PART B

TEXAS INSTRUMENTS TI-85/TI-86 GRAPHING CALCULATORS

Setup

When using this *Guide*, you should always, unless instructed otherwise, use the calculator setup specified below for both the TI-85 and TI-86. Before you begin, check the basic setup with 2nd MORE (MODE). Use the following instructions to choose the settings shown in Figure 1. Check the window format with GRAPH MORE F3 (FORMT). Choose the settings shown in Figure 2.

- If you do not have the darkened choices shown in Figures 1 and 2 (below), use the arrow keys to move the blinking cursor over the setting you want to choose and press ENTER.
- Press EXIT or 2nd EXIT (QUIT) to return to the home screen.

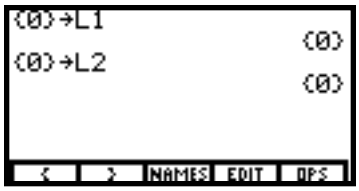
TI-85, 86 Basic Setup
FIGURE 1

TI-85, 86 Window Setup
FIGURE 2

For the TI-85 Return to the home screen with EXIT or 2nd EXIT (QUIT). Specify the statistical setup as shown in Figure 3 by pressing 2nd – (LIST) F1 ({) 0 F2 (}) STO 7 (L) ALPHA 1 ENTER 2nd ENTER (ENTRY) ◁ 2 ENTER. (See Figure 3.) Press EXIT or 2nd EXIT (QUIT) to return to the home screen.

For the TI-86 Return to the home screen with EXIT or 2nd EXIT (QUIT). Specify the statistical setup as shown in Figure 6 by pressing 2nd – (LIST) F5 (OPS) MORE MORE MORE F3 (SetLE) ALPHA 7 (L) 1 ,

ALPHA 7 (L) 2 , ALPHA 7 (L) 3 , ALPHA 7 (L) 4 , ALPHA 7 (L) 5 ENTER . (See Figure 4.) Press EXIT or 2nd EXIT (QUIT) to return to the home screen.



TI-85 Statistical Setup
FIGURE 3

TI- 86 Statistical Setup
FIGURE 4

TI-86 Note: Because appropriate TI-86 computer linking software had not yet been perfected at the time this material was written, TI-86 screens in this *Guide* appear smaller than the TI-85 screens. Screens that appear the same on both calculators are shown for the TI-85.

Basic Operation

You should be familiar with the basic operation of your calculator. With calculator in hand, go through each of the following.

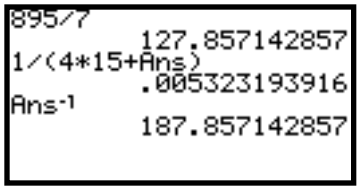
1. CALCULATING You can type in lengthy expressions; just make sure that you use parentheses when you are not sure of the calculator's order of operations. As a general rule, numerators and denominators of fractions and powers consisting of more than one term should be enclosed in parentheses.

Evaluate $\frac{1}{4*15+\frac{895}{7}}$. Evaluate $\frac{(-3)^4-5}{8+1.456}$. (Use (–) for the negative symbol and – for the subtraction sign.)	1/(4*15+895/7) .005323193916 ((-3)^4-5)/(8+1.456) 8.0372250423
Evaluate $e^{3}*0.027$ and $e^{3*0.027}$. The calculator will assume you mean the first expression unless you use parentheses around the two values in the exponent. (It is not necessary to type in the 0 before the decimal point.)	e^3*.027 .542309496926 e^(3*.027) 1.08437089657

2. USING THE *ANS* MEMORY Instead of again typing an expression that was evaluated immediately prior, use the answer memory by pressing 2nd (–) (ANS) .

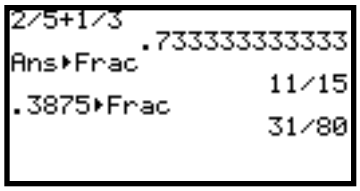
Calculate $\left(\frac{1}{4*15 + \frac{895}{7}}\right)^{-1}$ using this nice shortcut.

