Engineering Mathematics
Made Easy with TI-36X Pro
(All India Syllabus)

By PROF. AJOY K RAY

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TEXAS INSTRUMENTS
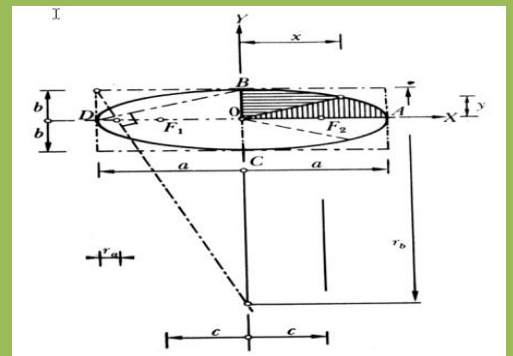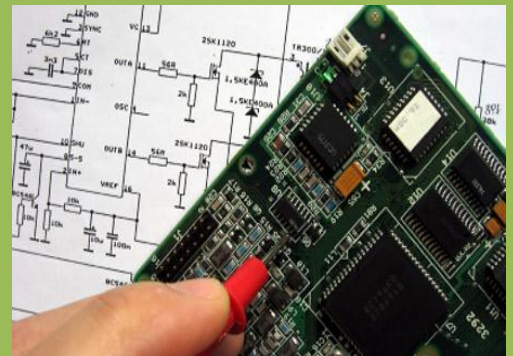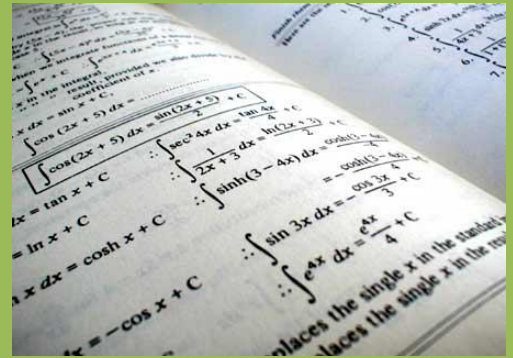
Engineering Mathematics Made Easy with TI-36X Pro (All India Syllabus)

Published by Statworks India Private Limited ("Statworks")

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FOREWORD

To all Academicians and Students

Are you expecting to solve Engineering Mathematics easily and quickly using the latest and leading Scientific Calculator in the market right now? If your answer is a resounding YES, then you are on the right path purchasing my book: **ENGINEERING MATHEMATICS MADE EASY WITH TI-36XPRO (All India Syllabus)**.

This book is an interesting combo package – It is designed to be used in conjunction with Engineering Mathematics Textbooks. The calculator selected for this workbook is the Texas Instruments Scientific Calculator – TI-36XPRO.

Each section in this workbook shows how to use the calculator to work out solutions to problems similar to those found in Engineering Mathematics textbooks. Most types of problems are covered. Emphasis is on the use of the scientific functions of the calculator. The only assumption is that students have the identical calculator. If another model of calculator is used, it is possible, or even likely, that other keystroke sequences will be needed to achieve correct results. Some calculators will not be able to do all the types of problems demonstrated in this workbook. This guidebook will enable students and academicians to be in a better position to assist others on the usage of the scientific calculator to solve Engineering Mathematics problems.

The students will find this book useful in assisting them to improve their skills on how to use the Scientific Calculator TI-36XPRO to efficiently solve their problems. The **ENGINEERING MATHEMATICS MADE EASY WITH TI-36XPRO (All India Syllabus)** provides the Engineering Mathematics examples, answers and step to be followed with the calculator to solve these mathematics problems. A student could either refer to the answer before following the steps or the student could follow the steps and check whether their methods and answers are correct in solving the problems. It is a way for the students to assess themselves with the help of the book.

Texas Instruments TI 36X Pro is the leading Advanced Scientific Calculator (non-programmable) which is widely used in US, Europe and other Asian Countries. It is fast gaining acceptance in India due to its unique features and ease of use. This guidebook helps academicians and students to learn to use TI-36XPRO thoroughly which in turn help students to save time in solving mathematical problems during their examinations.

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ABOUT THE AUTHOR

The Author, Prof. Ajoy Kumar Ray is the Vice Chancellor of the Bengal Engineering and Science University, Shibpur. Previously, he was a Professor of Electronics and Electrical Communication Engineering and Head, School of Medical Science and Technology at IIT Kharagpur. He has co-authored about 90 research papers well as has authored five books published by International Publishing Houses, such as John Wiley, Tata McGraw Hill, Prentice Hall of India etc. He is an expert in engineering mathematics and is experienced in using various types of calculators whereby which he has selected the TI-36XPRO Scientific Calculator and has written a Guidebook to assist students and academicians in solving Engineering Mathematics problems easily.

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Arithmetic

Overview / Introduction / Terminology:

Arithmetic is the study of numerical quantities. At least a basic understanding of arithmetic is fundamental in the study of algebra and all other mathematical studies. This book assumes that the reader already understands some mathematics but wishes to relearn it in a more formal manner.

Arithmetic Sequence

Arithmetic sequence is a sequence of numbers that has a constant difference between every two consecutive terms.

In other words, arithmetic sequence is a sequence of numbers in which each term except the first term is the result of adding the same number, called the common difference, to the preceding term. Example: The sequence $5, 11, 17, 23, 29, 35 \ldots$ is an arithmetic sequence; because the same number 6 (i.e. the common difference) is added to each term of the sequence to get the succeeding term.

Arithmetic Progressions

By an arithmetic progression of m terms, we mean a finite sequence of the form
$$a, a + d, a + 2d, a + 3d, \ldots, a + (m - 1)d$$

The real number a is called the first term of the arithmetic progression, and the real number d is called the difference of the arithmetic progression.

Arithmetic Series

A series such as $3 + 7 + 11 + 15 + \cdots + 99 \; or \; 10 + 20 + 30 + \cdots + 10000$ which has a constant difference between terms. The first term is a_1, the common difference is d, and the number of terms is n. The sum of an arithmetic series is found by multiplying the number of terms times the average of the first and last terms. Formula:

$$Sum = n\left(\frac{a_1 + a_n}{2}\right) \; or \; \frac{n}{2}[2a_1 + (n - 1)d]$$

Keystrokes:

`math` , `2nd` `num-solv` , `sto→`

Topic:	Math Concepts:
Basic Arithmetic	Arithmetic Sequence, Arithmetic Progressions, Arithmetic Series.

Example:

1. Fine the value of the following summation:

$$\sum_{n=15}^{47} 2n - 5$$

From the formula ("$2n - 5$") for the n-th term, each term will be two units larger than the previous term. (Plug in values for n if you're not sure about this.) So this is indeed an arithmetical sum. But this summation starts at $n = 15$, not at $n = 1$, and the summation formula applies to sums starting at $n = 1$. So how can it work with this summation?

The quickest way to find the value of this sum is to find the $14th$ and $47th$ partial sums, and then subtract the $14th$ from the $47th$. By doing this subtraction, it be left with the value of the sum of the $15th$ through $47th$ terms. The first term is $a_1 = 2(1) - 5 = -3$. The other necessary terms are $a_{14} = 2(14) - 5 = 23$ and $a_{47} = 2(47) - 5 = 89$.

$$\sum_{n=1}^{14} 2n - 5 = \left(\frac{14}{2}\right)(-3 + 23) = (7)(20) = 140$$

$$\sum_{n=1}^{14} 2n - 5 = \left(\frac{47}{2}\right)(-3 + 89) = \left(\frac{47}{2}\right)(86) = 2021$$

Subtracting:

$$\sum_{n=15}^{47} 2n - 5 = \left(\sum_{n=1}^{47} 2n - 5\right) - \left(\sum_{n=1}^{47} 2n - 5\right)$$
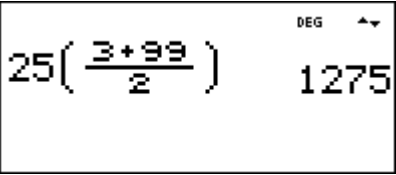
$$= 2021 - 140 = 1881$$

Topic:	Math Concepts:
Basic Arithmetic	Arithmetic Sequence, Arithmetic Progressions, Arithmetic Series.

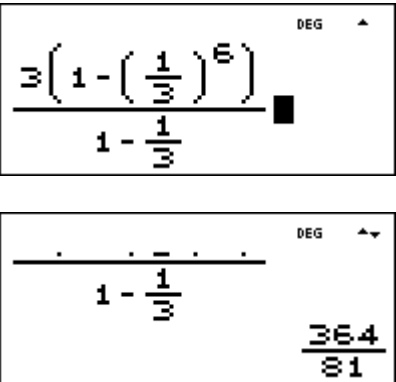| No	Keystrokes	Screenshots
1	Assume $n = x$, Firstly press button, math ⌄⌄⌄⌄ enter or 5:sum(,	
2	Insert value in the formulae, 1 5 ⌃ 4 7 ▷ 2 x^{yzt}_{abcd} − 5 ▷ enter The answer = 1881.	

2. $3 + 7 + 11 + 15 + \cdots + 99$ has $a\_1 = 3$ and $d = 4$. To find x, use the explicit formula for an arithmetic sequence.

No	Keystrokes	Screenshots
1	Firstly solve value for n, press 2nd [num-solv], Then insert value 3 + ((x^{yzt}_{abcd} − 1) 4 ▷ 9 9 enter	
2	Assume $x = 0$, and press enter enter Press ▷ enter to go to the home screen.	

Topic:	Math Concepts:
Basic Arithmetic	Arithmetic Sequence, Arithmetic Progressions, Arithmetic Series.

No	Keystrokes	Screenshots
3	And then, insert $x = 25$ in the formula and press 2 5 (□/□ 3 + 9 9 ⊝ 2 ▷) enter . The answer is 1275.	$25\left(\dfrac{3+99}{2}\right)$ DEG 1275

3. Solve, $3 + 1 + \dfrac{1}{3} + \dfrac{1}{9} + \dfrac{1}{27} + \dfrac{1}{81}$ has $a = 3, t = \dfrac{1}{3}$ and $x = 6$.

No	Keystrokes	Screenshots
1	Store the value in the variable a, t, x. Press 3 sto→ x^{yzt}_{abcd} x^{yzt}_{abcd} x^{yzt}_{abcd} x^{yzt}_{abcd} x^{yzt}_{abcd} (a) enter , □/□ 1 ⊝ 3 ▷ sto→ x^{yzt}_{abcd} x^{yzt}_{abcd} x^{yzt}_{abcd} x^{yzt}_{abcd} (t) enter and 6 sto→ x^{yzt}_{abcd} (x) enter . And press clear (home screen).	DEG $3 \rightarrow a$ 3 $\dfrac{1}{3} \rightarrow t$ $\dfrac{1}{3}$ $6 \rightarrow x$ 6
2	Then, insert value in the formula, press □/□ 3 (1 − (□/□ 1 ⊝ 3 ⊝) $x^{□}$ 6 ▷) ⊝ 1 − □/□ 1 ⊝ 3 ▷ enter .	DEG $\dfrac{3\left(1-\left(\dfrac{1}{3}\right)^{6}\right)}{1-\dfrac{1}{3}}$ ■ DEG $\dfrac{1-\dfrac{1}{3}}{}$ $\dfrac{364}{81}$

Exercise:

1. Holes are drilled 35.7 mm apart in a metal plate. If a row of 26 holes is drilled, determine the distance, in centimeters, between the centers of the first and last holes.
2. Determine the HCF and LCM of the numbers given:
   a) $12, 30, 45$
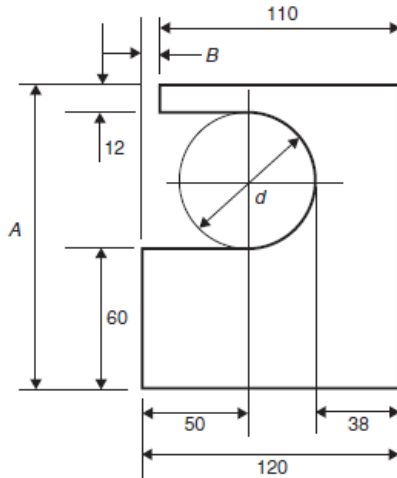   b) $10, 15, 70, 105$
   c) $196, 210, 910, 462$

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3. Simplify the expressions given in below:

   a) $\frac{112}{16} - 119 \div 17 + (3 \times 19)$

   b) $\frac{(50-14)}{3} + 7(16 - 7) - 7$

4. Calculate the diameter d and dimensions A and B for the template shown in the figure below. All dimensions are in the millimeters.

5. Expand the following series and find the sum :

$$\sum_{n=0}^{4} 2n$$

Answer:

1. $89.25\ cm$
2. a) $HCF = 3, LCM = 180$
   b) $HCF = 5, LCM = 210$
   c) $HCF = 14, LCM = 420$
3. a) 57
   b) 68
4. $d = 64\ \text{mm},\ A = 136\ \text{mm},\ B = 10\ \text{mm}.$
5. 20

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Fractions, Decimals and Percentages

Overview / Introduction / Terminology:

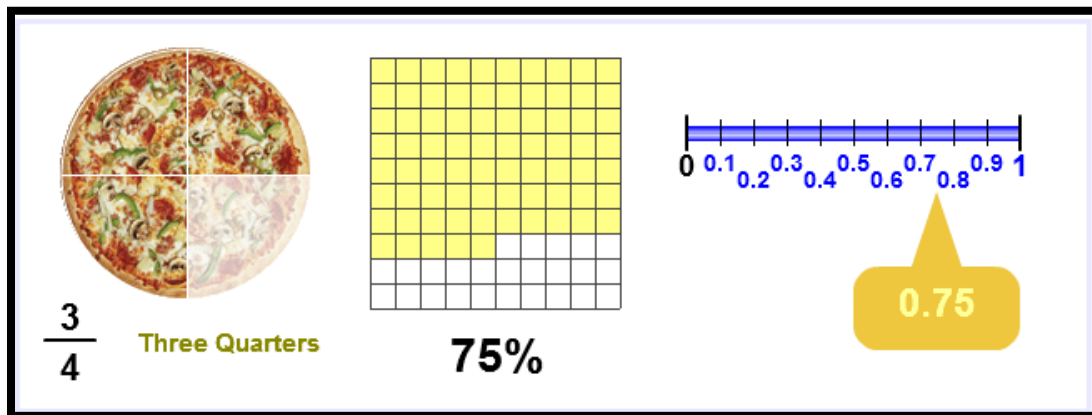Decimals, Fractions and Percentages are just different ways of showing the same value:

A Half can be written...

As a fraction: $\dfrac{1}{2}$

As a decimal: 0.5

As a percentage: 50%

A Quarter can be written...

As a fraction: $\dfrac{1}{4}$
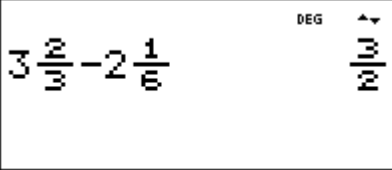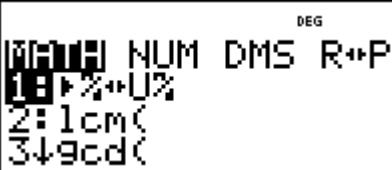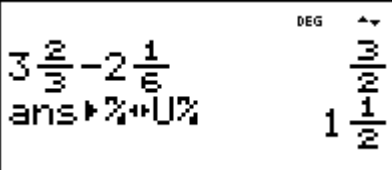
As a decimal: 0.25

As a percentage: 25%

$\dfrac{3}{4}$ **Three Quarters**   75%   0.75

Keystrokes:

▯, 2nd ▯▯ math , mode (DEC), 2nd % , ◂▸≈

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Topic:	Math Concepts:
Fractions, Decimals and Percentages	Fractions, Decimals, Percentages and Converter.

Example:
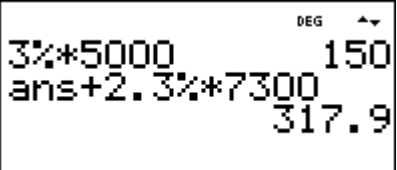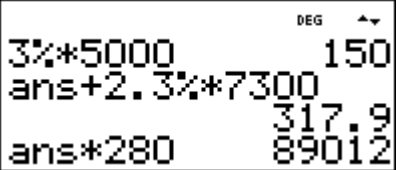
1. Find the value of $3\frac{2}{3} - 2\frac{1}{6}$

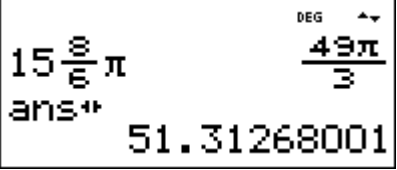| No	Keystrokes	Screenshots
1	To view a value as a mathprint mode, Press: mode ⌄ ⌄ ⌄ ⌄ ⌄ ⟩ enter . Go to a home screen with press clear .	DEG DEC HEX BIN OCT † CLASSIC MATHPRINT
2	Press 2nd 7 [□▯] to enter a mixed number. Insert value with press, 3 ⟩ 2 ⟩ 3 ⟩ ⟩ ☐ 2nd 7 2 ⟩ 1 ⟩ 6 ⟩ ⟩ enter	DEG ▲▾ $3\frac{2}{3} - 2\frac{1}{6}$ $\frac{3}{2}$
3	To chance the proper fractions to the mixed fraction, press math 1: enter .	DEG MATH NUM DMS R↔P 1:▶%↔U% 2:1cm(3↓gcd(
4	Press enter to continue, The answer is $1\frac{1}{2}$	DEG ▲▾ $3\frac{2}{3} - 2\frac{1}{6}$ ans▶%↔U% $\frac{3}{2}$ $1\frac{1}{2}$

2. A mining company extracts 5000 tons of ore with a concentration of metal of 3% and 7300 tons with a concentration of 2.3%. On the basis of these two extraction figures, what is the total quantity of metal obtained?

If one ton of metal is worth 280 dollars, what is the total value of the metal extracted?

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Topic:	Math Concepts:
Fractions, Decimals and Percentages	Fractions, Decimals, Percentages and Converter.

No	Keystrokes	Screenshots
1	Press 3 $2nd$ $[\%]$ $\times$ 5 0 0 0 $enter$ Then press $+$ 2 $\cdot$ 3 $2nd$ $[\%]$ $\times$ 7 3 0 0 $enter$	DEG 3%*5000 150 ans+2.3%*7300 317.9
2	After get a value, Press $\times$ 2 8 0 The two extractions represent a total of 317.9 tons of metal for a total value of **89012 dollars**.	DEG 3%*5000 150 ans+2.3%*7300 317.9 ans*280 89012

3. Express as decimal fraction: $15\frac{8}{6}\pi$

No	Keystrokes	Screenshots
1	Press $2nd$ 7 to get the template for mixed fraction Then insert 1 5 $\lozenge$ 8 $\lozenge$ 6 $\lozenge$ π^e_i $enter$	DEG $15\frac{8}{6}\pi$ $\frac{49\pi}{3}$
2	To change the answer as decimal, press $\leftrightarrow\approx$ Thus, $15\frac{8}{6}\pi = \mathbf{51.31}$	DEG $15\frac{8}{6}\pi$ $\frac{49\pi}{3}$ ans▸ 51.31268001

Topic:	Math Concepts:
Fractions, Decimals and Percentages	Fractions, Decimals, Percentages and Converter.

Exercise:

1. $\left(\frac{2}{3} \times 1\frac{1}{4}\right) \div \left(\frac{2}{3} + \frac{1}{4}\right) + 1\frac{3}{5}$

2. Determine the dimension marked x in the length of shaft shown in Figure below. The dimentions are in millimeters.
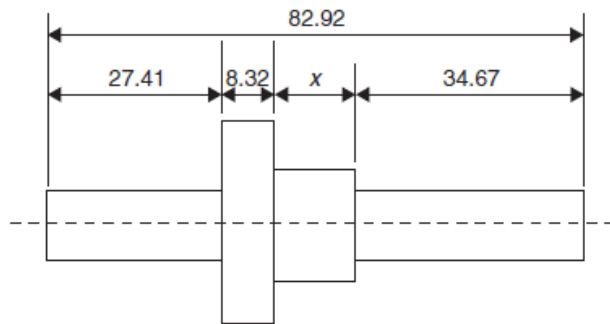
3. A concrete mixture contain seven parts by volume of ballast, four parts by volume of sand and two parts by volume of cement. Determine the percentage of each of these three constituents correct to the nearest 1% and the mass of cement in a two tonne dry mix, correct to 1 significant figure.

4. Convert to percentages:
   a) 0.057
   b) 0.374
   c) 1.285

5. In a sample of iron ore, 18% is iron. How much more is needed to produce 3600 kg of iron?

Answer:

1. $228/55$
2. $12.52 \ mm$
3. $54\%, 31\%, 15\%, 0.3 \ t$
4. (a) 5.7%
   (b) 37.4%
   (c) 128.5%
5. $20 \ 000 kg$

Indices, Standard Form and Engineering Notation

Overview / Introduction / Terminology:

General use of decimal notation for whole numbers and decimal fractions dates from 1585 when *Simon Stevin* (1548–1620) published his book, *Die Thiende*. Stevin used powers of 10 to introduce place value and showed how the algebra of powers (the *index laws*) led to relatively simple ways of doing arithmetic. We write a number such as: Three hundred and sixteen and a quarter in terms of powers of 10 as $3 \times 10^2 + 1 \times 10^1 + 2 \times 10^0 + 2 \times 10^{-1} + 5 \times 10^{-2}$ and shorten this to 316.25.

Here $10^n = 10 \times 10 \times \ldots \times 10$, (*n* times) when $n > 0, 10^0 = 1 \; and \; 10^{-n} = \frac{1}{10^n}$.
When we multiply 316.25 by 10 we use the index law, e $10^n \times 10 = 10^{n+1}$ (and the distributive law) to obtain the quick answer 3162.5.

The two basic *index laws:* $\mathbf{10^a \times 10^b = 10^{a+b}}$ **and** $(\mathbf{10^a})^{\mathbf{b}} = \mathbf{10^{ab}}$ can be easily checked from the definitions when *a* and *b* are positive integers. A little more thought is needed when *a* and/or *b* are negative integers (or fractions!).

The definitions of $10^{\,0}(= 1)$ and $10^{-n} \left(= \frac{1}{10^n}\right)$, and later of $10^{\frac{1}{2}}(= \sqrt{10})$, are chosen to ensure that the basic index laws

$$x^a \times x^b = x^{a+b}, (x^a)^b = x^{ab} \; and \; x^a . y^a = (xy)^a$$

The index laws allow us to write very large numbers in a compact and manageable form. For Example, the number of atoms in the universe is frighteningly large but elementary arguments show that this number is approximately 10^{50}. Scientific notation provides an agreed way of giving in standard form the approximate value of very large numbers which occur in science, e.g.

$$2^{10} = 1024 = 1.024 \times 10^3 \approx 1 \times 10^3$$

$$2^{20} = 1\,048576 \approx 1.05 \times 10^6$$

Writing numbers in this form makes it easy to do rough calculators. For example,

$$2^{40} = (2^{20})^2 = (1.05 \times 10^6)^2$$

$$(1.05 \times 10^6)^2 = (1.05)^2 \times (10^6)^2 = 1.1025 \times 10^2$$

$$2^{-20} = \frac{1}{2^{20}} = \frac{1}{1.05 \times 10^6} = 0.95 \times 10^{-6}$$

$$0.95 \times 10^{-6} = 9.5 \times 10^{-7}$$
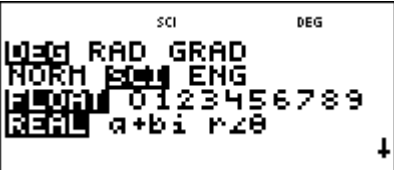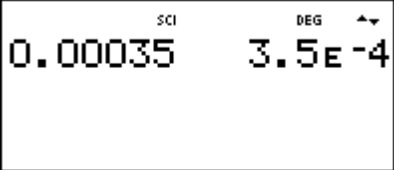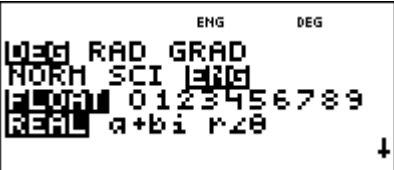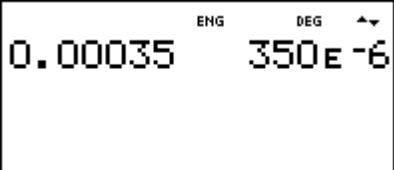
Keystrokes:
math [MATH][NUM], mode [SCI][ENG], 2nd [num-solv]

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Topic:	Math Concepts:
Calculations and Evaluation of Formulae	Calculations and Evaluation of Formulae

Example:

1. Engineering notation is similar to scientific notation except that the power of ten is always a multiple of 3. For example,

$0.00035 = 3.5 \times 10^{-4}$ in scientific notation,

$0.00035 = 0.35 \times 10^{-3}$ or 350×10^{-6} in engineering notation.

No	Keystrokes	Screenshots
1	In scientific notation, press [mode]⊙⟩[enter][clear], [0][·][0][0][0][3][5][enter] .	
2	In engineering notation, press [mode]⊙⟩⟩[enter][clear],[0][·][0][0][0][3][5][enter]. Thus, in scientific notation = 3.5E^{-4} and in engineering notation = 350 E^{-6}	

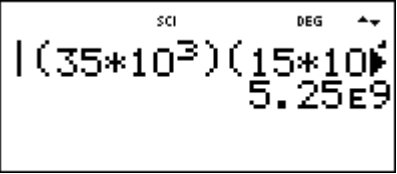2. Find the value of x on the following equation:

$$15x^2 + \frac{1}{2}\pi = 20$$

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Topic:	Math Concepts:
Calculations and Evaluation of Formulae	Calculations and Evaluation of Formulae

No	Keystrokes	Screenshots
1	To find the value of x, press 2nd [num-solv],	DEG ∷=∷ Enter equation to solve.
2	Insert equation in the formulae. Press 1 5 x^{yzt}_{abcd} x^2 + ☐/☐ 1 ⌄ 2 ⟩ π^e_i ⟩ 2 0. Then press enter.	DEG $15x^2+\frac{1}{2}\pi=20$▮
3	Assume that $x = 0$ and press enter enter	DEG ENTER AND SOLVE † X=0 SOLVE: X
4	So, value of $x = 1.1084$. Press ⟩ enter to back to home screen.	DEG SOLUTION † X=1.108428427 L - R =0 SOLVE AGAIN QUIT

3. Evaluate $|(35 \times 10^3)(15 \times 10^4)|$. Expressing answer is standard form.

No	Keystrokes	Screenshots
1	Firstly, Change mode calculator to the scientific mode with press, mode ⌄ ⟩ enter clear .	SCI DEG DEG RAD GRAD NORM SCI ENG FLOAT 0123456789 REAL a+bi r∠θ ↓

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No	Keystrokes	Screenshots
2	Press [math] [▷] [enter] to get the absolute value.	MATH **NUM** DMS R▸P **1**▪abs(2:round(3↓iPart(
3	Then, press ([3][5][×][1][0][x□][3][▷][)]([1][5][×][1][0][x□][4][▷][)][▷][enter] So the answer is **5.25E9**	$\|(35*10^3)(15*10$ 5.25E9

Exercise:

1. In problems (a) to (e), simplify the expressions given, expressing the answers in index form and with positive indices:
    a) $4^2 \times 4^3 \times 4^4$
    b) $(15^3)^5$
    c) $\dfrac{5^{-7} \times 5^2}{5^{-8} \times 5^3}$
2. $\|(9.293 \times 10^2) + (1.3 \times 10^3)\|$ Expressing the answers in standard form.
3. Use a calculator to evaluate the following in engineering notation:
    a) $4.5 \times 10^{-7} \times 3 \times 10^4$
    b) $\dfrac{\left(1.6 \times 10^{-5}(25 \times 10^3)\right)}{(100 \times 10^{-6})}$
4. Find the intercepts for this equation $5x + 8y = 20$
5. Find the value of $7.9 \times 10^{-2} - 5.4 \times 10^{-2}$

Answer:

1. a) 4^9
    b) 15^{15}
    c) 1
2. $2.2293\text{E}3$
3. a) 13.5×10^{-3}
    b) 4×10^3
4. $x = 4$ and $y = 2.5$
5. 2.5×10^{-2}

Calculations and Evaluation of Formulae

Overview / Introduction / Terminology:

One commonly distinguishes between the relative error and the absolute error. The absolute error is the magnitude of the difference between the exact value and the approximation. The relative error is the absolute error divided by the magnitude of the exact value. The percent error is the relative error expressed in terms of per 100.

As an example, if the exact value is 50 and the approximation is 49.9, then the absolute error is 0.1 and the relative error is $0.1/50 = 0.002$. The relative error is often used to compare approximations of numbers of widely differing size; for example, approximating the number 1,000 with an absolute error of 3 is, in most applications, much worse than approximating the number 1,000,000 with an absolute error of 3; in the first case the relative error is 0.003 and in the second it is only 0.000003.

Another example would be if you measured a beaker and read $5mL$. The correct reading would have been $6mL$. This means that your % error (Approximate error) would be 16.66666. % error.

