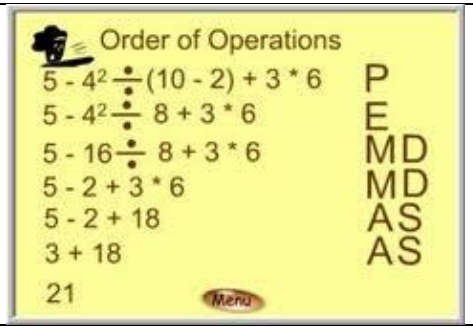
ACT Math Concepts to Know

Just Numbers

Terms

√?	Concept	Explanation	Example/Visual
	Undefined	An expression is undefined **when the denominator equals zero**.	If f(x) = $\frac{a+b}{x-c}$, for what value of x would this function be undefined? *Answer: **c**, because c - c = 0*
	Imaginary number	• To take the square root of a negative number, first take the square root of the number as if it were positive, **then add "i"** • $i^2 = -1$	Solve: $\sqrt{3x} = 6i$ 1) Square both sides → 3x = -6 2) Divide both sides by 3 → **x = -2**
	Integers	Whole numbers, including negative numbers and zero	-2, 0, 3, 7
	Rational/ Irrational number	Rational – Can be expressed as a decimal or fraction Irrational – Cannot be expressed as a decimal or fraction	Rational - .45, ¾ Irrational - $\sqrt{2}, \sqrt{3}, \pi$
	Adding and subtracting negative numbers	Adding a positive and a negative – 1) Ignore the signs 2) Subtract the smaller number from the larger number 3) Put on the sign from the larger number Subtracting negative numbers – 1) Change the subtraction into addition	-38 + 25 1) 38 - 25 = 13 2) -38 is larger than 25, so the answer is **-13** 13 - (-23) = 13 + (+23) = **36**
	Multiplying and dividing with negative numbers	1) Ignore the signs and do the problem without signs 2) If there is an odd number of negative signs, add a negative sign to the final answer	-3 x -5 x -2 1) 3 x 5 x 2 = 30 2) Odd number of negative signs → **-30**

	PEMDAS/ Order of operations	1) Parentheses 2) Exponents 3) Multiplication and Division 4) Addition and Subtraction	
	Absolute value	A number inside an absolute value sign becomes positive 1) Do what's inside the absolute value sign first 2) Make the result positive	|2 - 5| = |-3| = **3**
	The number of integers from one number to another	Subtract the two numbers and then add 1 We need to add 1 to include the first number	How many integers are there from 12 to 25? 25 - 12 = 13 13 + 1 = 14

Divisibility

√?	Concept	Explanation	Example/Visual
	Factors and Multiples	Factor – A number that divides into an integer with no remainder Multiple – A number that the integer divides into with no remainder	**Factors of 20:** 1, 2, 4, 5, 10, 20 **Multiples of 6:** 6, 12, 18, 24, 30, 36…
	Prime number	The only factors are 1 and the number. Note: 1 is NOT a prime number	2, 3, 5, 7, 13…
	Prime Factorization	Keep breaking a number into factors until all the factors are prime	Prime factorization of 48: 48 = 12 x 4 = (3 x 4) x (2 x 2) = **3 x 2 x 2 x 2 x 2**

	Least Common Multiple	The lowest number that is a multiple of both numbers 1) Find multiples of the larger number until you get to one that is also a multiple of the smaller number.	The LCM of 12 and 15: 15, 30, 45, **60** **60** is also a multiple of 12 The LCM is **60**
	Greatest Common Factor	The highest number that is a factor of both numbers 1) Figure it out mentally OR 1) Break each number into its prime factors 2) Multiple the prime factors they have in common	The GCM of 16 and 24 **Prime factorization of 16:** 2 x 2 x 2 x 2 **Prime factorization of 24:** 2 x 2 x 2 x 3 16 and 24 have **2 x 2 x 2** in common The GCM is 2 x 2 x 2 = **8**
	Even or Odd?	To find out if an answer will be even or odd, just plug in simple numbers like 1 and 2	Will $2x^3 + 1$ be odd or even? x = 1 → 2 + 1 = 3 x = 2 → 16 + 1 = 17 $2x^3 + 1$ will always be **odd**
	Is a number divisible by 2, 3, 4, 5, 9, and 10?	Divisible by… 2 – If the last digit is even 3 – If the sum of the digits is divisible by 3 4 – If the last two digits are divisible by 4 5 – If the last digit is 5 or 0 9 – If the sum of the digits is divisible by 9 10 – If the last digit is 0	 36 → the last digit, 6, is even 357 → 3 + 5 + 7 = 15 (15 is divisible by 3) 524 → 24 is divisible by 4 55 → the last digit is 5 396 → 3 + 9 + 6 = 18 (18 is divisible by 9) 730 → the last digit is 0
	Remainder	The whole number left over after division	14/4 = 3 Remainder 2

