular}{r|rr}
\hline \begin{tabular}{l} 
Repair \\
Record \\
1978
\end{tabular} & \multicolumn{2}{|c}{\begin{tabular}{r} 
Car
\end{tabular}} \\
\hline Poor & Domestic & Foreign \\
Fair & 21 & \\
Average & 19.125 & \\
Good & 18.4444 & 23.3333 \\
Excellent & 32 & 26.8889 \\
\hline
\end{tabular}

Note the missing cells. Certain combinations of repair record and car type do not exist in our dataset.
As with one-way tables, we can specify a display format for the cells and center the numbers within the cells if we wish.
. table rep78 foreign, c(mean mpg) format (\%9.2f) center
\begin{tabular}{r|rr}
\hline \begin{tabular}{l} 
Repair \\
Record \\
1978
\end{tabular} & \multicolumn{2}{|c}{\begin{tabular}{r} 
Car type \\
Domestic
\end{tabular}} \\
\hline Foreign \\
Foor & 21.00 & \\
Fair & 19.12 & \\
Average & 19.00 & 23.33 \\
Gxcellent & 18.44 & 24.89 \\
\hline
\end{tabular}

We can obtain row totals by specifying the row option and obtain column totals by specifying the col option. We specify both below:
```

. table rep78 foreign, c(mean mpg) format(\%9.2f) center row col

```
\begin{tabular}{r|ccc}
\hline \begin{tabular}{l} 
Repair \\
Record \\
1978
\end{tabular} & Domestic & \begin{tabular}{c} 
Car type \\
Foreign
\end{tabular} & Total \\
\hline Poor & 21.00 & & 21.00 \\
Fair & 19.12 & & 19.12 \\
Average & 19.00 & 23.33 & 19.43 \\
Good & 18.44 & 24.89 & 21.67 \\
Excellent & 32.00 & 26.33 & 27.36 \\
Total & 19.54 & 25.29 & 21.29 \\
\hline
\end{tabular}
table can display multiple statistics within cells, but once we move beyond one-way tables, the table becomes busy:
. table foreign rep78, \(\mathrm{c}(\) mean mpg n mpg ) format(\%9.2f) center
\begin{tabular}{c|rrrrr}
\hline & \multicolumn{5}{c}{ Repair Record 1978 } \\
Car type & Poor & Fair & Average & Good & Excellent \\
\hline Domestic & 21.00 & 19.12 & 19.00 & 18.44 & 32.00 \\
& 2 & 8 & 27 & 9 & 2 \\
Foreign & & & 23.33 & 24.89 & 26.33 \\
& & & 3 & 9 & 9 \\
\hline
\end{tabular}

This two-way table with two statistics per cell works well here. That was, in part, helped along by our interchanging the rows and columns. We turned the table around by typing table foreign rep78 rather than table rep78 foreign.

Another way to display two-way tables is to specify a row and superrow rather than a row and column. We do that below and display three statistics per cell:
. table foreign, by (rep78) c(mean mpg sd mpg \(n \mathrm{mpg}\) ) format(\%9.2f) center
\begin{tabular}{l|ccc}
\hline \begin{tabular}{l} 
Repair \\
Record \\
1978 and \\
Car type
\end{tabular} & mean(mpg) & sd(mpg) & \(\mathrm{N}(\mathrm{mpg})\) \\
\hline \begin{tabular}{c} 
Poor \\
Domestic \\
Foreign
\end{tabular} & 21.00 & 4.24 & \\
\hline \begin{tabular}{c} 
Fair \\
Domestic \\
Foreign
\end{tabular} & 19.12 & 3.76 & 2 \\
\hline \begin{tabular}{c} 
Average \\
Domestic \\
Foreign
\end{tabular} & 19.00 & 4.09 & 8 \\
\hline \begin{tabular}{c} 
Good \\
Domestic \\
Foreign
\end{tabular} & 23.33 & 2.52 & 27 \\
\hline \begin{tabular}{c} 
Excellent \\
Domestic \\
Foreign
\end{tabular} & 26.44 & 4.59 & 3 \\
\hline
\end{tabular}

\section*{Three-way tables}

\section*{> Example 3}

We have data on the prevalence of byssinosis, a form of pneumoconiosis to which workers exposed to cotton dust are susceptible. The dataset is on 5,419 workers in a large cotton mill. We know whether each worker smokes, his or her race, and the dustiness of the work area. The categorical variables are
\[
\begin{array}{ll}
\text { smokes } & \begin{array}{l}
\text { Smoker or nonsmoker in the last five years. } \\
\text { race }
\end{array} \\
\text { White or other. }
\end{array}
\]

Moreover, this dataset includes a frequency-weight variable pop. Here is a three-way table showing the fraction of workers with byssinosis:
\begin{tabular}{|c|c|c|c|c|}
\hline Dustiness of workplace &  & \[
\frac{\text { Race ar }}{\text { yes }}
\] & \[
\begin{array}{r}
\text { Smokes } \\
\text { no }
\end{array}
\] & yes \\
\hline least & . 0107527 & . 0101523 & . 0081549 & . 0162774 \\
\hline less & . 02 & . 0081633 & . 0136612 & . 0143149 \\
\hline most & . 0820896 & . 1679105 & . 0833333 & . 2295082 \\
\hline
\end{tabular}

This table would look better if we showed the fraction to four digits:
. table workplace smokes race [fw=pop], c(mean prob) format(\%9.4f)
\begin{tabular}{|c|c|c|c|c|}
\hline Dustiness of workplace & \multicolumn{4}{|l|}{\[
\begin{gathered}
\text { Race and Smokes } \\
\text { no } \begin{array}{c}
\text { other } \\
\text { nos }
\end{array} \quad \text { no } \quad \text { no } \quad \text { yes }
\end{gathered}
\]} \\
\hline least & 0.0108 & 0.0102 & 0.0082 & 0.0163 \\
\hline less & 0.0200 & 0.0082 & 0.0137 & 0.0143 \\
\hline most & 0.0821 & 0.1679 & 0.0833 & 0.2295 \\
\hline
\end{tabular}

In this table, the rows are the dustiness of the workplace, the columns are whether the worker smokes, and the supercolumns are the worker's race.

Now we request that the table include the supercolumn totals by specifying the sctotal option, which we can abbreviate as sc:
. table workplace smokes race [fw=pop], c(mean prob) format(\%9.4f) sc
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline Dustiness of workplace & \multicolumn{2}{|r|}{\[
\begin{array}{ll}
- \text { other } \\
\text { no } & \text { yes }
\end{array}
\]} & \multicolumn{2}{|l|}{\[
\begin{gathered}
\text { Race and Smokes } \\
\text { white }-\frac{\text { yes }}{\text { no }}
\end{gathered}
\]} & \[
\begin{aligned}
& \text { To } \\
& \text { no }
\end{aligned}
\] & \[
\text { tal } \overline{\text { yes }}
\] \\
\hline least & 0.0108 & 0.0102 & 0.0082 & 0.0163 & 0.0090 & 0.0145 \\
\hline less & 0.0200 & 0.0082 & 0.0137 & 0.0143 & 0.0159 & 0.0123 \\
\hline most & 0.0821 & 0.1679 & 0.0833 & 0.2295 & 0.0826 & 0.1929 \\
\hline
\end{tabular}

The supercolumn total is the total over race and is divided into its columns based on smokes. Here is the table with the column rather than the supercolumn totals:
. table workplace smokes race [fw=pop], c(mean prob) format(\%9.4f) col
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \multirow[t]{2}{*}{Dustiness of workplace} & & \multirow[b]{2}{*}{other yes} & \multicolumn{2}{|l|}{Race and Smokes} & \multirow[b]{2}{*}{white yes} & \\
\hline & no & & Total & no & & Total \\
\hline least & 0.0108 & 0.0102 & 0.0104 & 0.0082 & 0.0163 & 0.0129 \\
\hline less & 0.0200 & 0.0082 & 0.0135 & 0.0137 & 0.0143 & 0.0140 \\
\hline most & 0.0821 & 0.1679 & 0.1393 & 0.0833 & 0.2295 & 0.1835 \\
\hline
\end{tabular}

Here is the table with both column and supercolumn totals:
. table workplace smokes race [fw=pop], c(mean prob) format(\%9.4f) sc col
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline Dustin ess of workpl ace & no & other yes & Total & \[
\begin{gathered}
\text { Race } \\
\hline \text { no }
\end{gathered}
\] & and Smo white yes & \begin{tabular}{l}
kes \\
Total
\end{tabular} & no & Total
yes & Total \\
\hline least & 0.0108 & 0.0102 & 0.0104 & 0.0082 & 0.0163 & 0.0129 & 0.0090 & 0.0145 & 0.0122 \\
\hline less & 0.0200 & 0.0082 & 0.0135 & 0.0137 & 0.0143 & 0.0140 & 0.0159 & 0.0123 & 0.0138 \\
\hline most & 0.0821 & 0.1679 & 0.1393 & 0.0833 & 0.2295 & 0.1835 & 0.0826 & 0.1929 & 0.1570 \\
\hline
\end{tabular}
table is struggling to keep this table from becoming too wide—notice how it divided the words in the title in the top-left stub. Here, if the table had more columns, or, if we demanded more digits, table would be forced to segment the table and present it in pieces, which it would do:
. table workplace smokes race [fw=pop], c(mean prob) format (\%9.6f) sc col


Here three digits is probably enough, so here is the table including all the row, column, and supercolumn totals:
. table workplace smokes race [fw=pop], c(mean prob) format (\%9.3f) sc col row


We can show multiple statistics:
. table workplace smokes race [fw=pop], c(mean prob \(n\) prob) format(\% \(\%\).af) sc
> col row


\section*{Four-way and higher-dimensional tables}

\section*{> Example 4}

Let's pretend that our byssinosis dataset also recorded each worker's sex (it does not, and we have made up this extra information). We obtain a four-way table just as we would a three-way table, but we specify the fourth variable as a superrow by including it in the by () option:
```

. use http://www.stata-press.com/data/r13/byssin1
(Byssinosis incidence)
. table workplace smokes race [fw=pop], by(sex) c(mean prob) format(%9.3f) sc
> col row

```
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline Sex and Dustiness of workplace & no & other yes & Total & \multicolumn{3}{|l|}{Race and Smokes
\(\qquad\) white \(\qquad\)} & no & Total yes & Total \\
\hline \multicolumn{10}{|l|}{Female} \\
\hline least & 0.006 & 0.009 & 0.008 & 0.009 & 0.021 & 0.016 & 0.009 & 0.018 & 0.014 \\
\hline less & 0.020 & 0.008 & 0.010 & 0.015 & 0.015 & 0.015 & 0.016 & 0.012 & 0.014 \\
\hline most & 0.057 & 0.154 & 0.141 & & & & 0.057 & 0.154 & 0.141 \\
\hline Total & 0.017 & 0.051 & 0.043 & 0.011 & 0.020 & 0.016 & 0.012 & 0.032 & 0.024 \\
\hline \multicolumn{10}{|l|}{Male} \\
\hline least & 0.013 & 0.011 & 0.012 & 0.006 & 0.007 & 0.006 & 0.009 & 0.008 & 0.009 \\
\hline less & 0.020 & 0.000 & 0.019 & 0.000 & 0.013 & 0.011 & 0.016 & 0.013 & 0.014 \\
\hline most & 0.091 & 0.244 & 0.136 & 0.083 & 0.230 & 0.184 & 0.087 & 0.232 & 0.167 \\
\hline Total & 0.029 & 0.041 & 0.033 & 0.020 & 0.056 & 0.043 & 0.025 & 0.052 & 0.039 \\
\hline
\end{tabular}

If our dataset also included work group and we wanted a five-way table, we could include both the sex and work-group variables in the by () option. You may include up to four variables in by (), and so produce up to 7-way tables.

\section*{Video example}

Combining cross-tabulations and descriptives in Stata

\section*{Methods and formulas}

The contents of cells are calculated by collapse and are displayed by tabdisp; see [D] collapse and \([\mathrm{P}]\) tabdisp.

\section*{Also see}
\([\mathrm{R}]\) summarize - Summary statistics
[R] tabstat - Compact table of summary statistics
[R] tabulate oneway - One-way table of frequencies
[R] tabulate twoway - Two-way table of frequencies
[D] collapse - Make dataset of summary statistics
[P] tabdisp - Display tables

\section*{Title}
tabstat - Compact table of summary statistics
\begin{tabular}{llll} 
Syntax & Menu & Description & Options \\
Remarks and examples & Acknowledgments & Also see &
\end{tabular}

\section*{Syntax}
tabstat varlist [if] [in] [weight] [, options]
options
Description
Main
by (varname)
statistics(statname[...])
Options
labelwidth(\#)
varwidth(\#)
columns(variables)
columns(statistics)
format [ \(\%\) fmt \()\) ]
casewise
nototal
missing
noseparator
longstub
save
group statistics by variable report specified statistics
width for by () variable labels; default is labelwidth (16)
variable width; default is varwidth(12)
display variables in table columns; the default
display statistics in table columns
display format for statistics; default format is \(\% 9.0 \mathrm{~g}\)
perform casewise deletion of observations
do not report overall statistics; use with by ()
report statistics for missing values of by () variable
do not use separator line between by () categories
make left table stub wider
store summary statistics in \(r()\)
by is allowed; see [D] by.
aweights and fweights are allowed; see [U] 11.1.6 weight.

\section*{Menu}

Statistics \(>\) Summaries, tables, and tests \(>\) Other tables \(>\) Compact table of summary statistics

\section*{Description}
tabstat displays summary statistics for a series of numeric variables in one table, possibly broken down on (conditioned by) another variable.

Without the by () option, tabstat is a useful alternative to summarize (see [R] summarize) because it allows you to specify the list of statistics to be displayed.

With the by () option, tabstat resembles tabulate used with its summarize() option in that both report statistics of varlist for the different values of varname. tabstat allows more flexibility in terms of the statistics presented and the format of the table.
tabstat is sensitive to the linesize (see set linesize in [R] log); it widens the table if possible and wraps if necessary.

\section*{Options}
\(\qquad\) Main
by (varname) specifies that the statistics be displayed separately for each unique value of varname; varname may be numeric or string. For instance, tabstat height would present the overall mean of height. tabstat height, by (sex) would present the mean height of males, and of females, and the overall mean height. Do not confuse the by () option with the by prefix (see [D] by); both may be specified.
statistics (statname [...]) specifies the statistics to be displayed; the default is equivalent to specifying statistics(mean). (stats() is a synonym for statistics().) Multiple statistics may be specified and are separated by white space, such as statistics(mean sd). Available statistics are
\begin{tabular}{llll} 
statname & Definition & statname & Definition \\
\hline\(\underline{\text { mean }}\) & mean & p 1 & 1st percentile \\
\(\underline{\text { count }}\) & count of nonmissing observations & p 5 & 5th percentile \\
n & same as count & p 10 & 10th percentile \\
\(\underline{\text { sum }}\) & sum & p 25 & 25th percentile \\
\(\underline{\max }\) & maximum & \(\underline{\text { median }}\) & median (same as p50) \\
\(\underline{\text { min }}\) & minimum & p 50 & 50th percentile (same as median) \\
\(\underline{\text { range }}\) & range \(=\max -\min\) & p 75 & 75th percentile \\
sd & standard deviation & p 90 & 90th percentile \\
\(\underline{\text { variance }}\) & variance & p 95 & 95th percentile \\
cv & coefficient of variation \((\mathrm{sd} / \mathrm{mean})\) & p 99 & 99th percentile \\
semean & standard error of mean \((\mathrm{sd} / \sqrt{\mathrm{n}})\) & iqr & interquartile range \(=\mathrm{p} 75-\mathrm{p} 25\) \\
skewness & skewness & q & equivalent to specifying p25 p50 p75 \\
\(\underline{\text { kurtosis }}\) & kurtosis & &
\end{tabular}

Options
labelwidth(\#) specifies the maximum width to be used within the stub to display the labels of the by () variable. The default is labelwidth(16). \(8 \leq \# \leq 32\).
varwidth (\#) specifies the maximum width to be used within the stub to display the names of the variables. The default is varwidth(12). varwidth() is effective only with columns(statistics). Setting varwidth() implies longstub. \(8 \leq \# \leq 16\).
columns (variables \(\mid\) statistics) specifies whether to display variables or statistics in the columns of the table. columns (variables) is the default when more than one variable is specified.
format and format (\%fmt) specify how the statistics are to be formatted. The default is to use a \(\% 9.0 \mathrm{~g}\) format.
format specifies that each variable's statistics be formatted with the variable's display format; see [D] format.
format (\% fint) specifies the format to be used for all statistics. The maximum width of the specified format should not exceed nine characters.
casewise specifies casewise deletion of observations. Statistics are to be computed for the sample that is not missing for any of the variables in varlist. The default is to use all the nonmissing values for each variable.
nototal is for use with by (); it specifies that the overall statistics not be reported.
missing specifies that missing values of the by () variable be treated just like any other value and that statistics should be displayed for them. The default is not to report the statistics for the by ()\(==\) missing group. If the by () variable is a string variable, by ( \()==="\) is considered to mean missing. noseparator specifies that a separator line between the by () categories not be displayed.
longstub specifies that the left stub of the table be made wider so that it can include names of the statistics or variables in addition to the categories of by (varname). The default is to describe the statistics or variables in a header. longstub is ignored if by (varname) is not specified.
save specifies that the summary statistics be returned in \(r()\). The overall (unconditional) statistics are returned in matrix \(r\) (StatTotal) (rows are statistics, columns are variables). The conditional statistics are returned in the matrices \(r(S t a t 1), r(S t a t 2), \ldots\), and the names of the corresponding variables are returned in the macros \(r\) (name1), \(r\) (name2), \(\ldots\).

\section*{Remarks and examples}

This command is probably most easily understood by going through a series of examples.
\(>\) Example 1
We have data on the price, weight, mileage rating, and repair record of 22 foreign and 52 domestic 1978 automobiles. We want to summarize these variables for the different origins of the automobiles.
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. tabstat price weight mpg rep78, by (foreign)
Summary statistics: mean
by categories of: foreign (Car type)
foreign \(\quad\) price \begin{tabular}{r|rrrr} 
weight & mpg & rep78 \\
\hline Domestic & 6072.423 & 3317.115 & 19.82692 & 3.020833 \\
Foreign & 6384.682 & 2315.909 & 24.77273 & 4.285714 \\
\hline Total & 6165.257 & 3019.459 & 21.2973 & 3.405797 \\
\hline
\end{tabular}

More summary statistics can be requested via the statistics() option. The group totals can be suppressed with the nototal option.


Although the header of the table describes the statistics running vertically in the "cells", the table may become hard to read, especially with many variables or statistics. The longstub option specifies that a column be added describing the contents of the cells. The format option can be issued to
specify that tabstat display the statistics by using the display format of the variables rather than the overall default \(\% 9.0 \mathrm{~g}\).
. tabstat price weight mpg rep78, by(foreign) stat(mean sd min max) long format
\begin{tabular}{lr|rrrr} 
foreign & stats & price & weight & mpg & rep78 \\
\hline Domestic & mean & \(6,072.4\) & \(3,317.1\) & 19.8269 & 3.02083 \\
& sd & \(3,097.1\) & 695.364 & 4.7433 & .837666 \\
& \(\min\) & 3,291 & 1,800 & 12 & 1 \\
& \(\max\) & 15,906 & 4,840 & 34 & 5 \\
\hline Foreign & mean & \(6,384.7\) & \(2,315.9\) & 24.7727 & 4.28571 \\
& sd & \(2,621.9\) & 433.003 & 6.61119 & .717137 \\
& \(\min\) & 3,748 & 1,760 & 14 & 3 \\
& \(\max\) & 12,990 & 3,420 & 41 & 5 \\
& & & & & \\
& Total & \(\operatorname{mean}\) & \(6,165.3\) & \(3,019.5\) & 21.2973 \\
& sd & \(2,949.5\) & 777.194 & 5.7855 & .989932 \\
& \(\min\) & 3,291 & 1,760 & 12 & 1 \\
& \(\max\) & 15,906 & 4,840 & 41 & 5 \\
& & & & & \\
\hline
\end{tabular}

We can specify a layout of the table in which the statistics run horizontally and the variables run vertically by specifying the col(statistics) option.
\begin{tabular}{|c|c|c|c|c|}
\hline foreign & variable & min & mean & max \\
\hline \multirow[t]{4}{*}{Domestic} & price & 3291 & 6072.423 & 15906 \\
\hline & weight & 1800 & 3317.115 & 4840 \\
\hline & mpg & 12 & 19.82692 & 34 \\
\hline & rep78 & 1 & 3.020833 & 5 \\
\hline \multirow[t]{4}{*}{Foreign} & price & 3748 & 6384.682 & 12990 \\
\hline & weight & 1760 & 2315.909 & 3420 \\
\hline & mpg & 14 & 24.77273 & 41 \\
\hline & rep78 & 3 & 4.285714 & 5 \\
\hline \multirow[t]{4}{*}{Total} & price & 3291 & 6165.257 & 15906 \\
\hline & weight & 1760 & 3019.459 & 4840 \\
\hline & mpg & 12 & 21.2973 & 41 \\
\hline & rep78 & 1 & 3.405797 & 5 \\
\hline
\end{tabular}

Finally, tabstat can also be used to enhance summarize so we can specify the statistics to be displayed. For instance, we can display the number of observations, the mean, the coefficient of variation, and the \(25 \%, 50 \%\), and \(75 \%\) quantiles for a list of variables.
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline variable & N & mean & cv & p25 & p50 & p75 \\
\hline price & 74 & 6165.257 & . 478406 & 4195 & 5006.5 & 6342 \\
\hline weight & 74 & 3019.459 & . 2573949 & 2240 & 3190 & 3600 \\
\hline mpg & 74 & 21.2973 & . 2716543 & 18 & 20 & 25 \\
\hline rep78 & 69 & 3.405797 & . 290661 & 3 & 3 & 4 \\
\hline
\end{tabular}

Because we did not specify the by () option, these statistics were not displayed for the subgroups of the data formed by the categories of the by () variable.

\section*{Video example}

Descriptive statistics in Stata

\section*{Acknowledgments}

The tabstat command was written by Jeroen Weesie and Vincent Buskens both of the Department of Sociology at Utrecht University, The Netherlands.

\section*{Also see}
[R] summarize - Summary statistics
[R] table - Flexible table of summary statistics
\([R]\) tabulate, summarize() - One- and two-way tables of summary statistics
[D] collapse - Make dataset of summary statistics

\section*{Title}
tabulate oneway - One-way table of frequencies
\begin{tabular}{llll} 
Syntax & Menu & Description & Options \\
Remarks and examples & Stored results & References & Also see
\end{tabular}

\section*{Syntax}

One-way table
tabulate varname \([\) if \(][\) in \(]\) [weight \(]\) [, tabulatel_options]

One-way table for each variable-a convenience tool
tab1 varlist \([\) if \(][\) in \(][\) weight \(][\), tabl_options \(]\)
tabulate1_options
Description
Main
subpop (varname) exclude observations for which varname \(=0\)
missing
treat missing values like other values
nofreq
do not display frequencies
nolabel
display numeric codes rather than value labels
plot
produce a bar chart of the relative frequencies
sort
display the table in descending order of frequency
Advanced
generate (stubname) create indicator variables for stubname
matcell (matname)
matrow(matname)
save frequencies in matname; programmer's option
save unique values of varname in matname; programmer's option
tabl_options
Description
Main
subpop (varname)
exclude observations for which varname \(=0\)
missing
treat missing values like other values
nofreq
nolabel
plot
sort
do not display frequencies
display numeric codes rather than value labels
produce a bar chart of the relative frequencies
display the table in descending order of frequency
by is allowed with tabulate and tab1; see [D] by.
fweights, aweights, and iweights are allowed by tabulate. fweights are allowed by tab1. See [U] 11.1.6 weight.

\section*{Menu}

\section*{tabulate oneway}

Statistics \(>\) Summaries, tables, and tests \(>\) Frequency tables \(>\) One-way table
tabulate ..., generate()
Data \(>\) Create or change data \(>\) Other variable-creation commands \(>\) Create indicator variables
tab1
Statistics \(>\) Summaries, tables, and tests \(>\) Frequency tables \(>\) Multiple one-way tables

\section*{Description}
tabulate produces a one-way table of frequency counts.
For information about a two-way table of frequency counts along with various measures of association, including the common Pearson \(\chi^{2}\), the likelihood-ratio \(\chi^{2}\), Cramér's \(V\), Fisher's exact test, Goodman and Kruskal's gamma, and Kendall's \(\tau_{b}\), see [R] tabulate twoway.
tab1 produces a one-way tabulation for each variable specified in varlist.
Also see \([R]\) table and \([R]\) tabstat if you want one-, two-, or \(n\)-way table of frequencies and a wide variety of summary statistics. See \([R]\) tabulate, summarize() for a description of tabulate with the summarize() option; it produces a table (breakdowns) of means and standard deviations. table is better than tabulate, summarize(), but tabulate, summarize() is faster. See [ST] epitab for a \(2 \times 2\) table with statistics of interest to epidemiologists.

\section*{Options}

\footnotetext{
subpop (varname) excludes observations for which varname \(=0\) in tabulating frequencies. The mathematical results of tabulate ..., subpop(myvar) are the same as tabulate... if myvar \(!=0\), but the table may be presented differently. The identities of the rows and columns will be determined from all the data, including the myvar \(=0\) group, so there may be entries in the table with frequency 0 .
Consider tabulating answer, a variable that takes on values 1,2 , and 3 , but consider tabulating it just for the male==1 subpopulation. Assume that answer is never 2 in this group. tabulate answer if male==1 produces a table with two rows: one for answer 1 and one for answer 3. There will be no row for answer 2 because answer 2 was never observed. tabulate answer, subpop(male) produces a table with three rows. The row for answer 2 will be shown as having 0 frequency.
missing requests that missing values be treated like other values in calculations of counts, percentages, and other statistics.
nofreq suppresses the printing of the frequencies.
nolabel causes the numeric codes to be displayed rather than the value labels.
plot produces a bar chart of the relative frequencies in a one-way table. (Also see [R] histogram.)
sort puts the table in descending order of frequency (and ascending order of the variable within equal values of frequency).
}

Advanced
generate (stubname) creates a set of indicator variables (stubname 1 , stubname \(2, \ldots\) ) reflecting the observed values of the tabulated variable. The generate() option may not be used with the by prefix.
matcell (matname) saves the reported frequencies in matname. This option is for use by programmers. matrow (matname) saves the numeric values of the \(r \times 1\) row stub in matname. This option is for use by programmers. matrow () may not be specified if the row variable is a string.

\section*{Limits}

A one-way table may have a maximum of 12,000 rows (Stata/MP and Stata/SE), 3,000 rows (Stata/IC), or 500 rows (Small Stata).

\section*{Remarks and examples}

Remarks are presented under the following headings:
```

tabulate
tab1
Video example

```

For each value of a specified variable, tabulate reports the number of observations with that value. The number of times a value occurs is called its frequency.

\section*{tabulate}
> Example 1
We have data summarizing the speed limit and the accident rate per million vehicle miles along various Minnesota highways in 1973. The variable containing the speed limit is called spdlimit. If we summarize the variable, we obtain its mean and standard deviation:
. use http://www.stata-press.com/data/r13/hiway
(Minnesota Highway Data, 1973)
. summarize spdlimit
Variable

The average speed limit is 55 miles per hour. We can learn more about this variable by tabulating it:
\begin{tabular}{r|rrr}
. tabulate spdlimit \\
Speed Limit & Freq. & Percent & Cum. \\
\hline 40 & 1 & 2.56 & 2.56 \\
45 & 3 & 7.69 & 10.26 \\
50 & 7 & 17.95 & 28.21 \\
55 & 15 & 38.46 & 66.67 \\
60 & 11 & 28.21 & 94.87 \\
65 & 1 & 2.56 & 97.44 \\
70 & 1 & 2.56 & 100.00 \\
\hline Total & 39 & 100.00 &
\end{tabular}

We see that one highway has a speed limit of 40 miles per hour, three have speed limits of 45,7 of 50, and so on. The column labeled Percent shows the percentage of highways in the dataset that have the indicated speed limit. For instance, \(38.46 \%\) of highways in our dataset have a speed limit of 55 miles per hour. The final column shows the cumulative percentage. We see that \(66.67 \%\) of highways in our dataset have a speed limit of 55 miles per hour or less.

\section*{> Example 2}

The plot option places a sideways histogram alongside the table:
\begin{tabular}{|c|c|c|}
\hline Speed Limit & Freq. & \\
\hline 40 & 1 & * \\
\hline 45 & 3 & *** \\
\hline 50 & 7 & ******* \\
\hline 55 & 15 & *************** \\
\hline 60 & 11 & *********** \\
\hline 65 & 1 & * \\
\hline 70 & 1 & * \\
\hline Total & 39 & \\
\hline
\end{tabular}

Of course, graph can produce better-looking histograms; see [R] histogram.
> Example 3
tabulate labels tables using variable and value labels if they exist. To demonstrate how this works, let's add a new variable to our dataset that categorizes spdlimit into three categories. We will call this new variable spdcat:
```

. generate spdcat=recode(spdlimit,50,60,70)

```

The recode() function divides spdlimit into 50 miles per hour or below, 51-60, and above 60; see [D] functions. We specified the breakpoints in the arguments (spdlimit, 50, 60, 70). The first argument is the variable to be recoded. The second argument is the first breakpoint, the third argument is the second breakpoint, and so on. We can specify as many breakpoints as we wish.
recode () used our arguments not only as the breakpoints but also to label the results. If spdlimit is less than or equal to 50 , spdcat is set to 50 ; if spdlimit is between 51 and 60 , spdcat is 60 ; otherwise, spdcat is arbitrarily set to 70 . (See [U] 25 Working with categorical data and factor variables.)

Because we just created the variable spdcat, it is not yet labeled. When we make a table using this variable, tabulate uses the variable's name to label it:
\begin{tabular}{r|rrr}
. tabulate spdcat \\
spdcat & Freq. & Percent & Cum. \\
\hline 50 & 11 & 28.21 & 28.21 \\
60 & 26 & 66.67 & 94.87 \\
70 & 2 & 5.13 & 100.00 \\
\hline Total & 39 & 100.00 &
\end{tabular}

Even through the table is not well labeled, recode()'s coding scheme provides us with clues as to the table's meaning. The first line of the table corresponds to 50 miles per hour and below, the next to 51 through 60 miles per hour, and the last to above 60 miles per hour.

We can improve this table by labeling the values and variables:
```

. label define scat 50 "40 to 50" 60 "55 to 60" 70 "Above 60"
. label values spdcat scat
. label variable spdcat "Speed Limit Category"

```

We define a value label called scat that attaches labels to the numbers 50, 60, and 70 using the label define command; see [U] 12.6.3 Value labels. We label the value 50 as ' 40 to 50 ', because we looked back at our original tabulation in the first example and saw that the speed limit was never less than 40 . Similarly, we could have labeled the last category ' 65 to 70 ' because the speed limit is never greater than 70 miles per hour.

Next we requested that Stata label the values of the new variable spdcat using the value label scat. Finally, we labeled our variable Speed Limit Category. We are now ready to tabulate the result:
\begin{tabular}{r|rrr}
\begin{tabular}{r}
. tabulate spdcat \\
Speed Limit \\
Category
\end{tabular} & Freq. & Percent & Cum. \\
\hline 40 to 50 & 11 & 28.21 & 28.21 \\
55 to 60 & 26 & 66.67 & 94.87 \\
Above 60 & 2 & 5.13 & 100.00 \\
\hline Total & 39 & 100.00 &
\end{tabular}

\section*{Example 4}

If we have missing values in our dataset, tabulate ignores them unless we explicitly indicate otherwise. We have no missing data in our example, so let's add some:
```

. replace spdcat=. in 39
(1 real change made, 1 to missing)

```

We changed the first observation on spdcat to missing. Let's now tabulate the result:
\begin{tabular}{r|rrr}
\begin{tabular}{c} 
tabulate spdcat \\
Speed Limit \\
Category
\end{tabular} & Freq. & Percent & Cum. \\
\hline 40 to 50 & 11 & 28.95 & 28.95 \\
55 to 60 & 26 & 68.42 & 97.37 \\
Above 60 & 1 & 2.63 & 100.00 \\
\hline Total & 38 & 100.00 & \\
\end{tabular}

Comparing this output with that in the previous example, we see that the total frequency count is now one less than it was - 38 rather than 39 . Also, the 'Above 60 ' category now has only one observation where it used to have two, so we evidently changed a road with a high speed limit.

We want tabulate to treat missing values just as it treats numbers, so we specify the missing option:
\begin{tabular}{r|rrr}
\begin{tabular}{c} 
tabulate spdcat, missing \\
Speed Limit \\
Category
\end{tabular} & Freq. & Percent & Cum. \\
\hline 40 to 50 & 11 & 28.21 & 28.21 \\
55 to 60 & 26 & 66.67 & 94.87 \\
Above 60 & 1 & 2.56 & 97.44 \\
\(\cdot\) & 1 & 2.56 & 100.00 \\
\hline Total & 39 & 100.00 &
\end{tabular}

We now see our missing value-the last category, labeled '.', shows a frequency count of 1 . The table sum is once again 39.

Let's put our dataset back as it was originally:
```

. replace spdcat=70 in 39
(1 real change made)

```
- Technical note
tabulate also can automatically create indicator variables from categorical variables. We will briefly review that capability here, but see [U] \(\mathbf{2 5}\) Working with categorical data and factor variables for a complete description. Let's begin by describing our highway dataset:
\begin{tabular}{|c|c|c|c|c|}
\hline \multicolumn{5}{|l|}{Contains data from http://www.stata-press.com/data/r13/hiway.dta} \\
\hline obs: & \multicolumn{3}{|l|}{39} & \multirow[t]{2}{*}{Minnesota Highway Data, 1973
16 Nov 2012 12:39} \\
\hline vars: & 3 & & & \\
\hline size: & 351 & & & \\
\hline & storage & display & value & \\
\hline variable name & type & format & label & variable label \\
\hline spdlimit & byte & \(\% 8.0 \mathrm{~g}\) & & Speed Limit \\
\hline rate & float & \%9.0g & rcat & ```
Accident rate per million vehicle
    miles
``` \\
\hline spdcat & float & \% 9.0g & scat & Speed Limit Category \\
\hline
\end{tabular}

Sorted by:
Note: dataset has changed since last saved
Our dataset contains three variables. We will type tabulate spdcat, generate(spd), describe our data, and then explain what happened.
\begin{tabular}{r|rrr}
.\(\quad\) tabulate spdcat, generate(spd) \\
\begin{tabular}{r} 
Speed Limit \\
Category
\end{tabular} & Freq. & Percent & Cum. \\
\hline 40 to 50 & 11 & 28.21 & 28.21 \\
55 to 60 & 26 & 66.67 & 94.87 \\
Above 60 & 2 & 5.13 & 100.00 \\
\hline Total & 39 & 100.00 &
\end{tabular}
\begin{tabular}{|c|c|c|c|c|}
\hline ```
Contains data
    obs:
    vars:
    size:
``` & \[
\begin{gathered}
\text { from htt } \\
39 \\
6 \\
468
\end{gathered}
\] & //www.st & a-press & \begin{tabular}{l}
/data/r13/hiway.dta \\
Minnesota Highway Data, 1973 \\
16 Nov 2012 12:39
\end{tabular} \\
\hline variable name & storage type & \begin{tabular}{l}
display \\
format
\end{tabular} & \begin{tabular}{l}
value \\
label
\end{tabular} & variable label \\
\hline spdlimit & byte & \%8.0g & & Speed Limit \\
\hline rate & float & \%9.0g & rcat & ```
Accident rate per million vehicle
    miles
``` \\
\hline spdcat & float & \%9.0g & scat & Speed Limit Category \\
\hline spd1 & byte & \%8.0g & & spdcat==40 to 50 \\
\hline spd2 & byte & \%8.0g & & spdcat==55 to 60 \\
\hline spd3 & byte & \%8.0g & & spdcat==Above 60 \\
\hline
\end{tabular}

Sorted by:
Note: dataset has changed since last saved
When we typed tabulate with the generate() option, Stata responded by producing a one-way frequency table, so it appeared that the option did nothing. Yet when we describe our dataset, we find that we now have six variables instead of the original three. The new variables are named spd1, spd2, and spd3.

When we specify the generate() option, we are telling Stata to not only produce the table but also create a set of indicator variables that correspond to that table. Stata adds a numeric suffix to the name we specify in the parentheses. spd1 refers to the first line of the table, spd2 to the second line, and so on. Also Stata labels the variables so that we know what they mean. spd1 is an indicator variable that is true (takes on the value 1) when spdcat is between 40 and 50 ; otherwise, it is zero. (There is an exception: if spdcat is missing, so are the \(\operatorname{spd} 1\), \(s p d 2\), and \(\operatorname{spd} 3\) variables. This did not happen in our dataset.)

We want to prove our claim. Because we have not yet introduced two-way tabulations, we will use the summarize statement:
. summarize spdlimit if spd1==1
\begin{tabular}{|c|c|c|c|c|c|}
\hline Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline spdlimit & 11 & 47.72727 & 3.437758 & 40 & 50 \\
\hline \multicolumn{6}{|l|}{. summarize spdlimit if spd2==1} \\
\hline Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline spdlimit & 26 & 57.11538 & 2.519157 & 55 & 60 \\
\hline \multicolumn{6}{|l|}{. summarize spdlimit if spd3==1} \\
\hline Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline spdlimit & 2 & 67.5 & 3.535534 & 65 & 70 \\
\hline
\end{tabular}

Notice the indicated minimum and maximum in each of the tables above. When we restrict the sample to spd1, spdlimit is between 40 and 50 ; when we restrict the sample to spd2, spdlimit is between 55 and 60 ; when we restrict the sample to spd3, spdlimit is between 65 and 70 .

Thus tabulate provides an easy way to create indicator (sometimes called dummy) variables. For an overview of indicator and categorical variables, see [U] 25 Working with categorical data and factor variables.

\section*{tab1}
tab1 is a convenience tool. Typing
. tab1 myvar thisvar thatvar, plot
is equivalent to typing
. tabulate myvar, plot
. tabulate thisvar, plot
. tabulate thatvar, plot

\section*{Video example}

Tables and cross-tabulations in Stata

\section*{Stored results}
tabulate and tab1 store the following in \(r()\) :
Scalars
\(r(N)\) number of observations \(r(r) \quad\) number of rows

\section*{References}

Cox, N. J. 2009. Speaking Stata: I. J. Good and quasi-Bayes smoothing of categorical frequencies. Stata Journal 9: 306-314.

Harrison, D. A. 2006. Stata tip 34: Tabulation by listing. Stata Journal 6: 425-427.

\section*{Also see}
[R] table - Flexible table of summary statistics
[R] tabstat - Compact table of summary statistics
[R] tabulate twoway - Two-way table of frequencies
[R] tabulate, summarize() - One- and two-way tables of summary statistics
[D] collapse - Make dataset of summary statistics
[ST] epitab - Tables for epidemiologists
[SVY] svy: tabulate oneway - One-way tables for survey data
[SVY] svy: tabulate twoway - Two-way tables for survey data
[XT] xttab - Tabulate xt data
[U] 12.6.3 Value labels
[U] 25 Working with categorical data and factor variables

\section*{Title}
tabulate twoway - Two-way table of frequencies
\begin{tabular}{llll} 
Syntax & Menu & Description & Options \\
Remarks and examples & Stored results & Methods and formulas & References
\end{tabular}

Menu
Stored results

Methods and formulas

References

\section*{Syntax}

Two-way table
tabulate varname \({ }_{1}\) varname \(_{2}[\) if \(][\) in \(][\) weight \(][\), options \(]\)

Two-way table for all possible combinations-a convenience tool
tab2 varlist [if] [in] [weight] [, options]

Immediate form of two-way tabulations
tabi \(\#_{11} \#_{12}[\ldots] \backslash \#_{21} \#_{22}[\ldots][\backslash \ldots][\), options \(]\)
options
Description
Main
chi2
exact [(\#)]
gamma
lrchi2
taub
V
cchi2
column
row
clrchi2
cell
expected
nofreq
missing
wrap
[no]key
nolabel
nolog
* firstonly
report Pearson's \(\chi^{2}\)
report Fisher's exact test
report Goodman and Kruskal's gamma
report likelihood-ratio \(\chi^{2}\)
report Kendall's \(\tau_{b}\)
report Cramér's \(V\)
report Pearson's \(\chi^{2}\) in each cell
report relative frequency within its column of each cell
report relative frequency within its row of each cell
report likelihood-ratio \(\chi^{2}\) in each cell
report the relative frequency of each cell
report expected frequency in each cell
do not display frequencies
treat missing values like other values
do not wrap wide tables
report/suppress cell contents key
display numeric codes rather than value labels
do not display enumeration log for Fisher's exact test show only tables that include the first variable in varlist

Advanced
matcell (matname)
matrow (matname)
matcol (matname)
\(\ddagger\) replace
all
save frequencies in matname; programmer's option save unique values of varname \(_{1}\) in matname; programmer's option save unique values of varname \(_{2}\) in matname; programmer's option replace current data with given cell frequencies
equivalent to specifying chi2 lrchi2 V gamma taub
*firstonly is available only for tab2.
\(\ddagger_{\text {replace is available only for tabi. }}\) by is allowed with tabulate and tab2; see [D] by.
fweights, aweights, and iweights are allowed by tabulate. fweights are allowed by tab2. See [U] 11.1.6 weight. all does not appear in the dialog box.

\section*{Menu}

\section*{tabulate}

Statistics \(>\) Summaries, tables, and tests \(>\) Frequency tables \(>\) Two-way table with measures of association

\section*{tab2}

Statistics \(>\) Summaries, tables, and tests \(>\) Frequency tables \(>\) All possible two-way tables

\section*{tabi}

Statistics \(>\) Summaries, tables, and tests \(>\) Frequency tables \(>\) Table calculator

\section*{Description}
tabulate produces a two-way table of frequency counts, along with various measures of association, including the common Pearson's \(\chi^{2}\), the likelihood-ratio \(\chi^{2}\), Cramér's \(V\), Fisher's exact test, Goodman and Kruskal's gamma, and Kendall's \(\tau_{b}\).

Line size is respected. That is, if you resize the Results window before running tabulate, the resulting two-way tabulation will take advantage of the available horizontal space. Stata for Unix(console) users can instead use the set linesize command to take advantage of this feature.
tab2 produces all possible two-way tabulations of the variables specified in varlist.
tabi displays the \(r \times c\) table, using the values specified; rows are separated by ' \(\backslash\) '. If no options are specified, it is as if exact were specified for a \(2 \times 2\) table and chi2 were specified otherwise. See [U] 19 Immediate commands for a general description of immediate commands. See Tables with immediate data below for examples using tabi.

See [R] tabulate oneway if you want a one-way table of frequencies. See [R] table and [R] tabstat if you want one-, two-, or \(n\)-way table of frequencies and a wide variety of summary statistics. See \([\mathrm{R}]\) tabulate, summarize() for a description of tabulate with the summarize() option; it produces a table (breakdowns) of means and standard deviations. table is better than tabulate, summarize(), but tabulate, summarize() is faster. See [ST] epitab for a \(2 \times 2\) table with statistics of interest to epidemiologists.

\section*{Options}

\section*{Main}
chi2 calculates and displays Pearson's \(\chi^{2}\) for the hypothesis that the rows and columns in a two-way table are independent. chi2 may not be specified if aweights or iweights are specified.
exact \([(\#)]\) displays the significance calculated by Fisher's exact test and may be applied to \(r \times c\) as well as to \(2 \times 2\) tables. For \(2 \times 2\) tables, both one- and two-sided probabilities are displayed. For \(r \times c\) tables, one-sided probabilities are displayed. The optional positive integer \# is a multiplier on the amount of memory that the command is permitted to consume. The default is 1 . This option should not be necessary for reasonable \(r \times c\) tables. If the command terminates with error 910, try exact (2). The maximum row or column dimension allowed when computing Fisher's exact test is the maximum row or column dimension for tabulate (see [ R\(]\) limits).
gamma displays Goodman and Kruskal's gamma along with its asymptotic standard error. gamma is appropriate only when both variables are ordinal. gamma may not be specified if aweights or iweights are specified.
Irchi2 displays the likelihood-ratio \(\chi^{2}\) statistic. Irchi2 may not be specified if aweights or iweights are specified.
taub displays Kendall's \(\tau_{b}\) along with its asymptotic standard error. taub is appropriate only when both variables are ordinal. taub may not be specified if aweights or iweights are specified.

V (note capitalization) displays Cramér's \(V\). V may not be specified if aweights or iweights are specified.
cchi2 displays each cell's contribution to Pearson's chi-squared in a two-way table.
column displays the relative frequency of each cell within its column in a two-way table.
row displays the relative frequency of each cell within its row in a two-way table.
clrchi2 displays each cell's contribution to the likelihood-ratio chi-squared in a two-way table.
cell displays the relative frequency of each cell in a two-way table.
expected displays the expected frequency of each cell in a two-way table.
nofreq suppresses the printing of the frequencies.
missing requests that missing values be treated like other values in calculations of counts, percentages, and other statistics.
wrap requests that Stata take no action on wide, two-way tables to make them readable. Unless wrap is specified, wide tables are broken into pieces to enhance readability.
[no] key suppresses or forces the display of a key above two-way tables. The default is to display the key if more than one cell statistic is requested, and otherwise to omit it. key forces the display of the key. nokey suppresses its display.
nolabel causes the numeric codes to be displayed rather than the value labels.
nolog suppresses the display of the log for Fisher's exact test. Using Fisher's exact test requires counting all tables that have a probability exceeding that of the observed table given the observed row and column totals. The log counts down each stage of the network computations, starting from the number of columns and counting down to 1 , displaying the number of nodes in the network at each stage. A \(\log\) is not displayed for \(2 \times 2\) tables.
firstonly, available only with tab2, restricts the output to only those tables that include the first variable in varlist. Use this option to interact one variable with a set of others.
matcell (matname) saves the reported frequencies in matname. This option is for use by programmers. matrow (matname) saves the numeric values of the \(r \times 1\) row stub in matname. This option is for use by programmers. matrow () may not be specified if the row variable is a string.
matcol (matname) saves the numeric values of the \(1 \times c\) column stub in matname. This option is for use by programmers. matcol() may not be specified if the column variable is a string.
replace indicates that the immediate data specified as arguments to the tabi command be left as the current data in place of whatever data were there.

The following option is available with tabulate but is not shown in the dialog box:
all is equivalent to specifying chi2 lrchi2 V gamma taub. Note the omission of exact. When all is specified, no may be placed in front of the other options. all noV requests all association measures except Cramér's \(V\) (and Fisher's exact). all exact requests all association measures, including Fisher's exact test. all may not be specified if aweights or iweights are specified.

\section*{Limits}

Two-way tables may have a maximum of 1,200 rows and 80 columns (Stata/MP and Stata/SE), 300 rows and 20 columns (Stata/IC), or 160 rows and 20 columns (Small Stata). If larger tables are needed, see \([R]\) table.

\section*{Remarks and examples}

Remarks are presented under the following headings:

\author{
tabulate \\ Measures of association \\ \(N\)-way tables \\ Weighted data \\ Tables with immediate data \\ tab2 \\ Video examples
}

For each value of a specified variable (or a set of values for a pair of variables), tabulate reports the number of observations with that value. The number of times a value occurs is called its frequency.

\section*{tabulate}
\(>\) Example 1
tabulate will make two-way tables if we specify two variables following the word tabulate. In our highway dataset, we have a variable called rate that divides the accident rate into three categories: below 4, 4-7, and above 7 per million vehicle miles. Let's make a table of the speed limit category and the accident-rate category:
```

. use http://www.stata-press.com/data/r13/hiway2
(Minnesota Highway Data, 1973)
. tabulate spdcat rate

| Speed <br> Limit <br> Category | Accident rate per million <br> vehicle miles <br> $4-7$ |  |  | Above 7 |
| ---: | ---: | ---: | ---: | ---: | Total

```

The table indicates that three stretches of highway have an accident rate below 4 and a speed limit of 40 to 50 miles per hour. The table also shows the row and column sums (called the marginals). The number of highways with a speed limit of 40 to 50 miles per hour is 11 , which is the same result we obtained in our previous one-way tabulations.

Stata can present this basic table in several ways-16, to be precise-and we will show just a few below. It might be easier to read the table if we included the row percentages. For instance, of 11 highways in the lowest speed limit category, three are also in the lowest accident-rate category. Three-elevenths amounts to some \(27.3 \%\). We can ask Stata to fill in this information for us by using the row option:
. tabulate spdcat rate, row

\begin{tabular}{r|rrr|r}
\begin{tabular}{r} 
Speed \\
Limit \\
Category
\end{tabular} & \multicolumn{3}{|c}{\begin{tabular}{r} 
Accident rate per million \\
vehicle miles \\
Below 4
\end{tabular}} & \(4-7\)
\end{tabular} Above 7 \(\quad\) Total

The number listed below each frequency is the percentage of cases that each cell represents out of its row. That is easy to remember because we see \(100 \%\) listed in the "Total" column. The bottom row is also informative. We see that \(61.54 \%\) of all the highways in our dataset fall into the lowest accident-rate category, that \(28.21 \%\) are in the middle category, and that \(10.26 \%\) are in the highest.
tabulate can calculate column percentages and cell percentages, as well. It does so when we specify the column or cell options, respectively. We can even specify them together. Below is a table that includes everything:
. tabulate spdcat rate, row column cell

\begin{tabular}{|c|c|c|c|c|}
\hline \begin{tabular}{l}
Speed \\
Limit
\end{tabular} & \multicolumn{3}{|l|}{Accident rate per million vehicle miles} & \multirow[b]{2}{*}{Total} \\
\hline Category & Below 4 & 4-7 & Above 7 & \\
\hline \multirow[t]{4}{*}{40 to 50} & 3 & 5 & 3 & 11 \\
\hline & 27.27 & 45.45 & 27.27 & 100.00 \\
\hline & 12.50 & 45.45 & 75.00 & 28.21 \\
\hline & 7.69 & 12.82 & 7.69 & 28.21 \\
\hline \multirow[t]{4}{*}{55 to 50} & 19 & 6 & 1 & 26 \\
\hline & 73.08 & 23.08 & 3.85 & 100.00 \\
\hline & 79.17 & 54.55 & 25.00 & 66.67 \\
\hline & 48.72 & 15.38 & 2.56 & 66.67 \\
\hline \multirow[t]{4}{*}{Above 60} & 2 & 0 & 0 & 2 \\
\hline & 100.00 & 0.00 & 0.00 & 100.00 \\
\hline & 8.33 & 0.00 & 0.00 & 5.13 \\
\hline & 5.13 & 0.00 & 0.00 & 5.13 \\
\hline \multirow[t]{4}{*}{Total} & 24 & 11 & 4 & 39 \\
\hline & 61.54 & 28.21 & 10.26 & 100.00 \\
\hline & 100.00 & 100.00 & 100.00 & 100.00 \\
\hline & 61.54 & 28.21 & 10.26 & 100.00 \\
\hline
\end{tabular}

The number at the top of each cell is the frequency count. The second number is the row percentage - they sum to \(100 \%\) going across the table. The third number is the column percentage - they sum to \(100 \%\) going down the table. The bottom number is the cell percentage - they sum to \(100 \%\) going down all the columns and across all the rows. For instance, highways with a speed limit above 60 miles per hour and in the lowest accident rate category account for \(100 \%\) of highways with a speed limit above 60 miles per hour; \(8.33 \%\) of highways in the lowest accident-rate category; and \(5.13 \%\) of all our data.

A fourth option, nofreq, tells Stata not to print the frequency counts. To construct a table consisting of only row percentages, we type
\begin{tabular}{r|rrr|r}
. tabulate spdcat rate, row nofreq \\
\begin{tabular}{r} 
Speed \\
Limit \\
Category
\end{tabular} & \begin{tabular}{r} 
Accident rate per million \\
vehicle miles \\
Below 4
\end{tabular} & \(4-7\) & Above 7 & Total \\
\hline 40 to 50 & 27.27 & 45.45 & 27.27 & 100.00 \\
55 to 50 & 73.08 & 23.08 & 3.85 & 100.00 \\
Above 60 & 100.00 & 0.00 & 0.00 & 100.00 \\
\hline Total & 61.54 & 28.21 & 10.26 & 100.00
\end{tabular}

\section*{Measures of association}

\section*{Example 2}
tabulate will calculate the Pearson \(\chi^{2}\) test for the independence of the rows and columns if we specify the chi2 option. Suppose that we have 1980 census data on 956 cities in the United States and wish to compare the age distribution across regions of the country. Assume that agecat is the median age in each city and that region denotes the region of the country in which the city is located.
. use http://www.stata-press.com/data/r13/citytemp2
(City Temperature Data)
• tabulate region agecat, chi2
Census
Region

We obtain the standard two-way table and, at the bottom, a summary of the \(\chi^{2}\) test. Stata informs us that the \(\chi^{2}\) associated with this table has 6 degrees of freedom and is 61.29 . The observed differences are significant.

The table is, perhaps, easier to understand if we suppress the frequencies and print just the row percentages:
\(\left.\begin{array}{l}\text {. tabulate region agecat, row nofreq chi2 } \\ \begin{array}{r}\text { Census } \\ \text { Region }\end{array} \\ \hline \text { NE } \\ \text { agecat } \\ 30-34\end{array}\right)\)

\section*{Example 3}

We have data on dose level and outcome for a set of patients and wish to evaluate the association between the two variables. We can obtain all the association measures by specifying the all and exact options:
```

. use http://www.stata-press.com/data/r13/dose
. tabulate dose function, all exact
Enumerating sample-space combinations:
stage 3: enumerations = 1
stage 2: enumerations = 9
stage 1: enumerations = 0

| Dosage | Function |  |  | Total |
| :---: | :---: | :---: | :---: | :---: |
| 1/day | 20 | 10 | 2 | 32 |
| 2/day | 16 | 12 | 4 | 32 |
| 3/day | 10 | 16 | 6 | 32 |
| Total | 46 | 38 | 12 | 96 |
|  | Pearson chi2 $(4)=6.7780$ | 6.7780 | $\operatorname{Pr}=0.148$ |  |
| likelihood-ratio chi2(4) = |  | $=6.9844$ | $\operatorname{Pr}=0.137$ |  |
| Cramr's V = 0.1879 |  |  |  |  |
|  | gamm | 0.3689 | $\mathrm{ASE}=0$ |  |
| Kendall's tau |  | 0.2378 | ASE $=0.086$ |  |
| Fisher's exact = |  |  |  |  |

```

We find evidence of association but not enough to be truly convincing.
If we had not also specified the exact option, we would not have obtained Fisher's exact test. Stata can calculate this statistic both for \(2 \times 2\) tables and for \(r \times c\). For \(2 \times 2\) tables, the calculation is almost instant. On more general tables, however, the calculation can take longer.

We carefully constructed our example so that all would be meaningful. Kendall's \(\tau_{b}\) and Goodman and Kruskal's gamma are relevant only when both dimensions of the table can be ordered, say, from low to high or from worst to best. The other statistics, however, are always applicable.

\section*{- Technical note}

Be careful when attempting to compute the \(p\)-value for Fisher's exact test because the number of tables that contribute to the \(p\)-value can be extremely large and a solution may not be feasible. The errors that are indicative of this situation are errors 910, exceeded memory limitations, and 1401, integer overflow due to large row-margin frequencies. If execution terminates because of memory limitations, use exact (2) to permit the algorithm to consume twice the memory, exact (3) for three times the memory, etc. The default memory usage should be sufficient for reasonable tables.

\section*{N -way tables}

If you need more than two-way tables, your best alternative to is use table, not tabulate; see [R] table.

The technical note below shows you how to use tabulate to create a sequence of two-way tables that together form, in effect, a three-way table, but using table is easy and produces prettier results:
```

. use http://www.stata-press.com/data/r13/birthcat
(City data)

```
. table birthcat region agecat, c(freq)
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline \multirow{2}{*}{birthcat} & \multicolumn{9}{|c|}{agecat and Census Region} \\
\hline & NE & N Cntrl & South & West & NE & N & Cntrl & South & West \\
\hline 29-136 & 11 & 23 & 11 & 11 & 34 & & 27 & 10 & 8 \\
\hline 137-195 & 31 & 97 & 65 & 46 & 48 & & 58 & 45 & 42 \\
\hline 196-529 & 4 & 38 & 59 & 91 & 1 & & 3 & 12 & 21 \\
\hline
\end{tabular}
\begin{tabular}{r|rrrr}
\hline & \multicolumn{4}{c}{ agecat and Census Region } \\
\cline { 4 - 5 } birthcat & \multicolumn{2}{c}{\(35+\)} & \multicolumn{2}{c}{ West } \\
\cline { 4 - 6 } & NE & N Cntrl & South & West \\
\hline \(29-136\) & 34 & 26 & 27 & 18 \\
\(137-195\) & 3 & 4 & 7 & 4 \\
\(196-529\) & & & 4 & \\
\hline
\end{tabular}

\section*{Technical note}

We can make \(n\)-way tables by combining the by varlist: prefix with tabulate. Continuing with the dataset of 956 cities, say that we want to make a table of age category by birth-rate category by region of the country. The birth-rate category variable is named birthcat in our dataset. To make separate tables for each age category, we would type
. by agecat, sort: tabulate birthcat region
```

-> agecat = 19-29

```
\begin{tabular}{r|rrrrr|r} 
& \multicolumn{5}{|c}{ Census Region } & Total \\
birthcat & NE & N Cntrl & South & West & Tot \\
\hline \(29-136\) & 11 & 23 & 11 & 11 & 56 \\
\(137-195\) & 31 & 97 & 65 & 46 & 239 \\
\(196-529\) & 4 & 38 & 59 & 91 & 192 \\
\hline Total & 46 & 158 & 135 & 148 & 487
\end{tabular}
-> agecat \(=30-34\)
\begin{tabular}{r|rrrrrr} 
& \multicolumn{5}{|c}{ Census Region } & Total \\
birthcat & NE & N Cntrl & South & West & Tol \\
\hline \(29-136\) & 34 & 27 & 10 & 8 & 79 \\
\(137-195\) & 48 & 58 & 45 & 42 & 193 \\
\(196-529\) & 1 & 3 & 12 & 21 & 37 \\
\hline Total & 83 & 88 & 67 & 71 & 309
\end{tabular}
\begin{tabular}{r|rrrrrrr}
\(\rightarrow\) agecat \(=35+\) \\
birthcat & NE & N Census & Region & South & West & Total \\
\hline \(29-136\) & 34 & 26 & 27 & 18 & 105 \\
\(137-195\) & 3 & 4 & 7 & 4 & 18 \\
\(196-529\) & 0 & 0 & 4 & 0 & 4 \\
\hline Total & 37 & 30 & 38 & 22 & 127
\end{tabular}

\section*{Weighted data}

\section*{Example 4}
tabulate can process weighted as well as unweighted data. As with all Stata commands, we indicate the weight by specifying the [weight] modifier; see [U] 11.1.6 weight.

Continuing with our dataset of 956 cities, we also have a variable called pop, the population of each city. We can make a table of region by age category, weighted by population, by typing
\begin{tabular}{|c|c|c|c|c|}
\hline Census Region & 19-29 & \[
\begin{aligned}
& \text { agecat } \\
& 30-34
\end{aligned}
\] & 35+ & Total \\
\hline NE & 4,721,387 & 10,421,387 & 5,323,610 & 20,466,384 \\
\hline N Cntrl & 16,901,550 & 8,964,756 & 4,015,593 & 29,881,899 \\
\hline South & 13,894,254 & 7,686,531 & 4,141,863 & 25,722,648 \\
\hline West & 16,698,276 & 7,755,255 & 2,375,118 & 26,828,649 \\
\hline Total & 52,215,467 & 34,827,929 & 15,856,184 & 102899580 \\
\hline
\end{tabular}

If we specify the cell, column, or row options, they will also be appropriately weighted. Below we repeat the table, suppressing the counts and substituting row percentages:
\begin{tabular}{r|rrr|r}
. tabulate region agecat \begin{tabular}{r} 
[freq=pop], nofreq row \\
\begin{tabular}{r} 
Census \\
Region
\end{tabular} \\
\hline \(19-29\)
\end{tabular} \begin{tabular}{r} 
agecat \\
\(30-34\)
\end{tabular} & \(35+\) & Total \\
\hline NE & 23.07 & 50.92 & 26.01 & 100.00 \\
N Cntrl & 56.56 & 30.00 & 13.44 & 100.00 \\
South & 54.02 & 29.88 & 16.10 & 100.00 \\
West & 62.24 & 28.91 & 8.85 & 100.00 \\
\hline Total & 50.74 & 33.85 & 15.41 & 100.00
\end{tabular}

\section*{Tables with immediate data}

\section*{\(>\) Example 5}
tabi ignores the dataset in memory and uses as the table the values that we specify on the command line:


We may specify any of the options of tabulate and are not limited to \(2 \times 2\) tables:
```

. tabi 30 18 38 \ 13 7 22, chi2 exact
Enumerating sample-space combinations:
stage 3: enumerations = 1
stage 2: enumerations = 3
stage 1: enumerations = 0

```
\begin{tabular}{r|rrr|r} 
row & 1 & 2 & 3 & Total \\
\hline 1 & 30 & 18 & 38 & 86 \\
2 & 13 & 7 & 22 & 42 \\
\hline Total & 43 & 25 & 60 & 128 \\
Pearson chi2(2) \(=\) & 0.7967 & \(\operatorname{Pr}=0.671\) & \\
Fisher's exact \(=\) & & 0.707
\end{tabular}
. tabi \(3013 \backslash 187\) \ 3822 , all exact col
\begin{tabular}{|l|}
\hline Key \\
\hline frequency \\
column percentage \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|}
\hline \multicolumn{4}{|l|}{\multirow[t]{2}{*}{```
Enumerating sample-space combinations:
stage 3: enumerations = 1
stage 2: enumerations = 3
stage 1: enumerations = 0
```}} \\
\hline & & & \\
\hline \multirow[t]{2}{*}{1} & 30 & 13 & 43 \\
\hline & 34.88 & 30.95 & 33.59 \\
\hline \multirow[t]{2}{*}{2} & 18 & 7 & 25 \\
\hline & 20.93 & 16.67 & 19.53 \\
\hline \multirow[t]{2}{*}{3} & 38 & 22 & 60 \\
\hline & 44.19 & 52.38 & 46.88 \\
\hline \multirow[t]{2}{*}{Total} & 86 & 42 & 128 \\
\hline & 100.00 & 100.00 & 100.00 \\
\hline
\end{tabular}
```

    Pearson chi2(2) = 0.7967 Pr = 0.671
    likelihood-ratio chi2(2) = 0.7985 Pr = 0.671
Cramr's V = 0.0789
gamma = 0.1204 ASE = 0.160
Kendall's tau-b = 0.0630 ASE = 0.084
Fisher's exact = 0.707

```

For \(2 \times 2\) tables, both one- and two-sided Fisher's exact probabilities are displayed; this is true of both tabulate and tabi. See Cumulative incidence data and Case-control data in [ST] epitab for more discussion on the relationship between one- and two-sided probabilities.

\section*{Technical note}
tabi, as with all immediate commands, leaves any data in memory undisturbed. With the replace option, however, the data in memory are replaced by the data from the table:
. tabi 3018 \} 3 8 \text { 14, replace } \(\\{\text { row }} \end{array}\)
\begin{tabular}{l|rrr|}
\cline { 2 - 3 } & row & col & pop \\
\cline { 2 - 3 } 1. & 1 & 1 & 30 \\
2. & 1 & 2 & 18 \\
3. & 2 & 1 & 38 \\
4. & 2 & 2 & 14 \\
\cline { 2 - 3 } & & &
\end{tabular}

With this dataset, you could re-create the above table by typing
• tabulate row col [freq=pop], exact
row

\section*{tab2}
tab2 is a convenience tool. Typing
. tab2 myvar thisvar thatvar, chi2
is equivalent to typing
```

. tabulate myvar thisvar, chi2

```
. tabulate myvar thatvar, chi2
. tabulate thisvar thatvar, chi2

\section*{Video examples}

Pearson's chi2 and Fisher's exact test in Stata
Tables and cross-tabulations in Stata
Immediate commands in Stata: Cross-tabulations and chi-squared tests from summary data

\section*{Stored results}
tabulate, tab2, and tabi store the following in \(r()\) :
Scalars
\begin{tabular}{|c|c|c|c|}
\hline \(\mathrm{r}(\mathrm{N})\) & number of observations & r(p_exact) & Fisher's exact \(p\) \\
\hline \(\mathrm{r}(\mathrm{r})\) & number of rows & r (chi2_lr) & likelihood-ratio \(\chi^{2}\) \\
\hline r (c) & number of columns & r(p_lr) & significance of likelihood-ratio \(\chi^{2}\) \\
\hline \(r\) (chi2) & Pearson's \(\chi^{2}\) & \(r\) (CramersV) & Cramér's V \\
\hline \(\mathrm{r}(\mathrm{p})\) & significance of Pearson's \(\chi^{2}\) & r(ase_gam) & ASE of gamma \\
\hline r(gamma) & gamma & r(ase_taub) & ASE of \(\tau_{\mathrm{b}}\) \\
\hline r(p1_exact & one-sided Fisher's exact \(p\) & r (taub) & \(\tau_{\mathrm{b}}\) \\
\hline
\end{tabular}
\(\mathrm{r}\left(\mathrm{p} 1 \_\right.\)exact ) is defined only for \(2 \times 2\) tables. Also, the matrow(), matcol(), and matcell() options allow you to obtain the row values, column values, and frequencies, respectively.

\section*{Methods and formulas}

Let \(n_{i j}, i=1, \ldots, I\) and \(j=1, \ldots, J\), be the number of observations in the \(i\) th row and \(j\) th column. If the data are not weighted, \(n_{i j}\) is just a count. If the data are weighted, \(n_{i j}\) is the sum of the weights of all data corresponding to the \((i, j)\) cell.

Define the row and column marginals as
\[
n_{i}=\sum_{j=1}^{J} n_{i j} \quad n_{\cdot j}=\sum_{i=1}^{I} n_{i j}
\]
and let \(n=\sum_{i} \sum_{j} n_{i j}\) be the overall sum. Also, define the concordance and discordance as
\[
A_{i j}=\sum_{k>i} \sum_{l>j} n_{k l}+\sum_{k<i} \sum_{l<j} n_{k l} \quad D_{i j}=\sum_{k>i} \sum_{l<j} n_{k l}+\sum_{k<i} \sum_{l>j} n_{k l}
\]
along with twice the number of concordances \(P=\sum_{i} \sum_{j} n_{i j} A_{i j}\) and twice the number of discordances \(Q=\sum_{i} \sum_{j} n_{i j} D_{i j}\).

The Pearson \(\chi^{2}\) statistic with \((I-1)(J-1)\) degrees of freedom (so called because it is based on Pearson (1900); see Conover [1999, 240] and Fienberg [1980, 9]) is defined as
\[
X^{2}=\sum_{i} \sum_{j} \frac{\left(n_{i j}-m_{i j}\right)^{2}}{m_{i j}}
\]
where \(m_{i j}=n_{i \cdot n \cdot j} / n\).
The likelihood-ratio \(\chi^{2}\) statistic with \((I-1)(J-1)\) degrees of freedom (Fienberg 1980, 40) is defined as
\[
G^{2}=2 \sum_{i} \sum_{j} n_{i j} \ln \left(n_{i j} / m_{i j}\right)
\]

Cramér's \(V\) (Cramér 1946) is a measure of association designed so that the attainable upper bound is 1 . For \(2 \times 2\) tables, \(-1 \leq V \leq 1\), and otherwise, \(0 \leq V \leq 1\).
\[
V= \begin{cases}\left(n_{11} n_{22}-n_{12} n_{21}\right) /\left(n_{1} \cdot n_{2} \cdot n_{1} n_{\cdot 2}\right)^{1 / 2} & \text { for } 2 \times 2 \\ \left\{\left(X^{2} / n\right) / \min (I-1, J-1)\right\}^{1 / 2} & \text { otherwise }\end{cases}
\]

Gamma (Goodman and Kruskal 1954, 1959, 1963, 1972; also see Agresti [2010,186-188]) ignores tied pairs and is based only on the number of concordant and discordant pairs of observations, \(-1 \leq \gamma \leq 1\),
\[
\gamma=(P-Q) /(P+Q)
\]
with asymptotic variance
\[
16 \sum_{i} \sum_{j} n_{i j}\left(Q A_{i j}-P D_{i j}\right)^{2} /(P+Q)^{4}
\]

Kendall's \(\tau_{b}\) (Kendall 1945; also see Agresti 2010, 188-189), \(-1 \leq \tau_{b} \leq 1\), is similar to gamma, except that it uses a correction for ties,
\[
\tau_{b}=(P-Q) /\left(w_{r} w_{c}\right)^{1 / 2}
\]
with asymptotic variance
\[
\frac{\sum_{i} \sum_{j} n_{i j}\left(2 w_{r} w_{c} d_{i j}+\tau_{b} v_{i j}\right)^{2}-n^{3} \tau_{b}^{2}\left(w_{r}+w_{c}\right)^{2}}{\left(w_{r} w_{c}\right)^{4}}
\]
where
\[
\begin{aligned}
w_{r} & =n^{2}-\sum_{i} n_{i \cdot}^{2} \\
w_{c} & =n^{2}-\sum_{j} n_{\cdot j}^{2} \\
d_{i j} & =A_{i j}-D_{i j} \\
v_{i j} & =n_{i} \cdot w_{c}+n \cdot{ }^{2} w_{r}
\end{aligned}
\]

Fisher's exact test (Fisher 1935; Finney 1948; see Zelterman and Louis [1992, 293-301] for the \(2 \times 2\) case) yields the probability of observing a table that gives at least as much evidence of association as the one actually observed under the assumption of no association. Holding row and column marginals fixed, the hypergeometric probability \(P\) of every possible table \(A\) is computed, and the
\[
P=\sum_{T \in A} \operatorname{Pr}(T)
\]
where \(A\) is the set of all tables with the same marginals as the observed table, \(T^{*}\), such that \(\operatorname{Pr}(T) \leq \operatorname{Pr}\left(T^{*}\right)\). For \(2 \times 2\) tables, the one-sided probability is calculated by further restricting \(A\) to tables in the same tail as \(T^{*}\). The first algorithm extending this calculation to \(r \times c\) tables was Pagano and Halvorsen (1981); the one implemented here is the FEXACT algorithm by Mehta and Patel (1986). This is a search-tree clipping method originally published by Mehta and Patel (1983) with further refinements by Joe (1988) and Clarkson, Fan, and Joe (1993). Fisher's exact test is a permutation test. For more information on permutation tests, see Good (2005 and 2006) and Pesarin (2001).

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\section*{Also see}
[R] table - Flexible table of summary statistics
[R] tabstat - Compact table of summary statistics
[R] tabulate oneway - One-way table of frequencies
[R] tabulate, summarize() - One- and two-way tables of summary statistics
[D] collapse - Make dataset of summary statistics
[ST] epitab - Tables for epidemiologists
[SVY] svy: tabulate oneway - One-way tables for survey data
[SVY] svy: tabulate twoway - Two-way tables for survey data
[XT] xttab - Tabulate xt data
[U] 12.6.3 Value labels
[U] 25 Working with categorical data and factor variables

\section*{Title}
tabulate, summarize() - One- and two-way tables of summary statistics
\begin{tabular}{lll} 
Syntax & Menu & Description \\
Remarks and examples & Also see &
\end{tabular}

\section*{Syntax}
\({\text { tabulate } \text { varname }_{1}\left[\text { varname }_{2}\right][\text { if }][\mathrm{in}][\text { weight }][\text {, options }]}^{\text {, }}\)
options
Description
Main
summarize( varname \(_{3}\) )
[no] means
[no] standard
[no] freq
[no] obs
nolabel
wrap
missing
report summary statistics for varname \({ }_{3}\)
include or suppress means
include or suppress standard deviations
include or suppress frequencies
include or suppress number of observations
show numeric codes, not labels
do not break wide tables
treat missing values of varname \(_{1}\) and varname \(_{2}\) as categories
by is allowed; see [D] by.
aweights and fweights are allowed; see [U] 11.1.6 weight.

\section*{Menu}

Statistics \(>\) Summaries, tables, and tests \(>\) Other tables \(>\) Table of means, std. dev., and frequencies

\section*{Description}
tabulate, summarize() produces one- and two-way tables (breakdowns) of means and standard deviations. See \([R]\) tabulate oneway and \([R]\) tabulate twoway for one- and two-way frequency tables. See \([R]\) table for a more flexible command that produces one-, two-, and \(n\)-way tables of frequencies and a wide variety of summary statistics. table is better, but tabulate, summarize() is faster. Also see \([R]\) tabstat for yet another alternative.

\section*{Options}

\footnotetext{
Main
summarize ( varname \(_{3}\) ) identifies the name of the variable for which summary statistics are to be reported. If you do not specify this option, a table of frequencies is produced; see \([R]\) tabulate oneway and \([R]\) tabulate twoway. The description here concerns tabulate when this option is specified.
}
[no]means includes or suppresses only the means from the table.
The summarize() table normally includes the mean, standard deviation, frequency, and, if the data are weighted, number of observations. Individual elements of the table may be included or suppressed by the [no]means, [no]standard, [no]freq, and [no]obs options. For example, typing
```

. tabulate category, summarize(myvar) means standard

```
produces a summary table by category containing only the means and standard deviations of myvar. You could also achieve the same result by typing
```

. tabulate category, summarize(myvar) nofreq

```
[no]standard includes or suppresses only the standard deviations from the table; see [no]means option above.
[no]freq includes or suppresses only the frequencies from the table; see [no]means option above.
[no]obs includes or suppresses only the reported number of observations from the table. If the data are not weighted, the number of observations is identical to the frequency, and by default only the frequency is reported. If the data are weighted, the frequency refers to the sum of the weights. See [no]means option above.
nolabel causes the numeric codes to be displayed rather than the label values.
wrap requests that no action be taken on wide tables to make them readable. Unless wrap is specified, wide tables are broken into pieces to enhance readability.
missing requests that missing values of varname \({ }_{1}\) and varname \(_{2}\) be treated as categories rather than as observations to be omitted from the analysis.

\section*{Remarks and examples}
tabulate with the summarize() option produces one- and two-way tables of summary statistics. When combined with the by prefix, it can produce \(n\)-way tables as well.

Remarks are presented under the following headings:

> One-way tables
> Two-way tables

\section*{One-way tables}

\section*{> Example 1}

We have data on 74 automobiles. Included in our dataset are the variables foreign, which marks domestic and foreign cars, and mpg, the car's mileage rating. Typing tabulate foreign displays a breakdown of the number of observations we have by the values of the foreign variable.
\begin{tabular}{l}
. use http://www.stata-press.com/data/r13/auto \\
(1978 Automobile Data) \\
. tabulate foreign \\
\multicolumn{2}{c}{ Car type } & Freq. & Percent & Cum. \\
\hline Domestic \\
Foreign
\end{tabular}

We discover that we have 52 domestic cars and 22 foreign cars in our dataset. If we add the summarize (varname) option, however, tabulate produces a table of summary statistics for varname:
\begin{tabular}{r|rrr}
. tabulate foreign, summarize(mpg) & \\
Car type & \begin{tabular}{r} 
Summary of Mileage (mpg) \\
Mean
\end{tabular} & \begin{tabular}{rl} 
Std. Dev.
\end{tabular} & Freq. \\
\hline Domestic & 19.826923 & 4.7432972 & 52 \\
Foreign & 24.772727 & 6.6111869 & 22 \\
\hline Total & 21.297297 & 5.7855032 & 74
\end{tabular}

We also discover that the average gas mileage for domestic cars is about 20 mpg and the average foreign is almost 25 mpg . Overall, the average is 21 mpg in our dataset.

\section*{- Technical note}

We might now wonder if the difference in gas mileage between foreign and domestic cars is statistically significant. We can use the oneway command to find out; see [R] oneway. To obtain an analysis-of-variance table of mpg on foreign, we type
. oneway mpg foreign
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multicolumn{6}{|c|}{Analysis of Variance} \\
\hline Source & SS & df & MS & F & Prob > F \\
\hline Between groups & 378.153515 & 1 & 378.153515 & 13.18 & 0.0005 \\
\hline Within groups & 2065.30594 & 72 & 28.6848048 & & \\
\hline Total & 2443.45946 & 73 & 33.4720474 & & \\
\hline Bartlett's test & equal vari & : & i2(1) = & 8 Pro & chi2 \(=0\) \\
\hline
\end{tabular}

The \(F\) statistic is 13.18 , and the difference between foreign and domestic cars' mileage ratings is significant at the \(0.05 \%\) level.

There are several ways that we could have statistically compared mileage ratings-see, for instance, \([R]\) anova, \([R]\) oneway, \([R]\) regress, and \([R]\) ttest-but oneway seemed the most convenient.

\section*{Two-way tables}

\section*{\(>\) Example 2}
tabulate, summarize can be used to obtain two-way as well as one-way breakdowns. For instance, we obtained summary statistics on mpg decomposed by foreign by typing tabulate foreign, summarize (mpg). We can specify up to two variables before the comma:
. generate wgtcat \(=\) autocode(weight, \(4,1760,4840)\)
. tabulate wgtcat foreign, summarize(mpg)
\begin{tabular}{r|rr|r}
\multicolumn{3}{c}{ Means, Standard Deviations } & and Frequen \\
wgtcat & \multicolumn{2}{|c}{ Car } & type \\
& Domestic & Foreign & Total \\
\hline 2530 & 28.285714 & 27.0625 & 27.434783 \\
& 3.0937725 & 5.9829619 & 5.2295149 \\
& 7 & 16 & 23 \\
\hline 3300 & 21.75 & 19.6 & 21.238095 \\
& 2.4083189 & 3.4351128 & 2.7500819 \\
& 16 & 5 & 21 \\
\hline 4070 & 17.26087 & 14 & 17.125 \\
& 1.8639497 & 0 & 1.9406969 \\
& 23 & 1 & 24 \\
\hline 4840 & 14.666667 &. & 14.666667 \\
& 3.32666 &. & 3.32666 \\
& 6 & 0 & 6 \\
\hline Total & 19.826923 & 24.772727 & 21.297297 \\
& 4.7432972 & 6.6111869 & 5.7855032 \\
& 52 & 22 & 74
\end{tabular}

In addition to the means, standard deviations, and frequencies for each weight-mileage cell, also reported are the summary statistics by weight, by mileage, and overall. For instance, the last row of the table reveals that the average mileage of domestic cars is 19.83 and that of foreign cars is 24.77-domestic cars yield poorer mileage than foreign cars. But we now see that domestic cars yield better gas mileage within weight class - the reason domestic cars yield poorer gas mileage is because they are, on average, heavier.

