

# CLASS MANUAL

Last updated March 25, 2017

Generated by Doxygen 1.8.13



# Contents

- 1 CLASS: Cosmic Linear Anisotropy Solving System** **1**
  
- 2 Where to find information and documentation on CLASS?** **3**
  
- 3 CLASS overview (architecture, input/output, general principles)** **5**
  
- 4 Data Structure Documentation** **15**
  - 4.1 nonlinear Struct Reference . . . . . 15
    - 4.1.1 Detailed Description . . . . . 15
    - 4.1.2 Field Documentation . . . . . 16
      - 4.1.2.1 method . . . . . 16
      - 4.1.2.2 k\_size . . . . . 16
      - 4.1.2.3 k . . . . . 16
      - 4.1.2.4 tau\_size . . . . . 16
      - 4.1.2.5 tau . . . . . 16
      - 4.1.2.6 nl\_corr\_density . . . . . 16
      - 4.1.2.7 k\_nl . . . . . 16
      - 4.1.2.8 nonlinear\_verbose . . . . . 17
      - 4.1.2.9 error\_message . . . . . 17

|  |           |
|--|-----------|
| <b>5 File Documentation</b>                        | <b>19</b> |
| 5.1 background.c File Reference                    | 19        |
| 5.1.1 Detailed Description                         | 20        |
| 5.1.2 Function Documentation                       | 21        |
| 5.1.2.1 background_at_tau()                        | 21        |
| 5.1.2.2 background_tau_of_z()                      | 22        |
| 5.1.2.3 background_functions()                     | 23        |
| 5.1.2.4 background_w_fld()                         | 24        |
| 5.1.2.5 background_init()                          | 25        |
| 5.1.2.6 background_free()                          | 26        |
| 5.1.2.7 background_free_input()                    | 26        |
| 5.1.2.8 background_indices()                       | 27        |
| 5.1.2.9 background_ncdm_distribution()             | 27        |
| 5.1.2.10 background_ncdm_test_function()           | 28        |
| 5.1.2.11 background_ncdm_init()                    | 28        |
| 5.1.2.12 background_ncdm_momenta()                 | 29        |
| 5.1.2.13 background_ncdm_M_from_Omega()            | 30        |
| 5.1.2.14 background_solve()                        | 30        |
| 5.1.2.15 background_initial_conditions()           | 31        |
| 5.1.2.16 background_output_titles()                | 33        |
| 5.1.2.17 background_output_data()                  | 33        |
| 5.1.2.18 background_derivs()                       | 33        |
| 5.1.2.19 V_e_scf()                                 | 34        |
| 5.1.2.20 V_p_scf()                                 | 35        |
| 5.1.2.21 V_scf()                                   | 36        |
| 5.2 background.h File Reference                    | 36        |
| 5.2.1 Detailed Description                         | 37        |
| 5.2.2 Data Structure Documentation                 | 37        |
| 5.2.2.1 struct background                          | 37        |
| 5.2.2.2 struct background_parameters_and_workspace | 41        |

---

|          |  |    |
|----------|--|----|
| 5.2.2.3  | <a href="#">struct background_parameters_for_distributions</a> | 41 |
| 5.3      | <a href="#">class.c File Reference</a>                         | 41 |
| 5.3.1    | <a href="#">Detailed Description</a>                           | 41 |
| 5.4      | <a href="#">common.h File Reference</a>                        | 42 |
| 5.4.1    | <a href="#">Detailed Description</a>                           | 43 |
| 5.4.2    | <a href="#">Data Structure Documentation</a>                   | 43 |
| 5.4.2.1  | <a href="#">struct precision</a>                               | 43 |
| 5.4.3    | <a href="#">Enumeration Type Documentation</a>                 | 51 |
| 5.4.3.1  | <a href="#">evolver_type</a>                                   | 51 |
| 5.4.3.2  | <a href="#">pk_def</a>   | 51 |
| 5.4.3.3  | <a href="#">file_format</a>                                    | 52 |
| 5.5      | <a href="#">input.c File Reference</a>                         | 52 |
| 5.5.1    | <a href="#">Detailed Description</a>                           | 53 |
| 5.5.2    | <a href="#">Function Documentation</a>                         | 53 |
| 5.5.2.1  | <a href="#">input_init_from_arguments()</a>                    | 53 |
| 5.5.2.2  | <a href="#">input_init()</a>                                   | 54 |
| 5.5.2.3  | <a href="#">input_read_parameters()</a>                        | 55 |
| 5.5.2.4  | <a href="#">input_default_params()</a>                         | 57 |
| 5.5.2.5  | <a href="#">input_default_precision()</a>                      | 58 |
| 5.5.2.6  | <a href="#">get_machine_precision()</a>                        | 59 |
| 5.5.2.7  | <a href="#">class_fzero_ridder()</a>                           | 59 |
| 5.5.2.8  | <a href="#">input_try_unknown_parameters()</a>                 | 59 |
| 5.5.2.9  | <a href="#">input_get_guess()</a>                              | 60 |
| 5.5.2.10 | <a href="#">input_find_root()</a>                              | 60 |
| 5.6      | <a href="#">input.h File Reference</a>                         | 61 |
| 5.6.1    | <a href="#">Detailed Description</a>                           | 62 |
| 5.6.2    | <a href="#">Enumeration Type Documentation</a>                 | 62 |
| 5.6.2.1  | <a href="#">target_names</a>                                   | 62 |
| 5.7      | <a href="#">lensing.c File Reference</a>                       | 63 |
| 5.7.1    | <a href="#">Detailed Description</a>                           | 64 |

---

---

|          |                              |    |
|----------|------------------------------|----|
| 5.7.2    | Function Documentation       | 64 |
| 5.7.2.1  | lensing_cl_at_l()            | 64 |
| 5.7.2.2  | lensing_init()               | 65 |
| 5.7.2.3  | lensing_free()               | 66 |
| 5.7.2.4  | lensing_indices()            | 66 |
| 5.7.2.5  | lensing_lensed_cl_tt()       | 67 |
| 5.7.2.6  | lensing_addback_cl_tt()      | 67 |
| 5.7.2.7  | lensing_lensed_cl_te()       | 68 |
| 5.7.2.8  | lensing_addback_cl_te()      | 68 |
| 5.7.2.9  | lensing_lensed_cl_ee_bb()    | 68 |
| 5.7.2.10 | lensing_addback_cl_ee_bb()   | 69 |
| 5.7.2.11 | lensing_d00()                | 69 |
| 5.7.2.12 | lensing_d11()                | 70 |
| 5.7.2.13 | lensing_d1m1()               | 70 |
| 5.7.2.14 | lensing_d2m2()               | 71 |
| 5.7.2.15 | lensing_d22()                | 71 |
| 5.7.2.16 | lensing_d20()                | 71 |
| 5.7.2.17 | lensing_d31()                | 72 |
| 5.7.2.18 | lensing_d3m1()               | 72 |
| 5.7.2.19 | lensing_d3m3()               | 73 |
| 5.7.2.20 | lensing_d40()                | 73 |
| 5.7.2.21 | lensing_d4m2()               | 73 |
| 5.7.2.22 | lensing_d4m4()               | 74 |
| 5.8      | lensing.h File Reference     | 74 |
| 5.8.1    | Detailed Description         | 76 |
| 5.8.2    | Data Structure Documentation | 76 |
| 5.8.2.1  | struct lensing               | 76 |
| 5.9      | nonlinear.c File Reference   | 77 |
| 5.9.1    | Detailed Description         | 78 |
| 5.9.2    | Function Documentation       | 78 |

---

---

|           |                                |    |
|-----------|--------------------------------|----|
| 5.9.2.1   | nonlinear_init()               | 78 |
| 5.9.2.2   | nonlinear_halofit()            | 78 |
| 5.10      | nonlinear.h File Reference     | 79 |
| 5.10.1    | Detailed Description           | 80 |
| 5.10.2    | Macro Definition Documentation | 80 |
| 5.10.2.1  | _M_EV_TOO_BIG_FOR_HALOFIT_     | 80 |
| 5.11      | output.c File Reference        | 80 |
| 5.11.1    | Detailed Description           | 81 |
| 5.11.2    | Function Documentation         | 81 |
| 5.11.2.1  | output_init()                  | 81 |
| 5.11.2.2  | output_cl()                    | 82 |
| 5.11.2.3  | output_pk()                    | 83 |
| 5.11.2.4  | output_pk_nl()                 | 83 |
| 5.11.2.5  | output_tk()                    | 84 |
| 5.11.2.6  | output_print_data()            | 85 |
| 5.11.2.7  | output_open_cl_file()          | 85 |
| 5.11.2.8  | output_one_line_of_cl()        | 86 |
| 5.11.2.9  | output_open_pk_file()          | 86 |
| 5.11.2.10 | output_one_line_of_pk()        | 87 |
| 5.12      | output.h File Reference        | 87 |
| 5.12.1    | Detailed Description           | 89 |
| 5.12.2    | Data Structure Documentation   | 89 |
| 5.12.2.1  | struct output                  | 89 |
| 5.12.3    | Macro Definition Documentation | 89 |
| 5.12.3.1  | _Z_PK_NUM_MAX_                 | 89 |
| 5.13      | perturbations.c File Reference | 90 |
| 5.13.1    | Detailed Description           | 91 |
| 5.13.2    | Function Documentation         | 91 |
| 5.13.2.1  | perturb_sources_at_tau()       | 91 |
| 5.13.2.2  | perturb_init()                 | 92 |

|           |  |     |
|-----------|--|-----|
| 5.13.2.3  | <a href="#">perturb_free()</a>                                       | 93  |
| 5.13.2.4  | <a href="#">perturb_indices_of_perturbs()</a>                        | 94  |
| 5.13.2.5  | <a href="#">perturb_timesampling_for_sources()</a>                   | 95  |
| 5.13.2.6  | <a href="#">perturb_get_k_list()</a>                                 | 96  |
| 5.13.2.7  | <a href="#">perturb_workspace_init()</a>                             | 97  |
| 5.13.2.8  | <a href="#">perturb_workspace_free()</a>                             | 98  |
| 5.13.2.9  | <a href="#">perturb_solve()</a>                                      | 98  |
| 5.13.2.10 | <a href="#">perturb_prepare_output()</a>                             | 99  |
| 5.13.2.11 | <a href="#">perturb_find_approximation_number()</a>                  | 100 |
| 5.13.2.12 | <a href="#">perturb_find_approximation_switches()</a>                | 100 |
| 5.13.2.13 | <a href="#">perturb_vector_init()</a>                                | 102 |
| 5.13.2.14 | <a href="#">perturb_vector_free()</a>                                | 103 |
| 5.13.2.15 | <a href="#">perturb_initial_conditions()</a>                         | 104 |
| 5.13.2.16 | <a href="#">perturb_approximations()</a>                             | 108 |
| 5.13.2.17 | <a href="#">perturb_timescale()</a>                                  | 109 |
| 5.13.2.18 | <a href="#">perturb_einstein()</a>                                   | 110 |
| 5.13.2.19 | <a href="#">perturb_total_stress_energy()</a>                        | 111 |
| 5.13.2.20 | <a href="#">perturb_sources()</a>                                    | 112 |
| 5.13.2.21 | <a href="#">perturb_print_variables()</a>                            | 113 |
| 5.13.2.22 | <a href="#">perturb_derivs()</a>                                     | 114 |
| 5.13.2.23 | <a href="#">perturb_tca_slip_and_shear()</a>                         | 117 |
| 5.14      | <a href="#">perturbations.h File Reference</a>                       | 118 |
| 5.14.1    | <a href="#">Detailed Description</a>                                 | 120 |
| 5.14.2    | <a href="#">Data Structure Documentation</a>                         | 120 |
| 5.14.2.1  | <a href="#">struct <code>perturbs</code></a>                         | 120 |
| 5.14.2.2  | <a href="#">struct <code>perturb_vector</code></a>                   | 125 |
| 5.14.2.3  | <a href="#">struct <code>perturb_workspace</code></a>                | 127 |
| 5.14.2.4  | <a href="#">struct <code>perturb_parameters_and_workspace</code></a> | 128 |
| 5.14.3    | <a href="#">Macro Definition Documentation</a>                       | 129 |
| 5.14.3.1  | <a href="#">_SELECTION_NUM_MAX_</a>                                  | 129 |



---

|           |   |     |
|-----------|---|-----|
| 5.14.3.2  | <code>_MAX_NUMBER_OF_K_FILES_</code>                  | 129 |
| 5.14.4    | Enumeration Type Documentation                        | 129 |
| 5.14.4.1  | <code>tca_flags</code>                                | 129 |
| 5.14.4.2  | <code>tca_method</code>                               | 129 |
| 5.14.4.3  | <code>possible_gauges</code>                          | 129 |
| 5.15      | <code>primordial.c</code> File Reference              | 130 |
| 5.15.1    | Detailed Description                                  | 131 |
| 5.15.2    | Function Documentation                                | 131 |
| 5.15.2.1  | <code>primordial_spectrum_at_k()</code>               | 132 |
| 5.15.2.2  | <code>primordial_init()</code>                        | 133 |
| 5.15.2.3  | <code>primordial_free()</code>                        | 133 |
| 5.15.2.4  | <code>primordial_indices()</code>                     | 134 |
| 5.15.2.5  | <code>primordial_get_lnk_list()</code>                | 134 |
| 5.15.2.6  | <code>primordial_analytic_spectrum_init()</code>      | 135 |
| 5.15.2.7  | <code>primordial_analytic_spectrum()</code>           | 135 |
| 5.15.2.8  | <code>primordial_inflation_potential()</code>         | 136 |
| 5.15.2.9  | <code>primordial_inflation_hubble()</code>            | 136 |
| 5.15.2.10 | <code>primordial_inflation_indices()</code>           | 137 |
| 5.15.2.11 | <code>primordial_inflation_solve_inflation()</code>   | 137 |
| 5.15.2.12 | <code>primordial_inflation_analytic_spectra()</code>  | 138 |
| 5.15.2.13 | <code>primordial_inflation_spectra()</code>           | 139 |
| 5.15.2.14 | <code>primordial_inflation_one_wavenumber()</code>    | 139 |
| 5.15.2.15 | <code>primordial_inflation_one_k()</code>             | 140 |
| 5.15.2.16 | <code>primordial_inflation_find_attractor()</code>    | 141 |
| 5.15.2.17 | <code>primordial_inflation_evolve_background()</code> | 141 |
| 5.15.2.18 | <code>primordial_inflation_check_potential()</code>   | 142 |
| 5.15.2.19 | <code>primordial_inflation_check_hubble()</code>      | 143 |
| 5.15.2.20 | <code>primordial_inflation_get_epsilon()</code>       | 143 |
| 5.15.2.21 | <code>primordial_inflation_find_phi_pivot()</code>    | 145 |
| 5.15.2.22 | <code>primordial_inflation_derivs()</code>            | 146 |

---

|   |     |
|---|-----|
| 5.15.2.23 primordial_external_spectrum_init() | 147 |
| 5.16 primordial.h File Reference              | 147 |
| 5.16.1 Detailed Description                   | 149 |
| 5.16.2 Data Structure Documentation           | 149 |
| 5.16.2.1 struct primordial                    | 149 |
| 5.16.3 Enumeration Type Documentation         | 153 |
| 5.16.3.1 primordial_spectrum_type             | 153 |
| 5.16.3.2 linear_or_logarithmic                | 153 |
| 5.16.3.3 potential_shape                      | 153 |
| 5.16.3.4 target_quantity                      | 154 |
| 5.16.3.5 integration_direction                | 154 |
| 5.16.3.6 time_definition                      | 154 |
| 5.16.3.7 phi_pivot_methods                    | 154 |
| 5.16.3.8 inflation_module_behavior            | 154 |
| 5.17 spectra.c File Reference                 | 155 |
| 5.17.1 Detailed Description                   | 156 |
| 5.17.2 Function Documentation                 | 156 |
| 5.17.2.1 spectra_cl_at_l()                    | 156 |
| 5.17.2.2 spectra_pk_at_z()                    | 157 |
| 5.17.2.3 spectra_pk_at_k_and_z()              | 158 |
| 5.17.2.4 spectra_pk_nl_at_z()                 | 159 |
| 5.17.2.5 spectra_pk_nl_at_k_and_z()           | 160 |
| 5.17.2.6 spectra_tk_at_z()                    | 161 |
| 5.17.2.7 spectra_tk_at_k_and_z()              | 162 |
| 5.17.2.8 spectra_init()                       | 162 |
| 5.17.2.9 spectra_free()                       | 163 |
| 5.17.2.10 spectra_indices()                   | 163 |
| 5.17.2.11 spectra_cls()                       | 164 |
| 5.17.2.12 spectra_compute_cl()                | 165 |
| 5.17.2.13 spectra_k_and_tau()                 | 166 |

---

|  |     |
|--|-----|
| 5.17.2.14 spectra_pk()                               | 166 |
| 5.17.2.15 spectra_sigma()                            | 167 |
| 5.17.2.16 spectra_matter_transfers()                 | 168 |
| 5.17.2.17 spectra_output_tk_data()                   | 168 |
| 5.18 spectra.h File Reference                        | 169 |
| 5.18.1 Detailed Description                          | 170 |
| 5.18.2 Data Structure Documentation                  | 170 |
| 5.18.2.1 struct spectra                              | 170 |
| 5.19 thermodynamics.c File Reference                 | 174 |
| 5.19.1 Detailed Description                          | 176 |
| 5.19.2 Function Documentation                        | 177 |
| 5.19.2.1 thermodynamics_at_z()                       | 177 |
| 5.19.2.2 thermodynamics_init()                       | 177 |
| 5.19.2.3 thermodynamics_free()                       | 179 |
| 5.19.2.4 thermodynamics_indices()                    | 179 |
| 5.19.2.5 thermodynamics_helium_from_bbn()            | 180 |
| 5.19.2.6 thermodynamics_onthespot_energy_injection() | 181 |
| 5.19.2.7 thermodynamics_energy_injection()           | 181 |
| 5.19.2.8 thermodynamics_reionization_function()      | 183 |
| 5.19.2.9 thermodynamics_get_xe_before_reionization() | 184 |
| 5.19.2.10 thermodynamics_reionization()              | 184 |
| 5.19.2.11 thermodynamics_reionization_sample()       | 185 |
| 5.19.2.12 thermodynamics_recombination()             | 186 |
| 5.19.2.13 thermodynamics_recombination_with_hyrec()  | 187 |
| 5.19.2.14 thermodynamics_recombination_with_recast() | 187 |
| 5.19.2.15 thermodynamics_derivs_with_recast()        | 189 |
| 5.19.2.16 thermodynamics_merge_reco_and_reio()       | 190 |
| 5.19.2.17 thermodynamics_output_titles()             | 190 |
| 5.20 thermodynamics.h File Reference                 | 191 |
| 5.20.1 Detailed Description                          | 192 |

---

---

|           |  |     |
|-----------|--|-----|
| 5.20.2    | Data Structure Documentation                   | 192 |
| 5.20.2.1  | struct thermo                                  | 192 |
| 5.20.2.2  | struct recombination                           | 195 |
| 5.20.2.3  | struct reionization                            | 196 |
| 5.20.2.4  | struct thermodynamics_parameters_and_workspace | 197 |
| 5.20.3    | Macro Definition Documentation                 | 197 |
| 5.20.3.1  | f1   | 198 |
| 5.20.3.2  | f2   | 198 |
| 5.20.3.3  | _YHE_BIG_                                      | 198 |
| 5.20.3.4  | _YHE_SMALL_                                    | 198 |
| 5.20.4    | Enumeration Type Documentation                 | 198 |
| 5.20.4.1  | recombination_algorithm                        | 198 |
| 5.20.4.2  | reionization_parametrization                   | 198 |
| 5.20.4.3  | reionization_z_or_tau                          | 199 |
| 5.21      | transfer.c File Reference                      | 199 |
| 5.21.1    | Detailed Description                           | 200 |
| 5.21.2    | Function Documentation                         | 201 |
| 5.21.2.1  | transfer_functions_at_q()                      | 201 |
| 5.21.2.2  | transfer_init()                                | 202 |
| 5.21.2.3  | transfer_free()                                | 203 |
| 5.21.2.4  | transfer_indices_of_transfers()                | 204 |
| 5.21.2.5  | transfer_get_l_list()                          | 205 |
| 5.21.2.6  | transfer_get_q_list()                          | 205 |
| 5.21.2.7  | transfer_get_k_list()                          | 206 |
| 5.21.2.8  | transfer_get_source_correspondence()           | 206 |
| 5.21.2.9  | transfer_source_tau_size()                     | 207 |
| 5.21.2.10 | transfer_compute_for_each_q()                  | 207 |
| 5.21.2.11 | transfer_interpolate_sources()                 | 208 |
| 5.21.2.12 | transfer_sources()                             | 209 |
| 5.21.2.13 | transfer_selection_function()                  | 210 |

---

---

|           |  |            |
|-----------|--|------------|
| 5.21.2.14 | <code>transfer_dNdz_analytic()</code>      | 210        |
| 5.21.2.15 | <code>transfer_selection_sampling()</code> | 211        |
| 5.21.2.16 | <code>transfer_lensing_sampling()</code>   | 212        |
| 5.21.2.17 | <code>transfer_source_resample()</code>    | 212        |
| 5.21.2.18 | <code>transfer_selection_times()</code>    | 213        |
| 5.21.2.19 | <code>transfer_selection_compute()</code>  | 214        |
| 5.21.2.20 | <code>transfer_compute_for_each_l()</code> | 214        |
| 5.21.2.21 | <code>transfer_integrate()</code>          | 215        |
| 5.21.2.22 | <code>transfer_limber()</code>             | 218        |
| 5.21.2.23 | <code>transfer_limber_interpolate()</code> | 218        |
| 5.21.2.24 | <code>transfer_limber2()</code>            | 219        |
| 5.22      | <code>transfer.h</code> File Reference     | 220        |
| 5.22.1    | Detailed Description                       | 221        |
| 5.22.2    | Data Structure Documentation               | 221        |
| 5.22.2.1  | <code>struct transfers</code>              | 221        |
| 5.22.2.2  | <code>struct transfer_workspace</code>     | 225        |
| 5.22.3    | Enumeration Type Documentation             | 225        |
| 5.22.3.1  | <code>radial_function_type</code>          | 225        |
| <b>6</b>  | <b>The ‘external_Pk’ mode</b>              | <b>227</b> |
| <b>7</b>  | <b>Updating the manual</b>                 | <b>231</b> |
|           | <b>Index</b>                               | <b>233</b> |



# Chapter 1

## CLASS: Cosmic Linear Anisotropy Solving System

Authors: Julien Lesgourgues and Thomas Tram

with several major inputs from other people, especially Benjamin Audren, Simon Prunet, Jesus Torrado, Miguel Zumalacarregui, Francesco Montanari, etc.

For download and information, see <http://class-code.net>

### Compiling CLASS and getting started

(the information below can also be found on the webpage, just below the download button)

Download the code from the webpage and unpack the archive (tar -zxvf class\_vx.y.z.tar.gz), or clone it from [https://github.com/lesgourg/class\\_public](https://github.com/lesgourg/class_public). Go to the class directory (cd class/ or class\_public/ or class\_vx.y.z/) and compile (make clean; make class). You can usually speed up compilation with the option -j: make -j class. If the first compilation attempt fails, you may need to open the Makefile and adapt the name of the compiler (default: gcc), of the optimization flag (default: -O4 -ffast-math) and of the OpenMP flag (default: -fopenmp; this flag is facultative, you are free to compile without OpenMP if you don't want parallel execution; note that you need the version 4.2 or higher of gcc to be able to compile with -fopenmp). Many more details on the CLASS compilation are given on the wiki page

[https://github.com/lesgourg/class\\_public/wiki/Installation](https://github.com/lesgourg/class_public/wiki/Installation)

(in particular, for compiling on Mac >= 10.9 despite of the clang incompatibility with OpenMP).

To check that the code runs, type:

```
./class explanatory.ini
```

The explanatory.ini file is THE reference input file, containing and explaining the use of all possible input parameters. We recommend to read it, to keep it unchanged (for future reference), and to create for your own purposes some shorter input files, containing only the input lines which are useful for you. Input files must have a \*.ini extension.

If you want to play with the precision/speed of the code, you can use one of the provided precision files (e.g. cl\_permille.pre) or modify one of them, and run with two input files, for instance:

```
./class test.ini cl_permille.pre
```

The files \*.pre are supposed to specify the precision parameters for which you don't want to keep default values. If you find it more convenient, you can pass these precision parameter values in your \*.ini file instead of an additional \*.pre file.

The automatically-generated documentation is located in

```
doc/manual/html/index.html  
doc/manual/CLASS_manual.pdf
```

On top of that, if you wish to modify the code, you will find lots of comments directly in the files.

## Python

To use CLASS from python, or ipython notebooks, or from the Monte Python parameter extraction code, you need to compile not only the code, but also its python wrapper. This can be done by typing just 'make' instead of 'make class' (or for speeding up: 'make -j'). More details on the wrapper and its compilation are found on the wiki page

[https://github.com/lesgourg/class\\_public/wiki](https://github.com/lesgourg/class_public/wiki)

## Plotting utility

Since version 2.3, the package includes an improved plotting script called CPU.py (Class Plotting Utility), written by Benjamin Audren and Jesus Torrado. It can plot the Cl's, the P(k) or any other CLASS output, for one or several models, as well as their ratio or percentage difference. The syntax and list of available options is obtained by typing 'python CPU.py -h'. There is a similar script for MATLAB, written by Thomas Tram. To use it, once in MATLAB, type 'help plot\_CLASS\_output.m'

## Developing the code

If you want to develop the code, we suggest that you download it from the github webpage

[https://github.com/lesgourg/class\\_public](https://github.com/lesgourg/class_public)

rather than from class-code.net. Then you will enjoy all the feature of git repositories. You can even develop your own branch and get it merged to the public distribution. For related instructions, check

[https://github.com/lesgourg/class\\_public/wiki/Public-Contributing](https://github.com/lesgourg/class_public/wiki/Public-Contributing)

## Using the code

You can use CLASS freely, provided that in your publications, you cite at least the paper CLASS II↔ : Approximation schemes <<http://arxiv.org/abs/1104.2933>>. Feel free to cite more C↔ LASS papers!

## Support

To get support, please open a new issue on the

[https://github.com/lesgourg/class\\_public](https://github.com/lesgourg/class_public)

webpage!



## Chapter 2

# Where to find information and documentation on CLASS?

Author: Julien Lesgourgues

- **For what the code can actually compute:** all possible input parameters, all coded cosmological models, all functionalities, all observables, etc.: read the file `explanatory.ini` in the main CLASS directory: it is THE reference file where we keep track of all possible input and the definition of all input parameters. For that reason we recommend to leave it always unchanged and to work with copies of it, or with short input files written from scratch.
- **For the structure, style, and concrete aspects of the code:** this documentation, especially the CLASS `overview` chapter (the extensive automatically-generated part of this documentation is more for advanced users); plus the slides of our CLASS lectures, for instance those from Tokyo 2014 available at <http://lesgourg.github.io/class-tour-Tokyo.html> or the more recent and concise summary from the Narbonne 2016 lecture available at <http://lesgourg.github.io/class-tour/Narbonne.pdf>  
An updated overview of available CLASS lecture slides is always available at <http://lesgourg.github.io/courses.html> in the section `Courses on numerical tools`.
- **For the python wrapper of CLASS:** at the moment, the best are the last slides (pages 75-96) of the Narbonne 2016 lectures <http://lesgourg.github.io/class-tour/Narbonne.pdf>  
Later we will expand the wrapper documentation with a dedicated chapter here.
- **For the physics and equations used in the code:** mainly, the following papers:
  - *Cosmological perturbation theory in the synchronous and conformal Newtonian gauges*  
C. P. Ma and E. Bertschinger.  
<http://arxiv.org/abs/astro-ph/9506072>  
10.1086/176550  
Astrophys. J. **455**, 7 (1995)
  - *The Cosmic Linear Anisotropy Solving System (CLASS) II: Approximation schemes*  
D. Blas, J. Lesgourgues and T. Tram.  
<http://arxiv.org/abs/1104.2933> [astro-ph.CO]  
10.1088/1475-7516/2011/07/034  
JCAP **1107**, 034 (2011)

- *The Cosmic Linear Anisotropy Solving System (CLASS) IV: efficient implementation of non-cold relics*  
J. Lesgourgues and T. Tram.  
<http://arxiv.org/abs/1104.2935> [astro-ph.CO]  
10.1088/1475-7516/2011/09/032  
JCAP **1109**, 032 (2011)
- *Optimal polarisation equations in FLRW universes*  
T. Tram and J. Lesgourgues.  
<http://arxiv.org/abs/1305.3261> [astro-ph.CO]  
10.1088/1475-7516/2013/10/002  
JCAP **1310**, 002 (2013)
- *Fast and accurate CMB computations in non-flat FLRW universes*  
J. Lesgourgues and T. Tram.  
<http://arxiv.org/abs/1312.2697> [astro-ph.CO]  
10.1088/1475-7516/2014/09/032  
JCAP **1409**, no. 09, 032 (2014)
- *The CLASSgal code for Relativistic Cosmological Large Scale Structure*  
E. Di Dio, F. Montanari, J. Lesgourgues and R. Durrer.  
<http://arxiv.org/abs/1307.1459> [astro-ph.CO]  
10.1088/1475-7516/2013/11/044  
JCAP **1311**, 044 (2013)

plus also some latex notes on specific sectors:

- *Equations for perturbed recombination*  
(can be turned on optionally by the user since v2.1.0)  
L. Voruz.  
[http://lesgourg.github.io/class\\_public/perturbed\\_recombination.pdf](http://lesgourg.github.io/class_public/perturbed_recombination.pdf)
- *PPF formalism in Newtonian and synchronous gauge*  
(used by default for the fluid perturbations since v2.6.0)  
T. Tram.  
[http://lesgourg.github.io/class\\_public/PPF\\_formalism.pdf](http://lesgourg.github.io/class_public/PPF_formalism.pdf)

## Chapter 3

# CLASS overview (architecture, input/output, general principles)

Author: Julien Lesgourgues

### Overall architecture of `class`

#### Files and directories

After downloading CLASS, one can see the following files in the root directory contains:

- some example of input files, the most important being `explanatory.ini`. a reference input file containing all possible flags, options and physical input parameters. While this documentation explains the structure and use of the code, `explanatory.ini` can be seen as the *physical* documentation of CLASS. The other input file are alternative parameter input files (ending with `.ini`) and precision input files (ending with `.pre`)
- the Makefile, with which you can compile the code by typing `make clean; make`; this will create the executable `class` and some binary files in the directory `build/`. The Makefile contains other compilation options that you can view inside the file.
- `CPU.py` is a python script designed for plotting the CLASS output; for documentation type `python CPU.py --help`
- `plot_CLASS_output.m` is the counterpart of `CPU.py` for MatLab
- there are other input files for various applications: an example of a non-cold dark matter distribution functions (`psd_FD_single.dat`), and examples of evolution and selection functions for galaxy number count observables (`myevolution.dat`, `myselection.dat`).

Other files are split between the following directories:

- `source/` contains the C files for each CLASS module, i.e. each block containing some part of the physical equations and logic of the Boltzmann code.
- `tools/` contains purely numerical algorithms, applicable in any context: integrators, simple manipulation of arrays (derivation, integration, interpolation), Bessel function calculation, quadrature algorithms, parser, etc.

- `main/` contains the main module `class.c` with the main routine `class(...)`, to be used in interactive runs (but not necessarily when the code is interfaced with other ones).
- `test/` contains alternative main routines which can be used to run only some part of the code, to test its accuracy, to illustrate how it can be interfaced with other codes, etc.
- `include/` contains all the include files with a `.h` suffix.
- `output/` is where the output files will be written by default (this can be changed to another directory by adjusting the input parameter `root = <...>`)
- `python/` contains the python wrapper of CLASS, called `classy` (see `python/README`)
- `cpp/` contains the C++ wrapper of CLASS, called `ClassEngine` (see `cpp/README`)
- `doc/` contains the automatic documentation (manual and input files required to build it)
- `external_Pk/` contains examples of external codes that can be used to generate the primordial spectrum and be interfaced with CLASS, when one of the many options already built inside the code are not sufficient.
- `bbn/` contains interpolation tables produced by BBN codes, in order to predict e.g.  $Y_{\text{He}}(\omega_b, \Delta N_{\text{eff}})$ .
- `hyrec/` contains the recombination code HyRec of Yacine Ali-Haïmoud and Chris Hirata, that can be used as an alternative to the built-in Recfast (using the input parameter `recombination = <...>`).

## The ten-module backbone

### Ten tasks

The purpose of `class` consists in computing some background quantities, thermodynamical quantities, perturbation transfer functions, and finally 2-point statistics (power spectra) for a given set of cosmological parameters. This task can be decomposed in few steps or modules:

1. set input parameter values.
2. compute the evolution of cosmological background quantities.
3. compute the evolution of thermodynamical quantities (ionization fractions, etc.)
4. compute the evolution of source functions  $S(k, \tau)$  (by integrating over all perturbations).
5. compute the primordial spectra.
6. eventually, compute non-linear corrections at small redshift/large wavenumber.
7. compute transfer functions in harmonic space  $\Delta_l(k)$  (unless one needs only Fourier spectra  $P(k)$ 's and no harmonic spectra  $C_l$ 's).
8. compute the observable power spectra  $C_l$ 's (by convolving the primordial spectra and the harmonic transfer functions) and/or  $P(k)$ 's (by multiplying the primordial spectra and the appropriate source functions  $S(k, \tau)$ ).
9. eventually, compute the lensed CMB spectra (using second-order perturbation theory)
10. write results in files (when CLASS is used interactively. The python wrapper does not go through this step, after 1.-9. it just keeps the output stored internally).

---

## Ten structures

In `class`, each of these steps is associated with a structure:

1. `struct precision` for input precision parameters (input physical parameters are dispatched among the other structures listed below)
2. `struct background` for cosmological background,
3. `struct thermo` for thermodynamics,
4. `struct perturbs` for source functions,
5. `struct primordial` for primordial spectra,
6. `struct nonlinear` for nonlinear corrections,
7. `struct transfers` for transfer functions,
8. `struct spectra` for observable spectra,
9. `struct lensing` for lensed CMB spectra,
10. `struct output` for auxiliary variable describing the output format.

A given structure contains "everything concerning one step that the subsequent steps need to know" (for instance, `struct perturbs` contains everything about source functions that the transfer module needs to know). In particular, each structure contains one array of tabulated values (for `struct background`, background quantities as a function of time, for `struct thermo`, thermodynamical quantities as a function of redshift, for `struct perturbs`, sources  $S(k, \tau)$ , etc.). It also contains information about the size of this array and the value of the index of each physical quantity, so that the table can be easily read and interpolated. Finally, it contains any derived quantity that other modules might need to know. Hence, the communication from one module A to another module B consists in passing a pointer to the structure filled by A, and nothing else.

All "precision parameters" are grouped in the single structure `struct precision`. The code contains *no other arbitrary numerical coefficient*.

## Ten modules

Each structure is defined and filled in one of the following modules (and precisely in the order below):

1. `input.c`
2. `background.c`
3. `thermodynamics.c`
4. `perturbations.c`
5. `primordial.c`
6. `nonlinear.c`
7. `transfer.c`
8. `spectra.c`
9. `lensing.c`
10. `output.c`

Each of these modules contains at least three functions:

- `module_init(...)`
- `module_free(...)`
- `module_something_at_somevalue`

where *module* is one of `input`, `background`, `thermodynamics`, `perturb`, `primordial`, `nonlinear`, `transfer`, `spectra`, `lensing`, `output`.

The first function allocates and fills each structure. This can be done provided that the previous structures in the hierarchy have been already allocated and filled. In summary, calling one of `module_init(...)` amounts in solving entirely one of the steps 1 to 10.

The second function deallocates the fields of each structure. This can be done optionally at the end of the code (or, when the code is embedded in a sampler, this **must** be done between each execution of `class`, and especially before calling `module_init(...)` again with different input parameters).

The third function is able to interpolate the pre-computed tables. For instance, `background_init()` fills a table of background quantities for discrete values of conformal time  $\tau$ , but `background_at_tau(tau, *values)` will return these values for any arbitrary  $\tau$ .

Note that functions of the type `module_something_at_somevalue` are the only ones which are called from another module, while functions of the type `module_init(...)` and `module_free(...)` are the only one called by the main executable. All other functions are for internal use in each module.

When writing a C code, the ordering of the functions in the `*.c` file is in principle arbitrary. However, for the sake of clarity, we always respected the following order in each CLASS module:

1. all functions that may be called by other modules, i.e. "external functions", usually named like `module_↔something_at_somevalue(...)`
2. then, `module_init(...)`
3. then, `module_free()`
4. then, all functions used only internally by the module

## The `main()` function(s)

### The `main.c` file

The main executable of `class` is the function `main()` located in the file `main/main.c`. This function consist only in the following lines (not including comments and error-management lines explained later):

```
main() {
  struct precision pr;

  struct background ba;

  struct thermo th;

  struct perturbs pt;

  struct primordial pm;

  struct nonlinear nl;
```

```

struct transfers tr;

struct spectra sp;

struct lensing le;

struct output op;

input_init_from_arguments(argc, argv, &pr, &ba, &th, &pt, &tr, &pm, &sp, &nl, &le, &op, errmsg);

background_init(&pr, &ba);

thermodynamics_init(&pr, &ba, &th);

perturb_init(&pr, &ba, &th, &pt);

primordial_init(&pr, &pt, &pm);

nonlinear_init(&pr, &ba, &th, &pt, &pm, &nl);

transfer_init(&pr, &ba, &th, &pt, &nl, &tr);

spectra_init(&pr, &ba, &pt, &pm, &nl, &tr, &sp);

lensing_init(&pr, &pt, &sp, &nl, &le);

output_init(&ba, &th, &pt, &pm, &tr, &sp, &nl, &le, &op)

/***** done *****/

lensing_free(&le);

spectra_free(&sp);

transfer_free(&tr);

nonlinear_free(&nl);

primordial_free(&pm);

perturb_free(&pt);

thermodynamics_free(&th);

background_free(&ba);

```

We can come back on the role of each argument. The arguments above are all pointers to the 10 structures of the code, excepted `argc`, `argv` which contains the input files passed by the user, and `errmsg` which contains the output error message of the input module (error management will be described below).

`input_init_from_arguments` needs all structures, because it will set the precision parameters inside the precision structure, and the physical parameters in some fields of the respective other structures. For instance, an input parameter relevant for the primordial spectrum calculation (like the tilt  $n_s$ ) will be stored in the primordial structure. Hence, in `input_init_from_arguments`, all structures can be seen as output arguments.

Other `module_init()` functions typically need all previous structures, which contain the result of the previous modules, plus its own structures, which contain some relevant input parameters before the function is called, as well as all the result form the module when the function has been executed. Hence all passed structures can be seen as input argument, excepted the last one which is both input and output. An example is `perturb_←init(&pr, &ba, &th, &pt)`.

Each function `module_init()` does not need **all** previous structures, it happens that a module does not depend on a **all** previous one. For instance, the primordial module does not need information on the background and thermodynamics evolution in order to compute the primordial spectra, so the dependency is reduced: `primordial_←init(&pr, &pt, &pm)`.

Each function `module_init()` only deallocates arrays defined in the structure of their own module, so they need only their own structure as argument. (This is possible because all structures are self-contained, in the sense that when the structure contains an allocated array, it also contains the size of this array). The first and last module, `input` and `output`, have no `input_free()` or `output_free()` functions, because the structures `precision` and `output` do not contain arrays that would need to be de-allocated after the execution of the module.

#### The `test_<...>.c` files

For a given purpose, somebody could only be interested in the intermediate steps (only background quantities, only the thermodynamics, only the perturbations and sources, etc.) It is then straightforward to truncate the full hierarchy of modules 1, ... 10 at some arbitrary order. We provide several "reduced executables" achieving precisely this. They are located in `test/test_module_.` (like, for instance, `test/test_perturbations.c`) and they can be compiled using the Makefile, which contains the appropriate commands and definitions (for instance, you can type `make test_perturbations`).

The `test/` directory contains other useful example of alternative main functions, like for instance `test_↔ loops.c` which shows how to call CLASS within a loop over different parameter values. There is also a version `test/test_loops_omp.c` using a double level of openMP parallelisation: one for running several CLASS instances in parallel, one for running each CLASS instance on several cores. The comments in these files are self-explanatory.

## Input/output

### Input

There are two types of input:

- "precision parameters" (controlling the precision of the output and the execution time),
- "input parameters" (cosmological parameters, flags telling to the code what it should compute, ...)

The code can be executed with a maximum of two input files, e.g.

```
./class explanatory.ini cl_permille.pre
```

The file with a `.ini` extension is the cosmological parameter input file, and the one with a `.pre` extension is the precision file. Both files are optional: all parameters are set to default values corresponding to the "most usual choices", and are eventually replaced by the parameters passed in the two input files. For instance, if one is happy with default accuracy settings, it is enough to run with

```
./class explanatory.ini
```

Input files do not necessarily contain a line for each parameter, since many of them can be left to default value. The example file `explanatory.ini` is very long and somewhat indigestible, since it contains all possible parameters, together with lengthy explanations. We recommend to keep this file unchanged for reference, and to copy it in e.g. `test.ini`. In the latter file, the user can erase all sections in which he/she is absolutely not interested (e.g., all the part on isocurvature modes, or on tensors, or on non-cold species, etc.). Another option is to create an input file from scratch, copying just the relevant lines from `explanatory.ini`. For the simplest applications, the user will just need a few lines for basic cosmological parameters, one line for the `output` entry (where one can specifying which power spectra must be computed), and one line for the `root` entry (specifying the prefix of all output files).

The syntax of the input files is explained at the beginning of `explanatory.ini`. Typically, lines in those files look like:



```

parameter1 = value1

free comments

parameter2 = value2 # further comments

# commented_parameter = commented_value

```

and parameters can be entered in arbitrary order. This is rather intuitive. The user should just be careful not to put an "=" sign not preceded by a "#" sign inside a comment: the code would then think that one is trying to pass some unidentified input parameter.

The syntax for the cosmological and precision parameters is the same. It is clearer to split these parameters in the two files `.ini` and `.pre`, but there is no strict rule about which parameter goes into which file: in principle, precision parameters could be passed in the `.ini`, and vice-versa. The only important thing is not to pass the same parameter twice: the code would then complain and not run.

The CLASS input files are also user-friendly in the sense that many different cosmological parameter bases can be used. This is made possible by the fact that the code does not only read parameters, it "interprets them" with the level of logic which has been coded in the `input.c` module. For instance, the Hubble parameter, the photon density, the baryon density and the ultra-relativistic neutrino density can be entered as:

```

h = 0.7

T_cmb = 2.726      # Kelvin units

omega_b = 0.02

N_eff = 3.04

```

(in arbitrary order), or as

```

H0 = 70

omega_g = 2.5e-5   # g is the label for photons

Omega_b = 0.04

omega_ur = 1.7e-5  # ur is the label for ultra-relativistic species

```

or any combination of the two. The code knows that for the photon density, one should pass one (but not more than one) parameter out of `T_cmb`, `omega_g`, `Omega_g` (where small omega's refer to  $\omega_i \equiv \Omega_i h^2$ ). It searches for one of these values, and if needed, it converts it into one of the other two parameters, using also other input parameters. For instance, `omega_g` will be converted into `Omega_g` even if `h` is written later in the file than `omega_g`: the order makes no difference. Lots of alternatives have been defined. If the code finds that not enough parameters have been passed for making consistent deductions, it will complete the missing information with in-built default values. On the contrary, if it finds that there is too much information and no unique solution, it will complain and return an error.

In summary, the input syntax has been defined in such way that the user does not need to think too much, and can pass his preferred set of parameters in a nearly informal way.

Let us mention a two useful parameters defined at the end of `explanatory.ini`, that we recommend setting to `yes` in order to run the code in a safe way:

```
write parameters = [yes or no] (default: no)
```

When set to `yes`, all input/precision parameters which have been read are written in a file `<root>parameters.↵.ini`, to keep track all the details of this execution; this file can also be re-used as a new input file. Also, with this option, all parameters that have been passed and that the code did not read (because the syntax was wrong,

or because the parameter was not relevant in the context of the run) are written in a file `<root>unused_↵ parameters`. When you have doubts about your input or your results, you can check what is in there.

```
write warnings = [yes or no] (default: no)
```

When set to `yes`, the parameters that have been passed and that the code did not read (because the syntax was wrong, or because the parameter was not relevant in the context of the run) are written in the standard output as `[Warning:]....`

There is also a list of "verbose" parameters at the end of `explanatory.ini`. They can be used to control the level of information passed to the standard output (0 means silent; 1 means normal, e.g. information on age of the universe, etc.; 2 is useful for instance when you want to check on how many cores the run is parallelised; 3 and more are intended for debugging).

CLASS comes with a list of precision parameter files ending by `.pre`. Honestly we have not been updating all these files recently, and we need to do a bit of cleaning there. However you can trust `cl_ref.pre`. We have derived this file by studying both the convergence of the CMB output with respect to all CLASS precision parameters, and the agreement with CAMB. We consider that this file generates good reference CMB spectra, accurate up to the hundredth of per cent level, as explained in the CLASS IV paper and re-checked since then. You can try it with e.g.

```
./class explanatory.ini cl_ref.pre
```

but the run will be extremely long. This is an occasion to run a many-core machine with a lot of RAM. It may work also on your laptop, but in half an hour or so.

If you want a reference matter power spectrum  $P(k)$ , also accurate up to the hundredth of percent level, we recommend using the file `pk_ref.pre`, identical to `cl_ref.pre` excepted that the truncation of the neutrino hierarchy has been pushed to `l_max_ur=150`.

In order to increase moderately the precision to a tenth of percent, without prohibitive computing time, we recommend using `cl_permille.pre`.

## Output

The input file may contain a line

```
root = <root>
```

where `<root>` is a path of your choice, e.g. `output/test_`. Then all output files will start like this, e.↵ g. `output/test_cl.dat`, `output/test_cl_lensed.dat`, etc. Of course the number of output files depends on your settings in the input file. There can be input files for CMB, LSS, background, thermodynamics, transfer functions, primordial spectra, etc. All this is documented in `explanatory.ini`.

If you do not pass explicitly a `root = <root>`, the code will name the output in its own way, by concatenating `output/`, the name of the input parameter file, and the first available integer number, e.g.

```
output/explanatory03_cl.dat, etc.
```

## General principles

### Error management

Error management is based on the fact that all functions are defined as integers returning either `_SUCCESS_` or `_FAILURE_`. Before returning `_FAILURE_`, they write an error message in the structure of the module to which they belong. The calling function will read this message, append it to its own error message, and return a `_FAILURE_`; and so on and so forth, until the main routine is reached. This error management allows the user to see the whole nested structure of error messages when an error has been met. The structure associated to each module contains a field for writing error messages, called `structure_i.error_message`, where `structure_i` could be one of `background`, `thermo`, `perturbs`, etc. So, when a function from a module `i` is called within module `j` and returns an error, the goal is to write in `structure_j.error_message` a local error message, and to append to it the error message in `structure_i.error_message`. These steps are implemented in a macro `class_call()`, used for calling whatever function:

```
class_call(module_i_function(...,structure_i),
           structure_i.error_message,
           structure_j.error_message);
```

So, the first argument of `class_call()` is the function we want to call; the second argument is the location of the error message returned by this function; and the third one is the location of the error message which should be returned to the higher level. Usually, in the bulk of the code, we use pointer to structures rather than structure themselves; then the syntax is

```
class_call(module_i_function(...,pi),
           pi->error_message,
           pj->error_message);`
```

where in this generic example, `pi` and `pj` are assumed to be pointers towards the structures `structure_i` and `structure_j`.

The user will find in `include/common.h` a list of additional macros, all starting by `class_...()`, which are all based on this logic. For instance, the macro `class_test()` offers a generic way to return an error in a standard format if a condition is not fulfilled. A typical error message from `CLASS` looks like:

```
Error in module_j_function1

module_j_function1 (L:340) : error in module_i_function2(...)

module_i_function2 (L:275) : error in module_k_function3(...)

...

=> module_x_functionN (L:735) : your choice of input parameter blabla=30
is not consistent with the constraint blabla<1
```

where the `L`'s refer to line numbers in each file. These error messages are very informative, and are built almost entirely automatically by the macros. For instance, in the above example, it was only necessary to write inside the function `module_x_functionN()` a test like:

```
class_test(blabla >= 1,
           px->error_message,
           "your choice of input parameter blabla=%e
is not consistent with the constraint blabla<%e",
           blabla,blablamax);
```

All the rest was added step by step by the various `class_call()` macros.

## Dynamical allocation of indices

One might be tempted to decide that in a given array, matrix or vector, a given quantity is associated with an explicit index value. However, when modifying the code, extra entries will be needed and will mess up the initial scheme; the user will need to study which index is associated to which quantity, and possibly make an error. All this can be avoided by using systematically a dynamical index allocation. This means that all indices remain under a symbolic form, and in each run the code attributes automatically a value to each index. The user never needs to know this value.

Dynamical indexing is implemented in a very generic way in CLASS, the same rules apply everywhere. They are explained in these lecture slides:

[https://www.dropbox.com/sh/ma5muh76sggw8k/AAB1\\_DDUBEzAjjdymMjeTya2a?dl=0](https://www.dropbox.com/sh/ma5muh76sggw8k/AAB1_DDUBEzAjjdymMjeTya2a?dl=0)

in the folder `CLASS_Lecture_slides/lecture5_index_and_error.pdf`.

## No hard coding

Any feature or equation which could be true in one cosmology and not in another one should not be written explicitly in the code, and should not be taken as granted in several other places. Discretization and integration steps are usually defined automatically by the code for each cosmology, instead of being set to something which might be optimal for minimal models, and not sufficient for other ones. You will find many examples of this in the code. As a consequence, in the list of precision parameters, you rarely find actual stepsize. You find rather parameters representing the ratio between a stepsize and a physical quantity computed for each cosmology.

## Modifying the code

Implementing a new idea completely from scratch would be rather intimidating, even for the main developers of CLASS. Fortunately, we never have to work from scratch. Usually we want to code a new species, a new observable, a new approximation scheme, etc. The trick is to think of another species, observable, approximation scheme, etc., looking as close as possible to the new one.

Then, playing with the `grep` command and the `search` command of your editor, search for all occurrences of this nearest-as-possible other feature. This is usually easy thanks to our naming scheme. For each species, observable, approximation scheme, etc., we usually use the same sequence of few letters everywhere (for instance, `fld` for the fluid usually representing Dark Energy). Grep for `fld` and you'll get all the lines related to the fluid. There is another way: we use everywhere some conditional jumps related to a given feature. For instance, the lines related to the fluid are always in between `if (pba->has_fld == _TRUE_) { ... }` and the lines related to the cosmic shear observables are always in between `if (ppt->has_lensing_potential == _TRUE_) { ... }`. Locating these flags and conditional jumps shows you all the parts related to a given feature/ingredient.

Once you have localised your nearest-as-possible other feature, you can copy/paste these lines and adapt them to the case of your new feature! You are then sure that you didn't miss any step, even the smallest technical steps (definition of indices, etc.)

## Units

Internally, the code uses almost everywhere units of Mpc to some power, excepted in the inflation module, where many quantities are in natural units (wrt the true Planck mass).

# Chapter 4

## Data Structure Documentation

### 4.1 nonlinear Struct Reference

```
#include <nonlinear.h>
```

#### Data Fields

**- input parameters initialized by user in input module**

*(all other quantities are computed in this module, given these parameters and the content of the 'precision', 'background', 'thermo', 'primordial' and 'spectra' structures)*

- enum non\_linear\_method [method](#)

**- table non-linear corrections for matter density,  $\sqrt{P\_NL(k,z)/P\_NL(k,z)}$**

- int [k\\_size](#)
- double \* [k](#)
- int [tau\\_size](#)
- double \* [tau](#)
- double \* [nl\\_corr\\_density](#)
- double \* [k\\_nl](#)

**- technical parameters**

- short [nonlinear\\_verbose](#)
- ErrorMessage [error\\_message](#)

#### 4.1.1 Detailed Description

Structure containing all information on non-linear spectra.

Once initialized by [nonlinear\\_init\(\)](#), contains a table for all two points correlation functions and for all the ai,bj functions (containing the three points correlation functions), for each time and wave-number.

Structure containing all information on non-linear spectra.

Once initialised by [nonlinear\\_init\(\)](#), contains a table for all two points correlation functions and for all the ai,bj functions (containing the three points correlation functions), for each time and wave-number.

## 4.1.2 Field Documentation

### 4.1.2.1 method

```
enum non_linear_method nonlinear::method
```

method for computing non-linear corrections (none, Halogit, etc.)

### 4.1.2.2 k\_size

```
int nonlinear::k_size
```

k\_size = total number of k values

### 4.1.2.3 k

```
double * nonlinear::k
```

k[index\_k] = list of k values

### 4.1.2.4 tau\_size

```
int nonlinear::tau_size
```

tau\_size = number of values

### 4.1.2.5 tau

```
double * nonlinear::tau
```

tau[index\_tau] = list of time values

### 4.1.2.6 nl\_corr\_density

```
double * nonlinear::nl_corr_density
```

nl\_corr\_density[index\_tau \* ppt->k\_size + index\_k]

### 4.1.2.7 k\_nl

```
double * nonlinear::k_nl
```

wavenumber at which non-linear corrections become important, defined differently by different non\_linear\_method's

#### 4.1.2.8 nonlinear\_verbose

```
short nonlinear::nonlinear_verbose
```

amount of information written in standard output

#### 4.1.2.9 error\_message

```
ErrorMsg nonlinear::error_message
```

zone for writing error messages

The documentation for this struct was generated from the following files:

- [nonlinear.h](#)
- [nonlinear\\_exp.h](#)
- [nonlinear\\_test.h](#)





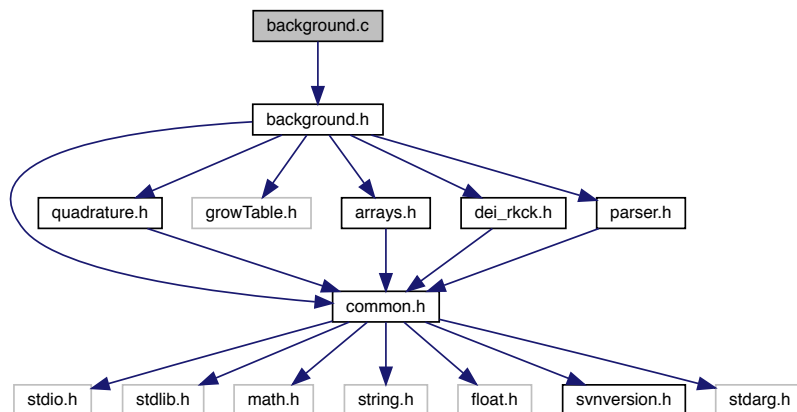
# Chapter 5

## File Documentation

### 5.1 background.c File Reference

```
#include "background.h"
```

Include dependency graph for background.c:



### Functions

- int [background\\_at\\_tau](#) (struct [background](#) \*pba, double tau, short return\_format, short intermode, int \*last\_index, double \*pvecback)
- int [background\\_tau\\_of\\_z](#) (struct [background](#) \*pba, double z, double \*tau)
- int [background\\_functions](#) (struct [background](#) \*pba, double \*pvecback\_B, short return\_format, double \*pvecback)
- int [background\\_w\\_fld](#) (struct [background](#) \*pba, double a, double \*w\_fld, double \*dw\_over\_da\_fld, double \*integral\_fld)
- int [background\\_init](#) (struct [precision](#) \*ppr, struct [background](#) \*pba)
- int [background\\_free](#) (struct [background](#) \*pba)
- int [background\\_free\\_input](#) (struct [background](#) \*pba)
- int [background\\_indices](#) (struct [background](#) \*pba)

- int [background\\_ncdm\\_distribution](#) (void \*pbadist, double q, double \*f0)
- int [background\\_ncdm\\_test\\_function](#) (void \*pbadist, double q, double \*test)
- int [background\\_ncdm\\_init](#) (struct [precision](#) \*ppr, struct [background](#) \*pba)
- int [background\\_ncdm\\_momenta](#) (double \*qvec, double \*wvec, int qsize, double M, double factor, double z, double \*n, double \*rho, double \*p, double \*drho\_dM, double \*pseudo\_p)
- int [background\\_ncdm\\_M\\_from\\_Omega](#) (struct [precision](#) \*ppr, struct [background](#) \*pba, int n\_ncdm)
- int [background\\_solve](#) (struct [precision](#) \*ppr, struct [background](#) \*pba)
- int [background\\_initial\\_conditions](#) (struct [precision](#) \*ppr, struct [background](#) \*pba, double \*pvecback, double \*pvecback\_integration)
- int [background\\_output\\_titles](#) (struct [background](#) \*pba, char titles[\_MAXTITLESTRINGLENGTH\_])
- int [background\\_output\\_data](#) (struct [background](#) \*pba, int number\_of\_titles, double \*data)
- int [background\\_derivs](#) (double tau, double \*y, double \*dy, void \*parameters\_and\_workspace, ErrorMessage error\_message)
- double [V\\_e\\_scf](#) (struct [background](#) \*pba, double phi)
- double [V\\_p\\_scf](#) (struct [background](#) \*pba, double phi)
- double [V\\_scf](#) (struct [background](#) \*pba, double phi)

### 5.1.1 Detailed Description

Documented background module

- Julien Lesgourgues, 17.04.2011
- routines related to ncdm written by T. Tram in 2011

Deals with the cosmological background evolution. This module has two purposes:

- at the beginning, to initialize the background, i.e. to integrate the background equations, and store all background quantities as a function of conformal time inside an interpolation table.
- to provide routines which allow other modules to evaluate any background quantity for a given value of the conformal time (by interpolating within the interpolation table), or to find the correspondence between redshift and conformal time.

The overall logic in this module is the following:

1. most background parameters that we will call {A} (e.g. rho\_gamma, ..) can be expressed as simple analytical functions of a few variables that we will call {B} (in simplest models, of the scale factor 'a'; in extended cosmologies, of 'a' plus e.g. (phi, phidot) for quintessence, or some temperature for exotic particles, etc...).
2. in turn, quantities {B} can be found as a function of conformal time by integrating the background equations.
3. some other quantities that we will call {C} (like e.g. the sound horizon or proper time) also require an integration with respect to time, that cannot be inferred analytically from parameters {B}.

So, we define the following routines:

- [background\\_functions\(\)](#) returns all background quantities {A} as a function of quantities {B}.
- [background\\_solve\(\)](#) integrates the quantities {B} and {C} with respect to conformal time; this integration requires many calls to [background\\_functions\(\)](#).

- the result is stored in the form of a big table in the background structure. There is one column for conformal time 'tau'; one or more for quantities {B}; then several columns for quantities {A} and {C}.

Later in the code, if we know the variables {B} and need some quantity {A}, the quickest and most precise way is to call directly `background_functions()` (for instance, in simple models, if we want H at a given value of the scale factor). If we know 'tau' and want any other quantity, we can call `background_at_tau()`, which interpolates in the table and returns all values. Finally it can be useful to get 'tau' for a given redshift 'z': this can be done with `background_tau_of_z()`. So if we are somewhere in the code, knowing z and willing to get background quantities, we should call first `background_tau_of_z()` and then `background_at_tau()`.

In order to save time, `background_at_tau()` can be called in three modes: `short_info`, `normal_info`, `long_info` (returning only essential quantities, or useful quantities, or rarely useful quantities). Each line in the interpolation table is a vector whose first few elements correspond to the `short_info` format; a larger fraction contribute to the `normal_info` format; and the full vector corresponds to the `long_info` format. The guideline is that `short_info` returns only geometric quantities like a, H, H'; `normal_info` returns quantities strictly needed at each step in the integration of perturbations; `long_info` returns quantities needed only occasionally.

In summary, the following functions can be called from other modules:

1. `background_init()` at the beginning
2. `background_at_tau()`, `background_tau_of_z()` at any later time
3. `background_free()` at the end, when no more calls to the previous functions are needed

## 5.1.2 Function Documentation

### 5.1.2.1 `background_at_tau()`

```
int background_at_tau (
    struct background * pba,
    double tau,
    short return_format,
    short intermode,
    int * last_index,
    double * pvecback )
```

Background quantities at given conformal time tau.

Evaluates all background quantities at a given value of conformal time by reading the pre-computed table and interpolating.

#### Parameters

|                      |   |
|----------------------|---|
| <i>pba</i>           | Input: pointer to background structure (containing pre-computed table)  |
| <i>tau</i>           | Input: value of conformal time  |
| <i>return_format</i> | Input: format of output vector (short, normal, long)  |
| <i>intermode</i>     | Input: interpolation mode (normal or closeby)   |
| <i>last_index</i>    | Input/Output: index of the previous/current point in the interpolation array (input only for closeby mode, output for both) |
| <i>pvecback</i>      | Output: vector (assumed to be already allocated)  |

**Returns**

the error status

**Summary:**

- define local variables
- check that tau is in the pre-computed range
- deduce length of returned vector from format mode
- interpolate from pre-computed table with `array_interpolate()` or `array_interpolate_growing_closeby()` (depending on interpolation mode)

**5.1.2.2 background\_tau\_of\_z()**

```
int background_tau_of_z (
    struct background * pba,
    double z,
    double * tau )
```

Conformal time at given redshift.

Returns tau(z) by interpolation from pre-computed table.

**Parameters**

|            |  |
|------------|--|
| <i>pba</i> | Input: pointer to background structure |
| <i>z</i>   | Input: redshift                        |
| <i>tau</i> | Output: conformal time                 |

**Returns**

the error status

**Summary:**

- define local variables
- check that  $z$  is in the pre-computed range
- interpolate from pre-computed table with `array_interpolate()`

## 5.1.2.3 background\_functions()

```
int background_functions (
    struct background * pba,
    double * pvecback_B,
    short return_format,
    double * pvecback )
```

Background quantities at given  $a$ .

Function evaluating all background quantities which can be computed analytically as a function of {B} parameters such as the scale factor 'a' (see discussion at the beginning of this file). In extended cosmological models, the pvecback\_B vector contains other input parameters than just 'a', e.g. (phi, phidot) for quintessence, some temperature of exotic relics, etc...

## Parameters

|                      |   |
|----------------------|---|
| <i>pba</i>           | Input: pointer to background structure                                    |
| <i>pvecback_B</i>    | Input: vector containing all {B} type quantities (scale factor, ...)      |
| <i>return_format</i> | Input: format of output vector  |
| <i>pvecback</i>      | Output: vector of background quantities (assumed to be already allocated) |

## Returns

the error status

## Summary:

- define local variables
- initialize local variables
- pass value of  $a$  to output
- compute each component's density and pressure
- compute expansion rate H from Friedmann equation: this is the only place where the Friedmann equation is assumed. Remember that densities are all expressed in units of  $[3c^2/8\pi G]$ , ie  $\rho_{class} = [8\pi G\rho_{physical}/3c^2]$
- compute derivative of H with respect to conformal time
- compute relativistic density to total density ratio
- compute other quantities in the exhaustive, redundant format
- compute critical density
- compute Omega\_m

#### 5.1.2.4 background\_w\_fld()

```
int background_w_fld (
    struct background * pba,
    double a,
    double * w_fld,
    double * dw_over_da_fld,
    double * integral_fld )
```

Single place where the fluid equation of state is defined. Parameters of the function are passed through the background structure. Generalisation to arbitrary functions should be simple.

