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Statistician Owners Handbook

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Introduction

Congratulations. Your purchase of the new Statistician calculator should give you many years of satisfactory use. You will find it an essential and enjoyable aid in your professional work.

But first you must acquire familiarity with, confidence in, and understanding of the potential of your new calculator. This manual will help. We encourage you to spend all the time you need going through the manual. You would then assure yourself of optimum use and enjoyment.

Your calculator is *dedicated* and *preprogrammed*; really the first of what we call the "Third Generation" of scientific calculators.*

Dedicated to your specific needs, as a professional or student, for handling large amounts of statistical data.

Preprogrammed with a large array of advanced functions such as linear regression, the distributions such as the hypergeometric, chi-square, F and t.

Yet it is very simple to operate. Every feature has been designed to cut down on operating time, to reduce "setting up" to an absolute minimum and to eliminate programming errors and the need to remember complicated input routines.

The manual is divided into five basic sections:

- Fundamental operations (Section (II) such as basic arithmetic, trigonometric and logarithmic functions.
- Operations of the statistical functions (Section III).
- Examples (Section IV) which will help you to learn the kinds of problems that can be solved with your calculator.
- To make your new calculator even more effective, we have included a complete keyboard layout and index (Section I), and an appendix (Section V).

We sincerely hope you enjoy using this calculator as much as we enjoyed designing and building it especially to suit your needs.

*We refer to "Third Generation." The First Generation of scientific calculators represents what we now call "basic scientifics." The Second is represented by the programmables — both card- and keyboard-actuated. The Third, we feel, is actually a major step forward for most applications.

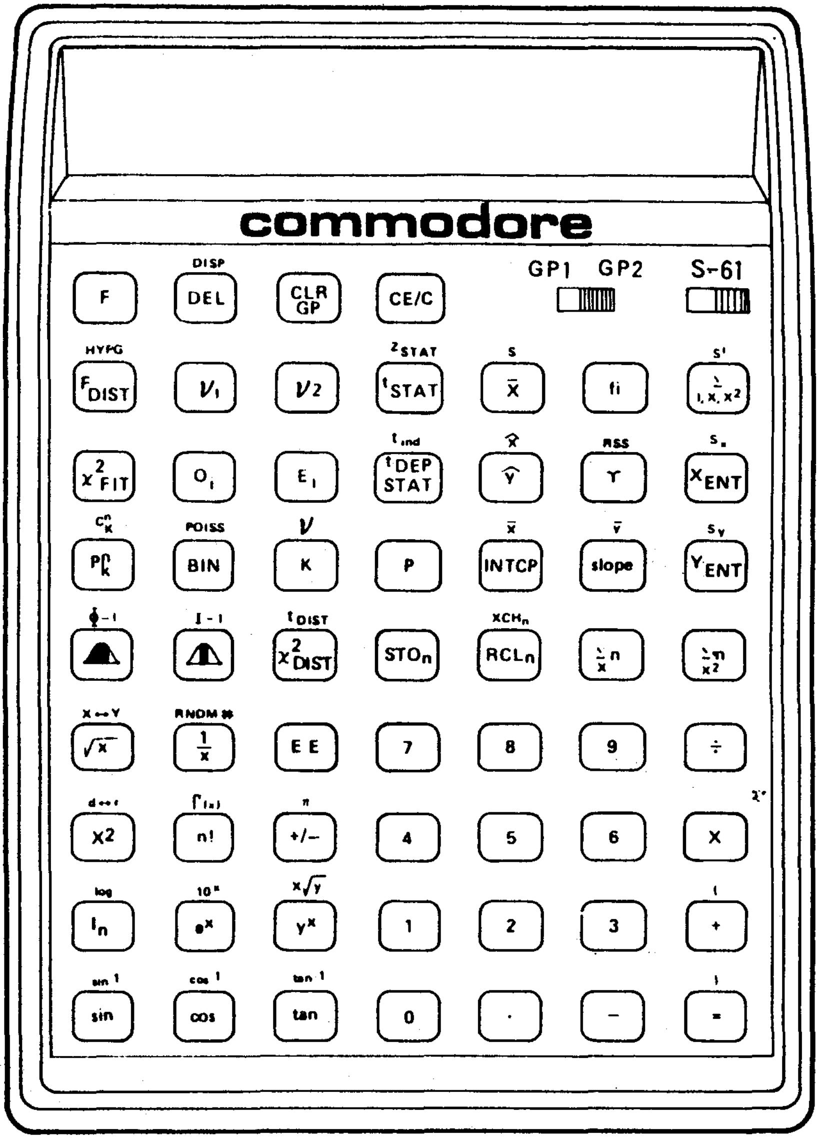


Figure I: Keyboard Layout

I. KEYBOARD LAYOUT AND KEYBOARD INDEX

NOTE: Functions appearing on keytops are designated by inclusion in a box: ☐. Those functions appearing above a keytop – and thus requiring the use of the Function key – appear as: ⌐¬.

KEY	DESCRIPTION	PAGE
ON ▭■ OFF	Power Switch	13
GP_1 GP_2 ■▭	Group Select Switch; registers 1 and 2	13
[CLR GP]	Clear group	15
[CE/C]	Clear entry/clear	15
[EE]	Exponent mode	15
[F]	Upper case key. Selects functions	15

A. Distributions and Other Statistical Functions

KEY	DESCRIPTION	PAGE
[FDIST]	F distribution	36
HYPG	Hypergeometric distribution	35
[ν_1]	Hypergeometric & F distribution degree of freedom No. 1	35, 36
[ν_2]	Hypergeometric & F distribution degree of freedom No. 2	35, 36
[χ^2 FIT]	Chi-square statistic	31
[0_i]	Observed frequency of chi-square	31
[E_i]	Expected frequency of chi-square	31
[P_k^n]	Permutation	33

		PAGE
C_k^n	Combination	33
$\boxed{\text{BIN}}$	Binomial density function	34
POISS	Poisson density function	34
$\boxed{\text{k}}$	Parameter for permutation, combination, binomial and Poisson distributions	33, 34
ν	Degrees of freedom (for Chi square cumulative distribution and t distribution	36, 40
$\boxed{\text{P}}$	Parameter for Hypergeometric and Binomial distributions	34, 35
	Gaussian probability cumulative distribution function	39
Φ^{-1}	Inverse Gaussian probability cumulative distribution function	40
	Gaussian cumulative distribution from $-x$ to x	39
I^{-1}	Inverse Gaussian cumulative distribution from $-x$ to x	40
$\boxed{\chi^2\text{ DIST}}$	Chi-square distribution	40
^{t}DIST	t-distribution	36

B. Mean and Standard Deviation

$\bar{x}$	Mean	27
s	Unbiased standard deviation	27
$\boxed{\text{fi}}$	frequency for grouped data	27
$\boxed{\Sigma\ i, x, x^2}$	For computing mean	27
s'	Biased standard deviation	27

C. Linear Regression

$\boxed{X_{ENT}}$	x entry for linear regression	23
$\boxed{Y_{ENT}}$	y entry for linear regression	23
$\boxed{\text{Slope}}$	the slope of the equation line	23
$\boxed{\text{INTCP}}$	y intercept of the equation line	23
$\boxed{\hat{y}}$	will give fitted value for corresponding x	23
$\hat{x}$	will give fitted value for corresponding y	23
$\boxed{r}$	coefficient of correlation	23
RSS	Residual Sum of Squares	23
Sx	Standard deviation of the x values	23
Sy	Standard deviation of the y values	23
$\bar{y}$	Mean of the y values	23
$\boxed{\bar{x}}$	Mean of the x values	23

D. Memory Functions

$\boxed{STO_n}$	Store display in user memory	20
$\boxed{RCL_n}$	Recall user memory	21
XCH_n	Exchange user memory with display	21
$\boxed{\Sigma n \; x}$	Sum x user memory and display	21
$\boxed{\Sigma n \; x^2}$	Sum x^2 user memory and display	22

E. Basic Functions

$\boxed{+}$ $\boxed{-}$ $\boxed{\div}$ $\boxed{\times}$ $\boxed{=}$ $\boxed{\cdot}$	Arithmetic functions, equals, decimal	15

[0] . . . [9]	Numbers	14
[$\sqrt{x}$]	Square root	17
$x \leftrightarrow y$	x exchange y	19
[1/x]	Reciprocal	17
[x^2]	Square of number	17
$d \leftrightarrow r$	Degree radian	20
[+/–]	Change sign key	15
π	Pi (not specifically described)	
[ln]	Natural log	18
log	Common log	18
[e^x]	Anti natural log	18
10^x	Anti common log	18
[y^x]	Power of a number	19
$x\sqrt{y}$	Root of a number	19
[sin] $\sin^{-1}$ [cos] $\cos^{-1}$ [tan] $\tan^{-1}$	Trigonometric functions and their inverses	18

F. Special Functions

Unique; necessary but rarely found in any personal calculator.

RNDM#	Random number generator	42
[DEL]	Deleter key in linear regression and standard deviation	23, 27
DISP	For fixed decimal point and significant digit	14
[n!]	Factorial	18
$\Gamma(x)$	Gamma Function	19

II. OPERATING INSTRUCTIONS – FUNDAMENTAL

A. Switches

On the upper right hand corner of the machine are two switches. One is the *Power-on* switch. The other is the *Group Select* switch. The Group Select switch chooses the three memory registers which are to be used when the functions indicated below are used.

	Group 1 [GP1]	Group 2 [GP2]
Memory Registers	3, 4, 5	6, 7, 8

The Group Select switch is useful with the following keys:

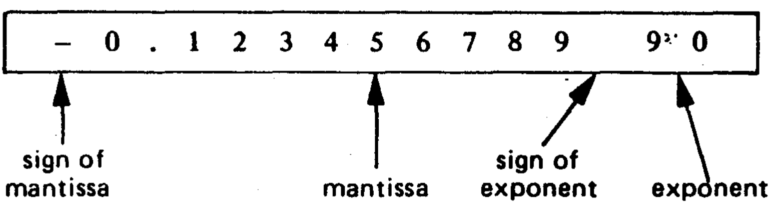
[CLR GP] [fi] [Σ i, x, x^2] [^{t}ind] [^{t}STAT] [^{Z}STAT] [$\bar{X}$] [s]

For more information on the Group Select key, refer to page 27.

B. Display Format

At most, fourteen digits (including signs) can be displayed on your calculator.

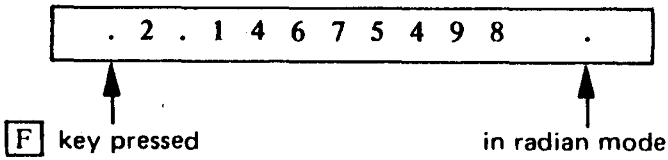
Sample display:



The mantissa is a maximum of ten digits with or without a decimal point. The sign of the mantissa is *positive* if the sign of the mantissa field is blank and *negative* if the sign of the mantissa field contains a "–" sign.
The exponent is a maximum of two digits. The sign of the exponent is *positive* if the sign of the exponent field is blank and *negative* if the sign of the exponent field contains a "–" sign.

Your calculator has two display dot indicators, one to signify radian mode and the other to indicate that the upper case function key [F] has been pressed:

Sample display:



1. Display Key

A special feature on your calculator is the DISP key which enables you to obtain the number of significant digits as well as fixed point. By doing this operation, the number will automatically be rounded off. Supposing we want to round off ***22.5681243*** to four digits. Depress

[F] [DISP] followed by 4 to obtain ***22.57***

on the display. If you switch off the calculator and switch it on again, the display returns to normal operation.
To display an answer to n decimal places

[F] DISP [·] n (where n is an integer between 0 and 9).

2. Busy Signal

Another feature of the calculator is the busy signal which appears at the left corner of the display. The busy signal is a dash which appears when a computation is being carried out. Depressing any key while the busy signal is showing does not affect the computation.

3. Error Display

If an improper operation is carried out, the word ERROR will appear on your display. To clear the ERROR display, press [C/CE]

C. Numerical Entry

1. Enter a positive number by pressing the digit keys in order, from left to right. When not entered, the decimal point is assumed to be to the right of the least significant digit, which is the last number entered.

2. *Enter negative numbers,* by entering a positive number and then depressing [+/–].

3. *Enter exponents* by entering the mantissa (maximum 10 digits) and then depressing [EE] and entering the exponent number (maximum 2 digits). To enter in a negative exponent, depress [+/–] after entering the exponent number.

D. Function Key

The [F] Key is depressed when an upper case function is required (functions above key tops).

E. Clearing

1. *Clear erroneous entry,* while keeping prior numerical entries intact by pressing [C/CE] once.

Example: 4 [÷] 2 [C/CE] 4 = 1

Pressing [C/CE] once clears the display.

2. *Clear a calculation* and allow for the entering of another calculation by depressing [C/CE] twice successively.

3. *Clear the memory registers* by depressing [CLR GP] with the group select switch on 1 (this clears memory registers 3, 4, 5) and on 2 (memory registers 6, 7, 8 are cleared.) To clear memory register 1, enter 0 [STO_n] 1. Memory register 2 is cleared similarly.

4. *Clear both the display and the memory registers* by switching off the power and switching it on again.

F. Simple Arithmetic

Four functions – [+] [–] [x] [÷]

To perform simple addition, subtraction, multiplication or division, simply enter as the problem appears: Example x + y + z

Key Entry	Display	Explanation
x	x	
$\boxed{+}$	x	for simple addition*
y	y	
$\boxed{+}$	x + y	
z	z	
$\boxed{=}$	x + y + z	

*For simple subtraction, multiplication or division, simply press the required key (i.e., $\boxed{-}$, $\boxed{\times}$, or $\boxed{\div}$)

NOTE: The $\boxed{=}$ key presents the final answer. There is no need to enter the $\boxed{=}$ key after the first operation since the result is displayed after the function key is depressed.

G. Chained Calculations

Chained calculations involving several operations such as the calculation of the sum of products or the product of sums can be carried out by using parentheses or memory. For an example of chained calculation involving parentheses, refer to page 17. Simple chaining can be carried out as follows:

Example: $\frac{x \times y}{z}$

Key Entry	Display
x $\boxed{\times}$	x
y	y
$\boxed{\div}$	$x \times y$
z	$\frac{x \times y}{z}$
$\boxed{\div}$ w	$\frac{x \times y}{z} \div w$

NOTE: Chaining can be carried out with most functions although it is not available for certain statistical functions.

H. Parentheses

The use of parentheses is very important in chained calculations because parentheses allow the user to enter the equation exactly as it is written. To illustrate this point, the following example is provided.

Example: $2 \times (5 + 3) =$

Enter as follows:

Key Entry	Display	Explanation
2 $\boxed{x}$	2	
$\boxed{F}$ (	2	
5 $\boxed{+}$	5	
3	3	
$\boxed{F}$)	8	5 + 3
$\boxed{=}$	16	2 x (5 + 3)

NOTE: Parentheses may not be available for some statistical functions.

I. Single Function Keys

1. Finding square of numbers

To find the square of a number, enter the number, then depress $\boxed{x^2}$

2. Finding square root of numbers

To obtain the square root of a number, enter the number, then depress $\boxed{F}$ $\sqrt{x}$.

NOTE: Valid for $x > 0$.

3. Finding reciprocal of numbers

The reciprocal of a number can be obtained by entering in the number and then depressing the key $\boxed{1/x}$

NOTE: Not valid for $x = 0$.

4. Finding natural logarithm of numbers

To find the natural logarithm of a number, enter the number, then depress $\boxed{\ln}$.

NOTE: $x > 0$.

5. Finding e to the power x

To obtain the e to the power x of a number, enter the number, then depress $\boxed{e^x}$.

6. Finding common logarithm of numbers

The common logarithm of a number can be obtained by entering in the number and then depressing $\boxed{F}$ log.

NOTE: $x > 0$.

7. Finding common antilog of numbers

To calculate the common antilog of a number, enter the number, then key in $\boxed{F}$ 10^x.

8. Finding Trigonometric Functions

To find the *sine* of a number in degrees, enter the number and then depress $\boxed{\sin}$. The *cosine* and *tangent* can be obtained similarly by depressing respectively, $\boxed{\cos}$ or $\boxed{\tan}$.