\section*{> Example 3}

If we do not specify the statistics to be included in a table, tabulate reports the mean, standard deviation, and frequency. We can specify the statistics that we want to see using the means, standard, and freq options:
```

. tabulate wgtcat foreign, summarize(mpg) means

```

Means of Mileage (mpg)
\begin{tabular}{r|rr|r}
\multirow{2}{*}{ wgtcat } & \multicolumn{2}{|c}{ Car type } & Domestic \\
& Foreign & Total \\
\hline 2530 & 28.285714 & 27.0625 & 27.434783 \\
3300 & 21.75 & 19.6 & 21.238095 \\
4070 & 17.26087 & 14 & 17.125 \\
4840 & 14.666667 & \(\cdot\) & 14.666667 \\
\hline Total & 19.826923 & 24.772727 & 21.297297
\end{tabular}

When we specify one or more of the means, standard, and freq options, only those statistics are displayed. Thus we could obtain a table containing just the means and standard deviations by typing means standard after the summarize (mpg) option. We can also suppress selected statistics by placing no in front of the option name. Another way of obtaining only the means and standard deviations is to add the nofreq option:
\begin{tabular}{|c|c|c|c|}
\hline \multirow[b]{2}{*}{wgtcat} & \multicolumn{2}{|l|}{Means and Standard Car type} & \multirow[b]{2}{*}{Total} \\
\hline & Domestic & Foreign & \\
\hline \multirow[t]{2}{*}{2530} & 28.285714 & 27.0625 & 27.434783 \\
\hline & 3.0937725 & 5.9829619 & 5.2295149 \\
\hline \multirow[t]{2}{*}{3300} & 21.75 & 19.6 & 21.238095 \\
\hline & 2.4083189 & 3.4351128 & 2.7550819 \\
\hline \multirow[t]{2}{*}{4070} & 17.26087 & 14 & 17.125 \\
\hline & 1.8639497 & 0 & 1.9406969 \\
\hline \multirow[t]{2}{*}{4840} & 14.666667 & . & 14.666667 \\
\hline & 3.32666 & . & 3.32666 \\
\hline \multirow[t]{2}{*}{Total} & 19.826923 & 24.772727 & 21.297297 \\
\hline & 4.7432972 & 6.6111869 & 5.7855032 \\
\hline
\end{tabular}

\section*{Also see}
[R] table - Flexible table of summary statistics
[R] tabstat - Compact table of summary statistics
[R] tabulate oneway - One-way table of frequencies
[R] tabulate twoway - Two-way table of frequencies
[D] collapse - Make dataset of summary statistics
[SVY] svy: tabulate oneway - One-way tables for survey data
[SVY] svy: tabulate twoway - Two-way tables for survey data
[U] 12.6 Dataset, variable, and value labels
[U] 25 Working with categorical data and factor variables

\section*{Title}
test - Test linear hypotheses after estimation
\begin{tabular}{llll} 
Syntax & Menu & Description & Options for testparm \\
Options for test & Remarks and examples & Stored results & Methods and formulas \\
Acknowledgment & References & Also see &
\end{tabular}

\section*{Syntax}

\section*{Basic syntax}
```

test coeflist
(Syntax 1)
test exp=exp[=...]
test [eqno] [: coeflist]
(Syntax 3)
test [eqno=eqno[=...]] [: coeflist]
(Syntax 4)
testparm varlist [, equal equation(eqno)]

```

Full syntax
```

test (spec) [(spec) ...] [, test_options]

```
test_options Description

Options
mtest [(opt)]
coef
accumulate
notest
common
constant
nosvyadjust
minimum
matvlc (matname)
test each condition separately report estimated constrained coefficients
test hypothesis jointly with previously tested hypotheses suppress the output
test only variables common to all the equations include the constant in coefficients to be tested compute unadjusted Wald tests for survey results perform test with the constant, drop terms until the test becomes nonsingular, and test without the constant on the remaining terms; highly technical
coeflist and varlist may contain factor variables and time-series operators; see [U] 11.4.3 Factor variables and [U] 11.4.4 Time-series varlists.
matvlc(matname) does not appear in the dialog box.

Syntax 1 tests that coefficients are 0 .
Syntax 2 tests that linear expressions are equal.
Syntax 3 tests that coefficients in eqno are 0.
Syntax 4 tests equality of coefficients between equations.
spec is one of
\[
\begin{aligned}
& \text { coeflist } \\
& \exp =\exp [=\exp ] \\
& {[\text { eqno }][: \text { coeflist }]} \\
& {\left[\text { eqno }{ }_{1}=\text { eqno }_{2}[=\ldots]\right][: \text { coeflist }]}
\end{aligned}
\]
coeflist is
\[
\begin{aligned}
& \operatorname{coef}[\operatorname{coef} \ldots] \\
& {[\text { eqno }] \operatorname{coef}[[\text { eqno }] \text { coef } \ldots]} \\
& \left.[\text { eqno }] \_\mathrm{b}[\text { coef }][\text { eqno }] \ldots \mathrm{b}[\text { coef }] \ldots\right]
\end{aligned}
\]
\(\exp\) is a linear expression containing
coef
_b [coef]
_b [eqno:coef]
[eqno]coef
[eqno]_b[coef]
eqno is
\#\#
name
coef identifies a coefficient in the model. coef is typically a variable name, a level indicator, an interaction indicator, or an interaction involving continuous variables. Level indicators identify one level of a factor variable and interaction indicators identify one combination of levels of an interaction; see [U] 11.4.3 Factor variables. coef may contain time-series operators; see [U] 11.4.4 Time-series varlists.
Distinguish between [], which are to be typed, and [], which indicate optional arguments.
Although not shown in the syntax diagram, parentheses around spec are required only with multiple specifications. Also, the diagram does not show that test may be called without arguments to redisplay the results from the last test.
anova and manova (see [R] anova and [MV] manova) allow the test syntax above plus more (see [R] anova postestimation for test after anova; see [MV] manova postestimation for test after manova).

\section*{Menu}

\section*{test}

Statistics \(>\) Postestimation \(>\) Tests \(>\) Test linear hypotheses

\section*{testparm}

Statistics \(>\) Postestimation \(>\) Tests \(>\) Test parameters

\section*{Description}
test performs Wald tests of simple and composite linear hypotheses about the parameters of the most recently fit model.
test supports svy estimators (see [SVY] svy estimation), carrying out an adjusted Wald test by default in such cases. test can be used with svy estimation results, see [SVY] svy postestimation.
testparm provides a useful alternative to test that permits varlist rather than a list of coefficients (which is often nothing more than a list of variables), allowing the use of standard Stata notation, including '-' and ' \(*\) ', which are given the expression interpretation by test.
test and testparm perform Wald tests. For likelihood-ratio tests, see [R] Irtest. For Wald-type tests of nonlinear hypotheses, see \([R]\) testnl. To display estimates for one-dimensional linear or nonlinear expressions of coefficients, see \([R]\) lincom and \([R]\) nlcom.

See \([R]\) anova postestimation for additional test syntax allowed after anova.
See [MV] manova postestimation for additional test syntax allowed after manova.

\section*{Options for testparm}
equal tests that the variables appearing in varlist, which also appear in the previously fit model, are equal to each other rather than jointly equal to zero.
equation (eqno) is relevant only for multiple-equation models, such as mvreg, mlogit, and heckman. It specifies the equation for which the all-zero or all-equal hypothesis is tested. equation(\#1) specifies that the test be conducted regarding the first equation \#1. equation(price) specifies that the test concern the equation named price.

\section*{Options for test}
\(\underset{\text { mtest }[(o p t)] \text { specifies that tests be performed for each condition separately. opt specifies the method }}{ }\) for adjusting \(p\)-values for multiple testing. Valid values for opt are
\begin{tabular}{cl} 
bonferroni & Bonferroni's method \\
holm & Holm's method \\
sidak & Šidák's method \\
noadjust & no adjustment is to be made
\end{tabular}

Specifying mtest without an argument is equivalent to mtest (noadjust).
coef specifies that the constrained coefficients be displayed.
accumulate allows a hypothesis to be tested jointly with the previously tested hypotheses.
notest suppresses the output. This option is useful when you are interested only in the joint test of several hypotheses, specified in a subsequent call of test, accumulate.
common specifies that when you use the \(\left[\right.\) eqno \(\left._{1}=e q n o_{2}[=\ldots]\right]\) form of \(s p e c\), the variables common to the equations \(e^{2 n n o} o_{1}\), eqno 2 , etc., be tested. The default action is to complain if the equations have variables not in common.
constant specifies that _cons be included in the list of coefficients to be tested when using the \(\left[e^{2}\right.\) no \(_{1}=\) eqno \(\left._{2}[=\ldots]\right]\) or \([\) eqno \(]\) forms of spec. The default is not to include _cons.
nosvyadjust is for use with svy estimation commands; see [SVY] svy estimation. It specifies that the Wald test be carried out without the default adjustment for the design degrees of freedom. That is, the test is carried out as \(W / k \sim F(k, d)\) rather than as \((d-k+1) W /(k d) \sim F(k, d-k+1)\), where \(k=\) the dimension of the test and \(d=\) the total number of sampled PSUs minus the total number of strata.
minimum is a highly technical option. It first performs the test with the constant added. If this test is singular, coefficients are dropped until the test becomes nonsingular. Then the test without the constant is performed with the remaining terms.

The following option is available with test but is not shown in the dialog box:
matvlc(matname), a programmer's option, saves the variance-covariance matrix of the linear combinations involved in the suite of tests. For the test of the linear constraints \(L b=c\), matname contains \(L \mathbf{V} L^{\prime}\), where \(\mathbf{V}\) is the estimated variance-covariance matrix of \(b\).

\section*{Remarks and examples}

Remarks are presented under the following headings:

\author{
Introductory examples \\ Special syntaxes after multiple-equation estimation \\ Constrained coefficients \\ Multiple testing
}

\section*{Introductory examples}
test performs \(F\) or \(\chi^{2}\) tests of linear restrictions applied to the most recently fit model (for example, regress or svy: regress in the linear regression case; logit, stcox, svy: logit,... in the single-equation maximum-likelihood case; and mlogit, mvreg, streg, ... in the multipleequation maximum-likelihood case). test may be used after any estimation command, although for maximum likelihood techniques, test produces a Wald test that depends only on the estimate of the covariance matrix - you may prefer to use the more computationally expensive likelihood-ratio test; see [U] 20 Estimation and postestimation commands and [R] Irtest.

There are several variations on the syntax for test. The second syntax,
\[
\text { test } \exp =\exp [=. .]
\]
is allowed after any form of estimation. After fitting a model of depvar on x 1 , x 2 , and x 3 , typing test \(x 1+x 2=x 3\) tests the restriction that the coefficients on \(x 1\) and \(x 2\) sum to the coefficient on \(x 3\). The expressions can be arbitrarily complicated; for instance, typing test \(x 1+2 *(x 2+x 3)=x 2+3 * x 3\) is the same as typing test \(\mathrm{x} 1+\mathrm{x} 2=\mathrm{x} 3\).

As a convenient shorthand, test also allows you to specify equality for multiple expressions; for example, test \(\mathrm{x} 1+\mathrm{x} 2=\mathrm{x} 3+\mathrm{x} 4=\mathrm{x} 5+\mathrm{x} 6\) tests that the three specified pairwise sums of coefficients are equal.
test understands that when you type x 1 , you are referring to the coefficient on x 1 . You could also more explicitly type test \(\quad \mathrm{b}[\mathrm{x} 1]+\_\mathrm{b}[\mathrm{x} 2]=\_\mathrm{b}[\mathrm{x} 3]\); or you could test _coef \([\mathrm{x} 1]+\) _coef \([\mathrm{x} 2]=\) _coef \([\mathrm{x} 3]\), or test \([\# 1] \mathrm{x} 1+[\# 1] \mathrm{x} 2=[\# 1] \mathrm{x} 3\), or many other things because there is more than one way to refer to an estimated coefficient; see [U] 13.5 Accessing coefficients and standard errors. The shorthand involves less typing. On the other hand, you must be more explicit
after estimation of multiple-equation models because there may be more than one coefficient associated with an independent variable. You might type, for instance, test [\#2] x1+[\#2] x2=[\#2] x3 to test the constraint in equation 2 or, more readably, test [ford] x1+[ford] x2=[ford] x3, meaning that Stata will test the constraint on the equation corresponding to ford, which might be equation 2 . ford would be an equation name after, say, sureg, or, after mlogit, ford would be one of the outcomes. For mlogit, you could also type test [2] x1+[2] x2=[2] x3-note the lack of the \#-meaning not equation 2, but the equation corresponding to the numeric outcome 2. You can even test constraints across equations: test [ford]x1+[ford]x2=[buick]x3.

The syntax

\section*{test coeflist}
is available after all estimation commands and is a convenient way to test that multiple coefficients are zero following estimation. A coeflist can simply be a list of variable names,

> test varname [varname ...]
and it is most often specified that way. After you have fit a model of depvar on x 1 , x 2 , and x 3 , typing test \(\mathrm{x} 1 \times 3\) tests that the coefficients on x 1 and x 3 are jointly zero. After multiple-equation estimation, this would test that the coefficients on \(x 1\) and \(x 3\) are zero in all equations that contain them. You can also be more explicit and type, for instance, test [ford]x1 [ford]x3 to test that the coefficients on x 1 and x 3 are zero in the equation for ford.

In the multiple-equation case, there are more alternatives. You could also test that the coefficients on x 1 and x 3 are zero in the equation for ford by typing test [ford]: x1 x3. You could test that all coefficients except the coefficient on the constant are zero in the equation for ford by typing test [ford]. You could test that the coefficients on \(x 1\) and \(x 3\) in the equation for ford are equal to the corresponding coefficients in the equation corresponding to buick by typing test[ford=buick]: x 1 x 3 . You could test that all the corresponding coefficients except the constant in three equations are equal by typing test [ford=buick=volvo].
testparm is much like the first syntax of test. Its usefulness will be demonstrated below.
The examples below use regress, but what is said applies equally after any single-equation estimation command (such as logistic). It also applies after multiple-equation estimation commands as long as references to coefficients are qualified with an equation name or number in square brackets placed before them. The convenient syntaxes for dealing with tests of many coefficients in multipleequation models are demonstrated in Special syntaxes after multiple-equation estimation below.
> Example 1: Testing for a single coefficient against zero
We have 1980 census data on the 50 states recording the birth rate in each state (brate), the median age (medage), and the region of the country in which each state is located.

The region variable is 1 if the state is in the Northeast, 2 if the state is in the North Central, 3 if the state is in the South, and 4 if the state is in the West. We estimate the following regression:
```

. use http://www.stata-press.com/data/r13/census3
(1980 Census data by state)
. regress brate medage c.medage\#c.medage i.region

```
\begin{tabular}{r|rrc} 
Source & \multicolumn{1}{|c}{ SS } & df & MS \\
\hline Model & 38803.4208 & 5 & 7760.68416 \\
Residual & 3393.39921 & 44 & 77.1227094 \\
\hline Total & 42196.82 & 49 & 861.159592
\end{tabular}
\begin{tabular}{lrr} 
Number of obs & \(=\) & 50 \\
\(\mathrm{~F}(5\), & \(44)\) & \(=100.63\) \\
Prob \(>\) F & \(=\) & 0.0000 \\
R-squared & \(=\) & 0.9196 \\
Adj R-squared & \(=\) & 0.9104 \\
Root MSE & \(=8.782\)
\end{tabular}
\begin{tabular}{c|rrrrrr}
\hline brate & Coef. & Std. Err. & t & \(\mathrm{P}>|\mathrm{t}|\) & [95\% Conf. Interval] \\
\hline medage & -109.0958 & 13.52452 & -8.07 & 0.000 & -136.3527 & -81.83892 \\
c.medage\# & & & & & & \\
c.medage & 1.635209 & .2290536 & 7.14 & 0.000 & 1.173582 & 2.096836 \\
region & & & & & & \\
N Cntrl & 15.00283 & 4.252067 & 3.53 & 0.001 & 6.433353 & 23.57231 \\
South & 7.366445 & 3.953335 & 1.86 & 0.069 & -.6009775 & 15.33387 \\
West & 21.39679 & 4.650601 & 4.60 & 0.000 & 12.02412 & 30.76946 \\
_cons & 1947.611 & 199.8405 & 9.75 & 0.000 & 1544.859 & 2350.363 \\
\hline
\end{tabular}
test can now be used to perform a variety of statistical tests. Specify the coeflegend option with your estimation command to see a legend of the coefficients and how to specify them; see [R] estimation options. We can test the hypothesis that the coefficient on 3.region is zero by typing
```

. test 3.region=0
( 1) 3.region = 0
F( 1, 44) = 3.47
Prob > F = 0.0691

```

The \(F\) statistic with 1 numerator and 44 denominator degrees of freedom is 3.47 . The significance level of the test is \(6.91 \%\) - we can reject the hypothesis at the \(10 \%\) level but not at the \(5 \%\) level.

This result from test is identical to one presented in the output from regress, which indicates that the \(t\) statistic on the 3 .region coefficient is 1.863 and that its significance level is 0.069 . The \(t\) statistic presented in the output can be used to test the hypothesis that the corresponding coefficient is zero, although it states the test in slightly different terms. The \(F\) distribution with 1 numerator degree of freedom is, however, identical to the \(t^{2}\) distribution. We note that \(1.863^{2} \approx 3.47\) and that the significance levels in each test agree, although one extra digit is presented by the test command.

\section*{Technical note}

After all estimation commands, including those that use the maximum likelihood method, the test that one variable is zero is identical to that reported by the command's output. The tests are performed in the same way-using the estimated covariance matrix-and are known as Wald tests. If the estimation command reports significance levels and confidence intervals using \(z\) rather than \(t\) statistics, test reports results using the \(\chi^{2}\) rather than the \(F\) statistic.

\section*{> Example 2: Testing the value of a single coefficient}

If that were all test could do, it would be useless. We can use test, however, to perform other

```

. test 2.region=21
( 1) 2.region = 21
F( 1, 44) = 1.99
Prob > F = 0.1654

```

We find that we cannot reject that hypothesis, or at least we cannot reject it at any significance level below \(16.5 \%\).
> Example 3: Testing the equality of two coefficients
The previous test is useful, but we could almost as easily perform it by hand using the results presented in the regression output if we were well read on our statistics. We could type
```

. display Ftail(1,44,((_coef[2.region]-21)/4.252068)^2)
. }1654487

```

So, now let's test something a bit more difficult: whether the coefficient on 2. region is the same as the coefficient on 4.region:
```

. test 2.region=4.region
( 1) 2.region - 4.region = 0
F( 1, 44) = 2.84
Prob > F = 0.0989

```

We find that we cannot reject the equality hypothesis at the \(5 \%\) level, but we can at the \(10 \%\) level.

\section*{> Example 4}

When we tested the equality of the 2.region and 4.region coefficients, Stata rearranged our algebra. When Stata displayed its interpretation of the specified test, it indicated that we were testing whether 2.region minus 4.region is zero. The rearrangement is innocuous and, in fact, allows Stata to perform much more complicated algebra, for instance,
```

. test 2*(2.region-3*(3.region-4.region))=3.region+2.region+6*(4.region-3.region)
(1) 2.region - 3.region = 0
F( 1, 44) = 5.06
Prob > F = 0.0295

```

Although we requested what appeared to be a lengthy hypothesis, once Stata simplified the algebra, it realized that all we wanted to do was test whether the coefficient on 2. region is the same as the coefficient on 3.region.
- Technical note

Stata's ability to simplify and test complex hypotheses is limited to linear hypotheses. If you attempt to test a nonlinear hypothesis, you will be told that it is not possible:
```

. test 2.region/3.region=2.region+3.region
not possible with test
r(131);

```

To test a nonlinear hypothesis, see [R] testnl.

\section*{> Example 5: Testing joint hypotheses}

The real power of test is demonstrated when we test joint hypotheses. Perhaps we wish to test whether the region variables, taken as a whole, are significant by testing whether the coefficients on 2.region, 3.region, and 4.region are simultaneously zero. test allows us to specify multiple conditions to be tested, each embedded within parentheses.
```

. test (2.region=0) (3.region=0) (4.region=0)
( 1) 2.region = 0
( 2) 3.region = 0
( 3) 4.region = 0
F( 3, 44) = 8.85
Prob > F = 0.0001

```
test displays the set of conditions and reports an \(F\) statistic of 8.85 . test also reports the degrees of freedom of the test to be 3 , the "dimension" of the hypothesis, and the residual degrees of freedom, 44. The significance level of the test is close to 0 , so we can strongly reject the hypothesis of no difference between the regions.

An alternative method to specify simultaneous hypotheses uses the convenient shorthand of conditions with multiple equality operators.
```

. test 2.region=3.region=4.region=0
( 1) 2.region - 3.region = 0
( 2) 2.region - 4.region = 0
( 3) 2.region = 0
F( 3, 44) = 8.85
Prob > F = 0.0001

```
- Technical note

Another method to test simultaneous hypotheses is to specify a test for each constraint and accumulate it with the previous constraints:
```

. test 2.region=0
(1) 2.region = 0
F( 1, 44) = 12.45
Prob > F = 0.0010
. test 3.region=0, accumulate
( 1) 2.region = 0
( 2) 3.region = 0
F( 2, 44) = 6.42
Prob > F = 0.0036
. test 4.region=0, accumulate
( 1) 2.region = 0
( 2) 3.region = 0
( 3) 4.region = 0
F( 3, 44) = 8.85
Prob > F = 0.0001

```

We tested the hypothesis that the coefficient on 2.region was zero by typing test 2. region \(=0\). We then tested whether the coefficient on 3.region was also zero by typing test 3.region=0, accumulate. The accumulate option told Stata that this was not the start of a new test but a continuation of a previous one. Stata responded by showing us the two equations and reporting an \(F\) statistic of 6.42 . The significance level associated with those two coefficients being zero is \(0.36 \%\).

When we added the last constraint test 4.region=0, accumulate, we discovered that the three region variables are significant. If all we wanted was the overall significance and we did not want to bother seeing the interim results, we could have used the notest option:
```

. test 2.region=0, notest
( 1) 2.region = 0
. test 3.region=0, accumulate notest
( 1) 2.region = 0
( 2) 3.region = 0
. test 4.region=0, accumulate
( 1) 2.region = 0
(2) 3.region = 0
( 3) 4.region = 0
F( 3, 44) = 8.85
Prob > F = 0.0001

```

\section*{Example 6: Quickly testing coefficients against zero}

Because tests that coefficients are zero are so common in applied statistics, the test command has a more convenient syntax to accommodate this case:
```

. test 2.region 3.region 4.region
( 1) 2.region = 0
( 2) 3.region = 0
( 3) 4.region = 0
F( 3, 44) = 8.85
Prob > F = 0.0001

```

\section*{> Example 7: Specifying varlists}

We will now show how to use testparm. In its first syntax, test accepts a list of variable names but not a varlist.
```

. test i(2/4).region
i not found
r(111);

```

In the varlist, i(2/4).region means all the level variables from 2.region through 4.region, yet we received an error. test does not actually understand varlists, but testparm does. In fact, it understands only varlists.
```

. testparm i(2/4).region
( 1) 2.region = 0
( 2) 3.region = 0
( 3) 4.region = 0
F( 3, 44) = 8.85
Prob > F = 0.0001

```

Another way to test all the region variables is to type testparm i.region.
That testparm accepts varlists has other advantages that do not involve factor variables. Suppose that we have a dataset that has dummy variables reg2, reg3, and reg4, rather than the categorical variable region.
```

. use http://www.stata-press.com/data/r13/census4
(birth rate, median age)
. regress brate medage c.medage\#c.medage reg2 reg3 reg4
(output omitted)
. test reg2-reg4

- not found
r(111);

```

In a varlist, reg2-reg4 means variables reg2 and reg4 and all the variables between, yet we received an error. test is confused because the - has two meanings: it means subtraction in an expression and "through" in a varlist. Similarly, '*' means "any set of characters" in a varlist and multiplication in an expression. testparm avoids this confusion-it allows only a varlist.
```

. testparm reg2-reg4
(1) reg2 = 0
( 2) reg3 = 0
( 3) reg4 = 0
F( 3, 44) = 8.85
Prob > F = 0.0001

```
testparm has another advantage. We have five variables in our dataset that start with the characters reg: region, reg1, reg2, reg3, and reg4. reg* thus means those five variables:
. describe reg*
\begin{tabular}{lllll} 
variable name & \begin{tabular}{c} 
storage \\
type
\end{tabular} & \begin{tabular}{l} 
display \\
format
\end{tabular} & \begin{tabular}{l} 
value \\
label
\end{tabular} & variable label \\
\hline region & int & \(\% 8.0 \mathrm{~g}\) & region & Census Region \\
reg1 & byte & \(\% 9.0 \mathrm{~g}\) & & region==NE \\
reg2 & byte & \(\% 9.0 \mathrm{~g}\) & & region==N Cntrl \\
reg3 & byte & \(\% 9.0 \mathrm{~g}\) & & region==South \\
reg4 & byte & \(\% 9.0 \mathrm{~g}\) & & region==West
\end{tabular}

We cannot type test reg* because, in an expression, '*' means multiplication, but here is what would happen if we attempted to test all the variables that begin with reg:
```

. test region reg1 reg2 reg3 reg4
region not found
r(111);

```

The variable region was not included in our model, so it was not found. However, with testparm,
```

. testparm reg*
( 1) reg2 = 0
( 2) reg3 = 0
( 3) reg4 = 0
F( 3, 44) = 8.85
Prob > F = 0.0001

```

That is, testparm took reg* to mean all variables that start with reg that were in our model.

\section*{- Technical note}

Actually, reg* means what it always does-all variables in our dataset that begin with reg-in this case, region reg1 reg2 reg3 reg4. testparm just ignores any variables you specify that are not in the model.
> Example 8: Replaying the previous test
We just used test (testparm, actually, but it does not matter) to test the hypothesis that reg2, reg3, and reg4 are jointly zero. We can review the results of our last test by typing test without arguments:
```

. test
(1) reg2 = 0
( 2) reg3 = 0
( 3) reg4 = 0
F( 3, 44) = 8.85
Prob > F = 0.0001

```

\section*{- Technical note}
test does not care how we build joint hypotheses; we may freely mix different forms of syntax. (We can even start with testparm, but we cannot use it thereafter because it does not have an accumulate option.)

Say that we type test reg2 reg3 reg4 to test that the coefficients on our region dummies are jointly zero. We could then add a fourth constraint, say, that medage \(=100\), by typing test medage \(=100\), accumulate. Or, if we had introduced the medage constraint first (our first test command had been test medage=100), we could then add the region dummy test by typing test reg2 reg3 reg4, accumulate or test (reg2=0) (reg3=0) (reg4=0), accumulate.

Remember that all previous tests are cleared when we do not specify the accumulate option. No matter what tests we performed in the past, if we type test medage c.medage\#c.medage, omitting the accumulate option, we would test that medage and c.medage\#c.medage are jointly zero.
\(>\) Example 9: Testing the equality of multiple coefficients
Let's return to our census3.dta dataset and test the hypothesis that all the included regions have the same coefficient - that the Northeast is significantly different from the rest of the nation:
```

. use http://www.stata-press.com/data/r13/census3
(1980 Census data by state)
. regress brate medage c.medage\#c.medage i.region
(output omitted)
. test 2.region=3.region=4.region
(1) 2.region - 3.region = 0
( 2) 2.region - 4.region = 0
F( 2, 44) = 8.23
Prob > F = 0.0009

```

We find that they are not all the same. The syntax 2.region=3.region=4.region with multiple \(=\) operators is just a convenient shorthand for typing that the first expression equals the second expression and that the first expression equals the third expression,
```

. test (2.region=3.region) (2.region=4.region)

```

We performed the test for equality of the three regions by imposing two constraints: region 2 has the same coefficient as region 3, and region 2 has the same coefficient as region 4. Alternatively, we could have tested that the coefficients on regions 2 and 3 are the same and that the coefficients on regions 3 and 4 are the same. We would obtain the same results in either case.

To test for equality of the three regions, we might, likely by mistake, type equality constraints for all pairs of regions:
```

. test (2.region=3.region) (2.region=4.region) (3.region=4.region)
( 1) 2.region - 3.region = 0
( 2) 2.region - 4.region = 0
( 3) 3.region - 4.region = 0
Constraint 3 dropped
F( 2, 44) = 8.23
Prob > F = 0.0009

```

Equality of regions 2 and 3 and of regions 2 and 4, however, implies equality of regions 3 and 4. test recognized that the last constraint is implied by the other constraints and hence dropped it.

\section*{- Technical note}

Generally, Stata uses \(=\) for assignment, as in gen newvar \(=\exp\), and \(==\) as the operator for testing equality in expressions. For your convenience, test allows both \(=\) and \(==\) to be used.

\section*{> Example 10}

The test for the equality of the regions is also possible with the testparm command. When we include the equal option, testparm tests that the coefficients of all the variables specified are equal:
```

. testparm i(2/4).region, equal
(1) - 2.region + 3.region = 0
(2) - 2.region + 4.region = 0
F(2, 44) = 8.23
Prob > F = 0.0009

```

We can also obtain the equality test by accumulating single equality tests.
```

. test 2.region=3.region, notest
(1) 2.region - 3.region = 0
. test 2.region=4.region, accum
( 1) 2.region - 3.region = 0
( 2) 2.region - 4.region = 0
F( 2, 44) = 8.23
Prob > F = 0.0009

```

\section*{- Technical note}

If we specify a set of inconsistent constraints, test will tell us by dropping the constraint or constraints that led to the inconsistency. For instance, let's test that the coefficients on region 2 and region 4 are the same, add the test that the coefficient on region 2 is 20 , and finally add the test that the coefficient on region 4 is 21 :
```

. test (2.region=4.region) (2.region=20) (4.region=21)
( 1) 2.region - 4.region = 0
( 2) 2.region = 20
( 3) 4.region = 21
Constraint 1 dropped
F( 2, 44) = 1.29
Prob > F = 0.2868

```
test informed us that it was dropping constraint 1 . All three equations cannot be simultaneously true, so test drops whatever it takes to get back to something that makes sense.

\section*{Special syntaxes after multiple-equation estimation}

Everything said above about tests after single-equation estimation applies to tests after multipleequation estimation, as long as you remember to specify the equation name. To demonstrate, let's estimate a seemingly unrelated regression by using sureg; see [R] sureg.
```

. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. sureg (price foreign mpg displ) (weight foreign length)
Seemingly unrelated regression

```
\begin{tabular}{lrrrrrr}
\hline Equation & Obs & Parms & RMSE & "R-sq" & chi2 & P \\
\hline price & 74 & 3 & 2165.321 & 0.4537 & 49.64 & 0.0000 \\
weight & 74 & 2 & 245.2916 & 0.8990 & 661.84 & 0.0000 \\
\hline
\end{tabular}
\begin{tabular}{r|rrrrrr}
\hline & Coef. & Std. Err. & \(z\) & \(\mathrm{P}>|\mathrm{z}|\) & [95\% Conf. Interval] \\
\hline price & & & & & & \\
foreign & 3058.25 & 685.7357 & 4.46 & 0.000 & 1714.233 & 4402.267 \\
mpg & -104.9591 & 58.47209 & -1.80 & 0.073 & -219.5623 & 9.644042 \\
displacement & 18.18098 & 4.286372 & 4.24 & 0.000 & 9.779842 & 26.58211 \\
_cons & 3904.336 & 1966.521 & 1.99 & 0.047 & 50.0263 & 7758.645 \\
\hline weight & & & & & & \\
foreign & -147.3481 & 75.44314 & -1.95 & 0.051 & -295.2139 & .517755 \\
length & 30.94905 & 1.539895 & 20.10 & 0.000 & 27.93091 & 33.96718 \\
_cons & -2753.064 & 303.9336 & -9.06 & 0.000 & -3348.763 & -2157.365 \\
\hline
\end{tabular}

To test the significance of foreign in the price equation, we could type
```

. test [price]foreign
( 1) [price]foreign = 0
chi2( 1) = 19.89
Prob > chi2 = 0.0000

```
which is the same result reported by sureg: \(4.460^{2} \approx 19.89\). To test foreign in both equations, we could type
```

. test [price]foreign [weight]foreign
( 1) [price]foreign = 0
( 2) [weight]foreign = 0
chi2( 2) = 31.61
Prob > chi2 = 0.0000

```
or
```

. test foreign
( 1) [price]foreign = 0
( 2) [weight]foreign = 0
chi2( 2) = 31.61
Prob > chi2 = 0.0000

```

This last syntax - typing the variable name by itself-tests the coefficients in all equations in which they appear. The variable length appears in only the weight equation, so typing
```

. test length
( 1) [weight]length = 0
chi2( 1) = 403.94
Prob > chi2 = 0.0000

```
yields the same result as typing test [weight]length. We may also specify a linear expression rather than a list of coefficients:
```

. test mpg=displ
( 1) [price]mpg - [price]displacement = 0
chi2( 1) = 4.85
Prob > chi2 = 0.0277

```
or
```

. test [price]mpg = [price]displ
( 1) [price]mpg - [price]displacement = 0
chi2( 1) = 4.85
Prob > chi2 = 0.0277

```

A variation on this syntax can be used to test cross-equation constraints:
```

. test [price]foreign = [weight]foreign
( 1) [price]foreign - [weight]foreign = 0
chi2( 1) = 23.07
Prob > chi2 = 0.0000

```

Typing an equation name in square brackets by itself tests all the coefficients except the intercept in that equation:
```

. test [price]
( 1) [price]foreign = 0
( 2) [price]mpg = 0
( 3) [price]displacement = 0
chi2( 3) = 49.64
Prob > chi2 = 0.0000

```

Typing an equation name in square brackets, a colon, and a list of variable names tests those variables in the specified equation:
```

. test [price]: foreign displ
( 1) [price]foreign = 0
( 2) [price]displacement = 0
chi2( 2) = 25.19
Prob > chi2 = 0.0000

```
test \(\left[\right.\) eqname \(_{1}=\) eqname \(_{2}\) ] tests that all the coefficients in the two equations are equal. We cannot use that syntax here because there are different variables in the model:
```

. test [price=weight]
variables differ between equations
(to test equality of coefficients in common, specify option common)
r(111);

```

The common option specifies a test of the equality coefficients common to the equations price and weight,
```

. test [price=weight], common
( 1) [price]foreign - [weight]foreign = 0
chi2( 1) = 23.07
Prob > chi2 = 0.0000

```

By default, test does not include the constant, the coefficient of the constant variable _cons, in the test. The cons option specifies that the constant be included.
```

. test [price=weight], common cons
( 1) [price]foreign - [weight]foreign = 0
( 2) [price]_cons - [weight]_cons = 0
chi2( 2) = 51.23
Prob > chi2 = 0.0000

```

We can also use a modification of this syntax with the model if we also type a colon and the names of the variables we want to test:
```

. test [price=weight]: foreign
( 1) [price]foreign - [weight]foreign = 0
chi2( 1) = 23.07
Prob > chi2 = 0.0000

```

We have only one variable in common between the two equations, but if there had been more, we could have listed them.

Finally, a simultaneous test of multiple constraints may be specified just as after single-equation estimation.
```

. test ([price]: foreign) ([weight]: foreign)
( 1) [price]foreign = 0
( 2) [weight]foreign = 0
chi2( 2) = 31.61
Prob > chi2 = 0.0000

```
test can also test for equality of coefficients across more than two equations. For instance, test [eq1=eq2=eq3] specifies a test that the coefficients in the three equations eq1, eq2, and eq3 are equal. This requires that the same variables be included in the three equations. If some variables are entered only in some of the equations, you can type test [eq1=eq2=eq3], common to test that the coefficients of the variables common to all three equations are equal. Alternatively, you can explicitly list the variables for which equality of coefficients across the equations is to be tested. For instance, test [eq1=eq2=eq3]: time money tests that the coefficients of the variables time and money do not differ between the equations.

\section*{- Technical note}
test [eq1=eq2=eq3], common tests the equality of the coefficients common to all equations, but it does not test the equality of all common coefficients. Consider the case where
\[
\begin{array}{ll}
\text { eq1 } & \text { contains the variables var1 var2 var3 } \\
\text { eq2 } & \text { contains the variables var1 var2 var4 } \\
\text { eq3 } & \text { contains the variables var1 var3 var4 }
\end{array}
\]

Obviously, only var1 is common to all three equations. Thus test [eq1=eq2=eq3], common tests that the coefficients of var1 do not vary across the equations, so it is equivalent to test [eq1=eq2=eq3]: var1. To perform a test of the coefficients of variables common to two equations, you could explicitly list the constraints to be tested,
```

. test ([eq1=eq2=eq3]:var1) ([eq1=eq2]:var2) ([eq1=eq3]:var3) ([eq2=eq3]:var4)

```
or use test with the accumulate option, and maybe also with the notest option, to form the appropriate joint hypothesis:
```

. test [eq1=eq2], common notest
. test [eq1=eq3], common accumulate notest
. test [eq2=eq3], common accumulate

```

\section*{Constrained coefficients}

If the test indicates that the data do not allow you to conclude that the constraints are not satisfied, you may want to inspect the constrained coefficients. The coef option specified that the constrained results, estimated by GLS, are shown.
```

. test [price=weight], common coef
( 1) [price]foreign - [weight]foreign = 0
chi2( 1) = 23.07
Prob > chi2 = 0.0000

```

Constrained coefficients
\begin{tabular}{r|rrrrrr}
\hline & Coef. & Std. Err. & \(z\) & P>|z| & [95\% Conf. Interval] \\
\hline price & & & & & & \\
foreign & -216.4015 & 74.06083 & -2.92 & 0.003 & -361.558 & -71.2449 \\
mpg & -121.5717 & 58.36972 & -2.08 & 0.037 & -235.9742 & -7.169116 \\
displacement & 7.632566 & 3.681114 & 2.07 & 0.038 & .4177148 & 14.84742 \\
_cons & 7312.856 & 1834.034 & 3.99 & 0.000 & 3718.215 & 10907.5 \\
\hline weight & & & & & & \\
foreign & -216.4015 & 74.06083 & -2.92 & 0.003 & -361.558 & -71.2449 \\
length & 30.34875 & 1.534815 & 19.77 & 0.000 & 27.34057 & 33.35693 \\
_cons & -2619.719 & 302.6632 & -8.66 & 0.000 & -3212.928 & -2026.51 \\
\hline
\end{tabular}

The constrained coefficient of foreign is -216.40 with standard error 74.06 in equations price and weight. The other coefficients and their standard errors are affected by imposing the equality constraint of the two coefficients of foreign because the unconstrained estimates of these two coefficients were correlated with the estimates of the other coefficients.

\section*{Technical note}

The two-step constrained coefficients \(b_{c}\) displayed by test, coef are asymptotically equivalent to the one-stage constrained estimates that are computed by specifying the constraints during estimation using the constraint () option of estimation commands (Gourieroux and Monfort 1995, chap. 10). Generally, one-step constrained estimates have better small-sample properties. For inspection and interpretation, however, two-step constrained estimates are a convenient alternative. Moreover, some estimation commands (for example, stcox, many xt estimators) do not have a constraint () option.

\section*{Multiple testing}

When performing the test of a joint hypothesis, you might want to inspect the underlying 1-degree-of-freedom hypotheses. Which constraint "is to blame"? test displays the univariate as well as the simultaneous test if the mtest option is specified. For example,
\begin{tabular}{l}
. test [price=weight], common cons mtest \\
\begin{tabular}{c} 
(1) \\
( 2 [price]foreign \\
[price]_cons - [weight]foreign
\end{tabular}\(=0\) \\
\hline
\end{tabular}
\# unadjusted p-values
Both coefficients seem to contribute to the highly significant result. The 1-degree-of-freedom test shown here is identical to those if test had been invoked to test just this simple hypotheses. There is, of course, a real risk in inspecting these simple hypotheses. Especially in high-dimensional hypotheses, you may easily find one hypothesis that happens to be significant. Multiple testing procedures are designed to provide some safeguard against this risk. \(p\)-values of the univariate hypotheses are modified so that the probability of falsely rejecting one of the null hypotheses is bounded. test provides the methods based on Bonferroni, Šidák, and Holm.
```

. test [price=weight], common cons mtest(b)
( 1) [price]foreign - [weight]foreign = 0
( 2) [price]_cons - [weight]_cons = 0

```
\begin{tabular}{c|ccc}
\hline & chi2 & df & p \\
\hline (1) & 23.07 & 1 & 0.0000 \# \\
(2) & 11.17 & 1 & 0.0017 \# \\
\hline all & 51.23 & 2 & 0.0000 \\
\hline
\end{tabular}
\# Bonferroni-adjusted p-values

\section*{Stored results}
test and testparm store the following in \(r()\) :
Scalars
```

        r(p) two-sided p-value
        r(F) F
        r(df) test constraints degrees of freedom
        r(df_r) residual degrees of freedom
        r(dropped_i) index of ith constraint dropped
    r(mtmethod) method of adjustment for multiple
                        testing
        r(mtest) multiple test results
    ```
```

r(chi2) \chi

```
r(chi2) \chi
r(ss) sum of squares (test)
r(ss) sum of squares (test)
r(rss) residual sum of squares
r(rss) residual sum of squares
r(drop) 1 if constraints were dropped, 0
r(drop) 1 if constraints were dropped, 0
    otherwise
```

    otherwise
    ```
Macros
Matrices
\(r(s s)\) and \(r\) (rss) are defined only when test is used for testing effects after anova.

\section*{Methods and formulas}
test and testparm perform Wald tests. Let the estimated coefficient vector be \(\mathbf{b}\) and the estimated variance-covariance matrix be \(\mathbf{V}\). Let \(\mathbf{R b}=\mathbf{r}\) denote the set of \(q\) linear hypotheses to be tested jointly.

The Wald test statistic is (Judge et al. 1985, 20-28)
\[
W=(\mathbf{R b}-\mathbf{r})^{\prime}\left(\mathbf{R V} \mathbf{R}^{\prime}\right)^{-1}(\mathbf{R b}-\mathbf{r})
\]

If the estimation command reports its significance levels using \(Z\) statistics, a chi-squared distribution with \(q\) degrees of freedom,
\[
W \sim \chi_{q}^{2}
\]
is used for computation of the significance level of the hypothesis test.
If the estimation command reports its significance levels using \(t\) statistics with \(d\) degrees of freedom, an \(F\) statistic,
\[
F=\frac{1}{q} W
\]
is computed, and an \(F\) distribution with \(q\) numerator degrees of freedom and \(d\) denominator degrees of freedom computes the significance level of the hypothesis test.

The two-step constrained estimates \(b_{c}\) displayed by test with the coef option are the GLS estimates of the unconstrained estimates \(b\) subject to the specified constraints \(R b=c\) (Gourieroux and Monfort 1995, chap. 10),
\[
\mathbf{b}_{\mathbf{c}}=\mathbf{b}-\mathbf{R}^{\prime}\left(\mathbf{R V R} \mathbf{R}^{\prime}\right)^{-1} \mathbf{R}(\mathbf{R b}-\mathbf{r})
\]
with variance-covariance matrix
\[
\mathbf{V}_{\mathbf{c}}=\mathbf{V}-\mathbf{V R}^{\prime}\left(\mathbf{R V R}^{\prime}\right)^{-1} \mathbf{R V}
\]

If test displays a Wald test for joint (simultaneous) hypotheses, it can also display all 1-degree-offreedom tests, with \(p\)-values adjusted for multiple testing. Let \(p_{1}, p_{2}, \ldots, p_{k}\) be the unadjusted \(p\)-values of these 1-degree-of-freedom tests. The Bonferroni-adjusted \(p\)-values are defined as \(p_{i}^{b}=\min \left(1, k p_{i}\right)\). The Šidák-adjusted \(p\)-values are \(p_{i}^{s}=1-\left(1-p_{i}\right)^{k}\). Holm's method for adjusting \(p\)-values is defined as \(p_{i}^{h}=\min \left(1, k_{i} p_{i}\right)\), where \(k_{i}\) is the number of \(p\)-values at least as large as \(p_{i}\). Note that \(p_{i}^{h}<p_{i}^{b}\), reflecting that Holm's method is strictly less conservative than the widely used Bonferroni method.

If test is used after a svy command, it carries out an adjusted Wald test-this adjustment should not be confused with the adjustment for multiple testing. Both adjustments may actually be combined. Specifically, the survey adjustment uses an approximate \(F\) statistic \((d-k+1) W /(k d)\), where \(W\) is the Wald test statistic, \(k\) is the dimension of the hypothesis test, and \(d=\) the total number of sampled PSUs minus the total number of strata. Under the null hypothesis, \((d-k+1) F /(k d) \sim F(k, d-k+1)\), where \(F(k, d-k+1)\) is an \(F\) distribution with \(k\) numerator degrees of freedom and \(d-k+1\) denominator degrees of freedom. If nosvyadjust is specified, the \(p\)-value is computed using \(W / k \sim F(k, d)\).

See Korn and Graubard (1990) for a detailed description of the Bonferroni adjustment technique and for a discussion of the relative merits of it and of the adjusted and unadjusted Wald tests.

\section*{Acknowledgment}

The svy adjustment code was adopted from another command developed in collaboration with John L. Eltinge of the Bureau of Labor Statistics.

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\footnotetext{
Also see
[R] anova - Analysis of variance and covariance
[R] anova postestimation - Postestimation tools for anova
[R] contrast - Contrasts and linear hypothesis tests after estimation
[R] lincom - Linear combinations of estimators
[R] Irtest - Likelihood-ratio test after estimation
[R] nestreg - Nested model statistics
[R] nlcom - Nonlinear combinations of estimators
[R] testnl - Test nonlinear hypotheses after estimation
[U] 13.5 Accessing coefficients and standard errors
[U] 20 Estimation and postestimation commands
}

\section*{Title}
testnl - Test nonlinear hypotheses after estimation
\begin{tabular}{llll} 
Syntax & Menu & Description & Options \\
Remarks and examples & Stored results & Methods and formulas & References
\end{tabular}

\section*{Syntax}
```

testnl $\exp =\exp [=\exp \ldots][$, options $]$
$\operatorname{testn} 1(\exp =\exp [=\exp \ldots])[(\exp =\exp [=\exp \ldots]) \ldots][$, options $]$

```
options
Description
\begin{tabular}{|c|c|}
\hline \(\underline{\text { mtest }[(o p t)] ~}\) & test each condition separately \\
\hline iterate(\#) & use maximum \# of iterations to find the optimal step size \\
\hline df (\#) & use \(F\) distribution with \# denominator degrees of freedom for the reference distribution of the test statistic \\
\hline nosvyadjust & carry out the Wald test as \(W / k \sim F(k, d)\); for use with svy estimation commands when the df() option is also specified \\
\hline
\end{tabular}
\(\mathrm{df}(\#)\) and nosvyadjust do not appear in the dialog box.
The second syntax means that if more than one expression is specified, each must be surrounded by parentheses.
exp is a possibly nonlinear expression containing
_b [coef]
_b [eqno:coef]
[eqno] coef
[eqno]_b[coef]
eqno is

\section*{\#\#}
name
coef identifies a coefficient in the model. coef is typically a variable name, a level indicator, an interaction indicator, or an interaction involving continuous variables. Level indicators identify one level of a factor variable and interaction indicators identify one combination of levels of an interaction; see [U] 11.4.3 Factor variables. coef may contain time-series operators; see [U] 11.4.4 Time-series varlists.

Distinguish between [], which are to be typed, and [], which indicate optional arguments.

\section*{Menu}

Statistics \(>\) Postestimation \(>\) Tests \(>\) Test nonlinear hypotheses

\section*{Description}
testnl tests (linear or nonlinear) hypotheses about the estimated parameters from the most recently fit model.
testnl produces Wald-type tests of smooth nonlinear (or linear) hypotheses about the estimated parameters from the most recently fit model. The \(p\)-values are based on the delta method, an approximation appropriate in large samples.
testnl can be used with svy estimation results; see [SVY] svy postestimation.
The format ( \(\exp _{1}=\exp _{2}=\exp _{3} \ldots\) ) for a simultaneous-equality hypothesis is just a convenient shorthand for a list \(\left(\exp _{1}=\exp _{2}\right)\left(\exp _{1}=\exp _{3}\right)\), etc.
testnl may also be used to test linear hypotheses. test is faster if you want to test only linear hypotheses; see \([R]\) test. testnl is the only option for testing linear and nonlinear hypotheses simultaneously.

\section*{Options}
mtest \([(o p t)]\) specifies that tests be performed for each condition separately. opt specifies the method for adjusting \(p\)-values for multiple testing. Valid values for opt are
\begin{tabular}{rl} 
bonferroni & Bonferroni's method \\
holm & Holm's method \\
sidak & Šidák's method \\
noadjust & no adjustment is to be made
\end{tabular}

Specifying mtest without an argument is equivalent to specifying mtest(noadjust).
iterate(\#) specifies the maximum number of iterations used to find the optimal step size in the calculation of numerical derivatives of the test expressions. By default, the maximum number of iterations is 100 , but convergence is usually achieved after only a few iterations. You should rarely have to use this option.

The following options are available with testnl but are not shown in the dialog box:
df (\#) specifies that the \(F\) distribution with \# denominator degrees of freedom be used for the reference distribution of the test statistic. With survey data, \# is the design degrees unless nosvyadjust is specified.
nosvyadjust is for use with svy estimation commands when the df() option is also specified; see [SVY] svy estimation. It specifies that the Wald test be carried out without the default adjustment for the design degrees of freedom. That is, the test is carried out as \(W / k \sim F(k, d)\) rather than as \((d-k+1) W /(k d) \sim F(k, d-k+1)\), where \(k=\) the dimension of the test and \(d=\) the design degrees of freedom specified in the df() option.

\section*{Remarks and examples}

Remarks are presented under the following headings:

\footnotetext{
Introduction
Using testnl to perform linear tests
Specifying constraints
Dropped constraints
Multiple constraints
Manipulability
}

\section*{Introduction}

\section*{> Example 1}

We have just estimated the parameters of an earnings model on cross-sectional time-series data using one of Stata's more sophisticated estimators:
\begin{tabular}{|c|c|c|c|c|}
\hline \multicolumn{2}{|l|}{GEE population-averaged model} & Number of obs & = & 1326 \\
\hline Group variable: & idcode & Number of groups & = & 269 \\
\hline Link: & identity & Obs per group: min & = & 1 \\
\hline Family: & Gaussian & avg & = & 4.9 \\
\hline Correlation: & exchangeable & max & = & 9 \\
\hline & & Wald chi2(3) & = & 327.33 \\
\hline Scale parameter: & . 0976738 & Prob > chi2 & = & 0.0000 \\
\hline
\end{tabular}
\begin{tabular}{r|rrrrrr}
\hline ln_wage & Coef. & Std. Err. & \multicolumn{1}{c}{z} & \(\mathrm{P}>|\mathrm{z}|\) & [95\% Conf. Interval] \\
\hline grade & .0749686 & .0066111 & 11.34 & 0.000 & .062011 & .0879261 \\
age & .1080806 & .0235861 & 4.58 & 0.000 & .0618526 & .1543086 \\
c.age\#c.age & -.0016253 & .0004739 & -3.43 & 0.001 & -.0025541 & -.0006966 \\
_cons & -.8788933 & .2830899 & -3.10 & 0.002 & -1.433739 & -.3240473 \\
\hline
\end{tabular}

An implication of this model is that peak earnings occur at age -_b[age]/(2*_b[c.age\#c.age]), which here is equal to 33.2 . Say that we have a theory that peak earnings should occur at age \(16+\) 1/_b [grade].
```

. testnl -_b[age]/(2*_b[c.age\#c.age]) = 16 + 1/_b[grade]
(1) __b[age]/(2*_b[c.age\#c.age]) = 16 + 1/_b[grade]
chi2(1) = 1.71
Prob > chi2 = 0.1914

```

These data do not reject our theory.

\section*{Using testnl to perform linear tests}
testnl may be used to test linear constraints, but test is faster; see [R] test. You could type
```

. testnl _b[x4] = _b[x1]

```
but it would take less computer time if you typed
```

. test _b[x4] = _b[x1]

```

\section*{Specifying constraints}

The constraints to be tested can be formulated in many different ways. You could type
```

. testnl _b[mpg]*_b[weight] = 1

```
or
```

. testnl _b[mpg] = 1/_b[weight]

```
or you could express the constraint any other way you wished. (To say that testnl allows constraints to be specified in different ways does not mean that the test itself does not depend on the formulation. This point is briefly discussed later.) In formulating the constraints, you must, however, exercise one caution: users of test often refer to the coefficient on a variable by specifying the variable name. For example,
```

. test mpg = 0

```

More formally, they should type
```

test _b[mpg] = 0

```
but test allows the _b [] surrounding the variable name to be omitted. testnl does not allow this shorthand. Typing
```

. testnl mpg=0

```
specifies the constraint that the value of variable mpg in the first observation is zero. If you make this mistake, sometimes testnl will catch it:
```

. testnl mpg=0
equation (1) contains reference to X rather than _b [X]
r(198);

```

In other cases, testnl may not catch the mistake; then the constraint will be dropped because it does not make sense:
```

. testnl mpg=0
Constraint (1) dropped

```
(There are reasons other than this for constraints being dropped.) The worst case, however, is
```

. testnl _b[weight]*mpg = 1

```
when what you mean is not that _b [weight] equals the reciprocal of the value of mpg in the first observation, but rather that
```

. testnl _b[weight]*_b[mpg] = 1

```

Sometimes this mistake will be caught by the "contains reference to X rather than \(\_\mathrm{b}[\mathrm{X}]\) " error, and sometimes it will not. Be careful.
testnl, like test, can be used after any Stata estimation command, including the survey estimators. When you use it after a multiple-equation command, such as mlogit or heckman, you refer to coefficients by using Stata's standard syntax: [eqname]_b[varname].

Stata's single-equation estimation output looks like this:
\begin{tabular}{r|rllll} 
& Coef & \(\ldots\) & & \\
\hline weight & 12.27 & \(\ldots\) & \(<-\) & coefficient is \(\quad\) b[weight] \\
mpg & 3.21 & \(\ldots\) & & & \\
\hline
\end{tabular}

Stata's multiple-equation output looks like this:
\begin{tabular}{|c|c|c|c|}
\hline & \multicolumn{3}{|l|}{Coef} \\
\hline \multicolumn{4}{|l|}{cat1} \\
\hline weight & 12.27 & <- coefficient is & [cat1]_b[weight] \\
\hline mpg & 3.21 & & \\
\hline \multicolumn{4}{|l|}{8} \\
\hline weight & 5.83 & <- coefficient is & [8]_b[weight] \\
\hline mpg & 7.43 & & \\
\hline
\end{tabular}

\section*{Dropped constraints}
testnl automatically drops constraints when
- They are nonbinding, for example, \(\_\mathrm{b}[\mathrm{mpg}]=\_\mathrm{b}[\mathrm{mpg}]\). More subtle cases include
```

_b[mpg]*_b[weight] = 4
_b[weight] = 2
_b[mpg] = 2

```

In this example, the third constraint is nonbinding because it is implied by the first two.
- They are contradictory, for example, \(\_\mathrm{b}[\mathrm{mpg}]=2\) and \(\_\mathrm{b}[\mathrm{mpg}]=3\). More subtle cases include
```

_b[mpg]*_b[weight] = 4
_b[weight] = 2
_b[mpg] = 3

```

The third constraint contradicts the first two.

\section*{Multiple constraints}
\(>\) Example 2
We illustrate the simultaneous test of a series of constraints using simulated data on labor-market promotion in a given year. We fit a probit model with separate effects for education, experience, and experience-squared for men and women.
. use http://www.stata-press.com/data/r13/promotion
. probit promo male male\#c. (yedu yexp yexp2), nolog
\begin{tabular}{lllr} 
Probit regression & Number of obs & \(=\) & 775 \\
& LR chi2 \((7)\) & \(=\) & 424.42 \\
Log likelihood \(=-245.42768\) & Prob > chi2 & \(=\) & 0.0000 \\
& Pseudo R2 & \(=\) & 0.4637
\end{tabular}
\begin{tabular}{c|rccccc}
\hline promo & Coef. & Std. Err. & \(z\) & \(\mathrm{P}>|\mathrm{z}|\) & [95\% Conf. Interval] \\
\hline male & .6489974 & .203739 & 3.19 & 0.001 & .2496763 & 1.048318 \\
male\#c.yedu & & & & & & \\
0 & .9730237 & .1056136 & 9.21 & 0.000 & .7660248 & 1.180023 \\
1 & 1.390517 & .1527288 & 9.10 & 0.000 & 1.091174 & 1.68986 \\
male\#c.yexp & & & & & & \\
0 & .4559544 & .0901169 & 5.06 & 0.000 & .2793285 & .6325803 \\
1 & 1.422539 & .1544255 & 9.21 & 0.000 & 1.11987 & 1.725207 \\
male\#c.yexp2 & & & & & & \\
0 & -.1027149 & .0573059 & -1.79 & 0.073 & -.2150325 & .0096026 \\
1 & -.3749457 & .1160113 & -3.23 & 0.001 & -.6023236 & -.1475677 \\
& & & & & & \\
cons & .9872018 & .1148215 & 8.60 & 0.000 & .7621559 & 1.212248 \\
\hline
\end{tabular}

Note: 1 failure and 2 successes completely determined.
The effects of human capital seem to differ between men and women. A formal test confirms this.
```

. test (yedu\#O.male = yedu\#1.male) (yexp\#0.male = yexp\#1.male)
> (yexp2\#0.male = yexp2\#1.male)
( 1) [promo]0b.male\#c.yedu - [promo]1.male\#c.yedu = 0
( 2) [promo]0b.male\#c.yexp - [promo]1.male\#c.yexp = 0
( 3) [promo]0b.male\#c.yexp2 - [promo]1.male\#c.yexp2 = 0
chi2( 3) = 35.43
Prob > chi2 = 0.0000

```

How do we interpret this gender difference? It has repeatedly been stressed (see, for example, Long [1997, 47-50]; Allison [1999]) that comparison of groups in binary response models, and similarly in other latent-variable models, is hampered by an identification problem: with \(\beta\) the regression coefficients for the latent variable and \(\sigma\) the standard deviation of the latent residual, only the \(\beta / \sigma\) are identified. In fact, in terms of the latent regression, the probit coefficients should be interpreted as \(\beta / \sigma\), not as the \(\beta\). If we cannot claim convincingly that the residual standard deviation \(\sigma\) does not vary between the sexes, equality of the regression coefficients \(\beta\) implies that the coefficients of the probit model for men and women are proportional but not necessarily equal. This is a nonlinear hypothesis in terms of the probit coefficients, not a linear one.
```

. testnl _b[yedu\#1.male]/_b[yedu\#0.male] = _b[yexp\#1.male]/_b[yexp\#0.male]
> = _b[yexp2\#1.male]/_b[yexp2\#0.male]
(1) _b[yedu\#1.male]/_b[yedu\#0.male] = _b[yexp\#1.male]/_b[yexp\#0.male]
(2) _b[yedu\#1.male]/_b[yedu\#0.male] = _b[yexp2\#1.male]/_b[yexp2\#0.male]

$$
\begin{array}{rll}
\operatorname{chi2}(2) & = & 9.21 \\
\text { Prob }>\text { chi2 } & = & 0.0100
\end{array}
$$

```

We conclude that we find fairly strong evidence against the proportionality of the coefficients, and hence we have to conclude that success in the labor market is produced in different ways by men and women. (But remember, these were simulated data.)

\section*{> Example 3}

The syntax for specifying the equality of multiple expressions is just a convenient shorthand for specifying a series of constraints, namely, that the first expression equals the second expression, the first expression also equals the third expression, etc. The Wald test performed and the output of testnl are the same whether we use the shorthand or we specify the series of constraints. The lengthy specification as a series of constraints can be simplified using the continuation symbols ///.
```

. testnl (_b[yedu\#1.male]/_b[yedu\#0.male] = ///
> _b[yexp\#1.male]/_b[yexp\#0.male]) ///
> (_b[yedu\#1.male]/_b[yedu\#0.male] = ///
> _b[yexp2\#1.male]/_b[yexp2\#0.male])
(1) _b[yedu\#1.male]/_b[yedu\#0.male] = _b[yexp\#1.male]/_b[yexp\#0.male]
(2) _b[yedu\#1.male]/_b[yedu\#0.male] = _b[yexp2\#1.male]/_b[yexp2\#0.male]
chi2(2) = 9.21
Prob > chi2 = 0.0100

```

Having established differences between men and women, we would like to do multiple testing between the ratios. Because we did not specify hypotheses in advance, we prefer to adjust the \(p\)-values of tests using, here, Bonferroni's method.


\section*{Manipulability}

Although testnl allows you to specify constraints in different ways that are mathematically equivalent, as noted above, this does not mean that the tests are the same. This difference is known as the manipulability of the Wald test for nonlinear hypotheses; also see \([R]\) boxcox. The test might even be significant for one formulation but not significant for another formulation that is mathematically equivalent. Trying out different specifications to find a formulation with the desired \(p\)-value is totally inappropriate, though it may actually be fun to try. There is no variance under representation because the nonlinear Wald test is actually a standard Wald test for a linearization of the constraint, which depends on the particular specification. We note that the likelihood-ratio test is not manipulable in this sense.

From a statistical point of view, it is best to choose a specification of the constraints that is as linear is possible. Doing so usually improves the accuracy of the approximation of the null-distribution of the test by a \(\chi^{2}\) or an \(F\) distribution. The example above used the nonlinear Wald test to test whether the coefficients of human capital variables for men were proportional to those of women. A specification of proportionality of coefficients in terms of ratios of coefficients is fairly nonlinear if the coefficients in the denominator are close to 0 . A more linear version of the test results from a bilinear formulation. Thus instead of
```

. testnl _b[yedu\#1.male]/_b[yedu\#0.male] = _b[yexp\#1.male]/_b[yexp\#0.male]
(1) _b[yedu\#1.male]/_b[yedu\#0.male] = _b[yexp\#1.male]/_b[yexp\#0.male]
chi2(1) = 6.89
Prob > chi2 = 0.0087

```
perhaps
```

testnl _b[yedu\#1.male]*_b[yexp\#0.male] = _b[yedu\#0.male]*_b[yexp\#1.male]
(1) _b[yedu\#1.male]*_b[yexp\#0.male] = _b[yedu\#0.male]*_b[yexp\#1.male]
chi2(1) = 13.95
Prob > chi2 = 0.0002

```
is better, and in fact it has been suggested that the latter version of the test is more reliable. This assertion is confirmed by performing simulations and is in line with theoretical results of Phillips and Park (1988). There is strong evidence against the proportionality of human capital effects between men and women, implying for this example that differences in the residual variances between the sexes can be ruled out as the explanation of the sex differences in the analysis of labor market participation.

\section*{Stored results}
testnl stores the following in \(r()\) :
Scalars
```

    r(df) degrees of freedom
    r(df_r) residual degrees of freedom
    ```

```

    r(p) significance
    r(F) F statistic
    ```
Macros
    \(r\) (mtmethod) method specified in mtest ()
Matrices
    \(r(G) \quad\) derivatives of \(R(\mathbf{b})\) with respect to \(\mathbf{b}\); see Methods and formulas below
    \(\mathrm{r}(\mathrm{R}) \quad R(\mathbf{b})-\mathbf{q}\); see Methods and formulas below
    \(r\) (mtest) multiple test results

\section*{Methods and formulas}

After fitting a model, define \(\mathbf{b}\) as the resulting \(1 \times k\) parameter vector and \(\mathbf{V}\) as the \(k \times k\) covariance matrix. The (linear or nonlinear) hypothesis is given by \(R(\mathbf{b})=\mathbf{q}\), where \(R\) is a function returning a \(j \times 1\) vector. The Wald test formula is (Greene 2012, 528)
\[
W=\{R(\mathbf{b})-\mathbf{q}\}^{\prime}\left(\mathbf{G V G}^{\prime}\right)^{-1}\{R(\mathbf{b})-\mathbf{q}\}
\]
where \(\mathbf{G}\) is the derivative matrix of \(R(\mathbf{b})\) with respect to \(\mathbf{b} . W\) is distributed as \(\chi^{2}\) if \(\mathbf{V}\) is an asymptotic covariance matrix. \(F=W / j\) is distributed as \(F\) for linear regression.

The adjustment methods for multiple testing are described in \([R]\) test. The adjustment for survey design effects is described in [SVY] svy postestimation.

\section*{References}

Allison, P. D. 1999. Comparing logit and probit coefficients across groups. Sociological Methods and Research 28 : 186-208.

Gould, W. W. 1996. cre43: Wald test of nonlinear hypotheses after model estimation. Stata Technical Bulletin 29: 2-4. Reprinted in Stata Technical Bulletin Reprints, vol. 5, pp. 15-18. College Station, TX: Stata Press.
Greene, W. H. 2012. Econometric Analysis. 7th ed. Upper Saddle River, NJ: Prentice Hall.
Long, J. S. 1997. Regression Models for Categorical and Limited Dependent Variables. Thousand Oaks, CA: Sage.
Phillips, P. C. B., and J. Y. Park. 1988. On the formulation of Wald tests of nonlinear restrictions. Econometrica 56: 1065-1083.

\section*{Also see}
[R] contrast - Contrasts and linear hypothesis tests after estimation
[R] lincom - Linear combinations of estimators
[R] Irtest - Likelihood-ratio test after estimation
[R] nlcom - Nonlinear combinations of estimators
[R] test - Test linear hypotheses after estimation
[U] 13.5 Accessing coefficients and standard errors
[U] 20 Estimation and postestimation commands

\section*{Title}
tetrachoric - Tetrachoric correlations for binary variables
\begin{tabular}{llll} 
Syntax & Menu & Description & Options \\
Remarks and examples & Stored results & Methods and formulas & References \\
Also see & &
\end{tabular}

\section*{Syntax}
tetrachoric varlist [if] [in] [weight] [, options]
options Description

Main
stats (statlist) list of statistics; select up to 4 statistics; default is stats (rho)
edwards use the noniterative Edwards and Edwards estimator; default is the maximum likelihood estimator
print(\#) significance level for displaying coefficients
star(\#) significance level for displaying with a star
bonferroni use Bonferroni-adjusted significance level
sidak use Šidák-adjusted significance level
pw calculate all the pairwise correlation coefficients by using all available data (pairwise deletion)
zeroadjust
matrix display output in matrix form
notable suppress display of correlations
posdef modify correlation matrix to be positive semidefinite
\begin{tabular}{ll} 
statlist & Description \\
\hline rho & tetrachoric correlation coefficient \\
se & standard error of rho \\
obs & number of observations \\
p & exact two-sided significance level
\end{tabular}
by is allowed; see [D] by.
fweights are allowed; see [U] 11.1.6 weight.

\section*{Menu}

Statistics \(>\) Summaries, tables, and tests \(>\) Summary and descriptive statistics \(>\) Tetrachoric correlations

\section*{Description}
tetrachoric computes estimates of the tetrachoric correlation coefficients of the binary variables in varlist. All of these variables should be 0,1 , or missing values.

Tetrachoric correlations assume a latent bivariate normal distribution ( \(X_{1}, X_{2}\) ) for each pair of variables \(\left(v_{1}, v_{2}\right)\), with a threshold model for the manifest variables, \(v_{i}=1\) if and only if \(X_{i}>0\). The means and variances of the latent variables are not identified, but the correlation, \(r\), of \(X_{1}\) and \(X_{2}\) can be estimated from the joint distribution of \(v_{1}\) and \(v_{2}\) and is called the tetrachoric correlation coefficient.
tetrachoric computes pairwise estimates of the tetrachoric correlations by (iterative) maximum likelihood estimator obtained from bivariate probit without explanatory variables (see [R] biprobit) by using the Edwards and Edwards (1984) noniterative estimator as the initial value.

The pairwise correlation matrix is returned as \(r\) (Rho) and can be used to perform a factor analysis or a principal component analysis of binary variables by using the factormat or pcamat commands; see [MV] factor and [MV] pca.

\section*{Options}
\(\qquad\)
stats (statlist) specifies the statistics to be displayed in the matrix of output. stats(rho) is the default. Up to four statistics may be specified. stats (rho se p obs) would display the tetrachoric correlation coefficient, its standard error, the significance level, and the number of observations. If varlist contains only two variables, all statistics are shown in tabular form. stats(), print(), and star() have no effect unless the matrix option is also specified.
edwards specifies that the noniterative Edwards and Edwards estimator be used. The default is the maximum likelihood estimator. If you analyze many binary variables, you may want to use the fast noniterative estimator proposed by Edwards and Edwards (1984). However, if you have skewed variables, the approximation does not perform well.
print (\#) specifies the maximum significance level of correlation coefficients to be printed. Correlation coefficients with larger significance levels are left blank in the matrix. Typing tetrachoric..., print (.10) would list only those correlation coefficients that are significant at the \(10 \%\) level or lower.
star(\#) specifies the maximum significance level of correlation coefficients to be marked with a star. Typing tetrachoric ..., star (.05) would "star" all correlation coefficients significant at the \(5 \%\) level or lower.
bonferroni makes the Bonferroni adjustment to calculated significance levels. This option affects printed significance levels and the print() and star() options. Thus tetrachoric..., print(.05) bonferroni prints coefficients with Bonferroni-adjusted significance levels of 0.05 or less.
sidak makes the Šidák adjustment to calculated significance levels. This option affects printed significance levels and the print() and star() options. Thus tetrachoric..., print (.05) sidak prints coefficients with Sidák-adjusted significance levels of 0.05 or less.
pw specifies that the tetrachoric correlation be calculated by using all available data. By default, tetrachoric uses casewise deletion, where observations are ignored if any of the specified variables in varlist are missing.
zeroadjust specifies that when one of the cells has a zero count, a frequency adjustment be applied in such a way as to increase the zero to one-half and maintain row and column totals.
matrix forces tetrachoric to display the statistics as a matrix, even if varlist contains only two variables. matrix is implied if more than two variables are specified.
notable suppresses the output.
posdef modifies the correlation matrix so that it is positive semidefinite, that is, a proper correlation matrix. The modified result is the correlation matrix associated with the least-squares approximation of the tetrachoric correlation matrix by a positive-semidefinite matrix. If the correlation matrix is modified, the standard errors and significance levels are not displayed and are returned in \(r()\).

\section*{Remarks and examples}

Remarks are presented under the following headings:
Association in 2-by-2 tables
Factor analysis of dichotomous variables
Tetrachoric correlations with simulated data

\section*{Association in 2-by-2 tables}

Although a wide variety of measures of association in cross tabulations have been proposed, such measures are essentially equivalent (monotonically related) in the special case of \(2 \times 2\) tables-there is only 1 degree of freedom for nonindependence. Still, some measures have more desirable properties than others. Here we compare two measures: the standard Pearson correlation coefficient and the tetrachoric correlation coefficient. Given asymmetric row or column margins, Pearson correlations are limited to a range smaller than -1 to 1 , although tetrachoric correlations can still span the range from -1 to 1 . To illustrate, consider the following set of tables for two binary variables, X and Z :
\begin{tabular}{l|cc|c} 
& \(\mathrm{Z}=0\) & \(\mathrm{Z}=1\) & \\
\hline \(\mathrm{X}=0\) & \(20-a\) & \(10+a\) & 30 \\
\(\mathrm{X}=1\) & \(a\) & \(10-a\) & 10 \\
\hline & 20 & 20 & 40
\end{tabular}

For \(a\) equal to \(0,1,2,5,8,9\), and 10 , the Pearson and tetrachoric correlations for the above table are
\begin{tabular}{r|ccccccc}
\(a\) & 0 & 1 & 2 & 5 & 8 & 9 & 10 \\
\hline Pearson & 0.577 & 0.462 & 0.346 & 0 & -0.346 & -0.462 & -0.577 \\
Tetrachoric & 1.000 & 0.792 & 0.607 & 0 & -0.607 & -0.792 & -1.000 \\
\hline
\end{tabular}

The restricted range for the Pearson correlation is especially unfortunate when you try to analyze the association between binary variables by using models developed for continuous data, such as factor analysis and principal component analysis.

The tetrachoric correlation of two variables \(\left(Y_{1}, Y_{2}\right)\) can be thought of as the Pearson correlation of two latent bivariate normal distributed variables \(\left(Y_{1}^{*}, Y_{2}^{*}\right)\) with threshold measurement models \(Y_{i}=\left(Y_{i}^{*}>c_{i}\right)\) for unknown cutpoints \(c_{i}\). Or equivalently, \(Y_{i}=\left(Y_{i}^{* *}>0\right)\) where the latent bivariate normal \(\left(Y_{1}^{* *}, Y_{2}^{* *}\right)\) are shifted versions of \(\left(Y_{1}^{*}, Y_{2}^{*}\right)\) so that the cutpoints are zero. Obviously, you must judge whether assuming underlying latent variables is meaningful for the data. If this assumption is justified, tetrachoric correlations have two advantages. First, you have an intuitive understanding of
the size of correlations that are substantively interesting in your field of research, and this intuition is based on correlations that range from -1 to 1 . Second, because the tetrachoric correlation for binary variables estimates the Pearson correlation of the latent continuous variables (assumed multivariate normal distributed), you can use the tetrachoric correlations to analyze multivariate relationships between the dichotomous variables. When doing so, remember that you must interpret the model in terms of the underlying continuous variables.

\section*{> Example 1}

To illustrate tetrachoric correlations, we examine three binary variables from the familyvalues dataset (described in example 2).


As usual, the tetrachoric correlation coefficients are larger (in absolute value) and more dispersed than the Pearson correlations.

\section*{Factor analysis of dichotomous variables}

\section*{\(>\) Example 2}

Factor analysis is a popular model for measuring latent continuous traits. The standard estimators are appropriate only for continuous unimodal data. Because of the skewness implied by Bernoullidistributed variables (especially when the probability is distributed unevenly), a factor analysis of a Pearson correlation matrix can be rather misleading when used in this context. A factor analysis of a matrix of tetrachoric correlations is more appropriate under these conditions (Uebersax 2000). We illustrate this with data on gender, relationship, and family attitudes of spouses using the Households in The Netherlands survey 1995 (Weesie et al. 1995). For attitude variables, it seems reasonable to assume that agreement or disagreement is just a coarse measurement of more nuanced underlying attitudes.