## Parameters

|                       |   |
|-----------------------|---|
| <i>pba</i>            | Input: pointer to background structure              |
| <i>a</i>              | Input: current value of scale factor                |
| <i>w_fld</i>          | Output: equation of state parameter $w_{fld}(a)$    |
| <i>dw_over_da_fld</i> | Output: function $dw_{fld}/da$                      |
| <i>integral_fld</i>   | Output: function $\int_a^{a_0} da 3(1 + w_{fld})/a$ |

## Returns

the error status

- first, define the function  $w(a)$
- then, give the corresponding analytic derivative  $dw/da$  (used by perturbation equations; we could compute it numerically, but with a loss of precision; as long as there is a simple analytic expression of the derivative of the previous function, let's use it!
- finally, give the analytic solution of the following integral:  $\int_a^{a_0} da 3(1 + w_{fld})/a$ . This is used in only one place, in the initial conditions for the background, and with  $a=a_{ini}$ . If your  $w(a)$  does not lead to a simple analytic solution of this integral, no worry: instead of writing something here, the best would then be to leave it equal to zero, and then in [background\\_initial\\_conditions\(\)](#) you should implement a numerical calculation of this integral only for  $a=a_{ini}$ , using for instance Romberg integration. It should be fast, simple, and accurate enough.

note: of course you can generalise these formulas to anything, defining new parameters  $pba \rightarrow w_{...fld}$ . Just remember that so far, HyRec explicitly assumes that  $w(a) = w_0 + w_a (1 - a/a_0)$ ; but Recfast does not assume anything

## 5.1.2.5 background\_init()

```
int background_init (
    struct precision * ppr,
    struct background * pba )
```

Initialize the background structure, and in particular the background interpolation table.

## Parameters

|            |   |
|------------|---|
| <i>ppr</i> | Input: pointer to precision structure                     |
| <i>pba</i> | Input/Output: pointer to initialized background structure |

## Returns

the error status

Summary:

- define local variables

- in verbose mode, provide some information
- if shooting failed during input, catch the error here
- assign values to all indices in vectors of background quantities with [background\\_indices\(\)](#)
- control that cosmological parameter values make sense
- this function integrates the background over time, allocates and fills the background table

#### 5.1.2.6 background\_free()

```
int background_free (  
    struct background * pba )
```

Free all memory space allocated by [background\\_init\(\)](#).

##### Parameters

|            |  |
|------------|--|
| <i>pba</i> | Input: pointer to background structure (to be freed) |
|------------|--|

##### Returns

the error status

Here is the call graph for this function:



#### 5.1.2.7 background\_free\_input()

```
int background_free_input (  
    struct background * pba )
```

Free pointers inside background structure which were allocated in [input\\_read\\_parameters\(\)](#)

##### Parameters

|            |  |
|------------|--|
| <i>pba</i> | Input: pointer to background structure |
|------------|--|



**Returns**

the error status

Here is the caller graph for this function:

**5.1.2.8 background\_indices()**

```
int background_indices (  
    struct background * pba )
```

Assign value to each relevant index in vectors of background quantities.

**Parameters**

|            |  |
|------------|--|
| <i>pba</i> | Input: pointer to background structure |
|------------|--|

**Returns**

the error status

Summary:

- define local variables
- initialize all flags: which species are present?
- initialize all indices

**5.1.2.9 background\_ncdm\_distribution()**

```
int background_ncdm_distribution (  
    void * pbadist,  
    double q,  
    double * f0 )
```

This is the routine where the distribution function  $f_0(q)$  of each ncdm species is specified (it is the only place to modify if you need a partlar  $f_0(q)$ )

## Parameters

|                |  |
|----------------|--|
| <i>pbadist</i> | Input: structure containing all parameters defining $f_0(q)$ |
| <i>q</i>       | Input: momentum  |
| <i>f0</i>      | Output: phase-space distribution                             |

- extract from the input structure *pbadist* all the relevant information
- shall we interpolate in file, or shall we use analytical formula below?
- a) deal first with the case of interpolating in files
- b) deal now with case of reading analytical function

Next enter your analytic expression(s) for the p.s.d.'s. If you need different p.s.d.'s for different species, put each p.s.d inside a condition, like for instance: if ( $n_{\text{ncdm}}==2$ ) { $*f_0=...$ }. Remember that  $n_{\text{ncdm}} = 0$  refers to the first species.

This form is only appropriate for approximate studies, since in reality the chemical potentials are associated with flavor eigenstates, not mass eigenstates. It is easy to take this into account by introducing the mixing angles. In the later part (not read by the code) we illustrate how to do this.

5.1.2.10 `background_ncdm_test_function()`

```
int background_ncdm_test_function (
    void * pbadist,
    double q,
    double * test )
```

This function is only used for the purpose of finding optimal quadrature weights. The logic is: if we can accurately convolve  $f_0(q)$  with this function, then we can convolve it accurately with any other relevant function.

## Parameters

|                |   |
|----------------|---|
| <i>pbadist</i> | Input: structure containing all background parameters |
| <i>q</i>       | Input: momentum                                       |
| <i>test</i>    | Output: value of the test function $test(q)$          |

Using  $a + bq$  creates problems for otherwise acceptable distributions which diverges as  $1/r$  or  $1/r^2$  for  $r \rightarrow 0$

5.1.2.11 `background_ncdm_init()`

```
int background_ncdm_init (
    struct precision * ppr,
    struct background * pba )
```

This function finds optimal quadrature weights for each ncdm species

## Parameters

|            |                                    |
|------------|------------------------------------|
| <i>ppr</i> | Input: precision structure         |
| <i>pba</i> | Input/Output: background structure |

- in verbose mode, inform user of number of sampled momenta for background quantities

#### 5.1.2.12 background\_ncdm\_momenta()

```
int background_ncdm_momenta (
    double * qvec,
    double * wvec,
    int qsize,
    double M,
    double factor,
    double z,
    double * n,
    double * rho,
    double * p,
    double * drho_dM,
    double * pseudo_p )
```

For a given ncdm species: given the quadrature weights, the mass and the redshift, find background quantities by a quick weighted sum over. Input parameters passed as NULL pointers are not evaluated for speed-up

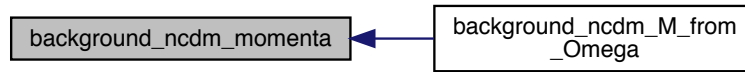
#### Parameters

|                 |  |
|-----------------|--|
| <i>qvec</i>     | Input: sampled momenta   |
| <i>wvec</i>     | Input: quadrature weights  |
| <i>qsize</i>    | Input: number of momenta/weights                                     |
| <i>M</i>        | Input: mass  |
| <i>factor</i>   | Input: normalization factor for the p.s.d.                           |
| <i>z</i>        | Input: redshift  |
| <i>n</i>        | Output: number density   |
| <i>rho</i>      | Output: energy density   |
| <i>p</i>        | Output: pressure   |
| <i>drho_dM</i>  | Output: derivative used in next function                             |
| <i>pseudo_p</i> | Output: pseudo-pressure used in perturbation module for fluid approx |

#### Summary:

- rescale normalization at given redshift
- initialize quantities
- loop over momenta
- adjust normalization

Here is the caller graph for this function:



### 5.1.2.13 background\_ncdm\_M\_from\_Omega()

```

int background_ncdm_M_from_Omega (
    struct precision * ppr,
    struct background * pba,
    int n_ncdm )
  
```

When the user passed the density fraction `Omega_ncdm` or `omega_ncdm` in input but not the mass, infer the mass with Newton iteration method.

#### Parameters

|               |                                    |
|---------------|------------------------------------|
| <i>ppr</i>    | Input: precision structure         |
| <i>pba</i>    | Input/Output: background structure |
| <i>n_ncdm</i> | Input: index of ncdm species       |

Here is the call graph for this function:



### 5.1.2.14 background\_solve()

```

int background_solve (
    struct precision * ppr,
    struct background * pba )
  
```

This function integrates the background over time, allocates and fills the background table

## Parameters

|            |                                    |
|------------|------------------------------------|
| <i>ppr</i> | Input: precision structure         |
| <i>pba</i> | Input/Output: background structure |

## Summary:

- define local variables
- allocate vector of quantities to be integrated
- initialize generic integrator with `initialize_generic_integrator()`
- impose initial conditions with `background_initial_conditions()`
- create a growTable with `gt_init()`
- loop over integration steps: call `background_functions()`, find step size, save data in growTable with `gt_add()`, perform one step with `generic_integrator()`, store new value of tau
- save last data in growTable with `gt_add()`
- clean up generic integrator with `cleanup_generic_integrator()`
- retrieve data stored in the growTable with `gt_getPtr()`
- interpolate to get quantities precisely today with `array_interpolate()`
- deduce age of the Universe
- allocate background tables
- In a loop over lines, fill background table using the result of the integration plus `background_functions()`
- free the growTable with `gt_free()`
- fill tables of second derivatives (in view of spline interpolation)
- compute remaining "related parameters"
  - so-called "effective neutrino number", computed at earliest time in interpolation table. This should be seen as a definition:  $N_{\text{eff}}$  is the equivalent number of instantaneously-decoupled neutrinos accounting for the radiation density, beyond photons
- done

5.1.2.15 `background_initial_conditions()`

```
int background_initial_conditions (
    struct precision * ppr,
    struct background * pba,
    double * pvecback,
    double * pvecback_integration )
```

Assign initial values to background integrated variables.

## Parameters

|                             |   |
|-----------------------------|---|
| <i>ppr</i>                  | Input: pointer to precision structure   |
| <i>pba</i>                  | Input: pointer to background structure  |
| <i>pvecback</i>             | Input: vector of background quantities used as workspace                                      |
| <i>pvecback_integration</i> | Output: vector of background quantities to be integrated, returned with proper initial values |

## Returns

the error status

## Summary:

- define local variables
- fix initial value of  $a$

If we have ncdm species, perhaps we need to start earlier than the standard value for the species to be relativistic. This could happen for some WDM models.

- We must add the relativistic contribution from NCDM species
  - $f$  is the critical density fraction of DR. The exact solution is:

$$f = -\Omega_{\text{rad}} + \text{pow}(\text{pow}(\Omega_{\text{rad}}, 3./2.) + 0.5 * \text{pow}(a/pba \rightarrow a_{\text{today}}, 6) * pvecback \leftarrow \_integration[pba \rightarrow \text{index\_bi\_rho\_dcdm}] * pba \rightarrow \text{Gamma\_dcdm} / \text{pow}(pba \rightarrow H_0, 3), 2./3.);$$

but it is not numerically stable for very small  $f$  which is always the case. Instead we use the Taylor expansion of this equation, which is equivalent to ignoring  $f(a)$  in the Hubble rate.

There is also a space reserved for a future case where dr is not sourced by dcdm

- Fix initial value of  $\phi, \phi'$  set directly in the radiation attractor => fixes the units in terms of rho\_ur

## TODO:

- There seems to be some small oscillation when it starts.
- Check equations and signs. Sign of phi\_prime?
- is rho\_ur all there is early on?
- -> If there is no attractor solution for scf\_lambda, assign some value. Otherwise would give a nan.
- -> If no attractor initial conditions are assigned, gets the provided ones.
- compute initial proper time, assuming radiation-dominated universe since Big Bang and therefore  $t = 1/(2H)$  (good approximation for most purposes)
- compute initial conformal time, assuming radiation-dominated universe since Big Bang and therefore  $\tau = 1/(aH)$  (good approximation for most purposes)
- compute initial sound horizon, assuming  $c_s = 1/\sqrt{3}$  initially
- set initial value of D and D' in RD. D will be renormalised later, but D' must be correct.

## 5.1.2.16 background\_output\_titles()

```
int background_output_titles (
    struct background * pba,
    char titles[_MAXTITLESTRINGLENGTH_] )
```

Subroutine for formatting background output

- Length of the column title should be less than *OUTPUTPRECISION*+6 to be indented correctly, but it can be as long as .

## 5.1.2.17 background\_output\_data()

```
int background_output_data (
    struct background * pba,
    int number_of_titles,
    double * data )
```

Stores quantities

## 5.1.2.18 background\_derivs()

```
int background_derivs (
    double tau,
    double * y,
    double * dy,
    void * parameters_and_workspace,
    ErrorMsg error_message )
```

Subroutine evaluating the derivative with respect to conformal time of quantities which are integrated (a, t, etc).

This is one of the few functions in the code which is passed to the `generic_integrator()` routine. Since `generic_integrator()` should work with functions passed from various modules, the format of the arguments is a bit special:

- fixed input parameters and workspaces are passed through a generic pointer. Here, this is just a pointer to the background structure and to a background vector, but `generic_integrator()` doesn't know its fine structure.
- the error management is a bit special: errors are not written as usual to `pba->error_message`, but to a generic `error_message` passed in the list of arguments.

## Parameters

|                                 |   |
|---------------------------------|---|
| <i>tau</i>                      | Input: conformal time                             |
| <i>y</i>                        | Input: vector of variable                         |
| <i>dy</i>                       | Output: its derivative (already allocated)        |
| <i>parameters_and_workspace</i> | Input: pointer to fixed parameters (e.g. indices) |
| <i>error_message</i>            | Output: error message                             |

Summary:

- define local variables
- calculate functions of  $a$  with `background_functions()`
- Short hand notation
- calculate  $a' = a^2 H$
- calculate  $t' = a$
- calculate  $rs' = c_s$
- solve second order growth equation  $[D''(\tau) = -aHD'(\tau) + 3/2a^2\rho_M D(\tau)]$
- compute dcdm density  $\rho' = -3aH\rho - a\Gamma\rho$
- Compute dr density  $\rho' = -4aH\rho - a\Gamma\rho$
- Compute fld density  $\rho' = -3aH(1 + w_{fld}(a))\rho$
- Scalar field equation:  $\phi'' + 2aH\phi' + a^2 dV = 0$  (note H is wrt cosmic time)

### 5.1.2.19 V\_e\_scf()

```
double V_e_scf (
    struct background * pba,
    double phi )
```

Scalar field potential and its derivatives with respect to the field \_scf For Albrecht & Skordis model: 9908085

- $V = V_{p_scf} * V_{e_scf}$
- $V_e = \exp(-\lambda\phi)$  (exponential)
- $V_p = (\phi - B)^\alpha + A$  (polynomial bump)

TODO:

- Add some functionality to include different models/potentials (tuning would be difficult, though)
- Generalize to Kessence/Horndeski/PPF and/or couplings
- A default module to numerically compute the derivatives when no analytic functions are given should be added.
- Numerical derivatives may further serve as a consistency check.

The units of phi, tau in the derivatives and the potential V are the following:

- phi is given in units of the reduced Planck mass  $m_{pl} = (8\pi G)^{-1/2}$
- tau in the derivative is given in units of Mpc.



- the potential  $V(\phi)$  is given in units of  $m_{pl}^2/Mpc^2$ . With this convention, we have  $\rho^{class} = (8\pi G)/3\rho^{physical} = 1/(3m_{pl}^2)\rho^{physical} = 1/3 * [1/(2a^2)(\phi')^2 + V(\phi)]$  and  $\rho^{class}$  has the proper dimension  $Mpc^{-2}$ .

Here is the caller graph for this function:



#### 5.1.2.20 V\_p\_scf()

```
double V_p_scf (
    struct background * pba,
    double phi )
```

parameters and functions for the polynomial coefficient  $V_p = (\phi - B)^\alpha + A(\text{polynomial bump})$

```
double scf_alpha = 2;
```

```
double scf_B = 34.8;
```

```
double scf_A = 0.01; (values for their Figure 2)
```

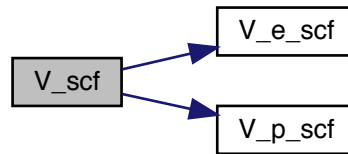
Here is the caller graph for this function:



## 5.1.2.21 V\_scf()

```
double V_scf (
    struct background * pba,
    double phi )
```

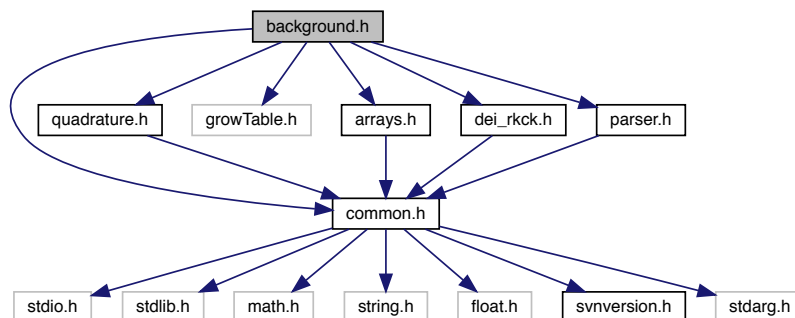
Fianlly we can obtain the overall potential  $V = V_p * V_e$  Here is the call graph for this function:



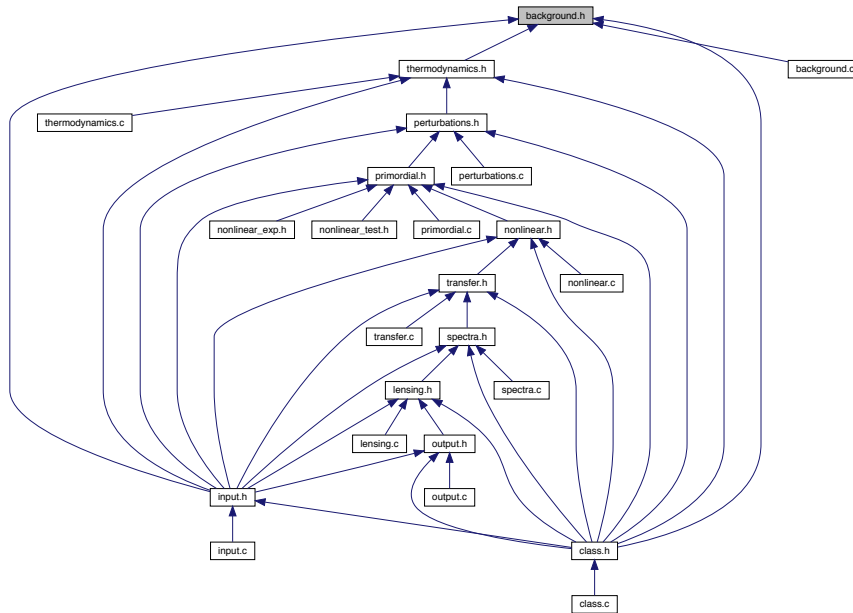
## 5.2 background.h File Reference

```
#include "common.h"
#include "quadrature.h"
#include "growTable.h"
#include "arrays.h"
#include "dei_rkck.h"
#include "parser.h"
```

Include dependency graph for background.h:



This graph shows which files directly or indirectly include this file:



## Data Structures

- struct [background](#)
- struct [background\\_parameters\\_and\\_workspace](#)
- struct [background\\_parameters\\_for\\_distributions](#)

### 5.2.1 Detailed Description

Documented includes for background module

### 5.2.2 Data Structure Documentation

#### 5.2.2.1 struct background

All background parameters and evolution that other modules need to know.

Once initialized by the `background_init()`, contains all necessary information on the background evolution (except thermodynamics), and in particular, a table of all background quantities as a function of time and scale factor, used for interpolation in other modules.

#### Data Fields

|        |          |  |
|--------|----------|--|
| double | H0       | $H_0$ : Hubble parameter (in fact, $[H_0/c]$ ) in $Mpc^{-1}$ |
| double | Omega0_g | $\Omega_{0\gamma}$ : photons                                 |
| double | T_cmb    | $T_{cmb}$ : current CMB temperature in Kelvins               |
| double | Omega0_b | $\Omega_{0b}$ : baryons                                      |

## Data Fields

|          |                     |  |
|----------|---------------------|--|
| double   | Omega0_cdm          | $\Omega_{0cdm}$ : cold dark matter   |
| double   | Omega0_lambda       | $\Omega_{0\Lambda}$ : cosmological constant  |
| double   | Omega0_fld          | $\Omega_{0de}$ : fluid   |
| double   | w0_fld              | $w_{0DE}$ : current fluid equation of state parameter  |
| double   | wa_fld              | $w_{aDE}$ : fluid equation of state parameter derivative   |
| double   | cs2_fld             | $c_{sDE}^2$ : sound speed of the fluid in the frame comoving with the fluid (so, this is not $[\Delta p/\Delta \rho]$ in the synchronous or newtonian gauge!!!)  |
| short    | use_ppf             | flag switching on PPF perturbation equations instead of true fluid equations for perturbations. It could have been defined inside perturbation structure, but we leave it here in such way to have all fld parameters grouped. |
| double   | c_gamma_over_c_fld  | ppf parameter defined in eq. (16) of 0808.3125 [astro-ph]  |
| double   | Omega0_ur           | $\Omega_{0ur}$ : ultra-relativistic neutrinos  |
| double   | Omega0_dcdm         | $\Omega_{0dcdm} + \Omega_{0dr}$ : decaying cold dark matter (dcdm) decaying to dark radiation (dr)   |
| double   | Gamma_dcdm          | $\Gamma_{dcdm}$ : decay constant for decaying cold dark matter   |
| double   | Omega_ini_dcdm      | $\Omega_{ini,dcdm}$ : rescaled initial value for dcdm density (see 1407.2418 for definitions)  |
| double   | Omega0_scf          | $\Omega_{0scf}$ : scalar field   |
| short    | attractor_ic_scf    | whether the scalar field has attractor initial conditions  |
| double   | phi_ini_scf         | $\phi(t_0)$ : scalar field initial value   |
| double   | phi_prime_ini_scf   | $d\phi(t_0)/d\tau$ : scalar field initial derivative wrt conformal time  |
| double * | scf_parameters      | list of parameters describing the scalar field potential   |
| int      | scf_parameters_size | size of scf_parameters   |
| int      | scf_tuning_index    | index in scf_parameters used for tuning  |
| double   | Omega0_k            | $\Omega_{0k}$ : curvature contribution   |
| int      | N_ncdm              | Number of distinguishable ncdm species   |
| double * | M_ncdm              | vector of masses of non-cold relic: dimensionless ratios $m_{ncdm}/T_{ncdm}$   |
| double * | Omega0_ncdm         |  |
| double   | Omega0_ncdm_tot     | Omega0_ncdm for each species and for the total Omega0_ncdm   |
| double * | deg_ncdm            |  |
| double   | deg_ncdm_default    | vector of degeneracy parameters in factor of p-s-d: 1 for one family of neutrinos (= one neutrino plus its anti-neutrino, total $g^*=1+1=2$ , so $deg = 0.5 g^*$ ); and its default value                                      |
| double * | T_ncdm              |  |
| double   | T_ncdm_default      | list of 1st parameters in p-s-d of non-cold relics: relative temperature $T_{ncdm1}/T_{gamma}$ ; and its default value   |
| double * | ksi_ncdm            |  |
| double   | ksi_ncdm_default    | list of 2nd parameters in p-s-d of non-cold relics: relative chemical potential $ksi_{ncdm1}/T_{ncdm1}$ ; and its default value  |
| double * | ncdm_psd_parameters | list of parameters for specifying/modifying ncdm p.s.d.'s, to be customized for given model (could be e.g. mixing angles)  |
| int *    | got_files           | list of flags for each species, set to true if p-s-d is passed through file  |
| char *   | ncdm_psd_files      | list of filenames for tabulated p-s-d  |
| double   | h                   | reduced Hubble parameter   |
| double   | age                 | age in Gyears  |

## Data Fields

|          |                         |   |
|----------|-------------------------|---|
| double   | conformal_age           | conformal age in Mpc  |
| double   | K                       | $K$ : Curvature parameter $K = -\Omega_{0k} * a_{today}^2 * H_0^2$ ;                    |
| int      | sgnK                    | $K/ K $ : -1, 0 or 1  |
| double * | m_ncdm_in_eV            | list of ncdm masses in eV (inferred from M_ncdm and other parameters above)             |
| double   | Neff                    | so-called "effective neutrino number", computed at earliest time in interpolation table |
| double   | Omega0_dcdm             | $\Omega_{0dcdm}$ : decaying cold dark matter  |
| double   | Omega0_dr               | $\Omega_{0dr}$ : decay radiation  |
| double   | a_today                 | scale factor today (arbitrary and irrelevant for most purposes)                         |
| int      | index_bg_a              | scale factor  |
| int      | index_bg_H              | Hubble parameter in $Mpc^{-1}$  |
| int      | index_bg_H_prime        | its derivative w.r.t. conformal time  |
| int      | index_bg_rho_g          | photon density  |
| int      | index_bg_rho_b          | baryon density  |
| int      | index_bg_rho_cdm        | cdm density   |
| int      | index_bg_rho_lambda     | cosmological constant density   |
| int      | index_bg_rho fld        | fluid density   |
| int      | index_bg_w fld          | fluid equation of state   |
| int      | index_bg_rho_ur         | relativistic neutrinos/relics density   |
| int      | index_bg_rho_dcdm       | dcdm density  |
| int      | index_bg_rho_dr         | dr density  |
| int      | index_bg_phi_scf        | scalar field value  |
| int      | index_bg_phi_prime_scf  | scalar field derivative wrt conformal time  |
| int      | index_bg_V_scf          | scalar field potential V  |
| int      | index_bg_dV_scf         | scalar field potential derivative V'  |
| int      | index_bg_ddV_scf        | scalar field potential second derivative V''  |
| int      | index_bg_rho_scf        | scalar field energy density   |
| int      | index_bg_p_scf          | scalar field pressure   |
| int      | index_bg_rho_ncdm1      | density of first ncdm species (others contiguous)                                       |
| int      | index_bg_p_ncdm1        | pressure of first ncdm species (others contiguous)                                      |
| int      | index_bg_pseudo_p_ncdm1 | another statistical momentum useful in ncdma approximation                              |
| int      | index_bg_Omega_r        | relativistic density fraction ( $\Omega_\gamma + \Omega_{\nu r}$ )                      |
| int      | index_bg_rho_crit       | critical density  |
| int      | index_bg_Omega_m        | non-relativistic density fraction ( $\Omega_b + \Omega_{cdm} + \Omega_{\nu nr}$ )       |
| int      | index_bg_conf_distance  | conformal distance (from us) in Mpc   |
| int      | index_bg_ang_distance   | angular diameter distance in Mpc  |
| int      | index_bg_lum_distance   | luminosity distance in Mpc  |
| int      | index_bg_time           | proper (cosmological) time in Mpc   |
| int      | index_bg_rs             | comoving sound horizon in Mpc   |
| int      | index_bg_D              | scale independent growth factor D(a) for CDM perturbations                              |
| int      | index_bg_f              | corresponding velocity growth factor $[d \ln D]/[d \ln a]$                              |
| int      | bg_size_short           | size of background vector in the "short format"   |
| int      | bg_size_normal          | size of background vector in the "normal format"  |
| int      | bg_size                 | size of background vector in the "long format"  |
| int      | bt_size                 | number of lines (i.e. time-steps) in the array  |

## Data Fields

|           |                          |  |
|-----------|--------------------------|--|
| double *  | tau_table                | vector tau_table[index_tau] with values of $\tau$ (conformal time)   |
| double *  | z_table                  | vector z_table[index_tau] with values of $z$ (redshift)  |
| double *  | background_table         | table background_table[index_tau*pba->bg_size+pba->index_bg] with all other quantities (array of size bg_size*bt_size)       |
| double *  | d2tau_dz2_table          | vector d2tau_dz2_table[index_tau] with values of $d^2\tau/dz^2$ (conformal time)   |
| double *  | d2background_dtau2_table | table d2background_dtau2_table[index_tau*pba->bg_size+pba->index_bg] with values of $d^2b_i/d\tau^2$ (conformal time)        |
| int       | index_bi_a               | {B} scale factor   |
| int       | index_bi_rho_dcdm        | {B} dcdm density   |
| int       | index_bi_rho_dr          | {B} dr density   |
| int       | index_bi_rho_fld         | {B} fluid density  |
| int       | index_bi_phi_scf         | {B} scalar field value   |
| int       | index_bi_phi_prime_scf   | {B} scalar field derivative wrt conformal time   |
| int       | index_bi_time            | {C} proper (cosmological) time in Mpc  |
| int       | index_bi_rs              | {C} sound horizon  |
| int       | index_bi_tau             | {C} conformal time in Mpc  |
| int       | index_bi_D               | {C} scale independent growth factor D(a) for CDM perturbations.  |
| int       | index_bi_D_prime         | {C} D satisfies $[D''(\tau) = -aHD'(\tau) + 3/2a^2\rho_M D(\tau)]$   |
| int       | bi_B_size                | Number of {B} parameters   |
| int       | bi_size                  | Number of {B}+{C} parameters   |
| short     | has_cdm                  | presence of cold dark matter?  |
| short     | has_dcdm                 | presence of decaying cold dark matter?   |
| short     | has_dr                   | presence of relativistic decay radiation?  |
| short     | has_scf                  | presence of a scalar field?  |
| short     | has_ncdm                 | presence of non-cold dark matter?  |
| short     | has_lambda               | presence of cosmological constant?   |
| short     | has_fld                  | presence of fluid with constant w and cs2?   |
| short     | has_ur                   | presence of ultra-relativistic neutrinos/relics?   |
| short     | has_curvature            | presence of global spatial curvature?  |
| double ** | q_ncdm_bg                | Pointers to vectors of background sampling in q  |
| double ** | w_ncdm_bg                | Pointers to vectors of corresponding quadrature weights w  |
| double ** | q_ncdm                   | Pointers to vectors of perturbation sampling in q  |
| double ** | w_ncdm                   | Pointers to vectors of corresponding quadrature weights w  |
| double ** | dlnf0_dlnq_ncdm          | Pointers to vectors of logarithmic derivatives of p-s-d  |
| int *     | q_size_ncdm_bg           | Size of the q_ncdm_bg arrays   |
| int *     | q_size_ncdm              | Size of the q_ncdm arrays  |
| double *  | factor_ncdm              | List of normalization factors for calculating energy density etc.  |
| short     | short_info               | flag for calling background_at_eta and return little information   |
| short     | normal_info              | flag for calling background_at_eta and return medium information   |
| short     | long_info                | flag for calling background_at_eta and return all information  |
| short     | inter_normal             | flag for calling background_at_eta and find position in interpolation table normally   |
| short     | inter_closeby            | flag for calling background_at_eta and find position in interpolation table starting from previous position in previous call |

## Data Fields

|          |                    |   |
|----------|--------------------|---|
| short    | shooting_failed    | flag is set to true if shooting failed.   |
| ErrorMsg | shooting_error     | Error message from shooting failed.   |
| short    | background_verbose | flag regulating the amount of information sent to standard output (none if set to zero) |
| ErrorMsg | error_message      | zone for writing error messages   |

## 5.2.2.2 struct background\_parameters\_and\_workspace

temporary parameters and workspace passed to the background\_derivs function

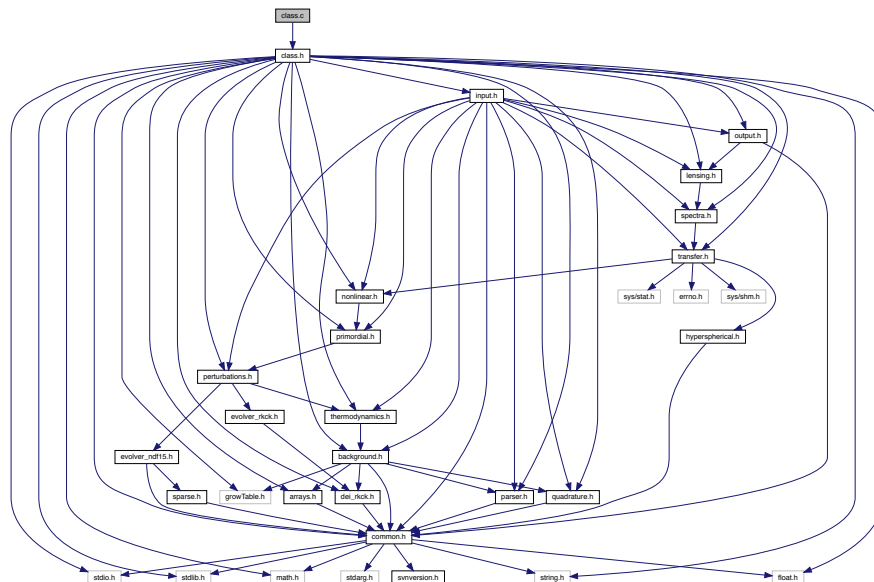
## 5.2.2.3 struct background\_parameters\_for\_distributions

temporary parameters and workspace passed to phase space distribution function

## 5.3 class.c File Reference

```
#include "class.h"
```

Include dependency graph for class.c:



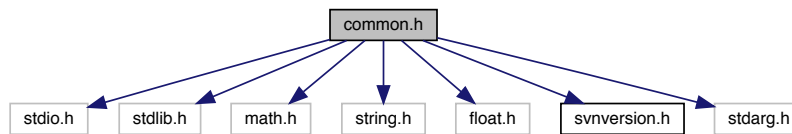
## 5.3.1 Detailed Description

Julien Lesgourgues, 17.04.2011

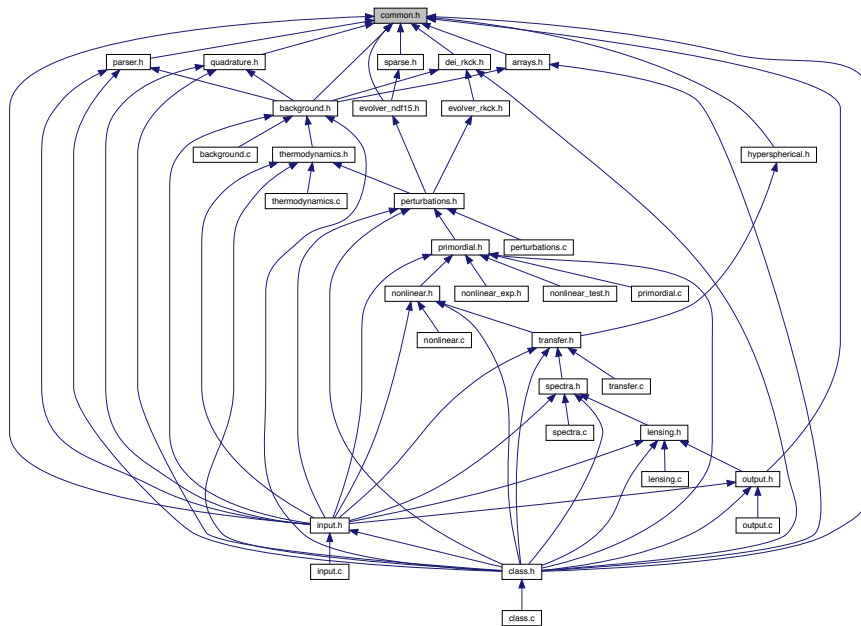
## 5.4 common.h File Reference

```
#include "stdio.h"
#include "stdlib.h"
#include "math.h"
#include "string.h"
#include "float.h"
#include "svnversion.h"
#include <stdarg.h>
```

Include dependency graph for common.h:



This graph shows which files directly or indirectly include this file:



### Data Structures

- struct [precision](#)

### Enumerations

- enum [evolver\\_type](#)
- enum [pk\\_def](#) { [delta\\_m\\_squared](#), [delta\\_tot\\_squared](#), [delta\\_bc\\_squared](#), [delta\\_tot\\_from\\_poisson\\_squared](#) }
- enum [file\\_format](#)



### 5.4.1 Detailed Description

Generic libraries, parameters and functions used in the whole code.

### 5.4.2 Data Structure Documentation

#### 5.4.2.1 struct precision

All precision parameters.

Includes integrations steps, flags telling how the computation is to be performed, etc.

#### Data Fields

|        |                            |  |
|--------|----------------------------|--|
| double | a_ini_over_a_today_default | default initial value of scale factor in background integration, in units of scale factor today  |
| double | back_integration_stepsize  | default step $d\tau$ in background integration, in units of conformal Hubble time ( $d\tau = \text{back\_integration\_stepsize} / aH$ )                            |
| double | tol_background_integration | parameter controlling precision of background integration  |
| double | tol_initial_Omega_r        | parameter controlling how deep inside radiation domination must the initial time be chosen   |
| double | tol_M_ncdm                 | parameter controlling relative precision of ncdm mass for given ncdm current density   |
| double | tol_ncdm_newtonian         | parameter controlling relative precision of integrals over ncdm phase-space distribution during perturbation calculation: value to be applied in Newtonian gauge   |
| double | tol_ncdm_synchronous       | parameter controlling relative precision of integrals over ncdm phase-space distribution during perturbation calculation: value to be applied in synchronous gauge |
| double | tol_ncdm                   | parameter controlling relative precision of integrals over ncdm phase-space distribution during perturbation calculation: value actually applied in chosen gauge   |
| double | tol_ncdm_bg                | parameter controlling relative precision of integrals over ncdm phase-space distribution during background evolution   |
| double | tol_ncdm_initial_w         | parameter controlling how relativistic must non-cold relics be at initial time   |
| double | safe_phi_scf               | parameter controlling the initial scalar field in background functions   |
| double | recfast_z_initial          | initial redshift in recfast  |
| int    | recfast_Nz0                | number of integration steps  |
| double | tol_thermo_integration     | precision of each integration step   |

## Data Fields

|                                   |                                |   |
|-----------------------------------|--------------------------------|---|
| int                               | recfast_Heswitch               | recfast 1.4 parameter   |
| double                            | recfast_fudge_He               | recfast 1.4 parameter   |
| int                               | recfast_Hswitch                | recfast 1.5 switching parameter   |
| double                            | recfast_fudge_H                | H fudge factor when recfast_Hswitch set to false (v1.4 fudging)   |
| double                            | recfast_delta_fudge_H          | correction to H fudge factor in v1.5  |
| double                            | recfast_AGauss1                | Amplitude of 1st Gaussian   |
| double                            | recfast_AGauss2                | Amplitude of 2nd Gaussian   |
| double                            | recfast_zGauss1                | $\ln(1+z)$ of 1st Gaussian  |
| double                            | recfast_zGauss2                | $\ln(1+z)$ of 2nd Gaussian  |
| double                            | recfast_wGauss1                | Width of 1st Gaussian   |
| double                            | recfast_wGauss2                | Width of 2nd Gaussian   |
| double                            | recfast_z_He_1                 | down to which redshift Helium fully ionized   |
| double                            | recfast_delta_z_He_1           | z range over which transition is smoothed   |
| double                            | recfast_z_He_2                 | down to which redshift first Helium recombination not complete  |
| double                            | recfast_delta_z_He_2           | z range over which transition is smoothed   |
| double                            | recfast_z_He_3                 | down to which redshift Helium singly ionized  |
| double                            | recfast_delta_z_He_3           | z range over which transition is smoothed   |
| double                            | recfast_x_He0_trigger          | value below which recfast uses the full equation for Helium   |
| double                            | recfast_x_He0_trigger2         | a second threshold used in derivative routine   |
| double                            | recfast_x_He0_trigger_delta    | x_He range over which transition is smoothed  |
| double                            | recfast_x_H0_trigger           | value below which recfast uses the full equation for Hydrogen   |
| double                            | recfast_x_H0_trigger2          | a second threshold used in derivative routine   |
| double                            | recfast_x_H0_trigger_delta     | x_H range over which transition is smoothed   |
| double                            | recfast_H_frac                 | governs time at which full equation of evolution for Tmat is used   |
| double                            | reionization_z_start_max       | maximum redshift at which reionization should start. If not, return an error.   |
| double                            | reionization_sampling          | control stepsize in z during reionization   |
| double                            | reionization_optical_depth_tol | fractional error on optical_depth   |
| double                            | reionization_start_factor      | parameter for CAMB-like parametrization   |
| int                               | thermo_rate_smoothing_radius   | plays a minor (almost aesthetic) role in the definition of the variation rate of thermodynamical quantities                       |
| enum <a href="#">evolver_type</a> | evolver                        | which type of evolver for integrating perturbations (Runge-Kutta? Stiff?...)  |
| double                            | k_min_tau0                     | number defining k_min for the computation of Cl's and P(k)'s (dimensionless): (k_min tau_0), usually chosen much smaller than one |

## Data Fields

|        |                                   |  |
|--------|-----------------------------------|--|
| double | k_max_tau0_over_l_max             | number defining k_max for the computation of Cl's (dimensionless): $(k\_max \tau_0)/l\_max$ , usually chosen around two  |
| double | k_step_sub                        | step in k space, in units of one period of acoustic oscillation at decoupling, for scales inside sound horizon at decoupling   |
| double | k_step_super                      | step in k space, in units of one period of acoustic oscillation at decoupling, for scales above sound horizon at decoupling  |
| double | k_step_transition                 | dimensionless number regulating the transition from 'sub' steps to 'super' steps. Decrease for more precision.   |
| double | k_step_super_reduction            | the step k_step_super is reduced by this amount in the $k \rightarrow 0$ limit (below scale of Hubble and/or curvature radius)   |
| double | k_per_decade_for_pk               | if values needed between kmax inferred from k_oscillations and k_kmax_for_pk, this gives the number of k per decade outside the BAO region   |
| double | k_per_decade_for_bao              | if values needed between kmax inferred from k_oscillations and k_kmax_for_pk, this gives the number of k per decade inside the BAO region (for finer sampling)   |
| double | k_bao_center                      | in $\ln(k)$ space, the central value of the BAO region where sampling is finer is defined as k_rec times this number (recommended: 3, i.e. finest sampling near 3rd BAO peak)  |
| double | k_bao_width                       | in $\ln(k)$ space, width of the BAO region where sampling is finer: this number gives roughly the number of BAO oscillations well resolved on both sides of the central value (recommended: 4, i.e. finest sampling from before first up to 3+4=7th peak)  |
| double | start_small_k_at_tau_c_over_tau_h | largest wavelengths start being sampled when universe is sufficiently opaque. This is quantified in terms of the ratio of thermo to hubble time scales, $\tau_c/\tau_H$ . Start when start_largek_at_tau_c_over_tau_h equals this ratio. Decrease this value to start integrating the wavenumbers earlier in time.   |
| double | start_large_k_at_tau_h_over_tau_k | largest wavelengths start being sampled when mode is sufficiently outside Hubble scale. This is quantified in terms of the ratio of hubble time scale to wavenumber time scale, $\tau_h/\tau_k$ which is roughly equal to $(k*\tau)$ . Start when this ratio equals start_large_k_at_tau_k_over_tau_h. Decrease this value to start integrating the wavenumbers earlier in time. |

## Data Fields

|        |   |   |
|--------|---|---|
| double | tight_coupling_trigger_tau_c_over_tau_h | when to switch off tight-coupling approximation: first condition: $\tau_c/\tau_H > \text{tight\_coupling\_trigger\_tau\_c\_over\_tau\_h}$ . Decrease this value to switch off earlier in time. If this number is larger than <code>start_sources_at_tau_c_over_tau_h</code> , the code returns an error, because the source computation requires tight-coupling to be switched off. |
| double | tight_coupling_trigger_tau_c_over_tau_k | when to switch off tight-coupling approximation: second condition: $\tau_c/\tau_k \equiv k\tau_c < \text{tight\_coupling\_trigger\_tau\_c\_over\_tau\_k}$ . Decrease this value to switch off earlier in time.  |
| double | start_sources_at_tau_c_over_tau_h       | sources start being sampled when universe is sufficiently opaque. This is quantified in terms of the ratio of thermo to hubble time scales, $\tau_c/\tau_H$ . Start when <code>start_sources_at_tau_c_over_tau_h</code> equals this ratio. Decrease this value to start sampling the sources earlier in time.   |
| int    | tight_coupling_approximation            | method for tight coupling approximation   |
| int    | l_max_g                                 | number of momenta in Boltzmann hierarchy for photon temperature (scalar), at least 4  |
| int    | l_max_pol_g                             | number of momenta in Boltzmann hierarchy for photon polarization (scalar), at least 4   |
| int    | l_max_dr                                | number of momenta in Boltzmann hierarchy for decay radiation, at least 4  |
| int    | l_max_ur                                | number of momenta in Boltzmann hierarchy for relativistic neutrino/relics (scalar), at least 4  |
| int    | l_max_ncdm                              | number of momenta in Boltzmann hierarchy for relativistic neutrino/relics (scalar), at least 4  |
| int    | l_max_g_ten                             | number of momenta in Boltzmann hierarchy for photon temperature (tensor), at least 4  |
| int    | l_max_pol_g_ten                         | number of momenta in Boltzmann hierarchy for photon polarization (tensor), at least 4   |
| double | curvature_ini                           | initial condition for curvature for adiabatic   |
| double | entropy_ini                             | initial condition for entropy perturbation for isocurvature   |
| double | gw_ini                                  | initial condition for tensor metric perturbation h  |
| double | perturb_integration_stepsize            | default step $d\tau$ in perturbation integration, in units of the timescale involved in the equations (usually, the min of $1/k$ , $1/aH$ , $1/\dot{\kappa}$ )  |

## Data Fields

|        |  |  |
|--------|--|--|
| double | perturb_sampling_stepsize                    | default step $d\tau$ for sampling the source function, in units of the timescale involved in the sources: $(\dot{\kappa} - \ddot{\kappa}/\dot{\kappa})^{-1}$   |
| double | tol_perturb_integration                      | control parameter for the precision of the perturbation integration  |
| double | tol_tau_approx                               | precision with which the code should determine (by bisection) the times at which sources start being sampled, and at which approximations must be switched on/off (units of Mpc)   |
| int    | radiation_streaming_approximation            | method for switching off photon perturbations  |
| double | radiation_streaming_trigger_tau_over_tau_k   | when to switch off photon perturbations, ie when to switch on photon free-streaming approximation (keep density and $t\tau$ , set shear and higher momenta to zero): first condition: $k\tau > \text{radiation\_streaming\_trigger\_tau\_h\_over\_tau\_k}$ |
| double | radiation_streaming_trigger_tau_c_over_tau_k | when to switch off photon perturbations, ie when to switch on photon free-streaming approximation (keep density and $\theta$ , set shear and higher momenta to zero): second condition:  |
| int    | ur_fluid_approximation                       | method for ultra relativistic fluid approximation  |
| double | ur_fluid_trigger_tau_over_tau_k              | when to switch off ur (massless neutrinos / ultra-relativistic relics) fluid approximation   |
| int    | ncdm_fluid_approximation                     | method for non-cold dark matter fluid approximation  |
| double | ncdm_fluid_trigger_tau_over_tau_k            | when to switch off ncdm (massive neutrinos / non-cold relics) fluid approximation  |
| double | neglect_CMB_sources_below_visibility         | whether CMB source functions can be approximated as zero when visibility function $g(\tau)$ is tiny  |
| double | k_per_decade_primordial                      | logarithmic sampling for primordial spectra (number of points per decade in $k$ space)   |
| double | primordial_inflation_ratio_min               | for each $k$ , start following wavenumber when $aH = k/\text{primordial\_inflation\_ratio\_min}$   |
| double | primordial_inflation_ratio_max               | for each $k$ , stop following wavenumber, at the latest, when $aH = k/\text{primordial\_inflation\_ratio\_max}$  |
| int    | primordial_inflation_phi_ini_maxit           | maximum number of iteration when searching a suitable initial field value $\phi_{ini}$ (value reached when no long-enough slow-roll period before the pivot scale)   |
| double | primordial_inflation_pt_stepsize             | controls the integration timestep for inflaton perturbations   |
| double | primordial_inflation_bg_stepsize             | controls the integration timestep for inflaton background  |
| double | primordial_inflation_tol_integration         | controls the precision of the ODE integration during inflation   |

## Data Fields

|        |  |   |
|--------|--|---|
| double | primordial_inflation_attractor_precision_pivot   | targeted precision when searching attractor solution near phi_pivot   |
| double | primordial_inflation_attractor_precision_initial | targeted precision when searching attractor solution near phi_ini   |
| int    | primordial_inflation_attractor_maxit             | maximum number of iteration when searching attractor solution   |
| double | primordial_inflation_tol_curvature               | for each k, stop following wavenumber, at the latest, when curvature perturbation R is stable up to to this tolerance   |
| double | primordial_inflation_aH_ini_target               | control the step size in the search for a suitable initial field value  |
| double | primordial_inflation_end_dphi                    | first bracketing width, when trying to bracket the value phi_end at which inflation ends naturally  |
| double | primordial_inflation_end_logstep                 | logarithmic step for updating the bracketing width, when trying to bracket the value phi_end at which inflation ends naturally  |
| double | primordial_inflation_small_epsilon               | value of slow-roll parameter epsilon used to define a field value phi_end close to the end of inflation (doesn't need to be exactly at the end):<br>epsilon(phi_end)=small_epsilon (should be smaller than one) |
| double | primordial_inflation_small_epsilon_tol           | tolerance in the search for phi_end   |
| double | primordial_inflation_extra_efolds                | a small number of efolds, irrelevant at the end, used in the search for the pivot scale (backward from the end of inflation)  |
| int    | l_linstep  | factor for logarithmic spacing of values of l over which bessel and transfer functions are sampled  |
| double | l_logstep  | maximum spacing of values of l over which Bessel and transfer functions are sampled (so, spacing becomes linear instead of logarithmic at some point)   |
| double | hyper_x_min                                      | flat case: lower bound on the smallest value of x at which we sample $\Phi_l^\nu(x)$ or $j_l(x)$  |
| double | hyper_sampling_flat                              | flat case: number of sampled points x per approximate wavelength $2\pi$   |
| double | hyper_sampling_curved_low_nu                     | open/closed cases: number of sampled points x per approximate wavelength $2\pi/\nu$ , when $\nu$ smaller than hyper_nu_sampling_step  |
| double | hyper_sampling_curved_high_nu                    | open/closed cases: number of sampled points x per approximate wavelength $2\pi/\nu$ , when $\nu$ greater than hyper_nu_sampling_step  |
| double | hyper_nu_sampling_step                           | open/closed cases: value of nu at which sampling changes  |
| double | hyper_phi_min_abs                                | small value of Bessel function used in calculation of first point x ( $\Phi_l^\nu(x)$ equals hyper_phi_min_abs)   |

## Data Fields

|        |                               |  |
|--------|-------------------------------|--|
| double | hyper_x_tol                   | tolerance parameter used to determine first value of $x$   |
| double | hyper_flat_approximation_nu   | value of $\nu$ below which the flat approximation is used to compute Bessel function   |
| double | q_linstep                     | asymptotic linear sampling step in $q$ space, in units of $2\pi/r_a(\tau_{rec})$ (comoving angular diameter distance to recombination)   |
| double | q_logstep_spline              | initial logarithmic sampling step in $q$ space, in units of $2\pi/r_a(\tau_{rec})$ (comoving angular diameter distance to recombination)   |
| double | q_logstep_open                | in open models, the value of $q\_logstep\_spline$ must be decreased according to curvature. Increasing this number will make the calculation more accurate for large positive $\Omega_k$   |
| double | q_logstep_trapzd              | initial logarithmic sampling step in $q$ space, in units of $2\pi/r_a(\tau_{rec})$ (comoving angular diameter distance to recombination), in the case of small $q$ 's in the closed case, for which one must use trapezoidal integration instead of spline (the number of $q$ 's for which this is the case decreases with curvature and vanishes in the flat limit) |
| double | q_numstep_transition          | number of steps for the transition from $q\_logstep\_trapzd$ steps to $q\_logstep\_spline$ steps (transition must be smooth for spline)  |
| double | transfer_neglect_delta_k_S_t0 | for temperature source function $T_0$ of scalar mode, range of $k$ values (in $1/\text{Mpc}$ ) taken into account in transfer function: for $l < (k - \delta_k) * \tau_0$ , ie for $k > (l/\tau_0 + \delta_k)$ , the transfer function is set to zero  |
| double | transfer_neglect_delta_k_S_t1 | same for temperature source function $T_1$ of scalar mode  |
| double | transfer_neglect_delta_k_S_t2 | same for temperature source function $T_2$ of scalar mode  |
| double | transfer_neglect_delta_k_S_e  | same for polarization source function $E$ of scalar mode   |
| double | transfer_neglect_delta_k_V_t1 | same for temperature source function $T_1$ of vector mode  |
| double | transfer_neglect_delta_k_V_t2 | same for temperature source function $T_2$ of vector mode  |
| double | transfer_neglect_delta_k_V_e  | same for polarization source function $E$ of vector mode   |
| double | transfer_neglect_delta_k_V_b  | same for polarization source function $B$ of vector mode   |
| double | transfer_neglect_delta_k_T_t2 | same for temperature source function $T_2$ of tensor mode  |
| double | transfer_neglect_delta_k_T_e  | same for polarization source function $E$ of tensor mode   |

## Data Fields

|        |                                     |  |
|--------|-------------------------------------|--|
| double | transfer_neglect_delta_k_T_b        | same for polarization source function B of tensor mode   |
| double | transfer_neglect_late_source        | value of l below which the CMB source functions can be neglected at late time, excepted when there is a Late ISW contribution  |
| double | l_switch_limber                     | when to use the Limber approximation for project gravitational potential cl's  |
| double | l_switch_limber_for_nc_local_over_z | when to use the Limber approximation for local number count contributions to cl's (relative to central redshift of each bin)   |
| double | l_switch_limber_for_nc_los_over_z   | when to use the Limber approximation for number count contributions to cl's integrated along the line-of-sight (relative to central redshift of each bin)  |
| double | selection_cut_at_sigma              | in sigma units, where to cut gaussian selection functions  |
| double | selection_sampling                  | controls sampling of integral over time when selection functions vary quicker than Bessel functions. Increase for better sampling.   |
| double | selection_sampling_bessel           | controls sampling of integral over time when selection functions vary slower than Bessel functions. Increase for better sampling   |
| double | selection_sampling_bessel_los       | controls sampling of integral over time when selection functions vary slower than Bessel functions. This parameter is specific to number counts contributions to CI integrated along the line of sight. Increase for better sampling   |
| double | selection_tophat_edge               | controls how smooth are the edge of top-hat window function ( $\ll 1$ for very sharp, 0.1 for sharp)   |
| double | halofit_dz                          | parameters relevant for HALOFIT computation spacing in redshift space defining values of z at which HALOFIT will be used. Intermediate values will be obtained by interpolation. Decrease for more precise interpolations, at the expense of increasing time spent in <a href="#">nonlinear_init()</a> |
| double | halofit_min_k_nonlinear             | value of k in 1/Mpc above which non-linear corrections will be computed  |
| double | halofit_sigma_precision             | a smaller value will lead to a more precise halofit result at the highest requested redshift, at the expense of requiring a larger k_max   |
| double | halofit_min_k_max                   | when halofit is used, k_max must be at least equal to this value (otherwise halofit could not find the scale of non-linearity)   |



## Data Fields

|          |                            |  |
|----------|----------------------------|--|
| double   | halofit_k_per_decade       | halofit needs to evaluate integrals (linear power spectrum times some kernels). They are sampled using this logarithmic step size.                                       |
| int      | accurate_lensing           | switch between Gauss-Legendre quadrature integration and simple quadrature on a subdomain of angles  |
| int      | num_mu_minus_lmax          | difference between num_mu and l_max, increase for more precision   |
| int      | delta_l_max                | difference between l_max in unlensed and lensed spectra  |
| double   | tol_gauss_legendre         | tolerance with which quadrature points are found: must be very small for an accurate integration (if not entered manually, set automatically to match machine precision) |
| double   | smallest_allowed_variation | machine-dependent, assigned automatically by the code  |
| ErrorMsg | error_message              | zone for writing error messages  |

## 5.4.3 Enumeration Type Documentation

5.4.3.1 `evolver_type`

```
enum evolver_type
```

parameters related to the precision of the code and to the method of calculation list of evolver types for integrating perturbations over time

5.4.3.2 `pk_def`

```
enum pk_def
```

List of ways in which matter power spectrum  $P(k)$  can be defined. The standard definition is the first one (`delta_m_squared`) but alternative definitions can be useful in some projects.

## Enumerator

|   |  |
|---|--|
| <code>delta_m_squared</code>                | normal definition ( <code>delta_m</code> includes all non-relativistic species at late times)  |
| <code>delta_tot_squared</code>              | <code>delta_tot</code> includes all species contributions to ( <code>delta rho</code> ), and only non-relativistic contributions to <code>rho</code> |
| <code>delta_bc_squared</code>               | <code>delta_bc</code> includes contribution of baryons and cdm only to ( <code>delta rho</code> ) and to <code>rho</code>                            |
| <code>delta_tot_from_poisson_squared</code> | use <code>delta_tot</code> inferred from gravitational potential through Poisson equation  |

### 5.4.3.3 file\_format

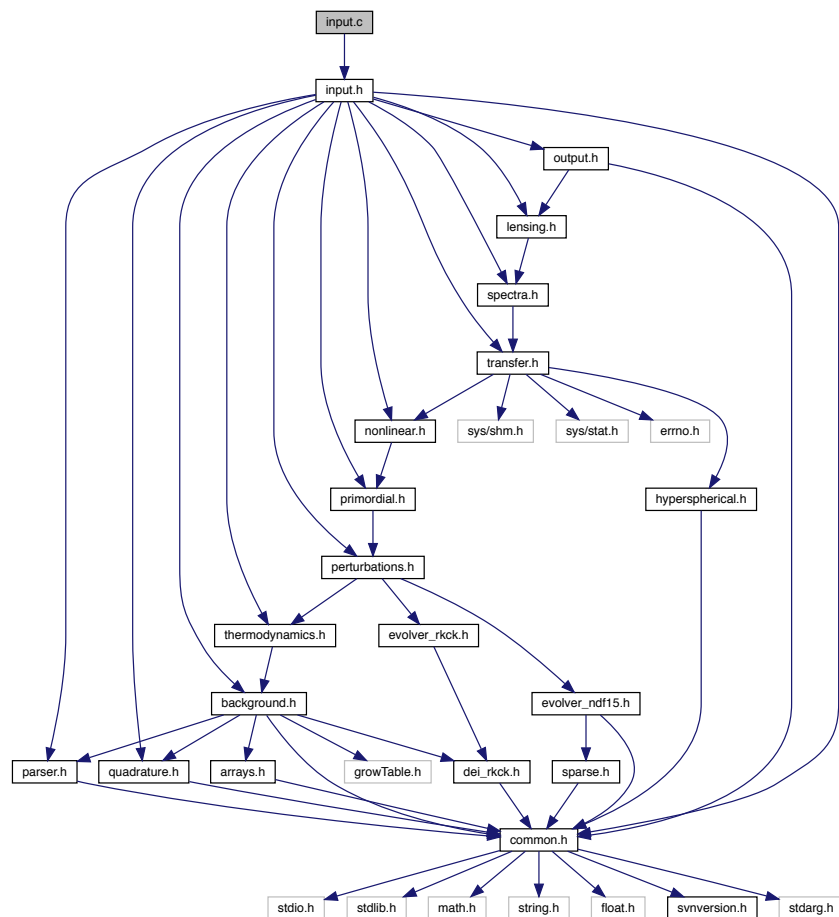
enum `file_format`

Different ways to present output files

## 5.5 input.c File Reference

```
#include "input.h"
```

Include dependency graph for input.c:



## Functions

- int `input_init_from_arguments` (int argc, char \*\*argv, struct `precision` \*ppr, struct `background` \*pba, struct `thermo` \*pth, struct `perturbs` \*ppt, struct `transfers` \*ptr, struct `primordial` \*ppm, struct `spectra` \*psp, struct `nonlinear` \*pnl, struct `lensing` \*ple, struct `output` \*pop, ErrorMsg errmsg)

- int `input_init` (struct `file_content` \*pfc, struct `precision` \*ppr, struct `background` \*pba, struct `thermo` \*pth, struct `perturbs` \*ppt, struct `transfers` \*ptr, struct `primordial` \*ppm, struct `spectra` \*psp, struct `nonlinear` \*pnl, struct `lensing` \*ple, struct `output` \*pop, `ErrMsg` errmsg)
- int `input_read_parameters` (struct `file_content` \*pfc, struct `precision` \*ppr, struct `background` \*pba, struct `thermo` \*pth, struct `perturbs` \*ppt, struct `transfers` \*ptr, struct `primordial` \*ppm, struct `spectra` \*psp, struct `nonlinear` \*pnl, struct `lensing` \*ple, struct `output` \*pop, `ErrMsg` errmsg)
- int `input_default_params` (struct `background` \*pba, struct `thermo` \*pth, struct `perturbs` \*ppt, struct `transfers` \*ptr, struct `primordial` \*ppm, struct `spectra` \*psp, struct `nonlinear` \*pnl, struct `lensing` \*ple, struct `output` \*pop)
- int `input_default_precision` (struct `precision` \*ppr)
- int `get_machine_precision` (double \*smallest\_allowed\_variation)
- int `class_fzero_ridder` (int(\*func)(double x, void \*param, double \*y, `ErrMsg` error\_message), double x1, double x2, double xtol, void \*param, double \*Fx1, double \*Fx2, double \*xzero, int \*fevals, `ErrMsg` error\_message)
- int `input_try_unknown_parameters` (double \*unknown\_parameter, int unknown\_parameters\_size, void \*voidpzw, double \*output, `ErrMsg` errmsg)
- int `input_get_guess` (double \*xguess, double \*dxdy, struct `fzerofun_workspace` \*pzw, `ErrMsg` errmsg)
- int `input_find_root` (double \*xzero, int \*fevals, struct `fzerofun_workspace` \*pzw, `ErrMsg` errmsg)

### 5.5.1 Detailed Description

Documented input module.

Julien Lesgourgues, 27.08.2010

### 5.5.2 Function Documentation

#### 5.5.2.1 input\_init\_from\_arguments()

```
int input_init_from_arguments (
    int argc,
    char ** argv,
    struct precision * ppr,
    struct background * pba,
    struct thermo * pth,
    struct perturbs * ppt,
    struct transfers * ptr,
    struct primordial * ppm,
    struct spectra * psp,
    struct nonlinear * pnl,
    struct lensing * ple,
    struct output * pop,
    ErrMsg errmsg )
```

Use this routine to extract initial parameters from files 'xxx.ini' and/or 'xxx.pre'. They can be the arguments of the main() routine.

If class is embedded into another code, you will probably prefer to call directly `input_init()` in order to pass input parameters through a 'file\_content' structure. Summary:

- define local variables

- -> the final structure with all parameters
- -> a temporary structure with all input parameters
- -> a temporary structure with all precision parameters
- -> a temporary structure with only the root name
- -> sum of `fc_inoput` and `fc_root`
- -> a pointer to either `fc_root` or `fc_inputroot`
- Initialize the two `file_content` structures (for input parameters and precision parameters) to some null content. If no arguments are passed, they will remain null and inform `init_params()` that all parameters take default values.
- If some arguments are passed, identify eventually some 'xxx.ini' and 'xxx.pre' files, and store their name.
- if there is an 'xxx.ini' file, read it and store its content.
- check whether a root name has been set
- if root has not been set, use `root=output/inputfilenameN_`
- if there is an 'xxx.pre' file, read it and store its content.
- if one or two files were read, merge their contents in a single 'file\_content' structure.
- Finally, initialize all parameters given the input 'file\_content' structure. If its size is null, all parameters take their default values.

