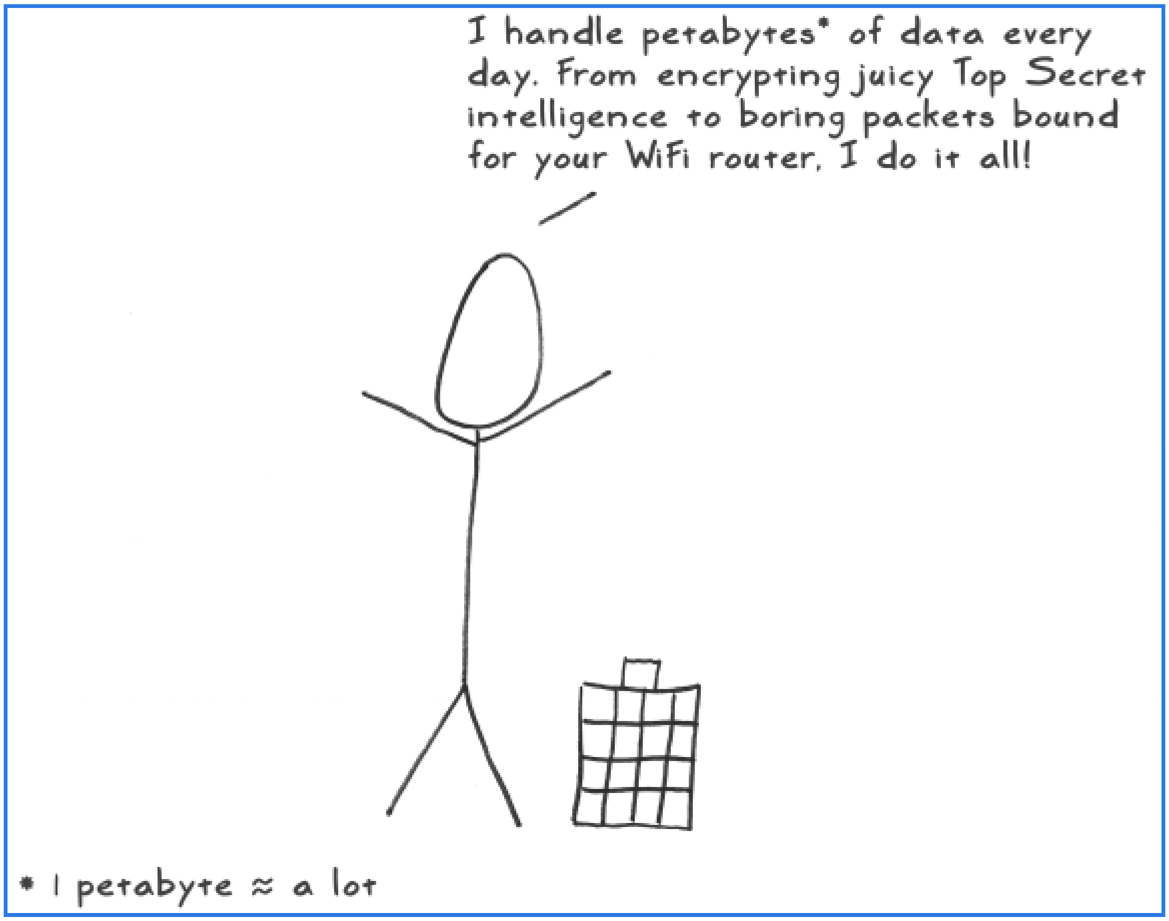
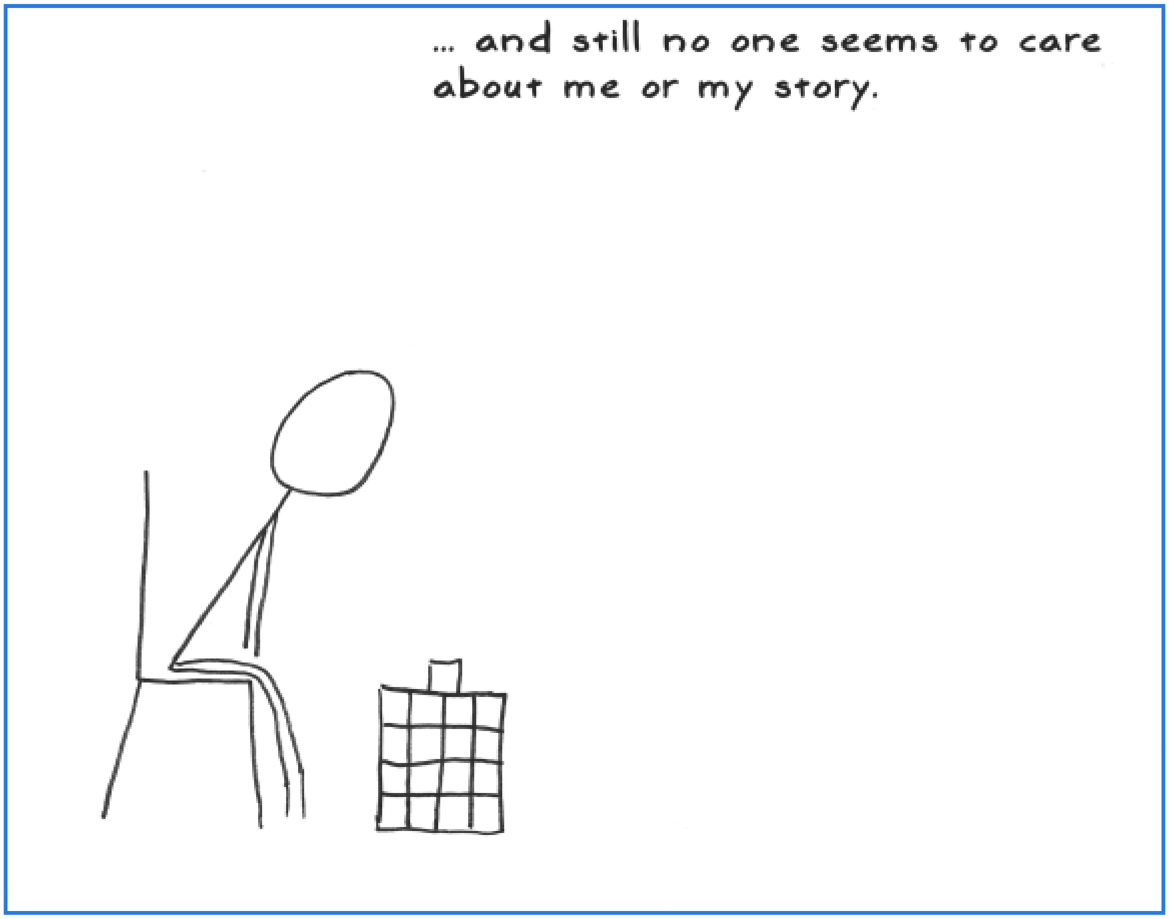
A Stick Figure Guide to the Advanced Encryption Standard (AES)

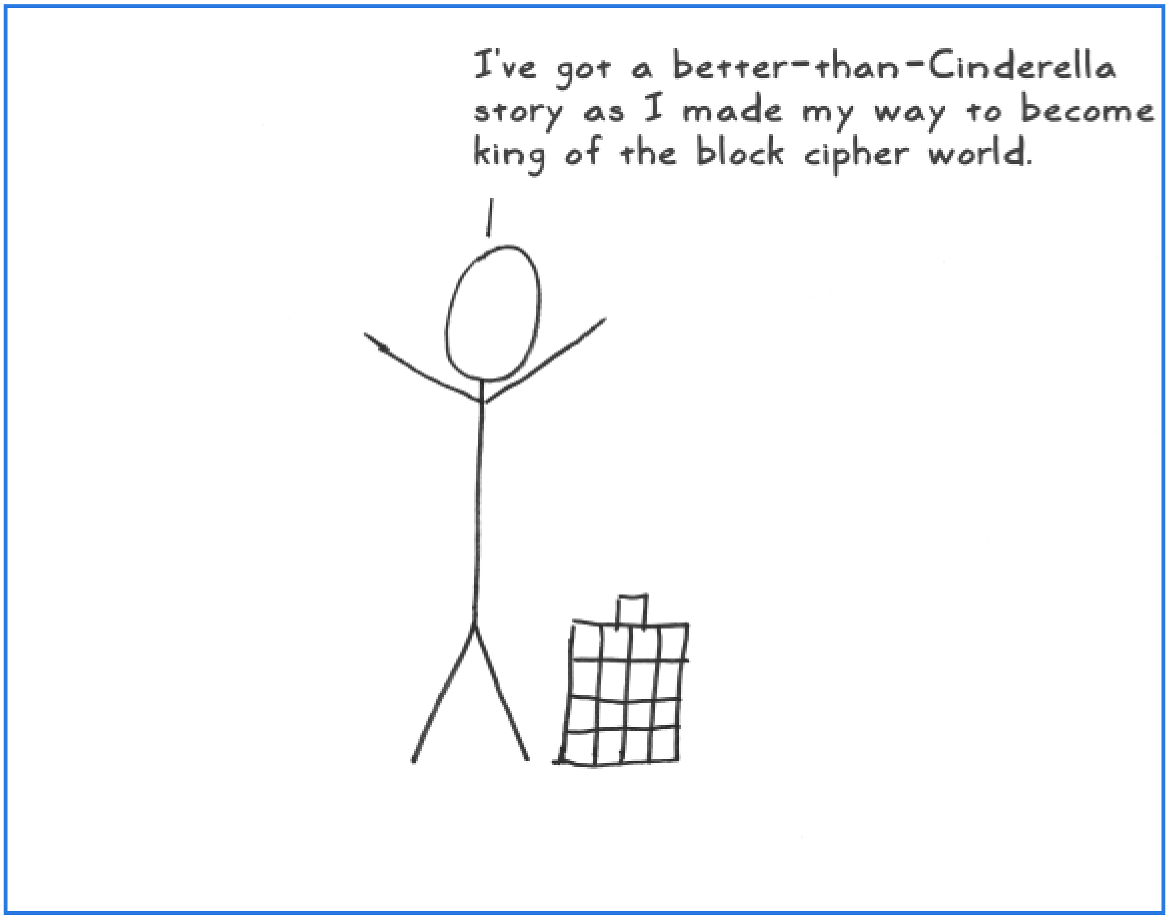
Sep 22, 2009

(A play in 4 acts. Please feel free to exit along with the stage character that best represents you. Take intermissions as you see fit. Click on the stage if you have a hard time seeing it. If you get bored, you can jump to the code. Most importantly, enjoy the show!)

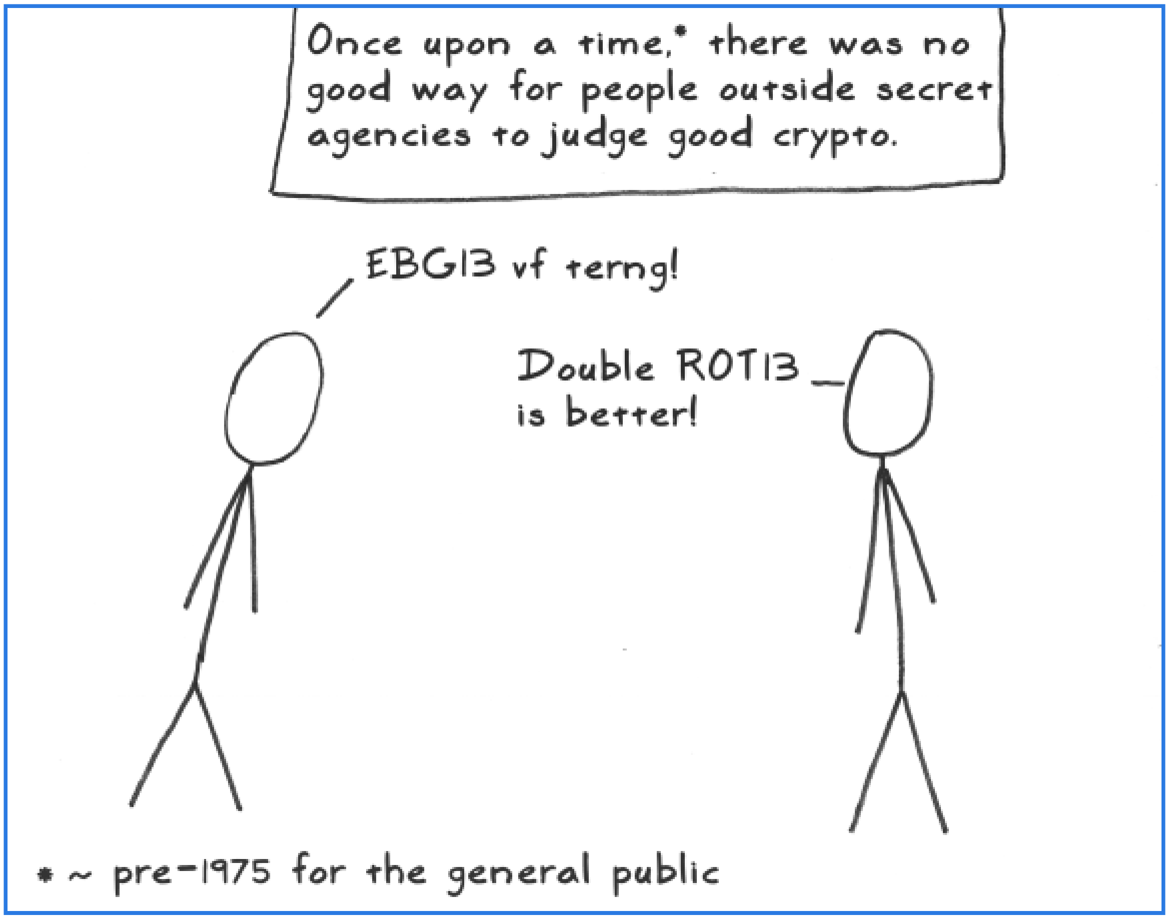
Act 1: Once Upon a Time…

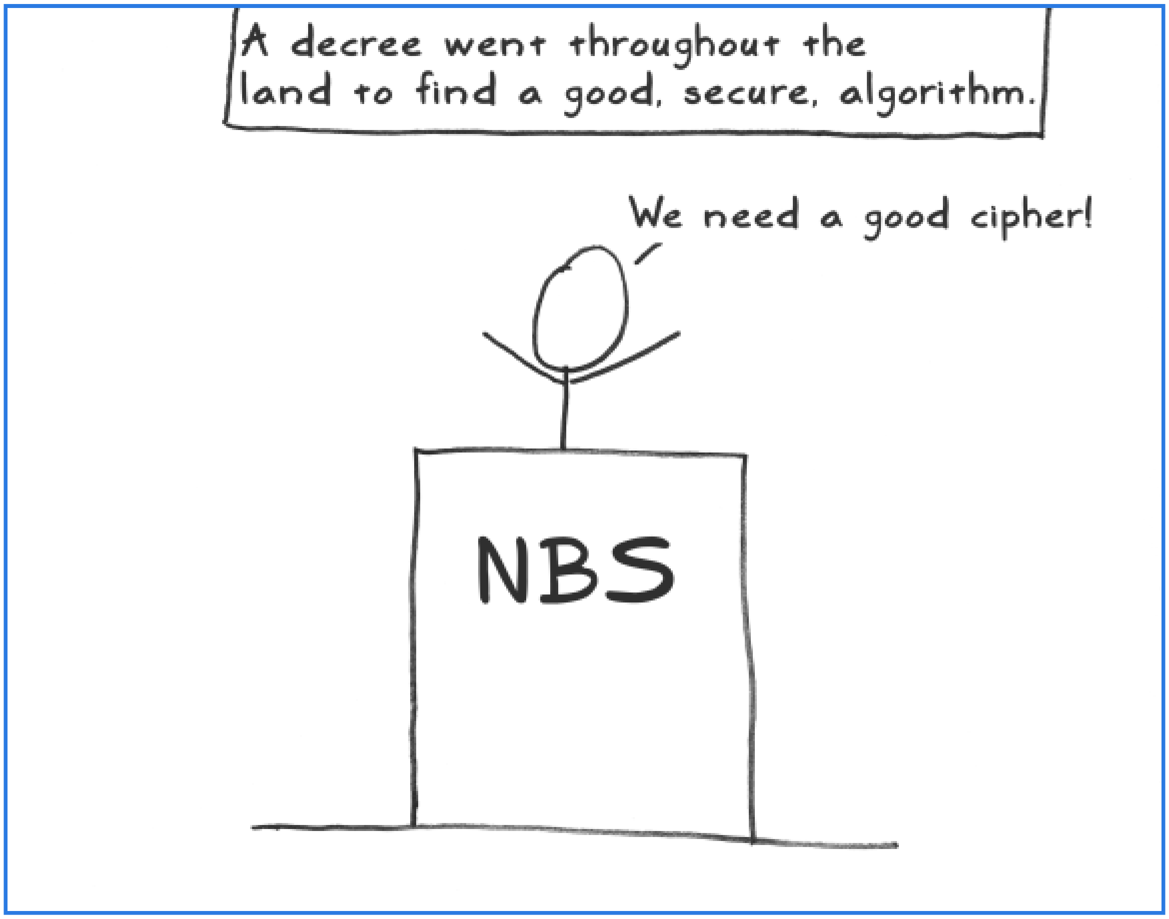

I handle petabytes* of data every day. From encrypting juicy Top Secret intelligence to boring packets bound for your WiFi router, I do it all!
* 1 petabyte ≈ a lot

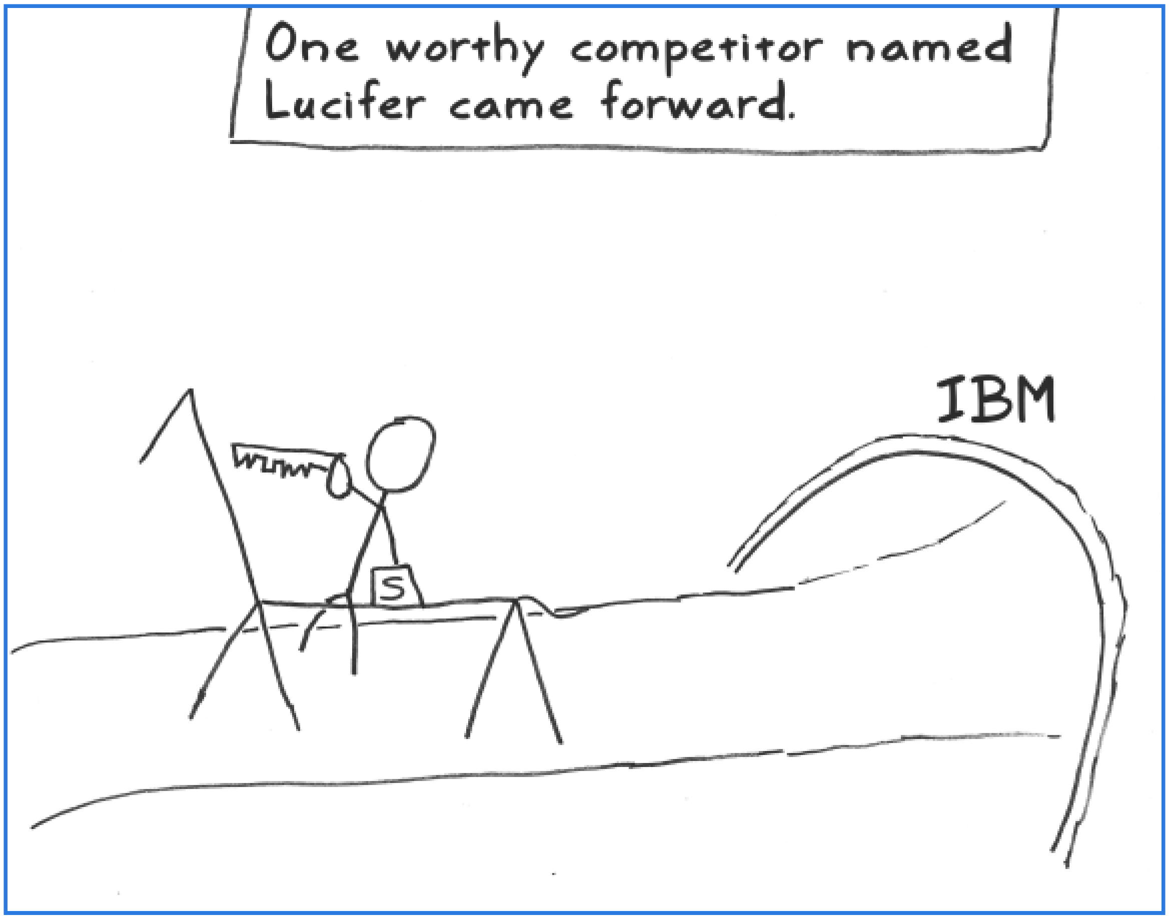

... and still no one seems to care about me or my story.

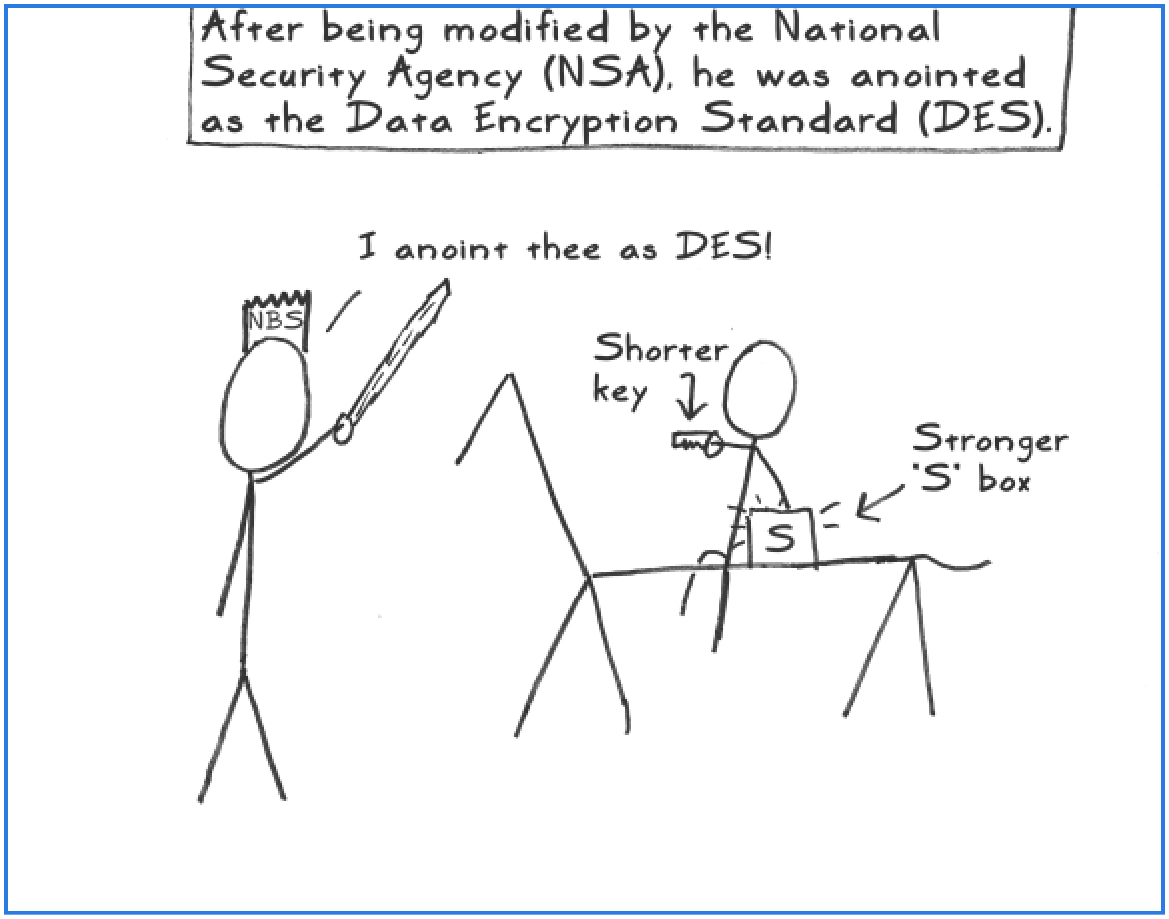

I've got a better-than-Cinderella story as I made my way to become king of the block cipher world.

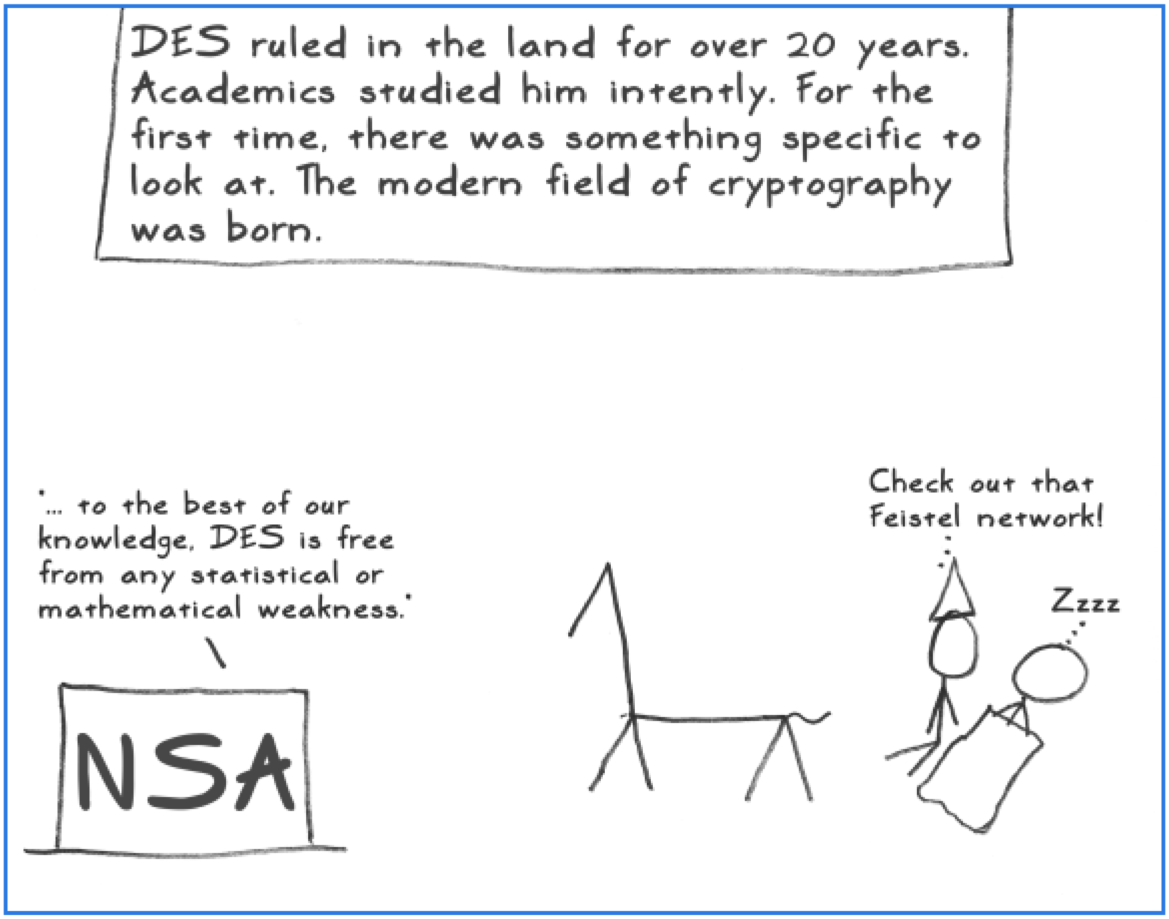
Whoa! You're still there. You want to hear it? Well let's get started...

