7. Lenses

A. Objectives

- Measure the deflection of light rays by a positive lens; determine the lens focal length.
- Study image formation by a positive lens; measure lens magnification and experimentally confirm the thin lens equation.
- Study virtual image formation.
- Measure the radius of curvature of the positive lens, and compare its calculated and observed focal lengths.
- Observe two common lens aberrations: spherical aberration and coma.

B. Equipment required

1. Optical breadboard with guide rail and beam stops
2. Diode laser in mount
3. Large positive lens in mount
4. Negative lens in mount
5. 4"x4" plate in mount, paper and tape
6. Ruler, yardstick, and protractor
7. Aluminum and cardboard aperture plates: 1 each of 1 mm, 2 mm, 50 mm, and 70 mm circular apertures, arrow shape, and blank
8. Two filter holders
9. Incandescent spot lamp, mount, and small optical breadboard

C. Introduction

Lenses are transparent dielectric objects used to refract light rays in image forming optical systems. The most common lens type is the *spherical glass lens,* as illustrated in Figure 7.1. This lens has two surfaces that are sections of a spherical surface; each surface has a radius of curvature and a center of curvature as shown in Figure 7.1(a). The *optical axis* or *lens axis* of the lens is the line passing through the two centers of curvature. ("Optical axis" and "lens axis" mean the same thing for a system consisting of one lens. For multiple lens systems, "optical axis" has a different definition, and the term "lens axis" should be used for the line joining the two centers of curvature of a given lens.)

Spherical lenses are produced by grinding two glass surfaces together. If one or both surfaces are non-spherical, there will always be isolated points of contact, as shown in Figure 7.1(b). These points of contact will tend to be ground down. The only surfaces for which all points are simultaneously in contact throughout a lateral grinding motion are spherical surfaces of the same radius of curvature as shown in Figure 7.1(c). Under this condition the lens surface grinds down evenly. For this reason, spherical lens surfaces are much easier to produce than non-spherical ones.

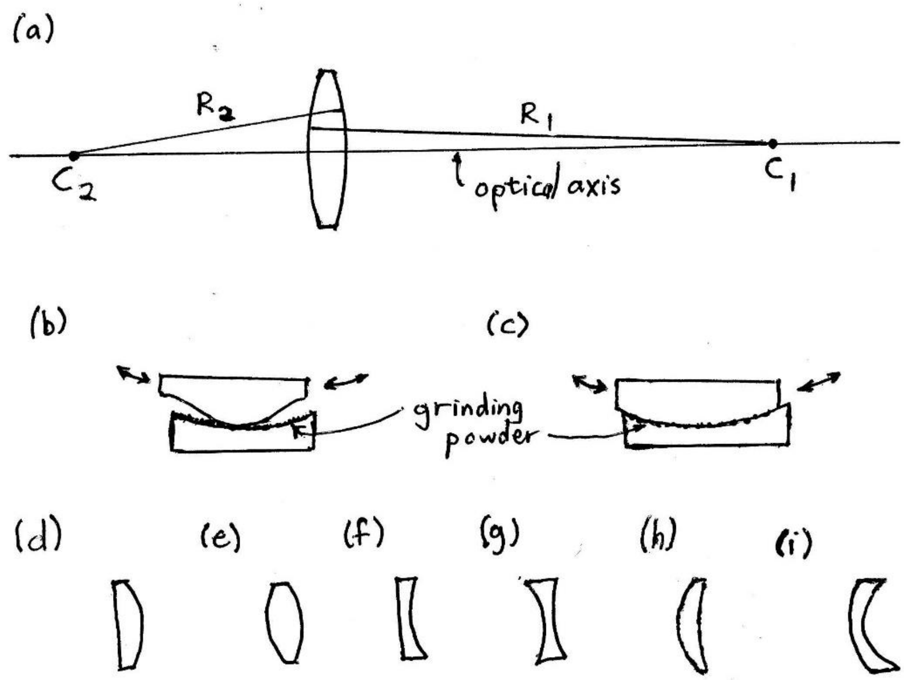

Figure 7.1. (a) Biconvex spherical lens, with radii of curvature R_1 and R_2, and centers of curvature C_1 and C_2. (b) Grinding in progress with a non-spherical surface. (c) Grinding in progress with matching spherical surfaces. (d) Plano-convex lens. (e) Biconvex lens. (f) Plano-concave lens. (g) Biconcave lens. (h) Positive meniscus lens. (i) Negative meniscus lens.