### 5.5.2.2 input\_init()

```
int input_init (
    struct file_content * pfc,
    struct precision * ppr,
    struct background * pba,
    struct thermo * pth,
    struct perturbs * ppt,
    struct transfers * ptr,
    struct primordial * ppm,
    struct spectra * psp,
    struct nonlinear * pnl,
    struct lensing * ple,
    struct output * pop,
    ErrorMessage errmsg )
```

Initialize each parameter, first to its default values, and then from what can be interpreted from the values passed in the input 'file\_content' structure. If its size is null, all parameters keep their default values. These two arrays must contain the strings of names to be searched for and the corresponding new parameter

- Do we need to fix unknown parameters?
- -> `input_auxillary_target_conditions()` takes care of the case where for instance `Omega_dcdm` is set to 0.0.
- case with unknown parameters
- -> go through all cases with unknown parameters:

- -> Read all parameters from tuned pfc
- -> Set status of shooting
- -> Free arrays allocated
- case with no unknown parameters
- -> just read all parameters from input pfc:
- eventually write all the read parameters in a file, unread parameters in another file, and warnings about unread parameters

### 5.5.2.3 input\_read\_parameters()

```
int input_read_parameters (
    struct file_content * pfc,
    struct precision * ppr,
    struct background * pba,
    struct thermo * pth,
    struct perturbs * ppt,
    struct transfers * ptr,
    struct primordial * ppm,
    struct spectra * psp,
    struct nonlinear * pnl,
    struct lensing * ple,
    struct output * pop,
    ErrorMessage errmsg )
```

#### Summary:

- define local variables
- set all parameters (input and precision) to default values
- if entries passed in file\_content structure, carefully read and interpret each of them, and tune the relevant input parameters accordingly

Knowing the gauge from the very beginning is useful (even if this could be a run not requiring perturbations at all: even in that case, knowing the gauge is important e.g. for fixing the sampling in momentum space for non-cold dark matter)

#### (a) background parameters

- scale factor today (arbitrary)
- $h$  (dimensionless) and  $[H_0/c]$  in  $Mpc^{-1} = h/2997.9... = h * 10^5/c$
- $\Omega_{0,g}$  (photons) and  $T_{cmb}$
- $\Omega_{0,g} = \rho_g / \rho_{c0}$ , each of them expressed in  $Kg/m/s^2$
- $\rho_g = (4 \sigma_B / c) T^4$
- $\rho_{c0} = 3c^2 H_0^2 / (8\pi G)$

- Omega\_0\_b (baryons)
- Omega\_0\_ur (ultra-relativistic species / massless neutrino)
- Omega\_0\_cdm (CDM)
- Omega\_0\_dcdm (DCDM)
- Read Omega\_ini\_dcdm or omega\_ini\_dcdm
- Read Gamma in same units as H0, i.e. km/(s Mpc)
- non-cold relics (ncdm)
- Omega\_0\_k (effective fractional density of curvature)
- Set curvature parameter K
- Set curvature sign
- Omega\_0\_lambda (cosmological constant), Omega0\_fld (dark energy fluid), Omega0\_scf (scalar field)
- -> (flag3 == FALSE) || (param3 >= 0.) explained: it means that either we have not read Omega\_scf so we are ignoring it (unlike lambda and fld!) OR we have read it, but it had a positive value and should not be used for filling. We now proceed in two steps: 1) set each Omega0 and add to the total for each specified component. 2) go through the components in order {lambda, fld, scf} and fill using first unspecified component.
- Test that the user have not specified Omega\_scf = -1 but left either Omega\_lambda or Omega\_fld unspecified:
- Read parameters describing scalar field potential
- Assign shooting parameter

(b) assign values to thermodynamics cosmological parameters

- primordial helium fraction
- recombination parameters
- reionization parametrization
- reionization parameters if reio\_parametrization=reio\_camb
- reionization parameters if reio\_parametrization=reio\_bins\_tanh
- reionization parameters if reio\_parametrization=reio\_many\_tanh
- reionization parameters if reio\_parametrization=reio\_many\_tanh
- energy injection parameters from CDM annihilation/decay

(c) define which perturbations and sources should be computed, and down to which scale

(d) define the primordial spectrum

(e) parameters for final spectra

(f) parameter related to the non-linear spectra computation

(g) amount of information sent to standard output (none if all set to zero)

(h) all precision parameters

- (h.1.) parameters related to the background
- (h.2.) parameters related to the thermodynamics
- (h.3.) parameters related to the perturbations
- —> Include ur and ncdm shear in tensor computation?
- —> derivatives of baryon sound speed only computed if some non-minimal tight-coupling schemes is requested
- (h.4.) parameter related to the primordial spectra
- (h.5.) parameter related to the transfer functions
- (h.6.) parameters related to nonlinear calculations
- (h.7.) parameter related to lensing

## (i) Write values in file

- (i.1.) shall we write background quantities in a file?
- (i.2.) shall we write thermodynamics quantities in a file?
- (i.3.) shall we write perturbation quantities in files?
- (i.4.) shall we write primordial spectra in a file?

## 5.5.2.4 input\_default\_params()

```
int input_default_params (
    struct background * pba,
    struct thermo * pth,
    struct perturbs * ppt,
    struct transfers * ptr,
    struct primordial * ppm,
    struct spectra * psp,
    struct nonlinear * pnl,
    struct lensing * ple,
    struct output * pop )
```

All default parameter values (for input parameters)

## Parameters

|            |  |
|------------|--|
| <i>pba</i> | Input: pointer to background structure     |
| <i>pth</i> | Input: pointer to thermodynamics structure |
| <i>ppt</i> | Input: pointer to perturbation structure   |
| <i>ptr</i> | Input: pointer to transfer structure       |
| <i>ppm</i> | Input: pointer to primordial structure     |
| <i>psp</i> | Input: pointer to spectra structure        |
| <i>pnl</i> | Input: pointer to nonlinear structure      |
| <i>ple</i> | Input: pointer to lensing structure        |
| <i>pop</i> | Input: pointer to output structure         |

**Returns**

the error status

Define all default parameter values (for input parameters) for each structure:

- background structure
- thermodynamics structure
- perturbation structure
- primordial structure
- transfer structure
- output structure
- spectra structure
- nonlinear structure
- lensing structure
- nonlinear structure
- all verbose parameters

**5.5.2.5 input\_default\_precision()**

```
int input_default_precision (
    struct precision * ppr )
```

Initialize the precision parameter structure.

All precision parameters used in the other modules are listed here and assigned here a default value.

**Parameters**

|            |  |
|------------|--|
| <i>ppr</i> | Input/Output: a precision_params structure pointer |
|------------|--|

**Returns**

the error status

Initialize precision parameters for different structures:

- parameters related to the background
- parameters related to the thermodynamics
- parameters related to the perturbations
- parameter related to the primordial spectra



- parameter related to the transfer functions
- parameters related to spectra module
- parameters related to nonlinear module
- parameter related to lensing
- automatic estimate of machine precision

#### 5.5.2.6 `get_machine_precision()`

```
int get_machine_precision (
    double * smallest_allowed_variation )
```

Automatically computes the machine precision.

##### Parameters

|                                   |   |
|-----------------------------------|---|
| <i>smallest_allowed_variation</i> | a pointer to the smallest allowed variation |
|-----------------------------------|---|

Returns the smallest allowed variation (minimum epsilon \* *TOLVAR*)

#### 5.5.2.7 `class_fzero_ridder()`

```
int class_fzero_ridder (
    int(*) (double x, void *param, double *y, ErrorMsg error_message) func,
    double x1,
    double x2,
    double xtol,
    void * param,
    double * Fx1,
    double * Fx2,
    double * xzero,
    int * fevals,
    ErrorMsg error_message )
```

Using Ridders' method, return the root of a function *func* known to lie between *x1* and *x2*. The root, returned as *zridder*, will be found to an approximate accuracy *xtol*.

#### 5.5.2.8 `input_try_unknown_parameters()`

```
int input_try_unknown_parameters (
    double * unknown_parameter,
    int unknown_parameters_size,
    void * voidpfzw,
    double * output,
    ErrorMsg errmsg )
```

Summary:

- Call the structures
- Do computations
- In case scalar field is used to fill, `pba->Omega0_scf` is not equal to `pfzw->target_value[i]`.
- Free structures
- Set filecontent to unread

### 5.5.2.9 input\_get\_guess()

```
int input_get_guess (
    double * xguess,
    double * dxdy,
    struct fzerofun_workspace * pfzw,
    ErrorMsg errmsg )
```

#### Summary:

- Here we should write reasonable guesses for the unknown parameters. Also estimate `dxdy`, i.e. how the unknown parameter responds to the known. This can simply be estimated as the derivative of the guess formula.
- Update `pb` to reflect guess
  - This guess is arbitrary, something nice using WKB should be implemented.
- Version 2: use a fit: `xguess[index_guess] = 1.77835*pow(ba.Omega0_scf,-2./7.);`  
`dxdy[index_guess] = -0.5081*pow(ba.Omega0_scf,-9./7.);`
- Version 3: use attractor solution
- This works since correspondence is `Omega_ini_dcdm -> Omega_dcdm` and `omega_ini_dcdm -> omega_dcdm`
- Deallocate everything allocated by `input_read_parameters`

### 5.5.2.10 input\_find\_root()

```
int input_find_root (
    double * xzero,
    int * fevals,
    struct fzerofun_workspace * pfzw,
    ErrorMsg errmsg )
```

#### Summary:

- First we do our guess
- Do linear hunt for boundaries
- root has been bracketed
- Find root using Ridders method. (Exchange for bisection if you are old-school.)

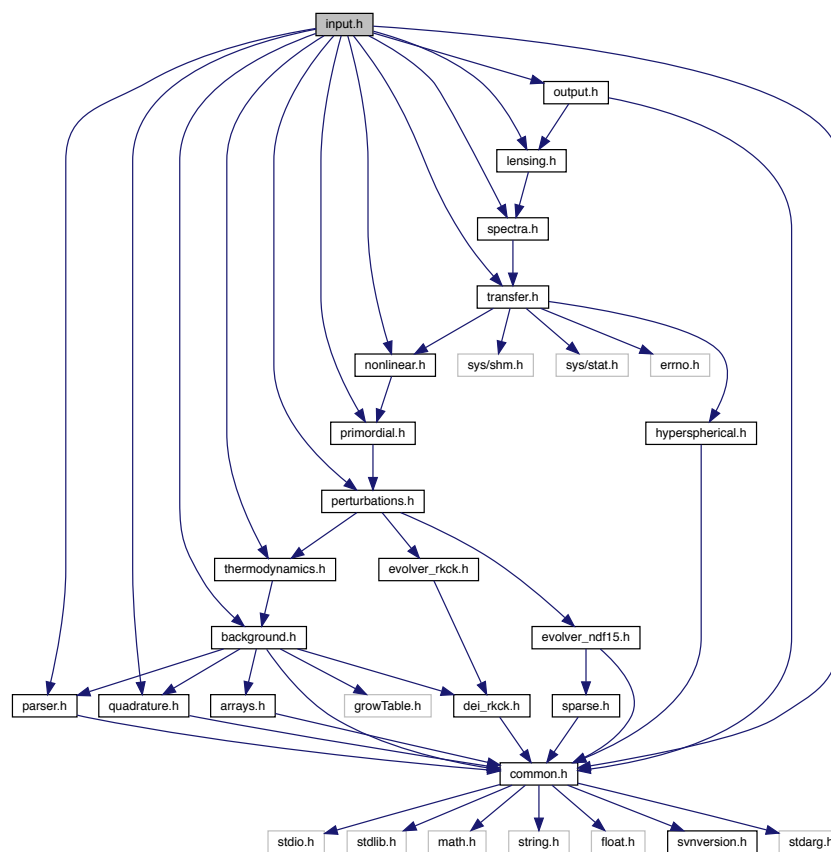
## 5.6 input.h File Reference

```

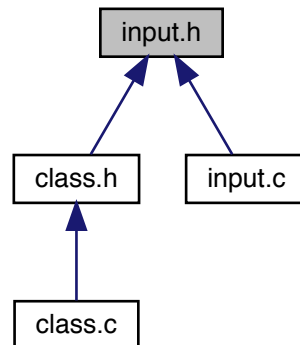
#include "common.h"
#include "parser.h"
#include "quadrature.h"
#include "background.h"
#include "thermodynamics.h"
#include "perturbations.h"
#include "transfer.h"
#include "primordial.h"
#include "spectra.h"
#include "nonlinear.h"
#include "lensing.h"
#include "output.h"

```

Include dependency graph for input.h:



This graph shows which files directly or indirectly include this file:



## Enumerations

- enum [target\\_names](#)

### 5.6.1 Detailed Description

Documented includes for input module

### 5.6.2 Enumeration Type Documentation

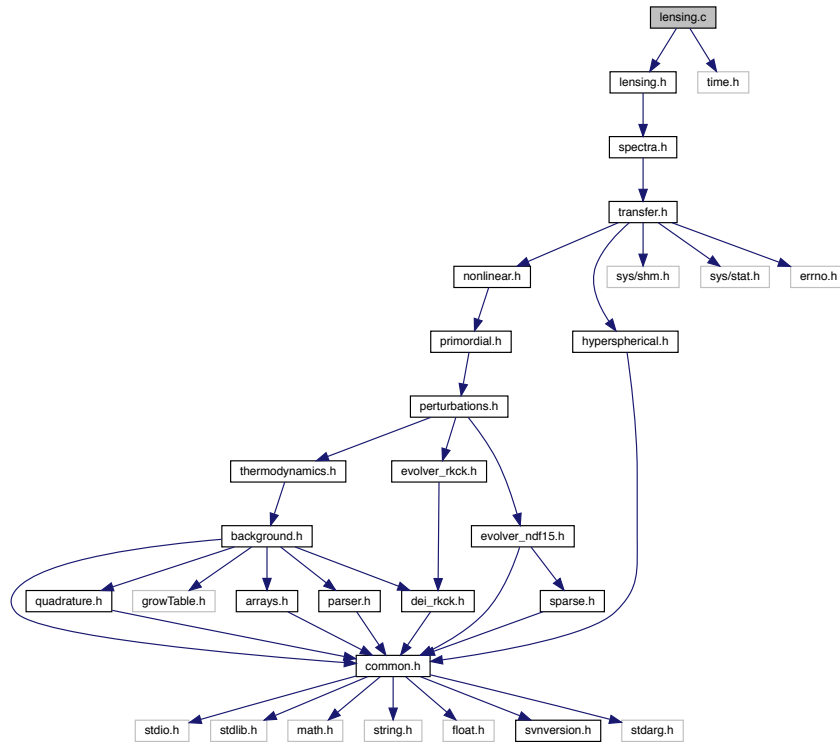
#### 5.6.2.1 target\_names

enum [target\\_names](#)

temporary parameters for background fzero function

## 5.7 lensing.c File Reference

```
#include "lensing.h"
#include <time.h>
Include dependency graph for lensing.c:
```



### Functions

- int [lensing\\_cl\\_at\\_l](#) (struct [lensing](#) \*ple, int l, double \*cl\_lensed)
- int [lensing\\_init](#) (struct [precision](#) \*ppr, struct [perturbs](#) \*ppt, struct [spectra](#) \*psp, struct [nonlinear](#) \*pnl, struct [lensing](#) \*ple)
- int [lensing\\_free](#) (struct [lensing](#) \*ple)
- int [lensing\\_indices](#) (struct [precision](#) \*ppr, struct [spectra](#) \*psp, struct [lensing](#) \*ple)
- int [lensing\\_lensed\\_cl\\_tt](#) (double \*ksi, double \*\*d00, double \*w8, int nm\_u, struct [lensing](#) \*ple)
- int [lensing\\_addback\\_cl\\_tt](#) (struct [lensing](#) \*ple, double \*cl\_tt)
- int [lensing\\_lensed\\_cl\\_te](#) (double \*ksiX, double \*\*d20, double \*w8, int nm\_u, struct [lensing](#) \*ple)
- int [lensing\\_addback\\_cl\\_te](#) (struct [lensing](#) \*ple, double \*cl\_te)
- int [lensing\\_lensed\\_cl\\_ee\\_bb](#) (double \*ksip, double \*ksim, double \*\*d22, double \*\*d2m2, double \*w8, int nm\_u, struct [lensing](#) \*ple)
- int [lensing\\_addback\\_cl\\_ee\\_bb](#) (struct [lensing](#) \*ple, double \*cl\_ee, double \*cl\_bb)
- int [lensing\\_d00](#) (double \*mu, int num\_mu, int lmax, double \*\*d00)
- int [lensing\\_d11](#) (double \*mu, int num\_mu, int lmax, double \*\*d11)
- int [lensing\\_d1m1](#) (double \*mu, int num\_mu, int lmax, double \*\*d1m1)
- int [lensing\\_d2m2](#) (double \*mu, int num\_mu, int lmax, double \*\*d2m2)
- int [lensing\\_d22](#) (double \*mu, int num\_mu, int lmax, double \*\*d22)
- int [lensing\\_d20](#) (double \*mu, int num\_mu, int lmax, double \*\*d20)
- int [lensing\\_d31](#) (double \*mu, int num\_mu, int lmax, double \*\*d31)

- int [lensing\\_d3m1](#) (double \*mu, int num\_mu, int lmax, double \*\*d3m1)
- int [lensing\\_d3m3](#) (double \*mu, int num\_mu, int lmax, double \*\*d3m3)
- int [lensing\\_d40](#) (double \*mu, int num\_mu, int lmax, double \*\*d40)
- int [lensing\\_d4m2](#) (double \*mu, int num\_mu, int lmax, double \*\*d4m2)
- int [lensing\\_d4m4](#) (double \*mu, int num\_mu, int lmax, double \*\*d4m4)

### 5.7.1 Detailed Description

Documented lensing module

Simon Prunet and Julien Lesgourgues, 6.12.2010

This module computes the lensed temperature and polarization anisotropy power spectra  $C_l^X, P(k), \dots$ 's given the unlensed temperature, polarization and lensing potential spectra.

Follows Challinor and Lewis full-sky method, astro-ph/0502425

The following functions can be called from other modules:

1. [lensing\\_init\(\)](#) at the beginning (but after [spectra\\_init\(\)](#))
2. [lensing\\_cl\\_at\\_l\(\)](#) at any time for computing  $Cl_{\text{lensed}}$  at any  $l$
3. [lensing\\_free\(\)](#) at the end

### 5.7.2 Function Documentation

#### 5.7.2.1 [lensing\\_cl\\_at\\_l\(\)](#)

```
int lensing_cl_at_l (
    struct lensing * ple,
    int l,
    double * cl_lensed )
```

Anisotropy power spectra  $C_l$ 's for all types, modes and initial conditions. SO FAR: ONLY SCALAR

This routine evaluates all the lensed  $C_l$ 's at a given value of  $l$  by picking it in the pre-computed table. When relevant, it also sums over all initial conditions for each mode, and over all modes.

This function can be called from whatever module at whatever time, provided that [lensing\\_init\(\)](#) has been called before, and [lensing\\_free\(\)](#) has not been called yet.

#### Parameters

|                  |   |
|------------------|---|
| <i>ple</i>       | Input: pointer to lensing structure                       |
| <i>l</i>         | Input: multipole number                                   |
| <i>cl_lensed</i> | Output: lensed $C_l$ 's for all types (TT, TE, EE, etc..) |

**Returns**

the error status

**5.7.2.2 lensing\_init()**

```
int lensing_init (
    struct precision * ppr,
    struct perturbs * ppt,
    struct spectra * psp,
    struct nonlinear * pnl,
    struct lensing * ple )
```

This routine initializes the lensing structure (in particular, computes table of lensed anisotropy spectra  $C_l^X$ )

**Parameters**

|            |   |
|------------|---|
| <i>ppr</i> | Input: pointer to precision structure   |
| <i>ppt</i> | Input: pointer to perturbation structure (just in case, not used in current version...) |
| <i>psp</i> | Input: pointer to spectra structure   |
| <i>pnl</i> | Input: pointer to nonlinear structure   |
| <i>ple</i> | Output: pointer to initialized lensing structure  |

**Returns**

the error status

**Summary:**

- Define local variables
- check that we really want to compute at least one spectrum
- initialize indices and allocate some of the arrays in the lensing structure
- put all precision variables here; will be stored later in precision structure
- Last element in  $\mu$  will be for  $\mu = 1$ , needed for sigma2. The rest will be chosen as roots of a Gauss-Legendre quadrature
- allocate array of  $\mu$  values, as well as quadrature weights
- Compute  $d_{mm'}^l(\mu)$
- Allocate main contiguous buffer
- compute  $C_{gl}(\mu)$ ,  $C_{gl2}(\mu)$  and  $\text{sigma2}(\mu)$
- Locally store unlensed temperature  $cl_{tt}$  and potential  $cl_{pp}$  spectra
- Compute  $\text{sigma2}(\mu)$  and  $\text{Cgl2}(\mu)$
- compute  $\text{ksi}$ ,  $\text{ksi+}$ ,  $\text{ksi-}$ ,  $\text{ksiX}$
- $\rightarrow$   $\text{ksi}$  is for TT

- -> ksiX is for TE
- -> ksip, ksim for EE, BB
- compute lensed  $C_l$ 's by integration
- spline computed  $C_l$ 's in view of interpolation
- Free lots of stuff
- Exit

### 5.7.2.3 `lensing_free()`

```
int lensing_free (
    struct lensing * ple )
```

This routine frees all the memory space allocated by `lensing_init()`.

To be called at the end of each run, only when no further calls to `lensing_cl_at_l()` are needed.

#### Parameters

|            |  |
|------------|--|
| <i>ple</i> | Input: pointer to lensing structure (which fields must be freed) |
|------------|--|

#### Returns

the error status

### 5.7.2.4 `lensing_indices()`

```
int lensing_indices (
    struct precision * ppr,
    struct spectra * psp,
    struct lensing * ple )
```

This routine defines indices and allocates tables in the lensing structure

#### Parameters

|            |  |
|------------|--|
| <i>ppr</i> | Input: pointer to precision structure      |
| <i>psp</i> | Input: pointer to spectra structure        |
| <i>ple</i> | Input/output: pointer to lensing structure |

#### Returns

the error status



5.7.2.5 `lensing_lensed_cl_tt()`

```
int lensing_lensed_cl_tt (
    double * ksi,
    double ** d00,
    double * w8,
    int nmu,
    struct lensing * ple )
```

This routine computes the lensed power spectra by Gaussian quadrature

**Parameters**

|            |   |
|------------|---|
| <i>ksi</i> | Input: Lensed correlation function (ksi[index_mu])    |
| <i>d00</i> | Input: Legendre polynomials ( $d_{00}^l$ [index_mu])  |
| <i>w8</i>  | Input: Legendre quadrature weights (w8[index_mu])     |
| <i>nmu</i> | Input: Number of quadrature points (0<=index_mu<=nmu) |
| <i>ple</i> | Input/output: Pointer to the lensing structure        |

**Returns**

the error status

Integration by Gauss-Legendre quadrature.

5.7.2.6 `lensing_addback_cl_tt()`

```
int lensing_addback_cl_tt (
    struct lensing * ple,
    double * cl_tt )
```

This routine adds back the unlensed  $cl_{tt}$  power spectrum Used in case of fast (and BB inaccurate) integration of correlation functions.

**Parameters**

|                        |  |
|------------------------|--|
| <i>ple</i>             | Input/output: Pointer to the lensing structure |
| <i>cl<sub>tt</sub></i> | Input: Array of unlensed power spectrum        |

**Returns**

the error status

### 5.7.2.7 `lensing_lensed_cl_te()`

```
int lensing_lensed_cl_te (
    double * ksiX,
    double ** d20,
    double * w8,
    int nmU,
    struct lensing * ple )
```

This routine computes the lensed power spectra by Gaussian quadrature

#### Parameters

|                   |  |
|-------------------|--|
| <code>ksiX</code> | Input: Lensed correlation function ( <code>ksiX[index_mu]</code> )               |
| <code>d20</code>  | Input: Wigner d-function ( <code>d<sub>20,l</sub>[l][index_mu]</code> )          |
| <code>w8</code>   | Input: Legendre quadrature weights ( <code>w8[index_mu]</code> )                 |
| <code>nmU</code>  | Input: Number of quadrature points ( $0 \leq \text{index\_mu} \leq \text{nmU}$ ) |
| <code>ple</code>  | Input/output: Pointer to the lensing structure                                   |

#### Returns

the error status

Integration by Gauss-Legendre quadrature.

### 5.7.2.8 `lensing_addback_cl_te()`

```
int lensing_addback_cl_te (
    struct lensing * ple,
    double * cl_te )
```

This routine adds back the unlensed  $cl_{te}$  power spectrum Used in case of fast (and BB inaccurate) integration of correlation functions.

#### Parameters

|                              |  |
|------------------------------|--|
| <code>ple</code>             | Input/output: Pointer to the lensing structure |
| <code>cl<sub>te</sub></code> | Input: Array of unlensed power spectrum        |

#### Returns

the error status

### 5.7.2.9 `lensing_lensed_cl_ee_bb()`

```
int lensing_lensed_cl_ee_bb (
    double * ksip,
```

```

double * ksim,
double ** d22,
double ** d2m2,
double * w8,
int nmu,
struct lensing * ple )

```

This routine computes the lensed power spectra by Gaussian quadrature

#### Parameters

|             |  |
|-------------|--|
| <i>ksip</i> | Input: Lensed correlation function ( $\kappa_{i+[\text{index\_mu}]}$ )           |
| <i>ksim</i> | Input: Lensed correlation function ( $\kappa_{i-[\text{index\_mu}]}$ )           |
| <i>d22</i>  | Input: Wigner d-function ( $d_{22}^l[l][\text{index\_mu}]$ )                     |
| <i>d2m2</i> | Input: Wigner d-function ( $d_{2-2}^l[l][\text{index\_mu}]$ )                    |
| <i>w8</i>   | Input: Legendre quadrature weights ( $w8[\text{index\_mu}]$ )                    |
| <i>nmu</i>  | Input: Number of quadrature points ( $0 \leq \text{index\_mu} \leq \text{nmu}$ ) |
| <i>ple</i>  | Input/output: Pointer to the lensing structure                                   |

#### Returns

the error status

Integration by Gauss-Legendre quadrature.

#### 5.7.2.10 `lensing_addback_cl_ee_bb()`

```

int lensing_addback_cl_ee_bb (
    struct lensing * ple,
    double * cl_ee,
    double * cl_bb )

```

This routine adds back the unlensed  $cl_{ee}$ ,  $cl_{bb}$  power spectra Used in case of fast (and BB inaccurate) integration of correlation functions.

#### Parameters

|              |  |
|--------------|--|
| <i>ple</i>   | Input/output: Pointer to the lensing structure |
| <i>cl_ee</i> | Input: Array of unlensed power spectrum        |
| <i>cl_bb</i> | Input: Array of unlensed power spectrum        |

#### Returns

the error status

#### 5.7.2.11 `lensing_d00()`

```

int lensing_d00 (
    double * mu,

```

```

int num_mu,
int lmax,
double ** d00 )

```

This routine computes the d00 term

#### Parameters

|               |                                     |
|---------------|-------------------------------------|
| <i>mu</i>     | Input: Vector of cos(beta) values   |
| <i>num_mu</i> | Input: Number of cos(beta) values   |
| <i>lmax</i>   | Input: maximum multipole            |
| <i>d00</i>    | Input/output: Result is stored here |

Wigner d-functions, computed by recurrence actual recurrence on  $\sqrt{(2l+1)/2}d_{mm}^l$ , for stability Formulae from Kostelec & Rockmore 2003

#### 5.7.2.12 lensing\_d11()

```

int lensing_d11 (
    double * mu,
    int num_mu,
    int lmax,
    double ** d11 )

```

This routine computes the d11 term

#### Parameters

|               |                                     |
|---------------|-------------------------------------|
| <i>mu</i>     | Input: Vector of cos(beta) values   |
| <i>num_mu</i> | Input: Number of cos(beta) values   |
| <i>lmax</i>   | Input: maximum multipole            |
| <i>d11</i>    | Input/output: Result is stored here |

Wigner d-functions, computed by recurrence actual recurrence on  $\sqrt{(2l+1)/2}d_{mm}^l$ , for stability Formulae from Kostelec & Rockmore 2003

#### 5.7.2.13 lensing\_d1m1()

```

int lensing_d1m1 (
    double * mu,
    int num_mu,
    int lmax,
    double ** d1m1 )

```

This routine computes the d1m1 term

#### Parameters

|               |                                     |
|---------------|-------------------------------------|
| <i>mu</i>     | Input: Vector of cos(beta) values   |
| <i>num_mu</i> | Input: Number of cos(beta) values   |
| <i>lmax</i>   | Input: maximum multipole            |
| <i>d1m1</i>   | Input/output: Result is stored here |

Wigner d-functions, computed by recurrence actual recurrence on  $\sqrt{(2l+1)/2}d_{mm}^l$ , for stability Formulae from Kostelec & Rockmore 2003

#### 5.7.2.14 lensing\_d2m2()

```
int lensing_d2m2 (
    double * mu,
    int num_mu,
    int lmax,
    double ** d2m2 )
```

This routine computes the d2m2 term

##### Parameters

|               |                                     |
|---------------|-------------------------------------|
| <i>mu</i>     | Input: Vector of cos(beta) values   |
| <i>num_mu</i> | Input: Number of cos(beta) values   |
| <i>lmax</i>   | Input: maximum multipole            |
| <i>d2m2</i>   | Input/output: Result is stored here |

Wigner d-functions, computed by recurrence actual recurrence on  $\sqrt{(2l+1)/2}d_{mm}^l$ , for stability Formulae from Kostelec & Rockmore 2003

#### 5.7.2.15 lensing\_d22()

```
int lensing_d22 (
    double * mu,
    int num_mu,
    int lmax,
    double ** d22 )
```

This routine computes the d22 term

##### Parameters

|               |                                     |
|---------------|-------------------------------------|
| <i>mu</i>     | Input: Vector of cos(beta) values   |
| <i>num_mu</i> | Input: Number of cos(beta) values   |
| <i>lmax</i>   | Input: maximum multipole            |
| <i>d22</i>    | Input/output: Result is stored here |

Wigner d-functions, computed by recurrence actual recurrence on  $\sqrt{(2l+1)/2}d_{mm}^l$ , for stability Formulae from Kostelec & Rockmore 2003

#### 5.7.2.16 lensing\_d20()

```
int lensing_d20 (
    double * mu,
    int num_mu,
```

```

int lmax,
double ** d20 )

```

This routine computes the d20 term

#### Parameters

|               |                                     |
|---------------|-------------------------------------|
| <i>mu</i>     | Input: Vector of cos(beta) values   |
| <i>num_mu</i> | Input: Number of cos(beta) values   |
| <i>lmax</i>   | Input: maximum multipole            |
| <i>d20</i>    | Input/output: Result is stored here |

Wigner d-functions, computed by recurrence actual recurrence on  $\sqrt{(2l+1)/2}d_{mm}^l$ , for stability Formulae from Kostelec & Rockmore 2003

#### 5.7.2.17 lensing\_d31()

```

int lensing_d31 (
    double * mu,
    int num_mu,
    int lmax,
    double ** d31 )

```

This routine computes the d31 term

#### Parameters

|               |                                     |
|---------------|-------------------------------------|
| <i>mu</i>     | Input: Vector of cos(beta) values   |
| <i>num_mu</i> | Input: Number of cos(beta) values   |
| <i>lmax</i>   | Input: maximum multipole            |
| <i>d31</i>    | Input/output: Result is stored here |

Wigner d-functions, computed by recurrence actual recurrence on  $\sqrt{(2l+1)/2}d_{mm}^l$ , for stability Formulae from Kostelec & Rockmore 2003

#### 5.7.2.18 lensing\_d3m1()

```

int lensing_d3m1 (
    double * mu,
    int num_mu,
    int lmax,
    double ** d3m1 )

```

This routine computes the d3m1 term

#### Parameters

|               |                                     |
|---------------|-------------------------------------|
| <i>mu</i>     | Input: Vector of cos(beta) values   |
| <i>num_mu</i> | Input: Number of cos(beta) values   |
| <i>lmax</i>   | Input: maximum multipole            |
| <i>d3m1</i>   | Input/output: Result is stored here |

Wigner d-functions, computed by recurrence actual recurrence on  $\sqrt{(2l+1)/2}d_{mm}^l$ , for stability Formulae from Kostelec & Rockmore 2003

#### 5.7.2.19 lensing\_d3m3()

```
int lensing_d3m3 (
    double * mu,
    int num_mu,
    int lmax,
    double ** d3m3 )
```

This routine computes the d3m3 term

##### Parameters

|               |                                     |
|---------------|-------------------------------------|
| <i>mu</i>     | Input: Vector of cos(beta) values   |
| <i>num_mu</i> | Input: Number of cos(beta) values   |
| <i>lmax</i>   | Input: maximum multipole            |
| <i>d3m3</i>   | Input/output: Result is stored here |

Wigner d-functions, computed by recurrence actual recurrence on  $\sqrt{(2l+1)/2}d_{mm}^l$ , for stability Formulae from Kostelec & Rockmore 2003

#### 5.7.2.20 lensing\_d40()

```
int lensing_d40 (
    double * mu,
    int num_mu,
    int lmax,
    double ** d40 )
```

This routine computes the d40 term

##### Parameters

|               |                                     |
|---------------|-------------------------------------|
| <i>mu</i>     | Input: Vector of cos(beta) values   |
| <i>num_mu</i> | Input: Number of cos(beta) values   |
| <i>lmax</i>   | Input: maximum multipole            |
| <i>d40</i>    | Input/output: Result is stored here |

Wigner d-functions, computed by recurrence actual recurrence on  $\sqrt{(2l+1)/2}d_{mm}^l$ , for stability Formulae from Kostelec & Rockmore 2003

#### 5.7.2.21 lensing\_d4m2()

```
int lensing_d4m2 (
    double * mu,
    int num_mu,
```

```

int lmax,
double ** d4m2 )

```

This routine computes the d4m2 term

#### Parameters

|               |                                     |
|---------------|-------------------------------------|
| <i>mu</i>     | Input: Vector of cos(beta) values   |
| <i>num_mu</i> | Input: Number of cos(beta) values   |
| <i>lmax</i>   | Input: maximum multipole            |
| <i>d4m2</i>   | Input/output: Result is stored here |

Wigner d-functions, computed by recurrence actual recurrence on  $\sqrt{(2l+1)/2}d_{mm}^l$ , for stability Formulae from Kostelec & Rockmore 2003

#### 5.7.2.22 lensing\_d4m4()

```

int lensing_d4m4 (
    double * mu,
    int num_mu,
    int lmax,
    double ** d4m4 )

```

This routine computes the d4m4 term

#### Parameters

|               |                                     |
|---------------|-------------------------------------|
| <i>mu</i>     | Input: Vector of cos(beta) values   |
| <i>num_mu</i> | Input: Number of cos(beta) values   |
| <i>lmax</i>   | Input: maximum multipole            |
| <i>d4m4</i>   | Input/output: Result is stored here |

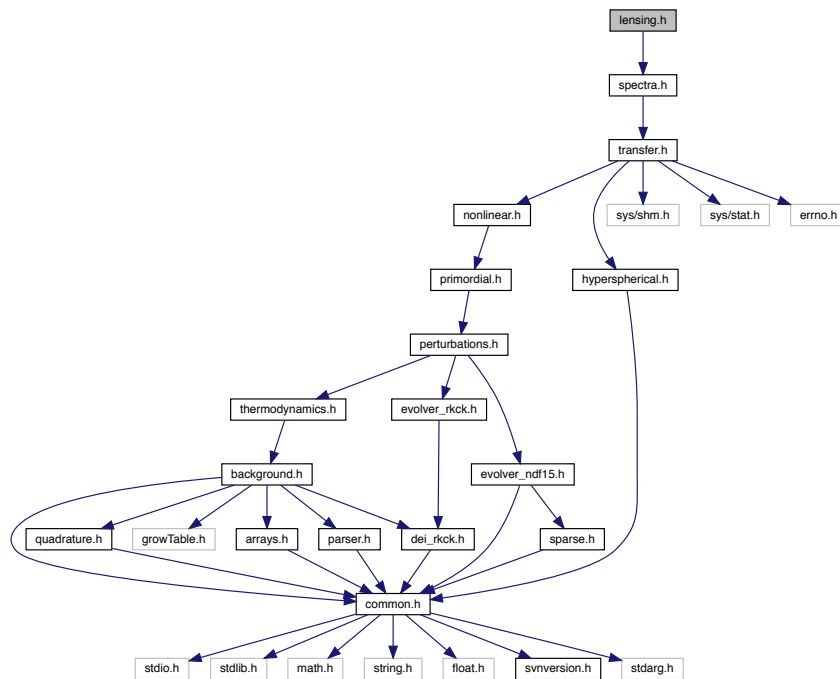
Wigner d-functions, computed by recurrence actual recurrence on  $\sqrt{(2l+1)/2}d_{mm}^l$ , for stability Formulae from Kostelec & Rockmore 2003

## 5.8 lensing.h File Reference

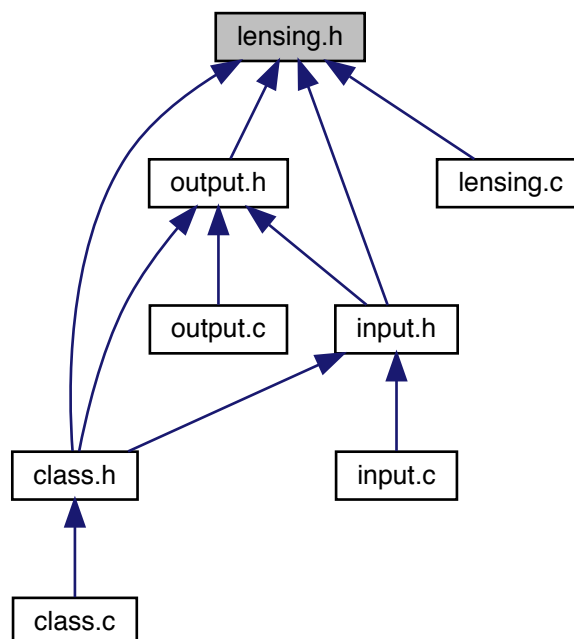
```
#include "spectra.h"
```



Include dependency graph for lensing.h:



This graph shows which files directly or indirectly include this file:



## Data Structures

- struct [lensing](#)

### 5.8.1 Detailed Description

Documented includes for spectra module

### 5.8.2 Data Structure Documentation

#### 5.8.2.1 struct lensing

Structure containing everything about lensed spectra that other modules need to know.

Once initialized by [lensing\\_init\(\)](#), contains a table of all lensed  $C_l$ 's for the all modes (scalar/tensor), all types (TT, TE...), and all pairs of initial conditions (adiabatic, isocurvatures...). FOR THE MOMENT, ASSUME ONLY SCALAR & ADIABATIC

#### Data Fields

|       |                |   |
|-------|----------------|---|
| short | has_lensed_cls | do we need to compute lensed $C_l$ 's at all ?                  |
| int   | has_tt         | do we want lensed $C_l^{TT}$ ? (T = temperature)                |
| int   | has_ee         | do we want lensed $C_l^{EE}$ ? (E = E-polarization)             |
| int   | has_te         | do we want lensed $C_l^{TE}$ ?                                  |
| int   | has_bb         | do we want $C_l^{BB}$ ? (B = B-polarization)                    |
| int   | has_pp         | do we want $C_l^{\phi\phi}$ ? ( $\phi$ = CMB lensing potential) |
| int   | has_tp         | do we want $C_l^{T\phi}$ ?                                      |
| int   | has_dd         | do we want $C_l^{dd}$ ? (d = matter density)                    |
| int   | has_td         | do we want $C_l^{Td}$ ?   |
| int   | has_ll         | do we want $C_l^{ll}$ ? (l = lensing potential)                 |
| int   | has_tl         | do we want $C_l^{Tl}$ ?   |
| int   | index_lt_tt    | index for type $C_l^{TT}$                                       |
| int   | index_lt_ee    | index for type $C_l^{EE}$                                       |
| int   | index_lt_te    | index for type $C_l^{TE}$                                       |
| int   | index_lt_bb    | index for type $C_l^{BB}$                                       |
| int   | index_lt_pp    | index for type $C_l^{\phi\phi}$                                 |
| int   | index_lt_tp    | index for type $C_l^{T\phi}$                                    |
| int   | index_lt_dd    | index for type $C_l^{dd}$                                       |
| int   | index_lt_td    | index for type $C_l^{Td}$                                       |
| int   | index_lt_ll    | index for type $C_l^{ll}$                                       |
| int   | index_lt_tl    | index for type $C_l^{Tl}$                                       |
| int   | lt_size        | number of $C_l$ types requested                                 |
| int   | l_unlensed_max | last multipole in all calculations (same as in spectra module)  |
| int   | l_lensed_max   | last multipole at which lensed spectra are computed             |

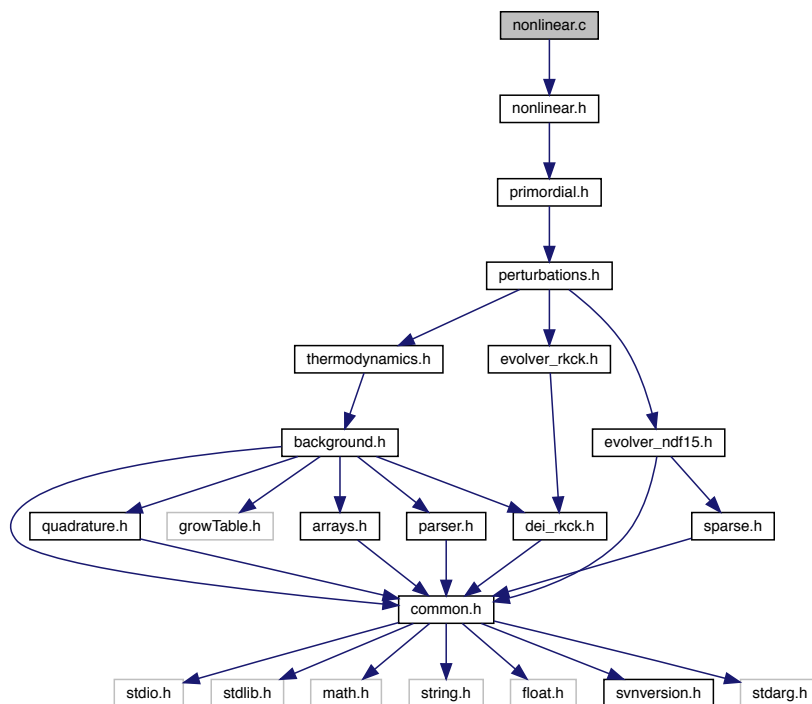
## Data Fields

|          |                 |   |
|----------|-----------------|---|
| int      | l_size          | number of l values  |
| int *    | l_max_lt        | last multipole (given as an input) at which we want to output $C_l$ 's for a given mode and type                |
| double * | l               | table of multipole values $l[index\_l]$   |
| double * | cl_lens         | table of anisotropy spectra for each multipole and types, $cl[index\_l * ple \rightarrow lt\_size + index\_lt]$ |
| double * | ddcl_lens       | second derivatives for interpolation  |
| short    | lensing_verbose | flag regulating the amount of information sent to standard output (none if set to zero)                         |
| ErrorMsg | error_message   | zone for writing error messages   |

## 5.9 nonlinear.c File Reference

```
#include "nonlinear.h"
```

Include dependency graph for nonlinear.c:



## Functions

- int `nonlinear_init` (struct `precision` \*ppr, struct `background` \*pba, struct `thermo` \*pth, struct `perturbs` \*ppt, struct `primordial` \*ppm, struct `nonlinear` \*pnl)
- int `nonlinear_halofit` (struct `precision` \*ppr, struct `background` \*pba, struct `primordial` \*ppm, struct `nonlinear` \*pnl, double tau, double \*pk\_l, double \*pk\_nl, double \*lnk\_l, double \*lnpk\_l, double \*ddlmpk\_l, double \*k\_nl)

## 5.9.1 Detailed Description

Documented nonlinear module

Julien Lesgourgues, 6.03.2014

New module replacing an older one present up to version 2.0 The new module is located in a better place in the main, allowing it to compute non-linear correction to  $C_l$ 's and not just  $P(k)$ . It will also be easier to generalize to new methods. The old implementation of one-loop calculations and TRG calculations has been dropped from this version, they can still be found in older versions.

## 5.9.2 Function Documentation

### 5.9.2.1 nonlinear\_init()

```
int nonlinear_init (
    struct precision * ppr,
    struct background * pba,
    struct thermo * pth,
    struct perturbs * ppt,
    struct primordial * ppm,
    struct nonlinear * pnl )
```

Summary

(a) First deal with the case where non non-linear corrections requested

(b) Compute for HALOFIT non-linear spectrum

- copy list of (k,tau) from perturbation module
- loop over time

### 5.9.2.2 nonlinear\_halofit()

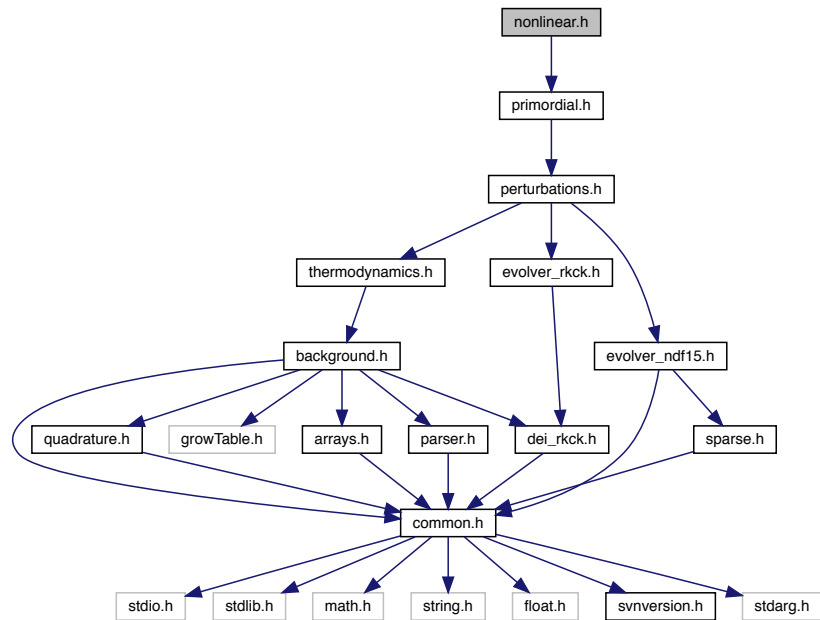
```
int nonlinear_halofit (
    struct precision * ppr,
    struct background * pba,
    struct primordial * ppm,
    struct nonlinear * pnl,
    double tau,
    double * pk_l,
    double * pk_nl,
    double * lnk_l,
    double * lnpk_l,
    double * ddlnpk_l,
    double * k_nl )
```

Determine non linear ratios (from pk)

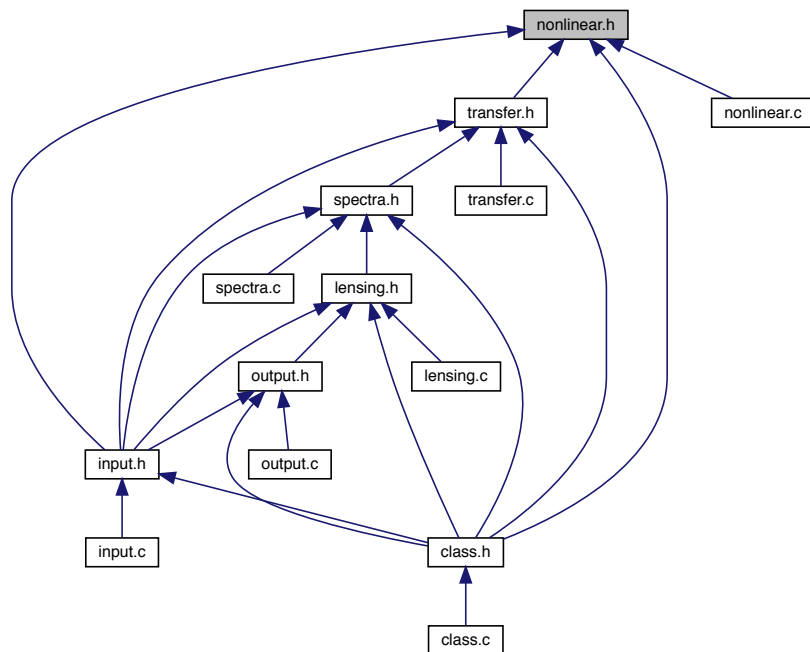
## 5.10 nonlinear.h File Reference

```
#include "primordial.h"
```

Include dependency graph for nonlinear.h:



This graph shows which files directly or indirectly include this file:



## Data Structures

- struct [nonlinear](#)

## Macros

- `#define \_M\_EV\_TOO\_BIG\_FOR\_HALOFIT\_ 10.`

### 5.10.1 Detailed Description

Documented includes for `trg` module

### 5.10.2 Macro Definition Documentation

#### 5.10.2.1 `_M_EV_TOO_BIG_FOR_HALOFIT_`

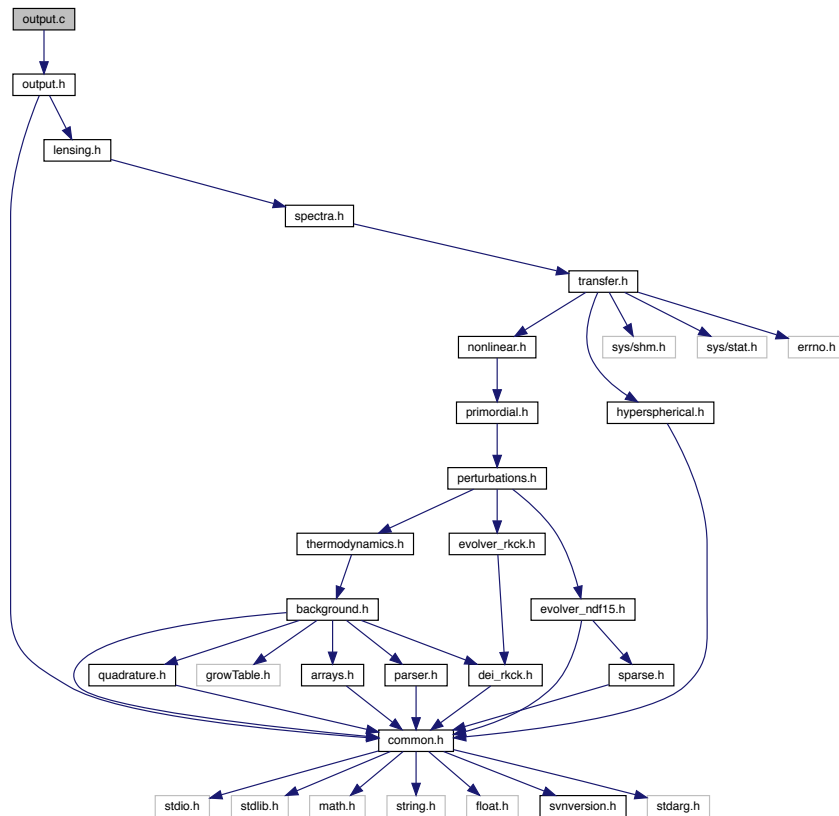
```
#define _M_EV_TOO_BIG_FOR_HALOFIT_ 10.
```

above which value of non-CDM mass (in eV) do we stop trusting halofit?

## 5.11 `output.c` File Reference

```
#include "output.h"
```

Include dependency graph for `output.c`:



## Functions

- int `output_init` (struct `background` \*pba, struct `thermo` \*pth, struct `perturbs` \*ppt, struct `primordial` \*ppm, struct `transfers` \*ptr, struct `spectra` \*psp, struct `nonlinear` \*pnl, struct `lensing` \*ple, struct `output` \*pop)
- int `output_cl` (struct `background` \*pba, struct `perturbs` \*ppt, struct `spectra` \*psp, struct `lensing` \*ple, struct `output` \*pop)
- int `output_pk` (struct `background` \*pba, struct `perturbs` \*ppt, struct `spectra` \*psp, struct `output` \*pop)
- int `output_pk_nl` (struct `background` \*pba, struct `perturbs` \*ppt, struct `spectra` \*psp, struct `output` \*pop)
- int `output_tk` (struct `background` \*pba, struct `perturbs` \*ppt, struct `spectra` \*psp, struct `output` \*pop)
- int `output_print_data` (FILE \*out, char titles[\_MAXTITLESTRINGLENGTH\_], double \*dataptr, int size\_↵ dataptr)
- int `output_open_cl_file` (struct `spectra` \*psp, struct `output` \*pop, FILE \*\*clfile, FileName filename, char \*first\_↵ \_line, int lmax)
- int `output_one_line_of_cl` (struct `background` \*pba, struct `spectra` \*psp, struct `output` \*pop, FILE \*clfile, double l, double \*cl, int ct\_size)
- int `output_open_pk_file` (struct `background` \*pba, struct `spectra` \*psp, struct `output` \*pop, FILE \*\*pkfile, File\_↵ Name filename, char \*first\_line, double z)
- int `output_one_line_of_pk` (FILE \*pkfile, double one\_k, double one\_pk)

### 5.11.1 Detailed Description

Documented output module

Julien Lesgourgues, 26.08.2010

This module writes the output in files.

The following functions can be called from other modules or from the main:

1. `output_init()` (must be called after `spectra_init()`)
2. `output_total_cl_at_l()` (can be called even before `output_init()`)

No memory needs to be deallocated after that, hence there is no `output_free()` routine like in other modules.

### 5.11.2 Function Documentation

#### 5.11.2.1 `output_init()`

```
int output_init (
    struct background * pba,
    struct thermo * pth,
    struct perturbs * ppt,
    struct primordial * ppm,
    struct transfers * ptr,
    struct spectra * psp,
    struct nonlinear * pnl,
    struct lensing * ple,
    struct output * pop )
```

This routine writes the output in files.

## Parameters

|            |  |
|------------|--|
| <i>pba</i> | Input: pointer to background structure (needed for calling <a href="#">spectra_pk_at_z()</a> ) |
| <i>pth</i> | Input: pointer to thermodynamics structure   |
| <i>ppt</i> | Input: pointer perturbation structure  |
| <i>ppm</i> | Input: pointer to primordial structure   |
| <i>ptr</i> | Input: pointer to transfer structure   |
| <i>psp</i> | Input: pointer to spectra structure  |
| <i>pnl</i> | Input: pointer to nonlinear structure  |
| <i>ple</i> | Input: pointer to lensing structure  |
| <i>pop</i> | Input: pointer to output structure   |

## Summary:

- check that we really want to output at least one file
- deal with all anisotropy power spectra  $C_l$ 's
- deal with all Fourier matter power spectra  $P(k)$ 's
- deal with density and matter power spectra
- deal with background quantities
- deal with thermodynamics quantities
- deal with perturbation quantities
- deal with primordial spectra

5.11.2.2 `output_cl()`

```
int output_cl (
    struct background * pba,
    struct perturbs * ppt,
    struct spectra * psp,
    struct lensing * ple,
    struct output * pop )
```

This routines writes the output in files for anisotropy power spectra  $C_l$ 's.

## Parameters

|            |  |
|------------|--|
| <i>pba</i> | Input: pointer to background structure (needed for $T_{cmb}$ ) |
| <i>ppt</i> | Input: pointer perturbation structure                          |
| <i>psp</i> | Input: pointer to spectra structure                            |
| <i>ple</i> | Input: pointer to lensing structure                            |
| <i>pop</i> | Input: pointer to output structure                             |

## Summary:



- define local variables
- first, allocate all arrays of files and  $C_l$ 's
- second, open only the relevant files, and write a heading in each of them
- third, perform loop over  $l$ . For each multipole, get all  $C_l$ 's by calling [spectra\\_cl\\_at\\_l\(\)](#) and distribute the results to relevant files
- finally, close files and free arrays of files and  $C_l$ 's

### 5.11.2.3 output\_pk()

```
int output_pk (
    struct background * pba,
    struct perturbs * ppt,
    struct spectra * psp,
    struct output * pop )
```

This routine writes the output in files for Fourier matter power spectra  $P(k)$ 's.

#### Parameters

|            |  |
|------------|--|
| <i>pba</i> | Input: pointer to background structure (needed for calling <a href="#">spectra_pk_at_z()</a> ) |
| <i>ppt</i> | Input: pointer perturbation structure  |
| <i>psp</i> | Input: pointer to spectra structure  |
| <i>pop</i> | Input: pointer to output structure   |

#### Summary:

- define local variables
- first, check that requested redshift  $z_{pk}$  is consistent
- second, open only the relevant files and write a heading in each of them
- third, compute  $P(k)$  for each  $k$  (if several  $ic$ 's, compute it for each  $ic$  and compute also the total); if  $z_{pk} = 0$ , this is done by directly reading inside the pre-computed table; if not, this is done by interpolating the table at the correct value of  $\tau$ .
- fourth, write in files
- fifth, free memory and close files

### 5.11.2.4 output\_pk\_nl()

```
int output_pk_nl (
    struct background * pba,
    struct perturbs * ppt,
    struct spectra * psp,
    struct output * pop )
```

This routine writes the output in files for Fourier non-linear matter power spectra  $P(k)$ 's.

## Parameters

|            |  |
|------------|--|
| <i>pba</i> | Input: pointer to background structure (needed for calling <a href="#">spectra_pk_at_z()</a> ) |
| <i>ppt</i> | Input: pointer perturbation structure  |
| <i>psp</i> | Input: pointer to spectra structure  |
| <i>pop</i> | Input: pointer to output structure   |

## Summary:

- define local variables
- first, check that requested redshift  $z_{pk}$  is consistent
- second, open only the relevant files, and write a heading in each of them
- third, compute  $P(k)$  for each  $k$  (if several  $ic$ 's, compute it for each  $ic$  and compute also the total); if  $z_{pk} = 0$ , this is done by directly reading inside the pre-computed table; if not, this is done by interpolating the table at the correct value of  $\tau$ .
- fourth, write in files
- fifth, free memory and close files

5.11.2.5 `output_tk()`

```
int output_tk (
    struct background * pba,
    struct perturbs * ppt,
    struct spectra * psp,
    struct output * pop )
```

This routines writes the output in files for matter transfer functions  $T_i(k)$ 's.

## Parameters

|            |  |
|------------|--|
| <i>pba</i> | Input: pointer to background structure (needed for calling <a href="#">spectra_pk_at_z()</a> ) |
| <i>ppt</i> | Input: pointer perturbation structure  |
| <i>psp</i> | Input: pointer to spectra structure  |
| <i>pop</i> | Input: pointer to output structure   |

## Summary:

- define local variables
- first, check that requested redshift  $z_{pk}$  is consistent
- second, open only the relevant files, and write a heading in each of them
- free memory and close files

## 5.11.2.6 output\_print\_data()

```
int output_print_data (
    FILE * out,
    char titles[_MAXTITLESTRINGLENGTH_],
    double * dataptr,
    int size_dataptr )
```

## Summary

- First we print the titles
- Then we print the data

## 5.11.2.7 output\_open\_cl\_file()

```
int output_open_cl_file (
    struct spectra * psp,
    struct output * pop,
    FILE ** clfile,
    FileName filename,
    char * first_line,
    int lmax )
```

This routine opens one file where some  $C_l$ 's will be written, and writes a heading with some general information concerning its content.

## Parameters

|                   |  |
|-------------------|--|
| <i>psp</i>        | Input: pointer to spectra structure                                  |
| <i>pop</i>        | Input: pointer to output structure                                   |
| <i>clfile</i>     | Output: returned pointer to file pointer                             |
| <i>filename</i>   | Input: name of the file  |
| <i>first_line</i> | Input: text describing the content (mode, initial condition..)       |
| <i>lmax</i>       | Input: last multipole in the file (the first one is assumed to be 2) |

## Returns

the error status

## Summary

- First we deal with the entries that are dependent of format type
- Next deal with entries that are independent of format type

### 5.11.2.8 output\_one\_line\_of\_cl()

```
int output_one_line_of_cl (
    struct background * pba,
    struct spectra * psp,
    struct output * pop,
    FILE * clfile,
    double l,
    double * cl,
    int ct_size )
```

This routine write one line with  $l$  and all  $C_l$ 's for all types (TT, TE...)

#### Parameters

|                |  |
|----------------|--|
| <i>pba</i>     | Input: pointer to background structure (needed for $T_{cmb}$ ) |
| <i>psp</i>     | Input: pointer to spectra structure                            |
| <i>pop</i>     | Input: pointer to output structure                             |
| <i>clfile</i>  | Input: file pointer  |
| <i>l</i>       | Input: multipole   |
| <i>cl</i>      | Input: $C_l$ 's for all types                                  |
| <i>ct_size</i> | Input: number of types   |

#### Returns

the error status

### 5.11.2.9 output\_open\_pk\_file()

```
int output_open_pk_file (
    struct background * pba,
    struct spectra * psp,
    struct output * pop,
    FILE ** pkfile,
    FileName filename,
    char * first_line,
    double z )
```

This routine opens one file where some  $P(k)$ 's will be written, and writes a heading with some general information concerning its content.