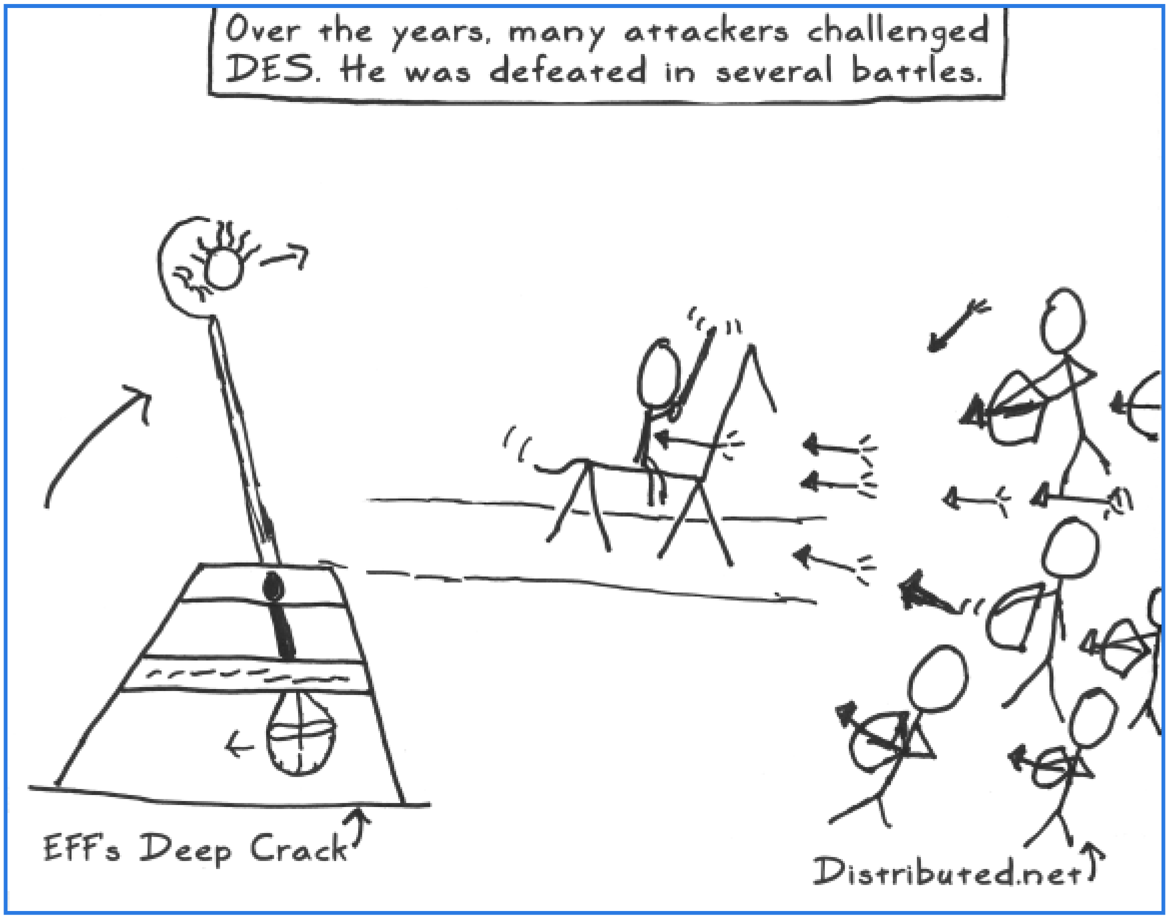

Once upon a time,* there was no good way for people outside secret agencies to judge good crypto.
EBG13 vf terng!
Double ROT13 is better!
* ~ pre-1975 for the general public

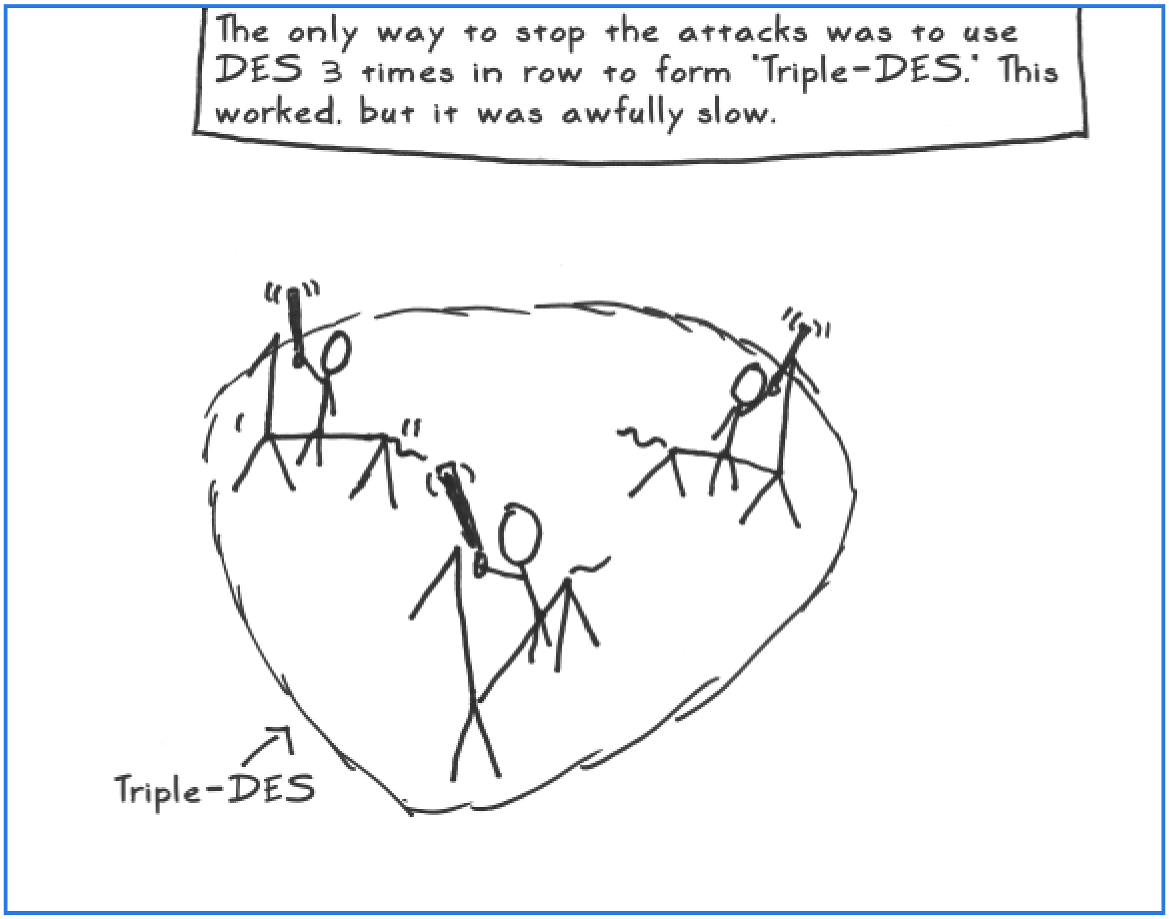

A decree went throughout the land to find a good, secure, algorithm.
We need a good cipher!
NBS

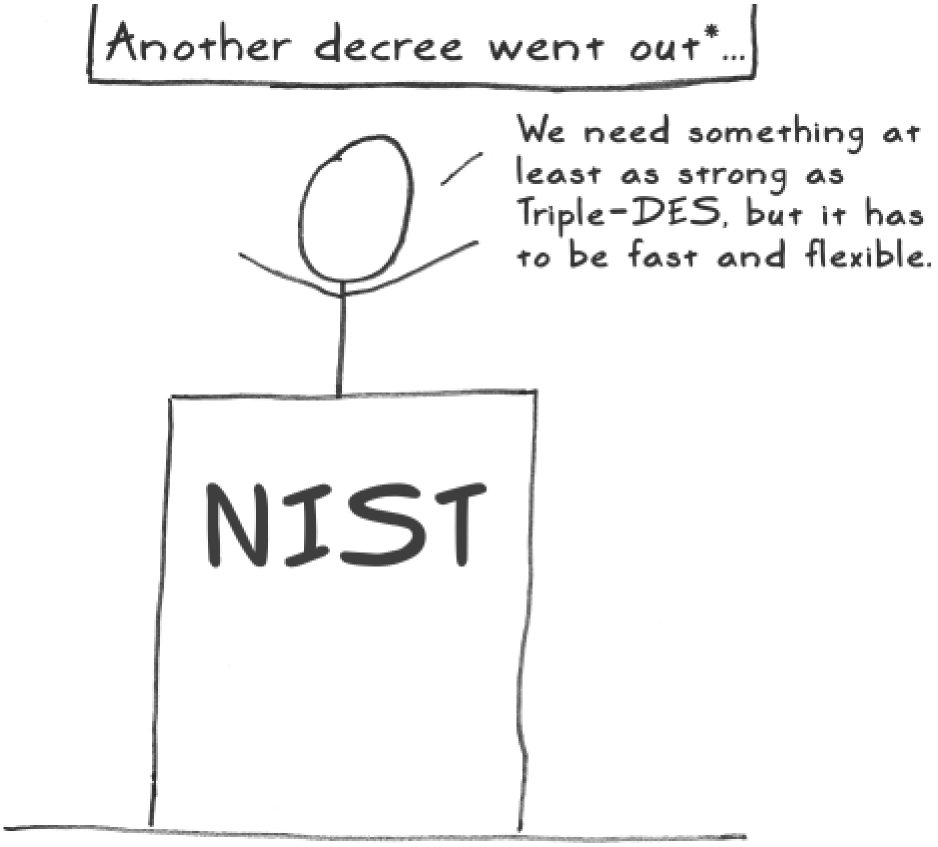

One worthy competitor named Lucifer came forward.
S
IBM


After being modified by the National Security Agency (NSA), he was anointed as the Data Encryption Standard (DES).
I anoint thee as DES!
NBS
Shorter key
Stronger 'S' box
S

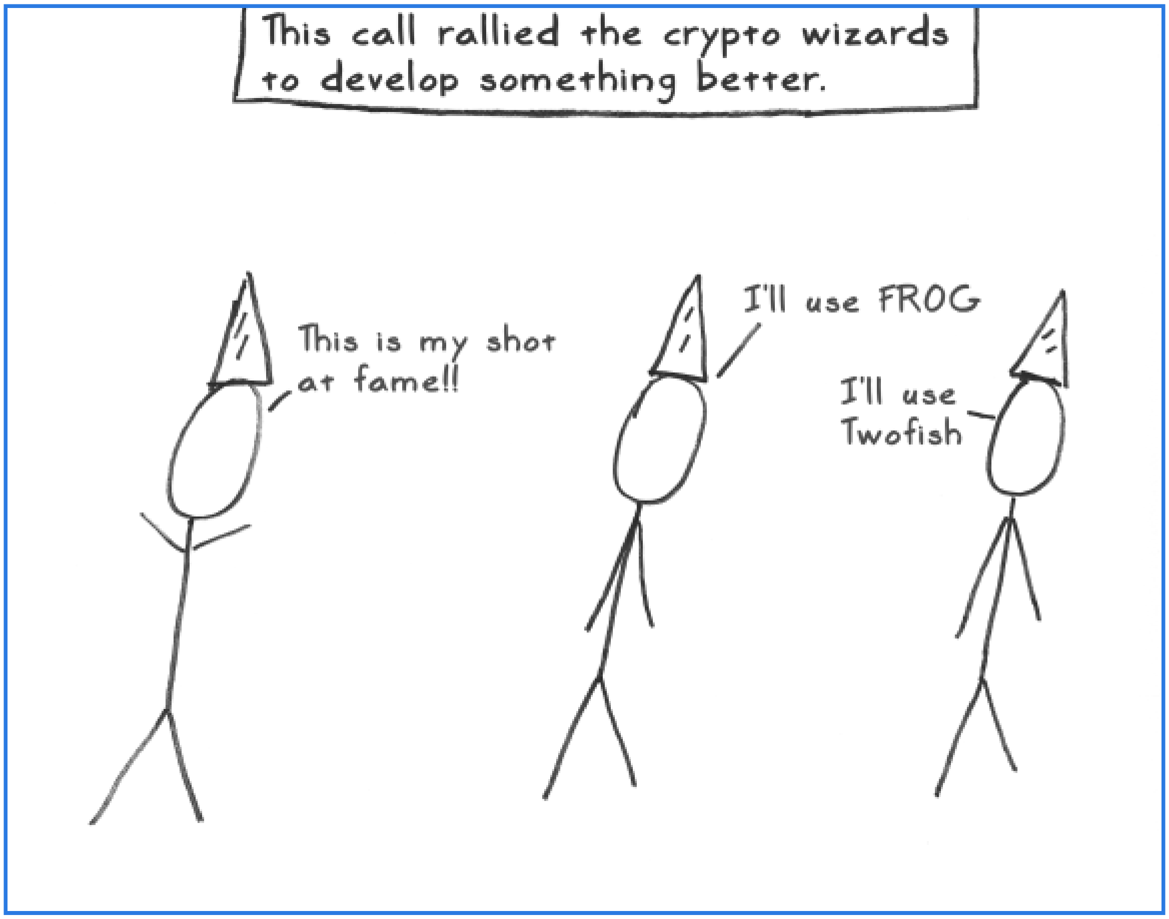

DES ruled in the land for over 20 years. Academics studied him intently. For the first time, there was something specific to look at. The modern field of cryptography was born.
"... to the best of our knowledge, DES is free from any statistical or mathematical weakness."
NSA
Check out that Feistel network!
Zzzz


Over the years, many attackers challenged DES. He was defeated in several battles.
EFF's Deep Crack
Distributed.net

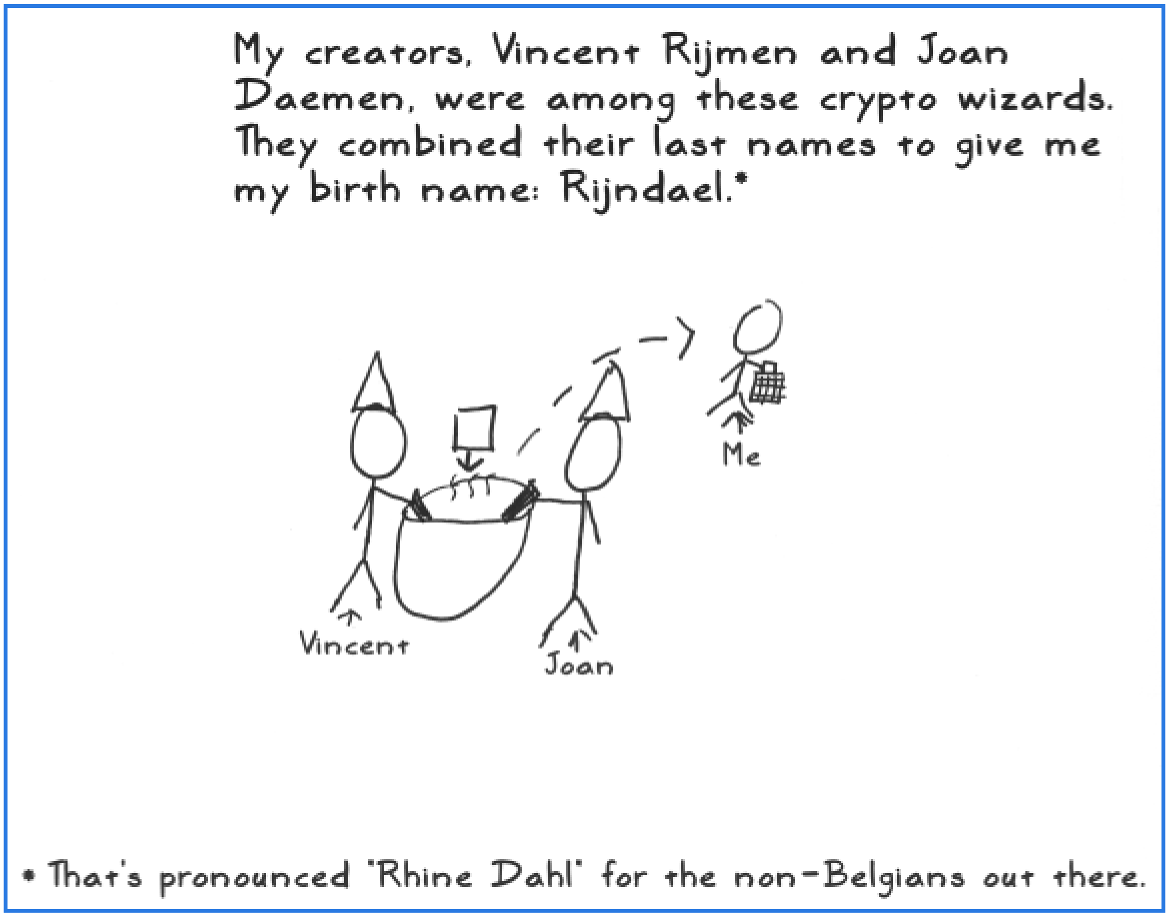

The only way to stop the attacks was to use DES 3 times in row to form "Triple-DES." This worked, but it was awfully slow.
Triple-DES


Another decree went out*...
We need something at least as strong as Triple-DES, but it has to be fast and flexible.
NIST

* ~early 1997

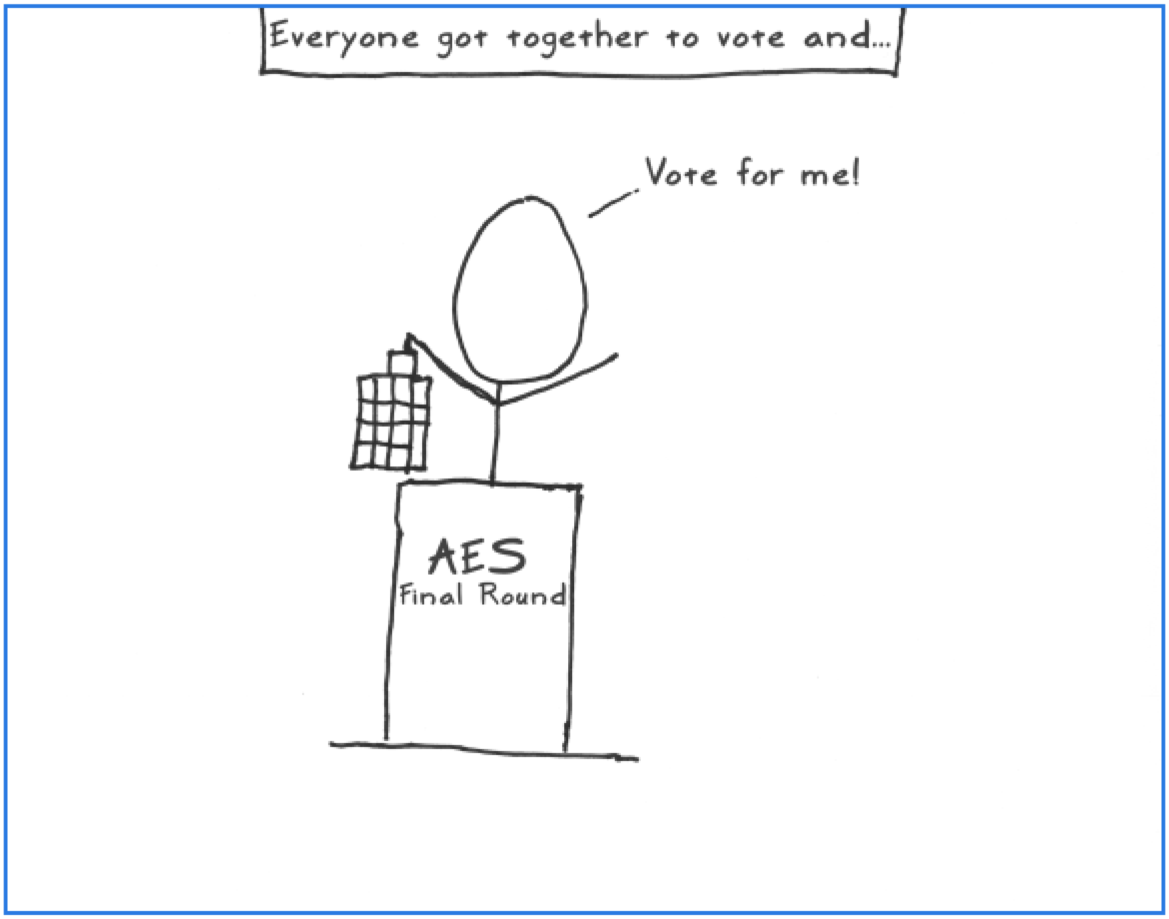


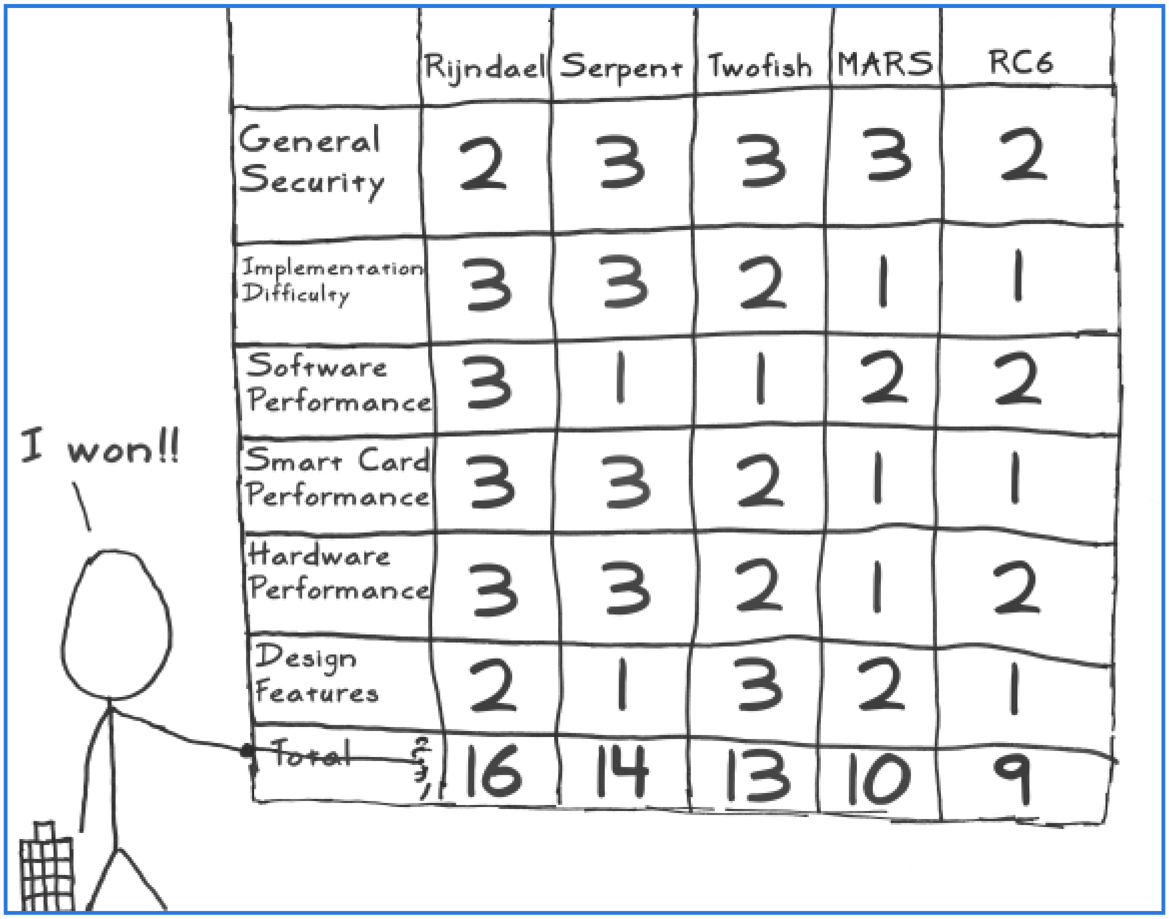
This call rallied the crypto wizards to develop something better.



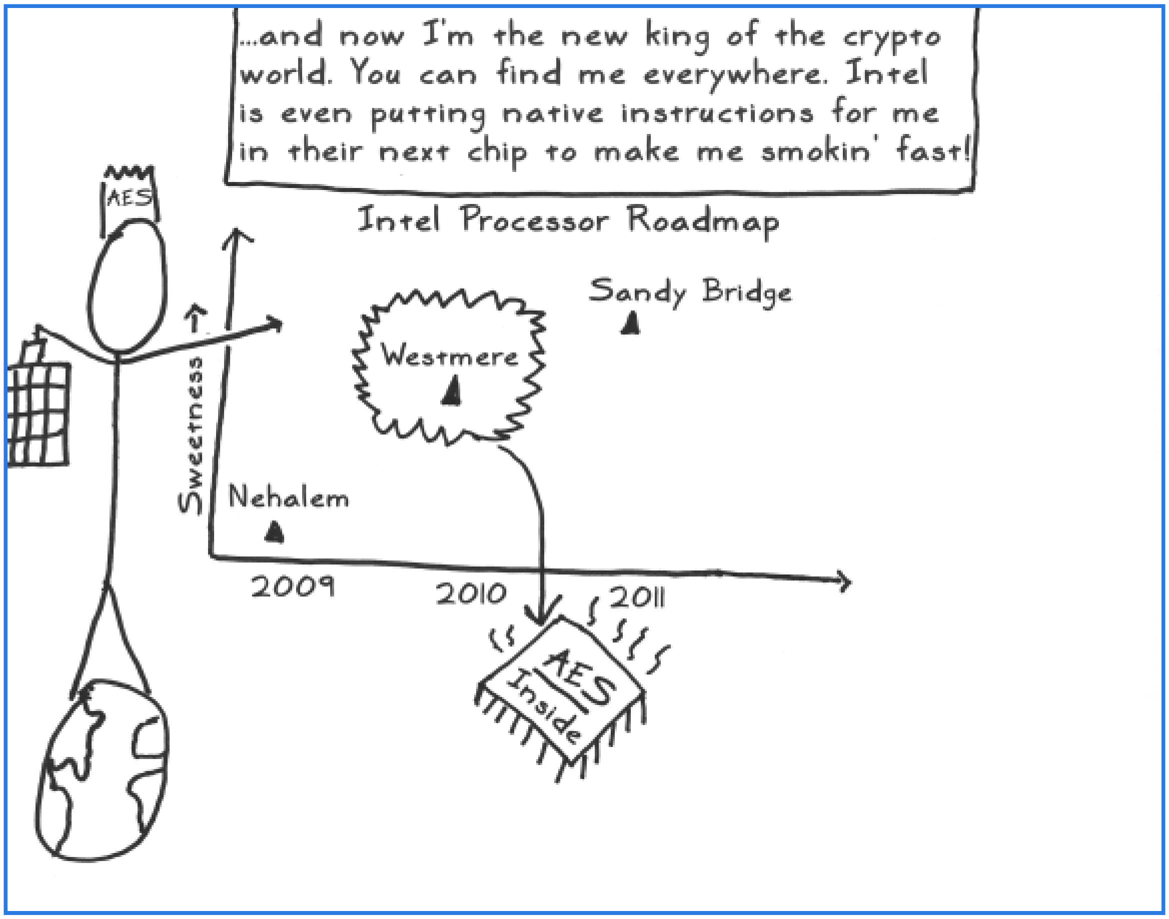
My creators, Vincent Rijmen and Joan Daemen, were among these crypto wizards. They combined their last names to give me my birth name: Rijndael.*

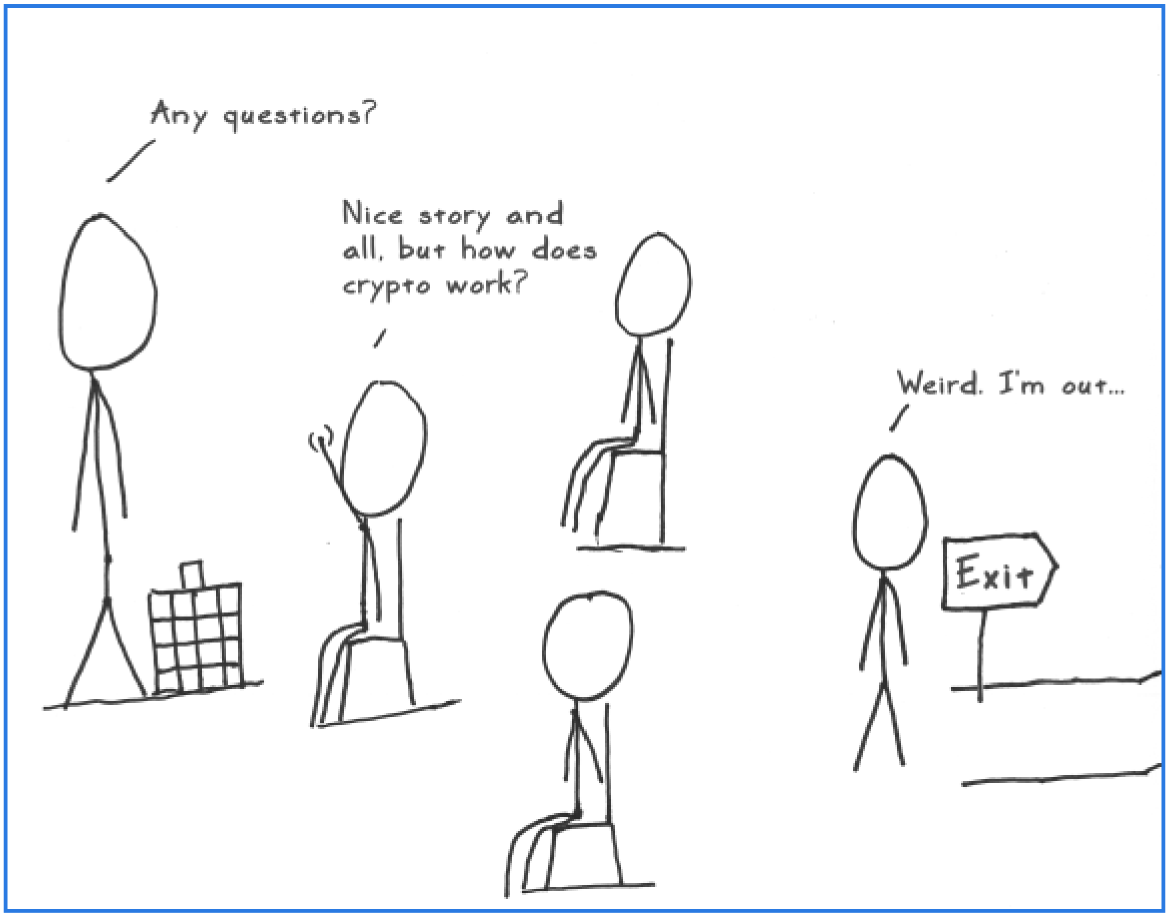
* That's pronounced "Rhine Dahl" for the non-Belgians out there.