NOTE. If you want to calculate the sine *of a number in radians, the calculator has to be set in the radian mode by depressing* $\boxed{F}$ d↔r *and then entering in the number, followed by* $\boxed{\sin}$. *The* cosine *and* tangent *can be found similarly.*

To find the inverse sine of a number, enter the number, then depress $\boxed{F}$ $\sin^{-1}$. The inverse of the cosine and tangent can be obtained similarly by depressing, respectively, $\boxed{F}$ $\cos^{-1}$ or $\boxed{F}$ $\tan^{-1}$.

NOTE: Inverse sine and cosine $\leqslant 1$. ¶ Also tan 90° or tan $\pi/2$ is invalid.

9. Finding factorials

To obtain the factorial of an integer on display, press $\boxed{n!}$.

NOTE: $\boxed{n!}$ *is obtained if $n < 69$.*

If the factorial of a non integer number is attempted the display will show "Error".

10. Finding gamma of x

To obtain the gamma function of x, enter x followed by $\boxed{F}$ $\Gamma(x)$.

J. Double Functions

1. Finding y to the power x

To raise a positive number to any power, enter as follows:

Key Entry	Display
y	y
$\boxed{y^x}$	y
x	x
$\boxed{=}$	y^x

NOTE: x can be an integer or a decimal, negative or positive.

2. Finding y to the root x

To obtain the root x, of any positive number y, enter as follows:

Key Entry	Display
y	y
$\boxed{F}$ $\sqrt[x]{y}$	y
x	x
$\boxed{=}$	$\sqrt[x]{y}$

NOTE: x can be negative or positive, an integer or a decimal. However, y is only positive.

3. Using the Exchange Register Key

The exchange key, $x \leftrightarrow y$ reverses the order of the operands. For instance, $x \div y$ will become $y \div x$. The exchange key can be used as followed:

Key Entry	Display
x	x
[÷]	x
y	y
[F] x↔y	x
[=]	y ÷ x

NOTE: You can use the exchange register key for the following operations: division, subtraction, power and root.

K. Degree/Radian Conversions and Modes

When you require either a degree/radian conversion or a change of degree/radian mode, press:

[F] d↔r

Pressing the above will both do the conversion and reset the mode. In other words, if your calculation is in degree mode and [F] d↔r is pressed, a degree-to-radian conversion is done and your calculator is put in *radian mode*. Likewise, if your calculator is in radian mode and [F] d↔r is pressed, a radian-to-degree conversion is done and your calculator is put in *degree mode*.

NOTE: Rules for determining your calculator's mode are: When turned on, your calculator is initially in degree mode. ¶ *If there is a decimal point in the exponent field of the display, your calculator is in radian mode. If not, your calculator is in degree mode.*

L. User Memories

There are a maximum of eight memories available to the user. The eight memories will be referenced to as registers from 1 to 8. All 8 memories may not be available to the user when certain advanced mathematical functions are being evaluated. (Refer to page 22). Many of the problems presented provide excellent use of the memory registers.

1. Storing the Display in User Memory

For storing a number on display in a memory, simply depress [STO_n] followed by an arbitrary number from 1 to 8 (these are the 8 memory registers available to the user).

For instance, if we want to store 234 in register 2, simply enter 234, then depress $\boxed{STO_n}$ 2.

2. *Recalling the Quantity Stored in User Memory*

For recalling a value stored in a memory register, simply depress $\boxed{RCL_n}$ followed by the memory register (number 1 to 8) in which the value is stored. *For instance,* if we want to recall the value stored in register 2, simply depress $\boxed{RCL_n}$ 2. Value obtained on the display is 234. (Refer to example above.)

3. *Exchanging User Memory and Display*

A very important operation available in the Statistician is the exchange in memory key $\boxed{XCH_n}$ combining the effects of storing a new value and recalling the value stored earlier in one single step. To show how the $\boxed{XCH_n}$ key is used, an example is presented below:

Key Entry	Display	Explanation
5 $\boxed{STO_n}$ 1	5	5 in register 1
150 $\boxed{\div}$	150	
25	25	
$\boxed{+}$	6	150 ÷ 25
$\boxed{F}$ $\underline{XCH_n}$ 1	5	6 in register 1 (new number)
$\boxed{=}$	11	6 + 5
$\boxed{RCL_n}$ 1	6	

4. *Using the sum X and X^2 user memories and displays*

To *add a* to the quantity present in memory register 1, press *a* $\boxed{\Sigma n \atop x}$ 1 followed by $\boxed{RCL_n}$ 1.

To *subtract a* from the quantity present in memory register 1, press *a* $\boxed{+/-}$ $\boxed{\Sigma n \atop x}$ 1 followed by $\boxed{RCL_n}$ 1.

To add a^2 to the quantity present in memory register 1, depress a $\boxed{\begin{matrix}\Sigma n \\ x^2\end{matrix}}$ 1 followed by $\boxed{\text{RCLn}}$ 1.

NOTE: The same operations can be carried out using any of the 8 memory registers.

5. User Memory Register Limitations

Not all user memory registers are available under certain conditions. The table below provides the list of the functions in which the memory registers are used in the computation. For some computations, the memory registers are not available because the data base is stored in these registers.

Function	Memory Register Used
Linear Regression	3, 4, 5, 6, 7, 8
Mean and Standard Deviation	Group 1 – 3, 4, 5
	Group 2 – 6, 7, 8
Chi-Square Statistic	8
Combination	8
Binomial Density Function	7, 8
Hypergeometric	5, 6, 7, 8
t-Distribution	1, 2
F-Distribution	1, 2

III. OPERATION INSTRUCTIONS – STATISTICAL

A. Linear Regression

Before data is entered for a linear regression, make sure memory registers 3, 4, 5, 6, 7, and 8 are cleared by depressing $\boxed{\begin{matrix}\text{CLR}\\\text{GP}\end{matrix}}$ with the switch on [GP1] and then on [GP2]. To enter the data, the x value is entered first followed by $\boxed{\text{XENT}}$. The y value is entered and $\boxed{\text{YENT}}$ is depressed. The display will now show the number of pairs of points entered at this time.

NOTE: The data must be entered in pairs with the x value first. Unwanted pairs of points can be deleted by depressing $\boxed{\text{DEL}}$, entering the x value and depressing $\boxed{\text{XENT}}$ and then the y value and depressing $\boxed{\text{YENT}}$. The display will show the number of valid points in the machine.

A very powerful feature of your calculator is its ability to *preserve the data base.* This allows the user to do a linear regression and calculate the relevant parameters, then remove certain points from the data base by using $\boxed{\text{DEL}}$ if desired, or continue to add more points. Performing linear regression calculations does *not* destroy the data base.

The following quantities are stored in the registers specified:

Memory Register	Quantity
3	$\sum_{i=1}^{N} i$
4	$\sum_{i=1}^{N} x_i$
5	$\sum_{i=1}^{N} x^2_i$

Memory Register	Quantity
6	$\sum_{i=1}^{N} x_i y_i$
7	$\sum_{i=1}^{N} y_i$
8	$\sum_{i=1}^{N} y^2{}_i$

Supposing we have a set of points (x_i, y_i) with which we want to fit a straight line

$$y = \alpha + \beta x$$

x	3	4	6	7
y	5	8	10	13

We want to calculate:

1) the slope b (best estimate of β)

2) the intercept a (best estimate of α)

3) the residual sum of squares, RSS

$$\text{where RSS} = \sum_{i=1}^{N} [y_i - (\alpha + \beta x_i)]^2$$

4) the coefficient of correlation,

$$\text{where } r = \frac{N \sum_{i=1}^{N} x_i y_i - (\sum_{i=1}^{N} x_i)(\sum_{i=1}^{N} y_i)}{\sqrt{[N \sum_{i=1}^{N} x_i^2 - (\sum_{i=1}^{N} x_i)^2]\,[N \sum_{i=1}^{N} y_i - (\sum_{i=1}^{N} y_i)^2]}}$$

$0 \leqslant r^2 \leqslant 1$

$r^2 = 1$ is perfect fit

5) The mean of the x values,

$$\text{where } \bar{x} = \frac{\sum_{i=1}^{N} x_i}{N}$$

6) The mean of the y values,

$$\text{where } \bar{y} = \frac{\sum_{i=1}^{N} y_i}{N}$$

7) fitted value of y for a corresponding x,

$$\text{where } \hat{y} = \alpha + \beta x$$

8) fitted value of x for a corresponding y,

$$\text{where } \hat{x} = \frac{y - \alpha}{\beta}$$

9) standard deviation of the x values,

$$\text{where } Sx = \sqrt{\frac{\Sigma x^2{}_i - N\bar{x}^2}{N-1}}$$

10) standard deviation of the y values,

$$\text{where } Sy = \sqrt{\frac{\Sigma y^2{}_i - N\bar{y}^2}{N-1}}$$

then the key sequence is as follows:

Key Entry	Display	Explanation
3	3	x value first
XENT	3	
5	5	
YENT	1	number of paired values entered
4	4	
XENT	4	

Key Entry	Display	Explanation
8	8	
yENT	2	
6	6	
xENT	6	
10	10	
yENT	3	
7	7	
xENT	7	
13	13	
yENT	4	
SLOPE	1.8	$\underline{\beta}$
INTCP	0	$\underline{\alpha}$
F RSS	1.6	Residual sum of squares
r	9.7618706 -01	Coefficient of correlation
F $\bar{x}$	5	Mean of the x values
F $\bar{y}$	9	Mean of the y values
8 $\hat{y}$	14.4	fitted value of y
11 F $\hat{x}$	6.11111...	fitted value of x for a corresponding y
F Sx	1.825741858	standard deviation of x values
F Sy	3.366501646	standard deviation of y values

NOTE: The linear regression calculations use memory registers 3 through 8. However, memory registers 1 and 2 *are not used here and* are available *to the user.* ¶ *Slope- and Intercept-chaining and use of parentheses is available. RSS- and r-chaining is allowed but use of parentheses is not allowed.*

B. Mean and Standard Deviation (Grouped and Ungrouped Data)

The calculator has a special feature which allows you to perform calculations with two sets of data independently by using group 1 for one set and group 2 for the second set.

Before data is entered for a mean and standard deviation calculation, make sure that the relevant data registers are cleared by depressing $\boxed{\begin{matrix}\text{CLR}\\ \text{GP}\end{matrix}}$. *Ungrouped data* is entered by entering the number and depressing $\boxed{\begin{matrix}\Sigma\\ i, x, x^2\end{matrix}}$. *Grouped data* is entered by first entering the frequency and depressing $\boxed{\text{fi}}$. The xi value is then entered and $\boxed{\begin{matrix}\Sigma\\ i, x, x^2\end{matrix}}$ is depressed.

Mixed calculation can be done as grouped and ungrouped. This is because fi for ungrouped data is taken to be 1. Unwanted points can be removed by depressing $\boxed{\text{DEL}}$ (deleted), entering the frequency (for grouped data), depressing $\boxed{\text{fi}}$, then entering the x value and depressing $\boxed{\begin{matrix}\Sigma\\ i, x, x^2\end{matrix}}$.

NOTE: If the data point to be deleted is not part of grouped data, it is not necessary to depress $\boxed{\text{fi}}$. *¶ As noted before, your calculator has the ability to* preserve the data base. ¶ *Depressing the* $\boxed{\bar{x}}$ *or the* $\boxed{\text{F}}$ $\underline{s}$ *or the* $\boxed{\text{F}}$ $\underline{s'}$ *to do a calculation does not destroy this data base.*

1. *The following quantities are stored in the memory registers specified:*

Quantity	Group 1 Memory Register	Group 2 Memory Register
$\sum_{i=1}^{N} f_i$	3	6
$\sum_{i=1}^{N} f_i x_i$	4	7
$\sum_{i=1}^{N} f_i x^2_i$	5	8

NOTE: $f_i = 1$ for Ungrouped data

f_i has to be entered first for grouped data.

2. *The sample mean, $\overline{x}$, is evaluated using*

$$\overline{x} = \frac{\sum_{i=1}^{N} f_i x_i}{\sum_{i=1}^{N} f_i}$$

where $f_i = 1$ for ungrouped data

3. *The standard deviation for a sample is evaluated using*

$$s = \sqrt{\frac{\sum_{i=1}^{N} f_i x^2{}_i - N\overline{x}^2}{\sum_{i=1}^{N} f_i - 1}}$$

NOTE: This is the unbiased *estimate of the standard deviation (s).*

4. *The standard deviation for a population, s′, is obtained by evaluating*

$$s' = \sqrt{\frac{\sum_{i=1}^{N} f_i x^2{}_i - N\overline{x}^2}{\sum_{i=1}^{N} f_i}}$$

5. *The standard error is given by* $S_{\overline{x}} = S/\sqrt{\Sigma f_i}$

Suppose we are given a set of numbers $\{3.5, 3.77, 3.4, 3.4, 3.9\}$, and are to evaluate their mean and standard deviation. The key sequence is as follows:

Key Entry	Display	Explanation
3.5	3.5	
$\boxed{\Sigma \; i, x, x^2}$	3.5	
3.77	3.77	
$\boxed{\Sigma \; i, x, x^2}$	3.77	
3.4	3.4	
$\boxed{\Sigma \; i, x, x^2}$	3.4	
$\boxed{\Sigma \; i, x, x^2}$	3.4	
3.9	3.9	
$\boxed{\Sigma \; i, x, x^2}$	3.9	
$\boxed{F}$ $\bar{x}$	3.594	Mean
$\boxed{F}$ s	2.2842942 -01	Standard deviation (unbiased)
$\boxed{F}$ s′	2.04313484 -01	Standard deviation for a population

Standard error can be obtained by depressing $\boxed{RCL_n}$ 3 for group 1 and $\boxed{RCL_n}$ 6 for group 2 to obtain Σf_i. Then $\boxed{\sqrt{x}}$ is depressed followed by $\boxed{1/x}$ to obtain $1/\sqrt{\Sigma f_i}$. This value can be multiplied to $\boxed{F}$ s or $\boxed{F}$ s′ to obtain Sx or S′x respectively.

NOTE: Chaining and use of parentheses is available. ¶ The mean and standard deviation can be obtained in Group 1 or 2. ¶ N is a positive integer, > 1. ¶ When the standard deviation is small compared with the mean, you may get "ERROR."

C. One-Sample Test Statistics: t statistic, z statistic, $\boxed{^{t}\text{STAT}}$, $\boxed{\text{F}}$ $\underline{^{Z}\text{STAT}}$

1. *To evaluate the t statistic,* enter the data as in mean and standard deviation. Then enter μ and depress ^{t}STAT

$\boxed{^{t}\text{STAT}}$ is evaluated using:

$$\boxed{^{t}\text{STAT} = \frac{\sqrt{N}\,(\bar{x} - \mu)}{s}}$$

2. *Similarly, for evaluating the z statistic,* the data is entered as above. To evaluate the z statistic, enter the data and then enter σ and depress $\boxed{\text{K}}$. Then enter μ and depress $\boxed{\text{F}}$ $\underline{^{Z}\text{STAT}}$.

$\boxed{^{Z}\text{STAT}}$ is evaluated using:

$$\boxed{^{Z}\text{STAT} = \frac{\sqrt{N}\,(\bar{x} - \mu)}{\sigma}}$$

NOTE: Chaining and parentheses for t statistic are available. However, parentheses are not available for z statistic, although chaining is.

D. Two-Sample Statistics: t dependent and t independent statistics, $\boxed{\begin{matrix}^{t}\text{DEP}\\ \text{STAT}\end{matrix}}$, $\boxed{\text{F}}$ $\underline{^{t}\text{IND}}$

1. *t dependent statistic*

Given a set of paired observations (x_i, y_i) from two populations, ^{t}DEP STAT is evaluated using:

$$\boxed{^{t}\text{DEP STAT} = \frac{\bar{x} - \bar{y}}{\sqrt{\dfrac{S_x{}^2 - S_y{}^2 - 2r\, S_x S_y}{N}}}}$$

where the symbols are the same as the symbols in the section on linear regression (refer to page 25).

To obtain ^{t}DEP STAT, data is first entered as in linear regression (using ^{X}ENT and ^{Y}ENT) and then $\boxed{\begin{matrix}^{t}\text{DEP}\\ \text{STAT}\end{matrix}}$ is depressed.