#### Parameters

|                   |  |
|-------------------|--|
| <i>pba</i>        | Input: pointer to background structure (needed for h)        |
| <i>psp</i>        | Input: pointer to spectra structure                          |
| <i>pop</i>        | Input: pointer to output structure                           |
| <i>pkfile</i>     | Output: returned pointer to file pointer                     |
| <i>filename</i>   | Input: name of the file                                      |
| <i>first_line</i> | Input: text describing the content (initial conditions, ...) |
| <i>z</i>          | Input: redshift of the output                                |

**Returns**

the error status

**5.11.2.10 output\_one\_line\_of\_pk()**

```
int output_one_line_of_pk (
    FILE * pkfile,
    double one_k,
    double one_pk )
```

This routine writes one line with  $k$  and  $P(k)$

**Parameters**

|               |                              |
|---------------|------------------------------|
| <i>pkfile</i> | Input: file pointer          |
| <i>one_k</i>  | Input: wavenumber            |
| <i>one_pk</i> | Input: matter power spectrum |

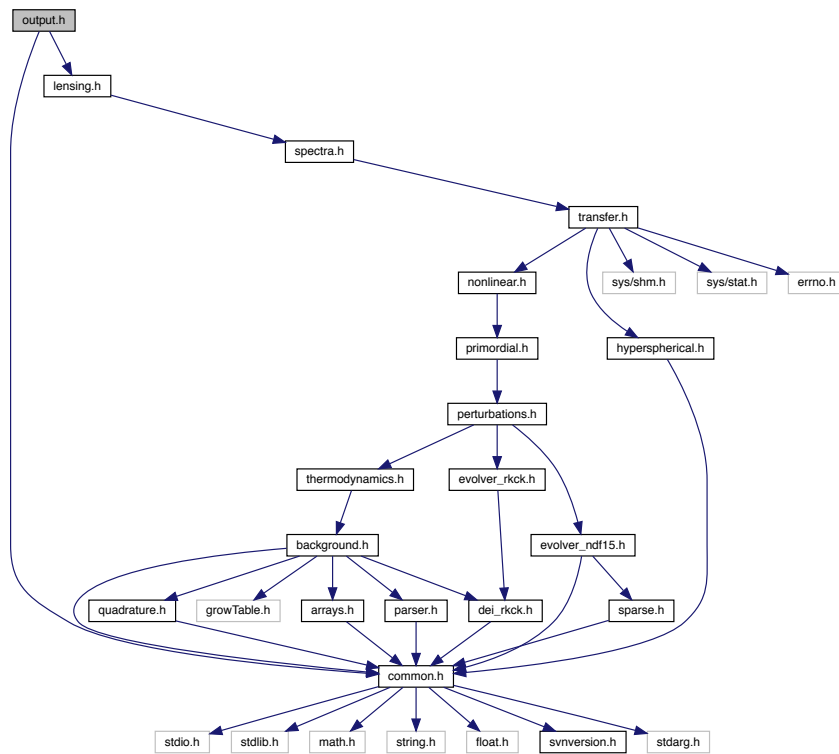
**Returns**

the error status

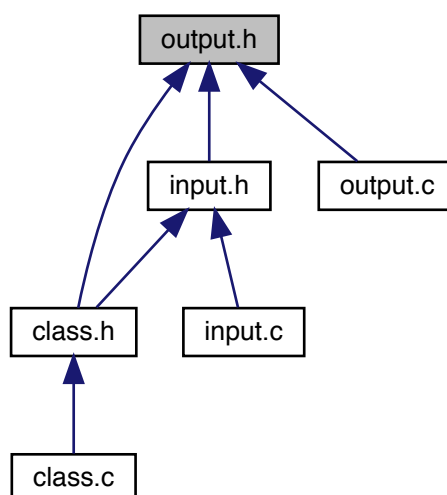
**5.12 output.h File Reference**

```
#include "common.h"
#include "lensing.h"
```

Include dependency graph for output.h:



This graph shows which files directly or indirectly include this file:



## Data Structures

- struct [output](#)

## Macros

- `#define \_Z\_PK\_NUM\_MAX\_ 100`

### 5.12.1 Detailed Description

Documented includes for output module

### 5.12.2 Data Structure Documentation

#### 5.12.2.1 struct output

Structure containing various informations on the output format, all of them initialized by user in input module.

#### Data Fields

|                                  |  |   |
|----------------------------------|--|---|
| FileName                         | root                                   | root for all file names   |
| int                              | z_pk_num                               | number of redshift at which P(k,z) and T <sub>i</sub> (k,z) should be written           |
| double                           | z_pk[ <a href="#">_Z_PK_NUM_MAX_</a> ] | value(s) of redshift at which P(k,z) and T <sub>i</sub> (k,z) should be written         |
| short                            | write_header                           | flag stating whether we should write a header in output files                           |
| enum <a href="#">file_format</a> | output_format                          | which format for output files (definitions, order of columns, etc.)                     |
| short                            | write_background                       | flag for outputting background evolution in file  |
| short                            | write_thermodynamics                   | flag for outputting thermodynamical evolution in file                                   |
| short                            | write_perturbations                    | flag for outputting perturbations of selected wavenumber(s) in file(s)                  |
| short                            | write_primordial                       | flag for outputting scalar/tensor primordial spectra in files                           |
| short                            | output_verbose                         | flag regulating the amount of information sent to standard output (none if set to zero) |
| ErrorMsg                         | error_message                          | zone for writing error messages   |

### 5.12.3 Macro Definition Documentation

#### 5.12.3.1 [\\_Z\\_PK\\_NUM\\_MAX\\_](#)

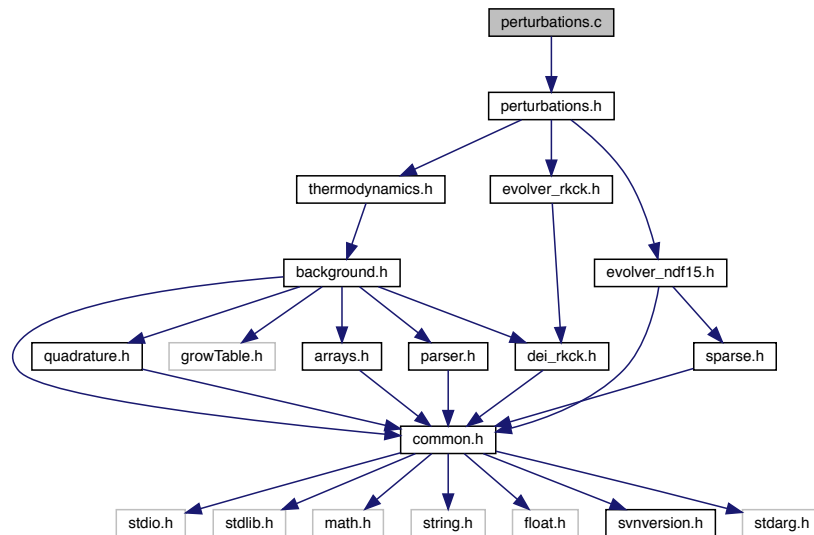
```
#define \_Z\_PK\_NUM\_MAX\_ 100
```

Maximum number of values of redshift at which the spectra will be written in output files

## 5.13 perturbations.c File Reference

```
#include "perturbations.h"
```

Include dependency graph for perturbations.c:



## Functions

- int [perturb\\_sources\\_at\\_tau](#) (struct [perturbs](#) \*ppt, int index\_md, int index\_ic, int index\_type, double tau, double \*psource)
- int [perturb\\_init](#) (struct [precision](#) \*ppr, struct [background](#) \*pba, struct [thermo](#) \*pth, struct [perturbs](#) \*ppt)
- int [perturb\\_free](#) (struct [perturbs](#) \*ppt)
- int [perturb\\_indices\\_of\\_perturbs](#) (struct [precision](#) \*ppr, struct [background](#) \*pba, struct [thermo](#) \*pth, struct [perturbs](#) \*ppt)
- int [perturb\\_timesampling\\_for\\_sources](#) (struct [precision](#) \*ppr, struct [background](#) \*pba, struct [thermo](#) \*pth, struct [perturbs](#) \*ppt)
- int [perturb\\_get\\_k\\_list](#) (struct [precision](#) \*ppr, struct [background](#) \*pba, struct [thermo](#) \*pth, struct [perturbs](#) \*ppt)
- int [perturb\\_workspace\\_init](#) (struct [precision](#) \*ppr, struct [background](#) \*pba, struct [thermo](#) \*pth, struct [perturbs](#) \*ppt, int index\_md, struct [perturb\\_workspace](#) \*ppw)
- int [perturb\\_workspace\\_free](#) (struct [perturbs](#) \*ppt, int index\_md, struct [perturb\\_workspace](#) \*ppw)
- int [perturb\\_solve](#) (struct [precision](#) \*ppr, struct [background](#) \*pba, struct [thermo](#) \*pth, struct [perturbs](#) \*ppt, int index\_md, int index\_ic, int index\_k, struct [perturb\\_workspace](#) \*ppw)
- int [perturb\\_prepare\\_output](#) (struct [background](#) \*pba, struct [perturbs](#) \*ppt)
- int [perturb\\_find\\_approximation\\_number](#) (struct [precision](#) \*ppr, struct [background](#) \*pba, struct [thermo](#) \*pth, struct [perturbs](#) \*ppt, int index\_md, double k, struct [perturb\\_workspace](#) \*ppw, double tau\_ini, double tau\_end, int \*interval\_number, int \*interval\_number\_of)
- int [perturb\\_find\\_approximation\\_switches](#) (struct [precision](#) \*ppr, struct [background](#) \*pba, struct [thermo](#) \*pth, struct [perturbs](#) \*ppt, int index\_md, double k, struct [perturb\\_workspace](#) \*ppw, double tau\_ini, double tau\_end, double [precision](#), int interval\_number, int \*interval\_number\_of, double \*interval\_limit, int \*\*interval\_approx)
- int [perturb\\_vector\\_init](#) (struct [precision](#) \*ppr, struct [background](#) \*pba, struct [thermo](#) \*pth, struct [perturbs](#) \*ppt, int index\_md, int index\_ic, double k, double tau, struct [perturb\\_workspace](#) \*ppw, int \*pa\_old)
- int [perturb\\_vector\\_free](#) (struct [perturb\\_vector](#) \*pv)
- int [perturb\\_initial\\_conditions](#) (struct [precision](#) \*ppr, struct [background](#) \*pba, struct [perturbs](#) \*ppt, int index\_md, int index\_ic, double k, double tau, struct [perturb\\_workspace](#) \*ppw)



- int `perturb_approximations` (struct `precision` \*ppr, struct `background` \*pba, struct `thermo` \*pth, struct `perturbs` \*ppt, int `index_md`, double `k`, double `tau`, struct `perturb_workspace` \*ppw)
- int `perturb_timescale` (double `tau`, void \*parameters\_and\_workspace, double \*timescale, `ErrMsg` error\_↔ message)
- int `perturb_einstein` (struct `precision` \*ppr, struct `background` \*pba, struct `thermo` \*pth, struct `perturbs` \*ppt, int `index_md`, double `k`, double `tau`, double \*y, struct `perturb_workspace` \*ppw)
- int `perturb_total_stress_energy` (struct `precision` \*ppr, struct `background` \*pba, struct `thermo` \*pth, struct `perturbs` \*ppt, int `index_md`, double `k`, double \*y, struct `perturb_workspace` \*ppw)
- int `perturb_sources` (double `tau`, double \*y, double \*dy, int `index_tau`, void \*parameters\_and\_workspace, `ErrMsg` error\_message)
- int `perturb_print_variables` (double `tau`, double \*y, double \*dy, void \*parameters\_and\_workspace, `ErrMsg` error\_message)
- int `perturb_derivs` (double `tau`, double \*y, double \*dy, void \*parameters\_and\_workspace, `ErrMsg` error\_↔ message)
- int `perturb_tca_slip_and_shear` (double \*y, void \*parameters\_and\_workspace, `ErrMsg` error\_message)

### 5.13.1 Detailed Description

Documented perturbation module

Julien Lesgourgues, 23.09.2010

Deals with the perturbation evolution. This module has two purposes:

- at the beginning; to initialize the perturbations, i.e. to integrate the perturbation equations, and store temporarily the terms contributing to the source functions as a function of conformal time. Then, to perform a few manipulations of these terms in order to infer the actual source functions  $S^X(k, \tau)$ , and to store them as a function of conformal time inside an interpolation table.
- at any time in the code; to evaluate the source functions at a given conformal time (by interpolating within the interpolation table).

Hence the following functions can be called from other modules:

1. `perturb_init()` at the beginning (but after `background_init()` and `thermodynamics_init()`)
2. `perturb_sources_at_tau()` at any later time
3. `perturb_free()` at the end, when no more calls to `perturb_sources_at_tau()` are needed

### 5.13.2 Function Documentation

#### 5.13.2.1 `perturb_sources_at_tau()`

```
int perturb_sources_at_tau (
    struct perturbs * ppt,
    int index_md,
    int index_ic,
    int index_type,
    double tau,
    double * psource )
```

Source function  $S^X(k, \tau)$  at a given conformal time tau.

Evaluate source functions at given conformal time tau by reading the pre-computed table and interpolating.

**Parameters**

|                   |  |
|-------------------|--|
| <i>ppt</i>        | Input: pointer to perturbation structure containing interpolation tables |
| <i>index_md</i>   | Input: index of requested mode   |
| <i>index_ic</i>   | Input: index of requested initial condition                              |
| <i>index_type</i> | Input: index of requested source function type                           |
| <i>tau</i>        | Input: any value of conformal time                                       |
| <i>psource</i>    | Output: vector (already allocated) of source function as a function of k |

**Returns**

the error status

**Summary:**

- interpolate in pre-computed table contained in ppt

**5.13.2.2 perturb\_init()**

```
int perturb_init (
    struct precision * ppr,
    struct background * pba,
    struct thermo * pth,
    struct perturbs * ppt )
```

Initialize the perturbs structure, and in particular the table of source functions.

**Main steps:**

- given the values of the flags describing which kind of perturbations should be considered (modes↔ : scalar/vector/tensor, initial conditions, type of source functions needed...), initialize indices and wavenumber list
- define the time sampling for the output source functions
- for each mode (scalar/vector/tensor): initialize the indices of relevant perturbations, integrate the differential system, compute and store the source functions.

**Parameters**

|            |  |
|------------|--|
| <i>ppr</i> | Input: pointer to precision structure      |
| <i>pba</i> | Input: pointer to background structure     |
| <i>pth</i> | Input: pointer to thermodynamics structure |
| <i>ppt</i> | Output: Initialized perturbation structure |

**Returns**

the error status

**Summary:**

- define local variables
- perform preliminary checks
- initialize all indices and lists in `perturbs` structure using [perturb\\_indices\\_of\\_perturbs\(\)](#)
- define the common time sampling for all sources using [perturb\\_timesampling\\_for\\_sources\(\)](#)
- if we want to store perturbations, write titles and allocate storage
- create an array of workspaces in multi-thread case
- loop over modes (scalar, tensors, etc). For each mode:
  - -> (a) create a workspace (one per thread in multi-thread case)
  - -> (b) initialize indices of vectors of perturbations with [perturb\\_indices\\_of\\_current\\_vectors\(\)](#)
  - -> (c) loop over initial conditions and wavenumbers; for each of them, evolve perturbations and compute source functions with [perturb\\_solve\(\)](#)

**5.13.2.3 perturb\_free()**

```
int perturb_free (
    struct perturbs * ppt )
```

Free all memory space allocated by [perturb\\_init\(\)](#).

To be called at the end of each run, only when no further calls to [perturb\\_sources\\_at\\_tau\(\)](#) are needed.

**Parameters**

|            |   |
|------------|---|
| <i>ppt</i> | Input: perturbation structure to be freed |
|------------|---|

**Returns**

the error status

Stuff related to perturbations output:

- Free non-NULL pointers

### 5.13.2.4 perturb\_indices\_of\_perturbs()

```
int perturb_indices_of_perturbs (
    struct precision * ppr,
    struct background * pba,
    struct thermo * pth,
    struct perturbs * ppt )
```

Initialize all indices and allocate most arrays in perturbs structure.

#### Parameters

|            |  |
|------------|--|
| <i>ppr</i> | Input: pointer to precision structure            |
| <i>pba</i> | Input: pointer to background structure           |
| <i>pth</i> | Input: pointer to thermodynamics structure       |
| <i>ppt</i> | Input/Output: Initialized perturbation structure |

#### Returns

the error status

#### Summary:

- define local variables
- count modes (scalar, vector, tensor) and assign corresponding indices
- allocate array of number of types for each mode, `ppt->tp_size[index_md]`
- allocate array of number of initial conditions for each mode, `ppt->ic_size[index_md]`
- allocate array of arrays of source functions for each mode, `ppt->source[index_md]`
- initialization of all flags to false (will eventually be set to true later)
- source flags and indices, for sources that all modes have in common (temperature, polarization, ...). For temperature, the term `t2` is always non-zero, while other terms are non-zero only for scalars and vectors. For polarization, the term `e` is always non-zero, while the term `b` is only for vectors and tensors.
- define `k` values with `perturb_get_k_list()`
- loop over modes. Initialize flags and indices which are specific to each mode.
  - (a) scalars
    - -> source flags and indices, for sources that are specific to scalars
    - -> count scalar initial conditions (for scalars: `ad`, `cdi`, `nid`, `niv`; for tensors: only one) and assign corresponding indices
  - (b) vectors
    - -> source flags and indices, for sources that are specific to vectors
    - -> initial conditions for vectors
  - (c) tensors
    - -> source flags and indices, for sources that are specific to tensors
    - -> only one initial condition for tensors
  - (d) for each mode, allocate array of arrays of source functions for each initial conditions and wavenumber, `(ppt->source[index_md])[index_ic][index_type]`

## 5.13.2.5 perturb\_timesampling\_for\_sources()

```
int perturb_timesampling_for_sources (
    struct precision * ppr,
    struct background * pba,
    struct thermo * pth,
    struct perturbs * ppt )
```

Define time sampling for source functions.

For each type, compute the list of values of tau at which sources will be sampled. Knowing the number of tau values, allocate all arrays of source functions.

## Parameters

|            |  |
|------------|--|
| <i>ppr</i> | Input: pointer to precision structure            |
| <i>pba</i> | Input: pointer to background structure           |
| <i>pth</i> | Input: pointer to thermodynamics structure       |
| <i>ppt</i> | Input/Output: Initialized perturbation structure |

## Returns

the error status

## Summary:

- define local variables
- allocate background/thermodynamics vectors
- first, just count the number of sampling points in order to allocate the array containing all values
- (a) if CMB requested, first sampling point = when the universe stops being opaque; otherwise, start sampling gravitational potential at recombination [however, if perturbed recombination is requested, we also need to start the system before recombination. Otherwise, the initial conditions for gas temperature and ionization fraction perturbations ( $\delta_T = 1/3 \delta_b$ ,  $\delta_{x_e}$ ) are not valid].
- (b) next sampling point = previous +  $ppr \rightarrow \text{perturb\_sampling\_stepsize} * \text{timescale\_source}$ , where:
  - $\rightarrow$  if CMB requested:  $\text{timescale\_source1} = |g/\dot{g}| = |\dot{\kappa} - \ddot{\kappa}/\dot{\kappa}|^{-1}$ ;  $\text{timescale\_source2} = |2\ddot{a}/a - (\dot{a}/a)^2|^{-1/2}$  (to sample correctly the late ISW effect; and  $\text{timescale\_source} = 1/(1/\text{timescale\_source1} + 1/\text{timescale\_source2})$ ; repeat till today.
  - $\rightarrow$  if CMB not requested:  $\text{timescale\_source} = 1/aH$ ; repeat till today.
- $\rightarrow$  infer total number of time steps,  $ppt \rightarrow \text{tau\_size}$
- $\rightarrow$  allocate array of time steps,  $ppt \rightarrow \text{tau\_sampling}[\text{index\_tau}]$
- $\rightarrow$  repeat the same steps, now filling the array with each tau value:
- $\rightarrow$  (b.1.) first sampling point = when the universe stops being opaque
- $\rightarrow$  (b.2.) next sampling point = previous +  $ppr \rightarrow \text{perturb\_sampling\_stepsize} * \text{timescale\_source}$ , where  $\text{timescale\_source1} = |g/\dot{g}| = |\dot{\kappa} - \ddot{\kappa}/\dot{\kappa}|^{-1}$ ;  $\text{timescale\_source2} = |2\ddot{a}/a - (\dot{a}/a)^2|^{-1/2}$  (to sample correctly the late ISW effect; and  $\text{timescale\_source} = 1/(1/\text{timescale\_source1} + 1/\text{timescale\_source2})$ ; repeat till today. If CMB not requested:  $\text{timescale\_source} = 1/aH$ ; repeat till today.
- last sampling point = exactly today
- loop over modes, initial conditions and types. For each of them, allocate array of source functions.

### 5.13.2.6 perturb\_get\_k\_list()

```
int perturb_get_k_list (
    struct precision * ppr,
    struct background * pba,
    struct thermo * pth,
    struct perturbs * ppt )
```

Define the number of comoving wavenumbers using the information passed in the precision structure.

#### Parameters

|            |  |
|------------|--|
| <i>ppr</i> | Input: pointer to precision structure      |
| <i>pba</i> | Input: pointer to background structure     |
| <i>pth</i> | Input: pointer to thermodynamics structure |
| <i>ppt</i> | Input: pointer to perturbation structure   |

#### Returns

the error status

#### Summary:

- allocate arrays related to k list for each mode
- scalar modes
  - → find k\_max (as well as k\_max\_cmb[ppt->index\_md\_scalars], k\_max\_cl[ppt->index\_md\_scalars])
  - → test that result for k\_min, k\_max make sense
- vector modes
  - → find k\_max (as well as k\_max\_cmb[ppt->index\_md\_vectors], k\_max\_cl[ppt->index\_md\_vectors])
  - → test that result for k\_min, k\_max make sense
- tensor modes
  - → find k\_max (as well as k\_max\_cmb[ppt->index\_md\_tensors], k\_max\_cl[ppt->index\_md\_tensors])
  - → test that result for k\_min, k\_max make sense
- If user asked for k\_output\_values, add those to all k lists:
  - → Find indices in ppt->k[index\_md] corresponding to 'k\_output\_values'. We are assuming that ppt->k is sorted and growing, and we have made sure that ppt->k\_output\_values is also sorted and growing.
  - → Decide if we should add k\_output\_value now. This has to be this complicated, since we can only compare the k-values when both indices are in range.
  - → The two MIN statements are here because in a normal run, the cl and cmb arrays contain a single k value larger than their respective k\_max. We are mimicking this behavior.
- finally, find the global k\_min and k\_max for the ensemble of all modes (scalars, vectors, tensors)

## 5.13.2.7 perturb\_workspace\_init()

```
int perturb_workspace_init (
    struct precision * ppr,
    struct background * pba,
    struct thermo * pth,
    struct perturbs * ppt,
    int index_md,
    struct perturb_workspace * ppw )
```

Initialize a `perturb_workspace` structure. All fields are allocated here, with the exception of the `perturb_vector` '→pv' field, which is allocated separately in `perturb_vector_init`. We allocate one such `perturb_workspace` structure per thread and per mode (scalar/./tensor). Then, for each thread, all initial conditions and wavenumbers will use the same workspace.

## Parameters

|                 |   |
|-----------------|---|
| <i>ppr</i>      | Input: pointer to precision structure   |
| <i>pba</i>      | Input: pointer to background structure  |
| <i>pth</i>      | Input: pointer to the thermodynamics structure  |
| <i>ppt</i>      | Input: pointer to the perturbation structure  |
| <i>index_md</i> | Input: index of mode under consideration (scalar/./tensor)  |
| <i>ppw</i>      | Input/Output: pointer to <code>perturb_workspace</code> structure which fields are allocated or filled here |

## Returns

the error status

## Summary:

- define local variables
- Compute maximum  $l_{\max}$  for any multipole
- Allocate  $s_l[ ]$  array for freestreaming of multipoles (see arXiv:1305.3261) and initialize to 1.0, which is the  $K=0$  value.
- define indices of metric perturbations obeying constraint equations (this can be done once and for all, because the vector of metric perturbations is the same whatever the approximation scheme, unlike the vector of quantities to be integrated, which is allocated separately in `perturb_vector_init`)
- allocate some workspace in which we will store temporarily the values of background, thermodynamics, metric and source quantities at a given time
- count number of approximations, initialize their indices, and allocate their flags
- For definiteness, initialize approximation flags to arbitrary values (correct values are overwritten in `perturb_↔ find_approximation_switches`)
- allocate fields where some of the perturbations are stored

### 5.13.2.8 perturb\_workspace\_free()

```
int perturb_workspace_free (
    struct perturbs * ppt,
    int index_md,
    struct perturb\_workspace * ppw )
```

Free the [perturb\\_workspace](#) structure (with the exception of the [perturb\\_vector](#) '→pv' field, which is freed separately in [perturb\\_vector\\_free](#)).

#### Parameters

|                 |   |
|-----------------|---|
| <i>ppt</i>      | Input: pointer to the perturbation structure                              |
| <i>index_md</i> | Input: index of mode under consideration (scalar/.../tensor)              |
| <i>ppw</i>      | Input: pointer to <a href="#">perturb_workspace</a> structure to be freed |

#### Returns

the error status

### 5.13.2.9 perturb\_solve()

```
int perturb_solve (
    struct precision * ppr,
    struct background * pba,
    struct thermo * pth,
    struct perturbs * ppt,
    int index_md,
    int index_ic,
    int index_k,
    struct perturb\_workspace * ppw )
```

Solve the perturbation evolution for a given mode, initial condition and wavenumber, and compute the corresponding source functions.

For a given mode, initial condition and wavenumber, this function finds the time ranges over which the perturbations can be described within a given approximation. For each such range, it initializes (or redistributes) perturbations using [perturb\\_vector\\_init\(\)](#), and integrates over time. Whenever a "source sampling time" is passed, the source terms are computed and stored in the source table using [perturb\\_sources\(\)](#).

#### Parameters

|                 |  |
|-----------------|--|
| <i>ppr</i>      | Input: pointer to precision structure  |
| <i>pba</i>      | Input: pointer to background structure   |
| <i>pth</i>      | Input: pointer to the thermodynamics structure   |
| <i>ppt</i>      | Input/Output: pointer to the perturbation structure (output source functions S(k,tau) written here)  |
| <i>index_md</i> | Input: index of mode under consideration (scalar/.../tensor)   |
| <i>index_ic</i> | Input: index of initial condition under consideration (ad, iso...)                                   |
| <i>index_k</i>  | Input: index of wavenumber   |
| <i>ppw</i>      | Input: pointer to <a href="#">perturb_workspace</a> structure containing index values and workspaces |



**Returns**

the error status

**Summary:**

- define local variables
- initialize indices relevant for back/thermo tables search
- get wavenumber value
- If non-zero curvature, update array of free-streaming coefficients `ppw->s_l`
- maximum value of tau for which sources are calculated for this wavenumber
- using bisection, compute minimum value of tau for which this wavenumber is integrated
- find the number of intervals over which approximation scheme is constant
- fill the structure containing all fixed parameters, indices and workspaces needed by `perturb_derivs`
- check whether we need to print perturbations to a file for this wavenumber
- loop over intervals over which approximation scheme is uniform. For each interval:
  - -> (a) fix the approximation scheme
  - -> (b) get the previous approximation scheme. If the current interval starts from the initial time `tau_ini`, the previous approximation is set to be a NULL pointer, so that the function `perturb_vector_init()` knows that perturbations must be initialized
  - -> (c) define the vector of perturbations to be integrated over. If the current interval starts from the initial time `tau_ini`, fill the vector with initial conditions for each mode. If it starts from an approximation switching point, redistribute correctly the perturbations from the previous to the new vector of perturbations.
  - -> (d) integrate the perturbations over the current interval.
- if perturbations were printed in a file, close the file
- fill the source terms array with zeros for all times between the last integrated time `tau_max` and `tau_today`.
- free quantities allocated at the beginning of the routine

**5.13.2.10 perturb\_prepare\_output()**

```
int perturb_prepare_output (
    struct background * pba,
    struct perturbs * ppt )
```

Write titles for all perturbations that we would like to print/store.

### 5.13.2.11 perturb\_find\_approximation\_number()

```
int perturb_find_approximation_number (
    struct precision * ppr,
    struct background * pba,
    struct thermo * pth,
    struct perturbs * ppt,
    int index_md,
    double k,
    struct perturb_workspace * ppw,
    double tau_ini,
    double tau_end,
    int * interval_number,
    int * interval_number_of )
```

For a given mode and wavenumber, find the number of intervals of time between tau\_ini and tau\_end such that the approximation scheme (and the number of perturbation equations) is uniform.

#### Parameters

|                                 |   |
|---------------------------------|---|
| <i>ppr</i>                      | Input: pointer to precision structure   |
| <i>pba</i>                      | Input: pointer to background structure  |
| <i>pth</i>                      | Input: pointer to the thermodynamics structure  |
| <i>ppt</i>                      | Input: pointer to the perturbation structure  |
| <i>index_md</i>                 | Input: index of mode under consideration (scalar/.../tensor)                                      |
| <i>k</i>                        | Input: index of wavenumber  |
| <i>ppw</i>                      | Input: pointer to <code>perturb_workspace</code> structure containing index values and workspaces |
| <i>tau_ini</i>                  | Input: initial time of the perturbation integration   |
| <i>tau_end</i>                  | Input: final time of the perturbation integration   |
| <i>interval_number</i>          | Output: total number of intervals   |
| <i>interval_number↔<br/>_of</i> | Output: number of intervals with respect to each particular approximation                         |

#### Returns

the error status

#### Summary:

- fix default number of intervals to one (if no approximation switch)
- loop over each approximation and add the number of approximation switching times

### 5.13.2.12 perturb\_find\_approximation\_switches()

```
int perturb_find_approximation_switches (
    struct precision * ppr,
    struct background * pba,
    struct thermo * pth,
```

```

struct perturbs * ppt,
int index_md,
double k,
struct perturb_workspace * ppw,
double tau_ini,
double tau_end,
double precision,
int interval_number,
int * interval_number_of,
double * interval_limit,
int ** interval_approx )

```

For a given mode and wavenumber, find the values of time at which the approximation changes.

#### Parameters

|                                 |   |
|---------------------------------|---|
| <code>ppr</code>                | Input: pointer to precision structure   |
| <code>pba</code>                | Input: pointer to background structure  |
| <code>pth</code>                | Input: pointer to the thermodynamics structure  |
| <code>ppt</code>                | Input: pointer to the perturbation structure  |
| <code>index_md</code>           | Input: index of mode under consideration (scalar/.../tensor)  |
| <code>k</code>                  | Input: index of wavenumber  |
| <code>ppw</code>                | Input: pointer to <code>perturb_workspace</code> structure containing index values and workspaces                                   |
| <code>tau_ini</code>            | Input: initial time of the perturbation integration   |
| <code>tau_end</code>            | Input: final time of the perturbation integration   |
| <code>precision</code>          | Input: tolerance on output values   |
| <code>interval_number</code>    | Input: total number of intervals  |
| <code>interval_number_of</code> | Input: number of intervals with respect to each particular approximation  |
| <code>interval_limit</code>     | Output: value of time at the boundary of the intervals: <code>tau_ini</code> , <code>tau_switch1</code> , ..., <code>tau_end</code> |
| <code>interval_approx</code>    | Output: value of approximations in each interval  |

#### Returns

the error status

#### Summary:

- write in output arrays the initial time and approximation
- if there are no approximation switches, just write final time and return
- if there are switches, consider approximations one after each other. Find switching time by bisection. Store all switches in arbitrary order in array `unsorted_tau_switch[ ]`
- now sort interval limits in correct order
- store each approximation in chronological order

## 5.13.2.13 perturb\_vector\_init()

```
int perturb_vector_init (
    struct precision * ppr,
    struct background * pba,
    struct thermo * pth,
    struct perturbs * ppt,
    int index_md,
    int index_ic,
    double k,
    double tau,
    struct perturb_workspace * ppw,
    int * pa_old )
```

Initialize the field '→pv' of a `perturb_workspace` structure, which is a `perturb_vector` structure. This structure contains indices and values of all quantities which need to be integrated with respect to time (and only them: quantities fixed analytically or obeying constraint equations are NOT included in this vector). This routine distinguishes between two cases:

→ the input `pa_old` is set to the NULL pointer:

This happens when we start integrating over a new wavenumber and we want to set initial conditions for the perturbations. Then, it is assumed that `ppw→pv` is not yet allocated. This routine allocates it, defines all indices, and then fills the vector `ppw→pv→y` with the initial conditions defined in `perturb_initial_conditions`.

→ the input `pa_old` is not set to the NULL pointer and describes some set of approximations:

This happens when we need to change approximation scheme while integrating over a given wavenumber. The new approximation described by `ppw→pa` is then different from `pa_old`. Then, this routine allocates a new vector with a new size and new index values; it fills this vector with initial conditions taken from the previous vector passed as an input in `ppw→pv`, and eventually with some analytic approximations for the new variables appearing at this time; then the new vector comes in replacement of the old one, which is freed.

## Parameters

|                 |  |
|-----------------|--|
| <i>ppr</i>      | Input: pointer to precision structure  |
| <i>pba</i>      | Input: pointer to background structure   |
| <i>pth</i>      | Input: pointer to the thermodynamics structure   |
| <i>ppt</i>      | Input: pointer to the perturbation structure   |
| <i>index_md</i> | Input: index of mode under consideration (scalar/.../tensor)   |
| <i>index_ic</i> | Input: index of initial condition under consideration (ad, iso...)   |
| <i>k</i>        | Input: wavenumber  |
| <i>tau</i>      | Input: conformal time  |
| <i>ppw</i>      | Input/Output: workspace containing in input the approximation scheme, the background/thermodynamics/metric quantities, and eventually the previous vector <i>y</i> ; and in output the new vector <i>y</i> . |
| <i>pa_old</i>   | Input: NULL is we need to set <i>y</i> to initial conditions for a new wavenumber; points towards a <code>perturb_approximations</code> if we want to switch of approximation.                               |

## Returns

the error status

## Summary:

- define local variables
- allocate a new `perturb_vector` structure to which `ppw->pv` will point at the end of the routine
- initialize pointers to NULL (they will be allocated later if needed), relevant for `perturb_vector_free()`
- define all indices in this new vector (depends on approximation scheme, described by the input structure `ppw->pa`)
- (a) metric perturbations  $V$  or  $h_v$  depending on gauge
- (b) metric perturbation  $h$  is a propagating degree of freedom, so  $h$  and  $h\dot{}$  are included in the vector of ordinary perturbations, no in that of metric perturbations
- allocate vectors for storing the values of all these quantities and their time-derivatives at a given time
- specify which perturbations are needed in the evaluation of source terms
- case of setting initial conditions for a new wavenumber
- -> (a) check that current approximation scheme is consistent with initial conditions
- -> (b) let `ppw->pv` points towards the `perturb_vector` structure that we just created
- -> (c) fill the vector `ppw->pv->y` with appropriate initial conditions
- case of switching approximation while a wavenumber is being integrated
- -> (a) for the scalar mode:
  - -> (a.1.) check that the change of approximation scheme makes sense (note: before calling this routine there is already a check that we wish to change only one approximation flag at a time)
  - -> (a.2.) some variables ( $b$ ,  $cdm$ ,  $fld$ , ...) are not affected by any approximation. They need to be reconducted whatever the approximation switching is. We treat them here. Below we will treat other variables case by case.
- -> (b) for the vector mode
  - -> (b.1.) check that the change of approximation scheme makes sense (note: before calling this routine there is already a check that we wish to change only one approximation flag at a time)
  - -> (b.2.) some variables ( $gw$ ,  $gwdot$ , ...) are not affected by any approximation. They need to be reconducted whatever the approximation switching is. We treat them here. Below we will treat other variables case by case.
- -> (c) for the tensor mode
  - -> (c.1.) check that the change of approximation scheme makes sense (note: before calling this routine there is already a check that we wish to change only one approximation flag at a time)
  - -> (c.2.) some variables ( $gw$ ,  $gwdot$ , ...) are not affected by any approximation. They need to be reconducted whatever the approximation switching is. We treat them here. Below we will treat other variables case by case.
- -> (d) free the previous vector of perturbations
- -> (e) let `ppw->pv` points towards the `perturb_vector` structure that we just created

#### 5.13.2.14 `perturb_vector_free()`

```
int perturb_vector_free (
    struct perturb_vector * pv )
```

Free the `perturb_vector` structure.

## Parameters

|           |   |
|-----------|---|
| <i>pv</i> | Input: pointer to <code>perturb_vector</code> structure to be freed |
|-----------|---|

## Returns

the error status

5.13.2.15 `perturb_initial_conditions()`

```
int perturb_initial_conditions (
    struct precision * ppr,
    struct background * pba,
    struct perturbs * ppt,
    int index_md,
    int index_ic,
    double k,
    double tau,
    struct perturb_workspace * ppw )
```

For each mode, wavenumber and initial condition, this function initializes in the vector all values of perturbed variables (in a given gauge). It is assumed here that all values have previously been set to zero, only non-zero values are set here.

## Parameters

|                 |  |
|-----------------|--|
| <i>ppr</i>      | Input: pointer to precision structure  |
| <i>pba</i>      | Input: pointer to background structure   |
| <i>ppt</i>      | Input: pointer to the perturbation structure   |
| <i>index_md</i> | Input: index of mode under consideration (scalar/.../tensor)   |
| <i>index_ic</i> | Input: index of initial condition under consideration (ad, iso...)   |
| <i>k</i>        | Input: wavenumber  |
| <i>tau</i>      | Input: conformal time  |
| <i>ppw</i>      | Input/Output: workspace containing in input the approximation scheme, the background/thermodynamics/metric quantities, and eventually the previous vector <i>y</i> ; and in output the new vector <i>y</i> . |

## Returns

the error status

## Summary:

→ Declare local variables

→ For scalars

- (a) compute relevant background quantities: compute `rho_r`, `rho_m`, `rho_nu` (= all relativistic except photons), and their ratio.

- (b) starts by setting everything in synchronous gauge. If another gauge is needed, we will perform a gauge transformation below.
- → (b.1.) adiabatic
- → Canonical field (solving for the perturbations): initial perturbations set to zero, they should reach the attractor soon enough.
- → TODO: Incorporate the attractor IC from 1004.5509.  $\delta\phi = -(a/k)^2/\phi'(\rho+p)\theta$ ,  $\delta\phi_{\text{prime}} = a^2/\phi'(\delta\rho_{\text{phi}} + V'\delta\phi)$ , and assume theta, delta\_rho as for perfect fluid with  $c_s^2 = 1$  and  $w = 1/3$  (ASSUMES radiation TRACKING)
- → (b.2.) Cold dark matter Isocurvature
- → (b.3.) Baryon Isocurvature
- → (b.4.) Neutrino density Isocurvature
- → (b.5.) Neutrino velocity Isocurvature
- (c) If the needed gauge is really the synchronous gauge, we need to affect the previously computed value of eta to the actual variable eta
- (d) If the needed gauge is the newtonian gauge, we must compute alpha and then perform a gauge transformation for each variable
- (e) In any gauge, we should now implement the relativistic initial conditions in ur and ncdm variables

→ For tensors

tensor initial conditions take into account the fact that scalar (resp. tensor)  $C_l$ 's are related to the real space power spectrum of curvature (resp. of the tensor part of metric perturbations)

$$\langle R(x)R(x) \rangle = \sum_{ij} \langle h_{ij}(x)h^{ij}(x) \rangle$$

In momentum space it is conventional to use the modes  $R(k)$  and  $h(k)$  where the quantity  $h$  obeying to the equation of propagation:

$$h'' + \frac{2a'}{a}h + [k^2 + 2K]h = 12\pi G a^2(\rho + p)\sigma = 8\pi G a^2 p \pi$$

and the power spectra in real space and momentum space are related through:

$$\begin{aligned} \langle R(x)R(x) \rangle &= \int \frac{dk}{k} \left[ \frac{k^3}{2\pi^2} \langle R(k)R(k)^* \rangle \right] = \int \frac{dk}{k} \mathcal{P}_R(k) \\ \sum_{ij} \langle h_{ij}(x)h^{ij}(x) \rangle &= \frac{dk}{k} \left[ \frac{k^3}{2\pi^2} F\left(\frac{k^2}{K}\right) \langle h(k)h(k)^* \rangle \right] = \int \frac{dk}{k} F\left(\frac{k^2}{K}\right) \mathcal{P}_h(k) \end{aligned}$$

where  $\mathcal{P}_R$  and  $\mathcal{P}_h$  are the dimensionless spectrum of curvature  $R$ , and  $F$  is a function of  $k^2/K$ , where  $K$  is the curvature parameter.  $F$  is equal to one in flat space ( $K=0$ ), and coming from the contraction of the laplacian eigentensor  $Q_{ij}$  with itself. We will give  $F$  explicitly below.

Similarly the scalar (S) and tensor (T)  $C_l$ 's are given by

$$C_l^S = 4\pi \int \frac{dk}{k} [\Delta_l^S(q)]^2 \mathcal{P}_R(k)$$

$$C_l^T = 4\pi \int \frac{dk}{k} [\Delta_l^T(q)]^2 F\left(\frac{k^2}{K}\right) \mathcal{P}_h(k)$$

The usual convention for the tensor-to-scalar ratio  $r = A_t/A_s$  at pivot scale = 16 epsilon in single-field inflation is such that for constant  $\mathcal{P}_R(k)$  and  $\mathcal{P}_h(k)$ ,

$$r = 6 \frac{\mathcal{P}_h(k)}{\mathcal{P}_R(k)}$$

so

$$\mathcal{P}_h(k) = \frac{\mathcal{P}_R(k)r}{6} = \frac{A_s r}{6} = \frac{A_t}{6}$$

A priori it would make sense to say that for a power-law primordial spectrum there is an extra factor  $(k/k_{pivot})^{n_t}$  (and eventually running and so on and so forth...)

However it has been shown that the minimal models of inflation in a negatively curved bubble lead to  $\mathcal{P}_h(k) = \tanh(\pi * \nu/2)$ . In open models it is customary to define the tensor tilt in a non-flat universe as a deviation from this behavior rather than from true scale-invariance in the above sense.

Hence we should have

$$\mathcal{P}_h(k) = \frac{A_t}{6} [\tanh(\pi * \frac{\nu}{2})] (k/k_{pivot})^{(n_t+\dots)}$$

where the brackets

[...]

mean "if  $K < 0$ "

Then

$$C_l^T = 4\pi \int \frac{dk}{k} [\Delta_l^T(q)]^2 F\left(\frac{k^2}{K}\right) \frac{A_t}{6} [\tanh(\pi * \frac{\nu}{2})] (k/k_{pivot})^{(n_t+\dots)}$$

In the code, it is then a matter of choice to write:

- In the primordial module:  $\mathcal{P}_h(k) = \frac{A_t}{6} \tanh(\pi * \frac{\nu}{2}) (k/k^*)^{n_T}$
- In the perturbation initial conditions:  $h = 1$
- In the spectra module:  $C_l^T = \frac{4}{\pi} \int \frac{dk}{k} [\Delta_l^T(q)]^2 F\left(\frac{k^2}{K}\right) \mathcal{P}_h(k)$

or:

- In the primordial module:  $\mathcal{P}_h(k) = A_t (k/k^*)^{n_T}$



- In the perturbation initial conditions:  $h = \sqrt{[F \left(\frac{k^2}{K}\right) / 6] \tanh\left(\pi * \frac{\nu}{2}\right)}$
- In the spectra module:  $C_l^T = \frac{4}{\pi} \int \frac{dk}{k} [\Delta_l^T(q)]^2 \mathcal{P}_h(k)$

We choose this last option, such that the primordial and spectra module differ minimally in flat and non-flat space. Then we must impose

$$h = \sqrt{\left(\frac{F}{6}\right) \tanh\left(\pi * \frac{\nu}{2}\right)}$$

The factor F is found to be given by:

$$\sum_{ij} \langle h_{ij}(x) h^{ij}(x) \rangle = \int \frac{dk}{k} \frac{k^2(k^2 - K)}{(k^2 + 3K)(k^2 + 2K)} \mathcal{P}_h(k)$$

Introducing as usual  $q^2 = k^2 - 3K$  and using  $qdk = kdk$  this gives

$$\sum_{ij} \langle h_{ij}(x) h^{ij}(x) \rangle = \int \frac{dq}{k} \frac{(q^2 - 3K)(q^2 - 4K)}{q^2(q^2 - K)} \mathcal{P}_h(k)$$

Using  $qdk = kdk$  this is equivalent to

$$\sum_{ij} \langle h_{ij}(x) h^{ij}(x) \rangle = \int \frac{dq}{q} \frac{q^2 - 4K}{q^2 - K} \mathcal{P}_h(k(q))$$

Finally, introducing  $\nu = q/\sqrt{|K|}$  and  $\text{sgn}K = \text{SIGN}(k) = \pm 1$ , this could also be written

$$\sum_{ij} \langle h_{ij}(x) h^{ij}(x) \rangle = \int \frac{d\nu}{\nu} \frac{(\nu^2 - 4\text{sgn}K)}{(\nu^2 - \text{sgn}K)} \mathcal{P}_h(k(\nu))$$

Equation (43,44) of Hu, Seljak, White, Zaldarriaga is equivalent to absorbing the above factor  $(\nu^2 - 4\text{sgn}K)/(\nu^2 - \text{sgn}K)$  in the definition of the primordial spectrum. Since the initial condition should be written in terms of k rather than nu, they should read

$$h = \sqrt{[k^2(k^2 - K)] / [(k^2 + 3K)(k^2 + 2K)] / 6 * \tanh\left(\pi * \frac{\nu}{2}\right)}$$

We leave the freedom to multiply by an arbitrary number  $\text{ppr} \rightarrow \text{gw\_ini}$ . The standard convention corresponding to standard definitions of  $r$ ,  $A_T$ ,  $n_T$  is however  $\text{ppr} \rightarrow \text{gw\_ini} = 1$ .

## 5.13.2.16 perturb\_approximations()

```
int perturb_approximations (
    struct precision * ppr,
    struct background * pba,
    struct thermo * pth,
    struct perturbs * ppt,
    int index_md,
    double k,
    double tau,
    struct perturb_workspace * ppw )
```

Evaluate background/thermodynamics at  $\tau$ , infer useful flags / time scales for integrating perturbations.

Evaluate background quantities at  $\tau$ , as well as thermodynamics for scalar mode; infer useful flags and time scales for integrating the perturbations:

- check whether tight-coupling approximation is needed.
- check whether radiation (photons, massless neutrinos...) perturbations are needed.
- choose step of integration:  $\text{step} = \text{ppr} \rightarrow \text{perturb\_integration\_stepsize} * \text{min\_time\_scale}$ , where  $\text{min\_time\_scale} \leftarrow \text{scale} = \text{smallest time scale involved in the equations}$ . There are three time scales to compare:
  1. that of recombination,  $\tau_c = 1/\kappa'$
  2. Hubble time scale,  $\tau_h = a/a'$
  3. Fourier mode,  $\tau_k = 1/k$

So, in general,  $\text{min\_time\_scale} = \min(\tau_c, \tau_b, \tau_h, \tau_k)$ .

However, if  $\tau_c \ll \tau_h$  and  $\tau_c \ll \tau_k$ , we can use the tight-coupling regime for photons and write equations in such way that the time scale  $\tau_c$  becomes irrelevant (no effective mass term in  $1/\tau_c$ ). Then, the smallest scale in the equations is only  $\min(\tau_h, \tau_k)$ . In practise, it is sufficient to use only the condition  $\tau_c \ll \tau_h$ .

Also, if  $\rho_{\text{matter}} \gg \rho_{\text{radiation}}$  and  $k \gg aH$ , we can switch off radiation perturbations (i.e. switch on the free-streaming approximation) and then the smallest scale is simply  $\tau_h$ .

## Parameters

|                 |  |
|-----------------|--|
| <i>ppr</i>      | Input: pointer to precision structure                                      |
| <i>pba</i>      | Input: pointer to background structure                                     |
| <i>pth</i>      | Input: pointer to thermodynamics structure                                 |
| <i>ppt</i>      | Input: pointer to the perturbation structure                               |
| <i>index_md</i> | Input: index of mode under consideration (scalar/.../tensor)               |
| <i>k</i>        | Input: wavenumber  |
| <i>tau</i>      | Input: conformal time  |
| <i>ppw</i>      | Input/Output: in output contains the approximation to be used at this time |

## Returns

the error status

Summary:

- define local variables
- compute Fourier mode time scale =  $\tau_k = 1/k$
- evaluate background quantities with `background_at_tau()` and Hubble time scale  $\tau_h = a/a'$
- for scalar modes:
  - → (a) evaluate thermodynamical quantities with `thermodynamics_at_z()`
  - → (b.1.) if  $\kappa' = 0$ , recombination is finished; tight-coupling approximation must be off
  - → (b.2.) if  $\kappa' \neq 0$ , recombination is not finished: check tight-coupling approximation
  - → (b.2.a) compute recombination time scale for photons,  $\tau_\gamma = 1/\kappa'$
  - → (b.2.b) check whether tight-coupling approximation should be on
- → (c) free-streaming approximations
- for tensor modes:
  - → (a) evaluate thermodynamical quantities with `thermodynamics_at_z()`
  - → (b.1.) if  $\kappa' = 0$ , recombination is finished; tight-coupling approximation must be off
  - → (b.2.) if  $\kappa' \neq 0$ , recombination is not finished: check tight-coupling approximation
  - → (b.2.a) compute recombination time scale for photons,  $\tau_\gamma = 1/\kappa'$
  - → (b.2.b) check whether tight-coupling approximation should be on

#### 5.13.2.17 perturb\_timescale()

```
int perturb_timescale (
    double tau,
    void * parameters_and_workspace,
    double * timescale,
    ErrorMsg error_message )
```

Compute typical timescale over which the perturbation equations vary. Some integrators (e.g. Runge-Kunta) benefit from calling this routine at each step in order to adapt the next step.

This is one of the few functions in the code which is passed to the `generic_integrator()` routine. Since `generic_integrator()` should work with functions passed from various modules, the format of the arguments is a bit special:

- fixed parameters and workspaces are passed through a generic pointer. `generic_integrator()` doesn't know the content of this pointer.
- the error management is a bit special: errors are not written as usual to `pth->error_message`, but to a generic `error_message` passed in the list of arguments.

#### Parameters

|                                 |   |
|---------------------------------|---|
| <i>tau</i>                      | Input: conformal time   |
| <i>parameters_and_workspace</i> | Input: fixed parameters (e.g. indices), workspace, approximation used, etc. |
| <i>timescale</i>                | Output: perturbation variation timescale (given the approximation used)     |
| <i>error_message</i>            | Output: error message   |

Summary:

- define local variables
- extract the fields of the `parameter_and_workspace` input structure
- compute Fourier mode time scale =  $\tau_k = 1/k$
- evaluate background quantities with `background_at_tau()` and Hubble time scale  $\tau_h = a/a'$
- for scalars modes:
  - -> compute recombination time scale for photons,  $\tau_\gamma = 1/\kappa'$
- for vector modes:
  - -> compute recombination time scale for photons,  $\tau_\gamma = 1/\kappa'$
- for tensor modes:
  - -> compute recombination time scale for photons,  $\tau_\gamma = 1/\kappa'$

#### 5.13.2.18 `perturb_einstein()`

```
int perturb_einstein (
    struct precision * ppr,
    struct background * pba,
    struct thermo * pth,
    struct perturbs * ppt,
    int index_md,
    double k,
    double tau,
    double * y,
    struct perturb_workspace * ppw )
```

Compute metric perturbations (those not integrated over time) using Einstein equations

#### Parameters

|                 |   |
|-----------------|---|
| <i>ppr</i>      | Input: pointer to precision structure   |
| <i>pba</i>      | Input: pointer to background structure  |
| <i>pth</i>      | Input: pointer to thermodynamics structure                                      |
| <i>ppt</i>      | Input: pointer to the perturbation structure                                    |
| <i>index_md</i> | Input: index of mode under consideration (scalar/.../tensor)                    |
| <i>k</i>        | Input: wavenumber   |
| <i>tau</i>      | Input: conformal time   |
| <i>y</i>        | Input: vector of perturbations (those integrated over time) (already allocated) |
| <i>ppw</i>      | Input/Output: in output contains the updated metric perturbations               |

#### Returns

the error status

Summary:

- define local variables
- define wavenumber and scale factor related quantities
- sum up perturbations from all species
- for scalar modes:
  - → infer metric perturbations from Einstein equations
- for vector modes
- for tensor modes

#### 5.13.2.19 perturb\_total\_stress\_energy()

```
int perturb_total_stress_energy (
    struct precision * ppr,
    struct background * pba,
    struct thermo * pth,
    struct perturbs * ppt,
    int index_md,
    double k,
    double * y,
    struct perturb_workspace * ppw )
```

Summary:

- define local variables
- wavenumber and scale factor related quantities
- for scalar modes
  - → (a) deal with approximation schemes
    - → (a.1.) photons
      - → (a.1.1.) no approximation
      - → (a.1.2.) radiation streaming approximation
      - → (a.1.3.) tight coupling approximation
    - → (a.2.) ur
  - → (b) compute the total density, velocity and shear perturbations
- for vector modes
  - → photon contribution to vector sources:
    - → baryons
- for tensor modes
  - → photon contribution to gravitational wave source:
  - → ur contribution to gravitational wave source:
  - → ncdm contribution to gravitational wave source:

5.13.2.20 `perturb_sources()`

```
int perturb_sources (
    double tau,
    double * y,
    double * dy,
    int index_tau,
    void * parameters_and_workspace,
    ErrorMsg error_message )
```

Compute the source functions (three terms for temperature, one for E or B modes, etc.)

This is one of the few functions in the code which is passed to the `generic_integrator()` routine. Since `generic_integrator()` should work with functions passed from various modules, the format of the arguments is a bit special:

- fixed parameters and workspaces are passed through a generic pointer. `generic_integrator()` doesn't know the content of this pointer.
- the error management is a bit special: errors are not written as usual to `pth->error_message`, but to a generic `error_message` passed in the list of arguments.

**Parameters**

|                                 |  |
|---------------------------------|--|
| <i>tau</i>                      | Input: conformal time  |
| <i>y</i>                        | Input: vector of perturbations   |
| <i>dy</i>                       | Input: vector of time derivative of perturbations  |
| <i>index_tau</i>                | Input: index in the array <code>tau_sampling</code>  |
| <i>parameters_and_workspace</i> | Input/Output: in input, all parameters needed by <code>perturb_derivs</code> , in output, source terms |
| <i>error_message</i>            | Output: error message  |

**Returns**

the error status

**Summary:**

- define local variables
- rename structure fields (just to avoid heavy notations)
- get background/thermo quantities in this point
- for scalars
- → compute metric perturbations
- → compute quantities depending on approximation schemes
- → for each type, compute source terms
- for tensors
- → compute quantities depending on approximation schemes

## 5.13.2.21 perturb\_print\_variables()

```
int perturb_print_variables (
    double tau,
    double * y,
    double * dy,
    void * parameters_and_workspace,
    ErrorMsg error_message )
```

When testing the code or a cosmological model, it can be useful to output perturbations at each step of integration (and not just the delta's at each source sampling point, which is achieved simply by asking for matter transfer functions). Then this function can be passed to the generic\_evolver routine.

By default, instead of passing this function to generic\_evolver, one passes a null pointer. Then this function is just not used.

## Parameters

|                                 |  |
|---------------------------------|--|
| <i>tau</i>                      | Input: conformal time                                |
| <i>y</i>                        | Input: vector of perturbations                       |
| <i>dy</i>                       | Input: vector of its derivatives (already allocated) |
| <i>parameters_and_workspace</i> | Input: fixed parameters (e.g. indices)               |
| <i>error_message</i>            | Output: error message                                |

## Summary:

- define local variables
- ncdm sector begins
- ncdm sector ends
- rename structure fields (just to avoid heavy notations)
- calculate perturbed recombination
- for scalar modes
  - -> Get delta, deltaP/rho, theta, shear and store in array
  - -> Do gauge transformation of delta, deltaP/rho (?) and theta using  $-= 3aH(1+w_{\text{ncdm}})$  alpha for delta.
  - -> Handle (re-)allocation
- for tensor modes:
  - -> Handle (re-)allocation

### 5.13.2.22 perturb\_derivs()

```
int perturb_derivs (
    double tau,
    double * y,
    double * dy,
    void * parameters_and_workspace,
    ErrorMsg error_message )
```

Compute derivative of all perturbations to be integrated

For each mode (scalar/vector/tensor) and each wavenumber  $k$ , this function computes the derivative of all values in the vector of perturbed variables to be integrated.

This is one of the few functions in the code which is passed to the `generic_integrator()` routine. Since `generic_integrator()` should work with functions passed from various modules, the format of the arguments is a bit special:

- fixed parameters and workspaces are passed through a generic pointer. `generic_integrator()` doesn't know what the content of this pointer is.
- errors are not written as usual in `pth->error_message`, but in a generic `error_message` passed in the list of arguments.

#### Parameters

|                                 |  |
|---------------------------------|--|
| <i>tau</i>                      | Input: conformal time  |
| <i>y</i>                        | Input: vector of perturbations   |
| <i>dy</i>                       | Output: vector of its derivatives (already allocated)  |
| <i>parameters_and_workspace</i> | Input/Output: in input, fixed parameters (e.g. indices); in output, background and thermo quantities evaluated at tau. |
| <i>error_message</i>            | Output: error message  |

#### Summary:

- define local variables
- rename the fields of the input structure (just to avoid heavy notations)
- get background/thermo quantities in this point
- get metric perturbations with [perturb\\_einstein\(\)](#)
- compute related background quantities
- Compute 'generalised cotK function of argument  $\sqrt{|K|} * \tau$ , for closing hierarchy. (see equation 2.34 in arXiv:1305.3261):
- for scalar modes:
  - -> (a) define short-cut notations for the scalar perturbations
  - -> (b) perturbed recombination
  - -> (c) compute metric-related quantities (depending on gauge; additional gauges can be coded below)
    - Each continuity equation contains a term in (theta+metric\_continuity) with `metric_continuity = (h_prime/2)` in synchronous gauge, `(-3 phi_prime)` in newtonian gauge



- Each Euler equation contains a source term `metric_euler` with `metric_euler = 0` in synchronous gauge,  $(k^2 \psi)$  in newtonian gauge
- Each shear derivative equation contains a source term `metric_shear` equal to `metric_shear = (h_prime + 6*eta_prime)/2` in synchronous gauge, 0 in newtonian gauge
- `metric_shear_prime` is the derivative of `metric_shear`
- In the `ufa_class` approximation, the leading-order source term is  $(h\_prime/2)$  in synchronous gauge,  $(-3(\psi\_prime + \psi\_prime))$  in newtonian gauge: we approximate the later by  $(-6 \psi\_prime)$
- → (d) if some approximation schemes are turned on, enforce a few `y[]` values computed in `perturb_einstein`
- → (e) BEGINNING OF ACTUAL SYSTEM OF EQUATIONS OF EVOLUTION
- → photon temperature density
- → baryon density
- → baryon velocity (depends on `tight-coupling approximation=tca`)
- → perturbed recombination has an impact
- → photon temperature higher momenta and photon polarization (depend on `tight-coupling approximation`)
- → if photon `tight-coupling` is off
- → define  $\Pi = G_{\gamma 0} + G_{\gamma 2} + F_{\gamma 2}$
- → photon temperature velocity
- → photon temperature shear
- → photon temperature  $l=3$
- → photon temperature  $l>3$
- → photon temperature  $l_{max}$
- → photon polarization  $l=0$
- → photon polarization  $l=1$
- → photon polarization  $l=2$
- → photon polarization  $l>2$
- → photon polarization  $l_{max\_pol}$
- → if photon `tight-coupling` is on:
- → in that case, only need photon velocity
- → `cdm`
- → newtonian gauge: `cdm` density and velocity
- → synchronous gauge: `cdm` density only (velocity set to zero by definition of the gauge)
- → `dcdm` and `dr`
- → `dcdm`
- → `dr`
- → `dr F0`
- → `dr F1`
- → exact `dr F2`
- → exact `dr l=3`

- —> exact dr  $l > 3$
- —> exact dr `lmax_dr`
- —> fluid (`fld`)
- —> factors `w`, `w_prime`, adiabatic sound speed `ca2` (all three background-related), plus actual sound speed in the fluid rest frame `cs2`
- —> fluid density
- —> fluid velocity
- —> scalar field (`scf`)
- —> field value
- —> Klein Gordon equation
- —> ultra-relativistic neutrino/relics (`ur`)
- —> if radiation streaming approximation is off
- —> ur density
- —> ur velocity
- —> exact ur shear
- —> exact ur  $l=3$
- —> exact ur  $l > 3$
- —> exact ur `lmax_ur`
- —> in fluid approximation (`ufa`): only ur shear needed
- —> non-cold dark matter (`ncdm`): massive neutrinos, WDM, etc.
- —> first case: use a fluid approximation (`ncdmfa`)
- —> loop over species
- —> define intermediate quantities
- —> exact continuity equation
- —> exact euler equation
- —> different ansatz for approximate shear derivative
- —> jump to next species
- —> second case: use exact equation (Boltzmann hierarchy on momentum grid)
- —> loop over species
- —> loop over momentum
- —> define intermediate quantities
- —> `ncdm` density for given momentum bin
- —> `ncdm` velocity for given momentum bin
- —> `ncdm` shear for given momentum bin
- —> `ncdm`  $l > 3$  for given momentum bin
- —> `ncdm` `lmax` for given momentum bin (truncation as in Ma and Bertschinger) but with curvature taken into account a la [arXiv:1305.3261](https://arxiv.org/abs/1305.3261)

- —> jump to next momentum bin or species
- —> metric
- —> eta of synchronous gauge
- vector mode
- -> baryon velocity
- tensor modes:
  - -> non-cold dark matter (ncdm): massive neutrinos, WDM, etc.
  - —> loop over species
  - —> loop over momentum
  - —> define intermediate quantities
  - —> ncdm density for given momentum bin
  - —> ncdm  $l > 0$  for given momentum bin
  - —> ncdm  $l_{max}$  for given momentum bin (truncation as in Ma and Bertschinger) but with curvature taken into account a la arXiv:1305.3261
  - —> jump to next momentum bin or species
  - -> tensor metric perturbation  $h$  (gravitational waves)
  - -> its time-derivative

### 5.13.2.23 perturb\_tca\_slip\_and\_shear()

```
int perturb_tca_slip_and_shear (
    double * y,
    void * parameters_and_workspace,
    ErrorMsg error_message )
```

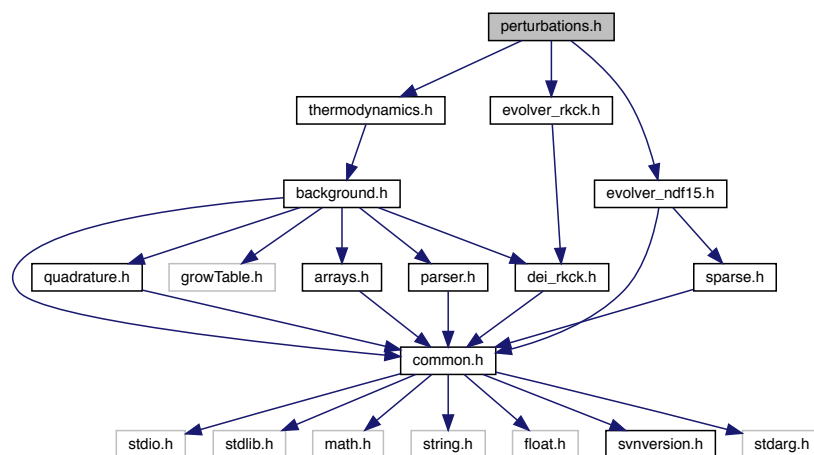
#### Summary:

- define local variables
- rename the fields of the input structure (just to avoid heavy notations)
- compute related background quantities
- -> (a) define short-cut notations for the scalar perturbations
- -> (b) define short-cut notations used only in tight-coupling approximation
- -> (c) compute metric-related quantities (depending on gauge; additional gauges can be coded below)
  - Each continuity equation contains a term in  $(\theta + \text{metric\_continuity})$  with  $\text{metric\_continuity} = (h_{\leftarrow} \text{prime}/2)$  in synchronous gauge,  $(-3 \text{ phi\_prime})$  in newtonian gauge
  - Each Euler equation contains a source term  $\text{metric\_euler}$  with  $\text{metric\_euler} = 0$  in synchronous gauge,  $(k^2 \text{ psi})$  in newtonian gauge
  - Each shear derivative equation contains a source term  $\text{metric\_shear}$  equal to  $\text{metric\_shear} = (h_{\leftarrow} \text{prime} + 6\text{eta\_prime})/2$  in synchronous gauge, 0 in newtonian gauge

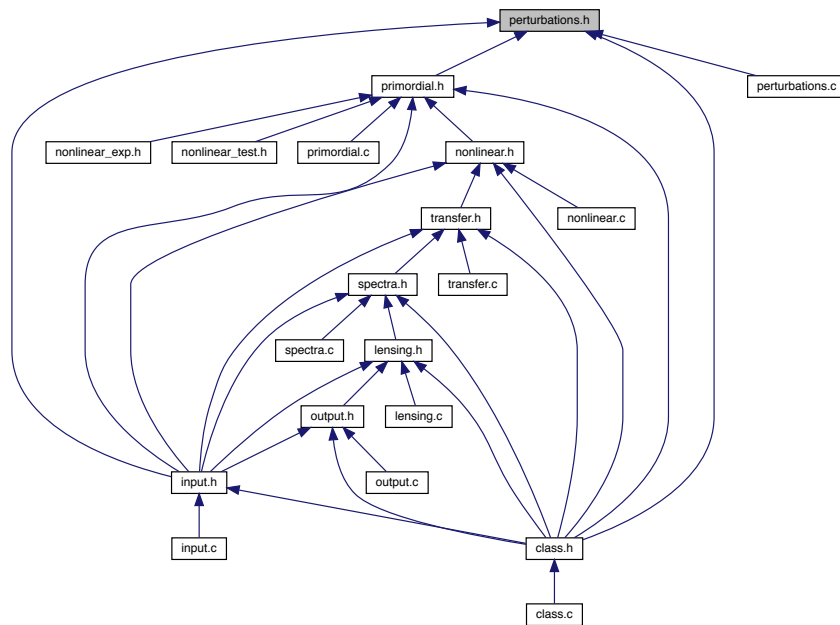
- metric\_shear\_prime is the derivative of metric\_shear
- In the ufa\_class approximation, the leading-order source term is (h\_prime/2) in synchronous gauge, (-3 (phi\_prime+psi\_prime)) in newtonian gauge: we approximate the later by (-6 phi\_prime)
- -> (d) if some approximation schemes are turned on, enforce a few y[ ] values computed in perturb\_einstein
- -> like Ma & Bertschinger
- -> relax assumption  $dkappa \sim a^{-2}$  (like in CAMB)
- -> also relax assumption  $cb2 \sim a^{-1}$
- -> intermediate quantities for 2nd order tca: shear\_g at first order in tight-coupling
- -> intermediate quantities for 2nd order tca: zero order for theta\_b' = theta\_g'
- -> perturbed recombination has an impact
- -> intermediate quantities for 2nd order tca: shear\_g\_prime at first order in tight-coupling
- -> 2nd order as in CRS
- -> 2nd order like in CLASS paper
- -> add only the most important 2nd order terms
- -> store tight-coupling values of photon shear and its derivative

## 5.14 perturbations.h File Reference

```
#include "thermodynamics.h"
#include "evolver_ndf15.h"
#include "evolver_rkck.h"
Include dependency graph for perturbations.h:
```



This graph shows which files directly or indirectly include this file:



## Data Structures

- struct [perturbs](#)
- struct [perturb\\_vector](#)
- struct [perturb\\_workspace](#)
- struct [perturb\\_parameters\\_and\\_workspace](#)

## Macros

- `#define` [\\_MAX\\_NUMBER\\_OF\\_K\\_FILES\\_30](#)

## Enumerations

- enum [tca\\_flags](#)
- enum [tca\\_method](#)
- enum [possible\\_gauges](#) { [newtonian](#), [synchronous](#) }
- `#define` [\\_SELECTION\\_NUM\\_MAX\\_100](#)

### 5.14.1 Detailed Description

Documented includes for perturbation module

### 5.14.2 Data Structure Documentation

#### 5.14.2.1 struct perturbs

Structure containing everything about perturbations that other modules need to know, in particular tabled values of the source functions  $S(k, \tau)$  for all requested modes (scalar/vector/tensor), initial conditions, types (temperature, E-polarization, B-polarization, lensing potential, etc), multipole  $l$  and wavenumber  $k$ .