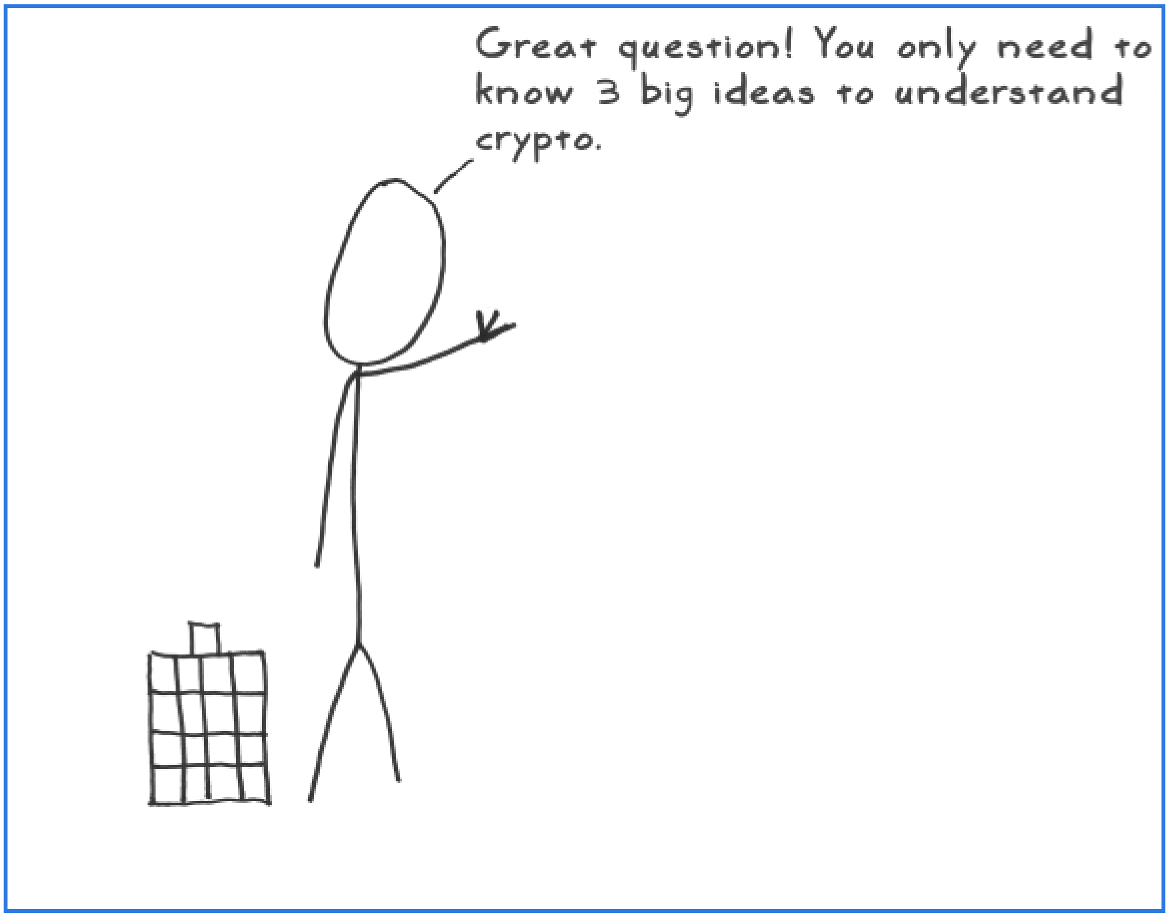


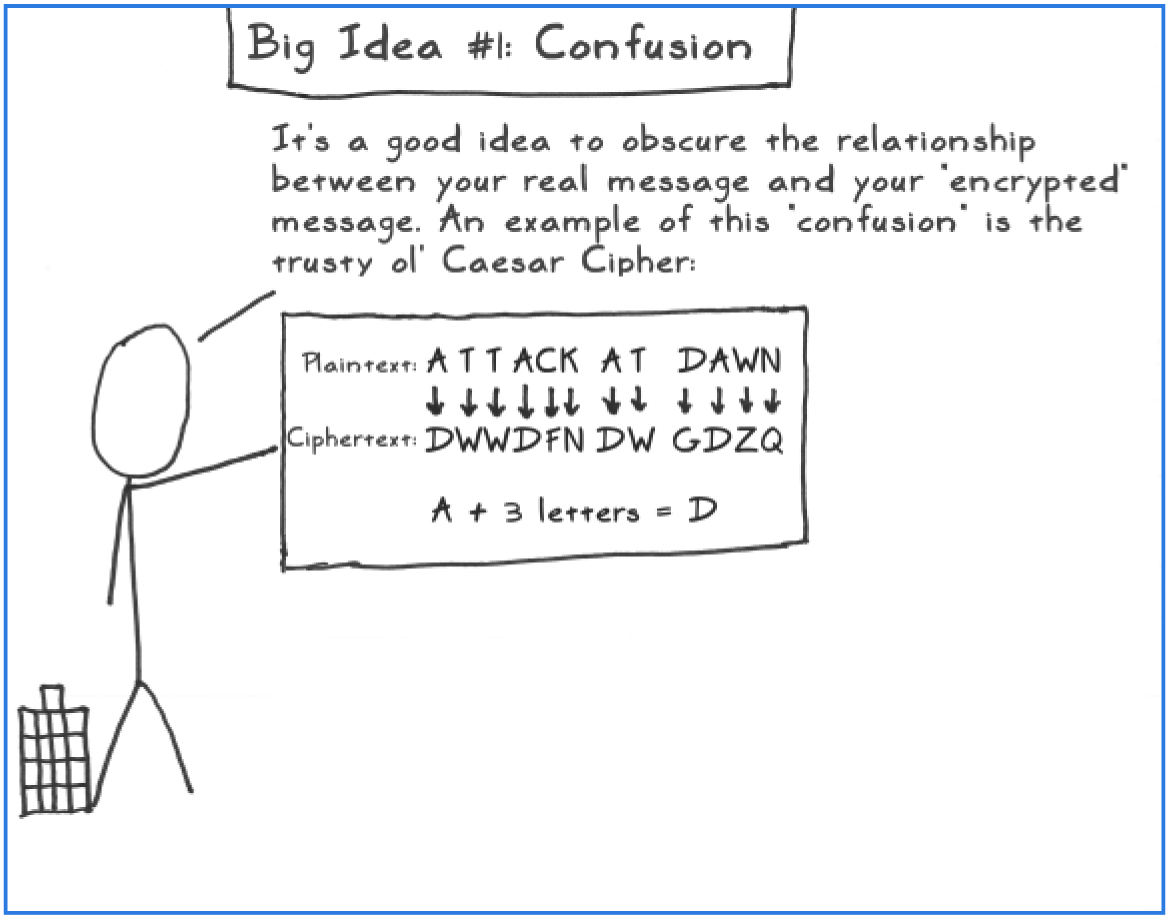
	Rijndael	Serpent	Twofish	MARS	RC6
General Security	2	3	3	3	2
Implementation Difficulty	3	3	2	1	1
Software Performance	3	1	1	2	2
Smart Card Performance	3	3	2	1	1
Hardware Performance	3	3	2	1	2
Design Features	2	1	3	2	1
~~Total~~	16	14	13	10	9

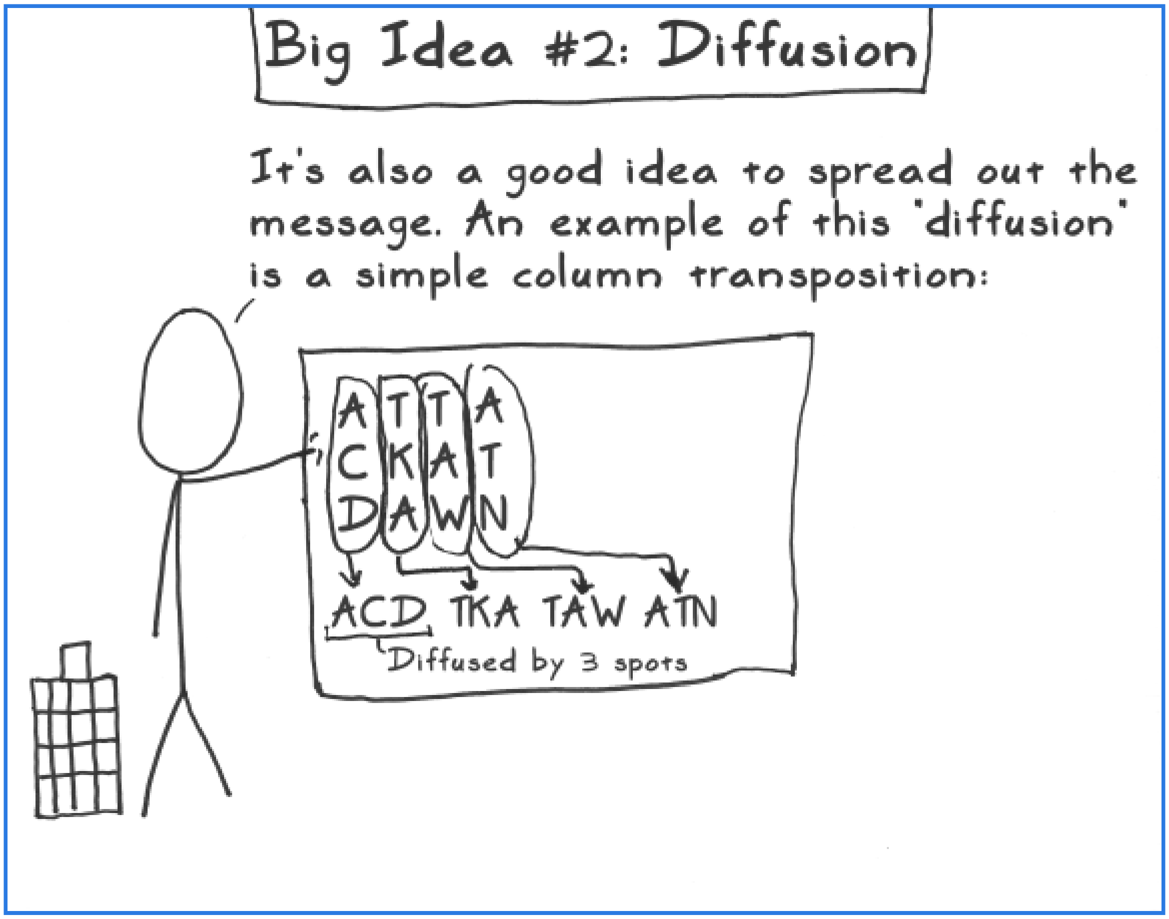

...and now I'm the new king of the crypto world. You can find me everywhere. Intel is even putting native instructions for me in their next chip to make me smokin' fast!
AES
Intel Processor Roadmap
Sweetness
Nehalem
Westmere
Sandy Bridge
2009
2010
2011
AES
Inside

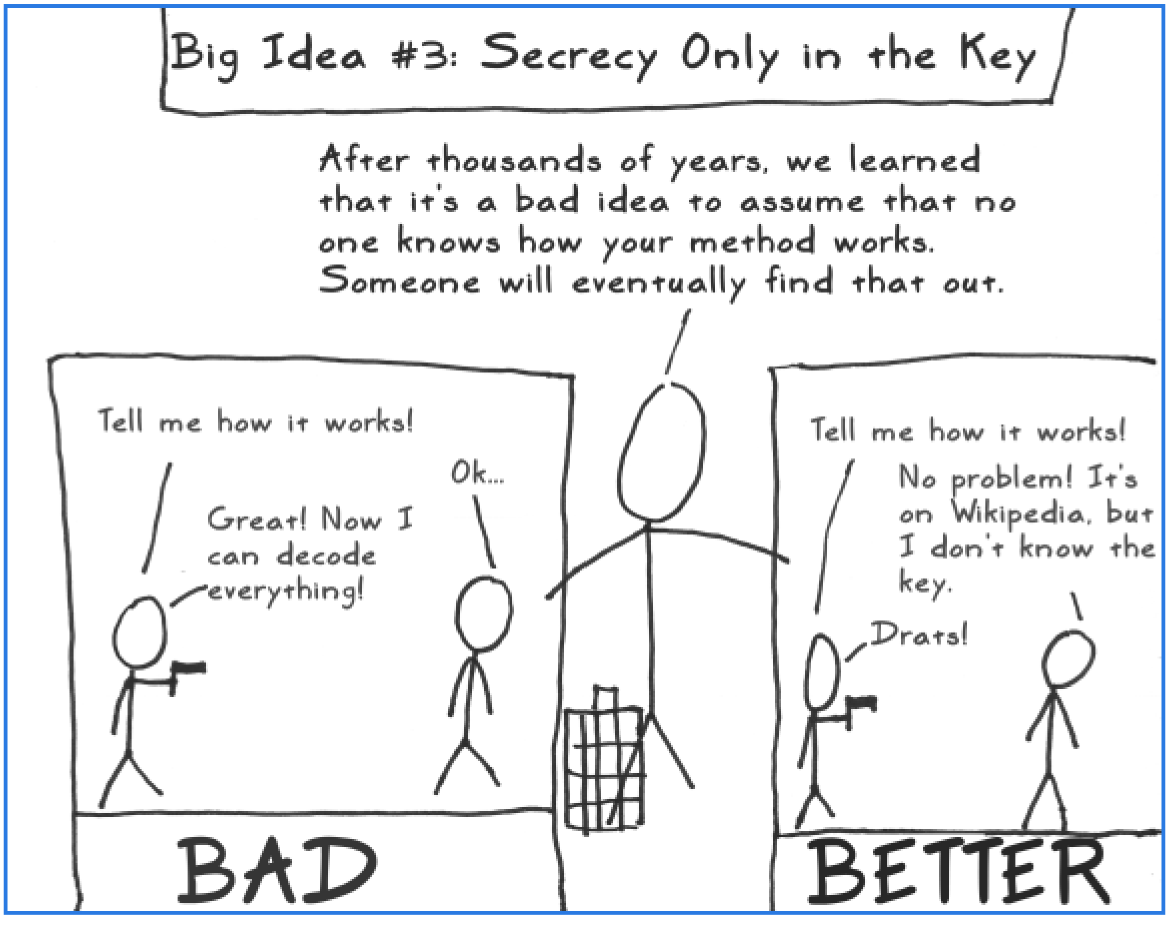

Any questions?
Nice story and all, but how does crypto work?
Weird. I'm out...
Exit

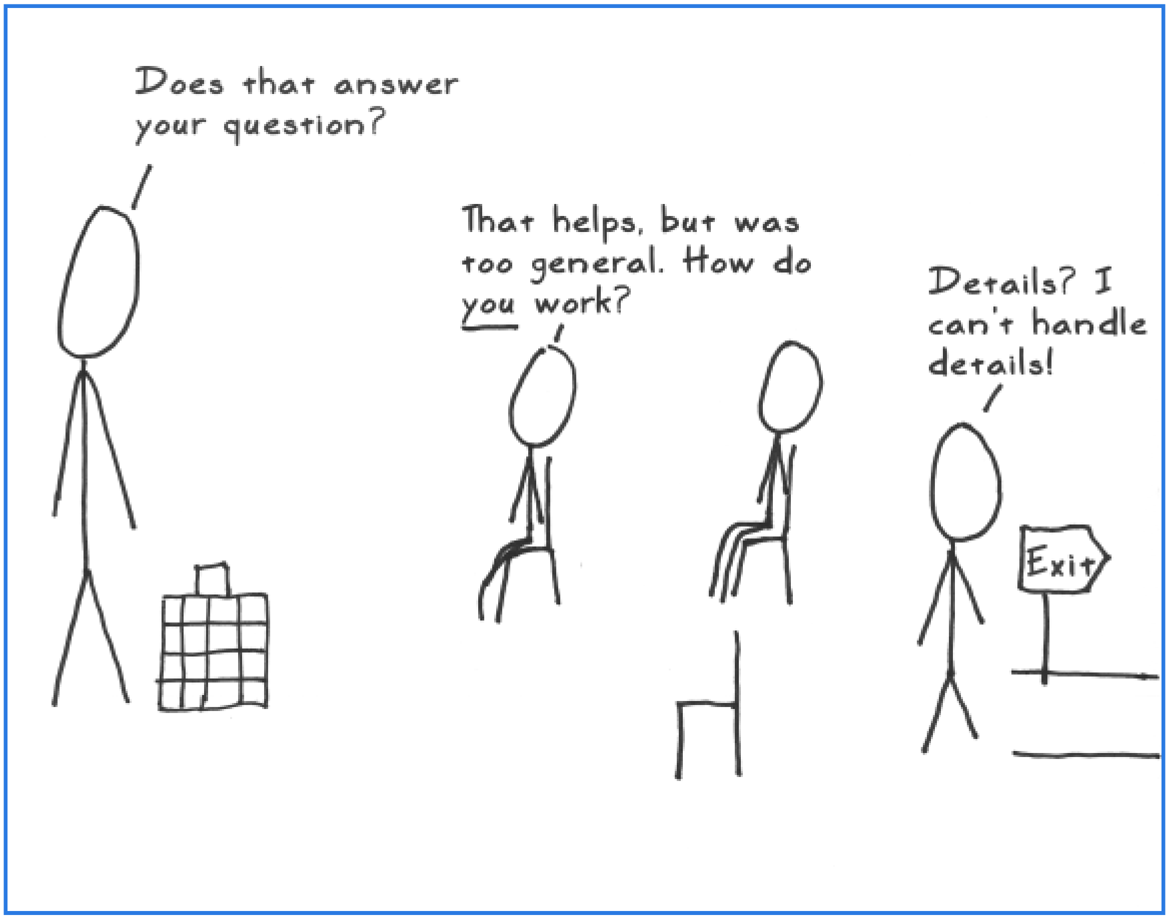
Act 2: Crypto Basics


Great question! You only need to know 3 big ideas to understand crypto.

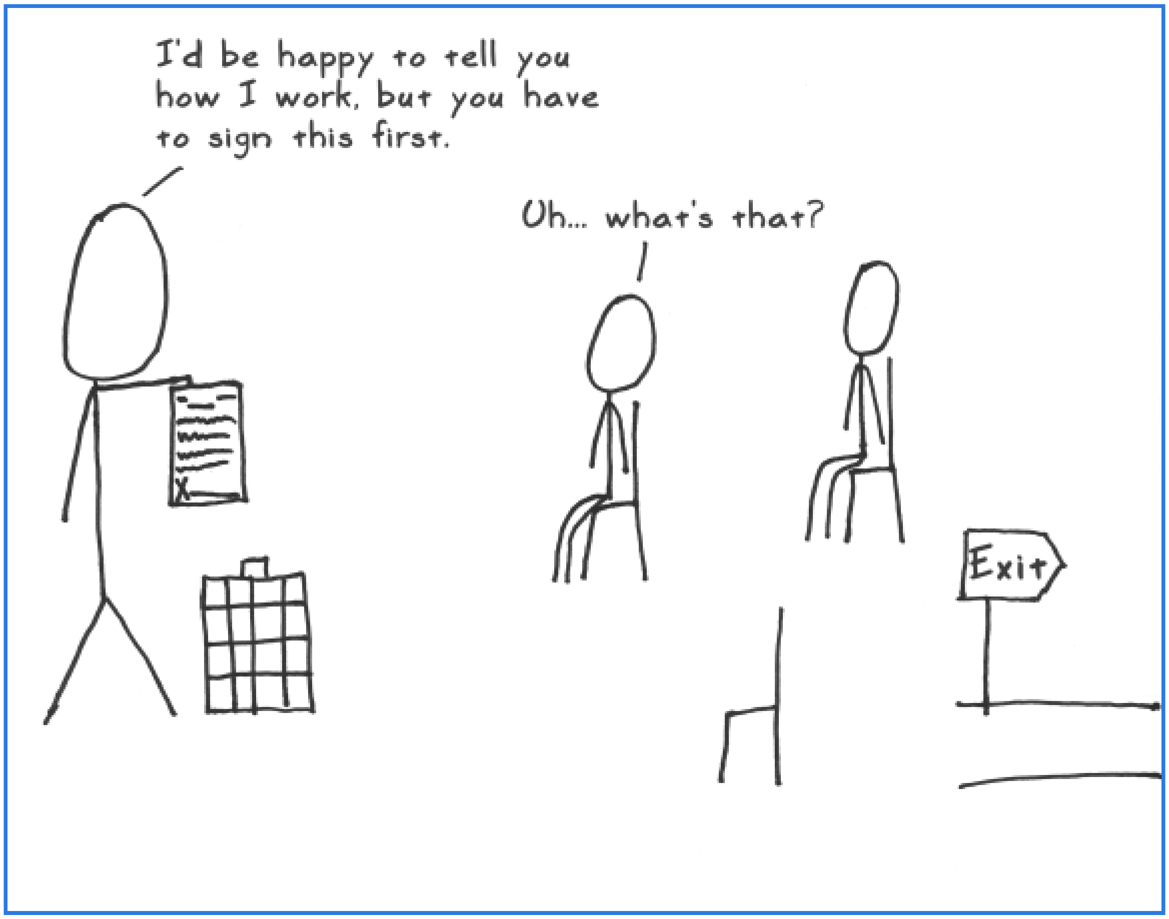

Big Idea #1: Confusion
It's a good idea to obscure the relationship between your real message and your "encrypted" message. An example of this "confusion" is the trusty ol' Caesar Cipher:
Plaintext: ATTACK AT DAWN
↓↓↓↓↓↓ ↓↓ ↓↓↓↓
Ciphertext: DWWDFN DW GDZQ
A + 3 letters = D

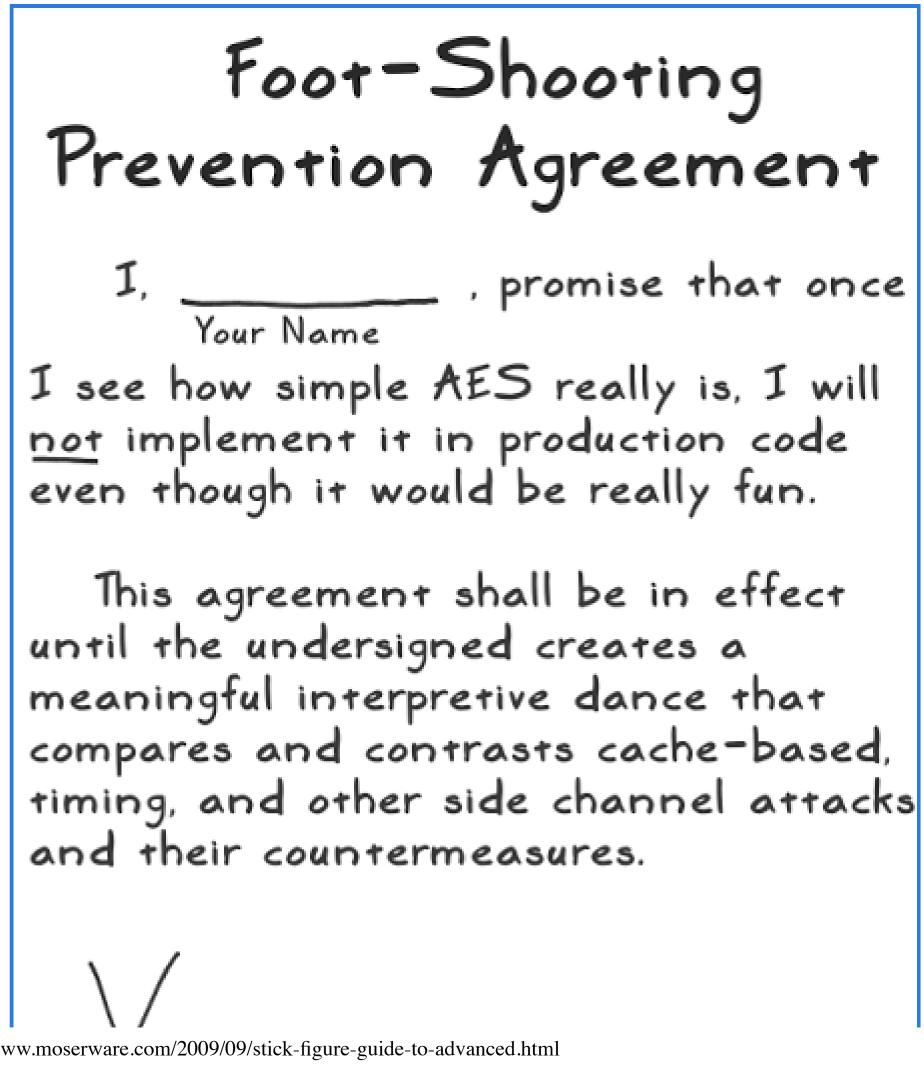

Big Idea #2: Diffusion
It's also a good idea to spread out the message. An example of this "diffusion" is a simple column transposition:
A T T A
C K A T
D A W N
ACD TKA TAW ATN
Diffused by 3 spots


Big Idea #3: Secrecy Only in the Key
After thousands of years, we learned that it's a bad idea to assume that no one knows how your method works. Someone will eventually find that out.
Tell me how it works!
Ok...
Great! Now I can decode everything!
BAD
Tell me how it works!
No problem! It's on Wikipedia, but I don't know the key.
Drats!
BETTER


Does that answer your question?
That helps, but was too general. How do you work?
Details? I can't handle details!
Exit

Act 3: Details

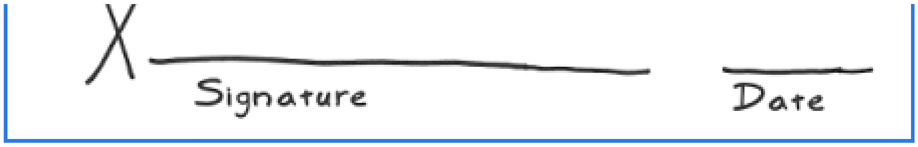

I'd be happy to tell you how I work, but you have to sign this first.
Oh... what's that?
Exit

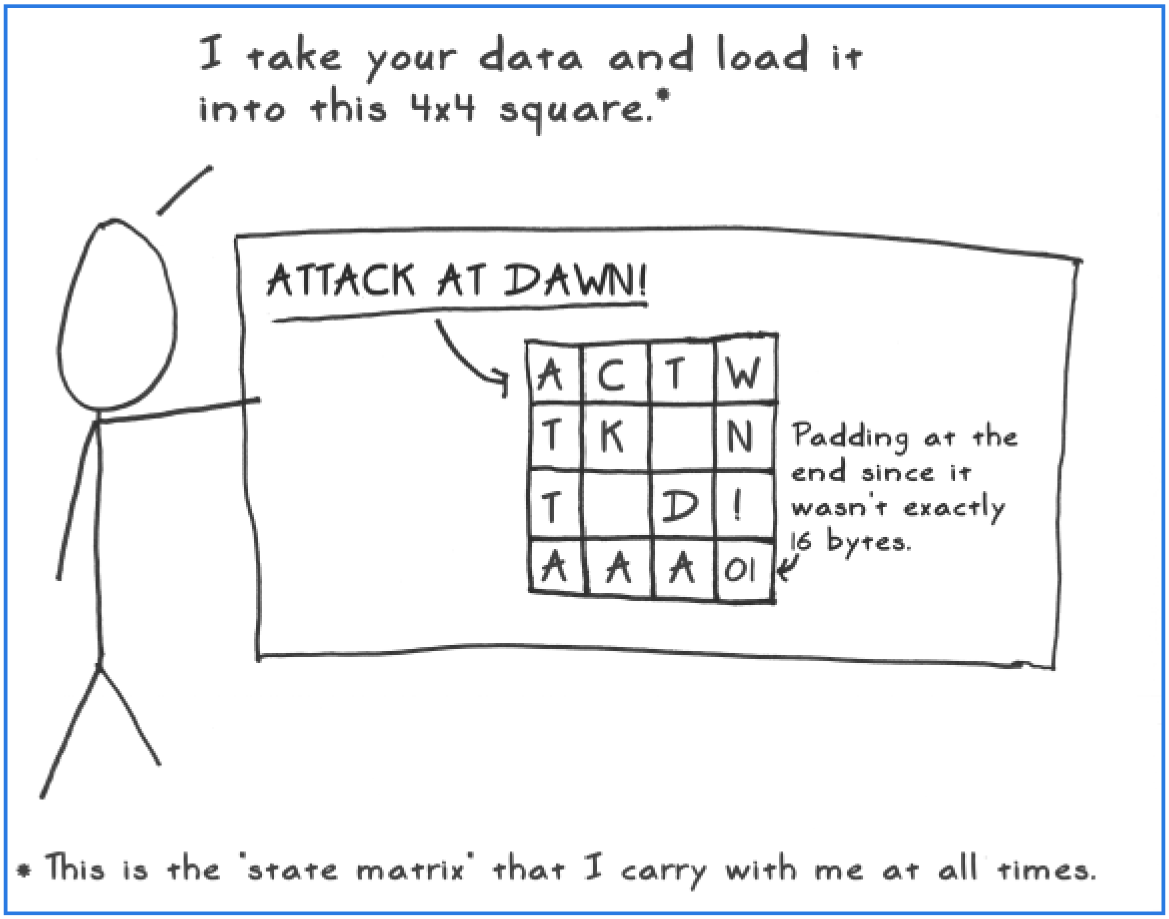


Foot-Shooting Prevention Agreement

I, ____________ , promise that once
Your Name
I see how simple AES really is, I will not implement it in production code even though it would be really fun.

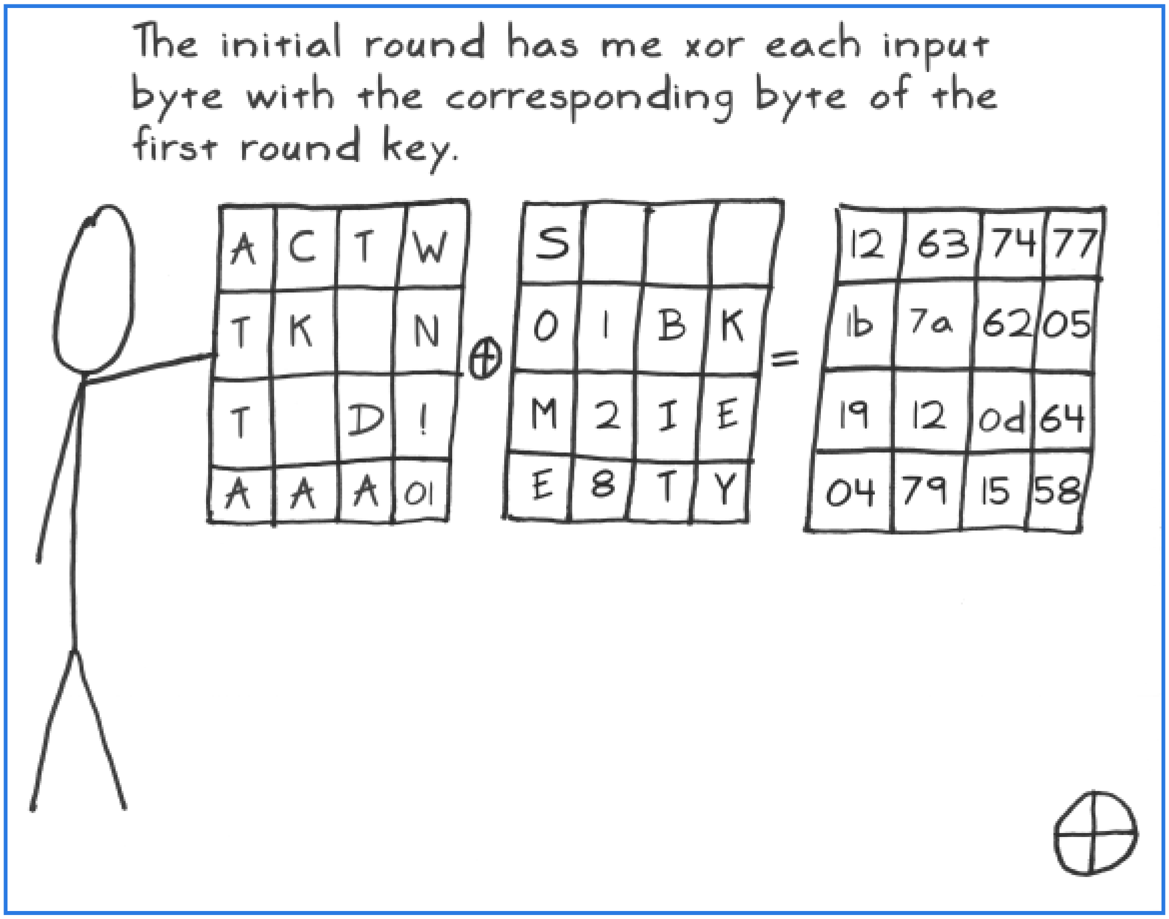
This agreement shall be in effect until the undersigned creates a meaningful interpretive dance that compares and contrasts cache-based, timing, and other side channel attacks and their countermeasures.




I take your data and load it into this 4x4 square.*
ATTACK AT DAWN!
Padding at the end since it wasn't exactly 16 bytes.

A	C	T	W
T	K		N
T		D	!
A	A	A	01

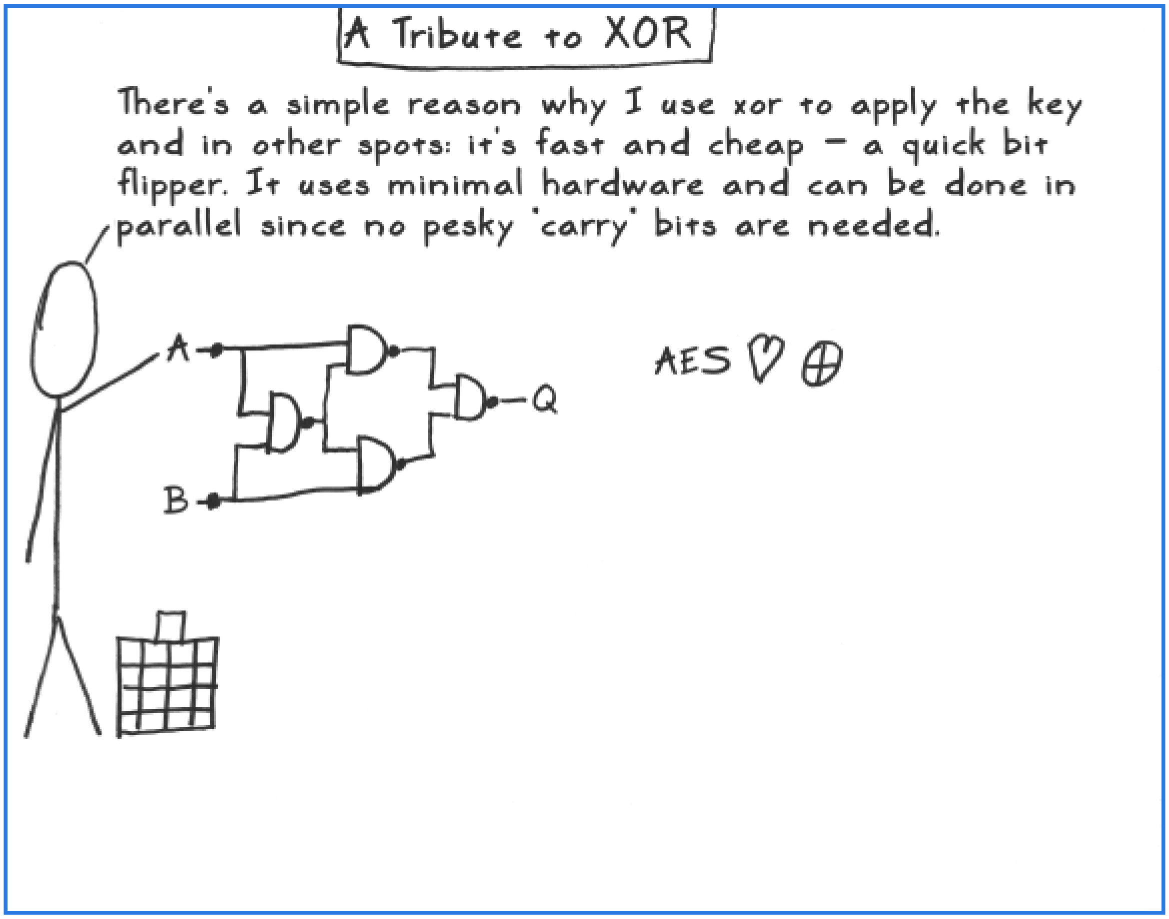
* This is the 'state matrix' that I carry with me at all times.