Given some value v and its approximation the **absolute error** is

$$\epsilon = |v - vapprox|$$

where the vertical bars denote the absolute value. If $v \neq 0$ the **relative error** is

$$n = \frac{|v - v_{approx}|}{|v|} = \left|\frac{v - v_{approx}}{v}\right| = \left|1 - \frac{v_{approx}}{v}\right|$$

and the **percent error** is

$$\delta = \frac{|v - v_{approx}|}{|v|} \times 100\% = \left|\frac{v - v_{approx}}{v}\right| \times 100\%$$

These definitions can be extended to the case when v and v_{approx} are n-dimensional vectors, by replacing the absolute value with an n-norm.
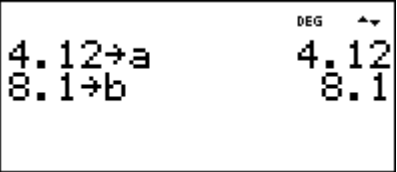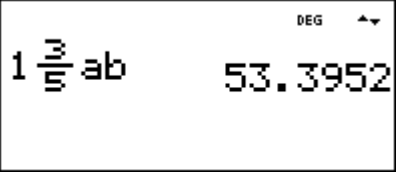
Keystrokes:

[2nd] [complex] , [sto→] [x^{yzt}_{abcd}] , [2nd] [$\sqrt{}$] , [π^e_i]

TEXAS INSTRUMENTS

$\delta\tau\alpha t\omega o\gamma\kappa\delta$
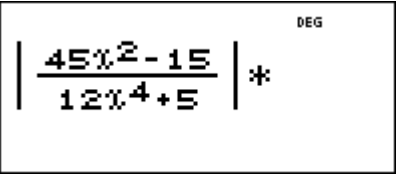Computational Analytics

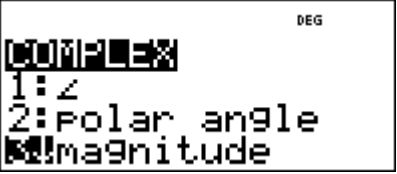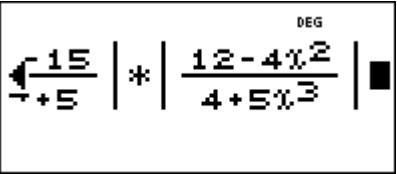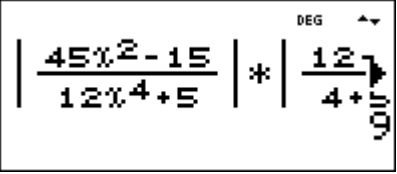| Topic:	Math Concepts:
Calculations and Evaluation of Formulae	Calculations and Evaluation of Formulae

Example:

1. The area A of a triangle is given by $A = 1\frac{3}{5}ab$. The base a when measures is found to be $4.12\ cm$, and the perpendicular height b is $8.1\ cm$. Determine the area of the triangle.

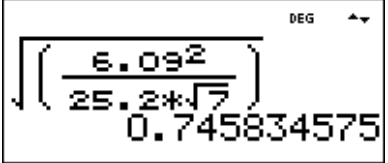| No	Keystrokes	Screenshots
1	Firstly, insert value in the variable using STO button. First variable $\boxed{4}\boxed{\cdot}\boxed{1}\boxed{2}\boxed{\text{sto}\rightarrow}\boxed{x^{yzt}_{abcd}}\boxed{x^{yzt}_{abcd}}\boxed{x^{yzt}_{abcd}}\boxed{x^{yzt}_{abcd}}\boxed{x^{yzt}_{abcd}}(a)\boxed{\text{enter}}$ Second variable $\boxed{8}\boxed{\cdot}\boxed{1}\boxed{\text{sto}\rightarrow}\boxed{x^{yzt}_{abcd}}\boxed{x^{yzt}_{abcd}}\boxed{x^{yzt}_{abcd}}\boxed{x^{yzt}_{abcd}}\boxed{x^{yzt}_{abcd}}\boxed{x^{yzt}_{abcd}}(b)\boxed{\text{enter}}$	DEG 4.12→a 4.12 8.1→b 8.1
2	After that, calculate $A = 1\frac{3}{5}ab$ with press button $\boxed{\text{2nd}}\boxed{7}\boxed{\square\frac{\square}{\square}}\boxed{1}\boxed{◊}\boxed{3}\boxed{⌄}\boxed{5}\boxed{◊}\boxed{x^{yzt}_{abcd}}\boxed{x^{yzt}_{abcd}}\boxed{x^{yzt}_{abcd}}\boxed{x^{yzt}_{abcd}}\boxed{x^{yzt}_{abcd}}(a)\boxed{◊}\boxed{x^{yzt}_{abcd}}$ $\boxed{x^{yzt}_{abcd}}\boxed{x^{yzt}_{abcd}}\boxed{x^{yzt}_{abcd}}\boxed{x^{yzt}_{abcd}}\boxed{x^{yzt}_{abcd}}(b)\boxed{\text{enter}}$. Thus, $A = 1\frac{3}{5}ab = \mathbf{53.4\ cm^2}$.	DEG $1\frac{3}{5}ab$ 53.3952

2. Calculate $\left|\frac{45x^2 - 15}{12x^4 + 5}\right| \times \left|\frac{12 - 4x^2}{4 + 5x^3}\right|$.

No	Keystrokes	Screenshots
1	Firstly, press $\boxed{\text{2nd}}\boxed{\text{complex}}\boxed{⌄}\boxed{⌄}\boxed{\text{enter}}$.	DEG **COMPLEX** 1:∠ 2:polar angle 3:magnitude
2	Insert the calculation. Press $\boxed{\frac{\square}{\square}}\boxed{4}\boxed{5}\boxed{x^{yzt}_{abcd}}\boxed{x^2}\boxed{-}\boxed{1}\boxed{5}\boxed{⌄}\boxed{1}\boxed{2}\boxed{x^{yzt}_{abcd}}\boxed{x^{\square}}\boxed{4}\boxed{◊}\boxed{+}\boxed{5}\boxed{◊}\boxed{◊}\boxed{\times}$	DEG $\left\|\dfrac{45x^2-15}{12x^4+5}\right\| *$

TEXAS INSTRUMENTS

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Computational Analytics

Topic: Calculations and Evaluation of Formulae	Math Concepts: Calculations and Evaluation of Formulae

No	Keystrokes	Screenshots
3	Then press 2nd [complex] ⊙⊙ enter	DEG **COMPLEX** 1:∠ 2:polar angle **3:magnitude**
4	Continue with the calculation. Press $\boxed{\frac{\square}{\square}}12-$ x$^{yzt}_{abcd}$ x²⊙4+5 x$^\square$3 ◐◐	DEG $\left\lvert\dfrac{-15}{?+5}\right\rvert * \left\lvert\dfrac{12-4x^2}{4+5x^3}\right\rvert \blacksquare$
5	Press enter. Thus, $\left\lvert\dfrac{45x^2-15}{12x^4+5}\right\rvert \times \left\lvert\dfrac{12-4x^2}{4+5x^3}\right\rvert = 9$	DEG $\left\lvert\dfrac{45x^2-15}{12x^4+5}\right\rvert * \left\lvert\dfrac{12}{4+5}\right\rvert$ 9

3. Evaluate the value of $\sqrt{\left[\dfrac{6.09^2}{25.2 \times \sqrt{7}}\right]}$

No	Keystrokes	Screenshots
1	Using the square root button Press 2nd [√] ($\boxed{\frac{\square}{\square}}$ x$^\square$ ◐6·09◐2◐◐25·2× 2nd [√] 7 ◐) ◐ enter. So, the answer is 0.745834575	DEG $\sqrt{\left(\dfrac{6.09^2}{25.2*\sqrt{7}}\right)}$ 0.745834575

4. The volume $V\ cm^3$ of a right circular cone is given by $V = \frac{1}{3}\pi x^2 a$. Given that $x = 4.321cm$ and $a = 18.35cm,$ find the volume V.

TEXAS INSTRUMENTS

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Computational Analytics

No	Keystrokes	Screenshots
1	Insert a value in the variable Press $\boxed{4}\boxed{\cdot}\boxed{3}\boxed{2}\boxed{1}\boxed{\text{sto→}}\boxed{x_{abcd}^{yzt}}\boxed{\text{enter}}$, $\boxed{1}\boxed{8}\boxed{\cdot}\boxed{3}\boxed{5}\boxed{\text{sto→}}\boxed{x_{abcd}^{yzt}}\boxed{x_{abcd}^{yzt}}\boxed{x_{abcd}^{yzt}}\boxed{x_{abcd}^{yzt}}\boxed{x_{abcd}^{yzt}}$ 4 times to get variable a.	DEG 4.321→x 4.321 18.35→a 18.35
2	Calculate $\qquad V = \frac{1}{3}\pi x^2 a \qquad$ with $\qquad$ press $\boxed{\frac{\square}{\square}}\boxed{1}\boxed{\lozenge}\boxed{3}\boxed{\lozenge}\boxed{\pi_i^e}\boxed{x_{abcd}^{yzt}}\boxed{x^2}\boxed{x_{abcd}^{yzt}}\boxed{x_{abcd}^{yzt}}\boxed{x_{abcd}^{yzt}}\boxed{x_{abcd}^{yzt}}\boxed{x_{abcd}^{yzt}}\boxed{\text{enter}}$. Hence volume, $V = 358.7841254\ cm^3$	DEG $\frac{1}{3}\pi x^2 a$ 358.7841254

Exercise:

1. For a gas $pV = c$. When pressure $p = 103400\ Pa$ and $V = 0.54m^3$ then c = 55836 Pa m³.
2. Use a calculator to evaluate correct to 4 significant figures:
    a) $\left(\frac{0.2681 \times 41.2^2}{32.6 \times 11.89}\right)^4$
    b) $\sqrt{(6.921^2 + 4.816^3 - 2.161^4)}$
3. Resistance R_2 is given by $R_2 = R_1(1+\propto t)$. Find R_2, correct to 4 significant figures, when $R^1 = 220, \propto= 0.00027$ and $t = 75.6$.
4. The area, A, of a circle is given by $A = \pi r^2$. Determine the area correct to 2 decimal place, given radius $r = 5.23\ m$.
5. The time of swing t seconds of a simple pendulum is given by $t = 2\pi\sqrt{\frac{l}{g}}$. Determine the time, correct to 3 decimal place, given that $l = 12.0$ and $g = 9.81$.

Answer:

1. Measured value, hence $c = 55\ 800\ Pa\ m^3$.
2. a) 1.900
    b) 11.74
3. 224.5
4. Hence area, $A = 85.93m^2$, correct to 2 decimal place.
5. Hence time $t = 6.950$ seconds, correct to 3 decimal place.

Topic:	Math Concepts:
Computer Numbering Systems	Binary, Decimal, Octal and Hexadecimal.

Computer Numbering Systems

Overview / Introduction / Terminology:

All computers do the amazing things that they do by simply manipulating ones and zeros. Computers can only do binary (base 2) arithmetic. It is able to use the binary number system in order to appreciate how the computer works. Since the computer works with the binary number system and since it is relatively easy to convert hexadecimal (base 16) and octal numbers (base 8) into binary, computer scientists need to be familiar with the octal and hexadecimal number systems.

I. Numbering Systems

1. Decimal System
   o base 10
   o valid digits $- 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$
2. Binary System
   o base 2
   o valid digits $- 0, 1$
3. Octal System
   o base 8
   o valid digits $- 0, 1, 2, 3, 4, 5, 6, 7$
4. Hexadecimal System
   o base 16
   o valid digits $- 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F$
   o A refers to decimal 10, B refers to decimal 11, etc.

II. To convert a number in a base other than 10 to the base 10

1. Easy way: To convert 345 (base 8) to decimal, it would be label the three columns as the $1's, 8's,$ and $64's$ columns **from right to left**. Then, multiply the digits by these column labels and compute the sum of the 3 products. Since a 3 is in the $64's$ column, multiply to obtain 192. Since 4 is in the 8's column, multiply $4 \ x \ 8$ to get 32. Then add 32 to the 192 from the step before to get a running total of 224. Finally, since a 5 is in the 1's column, multiply 1 x 5 to get the product of 5 and add it to the running toal of 224 to get 229. That final value, 229, is the decimal equivalent to the original number $345 \ (base \ 8)$.

2. Formal way: Expand the number $345 \ (base \ 8)$ into expanded form. Expanded form is written as

$$(3 \ x \ 8^2) + (4 \ x \ 8^1) + (5 \ x \ 8^0) = 229$$

III. To convert a decimal number into a base other than 10

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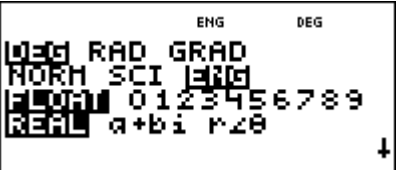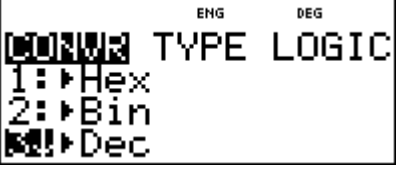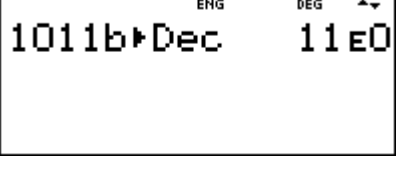| Topic:	Math Concepts:
Computer Numbering Systems	Binary, Decimal, Octal and Hexadecimal.

1. Write out the powers in the other base starting at the power of 0 until you reach a number higher than the given number.
2. Divide the highest power of the base that can divided at least once into the given number.
3. Put the quotient of that division into a column that will eventually be the leftmost digit of the final answer.
4. Continue steps 2 & 3 using the remainder of the previous division but add each successive quotient to the right of previous quotients in the final answer.
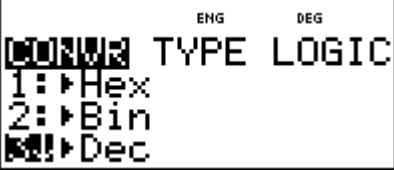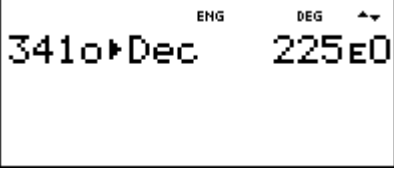
Keystrokes:

2nd [base n]

Example:

1. Convert 1011_2 into decimal number.

No	Keystrokes	Screenshots
1	Firstly, change the calculator mode to engineering mode. Press mode ⌄ ◊ ◊ enter and press clear to back to home screen.	DEG RAD GRAD NORM SCI ENG FLOAT 0123456789 REAL a+bi r∠θ
2	Press 1 0 1 1 in the calculator. After that, press 2nd [base n] ◊ ⌄ enter, to get the **b=binary.**	CONVR TYPE LOGIC 1:h 2:b 3:d
3	Then, to change to the decimal, press 2nd [base n] ⌄ ⌄ [▶Dec] enter enter .	CONVR TYPE LOGIC 1:▶Hex 2:▶Bin 3:▶Dec
4	Thus, 1011^2 to decimal = **11ᴇ0**	1011b▶Dec 11ᴇ0

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Topic:	Math Concepts:
Computer Numbering Systems	Binary, Decimal, Octal and Hexadecimal.
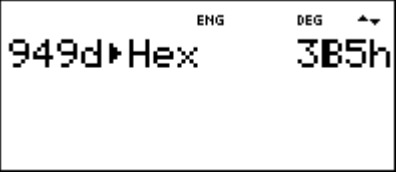
2. Convert 341_8 into decimal

No	Keystrokes	Screenshots
1	Firstly, press ③④①, in the calculator.	ENG DEG 341■
2	And followed by [2nd] [base n] ⊙⊙⊙⊙ [enter], to find the **o = octal**	ENG DEG CONVR **TYPE** LOGIC 2↑b 3:d **4:**o
3	Then, to convert to decimal, press [2nd] [base n] ⊙⊙ [enter] [enter].	ENG DEG **CONVR** TYPE LOGIC 1:▶Hex 2:▶Bin **3:**▶Dec
4	341_8 into decimal = **225 E0**	ENG DEG 341o▶Dec 225E0

3. Convert 949_{10} into hexadecimal number

No	Keystrokes	Screenshots
1	Press ⑨④⑨ in the calculator.	ENG DEG 949

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Computational Analytics

No	Keystrokes	Screenshots
2	Press [2nd][base n]⊙⊝⊝[:d]to find the **d=decimal** and press [enter],	
3	To change the decimal to hexadecimal number, Press [2nd][base n][enter][enter].	
4	Thus 949_{10} into $hexadecimal = \textbf{3B5h}$.	

Exercise:

1. Convert binary numbers given to decimal numbers:
   a) 11001_2
   b) 101101_2
2. Convert the decimal numbers to the octal numbers:
   a) 47.40625_{10}
   b) 535.90625_{10}
3. Convert octal numbers to their hexadecimal equations.
   a) 11010111_8
   b) 11101010_8
4. Convert the following hexadecimal numbers into their binary equivalents.
   a) $7B_{16}$
   b) $17D_{16}$
5. Convert binary to hexadecimal for 1110011110101001.

Answer:

1. a) 25_E0 b) 45_E0
2. a) $57o$ b) $1027o$
3. a) $241049h$ b) $248208h$
4. a) 1111011_2 b) 101111101_2
5. $E7A9_{16}$

TEXAS INSTRUMENTS

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Computational Analytics

Topic:	Math Concepts:
Algebra	Basic Algebra, Laws of Indices, Formula and Literal Equations.

Algebra

Overview / Introduction / Terminology:

Algebra is the branch of mathematics that uses letters in place of some unknown numbers. You've been using algebra since your early schooling, when you learned formulas like the **area of a rectangle**, with width w, height h:

$$A = w \times h$$

We used **letters to stand for numbers.** Once we knew the width and height, we could substitute them into the formula and find our area. Another one you may have seen is the **area of a square**, with sides s:

$$A = s^2$$

As soon as we know the length of the sides, we can find the area.

Literal numbers (the letters used in algebra) can either stand for **variables** (the value of the letter can change, like in the examples of the area of a rectangle and the area of a square) or **constants** (where the value does not change), for example e (which has a constant value of 2.781828...).

Keystrokes:

[sto→],[▯▯][▯▯],[π ᵉᵢ]

TEXAS INSTRUMENTS

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Topic:	Math Concepts:
Algebra	Basic Algebra, Laws of Indices, Formula and Literal Equations.

Example:

1. Simplify $\frac{x^{1/2}y^2z^{2/3}}{x^{1/4}y^{1/2}z^{1/6}}$ and evaluate when $x = 16, y = 9$ and $z = 4$, taking positive roots only.
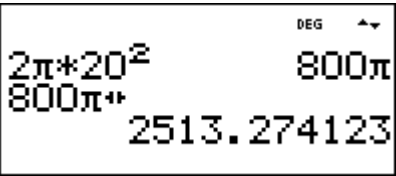
No	Keystrokes	Screenshots
1	Firstly you need to store a value into a variable Insert value $x = 16, y = 9$ and $z = 4$ Press $\boxed{1}\boxed{6}$ $\boxed{\text{sto}\rightarrow}$ $\boxed{x_{abcd}^{yzt}}$ $\boxed{\text{enter}}$ Press $\boxed{9}$ $\boxed{\text{sto}\rightarrow}$ $\boxed{x_{abcd}^{yzt}}$ $\boxed{x_{abcd}^{yzt}}$ $\boxed{\text{enter}}$, Press $\boxed{4}$ $\boxed{\text{sto}\rightarrow}$ $\boxed{x_{abcd}^{yzt}}$ $\boxed{x_{abcd}^{yzt}}$ $\boxed{x_{abcd}^{yzt}}$ $\boxed{\text{enter}}$.	DEG 16→x 16 9→y 9 4→z 4
2	Calculate the equation. Press $\boxed{\frac{\square}{\square}}\boxed{x_{abcd}^{yzt}}\boxed{x^\square}\boxed{\frac{\square}{\square}}\boxed{1}\boxed{0}\boxed{2}\boxed{0}\boxed{0}$, $\boxed{x_{abcd}^{yzt}}\boxed{x_{abcd}^{yzt}}\boxed{x^2}$, $\boxed{x_{abcd}^{yzt}}\boxed{x_{abcd}^{yzt}}\boxed{x_{abcd}^{yzt}}\boxed{x^\square}\boxed{\frac{\square}{\square}}\boxed{2}\boxed{0}\boxed{3}\boxed{0}\boxed{0}\boxed{0}$, $\boxed{x_{abcd}^{yzt}}\boxed{x^\square}\boxed{\frac{\square}{\square}}\boxed{1}\boxed{0}\boxed{4}\boxed{0}\boxed{0}$, $\boxed{x_{abcd}^{yzt}}\boxed{x_{abcd}^{yzt}}\boxed{x^\square}\boxed{\frac{\square}{\square}}\boxed{1}\boxed{0}\boxed{2}\boxed{0}\boxed{0}$ $\boxed{x_{abcd}^{yzt}}\boxed{x_{abcd}^{yzt}}\boxed{x_{abcd}^{yzt}}\boxed{x^\square}\boxed{\frac{\square}{\square}}\boxed{1}\boxed{0}\boxed{6}\boxed{0}\boxed{0}\boxed{0}$, $\boxed{\text{enter}}$	DEG 4→z 4 $\dfrac{x^{1/2}y^2z^{2/3}}{x^{1/4}y^{1/2}z^{1/6}}$ 108

2. What is the area of circle if the radius is $20\ cm$?
   Reminder: $A = 2\pi \times r^2$
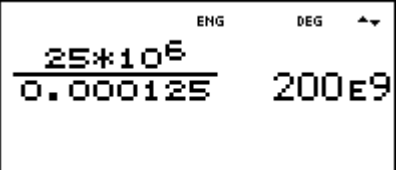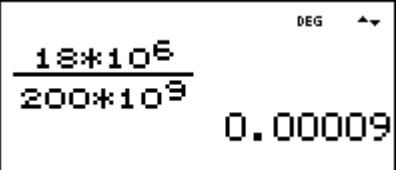
No	Keystrokes	Screenshots
1	Press $\boxed{2}\boxed{\pi_i^e}\boxed{\times}\boxed{2}\boxed{0}\boxed{x^2}\boxed{\text{enter}}$. The area of the circle is 800π square cm.	DEG $2\pi*20^2$ 800π

TEXAS INSTRUMENTS

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Computational Analytics

No	Keystrokes	Screenshots
2	Then press ⟨•≈⟩. The area of the circle is approximately 2513.27 square cm when rounded to one decimal place.	DEG 2π*20² 800π 800π•↔ 2513.274123

3. Hooke's law states that stress σ is directly proportional to strain ε within the elastic limit of a material. When, for mild steel, the stress is 25×10^6 pascals, the strain is 0.000125. Determine:
   a) The coefficient of proportionality
   b) The value of strain when the stress is 18×10^6 pascals.

No	Keystrokes	Screenshots
1	a) $\sigma \propto \varepsilon$, i.e. $\sigma = k\varepsilon$, from which $k = \sigma/\varepsilon$. Hence the coefficient of proportionality Firstly, change mode calculator in the engineering mode with press, mode ⊙ ⊙ ⊙	ENG DEG DEG RAD GRAD NORM SCI ENG FLOAT 0123456789 REAL a+bi r∠θ
2	Then press ▨ 2 5 × 1 0 $x^\square$ 6 ⊙ 0 · 0 0 0 1 2 5 ⊙ enter. So $k = 200 \times 10^9$ or 200ᴇ9	ENG DEG $\dfrac{25*10^6}{0.000125}$ 200ᴇ9
3	b) After that, change the calculator to the normal mode with press mode ⊙ enter.	DEG DEG RAD GRAD NORM SCI ENG FLOAT 0123456789 REAL a+bi r∠θ
4	Since $\sigma = k\varepsilon, \varepsilon = \sigma/k$. Hence when $\sigma = 18 \times 10^6$ Then press ▨ 1 8 × 1 0 $x^\square$ 6 ⊙ 2 0 0 × 1 0 $x^\square$ 9 ⊙ enter. So $\varepsilon = \dfrac{18\times10^6}{200\times10^9} = 0.00009$	DEG $\dfrac{18*10^6}{200*10^9}$ 0.00009

TEXAS INSTRUMENTS

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Computational Analytics

Topic:	Math Concepts:
Algebra	Basic Algebra, Laws of Indices, Formula and Literal Equations.

Exercise:

1. Subtract $\frac{3}{2}a - \frac{b}{3} + c$ from $\frac{b}{2} - 4a - 3c$

2. Simplify $\frac{a^5bc^3}{a^2b^3c^2}$ and evaluate when $a = \frac{3}{2}$, $b = \frac{1}{2}$ and $c = \frac{2}{3}$.

3. $\dfrac{\left(a^3b^{\frac{1}{2}}c^{-\frac{1}{2}}\right)(ab)^{\frac{1}{3}}}{\left(\sqrt{a^3}\ \sqrt{bc}\right)}$

4. If y is inversely proportional to x and $y = 15.3$ when $x = 0.6$, determine:
   a) The coefficient of proportionality
   b) The value of y when x is 1.5
   c) The value of x when y is 27.2

5. Boyle's law states that for a gas at constant temperature, the volume of a fixed mass of gas is inversely proportional to its absolute pressure. If a gas occupies a volume of $1.5m^3$ at a pressure of 200×10^3 pascals, determine
   a) The constant od proportionality
   b) The volume when the pressure is 800×10^3 pascals
   c) The pressure when the volume is $1.25m^3$.

Answer:

1. $-5\frac{1}{2}a + \frac{5}{6}b - 4$

2. $a^3b^{-2}c$, 9

3. $a^{11/6}b^{1/3}c^{-3/2}$ or $\dfrac{\sqrt[6]{a^{11}\sqrt[3]{b}}}{\sqrt{c^3}}$

4. a) 9.18
   b) 6.12
   c) 0.3375

5. a) 300×10^3
   b) $0..375m^2$
   c) 24×10^3

Logarithms

Overview / Introduction / Terminology:

If a number y can be written in the form a^x, then the index x is called the 'logarithm of y to the base of a',

$$\text{If } y = ax, \text{ then } x = \log_a y. \text{ Thus, since } 1000 = 10^3, \text{ then } 3 = \log 10^{1000}$$

Types of logarithms:

(a) Logarithms having a base of 10 are called common logarithms and $\log_{10}$ is usually abbreviated to lg.

For example: $lg\ 17.9 = 1.2528$

(b) Logarithms having a base of e (where 'e' is a mathematical constant approximately equal to 2.7183) are called hyperbolic, Napierian or natural logarithms, and $\log_e$ or usually abbreviated as ln.

For example: $ln\ 3.15 = 1.1474,\ ln\ 362.7 = 5.8935,\ ln\ 0.156 = -1.8578$

Laws of logarithms

(i)   To multiply two numbers $: log(A \times B) = log\ A + log B$

(ii)   To divide to numbers $: log\left(\frac{A}{B}\right) = log\ A - log\ B$

(iii)   To raise a number to the power $: log\ A^n = n\ log\ A$

A logarithmic scale is a scale of measurement that displays the value of a physical quantity using intervals corresponding to orders of magnitude, rather than a standard linear scale.

An exponential function is one which contains e^x, e being a constant called the exponent and having an approximate value of 2.7183. The exponent arises from the natural laws of growth and decay and is used as a base for natural or Napierian logarithms.

For example: $e^1 = 2.7182818,\quad e^{2.4} = 11.023176,$

The Laws of Growth and Decay

The laws of exponential growth and decay are of the form $y = Ae^{-kx}$ and $y = A\left(1 - e^{-kx}\right)$, where A and k are constants.

Keystrokes:

2nd   e□ 10□   ln log   [num-solv]   data   [stat-reg/distr]

TEXAS INSTRUMENTS

δτατШογκδ
Computational Analytics

Topic:	Math Concepts:
Number Sequences	Simple Sequence, Arithmetic Sequence, Geometric Sequence.

Example:
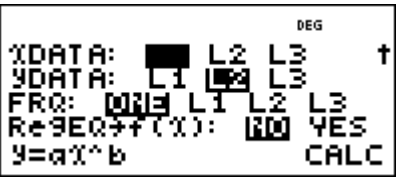
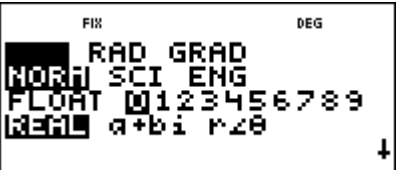1. The power dissipated by a resistor was measured for varying values of current flowing in the resistor and the results are as shown:

Current, I amperes	1.4	4.7	6.8	9.1	11.2	13.1
Power, P watts	49	552	1156	2070	3136	4290

Prove that the law relating current and power is of the form $P = RIn$, where R and n are constants, and determine the law. Hence calculate the power when the current is 12 amperes and the current when the power is 1000 $watts$.
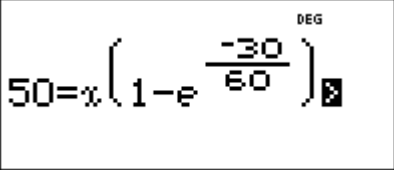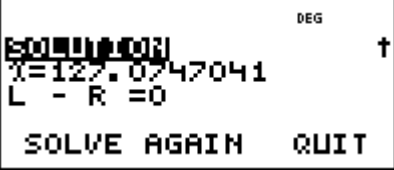
No	Keystrokes	Screenshots
1	To key in, the data, press ⌑data⌑ and key in the I values.	
2	To key in the data values in the table, press ⌑1⌑⌑·⌑⌑4⌑⌑enter⌑⌑4⌑⌑·⌑⌑7⌑⌑enter⌑⌑6⌑⌑·⌑⌑8⌑⌑enter⌑⌑9⌑⌑·⌑⌑1⌑⌑enter⌑ ⌑1⌑⌑1⌑⌑·⌑⌑2⌑⌑1⌑⌑3⌑⌑·⌑⌑1⌑⌑enter⌑	
3	To key in the data values in second table, press ⓞ and press ⌑4⌑⌑9⌑⌑enter⌑⌑5⌑⌑5⌑⌑2⌑⌑enter⌑⌑1⌑⌑1⌑⌑5⌑⌑6⌑⌑enter⌑⌑2⌑⌑0⌑⌑7⌑⌑0⌑⌑enter⌑ ⌑3⌑⌑1⌑⌑3⌑⌑6⌑⌑enter⌑⌑4⌑⌑2⌑⌑9⌑⌑0⌑⌑enter⌑	
4	To calculate the value of x and y, press ⌑2nd⌑⌑stat-reg/distr⌑	

TEXAS INSTRUMENTS

δτατ𝔴ογκδ
Computational Analytics

Topic:	Math Concepts:
Number Sequences	Simple Sequence, Arithmetic Sequence, Geometric Sequence.
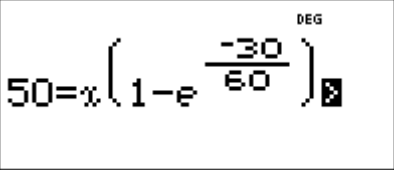
No	Keystrokes	Screenshots
5	Press ⊙⊙⊙⊙⊙⊙⊙ enter to choose Power Regression ax^b.	
6	Press ⊙⊙⊙⊙ enter to get the answer. Thus, value for $a = 24.996$ and $b = 2.000$	
7	To round the answer, press mode ⊙⊙◐ enter to choose rounded to 0.	
8	To get the answer, press 2nd [stat-reg/distr] enter to go back to the table.	
9	Thus, the answer is rounded to O. $$a = 25 \quad and \quad b = 2$$ So, the equation $P = RIn$ is $P = 25I^2$	

TEXAS INSTRUMENTS

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Computational Analytics

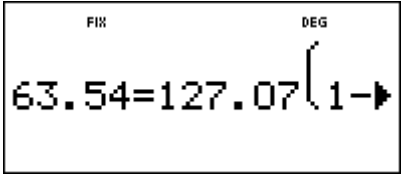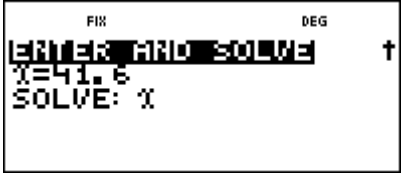| Topic:	Math Concepts:
Number Sequences	Simple Sequence, Arithmetic Sequence, Geometric Sequence.
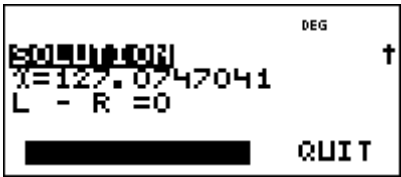
2. The temperature θ_2 of a winding which is being heated electrically at time t is given by:
   $\theta 2 = \theta 1 (1 − e\hat{} (−t/\tau))$ where θ_1 is the temperature (in degrees Celsius) at time t = 0 and τ is a constant. Calculate the value of θ_1, correct to the nearest degree, when θ_2 is 50°C, t is 30s and τ is 60s.