#### Data Fields

|                     |                              |   |
|---------------------|------------------------------|---|
| short               | has_perturbations            | do we need to compute perturbations at all ?  |
| short               | has_cls                      | do we need any harmonic space spectrum $C_l$ (and hence Bessel functions, transfer functions, ...)?                                     |
| short               | has_scalars                  | do we need scalars?   |
| short               | has_vectors                  | do we need vectors?   |
| short               | has_tensors                  | do we need tensors?   |
| short               | has_ad                       | do we need adiabatic mode?  |
| short               | has_bi                       | do we need isocurvature bi mode?  |
| short               | has_cdi                      | do we need isocurvature cdi mode?   |
| short               | has_nid                      | do we need isocurvature nid mode?   |
| short               | has_niv                      | do we need isocurvature niv mode?   |
| short               | has_perturbed_recombination  | Do we want to consider perturbed temperature and ionization fraction?   |
| enum tensor_methods | tensor_method                | Neutrino contribution to tensors way to treat neutrinos in tensor perturbations(neglect, approximate as massless, take exact equations) |
| short               | evolve_tensor_ur             | will we evolve ur tensor perturbations (either because we have ur species, or we have ncdm species with massless approximation) ?       |
| short               | evolve_tensor_ncdm           | will we evolve ncdm tensor perturbations (if we have ncdm species and we use the exact method) ?  |
| short               | has_cl_cmb_temperature       | do we need $C_l$ 's for CMB temperature?  |
| short               | has_cl_cmb_polarization      | do we need $C_l$ 's for CMB polarization?   |
| short               | has_cl_cmb_lensing_potential | do we need $C_l$ 's for CMB lensing potential?  |
| short               | has_cl_lensing_potential     | do we need $C_l$ 's for galaxy lensing potential?   |
| short               | has_cl_number_count          | do we need $C_l$ 's for density number count?   |
| short               | has_pk_matter                | do we need matter Fourier spectrum?   |
| short               | has_density_transfers        | do we need to output individual matter density transfer functions?  |

## Data Fields

|                     |                                      |   |
|---------------------|--------------------------------------|---|
| short               | has_velocity_transfers               | do we need to output individual matter velocity transfer functions?   |
| short               | has_metricpotential_transfers        | do we need to output individual transfer functions for scalar metric perturbations?   |
| short               | has_nl_corrections_based_on_delta_m  | do we want to compute non-linear corrections with an algorithm relying on delta_m (like halofit)?                                     |
| short               | has_nc_density                       | in dCl, do we want density terms ?  |
| short               | has_nc_rsd                           | in dCl, do we want redshift space distortion terms ?  |
| short               | has_nc_lens                          | in dCl, do we want lensing terms ?  |
| short               | has_nc_gr                            | in dCl, do we want gravity terms ?  |
| int                 | l_scalar_max                         | maximum l value for CMB scalars $C_l$ 's  |
| int                 | l_vector_max                         | maximum l value for CMB vectors $C_l$ 's  |
| int                 | l_tensor_max                         | maximum l value for CMB tensors $C_l$ 's  |
| int                 | l_lss_max                            | maximum l value for LSS $C_l$ 's (density and lensing potential in bins)  |
| double              | k_max_for_pk                         | maximum value of k in 1/Mpc in P(k) (if $C_l$ 's also requested, overseeded by value kmax inferred from l_scalar_max if it is bigger) |
| int                 | selection_num                        | number of selection functions (i.e. bins) for matter density $C_l$ 's   |
| enum selection_type | selection                            | type of selection functions   |
| double              | selection_mean[_SELECTION_NUM_MAX]   | centers of selection functions  |
| double              | selection_width[_SELECTION_NUM_MAX]  | widths of selection functions   |
| int                 | switch_sw                            | in temperature calculation, do we want to include the intrinsic temperature + Sachs Wolfe term?                                       |
| int                 | switch_eisw                          | in temperature calculation, do we want to include the early integrated Sachs Wolfe term?  |
| int                 | switch_lisw                          | in temperature calculation, do we want to include the late integrated Sachs Wolfe term?   |
| int                 | switch_dop                           | in temperature calculation, do we want to include the Doppler term?   |
| int                 | switch_pol                           | in temperature calculation, do we want to include the polarization-related term?  |
| double              | eisw_lisw_split_z                    | at which redshift do we define the cut between eisw and lisw ?  |
| int                 | store_perturbations                  | Do we want to store perturbations?  |
| int                 | k_output_values_num                  | Number of perturbation outputs (default=0)  |
| double              | k_output_values[_MAX_NUMBER_OF_K]    | List of k values where perturbation output is requested.  |
| int *               | index_k_output_values                | List of indices corresponding to k-values close to k_output_values for each mode. [index_md*k_output_values_num+ik]                   |
| char                | scalar_titles[_MAXTITLESTRINGLENGTH] | <i>DELIMITER</i> separated string of titles for scalar perturbation output files.   |

## Data Fields

|                                      |   |   |
|--------------------------------------|---|---|
| char                                 | vector_titles[_MAXTITLESTRINGLENGTH]                | <i>DELIMITER</i> separated string of titles for vector perturbation output files.   |
| char                                 | tensor_titles[_MAXTITLESTRINGLENGTH]                | <i>DELIMITER</i> separated string of titles for tensor perturbation output files.   |
| int                                  | number_of_scalar_titles                             | number of titles/columns in scalar perturbation output files  |
| int                                  | number_of_vector_titles                             | number of titles/columns in vector perturbation output files  |
| int                                  | number_of_tensor_titles                             | number of titles/columns in tensor perturbation output files  |
| double *                             | scalar_perturbations_data[_MAX_NUMBER_OF_FILES]     | Array of double pointers to perturbation output for scalars   |
| double *                             | vector_perturbations_data[_MAX_NUMBER_OF_FILES]     | Array of double pointers to perturbation output for vectors   |
| double *                             | tensor_perturbations_data[_MAX_NUMBER_OF_FILES]     | Array of double pointers to perturbation output for tensors   |
| int                                  | size_scalar_perturbation_data[_MAX_NUMBER_OF_FILES] | Array of sizes of scalar double pointers  |
| int                                  | size_vector_perturbation_data[_MAX_NUMBER_OF_FILES] | Array of sizes of vector double pointers  |
| int                                  | size_tensor_perturbation_data[_MAX_NUMBER_OF_FILES] | Array of sizes of tensor double pointers  |
| double                               | three_ceff2_ur                                      | 3 x effective squared sound speed for the ultrarelativistic perturbations   |
| double                               | three_cvis2_ur                                      | 3 x effective viscosity parameter for the ultrarelativistic perturbations   |
| double                               | z_max_pk  | when we compute only the matter spectrum / transfer functions, but not the CMB, we are sometimes interested to sample source functions at very high redshift, way before recombination. This z_max_pk will then fix the initial sampling time of the sources. |
| short                                | has_cmb   | do we need CMB-related sources (temperature, polarization) ?  |
| short                                | has_lss   | do we need LSS-related sources (lensing potential, ...) ?   |
| enum <a href="#">possible_gauges</a> | gauge   | gauge in which to perform this calculation  |
| int                                  | index_md_scalars                                    | index value for scalars   |
| int                                  | index_md_tensors                                    | index value for tensors   |
| int                                  | index_md_vectors                                    | index value for vectors   |
| int                                  | md_size   | number of modes included in computation   |
| int                                  | index_ic_ad   | index value for adiabatic   |
| int                                  | index_ic_cdi  | index value for CDM isocurvature  |
| int                                  | index_ic_bi   | index value for baryon isocurvature   |
| int                                  | index_ic_nid  | index value for neutrino density isocurvature   |
| int                                  | index_ic_niv  | index value for neutrino velocity isocurvature  |
| int                                  | index_ic_ten  | index value for unique possibility for tensors  |



## Data Fields

|       |                         |  |
|-------|-------------------------|--|
| int * | ic_size                 | for a given mode, ic_size[index_md] = number of initial conditions included in computation |
| short | has_source_t            | do we need source for CMB temperature?   |
| short | has_source_p            | do we need source for CMB polarization?  |
| short | has_source_delta_m      | do we need source for delta of total matter?   |
| short | has_source_delta_g      | do we need source for delta of gammas?   |
| short | has_source_delta_b      | do we need source for delta of baryons?  |
| short | has_source_delta_cdm    | do we need source for delta of cold dark matter?   |
| short | has_source_delta_dcdm   | do we need source for delta of DCDM?   |
| short | has_source_delta_fld    | do we need source for delta of dark energy?  |
| short | has_source_delta_scf    | do we need source for delta from scalar field?   |
| short | has_source_delta_dr     | do we need source for delta of decay radiation?  |
| short | has_source_delta_ur     | do we need source for delta of ultra-relativistic neutrinos/relics?                        |
| short | has_source_delta_ncdm   | do we need source for delta of all non-cold dark matter species (e.g. massive neutrinos)?  |
| short | has_source_theta_m      | do we need source for theta of total matter?   |
| short | has_source_theta_g      | do we need source for theta of gammas?   |
| short | has_source_theta_b      | do we need source for theta of baryons?  |
| short | has_source_theta_cdm    | do we need source for theta of cold dark matter?   |
| short | has_source_theta_dcdm   | do we need source for theta of DCDM?   |
| short | has_source_theta_fld    | do we need source for theta of dark energy?  |
| short | has_source_theta_scf    | do we need source for theta of scalar field?   |
| short | has_source_theta_dr     | do we need source for theta of ultra-relativistic neutrinos/relics?                        |
| short | has_source_theta_ur     | do we need source for theta of ultra-relativistic neutrinos/relics?                        |
| short | has_source_theta_ncdm   | do we need source for theta of all non-cold dark matter species (e.g. massive neutrinos)?  |
| short | has_source_phi          | do we need source for metric fluctuation phi?  |
| short | has_source_phi_prime    | do we need source for metric fluctuation phi'?   |
| short | has_source_phi_plus_psi | do we need source for metric fluctuation (phi+psi)?  |
| short | has_source_psi          | do we need source for metric fluctuation psi?  |

## Data Fields

|       |  |  |
|-------|--|--|
| short | has_source_h                               | do we need source for metric fluctuation h?  |
| short | has_source_h_prime                         | do we need source for metric fluctuation h'?   |
| short | has_source_eta                             | do we need source for metric fluctuation eta?  |
| short | has_source_eta_prime                       | do we need source for metric fluctuation eta'?                                       |
| int   | index_tp_t0                                | index value for temperature (j=0 term)   |
| int   | index_tp_t1                                | index value for temperature (j=1 term)   |
| int   | index_tp_t2                                | index value for temperature (j=2 term)   |
| int   | index_tp_p                                 | index value for polarization   |
| int   | index_tp_delta_m                           | index value for delta tot  |
| int   | index_tp_delta_g                           | index value for delta of gammas  |
| int   | index_tp_delta_b                           | index value for delta of baryons   |
| int   | index_tp_delta_cdm                         | index value for delta of cold dark matter  |
| int   | index_tp_delta_dcdm                        | index value for delta of DCDM  |
| int   | index_tp_delta_fld                         | index value for delta of dark energy   |
| int   | index_tp_delta_scf                         | index value for delta of scalar field  |
| int   | index_tp_delta_dr                          | index value for delta of decay radiation   |
| int   | index_tp_delta_ur                          | index value for delta of ultra-relativistic neutrinos/relics                         |
| int   | index_tp_delta_ncdm1                       | index value for delta of first non-cold dark matter species (e.g. massive neutrinos) |
| int   | index_tp_perturbed_recombination_delta_t0  | temp temperature perturbation  |
| int   | index_tp_perturbed_recombination_delta_chi | ionization fraction perturbation   |
| int   | index_tp_theta_m                           | index value for theta tot  |
| int   | index_tp_theta_g                           | index value for theta of gammas  |
| int   | index_tp_theta_b                           | index value for theta of baryons   |
| int   | index_tp_theta_cdm                         | index value for theta of cold dark matter  |
| int   | index_tp_theta_dcdm                        | index value for theta of DCDM  |
| int   | index_tp_theta_fld                         | index value for theta of dark energy   |
| int   | index_tp_theta_scf                         | index value for theta of scalar field  |
| int   | index_tp_theta_ur                          | index value for theta of ultra-relativistic neutrinos/relics                         |
| int   | index_tp_theta_dr                          | index value for F1 of decay radiation  |
| int   | index_tp_theta_ncdm1                       | index value for theta of first non-cold dark matter species (e.g. massive neutrinos) |
| int   | index_tp_phi                               | index value for metric fluctuation phi   |
| int   | index_tp_phi_prime                         | index value for metric fluctuation phi'  |
| int   | index_tp_phi_plus_psi                      | index value for metric fluctuation phi+psi   |
| int   | index_tp_psi                               | index value for metric fluctuation psi   |
| int   | index_tp_h                                 | index value for metric fluctuation h   |
| int   | index_tp_h_prime                           | index value for metric fluctuation h'  |
| int   | index_tp_eta                               | index value for metric fluctuation eta   |
| int   | index_tp_eta_prime                         | index value for metric fluctuation eta'  |

## Data Fields

|            |                          |   |
|------------|--------------------------|---|
| int *      | tp_size                  | number of types tp_size[index_md] included in computation for each mode   |
| int *      | k_size_cmb               | k_size_cmb[index_md] number of k values used for CMB calculations, requiring a fine sampling in k-space   |
| int *      | k_size_cl                | k_size_cl[index_md] number of k values used for non-CMB $C_l$ calculations, requiring a coarse sampling in k-space.                                   |
| int *      | k_size                   | k_size[index_md] = total number of k values, including those needed for P(k) but not for $C_l$ 's   |
| double **  | k                        | k[index_md][index_k] = list of values   |
| double     | k_min                    | minimum value (over all modes)  |
| double     | k_max                    | maximum value (over all modes)  |
| int        | tau_size                 | tau_size = number of values   |
| double *   | tau_sampling             | tau_sampling[index_tau] = list of tau values  |
| double     | selection_min_of_tau_min | used in presence of selection functions (for matter density, cosmic shear...)   |
| double     | selection_max_of_tau_max | used in presence of selection functions (for matter density, cosmic shear...)   |
| double     | selection_delta_tau      | used in presence of selection functions (for matter density, cosmic shear...)   |
| double *   | selection_tau_min        | value of conformal time below which W(tau) is considered to vanish for each bin   |
| double *   | selection_tau_max        | value of conformal time above which W(tau) is considered to vanish for each bin   |
| double *   | selection_tau            | value of conformal time at the center of each bin   |
| double *   | selection_function       | selection function W(tau), normalized to $\int W(\tau) d\tau = 1$ , stored in selection_function[bin*ppt->tau_↔size+index_tau]                        |
| double *** | sources                  | Pointer towards the source interpolation table sources[index_md] [index_ic * ppt->tp_size[index_md] + index_type] [index_tau * ppt->k_size + index_k] |
| short      | perturbations_verbose    | flag regulating the amount of information sent to standard output (none if set to zero)   |
| ErrorMsg   | error_message            | zone for writing error messages   |

## 5.14.2.2 struct perturb\_vector

Structure containing the indices and the values of the perturbation variables which are integrated over time (as well as their time-derivatives). For a given wavenumber, the size of these vectors changes when the approximation scheme changes.

## Data Fields

|       |   |   |
|-------|---|---|
| int   | index_pt_delta_g                            | photon density  |
| int   | index_pt_theta_g                            | photon velocity   |
| int   | index_pt_shear_g                            | photon shear  |
| int   | index_pt_l3_g                               | photon l=3  |
| int   | l_max_g                                     | max momentum in Boltzmann hierarchy (at least 3)  |
| int   | index_pt_pol0_g                             | photon polarization, l=0  |
| int   | index_pt_pol1_g                             | photon polarization, l=1  |
| int   | index_pt_pol2_g                             | photon polarization, l=2  |
| int   | index_pt_pol3_g                             | photon polarization, l=3  |
| int   | l_max_pol_g                                 | max momentum in Boltzmann hierarchy (at least 3)  |
| int   | index_pt_delta_b                            | baryon density  |
| int   | index_pt_theta_b                            | baryon velocity   |
| int   | index_pt_delta_cdm                          | cdm density   |
| int   | index_pt_theta_cdm                          | cdm velocity  |
| int   | index_pt_delta_dcdm                         | dcdm density  |
| int   | index_pt_theta_dcdm                         | dcdm velocity   |
| int   | index_pt_delta_fld                          | dark energy density in true fluid case  |
| int   | index_pt_theta_fld                          | dark energy velocity in true fluid case   |
| int   | index_pt_Gamma_fld                          | unique dark energy dynamical variable in PPF case   |
| int   | index_pt_phi_scf                            | scalar field density  |
| int   | index_pt_phi_prime_scf                      | scalar field velocity   |
| int   | index_pt_delta_ur                           | density of ultra-relativistic neutrinos/relics  |
| int   | index_pt_theta_ur                           | velocity of ultra-relativistic neutrinos/relics   |
| int   | index_pt_shear_ur                           | shear of ultra-relativistic neutrinos/relics  |
| int   | index_pt_l3_ur                              | l=3 of ultra-relativistic neutrinos/relics  |
| int   | l_max_ur                                    | max momentum in Boltzmann hierarchy (at least 3)  |
| int   | index_pt_perturbed_recombination_delta_temp | Gas temperature perturbation  |
| int   | index_pt_perturbed_recombination_delta_chi  | Ionization fraction perturbation  |
| int   | index_pt_F0_dr                              | The index to the first Legendre multipole of the DR expansion. Not that this is not exactly the usual delta, see Kaplinghat et al., astro-ph/9907388. |
| int   | l_max_dr                                    | max momentum in Boltzmann hierarchy for dr)   |
| int   | index_pt_psi0_ncdm1                         | first multipole of perturbation of first ncdm species, Psi_0  |
| int   | N_ncdm                                      | number of distinct non-cold-dark-matter (ncdm) species  |
| int * | l_max_ncdm                                  | mutipole l at which Boltzmann hierarchy is truncated (for each ncdm species)  |
| int * | q_size_ncdm                                 | number of discrete momenta (for each ncdm species)  |
| int   | index_pt_eta                                | synchronous gauge metric perturbation eta   |
| int   | index_pt_phi                                | newtonian gauge metric perturbation phi   |
| int   | index_pt_hv_prime                           | vector metric perturbation h_v' in synchronous gauge  |

## Data Fields

|          |                 |   |
|----------|-----------------|---|
| int      | index_pt_V      | vector metric perturbation V in Newtonian gauge   |
| int      | index_pt_gw     | tensor metric perturbation h (gravitational waves)  |
| int      | index_pt_gwdot  | its time-derivative   |
| int      | pt_size         | size of perturbation vector   |
| double * | y               | vector of perturbations to be integrated  |
| double * | dy              | time-derivative of the same vector  |
| int *    | used_in_sources | boolean array specifying which perturbations enter in the calculation of source functions |

## 5.14.2.3 struct perturb\_workspace

Workspace containing, among other things, the value at a given time of all background/perturbed quantities, as well as their indices. There will be one such structure created for each mode (scalar/.../tensor) and each thread (in case of parallel computing)

## Data Fields

|  |   |                         |  |
|--|---|-------------------------|--|
|  | int                                     | index_mt_psi            | psi in longitudinal gauge  |
|  | int                                     | index_mt_phi_prime      | (d phi/d conf.time) in longitudinal gauge  |
|  | int                                     | index_mt_h_prime        | h' (wrt conf. time) in synchronous gauge   |
|  | int                                     | index_mt_h_prime_prime  | h'' (wrt conf. time) in synchronous gauge  |
|  | int                                     | index_mt_eta_prime      | eta' (wrt conf. time) in synchronous gauge                                       |
|  | int                                     | index_mt_alpha          | $\alpha = (h' + 6\eta')/(2k^2)$ in synchronous gauge                             |
|  | int                                     | index_mt_alpha_prime    | $\alpha'$ wrt conf. time) in synchronous gauge                                   |
|  | int                                     | index_mt_gw_prime_prime | second derivative wrt conformal time of gravitational wave field, often called h |
|  | int                                     | index_mt_V_prime        | derivative of Newtonian gauge vector metric perturbation V                       |
|  | int                                     | index_mt_hv_prime_prime | Second derivative of Synchronous gauge vector metric perturbation $h_v$          |
|  | int                                     | mt_size                 | size of metric perturbation vector   |
|  | double *                                | pvecback                | background quantities  |
|  | double *                                | pvecthermo              | thermodynamics quantities  |
|  | double *                                | pvecmetric              | metric quantities  |
|  | struct <a href="#">perturb_vector</a> * | pv                      | pointer to vector of integrated perturbations and their time-derivatives         |
|  | double                                  | delta_rho               | total density perturbation (gives delta Too)                                     |
|  | double                                  | rho_plus_p_theta        | total (rho+p)*theta perturbation (gives delta Toi)                               |
|  | double                                  | rho_plus_p_shear        | total (rho+p)*shear (gives delta Tij)  |
|  | double                                  | delta_p                 | total pressure perturbation (gives Tii)  |
|  | double                                  | gw_source               | stress-energy source term in Einstein's tensor equations (gives Tij[tensor])     |
|  | double                                  | vector_source_pi        | first stress-energy source term in Einstein's vector equations                   |
|  | double                                  | vector_source_v         | second stress-energy source term in Einstein's vector equations                  |

## Data Fields

|          |                      |  |
|----------|----------------------|--|
| double   | tca_shear_g          | photon shear in tight-coupling approximation   |
| double   | tca_slip             | photon-baryon slip in tight-coupling approximation   |
| double   | rsa_delta_g          | photon density in radiation streaming approximation  |
| double   | rsa_theta_g          | photon velocity in radiation streaming approximation   |
| double   | rsa_delta_ur         | photon density in radiation streaming approximation  |
| double   | rsa_theta_ur         | photon velocity in radiation streaming approximation   |
| double * | delta_ncdm           | relative density perturbation of each ncdm species   |
| double * | theta_ncdm           | velocity divergence theta of each ncdm species   |
| double * | shear_ncdm           | shear for each ncdm species  |
| double   | delta_m              | relative density perturbation of all non-relativistic species  |
| double   | theta_m              | velocity divergence theta of all non-relativistic species  |
| double   | delta_rho_fld        | density perturbation of fluid, not so trivial in PPF scheme  |
| double   | rho_plus_p_theta_fld | velocity divergence of fluid, not so trivial in PPF scheme   |
| double   | S_fld                | S quantity sourcing Gamma_prime evolution in PPF scheme (equivalent to eq. 15 in 0808.3125)  |
| double   | Gamma_prime_fld      | Gamma_prime in PPF scheme (equivalent to eq. 14 in 0808.3125)  |
| FILE *   | perturb_output_file  | filepointer to output file   |
| int      | index_ikout          | index for output k value (when k_output_values is set)   |
| short    | inter_mode           | flag defining the method used for interpolation background/thermo quantities tables  |
| int      | last_index_back      | the background interpolation function <a href="#">background_at_tau()</a> keeps memory of the last point called through this index       |
| int      | last_index_thermo    | the thermodynamics interpolation function <a href="#">thermodynamics_at_z()</a> keeps memory of the last point called through this index |
| int      | index_ap_tca         | index for tight-coupling approximation   |
| int      | index_ap_rsa         | index for radiation streaming approximation  |
| int      | index_ap_ufa         | index for ur fluid approximation   |
| int      | index_ap_ncdmfa      | index for ncdm fluid approximation   |
| int      | ap_size              | number of relevant approximations for a given mode   |
| int *    | approx               | array of approximation flags holding at a given time: approx[index_ap]   |
| int      | max_l_max            | maximum l_max for any multipole  |
| double * | s_l                  | array of freestreaming coefficients<br>$s_l = \sqrt{1 - K * (l^2 - 1) / k^2}$  |

## 5.14.2.4 struct perturb\_parameters\_and\_workspace

Structure pointing towards all what the function that perturb\_derivs needs to know: fixed input parameters and indices contained in the various structures, workspace, etc.

## Data Fields

|  |          |  |
|--|----------|--|
| struct <a href="#">precision</a> *         | ppr      | pointer to the precision structure                         |
| struct <a href="#">background</a> *        | pba      | pointer to the background structure                        |
| struct <a href="#">thermo</a> *            | pth      | pointer to the thermodynamics structure                    |
| struct <a href="#">perturbs</a> *          | ppt      | pointer to the precision structure                         |
| int  | index_md | index of mode (scalar/.../vector/tensor)                   |
| int  | index_ic | index of initial condition (adiabatic/isocurvature(s)/...) |
| int  | index_k  | index of wavenumber  |
| double                                     | k        | current value of wavenumber in 1/Mpc                       |
| struct <a href="#">perturb_workspace</a> * | ppw      | workspace defined above                                    |

## 5.14.3 Macro Definition Documentation

5.14.3.1 `_SELECTION_NUM_MAX_`

```
#define _SELECTION_NUM_MAX_ 100
```

maximum number and types of selection function (for bins of matter density or cosmic shear)

5.14.3.2 `_MAX_NUMBER_OF_K_FILES_`

```
#define _MAX_NUMBER_OF_K_FILES_ 30
```

maximum number of k-values for perturbation output

## 5.14.4 Enumeration Type Documentation

5.14.4.1 `tca_flags`

```
enum tca\_flags
```

flags for various approximation schemes (tca = tight-coupling approximation, rsa = radiation streaming approximation, ufa = massless neutrinos / ultra-relativistic relics fluid approximation)

CAUTION: must be listed below in chronological order, and cannot be reversible. When integrating equations for a given mode, it is only possible to switch from left to right in the lists below.

5.14.4.2 `tca_method`

```
enum tca\_method
```

labels for the way in which each approximation scheme is implemented

5.14.4.3 `possible_gauges`

```
enum possible\_gauges
```

List of coded gauges. More gauges can in principle be defined.

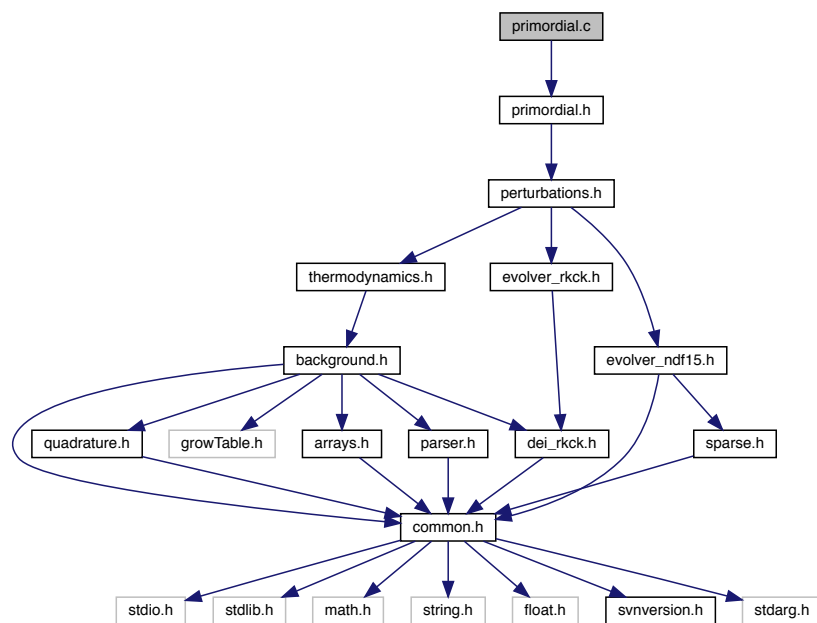
## Enumerator

|             |   |
|-------------|---|
| newtonian   | newtonian (or longitudinal) gauge                       |
| synchronous | synchronous gauge with $\theta_{cdm} = 0$ by convention |

## 5.15 primordial.c File Reference

```
#include "primordial.h"
```

Include dependency graph for primordial.c:



## Functions

- int `primordial_spectrum_at_k` (struct `primordial` \*ppm, int index\_md, enum `linear_or_logarithmic` mode, double input, double \*output)
- int `primordial_init` (struct `precision` \*ppr, struct `perturbs` \*ppt, struct `primordial` \*ppm)
- int `primordial_free` (struct `primordial` \*ppm)
- int `primordial_indices` (struct `perturbs` \*ppt, struct `primordial` \*ppm)
- int `primordial_get_lnk_list` (struct `primordial` \*ppm, double kmin, double kmax, double k\_per\_decade)
- int `primordial_analytic_spectrum_init` (struct `perturbs` \*ppt, struct `primordial` \*ppm)
- int `primordial_analytic_spectrum` (struct `primordial` \*ppm, int index\_md, int index\_ic1\_ic2, double k, double \*pk)
- int `primordial_inflation_potential` (struct `primordial` \*ppm, double phi, double \*V, double \*dV, double \*ddV)
- int `primordial_inflation_hubble` (struct `primordial` \*ppm, double phi, double \*H, double \*dH, double \*ddH, double \*dddH)
- int `primordial_inflation_indices` (struct `primordial` \*ppm)
- int `primordial_inflation_solve_inflation` (struct `perturbs` \*ppt, struct `primordial` \*ppm, struct `precision` \*ppr)



- int [primordial\\_inflation\\_analytic\\_spectra](#) (struct [perturbs](#) \*ppt, struct [primordial](#) \*ppm, struct [precision](#) \*ppr, double \*y\_ini)
- int [primordial\\_inflation\\_spectra](#) (struct [perturbs](#) \*ppt, struct [primordial](#) \*ppm, struct [precision](#) \*ppr, double \*y\_ini)
- int [primordial\\_inflation\\_one\\_wavenumber](#) (struct [perturbs](#) \*ppt, struct [primordial](#) \*ppm, struct [precision](#) \*ppr, double \*y\_ini, int index\_k)
- int [primordial\\_inflation\\_one\\_k](#) (struct [primordial](#) \*ppm, struct [precision](#) \*ppr, double k, double \*y, double \*dy, double \*curvature, double \*tensor)
- int [primordial\\_inflation\\_find\\_attractor](#) (struct [primordial](#) \*ppm, struct [precision](#) \*ppr, double phi\_0, double [precision](#), double \*y, double \*dy, double \*H\_0, double \*dphid\_0)
- int [primordial\\_inflation\\_evolve\\_background](#) (struct [primordial](#) \*ppm, struct [precision](#) \*ppr, double \*y, double \*dy, enum [target\\_quantity](#) target, double stop, short check\_epsilon, enum [integration\\_direction](#) direction, enum [time\\_definition](#) time)
- int [primordial\\_inflation\\_check\\_potential](#) (struct [primordial](#) \*ppm, double phi, double \*V, double \*dV, double \*ddV)
- int [primordial\\_inflation\\_check\\_hubble](#) (struct [primordial](#) \*ppm, double phi, double \*H, double \*dH, double \*ddH, double \*dddH)
- int [primordial\\_inflation\\_get\\_epsilon](#) (struct [primordial](#) \*ppm, double phi, double \*epsilon)
- int [primordial\\_inflation\\_find\\_phi\\_pivot](#) (struct [primordial](#) \*ppm, struct [precision](#) \*ppr, double \*y, double \*dy)
- int [primordial\\_inflation\\_derivs](#) (double tau, double \*y, double \*dy, void \*parameters\_and\_workspace, Error↔ Msg error\_message)
- int [primordial\\_external\\_spectrum\\_init](#) (struct [perturbs](#) \*ppt, struct [primordial](#) \*ppm)

### 5.15.1 Detailed Description

Documented primordial module.

Julien Lesgourgues, 24.08.2010

This module computes the primordial spectra. It can be used in different modes: simple parametric form, evolving inflaton perturbations, etc. So far only the mode corresponding to a simple analytic form in terms of amplitudes, tilts and runnings has been developed.

The following functions can be called from other modules:

1. [primordial\\_init\(\)](#) at the beginning (anytime after [perturb\\_init\(\)](#) and before [spectra\\_init\(\)](#))
2. [primordial\\_spectrum\\_at\\_k\(\)](#) at any time for computing  $P(k)$  at any  $k$
3. [primordial\\_free\(\)](#) at the end

### 5.15.2 Function Documentation

### 5.15.2.1 `primordial_spectrum_at_k()`

```
int primordial_spectrum_at_k (
    struct primordial * ppm,
    int index_md,
    enum linear_or_logarithmic mode,
    double input,
    double * output )
```

Primordial spectra for arbitrary argument and for all initial conditions.

This routine evaluates the primordial spectrum at a given value of k by interpolating in the pre-computed table.

When k is not in the pre-computed range but the spectrum can be found analytically, it finds it. Otherwise returns an error.

Can be called in two modes; linear or logarithmic:

- linear: takes k, returns P(k)
- logarithmic: takes ln(k), return ln(P(k))

One little subtlety: in case of several correlated initial conditions, the cross-correlation spectrum can be negative. Then, in logarithmic mode, the non-diagonal elements contain the cross-correlation angle  $P_{12}/\sqrt{P_{11}P_{22}}$  (from -1 to 1) instead of  $\ln P_{12}$

This function can be called from whatever module at whatever time, provided that `primordial_init()` has been called before, and `primordial_free()` has not been called yet.

#### Parameters

|                 |  |
|-----------------|--|
| <i>ppm</i>      | Input: pointer to primordial structure containing tabulated primordial spectrum  |
| <i>index_md</i> | Input: index of mode (scalar, tensor, ...)   |
| <i>mode</i>     | Input: linear or logarithmic   |
| <i>input</i>    | Input: wavenumber in 1/Mpc (linear mode) or its logarithm (logarithmic mode)   |
| <i>output</i>   | Output: for each pair of initial conditions, primordial spectra P(k) in $Mpc^3$ (linear mode), or their logarithms and cross-correlation angles (logarithmic mode) |

#### Returns

the error status

#### Summary:

- define local variables
- infer ln(k) from input. In linear mode, reject negative value of input k value.
- if ln(k) is not in the interpolation range, return an error, unless we are in the case of a analytic spectrum, for which a direct computation is possible
- otherwise, interpolate in the pre-computed table

## 5.15.2.2 primordial\_init()

```
int primordial_init (
    struct precision * ppr,
    struct perturbs * ppt,
    struct primordial * ppm )
```

This routine initializes the primordial structure (in particular, it computes table of primordial spectrum values)

## Parameters

|            |   |
|------------|---|
| <i>ppr</i> | Input: pointer to precision structure (defines method and precision for all computations) |
| <i>ppt</i> | Input: pointer to perturbation structure (useful for knowing k_min, k_max, etc.)          |
| <i>ppm</i> | Output: pointer to initialized primordial structure                                       |

## Returns

the error status

## Summary:

- define local variables
- check that we really need to compute the primordial spectra
- get kmin and kmax from perturbation structure. Test that they make sense.
- allocate and fill values of  $\ln k$ 's
- define indices and allocate tables in primordial structure
- deal with case of analytic primordial spectra (with amplitudes, tilts, runnings, etc.)
- deal with case of inflation with given  $V(\phi)$  or  $H(\phi)$
- deal with the case of external calculation of  $P_k$
- compute second derivative of each  $\ln P_k$  versus  $\ln k$  with spline, in view of interpolation
- derive spectral parameters from numerically computed spectra (not used by the rest of the code, but useful to keep in memory for several types of investigation)

- expression for  $\alpha_s$  comes from:

$$ns_2 = (\ln pk_{plus} - \ln pk_{pivot}) / (dlnk) + 1$$

$$ns_1 = (\ln pk_{pivot} - \ln pk_{minus}) / (dlnk) + 1$$

$$\alpha_s = dns / dlnk = (ns_2 - ns_1) / dlnk = (\ln pk_{plus} - \ln pk_{pivot} - \ln pk_{pivot} + \ln pk_{minus}) / (dlnk) / (dlnk)$$

- expression for  $\beta_s$ :

$$ppm \rightarrow \beta_s = (\alpha_{plus} - \alpha_{minus}) / dlnk = (\ln pk_{plusplus} - 2 * \ln pk_{pivot} + \ln pk_{minus} - (\ln pk_{pivot} - 2 * \ln pk_{minus} + \ln pk_{minusminus}) / \text{pow}(dlnk, 3))$$

## 5.15.2.3 primordial\_free()

```
int primordial_free (
    struct primordial * ppm )
```

This routine frees all the memory space allocated by `primordial_init()`.

To be called at the end of each run.

**Parameters**

|            |   |
|------------|---|
| <i>ppm</i> | Input: pointer to primordial structure (which fields must be freed) |
|------------|---|

**Returns**

the error status

**5.15.2.4 primordial\_indices()**

```
int primordial_indices (
    struct perturbs * ppt,
    struct primordial * ppm )
```

This routine defines indices and allocates tables in the primordial structure

**Parameters**

|            |   |
|------------|---|
| <i>ppt</i> | Input: pointer to perturbation structure      |
| <i>ppm</i> | Input/output: pointer to primordial structure |

**Returns**

the error status

**5.15.2.5 primordial\_get\_lnk\_list()**

```
int primordial_get_lnk_list (
    struct primordial * ppm,
    double kmin,
    double kmax,
    double k_per_decade )
```

This routine allocates and fills the list of wavenumbers k

**Parameters**

|                     |   |
|---------------------|---|
| <i>ppm</i>          | Input/output: pointer to primordial structure |
| <i>kmin</i>         | Input: first value                            |
| <i>kmax</i>         | Input: last value that we should encompass    |
| <i>k_per_decade</i> | Input: number of k per decade                 |

**Returns**

the error status

**5.15.2.6 primordial\_analytic\_spectrum\_init()**

```
int primordial_analytic_spectrum_init (
    struct perturbs * ppt,
    struct primordial * ppm )
```

This routine interprets and stores in a condensed form the input parameters in the case of a simple analytic spectra with amplitudes, tilts, runnings, in such way that later on, the spectrum can be obtained by a quick call to the routine `primordial_analytic_spectrum()`

**Parameters**

|            |   |
|------------|---|
| <i>ppt</i> | Input: pointer to perturbation structure      |
| <i>ppm</i> | Input/output: pointer to primordial structure |

**Returns**

the error status

**5.15.2.7 primordial\_analytic\_spectrum()**

```
int primordial_analytic_spectrum (
    struct primordial * ppm,
    int index_md,
    int index_ic1_ic2,
    double k,
    double * pk )
```

This routine returns the primordial spectrum in the simple analytic case with amplitudes, tilts, runnings, for each mode (scalar/tensor...), pair of initial conditions, and wavenumber.

**Parameters**

|                      |  |
|----------------------|--|
| <i>ppm</i>           | Input/output: pointer to primordial structure                          |
| <i>index_md</i>      | Input: index of mode (scalar, tensor, ...)                             |
| <i>index_ic1_ic2</i> | Input: pair of initial conditions (ic1, ic2)                           |
| <i>k</i>             | Input: wavenumber in same units as pivot scale, i.e. in 1/Mpc          |
| <i>pk</i>            | Output: primordial power spectrum $A (k/k_{\text{pivot}})^{(n+\dots)}$ |

**Returns**

the error status

**5.15.2.8 primordial\_inflation\_potential()**

```
int primordial_inflation_potential (
    struct primordial * ppm,
    double phi,
    double * V,
    double * dV,
    double * ddV )
```

This routine encodes the inflaton scalar potential

**Parameters**

|            |   |
|------------|---|
| <i>ppm</i> | Input: pointer to primordial structure                        |
| <i>phi</i> | Input: background inflaton field value in units of Mp         |
| <i>V</i>   | Output: inflaton potential in units of $Mp^4$                 |
| <i>dV</i>  | Output: first derivative of inflaton potential wrt the field  |
| <i>ddV</i> | Output: second derivative of inflaton potential wrt the field |

**Returns**

the error status

**5.15.2.9 primordial\_inflation\_hubble()**

```
int primordial_inflation_hubble (
    struct primordial * ppm,
    double phi,
    double * H,
    double * dH,
    double * ddH,
    double * dddH )
```

This routine encodes the function  $H(\phi)$

**Parameters**

|             |   |
|-------------|---|
| <i>ppm</i>  | Input: pointer to primordial structure                |
| <i>phi</i>  | Input: background inflaton field value in units of Mp |
| <i>H</i>    | Output: Hubble parameters in units of Mp              |
| <i>dH</i>   | Output: $dH/d\phi$                                    |
| <i>ddH</i>  | Output: $d^2H/d\phi^2$                                |
| <i>dddH</i> | Output: $d^3H/d\phi^3$                                |

**Returns**

the error status

**5.15.2.10 primordial\_inflation\_indices()**

```
int primordial_inflation_indices (
    struct primordial * ppm )
```

This routine defines indices used by the inflation simulator

**Parameters**

|            |   |
|------------|---|
| <i>ppm</i> | Input/output: pointer to primordial structure |
|------------|---|

**Returns**

the error status

**5.15.2.11 primordial\_inflation\_solve\_inflation()**

```
int primordial_inflation_solve_inflation (
    struct perturbs * ppt,
    struct primordial * ppm,
    struct precision * ppr )
```

Main routine of inflation simulator. Its goal is to check the background evolution before and after the pivot value  $\phi = \phi_{\text{pivot}}$ , and then, if this evolution is suitable, to call the routine [primordial\\_inflation\\_spectra\(\)](#).

**Parameters**

|            |   |
|------------|---|
| <i>ppt</i> | Input: pointer to perturbation structure      |
| <i>ppm</i> | Input/output: pointer to primordial structure |
| <i>ppr</i> | Input: pointer to precision structure         |

**Returns**

the error status

**Summary:**

- define local variables
- allocate vectors for background/perturbed quantities
- eventually, needs first to find  $\phi_{\text{pivot}}$

- compute  $H_{\text{pivot}}$  at  $\phi_{\text{pivot}}$
- check positivity and negative slope of potential in field pivot value, and find value of  $\phi_{\text{dot}}$  and  $H$  for field's pivot value, assuming slow-roll attractor solution has been reached. If no solution, code will stop there.
- check positivity and negative slope of  $H(\phi)$  in field pivot value, and get  $H_{\text{pivot}}$
- find  $a_{\text{pivot}}$ , value of scale factor when  $k_{\text{pivot}}$  crosses horizon while  $\phi=\phi_{\text{pivot}}$
- integrate background solution starting from  $\phi_{\text{pivot}}$  and until  $k_{\text{max}} \gg aH$ . This ensures that the inflationary model considered here is valid and that the primordial spectrum can be computed. Otherwise, if slow-roll brakes too early, model is not suitable and run stops.
- starting from this time, i.e. from  $y_{\text{ini}}[ ]$ , we run the routine which takes care of computing the primordial spectrum.
- before ending, we want to compute and store the values of  $\phi$  corresponding to  $k=aH$  for  $k_{\text{min}}$  and  $k_{\text{max}}$
- finally, we can de-allocate

#### 5.15.2.12 primordial\_inflation\_analytic\_spectra()

```
int primordial_inflation_analytic_spectra (
    struct perturbs * ppt,
    struct primordial * ppm,
    struct precision * ppr,
    double * y_ini )
```

Routine for the computation of an analytic approximation to the the primordial spectrum. In general, should be used only for comparing with exact numerical computation performed by [primordial\\_inflation\\_spectra\(\)](#).

#### Parameters

|                    |  |
|--------------------|--|
| <code>ppt</code>   | Input: pointer to perturbation structure   |
| <code>ppm</code>   | Input/output: pointer to primordial structure  |
| <code>ppr</code>   | Input: pointer to precision structure  |
| <code>y_ini</code> | Input: initial conditions for the vector of background/perturbations, already allocated and filled |

#### Returns

the error status

#### Summary

- allocate vectors for background/perturbed quantities
- initialize the background part of the running vector
- loop over Fourier wavenumbers
- read value of  $\phi$  at time when  $k=aH$
- get potential (and its derivatives) at this value
- calculate the analytic slow-roll formula for the spectra
- store the obtained result for curvature and tensor perturbations



5.15.2.13 `primordial_inflation_spectra()`

```
int primordial_inflation_spectra (
    struct perturbs * ppt,
    struct primordial * ppm,
    struct precision * ppr,
    double * y_ini )
```

Routine with a loop over wavenumbers for the computation of the primordial spectrum. For each wavenumber it calls `primordial_inflation_one_wavenumber()`

## Parameters

|                    |  |
|--------------------|--|
| <code>ppt</code>   | Input: pointer to perturbation structure   |
| <code>ppm</code>   | Input/output: pointer to primordial structure  |
| <code>ppr</code>   | Input: pointer to precision structure  |
| <code>y_ini</code> | Input: initial conditions for the vector of background/perturbations, already allocated and filled |

## Returns

the error status

5.15.2.14 `primordial_inflation_one_wavenumber()`

```
int primordial_inflation_one_wavenumber (
    struct perturbs * ppt,
    struct primordial * ppm,
    struct precision * ppr,
    double * y_ini,
    int index_k )
```

Routine coordinating the computation of the primordial spectrum for one wavenumber. It calls `primordial_inflation_one_k()` to integrate the perturbation equations, and then it stores the result for the scalar/tensor spectra.

## Parameters

|                      |  |
|----------------------|--|
| <code>ppt</code>     | Input: pointer to perturbation structure   |
| <code>ppm</code>     | Input/output: pointer to primordial structure  |
| <code>ppr</code>     | Input: pointer to precision structure  |
| <code>y_ini</code>   | Input: initial conditions for the vector of background/perturbations, already allocated and filled |
| <code>index_k</code> | Input: index of wavenumber to be considered  |

## Returns

the error status

## Summary

- allocate vectors for background/perturbed quantities
- initialize the background part of the running vector
  - evolve the background until the relevant initial time for integrating perturbations
  - evolve the background/perturbation equations from this time and until some time after Horizon crossing
- store the obtained result for curvature and tensor perturbations

### 5.15.2.15 primordial\_inflation\_one\_k()

```
int primordial_inflation_one_k (
    struct primordial * ppm,
    struct precision * ppr,
    double k,
    double * y,
    double * dy,
    double * curvature,
    double * tensor )
```

Routine integrating the background plus perturbation equations for each wavenumber, and returning the scalar and tensor spectrum.

#### Parameters

|                        |  |
|------------------------|--|
| <code>ppm</code>       | Input: pointer to primordial structure   |
| <code>ppr</code>       | Input: pointer to precision structure  |
| <code>k</code>         | Input: Fourier wavenumber  |
| <code>y</code>         | Input: running vector of background/perturbations, already allocated and initialized |
| <code>dy</code>        | Input: running vector of background/perturbation derivatives, already allocated      |
| <code>curvature</code> | Output: curvature perturbation   |
| <code>tensor</code>    | Output: tensor perturbation  |

#### Returns

the error status

#### Summary:

- define local variables
- initialize the generic integrator (same integrator already used in background, thermodynamics and perturbation modules)
- initialize variable used for deciding when to stop the calculation (= when the curvature remains stable)
- initialize conformal time to arbitrary value (here, only variations of tau matter: the equations that we integrate do not depend explicitly on time)
- compute derivative of initial vector and infer first value of adaptive time-step
- loop over time

- clean the generic integrator
- store final value of curvature for this wavenumber
- store final value of tensor perturbation for this wavenumber

#### 5.15.2.16 primordial\_inflation\_find\_attractor()

```
int primordial_inflation_find_attractor (
    struct primordial * ppm,
    struct precision * ppr,
    double phi_0,
    double precision,
    double * y,
    double * dy,
    double * H_0,
    double * dphidt_0 )
```

Routine searching for the inflationary attractor solution at a given  $\phi_0$ , by iterations, with a given tolerance. If no solution found within tolerance, returns error message. The principle is the following. The code starts integrating the background equations from various values of  $\phi$ , corresponding to earlier and earlier value before  $\phi_0$ , and separated by a small arbitrary step size, corresponding roughly to 1 e-fold of inflation. Each time, the integration starts with the initial condition  $\dot{\phi} = -V'/3H$  (slow-roll prediction). If the found value of  $\dot{\phi}$  in  $\phi_0$  is stable (up to the parameter "precision"), the code considers that there is an attractor, and stops iterating. If this process does not converge, it returns an error message.

#### Parameters

|                  |   |
|------------------|---|
| <i>ppm</i>       | Input: pointer to primordial structure  |
| <i>ppr</i>       | Input: pointer to precision structure   |
| <i>phi_0</i>     | Input: field value at which we wish to find the solution  |
| <i>precision</i> | Input: tolerance on output values (if too large, an attractor will always considered to be found) |
| <i>y</i>         | Input: running vector of background variables, already allocated and initialized                  |
| <i>dy</i>        | Input: running vector of background derivatives, already allocated                                |
| <i>H_0</i>       | Output: Hubble value at $\phi_0$ for attractor solution   |
| <i>dphidt_0</i>  | Output: field derivative value at $\phi_0$ for attractor solution                                 |

#### Returns

the error status

#### 5.15.2.17 primordial\_inflation\_evolve\_background()

```
int primordial_inflation_evolve_background (
    struct primordial * ppm,
    struct precision * ppr,
```

```

double * y,
double * dy,
enum target_quantity target,
double stop,
short check_epsilon,
enum integration_direction direction,
enum time_definition time )

```

Routine integrating background equations only, from initial values stored in *y*, to a final value (if target = *aH*, until  $aH = aH\_stop$ ; if target = *phi*, till  $\phi = \phi\_stop$ ; if target = *end\_inflation*, until  $d^2a/dt^2 = 0$  (here  $t = \text{proper time}$ )). In output, *y* contains the final background values. In addition, if `check_epsilon` is true, the routine controls at each step that the expansion is accelerated and that inflation holds ( $w_{\text{epsilon}} > 1$ ), otherwise it returns an error. Thanks to the last argument, it is also possible to specify whether the integration should be carried forward or backward in time. For the *inflation\_H* case, only a 1st order differential equation is involved, so the forward and backward case can be done exactly without problems. For the *inflation\_V* case, the equation of motion is 2nd order. What the module will do in the backward case is to search for an approximate solution, corresponding to the (first-order) attractor inflationary solution. This approximate backward solution is used in order to estimate some initial times, but the approximation made here will never impact the final result: the module is written in such a way that after using this approximation, the code always computes (and relies on) the exact forward solution.

#### Parameters

|                      |   |
|----------------------|---|
| <i>ppm</i>           | Input: pointer to primordial structure  |
| <i>ppr</i>           | Input: pointer to precision structure   |
| <i>y</i>             | Input/output: running vector of background variables, already allocated and initialized |
| <i>dy</i>            | Input: running vector of background derivatives, already allocated                      |
| <i>target</i>        | Input: whether the goal is to reach a given $aH$ or $\phi$                              |
| <i>stop</i>          | Input: the target value of either $aH$ or $\phi$  |
| <i>check_epsilon</i> | Input: whether we should impose inflation ( $\epsilon > 1$ ) at each step               |
| <i>direction</i>     | Input: whether we should integrate forward or backward in time                          |
| <i>time</i>          | Input: definition of time (proper or conformal)   |

#### Returns

the error status

#### 5.15.2.18 primordial\_inflation\_check\_potential()

```

int primordial_inflation_check_potential (
    struct primordial * ppm,
    double phi,
    double * V,
    double * dV,
    double * ddV )

```

Routine checking positivity and negative slope of potential. The negative slope is an arbitrary choice. Currently the code can only deal with monotonic variations of the inflaton during inflation. So the slope had to be always negative or always positive... we took the first option.

## Parameters

|            |   |
|------------|---|
| <i>ppm</i> | Input: pointer to primordial structure                        |
| <i>phi</i> | Input: field value where to perform the check                 |
| <i>V</i>   | Output: inflaton potential in units of $Mp^4$                 |
| <i>dV</i>  | Output: first derivative of inflaton potential wrt the field  |
| <i>ddV</i> | Output: second derivative of inflaton potential wrt the field |

## Returns

the error status

## 5.15.2.19 primordial\_inflation\_check\_hubble()

```
int primordial_inflation_check_hubble (
    struct primordial * ppm,
    double phi,
    double * H,
    double * dH,
    double * ddH,
    double * dddH )
```

Routine checking positivity and negative slope of  $H(\phi)$ . The negative slope is an arbitrary choice. Currently the code can only deal with monotonic variations of the inflaton during inflation. And H can only decrease with time. So the slope  $dH/d\phi$  has to be always negative or always positive... we took the first option: phi increases, H decreases.

## Parameters

|             |   |
|-------------|---|
| <i>ppm</i>  | Input: pointer to primordial structure        |
| <i>phi</i>  | Input: field value where to perform the check |
| <i>H</i>    | Output: Hubble parameters in units of Mp      |
| <i>dH</i>   | Output: $dH/d\phi$                            |
| <i>ddH</i>  | Output: $d^2H/d\phi^2$                        |
| <i>dddH</i> | Output: $d^3H/d\phi^3$                        |

## Returns

the error status

## 5.15.2.20 primordial\_inflation\_get\_epsilon()

```
int primordial_inflation_get_epsilon (
    struct primordial * ppm,
```

```
double phi,  
double * epsilon )
```

Routine computing the first slow-roll parameter  $\epsilon$

## Parameters

|                |   |
|----------------|---|
| <i>ppm</i>     | Input: pointer to primordial structure      |
| <i>phi</i>     | Input: field value where to compute epsilon |
| <i>epsilon</i> | Output: result                              |

## Returns

the error status

## 5.15.2.21 primordial\_inflation\_find\_phi\_pivot()

```
int primordial_inflation_find_phi_pivot (
    struct primordial * ppm,
    struct precision * ppr,
    double * y,
    double * dy )
```

Routine searching phi\_pivot when a given amount of inflation is requested.

## Parameters

|            |  |
|------------|--|
| <i>ppm</i> | Input/output: pointer to primordial structure                                    |
| <i>ppr</i> | Input: pointer to precision structure  |
| <i>y</i>   | Input: running vector of background variables, already allocated and initialized |
| <i>dy</i>  | Input: running vector of background derivatives, already allocated               |

## Returns

the error status

## Summary:

- define local variables
- check whether in vicinity of phi\_end, inflation is still ongoing
- case in which epsilon > 1: hence we must find the value phi\_stop < phi\_end where inflation ends up naturally
- -> find latest value of the field such that epsilon = primordial\_inflation\_small\_epsilon (default: 0.1)
- -> bracketing right-hand value is phi\_end (but the potential will not be evaluated exactly there, only closeby)
- -> bracketing left-hand value is found by iterating with logarithmic step until epsilon < primordial\_inflation\_small\_epsilon
- -> find value such that epsilon = primordial\_inflation\_small\_epsilon by bisection
- -> value found and stored as phi\_small\_epsilon
- -> find inflationary attractor in phi\_small\_epsilon (should exist since epsilon << 1 there)

- --> compute amount of inflation between this `phi_small_epsilon` and the end of inflation
- -> by starting from `phi_small_epsilon` and integrating an approximate solution backward in time, try to estimate roughly a value close to `phi_pivot` but a bit smaller. This is done by trying to reach an amount of inflation equal to the requested one, minus the amount after `phi_small_epsilon`, and plus `primordial_inflation_extra_efolds` e-folds (default: two). Note that it is not aggressive to require two extra e-folds of inflation before the pivot, since the calculation of the spectrum in the observable range will require even more.
- -> find attractor in `phi_try`
- -> check the total amount of inflation between `phi_try` and the end of inflation
- -> go back to `phi_try`, and now find `phi_pivot` such that the amount of inflation between `phi_pivot` and the end of inflation is exactly the one requested.
- case in which `epsilon < 1`:
- -> find inflationary attractor in `phi_small_epsilon` (should exist since `epsilon < 1` there)
- --> by starting from `phi_end` and integrating an approximate solution backward in time, try to estimate roughly a value close to `phi_pivot` but a bit smaller. This is done by trying to reach an amount of inflation equal to the requested one, minus the amount after `phi_small_epsilon`, and plus `primordial_inflation_extra_efolds` e-folds (default: two). Note that it is not aggressive to require two extra e-folds of inflation before the pivot, since the calculation of the spectrum in the observable range will require even more.
- -> we now have a value `phi_try` believed to be close to and slightly smaller than `phi_pivot`
- -> find attractor in `phi_try`
- -> check the total amount of inflation between `phi_try` and the end of inflation
- -> go back to `phi_try`, and now find `phi_pivot` such that the amount of inflation between `phi_pivot` and the end of inflation is exactly the one requested.
- -> In verbose mode, check that `phi_pivot` is correct. Done by restarting from `phi_pivot` and going again till the end of inflation.

#### 5.15.2.22 `primordial_inflation_derivs()`

```
int primordial_inflation_derivs (
    double tau,
    double * y,
    double * dy,
    void * parameters_and_workspace,
    ErrorMsg error_message )
```

Routine returning derivative of system of background/perturbation variables. Like other routines used by the generic integrator (`background_derivs`, `thermodynamics_derivs`, `perturb_derivs`), this routine has a generic list of arguments, and a slightly different error management, with the error message returned directly in an `ErrMsg` field.

#### Parameters

|                                 |   |
|---------------------------------|---|
| <i>tau</i>                      | Input: time (not used explicitly inside the routine, but requested by the generic integrator) |
| <i>y</i>                        | Input/output: running vector of background variables, already allocated and initialized       |
| <i>dy</i>                       | Input: running vector of background derivatives, already allocated                            |
| <i>parameters_and_workspace</i> | Input: all necessary input variables apart from <i>y</i>                                      |
| <i>error_message</i>            | Output: error message   |



**Returns**

the error status

**5.15.2.23 primordial\_external\_spectrum\_init()**

```
int primordial_external_spectrum_init (
    struct perturbs * ppt,
    struct primordial * ppm )
```

This routine reads the primordial spectrum from an external command, and stores the tabulated values. The sampling of the  $k$ 's given by the external command is preserved.

Author: Jesus Torrado ([torradocacho@lorentz.leidenuniv.nl](mailto:torradocacho@lorentz.leidenuniv.nl)) Date: 2013-12-20

**Parameters**

|                  |   |
|------------------|---|
| <code>ppt</code> | Input/output: pointer to perturbation structure |
| <code>ppm</code> | Input/output: pointer to primordial structure   |

**Returns**

the error status

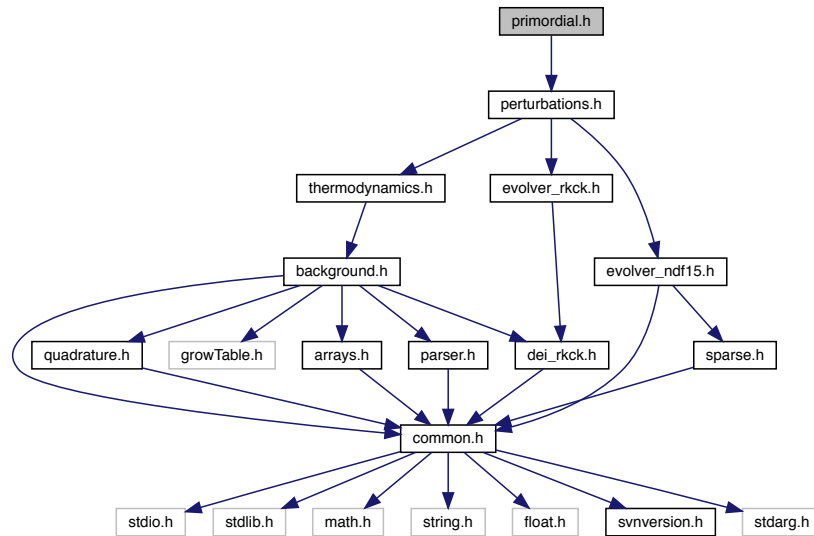
**Summary:**

- Initialization
- Launch the command and retrieve the output
- Store the read results into CLASS structures
- Make room
- Store values
- Release the memory used locally
- Tell CLASS that there are scalar (and tensor) modes

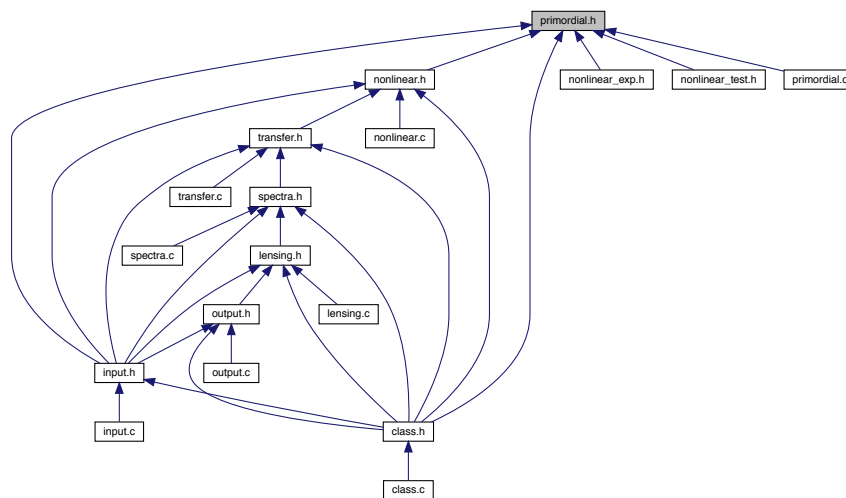
**5.16 primordial.h File Reference**

```
#include "perturbations.h"
```

Include dependency graph for `primordial.h`:



This graph shows which files directly or indirectly include this file:



## Data Structures

- struct [primordial](#)

## Enumerations

- enum [primordial\\_spectrum\\_type](#)
- enum [linear\\_or\\_logarithmic](#)

- enum [potential\\_shape](#)
- enum [target\\_quantity](#)
- enum [integration\\_direction](#)
- enum [time\\_definition](#)
- enum [phi\\_pivot\\_methods](#)
- enum [inflation\\_module\\_behavior](#)

### 5.16.1 Detailed Description

Documented includes for primordial module.