To demonstrate, we examine a few of the variables from the familyvalues dataset.
. use http://www.stata-press.com/data/r13/familyvalues
(Attitudes on gender, relationships and family)
. describe RS056-RS063
\begin{tabular}{lllll} 
variable name & \begin{tabular}{c} 
storage \\
type
\end{tabular} & \begin{tabular}{l} 
display \\
format
\end{tabular} & \begin{tabular}{c} 
value \\
label
\end{tabular} & variable label \\
\hline RS056 & byte & \(\% 9.0 \mathrm{~g}\) & & fam att: should be together \\
RS057 & byte & \(\% 9.0 \mathrm{~g}\) & & fam att: should fight for relat \\
RS058 & byte & \(\% 9.0 \mathrm{~g}\) & & fam att: should avoid conflict \\
RS059 & byte & \(\% 9.0 \mathrm{~g}\) & & fam att: woman better nurturer \\
RS060 & byte & \(\% 9.0 \mathrm{~g}\) & & fam att: both spouses money goo \\
RS061 & byte & \(\% 9.0 \mathrm{~g}\) & & fam att: woman techn school goo \\
RS062 & byte & \(\% 9.0 \mathrm{~g}\) & & fam att: man natural breadwinne \\
RS063 & byte & \(\% 9.0 \mathrm{~g}\) & & fam att: common leisure good
\end{tabular}
. summarize RS056-RS063
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline RS056 & 3298 & .5630685 & .4960816 & 0 & 1 \\
RS057 & 3296 & .5400485 & .4984692 & 0 & 1 \\
RS058 & 3283 & .6387451 & .4804374 & 0 & 1 \\
RS059 & 3308 & .654474 & .4756114 & 0 & 1 \\
RS060 & 3302 & .3906723 & .487975 & 0 & 1 \\
\hline RS061 & 3293 & .7102946 & .4536945 & 0 & 1 \\
RS062 & 3307 & .5857272 & .4926705 & 0 & 1 \\
RS063 & 3298 & .5379018 & .498637 & 0 & 1
\end{tabular}
. correlate RS056-RS063
(obs=3221)
\begin{tabular}{r|rrrrrrr} 
& RS056 & RS057 & RS058 & RS059 & RS060 & RS061 & RS062 \\
\hline RS056 & 1.0000 & & & & & & \\
RS057 & 0.1350 & 1.0000 & & & & & \\
RS058 & 0.2377 & 0.0258 & 1.0000 & & & \\
RS059 & 0.1816 & 0.0097 & 0.2550 & 1.0000 & & & \\
RS060 & -0.1020 & -0.0538 & -0.0424 & 0.0126 & 1.0000 & & \\
RS061 & -0.1137 & 0.0610 & -0.1375 & -0.2076 & 0.0706 & 1.0000 & \\
RS062 & 0.2014 & 0.0285 & 0.2273 & 0.4098 & -0.0793 & -0.2873 & 1.0000 \\
RS063 & 0.2057 & 0.1460 & 0.1049 & 0.0911 & 0.0179 & -0.0233 & 0.0975 \\
& RS063 & & & & & & \\
\hline RS063 & 1.0000 & & & & & & \\
& & & & & & &
\end{tabular}

Skewness in these data is relatively modest. For comparison, here are the tetrachoric correlations:
\begin{tabular}{l|rrrrrrr}
\begin{tabular}{l} 
(tetrachoric \\
(obs=3221)
\end{tabular} & RSO56-RS063 \\
& RS056 & RS057 & RS058 & RS059 & RS060 & RS061 & RS062 \\
\hline RS056 & 1.0000 & & & & & & \\
RS057 & 0.2114 & 1.0000 & & & & & \\
RS058 & 0.3716 & 0.0416 & 1.0000 & & & & \\
RS059 & 0.2887 & 0.0158 & 0.4007 & 1.0000 & & & \\
RS060 & -0.1620 & -0.0856 & -0.0688 & 0.0208 & 1.0000 & & \\
RS061 & -0.1905 & 0.1011 & -0.2382 & -0.3664 & 0.1200 & 1.0000 & \\
RS062 & 0.3135 & 0.0452 & 0.3563 & 0.6109 & -0.1267 & -0.4845 & 1.0000 \\
RS063 & 0.3187 & 0.2278 & 0.1677 & 0.1467 & 0.0286 & -0.0388 & 0.1538 \\
& RS063 & & & & & & \\
\hline RS063 & 1.0000 & & & & & & \\
\hline
\end{tabular}

Again we see that the tetrachoric correlations are generally larger in absolute value than the Pearson correlations. The bivariate probit and Edwards and Edwards estimators (the edwards option) implemented in tetrachoric may return a correlation matrix that is not positive semidefinite-a mathematical property of any real correlation matrix. Positive definiteness is required by commands for analyses of correlation matrices, such as factormat and pcamat; see [MV] factor and [MV] pca. The posdef option of tetrachoric tests for positive definiteness and projects the estimated correlation matrix to a positive-semidefinite matrix if needed.
```

. tetrachoric RS056-RS063, notable posdef
. matrix C = r(corr)

```

This time, we suppressed the display of the correlations with the notable option and requested that the correlation matrix be positive semidefinite with the posdef option. Had the correlation matrix not been positive definite, tetrachoric would have displayed a warning message and then adjusted the matrix to be positive semidefinite. We placed the resulting tetrachoric correlation matrix into a matrix, C, so that we can perform a factor analysis upon it.
tetrachoric with the posdef option asserted that C was positive definite because no warning message was displayed. We can verify this by using a familiar characterization of symmetric positivedefinite matrices: all eigenvalues are real and positive.
```

. matrix symeigen eigenvectors eigenvalues = C
. matrix list eigenvalues
eigenvalues[1,8]

```


We can proceed with a factor analysis on the matrix C. We use factormat and select iterated principal factors as the estimation method; see [MV] factor.
```

. factormat C, n(3221) ipf factor(2)
(obs=3221)

```
Factor analysis/correlation \(\quad\) Number of obs \(=3221\)
    Method: iterated principal factors
    Retained factors \(=\quad 2\)
    Rotation: (unrotated)
    Number of params \(=15\)
\begin{tabular}{c|rrrr}
\hline Factor & Eigenvalue & Difference & Proportion & Cumulative \\
\hline Factor1 & 2.06855 & 1.40178 & 0.7562 & 0.7562 \\
Factor2 & 0.66677 & 0.47180 & 0.2438 & 1.0000 \\
Factor3 & 0.19497 & 0.06432 & 0.0713 & 1.0713 \\
Factor4 & 0.13065 & 0.10967 & 0.0478 & 1.1191 \\
Factor5 & 0.02098 & 0.10085 & 0.0077 & 1.1267 \\
Factor6 & -0.07987 & 0.01037 & -0.0292 & 1.0975 \\
Factor7 & -0.09024 & 0.08626 & -0.0330 & 1.0645 \\
Factor8 & -0.17650 &. & -0.0645 & 1.0000 \\
\hline
\end{tabular}

LR test: independent vs. saturated: chi2(28) \(=4620.01\) Prob>chi2 \(=0.0000\)
Factor loadings (pattern matrix) and unique variances
\begin{tabular}{r|rr|r}
\hline Variable & Factor1 & Factor2 & Uniqueness \\
\hline RS056 & 0.5528 & 0.4120 & 0.5247 \\
RS057 & 0.1124 & 0.4214 & 0.8098 \\
RS058 & 0.5333 & 0.0718 & 0.7105 \\
RS059 & 0.6961 & -0.1704 & 0.4865 \\
RS060 & -0.1339 & -0.0596 & 0.9785 \\
RS061 & -0.5126 & 0.2851 & 0.6560 \\
RS062 & 0.7855 & -0.2165 & 0.3361 \\
RS063 & 0.2895 & 0.3919 & 0.7626 \\
\hline
\end{tabular}

\section*{Example 3}

We noted in example 2 that the matrix of estimates of the tetrachoric correlation coefficients need not be positive definite. Here is an example:
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline \multicolumn{8}{|l|}{```
matrix with tetrachoric correlations is not positive semidefinite;
    it has 2 negative eigenvalues
    maxdiff(corr,adj-corr) = 0.2346
    (adj-corr: tetrachoric correlations adjusted to be positive semidefinite)
```} \\
\hline adj-corr & RS056 & RS057 & RS058 & RS059 & RS060 & RS061 & RS062 \\
\hline RS056 & 1.0000 & & & & & & \\
\hline RS057 & 0.5284 & 1.0000 & & & & & \\
\hline RS058 & 0.3012 & 0.2548 & 1.0000 & & & & \\
\hline RS059 & 0.3251 & 0.2791 & 0.0550 & 1.0000 & & & \\
\hline RS060 & -0.5197 & -0.4222 & -0.7163 & 0.0552 & 1.0000 & & \\
\hline RS061 & 0.3448 & 0.4815 & -0.0958 & -0.1857 & -0.0980 & 1.0000 & \\
\hline RS062 & 0.1066 & -0.0375 & 0.0072 & 0.3909 & -0.2333 & -0.7654 & 1.0000 \\
\hline RS063 & 0.3830 & 0.4939 & 0.4336 & 0.0075 & -0.8937 & -0.0337 & 0.4934 \\
\hline adj-corr & RS063 & & & & & & \\
\hline RS063 & 1.0000 & & & & & & \\
\hline
\end{tabular}
```

. mata:
_ mata (type end to exit) -_
: C2 = st_matrix("r(corr)")
: eigenvecs = .
: eigenvals = .
: symeigensystem(C2, eigenvecs, eigenvals)
: eigenvals
1 2 3 4
1
4.156592567
1 .4845368741 . 2131895139 -8.80914e-19 -1.90196e-16
: end

```

The estimated tetrachoric correlation matrix is rank-2 deficient. With this C2 matrix, we can only use models of correlation that allow for singular cases.

\section*{Tetrachoric correlations with simulated data}

\section*{> Example 4}

We use drawnorm (see [D] drawnorm) to generate a sample of 1,000 observations from a bivariate normal distribution with means -1 and 1 , unit variances, and correlation 0.4.
```

. clear
. set seed 11000
. matrix m = (1, -1)
. matrix V = (1, 0.4 \ 0.4, 1)
. drawnorm c1 c2, n(1000) means(m) cov(V)
(obs 1000)

```

Now consider the measurement model assumed by the tetrachoric correlations. We observe only whether c1 and c2 are greater than zero,


We want to estimate the correlation of \(c 1\) and \(c 2\) from the binary variables d 1 and d2. Pearson's correlation of the binary variables d 1 and d 2 is 0.170 -a seriously biased estimate of the underlying correlation \(\rho=0.4\).
```

. correlate d1 d2
(obs=1000)

|  | d 1 | d 2 |
| ---: | ---: | ---: |
| d 1 | 1.0000 |  |
| d 2 | 0.1704 | 1.0000 |

```

The tetrachoric correlation coefficient of d 1 and d2 estimates the Pearson correlation of the latent continuous variables, c1 and c2.
```

. tetrachoric d1 d2
Number of obs = 1000
Tetrachoric rho = 0.4790
Std error = 0.0700
Test of Ho: d1 and d2 are independent
2-sided exact P = 0.0000

```

The estimate of the tetrachoric correlation of d 1 and \(\mathrm{d} 2,0.4790\), is much closer to the underlying correlation, 0.4 , between c 1 and c 2 .

\section*{Stored results}
tetrachoric stores the following in \(r()\) :
Scalars
\(r\) (rho) tetrachoric correlation coefficient between variables 1 and 2
\(r(N) \quad\) number of observations
\(r\) (nneg) number of negative eigenvalues (posdef only)
\(r\) (se_rho) standard error of \(r\) (rho)
\(r(p) \quad\) exact two-sided significance level
Macros
\(r\) (method) estimator used
Matrices
\(r\) (Rho) tetrachoric correlation matrix
\(r\) (Se_Rho) standard errors of \(r\) (Rho)
\(r\) (Nobs) number of observations used in computing correlation
\(r(P) \quad\) exact two-sided significance level matrix

\section*{Methods and formulas}
tetrachoric provides two estimators for the tetrachoric correlation \(\rho\) of two binary variables with the frequencies \(n_{i j}, i, j=0,1\). tetrachoric defaults to the slower (iterative) maximum likelihood estimator obtained from bivariate probit without explanatory variables (see [ R\(]\) biprobit) by using the Edwards and Edwards noniterative estimator as the initial value. A fast (noniterative) estimator is also available by specifying the edwards option (Edwards and Edwards 1984; Digby 1983)
\[
\widehat{\rho}=\frac{\alpha-1}{\alpha+1}
\]
where
\[
\alpha=\left(\frac{n_{00} n_{11}}{n_{01} n_{10}}\right)^{\pi / 4}(\pi=3.14 \ldots)
\]
if all \(n_{i j}>0\). If \(n_{00}=0\) or \(n_{11}=0, \widehat{\rho}=-1\); if \(n_{01}=0\) or \(n_{10}=0, \widehat{\rho}=1\).
The asymptotic variance of the Edwards and Edwards estimator of the tetrachoric correlation is easily obtained by the delta method,
\[
\operatorname{avar}(\widehat{\rho})=\left(\frac{\pi \alpha}{2(1+\alpha)^{2}}\right)^{2}\left(\frac{1}{n_{00}}+\frac{1}{n_{01}}+\frac{1}{n_{10}}+\frac{1}{n_{11}}\right)
\]
provided all \(n_{i j}>0\), otherwise it is left undefined (missing). The Edwards and Edwards estimator is fast, but may be inaccurate if the margins are very skewed.
tetrachoric reports exact \(p\)-values for statistical independence, computed by the exact option of \([R]\) tabulate twoway.

\section*{References}

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Brown, M. B., and J. K. Benedetti. 1977. On the mean and variance of the tetrachoric correlation coefficient. Psychometrika 42: 347-355.
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Edwards, J. H., and A. W. F. Edwards. 1984. Approximating the tetrachoric correlation coefficient. Biometrics 40 : 563.

Golub, G. H., and C. F. Van Loan. 1996. Matrix Computations. 3rd ed. Baltimore: Johns Hopkins University Press.
Uebersax, J. S. 2000. Estimating a latent trait model by factor analysis of tetrachoric correlations. http://ourworld.compuserve.com/homepages/jsuebersax/irt.htm.
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\section*{Also see}
[R] biprobit - Bivariate probit regression
[R] correlate - Correlations (covariances) of variables or coefficients
[R] spearman - Spearman's and Kendall's correlations
[R] tabulate twoway - Two-way table of frequencies
[MV] factor - Factor analysis
[MV] pca - Principal component analysis

\section*{Title}
tnbreg - Truncated negative binomial regression
\begin{tabular}{llll} 
Syntax & Menu & Description & Options \\
Remarks and examples & Stored results & Methods and formulas & Acknowledgment \\
References & Also see & &
\end{tabular}

\section*{Syntax}
tnbreg depvar \([\) indepvars \(][\) if \(][\) in \(][\) weight \(][\), options \(]\)
options
Description
Model
noconstant
ll (\#|varname)
dispersion(mean)
dispersion(constant)
exposure (varname \({ }_{e}\) )
offset ( varname \(_{o}\) )
constraints (constraints)
collinear
SE/Robust
vce (vcetype)

Reporting
level(\#)
nolrtest
irr
nocnsreport
display_options

Maximization
maximize_options
coeflegend
suppress constant term
truncation point; default value is \(1 l(0)\), zero truncation parameterization of dispersion; the default
constant dispersion for all observations
include \(\ln \left(\right.\) varname \(\left._{e}\right)\) in model with coefficient constrained to 1 include varname \(_{o}\) in model with coefficient constrained to 1 apply specified linear constraints keep collinear variables
vcetype may be oim, robust, cluster clustvar, opg, bootstrap, or jackknife
set confidence level; default is level (95)
suppress likelihood-ratio test
report incidence-rate ratios
do not display constraints
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
control the maximization process; seldom used
display legend instead of statistics
indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
depvar and indepvars may contain time-series operators; see [U] 11.4.4 Time-series varlists.
bootstrap, by, fp, jackknife, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the bootstrap prefix; see [R] bootstrap.
vce() and weights are not allowed with the svy prefix; see [SVY] svy.
fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

\section*{Menu}

Statistics \(>\) Count outcomes \(>\) Truncated negative binomial regression

\section*{Description}
tnbreg estimates the parameters of a truncated negative binomial model by maximum likelihood. The dependent variable depvar is regressed on indepvars, where depvar is a positive count variable whose values are all above the truncation point.

\section*{Options}
\(\qquad\) Model
noconstant; see \([R]\) estimation options.
ll (\#|varname) specifies the truncation point, which is a nonnegative integer. The default is zero truncation, 11 (0).
dispersion(mean \(\mid\) constant) specifies the parameterization of the model. dispersion(mean), the default, yields a model with dispersion equal to \(1+\alpha \exp \left(\mathbf{x}_{j} \boldsymbol{\beta}+\right.\) offset \(\left._{j}\right)\); that is, the dispersion is a function of the expected mean: \(\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}+\right.\) offset \(\left._{j}\right)\). dispersion(constant) has dispersion equal to \(1+\delta\); that is, it is a constant for all observations.
exposure (varname \({ }_{e}\) ), offset ( varname \(_{o}\) ), constraints (constraints), collinear; see [R] estimation options.

\section*{SE/Robust}
vce (vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

\section*{Reporting}
level (\#); see [R] estimation options.
nolrtest suppresses fitting the Poisson model. Without this option, a comparison Poisson model is fit, and the likelihood is used in a likelihood-ratio test of the null hypothesis that the dispersion parameter is zero.
irr reports estimated coefficients transformed to incidence-rate ratios, that is, \(e^{\beta_{i}}\) rather than \(\beta_{i}\). Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated or stored. irr may be specified at estimation or when replaying previously estimated results.
nocnsreport; see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% fmt), pformat (\% fmt), sformat (\% fmt), and nolstretch; see [R] estimation options.

Maximization
maximize_options: difficult, technique (algorithm_spec), iterate(\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance(\#), nonrtolerance, and from (init_specs); see [R] maximize. These options are seldom used.
Setting the optimization type to technique (bhhh) resets the default vcetype to vce (opg).
The following option is available with tnbreg but is not shown in the dialog box:
coeflegend; see \([R]\) estimation options.

\section*{Remarks and examples}

Grogger and Carson (1991) showed that overdispersion causes inconsistent estimation of the mean in the truncated Poisson model. To solve this problem, they proposed using the truncated negative binomial model as an alternative. If data are truncated but do not exhibit overdispersion, the truncated Poisson model is more appropriate; see [R] tpoisson. For an introduction to negative binomial regression, see Cameron and Trivedi (2005, 2010) and Long and Freese (2014). For an introduction to truncated negative binomial models, see Cameron and Trivedi (2013) and Long (1997, chap. 8).
tnbreg fits the mean-dispersion and the constant-dispersion parameterizations of truncated negative binomial models. These parameterizations extend those implemented in nbreg; see [R] nbreg.

\section*{> Example 1}

We illustrate the truncated negative binomial model using the 1997 MedPar dataset (Hilbe 1999). The data are from 1,495 patients in Arizona who were assigned to a diagnostic-related group (DRG) of patients having a ventilator. Length of stay (los), the dependent variable, is a positive integer; it cannot have zero values. The data are truncated because there are no observations on individuals who stayed for zero days.

The objective of this example is to determine whether the length of stay was related to the binary variables: died, hmo, type1, type2, and type3.

The died variable was recorded as a 0 unless the patient died, in which case, it was recorded as a 1 . The other variables also adopted this encoding. The hmo variable was set to 1 if the patient belonged to a health maintenance organization (HMO).

The type1-type3 variables indicated the type of admission used for the patient. The type 1 variable indicated an emergency admit. The type 2 variable indicated an urgent admit-that is, the first available bed. The type3 variable indicated an elective admission. Because type1-type3 were mutually exclusive, only two of the three could be used in the truncated negative binomial regression shown below.
```

. use http://www.stata-press.com/data/r13/medpar
. tnbreg los died hmo type2-type3, vce(cluster provnum) nolog
Truncated negative binomial regression
Truncation point: 0 Number of obs = 1495
Dispersion = mean Wald chi2(4) = 36.01
Log likelihood = -4737.535 Prob > chi2 = 0.0000

```
(Std. Err. adjusted for 54 clusters in provnum)
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline los & Coef. & Robust Std. Err. & z & \(P>|z|\) & [95\% Conf. & Interval] \\
\hline died & -. 2521884 & . 061533 & -4.10 & 0.000 & -. 3727908 & -. 1315859 \\
\hline hmo & -. 0754173 & . 0533132 & -1.41 & 0.157 & -. 1799091 & . 0290746 \\
\hline type2 & . 2685095 & . 0666474 & 4.03 & 0.000 & . 137883 & . 3991359 \\
\hline type3 & . 7668101 & . 2183505 & 3.51 & 0.000 & . 338851 & 1.194769 \\
\hline _cons & 2.224028 & . 034727 & 64.04 & 0.000 & 2.155964 & 2.292091 \\
\hline /lnalpha & -. 630108 & . 0764019 & & & -. 779853 & -. 480363 \\
\hline alpha & . 5325343 & . 0406866 & & & . 4584734 & . 6185588 \\
\hline
\end{tabular}

Because observations within the same hospital (provnum) are likely to be correlated, we specified the vce (cluster provnum) option. The results show that whether the patient died in the hospital and the type of admission have significant effects on the patient's length of stay.

\section*{> Example 2}

To illustrate truncated negative binomial regression with more complex data than the previous example, similar data were created from 100 hospitals. Each hospital had its own way of tracking patient data. In particular, hospitals only recorded data from patients with a minimum length of stay, denoted by the variable minstay.

Definitions for minimum length of stay varied among hospitals, typically, from 5 to 18 days. The objective of this example is the same as before: to determine whether the length of stay, recorded in los, was related to the binary variables: died, hmo, type1, type2, and type3.

The binary variables encode the same information as in example 1 above. The minstay variable was used to allow for varying truncation points.
. use http://www.stata-press.com/data/r13/medproviders
. tnbreg los died hmo type2-type3, ll(minstay) vce(cluster hospital) nolog
Truncated negative binomial regression
Truncation points: minstay Number of obs \(=\quad 2144\)
Dispersion = mean Wald chi2(4) = 15.22
Log likelihood \(=-7864.0928 \quad\) Prob > chi2 \(=0.0043\)
(Std. Err. adjusted for 100 clusters in hospital)
\begin{tabular}{r|rrrrrr}
\hline & & \multicolumn{4}{c}{ Robust } \\
los & Coef. & Std. Err. & \multicolumn{1}{c}{\(\mathbf{z}\)} & \(\mathrm{P}>|\mathrm{z}|\) & [95\% Conf. Interval] \\
\hline died & .0781044 & .0303596 & 2.57 & 0.010 & .0186006 & .1376081 \\
hmo & -.0731128 & .0368897 & -1.98 & 0.047 & -.1454152 & -.0008104 \\
type2 & .0294136 & .0390167 & 0.75 & 0.451 & -.0470578 & .1058849 \\
type3 & .0626352 & .054012 & 1.16 & 0.246 & -.0432265 & .1684969 \\
_cons & 3.014964 & .0290886 & 103.65 & 0.000 & 2.957951 & 3.071977 \\
\hline /lnalpha & -.9965131 & .082867 & & & -1.158929 & -.8340967 \\
\hline alpha & .3691645 & .0305916 & & & .313822 & .4342666 \\
\hline
\end{tabular}

In this analysis, two variables have a statistically significant relationship with length of stay. On average, patients who died in the hospital had longer lengths of stay \((p=0.01)\). Because the coefficient for HMO is negative, that is, \(b_{\mathrm{HMO}}=-0.073\), on average, patients who were insured by an HMO had shorter lengths of stay ( \(p=0.047\) ). The type of admission was not statistically significant ( \(p>0.05\) ).

\section*{Stored results}
tnbreg stores the following in e():
Scalars
e(N)
e(k)
e(k_aux)
e(k_eq)
e(k_eq_model)
e(k_dv)
e(df_m)
e(r2_p)
e(ll)
e(ll_0)
e(ll_c)
e(alpha)
e(N_clust)
e(chi2)
e(chi2_c)
e(p)
e(rank)
e(rank0)
e(ic)
e(rc)
e(converged)
Macros
```

e(cmd)
e(cmdline)
e(depvar)
e(llopt)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(offset)
e(chi2type)
e(chi2_ct)
e(dispers)
e(vce)
e(vcetype)
e(opt)
e(which)
e(ml_method)
e(user)
e(technique)
e(properties)
e(predict)
e(asbalanced)
e(asobserved)

```
Matrices
    e(b)
    e(Cns)
    e(ilog)
    e(gradient)
    e(V)
    e(V_modelbased)

Functions
e(sample) marks estimation sample

\section*{Methods and formulas}

Methods and formulas are presented under the following headings:
Mean-dispersion model
Constant-dispersion model

\section*{Mean-dispersion model}

A negative binomial distribution can be regarded as a gamma mixture of Poisson random variables. The number of times an event occurs, \(y_{j}\), is distributed as Poisson \(\left(\nu_{j} \mu_{j}\right)\). That is, its conditional likelihood is
\[
f\left(y_{j} \mid \nu_{j}\right)=\frac{\left(\nu_{j} \mu_{j}\right)^{y_{j}} e^{-\nu_{j} \mu_{j}}}{\Gamma\left(y_{j}+1\right)}
\]
where \(\mu_{j}=\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}+\right.\) offset \(\left._{j}\right)\) and \(\nu_{j}\) is an unobserved parameter with a \(\operatorname{Gamma}(1 / \alpha, \alpha)\) density:
\[
g(\nu)=\frac{\nu^{(1-\alpha) / \alpha} e^{-\nu / \alpha}}{\alpha^{1 / \alpha} \Gamma(1 / \alpha)}
\]

This gamma distribution has a mean of 1 and a variance of \(\alpha\), where \(\alpha\) is our ancillary parameter.
The unconditional likelihood for the \(j\) th observation is therefore
\[
f\left(y_{j}\right)=\int_{0}^{\infty} f\left(y_{j} \mid \nu\right) g(\nu) d \nu=\frac{\Gamma\left(m+y_{j}\right)}{\Gamma\left(y_{j}+1\right) \Gamma(m)} p_{j}^{m}\left(1-p_{j}\right)^{y_{j}}
\]
where \(p_{j}=1 /\left(1+\alpha \mu_{j}\right)\) and \(m=1 / \alpha\). Solutions for \(\alpha\) are handled by searching for \(\ln \alpha\) because \(\alpha\) must be greater than zero. The conditional probability of observing \(y_{j}\) events given that \(y_{j}\) is greater than the truncation point \(\tau_{j}\) is
\[
\operatorname{Pr}\left(Y=y_{j} \mid y_{j}>\tau_{j}, \mathbf{x}_{j}\right)=\frac{f\left(y_{j}\right)}{\operatorname{Pr}\left(Y>\tau_{j} \mid \mathbf{x}_{j}\right)}
\]

The log likelihood (with weights \(w_{j}\) and offsets) is given by
\[
\begin{aligned}
& m=1 / \alpha \quad p_{j}=1 /\left(1+\alpha \mu_{j}\right) \quad \mu_{j}=\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}+\text { offset }_{j}\right) \\
& \ln L=\sum_{j=1}^{n} w_{j}\left[\ln \left\{\Gamma\left(m+y_{j}\right)\right\}-\ln \left\{\Gamma\left(y_{j}+1\right)\right\}\right. \\
& \left.\quad-\ln \{\Gamma(m)\}+m \ln \left(p_{j}\right)+y_{j} \ln \left(1-p_{j}\right)-\ln \left\{\operatorname{Pr}\left(Y>\tau_{j} \mid p_{j}, m\right)\right\}\right]
\end{aligned}
\]

\section*{Constant-dispersion model}

The constant-dispersion model assumes that \(y_{j}\) is conditionally distributed as Poisson \(\left(\mu_{j}^{*}\right)\), where \(\mu_{j}^{*} \sim \operatorname{Gamma}\left(\mu_{j} / \delta, \delta\right)\) for some dispersion parameter \(\delta\) [by contrast, the mean-dispersion model assumes that \(\left.\mu_{j}^{*} \sim \operatorname{Gamma}\left(1 / \alpha, \alpha \mu_{j}\right)\right]\). The log likelihood is given by
\[
\begin{gathered}
m_{j}=\mu_{j} / \delta \quad p=1 /(1+\delta) \\
\ln L=\sum_{j=1}^{n} w_{j}\left[\ln \left\{\Gamma\left(m_{j}+y_{j}\right)\right\}-\ln \left\{\Gamma\left(y_{j}+1\right)\right\}\right. \\
\left.\quad-\ln \left\{\Gamma\left(m_{j}\right)\right\}+m_{j} \ln (p)+y_{j} \ln (1-p)-\ln \left\{\operatorname{Pr}\left(Y>\tau_{j} \mid p, m_{j}\right)\right\}\right]
\end{gathered}
\]
with everything else defined as shown above in the calculations for the mean-dispersion model.
This command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce (robust) and vce(cluster clustvar), respectively. See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas.
tnbreg also supports estimation with survey data. For details on variance-covariance estimates with survey data, see [SVY] variance estimation.

\section*{Acknowledgment}

We gratefully acknowledge the previous work by Joseph Hilbe (1999) at Arizona State University and past editor of the Stata Technical Bulletin and coauthor of the Stata Press book Generalized Linear Models and Extensions.

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\section*{Also see}
[R] tnbreg postestimation - Postestimation tools for tnbreg
[R] nbreg - Negative binomial regression
[R] poisson - Poisson regression
[R] tpoisson - Truncated Poisson regression
\([\mathrm{R}]\) zinb - Zero-inflated negative binomial regression
[R] zip - Zero-inflated Poisson regression
[SVY] svy estimation - Estimation commands for survey data
[XT] xtnbreg - Fixed-effects, random-effects, \& population-averaged negative binomial models
[U] 20 Estimation and postestimation commands

\section*{Title}
tnbreg postestimation - Postestimation tools for tnbreg
\begin{tabular}{ll} 
Description & Syntax for predict \\
Methods and formulas & Also see
\end{tabular}

\section*{Description}

The following postestimation commands are available after tnbreg:
\begin{tabular}{|c|c|}
\hline Command & Description \\
\hline contrast & contrasts and ANOVA-style joint tests of estimates \\
\hline estat ic & Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) \\
\hline estat summarize & summary statistics for the estimation sample \\
\hline estat vce & variance-covariance matrix of the estimators (VCE) \\
\hline estat (svy) & postestimation statistics for survey data \\
\hline estimates & cataloging estimation results \\
\hline forecast \({ }^{1}\) & dynamic forecasts and simulations \\
\hline lincom & point estimates, standard errors, testing, and inference for linear combinations of coefficients \\
\hline Irtest \({ }^{2}\) & likelihood-ratio test \\
\hline margins & marginal means, predictive margins, marginal effects, and average marginal effects \\
\hline marginsplot & graph the results from margins (profile plots, interaction plots, etc.) \\
\hline nlcom & point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients \\
\hline predict & predictions, residuals, influence statistics, and other diagnostic measures \\
\hline predictnl & point estimates, standard errors, testing, and inference for generalized predictions \\
\hline pwcompare & pairwise comparisons of estimates \\
\hline suest & seemingly unrelated estimation \\
\hline test & Wald tests of simple and composite linear hypotheses \\
\hline testnl & Wald tests of nonlinear hypotheses \\
\hline
\end{tabular}

\footnotetext{
\({ }^{1}\) forecast is not appropriate with svy estimation results.
\({ }^{2}\) lrtest is not appropriate with svy estimation results.
}

\section*{Syntax for predict}
```

predict [type] newvar [if] [in] [, statistic nooffset]
predict [type] {stub*|newvar reg newvar disp } [if ] [in], scores

```
statistic Description
\begin{tabular}{ll} 
Main & \\
n & number of events; the default \\
ir & incidence rate \\
cm & conditional mean, \(E\left(y_{j} \mid y_{j}>\tau_{j}\right)\) \\
\(\operatorname{pr}(n)\) & probability \(\operatorname{Pr}\left(y_{j}=n\right)\) \\
\(\operatorname{pr}(a, b)\) & probability \(\operatorname{Pr}\left(a \leq y_{j} \leq b\right)\) \\
\(\mathrm{cpr}(n)\) & conditional probability \(\operatorname{Pr}\left(y_{j}=n \mid y_{j}>\tau_{j}\right)\) \\
\(\mathrm{cpr}(a, b)\) & conditional probability \(\operatorname{Pr}\left(a \leq y_{j} \leq b \mid y_{j}>\tau_{j}\right)\) \\
xb & linear prediction \\
stdp & standard error of the linear prediction
\end{tabular}

These statistics are available both in and out of sample; type predict ... if e(sample)... if wanted only for the estimation sample.

\section*{Menu for predict}

Statistics \(>\) Postestimation \(>\) Predictions, residuals, etc.

\section*{Options for predict}
\(\qquad\) Main
n , the default, calculates the predicted number of events, which is \(\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}\right)\) if neither offset () nor exposure() was specified when the model was fit; \(\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}+\right.\) offset \(\left._{j}\right)\) if offset() was specified; or \(\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}\right) \times\) exposure \(_{j}\) if exposure() was specified.
ir calculates the incidence rate \(\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}\right)\), which is the predicted number of events when exposure is 1 . This is equivalent to specifying both the n and the nooffset options.
cm calculates the conditional mean,
\[
E\left(y_{j} \mid y_{j}>\tau_{j}\right)=\frac{E\left(y_{j}\right)}{\operatorname{Pr}\left(y_{j}>\tau_{j}\right)}
\]
where \(\tau_{j}\) is the truncation point found in e(llopt).
\(\operatorname{pr}(n)\) calculates the probability \(\operatorname{Pr}\left(y_{j}=n\right)\), where \(n\) is a nonnegative integer that may be specified as a number or a variable.
\(\operatorname{pr}(a, b)\) calculates the probability \(\operatorname{Pr}\left(a \leq y_{j} \leq b\right)\), where \(a\) and \(b\) are nonnegative integers that may be specified as numbers or variables;
\(b\) missing ( \(b \geq\).) means \(+\infty\);
\(\operatorname{pr}(20,\).\() calculates \operatorname{Pr}\left(y_{j} \geq 20\right)\);
\(\operatorname{pr}(20, b)\) calculates \(\operatorname{Pr}\left(y_{j} \geq 20\right)\) in observations for which \(b \geq\). and calculates
\(\operatorname{Pr}\left(20 \leq y_{j} \leq b\right)\) elsewhere.
\(\operatorname{pr}(., b)\) produces a syntax error. A missing value in an observation of the variable \(a\) causes a missing value in that observation for \(\mathrm{pr}(a, b)\).
\(\operatorname{cpr}(n)\) calculates the conditional probability \(\operatorname{Pr}\left(y_{j}=n \mid y_{j}>\tau_{j}\right)\), where \(\tau_{j}\) is the truncation point found in e(llopt). \(n\) is an integer greater than the truncation point that may be specified as a number or a variable.
\(\operatorname{cpr}(a, b)\) calculates the conditional probability \(\operatorname{Pr}\left(a \leq y_{j} \leq b \mid y_{j}>\tau_{j}\right)\), where \(\tau_{j}\) is the truncation point found in e(llopt). The syntax for this option is analogous to that used for \(\mathrm{pr}(a, b)\) except that \(a\) must be greater than the truncation point.
xb calculates the linear prediction, which is \(\mathbf{x}_{j} \boldsymbol{\beta}\) if neither offset () nor exposure() was specified when the model was fit; \(\mathbf{x}_{j} \boldsymbol{\beta}+\) offset \(_{j}\) if offset () was specified; or \(\mathbf{x}_{j} \boldsymbol{\beta}+\ln \left(\right.\) exposure \(\left._{j}\right)\) if exposure() was specified; see nooffset below.
stdp calculates the standard error of the linear prediction.
nooffset is relevant only if you specified offset() or exposure() when you fit the model. It modifies the calculations made by predict so that they ignore the offset or exposure variable; the linear prediction is treated as \(\mathbf{x}_{j} \boldsymbol{\beta}\) rather than as \(\mathbf{x}_{j} \boldsymbol{\beta}+\) offset \(_{j}\) or \(\mathbf{x}_{j} \boldsymbol{\beta}+\ln\) (exposure \({ }_{j}\) ). Specifying predict..., nooffset is equivalent to specifying predict..., ir.
scores calculates equation-level score variables.
The first new variable will contain \(\partial \ln L / \partial\left(\mathbf{x}_{j} \boldsymbol{\beta}\right)\).
The second new variable will contain \(\partial \ln L / \partial(\ln \alpha)\) for dispersion(mean).
The second new variable will contain \(\partial \ln L / \partial(\ln \delta)\) for dispersion(constant).

\section*{Methods and formulas}

In the following formulas, we use the same notation as in \([\mathrm{R}]\) tnbreg.
Methods and formulas are presented under the following headings:
Mean-dispersion model
Constant-dispersion model

\section*{Mean-dispersion model}

The equation-level scores are given by
\[
\begin{aligned}
\operatorname{score}(\mathbf{x} \boldsymbol{\beta})_{j}= & p_{j}\left(y_{j}-\mu_{j}\right)-\frac{p_{j}^{(m+1)} \mu_{j}}{\operatorname{Pr}\left(Y>\tau_{j} \mid p_{j}, m\right)} \\
\operatorname{score}(\omega)_{j}= & -m\left\{\frac{\alpha\left(\mu_{j}-y_{j}\right)}{1+\alpha \mu_{j}}-\ln \left(1+\alpha \mu_{j}\right)+\psi\left(y_{j}+m\right)-\psi(m)\right\} \\
& -\frac{p_{j}^{m}}{\operatorname{Pr}\left(Y>\tau_{j} \mid p_{j}, m\right)}\left\{m \ln \left(p_{j}\right)+\mu_{j} p_{j}\right\}
\end{aligned}
\]
where \(\omega_{j}=\ln \alpha_{j}, \psi(z)\) is the digamma function, and \(\tau_{j}\) is the truncation point found in e(llopt).

\section*{Constant-dispersion model}

The equation-level scores are given by
\[
\begin{aligned}
\operatorname{score}(\mathbf{x} \boldsymbol{\beta})_{j} & =m_{j}\left\{\psi\left(y_{j}+m_{j}\right)-\psi\left(m_{j}\right)+\ln (p)+\frac{p^{m_{j}} \ln (p)}{\operatorname{Pr}\left(Y>\tau_{j} \mid p, m_{j}\right)}\right\} \\
\operatorname{score}(\omega)_{j} & =y_{j}-\left(y_{j}+m_{j}\right)(1-p)-\operatorname{score}(\mathbf{x} \boldsymbol{\beta})_{j}-\frac{\mu_{j} p}{\operatorname{Pr}\left(Y>\tau_{j} \mid p, m_{j}\right)}
\end{aligned}
\]
where \(\omega_{j}=\ln \delta_{j}\) and \(\tau_{j}\) is the truncation point found in e(llopt).

\section*{Also see}
[R] tnbreg - Truncated negative binomial regression
[U] 20 Estimation and postestimation commands

\section*{Title}
tobit — Tobit regression
\begin{tabular}{llll} 
Syntax & Menu & Description & Options \\
Remarks and examples & Stored results & Methods and formulas & References
\end{tabular}

\section*{Syntax}
tobit depvar [indepvars] [if][in][weight], \(1 \mathrm{ll}[(\#)] \mathrm{ul}[(\#)][\) options \(]\)
options Description

Model
noconstant
* 11 [(\#)
*ul [(\#)
offset (varname)
SE/Robust
vce (vcetype)

Reporting
level(\#)
display_options

Maximization
maximize_options
coeflegend
suppress constant term
left-censoring limit
right-censoring limit
include varname in model with coefficient constrained to 1
vcetype may be oim, robust, cluster clustvar, bootstrap, or jackknife
*You must specify at least one of \(1 \mathrm{ll}[(\#)]\) or \(\mathrm{ul}[(\#)]\).
indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
depvar and indepvars may contain time-series operators; see [U] 11.4.4 Time-series varlists.
bootstrap, by, fp, jackknife, nestreg, rolling, statsby, stepwise, and svy are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the bootstrap prefix; see \([\mathrm{R}]\) bootstrap.
aweights are not allowed with the jackknife prefix; see \([R]\) jackknife.
vce() and weights are not allowed with the svy prefix; see [SVY] svy.
aweights, fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

\section*{Menu}

Statistics \(>\) Linear models and related \(>\) Censored regression \(>\) Tobit regression

\section*{Description}
tobit fits a model of depvar on indepvars where the censoring values are fixed.

\section*{Options}
\(\qquad\) Model
noconstant; see \([R]\) estimation options.
\(11[(\#)]\) and \(\mathrm{ul}[(\#)]\) indicate the lower and upper limits for censoring, respectively. You may specify one or both. Observations with depvar \(\leq 11\) () are left-censored; observations with depvar \(\geq \mathrm{ul}\) () are right-censored; and remaining observations are not censored. You do not have to specify the censoring values at all. It is enough to type ll, ul, or both. When you do not specify a censoring value, tobit assumes that the lower limit is the minimum observed in the data (if 11 is specified) and the upper limit is the maximum (if \(u l\) is specified).
offset (varname); see [R] estimation options.

\section*{SE/Robust}
vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
\(\qquad\) Reporting
level (\#); see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\%fmt), pformat (\%fmt), sformat (\%fmt), and nolstretch; see [R] estimation options.

\section*{Maximization}
maximize_options: iterate (\#), [no log, trace, tolerance(\#), ltolerance(\#), nrtolerance(\#), and nonrtolerance; see [R] maximize. These options are seldom used.

Unlike most maximum likelihood commands, tobit defaults to nolog-it suppresses the iteration log.

The following option is available with tobit but is not shown in the dialog box:
coeflegend; see [R] estimation options.

\section*{Remarks and examples}

Tobit estimation was originally developed by Tobin (1958). A consumer durable was purchased if a consumer's desire was high enough, where desire was measured by the dollar amount spent by the purchaser. If no purchase was made, the measure of desire was censored at zero.

\section*{> Example 1: Censored from below}

We will demonstrate tobit with an artificial example, which in the process will allow us to emphasize the assumptions underlying the estimation. We have a dataset containing the mileage ratings and weights of 74 cars. There are no censored variables in this dataset, but we are going to create one. Before that, however, the relationship between mileage and weight in our complete data is
```

. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)

```
- generate wgt = weight/1000
. regress mpg wgt
\begin{tabular}{r|crc} 
Source & SS & df & MS \\
\hline Model & 1591.99024 & 1 & 1591.99024 \\
Residual & 851.469221 & 72 & 11.8259614 \\
\hline Total & 2443.45946 & 73 & 33.4720474
\end{tabular}
\begin{tabular}{lrr} 
Number of obs & \(=\) & 74 \\
F 1, 72) & \(=134.62\) \\
Prob \(>\) F & \(=\) & 0.0000 \\
R-squared & \(=0.6515\) \\
Adj R-squared & \(=\) & 0.6467 \\
Root MSE & \(=3.4389\)
\end{tabular}
\begin{tabular}{r|rrrrrrr}
\hline mpg & Coef. & Std. Err. & t & P>|t| & [95\% Conf. Interval] \\
\hline wgt & -6.008687 & .5178782 & -11.60 & 0.000 & -7.041058 & -4.976316 \\
_cons & 39.44028 & 1.614003 & 24.44 & 0.000 & 36.22283 & 42.65774 \\
\hline
\end{tabular}
(We divided weight by 1,000 simply to make discussing the resulting coefficients easier. We find that each additional 1,000 pounds of weight reduces mileage by 6 mpg .)
mpg in our data ranges from 12 to 41 . Let us now pretend that our data were censored in the sense that we could not observe a mileage rating below 17 mpg . If the true mpg is 17 or less, all we know is that the mpg is less than or equal to 17 :
. replace mpg=17 if mpg<=17
(14 real changes made)
. tobit mpg wgt, ll
Tobit regression


Log likelihood \(=-164.25438\)\(\quad\)\begin{tabular}{ll} 
\\
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline mpg & Coef. & Std. Err. & t & \(P>|t|\) & [95\% Conf. & Interval] \\
\hline wgt & -6.87305 & . 7002559 & -9.82 & 0.000 & -8.268658 & -5.477442 \\
\hline _cons & 41.49856 & 2.05838 & 20.16 & 0.000 & 37.39621 & 45.6009 \\
\hline /sigma & 3.845701 & . 3663309 & & & 3.115605 & 4.575797 \\
\hline \multicolumn{2}{|l|}{Obs. summary: 18} & \multicolumn{5}{|l|}{\begin{tabular}{l}
left-censored observations at \(\mathrm{mpg}<=17\) uncensored observations \\
right-censored observations
\end{tabular}} \\
\hline
\end{tabular}

The replace before estimation was not really necessary - we remapped all the mileage ratings below 17 to 17 merely to reassure you that tobit was not somehow using uncensored data. We typed 11 after tobit to inform tobit that the data were left-censored. tobit found the minimum of mpg in our data and assumed that was the censoring point. We could also have dispensed with replace and typed 11 (17), informing tobit that all values of the dependent variable 17 and below are really censored at 17. In either case, at the bottom of the table, we are informed that there are, as a result, 18 left-censored observations.

On these data, our estimate is now a reduction of 6.9 mpg per 1,000 extra pounds of weight as opposed to 6.0. The parameter reported as /sigma is the estimated standard error of the regression; the resulting 3.8 is comparable with the estimated root mean squared error reported by regress of 3.4.

\section*{- Technical note}

You would never want to throw away information by purposefully censoring variables. The regress estimates are in every way preferable to those of tobit. Our example is designed solely to illustrate the relationship between tobit and regress. If you have uncensored data, use regress. If your data are censored, you have no choice but to use tobit.

\section*{> Example 2: Censored from above}
tobit can also fit models that are censored from above. This time, let's assume that we do not observe the actual mileage rating of cars yielding 24 mpg or better-we know only that it is at least 24. (Also assume that we have undone the change to mpg we made in the previous example.)
```

. use http://www.stata-press.com/data/r13/auto, clear
(1978 Automobile Data)
. generate wgt = weight/1000
. regress mpg wgt
(output omitted)
. tobit mpg wgt, ul(24)
Tobit regression Number of obs = 74
LR chi2(1) = 90.72
Prob > chi2 = 0.0000
Pseudo R2 = 0.2589

| mpg | Coef. | Std. Err. | t | P>\|t| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| wgt | -5.080645 | .43493 | -11.68 | 0.000 | -5.947459 | -4.213831 |
| _cons | 36.08037 | 1.432056 | 25.19 | 0.000 | 33.22628 | 38.93445 |
| /sigma | 2.385357 | .2444604 |  |  | 1.898148 | 2.872566 |

    Obs. summary:
    O left-censored observations
        uncensored observations
    23 right-censored observations at mpg>=24
    ```

\section*{> Example 3: Two-limit tobit model}
tobit can also fit models that are censored from both sides (the so-called two-limit tobit):
```

. tobit mpg wgt, ll(17) ul(24)
Tobit regression Number of obs = 74
LR chi2(1) = 77.60
Prob > chi2 = 0.0000
Pseudo R2 = 0.2712

```
\begin{tabular}{r|rcccrr}
\hline mpg & Coef. & Std. Err. & t & P>|t| & [95\% Conf. Interval] \\
\hline wgt & -5.764448 & .7245417 & -7.96 & 0.000 & -7.208457 & -4.320438 \\
_cons & 38.07469 & 2.255917 & 16.88 & 0.000 & 33.57865 & 42.57072 \\
\hline /sigma & 2.886337 & .3952143 & & & 2.098676 & 3.673998 \\
\hline
\end{tabular}

Obs. summary: 18 left-censored observations at \(\mathrm{mpg}<=17\)
33 uncensored observations
23 right-censored observations at mpg>=24

\section*{Stored results}
tobit stores the following in e():
Scalars
e(N)
e(N_unc)
e(N_lc)
e(N_rc)
e(llopt)
e(ulopt)
e(k_aux)
e(df_m)
e(df_r)
e(r2_p)
e(chi2)
e(11)
e(11_0)
e(N_clust)
e(F)
\(e(p)\)
e(rank)
\(e\) (converged)
number of observations
number of uncensored observations
number of left-censored observations
number of right-censored observations
contents of 11 (), if specified
contents of ul(), if specified
number of auxiliary parameters
model degrees of freedom
residual degrees of freedom
pseudo- \(R\)-squared
\(\chi^{2}\)
log likelihood
log likelihood, constant-only model
number of clusters
\(F\) statistic
significance
rank of e(V)
1 if converged, 0 otherwise

Macros
e(cmd)
e(cmdline)
e(depvar)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(offset)
e(chi2type)
e(vce)
e(vcetype)
e(properties)
e(predict)
e(footnote)
e(asbalanced)
e(asobserved)
Matrices
e(b)
e(V)
e(V_modelbased)
Functions
e(sample)
tobit
command as typed
name of dependent variable
weight type
weight expression title in estimation output name of cluster variable linear offset variable LR; type of model \(\chi^{2}\) test
vcetype specified in vce() title used to label Std. Err. b V
program used to implement predict
program and arguments to display footnote factor variables fvset as asbalanced factor variables fvset as asobserved
coefficient vector variance-covariance matrix of the estimators model-based variance
marks estimation sample

James Tobin (1918-2002) was an American economist who after education and research at Harvard moved to Yale, where he was on the faculty from 1950 to 1988 . He made many outstanding contributions to economics and was awarded the Nobel Prize in 1981 "for his analysis of financial markets and their relations to expenditure decisions, employment, production and prices". He trained in the U.S. Navy with the writer in Herman Wouk, who later fashioned a character after Tobin in the novel The Caine Mutiny (1951): "A mandarin-like midshipman named Tobit, with a domed forehead, measured quiet speech, and a mind like a sponge, was ahead of the field by a spacious percentage."

\section*{Methods and formulas}

See Methods and formulas in \([\mathrm{R}]\) intreg.
See Tobin (1958) for the original derivation of the tobit model. An introductory description of the tobit model can be found in, for instance, Wooldridge (2013, sec. 17.2), Davidson and MacKinnon (2004, 484-486), Long (1997, 196-210), and Maddala and Lahiri (2006, 333-336). Cameron and Trivedi (2010, chap. 16) discuss the tobit model using Stata examples.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce (robust) and vce (cluster clustvar), respectively. See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas.
tobit also supports estimation with survey data. For details on VCEs with survey data, see [SVY] variance estimation.

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\footnotetext{
Also see
[R] tobit postestimation - Postestimation tools for tobit
[ R ] heckman - Heckman selection model
\([R]\) intreg - Interval regression
[R] ivtobit - Tobit model with continuous endogenous regressors
\([R]\) regress - Linear regression
\([\mathrm{R}]\) truncreg - Truncated regression
[SVY] svy estimation - Estimation commands for survey data
[XT] xtintreg - Random-effects interval-data regression models
[XT] xttobit — Random-effects tobit models
[U] 20 Estimation and postestimation commands
}

\section*{Title}
tobit postestimation - Postestimation tools for tobit
\begin{tabular}{llll} 
Description & Syntax for predict & Menu for predict & Options for predict \\
Remarks and examples & References & Also see
\end{tabular}

\section*{Description}

The following postestimation commands are available after tobit:
\begin{tabular}{ll} 
Command & Description \\
\hline contrast & contrasts and ANOVA-style joint tests of estimates \\
estat ic & Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) \\
estat summarize & \begin{tabular}{l} 
summary statistics for the estimation sample \\
estat vce \\
estat (svy)
\end{tabular} \\
\begin{tabular}{l} 
variance-covariance matrix of the estimators (VCE) \\
postestimation statistics for survey data \\
cataloging estimation results
\end{tabular} \\
forecast \\
hausman & \begin{tabular}{l} 
dynamic forecasts and simulations
\end{tabular} \\
lincom & \begin{tabular}{l} 
Hausman's specification test \\
point estimates, standard errors, testing, and inference for linear \\
combinations of coefficients
\end{tabular} \\
linktest & \begin{tabular}{l} 
link test for model specification \\
likelihood-ratio test
\end{tabular} \\
margins & \begin{tabular}{l} 
marginal means, predictive margins, marginal effects, and average marginal \\
effects
\end{tabular} \\
marginsplot & \begin{tabular}{l} 
graph the results from margins (profile plots, interaction plots, etc.) \\
point estimates, standard errors, testing, and inference for nonlinear \\
combinations of coefficients
\end{tabular} \\
nlcom & \begin{tabular}{l} 
predictions, residuals, influence statistics, and other diagnostic measures \\
point estimates, standard errors, testing, and inference for generalized predictions \\
predict
\end{tabular} \\
predictnl & \begin{tabular}{l} 
pairwise comparisons of estimates \\
seemingly unrelated estimation
\end{tabular} \\
pwcompare & \begin{tabular}{l} 
Wald tests of simple and composite linear hypotheses
\end{tabular} \\
suest & Wald tests of nonlinear hypotheses
\end{tabular}

\footnotetext{
\({ }^{1}\) forecast is not appropriate with svy estimation results.
2 lrtest is not appropriate with svy estimation results.
}

\section*{Syntax for predict}
```

    predict [type] newvar [if] [in] [, statistic nooffset]
    predict [type] {stub*|newvar reg newvar sigma } [if ] [in], scores
statistic Description

```
Main
    \(\mathrm{xb} \quad\) linear prediction; the default
    stdp standard error of the linear prediction
    stdf standard error of the forecast
    \(\underline{\operatorname{pr}}(a, b) \quad \operatorname{Pr}\left(a<y_{j}<b\right)\)
    \(\mathrm{e}(a, b) \quad E\left(y_{j} \mid a<y_{j}<b\right)\)
    ystar \((a, b) \quad E\left(y_{j}^{*}\right), y_{j}^{*}=\max \left\{a, \min \left(y_{j}, b\right)\right\}\)

These statistics are available both in and out of sample; type predict ... if e(sample)... if wanted only for the estimation sample.
stdf is not allowed with svy estimation results.
where \(a\) and \(b\) may be numbers or variables; \(a\) missing ( \(a \geq\).) means \(-\infty\), and \(b\) missing ( \(b \geq\).) means \(+\infty\); see [U] 12.2.1 Missing values.

\section*{Menu for predict}

Statistics \(>\) Postestimation \(>\) Predictions, residuals, etc.

\section*{Options for predict}
\(\qquad\) Main
xb , the default, calculates the linear prediction.
stdp calculates the standard error of the prediction, which can be thought of as the standard error of the predicted expected value or mean for the observation's covariate pattern. The standard error of the prediction is also referred to as the standard error of the fitted value.
stdf calculates the standard error of the forecast, which is the standard error of the point prediction for 1 observation. It is commonly referred to as the standard error of the future or forecast value. By construction, the standard errors produced by stdf are always larger than those produced by stdp; see Methods and formulas in [R] regress postestimation.
\(\operatorname{pr}(a, b)\) calculates \(\operatorname{Pr}\left(a<\mathbf{x}_{j} \mathbf{b}+u_{j}<b\right)\), the probability that \(y_{j} \mid \mathbf{x}_{j}\) would be observed in the interval \((a, b)\).
\(a\) and \(b\) may be specified as numbers or variable names; \(l b\) and \(u b\) are variable names;
\(\operatorname{pr}(20,30)\) calculates \(\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)\);
\(\operatorname{pr}(l b, u b)\) calculates \(\operatorname{Pr}\left(l b<\mathbf{x}_{j} \mathbf{b}+u_{j}<u b\right)\); and
\(\operatorname{pr}(20, u b)\) calculates \(\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{j}<u b\right)\).
\(a\) missing \((a \geq\).\() means -\infty ; \operatorname{pr}(., 30)\) calculates \(\operatorname{Pr}\left(-\infty<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)\);
\(\operatorname{pr}(l b, 30)\) calculates \(\operatorname{Pr}\left(-\infty<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)\) in observations for which \(l b \geq\).
and calculates \(\operatorname{Pr}\left(l b<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)\) elsewhere.
\(b\) missing \((b \geq\).\() means +\infty\); \(\operatorname{pr}(20,\).\() calculates \operatorname{Pr}\left(+\infty>\mathbf{x}_{j} \mathbf{b}+u_{j}>20\right)\);
\(\operatorname{pr}(20, u b)\) calculates \(\operatorname{Pr}\left(+\infty>\mathbf{x}_{j} \mathbf{b}+u_{j}>20\right)\) in observations for which \(u b \geq\). and calculates \(\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{j}<u b\right)\) elsewhere.
\(\mathrm{e}(a, b)\) calculates \(E\left(\mathbf{x}_{j} \mathbf{b}+u_{j} \mid a<\mathbf{x}_{j} \mathbf{b}+u_{j}<b\right)\), the expected value of \(y_{j} \mid \mathbf{x}_{j}\) conditional on \(y_{j} \mid \mathbf{x}_{j}\) being in the interval \((a, b)\), meaning that \(y_{j} \mid \mathbf{x}_{j}\) is truncated.
\(a\) and \(b\) are specified as they are for \(\operatorname{pr}()\).
ystar \((a, b)\) calculates \(E\left(y_{j}^{*}\right)\), where \(y_{j}^{*}=a\) if \(\mathbf{x}_{j} \mathbf{b}+u_{j} \leq a, y_{j}^{*}=b\) if \(\mathbf{x}_{j} \mathbf{b}+u_{j} \geq b\), and \(y_{j}^{*}=\mathbf{x}_{j} \mathbf{b}+u_{j}\) otherwise, meaning that \(y_{j}^{*}\) is censored. \(a\) and \(b\) are specified as they are for pr() .
nooffset is relevant only if you specified offset (varname). It modifies the calculations made by predict so that they ignore the offset variable; the linear prediction is treated as \(\mathbf{x}_{j} \mathbf{b}\) rather than as \(\mathbf{x}_{j} \mathbf{b}+\) offset \(_{j}\).
scores calculates equation-level score variables.
The first new variable will contain \(\partial \ln L / \partial\left(\mathbf{x}_{j} \boldsymbol{\beta}\right)\).
The second new variable will contain \(\partial \ln L / \partial \sigma\).

\section*{Remarks and examples}

Following Cong (2000), write the tobit model as
\[
y_{i}^{*}= \begin{cases}y_{i}, & \text { if } a<y_{i}<b \\ a, & \text { if } y_{i} \leq a \\ b, & \text { if } y_{i} \geq b\end{cases}
\]
\(y_{i}\) is a latent variable; instead, we observe \(y_{i}^{*}\), which is bounded between \(a\) and \(b\) if \(y_{i}\) is outside those bounds.

There are four types of marginal effects that may be of interest in the tobit model, depending on the application:
1. The \(\beta\) coefficients themselves measure how the unobserved variable \(y_{i}\) changes with respect to changes in the regressors.
2. The marginal effects of the truncated expected value \(E\left(y_{i}^{*} \mid a<y_{i}^{*}<b\right)\) measure the changes in \(y_{i}\) with respect to changes in the regressors among the subpopulation for which \(y_{i}\) is not at a boundary.
3. The marginal effects of the censored expected value \(E\left(y_{i}^{*}\right)\) describe how the observed variable \(y_{i}^{*}\) changes with respect to the regressors.
4. The marginal effects of \(\operatorname{Pr}\left(\mathrm{a}<\mathrm{y}_{\mathrm{i}}^{*}<\mathrm{b}\right)\) describe how the probability of being uncensored changes with respect to the regressors.
In the next example, we show how to obtain each of these.

\section*{> Example 1}

In example 3 of \([R]\) tobit, we fit a two-limit tobit model of mpg on wgt.
```

. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. generate wgt = weight/1000
. tobit mpg wgt, ll(17) ul(24)

```
Tobit regression \(\quad\) Number of obs \(=\quad 74\)
LR chi2 (1) \(=77.60\)
Prob > chi2 \(=0.0000\)
Pseudo R2 \(=0.2712\)
\begin{tabular}{r|rrcrrr}
\hline mpg & Coef. & Std. Err. & \(t\) & P>|t| & [95\% Conf. Interval] \\
wgt & -5.764448 & .7245417 & -7.96 & 0.000 & -7.208457 & -4.320438 \\
_cons & 38.07469 & 2.255917 & 16.88 & 0.000 & 33.57865 & 42.57072 \\
\hline /sigma & 2.886337 & .3952143 & & & 2.098676 & 3.673998 \\
\hline
\end{tabular}
Obs. summary: \begin{tabular}{ll}
18 & left-censored observations at \(\mathrm{mpg}<=17\) \\
33 & uncensored observations \\
& 23 right-censored observations at \(\mathrm{mpg}>=24\)
\end{tabular}
tobit reports the \(\beta\) coefficients for the latent regression model. The marginal effect of \(x_{k}\) on \(y\) is simply the corresponding \(\beta_{k}\), because \(E(y \mid x)\) is linear in \(x\). Thus a 1,000 -pound increase in a car's weight (which is a 1 -unit increase in wgt) would lower fuel economy by 5.8 mpg .

To estimate the means of the marginal effects on the expected value of the censored outcome, conditional on weight being each of three values ( 2,\(000 ; 3,000\); and 4,000 pounds), we type
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \begin{tabular}{l}
Conditional \\
Model VCE
\end{tabular} & \begin{tabular}{l}
inal effec \\
IM
\end{tabular} & & & Numb & of obs & 74 \\
\hline Expression dy/dx w.r.t. & \multicolumn{6}{|l|}{\[
\begin{aligned}
& \mathrm{E}(\mathrm{mpg} * \mid 17<\mathrm{mpg}<24), \text { predict }(y s t a r(17,24)) \\
& \text { wgt }
\end{aligned}
\]} \\
\hline 1._at & & = & \multicolumn{4}{|l|}{2} \\
\hline 2._at & & = & \multicolumn{4}{|l|}{3} \\
\hline 3._at & & = & \multicolumn{4}{|l|}{4} \\
\hline & \multicolumn{5}{|c|}{Delta-method} & Interval] \\
\hline \multicolumn{7}{|l|}{wgt} \\
\hline _at & & & & & & \\
\hline 1 & -1.0861 & . 311273 & -3.49 & 0.000 & -1.696184 & -. 4760162 \\
\hline 2 & -4.45315 & . 4772541 & -9.33 & 0.000 & -5.388551 & -3.51775 \\
\hline 3 & -1.412822 & . 3289702 & -4.29 & 0.000 & -2.057591 & -. 768052 \\
\hline
\end{tabular}

The \(E\left(y^{*} \mid x\right)\) is nonlinear in \(x\), so the marginal effect for a continuous covariate is not the same as the change in \(y^{*}\) induced by a one-unit change in \(x\). Recall that the marginal effect at a point is the slope of the tangent line at that point. In our example, we estimate the mean of the marginal effects for different values of wgt. The estimated mean of the marginal effects is -1.1 mpg for a 2,000 pound car; -4.5 mpg for a 3,000 pound car; and -1.4 mpg for a 4,000 pound car.

To estimate the means of the marginal effects on the expected value of the truncated outcome at the same levels of wgt, we type
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline Conditional Model VCE & ginal effec OIM & & & Numb & of obs & 74 \\
\hline \begin{tabular}{l}
Expression \\
dy/dx w.r.t.
\end{tabular} & \multicolumn{6}{|l|}{```
E(mpg|17<mpg<24), predict(e(17,24))
wgt
```} \\
\hline 1._at & & = & \multicolumn{4}{|l|}{2} \\
\hline 2._at & wgt & = & \multicolumn{4}{|l|}{3} \\
\hline 3._at & wgt & = & \multicolumn{4}{|l|}{4} \\
\hline & \[
d y / d x
\] & \multicolumn{5}{|l|}{Delta-method} \\
\hline \multicolumn{7}{|l|}{wgt} \\
\hline _at & & & & & & \\
\hline 1 & -1.166572 & . 0827549 & -14.10 & 0.000 & -1.328768 & -1.004375 \\
\hline 2 & -2.308842 & . 4273727 & -5.40 & 0.000 & -3.146477 & -1.471207 \\
\hline 3 & -1.288896 & . 0889259 & -14.49 & 0.000 & -1.463188 & -1.114604 \\
\hline
\end{tabular}

The mean of the marginal effects of a change in wgt on \(y_{i}\) (which is bounded between 17 and 24) is about -1.2 mpg for a 2,000 pound car; -2.3 mpg for a 3,000 pound car; and -1.3 for a 4,000 pound car.

\section*{References}

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\section*{Also see}
[R] tobit - Tobit regression
[U] 20 Estimation and postestimation commands

\section*{Title}
\begin{tabular}{llll} 
Syntax & Menu & Description & Options \\
Remarks and examples & Stored results & Methods and formulas & References \\
Also see & &
\end{tabular}

\section*{Syntax}
total varlist \([\) if \(][\) in \(][\) weight \(][\), options \(]\)
options
iffin/over
over (varlist [, nolabel])

SE/Cluster
vce (vcetype)

Reporting
level(\#)
noheader
nolegend
display_options
coeflegend

Description
group over subpopulations defined by varlist; optionally, suppress group labels
bootstrap, jackknife, mi estimate, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands. vce(bootstrap) and vce(jackknife) are not allowed with the mi estimate prefix.
Weights are not allowed with the bootstrap prefix; see \([R]\) bootstrap.
vce() and weights are not allowed with the svy prefix; see [SVY] svy.
fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

\section*{Menu}

Statistics \(>\) Summaries, tables, and tests \(>\) Summary and descriptive statistics \(>\) Totals

\section*{Description}
total produces estimates of totals, along with standard errors.

\section*{Options}

\section*{if/in/over}
over (varlist [, nolabel]) specifies that estimates be computed for multiple subpopulations, which are identified by the different values of the variables in varlist.
When this option is supplied with one variable name, such as over (varname), the value labels of varname are used to identify the subpopulations. If varname does not have labeled values (or there are unlabeled values), the values themselves are used, provided that they are nonnegative integers. Noninteger values, negative values, and labels that are not valid Stata names are substituted with a default identifier.
When over() is supplied with multiple variable names, each subpopulation is assigned a unique default identifier.
nolabel specifies that value labels attached to the variables identifying the subpopulations be ignored.
\(\qquad\) SE/Cluster
vce (vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (analytic), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
vce(analytic), the default, uses the analytically derived variance estimator associated with the sample total.

Reporting \(\qquad\)
level(\#); see [R] estimation options.
noheader prevents the table header from being displayed. This option implies nolegend. nolegend prevents the table legend identifying the subpopulations from being displayed. display_options: cformat (\%fmt) and nolstretch; see [R] estimation options.

The following option is available with total but is not shown in the dialog box: coeflegend; see \([R]\) estimation options.

\section*{Remarks and examples}
> Example 1
Suppose that we collected data on incidence of heart attacks. The variable heartatk indicates whether a person ever had a heart attack ( 1 means yes; 0 means no). We can then estimate the total number of persons who have had heart attacks for each sex in the population represented by the data we collected.
\begin{tabular}{|c|c|c|c|c|}
\hline \multicolumn{2}{|l|}{Total estimation} & \multicolumn{2}{|r|}{Number of obs} & \multirow[t]{3}{*}{\(=4946\)} \\
\hline \multicolumn{4}{|c|}{\multirow[t]{2}{*}{\[
\begin{aligned}
\text { Male: sex } & =\text { Male } \\
\text { Female: sex } & =\text { Female }
\end{aligned}
\]}} & \\
\hline & & & & \\
\hline Over & Total & Std. Err. & [95\% Con & Interval] \\
\hline \multicolumn{5}{|l|}{heartatk} \\
\hline Male & 944559 & 104372.3 & 739943 & 1149175 \\
\hline Female & 581590 & 82855.59 & 419156.3 & 744023.7 \\
\hline
\end{tabular}

\section*{Stored results}
total stores the following in e():
Scalars
e(N)
e(N_over)
e(N_clust)
e(k_eq)
e(df_r)
\(e\) (rank)
Macros
e(cmd)
\(e(c m d l i n e)\)
e(varlist)
e(wtype)
e(wexp)
\(e(t i t l e)\)
e(cluster)
e(over)
e(over_labels)
e(over_namelist)
e(vce)
e(vcetype)
e(properties)
e(estat_cmd)
e(marginsnotok)
Matrices
e(b)
e(V)
e(_N)
e(error)
Functions
\[
e \text { (sample) } \quad \text { marks estimation sample }
\]

\section*{Methods and formulas}

Methods and formulas are presented under the following headings:
The total estimator
Survey data
The survey total estimator
The poststratified total estimator
Subpopulation estimation

\section*{The total estimator}

Let \(y\) denote the variable on which to calculate the total and \(y_{j}, j=1, \ldots, n\), denote an individual observation on \(y\). Let \(w_{j}\) be the frequency weight (or iweight), and if no weight is specified, define \(w_{j}=1\) for all \(j\). See the next section for pweighted data. The sum of the weights is an estimate of the population size:
\[
\widehat{N}=\sum_{j=1}^{n} w_{j}
\]

If the population values of \(y\) are denoted by \(Y_{j}, j=1, \ldots, N\), the associated population total is
\[
Y=\sum_{j=1}^{N} Y_{j}=N \bar{y}
\]
where \(\bar{y}\) is the population mean. The total is estimated as
\[
\widehat{Y}=\widehat{N} \bar{y}
\]

The variance estimator for the total is
\[
\widehat{V}(\widehat{Y})=\widehat{N}^{2} \widehat{V}(\bar{y})
\]
where \(\widehat{V}(\bar{y})\) is the variance estimator for the mean; see \([R]\) mean. The standard error of the total is the square root of the variance.

If \(x, x_{j}, \bar{x}\), and \(\widehat{X}\) are similarly defined for another variable (observed jointly with \(y\) ), the covariance estimator between \(\widehat{X}\) and \(\widehat{Y}\) is
\[
\widehat{\operatorname{Cov}}(\widehat{X}, \widehat{Y})=\widehat{N}^{2} \widehat{\operatorname{Cov}}(\bar{x}, \bar{y})
\]
where \(\widehat{\operatorname{Cov}}(\bar{x}, \bar{y})\) is the covariance estimator between two means; see \([\mathrm{R}]\) mean.

\section*{Survey data}

See [SVY] variance estimation and [SVY] poststratification for discussions that provide background information for the following formulas.

\section*{The survey total estimator}

Let \(Y_{j}\) be a survey item for the \(j\) th individual in the population, where \(j=1, \ldots, M\) and \(M\) is the size of the population. The associated population total for the item of interest is
\[
Y=\sum_{j=1}^{M} Y_{j}
\]

Let \(y_{j}\) be the survey item for the \(j\) th sampled individual from the population, where \(j=1, \ldots, m\) and \(m\) is the number of observations in the sample.

The estimator \(\widehat{Y}\) for the population total \(Y\) is
\[
\widehat{Y}=\sum_{j=1}^{m} w_{j} y_{j}
\]
where \(w_{j}\) is a sampling weight. The estimator for the number of individuals in the population is
\[
\widehat{M}=\sum_{j=1}^{m} w_{j}
\]

The score variable for the total estimator is the variable itself,
\[
z_{j}(\widehat{Y})=y_{j}
\]

\section*{The poststratified total estimator}

Let \(P_{k}\) denote the set of sampled observations that belong to poststratum \(k\), and define \(I_{P_{k}}(j)\) to indicate if the \(j\) th observation is a member of poststratum \(k\), where \(k=1, \ldots, L_{P}\) and \(L_{P}\) is the number of poststrata. Also, let \(M_{k}\) denote the population size for poststratum \(k . P_{k}\) and \(M_{k}\) are identified by specifying the poststrata() and postweight () options on svyset; see [SVY] svyset.

The estimator for the poststratified total is
\[
\widehat{Y}^{P}=\sum_{k=1}^{L_{P}} \frac{M_{k}}{\widehat{M}_{k}} \widehat{Y}_{k}=\sum_{k=1}^{L_{P}} \frac{M_{k}}{\widehat{M}_{k}} \sum_{j=1}^{m} I_{P_{k}}(j) w_{j} y_{j}
\]
where
\[
\widehat{M}_{k}=\sum_{j=1}^{m} I_{P_{k}}(j) w_{j}
\]

The score variable for the poststratified total is
\[
z_{j}\left(\widehat{Y}^{P}\right)=\sum_{k=1}^{L_{P}} I_{P_{k}}(j) \frac{M_{k}}{\widehat{M}_{k}}\left(y_{j}-\frac{\widehat{Y}_{k}}{\widehat{M}_{k}}\right)
\]

\section*{Subpopulation estimation}

Let \(S\) denote the set of sampled observations that belong to the subpopulation of interest, and define \(I_{S}(j)\) to indicate if the \(j\) th observation falls within the subpopulation.

The estimator for the subpopulation total is
\[
\widehat{Y}^{S}=\sum_{j=1}^{m} I_{S}(j) w_{j} y_{j}
\]
and its score variable is
\[
z_{j}\left(\widehat{Y}^{S}\right)=I_{S}(j) y_{j}
\]

The estimator for the poststratified subpopulation total is
\[
\widehat{Y}^{P S}=\sum_{k=1}^{L_{P}} \frac{M_{k}}{\widehat{M}_{k}} \widehat{Y}_{k}^{S}=\sum_{k=1}^{L_{P}} \frac{M_{k}}{\widehat{M}_{k}} \sum_{j=1}^{m} I_{P_{k}}(j) I_{S}(j) w_{j} y_{j}
\]
and its score variable is
\[
z_{j}\left(\widehat{Y}^{P S}\right)=\sum_{k=1}^{L_{P}} I_{P_{k}}(j) \frac{M_{k}}{\widehat{M}_{k}}\left\{I_{S}(j) y_{j}-\frac{\widehat{Y}_{k}^{S}}{\widehat{M}_{k}}\right\}
\]

\section*{References}

Cochran, W. G. 1977. Sampling Techniques. 3rd ed. New York: Wiley.
Stuart, A., and J. K. Ord. 1994. Kendall's Advanced Theory of Statistics: Distribution Theory, Vol I. 6th ed. London: Arnold.

\section*{Also see}
[R] total postestimation - Postestimation tools for total
[R] mean - Estimate means
\([\mathrm{R}]\) proportion - Estimate proportions
[ R\(]\) ratio - Estimate ratios
[MI] estimation - Estimation commands for use with mi estimate
[SVY] direct standardization - Direct standardization of means, proportions, and ratios
[SVY] poststratification - Poststratification for survey data
[SVY] subpopulation estimation - Subpopulation estimation for survey data
[SVY] svy estimation - Estimation commands for survey data
[SVY] variance estimation - Variance estimation for survey data
[U] 20 Estimation and postestimation commands

\section*{Title}
total postestimation - Postestimation tools for total

\section*{Description Remarks and examples Also see}

\section*{Description}

The following postestimation commands are available after total:
\begin{tabular}{ll} 
Command & Description \\
\hline estat vce & \begin{tabular}{l} 
variance-covariance matrix of the estimators (VCE) \\
estat (svy) \\
postestimation statistics for survey data \\
cataloging estimation results
\end{tabular} \\
lincom & \begin{tabular}{l} 
point estimates, standard errors, testing, and inference for linear combinations \\
of coefficients
\end{tabular} \\
nlcom & \begin{tabular}{l} 
of estimates, standard errors, testing, and inference for nonlinear combinations \\
of coefficients
\end{tabular} \\
test & \begin{tabular}{l} 
Wald tests of simple and composite linear hypotheses
\end{tabular} \\
testnl & Wald tests of nonlinear hypotheses
\end{tabular}

\section*{Remarks and examples}
\(>\) Example 1
Continuing with our data on incidence of heart attacks from example 1 in \([\mathrm{R}]\) total, we want to test whether there are twice as many heart attacks among men than women in the population.
```

. use http://www.stata-press.com/data/r13/total
. total heartatk [pw=swgt], over(sex)
(output omitted)
. test _b[Male] = 2*_b[Female]
( 1) [heartatk]Male - 2*[heartatk]Female = 0
F( 1, 4945) = 1.25
Prob > F = 0.2643

```

Thus we do not reject our hypothesis that the total number of heart attacks for men is twice that for women in the population.

\section*{Also see}
[R] total - Estimate totals
[U] 20 Estimation and postestimation commands

\section*{Title}
tpoisson - Truncated Poisson regression
\begin{tabular}{llll} 
Syntax & Menu & Description & Options \\
Remarks and examples & Stored results & Methods and formulas & Acknowledgment \\
References & Also see & &
\end{tabular}

\section*{Syntax}
tpoisson depvar \([\) indepvars \(][\) if \(][\) in \(][\) weight \(][\), options \(]\)
options Description
Model
noconstant
ll(\#|varname) exposure ( varname \(_{e}\) )
offset ( varname \(_{o}\) )
constraints (constraints)
collinear
SE/Robust
vce (vcetype)

Reporting
level(\#)
irr
nocnsreport
display_options

Maximization
maximize_options
coeflegend
suppress constant term
truncation point; default value is \(11(0)\), zero truncation include \(\ln \left(\right.\) varname \(\left._{e}\right)\) in model with coefficient constrained to 1 include varname \(_{o}\) in model with coefficient constrained to 1 apply specified linear constraints
keep collinear variables
vcetype may be oim, robust, cluster clustvar, opg, bootstrap, or jackknife
set confidence level; default is level (95)
report incidence-rate ratios
do not display constraints
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
control the maximization process; seldom used
display legend instead of statistics
indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
depvar and indepvars may contain time-series operators; see [U] 11.4.4 Time-series varlists.
bootstrap, by, fp, jackknife, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the bootstrap prefix; see [R] bootstrap.
vce() and weights are not allowed with the svy prefix; see [SVY] svy.
fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

\section*{Menu}

Statistics \(>\) Count outcomes \(>\) Truncated Poisson regression

\section*{Description}
tpoisson estimates the parameters of a truncated Poisson model by maximum likelihood. The dependent variable depvar is regressed on indepvars, where depvar is a positive count variable whose values are all above the truncation point.

\section*{Options}
```

Model
noconstant; see [R] estimation options.
ll(\#|varname) specifies the truncation point, which is a nonnegative integer. The default is zero
truncation, ll(0).
exposure(varname e), offset(varname o), constraints(constraints), collinear; see [R] esti-
mation options.

```

\section*{SE/Robust}
vce (vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

\section*{Reporting}
level(\#); see [R] estimation options.
irr reports estimated coefficients transformed to incidence-rate ratios, that is, \(e^{\beta_{i}}\) rather than \(\beta_{i}\). Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated. irr may be specified at estimation or when replaying previously estimated results.
nocnsreport; see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% fmt), pformat (\% fmt), sformat (\% fmt), and nolstretch; see [R] estimation options.

Maximization
maximize_options: difficult, technique (algorithm_spec), iterate(\#), [no \(]\) log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance(\#), nonrtolerance, and from(init_specs); see [R] maximize. These options are seldom used.
Setting the optimization type to technique (bhhh) resets the default vcetype to vce (opg).
The following option is available with tpoisson but is not shown in the dialog box:
coeflegend; see \([R]\) estimation options.

\section*{Remarks and examples}

Truncated Poisson regression is used to model the number of occurrences of an event when that number is restricted to be above the truncation point. If the dependent variable is not truncated, standard Poisson regression may be more appropriate; see \([R]\) poisson. Truncated Poisson regression was first proposed by Grogger and Carson (1991). For an introduction to Poisson regression, see Cameron and Trivedi \((2005,2010)\) and Long and Freese (2014). For an introduction to truncated Poisson models, see Cameron and Trivedi (2013) and Long (1997, chap. 8).

Suppose that the patients admitted to a hospital for a given condition form a random sample from a population of interest and that each admitted patient stays at least one day. You are interested in modeling the length of stay of patients in days. The sample is truncated at zero because you only have data on individuals who stayed at least one day. tpoisson accounts for the truncated sample, whereas poisson does not.

Truncation is not the same as censoring. Right-censored Poisson regression was implemented in Stata by Raciborski (2011).

\section*{> Example 1}

Consider the Simonoff (2003) dataset of running shoes for a sample of runners who registered an online running log. A running-shoe marketing executive is interested in knowing how the number of running shoes purchased relates to other factors such as gender, marital status, age, education, income, typical number of runs per week, average miles run per week, and the preferred type of running. These data are naturally truncated at zero. A truncated Poisson model is fit to the number of shoes owned on runs per week, miles run per week, gender, age, and marital status.

No options are needed because zero truncation is the default for tpoisson.
```

