K'NEX® Education™

TEACHER'S GUIDE

INTERMEDIATE MATH & GEOMETRY™
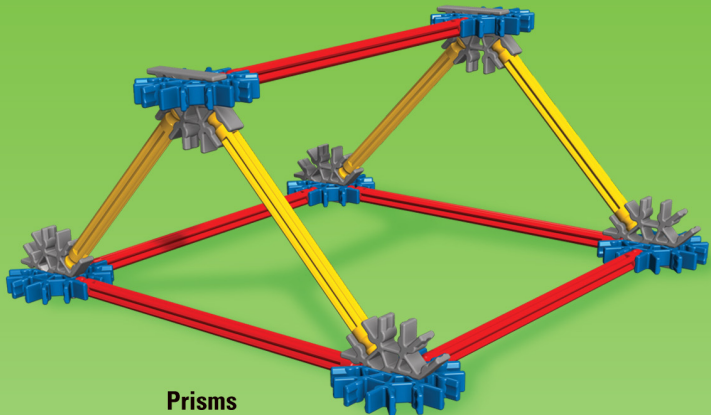
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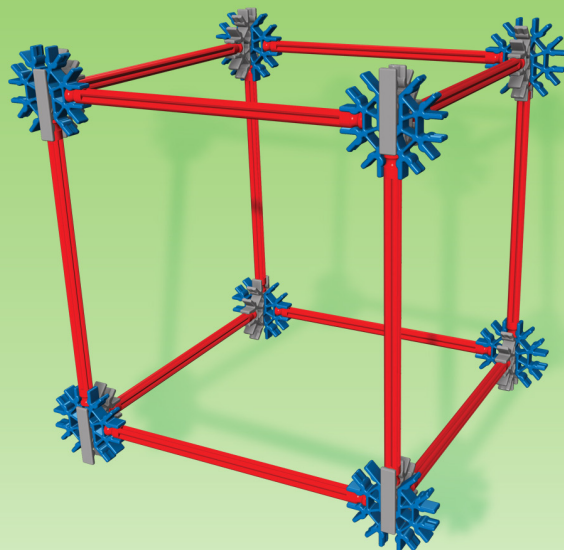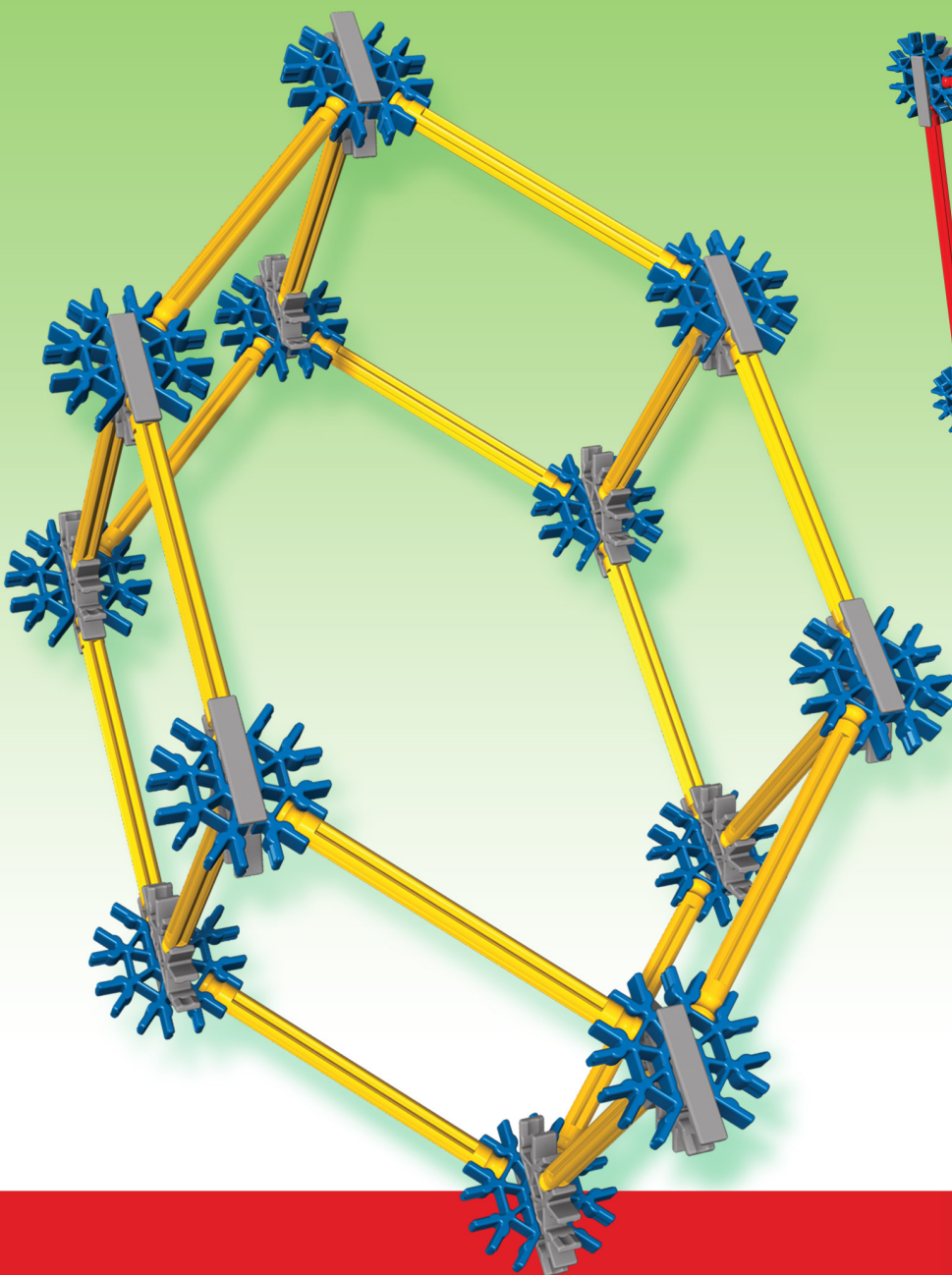
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INTERMEDIATE
MATH & GEOMETRY™
Teacher's Guide

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⚠ **WARNING:**   ⚠ **AVERTISSEMENT:**

CHOKING HAZARD - Small parts.   DANGER D'ÉTOUFFEMENT - Piéces de petite taille.
Not for children under 3 years.   Ne pas donner aux enfants de moins de 3 ans.

TABLE OF CONTENTS

INTRODUCTION

Overview:

The Teacher's Guide for the Intermediate Math and Geometry set was developed to provide the support you need to facilitate hands-on learning and the support your students need to investigate a wide variety of mathematics concepts that form the core of mathematics instruction in the upper elementary grades.

Good math does not just happen – it takes planning and the effective building of concepts using all of a student's senses. Research[1] has shown that the use of manipulatives to develop mathematics concepts has many benefits. It allows students to improve their conceptual understanding, achievement and problem solving skills; it encourages student reflections on retaining the information; it promotes a positive attitude toward mathematics; and it develops more self-confidence in students. K'NEX is a powerful manipulative for students to use in developing and maintaining mathematics concepts.

In these times of high-stakes testing and the national desire to make students competitive at the international level it becomes more important than ever to set a sound foundation and understanding for students. In order to do this, teachers need to utilize exciting and powerful tools to actively engage students as they explore the wonders of mathematics and use mathematics to model the world around them.

As the education community moves away from the rote memorization of the past and encourages teachers to offer students opportunities to explore the concepts behind the mathematics they are learning, the role, use and quality of manipulatives must change. The K'NEX Education Intermediate Math & Geometry set provides exciting, dynamic materials and curriculum that will help you guide your students as they explore the patterns and relationships in their world and discover how mathematical concepts can be used to understand and model those patterns and relationships.

Intermediate Math & Geometry Set:

This set is designed to address critical mathematics concepts in the upper elementary school classroom and provide instructional models that will enhance students' understanding of important concepts and algorithms.

- The K'NEX building system can trace its development to the world of mathematical relationships and thus shapes and structures made from K'NEX can be easily linked to mathematical concepts, as you will see in the 14 lessons included in this guide.
- Using K'NEX and the direction provided in this guide teachers are able to offer students a program of study that uses hands-on exploration in conjunction with an engaging inquiry-based approach to learning.
- As students work cooperatively they are encouraged to interact with each other as they build, investigate, discuss and evaluate mathematical concepts, ideas and models.
- This set also provides alternative assessment options to better assess what all the students know.

Teacher's Guide:

This guide is intended as a resource for teachers and students as they tackle meaningful mathematics content in the classroom. This series of comprehensive lessons Includes:

- Student objectives
- Lesson overviews
- Materials lists
- Extension activities provide teachers with all of the information they need to successfully work with their students.
- The Teacher Notes section at the beginning of this guide includes additional information that teachers will find invaluable as they seek to motivate students to explore, experience and learn mathematics.
- There is also an NCTM Standards alignment chart that includes reference to each of the lessons, assessment strategy ideas, suggested K'NEX mathematics conventions and more.

Student Journals:

We suggest that students maintain a journal for the activities they complete from the Intermediate Math & Geometry set. A loose-leaf format serves this purpose well. Students should include notes, drawings, and conjectures made by the students to provide a comprehensive record of the growth of individual students. This information is an excellent source for assessment data.

[1]References:

Johnson, L. (2005). Math Manipulatives. *Parents Teaching Overseas.* Retreived July 17, 2009 from ICHED Web site: http://www.iched.org/cms/scripts/page.php?site_id=iched&item_id=math_manipulatives.

Moyer, P. S. and Reimer, K. (2005). Third-Graders Learn About Fractions Using Virtual Manipulatives: A Classroom Study. *The Journal of Computers in Mathematics and Science Teaching.* 24:5-25.

Rust, A. L. (1999). A Study of the Benefits of Math Manipulatives Versus Standard Curriculum in the Comprehension of Mathematics Concepts, Master Action. 18.

Young, S. (2004). Controlling Choice: Teachers, Students and Manipulatives in Mathematics Classrooms. *School Science and Mathematics.* 104 (1): 16-31.

Teacher's Notes

This set of K'NEX has enough materials to serve four groups of 3 – 4 students each working cooperatively on any of the grade 3 – 5 activities outlined in this Teacher's Guide. The materials can also be divided into four smaller units and used to supply mathematics activity centers around the classroom.

The compartmentalized tray that holds the K'NEX pieces may serve as a resource center for each of the groups to draw from during activities or you may choose to separate the materials into four smaller sets before the students begin to work.

As with any classroom manipulative, it is suggested that you provide time for students to explore building with the K'NEX pieces on their own at the beginning of their first session with the materials. Students are curious and will want to explore and investigate so whether we provide time for that personal exploration or not, the students are going to take time to do it on their own.

When you first introduce K'NEX in the classroom, ask for a show of hands to indicate which students have used K'NEX in other classrooms or at home. When you form groups for instruction, include an experienced K'NEX builder in each group.

Also, assign groups carefully so that students of varying abilities are in each group.

Lesson Introduction:

The first page or two of each lesson provides an outline of the information you will be presenting and includes the following:

- **Lesson Length**: the suggested class time needed to complete the lesson.
- **Student Objectives**: objectives that you can expect your students to achieve through the successful completion of the activity.
- **Overview of Lesson**: a listing of what the lesson will convey to students and how the lesson will flow.
- **Materials and Equipment**: a list of the materials that will be needed to complete the lesson.

- **Development**: a brief outline of how the activity will progress. Additionally, an introductory question or statement to pique student interest is included.
- **Summary and Closure**: what types of information should the students provide to indicate the level of their understanding at the end of the lesson.
- **Assessment Activities**: suggestions for hands-on assessments of the mathematics concepts discussed in the lesson. These assessments may be done either individually or as a group.
- **Extensions**: listings of possible extension activities to use when appropriate.

National Council of Teachers of Mathematics Standards Alignments:

The K'NEX Education Intermediate Math and Geometry Set for Grades 3 – 5 was designed for use in a standards-based environment.

- The lessons and activities in this Teacher's Guide were all crafted to provide students with an opportunity not only to meet but to exceed the levels of mathematics understanding outlined in the NCTM Standards.
- The NCTM Standards & Expectations chart, found on pages 8 and 9 indicates which of the lessons address the various standards. This chart makes it

possible for the teacher to select a group of lessons that align with a particular set of standards or to plan a series of lessons to support the local curriculum.

- K'NEX Education maintains and updates a section of the www.knexeducation.com web site that includes alignments of each of its mathematics sets with NCTM as well as state standards.
- Go to **www.knexeducation.com**, click on your state on the web site's standards map, then scroll down to the standards alignments for the Intermediate Math & Geometry set for Grades 3 – 5.
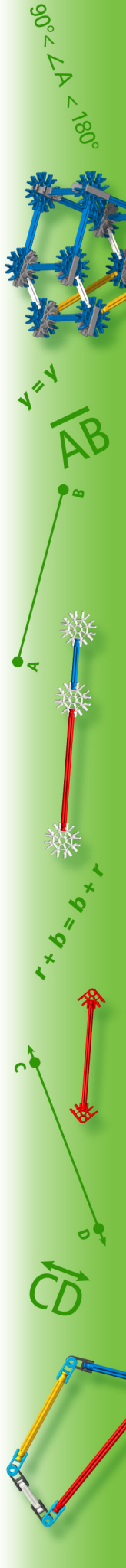
Mathematics Conventions with K'NEX:

The author of this Teacher's Guide and the staff at K'NEX Education have developed some mathematics conventions that will enable students to easily see relationships between their K'NEX creations and drawings, graphics, formulas and descriptions found in their textbooks, workbooks, quizzes, or tests.

Line, Line Segment, and Ray:

- Refer to page 2 in the Instructions Booklet. On the top of the page you will see models of a K'NEX line, line segment, and ray. These three models are made from red rods, white connectors, and red connectors.
  - The three models can be made from any of the K'NEX rods (green, white, blue, yellow, red, or silver).
  - Students can practice making a variety of these models using rods of various colors.
- The line segment on page 2 has two distinct endpoints which are highlighted with white connectors.
  - Students can transfer the red line segment to paper by 1) placing the model on a sheet of paper, 2) using a crayon or pencil to place a dot in the center of each endpoint and 3) after removing the model, connecting the two endpoints with a ruler. The same process can be used to transfer the ray and the line to paper.
  - In order to tell the three drawings apart once they are on paper, the two endpoints of the line segment and one endpoint of the ray should be made larger and darker. Arrow heads should be added to the drawings of the ray and the line where the red connectors had been. For naming purposes, points on the line and a point on the ray just before the arrow head should be darkened and enlarged.
- Any of the K'NEX line segments can be identified based on its color. Additionally, K'NEX has assigned a letter to serve as an abbreviation for each of the line segments. These abbreviations are lower case and they can be used to identify the line segment in written work or to identify the length of the line segment in formulas. The listing of the K'NEX line segments and their abbreviations can be found below. (If your curriculum or textbook places a bar above the abbreviations for line segments, please feel free to instruct students to do that when using these abbreviations.)

Green line segment	$= g$
White line segment	$= w$
Blue line segment	$= b$
Yellow line segment	$= y$
Red line segment	$= r$
Silver line segment	$= s$

Angles:

- Refer to page 2 in the K'NEX Instructions Booklet to see a variety of angles (two rays with a common endpoint) that can be formed using K'NEX pieces.
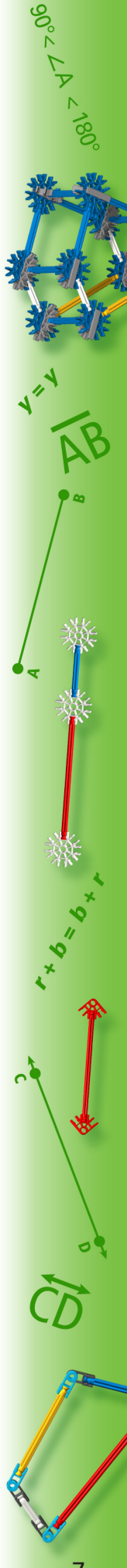
- The measures of fixed angles on standard K'NEX connector pieces are multiples of 45°. Notice on page 2 that there are some angles that can be formed with a two-piece hinge connector that allow you to form a wide range of angles. Any angle with measure from approximately 40° to 320° can be formed with the hinged connector pieces.
  - These K'NEX angles can be transferred to paper.
  - If the model is placed on a sheet of paper, a crayon or pencil can be used to place a dot in the center of the common endpoint and each arrow head. When the model is removed, the dots can be connected with a ruler and arrow heads can be drawn on the ends of the rays. The angle can now be measured or used for other activities requiring pencil and paper. (The opening in a hinged angle is small. A sharp pencil or mechanical pencil may be required to transfer one of these angles to paper.)

Perimeter and Area Notation:

- To describe the perimeter of a K'NEX shape students may use the abbreviations for the line segments that were introduced earlier. (See page 4, 5, 6, 7, and 13 of the Instructions Booklet for shape diagrams that can be used for activities where students determine perimeter.) For example:
  - A square with sides composed of 4 red rods would have a perimeter of $4r$.
  - A rectangle with a length composed of 2 blue rods and a width of 2 white rods would have a perimeter of $2b + 2w$.
  - An octagon with sides composed of 8 yellow rods would have a perimeter of $8y$.
- To describe the area of a K'NEX shape students can use rods of various colors, as they describe how many blue squares, red squares, or white squares, etc. that will describe the area of a polygon.
- Students should be able to determine the area of several different quadrilaterals that they build. (In higher grades, students will describe area in terms of b^2, r^2 or in other ways.) Some examples of areas might be:
  - A square with sides composed of 4 white rods would have an area of 1 "white square".
  - A square with sides composed of 4 red rods would have an area of 4 "blue squares" or 1 "red square".
  - A rectangle with a length composed of 1 red rod and 1 blue rod ($r + b$) and a width of 1 blue rod (b), will have an area of 3 "blue squares".

If students are given time to experiment with the concept of area using their K'NEX pieces they will identify many relationships that exist. They will also develop the ability to find the areas of many complex irregular polygons by decomposing the figure into simpler shapes. Refer to page 14 of the Instructions Booklet to find shapes that students can investigate to determine area using this strategy.

NCTM Standards & Expectations for Grades 3 – 5

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MEASUREMENT

Understand measurable attributes of objects and the units, systems, and processes of measurement:

- Understand such attributes as length, area, weight, volume, and size of angle and select the appropriate type of unit for measuring each attribute 2, 6, 9

- Explore what happens to measurements of a two-dimensional shape such as its perimeter and area when the shape is changed in some way 6, 9, 10

Apply appropriate techniques, tools, and formulas to determine measurements:

- Develop strategies for estimating the perimeters, areas, and volumes of irregular shapes 6, 9

DATA ANALYSIS AND PROBABILITY

Understand and apply basic concepts of probability:

- Describe events as likely or unlikely and discuss the degree of likelihood using such words as certain, equally likely, and impossible 8

- Predict the probability of outcomes of simple experiments and test the predictions 8

- Understand that the measure of the likelihood of an event can be represented by a number from 0 to 1 8

PROCESS STANDARDS

Problem Solving:

- Solve problems that arise in mathematics and in other contexts 9, 11, 13

- Apply and adapt a variety of appropriate strategies to solve problems 6, 8, 9, 10, 11, 13, 14

- Monitor and reflect on the process of mathematical problem solving 6, 8, 9, 10, 11, 13, 14

Reasoning and Proof:

- Make and investigate mathematical conjectures 3, 5, 6, 8, 9, 10, 11, 13, 14

- Develop and evaluate mathematical arguments and proofs 3, 5, 6, 8, 9, 10, 11, 13, 14

- Select and use various types of reasoning and methods of proof 3, 5, 6, 8, 9, 10, 11, 13, 14

Communication:

- Organize and consolidate their mathematical thinking through communication 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14

- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others 1, 2, 3,4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14

- Analyze and evaluate the mathematical thinking and strategies of others 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14

- Use the language of mathematics to express mathematical ideas precisely 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14

Connections:

- Recognize and use connections among mathematical ideas 3, 4, 5, 6, 7, 8, 12, 13, 14

- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14
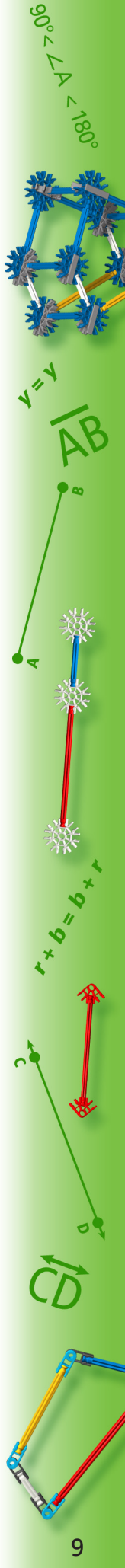
LESSON 1
Points, Lines, Line Segments, & Rays

Lesson Topics: Concept of Points, Lines, Segments, & Rays and the Relationship Between K'NEX Models, Drawn Models, and Geometry Notation.

Lesson Length: 60-minutes

Student Objectives:
Students will:
- Build and draw points, lines, line segments, and rays.
- Recognize, name, compare and sort points, lines, line segments, and rays.
- Transfer K'NEX points, lines, line segments, and rays to paper and label them following common mathematics conventions.
- Develop vocabulary and concepts related to points, lines, line segments, and rays.

Grouping for Instruction:
- Four groups of 3 – 4 students each for building models.
- Individual students for drawing models and labeling parts.
- Individual students for performance assessment and written assessment activities.

Materials and Equipment:
➤ *The K'NEX Intermediate Math & Geometry set and Instruction Booklets*

➤ *Pencils or crayons to share*

➤ *Journals and plain paper*

➤ *Rulers*

Overview of Lesson:
- Students use K'NEX materials to build, investigate and draw points, lines, line segments, and rays.
- Student groups will combine the models they have built and complete a round-robin activity to name and determine the attributes of each model according to standard mathematics conventions.
- Individual students will transfer a point, line, line segment, and ray to paper and add appropriate symbols and labels to demonstrate their individual understanding of the relationship between the K'NEX models and standard geometry notation conventions.
- Students will include drawings, names and attributes of their models in their notebooks for reference.