### 5.16.2 Data Structure Documentation

#### 5.16.2.1 struct primordial

Structure containing everything about primordial spectra that other modules need to know.

Once initialized by [primordial\\_init\(\)](#), contains a table of all primordial spectra as a function of wavenumber, mode, and pair of initial conditions.

#### Data Fields

|   |                      |   |
|---|----------------------|---|
| double  | k_pivot              | pivot scale in $Mpc^{-1}$   |
| enum <a href="#">primordial_spectrum_type</a> | primordial_spec_type | type of primordial spectrum (simple analytic from, integration of inflationary perturbations, etc.) |
| double  | A_s                  | usual scalar amplitude = curvature power spectrum at pivot scale                                    |
| double  | n_s                  | usual scalar tilt = [curvature power spectrum tilt at pivot scale -1]                               |
| double  | alpha_s              | usual scalar running  |
| double  | beta_s               | running of running  |
| double  | r                    | usual tensor to scalar ratio of power spectra,<br>$r = A_T/A_S = P_h/P_R$                           |
| double  | n_t                  | usual tensor tilt = [GW power spectrum tilt at pivot scale]   |
| double  | alpha_t              | usual tensor running  |
| double  | f_bi                 | baryon isocurvature (BI) entropy-to-curvature ratio<br>$S_{bi}/R$                                   |
| double  | n_bi                 | BI tilt   |
| double  | alpha_bi             | BI running  |
| double  | f_cdi                | CDM isocurvature (CDI) entropy-to-curvature ratio<br>$S_{cdi}/R$                                    |
| double  | n_cdi                | CDI tilt  |
| double  | alpha_cdi            | CDI running   |
| double  | f_nid                | neutrino density isocurvature (NID) entropy-to-curvature ratio<br>$S_{nid}/R$                       |
| double  | n_nid                | NID tilt  |
| double  | alpha_nid            | NID running   |
| double  | f_niv                | neutrino velocity isocurvature (NIV) entropy-to-curvature ratio<br>$S_{niv}/R$                      |

## Data Fields

|  |        |                 |   |
|--|--------|-----------------|---|
|  | double | n_niv           | NIV tilt  |
|  | double | alpha_niv       | NIV running   |
|  | double | c_ad_bi         | ADxBI cross-correlation at pivot scale, from -1 to 1              |
|  | double | n_ad_bi         | ADxBI cross-correlation tilt                                      |
|  | double | alpha_ad_bi     | ADxBI cross-correlation running                                   |
|  | double | c_ad_cdi        | ADxCDI cross-correlation at pivot scale, from -1 to 1             |
|  | double | n_ad_cdi        | ADxCDI cross-correlation tilt                                     |
|  | double | alpha_ad_cdi    | ADxCDI cross-correlation running                                  |
|  | double | c_ad_nid        | ADxNID cross-correlation at pivot scale, from -1 to 1             |
|  | double | n_ad_nid        | ADxNID cross-correlation tilt                                     |
|  | double | alpha_ad_nid    | ADxNID cross-correlation running                                  |
|  | double | c_ad_niv        | ADxNIV cross-correlation at pivot scale, from -1 to 1             |
|  | double | n_ad_niv        | ADxNIV cross-correlation tilt                                     |
|  | double | alpha_ad_niv    | ADxNIV cross-correlation running                                  |
|  | double | c_bi_cdi        | BlxCDI cross-correlation at pivot scale, from -1 to 1             |
|  | double | n_bi_cdi        | BlxCDI cross-correlation tilt                                     |
|  | double | alpha_bi_cdi    | BlxCDI cross-correlation running                                  |
|  | double | c_bi_nid        | BlxNIV cross-correlation at pivot scale, from -1 to 1             |
|  | double | n_bi_nid        | BlxNIV cross-correlation tilt                                     |
|  | double | alpha_bi_nid    | BlxNIV cross-correlation running                                  |
|  | double | c_bi_niv        | BlxNIV cross-correlation at pivot scale, from -1 to 1             |
|  | double | n_bi_niv        | BlxNIV cross-correlation tilt                                     |
|  | double | alpha_bi_niv    | BlxNIV cross-correlation running                                  |
|  | double | c_cdi_nid       | CDIxNID cross-correlation at pivot scale, from -1 to 1            |
|  | double | n_cdi_nid       | CDIxNID cross-correlation tilt                                    |
|  | double | alpha_cdi_nid   | CDIxNID cross-correlation running                                 |
|  | double | c_cdi_niv       | CDIxNIV cross-correlation at pivot scale, from -1 to 1            |
|  | double | n_cdi_niv       | CDIxNIV cross-correlation tilt                                    |
|  | double | alpha_cdi_niv   | CDIxNIV cross-correlation running                                 |
|  | double | c_nid_niv       | NIDxNIV cross-correlation at pivot scale, from -1 to 1            |
|  | double | n_nid_niv       | NIDxNIV cross-correlation tilt                                    |
|  | double | alpha_nid_niv   | NIDxNIV cross-correlation running                                 |
|  | enum   | potential_shape | potential   |
|  |        |                 | parameters describing the case primordial_spec_type = inflation_V |
|  | double | V0              | one parameter of the function V(phi)                              |
|  | double | V1              | one parameter of the function V(phi)                              |
|  | double | V2              | one parameter of the function V(phi)                              |
|  | double | V3              | one parameter of the function V(phi)                              |
|  | double | V4              | one parameter of the function V(phi)                              |
|  | double | H0              | one parameter of the function H(phi)                              |
|  | double | H1              | one parameter of the function H(phi)                              |

## Data Fields

|  |  |                  |   |
|--|--|------------------|---|
|  | double   | H2               | one parameter of the function H(phi)  |
|  | double   | H3               | one parameter of the function H(phi)  |
|  | double   | H4               | one parameter of the function H(phi)  |
|  | double   | phi_end          | value of inflaton at the end of inflation   |
|  | enum <a href="#">phi_pivot_methods</a>         | phi_pivot_method | flag for method used to define and find the pivot scale   |
|  | double   | phi_pivot_target | For each of the above methods, critical value to be reached between pivot and end of inflation (N_star, [aH]ratio, etc.)                          |
|  | enum <a href="#">inflation_module_behavior</a> | behavior         | Specifies if the inflation module computes the primordial spectrum numerically (default) or analytically  |
|  | char *   | command          | 'external_Pk' mode: command generating the table of Pk and custom parameters to be passed to it string with the command for calling 'external_Pk' |
|  | double   | custom1          | one parameter of the primordial computed in 'external_Pk'   |
|  | double   | custom2          | one parameter of the primordial computed in 'external_Pk'   |
|  | double   | custom3          | one parameter of the primordial computed in 'external_Pk'   |
|  | double   | custom4          | one parameter of the primordial computed in 'external_Pk'   |
|  | double   | custom5          | one parameter of the primordial computed in 'external_Pk'   |
|  | double   | custom6          | one parameter of the primordial computed in 'external_Pk'   |
|  | double   | custom7          | one parameter of the primordial computed in 'external_Pk'   |
|  | double   | custom8          | one parameter of the primordial computed in 'external_Pk'   |
|  | double   | custom9          | one parameter of the primordial computed in 'external_Pk'   |
|  | double   | custom10         | one parameter of the primordial computed in 'external_Pk'   |
|  | int  | md_size          | number of modes included in computation   |
|  | int *  | ic_size          | for a given mode, ic_size[index_md] = number of initial conditions included in computation  |
|  | int *  | ic_ic_size       | number of ordered pairs of (index_ic1, index_ic2); this number is just N(N+1)/2 where N = ic_size[index_md]                                       |
|  | int  | lnk_size         | number of ln(k) values  |
|  | double *                                       | lnk              | list of ln(k) values lnk[index_k]   |

## Data Fields

|           |                  |  |
|-----------|------------------|--|
| double ** | lnpk             | <p>depends on indices <code>index_md</code>, <code>index_ic1</code>, <code>index_ic2</code>, <code>index_k</code> as: <code>lnpk[index_md][index_ic1 -&gt; k * ppm -&gt; ic_ic_size[index_md] + index_ic1_ic2]</code> where <code>index_ic1_ic2</code> labels ordered pairs (<code>index_ic1</code>, <code>index_ic2</code>) (since the primordial spectrum is symmetric in (<code>index_ic1</code>, <code>index_ic2</code>)).</p> <ul style="list-style-type: none"> <li>for diagonal elements (<code>index_ic1 = index_ic2</code>) this arrays contains <math>\ln[P(k)]</math> where <math>P(k)</math> is positive by construction.</li> <li>for non-diagonal elements this arrays contains the <math>k</math>-dependent cosine of the correlation angle, namely <math>P(k)_{(index\_ic1, index\_ic2)} / \sqrt{P(k)_{index\_ic1} P(k)_{index\_ic2}}</math> This choice is convenient since the sign of the non-diagonal cross-correlation is arbitrary. For fully correlated or anti-correlated initial conditions, this non -diagonal element is independent on <math>k</math>, and equal to <math>+1</math> or <math>-1</math>.</li> </ul> |
| double ** | ddlmpk           | <p>second derivative of above array, for spline interpolation. So:</p> <ul style="list-style-type: none"> <li>for <code>index_ic1 = index_ic2</code>, we spline <math>\ln[P(k)]</math> vs. <math>\ln(k)</math>, which is good since this function is usually smooth.</li> <li>for non-diagonal coefficients, we spline <math>P(k)_{(index\_ic1, index\_ic2)} / \sqrt{P(k)_{index\_ic1} P(k)_{index\_ic2}}</math> vs. <math>\ln(k)</math>, which is fine since this quantity is often assumed to be constant (e.g for fully correlated/anticorrelated initial conditions) or nearly constant, and with arbitrary sign.</li> </ul>   |
| short **  | is_non_zero      | <p><code>is_non_zero[index_md][index_ic1_ic2]</code> set to false if pair (<code>index_ic1</code>, <code>index_ic2</code>) is uncorrelated (ensures more precision and saves time with respect to the option of simply setting <math>P(k)_{(index\_ic1, index\_ic2)}</math> to zero)</p>   |
| double ** | amplitude        | <p>all amplitudes in matrix form:<br/><code>amplitude[index_md][index_ic1_ic2]</code></p>  |
| double ** | tilt             | <p>all tilts in matrix form: <code>tilt[index_md][index_ic1_ic2]</code></p>  |
| double ** | running          | <p>all runnings in matrix form:<br/><code>running[index_md][index_ic1_ic2]</code></p>  |
| int       | index_in_a       | scale factor   |
| int       | index_in_phi     | inflaton vev   |
| int       | index_in_dphi    | its time derivative  |
| int       | index_in_ksi_re  | Mukhanov variable (real part)  |
| int       | index_in_ksi_im  | Mukhanov variable (imaginary part)   |
| int       | index_in_dksi_re | Mukhanov variable (real part, time derivative)   |
| int       | index_in_dksi_im | Mukhanov variable (imaginary part, time derivative)  |

## Data Fields

|  |        |                    |   |
|--|--------|--------------------|---|
|  | int    | index_in_ah_re     | tensor perturbation (real part)   |
|  | int    | index_in_ah_im     | tensor perturbation (imaginary part)  |
|  | int    | index_in_dah_re    | tensor perturbation (real part, time derivative)  |
|  | int    | index_in_dah_im    | tensor perturbation (imaginary part, time derivative)   |
|  | int    | in_bg_size         | size of vector of background quantities only  |
|  | int    | in_size            | full size of vector   |
|  | double | phi_pivot          | in inflationary module, value of phi_pivot (set to 0 for inflation_V, inflation_H; found by code for inflation_V_end) |
|  | double | phi_min            | in inflationary module, value of phi when $k_{min} = aH$  |
|  | double | phi_max            | in inflationary module, value of phi when $k_{max} = aH$  |
|  | double | phi_stop           | in inflationary module, value of phi at the end of inflation  |
|  | short  | primordial_verbose | flag regulating the amount of information sent to standard output (none if set to zero)                               |
|  | ErrMsg | error_message      | zone for writing error messages   |

## 5.16.3 Enumeration Type Documentation

## 5.16.3.1 primordial\_spectrum\_type

```
enum primordial_spectrum_type
```

enum defining how the primordial spectrum should be computed

## 5.16.3.2 linear\_or\_logarithmic

```
enum linear_or_logarithmic
```

enum defining whether the spectrum routine works with linear or logarithmic input/output

## 5.16.3.3 potential\_shape

```
enum potential_shape
```

enum defining the type of inflation potential function  $V(\phi)$

#### 5.16.3.4 target\_quantity

enum `target_quantity`

enum defining which quantity plays the role of a target for evolving inflationary equations

#### 5.16.3.5 integration\_direction

enum `integration_direction`

enum specifying if we want to integrate equations forward or backward in time

#### 5.16.3.6 time\_definition

enum `time_definition`

enum specifying if we want to evolve quantities with conformal or proper time

#### 5.16.3.7 phi\_pivot\_methods

enum `phi_pivot_methods`

enum specifying how, in the `inflation_V_end` case, the value of `phi_pivot` should be calculated

#### 5.16.3.8 inflation\_module\_behavior

enum `inflation_module_behavior`

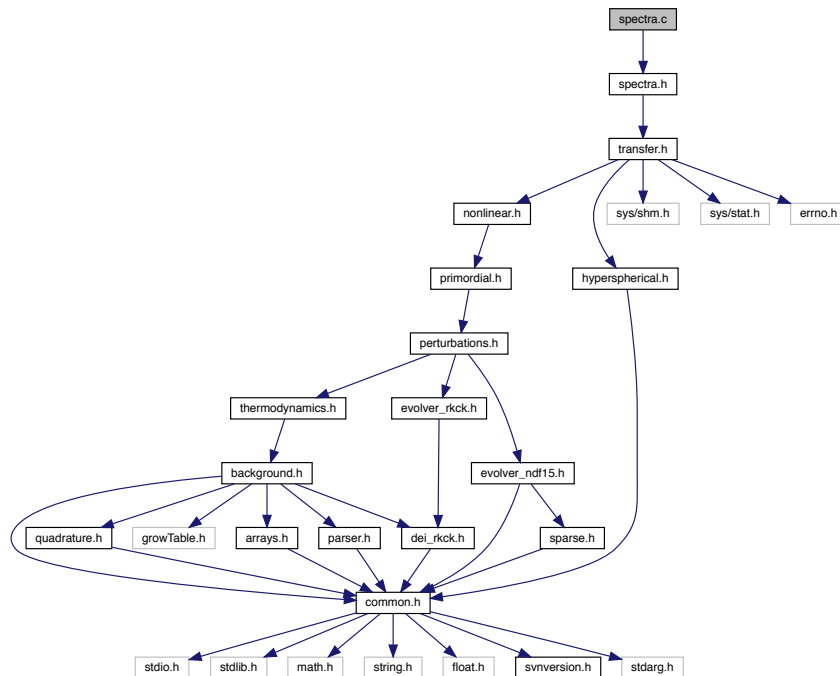
enum specifying how the inflation module computes the primordial spectrum (default: numerical)



## 5.17 spectra.c File Reference

```
#include "spectra.h"
```

Include dependency graph for spectra.c:



### Functions

- int [spectra\\_cl\\_at\\_l](#) (struct [spectra](#) \*psp, double l, double \*cl\_tot, double \*\*cl\_md, double \*\*cl\_md\_ic)
- int [spectra\\_pk\\_at\\_z](#) (struct [background](#) \*pba, struct [spectra](#) \*psp, enum [linear\\_or\\_logarithmic](#) mode, double z, double \*output\_tot, double \*output\_ic)
- int [spectra\\_pk\\_at\\_k\\_and\\_z](#) (struct [background](#) \*pba, struct [primordial](#) \*ppm, struct [spectra](#) \*psp, double k, double z, double \*pk\_tot, double \*pk\_ic)
- int [spectra\\_pk\\_nl\\_at\\_z](#) (struct [background](#) \*pba, struct [spectra](#) \*psp, enum [linear\\_or\\_logarithmic](#) mode, double z, double \*output\_tot)
- int [spectra\\_pk\\_nl\\_at\\_k\\_and\\_z](#) (struct [background](#) \*pba, struct [primordial](#) \*ppm, struct [spectra](#) \*psp, double k, double z, double \*pk\_tot)
- int [spectra\\_tk\\_at\\_z](#) (struct [background](#) \*pba, struct [spectra](#) \*psp, double z, double \*output)
- int [spectra\\_tk\\_at\\_k\\_and\\_z](#) (struct [background](#) \*pba, struct [spectra](#) \*psp, double k, double z, double \*output)
- int [spectra\\_init](#) (struct [precision](#) \*ppr, struct [background](#) \*pba, struct [perturbs](#) \*ppt, struct [primordial](#) \*ppm, struct [nonlinear](#) \*pnl, struct [transfers](#) \*ptr, struct [spectra](#) \*psp)
- int [spectra\\_free](#) (struct [spectra](#) \*psp)
- int [spectra\\_indices](#) (struct [background](#) \*pba, struct [perturbs](#) \*ppt, struct [transfers](#) \*ptr, struct [primordial](#) \*ppm, struct [spectra](#) \*psp)
- int [spectra\\_cls](#) (struct [background](#) \*pba, struct [perturbs](#) \*ppt, struct [transfers](#) \*ptr, struct [primordial](#) \*ppm, struct [spectra](#) \*psp)
- int [spectra\\_compute\\_cl](#) (struct [background](#) \*pba, struct [perturbs](#) \*ppt, struct [transfers](#) \*ptr, struct [primordial](#) \*ppm, struct [spectra](#) \*psp, int index\_md, int index\_ic1, int index\_ic2, int index\_l, int cl\_integrand\_num\_↔ columns, double \*cl\_integrand, double \*primordial\_pk, double \*transfer\_ic1, double \*transfer\_ic2)
- int [spectra\\_k\\_and\\_tau](#) (struct [background](#) \*pba, struct [perturbs](#) \*ppt, struct [spectra](#) \*psp)

- int `spectra_pk` (struct `background` \*pba, struct `perturbs` \*ppt, struct `primordial` \*ppm, struct `nonlinear` \*pnl, struct `spectra` \*psp)
- int `spectra_sigma` (struct `background` \*pba, struct `primordial` \*ppm, struct `spectra` \*psp, double R, double z, double \*sigma)
- int `spectra_matter_transfers` (struct `background` \*pba, struct `perturbs` \*ppt, struct `spectra` \*psp)
- int `spectra_output_tk_data` (struct `background` \*pba, struct `perturbs` \*ppt, struct `spectra` \*psp, enum `file_↔format` output\_format, double z, int number\_of\_titles, double \*data)

### 5.17.1 Detailed Description

Documented spectra module

Julien Lesgourgues, 25.08.2010

This module computes the anisotropy and Fourier power spectra  $C_l^X, P(k), \dots$ 's given the transfer and Bessel functions (for anisotropy spectra), the source functions (for Fourier spectra) and the primordial spectra.

The following functions can be called from other modules:

1. `spectra_init()` at the beginning (but after `transfer_init()`)
2. `spectra_cl_at_l()` at any time for computing  $C_l$  at any l
3. `spectra_spectrum_at_z()` at any time for computing P(k) at any z
4. `spectra_spectrum_at_k_and_z()` at any time for computing P at any k and z
5. `spectra_free()` at the end

### 5.17.2 Function Documentation

#### 5.17.2.1 `spectra_cl_at_l()`

```
int spectra_cl_at_l (
    struct spectra * psp,
    double l,
    double * cl_tot,
    double ** cl_md,
    double ** cl_md_ic )
```

Anisotropy power spectra  $C_l$ 's for all types, modes and initial conditions.

This routine evaluates all the  $C_l$ 's at a given value of l by interpolating in the pre-computed table. When relevant, it also sums over all initial conditions for each mode, and over all modes.

This function can be called from whatever module at whatever time, provided that `spectra_init()` has been called before, and `spectra_free()` has not been called yet.

#### Parameters

|                       |  |
|-----------------------|--|
| <code>psp</code>      | Input: pointer to spectra structure (containing pre-computed table)  |
| <code>l</code>        | Input: multipole number  |
| <code>cl_tot</code>   | Output: total $C_l$ 's for all types (TT, TE, EE, etc..)   |
| <code>cl_md</code>    | Output: $C_l$ 's for all types (TT, TE, EE, etc..) decomposed mode by mode (scalar, tensor, ...) when relevant |
| <code>cl_md_ic</code> | Output: $C_l$ 's for all types (TT, TE, EE, etc..) decomposed by pairs of initial conditions (adiabatic        |

**Returns**

the error status

**Summary:**

- define local variables
- (a) treat case in which there is only one mode and one initial condition. Then, only `cl_tot` needs to be filled.
- (b) treat case in which there is only one mode with several initial condition. Fill `cl_md_ic[index_md=0]` and sum it to get `cl_tot`.
- (c) loop over modes
- -> (c.1.) treat case in which the mode under consideration has only one initial condition. Fill `cl_md[index_md]`.
- -> (c.2.) treat case in which the mode under consideration has several initial conditions. Fill `cl_md_ic[index_md]` and sum it to get `cl_md[index_md]`
- -> (c.3.) add contribution of `cl_md[index_md]` to `cl_tot`

**5.17.2.2 spectra\_pk\_at\_z()**

```
int spectra_pk_at_z (
    struct background * pba,
    struct spectra * psp,
    enum linear_or_logarithmic mode,
    double z,
    double * output_tot,
    double * output_ic )
```

Matter power spectrum for arbitrary redshift and for all initial conditions.

This routine evaluates the matter power spectrum at a given value of  $z$  by interpolating in the pre-computed table (if several values of  $z$  have been stored) or by directly reading it (if it only contains values at  $z=0$  and we want  $P(k,z=0)$ )

Can be called in two modes: linear or logarithmic.

- linear: returns  $P(k)$  (units:  $Mpc^3$ )
- logarithmic: returns  $\ln P(k)$

One little subtlety: in case of several correlated initial conditions, the cross-correlation spectrum can be negative. Then, in logarithmic mode, the non-diagonal elements contain the cross-correlation angle  $P_{12}/\sqrt{P_{11}P_{22}}$  (from -1 to 1) instead of  $\ln P_{12}$

This function can be called from whatever module at whatever time, provided that `spectra_init()` has been called before, and `spectra_free()` has not been called yet.

## Parameters

|                   |  |
|-------------------|--|
| <i>pba</i>        | Input: pointer to background structure (used for converting z into tau)  |
| <i>psp</i>        | Input: pointer to spectra structure (containing pre-computed table)  |
| <i>mode</i>       | Input: linear or logarithmic   |
| <i>z</i>          | Input: redshift  |
| <i>output_tot</i> | Output: total matter power spectrum $P(k)$ in $Mpc^3$ (linear mode), or its logarithms (logarithmic mode)  |
| <i>output_ic</i>  | Output: for each pair of initial conditions, matter power spectra $P(k)$ in $Mpc^3$ (linear mode), or their logarithms and cross-correlation angles (logarithmic mode) |

## Returns

the error status

## Summary:

- define local variables
- first step: convert z into  $\ln \tau$
- second step: for both modes (linear or logarithmic), store the spectrum in logarithmic format in the output array(s)
- → (a) if only values at  $\tau=\tau_{\text{today}}$  are stored and we want  $P(k, z=0)$ , no need to interpolate
- → (b) if several values of tau have been stored, use interpolation routine to get spectra at correct redshift
- third step: if there are several initial conditions, compute the total  $P(k)$  and set back all uncorrelated coefficients to exactly zero. Check positivity of total  $P(k)$ .
- fourth step: depending on requested mode (linear or logarithmic), apply necessary transformation to the output arrays
- → (a) linear mode: if only one initial condition, convert *output\_pk* to linear format; if several initial conditions, convert *output\_ic* to linear format, *output\_tot* is already in this format
- → (b) logarithmic mode: if only one initial condition, nothing to be done; if several initial conditions, convert *output\_tot* to logarithmic format, *output\_ic* is already in this format

## 5.17.2.3 spectra\_pk\_at\_k\_and\_z()

```
int spectra_pk_at_k_and_z (
    struct background * pba,
    struct primordial * ppm,
    struct spectra * psp,
    double k,
    double z,
    double * pk_tot,
    double * pk_ic )
```

Matter power spectrum for arbitrary wavenumber, redshift and initial condition.

This routine evaluates the matter power spectrum at a given value of k and z by interpolating in a table of all  $P(k)$ 's computed at this z by [spectra\\_pk\\_at\\_z\(\)](#) (when  $k_{\text{min}} \leq k \leq k_{\text{max}}$ ), or eventually by using directly the primordial spectrum (when  $0 \leq k < k_{\text{min}}$ ): the latter case is an approximation, valid when  $k_{\text{min}} \ll$  comoving Hubble scale today. Returns zero when  $k=0$ . Returns an error when  $k < 0$  or  $k > k_{\text{max}}$ .

This function can be called from whatever module at whatever time, provided that [spectra\\_init\(\)](#) has been called before, and [spectra\\_free\(\)](#) has not been called yet.

## Parameters

|               |   |
|---------------|---|
| <i>pba</i>    | Input: pointer to background structure (used for converting z into tau)             |
| <i>ppm</i>    | Input: pointer to primordial structure (used only in the case $0 < k < k_{\min}$ )  |
| <i>psp</i>    | Input: pointer to spectra structure (containing pre-computed table)                 |
| <i>k</i>      | Input: wavenumber in 1/Mpc  |
| <i>z</i>      | Input: redshift   |
| <i>pk_tot</i> | Output: total matter power spectrum $P(k)$ in $Mpc^3$                               |
| <i>pk_ic</i>  | Output: for each pair of initial conditions, matter power spectra $P(k)$ in $Mpc^3$ |

## Returns

the error status

## Summary:

- define local variables
- first step: check that  $k$  is in valid range  $[0:k_{\max}]$  (the test for  $z$  will be done when calling [spectra\\_pk\\_at\\_z\(\)](#))
- deal with case  $0 \leq k < k_{\min}$
- $\rightarrow$  (a) subcase  $k=0$ : then  $P(k)=0$
- $\rightarrow$  (b) subcase  $0 < k < k_{\min}$ : in this case we know that on super-Hubble scales:  $P(k) = [\text{some number}] * k * P_{\text{primordial}}(k)$  so  $P(k) = P(k_{\min}) * (k P_{\text{primordial}}(k)) / (k_{\min} P_{\text{primordial}}(k_{\min}))$  (note that the result is accurate only if  $k_{\min}$  is such that  $[a0 k_{\min}] \ll H_0$ )
- deal with case  $k_{\min} \leq k \leq k_{\max}$
- last step: if more than one condition, sum over *pk\_ic* to get *pk\_tot*, and set back coefficients of non-correlated pairs to exactly zero.

## 5.17.2.4 spectra\_pk\_nl\_at\_z()

```
int spectra_pk_nl_at_z (
    struct background * pba,
    struct spectra * psp,
    enum linear_or_logarithmic mode,
    double z,
    double * output_tot )
```

Non-linear total matter power spectrum for arbitrary redshift.

This routine evaluates the non-linear matter power spectrum at a given value of  $z$  by interpolating in the pre-computed table (if several values of  $z$  have been stored) or by directly reading it (if it only contains values at  $z=0$  and we want  $P(k,z=0)$ )

Can be called in two modes: linear or logarithmic.

- linear: returns  $P(k)$  (units:  $Mpc^3$ )
- logarithmic: returns  $\ln(P(k))$

This function can be called from whatever module at whatever time, provided that [spectra\\_init\(\)](#) has been called before, and [spectra\\_free\(\)](#) has not been called yet.

## Parameters

|                   |   |
|-------------------|---|
| <i>pba</i>        | Input: pointer to background structure (used for converting z into tau)                                 |
| <i>psp</i>        | Input: pointer to spectra structure (containing pre-computed table)                                     |
| <i>mode</i>       | Input: linear or logarithmic  |
| <i>z</i>          | Input: redshift   |
| <i>output_tot</i> | Output: total matter power spectrum P(k) in $Mpc^3$ (linear mode), or its logarithms (logarithmic mode) |

## Returns

the error status

## Summary:

- define local variables
- first step: convert z into ln(tau)
- second step: for both modes (linear or logarithmic), store the spectrum in logarithmic format in the output array(s)
- → (a) if only values at tau=tau\_today are stored and we want P(k,z=0), no need to interpolate
- → (b) if several values of tau have been stored, use interpolation routine to get spectra at correct redshift
- fourth step: eventually convert to linear format

## 5.17.2.5 spectra\_pk\_nl\_at\_k\_and\_z()

```
int spectra_pk_nl_at_k_and_z (
    struct background * pba,
    struct primordial * ppm,
    struct spectra * psp,
    double k,
    double z,
    double * pk_tot )
```

Non-linear total matter power spectrum for arbitrary wavenumber and redshift.

This routine evaluates the matter power spectrum at a given value of k and z by interpolating in a table of all P(k)'s computed at this z by [spectra\\_pk\\_nl\\_at\\_z\(\)](#) (when  $k_{min} \leq k \leq k_{max}$ ), or eventually by using directly the primordial spectrum (when  $0 \leq k < k_{min}$ ): the latter case is an approximation, valid when  $k_{min} \ll$  comoving Hubble scale today. Returns zero when k=0. Returns an error when  $k < 0$  or  $k > k_{max}$ .

This function can be called from whatever module at whatever time, provided that [spectra\\_init\(\)](#) has been called before, and [spectra\\_free\(\)](#) has not been called yet.

## Parameters

|               |   |
|---------------|---|
| <i>pba</i>    | Input: pointer to background structure (used for converting z into tau)           |
| <i>ppm</i>    | Input: pointer to primordial structure (used only in the case $0 < k < k_{min}$ ) |
| <i>psp</i>    | Input: pointer to spectra structure (containing pre-computed table)               |
| <i>k</i>      | Input: wavenumber in 1/Mpc  |
| <i>z</i>      | Input: redshift   |
| <i>pk_tot</i> | Output: total matter power spectrum P(k) in $Mpc^3$                               |

**Returns**

the error status

**Summary:**

- define local variables
- check that k is in valid range [0:kmax] (the test for z will be done when calling [spectra\\_pk\\_at\\_z\(\)](#))
- compute P(k,z) (in logarithmic format for more accurate interpolation)
- get its second derivatives with spline, then interpolate, then convert to linear format

**5.17.2.6 spectra\_tk\_at\_z()**

```
int spectra_tk_at_z (
    struct background * pba,
    struct spectra * psp,
    double z,
    double * output )
```

Matter transfer functions  $T_i(k)$  for arbitrary redshift and for all initial conditions.

This routine evaluates the matter transfer functions at a given value of z by interpolating in the pre-computed table (if several values of z have been stored) or by directly reading it (if it only contains values at z=0 and we want  $T_i(k, z = 0)$ )

This function can be called from whatever module at whatever time, provided that [spectra\\_init\(\)](#) has been called before, and [spectra\\_free\(\)](#) has not been called yet.

**Parameters**

|               |   |
|---------------|---|
| <i>pba</i>    | Input: pointer to background structure (used for converting z into tau) |
| <i>psp</i>    | Input: pointer to spectra structure (containing pre-computed table)     |
| <i>z</i>      | Input: redshift   |
| <i>output</i> | Output: matter transfer functions                                       |

**Returns**

the error status

**Summary:**

- define local variables
- first step: convert z into ln(tau)
- second step: store the matter transfer functions in the output array
- → (a) if only values at tau=tau\_today are stored and we want  $T_i(k, z = 0)$ , no need to interpolate
- → (b) if several values of tau have been stored, use interpolation routine to get spectra at correct redshift

### 5.17.2.7 spectra\_tk\_at\_k\_and\_z()

```
int spectra_tk_at_k_and_z (
    struct background * pba,
    struct spectra * psp,
    double k,
    double z,
    double * output )
```

Matter transfer functions  $T_i(k)$  for arbitrary wavenumber, redshift and initial condition.

This routine evaluates the matter transfer functions at a given value of  $k$  and  $z$  by interpolating in a table of all  $T_i(k, z)$ 's computed at this  $z$  by [spectra\\_tk\\_at\\_z\(\)](#) (when  $k_{min} \leq k \leq k_{max}$ ). Returns an error when  $k < k_{min}$  or  $k > k_{max}$ .

This function can be called from whatever module at whatever time, provided that [spectra\\_init\(\)](#) has been called before, and [spectra\\_free\(\)](#) has not been called yet.

#### Parameters

|               |   |
|---------------|---|
| <i>pba</i>    | Input: pointer to background structure (used for converting $z$ into $\tau$ ) |
| <i>psp</i>    | Input: pointer to spectra structure (containing pre-computed table)           |
| <i>k</i>      | Input: wavenumber in 1/Mpc  |
| <i>z</i>      | Input: redshift   |
| <i>output</i> | Output: matter transfer functions   |

#### Returns

the error status

#### Summary:

- define local variables
- check that  $k$  is in valid range  $[0:k_{max}]$  (the test for  $z$  will be done when calling [spectra\\_tk\\_at\\_z\(\)](#))
- compute  $T_i(k, z)$
- get its second derivatives w.r.t.  $k$  with spline, then interpolate

### 5.17.2.8 spectra\_init()

```
int spectra_init (
    struct precision * ppr,
    struct background * pba,
    struct perturbs * ppt,
    struct primordial * ppm,
    struct nonlinear * pnl,
    struct transfers * ptr,
    struct spectra * psp )
```

This routine initializes the spectra structure (in particular, computes table of anisotropy and Fourier spectra  $C_l^X, P(k), \dots$ )



## Parameters

|            |  |
|------------|--|
| <i>ppr</i> | Input: pointer to precision structure  |
| <i>pba</i> | Input: pointer to background structure (will provide H, Omega_m at redshift of interest) |
| <i>ppt</i> | Input: pointer to perturbation structure   |
| <i>ptr</i> | Input: pointer to transfer structure   |
| <i>ppm</i> | Input: pointer to primordial structure   |
| <i>pnl</i> | Input: pointer to nonlinear structure  |
| <i>psp</i> | Output: pointer to initialized spectra structure   |

## Returns

the error status

## Summary:

- check that we really want to compute at least one spectrum
- initialize indices and allocate some of the arrays in the spectra structure
- deal with  $C_l$ 's, if any
- deal with  $P(k, \tau)$  and  $T_i(k, \tau)$

## 5.17.2.9 spectra\_free()

```
int spectra_free (
    struct spectra * psp )
```

This routine frees all the memory space allocated by [spectra\\_init\(\)](#).

To be called at the end of each run, only when no further calls to [spectra\\_cls\\_at\\_l\(\)](#), [spectra\\_pk\\_at\\_z\(\)](#), [spectra\\_pk\\_at\\_k\\_and\\_z\(\)](#) are needed.

## Parameters

|            |  |
|------------|--|
| <i>psp</i> | Input: pointer to spectra structure (which fields must be freed) |
|------------|--|

## Returns

the error status

## 5.17.2.10 spectra\_indices()

```
int spectra_indices (
    struct background * pba,
```

```

struct perturbs * ppt,
struct transfers * ptr,
struct primordial * ppm,
struct spectra * psp )

```

This routine defines indices and allocates tables in the spectra structure

#### Parameters

|                  |  |
|------------------|--|
| <code>pba</code> | Input: pointer to background structure     |
| <code>ppt</code> | Input: pointer to perturbation structure   |
| <code>ptr</code> | Input: pointer to transfers structure      |
| <code>ppm</code> | Input: pointer to primordial structure     |
| <code>psp</code> | Input/output: pointer to spectra structure |

#### Returns

the error status

#### 5.17.2.11 `spectra_cls()`

```

int spectra_cls (
    struct background * pba,
    struct perturbs * ppt,
    struct transfers * ptr,
    struct primordial * ppm,
    struct spectra * psp )

```

This routine computes a table of values for all harmonic spectra  $C_i$ 's, given the transfer functions and primordial spectra.

#### Parameters

|                  |  |
|------------------|--|
| <code>pba</code> | Input: pointer to background structure     |
| <code>ppt</code> | Input: pointer to perturbation structure   |
| <code>ptr</code> | Input: pointer to transfers structure      |
| <code>ppm</code> | Input: pointer to primordial structure     |
| <code>psp</code> | Input/Output: pointer to spectra structure |

#### Returns

the error status

#### Summary:

- define local variables
- allocate pointers to arrays where results will be stored

- store values of  $l$
- loop over modes (scalar, tensors, etc). For each mode:
  - → (a) store number of  $l$  values for this mode
  - → (b) allocate arrays where results will be stored
  - → (c) loop over initial conditions
  - → loop over  $l$  values defined in the transfer module. For each  $l$ , compute the  $C_l$ 's for all types (TT, TE, ...) by convolving primordial spectra with transfer functions. This elementary task is assigned to `spectra_compute_cl()`
  - → (d) now that for a given mode, all possible  $C_l$ 's have been computed, compute second derivative of the array in which they are stored, in view of spline interpolation.

#### 5.17.2.12 spectra\_compute\_cl()

```
int spectra_compute_cl (
    struct background * pba,
    struct perturbs * ppt,
    struct transfers * ptr,
    struct primordial * ppm,
    struct spectra * psp,
    int index_md,
    int index_ic1,
    int index_ic2,
    int index_l,
    int cl_integrand_num_columns,
    double * cl_integrand,
    double * primordial_pk,
    double * transfer_ic1,
    double * transfer_ic2 )
```

This routine computes the  $C_l$ 's for a given mode, pair of initial conditions and multipole, but for all types (TT, TE...), by convolving the transfer functions with the primordial spectra.

#### Parameters

|                                 |   |
|---------------------------------|---|
| <i>pba</i>                      | Input: pointer to background structure                                |
| <i>ppt</i>                      | Input: pointer to perturbation structure                              |
| <i>ptr</i>                      | Input: pointer to transfers structure                                 |
| <i>ppm</i>                      | Input: pointer to primordial structure                                |
| <i>psp</i>                      | Input/Output: pointer to spectra structure (result stored here)       |
| <i>index_md</i>                 | Input: index of mode under consideration                              |
| <i>index_ic1</i>                | Input: index of first initial condition in the correlator             |
| <i>index_ic2</i>                | Input: index of second initial condition in the correlator            |
| <i>index_l</i>                  | Input: index of multipole under consideration                         |
| <i>cl_integrand_num_columns</i> | Input: number of columns in <i>cl_integrand</i>                       |
| <i>cl_integrand</i>             | Input: an allocated workspace   |
| <i>primordial_pk</i>            | Input: table of primordial spectrum values                            |
| <i>transfer_ic1</i>             | Input: table of transfer function values for first initial condition  |
| <i>transfer_ic2</i>             | Input: table of transfer function values for second initial condition |

**Returns**

the error status

**5.17.2.13 spectra\_k\_and\_tau()**

```
int spectra_k_and_tau (
    struct background * pba,
    struct perturbs * ppt,
    struct spectra * psp )
```

This routine computes the values of  $k$  and  $\tau$  at which the matter power spectra  $P(k, \tau)$  and the matter transfer functions  $T_i(k, \tau)$  will be stored.

**Parameters**

|            |   |
|------------|---|
| <i>pba</i> | Input: pointer to background structure (for $z$ to $\tau$ conversion) |
| <i>ppt</i> | Input: pointer to perturbation structure (contain source functions)   |
| <i>psp</i> | Input/Output: pointer to spectra structure                            |

**Returns**

the error status

**Summary:**

- define local variables
- check the presence of scalar modes
- check the maximum redshift  $z_{\text{max\_pk}}$  at which  $P(k, z)$  and  $T_i(k, z)$  should be computable by interpolation. If it is equal to zero, only  $P(k, z = 0)$  needs to be computed. If it is higher, we will store in a table various  $P(k, \tau)$  at several values of  $\tau$  generously encompassing the range  $0 < z < z_{\text{max\_pk}}$
- allocate and fill table of  $\tau$  values at which  $P(k, \tau)$  and  $T_i(k, \tau)$  are stored
- allocate and fill table of  $k$  values at which  $P(k, \tau)$  is stored

**5.17.2.14 spectra\_pk()**

```
int spectra_pk (
    struct background * pba,
    struct perturbs * ppt,
    struct primordial * ppm,
    struct nonlinear * pnl,
    struct spectra * psp )
```

This routine computes a table of values for all matter power spectra  $P(k)$ , given the source functions and primordial spectra.

## Parameters

|            |  |
|------------|--|
| <i>pba</i> | Input: pointer to background structure (will provide H, Omega_m at redshift of interest) |
| <i>ppt</i> | Input: pointer to perturbation structure (contain source functions)                      |
| <i>ppm</i> | Input: pointer to primordial structure   |
| <i>pnl</i> | Input: pointer to nonlinear structure  |
| <i>psp</i> | Input/Output: pointer to spectra structure   |

## Returns

the error status

## Summary:

- define local variables
- check the presence of scalar modes
- allocate temporary vectors where the primordial spectrum and the background quantities will be stored
- allocate and fill array of  $P(k, \tau)$  values
- if interpolation of  $P(k, \tau)$  will be needed (as a function of tau), compute array of second derivatives in view of spline interpolation
- if interpolation of  $P_{NL}(k, \tau)$  will be needed (as a function of tau), compute array of second derivatives in view of spline interpolation

## 5.17.2.15 spectra\_sigma()

```
int spectra_sigma (
    struct background * pba,
    struct primordial * ppm,
    struct spectra * psp,
    double R,
    double z,
    double * sigma )
```

This routine computes sigma(R) given P(k) (does not check that k\_max is large enough)

## Parameters

|              |  |
|--------------|--|
| <i>pba</i>   | Input: pointer to background structure                   |
| <i>ppm</i>   | Input: pointer to primordial structure                   |
| <i>psp</i>   | Input: pointer to spectra structure                      |
| <i>z</i>     | Input: redshift  |
| <i>R</i>     | Input: radius in Mpc                                     |
| <i>sigma</i> | Output: variance in a sphere of radius R (dimensionless) |

### 5.17.2.16 spectra\_matter\_transfers()

```
int spectra_matter_transfers (
    struct background * pba,
    struct perturbs * ppt,
    struct spectra * psp )
```

This routine computes a table of values for all matter power spectra  $P(k)$ , given the source functions and primordial spectra.

#### Parameters

|            |   |
|------------|---|
| <i>pba</i> | Input: pointer to background structure (will provide density of each species) |
| <i>ppt</i> | Input: pointer to perturbation structure (contain source functions)           |
| <i>psp</i> | Input/Output: pointer to spectra structure                                    |

#### Returns

the error status

#### Summary:

- define local variables
- check the presence of scalar modes
- allocate and fill array of  $T_i(k, \tau)$  values
- allocate temporary vectors where the background quantities will be stored
- if interpolation of  $P(k, \tau)$  will be needed (as a function of tau), compute array of second derivatives in view of spline interpolation

### 5.17.2.17 spectra\_output\_tk\_data()

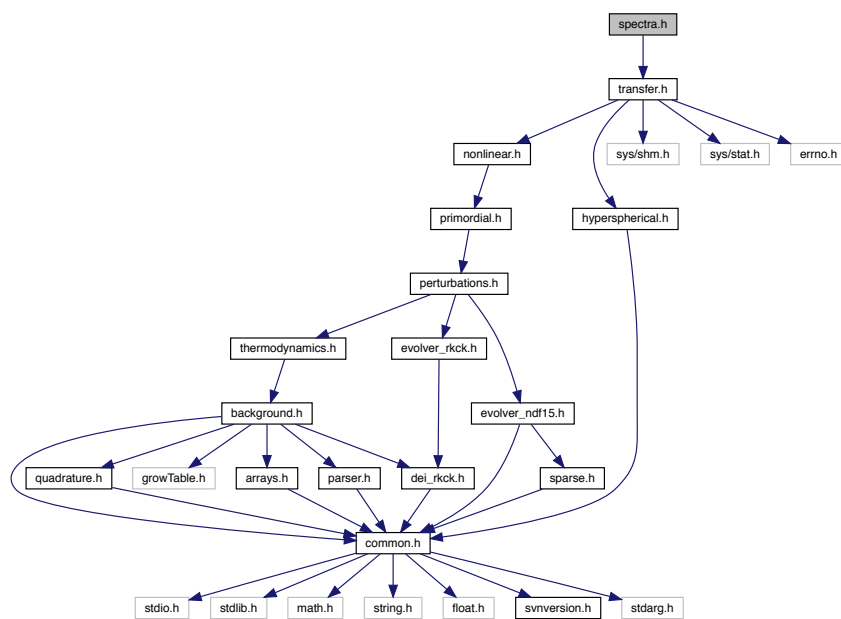
```
int spectra_output_tk_data (
    struct background * pba,
    struct perturbs * ppt,
    struct spectra * psp,
    enum file_format output_format,
    double z,
    int number_of_titles,
    double * data )
```

- compute  $T_i(k)$  for each k (if several ic's, compute it for each ic; if  $z_{pk} = 0$ , this is done by directly reading inside the pre-computed table; if not, this is done by interpolating the table at the correct value of tau.
- store data

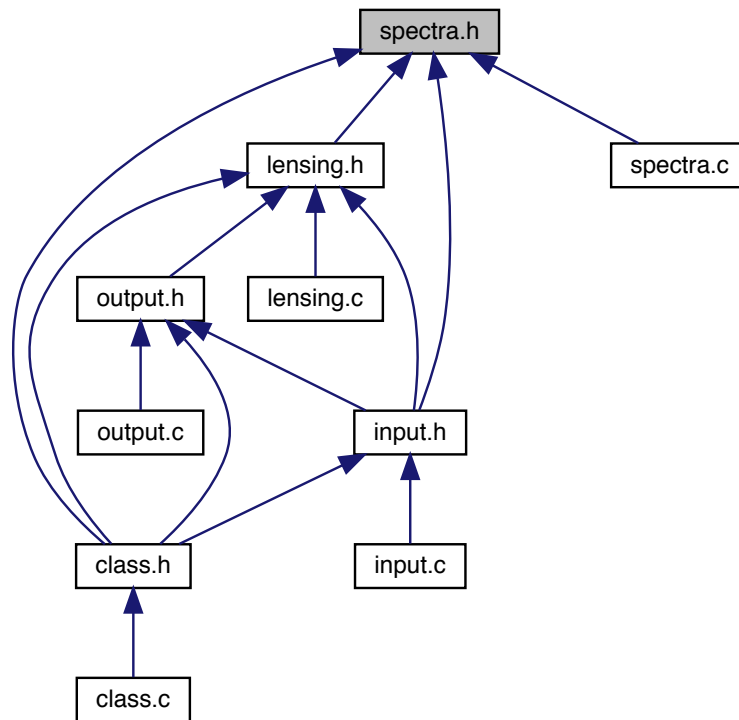
## 5.18 spectra.h File Reference

```
#include "transfer.h"
```

Include dependency graph for spectra.h:



This graph shows which files directly or indirectly include this file:



## Data Structures

- struct [spectra](#)

### 5.18.1 Detailed Description

Documented includes for spectra module

### 5.18.2 Data Structure Documentation

#### 5.18.2.1 struct spectra

Structure containing everything about anisotropy and Fourier power spectra that other modules need to know.

Once initialized by [spectra\\_init\(\)](#), contains a table of all  $C_l$ 's and  $P(k)$  as a function of multipole/wavenumber, mode (scalar/tensor...), type (for  $C_l$ 's: TT, TE...), and pairs of initial conditions (adiabatic, isocurvatures...).



## Data Fields

|          |                  |   |
|----------|------------------|---|
| double   | z_max_pk         | maximum value of z at which matter spectrum P(k,z) will be evaluated; keep fixed to zero if P(k) only needed today  |
| int      | non_diag         | sets the number of cross-correlation spectra that you want to calculate: 0 means only auto-correlation, 1 means only adjacent bins, and number of bins minus one means all correlations           |
| int      | md_size          | number of modes (scalar, tensor, ...) included in computation   |
| int      | index_md_scalars | index for scalar modes  |
| int *    | ic_size          | for a given mode, ic_size[index_md] = number of initial conditions included in computation  |
| int *    | ic_ic_size       | for a given mode, ic_ic_size[index_md] = number of pairs of (index_ic1, index_ic2) with index_ic2 >= index_ic1; this number is just N(N+1)/2 where N = ic_size[index_md]                          |
| short ** | is_non_zero      | for a given mode, is_non_zero[index_md][index_ic1_ic2] is set to true if the pair of initial conditions (index_ic1, index_ic2) are statistically correlated, or to false if they are uncorrelated |
| int      | has_tt           | do we want $C_l^{TT}$ ? (T = temperature)   |
| int      | has_ee           | do we want $C_l^{EE}$ ? (E = E-polarization)  |
| int      | has_te           | do we want $C_l^{TE}$ ?   |
| int      | has_bb           | do we want $C_l^{BB}$ ? (B = B-polarization)  |
| int      | has_pp           | do we want $C_l^{\phi\phi}$ ? ( $\phi$ = CMB lensing potential)   |
| int      | has_tp           | do we want $C_l^{T\phi}$ ?  |
| int      | has_ep           | do we want $C_l^{E\phi}$ ?  |
| int      | has_dd           | do we want $C_l^{dd}$ ? (d = density)   |
| int      | has_td           | do we want $C_l^{Td}$ ?   |
| int      | has_pd           | do we want $C_l^{\phi d}$ ?   |
| int      | has_ll           | do we want $C_l^{ll}$ ? (l = galaxy lensing potential)  |
| int      | has_tl           | do we want $C_l^{Tl}$ ?   |
| int      | has_dl           | do we want $C_l^{dl}$ ?   |
| int      | index_ct_tt      | index for type $C_l^{TT}$   |
| int      | index_ct_ee      | index for type $C_l^{EE}$   |
| int      | index_ct_te      | index for type $C_l^{TE}$   |
| int      | index_ct_bb      | index for type $C_l^{BB}$   |
| int      | index_ct_pp      | index for type $C_l^{\phi\phi}$   |
| int      | index_ct_tp      | index for type $C_l^{T\phi}$  |
| int      | index_ct_ep      | index for type $C_l^{E\phi}$  |
| int      | index_ct_dd      | first index for type $C_l^{dd}((d\_size*d\_size-(d\_size-non\_diag))*(d\_size-non\_diag-1)/2)$ values)  |
| int      | index_ct_td      | first index for type $C_l^{Td}$ (d_size values)   |
| int      | index_ct_pd      | first index for type $C_l^{pd}$ (d_size values)   |
| int      | index_ct_ll      | first index for type $C_l^{ll}((d\_size*d\_size-(d\_size-non\_diag))*(d\_size-non\_diag-1)/2)$ values)  |
| int      | index_ct_tl      | first index for type $C_l^{Tl}$ (d_size values)   |
| int      | index_ct_dl      | first index for type $C_l^{dl}$ (d_size values)   |

## Data Fields

|           |                   |  |
|-----------|-------------------|--|
| int       | d_size            | number of bins for which density Cl's are computed   |
| int       | ct_size           | number of $C_l$ types requested  |
| int *     | l_size            | number of multipole values for each requested mode, l_size[index_md]   |
| int       | l_size_max        | greatest of all l_size[index_md]   |
| double *  | l                 | list of multipole values l[index_l]  |
| int **    | l_max_ct          | last multipole (given as an input) at which we want to output $C_l$ 's for a given mode and type; l[index_md][l_size[index_md]-1] can be larger than l_max[index_md], in order to ensure a better interpolation with no boundary effects               |
| int *     | l_max             | last multipole (given as an input) at which we want to output $C_l$ 's for a given mode (maximized over types); l[index_md][l_size[index_md]-1] can be larger than l_max[index_md], in order to ensure a better interpolation with no boundary effects |
| int       | l_max_tot         | last multipole (given as an input) at which we want to output $C_l$ 's (maximized over modes and types); l[index_md][l_size[index_md]-1] can be larger than l_max[index_md], in order to ensure a better interpolation with no boundary effects        |
| double ** | cl                | table of anisotropy spectra for each mode, multipole, pair of initial conditions and types, cl[index_md][(index_l * psp->ic_size[index_md] + index_ic1_ic2) * psp->ct_size + index_ct]   |
| double ** | ddcl              | second derivatives of previous table with respect to l, in view of spline interpolation  |
| double    | alpha_II_2_20     | parameter describing adiabatic versus isocurvature contribution in mutipole range [2,20] (see Planck parameter papers)   |
| double    | alpha_RI_2_20     | parameter describing adiabatic versus isocurvature contribution in mutipole range [2,20] (see Planck parameter papers)   |
| double    | alpha_RR_2_20     | parameter describing adiabatic versus isocurvature contribution in mutipole range [2,20] (see Planck parameter papers)   |
| double    | alpha_II_21_200   | parameter describing adiabatic versus isocurvature contribution in mutipole range [21,200] (see Planck parameter papers)   |
| double    | alpha_RI_21_200   | parameter describing adiabatic versus isocurvature contribution in mutipole range [21,200] (see Planck parameter papers)   |
| double    | alpha_RR_21_200   | parameter describing adiabatic versus isocurvature contribution in mutipole range [21,200] (see Planck parameter papers)   |
| double    | alpha_II_201_2500 | parameter describing adiabatic versus isocurvature contribution in mutipole range [201,2500] (see Planck parameter papers)   |
| double    | alpha_RI_201_2500 | parameter describing adiabatic versus isocurvature contribution in mutipole range [201,2500] (see Planck parameter papers)   |
| double    | alpha_RR_201_2500 | parameter describing adiabatic versus isocurvature contribution in mutipole range [201,2500] (see Planck parameter papers)   |
| double    | alpha_II_2_2500   | parameter describing adiabatic versus isocurvature contribution in mutipole range [2,2500] (see Planck parameter papers)   |
| double    | alpha_RI_2_2500   | parameter describing adiabatic versus isocurvature contribution in mutipole range [2,2500] (see Planck parameter papers)   |
| double    | alpha_RR_2_2500   | parameter describing adiabatic versus isocurvature contribution in mutipole range [2,2500] (see Planck parameter papers)   |
| double    | alpha_kp          | parameter describing adiabatic versus isocurvature contribution at pivot scale (see Planck parameter papers)   |
| double    | alpha_k1          | parameter describing adiabatic versus isocurvature contribution at scale k1 (see Planck parameter papers)  |

## Data Fields

|          |                      |  |
|----------|----------------------|--|
| double   | alpha_k2             | parameter describing adiabatic versus isocurvature contribution at scale k2 (see Planck parameter papers)  |
| int      | ln_k_size            | number ln(k) values  |
| double * | ln_k                 | list of ln(k) values ln_k[index_k]   |
| int      | ln_tau_size          | number ln(tau) values (only one if z_max_pk = 0)   |
| double * | ln_tau               | list of ln(tau) values ln_tau[index_tau]   |
| double * | ln_pk                | Matter power spectrum. depends on indices index_md, index_ic1, index_ic2, index_k, index_tau as: ln_pk[(index_tau * psp->k_size + index_k) * psp->ic_ic_size[index_md] + index_ic1_ic2] where index_ic1_ic2 labels ordered pairs (index_ic1, index_ic2) (since the primordial spectrum is symmetric in (index_ic1, index_ic2)). <ul style="list-style-type: none"> <li>• for diagonal elements (index_ic1 = index_ic2) this arrays contains ln[P(k)] where P(k) is positive by construction.</li> <li>• for non-diagonal elements this arrays contains the k-dependent cosine of the correlation angle, namely <math>P(k)_{(index\_ic1, index\_ic2)} / \sqrt{P(k)_{index\_ic1} P(k)_{index\_ic2}}</math> This choice is convenient since the sign of the non-diagonal cross-correlation is arbitrary. For fully correlated or anti-correlated initial conditions, this non-diagonal element is independent on k, and equal to +1 or -1.</li> </ul> |
| double * | ddln_pk              | second derivative of above array with respect to log(tau), for spline interpolation. So: <ul style="list-style-type: none"> <li>• for index_ic1 = index_ic, we spline ln[P(k)] vs. ln(k), which is good since this function is usually smooth.</li> <li>• for non-diagonal coefficients, we spline <math>P(k)_{(index\_ic1, index\_ic2)} / \sqrt{P(k)_{index\_ic1} P(k)_{index\_ic2}}</math> vs. ln(k), which is fine since this quantity is often assumed to be constant (e.g for fully correlated/anticorrelated initial conditions) or nearly constant, and with arbitrary sign.</li> </ul>   |
| double   | sigma8               | sigma8 parameter   |
| double * | ln_pk_nl             | Non-linear matter power spectrum. depends on indices index_k, index_tau as: ln_pk_nl[index_tau * psp->k_size + index_k]  |
| double * | ddln_pk_nl           | second derivative of above array with respect to log(tau), for spline interpolation.   |
| int      | index_tr_delta_g     | index of gamma density transfer function   |
| int      | index_tr_delta_b     | index of baryon density transfer function  |
| int      | index_tr_delta_cdm   | index of cold dark matter density transfer function  |
| int      | index_tr_delta_dcdm  | index of decaying cold dark matter density transfer function   |
| int      | index_tr_delta_scf   | index of scalar field phi transfer function  |
| int      | index_tr_delta_fld   | index of dark energy fluid density transfer function   |
| int      | index_tr_delta_ur    | index of ultra-relativistic neutrinos/relics density transfer function   |
| int      | index_tr_delta_dr    | index of decay radiation density transfer function   |
| int      | index_tr_delta_ncdm1 | index of first species of non-cold dark matter (massive neutrinos, ...) density transfer function  |
| int      | index_tr_delta_tot   | index of total matter density transfer function  |
| int      | index_tr_theta_g     | index of gamma velocity transfer function  |
| int      | index_tr_theta_b     | index of baryon velocity transfer function   |
| int      | index_tr_theta_cdm   | index of cold dark matter velocity transfer function   |

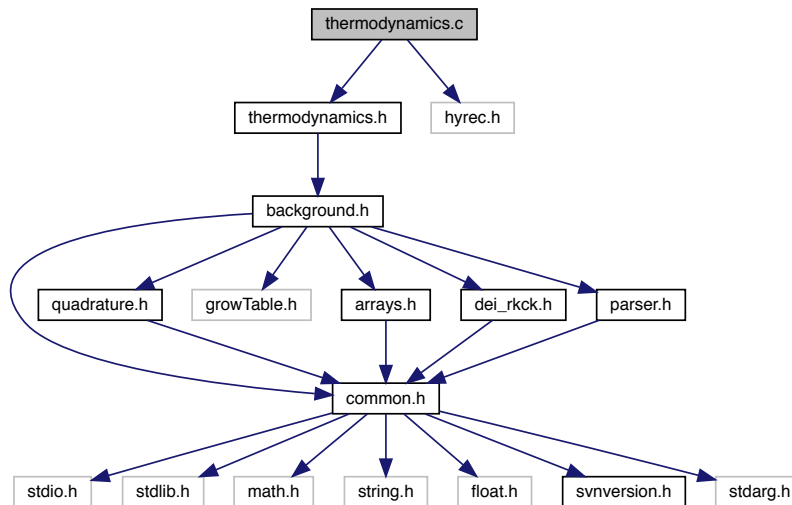
## Data Fields

|          |                      |   |
|----------|----------------------|---|
| int      | index_tr_theta_dcdm  | index of decaying cold dark matter velocity transfer function   |
| int      | index_tr_theta_scf   | index of derivative of scalar field phi transfer function   |
| int      | index_tr_theta_fld   | index of dark energy fluid velocity transfer function   |
| int      | index_tr_theta_ur    | index of ultra-relativistic neutrinos/relics velocity transfer function   |
| int      | index_tr_theta_dr    | index of decay radiation velocity transfer function   |
| int      | index_tr_theta_ncdm1 | index of first species of non-cold dark matter (massive neutrinos, ...) velocity transfer function  |
| int      | index_tr_theta_tot   | index of total matter velocity transfer function  |
| int      | index_tr_phi         | index of Bardeen potential phi  |
| int      | index_tr_psi         | index of Bardeen potential psi  |
| int      | index_tr_phi_prime   | index of derivative of Bardeen potential phi  |
| int      | index_tr_h           | index of synchronous gauge metric perturbation h  |
| int      | index_tr_h_prime     | index of synchronous gauge metric perturbation h'   |
| int      | index_tr_eta         | index of synchronous gauge metric perturbation eta  |
| int      | index_tr_eta_prime   | index of synchronous gauge metric perturbation eta'   |
| int      | tr_size              | total number of species in transfer functions   |
| double * | matter_transfer      | Matter transfer functions. Depends on indices index_md, index_tau, index_ic, index_k, index_tr as:<br>matter_transfer[((index_tau*psp->ln_k_size + index_k) * psp->ic_size[index_md] + index_ic) * psp->tr_size + index_tr] |
| double * | ddmatter_transfer    | second derivative of above array with respect to log(tau), for spline interpolation.  |
| short    | spectra_verbose      | flag regulating the amount of information sent to standard output (none if set to zero)   |
| ErrorMsg | error_message        | zone for writing error messages   |

## 5.19 thermodynamics.c File Reference

```
#include "thermodynamics.h"
#include "hyrec.h"
```

Include dependency graph for thermodynamics.c:



## Functions

- int `thermodynamics_at_z` (struct `background` \*pba, struct `thermo` \*pth, double z, short inter\_mode, int \*last\_index, double \*pvecback, double \*pvecthermo)
- int `thermodynamics_init` (struct `precision` \*ppr, struct `background` \*pba, struct `thermo` \*pth)
- int `thermodynamics_free` (struct `thermo` \*pth)
- int `thermodynamics_indices` (struct `thermo` \*pth, struct `recombination` \*preco, struct `reionization` \*preio)
- int `thermodynamics_helium_from_bbn` (struct `precision` \*ppr, struct `background` \*pba, struct `thermo` \*pth)
- int `thermodynamics_onthespot_energy_injection` (struct `precision` \*ppr, struct `background` \*pba, struct `recombination` \*preco, double z, double \*energy\_rate, `ErrorMsg` error\_message)
- int `thermodynamics_energy_injection` (struct `precision` \*ppr, struct `background` \*pba, struct `recombination` \*preco, double z, double \*energy\_rate, `ErrorMsg` error\_message)
- int `thermodynamics_reionization_function` (double z, struct `thermo` \*pth, struct `reionization` \*preio, double \*xe)
- int `thermodynamics_get_xe_before_reionization` (struct `precision` \*ppr, struct `thermo` \*pth, struct `recombination` \*preco, double z, double \*xe)
- int `thermodynamics_reionization` (struct `precision` \*ppr, struct `background` \*pba, struct `thermo` \*pth, struct `recombination` \*preco, struct `reionization` \*preio, double \*pvecback)
- int `thermodynamics_reionization_sample` (struct `precision` \*ppr, struct `background` \*pba, struct `thermo` \*pth, struct `recombination` \*preco, struct `reionization` \*preio, double \*pvecback)
- int `thermodynamics_recombination` (struct `precision` \*ppr, struct `background` \*pba, struct `thermo` \*pth, struct `recombination` \*preco, double \*pvecback)
- int `thermodynamics_recombination_with_hyrec` (struct `precision` \*ppr, struct `background` \*pba, struct `thermo` \*pth, struct `recombination` \*preco, double \*pvecback)
- int `thermodynamics_recombination_with_refcast` (struct `precision` \*ppr, struct `background` \*pba, struct `thermo` \*pth, struct `recombination` \*preco, double \*pvecback)
- int `thermodynamics_derivs_with_refcast` (double z, double \*y, double \*dy, void \*parameters\_and\_workspace, `ErrorMsg` error\_message)
- int `thermodynamics_merge_reco_and_reio` (struct `precision` \*ppr, struct `thermo` \*pth, struct `recombination` \*preco, struct `reionization` \*preio)
- int `thermodynamics_output_titles` (struct `background` \*pba, struct `thermo` \*pth, char titles[\_MAXTITLESTR\_INGLENGTH\_])

### 5.19.1 Detailed Description

Documented thermodynamics module

Julien Lesgourgues, 6.09.2010

Deals with the thermodynamical evolution. This module has two purposes:

- at the beginning, to initialize the thermodynamics, i.e. to integrate the thermodynamical equations, and store all thermodynamical quantities as a function of redshift inside an interpolation table. The current version of recombination is based on RECFAST v1.5. The current version of reionization is based on exactly the same reionization function as in CAMB, in order to make allow for comparison. It should be easy to generalize the module to more complicated reionization histories.
- to provide a routine which allow other modules to evaluate any thermodynamical quantities at a given redshift value (by interpolating within the interpolation table).