. use http://www.stata-press.com/data/r13/runshoes
. tpoisson shoes rpweek mpweek male age married
Iteration 0: log likelihood = -88.328151
Iteration 1: log likelihood = -86.272639
Iteration 2: log likelihood = -86.257999
Iteration 3: log likelihood = -86.257994

| Truncated Poisson regression | Number of obs | $=$ | 60 |
| :--- | :--- | :--- | :--- |
| Truncation point: 0 | LR chi2(5) | $=$ | 22.75 |
|  | Prob > chi2 | $=$ | 0.0004 |
| Log likelihood $=-86.257994$ |  | Pseudo R2 | $=$ |

```
\begin{tabular}{r|rrccrr}
\hline shoes & Coef. & Std. Err. & z & P>|z| & [95\% Conf. Interval] \\
\hline rpweek & .1575811 & .1097893 & 1.44 & 0.151 & -.057602 & .3727641 \\
mpweek & .0210673 & .0091113 & 2.31 & 0.021 & .0032094 & .0389252 \\
male & .0446134 & .2444626 & 0.18 & 0.855 & -.4345246 & .5237513 \\
age & .0185565 & .0137786 & 1.35 & 0.178 & -.008449 & .045562 \\
married & -.1283912 & .2785044 & -0.46 & 0.645 & -.6742498 & .4174674 \\
_cons & -1.205844 & .6619774 & -1.82 & 0.069 & -2.503296 & .0916078 \\
\hline
\end{tabular}

Using the zero-truncated Poisson regression with these data, only the coefficient on average miles per week is statistically significant at the \(5 \%\) level.

\section*{> Example 2}

Semiconductor manufacturing requires that silicon wafers be coated with a layer of metal oxide. The depth of this layer is strictly controlled. In this example, a critical oxide layer is designed for \(300 \pm 20\) angstroms ( \(\AA\) ).

After the oxide layer is coated onto a wafer, the wafer enters a photolithography step in which the lines representing the electrical connections are printed on the oxide and later etched and filled with metal. The widths of these lines are measured. In this example, they are controlled to \(90 \pm 5\) micrometers ( \(\mu \mathrm{m}\) ).

After these and other steps, each wafer is electrically tested at probe. If too many failures are discovered, the wafer is rejected and sent for engineering analysis. In this example, the maximum number of probe failures tolerated for this product is 10.

A major failure at probe has been encountered- 88 wafers had more than 10 failures each. The 88 wafers that failed were tested using 4 probe machines. The engineer suspects that the failures were a result of faulty probe machines, poor depth control, or poor line widths. The line widths and depths in these data are the actual measurement minus its specification target, \(300 \AA\) for the oxide depths and \(90 \mu \mathrm{~m}\) for the line widths.

The following table tabulates the average failure rate for each probe using Stata's mean command; see [R] mean.
\begin{tabular}{|c|c|c|c|c|}
\hline \multicolumn{2}{|l|}{Mean estimation} & \multicolumn{2}{|r|}{Number of obs} & \(=88\) \\
\hline Over & Mean & Std. Err & [95\% Conf & Interval] \\
\hline \multicolumn{5}{|l|}{failures} \\
\hline 1 & 15.875 & 1.186293 & 13.51711 & 18.23289 \\
\hline 2 & 14.95833 & . 5912379 & 13.78318 & 16.13348 \\
\hline 3 & 16.47059 & . 9279866 & 14.62611 & 18.31506 \\
\hline 4 & 23.09677 & . 9451117 & 21.21826 & 24.97529 \\
\hline
\end{tabular}

The \(95 \%\) confidence intervals in this table suggest that there are about 5-11 additional failures per wafer on probe 4 . These are unadjusted for varying line widths and oxide depths. Possibly, probe 4 received the wafers with larger line widths or extreme oxide depths.

Truncated Poisson regression more clearly identifies the root causes for the increased failures by estimating the differences between probes adjusted for the line widths and oxide depths. It also allows us to determine whether the deviations from specifications in line widths or oxide depths might be contributing to the problem.
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline \multicolumn{3}{|l|}{\multirow[t]{3}{*}{\begin{tabular}{l}
Truncated Poisson regression \\
Truncation point: 10
\end{tabular}}} & & \multicolumn{3}{|l|}{\multirow[t]{2}{*}{Number of obs LR chi2(5)}} & \(\begin{array}{ll}= & 88\end{array}\) \\
\hline & & & & & & & LR chi2(5) \(\quad=\quad 73.70\) \\
\hline & & & & Prob & chi2 & & 0.0000 \\
\hline \multicolumn{4}{|l|}{Log likelihood \(=-239.35746\)} & \multicolumn{3}{|l|}{Pseudo R2} & 0.1334 \\
\hline failures & Coef. & Std. Err. & z & \(\mathrm{P}>|\mathrm{z}|\) & \multicolumn{3}{|l|}{[95\% Conf. Interval]} \\
\hline \multicolumn{8}{|l|}{probe} \\
\hline 2 & -. 1113037 & . 1019786 & -1.09 & 0.275 & -. 3111 & & . 0885707 \\
\hline 3 & . 0114339 & . 1036032 & 0.11 & 0.912 & -. 1916 & & . 2144924 \\
\hline 4 & . 4254115 & . 0841277 & 5.06 & 0.000 & . 2605 & & . 5902989 \\
\hline depth & -. 0005034 & . 0033375 & -0.15 & 0.880 & -. 0070 & & . 006038 \\
\hline width & . 0330225 & . 015573 & 2.12 & 0.034 & . 0025 & & . 063545 \\
\hline _cons & 2.714025 & . 0752617 & 36.06 & 0.000 & 2.566 & & 2.861536 \\
\hline
\end{tabular}

The coefficients listed for the probes are testing the null hypothesis: \(H_{0}:\) probe \(_{i}=\) probe \(_{1}\), where \(i\) equals 2,3 , and 4 . Because the only coefficient that is statistically significant is the one for testing for \(H_{0}:\) probe \(_{4}=\) probe \(_{1}, p<0.001\), and because the \(p\)-values for the other probes are not statistically significant, that is, \(p \geq 0.275\), the implication is that there is a difference between probe 4 and the other machines. Because the coefficient for this test is positive, 0.425 , the conclusion is that the average failure rate for probe 4 , after adjusting for line widths and oxide depths, is higher than the other probes. Possibly, probe 4 needs calibration or the head used with this machine is defective.

Line-width control is statistically significant, \(p=0.034\), but variation in oxide depths is not causing the increased failure rate. The engineer concluded that the sudden increase in failures is the result of two problems. First, probe 4 is malfunctioning, and second, there is a possible lithography or etching problem.

\section*{Stored results}
tpoisson stores the following in \(e()\) :
Scalars
```

e(N)
e(k)
e(k_eq)
e(k_eq_model)
e(k_dv)
e(df_m)
e(r2_p)
e(ll)
e(ll_0)
e(N_clust)
e(chi2)
e(p)
e(rank)
e(ic)
e(rc)
e(converged)

```

Macros
```

e(cmd)
e(cmdline)
e(depvar)
e(llopt)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(offset)
e(chi2type)
e(vce)
e(vcetype)
e(opt)
e(which)
e(ml_method)
e(user)
e(technique)
e(properties)
e(predict)
e(asbalanced)
e(asobserved)

```

Matrices
```

e(b)
e(Cns)
e(ilog)
e(gradient)
e(V)
e(V_modelbased)

```

Functions
e(sample)
number of observations
number of parameters
number of equations in e(b)
number of equations in overall model test
number of dependent variables
model degrees of freedom
pseudo- \(R\)-squared
log likelihood
log likelihood, constant-only model
number of clusters
\(\chi^{2}\)
significance
rank of e(V)
number of iterations
return code
1 if converged, 0 otherwise
tpoisson
command as typed
name of dependent variable
contents of 11() , or 0 if not specified
weight type
weight expression
title in estimation output
name of cluster variable
linear offset variable
Wald or LR; type of model \(\chi^{2}\) test
vcetype specified in vce()
title used to label Std. Err.
type of optimization
max or min; whether optimizer is to perform maximization or minimization
type of ml method
name of likelihood-evaluator program
maximization technique
b V
program used to implement predict
factor variables fvset as asbalanced
factor variables fvset as asobserved
coefficient vector
constraints matrix
iteration \(\log\) (up to 20 iterations)
gradient vector
variance-covariance matrix of the estimators
model-based variance
marks estimation sample

\section*{Methods and formulas}

The conditional probability of observing \(y_{j}\) events given that \(y_{j}>\tau_{j}\), where \(\tau_{j}\) is the truncation point, is given by
\[
\operatorname{Pr}\left(Y=y_{j} \mid y_{j}>\tau_{j}, \mathbf{x}_{j}\right)=\frac{\exp (-\lambda) \lambda^{y_{j}}}{y_{j}!\operatorname{Pr}\left(Y>\tau_{j} \mid \mathbf{x}_{j}\right)}
\]

The \(\log\) likelihood (with weights \(w_{j}\) and offsets) is given by
\[
\begin{aligned}
\xi_{j} & =\mathbf{x}_{j} \boldsymbol{\beta}+\text { offset }_{j} \\
f\left(y_{j}\right) & =\frac{\exp \left\{-\exp \left(\xi_{j}\right)\right\} \exp \left(\xi_{j} y_{j}\right)}{y_{j}!\operatorname{Pr}\left(Y>\tau_{j} \mid \xi_{j}\right)} \\
\ln L & =\sum_{j=1}^{n} w_{j}\left[-\exp \left(\xi_{j}\right)+\xi_{j} y_{j}-\ln \left(y_{j}!\right)-\ln \left\{\operatorname{Pr}\left(Y>\tau_{j} \mid \xi_{j}\right)\right\}\right]
\end{aligned}
\]

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce (robust) and vce(cluster clustvar), respectively. See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas.
tpoisson also supports estimation with survey data. For details on variance-covariance estimates with survey data, see [SVY] variance estimation.

\section*{Acknowledgment}

We gratefully acknowledge the previous work by Joseph Hilbe (1999) at Arizona State University and past editor of the Stata Technical Bulletin and coauthor of the Stata Press book Generalized Linear Models and Extensions.

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\section*{Also see}
[R] tpoisson postestimation - Postestimation tools for tpoisson
[R] poisson - Poisson regression
[R] nbreg - Negative binomial regression
\([\mathrm{R}]\) tnbreg - Truncated negative binomial regression
[R] zinb - Zero-inflated negative binomial regression
[R] zip - Zero-inflated Poisson regression
[SVY] svy estimation - Estimation commands for survey data
[XT] xtpoisson - Fixed-effects, random-effects, and population-averaged Poisson models
[U] 20 Estimation and postestimation commands

\section*{Title}
tpoisson postestimation - Postestimation tools for tpoisson

Description Syntax for predict Menu for predict Options for predict
Methods and formulas Also see

\section*{Description}

The following postestimation commands are available after tpoisson:
\begin{tabular}{ll} 
Command & Description \\
\hline contrast & contrasts and ANOVA-style joint tests of estimates \\
estat ic & Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) \\
estat summarize & \begin{tabular}{l} 
summary statistics for the estimation sample \\
variance-covariance matrix of the estimators (VCE)
\end{tabular} \\
estat vce & \begin{tabular}{l} 
pstat (svy) \\
postestimation statistics for survey data
\end{tabular} \\
estimates & \begin{tabular}{l} 
cataloging estimation results \\
dynamic forecasts and simulations
\end{tabular} \\
forecast \({ }^{1}\) & \begin{tabular}{l} 
point estimates, standard errors, testing, and inference for linear combinations \\
of coefficients
\end{tabular} \\
lincom & \begin{tabular}{l} 
likelihood-ratio test
\end{tabular} \\
marginal means, predictive margins, marginal effects, and average marginal \\
margins & \begin{tabular}{l} 
effects
\end{tabular} \\
marginsplot & \begin{tabular}{l} 
graph the results from margins (profile plots, interaction plots, etc.) \\
point estimates, standard errors, testing, and inference for nonlinear combinations \\
of coefficients
\end{tabular} \\
nlcom & \begin{tabular}{l} 
predictions, residuals, influence statistics, and other diagnostic measures \\
point estimates, standard errors, testing, and inference for generalized predictions
\end{tabular} \\
predict & \begin{tabular}{l} 
pairwise comparisons of estimates \\
predictnl \\
swcompare \\
suest
\end{tabular} \\
test & \begin{tabular}{l} 
Wald tests of simple and composite linear hypotheses
\end{tabular} \\
testnl & Wald tests of nonlinear hypotheses
\end{tabular}

\footnotetext{
\({ }^{1}\) forecast is not appropriate with svy estimation results.
\({ }^{2}\) lrtest is not appropriate with svy estimation results.
}

\section*{Syntax for predict}
predict [type] newvar [if] [in] [, statistic nooffset]
statistic Description
Main
n
ir incidence rate
\(\mathrm{cm} \quad\) conditional mean, \(E\left(y_{j} \mid y_{j}>\tau_{j}\right)\)
\(\operatorname{pr}(n)\)
probability \(\operatorname{Pr}\left(y_{j}=n\right)\)
\(\operatorname{pr}(a, b) \quad\) probability \(\operatorname{Pr}\left(a \leq y_{j} \leq b\right)\)
\(\operatorname{cpr}(n) \quad\) conditional probability \(\operatorname{Pr}\left(y_{j}=n \mid y_{j}>\tau_{j}\right)\)
\(\operatorname{cpr}(a, b) \quad\) conditional probability \(\operatorname{Pr}\left(a \leq y_{j} \leq b \mid y_{j}>\tau_{j}\right)\)
xb
stdp
linear prediction
score
standard error of the linear prediction
first derivative of the log likelihood with respect to \(\mathbf{x}_{j} \boldsymbol{\beta}\)
These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

\section*{Menu for predict}

Statistics \(>\) Postestimation \(>\) Predictions, residuals, etc.

\section*{Options for predict}
\(\qquad\) Main
n , the default, calculates the predicted number of events, which is \(\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}\right)\) if neither offset () nor exposure() was specified when the model was fit; \(\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}+\right.\) offset \(\left._{j}\right)\) if offset () was specified; or \(\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}\right) \times\) exposure \(_{j}\) if exposure() was specified.
ir calculates the incidence rate \(\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}\right)\), which is the predicted number of events when exposure is 1 . This is equivalent to specifying both the n and the nooffset options.
cm calculates the conditional mean,
\[
E\left(y_{j} \mid y_{j}>\tau_{j}\right)=\frac{E\left(y_{j}\right)}{\operatorname{Pr}\left(y_{j}>\tau_{j}\right)}
\]
where \(\tau_{j}\) is the truncation point found in e(llopt).
\(\operatorname{pr}(n)\) calculates the probability \(\operatorname{Pr}\left(y_{j}=n\right)\), where \(n\) is a nonnegative integer that may be specified as a number or a variable.
\(\operatorname{pr}(a, b)\) calculates the probability \(\operatorname{Pr}\left(a \leq y_{j} \leq b\right)\), where \(a\) and \(b\) are nonnegative integers that may be specified as numbers or variables;
\(b\) missing ( \(b \geq\).) means \(+\infty\);
\(\operatorname{pr}(20,\).\() calculates \operatorname{Pr}\left(y_{j} \geq 20\right)\);
\(\operatorname{pr}(20, b)\) calculates \(\operatorname{Pr}\left(y_{j} \geq 20\right)\) in observations for which \(b \geq\). and calculates
\(\operatorname{Pr}\left(20 \leq y_{j} \leq b\right)\) elsewhere.
\(\operatorname{pr}(., b)\) produces a syntax error. A missing value in an observation of the variable \(a\) causes a missing value in that observation for \(\mathrm{pr}(a, b)\).
\(\operatorname{cpr}(n)\) calculates the conditional probability \(\operatorname{Pr}\left(y_{j}=n \mid y_{j}>\tau_{j}\right)\), where \(\tau_{j}\) is the truncation point found in e(llopt). \(n\) is an integer greater than the truncation point that may be specified as a number or a variable.
\(\operatorname{cpr}(a, b)\) calculates the conditional probability \(\operatorname{Pr}\left(a \leq y_{j} \leq b \mid y_{j}>\tau_{j}\right)\), where \(\tau_{j}\) is the truncation point found in e(llopt). The syntax for this option is analogous to that used for \(\mathrm{pr}(a, b)\) except that \(a\) must be greater than the truncation point.
xb calculates the linear prediction, which is \(\mathbf{x}_{j} \boldsymbol{\beta}\) if neither offset () nor exposure() was specified when the model was fit; \(\mathbf{x}_{j} \boldsymbol{\beta}+\) offset \(_{j}\) if offset () was specified; or \(\mathbf{x}_{j} \boldsymbol{\beta}+\ln \left(\right.\) exposure \(\left._{j}\right)\) if exposure() was specified; see nooffset below.
stdp calculates the standard error of the linear prediction.
score calculates the equation-level score, \(\partial \ln L / \partial\left(\mathbf{x}_{j} \boldsymbol{\beta}\right)\).
nooffset is relevant only if you specified offset() or exposure() when you fit the model. It modifies the calculations made by predict so that they ignore the offset or exposure variable; the linear prediction is treated as \(\mathbf{x}_{j} \boldsymbol{\beta}\) rather than as \(\mathbf{x}_{j} \boldsymbol{\beta}+\) offset \(_{j}\) or \(\mathbf{x}_{j} \boldsymbol{\beta}+\ln\) (exposure \({ }_{j}\) ). Specifying predict..., nooffset is equivalent to specifying predict..., ir.

\section*{Methods and formulas}

In the following formula, we use the same notation as in \([R]\) tpoisson.
The equation-level scores are given by
\[
\operatorname{score}(\mathbf{x} \boldsymbol{\beta})_{j}=y_{j}-e^{\xi_{j}}-\frac{e^{-e^{\xi_{j}}} e^{\xi_{j}}}{\operatorname{Pr}\left(Y>\tau_{j} \mid \xi_{j}\right)}
\]
where \(\tau_{j}\) is the truncation point found in e (llopt).

\section*{Also see \\ [R] tpoisson - Truncated Poisson regression \\ [U] 20 Estimation and postestimation commands}

\section*{Title}
translate - Print and translate logs
\begin{tabular}{llll} 
Syntax & Description & Options for print & Options for translate \\
Remarks and examples & Stored results & Also see
\end{tabular}

\section*{Syntax}

Print log and SMCL files
```

print filename [, like(ext) name(windowname) override_options]

```

Translate \(\log\) files to SMCL files and vice versa
```

translate filename in filename out [, translator(tname) name(windowname)
override_options replace]

```

View translator parameter settings
```

translator query [tname]

```

Change translator parameter settings translator set [tname setopt setval]

Return translator parameter settings to default values
translator reset tname

List current mappings from one extension to another
transmap query [.ext]
Specify that files with one extension be treated the same as files with another extension transmap define .ext \({ }_{\text {new }}\).ext \({ }_{\text {old }}\)
filename in print, in addition to being a filename to be printed, may be specified as @Results to mean the Results window and @Viewer to mean the Viewer window.
filename \(_{\text {in }}\) in translate may be specified just as filename in print.
thame in translator specifies the name of a translator; see the translator () option under Options for translate.

\section*{Description}
print prints log, SMCL, and text files. Although there is considerable flexibility in how print (and translate, which print uses) can be set to work, they have already been set up and should just work:
```

. print mylog.smcl
. print mylog.log

```

Unix users may discover that they need to do a bit of setup before print works; see Printing files, Unix below. International Unix users may also wish to modify the default paper size. All users can tailor print and translate to their needs.
print may also be used to print the current contents of the Results window or the Viewer. For instance, the current contents of the Results window could be printed by typing
```

. print @Results

```
translate translates \(\log\) and SMCL files from one format to another, the other typically being suitable for printing. translate can also translate SMCL logs (logs created by typing, say, log using mylog) to plain text:
. translate mylog.smcl mylog.log
You can use translate to recover a log when you have forgotten to start one. You may type
```

. translate @Results mylog.txt

```
to capture as plain text what is currently shown in the Results window.
This entry provides a general overview of print and translate and covers in detail the printing and translation of text (nongraphic) files.
translator query, translator set, and translator reset show, change, and restore the default values of the settings for each translator.
transmap define and transmap query create and show mappings from one file extension to another for use with print and translate.

For example, print myfile.txt knows to use a translator appropriate for printing text files because of the .txt extension. However, it does not know what to do with .xyz files. If you have .xyz files and always wish to treat them as .txt files, you can type transmap define .xyz .txt.

\section*{Options for print}
like (ext) specifies how the file should be translated to a form suitable for printing. The default is to determine the translation method from the extension of filename. Thus mylog.smcl is translated according to the rule for translating smcl files, myfile.txt is translated according to the rule for translating txt files, and so on. (These rules are, in fact, translate's smcl2prn and txt2prn translators, but put that aside for the moment.)

Rules for the following extensions are predefined:
\[
\begin{array}{ll}
. \text { txt } & \text { assume input file contains plain text } \\
. \log & \text { assume input file contains Stata log text } \\
. \text { smcl } & \text { assume input file contains SMCL }
\end{array}
\]

To print a file that has an extension different from those listed above, you can define a new extension, but you do not have to do that. Assume that you wish to print the file read.me, which you know to contain plain text. If you were just to type print read.me, you would be told that Stata cannot translate .me files. (You would actually be told that the translator for me2prn was not found.) You could type print read.me, like(txt) to tell print to print read.me like a .txt file.

On the other hand, you could type
to tell Stata that .me files are always to be treated like .txt files. If you did that, Stata would remember the new rule, even in future sessions.
When you specify the like() option, you override the recorded rules. So, if you were to type print mylog.smcl, like(txt), the file would be printed as plain text (meaning that all the SMCL commands would show).
name (windowname) specifies which window to print when printing a Viewer. The default is for Stata to print the topmost Viewer [Unix(GUI) users: See the second technical note in Printing files, Unix]. The name () option is ignored when printing the Results window.

The window name is located inside parentheses in the window title. For example, if the title for a Viewer window is Viewer (\#1) [help print], the name for the window is \#1.
override_options refers to translate's options for overriding default values. print uses translate to translate the file into a format suitable for sending to the printer, and thus translate's override_options may also be used with print. The settings available vary between each translator (for example, smcl2ps will have different settings than smcl2txt) and may also differ across operating systems (for example, Windows may have different printing options than Mac OS X). To find out what you can override when printing .smcl files, type
```

. translator query smcl2prn
(output omitted)

```

In the omitted output, you might learn that there is an rmargin \# tunable value, which specifies the right margin in inches. You could specify the override_option rmargin(\#) to temporarily override the default value, or you could type translator set smcl2prn rmargin \# beforehand to permanently reset the value.

Alternatively, on some computers with some translators, you might discover that nothing can be set.

\section*{Options for translate}
translator (tname) specifies the name of the translator to be used to translate the file. The available translators are
\begin{tabular}{lll} 
tname & Input & Output \\
\hline smcl2ps & SMCL & PostScript \\
log2ps & Stata text log & PostScript \\
txt2ps & generic text file & PostScript \\
Viewer2ps & Viewer window & PostScript \\
Results2ps & Results window & PostScript \\
smcl2prn & SMCL & default printer format \\
log2prn & Stata text log & default printer format \\
txt2prn & generic text log & default printer format \\
Results2prn & Results window & default printer format \\
Viewer2prn & Viewer window & default printer format \\
smcl2txt & SMCL & generic text file \\
smcl2log & SMCL & Stata text log \\
Results2txt & Results window & generic text file \\
Viewer2txt & Viewer window & generic text file \\
smcl2pdf & SMCL & PDF \\
log2pdf & Stata text log & PDF \\
txt2pdf & generic text log & PDF \\
Results2pdf & Results window & PDF \\
Viewer2pdf & Viewer window & PDF \\
\hline
\end{tabular}

If translator () is not specified, translate determines which translator to use from extensions of the filenames specified. Typing translate myfile.smcl myfile.ps would use the smcl2ps translator. Typing translate myfile.smcl myfile.ps, translate (smcl2prn) would override the default and use the smcl2prn translator.
Actually, when you type translate \(a . b c . d\), translate looks up.\(b\) in the transmap extensionsynonym table. If.\(b\) is not found, the translator \(b 2 d\) is used. If.\(b\) is found in the table, the mapped extension is used (call it \(b^{\prime}\) ), and then the translator \(b^{\prime} 2 d\) is used. For example,
\begin{tabular}{ll} 
Command & Translator used \\
\hline . translate myfile.smcl myfile.ps & smcl2ps \\
. translate myfile.odd myfile.ps & odd2ps, which does not exist, so error \\
. transmap define .odd .txt & \\
. translate myfile.odd myfile.ps & txt2ps
\end{tabular}

You can list the mappings that translate uses by typing transmap query.
name (windowname) specifies which window to translate when translating a Viewer. The default is for Stata to translate the topmost Viewer. The name() option is ignored when translating the Results window.

The window name is located inside parentheses in the window title. For example, if the title for a Viewer window is Viewer (\#1) [help print], the name for the window is \#1.
override_options override any of the default options of the specified or implied translator. To find out what you can override for, say, log2ps, type
```

. translator query log2ps
(output omitted)

```

In the omitted output, you might learn that there is an rmargin \# tunable value, which, for \(\log 2 \mathrm{ps}\), specifies the right margin in inches. You could specify the override_option rmargin(\#) to temporarily override the default value or type translator set log2ps rmargin \# beforehand to permanently reset the value.
replace specifies that filename \(_{\text {out }}\) be replaced if it already exists.

\section*{Remarks and examples}

Remarks are presented under the following headings:

\author{
Printing files \\ Printing files, Mac and Windows \\ Printing files, Unix \\ Translating files from one format to another
}

\section*{Printing files}

Printing should be easy; just type
. print mylog.smcl
. print mylog.log
You can use print to print SMCL files, plain text files, and even the contents of the Results and Viewer windows:
. print @Results
- print @Viewer
. print @Viewer, name(\#2)
For information about printing and translating graph files, see [G-2] graph print and see [G-2] graph export.

\section*{Printing files, Mac and Windows}

When you type print, you are using the same facility that you would be using if you had selected Print from the File menu. If you try to print a file that Stata does not know about, Stata will complain:
```

. print read.me
translator me2prn not found
(perhaps you need to specify the like() option)
r(111);

```

Then you could type
```

. print read.me, like(txt)

```
to indicate that you wanted read.me sent to the printer in the same fashion as if the file were named readme.txt, or you could type
```

. transmap define .me .txt

```
. print read.me
Here you are telling Stata once and for all that you want files ending in .me to be treated in the same way as files ending in .txt. Stata will remember this mapping, even across sessions. To clear the .me mapping, type
```

. transmap define .me

```

To see all the mappings, type
```

. transmap query

```

To print to a file, use the translate command, not print:
```

. translate mylog.smcl mylog.prn

```
translate prints to a file by using the Windows print driver when the new filename ends in . prn. Under Mac, the prn translators are the same as the pdf translators. We suggest that you simply use the .pdf file extension when printing to a file.

\section*{Printing files, Unix}

Stata assumes that you have a PostScript printer attached to your Unix computer and that the Unix command \(\operatorname{lpr}(1)\) can be used to send PostScript files to it, but you can change this. On your Unix system, typing
```

mycomputer\$ lpr < flename

```
may not be sufficient to print PostScript files. For instance, perhaps on your system you would need to type
```

mycomputer\$ lpr -Plexmark < filename

```
or
mycomputer\$ lpr -Plexmark filename
or something else. To set the print command to be lpr -Plexmark filename and to state that the printer expects to receive PostScript files, type
. printer define prn ps "lpr -Plexmark @"
To set the print command to lpr -Plexmark < filename and to state that the printer expects to receive plain text files, type
. printer define prn txt "lpr -Plexmark < @"
That is, just type the command necessary to send files to your printer and include an @ sign where the filename should be substituted. Two file formats are available: ps and txt. The default setting, as shipped from the factory, is
```

. printer define prn ps "lpr < @"

```

We will return to the printer command in the technical note that follows because it has some other capabilities you should know about.

In any case, after you redefine the default printer, the following should just work:
. print mylog.smcl
. print mylog.log
If you try to print a file that Stata does not know about, it will complain:
. print read.me
translator me2prn not found
r(111) ;
Here you could type
. print read.me, like(txt)
to indicate that you wanted read.me sent to the printer in the same fashion as if the file were named readme.txt, or you could type
```

. transmap define .me .txt

```
. print read.me

Here you are telling Stata once and for all that you want files ending in .me to be treated in the same way as files ending in .txt. Stata will remember this setting for .me, even across sessions.

If you want to clear the .me setting, type
. transmap define .me
If you want to see all your settings, type
. transmap query

\section*{- Technical note}

The syntax of the printer command is
printer define printername \([\{\mathrm{ps} \mid \mathrm{txt}\}\) "Unix command with @" \(]\)
printer query [printername]

You may define multiple printers. By default, print uses the printer named prn, but print has the syntax
```

print filename [, like(ext) printer(printername) override_options ]

```
so, if you define multiple printers, you may route your output to them.
For instance, if you have a second printer on your system, you might type
```

. printer define lexmark ps "lpr -Plexmark < @"

```

After doing that, you could type
. print myfile.smcl, printer(lexmark)
Any printers that you set will be remembered even across sessions. You can delete printers:
- printer define lexmark

You can list all the defined printers by typing printer query, and you can list the definition of a particular printer, say, prn, by typing printer query prn.

The default printer prn we have predefined for you is
```

. printer define prn ps "lpr < @"

```
meaning that we assume that it is a PostScript printer and that the Unix command \(\operatorname{lpr}(1)\), without options, is sufficient to cause files to print. Feel free to change the default definition. If you change it, the change will be remembered across sessions.

\section*{- Technical note}

Unix(GUI) users should note that X-Windows does not have the concept of a window z-order, which prevents Stata from determining which window is the topmost window. Instead, Stata determines which window is topmost based on which window has the focus. However, some window managers will set the focus to a window without bringing the window to the top. What Stata considers the topmost window may not appear topmost visually. For this reason, you should always use the name () option to ensure that the correct window is printed.

\section*{Technical note}

When you select the Results window to print from the Print menu or toolbar button, the result is the same as if you were to issue the print command. When you select a Viewer window to print from the Print menu or toolbar button, the result is the same as if you were to issue the print command with a name() option.

The translation to PostScript format is done by translate and, in particular, is performed by the translators smcl2ps, log2ps, and txt2ps. There are many tunable parameters in each of these translators. You can display the current values of these tunable parameters for, say, smcl2ps by typing
and you can set any of the tunable parameters (for instance, setting smcl2ps's rmargin value to 1 ) by typing
```

- translator set smcl2ps rmargin 1
(output omitted)

```

Any settings you make will be remembered across sessions. You can reset smcl2ps to be as it was when Stata was shipped by typing
```

. translator reset smcl2ps

```

\section*{Translating files from one format to another}

If you have a SMCL log, which you might have created by previously typing log using mylog, you can translate it to an text log by typing
. translate myfile.smcl myfile.log
and you can translate it to a PostScript file by typing
. translate myfile.smcl myfile.ps
translate translates files from one format to another, and, in fact, print uses translate to produce a file suitable for sending to the printer.

When you type
. translate \(a . b\) c.d
translate looks for the predefined translator \(b 2 d\) and uses that to perform the translation. If there is a transmap synonym for \(b\), however, the mapped value \(b^{\prime}\) is used: \(b^{\prime} 2 d\).

Only certain translators exist, and they are listed under the description of the translate() option in Options for translate above, or you can type
. translator query
for a complete (and perhaps more up-to-date) list.
Anyway, translate forms the name \(b 2 d\) or \(b^{\prime} 2 d\), and if the translator does not exist, translate issues an error message. With the translator() option, you can specify exactly which translator to use, and then it does not matter how your files are named.

The only other thing to know is that some translators have tunable parameters that affect how they perform their translation. You can type
. translator query translator_name
to find out what those parameters are. Some translators have no tunable parameters, and some have many:
. translator query smcl2ps
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline header
headertext
logo
user
projecttext
cmdnumber & on
on
on & & & & & & \\
\hline fontsize pagesize pagewidth pageheight & \begin{tabular}{l}
9 \\
lett \\
8.50 \\
11.0
\end{tabular} & & & \begin{tabular}{l}
lmargin \\
rmargin \\
tmargin \\
bmargin
\end{tabular} & \[
\begin{aligned}
& 1.00 \\
& 1.00 \\
& 1.00 \\
& 1.00
\end{aligned}
\] & & \\
\hline scheme & \multicolumn{7}{|l|}{monochrome} \\
\hline cust1_result_color & 0 & 0 & 0 & cust2_result_color & 0 & 0 & 0 \\
\hline cust1_standard_color & 0 & 0 & 0 & cust2_standard_color & 0 & 0 & 0 \\
\hline cust1_error_color & 0 & 0 & 0 & cust2_error_color & 255 & 0 & 0 \\
\hline cust1_input_color & 0 & 0 & 0 & cust2_input_color & 0 & 0 & 0 \\
\hline cust1_link_color & 0 & 0 & 0 & cust2_link_color & 0 & 0 & 255 \\
\hline cust1_hilite_color & 0 & 0 & 0 & cust2_hilite_color & 0 & 0 & 0 \\
\hline cust1_result_bold & on & & & cust2_result_bold & on & & \\
\hline cust1_standard_bold & off & & & cust2_standard_bold & off & & \\
\hline cust1_error_bold & on & & & cust2_error_bold & on & & \\
\hline cust1_input_bold & off & & & cust2_input_bold & off & & \\
\hline cust1_link_bold & off & & & cust2_link_bold & off & & \\
\hline cust1_hilite_bold & on & & & cust2_hilite_bold & on & & \\
\hline cust1_link_underline & on & & & cust2_link_underline & on & & \\
\hline cust1_hilite_underline & off & & & ust2_hilite_underline & off & & \\
\hline
\end{tabular}

You can temporarily override any setting by specifying the setopt (setval) option on the translate (or print) command. For instance, you can type
. translate ..., ... cmdnumber (off)
or you can reset the value permanently by typing
. translator set smcl2ps setopt setval
For instance,
. translator set smcl2ps cmdnumber off
If you reset a value, Stata will remember the change, even in future sessions.
Mac and Windows users: The smcl2ps (and the other \(* 2 \mathrm{ps}\) translators) are not used by print, even when you have a PostScript printer attached to your computer. Instead, the Mac or Windows print driver is used. Resetting smcl2ps values will not affect printing; instead, you change the defaults in the Printers Control Panel in Windows and by selecting Page Setup... from the File menu in Mac. You can, however, translate files yourself using the smcl2ps translator and the other \(* 2 \mathrm{ps}\) translators.

\section*{Stored results}
transmap query .ext stores in macro \(r\) (suffix) the mapped extension (without the leading period) or stores ext if the ext is not mapped.
translator query translatorname stores setval in macro r(setopt) for every setopt, setval pair.
printer query printername (Unix only) stores in macro \(r\) (suffix) the "filetype" of the input that the printer expects (currently "ps" or "txt") and, in macro r(command), the command to send output to the printer.

\section*{Also see}
[R] \(\log\) - Echo copy of session to file
[G-2] graph export - Export current graph
[G-2] graph print - Print a graph
[G-2] graph set - Set graphics options
[P] smcl - Stata Markup and Control Language
[U] 15 Saving and printing output-log files

\section*{Title}
truncreg - Truncated regression
\begin{tabular}{llll} 
Syntax & Menu & Description & Options \\
Remarks and examples & Stored results & Methods and formulas & References
\end{tabular}

\section*{Syntax}
truncreg depvar [indepvars] [if] [in] [weight] [, options]
options Description

Model

\section*{noconstant}
ll(varname|\#)
ul(varname|\#)
offset (varname)
constraints (constraints)
collinear
SE/Robust
vce (vcetype)

Reporting
level(\#)
noskip
nocnsreport
display_options

Maximization
maximize_options
coeflegend
suppress constant term
lower limit for left-truncation
upper limit for right-truncation
include varname in model with coefficient constrained to 1
apply specified linear constraints
keep collinear variables
vcetype may be oim, robust, cluster clustvar, opg, bootstrap, or jackknife
indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
depvar and indepvars may contain time-series operators; see [U] 11.4.4 Time-series varlists.
bootstrap, by, fp, jackknife, mi estimate, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands.
vce(bootstrap) and vce(jackknife) are not allowed with the mi estimate prefix; see [MI] mi estimate.
Weights are not allowed with the bootstrap prefix; see \([\mathrm{R}]\) bootstrap.
aweights are not allowed with the jackknife prefix; see [R] jackknife.
vce(), noskip, and weights are not allowed with the svy prefix; see [SVY] svy.
aweights, fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

\section*{Menu}

Statistics \(>\) Linear models and related \(>\) Truncated regression

\section*{Description}
truncreg fits a regression model of depvar on indepvars from a sample drawn from a restricted part of the population. Under the normality assumption for the whole population, the error terms in the truncated regression model have a truncated normal distribution, which is a normal distribution that has been scaled upward so that the distribution integrates to one over the restricted range.

\section*{Options}

> Model
> noconstant; see \([R]\) estimation options.
> ll (varname \(\mid \#\) ) and ul (varname \(\mid \#\) ) indicate the lower and upper limits for truncation, respectively. You may specify one or both. Observations with depvar \(\leq 11\) () are left-truncated, observations with depvar \(\geq u l()\) are right-truncated, and the remaining observations are not truncated. See [R] tobit for a more detailed description.
> offset (varname), constraints (constraints), collinear; see [R] estimation options.

\section*{SE/Robust}
vce (vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
level (\#); see [R] estimation options.
noskip specifies that a full maximum-likelihood model with only a constant for the regression equation be fit. This model is not displayed but is used as the base model to compute a likelihood-ratio test for the model test statistic displayed in the estimation header. By default, the overall model test statistic is an asymptotically equivalent Wald test of all the parameters in the regression equation being zero (except the constant). For many models, this option can substantially increase estimation time.
nocnsreport; see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% \(f m t\) ), pformat (\% fmt), sformat (\%fmt), and nolstretch; see [R] estimation options.

Maximization
maximize_options: difficult, technique (algorithm_spec), iterate(\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#),
nrtolerance (\#), nonrtolerance, and from(init_specs); see [R] maximize. These options are seldom used, but you may use the ltol(\#) option to relax the convergence criterion; the default is \(1 \mathrm{e}-6\) during specification searches.
Setting the optimization type to technique (bhhh) resets the default vcetype to vce (opg).

The following option is available with truncreg but is not shown in the dialog box: coeflegend; see \([R]\) estimation options.

\section*{Remarks and examples}

Truncated regression fits a model of a dependent variable on independent variables from a restricted part of a population. Truncation is essentially a characteristic of the distribution from which the sample data are drawn. If \(x\) has a normal distribution with mean \(\mu\) and standard deviation \(\sigma\), the density of the truncated normal distribution is
\[
\begin{aligned}
f(x \mid a<x<b) & =\frac{f(x)}{\Phi\left(\frac{b-\mu}{\sigma}\right)-\Phi\left(\frac{a-\mu}{\sigma}\right)} \\
& =\frac{\frac{1}{\sigma} \phi\left(\frac{x-\mu}{\sigma}\right)}{\Phi\left(\frac{b-\mu}{\sigma}\right)-\Phi\left(\frac{a-\mu}{\sigma}\right)}
\end{aligned}
\]
where \(\phi\) and \(\Phi\) are the density and distribution functions of the standard normal distribution.
Compared with the mean of the untruncated variable, the mean of the truncated variable is greater if the truncation is from below, and the mean of the truncated variable is smaller if the truncation is from above. Moreover, truncation reduces the variance compared with the variance in the untruncated distribution.

\section*{\(>\) Example 1}

We will demonstrate truncreg with part of the Mroz dataset distributed with Berndt (1996). This dataset contains 753 observations on women's labor supply. Our subsample is of 250 observations, with 150 market laborers and 100 nonmarket laborers.
```

. use http://www.stata-press.com/data/r13/laborsub
. describe
Contains data from http://www.stata-press.com/data/r13/laborsub.dta
obs: 250
vars: 6 25 Sep 2012 18:36
size: 1,750

```
\begin{tabular}{lllll} 
variable name \begin{tabular}{c} 
storage \\
type
\end{tabular} & \begin{tabular}{l} 
display \\
format
\end{tabular} & \begin{tabular}{c} 
value \\
label
\end{tabular} & variable label \\
\hline lfp & byte & \(\% 9.0 \mathrm{~g}\) & & 1 if woman worked in 1975 \\
whrs & int & \(\% 9.0 \mathrm{~g}\) & & Wife's hours of work \\
kl6 & byte & \(\% 9.0 \mathrm{~g}\) & & \# of children younger than 6 \\
k618 & byte & \(\% 9.0 \mathrm{~g}\) & & \# of children between 6 and 18 \\
wa & byte & \(\% 9.0 \mathrm{~g}\) & & Wife's age \\
we & byte & \(\% 9.0 \mathrm{~g}\) & & Wife's educational attainment \\
\hline
\end{tabular}

Sorted by:
. summarize, sep(0)
\begin{tabular}{r|rrrrr} 
Variable & Obs & Mean & Std. Dev. & Min & Max \\
\hline lfp & 250 & .6 & .4908807 & 0 & 1 \\
whrs & 250 & 799.84 & 915.6035 & 0 & 4950 \\
kl6 & 250 & .236 & .5112234 & 0 & 3 \\
k618 & 250 & 1.364 & 1.370774 & 0 & 8 \\
wa & 250 & 42.92 & 8.426483 & 30 & 60 \\
we & 250 & 12.352 & 2.164912 & 5 & 17
\end{tabular}

We first perform ordinary least-squares estimation on the market laborers.
. regress whrs kl6 k618 wa we if whrs > 0
\begin{tabular}{r|rrr} 
Source & \multicolumn{1}{|c}{ SS } & df & MS \\
\hline Model & 7326995.15 & 4 & 1831748.79 \\
Residual & 94793104.2 & 145 & 653745.546 \\
\hline Total & 102120099 & 149 & 685369.794
\end{tabular}
Number of obs \(=150\)
\(F(4,145)=2.80\)
Prob > F \(=0.0281\)
R-squared \(=0.0717\)
Adj R-squared \(=0.0461\)
Root MSE \(=808.55\)
\begin{tabular}{r|rlccrr}
\hline whrs & Coef. & Std. Err. & t & P>|t| & [95\% Conf. Interval] \\
\hline k16 & -421.4822 & 167.9734 & -2.51 & 0.013 & -753.4748 & -89.48953 \\
k618 & -104.4571 & 54.18616 & -1.93 & 0.056 & -211.5538 & 2.639668 \\
wa & -4.784917 & 9.690502 & -0.49 & 0.622 & -23.9378 & 14.36797 \\
we & 9.353195 & 31.23793 & 0.30 & 0.765 & -52.38731 & 71.0937 \\
_cons & 1629.817 & 615.1301 & 2.65 & 0.009 & 414.0371 & 2845.597 \\
\hline
\end{tabular}

Now we use truncreg to perform truncated regression with truncation from below zero.
```

. truncreg whrs kl6 k618 wa we, ll(0)
(note: 100 obs. truncated)
Fitting full model:

| Iteration 0: | log likelihood $=-1205.6992$ |
| :--- | :--- |
| Iteration 1: | log likelihood $=-1200.9873$ |
| Iteration 2: | log likelihood $=-1200.9159$ |
| Iteration 3: | log likelihood $=-1200.9157$ |
| Iteration 4: | log likelihood $=-1200.9157$ |

```

Truncated regression
Limit: lower \(=\quad 0 \quad\) Number of obs \(=150\)
upper \(=\quad\) +inf \(\quad\) Wald chi2(4) \(=10.05\)
Log likelihood \(=-1200.9157 \quad\) Prob \(>\) chi2 \(=0.0395\)
\begin{tabular}{r|rrcccr}
\hline whrs & Coef. & Std. Err. & z & P>|z| & [95\% Conf. Interval] \\
\hline kl6 & -803.0042 & 321.3614 & -2.50 & 0.012 & -1432.861 & -173.1474 \\
k 618 & -172.875 & 88.72898 & -1.95 & 0.051 & -346.7806 & 1.030579 \\
wa & -8.821123 & 14.36848 & -0.61 & 0.539 & -36.98283 & 19.34059 \\
we & 16.52873 & 46.50375 & 0.36 & 0.722 & -74.61695 & 107.6744 \\
_cons & 1586.26 & 912.355 & 1.74 & 0.082 & -201.9233 & 3374.442 \\
\hline /sigma & 983.7262 & 94.44303 & 10.42 & 0.000 & 798.6213 & 1168.831 \\
\hline
\end{tabular}

If we assume that our data were censored, the tobit model is
\begin{tabular}{llll}
. tobit whrs kl6 k618 wa we, ll (0) & & \\
Tobit regression & Number of obs & \(=\) & 250 \\
& LR chi2 (4) & \(=\) & 23.03 \\
& Prob > chi2 & \(=\) & 0.0001 \\
Log likelihood \(=-1367.0903\) & & Pseudo R2 & \(=\)
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline whrs & Coef. & Std. Err. & t & \(P>|t|\) & [95\% Con & Interval] \\
\hline kl6 & -827.7657 & 214.7407 & -3.85 & 0.000 & -1250.731 & -404.8008 \\
\hline k618 & -140.0192 & 74.22303 & -1.89 & 0.060 & -286.2129 & 6.174547 \\
\hline wa & -24.97919 & 13.25639 & -1.88 & 0.061 & -51.08969 & 1.131317 \\
\hline we & 103.6896 & 41.82393 & 2.48 & 0.014 & 21.31093 & 186.0683 \\
\hline _cons & 589.0001 & 841.5467 & 0.70 & 0.485 & -1068.556 & 2246.556 \\
\hline /sigma & 1309.909 & 82.73335 & & & 1146.953 & 1472.865 \\
\hline \multicolumn{2}{|l|}{Obs. summary: 100} & \multicolumn{5}{|l|}{\begin{tabular}{l}
left-censored observations at whrs<=0 uncensored observations \\
right-censored observations
\end{tabular}} \\
\hline
\end{tabular}

\section*{Technical note}

Whether truncated regression is more appropriate than the ordinary least-squares estimation depends on the purpose of that estimation. If we are interested in the mean of wife's working hours conditional on the subsample of market laborers, least-squares estimation is appropriate. However if we are interested in the mean of wife's working hours regardless of market or nonmarket labor status, least-squares estimates could be seriously misleading.

Truncation and censoring are different concepts. A sample has been censored if no observations have been systematically excluded but some of the information contained in them has been suppressed. In a truncated distribution, only the part of the distribution above (or below, or between) the truncation points is relevant to our computations. We need to scale it up by the probability that an observation falls in the range that interests us to make the distribution integrate to one. The censored distribution used by tobit, however, is a mixture of discrete and continuous distributions. Instead of rescaling over the observable range, we simply assign the full probability from the censored regions to the censoring points. The truncated regression model is sometimes less well behaved than the tobit model. Davidson and MacKinnon (1993) provide an example where truncation results in more inconsistency than censoring.

\section*{Stored results}
truncreg stores the following in e():
Scalars
```

        e(N)
        e(N_bf)
        e(chi2)
        e(k_eq)
        e(k_eq_model)
        e(k_aux)
        e(df_m)
        e(ll)
        e(1l_0)
        e(N_clust)
        e(sigma)
        e(p)
        e(rank)
        e(ic)
        e(rc)
        e(converged)
    ```
Macros
    e(cmd)
    e(cmdline)
    e(llopt)
    e(ulopt)
    e(depvar)
    e(wtype)
    e(wexp)
    e(title)
    e(clustvar)
    e(offset1)
    e(chi2type)
    e(vce)
    e(vcetype)
    e(opt)
    \(e\) (which)
    e(ml_method)
    e(user)
    e(technique)
    e(properties)
    e(predict)
    e(asbalanced)
    e(asobserved)
Matrices
    e(b)
    e(Cns)
    e(ilog)
    e(gradient)
    e(V)
    e(V_modelbased)
    \(e\) (means)
    e(dummy)
Functions
    e(sample)
    number of observations
    number of obs. before truncation
    model \(\chi^{2}\)
    number of equations in \(e(b)\)
    number of equations in overall model test
    number of auxiliary parameters
    model degrees of freedom
    log likelihood
    log likelihood, constant-only model
    number of clusters
    estimate of sigma
    significance
    rank of e(V)
    number of iterations
    return code
    1 if converged, 0 otherwise

\section*{truncreg}
command as typed
contents of 11() , if specified
contents of \(u l()\), if specified
name of dependent variable
weight type
weight expression
title in estimation output
name of cluster variable
offset
Wald or LR; type of model \(\chi^{2}\) test
vcetype specified in vce()
title used to label Std. Err.
type of optimization
max or min; whether optimizer is to perform maximization or minimization
type of ml method
name of likelihood-evaluator program
maximization technique
b V
program used to implement predict
factor variables fvset as asbalanced
factor variables fvset as asobserved
coefficient vector
constraints matrix
iteration \(\log\) (up to 20 iterations)
gradient vector
variance-covariance matrix of the estimators
model-based variance
means of independent variables
indicator for dummy variables
marks estimation sample

\section*{Methods and formulas}

Greene (2012, 833-839) and Davidson and MacKinnon (1993, 534-537) provide introductions to the truncated regression model.

Let \(\mathbf{y}=\mathbf{X} \boldsymbol{\beta}+\boldsymbol{\epsilon}\) be the model. \(\mathbf{y}\) represents continuous outcomes either observed or not observed. Our model assumes that \(\epsilon \sim N\left(\mathbf{0}, \sigma^{2} \mathbf{I}\right)\).

Let \(a\) be the lower limit and \(b\) be the upper limit. The log likelihood is
\[
\ln L=-\frac{n}{2} \log \left(2 \pi \sigma^{2}\right)-\frac{1}{2 \sigma^{2}} \sum_{j=1}^{n}\left(y_{j}-\mathbf{x}_{j} \boldsymbol{\beta}\right)^{2}-\sum_{j=1}^{n} \log \left\{\Phi\left(\frac{b-\mathbf{x}_{j} \boldsymbol{\beta}}{\sigma}\right)-\Phi\left(\frac{a-\mathbf{x}_{j} \boldsymbol{\beta}}{\sigma}\right)\right\}
\]

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce (robust) and vce (cluster clustvar), respectively. See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas.
truncreg also supports estimation with survey data. For details on VCEs with survey data, see [SVY] variance estimation.

\section*{References}

Berndt, E. R. 1996. The Practice of Econometrics: Classic and Contemporary. New York: Addison-Wesley.
Cong, R. 1999. sg122: Truncated regression. Stata Technical Bulletin 52: 47-52. Reprinted in Stata Technical Bulletin Reprints, vol. 9, pp. 248-255. College Station, TX: Stata Press.
Davidson, R., and J. G. MacKinnon. 1993. Estimation and Inference in Econometrics. New York: Oxford University Press.
Greene, W. H. 2012. Econometric Analysis. 7th ed. Upper Saddle River, NJ: Prentice Hall.

\section*{Also see}
[R] truncreg postestimation - Postestimation tools for truncreg
\([R]\) regress - Linear regression
[R] tobit - Tobit regression
[MI] estimation - Estimation commands for use with mi estimate
[SVY] svy estimation - Estimation commands for survey data
[U] 20 Estimation and postestimation commands

\section*{Title}
truncreg postestimation - Postestimation tools for truncreg
Description Syntax for predict Menu for predict Options for predict Also see

\section*{Description}

The following postestimation commands are available after truncreg:
\begin{tabular}{|c|c|}
\hline Command & Description \\
\hline contrast & contrasts and ANOVA-style joint tests of estimates \\
\hline estat ic & Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) \\
\hline estat summarize & summary statistics for the estimation sample \\
\hline estat vce & variance-covariance matrix of the estimators (VCE) \\
\hline estat (svy) & postestimation statistics for survey data \\
\hline estimates & cataloging estimation results \\
\hline forecast \({ }^{1}\) & dynamic forecasts and simulations \\
\hline lincom & point estimates, standard errors, testing, and inference for linear combinations of coefficients \\
\hline lrtest \({ }^{2}\) & likelihood-ratio test \\
\hline margins & marginal means, predictive margins, marginal effects, and average marginal effects \\
\hline marginsplot & graph the results from margins (profile plots, interaction plots, etc.) \\
\hline nlcom & point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients \\
\hline predict & predictions, residuals, influence statistics, and other diagnostic measures \\
\hline predictnl & point estimates, standard errors, testing, and inference for generalized predictions \\
\hline pwcompare & pairwise comparisons of estimates \\
\hline suest & seemingly unrelated estimation \\
\hline test & Wald tests of simple and composite linear hypotheses \\
\hline testnl & Wald tests of nonlinear hypotheses \\
\hline
\end{tabular}

\footnotetext{
\({ }^{1}\) forecast is not appropriate with mi or svy estimation results.
2 lrtest is not appropriate with svy estimation results.
}

\section*{Syntax for predict}
```

predict [type] newvar [if] [in] [, statistic nooffset]
predict [type] {stub*|newvar reg newvar lnsigma } [if] [in], scores

```
\begin{tabular}{ll} 
Main & \\
xb & linear prediction; the default \\
stdp & standard error of the prediction \\
stdf & standard error of the forecast \\
\(\mathrm{pr}(a, b)\) & \(\operatorname{Pr}\left(a<y_{j}<b\right)\) \\
\(\mathrm{e}(a, b)\) & \(E\left(y_{j} \mid a<y_{j}<b\right)\) \\
\(\mathrm{ystar}(a, b)\) & \(E\left(y_{j}^{*}\right), y_{j}^{*}=\max \left\{a, \min \left(y_{j}, b\right)\right\}\)
\end{tabular}

These statistics are available both in and out of sample; type predict ... if e(sample)... if wanted only for the estimation sample.
stdf is not allowed with svy estimation results.
where \(a\) and \(b\) may be numbers or variables; \(a\) missing ( \(a \geq\).) means \(-\infty\), and \(b\) missing ( \(b \geq\).) means \(+\infty\); see [U] 12.2.1 Missing values.

\section*{Menu for predict}

Statistics \(>\) Postestimation \(>\) Predictions, residuals, etc.

\section*{Options for predict}

Main
xb , the default, calculates the linear prediction.
stdp calculates the standard error of the prediction, which can be thought of as the standard error of the predicted expected value or mean for the observation's covariate pattern. The standard error of the prediction is also referred to as the standard error of the fitted value.
stdf calculates the standard error of the forecast, which is the standard error of the point prediction for 1 observation. It is commonly referred to as the standard error of the future or forecast value. By construction, the standard errors produced by stdf are always larger than those produced by stdp; see Methods and formulas in [R] regress postestimation.
\(\operatorname{pr}(a, b)\) calculates \(\operatorname{Pr}\left(a<\mathbf{x}_{j} \mathbf{b}+u_{j}<b\right)\), the probability that \(y_{j} \mid \mathbf{x}_{j}\) would be observed in the interval \((a, b)\).
\(a\) and \(b\) may be specified as numbers or variable names; \(l b\) and \(u b\) are variable names;
\(\operatorname{pr}(20,30)\) calculates \(\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)\);
\(\operatorname{pr}(l b, u b)\) calculates \(\operatorname{Pr}\left(l b<\mathbf{x}_{j} \mathbf{b}+u_{j}<u b\right)\); and
\(\operatorname{pr}(20, u b)\) calculates \(\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{j}<u b\right)\).
\(a\) missing ( \(a \geq\). ) means \(-\infty\); \(\operatorname{pr}(., 30)\) calculates \(\operatorname{Pr}\left(-\infty<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)\);
\(\operatorname{pr}(l b, 30)\) calculates \(\operatorname{Pr}\left(-\infty<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)\) in observations for which \(l b \geq\).
and calculates \(\operatorname{Pr}\left(l b<\mathbf{x}_{j} \mathbf{b}+u_{j}<30\right)\) elsewhere.
\(b\) missing \((b \geq\).\() means +\infty\); \(\operatorname{pr}(20,\).\() calculates \operatorname{Pr}\left(+\infty>\mathbf{x}_{j} \mathbf{b}+u_{j}>20\right)\); \(\operatorname{pr}(20, u b)\) calculates \(\operatorname{Pr}\left(+\infty>\mathbf{x}_{j} \mathbf{b}+u_{j}>20\right)\) in observations for which \(u b \geq\). and calculates \(\operatorname{Pr}\left(20<\mathbf{x}_{j} \mathbf{b}+u_{j}<u b\right)\) elsewhere.
\(\mathrm{e}(a, b)\) calculates \(E\left(\mathbf{x}_{j} \mathbf{b}+u_{j} \mid a<\mathbf{x}_{j} \mathbf{b}+u_{j}<b\right)\), the expected value of \(y_{j} \mid \mathbf{x}_{j}\) conditional on \(y_{j} \mid \mathbf{x}_{j}\) being in the interval \((a, b)\), meaning that \(y_{j} \mid \mathbf{x}_{j}\) is truncated.
\(a\) and \(b\) are specified as they are for \(\operatorname{pr}()\).
ystar \((a, b)\) calculates \(E\left(y_{j}^{*}\right)\), where \(y_{j}^{*}=a\) if \(\mathbf{x}_{j} \mathbf{b}+u_{j} \leq a, y_{j}^{*}=b\) if \(\mathbf{x}_{j} \mathbf{b}+u_{j} \geq b\), and \(y_{j}^{*}=\mathbf{x}_{j} \mathbf{b}+u_{j}\) otherwise, meaning that \(y_{j}^{*}\) is censored. \(a\) and \(b\) are specified as they are for pr() .
nooffset is relevant only if you specified offset (varname). It modifies the calculations made by predict so that they ignore the offset variable; the linear prediction is treated as \(\mathbf{x}_{j} \mathbf{b}\) rather than as \(\mathbf{x}_{j} \mathbf{b}+\) offset \(_{j}\).
scores calculates equation-level score variables.
The first new variable will contain \(\partial \ln L / \partial\left(\mathbf{x}_{j} \boldsymbol{\beta}\right)\).
The second new variable will contain \(\partial \ln L / \partial \sigma\).

\section*{Also see}
[R] truncreg - Truncated regression
[U] 20 Estimation and postestimation commands

\section*{Title}
ttest \(-t\) tests (mean-comparison tests)
\begin{tabular}{llll} 
Syntax & Menu & Description & Options \\
Remarks and examples & Stored results & Methods and formulas & References
\end{tabular}

\section*{Syntax}

One-sample \(t\) test
ttest varname \(==\#[\) if \(][\) in \(][\), level (\#) \(]\)
Two-sample \(t\) test using groups
```

ttest varname [if][in], by(groupvar) [options (1]

```

Two-sample \(t\) test using variables
```

ttest varname}\mp@subsup{1}{1}{==}\mp@subsup{\mathrm{ varname }}{2}{}[\mathrm{ [if] [in], unpaired [unequal welch level(\#)]

```

\section*{Paired t test}
```

ttest varname

```

Immediate form of one-sample \(t\) test
```

ttesti \# obs \#mean \# sd \#

```

Immediate form of two-sample \(t\) test
```

ttesti \# (obs1 \#mean1 \# sd1 \# obs2 \#mean2 \# \#sd2 [, options 2 ]

```
options \({ }_{1}\)
Main
*by (groupvar) variable defining the groups
unequal unpaired data have unequal variances
welch use Welch's approximation
level(\#)
set confidence level; default is level (95)
\({ }^{*}\) by (groupvar) is required.
options \({ }_{2}\)
Description
Main
unequal
welch
level(\#)
unpaired data have unequal variances
use Welch's approximation
set confidence level; default is level (95)
by is allowed with ttest; see [D] by.

\section*{Menu}
ttest
Statistics \(>\) Summaries, tables, and tests \(>\) Classical tests of hypotheses \(>\mathrm{t}\) test (mean-comparison test)
ttesti
Statistics \(>\) Summaries, tables, and tests \(>\) Classical tests of hypotheses \(>\mathrm{t}\) test calculator

\section*{Description}
ttest performs \(t\) tests on the equality of means. In the first form, ttest tests that varname has a mean of \#. In the second form, ttest tests that varname has the same mean within the two groups defined by groupvar. In the third form, ttest tests that varname \({ }_{1}\) and varname \({ }_{2}\) have the same mean, assuming unpaired data. In the fourth form, ttest tests that varname \({ }_{1}\) and varname \({ }_{2}\) have the same mean, assuming paired data.
ttesti is the immediate form of ttest; see [U] 19 Immediate commands.
For the equivalent of a two-sample \(t\) test with sampling weights (pweights), use the svy: mean command with the over () option, and then use lincom; see [R] mean and [SVY] svy postestimation.

\section*{Options}
by (groupvar) specifies the groupvar that defines the two groups that ttest will use to test the hypothesis that their means are equal. Specifying by (groupvar) implies an unpaired (two sample) \(t\) test. Do not confuse the by () option with the by prefix; you can specify both.
unpaired specifies that the data be treated as unpaired. The unpaired option is used when the two sets of values to be compared are in different variables.
unequal specifies that the unpaired data not be assumed to have equal variances.
welch specifies that the approximate degrees of freedom for the test be obtained from Welch's formula (1947) rather than from Satterthwaite's approximation formula (1946), which is the default when unequal is specified. Specifying welch implies unequal.
level(\#) specifies the confidence level, as a percentage, for confidence intervals. The default is level (95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals.

\section*{Remarks and examples}

Remarks are presented under the following headings:

\footnotetext{
One-sample \(t\) test
Two-sample \(t\) test
Paired \(t\) test
Two-sample \(t\) test compared with one-way ANOVA
Immediate form
Video examples
}

\section*{One-sample t test}

\section*{> Example 1}

In the first form, ttest tests whether the mean of the sample is equal to a known constant under the assumption of unknown variance. Assume that we have a sample of 74 automobiles. We know each automobile's average mileage rating and wish to test whether the overall average for the sample is 20 miles per gallon.
```

. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. ttest mpg==20
One-sample t test

```
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline Variable & Obs & Mean & Std. Err. & Std. Dev. & [95\% Conf & Interval] \\
\hline mpg & 74 & 21.2973 & . 6725511 & 5.785503 & 19.9569 & 22.63769 \\
\hline \multicolumn{3}{|l|}{mean \(=\) mean \((\mathrm{mpg})\)} & & & & 1.9289 \\
\hline \multicolumn{3}{|l|}{Ho: mean \(=20\)} & & \multicolumn{2}{|l|}{degrees of freedom} & 73 \\
\hline \multicolumn{2}{|l|}{\multirow[t]{2}{*}{\[
\begin{gathered}
\text { Ha: mean }<20 \\
\operatorname{Pr}(T<t)=0.9712
\end{gathered}
\]}} & \multicolumn{3}{|c|}{Ha: mean \(!=20\)} & \multicolumn{2}{|l|}{\multirow[t]{2}{*}{\(\begin{aligned} \text { Ha: mean } & >20 \\ \operatorname{Pr}(\mathrm{~T}>\mathrm{t}) & =0.0288\end{aligned}\)}} \\
\hline & & \multicolumn{3}{|r|}{\(\operatorname{Pr}(|T|>|t|)=0.0576\)} & & \\
\hline
\end{tabular}

The test indicates that the underlying mean is not 20 with a significance level of \(5.8 \%\).

\section*{Two-sample t test}

\section*{Example 2: Two-sample \(t\) test using groups}

We are testing the effectiveness of a new fuel additive. We run an experiment in which 12 cars are given the fuel treatment and 12 cars are not. The results of the experiment are as follows:
\begin{tabular}{cc} 
treated & mpg \\
\hline 0 & 20 \\
0 & 23 \\
0 & 21 \\
0 & 25 \\
0 & 18 \\
0 & 17 \\
0 & 18 \\
0 & 24 \\
0 & 20 \\
0 & 24 \\
0 & 23 \\
0 & 19 \\
1 & 24 \\
1 & 25 \\
1 & 21 \\
1 & 22 \\
1 & 23 \\
1 & 18 \\
1 & 17 \\
1 & 28 \\
1 & 24 \\
1 & 27 \\
1 & 21 \\
1 & 23
\end{tabular}

The treated variable is coded as 1 if the car received the fuel treatment and 0 otherwise.
We can test the equality of means of the treated and untreated group by typing
. use http://www.stata-press.com/data/r13/fuel3
. ttest mpg, by(treated)
Two-sample \(t\) test with equal variances


We do not find a statistically significant difference in the means.
If we were not willing to assume that the variances were equal and wanted to use Welch's formula, we could type
. ttest mpg, by (treated) welch
Two-sample t test with unequal variances
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline Group & Obs & Mean & Std. Err & Std. Dev. & [95\% Con & Interval] \\
\hline 0 & 12 & 21 & . 7881701 & 2.730301 & 19.26525 & 22.73475 \\
\hline 1 & 12 & 22.75 & . 9384465 & 3.250874 & 20.68449 & 24.81551 \\
\hline combined & 24 & 21.875 & . 6264476 & 3.068954 & 20.57909 & 23.17091 \\
\hline diff & & -1.75 & 1.225518 & & -4.28369 & . 7836902 \\
\hline \multicolumn{3}{|l|}{\multirow[t]{2}{*}{\(\begin{aligned} \text { Hiff } & =\text { mean }(0)-\operatorname{mean}(1) \\ \text { Hiff } & =0\end{aligned}\)}} & & & & -1.4280 \\
\hline & & & \multicolumn{4}{|r|}{Welch's degrees of freedom \(=23.2465\)} \\
\hline \multicolumn{2}{|l|}{\multirow[t]{2}{*}{\[
\begin{gathered}
\text { Ha: diff }<0 \\
\operatorname{Pr}(\mathrm{~T}<\mathrm{t})=0.0833
\end{gathered}
\]}} & \multicolumn{3}{|c|}{Ha: diff != 0} & \multicolumn{2}{|l|}{\multirow[t]{2}{*}{\[
\begin{gathered}
H a: \operatorname{diff}>0 \\
\operatorname{Pr}(\mathrm{~T}>\mathrm{t})=0.9167
\end{gathered}
\]}} \\
\hline & & \multicolumn{3}{|r|}{\(\operatorname{Pr}(|T|>|t|)=0.1666\)} & & \\
\hline
\end{tabular}
- Technical note

In two-sample using groups randomized designs, subjects will sometimes refuse the assigned treatment but still be measured for an outcome. In this case, take care to specify the group properly. You might be tempted to let varname contain missing where the subject refused and thus let ttest drop such observations from the analysis. Zelen (1979) argues that it would be better to specify that the subject belongs to the group in which he or she was randomized, even though such inclusion will dilute the measured effect.

\section*{> Example 3: Two-sample \(t\) test using variables}

There is a second, inferior way to organize the data in the preceding example. We ran a test on 24 cars, 12 without the additive and 12 with. We now create two new variables, mpg1 and mpg2.
\begin{tabular}{cl} 
mpg1 & mpg2 \\
\hline 20 & 24 \\
23 & 25 \\
21 & 21 \\
25 & 22 \\
18 & 23 \\
17 & 18 \\
18 & 17 \\
24 & 28 \\
20 & 24 \\
24 & 27 \\
23 & 21 \\
19 & 23
\end{tabular}

This method is inferior because it suggests a connection that is not there. There is no link between the car with 20 mpg and the car with 24 mpg in the first row of the data. Each column of data could be arranged in any order. Nevertheless, if our data are organized like this, ttest can accommodate us.
```

. use http://www.stata-press.com/data/r13/fuel
. ttest mpg1==mpg2, unpaired
Two-sample t test with equal variances

```
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline Variable & Obs & Mean & Std. Err. & Std. Dev. & [95\% Con & Interval] \\
\hline mpg1 & 12 & 21 & . 7881701 & 2.730301 & 19.26525 & 22.73475 \\
\hline mpg2 & 12 & 22.75 & . 9384465 & 3.250874 & 20.68449 & 24.81551 \\
\hline combined & 24 & 21.875 & . 6264476 & 3.068954 & 20.57909 & 23.17091 \\
\hline diff & & -1.75 & 1.225518 & & -4.291568 & . 7915684 \\
\hline \multicolumn{4}{|l|}{diff \(=\) mean (mpg1) - mean (mpg2)} & \multicolumn{3}{|r|}{\(\mathrm{t}=-1.4280\)} \\
\hline \multicolumn{2}{|l|}{Ho: diff \(=0\)} & & & \multirow[t]{3}{*}{degree} & of freedom & 22 \\
\hline Ha: & & & Ha: diff ! & & Ha & ff > 0 \\
\hline \(\operatorname{Pr}(\mathrm{T}<\mathrm{t})\) & 0837 & & | \(>\) |t|) = & & \(\operatorname{Pr}(\mathrm{T}>\) & \(=0.9163\) \\
\hline
\end{tabular}

\section*{Paired t test}

\section*{> Example 4}

Suppose that the preceding data were actually collected by running a test on 12 cars. Each car was run once with the fuel additive and once without. Our data are stored in the same manner as in example 3, but this time, there is most certainly a connection between the mpg values that appear in the same row. These come from the same car. The variables mpg1 and mpg2 represent mileage without and with the treatment, respectively.
. use http://www.stata-press.com/data/r13/fuel
. ttest mpg1==mpg2
Paired t test
\begin{tabular}{r|rrrrrr}
\hline Variable & Obs & Mean & Std. Err. & Std. Dev. & [95\% Conf. Interval] \\
\hline mpg1 & 12 & 21 & .7881701 & 2.730301 & 19.26525 & 22.73475 \\
mpg2 & 12 & 22.75 & .9384465 & 3.250874 & 20.68449 & 24.81551 \\
\hline diff & 12 & -1.75 & .7797144 & 2.70101 & -3.46614 & -.0338602 \\
\hline
\end{tabular}


We find that the means are statistically different from each other at any level greater than \(4.6 \%\).

\section*{Two-sample t test compared with one-way ANOVA}

\section*{- Example 5}

In example 2, we saw that ttest can be used to test the equality of a pair of means; see \([R]\) oneway for an extension that allows testing the equality of more than two means.

Suppose that we have data on the 50 states. The dataset contains the median age of the population (medage) and the region of the country (region) for each state. Region 1 refers to the Northeast, region 2 to the North Central, region 3 to the South, and region 4 to the West. Using oneway, we can test the equality of all four means.
\begin{tabular}{|c|c|c|c|c|c|}
\hline Source & \multicolumn{3}{|l|}{Analysis of Variance} & F & Prob > F \\
\hline Between groups & 46.3961903 & 3 & 15.4653968 & 7.56 & 0.0003 \\
\hline Within groups & 94.1237947 & 46 & 2.04616945 & & \\
\hline Total & 140.519985 & 49 & 2.8677548 & & \\
\hline Bartlett's test & equal varia & : & i2 (3) = 10 & 7 Pr & \(>\operatorname{chi2}=0\) \\
\hline
\end{tabular}

We find that the means are different, but we are interested only in testing whether the means for the Northeast (region==1) and West (region==4) are different. We could use oneway:
\begin{tabular}{|c|c|c|c|c|c|}
\hline Source & Analysi SS & \[
\begin{gathered}
\text { f } \mathrm{Va} \\
\mathrm{df}
\end{gathered}
\] & iance MS & F & Prob > F \\
\hline Between groups & 46.241247 & 1 & 46.241247 & 20.02 & 0.0002 \\
\hline Within groups & 46.1969169 & 20 & 2.30984584 & & \\
\hline Total & 92.4381638 & 21 & 4.40181733 & & \\
\hline Bartlett's test & equal varia & : & i2(1) = & 79 Pr & \(>\operatorname{chi2}=0\) \\
\hline
\end{tabular}

We could also use ttest:
. ttest medage if region==1 | region==4, by(region)
Two-sample t test with equal variances


The significance levels of both tests are the same.

\section*{Immediate form}

\section*{> Example 6}
ttesti is like ttest, except that we specify summary statistics rather than variables as arguments. For instance, we are reading an article that reports the mean number of sunspots per month as 62.6 with a standard deviation of 15.8 . There are 24 months of data. We wish to test whether the mean is 75 :
. ttesti 2462.615 .875
One-sample t test


\section*{> Example 7}

There is no immediate form of ttest with paired data because the test is also a function of the covariance, a number unlikely to be reported in any published source. For nonpaired data, however, we might type
. ttesti 2020532154
Two-sample t test with equal variances


If we had typed ttesti 2020532154 , unequal, the test would have assumed unequal variances.

\section*{Video examples}

One-sample \(t\) test in Stata
\(t\) test for two independent samples in Stata
\(t\) test for two paired samples in Stata
Immediate commands in Stata: One-sample \(t\) test from summary data
Immediate commands in Stata: Two-sample \(t\) test from summary data

\section*{Stored results}
ttest and ttesti store the following in \(r()\) :
Scalars
\begin{tabular}{|c|c|c|c|}
\hline \(r\) (N_1) & sample size \(n_{1}\) & r(sd_1) & standard deviation for first variable \\
\hline \(\mathrm{r}\left(\mathrm{N} \_2\right)\) & sample size \(n_{2}\) & r(sd_2) & standard deviation for second variable \\
\hline r (p_l) & lower one-sided \(p\)-value & r (sd) & combined standard deviation \\
\hline \(r\) (p_u) & upper one-sided \(p\)-value & r(mu_1) & \(\bar{x}_{1}\) mean for population 1 \\
\hline \(r\) (p) & two-sided \(p\)-value & r(mu_2) & \(\bar{x}_{2}\) mean for population 2 \\
\hline \(r\) (se) & estimate of standard error & r (df_t) & degrees of freedom \\
\hline \(r\) ( t ) & \(t\) statistic & r(level) & confidence level \\
\hline
\end{tabular}

\section*{Methods and formulas}

See, for instance, Hoel (1984, 140-161) or Dixon and Massey (1983, 121-130) for an introduction and explanation of the calculation of these tests. Acock (2014, 162-173) and Hamilton (2013, 145-150) describe \(t\) tests using applications in Stata.

The test for \(\mu=\mu_{0}\) for unknown \(\sigma\) is given by
\[
t=\frac{\left(\bar{x}-\mu_{0}\right) \sqrt{n}}{s}
\]

The statistic is distributed as Student's \(t\) with \(n-1\) degrees of freedom (Gosset [Student, pseud.] 1908).

The test for \(\mu_{x}=\mu_{y}\) when \(\sigma_{x}\) and \(\sigma_{y}\) are unknown but \(\sigma_{x}=\sigma_{y}\) is given by
\[
t=\frac{\bar{x}-\bar{y}}{\left\{\frac{\left(n_{x}-1\right) s_{x}^{2}+\left(n_{y}-1\right) s_{y}^{2}}{n_{x}+n_{y}-2}\right\}^{1 / 2}\left(\frac{1}{n_{x}}+\frac{1}{n_{y}}\right)^{1 / 2}}
\]

The result is distributed as Student's \(t\) with \(n_{x}+n_{y}-2\) degrees of freedom.
You could perform ttest (without the unequal option) in a regression setting given that regression assumes a homoskedastic error model. To compare with the ttest command, denote the underlying observations on \(x\) and \(y\) by \(x_{j}, j=1, \ldots, n_{x}\), and \(y_{j}, j=1, \ldots, n_{y}\). In a regression framework, typing ttest without the unequal option is equivalent to
1. creating a new variable \(z_{j}\) that represents the stacked observations on \(x\) and \(y\) (so that \(z_{j}=x_{j}\) for \(j=1, \ldots, n_{x}\) and \(z_{n_{x}+j}=y_{j}\) for \(j=1, \ldots, n_{y}\) )
2. and then estimating the equation \(z_{j}=\beta_{0}+\beta_{1} d_{j}+\epsilon_{j}\), where \(d_{j}=0\) for \(j=1, \ldots, n_{x}\) and \(d_{j}=1\) for \(j=n_{x}+1, \ldots, n_{x}+n_{y}\) (that is, \(d_{j}=0\) when the \(z\) observations represent \(x\), and \(d_{j}=1\) when the \(z\) observations represent \(y\) ).
The estimated value of \(\beta_{1}, b_{1}\), will equal \(\bar{y}-\bar{x}\), and the reported \(t\) statistic will be the same \(t\) statistic as given by the formula above.

The test for \(\mu_{x}=\mu_{y}\) when \(\sigma_{x}\) and \(\sigma_{y}\) are unknown and \(\sigma_{x} \neq \sigma_{y}\) is given by
\[
t=\frac{\bar{x}-\bar{y}}{\left(s_{x}^{2} / n_{x}+s_{y}^{2} / n_{y}\right)^{1 / 2}}
\]

The result is distributed as Student's \(t\) with \(\nu\) degrees of freedom, where \(\nu\) is given by (with Satterthwaite's [1946] formula)
\[
\frac{\left(s_{x}^{2} / n_{x}+s_{y}^{2} / n_{y}\right)^{2}}{\frac{\left(s_{x}^{2} / n_{x}\right)^{2}}{n_{x}-1}+\frac{\left(s_{y}^{2} / n_{y}\right)^{2}}{n_{y}-1}}
\]

With Welch's formula (1947), the number of degrees of freedom is given by
\[
-2+\frac{\left(s_{x}^{2} / n_{x}+s_{y}^{2} / n_{y}\right)^{2}}{\frac{\left(s_{x}^{2} / n_{x}\right)^{2}}{n_{x}+1}+\frac{\left(s_{y}^{2} / n_{y}\right)^{2}}{n_{y}+1}}
\]

The test for \(\mu_{x}=\mu_{y}\) for matched observations (also known as paired observations, correlated pairs, or permanent components) is given by
\[
t=\frac{\bar{d} \sqrt{n}}{s_{d}}
\]
where \(\bar{d}\) represents the mean of \(x_{i}-y_{i}\) and \(s_{d}\) represents the standard deviation. The test statistic \(t\) is distributed as Student's \(t\) with \(n-1\) degrees of freedom.

You can also use ttest without the unpaired option in a regression setting because a paired comparison includes the assumption of constant variance. The ttest with an unequal variance assumption does not lend itself to an easy representation in regression settings and is not discussed here. \(\left(x_{j}-y_{j}\right)=\beta_{0}+\epsilon_{j}\).

William Sealy Gosset (1876-1937) was born in Canterbury, England. He studied chemistry and mathematics at Oxford and worked as a chemist with the brewers Guinness in Dublin. Gosset became interested in statistical problems, which he discussed with Karl Pearson and later with Fisher and Neyman. He published several important papers under the pseudonym "Student", and he lent that name to the \(t\) test he invented.

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Also see
[R] bitest - Binomial probability test
\([\mathrm{R}]\) ci - Confidence intervals for means, proportions, and counts
[R] esize - Effect size based on mean comparison
[R] mean - Estimate means
[R] oneway - One-way analysis of variance
[R] prtest - Tests of proportions
[R] sdtest - Variance-comparison tests
[MV] hotelling - Hotelling's T-squared generalized means test

\section*{Title}
update - Check for official updates
\begin{tabular}{llll} 
Syntax & Menu & Description & Options \\
Remarks and examples & Stored results & Also see &
\end{tabular}

\section*{Syntax}

Report on update level of currently installed Stata update

Set update source update from location

Compare update level of currently installed Stata with that of source
```

update query [, from(location)]

```

\section*{Perform update if necessary}
```

update all [, from(location) detail force exit]

```

Set automatic updates (Mac and Windows only)
```

set update_query { on|off }
set update_interval \#
set update_prompt { on|off }

```

\section*{Menu}

Help \(>\) Check for Updates

\section*{Description}

The update command reports on the current update level and installs official updates to Stata. Official updates are updates to Stata as it was originally shipped from StataCorp, not the additions to Stata published in, for instance, the Stata Journal (SJ). Those additions are installed using the net command and updated using the adoupdate command; see \([R]\) net and \([R]\) adoupdate.
update without arguments reports on the update level of the currently installed Stata.
update from sets an update source, where location is a directory name or URL. If you are on the Internet, type update from http://www.stata.com.
update query compares the update level of the currently installed Stata with that available from the update source and displays a report.
update all updates all necessary files. This is what you should type to check for and install updates.
set update_query determines if update query is to be automatically performed when Stata is launched. Only Mac and Windows platforms can be set for automatic updating.
set update_interval \# sets the number of days to elapse before performing the next automatic update query. The default \# is 7 . The interval starts from the last time an update query was performed (automatically or manually). Only Mac and Windows platforms can be set for automatic updating.
set update_prompt determines whether a dialog is to be displayed before performing an automatic update query. The dialog allows you to perform an update query now, perform one the next time Stata is launched, perform one after the next interval has passed, or disable automatic update query. Only Mac and Windows platforms can be set for automatic updating.