Background Information:
➤ *Students should have had exposure to the basics of points, lines, line segments, and rays as they are traditionally presented.*

A – Motivation and Introduction:
1. Introduce K'NEX rods and connectors to the students.
2. Provide time for creative exploration and play with the materials.
3. Select a student's collection of materials and identify a connector, rod, and a hinge pair.

B – Development (including discussion points and feedback):

POINTS:
1. Instruct each student to select a white connector. Introduce the concept that a connector with a circular hole in it will represent what in geometry we call, a point. Ask students to hold the connector firmly on a sheet of paper and to place a crayon or pencil in the hole on the connector to place a dot on the paper.
2. Have students darken their dot when they remove the connector. This dot represents a point. Ask students to label their point with a letter "A" using their pencil or crayon. (Note that when labeling a point a capital letter should be used.)
3. Help students to understand that when they make models using white connectors, the connectors can represent points and can be shown on paper as dots that are labeled with capital letters.
4. Instruct students to find other connectors in their sets that have a circular hole in them (look for orange,
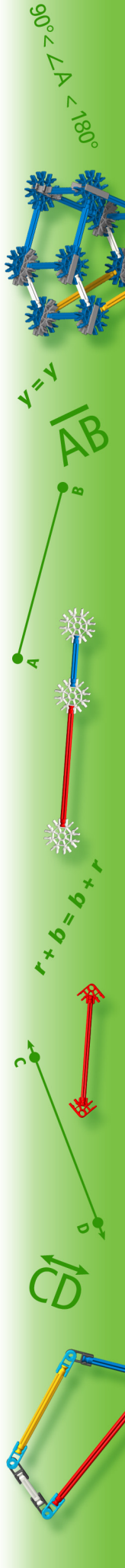
light gray, red, green, and yellow connectors).

5. Each of these connectors can be used to represent and draw points.

LINES, line segments, and rays:

1. Instruct the students to turn to page 2 in their Instructions Booklet. Ask students to make the three models they see on the left side of the page.

2. Ask students if the models look familiar. If someone remembers that lines, line segments, and rays look like these models, use the input to introduce these three models as K'NEX lines, line segments, and rays. If not, refer to a presentation of lines, line segments, and rays in the students' math textbooks and ask if the models remind them of anything they see in their textbooks.

3. Ask students to hold up the model they think represents a line, segment, or ray.

4. Instruct students to hold each model firmly on a piece of paper and to use a crayon or pencil to put a dot on the page for each point on the models. Ask students to use a ruler to connect the pairs of dots.

5. Ask students to place a model next to each of the diagrams and to place symbols on the drawings to represent the points and arrows on the shapes.

6. Ask students to label their drawings. Have groups discuss each of the drawings and to determine what the arrows on the shapes indicate. (An arrow represents an end that continues to infinity. The points represent end points which indicate the starting point or ending point for a line segment or ray.)

7. Ask students to label the end points on their segment: "A" and "B" respectively.

8. Instruct them to label the line with a "C" and a "D" respectively. There are no points on the drawing of the line to label. Instruct students that they need to add a point just before each of the arrows on their line drawing. The points provide locations to use as they identify the "C" and "D" points on their line. Repeat the activity for the ray using "E" and "F" with the students adding only one additional point to correctly label their ray drawing.

9. Introduce students to the concept of describing a line in a sentence, it is represented by two capital letters and a small line with a two headed arrow drawn above the letters. $\overleftrightarrow{CD}$

A segment is represented by two capital letters with a line above them. $\overline{AB}$

A ray is represented by two capital letters with a one headed arrow above them. $\overrightarrow{EF}$
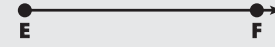
10. Help students to understand that the drawings they see in their textbooks and the K'NEX models they have built represent the same things.

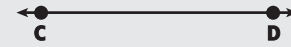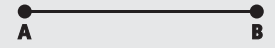11. Ask students to make a model of a blue line, a yellow segment and a red ray, etc. to ensure they have internalized the relationship between the models and their oral descriptions. Draw various lines, line segments, and rays on the board with various colors of chalk. Ask students to make the appropriate models using K'NEX. Write out several descriptions of lines, line segments, and rays as they might appear in a textbook, quiz, or test and instruct students to make the appropriate models.

12. Use a vocabulary game to help strengthen and practice vocabulary skills. Have students name one of the models in their group and pass the model to a classmate who must add additional information about the model and then pass it on to another student for even more information. (Possible student answers: red segment; two end points; defined length; written in text with two capital letters with a bar above them; drawn as a section of a line with two distinct end points on the line; etc.)

C – Summary and closure:

1. Have a collection of various line, line segment and ray models made up in advance.

2. Hold up the models one at a time. Ask students to name the model (blue ray, yellow line, etc.).

3. Ask students what they learned in this lesson.

4. Have students share their statements with others in their group.

5. Have students share with the class.

Assessment:

• Place the K'NEX line, line segment, and ray models, or loose K'NEX pieces at locations around the room and provide directions and or questions at each station. Have the students move from station to station.

For example:

• Use the pieces provided to make a red line segment model. When the model is built, wrap a piece of masking tape on it; add your name and put it in the collection box (provided by teacher).

• Build a yellow ray and draw it on your paper. Label your drawing and describe the ray using the letters "F" and "G".

• There are three models at this station. Draw and label each on your paper.

• There is a line segment model at this station. It can be described by its end points (A and B). What symbol would you use to describe the line segment in a sentence?
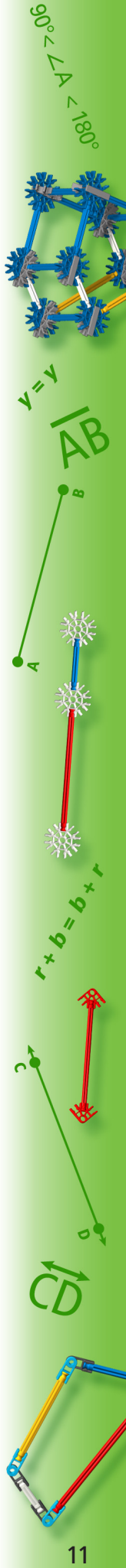
Lesson 2
Angle Conventions

Lesson Topics: Concept of Angles and the Relationship Between K'NEX Models, Drawn Angles and Geometry Notation.

Lesson Length: 60-minutes

Student Objectives:
Students will:
- Build and draw angles.
- Recognize, name, measure and compare angles and angle models.
- Transfer K'NEX angles to paper and label them following common mathematics conventions.
- Develop vocabulary and concepts related to angles.

Grouping for Instruction:
- Four groups of 3 – 4 students each for building models.
- Individual students for drawing models and labeling parts.
- Individual students for performance assessment and written assessment activities.

Materials and Equipment:
➤ *The K'NEX Intermediate Math & Geometry set and Instruction Booklets*

➤ *Pencils or crayons to share*

➤ *Journals and plain paper*

➤ *Rulers*

Overview of Lesson:
- Students use K'NEX materials to build, investigate and draw angles.
- Student groups will combine the models they have built and complete a round-robin activity to name and determine the attributes of each model according to standard mathematics conventions.
- Individual students will transfer angles to paper and add appropriate symbols and labels to demonstrate their individual understanding of the relationship between the K'NEX models and standard geometry notation conventions.
- Students will include drawings, names and attributes of their models in their notebooks for reference.

Background Information:
➤ *Students should have had exposure to the basics of points, lines, line segments, and rays as they are traditionally presented. Additionally, students should have a basic understanding of concepts related to angles.*

A – Motivation and Introduction:
1. Ask student groups to prepare a collection of different ray models for today's activities using a variety of rod colors.
2. Provide time for creative exploration with the rays.
3. Select a student's collection of materials and review the attributes of a ray.
4. Ask students to keep their rays for a later activity.

B – Development (including discussion points and feedback):
ANGLES:
1. Instruct students to turn to page 2 in their Instructions Booklets. Ask students to make the three models on the right side of the page.
2. Ask students if the models look familiar. If someone remembers that pictures they have seen of angles look like these models, use their input to introduce these models as K'NEX angles. If not, refer to a presentation of angles in the students' math textbooks and ask if the models remind them of anything they see in their textbooks.
3. Ask students to hold up the model they think represents a right angle, an obtuse angle and finally, an acute angle.
4. Ask students to describe why rays are important to the discussion of angles, angle drawings, or angle models. (An angle is made from two rays which share a common end point. Students can use the ray

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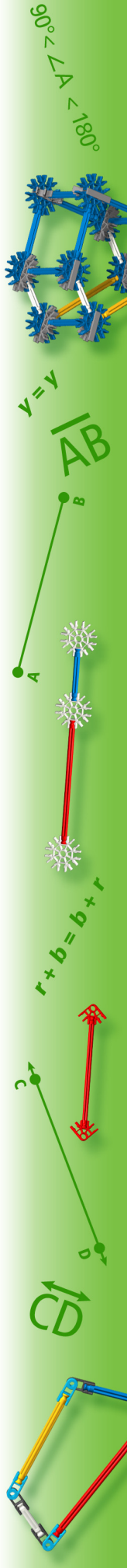
models they produced earlier to demonstrate the relationship between rays and angles.)

5. Can students describe the angles they have built in more detail? (One is a right angle, one is an obtuse angle and one is an angle that can be adjusted to a variety of angle measurements.)

6. Instruct students to hold each model firmly on a piece of paper and to use a crayon or pencil to put a dot on the page for each point on the model. Ask students to use a ruler to connect the three dots for each angle. (The circular openings in the hinges are narrow. Students will need to use sharp pencils or mechanical pencils to place dots for the hinged angle system they are using.)

7. Ask students to place a model next to each of the diagrams and to place symbols on the drawings to represent the points and arrows on the shapes.

8. Ask students to label their drawings. Have groups discuss each of the drawings and to determine what the arrows on the shapes indicate. (The arrows indicate that the ray continues to infinity. The points represent common end points that indicate the starting point for the rays that make up their angles.)

9. Based on the previous lesson, students should realize that each of the rays that make up an angle requires the addition of a point near the arrowhead to be used for naming purposes.

10. Instruct them to label their right angle with the letters A-B-C, the obtuse angle with the letters D-E-F, and the adjustable angle with the letters G-H-I.

11. Introduce students to the concept that when an angle is described in a sentence, it is described by three capital letters preceded by a small angle symbol.

The center letter is always the vertex of the angle. (∠ACB)

12. Help students to understand the relationship between what they see in their textbook and the K'NEX models that represent those same things.

13. Ask students to make a model of the following angles:

- 45°
- 90°
- 180°
- 225°
- 270° ... etc.

Do this to ensure the students have internalized the relationship between the models and their oral descriptions. Students can use protractors to verify that their models demonstrate the assigned angles. Write out several descriptions of angles as they might appear in a textbook, quiz, or test. Ask students to make appropriate angle drawings from the models and to label those drawings. (If students do not include a small arc as a part of the drawings for the 225 degree and 270 degree angles, demonstrate the importance of these symbols. Based on the notation used in your textbook you may wish to require the small arc notations for all angle drawings.)

C – Summary and closure:

1. Have a collection of angle models made up in advance.

2. Hold up the models one at a time. Ask students to name the model and provide as much detail as possible.

3. Ask students to think about what they learned in class today.

4. Have them share their statements students in their group.

5. Have selected students share with the class.

Assessment:

- Place K'NEX angle models or loose K'NEX pieces at locations around the room and have students move from station to station. Provide directions and or questions at each station.

For example:

- Use the pieces provided to make a model of a right angle. Place a masking tape tag on the model with your name on it and place the model in the box.
- Build a 135 degree angle and draw it on your paper. Label your drawing and describe the angle using the letters D-E-F.
- There are three models at this station. Draw and label each one on your paper.

- There is an angle model at this station. It can be described by three points:

  1. A at the arrow end of one ray.

  2. B at the arrow end of the other ray.

  3. C at the vertex of the angle.

- If you were describing this angle in a sentence, how would you abbreviate the angle? (∠ACB)
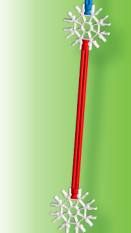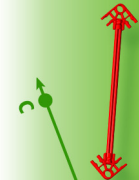
Length Conventions

Lesson Topics: Conventions and symbols that will enable students to measure and report lengths using K'NEX.

Lesson Length: 60-minutes

Student Objectives:

Students will:

- Build K'NEX models of various lengths and describe them with common math notation, terms and symbols.
- Describe lengths in letter form using appropriate mathematics notation.
- Describe lengths of previously unknown K'NEX rod sequences with letter symbols.
- Develop vocabulary and concepts related to length and length measurement.

Grouping for Instruction:

- Four groups of 3 – 4 students each for building models. (There are sufficient materials in your set that most students should be able to work individually or in pairs for this lesson.)
- Individual students for drawing models, labeling parts describing measurements.
- Individual students for performance assessment and written assessment activities.

Materials and Equipment:

➤ K'NEX Intermediate Math & Geometry set and Instructions Booklets

➤ 3" X 5" cards (cut in quarters with a math symbol on each card: =, <, and >). You may need as many as 16 sets of these cards depending on how the class is organized for instruction.

➤ Pencils or crayons to share

➤ Journals and plain paper

➤ Rulers

Overview of Lesson:

- Students use K'NEX materials to build, investigate, draw, and describe lengths. K'NEX conventions for the length of various rods will be presented to students and they will complete a series of activities to ensure that they have internalized the conventions and can use them in describing length measurements in future lessons.
- Individual students will transfer rod lengths to paper and describe the resulting line segments with appropriate mathematics symbolism and labels to demonstrate their individual understanding of the relationship between the K'NEX models and standard geometry notation conventions.
- Students will include drawings, conventions, and symbols for representative length models in their notebooks for reference.

Background Information:

➤ *Students should have had exposure to the basics of length measurement. They will explore a non-standard measurement system that will help them to better grasp the concept of length measurement. Actual number values can be attached to the length of the various K'NEX line segments in future years after students have internalized a working understanding of measurement.*

A – Motivation and Introduction:

1. Ask student groups to prepare a collection of different line segments for today's activities using all six basic K'NEX rod colors.
2. Provide time for creative exploration with the line segments. Do students sequence the line segments? Have they created all six of the models that are possible? If not, help them to identify any missing line segments that they can build.
3. Select a student's collection of materials and review the attributes of the various line segments.

B – Development (including discussion points and feedback):

LINE SEGMENT:

1. Instruct students to organize their six line segments in vertical rows in front of them. The shortest line segment (made from a green rod) should be to the left
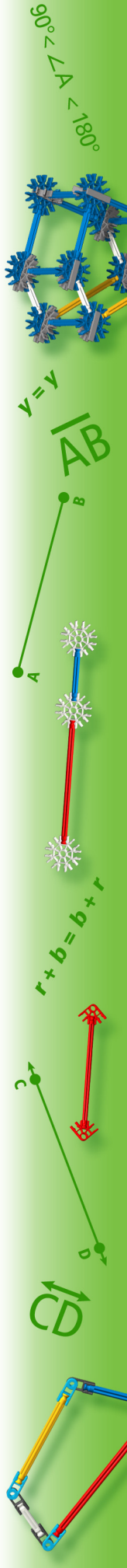
and the longest line segment (made with a silver rod) should be to the right.

2. Ask students if they can make any true statements about the relationships between the six models they have in front of them. "Write your ideas first and then we will share them as a class." Students may respond that the green line segment is the shortest, the blue line segment is shorter than the red line segment, etc.

3. Ellicit student responses and check with the rest of the class to determine if they are in agreement with statements that are offered.

4. Inform students that we can use symbols rather than words to describe some of the relationships they identified between the line segments.

First, they will need symbols (letters) to represent the line segments. With K'NEX, the following represent the standard letters for the line segments. These conventions hold true for line segments made with the following K'NEX connectors: purple, orange, light gray, red, green, yellow, white and hinge pairs.

- The letter "g" represents a green K'NEX line segment (the shortest K'NEX line segment). Note that when using a single letter to represent a line segment, a lower-case letter should be used.

- The letter "w" represents a white K'NEX line segment.

- The letter "b" represents a blue K'NEX line segment.

- The letter "y" represents a yellow K'NEX line segment.

- The letter "r" represents a red K'NEX line segment.

- The letter "s" represents a silver K'NEX line segment (the longest K'NEX line segment).

Second, students will need common mathematics symbols that are used to describe mathematical relationships. Distribute a collection of =, <, and > cards to each student or group of students.

5. Ask students if they can describe some of the statements they made about the relationships between the K'NEX line segments using the models and the =, <, and > cards? Expect students to place a sequence of a green line segment, a < symbol and a blue line segment in front of them on the table.

You may need to demonstrate the skill initially. Ask groups to combine their models and symbols to see how many different true math statements they can create with their cards and models.

6. Challenge students to transfer their line segments to paper and to use pencil and paper to draw their line segments and symbols in standard mathematics form. Students will draw a sequence of a line segment, a symbol and a second line segment.

7. Instruct students to describe each of the relationships they have drawn on their papers using letter and symbol codes. They can place their coded information under each of their sequence of drawings and symbols (For example: $w < r$, $y = y$, $r > w$). You may wish to require students to place a small bar above each of the letters to clearly identify the fact that the letters represent line segments. Let your standards, curriculum, and textbook conventions be your guide.

8. Provide challenges for children to solve using their models along with the letters and symbols they have to choose from. You might provide them a list of relationships and ask them to demonstrate the relationships described using models and symbol cards (For example: $b < r$, $r = r$, $r > w$) or (the blue line segment is shorter than the red line segment; the yellow line segment is longer than the white line segment)

C – Summary and Closure

1. Have a collection of line segment models made up in advance.

2. Hold up the models two at a time. Ask students to write down the appropriate letters and a symbol that could be used to describe the relationship between the line segments. (Remember, if you hold the models in from of you they will be in reverse order as seen by the students.)

3. Ask the students to think about what they learned in class today.

4. Have the students share their statements with their group.

5. Have selected students share with the class.

Assessment:

- Place K'NEX line segment models or loose K'NEX pieces at locations around the room and have students move from station to station. Provide directions and or questions at each station.

For example:

- Use the pieces provided to make a model of a red line segment and a white line segment. Have the students draw them in sequence on their paper with an appropriate symbol of their choice. Student responses may differ as they can sequence the drawings in either order.

- List several letter pairs and ask students to place them on their paper in order and to connect each pair with an appropriate symbol.

- List several letter pairs and ask students to build the appropriate line segment models and to transfer them to paper and place a symbol between each of the drawn line segments.
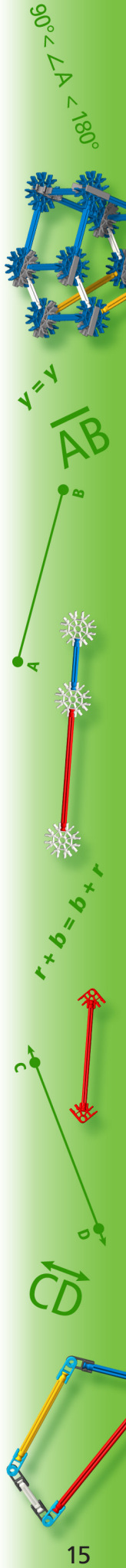
Lesson 4
Equalities and Inequalities

Student Objectives:
Students will:
- Build K'NEX line segment models made up of two or more rods and associated connectors to represent lengths.
- Use letter notation and "+" symbols to represent the length of longer line segment models. Relate letter notation to algebraic notation and formulas.
- State the length of models in a variety of equivalent letter units.
- Develop vocabulary and concepts related to length, algebraic notation, and length measurement.

Grouping for Instruction:
- Four groups of 3 – 4 students each for building models.
- Individual students for drawing models, describing measurements in a variety of formats and using proper mathematics (algebraic) notation.
- Individual students for performance assessment and written assessment activities.

Materials and Equipment:
➤ *K'NEX Intermediate Math & Geometry set and Instructions Booklets*

➤ *All four groups receive: 10 orange connectors, 10 yellow connectors, and 12 white connectors*

➤ *Two groups receive: 10 silver rods, 52 yellow rods, and 24 white rods*

➤ *Two groups receive: 48 red rods, 28 blue rods, and 10 green rods*

➤ *3" X 5" cards (cut in quarters with a math symbol on each card: =, <, and >). You may need as many as 16 sets of these cards depending on how the class is organized for instruction.*

➤ *Pencils or crayons to share*

➤ *Journals and plain paper*

➤ *Rulers*

Overview of Lesson:
- Students use K'NEX materials to build, investigate, draw, and describe long linear models. K'NEX conventions assign letter codes to describe the length of models made up of multiple rods and connectors. These letter codes are essentially algebraic descriptions of the length of the models in question.
- The students seek to develop an understanding of algebraic code through the application of this code to describe the length of linear models. As students move on to further lessons related to the perimeter and area of polygons, they will use this same algebraic language to quantify their descriptions of polygons and their attributes.
- Students will exercise their problem-solving skills as they identify numerical relationships between the lengths of some K'NEX line segments and use that information to state the length of various models in a variety of terms that are all equivalent.
- Individual students will transfer rod lengths to paper and describe the resulting line segments with appropriate mathematics symbolism and labels to demonstrate their individual understanding of the relationship between the K'NEX linear models and standard geometry notation conventions.
- Students will include drawings, conventions, and symbols as they represent length models in their notebooks for reference.

Background Information:

➤ *Students should have had exposure to the basics of length measurement. They will explore a non-standard measurement system that will enable them to better understand the concept of length measurement. Actual numbers will be attached to the letter codes in future years to quantify the length of various K'NEX line segments.*

A – Motivation and Introduction:

1. Ask student groups to use any rods they would like and orange, yellow, and white connectors to make some longer line segments that are made with endpoints and two or three rods arranged in a linear fashion.

2. Provide time for creative exploration with the line segments. How do the students sequence the line segments? How many models is each group able to make?

3. Select a student's collection of materials and review the attributes of the various line segments.

4. Hold up a line segment model that includes a blue rod, silver rod and a red rod. Ask students to describe the length of the total model. Work toward a response of $b + s + r$. Repeat the same discussion with a variety of other models made up of two or three K'NEX rods.

TEACHER NOTE:

It was established in an earlier lesson that: g = green line segment, w = white line segment, b = blue line segment, y = yellow line segment, r = red line segment and s = silver line segment.

B – Development (including discussion points and feedback):

EQUALITIES:

1. Instruct half of the groups (the groups with red, green and blue rods) to construct the two blue and red line segments on page 3 in the Instructions Booklet. Instruct the other half of the groups (the groups with white, yellow and silver rods) to construct the two silver and yellow line segments on that same page.