No	Keystrokes	Screenshots
1	Based on the question, $$50 = \theta 1 (1 − e^{−30/60})$$ To enter the equation and values, press [2nd] [num-solv] [5] [0] [▷] [▷] [x_{abcd}^{yzt}] [(] [(] [1] [(−)] [e□ 10□] [□/□] [(−)] [3] [0] [⌄] [6] [0] [▷] [▷]	$50=x\left(1-e^{\frac{-30}{60}}\right)$■
2	To get the answer, press [enter] [enter] Thus, $x = 127.07$°C	SOLUTION X=127.0747041 L - R =0 SOLVE AGAIN QUIT

3. The temperature θ_2 of a winding which is being heated electrically at time t is given by
   $\theta 2 = \theta\_1 (1 − e\hat{}((−t)/\tau))$ where θ_1 is the temperature (in degrees Celsius) at time $t = 0$ and τ is a constant. Calculate:
   (a) θ_1, correct to the nearest degree, when θ_2 is 50°C, t is 30s and τ is 60s.
   (b) The time t, correct to 1 decimal place, for θ_2 to be half the value of θ_1.

No	Keystrokes	Screenshots
1	To enter the values in the equation, press $$50 = x (1 − e^{\frac{−30}{60}})$$ [2nd] [num-solv] [5] [0] [▷] [x_{abcd}^{yzt}] [(] [(] [1] [−] [e□ 10□] [□/□] [(−)] [3] [0] [⌄] [6] [0] [▷] [▷] [)]	$50=x\left(1-e^{\frac{-30}{60}}\right)$■

TEXAS INSTRUMENTS

δτατ𝓌ογκδ
Computational Analytics

Topic:	Math Concepts:
Number Sequences	Simple Sequence, Arithmetic Sequence, Geometric Sequence.

No	Keystrokes	Screenshots
2	To get the answer, press [enter] [enter] [enter] Thus, $x @ \theta\_1 = 127.07℃$	DEG SOLUTION X=127.0747041 L - R =0 QUIT
3	Quit the previous calculation by press [2nd] [quit]. Before do the calculation, set the decimal place to 1. Press [mode] ⊙ ⊙ ⊙ ⊙ [enter] Press [clear] to exit from the mode screen.	FIX DEG DEG RAD GRAD NORM SCI ENG FLOAT 0■23456789 REAL a+bi r∠θ
4	Since $\theta_2 = \frac{1}{2}\theta_1$, thus $\theta_2 = 63.54$ From equation, $\theta2 = \theta\_1 (1 - e\char94((-t)/\tau))$ let $t = x$ $63.54 = 127.07 (1 - e^{\frac{-x}{60}})$ To enter the new values in the equation, press [2nd] [num-solv] to go back to the previous equation. Enter the new values in the equation, press [delete] [delete] [2nd] [insert] [6][3][·][5][4] ⊙ [delete] [2nd] [insert] [1][2][7][·][0][7] ⊙⊙⊙⊙ [delete] [delete] [delete] $[x_{abcd}^{yzt}]$ ⊙⊙⊙⊙	FIX DEG $63.54=127.07\left(1\rightarrow\right.$
5	To get the answer, press [enter] [enter] [enter] Thus, $t = 41.6$	FIX DEG ENTER AND SOLVE X=41.6 SOLVE: X

TEXAS INSTRUMENTS

δτατѠογκδ
Computational Analytics

Topic:	Math Concepts:
Number Sequences	Simple Sequence, Arithmetic Sequence, Geometric Sequence.

Exercise:

1. Atmospheric pressure p is measured at varying altitudes h and the results are as shown below:

Altitude, h cm	500	1500	3000	5000	8000
Pressure, p cm	73.39	68.42	61.60	53.56	43.41

    Show that the quantities are related by the $aw = ae^{kh}$, where a and k are constants. Determine the values of a and k and state the law. Find also the atmospheric pressure at $10000\ m$.

2. When a chain of length $2L$ is suspended from two points, $2D$ metres apart, on the same horizontal level
    $$D = k \left\{ \ln \left(\frac{L + \sqrt{L^2 + k^2}}{k} \right) \right\}$$
    Evaluate D when k = 75m and L = 180 m.

3. Choose K = 10 in the expression $x^K e^{-x}$ and calculate $x^K e^{-x}$ using your calculator for $x = 5, 10, 15, 20, 25, 30, 35$.

4. The amount of product x (in mol/cm^3) found in a chemical reaction starting with 2.5 mol/cm^3 of reactant is given by $x = 2.5\ (1 - e^{\wedge}(-4t))$ where t is the time, in minutes, to form product x. Plot a graph at 30 second intervals up to 2.5 minutes and determine x after 1 minute.

5. The amount A after n years of a sum invested P is given by the compound interest law: $A = Pe - rn/100$ when the per unit interest rate r is added continuously. Determine, correct to the nearest pounds, the amount after 8 years for a sum of £1500 invested if the interest rate is 6% per annum.

Answer:

1. $a = 76,\ \ k = -7\ x\ 10^{-5}$, $p = 76e^{-7\ x\ 10^{-5}}$, $37.74\ cm$

2. $120.7\ m$

3.

x	5	10	15	20	25	30	35
$x^{\wedge}10\ e^{\wedge}(-x)$	6.5 x104	4.5 x 105	1.7 x 105	2.1 x 104	1324	55	1.7

4. $2.45\ mol/cm3$

5. £2424

TEXAS INSTRUMENTS

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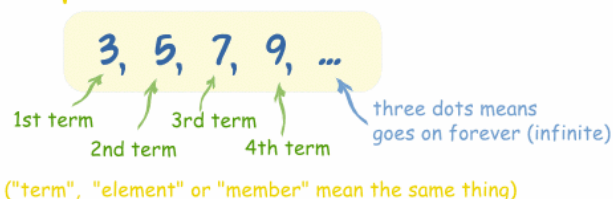
Number Sequences

Overview / Introduction / Terminology:

Number sequences can be used as a tool to practice and improve any numerical reasoning skills. By practicing it's can improve the numerical reasoning ability which can be of great help in daily life activities like loan calculations, groceries or during job applications in assessments.

Simple Sequence

If the sequence is simple enough one can look at the first few terms and guess the general rule for computing the nth term. For instance:

The graph of a sequence will look like a collection of dots whose x-coordinates are spaced one apart. Some examples of graphs of sequences are shown in Figure below.

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots \ldots a_n = \frac{1}{n}$$

Arithmetic Sequence

An arithmetic sequence is a mathematical sequence consisting of a sequence in which the next term originates by adding a constant to its predecessor. When the first term x1 and the difference of the sequence d is known, the whole sequence is fixed, or in formula:

$$X_n = a + d(n-1)$$

An example of this type of number sequence could be the following:

3, 8, 13, 18, 23, 28, 33, 38, …

This common difference is -2. The pattern is continued by subtracting 2 each time.

Geometric Sequence

A Geometric sequence is a mathematical sequence consisting of a sequence in which the next term originates by multiplying the predecessor with a constant, better known as the common ratio. When the first term x1 and the common ratio r are known, the whole sequence is fixed, or in formula:
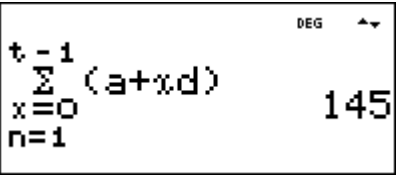
$$X_n = ar^{n-1}$$

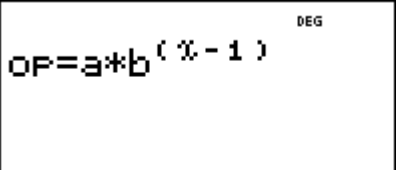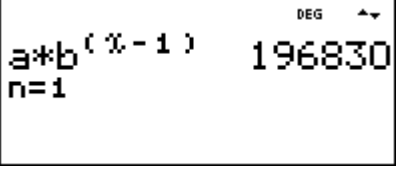An example of this type of number sequence could be the following:

2, 4, 8, 16, 32, 64, 128, 256, …

This sequence has a factor of 3 between each number, however as can be seen the sequence can work both by increasing as well as decreasing the value of numbers. The pattern is continued by dividing the last number by 3 each time.

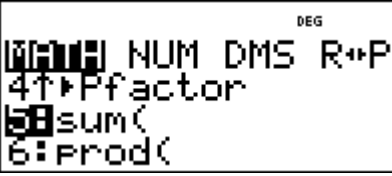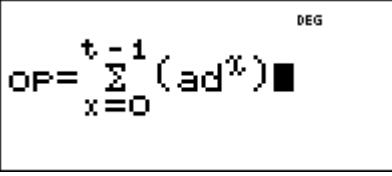| Topic: Number Sequences	Math Concepts: Simple Sequence, Arithmetic Sequence, Geometric Sequence.

Keystrokes:

2nd [set op] , 2nd [op] , sto→ x_{abcd}^{yzt} , math [sum(].

Example:

1. Add up the first 10 terms of the arithmetic sequence:

$$\{\ 1, 4, 7, 10, 13, \ldots\ \}$$

The value of $a = 1$ (the first term), $d = 3$ (the "common difference")and
$t = 10$ (how many terms to add up).

No	Keystrokes	Screenshots
1	Set operation with press 2nd [set op] ,	DEG OP=█
2	Then, press button math ⊙⊙⊙⊙ [sum(] enter .	DEG MATH NUM DMS R▸P 4↑▸Pfactor 5▸sum(6:prod(
3	Insert value, 0 ▷ x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} (t) − 1 ▷ x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} (a) + x_{abcd}^{yzt} ▷ x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} (d) ▷ enter clear .	DEG OP=$\sum_{x=0}^{t-1}(a+xd)$█
4	Insert value for variable, 1 sto→ x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} (a) enter , 3 sto→ x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} (d) enter , and 1 0 sto→ x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} (t) enter clear .	DEG ▲▾ 1→a 1 3→d 3 10→t 10

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No	Keystrokes	Screenshots
5	Recall the operation with press [2nd][op] So, the answer is **145.**	$\sum\limits_{x=0}^{t-1}(a+xd)$ 145 n=1

2. Find the 10th term of the geometric sequence for $10, 30, 90, 270, 810, 2430, …$ This sequence has a factor of 3 between each number.
   The value of $a = 10$ (first term) and $b = 3$ (common ratio)
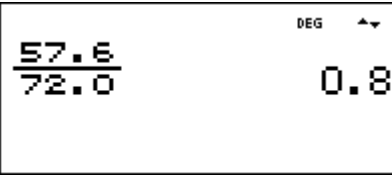
No	Keystrokes	Screenshots
1	Firstly, set the operation with press button [2nd][set op]. Insert value, $x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt}$ (a) [×] $x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt}$ $(b) x^{\square}$ [(] x_{abcd}^{yzt} [−] [1] [)] [▷] [enter] [clear].	OP=a*b$^{(x-1)}$
2	After that, store value in the variable, [1][0] [sto→] $x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt}$ (a) [enter], [3] [sto→] $x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt}$ (b) [enter], [1][0] [sto→] x_{abcd}^{yzt} [enter] [clear].	10→a 10 3→b 3 10→x 10
3	After finish, recall the operation, press [2nd][op] And the answer is **196830**	a*b$^{(x-1)}$ 196830 n=1

3. Sum the first 4 terms of $10, 30, 90, 270, 810, 2430, …$ This sequence has a factor of 3 between each number.
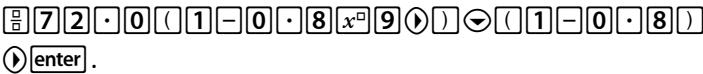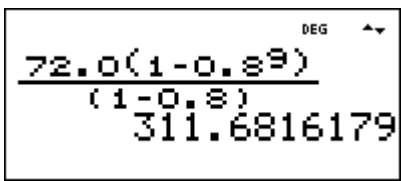   The value of $a = 10, d = 3 \; and \; t = 4$ are:

Topic:	Math Concepts:
Number Sequences	Simple Sequence, Arithmetic Sequence, Geometric Sequence.

No	Keystrokes	Screenshots
1	Set the operation in the calculation, press 2nd [set op].	DEG OP=■
2	Then press math ⊙⊙⊙⊙ [sum(],	DEG **MATH** NUM DMS R↔P 4↑▶Pfactor **5**▪sum(6:prod(
3	Insert value and press 0 ▷ x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} (t) $-$ 1 ▷ x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} (a) ▷ x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} (d) $x^{□}$ x_{abcd}^{yzt} (x) ▷ enter clear .	DEG $OP = \sum_{x=0}^{t-1}(ad^x)$■
4	After that, insert value in variable, 1 0 sto→ x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} (a) enter 3 sto→ x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} (d) enter 4 sto→ x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} (t) enter clear .	DEG ▲▾ 10→a 10 3→d 3 4→t 4
5	Press 2nd [op] to get the answer. The answer is 400	DEG ▲▾ $\sum_{x=0}^{t-1}(ad^x)$ n=1 400

4. Find the sum of the first 9 terms of the series 72.0, 57.6, 46.08, …

No	Keystrokes	Screenshots
1	For the common ratio, Press ▦ 5 7 $·$ 6 ⊙ 7 2 $·$ 0 enter $a = \mathbf{72.0}, \qquad r = \dfrac{57.6}{72.0} = \mathbf{0.8}$	DEG ▲▾ $\dfrac{57.6}{72.0}$ 0.8

TEXAS INSTRUMENTS

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No	Keystrokes	Screenshots
2	For the sum of 0 terms = $S_9 = \frac{a(1-r^n)}{(1-r)}$ Press ▣ 7 2 · 0 ((1 − 0 · 8 $x^\square$ 9 ◊)) ⊙ (1 − 0 · 8) ◊ enter . The answer is 311.7	$\dfrac{72.0(1-0.8^9)}{(1-0.8)}$ 311.6816179

Exercise:

1. The seventh term of a series is 29 and the eleventh term is 54. Determine the sixteenth term.
2. The sum of 15 terms of an arithmetic progression is 202.5 and the common difference is 2. Find the term of the series.
3. Which term of the series $3, 9, 27, \ldots$ is 59049?
4. The first, twelfth and last term of an arithmetic progression are $4, 31\frac{1}{2}$ and $376\frac{1}{2}$ respectively. Determine,
   a) The number of terms in the series
   b) The sum of all the terms
   c) The 80'th term.
5. In a geometric progression the 5th term is 9 times the 3th term and the sum of the 6th and 7th terms is 1944. Determine,
   a) The common ratio,
   b) The first term,
   c) The sum of the $4th$ to $10th$ terms inclusive.

Answer:

1. 85.25
2. $-\frac{1}{2}$
3. $10th$
4. (a) 150
   (b) 28537.5
   (c) $201\frac{1}{2}$
5. (a) 3
   (b) 2
   (c) 59022

TEXAS INSTRUMENTS

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Computational Analytics

Complex Number

Overview / Introduction / Terminology:

A complex number is a number that can be expressed in the form

$$a + bi$$

where a and b are real numbers and i is the _imaginary unit_, satisfying $i2 = -1$. For example, $-3.5 + 2i$ is a complex number. It is common to write a for $a + 0i$ and bi for $0 + bi$. Moreover, when the imaginary part is negative, it is common to write $a - bi$ with $b > 0$ instead of $a + (-b)i$, for example $3 - 4i$ instead of $3 + (-4)i$.
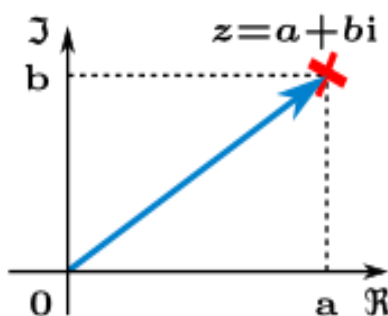
Complex Conjugates are a pair of complex numbers, both having the same real part, but with imaginary parts of equal magnitude and opposite signs. For example, $3 + 4i$ and $3 - 4i$ are complex conjugates.

The conjugate of the complex number z

$$z = a + ib$$

where a and b are real numbers, is

$$\overline{z} = a - ib$$
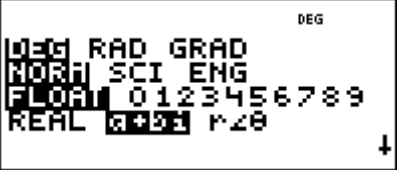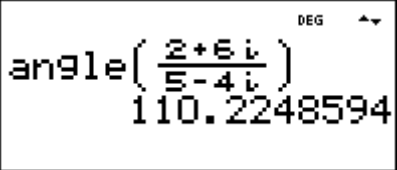
Keystrokes:
[complex] [cos / cos⁻¹] [sin / sin⁻¹] [enter]

Topic:	Math Concepts:
Complex Number	Complex Conjugate, magnitude, Polar form of complex number and distance between 2 complex numbers

Example:

1. Solve $\frac{2+6i}{5-4i}$ in polar form
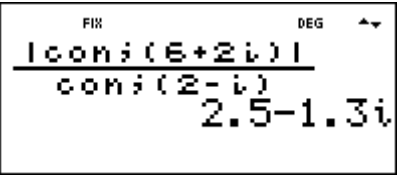
No	Keystrokes	Screenshots
1	Change REAL mode to COMPLEX mode Press [mode] ⊙⊙⊙⊙ [enter]	
2	Press [2nd][complex] ⊙ [enter] [▯/▯] [2] [+] [6] [π ⁿ/ᵢ] [π ⁿ/ᵢ][π ⁿ/ᵢ] ⊙ [5] [−] [4] [π ⁿ/ᵢ][π ⁿ/ᵢ][π ⁿ/ᵢ] ⊙ [)] [enter]	

2. Solve $\frac{3+2i}{6-7i}$ in $r < \theta$

No	Keystrokes	Screenshots
1	Press [(] [▯/▯] [3] [+] [2] [π ⁿ/ᵢ][π ⁿ/ᵢ][π ⁿ/ᵢ] ⊙ [6] [−] [7] [π ⁿ/ᵢ][π ⁿ/ᵢ][π ⁿ/ᵢ] ⊙ [)] [2nd] [π ⁿ/ᵢ] ⊙⊙⊙ [enter]	

TEXAS INSTRUMENTS

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Computational Analytics

Topic: Complex Number	Math Concepts: Complex Conjugate, magnitude, Polar form of complex number and distance between 2 complex numbers

3. Solve $\frac{|conj(6+2i|}{conj\,(2-i)}$, answer must in the form of $a + bi$

No	Keystrokes	Screenshots
1	Press ⊟ 2nd $\pi\,^e_i$ 3 2nd $\pi\,^e_i$ 6 6 + 2 $\pi\,^e_i$ $\pi\,^e_i$ $\pi\,^e_i$) ⌄ 2nd $\pi\,^e_i$ 6 2 ⊟ $\pi\,^e_i$ $\pi\,^e_i$ $\pi\,^e_i$) enter	FIX DEG ▲▼ $\frac{\|conj(6+2i)\|}{conj(2-i)}$ 2.5-1.3i

Exercise:

1. Solve magnitude of $\frac{|2+5i|}{|6-4i|}$
2. Solve $\frac{|4-i|}{2+2i}$ in $r < \theta$
3. Solve $6 - 6i$ in polar form
4. Solve $|conj(2 + 6i)|$
5. Solve magnitude of $|3 + 3i|.|2 + i|$

Answer:

1. 0.75
2. $1.46 < -45$
3. -45
4. 6.32
5. 9.49

Topic:	Math Concepts:
Simple Equations	Linear, Absolute Value, Quadratic Equations.

Simple Equations

Overview / Introduction / Terminology:

A mathematical **equation** is a formula containing an equals sign (=) with a mathematical expression on each side of it. The equals sign says that both sides are exactly the same.

There are two kinds of mathematical equations:

- The kind of equation that is either true or false; these are also called identities
  Example: $2 . (x + 4) = 2x + 8 \rightarrow true$
- The kind of equation that lets you calculates the value of one or several variables. The equation is only true if the variable(s) have that value.
  Example: $2 . x = 8 \rightarrow x = true$

  The second kind is often used to solve problems in which you have to know the value of some variables. For example, if $2x = 8, x = 8 / 2 = 4$.

  The second kind of equation is used in algebra. For example, to solve the equation $2x = 8$ by finding x you would follow an algebraic rule. Then you can work out that $x = 4$.

An equation is like a weighing scale - both sides should always be perfectly balanced. To solve the equation you need to find the value of missing numbers and perform the same operation to each side.

For example, suppose you are trying to find out how many sweets are in the bag shown here.
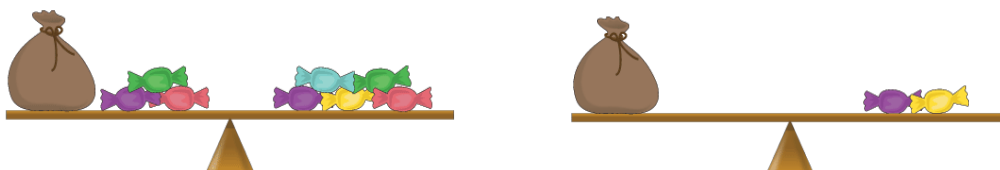
By subtracting three sweets from each side, the scales remain balanced. You can now see that one bag is equivalent to two sweets. Written algebraically, this is:

$$x + 3 = 5$$

Subtract 3 from both sides, to give:

$$x = 2$$

Keystrokes:

2nd [sys-solv], math NUM, 2nd [poly-solv]

TEXAS INSTRUMENTS

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Topic:	Math Concepts:
Simple Equations	Linear, Absolute Value, Quadratic Equations.

Example:

1. Find value for $\left|\frac{\sin(2^{x^2}-6x+1)}{3x+5\pi}\right|$

No	Keystrokes	Screenshots
1	Firstly, press [math] [▷] [NUM] [enter] [▯/▯] [sin⁻¹]	DEG MATH **NUM** DMS R↔P 1▮abs(2:round(3↓iPart(
2	Insert value in the equation, [2] [x□] [x_{abcd}^{yzt}] [x²] [▷] [▷] [−] [6] [x_{abcd}^{yzt}] [+] [1] [)] [▷] [3] [x_{abcd}^{yzt}] [+] [5] [π_i^e] [▷] [▷] [enter]	DEG $\left\|\frac{\sin\left(2^{x^2}-6x+1\right)}{3x+5\pi}\right\|$
3	Thus, $\left\|\frac{\sin(2^{x^2}-6x+1)}{3x+5\pi}\right\| = \mathbf{0.002221771}$	DEG ▲▾ $\left\|\frac{\sin\left(2^{x^2}-6x+1\right)}{3x+5\pi}\right\|$ 0.002221771

2. In the equation $9x^2 - 86x + 6 = 0$, a is the coefficient of the x^2 term, b is the coefficient of the x term, and c is the constant. Substitute 9 for a, -86 for b, and 6 for c in the quadratic formula and simplify.
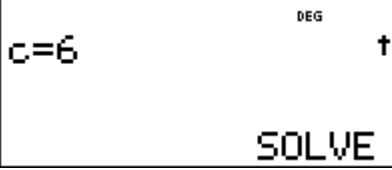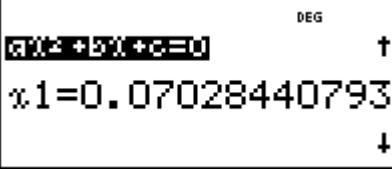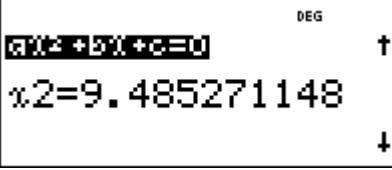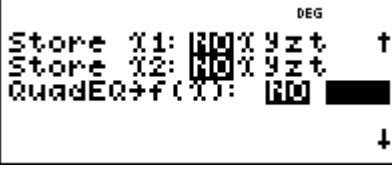
The quadratic formula is:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

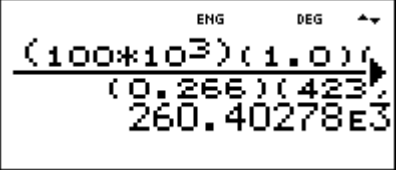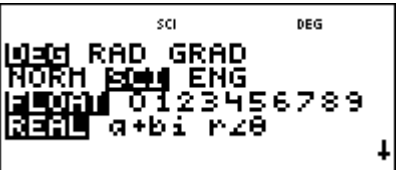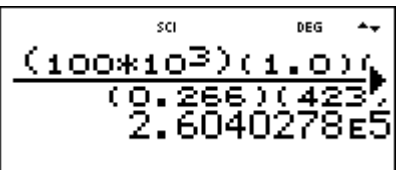| No	Keystrokes	Screenshots
1	If you have already defined variables, the solver will assume those values. To solve this equation, press [2nd] [poly-solv] [enter]	DEG **POLY SOLVER** 1▮ax²+bx+c=0 2: ax³+bx²+cx+d=0

TEXAS INSTRUMENTS

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Computational Analytics
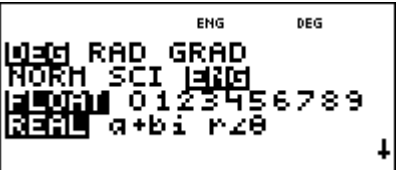
Topic:	Math Concepts:
Simple Equations	Linear, Absolute Value, Quadratic Equations.

No	Keystrokes	Screenshots
2	Then, insert value with press $\boxed{9}$ $\boxed{\text{enter}}$ $\boxed{(-)}$ $\boxed{8}$ $\boxed{6}$ $\boxed{\text{enter}}$ $\boxed{6}$ $\boxed{\text{enter}}$ $\boxed{\text{enter}}$.	a=9 DEG ↑ ↓ b=-86 DEG ↑ ↓ c=6 DEG ↑ SOLVE
3	Press $\boxed{\text{enter}}$, $\boxed{\text{enter}}$, $\boxed{\text{enter}}$, $\boxed{\text{enter}}$, $\boxed{()}$ (YES) $\boxed{\text{enter}}$ So, $x1 = 0.07028440793$ and $$x2 = 9.485271148$$	ax²+bx+c=0 DEG ↑ x1=0.07028440793 ↓ ax²+bx+c=0 DEG ↑ x2=9.485271148 ↓ Store X1: NO X y z t ↑ Store X2: NO X y z t QuadEQ→f(X): NO ↓ FORM: a(x-h)²+k=0 DEG ↑ a=9 h=43/9 k=-199.4444444 SOLVE AGAIN QUIT

Topic:	Math Concepts:
Simple Equations	Linear, Absolute Value, Quadratic Equations.

3. A formula relating initial and final states of pressures, P_1 and , P_2, volumes , V_1 and , V_2 and absolute temperatures, , T_1 and , T_2, of an ideal gas is $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$. Find the value of P_2 given $P_1 = 100 \times 10^3, V_1 = 1.0, V_2 = 0.266, T_1 = 423 \ and \ T_2 293$.

No	Keystrokes	Screenshots
1	Since is $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$ then $\frac{(100 \times 10^3)(1.0)}{423} = \frac{P_2(0.266)}{293}$ 'Cross-multiplying' gives: Firstly, press mode ⊙①①① enter to change to engineering mode. Then press clear to go to home screen.	
2	After that, insert the equation, press ▢(①⓪⓪×①⓪$x^□$③①)(①·⓪)(②⑨③)⊙(⓪·②⑥⑥)(④②③). Then press enter Try change to the scientific mode with press mode⊙①enter clear. Press ⊙⊙ to select the last equation and press enter to get the new answer. Hence $P_2 = 260 \times 10^3$ or 2.6×10^5	

TEXAS INSTRUMENTS

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Topic:	Math Concepts:
Simple Equations	Linear, Absolute Value, Quadratic Equations.

Exercise:

1. Solve $4(3x + 1) = 7(x + 4) - 2(x + 5)$

2. Solve this equation $\sqrt{\left(\frac{y+2}{y-2}\right)} = \frac{1}{2}$

3. A rectangle has a length of 20 cm and width b cm. When its width is reduced by 4 cm its area becomes $160\ cm^2$. Find the original width and area of the rectangle.

4. If $t = 2\pi\sqrt{(w/Sg)}$, find the value of S given $w = 1.219, g = 9.81$ and $t = 0.3132$.

5. The extension x m of an aluminum tie bar of length l m and cross-sectional area $A\ m^2$ when carrying a load of F newton is given by the modulus of elasticity $E = Fl/Ax$. Find the extension of the tie bar (in mm) if $E = 70 \times 10^9\ \frac{N}{m^2}, F = 20 \times 10^6\ N, A = 0.1\ m^2$ and $l = 1.4m$.

Answer:

1. 2
2. $-3\frac{1}{3}$
3. $12\ cm, 240\ cm2$
4. 50
5. $x = 4\ mm$

TEXAS INSTRUMENTS

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Simultaneous Equation

Overview / Introduction / Terminology:

The terms **simultaneous equations** and systems of equations refer to conditions where two or more unknown variables are related to each other through an equal number of equations. Consider the following example:

$$x + y = 24$$
$$2x - y = -6$$

The intersection point has been defined at point (6,18) which is $x = 6$ and $y = 18$.