\section*{Options}
from(location) specifies the location of the update source. You can specify the from() option on the individual update commands or use the update from command. Which you do makes no difference. You typically do not need to use this option.
detail specifies to display verbose output during the update process.
force specifies to force downloading of all files even if, based on the date comparison, Stata does not think it is necessary. There is seldom a reason to specify this option.
exit instructs Stata to exit when the update has successfully completed. There is seldom a reason to specify this option.

\section*{Remarks and examples}
update updates the official components of Stata from the official source: http://www.stata.com. If you are connected to the Internet, the easy thing to do is to type
. update all
and follow the instructions. If Stata is up to date, update all will do nothing. Otherwise, it will download whatever is necessary and install the files. If you just want to know what updates are available, type
. update query
update query will check if any updates are available and report that information. If updates are available, it will recommend that you type update all.

If you want to report the current update level, type
. update
update will report the update level of the Stata installation. update will also show you the date that updates were last checked and if any updates were available at that time.

\section*{Stored results}
update without a subcommand, update from, and update query store the following in \(r()\) :
Scalars
```

r(inst_exe) date of executable installed (*)
r(avbl_exe) date of executable available over web (*) (**)
r(inst_ado) date of ado-files installed (*)
r(avbl_ado) date of ado-files available over web (*) (**)
r(inst_utilities) date of utilities installed (*)
r(avbl_utilities) date of utilities available over web (*) (**)
r(inst_docs) date of documentation installed (*)
r(avbl_docs) date of documentation available over web (*)(**)
r(name_exe) name of the Stata executable
r(dir_exe) directory in which executable is stored
r(dir_ado) directory in which ado-files are stored
r(dir_utilities) directory in which utilities are stored
r(dir_docs) directory in which PDF documentation is stored

```
Macros

Notes:
* Dates are stored as integers counting the number of days since January 1, 1960; see [D] datetime.
** These dates are not stored by update without a subcommand because update by itself reports information solely about the local computer and does not check what is available on the web.

\section*{Also see}
[R] adoupdate - Update user-written ado-files
[R] net - Install and manage user-written additions from the Internet
[R] ssc - Install and uninstall packages from SSC
[P] sysdir - Query and set system directories
[U] 28 Using the Internet to keep up to date
[GSM] 19 Updating and extending Stata-Internet functionality
[GSU] 19 Updating and extending Stata-Internet functionality
[GSW] 19 Updating and extending Stata-Internet functionality

\section*{Title}
\(\boldsymbol{v c} \boldsymbol{c}\) _option - Variance estimators
\begin{tabular}{llll} 
Syntax & Description Options & Remarks and examples \\
Methods and formulas & Also see & &
\end{tabular}

\section*{Syntax}
\[
\begin{aligned}
& \qquad \text { estimation_cmd } \ldots[, \text { vce }(\text { vcetype }) \ldots] \\
& \text { vcetype }
\end{aligned}
\]

Likelihood based
oim observed information matrix (OIM)
opg outer product of the gradient (OPG) vectors

Sandwich estimators
robust
cluster clustvar

Replication based
bootstrap [, bootstrap_options] bootstrap estimation
jackknife [, jackknife_options] jackknife estimation

\section*{Description}

This entry describes the vce() option, which is common to most estimation commands. vce() specifies how to estimate the variance-covariance matrix (VCE) corresponding to the parameter estimates. The standard errors reported in the table of parameter estimates are the square root of the variances (diagonal elements) of the VCE.

\section*{Options}

\footnotetext{
SE/Robust
vce (oim) is usually the default for models fit using maximum likelihood. vce (oim) uses the observed information matrix (OIM); see [R] ml.
vce (opg) uses the sum of the outer product of the gradient (OPG) vectors; see [R] ml. This is the default VCE when the technique (bhhh) option is specified; see [ \(R\) ] maximize.
vce (robust) uses the robust or sandwich estimator of variance. This estimator is robust to some types of misspecification so long as the observations are independent; see [U] 20.21 Obtaining robust variance estimates.
}

If the command allows pweights and you specify them, vce(robust) is implied; see [U] 20.23.3 Sampling weights.
vce (cluster clustvar) specifies that the standard errors allow for intragroup correlation, relaxing the usual requirement that the observations be independent. That is, the observations are independent across groups (clusters) but not necessarily within groups. clustvar specifies to which group each observation belongs, for example, vce(cluster personid) in data with repeated observations on individuals. vce (cluster clustvar) affects the standard errors and variance-covariance matrix of the estimators but not the estimated coefficients; see [U] 20.21 Obtaining robust variance estimates.
vce (bootstrap [, bootstrap_options]) uses a bootstrap; see [R] bootstrap. After estimation with vce(bootstrap), see [R] bootstrap postestimation to obtain percentile-based or bias-corrected confidence intervals.
vce(jackknife [, jackknife_options]) uses the delete-one jackknife; see [R] jackknife.

\section*{Remarks and examples}

Remarks are presented under the following headings:

\section*{Prefix commands}

Passing options in vce()

\section*{Prefix commands}

Specifying vce(bootstrap) or vce(jackknife) is often equivalent to using the corresponding prefix command. Here is an example using jackknife with regress.
```

. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. regress mpg turn trunk, vce(jackknife)
(running regress on estimation sample)
Jackknife replications (74)

```


Linear regression
\begin{tabular}{llr} 
Number of obs & \(=\) & 74 \\
Replications & \(=\) & 74 \\
F( 2, 73) & \(=\) & 66.26 \\
Prob > F & \(=\) & 0.0000 \\
R-squared & \(=\) & 0.5521 \\
Adj R-squared & \(=\) & 0.5395 \\
Root MSE & \(=\) & 3.9260
\end{tabular}
\begin{tabular}{r|rrrrrr}
\hline & Coef. & \begin{tabular}{l} 
Jackknife \\
Std. Err.
\end{tabular} & t & \(\mathrm{P}>|\mathrm{t}|\) & [95\% Conf. Interval] \\
\hline mpg & turn & -.7610113 & .150726 & -5.05 & 0.000 & -1.061408 \\
trunk & -.3161825 & .1282326 & -2.47 & 0.016 & -.5717498 & -.0606147 \\
_cons & 55.82001 & 5.031107 & 11.09 & 0.000 & 45.79303 & 65.84699 \\
\hline
\end{tabular}


Here it does not matter whether we specify the vce(jackknife) option or instead use the jackknife prefix.

However, vce(jackknife) should be used in place of the jackknife prefix whenever available because they are not always equivalent. For example, to use the jackknife prefix with clogit properly, you must tell jackknife to omit whole groups rather than individual observations. Specifying vce(jackknife) does this automatically.
```

. use http://www.stata-press.com/data/r13/clogitid
. jackknife, cluster(id): clogit y x1 x2, group(id)
(output omitted)

```

This extra information is automatically communicated to jackknife by clogit when the vce() option is specified.
```

. clogit y x1 x2, group(id) vce(jackknife)
(running clogit on estimation sample)
Jackknife replications (66)
............................................................. 50
Conditional (fixed-effects) logistic regression

```
\begin{tabular}{llr} 
Number of obs & \(=\) & 369 \\
Replications & \(=\) & 66 \\
F 2, & 4.58 \\
Prob \(>\) F & \(=\) & 0.0137 \\
Pseudo R2 & \(=\) & 0.0355
\end{tabular}
(Replications based on 66 clusters in id)
\begin{tabular}{c|rcccrr}
\hline & & \multicolumn{4}{c}{ Jackknife } \\
y & Coef. & Std. Err. & t & \(\mathrm{P}>|\mathrm{t}|\) & [95\% Conf. Interval] \\
\hline x 1 & .653363 & .3010608 & 2.17 & 0.034 & .052103 & 1.254623 \\
x 2 & .0659169 & .0487858 & 1.35 & 0.181 & -.0315151 & .1633489 \\
\hline
\end{tabular}

\section*{Passing options in vce()}

If you wish to specify more options to the bootstrap or jackknife estimation, you can include them within the vce() option. Below we request 300 bootstrap replications and save the replications in bsreg.dta:

\begin{tabular}{r|rrrrrr}
\hline & \begin{tabular}{r} 
Observed \\
Coef.
\end{tabular} & \begin{tabular}{c} 
Bootstrap \\
Std. Err.
\end{tabular} & z & \(\mathrm{P}>|\mathrm{z}|\) & \multicolumn{2}{c}{\begin{tabular}{r} 
Normal-based \\
[95\% Conf. Interval]
\end{tabular}} \\
\hline turn & -.7610113 & .1361786 & -5.59 & 0.000 & -1.027916 & -.4941062 \\
trunk & -.3161825 & .1145728 & -2.76 & 0.006 & -.540741 & -.0916239 \\
_cons & 55.82001 & 4.69971 & 11.88 & 0.000 & 46.60875 & 65.03127 \\
\hline
\end{tabular}
. bstat using bsreg
Bootstrap results \(\quad\) Number of obs \(\quad 74\)
Replications \(=300\)
command: regress mpg turn trunk
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline & Observed Coef. & \begin{tabular}{l}
Bootstrap \\
Std. Err.
\end{tabular} & 2 & \(\mathrm{P}>|\mathrm{z}|\) & \multicolumn{2}{|l|}{Normal-based} \\
\hline turn & -. 7610113 & . 1361786 & -5.59 & 0.000 & -1.027916 & -. 4941062 \\
\hline trunk & -. 3161825 & . 1145728 & -2.76 & 0.006 & -. 540741 & -. 0916239 \\
\hline _cons & 55.82001 & 4.69971 & 11.88 & 0.000 & 46.60875 & 65.03127 \\
\hline
\end{tabular}

\section*{Methods and formulas}

By default, Stata's maximum likelihood estimators display standard errors based on variance estimates given by the inverse of the negative Hessian (second derivative) matrix. If vce (robust), vce (cluster clustvar), or pweights is specified, standard errors are based on the robust variance estimator (see [U] 20.21 Obtaining robust variance estimates); likelihood-ratio tests are not appropriate here (see [SVY] survey), and the model \(\chi^{2}\) is from a Wald test. If vce (opg) is specified, the standard errors are based on the outer product of the gradients; this option has no effect on likelihood-ratio tests, though it does affect Wald tests.

If vce(bootstrap) or vce(jackknife) is specified, the standard errors are based on the chosen replication method; here the model \(\chi^{2}\) or \(F\) statistic is from a Wald test using the respective replicationbased covariance matrix. The \(t\) distribution is used in the coefficient table when the vce (jackknife) option is specified. vce(bootstrap) and vce(jackknife) are also available with some commands that are not maximum likelihood estimators.

\section*{Also see}
[R] bootstrap - Bootstrap sampling and estimation
[R] jackknife - Jackknife estimation
[XT] vce_options - Variance estimators
[U] 20 Estimation and postestimation commands

\section*{Title}
view - View files and logs
\begin{tabular}{llll} 
Syntax & Menu & Description & Options \\
Remarks and examples & Also see & &
\end{tabular}

\section*{Syntax}

Display file in Viewer
\[
\text { view [file] ["]filename["] [, asis adopath }]
\]

Bring up browser pointed to specified URL
view browse ["]url["]

Display help results in Viewer [view] help [topic_or_command_name]

Display search results in Viewer [view] search keywords

Display news results in Viewer view news

Display net results in Viewer view net [netcmd]

Display ado-results in Viewer view ado [adocmd]

Display update results in Viewer view update [updatecmd]

\section*{Menu}

File > View...

\section*{Description}
view displays file contents in the Viewer.
view file displays the specified file. file is optional, so if you had a SMCL session log created by typing log using mylog, you could view it by typing view mylog.smcl. view file can properly display .smcl files (logs and the like), .sthlp files, and text files. view file's asis option specifies that the file be displayed as plain text, regardless of the filename's extension.
view browse opens your browser pointed to url. Typing
view browse http://www.stata.com would bring up your browser pointed to the http://www.stata.com website.
[view] help displays the specified topic in the Viewer. For example, to review the help for Stata's print command, you could type help print. See [R] help for more details.
[view] search displays the results of the search command in the Viewer. For instance, to search the system help for information on robust regression, you could type search robust regression. See \([R]\) search for more details.
view news does the same as the news command-see [R] news-but displays the results in the Viewer. (news displays the latest news from http://www.stata.com.)
view net does the same as the net command-see [R] net-but displays the result in the Viewer. For instance, typing view net search hausman test would search the Internet for additions to Stata related to the Hausman test. Typing view net from http://www.stata.com would go to the Stata additions download site at http://www.stata.com.
view ado does the same as the ado command-see [R] net-but displays the result in the Viewer. For instance, typing view ado dir would show a list of files you have installed.
view update does the same as the update command-see [R] update-but displays the result in the Viewer. Typing view update would show the dates of what you have installed, and from there you could click to compare those dates with the latest updates available. Typing view update query would skip the first step and show the comparison.

\section*{Options}
asis, allowed with view file, specifies that the file be displayed as text, regardless of the filename's extension. view file's default action is to display files ending in .smcl and .sthlp as SMCL; see [P] smcl.
adopath, allowed with view file, specifies that Stata search the S_ADO path for filename and display it, if found.

\section*{Remarks and examples}

Most users access the Viewer by selecting File \(>\) View... and proceeding from there. Some commands allow you to skip that step. Some common interactive uses of commands that display their results in the Viewer are the following:
. view mysession.smcl
. view mysession.log
. help print
. help regress
. view news
. view browse http://www.stata.com
. search hausman test
. view net
. view ado
. view update query

\section*{Also see}
[R] help - Display help in Stata
[R] net - Install and manage user-written additions from the Internet
[R] news - Report Stata news
[R] search - Search Stata documentation and other resources
[R] update - Check for official updates
[D] type - Display contents of a file
[GSM] 3 Using the Viewer
[GSU] 3 Using the Viewer
[GSW] 3 Using the Viewer

\section*{Title}
vwls - Variance-weighted least squares
\begin{tabular}{llll} 
Syntax & Menu & Description & Options \\
Remarks and examples & Stored results & Methods and formulas & References
\end{tabular}

\section*{Syntax}
\[
\text { vwls depvar indepvars }[\text { if }][\text { in }][\text { weight }][\text {, options }]
\]
options
Description

Model

\section*{noconstant}
sd(varname)
Reporting
level(\#)
display_options
coeflegend
suppress constant term
variable containing estimate of conditional standard deviation
indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
bootstrap, by, jackknife, rolling, and statsby are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the bootstrap prefix; see \([R]\) bootstrap.
fweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

\section*{Menu}

Statistics \(>\) Linear models and related \(>\) Other \(>\) Variance-weighted least squares

\section*{Description}
vwls estimates a linear regression using variance-weighted least squares. It differs from ordinary least-squares (OLS) regression in that it does not assume homogeneity of variance, but requires that the conditional variance of depvar be estimated prior to the regression. The estimated variance need not be constant across observations. vwls treats the estimated variance as if it were the true variance when it computes the standard errors of the coefficients.

You must supply an estimate of the conditional standard deviation of depvar to vwls by using the sd(varname) option, or you must have grouped data with the groups defined by the indepvars variables. In the latter case, vwls treats all indepvars as categorical variables, computes the mean and standard deviation of depvar separately for each subgroup, and computes the regression of the subgroup means on indepvars.
regress with analytic weights can be used to produce another kind of "variance-weighted least squares"; see Remarks and examples for an explanation of the difference.

\section*{Options}
\(\qquad\) Model
noconstant; see [R] estimation options.
\(\operatorname{sd}\) (varname) is an estimate of the conditional standard deviation of depvar (that is, it can vary observation by observation). All values of varname must be \(>0\). If you specify sd(), you cannot use fweights.
If \(\operatorname{sd}()\) is not given, the data will be grouped by indepvars. Here indepvars are treated as categorical variables, and the means and standard deviations of depvar for each subgroup are calculated and used for the regression. Any subgroup for which the standard deviation is zero is dropped.

\section*{Reporting}
level(\#); see [R] estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% fint), pformat (\% fint), sformat (\% fint), and nolstretch; see [R] estimation options.

The following option is available with vwls but is not shown in the dialog box:
coeflegend; see [R] estimation options.

\section*{Remarks and examples}

The vwls command is intended for use with two special - and different - types of data. The first contains data that consist of measurements from physical science experiments in which all error is due solely to measurement errors and the sizes of the measurement errors are known.

You can also use variance-weighted least-squares linear regression for certain problems in categorical data analysis, such as when all the independent variables are categorical and the outcome variable is either continuous or a quantity that can sensibly be averaged. If each of the subgroups defined by the categorical variables contains a reasonable number of subjects, then the variance of the outcome variable can be estimated independently within each subgroup. For the purposes of estimation, vwls treats each subgroup as one observation, with the dependent variable being the subgroup mean of the outcome variable.

The vwls command fits the model
\[
y_{i}=\mathbf{x}_{i} \boldsymbol{\beta}+\varepsilon_{i}
\]
where the errors \(\varepsilon_{i}\) are independent normal random variables with the distribution \(\varepsilon_{i} \sim N\left(0, \nu_{i}\right)\). The independent variables \(\mathbf{x}_{i}\) are assumed to be known without error.

As described above, vwls assumes that you already have estimates \(s_{i}^{2}\) for the variances \(\nu_{i}\). The error variance is not estimated in the regression. The estimates \(s_{i}^{2}\) are used to compute the standard errors of the coefficients; see Methods and formulas below.

In contrast, weighted oLS regression assumes that the errors have the distribution \(\varepsilon_{i} \sim N\left(0, \sigma^{2} / w_{i}\right)\), where the \(w_{i}\) are known weights and \(\sigma^{2}\) is an unknown parameter that is estimated in the regression. This is the difference from variance-weighted least squares: in weighted OLS, the magnitude of the error variance is estimated in the regression using all the data.

\section*{Example 1}

An artificial, but informative, example illustrates the difference between variance-weighted least squares and weighted OLS.

We measure the quantities \(x_{i}\) and \(y_{i}\) and estimate that the standard deviation of \(y_{i}\) is \(s_{i}\). We enter the data into Stata:
```

. use http://www.stata-press.com/data/r13/vwlsxmpl

```
. list
\begin{tabular}{l|rrr|}
\cline { 2 - 3 } & x & y & s \\
\hline 1. & 1 & 1.2 & .5 \\
2. & 2 & 1.9 & .5 \\
3. & 3 & 3.2 & 1 \\
4. & 4 & 4.3 & 1 \\
5. & 5 & 4.9 & 1 \\
\hline 6. & 6 & 6.0 & 2 \\
7. & 7 & 7.2 & 2 \\
8. & 8 & 7.9 & 2 \\
\hline
\end{tabular}

Because we want observations with smaller variance to carry larger weight in the regression, we compute an OLS regression with analytic weights proportional to the inverse of the squared standard deviations:
\begin{tabular}{|c|c|c|c|c|c|}
\hline Source & SS & df & MS & Number of obs & 8 \\
\hline & & & & F ( 1, 6) & 702.26 \\
\hline Model & 22.6310183 & 1 & 22.6310183 & Prob > F & 0.0000 \\
\hline Residual & . 193355117 & 6 & . 032225853 & R -squared & 0.9915 \\
\hline & & & & Adj R-squared & 0.9901 \\
\hline Total & 22.8243734 & 7 & 3.26062477 & Root MSE & . 17952 \\
\hline
\end{tabular}
\begin{tabular}{r|rrrrrr}
\hline y & Coef. & Std. Err. & t & \(\mathrm{P}>|\mathrm{t}|\) & [95\% Conf. Interval] \\
\hline x & .9824683 & .0370739 & 26.50 & 0.000 & .8917517 & 1.073185 \\
_cons & .1138554 & .1120078 & 1.02 & 0.349 & -.1602179 & .3879288 \\
\hline
\end{tabular}

If we compute a variance-weighted least-squares regression by using vwls, we get the same results for the coefficient estimates but very different standard errors:
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline \multicolumn{4}{|l|}{Variance-weighted least-squares regression} & \multicolumn{2}{|r|}{\multirow[t]{2}{*}{Number of obs Model chi2(1)}} & \(=\) & 8 \\
\hline Goodness-of-f & hi2(6) & \multicolumn{2}{|l|}{0.28} & & & = & 33.24 \\
\hline Prob > chi2 & & \(=0.9996\) & & \multicolumn{2}{|r|}{Prob > chi2} & = & 0.0000 \\
\hline y & Coef. & Std. Err. & z & \(\mathrm{P}>|z|\) & [95\% Conf & In & terval] \\
\hline x & . 9824683 & . 170409 & 5.77 & 0.000 & . 6484728 & & . 316464 \\
\hline _cons & . 1138554 & . 51484 & 0.22 & 0.825 & -. 8952124 & & . 122923 \\
\hline
\end{tabular}

Although the values of \(y_{i}\) were nicely linear with \(x_{i}\), the vwls regression used the large estimates for the standard deviations to compute large standard errors for the coefficients. For weighted OLS regression, however, the scale of the analytic weights has no effect on the standard errors of the coefficients-only the relative proportions of the analytic weights affect the regression.

If we are sure of the sizes of our error estimates for \(y_{i}\), using vwls is valid. However, if we can estimate only the relative proportions of error among the \(y_{i}\), then vwls is not appropriate.

\section*{> Example 2}

Let's now consider an example of the use of vwls with categorical data. Suppose that we have blood pressure data for \(n=400\) subjects, categorized by gender and race (black or white). Here is a description of the data:
. use http://www.stata-press.com/data/r13/bp
. table gender race, \(c\) (mean bp sd bp freq) row col format(\%8.1f)
\begin{tabular}{r|rrr}
\hline & \multicolumn{3}{|c}{\begin{tabular}{c} 
Race \\
Gender
\end{tabular}} \\
\hline Female & White & Black & Total \\
\hline & 117.1 & 118.5 & 117.8 \\
& 100.0 & 11.6 & 10.9 \\
Male & 122.1 & 125.8 & 124.0 \\
& 10.6 & 15.5 & 13.3 \\
& 100.0 & 100.0 & 200.0 \\
Total & 119.6 & 122.2 & 120.9 \\
& 10.7 & 14.1 & 12.6 \\
& 200.0 & 200.0 & 400.0 \\
\hline
\end{tabular}

Performing a variance-weighted regression using vwls gives
\begin{tabular}{lllll}
. vwls bp gender race & & & \\
Variance-weighted least-squares & regression & Number of obs & \(=\) & 400 \\
Goodness-of-fit chi2(1) & \(=\) & 0.88 & Model chi2 \((2)\) & \(=\) \\
Prob > chi2 & \(=\) & 0.3486 & Prob > chi2 & \(=0.0000\)
\end{tabular}
\begin{tabular}{r|rrcccc}
\hline bp & Coef. & Std. Err. & \(z\) & P>|z| & [95\% Conf. Interval] \\
\hline gender & 5.876522 & 1.170241 & 5.02 & 0.000 & 3.582892 & 8.170151 \\
race & 2.372818 & 1.191683 & 1.99 & 0.046 & .0371631 & 4.708473 \\
_cons & 116.6486 & .9296297 & 125.48 & 0.000 & 114.8266 & 118.4707 \\
\hline
\end{tabular}

By comparison, an OLS regression gives the following result:
. regress bp gender race
\begin{tabular}{r|rrr} 
Source & SS & df & MS \\
\hline Model & 4485.66639 & 2 & 2242.83319 \\
Residual & 58442.7305 & 397 & 147.210908 \\
\hline Total & 62928.3969 & 399 & 157.71528
\end{tabular}

Number of obs \(=400\)
F( 2, 397) \(=15.24\)
Prob > F \(=0.0000\)
\(R\)-squared \(=0.0713\)
Adj R-squared \(=0.0666\)
Root MSE \(=12.133\)
\begin{tabular}{r|rcccrr}
\hline bp & Coef. & Std. Err. & t & \(\mathrm{P}>|\mathrm{t}|\) & {\([95 \%\) Conf. Interval] } \\
\hline gender & 6.1775 & 1.213305 & 5.09 & 0.000 & 3.792194 & 8.562806 \\
race & 2.5875 & 1.213305 & 2.13 & 0.034 & .2021938 & 4.972806 \\
_cons & 116.4862 & 1.050753 & 110.86 & 0.000 & 114.4205 & 118.552 \\
\hline
\end{tabular}

Note the larger value for the race coefficient (and smaller \(p\)-value) in the OLS regression. The assumption of homogeneity of variance in OLS means that the mean for black men pulls the regression line higher than in the vwls regression, which takes into account the larger variance for black men and reduces its effect on the regression.

\section*{Stored results}
vwls stores the following in e():
Scalars
e(N)
e(df_m)
e(chi2)
e(df_gf)
e(chi2_gf)
\(e\) (rank)
Macros
e(cmd)
e(cmdline)
e(depvar)
e(properties)
e(predict)
e(asbalanced)
e(asobserved)
Matrices
e(b)
e(V)
Functions
\(e\) (sample) marks estimation sample

\section*{Methods and formulas}

Let \(\mathbf{y}=\left(y_{1}, y_{2}, \ldots, y_{n}\right)^{\prime}\) be the vector of observations of the dependent variable, where \(n\) is the number of observations. When \(\operatorname{sd}()\) is specified, let \(s_{1}, s_{2}, \ldots, s_{n}\) be the standard deviations supplied by sd(). For categorical data, when sd() is not given, the means and standard deviations of \(y\) for each subgroup are computed, and \(n\) becomes the number of subgroups, \(\mathbf{y}\) is the vector of subgroup means, and \(s_{i}\) are the standard deviations for the subgroups.

Let \(\mathbf{V}=\operatorname{diag}\left(s_{1}^{2}, s_{2}^{2}, \ldots, s_{n}^{2}\right)\) denote the estimate of the variance of \(\mathbf{y}\). Then the estimated regression coefficients are
\[
\mathbf{b}=\left(\mathbf{X}^{\prime} \mathbf{V}^{-1} \mathbf{X}\right)^{-1} \mathbf{X}^{\prime} \mathbf{V}^{-1} \mathbf{y}
\]
and their estimated covariance matrix is
\[
\widehat{\operatorname{Cov}}(\mathbf{b})=\left(\mathbf{X}^{\prime} \mathbf{V}^{-1} \mathbf{X}\right)^{-1}
\]

A statistic for the goodness of fit of the model is
\[
Q=(\mathbf{y}-\mathbf{X b})^{\prime} \mathbf{V}^{-1}(\mathbf{y}-\mathbf{X} \mathbf{b})
\]
where \(Q\) has a \(\chi^{2}\) distribution with \(n-k\) degrees of freedom ( \(k\) is the number of independent variables plus the constant, if any).

\section*{References}

Gini, R., and J. Pasquini. 2006. Automatic generation of documents. Stata Journal 6: 22-39.
Grizzle, J. E., C. F. Starmer, and G. G. Koch. 1969. Analysis of categorical data by linear models. Biometrics 25: 489-504.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery. 2007. Numerical Recipes: The Art of Scientific Computing. 3rd ed. New York: Cambridge University Press.

\section*{Also see}
[R] vwls postestimation - Postestimation tools for vwls
\([\mathrm{R}]\) regress - Linear regression
[U] 11.1.6 weight
[U] 20 Estimation and postestimation commands

\section*{Title}
vwls postestimation - Postestimation tools for vwls
Description Syntax for predict Menu for predict Options for predict Also see

\section*{Description}

The following postestimation commands are available after vwls:
\begin{tabular}{ll} 
Command & Description \\
\hline contrast & \begin{tabular}{l} 
contrasts and ANOVA-style joint tests of estimates \\
estat summarize \\
estat vce
\end{tabular} \\
\begin{tabular}{l} 
summary statistics for the estimation sample \\
variance-covariance matrix of the estimators (VCE) \\
estalatoging estimation results
\end{tabular} \\
forecast & \begin{tabular}{l} 
dynamic forecasts and simulations \\
point estimates, standard errors, testing, and inference for linear \\
combinations of coefficients
\end{tabular} \\
lincom & \begin{tabular}{l} 
link test for model specification \\
marginal means, predictive margins, marginal effects, and average marginal \\
effects \\
graph the results from margins (profile plots, interaction plots, etc.)
\end{tabular} \\
margins & \begin{tabular}{l} 
point estimates, standard errors, testing, and inference for nonlinear \\
combinations of coefficients
\end{tabular} \\
marginsplot & predictions, residuals, influence statistics, and other diagnostic measures \\
nlcom & \begin{tabular}{l} 
point estimates, standard errors, testing, and inference for generalized predictions
\end{tabular} \\
predict & pairwise comparisons of estimates \\
predictnl & Wald tests of simple and composite linear hypotheses \\
pwcompare & Wald tests of nonlinear hypotheses
\end{tabular}

\section*{Syntax for predict}
predict [type] newvar [if] [in] [, xb stdp]
These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

\section*{Menu for predict}

Statistics \(>\) Postestimation \(>\) Predictions, residuals, etc.

\section*{Options for predict}
\(\qquad\) Main
xb , the default, calculates the linear prediction.
stdp calculates the standard error of the linear prediction.

\section*{Also see}
[R] vwls - Variance-weighted least squares
[U] 20 Estimation and postestimation commands

\section*{Title}
which - Display location and version for an ado-file

> Syntax Description Option Remarks and examples Also see

\section*{Syntax}
which fname[.ftype] [, all]

\section*{Description}
which looks for fname.ftype along the S_ADO path. If Stata finds the file, which displays the full path and filename, along with, if the file is text, all lines in the file that begin with "*!" in the first column. If Stata cannot find the file, which issues the message "file not found along ado-path" and sets the return code to 111. ftype must be a file type for which Stata usually looks along the ado-path to find. Allowable ftypes are
```

.ado,.class,.dlg, .idlg, .sthlp, .ihlp, .hlp, .key, .maint, .mata, .mlib, .mo, .mnu,
.plugin,.scheme,.stbcal, and .style

```

If ftype is omitted, which assumes .ado. When searching for . ado files, if Stata cannot find the file, Stata then checks to see if fname is a built-in Stata command, allowing for valid abbreviations. If it is, the message "built-in command" is displayed; if not, the message "command not found as either built-in or ado-file" is displayed and the return code is set to 111 .

For information about internal version control, see \([\mathrm{P}]\) version.

\section*{Option}
all forces which to report the location of all files matching the fname.ftype found along the search path. The default is to report just the first one found.

\section*{Remarks and examples}

If you write programs, you know that you make changes to the programs over time. If you are like us, you also end up with multiple versions of the program stored on your disk, perhaps in different directories. You may even have given copies of your programs to other Stata users, and you may not remember which version of a program you or your friends are using. The which command helps you solve this problem.

\section*{> Example 1}

The which command displays the path for filename.ado and any lines in the code that begin with "*!". For example, we might want information about the test command, described in [R] test, which is an ado-file written by StataCorp. Here is what happens when we type which test:
```

. which test
C:\Program Files\Stata13\ado\base\t\test.ado
*! version 2.2.2 07feb2012

```
which displays the path for the test.ado file and also a line beginning with " \(*\) !" that indicates the version of the file. This is how we, at StataCorp, do version control-see [U] 18.11.1 Version for an explanation of our version control numbers.

We do not need to be so formal. which will display anything typed after lines that begin with ‘*!’. For instance, we might write myprog.ado:
```

. which myprog
.\myprog.ado
*! first written 1/03/2013
*! bug fix on 1/05/2013 (no variance case)
*! updated 1/24/2013 to include noconstant option
*! still suspicious if variable takes on only two values

```

It does not matter where in the program the lines beginning with \(*!\) are-which will list them (in particular, our "still suspicious" comment was buried about 50 lines down in the code). All that is important is that the \(*!\) marker appear in the first two columns of a line.

\section*{> Example 2}

If we type which command, where command is a built-in command rather than an ado-file, Stata responds with
```

. which summarize
built-in command: summarize

```

If command was neither a built-in command nor an ado-file, Stata would respond with
```

. which junk
command junk not found as either built-in or ado-file
r(111);

```

\section*{Also see}
[P] findfile - Find file in path
\([\mathrm{P}]\) version - Version control
[U] 17 Ado-files
[U] 18.11.1 Version

\section*{Title}
xi - Interaction expansion
\begin{tabular}{llll} 
Syntax & Menu & Description & Options \\
Remarks and examples & Stored results & References & Also see
\end{tabular}

\section*{Syntax}
```

xi [, prefix(string) noomit] term(s)
xi [, prefix(string) noomit] : any_stata_command varlist_with_terms ...

```
where a term has the form
\begin{tabular}{lll} 
i. varname & or & I. varname \\
i. varname \({ }_{1} *\) i. varname \(_{2}\) & & I.varname \({ }_{1} *\) I. varname \\
2
\end{tabular}
varname, varname \(_{1}\), and varname \({ }_{2}\) denote numeric or string categorical variables. varname \({ }_{3}\) denotes a continuous, numeric variable.

\section*{Menu}

Data \(>\) Create or change data \(>\) Other variable-creation commands \(>\) Interaction expansion

Most commands in Stata now allow factor variables; see [U] 11.4.3 Factor variables. To determine if a command allows factor variables, see the information printed below the options table for the command. If the command allows factor variables, it will say something like "indepvars may contain factor variables".

We recommend that you use factor variables instead of xi if a command allows factor variables. We include \([R]\) xi in our documentation so that readers can consult it when using a Stata command that does not allow factor variables.

\section*{Description}
xi expands terms containing categorical variables into indicator (also called dummy) variable sets by creating new variables and, in the second syntax (xi: any_stata_command), executes the specified command with the expanded terms. The dummy variables created are
\begin{tabular}{|c|c|}
\hline i. varname & creates dummies for categorical variable varname \\
\hline i. varname \(_{1} *\) i. varname \(_{2}\) & \begin{tabular}{l}
creates dummies for categorical variables varname \(_{1}\) and varname \({ }_{2}\) : \\
all interactions and main effects
\end{tabular} \\
\hline i. varname \(_{1}\) * arname \(_{3}\) & \begin{tabular}{l}
creates dummies for categorical variable varname \({ }_{1}\) and continuous variable varname \(3_{3}\) : \\
all interactions and main effects
\end{tabular} \\
\hline i. varname \(_{1}\) | arrname \(_{3}\) & \begin{tabular}{l}
creates dummies for categorical variable varname \({ }_{1}\) and continuous variable varname \({ }_{3}\) : \\
all interactions and main effect of varname \(3_{3}\), but no main effect of varname \(_{1}\)
\end{tabular} \\
\hline
\end{tabular}

\section*{Options}
prefix (string) allows you to choose a prefix other than _I for the newly created interaction variables. The prefix cannot be longer than four characters. By default, xi will create interaction variables starting with _I. When you use xi, it drops all previously created interaction variables starting with the prefix specified in the prefix (string) option or with _I by default. Therefore, if you want to keep the variables with a certain prefix, specify a different prefix in the prefix(string) option.
noomit prevents xi from omitting groups. This option provides a way to generate an indicator variable for every category having one or more variables, which is useful when combined with the noconstant option of an estimation command.

\section*{Remarks and examples}

Remarks are presented under the following headings:
```

Background
Indicator variables for simple effects
Controlling the omitted dummy
Categorical variable interactions
Interactions with continuous variables
Using xi: Interpreting output
How xi names variables
xi as a command rather than a command prefix
Warnings

```
xi provides a convenient way to include dummy or indicator variables when fitting a model (say, with regress or logistic). For instance, assume that the categorical variable agegrp contains 1 for ages 20-24, 2 for ages \(25-39\), 3 for ages \(40-44\), etc. Typing
```

. xi: logistic outcome weight i.agegrp bp

```
estimates a logistic regression of outcome on weight, dummies for each agegrp category, and bp. That is, xi searches out and expands terms starting with "i." or "I." but ignores the other variables. xi will expand both numeric and string categorical variables, so if you had a string variable race containing "white", "black", and "other", typing
```

. xi: logistic outcome weight bp i.agegrp i.race

```
would include indicator variables for the race group as well.
The i. indicator variables xi expands may appear anywhere in the varlist, so
```

. xi: logistic outcome i.agegrp weight i.race bp

```
would fit the same model.
You can also create interactions of categorical variables; typing
```

xi: logistic outcome weight bp i.agegrp*i.race

```
fits a model with indicator variables for all agegrp and race combinations, including the agegrp and race main-effect terms (that is, the terms that are created when you just type i.agegrp i.race).

You can interact dummy variables with continuous variables; typing
```

xi: logistic outcome bp i.agegrp*weight i.race

```
fits a model with indicator variables for all agegrp categories interacted with weight, plus the main-effect terms weight and i. agegrp.

You can get the interaction terms without the agegrp main effect (but with the weight main effect) by typing
```

xi: logistic outcome bp i.agegrp|weight i.race

```

You can also include multiple interactions:
xi: logistic outcome bp i.agegrp*weight i.agegrp*i.race
We will now back up and describe the construction of dummy variables in more detail.

\section*{Background}

The terms continuous, categorical, and indicator or dummy variables are used below. Continuous variables measure something - such as height or weight - and at least conceptually can take on any real number over some range. Categorical variables, on the other hand, take on a finite number of values, each denoting membership in a subclass-for example, excellent, good, and poor, which might be coded 0, 1, 2, or 1, 2, 3, or even "Excellent", "Good", and "Poor". An indicator or dummy variable-the terms are used interchangeably-is a special type of two-valued categorical variable that contains values 0 , denoting false, and 1 , denoting true. The information contained in any \(k\)-valued categorical variable can be equally well represented by \(k\) indicator variables. Instead of one variable recording values representing excellent, good, and poor, you can have three indicator variables, indicating the truth or falseness of "result is excellent", "result is good", and "result is poor".
xi provides a convenient way to convert categorical variables to dummy or indicator variables when you fit a model (say, with regress or logistic).

\section*{> Example 1}

For instance, assume that the categorical variable agegrp contains 1 for ages 20-24, 2 for ages \(25-39\), and 3 for ages \(40-44\). (There is no one over 44 in our data.) As it stands, agegrp would be a poor candidate for inclusion in a model even if we thought age affected the outcome. The reason is that the coding would restrict the effect of being in the second age group to be twice the effect of being in the first, and, similarly, the effect of being in the third to be three times the first. That is, if we fit the model,
\[
y=\beta_{0}+\beta_{1} \text { agegrp }+X \beta_{2}
\]
the effect of being in the first age group is \(\beta_{1}\), the second \(2 \beta_{1}\), and the third \(3 \beta_{1}\). If the coding 1,2 , and 3 is arbitrary, we could just as well have coded the age groups 1,4 , and 9 , making the effects \(\beta_{1}, 4 \beta_{1}\), and \(9 \beta_{1}\).

The solution is to convert the categorical variable agegrp to a set of indicator variables, \(a_{1}, a_{2}\), and \(a_{3}\), where \(a_{i}\) is 1 if the individual is a member of the \(i\) th age group and 0 otherwise. We can then fit the model
\[
y=\beta_{0}+\beta_{11} a_{1}+\beta_{12} a_{2}+\beta_{13} a_{3}+X \beta_{2}
\]

The effect of being in age group 1 is now \(\beta_{11} ; 2, \beta_{12}\); and \(3, \beta_{13}\); and these results are independent of our (arbitrary) coding. The only difficulty at this point is that the model is unidentified in the sense that there are an infinite number of \(\left(\beta_{0}, \beta_{11}, \beta_{12}, \beta_{13}\right)\) that fit the data equally well.

To see this, pretend that \(\left(\beta_{0}, \beta_{11}, \beta_{12}, \beta_{13}\right)=(1,1,3,4)\). The predicted values of \(y\) for the various age groups are
\[
y= \begin{cases}1+1+X \beta_{2}=2+X \beta_{2} & (\text { age group } 1) \\ 1+3+X \beta_{2}=4+X \beta_{2} & (\text { age group } 2) \\ 1+4+X \beta_{2}=5+X \beta_{2} & (\text { age group } 3)\end{cases}
\]

Now pretend that \(\left(\beta_{0}, \beta_{11}, \beta_{12}, \beta_{13}\right)=(2,0,2,3)\). The predicted values of \(y\) are
\[
y= \begin{cases}2+0+X \beta_{2}=2+X \beta_{2} & (\text { age group 1) } \\ 2+2+X \beta_{2}=4+X \beta_{2} & (\text { age group } 2) \\ 2+3+X \beta_{2}=5+X \beta_{2} & (\text { age group } 3)\end{cases}
\]

These two sets of predictions are indistinguishable: for age group \(1, y=2+X \beta_{2}\) regardless of the coefficient vector used, and similarly for age groups 2 and 3 . This arises because we have three equations and four unknowns. Any solution is as good as any other, and, for our purposes, we merely need to choose one of them. The popular selection method is to set the coefficient on the first indicator variable to 0 (as we have done in our second coefficient vector). This is equivalent to fitting the model
\[
y=\beta_{0}+\beta_{12} a_{2}+\beta_{13} a_{3}+X \beta_{2}
\]

How we select a particular coefficient vector (identifies the model) does not matter. It does, however, affect the interpretation of the coefficients.

For instance, we could just as well choose to omit the second group. In our artificial example, this would yield \(\left(\beta_{0}, \beta_{11}, \beta_{12}, \beta_{13}\right)=(4,-2,0,1)\) instead of \((2,0,2,3)\). These coefficient vectors are the same in the sense that
\[
y= \begin{cases}2+0+X \beta_{2}=2+X \beta_{2}=4-2+X \beta_{2} & \text { (age group 1) } \\ 2+2+X \beta_{2}=4+X \beta_{2}=4+0+X \beta_{2} & \text { (age group 2) } \\ 2+3+X \beta_{2}=5+X \beta_{2}=4+1+X \beta_{2} & \text { (age group 3) }\end{cases}
\]

But what does it mean that \(\beta_{13}\) can just as well be 3 or 1 ? We obtain \(\beta_{13}=3\) when we set \(\beta_{11}=0\), so \(\beta_{13}=\beta_{13}-\beta_{11}\) and \(\beta_{13}\) measures the difference between age groups 3 and 1 .

In the second case, we obtain \(\beta_{13}=1\) when we set \(\beta_{12}=0\), so \(\beta_{13}-\beta_{12}=1\) and \(\beta_{13}\) measures the difference between age groups 3 and 2 . There is no inconsistency. According to our \(\beta_{12}=0\) model, the difference between age groups 3 and 1 is \(\beta_{13}-\beta_{11}=1-(-2)=3\), the same result we got in the \(\beta_{11}=0\) model.

\section*{> Example 2}

The issue of interpretation is important because it can affect the way we discuss results. Imagine that we are studying recovery after a coronary bypass operation. Assume that the age groups are children under 13 (we have two of them), young adults under 25 (we have a handful of them), adults under 46 (of which we have even more), mature adults under 56 , older adults under 65 , and elderly adults. We follow the prescription of omitting the first group, so all our results are reported relative to children under 13. While there is nothing statistically wrong with this, readers will be suspicious when we make statements like "compared with young children, older and elder adults . . .". Moreover, we will probably have to end each statement with "although results are not statistically significant" because we have only two children in our comparison group. Of course, even with results reported in this way, we can do reasonable comparisons (say, with mature adults), but we will have to do extra work to perform the appropriate linear hypothesis test using Stata's test command.

Here it would be better to force the omitted group to be more reasonable, such as mature adults. There is, however, a generic rule for automatic comparison group selection that, although less popular, tends to work better than the omit-the-first-group rule. That rule is to omit the most prevalent group. The most prevalent is usually a reasonable baseline.

In any case, the prescription for categorical variables is
1. Convert each \(k\)-valued categorical variable to \(k\) indicator variables.
2. Drop one of the \(k\) indicator variables; any one will do, but dropping the first is popular, dropping the most prevalent is probably better in terms of having the computer guess at a reasonable interpretation, and dropping a specified one often eases interpretation the most.
3. Fit the model on the remaining \(k-1\) indicator variables.
xi automates this procedure.
We will now consider each of xi's features in detail.

\section*{Indicator variables for simple effects}

When you type i.varname, xi internally tabulates varname (which may be a string or a numeric variable) and creates indicator (dummy) variables for each observed value, omitting the indicator for the smallest value. For instance, say that agegrp takes on the values 1, 2, 3, and 4. Typing
```

xi: logistic outcome i.agegrp

```
creates indicator variables named _Iagegrp_2, _Iagegrp_3, and _Iagegrp_4. (xi chooses the names and tries to make them readable; xi guarantees that the names are unique.) The expanded logistic model is
```

. logistic outcome _Iagegrp_2 _Iagegrp_3 _Iagegrp_4

```

Afterward, you can drop the new variables xi leaves behind by typing 'drop _I*' (note the capitalization).
xi provides the following features when you type i.varname:
- varname may be string or numeric.
- Dummy variables are created automatically.
- By default, the dummy-variable set is identified by dropping the dummy corresponding to the smallest value of the variable (how to specify otherwise is discussed below).
- The new dummy variables are left in your dataset. By default, the names of the new dummy variables start with _I; therefore, you can drop them by typing 'drop _I*'. You do not have to do this; each time you use xi, any automatically generated dummies with the same prefix as the one specified in the prefix(string) option, or _I by default, are dropped and new ones are created.
- The new dummy variables have variable labels so that you can determine what they correspond to by typing 'describe'.
- xi may be used with any Stata command (not just logistic).

\section*{Controlling the omitted dummy}

By default, i.varname omits the dummy corresponding to the smallest value of varname; for a string variable, this is interpreted as dropping the first in an alphabetical, case-sensitive sort. xi provides two alternatives to dropping the first: xi will drop the dummy corresponding to the most prevalent value of varname, or xi will let you choose the particular dummy to be dropped.

To change xi's behavior to dropping the most prevalent dummy, type
```

. char _dta[omit] prevalent

```
although whether you type "prevalent" or "yes" or anything else does not matter. Setting this characteristic affects the expansion of all categorical variables in the dataset. If you resave your dataset, the prevalent preference will be remembered. If you want to change the behavior back to the default drop-the-first rule, type
```

. char _dta[omit]

```
to clear the characteristic.
Once you set _dta[omit], i.varname omits the dummy corresponding to the most prevalent value of varname. Thus the coefficients on the dummies have the interpretation of change from the most prevalent group. For example,
. char _dta[omit] prevalent
. xi: regress y i.agegrp
might create _Iagegrp_1 through _Iagegrp_4, resulting in _Iagegrp_2 being omitted if agegrp \(=\) 2 is most common (as opposed to the default dropping of _Iagegrp_1). The model is then
\[
\mathrm{y}=b_{0}+b_{1} \text { _Iagegrp_1 }+b_{3} \text { _Iagegrp_3 }+b_{4} \text { _Iagegrp_ } 4+u
\]

Then
\[
\begin{array}{ll}
\text { Predicted y for agegrp } 1=b_{0}+b_{1} & \text { Predicted y for agegrp } 3=b_{0}+b_{3} \\
\text { Predicted y for agegrp } 2=b_{0} & \text { Predicted y for agegrp } 4=b_{0}+b_{4}
\end{array}
\]

Thus the model's reported \(t\) or \(Z\) statistics are for a test of whether each group is different from the most prevalent group.

Perhaps you wish to omit the dummy for agegrp 3 instead. You do this by setting the variable's omit characteristic:
. char agegrp[omit] 3
This overrides _dta[omit] if you have set it. Now when you type
. xi: regress y i.agegrp
_Iagegrp_3 will be omitted, and you will fit the model
\[
\mathrm{y}=b_{0}^{\prime}+b_{1}^{\prime} \text { _Iagegrp_1 }+b_{2}^{\prime} \text { _Iagegrp_2 }+b_{4}^{\prime} \text { _Iagegrp_ } 4+u
\]

Later if you want to return to the default omission, type
```

. char agegrp[omit]

```
to clear the characteristic.
In summary, i.varname omits the first group by default, but if you define
```

. char _dta[omit] prevalent

```
the default behavior changes to dropping the most prevalent group. Either way, if you define a characteristic of the form
. char varname[omit] \#
or, if varname is a string,
- char varname[omit] string-literal
the specified value will be omitted.
Examples: . char agegrp[omit] 1
. char race[omit] White (for race, a string variable)
. char agegrp[omit]

\section*{Categorical variable interactions}
i. varname \({ }_{1} *\) i. varname \(_{2}\) creates the dummy variables associated with the interaction of the categorical variables varname \(1_{1}\) and varname \(_{2}\). The identification rules - which categories are omitted -are the same as those for i.varname. For instance, assume that agegrp takes on four values and race takes on three values. Typing
```

. xi: regress y i.agegrp*i.race

```
results in
model:
\[
\begin{array}{lrl}
\text { model }: & & \text { dummies for: } \\
\begin{array}{rlrl}
y=a & +b_{2} \text { _Iagegrp_2 }+b_{3} \text { _Iagegrp_3 }+b_{4} \text { _Iagegrp_4 } & & \text { (agegrp) } \\
& +c_{2} \text { _Irace_2 }+c_{3} \text { _Irace_3 } & & \text { (race) } \\
& +d_{22} \text { _IageXrac_2_2 }+d_{23} \text {-IageXrac_2_3 } & \\
& +d_{32} \text { _IageXrac_3_2 }+d_{33} \text { _IageXrac_3_3 } & & \text { (agegrp*race) } \\
& +d_{42} \text { _IageXrac_4_2 }+d_{43} \text { _IageXrac_4_3 } & & \\
& +u & &
\end{array}
\end{array}
\]

That is, typing
```

. xi: regress y i.agegrp*i.race

```
is the same as typing
. xi: regress y i.agegrp i.race i.agegrp*i.race
Although there are many other ways the interaction could have been parameterized, this method has the advantage that you can test the joint significance of the interactions by typing
. testparm _IageXrac*
When you perform the estimation step, whether you specify i.agegrp*i.race or i.race*i.agegrp makes no difference (other than in the names given to the interaction terms; in the first case, the names will begin with _IageXrac; in the second, _IracXage). Thus
```

. xi: regress y i.race*i.agegrp

```
fits the same model.
You may also include multiple interactions simultaneously:
```

. xi: regress y i.agegrp*i.race i.agegrp*i.sex

```

The model fit is
model :
dummies for:
\[
\begin{array}{rlrl}
y=a & +b_{2} \text { _Iagegrp_2 }+b_{3} \text { _Iagegrp_3 }+b_{4} \text { _Iagegrp_4 } & & \text { (agegrp) } \\
& +c_{2} \text { _Irace_2 }+c_{3} \_ \text {Irace_3 } & & \text { (race) } \\
& +d_{22} \text {-IageXrac_2_2 }+d_{23} \text { _IageXrac_2_3 } & \\
& +d_{32} \text {-IageXrac_3_2 }+d_{33} \text {-IageXrac_3_3 } & & \text { (agegrp*race) } \\
& +d_{42} \text { _IageXrac_4_2 }+d_{43} \text { _IageXrac_4_3 } & \\
& +e_{2} \text {-Isex_2 } & & \text { (sex) } \\
& +f_{22} \text { _IageXsex_2_2 }+f_{23} \text { _IageXsex_2_3 }+f_{24} \text { _IageXsex_2_4 } & \text { (agegrp*sex) } \\
& +u & &
\end{array}
\]

The agegrp dummies are (correctly) included only once.

\section*{Interactions with continuous variables}
i.varname \({ }_{1} *\) varname \(_{2}\) (as distinguished from i.varname \({ }_{1} *\) i.varname \({ }_{2}\) —note the second i.) specifies an interaction of a categorical variable with a continuous variable. For instance,
. xi: regress y i.agegr*wgt
results in the model
\[
\begin{aligned}
y=a & +b_{2} \_ \text {Iagegrp_2 }+b_{3} \_ \text {Iagegrp_3 }+b_{4} \text { _Iagegrp_4 } & & \text { (agegrp dummies) } \\
& +c \text { wgt } & & \text { (continuous wgt effect) } \\
& +d_{2} \text { _IageXwgt_2 }+d_{3} \text { _IageXwgt_3 }+d_{4} \text { _IageXwgt_4 } & & \text { (agegrp*wgt interactions) } \\
& +u & &
\end{aligned}
\]

A variation on this notation, using | rather than *, omits the agegrp dummies. Typing . xi: regress y i.agegrplwgt
fits the model
\[
\begin{aligned}
y=a^{\prime} & +c^{\prime} \text { wgt } & & \text { (continuous wgt effect) } \\
& +d_{2}^{\prime} \_ \text {IageXwgt_2 }+d_{3}^{\prime} \_ \text {IageXwgt_3 }+d_{4}^{\prime} \_ \text {IageXwgt_4 } & & \text { (agegrp*wgt interactions) } \\
& +u^{\prime} & &
\end{aligned}
\]

The predicted values of \(y\) are
\begin{tabular}{cll} 
agegrp*wgt model & agegrplwgt model & \\
\hline\(y=a+c \mathrm{wgt}\) & \(a^{\prime}+c^{\prime} \mathrm{wgt}\) & if agegrp \(=1\) \\
\(a+c \mathrm{wgt}+b_{2}+d_{2} \mathrm{wgt}\) & \(a^{\prime}+c^{\prime} \mathrm{wgt}+d_{2}^{\prime} \mathrm{wgt}\) & if agegrp \(=2\) \\
\(a+c \mathrm{wgt}+b_{3}+d_{3} \mathrm{wgt}\) & \(a^{\prime}+c^{\prime} \mathrm{wgt}+d_{3}^{\prime} \mathrm{wgt}\) & if agegrp \(=3\) \\
\(a+c \mathrm{wgt}+b_{4}+d_{4} \mathrm{wgt}\) & \(a^{\prime}+c^{\prime}\) wgt \(+d_{4}^{\prime} \mathrm{wgt}\) & if agegrp \(=4\)
\end{tabular}

That is, typing
is equivalent to typing
```

. xi: regress y i.agegrp i.agegrplwgt

```

In either case, you do not need to specify separately the continuous variable wgt; it is included automatically.

\section*{Using xi: Interpreting output}
```

. xi: regress mpg i.rep78
i.rep78 _Irep78_1-5 (naturally coded; _Irep78_1 omitted)
(output from regress appears)

```

Interpretation: i.rep78 expanded to the dummies _Irep78_1,_Irep78_2, ...,_Irep78_5. The numbers on the end are "natural" in the sense that _Irep78_1 corresponds to rep78 = 1, _Irep78_2 to \(\operatorname{rep} 78=2\), and so on. Finally, the dummy for rep78 \(=1\) was omitted.
```

. xi: regress mpg i.make
i.make _Imake_1-74 (_Imake_1 for make==AMC Concord omitted)
(output from regress appears)

```

Interpretation: i.make expanded to _Imake_1, _Imake_2, ..., _Imake_74. The coding is not natural because make is a string variable. _Imake_1 corresponds to one make, _Imake_2 to another, and so on. You can find out the coding by typing describe. _Imake_1 for the AMC Concord was omitted.

\section*{How xi names variables}

By default, xi assigns to the dummy variables it creates names having the form
_Istub_groupid

You may subsequently refer to the entire set of variables by typing 'Istub*'. For example,
\begin{tabular}{lllll} 
name & \(=\) _I + stub & \(+\ldots+\) groupid & Entire set \\
\hline -Iagegrp_1 & _I & agegrp & -1 & -Iagegrp* \\
-Iagegrp_2 & -I agegrp & -2 & -Iagegrp* \\
-IageXwgt_1 & -I ageXwgt & -1 & -IageXwgt* \\
_IageXrac_1_2 & -I ageXrac & \(-1 \_2\) & -IageXrac* \\
_IageXrac_2_1 & _I ageXrac & \(-2 \_1\) & _IageXrac*
\end{tabular}

If you specify a prefix in the prefix(string) option, say, \(\quad\) S, then \(x i\) will name the variables starting with the prefix

\section*{xi as a command rather than a command prefix}
xi can be used as a command prefix or as a command by itself. In the latter form, xi merely creates the indicator and interaction variables. Typing
```

. xi: regress y i.agegrp*wgt
i.agegrp _Iagegrp_1-4 (naturally coded; _Iagegrp_1 omitted)
i.agegrp*wgt _IageXwgt_1-4 (coded as above)
(output from regress appears)

```
is equivalent to typing
```

. xi i.agegrp*wgt
i.agegrp _Iagegrp_1-4 (naturally coded; _Iagegrp_1 omitted)
i.agegrp*wgt _IageXwgt_1-4 (coded as above)
. regress y _Iagegrp* _IageXwgt*
(output from regress appears)

```

\section*{Warnings}
1. xi creates new variables in your dataset; most are bytes, but interactions with continuous variables will have the storage type of the underlying continuous variable. You may get the message "insufficient memory". If so, you will need to increase the amount of memory allocated to Stata's data areas; see [U] 6 Managing memory.
2. When using xi with an estimation command, you may get the message "matsize too small". If so, see \([R]\) matsize.

\section*{Stored results}
xi stores the following characteristics:
```

_dta[__xi__Vars___Prefix__] prefix names
_dta[__xi__Vars__To__Drop__] variables created

```

\section*{References}

Hendrickx, J. 1999. dm73: Using categorical variables in Stata. Stata Technical Bulletin 52: 2-8. Reprinted in Stata Technical Bulletin Reprints, vol. 9, pp. 51-59. College Station, TX: Stata Press.
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\section*{Also see}
[U] 11.1.10 Prefix commands
[U] 20 Estimation and postestimation commands

\section*{Title}
zinb - Zero-inflated negative binomial regression
\begin{tabular}{llll} 
Syntax & Menu & Description & Options \\
Remarks and examples & Stored results & Methods and formulas & References
\end{tabular}

\section*{Syntax}
\[
\begin{aligned}
& \text { zinb depvar }[\text { indepvars }][\text { if }][\text { in }][\text { weight }], \\
& \text { inflate (varlist }[, \underline{\text { offset }(\text { varname })] \mid \text { _cons) }[\text { options }]}
\end{aligned}
\]

\section*{options}

Model
*inflate ()
noconstant
exposure ( varname \(_{e}\) )
offset ( varname \(_{o}\) )
constraints (constraints)
collinear
probit
Description

SE/Robust
vce (vcetype)

Reporting
level(\#)
irr
vuong
zip
nocnsreport
display_options

\section*{Maximization}
maximize_options
coeflegend or jackknife
equation that determines whether the count is zero suppress constant term
include \(\ln \left(\right.\) varname \(\left._{e}\right)\) in model with coefficient constrained to 1 include varname \(_{o}\) in model with coefficient constrained to 1 apply specified linear constraints
keep collinear variables
use probit model to characterize excess zeros; default is logit
vcetype may be oim, robust, cluster clustvar, opg, bootstrap,
set confidence level; default is level (95)
report incidence-rate ratios
perform Vuong test
perform ZIP likelihood-ratio test
do not display constraints
control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
\({ }^{*}\) inflate(varlist \([\), offset(varname) \(] \mid\) _cons) is required.
indepvars and varlist may contain factor variables; see [U] 11.4.3 Factor variables.
bootstrap, by, fp, jackknife, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the bootstrap prefix; see [R] bootstrap.
vce(), vuong, zip, and weights are not allowed with the svy prefix; see [SVY] svy.
fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

\section*{Menu}

Statistics \(>\) Count outcomes \(>\) Zero-inflated negative binomial regression

\section*{Description}
zinb estimates a zero-inflated negative binomial (ZINB) regression of depvar on indepvars, where depvar is a nonnegative count variable.

\section*{Options}

Model
inflate (varlist [, offset(varname) \(] \mid\) _cons) specifies the equation that determines whether the observed count is zero. Conceptually, omitting inflate() would be equivalent to fitting the model with nbreg.
inflate (varlist [, offset (varname) \(]\) ) specifies the variables in the equation. You may optionally include an offset for this varlist.
inflate (_cons) specifies that the equation determining whether the count is zero contains only an intercept. To run a zero-inflated model of depvar with only an intercept in both equations, type zinb depvar, inflate(_cons).
noconstant, exposure (varname \()\), offset (varname \(e_{o}\) ), constraints (constraints), collinear; see \([R]\) estimation options.
probit requests that a probit, instead of logit, model be used to characterize the excess zeros in the data.

SE/Robust
vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

\section*{Reporting}
level(\#); see [R] estimation options.
irr reports estimated coefficients transformed to incidence-rate ratios, that is, \(e^{\beta_{i}}\) rather than \(\beta_{i}\). Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated or stored. irr may be specified at estimation or when replaying previously estimated results.
vuong specifies that the Vuong (1989) test of ZINB versus negative binomial be reported. This test statistic has a standard normal distribution with large positive values favoring the ZINB model and large negative values favoring the negative binomial model.
zip requests that a likelihood-ratio test comparing the ZINB model with the zero-inflated Poisson model be included in the output.
nocnsreport; see \([\mathrm{R}]\) estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% \(f m t\) ), pformat (\% fmt), sformat (\%fmt), and nolstretch; see [R] estimation options.

Maximization
maximize_options: difficult, technique (algorithm_spec), iterate(\#), [no \(]\) log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance (\#), nonrtolerance, and from(init_specs); see [R] maximize. These options are seldom used.
Setting the optimization type to technique (bhhh) resets the default vcetype to vce (opg).
The following option is available with zinb but is not shown in the dialog box:
coeflegend; see [R] estimation options.

\section*{Remarks and examples}

See Long \((1997,242-247)\) and Greene \((2012,821-826)\) for a discussion of zero-modified count models. For information about the test developed by Vuong (1989), see Greene \((2012,823-824)\) and Long (1997). Greene (1994) applied the test to zero-inflated Poisson and negative binomial models, and there is a description of that work in Greene (2012).

Negative binomial regression fits models of the number of occurrences (counts) of an event. You could use nbreg for this (see [R] nbreg), but in some count-data models, you might want to account for the prevalence of zero counts in the data.

For instance, you could count how many fish each visitor to a park catches. Many visitors may catch zero, because they do not fish (as opposed to being unsuccessful). You may be able to model whether a person fishes depending on several covariates related to fishing activity and model how many fish a person catches depending on several covariates having to do with the success of catching fish (type of lure/bait, time of day, temperature, season, etc.). This is the type of data for which the zinb command is useful.

The zero-inflated (or zero-altered) negative binomial model allows overdispersion through the splitting process that models the outcomes as zero or nonzero.
\(>\) Example 1
We have data on the number of fish caught by visitors to a national park. Some of the visitors do not fish, but we do not have the data on whether a person fished; we have data merely on how many fish were caught, together with several covariates. Because our data have a preponderance of zeros ( 142 of 250 ), we use the zinb command to model the outcome.
. use http://www.stata-press.com/data/r13/fish
. zinb count persons livebait, inf(child camper) vuong
Fitting constant-only model:
Iteration 0: log likelihood \(=-519.33992\)
(output omitted)
Iteration 8: log likelihood \(=-442.66299\)
Fitting full model:
Iteration 0: log likelihood \(=-442.66299\) (not concave) (output omitted)
Iteration 8: log likelihood \(=-401.54776\)
\begin{tabular}{lllr} 
Zero-inflated negative binomial regression & Number of obs & \(=\) & 250 \\
& Nonzero obs & \(=\) & 108 \\
& Zero obs & \(=\) & 142 \\
Inflation model = logit & LR chi2 (2) & \(=\) & 82.23 \\
Log likelihood \(=-401.5478\) & Prob \(>\) chi2 & \(=\) & 0.0000
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline count & Coef. & Std. Err & z & \(P>|z|\) & [95\% Conf & Interval] \\
\hline \multicolumn{7}{|l|}{count} \\
\hline persons & . 9742984 & . 1034938 & 9.41 & 0.000 & . 7714543 & 1.177142 \\
\hline livebait & 1.557523 & . 4124424 & 3.78 & 0.000 & . 7491503 & 2.365895 \\
\hline _cons & -2.730064 & . 476953 & -5.72 & 0.000 & -3.664874 & -1.795253 \\
\hline \multicolumn{7}{|l|}{inflate} \\
\hline child & 3.185999 & . 7468551 & 4.27 & 0.000 & 1.72219 & 4.649808 \\
\hline camper & -2.020951 & . 872054 & -2.32 & 0.020 & -3.730146 & -. 3117567 \\
\hline _cons & -2.695385 & . 8929071 & -3.02 & 0.003 & -4.44545 & -. 9453189 \\
\hline /lnalpha & . 5110429 & . 1816816 & 2.81 & 0.005 & . 1549535 & . 8671323 \\
\hline alpha & 1.667029 & . 3028685 & & & 1.167604 & 2.380076 \\
\hline
\end{tabular}

In general, Vuong test statistics that are significantly positive favor the zero-inflated models, whereas those that are significantly negative favor the non-zero-inflated models. Thus, in the above model, the zero inflation is significant.

\section*{Stored results}
zinb stores the following in e():
Scalars
```

e(N)
e(N_zero)
e(k)
e(k_eq)
e(k_eq_model)
e(k_aux)
e(k_dv)
e(df_m)
e(11)
e(ll_0)
e(df_c)
e(N_clust)
e(chi2)
e(p)
e(chi2_cp)
e(vuong)
e(rank)
e(ic)
e(rc)
e(converged)

```

Macros
e(cmd)
e(cmdline)
e(depvar)
\(e\) (inflate)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(offset1)
e(offset2)
e(chi2type)
e(chi2_cpt)
e(vce)
e(vcetype)
e(opt)
\(e\) (which)
\(e\left(m l \_\right.\)method)
e(user)
e(technique)
e(properties)
e(predict)
e(asbalanced)
e(asobserved)

Matrices
```

e(b)
e(Cns)
e(ilog)
e(gradient)
e(V)
e(V_modelbased)

```

Functions
e(sample)
number of observations
number of zero observations
number of parameters
number of equations in \(e(b)\)
number of equations in overall model test
number of auxiliary parameters
number of dependent variables
model degrees of freedom
log likelihood
log likelihood, constant-only model
degrees of freedom for comparison test
number of clusters
\(\chi^{2}\)
significance of model test
\(\chi^{2}\) for test of \(\alpha=0\)
Vuong test statistic
rank of e(V)
number of iterations
return code
1 if converged, 0 otherwise
zinb
command as typed
name of dependent variable
logit or probit
weight type
weight expression
title in estimation output
name of cluster variable
offset
offset for inflate()
Wald or LR; type of model \(\chi^{2}\) test
Wald or LR; type of model \(\chi^{2}\) test corresponding to e(chi2_cp)
vcetype specified in vce()
title used to label Std. Err.
type of optimization
max or min; whether optimizer is to perform maximization or minimization
type of ml method
name of likelihood-evaluator program
maximization technique
b V
program used to implement predict
factor variables fvset as asbalanced
factor variables fvset as asobserved
coefficient vector
constraints matrix
iteration \(\log\) (up to 20 iterations)
gradient vector
variance-covariance matrix of the estimators
model-based variance
marks estimation sample

\section*{Methods and formulas}

Several models in the literature are (correctly) described as zero inflated. The zinb command maximizes the \(\log\) likelihood \(\ln L\), defined by
\[
\begin{aligned}
m & =1 / \alpha \\
p_{j} & =1 /\left(1+\alpha \mu_{j}\right) \\
\xi_{j}^{\beta} & =\mathbf{x}_{j} \boldsymbol{\beta}+\operatorname{offset}_{j}^{\beta} \\
\xi_{j}^{\gamma} & =\mathbf{z}_{j} \gamma+\operatorname{offset}_{j}^{\gamma} \\
\mu_{j} & =\exp \left(\xi_{j}^{\beta}\right) \\
\ln L & =\sum_{j \in S} w_{j} \ln \left[F\left(\xi_{j}^{\gamma}\right)+\left\{1-F\left(\xi_{j}^{\gamma}\right)\right\} p_{j}^{m}\right] \\
& \quad+\sum_{j \notin S} w_{j}\left[\ln \left\{1-F\left(\xi_{j}^{\gamma}\right)\right\}+\ln \Gamma\left(m+y_{j}\right)-\ln \Gamma\left(y_{j}+1\right)\right. \\
& \left.\quad-\ln \Gamma(m)+m \ln p_{j}+y_{j} \ln \left(1-p_{j}\right)\right]
\end{aligned}
\]
where \(w_{j}\) are the weights, \(F\) is the inverse of the logit link (or the inverse of the probit link if probit was specified), and \(S\) is the set of observations for which the outcome \(y_{j}=0\).

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce(robust) and vce(cluster clustvar), respectively. See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas.
zinb also supports estimation with survey data. For details on VCEs with survey data, see [SVY] variance estimation.

\section*{References}

Desmarais, B. A., and J. J. Harden. 2013. Testing for zero inflation in count models: Bias correction for the Vuong test. Stata Journal 13: 810-835.

Greene, W. H. 1994. Accounting for excess zeros and sample selection in Poisson and negative binomial regression models. Working paper EC-94-10, Department of Economics, Stern School of Business, New York University. http://ideas.repec.org/p/ste/nystbu/94-10.html.
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Long, J. S. 1997. Regression Models for Categorical and Limited Dependent Variables. Thousand Oaks, CA: Sage.
Long, J. S., and J. Freese. 2001. Predicted probabilities for count models. Stata Journal 1: 51-57.
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Mullahy, J. 1986. Specification and testing of some modified count data models. Journal of Econometrics 33: 341-365.
Vuong, Q. H. 1989. Likelihood ratio tests for model selection and non-nested hypotheses. Econometrica 57: 307-333.

\section*{Also see}
[R] zinb postestimation - Postestimation tools for zinb
[R] zip — Zero-inflated Poisson regression
[R] nbreg - Negative binomial regression
[R] poisson - Poisson regression
[R] tnbreg - Truncated negative binomial regression
[R] tpoisson - Truncated Poisson regression
[SVY] svy estimation - Estimation commands for survey data
[XT] xtnbreg - Fixed-effects, random-effects, \& population-averaged negative binomial models
[U] 20 Estimation and postestimation commands

\section*{Title}
zinb postestimation - Postestimation tools for zinb
\begin{tabular}{llll} 
Description & Syntax for predict & Menu for predict & Options for predict \\
Methods and formulas & Reference & Also see &
\end{tabular}

\section*{Description}

The following postestimation commands are available after zinb:
Command Description
\begin{tabular}{|c|c|}
\hline contrast & contrasts and ANOVA-style joint tests of estimates \\
\hline estat ic & Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) \\
\hline estat summarize & summary statistics for the estimation sample \\
\hline estat vce & variance-covariance matrix of the estimators (VCE) \\
\hline estat (svy) & postestimation statistics for survey data \\
\hline estimates & cataloging estimation results \\
\hline forecast \({ }^{1}\) & dynamic forecasts and simulations \\
\hline lincom & point estimates, standard errors, testing, and inference for linear combinations of coefficients \\
\hline lrtest \({ }^{2}\) & likelihood-ratio test \\
\hline margins & marginal means, predictive margins, marginal effects, and average marginal effects \\
\hline marginsplot & graph the results from margins (profile plots, interaction plots, etc.) \\
\hline nlcom & point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients \\
\hline predict & predictions, residuals, influence statistics, and other diagnostic measures \\
\hline predictnl & point estimates, standard errors, testing, and inference for generalized predictions \\
\hline pwcompare & pairwise comparisons of estimates \\
\hline suest & seemingly unrelated estimation \\
\hline test & Wald tests of simple and composite linear hypotheses \\
\hline testnl & Wald tests of nonlinear hypotheses \\
\hline
\end{tabular}

\footnotetext{
1 forecast is not appropriate with svy estimation results.
2 lrtest is not appropriate with svy estimation results.
}

\section*{Syntax for predict}
```

predict [type] newvar [if] [in] [, statistic nooffset]
predict [type] {stub*| newvar reg newvarinflate newvar lnalpha } [if] [in], scores

```
statistic
Main
n
ir
pr
\(\operatorname{pr}(n)\)
\(\operatorname{pr}(a, b)\)
xb
stdp

Description
number of events; the default
incidence rate
probability of a degenerate zero
probability \(\operatorname{Pr}\left(y_{j}=n\right)\)
probability \(\operatorname{Pr}\left(a \leq y_{j} \leq b\right)\)
linear prediction
standard error of the linear prediction

These statistics are available both in and out of sample; type predict ... if e(sample)... if wanted only for the estimation sample.

\section*{Menu for predict}

Statistics \(>\) Postestimation \(>\) Predictions, residuals, etc.

\section*{Options for predict}
\(\qquad\) Main
n , the default, calculates the predicted number of events, which is \(\left(1-p_{j}\right) \exp \left(\mathbf{x}_{j} \boldsymbol{\beta}\right)\) if neither offset() nor exposure() was specified when the model was fit, where \(p_{j}\) is the predicted probability of a zero outcome; \(\left(1-p_{j}\right) \exp \left\{\left(\mathbf{x}_{j} \boldsymbol{\beta}\right)+\operatorname{offset}_{j}\right\}\) if offset() was specified; or \(\left(1-p_{j}\right)\left\{\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}\right) \times \operatorname{exposure}_{j}\right\}\) if exposure() was specified.
ir calculates the incidence rate \(\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}\right)\), which is the predicted number of events when exposure is 1 . This is equivalent to specifying both the n and the nooffset options.
pr calculates the probability \(\operatorname{Pr}\left(y_{j}=0\right)\), where this zero was obtained from the degenerate distribution \(F\left(\mathbf{z}_{j} \gamma\right)\). If offset() was specified within the inflate() option, then \(F\left(\mathbf{z}_{j} \gamma+\operatorname{offset}_{j}^{\gamma}\right)\) is calculated.
\(\operatorname{pr}(n)\) calculates the probability \(\operatorname{Pr}\left(y_{j}=n\right)\), where \(n\) is a nonnegative integer that may be specified as a number or a variable. Note that pr is not equivalent to \(\mathrm{pr}(0)\).
\(\operatorname{pr}(a, b)\) calculates the probability \(\operatorname{Pr}\left(a \leq y_{j} \leq b\right)\), where \(a\) and \(b\) are nonnegative integers that may be specified as numbers or variables;
\(b\) missing ( \(b \geq\).) means \(+\infty\);
\(\operatorname{pr}(20,\).\() calculates \operatorname{Pr}\left(y_{j} \geq 20\right)\);
\(\operatorname{pr}(20, b)\) calculates \(\operatorname{Pr}\left(y_{j} \geq 20\right)\) in observations for which \(b \geq\). and calculates \(\operatorname{Pr}\left(20 \leq y_{j} \leq b\right)\) elsewhere.
\(\operatorname{pr}(., b)\) produces a syntax error. A missing value in an observation of the variable \(a\) causes a missing value in that observation for \(\mathrm{pr}(a, b)\).
xb calculates the linear prediction, which is \(\mathbf{x}_{j} \boldsymbol{\beta}\) if neither offset () nor exposure() was specified; \(\mathbf{x}_{j} \boldsymbol{\beta}+\) offset \(_{j}\) if offset () was specified; or \(\mathbf{x}_{j} \boldsymbol{\beta}+\ln\) (exposure \(_{j}\) ) if exposure() was specified; see nooffset below.
stdp calculates the standard error of the linear prediction.
nooffset is relevant only if you specified offset() or exposure() when you fit the model. It modifies the calculations made by predict so that they ignore the offset or exposure variable; the linear prediction is treated as \(\mathbf{x}_{j} \boldsymbol{\beta}\) rather than as \(\mathbf{x}_{j} \boldsymbol{\beta}+\) offset \(_{j}\) or \(\mathbf{x}_{j} \boldsymbol{\beta}+\ln\) (exposure \(\left.{ }_{j}\right)\). Specifying predict..., nooffset is equivalent to specifying predict..., ir.
scores calculates equation-level score variables.
The first new variable will contain \(\partial \ln L / \partial\left(\mathbf{x}_{j} \boldsymbol{\beta}\right)\).
The second new variable will contain \(\partial \ln L / \partial\left(\mathbf{z}_{j} \gamma\right)\).
The third new variable will contain \(\partial \ln L / \partial \ln \alpha\).

\section*{Methods and formulas}

The probabilities calculated using the \(\operatorname{pr}(n)\) option are the probability \(\operatorname{Pr}\left(y_{i}=n\right)\). These are calculated using
\[
\begin{array}{ll}
\operatorname{Pr}\left(0 \mid \mathbf{x}_{i}\right)=\omega_{i}+\left(1-\omega_{i}\right) p_{2}\left(0 \mid \mathbf{x}_{i}\right) \\
\operatorname{Pr}\left(n \mid \mathbf{x}_{i}\right)=\left(1-\omega_{i}\right) p_{2}\left(n \mid \mathbf{x}_{i}\right) \quad \text { for } n=1,2, \ldots
\end{array}
\]
where \(\omega_{i}\) is the probability of obtaining an observation from the degenerate distribution whose mass is concentrated at zero, and \(p_{2}\left(n \mid \mathbf{x}_{i}\right)\) is the probability of \(y_{i}=n\) from the nondegenerate, negative binomial distribution. \(\omega_{i}\) can be obtained from the pr option.

See Cameron and Trivedi (2013, sec. 4.6) for further details.

\section*{Reference}

Cameron, A. C., and P. K. Trivedi. 2013. Regression Analysis of Count Data. 2nd ed. New York: Cambridge University Press.