2. Ask students to write down a mathematical statement with letters and symbols to describe the models that they have constructed. (For example: one group may respond: $r + b$ and $b + r$) Ask students if they can make any true statements about the relationships between their two line segments. "Write your ideas first and then we will share them as a class."

3. Solicit student responses and check with the rest of the class to determine if they are in agreement with statements that are offered. Work with students until they realize that $r + b = b + r$ for one group and $s + y = y + s$ for the other group. If students are unsure, request that they hold the models on top of each other to see if the circular holes in the endpoints of the two long line segments line up perfectly in order to verify that they are in fact equal in length.

4. Instruct students to use as many rods and orange, white and yellow connectors as they can from their sets to build as many rod and connector combinations as they can that also equal their original line segment. One half of the groups will be working with the $r + b$ line segment and half of the groups will be working with the $s + y$ line segment. Ask students to describe

each of the combinations they find in terms of letters, "=" signs, and "+" signs. Students will eventually determine the relationships among the rods they are using and will be able to write the descriptions of the equivalent line segments without having to make all of the models.

For the teacher, a list of some of the relationships:

$$
\begin{aligned}
r + b \ = \ & b + r \\
= \ & b + b + b \\
= \ & b + b + g + g \\
= \ & b + g + g + b \\
= \ & g + g + g + g + g + g \\
= \ & \text{etc} \dots
\end{aligned}
$$

$$
\begin{aligned}
s + y \ = \ & y + s \\
= \ & y + y + y \\
= \ & y + y + w + w \\
= \ & y + w + w + y \\
= \ & w + w + w + w + w + w \\
= \ & \text{etc} \dots
\end{aligned}
$$

Students respond very well to the challenge to determine the highest number of possible equalities that they can make for their beginning line segment.

TEACHER NOTES:

- Students using the $r + b$ line segment should quickly realize that $r = b + b$ and $b = g + g$. With that information in hand, they can quickly list the various line segment combinations that equal $r + b$. No white, yellow, or silver rod combinations will equal $r + b$. Students using the $s + y$ line segment should quickly realize that $s = y + y$ and $y = w + w$. With that information in hand, they can quickly list the various line segment combinations that equal $s + y$ No green, blue, or red rod combinations will equal $s + y$.

- You will notice that all of the solutions for $b + r$ include red, blue and green rods while solutions for $s + y$ include only silver, yellow and white rods. This is a function of the characteristic of the K'NEX matrix of pieces. For your purposes, it means that students can be working side-by-side on identical relationships and using completely different sets of rods. This is a very efficient use of the materials in your Intermediate Math and Geometry Set.

- The two statements: $r + b = b + r$ and $s + y = y + s$ are excellent demonstrations of the commutative property of addition.

- Ask some of the students to use models, "=" symbols, and "+" symbols to demonstrate some of the statements of equality that they discovered.
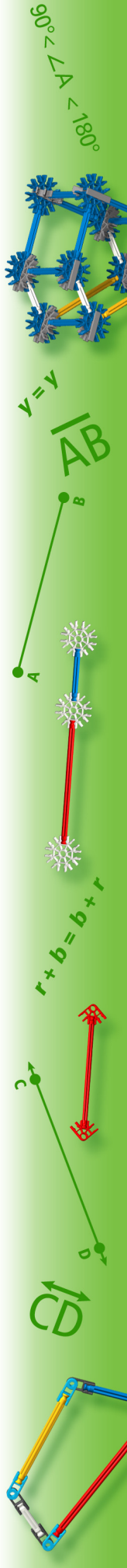
6. This is an excellent opportunity for you to work with some of the students' responses to see if there is an easier way to express some of their answers to #4 above. Can they come up with a better way to express $b + b + b$ or $y + y + w + w$? They will soon describe these as $b + b + b = 3b$ or $y + y + w + w = 2y + 2w$. To most observers, it appears as though the students are completing algebra activities and working with algebraic notation. They are in actuality, but in the students' minds, they are working with letters that describe colors. They are, in a sense, demystifying some of the algebraic code that they will investigate in an abstract form in the future. The establishment of an understanding of concrete examples now will provide the foundation for a better understanding of abstractions in the future.

7. Challenge students to develop some linear models that are made up of a combination of rods from each of the two groups (Group #1: red, blue, and green; Group #2 silver, yellow, and white). (To complete this activity, groups with red, green, and blue rods will have to exchange half of their rods with a group that has silver, white, and yellow rods) Are the students able to describe each of the line segments with a variety of equivalent algebraic expressions using their six-letter code and numbers where appropriate?

8. Provide multiple opportunities for students to practice their problem solving skills as they try to determine various ways to express lengths in equivalent forms. Some students will initially rely on trial and error strategies until they realize how the patterns and relationships between the various rod lengths work. You will see their skills and performance improve right before your eyes.

EQUALITIES:

1. In addition to the importance of equalities, inequalities provide another challenge for students as they practice their skills with problem solving and mathematical, geometric and algebraic relationships. Ask groups to create two long line segments that may or may not be equal in length.

2. Have the various groups lay out the models in a horizontal arrangement in pairs. They can then move to another groups' area and place either an "=" equal sign or a "≠" not equal sign between the two models. During the activity, the students may not touch the long, linear models or use other K'NEX pieces to recreate them. This is a mental challenge that the students enjoy very much. They love the opportunity to stump their classmates and when the symbols have been decided upon it is easy to place one of the models on top of the other to see if the line segments are actually equal in length or not.

C – Summary and Closure

1. Have a collection of long line segment models made up in advance.

2. Hold up a model. Ask students to write down the appropriate letters and symbols that could be used to describe the line segment. Can the students list one or two equivalent descriptions of the same line segment?

3. Ask students to think about what they learned in class today.

4. Have students share their statements to students in their group.

5. Have selected students share with the class.

Assessment:

• Place K'NEX line segment models or loose K'NEX pieces at locations around the room and have students move from station to station. Provide directions and or questions at each station.

For example:

• Place this message at the first station, "There is a long line segment at this station. Build a model of an equivalent expression that uses 2 fewer rods than the model that is given." (Be certain that an equivalent model can be made with 2 fewer rods.) Students can place a masking tape flag on their model that includes their name and place the solution in a tall box that is provided.
(For example: given $2b + s + 2w$, the students may build a model that can be described as $r + s + y$ because $2b + s + 2w = r + s + y$.)

• Place several very long line segments at a station and ask the students to describe an equivalent model with the least number of letters possible. Numbers like 3, 4, or 2 that precede a letter in a formula for a solution are not counted as letters for this activity.

• Ask students to design a series of challenge activities that would enable them to demonstrate their understanding of equalities and inequalities using models and formulas to describe the models they include in the activities.

• Complete a series of mental challenge activities with the entire class or with the various groups one at a time. Use a collection of pre-built models for the activity. Hold up a model and see who can describe a solution that includes none of the colors in the original model.
(For example: if you hold up $y + r + b + b$ someone may offer $2w + 8g$. Be certain that the models you offer can be solved with the restriction you place on the students.)

EF

$y + s = s + y$

$y + y + w + w = 2y + 2w$

$w < r$

$90° < \angle A < 180°$

$\angle A < 180°$
$90° <$

$v = v$

AB

$r + b = b + r$

CD
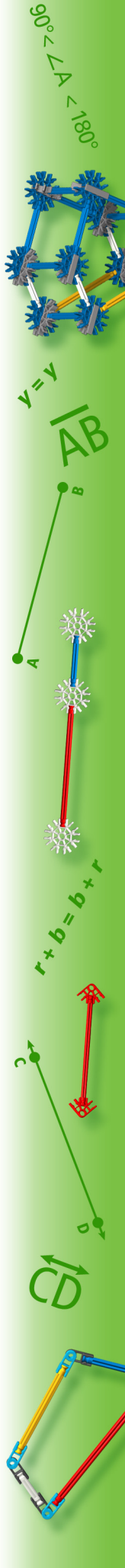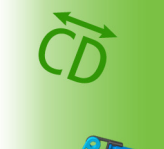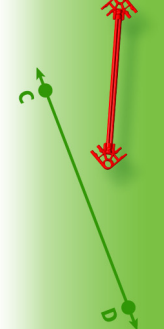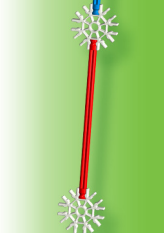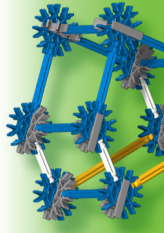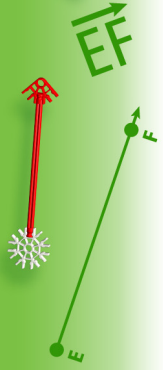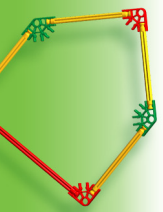
LESSON 5
Can You Name That Shape?

Lesson Topics: Basic Polygons.

Lesson Length: 90-minutes
(two 45 minute time periods)

Student Objectives:
Students will:
- Build and draw geometry objects.
- Recognize, name, build, draw, compare, and sort shapes.
- Describe attributes and parts of shapes: circle, rectangle, square, triangle, parallelogram (sides and vertices); locate interior (inside) angles and exterior (outside) angles.
- Identify right angles in polygons.
- Develop vocabulary and concepts related to two-dimensional and three-dimensional geometric shapes.

Grouping for Instruction:
- Individual students for building shapes, drawing shapes, and making a shape poster.
- Whole class for a shapes round-robin activity.

Materials and Equipment:
➤ K'NEX Intermediate Math & Geometry set and Instructions Booklets

➤ Chart or easel paper (one sheet per group)

➤ Pencils and crayons to share

➤ Journals and plain paper

➤ Rulers

Overview of Lesson:
- Students use K'NEX materials to build, investigate, and draw two-dimensional shapes (polygons).
- The class will combine the shapes they have built and draw from the pile to begin a round-robin activity to name and determine the attributes of a selection of the shapes.
- Individual students will return to their desks with a single shape to draw on poster paper.
- Students will include lists of: names, attributes and real world examples of their shapes.

Background Information:
➤ Students should have had exposure to basic shape names and have some knowledge of attributes.

A – Motivation and Introduction:
1. Introduce K'NEX rods and connectors to students and ask students to make some flat creations from the pieces in their sets.
2. Provide time for creative exploration with the materials.
3. Select a student's model or creation that is a polygon. Instruct students that the rods represent sides and the connectors represent vertices.

B – Development (including discussion points and feedback):
1. Instruct each student to build three to four different polygons. If students are having problems, refer them to polygons found on pages 4, 5, 6, and 7 of the

Instructions Booklet.
2. Have students take their shapes to an open area in the classroom. Have students stand in a circle and place their shapes inside the circle.
3. Select a student to pick up someone's shape (other than his or her own) that he/she can name.
4. Have the student name the shape. Pass the shape to the person on their right.
5. Have that person give one of the following:
   - An attribute of the shape
   - Another name for the shape
   - A real world example of the shape (an object with the same shape that you would find somewhere outside the classroom? Limit real world examples

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to three per shape as the game continues).

6. The shape continues moving to the right in round-robin style until students are stumped.

7. The next student picks a new shape. Feel free to move some shapes to the left.

8. Continue for about 15 to 20 minutes. Students will learn from each other's observations and the quality of responses will improve as the activity continues.

9. Students will now complete a follow-up activity based on this game. Tell students that they will each complete this activity with a single K'NEX polygon on their own.

10. Assign each student a shape from the floor and provide them with a large sheet of chart or easel paper.

11. Instruct students to:
   • Place the shape at the top of the page.
   • Use a pencil or crayon point to make a dot on the paper below the circular opening in each K'NEX connector (vertex) of their shape. If students have a hinged shape they may need a very sharp pencil or mechanical pencil to transfer their shape to paper.
   • Remove the shape and use a ruler to draw the shape by connecting the dots.
   • Use crayons to color the sides of the shapes.

• Leaving room below each shape, have students write the answers to these three questions below their shape:
   1. Names
   2. Attributes
   3. Real-world examples

• Begin their posters and fill in the three columns using information from the game or from their knowledge of geometry.

12. Have students share their posters with others and discuss their lists.

13. Select a few posters and review them with the class.

14. Ask students to question entries that they do not believe are correct.

C – Summary and Closure

1. Ask students to write three things that they learned today.

2. Have them share their statements with another student or students in their group.

3. Have students share with the class.

D. Assignment

Have students use other resources (text, Internet, library) to find three things to add to their lists during the next class session.

Assessment:

• Place K'NEX two-dimensional shapes (polygons) at locations around the room and have students move from station to station. Provide questions at each station.

For example:

• How many vertices does the shape have?
• List three attributes of the shape.
• List two names for the shape.
• Does the shape have any right angles? How many?

Alternate Version for Lesson 5:

• This alternate version of lesson #5 would take place during the students' study of polyhedra.

• Repeat the activity using three-dimensional models rather than the two-dimensional models described above.

• This activity provides you an excellent opportunity to introduce students to the terms "edge", "face", and "polyhedron".

• At the conclusion of the activity when the students are asked to transfer their shape to paper, an alternative to drawing their three-dimensional shape on paper is to have the students write the proper name of the shape at the top of their charts. They could also take digital pictures of their shape and tape a printed copy to the top of their charts.

• Students can refer to pages 10, 11 and 15 of the Instructions Booklet to find a variety of polyhedra to use during this activity.

• Also, encourage the students to create other polyhedra shapes on their own.
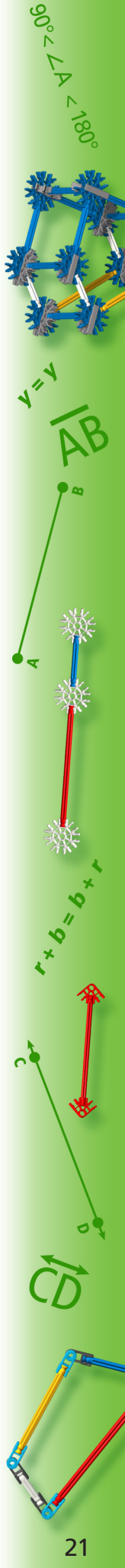
Lesson 6
Perimeter Concepts

Lesson Topics: The measurement and reporting of perimeter using K'NEX models of polygons.

Lesson Length: 60-minutes

Materials and Equipment:

➤ K'NEX Intermediate Math & Geometry set and Instructions Booklets

➤ Each group receives: 9 light gray, 20 red, 21 green, 10 yellow, and 12 white connectors in addition to 15 hinge pairs.

➤ Each group receives: 5 silver, 24 red, 26 yellow, 14 blue, 12 white, and 5 green rods.

➤ Chart or easel paper (one sheet per group)

➤ Journals and drawing paper

➤ Pencils and crayons to share

➤ Rulers

Student Objectives:

Students will:
- Build polygon models of various shapes and describe them in common mathematics terms and symbols.
- Follow appropriate mathematics conventions and algebraic expressions as they express the perimeter of polygons.
- Describe the perimeter of previously unknown K'NEX polygons using letter symbols.
- Develop and refine vocabulary and concepts related to perimeter and length measurement.

Grouping for Instruction:
- Four groups of 3 – 4 students each for building models.
- Individual students for drawing models, labeling sides, and listing a formula to represent perimeter measurements.
- Individual students for performance assessment and written assessment activities.

Overview of Lesson:
- Students will use K'NEX materials to build, investigate, draw, and describe the perimeter of polygons.
- Students will use K'NEX mathematics conventions presented and mastered in Lesson #3 as they describe the perimeter of polygons.
- Individual students will transfer the polygons they construct to paper and describe the perimeter of these polygons with appropriate mathematics symbolism and labels to demonstrate their individual understanding of the relationship between the K'NEX models, drawings and their perimeter.
- Students will include representative polygon drawings and perimeter solutions for a variety of models they or their classmates create.

Background Information:

➤ Students should have had exposure to the basics of length measurement and the concepts investigated in Lesson #3 and Lesson #4. They will use a non-standard measurement system to describe the perimeter of polygons and express the values in algebraic form. Actual number values can be attached to the length of the various K'NEX line segments in future years after students have internalized a working understanding of measurement and perimeter concepts.

A – Motivation and Introduction:

1. Ask student groups to prepare a collection of two-dimensional shapes (polygons) using all six K'NEX rods for the sides of the polygons. (For this activity, students can use light gray, red, green, yellow, or white K'NEX connectors or hinge pairs.) Refer to pages 4, 5, 6, and 7 of the Instructions Booklet.

2. Provide time for creative exploration with the polygons. Do students sequence the polygons? If so, how do they sequence them? How many different polygons have been created in the classroom?

TEACHER NOTES:

If there are four groups in the classroom, each group should be able to build nine different quadrilaterals, a pentagon, a hexagon, and an octagon at a minimum. This will provide a wide array of polygons for perimeter investigations.)

3. Lead students to realize that one characteristic of their polygons is perimeter, the distance around the figure.

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4. Have students indicate how they can express the perimeter of the polygons they have in front of them. (By using the letter code conventions they learned in Lesson #3.)

B – Development (including discussion points and feedback):

POLYGONS:

1. Instruct students to place all of the polygons that their group has constructed on one or two sheets of easel paper at their work station.

2. Ask students to work as a group and to transfer all of their polygons to the easel paper. (One student holds the model firmly to the page while another student uses a crayon or pencil to place a dot at each vertex of the polygon. Each of the connectors used as vertices has a circular hole in it at the exact location of the vertex.) When finished, students can use a ruler to connect the vertices to form exact drawings of their polygon models.

   CAUTION FOR STUDENTS:

   Complete the drawing of each polygon before moving on to the next. It is very easy to make drawing errors when facing a sheet of 20 or 30 vertices [points] and not being able to remember which ones go with which polygon.

3. Instruct students to discuss the perimeter of each model and to use a pencil to describe the perimeter symbolically next to the drawing of each polygon.

4. Use a round-robin-response format and ask groups to hold up one of their polygons and describe its perimeter orally.

   For example: students may describe the perimeters as shown below.

   • A parallelogram made of red and yellow rods is:

   $$r + y + r + y$$

   or

   $$2r + 2y$$

   • An equilateral hexagon with blue rods is:

   $$b + b + b + b + b + b$$

   or

   $$6b$$

   • This provides an excellent opportunity to help students to understand that the expression:

   $2r + 2y$ is equal to $r + y + r + y$

   and

   $2r + 2y = r + y + r + y$

   and

   $b + b + b + b + b + b = 6b$

TEACHER NOTES:

You can help reinforce students' oral information by writing student responses on the chalk board or white board.

5. Ask the other groups to verify the information provided.

6. Continue taking responses from the various groups until you are confident that the students are providing informed responses that demonstrate both an understanding of perimeter and proper algebraic expression of perimeter values.

8. *Variation activity:*

   Provide students with the perimeter of a polygon and see if they are able to build a polygon to match the given perimeter. This requires that students transfer what they have learned and practiced to a new learning situation. The ability to successfully complete these activities will provide very useful diagnostic data that you can use to understand the level of student understanding.

   Some suggested perimeters to provide students when you ask them to make the model to match the given information.

   • Perimeter = $4y$

     ANS: forms a yellow square or a yellow rhombus.

   • Perimeter = $2y + r$

     ANS: forms an isosceles right triangle with a red hypotenuse.

   • Perimeter = $3r + 2y$

     ANS: forms a pentagon that looks like the silhouette of a house or a pentagon that is concave.

   Let your imagination and the skill level of your students determine other appropriate perimeter challenges to provide for students.

9. Provide perimeter challenges for students to solve.

   • Ask Each group to make the models shown on **page 4** of the instructions:

     Transfer the models to paper and provide a perimeter for each.

   • Ask each group to make the models shown on **page 5** of the instructions.

     Transfer the models to paper and provide a perimeter for each.

   • Ask each group to make the models shown on **page 6** of the instructions.

     Transfer the models to paper and provide a perimeter for each.

   • Ask each group to make the models shown on **page 7** of the instructions.

     Transfer the models to paper and provide a perimeter for each.
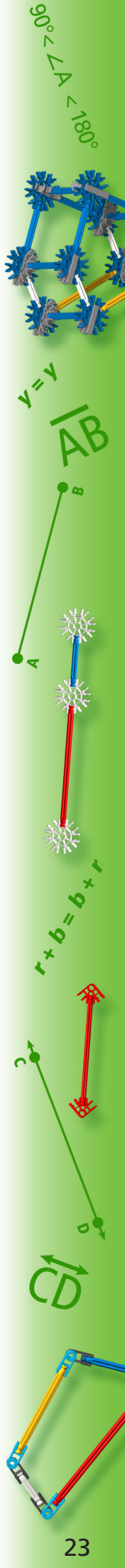
- Ask each group to make the models shown on **page 13** of the instructions.
- Transfer the models to paper and provide a perimeter for each.

TEACHER NOTES:

Based on the abilities that your students demonstrate, you may use all of the challenges above or you may select individual pages for your students. You may also choose to use some of these challenges for quizzes, tests, or other performance assessment activities.

C – Summary and Closure

1. Have a collection of polygon models made up in advance.
2. Hold up the models one at a time. Ask students to write down the appropriate letters and symbols that could be used to describe the perimeter of the polygons.
3. Ask students to write down three things that they learned in class today.
4. Have them share their statements with students in their group.
5. Have each group share at least one observation with the class.

Assessment:

- Establish a series of four or five performance assessment stations around the room that students can visit during a pencil and paper test or quiz that they are working on at their seats. Have one row of students at a time visit the performance assessment stations and provide one to one and one-half minutes for them to work at each station. Place polygon models or a collection of loose K'NEX pieces at stations around the room. Provide directions and or questions at each station.

For example:

- Use the pieces provided to make a polygon that has a perimeter of 6*b*. (When students have made the model, they can use a piece of masking tape to place their name on their creation before they place the model in a tall box provided by the teacher.)
- Place polygon models at several stations and ask:
- What is the perimeter of the polygon?

Challenge students:

Have students build models of polygons to use in the assessment scenarios. Student models can be placed around the room or hung from hooks above the board for other students to solve. This activity works best if the polygons are numbered. That allows the students to hand in a sheet with the polygon numbers and their appropriate perimeters.