Three methods of solving simultaneous equations analytically are:

   (a) by **substitution**, and
   (b) by **elimination**.
   (c)   using formula, $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Quadratic equation is the equation contains power 2 in general form:
$$ax^2 + bx + c = 0$$

Ex: $x2 - 5x + 6 = 0$ with **two roots** which is $x1 = 2$, $x2 = 3$

There are **four** methods of **solving quadratic equations**.
These are: (i) by factorization (where possible)
               (ii) by 'completing the square'
               (iii) by using the 'quadratic formula'
               (iv) graphically

The **cubic equation** which have the equation up to power 3 in general form:
$$ax^3 + bx^2 + cx + d = 0$$

Ex: $x^3 - 27x + 54 = 0$ with three roots which are $x1 = -6$, $x2 = 3$, $x3 = 3$

Keystrokes:
2nd [num-solv] [sys-solv]
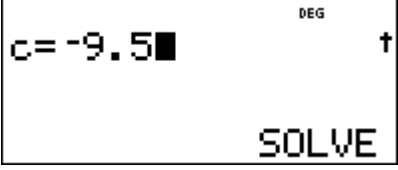
Topic:	Math Concepts:
Simultaneous Equation	Transposition of Formulae, Simultaneous Equation, Quadratic Equation, Cubic Equation

Example:

1. A shed is 4.0 m long and 2.0 m wide. A concrete path of constant width is laid all the way around the shed. If the area of the path is 9.50 m^2, calculate its width.

No	Keystrokes	Screenshots
1	From the above figure, it shows a plan view of the shed with its surrounding path of width **t** metres. $$Area\ of\ path\ =\ 2(2.0\ \times t)\ +\ 2t\ (4.0\ +\ 2t)$$ Or $$9.50\ =\ 4.0t\ +\ 8.0t\ +\ 4t^2$$ We can rearrange the equation: $$4t^2 + 12t - 9.5 = 0$$ Therefore; a = 4 , b = 12 , c = -9.5 To enter the quadratic equation solver, press : 2nd [poly-solv] choose no 1 and then press enter To enter coefficient of **a**: 4 enter To enter coefficient for **b** : 1 2 enter To enter coefficient for **c** : (−) 9 · 5 enter	DEG **POLY SOLVER** 1: ax²+bx+c=0 2: ax³+bx²+cx+d=0 DEG a=4█ DEG b=12█ DEG c=⁻9.5█ SOLVE

TEXAS INSTRUMENTS

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Topic:	Math Concepts:
Simultaneous Equation	Transposition of Formulae, Simultaneous Equation, Quadratic Equation, Cubic Equation

No	Keystrokes	Screenshots
2	To find the answer. Root 1: enter ⊙	FIX DEG $ax^2+bx+c=0$ $x1=0.6505813168$
3	To find Root 2: ⊙ Thus, t_1 = 0.6505813168 m t_1 = - 3.650581317 m	FIX DEG $ax^2+bx+c=0$ $x2=-3.650581317$

2. When Kirchhoff's laws are applied to the electrical circuit shown in Figure 9.1 the currents I_1 and I_2 are connected by the equations:
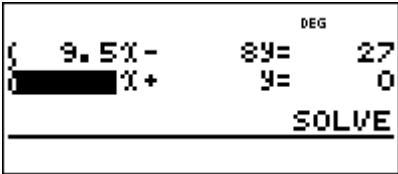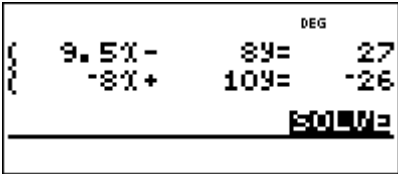
$$1.5 I1 + 8(I1 - I2) = 27 \text{-------------- (1)}$$
$$2 I2 - 8(I1 - I2) = -26 \text{------------ (2)}$$

Solve the equations to find the values of currents I_1 and I_2.
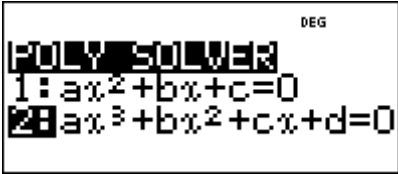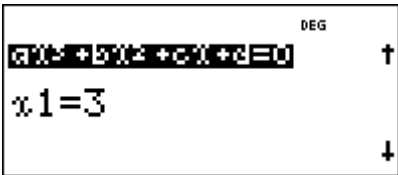
No	Keystrokes	Screenshots
1	From the equation, $$9.5\, l1 - 8\, l2 = 27 \text{-------------- (1)}$$ $$-8\, l1 + 10\, l2 = -26 \text{-------------- (2)}$$ To enter to system solver for simultaneous equation, Press 2nd [sys-solv] and choose 1, enter .	DEG SYSTEM SOLVER 1:2×2 LIN EQUs 2:3×3 LIN SYSTEM

TEXAS INSTRUMENTS

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Topic:	Math Concepts:
Simultaneous Equation	Transposition of Formulae, Simultaneous Equation, Quadratic Equation, Cubic Equation

No	Keystrokes	Screenshots
2	Assume $x = l_1$, $y = l_2$ To enter the values in first equation, $\boxed{9}\boxed{\cdot}\boxed{5}\boxed{enter}\boxed{-}\boxed{8}\boxed{enter}\boxed{2}\boxed{7}\boxed{enter}$	
3	To enter the values in second equation, $\boxed{-}\boxed{8}\boxed{enter}\boxed{+}\boxed{enter}\boxed{1}\boxed{0}\boxed{enter}\boxed{(-)}\boxed{2}\boxed{6}\boxed{enter}$	
4	To get the answer, press $\boxed{enter}$ Thus, $x = 2$ @ $\mathbf{l1} = \mathbf{2}$ $y = -1$ @ $\mathbf{l2} = \mathbf{-1}$	

3. If one of the roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$ is 2, then the other two roots are:

No	Keystrokes	Screenshots
1	To enter to equation solver, press $\boxed{2nd}\boxed{poly\text{-}solv}\boxed{\ominus}\boxed{enter}$	
2	To enter the values in the equation, $\boxed{1}\boxed{enter}\boxed{(-)}\boxed{6}\boxed{enter}\boxed{1}\boxed{1}\boxed{enter}\boxed{(-)}\boxed{6}\boxed{enter}\boxed{enter}$ To get the answers, press	

Topic:	Math Concepts:
Simultaneous Equation	Transposition of Formulae, Simultaneous Equation, Quadratic Equation, Cubic Equation

enter enter enter enter Thus the answers are: $x = 3, 2, 1$	DEG $ax^3 + bx^2 + cx + d = 0$ ↑ $x2 = 2$ ↓ DEG $ax^3 + bx^2 + cx + d = 0$ ↑ $x3 = 1$ ↓

Exercise:

1. In an electrical alternating current circuit the impedance Z is given by:
$$Z = \sqrt{\left\{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right\}}.$$
Transpose the formula to make C the subject and hence evaluate C when $Z = 130, R = 120,$ $\omega = 314 \ and \ L = 0.32.$

2. An approximate relationship between the number of teeth, T, on a milling cutter, the diameter of cutter, D, and the depth of cut, d, is given by: $T = \frac{12.5D}{D+4d}$. Determine the value of D when $T = 10$ and $d = 4 \ mm$.

3. A rectangular building is $15 \ m$ long by $11 \ m$ wide. A concrete path of constant width is laid all the way around the building. If the area of the path is $60.0 \ m^2$, calculate its width correct to the nearest millimeter.

4. The resistance $R \ ohms$ of copper wire at t$°$C is given by $R = R_0 (1 + \alpha t)$, where R_0 is the resistance at $0°$C and α is the temperature coefficient of resistance. If $R = 25.44$ at $30°C$ and $R = 32.17$ at $100°C$, find α and R_0.

5. One real root of the equation $x^3 + x - 5 = 0$ lies in the interval?

Answer:

1. $C = \dfrac{1}{\omega\left\{\omega L - \sqrt{Z^2 - R^2}\right\}}$, 63.1×10^{-6}

2. $64mm$

3. $1.066m$

4. $\alpha = 0.00\ 426, \ R_0 = 22.56\Omega$

5. $(1,2)$

TEXAS INSTRUMENTS

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Computational Analytics

Determinants and Matrices

Overview / Introduction / Terminology:

The expression is called a ***determinant of the second order*** and stands for '$a_1b_2 - a_2b_1$'. It contains 4 number a_1, b_1, a_2, b_2 (called **elements**) which are arranged along two horizontal lines (called **rows**) and two vertical lines (called **columns**).

$$A = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

*A system of mn numbers arranged in a rectangular formation along m rows and n columns and bounded by the brackets [ ] is called an m by n **matrix**; which is written m x n matrix.*

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Matrix Operations

1. Equality of Matrices

    Two matrices A and B are said to equal if and only if

    (i)   They are of the same order

    (ii)   Each element of A is equal to the corresponding element of B

2. Addition and Subtraction of matrices : $\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} + \begin{bmatrix} c_1 & d_1 \\ c_2 & d_2 \end{bmatrix} = \begin{bmatrix} a_1c_1 & b_1d_1 \\ a_2c_2 & b_2d_2 \end{bmatrix}$

3. Multiplication of matrix by a scalar : $k\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} = \begin{bmatrix} ka_1 & kb_1 \\ ka_2 & kb_2 \end{bmatrix}$

4. Multiplication of matrices : $\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \times \begin{bmatrix} c_1 & d_1 \\ c_2 & d_2 \end{bmatrix} = \begin{bmatrix} a_1c_1 + b_1c_2 & a_1d_1 + b_1d_2 \\ a_2c_1 + b_2c_2 & a_2d_1 + b_2d_2 \end{bmatrix}$

Types of matrices:

1. Transpose of a matrix *: **A = A'***

    $$A = \begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix} \qquad A' = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \end{bmatrix}$$
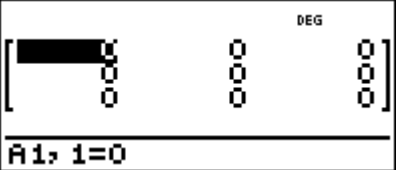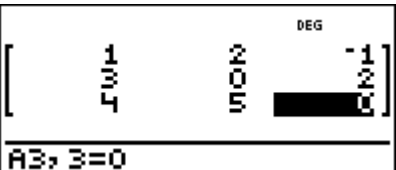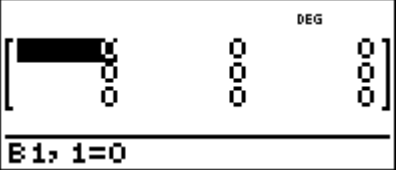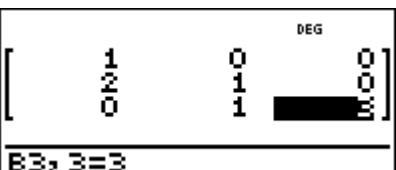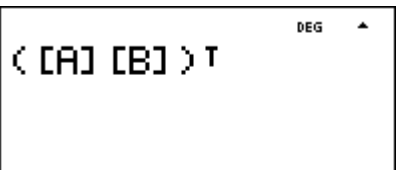
2. Adjoint of a square matrix : ***Adjoint of the matrix A = Adj. A***

3. Inverse of a matrix : ***A = A^{-1} = $\frac{Adj.A}{|A|}$***

Keystrokes:

2nd [matrix]

Topic:	Math Concepts:
Determinants and Matrices	Basic Algebra, Matrices

Example:

1. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$, verify that $(AB)' = B'A'$, where A' is the transpose of A.=
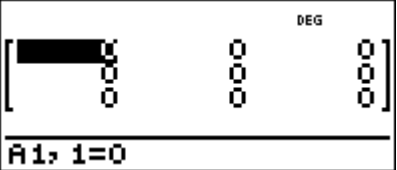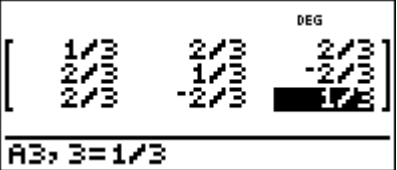
No	Keystrokes	Screenshots
1	To enter to matrices screen, press 2nd [matrix] ◀. Press ⊙ enter to choose first matrix, A. Press ▶▶ enter ▶▶ enter enter to enter the matrix screen.	
2	To enter the values, key in the values in matrix A. 1 enter 2 enter (–) 1 enter 3 enter 0 enter 2 enter 4 enter 5 enter 0 enter	
3	To enter the values in matrix B, press 2nd [matrix] ◀ ⊙ enter . Press ▶▶ enter ▶▶ enter enter to enter the matrix screen	
4	To enter the values, key in the values in matrix B. 1 enter 0 enter 0 enter 2 enter 1 enter 0 enter 0 enter 1 enter 3 enter	
5	To get the answer for (AB)' press (2nd [matrix] enter 2nd [matrix] ⊙ enter) 2nd [matrix] ▶ ⊙ enter enter	

TEXAS INSTRUMENTS
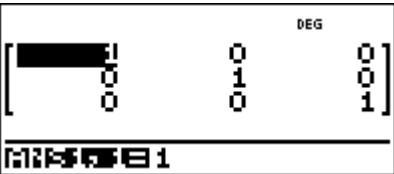
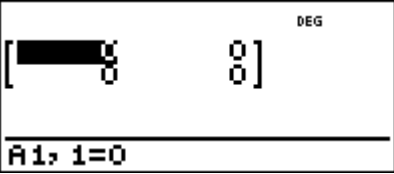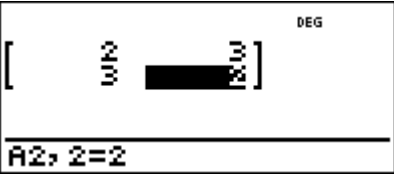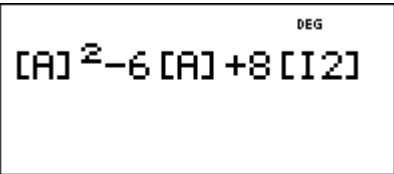|		
		DEG
6	To get the answer for B'A', press 2nd [matrix] ⊙ enter 2nd [matrix] ◊ ⊙ enter 2nd [matrix] enter 2nd [matrix] ◊ ⊙ enter Thus, $(AB)' \;=\; B'A'$ is proved.	DEG [B] ᵀ [A] ᵀ■ DEG

3. Verify that the following matrix is orthogonal.

$$\begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix}$$

No	Keystrokes	Screenshots
1	To enter to matrices screen, press 2nd [matrix] ◊. Press ⊙ enter to choose first matrix, A. Press ◊◊ enter ◊◊ enter enter to enter the matrix screen.	DEG A1, 1=0
2	To enter the values, 1 ▯ 3 enter 2 ▯ 3 enter 2 ▯ 3 enter 2 ▯ 3 enter 1 ▯ 3 enter (−) 2 ▯ 3 enter 2 ▯ 3 enter (−) 2 ▯ 3 enter 1 ▯ 3 enter	DEG 1/3 2/3 2/3 2/3 1/3 -2/3 2/3 -2/3 1/3 A3, 3=1/3

Topic:	Math Concepts:
Determinants and Matrices	Basic Algebra, Matrices

No	Keystrokes	Screenshots
3	To define the matrix is orthogonal. Press, 2nd quit to quit from the matrix template. To do calculation, press 2nd matrix ▶ ⌃ 2nd matrix enter)	DEG rref([A])■
4	To get the answer, press enter . Thus, the matrix is orthogonal.	DEG $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

4. If $= \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$, find the value of $A^2 - 6A + 8I$, where I is a unit matrix of second order.

No	Keystrokes	Screenshots
1	To enter to matrices screen, press 2nd matrix ◀. Press ⌄ enter to choose first matrix, A. Press ▶ enter ▶ enter enter to enter the matrix screen.	DEG $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ A1, 1=0
2	To enter the values, 3 enter 2 enter 2 enter 3 enter	DEG $\begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$ A2, 2=2
3	Press 2nd quit to quit from the matrix form. To get the answer, key in the equation, $A^2 - 6A + 8I$. 2nd matrix enter x² − 6 2nd matrix enter + 8 2nd enter + 8 2nd matrix ⌃ ⌃ enter	DEG [A]²−6[A]+8[I2]

TEXAS INSTRUMENTS

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Computational Analytics

No	Keystrokes	Screenshots
4	To get the answer, press $\boxed{\text{enter}}$. Thus, the answer is **3I**.	

Exercise:

1. If $= \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$, and $C = \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}$, verify that $(AB)C = A(BC)$ and $A(B + C) = AB + AC$

2. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$, and I is the unit matrix of order 3, evaluate $A^2 - 3A + 9\,I$.

3. If $= \begin{bmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 5 & 3 & 4 \end{bmatrix}$, compute $adj\ A$ and A^{-1}. Also find B such that $AB = \begin{bmatrix} 3 & 4 & 2 \\ 1 & 6 & 1 \\ 5 & 6 & 4 \end{bmatrix}$

4. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ find $A^{-1.}$ Also find two non-singular matrices P and Q such that $PAQ = I$.
   Where I is the unit matrix and verify that $A^{-1 =}QP$.

5. Find the inverse of the following matrix.
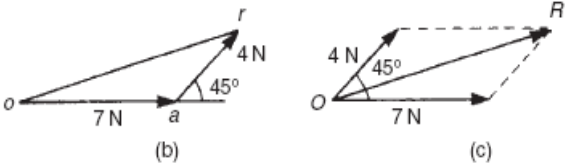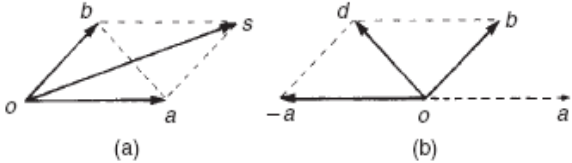 $\begin{bmatrix} 3 & 2 & 4 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}$

Answer:

1. Proved.

2. $\begin{bmatrix} -6 & 1 & 2 \\ 5 & 4 & 4 \\ 2 & 8 & -3 \end{bmatrix}$

3. $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

4. $\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

5. $\frac{1}{8}\begin{bmatrix} 2 & 2 & -2 \\ -9 & 11 & 5 \\ 5 & -7 & -1 \end{bmatrix}$

TEXAS INSTRUMENTS

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Vector

## Overview / Introduction / Terminology:

Some physical quantities are entirely defined by a numerical value and are called **scalar quantities** or **scalars**. Other physical quantities are defined by both a numerical value and a direction in space and these are called **vector quantities** or **vectors**.

There are many types of symbol for vector such as $\overrightarrow{AB}, \vec{A}, \bar{a}, \underline{a}, xi + jy$ and in column matric $\begin{pmatrix} a \\ b \end{pmatrix}$.

Vector Addition	Vector Subtraction
Figure (b) shows the resultant of vector addition is : $$or = oa + ar.$$	Figure (a) shows the resultant of vector addition is $$os = oa + ob.$$ Figure (b) shows vectors $ob + (-oa)$, that is, $ob - oa$ and the vector equation is $$ob - oa = od.$$

Resolution of Vectors

A vector can be resolved into two component parts such that the vector addition of the component parts is equal to the original vector.
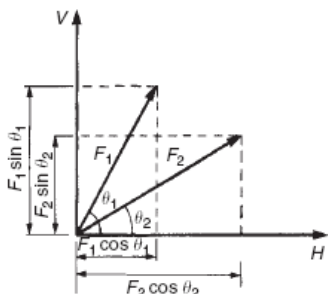
For the vector F_1 and F_2 shown in the figure, the horizontal component of vector addition is:
$$H = F1 \cos \theta 1 + F2 \cos \theta 2$$

And the vertical component of vector addition is:
$$V = F1 \sin \theta 1 + F2 \sin \theta 2$$

TEXAS INSTRUMENTS

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Topic:	Math Concepts:
Vector	Basic Algebra, Vector Algebra

Relative velocity

In any vector equation, only the start and finish points affect the resultant vector of a system. Two different systems are shown in figure below, but in each of the systems, the resultant vector is **ad**.
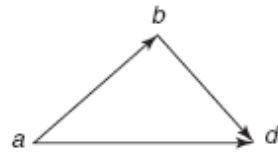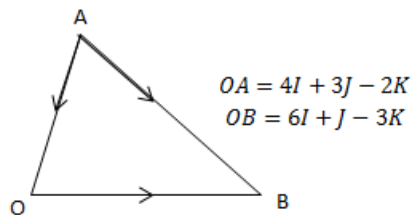
$$ad = ab + bd$$

Keystrokes:

[2nd] [matrix] [vector]

Example:

1. If the equation $A = I + 2J + 3K$, $B = -I + 2J + K$. Find the dot product for AB.

No	Keystrokes	Screenshots
1	To enter to vector screen, press [2nd] [vector] ⓐ [enter] ⓑⓑ [enter] [enter]	DEG [0 0 0] w1=0
2	Key in the values, [1] [enter] [2] [enter] [3] [enter]	DEG [1 2 3] w3=3
3	To key in the values in vector B, press [2nd] [vector] ⓐⓥ [enter] ⓑⓑ [enter] [enter] And key in the values, [(−)] [1] [enter] [2] [enter] [1] [enter]	DEG [-1 2 1] v1= -1
4	To do calculation, A.B press [2nd] [vector] ⓑ [enter] [2nd] [vector] [enter] [2nd] [,] [2nd] [vector] ⓥ [enter] [)] [enter] Thus, the answer is 6.	DEG DotP([u] , [v]) 6

TEXAS INSTRUMENTS

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Topic:	Math Concepts:
Vector	Basic Algebra, Vector Algebra

2. Find the AB.

$$OA = 4I + 3J - 2K$$
$$OB = 6I + J - 3K$$
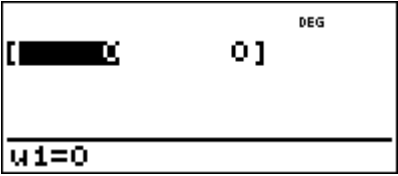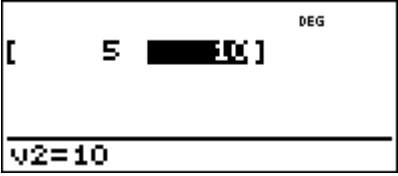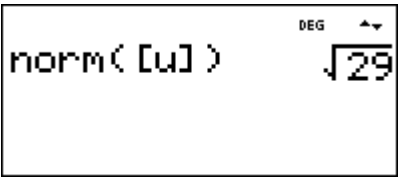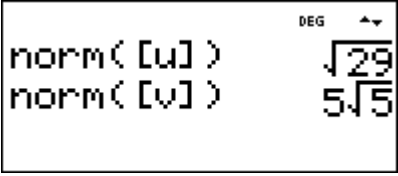
No	Keystrokes	Screenshots
1	$$AB = OA + OB$$ To enter to vector screen, press 2nd vector ◁ enter ▷ ▷ enter enter	
2	Key in the values, 4 enter 3 enter (−) 2 enter	
3	key in the values in vector B, press 2nd vector ◁ ⊙ enter ▷ ▷ enter enter And key in the values, 6 enter 1 enter (−) 3 enter	
4	To do calculation, 2nd vector enter + 2nd vector ⊙ enter	

Topic:	Math Concepts:
Vector	Basic Algebra, Vector Algebra

3. Find the norm magnitude for vector $A = 2I + 5K$, and $B = 5I + 10K$

No	Keystrokes	Screenshots
1	To enter to vector screen, press 2nd [vector] ◁ enter ▷ enter enter	
2	Key in the values, 2 enter 5 enter	
3	Key in the values in vector B, press 2nd [vector] ◁ ⊖ enter ▷ enter enter And key in the values, 5 enter 1 0 enter	
4	Press 2nd [quit] to quit from the vector template. To do calculation, 2nd [vector] ▷ ⊖ enter 2nd [vector] ⟩	norm([u]) √29
5	To calculate norm magnitude for B, press 2nd [vector] ▷ ⊖ enter 2nd [vector] ⊖ ⟩ Thus, norm magnitude for $A = \sqrt{29}$ $$B = 5\sqrt{5}$$	norm([u]) √29 norm([v]) 5√5

TEXAS INSTRUMENTS

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Computational Analytics

Topic: Vector	Math Concepts: Basic Algebra, Vector Algebra

Exercise:

1. If $A = I - 2J - 3K, \quad B = 2I + J - K, \quad C = I + 3J - K$, find:
   (i) $A \ x \ (B \ x \ C)$ (ii) $(A \ x \ B) \ x \ (B \ x \ C)$

2. Find the moment about a line through the origin having direction of $2I + 2J + K$, due to a 30 kg force acting at a point $(-4,2,5)$ in the direction of $12I - 4J - 3K$.
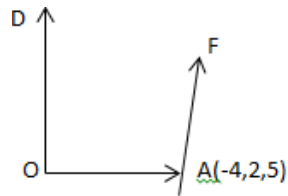
3. Show that the volume of the tetrahedron $ABCD$ is $\frac{1}{6}[\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}]$.

   Hence find the volume of the tetrahedron formed by the points $(1,1,1), (2,1,3), (3,2,2)$ and $(3,3,4)$.
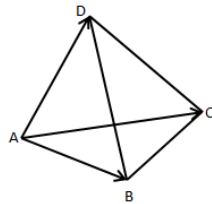
4. Show that the points $-6I + 3J + 2K, \quad 3I - 2J + 4K, \quad 5I + 7J \ 3K$ and $-13I + 17J - K$ are coplanar.

5. Find the torque about the point $2I + J - K$ of a force represented by $4I + K$ acting through the point $I - J + 2K$.
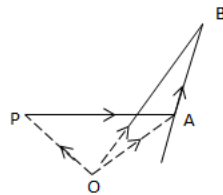
Answer:

1. (i) $-7 \ I - 11J + 5K$ (ii) $-30I - 15J + 15K$
2. 89.23
3. $\frac{5}{6}$
4. $[\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}] = 0$. Thus, A,B,C,D are coplanar.
5. Magnitude of the moment $= 15.4$

TEXAS INSTRUMENTS

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Computational Analytics

Introduction to Trigonometry

Overview / Introduction / Terminology:

Trigonometry is a branch of mathematics that studies triangles and the relationships between their sides and the angles between these sides.

Pythagorean theorem:

The theorem can be written as an equation relating the lengths of the sides a, b and c, often called the Pythagorean equation:
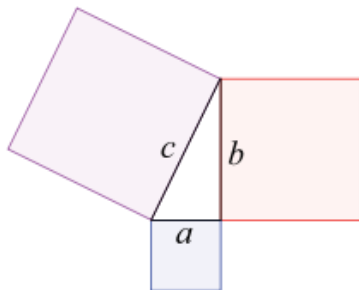
$$a^2 + b^2 = c^2$$

where c represents the length of the hypotenuse, and a and b represent the lengths of the other two sides.

Trigonometric ratios are given by the following trigonometric functions of the known angle A, where a, b and c refer to the lengths of the sides in the accompanying figure:

Sine function (*sin*), defined as the ratio of the side opposite the angle to the hypotenuse.
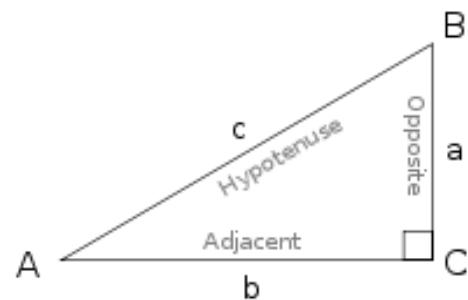
$$\sin A = \frac{opposite}{hypotenuse} = \frac{a}{c}$$

Cosine function (*cos*), defined as the ratio of the adjacent leg to the hypotenuse.

$$\cos A = \frac{adjacent}{hypotenuse} = \frac{b}{c}$$

Tangent function (*tan*), defined as the ratio of the opposite leg to the adjacent leg.

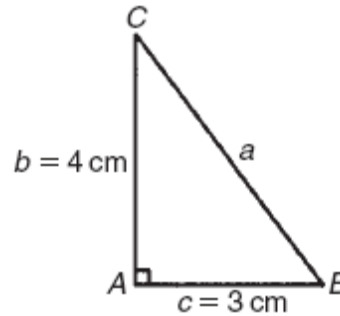$$\tan A = \frac{opposite}{adjacent} = \frac{a}{b}$$

TEXAS INSTRUMENTS

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Topic:	Math Concepts:
Introduction to Trigonometry	sin, cos, tan, angle, Pythagorean theorem, trigonometric ratios

Keystrokes: [sin / sin⁻¹] [cos / cos⁻¹] [tan / tan⁻¹]

Example:

1. Find the length of BC

No	Instructions / Keystrokes	Screenshots
1	By Pythagoras' theorem, $a^2 = b^2 + c^2$. We can rearrange the equation as $a = \sqrt{b^2 + c^2}$. From the figure we know that $b = 4$ and $c = 3$. Key-in: [2nd][√][4][x²][+][3][x²][enter] Thus $BC = 5\ cm$	

2. Determine the values of $\sin\theta$, $\cos\theta$ and $\tan\theta$ for the right-angled triangle ABC
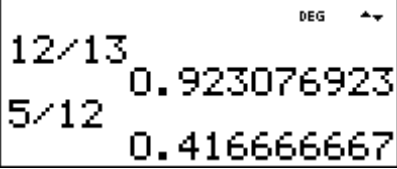
TEXAS INSTRUMENTS

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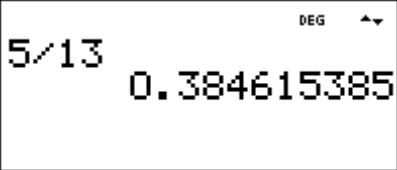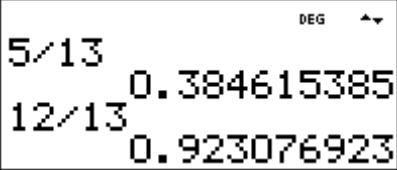| No	Instructions / Keystrokes	Screenshots
1	By definition: $$\sin \theta = \frac{opposite}{hypotenuse},$$ $$\cos \theta = \frac{adjacent}{hypotenuse} \text{ and}$$ $$\tan \theta = \frac{opposite}{adjacent}$$ Key-in: $\boxed{5}\boxed{\div}\boxed{1}\boxed{3}\boxed{\text{enter}}$ for $\sin \theta$ $\boxed{1}\boxed{2}\boxed{\div}\boxed{1}\boxed{3}\boxed{\text{enter}}$ for $\cos \theta$ $\boxed{5}\boxed{\div}\boxed{1}\boxed{2}\boxed{\text{enter}}$ for $\tan \theta$	DEG 5/13 0.384615385 DEG 5/13 0.384615385 12/13 0.923076923 DEG 12/13 0.923076923 5/12 0.416666667

TEXAS INSTRUMENTS

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Topic:	Math Concepts:
Introduction to Trigonometry	sin, cos, tan, angle, Pythagorean theorem, trigonometric ratios

3. Find angle A of the right triangle below. Then calculate angle
   B and the length of the hypotenuse *c*. Lengths are in meters.
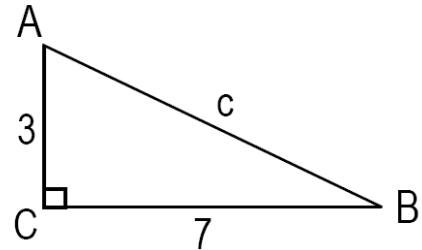
No	Instructions / Keystrokes	Screenshots
1	$\tan A = \dfrac{7}{3}, therefore\ angle\ A = tan^{-1}\left(\dfrac{7}{3}\right)$ To get the answer, press [tan⁻¹][tan⁻¹][7][☐/☐][3][▶][)][enter]	DEG tan⁻¹(7/3) 66.80140949
2	Key-in [9][0][−][2nd][answer][enter] to calculate the value of angle B	DEG tan⁻¹(3) 66.80140949 90−ans 23.19859051
3	For length of hypotenuse *c*, press [2nd][√][3][x²][+][7][x²][enter] Press [◀▶≈] to convert the answer in decimal mode.	DEG √3²+7² √58 √58▸ 7.615773106

TEXAS INSTRUMENTS

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Topic:	Math Concepts:
Introduction to Trigonometry	sin, cos, tan, angle, Pythagorean theorem, trigonometric ratios

Exercises:

1. When driving, a steep hill is typically only 12°. What is the cosine of this angle?

2. The angle that waves hit a shoreline is 75°. What is the tangent of this angle?

3. The angle of repose is the steepest angle at which dry, unconsolidated sediment is stable. You create a conical pile of sand that is as steep as you can make it. The pile is 11 cm high and has a radius of 16 cm. What is the angle of repose of this sand?