2. *t independent statistic*

To test two sets of *independent* random samples having means μ_1, μ_2 (unknown) and the *same* unknown variance σ^2, the t_{ind} statistic is evaluated. The formula for the t_{IND} statistic is

$$t_{IND} = \frac{\bar{x} - \bar{y} - (\mu_1 - \mu_2)}{\sqrt{\left(\frac{1}{N_1} + \frac{1}{N_2}\right)\left(\frac{\Sigma x_i^2 - N_1\bar{x}^2 + y^2{}_i - N_2\bar{y}^2}{N_1 + N_2 - 2}\right)}}$$

The x data is first entered with the switch turned to [GP1] and using $\boxed{\begin{matrix}\Sigma\\ i,x,x^2\end{matrix}}$

The y data is then entered using the switch turned to [GP2] and using $\boxed{\begin{matrix}\Sigma\\ i,x,x^2\end{matrix}}$

Enter $(\mu_1 - \mu_2)$ into the machine and depress $\boxed{F}$ t_{IND} to evaluate the t_{IND} statistic.

NOTE: Make sure that the registers are clear by depressing $\boxed{\begin{matrix}\text{CLR}\\ \text{GP}\end{matrix}}$ *before commencing to enter data.* ¶ *You can do chaining, but you cannot use parentheses for both* $\boxed{\begin{matrix}t\text{dep}\\ \text{STAT}\end{matrix}}$ *and t_{IND} STAT.* ¶ See the section on mean and standard deviation for entry of data.

E. Chi-Square Statistic: Goodness of fit, $\boxed{\chi^2 \text{FIT}}$, $\boxed{O_i}$, $\boxed{E_i}$

The data is in pairs of (O_i, E_i) where O_i is the observed frequency and E_i the expected frequency. The χ^2 statistic is evaluated using the equation

$$\chi^2 = \sum_{i=1}^{N} \frac{(O_i - E_i)^2}{E_i}$$

Register number 8 is used in this function and so must be cleared before entering the numbers. To find the value of Chi-Square statistic, enter as follows:

Key Entry	Display	Explanation
0 [STOn] 8	0	To clear register 8
0_i [0_i]	0_i	Value of 0_i
E_i [E_i]	E_i	Value of E_i
To remove erroneous data:		
DEL 0_i [0_i]	0_i	Value of 0_i
E_i [E_i]	E_i	Value of E_i
[x^2 FIT]	–	x^2 FIT statistic

NOTE: Data base preserved in memory register 8. You can do chaining and use parentheses.

1. Contingency Tables

The 2 x 2 contingency table can be obtained by the formula,

$$\boxed{x^2 = \frac{n\,(f_{11}f_{22} - f_{12}f_{21})^2}{C_1C_2R_1R_2}}$$

where f_{ij} = observed frequency in category i and j

C_j = Column total

R_i = Row total

The statistic can be tested using the x^2 distribution.

To obtain an n x m contingency table, find the expected frequency by using the formula,

$$\boxed{F_{ij} = \frac{(R_i)\,(C_j)}{n}}$$

Then χ^2 goodness of fit can be obtained by entering the observed frequency (f_{ij}) followed by $\boxed{0_i}$. The expected frequency (F_{ij}) can be entered next followed by $\boxed{E_i}$. Then enter $\boxed{\chi^2 FIT}$. The χ^2 statistic can be tested using the χ^2 distribution.
Refer to Example G, page 64.

F. Permutation and Combinations, Binomial, Poisson and Hypergeometric Density Functions

1. Permutation and Combinations $\boxed{P_k^n}$, $\boxed{F}$ C_k^n

These are evaluated using

$$\boxed{P_k^n = \frac{n!}{(n-k)!}}$$

$$\boxed{C_k^n = \frac{n!}{(n-k)!\,k!}}$$

where n, k are integers and $0 \leq k \leq n$

To find P_k^n, enter as follows:

Key Entry	Display
k $\boxed{k}$	k
n	n
$\boxed{P_k^n}$	P_k^n value

To find C_k^n, enter as follows:

Key Entry	Display
k $\boxed{k}$	k
n	n
$\boxed{F}$ C_k^n	C_k^n

NOTE: In the computation of C_k^n, Memory Register 8 is not available to the user. ¶ Chaining and parentheses are not available for both P_k^n and C_k^n.

2. *Binomial Density Function* BIN, P, k

The Binomial density function is evaluated using

$$\boxed{BIN(k) = C_k^n \cdot p^k \cdot q^{n-k}} \qquad q = 1 - p$$

where n is a positive integer and $0 < p < 1$ and $k = 0, 1, 2, \ldots n$

To evaluate the binomial probability mass function, enter as follows:

Key Entry	Display	Explanation
p P	p	
k k	k	
n	n	
BIN	—	BIN (k)

NOTE: Memory registers 7 and 8 are used in the calculation above and thus are not available for the user. ¶ Chaining and parentheses are not available. ¶ For Cumulative Binomial distribution, refer to page 93.

3. *Poisson Density Function* F POISS

The poisson probability mass function is evaluated using

$$\boxed{POISS(k) = \frac{\lambda^k e^{-\lambda}}{k!}} \qquad \text{where } \lambda > 0 \text{ and } k = 0, 1, 2, \ldots$$

To obtain the Poisson probability mass function, enter as follows:

Key Entry	Display	Explanation
k k	k	
λ		
F POISS	—	POISS (k)

NOTE: For Poisson Cumulative distribution, *refer to page* ¶ *Chaining and parentheses are not available.*

4. *Hypergeometric Distribution* $\boxed{\mathrm{F}}$ $\underbrace{\mathrm{HYPG}}$, $\boxed{\nu_1}$, $\boxed{\nu_2}$, $\boxed{\mathrm{p}}$

The parameters for the hypergeometric distribution are ν_1, ν_2, p, and n. The Hypergeometric probability mass function is given by

$$\boxed{\mathrm{HYPG} = \frac{C_p^{\nu_1} \cdot C_{n-p}^{\nu_2}}{C_n^{\nu_1+\nu_2}}}$$

where ν_1, ν_2, n are positive integers $p \leqslant \nu_1$, $n - p \leqslant \nu_2$ and $p = 0, 1, 2, \ldots n$

To find hypergeometric distribution, enter as follows:

Key Entry	Display	Explanation
ν_1 $\boxed{\nu_1}$	ν_1	
ν_2 $\boxed{\nu_2}$	ν_2	
p $\boxed{\mathrm{p}}$	p	
n	n	
$\boxed{\mathrm{F}}$ $\underbrace{\mathrm{HYPG}}$	–	HYPG probability mass function

NOTE: ν_1, ν_2 may be entered in any order. ¶ Memory Registers 5, 6, 7 and 8 are used in the calculation and hence are not available to the user. ¶ Chaining and parentheses are not available. ¶ In some books, the hypergeometric distribution is also defined as follows:

$$\boxed{\mathrm{HYPG} = \frac{C_d^D \, C_{n-d}^{N-D}}{C_n^N}}$$

where $d < D$
$n < N$

and $d < n$
$D < N$

- $N = \nu_1 + \nu_2$
  $D = \nu_1$
  $d = p$
  $n = n$

G. t-Distribution [F] ^{t}DIST , [F] ν

This function evaluates the integral for the t-distribution using

$$^t\text{DIST}(x,\nu) = \int_{-x}^{x} \frac{\Gamma\left(\frac{\nu+1}{2}\right)\left(1+\frac{y^2}{\nu}\right)^{-\frac{\nu+1}{2}}}{\sqrt{\pi\nu}\;\Gamma\left(\frac{\nu}{2}\right)}\,dy$$

where $x > 0$ and ν is the degrees of freedom.

To obtain required interval for t-distribution, enter as follows:

Key Entry	Display	Explanation
ν [F] ν	ν	degrees of freedom
x	x	
[F] ^{t}DIST	—	^{t}DIST (x, ν)

NOTE: Memory Registers 1 and 2 are used in this calculation and hence are not available for user. ¶ As a result of the calculation, depressing the [F] ^{t}DIST converts it to radian mode. ¶ Chaining and parentheses are not available.

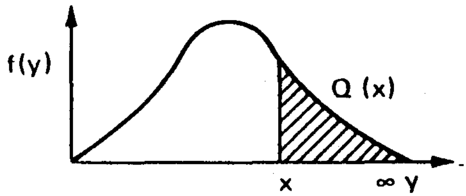
H. F Distribution Function [^{F}DIST], [ν_1], [ν_2]

The integral of the F distribution with degrees of freedom ν_1, ν_2 is evaluated using:

$$Q(x;\nu_1,\nu_2) = \int_{x}^{\infty} \frac{\Gamma\left(\frac{\nu_1+\nu_2}{2}\right) y^{\frac{\nu_1}{2}-1}\left(\frac{\nu_1}{\nu_2}\right)^{\frac{\nu_1}{2}}}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)\left(1+\frac{\nu_1}{\nu_2}y\right)^{\frac{\nu_1+\nu_2}{2}}}\,dy$$

where $x > 0$ and ν_1, ν_2 are positive integers

$f(y)$ = probability density of the F distribution



To obtain the F distribution, enter as follows:

Key Entry	Display	Explanation
ν_1 $\boxed{\nu_1}$	ν_1	
ν_2 $\boxed{\nu_2}$	ν_2	
x	x	
$\boxed{\text{FDIST}}$	--	$Q\,(x;\ \nu_1,\nu_2)$

NOTE: Memory Registers 1 and 2 are used in this calculation and hence are not available for the user. ¶ As a result of the calculation, depressing $\boxed{\text{FDIST}}$ *converts the calculator to a radian mode. ¶ Chaining and parentheses are not available.*

I. Binomial Cumulative Distribution

The Binomial Cumulative Distribution can be obtained by using the following relationship:

$$\boxed{\sum_{s=a}^{n} C_s^n \cdot p^s\,(1-p)^{n-s} = 1 - P\,(F|\nu_1,\nu_2)}$$

where $\nu_1 = 2\,(n-a+1)$, $\nu_2 = 2\,(a)$,

and $F = \dfrac{a-ap}{(n-a+1)\,p}$, also $\underline{a = k+1}$

To obtain the Binomial Cumulative Distribution where n = 10, k = 4 and p = 0.65; enter as follows:

Key Entry	Display	Explanation
10	10	
$\boxed{-}$	10	
5	5	k + 1 = a
$\boxed{+}$	5	
1	1	
$\boxed{\times}$	6	
.65	.65	
$\boxed{=}$	3.9	
$\boxed{1/x}$	2.56410256 -01	
$\boxed{X}$	2.56410256 -01	
5	5	
$\boxed{\times}$	1.282051282	
·35	0.35	
$\boxed{=}$	4.487179487-01	
$\boxed{\text{STOn}}$ 1	4.487179487-01	F stored in memory register 1.
2 $\boxed{\times}$	2	
5	5	
$\boxed{=}$	10	ν_2
$\boxed{\text{STOn}}$ 2	10	
$\boxed{-}$	10	
5	5	
$\boxed{+}$	5	
1	1	
$\boxed{\times}$	6	
2	2	
$\boxed{=}$	12	ν_1

Key Entry	Display	Explanation
[ν_1]	12	enter ν_1
[RCLn] 2	10	
[ν_2]	10	
[RCLn] 1	4.487179487 -01	
[FDIST]	9.050659198 -01	
[+/−]	- 9.050659198 -01	
[+] 1	1	
[=]	9.49340817 -02	

∴ the probability is 0.09 to 2 decimal places

J. Gaussian Probability Distribution , 

The Gaussian probability cumulative distribution function (Φ) is evaluated using

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}}\, e^{\frac{-y^2}{2}}\, dy \qquad \text{where } -\infty < x < \infty.$$

To evaluate Φ (x), enter x then depress 

Another integral which is very useful in statistical work for evaluating the probability of a normally distributed random variable X lying between x and −x is given by

$$I(x) = \int_{-x}^{x} \frac{1}{\sqrt{2\pi}}\, e^{\frac{-y^2}{2}}\, dy \qquad \text{where } x > 0$$

To evaluate I(x), enter x then depress 

NOTE: Chaining and parentheses are not available.

1. Inverse of the Gaussian Probability Distribution $\boxed{\mathrm{F}}$ Φ^{-1}, $\boxed{\mathrm{F}}$ I^{-1}

The inverse of the Gaussian probability cumulative distribution gives the value x such that

$$\Phi = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{\frac{-t^2}{2}} dt$$

To obtain the value x, enter Φ and then depress $\boxed{\mathrm{F}}$ Φ^{-1}.

When dealing with confidence intervals, it is often useful to find the inverse of the integral I (x) defined above, and this is given by the value x, such that

$$I = \int_{-x}^{x} \frac{1}{\sqrt{2\pi}} e^{\frac{-t^2}{2}} dt$$

To obtain the value x enter I then depress $\boxed{\mathrm{F}}$ I^{-1}

NOTE: Entering Φ and I outside the range 0,1 will result in an error. ¶ Chaining is available, but not parentheses, for both I^{-1}, and Φ^{-1}.

K. Chi-Square Probability Cumulative Distribution Function $\boxed{\chi^2\ \mathrm{DIST}}$, $\boxed{\mathrm{F}}$ ν

The χ^2 probability distribution function is evaluated using:

$$P(\chi^2|\nu) = \chi^2(x) = \int_{0}^{x} \frac{1}{2^{\frac{\nu}{2}}\,\Gamma(\frac{\nu}{2})} y^{\frac{\nu}{2}-1} e^{\frac{-y}{2}} dy$$

where $\chi \geq 0$ and ν is the degrees of freedom (ν is positive integer).

Key Entry	Display	Explanation
ν $\boxed{F}$ ν	ν	
x	x	
$\boxed{\chi^2\ \mathrm{DIST}}$	–.	χ^2 (x)

NOTE: Chaining and parentheses are not available.

L. Poisson Cumulative Distribution

The Poisson Cumulative Distribution is obtained by using the following relationship:

$$\sum_{d=0}^{C} \frac{e^{-\lambda}\lambda^{d}}{d!} = 1 - P(\chi^2 | \nu) \quad c + 1 = \frac{\nu}{2} \quad \lambda = \frac{\chi^2}{2}\ \text{(even)}$$

Supposing we want to find the Poisson Cumulative distribution for c = 5 and λ = 2.4. We enter as follows:

Key Entry	Display	Explanation
2.4 $\boxed{\times}$	2.4	
2	2	
$\boxed{=}$	4.8	
$\boxed{\mathrm{STOn}}$ 1	4.8	
5 $\boxed{+}$	5	
1	1	
$\boxed{\times}$	6	
2	2	
$\boxed{=}$	12	degrees of freedom

Key Entry	Display	Explanation
F ν	12	
RCLn 1	4.8	
x^2 DIST	3.56725 . . . -02	x^2 (4.8|12)
+/-	-3.56725 . . . -02	
+ 1	1	
=	9.64327 . . . -01	
F DISP 3	9.64 -01	to 3 significant digits

M. Pseudo Random Number

Enter any number up to 5 digits at random into the machine and then depress F RNDM# successively to get a sequence of pseudo-random numbers.

N. General Curve Fitting

Use transformation for dependent variable y given by

$$W_{(k)} = \frac{y^k - 1}{k}$$

where $0 \leqslant k \leqslant 1$

Since $\lim\limits_{k \to 0} W_{(k)} = \ln y$

k = 1 gives a linear fit and k = 0 gives an exponential curve fit. k = 0:5 gives a quadratic curve fit and so on. Thus the above transformation gives a wide range of general curve fittings ranging from the linear to the exponential case.

Suggested procedure to be followed for practical examples:
Do a linear regression without using a transformation. Find the Residual Sum of Squares (RSS). Pick a value of k between 0 and 1 and use the above transformation for the y values. Find the RSS. Choose the k which gives the smallest RSS. Usually it is sufficient to enter the data only three to four times to get a good value of k. See example J, Section IV.

IV. APPLICATION EXAMPLES

A. Linear Regression Example

The following table shows the ages "x" and systolic blood pressure "y" of 12 women.

1) Determine the least square regression equation of y on x, $y = \alpha + \beta x$.
2) Find the coefficient of correlation between x and y.
3) Find 95% confidence interval for α.
4) Find 95% confidence interval for β.
5) Estimate the blood pressure of a woman whose age is 45 years.
6) Find 95% confidence limits for the blood pressure of a 45 year old woman.