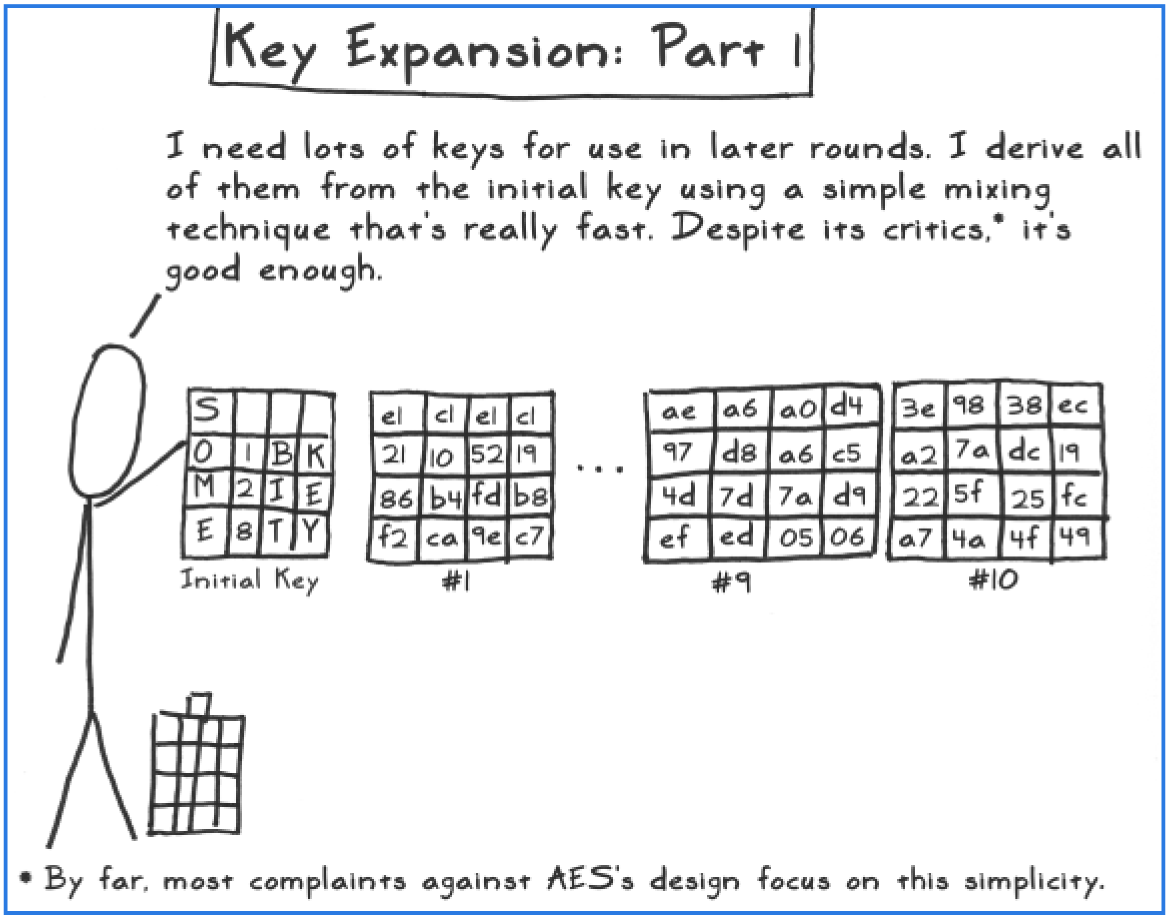

The initial round has me xor each input byte with the corresponding byte of the first round key.
⊕
=
⊕

A	C	T	W
T	K		N
T		D	!
A	A	A	01

S			
O	1	B	K
M	2	I	E
E	8	T	Y

12	63	74	77
1b	7a	62	05
19	12	0d	64
04	79	15	58


A Tribute to XOR
There's a simple reason why I use xor to apply the key and in other spots: it's fast and cheap – a quick bit flipper. It uses minimal hardware and can be done in parallel since no pesky 'carry' bits are needed.
A
B
Q
AES ♡ ⊕


Key Expansion: Part 1
I need lots of keys for use in later rounds. I derive all of them from the initial key using a simple mixing technique that's really fast. Despite its critics,* it's good enough.
Initial Key
#1
...
#9
#10
* By far, most complaints against AES's design focus on this simplicity.

Initial Key

S			
0	1	B	K
M	2	I	E
E	8	T	Y

#1

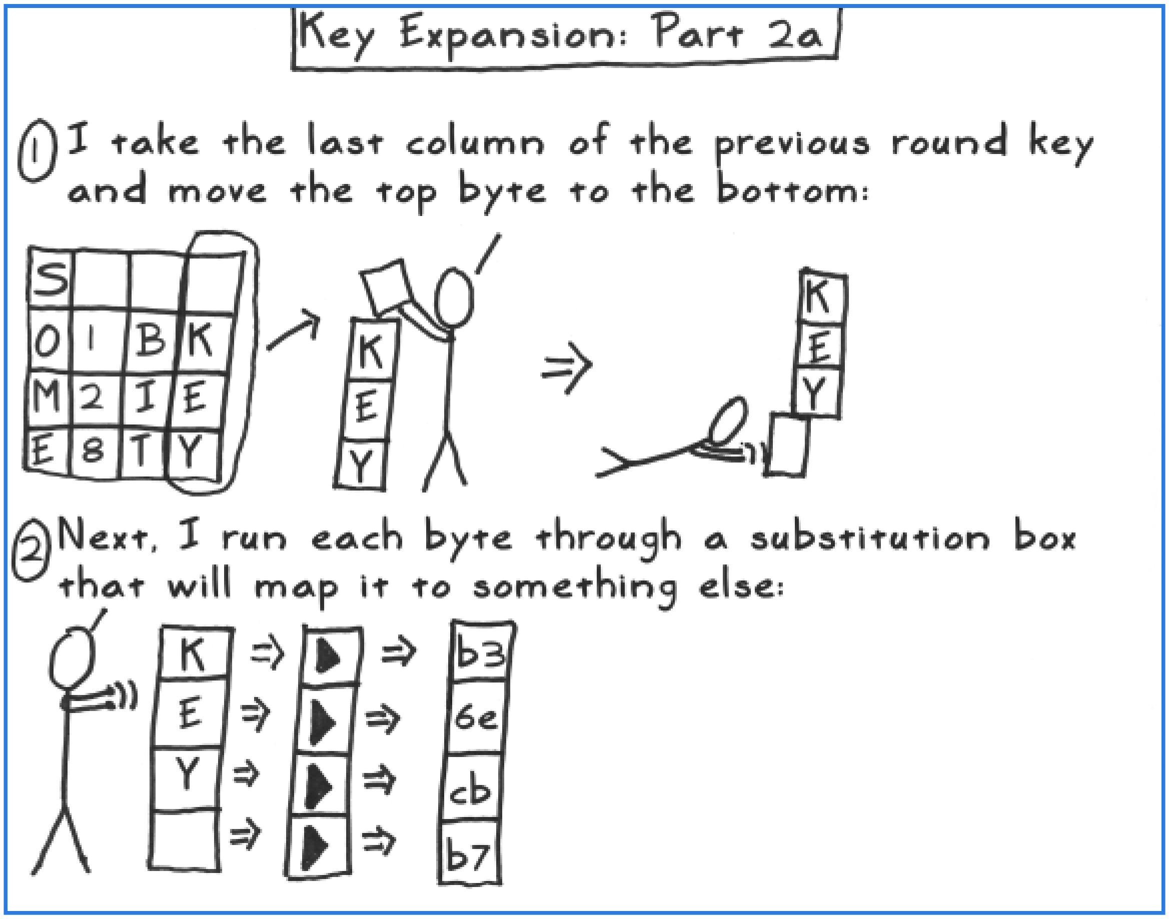
e1	c1	e1	c1
21	10	52	19
86	b4	fd	b8
f2	ca	9e	c7

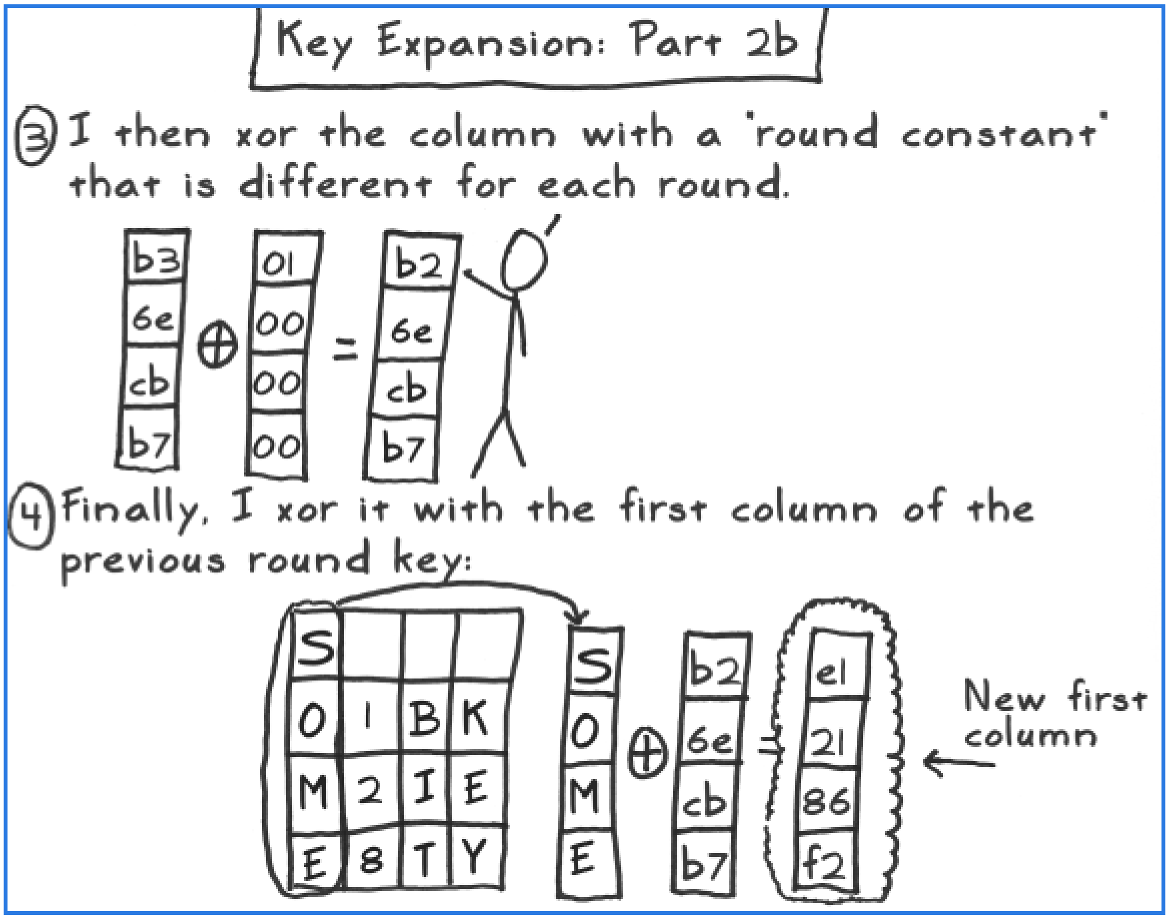
#9

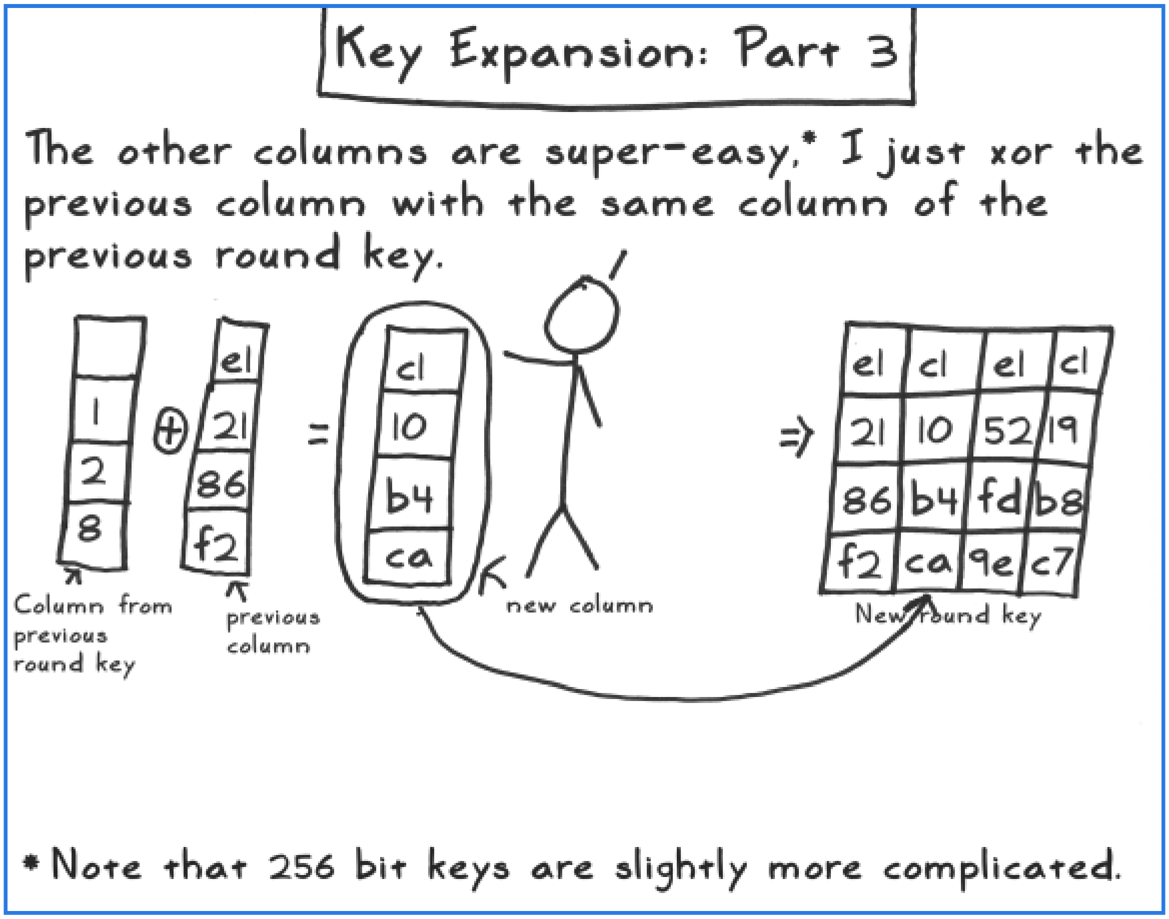
ae	a6	a0	d4
97	d8	a6	c5
4d	7d	7a	d9
ef	ed	05	06

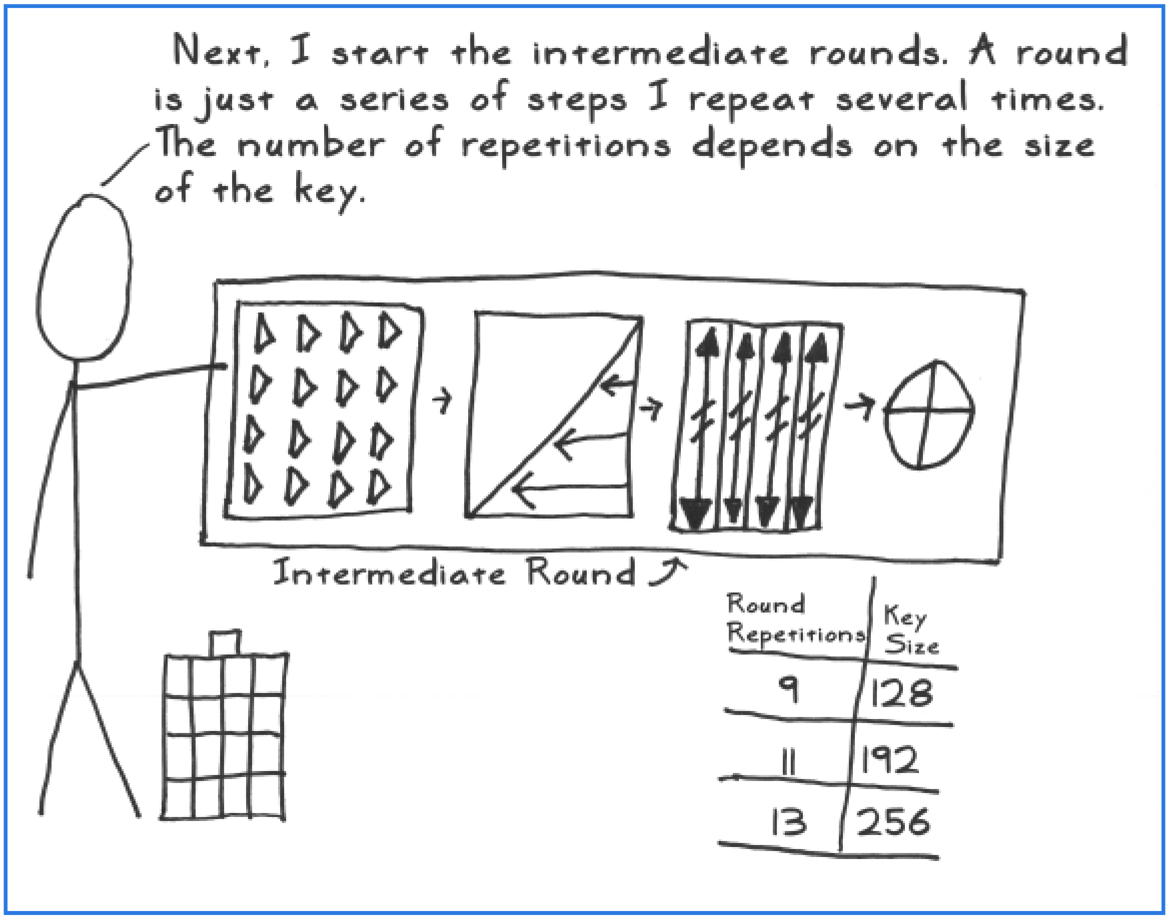
#10

3e	98	38	ec
a2	7a	dc	19
22	5f	25	fc
a7	4a	4f	49


Key Expansion: Part 2a
(1) I take the last column of the previous round key and move the top byte to the bottom:
S
0 1 B K
M 2 I E
E 8 T Y
K
E
Y
K
E
Y
(2) Next, I run each byte through a substitution box that will map it to something else:
K
E
Y
b3
6e
cb
b7

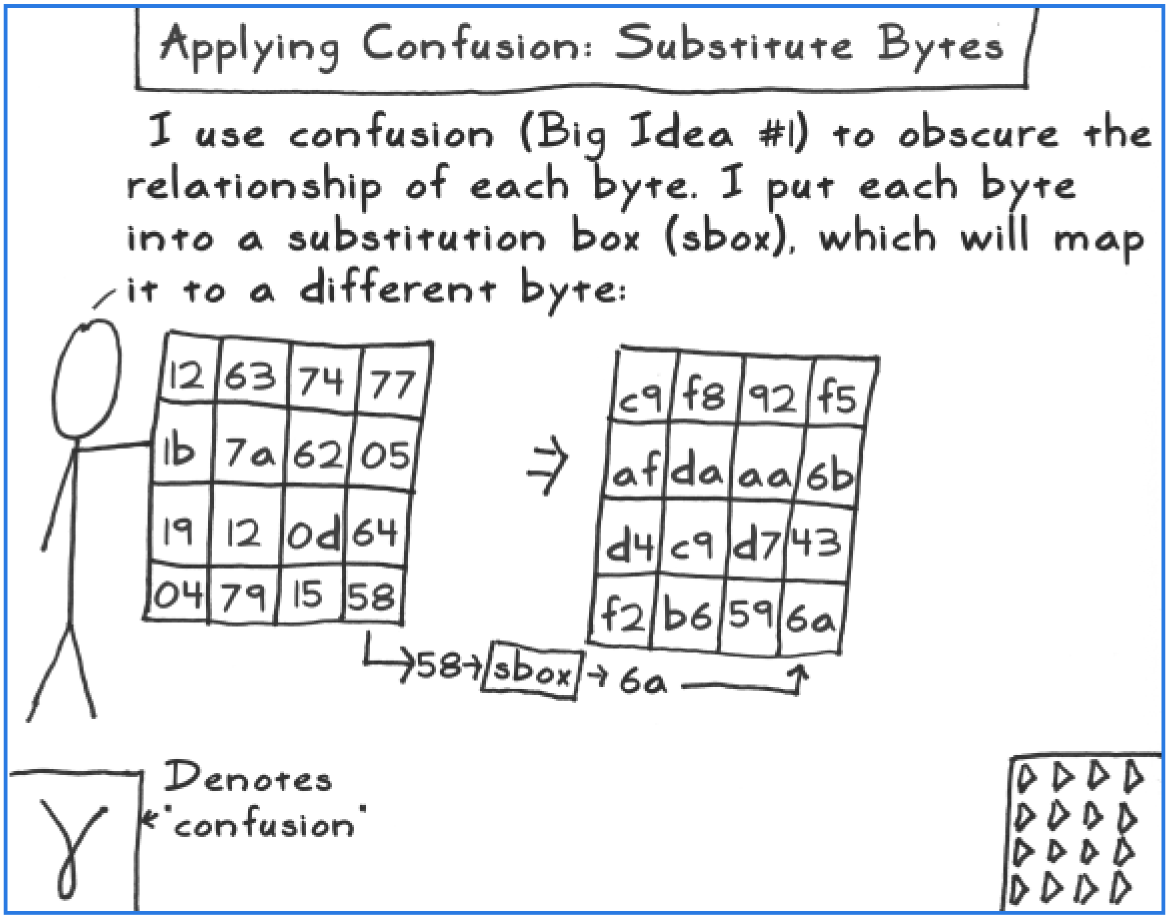

Key Expansion: Part 2b
③ I then xor the column with a 'round constant' that is different for each round.
b3
6e
cb
b7
⊕
01
00
00
00
=
b2
6e
cb
b7
④ Finally, I xor it with the first column of the previous round key:
S
O 1 B K
M 2 I E
E 8 T Y
S
O
M
E
⊕
b2
6e
cb
b7
=
e1
21
86
f2
New first column


Key Expansion: Part 3
The other columns are super-easy,* I just xor the previous column with the same column of the previous round key.
1
2
8
⊕
e1
21
86
f2
=
c1
10
b4
ca
Column from previous round key
previous column
new column
⇒
e1 c1 e1 c1
21 10 52 19
86 b4 fd b8
f2 ca 9e c7
New round key
* Note that 256 bit keys are slightly more complicated.



Next, I start the intermediate rounds. A round is just a series of steps I repeat several times. The number of repetitions depends on the size of the key.

Round Repetitions	Key Size
9	128
11	192
13	256



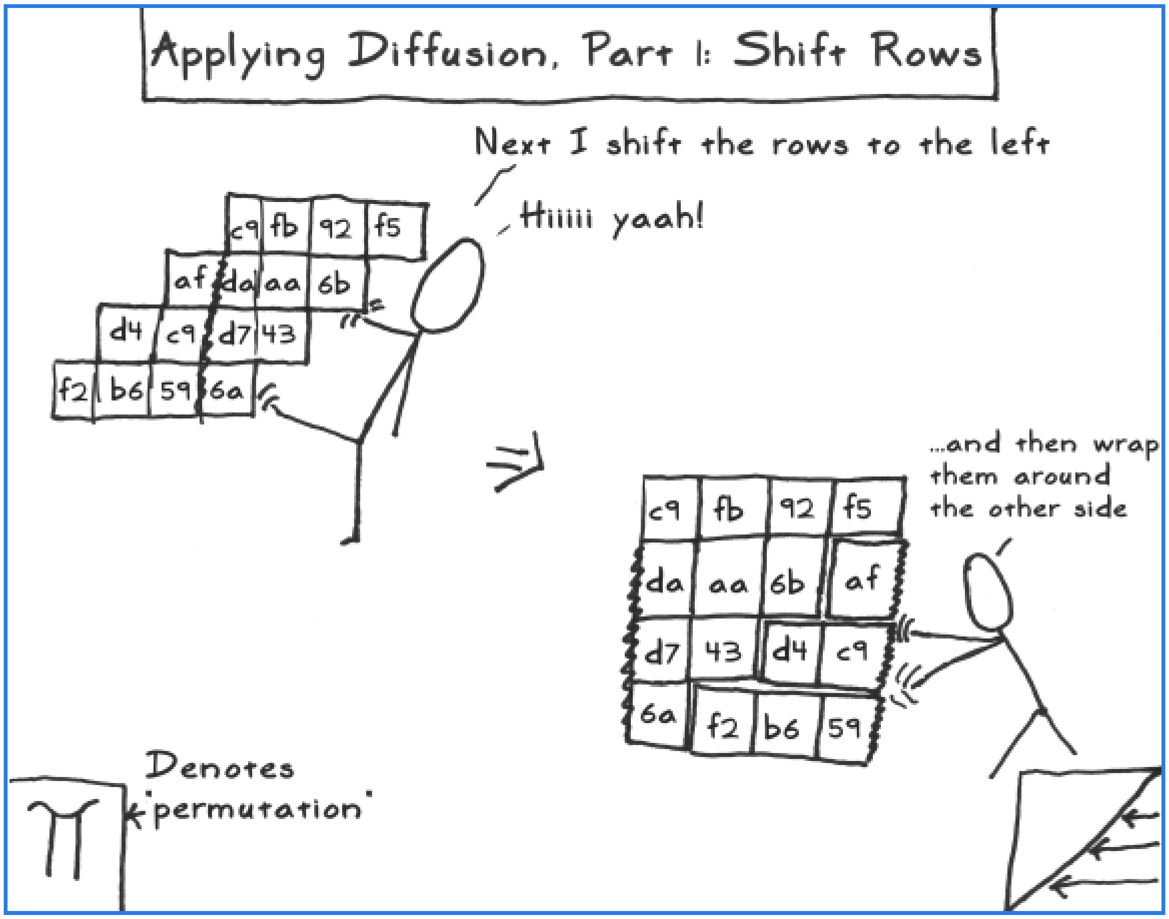
Applying Confusion: Substitute Bytes

I use confusion (Big Idea #1) to obscure the relationship of each byte. I put each byte into a substitution box (sbox), which will map it to a different byte:

12	63	74	77
1b	7a	62	05
19	12	0d	64
04	79	15	58

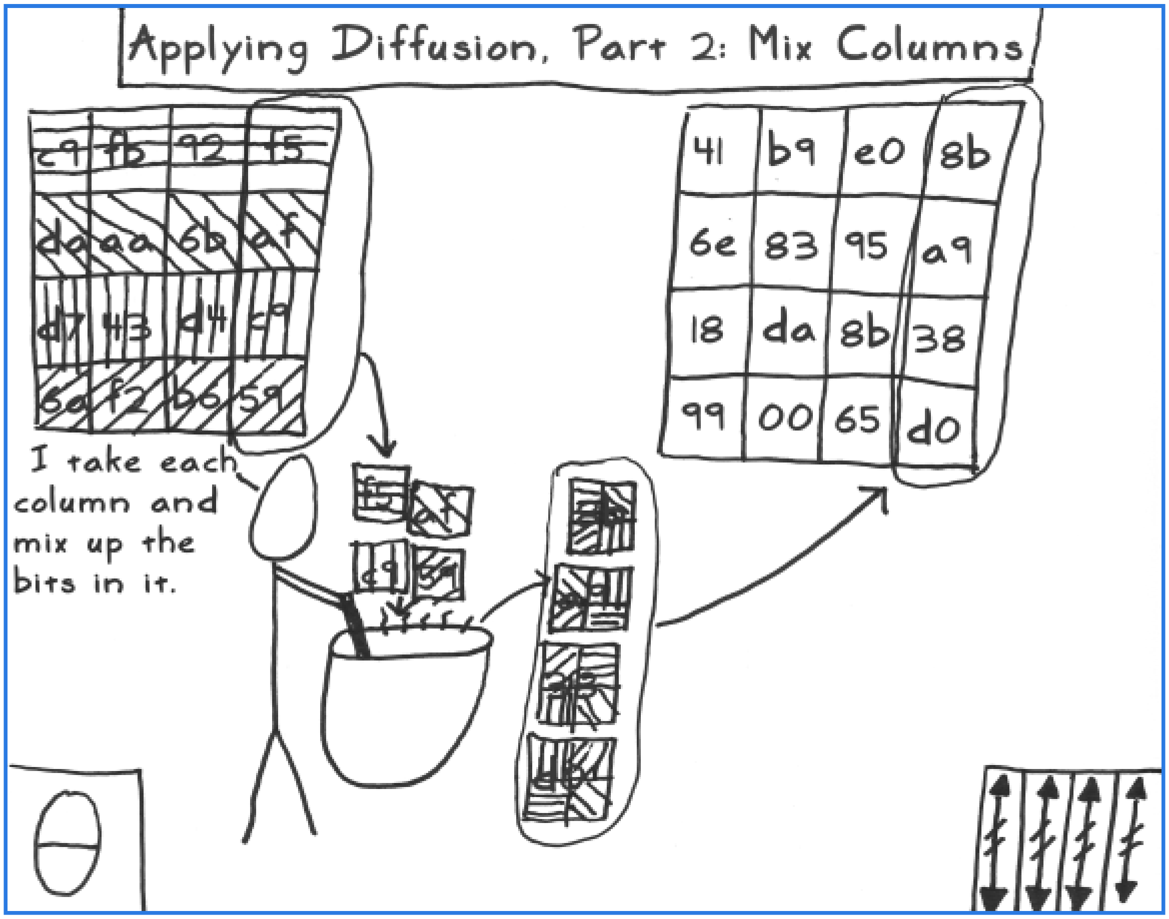
→

c9	f8	92	f5
af	da	aa	6b
d4	c9	d7	43
f2	b6	59	6a


Applying Diffusion, Part 1: Shift Rows
Next I shift the rows to the left
Hiiiii yaah!
...and then wrap them around the other side
Denotes "permutation"
π