Each spherical lens surface may be either convex, plano (flat), or concave. This leads to a total of six spherical lens types, as shown in Figure 7.1(d) through (i). A *positive lens* is thicker in the center than at the edge. A *negative lens* is thinner in the center than at the edge. There are three positive lens types: plano-convex, bi-convex, and positive meniscus. And, there are three negative lens types: plano-concave, bi-concave, and negative meniscus.

Non-spherical lens surfaces can also be produced with a variety of techniques. Such non-spherical lenses can offer powerful performance advantages over spherical ones. Historically, the fabrication cost of these non-spherical lenses tended to be prohibitively high. But over the last twenty years, techniques for producing non-spherical lenses have advanced a great deal, and they are now cost-effective in many applications. Molded aspherical lenses are now widely used in cell phone cameras, and are largely responsible for their relatively good performance.

In this lab, we'll study only spherical lenses, because they're easier to understand, and because you need to understand spherical lenses before you study non-spherical ones anyway.

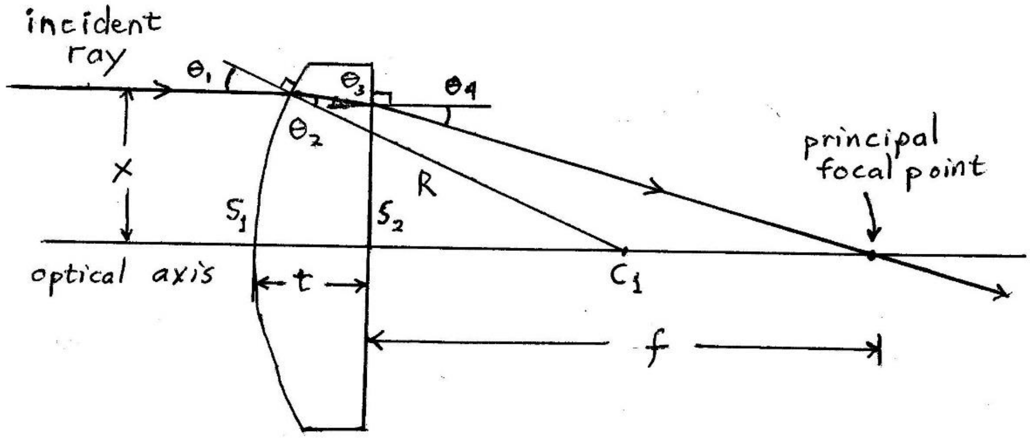

Figure 7.2. Refraction of a light ray by a plano-convex lens.

Figure 7.2 illustrates the refraction of a light ray by a plano-convex lens, for the case that the incident light ray is parallel to the optical axis, displaced from it by distance *x*, and that the curved side of the lens faces the incident light. We denote the two lens surfaces as S_1 and S_2. S_1 has a radius of curvature *R*, and a center of curvature located at position C_1.

The *paraxial approximation* is the approximation that all angles of incidence and refraction θ in an optical system are small compared to 1, so that we may use the approximation $\sin\theta \approx \theta.$ The paraxial approximation will be valid for rays which remain near the optical axis throughout the optical system.

The *thin lens approximation* is the approximation that we can neglect the thickness *t* of the lens. This will be valid when *t* is much less than the focal length of the lens. (Focal length is defined below.)

Except for the section on lens aberrations, in this lab we'll assume that the paraxial and thin lens approximations are valid.