EF

F

E

$y + s = s + y$

$w < r$

$y + y + w + w = 2y + 2w$

$90° < \angle A < 180°$

$v = v$

AB

B

A

$r + b = b + r$

C

D
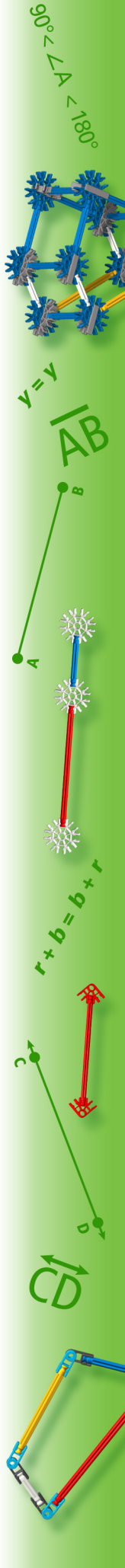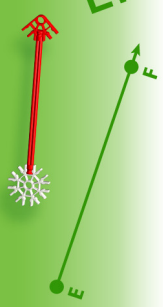
CD

LESSON 7

Symmetry

Lesson Topics: Defining symmetry and identifying lines of symmetry using K'NEX models.

Lesson Length: 60-minutes

Student Objectives:

Students will:

- Recognize symmetry in a line segment or in a two-dimensional figure.
- Explain what it means for a figure to have a point or line of symmetry.
- Identify figures that have symmetry and figures that do not have symmetry.
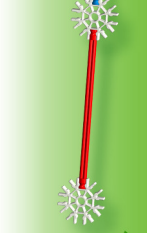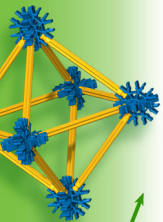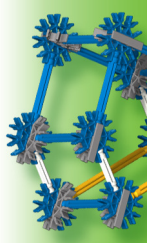- Identify lines or points of symmetry for figures, models, or drawing that have been presented to them.
- Build polygons with 0, 1, 2, or more lines of symmetry
- Develop vocabulary and concepts related to symmetry.

Grouping for Instruction:

- Four groups of 3 – 4 students each for building models. (There are sufficient materials in your set that will allow students to work individually or in pairs for many segments of this lesson as many of the models they build will be quadrilaterals.)
- Individual students for constructing models, labeling models as symmetrical or non-symmetrical and indicating a point or line of symmetry.
- Individual students for performance assessment and written assessment activities.

Materials and Equipment:

➤ *K'NEX Intermediate Math & Geometry set and Instructions Booklets*

➤ *11 prepared shapes/figures made from K'NEX, six symmetrical shapes and five non-symmetrical shapes. Refer to pages 4, 5, 6, and 7 in the Instructions Booklet for shape ideas. One of these figures should be a line and one should be a ray. The others should be polygons.*

➤ *Pencils or crayons to share*

➤ *Journals and drawing paper*

➤ *Rulers*

Overview of Lesson:

- Students will use K'NEX materials to identify, investigate, build, draw, and describe figures as having symmetry or not having symmetry.
- Students will transfer the figures to paper and will show a point or line of symmetry for each figure that they have identified as having symmetry.
- Students will demonstrate their individual understanding of the relationship between the K'NEX models and drawings that are or are not symmetrical.
- Students will include representative drawings, for a variety of models they or their classmates create, in their journals for reference.

Background Information:

➤ *Participants should be familiar with the basic geometric terms point, line, ray, and plane. Since the students will be working with many polygon models, a familiarity with polygons would be helpful.*

A – Motivation and Introduction:

1. Show students six of the prepared K'NEX shapes one at a time – three symmetrical and three non-symmetrical. As you show each shape, announce: "This shape has symmetry" or "This shape does not have symmetry". Post all symmetrical shapes in one area on the board and all non-symmetrical shapes in a different area that students can clearly see.

2. Ask students if they think they know what gives a shape the property of symmetry. Have them respond with thumbs-up, thumbs-down, or hands-neutral (don't know) – but do not take any conjectures.

3. Show the next shape (the seventh) and ask students to put their thumb-up if they think the shape has symmetry, thumb-down if they think it does not have symmetry or show hand-neutral if they are not sure. Continue in this fashion for one more shape. Post these shapes with the correct collection of symmetrical or non-symmetrical shapes. Have students look at the posted figures and see if they can determine why some shapes are symmetrical and others are non-symmetrical.

4. Ask again for thumbs-up, thumbs-down, or hands-neutral to indicate whether they know what makes a shape have symmetry.
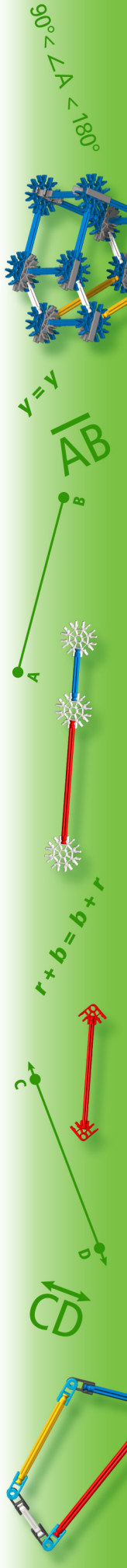
5. Show one more figure and have the students give thumbs-up, thumbs-down, or hands-neutral to indicate symmetry on non-symmetry. Ask one student who has their thumb-up to explain his or her thinking. Ask other students to tell you what they think symmetry means.

6. Explain that symmetry is when a figure has two halves that are exactly alike or that are mirror images of one another. Tell them that you can draw a line through the middle of a shape and both sides of the image will be mirror images. Explain that this line would be called a line of symmetry. Illustrate this with all of the figures in the symmetry display.

7. Show two more shapes, one at a time, and have the students give thumbs-up, thumbs-down, or hands-neutral to indicate symmetry or non-symmetry. Hopefully, all the students will be indicating the correct answer. If not, then additional practice would be in order.

B – Development (including discussion points and feedback):

1. Tell the students that they are going to make symmetrical figures with K'NEX.

   Review: symmetry means that a figure can be divided in such a way as to have two halves that are mirror images of each other.

2. Illustrate with a line segment made of K'NEX, that a point in the middle would divide the segment into two parts so that each is a mirror image of the other.

3. Show the students a ray made of K'NEX and ask if this figure has symmetry. Have a student explain why the shape does not have symmetry. (No matter where a point is identified on a ray, a comparison of each side of the ray would show that one is finite and the other side of the point is infinite.)

4. Illustrate symmetry with an isosceles trapezoid made of K'NEX and show that a line through the midpoints of the two bases would divide the segment into two parts so that each is a mirror image of the other.

5. Show a scalene triangle made of K'NEX and ask if the shape has symmetry. Have a student explain why the shape does not have symmetry. Explain that shapes that have symmetry are said to be symmetrical.

6. Organize the class into partners. The first partner in each group will explain to his or her partner in his or her own words what it means for a shape to have symmetry. The second person may comment on that description. Then, the second person explains to the first in his or her own words what it means for a shape to have symmetry. The first person may comment on that description.

7. Ask the students to individually make two K'NEX shapes and to construct them so that one shape has symmetry and one shape does not have symmetry. Encourage the students to be creative.

8. In their groups, have students show their shapes to others and to identify which shape has symmetry and which shape does not have symmetry. With each shape, explain why the shape is symmetrical or not symmetrical.

   As students work, look for some shapes that have more than one line of symmetry, such as an equilateral triangle, a square, a rectangle, a hexagon, etc. Make a note of who has these shapes and use them later in procedure #10 below.

9. Have each group show one shape that has symmetry and indicate the line of symmetry.

10. Ask students that you observed with shapes having more than one line of symmetry to please give you their shapes. Show one figure with two lines of symmetry and ask a student to identify a line of symmetry.

11. Ask other students if they agree. Someone might note that there is another line of symmetry. Or if no one notices, ask if anyone sees another line of symmetry.

12. Using student input, or your nudging through creative questioning, show that this figure actually has two lines of symmetry.

13. Show a shape with three (equilateral triangle) or four lines of symmetry (square). Tell the students that the figure has three (or four) lines of symmetry, can they identify them?

14. Have the students build one more figure and to make that figure have more than one line of symmetry.

15. Have students share their creation in their groups and to identify the different lines of symmetry.

16. Have each group select one shape from their group that has more than one line of symmetry. They may want to select the shape that has the most lines of symmetry.

17. Have the group transfer that shape to chart paper and then use a ruler to draw in the lines of symmetry.

18. Have each group show their shape and their drawing to the class. Have them explain why it is symmetrical and to show the lines of symmetry.

19. Ask the students to again get a partner and have the first person explain in his or her own words what it means for a shape to have symmetry. Then, the second person explains to the first in his or her own words what it means for a shape to have symmetry. As you listen to the various students you should notice that the students' definitions of symmetry are becoming more detailed.
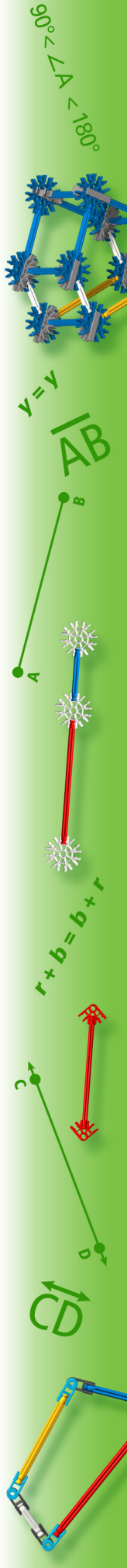
C – Summary and Closure

1. Have a collection of six K'NEX models made up in advance. Include a line segment, ray, triangle, hexagon, and several quadrilaterals.
2. Hold up the models one at a time. Ask students to write down whether or not the models are symmetrical and if they are, have them write how many points or lines of symmetry they have.
3. Ask students to write down three things that they learned in class today.
4. Have them share their statements with students in their group.
5. Have each group share at least one observation with the class.

Assessment:

Informal:

• Observe participants during the group work.

• Use a checklist to record what the participants know and any concepts that need to be revisited.

• Give each team a group grade for each segment of the activity you observe them completing.

• Ask the participants to explain in writing (Math Journal) what they learned during the lesson and any concepts that are still unclear.
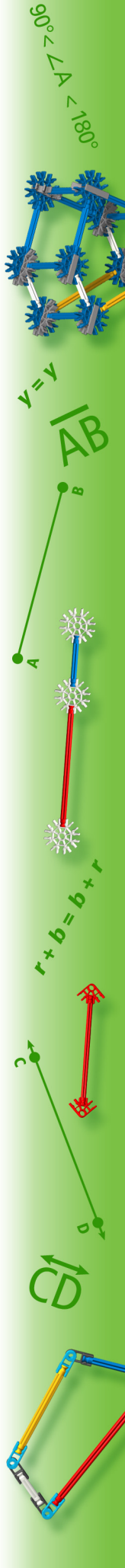
Formal:

• Establish a series of four or five performance assessment stations around the room that students can visit during a pencil and paper test or quiz that they are working on at their seats.

• Have one row of students at a time visit the performance assessment stations and provide one to one and one-half minutes for them to work at each station.

• Place polygon models or a collection of loose K'NEX pieces at stations around the room. Provide directions and or questions at each station.
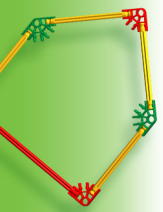
For example:

• Use the pieces provided to make a polygon that has exactly one line of symmetry. (When students have made the model, they can use a piece of masking tape to place their name on their creation before they place the model in a tall box provided by the teacher.)

• Place polygon models at several stations and ask:
  - Is the polygon symmetrical?
  - If so, how many points or lines of symmetry does it have?
  - Transfer the shape to the back of your paper and draw lines of symmetry if it is symmetrical. If it is not symmetrical, write that below the drawing.

Challenge students:

To build models of polygons to use in assessment scenarios. Student models can be placed about the room or hung from hooks above the board for other students to solve. This activity works best if the polygons are numbered. That allows students to hand in a sheet with the polygon numbers and an identification of whether the polygon is symmetrical or not symmetrical and if it is symmetrical, how many lines of symmetry it has.

EF

F

E

$y + s = s + y$

$y + y + w + w = 2y + 2w$

$w < r$

$90° < \angle A < 180°$

$90° < \angle A < 180°$

AB

$v = v$

B

A

$r + b = b + r$

C

D

CD

Lesson 8

Probability Concepts

Lesson Topics: Probability and the demonstration of probability concepts using manipulatives.

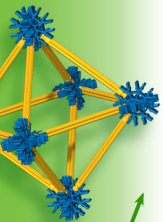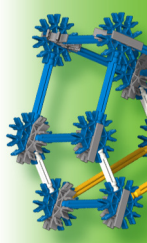**Lesson Length:** 45-minutes

Student Objectives:

Students will:
- Describe probability as likely, unlikely, impossible, or certain.
- Understand and apply the basic concepts of probability.
- Use a fraction to express probability.

Grouping for Instruction:
- Four groups of 3 – 4 students each for building models. (There are sufficient materials in your set that will allow students to work individually or in pairs for this lesson as many of the models they build will be quadrilaterals.)
- Individual students for drawing models and preparing data charts.
- Individual students for performance assessment and written assessment activities.

Materials and Equipment:

➤ *K'NEX Intermediate Math & Geometry set and Instructions Booklets*

➤ *A large jar or other transparent container*

➤ *Pencils or crayons to share*

➤ *Journals and drawing paper*

Overview of Lesson:
- Students will use K'NEX materials to explore the basic concepts of probability.
- They will build models, investigate, draw, and describe probability.

Background Information:

➤ *Students should have had exposure to the basics of fractions and decimals (during grades 4 and 5).*

A – Motivation and Introduction:

1. Assign numbers to each of the four groups... 1 - 2 - 3 - 4.
2. Assign each group to prepare a specific type of polygons using K'NEX rods for the sides of the polygons. The students can use light gray, red, green, yellow, or white K'NEX connectors. Provide ample time for the building of these shapes.
   - Group 1 will make 4 triangles
   - Group 2 will make 4 squares
   - Group 3 will make 3 rectangles
   - Group 4 will make 5 trapezoids.
3. Tell the students to place their shapes in the center of their table for use later in the lesson.

B – Development (including discussion points and feedback):

1. Tell the class that today they are going to work with probability or the likelihood that an event will occur.

2. Show the students the large jar or container in the front of the room. Show them that the container is empty.
   - Ask Group 1 to bring five green rods and place them into the large jar or see-through container.
   - Ask Group 2 to bring four red rods and place them into the container.
   - Ask Group 3 to bring five yellow rods and three blue rods and place them into the container.
   - Ask Group 4 to bring three white connectors and place them into the container.
4. Ask the students to raise their hand if they think that Sunday will come after Saturday this week. (All students should raise their hands.) Agree that this will certainly happen, so the event is said to be certain. Ask the students if they can think of another event that is certain. (We will go down and have lunch today; it will be dark outside at 10:00 PM tonight, etc.)

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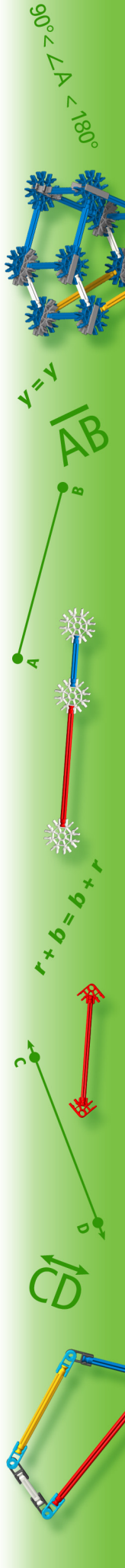
5. Ask students to raise their hand if they think I can hold my breath for 10 minutes. (No student should raise his or her hand.) Emphatically agree with them. So this is not possible or it is an impossible event. Ask students if they can think of another impossible event. (That a student would be able to lift up one end of the school bus; that it will snow on the 4th of July, etc.)

6. Ask students about wearing shorts in the winter. Is this likely or unlikely? (Unlikely.)

   Ask students about wearing a coat in the winter. Is that likely or unlikely? (Likely.)

7. So when you think about things that might happen, things that are called "events", you can predict that the event is a certain event if it definitely will happen; it is an impossible event if it definitely will not happen; it is a likely event if it will probably happen; and it is an unlikely event if it could happen but probably will not.

   As you say these terms write them on the board or on chart paper.
   • Certain event
   • Impossible event
   • Likely event
   • Unlikely event

8. Explain to students that this is called probability. How likely it is that something happens.
   • Explain that we use 0 (zero) to represent an impossible event.
   • That we use 1 (one) to represent a certain event.
   • And that we use a fraction between 0 and 1 to represent a likely or unlikely event.

9. Ask the students to turn to a partner and take turns explaining to each other the four different events that could happen and to assign a number, if they can, to each.

10. Have each group state one of the events and the number that would be assigned to it.
    • Certain event: 0
    • Impossible event: 1
    • Likely event: number between 0 and 1
    • Unlikely event: number between 0 and 1

11. Have the students look at the clear container in which the K'NEX pieces have been placed. Have them summarize what pieces are in the jar and as they do, write them on the front board or on chart paper.
    • 5 green rods
    • 4 red rods
    • 5 yellow rods
    • 3 blue rods
    • 3 white connectors

12. Ask the students to answer these questions:
    • If I took a one thing out of the container, what kind

of event is it and what is the probability that it would be a K'NEX piece?
    • And Why?

13. Ask:
    • If I took one thing out of the container, what kind of event is it and what is the probability that it would be a candy bar?
    • Why?

14. Ask:
    • If I took one thing out of the container, how likely is it that it would be a rod?
    • Why do you think that?
    • What kind of number could represent that event?
    • Can anyone think of a number that might work to represent that?

15. Ask:
    • How many K'NEX pieces were in the container? (20 pieces total)
    • How many of them were rods? (17 rods)
    • How many were connectors? (3 connectors)

16. Explain that the fraction between 0 and 1 that represents an event is a ratio of the number of possible successes to the number or possible events or ways that we could have chosen a piece. So in this case the ratio would be:

$$\frac{\text{rods}}{\text{total pieces}} \quad \text{or} \quad \frac{17}{20}$$

17. Ask:
    • Now, if I took one thing out of the container, how likely is it that it would be a connector?
    • Why do you think that?
    • Now, thinking about what we just did a minute ago, who knows what number should be used to represent that? ($3/20$)

18. Ask:
    • Suppose I took out a yellow rod? What would that event be called? (Unlikely.)
    • What fraction would represent that event? ($5/20$ or $1/4$)

19. Ask individuals or pairs of students to select one of the polygon models that was prepared at the beginning of the lesson.

20. Instruct the students to hold each model firmly on a piece of paper and to use a crayon or pencil to put a dot on the page for each vertex on the models. Ask the students to use a ruler to connect the dots to make the polygon. Students should draw each side of their polygon the same color as the K'NEX rod that makes up that side. Students should color a dot for each of the vertices the same color as the connector they used for that vertex.

21. Have the students list the colors of the rods and
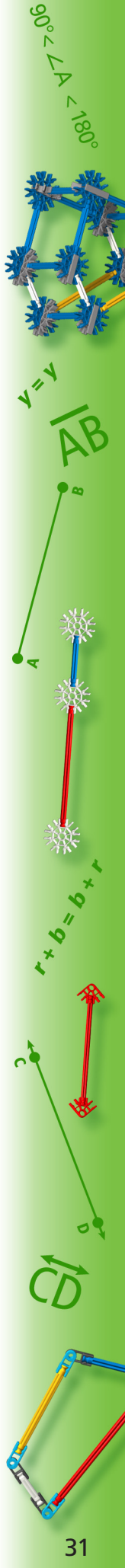
connectors that were used along with the number of each that was used to form their polygon model. For example: if a pair had made a pentagon consisting of three yellow rods, a red rod, three white connectors and two green connectors, they would write:

- 4 yellow rods
- 1 red rod
- 3 white connectors
- 2 green connectors
- 10 K'NEX pieces

22. Instruct the students to consider each piece and to list the probability that someone would select that piece if he or she were to take a piece out of their model. For the example above:

- yellow rod: $^4/_{10} = {^2/_5}$
- red rod: $^1/_{10}$
- white connector: $^3/_{10}$
- green connector: $^2/_{10} = {^1/_5}$

23. When students have completed this task, instruct each group to combine with another group and to explain what they have discovered using their model, drawing and chart of probabilities.

24. Have several pairs (as time allows) share with the entire class.

C – Summary and Closure

1. Have a collection of three polygon models made up in advance.
2. Hold up the models one at a time. For each ask how likely is it that if someone were to take a piece from the model it would be the (name a piece) one.
3. Ask the students to write down three things that they learned in class today and to write a number 0, 1, 2, 3, 4, or 5 to indicate how well they understood the things we learned today.

- 0 = student does not understand anything
- 1 - 4 = student understands some - most
- 5 = student understands completely and can explain it to someone else

4. Have them share their statements and numbers with students in their group.
5. Have each group share at least one observation with the class.

Assessment:

- Use six of the models that students made in class for this assessment. Number the polygons and place them at different stations. Have students visit each station to answer probability questions related to that polygon.

For example:

- Place a polygon model at a station and ask:
  - What is the probability that you would choose a white connector from this model if you had to remove one piece?
- Place another polygon at a second station and ask:
  - What is the probability that you would choose a green piece from this model if you had to remove one piece?

- Place loose K'NEX pieces at a third station and ask:
  - Build a polygon that would have a probability of $^1/_4$ that someone would pick a yellow piece from the model if you had to remove one piece.