\section*{Also see}
[R] zinb - Zero-inflated negative binomial regression
[U] 20 Estimation and postestimation commands

\section*{Title}
zip — Zero-inflated Poisson regression
\begin{tabular}{llll} 
Syntax & Menu & Description & Options \\
Remarks and examples & Stored results & Methods and formulas & References
\end{tabular}

\section*{Syntax}
```

zip depvar [indepvars] [if] [in] [weight],

```
    inflate(varlist[, offset(varname)]|-cons) [options]
options
Model
*inflate() noconstant exposure (varname \({ }_{e}\) ) offset ( varname \(_{o}\) ) constraints (constraints)
collinear
probit
SE/Robust
vce (vcetype)

Reporting
level(\#)
irr
vuong
nocnsreport
display_options

Maximization
maximize_options
coeflegend
equation that determines whether the count is zero suppress constant term include \(\ln \left(\right.\) varname \(\left._{e}\right)\) in model with coefficient constrained to 1 include varname \(_{o}\) in model with coefficient constrained to 1 apply specified linear constraints
keep collinear variables use probit model to characterize excess zeros; default is logit
vcetype may be oim, robust, cluster clustvar, opg, bootstrap, or jackknife
\({ }^{*}\) inflate(varlist \([\), offset(varname) \(] \mid\) _cons) is required.
indepvars and varlist may contain factor variables; see [U] 11.4.3 Factor variables.
bootstrap, by, fp, jackknife, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the bootstrap prefix; see \([R]\) bootstrap.
vce(), vuong, and weights are not allowed with the svy prefix; see [SVY] svy.
fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

\section*{Menu}

Statistics \(>\) Count outcomes \(>\) Zero-inflated Poisson regression

\section*{Description}
zip estimates a zero-inflated Poisson (ZIP) regression of depvar on indepvars, where depvar is a nonnegative count variable.

\section*{Options}

inflate(varlist [, offset(varname)]|_cons) specifies the equation that determines whether the observed count is zero. Conceptually, omitting inflate() would be equivalent to fitting the model with poisson; see [R] poisson.
inflate (varlist [, offset (varname) \(]\) ) specifies the variables in the equation. You may optionally include an offset for this varlist.
inflate (_cons) specifies that the equation determining whether the count is zero contains only an intercept. To run a zero-inflated model of depvar with only an intercept in both equations, type zip depvar, inflate(_cons).
noconstant, exposure (varname \({ }_{e}\) ), offset (varname \({ }_{o}\) ), constraints (constraints), collinear; see [R] estimation options.
probit requests that a probit, instead of logit, model be used to characterize the excess zeros in the data.

SE/Robust
vce (vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

Reporting
level (\#); see [R] estimation options.
irr reports estimated coefficients transformed to incidence-rate ratios, that is, \(e^{b}\) rather than \(b\). Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated or stored. irr may be specified at estimation or when replaying previously estimated results.
vuong specifies that the Vuong (1989) test of ZIP versus Poisson be reported. This test statistic has a standard normal distribution with large positive values favoring the ZIP model and large negative values favoring the Poisson model.
nocnsreport; see \([\mathrm{R}]\) estimation options.
display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(\#), fvwrapon(style), cformat (\% fmt), pformat (\% fmt), sformat (\% fmt), and nolstretch; see [R] estimation options.

Maximization
maximize_options: difficult, technique (algorithm_spec), iterate(\#), [no log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance (\#), nonrtolerance, and from(init_specs); see [R] maximize. These options are seldom used.
Setting the optimization type to technique (bhhh) resets the default vcetype to vce (opg).
The following option is available with zip but is not shown in the dialog box:
coeflegend; see [R] estimation options.

\section*{Remarks and examples}

See Long \((1997,242-247)\) and Greene \((2012,821-826)\) for a discussion of zero-modified count models. For information about the test developed by Vuong (1989), see Greene \((2012,823-824)\) and Long (1997). Greene (1994) applied the test to ZIP and ZINB models, as described in Greene (2012, 824).

Poisson regression fits models of the number of occurrences (counts) of an event. You could use poisson for this (see [R] poisson), but in some count-data models, you might want to account for the prevalence of zero counts in the data.

For instance, you might count how many fish each visitor to a park catches. Many visitors may catch zero, because they do not fish (as opposed to being unsuccessful). You may be able to model whether a person fishes depending on several covariates related to fishing activity and model how many fish a person catches depending on several covariates having to do with the success of catching fish (type of lure/bait, time of day, temperature, season, etc.). This is the type of data for which the zip command is useful.

The zero-inflated (or zero-altered) Poisson model allows overdispersion through the splitting process that models the outcomes as zero or nonzero.

\section*{> Example 1}

We have data on the number of fish caught by visitors to a national park. Some of the visitors do not fish, but we do not have the data on whether a person fished; we merely have data on how many fish were caught together with several covariates. Because our data have a preponderance of zeros (142 of 250), we use the zip command to model the outcome.
. use http://www.stata-press.com/data/r13/fish
. zip count persons livebait, inf(child camper) vuong
Fitting constant-only model:
Iteration 0: log likelihood \(=-1347.807\) (output omitted)
Iteration 4: log likelihood \(=-1103.9425\)
Fitting full model:
Iteration 0: log likelihood \(=-1103.9425\) (output omitted)
Iteration 5: log likelihood \(=-850.70142\)
\begin{tabular}{lllr} 
Zero-inflated Poisson regression & Number of obs & \(=\) & 250 \\
& Nonzero obs & \(=\) & 108 \\
& Zero obs & \(=\) & 142 \\
Inflation model \(=\) logit & LR chi2 (2) & \(=\) & 506.48 \\
Log likelihood \(=-850.7014\) & Prob > chi2 & \(=\) & 0.0000
\end{tabular}
\begin{tabular}{r|rrrrrrr}
\hline count & Coef. & Std. Err. & z & P>|z| & [95\% Conf. Interval] \\
\hline count & & & & & & \\
persons & .8068853 & .0453288 & 17.80 & 0.000 & .7180424 & .8957281 \\
livebait & 1.757289 & .2446082 & 7.18 & 0.000 & 1.277866 & 2.236713 \\
_cons & -2.178472 & .2860289 & -7.62 & 0.000 & -2.739078 & -1.617865 \\
\hline inflate & & & & & & \\
child & 1.602571 & .2797719 & 5.73 & 0.000 & 1.054228 & 2.150913 \\
camper & -1.015698 & .365259 & -2.78 & 0.005 & -1.731593 & -.2998038 \\
_cons & -.4922872 & .3114562 & -1.58 & 0.114 & -1.10273 & .1181558 \\
\hline Vuong test of zip vs. standard Poisson: & & \(z=\) & 3.95 & \(\operatorname{Pr}>z=0.0000\)
\end{tabular}

In general, Vuong test statistics that are significantly positive favor the zero-inflated models, while those that are significantly negative favor the non-zero-inflated models. Thus, in the above model, the zero inflation is significant.

\section*{Stored results}
zip stores the following in e():
Scalars
```

e(N)
e(N_zero)
e(k)
e(k_eq)
e(k_eq_model)
e(k_dv)
e(df_m)
e(ll)
e(ll_0)
e(ll_c)
e(df_c)
e(N_clust)
e(chi2)
e(p)
e(vuong)
e(rank)
e(ic)
e(rc)
e(converged)

```

Macros
e(cmd)
e(cmdline)
e(depvar)
e(inflate)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(offset1)
e(offset2)
e(chi2type)
e(vce)
e(vcetype)
e(opt)
e(which)
e(ml_method)
e(user)
e(technique)
e(properties)
e(predict)
e(asbalanced)
e(asobserved)

Matrices
e(b)
e(Cns)
e(ilog)
e(gradient)
e(V)
e(V_modelbased)

Functions
```

e(sample)
marks estimation sample

```
number of observations
number of zero observations
number of parameters
number of equations in \(e(b)\)
number of equations in overall model test
number of dependent variables
model degrees of freedom
log likelihood
log likelihood, constant-only model
log likelihood, comparison model
degrees of freedom for comparison test
number of clusters
\(\chi^{2}\)
significance of model test
Vuong test statistic
rank of \(e(V)\)
number of iterations
return code
1 if converged, 0 otherwise
zip
command as typed
name of dependent variable
logit or probit
weight type
weight expression
title in estimation output
name of cluster variable
offset
offset for inflate()
Wald or LR; type of model \(\chi^{2}\) test
vcetype specified in vce()
title used to label Std. Err.
type of optimization
max or min; whether optimizer is to perform maximization or minimization
type of ml method
name of likelihood-evaluator program
maximization technique
b V
program used to implement predict
factor variables fvset as asbalanced
factor variables fvset as asobserved
coefficient vector
constraints matrix
iteration \(\log\) (up to 20 iterations)
gradient vector
variance-covariance matrix of the estimators
model-based variance

\section*{Methods and formulas}

Several models in the literature are (correctly) described as zero inflated. The zip command maximizes the \(\log\)-likelihood \(\ln L\), defined by
\[
\begin{aligned}
\xi_{j}^{\beta} & =\mathbf{x}_{j} \boldsymbol{\beta}+\operatorname{offset}_{j}^{\beta} \\
\xi_{j}^{\gamma} & =\mathbf{z}_{j} \gamma+\operatorname{offset}_{j}^{\gamma} \\
\ln L & =\sum_{j \in S} w_{j} \ln \left[F\left(\xi_{j}^{\gamma}\right)+\left\{1-F\left(\xi_{j}^{\gamma}\right)\right\} \exp \left(-\lambda_{j}\right)\right]+ \\
& \quad \sum_{j \notin S} w_{j}\left[\ln \left\{1-F\left(\xi_{j}^{\gamma}\right)\right\}-\lambda_{j}+\xi_{j}^{\beta} y_{j}-\ln \left(y_{j}!\right)\right]
\end{aligned}
\]
where \(w_{j}\) are the weights, \(F\) is the inverse of the logit link (or the inverse of the probit link if probit was specified), and \(S\) is the set of observations for which the outcome \(y_{j}=0\).

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce (robust) and vce(cluster clustvar), respectively. See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas.
zip also supports estimation with survey data. For details on VCEs with survey data, see [SVY] variance estimation.

\section*{References}

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Vuong, Q. H. 1989. Likelihood ratio tests for model selection and non-nested hypotheses. Econometrica 57: 307-333.

\section*{Also see}
[R] zip postestimation - Postestimation tools for zip
\([R]\) zinb - Zero-inflated negative binomial regression
[R] nbreg - Negative binomial regression
[R] poisson - Poisson regression
[R] tnbreg - Truncated negative binomial regression
[R] tpoisson - Truncated Poisson regression
[SVY] svy estimation - Estimation commands for survey data
[XT] xtpoisson - Fixed-effects, random-effects, and population-averaged Poisson models
[U] 20 Estimation and postestimation commands

\section*{Title}
zip postestimation - Postestimation tools for zip
\begin{tabular}{llll} 
Description & Syntax for predict & Menu for predict & Options for predict \\
Remarks and examples & Methods and formulas & Reference & Also see
\end{tabular}

\section*{Description}

The following postestimation commands are available after zip:
\begin{tabular}{ll} 
Command & Description \\
\hline contrast & contrasts and ANOVA-style joint tests of estimates \\
estat ic & Akaike's and Schwarz's Bayesian information criteria (AIC and BIC) \\
estat summarize & \begin{tabular}{l} 
summary statistics for the estimation sample \\
variance-covariance matrix of the estimators (VCE)
\end{tabular} \\
estat vce & \begin{tabular}{l} 
varian (svy) \\
postestimation statistics for survey data
\end{tabular} \\
estimates & \begin{tabular}{l} 
cataloging estimation results \\
dynamic forecasts and simulations
\end{tabular} \\
forecast \({ }^{1}\) & \begin{tabular}{l} 
point estimates, standard errors, testing, and inference for linear \\
lincom
\end{tabular} \\
combinations of coefficients
\end{tabular}

\footnotetext{
\({ }^{1}\) forecast is not appropriate with svy estimation results.
2 lrtest is not appropriate with svy estimation results.
}

\section*{Syntax for predict}
```

predict [type] newvar [if] [in] [, statistic nooffset]
predict [type] {stub* |ewvar reg newvar }\mp@subsup{\mp@code{minflate }}{}{\mathrm{ n }

```
statistic
Main
n
ir
pr
\(\operatorname{pr}(n)\)
\(\operatorname{pr}(a, b)\)
xb
stdp

Description
number of events; the default
incidence rate
probability of a degenerate zero
probability \(\operatorname{Pr}\left(y_{j}=n\right)\)
probability \(\operatorname{Pr}\left(a \leq y_{j} \leq b\right)\)
linear prediction
standard error of the linear prediction

These statistics are available both in and out of sample; type predict . . . if e (sample) ... if wanted only for the estimation sample.

\section*{Menu for predict}

Statistics \(>\) Postestimation \(>\) Predictions, residuals, etc.

\section*{Options for predict}
\(\qquad\) Main
n , the default, calculates the predicted number of events, which is \(\left(1-p_{j}\right) \exp \left(\mathbf{x}_{j} \boldsymbol{\beta}\right)\) if neither offset() nor exposure() was specified when the model was fit, where \(p_{j}\) is the predicted probability of a zero outcome; \(\left(1-p_{j}\right) \exp \left\{\left(\mathbf{x}_{j} \boldsymbol{\beta}\right)+\right.\) offset \(\left._{j}\right\}\) if offset() was specified; or \(\left(1-p_{j}\right)\left\{\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}\right) \times \operatorname{exposure}_{j}\right\}\) if exposure() was specified.
ir calculates the incidence rate \(\exp \left(\mathbf{x}_{j} \boldsymbol{\beta}\right)\), which is the predicted number of events when exposure is 1 . This is equivalent to specifying both the n and the nooffset options.
pr calculates the probability \(\operatorname{Pr}\left(y_{j}=0\right)\), where this zero was obtained from the degenerate distribution \(F\left(\mathbf{z}_{j} \gamma\right)\). If offset() was specified within the inflate() option, then \(F\left(\mathbf{z}_{j} \gamma+\operatorname{offset}_{j}^{\gamma}\right)\) is calculated.
\(\operatorname{pr}(n)\) calculates the probability \(\operatorname{Pr}\left(y_{j}=n\right)\), where \(n\) is a nonnegative integer that may be specified as a number or a variable. Note that pr is not equivalent to \(\mathrm{pr}(0)\).
\(\operatorname{pr}(a, b)\) calculates the probability \(\operatorname{Pr}\left(a \leq y_{j} \leq b\right)\), where \(a\) and \(b\) are nonnegative integers that may be specified as numbers or variables;
\(b\) missing ( \(b \geq\).) means \(+\infty\);
\(\operatorname{pr}(20,\).\() calculates \operatorname{Pr}\left(y_{j} \geq 20\right)\);
\(\operatorname{pr}(20, b)\) calculates \(\operatorname{Pr}\left(y_{j} \geq 20\right)\) in observations for which \(b \geq\). and calculates \(\operatorname{Pr}\left(20 \leq y_{j} \leq b\right)\) elsewhere.
\(\operatorname{pr}(., b)\) produces a syntax error. A missing value in an observation of the variable \(a\) causes a missing value in that observation for \(\operatorname{pr}(a, b)\).
xb calculates the linear prediction, which is \(\mathbf{x}_{j} \boldsymbol{\beta}\) if neither offset () nor exposure() was specified; \(\mathbf{x}_{j} \boldsymbol{\beta}+\) offset \(_{j}\) if offset () was specified; or \(\mathbf{x}_{j} \boldsymbol{\beta}+\ln\) (exposure \(_{j}\) ) if exposure() was specified; see nooffset below.
stdp calculates the standard error of the linear prediction.
nooffset is relevant only if you specified offset() or exposure() when you fit the model. It modifies the calculations made by predict so that they ignore the offset or exposure variable; the linear prediction is treated as \(\mathbf{x}_{j} \boldsymbol{\beta}\) rather than as \(\mathbf{x}_{j} \boldsymbol{\beta}+\) offset \(_{j}\) or \(\mathbf{x}_{j} \boldsymbol{\beta}+\ln\) (exposure \(\left.{ }_{j}\right)\). Specifying predict..., nooffset is equivalent to specifying predict..., ir.
scores calculates equation-level score variables.
The first new variable will contain \(\partial \ln L / \partial\left(\mathbf{x}_{j} \boldsymbol{\beta}\right)\).
The second new variable will contain \(\partial \ln L / \partial\left(\mathbf{z}_{j} \gamma\right)\).

\section*{Remarks and examples}

\section*{> Example 1}

Continuing with example 1 from \([\mathrm{R}]\) zip, we will use predict to compute the predicted number of fish captured by each individual.
```

. use http://www.stata-press.com/data/r13/fish
. zip count persons livebait, inf(child camper) vuong
(output omitted)
. predict numfished
(option n assumed; predicted number of events)

```
predict with the pr option computes the probability that an individual does not fish.
```

. predict pr, pr

```

On the other hand, predict with the \(\operatorname{pr}(n)\) option computes the probability of catching \(n\) fish; particularly, the probability of catching zero fish will be
```

. predict pr0, pr(0)

```
. list pr pr0 in 1
1.
\begin{tabular}{|rr|}
\hline pr & pr0 \\
\hline .3793549 & .8609267 \\
\hline
\end{tabular}

Notice that pr0 is always equal to or greater than pr. For example, for the first individual, the probability of not fishing is 0.38 ; on the other hand, the probability of catching zero fish (0.86) is equal to the sum of the probability of not fishing and the probability of fishing but not catching any fish. pr0 can be also computed as one minus the probability of catching at least one fish, that is:
```

. predict pr_catch, pr(1,.)
. gen prOb = 1-pr_catch

```

\section*{Methods and formulas}

The probabilities calculated using the \(\operatorname{pr}(n)\) option are the probability \(\operatorname{Pr}\left(y_{i}=n\right)\). These are calculated using
\[
\begin{aligned}
& \operatorname{Pr}\left(0 \mid \mathbf{x}_{i}\right)=\omega_{i}+\left(1-\omega_{i}\right) \exp \left(-\lambda_{i}\right) \\
& \operatorname{Pr}\left(n \mid \mathbf{x}_{i}\right)=\left(1-\omega_{i}\right) \frac{\lambda_{i}^{n} \exp \left(-\lambda_{i}\right)}{n!} \quad \text { for } n=1,2, \ldots
\end{aligned}
\]
where \(\omega_{i}\) is the probability of obtaining an observation from the degenerate distribution whose mass is concentrated at zero. \(\omega_{i}\) can be obtained from the pr option.

See Cameron and Trivedi (2013, sec. 4.6) for further details.

\section*{Reference}

Cameron, A. C., and P. K. Trivedi. 2013. Regression Analysis of Count Data. 2nd ed. New York: Cambridge University Press.

\section*{Also see}
[R] zip - Zero-inflated Poisson regression
[U] 20 Estimation and postestimation commands

\section*{Author index}

This is the author index for the Stata Base Reference Manual.

\section*{A}

Abramowitz, M., \([R]\) contrast, \([R]\) orthog
Abrams, K. R., [R] meta
Abramson, J. H., [R] kappa
Abramson, Z. H., [R] kappa
Abrevaya, J., [R] boxcox postestimation
Achen, C. H., [R] scobit
Acock, A. C., [R] anova, [R] correlate, [R] nestreg, \([R]\) oneway, \([R]\) prtest, \([R]\) ranksum, \([R]\) ttest
Adkins, L. C., \([\mathrm{R}]\) heckman, \([\mathrm{R}]\) regress, \([\mathrm{R}]\) regress postestimation
Afifi, A. A., [R] anova, [R] stepwise
Agresti, A., [R] ci, [R] expoisson, [R] tabulate twoway
Aigner, D. J., [R] frontier
Aiken, L. S., [R] pcorr
Aitchison, J., [R] ologit, [R] oprobit
Aitken, A. C., \([R]\) reg3
Aivazian, S. A., \([\mathrm{R}]\) ksmirnov
Akaike, H., [R] BIC note, \([R]\) estat ic, \([R]\) glm
Aldrich, J. H., [R] logit, [R] probit
Alexandersson, A., \([\mathrm{R}]\) regress
Alf, E., Jr., [R] rocfit, [R] rocreg
Algina, J., [R] esize
Alldredge, J. R., [R] pk, [R] pkcross
Allison, P. D., [R] rologit, [R] testnl
Almås, \(\mathrm{I} .,[\mathrm{R}]\) inequality
Alonzo, T. A., \([\mathrm{R}]\) rocreg, \([\mathrm{R}]\) rocreg postestimation, [R] rocregplot
Altman, D. G., [R] anova, [R] fp, [R] kappa, \([R]\) kwallis, \([R]\) meta, \([R]\) mfp, \([R]\) nptrend, [R] oneway
Ambler, G., [R] fp, [R] fp postestimation, [R] mfp, [ \(R\) ] regress
Amemiya, T., [R] glogit, [R] intreg, [R] ivprobit, \([R]\) nlogit, \([R]\) tobit
Andersen, E. B., [R] clogit
Andersen, P. K., [R] glm
Anderson, J. A., [R] ologit, [R] slogit
Anderson, R. E., [R] rologit
Anderson, R. L., [R] anova
Anderson, S., [R] pkequiv
Anderson, T. W., [R] ivregress postestimation
Andrews, D. F., [R] rreg
Andrews, D. W. K., [R] ivregress
Ängquist, L., \([R]\) bootstrap, \([R]\) permute
Angrist, J. D., [R] ivregress, \([R]\) ivregress postestimation, \([R]\) qreg, \([R]\) regress
Anscombe, F. J., [R] binreg postestimation, [R] glm, \([R]\) glm postestimation
Arbuthnott, J., [R] signrank

Archer, K. J., [R] estat gof, [R] logistic, [R] logit Arellano, M., [R] areg postestimation, \([R]\) gmm
Arminger, G., \([R]\) suest
Armitage, P., [R] ameans, [R] expoisson, [R] pkcross, [R] sdtest
Armstrong, R. D., [R] qreg
Arthur, M., [R] symmetry
Atella, V., [R] frontier
Atkinson, A. C., [R] boxcox, [R] nl
Azen, S. P., [R] anova

\section*{B}

Babin, B. J., [R] rologit
Baker, R. J., [R] glm
Baker, R. M., [R] ivregress postestimation
Bakker, A., [R] mean
Balaam, L. N., [R] pkcross
Baltagi, B. H., [R] hausman
Bamber, D., \([R]\) rocfit, \([R]\) rocregplot, \([R]\) roctab
Bancroft, T. A., [R] stepwise
Barnard, G. A., [R] spearman, [R] ttest
Barnett, A. G., [R] glm
Barrison, I. G., [R] binreg
Bartlett, M. S., [R] oneway
Bartus, T., \([\mathrm{R}]\) margins
Basmann, R. L., [R] ivregress, [R] ivregress postestimation
Basu, A., [R] glm
Bauldry, S., [R] ivregress
Baum, C. F., [R] gmm, [R] heckman, \([R]\) heckoprobit, \([R]\) heckprobit, \([R]\) ivregress, \([R]\) ivregress postestimation, \([R]\) margins, \([R]\) net, \([R]\) net search, \([R]\) regress postestimation, \([R]\) regress postestimation time series, \([R]\) ssc
Bayart, D., [R] qc
Beale, E. M. L., [R] stepwise, [R] test
Beaton, A. E., \([\mathrm{R}]\) rreg
Becketti, S., \([R] \mathbf{f p},[R] \mathbf{f p}\) postestimation, \([R]\) regress, \([R]\) runtest, \([R]\) spearman
Beggs, S., [R] rologit
Belanger, A. J., [R] sktest
Bellocco, R., [R] glm, [R] logit
Belotti, F., [R] frontier
Belsley, D. A., [R] regress postestimation, [R] regress postestimation diagnostic plots
Bendel, R. B., [R] stepwise
Benedetti, J. K., [R] tetrachoric
Beniger, J. R., [R] cumul
Bera, A. K., [R] sktest
Beran, R. J., [R] regress postestimation time series
Berk, K. N., [R] stepwise
Berk, R. A., [R] rreg
Berkson, J., [R] logit, [R] probit
Bern, P. H., [R] nestreg
Bernasco, W., [R] tetrachoric
Berndt, E. K., [R] glm
Berndt, E. R., [R] truncreg

Berry, G., \([\mathrm{R}]\) ameans, \([\mathrm{R}]\) expoisson, \([\mathrm{R}]\) sdtest
Berry, K. J., [R] ranksum
Bewley, R., [R] reg3
Beyer, W. H., [R] qc
Bickeböller, H., [R] symmetry
Bickel, P. J., [R] rreg
Birdsall, T. G., [R] Iroc
Black, W. C., \([\mathrm{R}]\) rologit
Blackwell, J. L., III, [R] areg
Bland, M., \([\mathrm{R}]\) ranksum, \([\mathrm{R}]\) sdtest, \([\mathrm{R}]\) signrank, [R] spearman
Blevins, J. R., \([R]\) hetprobit
Bliese, P. D., \([\mathrm{R}]\) icc
Bliss, C. I., [R] probit
Bloch, D. A., [R] brier
Bloomfield, P., \([\mathrm{R}]\) qreg
Blundell, R., [R] gmm, [R] ivprobit
BMDP, \([R]\) symmetry
Bofinger, E., [R] qreg
Boice, J. D., Jr., [R] bitest
Boland, P. J., [R] ttest
Bolduc, D., [R] asmprobit
Bollen, K. A., \([\mathrm{R}]\) regress postestimation
Bond, S., \([R]\) gmm
Bonferroni, C. E., \([\mathrm{R}]\) correlate
Borenstein, M., \([R]\) meta
Bottai, M., [R] qreg
Bound, J., \([R]\) ivregress postestimation
Bowker, A. H., [R] symmetry
Box, G. E. P., \([R]\) anova, \([R]\) boxcox, \([R]\) Inskew0
Box, J. F., \([R]\) anova
Boyd, N. F., [R] kappa
Brackstone, G. J., [R] diagnostic plots, [R] swilk
Bradley, R. A., [R] signrank
Brady, A. R., \([R]\) logistic, \([R]\) spikeplot
Brant, R., [R] ologit
Breslow, N. E., \([R]\) clogit, \([R]\) dstdize, \([R]\) symmetry
Breusch, T. S., \([R]\) regress postestimation, \([R]\) regress postestimation time series, \([R]\) sureg
Brier, G. W., \([\mathrm{R}]\) brier
Brillinger, D. R., \([R]\) jackknife
Brook, R. H., [R] brier
Brown, D. R., \([R]\) anova, \([R]\) contrast, \([R]\) loneway, \([\mathrm{R}]\) oneway, \([\mathrm{R}]\) pwcompare
Brown, L. D., [R] ci
Brown, M. B., \([\mathrm{R}]\) sdtest, \([\mathrm{R}]\) tetrachoric
Brown, S. E., [R] symmetry
Brown, W., [R] icc
Bru, B., [R] poisson
Brzezinski, M., \([\mathrm{R}]\) swilk
Buchner, D. M., [R] ladder
Buis, M. L., \([\mathrm{R}]\) constraint, \([\mathrm{R}]\) eform_option, \([R]\) logistic, \([R]\) logit, \([R]\) margins
Bunch, D. S., \([\mathrm{R}]\) asmprobit
Burke, W. J., [R] tobit

Burnam, M. A., \([\mathrm{R}]\) lincom, \([\mathrm{R}]\) mlogit, \([\mathrm{R}]\) mprobit, \([R]\) mprobit postestimation, \([R]\) predictnl, [R] slogit
Burr, I. W., [R] qc
Buskens, V., [R] tabstat

\section*{C}

Cai, T., \([\mathrm{R}]\) rocreg
Cai, T. T., [R] ci
Cameron, A. C., \([\mathrm{R}]\) asclogit, \([\mathrm{R}]\) asmprobit,
\([R]\) bootstrap, \([R]\) gmm, \([R]\) heckman,
\([R]\) heckoprobit, \([R]\) intreg, \([R]\) ivpoisson,
\([R]\) ivregress, \([R]\) ivregress postestimation,
\([R]\) logit, \([R]\) mprobit, \([R]\) nbreg, \([R]\) ologit,
\([R]\) oprobit, \([R]\) poisson, \([R]\) probit, \([R]\) qreg,
\([R]\) regress, \([R]\) regress postestimation,
\([R]\) simulate, \([R]\) sureg, \([R]\) tnbreg, \([R]\) tobit,
\([R]\) tpoisson, \([R]\) zinb postestimation, \([R]\) zip postestimation
Campbell, M. J., \([R]\) ci, \([R]\) kappa, \([R]\) poisson, \([\mathrm{R}]\) tabulate twoway
Canette, \(I\)., \([R]\) nl, \([R]\) nlsur
Cappellari, L., \([\mathrm{R}]\) asmprobit
Cardell, S., \([R]\) rologit
Carlile, T., \([R]\) kappa
Carlin, J. B., \([R]\) ameans
Carpenter, J. R., \([\mathrm{R}]\) bootstrap, \([\mathrm{R}]\) bstat
Carroll, R. J., \([R]\) boxcox, \([R]\) rreg, \([R]\) sdtest
Carson, R. T., \([R]\) tnbreg, \([R]\) tpoisson
Carter, S. L., \([R]\) frontier, \([R]\) lrtest, \([R]\) nbreg
Caudill, S. B., \([R]\) frontier
Caulcutt, R., \([R]\) qc
Chadwick, J., [R] poisson
Chaimani, A., \([\mathrm{R}]\) meta
Chakraborti, S., \([\mathrm{R}]\) ksmirnov
Chamberlain, \(G .,[R]\) clogit, \([R]\) gmm, \([R]\) qreg
Chambers, J. M., \([R]\) diagnostic plots, \([R]\) grmeanby, \([R]\) lowess
Chang, I. M., [R] margins
Charlett, A., [R] fp
Chatfield, M., \([R]\) anova
Chatterjee, \(S\)., \([R]\) poisson, \([R]\) regress, \([R]\) regress postestimation, \([R]\) regress postestimation diagnostic plots
Chen, X., \([R]\) logistic, \([R]\) logistic postestimation, [R] logit
Chiburis, R., \([R]\) heckman, \([R]\) heckoprobit, \([R]\) heckprobit, \([R]\) oprobit
Choi, B. C. K., \([R]\) rocfit, \([R]\) rocreg postestimation, \([R]\) rocregplot, \([R]\) roctab
Chow, G. C., \([\mathrm{R}]\) contrast
Chow, S.-C., \([R] \mathbf{p k},[R]\) pkcross, \([R]\) pkequiv, \([R]\) pkexamine, \([R]\) pkshape
Christakis, N., \([R]\) rologit
Clark, V. A., \([R]\) stepwise
Clarke, R. D., [R] poisson
Clarke-Pearson, D. L., \([\mathrm{R}]\) roccomp, \([\mathrm{R}]\) rocreg, \([\mathrm{R}]\) roctab

Clarkson, D. B., [R] tabulate twoway
Clayton, D. G., [R] cloglog, [R] cumul
Clerget-Darpoux, F., [R] symmetry
Cleveland, W. S., [R] diagnostic plots, [R] lowess, \([R]\) lpoly, \([R]\) sunflower
Cleves, M. A., [R] binreg, [R] dstdize, [R] logistic,
\([R]\) logit, \([R]\) roccomp, \([R]\) rocfit, \([R]\) rocreg,
\([R]\) rocreg postestimation, \([R]\) rocregplot,
\([R]\) roctab, \([R]\) sdtest, \([R]\) symmetry
Clogg, C. C., [R] suest
Clopper, C. J., [R] ci
Cobb, G. W., [R] anova
Cochran, W. G., [R] ameans, [R] anova, [R] correlate, \([R]\) dstdize, \([R]\) mean, \([R]\) oneway, \([R]\) poisson,
\([R]\) probit, \([R]\) proportion, \([R]\) ranksum,
\([R]\) ratio, \([R]\) signrank, \([R]\) total
Coelli, T. J., [R] frontier
Cohen, J., [R] esize, [R] kappa, [R] pcorr
Cohen, P., [R] pcorr
Coleman, J. S., [R] poisson
Collett, D., [R] clogit, [R] logistic, [R] logistic postestimation
Cone-Wesson, B., \([R]\) rocreg, \([R]\) rocreg postestimation, \([R]\) rocregplot
Cong, R., [R] tobit, [R] tobit postestimation, \([R]\) truncreg
Conover, W. J., [R] centile, [R] ksmirnov, [R] kwallis, \([R]\) nptrend, \([R]\) sdtest, \([R]\) spearman, \([R]\) tabulate twoway
Conroy, R. M., [R] intreg, [R] ranksum
Consonni, D., [R] dstdize
Cook, A., [R] ci
Cook, N. R., [R] rocreg
Cook, R. D., \([R]\) boxcox, \([R]\) regress postestimation
Coster, D., \([\mathrm{R}]\) contrast
Coull, B. A., \([\mathrm{R}]\) ci
Cox, D. R., [R] boxcox, [R] exlogistic, [R] expoisson, [R] Inskew0
Cox, G. M., [R] anova
Cox, N. J., [R] ci, [R] cumul, [R] diagnostic plots, [ \(R\) ] histogram, \([R]\) inequality, \([R]\) kappa, \([R]\) kdensity, \([R]\) ladder, \([R]\) lowess, \([R]\) lpoly, \([R]\) net, \([R]\) net search, \([R]\) regress postestimation, \([R]\) regress postestimation diagnostic plots, \([R]\) search, \([R]\) serrbar, \([R]\) sktest, \([R]\) smooth, \([R]\) spikeplot, \([R]\) ssc, \([R]\) stem, \([R]\) summarize, \([R]\) sunflower, \([R]\) tabulate oneway, \([R]\) tabulate twoway
Cragg, J. G., [R] ivregress postestimation
Cramér, H., [R] tabulate twoway
Cramer, J. S., [R] logit
Cronbach, L. J., [R] icc
Croux, C., [R] rreg
Crowther, M. J., [R] meta
Cui, J., [R] symmetry
Cumming, \(G .,[R]\) esize, \([R]\) regress postestimation
Cummings, \(P\)., [ \(R\) ] binreg, \([R]\) glm, \([R]\) margins
Curts-García, J., [R] smooth
Cuzick, J., [R] kappa, [R] nptrend

\section*{D}

D'Agostino, R. B., [R] sktest
D'Agostino, R. B., Jr., [R] sktest
Daidone, S., \([R]\) frontier
Daniel, C., \([R]\) diagnostic plots, \([R]\) oneway
Danuso, F., [R] nl
DasGupta, A., [R] ci
Davey Smith, G., [R] meta
David, F. N., [R] correlate
David, H. A., [R] spearman, [R] summarize
Davidson, R., \([R]\) boxcox, \([R]\) ensreg, \([R]\) gmm, \([R]\) intreg, \([R]\) ivregress, \([R]\) ivregress postestimation, \([R]\) mlogit, \([R]\) nl, \([R]\) nlsur, \([R]\) reg3, \([R]\) regress, \([R]\) regress postestimation time series, \([R]\) tobit, \([R]\) truncreg
Davison, A. C., [R] bootstrap
Day, N. E., [R] clogit, [R] dstdize, [R] symmetry
de Irala-Estévez, J., [R] logistic
De Luca, G., [R] biprobit, \([R]\) heckoprobit, \([R]\) heckprobit, \([R]\) oprobit, \([R]\) probit
de Wolf, I., \([\mathrm{R}]\) rologit
Deaton, A. S., [R] nlsur
Deb, P., [R] nbreg
Debarsy, N., [R] lpoly
Dehon, C., [R] correlate
DeLong, D. M., [R] roccomp, \([R]\) rocreg, \([R]\) roctab
DeLong, E. R., \([R]\) roccomp, \([R]\) rocreg, \([R]\) roctab
DeMaris, A., \([\mathrm{R}]\) regress postestimation
Desbordes, R., [R] ivregress
Desmarais, B. A., [R] zinb, [R] zip
Dewey, M. E., [R] correlate
Didelez, V., [R] ivregress
Digby, P. G. N., [R] tetrachoric
Dixon, W. J., [R] ttest
Djulbegovic, B., [R] meta
Dobson, A. J., [R] glm
Dodd, L. E., [R] rocreg
Dohoo, I., [R] regress
Doll, R., [R] poisson
Donald, A., [R] meta
Donald, S. G., [R] ivregress postestimation
Donner, A., [R] loneway
Donoho, D. L., [R] lpoly
Dore, C. J., \([R]\) fp
Dorfman, D. D., \([R]\) rocfit, \([R]\) rocreg
Doris, A., \([\mathrm{R}]\) gmm, \([\mathrm{R}]\) inequality
Draper, N., \([R]\) eivreg, \([R]\) oneway, \([R]\) regress,
\([R]\) stepwise
Drukker, D. M., [R] asmprobit, [R] boxcox,
\([R]\) frontier, \([R]\) lrtest, \([R]\) nbreg, \([R]\) tobit
Duan, N., [R] boxcox postestimation, [R] heckman
Duncan, A. J., [R] qc
Dunn, G., [R] kappa
Dunnett, C. W., [R] mprobit, [R] pwcompare
Dunnington, G. W., \([\mathrm{R}]\) regress
Dupont, W. D., [R] logistic, [R] mkspline,
[R] sunflower

Durbin, J., [R] ivregress postestimation, [R] regress postestimation time series
Duren, P., [R] regress
Duval, R. D., [R] bootstrap, [R] jackknife, [R] rocreg, \([R]\) rocregplot

\section*{E}

Edgington, E. S., [R] runtest
Edwards, A. L., \([\mathrm{R}]\) anova
Edwards, A. W. F., [R] tetrachoric
Edwards, J. H., [R] tetrachoric
Efron, B., [R] bootstrap, [R] qreg
Efroymson, M. A., [R] stepwise
Egger, M., [R] meta
Eisenhart, C., [R] correlate, \([\mathrm{R}]\) runtest
Ellis, C. D., \([\mathrm{R}]\) poisson
Ellis, P. D., \([R]\) esize, \([R]\) regress postestimation
Eltinge, J. L., [R] test
Emerson, J. D., \([R] \mathbf{l v},[R]\) stem
Ender, P. B., \([R]\) marginsplot
Engel, A., [R] boxcox, [R] marginsplot
Engle, R. F., [R] regress postestimation time series
Erdreich, L. S., [R] roccomp, [R] rocfit, [R] roctab
Eubank, R. L., [R] lpoly
Evans, M. A., \([R]\) pk, \([R]\) pkcross
Everitt, B. S., [R] gllamm, [R] glm
Ewens, W. J., [R] symmetry
Ezekiel, M., [R] regress postestimation diagnostic plots

\section*{F}

Fagerland, M. W., [R] estat gof, [R] mlogit postestimation
Fan, J., [R] lpoly
Fan, Y.-A., [R] tabulate twoway
Fang, K.-T., [R] asmprobit
Farbmacher, H., [R] tpoisson
Feiveson, A. H., \([\mathrm{R}]\) nlcom, \([\mathrm{R}]\) ranksum
Feldt, L. S., [R] anova
Ferri, H. A., [R] kappa
Festinger, L., [R] ranksum
Field, C. A., [R] bootstrap
Fieller, E. C., [R] pkequiv
Fienberg, S. E., \([\mathrm{R}]\) kwallis, \([\mathrm{R}]\) tabulate twoway
Filon, L. N. G., [R] correlate
Filoso, V., [R] regress
Finch, S., [R] esize
Findley, D. F., [R] estat ic
Findley, T. W., [R] ladder
Finlay, K., [R] ivprobit, [R] ivregress, [R] ivtobit
Finney, D. J., \([R]\) probit, \([R]\) tabulate twoway
Fiorio, C. V., [R] kdensity
Fiser, D. H., [R] estat gof, [R] Iroc
Fishell, E., [R] kappa
Fisher, L. D., \([R]\) anova, \([R]\) dstdize, \([R]\) oneway

Fisher, N. I., [R] regress postestimation time series
Fisher, R. A., [R] anova, [R] anova, [R] esize,
\([R]\) ranksum, \([R]\) signrank, \([R]\) tabulate twoway
Flannery, B. P., [R] dydx, [R] vwls
Fleiss, J. L., [R] dstdize, [R] icc, [R] kappa
Fletcher, K., [R] rocreg, [R] rocreg postestimation, \([R]\) rocregplot
Flynn, Z. L., [R] gmm
Folsom, R. C., [R] rocreg, [R] rocreg postestimation, \([R]\) rocregplot
Ford, J. M., [R] frontier
Forsythe, A. B., [R] sdtest
Forthofer, R. N., [R] dstdize
Foster, A., [R] regress
Fouladi, R. T., [R] esize
Fourier, J. B. J., [R] cumul
Fox, J., [R] kdensity, [R] lv
Fox, W. C., [R] Iroc
Francia, R. S., [R] swilk
Freese, J., [R] asroprobit, [R] clogit, [R] cloglog,
\([R]\) logistic, \([R]\) logit, \([R]\) mlogit, \([R]\) mprobit,
\([R]\) nbreg, \([R]\) ologit, [ \(R\) ] oprobit, \([R]\) poisson,
\([R]\) probit, \([R]\) regress, \([R]\) regress postestimation, \([R]\) tnbreg, \([R]\) tpoisson, [R] zinb, [R] zip
Frölich, M., [R] qreg
Frome, E. L., [R] qreg
Frydenberg, M., [R] dstdize, \([R]\) roccomp, \([R]\) roctab
Fu, V. K., [R] ologit
Fuller, W. A., \([\mathrm{R}]\) regress, \([\mathrm{R}]\) spearman

\section*{G}

Gail, M. H., [R] rocreg, [R] rocreg postestimation
Gall, J.-R. L., \([\mathrm{R}]\) estat gof, \([\mathrm{R}]\) logistic
Gallant, A. R., [R] ivregress, [R] nl
Gallup, J. L., \([\mathrm{R}]\) estimates table
Galton, F., [R] correlate, [R] cumul, [R] regress, \([R]\) summarize
Gan, F. F., [R] diagnostic plots
Garrett, J. M., [R] logistic, [R] logistic postestimation,
\([R]\) regress postestimation
Garsd, A., [R] exlogistic
Gasser, T., [R] lpoly
Gastwirth, J. L., [R] sdtest
Gates, R., [R] asmprobit
Gauss, J. C. F., [R] regress
Gauvreau, K., [R] dstdize, [R] logistic
Geisser, S., [R] anova
Gel, Y. R., [R] sdtest
Gelbach, J., [R] ivprobit, [R] ivtobit
Gelman, R., [R] margins
Genest, C., \([R]\) diagnostic plots, \([R]\) swilk
Gentle, J. E., \([\mathrm{R}]\) anova, \([\mathrm{R}] \mathrm{nl}\)
Genton, M. G., [R] sktest
Genz, A., [R] asmprobit
Gerkins, V. R., [R] symmetry

Geweke, J., [R] asmprobit
Gibbons, J. D., [R] ksmirnov, [R] spearman
Giesen, D., [R] tetrachoric
Gijbels, I., [R] lpoly
Gillham, N. W., [R] regress
Gillispie, C. C., [R] regress
Gini, R., [R] vwls
Glass, G. V., [R] esize
Gleason, J. R., [R] anova, [R] bootstrap, \([\mathrm{R}]\) ci, \([R]\) correlate, \([R]\) loneway, \([R]\) summarize, [R] ttest
Glidden, D. V., [R] logistic
Gnanadesikan, R., [R] cumul, \([\mathrm{R}]\) diagnostic plots
Godfrey, L. G., [R] regress postestimation time series
Goeden, G. B., [R] kdensity
Goerg, S. J., [R] ksmirnov
Goldberger, A. S., [R] intreg, [R] mlexp, [R] tobit
Goldstein, R., [R] brier, [R] correlate, [R] inequality, \([R]\) nl, [R] ologit, [R] oprobit, [R] ranksum, \([R]\) regress postestimation
Golub, G. H., [R] orthog, [R] tetrachoric
Good, P. I., [R] permute, [R] symmetry, \([R]\) tabulate twoway
Goodall, C., [R] lowess, [R] rreg
Goodman, L. A., \([\mathrm{R}]\) tabulate twoway
Gordon, M. G., [R] binreg
Gorga, M. P., [R] rocreg, [R] rocreg postestimation, [R] rocregplot
Gorman, J. W., [R] stepwise
Gosset [Student, pseud.], W. S., [R] ttest
Gosset, W. S., [R] ttest
Gould, W. W., [R] bootstrap, [R] bsample, [R] dydx,
[R] frontier, \([R]\) gmm, [R] grmeanby,
[R] jackknife, [R] kappa, [R] logistic,
\([R]\) margins, \([R]\) maximize, \([R]\) mkspline,
\([R] \mathbf{m l},[R]\) mlexp, \([R]\) net search, \([R]\) nlcom,
\([R]\) ologit, \([R]\) oprobit, \([R]\) poisson,
\([R]\) predictnl, \([R]\) qreg, \([R]\) regress, \([R]\) rreg,
\([R]\) simulate, \([R]\) sktest, \([R]\) smooth, \([R]\) swilk, \([R]\) testnl
Gourieroux, C. S., \([R]\) hausman, \([R]\) suest, \([R]\) test
Graubard, B. I., \([R]\) margins, \([R] \mathrm{ml},[R]\) test
Graybill, F. A., [R] centile
Green, D. M., [R] Iroc
Greene, W. H., [R] asclogit, [R] asmprobit,
[R] biprobit, [R] clogit, [R] cnsreg, \([R]\) frontier, \([R]\) gmm, \([R]\) heckman, \([R]\) heckoprobit,
\([R]\) heckprobit, \([R]\) hetprobit, \([R]\) ivregress,
\([R]\) logit, \([R]\) Irtest, \([R]\) margins, \([R]\) mkspline,
\([R]\) mlexp, \([R]\) mlogit, \([R]\) nlogit, \([R]\) nlsur,
\([R]\) pcorr, \([R]\) probit, \([R]\) reg3, \([R]\) regress,
\([R]\) regress postestimation time series,
\([R]\) sureg, \([R]\) testnl, \([R]\) truncreg, \([R]\) zinb, [R] zip
Greenfield, S., [R] lincom, [R] mlogit, [R] mprobit, \([R]\) mprobit postestimation, \([R]\) predictnl, [R] slogit
Greenhouse, S. W., [R] anova

Greenland, S., [R] ci, [R] glogit, [R] mkspline, \([R]\) ologit, \([R]\) poisson
Gregoire, A., [R] kappa
Grieve, R., [R] bootstrap, [R] bstat
Griffith, J. L., [R] brier
Griffith, R., [R] gmm
Griffiths, W. E., [R] cnsreg, [R] estat ic, [R] glogit,
\([R]\) ivregress, \([R]\) ivregress postestimation,
\([R]\) logit, \([R]\) probit, \([R]\) regress, \([R]\) regress postestimation, \([\mathrm{R}]\) test
Grissom, R. J., \([R]\) esize, \([R]\) regress postestimation
Grizzle, J. E., [R] vwls
Grogger, J. T., [R] tnbreg, [R] tpoisson
Gronau, R., [R] heckman
Gropper, D. M., [R] frontier
Guan, W., [R] bootstrap
Gutierrez, R. G., [R] frontier, [R] lpoly, [R] lrtest, \([R]\) nbreg

\section*{H}

Haan, P., [R] asmprobit, [R] mlogit, [R] mprobit
Hadi, A. S., [R] poisson, [R] regress, [R] regress postestimation, \([R]\) regress postestimation diagnostic plots
Hadorn, D. C., [R] brier
Hahn, J., [R] ivregress postestimation
Hair, J. F., Jr., [R] rologit
Hajian-Tilaki, K. O., [R] rocreg
Hajivassiliou, V. A., [R] asmprobit
Hald, A., \([R]\) qreg, \([R]\) regress, \([R]\) signrank, \([R]\) summarize
Haldane, J. B. S., [R] ranksum
Hall, A. D., [R] frontier
Hall, A. R., [R] gmm, [R] gmm postestimation, \([R]\) ivpoisson, \([R]\) ivpoisson postestimation,
\([R]\) ivregress, \([R]\) ivregress postestimation
Hall, B. H., [R] glm
Hall, N. S., [R] anova
Hall, P., [R] bootstrap, [R] qreg, [R] regress postestimation time series
Hall, R. E., [R] glm
Hall, W. J., [R] roccomp, [R] rocfit, \([\mathrm{R}]\) roctab
Hallock, K., [R] qreg
Halvorsen, K. T., [R] tabulate twoway
Hamerle, A., [R] clogit
Hamilton, J. D., [R] gmm
Hamilton, L. C., [R] bootstrap, [R] diagnostic plots, \([R]\) estat vce, \([R]\) ladder, \([R]\) lv, \([R]\) mlogit,
\([R]\) regress, \([R]\) regress postestimation,
\([R]\) regress postestimation diagnostic plots,
\([R]\) rreg, \([R]\) simulate, \([R]\) summarize, \([R]\) ttest
Hampel, F. R., [R] rreg
Hanley, J. A., [R] roccomp, [R] rocfit, [R] rocreg, \([R]\) rocreg postestimation, \([R]\) rocregplot, \([R]\) roctab
Hansen, L. P., [R] gmm, [R] ivregress, [R] ivregress postestimation
Hao, L., [R] qreg

Harbord, R. M., [R] roccomp, [R] roctab
Harden, J. J., [R] zinb, [R] zip
Hardin, J. W., [R] binreg, [R] biprobit, [R] estat ic, \([R]\) glm, \([R]\) glm postestimation, \([R]\) Iroc, \([R]\) poisson, \([R]\) ranksum, \([R]\) regress postestimation, \([R]\) signrank
Haritou, A., [R] suest
Harkness, J., [R] ivprobit, [R] ivtobit
Harrell, F. E., Jr., [R] mkspline, [R] ologit
Harris, R. L., [R] qc
Harris, T., [R] poisson, [R] qreg, [R] ranksum, [ \(R\) ] signrank
Harrison, D. A., [R] histogram, [R] tabulate oneway, [R] tabulate twoway
Harrison, J. A., [R] dstdize
Hartmann, D. P., [R] icc
Harvey, A. C., \([R]\) hetprobit
Hastie, T. J., [R] grmeanby, [R] slogit
Hauck, W. W., [R] pkequiv
Haughton, J. H., [R] inequality
Hausman, J. A., [R] glm, [R] hausman, [R] ivregress postestimation, [ \(R\) ] nlogit, \([R]\) rologit, \([R]\) suest
Havnes, T., [R] inequality
Hayashi, F., \([R]\) gmm, \([R]\) ivpoisson, \([R]\) ivregress, \([R]\) ivregress postestimation
Hayes, R. J., [R] permute
Hays, R. D., \([\mathrm{R}]\) lincom, \([\mathrm{R}]\) mlogit, \([\mathrm{R}]\) mprobit, \([R]\) mprobit postestimation, \([R]\) predictnl, [R] slogit
Hays, W. L., \([R]\) esize, \([R]\) regress postestimation
Heagerty, P. J., [R] anova, \([R]\) dstdize, \([R]\) oneway
Heckman, J., [R] biprobit, \([R]\) heckman, \([R]\) heckman postestimation, \([R]\) heckoprobit, \([R]\) heckprobit
Hedges, L. V., [R] esize, [R] meta
Heiss, F., [R] nlogit
Henderson, B. E., [R] symmetry
Hendrickx, J., [R] mlogit, [R] xi
Hensher, D. A., [R] nlogit
Hickam, D. H., [R] brier
Higgins, J. E., [R] anova
Higgins, J. P. T., [R] meta
Hilbe, J. M., [R] cloglog, [R] estat ic, [R] glm, [R] glm postestimation, \([R]\) logistic, \([R]\) logit, \([R]\) lroc, \([R]\) nbreg, \([R]\) poisson, \([R]\) probit, \([R]\) simulate, [R] tnbreg, [R] tpoisson
Hill, A. B., [R] poisson
Hill, R. C., [R] ensreg, [R] estat ic, [R] glogit, \([R]\) heckman, \([R]\) ivregress, \([R]\) ivregress postestimation, [R] logit, [R] probit, \([R]\) regress, \([R]\) regress postestimation, \([R]\) test
Hills, M., [R] cloglog, [R] cumul
Hinchliffe, S. R., [R] meta
Hinkley, D. V., [R] bootstrap
Hirji, K. F., [R] exlogistic, [R] expoisson
Hoaglin, D. C., [R] diagnostic plots, [R] lv, [R] regress postestimation, \([R]\) regress postestimation diagnostic plots, \([R]\) smooth, \([R]\) stem
Hochberg, Y., [R] oneway

Hocking, R. R., [R] stepwise
Hoel, P. G., [R] bitest, [R] ttest
Hoffmann, J. P., [R] glm
Hole, A. R., [R] asmprobit, [R] clogit, [R] mlogit, [R] mprobit
Holloway, L., [R] brier
Holm, S., [R] test
Holmes, S., [R] bootstrap
Hood, W. C., [R] ivregress
Hosmer, D. W., Jr., [R] clogit, [R] clogit postestimation, \([R]\) estat classification, \([R]\) estat gof, \([R]\) glm, \([R]\) glogit, \([R]\) lincom, \([R]\) logistic, [R] logistic postestimation, [R] logit, [R] logit postestimation, [R] Iroc, [R] Irtest, [R] Isens,
\([R]\) mlogit, \([R]\) mlogit postestimation,
\([R]\) predictnl, \([R]\) stepwise
Hotelling, H., [R] roccomp, [R] rocfit, \([R]\) roctab
Hozo, I., [R] meta
Huang, C., [R] sunflower
Huang, D. S., [R] nlsur, [R] sureg
Huber, C., \([R]\) esize, \([R]\) regress postestimation
Huber, P. J., [R] qreg, [R] rreg, [R] suest
Hunter, D. R., [R] qreg
Hurd, M., \([R]\) intreg, \([R]\) tobit
Hutto, C., [R] exlogistic
Huynh, H., [R] anova

\section*{I}

Iglewicz, B., [R] lv
Ilardi, G., [R] frontier
Isaacs, D., [R] fp
Ishiguro, M., [R] BIC note

\section*{J}

Jackman, R. W., [R] regress postestimation
Jacobs, K. B., [R] symmetry
Jaeger, D. A., \([R]\) ivregress postestimation
James, B. R., \([\mathrm{R}]\) rocreg, \([\mathrm{R}]\) rocreg postestimation
James, K. L., [R] rocreg, [R] rocreg postestimation
Janes, H., [R] rocfit, [ \(R\) ] rocreg, \([R]\) rocreg postestimation, \([R]\) rocregplot
Jann, B., [R] estimates store, [R] ksmirnov, [R] stored results, \([R]\) tabulate twoway
Jarque, C. M., [R] sktest
Jeffreys, H., [R] ci, [R] spearman
Jenkins, S. P., [R] asmprobit, \([R]\) do, \([R]\) inequality
Joe, H., \([R]\) tabulate twoway
Johnson, D. E., [R] anova, \([\mathrm{R}]\) contrast,
[R] pwcompare
Johnson, M. E., [R] sdtest
Johnson, M. M., [R] sdtest
Johnson, N. L., \([\mathrm{R}]\) ksmirnov, \([\mathrm{R}]\) nbreg, \([\mathrm{R}]\) poisson
Johnston, J. E., [R] ranksum
Jolliffe, D., \([R]\) inequality, \([R]\) qreg, \([R]\) regress
Jolliffe, I. T., [R] brier
Jones, A., [R] heckman, [R] logit, [R] probit

Jones, D. R., [R] meta
Jones, M. C., [R] kdensity, [R] lpoly
Judge, G. G., [R] estat ic, [R] glogit, [R] ivregress,
\([R]\) ivregress postestimation, \([R]\) logit,
\([R]\) probit, \([R]\) regress postestimation, \([R]\) test
Judson, D. H., \([R]\) poisson, \([R]\) tabulate twoway, [R] tpoisson
Juul, S., \([\mathrm{R}]\) dstdize, \([\mathrm{R}]\) roccomp, \([\mathrm{R}]\) roctab

\section*{K}

Kahn, H. A., [R] dstdize
Kaiser, J., [R] ksmirnov, [R] permute, [R] signrank
Kalmijn, M., [R] tetrachoric
Keane, M. P., [R] asmprobit
Keeler, E. B., [R] brier
Kelley, K., [R] esize, [R] regress postestimation
Kemp, A. W., [R] nbreg, [R] poisson
Kempthorne, P. J., \([\mathrm{R}]\) regress postestimation
Kendall, M. G., [R] centile, [R] spearman,
[R] tabulate twoway
Kennedy, W. J., Jr., [R] anova, [R] nl, [R] regress, [R] stepwise
Kerlinger, F. N., [R] esize, [R] regress postestimation
Keselman, H. J., [R] esize
Kettenring, J. R., [R] diagnostic plots
Keynes, J. M., [R] ameans
Khan, S., [R] hetprobit
Khandker, S. R., [R] inequality
Kiernan, M., [R] kappa
Kim, J. J., \([\mathrm{R}]\) esize, \([\mathrm{R}]\) regress postestimation
Kirk, R. E., [R] esize, [R] regress postestimation
Kirkwood, B. R., [R] dstdize, [R] summarize
Kish, L., [R] loneway
Kitagawa, G., [R] BIC note
Klar, J., [R] estat gof
Kleiber, C., [R] inequality
Klein, L. R., [R] reg3, [R] reg3 postestimation,
\([R]\) regress postestimation time series
Klein, M., [R] binreg, [R] clogit, [R] logistic,
\([R]\) Irtest, \([R]\) mlogit, \([R]\) ologit
Kleinbaum, D. G., [R] binreg, [R] clogit, [R] logistic, \([R]\) Irtest, \([R]\) mlogit, [R] ologit
Kleiner, B., [R] diagnostic plots, [R] lowess
Kline, R. B., \([R]\) esize, \([R]\) regress postestimation
Kmenta, J., \([R]\) eivreg, \([R]\) ivregress, \([R]\) regress
Koch, G. G., [ R\(]\) anova, \([\mathrm{R}]\) kappa, \([\mathrm{R}]\) vwls
Koehler, K. J., [R] diagnostic plots
Koenker, R., [ R ] qreg, \([\mathrm{R}]\) regress postestimation
Kohler, U., [R] estat classification, \([R]\) kdensity,
\([R]\) regress, \([R]\) regress postestimation,
\([R]\) regress postestimation diagnostic plots
Kolmogorov, A. N., [R] ksmirnov
Kontopantelis, E., [R] meta
Koopman, S. J., [R] regress postestimation time series
Koopmans, T. C., [R] ivregress
Korn, E. L., [R] margins, [R] ml, [R] test

Kotz, S., [R] inequality, \([\mathrm{R}]\) ksmirnov, \([\mathrm{R}]\) nbreg, [ \(R\) ] nlogit, \([R]\) poisson
Kreuter, F., [R] estat classification, [R] kdensity, \([R]\) regress, \([R]\) regress postestimation, \([R]\) regress postestimation diagnostic plots
Krushelnytskyy, B., [R] inequality, [R] qreg
Kruskal, W. H., [R] kwallis, [R] ranksum,
\([R]\) spearman, \([R]\) tabulate twoway
Kuehl, R. O., \([R]\) anova, \([R]\) contrast, \([R]\) icc, \([R]\) oneway
Kuh, E., [R] regress postestimation, [R] regress postestimation diagnostic plots
Kumbhakar, S. C., [R] frontier, [R] frontier postestimation
Kung, D. S., [R] qreg
Kutner, M. H., [R] pkcross, [R] pkequiv, [R] pkshape, \([R]\) regress postestimation

\section*{L}

Lachenbruch, P. A., [R] diagnostic plots
Lacy, M. G., [R] permute
Lafontaine, F., [R] boxcox
Lahiri, K., \([R]\) tobit
Lai, S., [R] exlogistic
Laird, N. M., [R] expoisson
Lambert, D., [R] zip
Lambert, P. C., [R] poisson
Landis, J. R., [R] kappa
Lane, P. W., [R] margins
Langan, D., \([\mathrm{R}]\) meta
Lange, K., [R] qreg
Laplace, P.-S., \([\mathrm{R}]\) regress
Larsen, W. A., [R] regress postestimation diagnostic plots
Lash, T. L., \([\mathrm{R}]\) ci, [ R\(]\) glogit, \([\mathrm{R}]\) poisson
Lauritzen, S. L., \([R]\) summarize
Lee, E. S., [R] dstdize
Lee, E. T., \([\mathrm{R}]\) roccomp, \([\mathrm{R}]\) rocfit, \([\mathrm{R}]\) roctab
Lee, T.-C., [R] estat ic, \([R]\) glogit, [R] ivregress, \([R]\) ivregress postestimation, \([R]\) logit,
\([R]\) probit, \([R]\) regress postestimation, \([R]\) test
Lee, W. C., [R] roctab
Legendre, A.-M., [R] regress
Lehmann, E. L., [R] oneway
Lemeshow, S. A., [R] clogit, [R] clogit postestimation, \([R]\) estat classification, \([R]\) estat gof,
\([R]\) glm, \([R]\) glogit, \([R]\) lincom, \([R]\) logistic,
\([R]\) logistic postestimation, \([R]\) logit, \([R]\) logit postestimation, \([R]\) lroc, \([R]\) Irtest, \([R]\) Isens, \([R]\) mlogit, [ \(R\) ] predictnl, [ \(R\) ] stepwise
Leroy, A. M., \([R]\) qreg, \([R]\) regress postestimation, [ \(R\) ] rreg
Levene, H., [R] sdtest
Levin, B., \([\mathrm{R}]\) dstdize, \([\mathrm{R}]\) kappa
Levinsohn, J. A., \([\mathrm{R}]\) frontier
Levy, D. E., [R] sunflower
Lewis, H. G., [R] heckman
Lewis, I. G., [R] binreg

Lewis, J. D., [R] fp
Li, G., [R] rreg
Li, W., [R] pkcross, [R] pkequiv, [R] pkshape
Libois, F., [R] fp
Lim, G. C., \([\mathrm{R}]\) cnsreg, \([\mathrm{R}]\) regress, \([\mathrm{R}]\) regress postestimation
Lindley, D. V., [R] ci
Lindsey, C., [R] boxcox, [R] lowess, [R] regress postestimation, \([R]\) regress postestimation diagnostic plots, \([R]\) stepwise
Linhart, J. M., [R] lpoly
Lipset, S. M., [R] histogram
Liu, J.-P., [R] pk, [R] pkcross, [R] pkequiv, [R] pkexamine, [R] pkshape
Locke, C. S., [R] pkequiv
Lockwood, J. R., [R] areg
Lokshin, M., [R] biprobit, [R] heckman, [R] heckoprobit, [R] heckprobit, [R] oprobit
Long, J. S., \([R]\) asroprobit, \([R]\) clogit, \([R]\) cloglog, \([R]\) intreg, \([R]\) logistic, \([R]\) logit, \([R]\) mlogit, [R] mprobit, [R] nbreg, [R] ologit, [R] oprobit, \([R]\) poisson, \([R]\) probit, \([R]\) regress, \([R]\) regress postestimation, \([R]\) testnl, \([R]\) tnbreg, \([R]\) tobit, [ R ] tpoisson, [ R ] zinb, [ R ] zip
Longest, K. C., \([R]\) tabulate twoway
Longley, J. D., [R] kappa
Longton, G. M., [R] rocfit, [R] rocreg, [R] rocreg postestimation, \([R]\) rocregplot
López-Feldman, A., \([\mathrm{R}]\) inequality
Lorenz, M. O., \([\mathrm{R}]\) inequality
Louis, T. A., [R] tabulate twoway
Lovell, C. A. K., [R] frontier, [R] frontier postestimation
Lovie, A. D., [R] spearman
Lovie, P., [R] spearman
Lucas, H. L., [R] pkcross
Luce, R. D., [R] rologit
Lumley, T. S., \([R]\) anova, \([R]\) dstdize, \([R]\) oneway
Lunt, M., [R] ologit, [R] slogit
Lütkepohl, H., [R] estat ic, [R] glogit, [R] ivregress, \([R]\) ivregress postestimation, \([R]\) logit, \([R]\) probit, \([R]\) regress postestimation, \([R]\) test

\section*{M}

Ma, G., [R] roccomp, \([R]\) rocfit, \([R]\) roctab
Machin, D., \([R]\) ci, \([R]\) kappa, \([R]\) tabulate twoway
Mack, T. M., [R] symmetry
MacKinnon, J. G., [R] boxcox, [R] ensreg, [R] gmm, \([R]\) intreg, \([R]\) ivregress, \([R]\) ivregress postestimation, \([\mathrm{R}]\) mlogit, \([\mathrm{R}] \mathrm{nl},[\mathrm{R}]\) nlsur, \([R]\) reg3, \([R]\) regress, \([R]\) regress postestimation time series, \([R]\) tobit, \([R]\) truncreg
MacRae, K. D., [R] binreg
Madansky, A., [R] runtest
Maddala, G. S., [R] nlogit, [R] tobit
Magnusson, L. M., \([R]\) gmm, \([R]\) ivprobit, [R] ivregress, \([R]\) ivtobit

Mallows, C. L., [R] regress postestimation diagnostic plots
Mander, A. P., [R] anova, [R] symmetry
Mann, H. B., [R] kwallis, [R] ranksum
Manning, W. G., [R] heckman
Manski, C. F., [R] gmm
Mantel, N., [R] stepwise
Marchenko, Y. V., [R] anova, [R] loneway, \([R]\) oneway, \([R]\) sktest
Marden, J. I., [R] rologit
Markowski, C. A., [R] sdtest
Markowski, E. P., \([R]\) sdtest
Marschak, J., [R] ivregress
Martin, W., [R] regress
Martínez, M. A., [R] logistic
Mascher, K., [R] rocreg, [R] rocreg postestimation, \([R]\) rocregplot
Massey, F. J., Jr., [R] ttest
Massey, J. T., [R] boxcox, [R] marginsplot
Master, I. M., [R] exlogistic
Mastrucci, M. T., [R] exlogistic
Matthews, J. N. S., [R] ameans, [R] expoisson, [R] sdtest
Mátyás, L., [R] gmm
Maurer, K., [R] boxcox, [R] marginsplot
Maxwell, A. E., [R] symmetry
May, S., [R] stepwise
McCaffrey, D. F., [R] areg
McCleary, S. J., [R] regress postestimation diagnostic plots
McClish, D. K., \([\mathrm{R}]\) rocreg
McCullagh, P., \([R]\) binreg, \([R]\) binreg postestimation, \([R]\) glm, \([R]\) glm postestimation, \([R]\) ologit, \([R]\) rologit
McCulloch, C. E., [R] logistic
McDonald, J. A., [R] sunflower
McDonald, J. F., \([\mathrm{R}]\) tobit, \([\mathrm{R}]\) tobit postestimation
McDowell, A., [R] boxcox, [R] marginsplot
McDowell, A. W., [R] sureg
McFadden, D. L., [R] asclogit, [R] asmprobit, \([R]\) clogit, \([R]\) hausman, \([R]\) maximize, \([R]\) nlogit, \([R]\) suest
McGill, R., [R] sunflower
McGinnis, R. E., [R] symmetry
McGraw, K. O., [R] icc
McGuire, T. J., [R] dstdize
McKelvey, R. D., [R] ologit
McNeil, B. J., [R] roccomp, [R] rocfit, [R] rocreg, \([R]\) rocreg postestimation, \([R]\) rocregplot, \([R]\) roctab
McNeil, D., [R] poisson
Meeusen, W., [R] frontier
Mehta, C. R., [R] exlogistic, [R] exlogistic postestimation, \([R]\) expoisson, \([R]\) tabulate twoway
Melly, B., [R] qreg
Mensing, R. W., [R] anova postestimation
Metz, C. E., [R] Iroc

Miao, W., [R] sdtest
Michels, K. M., [R] anova, [R] contrast, [R] loneway, \([R]\) oneway, \([R]\) pwcompare
Mielke, P. W., Jr., [R] brier, [R] ranksum
Mihaly, K., [R] areg
Miladinovic, B., [R] meta
Miller, A. B., [R] kappa
Miller, R. G., Jr., [R] diagnostic plots, [R] oneway, [R] pwcompare
Milliken, G. A., \([R]\) anova, \([R]\) contrast, \([R]\) margins, [R] pwcompare
Miranda, A., \([R]\) gllamm, \([R]\) heckoprobit, \([R]\) heckprobit, \([R]\) ivprobit, \([R]\) ivtobit, \([R]\) logistic, \([R]\) logit, \([R]\) nbreg, \([R]\) ologit, \([R]\) oprobit, \([R]\) poisson, \([R]\) probit
Mitchell, C., [R] exlogistic
Mitchell, M. N., \([R]\) anova, \([R]\) anova postestimation, \([R]\) contrast, \([R]\) logistic, \([R]\) logistic postestimation, \([R]\) logit, \([R]\) margins, \([R]\) marginsplot, \([R]\) pwcompare, \([R]\) regress
Moffitt, R. A., [R] tobit, [R] tobit postestimation
Mogstad, M., [R] inequality
Monfort, A., \([R]\) hausman, \([R]\) suest, \([R]\) test
Monson, R. R., [R] bitest
Montoya, \(D .,[R]\) rocreg, \([R]\) rocreg postestimation, \([\mathrm{R}]\) rocregplot
Mood, A. M., [R] centile
Mooney, C. Z., [R] bootstrap, [R] jackknife, \([R]\) rocreg, \([R]\) rocregplot
Moran, J. L., \([\mathrm{R}]\) dstdize
Morris, C., \([\mathrm{R}]\) bootstrap
Morris, N. F., [R] binreg
Moskowitz, M., [R] kappa
Mosteller, C. F., \([\mathrm{R}]\) jackknife, \([\mathrm{R}]\) regress, \([\mathrm{R}]\) regress postestimation diagnostic plots, \([\mathrm{R}]\) rreg
Moulton, L. H., \([R]\) permute
Muellbauer, J., \([\mathrm{R}]\) nlsur
Mullahy, J., \([R]\) gmm, \([R]\) ivpoisson, \([R]\) zinb, \([R]\) zip
Müller, H.-G., [R] Ipoly
Muro, J., [R] heckoprobit, \([\mathrm{R}]\) heckprobit
Murphy, A. H., \([\mathrm{R}]\) brier
Murray-Lyon, I. M., [R] binreg
Muñoz, J., [R] exlogistic

\section*{N}

Nachtsheim, C. J., \([\mathrm{R}]\) pkcross, \([\mathrm{R}]\) pkequiv,
\([R]\) pkshape, \([R]\) regress postestimation
Nadarajah, S., \([R]\) nlogit
Nadaraya, E. A., [R] lpoly
Nagler, J., [R] scobit
Naiman, D. Q., [R] qreg
Narula, S. C., [R] qreg
Nee, J. C. M., [R] kappa
Neely, S. T., [R] rocreg, [R] rocreg postestimation, \([\mathrm{R}]\) rocregplot
Nelder, J. A., \([R]\) binreg, \([R]\) binreg postestimation, \([R]\) glm, \([R]\) glm postestimation, \([R]\) margins, [R] ologit

Nelson, C. R., \([\mathrm{R}]\) ivregress postestimation
Nelson, E. C., [R] lincom, [R] mlogit, [R] mprobit, \([R]\) mprobit postestimation, \([R]\) predictnl, [R] slogit
Nelson, F. D., [R] logit, [R] probit
Neter, J., [R] pkcross, \([R]\) pkequiv, \([\mathrm{R}]\) pkshape, \([R]\) regress postestimation
Newey, W. K., [R] glm, [R] gmm, [R] ivpoisson, \([R]\) ivprobit, \([R]\) ivregress, \([R]\) ivtobit
Newman, S. C., \([R]\) poisson
Newson, R. B., \([R]\) centile, \([R]\) glm, \([R]\) glm postestimation, \([R]\) inequality, \([R]\) kwallis, \([\mathrm{R}]\) logistic postestimation, \([\mathrm{R}]\) logit postestimation, \([R]\) margins, \([R]\) mkspline, \([R]\) ranksum, \([R]\) signrank, \([R]\) spearman, [R] tabulate twoway
Newton, H. J., \([\mathrm{R}]\) kdensity
Neyman, J., [R] ci
Ng, E. S.-W., [R] bootstrap, [R] bstat
Nicewander, W. A., [R] correlate
Nichols, A., [R] ivregress, [R] reg3
Nickell, S. J., [R] gmm
Nolan, D., [R] diagnostic plots
Norton, S. J., [R] rocreg, [R] rocreg postestimation, \([\mathrm{R}]\) rocregplot

\section*{0}

O'Fallon, W. M., [R] logit
O'Neill, D., [R] gmm, [R] inequality
Oehlert, G. W., \([\mathrm{R}]\) nlcom, \([\mathrm{R}]\) rocreg postestimation, [R] rocregplot
Olivier, D., [R] expoisson
Olkin, I., [R] kwallis
Olson, J. M., [R] symmetry
Ord, J. K., \([R]\) centile, \([R]\) mean, \([R]\) proportion, \([R]\) qreg, \([R]\) ratio, \([R]\) summarize, \([R]\) total
Orsini, N., \([R]\) glm, \([R]\) logit, \([R]\) mkspline, \([R]\) qreg
Ostle, B., \([R]\) anova postestimation
Over, M., \([\mathrm{R}]\) regress

\section*{P}

Pacheco, J. M., [R] dstdize
Pagan, A. R., \([R]\) frontier, \([R]\) regress postestimation, \([\mathrm{R}]\) sureg
Pagano, M., \([R]\) dstdize, \([R]\) logistic, \([R]\) margins, \([\mathrm{R}]\) tabulate twoway
Paik, M. C., \([\mathrm{R}]\) dstdize, \([\mathrm{R}]\) kappa
Palmer, T. M., \([\mathrm{R}]\) ivregress
Pampel, F. C., \([\mathrm{R}]\) logistic, \([\mathrm{R}]\) logit, \([\mathrm{R}]\) probit
Panis, C., [R] mkspline
Park, H. J., [R] regress
Park, J. Y., [R] boxcox, [R] margins, [R] nlcom, \([R]\) predictnl, \([R]\) rocreg postestimation, \([R]\) rocregplot, \([R]\) testnl
Parks, W. P., [R] exlogistic
Parner, E. T., [R] glm

Parzen, E., [R] estat ic, [R] kdensity
Pasquini, J., [R] vwls
Patel, N. R., [R] exlogistic, [R] exlogistic postestimation, [R] expoisson, \([R]\) tabulate twoway
Patterson, H. D., [R] pkcross
Paul, C., [R] logistic
Pearce, M. S., [R] logistic
Pearson, E. S., [R] ci, [R] ttest
Pearson, K., [R] correlate, \([R]\) correlate, \([R]\) esize, [R] tabulate twoway
Penfield, R. D., [R] esize
Pepe, M. S., [R] roc, [R] roccomp, [R] rocfit,
\([R]\) rocreg, \([R]\) rocreg postestimation,
\([R]\) rocregplot, \([R]\) roctab
Peracchi, F., [R] regress, [R] regress postestimation
Pérez-Hernández, M. A., [R] kdensity
Pérez-Hoyos, S., [R] Irtest
Perkins, A. M., [R] ranksum
Perotti, V., \([R]\) heckoprobit, \([R]\) heckprobit, [R] oprobit
Perrin, E., \([R]\) lincom, \([R]\) mlogit, \([R]\) mprobit, \([R]\) mprobit postestimation, \([R]\) predictnl, [R] slogit
Pesarin, F., [R] tabulate twoway
Peterson, B., \([\mathrm{R}]\) ologit
Peterson, W. W., [R] Iroc
Petitclerc, M., [R] kappa
Petkova, E., [R] suest
Petrin, A. K., [R] frontier
Pfeffer, R. I., [R] symmetry
Phillips, P. C. B., [R] boxcox, [R] margins, [R] nlcom, \([R]\) predictnl, \([R]\) regress postestimation time series, \([R]\) rocreg postestimation, \([R]\) rocregplot, \([R]\) testnl
Pickles, A., [R] gllamm, [R] glm
Pike, M. C., [R] symmetry
Pindyck, R. S., [R] biprobit, [R] heckprobit
Pischke, J.-S., [R] ivregress, \([R]\) ivregress postestimation, \([R]\) qreg, \([R]\) regress
Pitblado, J. S., [R] frontier, \([R]\) gmm, \([R]\) lpoly, \([R]\) maximize, \([R]\) ml, \([R]\) mlexp
Plackett, R. L., [R] ameans, [R] regress, [R] rologit, \([R]\) summarize, \([R]\) ttest
Plummer, W. D., Jr., [R] sunflower
Poi, B. P., [R] bootstrap, [R] bstat, [R] frontier, \([R]\) gmm, \([R]\) ivregress, \([R]\) ivregress postestimation, \([\mathrm{R}]\) maximize, \([\mathrm{R}] \mathrm{ml}\), \([R]\) mlexp, \([R]\) nl, \([R]\) nlsur, \([R]\) reg3
Poirier, D. J., [R] biprobit
Poisson, S. D., \([\mathrm{R}]\) poisson
Pollock, P. H., III, [R] histogram
Ponce de Leon, A., [R] roccomp, [R] roctab
Porter, T. M., [R] correlate
Powers, D. A., [R] logistic postestimation, [R] logit, [R] logit postestimation, [R] probit
Preacher, K. J., \([R]\) esize, \([R]\) regress postestimation Preece, D. A., [R] ttest

Pregibon, D., [R] glm, [R] linktest, [R] logistic, \([R]\) logistic postestimation, [R] logit, [R] logit postestimation
Press, W. H., [R] dydx, [R] vwls
Punj, G. N., [R] rologit

\section*{R}

Rabe-Hesketh, S., [R] gllamm, [R] glm,
\([R]\) heckoprobit, \([R]\) heckprobit, \([R]\) ivprobit,
\([R]\) ivtobit, [ \(R\) ] logistic, [ \(R\) ] logit, [ \(R\) ] nbreg,
\([R]\) ologit, \([R]\) oprobit, \([R]\) poisson, \([R]\) probit
Raciborski, R., [R] poisson, [R] tpoisson
Raftery, A. E., [R] BIC note, [R] estat ic, [R] glm
Ramalheira, C., [R] ameans
Ramsahai, R. R., [R] ivregress
Ramsey, J. B., [R] regress postestimation
Ratkowsky, D. A., [R] nl, [R] pk, [R] pkcross
Redelmeier, D. A., [R] brier
Reeves, D., [R] meta
Reichenheim, M. E., [R] kappa, \([\mathrm{R}]\) roccomp, \([R]\) roctab
Reid, C., \([\mathrm{R}]\) ci
Reilly, M., [R] logistic
Relles, D. A., [R] rreg
Rencher, A. C., [R] anova postestimation
Revankar, N. S., [R] frontier
Richardson, W., [R] ttest
Riffenburgh, R. H., [R] ksmirnov, [R] kwallis
Riley, A. R., [R] net search
Ringquist, E. J., [R] meta
Rivers, D., [R] ivprobit
Roberson, P. K., [R] estat gof, [R] Iroc
Robyn, D. L., [R] cumul
Rodgers, J. L., [R] correlate
Rodríguez, G., \([R]\) nbreg, \([R]\) poisson
Rogers, W. H., [R] brier, [R] glm, [R] heckman,
\([R]\) lincom, \([R]\) mlogit, \([R]\) mprobit,
\([R]\) mprobit postestimation, \([R]\) nbreg,
\([R]\) poisson, \([R]\) predictnl, \([R]\) qreg, \([R]\) regress,
\([R]\) rocreg, \([R]\) rreg, \([R]\) sktest, \([R]\) slogit,
[R] suest
Ronning, G., [R] clogit
Rose, J. M., [R] nlogit
Rosenthal, R., [R] contrast
Rosnow, R. L., [R] contrast
Ross, G. J. S., [R] nl
Rossi, P. E., [R] sureg
Rothman, K. J., [R] ci, [R] dstdize, [R] glogit, \([R]\) poisson
Rothstein, H. R., [R] meta
Rousseeuw, P. J., \([R]\) qreg, \([R]\) regress postestimation, [R] rreg
Rovine, M. J., [R] correlate