The angle of incidence of the ray on surface S_1 is $\theta_1 \approx x / R.$ (Again, we are using the paraxial approximation throughout this section.) According to Snell's law, the angle of refraction on S_1 is $\theta_2 \approx \theta_1 / n,$ where *n* is the index of refraction of the lens glass. The ray is bent towards the optical axis. The ray strikes the plano surface S_2 with angle of incidence $\theta_3 = \theta_1 - \theta_2 \approx \theta_1\left(1-\frac{1}{n}\right).$

Finally, it is refracted out of the lens with refraction angle $\theta_4 \approx n\theta_3 \approx \theta_1\left(n-1\right) \approx \frac{x}{R}\left(n-1\right).$ The ray is bent still further toward the optical axis by this second refraction. This light ray crosses the optical axis at a point called the *principal focal point* of the lens. In the paraxial and thin lens approximations, this is located at a distance $f \approx \frac{x}{\theta_4} \approx \frac{R}{n-1}$ from the lens. The quantity *f* is called the *focal length* of the lens.

Because the displacement *x* does not appear in the expression for *f*, *any* ray parallel to the optical axis will be directed through though the principal focal point, as shown in Figure 7.3(a). Thus, a bundle of parallel rays is focused to a point by a positive lens. Figure 7.3(b) shows what happens for a negative lens. In this case, the rays are bent away from the optical axis. The rays emerging from the lens appear to come from a common point in front of the lens. This point is the principal focal point for the negative lens. The distance of this point from the lens is the focal length for the negative lens. Actually, it is conventional to define *two* principal focal points for a lens, as illustrated in Figure 7.3(c) and Figure 7.3(d). These two principal focal points lie on the optical axis, at a distance *f* from the lens, on either side of the lens.

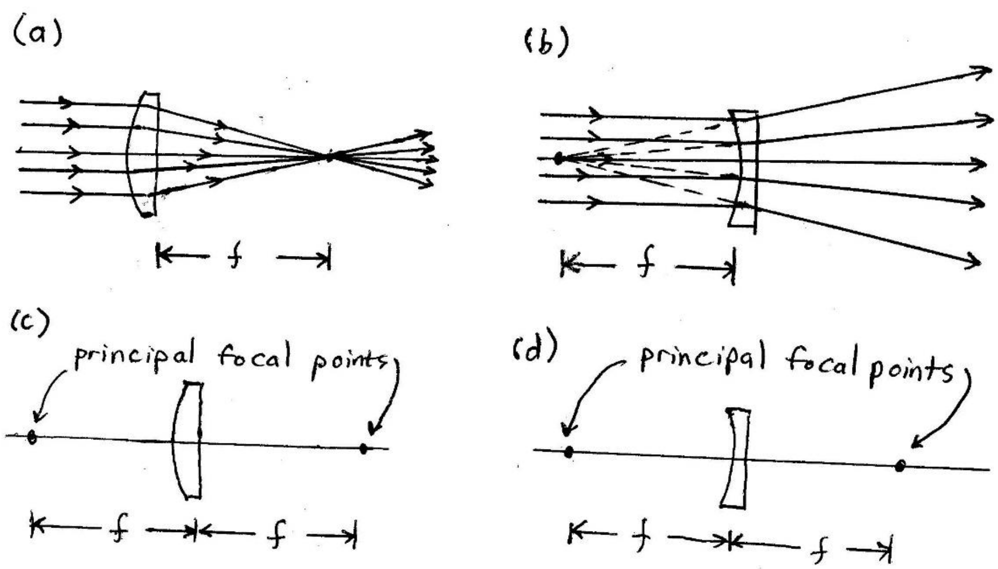

Figure 7.3. (a) Focusing of rays parallel to the optical axis for a positive lens. (b) Bending of light rays parallel to the optical axis for a negative lens. (c) Principal focal points for a positive lens. (d) Principal focal points for a negative lens.