  Have the students write their names on a piece of masking tape, attach it to their model, then place it in the box provided.
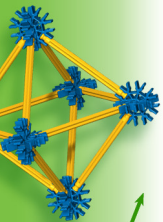- Variations of these examples can form an endless array of possible assessment activities.
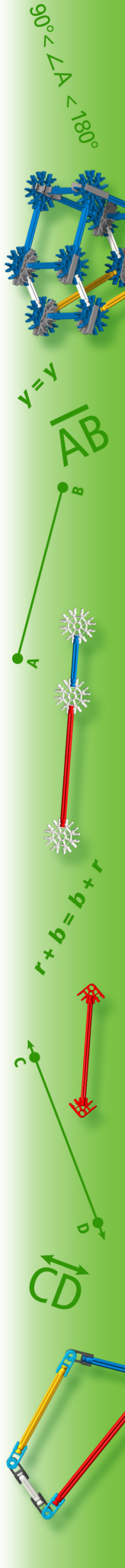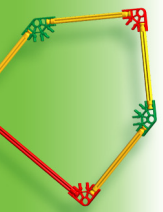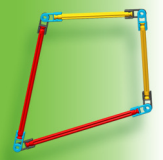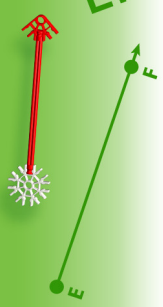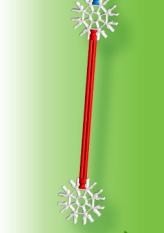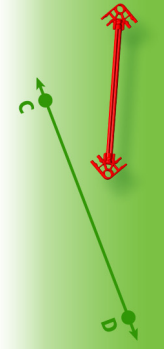
LESSON 9
Area Concepts: Polygons

Lesson Topics: The Measurement, Computing and Reporting of the Area of Polygons Using K'NEX.

Lesson Length: 60-minutes

Student Objectives:
Students will:
- Describe the area of polygons using math symbols.
- Investigate and explore the concept of area.
- Determine and describe the area of polygons.

Grouping for Instruction:
- Four groups of 3 – 4 students each for building models.
- Individual students for drawing models, labeling sides, and listing a representation of area measurements.
- Individual students for performance assessment and written assessment activities.

Materials and Equipment:
➤ *K'NEX Intermediate Math & Geometry set and Instructions Booklets*

➤ *Chart paper (one sheet per group)*

➤ *Pencils*

➤ *Colored pencils or crayons*

➤ *Journals and drawing paper*

Overview of Lesson:
- Students will use K'NEX materials to build, investigate, draw, and describe the area of polygons.
- Students will use K'NEX mathematics conventions as they describe the area of polygons.
- Individual students will transfer the polygons they construct to paper and describe the area of these polygons with appropriate mathematics terminology to demonstrate their individual understanding of the relationship between the K'NEX models, drawings and their areas.
- Students will include representative polygon drawings and area solutions for a variety of models they or their classmates create.

Background Information:
➤ *Students should have had exposure to the basics of length measurement and concepts they investigated in Lessons #3 and #4. They will use a non-standard measurement system to describe the area of polygons. Actual number values may be attached to the length of the various K'NEX line segments in future years after the students have internalized a working understanding of measurement and area concepts.*

A – Motivation and Introduction:
1. Ask student groups to prepare a collection of at least four two-dimensional rectangular shapes using the blue and red rods. (For this activity, students can use red, green, yellow, or white connectors.)
2. Provide time for creative exploration with the rectangles. Do students only stay with one-colored rod to make squares that are also rectangles? Do they connect blue rods to red rods to make larger rectangles that are not squares? If so, how do they sequence them? How many different rectangles have been created in the classroom?
3. Lead students to realize that one characteristic of their rectangle is area, the space within the figure and that this space is measured by the number of square units that are needed to cover the figure.
4. Have students discuss how they might express the area of the rectangles that they have made. Ask several groups to share how they would describe the area of a given rectangle. Lead students to realize that they can express the area of each of their rectangles by how many square rectangles with four blue sides are needed to cover a given rectangle that they have made. Have students call this unit a "blue square" and the smallest rectangle they have made would be a blue square, so its area would be one blue square. (If your students are familiar with exponents, they may express one blue square using K'NEX conventions with the term b^2.)
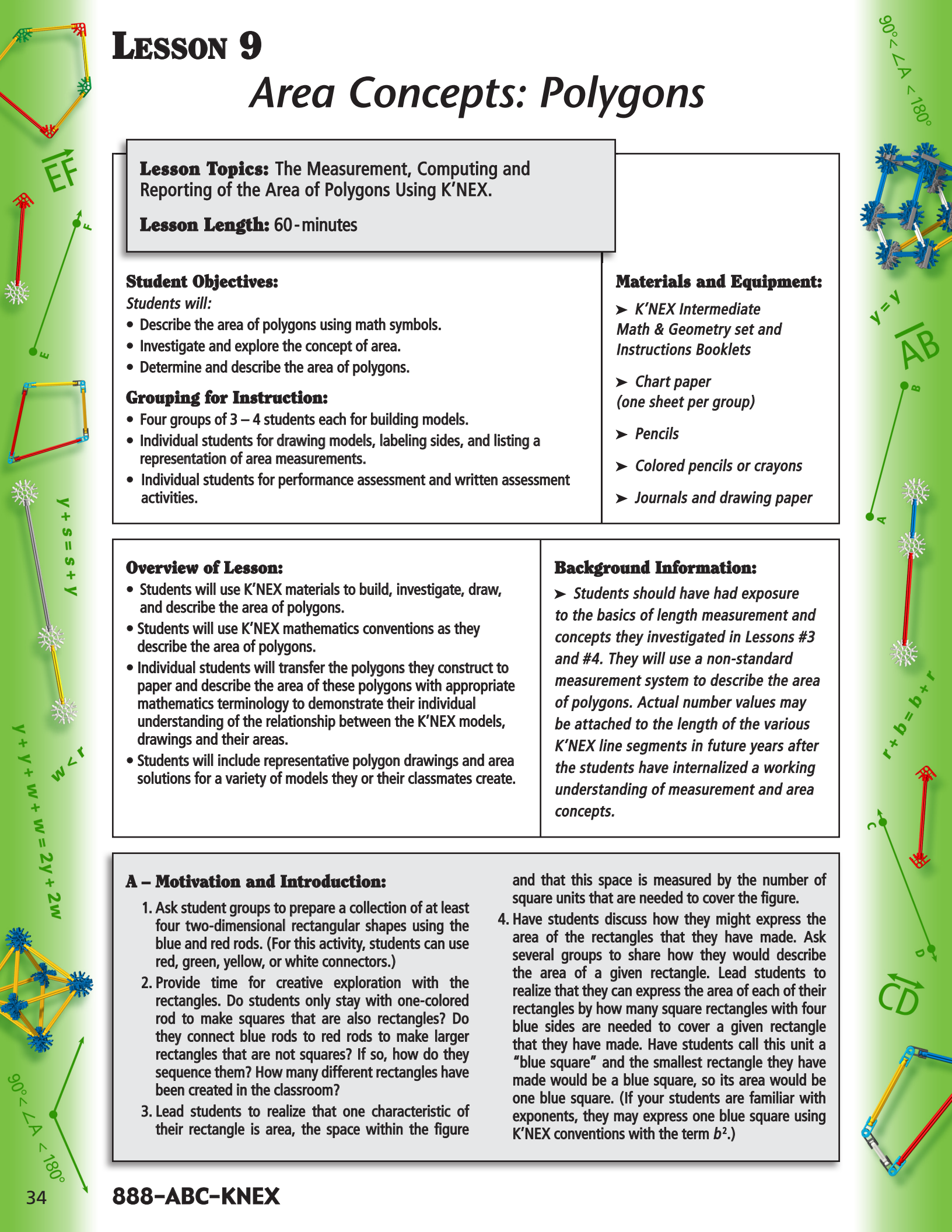
B – Development (including discussion points and feedback):

1. Instruct students to place all the rectangles that their group has constructed on a single sheet of chart paper.

2. Ask students to work as a group to transfer all of their polygons to the paper using pencils or crayons and rulers. Students should use a blue crayon/pencil to draw the blue rods and a red crayon/pencil to draw the red rods.

   CAUTION FOR STUDENTS:

   Complete the drawing of each polygon before moving on to the next. It is very easy to make drawing errors when facing a sheet of 20 or 30 vertices [points] and not being able to remember which ones go with which rectangle.

3. Instruct students to discuss the area of each model and to use a pencil or possibly a blue square model to determine the area of the rectangles they have created. List the number of blue squares in each model to the side of the shape.

4. Use a round-robin-response format and ask groups to hold up one of their polygons and to describe its area orally.

   TEACHER NOTES:

   Reinforce students' oral information by sketching each figure and writing the student's response on the chalk board or white board.

5. Ask the other groups to verify their understanding of the information provided by each group.

6. Continue taking responses from the various groups until you are confident that students are providing informed responses that demonstrate both an understanding of area and a proper description of area values expressed in blue squares.

7. When students seem comfortable with this activity, provide them with the area of a rectangle and see if they are able to build a rectangle to match the given area. This requires that students transfer what they have learned and practiced to a new learning situation. The ability to successfully complete these activities will provide very useful diagnostic data that you can use to determine the level of your students' understanding.

8. Here are suggested areas (with answers) that you can provide to the students when you ask them to make a model that matches the given information.

   Possible Questions and Answers:

   - Area = 4 blue squares (multiple answers)
     - 1 red square
     - 1 rectangle with two b sides and two sides of $2r$
   - Area = 6 blue squares
     - 1 rectangle with two sides of r and two sides of $r + b$
   - 1 rectangle with two sides of b and two sides of $3r$
   - Area = 8 blue squares
     - 1 rectangle with two sides of r and two sides of $2r$
     - 1 rectangle with two sides of b and two sides of $4r$

   Let your imagination and the skill level of your students determine any other appropriate area challenges to use.

9. Provide area challenges of irregular polygons for students to solve.
   - Find the area of the L-shaped polygon in blue squares (on page 13 of the Instructions Booklet):
     - Solution: $A = 3b^2$
   - Find the area of the L-shaped polygon in blue squares (on page 14 of the Instructions Booklet):
     - Solution: $A = 4b^2$
   - Find the area of the cross-shaped polygon in white squares (on page 13 of the Instructions Booklet):
   - Solution: $A = 5w^2$

   Ask each group to make models of three different figures that have an area of 9 blue squares. These figures may be either rectangles or irregular polygons. Instruct students to transfer the models to paper and show the area for each by drawing in the 9 blue squares using a blue crayon or blue pencil.

   TEACHER NOTES:

   Based on the abilities that your students demonstrate, you may use all of the challenges above or you may select individual challenges for your students. You may also choose to use some of these challenges for performance assessment questions on a quiz or test.

10. Ask students to construct the model of the concave hexagon on page 13 of the Instructions Booklet. Challenge the students to see if they can discover a way that they can find the area of that shape in blue squares. Provide ample time for them to explore and move about the groups to provide encouragement.
    - If students experience difficulty with the challenge, ask them if there are any rods in the shape that they can replace with blue rods.
      - Yes! The red rods can be replaced by 2 blue rods with a connector in between.
    - Also, is it possible to place blue rods in the shape to break it down into smaller units?
      - Yes, 2 blue rods and 1 connector can be used to connect the green connectors that form two of the vertices.
    - A bit more experimentation should help students to see that they can fill the shape in with either:
      - 8 triangles that are ½ blue squares each

        or
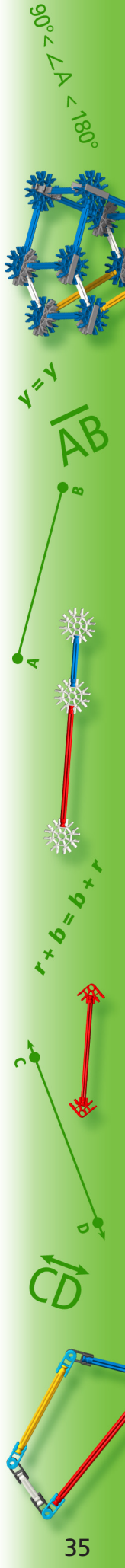
- 2 blue squares and 4 triangles that are ½ of a blue square each
- In either case, the area of the shape is $4b^2$.

11. Ask the students to construct the pentagon model on page 14 of the Instructions Booklet. Challenge the students to find the area of the shape in blue squares.
   - They will use the same strategy that they did with the shape in #10 above.
     - Solution: $A = 3b^2$.
   - Further challenge the students to find the area of the same shape in yellow squares.
     - Solution: $A = 1 ½ y^2$.

C – Summary and Closure

1. Have a collection of polygon models made up in advance, some rectangles and some irregular polygons.
2. Hold up the models one at a time. Ask students to write down the appropriate letters and symbols that could be used to describe the area of the rectangle or irregular polygons.
3. Ask students to write down three things that they learned in class today.
4. Have students share their statements with students in their group.
5. Have each group share at least one observation with the class.

Assessment:

- Establish a series of four or five performance assessment stations around the room where students can visit during a pencil and paper test or quiz that they are working on at their seats.

Questions may include:

- Use the pieces provided to make a polygon that has an area of 3 yellow squares. (When a student has made the model, he or she can use a piece of masking tape to place his or her name on their creation before placing the model in the box.)

- Place polygon models at several stations. Some models should be rectangles and some should be irregular polygons. Provide a question at each station. What is the area of the polygon?

Extensions:

- Challenge the students to build models of polygons to use in the assessment scenarios.
- Student models can be placed about the room or hung from hooks above the board for other students to solve. This activity works best if the polygons are

numbered. That allows the students to hand in a sheet with the polygon numbers and their appropriate areas.
- As students develop their skills with the determination of area they will eventually be able to express the area of many shapes
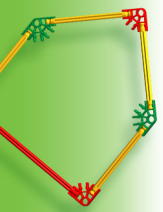
in terms of green, white, blue, yellow, red, or silver squares.
- They will also discover that they can build some shapes that include angles less than 90 degrees which they will be unable to solve.
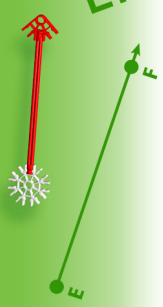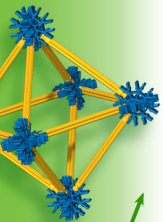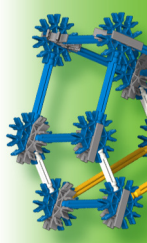
2 Examples

EF

$y + s = s + y$

$w < r$

$y + y + w + w = 2y + 2w$

$90° < \angle A < 180°$

$v = v$
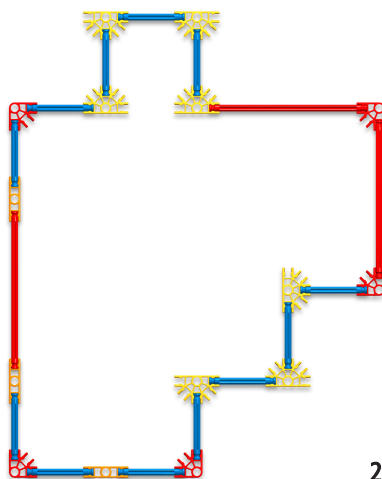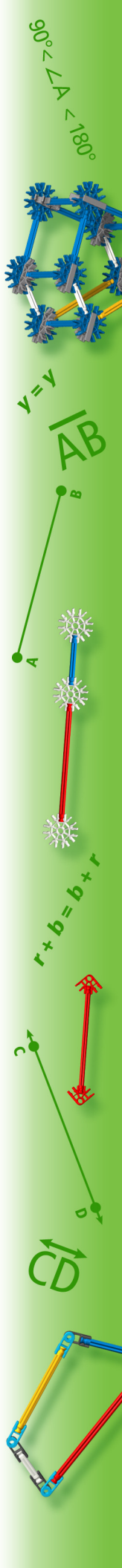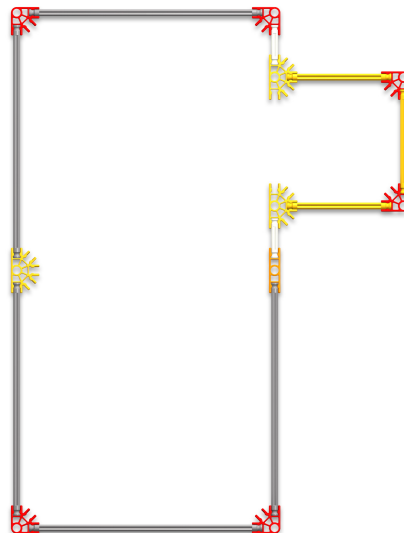
AB

$r + b = b + r$

CD

LESSON 10
Area and Perimeter of Polygons

Lesson Topics: The Measurement, Computation and Reporting of Perimeter and Area Using K'NEX Polygons.

Lesson Length: 90-minutes
(two or more 45 minute time periods)

Student Objectives:
Students will:
- Build polygon models of various shapes and describe them using common mathematics terms and symbols.
- Follow appropriate mathematics conventions and algebraic expressions as they express the perimeter and area of polygons.
- Describe K'NEX polygons in terms of their area and perimeter.
- Investigate the relationship between the perimeter and area of polygons.

Grouping for Instruction:
- Group students into groups of 3 – 4 students each for building models.
- Individual students for drawing models, labeling sides and listing a formula to represent perimeter measurements.
- Individual students for performance assessment and written assessment activities.

Materials and Equipment:
➤ K'NEX Intermediate Math & Geometry set and Instructions Booklets

➤ Chart paper (one sheet per group)

➤ Colored pencils or crayons

➤ Journals and drawing paper

➤ Rulers

Overview of Lesson:
- Students will build, investigate, draw and describe the relationship between the perimeter and area of polygons.
- Students will use K'NEX mathematics conventions as they describe the perimeter and area of polygons. They will understand that polygons with the same perimeter may have different areas and polygons with the same area may have different perimeters.
- Individual students will transfer the polygons they construct to paper and describe the perimeter and area of these polygons with appropriate mathematics symbolism and labels to demonstrate their individual understanding of the relationship between the K'NEX models, drawings and their areas.
- Students will include representative polygon drawings along with perimeter and area solutions for a variety of models they or their classmates create.

Background Information:
➤ *Students should have had exposure to the basics of perimeter and area concepts they investigated in Lessons #5 and #9. They will use a non-standard measurement system to describe the perimeter and area of polygons and express the values in algebraic form. Actual number values may be attached to the length of the various K'NEX line segments in future years after students have internalized a working understanding of measurement and perimeter concepts.*

A – Motivation and Introduction:
1. Ask student groups to prepare a collection of at least four two-dimensional polygons using the blue and red K'NEX rods for the sides of the polygons. In making these polygons, all angles should be right angles. (For this activity, students can use red, green, yellow, or white connectors.)
2. Provide time for creative exploration with the polygons. Do students make all squares and or rectangles? Do they make some irregular polygons? How do they sequence them? How many different polygons have the students created?
3. Have the students consider one of their figures and discuss with a partner the meaning of perimeter and the meaning of area as it applies to their polygon. What are they? How are they alike and how are they different?
4. Ask several student pairs to share their discussions.

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Summarize that the perimeter of a polygon is the distance around the figure and that area is the space within the figure, with the space within the figure being measured by the number of square units that are needed to cover the figure. In working with these concepts, use the conventions discussed in Lessons #5 and #9.

5. Have students represent the perimeter and area of two of the polygons that they have made. Ask several groups that have made rectangles (including squares) to share how they would describe the perimeter and the area of one of their rectangles. Remind students to express the perimeter in terms of r and b (red rods and blue rods) and that they should express the area of each of their rectangles by how many square rectangles with four blue sides are needed to cover a given rectangle that they have made. Have students call this unit a "blue square". Students can report their perimeters and areas using the standard mathematics symbols:

A = for area and P = for perimeter.

B. Development (including discussion points and feedback)

1. Instruct students to place two of the polygons that their group has constructed on a sheet of chart paper at their work station. Encourage them to elect at least one polygon that is irregular square.

2. Ask students to work as a group and to transfer these two polygons to the easel paper. When finished, the students can use a ruler and blue or red crayons/pencils to connect the vertices to form exact drawings of their polygons.
   CAUTION FOR STUDENTS:
   Complete the drawing of the first polygon before moving on to the next one.)

3. Instruct students to discuss the perimeter and the area of each model. Have them use a space next to each figure to write the perimeter in terms of b and r and to write the area in terms of blue squares. (For example: $P = 2r + 2b$ or $P = 6b$ and $A = 2$ blue squares or $A = 2b^2$.)

4. Use a round-robin-response format and ask each group to hold up one of their polygons and to describe its perimeter and its area orally. (The teacher should reinforce students' oral information by sketching each figure and writing the student's response on the chalk board or white board … Perimeter = … Area = …)

5. Ask the other groups to verify the information provided.

6. Continue moving from group to group until each group has described one of their polygons in terms of perimeter and area. Encourage one or two groups to show one of their irregular polygons.

7. Tell the students: "Suppose you had a little toy dog and wanted to make a pen for him. If you had 8 red rods and 4 blue rods, how would you make that little pen so that your toy pet had a nice area in which to play? The pen must be a rectangle."
   Ask students if they are able to build at least two different pens that may be built with 8 red rods and 4 blue rods each.

8. Have the students transfer the rectangular K'NEX dog pens to chart paper and to determine the perimeter and area of each pen.

9. Have several groups share their pens and the perimeter and area of these pens.

10. Ask students what they noticed about the areas and perimeters of the pens that they had built. (All of the pens had the same perimeter but different areas.)

11. Discuss with students the various pen shapes. Which pen would give the most area? Which pen would give their pet the longest running distance? Which pen do they think their pet would like best? Why?

12. Emphasize with students that polygons with the same perimeter may have different areas.
    If this lesson is taught over two periods, this is a good place to break. When coming back to this lesson, go back and review the meaning of area and perimeter. (Part A, steps 3 and 4 above.) Ask students to discuss with a partner what they learned in the last lesson.

13. Tell students that now they are going to build a pen for their little pet dog as they did before, but this time you are going to tell them the area of the pen and they will build at least two pens that have that area but that have different shapes.

14. Tell students that the area of their little dog's pen will now be 24 blue squares. The students may use red or blue rods to construct their pens. Ask the groups to build at least two different pens, one at a time, with an area of 24 blue squares.

15. Have the students transfer their rectangular K'NEX dog pens to chart paper and to determine the area (24 blue squares) and the perimeter of each pen.

16. Have several groups share their pens and the area and perimeter of these rectangular pens. (Remember that squares are a special type of rectangle, so they may also be included.)

17. Discuss the various pen shapes with students. Which pen would give the most area? Which pen would give their pet the longest running distance? Which pen do they think their pet would like best? Why?

18. Ask students what they noticed about the areas and perimeters of the pens that they had built. (All of the pens had the same area but many different perimeters were possible.)