Table I: Systolic pressure of 12 women

Age (x) years											
55	41	71	35	62	46	54	48	37	41	67	59

Blood Pressure (y) mm Hg											
146	124	159	119	145	130	147	144	118	139	151	157

1. To find slope (β) and intercept (α), enter as follows:

Key Entry	Display	Explanation
[CLR GP] GP₁ GP₂ (switch left)	0	clear memory registers 3, 4, and 5
[CLR GP] GP₁ GP₂ (switch right)		clear memory registers 6, 7 and 8 for linear regression
55	55	
[XENT]	55	

Key Entry	Display	Explanation
146	146	
[Y_{ENT}]	1	Have entered 1 pair so far
41	41	
[X_{ENT}]	41	
124	124	
[Y_{ENT}]	2	
.	.	
.	.	Continue sequence
.	.	
59	59	
[X_{ENT}]	59	
157	157	
[Y_{ENT}]	12	
[INTCP]	85.48258813	
[SLOPE]	1.060404127	
[F] DISP 5	1.0604	To obtain 5 significant digits
[INTCP]	85.483	

Therefore, regression equation is $y = 85.483 + 1.0604x$ or *x = 85.483 mmHg and β = 1.0604 mmHg/yr.*

2. To find coefficient of correlation, enter:

Key Entry	Display	Explanation
[r]	8.9456 -01	

The coefficient of correlation between x and y is 0.89456.

3. To find the 95% confidence interval for α, we need to find the standard error of estimate α ($s_{\bar{\alpha}}$)

An estimator of σ^2 is

$$s^2 = \frac{RSS}{n-2}$$

$$s^2_{\bar{\alpha}} = \frac{RSS}{n(n-2)}$$

To obtain $s_{\bar{\alpha}}$, enter as follows:

Key Entry	Display	Explanation
F RSS	435.26	
÷	435.26	
12	12	$n = 12$
÷	36.272	
10	10	$n - 2 = 10$
=	3.6272	$s^2_{\bar{\alpha}}$
$\sqrt{x}$	1.9045	$s_{\bar{\alpha}}$
STOn 1	1.9045	Store value in memory register 1

The confidence interval for $\bar{\alpha}$ is given by:

$\bar{\alpha} \pm t(n-2)(.95)s_{\bar{\alpha}}$ where $t(n-2)(.95)$ is the value of t with $(n-2)$ degrees of freedom.

We need to find x for which ^{t}DIST is 0.95.

We try $t = 2$ and get:

Key Entry	Display	Explanation
10	10	$n-2$
F ν	10	degrees of freedom
2	2	
F ^{t}DIST	9.2661 -01	

Key Entry	Display	Explanation

Since this is smaller than 0.95 try 2.25, this will give:

tDIST	9.5182 -01	

so try 2.21, this will give:

tDIST	9.4844 -01	

try 2.235, this will give:

tDIST	9.5058 -01	

try 2.228, this will give:

tDIST	9.4999 -01	

$\therefore t(10)\,(.95) = 2.228$

Then α lies in the range $85.483 \pm 2.228 \times 1.9045$
(i.e., 85.483 ± 4.2432

α is in the range (81.24, 89.726) with a 95% confidence level.

4. To find the 95% confidence for the β, we need to find the standard error of estimate β, which is given by:

$$S_\beta^2 = \frac{\sigma^2}{\Sigma\,(x_i-\bar{x})^2}$$

and $$S_{\bar{\beta}}^2 = \frac{RSS}{(n-2)\,(n-1)\,S_x^2}$$

where S_x = standard deviation of the x values

To obtain $S_{\bar{\alpha}}$, enter as follows:

Key Entry	Display	Explanation
F RSS	435.26	RSS
÷	435.26	

Key Entry	Display	Explanation
10	11	
$\boxed{\div}$	43.526	
11	11	
$\boxed{\div}$	3.9569	
$\boxed{F}$ s_x	11.873	
$\boxed{x^2}$	140.97	
$\boxed{=}$	2.8069 -02	
$\boxed{\sqrt{x}}$	1.6754 -01	

Then β lies in the range: 1.0604 ± 2.228 x 0.16754.
β is in the range (0.68712, 1.4337) with a 95% confidence.

5. To estimate the blood pressure of a woman whose age is 45 years the key sequence is:

Key Entry	Display	Explanation
45	45	
$\boxed{\hat{y}}$	133.2	

Thus the blood pressure is 133.2mmHg.

6. To find 95% confidence limits for the blood pressure of a 45 year old woman.

The 95% confidence limits are given by:

$$Y_O \pm \frac{t_{(10)}\,(.95)}{\sqrt{n-2}} \quad Sy|x \sqrt{\frac{1}{n} + \frac{x_O - \bar{x}}{S_x^2\,(n-1)}}$$

$$t_{(10)}\,(.95) = 2.228 \text{ and } Sy/x = \sqrt{\frac{RSS}{n-2}}$$

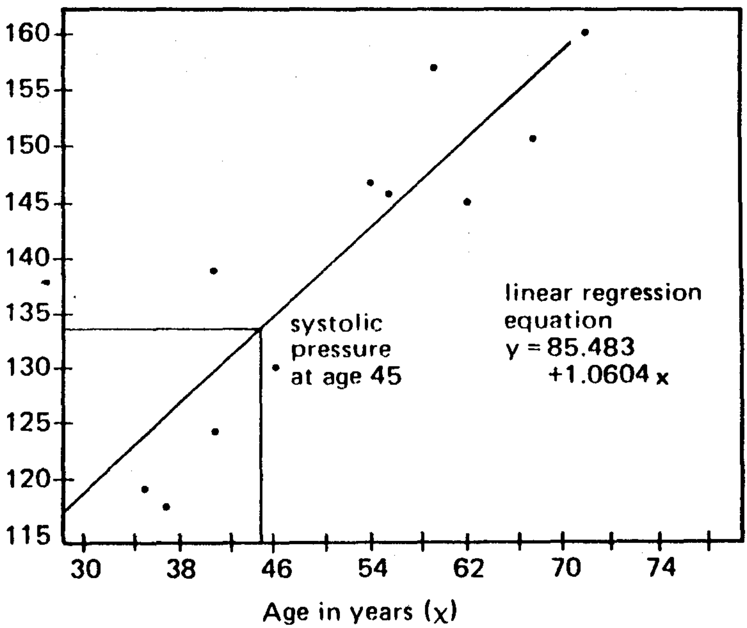
$$\text{If } \Delta Y_0 \triangleq \frac{t_{(10)}(0.95)}{\sqrt{n-2}} \sqrt{\frac{RSS}{n-2}\left[\frac{1}{n} + \frac{(X_0-\bar{X})^2}{S_x^2(n-1)}\right]}$$

Key Entry	Display	Explanation
45	45	
$\boxed{-}$	45	
$\boxed{F}$ $\bar{x}$	51.333	
$\boxed{=}$	–6.333	
$\boxed{x^2}$	40.111	
$\boxed{\div}$	40.111	
$\boxed{F}$ s_x	11.873	
$\boxed{x^2}$	140.97	
$\boxed{\div}$	2.8454 -01	
11	11	
$\boxed{+}$	2.5867 -02	
12	12	
$\boxed{1/x}$	8.3333 -02	
$\boxed{\div}$	1.092 -01	
10	10	
$\boxed{x}$	1.09 02	
$\boxed{F}$ RSS	435.26	

Key Entry	Display	Explanation
$\boxed{\times}$	4.7531	
2.228	2.228	
$\boxed{\div}$	10.590	
10	10	
$\boxed{\sqrt{x}}$	3.1623	
$\boxed{=}$	3.3488	

Thus the 95% confidence limits are given by 132 ± 3.35 or (128.65, 135.35).

Graph I: Systolic Pressure vs. Age in Women



B. One Sample Statistics

1. Mean and Standard Deviation

A physical Education instructor is interested in obtaining information about the heights of twelve-year old boys. To do

this, he collects a sample of data (the heights of twenty randomly selected twelve-year old boys).
[64, 40, 58, 63, 70, 55, 61, 62, 63, 60, 59, 46, 65, 72, 66, 54, 57, 65, 60, 59]

Find the mean and standard deviation of the sample.

Key Entry	Display	Explanation
CLR GP		Clear the registers before starting calculations.
GP_1 or GP_2		
64	64	
Σ i, x, x^2	64	
.	.	(continue entering data of 20 heights, alternating with depression of mean-and-standard deviation entry key)
.	.	
.	.	
.	.	
59	59	
Σ i, x, x^2	59	
$\bar{X}$	59.95	
F S	7,401813434	
F DISP .3	7.402	Set the mode to 3 decimal points

The mean is 59.95 and the standard deviation is 7.402.

2. *Find the 95% confidence interval for the mean.*

To do this we need the value of t with degree of freedom = 20 – 1 = 19 which gives a probability of 0.95.

try t = 2,

The key sequence is as follows:

Key Entry	Display	Explanation
19	19	
$\boxed{F}$ ν	19.000	3 decimal points
2	2	
$\boxed{F}$ ^{t}DIST	0.940	try 2.1
19	19	
$\boxed{F}$ ν	19.000	
2.1	2.1	
$\boxed{F}$ ^{t}DIST	0.951	try 2.088
19	19	
$\boxed{F}$ ν	19.000	
2.088	2.088	
$\boxed{F}$ ^{t}DIST	0.950	

Therefore t (95%) = 2.088.

Then the 95% confidence intervals for μ, the mean of the heights of the twelve-year olds is given by

$$\frac{\overline{X} \pm 2.088 \times s}{\sqrt{n}}$$

or 59.95 ± 3.456

or $(56.49\ ,\ 63.40)$ (to 4 significant figures)

Thus from the data gathered, the instructor is 95% confident that the true mean lies in the interval 56.49–63.41 inches.

3. Confidence Interval estimates for σ^2

Find the 95% confidence interval for the variance. (For the data above.) To do this we need the values of χ^2 DIST with degree of freedom = 20–1 = 19 which gives $\chi^2_{19} = 2.5 \times 10^{-2}$

and $\chi^2_{19} = 0.975$

try $\chi^2 = 2$

The key sequence is as follows:

Key Entry	Display	Explanation
19	19	
F ν	19.000	
2	2	
χ^2 DIST	0.000	
19	19	
F ν	19.000	
8	8	Try $\chi^2 = 8$
χ^2 DIST	0.013	
19	19	
F ν	19.000	
8.6	8.6	Try $\chi^2 = 8.6$
χ^2 DIST	0.020	
19	19	
F ν	19.000	
9	9	Try $\chi^2 = 9.0$
χ^2 DIST	0.027	
19	19	
F ν	19.000	
8.9	8.9	Try $\chi^2 = 8.9$
χ^2 DIST	0.025	

Thus the value we want is 8.9.

Now try $\chi^2 = 20$

Key Entry	Display	Explanation
19	19	
$\boxed{F}$ ν	19.00	
20	20	
$\boxed{\chi^2 \text{DIST}}$	6.05 . . . -01	
19	19	
$\boxed{F}$ ν	19.000	
		Try $\chi^2 = 30$
30	30	
$\boxed{\chi^2 \text{DIST}}$	9.48 -01	
19	19	
$\boxed{F}$ ν	19.000	
		Try $\chi^2 = 35$
35	35	
$\boxed{\chi^2 \text{DIST}}$	9.86 -01	
19	19	
$\boxed{F}$ ν	19.000	
		Try $\chi^2 = 33$
33	33	
$\boxed{\chi^2 \text{DIST}}$	9.75959837 -01	

Thus the value we want is 33. Try $\chi^2 = 32.8$
obtain .975.

The 95%-confidence interval is

$$\left\{ \frac{(n-1)s^2}{32.8}, \frac{(n-1)s^2}{8.9} \right\}$$

Key Entry	Display	Explanation
$\boxed{F}$ $\underset{\sqcap}{s}$	7.402	
$\boxed{x^2}$	54.787	
$\boxed{x}$	54.787	
19	19	
$\boxed{=}$	1040.950	
$\boxed{STOn}$	1040.950	
1	1040.950	
$\boxed{\div}$		
32.8	32.8	
$\boxed{=}$	31.736	
$\boxed{RCLn}$	28.2	
1	1040.95	
$\boxed{\div}$	1040.95	
8.9	8.9	
$\boxed{=}$	116.961	

The 95% confidence interval on σ is (31.736, 116.961)

4. t statistic

In the above example, the instructor notices that while the standard deviation is large, the average height of the children in the school is above the population average for twelve-year olds. He can use the t-test to determine whether this difference is significant.

NOTE: the data was already entered above.

The population mean μ, read from growth curve tables is 56 inches. Then compute t statistic:

Key Entry	Display	Explanation
56	56	
$\boxed{{}^{t}\text{STAT}}$	2.387	

We have to find out if the t value is significant. We have 19 degrees of freedom.

Key Entry	Display	Explanation
19	19	
$\boxed{\text{F}}$ ν	19.000	
2.387	2.387	
$\boxed{\text{F}}$ ^{t}DIST	0.972	

Since 1-^{t}DIST = 0.028 < 0.05 and we are looking for significance at the 0.05 level, we conclude that there is significant difference between this sample mean and the population mean.

C. Z Statistic

A production process gives components whose strengths are normally distributed with mean 401bf and standard deviation 1.21 lb. The process is modified and 12 components are selected at random giving strengths (in 1bf)

39.9	40.2	43.2	39.5	41.8	39.1
41.1	41.7	41.8	42.1	40.6	42.5

If the modified process gives on average stronger components than the unmodified process, it will be preferred for general use (the standard deviation can be assumed unchanged). Does the data give any evidence that the modified process is stronger?

Solution: enter the data first and find the z stat.

Key Entry	Display	Explanation
CLR GP		Clear registers
GP_1 or GP_2		
39.9	39.9	
Σ i, x, x^2	39.9	
40.2	40.2	
Σ i, x, x^2	40.2	
.	.	Continue data entry
.	.	
.	.	
40.6	40.6	
Σ i, x, x^2	40.6	
42.5	42.5	
Σ i, x, x^2	42.5	
1.21	1.21	enter σ
k	1.21	
40	40	enter μ
F ZSTAT	3.220755634	
(normal curve key)	9.993607344 -01	
+/−	-9.993607344 -01	
+	-9.993607344 -01	
1	1	
=	6.3926565 -04	

The significance level of this is 0.0006 . . . Thus the data give very strong evidence (significant at the 0.06% level) that the modification increases the average strength of the process.

D. Two-Sample Problems

The t-test for a difference between two independent means

An educator wishes to determine whether there is a significant difference between the IQ (Math ability) scores of boys and girls in a particular grade. The score recorded for each student is his score on the Bennet Test for Children.

Group 1 (Boys) Subject	Score	Group 2 (Girls) Subject	Score
s 1	109	s 13	107
s 2	105	s 14	97
s 3	127	s 15	88
s 4	130	s 16	80
s 5	119	s 17	105
s 6	94	s 18	120
s 7	99	s 19	87
s 8	106	s 20	102
s 9	108	s 21	91
s 10	92	s 22	101
s 11	96	s 23	106
s 12	111	s 24	106
		s 25	110
		s 26	108

Solution:

Key Entry	Display	Explanation
[CLR GP]		
GP_1		
109	109	
[Σ i, x, x²]	109	
105	105	
[Σ i, x, x²]	105	
	.	Continue data entry
	.	
	.	

Key Entry	Display	Explanation
111	111	
[Σ i, x, x^2]	111	

Now put GP switch on GP_2

Key Entry	Display	Explanation
[CLR GP]		
GP_2		
107	107	
[Σ i, x, x^2]	107	
.	.	Continue data entry
.	.	
.	.	
108	108	
[Σ i, x, x^2]	108	
0	0	$\mu_1 - \mu_2 = 0$
[F] t_{ind}	1.640606225	
[STOn] 1	1.640606225	

The degree of freedom is given by 26-2 = 24

Key Entry	Display	Explanation
24	24	
[F] ν	24	
[RCLn] 1	1.640606225	
[F] t_{DIST}	8.860784604 -01.	
[STOn] 2	8.860784604 -01.	
[C/CE]	0.	
1 [−]	1.	

Key Entry	Display	Explanation
$\boxed{\text{RCLn}}$ 2	8.860784604 -01.	
$\boxed{=}$	1.139215396 -01.	

We are looking for significance at the 0.05 level and since 0.11 > 0.05, we conclude that there is no significant difference between the IQ scores of boys and girls in this particular class.

E. The t-test for Related Measures

An educator is interested in determining the effects of a special education program on the intelligence test scores of underprivileged children. His first step is to match several pairs of students on the basis of their Wechsler Intelligence Test Scores. One student from each pair is randomly assigned to either the special training group or to the control group that receives no special treatment; the remaining student in each pair is assigned to the other group. After six weeks of training, an alternate form of the Wechsler test is given to the students in both groups to determine the effects of the program. The score recorded for each student is his score on this second test.