Fractions and Decimals

√?	Concept	Explanation	Example/Visual
	Reducing fractions	Cancel out all the factors that the numerator (top number) and the denominator (bottom number) have in common	$\frac{15}{20} = \frac{3 \times \cancel{5}}{4 \times \cancel{5}} = \mathbf{\frac{3}{4}}$
	Adding and subtracting fractions	1) Find a common denominator 2) Add or subtract the numerators	$\frac{1}{4} + \frac{2}{5} = \frac{5}{20} + \frac{8}{20} = \mathbf{\frac{13}{20}}$

	Multiplying fractions	1) Multiply the numerators 2) Multiply the denominators	$\frac{3}{8} \times \frac{5}{7} = \frac{15}{56}$
	Dividing fractions	1) Flip the second fraction 2) Multiple the two fractions	$\frac{2}{5} \div \frac{7}{9} = \frac{2}{5} \div \frac{9}{7} = \mathbf{\frac{18}{35}}$
	Changing a mixed number to an improper fraction	1) Multiply the whole number by the denominator, then add the numerator 2) Put the number from #1 over the same denominator	$3\frac{3}{7} = \frac{(3 \times 7) + 3}{7} = \mathbf{\frac{24}{7}}$
	Changing an improper fraction to a mixed number	1) Divide the denominator into the numerator to get a whole number and a remainder 2) The whole number remains a whole number, and the remainder is the numerator. The denominator stays the same	$\frac{23}{5}$ 1) $23 \div 5 = 4 \text{ R } 3$ 2) $\mathbf{4\frac{3}{5}}$
	Reciprocal	Flip the numerator and the denominator	$\frac{2}{5} \rightarrow \mathbf{\frac{5}{2}}$
	Which fraction is greater?	1) Convert both fractions so they have a common denominator OR 2) Convert both fractions to decimals	$\frac{5}{8}$ or $\frac{9}{16} \rightarrow \frac{10}{16}$ or $\frac{9}{16} \rightarrow \mathbf{\frac{5}{8}}$ **is greater** **5 ÷ 8 = .625** 9 ÷ 16 = .5625
	Converting fractions to decimals	Divide the top number by the bottom number	9 ÷ 16 = .5625
	Finding a particular digit in a repeating decimal	1) Which digit are you trying to find? 2) How many digits are repeating? 3) Find the multiple of #2 that is closest to the digit you are trying to find, and then count up or down to find the digit you are looking for	1) To find the 101th digit of $\frac{6}{11}$ 2) $\frac{6}{11}$ = .54545454 (2 digits are repeating) 3) The digit of every multiple of 2 is 4. The 100th digit is 4. **The 101th digit is 5.**
	Percent formula	1) Change the percent into a decimal (divide it by 100) 2) Change "of" into a multiplication sign, and Change "is/are" into an equal sign 3) Complete the problem	70% of 50 is what? 1) 70% ÷ 100 = .7 2) .7 x 50 = ? 3) .7 x 50 = **35**

	Percent increase and decrease	**Percent increase –** 1) Convert the percent to a decimal, add 1 2) Multiply **Percent decrease –** 1) Convert the percent to a decimal 2) Multiply 3) Check if you need to subtract your answer from the original	$120 increased by 25% 1) 25% = .25 .25 + 1 = 1.25 1.25 x $120 = **$150** $120 decreased by 25% 1) 25% = .25 2) .25 x $120 = $30 3) $120 - $30 = **$90**
	Multiple increases and decreases	1) Start with 100 and then apply the increases and decreases	A price is increased by 20% and then the new price is decreased by 30%. What is the net change? **20% increase** → 100 x 1.20 = 120 **30% decrease** → 120 x .30 = 36 120 - 36 = 84 **Net change** → 100 - 84 = 16 **16% decrease**