For completeness, the following formula gives the focal length of a lens in the general case of two curved surfaces:

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \tag{7.1}$$

Here, R_1 and R_2 are the two radii of curvatures illustrated in Figure 7.1(a). The sign convention is that the radius of curvature is positive if the center of curvature lies to the right of the lens, and negative if the center of curvature lies to the left of the lens. The focal length *f* is positive for a positive lens, and negative for a negative lens. With these sign conventions, equation (7.1) gives correct results for all six lens types shown in Figure 7.1. Equation (7.1) is called the *lens maker's equation.* This equation can be derived with a straightforward extension of the calculation we carried out above for the plano-convex lens.

Image formation

Suppose a positive lens is used to collect light from a point P on an object, and suppose that the distance *s* of the object from the lens is greater than its focal length, as shown in Figure 7.4. We can analyze the effect of the lens by *ray tracing,* in which we take a set of light rays emitted from point P and follow them through the optical system. Usually, we begin with the three *principal light rays* shown in Figure 7.4. These are (1) a ray from point P parallel to the optical axis, (2) a ray from point P through the center of the lens, and (3) a ray from point P through the front focal point of the lens.

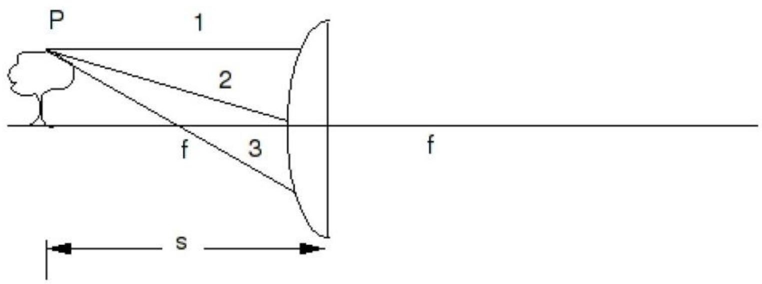

Figure 7.4. First half of the three principal light rays from a point P on an object.

It is easy to trace the principal light rays through the lens. According to what we have learned, the ray parallel to the optical axis is refracted so that it passes through the back focal point. The ray through the center of the lens is undeflected. And the ray through the front focal point is refracted to emerge from the lens parallel to the optical axis. This gives the complete ray trace for the principal light rays shown in Figure 7.5.

We see that rays emerging from object at point P, located a distance *s* from the lens, are focused by the lens to a common point P', at a distance s' from the lens. If all points on the object lie in the same plane, then the same statement applies to the entire object: light from each point on the object is focused to a single point in the plane a distance s' from the lens. If we put a screen in that plane, we will see an image of the object. The plane of the object is called the *object plane* and the plane of the image is called the *image plane.*

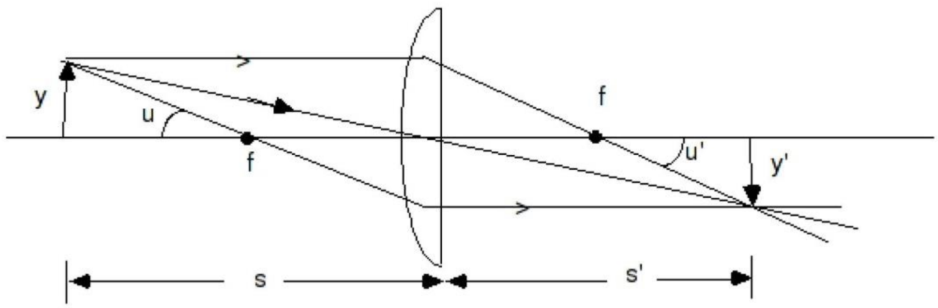

Figure 7.5. An object point displaced from the optical axis by distance *y*, outside the front principal focal point of the lens, produces an image point displaced from the optical axis by distance y', outside the back principal focal point of the lens.