Group 1 (No Special Treatment)		Group 2 (Special Treatment)	
Subject	Score	Subject	Score
s_1	88	$s_{1'}$	92
s_2	88	$s_{2'}$	92
s_3	95	$s_{3'}$	100
s_4	108	$s_{4'}$	115
s_5	95	$s_{5'}$	110
s_6	87	$s_{6'}$	82
s_7	82	$s_{7'}$	80
s_8	83	$s_{8'}$	98
s_9	84	$s_{9'}$	91
s_{10}	88	$s_{10'}$	98
s_{11}	110	$s_{11'}$	105
s_{12}	98	$s_{12'}$	99
s_{13}	107	$s_{13'}$	111
s_{14}	106	$s_{14'}$	118

Solution:

Key Entry	Display	Explanation
CLR GP		
GP_1		
CLR GP		
GP_2		
88	88	
XENT	88	
92	92	
YENT	1	
.	.	Continue data entry
.	.	
.	.	
106	106	
XENT	106	
118	118	
YENT	14	
F $\bar{X}$	94.21428571	
F $\bar{Y}$	99.35714286	
tDEP STAT	–2.959181773	
+/– STOn 1	2.959181773	

Let us determine if the t-value is significant.

We have 14-1 = 13 degrees of freedom.

Key Entry	Display	Explanation
13	13	
F $\underbrace{\nu}$	13	
RCLn 1.	2.959181773	
F $\underbrace{\text{t DIST}}$	9.8889252838 -01.	
+/-	-9.8889252838 -01.	
1	1	
=	1.107471623 -02.	

Since this is less than 5×10^{-2}, it is concluded that the special training program improved the IQ test scores.

F. Example using The Coefficient of Correlation r (The Pearson product-moment correlation.)

An experimenter wishes to determine whether there is a relationship between the grade point averages (GPA's) and the scores on a reading comprehension test of 15 first-year college students.

Student	Reading Score (x)	GPA (y)
s_1	39	2.1
s_2	55	2.9
s_3	42	3.0
s_4	44	2.3
s_5	52	2.6
s_6	60	3.7
s_7	59	3.2
s_8	23	1.3
s_9	38	1.8
s_{10}	35	2.5
s_{11}	47	3.4
s_{12}	47	2.6
s_{13}	42	2.4
s_{14}	38	2.5
s_{15}	49	3.3

Solution:

Key Entry	Display	Explanation
CLR GP		
GP_1		
CLR GP		
GP_2		
39	39	
X_{ENT}	39	
2.1	2.1	
Y_{ENT}	1	
.	.	Continue data entry
.	.	
.	.	
49	49	
X_{ENT}	49	
3.3	3.3	
Y_{ENT}	15	

Since N is smaller than 30, compute $t = r\sqrt{(N-2)/(1-r^2)}$ *

r	8.199699803 -01	
x^2	6.723507687 -01	
+/–	6.723507687 -01	
+ 1	1	
=	3.276492313 -01	
1/x	3.052044395	

Key Entry	Display	Explanation
$\boxed{\times}$	3.052044395	
13	13	
$\boxed{=}$	39.67657713	
$\boxed{\sqrt{x}}$	6.298934603	
$\boxed{\text{STOn}}$ 1	6.298934603	
$\boxed{r}$	8.199699803 -01	
$\boxed{\times}$ $\boxed{\text{RCLn}}$ 1	6.298934603	
$\boxed{=}$	5.164937283	
$\boxed{\text{STOn}}$ 1	5.164937283	
13	13	
$\boxed{F}$ ν	13	
$\boxed{\text{RCLn}}$ 1	5.164937283	
$\boxed{F}$ tDIST	9.998181319 -01	
$\boxed{+/-}$	-9.998181319 -01	
$\boxed{+}$	-9.998181319 -01	
1	1	
$\boxed{=}$	1.818681 -04	

Since this is less than 5×10^{-4}, the data is significant at the 0.05 level. In other words, there is a relationship between GPA's and the scores on a reading comprehension test for the 15 college students.

**NOTE: Testing the significance of r: Two different procedures are used to test the hypothesis that $r = 0$. If N (the number of pairs) is 30 or larger, a critical-ratio z-test should be done where $z = r\sqrt{N-1}$ (use the Gaussian distribution). For N smaller than 30, a t-test should be done where $t = r\sqrt{(N-2)/(1-r^2)}$.*

G. χ^2 Goodness of Fit Test

In Mendel's experiments with peas, he observed 315 round and yellow, 108 round and green, 101 wrinkled and yellow, and 32 wrinkled and green. According to his theory of heredity, the numbers should be in the proportion 9:3:3:1. Is there any evidence to doubt his theory at the

0.01 level
0.05 level of significance?

Solution:
Total number of peas = 556.

Since the expected numbers are in the proportion 9:3:3:1 (and 9 + 3 + 3 + 1 = 16), we get the following:

to obtain Ei, 556 x 9/16 = 312.75 556 x 3/16 = 104.25
556 x 1/16 = 34.75

	Round and yellow	Wrinkled and yellow	Round and green	Wrinkled and green
Oi	315	101	108	32
Ei	312.75	104.25	104.25	34.75

Key Entry	Display	Explanation
0	0.	
$\boxed{\text{STOn}}$	0.	
8	0.	
315	315.	
$\boxed{\text{Oi}}$	315	
312.75	312.75	
$\boxed{\text{Ei}}$	312.75	
101	101	

Key Entry	Display	Explanation
$\boxed{\text{0i}}$	101	
104.25	104.25	
$\boxed{\text{Ei}}$	104.25	
108	108	
$\boxed{\text{0i}}$	108	
104.25	104.25	
$\boxed{\text{Ei}}$	104.25	
32	32.	
$\boxed{\text{0i}}$	32.	
34.75	34.75	
$\boxed{\text{Ei}}$	34.75	
$\boxed{\chi^2\text{FIT}}$	0.4700239808	

Since there are 4 categories, the number of degrees of freedom is $\nu = 4 - 1 = 3$.

3	3	
$\boxed{\text{F}}$ ν	3	
$\boxed{\chi^2\text{FIT}}$	0.47002398	
$\boxed{\chi^2\text{DIST}}$	$7.45741049 \times 10^{-2}$	

Since χ^2 DIST is less than 0.99 or 0.95, we cannot reject the theory at the 0.01 level or at the 0.05 level. Also, note that although the agreement is good, the results obtained are subject to a reasonable amount of sampling error.

H. Analysis of Variance (one way)

This is an example drawn from psychology.

Assume that the experimenter is interested in determining the effect of shock on the time required to solve a set of difficult problems. Subjects are randomly assigned to four experimental conditions. Subjects in Group 1 receive no shock; Group 2, very-low-intensity shocks; Group 3, medium shocks; and Group 4, high-intensity shocks. The total time required to solve all the problems is the measure recorded for each subject.

Group 1 (No Shock)		Group 2 (Low Shock)		Group 3 (Med. Shock)		Group 4 (High Shock)	
Subject	Time (min.)	Subject	Time (min.)	Subject	Time (min.)	Subject	Time (min.)
s_1	10	s_{13}	3	s_{25}	19	s_{37}	23
s_2	7	s_{14}	8	s_{26}	12	s_{38}	14
s_3	9	s_{15}	7	s_{27}	16	s_{39}	16
s_4	8	s_{16}	5	s_{28}	14	s_{40}	18
s_5	15	s_{17}	6	s_{29}	7	s_{41}	12
s_6	3	s_{18}	10	s_{30}	8	s_{42}	13
s_7	8	s_{19}	12	s_{31}	13	s_{43}	16
s_8	9	s_{20}	4	s_{32}	10	s_{44}	17
s_9	11	s_{21}	7	s_{33}	19	s_{45}	19
s_{10}	9	s_{22}	6	s_{34}	9	s_{46}	14
s_{11}	5	s_{23}	5	s_{35}	15	s_{47}	16
s_{12}	17	s_{24}	15	s_{36}	14	s_{48}	17

Suppose we have g groups of observations to compare. Group i ($i = 1, 2, \ldots, g$) has n_i observations (treatment group may have equal or unequal number of observations).

Then, Sum = $\sum_{j=1}^{n_i} X_{ij}$ — The sum of observations in treatment group i.

$$SS_t \text{ (sum of squares) total} = \sum_{i=1}^{g} \sum_{j=1}^{n_i} X^2_{ij} - \frac{\left(\sum_{i=1}^{g} \sum_{j=1}^{n_i} X_{ij}\right)^2}{\sum_{i=1}^{g} n_i}$$

$$\text{Treat SS} = \sum_{i=1}^{g} \frac{(\sum^{n_i} X_{ij})^2}{n_i} - \frac{\left(\sum_{i=1}^{g} \sum_{j=1}^{n_i} X_{ij}\right)^2}{\sum_{i=1}^{g} n_i}$$

Error SS = Total SS -- Treat SS

d_{f_1} = Treat $d_f = g - 1$

d_{f_2} = Error $d_f = \sum_{i=1}^{g} n_i - g$

$$\text{Treat MS} = \frac{\text{Treat SS}}{\text{Treat } d_f}$$

$$\text{Error MS} = \frac{\text{Error SS}}{\text{Error } d_f}$$

$F = \dfrac{\text{Treat MS}}{\text{Error MS}}$ (with $g - 1$ and $\sum_{i=1}^{g} n_i - g$ degrees of freedom)

Key Entry	Display	Explanation
Turn switch to GP_1		
$\boxed{\text{CLR GP}}$		
GP_1		
10	10	Enter s_1
$\boxed{\Sigma\ i, x, x^2}$	10	
7	7	Enter s_2
$\boxed{\Sigma\ i, x, x^2}$	7	
9	9	Enter s_3
$\boxed{\Sigma\ i, x, x^2}$	9	
.	.	Continue data entry
.	.	

Key Entry	Display	Explanation
5	5	Enter s_{11}
$\boxed{\Sigma \; i, x, x^2}$	5	
17	17	Enter s_{12}
$\boxed{\Sigma \; i, x, x^2}$	17	
$\boxed{RCLn}$	17	
4	111.	Recall Σx_{1j} for gp. 1
$\boxed{x^2}$	12321.	
$\boxed{\div}$	12321.	
$\boxed{RCLn}$	12321.	
3	12	
$\boxed{=}$	1026.75	$\frac{(\Sigma x_{1j})^2}{n_1}$
$\boxed{STOn}$	1026.75	
1	1026.75	
Turn switch to GP$_2$		
$\boxed{CLR \; GP}$	1026.75	
GP$_2$ 		
3	3	Enter s_{13}
$\boxed{\Sigma \; i, x, x^2}$	3	
8	8	Enter s_{14}
$\boxed{\Sigma \; i, x, x^2}$	8	
.	.	Continue data entry
.	.	
.	.	

Key Entry	Display	Explanation
5	5	Enter s_{23}
$\boxed{\Sigma \; i, x, x^2}$	5	
15	15	Enter s_{24}
$\boxed{\Sigma \; i, x, x^2}$	15	
$\boxed{RCLn}$	15	
7	88	Recall Σx_{2j} for gp. 2
$\boxed{x^2}$	7744	
$\boxed{\div}$	7744	
$\boxed{RCLn}$	7744	
6	12	
$\boxed{=}$	645.3333333	$\frac{(\Sigma x_{2j})^2}{n_2}$
$\boxed{\Sigma n \; x}$	645.3333333	
1	645.3333333	Reg. 1 has $\frac{(\sum_{j=1}^{n_2} X_{ij})^2}{n_i}$
$\boxed{RCLn}$	645.3333333	
6	12.	
$\boxed{\Sigma n \; x}$	12.	
3	12.	Reg. 3 has $\sum_{i=1}^{2} \sum_{j=1}^{n_i} [n_1 + n_2]$
$\boxed{RCLn}$	12.	
7	88.	
$\boxed{\Sigma n \; x}$	88.	

Key Entry	Display	Explanation
4	88.	Reg. 4 has $\sum_{i=1}^{2} \sum_{j=1}^{n_i} \Sigma_{ij}$
$\boxed{\text{RCLn}}$	88.	
8	778	Recall $\Sigma(x_{2j})^2$
$\boxed{\Sigma n \; x}$	778	
5	778	Reg. 5 has $\sum_{i=1}^{2} \sum_{j=1}^{n_i} (x_{ij})^2$
$\boxed{\text{CLR GP}}$		
19	19	
$\boxed{\Sigma \; i, x, x^2}$	19	Enter s_{25}
12	12	
$\boxed{\Sigma \; i, x, x^2}$	12	Enter s_{26}
.	.	Continue data entry
.	.	
.	.	
15	15	Enter s_{35}
$\boxed{\Sigma \; i, x, x^2}$	15	
14	14	Enter s_{36}
$\boxed{\Sigma \; i, x, x^2}$	14	
$\boxed{\text{RCLn}}$	14	
7	156	Recall Σx_{3j}
$\boxed{x^2}$	24336	
$\boxed{\div}$	24336	
$\boxed{\text{RCLn}}$	24336	
6	12	

Key Entry	Display	Explanation
$\boxed{=}$	2028.	$\dfrac{(\Sigma x_{3j})^2}{n_3}$
$\boxed{\begin{matrix}\Sigma n \\ x\end{matrix}}$	2028.	
1	2028.	Reg. 1 has $\dfrac{\sum_{i=1}^{3} (\sum_{j=1}^{n_i} x_{ij})^2}{n_i}$
$\boxed{RCLn}$	2028.	
6	12.	
$\boxed{\begin{matrix}\Sigma n \\ x\end{matrix}}$	12.	
3	12	Reg. 3 has $\sum_{i=1}^{3} \sum_{j=1}^{n_i} {}_{ij}$
$\boxed{RCLn}$	12	
7	156	
$\boxed{\begin{matrix}\Sigma n \\ x\end{matrix}}$	156	
4	156	Reg. 4 has $\sum_{i=1}^{3} \sum_{j=1}^{n_i} x_{ij}$
$\boxed{RCLn}$	156	
8	2202	
$\boxed{\begin{matrix}\Sigma n \\ x\end{matrix}}$	2202	
5	2202	Reg. 5 has $\sum_{i=1}^{3} \sum_{j=1}^{n_i} (x_{ij})^2$
$\boxed{\begin{matrix}CLR \\ GP\end{matrix}}$	2202	
GP_2		
23	23	Enter s_{37}
$\boxed{\begin{matrix}\Sigma \\ i, x, x^2\end{matrix}}$	23	
14	14	Enter s_{38}

Key Entry	Display	Explanation
$\boxed{\Sigma \; i, x, x^2}$	14	
.	.	Continue data entry
.	.	
.	.	
16	16	Enter s_{47}
$\boxed{\Sigma \; i, x, x^2}$	16	
17	17	Enter s_{48}
$\boxed{\Sigma \; i, x, x^2}$	17	
$\boxed{RCL_n}$	17	
7	195	Recall Σx_{ij}
$\boxed{x^2}$	38025	
$\boxed{\div}$	38025	
$\boxed{RCL_n}$	38025	
6	12	
$\boxed{=}$	3168.75	$\frac{(\Sigma x_{ij})^2}{n_i}$
$\boxed{\Sigma n \; x}$	3168.75	
1	3168.75	Reg. 1 has $\sum_{i=1}^{4} \frac{(\sum_{j=1}^{n_i} x_{ij})^2}{n_i}$
$\boxed{RCL_n}$	3168.75	
6	12	
$\boxed{\Sigma n \; x}$	12	
3	12	Reg. 3 has $\sum_{i=1}^{4} \sum_{j=1}^{n_i} ij$
$\boxed{RCL_n}$	12	
7	195	

Key Entry	Display	Explanation
$\boxed{\begin{array}{l}\Sigma n\\ x\end{array}}$	195	
4	195	Reg. 4 has $\sum_{i=1}^{4} \sum_{j=1}^{n_i} x_{ij}$
$\boxed{\text{RCLn}}$	195	
8	3265	
$\boxed{\begin{array}{l}\Sigma n\\ x\end{array}}$	3265	
5	3265	Reg. 5 has $\sum_{i=1}^{4} \sum_{j=1}^{n_i} (x_{ij})^2$
$\boxed{\text{RCLn}}$	3265	
4	550.	
$\boxed{x^2}$	302500.	
$\boxed{\div}$	302500.	
$\boxed{\text{RCLn}}$	302500.	
3	48	
$\boxed{=}$	6302.083333	$\dfrac{(\sum_i \sum_j x_{ij})^2}{N}$
$\boxed{+/-}$	–6302.083333	
$\boxed{+}$	–6302.083333	
$\boxed{\begin{array}{l}\Sigma n\\ x\end{array}}$	–6302.083333	
1	–6302.083333	Reg. 1 has Treat SS
$\boxed{\text{RCLn}}$	–6302.083333	
5	7434.	
$\boxed{=}$	1131.916667	Total SS
$\boxed{\text{STOn}}$	1131.916667	
2	1131.916667	Total Sum of Squares is in Reg. 2
$\boxed{\text{RCLn}}$	1131.916667	

Key Entry	Display	Explanation
1	566.75	
+/−	−566.75	
Σn x	−566.75	
2	−566.75	Reg. 2 has Error SS
RCLn	−566.75	
1	566.75	
÷	566.75	
RCLn	566.75	
2	565.1666667	
÷	1.002801533	$\frac{\text{Treat SS}}{\text{Error SS}}$
3	3	df_1
x	0.334267177	
44	44	df_2
=	14.70775582	This is the F value or $\frac{\text{Treat MS}}{\text{Error MS}}$
STOn	14.70775582	
1	14.70775582	
3	3	
ν_1	3	
44	44	
ν_2	44	
RCLn	44	
1	14.70775582	
F_{DIST}	8.9919×10^{-7}	

Since this is less than 1×10^{-5}, it is concluded that level of shock intensity does affect the time required to solve these problems

I. The Comparison of Two Independent Sample Variances

In order to assess the effect that washing would have on the extensibility of yarn, 20 lengths were taken and 10 selected at random to serve as controls. All are measured for breaking extension, but two observations are missing because of a defect in the measuring apparatus. The results are as follows:

Group	% Breaking Extension									
Washed	12	15	9	11	15	12	11	15		
Control	13	11	14	10	17	8	16	5	16	7

The differences between the two groups of results can be ascribed to a treatment effect, if the lengths of yarn are otherwise treated identically in the experiment.