4. You are walking up a 500 meter high hill. The trail has an incline of 12 degrees. How far will you walk to get to the top?

5. You note that a bed coal is tilted at 12 degrees and comes to the surface 6 kilometers from your property. How deep will you have to dig to get to the coal bed on your property?

Answers:

1. 0.978
2. 0.0879
3. $x = 34.5\ degrees$, the angle of repose for this sand.
4. The trail up the hill is $2400\ m\ long$
5. the depth of the coal bed, is $1.275\ km$, or $1275\ meters$.

TEXAS INSTRUMENTS

Topic:	**Math Concepts:**
Cartesian and polar co-ordinates	Radius, Angle, Cartesian coordinates, Polar coordinates

Cartesian and polar co-ordinates

Overview / Introduction / Terminology:

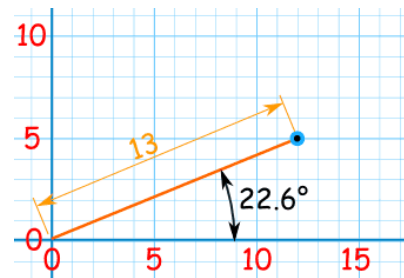### Type of coordinates

Cartesian Coordinates (x, y)

Polar Coordinates (r, θ)

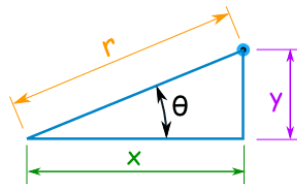Using Cartesian Coordinates you mark a point by **how far along** and **how far up** it is:
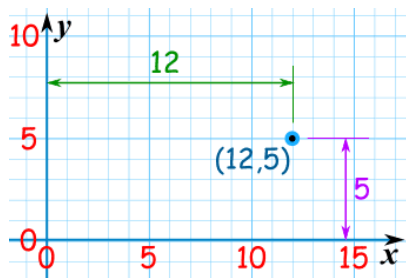
Using Polar Coordinates you mark a point by **how far away**, and **what angle** it is:

Converting

To convert from one to the other, you need to solve the triangle:

To convert from Cartesian to Polar

The Cartesian coordinates (also called Rectangular coordinates) x and y can be converted to polar coordinates r and θ with $r \geq 0$ and θ in the interval $(-\pi, \pi)$ by:

$$radius, r = \sqrt{x^2 + y^2} \qquad\qquad angle, \theta = tan^{-1}\frac{y}{x}$$

To convert from Polar to Cartesian

The two polar coordinates r and θ can be converted to the two Cartesian coordinates x and y by using the trigonometric functions sine and cosine:
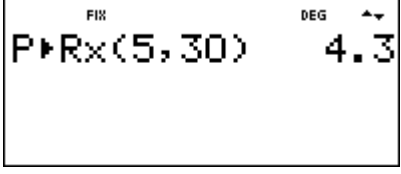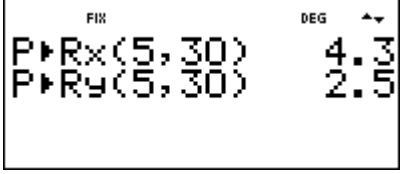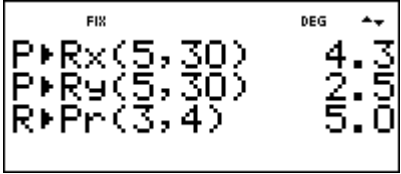
$$x = r\cos\theta \qquad\qquad y = r\sin\theta$$

Keystrokes: math > R◀▶P

TEXAS INSTRUMENTS

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Topic:	Math Concepts:
Cartesian and polar co-ordinates	Radius, Angle, Cartesian coordinates, Polar coordinates

Example:

1. Convert polar coordinates $(r, \theta) = (5,30)$ into rectangular coordinates. Then convert rectangular coordinates $(x, y) = (3,4)$ into polar coordinates. Round the results to one decimal place.

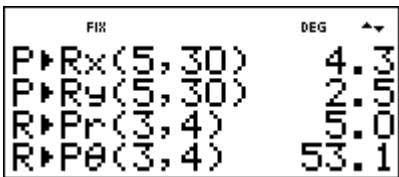| No	Instructions / Keystrokes	Screenshots
1	To set mode to one decimal place, press [clear] [mode] ⊙⊙⊙◑◑ [enter]. Then press [clear] to go back to home screen.	FIX DEG DEG RAD GRAD NORM SCI ENG FLOAT 0123456789 REAL a+bi r∠θ
2	Press [math] and using tight cursor, choose R↔P.	FIX DEG MATH NUM DMS CPX 1:P▶Rx(2:P▶Ry(3↓R▶Pr(
3	To convert to polar coordinates and display x choose 1. P ▶Rx(. Press [1] or [enter]. Key-in this value in your calculator: [5] [2nd] [,] [3] [0] [)] [enter]	FIX DEG ▲▼ P▶Rx(5,30) 4.3
4	To convert to polar coordinates and display y choose 2. R ▶Py(. Press [2] or [enter]. Key-in this value in your calculator: [5] [2nd] [,] [3] [0] [)] [enter] Converting $(r, \theta) = (5, 30)$ gives $(x, y) = (4.3, 2.5)$	FIX DEG ▲▼ P▶Rx(5,30) 4.3 P▶Ry(5,30) 2.5
5	To convert to polar coordinates and display r choose 3. R ▶Pr(. Press [3] or [enter]. Key-in this value in your calculator: [3] [2nd] [,] [4] [)] [enter]	FIX DEG ▲▼ P▶Rx(5,30) 4.3 P▶Ry(5,30) 2.5 R▶Pr(3,4) 5.0

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Computational Analytics

No	Instructions / Keystrokes	Screenshots
6	To convert to polar coordinates and display θ choose 4. R ▸Pθ(. Press 4 or enter . Key-in this value in your calculator: 3 2nd [,] 4 [)] enter Converting $(x, y) = (3, 4)$ gives $(r, \theta) = (5.0, 53.1)$	FIX DEG P▸Rx(5,30) 4.3 P▸Ry(5,30) 2.5 R▸Pr(3,4) 5.0 R▸Pθ(3,4) 53.1

Exercises:

1. Change the Cartesian co-ordinates into polar co-ordinates.
   a. $(3, 4)$
   b. $(-4, 3)$
   c. $(-5, -12)$
   d. $(2, -5)$

2. Change the polar co-ordinates into Cartesian co-ordinates.
   a. $(4, 32°)$
   b. $(6, 137°)$
   c. $(4.5, 5.16 \, rad)$

Answers:

1. Question 1:
   a. $(5, 53.13°)$ or $(5, 0.927 \, rad)$
   b. $(5, 143.13°)$ or $(5, 2.498 \, rad)$
   c. $(13, 247.38°)$ or $(13, 4.318 \, rad)$
   d. $(5.385, 291.80°)$ or $(5.385, 5.093 \, rad)$
2. Question 2
   a. $(3.39, 2.12)$
   b. $(-4.388, 4.092)$
   c. $(1.948, -4.057)$

TEXAS INSTRUMENTS

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Geometry and Triangles

Overview / Introduction / Terminology:

Geometry is a part of mathematics in which the properties of points, lines, surfaces and solids are investigated.

An **angle** is the amount of rotation between two straight lines. Angles may be measured in either **degrees (^{0})** or **radians.**
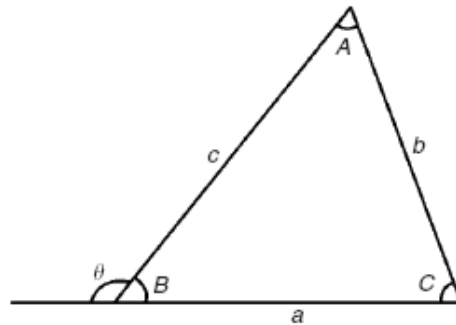
1 revolution = 360 degrees, thus 1 degree = $\frac{1}{360}$ th of one revolution.

1 minute = $\frac{1}{60}$ th of a degree   is written as **1'**

1 second = $\frac{1}{60}$ th of a minute   is written as **1"**
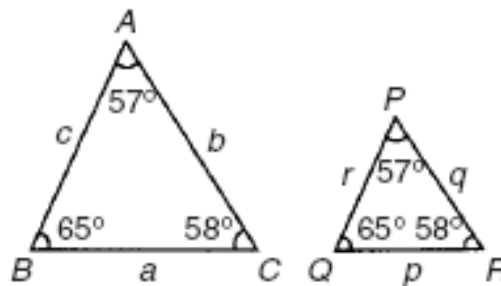
Properties of triangles
A triangle is a figure enclosed by three straight lines. The sum of the three angles of a triangle is equal to 180°.

Congruent triangles : Two triangles are said to be congruent if they are **equal in all respects**.
Similar triangles   : Two triangles are said to be **similar** if the angles of one triangle are equal to the angles of the other triangle.
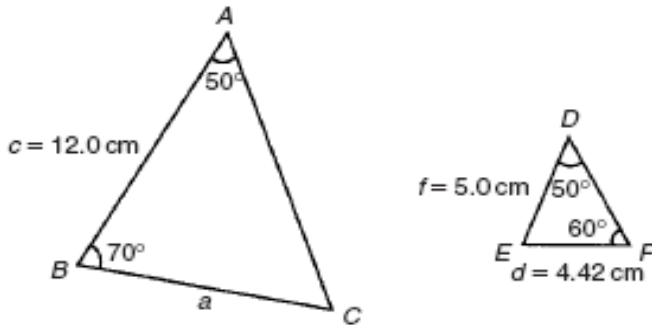

$$\frac{p}{a} = \frac{q}{b} = \frac{r}{c}$$

TEXAS INSTRUMENTS

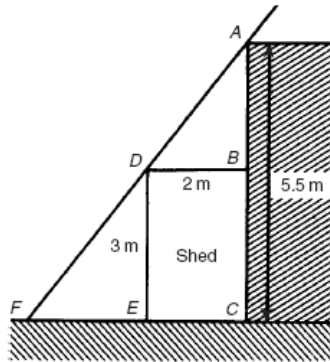$\delta\tau\alpha\tau\omega o\gamma\kappa\delta$
Computational Analytics

Topic:	Math Concepts:
Geometry and Triangles	Basic Algebra, Triangle, Degree, Radian,

Keystrokes:

math 2nd num-solv

Example:

1. In the below figure, find the length of side a.

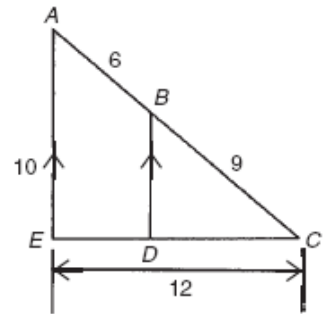| No	Keystrokes	Screenshots
1	Based on the figure, it shows the similar triangle. $C = 60°$. To get the answer, calculation will be: $\frac{a}{d} = \frac{c}{f}$, $\frac{a}{4.42} = \frac{12.0}{5.0}$ To enter the equation, press 2nd num-solv	DEG ∷=∷ Enter equation to solve.
2	Key in the values, $\frac{\square}{\square}$ x^{yzt}_{abcd} x^{yzt}_{abcd} x^{yzt}_{abcd} x^{yzt}_{abcd} x^{yzt}_{abcd} ⊙ 4 · 4 2 ◐ ◐ $\frac{\square}{\square}$ 1 2 · 0 ⊙ 5 · 0	DEG ▶ $\frac{a}{4.42} = \frac{12.0}{5.0}$
3	To get the answer, press x^{yzt}_{abcd} x^{yzt}_{abcd} . Thus, the answer is $a = 10.608m$	DEG † SOLUTION a=10.608 L - R =0 SOLVE AGAIN QUIT

TEXAS INSTRUMENTS

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Topic:	Math Concepts:
Geometry and Triangles	Basic Algebra, Triangle, Degree, Radian,

2. A rectangular shed 2m wide and 3m high stands against a perpendicular building of height $5.5\,m$. A ladder is used to gain access to the roof of the building. Determine the minimum distance between the bottom of the ladder and the shed.
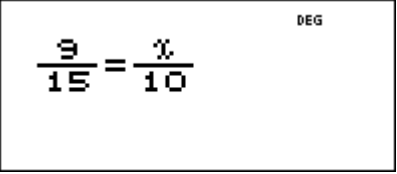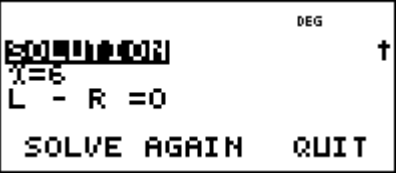

No	Keystrokes	Screenshots
1	Based on the figure, value $AB = 3.0m$. So, the equation for calculation will be: $\frac{EF}{BD} = \frac{DE}{AB}$, assume $EF = a$, so, $\frac{a}{2} = \frac{3.0}{2.5}$ To enter the equation, press 2nd [num-solv]	DEG ▒=▒ Enter equation to solve.
2	Key in the values, ▯ x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} ⌄ 2 ▷ ▷ ▯ 3 ⌄ 2 · 5	DEG $\frac{a}{2} = \frac{3}{2.5▯}$
3	To get the answer, press x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} . Thus, the answer is $a = 2.4m$ @ $EF = 2.4m$	DEG SOLUTION ↑ a=2.4 L - R =0 ▬▬▬▬▬ QUIT

TEXAS INSTRUMENTS

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Topic:	Math Concepts:
Geometry and Triangles	Basic Algebra, Triangle, Degree, Radian,

3. In figure at the right, show the triangles CBD and CAE are similar and hence find the length of CD and BD.

No	Keystrokes	Screenshots
1	Based on the figure, the proportion will be: $$\frac{CB}{CA} = \frac{CD}{CE} \left(= \frac{BD}{AE}\right)$$ To enter the equation, press 2nd [num-solv]	DEG ▒=▒ Enter equation to solve.
2	To enter the values, press ▦ 9 ⊝ 1 5 ◐ x_{abcd}^{yzt} ⊝ 1 2	DEG $\frac{9}{15} = \frac{x}{12}$ ▮
3	To get the answer, press enter enter enter Thus, length of $CD = 7.2cm$	DEG SOLUTION ↑ X=7.2 L - R =0 � QUIT

TEXAS INSTRUMENTS

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Computational Analytics

No	Keystrokes	Screenshots
4	To get the answer for length of BD, $\dfrac{9}{15} = \dfrac{BD}{10}$ To change the value, press 2nd [num-solv] again, and change the value. Press ⓥⓥⓥⓥⓥⓥⓥⓥⓥⓥⓥ 0	DEG $\dfrac{9}{15} = \dfrac{x}{10}$
5	To get the answer, press enter enter enter Thus, $BD = 6cm$	DEG SOLUTION † x=6 L - R =0 SOLVE AGAIN QUIT

## Exercise:

1. Add together the angles : 48°11'19'', 31°41'27'' and 9°9'37''
2. Convert the following angles to degrees and decimals of a degree, correct to 3 decimal places:
   (a) 15°11' (b) 29°53' (c) 49°42'17'' (d) 135°7'19''
3. Find the lengths x and y.

14.58 mm

25.69 mm

111°

x

32°

4.74 mm   y

32°   37°

7.36 mm
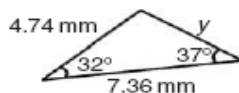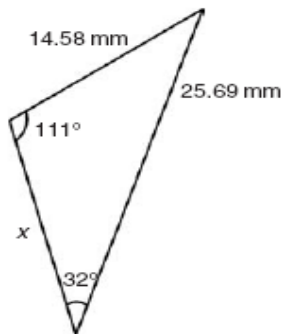
4. *PQR* is an equilateral triangle of side $4\ cm$. When *PQ* and *PR* are produced to *S* and *T*, respectively, *ST* is found to be parallel with *QR*. If *PS* is 9 cm, find the length of *ST*. *X* is a point on *ST* between *S* and *T* such that the line *PX* is the bisector of ∠*SPT*. Find the length of *PX* .

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5. Find (a) the length of BC when $AB = 6cm, DE = 8cm$ and $DC = 3cm$, (b) the length of DE when $EC = 2cm, AC = 5cm$ and $AB = 10cm$.
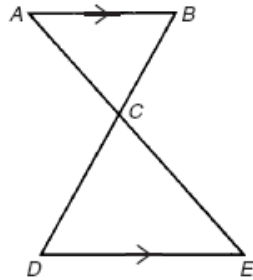
Answer:

1. 89°2'23"
2. (a) 15.183°   (b) 29.883°   (c) 49.705°   (d) 135.122°
3. $x = 16.54\ mm, \quad y = 4.18mm$
4. $9\ cm, 7.79\ cm$
5. (a) 2.25 cm (b) 4 cm

Triangles and some practical applications

Overview / Introduction / Terminology:

Sine and Cosine Rules:

For triangles which are *not* right-angled we use the sine and cosine rules. The triangle on the right has the conventional notation of small letters for the lengths of sides and capital letters for the angles opposite. To find lengths and angles, use:
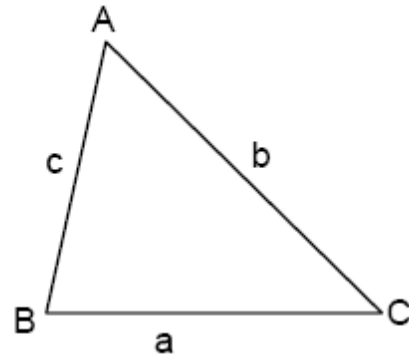
The sine rule may be used only when:

- 1 side and any 2 angles are initially given, or
- 2 sides and an angle (not the included angle) are initially given.

The cosine rule may be used only when:

- 2 sides and the included angle are initially given, or
- 3 sides are initially given.

<table>
<tr><td align="center">SINE RULE</td><td align="center">COSINE RULE</td></tr>
<tr><td align="center">

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

</td><td align="center">

$$a^2 = b^2 + c^2 - 2bc\,cosA \;(for\ a\ side)$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \;(for\ an\ angle)$$

</td></tr>
</table>

Don't be put off by the letters. Basically, the sine rule says the ratio of side/sine is the same for each pair of sides and angles. And in the cosine rule, ensure that the side on the LHS of the equation matches the angle on the RHS.

Area of a non-right angled triangle:

If you know two sides of a triangle, and the size of the angle between the two sides, then the area of the triangle can be found using:

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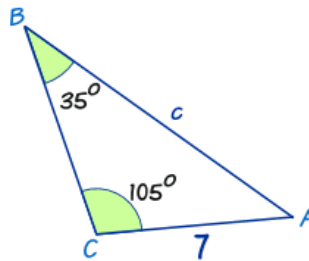Computational Analytics

$$Area \; = \frac{1}{2}ab\sin C \; or \; \frac{1}{2}ac\sin B \; or \; \frac{1}{2}bc\sin A$$
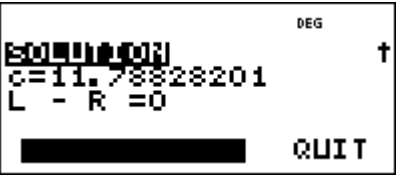
Keystrokes

basic operational keys

Example

1. Calculate side C

No	Instructions / Keystrokes	Screenshots
1	Law of sine: $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$ We can only find angle C at the moment, by using: $$\frac{7}{\sin 35} = \frac{c}{\sin 105}$$ We can use numerical solver from scientific calculator. [2nd] [num-solv] [7] [▢/▢] [sin⁻¹] [3] [5] [)] [▷] [▷] [x ʸᶻᵗ₍ₐᵦ𝒸𝒹₎] [x ʸᶻᵗ₍ₐᵦ𝒸𝒹₎] [x ʸᶻᵗ₍ₐᵦ𝒸𝒹₎] [x ʸᶻᵗ₍ₐᵦ𝒸𝒹₎] [x ʸᶻᵗ₍ₐᵦ𝒸𝒹₎] [x ʸᶻᵗ₍ₐᵦ𝒸𝒹₎] [x ʸᶻᵗ₍ₐᵦ𝒸𝒹₎] [▢/▢] [sin⁻¹] [1] [0] [5] [)] [enter]	

No	Keystrokes	Screenshots

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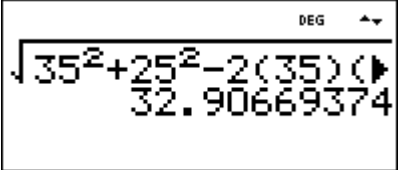
Topic:	Math Concepts:
Adding of Waveforms	Determining Resultant Phasors by Calculation

2	Bring you cursor to C and press [enter] to get the answer. Thus, side C is equal to 11.79	DEG ┃ **SOLUTION** c=11.78828201 L - R =0 ▬▬▬▬▬ QUIT

2. Solve triangle *DEF* and find its area given that *EF* =35.0 mm, *DE* =25.0mm and ∠*E* =64∘
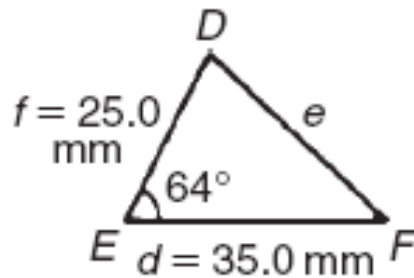
No	Instructions / Keystrokes	Screenshots
1	Applying the cosine rule $a^2 = b^2 + c^2 - 2bc\ cosA$ $e^2 = d^2 + f^2 - 2df\ cosE$ In your calculator, key-in this equation: $e = \sqrt{35^2 + 25^2 - 2(35)(25)\cos 64}$	DEG ▲▾ $\sqrt{35^2 + 25^2 - 2(35)(}$ ▶ 32.90669374

3. The diagram shows a triangle with sides 5, 7 and 8. Find the size of the smallest angle.

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Topic:	Math Concepts:
Adding of Waveforms	Determining Resultant Phasors by Calculation

No	Instructions / Keystrokes	Screenshots
1	The smallest angle is opposite the smallest side, 5. Using cosine rule, fill in the values you know, and the unknown length. Press: ▤ $5\,x^2\,+\,8\,x^2\,-\,7\,x^2\,\odot$ $2\,\times\,5\,\times\,8$ enter	
2	Press ◂▸≈ to convert the answer in decimal mode. Thus, $\cos P° = 0.5$	
3	Evaluate the right-hand-side and then use inverse-cosine cos^{-1} to find the angle $P°$ cos⁻¹ cos⁻¹ ⌃ enter) enter So, the angle $P°$ is 60°	

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4. Find the area for below triangle:

No	Instructions / Keystrokes	Screenshots
1	Given that $a = 8, c = 7\ and\ < B\ = 30°$. Using the formula for area, $Area = \frac{1}{2} ac \sin B$ key in 1 ▢/▢ 2 ◊ × 8 × 7 × sin⁻¹ 3 0) enter Thus, the area of triangle is $14 cm^2$	

Exercises

Find the unknown side or angle in each of the following diagrams. Round the answer to 3 significant features:

1.

2.

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3.

4.

5.

6.

7.

8.
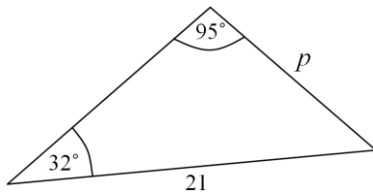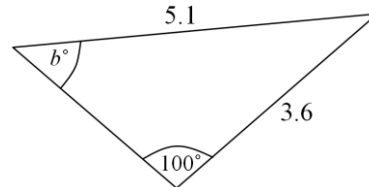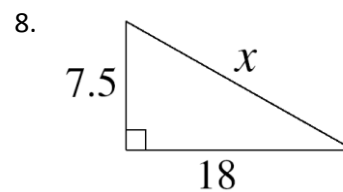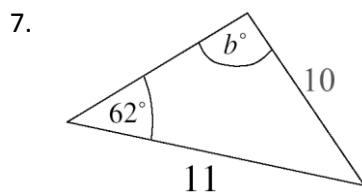
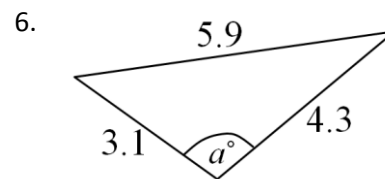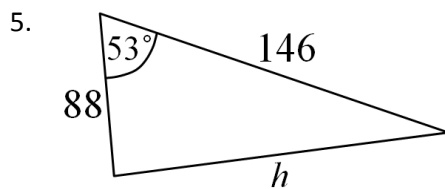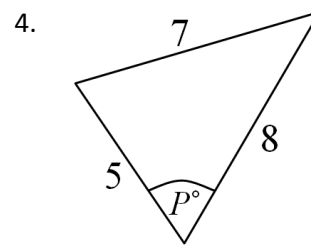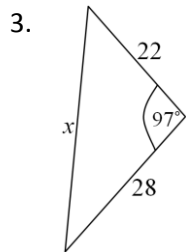9. Farmer Jones owns a triangular piece of land. The length of fence AB is $150m$. The length of the fence BC is $231m$. The angle between fence AB and fence BC is 123^0. How much land does Farmer Jones own?

Answers

1. $p = 11.2$
2. $b° = 44.0°$
3. $x = 37.7$
4. $P° = 60°$
5. $h = 117$
6. $a° = 105°$
7. $b° = 76.2°$
8. $x = 19.5$
9. Farmer Jones has $14,530\ m^2$ of land

Adding of Waveforms

Overview / Introduction / Terminology:

Determining Resultant Phasors by Calculation

This section will cover how to:

- Use the Sine Rule to find unknown sides and angles
- Use the Cosine Rule to find unknown sides and angles
- Combine trigonometry skills to solve problems

Each topic is introduced with a theory section including examples and then some practice questions. At the end of the page there is an exercise where you can test your understanding of all the topics covered in this page. You should already know each of the following formulae:

Formulae for right-angled triangles.
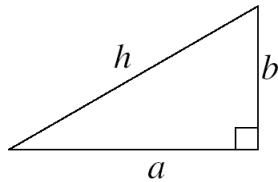
Pythagoras's Theorem

$$a^2 + b^2 = h^2$$

Trigonometric Ratios

$$\sin(\theta) = \frac{opp}{hyp}$$

$$\cos(\theta) = \frac{adj}{hyp}$$
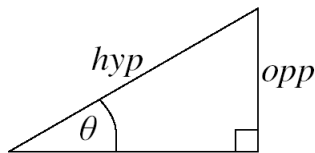
Formulae for all triangles.
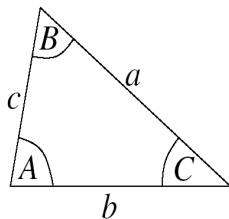
Sine Rule

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} \qquad \text{or} \qquad \frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

(for finding sides)   (for finding angles)

Cosine Rule

$$a^2 = b^2 + c^2 - 2bc\cos(A) \qquad \text{or} \qquad \cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

(for finding sides)   (for finding angles)

Keystrokes:

2nd [num-solv] , [▢/▢] , [sin⁻¹] , [cos⁻¹] , [math]

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Topic:	Math Concepts:
Adding of Waveforms	Determining Resultant Phasors by Calculation

Example:
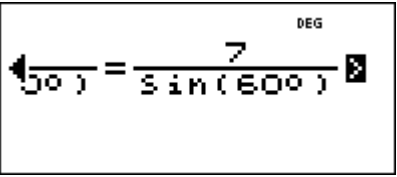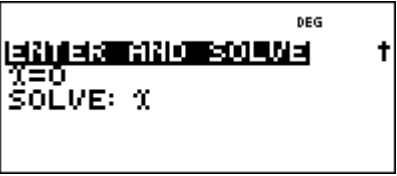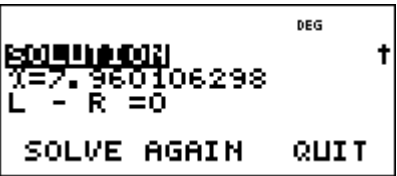
1. The Sine Rule can be used in any triangle (not just right-angled triangles) where a side and its opposite angle are known.

   Finding Sides: Work out the length of x in the diagram below,
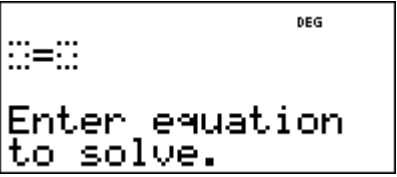
No	Keystrokes	Screenshots
1	Press 2nd [num-solv],	
2	Insert equation in the number solving, press $\frac{\square}{\square}$ x_{abcd}^{yzt} ⊖ $\frac{\sin}{\sin^{-1}}$ 8 0 math ⟩ ⟩ enter ⟩ .	
3	Then press ⟩ ⟩ $\frac{\square}{\square}$ 7 ⊖ $\frac{\sin}{\sin^{-1}}$ 6 0 math ⟩ ⟩ enter ⟩ ⟩ .	