Let's see if we can relate the object distance *s* and image distance s' for a lens. From Figure 7.5, you can find two triangles that lead to the expressions

$$\tan u = \frac{y'}{f} = \frac{y}{s-f} \tag{7.2}$$

and

$$\tan u' = \frac{y}{f} = \frac{y'}{s'-f} \tag{7.3}$$

From equations (7.2) and (7.3), you can show that

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \tag{7.4}$$

Equation (7.4) is called the *thin lens equation.*

The *magnification M* of the lens is the ratio of the image size to the object size, *i.e.* the magnitude of the magnification is

$$|M| = \left|\frac{y'}{y}\right| = \left|\frac{s'}{s}\right| \tag{7.5}$$

The magnification is positive if the image is erect $(y'/y > 0)$, and negative if the image is inverted $(y'/y < 0)$.

D. Experimental Procedure

The experimental arrangement is illustrated in Figure 7.6. You are supplied with a 100 mm diameter plano-convex positive lens in a large lens mount. You'll use the same diode laser and mount as in the last few experiments to generate a laser beam that will allow you to study the ray optics of the lenses. A good laser beam height for this experiment is about 6". This is high enough that you can get the beam through the center of the lens, but low enough that you can reach this beam height with the other components. You should use the short, 2" post mount to hold the lens, so that it is not too high above the board.

You are provided also with a 6" long guide rail. This should be secured to one end of the board as shown in Figure 7.6. The purpose of this rail is to allow you to slide the laser in its mount with a pure sideways motion (*i.e.* without any rotation of the mount). Finally, you are provided with a 4" x 4" metal plate on a mount. Tape a piece of white paper to the front of this plate to use as a screen.

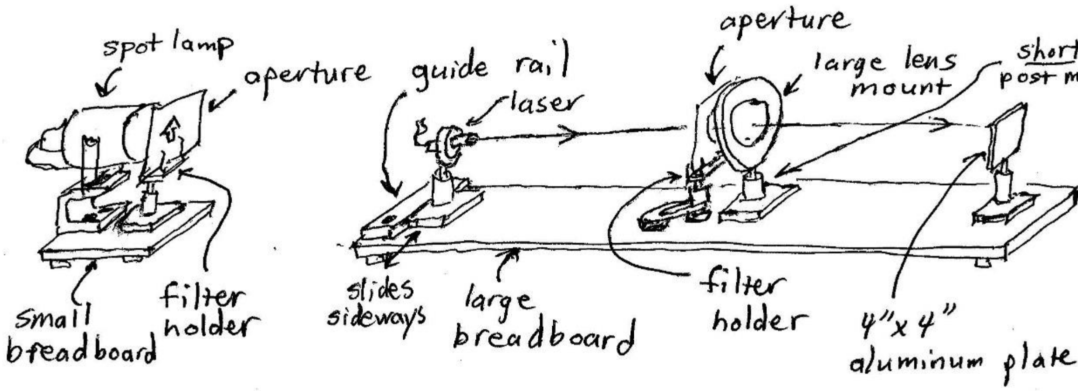

Figure 7.6. Experimental arrangement.

You are also provided with a spot lamp, which you'll use as an alternative light source. This should be mounted onto the small breadboard as shown in Figure 7.6. Finally, you have two filter holders in mounts. These will be used to hold the aluminum and cardboard aperture plates that are provided. One should be mounted close to the spotlamp, and the other should be mounted close to the lens. However, **make sure you have a gap of at least 1.5 cm between the aperture plate and the spotlamp.** This gap is necessary to provide adequate air cooling of the lamp and aperture. Without this gap, you might overheat the cardboard or the lamp. Also, mount the aperture for the lens as close as possible to the lens. Two steps you can take to do this are to (i) orient the baseplate with its wide axis perpendicular to the optical axis, and (ii) use the clamp-type post holder on the filter holder. With this orientation, you won't be able to bolt the lens baseplate into the breadboard (with the lens centered in the plate). However we are also providing a clamp that you can use to secure the plate to the board.

For this experiment, you must again follow the safety rules for using the laser given in the instructions for Experiment 3. Since you will not be spraying beams off to the side, you do not have to use a long beam block in front of the experiment, although you can if you want to. However **you must block all laser beams from leaving the work area**. To do this, you will need smaller beam blocks on the ends of the experiment.