A significance test of the hypothesis H_O: $\sigma_1^2 = \sigma_2^2$ is obtained by calculating the variance ratio

v = (larger variance) / (smaller variance)

and then using the F_{DIST} with ν_1 and ν_2 equal to the degrees of freedom of the numerator and denominator respectively.

Key Entry	Display	Explanation
[CLR GP]		
GP_1 [■□]		
12	12	
[Σ i, x, x²]	12	Enter data for group 1
15	15	
[Σ i, x, x²]	15	
.	.	Continue data entry
.	.	
.	.	

Key Entry	Display	Explanation
11	11	
$\boxed{\Sigma \; i, x, x^2}$	11	
15	15	
$\boxed{\Sigma \; i, x, x^2}$	15	
Now have switch on GP_2		
$\boxed{\text{CLR GP}}$	15	
GP_2		
13	13	Enter data for group 2
$\boxed{\Sigma \; i, x, x^2}$	13	
11	11	
$\boxed{\Sigma \; i, x, x^2}$	11	
.	.	Continue data entry
.	.	
.	.	
16	16	
$\boxed{\Sigma \; i, x, x^2}$	16	
7.	7	
$\boxed{\Sigma \; i, x, x^2}$	7	
$\boxed{F}$ s	4.164666186	std. dev. for gp. 2
Turn switch to GP_1		
$\boxed{F}$ s	2.267786838	
(Group 2 has higher std. deviation)		
$\boxed{\div}$	2.267786838	

Key Entry	Display	Explanation
Put switch on GP_2		
$\boxed{F}$ $\underline{s}$	4.164666186	
$\boxed{F}$ $\underline{x \leftrightarrow y}$		
$\boxed{=}$	1.83644517	
$\boxed{STOn}$	1.83644517	
1	1.83644517	
9	9	
$\boxed{\nu_1}$	9	
7	7	
$\boxed{\nu_2}$	7	
$\boxed{RCLn}$	7	
1	1.83644517	
$\boxed{F_{DIST}}$	0.2175393016	

Thus we conclude that the F-ratio is not significant at the 5% level.

In other words, the difference in standard deviations is not significant.

J. General Curve Fitting

The following data were obtained when measuring sugar content in peaches by two methods: a direct method and an indirect chemical method. Derive a relationship between the two methods of measurement using curve fitting. Also find the measurement by the direct method if the chemical method gives the value 10.

Two methods of measuring sugar content in peaches

No. of measurements	Direct method (x)	Indirect method (y)
1	11.0	12.0
2	11.1	12.1
3	7.2	7.5
4	8.3	8.0
5	12.4	16.0
6	14.7	24.5
7	5.1	5.0
8	21.7	47.9
9	20.2	43.1
10	19.1	38.2
11	25.0	69.0
12	10.2	11.8
13	13.3	20.0
14	23.6	57.6
15	12.0	15.0

Let: x = direct method since it is an independent variable.
y = indirect method

[illegible]

do a linear regression (y values remain unchanged) and find RSS

Next we try the relationship $y \alpha e^x$. We transform the y values by using $y'' = \frac{y^{o}-1}{0} = 1n\ y$, and do a linear regression and find RSS. The linear regression with the smallest RSS is chosen.

The data may be entered as follows:

Key Entry	Display	Explanation
[CLR GP] · GP_1 GP_2	0	
[CLR GP] GP_1 GP_2	0	
[F] DISP 3	0	to work with 3 significant digits
11	11	
[X_{ENT}]	11	
12	12	
[y^X]	12	
.5	0.5	
[−]	3.46	
1	1	Number of pairs entered
[÷]	2.46	
.5	0.5	
[=]	4.93	
[Y_{ENT}]	1	
.		Continue data entry
.		
.		
12	12	
[X_{ENT}]	12	
15	15	
[y^X]	15	
.5	0.5	

Key Entry	Display	Explanation
$\boxed{-}$	3.87	
1	2.87	
.5	0.5	
$\boxed{=}$	5.75	
$\boxed{\text{Y}_{\text{ENT}}}$	15	Number of pairs entered
$\boxed{\text{F}}$ RSS	1.71	
$\boxed{\text{INTCP}}$	−1.37	
$\boxed{\text{SLOPE}}$	0.618	

try the relationship y α e^x (i.e., y″ = 1n y)

Key Entry	Display	Explanation
$\boxed{\text{CLR GP}}$ GP_1, GP_2	0	
11	11	
$\boxed{\text{X}_{\text{ENT}}}$	11	
12	12	
$\boxed{\text{ln}}$	2.48	
$\boxed{\text{Y}_{\text{ENT}}}$	1	
.	.	Continue data entry
.	.	
.	.	
12	12	
$\boxed{\text{X}_{\text{ENT}}}$	12	
15	15	
$\boxed{\text{ln}}$	2.71	
$\boxed{\text{Y}_{\text{ENT}}}$	15	Number of pairs entered
$\boxed{\text{F}}$ RSS	1.43 . . . -01	
$\boxed{\text{SLOPE}}$	1.3 . . . -01	
$\boxed{\text{INTCP}}$	1.1	

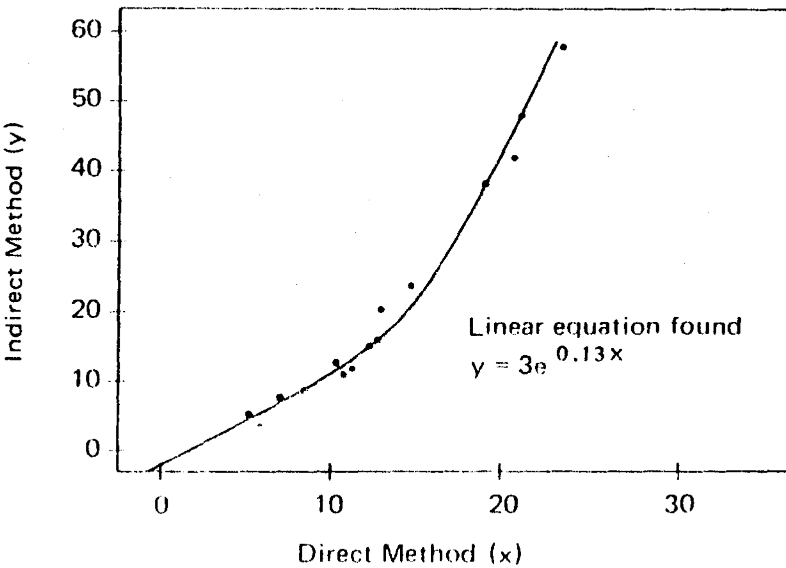
Since RSS for $y' = \ln y$ is smaller than the other transformed y, the best fit is:

$$y' = 1.1 + 0.13x$$

$$\ln y = 1.1 + 0.13x$$

$$y = e^{1.1 + 0.13x} = 3e^{0.13x}$$

Sugar Content in Peaches
Graph I: Direct measurement vs. Indirect measurement



Linear equation found.

K. Quality Inspection Using Hypergeometric Distribution

Find the probability of finding one defective in a sample of 10 if 5 defects are found in the lot of 100.

Solution:

We can use the hypergeometric distribution to find the probability, where

N = 100 D = 5
n = 10 d = 1

we have to find P where d = 0 and d = 1.

To find the probability, enter as follows:

Key Entry	Display	Explanation
100 [−]	100	
5	5	
[=]	95	ν_2
[ν_2]	95	
5 [ν_1]	5	
0 [P]	0	
10 [F] HYPG	0.583752367	P(0IN, D, n)
[STOn] 1	0.583752367	
95	95	
[ν_2]	95	
5	5	
[ν_1]	5	
1	1	
[P]	1	
10	10	
[F] HYPG	0.33939091	P(1IN, D, n)
[+]	0.33939091	
[RCLn] 1	0.583752367	
[=]	0.923143277	
[F] DISP .2	0.92	P(0IN, D, n) + P(1IN, D, n

Therefore the probability of finding one defective from a sample of 10 is 0.92.

L. Control Charts for Variables Using Inverse Gaussian Distribution

A manufacturer makes paper bags with a mean breaking strength of 12.5 lbs and a standard deviation of 1.06 lbs. To determine whether the product is conforming to standards, a sample of 20 paper bags is taken every hour and the mean breaking strength is determined. Find the (1) 99%; (2) 95% and (3) 90% control limits on a quality control chart.

Solution:

$$\overline{X} = 12.5 \text{ lbs} \quad \sigma = 1.06 \text{ lbs}$$

$$UCL_{\overline{x}} = 12.5 + k\sigma/\sqrt{N}$$

$$LCL_{\overline{x}} = 12.5 - k\sigma/\sqrt{N}$$

(1) Enter as follows to obtain the 99% Control Limits:

Key Entry	Display	Explanation
.99	0.99	
$\boxed{F}$ I^{-1}	2.5762	σ units = k
$\boxed{X}$	2.5762	
1.06	1.06	
$\boxed{\div}$	2.730772	
20	20	N
$\boxed{\sqrt{x}}$	4.472135955	$\sqrt{N}$
$\boxed{=}$	6.106191823 -01	
$\boxed{F}$ DISP 3	6.11 -01	

Therefore 99% control limits are:

$$UCL_{\overline{x}} = 12.5 + 0.611 = 13.111 \text{ lbs}$$

$$LCL_{\overline{x}} = 12.5 - 0.611 = 11.899 \text{ lbs}$$

N,D,

mple

(2) To obtain 95% control limits, enter as follows:

Key Entry	Display	Explanation
.95	0.95	
$\boxed{F}$ I^{-1}	1.9603	get k
$\boxed{X}$	1.9603	
1.06	1.06	
$\boxed{\div}$	2.077918	
20	20	
$\boxed{\sqrt{x}}$	4.472135955	
$\boxed{=}$	4.6463659 -01	
$\boxed{F}$ DISP 3	4.65 -01	

Therefore 95% control limits are:

$$UCL_{\bar{x}} = 12.5 + 0.465 = 13 \text{ lbs}$$
$$LCL_{\bar{x}} = 12.5 - 0.465 = 12 \text{ lbs}$$

(3) To obtain 90% control limits, enter as follows:

Key Entry	Display	Explanation
.90	0.90	
$\boxed{F}$ I^{-1}	1.65	get k
$\boxed{X}$	1.65	
1.06	1.06	
$\boxed{\div}$	1.74	
20	20	
$\boxed{\sqrt{x}}$	4.47	
$\boxed{=}$	3.9 -01	

Therefore 90% control limits are:

$$UCL_{\bar{x}} = 12.5 + 0.39 = 12.89 \text{ lbs}$$

$$LCL_{\bar{x}} = 12.5 - 0.39 = 12.11 \text{ lbs}$$

M. Derivation of OC Curve Using χ^2 Distribution (Related to Cumulative Poisson Distribution)

Derive an operation characteristics curve with a lot size, N = 6,000, a sample size of n = 100 and a maximum acceptable number of defects = 8. Compute P_a for (p = 0, 0.02, 0.04, 0.06, 0.08, 0.10, 0.12, 0.16).

Solution:

We use the Cumulative Poisson distribution to find P_a. The Cumulative Poisson distribution is related to the χ^2 distribution as follows:

$$1 - P(\chi^2 | \nu) = \sum_{j=0} \frac{e^{-\lambda} \lambda^j}{j!}, \quad C + 1 = \frac{\nu}{2} \quad \lambda = \frac{\chi^2}{2}$$

$$\nu = 8 + 1 \times 2 = 18$$

$$\lambda = np = \frac{\chi^2}{2} \qquad P_a = 1 - P(\chi^2/\nu)$$

2n = 2 x 100 = 200

Enter number as follows:

Key Entry	Display	Explanation
8 $\boxed{+}$	8	
1	1	
$\boxed{\times}$	9	
2	2	
$\boxed{=}$	18	ν.
$\boxed{\text{STO}_n}$ 1	18	
2 $\boxed{\times}$	2	

or

Key Entry	Display	Explanation
100	100	
[=]	200	2 x n
[STOn] 8	200	
[X] 0.02	2 -02	
[=]	4	$2np_1 = \chi_1^2$
[STOn] 2	4	
0.04 [X]	4 -02	
[RCLn] 8	200	
[=]	8	$2np_2 = \chi_2^2$
[STOn] 3	8	
0.06 [X]	6 -02	
[RCLn] 8	200	
[=]	12	$2np_1 = \chi_1^2$
[STOn] 4	12	
.	.	and so on for
.	.	p = 0.08, 0.10,
.	.	0.12, 0.16

To find P_a, enter:

Key Entry	Display	Explanation
[RCLn] 1	18	
[F] ν	18	
[RCLn] 2	4	
[χ^2 DIST]	2.37447. . . -04	$P(\chi^2/\nu) = 1 - P_a$

To work with 3 significant digits,

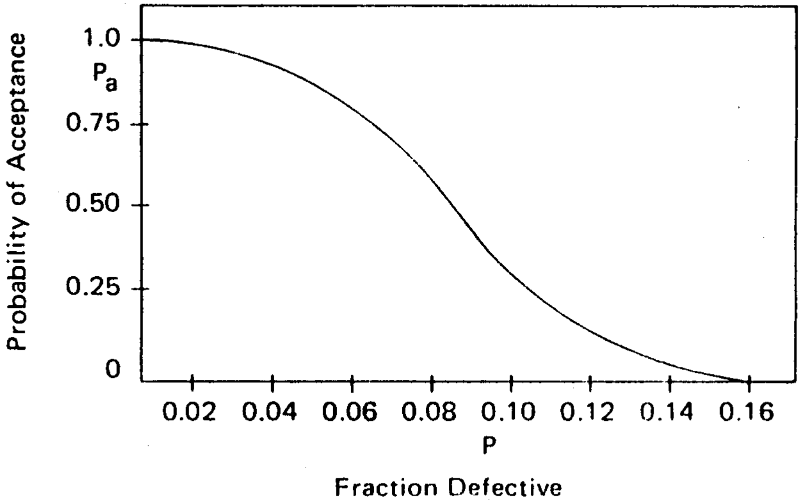
Key Entry	Display	Explanation
[F] DISP 3	2.37 -04	
[illegible]	2.37 04	

Key Entry	Display	Explanation
[+] 1	1	
[=]	1	P_a
[RCLn] 1	18	
[F] ν	18	
[RCLn] 3	8	
[χ^2 DIST]	2.14 -02	
[+/−]	-2.14 -02	
[+]	-2.14 -02	
1	1	
[=]	9.79 -01	P_a
[RCLn]	18	
[F] ν	18	
[RCLn] 4	12	
[χ^2 DIST]	1.53 -01	
[+/−]	-1.53 -01	
[+] 1	1	
[=]	8.47 -01	and so on for $P = 0.08 \ldots, 0.16$

We obtain the following:

P	$2nP = \chi^2$	$1 - P(\chi^2/\nu) = P_a$
0	0	1
0.02	4	0.99
0.04	8	0.97
0.06	12	0.85
0.08	16	0.59
0.10	20	0.33
0.12	24	0.16
0.16	32	0.02

O.C. Curve for n = 100 and c = 8



N. Target Hitting Example Using Poisson Probability Function

The probability of hitting a target at each firing is 0.0017. Find the probability of hitting a target with two or more bullets if the total number of shots fired is 3,000.