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Topic:	Math Concepts:
Adding of Waveforms	Determining Resultant Phasors by Calculation

No	Keystrokes	Screenshots
4	enter to solve the equation.	$$\frac{}{(50)} = \frac{7}{\sin(60°)}$$ DEG
5	Assume that $x = 0$,	DEG ENTER AND SOLVE † X=0 SOLVE: X
6	Press enter enter The answer is **7.96** (accurate to 3 significant figures)	DEG SOLUTION † X=7.960106298 L - R =0 SOLVE AGAIN QUIT
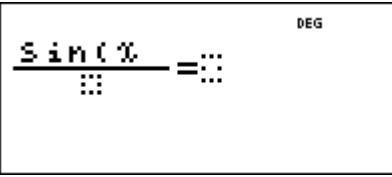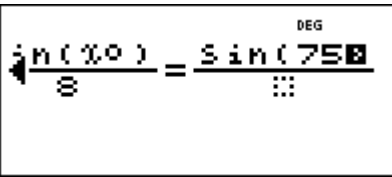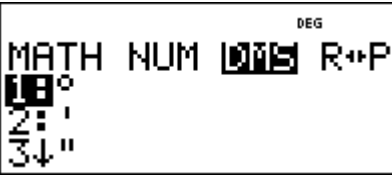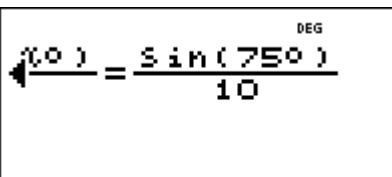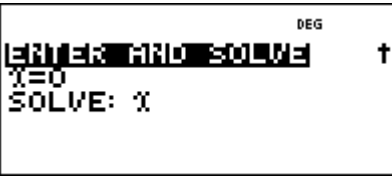
2. Finding Angles: Work out angle $m°$ in the diagram below: (Note, Change $m°$ to $x°$.)
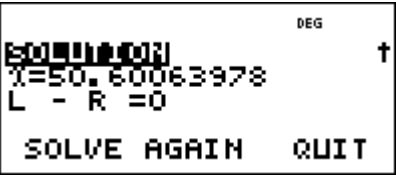
No	Keystrokes	Screenshots
1	Press 2nd num-solv ,	DEG ∷=∷ Enter equation to solve.

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Topic:	Math Concepts:
Adding of Waveforms	Determining Resultant Phasors by Calculation
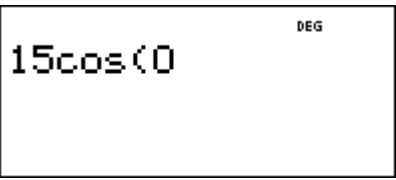
No	Keystrokes	Screenshots
2	Insert equation in the number solving, press ☐/☐ sin⁻¹ x^{yzt}_{abcd} math ► ► enter) ▼ 8 .	$$\frac{Sin(X}{\vdots} = \vdots$$ DEG MATH NUM DMS R↔P DEG 1 ° 2 ' 3↓ " $$\frac{Sin(X°)}{8} = \vdots$$ DEG
3	Then press ► ► ☐/☐ sin⁻¹ 7 5 math ► ► enter) ▼ 1 0 ► .	$$\frac{in(X°)}{8} = \frac{Sin(75█)}{\vdots}$$ DEG MATH NUM DMS R↔P DEG 1 ° 2 ' 3↓ "
4	enter to solve the equation.	$$\frac{(X°)}{} = \frac{Sin(75°)}{10}$$ DEG
5	Assume that $x = 0$,	ENTER AND SOLVE DEG † X=0 SOLVE: X

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Computational Analytics

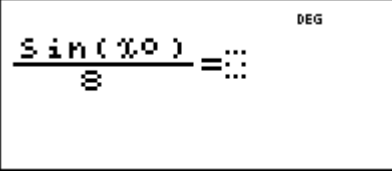| No	Keystrokes	Screenshots
6	Press enter enter The answer is $50.6°$ (accurate to 3 significant figures)	DEG SOLUTION X=50.60063978 L - R =0 SOLVE AGAIN QUIT
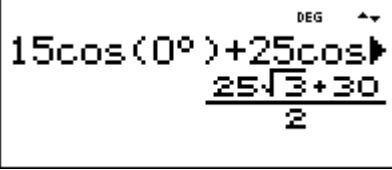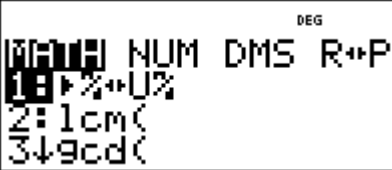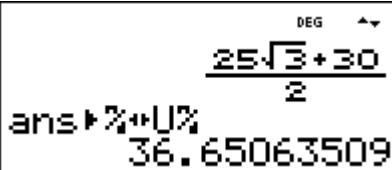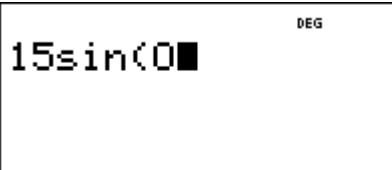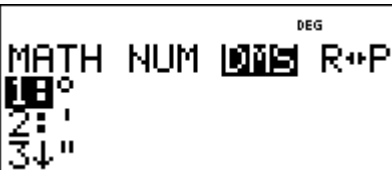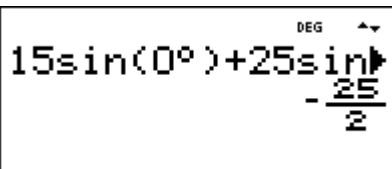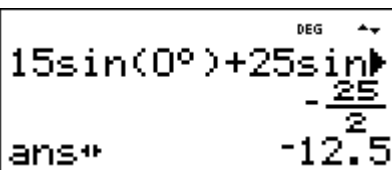
3. Two alternating voltages are given by $v_1 = 15 \sin \omega t$ volts and $v_2 = 25 \sin(\omega t - \pi/6)$ volts. Determine a sinusoidal expression for the resultant $v_R = v_1 + v_2$ by finding horizontal and vertical components.

    a) The relative positions of v_1 and v_2 at time $t = 0$

    b) The phasor diagram

No	Keystrokes	Screenshots
1	Firstly, for the horizontal component of v_R, Press 1 5 cos⁻¹ 0 , Then press math ▶ ▶ enter Then continue press) + 2 5 cos⁻¹ (−) 3 0 math ▶ ▶ enter) enter .	DEG 15cos(0 DEG MATH NUM DMS R↔P 1:° 2:' 3:↓"
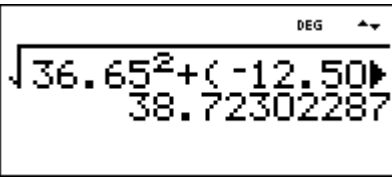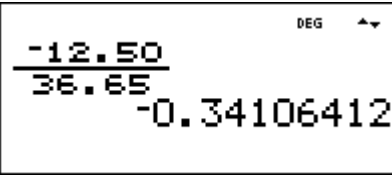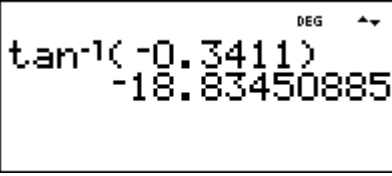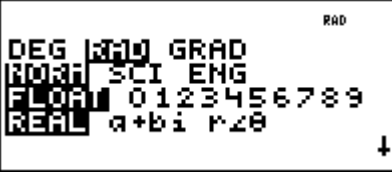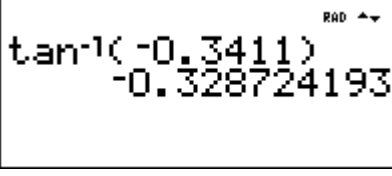
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	Change to the decimal, press math enter . $H = oa + ab = 36.65\,V$	DEG 15cos(0°)+25cos▶ $\dfrac{25\sqrt{3}+30}{2}$ DEG **MATH** NUM DMS R↔P **1:**▶%↔U% 2:1cm(3↓gcd(DEG $\dfrac{25\sqrt{3}+30}{2}$ ans▶%↔U% 36.65063509
2	For the vertical component of v_R, Press 1 5 sin⁻¹ 0 Then press math ▷ ▷ enter Then continue press) + 2 5 sin⁻¹ (−) 3 0 math ▷ ▷ enter) enter . Change to the decimal, press 2nd [answer] ◄▸≈ enter $V = bc = -12.50\,V$	DEG 15sin(0■ DEG MATH NUM **DMS** R↔P **1:**° 2:' 3↓" DEG 15sin(0°)+25sin▶ $-\dfrac{25}{2}$ DEG 15sin(0°)+25sin▶ $-\dfrac{25}{2}$ ans↔ -12.5

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No	Keystrokes	Screenshots
3	Press [2nd][√⁻][3][6][·][6][5][x^2][+][(][(][(-)][1][2][·][5][0][)][x^2][▷][enter] Hence $v_R (= oc) = 38.72$ volts	DEG ▲▾ $\sqrt{36.65^2 + (-12.50}$▶ 38.72302287
4	Press [▤][(-)][1][2][·][5][0][⊖][3][6][·][6][5][▷][enter] $\tan\emptyset = \dfrac{V}{H}\left(= \dfrac{bc}{ob}\right) = -0.3411$	DEG ▲▾ $\dfrac{-12.50}{36.65}$ -0.34106412
5	Press [tan/tan⁻¹][tan⁻¹][(][(-)][0][·][3][4][1][1][)][enter] , Change to Radians mode. Press [mode][▷][enter][clear] Recall or select the last equation. Press [▲][▲][enter][enter] $\emptyset = -\mathbf{18.83°}$ or $-\,\mathbf{0.329}$ **radians** Hence $v_R = v_1 + v_2 = \mathbf{38.72\sin(\omega t - 0.329)\,V}$	DEG ▲▾ $\tan^{-1}(-0.3411)$ -18.83450885 RAD DEG **RAD** GRAD **NORM** SCI ENG **FLOAT** 0123456789 **REAL** a+bi r∠θ ↓ RAD ▲▾ $\tan^{-1}(-0.3411)$ -0.328724193

Topic:	Math Concepts:
Adding of Waveforms	Determining Resultant Phasors by Calculation

Exercise:

1. Find the missing side in the diagram below: (Use Sine Rule)

2. Find the missing angle in the diagram below: (Use Cosine Rule)

3. Work out the value of x, y and z for each of the diagram below. The diagrams are not to scale. Try and use the most efficient method you can and give your answer to 3 significant figures.

    a)

    b)

4. Express the combination of periodic functions $i = 25 \sin \omega t - 15 \sin \left(\omega t + \frac{\pi}{3} \right)$ A in the form $A \sin(\omega t \pm \alpha)$.

5. Express the combination of periodic functions $x = 9 \sin \omega t + \frac{\pi}{7} - 7 \sin \left(\omega t - \frac{3\pi}{8} \right) cm$ in the form $A \sin(\omega t \pm \alpha)$.

Answer:

1. $p = 11.2$ (accurate to 3 significant figures)
2. $a° = \cos^{-1}(-0.252) = 105°$ (accurate to 3 significant figures)
3. (a) $x = 31°$ (b) $x = 5.64$
    $\quad\quad y = 7.39$ $y = 45.2$
    $\quad\quad z = 8.02$ $z = 44.8°$
4. $i = 21.79 \sin(\omega t - 0.639)A$
5. $x = 14.38 \sin(\omega t + 1.444)cm$

TEXAS INSTRUMENTS

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Topic:	Math Concepts:
Areas of plane figures	area, width, height, square, rectangle, parallelogram, triangle, trapezium

Areas of Plane Figures

Overview / Introduction / Terminology:

Area is the measurement of the amount of space occupied by a closed flat surface and is measured in square units. Every unit of length has a corresponding unit of area, namely the area of a square with the given side length. Thus areas can be measured in square meters (m^2), square centimeters (cm^2), square millimeters (mm^2), square kilometers (km^2), square feet (ft^2), square yards (yd^2), square miles (mi^2), and so forth. Algebraically, these units can be thought of as the squares of the corresponding length units. The SI unit of area is the square meter, which is considered an SI derived unit.

A plane is a flat or level surface in two dimensions. Figures such as circles or squares have all of their parts lying on a plane and thus, are examples of plane figures.

	Triangle $$Area = \frac{1}{2} \times b \times h$$ $b = base$ $h = vertical\ height$			Square $$Area = a^2$$ $a = length\ of\ side$
	Rectangle $$Area = w \times h$$ $w = width$ $h = height$			Parallelogram $$Area = b \times h$$ $b = base$ $h = vertical\ height$
	Trapezium $$Area = \frac{1}{2}(a + b) \times h$$ $h = vertical\ height$ $a\ and\ b = base$			Circle $$Area = \pi \times r^2$$ $$Circumference = 2 \times \pi \times r$$ $r = radius$
	Ellipse $$Area = \pi ab$$			Sector $$Area = \frac{1}{2} \times r^2 \times \theta$$ $r = radius$ $\theta = angle\ in\ radians$

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Topic:	Math Concepts:
Areas of plane figures	area, width, height, square, rectangle, parallelogram, triangle, trapezium

Keystrokes: Numbers and basic operational keys

Example:

1. Find the area of:
   a. A square flower-bed of side $17\ m$
   b. A rectangular field 45 m long and $40\ m$ wide
   c. A triangle of base length $30\ m$ and height $25\ m$

No	Instructions / Keystrokes	Screenshots
1	$Area\ of\ square = s^2$ Key in: 1 7 × 1 7 enter $s = 17\ m$	17*17 289
2	$Area\ of\ rectangle = w\ \times l$ Key in: 4 0 × 4 5 enter $w = 40\ m$ $l = 45\ m$	40*45 1800

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No	Instructions / Keystrokes	Screenshots
3	$Area\ of\ triangle = \dfrac{1}{2} \times b \times h$ Key in: 1 ▤ 2 ◐ × 3 0 × 2 5 enter $h = 25$ m $b = 30$ m	 DEG $\frac{1}{2} * 30 * 25$ 375

2. Sam cuts grass at RM0.10 per square meter. How much does Sam earn cutting this area:
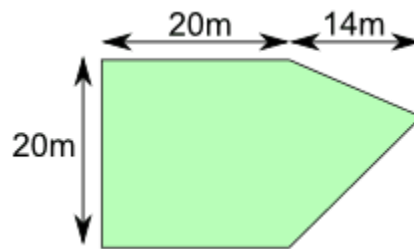
Let's break the area into two parts:

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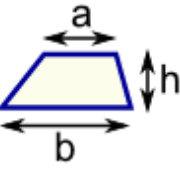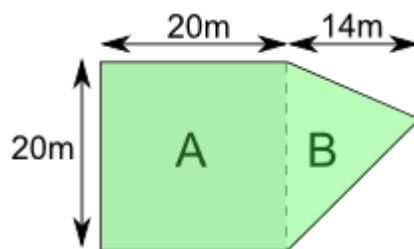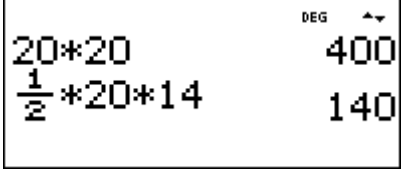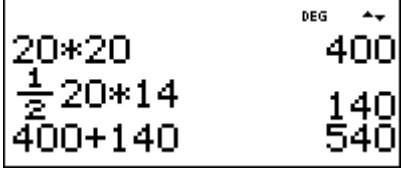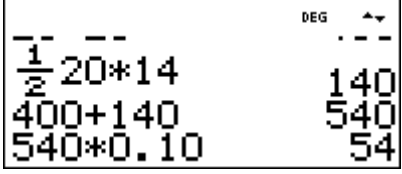
Topic: Areas of plane figures	Math Concepts: area, width, height, square, rectangle, parallelogram, triangle, trapezium

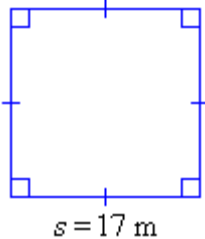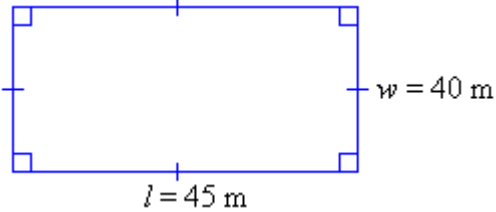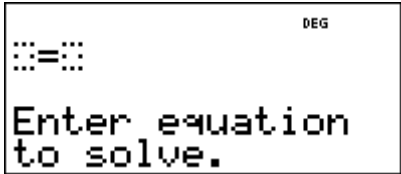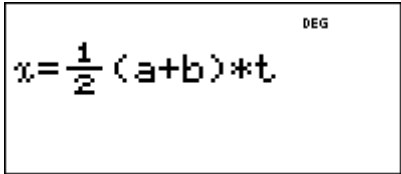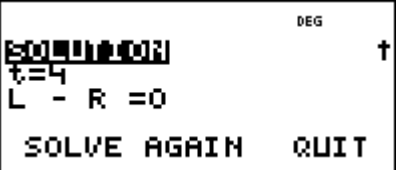| No	Instructions / Keystrokes	Screenshots
1	Part A is a square Area of A = $a^2 = 20m \times 20m$ 20×20 enter	DEG 20*20 400
2	Part B is a triangle. Viewed sideways it has a base of $20m$ and a height of $14m$ Area of B = $\frac{1}{2} \times b \times h = \frac{1}{2} \times 20m \times 14m$ 1⊟2⊙×20×14 enter	DEG 20*20 400 $\frac{1}{2}$*20*14 140
3	So total area is: $Area\ A\ +\ Area\ B\ =\ 400m^2 + 140m^2$ To calculate the total area: Copy answer Area A by pressing ⊙⊙⊙ enter Enter plus sign, ⊞ and copy answer Area B by pressing ⊙ enter Press enter to get the total area	DEG 20*20 400 $\frac{1}{2}$20*14 140 400+140 540
4	Sam earns RM0.10 per square meter For $540m^2$, Sam earns = RM0.10 × $540m^2$ ⊙ enter ×0·10 enter	DEG $\frac{1}{2}$20*14 140 400+140 540 540*0.10 54

Topic:	Math Concepts:
Areas of plane figures	area, width, height, square, rectangle, parallelogram, triangle, trapezium

3. The area of a trapezium is $52\ square\ inches$ and the bases are $11\ inches$ and $15\ inches$. Find the $height$.

No	Instructions / Keystrokes	Screenshots
1	Area of trapezium $= \frac{1}{2}(a+b) \times h$ To solve this problem we can use numeric equation solver (num-solv) function. Press 2nd [num-solv]	DEG ∷=∷ Enter equation to solve.
2	Key-in the equation by pressing: x_{abcd}^{yzt} ⓓⓓ $(variable\ x\ =\ area)$ 1 ▤ 2 ⓓ (x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} $(variable\ a\ =\ b_1)$ + x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt}) $(variable\ b\ =\ b_2)$ × x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} enter $(variable\ t\ =\ h)$	DEG $x = \frac{1}{2}(a+b) \ast t$
3	Insert known value for x, a and b and press enter. Leave blank for value t.	DEG ENTER AND SOLVE ↑ X=52 a=11 b=15 t=0 ↓

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No	Instructions / Keystrokes	Screenshots
1	Press ⊙ to go to next page. Bring your cursor to variable t by pressing right arrow.	DEG SOLVE: %ab■ †
2	Press enter get the answer. Thus, the height of trapezium is $4\ inches$.	DEG SOLUTION † t=4 L - R =0 SOLVE AGAIN QUIT

Exercises:

1. The height of a parallelogram is 6 more units than the base. What is the area of the parallelogram if the base is $15cm$?
2. Find the area of a trapezium with bases of $9cm$ and $7cm$, and a height of $3cm$.
3. The area of a trapezoid is 52 square inches and the bases are 11 inches and 15 inches. Find the height.
4. A triangular-shaped piece of paper has an area of 36 square centimeters and a base of $6cm$. Find the height.
5. The perimeter of a square-shaped rug is $36m$. Find the area.

Answers:

1. 315
2. $24\ cm^2$
3. $4\ in$
4. 12
5. 81

TEXAS INSTRUMENTS

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Topic:	Math Concepts:
The circle	circle, radius, circumference, diameter, semicircle, quadrant, tangent, sector, chord, segment, arc, angle, equation of a circle

The Circle

Overview / Introduction / Terminology:
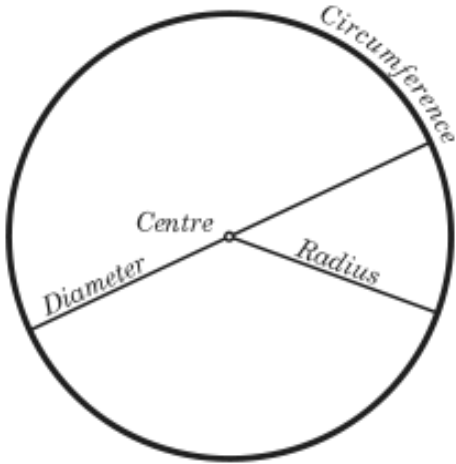
A **circle** is a simple shape of Euclidean geometry consisting of those points in a plane that are equidistant from a given point, the centre. The distance between any of the points and the centre is called the radius.

A circle's diameter is the length of a line segment whose endpoints lie on the circle and which passes through the centre. This is the largest distance between any two points on the circle. The diameter of a circle is twice the radius, or distance from the centre to the circle's boundary. The term "diameter" and "radius" also refer to the line segment which fit these descriptions. The circumference is the distance around the outside of a circle.

A chord is a line segment whose endpoints lie on the circle. A diameter is the longest chord in a circle. A tangent to a circle is a straight line that touches the circle at a single point, while a secant is an extended chord: a straight line cutting the circle at two points.

An arc of a circle is any connected part of the circle's circumference. A sector is a region bounded by two radii and an arc lying between the radii, and a segment is a region bounded by a chord and an arc lying between the chord's endpoints.
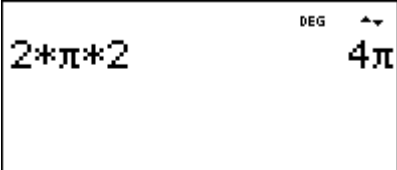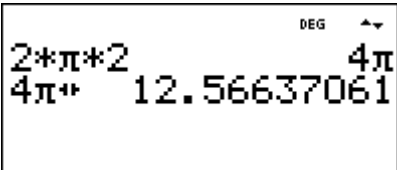
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Topic:	Math Concepts:
The circle	circle, radius, circumference, diameter, semicircle, quadrant, tangent, sector, chord, segment, arc, angle, equation of a circle

Keystrokes:
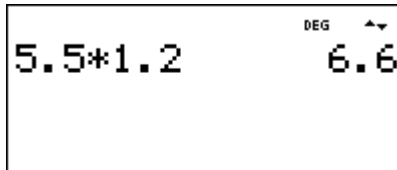
Numbers and basic operational keys

Example:

1. If a circle has a diameter of 4, find the circumference of a circle.
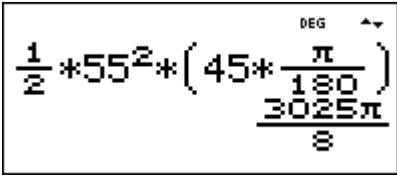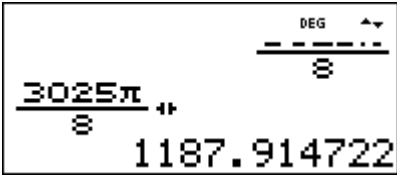
No	Instructions / Keystrokes	Screenshots
1	Formula of circumference, $c = 2\pi r\ where\ r\ is\ radius$ We already know that $d = 4$, so $r = 2$. Substitute the value in the formula. Key-in 2 × π^e_i × 2 enter	DEG 2*π*2 4π
2	To get the answer in decimal mode, press ◀▶≈ Thus, the answer is $12.57cm$	DEG 2*π*2 4π 4π◀ 12.56637061

2. Find the length of arc of a circle of radius 5.5 cm when the angle subtended at the centre is 1.20 radians.

No	Instructions / Keystrokes	Screenshots
1	Formula length of arc, $s = r\theta$, where θ is in radians Press 5 · 5 × 1 · 2	DEG 5.5*1.2 6.6

Topic:	Math Concepts:
The circle	circle, radius, circumference, diameter, semicircle, quadrant, tangent, sector, chord, segment, arc, angle, equation of a circle

3. A football stadium floodlight can spread its illumination over an angle of $45°$ to a distance of $55m$. Determine the maximum area that is floodlit.

No	Instructions / Keystrokes	Screenshots
1	Formula for area of sector $=\frac{1}{2}r^2\theta$ Press: 1 ▯ 2 ◊ × 5 5 x² × (4 5 × π ᵉᵢ ▯ 1 8 0 ◊) enter	$\frac{1}{2}*55^2*\left(45*\frac{\pi}{180}\right)$ $\frac{3025\pi}{8}$
2	Press ◄►≈ to get the answer in decimal mode. Thus, the answer is $11.88m^2$	$\frac{3025\pi}{8}$ 1187.914722

Exercises:

1. If the diameter of a circle is $75mm$, find its circumference.
2. Determine the radius of a circle if its perimeter is $112cm$.
3. Find the circumference of a circle of radius $12cm$.
4. Determine the diameter and circumference of a circle if an arc of length $4.75cm$ subtends an angle of $0.91 radians$.
5. An automatic garden sprays produces a spray to a distance of $1.8m$ and revolves through an angle α which may be varied. If the desired spray catchment area is to be $2.52m^2$, to what should angle α be set, correct to the nearest degree.

Answers:
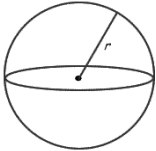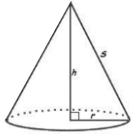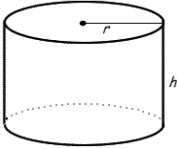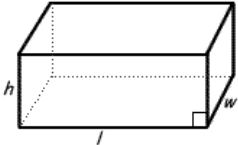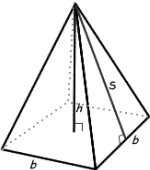
1. $235.6mm$
2. $17.83\ cm$
3. $75.40\ cm$
4. $Diameter = 10.44cm, Circumference = 32.80cm$
5. $angle\ \alpha = 88°$

Topic:	Math Concepts:
Volumes of common solids	Solid figures, area, volume, height, base, sphere, cone, cylinder, cuboid, pyramid

Volumes of common solids

Overview / Introduction / Terminology:

The volume is a measure of the amount of space inside the figure. Different figures lead to different computation. Common figures where the volume is calculated are shown below

Regular Solids	Surface Area	Volume
Sphere	$A = 4\pi r^2$	$V = \dfrac{4}{3}\pi r^3$
Cone	Area of the cone is $\pi r s$ Area of the base is πr^2 Therefore the formula is $SA = \pi r s + \pi r^2$	$V = \dfrac{1}{3}\pi r^2 h$
Cylinder	Area of the top is πr^2 Area of the base is πr^2 Area of the side is $2\pi r h$ Therefore the formula is $SA = 2\pi r^2 + 2\pi r h$	$V = \pi r^2 h$
Rectangular Prism (Cuboid)	$A = 2(wh + lw + lh)$	$V = lwh$
Square Based Pyramid	$A = 2bs + b^2$	$V = \dfrac{1}{3}b^2 h$

TEXAS INSTRUMENTS

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Topic:	Math Concepts:
Volumes of common solids	Solid figures, area, volume, height, base, sphere, cone, cylinder, cuboid, pyramid

Keystrokes:

Numbers and basic operational keys

Example:

1. What is the surface area of a cylinder with a height of 8 m and a base with a radius of $6m$? (Leave your answer in terms of π.)
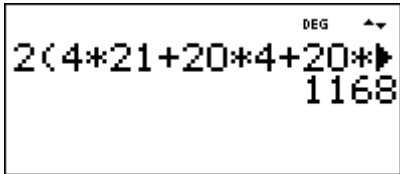
No	Instructions / Keystrokes	Screenshots
1	$SA \ for \ cylinder = 2\pi r^2 + 2\pi rh$ Substitute the values into formula. Key-in $\boxed{2}\boxed{\times}\boxed{\pi_i^e}\boxed{\times}\boxed{6}\boxed{x^2}\boxed{+}\boxed{2}\boxed{\times}\boxed{\pi_i^e}\boxed{\times}\boxed{6}\boxed{\times}\boxed{8}$ Press $\boxed{enter}$ to get the answer.	DEG $2*\pi*6^2+2*\pi*6*8$ 168π

2. What is the surface area of a sphere with a radius of $3m$? Leave your answer in terms of π.

No	Instructions / Keystrokes	Screenshots
1	$SA \ for \ sphere = 4\pi r^2$ Substitute the values into formula. Key-in $\boxed{4}\boxed{\times}\boxed{\pi_i^e}\boxed{\times}\boxed{3}\boxed{x^2}$ Press $\boxed{enter}$ to get the answer.	DEG $4*\pi*3^2$ 36π

3. What is the surface area of a rectangular prism with a length of $20m$, a height of $21m$, and a width of $4m$?
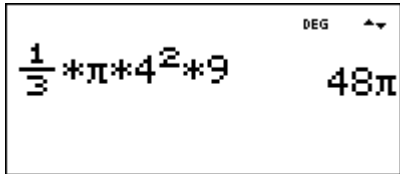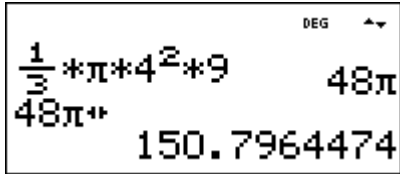
Topic:	Math Concepts:
Volumes of common solids	Solid figures, area, volume, height, base, sphere, cone, cylinder, cuboid, pyramid

No	Instructions / Keystrokes	Screenshots
1	$SA\ for\ rectangular\ prism = 2(wh + lw + lh)$ Substitute the values into formula. Key-in 2 (4 × 2 1 + 2 0 × 4 + 2 0 × 2 1) Press enter to get the answer. Thus the answer is $1168 m^2$	DEG 2(4*21+20*4+20*▶ 1168
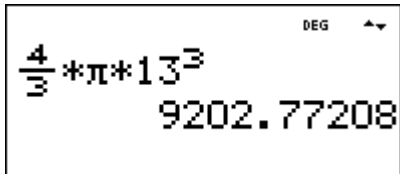
4. Let's find the volume of this cone.