Ratios, Proportions, and Rates

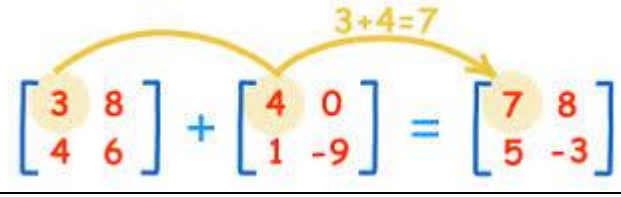
√?	Concept	Explanation	Example/Visual
	Setting up a ratio	1) Put the number after "of" on top 2) Put the number after "to" on bottom 3) Reduce	What is the ratio of 3 cats to 5 dogs? **Answer → 3:5**
	Solving a proportion	1) Cross multiply	$\frac{2}{5} = \frac{x}{15}$ 5x = 30 **x = 6**
	Solving rate problems	Rate x Time = Distance Average Rate = Total Distance / Total Time	A car travels 294 miles in 6 hours. What is the rate at which it is traveling? Rate x 6 = 294 **Rate = 49mph**

Averages

√?	Concept	Explanation	Example/Visual
	Average formula	Average = Sum of terms / Number of terms	What is the average of 25, 39, and 42? $\frac{25+39+44}{3} \rightarrow \frac{108}{3} \rightarrow \mathbf{36}$
	Average of evenly spaced numbers	Just average the smallest and largest numbers	What is the average of all the even numbers from 12 to 36? $\frac{12+36}{2} \rightarrow \frac{48}{2} \rightarrow \mathbf{24}$
	Using the average to find the sum	Sum = Average x Number of terms Then, subtract the numbers you already have to find the answer	Jim's average score after four tests is 88. What score on the fifth test would bring Martin's average up to exactly 90? **Answer:** 90 x 5 = 450 450 - 88 - 88 - 88 - 88 = **98**
	Counting the possibilities	Multiply the number of choices for the first thing by the number of choices for the second thing	John has 5 different shirts and 7 different pairs of pants. How many different combinations of shirts and pants can he have? **Answer → 5 x 7 = 35 different combinations**
	Probability	Number of items / Total number of items = Probability/Percentage	In a group of 30 students, 12 are male. What percentage of the group is male? **Answer → 12 / 30 = .4 → 40% are male**

Roots

√?	Concept	Explanation	Example/Visual
	Simplifying square roots	1) Factor out the perfect squares 2) Put the square root of the perfect square(s) in front of the radical	$\sqrt{27}$ $\sqrt{9 \times 3}$ $\mathbf{3\sqrt{3}}$
	Adding and subtracting roots	If the number under the radical is the same, you can add or subtract them	$2\sqrt{5} + 6\sqrt{5} = 8\sqrt{5}$

	Multiplying and dividing roots	You can multiply two different roots by first multiplying the numbers under the roots You can divide two different roots by first dividing the numbers under the roots	$\sqrt{3} \times \sqrt{5} = \sqrt{15}$ $\sqrt{15} \div \sqrt{5} = \sqrt{3}$
	Matrices – Adding and subtracting	Simply add or subtract the spaces that correspond to each other	

Algebra

Algebraic Expressions

√?	Concept	Explanation	Example/Visual
	Multiplying and Dividing Powers	To **multiply** powers with the same base: **add the exponents** To **divide** powers with the same base, **subtract the exponents**	$x^3 \times x^4 = x^7$ $x^5 \div x^3 = x^2$
	Raising powers to powers	**Multiply** the exponents	$x^3 \times x^4 = x^{12}$
	Evaluating an algebraic expression	Plug in the values for the unknown	If f(x) = $x^3 - x^2 + x$, what is the value of f(-2)? *Answer:* $(-2)^3 - (-2)^2 + (-2)$ (-8) - (4) + (-2) = **-14**
	Adding and subtracting algebraic expressions	Add and subtract like terms	2x + 3x = 5x $x^2 + 3x^2 = 4x^2$
	Multiplying monomials (one term by one term)	Multiply the coefficients and the variables separately	$3x^2 \times 5x^3 = 15x^5$
	Multiplying binomials (two terms by two terms) using FOIL	In this order, multiply the: 1) **First** terms 2) **Outside** terms 3) **Inside** terms 4) **Last** terms	(2x + 2)(x – 2) = (2x)(x) + (2x)(-2) + (2)(x) + (2)(-2) = $2x^2 + (-4x) + 2x + (-4)$ = **$2x^2 - 2x - 4$**