Solution:

We can use the Poisson distribution for calculating the probability $P(x \geq 2)$, where

$$\lambda = 0.0017 \times 3{,}000, \quad j = 0, 1$$

The required probability is

$$P(x \geq 2) = 1 - P_n(0) - P_n(1)$$

Enter as follows to obtain $P(x \geq 2)$:

Key Entry	Display	Explanation
0	0	
$\boxed{k}$	0	
.0017	0.0017	

Key Entry	Display	Explanation
X	0.0017	
3,000	3,000	
=	5.1	
STOn 1	5.1	
F POISS	6.096746565 -03	
STOn 2	6.096746565 -03	
1	1	
k	1	
RCLn 1	5.1	
F POISS	3.109340748 -02	
Σn x 2	0.609674657 -02	
RCLn 2	3.719015405 -02	
+/−	−3.719015405 -02	
+	−3.719015405 -02	
1	1	
=	9.62809846 -01	
F DISP 3	9.63 -01	

Therefore the probability of hitting a target with two or more bullets is 0.963.

O. Microbiological Example Using Poisson Distribution

A solution contains bacterial viruses in a concentration of 6×10^9 viruses/ml. In the same solution are 2×10^9 bacteria/ml. Assuming random distribution of viruses among the bacteria.

(1) What proportion of the bacteria will have no virus particles?

(2) What proportion of the bacteria will have virus particles?

(3) What proportion of the bacteria will have at least two virus particles?

(4) What proportion of the bacteria will have three virus particles?

Solution:

We can use the Poisson probability distribution to find the proportions.

$$\lambda = \frac{6 \times 10^9}{2 \times 10^9} = 3 \text{ viruses/bacterium}$$

(1) The proportion of the bacteria that will have no virus particles is given by

$$P(X = 0, \lambda)$$

To find the probability, enter as follows:

Key Entry	Display	Explanation
0	0	
[k]	0	
3	3	
[F] POISS	4.978706836 -02	
[STOn] 1	4.978706836 -02	

(2) The proportion of bacteria that will have virus particles is given by

$$1 - P(X = 0, \lambda)$$

Enter as follows:

Key Entry	Display	Explanation
[+/–]	-4.978706836 -02	
[+]	-4.978706836 -02	

Key Entry	Display	Explanation
1	1	
$\boxed{=}$	0.9502129316	

The proportion of bacteria that will have at least 1 virus particle is 0.950.

(3) The proportion of bacteria that will have at least 2 virus particles is given by

$$1 - P(X = 0) - P(X = 1) = P(X \geq 2)$$

Key Entry	Display	Explanation
1	1	
$\boxed{k}$	1	
3	3	
$\boxed{F}$ POISS	1.493612051 -01	
$\boxed{\Sigma n \; x}$ 1	1.493612051 -01	
$\boxed{RCLn}$ 1	1.991482735 -01	
$\boxed{+/-}$	-1.991482735 -01	
$\boxed{+}$	-1.991482735 -01	
1	1	
$\boxed{=}$	8.008517265 -01	

The proportion of bacteria to have at least 2 virus particles is 0.801.

(4) The proportion of the bacteria to have 3 virus particles is

$$P(X = 3, \lambda)$$

To obtain P (X = 3, λ), enter as follows:

Key Entry	Display	Explanation
3	3	
$\boxed{\text{k}}$	3	
3	3	
$\boxed{\text{F}}$ POISS	2.240418076 -01	

Therefore the proportion of the bacteria to have 3 virus particles is 0.224.

P. Binomial Distribution Example

Find the probability that in a family of 6 children there will be

- at least 1 boy
- at least 1 boy and 1 girl. (Assume that the probability of a male birth is 0.5.)

Solution: We use the binomial distribution.

The probability that in a family of 6 children, there will be at least 1 boy is

1 — Pr (no boy)   Pr = probability

Pr (no boy) = $C_0^6 \cdot P^0 (1-P)^6$; p = 1/2

To obtain 1 – Pr (no boy), enter as follows:

Key Entry	Display	Explanation
0	0	
$\boxed{\text{k}}$	0	
.5	0.5	
$\boxed{\text{P}}$	0.5	
6	6	
$\boxed{\text{BIN}}$	1.5625 -02	
$\boxed{\text{STOn}}$ 1	1.5625 -02	

Key Entry	Display	Explanation
+/-	-1.5625 -02	
+	-1.5625 -02	
1	1	
=	9.84375 -01	

The probability that there will be at least one boy and one girl among six children is given by:

(1 – Pr (no boy) x (1 – Pr (no girl))

but since Pr (no boy) = Pr (no girl)

Probability = (1 – Pr (no boy))^2

Key Entry	Display	Explanation
1	1	
−	1	
RCLn 1	1.5625 -02	
=	9.84375 -01.	
x^2	9.689941406 -01	

Therefore the probability of having at least one boy and one girl from among six children is 0.969.

Q. Binomial Cumulative Distribution Using F Distribution

If 25% of the bolts produced by a machine are defective, determine the probability that out of 8 bolts chosen at random, at most 2 bolts will be defective.

Solution: p = 0.25, n = 8, and k = 2

We use the following relationship:

$$\sum_{s=a}^{n} C_s^n \; P^s(1-P)^{n-s} = 1 - P(F|\nu_1, \nu_2)$$

where $\nu_1 = 2(n-a+1)$ $\quad \nu_2 = 2a$

and $F = \dfrac{a - aP}{(n-a+1)P}$ $\quad$ also $a = k + 1$

To solve problem, enter as follows;

Key Entry	Display	Explanation
8 $\boxed{-}$	8	
3	3	
$\boxed{+}$	5	
1	1	
$\boxed{X}$	6	
.25	.25	
$\boxed{=}$	1.5	
$\boxed{\text{STOn}}$ 1	1.5	
2 $\boxed{X}$	2	
3	3	
$\boxed{=}$	6	
$\boxed{\text{STOn}}$ 2	6	
8 $\boxed{-}$	8	
3	3	
$\boxed{+}$	5	
1	1	
$\boxed{X}$	6	
2	2	
$\boxed{=}$	12	

Key Entry	Display	Explanation
[ν_1]	12	
[RCLn] 2	6	
[ν_2]	6	
[RCLn] 1	1.5	
[FDIST]	3.214569092 -01	
[F] DISP [ ] 3	0.321	$Q(F\|\nu_1, \nu_2)$
[+/-]	-0.321	
[+] 1	1	
[=]	0.679	1 $1 - P(F\|\nu_1, \nu_2)$

Therefore the probability that out of 8 bolts, 2 at most will be defective is 0.679.

R. Example on Contingency Tables

Test the Null Hypothesis that human hair color is independent of sex at α = 0.05 level. The following data is provided.

	Hair Color				
Sex	Black	Brown	Blond	Red	Total
Male (Oi) (Ei)	30 27.40	42 37.67	18 27.05	10 7.88	100
Female (Oi) (Ei)	50 52.60	68 72.33	61 51.95	13 15.12	192
Total	80	110	79	23	292

Let Ho: Human hair color is independent of sex.

H_A: Human hair color is not independent of sex.

and α = 0.05 is given.

To test the data, enter as follows:

Key Entry	Display	Explanation
30	30	
[0i]	30	
27.4	27.4	
[Ei]	27.4	
42	42	
[0i]	42	
37.67	37.67	
[Ei]	37.67	
.	.	Continue data entry
.	.	
.	.	
13	13	
[0i]	13	
15.12	15.12	
[Ei]	15.12	
[X^2 FIT]	6.604146993	

Degrees of freedom = (4 - 1) (2 - 1) = 3

Key Entry	Display	Explanation
3	3	
[F] ν	3	
[RCLn] 8	6.604146993	Data base stored in memory register 8
[X^2 DIST]	9.143555377 -01	
[F] DISP [·] 2	0.91	
[+/-]	-0.91	

Key Entry	Display	Explanation
$\boxed{+}$	-0.91	
1	1	
$\boxed{=}$	0.09	

Since 0.09 > 0.05, we do not reject the Null Hypothesis.

V. APPENDIX

A. Discrete and Continuous Probability Distribution Laws

Name	Parameters	Probability Mass Function
Binomial	n a positive integer, $0 \leqslant p \leqslant 1$	$C_k^n p^k (1-p)^{n-k}$ $k = 0, 1, 2, \ldots n$
Poisson	$\lambda > 0$	$\frac{e^{-\lambda}\lambda^k}{k!}$ $k = 0, 1, \ldots$ 0 Otherwise
Hypergeo-metric	n, ν_1, ν_2 positive integers $n \leqslant \nu_1 + \nu_2$	$\frac{C_p^{\nu_1} \; C_{n-p}^{\nu_2}}{C_n^{\nu_1+\nu_2}}$
Normal or Gaussian	$-\infty < \mu < \infty$ $\sigma > 0$	$\frac{1}{\sqrt{2\pi\sigma}} e^{-(x-\mu)^2/2\sigma^2}$
Chi Square or χ^2	ν is a positive integer (degrees of freedom)	$\frac{x^{(\nu-2/2)} e^{-x/2}}{2^{\nu/2}\,\Gamma(\nu/2)}$ $x > 0$ 0 Otherwise
F or Snedecor's F	positive integers (degrees of freedom)	$\frac{\Gamma\left(\frac{\nu_1+\nu_2}{2}\right)\left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2} x^{\left(\frac{\nu_1-2}{2}\right)}}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)\left(1+\frac{\nu_1}{\nu_2 x}\right)^{\frac{\nu_1+\nu_2}{2}}}$ $x > 0$ 0 Otherwise
t or student's t	positive integer (degrees of freedom)	$\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)\left(1+\frac{x^2}{\nu}\right)^{\frac{\nu+1}{2}}}$

	Mean	Variance	Moment Generating Function
	DISCRETE PROBABILITY DISTRIBUTIONS		
	np	$np(1-p)$	$(pe^t + 1 - p)^n$
	λ	λ	$e^{\lambda(e^t - 1)}$
	$\frac{n\nu_1}{\nu_1 + \nu_2}$	$\frac{\nu_1 \nu_2 n(\nu_1 + \nu_2 - n)}{(\nu_1 + \nu_2)^2 (\nu_1 + \nu_2 - 1)}$	$\frac{C_n^{\nu_1}}{C_n^{(\nu_1 + \nu_2)}}$
	CONTINUOUS PROBABILITY DISTRIBUTIONS		
	μ	σ^2	$e^{t\mu + (\sigma^2 t^2/2)}$
	ν	2ν	$(1-2t)^{-\nu/2}$
> 0	$\frac{\nu_2}{\nu_2 - 2}, \nu_2 > 2$	$\frac{2\nu_2^2 (\nu_1 + \nu_2 - 2)}{\nu_2 (\nu_2 - 2)^2 (\nu_2 - 4)}$	
	$0, \nu > 1$	$\frac{\nu}{\nu - 2}, \nu > 2$	Characteristic Function

B. Important Formulas and Useful Relationships

1. *Central Limit Theorem in one dimension*

Let $x_1, x_2, \ldots$ be independent identically distributed random variables with mean μ and variance $\sigma^2 > 0$. Let $-\infty < a < b < +\infty$ then

$$\lim_{k \to \infty} P\left(a \leq \sqrt{k}\left(\sum_{j=1}^{k} x_j/k - \mu\right) \leq b \right) = \int_a^b \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$$

2. *Poisson approximation to the binomial distribution*

Let X_λ be a Poisson random variable with parameter λ and for each n let Xn be a binomial random variable with parameters n and Pn. If $\lim_{n \to \infty} np_n = \lambda$, then

$$\lim_{n \to \infty} P(X_n = k) = P(X_\lambda = k) \quad k = 0, 1, 2, \ldots$$

3. *Error function*

$\operatorname{erf} x = 2\Phi(x\sqrt{2}) - 1$ where $\Phi(x)$ = Gaussian probability cumulative distribution function

4. *Incomplete gamma function*

$$\frac{\gamma(a, x)}{\Gamma(a)} = P(\chi^2 | \nu) \quad \text{where } \nu = 2a,\ \chi^2 = 2x$$

$P(\chi^2 | \nu)$ = Chi-square distribution function

$$0 \leq \chi^2 < \infty$$

5. *Pearson's incomplete gamma function*

$$I(u,p) = \frac{1}{\Gamma(p+1)} \int_0^{\mu\sqrt{p+1}} t^p e^{-t} dt = P(\chi^2 | \nu) \qquad \begin{array}{l} \nu = 2(p+1) \\ \chi^2 = 2\mu\sqrt{p+1} \end{array}$$

6. *Poisson distribution*

$$1 - P(\chi^2 | \nu) = \sum_{j=0}^{C} e^{-\lambda} \frac{\lambda}{j!} \quad C + 1 = \frac{\nu}{2}, \lambda = \frac{\chi^2}{2}, \ (\nu \text{ even})$$

$$P(\chi^2 | \nu - 2) - P(\chi^2 | \nu) = \frac{e^{-\lambda} \lambda^C}{C!}$$

7. *Generalized Laguerre Polynomials*

$$n! \, L_n^{(\alpha)}(x) = \frac{\sum_{j=0}^{n+1} (-1)^{n+j} C_j^{n+1} (1 - P(\chi^2 | \nu + 2 - 2j))}{2^n \, [P(\chi^2 | \nu) - P(\chi^2 | \nu + 2)]}$$

8. *Pearson type III*

$$\left(\frac{ab}{e}\right)^{ab} \int_{-a}^{x} \left(1 + \frac{t}{a}\right)^{ab} e^{-bt} \, dt = P(\chi^2 | \nu)$$

$$\nu = 2ab + 2$$

$$\chi^2 = 2b(x + a)$$

9. *Binomial cumulative distribution related to F distribution*

$$\sum_{s=a}^{n} C_s^n \, p^s (1-p)^{n-s} = 1 - P(F | \nu_1, \nu_2) \quad \text{where } a = C + 1, \ C = \text{no. of defectives}$$

$$F = \frac{a - aP}{(n - a + 1)P} \qquad \nu_1 = 2(n - a + 1), \quad \nu_2 = 2(a)$$

10. *Relation between Binomial and Normal distributions*

If N is large and if neither p nor q is too close to zero, the binomial distribution can be closely approximated by a normal distribution with standardized variable given by

$Z = \dfrac{x - Nq}{\sqrt{Npq}}$. The approximation becomes better with increasing N, and in the limiting case is exact.