Royston, P., [R] bootstrap, [R] centile, [R] cusum,
\([R]\) diagnostic plots, \([R]\) dotplot, \([R]\) dydx,
\([R]\) estat ic, \([R] \mathbf{f p},[R] \mathbf{f p}\) postestimation,
\([R]\) glm, \([R]\) kdensity, \([R]\) Inskew \(0,[R]\) lowess,
\([R]\) marginsplot, \([R] \mathbf{m f p},[R] \mathbf{m l},[R] \mathbf{n l}\),
\([R]\) regress, \([R]\) sktest, \([R]\) smooth, \([R]\) swilk
Rubin, D. B., \([\mathrm{R}]\) contrast
Rubin, H., [R] ivregress postestimation
Rubinfeld, D. L., [R] biprobit, [R] heckprobit
Rudebusch, G. D., [R] ivregress postestimation
Ruppert, D., [R] boxcox, [R] rreg
Rutherford, E., [R] poisson
Rutherford, M. J., [R] poisson
Ruud, P. A., [R] gmm, [R] rologit, [R] suest
Ryan, T. P., [R] qc

\section*{S}

Sajaia, Z., [R] biprobit, [R] heckprobit
Sakamoto, Y., [R] BIC note
Salgado-Ugarte, I. H., [R] kdensity, [R] lowess, [R] smooth
Salim, A., [R] logistic
Sanders, F., [R] brier
Santos Silva, J. M. C., [R] gmm, [R] ivpoisson
Sargan, J. D., [R] ivregress postestimation
Sasieni, P. D., [R] dotplot, [R] glm, [R] lowess,
\([R]\) nptrend, \([R]\) poisson, \([R]\) smooth
Sass, T. R., [R] areg
Satterthwaite, F. E., \([R]\) esize, \([R]\) ttest
Sauerbrei, W., [R] bootstrap, [R] estat ic, [R] fp, \([R] \mathbf{m f p}\)
Savin, N. E., [R] regress postestimation time series
Saw, S. L. C., [R] qc
Sawa, T., [R] estat ic
Saxl, I., [R] correlate
Schaalje, G. B., [R] anova postestimation
Schaffer, M. E., [R] ivregress, [R] ivregress postestimation
Scheffé, H., [R] anova, [R] oneway
Schlesselman, J. J., [R] boxcox
Schlossmacher, E. J., [R] qreg
Schmidt, C. H., [R] brier
Schmidt, P., [R] frontier, \([R]\) regress postestimation
Schneider, H., [R] sdtest
Schnell, D., [R] regress
Schonlau, M., \([R]\) glm, \([R]\) logistic, \([R]\) logit, \([R]\) poisson, \([R]\) regress
Schuirmann, D. J., \([R]\) pkequiv
Schwarz, G., [R] BIC note, [R] estat ic
Scott, D. W., [R] kdensity
Scott, E. L., [R] intro
Scott, G. B., [R] exlogistic
Scotto, M. G., [R] diagnostic plots
Searle, S. R., [R] contrast, [R] margins, [R] pwcompare, \([R]\) pwmean
Seed, P. T., [R] ci, [R] correlate, [R] roccomp, \([R]\) roctab, [R] sdtest, [R] spearman

Seidler, J., [R] correlate
Selvin, S., [R] poisson
Sempos, C. T., [R] dstdize
Semykina, A., [R] inequality, [R] qreg
Seneta, E., [R] correlate
Senn, S. J., [R] glm, [R] ttest
Shapiro, S. S., [R] swilk
Shea, J. S., [R] ivregress postestimation
Sheather, S. J., [R] boxcox, [R] lowess, [R] lpoly, \([R]\) qreg, \([R]\) regress postestimation, \([R]\) regress postestimation diagnostic plots, \([R]\) stepwise
Sheehan, N. A., [R] ivregress
Sheldon, T. A., \([\mathrm{R}]\) meta
Shewhart, W. A., [R] qc
Shiboski, S. C., [R] logistic
Shiller, R. J., [R] tobit
Shimizu, M., [R] kdensity, [R] lowess
Shrout, P. E., [R] icc, [R] kappa
Šidák, \(Z\)., \([R]\) correlate, \([R]\) oneway
Silverman, B. W., [R] kdensity, [R] qreg
Silvey, S. D., [R] ologit, [R] oprobit
Simonoff, J. S., [R] kdensity, \([R]\) tnbreg, \([R]\) tpoisson
Simor, I. S., [R] kappa
Singleton, K. J., [R] gmm
Sininger, Y., \([R]\) rocreg, \([R]\) rocreg postestimation, \([R]\) rocregplot
Sitgreaves, R., [R] icc
Sjölander, P. C., [R] glm, [R] logit
Skrondal, A., [R] gllamm, [R] glm
Smeeton, N. C., [R] ranksum, [R] signrank
Smirnov, N. V., [R] ksmirnov
Smith, C. A. B., [R] ranksum
Smith, H., [R] eivreg, [R] oneway, [R] regress, \([R]\) stepwise
Smith, J. M., [R] fp
Smith, M. L., [R] esize
Smith, R. J., [R] ivprobit
Smithson, M., [R] esize, [R] regress postestimation
Snedecor, G. W., [R] ameans, [R] anova, [R] correlate, \([R]\) oneway, \([R]\) ranksum, \([R]\) signrank
Snell, E. J., [R] exlogistic, [R] expoisson
Song, F., [R] meta
Soon, T. W., [R] qc
Spearman, C. E., [R] icc, [R] spearman
Speed, F. M., [R] margins
Speed, T., [R] diagnostic plots
Spiegelhalter, D. J., [R] brier
Spieldman, R. S., [R] symmetry
Spitzer, J. J., [R] boxcox
Sprent, P., [R] ranksum, [R] signrank
Sribney, W. M., [R] orthog, [R] ranksum, \([R]\) signrank, \([R]\) stepwise, \([R]\) test
Staelin, R., [R] rologit
Staiger, D. O., [R] ivregress postestimation
Starmer, C. F., [R] vwls
Startz, R., [R] ivregress postestimation
Stegun, I. A., [R] contrast, [R] orthog

Steichen, T. J., [R] kappa, [R] kdensity, [R] sunflower
Steiger, J. H., [R] esize
Steiger, W., [R] qreg
Stein, C., [R] bootstrap
Stephenson, D. B., [R] brier
Stepniewska, K. A., [R] nptrend
Sterne, J. A. C., \([R]\) dstdize, \([R]\) meta, \([R]\) summarize
Stevenson, R. E., [R] frontier
Stewart, M. B., \([\mathrm{R}]\) intreg, \([\mathrm{R}]\) oprobit, \([\mathrm{R}]\) tobit
Stigler, S. M., [R] ameans, [R] ci, [R] correlate,
\([R]\) kwallis, \([R]\) qreg, \([R]\) regress,
\([R]\) summarize
Stillman, S., [R] ivregress, [R] ivregress postestimation
Stine, R., [R] bootstrap
Stock, J. H., [R] areg postestimation, [R] ivregress,
\([R]\) ivregress postestimation
Stoto, M. A., [R] lv
Stover, L., [R] rocreg, [R] rocreg postestimation,
\([R]\) rocregplot
Street, J. O., \([\mathrm{R}]\) rreg
Stryhn, H., [R] regress
Stuart, A., \([R]\) centile, \([R]\) mean, \([R]\) proportion,
\([R]\) qreg, \([R]\) ratio, \([R]\) summarize,
\([R]\) symmetry, \([R]\) total
Student, see Gosset, W. S.
Stuetzle, W., [R] sunflower
Sturdivant, R. X., [R] clogit, [R] clogit postestimation,
\([R]\) estat classification, \([R]\) estat gof,
\([R]\) glm, \([R]\) glogit, \([R]\) lincom, \([R]\) logistic,
\([R]\) logistic postestimation, \([R]\) logit, \([R]\) logit postestimation, \([R]\) Iroc, \([R]\) Irtest, \([R]\) Isens, \([R]\) mlogit, \([R]\) predictnl, \([R]\) stepwise
Suárez, C., [R] heckoprobit, [R] heckprobit
Suen, H. K., [R] icc
Sullivan, G., [R] regress
Sutton, A. J., [R] meta
Swed, F. S., [R] runtest
Sweetman, O., [R] gmm, [R] inequality
Swets, J. A., [R] lroc
Szroeter, J., [R] regress postestimation

\section*{T}

Taka, M. T., [R] pkcross
Tamhane, A. C., [R] oneway
Taniuchi, T., [R] kdensity
Tanner, W. P., Jr., [R] Iroc
Tanur, J. M., [R] kwallis
Tapia, R. A., [R] kdensity
Tarlov, A. R., [R] lincom, [R] mlogit, \([R]\) mprobit,
\([R]\) mprobit postestimation, \([R]\) predictnl, [R] slogit
Tauchmann, H., [R] frontier
Taylor, C., [R] gllamm, [R] glm
Teukolsky, S. A., [R] dydx, [R] vwls
Theil, H., [R] ivregress, [R] reg3
Thiele, T. N., [R] summarize
Thompson, B., \([\mathrm{R}]\) esize, \([\mathrm{R}]\) regress postestimation

Thompson, J. C., [R] diagnostic plots
Thompson, J. R., [R] kdensity, [R] poisson
Thompson, M. L., [R] rocreg
Thorndike, F., [R] poisson
Thurstone, L. L., [R] rologit
Tibshirani, R. J., [R] bootstrap, [R] qreg
Tidmarsh, C. E., [R] fp
Tilford, J. M., \([R]\) estat gof, \([R]\) Iroc
Tobías, A., \([R]\) lrtest, \([R]\) poisson, \([R]\) roccomp, \([R]\) roctab, \([R]\) sdtest
Tobin, J., \([\mathrm{R}]\) tobit
Toman, R. J., [R] stepwise
Tong, H., [R] estat ic
Toplis, P. J., [R] binreg
Tosetto, A., [R] logistic, [R] logit
Train, K. E., [R] asmprobit
Trapido, E., [R] exlogistic
Treiman, D. J., [R] eivreg, [R] mlogit
Trivedi, P. K., [R] asclogit, [R] asmprobit, \([R]\) bootstrap, \([R]\) gmm, \([R]\) heckman, \([R]\) heckoprobit, \([R]\) intreg, \([R]\) ivpoisson, \([R]\) ivregress, \([R]\) ivregress postestimation, \([R]\) logit, \([R]\) mprobit, \([R]\) nbreg, \([R]\) ologit, \([R]\) oprobit, \([R]\) poisson, \([R]\) probit, \([R]\) qreg, \([R]\) regress, \([R]\) regress postestimation, \([R]\) simulate, \([R]\) sureg, \([R]\) tnbreg, \([R]\) tobit, \([R]\) tpoisson, \([R]\) zinb postestimation, \([R]\) zip postestimation
Tsiatis, A. A., [R] exlogistic
Tufte, E. R., [R] stem
Tukey, J. W., [R] jackknife, [R] ladder, [R] linktest, \([R] \operatorname{lv},[R]\) regress, \([R]\) regress postestimation diagnostic plots, \([R]\) rreg, \([R]\) smooth, [R] spikeplot, [R] stem
Tukey, P. A., [R] diagnostic plots, \([R]\) lowess
Tyler, J. H., [R] regress

\section*{U}

Uebersax, J. S., [R] tetrachoric
Uhlendorff, A., \([\mathrm{R}]\) asmprobit, \([\mathrm{R}]\) mlogit, \([\mathrm{R}]\) mprobit University Group Diabetes Program, [R] glogit Utts, J. M., [R] ci

\section*{V}

Valman, H. B., [R] fp
van Belle, \(G .,[R]\) anova, \([R]\) dstdize, \([R]\) oneway
Van de Ven, W. P. M. M., [R] biprobit,
\([R]\) heckoprobit, \([R]\) heckprobit
van den Broeck, J., \([R]\) frontier
Van der Reyden, D., [R] ranksum
Van Kerm, P., \([R]\) inequality, \([R]\) kdensity
Van Loan, C. F., [R] orthog, [R] tetrachoric
Van Pragg, B. M. S., [R] biprobit, [R] heckoprobit, \([R]\) heckprobit
Velleman, P. F., [R] regress postestimation, [R] smooth Venables, W., [R] esize

Verardi, V., [R] correlate, [R] fp, [R] ivregress, [R] lpoly, [R] rreg
Vetterling, W. T., [R] dydx, [R] vwls
Vidmar, S., [R] ameans
Vittinghoff, E., [R] logistic
Vohr, B. R., [R] rocreg, [R] rocreg postestimation, [R] rocregplot
von Bortkiewicz, L., [R] poisson
von Eye, A., [R] correlate
Von Storch, H., [R] brier
Vondráček, J., \([\mathrm{R}]\) correlate
Vuong, Q. H., [R] ivprobit, [R] zinb, [R] zip

\section*{W}

Wacholder, S., [R] binreg
Wagner, H. M., [R] qreg
Wallis, W. A., [R] kwallis
Walters, S. J., [R] ci, [R] kappa, [R] tabulate twoway
Wand, M. P., [R] kdensity
Wang, D., [R] ci, [R] dstdize, [R] prtest
Wang, Q., [R] ivregress
Wang, Y., [R] asmprobit
Wang, Z., [R] logistic postestimation, [R] Irtest, \([R]\) stepwise
Ware, J. E., Jr., [R] lincom, [R] mlogit, \([\mathrm{R}]\) mprobit, \([R]\) mprobit postestimation, \([R]\) predictnl, [R] slogit
Waterson, E. J., [R] binreg
Watson, G. S., [R] lpoly, [R] regress postestimation time series
Watson, M. W., [R] areg postestimation, [R] ivregress
Weber, S., [R] correlate
Webster, A. D., [R] fp
Wedderburn, R. W. M., [R] glm
Weesie, J., [R] constraint, [R] hausman, \([R]\) ladder, \([R]\) reg3, \([R]\) regress, \([R]\) regress postestimation, \([R]\) rologit, \([R]\) simulate, \([R]\) suest, \([R]\) sureg, \([R]\) tabstat, \([R]\) tabulate twoway, \([R]\) test, \([R]\) tetrachoric
Weisberg, H. F., [R] summarize
Weisberg, S., [R] boxcox, \([R]\) regress, \([R]\) regress postestimation
Weiss, M., \([\mathrm{R}]\) estimates table
Weisstein, E. W., [R] rocreg postestimation
Welch, B. L., [R] esize, [R] ttest
Wellington, J. F., [R] qreg
Wells, K. B., [R] lincom, [R] mlogit, [R] mprobit, \([R]\) mprobit postestimation, \([R]\) predictnl, [R] slogit
Welsch, R. E., \([R]\) regress postestimation, \([R]\) regress postestimation diagnostic plots
Welsh, A. H., [R] bootstrap
West, K. D., \([\mathrm{R}]\) glm, \([\mathrm{R}]\) gmm, \([\mathrm{R}]\) ivregress
West, S. G., [R] pcorr
Westlake, W. J., [R] pkequiv
White, H. L., Jr., [R] regress, [R] regress postestimation, \([R]\) rocreg, \([R]\) suest

White, I. R., [R] meta, [R] simulate
White, K. J., \([R]\) boxcox, \([R]\) regress postestimation time series
Whitehouse, E., [R] inequality
Whitfield, J. W., [R] ranksum
Whiting, P., \([R]\) roccomp, \([R]\) roctab
Whitney, D. R., [R] kwallis, [R] ranksum
Widen, J. E., [R] rocreg, \([R]\) rocreg postestimation, \([\mathrm{R}]\) rocregplot
Wieand, S., [R] rocreg, [R] rocreg postestimation
Wiggins, V. L., [R] regress postestimation, [R] regress postestimation time series
Wilcox, D. W., [R] ivregress postestimation
Wilcoxon, F., [R] kwallis, [R] ranksum, [R] signrank
Wilde, J., [R] gmm
Wilk, M. B., \([\mathrm{R}]\) cumul, \([\mathrm{R}]\) diagnostic plots, \([R]\) swilk
Wilks, D. S., [R] brier
Williams, R., \([\mathrm{R}]\) glm, \([\mathrm{R}]\) margins, \([\mathrm{R}]\) marginsplot, \([R]\) ologit, \([R]\) oprobit, \([R]\) pcorr, \([R]\) stepwise
Wilson, E. B., [R] ci
Wilson, S. R., [R] bootstrap
Windmeijer, \(F\)., \([R]\) gmm, \([R]\) ivpoisson
Winer, B. J., [R] anova, [R] contrast, [R] loneway, \([R]\) oneway, \([R]\) pwcompare
Wolfe, F., [R] correlate, [R] spearman
Wolfe, R., [R] ologit, [R] oprobit, [R] tabulate twoway
Wolfson, C., [R] kappa
Wolpin, K. I., [R] asmprobit
Wong, S. P., [R] icc
Wood, F. S., [R] diagnostic plots
Woodard, D. E., [R] contrast
Wooldridge, J. M., [R] areg postestimation, \([\mathrm{R}]\) gmm, \([R]\) heckoprobit, \([R]\) intreg, \([R]\) ivpoisson, \([R]\) ivprobit, \([R]\) ivregress, \([R]\) ivregress postestimation, \([R]\) ivtobit, \([R]\) margins, \([R]\) margins, contrast, \([R]\) qreg, \([R]\) regress, \([R]\) regress postestimation, \([R]\) regress postestimation time series, \([R]\) tobit
Working, H., [R] roccomp, [R] rocfit, [R] roctab
Wright, J. H., [R] ivregress, [R] ivregress postestimation
Wright, J. T., [R] binreg
Wright, P. G., [R] ivregress
Wu, C. F. J., [R] qreg
Wu, D.-M., [R] ivregress postestimation
Wu, N., [R] ivregress

\section*{X}

Xie, Y., [R] logit, [R] probit
Xu, J., [R] cloglog, [R] logistic, [R] logit, [R] mlogit, [R] ologit, [R] oprobit, [R] probit

\section*{Y}

Yang, Z., [R] poisson
Yates, J. F., [R] brier
Yee, T. W., [R] slogit

Yellott, J. I., Jr., [R] rologit
Yogo, M., [R] ivregress, [R] ivregress postestimation
Yoshioka, H., [R] logistic postestimation, [R] logit postestimation
Yun, M.-S., [R] logistic postestimation, [R] logit postestimation

\section*{Z}

Zabell, S. L., [R] kwallis
Zamora, M., [R] heckoprobit, [R] heckprobit
Zavoina, W., [R] ologit
Zelen, M., [R] ttest
Zellner, A., \([R]\) frontier, \([R]\) nlsur, \([R]\) reg3, \([R]\) sureg
Zelterman, D., \([\mathrm{R}]\) tabulate twoway
Zheng, X., [R] gllamm
Zimmerman, F., [R] regress
Zubkoff, M., [R] lincom, [R] mlogit, [R] mprobit,
\([R]\) mprobit postestimation, \([R]\) predictnl,
[R] slogit
Zucchini, W., [R] rocreg
Zwiers, F. W., [R] brier

\section*{Subject index}

This is the subject index for the Base Reference Manual. Readers may also want to consult the combined subject index (and the combined author index) in the Glossary and Index.

\section*{A}
about command, \([\mathrm{R}]\) about
absorption in regression, \([R]\) areg
acprplot command, \([R]\) regress postestimation diagnostic plots
added-variable plots, \([\mathrm{R}]\) regress postestimation diagnostic plots
adjusted
margins, \([R]\) margins, \([R]\) marginsplot
means, [R] contrast, [R] margins, [R] marginsplot
partial residual plot, \([R]\) regress postestimation diagnostic plots
ado command, \([R]\) net
ado describe command, [R] net
ado dir command, [R] net
ado uninstall command, \([R]\) net
ado, view subcommand, \([R]\) view
ado_d, view subcommand, \([R]\) view
ado-files,
editing, \([R]\) doedit
installing, \([R]\) net, \([R]\) sj, \([R]\) ssc
location of, \([R]\) which
official, \([R]\) update
searching for, \([R]\) search, \([R]\) ssc
updating user-written, \([R]\) adoupdate
adosize, set subcommand, [R] set
adoupdate command, \([R]\) adoupdate
agreement, interrater, [R] kappa
AIC, see Akaike information criterion
Akaike information criterion, \([R]\) BIC note, \([R]\) estat, \([R]\) estat ic, \([R]\) estimates stats, \([R]\) glm, [R] Irtest
all, update subcommand, [R] update
alternative-specific
conditional logit (McFadden's choice) model, \([R]\) asclogit
multinomial probit regression, [R] asmprobit
rank-ordered probit regression, [R] asroprobit
alternatives, estat subcommand, \([R]\) asclogit postestimation, \([R]\) asmprobit postestimation, [R] asroprobit postestimation, [R] nlogit postestimation
ameans command, \([R]\) ameans
analysis of covariance, \([R]\) anova
analysis of variance, \([R]\) anova, \([R]\) contrast, \([R]\) icc, \([R]\) loneway, \([R]\) oneway
Kruskal-Wallis, [R] kwallis
plots, \([R]\) marginsplot
repeated measures, \([R]\) anova
analysis-of-variance test of normality, \([R]\) swilk

ANCOVA, see analysis of covariance
ANOVA, see analysis of variance
anova command, \([R]\) anova, \([R]\) anova postestimation
ARCH effects, testing for, \([\mathrm{R}]\) regress postestimation time series
archlm, estat subcommand, [R] regress
postestimation time series
area under the curve, \([R]\) lroc, also see pharmacokinetic data, also see receiver operating characteristic analysis
areg command, \([R]\) areg, \([R]\) areg postestimation
asclogit command, [R] asclogit, [R] asclogit postestimation
asmprobit command, [R] asmprobit, [R] asmprobit postestimation
asroprobit command, [R] asroprobit, [R] asroprobit postestimation
association test, \([R]\) correlate, \([R]\) spearman, \([R]\) tabulate twoway, \([R]\) tetrachoric
association, measures of, \([R]\) tabulate twoway
asymmetry, see skewness
AUC, also see area under the curve
augmented
component-plus-residual plot, \([\mathrm{R}]\) regress postestimation diagnostic plots
partial residual plot, [R] regress postestimation diagnostic plots
autocorrelation, \([\mathrm{R}]\) regress postestimation time series, also see HAC variance estimate
autoregressive conditional heteroskedasticity test, [R] regress postestimation time series
autotabgraphs, set subcommand, [R] set
average
marginal effects, \([R]\) margins, \([R]\) marginsplot
partial effects (APEs), \([R]\) margins, \([R]\) marginsplot
predictions, \([R]\) margins, \([R]\) marginsplot
averages, see means
avplot and avplots commands, [R] regress postestimation diagnostic plots

\section*{B}
backed up message, \([R]\) maximize
Bartlett's test for equal variances, \([R]\) oneway
base, fvset subcommand, \([R]\) fvset
Bayesian information criterion, [R] BIC note, \([R]\) estat, \([R]\) estat ic, \([R]\) estimates stats, \([R]\) glm, [R] lrtest
bcskew0 command, [R] Inskew0
Berndt-Hall-Hall-Hausman algorithm, [R] ml
beta coefficients, \([R]\) regress
BFGS algorithm, see Broyden-Fletcher-GoldfarbShanno algorithm
bgodfrey, estat subcommand, [R] regress postestimation time series
BHHH algorithm, see Berndt-Hall-Hall-Hausman algorithm
bias corrected and accelerated, \([\mathrm{R}]\) bootstrap postestimation, [R] bstat

BIC, see Bayesian information criterion
Bickenböller test statistic, [R] symmetry
binary outcome model, see outcomes, binary
binomial
distribution, confidence intervals, \([R]\) ci
family regression, \([R]\) binreg
probability test, [R] bitest
binreg command, \([\mathrm{R}]\) binreg, \([\mathrm{R}]\) binreg postestimation
bioequivalence test, \([\mathrm{R}] \mathbf{p k},[\mathrm{R}]\) pkequiv
biopharmaceutical data, see pharmacokinetic data
biprobit command, [R] biprobit, [R] biprobit postestimation
bitest and bitesti commands, [R] bitest
bivariate probit regression, \([R]\) biprobit
biweight kernel function, \([R]\) kdensity, \([R]\) Ipoly, [R] qreg
biweight regression estimates, \([R]\) rreg
blogit command, [R] glogit, [R] glogit postestimation
Bonferroni's multiple-comparison adjustment, see multiple comparisons, Bonferroni's method
bootstrap
sampling and estimation, \([\mathrm{R}]\) bootstrap, \([R]\) bsample, \([R]\) bstat, \([R]\) qreg, \([R]\) rocreg, \([\mathrm{R}]\) simulate
standard errors, [R] vce_option
bootstrap prefix command, \([\mathrm{R}]\) bootstrap, \([\mathrm{R}]\) bootstrap postestimation
bootstrap, estat subcommand, [R] bootstrap postestimation
Boston College archive, see Statistical Software Components archive
Box-Cox
power transformations, [R] lnskew0
regression, [R] boxcox
boxcox command, [R] boxcox, [R] boxcox postestimation
Box's conservative epsilon, [R] anova
bprobit command, [R] glogit, [R] glogit postestimation
Breusch-Godfrey test, [R] regress postestimation time series
Breusch-Pagan test, [R] sureg
Breusch-Pagan/Cook-Weisberg test for heteroskedasticity, \([R]\) regress postestimation
brier command, [R] brier
Brier score decomposition, \([\mathrm{R}]\) brier
browse, view subcommand, [R] view
Broyden-Fletcher-Goldfarb-Shanno algorithm, [R] ml bsample command, [R] bsample
bsqreg command, [R] qreg, [R] qreg postestimation bstat command, [R] bstat

\section*{C}
\(c\) (cformat) c-class value, \([R]\) set cformat
\(c\) (pformat) c-class value, \([R]\) set cformat
\(c\) (seed) c-class value, \([R]\) set emptycells, \([R]\) set seed \(c\) (sformat) c-class value, \([R]\) set cformat
c(showbaselevels) c-class value, \([\mathrm{R}]\) set showbaselevels
c (showemptycells) c-class value, \([\mathrm{R}]\) set showbaselevels
\(c\) (showomitted) c-class value, \([R]\) set showbaselevels calculator, \([R]\) display
carryover effects, [R] pk, [R] pkcross, [R] pkshape
case-control data, \([R]\) clogit, \([R]\) logistic, \([R]\) rocreg, [R] symmetry
categorical, also see factor variables
contrasts after anova, [R] contrast
covariates, \([\mathrm{R}]\) anova
data, agreement, measures for, [R] kappa
graphs, \([R]\) grmeanby, \([R]\) spikeplot
outcomes, see outcomes, categorical, also see outcomes, binary, also see outcomes, ordinal
regression, also see outcomes subentry absorbing one categorical variable, \([\mathrm{R}]\) areg
tabulations, \([R]\) table, \([R]\) tabstat, \([R]\) tabulate oneway, \([R]\) tabulate twoway, \([R]\) tabulate, summarize()
variable creation, \([R]\) tabulate oneway, \([R]\) xi
cchart command, [R] qc
\(c d\), net subcommand, \([R]\) net
censored observations, [R] heckman, \([R]\) heckoprobit, \([R]\) heckprobit, \([R]\) intreg, \([R]\) ivtobit, \([R]\) tobit, also see truncated observations
censored-normal regression, see interval regression
centile command, \([\mathrm{R}]\) centile
centiles, see percentiles, displaying
central tendency, measures of, see means, see medians
cformat, set subcommand, \([R]\) set, \([R]\) set cformat
charset, set subcommand, \([R]\) set
check, ml subcommand, [R] ml
checksum, set subcommand, [R] set
chi-squared
hypothesis test, \([R]\) hausman, \([R]\) lrtest, \([R]\) sdtest,
\([R]\) tabulate twoway, \([R]\) test, \([R]\) testnl
probability plot, \([R]\) diagnostic plots
quantile plot, \([R]\) diagnostic plots
test for marginal homogeneity, \([R]\) symmetry
test of independence, \([R]\) tabulate twoway
choice models, \([R]\) asclogit, \([R]\) asmprobit,
\([R]\) asroprobit, [R] clogit, [R] cloglog,
\([R]\) exlogistic, \([R]\) glm, \([R]\) glogit,
\([R]\) heckoprobit, \([R]\) heckprobit, \([R]\) hetprobit,
\([R]\) ivprobit, [ \(R\) ] logistic, [ \(R\) ] logit, [ \(R\) ] mlogit,
\([R]\) mprobit, [ \(R\) ] nlogit, \([R]\) ologit, \([R]\) oprobit,
[R] probit, [R] rologit, [R] scobit, [R] slogit,
[R] suest
Chow test, \([\mathrm{R}]\) anova, \([\mathrm{R}]\) contrast, \([\mathrm{R}]\) Irtest
ci and cii commands, [R] ci
classification
data, see receiver operating characteristic analysis
interrater agreement, [R] kappa
table, \([R]\) estat classification
classification, estat subcommand, [R] estat classification
clear,
estimates subcommand, [R] estimates store
fvset subcommand, [R] fvset
ml subcommand, [R] ml
clearing estimation results, \([\mathrm{R}]\) estimates store
clogit command, [R] bootstrap, [R] clogit, [R] clogit
postestimation, \([R]\) exlogistic, \([R]\) rologit
cloglog command, \([\mathrm{R}]\) cloglog, \([\mathrm{R}]\) cloglog postestimation
close,
cmdlog subcommand, [R] log
log subcommand, \([R] \log\)
cls command, \([\mathrm{R}]\) cls
cluster estimator of variance, \([\mathrm{R}]\) vee_option
alternative-specific
conditional logit model, \([\mathrm{R}]\) asclogit
multinomial probit regression, \([\mathrm{R}]\) asmprobit
rank-ordered probit regression, [R] asroprobit
complementary log-log regression, [R] cloglog
generalized linear models, [R] glm
for binomial family, \([R]\) binreg
generalized method of moments, \([R]\) gmm, [R] ivpoisson
heckman selection model, \([\mathrm{R}]\) heckman
instrumental-variables regression, [R] ivregress
interval regression, \([R]\) intreg
linear regression, \([R]\) regress constrained, \([\mathrm{R}]\) ensreg truncated, \([R]\) truncreg with dummy-variable set, \([\mathrm{R}]\) areg
logistic regression, [R] logistic, [R] logit, also see logit regression subentry conditional, [R] clogit multinomial, \([\mathrm{R}]\) mlogit ordered, \([\mathrm{R}]\) ologit rank-ordered, [R] rologit skewed, [R] scobit stereotype, [R] slogit
logit regression, \([\mathrm{R}]\) logit, also see logistic regression subentry
for grouped data, \([R]\) glogit nested, [R] nlogit
maximum likelihood estimation, \([R] \mathbf{m l},[\mathrm{R}]\) mlexp multinomial logistic regression, \([\mathrm{R}]\) mlogit probit regression, \([\mathrm{R}]\) mprobit
negative binomial regression truncated, \([R]\) nbreg zero-inflated, [R] zinb
nonlinear
least-squares estimation, [R] nl systems of equations, \([R]\) nlsur
Poisson regression, \([R]\) poisson truncated, \([R]\) tpoisson with endogenous regressors, \([R]\) ivpoisson zero-inflated, [R] zip
cluster estimator of variance, continued
probit regression, \([\mathrm{R}]\) probit
bivariate, \([\mathrm{R}]\) biprobit
for grouped data, \([\mathrm{R}]\) glogit
heteroskedastic, \([R]\) hetprobit
multinomial, \([\mathrm{R}]\) mprobit
ordered, [R] oprobit ordered heckman selection model,
[R] heckoprobit with endogenous regressors, \([R]\) ivprobit with sample selection, \([R]\) heckprobit
summary statistics,
mean, \([R]\) mean
proportion, \([\mathrm{R}]\) proportion
ratio, \([R]\) ratio
total, \([\mathrm{R}]\) total
tobit model, \([\mathrm{R}]\) tobit
with endogenous regressors, \([\mathrm{R}]\) ivtobit
truncated
negative binomial regression, \([\mathrm{R}]\) tnbreg
Poisson regression, [R] tpoisson
regression, \([R]\) truncreg
with endogenous regressors,
instrumental-variables regression, \([R]\) ivregress
Poisson regression, \([R]\) ivpoisson
probit model, \([R]\) ivprobit
tobit model, [R] ivtobit
zero-inflated
negative binomial regression, \([R]\) zinb
Poisson regression, [R] zip
cluster sampling, \([R]\) bootstrap, \([R]\) bsample, [R] jackknife
cmdlog
close command, \([\mathrm{R}] \log\)
command, \([\mathrm{R}] \log\)
off command, \([R] \log\)
on command, \([\mathrm{R}] \log\)
using command, [R] log
cnsreg command, \([R]\) ensreg, \([R]\) ensreg postestimation
coefficient of variation, \([\mathrm{R}]\) tabstat
coefficients (from estimation),
cataloging, \([R]\) estimates
linear combinations of, see linear combinations of estimators
nonlinear combinations of, see nonlinear combinations of estimators
testing equality of, \([R]\) test, \([R]\) testnl
coeftabresults, set subcommand, \([R]\) set
collinearity,
display of omitted variables, [R] set showbaselevels
handling by regress, \([R]\) regress
retaining collinear variables, \([\mathrm{R}]\) estimation options, [R] orthog
variance inflation factors, \([R]\) regress postestimation
command line, launching dialog box from, \([\mathrm{R}] \mathbf{d b}\)
commands, reviewing, [R] \#review
comparative scatterplot, [R] dotplot
comparison test between nested models, \([R]\) nestreg
complementary log-log regression, [R] cloglog, [R] glm
completely determined outcomes, \([R]\) logit
component-plus-residual plot, \([\mathrm{R}]\) regress
postestimation diagnostic plots
conditional
logistic regression, \([R]\) asclogit, \([R]\) clogit,
\([R]\) rologit, \([R]\) slogit
marginal effects, \([R]\) margins, \([R]\) marginsplot
margins, \([R]\) margins, \([R]\) marginsplot
confidence interval
for bioequivalence, \([R]\) pkequiv
for bootstrap statistics, \([\mathrm{R}]\) bootstrap
postestimation, \([R]\) rocreg, \([R]\) rocreg
postestimation
for combinations of coefficients,
linear, \([R]\) lincom
nonlinear, \([R]\) nlcom
for contrasts, \([R]\) contrast
for counts, \([\mathrm{R}]\) ci
for false-positive rates, \([R]\) rocregplot
for incidence-rate ratios, \([R]\) expoisson, \([R]\) glm, \([R]\) nbreg, \([R]\) poisson, \([R]\) tnbreg, \([R]\) tpoisson, [R] zinb, [R] zip
for intragroup correlations, \([R]\) loneway
for margins, \([R]\) margins
for means, \([R]\) ci, \([R]\) ameans, \([R]\) esize, \([R]\) mean, [R] ttest
for medians and percentiles, \([R]\) centile
for odds ratios, \([R]\) exlogistic, \([R]\) glm, \([R]\) glogit,
\([R]\) logistic, [R] logit, [R] ologit, [R] scobit
for proportions, \([R] \mathbf{c i},[R]\) proportion
for ratios, \([R]\) ratio
for relative-risk ratios, \([R]\) mlogit
for \(R O C\) area, \([R]\) roccomp, \([R]\) rocfit, \([R]\) rocreg, [R] roctab
for ROC values, \([R]\) rocregplot
for standardized mortality ratios, \([\mathrm{R}]\) dstdize
for totals, \([R]\) total
confidence interval, set default, \([R]\) level
confidence levels, \([\mathrm{R}]\) level
conjoint analysis, \([R]\) rologit
conren, set subcommand, \([R]\) set
console, controlling scrolling of output, \([R]\) more
constrained estimation, \([\mathrm{R}]\) constraint, \([\mathrm{R}]\) estimation options
alternative-specific
conditional logistic model, \([\mathrm{R}]\) asclogit
multinomial probit regression, [R] asmprobit
rank-ordered probit regression, \([R]\) asroprobit
complementary log-log regression, [R] cloglog
generalized linear models, \([R]\) glm
for binomial family, [R] binreg
generalized negative binomial regression, \([R]\) nbreg
heckman selection model, \([R]\) heckman,
[R] heckoprobit
interval regression, \([\mathrm{R}]\) intreg
constrained estimation, continued
linear regression, \([\mathrm{R}]\) cnsreg seemingly unrelated, \([R]\) sureg stochastic frontier, [R] frontier three-stage least squares, \([R]\) reg3 truncated, \([R]\) truncreg
logistic regression, [R] logistic, [R] logit, also see logit regression subentry conditional, [R] clogit multinomial, \([R]\) mlogit
ordered, \([R]\) ologit
skewed, [R] scobit
stereotype, \([R]\) slogit
logit regression, \([\mathrm{R}]\) logit, also see logistic regression subentry
for grouped data, \([R]\) glogit
nested, [R] nlogit
maximum likelihood estimation, \([\mathrm{R}] \mathrm{ml}\)
multinomial
logistic regression, [R] mlogit
probit regression, \([\mathrm{R}]\) mprobit
negative binomial regression, \([R]\) nbreg
truncated, \([\mathrm{R}]\) tnbreg
zero-inflated, \([R]\) zinb
Poisson regression, \([R]\) poisson
truncated, \([R]\) tpoisson
zero-inflated, [R] zip
probit regression, \([R]\) probit
bivariate, \([R]\) biprobit
for grouped data, \([R]\) glogit
heteroskedastic, \([R]\) hetprobit
multinomial, \([R]\) mprobit
ordered, [R] oprobit
with endogenous regressors, [R] ivprobit
with sample selection, \([R]\) heckprobit
tobit model with endogenous regressors, \([R]\) ivtobit
truncated
negative binomial regression, \([R]\) tnbreg
Poisson regression, \([R]\) tpoisson
regression, \([R]\) truncreg
with endogenous regressors
probit regression, \([R]\) ivprobit
tobit model, \([\mathrm{R}]\) ivtobit
zero-inflated
negative binomial regression, \([R]\) zinb
Poisson regression, [R] zip
constraint
command, \([\mathrm{R}]\) constraint
define command, \([R]\) constraint
dir command, \([\mathrm{R}]\) constraint
drop command, \([R]\) constraint
free command, \([R]\) constraint
get command, \([\mathrm{R}]\) constraint
list command, \([R]\) constraint
contingency tables, \([R]\) roctab, \([R]\) symmetry,
\([R]\) table, \([R]\) tabulate twoway
contrast command, \([\mathrm{R}]\) anova postestimation, \([R]\) contrast, \([R]\) contrast postestimation, \([R]\) margins, contrast
contrasts, \([R]\) contrast, \([R]\) margins, contrast, \([R]\) marginsplot
control charts, \([\mathrm{R}]\) qc
convergence criteria, \([\mathrm{R}]\) maximize
Cook-Weisberg test for heteroskedasticity, \([\mathrm{R}]\) regress postestimation
Cook's \(D,[\mathrm{R}]\) glm postestimation, \([\mathrm{R}]\) regress postestimation
copy, ssc subcommand, \([R]\) ssc
copycolor, set subcommand, [R] set
copyright
Apache, \([\mathrm{R}]\) copyright apache
boost, \([R]\) copyright boost
freetype, \([R]\) copyright freetype
icu, \([R]\) copyright icu
JagPDF, [R] copyright jagpdf
lapack, \([R]\) copyright lapack
libpng, \([R]\) copyright libpng
MiG Layout, \([\mathrm{R}]\) copyright miglayout
scintilla, \([R]\) copyright scintilla
ttf2pt1, [R] copyright ttf2pt1
zlib, \([R]\) copyright zlib
copyright command, \([\mathrm{R}]\) copyright
correlate command, \([\mathrm{R}]\) correlate
correlated errors, see robust, Huber/White/sandwich estimator of variance, also see autocorrelation
correlation, \([\mathrm{R}]\) correlate
binary variables, \([R]\) tetrachoric
continuous variables, \([\mathrm{R}]\) correlate
intraclass, \([R]\) icc
intracluster, \([\mathrm{R}]\) loneway
Kendall's rank, \([R]\) spearman
matrices, \([R]\) correlate, \([R]\) estat, \([R]\) estat vce
pairwise, \([\mathrm{R}]\) correlate
partial and semipartial, \([R]\) pcorr
serial, \([\mathrm{R}]\) runtest
Spearman's rank, \([R]\) spearman
structure, \([R]\) asmprobit, \([R]\) asroprobit, \([R]\) reg3
tetrachoric, \([\mathrm{R}]\) tetrachoric
correlation, estat subcommand, \([R]\) asmprobit postestimation, \([R]\) asroprobit postestimation
cosine kernel function, \([\mathrm{R}]\) kdensity, \([\mathrm{R}]\) lpoly, [R] qreg
cost frontier model, \([\mathrm{R}]\) frontier
count data,
confidence intervals for counts, \([\mathrm{R}]\) ci
estimation, \([R]\) expoisson, \([R]\) glm, \([R]\) gmm, \([R]\) ivpoisson, \([R]\) nbreg, \([R]\) poisson, \([R]\) tnbreg, \([R]\) tpoisson, \([R]\) zinb, \([R]\) zip graphs, \([R]\) histogram, \([R]\) kdensity, \([R]\) spikeplot interrater agreement, \([R]\) kappa
summary statistics of, \([R]\) table, \([R]\) tabstat, \([R]\) tabulate oneway, \([R]\) tabulate twoway, \([\mathrm{R}]\) tabulate, summarize()
count data, continued
symmetry and marginal homogeneity tests, [R] symmetry
count, ml subcommand, \([\mathrm{R}] \mathrm{ml}\)
covariance
matrix of estimators, \([R]\) estat, \([R]\) estat vce, \([R]\) estimates store
of variables or coefficients, [R] correlate covariance, analysis of, \([R]\) anova covariance, estat subcommand, \([R]\) asmprobit postestimation, \([R]\) asroprobit postestimation
covariate patterns, \([R]\) logistic postestimation, \([R]\) logit postestimation, \([R]\) probit postestimation
COVRATIO, \([R]\) regress postestimation
cprplot command, \([R]\) regress postestimation diagnostic plots
Cramér's \(V,[\mathrm{R}]\) tabulate twoway
crossover designs, \([R] \mathbf{p k},[R]\) pkcross, \([R]\) pkshape
cross-tabulations, see tables
cumul command, \([R]\) cumul
cumulative distribution, empirical, \([\mathrm{R}]\) cumul
cumulative incidence data, \([R]\) poisson
cusum command, \([R]\) cusum
cusum test, \([\mathrm{R}]\) cusum

\section*{D}
data,
autocorrelated, see autocorrelation
case-control, see case-control data
categorical, see categorical data, agreement, measures for
experimental, see experimental data
matched case-control, see matched case-control data
observational, see observational data
range of, see range of data
ranking, see ranking data
sampling, see sampling
summarizing, see summarizing data
survival-time, see survival analysis
time-series, see time-series analysis
data manipulation, \([R]\) fvrevar, \([R]\) fvset
Davidon-Fletcher-Powell algorithm, \([\mathrm{R}] \mathbf{~ m l}\)
db command, \([R] \mathbf{d b}\)
default settings of system parameters, \([R]\) query, \([R]\) set_defaults
define,
constraint subcommand, \([R]\) constraint
transmap subcommand, \([R]\) translate
delta beta influence statistic, \([\mathrm{R}]\) clogit postestimation,
\([R]\) logistic postestimation, \([R]\) logit postestimation
delta chi-squared influence statistic, \([R]\) clogit postestimation, \([\mathrm{R}]\) logistic postestimation, [R] logit postestimation
delta deviance influence statistic, \([\mathrm{R}]\) clogit postestimation, \([\mathrm{R}]\) logistic postestimation, [R] logit postestimation
delta method, [R] margins, \([\mathrm{R}]\) nlcom, \([\mathrm{R}]\) predictnl, \([R]\) testnl
density estimation, kernel, \([\mathrm{R}]\) kdensity
density-distribution sunflower plot, [R] sunflower
derivatives, numeric, \([R]\) dydx, \([R]\) testnl
describe,
ado subcommand, \([R]\) net
estimates subcommand, \([R]\) estimates describe
net subcommand, [R] net
ssc subcommand, [R] ssc
descriptive statistics,
CIs for means, proportions, and counts, [R] ci
correlations, \([R]\) correlate, \([R]\) pcorr,
\([R]\) tetrachoric
displaying, \([R]\) grmeanby, \([R]\) lv, \([R]\) summarize
estimation, \([R]\) mean, \([R]\) proportion, \([R]\) ratio, \([R]\) total
means, \([R]\) ameans, \([R]\) summarize
percentiles, \([R]\) centile
pharmacokinetic data,
make dataset of, [R] pkcollapse
summarize, \([R]\) pksumm
tables, \([R]\) table, \([R]\) tabstat, \([R]\) tabulate oneway,
\([R]\) tabulate twoway, \([R]\) tabulate, summarize()
design, fvset subcommand, [R] fvset
design effects, [R] loneway
deviance residual, \([R]\) binreg postestimation, \([R]\) fp postestimation, \([\mathrm{R}]\) glm postestimation, \([R]\) logistic postestimation, \([R]\) logit postestimation, \([\mathrm{R}]\) probit postestimation
DFBETA, [R] regress postestimation
dfbeta command, \([R]\) regress postestimation
DFITS, [R] regress postestimation
DFP algorithm, [R] ml
diagnostic plots, \([\mathrm{R}]\) diagnostic plots, \([\mathrm{R}]\) logistic postestimation, \([R]\) regress postestimation diagnostic plots
diagnostics, regression, see regression diagnostics
dialog box, [R] db
dichotomous outcome model, see outcomes, binary
difference of estimated coefficients, see linear combinations of estimators
difficult option, \([\mathrm{R}]\) maximize
dir,
ado subcommand, \([\mathrm{R}]\) net
constraint subcommand, \([R]\) constraint
estimates subcommand, [R] estimates store
direct standardization, \([R]\) dstdize, \([R]\) mean, \([R]\) proportion, \([R]\) ratio
dispersion, measures of, see percentiles, displaying, see range of data, see standard deviations, displaying, see variance, displaying
display
settings, \([R]\) set showbaselevels
width and length, [R] log
display command, as a calculator, [ R ] display
display, ml subcommand, [R] ml
displaying, also see printing, logs (output)
previously typed lines, [R] \#review
stored results, \([R]\) stored results
distributions,
examining, \([R]\) ameans, \([R]\) centile, \([R]\) kdensity, \([R]\) mean, \([R]\) pksumm, \([R]\) summarize,
[R] total
income, \([R]\) inequality
plots, \([R]\) cumul, \([R]\) cusum, \([R]\) diagnostic plots,
\([R]\) dotplot, \([R]\) histogram, \([R]\) kdensity,
\([R]\) ladder, \([R]\) lv, \([R]\) spikeplot, \([R]\) stem
standard population, \([R]\) dstdize
testing equality of, [R] ksmirnov, [R] kwallis,
\([R]\) ranksum, \([R]\) signrank
testing for normality, \([R]\) sktest, \([R]\) swilk
transformations
to achieve normality, \([R]\) boxcox, \([R]\) ladder to achieve zero skewness, [R] Inskew0
do command, \([\mathrm{R}]\) do
dockable, set subcommand, [R] set
dockingguides, set subcommand, [R] set
documentation, keyword search on, \([R]\) search
doedit command, [R] doedit
do-files, \([R]\) do
editing, \([R]\) doedit
dose-response models, \([\mathrm{R}]\) binreg, \([\mathrm{R}]\) glm, \([\mathrm{R}]\) logistic
dotplot command, [R] dotplot
doublebuffer, set subcommand, [R] set
dp, set subcommand, [R] set
drop,
constraint subcommand, \([\mathrm{R}]\) constraint
estimates subcommand, \([R]\) estimates store
dstdize command, \([R]\) dstdize
dummy variables, see indicator variables
Duncan's multiple-comparison adjustment, see multiple comparisons, Duncan's method
Dunnett's multiple comparison adjustment, see multiple comparisons, Dunnett's method
duration analysis, see survival analysis
Durbin-Watson statistic, [R] regress postestimation time series
durbinalt, estat subcommand, [R] regress postestimation time series
Durbin's alternative test, \([R]\) regress postestimation time series
dwatson, estat subcommand, [R] regress postestimation time series
dydx command, [R] dydx

\section*{E}
\(e()\) stored results, \([R]\) stored results
\(e\) (sample), resetting, \([R]\) estimates save
e-class command, [R] stored results
editing
ado-files and do-files, \([R]\) doedit
files while in Stata, [R] doedit
efficiency, query subcommand, [R] query
eform_option, [R] eform_option
eivreg command, [R] eivreg, [R] eivreg postestimation
empirical cumulative distribution function, \([R]\) cumul
emptycells, set subcommand, \([R]\) set, \([R]\) set emptycells
ending a Stata session, \([\mathrm{R}]\) exit
endless loop, see loop, endless
endogeneity test, \([R]\) ivregress postestimation
endogenous covariates, \([R]\) gmm, \([R]\) ivpoisson,
\([R]\) ivprobit, \([R]\) ivregress, \([R]\) ivtobit, \([R]\) reg3
endogenous, estat subcommand, \([R]\) ivregress postestimation
Engle's LM test, \([\mathrm{R}]\) regress postestimation time series eolchar, set subcommand, [R] set
Epanechnikov kernel function, [R] kdensity, [R] lpoly, [R] qreg
epidemiology and related,
Brier score decomposition, [R] brier
interrater agreement, [R] kappa
meta-analysis, \([\mathrm{R}]\) meta
pharmacokinetic data, see pharmacokinetic data
ROC analysis, see receiver operating characteristic analysis
standardization, \([R]\) dstdize
symmetry and marginal homogeneity tests,
[R] symmetry
tables, [R] tabulate twoway
equality test of
binomial proportions, \([\mathrm{R}]\) bitest
coefficients, \([R]\) pwcompare, \([R]\) sureg, \([R]\) test, [R] testnl
distributions, [R] ksmirnov, [R] kwallis, \([R]\) ranksum, \([R]\) signrank
margins, \([R]\) margins, \([R]\) pwcompare
means, \([R]\) contrast, \([R]\) esize, \([R]\) pwmean,
[R] ttest
medians, \([\mathrm{R}]\) ranksum
proportions, [R] bitest, [R] prtest
ROC areas, \([R]\) roccomp, \([R]\) rocreg
variances, \([R]\) sdtest
equivalence test, \([R] \mathbf{p k},[R]\) pkequiv
ereturn list command, \([R]\) stored results
error messages and return codes, \([R]\) error messages
searching, \([R]\) search
error-bar charts, \([R]\) serrbar
errors-in-variables regression, [R] eivreg esample, estimates subcommand, [R] estimates save
esize and esizei commands, [R] esize
esize, estat subcommand, \([R]\) regress
postestimation
estat
alternatives command, \([R]\) asclogit postestimation, [R] asmprobit postestimation, \([\mathrm{R}]\) asroprobit postestimation, \([\mathrm{R}]\) nlogit postestimation
archlm command, \([\mathrm{R}]\) regress postestimation time series
estat, continued
bgodfrey command, [R] regress postestimation time series
bootstrap command, [R] bootstrap postestimation
classification command, [R] estat classification
correlation command, [R] asmprobit postestimation, \([R]\) asroprobit postestimation
covariance command, \([R]\) asmprobit postestimation, \([R]\) asroprobit postestimation
durbinalt command, \([R]\) regress postestimation time series
dwatson command, \([\mathrm{R}]\) regress postestimation time series
endogenous command, [R] ivregress postestimation
esize command, \([R]\) regress postestimation
facweights command, [R] asmprobit postestimation, \([R]\) asroprobit postestimation
firststage command, [R] ivregress postestimation
gof command, \([R]\) estat gof, \([R]\) poisson postestimation
hettest command, \([R]\) regress postestimation
ic command, [R] estat, [R] estat ic
imtest command, \([R]\) regress postestimation
\(m f x\) command, \([R]\) asclogit postestimation,
[R] asmprobit postestimation, [R] asroprobit postestimation
nproc command, [R] rocreg postestimation
overid command, \([R]\) gmm postestimation, \([R]\) ivpoisson postestimation, \([R]\) ivregress postestimation
ovtest command, [ R ] regress postestimation
predict command, \([R]\) exlogistic postestimation
se command, \([R]\) exlogistic postestimation,
\([R]\) expoisson postestimation
summarize command, \([R]\) estat, \([R]\) estat summarize
szroeter command, [R] regress postestimation
vce command, \([R]\) estat, \([R]\) estat vce
vif command, \([R]\) regress postestimation
estimates
clear command, \([R]\) estimates store
command, \([R]\) suest introduction, \([R]\) estimates
describe command, \([R]\) estimates describe
dir command, \([R]\) estimates store
drop command, \([R]\) estimates store
esample command, [R] estimates save
for command, \([R]\) estimates for
notes command, \([R]\) estimates notes
query command, \([R]\) estimates store
replay command, \([R]\) estimates replay
restore command, \([R]\) estimates store
save command, \([R]\) estimates save
stats command, \([R]\) estimates stats
store command, \([R]\) estimates store
table command, \([R]\) estimates table
estimates, continued
title command, [R] estimates title
use command, \([R]\) estimates save
estimation
options, [R] estimation options
results,
clearing, \([R]\) estimates store
storing and restoring, \([R]\) estimates store
tables of, \([R]\) estimates table
sample, summarizing, \([R]\) estat, \([R]\) estat summarize
estimators,
covariance matrix of, \([R]\) correlate, \([R]\) estat, [R] estat vce
linear combinations of, \([R]\) lincom
nonlinear combinations of, [R] nlcom
event history analysis, see survival analysis
exact statistics,
binary confidence intervals, \([R]\) ci, \([R]\) exlogistic, [R] roctab
centiles, [R] centile
indirect standardization, \([R]\) dstdize
one-way anova, \([R]\) loneway
regression, \([R]\) exlogistic, \([R]\) expoisson
test,
binomial probability, [R] bitest
equality of distributions, \([R]\) ksmirnov
equality of medians, \([R]\) ranksum
Fisher's, [R] tabulate twoway
symmetry and marginal homogeneity, [R] symmetry
tetrachoric correlations, \([R]\) tetrachoric
exit command, \([R]\) exit
exiting Stata, see exit command
exlogistic command, \([R]\) exlogistic, [ \(R\) ] exlogistic postestimation
exogeneity test, see endogeneity test
experimental data, \([R]\) anova, \([R]\) contrast, \([R]\) correlate, \([R]\) kwallis, \([R]\) logit, \([R]\) mean, \([R]\) regress, \([R]\) summarize, \([R]\) tabulate oneway, \([R]\) tabulate twoway, \([R]\) ttest
exploded logit model, \([\mathrm{R}]\) rologit
expoisson command, \([R]\) expoisson, \([R]\) expoisson postestimation
exponentiated coefficients, [R] eform_option

\section*{F}
factor variables, \([R]\) fvrevar, \([R]\) fvset
factorial design, \([R]\) anova
factor-variable settings, [R] fvset
facweights, estat subcommand, [R] asmprobit
postestimation, [R] asroprobit postestimation
failure-time model, see survival analysis
false-positive rate, \([R]\) estat classification, \([R]\) roc,
\([R]\) rocreg, \([R]\) rocreg postestimation,
\([R]\) rocregplot
fastscroll, set subcommand, \([R]\) set
feasible generalized least squares, \([R]\) reg3, \([R]\) sureg feasible generalized nonlinear least squares, \([\mathrm{R}]\) nlsur fences, [R] lv
FGLS, see feasible generalized least squares
FGNLS, see feasible generalized nonlinear least squares
files, downloading, \([R]\) adoupdate, \([R]\) net, \([R]\) sj,
\([R]\) ssc, [R] update
firststage, estat subcommand, [R] ivregress postestimation
Fisher's exact test, [R] tabulate twoway
fixed-effects model, \([R]\) anova, \([R]\) areg, \([R]\) asclogit, [R] clogit
flexible functional form, \([R]\) boxcox, \([R] \mathbf{f p},[R] \mathbf{m f p}\)
floatwindows, set subcommand, \([R]\) set
footnote, ml subcommand, \([\mathrm{R}] \mathrm{ml}\)
for, estimates subcommand, \([R]\) estimates for
forecast, standard error of, \([R]\) regress postestimation
format settings, \([R]\) set cformat
fp generate command, [R] fp
fp plot command, \([\mathrm{R}] \mathbf{f p}\) postestimation
fp predict command, [R] fp postestimation
fp prefix command, \([R] \mathbf{f p},[R] \mathbf{f p}\) postestimation
fraction defective, \([\mathrm{R}] \mathbf{q c}\)
fractional polynomial regression, \([\mathrm{R}] \mathbf{f p}\)
multivariable, \([\mathrm{R}] \mathbf{m f p}\)
free, constraint subcommand, \([R]\) constraint
frequencies,
graphical representation, \([\mathrm{R}]\) histogram, [R] kdensity
table of, \([R]\) table, \([R]\) tabstat, \([R]\) tabulate oneway, \([R]\) tabulate twoway, \([R]\) tabulate, summarize()
from,
net subcommand, \([R]\) net
update subcommand, \([R]\) update
from() option, [R] maximize
frontier command, \([R]\) frontier, \([R]\) frontier postestimation
frontier model, see stochastic frontier model
functions,
combinations of estimators, \([R]\) lincom, \([R]\) nlcom
cumulative distribution, \([R]\) cumul
derivatives and integrals of, \([R]\) dydx
estimable, [R] margins
evaluator program, \([R]\) gmm, \([R]\) nl, \([R]\) nlsur
fractional polynomial, \([R] \mathbf{f p},[R] \mathbf{m f p}\)
index, \([R]\) logistic postestimation, \([R]\) logit
postestimation, \([R]\) probit postestimation
kernel, [R] kdensity, [R] lpoly
link, \([R]\) glm
maximizing likelihood, \([\mathrm{R}]\) maximize, \([\mathrm{R}] \mathrm{ml}\)
obtaining help for, \([R]\) help
orthogonalization, \([R]\) orthog
parameters, \([R]\) nlcom
piecewise cubic and piecewise linear, \([R]\) mkspline
prediction, \([R]\) predict, \([R]\) predictnl
functions, continued
production and cost, \([\mathrm{R}]\) frontier
variance, \([\mathrm{R}]\) glm
fvlabel, set subcommand, [R] set, [R] set
showbaselevels
fvrevar command, [R] fvrevar
fvset
base command, \([R]\) fvset
clear command, \([R]\) fvset
design command, \([R]\) fvset
report command, \([R]\) fvset
fvwrap, set subcommand, \([R]\) set, \([R]\) set showbaselevels
fvwrapon, set subcommand, \([R]\) set, \([R]\) set showbaselevels

\section*{G}

Gaussian kernel function, \([R]\) kdensity, \([R]\) lpoly, [R] qreg
generalized
least squares,
feasible, see feasible generalized least squares
linear latent and mixed models, \([\mathrm{R}]\) gllamm
linear models, \([R]\) binreg, \([R]\) glm
method of moments, see gmm command
negative binomial regression, \([\mathrm{R}]\) nbreg
get,
constraint subcommand, \([R]\) constraint
net subcommand, \([R]\) net
gladder command, \([R]\) ladder
GLLAMM, see generalized linear latent and mixed models
gllamm command, [R] gllamm
GLM, see generalized linear models
glm command, \([\mathrm{R}]\) glm, \([\mathrm{R}]\) glm postestimation
glogit command, \([R]\) glogit, \([R]\) glogit postestimation
gmm command, \([R]\) gmm, \([R]\) gmm postestimation
gnbreg command, \([\mathrm{R}]\) nbreg, \([\mathrm{R}]\) nbreg postestimation
gof, estat subcommand, \([R]\) estat gof, \([R]\) poisson postestimation
Goodman and Kruskal's gamma, \([\mathrm{R}]\) tabulate twoway
goodness of fit, \([R]\) brier, \([R]\) diagnostic plots, \([R]\) estat gof, \([R]\) ksmirnov, \([R]\) linktest, \([R]\) logistic postestimation, \([R]\) Irtest, \([R]\) poisson postestimation, \([R]\) regress postestimation, also see deviance residual, also see normal distribution and normality, test for
gprobit command, \([R]\) glogit, \([R]\) glogit postestimation
gradient option, \([\mathrm{R}]\) maximize
graph, ml subcommand, \([\mathrm{R}] \mathrm{ml}\)
graphics,
query subcommand, \([R]\) query
set subcommand, \([R]\) set
graphs,
added-variable plot, \([R]\) regress postestimation diagnostic plots
graphs, continued
adjusted partial residual plot, \([\mathrm{R}]\) regress
postestimation diagnostic plots
augmented component-plus-residual plot, \([R]\) regress
postestimation diagnostic plots
augmented partial residual plot, \([\mathrm{R}]\) regress
postestimation diagnostic plots
binary variable cumulative sum, \([\mathrm{R}]\) cusum
component-plus-residual, \([R]\) regress postestimation
diagnostic plots
cumulative distribution, \([\mathrm{R}]\) cumul
density, \([\mathrm{R}]\) kdensity
density-distribution sunflower, \([R]\) sunflower
derivatives, \([R]\) dydx, \([R]\) testnl
diagnostic, \([R]\) diagnostic plots
dotplot, \([R]\) dotplot
error-bar charts, \([R]\) serrbar
fractional polynomial, \([R]\) fp postestimation
histograms, \([R]\) histogram, \([R]\) kdensity
integrals, \([R]\) dydx
interaction plots, \([R]\) marginsplot
ladder-of-power histograms, \([\mathrm{R}]\) ladder
letter-value display, \([\mathrm{R}]\) Iv
leverage-versus-(squared)-residual, \([\mathrm{R}]\) regress postestimation diagnostic plots
logistic diagnostic, \([R]\) logistic postestimation, [R] Isens
lowess smoothing, \([\mathrm{R}]\) lowess
margins plots, \([R]\) marginsplot
means and medians, \([R]\) grmeanby
normal probability, \([\mathrm{R}]\) diagnostic plots
partial residual, \([R]\) regress postestimation diagnostic plots
partial-regression leverage, \([R]\) regress postestimation diagnostic plots
profile plots, \([\mathrm{R}]\) marginsplot
quality control, \([R]\) qc
quantile, \([R]\) diagnostic plots
quantile-normal, \([R]\) diagnostic plots
quantile-quantile, \([R]\) diagnostic plots
regression diagnostic, \([\mathrm{R}]\) regress postestimation diagnostic plots
residual versus fitted, \([R]\) regress postestimation diagnostic plots
residual versus predictor, \([\mathrm{R}]\) regress postestimation diagnostic plots
ROC curve, \([R]\) Iroc, \([R]\) roccomp, \([R]\) rocfit postestimation, \([R]\) rocregplot, \([R]\) roctab
rootograms, \([R]\) spikeplot
smoothing, \([R]\) kdensity, \([R]\) lowess, \([R]\) lpoly
spike plot, \([R]\) spikeplot
stem-and-leaf, \([R]\) stem
sunflower, \([\mathrm{R}]\) sunflower
symmetry, \([R]\) diagnostic plots
time-versus-concentration curve, \([\mathrm{R}] \mathbf{p k}\), [R] pkexamine
Greenhouse-Geisser epsilon, \([R]\) anova
grmeanby command, [R] grmeanby
group-data regression, \([R]\) glogit, \([R]\) intreg

\section*{H}

HAC variance estimate, [R] binreg, [R] glm, [R] gmm, \([R]\) ivregress, \([R]\) nl
Hansen's \(J\) statistic, [R] gmm, [R] gmm postestimation, \([R]\) ivpoisson, \([R]\) ivpoisson postestimation, \([\mathrm{R}]\) ivregress
harmonic mean, [R] ameans
hat matrix, see projection matrix, diagonal elements of hausman command, [R] hausman
Hausman specification test, [R] hausman
haverdir, set subcommand, [R] set
hazard ratio, \([\mathrm{R}]\) eform_option, \([\mathrm{R}]\) lincom
health ratio, \([\mathrm{R}]\) binreg
heckman command, \([R]\) heckman, \([R]\) heckman postestimation
Heckman selection model, \([R]\) heckman,
[ \(R\) ] heckoprobit, \([R]\) heckprobit
heckoprobit command, [R] heckoprobit,
[R] heckoprobit postestimation
heckprobit command, [R] heckprobit,
[R] heckprobit postestimation
Helmert contrasts, \([R]\) contrast
help command, \([\mathrm{R}]\) help
help, view subcommand, \([R]\) view
help_d, view subcommand, \([R]\) view
hessian option, \([\mathrm{R}]\) maximize
heteroskedastic probit regression, \([\mathrm{R}]\) hetprobit
heteroskedasticity, also see HAC variance estimate
conditional, \([\mathrm{R}]\) regress postestimation time series
robust variances, see robust, Huber/White/sandwich estimator of variance
test, \([R]\) hetprobit, \([R]\) regress postestimation,
\([R]\) regress postestimation time series
heteroskedasticity test, [R] sdtest
hetprobit command, [R] hetprobit, [R] hetprobit postestimation
hettest, estat subcommand, \([\mathrm{R}]\) regress postestimation
hierarchical
regression, \([R]\) nestreg, \([R]\) stepwise
samples, \([R]\) anova, \([R]\) gllamm, \([R]\) loneway, [R] areg
histogram command, [R] histogram
histograms, [R] histogram
dotplots, [R] dotplot
kernel density estimator, [R] kdensity
ladder-of-powers, [R] ladder
of categorical variables, \([R]\) histogram
rootograms, \([R]\) spikeplot
stem-and-leaf, \([\mathrm{R}]\) stem
Holm's multiple-comparison adjustment, see multiple comparisons, Holm's method
homogeneity of variances, \([R]\) oneway, \([R]\) sdtest
homoskedasticity tests, \([\mathrm{R}]\) regress postestimation

Hosmer-Lemeshow
delta chi-squared influence statistic, see delta chisquared influence statistic
delta deviance influence statistic, see delta deviance influence statistic
goodness-of-fit test, [R] estat gof
hot, ssc subcommand, \([R]\) ssc
httpproxy, set subcommand, \([R]\) netio, \([R]\) set
httpproxyauth, set subcommand, \([R]\) netio, \([R]\) set
httpproxyhost, set subcommand, \([R]\) netio, \([R]\) set
httpproxyport, set subcommand, \([R]\) netio, \([R]\) set
httpproxypw, set subcommand, \([R]\) netio, \([R]\) set
httpproxyuser, set subcommand, \([R]\) netio, \([R]\) set
Huber weighting, [R] rreg
Huber/White/sandwich estimator of variance, see robust, Huber/White/sandwich estimator of variance
Huynh-Feldt epsilon, [R] anova
hypertext help, [R] help

\section*{I}
ic, estat subcommand, \([R]\) estat, \([R]\) estat ic icc command, \([R]\) icc
IIA, see independence of irrelevant alternatives
immediate commands, \([R]\) bitest, \([R]\) ci, \([R]\) esize,
\([R]\) prtest, \([R]\) sdtest, [R] symmetry,
\([R]\) tabulate twoway, [R] ttest
imtest, estat subcommand, [R] regress postestimation
incidence rate,
negative binomial regression, \([\mathrm{R}]\) nbreg postestimation, \([\mathrm{R}]\) tnbreg postestimation, \([R]\) zinb postestimation
Poisson regression, \([R]\) poisson postestimation, \([R]\) tpoisson postestimation, \([R]\) zip postestimation
incidence-rate ratio, [R] eform_option
estimation, negative binomial regression, \([R]\) nbreg,
\([R]\) tnbreg, \([R]\) zinb Poisson regression, \([R]\) expoisson, \([R]\) ivpoisson,
\([R]\) poisson, \([R]\) tpoisson, \([R]\) zip
postestimation, \([R]\) contrast, \([R]\) expoisson postestimation, \([R]\) lincom
include_bitmap, set subcommand, \([R]\) set
income distributions, [R] inequality
independence of irrelevant alternatives,
assumption, [R] clogit, [R] mlogit
relaxing assumption, \([R]\) asclogit, \([R]\) asmprobit, \([R]\) asroprobit, \([R]\) nlogit
test for, \([R]\) hausman, \([R]\) nlogit, \([R]\) suest
independence test, \([R]\) correlate, \([R]\) spearman, \([R]\) tabulate twoway
index of probit and logit, \([R]\) logit postestimation, [R] predict, [R] probit postestimation
index search, \([R]\) search
indicator variables, \([R]\) tabulate oneway, \([R]\) xi, also see factor variables
indirect standardization, \([R]\) dstdize
inequality measures, \([R]\) inequality
influence statistics, see delta beta influence statistic, see delta chi-squared influence statistic, see delta deviance influence statistic, see DFBETA
information
criteria, see Akaike information criterion, see Bayesian information criterion
matrix, \([R]\) correlate, \([R]\) maximize
matrix test, \([R]\) regress postestimation
init, ml subcommand, [R] ml
inner fence, [R] lv
install,
net subcommand, \([R]\) net
ssc subcommand, [R] ssc
installation
of official updates, \([R]\) update
of SJ and STB, \([R]\) net, \([R]\) sj
of user-written commands (updating), \([R]\) adoupdate
instrumental-variables regression, \([R]\) gmm,
\([R]\) ivpoisson, \([R]\) ivprobit, \([R]\) ivregress,
\([R]\) ivtobit, \([R]\) reg3
integ command, \([R]\) dydx
integrals, numeric, \([R]\) dydx
interaction, \([R]\) anova, \([R]\) contrast, \([R]\) fvrevar,
\([R]\) margins, \([R]\) margins, contrast,
\([R]\) margins, pwcompare, \([R]\) marginsplot,
\([R]\) pwcompare, \([R]\) set emptycells, \([R]\) xi
interaction expansion, \([R] \mathbf{x i}\)
interaction plots, \([R]\) marginsplot
interface, query subcommand, [R] query
Internet,
commands to control connections to, \([R]\) netio
installation of updates from, \([R]\) adoupdate, \([R]\) net,
\([R] \mathbf{s j},[R]\) update
search, \([R]\) net search
interquantile range, \([R]\) qreg
interquartile range, \([R]\) lv, \([R]\) table, \([R]\) tabstat
interrater agreement, \([\mathrm{R}]\) kappa
interval regression, \([R]\) intreg
intraclass correlation, see correlation, intraclass
intracluster correlation, see correlation, intracluster
intreg command, \([\mathrm{R}]\) intreg, \([\mathrm{R}]\) intreg postestimation
IQR , see interquartile range
iqreg command, \([\mathrm{R}]\) qreg, \([\mathrm{R}]\) qreg postestimation
IRLS, see iterated, reweighted least squares
istdize command, [R] dstdize
iterate () option, \([\mathrm{R}]\) maximize
iterated, reweighted least squares, \([R]\) binreg, \([R]\) glm, \([R]\) reg3, \([R]\) sureg
iterations, controlling the maximum number,
[R] maximize
ivpoisson command, [R] ivpoisson, [R] ivpoisson postestimation
ivprobit command, [R] ivprobit, [R] ivprobit postestimation
ivregress command, [R] ivregress, [R] ivregress postestimation
ivtobit command, [R] ivtobit, [R] ivtobit postestimation

\section*{J}
jackknife
estimation, [R] jackknife
standard errors, [R] vce_option
jackknife prefix command, [R] jackknife, [R] jackknife postestimation
jackknifed residuals, \([R]\) regress postestimation

\section*{K}
kap command, [R] kappa
kappa command, [R] kappa
kapwgt command, [R] kappa
kdensity command, [R] kdensity
Kendall's tau, [R] spearman, [R] tabulate twoway
kernel density estimator, [R] kdensity
kernel-weighted local polynomial estimator, [R] lpoly
Kish design effects, [R] loneway
Kolmogorov-Smirnov test, [R] ksmirnov
Kruskal-Wallis test, [R] kwallis
ksmirnov command, [R] ksmirnov
ktau command, \([R]\) spearman
kurtosis, \([R]\) lv, \([R]\) pksumm, \([R]\) regress postestimation, \([R]\) sktest, \([R]\) summarize, [R] tabstat
kwallis command, [R] kwallis

\section*{L}

L1-norm models, [R] qreg
LAD regression, \([\mathrm{R}]\) qreg
ladder command, [ R ] ladder
ladder of powers, \([\mathrm{R}]\) ladder
Lagrange multiplier test, [R] regress postestimation time series
Latin-square designs, \([\mathrm{R}]\) anova, \([\mathrm{R}]\) pkshape
LAV regression, [R] qreg
least absolute
deviations, [R] qreg
residuals, \([\mathrm{R}]\) qreg
value regression, \([\mathrm{R}]\) qreg
least squared deviations, see linear regression
least squares, see linear regression
generalized, see feasible generalized least squares
least-squares means, \([R]\) margins, \([R]\) marginsplot
letter values, \([\mathrm{R}]\) lv
level, set subcommand, \([R]\) level, \([R]\) set
Levene's robust test statistic, [R] sdtest
leverage, \([R]\) logistic postestimation, \([R]\) regress postestimation diagnostic plots
leverage-versus-(squared)-residual plot, [R] regress postestimation diagnostic plots
license, [R] about
likelihood, see maximum likelihood estimation
likelihood-ratio
chi-squared of association, \([\mathrm{R}]\) tabulate twoway test, [R] Irtest
limited dependent variables, [R] asclogit,
\([R]\) asmprobit, \([R]\) asroprobit, \([R]\) binreg,
\([R]\) biprobit, \([R]\) brier, \([R]\) clogit, [ \(R\) ] cloglog,
\([R]\) cusum, \([R]\) exlogistic, \([R]\) expoisson,
\([R]\) glm, \([R]\) glogit, \([R]\) heckoprobit,
\([R]\) heckprobit, \([R]\) hetprobit, \([R]\) ivpoisson,
\([R]\) ivprobit, \([R]\) logistic, \([R]\) logit, \([R]\) mlogit,
\([R]\) mprobit, \([R]\) nbreg, \([R]\) nlogit, \([R]\) ologit,
\([R]\) oprobit, \([R]\) poisson, \([R]\) probit, \([R]\) rocfit,
\([R]\) rocreg, \([R]\) rologit, \([R]\) scobit, \([R]\) slogit,
\([R]\) tnbreg, \([R]\) tpoisson, \([R]\) zinb, \([R]\) zip
limits, \([R]\) limits, \([R]\) matsize
lincom command, \([R]\) lincom
linear
combinations of estimators, \([R]\) lincom
hypothesis test after estimation, \([R]\) contrast,
\([R]\) Irtest, \([R]\) margins, \([R]\) margins, contrast,
\([R]\) margins, pwcompare, \([R]\) pwcompare,
[R] test
regression, \([R]\) anova, \([R]\) areg, \([R]\) binreg,
\([R]\) cnsreg, \([R]\) eivreg, \([R]\) frontier,
\([R]\) glm, \([R]\) gmm, \([R]\) heckman, \([R]\) intreg,
\([R]\) ivregress, \([R]\) ivtobit, \([R]\) qreg, \([R]\) reg3,
\([R]\) regress, \([R]\) rreg, \([R]\) sureg, \([R]\) tobit,
[R] vwls
splines, [R] mkspline
linegap, set subcommand, \([R]\) set
linesize, set subcommand, [R] log, [R] set
link function, \([\mathrm{R}]\) glm
link, net subcommand, [R] net
linktest command, [R] linktest
list,
constraint subcommand, \([R]\) constraint
ereturn subcommand, \([R]\) stored results
return subcommand, \([R]\) stored results
sreturn subcommand, \([R]\) stored results
Inskew0 command, [R] Inskew0
local linear, [R] lpoly
local polynomial, [R] lpoly
locally weighted smoothing, \([R]\) lowess
location, measures of, \([R]\) lv, \([R]\) summarize, \([R]\) table
locksplitters, set subcommand, [R] set
log
close command, [R] log
command, \([R] \log ,[R]\) view
off command, \([R] \log\)
on command, \([R] \log\)
query command, \([R] \log\)
using command, \([\mathrm{R}] \log\)
\(\log\) files, printing, \([R]\) translate, also see log command log or nolog option, \([R]\) maximize
log transformations, [R] boxcox, [R] Inskew0
logistic and logit regression, [R] logistic, [R] logit complementary log-log, \([\mathrm{R}]\) cloglog
conditional, \([\mathrm{R}]\) asclogit, \([\mathrm{R}]\) clogit, \([\mathrm{R}]\) rologit exact, \([R]\) exlogistic
fixed-effects, [R] asclogit, [R] clogit
fractional polynomial, \([R] \mathbf{f p}\)
generalized linear model, [R] glm
multinomial, [R] asclogit, [R] clogit, [R] mlogit
nested, \([R]\) nlogit
ordered, [R] ologit
polytomous, \([R]\) mlogit
rank-ordered, \([\mathrm{R}]\) rologit
skewed, \([R]\) scobit
stereotype, [R] slogit
with grouped data, \([R]\) glogit
logistic command, [R] logistic, [R] logistic postestimation
logit command, [R] logit, [R] logit postestimation
logit regression, see logistic and logit regression
log-linear model, \([R]\) expoisson, \([R]\) glm, \([R]\) ivpoisson, \([R]\) poisson, \([R]\) tpoisson, \([R]\) zip
logtype, set subcommand, \([R] \log ,[R]\) set
loneway command, \([R]\) loneway
loop, endless, see endless loop
Lorenz curve, \([\mathrm{R}]\) inequality
lowess, see locally weighted smoothing
lowess command, [R] lowess
lpoly command, [R] lpoly
L-R plots, \([R]\) regress postestimation diagnostic plots
lroc command, [R] Iroc
lrtest command, [R] Irtest
lsens command, [R] Isens
lstat command, see estat classification command
lstretch, set subcommand, [R] set
ltolerance() option, [R] maximize
lv command, [R] lv
lvr2plot command, \([R]\) regress postestimation diagnostic plots