Factoring Algebraic Expressions

√?	Concept	Explanation	Example/Visual
	Factoring out a common divisor	If all the terms have a common factor, it can be factored out	$2x^2 - 8x$ = **2x(x - 4)**
	Factoring the difference of squares	The ACT likes to test this. x^2 - (number)^2 = (x - number)(x + number)	$x^2 - 9$ = **(x - 3)(x + 3)**

	Factoring the square of a binomial $(a + b)^2$ or $(a - b)^2$	The ACT likes to test this If the last number is a perfect square, check if the algebraic expression is a square of a binomial	$x^2 + 8x + 16 =$ **$(x + 4)^2$**
	Factoring other algebraic expressions	Think about what binomials you could use FOIL on to result with the algebraic expression 1) What **first** terms could get you the squared term? 2) What **last** terms could get you the number? 3) What combinations of first and last terms could get you the middle term?	$6x^2 - 16x + 8 =$ 1) 3 x 2 or 6 x 1 2) 4 x 2 or 8 x 1 3) (3 x 4) + (2 x 2) = 16 **$(3x^2 - 2)(2x^2 - 4)$**
	Simplifying an algebraic fraction	1) Factor the numerator and denominator 2) Cancel out factors that are in both the numerator and denominator	$\frac{x + 3}{x^2 + 5x + 6}$ $\frac{\cancel{(x + 3)}}{\cancel{(x + 3)}(x + 2)}$ $\frac{\mathbf{1}}{\boldsymbol{x + 2}}$

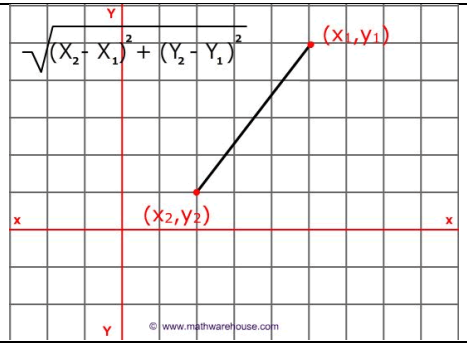
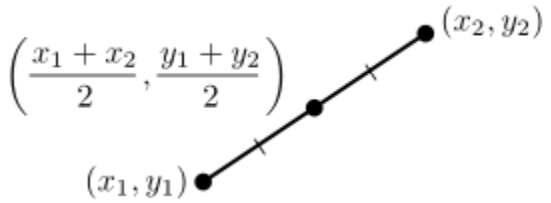
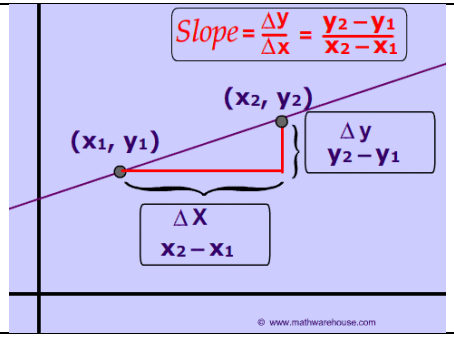
Solving Equations

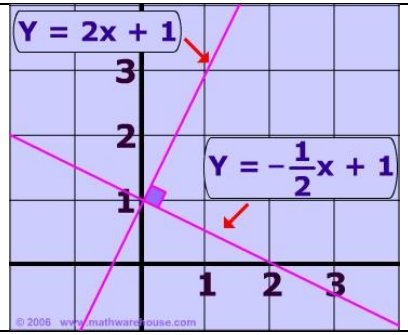
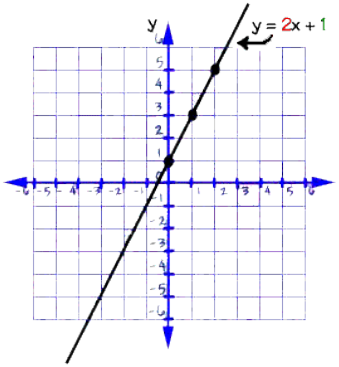
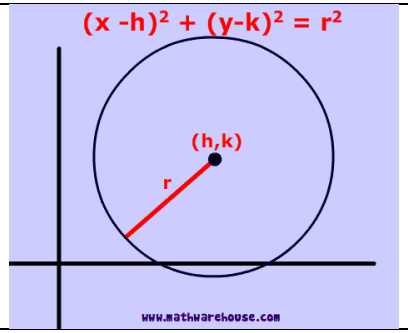
√?	Concept	Explanation	Example/Visual
	Solving a linear equation	1) Add and subtract terms to get the x terms on one side 2) Divide (or multiply) to solve for x	2x + 6 = 5x * Subtract 2x from both sides 6 = 3x **2 = x**
	Solving "in terms of"	To solve for one variable **in terms of** another, do the same thing above for the variable you are solving for. The other side will have the variable you are solving **in terms of**.	Solve for x in terms of y: 2x - y - 4 = y * add y to both sides 2x – 4 = 2y * divide both sides by 2 **x – 2 = y**