We can substitute the values into the volume formula.

h = 9 cm
r = 4 cm
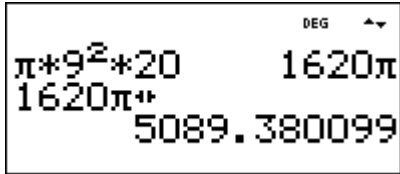
No	Instructions / Keystrokes	Screenshots
1	$Volume\ of\ cone\ = \dfrac{1}{3}\pi r^2 h$ Substitute the values into the volume formula. Key-in 1 ▤ 3 ▷ × πᵉᵢ × 4 x² × 9 enter	DEG $\frac{1}{3}$*π*4²*9 48π
2	Press ◀▸≈ to get the answer in decimal mode. When we perform the calculations, we find that the volume is $150.80 cm^3$.	DEG $\frac{1}{3}$*π*4²*9 48π 48π▸ 150.7964474

TEXAS INSTRUMENTS

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Topic:	Math Concepts:
Volumes of common solids	Solid figures, area, volume, height, base, sphere, cone, cylinder, cuboid, pyramid

No	Instructions / Keystrokes	Screenshots
1	$Volume\ of\ cone\ = \frac{4}{3}\pi r^3$ Substitute the values into the volume formula. Key-in 4 ☐/☐ 3 ▷ × π$_i^e$ × 1 3 x☐ 3 enter Thus, the answer is 9202.77 $cubic\ feet$.	DEG $\frac{4}{3}*\pi*13^3$ 9202.77208

5. Find the volume of this can of potato chips.

No	Instructions / Keystrokes	Screenshots
1	To find the volume of cylinder, here is the formula: $Volume\ of\ cylinder = \pi r^2 h$ Substitute the values into the formula. Key-in π$_i^e$ × 9 x² × 2 0 enter Press ◄►≈ to get the answer in decimal mode.	DEG $\pi*9^2*20$　　1620π 1620π▪ 5089.380099

Exercises:

1. What is the surface area of a cylinder with a height of $5m$ and a base radius of $4m$? (Leave your answer in terms of π.).=

2. What is the surface area of a cylinder with a base area of $25\pi\ in.$ and a height of $10\ in.$? (Leave your answer in terms of π.)==

3. What is the surface area of a sphere with a radius of $8cm$? (Leave your answer in terms of π.) $256\pi\ cm^2$

TEXAS INSTRUMENTS

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Computational Analytics

Topic:	Math Concepts:
Volumes of common solids	Solid figures, area, volume, height, base, sphere, cone, cylinder, cuboid, pyramid

4. What is the measure of the radius of a sphere with a total surface area of $16\pi\ cm$?

5. What is the volume of a pyramid with a square base with sides of $5cm$, and a height of $3cm$?

6. Find the volume of a sphere with a diameter of 5 in. us 3.14 for π and round your answer to the nearest inch.

7. Find the volume of a rectangular prism that has a base measuring 6 in by 4 in and a height of 8 in.

Answers:

1. 72π
2. 150π
3. The radius is $2\ cm$.
4. 25
5. 65
6. 192

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Integration

Overview / Introduction / Terminology:

The process of integration reverses the process of differentiation. In differentiation, if $f(x) = 2x2$ then $f\_(x) = 4x$. Thus the integral of $4x$ is $2x2$, i.e. integration is the process of moving from $f\_(x)$ to $f(x)$. B similar reasoning, the integral of $2t$ is $t2$.

The general solution of integrals of the form where a and n are constants is given by:

$$\int ax^n \, dx = \frac{ax^{n+1}}{n+1} + c$$

Definite integrals are those in which limits are applied. If an expression is written as $[x]ba$, 'b' is called the upper limit and 'a' the lower limit. The operation of applying the limits is defined as: $[x]ba = (b) - (a)$

Functions that require integrating are not always in the 'standard form'. However, it is often possible to change a function into a form which can be integrated by using either:
   a) an algebraic substitution
   b) trigonometric substitutions
   c) integration by parts

Keystrokes:
2nd [∫□□dx]

TEXAS INSTRUMENTS

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Topic:	Math Concepts:
Integration	Integration

Example:

1. Evaluate $\int_{x^2}^{2}(\sqrt[x]{2x^{2+x}})\,dx$

No	Instructions / Keystrokes	Screenshots
1	Press $\boxed{2nd}$ $\boxed{\int_{\square}^{\square}\square dx}$ $\boxed{x_{abcd}^{yzt}}$ $\boxed{x^2}$ $\boxed{\wedge}$ $\boxed{2}$ $\boxed{\triangleright}$ $\boxed{2nd}$ $\boxed{\sqrt[\square]{}}$ $\boxed{x_{abcd}^{yzt}}$ $\boxed{\triangleright}$ $\boxed{2}$ $\boxed{x_{abcd}^{yzt}}$ $\boxed{x^\square}$ $\boxed{2}$ $\boxed{+}$ $\boxed{x_{abcd}^{yzt}}$ Thus, $X = 4.47568$	
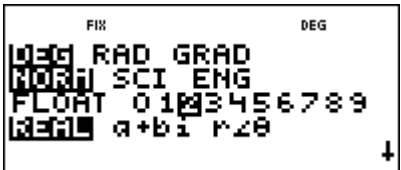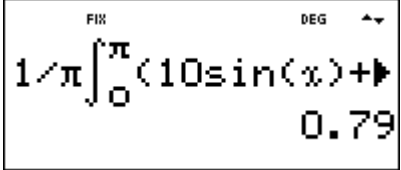
2. Find x when $\int_{2}^{5} \sqrt[x]{2x^{2-x}}\,dx = 4x^{3+6x}$

No	Instructions / Keystrokes	Screenshots
1	At the left hand side, Press $\boxed{2nd}$ $\boxed{num\text{-}solv}$ $\boxed{2nd}$ $\boxed{\int_{\square}^{\square}\square dx}$ $\boxed{2}$ $\boxed{\wedge}$ $\boxed{5}$ $\boxed{\triangleright}$ $\boxed{2nd}$ $\boxed{\sqrt[\square]{}}$ $\boxed{x_{abcd}^{yzt}}$ $\boxed{\triangleright}$ $\boxed{2}$ $\boxed{x_{abcd}^{yzt}}$ $\boxed{x^\square}$ $\boxed{2}$ $\boxed{-}$ $\boxed{x_{abcd}^{yzt}}$ $\boxed{\triangleright}$	
2	At the right hand side, Press $\boxed{4}$ $\boxed{x_{abcd}^{yzt}}$ $\boxed{x^\square}$ $\boxed{3}$ $\boxed{+}$ $\boxed{6}$ $\boxed{x_{abcd}^{yzt}}$ $\boxed{enter}$	
3	Press $\boxed{enter}$ $\boxed{enter}$	
4	Thus, $X = 0.93995$	

TEXAS INSTRUMENTS

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Computational Analytics

Topic:	Math Concepts:
Integration	Integration
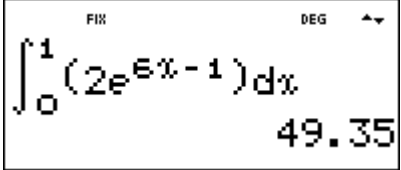
3. The average value of a complex voltage waveform is given by the following. Evaluate V correct to 2 decimal places

$$V = \frac{1}{\pi} \int_0^\pi (10 \sin x + 3 \sin 3x + 2 \sin 5x)dx$$

No	Instructions / Keystrokes	Screenshots
1	Change Angle to Degree [mode] [enter]	
2	Change Float to 2 decimal places ⊙⊙⊙⊙⊙ [enter]	
3	Press 1 $\div$ π_i^e [2nd] $[\int_\Box^\Box \Box dx]$ 0 ⊙ π_i^e ⊙ 1 0 $\frac{\sin}{\sin^{-1}}$ x_{abcd}^{yzt} $)$ $+$ 3 $\frac{\sin}{\sin^{-1}}$ 3 x_{abcd}^{yzt} $)$ $+$ 2 $\frac{\sin}{\sin^{-1}}$ 5 x_{abcd}^{yzt} $)$ $)$ ⊙ [enter]	

4. Evaluate the following

$$\int_0^1 2e^{6x-1}\,dx$$

No	Instructions / Keystrokes	Screenshots
1	Press [2nd] $[\int_\Box^\Box \Box dx]$ 0 ⊙ 1 ⊙ 2 π_i^e π_i^e $x^\Box$ 6 x_{abcd}^{yzt} $-$ 1 $)$ ⊙ [enter]	

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Topic:	Math Concepts:
Integration	Integration

5. Evaluate the following

$$\int_{0}^{\pi/2} sinx \cos x \, dx$$

No	Instructions / Keystrokes	Screenshots
1	Press 2nd [∫□dx] 0 ⏶ π $_i^e$ ÷ 2 ▶ sin⁻¹ x_{abcd}^{yzt}) cos⁻¹ x_{abcd}^{yzt}) ▶ enter	FIX DEG ▲▼ ∫$_0^{\pi/2}$(sin(x)cos(▶ 0.02

Exercise:

1. Determine the area enclosed by $y = 2x + 3$, the x-axis and coordinates at $x = 1$ and $x = 4$

2. Find x when the equation is given,

$$\int_{2}^{x} (\cos \sqrt{x^2} - 2x^2) = \sin x^2$$

3. Evaluate the following definite integral, correct to 4 significant figures

$$\int_{0}^{\pi/2} 3 \sin 2 t \, dt$$

Answer:
1. $24 \, square \, units$
2. 2.0756
3. 0.13

TEXAS INSTRUMENTS

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Differentiation

Overview / Introduction / Terminology:

Calculus is a branch of mathematics involving or leading to calculations dealing with continuously varying functions. Calculus is a subject that falls into two parts:

    a) **differential calculus** (or **differentiation**) and
    b) **integral calculus** (or **integration**).

Differentiation is used in calculations involving velocity and acceleration, rates of change and maximum and minimum values of curves.

A and *B* are two points very close together on a curve, *δx* (delta x) and *δy* (delta y) representing small increments in the *x* and *y* directions, respectively.
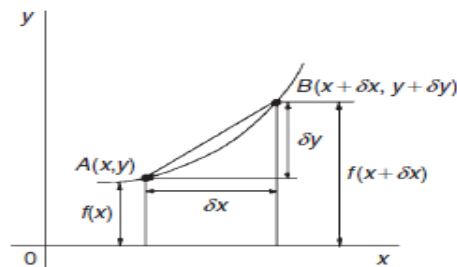
Summarizing the differential coefficient:

$$\frac{dy}{dx} = f'(x) = \lim_{n \to 0} \frac{\partial y}{\partial x}$$

$$= \lim_{n \to 0} \frac{f(x + \partial x) - f(x)}{\partial x}$$

From differentiation by first principles, a general rule for differentiating *axn* emerges where *a* and *n* are any constants. This rule is:
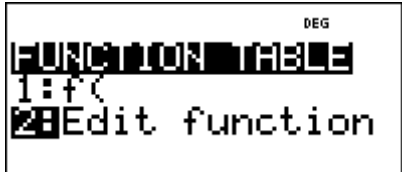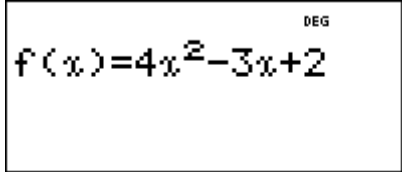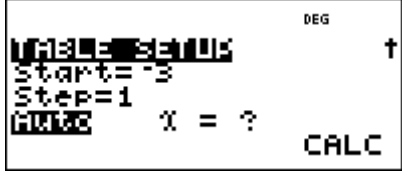
$y = ax^n$, then $\frac{dy}{dx} = anx^{n-1}$

Keystrokes:

[table] [2nd] [d/dx□]

TEXAS INSTRUMENTS

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Computational Analytics

Topic:	Math Concepts:
Differentiation	Differentiation
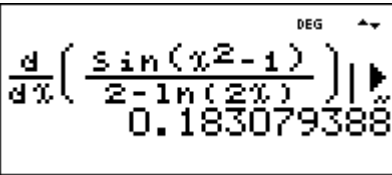
Example:

1. If $f(x) = 4x2 - 3x + 2$ find:
   a. $f(0), f(3), f(-1)$
   b. $f(3) - f(-1)$
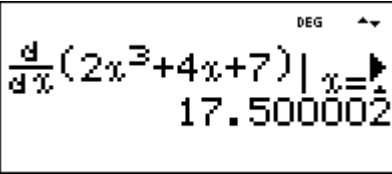
No	Instructions / Keystrokes	Screenshots
1	Press table . Select option No 2: Edit Function Press enter	**FUNCTION TABLE** DEG 1:f(2:Edit function
2	Key in 4 x_{abcd}^{yzt} x^2 − 3 x_{abcd}^{yzt} + 2 Press enter	$f(x)=4x^2-3x+2$ DEG
3	Key in table setup: Start: (−) 3 Press enter Step: 1 Press enter Press enter enter	**TABLE SETUP** DEG ↑ Start=⁻3 Step=1 **Auto** x = ? CALC
4	Press table . Select option No 1: f(Press enter Press 0) enter Press (−) 1) enter	**FUNCTION TABLE** DEG 1:f(2:Edit function DEG ▲ f(0) 2 f(-1) ▼ 9

TEXAS INSTRUMENTS

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Topic:	Math Concepts:
Differentiation	Differentiation

2. Find the differential coefficient of $y = \frac{\sin x^2 - 1}{2 - \ln 2x}$ when $x = 2$
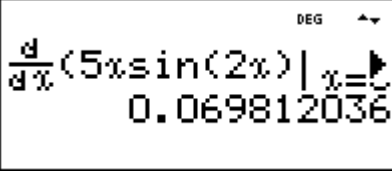
No	Instructions / Keystrokes	Screenshots
1	Press 2nd [d/dx□] $\frac{□}{□}$ sin sin⁻¹ x^{yzt}_{abcd} x^2 ─ 1) $\ominus$ 2 ─ ln log 2) enter Thus, $x = 0.1830$	$\frac{d}{dx}\left(\frac{\sin(x^2-1)}{2-\ln(2x)}\right)\|\blacktriangleright$ 0.183079388

3. Determine the derivative of the following and determine the gradient of the curve at $x = 1.5$
$$y = 2x^3 + 4x + 7$$

No	Instructions / Keystrokes	Screenshots
1	Press 2nd [d/dx□] 2 x^{yzt}_{abcd} $x^□$ 3 $\circledcirc$ + 4 x^{yzt}_{abcd} + 7 $\circledcirc$ 1 · 5 enter	$\frac{d}{dx}(2x^3+4x+7)\|_{x=}\blacktriangleright$ 17.500002

4. Determine the rate of change of voltage given the following when $t = 0.2s$

$$v = 5t \sin 2t \ volts$$

No	Instructions / Keystrokes	Screenshots
1	Press 2nd [d/dx□] 5 x^{yzt}_{abcd} sin sin⁻¹ 2 x^{yzt}_{abcd} $\circledcirc$ 0 · 2 enter Replace variable t as x	$\frac{d}{dx}(5x\sin(2x))\|_{x=}\blacktriangleright$ 0.069812036

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Computational Analytics

Topic:	Math Concepts:
Differentiation	Differentiation

5. If $y = \frac{\cos(2x+1)}{\ln(x^2-1)+\sqrt{x}}$, find dy/dx and solve the equality for $\frac{dy}{dx} = \int_x^2 \frac{\ln x - \sin 2x}{2}$

No	Instructions / Keystrokes	Screenshots
1	At left hand side, Press [num-solv]$[d/dx\square]$ $\boxed{\frac{\square}{\square}}$ $\boxed{\substack{\cos\\\cos^{-1}}}$ $\boxed{2}$ $\boxed{x_{abcd}^{yzt}}$ $\boxed{+}$ $\boxed{1}$ $\boxed{)}$ $\boxed{\ominus}$ $\boxed{\ln\log}$ $\boxed{x_{abcd}^{yzt}}$ $\boxed{x^2}$ $\boxed{-}$ $\boxed{1}$ $\boxed{)}$ $\boxed{\sqrt{}}$ $\boxed{x_{abcd}^{yzt}}$	
2	At right hand side, Press $\boxed{2nd}$ $[\int_{\square}^{\square}\square dx]$ $\boxed{x_{abcd}^{yzt}}$ $\boxed{2}$ $\boxed{\lozenge}$ $\boxed{\frac{\square}{\square}}$ $\boxed{\ln\log}$ $\boxed{x_{abcd}^{yzt}}$ $\boxed{)}$ $\boxed{-}$ $\boxed{\substack{\sin\\\sin^{-1}}}$ $\boxed{2}$ $\boxed{x_{abcd}^{yzt}}$ $\boxed{)}$ $\boxed{\ominus}$ $\boxed{2}$ $\boxed{enter}$	
3	Press $\boxed{enter}$ $\boxed{enter}$	
4	Thus, $x = 2.696178379$	

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Exercise:

1. Differentiate when $x = 2$, given $y = \frac{\cos(2x+1)^2 - \ln(x^2+1)}{\sin(2x-1)^2 + \ln(x^2-1)}$. Give the answer correct to 3 significant figures.

2. Find the gradient of the curve $y = 3x4 - 2x2 + 5x - 2$ at the points $(0, -2)$ and $(1, 4)$

3. An alternating current, i amperes, is given by $i = 10 \sin 2\pi f t$, where f is the frequency in hertz and t the time in seconds. Determine the rate of change of current when $t = 20 \, ms$, given that $f = 150 \, Hz$.

4. The distance x metres moved by a car in a time t seconds is given by: $x = 3t3 - 2t2 + 4t - 1$. Determine the velocity and acceleration when $t = 1.5 \, s$

Answer:

1. -0.0517
2. $12x3 - 4x + 5, 5, 13$
3. $3000 \pi \, A/s$
4. $18.25 \, m/s, 23m/s$

Correlation and Regression

Overview / Introduction / Terminology:

Correlation is a measure of the amount of association existing between two variables. For linear correlation, if points are plotted on a graph and all the points lie on a straight line, then **perfect linear correlation** is said to exist. When a straight line having a positive gradient can reasonably be drawn through points on a graph **positive or direct linear correlation** exists, similarly, when a straight line having a negative gradient can reasonably be drawn through points on a graph, **negative or inverse linear correlation** exists
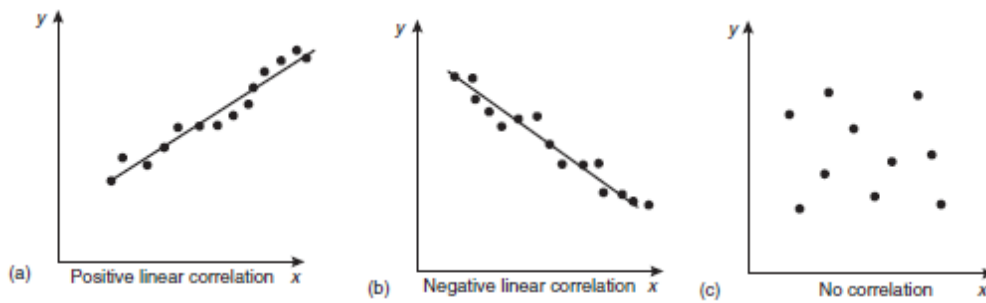
The amount of linear correlation between two variablesis expressed by a **coefficient of correlation**, given the symbol r. This is defined in terms of the derivations of the co-ordinates of two variables from their mean values and is given by the **product-moment formula** which states:

$$r = \frac{\sum xy}{\sqrt{\{(\sum x^2 (\sum y^2))\}}}$$

The results of this determination give values of r lyingbetween +1 and −1, where +1 indicates perfect direct correlation, −1 indicates perfect inverse correlation and 0 indicates that no correlation exists. Between these values, the smaller the value of r, the less is the amount of correlation which exists. Generally, values of r in the ranges 0.7 and 1 and $-0.7\ to -1$ show that a fair amount of correlation exists.

Regression analysis, usually termed **regression**, is used to draw the line of 'best fit' through co-ordinates on a graph. The techniques used enable a mathematical equation of the straight line form $y = mx + c$ to be deduced for a given set of co-ordinate values, the line being such that the sum of the deviations of the co-ordinate values from the line is a minimum, i.e. it is the line of 'best fit'. When a regression

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analysis is made, it is possible to obtain two lines of best fit, depending on which variable is selected as the dependent variable and which variable is the independent variable.

The equation of the least-squares regression line is usually written as $Y = a0 + a1X$, where $a0$ is the Y-axis intercept value and $a1$ is the gradient of the line (analogous to c and m in the equation $y=mx +c$). The regression line of Y on X is used to estimate values of Y for given value of X. By using the regression line of X on Y, values of X corresponding to given values of Y may be found by either interpolation or extrapolation.

The equation of the regression line is of the form: $X = b0 + b1Y$ and the normal equations become:

$$\sum X = b_0 N + b_1 \sum Y$$
$$\sum (XY) = b_0 \sum Y + b_1 \sum Y^2$$

The regression line of X on Y is used to estimated values of X for given values of Y. The regression line of Y on X is used to determine any value of Y corresponding to a given value of X. If the value of Y lies within the range of Y-values of the extreme co-ordinates, the process of finding the corresponding value of X is called **linear interpolation**. If it lies outside of the range of Y-values of the extreme co-ordinates than the process is called **linear extrapolation** and the assumption must be made that the line of best fit extends.

Keystrokes:

2nd data [stat-reg/distr]

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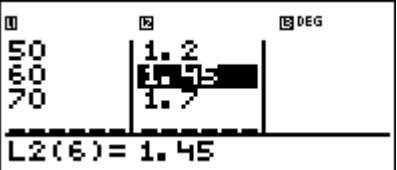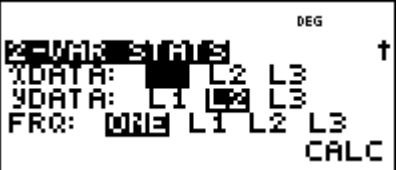
Topic:	Math Concepts:
Correlation and Regression	Correlation and Regression

Example:

1. In an experiment to determine the relationship between force on a wire and the resulting extension, the following data is obtained:

Force (N)	10	20	30	40	50	60	70
Extension (mm)	0.22	0.40	0.61	0.85	1.20	1.45	1.70
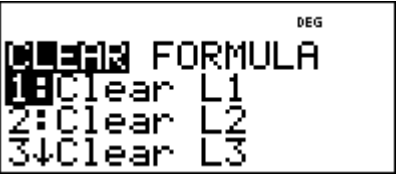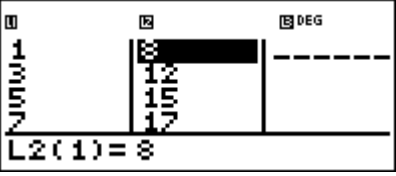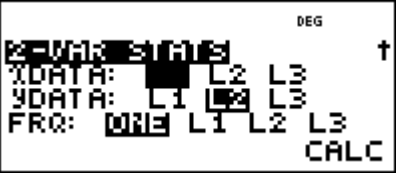
Determine the linear coefficient of correlation for this data.

No	Instructions / Keystrokes	Screenshots
1	Press [data] .Key in the data in list L1and L2	
2	Select Press [2nd] [data] Select option No 3: 2-Var Stats by using ⊙ Press [enter]	
3	Select default option : xData: L1 yData: L2 Frq: One	
4	Use ⊙to move to CALC Press [enter] . Scroll using ⊙to access the r statistics which shows the linear correlation	

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Topic:	Math Concepts:
Correlation and Regression	Correlation and Regression

2. Find the coefficient of correlation between the values X and Y

X	1	3	5	7	8	10
B	8	12	15	17	18	20

No	Instructions / Keystrokes	Screenshots
1	Press [data] [data] twice to clear list L1 by selecting the option to clear L1. Press [enter] Press [data] again and scroll using ⊙ to select list L2 to clear. Press [enter] Key in the new data in list L1 and L2	DEG **CLEAR** FORMULA 1:Clear L1 2:Clear L2 3↓Clear L3 L1 L2 L3 DEG 1 8 3 12 5 15 7 17 L2(1)= 8
2	Select Press [2nd] [data] Select option No 3: 2-Var Stats by using ⊙ Press [enter]	DEG **STAT-REG** DISTR 1:StatVars 2:1-Var Stats **3:**2-Var Stats
3	Select default option : xData: L1, yData: L2 Frq: One	DEG **2-VAR STATS** † XDATA: **L1** L2 L3 YDATA: L1 **L2** L3 FRQ: **ONE** L1 L2 L3 CALC
4	Use ⊙ to move to CALC Press [enter]. Scroll using ⊙ to access the r statistics which shows the linear correlation	DEG **2-Var:L1,L2,1** ↑a=1.301204819 :b=7.626506024 ↓r=0.9878783399

TEXAS INSTRUMENTS

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Computational Analytics

Topic:	Math Concepts:
Correlation and Regression	Correlation and Regression

3. In an experiment to determine the relationship between frequency and the inductive reactance of an electrical circuit, the following results were obtained: Determine the equation of the regression line of inductive reactance on frequency, assuming a linear relationship.

Freq	50	100	150	200	250	300
Ohms	30	65	90	130	150	190

No	Instructions / Keystrokes	Screenshots
1	Press [data][data] twice to clear list L1 by selecting the option to clear L1. Press [enter] Press[data] again and scroll using ⊙to select list L2 to clear. Press [enter] Key in the new data in list L1and L2	
2	Select Press [2nd][data] Select option No 4: LinReg by using ⊙ Press [enter]	
3	Select default option : xData: L1 yData: L2 Frq: One RegEQ :Use ⊙to move to Yes. Press [enter]	
4	Use ⊙to move to CALC Press [enter] . Scroll using ⊙to access the regression equation coefficients and r statistics	

Topic:	Math Concepts:
Correlation and Regression	Correlation and Regression

4. The experimental values relating centripetal force and radius, for a mass travelling at constant velocity in a circle, are as shown: Determine the equations of
   a) the regression line of force on radius and
   b) calculate the force at a radius of $40cm$
   c) the regression line of radius on force.
   d) calculate the radius corresponding to a force of $32N$

Force(Y)	5	10	15	20	25	30	35	40
Radius (X)	55	30	16	12	11	9	7	5

Solve 4a& b

No	Instructions / Keystrokes	Screenshots
1	Press ⌨data ⌨data twice to clear all lists by selecting the option to clear All. Press ⌨enter Key in the new data in list L1and L2	DEG **CLEAR** FORMULA 2↑Clear L2 3:Clear L3 4█Clear ALL L1 L2 L3 DEG **55** 5 30 10 16 15 12 20 L1(1)=55
2	Select Press ⌨2nd ⌨data Select option No 4: LinReg by using ⊙ Press ⌨enter	DEG **STAT-REG** DISTR 3↑2-Var Stats 4█LinReg ax+b 5↓QuadraticReg
3	Select default option : xData: L1, yData: L2 Frq: One RegEQ : Use ⊙to move to Yes Press ⌨enter	DEG XDATA: **L1** L2 L3 ↑ YDATA: L1 **L2** L3 FRQ: **ONE** L1 L2 L3 RegEQ→f(X): NO **YES** y=aX+b CALC

No	Instructions / Keystrokes	Screenshots
4	Use ⌄to move to CALCPress enter . Scroll using ⌄to access the regression equation coefficients and r statistics	ax+b:L1,L2,1 1:a=-1.15952381 2:b=44.21428571 3↓r²=0.715564969
5	Select Press 2nd data Select option No 1: StatsVar Press enter	STAT-REG DISTR 1:StatVars 2:1-Var Stats 3↓2-Var Stats
6	Scroll using ⌄to access the function variable force when $radius = 40cm$: y'(Press enter	ax+b:L1,L2,1 ↑Σxy=2045 :x'(:y'(
7	Key in 40 and Press) Press enter .	y'(40) 9.000506874
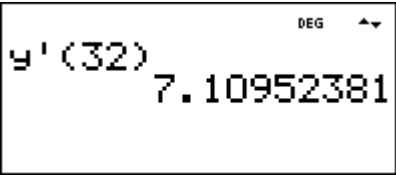
Solve 4c& d

No	Instructions / Keystrokes	Screenshots
1	Press data data twice to clear all lists by selecting the option to clear All. Press enter Key in the new data in list L1and L2	CLEAR FORMULA 2↑Clear L2 3:Clear L3 4:Clear ALL L1 L2 L3 55 5 30 10 16 15 12 20 L1(1)=55

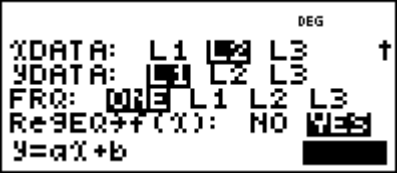| No	Instructions / Keystrokes	Screenshots
2	Select Press 2nd data Select option No 4: LinReg by using ⊙ Press enter	
3	Select default option : xData: L2 yData: L1 Frq: One RegEQ : Use ⊙to move to Yes Press enter	
4	Use ⊙to move to CALC Press enter . Scroll using ⊙to access the regression equation coefficients and r statistics	
5	Select Press 2nd data Select option No 1: StatsVar Press enter	
6	Scroll using ⊙to access the function variable radius when $force = 32\ N$: y'(Press enter	

TEXAS INSTRUMENTS

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Computational Analytics
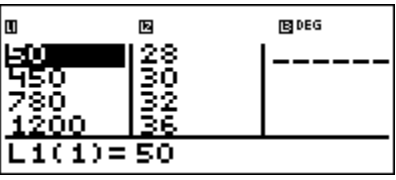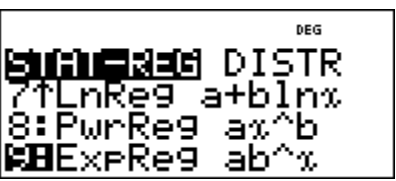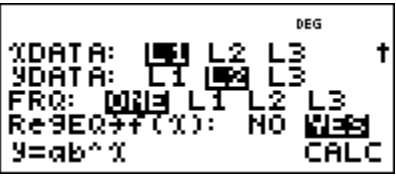
No	Instructions / Keystrokes	Screenshots
7	Key in 32 and Press) Press enter .	y'(32) 7.10952381

5. Predict the mean radiation dose at an altitude of $3000ft$ by fitting an exponential curve to the given data
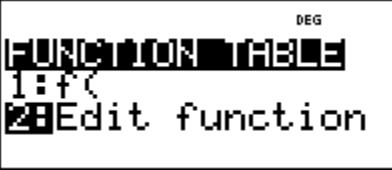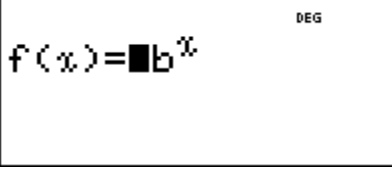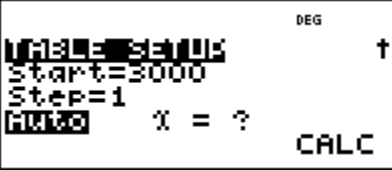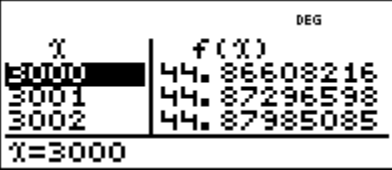
Altitude (x)	50	450	780	1200	4400	4800	5300
Radiation (y)	28	30	32	36	51	58	69

Let $y = ab^x$ be the exponential curve. Then

$$\log y = \log a + x \log b$$

No	Instructions / Keystrokes	Screenshots
1	Press data data twice to clear all lists. Scroll using ⊙to select all Lists to clear. Press enter Key in the new data in list L1and L2	L1 / L2 50 / 28 450 / 30 780 / 32 1200 / 36 L1(1)=50
2	Select Press 2nd data Select option No 9: ExpReg by using ⊙ Press enter	STAT-REG DISTR 7↑LnReg a+blnx 8:PwrReg ax^b 9█ExpReg ab^x
3	Select default option : xData: L1 yData: L2	XDATA: L1 L2 L3 YDATA: L1 L2 L3 FRQ: ONE L1 L2 L3 RegEQ→f(x): NO YES y=ab^x CALC

TEXAS INSTRUMENTS

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	Frq: One	
	RegEQ :Use ⊙to move to Yes. Press enter	
4	Use ⊙to move to CALC Press enter . Scroll using ⊙to access the regression equation coefficients and r statistics	ab^x:L1,L2,1 1:a=28.31597858 2:b=1.00015343 3↓r²=0.976948542
5	To get the the value of the radiation dose at an altitude of 3000ft . Press table to access the exponential function. Scroll ⊙to select Edit Function. Press enter Press enter	FUNCTION TABLE 1:f(2:Edit function $f(x)=\blacksquare b^x$
6	Key in 3000 at Start. Use ⊙to move to CALC Press enter .	TABLE SETUP Start=3000 Step=1 AUTO X = ? CALC
7	Therefore at an altitude of 3000, the radiation dose is at 44.866	X f(X) 3000 44.86608216 3001 44.87296598 3002 44.87985085 X=3000

TEXAS INSTRUMENTS

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Exercise:

1. In an experiment to determine the relationship between the current flowing in an electrical circuit and the applied voltage, the results obtained are:

Current (mA)	5	11	15	19	24	28	33
Applied voltage (V)	2	4	6	8	10	12	14

Find the coefficient of correlation for these results.