Focal length of a plano-convex lens

Set the laser mount onto the breadboard up against the guide rail, and turn on the laser. Do not bolt the mount to the breadboard. You should be able to slide the mount sideways. Put the screen (4" x 4" plate with paper) at the other end of the breadboard. You do not need to bolt the screen down either. Position the laser and screen so that the laser beam travels straight down the center of the board, about 6" above the breadboard, and strikes the center of the screen.

Put the large plano-convex lens into its mount near the center of the breadboard, with the curved side facing the laser. (The curved side is the one that is deeper in the mount, also the side with the retaining ring.) Adjust the position of the lens so that the laser beam passes through the center of the lens. The laser beam should hit the screen at about the same place as before.

Measurement: Determine the focal length of your lens. To do this, translate the laser back and forth laterally, and observe the movement of the laser spot on the screen. Do this for a screen position close to the lens, and another screen position far from the lens. You should find that the spot moves in the same direction as the laser for one of the screen positions, and in the opposite direction as the laser for the other screen position. This implies there is some screen position where the spot does not move. (Refer to Figure 7.3(a).) Find this position, and use it to determine the lens focal length.

Observations: Check whether the following lens properties are true:

If the lens is tilted at a small angle, the focal spot position and focal length are the same. (You can just rotate the lens in its mount a little bit, about a vertical axis of rotation.)

If the plano side of the lens faces the laser, the focal length is the same.

Image formation by a lens

Next, turn the laser off, set it aside, and position the spot lamp about 1 meter away from the lens. Put the arrow aperture in front of the lamp, and turn the lamp on. The light emerging from the arrow aperture will now serve as your "object." Tape the 50 mm aperture to the lens, so that the aperture is centered on the lens. (This is done to minimize the effect of aberrations on the following measurements.) Move the screen until you can see a focused image of the arrow. The ray optics of this image formation is illustrated in Figure 7.5.

Question: Is the image erect (oriented the same way as the object) or inverted? Explain why.

Question: Is the image larger than or smaller than the object? Explain why.

Observation: Move the object to lens distance, keeping the image in focus on the screen as you do this. (You may want to set the small breadboard onto the large one during this procedure, and possibly to raise the lens when you do this.) What happens to the magnification as you do this? Can you make the magnitude of the magnification smaller than 1? If so, what is the condition to make it smaller than 1? Can you make the magnitude of the magnification larger than 1? If so, what is the condition to make it larger than 1? Also, what happens to the lens to image distance s' as you vary the object to lens distance s?

Measurement: Check that the thin lens equation is correct. To do this, measure s, s', and the magnification M for three or four different object-lens distances s. Check how well your measurements agree with the thin lens equation. Also check whether your observed magnifications are equal to the theoretical values $|M| = |s'/s|$.

Virtual images

So far, we have studied only *real images*, which are images that can appear on a screen. *Virtual images* are images that cannot appear on a screen, but can appear to another optical instrument (such as your eye) peering into the optical system. A single negative lens will produce *only* virtual images. As you've seen, a single positive lens will produce real images for object-to-lens distances $s > f$. For $s < f$, a positive lens will produce only a virtual image.

Observation. Turn the laser off. Pick up the large positive lens mount in your hand, and look through it at some object. Verify that you can see a virtual image for $s < f$. Is the virtual image larger or smaller than the image you see without the lens?

Next, look through the negative lens at the same object. Is there any value of s for which you cannot see a virtual image? Is the virtual image larger or smaller than the image you see without the lens?

Be sure to explain these observations in your report.

Verification of the lens maker's equation

In this part of the lab, we will measure the curvature of the convex surface of the plano-convex lens, and use that to check the lens maker's equation. To do this, we'll use a spherometer, as depicted in Figure 7.7. In order to use the spherometer, you place it on the lens and turn the middle screw until the central point is just resting on the lens surface. Then you place the spherometer on a flat surface, and turn the screw until it contacts the flat surface, being careful to keep track of how many turns this takes, including fractional turns. Knowing the number of turns, the pitch of the screw, and some basic geometry, you can then determine the radius of curvature R. Additional instructions for how to use the spherometer are posted on the wall of the lab near the equipment storage room, and may be helpful if you don't quickly see how to work out the geometry.