\section*{M}

MAD regression, [R] qreg
main effects, \([R]\) anova
man command, [R] help
Mann-Whitney two-sample statistics, [R] ranksum marginal
effects, [R] margins, [R] marginsplot
homogeneity, test of, \([R]\) symmetry
means, \([R]\) contrast, \([R]\) margins, \([R]\) margins, contrast, \([R]\) margins, pwcompare, \([R]\) marginsplot, [R] pwcompare
margins command, \([R]\) margins, \([R]\) margins postestimation, \([R]\) margins, contrast, \([R]\) margins, pwcompare, \([R]\) marginsplot margins test, [R] margins, [R] pwcompare marginsplot command, [R] marginsplot
mata
query command, [R] set
set matacache command, [R] set
set matafavor command, [R] set
set matalibs command, \([R]\) set
set matalnum command, \([R]\) set
set matamofirst command, [R] set
set mataoptimize command, \([R]\) set
set matastrict command, \([\mathrm{R}]\) set
mata, query subcommand, \([R]\) query
matched case-control data, \([\mathrm{R}]\) asclogit, \([\mathrm{R}]\) clogit,
[R] symmetry
matched-pairs tests, \([R]\) signrank, \([R]\) ttest
matsize, set subcommand, \([R]\) matsize, \([R]\) set
max_memory, set subcommand, \([R]\) set
maxdb, set subcommand, \([R] \mathbf{d b},[R]\) set
maximization technique explained, \([R]\) maximize, [R] ml
maximize, ml subcommand, \([\mathrm{R}] \mathrm{ml}\)
maximum
likelihood estimation, \([R]\) maximize, \([R] \mathbf{m l}\), [R] mlexp
limits, [R] limits
number of variables in a model, \([R]\) matsize maximums and minimums, reporting, \([\mathrm{R}] \mathbf{l v}\), \([R]\) summarize, \([R]\) table
maxiter, set subcommand, [R] maximize, \([R]\) set
maxvar, set subcommand, \([R]\) set
McFadden's choice model, [R] asclogit
McNemar's chi-squared test, \([\mathrm{R}]\) clogit
mean command, \([R]\) mean, \([R]\) mean postestimation means,
arithmetic, geometric, and harmonic, \([R]\) ameans
confidence interval and standard error, \([R]\) ci
displaying, \([R]\) ameans, \([R]\) summarize, \([R]\) table,
\([R]\) tabstat, \([R]\) tabulate, summarize()
estimating, \([R]\) mean
graphing, \([R]\) grmeanby
marginal, [R] margins
pairwise comparisons of, \([\mathrm{R}]\) pwmean
pharmacokinetic data, [R] pksumm
robust, [R] rreg
testing equality of, see equality test of means
measurement error, \([R]\) vwls
measures of
association, \([R]\) tabulate twoway
central tendency, see means, see medians
dispersion, see percentiles, displaying, see range of data, see standard deviations, displaying, see variance, displaying
inequality, \([R]\) inequality
location, \([R]\) lv, \([R]\) summarize
median command, \([R]\) ranksum
median regression, \([\mathrm{R}]\) qreg
median test, \([R]\) ranksum
medians,
displaying, \([R]\) centile, \([R]\) lv, \([R]\) summarize,
[R] table, [R] tabstat
graphing, \([R]\) grmeanby
testing equality of, see equality test of medians
memory, matsize, see matsize, set subcommand
memory, query subcommand, [R] query
messages and return codes, see error messages and return codes
meta-analysis, \([R]\) meta
mfp prefix command, \([R] \mathbf{m f p},[R] \mathbf{m f p}\) postestimation
\(m f x\), estat subcommand, \([R]\) asclogit postestimation, [R] asmprobit postestimation, [R] asroprobit postestimation
midsummaries, \([\mathrm{R}]\) lv
mild outliers, \([\mathrm{R}]\) lv
Mills' ratio, \([R]\) heckman, \([R]\) heckman postestimation
min_memory, set subcommand, \([R]\) set
minimum
absolute deviations, [R] qreg
squared deviations, \([R]\) areg, \([R]\) cnsreg, \([R]\) nl, \([R]\) regress, \([R]\) regress postestimation
minimums and maximums, see maximums and minimums, reporting
missing values, \([\mathrm{R}]\) misstable
misstable
nested command, \([R]\) misstable
patterns command, [R] misstable
summarize command, [R] misstable
tree command, \([R]\) misstable
mixed designs, \([R]\) anova
mkspline command, [R] mkspline
ml
check command, \([\mathrm{R}] \mathrm{ml}\)
clear command, \([\mathrm{R}] \mathrm{ml}\)
count command, \([\mathrm{R}] \mathrm{ml}\)
display command, [R] ml
footnote command, [R] ml
graph command, [R] ml
init command, [R] ml
maximize command, \([R] \mathrm{ml}\)
model command, [R] ml
plot command, \([\mathrm{R}] \mathrm{ml}\)
query command, [R] ml
report command, \([\mathrm{R}] \mathrm{ml}\)
score command, [R] ml
search command, \([R] \mathrm{ml}\)
trace command, \([R] \mathrm{ml}\)
mleval command, \([\mathrm{R}] \mathrm{ml}\)
mlexp command, [R] mlexp, [R] mlexp postestimation
mlmatbysum command, [R] ml
mlmatsum command, [R] ml
mlogit command, [R] mlogit, [R] mlogit
postestimation
ml sum command, \([\mathrm{R}] \mathrm{ml}\)
mlvecsum command, [R] ml

MNP, see outcomes, multinomial
model coefficients test, [R] lrtest, [R] test, [R] testnl model specification test, see specification test model, ml subcommand, [R] ml
models, maximum number of variables in, \([R]\) matsize modulus transformations, [R] boxcox
monotone-missing pattern, \([\mathrm{R}]\) misstable
Monte Carlo simulations, \([R]\) permute, \([R]\) simulate
more command and parameter, \([\mathrm{R}]\) more
more, set subcommand, \([R]\) more, \([R]\) set
mprobit command, \([R]\) mprobit, \([R]\) mprobit postestimation
multilevel model, \([\mathrm{R}]\) gllamm
multinomial outcome model, see outcomes, multinomial
multiple comparisons, \([R]\) contrast, \([R]\) margins,
\([R]\) pwcompare, \([R]\) pwmean, \([R]\) anova postestimation, \([R]\) correlate, \([R]\) oneway,
\([R]\) regress postestimation, \([R]\) roccomp,
\([R]\) spearman, \([R]\) test, \([R]\) testnl,
\([R]\) tetrachoric
Bonferroni's method, \([R]\) contrast, \([R]\) margins, \([R]\) pwcompare, \([R]\) pwmean, \([R]\) anova postestimation, \([R]\) correlate, \([R]\) oneway, \([R]\) regress postestimation, \([R]\) roccomp, \([R]\) spearman, \([R]\) test, \([R]\) testnl, [R] tetrachoric
Duncan's method, \([R]\) pwcompare, \([R]\) pwmean
Dunnett's method, \([R]\) pwcompare, \([R]\) pwmean
Holm's method, [R] anova postestimation, \([R]\) regress postestimation, \([R]\) test, \([R]\) testnl
multiple-range method, see Dunnett's method subentry
Scheffé's method, [R] contrast, [R] margins, \([R]\) pwcompare, \([R]\) pwmean, \([R]\) oneway
Šidák's method, \([R]\) contrast, \([R]\) margins, \([R]\) pwcompare, \([R]\) pwmean, \([R]\) anova postestimation, \([\mathrm{R}]\) correlate, \([\mathrm{R}]\) oneway, \([R]\) regress postestimation, [ \(R\) ] roccomp, \([R]\) spearman, \([R]\) test, \([R]\) testnl, \([R]\) tetrachoric
Studentized-range method, see Tukey's method subentry
Student-Newman-Keuls' method, \([\mathrm{R}]\) pwcompare, [R] pwmean
Tukey's method, \([R]\) pwcompare, \([R]\) pwmean
multiple regression, see linear regression
multiple-range multiple-comparison adjustment, see multiple comparisons, Dunnett's method
multivariable fractional polynomial regression, \([\mathrm{R}] \mathbf{m f p}\) multivariate analysis,
bivariate probit, \([\mathrm{R}]\) biprobit
three-stage least squares, \([R]\) reg3
Zellner's seemingly unrelated, \([\mathrm{R}]\) nlsur, \([\mathrm{R}]\) sureg

\section*{N}
natural splines, \([R]\) mkspline
nbreg command, [R] nbreg, [R] nbreg postestimation needle plot, \([R]\) spikeplot
negative binomial regression, \([\mathrm{R}]\) nbreg
generalized linear models, \([\mathrm{R}]\) glm
truncated, [R] tnbreg
zero-inflated, \([R]\) zinb
nested
designs, \([\mathrm{R}]\) anova
effects, \([R]\) anova
logit, [R] nlogit
model statistics, \([R]\) nestreg
regression, \([R]\) nestreg
nested, misstable subcommand, \([R]\) misstable
nestreg prefix command, \([R]\) nestreg
net
cd command, \([R]\) net
describe command, \([R]\) net
from command, [R] net
get command, \([R]\) net
install command, \([R]\) net
link command, [R] net
query command, \([R]\) net
search command, \([R]\) net search
set ado command, \([R]\) net
set other command, \([R]\) net
sj command, \([R]\) net
stb command, \([R]\) net
net, view subcommand, \([R]\) view
net_d, view subcommand, \([R]\) view
network, query subcommand, \([R]\) query
new, ssc subcommand, \([R]\) ssc
Newey-West standard errors, [R] glm
news command, [R] news
news, view subcommand, \([R]\) view
Newton-Raphson algorithm, \([\mathrm{R}] \mathrm{ml}\)
niceness, set subcommand, \([R]\) set
nl command, \([\mathrm{R}] \mathrm{nl}\), \([\mathrm{R}]\) nl postestimation
nlcom command, \([R]\) nlcom
nlogit command, [R] nlogit, [R] nlogit postestimation
nlogitgen command, [R] nlogit
nlogittree command, [R] nlogit
nlsur command, \([R]\) nlsur, \([R]\) nlsur postestimation
nolog or log option, \([\mathrm{R}]\) maximize
nonconformities, quality control, \([\mathrm{R}]\) qc
nonconstant variance, see robust, Huber/White/sandwich estimator of variance
nonlinear
combinations of estimators, \([\mathrm{R}]\) nlcom
hypothesis test after estimation, [R] lrtest,
\([R]\) margins, \([R]\) margins, contrast,
\([R]\) margins, pwcompare, \([R]\) nlcom,
\([R]\) predictnl, [R] testnl
least squares, \([\mathrm{R}] \mathbf{n l}\)
regression, \([R]\) boxcox, \([R]\) nl, \([R]\) nlsur
nonparametric analysis,
hypothesis tests,
agreement, \([R]\) kappa
association, \([R]\) spearman, \([R]\) tabulate twoway
nonparametric analysis, hypothesis tests, continued cusum, [R] cusum equality of distributions, \([R]\) ksmirnov,
\([R]\) kwallis, \([R]\) ranksum, \([R]\) signrank medians, \([\mathrm{R}]\) ranksum proportions, [R] bitest, [R] prtest random order, \([\mathrm{R}]\) runtest trend, \([R]\) nptrend
percentiles, \([R]\) centile
quantile regression, \([R]\) qreg
ROC analysis, [ R\(]\) roc estimation, \([R]\) rocreg graphs, \([R]\) rocregplot test equality of areas, \([R]\) roccomp without covariates, \([R]\) roctab
smoothing, \([R]\) kdensity, \([R]\) lowess, \([R]\) Ipoly, [R] smooth
nonrtolerance option, \([\mathrm{R}]\) maximize
nonselection hazard, \([R]\) heckman, \([R]\) heckman postestimation
normal distribution and normality,
examining distributions for, \([\mathrm{R}]\) diagnostic plots, [R] lv
probability and quantile plots, \([\mathrm{R}]\) diagnostic plots
test for, \([R]\) sktest, \([R]\) swilk
transformations to achieve, \([\mathrm{R}]\) boxcox, [R] ladder, [R] Inskew0
not concave message, \([R]\) maximize
notes on estimation results, \([R]\) estimates notes
notes, estimates subcommand, \([R]\) estimates notes
notifyuser, set subcommand, [R] set
nproc, estat subcommand, \([R]\) rocreg postestimation
nptrend command, [R] nptrend
NR algorithm, \([\mathrm{R}] \mathrm{ml}\)
nrtolerance() option, [R] maximize
\(N\)-way analysis of variance, \([\mathrm{R}]\) anova

\section*{0}
obs, set subcommand, \([R]\) set
observational data, \([R]\) correlate, \([R]\) heckman,
\([R]\) ivregress, \([R]\) logit, \([R]\) mean, \([R]\) regress,
\([R]\) summarize, \([R]\) tabulate oneway,
\([R]\) tabulate twoway, \([R]\) ttest
observed information matrix, \([\mathrm{R}] \mathbf{m l},[\mathrm{R}]\) vce_option
odbcmgr, set subcommand, \([R]\) set
odds ratio, \([\mathrm{R}]\) eform_option
estimation, \([R]\) asclogit, \([R]\) binreg, \([R]\) clogit, \([R]\) cloglog, \([R]\) exlogistic, \([R]\) glm, \([R]\) glogit, \([R]\) logistic, \([R]\) logit, \([R]\) mlogit, \([R]\) scobit
postestimation, \([R]\) contrast, \([R]\) exlogistic postestimation, \([\mathrm{R}]\) lincom
off,
cmdlog subcommand, [R] log
log subcommand, [R] log
OIM, see observed information matrix
ologit command, [R] ologit, [R] ologit postestimation

OLS regression, see linear regression
omitted variables test, \([\mathrm{R}]\) regress postestimation, also see specification test
on,
cmdlog subcommand, [R] log
log subcommand, [R] \(\log\)
one-way analysis of variance, [R] kwallis, [R] loneway, [R] oneway
oneway command, \([R]\) oneway
OPG, see outer product of the gradient
oprobit command, [R] oprobit, [R] oprobit postestimation
order statistics, [R] lv
ordered
logit, [R] ologit
probit, [R] heckoprobit, [R] oprobit
ordinal outcome model, see outcomes, ordinal
ordinary least squares, see linear regression
orthog command, \([R]\) orthog
orthogonal polynomial, \([\mathrm{R}]\) contrast, \([\mathrm{R}]\) margins, contrast, \([\mathrm{R}]\) orthog
orthpoly command, \([R]\) orthog
other, query subcommand, \([R]\) query
outcomes,
binary,
complementary log-log, \([\mathrm{R}]\) cloglog
glm for binomial family, \([R]\) binreg, \([R]\) glm
grouped data, [R] glogit
logistic, \([R]\) exlogistic, \([R]\) logistic, \([R]\) logit, [R] scobit
probit, \([R]\) biprobit, \([R]\) heckprobit,
\([R]\) hetprobit, \([R]\) ivprobit, \([R]\) probit
\(R O C\) analysis, \([R]\) rocfit, \([R]\) rocreg
categorical,
logistic, [R] asclogit, [R] clogit, [R] mlogit,
\([R]\) nlogit, \([R]\) slogit
probit, [R] asmprobit, [R] mprobit
count,
negative binomial, \([R]\) nbreg, \([R]\) tnbreg, [R] zinb
Poisson, \([R]\) expoisson, \([R]\) ivpoisson,
\([R]\) poisson, \([R]\) tpoisson, \([R]\) zip
multinomial, see categorical subentry, see ordinal subentry, see rank subentry
ordinal,
logistic, [R] ologit, [R] slogit
probit, \([R]\) heckoprobit, \([R]\) oprobit
polytomous, see categorical subentry, see ordinal subentry, see rank subentry
rank,
logistic, [R] rologit
probit, [R] asroprobit
outer fence, \([\mathrm{R}] \mathbf{l v}\)
outer product of the gradient, \([\mathrm{R}] \mathbf{m l},[\mathrm{R}]\) vce_option
outliers, \([R]\) lv, \([R]\) qreg, \([R]\) regress postestimation, [R] rreg
out-of-sample predictions, [R] predict, [R] predictnl
output,
query subcommand, [R] query
set subcommand, \([R]\) set
output,
coefficient table,
automatically widen, \([R]\) set
display settings, \([R]\) set showbaselevels
format settings, \([R]\) set cformat
controlling the scrolling of, \([\mathrm{R}]\) more
printing, \([R]\) translate
recording, \([R] \log\)
outside values, [R] lv
overid, estat subcommand, [R] gmm postestimation,
\([R]\) ivpoisson postestimation, \([R]\) ivregress postestimation
overidentifying restrictions test, \([\mathrm{R}]\) gmm postestimation, \([R]\) ivpoisson postestimation, \([R]\) ivregress postestimation
ovtest, estat subcommand, \([\mathrm{R}]\) regress postestimation

\section*{P}
\(\mathrm{P}-\mathrm{P}\) plot, [R] diagnostic plots
pagesize, set subcommand, \([R]\) more, \([R]\) set paging of screen output, controlling, \([\mathrm{R}]\) more pairwise comparisons, \([\mathrm{R}]\) margins, pwcompare,
\([R]\) marginsplot, \([R]\) pwcompare, \([R]\) pwmean
pairwise correlation, \([R]\) correlate
parameters, system, see system parameters
partial
correlation, \([\mathrm{R}]\) pcorr
effects, [R] margins, [R] marginsplot
regression leverage plot, \([R]\) regress postestimation diagnostic plots
regression plot, \([\mathrm{R}]\) regress postestimation diagnostic plots
residual plot, \([R]\) regress postestimation diagnostic plots
Parzen kernel function, [R] kdensity, [R] lpoly, [R] qreg
pattern of missing values, \([\mathrm{R}]\) misstable
patterns, misstable subcommand, \([R]\) misstable
pausing until key is pressed, \([R]\) more
pchart command, [R] qc
pchi command, \([R]\) diagnostic plots
pcorr command, \([\mathrm{R}]\) pcorr
PDF, [R] translate
Pearson goodness-of-fit test, [R] estat gof, [R] logistic postestimation, \([\mathrm{R}]\) poisson postestimation
Pearson product-moment correlation coefficient, [R] correlate
Pearson residual, \([R]\) binreg postestimation, \([R]\) estat gof, \([R]\) glm postestimation, \([R]\) logistic postestimation, \([\mathrm{R}]\) logit postestimation
percentiles, displaying, \([R]\) centile, \([R] \mathbf{l v}\), \([R]\) summarize, \([R]\) table, \([R]\) tabstat
permutation test, \([\mathrm{R}]\) permute
permute prefix command, \([\mathrm{R}]\) permute
pformat, set subcommand, \([R]\) set, \([R]\) set cformat pharmaceutical statistics, [R] pk, [R] pksumm
pharmacokinetic data, \([\mathrm{R}] \mathbf{p k},[\mathrm{R}]\) pkcollapse,
[ \(R\) ] pkcross, [ R\(]\) pkequiv, \([\mathrm{R}]\) pkexamine,
\([R]\) pkshape, \([R]\) pksumm
piecewise
cubic functions, \([R]\) mkspline
linear functions, \([R]\) mkspline
pinnable, set subcommand, \([R]\) set
pk, see pharmacokinetic data
pkcollapse command, [R] pkcollapse
pkcross command, [R] pkcross
pkequiv command, [R] pkequiv
pkexamine command, \([R]\) pkexamine
.pkg filename suffix, \([R]\) net
pkshape command, \([\mathrm{R}]\) pkshape
pksumm command, [R] pksumm
Plackett-Luce model, [R] rologit
playsnd, set subcommand, [R] set
plot, ml subcommand, [R] ml
pnorm command, \([R]\) diagnostic plots
poisson command, [R] nbreg, \([R]\) poisson,
\([R]\) poisson postestimation
Poisson distribution,
confidence intervals, \([\mathrm{R}]\) ci
regression, see Poisson regression
Poisson regression, \([\mathrm{R}]\) nbreg, \([\mathrm{R}]\) poisson
generalized linear model, \([\mathrm{R}]\) glm
truncated, \([R]\) tpoisson
zero-inflated, [R] zip
polynomials,
fractional, \([\mathrm{R}] \mathbf{f p},[\mathrm{R}] \mathbf{m f p}\)
orthogonal, \([R]\) orthog
smoothing, see local polynomial
polytomous outcome model, see outcomes, polytomous
populations,
diagnostic plots, \([R]\) diagnostic plots
examining, \([R]\) histogram, \([R]\) lv, \([R]\) spikeplot, \([R]\) stem, \([R]\) summarize, \([R]\) table
standard, \([R]\) dstdize
testing equality of, see distributions, testing equality of
testing for normality, \([R]\) sktest, \([R]\) swilk
postestimation command, \([R]\) contrast, \([R]\) estat,
\([R]\) estat ic, \([R]\) estat summarize, \([R]\) estat vce, \([R]\) estimates, \([R]\) hausman, \([R]\) lincom,
\([R]\) linktest, [ \(R\) ] lrtest, [ \(R\) ] margins,
\([R]\) margins, contrast, \([R]\) margins, pwcompare, \([\mathrm{R}]\) marginsplot, \([\mathrm{R}]\) nlcom, \([\mathrm{R}]\) predict, \([\mathrm{R}]\) predictnl, \([\mathrm{R}]\) pwcompare, \([R]\) suest, \([R]\) test, \([R]\) testnl
poverty indices, \([R]\) inequality
power transformations, \([\mathrm{R}]\) boxcox, \([\mathrm{R}]\) Inskew0
predict command, \([\mathrm{R}]\) predict, \([\mathrm{R}]\) regress postestimation
predict, estat subcommand, [R] exlogistic postestimation
predictions, \([\mathrm{R}]\) predict, \([\mathrm{R}]\) predictnl
predictions, standard error of, \([\mathrm{R}]\) glm, \([\mathrm{R}]\) predict, \([R]\) regress postestimation
predictnl command, \([\mathrm{R}]\) predictnl
prefix command, \([R]\) bootstrap, \([R]\) fp, \([R]\) jackknife, \([R]\) mfp, \([R]\) nestreg, \([R]\) permute, \([R]\) simulate, \([R]\) stepwise, \([R]\) xi
Pregibon delta beta influence statistic, see delta beta influence statistic
preprocessor commands, [R] \#review
prevalence studies, see case-control data
print command, \([R]\) translate
printcolor, set subcommand, [R] set
printing, logs (output), \([\mathrm{R}]\) translate
probit command, \([\mathrm{R}]\) probit, \([\mathrm{R}]\) probit postestimation
probit regression, [R] probit alternative-specific multinomial probit, \([\mathrm{R}]\) asmprobit
alternative-specific rank-ordered, \([\mathrm{R}]\) asroprobit
bivariate, \([\mathrm{R}]\) biprobit
generalized linear model, \([\mathrm{R}]\) glm
heteroskedastic, \([\mathrm{R}]\) hetprobit
multinomial, \([\mathrm{R}]\) mprobit
ordered, \([R]\) heckoprobit, \([R]\) oprobit
two-equation, \([R]\) biprobit
with endogenous regressors, \([R]\) ivprobit
with grouped data, \([\mathrm{R}]\) glogit
with sample selection, \([\mathrm{R}]\) heckprobit
processors, set subcommand, \([R]\) set
production frontier model, [R] frontier
product-moment correlation, \([\mathrm{R}]\) correlate between ranks, \([R]\) spearman
profile plots, \([R]\) marginsplot
programming, limits, \([R]\) limits
programs, user-written, see ado-files
projection matrix, diagonal elements of, \([R]\) binreg postestimation, \([R]\) clogit postestimation, \([R]\) glm postestimation, \([R]\) logistic postestimation, \([R]\) logit postestimation, \([R]\) regress postestimation, \([R]\) rreg postestimation
proportion command, \([\mathrm{R}]\) proportion, \([\mathrm{R}]\) proportion postestimation
proportional
hazards models, see survival analysis
odds assumption, \([R]\) ologit relaxed, \([R]\) slogit
odds model, \([\mathrm{R}]\) ologit
sampling, \([R]\) bootstrap
proportions,
confidence intervals for, \([\mathrm{R}]\) ci
estimating, \([\mathrm{R}]\) proportion
testing equality of, \([\mathrm{R}]\) bitest, \([\mathrm{R}]\) prtest
prtest command, \([\mathrm{R}]\) prtest
prtesti command, \([\mathrm{R}]\) prtest
pseudo \(R\)-squared, \([\mathrm{R}]\) maximize
pseudosigmas, \([\mathrm{R}]\) Iv
pwcompare command, \([\mathrm{R}]\) pwcompare,
[R] pwcompare postestimation
pwcorr command, \([\mathrm{R}]\) correlate
pwmean command, \([R]\) pwmean, \([R]\) pwmean postestimation

\section*{Q}

Q-Q plot, [R] diagnostic plots
qc charts, see quality control charts
qchi command, \([\mathrm{R}]\) diagnostic plots
qladder command, \([R]\) ladder
qnorm command, \([R]\) diagnostic plots
qqplot command, \([\mathrm{R}]\) diagnostic plots
qreg command, \([\mathrm{R}]\) qreg, \([\mathrm{R}]\) qreg postestimation
qtolerance() option, \([R]\) maximize
qualitative dependent variables, \([R]\) asclogit,
\([R]\) asmprobit, \([R]\) asroprobit, \([R]\) binreg,
\([R]\) biprobit, \([R]\) brier, \([R]\) clogit, [R] cloglog,
\([R]\) cusum, \([R]\) exlogistic, \([R]\) glm, \([R]\) glogit,
\([R]\) heckoprobit, \([R]\) heckprobit, \([R]\) hetprobit,
\([R]\) ivprobit, \([R]\) logistic, \([R]\) logit, \([R]\) mlogit,
\([R]\) mprobit, \([R]\) nlogit, \([R]\) ologit, \([R]\) oprobit,
\([R]\) probit, \([R]\) rocfit, \([R]\) rocreg, \([R]\) rologit,
[R] scobit, [R] slogit
quality control charts, \([R]\) qc, \([R]\) serrbar
quantile command, \([R]\) diagnostic plots
quantile-normal plots, \([\mathrm{R}]\) diagnostic plots
quantile plots, \([R]\) diagnostic plots
quantile-quantile plots, \([R]\) diagnostic plots
quantile regression, \([\mathrm{R}]\) qreg
quantiles, see percentiles, displaying
query
command, \([\mathrm{R}]\) query
efficiency command, [R] query
graphics command, \([\mathrm{R}]\) query
interface command, \([R]\) query
mata command, \([\mathrm{R}]\) query
memory command, \([\mathrm{R}]\) query
network command, \([R]\) query
other command, \([\mathrm{R}]\) query
output command, \([R]\) query
trace command, \([\mathrm{R}]\) query
update command, \([R]\) query
query,
estimates subcommand, [R] estimates store
log subcommand, [R] log
ml subcommand, \([\mathrm{R}] \mathrm{ml}\)
net subcommand, \([R]\) net
translator subcommand, \([\mathrm{R}]\) translate
transmap subcommand, \([R]\) translate
update subcommand, \([\mathrm{R}]\) update
quitting Stata, see exit command

\section*{R}
\(r()\) stored results, \([R]\) stored results
Ramsey test, \([\mathrm{R}]\) regress postestimation
random
order, test for, \([\mathrm{R}]\) runtest
sample, [R] bootstrap
random-effects model, \([R]\) anova, \([R]\) loneway
random-order test, \([R]\) runtest
range chart, \([\mathrm{R}]\) qc
range of data, \([R]\) lv, \([R]\) stem, \([R]\) summarize,
[R] table, \([R]\) tabstat
rank correlation, \([\mathrm{R}]\) spearman
ranking data, \([\mathrm{R}]\) rologit
rank-order statistics, \([R]\) signrank, \([R]\) spearman rank-ordered logistic regression, see outcomes, rank
ranksum command, \([\mathrm{R}]\) ranksum
rate ratio, see incidence-rate ratio
ratio command, \([\mathrm{R}]\) ratio, \([\mathrm{R}]\) ratio postestimation
ratios, estimating, \([\mathrm{R}]\) ratio
rc (return codes), see error messages and return codes
rchart command, [R] qc
receiver operating characteristic analysis, \([R]\) roc
area under ROC curve, \([\mathrm{R}]\) lroc
nonparametric analysis without covariates,
[R] roctab
parametric analysis without covariates, \([\mathrm{R}]\) rocfit regression models, \([\mathrm{R}]\) rocreg
ROC curves after rocfit, \([\mathrm{R}]\) rocfit postestimation
ROC curves after rocreg, \([R]\) rocregplot
test equality of ROC areas, see equality test of ROC areas
rectangle kernel function, \([R]\) kdensity, \([R]\) lpoly, [R] qreg
reexpression, \([R]\) boxcox, \([R]\) ladder, \([R]\) Inskew0
reg3 command, \([R]\) reg3, \([R]\) reg3 postestimation
regress command, \([R]\) regress, \([R]\) regress postestimation, \([R]\) regress postestimation diagnostic plots, \([\mathrm{R}]\) regress postestimation time series
regression
diagnostics, \([R]\) estat classification, \([R]\) estat gof, \([R]\) logistic postestimation, \([R]\) lroc, \([R]\) lsens, \([R]\) poisson postestimation, \([R]\) predict, \([R]\) predictnl, \([R]\) regress postestimation diagnostic plots, \([R]\) regress postestimation time series
function, estimating, \([R]\) lpoly
regression,
constrained, \([R]\) cnsreg
creating orthogonal polynomials for, \([\mathrm{R}]\) orthog
dummy variables, with, \([R]\) anova, \([R]\) areg, \([R]\) xi
fixed-effects, \([R]\) areg
fractional polynomial, \([R] \mathbf{f p},[R] \mathbf{m f p}\)
graphing, \([R]\) logistic, \([R]\) regress postestimation diagnostic plots
grouped data, \([R]\) intreg
increasing number of variables allowed, \([R]\) matsize instrumental variables, \([R]\) gmm, \([R]\) ivpoisson, \([R]\) ivprobit, \([R]\) ivregress, \([R]\) ivtobit
regression, continued
system, \([R]\) gmm, \([R]\) ivpoisson, \([R]\) ivregress, \([R]\) nlsur, \([R]\) reg3, \([R]\) sureg
truncated, \([\mathrm{R}]\) truncreg
relative-risk ratio, \([R]\) eform_option, \([R]\) lincom, \([\mathrm{R}]\) mlogit
reliability, \([R]\) brier, \([R]\) eivreg, \([R]\) icc, \([R]\) intreg,
\([R]\) loneway, \([R]\) poisson
reliability theory, see survival analysis
repeated-measures ANOVA, \([\mathrm{R}]\) anova
repeating and editing commands, \([\mathrm{R}]\) \#review
replay, estimates subcommand, \([R]\) estimates replay
report, fvset subcommand, \([R]\) fvset
report, ml subcommand, [R] ml
RESET test, \([\mathrm{R}]\) regress postestimation
reset, translator subcommand, \([\mathrm{R}]\) translate
residuals, \([R]\) logistic, \([R]\) predict, \([R]\) regress postestimation diagnostic plots, \([\mathrm{R}]\) rreg postestimation
residual-versus-fitted plot, \([\mathrm{R}]\) regress postestimation diagnostic plots
residual-versus-predictor plot, \([\mathrm{R}]\) regress postestimation diagnostic plots
resistant smoothers, \([\mathrm{R}]\) smooth
restore, estimates subcommand, \([R]\) estimates store
restricted cubic splines, \([\mathrm{R}]\) mkspline
Results window, clearing, \([\mathrm{R}]\) cls
results,
saving, \([R]\) estimates save
stored, \([R]\) stored results
return codes, see error messages and return codes return list command, [R] stored results
reventries, set subcommand, [R] set
\#review command, [R] \#review
revkeyboard, set subcommand, [R] set
risk ratio, \([\mathrm{R}]\) binreg
rmsg, set subcommand, \([R]\) set
robust regression, \([R]\) regress, \([R]\) rreg, also see robust, Huber/White/sandwich estimator of variance
robust test for equality of variance, \([\mathrm{R}]\) sdtest
robust, Huber/White/sandwich estimator of variance, [R] vce_option
alternative-specific
conditional logit model, \([\mathrm{R}]\) asclogit multinomial probit regression, \([\mathrm{R}]\) asmprobit rank-ordered probit regression, \([\mathrm{R}]\) asroprobit
complementary \(\log\)-log regression, \([R]\) cloglog
generalized linear models, \([\mathrm{R}] \mathrm{glm}\)
for binomial family, [R] binreg
generalized method of moments, \([R]\) gmm, \([\mathrm{R}]\) ivpoisson
heckman selection model, [R] heckman instrumental-variables regression, \([\mathrm{R}]\) ivregress interval regression, \([\mathrm{R}]\) intreg
robust, Huber/White/sandwich estimator of variance, continued
linear regression, \([\mathrm{R}]\) regress constrained, \([\mathrm{R}]\) cnsreg truncated, \([\mathrm{R}]\) truncreg with dummy-variable set, \([R]\) areg
logistic regression, [R] logistic, [R] logit, also see logit regression subentry conditional, \([\mathrm{R}]\) clogit multinomial, \([R]\) mlogit ordered, \([R]\) ologit rank-ordered, \([\mathrm{R}]\) rologit skewed, \([R]\) scobit stereotype, \([R]\) slogit
logit regression, \([\mathrm{R}]\) logistic, [R] logit, also see
logistic regression subentry
for grouped data, \([R]\) glogit
nested, [R] nlogit
maximum likelihood estimation, \([\mathrm{R}] \mathbf{m l},[\mathrm{R}]\) mlexp multinomial
logistic regression, \([\mathrm{R}]\) mlogit
probit regression, \([R]\) mprobit
negative binomial regression, \([\mathrm{R}]\) nbreg
truncated, \([R]\) tnbreg
zero-inflated, [R] zinb
nonlinear
least-squares estimation, \([R] \mathbf{n l}\)
systems of equations, [R] nlsur
Poisson regression, \([R]\) poisson
truncated, \([R]\) tpoisson
with endogenous regressors, \([R]\) ivpoisson
zero-inflated, [R] zip
probit regression, \([R]\) probit
bivariate, \([R]\) biprobit
for grouped data, \([R]\) glogit
heteroskedastic, [R] hetprobit
multinomial, \([\mathrm{R}]\) mprobit
ordered, \([R]\) heckoprobit, \([R]\) oprobit
with endogenous regressors, [R] ivprobit
with sample selection, \([R]\) heckprobit
quantile regression, \([\mathrm{R}]\) qreg
summary statistics,
mean, \([R]\) mean
proportion, \([R]\) proportion
ratio, \([R]\) ratio
total, \([R]\) total
tobit model, \([R]\) tobit
with endogenous regressors, \([R]\) ivtobit
truncated
negative binomial regression, \([R]\) tnbreg
Poisson regression, \([R]\) tpoisson
regression, \([R]\) truncreg
with endogenous regressors,
instrumental-variables regression, \([R]\) ivregress
Poisson regression, \([R]\) ivpoisson
probit regression, \([R]\) ivprobit
tobit regression, \([R]\) ivtobit
robust, Huber/White/sandwich estimator of variance, continued
zero-inflated
negative binomial regression, \([R]\) zinb
Poisson regression, [R] zip
robust, other methods of, \([R]\) rreg, \([R]\) smooth robvar command, [R] sdtest
ROC, see receiver operating characteristic analysis
roccomp command, \([R]\) roc, \([R]\) roccomp
rocfit command, \([R]\) rocfit, \([R]\) rocfit postestimation
rocgold command, \([R]\) roc, \([R]\) roccomp
rocplot command, \([\mathrm{R}]\) rocfit postestimation
rocreg command, \([R]\) rocreg, \([R]\) rocreg
postestimation, \([R]\) rocregplot
rocregplot command, \([R]\) rocregplot
roctab command, \([R]\) roc, \([R]\) roctab
roh, [R] loneway
rologit command, \([\mathrm{R}]\) rologit, \([\mathrm{R}]\) rologit postestimation
rootograms, \([R]\) spikeplot
rreg command, \([R]\) rreg, \([R]\) rreg postestimation run command, \([R]\) do
runiform() function, [R] set seed
runtest command, \([R]\) runtest
rvfplot command, \([R]\) regress postestimation diagnostic plots
rvpplot command, [ R ] regress postestimation diagnostic plots

\section*{S}
s() stored results, \([R]\) stored results
sample, random, see random sample
sampling, \([\mathrm{R}]\) bootstrap, \([\mathrm{R}]\) bsample, also see cluster sampling
sandwich/Huber/White estimator of variance, see robust, Huber/White/sandwich estimator of variance
save, estimates subcommand, [R] estimates save
saved results, see stored results
saving results, \([\mathrm{R}]\) estimates save
Scheffé's multiple-comparison adjustment, see multiple comparisons, Scheffé's method
scheme, set subcommand, [R] set
Schwarz information criterion, see Bayesian information criterion
s-class command, \([R]\) stored results
scobit command, [R] scobit, [R] scobit postestimation
score, ml subcommand, [R] ml
scores, [R] predict
scrollbufsize, set subcommand, [R] set
scrolling of output, controlling, \([R]\) more
sdtest command, [R] sdtest
sdtesti command, [R] sdtest
se, estat subcommand, \([R]\) exlogistic postestimation, \([R]\) expoisson postestimation
search,
ml subcommand, [R] ml
net subcommand, [R] net
view subcommand, [R] view
search command, \([R]\) search
search Internet, \([R]\) net search
search_d, view subcommand, [R] view
searchdefault, set subcommand, \([R]\) search, \([R]\) set
seed, set subcommand, \([R]\) set, \([R]\) set seed
seemingly unrelated
estimation, [R] suest
regression, \([R]\) nlsur, \([R]\) reg3, \([R]\) sureg
segmentsize, set subcommand, \([R]\) set
selection models, \([\mathrm{R}]\) heckman, \([\mathrm{R}]\) heckoprobit, [R] heckprobit
sensitivity, [R] estat classification, [R] lroc, [R] Isens, also see receiver operating characteristic analysis
model, \([\mathrm{R}]\) regress postestimation, \([\mathrm{R}]\) rreg
serial correlation, see autocorrelation
serial independence test, \([R]\) runtest
serrbar command, [R] serrbar
session, recording, \([R] \log\)
set
adosize command, [R] set
autotabgraphs command, [R] set
cformat command, \([R]\) set, \([R]\) set cformat
charset command, \([R]\) set
checksum command, \([R]\) set
coeftabresults command, \([R]\) set
command, \([R]\) query, \([R]\) set
conren command, [R] set
copycolor command, [R] set
dockable command, \([R]\) set
dockingguides command, [R] set
doublebuffer command, [R] set
dp command, [R] set
emptycells command, [R] set, [R] set emptycells
eolchar command, \([R]\) set
fastscroll command, [R] set
floatwindows command, \([R]\) set
fvlabel command, \([R]\) set, \([R]\) set showbaselevels
fvwrap command, [R] set, [R] set showbaselevels
fvwrapon command, [R] set, [R] set
showbaselevels
graphics command, [R] set
haverdir command, [R] set
httpproxy command, \([R]\) netio, \([R]\) set httpproxyauth command, \([R]\) netio, \([R]\) set httpproxyhost command, [R] netio, [R] set httpproxyport command, [R] netio, [R] set httpproxypw command, \([R]\) netio, \([R]\) set httpproxyuser command, [ \(R\) ] netio, \([R]\) set include_bitmap command, [R] set level command, [R] level, [R] set linegap command, [R] set linesize command, [R] log, [R] set locksplitters command, [R] set
set, continued
logtype command, \([R] \log ,[R]\) set
lstretch command, \([R]\) set
matsize command, \([R]\) matsize, \([R]\) set
maxdb command, \([R] \mathbf{d b},[R]\) set
maxiter command, \([R]\) maximize, \([R]\) set
max_memory command, \([R]\) set
maxvar command, \([R]\) set
min_memory command, \([R]\) set
more command, \([R]\) more, \([R]\) set
niceness command, \([\mathrm{R}]\) set
notifyuser command, [R] set
obs command, \([R]\) set
odbcmgr command, [R] set
output command, [R] set
pagesize command, \([R]\) more, \([R]\) set
pformat command, \([R]\) set, \([R]\) set cformat
pinnable command, \([R]\) set
playsnd command, [R] set
printcolor command, \([R]\) set
processors command, [R] set
reventries command, \([R]\) set
revkeyboard command, [R] set
rmsg command, [R] set
scheme command, [R] set
scrollbufsize command, [R] set
searchdefault command, [R] search, [R] set
seed command, \([R]\) set, \([R]\) set seed
segmentsize command, [R] set
sformat command, \([R]\) set, \([R]\) set cformat
showbaselevels command, [R] set, [R] set showbaselevels
showemptycells command, [R] set, [R] set showbaselevels
showomitted command, \([R]\) set, \([R]\) set showbaselevels
smoothfonts command, [R] set
timeout1 command, \([R]\) netio, \([R]\) set
timeout2 command, \([R]\) netio, \([R]\) set
trace command, [R] set
tracedepth command, [R] set
traceexpand command, [R] set
tracehilite command, [R] set
traceindent command, [R] set
tracenumber command, [R] set
tracesep command, [R] set
type command, [R] set
update_interval command, [R] set, [R] update
update_prompt command, [R] set, [R] update
update_query command, \([R]\) set, \([R]\) update
varabbrev command, [R] set
varkeyboard command, \([R]\) set
set ado, net subcommand, \([R]\) net
set matacache, mata subcommand, \([R]\) set
set matafavor, mata subcommand, \([R]\) set
set matalibs, mata subcommand, [R] set
set matalnum, mata subcommand, \([R]\) set
set matamofirst, mata subcommand, [R] set
set mataoptimize, mata subcommand, [R] set
set matastrict, mata subcommand, [R] set
set other, net subcommand, \([R]\) net
set, translator subcommand, \([R]\) translate
set_defaults command, [R] set_defaults
settings,
display, \([R]\) set showbaselevels
format, \([\mathrm{R}]\) set cformat
sformat, set subcommand, \([R]\) set, \([R]\) set cformat
sfrancia command, \([\mathrm{R}]\) swilk
Shapiro-Francia test for normality, [R] swilk
Shapiro-Wilk test for normality, \([R]\) swilk
shewhart command, [R] qc
showbaselevels, set subcommand, [R] set, [R] set showbaselevels
showemptycells, set subcommand, [R] set, [R] set showbaselevels
shownrtolerance option, [R] maximize
showomitted, set subcommand, [R] set, [R] set showbaselevels
showstep option, [R] maximize
showtolerance option, [R] maximize
Šidák's multiple-comparison adjustment, see multiple comparisons, Sidák's method signrank command, [R] signrank
signtest command, \([R]\) signrank
simulate prefix command, \([R]\) simulate
simulation, Monte Carlo, \([R]\) permute, \([R]\) simulate
simultaneous
quantile regression, \([\mathrm{R}]\) qreg
systems, [R] reg3
SIR, see standardized incidence ratio
SJ, see Stata Journal and Stata Technical Bulletin
sj, net subcommand, \([R]\) net
skewed logistic regression, \([R]\) scobit
skewness, \([R]\) ladder, \([R]\) regress postestimation,
\([R]\) summarize, \([R]\) lnskew0, \([R]\) lv,
\([R]\) pksumm, [R] sktest, [R] tabstat
sktest command, [R] sktest
slogit command, [R] slogit, [R] slogit postestimation
Small Stata, [R] limits
smooth command, [R] smooth
smoothfonts, set subcommand, \([R]\) set
smoothing, \([R]\) lpoly, \([R]\) smooth
graphs, \([R]\) kdensity, \([R]\) lowess
SMR, see standardized mortality ratio
spearman command, [R] spearman
Spearman's rho, [R] spearman
specification test, \([R]\) gmm postestimation,
\([R]\) hausman, \([R]\) ivpoisson postestimation,
\([R]\) ivregress postestimation, \([R]\) linktest,
\([R]\) Inskew \(0,[R]\) regress postestimation, \([R]\) suest
specificity, \([R]\) estat classification, \([R]\) lroc, \([R]\) Isens, also see receiver operating characteristic analysis
Spiegelhalter's \(Z\) statistic, [R] brier
spike plot, \([\mathrm{R}]\) spikeplot
spikeplot command, [R] spikeplot
splines
linear, \([\mathrm{R}]\) mkspline
restricted cubic, \([R]\) mkspline
split-plot designs, \([R]\) anova
spread, [R] lv
sqreg command, \([R]\) qreg, \([R]\) qreg postestimation
sreturn list command, \([R]\) stored results
ssc
copy command, [R] ssc
describe command, \([R]\) ssc
hot command, [R] ssc
install command, [R] ssc
new command, \([R]\) ssc
type command, [R] ssc
uninstall command, [R] ssc
SSC archive, see Statistical Software Components archive
standard deviations,
displaying, \([R]\) lv, \([R]\) summarize, \([R]\) table, \([R]\) tabstat, \([R]\) tabulate, summarize()
testing equality of, \([R]\) sdtest
standard errors,
for general predictions, \([R]\) predictnl
forecast, \([R]\) predict, \([R]\) regress postestimation
mean, \([R]\) ci, \([R]\) mean
prediction, \([R]\) glm, \([R]\) predict, \([R]\) regress postestimation
residuals, \([R]\) predict, \([R]\) regress postestimation
robust, see robust, Huber/White/sandwich estimator of variance
standardized
incidence ratio, \([R]\) dstdize
margins, [ \(R\) ] margins
means, \([\mathrm{R}]\) mean
mortality ratio, \([\mathrm{R}]\) dstdize
proportions, \([\mathrm{R}]\) proportion
rates, \([R]\) dstdize
ratios, \([\mathrm{R}]\) ratio
residuals, [R] binreg postestimation, [R] glm postestimation, \([R]\) logistic postestimation, \([R]\) logit postestimation, \([R]\) predict, \([R]\) regress postestimation
Stata Journal and Stata Technical Bulletin
installation of, \([R]\) net, \([R]\) sj
keyword search of, \([R]\) search
Stata limits, [R] limits
Stata/IC, [R] limits
Stata/MP, [R] limits
Stata/SE, [R] limits
stata.key file, [R] search
Statistical Software Components archive, [R] ssc stats, estimates subcommand, [R] estimates stats
STB, see Stata Journal and Stata Technical Bulletin
stb, net subcommand, [R] net
stcox, fractional polynomials, \([R]\) fp, [R] mfp
stem command, \([R]\) stem
stem-and-leaf displays, [R] stem
stepwise estimation, \([\mathrm{R}]\) stepwise
stepwise prefix command, \([R]\) stepwise
stereotype logistic regression, \([\mathrm{R}]\) slogit
stochastic frontier model, [R] frontier
store, estimates subcommand, [R] estimates store
stored results, \([\mathrm{R}]\) stored results
storing and restoring estimation results, \([R]\) estimates store
stratified
graphs, \([R]\) dotplot
models, \([R]\) asclogit, \([R]\) asmprobit, \([R]\) asroprobit, \([R]\) clogit, \([R]\) exlogistic, \([R]\) expoisson, \([R]\) rocreg, \([R]\) rologit
resampling, \([\mathrm{R}]\) bootstrap, \([\mathrm{R}]\) bsample, \([\mathrm{R}]\) bstat, \([\mathrm{R}]\) permute
standardization, \([\mathrm{R}]\) dstdize
summary statistics, \([R]\) mean, \([R]\) proportion, \([R]\) ratio, \([R]\) total
structural vector autoregressive
postestimation, \([R]\) regress postestimation time series
Stuart-Maxwell test statistic, \([R]\) symmetry
Studentized residuals, [ \(R\) ] predict, [ \(R\) ] regress postestimation
Studentized-range multiple-comparison adjustment, see multiple comparisons, Tukey's method
Student-Newman-Keuls' multiple-comparison adjustment, see multiple comparisons, Student-Newman-Keuls' method
Student's \(t\) distribution, see \(t\) distribution
subhazard ratio, \([\mathrm{R}]\) eform_option, \([\mathrm{R}]\) lincom
suest command, \([R]\) suest
summarize,
estat subcommand, \([R]\) estat, \([R]\) estat summarize
misstable subcommand, \([R]\) misstable
summarize command, \([R]\) summarize, \([R]\) tabulate, summarize()
summarizing data, \([R]\) summarize, \([R]\) tabstat, \([R]\) lv, \([R]\) table, \([R]\) tabulate oneway, \([R]\) tabulate twoway, \([R]\) tabulate, summarize()
summary statistics, see descriptive statistics
sums, over observations, [R] summarize
sunflower command, \([R]\) sunflower
sunflower plots, [R] sunflower
sureg command, \([R]\) sureg, \([R]\) sureg postestimation
survey sampling, see cluster sampling
survival analysis, [R] intreg, [R] logistic, [R] poisson
survival-time data, see survival analysis
swilk command, \([R]\) swilk
symbolic forms, [R] anova
symmetry command, [R] symmetry
symmetry plots, \([R]\) diagnostic plots
symmetry test, \([R]\) symmetry
symmi command, \([\mathrm{R}]\) symmetry
symplot command, [R] diagnostic plots
syntax diagrams explained, \([R]\) intro
system
estimators, [R] gmm, [R] ivpoisson, [R] ivregress, \([R]\) nlsur, \([R]\) reg3, \([R]\) sureg
parameters, \([R]\) query, \([R]\) set, \([R]\) set_defaults szroeter, estat subcommand, \([R]\) regress postestimation
Szroeter's test for heteroskedasticity, [R] regress postestimation

\section*{T}
\(t\) distribution
confidence interval for mean, \([R]\) ci, \([R]\) mean
testing equality of means, \([R]\) esize, \([R]\) ttest
tab1 command, \([R]\) tabulate oneway
tab2 command, \([R]\) tabulate twoway
tabi command, \([R]\) tabulate twoway
table command, [R] table
table, estimates subcommand, \([R]\) estimates table tables,
coefficient,
display in exponentiated form, \([\mathrm{R}]\) eform_option
display settings, \([R]\) estimation options, \([R]\) set showbaselevels
format settings, [R] set cformat maximum likelihood display options, \([R] \mathbf{m l}\) system parameter settings, \([R]\) set
contingency, \([R]\) table, \([R]\) tabulate twoway
estimation results, \([R]\) estimates table
frequency, \([R]\) tabulate oneway, \([R]\) tabulate twoway, \([R]\) table, \([R]\) tabstat, \([R]\) tabulate, summarize()
missing values, \([R]\) misstable
summary statistics, \([R]\) table, \([R]\) tabstat,
\([R]\) tabulate, summarize()
tabstat command, [R] tabstat
tabulate command, \([R]\) tabulate oneway,
[R] tabulate twoway
summarize(), [R] tabulate, summarize()
tau, [R] spearman
TDT test, see transmission-disequilibrium test technique() option, \([\mathrm{R}]\) maximize
test,
ARCH, see autoregressive conditional heteroskedasticity test
association, see association test
autoregressive conditional heteroskedasticity, see autoregressive conditional heteroskedasticity test
binomial probability, see binomial probability test
bioequivalence, see bioequivalence test
Breusch-Godfrey, see Breusch-Godfrey test
Breusch-Pagan, see Breusch-Pagan test
chi-squared hypothesis, see chi-squared hypothesis test
Chow, see Chow test
comparison (between nested models), see
comparison test between nested models
cusum, see cusum test
test, continued
Durbin's alternative, see Durbin's alternative test endogeneity, see endogeneity test
Engle's LM, see Engle's LM test equality of
binomial proportions, see equality test of binomial proportions
coefficients, see equality test of coefficients distributions, see distributions, testing equality of margins, see equality test of margins means, see equality test of means medians, see equality test of medians proportions, see equality test of proportions ROC areas, see equality test of ROC areas variances, see equality test of variances equivalence, see equivalence test
exogeneity, see endogeneity test
Fisher's exact, see Fisher's exact test
goodness-of-fit, see goodness of fit
Hausman specification, see Hausman specification test
heteroskedasticity, see heteroskedasticity test independence, see independence test, also see Breusch-Pagan test
independence of irrelevant alternatives, see independence of irrelevant alternatives
information matrix, see information matrix test interrater agreement, see interrater agreement
Kolmogorov-Smirnov, see Kolmogorov-Smirnov test
Kruskal-Wallis, see Kruskal-Wallis test
kurtosis, see kurtosis
likelihood-ratio, see likelihood-ratio test
linear hypotheses after estimation, see linear hypothesis test after estimation
marginal homogeneity, see marginal homogeneity, test of
margins, see margins test
model coefficients, see model coefficients test
model specification, see specification test
nonlinear hypotheses after estimation, see nonlinear hypothesis test after estimation
normality, see normal distribution and normality omitted variables, see omitted variables test
overidentifying restrictions, see overidentifying restrictions test
permutation, see permutation test
Ramsey, see Ramsey test
random-order, see random-order test
RESET, see RESET test
serial correlation, see autocorrelation
serial independence, see serial independence test
Shapiro-Francia, see Shapiro-Francia test for normality
Shapiro-Wilk, see Shapiro-Wilk test for normality skewness, see skewness
symmetry, see symmetry test
Szroeter's, see Szroeter's test for heteroskedasticity
TDT, see transmission-disequilibrium test
test, continued
transmission-disequilibrium test, see transmissiondisequilibrium test
trend, see trend, test for
variance-comparison, see variance-comparison test
Vuong, see Vuong test
weak instrument, see weak instrument test
test command, \([R]\) anova postestimation, \([R]\) test
testnl command, [R] testnl
testparm command, \([R]\) test
tetrachoric command, \([R]\) tetrachoric
three-stage least squares, \([R]\) reg3
timeout1, set subcommand, \([R]\) netio, \([R]\) set
timeout2, set subcommand, \([R]\) netio, \([R]\) set
time-series analysis, \([R]\) regress postestimation time series
time-versus-concentration curve, \([\mathrm{R}] \mathbf{p k}\) title, estimates subcommand, [R] estimates title tnbreg command, \([R]\) tnbreg, \([R]\) tnbreg
postestimation
tobit command, \([R]\) tobit, \([R]\) tobit postestimation
tobit regression, \([\mathrm{R}]\) ivtobit, \([\mathrm{R}]\) tobit, also see intreg
command, also see truncreg command
.toc filename suffix, \([R]\) net
tolerance () option, [R] maximize
total command, \([R]\) total, \([R]\) total postestimation
totals, estimation, \([R]\) total
tpoisson command, \([R]\) tpoisson, \([R]\) tpoisson postestimation
trace,
ml subcommand, \([\mathrm{R}] \mathrm{ml}\)
query subcommand, \([R]\) query
set subcommand, \([R]\) set
trace option, [R] maximize
tracedepth, set subcommand, [R] set
traceexpand, set subcommand, [R] set
tracehilite, set subcommand, [R] set
traceindent, set subcommand, [R] set
tracenumber, set subcommand, [R] set
tracesep, set subcommand, [R] set
tracing iterative maximization process, \([\mathrm{R}]\) maximize transformations
\(\log ,[R]\) Inskew0
modulus, \([R]\) boxcox
power, \([R]\) boxcox, \([R]\) Inskew0
to achieve normality, \([R]\) boxcox, \([R]\) ladder
to achieve zero skewness, \([R]\) Inskew0
translate command, \([\mathrm{R}]\) translate
translate logs, \([\mathrm{R}]\) translate
translator
query command, \([R]\) translate
reset command, \([R]\) translate
set command, [R] translate
transmap
define command, \([R]\) translate
query command, \([R]\) translate transmission-disequilibrium test, \([R]\) symmetry
tree, misstable subcommand, [R] misstable
trend, test for, \([R]\) nptrend, \([R]\) symmetry
triangle kernel function, \([\mathrm{R}]\) kdensity, \([\mathrm{R}]\) lpoly, [R] qreg
truncated
negative binomial regression, \([\mathrm{R}]\) tnbreg
observations, \([R]\) truncreg, also see censored observations
Poisson regression, [R] tpoisson
regression, \([R]\) truncreg
truncreg command, \([R]\) truncreg, \([R]\) truncreg postestimation
ttest and ttesti commands, [R] ttest
Tukey's multiple-comparison adjustment, see multiple comparisons, Tukey's method
tuning constant, \([\mathrm{R}]\) rreg
two-stage least squares, \([R]\) ivregress
two-way
analysis of variance, \([R]\) anova
scatterplots, \([R]\) lowess
type,
set subcommand, \([R]\) set
ssc subcommand, \([R]\) ssc

\section*{U}
\(U\) statistic, [R] ranksum
uniformly distributed random-number function, \([R]\) set seed
uninstall,
net subcommand, \([R]\) net
ssc subcommand, [R] ssc
unique values, counting, \([R]\) table, \([R]\) tabulate oneway
univariate
distributions, displaying, \([R]\) cumul, \([R]\) diagnostic plots, \([R]\) histogram, \([R]\) ladder, \([R]\) lv, [R] stem
kernel density estimation, \([\mathrm{R}]\) kdensity
update
all command, [R] update
command, \([\mathrm{R}]\) update
from command, \([R]\) update
query command, \([R]\) update
update,
query subcommand, [R] query
view subcommand, \([R]\) view
update_d, view subcommand, \([R]\) view
update_interval, set subcommand, [R] set, [R] update
update_prompt, set subcommand, [R] set,
[R] update
update_query, set subcommand, \([R]\) set, \([R]\) update
updates to Stata, \([R]\) adoupdate, \([R]\) net, \([R]\) sj, [R] update
use, estimates subcommand, \([R]\) estimates save
user-written additions, installing, \([R]\) net, \([R]\) ssc searching for, \([R]\) net search, \([R]\) ssc
using,
cmdlog subcommand, [R] log
log subcommand, [R] log

\section*{v}
varabbrev, set subcommand, [R] set variables,
categorical, see categorical data, agreement, measures for
dummy, see indicator variables factor, see factor variables in model, maximum number, \([R]\) matsize orthogonalize, \([R]\) orthog
variance,
analysis of, \([R]\) anova, \([R]\) loneway, \([R]\) oneway
displaying, \([R]\) summarize, \([R]\) tabstat
estimators, [R] vce_option
Huber/White/sandwich estimator, see robust, Huber/White/sandwich estimator of variance
inflation factors, \([\mathrm{R}]\) regress postestimation
nonconstant, see robust, Huber/White/sandwich estimator of variance
stabilizing transformations, [R] boxcox
testing equality of, \([R]\) sdtest
variance-covariance matrix of estimators, \([\mathrm{R}]\) correlate, \([R]\) estat, \([R]\) estat vce
variance-comparison test, \([R]\) sdtest
variance-weighted least squares, \([R]\) vwls
varkeyboard, set subcommand, [R] set
vce, estat subcommand, \([R]\) estat, \([R]\) estat vce
vce() option, [R] vce_option
version of ado-file, [R] which
version of Stata, [R] about

\section*{view}
ado command, \([R]\) view
ado_d command, \([R]\) view
browse command, \([R]\) view
command, \([R]\) view
help command, \([R]\) view
help_d command, \([R]\) view
net command, \([R]\) view
net_d command, \([R]\) view
news command, \([R]\) view
search command, \([R]\) view
search_d command, \([R]\) view
update command, \([R]\) view
update_d command, [R] view
view_d command, [R] view
view_d, view subcommand, \([R]\) view
viewing previously typed lines, \([R]\) \#review
vif, estat subcommand, \([R]\) regress postestimation
Vuong test, [R] zinb, [R] zip
vwls command, \([R]\) vwls, \([R]\) vwls postestimation

\section*{W}

Wald test, [R] contrast, [R] predictnl, [R] test, [R] testnl
weak instrument test, \([R]\) ivregress postestimation
weighted least squares, \([R]\) regress
for grouped data, \([R]\) glogit
generalized linear models, \([\mathrm{R}]\) glm
generalized method of moments estimation,
\([R]\) gmm, \([R]\) ivpoisson
instrumental-variables regression, \([R]\) gmm,
[ \(R\) ] ivregress
nonlinear least-squares estimation, \([\mathrm{R}] \mathbf{n l}\)
nonlinear systems of equations, \([R]\) nlsur
variance, \([R]\) vwls
Welsch distance, \([R]\) regress postestimation
which command, [R] which
White/Huber/sandwich estimator of variance, see robust,
Huber/White/sandwich estimator of variance
White's test for heteroskedasticity, [R] regress
postestimation
Wilcoxon
rank-sum test, \([\mathrm{R}]\) ranksum
signed-ranks test, \([\mathrm{R}]\) signrank

\section*{X}
xchart command, [R] qc
xi prefix command, \([R]\) xi

\section*{Z}

Zellner's seemingly unrelated regression, \([R]\) sureg,
\([R]\) reg3, [ \(R\) ] suest
zero-altered, see zero-inflated
zero-inflated
negative binomial regression, \([\mathrm{R}]\) zinb
Poisson regression, [R] zip
zero-skewness transform, \([R]\) Inskew0
zinb command, \([R]\) zinb, \([R]\) zinb postestimation
zip command, [R] zip, [R] zip postestimation```


[^0]:    Main
    add(\#) adds the value \# to each variable in varlist before computing the means and confidence intervals. This option is useful when analyzing variables with nonpositive values.
    only modifies the action of the add(\#) option so that it adds \# only to variables with at least one nonpositive value.
    level(\#) specifies the confidence level, as a percentage, for confidence intervals. The default is level (95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals.

[^1]:    Introduction
    One-way ANOVA
    Two-way ANOVA
    $N$-way ANOVA
    Weighted data
    ANCOVA
    Nested designs
    Mixed designs
    Latin-square designs
    Repeated-measures ANOVA
    Video examples

[^2]:    ${ }^{1}$ forecast is not appropriate with mi estimation results.

[^3]:    * case(varname) and alternatives (varname) are required.

[^4]:    Introduction
    Variance structures

[^5]:    Background
    The problem of determining $N$
    The problem of conformable likelihoods
    The first problem does not arise with AIC; the second problem does Calculating BIC correctly

[^6]:    Also see
    [R] binreg postestimation - Postestimation tools for binreg
    [R] glm - Generalized linear models
    [ME] mecloglog - Multilevel mixed-effects complementary log-log regression
    [ME] meglm - Multilevel mixed-effects generalized linear model
    [ME] melogit - Multilevel mixed-effects logistic regression
    [ME] meprobit - Multilevel mixed-effects probit regression
    [MI] estimation - Estimation commands for use with mi estimate
    [U] 20 Estimation and postestimation commands

[^7]:    1 forecast may not be used with mi estimation results.

[^8]:    Main
    mu , the default, specifies that predict calculate the expected value of $y$, equal to $g^{-1}(\mathbf{x} \widehat{\boldsymbol{\beta}})\left[n g^{-1}(\mathbf{x} \widehat{\boldsymbol{\beta}})\right.$ for the binomial family].
    xb calculates the linear prediction $\eta=\mathbf{x} \widehat{\boldsymbol{\beta}}$.
    eta is a synonym for $x b$.
    stdp calculates the standard error of the linear prediction.

[^9]:    1 lrtest is not appropriate with svy estimation results.

[^10]:    *poisson is required.

[^11]:    ${ }^{1}$ forecast is not appropriate with mi or svy estimation results.
    2 lrtest is not appropriate with svy estimation results.

[^12]:    ${ }^{1}$ forecast is not appropriate with mi or svy estimation results.
    2 lrtest is not appropriate with svy estimation results.

[^13]:    Note: Bonferroni-adjusted p-values are reported for tests on individual contrasts only.

[^14]:    Also see
    [R] doedit - Edit do-files and other text files
    [P] include - Include commands from file
    [GSM] 13 Using the Do-file Editor-automating Stata
    [GSU] 13 Using the Do-file Editor-automating Stata
    [GSW] 13 Using the Do-file Editor-automating Stata
    [U] 15 Saving and printing output-log files
    [U] 16 Do-files

[^15]:    Also see
    [R] logistic - Logistic regression, reporting odds ratios
    [R] logit - Logistic regression, reporting coefficients
    [R] probit - Probit regression
    [R] ivprobit - Probit model with continuous endogenous regressors
    [R] lroc - Compute area under ROC curve and graph the curve
    [R] Isens - Graph sensitivity and specificity versus probability cutoff
    [R] estat gof - Pearson or Hosmer-Lemeshow goodness-of-fit test
    $[\mathrm{R}]$ roc - Receiver operating characteristic (ROC) analysis
    [U] 20 Estimation and postestimation commands

[^16]:    eqlist is rarely used and specifies the variables, with optional equation name, to be summarized. eqlist may be varlist or (eqname ${ }_{1}$ : varlist) (eqname ${ }_{2}$ : varlist) .... varlist may contain time-series operators; see

[^17]:    Integration
    intmethod(intmethod) specifies the integration method to be used for the random-effects model. It accepts one of four arguments: mvaghermite, the default for all but a crossed random-effects model, performs mean and variance adaptive Gauss-Hermite quadrature; mcaghermite performs mode and curvature adaptive Gauss-Hermite quadrature; ghermite performs nonadaptive GaussHermite quadrature; and laplace, the default for crossed random-effects models, performs the Laplacian approximation.
    intpoints(\#) specifies the number of integration points to use for integration by quadrature. The default is intpoints (12); the maximum is intpoints(195). Increasing this value improves the accuracy but also increases computation time. Computation time is roughly proportional to its value.

[^18]:    Main
    residuals(res_option) specifies what type of residuals to plot in the component-plus-residual plot. res_option is the same option that would be specified to predict after est_cmd. Residuals can be omitted from the plot by specifying residuals (none). residuals() is required.
    equation (eqno) is relevant only when you have previously fit a multiple-equation model in est_cmd. It specifies the equation to which you are referring.

[^19]:    . use http://www.stata-press.com/data/r13/auto2
    . fvrevar i.rep78 mpg turn

[^20]:    ${ }^{1}$ forecast is not appropriate with mi or svy estimation results.
    2 lrtest is not appropriate with svy estimation results.

[^21]:    Also see
    [R] glogit postestimation - Postestimation tools for glogit, gprobit, blogit, and bprobit
    [R] logistic - Logistic regression, reporting odds ratios
    [R] logit - Logistic regression, reporting coefficients
    [R] probit - Probit regression
    [R] scobit - Skewed logistic regression
    [U] 20 Estimation and postestimation commands

[^22]:    * You may not specify both hasderivatives and haslfderivatives.
    $\dagger$ You must specify equations(namelist) or nequations(\#); you may specify both.
    $\ddagger$
    You must specify parameters(namelist) or nparameters(\#); you may specify both.

[^23]:    Instruments for equation 1: private chronic female income _cons

[^24]:    Advanced
    tconsistent (string) and tefficient (string) are formatting options. They allow you to specify the headers of the columns of coefficients that default to the names of the models. These options will be of interest primarily to programmers.

[^25]:    1 estat ic and suest are not appropriate after heckman, twostep.
    2 lrtest is not appropriate with svy estimation results.

[^26]:    1 lrtest is not appropriate with svy estimation results.

[^27]:    1 lrtest is not appropriate with svy estimation results.

[^28]:    ${ }^{1}$ forecast is not appropriate with svy estimation results.
    2 lrtest is not appropriate with svy estimation results.

[^29]:    1 lrtest is not appropriate with svy estimation results.

[^30]:    * You can specify at most one of these options.
    $\ddagger$ These options may be specified only when igmm is specified. varlist $_{1}$ and varlist $_{\mathrm{iv}}$ may contain factor variables; see [U] 11.4.3 Factor variables. depvar, varlist ${ }_{1}$, varlist $t_{2}$, and varlist ${ }_{\mathrm{iv}}$ may contain time-series operators; see [U] 11.4.4 Time-series varlists. bootstrap, by, jackknife, rolling, and statsby are allowed; see [U] 11.1.10 Prefix commands.
    Weights are not allowed with the bootstrap prefix; see [R] bootstrap.
    aweights are not allowed with the jackknife prefix; see [R] jackknife. aweights, fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.
    See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

[^31]:    Also see
    [R] ivprobit postestimation - Postestimation tools for ivprobit
    [R] gmm - Generalized method of moments estimation
    [R] ivregress - Single-equation instrumental-variables regression
    [R] ivtobit - Tobit model with continuous endogenous regressors
    [R] probit - Probit regression
    [SVY] svy estimation - Estimation commands for survey data
    [XT] xtprobit - Random-effects and population-averaged probit models
    [U] 20 Estimation and postestimation commands

[^32]:    1 estat ic and suest are not appropriate after ivprobit, twostep.
    2 forecast is not appropriate with svy estimation results or after ivprobit, twostep.
    3 lrtest is not appropriate with svy estimation results.

[^33]:    ${ }^{1}$ forecast is not appropriate with svy estimation results.

[^34]:    *You must specify at least one of $11[(\#)]$ and $\mathrm{ul}[(\#)]$.

[^35]:    Also see
    [R] ivtobit postestimation - Postestimation tools for ivtobit
    [R] gmm - Generalized method of moments estimation
    [R] ivprobit - Probit model with continuous endogenous regressors
    $[R]$ ivregress - Single-equation instrumental-variables regression
    $[\mathrm{R}]$ regress - Linear regression
    [R] tobit - Tobit regression
    [SVY] svy estimation - Estimation commands for survey data
    [XT] xtintreg - Random-effects interval-data regression models
    [XT] xttobit — Random-effects tobit models

[^36]:    1 estat ic and suest are not appropriate after ivtobit, twostep.
    2 forecast is not appropriate with svy estimation results or after ivtobit, twostep.
    3 lrtest is not appropriate with svy estimation results.

[^37]:    Add plots
    addplot (plot) provides a way to add other plots to the generated graph. See [G-3] addplot_option.

[^38]:    ${ }^{1}$ forecast is not appropriate with mi or svy estimation results.
    2 lrtest is not appropriate with svy estimation results.

[^39]:    Maximization
    maximize_options: difficult, technique (algorithm_spec), iterate(\#), [no] log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance(\#), nrtolerance(\#), nonrtolerance, and from(init_specs); see [R] maximize. These options are seldom used.

[^40]:    Also see
    [R] logit postestimation - Postestimation tools for logit
    [R] brier - Brier score decomposition
    [R] cloglog - Complementary log-log regression
    [R] exlogistic - Exact logistic regression
    $[\mathrm{R}]$ glogit - Logit and probit regression for grouped data
    $[\mathrm{R}]$ logistic - Logistic regression, reporting odds ratios
    [R] probit - Probit regression
    $[R]$ roc - Receiver operating characteristic (ROC) analysis
    [ME] melogit - Multilevel mixed-effects logistic regression
    [MI] estimation - Estimation commands for use with mi estimate
    [SVY] svy estimation - Estimation commands for survey data
    [XT] xtlogit - Fixed-effects, random-effects, and population-averaged logit models
    [U] 20 Estimation and postestimation commands

[^41]:    ${ }^{1}$ forecast is not appropriate with mi or svy estimation results.
    2 lrtest is not appropriate with svy estimation results.

[^42]:    . use http://www.stata-press.com/data/r13/lbw, clear
    (Hosmer \& Lemeshow data)

[^43]:    Also see
    [R] logit - Logistic regression, reporting coefficients
    [R] estat classification - Classification statistics and table
    [R] estat gof - Pearson or Hosmer-Lemeshow goodness-of-fit test
    $[\mathrm{R}]$ Iroc - Compute area under ROC curve and graph the curve
    [R] Isens - Graph sensitivity and specificity versus probability cutoff
    [U] 20 Estimation and postestimation commands

[^44]:    Introduction
    Local polynomial smoothing
    Choice of a bandwidth
    Confidence bands

[^45]:    Also see
    [R] logistic - Logistic regression, reporting odds ratios
    [R] logit - Logistic regression, reporting coefficients
    [R] probit - Probit regression
    [R] ivprobit - Probit model with continuous endogenous regressors
    $[\mathrm{R}]$ Iroc - Compute area under ROC curve and graph the curve
    [R] estat classification - Classification statistics and table
    [R] estat gof - Pearson or Hosmer-Lemeshow goodness-of-fit test
    $[R]$ roc - Receiver operating characteristic (ROC) analysis
    [U] 20 Estimation and postestimation commands

[^46]:    iffin/over
    over (varlist) specifies that separate sets of margins be estimated for the groups defined by varlist. The variables in varlist must contain nonnegative integer values. The variables need not be covariates in your model. When over () is combined with the vce (unconditional) option, each group is treated as a subpopulation; see [SVY] subpopulation estimation.
    $\operatorname{subpop}([$ varname $][$ if $]$ ) is intended for use with the vce (unconditional) option. It specifies that margins be estimated for the single subpopulation identified by the indicator variable or by the if expression or by both. Zero indicates that the observation be excluded; nonzero, that it be included; and missing value, that it be treated as outside of the population (and so ignored). See [SVY] subpopulation estimation for why subpop() is preferred to if expressions and in ranges when also using vce(unconditional). If subpop() is used without vce(unconditional), it is treated merely as an additional if qualifier.

[^47]:    SE/Cluster
    vce (vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (analytic), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
    vce(analytic), the default, uses the analytically derived variance estimator associated with the sample mean.

[^48]:    ${ }^{1}$ forecast is not appropriate with mi or svy estimation results.
    2 lrtest is not appropriate with svy estimation results.

[^49]:    ${ }^{1}$ forecast is not appropriate with mi or svy estimation results.
    2 lrtest is not appropriate with svy estimation results.

[^50]:    ${ }^{1}$ forecast is not appropriate with mi or svy estimation results.
    2 lrtest is not appropriate with svy estimation results.

[^51]:    Model 2
    lnlsq(\#) fits the model by using log least-squares, which we define as least squares with shifted lognormal errors. In other words, $\ln ($ depvar - \#) is assumed to be normally distributed. Sums of squares and deviance are adjusted to the same scale as depvar.

[^52]:    ${ }^{1}$ forecast is not appropriate with svy estimation results.
    2 lrtest is not appropriate with svy estimation results.
    3 You must specify the variables() option with nl .

[^53]:    Introduction
    Basics
    Using the post option
    Reparameterizing ML estimators for univariate data
    nlcom versus eform

[^54]:    Two-level nested logit model
    Three-level nested logit model

[^55]:    1 forecast is not appropriate with mi or svy estimation results.
    2 lrtest is not appropriate with svy estimation results.

[^56]:    pr , the default, calculates the predicted probabilities. If you do not also specify the outcome() option, you specify $k$ new variables, where $k$ is the number of categories of the dependent variable. Say that you fit a model by typing ologit result x 1 x 2 , and result takes on three values. Then you could type predict p1 p2 p3 to obtain all three predicted probabilities. If you specify the outcome() option, you must specify one new variable. Say that result takes on the values 1,2 , and 3 . Typing predict p 1 , outcome (1) would produce the same p 1 .
    xb calculates the linear prediction. You specify one new variable, for example, predict linear, xb . The linear prediction is defined, ignoring the contribution of the estimated cutpoints.
    stdp calculates the standard error of the linear prediction. You specify one new variable, for example, predict se, stdp.
    outcome(outcome) specifies for which outcome the predicted probabilities are to be calculated. outcome () should contain either one value of the dependent variable or one of \#1, \#2, ..., with \#1 meaning the first category of the dependent variable, \#2 meaning the second category, etc.
    nooffset is relevant only if you specified offset (varname) for ologit. It modifies the calculations made by predict so that they ignore the offset variable; the linear prediction is treated as $\mathbf{x}_{j} \mathbf{b}$ rather than as $\mathbf{x}_{j} \mathbf{b}+$ offset $_{j}$.

[^57]:    ${ }^{1}$ forecast is not appropriate with mi or svy estimation results.
    2 lrtest is not appropriate with svy estimation results.

[^58]:    Options
    compare (string) specifies the two treatments to be tested for equivalence. Sometimes there may be more than two treatments, but the equivalence can be determined only between any two treatments.
    limit(\#) specifies the equivalence limit. The default is 0.2 . The equivalence limit can be changed only symmetrically; that is, it is not possible to have a 0.15 lower limit and a 0.2 upper limit in the same test.
    level(\#) specifies the confidence level, as a percentage, for confidence intervals. The default is level(90). This setting is not controlled by the set level command.

[^59]:    Y axis, X axis, Titles, Legend, Overall

[^60]:    ${ }^{1}$ forecast is not appropriate with mi or svy estimation results.
    2 lrtest is not appropriate with svy estimation results.

[^61]:    1 forecast is not appropriate with mi or svy estimation results.
    2 lrtest is not appropriate with svy estimation results.

[^62]:    SE/Cluster
    vce (vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (analytic), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
    vce(analytic), the default, uses the analytically derived variance estimator associated with the sample proportion.

[^63]:    * tukey, snk, duncan, and dunnett are only allowed with results from anova, manova, regress, and mvreg. tukey, snk, duncan, and dunnett are not allowed with results from svy.

[^64]:    Sorted by:

[^65]:    ${ }^{1}$ forecast is not appropriate with mi estimation results.

[^66]:    The ratio estimator
    Survey data
    The survey ratio estimator
    The standardized ratio estimator
    The poststratified ratio estimator
    The standardized poststratified ratio estimator
    Subpopulation estimation

[^67]:    ${ }^{1}$ forecast is not appropriate with mi or svy estimation results.
    2 lrtest is not appropriate with svy estimation results.

[^68]:    Reference line
    rlopts (cline_options) affects the rendition of the reference line. See [G-3] cline_options.

[^69]:    Statistics $>$ Linear models and related $>$ Regression diagnostics $>$ Residual-versus-predictor plot

[^70]:    Main
    confband specifies that simultaneous confidence bands be plotted around the ROC curve.
    norefline suppresses plotting the 45 -degree reference line from the graphical output of the ROC curve.
    level(\#) specifies the confidence level, as a percentage, for the confidence bands. The default is level (95) or as set by set level; see [R] level.

[^71]:    *probit is required.

[^72]:    *probit and ml are required.

[^73]:    Parametric model: Summary parameter definition
    Maximum likelihood estimation
    Estimating equations estimation

[^74]:    * Only one of roc() or invroc() may be specified.

[^75]:    _-. 2000. sg120.1: Two new options added to rocfit command. Stata Technical Bulletin 53: 18-19. Reprinted in Stata Technical Bulletin Reprints, vol. 9, pp. 230-231. College Station, TX: Stata Press.

[^76]:    ${ }^{1}$ forecast is not appropriate with svy estimation results.
    ${ }^{2}$ lrtest is not appropriate with svy estimation results.

[^77]:    . set showemptycells off