The logic is the following:

- in a first step, the code assumes that there is no reionization, and computes the ionization fraction, Thomson scattering rate, baryon temperature, etc., using RECFAST. The result is stored in a temporary table 'recombination\_table' (within a temporary structure of type 'recombination') for each redshift in a range  $0 < z < z_{\text{initial}}$ . The sampling in  $z$  space is done with a simple linear step size.
- in a second step, the code adds the reionization history, starting from a redshift  $z_{\text{reio\_start}}$ . The ionization fraction at this redshift is read in the previous recombination table in order to ensure a perfect matching. The code computes the ionization fraction, Thomson scattering rate, baryon temperature, etc., using a given parametrization of the reionization history. The result is stored in a temporary table 'reionization\_table' (within a temporary structure of type 'reionization') for each redshift in the range  $0 < z < z_{\text{reio\_start}}$ . The sampling in  $z$  space is found automatically, given the precision parameter 'reionization\_sampling'.
- in a third step, the code merges the two tables 'recombination\_table' and 'reionization\_table' inside the table 'thermodynamics\_table', and the temporary structures 'recombination' and 'reionization' are freed. In 'thermodynamics\_table', the sampling in  $z$  space is the one defined in the recombination algorithm for  $z_{\text{reio\_start}} < z < z_{\text{initial}}$ , and the one defined in the reionization algorithm for  $0 < z < z_{\text{reio\_start}}$ .
- at this stage, only a few columns in the table 'thermodynamics\_table' have been filled. In a fourth step, the remaining columns are filled, using some numerical integration/derivation routines from the 'array.c' tools module.
- small detail: one of the columns contains the maximum variation rate of a few relevant thermodynamical quantities. This rate will be used for defining automatically the sampling step size in the perturbation module. Hence, the exact value of this rate is unimportant, but its order of magnitude at a given  $z$  defines the sampling precision of the perturbation module. Hence, it is harmless to use a smoothing routine in order to make this rate look nicer, although this will not affect the final result significantly. The last step in the thermodynamics↔\_init module is to perform this smoothing.

In summary, the following functions can be called from other modules:

1. [thermodynamics\\_init\(\)](#) at the beginning (but after [background\\_init\(\)](#))
2. [thermodynamics\\_at\\_z\(\)](#) at any later time
3. [thermodynamics\\_free\(\)](#) at the end, when no more calls to [thermodynamics\\_at\\_z\(\)](#) are needed

## 5.19.2 Function Documentation

### 5.19.2.1 thermodynamics\_at\_z()

```
int thermodynamics_at_z (
    struct background * pba,
    struct thermo * pth,
    double z,
    short inter_mode,
    int * last_index,
    double * pvecback,
    double * pvecthermo )
```

Thermodynamics quantities at given redshift z.

Evaluates all thermodynamics quantities at a given value of the redshift by reading the pre-computed table and interpolating.

#### Parameters

|                   |   |
|-------------------|---|
| <i>pba</i>        | Input: pointer to background structure  |
| <i>pth</i>        | Input: pointer to the thermodynamics structure (containing pre-computed table)  |
| <i>z</i>          | Input: redshift   |
| <i>inter_mode</i> | Input: interpolation mode (normal or growing_closeby)   |
| <i>last_index</i> | Input/Output: index of the previous/current point in the interpolation array (input only for closeby mode, output for both)   |
| <i>pvecback</i>   | Input: vector of background quantities (used only in case $z > z_{\text{initial}}$ for getting ddkappa and dddkappa; in that case, should be already allocated and filled, with format short_info or larger; in other cases, will be ignored) |
| <i>pvecthermo</i> | Output: vector of thermodynamics quantities (assumed to be already allocated)   |

#### Returns

the error status

Summary:

- define local variables
- interpolate in table with array\_interpolate\_spline() (normal mode) or array\_interpolate\_spline\_growing\_closeby() (closeby mode)

### 5.19.2.2 thermodynamics\_init()

```
int thermodynamics_init (
    struct precision * ppr,
    struct background * pba,
    struct thermo * pth )
```

Initialize the thermo structure, and in particular the thermodynamics interpolation table.

## Parameters

|            |   |
|------------|---|
| <i>ppr</i> | Input: pointer to precision structure                 |
| <i>pba</i> | Input: pointer to background structure                |
| <i>pth</i> | Input/Output: pointer to initialized thermo structure |

## Returns

the error status

## Summary:

- define local variables
- initialize pointers, allocate background vector
- compute and check primordial Helium fraction
- check energy injection parameters
- assign values to all indices in the structures with [thermodynamics\\_indices\(\)](#)
- solve recombination and store values of  $z, x_e, d\kappa/d\tau, T_b, c_b^2$  with [thermodynamics\\_recombination\(\)](#)
- if there is reionization, solve reionization and store values of  $z, x_e, d\kappa/d\tau, T_b, c_b^2$  with [thermodynamics\\_reionization\(\)](#)
- merge tables in recombination and reionization structures into a single table in thermo structure
- compute table of corresponding conformal times
- store initial value of conformal time in the structure
- fill missing columns (quantities not computed previously but related)
- → baryon drag interaction rate time minus one,  $-\text{[R * kappa]}$ , stored temporarily in column `ddkappa`
- → second derivative of this rate,  $-\text{[R * kappa]'}$ , stored temporarily in column `dddkappa`
- → compute  $\tau\_d = [\text{int}_{\{\tau\_today\}}^{\{\tau\}} d\tau - d\text{kappa}_d/d\tau]$
- → compute  $r\_d = [\text{int}_{\{\tau\_ini\}}^{\{\tau\}} d\tau [1/\text{kappa}]$
- → second derivative with respect to  $\tau$  of  $d\text{kappa}$  (in view of spline interpolation)
- → first derivative with respect to  $\tau$  of  $d\text{kappa}$  (using spline interpolation)
- → compute  $-\text{kappa} = [\text{int}_{\{\tau\_today\}}^{\{\tau\}} d\tau d\text{kappa}/d\tau]$ , store temporarily in column "g"
- → derivatives of baryon sound speed (only computed if some non-minimal tight-coupling schemes is requested)
- → second derivative with respect to  $\tau$  of  $cb2$
- → first derivative with respect to  $\tau$  of  $cb2$  (using spline interpolation)
- → compute visibility:  $g = (d\kappa/d\tau)e^{-\kappa}$
- → compute  $g$
- → compute  $\exp(-\text{kappa})$
- → compute  $g'$  (the plus sign of the second term is correct, see def of  $-\text{kappa}$  in thermodynamics module!)



- —> compute  $g''$
- —> store  $g$
- —> compute variation rate
- smooth the rate (details of smoothing unimportant: only the order of magnitude of the rate matters)
- fill tables of second derivatives with respect to  $z$  (in view of spline interpolation)
- find maximum of  $g$
- find conformal recombination time using [background\\_tau\\_of\\_z\(\)](#)
- find damping scale at recombination (using linear interpolation)
- find time (always after recombination) at which  $\tau_c/\tau$  falls below some threshold, defining  $\tau_{\text{free}}$   $\leftrightarrow$  streaming
- find baryon drag time (when  $\tau_d$  crosses one, using linear interpolation) and sound horizon at that time
- find time above which visibility falls below a given fraction of its maximum
- if verbose flag set to next-to-minimum value, print the main results

### 5.19.2.3 thermodynamics\_free()

```
int thermodynamics_free (
    struct thermo * pth )
```

Free all memory space allocated by [thermodynamics\\_init\(\)](#).

#### Parameters

|            |   |
|------------|---|
| <i>pth</i> | Input/Output: pointer to thermo structure (to be freed) |
|------------|---|

#### Returns

the error status

### 5.19.2.4 thermodynamics\_indices()

```
int thermodynamics_indices (
    struct thermo * pth,
    struct recombination * preco,
    struct reionization * preio )
```

Assign value to each relevant index in vectors of thermodynamical quantities, as well as in vector containing reionization parameters.

**Parameters**

|              |  |
|--------------|--|
| <i>pth</i>   | Input/Output: pointer to thermo structure        |
| <i>preco</i> | Input/Output: pointer to recombination structure |
| <i>preio</i> | Input/Output: pointer to reionization structure  |

**Returns**

the error status

**Summary:**

- define local variables
- initialization of all indices and flags in thermo structure
- initialization of all indices and flags in recombination structure
- initialization of all indices and flags in reionization structure
- same with parameters of the function  $X_e(z)$

**5.19.2.5 thermodynamics\_helium\_from\_bbn()**

```
int thermodynamics_helium_from_bbn (
    struct precision * ppr,
    struct background * pba,
    struct thermo * pth )
```

Infer the primordial helium fraction from standard BBN, as a function of the baryon density and expansion rate during BBN.

This module is simpler than the one used in arXiv:0712.2826 because it neglects the impact of a possible significant chemical potentials for electron neutrinos. The full code with `xi_nu_e` could be introduced here later.

**Parameters**

|            |   |
|------------|---|
| <i>ppr</i> | Input: pointer to precision structure                 |
| <i>pba</i> | Input: pointer to background structure                |
| <i>pth</i> | Input/Output: pointer to initialized thermo structure |

**Returns**

the error status

**Summary:**

- Infer effective number of neutrinos at the time of BBN

- 8.6173e-11 converts from Kelvin to MeV. We randomly choose 0.1 MeV to be the temperature of BBN
- compute Delta  $N_{eff}$  as defined in bbn file, i.e.  $\Delta N_{eff} = 0$  means  $N_{eff} = 3.046$
- spline in one dimension (along deltaN)
- interpolate in one dimension (along deltaN)
- spline in remaining dimension (along omegab)
- interpolate in remaining dimension (along omegab)
- deallocate arrays

#### 5.19.2.6 thermodynamics\_onthespot\_energy\_injection()

```
int thermodynamics_onthespot_energy_injection (
    struct precision * ppr,
    struct background * pba,
    struct recombination * preco,
    double z,
    double * energy_rate,
    ErrorMsg error_message )
```

In case of non-minimal cosmology, this function determines the energy rate injected in the IGM at a given redshift  $z$  (= on-the-spot annihilation). This energy injection may come e.g. from dark matter annihilation or decay.

##### Parameters

|                      |   |
|----------------------|---|
| <i>ppr</i>           | Input: pointer to precision structure     |
| <i>pba</i>           | Input: pointer to background structure    |
| <i>preco</i>         | Input: pointer to recombination structure |
| <i>z</i>             | Input: redshift                           |
| <i>energy_rate</i>   | Output: energy density injection rate     |
| <i>error_message</i> | Output: error message                     |

##### Returns

the error status

#### 5.19.2.7 thermodynamics\_energy\_injection()

```
int thermodynamics_energy_injection (
    struct precision * ppr,
    struct background * pba,
    struct recombination * preco,
    double z,
    double * energy_rate,
    ErrorMsg error_message )
```

In case of non-minimal cosmology, this function determines the effective energy rate absorbed by the IGM at a given redshift (beyond the on-the-spot annihilation). This energy injection may come e.g. from dark matter annihilation or decay.

## Parameters

|                      |   |
|----------------------|---|
| <i>ppr</i>           | Input: pointer to precision structure     |
| <i>pba</i>           | Input: pointer to background structure    |
| <i>preco</i>         | Input: pointer to recombination structure |
| <i>z</i>             | Input: redshift                           |
| <i>energy_rate</i>   | Output: energy density injection rate     |
| <i>error_message</i> | Output: error message                     |

## Returns

the error status

## 5.19.2.8 thermodynamics\_reionization\_function()

```
int thermodynamics_reionization_function (
    double z,
    struct thermo * pth,
    struct reionization * preio,
    double * xe )
```

This subroutine contains the reionization function  $X_e(z)$  (one for each scheme; so far, only the function corresponding to the reio\_camb scheme is coded)

## Parameters

|              |  |
|--------------|--|
| <i>z</i>     | Input: redshift  |
| <i>pth</i>   | Input: pointer to thermo structure, to know which scheme is used                             |
| <i>preio</i> | Input: pointer to reionization structure, containing the parameters of the function $X_e(z)$ |
| <i>xe</i>    | Output: $X_e(z)$   |

## Summary:

- define local variables
- implementation of ionization function similar to the one in CAMB
- → case  $z > z_{\text{reio\_start}}$
- → case  $z < z_{\text{reio\_start}}$ : hydrogen contribution (tanh of complicated argument)
- → case  $z < z_{\text{reio\_start}}$ : helium contribution (tanh of simpler argument)
- implementation of binned ionization function similar to astro-ph/0606552
- → case  $z > z_{\text{reio\_start}}$
- implementation of many tanh jumps
- → case  $z > z_{\text{reio\_start}}$

- implementation of `reio_inter`
- $\rightarrow$  case  $z > z_{\text{reio\_start}}$

### 5.19.2.9 `thermodynamics_get_xe_before_reionization()`

```
int thermodynamics_get_xe_before_reionization (
    struct precision * ppr,
    struct thermo * pth,
    struct recombination * preco,
    double z,
    double * xe )
```

This subroutine reads  $X_e(z)$  in the recombination table at the time at which reionization starts. Hence it provides correct initial conditions for the reionization function.

#### Parameters

|              |   |
|--------------|---|
| <i>ppr</i>   | Input: pointer to precision structure     |
| <i>pth</i>   | Input: pointer to thermo structure        |
| <i>preco</i> | Input: pointer to recombination structure |
| <i>z</i>     | Input: redshift $z_{\text{reio\_start}}$  |
| <i>xe</i>    | Output: $X_e(z)$ at $z$                   |

### 5.19.2.10 `thermodynamics_reionization()`

```
int thermodynamics_reionization (
    struct precision * ppr,
    struct background * pba,
    struct thermo * pth,
    struct recombination * preco,
    struct reionization * preio,
    double * pvecback )
```

This routine computes the reionization history. In the `reio_camb` scheme, this is straightforward if the input parameter is the reionization redshift. If the input is the optical depth, need to find  $z_{\text{reio}}$  by dichotomy (trying several  $z_{\text{reio}}$  until the correct  $\tau_{\text{reio}}$  is approached).

#### Parameters

|                 |  |
|-----------------|--|
| <i>ppr</i>      | Input: pointer to precision structure  |
| <i>pba</i>      | Input: pointer to background structure   |
| <i>pth</i>      | Input: pointer to thermo structure   |
| <i>preco</i>    | Input: pointer to filled recombination structure   |
| <i>preio</i>    | Input/Output: pointer to reionization structure (to be filled)   |
| <i>pvecback</i> | Input: vector of background quantities (used as workspace: must be already allocated, with format <code>short_info</code> or larger, but does not need to be filled) |

**Returns**

the error status

**Summary:**

- define local variables
- allocate the vector of parameters defining the function  $X_e(z)$
- (a) if reionization implemented like in CAMB
  - → set values of these parameters, excepted those depending on the reionization redshift
  - → if reionization redshift given as an input, initialize the remaining values and fill reionization table
  - → if reionization optical depth given as an input, find reionization redshift by dichotomy and initialize the remaining values
- (b) if reionization implemented with reio\_bins\_tanh scheme
- (c) if reionization implemented with reio\_many\_tanh scheme
- (d) if reionization implemented with reio\_inter scheme

**5.19.2.11 thermodynamics\_reionization\_sample()**

```
int thermodynamics_reionization_sample (
    struct precision * ppr,
    struct background * pba,
    struct thermo * pth,
    struct recombination * preco,
    struct reionization * preio,
    double * pvecback )
```

For fixed input reionization parameters, this routine computes the reionization history and fills the reionization table.

**Parameters**

|                 |   |
|-----------------|---|
| <i>ppr</i>      | Input: pointer to precision structure   |
| <i>pba</i>      | Input: pointer to background structure  |
| <i>pth</i>      | Input: pointer to thermo structure  |
| <i>preco</i>    | Input: pointer to filled recombination structure  |
| <i>preio</i>    | Input/Output: pointer to reionization structure (to be filled)  |
| <i>pvecback</i> | Input: vector of background quantities (used as workspace: must be already allocated, with format short_info or larger, but does not need to be filled) |

**Returns**

the error status

**Summary:**

- define local variables
- (a) allocate vector of values related to reionization
- (b) create a growTable with gt\_init()
- (c) first line is taken from thermodynamics table, just before reionization starts
- -> look where to start in current thermodynamics table
- -> get redshift
- -> get  $X_e$
- -> get  $d\kappa/dz = (d\kappa/d\tau) * (d\tau/dz) = -(d\kappa/d\tau)/H$
- -> get baryon temperature
- -> after recombination,  $T_b$  scales like  $(1+z)**2$ . Compute constant factor  $T_b/(1+z)**2$ .
- -> get baryon sound speed
- -> store these values in growing table
- (d) set the maximum step value (equal to the step in thermodynamics table)
- (e) loop over redshift values in order to find values of  $z$ ,  $x_e$ ,  $\kappa'$  ( $T_b$  and  $cb_2$  found later by integration). The sampling in  $z$  space is found here.
- (f) allocate reionization\_table with correct size
- (g) retrieve data stored in the growTable with gt\_getPtr()
- (h) copy growTable to reionization\_temporary\_table (invert order of lines, so that redshift is growing, like in recombination table)
- (i) free the growTable with gt\_free() , free vector of reionization variables
- (j) another loop on  $z$ , to integrate equation for  $T_b$  and to compute  $cb_2$
- -> derivative of baryon temperature
- -> increment baryon temperature
- -> get baryon sound speed
- -> spline  $d\tau/dz$  with respect to  $z$  in view of integrating for optical depth
- -> integrate for optical depth

#### 5.19.2.12 thermodynamics\_recombination()

```
int thermodynamics_recombination (
    struct precision * ppr,
    struct background * pba,
    struct thermo * pth,
    struct recombination * preco,
    double * pvecback )
```

Integrate thermodynamics with your favorite recombination code.



## 5.19.2.13 thermodynamics\_recombination\_with\_hyrec()

```
int thermodynamics_recombination_with_hyrec (
    struct precision * ppr,
    struct background * pba,
    struct thermo * pth,
    struct recombination * preco,
    double * pvecback )
```

Integrate thermodynamics with HyRec.

Integrate thermodynamics with HyRec, allocate and fill the part of the thermodynamics interpolation table (the rest is filled in `thermodynamics_init()`). Called once by `thermodynamics_recombination()`, from `thermodynamics_init()`.

HYREC: Hydrogen and Helium Recombination Code  
Written by Yacine Ali-Haimoud and Chris Hirata (Caltech)

## Parameters

|                 |   |
|-----------------|---|
| <i>ppr</i>      | Input: pointer to precision structure                                     |
| <i>pba</i>      | Input: pointer to background structure                                    |
| <i>pth</i>      | Input: pointer to thermodynamics structure                                |
| <i>preco</i>    | Output: pointer to recombination structure                                |
| <i>pvecback</i> | Input: pointer to an allocated (but empty) vector of background variables |

Summary:

- Fill hyrec parameter structure
- Build effective rate tables
- distribute addresses for each table
- Normalize 2s–1s differential decay rate to L2s1s (can be set by user in hydrogen.h)
- Compute the recombination history by calling a function in hyrec (no CLASS-like error management here)
- fill a few parameters in preco and pth
- allocate memory for thermodynamics interpolation tables (size known in advance) and fill it
- → get redshift, corresponding results from hyrec, and background quantities
- → store the results in the table

## 5.19.2.14 thermodynamics\_recombination\_with\_recast()

```
int thermodynamics_recombination_with_recast (
    struct precision * ppr,
    struct background * pba,
    struct thermo * pth,
```

```

struct recombination * preco,
double * pvecback )

```

Integrate thermodynamics with RECFAST.

Integrate thermodynamics with RECFAST, allocate and fill the part of the thermodynamics interpolation table (the rest is filled in [thermodynamics\\_init\(\)](#)). Called once by `thermodynamics_recombination`, from [thermodynamics\\_init\(\)](#).

RECFAST is an integrator for Cosmic Recombination of Hydrogen and Helium, developed by Douglas Scott ([dscott@astro.ubc.ca](mailto:dscott@astro.ubc.ca)) based on calculations in the paper Seager, Sasselov & Scott (ApJ, 523, L1, 1999). and "fudge" updates in Wong, Moss & Scott (2008).

Permission to use, copy, modify and distribute without fee or royalty at any tier, this software and its documentation, for any purpose and without fee or royalty is hereby granted, provided that you agree to comply with the following copyright notice and statements, including the disclaimer, and that the same appear on ALL copies of the software and documentation, including modifications that you make for internal use or for distribution:

Copyright 1999-2010 by University of British Columbia. All rights reserved.

THIS SOFTWARE IS PROVIDED "AS IS", AND U.B.C. MAKES NO REPRESENTATIONS OR WARRANTIES, EXPRESS OR IMPLIED. BY WAY OF EXAMPLE, BUT NOT LIMITATION, U.B.C. MAKES NO REPRESENTATIONS OR WARRANTIES OF MERCHANTABILITY OR FITNESS FOR ANY PARTICULAR PURPOSE OR THAT THE USE OF THE LICENSED SOFTWARE OR DOCUMENTATION WILL NOT INFRINGE ANY THIRD PARTY PATENTS, COPYRIGHTS, TRADEMARKS OR OTHER RIGHTS.

Version 1.5: includes extra fitting function from Rubino-Martin et al. arXiv:0910.4383v1 [astro-ph.CO]

#### Parameters

|                 |   |
|-----------------|---|
| <i>ppr</i>      | Input: pointer to precision structure                                     |
| <i>pba</i>      | Input: pointer to background structure                                    |
| <i>pth</i>      | Input: pointer to thermodynamics structure                                |
| <i>preco</i>    | Output: pointer to recombination structure                                |
| <i>pvecback</i> | Input: pointer to an allocated (but empty) vector of background variables |

#### Returns

the error status

#### Summary:

- define local variables
- allocate memory for thermodynamics interpolation tables (size known in advance)
- initialize generic integrator with `initialize_generic_integrator()`
- read a few precision/cosmological parameters
- define the fields of the 'thermodynamics parameter and workspace' structure
- impose initial conditions at early times
- loop over redshift steps  $N_z$ ; integrate over each step with `generic_integrator()`, store the results in the table using [thermodynamics\\_derivs\\_with\\_recfast\(\)](#)

- -> first approximation: H and Helium fully ionized
- -> second approximation: first Helium recombination (analytic approximation)
- -> third approximation: first Helium recombination completed
- -> fourth approximation: second Helium recombination starts (analytic approximation)
- -> fifth approximation: second Helium recombination (full evolution for Helium), H recombination starts (analytic approximation)
- -> last case: full evolution for H and Helium
- -> store the results in the table
- cleanup generic integrator with `cleanup_generic_integrator()`

#### 5.19.2.15 `thermodynamics_derivs_with_recast()`

```
int thermodynamics_derivs_with_recast (
    double z,
    double * y,
    double * dy,
    void * parameters_and_workspace,
    ErrorMsg error_message )
```

Subroutine evaluating the derivative with respect to redshift of thermodynamical quantities (from RECFast version 1.4).

Computes derivatives of the three variables to integrate:  $dx_H/dz$ ,  $dx_{He}/dz$ ,  $dT_{mat}/dz$ .

This is one of the few functions in the code which are passed to the `generic_integrator()` routine. Since `generic_integrator()` should work with functions passed from various modules, the format of the arguments is a bit special:

- fixed parameters and workspaces are passed through a generic pointer. Here, this pointer contains the precision, background and recombination structures, plus a background vector, but `generic_integrator()` doesn't know its fine structure.
- the error management is a bit special: errors are not written as usual to `pth->error_message`, but to a generic `error_message` passed in the list of arguments.

#### Parameters

|                                       |   |
|---------------------------------------|---|
| <code>z</code>                        | Input: redshift   |
| <code>y</code>                        | Input: vector of variable to integrate  |
| <code>dy</code>                       | Output: its derivative (already allocated)  |
| <code>parameters_and_workspace</code> | Input: pointer to fixed parameters (e.g. indices) and workspace (already allocated) |
| <code>error_message</code>            | Output: error message   |

### 5.19.2.16 thermodynamics\_merge\_reco\_and\_reio()

```
int thermodynamics_merge_reco_and_reio (
    struct precision * ppr,
    struct thermo * pth,
    struct recombination * preco,
    struct reionization * preio )
```

This routine merges the two tables 'recombination\_table' and 'reionization\_table' inside the table 'thermodynamics\_table', and frees the temporary structures 'recombination' and 'reionization'.

#### Parameters

|              |  |
|--------------|--|
| <i>ppr</i>   | Input: pointer to precision structure            |
| <i>pth</i>   | Input/Output: pointer to thermo structure        |
| <i>preco</i> | Input: pointer to filled recombination structure |
| <i>preio</i> | Input: pointer to reionization structure         |

#### Returns

the error status

#### Summary:

- define local variables
- first, a little check that the two tables match each other and can be merged
- find number of redshift in full table = number in reco + number in reio - overlap
- allocate arrays in thermo structure
- fill these arrays
- free the temporary structures

### 5.19.2.17 thermodynamics\_output\_titles()

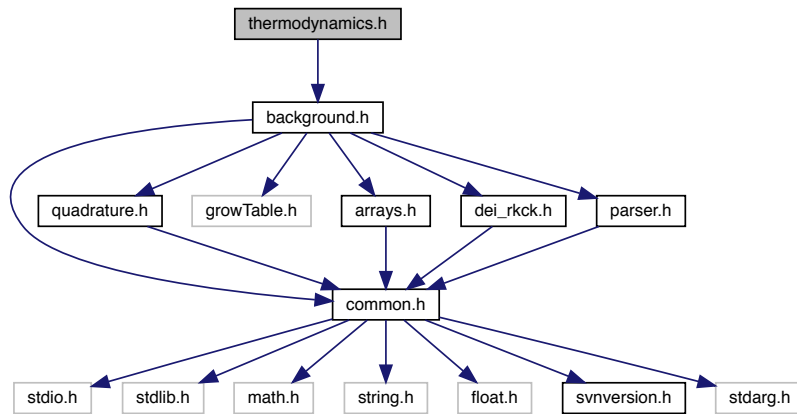
```
int thermodynamics_output_titles (
    struct background * pba,
    struct thermo * pth,
    char titles[_MAXTITLESTRINGLENGTH_] )
```

Subroutine for formatting thermodynamics output

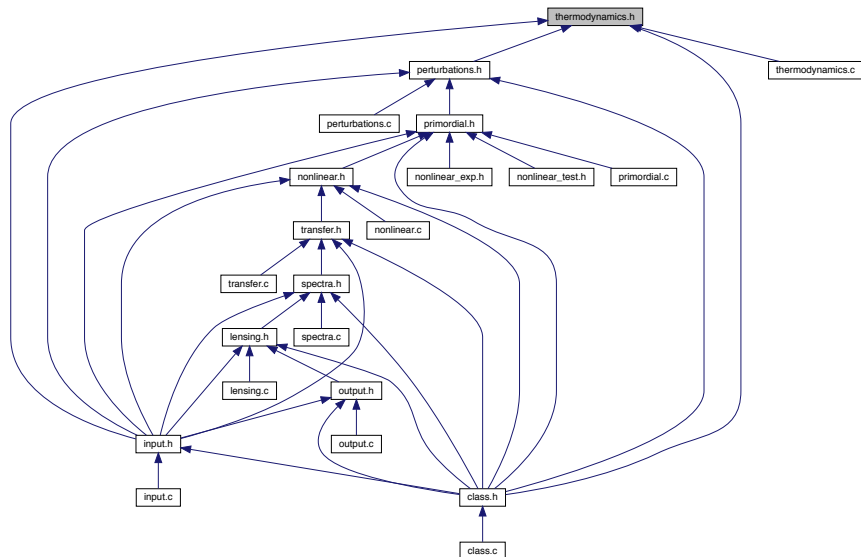
## 5.20 thermodynamics.h File Reference

```
#include "background.h"
```

Include dependency graph for thermodynamics.h:



This graph shows which files directly or indirectly include this file:



### Data Structures

- struct [thermo](#)
- struct [recombination](#)
- struct [reionization](#)
- struct [thermodynamics\\_parameters\\_and\\_workspace](#)

## Macros

- `#define f1(x) (-0.75*x*(x*x/3.-1.)+0.5)`
- `#define f2(x) (x*x*(0.5-x/3.)*6.)`
  
- `#define _YHE_BIG_ 0.5`
- `#define _YHE_SMALL_ 0.01`

## Enumerations

- enum `recombination_algorithm`
- enum `reionization_parametrization` {  
`reio_none`, `reio_camb`, `reio_bins_tanh`, `reio_half_tanh`,  
`reio_many_tanh`, `reio_inter` }
- enum `reionization_z_or_tau` { `reio_z`, `reio_tau` }

### 5.20.1 Detailed Description

Documented includes for thermodynamics module

### 5.20.2 Data Structure Documentation

#### 5.20.2.1 struct thermo

All thermodynamics parameters and evolution that other modules need to know.

Once initialized by `thermodynamics_init()`, contains all the necessary information on the thermodynamics, and in particular, a table of thermodynamical quantities as a function of the redshift, used for interpolation in other modules.

#### Data Fields

|  |  |                         |   |
|--|--|-------------------------|---|
|  | double   | YHe                     | $Y_{He}$ : primordial helium fraction   |
|  | enum <code>recombination_algorithm</code>      | recombination           | recombination code  |
|  | enum <code>reionization_parametrization</code> | reio_parametrization    | reionization scheme   |
|  | enum <code>reionization_z_or_tau</code>        | reio_z_or_tau           | is the input parameter the reionization redshift or optical depth?                                    |
|  | double   | tau_reio                | if above set to tau, input value of reionization optical depth  |
|  | double   | z_reio                  | if above set to z, input value of reionization redshift   |
|  | short  | compute_cb2_derivatives | do we want to include in computation derivatives of baryon sound speed?                               |
|  | short  | compute_damping_scale   | do we want to compute the simplest analytic approximation to the photon damping (or diffusion) scale? |
|  | double   | reionization_width      | parameters for reio_camb width of H reionization  |

## Data Fields

|          |                            |  |
|----------|----------------------------|--|
| double   | reionization_exponent      | shape of H reionization  |
| double   | helium_fullreio_redshift   | redshift for of helium reionization  |
| double   | helium_fullreio_width      | width of helium reionization   |
| int      | binned_reio_num            | parameters for reio_bins_tanh with how many bins do we want to describe reionization?  |
| double * | binned_reio_z              | central z value for each bin   |
| double * | binned_reio_xe             | imposed $X_e(z)$ value at center of each bin   |
| double   | binned_reio_step_sharpness | sharpness of tanh() step interpolating between binned values   |
| int      | many_tanh_num              | parameters for reio_many_tanh with how many jumps do we want to describe reionization?   |
| double * | many_tanh_z                | central z value for each tanh jump   |
| double * | many_tanh_xe               | imposed $X_e(z)$ value at the end of each jump (ie at later times)   |
| double   | many_tanh_width            | sharpness of tanh() steps  |
| int      | reio_inter_num             | parameters for reio_inter with how many jumps do we want to describe reionization?   |
| double * | reio_inter_z               | discrete z values  |
| double * | reio_inter_xe              | discrete $X_e(z)$ values   |
| double   | annihilation               | parameters for energy injection  |
| short    | has_on_the_spot            | parameter describing CDM annihilation ( $f \langle \sigma v \rangle / m_{\text{cdm}}$ , see e.g. 0905.0003)  |
| double   | decay                      | flag to specify if we want to use the on-the-spot approximation  |
| double   | annihilation_variation     | parameter describing CDM decay ( $f/\tau$ , see e.g. 1109.6322)  |
| double   | annihilation_z             | if this parameter is non-zero, the function $F(z)=(f \langle \sigma v \rangle / m_{\text{cdm}})(z)$ will be a parabola in log-log scale between $z_{\text{min}}$ and $z_{\text{max}}$ , with a curvature given by $\text{annihilation\_variation}$ (must be negative), and with a maximum in $z_{\text{max}}$ ; it will be constant outside this range |
| double   | annihilation_zmax          | if $\text{annihilation\_variation}$ is non-zero, this is the value of z at which the parameter $\text{annihilation}$ is defined, i.e. $F(\text{annihilation\_z})=\text{annihilation}$  |
| double   | annihilation_zmin          | if $\text{annihilation\_variation}$ is non-zero, redshift above which annihilation rate is maximal   |
| double   | annihilation_f_halo        | if $\text{annihilation\_variation}$ is non-zero, redshift below which annihilation rate is constant  |
| double   | annihilation_z_halo        | takes the contribution of DM annihilation in halos into account  |
| int      | index_th_xe                | ionization fraction $x_e$  |

## Data Fields

|          |                            |  |
|----------|----------------------------|--|
| int      | index_th_dkappa            | Thomson scattering rate $d\kappa/d\tau$<br>(units 1/Mpc)   |
| int      | index_th_tau_d             | Baryon drag optical depth  |
| int      | index_th_ddkappa           | scattering rate derivative $d^2\kappa/d\tau^2$   |
| int      | index_th_dddkappa          | scattering rate second derivative<br>$d^3\kappa/d\tau^3$   |
| int      | index_th_exp_m_kappa       | $exp^{-\kappa}$  |
| int      | index_th_g                 | visibility function<br>$g = (d\kappa/d\tau) * exp^{-\kappa}$   |
| int      | index_th_dg                | visibility function derivative $(dg/d\tau)$  |
| int      | index_th_ddg               | visibility function second derivative<br>$(d^2g/d\tau^2)$  |
| int      | index_th_Tb                | baryon temperature $T_b$   |
| int      | index_th_cb2               | squared baryon sound speed $c_b^2$   |
| int      | index_th_dcb2              | derivative wrt conformal time of<br>squared baryon sound speed<br>$d[c_b^2]/d\tau$ (only computed if some<br>non-minimal tight-coupling schemes<br>is requested)   |
| int      | index_th_ddcb2             | second derivative wrt conformal time<br>of squared baryon sound speed<br>$d^2[c_b^2]/d\tau^2$ (only computed if some<br>non0-minimal tight-coupling schemes<br>is requested)                             |
| int      | index_th_rate              | maximum variation rate of $exp^{-\kappa}$ , $g$<br>and $(dg/d\tau)$ , used for computing<br>integration step in perturbation<br>module   |
| int      | index_th_r_d               | simple analytic approximation to the<br>photon comoving damping scale  |
| int      | th_size                    | size of thermodynamics vector  |
| int      | tt_size                    | number of lines (redshift steps) in the<br>tables  |
| double * | z_table                    | vector $z\_table[index\_z]$ with values of<br>redshift (vector of size $tt\_size$ )  |
| double * | thermodynamics_table       | table $thermodynamics\_table[index\_z \leftrightarrow$<br>$\_z * pth \rightarrow tt\_size + pba \rightarrow index\_th]$<br>with all other quantities (array of size<br>$th\_size * tt\_size$ )           |
| double * | d2thermodynamics_dz2_table | table $d2thermodynamics\_dz2\_z \leftrightarrow$<br>$table[index\_z * pth \rightarrow tt\_size + pba \rightarrow$<br>$index\_th]$ with values of $d^2t_i/dz^2$<br>(array of size $th\_size * tt\_size$ ) |
| double   | z_rec                      | $z$ at which the visibility reaches its<br>maximum (= recombination redshift)  |
| double   | tau_rec                    | conformal time at which the visibility<br>reaches its maximum (= recombination<br>time)  |
| double   | rs_rec                     | comoving sound horizon at<br>recombination   |



## Data Fields

|          |                        |  |
|----------|------------------------|--|
| double   | ds_rec                 | physical sound horizon at recombination  |
| double   | ra_rec                 | conformal angular diameter distance to recombination   |
| double   | da_rec                 | physical angular diameter distance to recombination  |
| double   | rd_rec                 | comoving photon damping scale at recombination   |
| double   | z_d                    | baryon drag redshift   |
| double   | tau_d                  | baryon drag time   |
| double   | ds_d                   | physical sound horizon at baryon drag  |
| double   | rs_d                   | comoving sound horizon at baryon drag  |
| double   | tau_cut                | at at which the visibility goes below a fixed fraction of the maximum visibility, used for an approximation in perturbation module |
| double   | angular_rescaling      | [ratio ra_rec / (tau0-tau_rec)]: gives CMB rescaling in angular space relative to flat model (=1 for curvature K=0)                |
| double   | tau_free_streaming     | minimum value of tau at which sfree-streaming approximation can be switched on   |
| double   | tau_ini                | initial conformal time at which thermodynamical variables have been be integrated  |
| double   | n_e                    | total number density of electrons today (free or not)  |
| short    | inter_normal           | flag for calling thermodynamics_at_z and find position in interpolation table normally   |
| short    | inter_closeby          | flag for calling thermodynamics_at_z and find position in interpolation table starting from previous position in previous call     |
| short    | thermodynamics_verbose | flag regulating the amount of information sent to standard output (none if set to zero)  |
| ErrorMsg | error_message          | zone for writing error messages  |

## 5.20.2.2 struct recombination

Temporary structure where all the recombination history is defined and stored.

This structure is used internally by the thermodynamics module, but never passed to other modules.

## Data Fields

|     |            |              |
|-----|------------|--------------|
| int | index_re_z | redshift $z$ |
|-----|------------|--------------|

## Data Fields

|          |                        |  |
|----------|------------------------|--|
| int      | index_re_xe            | ionization fraction $x_e$  |
| int      | index_re_Tb            | baryon temperature $T_b$   |
| int      | index_re_cb2           | squared baryon sound speed $c_b^2$   |
| int      | index_re_dkappadtau    | Thomson scattering rate $d\kappa/d\tau$ (units 1/Mpc)  |
| int      | re_size                | size of this vector  |
| int      | rt_size                | number of lines (redshift steps) in the table  |
| double * | recombination_table    | table recombination_table[index_z*preco->re_size+index_re] with all other quantities (array of size preco->rt_size*preco->re_size)   |
| double   | CDB                    | defined as in RECFAST  |
| double   | CR                     | defined as in RECFAST  |
| double   | CK                     | defined as in RECFAST  |
| double   | CL                     | defined as in RECFAST  |
| double   | CT                     | defined as in RECFAST  |
| double   | fHe                    | defined as in RECFAST  |
| double   | CDB_He                 | defined as in RECFAST  |
| double   | CK_He                  | defined as in RECFAST  |
| double   | CL_He                  | defined as in RECFAST  |
| double   | fu                     | defined as in RECFAST  |
| double   | H_frac                 | defined as in RECFAST  |
| double   | Tnow                   | defined as in RECFAST  |
| double   | Nnow                   | defined as in RECFAST  |
| double   | Bfact                  | defined as in RECFAST  |
| double   | CB1                    | defined as in RECFAST  |
| double   | CB1_He1                | defined as in RECFAST  |
| double   | CB1_He2                | defined as in RECFAST  |
| double   | H0                     | defined as in RECFAST  |
| double   | YHe                    | defined as in RECFAST  |
| double   | annihilation           | parameter describing CDM annihilation ( $f \langle \sigma v \rangle / m_{\text{cdm}}$ , see e.g. 0905.0003)  |
| short    | has_on_the_spot        | flag to specify if we want to use the on-the-spot approximation  |
| double   | decay                  | parameter describing CDM decay ( $f/\tau$ , see e.g. 1109.6322)  |
| double   | annihilation_variation | if this parameter is non-zero, the function $F(z)=(f \langle \sigma v \rangle / m_{\text{cdm}})(z)$ will be a parabola in log-log scale between $z_{\text{min}}$ and $z_{\text{max}}$ , with a curvature given by annihilation_variation (must be negative), and with a maximum in $z_{\text{max}}$ ; it will be constant outside this range |
| double   | annihilation_z         | if annihilation_variation is non-zero, this is the value of $z$ at which the parameter annihilation is defined, i.e. $F(\text{annihilation}_z)=\text{annihilation}$  |
| double   | annihilation_zmax      | if annihilation_variation is non-zero, redshift above which annihilation rate is maximal   |
| double   | annihilation_zmin      | if annihilation_variation is non-zero, redshift below which annihilation rate is constant  |
| double   | annihilation_f_halo    | takes the contribution of DM annihilation in halos into account  |
| double   | annihilation_z_halo    | characteristic redshift for DM annihilation in halos   |

## 5.20.2.3 struct reionization

Temporary structure where all the reionization history is defined and stored.

This structure is used internally by the thermodynamics module, but never passed to other modules.

## Data Fields

|          |                                |   |
|----------|--------------------------------|---|
| int      | index_re_z                     | redshift $z$  |
| int      | index_re_xe                    | ionization fraction $x_e$   |
| int      | index_re_Tb                    | baryon temperature $T_b$  |
| int      | index_re_cb2                   | squared baryon sound speed $c_b^2$  |
| int      | index_re_dkappadtau            | Thomson scattering rate $d\kappa/d\tau$ (units 1/Mpc)   |
| int      | index_re_dkappadz              | Thomson scattering rate with respect to redshift $d\kappa/dz$ (units 1/Mpc)   |
| int      | index_re_d3kappadz3            | second derivative of previous quantity with respect to redshift   |
| int      | re_size                        | size of this vector   |
| int      | rt_size                        | number of lines (redshift steps) in the table   |
| double * | reionization_table             | table reionization_table[index_z*preio->re_size+index_re] with all other quantities (array of size preio->rt_size*preio->re_size) |
| double   | reionization_optical_depth     | reionization optical depth inferred from reionization history   |
| int      | index_reio_redshift            | hydrogen reionization redshift  |
| int      | index_reio_exponent            | an exponent used in the function $x_e(z)$ in the reio_camb scheme   |
| int      | index_reio_width               | a width defining the duration of hydrogen reionization in the reio_camb scheme  |
| int      | index_reio_xe_before           | ionization fraction at redshift 'reio_start'  |
| int      | index_reio_xe_after            | ionization fraction after full reionization   |
| int      | index_helium_fullreio_fraction | helium full reionization fraction inferred from primordial helium fraction  |
| int      | index_helium_fullreio_redshift | helium full reionization redshift   |
| int      | index_helium_fullreio_width    | a width defining the duration of helium full reionization in the reio_camb scheme   |
| int      | reio_num_z                     | number of reionization jumps  |
| int      | index_reio_first_z             | redshift at which we start to impose reionization function  |
| int      | index_reio_first_xe            | ionization fraction at redshift first_z (inferred from recombination code)  |
| int      | index_reio_step_sharpness      | sharpness of tanh jump  |
| int      | index_reio_start               | redshift above which hydrogen reionization neglected  |
| double * | reionization_parameters        | vector containing all reionization parameters necessary to compute $x_e(z)$   |
| int      | reio_num_params                | length of vector reionization_parameters  |
| int      | index_reco_when_reio_start     | index of line in recombination table corresponding to first line of reionization table  |

## 5.20.2.4 struct thermodynamics\_parameters\_and\_workspace

temporary parameters and workspace passed to the thermodynamics\_derivs function

## 5.20.3 Macro Definition Documentation

### 5.20.3.1 f1

```
#define f1(
    x ) (-0.75*x*(x*x/3.-1.)+0.5)
```

Two useful smooth step functions, for smoothing transitions in `recfast`. goes from 0 to 1 when `x` goes from -1 to 1

### 5.20.3.2 f2

```
#define f2(
    x ) (x*x*(0.5-x/3.)*6.)
```

goes from 0 to 1 when `x` goes from 0 to 1

### 5.20.3.3 \_YHE\_BIG\_

```
#define _YHE_BIG_ 0.5
```

maximal  $Y_{He}$

### 5.20.3.4 \_YHE\_SMALL\_

```
#define _YHE_SMALL_ 0.01
```

minimal  $Y_{He}$

## 5.20.4 Enumeration Type Documentation

### 5.20.4.1 recombination\_algorithm

```
enum recombination_algorithm
```

List of possible recombination algorithms.

### 5.20.4.2 reionization\_parametrization

```
enum reionization_parametrization
```

List of possible reionization schemes.

#### Enumerator

|                             |  |
|-----------------------------|--|
| <code>reio_none</code>      | no reionization  |
| <code>reio_camb</code>      | reionization parameterized like in CAMB                          |
| <code>reio_bins_tanh</code> | binned reionization history with tanh interpolation between bins |
| <code>reio_half_tanh</code> | half a tanh, instead of the full tanh                            |
| <code>reio_many_tanh</code> | similar to <code>reio_camb</code> but with more than one tanh    |
| <code>reio_inter</code>     | linear interpolation between specified points                    |

## 5.20.4.3 reionization\_z\_or\_tau

```
enum reionization_z_or_tau
```

Is the input parameter the reionization redshift or optical depth?

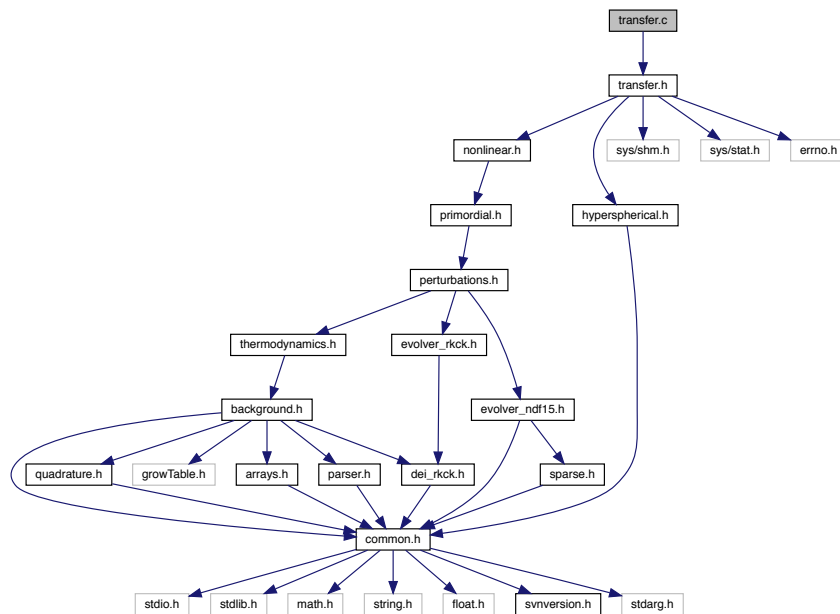
## Enumerator

|          |                  |
|----------|------------------|
| reio_z   | input = redshift |
| reio_tau | input = tau      |

## 5.21 transfer.c File Reference

```
#include "transfer.h"
```

Include dependency graph for transfer.c:



## Functions

- int `transfer_functions_at_q` (struct `transfers` \*ptr, int index\_md, int index\_ic, int index\_tt, int index\_l, double q, double \*transfer\_function)
- int `transfer_init` (struct `precision` \*ppr, struct `background` \*pba, struct `thermo` \*pth, struct `perturbs` \*ppt, struct `nonlinear` \*pnl, struct `transfers` \*ptr)
- int `transfer_free` (struct `transfers` \*ptr)
- int `transfer_indices_of_transfers` (struct `precision` \*ppr, struct `perturbs` \*ppt, struct `transfers` \*ptr, double q\_← period, double K, int sgnK)

- int `transfer_get_l_list` (struct `precision` \*ppr, struct `perturbs` \*ppt, struct `transfers` \*ptr)
- int `transfer_get_q_list` (struct `precision` \*ppr, struct `perturbs` \*ppt, struct `transfers` \*ptr, double q\_period, double K, int sgnK)
- int `transfer_get_k_list` (struct `perturbs` \*ppt, struct `transfers` \*ptr, double K)
- int `transfer_get_source_correspondence` (struct `perturbs` \*ppt, struct `transfers` \*ptr, int \*\*tp\_of\_tt)
- int `transfer_source_tau_size` (struct `precision` \*ppr, struct `background` \*pba, struct `perturbs` \*ppt, struct `transfers` \*ptr, double tau\_rec, double tau0, int index\_md, int index\_tt, int \*tau\_size)
- int `transfer_compute_for_each_q` (struct `precision` \*ppr, struct `background` \*pba, struct `perturbs` \*ppt, struct `transfers` \*ptr, int \*\*tp\_of\_tt, int index\_q, int tau\_size\_max, double tau\_rec, double \*\*\*pert\_sources, double \*\*\*pert\_sources\_spline, struct `transfer_workspace` \*ptw)
- int `transfer_interpolate_sources` (struct `perturbs` \*ppt, struct `transfers` \*ptr, int index\_q, int index\_md, int index\_ic, int index\_type, double \*pert\_source, double \*pert\_source\_spline, double \*interpolated\_sources)
- int `transfer_sources` (struct `precision` \*ppr, struct `background` \*pba, struct `perturbs` \*ppt, struct `transfers` \*ptr, double \*interpolated\_sources, double tau\_rec, int index\_q, int index\_md, int index\_tt, double \*sources, double \*tau0\_minus\_tau, double \*w\_trapz, int \*tau\_size\_out)
- int `transfer_selection_function` (struct `precision` \*ppr, struct `perturbs` \*ppt, struct `transfers` \*ptr, int bin, double z, double \*selection)
- int `transfer_dNdz_analytic` (struct `transfers` \*ptr, double z, double \*dNdz, double \*dln\_dNdz\_dz)
- int `transfer_selection_sampling` (struct `precision` \*ppr, struct `background` \*pba, struct `perturbs` \*ppt, struct `transfers` \*ptr, int bin, double \*tau0\_minus\_tau, int tau\_size)
- int `transfer_lensing_sampling` (struct `precision` \*ppr, struct `background` \*pba, struct `perturbs` \*ppt, struct `transfers` \*ptr, int bin, double tau0, double \*tau0\_minus\_tau, int tau\_size)
- int `transfer_source_resample` (struct `precision` \*ppr, struct `background` \*pba, struct `perturbs` \*ppt, struct `transfers` \*ptr, int bin, double \*tau0\_minus\_tau, int tau\_size, int index\_md, double tau0, double \*interpolated\_sources, double \*sources)
- int `transfer_selection_times` (struct `precision` \*ppr, struct `background` \*pba, struct `perturbs` \*ppt, struct `transfers` \*ptr, int bin, double \*tau\_min, double \*tau\_mean, double \*tau\_max)
- int `transfer_selection_compute` (struct `precision` \*ppr, struct `background` \*pba, struct `perturbs` \*ppt, struct `transfers` \*ptr, double \*selection, double \*tau0\_minus\_tau, double \*w\_trapz, int tau\_size, double \*pvecback, double tau0, int bin)
- int `transfer_compute_for_each_l` (struct `transfer_workspace` \*ptw, struct `precision` \*ppr, struct `perturbs` \*ppt, struct `transfers` \*ptr, int index\_q, int index\_md, int index\_ic, int index\_tt, int index\_l, double l, double q\_max, `radial_function_type` radial\_type)
- int `transfer_integrate` (struct `perturbs` \*ppt, struct `transfers` \*ptr, struct `transfer_workspace` \*ptw, int index\_q, int index\_md, int index\_tt, double l, int index\_l, double k, `radial_function_type` radial\_type, double \*trsf)
- int `transfer_limber` (struct `transfers` \*ptr, struct `transfer_workspace` \*ptw, int index\_md, int index\_q, double l, double q, `radial_function_type` radial\_type, double \*trsf)
- int `transfer_limber_interpolate` (struct `transfers` \*ptr, double \*tau0\_minus\_tau, double \*sources, int tau\_size, double tau0\_minus\_tau\_limber, double \*S)
- int `transfer_limber2` (int tau\_size, struct `transfers` \*ptr, int index\_md, int index\_k, double l, double k, double \*tau0\_minus\_tau, double \*sources, `radial_function_type` radial\_type, double \*trsf)

### 5.21.1 Detailed Description

Documented transfer module.

Julien Lesgourgues, 28.07.2013

This module has two purposes:

- at the beginning, to compute the transfer functions  $\Delta_l^X(q)$ , and store them in tables used for interpolation in other modules.
- at any time in the code, to evaluate the transfer functions (for a given mode, initial condition, type and multipole l) at any wavenumber q (by interpolating within the interpolation table).

Hence the following functions can be called from other modules:

1. [transfer\\_init\(\)](#) at the beginning (but after [perturb\\_init\(\)](#) and [bessel\\_init\(\)](#))
2. [transfer\\_functions\\_at\\_q\(\)](#) at any later time
3. [transfer\\_free\(\)](#) at the end, when no more calls to [transfer\\_functions\\_at\\_q\(\)](#) are needed

Note that in the standard implementation of CLASS, only the pre-computed values of the transfer functions are used, no interpolation is necessary; hence the routine [transfer\\_functions\\_at\\_q\(\)](#) is actually never called.

## 5.21.2 Function Documentation

### 5.21.2.1 transfer\_functions\_at\_q()

```
int transfer_functions_at_q (
    struct transfers * ptr,
    int index_md,
    int index_ic,
    int index_tt,
    int index_l,
    double q,
    double * transfer_function )
```

Transfer function  $\Delta_i^X(q)$  at a given wavenumber  $q$ .

For a given mode (scalar, vector, tensor), initial condition, type (temperature, polarization, lensing, etc) and multipole, computes the transfer function for an arbitrary value of  $q$  by interpolating between pre-computed values of  $q$ . This function can be called from whatever module at whatever time, provided that [transfer\\_init\(\)](#) has been called before, and [transfer\\_free\(\)](#) has not been called yet.

Wavenumbers are called  $q$  in this module and  $k$  in the perturbation module. In flat universes  $k=q$ . In non-flat universes  $q$  and  $k$  differ through  $q^2 = k^2 + K(1 + m)$ , where  $m=0,1,2$  for scalar, vector, tensor.  $q$  should be used throughout the transfer module, excepted when interpolating or manipulating the source functions  $S(k,\tau)$  calculated in the perturbation module: for a given value of  $q$ , this should be done at the corresponding  $k(q)$ .

#### Parameters

|                          |   |
|--------------------------|---|
| <i>ptr</i>               | Input: pointer to transfer structure        |
| <i>index_md</i>          | Input: index of requested mode              |
| <i>index_ic</i>          | Input: index of requested initial condition |
| <i>index_tt</i>          | Input: index of requested type              |
| <i>index_l</i>           | Input: index of requested multipole         |
| <i>q</i>                 | Input: any wavenumber                       |
| <i>transfer_function</i> | Output: transfer function                   |

**Returns**

the error status

**Summary:**

- interpolate in pre-computed table using `array_interpolate_two()`

**5.21.2.2 transfer\_init()**

```
int transfer_init (
    struct precision * ppr,
    struct background * pba,
    struct thermo * pth,
    struct perturbs * ppt,
    struct nonlinear * pnl,
    struct transfers * ptr )
```

This routine initializes the transfers structure, (in particular, computes table of transfer functions  $\Delta_t^X(q)$ )

**Main steps:**

- initialize all indices in the transfers structure and allocate all its arrays using `transfer_indices_of_transfers()`.
- for each thread (in case of parallel run), initialize the fields of a memory zone called the `transfer_workspace` with `transfer_workspace_init()`
- loop over q values. For each q, compute the Bessel functions if needed with `transfer_update_HIS()`, and defer the calculation of all transfer functions to `transfer_compute_for_each_q()`
- for each thread, free the the workspace with `transfer_workspace_free()`

**Parameters**

|            |  |
|------------|--|
| <i>ppr</i> | Input: pointer to precision structure              |
| <i>pba</i> | Input: pointer to background structure             |
| <i>pth</i> | Input: pointer to thermodynamics structure         |
| <i>ppt</i> | Input: pointer to perturbation structure           |
| <i>pnl</i> | Input: pointer to nonlinear structure              |
| <i>ptr</i> | Output: pointer to initialized transfers structure |

**Returns**

the error status

**Summary:**

- define local variables



- array with the correspondence between the index of sources in the perturbation module and in the transfer module, `tp_of_tt[index_md][index_tt]`
- check whether any spectrum in harmonic space (i.e., any  $C_l$ 's) is actually requested
- get number of modes (scalars, tensors...)
- get conformal age / recombination time from background / thermodynamics structures (only place where these structures are used in this module)
- correspondence between  $k$  and  $l$  depend on angular diameter distance, i.e. on curvature.
- order of magnitude of the oscillation period of transfer functions
- initialize all indices in the transfers structure and allocate all its arrays using [transfer\\_indices\\_of\\_transfers\(\)](#)
- copy sources to a local array `sources` (in fact, only the pointers are copied, not the data), and eventually apply non-linear corrections to the sources
- spline all the sources passed by the perturbation module with respect to  $k$  (in order to interpolate later at a given value of  $k$ )
- allocate and fill array describing the correspondence between perturbation types and transfer types
- evaluate maximum number of sampled times in the transfer sources: needs to be known here, in order to allocate a large enough workspace
- compute flat spherical bessel functions
- eventually read the selection and evolution functions
- loop over all wavenumbers (parallelized).
- finally, free arrays allocated outside parallel zone

### 5.21.2.3 transfer\_free()

```
int transfer_free (
    struct transfers * ptr )
```

This routine frees all the memory space allocated by [transfer\\_init\(\)](#).

To be called at the end of each run, only when no further calls to `transfer_functions_at_k()` are needed.

#### Parameters

|            |  |
|------------|--|
| <i>ptr</i> | Input: pointer to transfers structure (which fields must be freed) |
|------------|--|

#### Returns

the error status

#### 5.21.2.4 transfer\_indices\_of\_transfers()

```
int transfer_indices_of_transfers (
    struct precision * ppr,
    struct perturbs * ppt,
    struct transfers * ptr,
    double q_period,
    double K,
    int sgnK )
```

This routine defines all indices and allocates all tables in the transfers structure

Compute list of (k, l) values, allocate and fill corresponding arrays in the transfers structure. Allocate the array of transfer function tables.

##### Parameters

|                 |   |
|-----------------|---|
| <i>ppr</i>      | Input: pointer to precision structure                                     |
| <i>ppt</i>      | Input: pointer to perturbation structure                                  |
| <i>ptr</i>      | Input/Output: pointer to transfer structure                               |
| <i>q_period</i> | Input: order of magnitude of the oscillation period of transfer functions |
| <i>K</i>        | Input: spatial curvature (in absolute value)                              |
| <i>sgnK</i>     | Input: spatial curvature sign (open/closed/flat)                          |

##### Returns

the error status

##### Summary:

- define local variables
- define indices for transfer types
- type indices common to scalars and tensors
- type indices for scalars
- type indices for vectors
- type indices for tensors
- allocate arrays of (k, l) values and transfer functions
- get q values using [transfer\\_get\\_q\\_list\(\)](#)
- get k values using [transfer\\_get\\_k\\_list\(\)](#)
- get l values using [transfer\\_get\\_l\\_list\(\)](#)
- loop over modes (scalar, etc). For each mode:
- allocate arrays of transfer functions, (ptr->transfer[index\_md])[index\_ic][index\_tt][index\_l][index\_k]

## 5.21.2.5 transfer\_get\_l\_list()

```
int transfer_get_l_list (
    struct precision * ppr,
    struct perturbs * ppt,
    struct transfers * ptr )
```

This routine defines the number and values of multipoles  $l$  for all modes.

## Parameters

|            |  |
|------------|--|
| <i>ppr</i> | Input: pointer to precision structure                          |
| <i>ppt</i> | Input: pointer to perturbation structure                       |
| <i>ptr</i> | Input/Output: pointer to transfers structure containing $l$ 's |

## Returns

the error status

## Summary:

- allocate and fill  $l$  array
- start from  $l = 2$  and increase with logarithmic step
- when the logarithmic step becomes larger than some linear step, stick to this linear step till  $l_{\max}$
- last value set to exactly  $l_{\max}$
- so far we just counted the number of values. Now repeat the whole thing but fill array with values.

## 5.21.2.6 transfer\_get\_q\_list()

```
int transfer_get_q_list (
    struct precision * ppr,
    struct perturbs * ppt,
    struct transfers * ptr,
    double q_period,
    double K,
    int sgnK )
```

This routine defines the number and values of wavenumbers  $q$  for each mode (goes smoothly from logarithmic step for small  $q$ 's to linear step for large  $q$ 's).

## Parameters

|                 |   |
|-----------------|---|
| <i>ppr</i>      | Input: pointer to precision structure                                     |
| <i>ppt</i>      | Input: pointer to perturbation structure                                  |
| <i>ptr</i>      | Input/Output: pointer to transfers structure containing $q$ 's            |
| <i>q_period</i> | Input: order of magnitude of the oscillation period of transfer functions |
| <i>K</i>        | Input: spatial curvature (in absolute value)                              |
| <i>sgnK</i>     | Input: spatial curvature sign (open/closed/flat)                          |

**Returns**

the error status

**5.21.2.7 transfer\_get\_k\_list()**

```
int transfer_get_k_list (
    struct perturbs * ppt,
    struct transfers * ptr,
    double K )
```

This routine infers from the q values a list of corresponding k values for each mode.