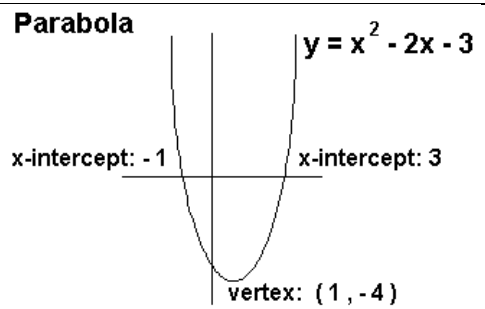
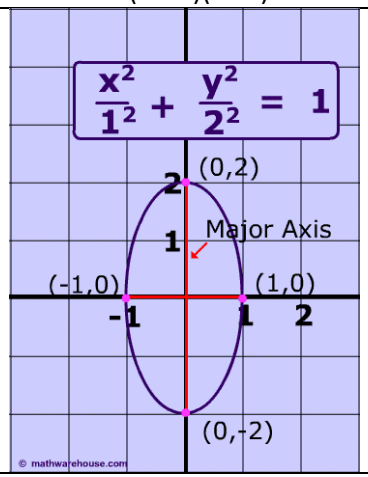
	Translating from English into algebra	1) Break the word problem into parts 2) Write out the algebraic expression for the different parts and then put them together according to what the problem is asking for	The toll for driving a segment of a certain freeway is $1.20 plus 20 cents for each mile traveled. John paid a $25.00 toll for driving a segment of the freeway. How many miles did he travel? **1.20 + .20(m) = 25** **.20(m) = 23.80** **m = 119 miles**
	Solving a quadratic equation	1) Factor the algebraic expression 2) Find out what values will make the expression equal zero OR 1) Use the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$x^2 + 5x + 6 = 0$ $(x + 2)(x + 3) = 0$ **x = -2 or -3** $x = \frac{-5 \pm \sqrt{5^2 - 4(1)(6)}}{2(1)}$ $x = \frac{-5 \pm \sqrt{1}}{2}$ $x = \frac{-4}{2}\ or\ \frac{-6}{2}$ $\boldsymbol{x = -2 \text{ or } -3}$
	Solving a system of equations	Combine the equations in a way that one of the variables cancels out	$2x + 3y = 20$ $+\ -2x + y = 4$ $0 + 4y = 24$ $4y = 24$ $y = 6$ Elimination $y = x + 5 \quad y = 2x + 2$ $x + 5 = 2x + 2$ $-x \quad -x$ $5 = x + 2$ $-2 \quad -2$ $3 = x$ $y = 2x + 2$ $y = 2(3) + 2 = 8$ **Solution: (3, 8)** Substitution
	Solving an equation that has absolute value signs	There will be two different answers: one that results in a positive number and one that results in a negative number	$\lvert 4x + 5 \rvert = 21$ $4x + 5 = 21$ $4x = 16$ $x = 4$ $4x + 5 = -21$ $4x = -26$ $x = -6.5$

	Solving an inequality	1) Solve for the variable as if it is a linear equation 2) If you multiply or divide by a negative number, flip the inequality sign	$-3n > 12$ If you divide or multiply by a negative number $\frac{-3n}{-3} > \frac{12}{-3}$ reverse the inequality symbol $n < -4$ Solution: all numbers less than -4
	Graphic inequalities on a number line	Use a solid circle if the point is included and an open circle of the point is not included	$<$ or $>$ use open circle $\leq$ or $\geq$ use closed circle Examples $X > 3$ $X < -7$ $X \geq 10$ $X \leq -2$
	Graphing inequalities on a grid	1) Choose a coordinate to plug into the equation 2) If the coordinate makes the inequality true, shade in that side of the line/curve. If not, shade in the other side.	Graph using same steps as linear inequalities $y < 2x^2 + 4x - 8$ boarder is - - - - Test a point, I like (0,0) $y < 2x^2 + 4x - 8$ $0 < 2(0)^2 + 4(0) - 8$ $0 < -8$ False statement- shade under parabola