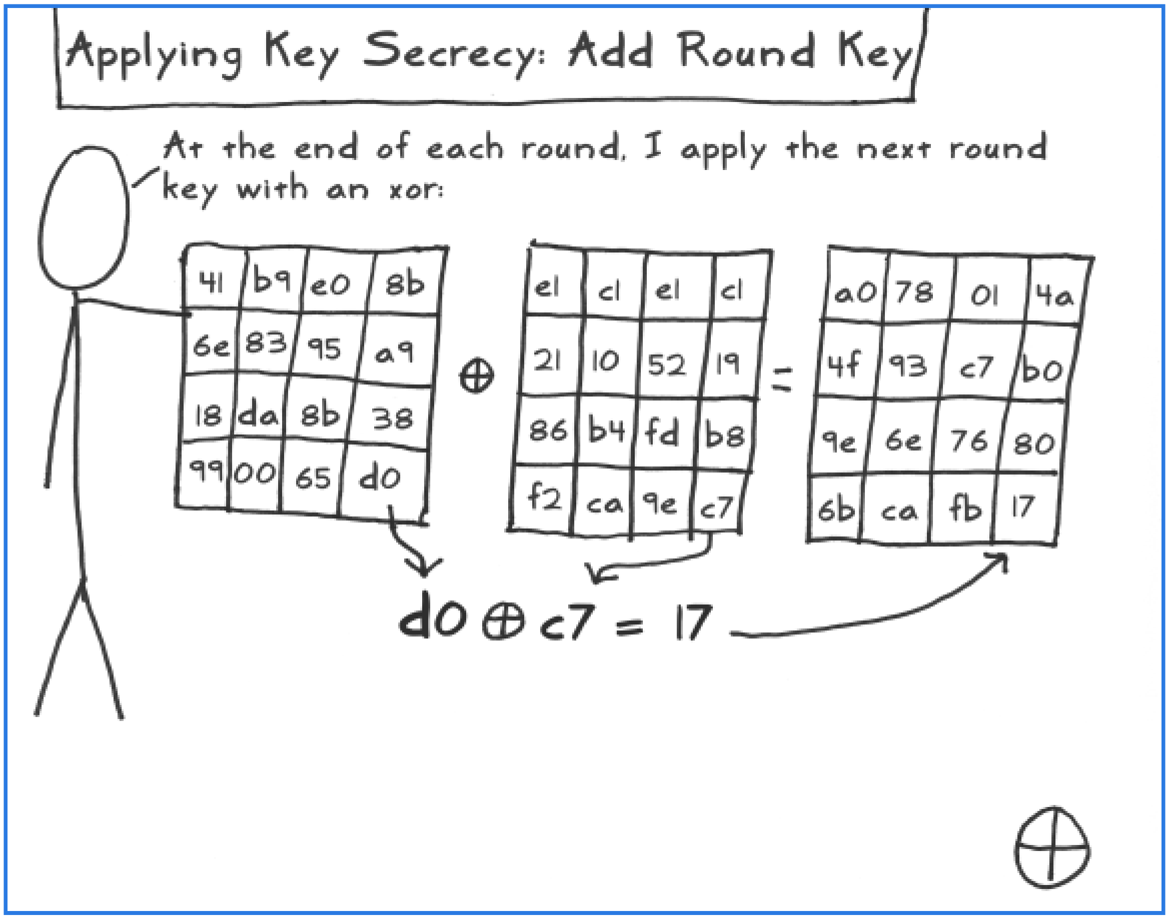
c9	fb	92	f5
af	da	aa	6b
d4	c9	d7	43
f2	b6	59	6a

c9	fb	92	f5
da	aa	6b	af
d7	43	d4	c9
6a	f2	b6	59


Applying Diffusion, Part 2: Mix Columns
I take each column and mix up the bits in it.
f5
af
c9
59
θ

c9	fb	92	f5
da	aa	6b	af
d7	43	d4	c9
6a	f2	b6	59

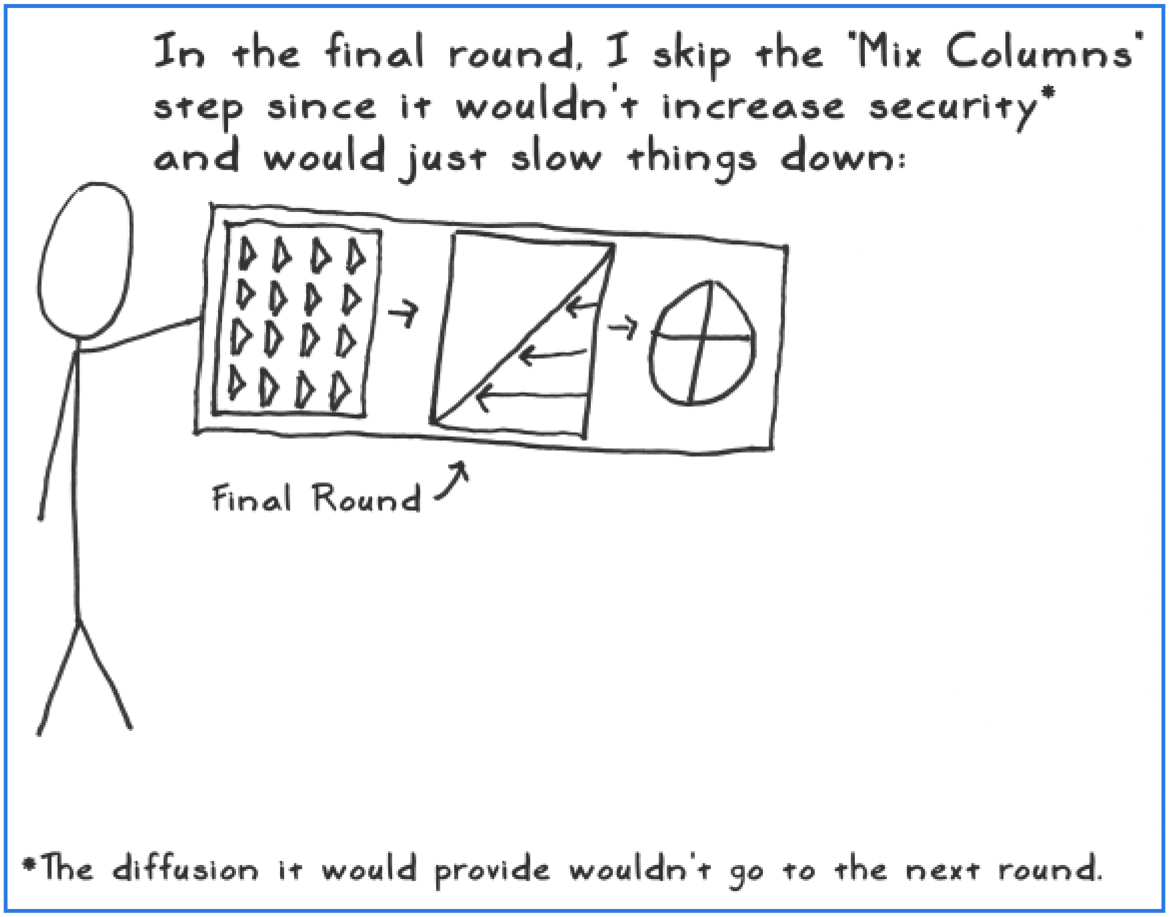
41	b9	e0	8b
6e	83	95	a9
18	da	8b	38
99	00	65	d0

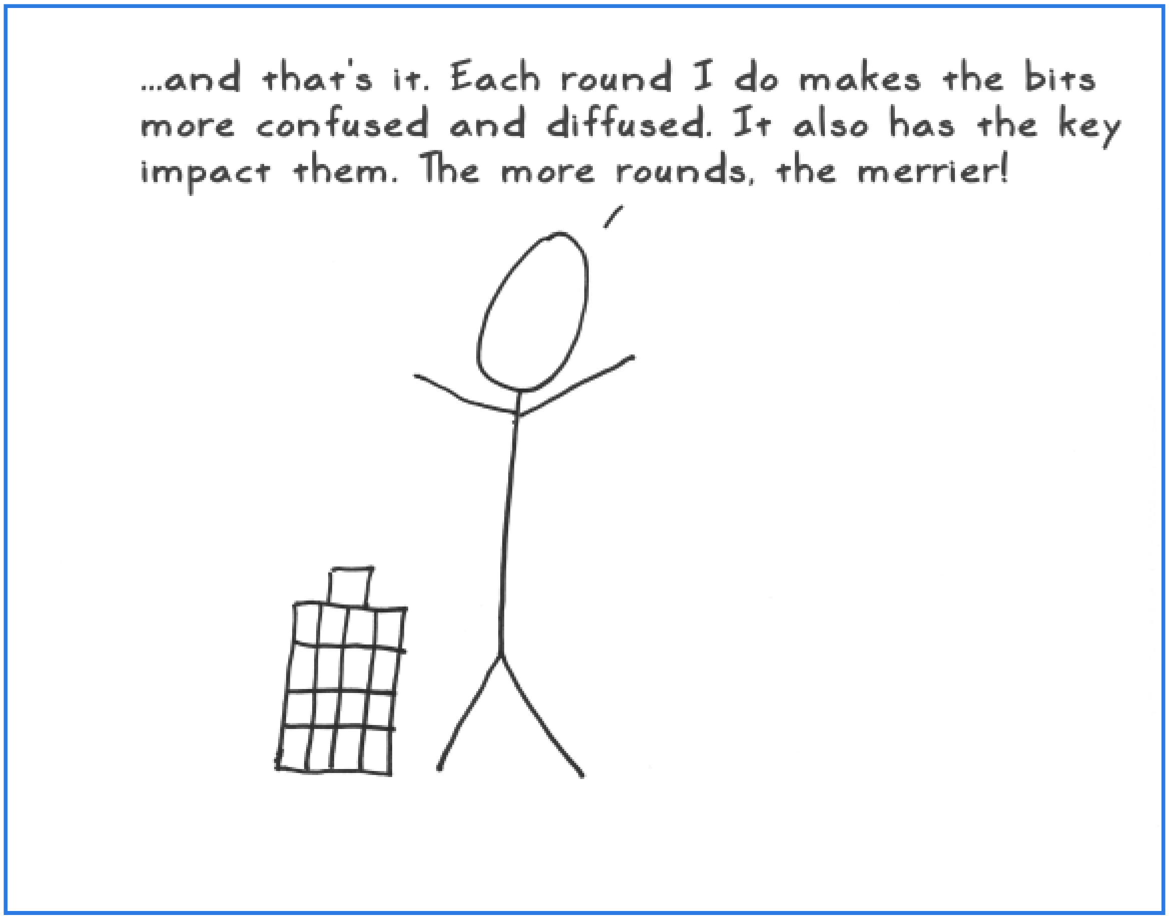

Applying Key Secrecy: Add Round Key
At the end of each round, I apply the next round key with an xor:
⊕
=
d0 ⊕ c7 = 17
⊕

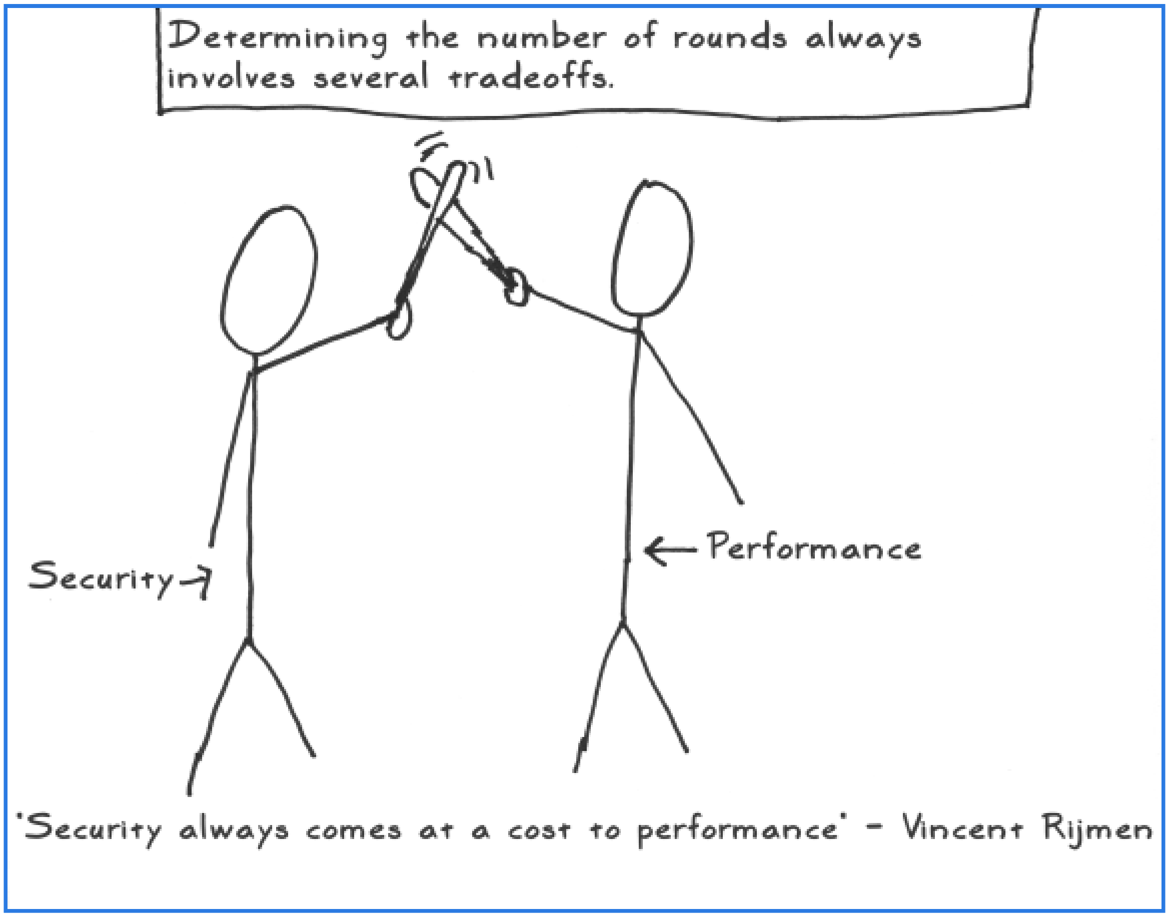
41	b9	e0	8b
6e	83	95	a9
18	da	8b	38
99	00	65	d0

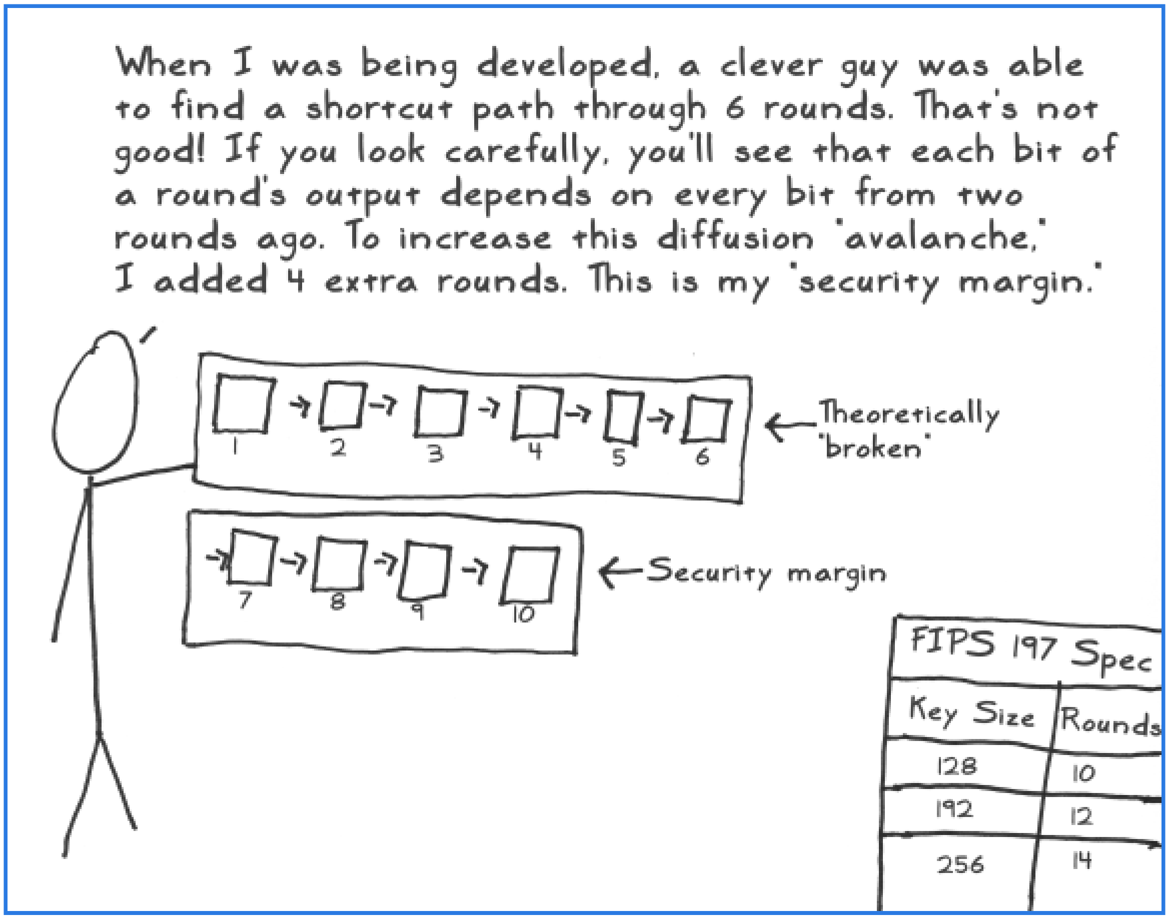
e1	c1	e1	c1
21	10	52	19
86	b4	fd	b8
f2	ca	9e	c7

a0	78	01	4a
4f	93	c7	b0
9e	6e	76	80
6b	ca	fb	17

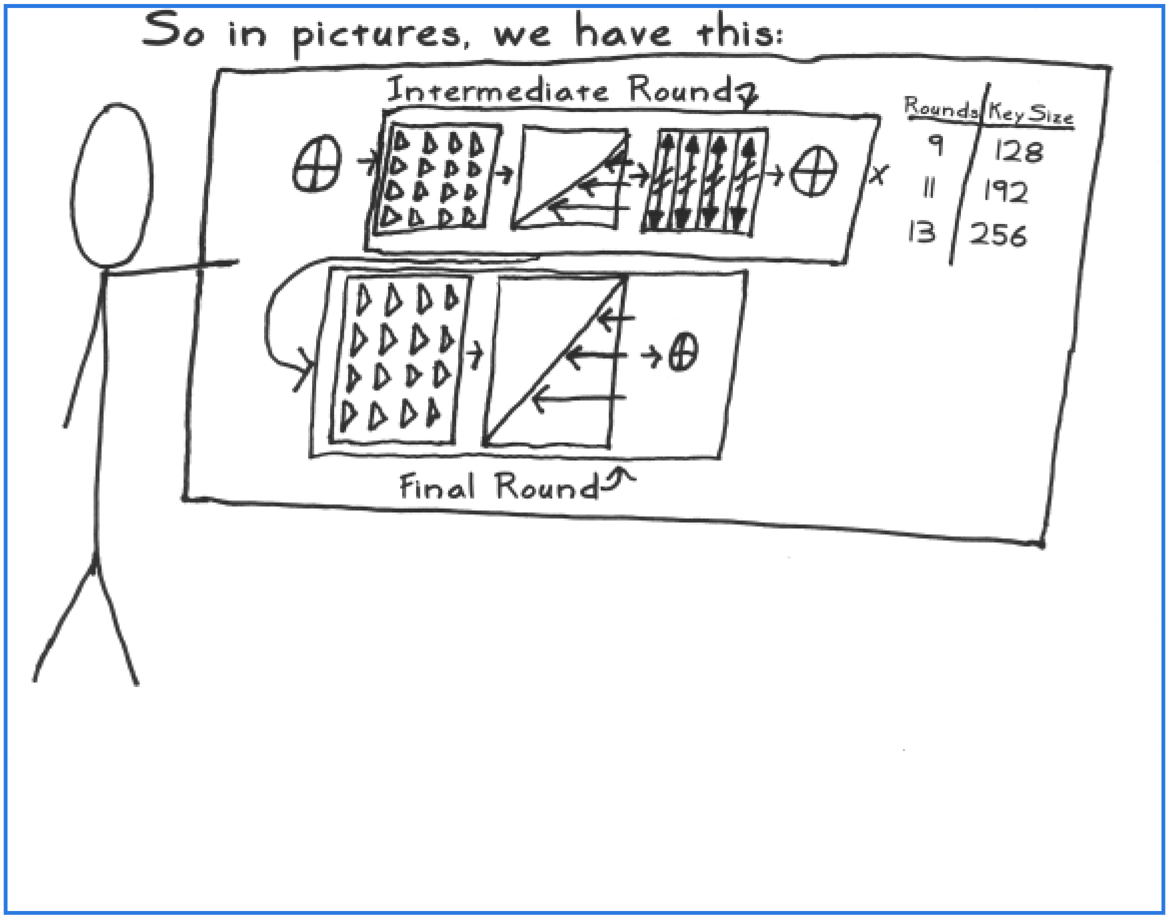

In the final round, I skip the 'Mix Columns' step since it wouldn't increase security* and would just slow things down:
Final Round
*The diffusion it would provide wouldn't go to the next round.


...and that's it. Each round I do makes the bits more confused and diffused. It also has the key impact them. The more rounds, the merrier!

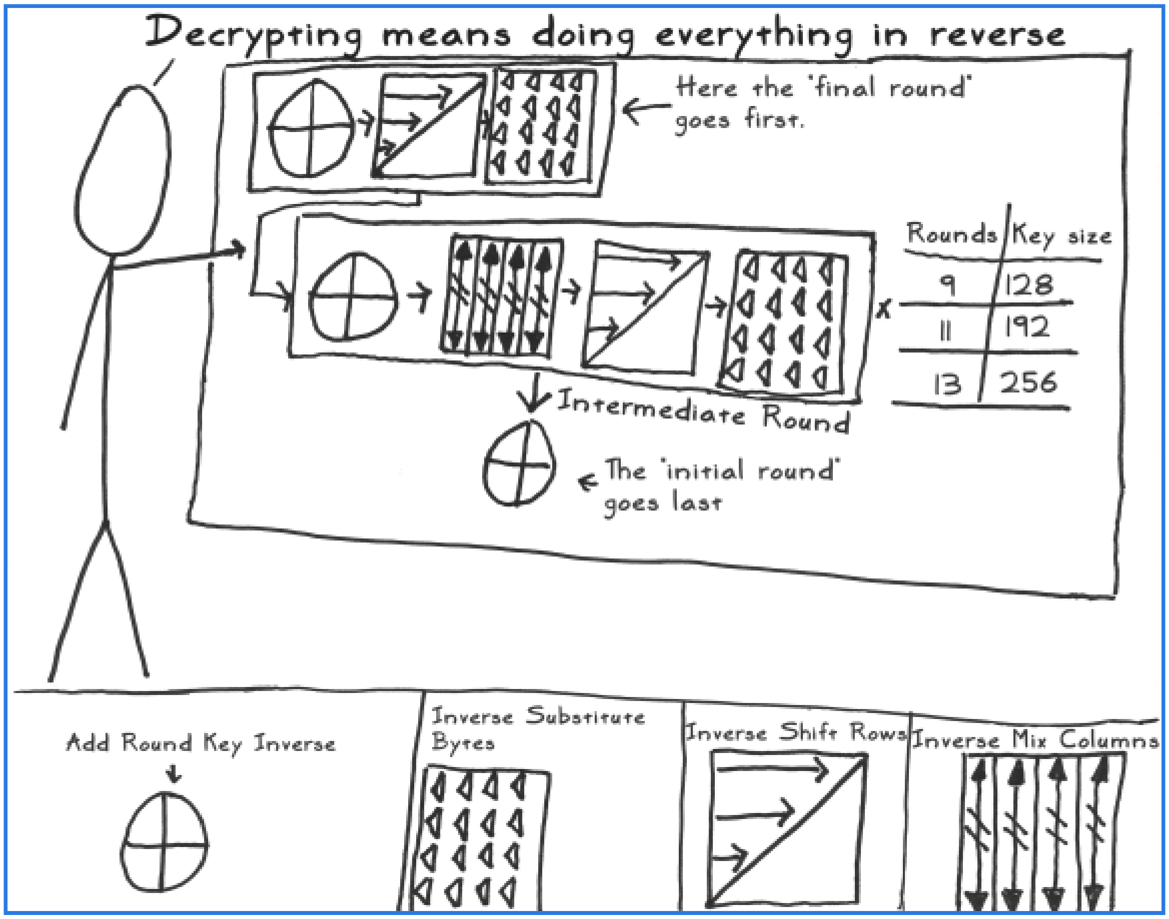

Determining the number of rounds always involves several tradeoffs.
Security
Performance
'Security always comes at a cost to performance' - Vincent Rijmen

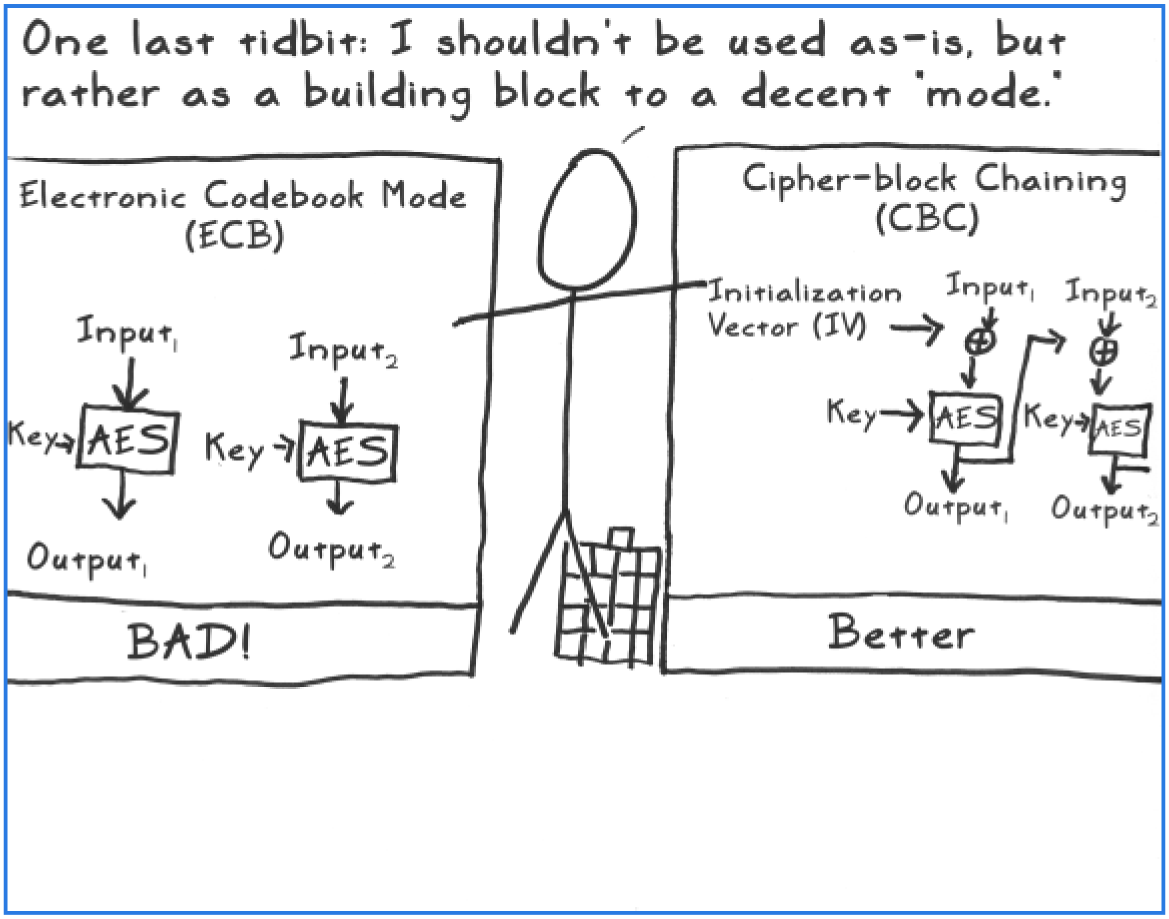

When I was being developed, a clever guy was able to find a shortcut path through 6 rounds. That's not good! If you look carefully, you'll see that each bit of a round's output depends on every bit from two rounds ago. To increase this diffusion "avalanche," I added 4 extra rounds. This is my "security margin."
1
2
3
4
5
6
Theoretically "broken"
7
8
9
10
Security margin
FIPS 197 Spec