Measurement: Use the spherometer to find the radius of curvature R of the plano-convex lens surface. Use your measured value of R to check whether your measure R and f are consistent with the lens maker's equation (7.1). Your lens is made of N-BK7 optical glass, which has an index of refraction $n = 1.51$ at the laser wavelength of 635 nm.

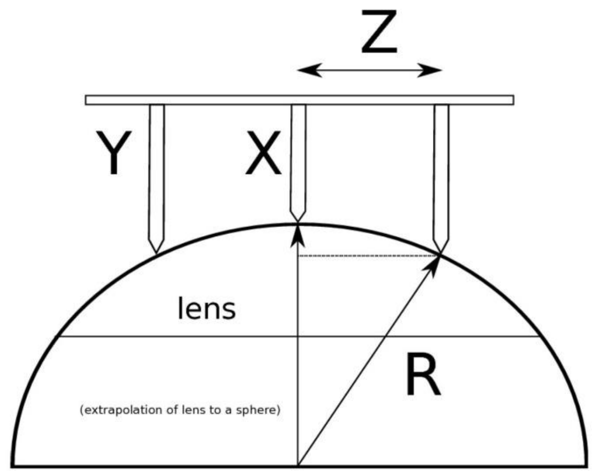

Figure 7.7. A spherometer can be used to find the radius of curvature of a lens surface, using a little geometry.

Lens aberrations: chromatic aberration, spherical aberration, and coma.

It is physically impossible for a lens system to produce an absolutely perfect image of an object. Image quality is often limited in practice by lens aberrations. The lens aberrations we'll study today are effects that cause rays from an object point to fail to arrive at exactly the same image point. In highly optimized lens systems (often very expensive), it is possible to eliminate the effects of lens aberration for all practical purposes. However, even in this case, image quality is limited by diffraction effects. We'll study diffraction later in this course.

Aberration in optical systems is a tremendously complex topic, and true lens design experts spend an entire career learning how to minimize them. There are three aberrations that tend to be the most important, and are also observable in this experiment.

The first of these is *chromatic aberration.* This refers to the tendency of a lens to have a different focal length for different wavelengths. This happens because the index of refraction of glass depends a little on wavelength, as we observed in Experiment 6. Chromatic aberration is not very prominent in this experiment. However, if you look very carefully, you may notice that there is a bit of rainbow coloration near the edges of the spots in the following part of the experiment. This is a small effect of chromatic aberration. Watch for this effect, and if you see it, note the conditions where you saw it in your report.

The other two aberrations we'll look for today are so-called "third-order" aberrations. The term "third-order" comes from the fact that the power series expansion of a sine function is $\sin\theta = \theta - \frac{\theta^3}{3!} + \cdots$ As far as ray optics is concerned, imaging would be perfect if only the first term, θ, were important (*i.e.* if the paraxial approximation were essentially exact). "Third-order" aberrations are ones that arise from the $-\frac{\theta^3}{3!}$ terms in the series expansion for $\sin\theta$.

They are also called "Seidel aberrations", because the German Mathematician Phillip L. von Seidel showed, in 1857, that the third order aberrations of a lens can be decomposed into five constituent aberrations: spherical aberration, coma, astigmatism, field curvature, and distortion. We'll study the first two of these today.

To see these aberrations, remove the 50 mm aperture from the positive lens. Set the experiment back up with the spotlamp, positive lens, and screen. Put the 2 mm diameter aluminum aperture in front of the spot lamp. This will make something approximating a "point" source of light. Place the source of light about 1 m away from the lens.

Next, study the image of the 2 mm aperture on the screen. If there were no aberrations, you would see a clean image of the 2 mm hole, which would be about 2 mm$\times$0.3 ~ 0.7 mm in diameter. But in practice, you should see that no matter where you place the screen, the diameter of the spot image is larger than this.