Coordinate geometry

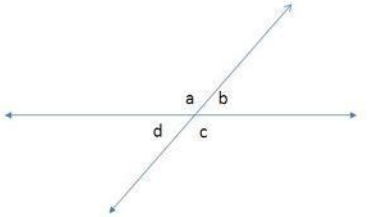
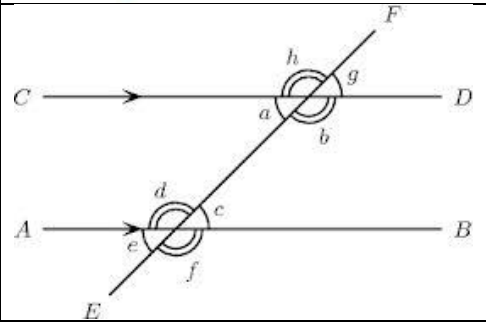
√?	Concept	Explanation	Example/Visual
	Finding the distance between two points	1) Use the Pythagorean Theorem $(x_1 - x_2)^2 + (y_1 - y_2)^2 = c^2$ OR 2) Use special right triangles	
	Finding the midpoint	$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$	
	Using two points to find the slope	Slope = Change in y / Change in x	

	A line perpendicular to the slope	Take the reciprocal of the slope and change the sign	
	Using an equation to find the slope	Use the slope-intercept form y = mx + b m is the slope, b is the y-intercept	
	Using an equation to find the intercept	To find the y-intercept: Plug in 0 for x and solve for y To find the x-intercept: Plug in 0 for y and solve for x	
	Equation for a circle	$(x - h)^2 + (y - k)^2 = r^2$ r = radius (h,k) is the center of the circle	

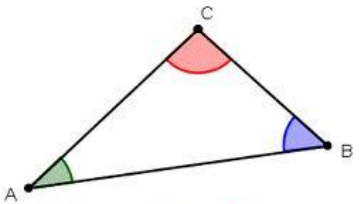
	Equation for a parabola	$y = ax^2 + bx + c$	 Factored: (x + 1)(x - 3)
	Equation for an ellipse	$x^2/a^2 + y^2/b^2 = 1$ a = x-intercepts b = y-intercepts	

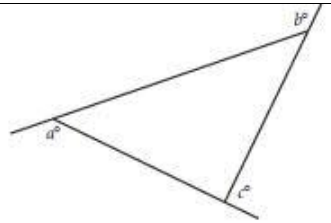
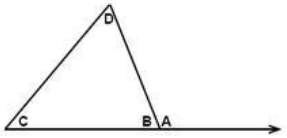
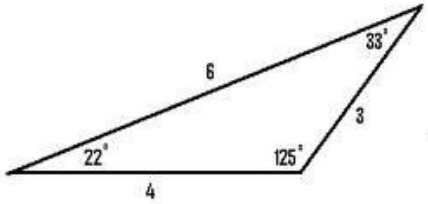
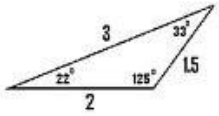
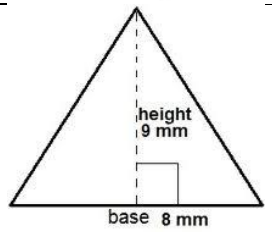
Plane Geometry

Lines and Angles

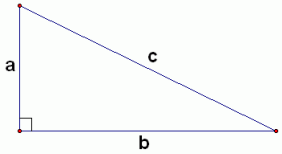
√?	Concept	Explanation	Example/Visual
	Intersecting lines	When two lines intersect: • **Adjacent angles**, or angles next to each other, are **supplementary and add up to 180 degrees** • **Vertical angles**, or angles across from each other, **are equal**	 ∠a + ∠b = 180° ∠b = ∠d
	Parallel lines and transversal (a line that crosses through parallel lines)	Forms **four equal acute angles** and **four equal obtuse angles**	