19. Hmmmm … Have students think about the pens that they have made today and compare them to what they had just done previously (or the lesson before). What do they notice? (That now they have polygons
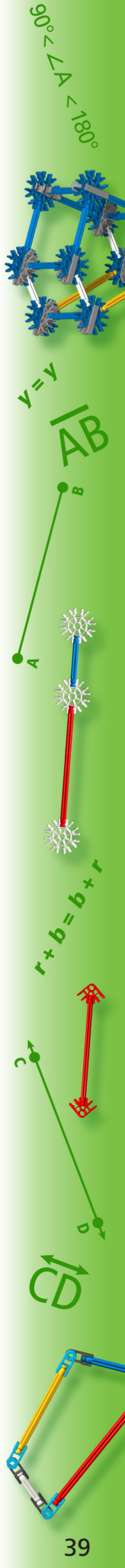
with the same area but different perimeters – and before [or yesterday] they built polygons that had the same perimeter but different areas.)

20. Emphasize with students that rectangles with the same area might have different perimeters, as was shown in this part of the lesson.

21. Reinforce what was learned before: **that polygons with the same perimeter may have different areas.**

C – Summary and Closure

1. Have a collection of paired models made up in advance. These pairs should illustrate the two facts above, that rectangles with the same perimeter might have different areas and that rectangles with the same area might have different perimeters.

2. Hold up the model-pairs one at a time. Ask students to write down the appropriate letters and symbols that could be used to describe the perimeter and area of the models. Ask students to describe whether the pair of models demonstrate that:

   - polygons with the same perimeter
     may have different areas

                or

   - polygons with the same area
     may have different perimeters.

3. Ask students to write down three things that they learned in class today.

4. Have students share their statements with students in their group.

5. Have each group share at least one observation with the class.

Assessment:

• Challenge students to build models of polygons to use in assessment scenarios. Student models can be placed about the room or hung from hooks above the board for other students to solve. This activity works best if the polygons are numbered. That allows students to hand in a sheet with the polygon numbers and their appropriate areas and perimeters.

• Establish a series of 4 or 5 performance assessment stations around the room where students can visit during a pencil and paper test or quiz that they are working on at their seats. Place polygon models or a collection of loose K'NEX pieces at stations around the room. Provide directions and or questions at each station.

Questions may include:

• Place polygon models at several stations. Some models should be squares and some should be irregular polygons. Provide a question related to perimeter or area at each station.

   - What is the area of the polygon?
   - What is the perimeter of the polygon?

• Use the pieces provided to make two rectangles that have the same area but that have different perimeters. (When a student has made the models, he or she can use a piece of masking tape to tape them together and place his or her name on the creations before placing the model in a tall box provided by the teacher.)

Extensions:

• Challenge students to create a K'NEX polygon that has the smallest possible area for a perimeter that you provide.

• Challenge students to create a K'NEX polygon that has the largest possible area for a perimeter that you provide. (Use a perimeter of 12 for these two challenges the first time.)

• Challenge students to find the perimeter and area (in yellow squares) of the concave hexagon shown on page 14

of the Instructions booklet. ($P = 3r + 2y + s$ and the area is $2\frac{1}{2} y^2$ (students will have to discover that five triangles with an area of ½ a yellow square will fit inside the hexagon shape). Once students have solved the challenge, ask them to create a new shape with an area of $2\frac{1}{2} y^2$ and to report its perimeter.

• Challenge students to extend the concepts they have learned in the discussion of area and perimeter and determine a

volume for the pentagonal prism found on page 15 of the Instructions Booklet in blue cubes. (Hint: Students should consider rebuilding the model to see if they can incorporate a blue cube into the model. If they replace the two yellow rods with pairs of white rods and pairs of dark gray and blue connectors, they can add blue rods to make one blue cube and two triangular prisms that are ¼ of a blue cube each.) (Solution: $V = 1\frac{1}{2}$ blue cubes or $V = 1\frac{1}{2} b^3$.)
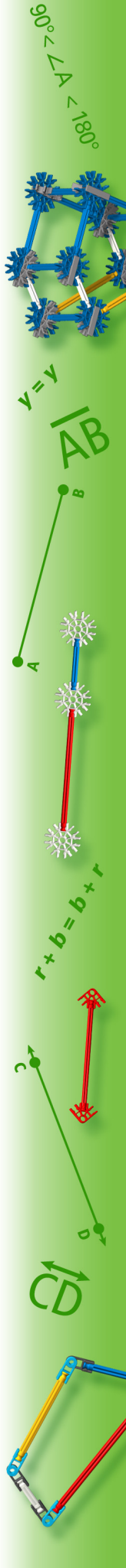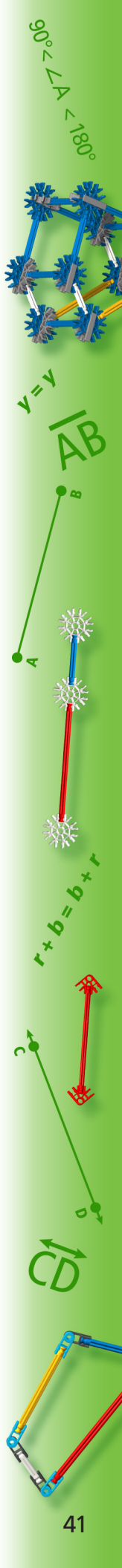
LESSON 11
Exploring Fractions

Lesson Topics: Exploring Fraction Concepts Using K'NEX Models.

Lesson Length: 60-minutes

Student Objectives:

Students will:

- Demonstrate understanding that a fraction can be represented as part of a group or set.
- Define the terms numerator and denominator.
- Use fraction names and symbols to describe fractional parts of whole objects or sets of K'NEX objects.
- Identify fractions when the whole (set) and part of the set is indicated.
- Describe K'NEX figures in terms of fractions.
- Construct concrete K'NEX models of fractions.
- Explore the concept of equivalent fractions.

Grouping for Instruction:

- Group students into groups of 3 – 4 students each.
- Individual students for drawing models and writing fractions to describe the parts of objects.
- Individual students for performance assessment and written assessment activities.

Materials and Equipment:

➤ K'NEX Intermediate Math & Geometry set and Instructions Booklets

➤ Chart paper (one sheet per group)

➤ Colored pencils or crayons

➤ Journals and drawing paper

➤ Rulers

Overview of Lesson:

- Students will use K'NEX materials to examine fractions as part of a set.
- They will use K'NEX materials to build, investigate, draw, and describe the relationship between different fractions and the whole.
- Students will use K'NEX as they describe the parts of fractions and they will use correct mathematics terminology as they discuss and describe fractions.
- Students will name the fractions that are represented in different models and sets and they will build models that illustrate fractions that have been given to them.
- This lesson helps students develop their problem solving and reasoning skills as they examine relationships among the fractions used to describe the parts of a set.

Background Information:

➤ Students should have had exposure to the basic concept of a fraction.

A – Motivation and Introduction:

1. Ask each student in each of the groups to pick out a set of different-colored rods. When they do this they should take at least four rods and they should include at least three different colors.

2. Have each student describe his or her set of rods to the other students in the group. Encourage the students to be as descriptive as they can. Observe: Do students simply count the rods and describe their set in terms of one number? Do students give a total number of rods in their set and also state how many rods they have of each of the different colors? Do any students describe their set in terms of fractional parts?

3. Ask each group to have someone in the group share a description of their set of rods. If desired, you may call each group for a second time to describe one of their sets. If any student described his or her set in terms of fractions, have that (those) student(s) describe their sets last.
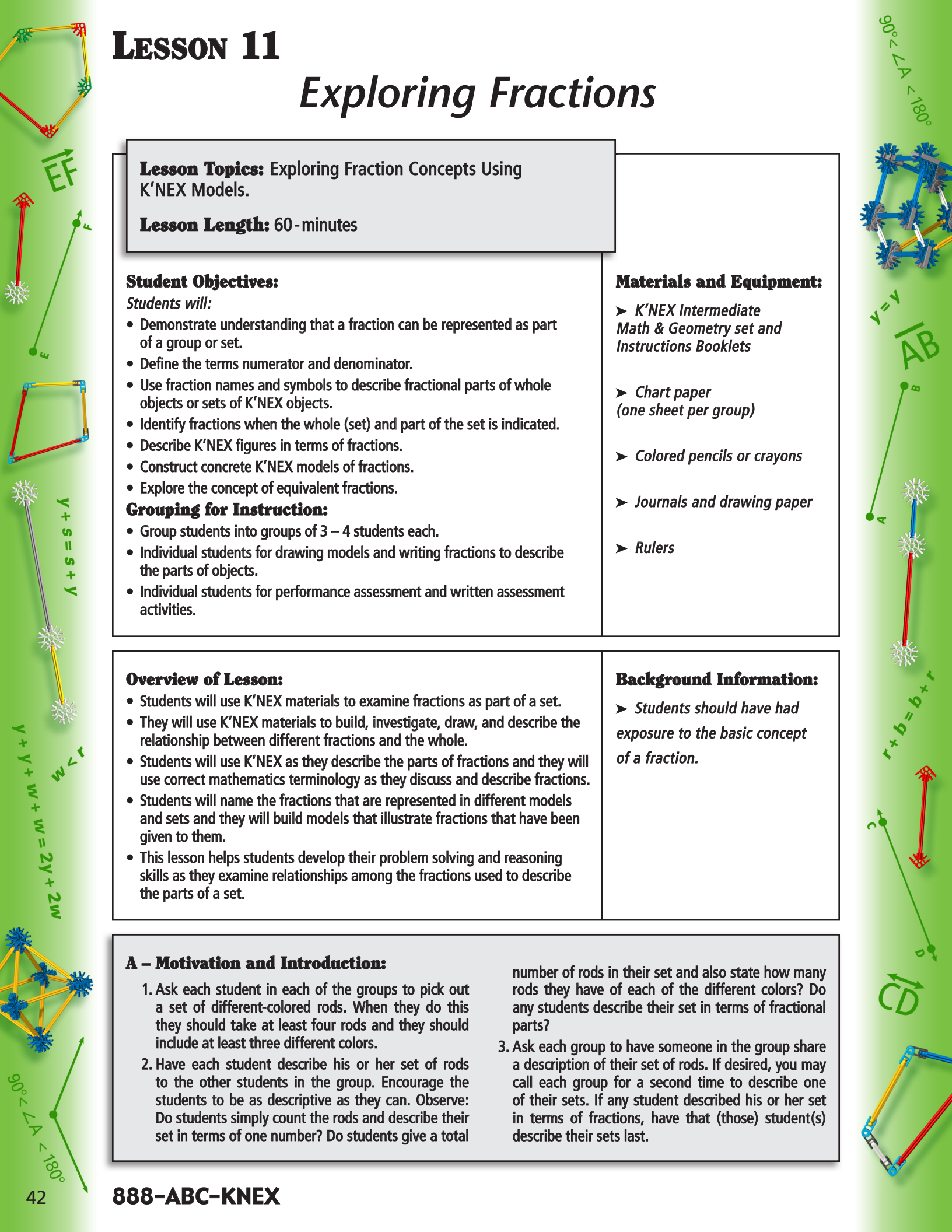
4. Discuss with the students how they may describe their sets in three ways:

    1 - The number of objects in the entire set.

    2 - The number of rods in each of the groups of one color of rods.

    3 - The part of the set represented by each of the groups of different colors in the set.

B – Development (including discussion points and feedback):

1. Illustrate the concept of fractions by taking a set of different colored rods from one of the groups. Illustrate the three different ways using this set.

   For example a set of rods consisting of 2 silver, 3 yellow, and 5 green rods would be: *s, s, y, y, y, g, g, g, g, g.*

   • The representations would be:

   i. Number in the set: 10

   ii. Number of rods in each of the 1-colored groups: 2*s*, 3*y*, 5*g*

   iii. Part of the set represented by each 1-colored group of rods:

   silver → $\frac{2}{10}$; yellow → $\frac{3}{10}$; green → $\frac{5}{10}$

2. Illustrate again with another set, possibly from one of the groups.

   For example: a set of rods consisting of 4 blue, 1 white and 2 green rods would be: *b, b, b, b, w, g, g.*

   • The representations would be:

   i. Number in the set: 7

   ii. Number of rods in each of the 1-colored groups: 4*b*, 1*w*, 2*g*.

   iii. Part of the set represented by each 1-colored group of rods:

   blue → $\frac{4}{7}$; white → $\frac{1}{7}$; green → $\frac{2}{7}$

3. Ask the students what they notice about these fractions. (In the fraction, the number of objects in the entire set is the bottom number; and, for each color, the number on the top is the number of rods of that color.)

4. Have students work in their groups to consider each set in their group and to determine the fraction that would represent each of the different-colored rods in that set.

5. Explain to students that in a fraction the top number is called the numerator and the bottom number is called the denominator.

6. Have students repeat these words several times.

7. Have students turn to a partner and explain to their partner what they understand about fractions, including the technical math names for the top and bottom numbers in a fraction. After a few minutes have the students switch roles and have the other person in the pair explain their understanding of a fraction, including the technical math names of the top and bottom numbers in a fraction.

8. Ask students to turn to pages 8 and 9 in their Instructions Booklets to find instructions to build a helicopter, a windmill and a sailboat. Have each group make each of these models.

9. Ask each to make a table with three columns and 10 rows on their chart paper. At the top of the chart write: Helicopter. Fill in the column labels as shown below. In the right-hand column, write the fraction that represents the part of the figure for that color. Ask students to analyze their Helicopter model and have them:

   • Fill in the colors found in their Helicopter (in the left column).

   • Fill in the number of pieces of that color (in the middle column).

   • Fill in the fraction of the total pieces in the model that are that particular color (in the right column).

For example:

Helicopter

Color of Piece	Number of Pieces	Fraction of the Whole
White		
Blue		
Etc...		

(Give students ample time to complete this part of the task. When the groups have finished this part, continue the instructions.)

Tell the students: "Now go back to each of your colors and look at the fraction that you wrote. Next to each fraction label the top number and the bottom number as 'numerator' and 'denominator'."

10. Have two groups join together and compare what they have done – Are their charts alike? If there are differences have the groups discuss the differences and come to agreement,

11. Ask one group to share their Helicopter chart and to explain why they completed the chart the way that they did.

12. Ask other groups/students to comment or to add any other points.

13. Have each group make a similar chart and write Windmill at the top.

14. Follow steps 9-12 in completing the information for the Windmill.

15. Ask students what fractions they thought they would get if they combined colors. Supposed the wanted to combine blue and yellow. What fraction of the entire figure would that be? Have the students revisit the Windmill and make fractions to represent some of the different color combinations that could be possible.
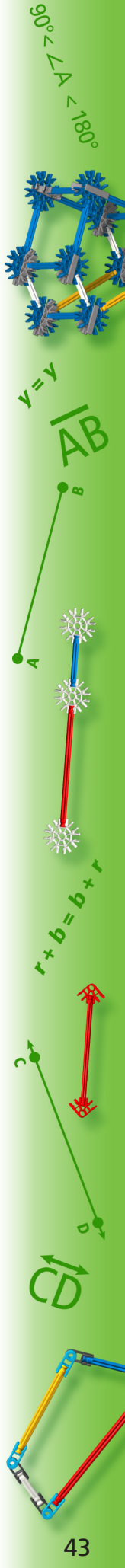
16. When groups have finished, have two groups join together and compare what they have done – Are their charts identical for the Windmill results? If there are differences have the groups discuss the differences and come to agreement.

17. Ask one group to share their Windmill chart and to explain why they completed the chart the way that they did.

18. Ask other groups/students to comment, or to add any other points.

19. Ask one or more groups to describe the fractions that resulted when they used various color combinations and found the fractions of the whole model that they represented.

20. Have groups make a chart for the Sailboat and follow instructions as above.

C – Summary and Closure

1. Have a collection of four K'NEX models of your own creation that are made of different colors.

2. Distribute one model to each group of students. Ask students to write the fractions that would represent each of the different colors in the model. You may also want to ask them to write the fraction that would represent some of the color-pairs. When the groups are finished they can exchange models for additional practice if time allows.

3. Ask students to write down three things that they learned in class today.

4. Have students share their statements with students in their group.

5. Have each group share at least one observation with the class.

Assessment:

• Challenge students to build K'NEX models of their own design to use in assessment scenarios. Student models can be placed about the room for other students to select, solve, and analyze in terms of their fractional characteristics using a chart like they used during the activity. Number the models for convenience, so that students may refer to the model by its number when they submit a solution chart.

• Establish a series of four or five performance assessment stations around the room where students can visit during a pencil and paper test or quiz that they are working on at their seats. Place models or a collection of loose K'NEX pieces at stations around the room. Provide directions and or questions at each station.

Questions may include:

• Place one or two models at several stations. Each model should be numbered so that students may refer to the number in providing a response. In visiting a station, each student selects one of the numbered models (if there are more than one) and would need only answer the station's questions in reference to that model. Provide a question related to fractional representations at each station. Be sure to have students indicate the terms numerator and denominator as related to fractions.

• Leave free K'NEX pieces on at least one of the stations. Ask students to make a K'NEX model that has different colored pieces representing different fractional parts of the model. (For example: make a model that is $2/10$ red, $4/10$ yellow, and $4/10$ blue.) When students complete their model, have them tag it with their name and place it in the collection box.

Extensions:

• Challenge students to review their charts for the Helicopter, Windmill and Sailboat. Did they simplify the fractions that they discovered for the various colors in the models? If not, instruct them to complete that task.
  - Observe students as they work. Are they working as a team? Is anyone not participating? Do students seem to understand what they are doing?
  - Have a few groups share their findings.

• Challenge students to build a house using any 48 rods and connectors.
  - Write a fraction showing how many parts of the house are rods and how many are connectors.
  - Write fractions for each of the different colored pieces as they relate to the whole house: 4 red pieces = $4/48$ = $1/12$.

• Challenge students to build a model which is: $1/8$ red, $1/16$ white, $1/16$ green, $3/8$ blue, $1/16$ purple, $1/8$ gray, $1/8$ yellow, and $1/16$ orange. They should prepare a chart to represent the fractional characteristics of their model. Instruct students to be prepared to identify the numerator and the denominator for each of the fractions they have created when you visit their group.
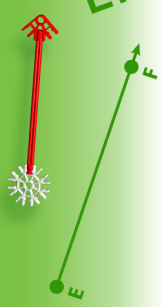
TEACHER NOTES:

It is suggested that you require students to simplify all of the fractions they report during the extension activities outlined above.
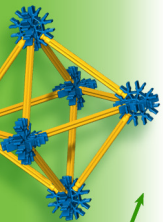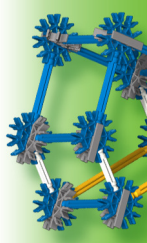
EF

F

E

y + s = s + y

y + y + w + w = 2y + 2w

w < r
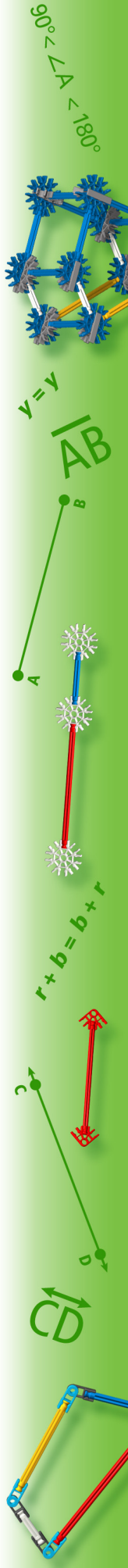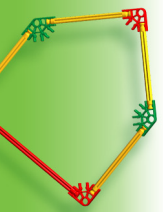
90° < A < 180°

90° < A < 180°

v = v

AB

B

A

r + b = b + r

C

D

CD

Transformation: Translation & Reflection

Lesson Topics: Exploring Transformations of Translation (Slide) and Reflection (Flip) Using K'NEX Polygons.

Lesson Length: 90-minutes
(two or more 45 minute time periods)

Student Objectives:

Students will:

- Review the concept of congruence.
- Identify a translation (slide) and create slides of models.
- Identify a reflection (flip) and create flips of models.
- Recognize the different geometric transformations of translations (slides) and reflections (flips).
- Draw the reflected image of figures across a line.
- Identify and draw lines of symmetry.

Grouping for Instruction:

- Group students into groups of 3 – 4 students each for building models. (There are sufficient materials in your set that will allow students to work in smaller groups for this lesson as many of the models they build will be small quadrilaterals.)
- Individual students or student pairs for creating and drawing models, labeling sides and writing directions for the transformation performed and for identifying that transformation.
- Individual students for performance assessment and written assessment activities.

Materials and Equipment:

➤ *K'NEX Intermediate Math & Geometry set and Instructions Booklets*

➤ *Overhead projector or document projector*

➤ *Three 11" x 14" sheets of paper (for each group) to serve as work mats. They should be oriented portrait style and divided into 4 equal parts with a large plus sign on each of three pages.*

➤ *Journals and drawing paper*

➤ *Rulers*

Overview of Lesson:

- Students will use K'NEX to build, investigate, draw and describe two of the three transformations of translation (slide) and reflection (flip). They will also identify the line of symmetry in models that are symmetrical.
- Students will use K'NEX mathematics conventions as they describe figures.
- Students will create a design that has at least one slide and one flip and if appropriate, they will also identify a line of symmetry. Students will transfer the K'NEX designs they construct to paper and label and describe the flips and slides in that model.
- In addition, given a design using slides and flips made of K'NEX shapes, the students will identify all slides, flips, and all lines of symmetry, if appropriate.

Background Information:

➤ *Students should have had exposure to basic conventions of creating K'NEX shapes. They should be familiar with the three basic transformations of translation (slide), rotation (turn), and reflection (flip), as well as the concepts of congruency and symmetry.*

A – Motivation and Introduction:

1. Ask student groups to make two each of the models shown on page 12 in the Instructions Booklet.
2. Provide ample time for creative exploration with the polygons. Do they place them on top of each other to check for congruence?
3. Ask students to discuss with a partner what they know about transformations. How many are there?

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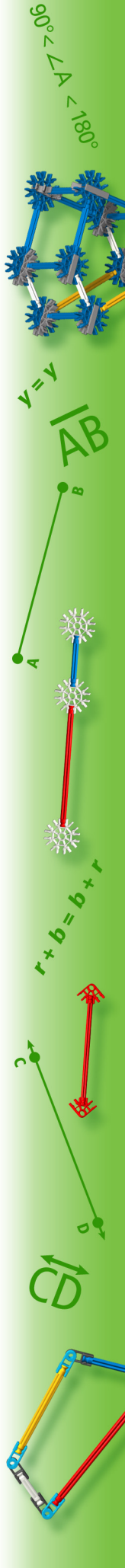
Can they name them? What does each transformation do? Have students take notes on their conversations for later discussion.