2. The data shown below refers to the number of times machine tools had to be taken out of service, in equal time periods, due to faults occurring and the number of hours worked by maintenance teams.

Machines out of service	4	13	2	9	16	8	7
Maintenance hours	400	515	360	440	570	380	415

Calculate the coefficientof correlation for this data.

3. Calculate the coeffiecient of the regression equation

Years	10	20	30	40	50	60
No of pax	15	32	51	78	79	109

Answer:

1. Correlation $r = 0.999$
2. Correlation $r = 0.937$
3. Regression : $a = 1.822, b = 3.1333$

TEXAS INSTRUMENTS

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Computational Analytics

Topic:	Math Concepts:
Measures of Central Tendency and Dispersion	Measures of Central Tendency and Dispersion

Measures of Central Tendency and Dispersion

Overview / Introduction / Terminology:

Statistics is the area of science that deals with collection, organization, analysis, and interpretation of data. It also deals with methods and techniques that can be used to draw conclusions about the characteristics of a large number of data points--commonly called a population. If the data set is based on a sample of a larger population, then the analyst can extend inferences onto the population based on the statistical results from the sample. Some statistical measures include regression analysis, mean, kurtosis, skewness, analysis of variance and variance.

Data are obtained largely by two methods:
  ➢ by counting—for example, the number of engines sold in equal periods of time, and
  ➢ by measurement — for example, the heights of a group of people.

A **quantitative variable** has a value or numerical measurement for which operations such as addition or averaging make sense. A **qualitative variable** describes and individual by placing the individual into a category or group such as male or female
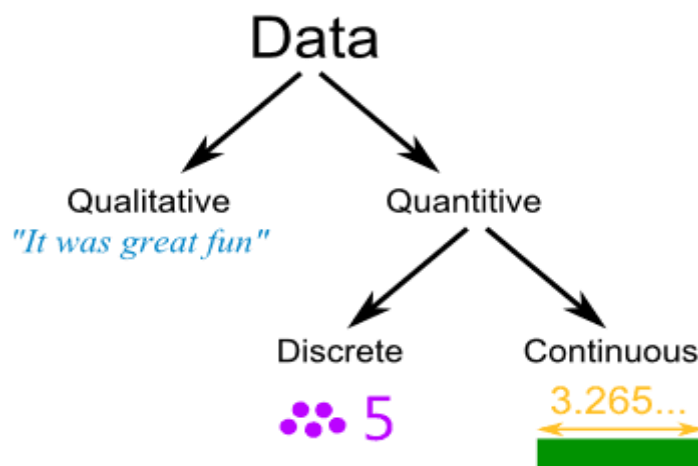
T h e r e are two types of data:
  ➢ Attribute data are countable data or data that can be put into categories
  ➢ Variable data are measurement data, based on some continuous scale

Topic:	Math Concepts:
Measures of Central Tendency and Dispersion	Measures of Central Tendency and Dispersion

Descriptive statistics enable us to understand data through summary values and graphical presentations. Summary values not only include the average, but also the spread, median, mode, range, interquartiles range, standard deviation and variances.

Arithmetic Mean

$$A = \frac{1}{n} * \sum_{i=1}^{n} x_i$$

Median – The number that is located in the middle of a set of numbers when that set is ordered sequentially from the smallest to the largest.

Range $Max(x) - Min(x)$

Interquartiles

$Q_1 = \left(\frac{n+1}{4}\right)^{th}$ $Q_3 = \left(\frac{3(n+1)}{4}\right)^{th}$

Standard Deviation

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \mu)^2}{n}}$$

Variances

$$\sigma^2 = \frac{\sum(X - \mu)^2}{N}$$

Keystrokes:
2nd data [stat-reg/distr]
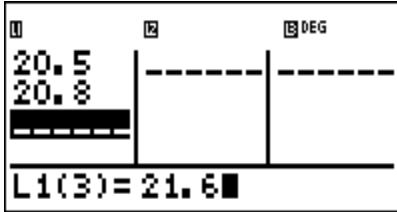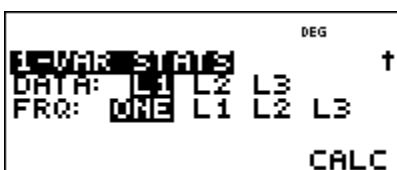
TEXAS INSTRUMENTS

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Computational Analytics

Topic:	Math Concepts:
Measures of Central Tendency and Dispersion	Measures of Central Tendency and Dispersion

Example:

1. The frequency distribution for the value of resistance in ohms of 48 resistors is as shown. Determine the mean, median, range and standard deviation value of resistance.
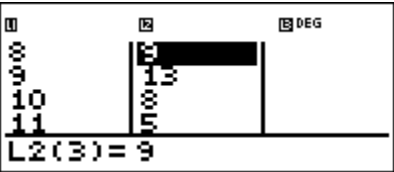
    $20.5, 20.8, 21.6, 22.3, 22.8, 22.9, 23.5, 24.6, 25.7, 26.8, 27.8, 25.9, 24.3, 22.5, 26.3$

No	Instructions / Keystrokes	Screenshots
1	Press [data] .Key in the data in list L1	
2	Select Press [2nd] [data] Select option No 2: 1-Var Stats. Press [enter]	
3	Select default option : Data: L1 Frq: One	
4	Use ⊙ to move to CALC Press [enter] . Scroll using ⊙ to access all parameters like mean, sample standard deviation etc	

TEXAS INSTRUMENTS

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Topic:	Math Concepts:
Measures of Central Tendency and Dispersion	Measures of Central Tendency and Dispersion

2. Calculate the mean, median, interquartile range, standard deviation and variance for the following:

Size of Items	6	7	8	9	10	11	12
Frequency	3	6	9	13	8	5	4

No	Keystrokes	Screenshots
1	Press[reset] to clear all previous entries. Press [data]. Key in the data in list L1 and list L2	
2	Select Press [2nd] [data] Select option No 2: 1-Var Stats. Press[enter]	
3	Select option : Data: L1. Use ⊙ to move down Frq: Use ⊙to select L2 :	
4	Use⊙to move to CALC Press [enter]. Scroll using ⊙ to access all parameters like mean, median, sample standard deviation etc	

TEXAS INSTRUMENTS

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Computational Analytics

Topic:	Math Concepts:
Measures of Central Tendency and Dispersion	Measures of Central Tendency and Dispersion

3. Calculate the mean, median, quartiles and the quartile coefficient of skewness for the following:

Weight	$70 - 80$	$80 - 90$	$90 - 100$	$100 - 110$	$110 - 120$	$120 - 130$	$130 - 140$	$140 - 150$
Freq	12	18	35	42	50	45	20	8

No	Instructions / Keystrokes	Screenshots
1	Press[reset] to clear all previous entries. Press [data]. Key in the data in list L1 (take the midpoint − example : (70+80)/2 = 75 and list L2	
2	Select Press [2nd][data] Select option No 2: 1-Var Stats. Press[enter]	
3	Select option : Data: L1. Use ⊙ to move down Frq: Use ⊙ to select L2 :	
4	Use⊙to move to CALC Press [enter]. Scroll using ⊙ to access all parameters	
5	The quartile coefficient of skewness $= \frac{Q1+Q3-2Q2}{Q3-Q1}$ Press (. Press [2nd][stat-reg/distr]. Select 1: StatVars. Scroll ⊙ to access Q1. Press [enter]. Repeat the steps to acess Q3 and Q2	

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Computational Analytics

Topic:	Math Concepts:
Measures of Central Tendency and Dispersion	Measures of Central Tendency and Dispersion

Exercise:

1. The scores of two golfers for 7 rounds each are

A	58	59	60	54	65	66	52
B	69	52	84	56	92	65	86

Who may be regarded as the more consistent player?

2. Given below are the marks obtained by a batch of 20 students in a certain class test in statistics and mathematics. In which subject is the level of knowledge of students higher?

Role No. Students	1	2	3	4	5	6	7	8	9	10
Marks in Statistics	53	54	52	32	30	60	47	46	35	28
Marks in Math	58	55	25	32	26	85	44	80	33	72

Role No. Students	11	12	13	14	15	16	17	18	19	20
Marks in Statistics	25	42	33	48	72	51	45	33	65	72
Marks in Math	10	42	15	46	50	64	39	38	80	85

Answer:
1. Golfer A
2. Statistics

Distributions

Overview / Introduction / Terminology:

The **binomial distribution** is a discrete probability distribution which is used when there are exactly two mutually exclusive outcomes of trials. These outcomes are appropriately labeled success and failure. This is used to obtain the probability of observing x successes in n trials, with the probability of success on a single trial denoted by p.

Binomial Formula. Suppose a binomial experiment consists of n trials and results in x successes. If the probability of success on an individual trial is P, then the binomial probability is:

$$b(x; n, P) = nCx * P^x * (1 - P)^{n-x}$$

A **Poisson distribution** is the probability distribution that results from a Poisson experiment that results in in outcomes that can be classified as successes or failures. Note that the specified region could take many forms. For instance, it could be a length, an area, a volume, a period of time, etc.

Poisson Formula. Suppose we conduct a Poisson experiment, in which the average number of successes within a given region is μ. Then, the Poisson probability is:

$$P(x; \mu) = (e^{-\mu})(\mu^x) / x!$$

If n is large and p is small, and the product np is less than 5, a very good approximation to a binomial distribution is given by the corresponding Poisson distribution

The **normal distribution** refers to a family of continuous probability distributions described by the normal equation

**Normal equation.** The value of the random variable Y is:

$$Y = \{ 1/[\sigma * sqrt(2\pi)] \} * e^{-(x-\mu)2/2\sigma2}$$

where X is a normal random variable, μ is the mean, σ is the standard deviation, π is approximately 3.14159, and e is approximately 2.71828.

The graph of the normal distribution depends on two factors - the mean and the standard deviation. The mean of the distribution determines the location of the center of the graph, and the standard deviation determines the height and width of the graph. When the standard deviation is large, the curve is short and wide; when the standard deviation is small, the curve is tall and narrow. All normal distributions look like a symmetric, bell-shaped curve.
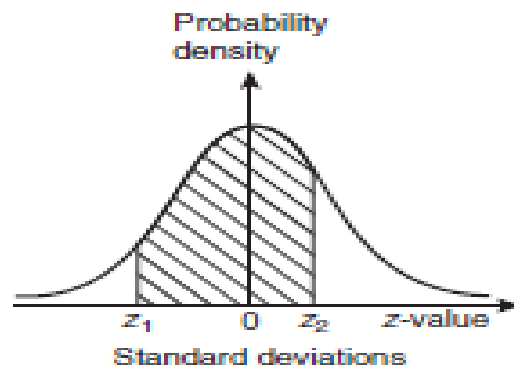
Topic:	Math Concepts:
Distribution	Distribution
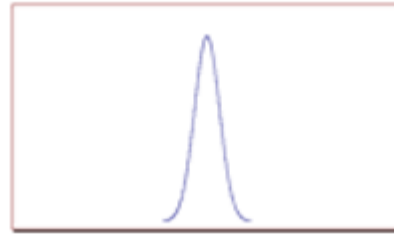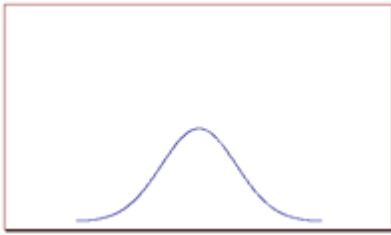
When a normal distribution curve has been standardized, the normal curve is called a **standardized normal curve** or a **normal probability curve**, and any normally distributed data may be represented by the **same** normal probability curve. The area under part of a normal probability curve is directly proportional to probability and the value of the shaded area.
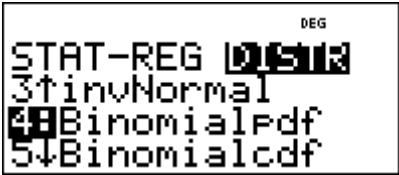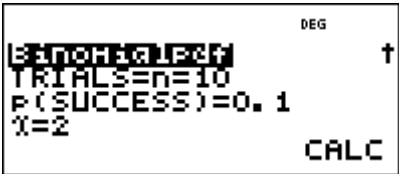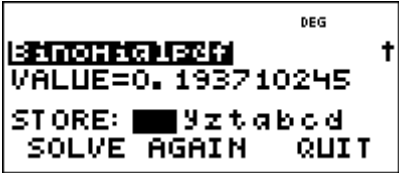
$$\int 1\frac{1}{\sqrt{2}} e^{x^2/2)} \, dz$$

Keystrokes:

2nd data [stat-reg/distr]

TEXAS INSTRUMENTS

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Computational Analytics

Topic:	Math Concepts:
Distribution	Distribution

Example:
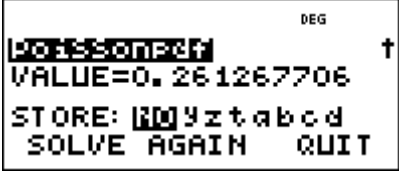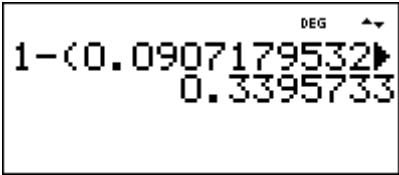
1. 10% of screws produced in a certain factory turn out to be defective. Find the probability that in a sample of 10 screws chosen at random, exactly two will be defective.

No	Instructions / Keystrokes	Screenshots
1	Press [2nd] [data] [stat-reg/distr] [enter]. Press⊙ to select distributions option. Then press ⊙ to scroll to option 4: Binomialpdf. Press [enter]	STAT-REG **DISTR** 3↑invNormal 4**■**Binomialpdf 5↓Binomialcdf
2	Press [enter] to select the x: single option	Binomialpdf X: **SINGLE** LIST ALL
3	Key in the following: n = [1][0]. Use ⊙ to go to the next option p = [0][·][1]. Use ⊙ to go to the next option x = [2]. Use ⊙ to go to CALC Press [enter]	Binomialpdf TRIALS=n=10 p(SUCCESS)=0.1 X=2 CALC Binomialpdf VALUE=0.193710245 STORE: **■** yztabcd SOLVE AGAIN QUIT

2. If 3% of the gearwheels produced by a company are defective, determine the probabilities that in a sample of 80 gearwheels
    a. (a) two and
    b. more than two will be defective.

    The solution is to determine that λ = np = 80 x 0.03 = 2.4< 5 – therefore use Poisson distribution
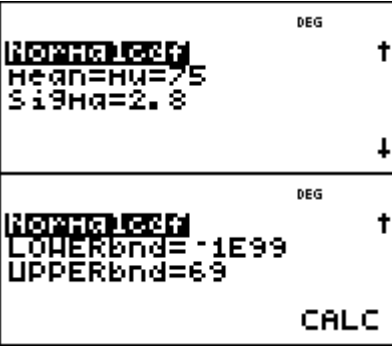
TEXAS INSTRUMENTS

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No	Instructions / Keystrokes	Screenshots
1	Press [2nd] [data] [stat-reg/distr] [enter]. Press ⊙ to select distributions option. Then keep press ⊙ again and again to scroll to option 6: Poissonpdf. Press [enter]	STAT-REG **DISTR** 5↑Binomialcdf **6:**Poissonpdf 7:Poissoncdf

No	Instructions / Keystrokes	Screenshots
2	Press [enter] to select the x: single option	**Poissonpdf** X: **SINGLE** LIST
3	Key in the following: mu = [2] [·] [4]. Use ⊙ to go to the next option x = [2]. Use ⊙ to go to CALC Press [enter]	**Poissonpdf** Mean=Mu=2.4 X=2 CALC **Poissonpdf** VALUE=0.261267706 STORE: **RD** yztabcd SOLVE AGAIN QUIT
4	The probability of having more than 2 defective is 1−(the sum of the probabilities of having 0, 1, and 2 defective gearwheels), i.e. 1 − (0.0907 + 0.2177 + 0.2613) Key in the following: Use ⊙ to go to Solve Again mu = [2] [·] [4]. Use ⊙ to go to the next option x = [0]. Use ⊙ to go to CALC Press [enter]	1−(0.0907179532▶ 0.3395733

Use ⊝ to go to Solve Again mu = ②·④. Use ⊝ to go to the next option x = ①. Use ⊝ to go to CALC Press enter	

3. A component is classed as defective if it has a diameter of less than 69 mm. In a batch of 350 components, the mean diameter is 75mm and the standard deviation is 2.8 mm.Determinehow many are likely to be classed as defective

No	Instructions / Keystrokes	Screenshots
1	Press 2nd data stat-reg/distr enter. Press ⊙ to select distributions option. Then press ⊝ to scroll to option 2: Normalcdf. Press enter	STAT-REG DISTR 1:Normalpdf 2:Normalcdf 3↓invNormal
2	Press ⑦⑤ enter to enter for mean Press②·⑧ to enter for sigma Press⊝⊝ Press ⑥⑨ enter enter enter	Normalcdf Mean=MU=75 Sigma=2.8 Normalcdf LOWERbnd=-1E99 UPPERbnd=69 CALC
3	Key in the following: Use ⊙ to go to the variable a under STORE Use ⊝⊙ to Qui	Normalcdf VALUE=0.016062228 STORE: NOXYzt■bcd SOLVE AGAIN ■

TEXAS INSTRUMENTS

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No	Instructions / Keystrokes	Screenshots
4	Press ③⑤⓪✕ [2nd] [recall] Use ⊙ to scroll to variable a Press [enter] [enter] The number of components that will be classified as defective is 6.	DEG RECALL VAR 3↑z=0 4:t=0 5▪a=0.016062228 DEG ▲▾ 350*0.016062227▶ 5.621779754

Exercise:

1. 2 percent of the electric bulbs produced by a company are defective. Find the probability that in a sample of 200 bulbs
   a. less than bulbs are defective and
   b. more than 3 bulbs are defective.

2. An average light bulb manufactured by the Acme Corporation lasts 300 days with a standard deviation of 50 days. Assuming that bulb life is normally distributed, what is the probability that an Acme light bulb will last at most 365 days

Answer:
   1. 210
   2. 3003

TEXAS INSTRUMENTS

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Computational Analytics

Probability

Overview / Introduction / Terminology:

Probabilities, permutations and combinations are used daily in many different fields that range from gambling and games, to mechanical or structural failure rates, to rates of detection in medical screening.

The **probability** of something happening is the likelihood or chance of it happening. Values of probability lie between 0 and 1, where 0 represents an absolute impossibility and 1 represents an absolute certainty. The probability of an event happening usually lies somewhere between these two extreme values and is expressed either as a proper or decimal fraction

$$P(A) = \frac{\text{The Number Of Ways Event A Can Occur}}{\text{The total number Of Possible Outcomes}}$$

Permutation: Use the equation to calculate the number of permutations of a given set of data, where *n* is the total number of elements in the data set, and *k* is the number of elements selected at one time. Notice that the definition of a permutation states that the ordering of the element selection *does* matter.

$$\# \ of \ permutations = \frac{n!}{(n-k)!}$$

Combination: Use the equation to calculate the number of combinations of a given set of data, where *n* is the total number of elements in the data set, and *k* is the number of elements selected at one time. Notice that the definition of a combination states that the ordering of the element selection *does not* matter.
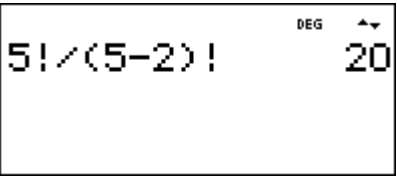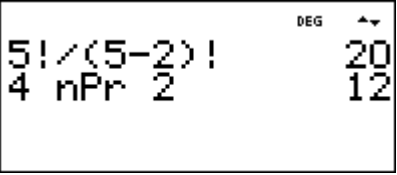
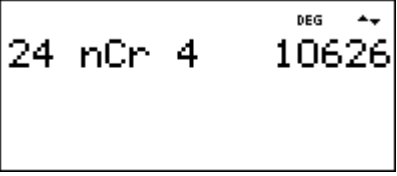$$\# \ of \ combinations = \frac{n!}{k! \ (n-k)!}$$

Keystrokes:

! nCr
nPr

TEXAS INSTRUMENTS

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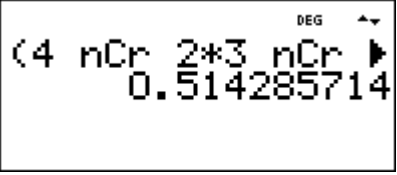| Topic:	Math Concepts:
Probability	Probability

Example:

1. Calculate the number of permutations there are of
   a. 5 distinct objects taken 2 at a time
   b. 4 distinct objects taken2 at a time

No	Instructions / Keystrokes	Screenshots
1	Press [5] [! nCr/nPr] [÷] [(] [5] [−] [2] [)] [! nCr/nPr] [enter]	DEG 5!/(5−2)! 20
2	Press [4] [! nCr/nPr] [! nCr/nPr] [! nCr/nPr] [2] [enter]	DEG 5!/(5−2)! 20 4 nPr 2 12

2. A class has 24 students. 4 can represent the class at an exam board. How many combinations are possible when choosing this group?

No	Instructions / Keystrokes	Screenshots
1	Press [2] [4] [! nCr/nPr] [! nCr/nPr] [4] [enter]	DEG 24 nCr 4 10626

3. There are 4 Engineers and 3 Managers in a firm. 3 of them are put on duty at a time. What is the probability that there are 2 Engineers and 1 Manager?

No	Instructions / Keystrokes	Screenshots
1	Total no of people = 7 Total number of combinations of taking 3 out of 7 is $^7C_3$ The combination of 2 Engineers and 1 Manager is $4_{C_2} \; X^3 C_1$	DEG (4 nCr 2*3 nCr ▶ 0.514285714

Topic: Probability	Math Concepts: Probability

Probability $= \dfrac{4\,C\,2\ \ X\,3\,C\,1}{7\,C\,3}$ Press $($ 4 $!\genfrac{}{}{0pt}{}{nCr}{nPr}$ $!\genfrac{}{}{0pt}{}{nCr}{nPr}$ 2 $\times$ 3 $!\genfrac{}{}{0pt}{}{nCr}{nPr}$ $!\genfrac{}{}{0pt}{}{nCr}{nPr}$ 1 $)$ $\div$ 7 $!\genfrac{}{}{0pt}{}{nCr}{nPr}$ $!\genfrac{}{}{0pt}{}{nCr}{nPr}$ 3 enter	DEG ans⁺⁺ $\dfrac{18}{35}$

Exercise:

1. In how many ways can a team of six be picked from ten possible players?

2. 15 boxes can each hold one object. In how many ways can 10 identical objects be placed in the boxes?

Answer:

1. 210

2. 3003

TEXAS INSTRUMENTS

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Computational Analytics

Sampling and Inference

Overview / Introduction / Terminology:

The sampling of attributes maybe regarded as the selection of samples from a population whose members possess the attributes K or not K. The presence of K maybe called a success and its absence a failure.

Suppose we draw a simple sample of n items. Clearly it is same as a series of n independent trials with the same probability p of success. The probabilities of $0, 1, 2, \ldots, n$ successes are the terms in the binomial expansion of $(q + p)^n$ where $q = 1 - p$.

If consider the proportion of successes, then:

1. Mean proposrtion of successes = $np/(n) = $ p

2. Standard error of the proposrtion of successes = $\sqrt{((n \cdot \frac{p}{n} \cdot (\frac{q}{n}))} = \sqrt{\frac{pq}{n}}$

3. Precision of the proportion of successes = $\sqrt{\frac{n}{pq}}$

Tests of significance:

1) Large samples :
$$z = \frac{(x - \mu)}{\sigma}, \quad \text{if } z > 1.96 \text{ ---it is significant}$$

2) Small samples :
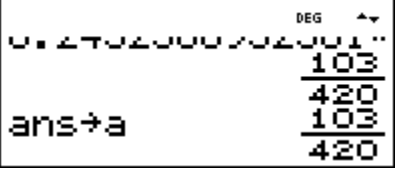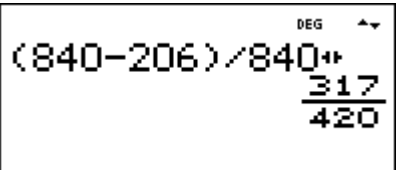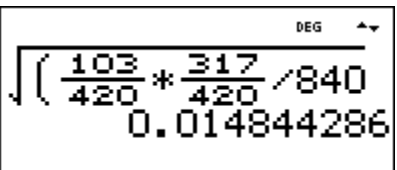$$t = \frac{(x - \mu)}{\sigma}, \text{ if } t > 1.96 \text{---it is significant}$$

Keystrokes:

[data] [2nd] [stat-reg/distr]

TEXAS INSTRUMENTS

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Topic:	Math Concepts:
Sampling and Inference	Sampling and Inference

Example:

1. In a locality containing 18000 families, a sample of 840 families was selected at random. Of these 840 families, 206 families were found to have a monthly income of $Rs.\,250$ or less. It is desired to estimate how many out of 18,000 families have a monthly income of $Rs.\,250$ or less. Within what limits would you place your estimate?
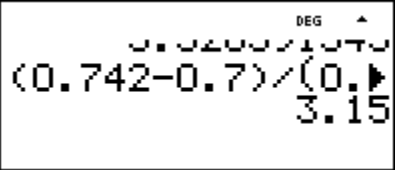
No	Instructions / Keystrokes	Screenshots
1	Define a. Press 2 0 6 ÷ 8 4 0 . Press enter Press ◄► ≈ to convert to fraction. Press 2nd [answer] sto→ x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt}	--- --- 0.245238095 0.2452380952381◄► $\frac{103}{420}$ 0.2452380952381◄► $\frac{103}{420}$ ans→a $\frac{103}{420}$
2	Define b. Press (8 4 0 − 2 0 6) ÷ 8 4 0 . Press enter Press ◄► ≈ to convert to fraction. Press 2nd [answer] sto→ x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt} x_{abcd}^{yzt}	(840−206)/840◄► $\frac{317}{420}$
3	Standard error of population: Press 2nd [√] ((2nd [recall] Scroll ⌄ until a Press enter Press × 2nd [recall] Scroll ⌄ until b ÷ 8 4 0 enter	$\sqrt{\left(\frac{103}{420} * \frac{317}{420}/840\right)}$ 0.01844286

Hence taking $103/420$ or 24.5% to be the estimate of families having a monthly income of $Rs\,250$ or less in the locality, the limits are $(24.5 +- 3\ x\ 1.5)\%$ which is 20% and 29% respectively.

TEXAS INSTRUMENTS

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Computational Analytics

Topic:	Math Concepts:
Sampling and Inference	Sampling and Inference

2. A machinist is making engine parts with axle diameter of $0.7\ inches$. A random sample of 10 parts shows mean diameter $0.842\ inch$ with a standard deviation of $0.04\ inch$. On the basis of this sample, would you say that work is inferior?

Taking the hypothesis that the product is not inferior, there is no significant differences between the means

No	Keystrokes	Screenshots
1	$\mu = 0.7\ ;\ x = 0.742,\ \sigma = 0.040, n = 10$ Press (0 · 7 4 2 − 0 · 7) ÷ (0 · 0 4 ÷ 2nd √ 1 0 − 1)	DEG (0.742−0.7)/(0.▶ 3.15

The value of t $_{0.05}$ = 2.262. Therefore t = 3.15 > 2.262, the value of t is significant. This implies that sample mean differs significantly from μ and the hypothesis is rejected. Hence the work is inferior.

Exercise:

1. 2 A group of boys and girls were given an intelligence test. The mean score, standard deviation and numbers in each group are as follows:

	Mean	SD	N
Girls	121	10	14
Boys	124	12	18

Is the mean score of boys significantly different from the girls?

Answer:

1. No

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