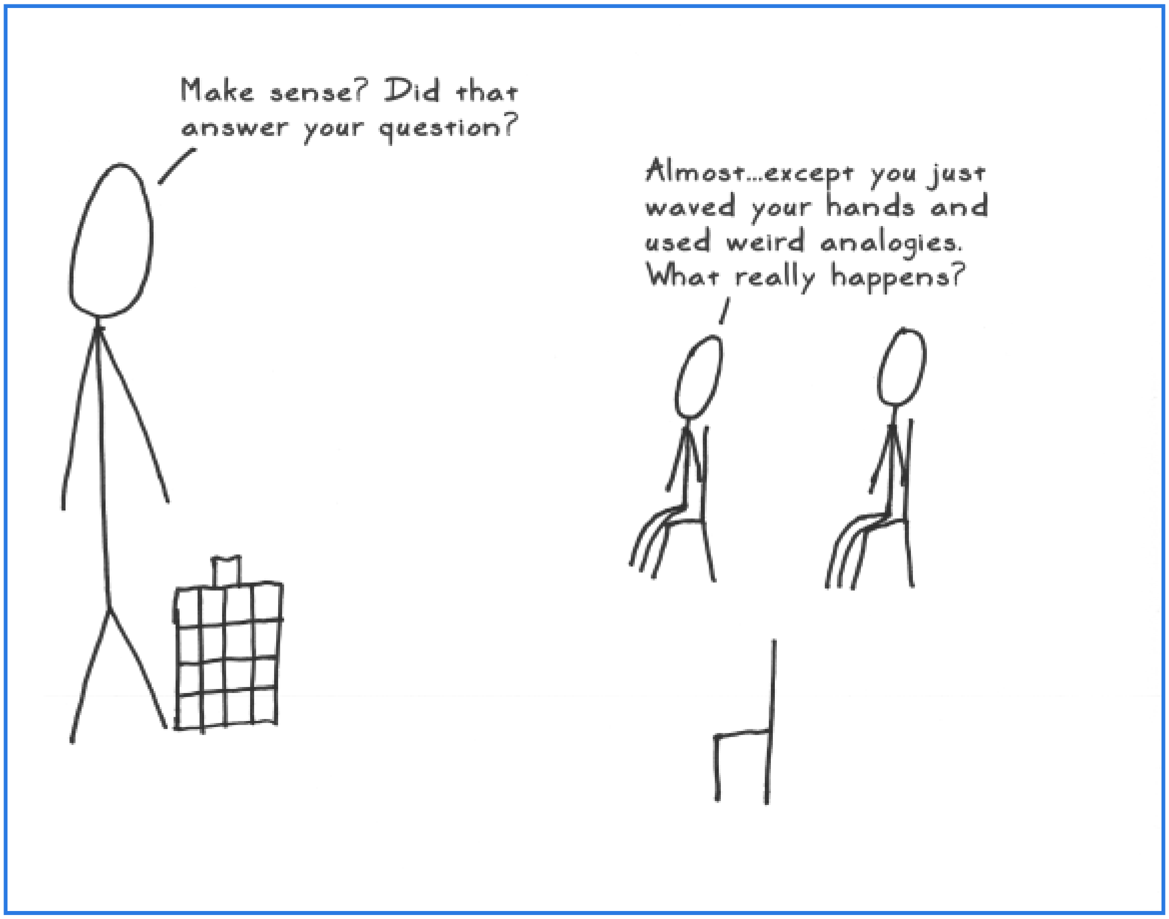
Key Size	Rounds
128	10
192	12
256	14

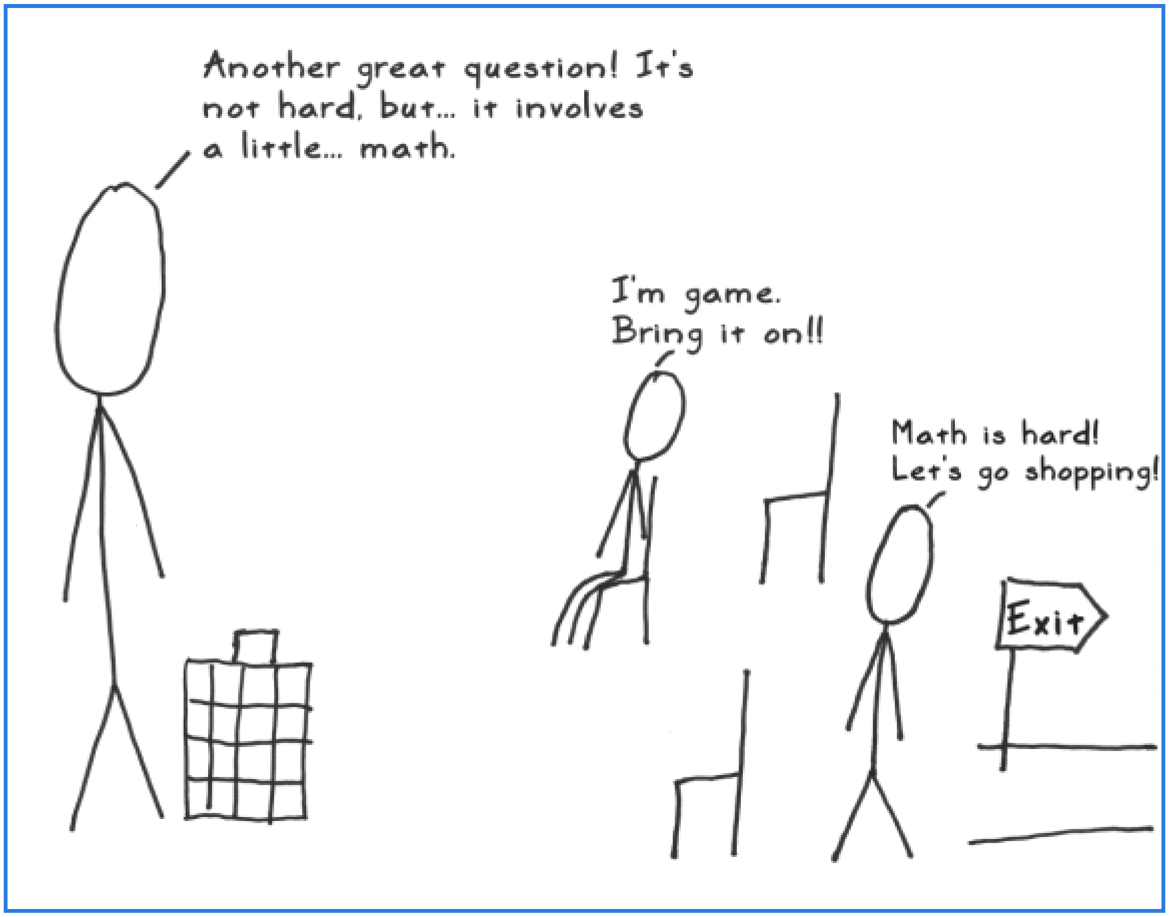


Rounds	Key Size
9	128
11	192
13	256

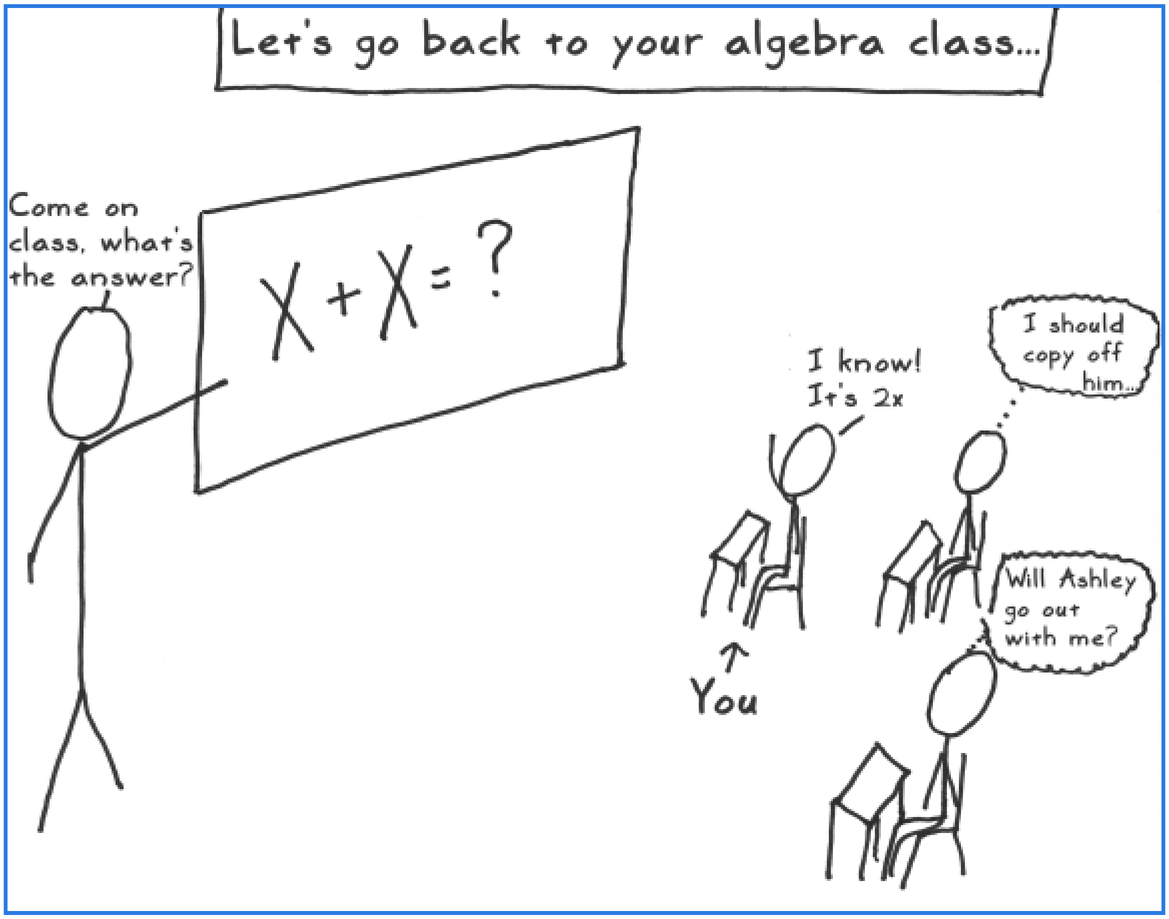

Decrypting means doing everything in reverse
Here the 'final round' goes first.
Rounds | Key size
9 | 128
11 | 192
13 | 256
x
Intermediate Round
The 'initial round' goes last
Add Round Key Inverse
Inverse Substitute Bytes
Inverse Shift Rows
Inverse Mix Columns

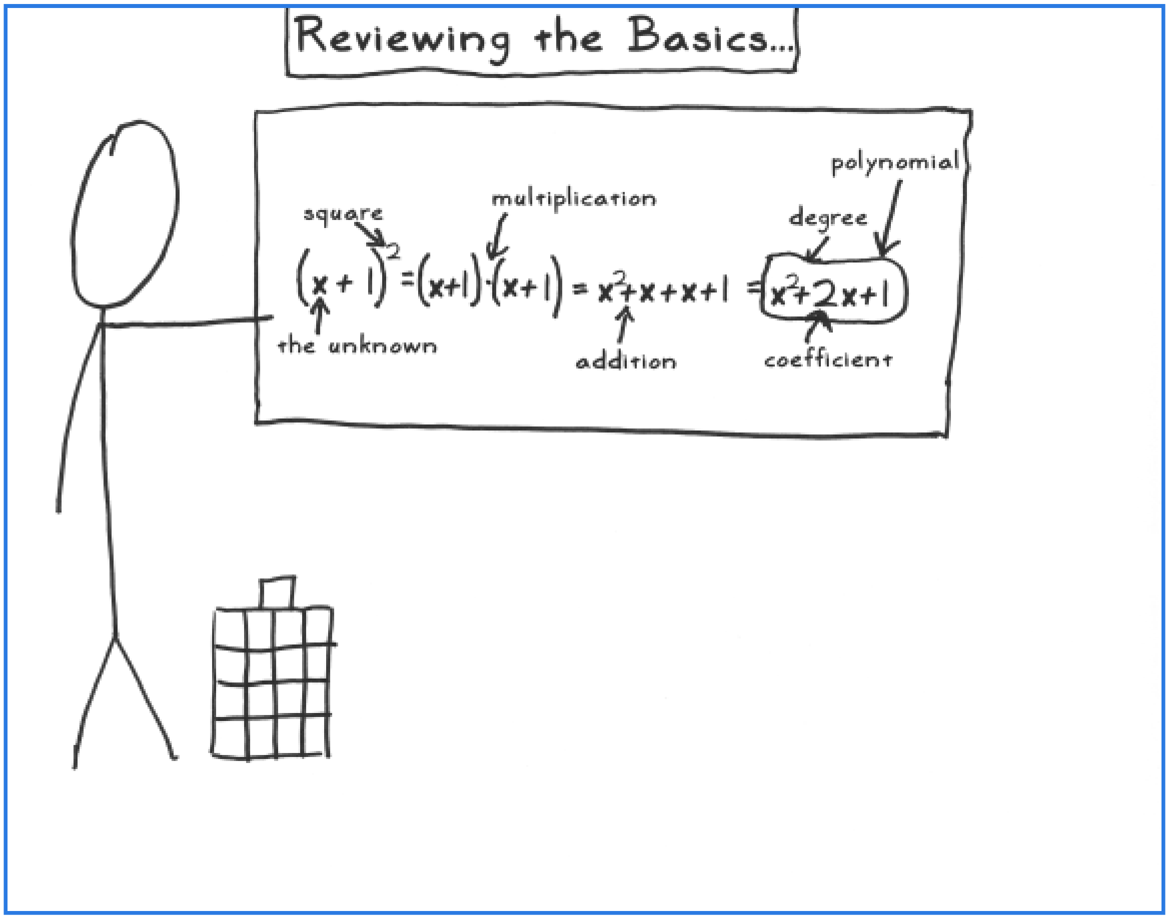

One last tidbit: I shouldn't be used as-is, but rather as a building block to a decent 'mode.'
Electronic Codebook Mode (ECB)
Input₁
Input₂
Key
AES
Key
AES
Output₁
Output₂
BAD!
Cipher-block Chaining (CBC)
Initialization Vector (IV)
Input₁
Input₂
Key
AES
Key
AES
Output₁
Output₂
Better

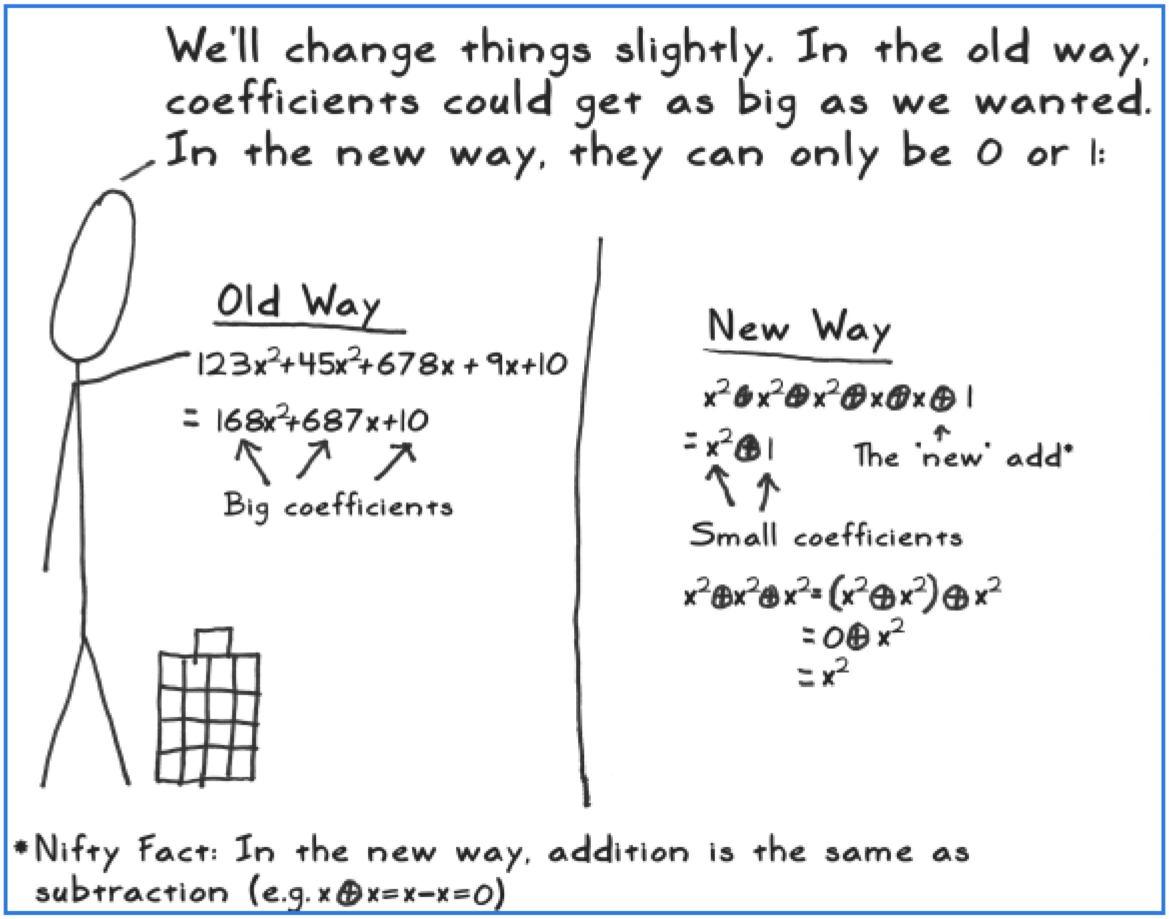

Make sense? Did that answer your question?
Almost...except you just waved your hands and used weird analogies. What really happens?

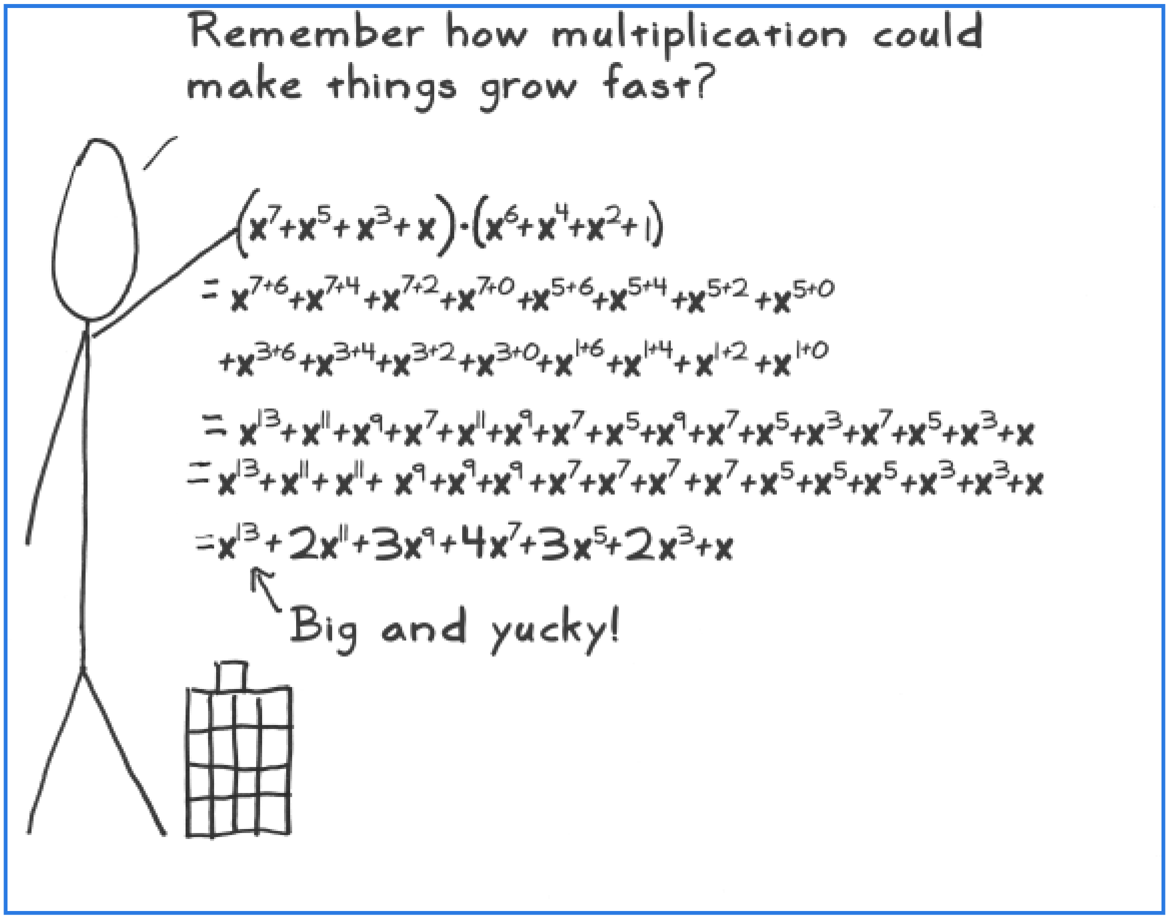

Another great question! It's not hard, but... it involves a little... math.
I'm game. Bring it on!!
Math is hard! Let's go shopping!
Exit

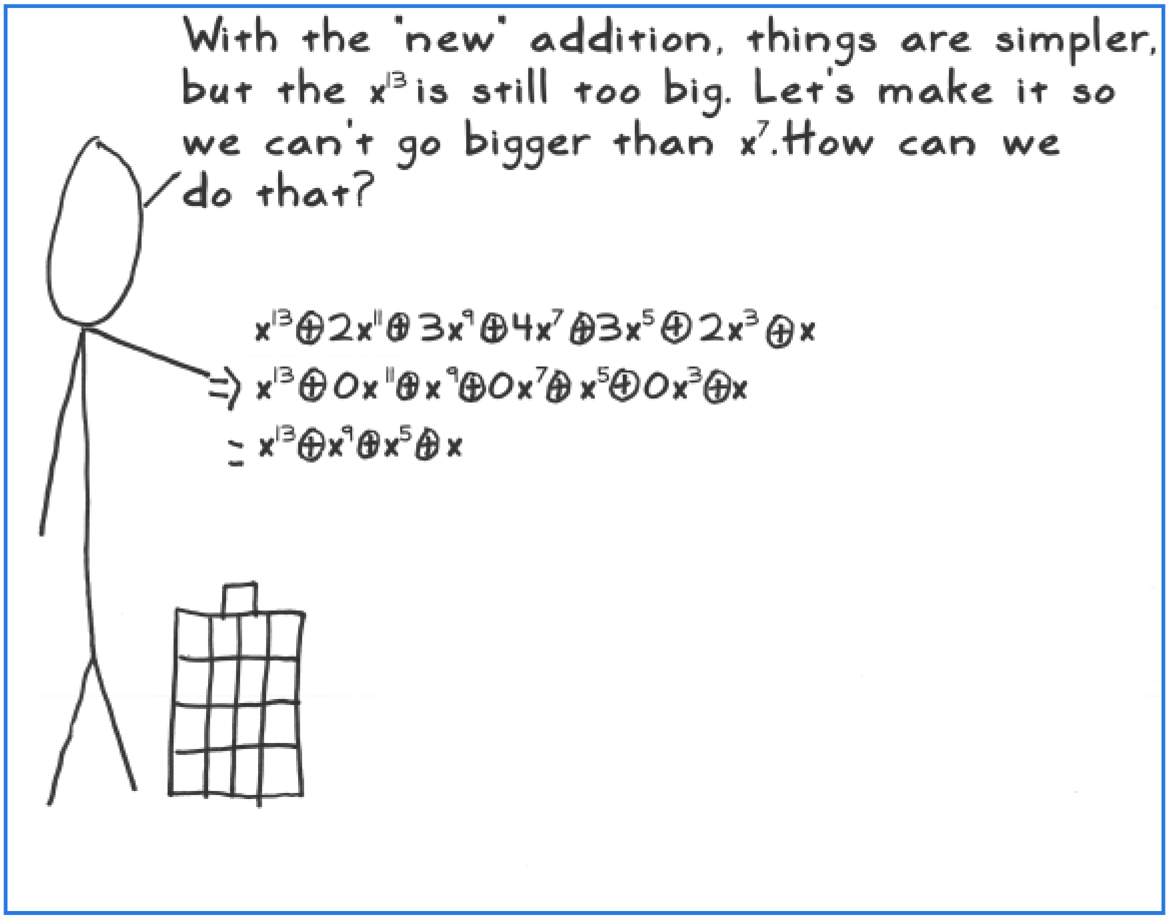
Act 4: Math!

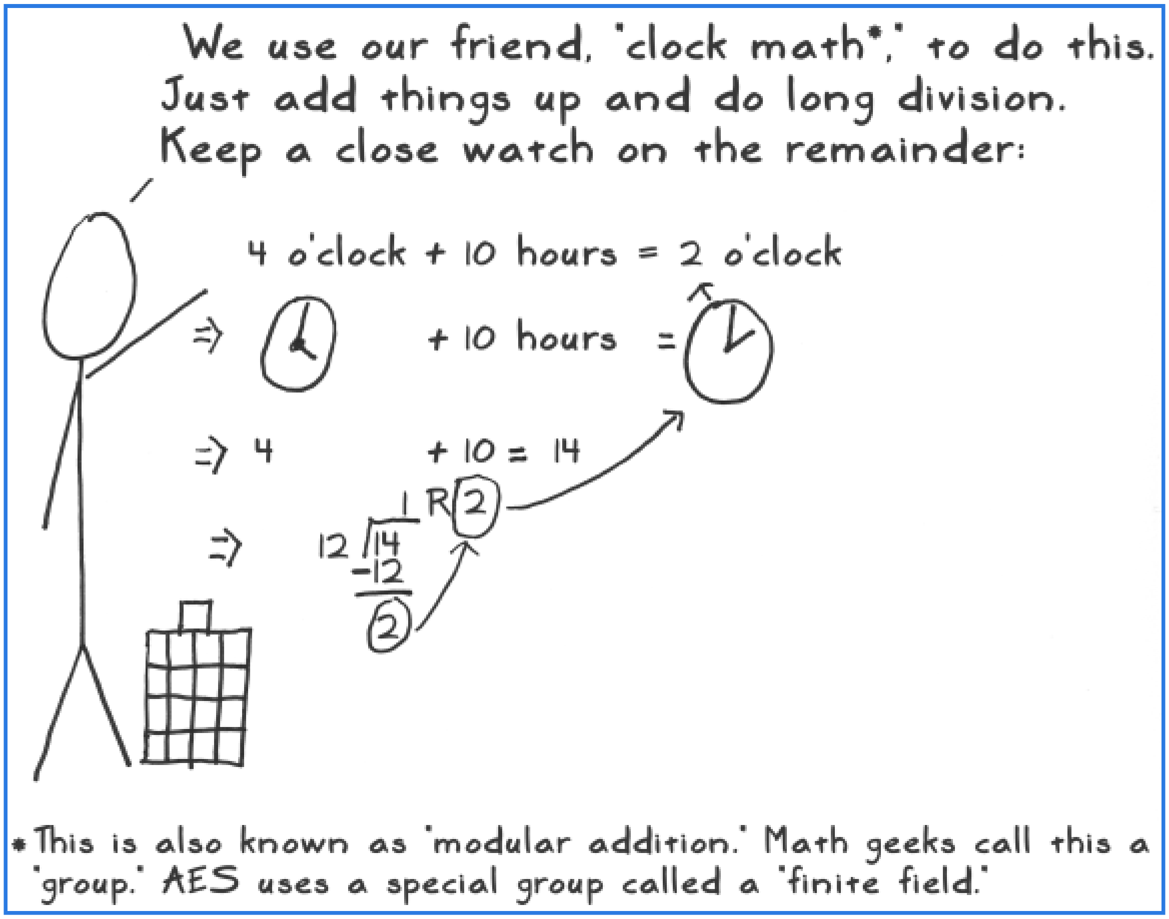

Let's go back to your algebra class...
Come on class, what's the answer?
$X+X=?$
I know! It's 2x
I should copy off him...
Will Ashley go out with me?
You

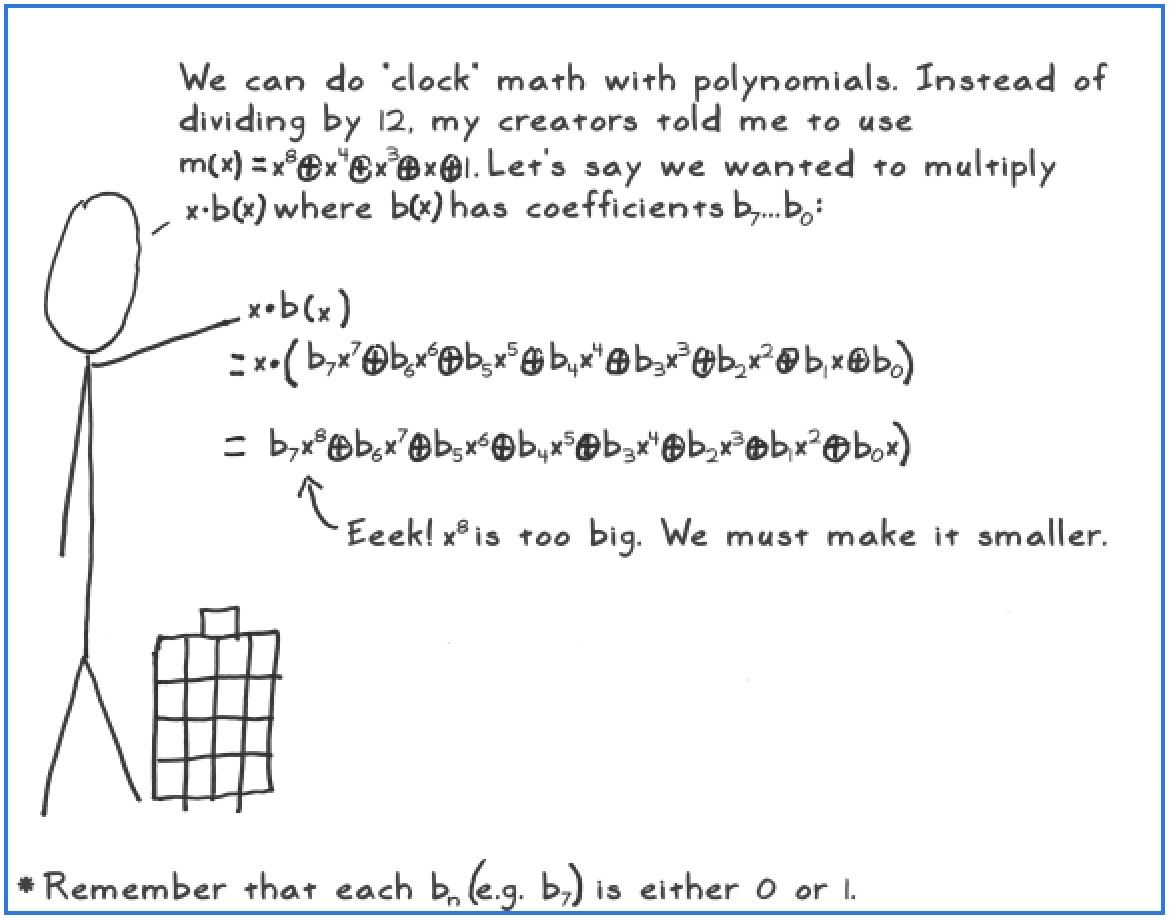

Reviewing the Basics...
square
multiplication
polynomial
degree
$(x+1)^2=(x+1)\cdot(x+1)=x^2+x+x+1=\boxed{x^2+2x+1}$
the unknown
addition
coefficient

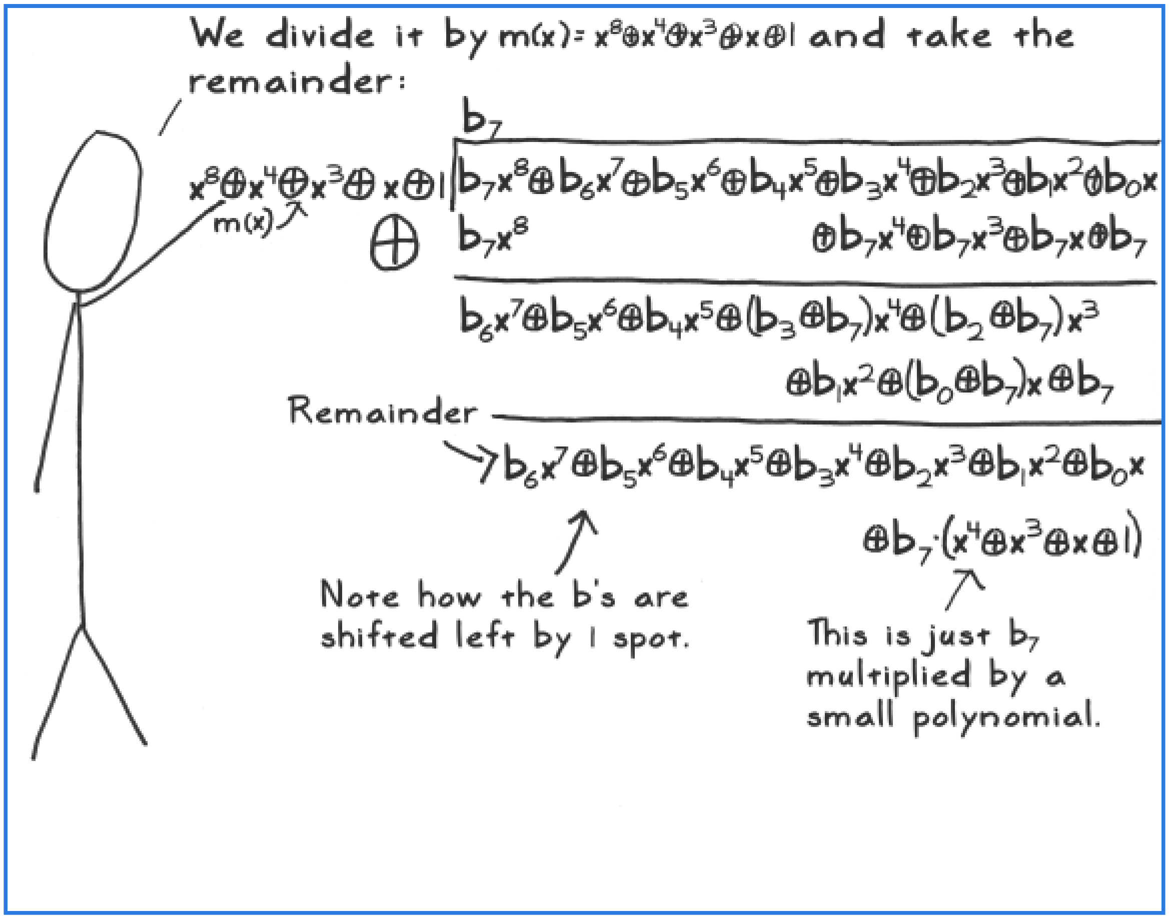

We'll change things slightly. In the old way, coefficients could get as big as we wanted. In the new way, they can only be 0 or 1:
Old Way
$123x^2+45x^2+678x+9x+10$
$=168x^2+687x+10$
Big coefficients
New Way
$x^2\oplus x^2\oplus x^2\oplus x\oplus x\oplus 1$
$=x^2\oplus 1$
The "new" add*
Small coefficients
$x^2\oplus x^2\oplus x^2=(x^2\oplus x^2)\oplus x^2$
$=0\oplus x^2$
$=x^2$
*Nifty Fact: In the new way, addition is the same as subtraction (e.g. $x\oplus x=x-x=0$)


Remember how multiplication could make things grow fast?
$(x^7+x^5+x^3+x)\cdot(x^6+x^4+x^2+1)$
$=x^{7+6}+x^{7+4}+x^{7+2}+x^{7+0}+x^{5+6}+x^{5+4}+x^{5+2}+x^{5+0}$
$+x^{3+6}+x^{3+4}+x^{3+2}+x^{3+0}+x^{1+6}+x^{1+4}+x^{1+2}+x^{1+0}$
$=x^{13}+x^{11}+x^9+x^7+x^{11}+x^9+x^7+x^5+x^9+x^7+x^5+x^3+x^7+x^5+x^3+x$
$=x^{13}+x^{11}+x^{11}+x^9+x^9+x^9+x^7+x^7+x^7+x^7+x^5+x^5+x^5+x^3+x^3+x$
$=x^{13}+2x^{11}+3x^9+4x^7+3x^5+2x^3+x$
Big and yucky!