4. When the students seem to have finished their discussions, ask one group to volunteer to share what they discussed:

    a. What are transformations?

    b. How many are there?
       How many do you know?

    c. What are their names?

    d. What does each transformation do?

5. Ask the other group(s) to comment on what the first group shared... Are there other modifications or additions to add to the discussion?

6. Using teacher models, review each of the three transformations: translation (slide), rotation (turn) and reflection (flip). Illustrate these transformations for the students on the overhead projector.

B. Development (including discussion points and feedback)

1. Tell students that today we are going to explore translation and reflection transformations using K'NEX models. The first transformation that we will consider is a translation, or a slide. (Be sure to give students both names – the formal mathematics name of translation and the informal name of slide.)

2. Instruct the students to assign each person in their group to play one of these roles: 1) Facilitator, 2) Model-Movers, 3) Note-Takers/Reporters and 4) Taskmasters (to keep the team on task).

3. Have each group place one 11" x 14" mat on their tables in a "portrait" position. Have them label the four areas starting from the top left and going clockwise. The top positions, therefore, are numbered 1 and 2, position 3 will be under position 2 and position 4 will be under position 1. (The numbers can be small and in the corners of the various sections of the page.)

1	2
4	3

4. Have the students replace one of the light gray connectors on their triangle models with a white connector. They can then place one of their triangle models in the center of area 1 and place the second one on top of it so that the congruent parts are touching.

5. Instruct students to slide the top model over to area 2 on their mat so that it is directly to the right of the model in area 1. This is a "right slide".

6. Instruct students to take the model from area 2 and place it back on top of the model in area 1 so that congruent parts are touching.

7. Instruct students to slide the top model down to area 4 on their mat so that it is directly under the model in area 1. This is a "down slide".

8. Ask students what they think an "up slide" would be? What about a "left slide"?

9. Ask students to take the model from area 4 and place it on top of the model in area 1 so that congruent parts are touching.

10. Ask students to move their model to area 3 using only the slides defined above (right, down, up, left)? How many slides must they complete? Can they find more than one way to slide the model on top of the model in space 1 so that it ends up in space 3? The note-taker should record the moves that their group makes in order to accomplish this task.

11. Ask one group to describe how they moved the model from area 1 to area 3. Ask if any other group to offer a second (different) solution.

12. Instruct students to place both triangle models in area 2 on top of one another. Challenge them to find a way to move the top model from area 1 to area 2 in a way other than a simple right slide.

13. Ask groups to raise their hands if they were able to move a model from area 1 to area 2 in a way other than a right-slide. Have one group describe what slides they could use to move a model from area 1 to area 2 (other than the right slide). Ask if any other group had done it that way. Ask if any group had done this is a different way.

14. Tell students that slide is one type of transformation. Its formal mathematics name is "translation". Another transformation is called "reflection", or "flip".

15. Instruct students to make two additional rectangles as shown on page 12 of the Instructions Booklet. Have students replace one of the red connectors on each of the models with a white connector. (This will help students to distinguish the orientation of their models when they are reflected/flipped.)

16. Using the 11" x 14" mat on their tables, have students place one of their rectangle models in the center of area 1 and then place a congruent (identical) model on top of it so that congruent parts are above one another. Make sure that the white connector is on the bottom left-hand corner of each model. (Model this for students on the overhead projector and then have students complete the direction on their papers.)

17. Instruct students to flip the top model over and place it in the center of area 2 on their mat. This model will be directly to the right of the model in area 1. This is a flip to the right. (Model this for students on the overhead projector.)
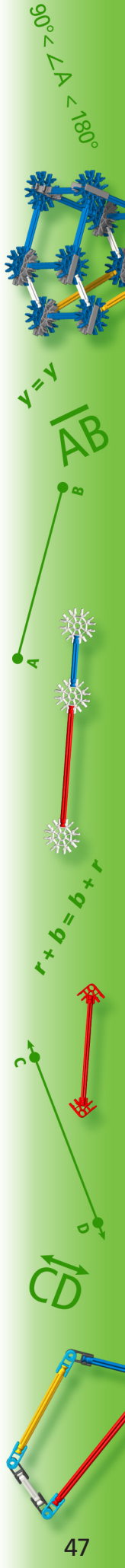
18. Ask students what they notice about the two models. (After the flip, the white connector is on the right side of the model.)

19. Instruct students to take their third rectangle model and place this congruent shape on top of the model in area 1 so that congruent parts are directly above one another.

20. Instruct students to flip the top model down to the center of area 4 on their mat so that it is directly under the model in area 1. This is a flip that goes down.

21. Ask students what they notice now about the result. (After the flip, the white connector on the model in area 4 is on the top left corner of the model.)

22. Ask students to take their third model and place this congruent shape on top of the model in area 1 so that congruent parts are touching.

23. Ask students to move their model to the center of space 3 using as many left-right-up-down flips as necessary. Can they find more than one way to do flip so that the model that is on top of the model in space 1 ends up in area 3? The note-taker should record the order and direction of the flips taken. They should also keep track of the location of the white connector during the series of moves.

24. Ask one group to describe how they moved the model from area 1 to area 3. Ask if any other group had done it that way.

25. Ask if any group had done it in a different way. Ask one of the groups who responded yes to describe how they had moved the model from area 1 to area 3. Ask if any other group had done it that way.

26. Ask students to compare the models in each of the 4 areas. What do they notice? (They are all in the center of the named space, but each model is in a different orientation.)

27. Ask the groups how many times they would have to flip the rectangle model to get it into the same orientation as it started with (the white connector in the lower left-hand corner). Allow time for experimentation. (It will require four flips and the model will be returned to its original position. Some students may suggest flipping the model in one direction and then back again to get the same results so you may wish to suggest that there will be no reverse flips.)

28. Ask students to work as a group to transfer the models in areas 1 and 4 to their paper. They will transfer one model first and then use a new technique to draw their second model. When they have finished drawing the rectangle in area 1 on their paper, they should place a large dot at the vertex where the white connector is located. They can then fold their paper carefully along the line that separates areas 1 and 4 so that their drawing is clearly visible and not covered when the fold is completed.

29. Identify the fold and the line that separates areas 1 and 4 as a "line of reflection". The image of the first rectangle will soon be reflected across this "line of reflection" from area 1 into area 4.

30. Instruct students to take their papers to the window, a few at a time and to place the side of the page with the rectangle drawing against the window. They should then be able to trace the rectangle onto area 4 of their paper including the large dot.

31. When students unfold their papers, they will have a perfect reflection of their rectangle across the "line of reflection" that includes the fold they made in their paper. They location of the reflection is in its exact position in area 4 and is referred to as a "mirror image" of the rectangle in area 1.

32. When you completed these activities with the triangles and rectangles, how could you tell whether the model was slid? That it was flipped?

33. If the models completed a slide from one area to another did the white connector remain in the same position? Why did that happen? (Yes, because the model was never turned or flipped.)

34. If the models completed a flip from one area to another did the white connector remain in the same position? Why did that happen? (No, because the flipping of the model caused a mirror image of the model to be formed in the new area.)

C – Summary and Closure

1. Have a collection of pairs of different K'NEX models made up in advance that can be used to demonstrate slides and flips on the overhead. Include one white connector on each so the results of the slides and flips can be clearly seen.

2. On an overhead transparency clearly outlined with areas 1, 2, 3, and 4 show the results of several different slides and flips. Place the original model in one area and then place the resulting image in another area. Ask students to write down the appropriate transformations that could have been followed to achieve those results. Repeat the activity several times.

3. Ask students to write down three things that they learned in class today.

4. Have students share their statements with students in their group.

5. Have each group share at least one observation with the class.
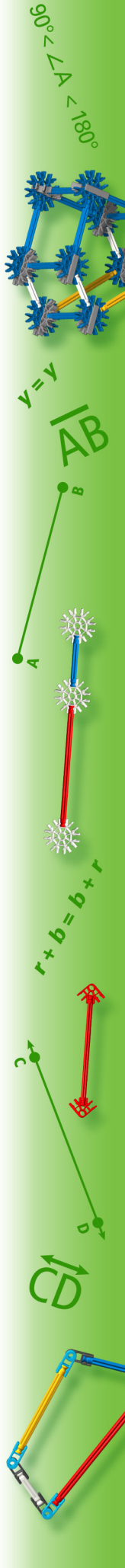
Assessment:

• Challenge students to build creative models to use in assessment scenarios for transformations. They should build two of each model. Student models may be used at different stations that the students visit.

• Establish a series of four to six performance assessment stations around the room that students can visit during a pencil and paper test or quiz that they are working on at their seats. Place K'NEX models or pieces at stations around the room. Provide directions and or questions related to transformations at each station.

For Example:

• Place identical pairs of polygon models at several stations. The model pairs could be rectangles, triangles, rhombi, pentagons or irregular convex and concave polygons. Provide a question related to transformations (translations [slides] and reflections [flips]) at each station. Compare and contrast results of slides and flips and question to see if students can offer alternate ways to approach problems.

• Have students show flips and identify a line of reflection.

• Have students explain the difference between a slide and a flip.

• Have students draw a shape and a mirror image of that shape across a line of reflection.

Extensions:

• Challenge students to make pairs of models that will be useful in demonstrating concepts they have learned about translations and reflections.

• Instruct the students to produce careful drawings that show their models in one area and then show an image of the model in another area. The graphic representations should include a listing of the series of slides and or flips required to complete the transformations.
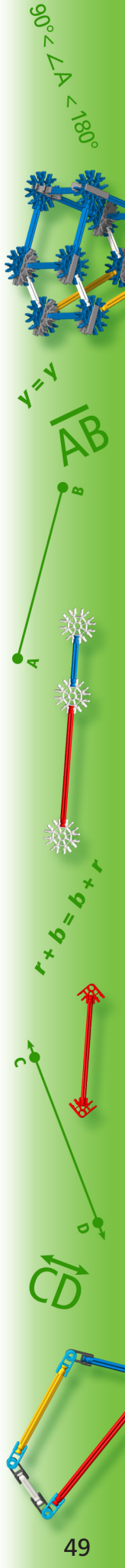
LESSON 13
Transformation: Rotation

Lesson Topics: Transformations: Rotations (Turns) and a Coordination of All Transformations Using K'NEX.

Lesson Length: Two or Three 45-minute classes

Materials and Equipment:
➤ *K'NEX Intermediate Math & Geometry set and Instructions Booklets*

➤ *Overhead projector or document projector*

➤ *Three 11" x 14" sheets of paper (for each group) to serve as work mats. They should be oriented portrait style and divided into 4 equal parts with a large plus sign on each of three pages.*

➤ *Chart paper (one sheet per group)*

➤ *Colored pencils or crayons*

➤ *Journals and drawing paper*

➤ *Rulers*

Student Objectives:
Students will:
* Explore the results of turning a K'NEX model into a new position.
* Explore the results of a sequence of turns.
* Identify the geometric transformations used to create a given design using informal language.
* Illustrate and define each of the three transformations: reflection (flip), translation (slide), and rotation (turn).
* Make a design using congruent models and transformations.

Grouping for Instruction:
* Place students into groups of 3 – 4 students each for building models.
* Individual students or student pairs for creating and drawing models, labeling sides and writing directions for the transformation performed and for identifying that transformation.
* Individual students for performance assessment and written assessment activities.

Overview of Lesson:
* This lesson encourages students to explore the geometric transformation of rotation. While the formal term is rotation, the informal term "turn" is used at this grade band.
* Students will use K'NEX materials to build, investigate, draw and describe the transformation of rotation (turn) and to compare and contrast the three transformations of translation (slide), rotation (turn) and reflection (flip).
* Students will use K'NEX mathematics conventions as they describe these operations and the models used.
* Students will create a design that has at least one slide, one flip, and one turn; and if appropriate, they will also identify a line of symmetry.
* Students will transfer the K'NEX designs they construct to paper and label and describe the flips, slides and turns in that model. In addition, given a design using slides, flips and turns made of K'NEX shapes, students will identify all slides, flips and turns and all lines of symmetry, if appropriate.

Background Information:
➤ *Students should understand the basic ways of building with K'NEX. They should be familiar with the 3 basic transformations of translation (slide), rotation (turn), and reflection (flip), as well as with the concepts of congruence and symmetry. If students completed Lesson #12 before this lesson, they will be well prepared for these activities.*

A – Motivation and Introduction:
1. Ask student groups to make four of the rhombi pictured on page 12 of the Instructions Booklet.
2. Ask students to discuss with a partner what they know about transformations that they learned previously using K'NEX. Can they name all three transformations? What does each transformation do? Have students take notes on their conversations for later discussion.
3. When the students seem to have finished their discussions, ask one group to volunteer to share what they discussed:
   a. What are transformations?
   b. How many are there? How many do you know?
   c. What are their names?
   d. What does each transformation do?
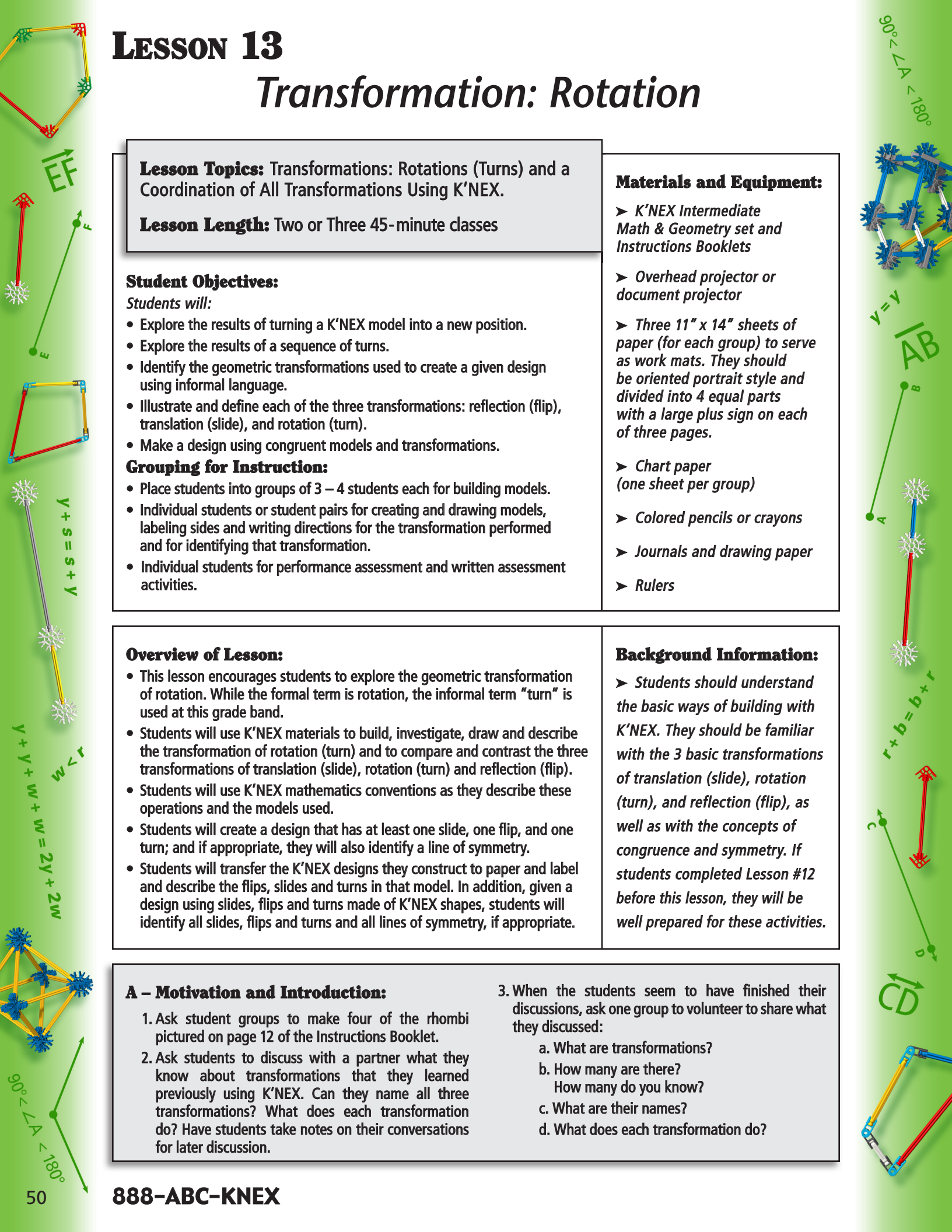
4. Ask another group(s) to comment on what the first group had shared. Are there other modifications or additions to add to the discussion?

5. Using teacher models on the overhead projector, review each of the two transformations: translation (slide) and rotation (turn) that students have already explored. Have the students come to the overhead projector to model each of these.

B. Development (including discussion points and feedback)

1. Tell students that today we are going to explore the transformation of rotation (turn). Rotation is the formal mathematics name and the informal name is "turn". We will be using the term "turn". (Be sure to give students both names – the formal mathematics name of rotation and the informal name of turn.)

2. Instruct the students to assign each person in their group to play one of these roles: 1) Facilitator, 2) Model-Movers, 3) Note-Takers/Reporters, and 4) Taskmasters (to keep the team on task).

3. Have each group place one 11" x 14" mat on their tables in a "portrait" position. Have them label the four areas starting from the top left and going clockwise. The top positions, therefore, are numbered 1 and 2, position 3 will be under position 2 and position 4 will be under position 1. (The numbers can be small and in the corners of the various sections of the page.)

1	2
4	3

4. Have the students replace one of the light gray connectors on their rhombi models with a white connector. They can then place one of their rhombi models in the center of area 1 and place the second one on top of it so that congruent parts are touching. (Orient the models so the white connectors are closest to the top of the page.)

5. Instruct students to turn the top model a half a turn (i.e. 180°) so that they may observe this top (turned) model more easily, have them place that model in the center of area 2 on their mat so that it is directly to the right of the model in area 1.

TEACHER NOTES:

Help students to see that after they rotated or turned their model, they slid it to area number 2 so that the results of the turn would be easier to see and to compare to their original rhombus.

6. Ask students to discuss in their groups what they notice. In what ways are the two models alike and

how are they different? Make as many observations as you can.

7. Instruct students to take their third model and place this congruent shape on top of the model in area 2 so that all congruent parts are touching.

8. Instruct students to turn the top model a half a turn (i.e. 180°) so that they may observe this model more easily, have them place that top (turned) model in the center of area 3.

9. Ask students to discuss in their groups what they notice. How is this newly-turned model the same as model in area 2? The same as the model in area 1? How is this newly-turned model different from the model in area 2? How is it different from the model in area 1? Make as many observations as you can. (You may want to have the students make a table in which to make these observations.)

10. Instruct students to take their fourth model and place this congruent shape on top of the model in area 3 so that all congruent parts are touching.

11. Instruct students to turn the top model a half a turn (i.e. 180°) so that they may observe this model more easily. Have them place that top (turned) model in the center of area 4.

12. Ask students to discuss in their groups what they notice. What are the likenesses and what are the differences that they notice when they view the complete set of four models? (The models in areas 2 and 4 will be identical and the models in areas 1 and 3 will be identical.)

13. Have students transfer the four rotated images onto their mat using pencils and rulers. Have students write "Half Turns" at the top of the mat.

14. Instruct students to use a new mat. Have them follow the procedure as above, but this time have them turn the model just a quarter of the way around (i.e. 90°). Instruct them do this for all four models going from area 1 to 2, area 2 to 3, and area 3 to 4.

15. As above, have students transfer these models to their mats. Have students write "Quarter Turns" at the top of the mat.

16. In groups, ask students explore the similarities and differences between the two mats. What likenesses do they notice? What differences do they notice?

17. Have students take their third mat. Ask how many can remember the three different transformations that we have just studied. (This assumes that Lesson #12 had been done previous to this lesson.) Have them discuss this in their groups and to model each on the new mat in front of them. Observe what the students are saying and doing.

18. Inform the students that they are going to make a design using a model that they will create. Once they have a model, they will use each of the three transformations – slide, flip, or turn to move their
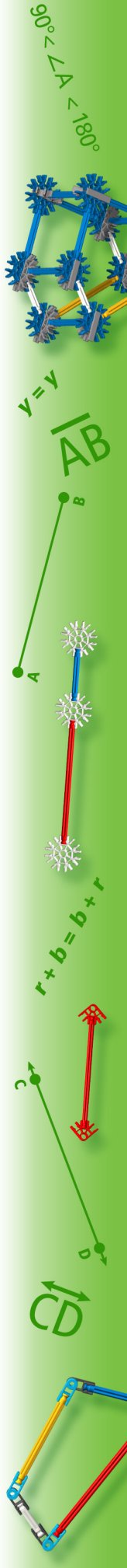
shape from location to location on their paper. They will draw the shape at each of these locations. When the models are removed from the page, a pattern will result.

19. Give students these directions:
   • Groups may work on their table or on the floor.
   • Designs will be built and transferred to chart paper.
   • Make a new K'NEX model using at least two different color rods. Make a collection of these models so you can plan your transformations to form a pattern you like. Connectors may be any color, but they should be the same for each model. (Keep the models simple and congruent.)
   • Put the first model on the paper and transfer it (as above) to the paper. Identify this model as "original model".
   • Place the model on top of the drawn model.
   • Use the additional models and transformations to create a design. Groups must use all three transformations in the design and they may use each more than once.
   • When the design is completed, the group should transfer it to the chart paper and identify each transformation used in making the design.
   • When all designs are completed, groups will be asked to share their design with the whole class and to discuss how they developed their design. (Suggested guiding questions for this discussion might be: What types of moves did you do first? Show us. Did you do a given transformation from the original piece? Or did you do each transformation from the last piece that was just added to the design? When you did a turn, did you also do a slide so that you moved the transformed model away from the original model?)

20. While students are working, observe what they are saying and what they are doing. Also observe the interaction within the group. Is one person doing all of the work? Are all students participating equally? Are all students' opinions listened to and respected? If a group has trouble getting started or gets stuck, ask them some prompting questions that will help to move them along.