**Parameters**

|            |   |
|------------|---|
| <i>ppt</i> | Input: pointer to perturbation structure                    |
| <i>ptr</i> | Input/Output: pointer to transfers structure containing q's |
| <i>K</i>   | Input: spatial curvature                                    |

**Returns**

the error status

**5.21.2.8 transfer\_get\_source\_correspondence()**

```
int transfer_get_source_correspondence (
    struct perturbs * ppt,
    struct transfers * ptr,
    int ** tp_of_tt )
```

This routine defines the correspondence between the sources in the perturbation and transfer module.

**Parameters**

|                 |   |
|-----------------|---|
| <i>ppt</i>      | Input: pointer to perturbation structure                                    |
| <i>ptr</i>      | Input: pointer to transfers structure containing l's                        |
| <i>tp_of_tt</i> | Input/Output: array with the correspondence (allocated before, filled here) |

**Returns**

the error status

Summary:

- running index on modes
- running index on transfer types
- which source are we considering? Define correspondence between transfer types and source types

#### 5.21.2.9 transfer\_source\_tau\_size()

```
int transfer_source_tau_size (
    struct precision * ppr,
    struct background * pba,
    struct perturbs * ppt,
    struct transfers * ptr,
    double tau_rec,
    double tau0,
    int index_md,
    int index_tt,
    int * tau_size )
```

the code makes a distinction between "perturbation sources" (e.g. gravitational potential) and "transfer sources" (e.g. total density fluctuations, obtained through the Poisson equation, and observed with a given selection function).

This routine computes the number of sampled time values for each type of transfer sources.

#### Parameters

|                 |  |
|-----------------|--|
| <i>ppr</i>      | Input: pointer to precision structure      |
| <i>pba</i>      | Input: pointer to background structure     |
| <i>ppt</i>      | Input: pointer to perturbation structure   |
| <i>ptr</i>      | Input: pointer to transfers structure      |
| <i>tau_rec</i>  | Input: recombination time                  |
| <i>tau0</i>     | Input: time today                          |
| <i>index_md</i> | Input: index of the mode (scalar, tensor)  |
| <i>index_tt</i> | Input: index of transfer type              |
| <i>tau_size</i> | Output: pointer to number of sampled times |

#### Returns

the error status

#### 5.21.2.10 transfer\_compute\_for\_each\_q()

```
int transfer_compute_for_each_q (
    struct precision * ppr,
    struct background * pba,
    struct perturbs * ppt,
    struct transfers * ptr,
```

```

int ** tp_of_tt,
int index_q,
int tau_size_max,
double tau_rec,
double *** pert_sources,
double *** pert_sources_spline,
struct transfer_workspace * ptw )

```

Summary:

- define local variables
  - we deal with workspaces, i.e. with contiguous memory zones (one per thread) containing various fields used by the integration routine
- for a given  $l$ , maximum value of  $k$  such that we can convolve the source with Bessel functions  $j_l(x)$  without reaching  $x_{\max}$
- store the sources in the workspace and define all fields in this workspace
- loop over all modes. For each mode
- loop over initial conditions.
- check if we must now deal with a new source with a new index  $ppt \rightarrow \text{index\_type}$ . If yes, interpolate it at the right values of  $k$ .
- Select radial function type

#### 5.21.2.11 `transfer_interpolate_sources()`

```

int transfer_interpolate_sources (
    struct perturbs * ppt,
    struct transfers * ptr,
    int index_q,
    int index_md,
    int index_ic,
    int index_type,
    double * pert_source,
    double * pert_source_spline,
    double * interpolated_sources )

```

This routine interpolates sources  $S(k, \tau)$  for each mode, initial condition and type (of perturbation module), to get them at the right values of  $k$ , using the spline interpolation method.

#### Parameters

|                             |   |
|-----------------------------|---|
| <i>ppt</i>                  | Input: pointer to perturbation structure  |
| <i>ptr</i>                  | Input: pointer to transfers structure   |
| <i>index_q</i>              | Input: index of wavenumber  |
| <i>index_md</i>             | Input: index of mode  |
| <i>index_ic</i>             | Input: index of initial condition   |
| <i>index_type</i>           | Input: index of type of source (in perturbation module)   |
| <i>pert_source</i>          | Input: array of sources   |
| <i>pert_source_spline</i>   | Input: array of second derivative of sources  |
| <i>interpolated_sources</i> | Output: array of interpolated sources (filled here but allocated in <code>transfer_init()</code> numerous reallocation) |

**Returns**

the error status

**Summary:**

- define local variables
- interpolate at each k value using the usual spline interpolation algorithm.

**5.21.2.12 transfer\_sources()**

```
int transfer_sources (
    struct precision * ppr,
    struct background * pba,
    struct perturbs * ppt,
    struct transfers * ptr,
    double * interpolated_sources,
    double tau_rec,
    int index_q,
    int index_md,
    int index_tt,
    double * sources,
    double * tau0_minus_tau,
    double * w_trapz,
    int * tau_size_out )
```

The code makes a distinction between "perturbation sources" (e.g. gravitational potential) and "transfer sources" (e.g. total density fluctuations, obtained through the Poisson equation, and observed with a given selection function).

This routine computes the transfer source given the interpolated perturbation source, and copies it in the workspace.

**Parameters**

|                             |   |
|-----------------------------|---|
| <i>ppr</i>                  | Input: pointer to precision structure                               |
| <i>pba</i>                  | Input: pointer to background structure                              |
| <i>ppt</i>                  | Input: pointer to perturbation structure                            |
| <i>ptr</i>                  | Input: pointer to transfers structure                               |
| <i>interpolated_sources</i> | Input: interpolated perturbation source                             |
| <i>tau_rec</i>              | Input: recombination time   |
| <i>index_q</i>              | Input: index of wavenumber  |
| <i>index_md</i>             | Input: index of mode  |
| <i>index_tt</i>             | Input: index of type of (transfer) source                           |
| <i>sources</i>              | Output: transfer source   |
| <i>tau0_minus_tau</i>       | Output: values of (tau0-tau) at which source are sample             |
| <i>w_trapz</i>              | Output: trapezoidal weights for integration over tau                |
| <i>tau_size_out</i>         | Output: pointer to size of previous two arrays, converted to double |

**Returns**

the error status

**Summary:**

- define local variables
  - in which cases are perturbation and transfer sources are different? I.e., in which case do we need to multiply the sources by some background and/or window function, and eventually to resample it, or redefine its time limits?
  - case where we need to redefine by a window function (or any function of the background and of k)
- case where we do not need to redefine
  - return tau\_size value that will be stored in the workspace (the workspace wants a double)

**5.21.2.13 transfer\_selection\_function()**

```
int transfer_selection_function (
    struct precision * ppr,
    struct perturbs * ppt,
    struct transfers * ptr,
    int bin,
    double z,
    double * selection )
```

Arbitrarily normalized selection function  $dN/dz(z,bin)$

**Parameters**

|                  |  |
|------------------|--|
| <i>ppr</i>       | Input: pointer to precision structure    |
| <i>ppt</i>       | Input: pointer to perturbation structure |
| <i>ptr</i>       | Input: pointer to transfers structure    |
| <i>bin</i>       | Input: redshift bin number               |
| <i>z</i>         | Input: one value of redshift             |
| <i>selection</i> | Output: pointer to selection function    |

**Returns**

the error status

**5.21.2.14 transfer\_dNdz\_analytic()**

```
int transfer_dNdz_analytic (
    struct transfers * ptr,
```



```
double z,
double * dNdz,
double * dln_dNdz_dz )
```

Analytic form for dNdz distribution, from arXiv:1004.4640

#### Parameters

|                    |   |
|--------------------|---|
| <i>ptr</i>         | Input: pointer to transfer structure                            |
| <i>z</i>           | Input: redshift   |
| <i>dNdz</i>        | Output: density per redshift, dN/dZ                             |
| <i>dln_dNdz_dz</i> | Output: dln(dN/dz)/dz, used optionally for the source evolution |

#### Returns

the error status

#### 5.21.2.15 transfer\_selection\_sampling()

```
int transfer_selection_sampling (
    struct precision * ppr,
    struct background * pba,
    struct perturbs * ppt,
    struct transfers * ptr,
    int bin,
    double * tau0_minus_tau,
    int tau_size )
```

For sources that need to be multiplied by a selection function, redefine a finer time sampling in a small range

#### Parameters

|                       |   |
|-----------------------|---|
| <i>ppr</i>            | Input: pointer to precision structure                   |
| <i>pba</i>            | Input: pointer to background structure                  |
| <i>ppt</i>            | Input: pointer to perturbation structure                |
| <i>ptr</i>            | Input: pointer to transfers structure                   |
| <i>bin</i>            | Input: redshift bin number                              |
| <i>tau0_minus_tau</i> | Output: values of (tau0-tau) at which source are sample |
| <i>tau_size</i>       | Output: pointer to size of previous array               |

#### Returns

the error status

### 5.21.2.16 transfer\_lensing\_sampling()

```
int transfer_lensing_sampling (
    struct precision * ppr,
    struct background * pba,
    struct perturbs * ppt,
    struct transfers * ptr,
    int bin,
    double tau0,
    double * tau0_minus_tau,
    int tau_size )
```

For lensing sources that need to be convolved with a selection function, redefine the sampling within the range extending from the tau\_min of the selection function up to tau0

#### Parameters

|                       |   |
|-----------------------|---|
| <i>ppr</i>            | Input: pointer to precision structure                   |
| <i>pba</i>            | Input: pointer to background structure                  |
| <i>ppt</i>            | Input: pointer to perturbation structure                |
| <i>ptr</i>            | Input: pointer to transfers structure                   |
| <i>bin</i>            | Input: redshift bin number                              |
| <i>tau0</i>           | Input: time today                                       |
| <i>tau0_minus_tau</i> | Output: values of (tau0-tau) at which source are sample |
| <i>tau_size</i>       | Output: pointer to size of previous array               |

#### Returns

the error status

### 5.21.2.17 transfer\_source\_resample()

```
int transfer_source_resample (
    struct precision * ppr,
    struct background * pba,
    struct perturbs * ppt,
    struct transfers * ptr,
    int bin,
    double * tau0_minus_tau,
    int tau_size,
    int index_md,
    double tau0,
    double * interpolated_sources,
    double * sources )
```

For sources that need to be multiplied by a selection function, redefine a finer time sampling in a small range, and resample the perturbation sources at the new value by linear interpolation

#### Parameters

|            |                                       |
|------------|---------------------------------------|
| <i>ppr</i> | Input: pointer to precision structure |
|------------|---------------------------------------|

## Parameters

|                             |   |
|-----------------------------|---|
| <i>pba</i>                  | Input: pointer to background structure                  |
| <i>ppt</i>                  | Input: pointer to perturbation structure                |
| <i>ptr</i>                  | Input: pointer to transfers structure                   |
| <i>bin</i>                  | Input: redshift bin number                              |
| <i>tau0_minus_tau</i>       | Output: values of (tau0-tau) at which source are sample |
| <i>tau_size</i>             | Output: pointer to size of previous array               |
| <i>index_md</i>             | Input: index of mode                                    |
| <i>tau0</i>                 | Input: time today                                       |
| <i>interpolated_sources</i> | Input: interpolated perturbation source                 |
| <i>sources</i>              | Output: resampled transfer source                       |

## Returns

the error status

## 5.21.2.18 transfer\_selection\_times()

```
int transfer_selection_times (
    struct precision * ppr,
    struct background * pba,
    struct perturbs * ppt,
    struct transfers * ptr,
    int bin,
    double * tau_min,
    double * tau_mean,
    double * tau_max )
```

For each selection function, compute the min, mean and max values of conformal time (associated to the min, mean and max values of redshift specified by the user)

## Parameters

|                 |   |
|-----------------|---|
| <i>ppr</i>      | Input: pointer to precision structure           |
| <i>pba</i>      | Input: pointer to background structure          |
| <i>ppt</i>      | Input: pointer to perturbation structure        |
| <i>ptr</i>      | Input: pointer to transfers structure           |
| <i>bin</i>      | Input: redshift bin number                      |
| <i>tau_min</i>  | Output: smallest time in the selection interval |
| <i>tau_mean</i> | Output: time corresponding to z_mean            |
| <i>tau_max</i>  | Output: largest time in the selection interval  |

## Returns

the error status

5.21.2.19 `transfer_selection_compute()`

```
int transfer_selection_compute (
    struct precision * ppr,
    struct background * pba,
    struct perturbs * ppt,
    struct transfers * ptr,
    double * selection,
    double * tau0_minus_tau,
    double * w_trapz,
    int tau_size,
    double * pvecback,
    double tau0,
    int bin )
```

Compute and normalize selection function for a set of time values

## Parameters

|                       |  |
|-----------------------|--|
| <i>ppr</i>            | Input: pointer to precision structure                  |
| <i>pba</i>            | Input: pointer to background structure                 |
| <i>ppt</i>            | Input: pointer to perturbation structure               |
| <i>ptr</i>            | Input: pointer to transfers structure                  |
| <i>selection</i>      | Output: normalized selection function                  |
| <i>tau0_minus_tau</i> | Input: values of (tau0-tau) at which source are sample |
| <i>w_trapz</i>        | Input: trapezoidal weights for integration over tau    |
| <i>tau_size</i>       | Input: size of previous two arrays                     |
| <i>pvecback</i>       | Input: allocated array of background values            |
| <i>tau0</i>           | Input: time today                                      |
| <i>bin</i>            | Input: redshift bin number                             |

## Returns

the error status

5.21.2.20 `transfer_compute_for_each_l()`

```
int transfer_compute_for_each_l (
    struct transfer_workspace * ptw,
    struct precision * ppr,
    struct perturbs * ppt,
    struct transfers * ptr,
    int index_q,
    int index_md,
    int index_ic,
    int index_tt,
    int index_l,
    double l,
    double q_max_bessel,
    radial_function_type radial_type )
```

This routine computes the transfer functions  $\Delta_i^X(k)$  as a function of wavenumber  $k$  for a given mode, initial condition, type and multipole  $l$  passed in input.

For a given value of  $k$ , the transfer function is inferred from the source function (passed in input in the array `interpolated_sources`) and from Bessel functions (passed in input in the `bessels` structure), either by convolving them along  $\tau$ , or by a Limber approximation. This elementary task is distributed either to `transfer_integrate()` or to `transfer_limber()`. The task of this routine is mainly to loop over  $k$  values, and to decide at which  $k_{\max}$  the calculation can be stopped, according to some approximation scheme designed to find a compromise between execution time and precision. The approximation scheme is defined by parameters in the precision structure.

#### Parameters

|                     |  |
|---------------------|--|
| <i>ptw</i>          | Input: pointer to <code>transfer_workspace</code> structure (allocated in <code>transfer_init()</code> to avoid numerous reallocation) |
| <i>ppr</i>          | Input: pointer to precision structure  |
| <i>ppt</i>          | Input: pointer to perturbation structure   |
| <i>ptr</i>          | Input/output: pointer to transfers structure (result stored there)   |
| <i>index_q</i>      | Input: index of wavenumber   |
| <i>index_md</i>     | Input: index of mode   |
| <i>index_ic</i>     | Input: index of initial condition  |
| <i>index_tt</i>     | Input: index of type of transfer   |
| <i>index_l</i>      | Input: index of multipole  |
| <i>l</i>            | Input: multipole   |
| <i>q_max_bessel</i> | Input: maximum value of argument $q$ at which Bessel functions are computed  |
| <i>radial_type</i>  | Input: type of radial (Bessel) functions to convolve with  |

#### Returns

the error status

#### Summary:

- define local variables
- return zero transfer function if  $l$  is above  $l_{\max}$
- store transfer function in transfer structure

#### 5.21.2.21 transfer\_integrate()

```
int transfer_integrate (
    struct perturbs * ppt,
    struct transfers * ptr,
    struct transfer_workspace * ptw,
    int index_q,
    int index_md,
    int index_tt,
    double l,
    int index_l,
```

```
double k,  
radial_function_type radial_type,  
double * trsf )
```

This routine computes the transfer functions  $\Delta_l^X(k)$  for each mode, initial condition, type, multipole  $l$  and wavenumber  $k$ , by convolving the source function (passed in input in the array `interpolated_sources`) with Bessel functions (passed in input in the `bessels` structure).

## Parameters

|                    |  |
|--------------------|--|
| <i>ppt</i>         | Input: pointer to perturbation structure   |
| <i>ptr</i>         | Input: pointer to transfers structure  |
| <i>ptw</i>         | Input: pointer to <a href="#">transfer_workspace</a> structure (allocated in <a href="#">transfer_init()</a> to avoid numerous reallocation) |
| <i>index_q</i>     | Input: index of wavenumber   |
| <i>index_md</i>    | Input: index of mode   |
| <i>index_tt</i>    | Input: index of type   |
| <i>l</i>           | Input: multipole   |
| <i>index_l</i>     | Input: index of multipole  |
| <i>k</i>           | Input: wavenumber  |
| <i>radial_type</i> | Input: type of radial (Bessel) functions to convolve with  |
| <i>trsf</i>        | Output: transfer function $\Delta_l(k)$  |

## Returns

the error status

## Summary:

- define local variables
- find minimum value of (tau0-tau) at which  $j_l(k[\tau_0 - \tau])$  is known, given that  $j_l(x)$  is sampled above some finite value  $x_{\min}$  (below which it can be approximated by zero)
- if there is no overlap between the region in which bessels and sources are non-zero, return zero
- if there is an overlap:
  - -> trivial case: the source is a Dirac function and is sampled in only one point
  - -> other cases
    - -> (a) find index in the source's tau list corresponding to the last point in the overlapping region. After this step, `index_tau_max` can be as small as zero, but not negative.
    - -> (b) the source function can vanish at large  $\tau$ . Check if further points can be eliminated. After this step and if we did not return a null transfer function, `index_tau_max` can be as small as zero, but not negative.
- Compute the radial function:
- Now we do most of the convolution integral:
  - This integral is correct for the case where no truncation has occurred. If it has been truncated at some `index_tau_max` because `f[index_tau_max+1]==0`, it is still correct. The 'mistake' in using the wrong weight `w_trapz[index_tau_max]` is exactly compensated by the triangle we miss. However, for the Bessel cut off, we must subtract the wrong triangle and add the correct triangle.

## 5.21.2.22 transfer\_limber()

```
int transfer_limber (
    struct transfers * ptr,
    struct transfer_workspace * ptw,
    int index_md,
    int index_q,
    double l,
    double q,
    radial_function_type radial_type,
    double * trsf )
```

This routine computes the transfer functions  $\Delta_l^X(k)$  for each mode, initial condition, type, multipole  $l$  and wavenumber  $k$ , by using the Limber approximation, i.e by evaluating the source function (passed in input in the array `interpolated_sources`) at a single value of  $\tau$  (the Bessel function being approximated as a Dirac distribution).

## Parameters

|                    |   |
|--------------------|---|
| <i>ptr</i>         | Input: pointer to transfers structure                     |
| <i>ptw</i>         | Input: pointer to transfer workspace structure            |
| <i>index_md</i>    | Input: index of mode                                      |
| <i>index_q</i>     | Input: index of wavenumber                                |
| <i>l</i>           | Input: multipole  |
| <i>q</i>           | Input: wavenumber   |
| <i>radial_type</i> | Input: type of radial (Bessel) functions to convolve with |
| <i>trsf</i>        | Output: transfer function $\Delta_l(k)$                   |

## Returns

the error status

## Summary:

- define local variables
- get  $k$ ,  $l$  and infer  $\tau$  such that  $k(\tau_0 - \tau) = l + 1/2$ ; check that  $\tau$  is in appropriate range
- get  $\text{transfer} = \text{source} * \sqrt{\pi/(2l + 1)}/q = \text{source} * [\tau_0 - \tau] * \sqrt{\pi/(2l + 1)}/(l + 1/2)$

## 5.21.2.23 transfer\_limber\_interpolate()

```
int transfer_limber_interpolate (
    struct transfers * ptr,
    double * tau0_minus_tau,
    double * sources,
    int tau_size,
    double tau0_minus_tau_limber,
    double * S )
```



- find bracketing indices. `index_tau` must be at least 1 (so that `index_tau-1` is at least 0) and at most `tau_size-2` (so that `index_tau+1` is at most `tau_size-1`).
- interpolate by fitting a polynomial of order two; get source and its first two derivatives. Note that we are not interpolating `S`, but the product `S*(tau0-tau)`. Indeed this product is regular in `tau=tau0`, while `S` alone diverges for lensing.

#### 5.21.2.24 transfer\_limber2()

```
int transfer_limber2 (
    int tau_size,
    struct transfers * ptr,
    int index_md,
    int index_k,
    double l,
    double k,
    double * tau0_minus_tau,
    double * sources,
    radial_function_type radial_type,
    double * trsf )
```

This routine computes the transfer functions  $\Delta_l^X(k)$  for each mode, initial condition, type, multipole `l` and wavenumber `k`, by using the Limber approximation at order two, i.e as a function of the source function and its first two derivatives at a single value of `tau`

#### Parameters

|                       |  |
|-----------------------|--|
| <i>tau_size</i>       | Input: size of conformal time array                        |
| <i>ptr</i>            | Input: pointer to transfers structure                      |
| <i>index_md</i>       | Input: index of mode                                       |
| <i>index_k</i>        | Input: index of wavenumber                                 |
| <i>l</i>              | Input: multipole   |
| <i>k</i>              | Input: wavenumber  |
| <i>tau0_minus_tau</i> | Input: array of values of ( <code>tau_today - tau</code> ) |
| <i>sources</i>        | Input: source functions                                    |
| <i>radial_type</i>    | Input: type of radial (Bessel) functions to convolve with  |
| <i>trsf</i>           | Output: transfer function $\Delta_l(k)$                    |

#### Returns

the error status

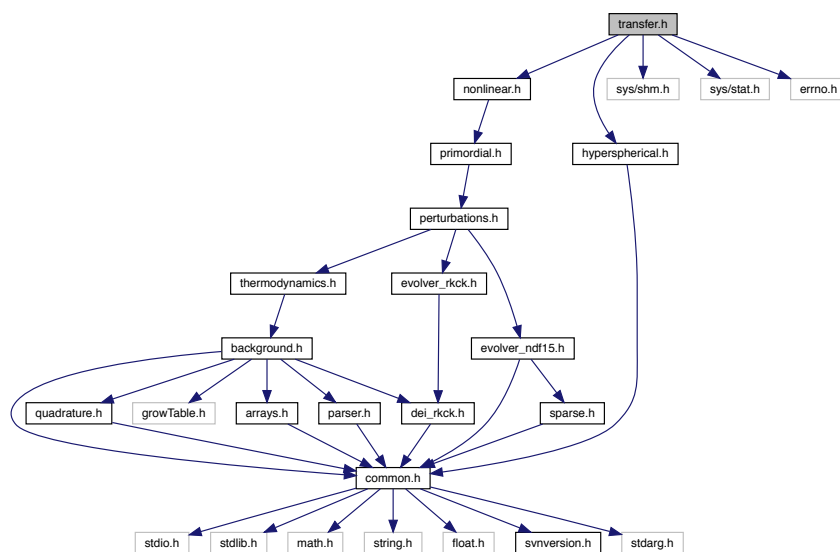
#### Summary:

- define local variables
- get `k`, `l` and infer `tau` such that  $k(\tau_0-\tau)=l+1/2$ ; check that `tau` is in appropriate range
- find bracketing indices
- interpolate by fitting a polynomial of order two; get source and its first two derivatives
- get transfer from 2nd order Limber approx (inferred from 0809.5112 [astro-ph])

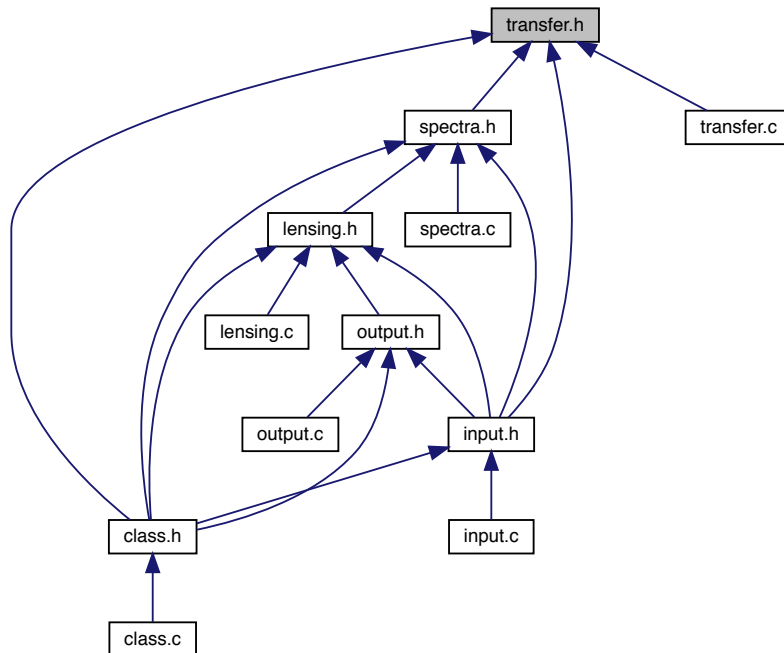
## 5.22 transfer.h File Reference

```
#include "nonlinear.h"
#include "hyperspherical.h"
#include <sys/shm.h>
#include <sys/stat.h>
#include "errno.h"
```

Include dependency graph for transfer.h:



This graph shows which files directly or indirectly include this file:



## Data Structures

- struct [transfers](#)
- struct [transfer\\_workspace](#)

## Enumerations

- enum [radial\\_function\\_type](#)

### 5.22.1 Detailed Description

Documented includes for transfer module.

### 5.22.2 Data Structure Documentation

#### 5.22.2.1 struct transfers

Structure containing everything about transfer functions in harmonic space  $\Delta_l^X(q)$  that other modules need to know.

Once initialized by [transfer\\_init\(\)](#), contains all tables of transfer functions used for interpolation in other modules, for all requested modes (scalar/vector/tensor), initial conditions, types (temperature, polarization, etc), multipoles  $l$ , and wavenumbers  $q$ .

Wavenumbers are called  $q$  in this module and  $k$  in the perturbation module. In flat universes  $k=q$ . In non-flat universes  $q$  and  $k$  differ through  $q^2 = k^2 + K(1+m)$ , where  $m=0,1,2$  for scalar, vector, tensor.  $q$  should be used throughout the transfer module, except when interpolating or manipulating the source functions  $S(k,\tau)$  calculated in the perturbation module: for a given value of  $q$ , this should be done at the corresponding  $k(q)$ .

The content of this structure is entirely computed in this module, given the content of the 'precision', 'bessels', 'background', 'thermodynamics' and 'perturbation' structures.

## Data Fields

|          |   |   |
|----------|---|---|
| double   | lcmb_rescale                                      | normally set to one, can be used exceptionally to rescale by hand the CMB lensing potential |
| double   | lcmb_tilt   | normally set to zero, can be used exceptionally to tilt by hand the CMB lensing potential   |
| double   | lcmb_pivot  | if lcmb_tilt non-zero, corresponding pivot scale  |
| double   | selection_bias[_SELECTION_NUM_MAX_]               | light-to-mass bias in the transfer function of density number count                         |
| double   | selection_magnification_bias[_SELECTION_NUM_MAX_] | magnification bias in the transfer function of density number count                         |
| short    | has_nz_file                                       | Has dN/dz (selection function) input file?  |
| short    | has_nz_analytic                                   | Use analytic form for dN/dz (selection function) distribution?                              |
| FileName | nz_file_name                                      | dN/dz (selection function) input file name  |
| int      | nz_size   | number of redshift values in input tabulated selection function                             |
| double * | nz_z  | redshift values in input tabulated selection function                                       |
| double * | nz_nz   | input tabulated values of selection function  |
| double * | nz_ddnz   | second derivatives in splined selection function  |
| short    | has_nz_evo_file                                   | Has dN/dz (evolution function) input file?  |
| short    | has_nz_evo_analytic                               | Use analytic form for dN/dz (evolution function) distribution?                              |
| FileName | nz_evo_file_name                                  | dN/dz (evolution function) input file name  |
| int      | nz_evo_size                                       | number of redshift values in input tabulated evolution function                             |
| double * | nz_evo_z  | redshift values in input tabulated evolution function                                       |
| double * | nz_evo_nz   | input tabulated values of evolution function  |
| double * | nz_evo_dlog_nz                                    | log of tabulated values of evolution function   |
| double * | nz_evo_dd_dlog_nz                                 | second derivatives in splined log of evolution function                                     |
| short    | has_cls   | copy of same flag in perturbation structure   |
| int      | md_size   | number of modes included in computation   |
| int      | index_tt_t0                                       | index for transfer type = temperature (j=0 term)  |
| int      | index_tt_t1                                       | index for transfer type = temperature (j=1 term)  |
| int      | index_tt_t2                                       | index for transfer type = temperature (j=2 term)  |
| int      | index_tt_e  | index for transfer type = E-polarization  |
| int      | index_tt_b  | index for transfer type = B-polarization  |
| int      | index_tt_lcmb                                     | index for transfer type = CMB lensing   |
| int      | index_tt_density                                  | index for first bin of transfer type = matter density                                       |
| int      | index_tt_lensing                                  | index for first bin of transfer type = galaxy lensing                                       |
| int      | index_tt_rsd                                      | index for first bin of transfer type = redshift space distortion of number count            |
| int      | index_tt_d0                                       | index for first bin of transfer type = doppler effect for of number count (j=0 term)        |
| int      | index_tt_d1                                       | index for first bin of transfer type = doppler effect for of number count (j=1 term)        |

## Data Fields

|           |                            |   |
|-----------|----------------------------|---|
| int       | index_tt_nc_lens           | index for first bin of transfer type = lensing for of number count  |
| int       | index_tt_nc_g1             | index for first bin of transfer type = gravity term G1 for of number count  |
| int       | index_tt_nc_g2             | index for first bin of transfer type = gravity term G2 for of number count  |
| int       | index_tt_nc_g3             | index for first bin of transfer type = gravity term G3 for of number count  |
| int       | index_tt_nc_g4             | index for first bin of transfer type = gravity term G3 for of number count  |
| int       | index_tt_nc_g5             | index for first bin of transfer type = gravity term G3 for of number count  |
| int *     | tt_size                    | number of requested transfer types<br>tt_size[index_md] for each mode   |
| int **    | l_size_tt                  | number of multipole values for which we effectively compute the transfer function, l_size_tt[index_md][index_tt]  |
| int *     | l_size                     | number of multipole values for each requested mode, l_size[index_md]  |
| int       | l_size_max                 | greatest of all l_size[index_md]  |
| int *     | l                          | list of multipole values l[index_l]   |
| double    | angular_rescaling          | correction between l and k space due to curvature (= comoving angular diameter distance to recombination / comoving radius to recombination)  |
| size_t    | q_size                     | number of wavenumber values   |
| double *  | q                          | list of wavenumber values, q[index_q]   |
| double ** | k                          | list of wavenumber values for each requested mode, k[index_md][index_q]. In flat universes k=q. In non-flat universes q and k differ through $q^2 = k^2 + K(1+m)$ , where m=0,1,2 for scalar, vector, tensor. q should be used throughout the transfer module, excepted when interpolating or manipulating the source functions S(k,tau): for a given value of q this should be done in k(q). |
| int       | index_q_flat_approximation | index of the first q value using the flat rescaling approximation   |
| double ** | transfer                   | table of transfer functions for each mode, initial condition, type, multipole and wavenumber, with argument transfer[index_md][((index_ic * ptr->tt_size[index_md] + index_tt) * ptr->l_size[index_md] + index_l) * ptr->q_size + index_q]  |
| short     | initialise_HIS_cache       | only true if we are using CLASS for setting up a cache of HIS structures  |
| short     | transfer_verbose           | flag regulating the amount of information sent to standard output (none if set to zero)   |
| ErrorMsg  | error_message              | zone for writing error messages   |

## 5.22.2.2 struct transfer\_workspace

Structure containing all the quantities that each thread needs to know for computing transfer functions (but that can be forgotten once the transfer functions are known, otherwise they would be stored in the transfer module)

## Data Fields

|                     |                      |  |
|---------------------|----------------------|--|
| HyperInterpStruct   | HIS                  | structure containing all hyperspherical bessel functions (flat case) or all hyperspherical bessel functions for a given value of $\beta = q/\sqrt{ K }$ (non-flat case). HIS = Hyperspherical Interpolation Structure. |
| int                 | HIS_allocated        | flag specifying whether the previous structure has been allocated  |
| HyperInterpStruct * | pBIS                 | pointer to structure containing all the spherical bessel functions of the flat case (used even in the non-flat case, for approximation schemes). pBIS = pointer to Bessel Interpolation Structure.                     |
| int                 | l_size               | number of l values   |
| int                 | tau_size             | number of discrete time values for a given type  |
| int                 | tau_size_max         | maximum number of discrete time values for all types   |
| double *            | interpolated_sources | interpolated_sources[index_tau]: sources interpolated from the perturbation module at the right value of k   |
| double *            | sources              | sources[index_tau]: sources used in transfer module, possibly differing from those in the perturbation module by some resampling or rescaling  |
| double *            | tau0_minus_tau       | tau0_minus_tau[index_tau]: values of (tau0 - tau)  |
| double *            | w_trapz              | w_trapz[index_tau]: values of weights in trapezoidal integration (related to time steps)   |
| double *            | chi                  | chi[index_tau]: value of argument of bessel function: $k(\tau_0 - \tau)$ (flat case) or $\sqrt{ K }(\tau_0 - \tau)$ (non-flat case)  |
| double *            | cscKgen              | cscKgen[index_tau]: useful trigonometric function  |
| double *            | cotKgen              | cotKgen[index_tau]: useful trigonometric function  |
| double              | K                    | curvature parameter (see background module for details)  |
| int                 | sgnK                 | 0 (flat), 1 (positive curvature, spherical, closed), -1 (negative curvature, hyperbolic, open)   |
| double              | tau0_minus_tau_cut   | critical value of (tau0-tau) in time cut approximation for the wavenumber at hand  |
| short               | neglect_late_source  | flag stating whether we use the time cut approximation for the wavenumber at hand  |

## 5.22.3 Enumeration Type Documentation

## 5.22.3.1 radial\_function\_type

```
enum radial_function_type
```

enumeration of possible source types. This looks redundant with respect to the definition of indices index\_tt\_... This definition is however convenient and time-saving: it allows to use a "case" statement in transfer\_radial\_function()





## Chapter 6

# The ‘external\_Pk’ mode

- Author: Jesus Torrado (torradocacho [at] lorentz.leidenuniv.nl)
- Date: 2013-12-20

### Introduction

This mode allows for an arbitrary primordial spectrum  $P(k)$  to be calculated by an external command and passed to CLASS. That external command may be anything that can be run in the shell: a python script, some compiled C or Fortran code... This command is executed from within CLASS, and CLASS is able to pass it a number of parameters defining the spectrum (an amplitude, a tilt...). Those parameters can be used in a Markov chain search performed by MontePython.

This mode includes the simple case of a precomputed primordial spectrum stored in a text file. In that case, the `cat` shell command will do the trick (see below).

Currently, scalar and tensor spectra of perturbations of adiabatic modes are supported.

### Use case #1: reading the spectrum from a table

In this case, say the file with the table is called `spectrum.txt`, located under `/path/to`, simply include in the `.ini` file

```
command = cat path/to/spectrum.txt
```

It is necessary that 1st 4 characters are exactly `cat`.

## Use case #2: getting the spectrum from an external command

Here an external command is called to generate the spectrum; it may be some compiled C or Fortran code, a python script... This command may be passed up to 10 floating point arguments, named `custom1` to `custom10`, which are assigned values inside the `.ini` file of CLASS. The `command` parameter would look like

```
command = /path/to/example.py
```

if it starts with `#!/usr/bin/python`, otherwise

```
command = python /path/to/example.py
```

As an example of the 1st use case, one may use the included script `generate_Pk_example.py`, which implements a single-field slow-roll spectrum without running, and takes 3 arguments:

- `custom1` – the pivot scale ( $k_0 = 0.05 \text{ 1/Mpc}$  for Planck).
- `custom2` – the amplitude of the scalar power spectrum.
- `custom3` – the scalar spectral index.

In order to use it, the following lines must be present in the parameter file:

```
P_k_ini type = external_Pk
command = /path/to/CLASS/external_Pk/generate_Pk_example.py
custom1 = 0.05
custom2 = 2.2e-9
custom3 = 1.
```

Defined or not (in that case, 0-valued), parameters from `custom4` to `custom10` will be passed to the example script, which should ignore them. In this case, CLASS will run in the shell the command

```
/path/to/CLASS/external_Pk/generate_Pk_example.py 0.05 2.2e-9 1. 0 0 0 0 0 0 0
```

If CLASS fails to run the command, try to do it directly yourself by hand, using exactly the same string that was given in `command`.

## Output of the command / format of the table

The command must generate an output separated into lines, each containing a tuple  $(k, P(k))$ . The following requirements must be fulfilled:

- Each line must contain 2 (3, if tensors) floating point numbers:  $k$  (in  $1/\text{Mpc}$  units) and  $P_s(k)$  (and  $P_{\leftrightarrow t}(k)$ , if tensors), separated by any number of spaces or tabs. The numbers can be in scientific notation, e.g.  $1.4e-3$ .
- The lines must be sorted in increasing values of  $k$ .
- There must be at least two points  $(k, P(k))$  before and after the interval of  $k$  requested by CLASS, in order not to introduce unnecessary interpolation error. Otherwise, an error will be raised. In most of the cases, generating the spectrum between  $1e-6$  and  $1 \text{ 1/Mpc}$  should be more than enough.

## Precision

This implementation properly handles double-precision floating point numbers (i.e. about 17 significant figures), both for the input parameters of the command and for the output of the command (or the table).

The sampling of  $k$  given by the command (or table) is preserved to be used internally by CLASS. It must be fine enough a sampling to clearly show the features of the spectrum. The best way to test this is to plot the output/table and check it with the naked eye.

Another thing to have in mind arises at the time of convolving with the transfer functions. Two precision parameters are implied: the sampling of  $k$  in the integral, given by `k_step_trans`, and the sampling of the transfer functions in  $l$ , given by `l_logstep` and `l_linstep`. In general, it will be enough to reduce the values of the first and the third parameters. A good start is to give them rather small values, say `k_step_trans=0.01` and `l_linstep=1`, and to increase them slowly until the point at which the effect of increasing them gets noticeable.

## Parameter fit with MontePython

(MontePython)[<http://montepython.net/>] is able to interact with the `external_Pk` mode transparently, using the `custom` parameters in an MCMC fit. One must just add the appropriate lines to the input file of MontePython. For our example, if we wanted to fit the amplitude and spectral index of the primordial spectrum, it would be:

```
data.cosmo_arguments['P_k_ini type'] = 'external_Pk'
data.cosmo_arguments['command'] = '/path/to/CLASS/external_Pk/generate_Pk_example.py'
data.cosmo_arguments['custom1'] = 0.05 # k_pivot
data.parameters['custom2'] = [ 2.2, 0, -1, 0.055, 1.e-9, 'cosmo'] # A_s
data.parameters['custom3'] = [ 1., 0, -1, 0.0074, 1, 'cosmo'] # n_s
```

Notice that since in our case `custom1` represents the pivot scale, it is passed as a (non-varying) argument, instead of as a (varying) parameter.

In this case, one would not include the corresponding lines for the primordial parameters of CLASS: `k_pivot`, `A_s`, `n_s`, `alpha_s`, etc. They would simply be ignored.

## Limitations

- So far, this mode cannot handle vector perturbations, nor isocurvature initial conditions.
- The external script knows nothing about the rest of the CLASS parameters, so if it needs, e.g., `k_pivot`, it should be either hard coded, or its value passed as one of the `custom` parameters.



# Chapter 7

## Updating the manual

Author: D. C. Hooper ([hooper@physik.rwth-aachen.de](mailto:hooper@physik.rwth-aachen.de))

This pdf manual and accompanying web version have been generated using the `doxygen` software (<http://www.doxygen.org>). This software directly reads the code and extracts the necessary comments to form the manual, meaning it is very easy to generate newer versions of the manual as desired.

### For CLASS developers:

To maintain the usefulness of the manual, a new version should be generated after any major upgrade to `CLASS`. To keep track of how up-to-date the manual is the title page also displays the last modification date. The manual is generated automatically from the code, excepted a few chapters written manually in the files

```
README.md
doc/input/chap2.md
doc/input/chap3.md
doc/input/mod.md
external_Pk/README.md
```

You can update these files, or add new ones that should be declared in the `INPUT=` field of `doc/input/doxyconf`.

Generating a new version of this manual is straightforward. First, you need to install the `doxygen` software, which can be done by following the instructions on the software's webpage. The location where you install this software is irrelevant; it doesn't need to be in the same folder as `CLASS`. For Mac OSX, homebrew users can install the software with `brew install doxygen --with-graphviz`.

Once installed, navigate to the `class/doc/input` directory and run the first script

```
. make1.sh
```

This will generate a new version of the html manual and the necessary files to make the pdf version. Unfortunately, `doxygen` does not yet offer the option to automatically order the output chapters in the pdf version of the manual. Hence, before compiling the pdf, this must be done manually. To do this you need to find the `refman.tex` file in `class/doc/manual/latex`. With this file you can modify the title page, headers, footers, and chapter ordering for the final pdf. Usually we just make two things: add manually the line

```
\vspace*{1cm}
{\large Last updated \today}\}
```

after

```
{\Large C\+L\+A\+SS M\+A\+N\+U\+AL }\\
```

and move manually the chapters "The external Pk mode" and "Updating the manual" to the end, after the automatically generated part. Once you have this file with your desired configuration, navigate back to the `class/doc/input` directory, and run the second script

```
. make2.sh
```

You should now be able to find the finished pdf in `class/doc/manual/CLASS_MANUAL.pdf`. Finally you can commit the changes to git, but not all the content of `doc/` is necessary: only `doc/README`, `doc/input/`, `doc/manual/CLASS_MANUAL.pdf`, `doc/manual/html/`. This means that before committing you will have to do a: `git add doc/manual/html/`, but NOT a: `git add doc/manual/latex/!`

As a final comment, doxygen uses two main configuration files: `doxyconf` and `doxygen.sty`, both located in `class/doc/input`. Changes to these files can dramatically impact the outcome, so any modifications to these files should be done with great care.

# Index

- [\\_MAX\\_NUMBER\\_OF\\_K\\_FILES\\_](#)
  - [perturbations.h, 129](#)
- [\\_M\\_EV\\_TOO\\_BIG\\_FOR\\_HALOFIT\\_](#)
  - [nonlinear.h, 80](#)
- [\\_SELECTION\\_NUM\\_MAX\\_](#)
  - [perturbations.h, 129](#)
- [\\_YHE\\_BIG\\_](#)
  - [thermodynamics.h, 198](#)
- [\\_YHE\\_SMALL\\_](#)
  - [thermodynamics.h, 198](#)
- [\\_Z\\_PK\\_NUM\\_MAX\\_](#)
  - [output.h, 89](#)
- [background, 37](#)
- [background.c, 19](#)
  - [background\\_at\\_tau, 21](#)
  - [background\\_derivs, 33](#)
  - [background\\_free, 26](#)
  - [background\\_free\\_input, 26](#)
  - [background\\_functions, 22](#)
  - [background\\_indices, 27](#)
  - [background\\_init, 25](#)
  - [background\\_initial\\_conditions, 31](#)
  - [background\\_ncdm\\_M\\_from\\_Omega, 30](#)
  - [background\\_ncdm\\_distribution, 27](#)
  - [background\\_ncdm\\_init, 28](#)
  - [background\\_ncdm\\_momenta, 29](#)
  - [background\\_ncdm\\_test\\_function, 28](#)
  - [background\\_output\\_data, 33](#)
  - [background\\_output\\_titles, 32](#)
  - [background\\_solve, 30](#)
  - [background\\_tau\\_of\\_z, 22](#)
  - [background\\_w fld, 23](#)
  - [V\\_e\\_scf, 34](#)
  - [V\\_p\\_scf, 35](#)
  - [V\\_scf, 35](#)
- [background.h, 36](#)
- [background\\_at\\_tau](#)
  - [background.c, 21](#)
- [background\\_derivs](#)
  - [background.c, 33](#)
- [background\\_free](#)
  - [background.c, 26](#)
- [background\\_free\\_input](#)
  - [background.c, 26](#)
- [background\\_functions](#)
  - [background.c, 22](#)
- [background\\_indices](#)
  - [background.c, 27](#)
- [background\\_init](#)
  - [background.c, 25](#)
- [background\\_initial\\_conditions](#)
  - [background.c, 31](#)
- [background\\_ncdm\\_M\\_from\\_Omega](#)
  - [background.c, 30](#)
- [background\\_ncdm\\_distribution](#)
  - [background.c, 27](#)
- [background\\_ncdm\\_init](#)
  - [background.c, 28](#)
- [background\\_ncdm\\_momenta](#)
  - [background.c, 29](#)
- [background\\_ncdm\\_test\\_function](#)
  - [background.c, 28](#)
- [background\\_output\\_data](#)
  - [background.c, 33](#)
- [background\\_output\\_titles](#)
  - [background.c, 32](#)
- [background\\_parameters\\_and\\_workspace, 41](#)
- [background\\_parameters\\_for\\_distributions, 41](#)
- [background\\_solve](#)
  - [background.c, 30](#)
- [background\\_tau\\_of\\_z](#)
  - [background.c, 22](#)
- [background\\_w fld](#)
  - [background.c, 23](#)
- [class.c, 41](#)
- [class\\_fzero\\_ridder](#)
  - [input.c, 59](#)
- [common.h, 42](#)
  - [evolver\\_type, 51](#)
  - [file\\_format, 52](#)
  - [pk\\_def, 51](#)
- [error\\_message](#)
  - [nonlinear, 17](#)
- [evolver\\_type](#)
  - [common.h, 51](#)
- [f1](#)
  - [thermodynamics.h, 197](#)
- [f2](#)
  - [thermodynamics.h, 198](#)
- [file\\_format](#)
  - [common.h, 52](#)
- [get\\_machine\\_precision](#)
  - [input.c, 59](#)
- [inflation\\_module\\_behavior](#)
  - [primordial.h, 154](#)

- input.c, 52
  - class\_fzero\_ridder, 59
  - get\_machine\_precision, 59
  - input\_default\_params, 57
  - input\_default\_precision, 58
  - input\_find\_root, 60
  - input\_get\_guess, 60
  - input\_init, 54
  - input\_init\_from\_arguments, 53
  - input\_read\_parameters, 55
  - input\_try\_unknown\_parameters, 59
- input.h, 61
  - target\_names, 62
- input\_default\_params
  - input.c, 57
- input\_default\_precision
  - input.c, 58
- input\_find\_root
  - input.c, 60
- input\_get\_guess
  - input.c, 60
- input\_init
  - input.c, 54
- input\_init\_from\_arguments
  - input.c, 53
- input\_read\_parameters
  - input.c, 55
- input\_try\_unknown\_parameters
  - input.c, 59
- integration\_direction
  - primordial.h, 154
- k
  - nonlinear, 16
- k\_nl
  - nonlinear, 16
- k\_size
  - nonlinear, 16
- lensing, 76
- lensing.c, 63
  - lensing\_addback\_cl\_ee\_bb, 69
  - lensing\_addback\_cl\_te, 68
  - lensing\_addback\_cl\_tt, 67
  - lensing\_cl\_at\_l, 64
  - lensing\_d00, 69
  - lensing\_d11, 70
  - lensing\_d1m1, 70
  - lensing\_d20, 71
  - lensing\_d22, 71
  - lensing\_d2m2, 71
  - lensing\_d31, 72
  - lensing\_d3m1, 72
  - lensing\_d3m3, 73
  - lensing\_d40, 73
  - lensing\_d4m2, 73
  - lensing\_d4m4, 74
  - lensing\_free, 66
  - lensing\_indices, 66
  - lensing\_init, 65
  - lensing\_lensed\_cl\_ee\_bb, 68
  - lensing\_lensed\_cl\_te, 67
  - lensing\_lensed\_cl\_tt, 67
  - linear\_or\_logarithmic
    - primordial.h, 153
  - method
    - nonlinear, 16
  - nl\_corr\_density
    - nonlinear, 16
  - nonlinear, 15
    - error\_message, 17



- k, 16
- k\_nl, 16
- k\_size, 16
- method, 16
- nl\_corr\_density, 16
- nonlinear\_verbose, 16
- tau, 16
- tau\_size, 16
- nonlinear.c, 77
  - nonlinear\_halofit, 78
  - nonlinear\_init, 78
- nonlinear.h, 79
  - \_M\_EV\_TOO\_BIG\_FOR\_HALOFIT\_, 80
- nonlinear\_halofit
  - nonlinear.c, 78
- nonlinear\_init
  - nonlinear.c, 78
- nonlinear\_verbose
  - nonlinear, 16
- output, 89
- output.c, 80
  - output\_cl, 82
  - output\_init, 81
  - output\_one\_line\_of\_cl, 85
  - output\_one\_line\_of\_pk, 87
  - output\_open\_cl\_file, 85
  - output\_open\_pk\_file, 86
  - output\_pk, 83
  - output\_pk\_nl, 83
  - output\_print\_data, 84
  - output\_tk, 84
- output.h, 87
  - \_Z\_PK\_NUM\_MAX\_, 89
- output\_cl
  - output.c, 82
- output\_init
  - output.c, 81
- output\_one\_line\_of\_cl
  - output.c, 85
- output\_one\_line\_of\_pk
  - output.c, 87
- output\_open\_cl\_file
  - output.c, 85
- output\_open\_pk\_file
  - output.c, 86
- output\_pk
  - output.c, 83
- output\_pk\_nl
  - output.c, 83
- output\_print\_data
  - output.c, 84
- output\_tk
  - output.c, 84
- perturb\_approximations
  - perturbations.c, 107
- perturb\_derivs
  - perturbations.c, 113
- perturb\_einstein
  - perturbations.c, 110
- perturb\_find\_approximation\_number
  - perturbations.c, 99
- perturb\_find\_approximation\_switches
  - perturbations.c, 100
- perturb\_free
  - perturbations.c, 93
- perturb\_get\_k\_list
  - perturbations.c, 95
- perturb\_indices\_of\_perturbs
  - perturbations.c, 93
- perturb\_init
  - perturbations.c, 92
- perturb\_initial\_conditions
  - perturbations.c, 104
- perturb\_parameters\_and\_workspace, 128
- perturb\_prepare\_output
  - perturbations.c, 99
- perturb\_print\_variables
  - perturbations.c, 112
- perturb\_solve
  - perturbations.c, 98
- perturb\_sources
  - perturbations.c, 111
- perturb\_sources\_at\_tau
  - perturbations.c, 91
- perturb\_tca\_slip\_and\_shear
  - perturbations.c, 117
- perturb\_timesampling\_for\_sources
  - perturbations.c, 94
- perturb\_timescale
  - perturbations.c, 109
- perturb\_total\_stress\_energy
  - perturbations.c, 111
- perturb\_vector, 125
- perturb\_vector\_free
  - perturbations.c, 103
- perturb\_vector\_init
  - perturbations.c, 101
- perturb\_workspace, 127
- perturb\_workspace\_free
  - perturbations.c, 97
- perturb\_workspace\_init
  - perturbations.c, 96
- perturbations.c, 90
  - perturb\_approximations, 107
  - perturb\_derivs, 113
  - perturb\_einstein, 110
  - perturb\_find\_approximation\_number, 99
  - perturb\_find\_approximation\_switches, 100
  - perturb\_free, 93
  - perturb\_get\_k\_list, 95
  - perturb\_indices\_of\_perturbs, 93
  - perturb\_init, 92
  - perturb\_initial\_conditions, 104
  - perturb\_prepare\_output, 99
  - perturb\_print\_variables, 112

- perturb\_solve, 98
- perturb\_sources, 111
- perturb\_sources\_at\_tau, 91
- perturb\_tca\_slip\_and\_shear, 117
- perturb\_timesampling\_for\_sources, 94
- perturb\_timescale, 109
- perturb\_total\_stress\_energy, 111
- perturb\_vector\_free, 103
- perturb\_vector\_init, 101
- perturb\_workspace\_free, 97
- perturb\_workspace\_init, 96
- perturbations.h, 118
  - \_MAX\_NUMBER\_OF\_K\_FILES\_, 129
  - \_SELECTION\_NUM\_MAX\_, 129
  - possible\_gauges, 129
  - tca\_flags, 129
  - tca\_method, 129
- perturbs, 120
- phi\_pivot\_methods
  - primordial.h, 154
- pk\_def
  - common.h, 51
- possible\_gauges
  - perturbations.h, 129
- potential\_shape
  - primordial.h, 153
- precision, 43
- primordial, 149
- primordial.c, 130
  - primordial\_analytic\_spectrum, 135
  - primordial\_analytic\_spectrum\_init, 135
  - primordial\_external\_spectrum\_init, 147
  - primordial\_free, 133
  - primordial\_get\_lnk\_list, 134
  - primordial\_indices, 134
  - primordial\_inflation\_analytic\_spectra, 138
  - primordial\_inflation\_check\_hubble, 143
  - primordial\_inflation\_check\_potential, 142
  - primordial\_inflation\_derivs, 146
  - primordial\_inflation\_evolve\_background, 141
  - primordial\_inflation\_find\_attractor, 141
  - primordial\_inflation\_find\_phi\_pivot, 145
  - primordial\_inflation\_get\_epsilon, 143
  - primordial\_inflation\_hubble, 136
  - primordial\_inflation\_indices, 137
  - primordial\_inflation\_one\_k, 140
  - primordial\_inflation\_one\_wavenumber, 139
  - primordial\_inflation\_potential, 136
  - primordial\_inflation\_solve\_inflation, 137
  - primordial\_inflation\_spectra, 138
  - primordial\_init, 132
  - primordial\_spectrum\_at\_k, 131
- primordial.h, 147
  - inflation\_module\_behavior, 154
  - integration\_direction, 154
  - linear\_or\_logarithmic, 153
  - phi\_pivot\_methods, 154
  - potential\_shape, 153
  - primordial\_spectrum\_type, 153
  - target\_quantity, 153
  - time\_definition, 154
- primordial\_analytic\_spectrum
  - primordial.c, 135
- primordial\_analytic\_spectrum\_init
  - primordial.c, 135
- primordial\_external\_spectrum\_init
  - primordial.c, 147
- primordial\_free
  - primordial.c, 133
- primordial\_get\_lnk\_list
  - primordial.c, 134
- primordial\_indices
  - primordial.c, 134
- primordial\_inflation\_analytic\_spectra
  - primordial.c, 138
- primordial\_inflation\_check\_hubble
  - primordial.c, 143
- primordial\_inflation\_check\_potential
  - primordial.c, 142
- primordial\_inflation\_derivs
  - primordial.c, 146
- primordial\_inflation\_evolve\_background
  - primordial.c, 141
- primordial\_inflation\_find\_attractor
  - primordial.c, 141
- primordial\_inflation\_find\_phi\_pivot
  - primordial.c, 145
- primordial\_inflation\_get\_epsilon
  - primordial.c, 143
- primordial\_inflation\_hubble
  - primordial.c, 136
- primordial\_inflation\_indices
  - primordial.c, 137
- primordial\_inflation\_one\_k
  - primordial.c, 140
- primordial\_inflation\_one\_wavenumber
  - primordial.c, 139
- primordial\_inflation\_potential
  - primordial.c, 136
- primordial\_inflation\_solve\_inflation
  - primordial.c, 137
- primordial\_inflation\_spectra
  - primordial.c, 138
- primordial\_init
  - primordial.c, 132
- primordial\_spectrum\_at\_k
  - primordial.c, 131
- primordial\_spectrum\_type
  - primordial.h, 153
- radial\_function\_type
  - transfer.h, 225
- recombination, 195
  - recombination\_algorithm
    - thermodynamics.h, 198
- reionization, 196
- reionization\_parametrization

- thermodynamics.h, 198
- reionization\_z\_or\_tau
  - thermodynamics.h, 199
- spectra, 170
- spectra.c, 155
  - spectra\_cl\_at\_l, 156
  - spectra\_cls, 164
  - spectra\_compute\_cl, 165
  - spectra\_free, 163
  - spectra\_indices, 163
  - spectra\_init, 162
  - spectra\_k\_and\_tau, 166
  - spectra\_matter\_transfers, 168
  - spectra\_output\_tk\_data, 168
  - spectra\_pk, 166
  - spectra\_pk\_at\_k\_and\_z, 158
  - spectra\_pk\_at\_z, 157
  - spectra\_pk\_nl\_at\_k\_and\_z, 160
  - spectra\_pk\_nl\_at\_z, 159
  - spectra\_sigma, 167
  - spectra\_tk\_at\_k\_and\_z, 161
  - spectra\_tk\_at\_z, 161
- spectra.h, 169
- spectra\_cl\_at\_l
  - spectra.c, 156
- spectra\_cls
  - spectra.c, 164
- spectra\_compute\_cl
  - spectra.c, 165
- spectra\_free
  - spectra.c, 163
- spectra\_indices
  - spectra.c, 163
- spectra\_init
  - spectra.c, 162
- spectra\_k\_and\_tau
  - spectra.c, 166
- spectra\_matter\_transfers
  - spectra.c, 168
- spectra\_output\_tk\_data
  - spectra.c, 168
- spectra\_pk
  - spectra.c, 166
- spectra\_pk\_at\_k\_and\_z
  - spectra.c, 158
- spectra\_pk\_at\_z
  - spectra.c, 157
- spectra\_pk\_nl\_at\_k\_and\_z
  - spectra.c, 160
- spectra\_pk\_nl\_at\_z
  - spectra.c, 159
- spectra\_sigma
  - spectra.c, 167
- spectra\_tk\_at\_k\_and\_z
  - spectra.c, 161
- spectra\_tk\_at\_z
  - spectra.c, 161
- target\_names
  - input.h, 62
- target\_quantity
  - primordial.h, 153
- tau
  - nonlinear, 16
- tau\_size
  - nonlinear, 16
- tca\_flags
  - perturbations.h, 129
- tca\_method
  - perturbations.h, 129
- thermo, 192
- thermodynamics.c, 174
  - thermodynamics\_at\_z, 177
  - thermodynamics\_derivs\_with\_recfast, 189
  - thermodynamics\_energy\_injection, 181
  - thermodynamics\_free, 179
  - thermodynamics\_get\_xe\_before\_reionization, 184
  - thermodynamics\_helium\_from\_bbn, 180
  - thermodynamics\_indices, 179
  - thermodynamics\_init, 177
  - thermodynamics\_merge\_reco\_and\_reio, 189
  - thermodynamics\_onthespot\_energy\_injection, 181
  - thermodynamics\_output\_titles, 190
  - thermodynamics\_recombination, 186
  - thermodynamics\_recombination\_with\_hyrec, 186
  - thermodynamics\_recombination\_with\_recfast, 187
  - thermodynamics\_reionization, 184
  - thermodynamics\_reionization\_function, 183
  - thermodynamics\_reionization\_sample, 185
- thermodynamics.h, 191
  - \_YHE\_BIG\_, 198
  - \_YHE\_SMALL\_, 198
  - f1, 197
  - f2, 198
  - recombination\_algorithm, 198
  - reionization\_parametrization, 198
  - reionization\_z\_or\_tau, 199
- thermodynamics\_at\_z
  - thermodynamics.c, 177
- thermodynamics\_derivs\_with\_recfast
  - thermodynamics.c, 189
- thermodynamics\_energy\_injection
  - thermodynamics.c, 181
- thermodynamics\_free
  - thermodynamics.c, 179
- thermodynamics\_get\_xe\_before\_reionization
  - thermodynamics.c, 184
- thermodynamics\_helium\_from\_bbn
  - thermodynamics.c, 180
- thermodynamics\_indices
  - thermodynamics.c, 179
- thermodynamics\_init
  - thermodynamics.c, 177
- thermodynamics\_merge\_reco\_and\_reio
  - thermodynamics.c, 189
- thermodynamics\_onthespot\_energy\_injection

- thermodynamics.c, 181
- thermodynamics\_output\_titles
  - thermodynamics.c, 190
- thermodynamics\_parameters\_and\_workspace, 197
- thermodynamics\_recombination
  - thermodynamics.c, 186
- thermodynamics\_recombination\_with\_hyrec
  - thermodynamics.c, 186
- thermodynamics\_recombination\_with\_recast
  - thermodynamics.c, 187
- thermodynamics\_reionization
  - thermodynamics.c, 184
- thermodynamics\_reionization\_function
  - thermodynamics.c, 183
- thermodynamics\_reionization\_sample
  - thermodynamics.c, 185
- time\_definition
  - primordial.h, 154
- transfer.c, 199
  - transfer\_compute\_for\_each\_l, 214
  - transfer\_compute\_for\_each\_q, 207
  - transfer\_dNdz\_analytic, 210
  - transfer\_free, 203
  - transfer\_functions\_at\_q, 201
  - transfer\_get\_k\_list, 206
  - transfer\_get\_l\_list, 204
  - transfer\_get\_q\_list, 205
  - transfer\_get\_source\_correspondence, 206
  - transfer\_indices\_of\_transfers, 203
  - transfer\_init, 202
  - transfer\_integrate, 215
  - transfer\_interpolate\_sources, 208
  - transfer\_lensing\_sampling, 211
  - transfer\_limber, 217
  - transfer\_limber2, 219
  - transfer\_limber\_interpolate, 218
  - transfer\_selection\_compute, 213
  - transfer\_selection\_function, 210
  - transfer\_selection\_sampling, 211
  - transfer\_selection\_times, 213
  - transfer\_source\_resample, 212
  - transfer\_source\_tau\_size, 207
  - transfer\_sources, 209
- transfer.h, 220
  - radial\_function\_type, 225
- transfer\_compute\_for\_each\_l
  - transfer.c, 214
- transfer\_compute\_for\_each\_q
  - transfer.c, 207
- transfer\_dNdz\_analytic
  - transfer.c, 210
- transfer\_free
  - transfer.c, 203
- transfer\_functions\_at\_q
  - transfer.c, 201
- transfer\_get\_k\_list
  - transfer.c, 206
- transfer\_get\_l\_list
  - transfer.c, 204
- transfer.c, 204
  - transfer\_get\_q\_list
    - transfer.c, 205
  - transfer\_get\_source\_correspondence
    - transfer.c, 206
  - transfer\_indices\_of\_transfers
    - transfer.c, 203
  - transfer\_init
    - transfer.c, 202
  - transfer\_integrate
    - transfer.c, 215
  - transfer\_interpolate\_sources
    - transfer.c, 208
  - transfer\_lensing\_sampling
    - transfer.c, 211
  - transfer\_limber
    - transfer.c, 217
  - transfer\_limber2
    - transfer.c, 219
  - transfer\_limber\_interpolate
    - transfer.c, 218
  - transfer\_selection\_compute
    - transfer.c, 213
  - transfer\_selection\_function
    - transfer.c, 210
  - transfer\_selection\_sampling
    - transfer.c, 211
  - transfer\_selection\_times
    - transfer.c, 213
  - transfer\_source\_resample
    - transfer.c, 212
  - transfer\_source\_tau\_size
    - transfer.c, 207
  - transfer\_sources
    - transfer.c, 209
  - transfer\_workspace, 224
- transfers, 221
- V\_e\_scf
  - background.c, 34
- V\_p\_scf
  - background.c, 35
- V\_scf
  - background.c, 35