With the 'new' addition, things are simpler, but the x^{13} is still too big. Let's make it so we can't go bigger than x^7. How can we do that?
$x^{13} \oplus 2x^{11} \oplus 3x^9 \oplus 4x^7 \oplus 3x^5 \oplus 2x^3 \oplus x$
$\Rightarrow x^{13} \oplus 0x^{11} \oplus x^9 \oplus 0x^7 \oplus x^5 \oplus 0x^3 \oplus x$
$= x^{13} \oplus x^9 \oplus x^5 \oplus x$


We use our friend, 'clock math*,' to do this. Just add things up and do long division. Keep a close watch on the remainder:
4 o'clock + 10 hours = 2 o'clock
⇒ + 10 hours =
⇒ 4 + 10 = 14
⇒ 12 /14 1 R 2
-12
2
* This is also known as 'modular addition.' Math geeks call this a 'group.' AES uses a special group called a 'finite field.'


We can do "clock" math with polynomials. Instead of dividing by 12, my creators told me to use $m(x) = x^8 \oplus x^4 \oplus x^3 \oplus x \oplus 1$. Let's say we wanted to multiply $x \cdot b(x)$ where $b(x)$ has coefficients $b_7 \ldots b_0$:
$x \cdot b(x)$
$= x \cdot (b_7x^7 \oplus b_6x^6 \oplus b_5x^5 \oplus b_4x^4 \oplus b_3x^3 \oplus b_2x^2 \oplus b_1x \oplus b_0)$
$= b_7x^8 \oplus b_6x^7 \oplus b_5x^6 \oplus b_4x^5 \oplus b_3x^4 \oplus b_2x^3 \oplus b_1x^2 \oplus b_0x)$
Eeek! x^8 is too big. We must make it smaller.
* Remember that each b_n (e.g. b_7) is either 0 or 1.


We divide it by $m(x) = x^8 \oplus x^4 \oplus x^3 \oplus x \oplus 1$ and take the remainder:
b_7
$x^8 \oplus x^4 \oplus x^3 \oplus x \oplus 1$
$m(x)$
$b_7x^8 \oplus b_6x^7 \oplus b_5x^6 \oplus b_4x^5 \oplus b_3x^4 \oplus b_2x^3 \oplus b_1x^2 \oplus b_0x$
$\oplus$
$b_7x^8 \oplus b_7x^4 \oplus b_7x^3 \oplus b_7x \oplus b_7$
$b_6x^7 \oplus b_5x^6 \oplus b_4x^5 \oplus (b_3 \oplus b_7)x^4 \oplus (b_2 \oplus b_7)x^3 \oplus b_1x^2 \oplus (b_0 \oplus b_7)x \oplus b_7$
Remainder
$b_6x^7 \oplus b_5x^6 \oplus b_4x^5 \oplus b_3x^4 \oplus b_2x^3 \oplus b_1x^2 \oplus b_0x \oplus b_7 \cdot (x^4 \oplus x^3 \oplus x \oplus 1)$
Note how the b's are shifted left by 1 spot.
This is just b_7 multiplied by a small polynomial.

Now we're ready for the hardest blast from the past: logarithms. After logarithms, everything else is cake! Logarithms let us turn multiplication into addition:

$$\log(x\cdot y)=\log(x)+\log(y)$$

$$\text{So... } \log(10\cdot 100)=\log(10^1)+\log(10^2)$$

$$=2+1=3$$

In reverse:

$$\log^{-1}(1)=10^1=10$$
$$\log^{-1}(2)=10^2=100$$
$$\log^{-1}(3)=10^3=1{,}000$$

$$\Rightarrow 10\cdot 100=1{,}000$$

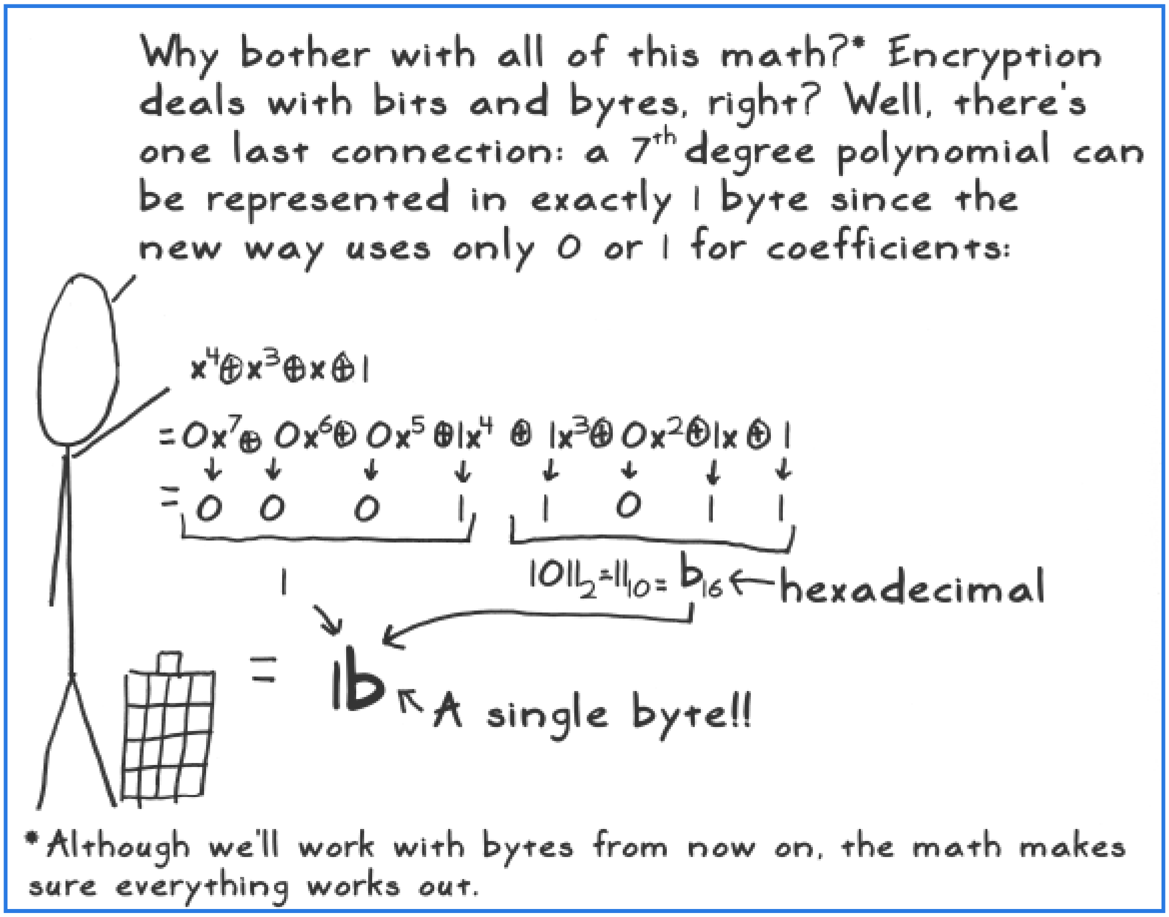
We can use logarithms in our new world. Instead of using 10 as the base, we can use the simple polynomial of $x\oplus1$ and watch the magic unravel.*

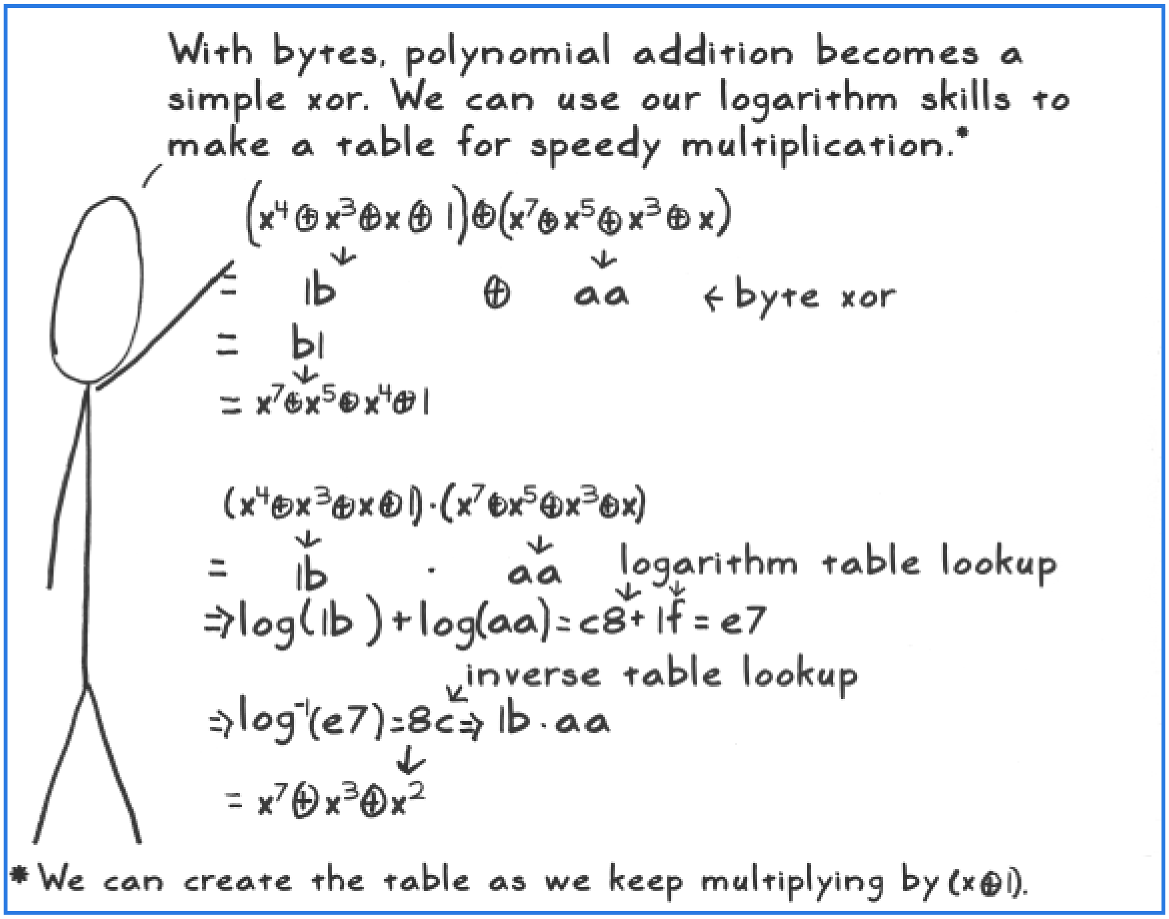
$$(x\oplus1)^1 = x\oplus1$$
$$(x\oplus1)^2 = (x\oplus1)(x\oplus1) = x^2\oplus x\oplus x\oplus1 = x^2\oplus1$$
$$(x\oplus1)^3 = (x\oplus1)^2\cdot(x\oplus1) = x^3\oplus x^2\oplus x\oplus1$$

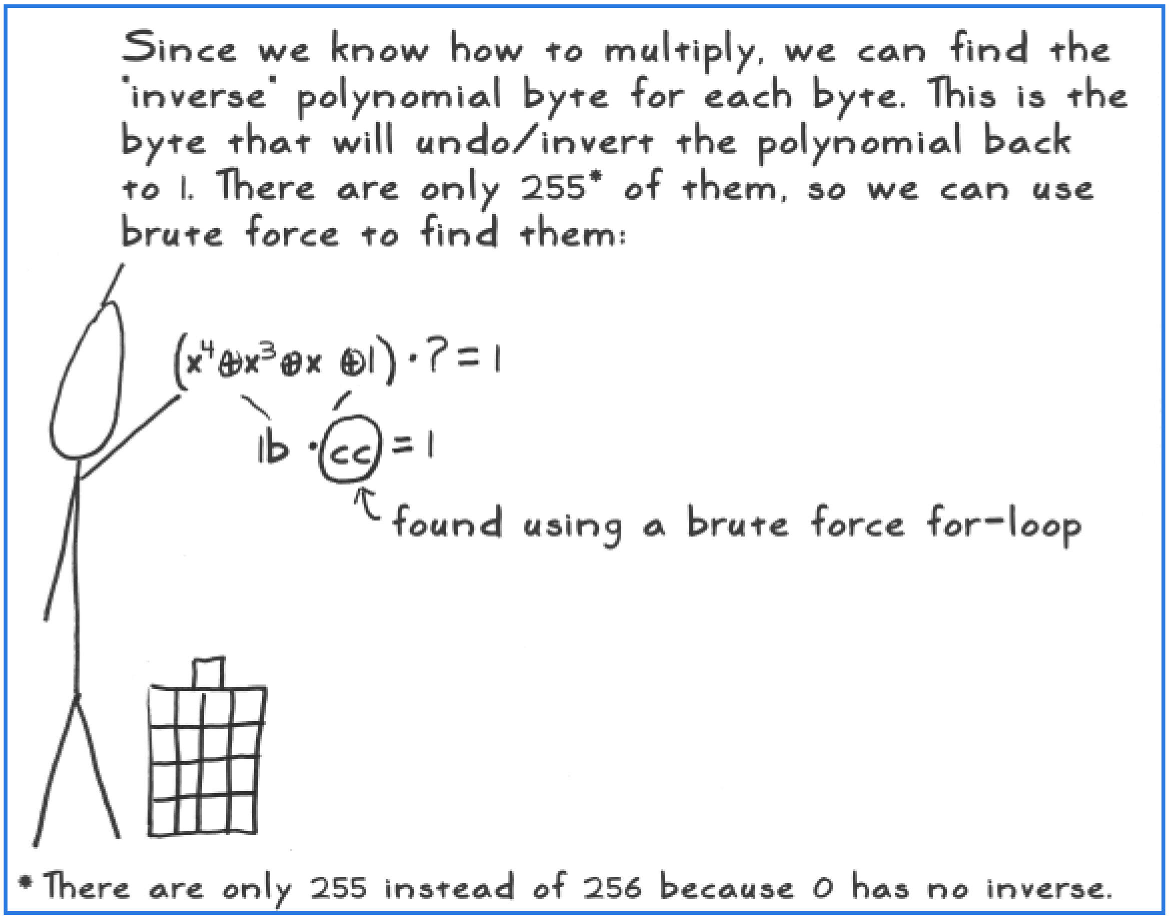
So...

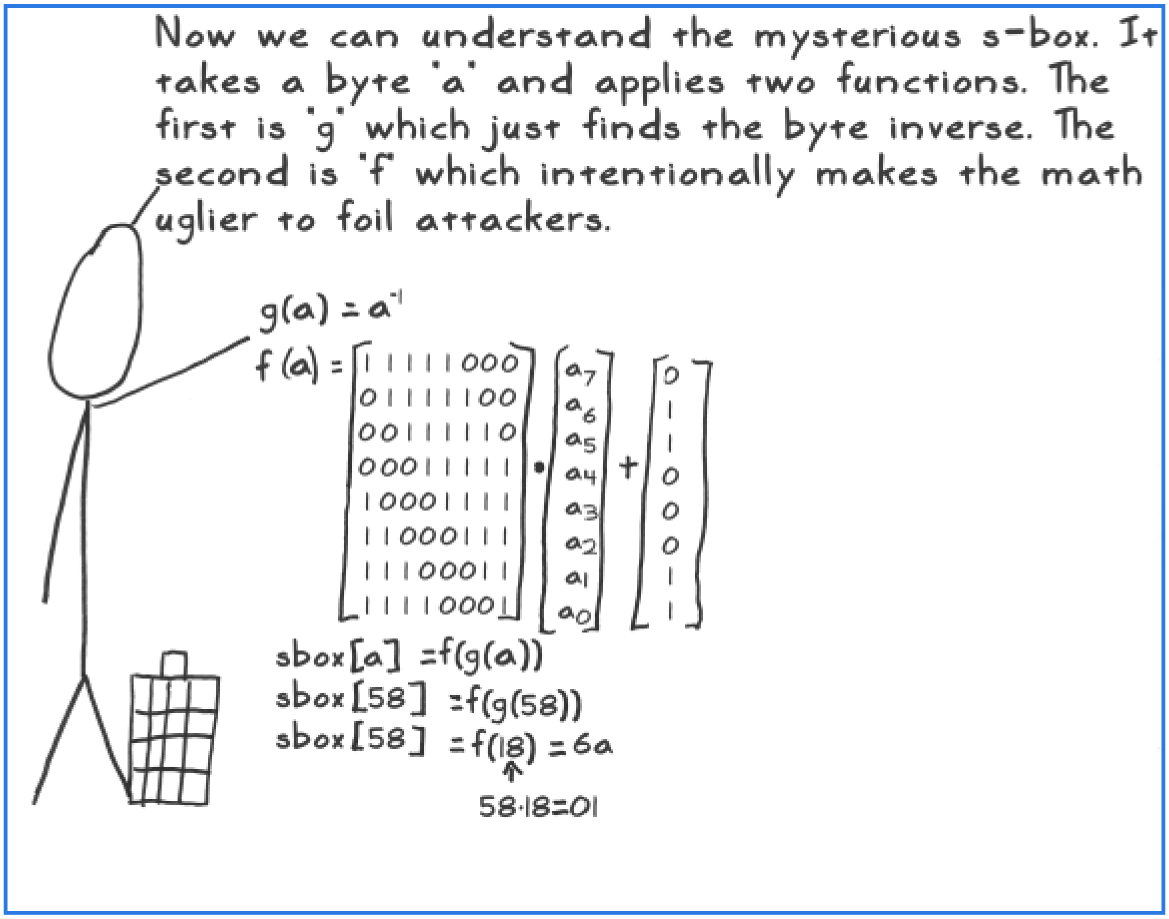
$$\log_{x\oplus1}(x\oplus1)=1,\ \log_{x\oplus1}(x^2\oplus1)=2,\ \log_{x\oplus1}(x^3\oplus x^2\oplus x\oplus1)=3$$

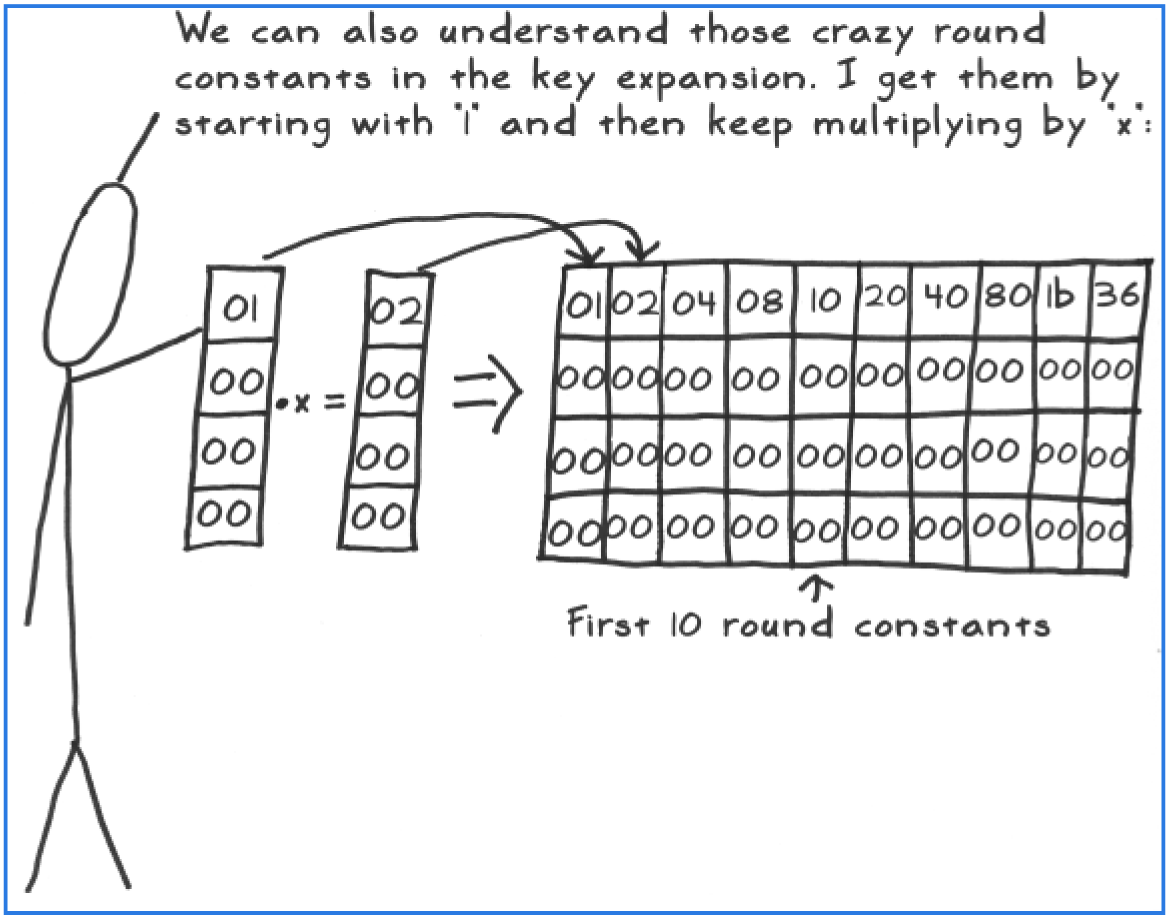
*If you keep multiplying by $(x\oplus1)$ and then take the remainder after dividing by $m(x)$, you'll see that you generate all possible polynomial below x^8. This is very important!

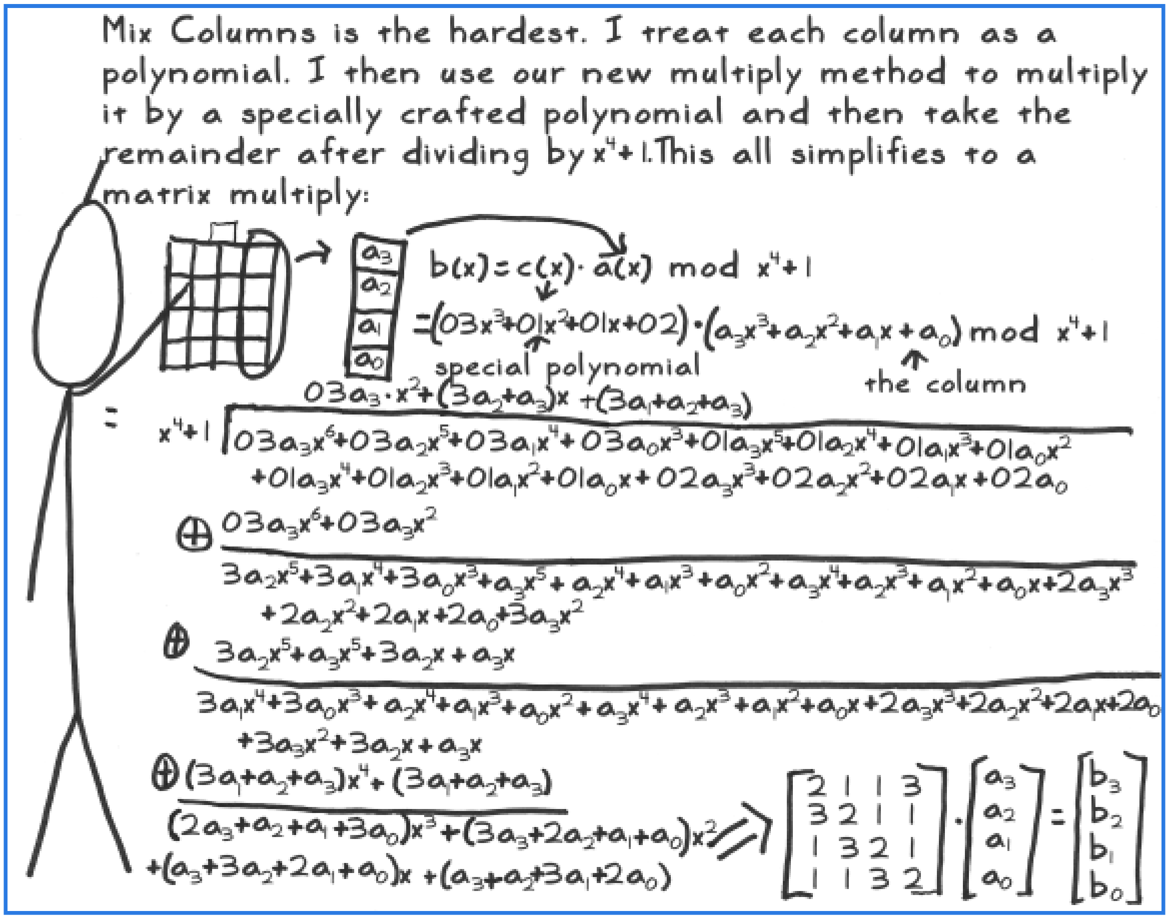

Why bother with all of this math?* Encryption deals with bits and bytes, right? Well, there's one last connection: a 7th degree polynomial can be represented in exactly 1 byte since the new way uses only 0 or 1 for coefficients:
$x^4 \oplus x^3 \oplus x \oplus 1$
$= 0x^7 \oplus 0x^6 \oplus 0x^5 \oplus 1x^4 \oplus 1x^3 \oplus 0x^2 \oplus 1x \oplus 1$
$= \underbrace{0\ 0\ 0\ 1}_{1}\ \underbrace{1\ 0\ 1\ 1}_{1011_2 = 11_{10} = b_{16}}$
← hexadecimal
$= 1b$
A single byte!!
* Although we'll work with bytes from now on, the math makes sure everything works out.

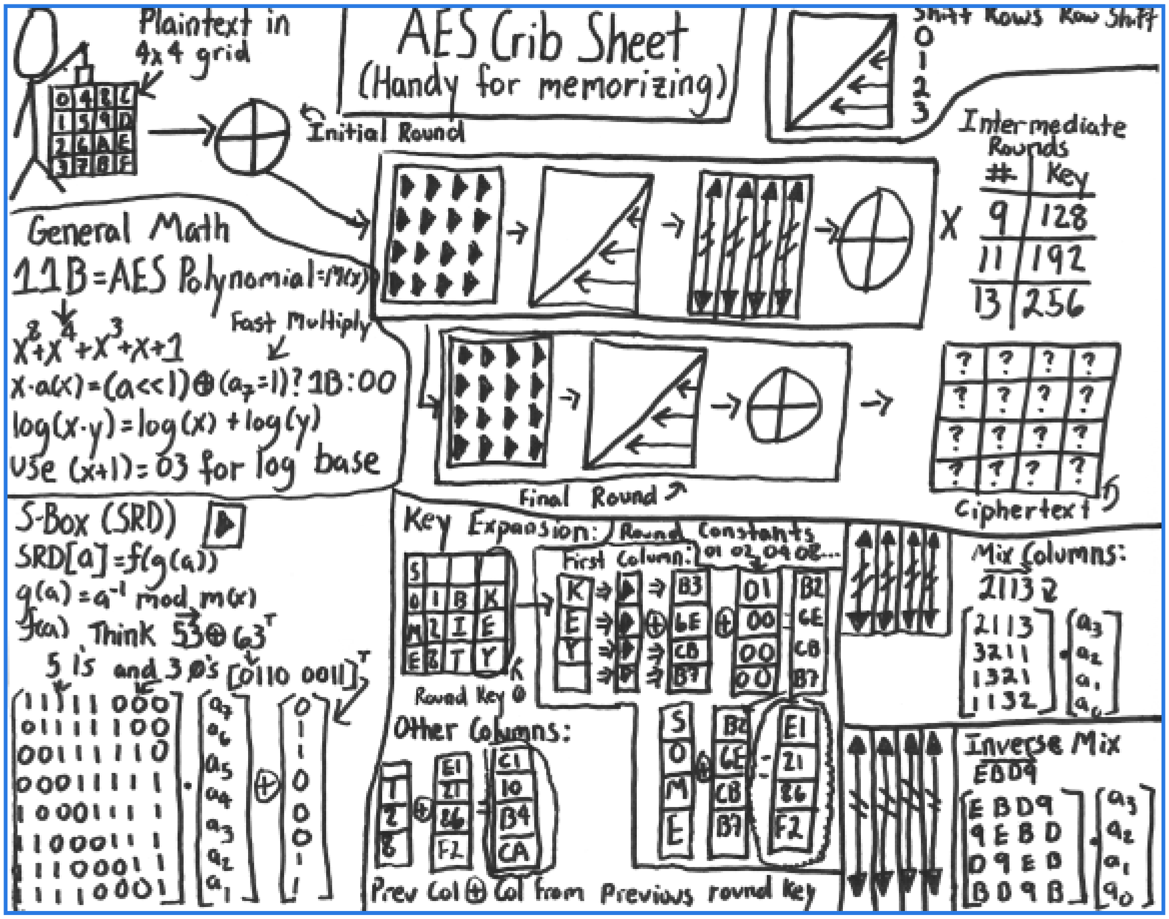

With bytes, polynomial addition becomes a simple xor. We can use our logarithm skills to make a table for speedy multiplication.*
$(x^4 \oplus x^3 \oplus x \oplus 1) \oplus (x^7 \oplus x^5 \oplus x^3 \oplus x)$
$= 1b \oplus aa$
← byte xor
$= b1$
$= x^7 \oplus x^5 \oplus x^4 \oplus 1$
$(x^4 \oplus x^3 \oplus x \oplus 1) \cdot (x^7 \oplus x^5 \oplus x^3 \oplus x)$
$= 1b \cdot aa$
logarithm table lookup
$\Rightarrow \log(1b) + \log(aa) = c8 + 1f = e7$
inverse table lookup
$\Rightarrow \log^{-1}(e7) = 8c \Rightarrow 1b \cdot aa$
$= x^7 \oplus x^3 \oplus x^2$
* We can create the table as we keep multiplying by $(x \oplus 1)$.