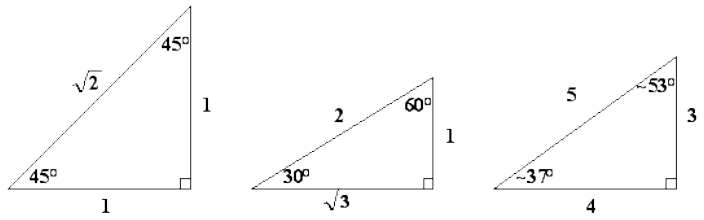
Triangles – General

√?	Concept	Explanation	Example/Visual
	Interior angles of a triangle	The three angles **add up to 180 degrees**	 $m\angle A + m\angle B + m\angle C = 180$

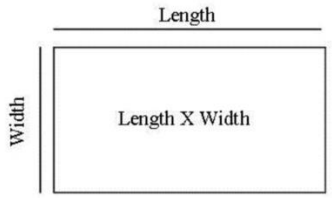
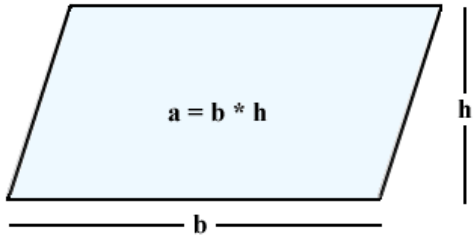
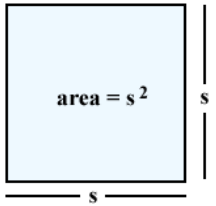
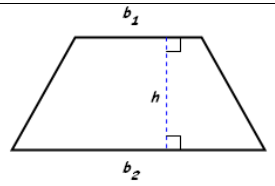
	Exterior angle of a triangle	• An exterior angle **equals the sum of the two angles it is not next to** • The **three exterior angles of a triangle add up to 360 degrees**	 ∠a + ∠b + ∠c = 360°  ∠a = ∠C + ∠D
	Similar triangles	Have the **same shape, but different size** • Corresponding **angles are equal** • Corresponding **sides are proportional**	 
	Area of a triangle	• Area = ½(Base)(Height) • For non-right triangles, the height is inside the triangle	

Right Triangles

√?	Concept	Explanation	Example/Visual
	Pythagorean Theorem	$(leg_1)^2 + (leg_2)^2 = (hypotenuse)^2$	 $a^2 + b^2 = c^2$

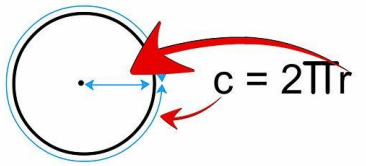
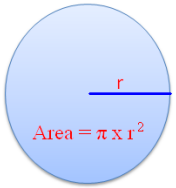
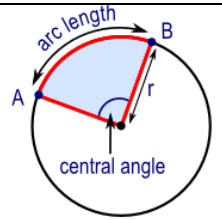
	Special right triangles	**3-4-5** (and multiples) **5-12-13** (and multiples) **30-60-90 – 1:$\sqrt{3}$:2** **45-45-90 – 1:1: $\sqrt{2}$**	

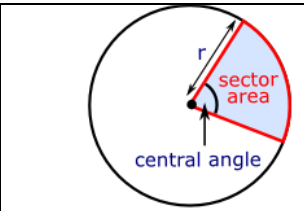
Other Polygons

√?	Concept	Explanation	Example/Visual
	Rectangle	• Has 4 right angles • Opposite sides are equal • Diagonals are equal • Area = Length x Width	
	Parallelogram	• Has 2 pairs of parallel sides • Opposite sides are equal • Opposite angles are equal • Angles next to each other add up to 180 degrees • Area = Base x Height	
	Square	• Rectangle with 4 equal dies • Area = (Side)^2	
	Trapezoid	• One pair of parallel sides • One pair of non-parallel sides • Area = $\frac{\text{base1} + \text{base2}}{2} \times height$	$A = \frac{1}{2}(b_1 + b_2) \cdot h$ 

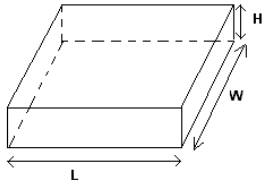
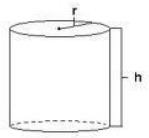
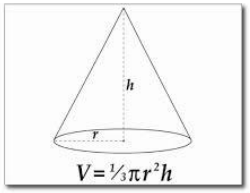
	Interior angles of a polygon	• $\frac{(n-2)x\ 180}{n}$ = Interior angle ○ n is the number of sides	$\frac{(n-2)(180)}{n}$ $\frac{(8-2)(180)}{8}$ $\frac{(6)(180)}{8} = 135$ for a polygon with 8 sides