21. As students finish their designs and have them transferred to chart paper call upon groups to show their design and explain how they were formed. (Questions above may guide this discussion.)

22. When students are finished, post their designs on the wall or board. (If some students complete the tasks quickly, provide them with a variety of crayons and ask them to add color to their designs.)

C – Summary and Closure

1. Have a collection of four congruent models made up in advance.
2. Take two of the models at a time. Place one against the white board or put it on the overhead projector. Take the second congruent model and put it on the first then do a transformation. Ask students to write down the name of the transformation that could be used to describe the action that was done. Do this several times.
3. Ask students to write down three things that they learned in class today.
4. Have students share their statements with students in their group.
5. Have each group share at least one observation with the class.
6. Ask the students to give themselves a rating of 0, 1, 2, 3, 4, or 5 to indicate how well they think they understand transformations.
   • 0 = student does not understand anything
   • 1 - 4 = student understands some - most
   • 5 = student understands completely and can explain it to someone else
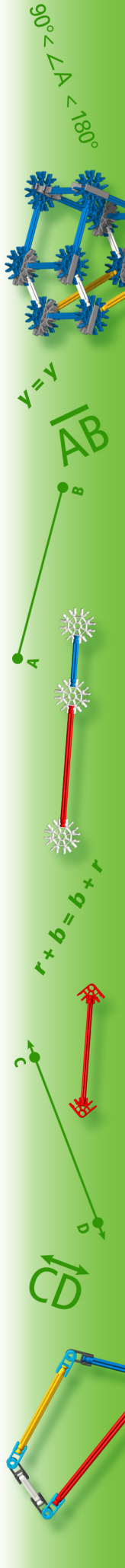
Assessment:

• Challenge students to build a model that may be used in an assessment to show different transformations. Student models illustrating transformations may be placed about the room for other students to identify. That allows students to hand in a sheet identifying and explaining the different transformations.

• Establish a series of four or five performance assessment stations around the room where students can visit during a pencil and paper test or quiz that they are working on at their seats. Have one group of students at a time visit the performance assessment stations and provide time for them to work at each station. Place polygon models or a collection of loose K'NEX pieces at stations around the room. Provide directions and or questions at each station.

Questions may include:

• Place groups of congruent polygon models at several stations. Each group should be different, but the models in each group should be congruent. Provide a question related to transformations at each station. For example: use the models and make a flip to the right. Draw and label both the original position and the position when the transformation is completed.

• Make a small model using K'NEX rods. Make three additional congruent models. Use these four models and make a design using the three transformations. Transfer your design to paper and identify the original piece and what transformation(s) was used to change the position of the models.

Extensions:

• Ask students to make a series of rotations of the Nonagon from page 7 of the Instructions Booklet and to transfer the results to a large sheet of paper in a single row rather than on a grid with four areas. Does the row of transfers form a pattern? Explain why or why not.

• Rotations, reflections and translations can be made using any of the models found on pages 4, 5, 6, 7, 12, 13, and 14 of the Instructions Booklet.

• Diagram the rotation of each of the three triangles found on page 4 of the Instructions Booklet. Make four diagrams of each to form three rows on a large sheet of paper.
  - Turn the Isosceles triangle a 1/4 turn or 90° each time.
  - Turn the Equilateral triangle 1/2 turn or 180° each time.
  - Turn the Acute triangle 3/4 turn or 270° each time.
  - Describe the results and indicate any patterns that are formed.

• Make a series of four diagrams of 1/4 rotations of either the cross-shaped polygon on page 13 or the L – shaped polygon on page 14 of the Instructions Booklet. What pattern(s) do you identify?
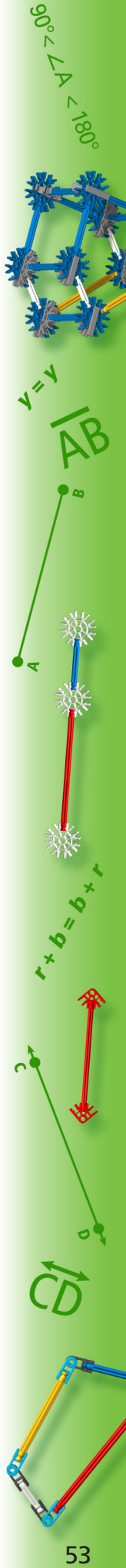
LESSON 14
Rules Make Patterns

Lesson Topics: Patterns.

Lesson Length: 45-minutes

Student Objectives:
Students will:
- Identify attributes that determine a pattern.
- Extend a pattern and justify the extension both orally and in writing.
- View patterns from several perspectives.
- Identify similarities and differences among items or objects.
- Examine collections of objects from different perspectives.
- Use elementary number theory to formulate conjectures.

Grouping for Instruction:
- Group students for building models, defining rules and making patterns.
- Individual students for drawing models, labeling sides, and listing a formula to represent perimeter measurements.
- Individual students for performance assessment and written assessment activities.

Materials and Equipment:
➤ K'NEX Intermediate Math & Geometry set and Instructions Booklets

➤ Chart Paper (two sheets per group)

➤ Pencils and crayons to share

➤ Journals and drawing paper

➤ Rulers

Overview of Lesson:
- Students will use K'NEX materials to identify patterns and to determine and justify what comes next in an identified pattern.
- They will build, investigate, draw, and describe the relationship between a term in a pattern and the previous and next term.
- Students will view patterns from different perspectives and identify similarities and differences between items or objects.
- They will use K'NEX mathematics conventions as they describe the patterns and use elementary number theory as they make conjectures.

Background Information:
➤ *Students should understand the basic ways of building with K'NEX and an understanding of simple patterns.*

A – Motivation and Introduction:
1. Tell the students that you were designing a pattern last night and these were the first five models that you put into your pattern. Tape to the white board – one at a time and in order. (Do this as the students watch.)
   - Figure 1 – 1 blue rod with 1 orange connector
   - Figure 2 – 2 white rods with 1 red connector
   - Figure 3 – 3 blue rods with 1 green connector
   - Figure 4 – 4 red rods with 1 white connector
   - Figure 5 – 5 yellow rods with 1 blue connector.

2. Ask the students this question:
   - Can you suggest what the next model should be? Think about it by yourself for a minute – then talk it over with a partner and make the model that you think should come next.
3. Some students might notice that each model has one rod more than the previous model. Other students might notice that the odd models in the sequence each has at least one blue K'NEX piece and the even models in the sequence each has at least one white and one red K'NEX piece. Other students might notice that in every model all of the spokes are of the same
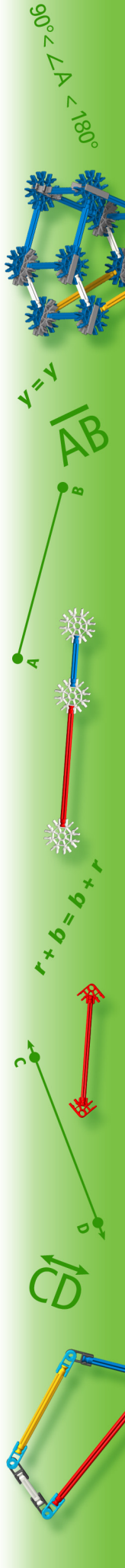
color. During this activity; if students ask questions to clarify one of their observations, turn the question back to them with a "Hmmmm ... What do you think?" As students work, observe their discussions and how students go about making the next model for this sequence. Make a note of any pairs who are coming up with an unusual solution.

4. As student-pairs have their models made, ask each pair to hold up the model they have made. Observe the differences in models students have designed.

5. Ask several pairs to show their models and explain what they had noticed about the pattern and why they had made the model that they made.

B. Development (including discussion points and feedback)

1. Instruct students to make the following polygons:
   a. 2 quadrilaterals from page 5 of the Instructions Booklet.
   b. 2 triangles from page 4 of the Instructions Booklet
   c. 2 pentagons from page 6 of the Instructions Booklet

2. Ask the student to note the different attributes of the models they have built. A short list will be beneficial as students continue to the next phase of the activity.

3. Have the groups discuss the models and then place them in order to make a pattern based on a "rule" that they have agreed upon.

4. Have groups transfer the figures onto a half-piece of drawing/chart paper in an order based on the "rule" they developed. Use the colors (that match the rods) to connect the dots and draw the rods on the mat.

5. Ask students to make a model of the next piece of the pattern, using the "rule" that they agreed upon. Have each group write their rule and a short explanation of their rule on a separate sheet of paper. Place their initials on their drawing page and on the page with their rule.

6. Ask groups to exchange their drawings. Each group

is to review the drawings to see if they are able to determine the rule that was used to make the pattern. They should write their ideas on a sheet of paper and place the initials of the team that provided the drawings next to the rule. Repeat this process until all papers have been reviewed and rules have been recorded.

7. Have the groups present what they had done and describe how they did it. Some questions to focus the discussion could be:
   a. How did you decide to sort your models?
   b. What was your rule?
   c. At first, did everyone in the group see this pattern or did some of you see things differently?
   d. Why is it important to collaborate when completing an activity such as this?

As each pair finishes presenting their pattern and explanation, provide the opportunity for others in the class to indicate if they identified the correct rule for the group's pattern.

8. Note that students often approach this activity in unique ways. Rules that the groups devised were different, so the patterns that they made were different. Emphasize that it is important to examine models or data when looking for a pattern and to use all observations to make patterns and make conjectures.

C – Summary and Closure

1. Show a collection of 4-5 models to the students and have them order the models according to a rule, one that they are able to describe

2. Ask students to write down three things that they learned in class today.

3. Have students share their statements with students in their group.

4. Have each group share at least one observation with the class.

5. Have students reflect on their understanding of this concept.
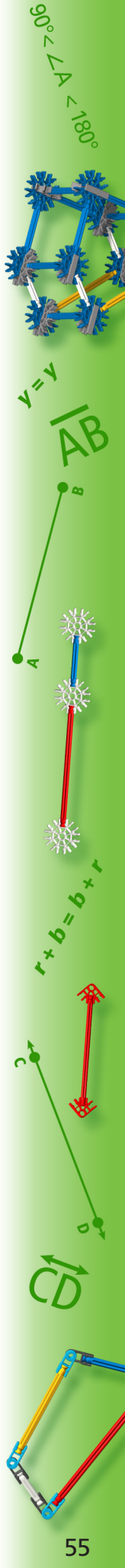
Assessment:

• Challenge the students to build a series of models which have different characteristics that will make them easy to organize and connect into a pattern. These models may be used in assessment scenarios. Student models may be placed at individual stations so that other students might visit and write a rule and create the next two models in the sequence.

• This activity works best if the sets of models are numbered so that the students can identify which set of models they were analyzing and writing rules to explain.

Questions may include:
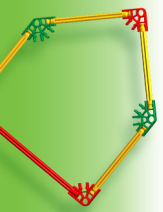
• Ask students to write some strategies that they used in observing models and writing rules.

• Have students write why it is important to observe different aspects of a set of models or data as they go about designing a rule and creating or extending a pattern sequence.

Extensions:

• Ask students to think of a rule for a pattern and make a set of 4 – 5 models or pieces that could be the first four or five objects in that pattern.

• Have them exchange their creations with another group and see how another group might order those same objects or pieces. Did their group come up with the same rule that was used to design the models or organize the pieces?

• Design a complex pattern for students to analyze. Place the sequence of models somewhere in the classroom at a location that will enable small groups of students to easily review the pattern. When groups have figured out the pattern, they can submit their rule to explain your pattern in writing.
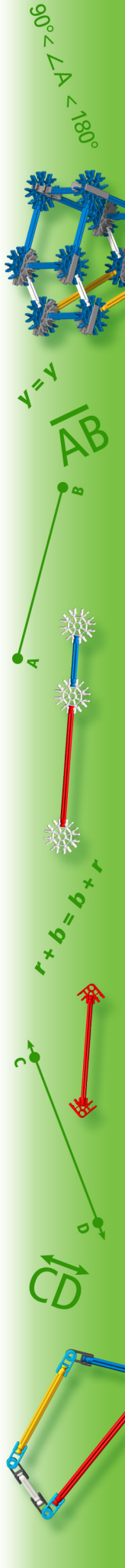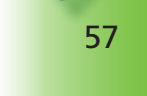
Glossary of Terms

Acute angle: an angle that measures between 0° and 90°.

Acute triangle: a triangle with three acute angles.

Adjacent sides: in a polygon, two sides that share a common endpoint.

adjacent sides
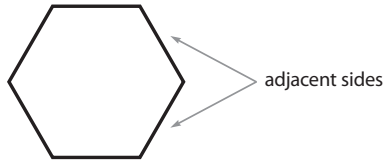
Altitude: height; the perpendicular distance from a vertex of a polygon to its opposite side.

Angle: the union of two rays with the same endpoint (its vertex); the amount of rotation of a ray about a fixed ray.

$0° < ∠A < 90°$
(acute angle)

$∠A = 90°$
(right angle)

$90° < ∠A < 180°$
(obtuse angle)

$∠A = 180°$
(straight angle)
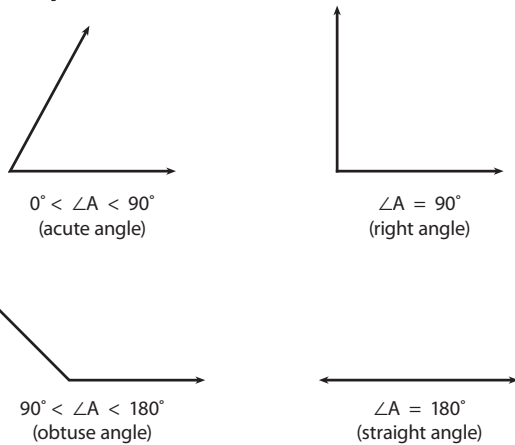
Apex: the point off the base of a pyramid where the triangular sides meet.

Area: the number of square units in a closed two-dimensional or plane shape.

Axis of symmetry: A line that divides a shape into two congruent halves.

axis of symmetry
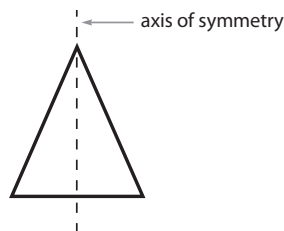
Base: the side of a shape used as the foundation for the shape; the face of a solid used as the foundation for the solid.

Bisect: to cut something (such as a line segment or an angle) into two equal parts.

Clockwise rotation: rotation about a point in a clockwise direction.

Collinear points: three or more points on the same line in a plane or in space.

Congruence: the relationship between two geometric shapes having the same size and shape (congruent shapes).

Corresponding parts (angles and sides): angles and sides in the same position on two shapes. In congruent shapes, the corresponding angles and sides are congruent.
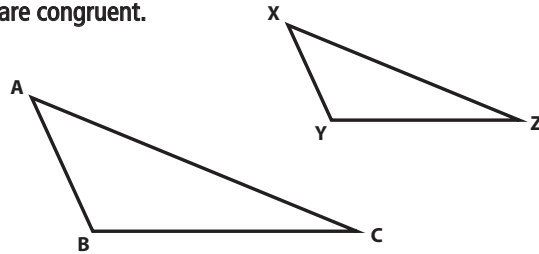
corresponding **sides**: $\overline{AB}$ and $\overline{XY}$; $\overline{AC}$ and $\overline{XZ}$; and $\overline{BC}$ and $\overline{YZ}$

corresponding **angles**: $∠A$ and $∠X$; $∠B$ and $∠Y$; and $∠C$ and $∠Z$

**Counterclockwise rotation**: rotation about a point opposite the rotation of a clock hand.

Cube: a rectangular polyhedron composed of six congruent squares.

cube
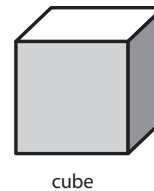
Decagon: a polygon of ten sides.

Degree: the standard unit for angle measure; one revolution is 360° ; a unit for measuring temperature.

Diagonal: a segment connecting two non-adjacent vertices of a polygon.
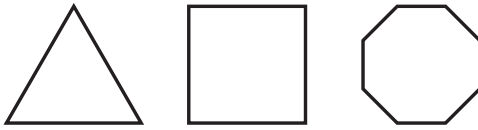
diagonal
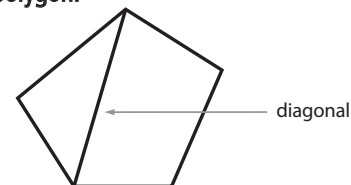
Dimensions: length, width and/or height of a plane, or solid shape.

Edge: the line of a three-dimensional shape where two plane faces meet.

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Equiangular: a term used to indicate that all angles of a polygon have the same measure.

Equilateral: a term used to indicate that all the sides of a polygon are equal in length.
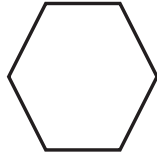
equilateral polygons

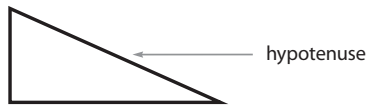**Face**: one of the plane surfaces of a polyhedron bounded by edges.

Geometry: the study of space and the properties of shapes in space.

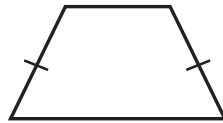**Hexagon**: a polygon with six sides.

hexagon

Hypotenuse: the side in a right triangle that is opposite the right angle.

hypotenuse

Isosceles trapezoid: a quadrilateral with one pair of sides that are parallel and another pair of sides that are not parallel but have equal lengths; the base angles are equal in measurement.

isosceles trapezoid

Isosceles triangle: a triangle with two congruent sides.

isosceles triangle

Kite: a quadrilateral with two distinct pairs of adjacent sides that are congruent.

Leg: in a right triangle, it is a side that is not the hypotenuse.

legs
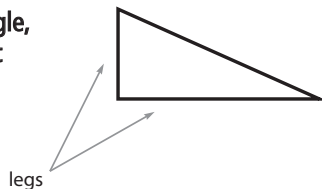
Length of a segment: the distance between the endpoints of a segment.

Line: an undefined term for an infinite, one-dimensional object.

Line of reflection: a line used to create a reflection of a shape.

Line segment: part of a line consisting of two endpoints and all the points on the line between them.

Line of symmetry: a line that divides a shape into two congruent halves.

Non-collinear points: points that do not lie on the same line.
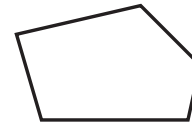
Obtuse angle: an angle with measure of between 90° and 180°.

Obtuse triangle: a triangle with one interior angle that is obtuse.

Octagon: a polygon of eight sides.

One-dimensional: having length but no width, e.g., lines, rays and segments.

Pentagon: a polygon with five sides.

pentagon

Perimeter of a polygon: the sum of the lengths of all sides of a polygon; the distance around a closed plane shape.

Perpendicular: forming a right angle.

right angle

**Plane**: an undefined term for a flat and infinite two-dimensional shape.

Point: a location on a line, in a coordinate plane, or in space; an undefined term for a zero-dimensional object (having no length, width, or height).

Polygon: a simple closed shape composed of a finite number of line segments, each of which intersects exactly two of the other segments, one at each endpoint.

Polyhedron: a simple closed three-dimensional shape formed by plane polygons.
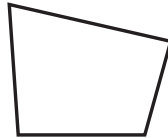
Protractor: a tool used to measure angles.
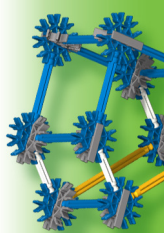
Pyramid: a polyhedron that has one base and a set of edges that meet at a single point (apex) that is not in the base; all faces except the base are triangles.

Quadrilateral: a polygon with four sides.

quadrilateral

**Ray**: a part of a line with a single endpoint and that extends infinitely in one direction.

Rectangle: a quadrilateral with all interior right angles.

rectangle

Reflection (flip): a transformation that maps each point in a plane to a new point that is the same distance from a fixed line (called the line of reflection) but on the opposite side of the line; informally, a geometric shape that can be flipped over a line so that the new shape is a mirror image of the original.

Regular polygon: a polygon that is equilateral and equiangular.
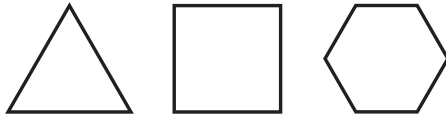
regular polygons

Rhombus: a parallelogram with four congruent sides.
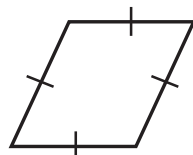
rhombus

**Right angle**: an angle with a measure of 90°.

Right triangle: a triangle that has one right angle.

right triangle

Rotation (turn): a transformation that maps each point in a plane shape to its image by rotating the plane through an angle (called the turn angle) around a fixed point (called the turn center or center of rotation).

Scalene triangle: a triangle with no two sides of the same length.
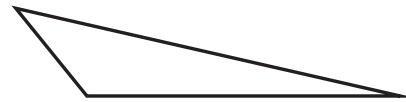
scalene triangle

Segment: a set of points containing two endpoints and all points along the straight line between the two endpoints; a part of a circle between an arc and its chord.

Similar shapes: two shapes that have: the same shape; corresponding angles that are congruent; and corresponding sides that are proportional.
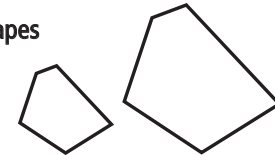
similar shapes

Slide: (see **translation**)

Solid geometry: geometry that deals with shapes and their properties in three-dimensional space.

Square: an equilateral and equiangular quadrilateral.

square

Straight angle: an angle with measure of 180°.

Symmetry: correspondence in size, shape and relative position of parts on opposite sides of a dividing line or median plane, or about a center or axis.

Three-dimensional: having length, width, and thickness or depth.
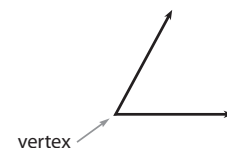
Translation (slide): a transformation of a plane where every point P is moved in the same direction and the same distance to a new point P; moving a shape along a straight line without it flipping, rotating, or reflecting.

Trapezoid: a quadrilateral with exactly one pair of parallel sides.
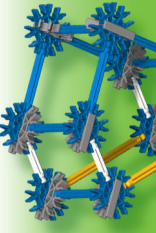
Triangle: a polygon with three sides.

Two-dimensional: having both length and width, but no thickness or depth.

Vertex: the point where two rays forming an angle meet, the point where two sides of a polygon meet, or the point where three or more faces of a polyhedron meet.

vertex

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