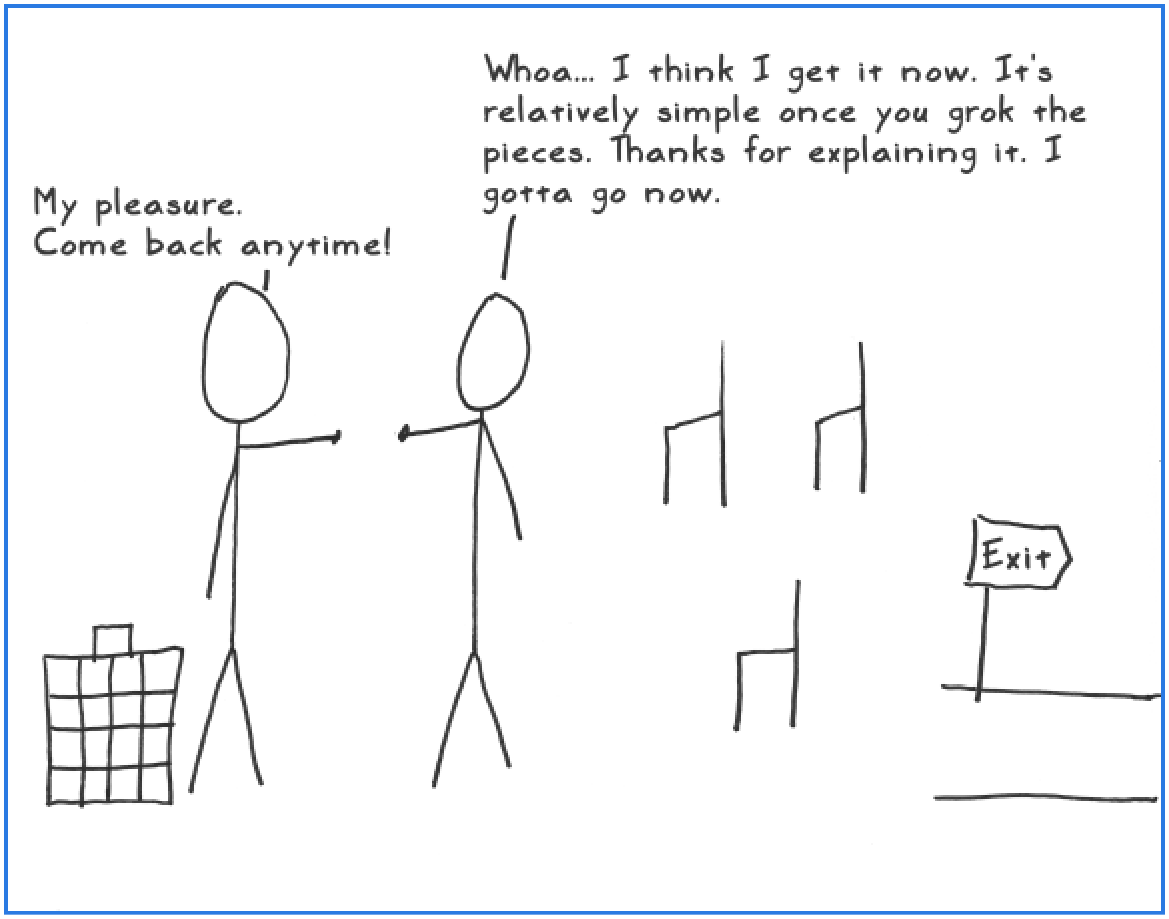

Since we know how to multiply, we can find the "inverse" polynomial byte for each byte. This is the byte that will undo/invert the polynomial back to 1. There are only 255* of them, so we can use brute force to find them:
$(x^4 \oplus x^3 \oplus x \oplus 1) \cdot ? = 1$
$1b \cdot \text{cc} = 1$
found using a brute force for-loop
* There are only 255 instead of 256 because 0 has no inverse.

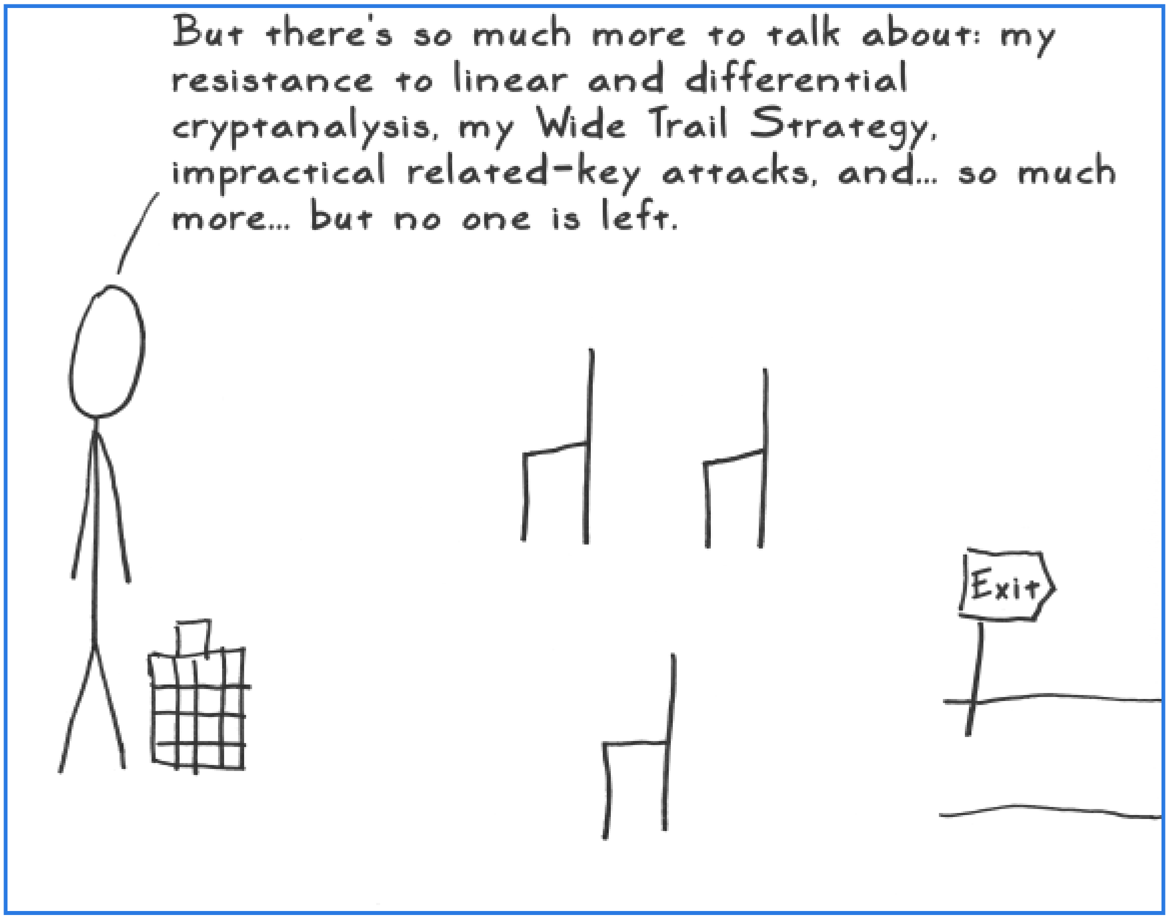

Now we can understand the mysterious s-box. It takes a byte "a" and applies two functions. The first is "g" which just finds the byte inverse. The second is "f" which intentionally makes the math uglier to foil attackers.
$g(a) = a^{-1}$
$f(a) = \begin{bmatrix} 11111000 \\ 01111100 \\ 00111110 \\ 00011111 \\ 10001111 \\ 11000111 \\ 11100011 \\ 11110001 \end{bmatrix} \bullet \begin{bmatrix} a_7 \\ a_6 \\ a_5 \\ a_4 \\ a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$
sbox[a] = f(g(a))
sbox[58] = f(g(58))
sbox[58] = f(18) = 6a
58·18=01

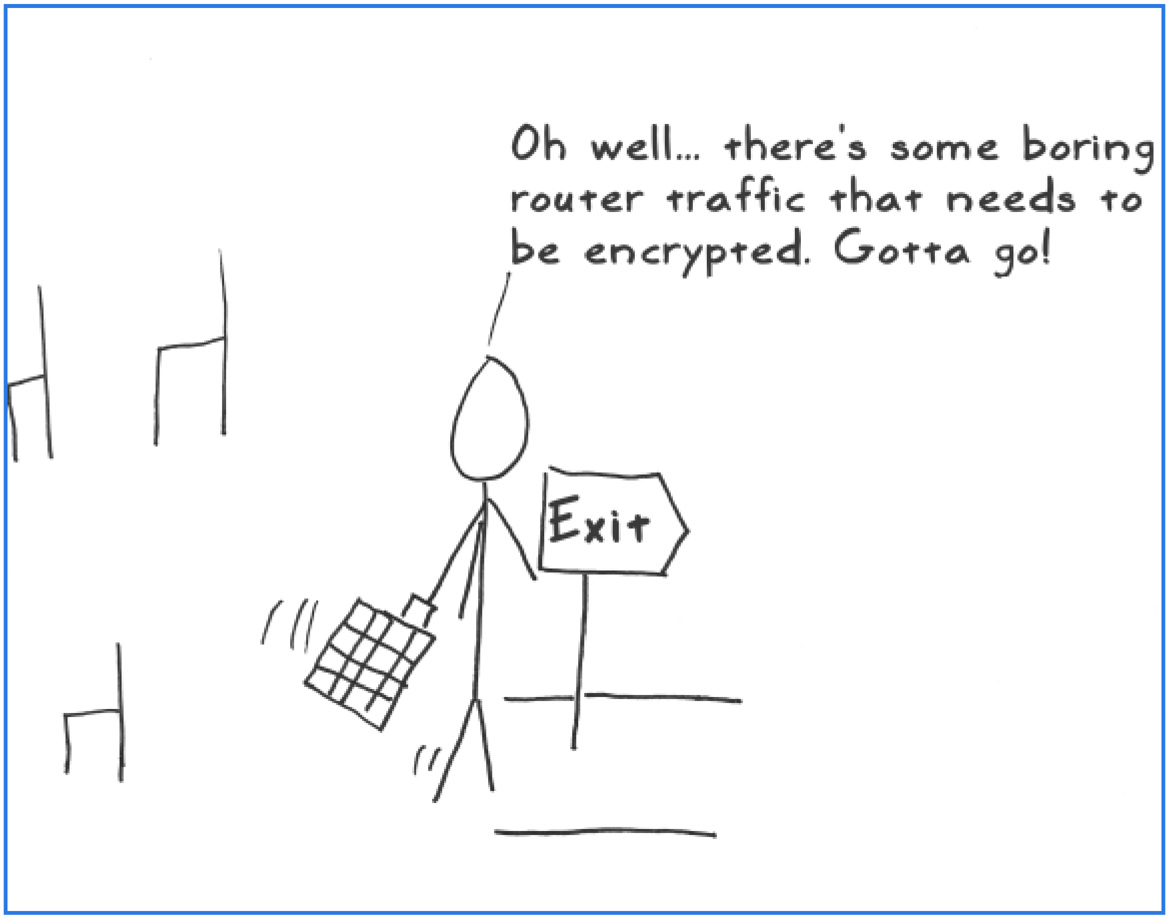

We can also understand those crazy round constants in the key expansion. I get them by starting with "1" and then keep multiplying by "x":
01 00 00 00
$\bullet x =$
02 00 00 00
$\Rightarrow$
01 02 04 08 10 20 40 80 1b 36
00 00 00 00 00 00 00 00 00 00
00 00 00 00 00 00 00 00 00 00
00 00 00 00 00 00 00 00 00 00
First 10 round constants

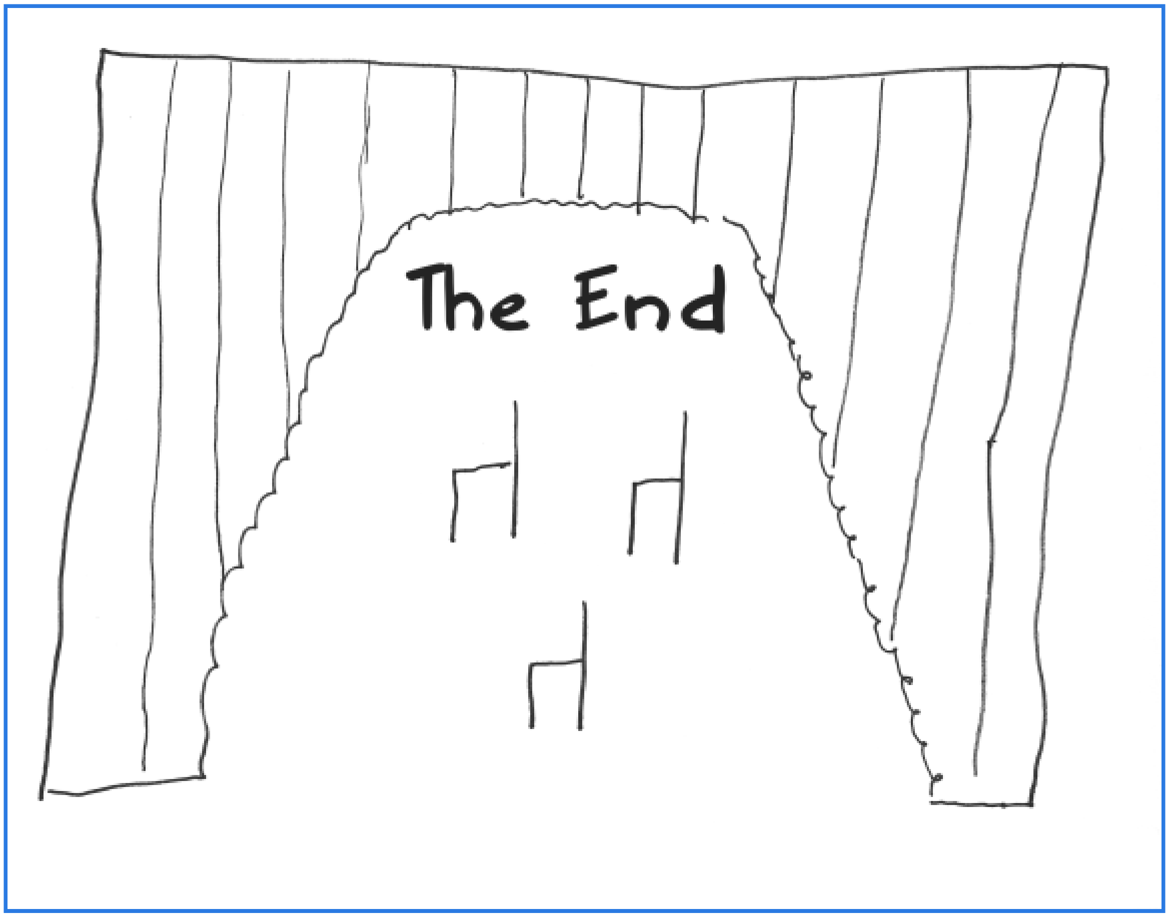

Mix Columns is the hardest. I treat each column as a polynomial. I then use our new multiply method to multiply it by a specially crafted polynomial and then take the remainder after dividing by x^4+1. This all simplifies to a matrix multiply:
a_3 a_2 a_1 a_0
$b(x) = c(x) \cdot a(x) \bmod x^4+1$
$= (03x^3+01x^2+01x+02) \cdot (a_3x^3+a_2x^2+a_1x+a_0) \bmod x^4+1$
special polynomial
the column
$03a_3 \cdot x^2 + (3a_2+a_3)x + (3a_1+a_2+a_3)$
$= x^4+1 \,\overline{)\, 03a_3x^6+03a_2x^5+03a_1x^4+03a_0x^3+01a_3x^5+01a_2x^4+01a_1x^3+01a_0x^2}$
$+01a_3x^4+01a_2x^3+01a_1x^2+01a_0x+02a_3x^3+02a_2x^2+02a_1x+02a_0$
$\oplus\ 03a_3x^6+03a_3x^2$
$3a_2x^5+3a_1x^4+3a_0x^3+a_3x^5+a_2x^4+a_1x^3+a_0x^2+a_3x^4+a_2x^3+a_1x^2+a_0x+2a_3x^3$
$+2a_2x^2+2a_1x+2a_0+3a_3x^2$
$\oplus\ 3a_2x^5+a_3x^5+3a_2x+a_3x$
$3a_1x^4+3a_0x^3+a_2x^4+a_1x^3+a_0x^2+a_3x^4+a_2x^3+a_1x^2+a_0x+2a_3x^3+2a_2x^2+2a_1x+2a_0$
$+3a_3x^2+3a_2x+a_3x$
$\oplus\ (3a_1+a_2+a_3)x^4+(3a_1+a_2+a_3)$
$(2a_3+a_2+a_1+3a_0)x^3+(3a_3+2a_2+a_1+a_0)x^2$
$+(a_3+3a_2+2a_1+a_0)x+(a_3+a_2+3a_1+2a_0)$
$\Rightarrow \begin{bmatrix} 2 & 1 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 2 & 1 \\ 1 & 1 & 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} b_3 \\ b_2 \\ b_1 \\ b_0 \end{bmatrix}$


Plaintext in 4x4 grid
0 4 8 C
1 5 9 D
2 6 A E
3 7 B F
Initial Round
AES Crib Sheet (Handy for memorizing)
Shift Rows Row Shift
0
1
2
3
Intermediate Rounds
# Key
9 128
11 192
13 256
X
General Math
11B = AES Polynomial = m(x)
x^8+x^4+x^3+x+1
Fast Multiply
x·a(x) = (a<<1) ⊕ (a_7=1) ? 1B : 00
log(x·y) = log(x) + log(y)
Use (x+1) = 03 for log base
Final Round
? ? ? ?
Ciphertext
S-Box (SRD)
SRD[a] = f(g(a))
g(a) = a^-1 mod m(x)
f(a) Think 53 ⊕ 63^T
5 1's and 3 0's [0110 0011]^T
11111000
01111100
00111110
00011111
10001111
11000111
11100011
11110001
a7 a6 a5 a4 a3 a2 a1
0 1 1 0 0 0 1 1
Key Expansion:
Round Constants 01 02 04 08...
S
0 1 B K
M 2 I E
E B T Y
K
Round Key 0
First Column:
K E Y
B3 6E CB B7
01 00 00 00
B2 6E CB B7
Other Columns:
1 2 6
E1 21 86 F2
C1 10 B4 CA
S O M E
B2 6E CB B7
E1 21 86 F2
Prev Col ⊕ Col from Previous round key
Mix Columns:
2113
2113
3211
1321
1132
a3 a2 a1 a0
Inverse Mix
EBD9
E B D 9
9 E B D
D 9 E B
B D 9 B
a3 a2 a1 a0


Whoa... I think I get it now. It's relatively simple once you grok the pieces. Thanks for explaining it. I gotta go now.
My pleasure. Come back anytime!
Exit

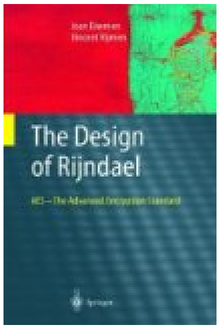

But there's so much more to talk about: my resistance to linear and differential cryptanalysis, my Wide Trail Strategy, impractical related-key attacks, and... so much more... but no one is left.
Exit


Oh well... there's some boring router traffic that needs to be encrypted. Gotta go!
Exit



Epilogue

I created a heavily-commented AES/Rijndael implementation to go along with this post and put it on GitHub. In keeping with the Foot-Shooting Prevention Agreement, it shouldn't be used for production code, but it should be helpful in seeing exactly where all the numbers came from in this play. Several resources were useful in creating this:



- The Design of Rijndael is *the* book on the subject, written by the Rijndael creators. It was helpful in understanding specifics, especially the math (although some parts were beyond me). It's also where I got the math notation and graphical representation in the left and right corners of the scenes describing the layers (SubBytes, ShiftRows, MixColumns, and AddRoundKey).
- The FIPS-197 specification formally defines AES and provides a good overview.
- The Puzzle Palace, especially chapter 9, was helpful while creating Act 1. For more on how the NSA modified DES, see this.
- More on Intel's (and now AMD) inclusion of native AES instructions can be found here and in detail here. - Other helpful resources include Wikipedia, Sam Trenholme's AES math series, and this animation.

Please leave a comment if you notice something that can be better explained.

Update #1: Several scenes were updated to fix some errors mentioned in the comments.
Update #2: By request, I’ve created a slide show presentation of this play in both PowerPoint and PDF formats. I’ve licensed them under the Creative Commons Attribution License so that you can use them as you see fit. If you’re teaching a class, consider giving extra credit to any student giving a worthy interpretive dance rendition in accordance with the Foot-Shooting Prevention Agreement.

251 Comments **Moserware** 1 **Login**

♡ **Recommend** 43 **Share** **Sort by Best**

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OR SIGN UP WITH DISQUS ?

Name

Chandan Jha • 3 months ago

Wow! That was an awesome explanation. Thanks for the PPT and PDF... Good contribution... Cheers!

PS: couldn't manage with the Maths part but still went through it. ;-) need to improve my maths skills.

6 ⌃ | ⌄ • Reply • Share ›

Anonymous • 8 years ago

Should be be in the preface of CS cryptography textbooks! Terrific

4 ⌃ | ⌄ • Reply • Share ›

Adam M. Erickson • 4 years ago

This should be in CS and ISM textbooks. A+ Moser.

1 ⌃ | ⌄ • Reply • Share ›

James McCune • 14 days ago

Awesome content man!

Really helped me out

⌃ | ⌄ • Reply • Share ›

Leonard Michael Hurlocker • 2 months ago

Love it! Thx.

⌃ | ⌄ • Reply • Share ›

^ | v • Reply • Share ›

Hope Ordu • 3 months ago

Lovely explanation. Couldn't have understood any better

^ | v • Reply • Share ›

Matthew Ewer • 5 months ago

In the big chart, bottom right, in the bottom right of the Inverse Mix table, I think that B should be an E.

^ | v • Reply • Share ›

Lightning342 • 5 months ago

Great epilogue, I especially liked the math parts that visualize the operations. Just one issue; in both encryption and decryption the last round is always the one with three steps (without mixcolumns). Check the pseudo code in NIST.FIPS.197 (figure 5 & 12), or search the Internet for images on "simplified aes decrypt" . So aes_act_3_scene_20_decrypting_1100 is incorrect.

^ | v • Reply • Share ›

Sleem A. Lama • 6 months ago

This is awesome.. Thank you

^ | v • Reply • Share ›

ATA UR REHMAN • 10 months ago

Seriously One great article.. Loved it (Y)

^ | v • Reply • Share ›

Rami Stefanidis • a year ago

Very cool article. Thank you

^ | v • Reply • Share ›

Gidz Paul • a year ago

Awesome.. :o

^ | v • Reply • Share ›

Walter Zambotti • a year ago

Very nice. Did I miss where in the presentation you display the final encrypted text???

^ | v • Reply • Share ›

netdeamon • 2 years ago

Cool !!! Keep it up!

^ | v • Reply • Share ›

Felipe S Mattos • 2 years ago

this is rocksome! very nice work!

^ | v • Reply • Share ›

⌃ ⌄ • Reply • Share ›

assignment writing • 2 years ago

It's actually a good work that you have shared which covers up some topics about that kind of subject that was being tackled in school for those students who studies in the field of engineering. Through this, they can learn something that they will going to use as their guide.

⌃ ⌄ • Reply • Share ›

Benjamin Barenblat • 3 years ago

A minor error in act 3, scene 11: entry a_{01} of the right block should read ‘fb’, not ‘f8’. (This has already been corrected in scenes 12 and 13.)

⌃ ⌄ • Reply • Share ›

Kevin • 4 years ago

Thanks man!! :)

⌃ ⌄ • Reply • Share ›

Raymond Starkey • 4 years ago

Talent, talent, talent, talent. Art - must try harder.

⌃ ⌄ • Reply • Share ›

Theuns Alberts • 4 years ago

Brilliantly explained! And thanks for the effort with the accompanied code.

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Todd • 4 years ago

Awesome. But where is the part where the NSA builds in a back door to circumvent all the cryptology?

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Anonymous • 4 years ago

was fun........gr8.......learning it and signng the letter....

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Anonymous • 5 years ago

Can't figure out if ALL these have any relation at all with this one:

"The key to this encryption rule is given by two numbers n and r. The number n is chosen in a very particular way: n is the product of two primes p and q. where n is the product of two primes p and q. To encrypt x we just compute: y = x^r(mod n).
The decryption then works via a simple formula, analogous to the encryption: we compute y^s(mod n).
Suppose we choose n = p * q = 29 * 37 = 1073. Let's take r = 25 with this choice of n and r, the choice s = 121 is an appropriate decryption key...
The decryption illustrated on the previous page is possible because r and s have a very special relationship. With p = 29. and q = 37. we compute:

m = (p-1) * (q-1) = 1008 and then we have chosen r and s so that:
r*s = 25 * 121 = 3025 = 1 (mod m)..."

Link: http://web.math.princeton.e...

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otis root ➦ Anonymous • a year ago

That is RSA

⌃ | ⌄ • Reply • Share ›

Cường Đặng Đình • 5 years ago

thanks very much!!!!!!!!!

⌃ | ⌄ • Reply • Share ›

Trung Le • 5 years ago

That was a really good work you have done! Helped me a ton to understand AES in a visual way. Keep it up!

⌃ | ⌄ • Reply • Share ›

KF • 5 years ago

Thank you for your awesome job... really let me understand much. This is fun too. Appreciate it very much.

⌃ | ⌄ • Reply • Share ›

Jeff Moser • 5 years ago

Thanks everyone for the kind feedback!

Anonymous: For details about being resistant to cryptanalysis, I recommend reading the book but it's quite a bit more complicated than this comic. Perhaps try the Wikipedia page. Regarding modes, Wikipedia's page on it is pretty good.

⌃ | ⌄ • Reply • Share ›

Charulatha Jain • 5 years ago

Its an awesome presentation and quite helpful for beginners to understand the concepts. Thank you for the awesome presentation.

⌃ | ⌄ • Reply • Share ›

Anonymous • 5 years ago

This is one of the most superb explanations of AES I've ever seen. So easy to understand, it's brilliant. But I'd love to know more about the resistance to cryptanalysis, I understand what it is but I have no idea how AES resists it. Plus I'd like to know more about the various "modes" you touched on!

In summary: encore!

⌃ | ⌄ • Reply • Share ›

durdave • 5 years ago

Great! But I'm still left with the question "Will Ashley go out with me?" Maybe if I get a Ferrari?

^ | ˅ • Reply • Share ›

Anonymous • 5 years ago

Superb. Keep it up Brother.

^ | ˅ • Reply • Share ›

Anonymous • 5 years ago

Amazing would be an understatement!

^ | ˅ • Reply • Share ›

naveregnide • 6 years ago

Hey! Just stopping by to say that this lovely comic really helped me understand AES a lot more. Such a great sense of humour used in it too! Thanks!

^ | ˅ • Reply • Share ›

Jeff Moser • 6 years ago

Thanks everyone for the kind feedback over the years.

Special thanks to **txipxi** for the Spanish translation.

Vincent: For details on how all S-Box values are calculated, follow along with the S-Box demonstration section my example program and the f(x) and g(x) section as well. For even more details, see how I implemented f and g. Note specifically how "f" is a multiply then an add/xor. Each bit of the result of the multiply is obtained through 8 boolean multiplies/ands.

Hope that helps!

^ | ˅ • Reply • Share ›

Vincent • 6 years ago

Hi Jeff, is it possible to show your working on f({18})={6a}? I have been trying to work out the rotational matrix part but still getting the answer wrong. Did you sum up all the 8 rows of multiplication before you XOR with {63}?

Is it possible can anyone show the working for multiple inverse for {58} is {18}?

^ | ˅ • Reply • Share ›

Krishnaprasad • 6 years ago

This is great.. lot of effort to explain, very easy to understand!! awesome..

^ | ˅ • Reply • Share ›

Anonymous • 6 years ago

Wonderful! Great work!

Wonderful! Great work!
Makes understanding AES a lot of fun!
Genius!

^ | ⌄ • Reply • Share ›

txipi • 6 years ago

Awesome!!!

My humble contribution to your great work, a Spanish translation:
http://www.slideshare.net/t...

^ | ⌄ • Reply • Share ›

The Grey Man • 6 years ago

WOW. Thanks for the excellent, amusing and visual run down on AES.

^ | ⌄ • Reply • Share ›

SherryL • 6 years ago

Wow, it's great to explain the AES this way. I've tried much to understand AES before I found this. BRAVO! Thanks!

^ | ⌄ • Reply • Share ›

Anonymous • 6 years ago

I would like to read more like this..it was a fun way of learning for an adult.

^ | ⌄ • Reply • Share ›

Anonymous • 6 years ago

Thanks for this.

^ | ⌄ • Reply • Share ›

Jeff Moser • 6 years ago

20 box: It's a decent book if you want a very academic overview of AES. It was a bit *too* academic for my tastes and was one of the reasons why I created this post.

If you're looking for a more hands-on/pragmatic understanding of AES, I'd recommend you just read and understand the source code that I included with this post.

^ | ⌄ • Reply • Share ›

20 box • 6 years ago

Is that book you recommended is good? I am considering buying it here in India but it is too costly a book for Indian standards and is not available on local markets.. so have to buy it from amazon or something.. but too costly..

Just want to know if it is worth $100 or not? as it would be the price for me including shipping...

^ | ⌄ • Reply • Share ›

Sandoval • 7 years ago

wow!

99.9% of comp professionals never can talk in simple English; BUT you've just proved that you are the 0.1%!

⌃ | ⌄ • Reply • Share ›

&lt;Martani/&gt; Fakhrou • 7 years ago

Totally awesome,

this is gonna help me in my crypto exam tomorrow :D

⌃ | ⌄ • Reply • Share ›

arun • 7 years ago

hey man..a very well explained..good job..
well m also working on AES and m really a bad prograamer..My work is with Equivalent Inverse Cipher have you worked on it...got nay code of it????It's really urgent......pls

⌃ | ⌄ • Reply • Share ›

Viviana • 7 years ago

Thanks a LOT for this explanation!
I nearly had given up trying to understand AES when I found that..!

⌃ | ⌄ • Reply • Share ›

Anonymous • 7 years ago

After reading this I am not sure if I successfully crypt-ed or decrypt-ed my knowledge about AES.