11. *F distribution related to the incomplete beta function*

$$Q(F \mid \nu_1, \nu_2) = Ix\left(\frac{\nu_2}{2}, \frac{\nu_1}{2}\right) \text{ where}$$

$$Ix(a, b) = \frac{1}{B(a, b)} \int_0^x t^{a-1} (1-t)^{b-1} \, dt$$

12. *The Negative binomial distribution approximated to the Poisson distribution*

$$f(k; r, P) \to \frac{e^{-\lambda} \lambda^k}{k!} \quad \text{if } r \to \infty \text{ and } rq = \lambda$$

13. *Normal approximation to the hypergeometric distribution*

$$\frac{C_k^{\nu_1} \, C_{n-k}^{\nu_2}}{C_n^{\nu_1 + \nu_2}} \sim h \, \Phi(x)$$

where

$$\frac{n}{\nu_1 + \nu_2} \to t, \quad \frac{\nu_1}{\nu_1 + \nu_2} \to p, \quad \frac{\nu_2}{\nu_1 + \nu_2} \to q$$

$$h \left\{ k - np \right\} \to x$$

$$h = \frac{1}{\sqrt{(\nu_1 + \nu_2) \, pqt(1-t)}}$$

14. *Incomplete moments of Normal Distribution*

$$\int_0^x t^n Z(t)\,dt = \begin{cases} (n-1)!!\ \dfrac{P(\chi^2|\nu)}{2} & (n \text{ even}) \\ \dfrac{(n-1)!!}{\sqrt{2\pi}}\ P(\chi^2|\nu) & (n \text{ odd}) \end{cases}$$

$\chi^2 = x^2,\ \nu = n+1$

15. *Special cases of confluent hypergeometric function*

$$M\left(\frac{1}{2}, \frac{3}{2}, -\frac{x^2}{2}\right) = \frac{\sqrt{2\pi}}{x}\left\{\Phi(x) - \frac{1}{2}\right\} \qquad x > 0$$

$$M\left(1, \frac{3}{2}, \frac{x^2}{2}\right) = \frac{1}{x\,Z(x)}\left\{\Phi(x) - \frac{1}{2}\right\}$$

where $Z(x) = \dfrac{1}{\sqrt{2\pi}}\, e^{-x^2/2}$

$x \cdot 0$

ed

C. Glossary of Symbols

df	degrees of freedom
df_1	treat degrees of freedom
df_2	Error degrees of freedom
E_i	A residual (i.e., $y_i - \hat{y}_i$)
E_i	Expected frequency
f_i	Observed frequency $[0_i]$
Ho	Null hypothesis
H_A	Alternate hypothesis
N	The size of a sample [n]
p	proportion in a binomial population
r	Sample correlation coefficient
r^2	Sample coefficient of determination in simple regression
RSS	Residual Sum of Squares
SS	Sum of Squares
SD	Standard deviation $[s, \sigma]$
SE	Standard error $[S_{\bar{x}}, \sigma_{\bar{x}}]$
S	Sample standard deviation, estimates σ [s]
S_x	Standard deviation of x values $[s_x]$
S_y	Standard deviation of y values $[s_y]$
$S_{\bar{x}}$	Standard error of mean $[s_{\bar{x}}]$
$S_{\bar{\beta}}$	Sample standard error of y intercept $[s_{\bar{\beta}}]$
$S_{\bar{\alpha}}$	Standard error of regression coefficient V $[\alpha]$
$S_{y \cdot x}$	Standard error of a regression $[s_{y \cdot x}]$
S_r	Sample standard error of correlation $[s_r]$
S^2	Sample variance; estimates σ^2 [V]
$S^2_{\bar{x}}$	Sample variance of the mean, estimates $\sigma^2_{\bar{x}}$
t	"students t," a test statistic
Xi	A sample variable, the independent variable in linear regression $[x_i]$
(Xi, Yi)	A data point of a simple linear regression line
Y_i	The dependent variable in regression $[y_i]$
$\bar{X}$	Sample mean of x, estimates μ $[\bar{x}]$
$\hat{x}_i$	predicted value of an independent variable, x, in regression (i.e., the value of x on a regression line)
Z	A normal deviate
α	The Y intercept in regression

β	The regression coefficient (slope)
μ	population mean
ν	degrees of freedom [df, DF]
Σ	Taking the sum
σ	Population standard deviation (SD)
$\sigma_{\overline{X}}$	Population standard error of mean
σ^2	Population variance
χ^2	Chi-square, a test statistic
X_{ij}	The jth measurement of a variable in group i

D. Summary of Important Topics in Statistics

1. *Single Factor Analysis of Variance*

To test the null hypothesis Ho: $\mu_1 = \mu_1 \ldots = \mu k$ where k is the number of experimental groups, or samples. Goup i (i = 1, 2, . . . k) has n_i observations (treatment group may have equal or unequal number of observations).

Sum_i = sum of observations in treatment group i.

$$= \Sigma X_{ij}$$

$$\text{Total SS} = \sum_{i=1}^{k} \sum_{j=1}^{n_i} X_{ij}^2 - \frac{\left(\sum_{i=1}^{k} \sum_{j=1}^{n_i} X_{ij} \right)^2}{\sum_{i=1}^{k} n_i}$$

$$\text{Treat SS} = \sum_{i=1}^{k} \frac{\left(\sum_{j=1}^{n_i} X_{ij} \right)^2}{n_i} - \frac{\left(\sum_{i=1}^{k} \sum_{i=1}^{n_i} X_{ij} \right)^2}{\sum_{i=1}^{k} n_i}$$

Error SS = Total SS - Treat SS

df_1 = Treat df = k - 1

df_2 = Error df = $\Sigma n_i - k$

$$\text{Treat MS} = \frac{\text{Treat SS}}{\text{Treat df}}$$

$$\text{Error MS} = \frac{\text{Error SS}}{\text{Error df}}$$

$$F = \frac{\text{Treat MS}}{\text{Error MS}} \quad \text{(with } k-1 \text{ and } \Sigma n_i - k \text{ degrees of freedom)}$$

2. *Linear Regression*

The simple linear regression equation is given as:

$$y_i = \alpha + \beta x_i$$

let a, b be the estimates of α and β respectively. Then,

$$\Sigma xy = (x_i - \bar{x})(y_i - \bar{y}) = \Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{N}$$

$$\Sigma x^2 = \Sigma (x_i - \bar{x})^2 = \Sigma x_i^2 - (\Sigma x_i)^2 / N =$$

$$\Sigma y^2 = \Sigma (y_i - \bar{y}) = \Sigma y_i^2 - (\Sigma y_i)^2 / N = \text{Total SS}$$

a) slope b

$$b = \frac{\Sigma x_i y_i - \dfrac{(\Sigma x_i)(\Sigma y_i)}{N}}{\Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{N}}$$

b) intercept a

$$a = \bar{y} - b\bar{x} \quad \text{where } \bar{x} = \frac{\Sigma x_i}{N} \text{ and } y = \frac{\Sigma y_i}{N}$$

c) coefficient of determination r^2

$$r^2 = \frac{\left[\Sigma x_i y_i - \dfrac{\Sigma x_i y_i}{N}\right]^2}{\left[\Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{N}\right]\left[\Sigma y_i^2 - \dfrac{(\Sigma y_i)^2}{N}\right]} = \frac{(\Sigma xy)^2}{(\Sigma x^2)(\Sigma y^2)}$$

d) estimated value $\hat{y}$ on the regression line for any given x

$$\hat{y} = a + bx$$

e) Regression SS $= \dfrac{(\Sigma xy)^2}{\Sigma x^2} = \dfrac{\Sigma x_i y_i - \dfrac{(\Sigma x_i)(\Sigma y_i)}{N}}{\Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{N}}$

f) RSS = TSS–RegSS $= \Sigma (y_i - \hat{y})^2$

g) standard error of estimate of y on x

$$S_{y \cdot x} = \frac{\Sigma (y_i - \hat{y})^2}{N-2} = \sqrt{\frac{RSS}{N-2}}$$

h) standard error of the regression coefficient, a (the intercept)

$$S_{\bar{a}} = S_{y \cdot x} \sqrt{\frac{\Sigma x_i^2}{N \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{N}}} = \sqrt{S^2{}_{yx} \quad \frac{\Sigma x_i^2}{N \Sigma x^2}}$$

i) standard error of slope, b

$$S_{\bar{b}} = \frac{S_{y \cdot x}}{\sqrt{\Sigma x_i^2 - \dfrac{\Sigma (x_i)^2}{N}}}$$

j) Linear Regression Mean Square $= \dfrac{\text{Reg SS}}{\text{Reg df}} = \dfrac{\text{Reg SS}}{1}$

k) Residual Mean Square $= \dfrac{\text{RSS}}{\text{Residual df}} = \dfrac{\text{RSS}}{N-2} = S^2_{y \cdot x}$

l) To test for Ho: $\beta = 0$ $\quad F = \dfrac{\text{MS regression}}{\text{MS residual}}$
H_A: $\beta \neq 0$

which is compared with the critical value, $F\alpha, \nu_1, \nu_2$ where ν_1 = df regression = 1 and ν_2 = df residual = N–2.

m) standard deviation of the x values

$$S_x = \sqrt{\frac{\Sigma x_i^2 - N\bar{x}^2}{N-1}}$$

n) standard deviation of the y values

$$S_y = \sqrt{\frac{\Sigma y_i^2 - N\bar{y}^2}{N-1}}$$

3. *Confidence Intervals* (for (1−α) confidence limits)

a) for mean:

$$\bar{X} - t\alpha, \nu\, S\bar{x} \leqslant \bar{X} + t\alpha, \nu\, S\bar{x}$$

b) for variance:

$$\frac{\nu S^2}{\chi^2_{\alpha/2,\nu}} \leqslant \sigma^2 \leqslant \frac{\nu S^2}{\chi^2_{(1-\alpha/2),\nu}}$$

d) for population standard deviation:

$$\sqrt{\frac{\nu S^2}{\chi^2_{\alpha/2,\nu}}} \leqslant S' \leqslant \sqrt{\frac{\nu S^2}{\chi^2_{(1-\alpha/2),\nu}}}$$

d) for regression coefficient (r):

$$\beta \pm t\alpha, N-2.\ S\beta$$

e) for an estimated y:

$$\hat{y}_i \pm t\alpha, \nu S\hat{y}_i$$

4. *Testing the Significance of r*

Two different procedures are used to test the hypothesis that r = 0. If N (the number of pairs) is 30 or larger, a critical-ratio z-test can easily be done. If N is smaller than 30, t-test should be done.

a) If $N \leq 30$, compute $z = z = r\sqrt{N-1}$. For example suppose $r = .56$ and $N = 37$ then

$$z = (-.56)\sqrt{37-1} = (-.56)\sqrt{36} = -.56 \times 6 = -3.36$$

If z is greater than $\pm$ 1.96, then r is significant at the 0.05 level using a 2 tailed test.

b) If $N < 30$, compute $t = r\sqrt{(N-2)/(1-r^2)}$
Supposing we have $r = +.87$ and $N = 15$. Thus,

$$t = (+.87)\sqrt{\frac{(15-2)}{1-.87^2}} = .87\sqrt{\frac{13}{1-.76}} = .87\sqrt{54.16} =$$

$$.87 \times 7.36 = 6.40$$

$$df = 15-2 = 13$$

E. Operating Accuracy

The precision of your calculator depends upon the operation being performed. Basic addition, subtraction, multiplication, division and reciprocal assignments have a maximum error of $\pm$ one count in the tenth or least significant digit.

While countless computations may be performed with complete accuracy, the accuracy limits of particular operations depend upon the input argument as shown below.

Function	Input Argument	Mantissa Error (Max.)
$\boxed{F}$ $\sqrt{x}$		2 counts in D_{10}
$\boxed{\ln}$ x		1 count in D_{10}
$\boxed{\log}$ x		1 count in D_{10}
$\boxed{F}$ e^x		3 counts in D_{10}
$\boxed{y^x}$		1 count in D_{10}
$\boxed{\sin}$ ϕ	$0° \leq \lvert\phi\rvert \leq 360°$ or $0 \leq \lvert\phi\rvert \leq 2\pi$	8 counts in D_{10}

Function	Input Argument	Mantissa Error (Max.)
$\boxed{\cos}$ ϕ	$0^\circ \leq \lvert\phi\rvert \leq 360^\circ$ or $0 \leq \lvert\phi\rvert \leq 2\pi$	8 counts in D_{10}
$\boxed{\tan}$ ϕ	$0 \leq \lvert\phi\rvert \leq 89^\circ$ $89^\circ \leq \lvert\phi\rvert \leq 89.95^\circ$	4 counts in D_9 1 count in D_6
$\boxed{F}$ $\sin^{-1}$ x	$10^{-10} \leq \lvert x\rvert \leq 1$	$E < 5 \times 10^{-8}$
$\boxed{F}$ $\cos^{-1}$ x	$10^{-10} \leq \lvert x\rvert \leq 1$	$E < 5 \times 10^{-8}$
$\boxed{F}$ $\tan^{-1}$ x		$E < 5 \times 10^{-8}$
Linear regression (all linear regression parameters)		5 counts in D_{10}
Mean and standard deviation		5 counts in D_{10}
t_{STAT}, Z_{STAT}, $t_{IND\ STAT}$, $t_{DEP\ STAT}$, χ^2 FIT		5 counts in D_{10}
Combination, Permutation, Hypergeometric, Binomial, Poisson, Chi-Square, F and t distribution.		5 counts in D_{10}
Gaussian		$\pm 3.5 \times 10^{-7}$
Inverse Gaussian distribution		$\pm 4.5 \times 10^{-4}$
n!	$n < 69$	2 counts in D_{10}

DN = Nth display digit assuming a left justified 10 digit result.

F. Error Conditions

An error condition results when an improper operation is performed or when the result of an operation overflows or underflows the absolute range of the calculator.

When an error condition occurs, the word ERROR is displayed on the calculator. To clear Error from display, depress $\boxed{C/CE}$

1. Overflow

Overflow occurs when a computed result is greater than $9.999999999 \times 10^{99}$.

2. Underflow

Underflow occurs when a computed result is less than 1.0×10^{-99}.

G. Rechargeable Battery

1. AC Operation

Connect the charger to any standard electrical outlet and plug the jack into the Calculator. After the above connections have been made, the power switch may be turned "ON".

2. Battery Operation

Disconnect the charger cord and push the power switch, "ON". With normal use a full battery charge can be expected to supply up to 2 hours of working time.

When the batter is low, figures on display will dim. Do not continue battery operation, this indicates the need for a battery charge. Use of the calculator can be continued during the charge cycle.

3. Battery Charging

Simply follow the same procedure as in AC operation. The calculator must be switched "OFF" to recharge the batteries. If however, it is left "ON" the calculator will act as a mains machine but will not recharge. If a power cell has completely discharged, the calculator should not be operated on battery power until it has been recharged for at least 4 - 6 hours, unless otherwise instructed by a notice accompanying your machine. Batteries will reach full efficiency after 2 or 3 charge cycles.

4. Adaptors

Use proper Commodore/CBM adaptor-recharger for AC operation and recharging.

Adaptor 640 or 707 North America

Adaptor 708 England

Adaptor 709 Continental Europe

5. Low Power

If battery is low calculator will:

a. Display will appear erratic

b. Display will dim

c. Display will fail to accept numbers

If one or all of the above conditions occur, you may check for a low battery condition by entering a series of 8's. If 8's fail to appear, operations should not be continued on battery power. Unit may be operated on AC power. See battery charging explanation. If machine continues to be inoperative see guarantee section.

6. CAUTION

A strong static discharge will damage your machine.

H. Service

A defective machine should be returned to the authorized service center nearest you.

See listing of service centers on back cover.

J. Temperature Range

Mode	Temperature °C	Temperature °F
Operating	0° to 50°	32° to 122°
Charging	10° to 40°	50° to 104°
Storage	–40° to 55°	–40° to 131°

K. Warranty

Your new electronic calculator carries a parts and labour warranty for 12 months from date of purchase.

We reserve the right to repair a damaged component, replace it entirely, or, if necessary, exchange your machine.

This warranty is valid only when a copy of your original sales slip or similar proof of purchase accompanies your defective machine.

This warranty applies only to the original owner. It does not cover damage or malfunctions resulting from fire, accident, neglect, abuse or other causes beyond our control.

The warranty does not cover the repair or replacement of plastic housings or transformers damaged by the use of improper voltage. Nor does it cover the replacement of expendable accessories and disposable batteries.

The warranty will also be automatically void if your machine is repaired or tampered with by any unauthorised person or agency.

This warranty supersedes, and is in lieu of, all other expressed warranties.

Sales and Service Centres

Commodore Business Machines Inc.,
901 California Avenue, Palo Alto, California 94304, U.S.A.

Commodore Business Machines (Canada) Ltd.,
946 Warden Avenue, Scarborough, Ontario, Canada.

Commodore Business Machines (U.K.) Ltd.,
(U.K. Sales Office), 446 Bath Road, Slough, Berkshire, England.

Commodore Business Machines (U.K.) Ltd.,
(Service Centre), Eaglescliffe Industrial Estate, Stockton on Tees, Cleveland, TS16 0BR, England.

Commodore Austria,
Nikolaygasse 1/2/1, Post Box 238, 1024 Vienna, Austria.

Commodore France S.A.,
Zone Industrielle, Departmentale M14, 06510 Carros, France.

Commodore Buromaschinen GmbH,
6072 Dreieich 1, Robert Bosch Str 12a, West Germany.

Commodore S.P.A.,
Divisione Italiana, Via Hesinone 6, 18038 San Remo (IM), Italy

Commodore AG Schweiz,
Bahnhof Strausse 29-31, CH-5000 Aarau, Switzerland.

Commodore Japan Ltd.,
Taisei Denshi Bldg., 8-14, Ikue 1-Chome, Asani-Ku, Osaka 535, Japan.

Commodore International (HK),
Floor 11, Block C, Watson's Estate, Hong Kong.