The spot is larger than the image of the 2 mm aperture due to the effects of *spherical aberration*, illustrated in Figure 7.8. What happens is that the rays passing through the center of the lens come to a focus further away from the lens than rays passing through the edges of the lens. As a result, the bundle of rays near the focus always has a non-zero diameter. The minimum diameter of the bundle is called the *circle of least confusion*, and corresponds approximately to the minimum spot diameter that you observe in the experiment.

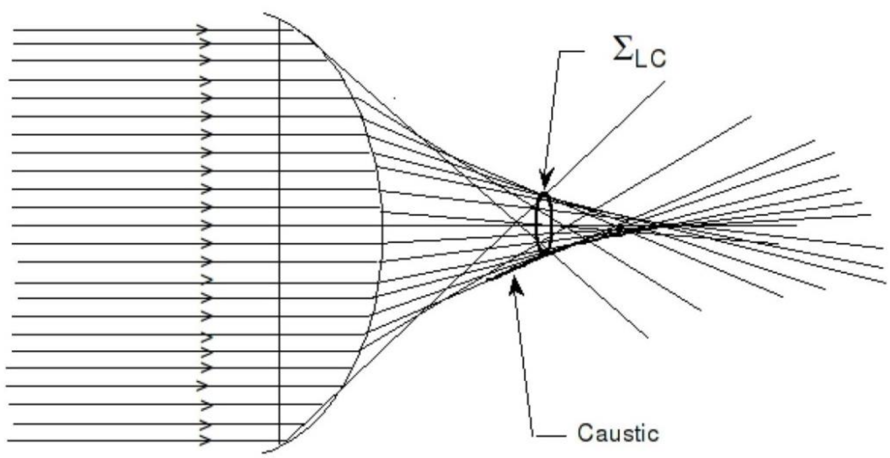

Figure 7.8. Focusing of light rays by a plano-convex lens, illustrating the origin of spherical aberration. The "circle of least confusion" Σ_{LC} is the circle that encompasses the minimum diameter bundle of rays.

When you look for the minimum diameter spot, you may see what appears to be a very small spot near the focal point of the central rays (the *paraxial focus*). But if you look carefully at this spot, you should also see a faint, larger diameter circle of light. To see this, it may help to turn the room lights out. *For purposes of this lab, spot "diameter" should be taken as the diameter of this larger circle – i.e. it should enclose* <u>*all*</u> *light rays.* The circle of least confusion has the smallest such diameter, and lies closer to the lens than the paraxial focus, as shown in Figure 7.8.

Once you are used to looking at these spots, replace the 2 mm aperture with the 1 mm aperture. The 2 mm aperture has the advantage of giving more light, which makes the larger diameter circle easier to see. But the 1 mm aperture has the advantage of looking more like a point source, *i.e.* the "geometric" size of its image is smaller - about 1/3 mm.

Once you have the 1 mm aperture in place, measure the minimum spot size for the following situations:

1. No aperture on lens, curved side of lens facing lamp.

2. No aperture on lens, flat side of lens facing lamp.

Question: Is the minimum spot size the same for these two lens orientations? If one is smaller than the other, which one is smaller? Also, if one is smaller than the other, see if you can give an explanation for that.

3. 70 mm aperture in front of lens, curved side of lens facing lamp.

4. 50 mm aperture in front of lens, curved side of lens facing lamp.

You should observe that the minimum spot size decreases with decreasing lens aperture size. Can you give an explanation for this? For the 50 mm aperture, the minimum spot size may be very close to the ~ 0.3 mm geometric size of the image.

Finally, leave the 50 mm aperture in place, and tilt the lens in its mount. You should observe something that likes like a comet, with a sharp "head" and a long "tail". This is *coma.*

Observation: At what angle of lens tilt does coma begin to appreciably affect the image of the 1 mm aperture?

- End of Experiment 7 -