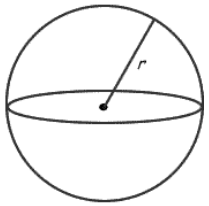
Circles

√?	Concept	Explanation	Example/Visual
	Circumference (outside) of a circle	Circumference = **2πr**	
	Area of a circle	Area = **πr^2**	
	Length of an arc (piece of the circumference)	Arc = **n/360 * (2πr)** n = the arc's central angle	 **Arc length = 2πr x $\frac{\text{central angle}}{360°}$**

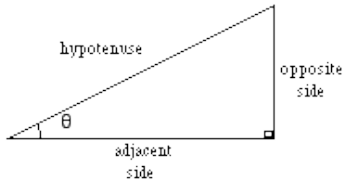
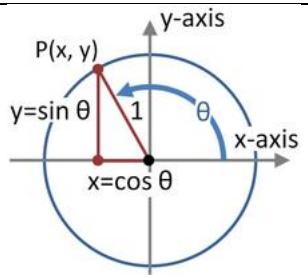
	Area of a sector (piece of the area of a circle)	Sector = **n/360 x (πr^2)** n = the sector's central angle	 **Sector area = πr^2 x $\frac{\text{central angle}}{360°}$**

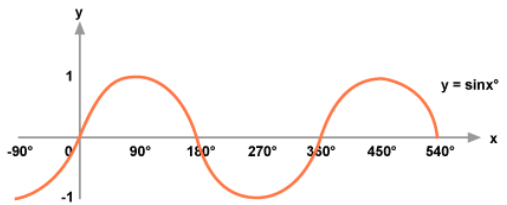
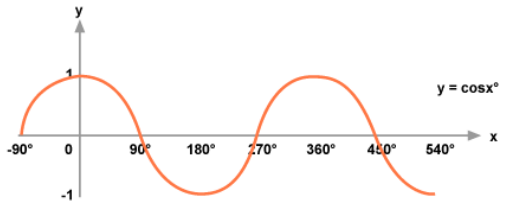
Solids

√?	Concept	Explanation	Example/Visual
	Surface area of a rectangular solid **Volume of a rectangular solid**	Surface area = **2(lw + wh + lh)** Volume = **lwh**	 Surface area=2(L*W+L*H+W*H) Volume = L*W*H
	Volume of a cylinder	Volume of a cylinder = **$\pi r^2 H$**	 $V = \pi r^2 h$
	Volume of a cone	Volume of a cone = **1/3$\pi r^2 H$**	 $V = \frac{1}{3}\pi r^2 h$

	Volume of a sphere **Surface area of a sphere**	Volume of a sphere = $\mathbf{4/3\pi r^3}$ Surface area of a sphere = $\mathbf{4\pi r^2}$	Sphere Surface Area $A = 4\pi r^2$  Volume $V = \frac{4}{3}\pi r^3$

Trigonometry

√?	Concept	Explanation	Example/Visual
	Sine, Cosine, and Tangent of acute angles	SOH-CAH-TOA Sine = Opposite/Hypotenuse Cosine = Adjacent/Hypotenuse Tangent = Opposite/Adjacent	SOH $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ CAH $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ TOA $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ 
	Cotangent, Secant, and Cosecant of acute angles	These are the reciprocals of Sine, Cosine, and Tangent Cotangent = 1/Tangent = Adjacent/Opposite Secant = 1/Cosine = Hypotenuse/Adjacent Cosecant = 1/Sine = Hypotenuse/Opposite	$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$
	Trigonometric functions of other angles	For angles greater than 90 degrees 1) Draw a circle with radius 1 centered on the coordinate grid 2) Rotate the appropriate number of degrees counterclockwise 3) Draw an acute triangle depending on where the angle lands 4) Find the answer	
	Simplifying trigonometric expressions	$\sin^2x + \cos^2x = 1$ Also, use the trigonometric properties to simplify	$\cot x + \tan x = \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$ $= \frac{\cos^2 x + \sin^2 x}{\sin x \cos x}$ $= \frac{1}{\sin x \cos x}$ $= cosec\ x \sec x \quad (Proven)$

	Graphic trigonometric functions	x-axis → angle y-axis → value of the trigonometric function Use special angles like 0, 30, 45, 60, 90, etc. to plot key points	 
	Radians	π = 180°	2π = 360° π = 180° π/2 = 90°