(If you wish to clear the home screen, press CLEAR .)



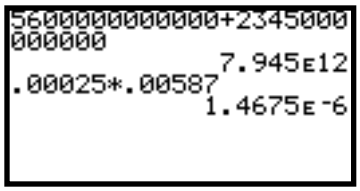
3. ANSWER DISPLAY When the denominator of a fraction has no more than three digits, your calculator can provide the answer in fraction form. When an answer is very large or very small, the calculator displays the result in scientific notation.

The "to a fraction" key is obtained by pressing
2nd X (MATH) F5 (MISC) MORE F1 (Fr



The calc r's syml "nes 10^{12}" is 1
7.945 1 ,000,000.

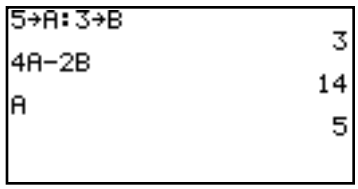
The result 1.4675 – 75*10^−6, the scientific notation expression for 0.0000014675.



4. STORING VALUES Sometimes it is beneficial to store numbers ex ssions for later recall. To store a number, type the number on the display and press S . (Note that the cursor automatically changes to alphabetic mode when you press STO .) Next, press the key corresponding to the letter in which you wish to store the value, and then press ENTER . To join several short commands together, use 2nd · (:).

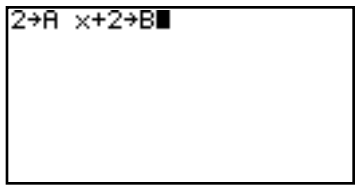
Store 5 in A and 3 in B, and then calculate $4A - 2B$.

To recall a value stored in a variable, use ALPHA to type the letter in which the expression or value is stored and then press ENTER . The value stays stored until you change it.



5. ERROR MESSAGES When your input is incorrect, an error message is displayed.

If you have more than one command on a line without the commands separated by a colon (:), an error message results when you press ENTER .



Choose F1 (Goto) to position the cursor to the place the error occurred so that you can correct the mistake or choose F5 (Quit) to begin a new line on the home screen.	

Chapter 1 Ingredients of Change: Functions and Linear Models

1.1 Fundamentals of Modeling

There are many uses for a function that is entered in the graphing list. Graphing the function in an appropriate viewing window is one of these. Because you must enter all functions on one line (that is, you cannot write fractions and exponents the same way you do on paper) it is very important to have a good understanding of the calculator's order of operations and to use parentheses whenever they are needed.

1.1.1 ENTERING AN EQUATION IN THE GRAPHING LIST

Press GRAPH F1 (y(x)=) to access the graphing list. Up to 99 equations can be entered in the graphing list, and the output variables are called by the names y1, y2, etc. When you intend to graph an equation you enter in the list, you must use x as the input variable.

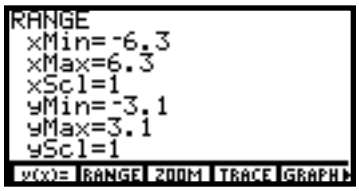
If there are any previously entered equations that you will no longer use, clear them out of the graphing list.	Position the cursor on the line containing the equation and press CLEAR or F4 (DELf) .

Suppose you want to graph $A = 1000(1 + 0.05)^t$. Because we intend to graph this equation, the input must be called x. Type x by pressing X-VAR or F1 (x) , not the times sign X .

TI 85 For convenience, we use the first, or y1, location in the graphing list. Enter the right hand side as 1000(1 + 0.05)^x	
TI 86 For convenience, we use the first, or y1, location in the graphing list. Enter the right hand side as 1000(1 + 0.05)^x Plot1, Plot2, and Plot3 at the top of the y(x)= list should not be darkened. If any of them are, use ▲ until you are on the darkened plot name. Press ENTER to make the name(s) not dark.	

1.1.2 DRAWING A GRAPH

If you have not already done so, enter the equation in the y(x)= list using x as the input variable before drawing a graph. We now draw the graph of $y = 1000(1 + 0.05)^x$.

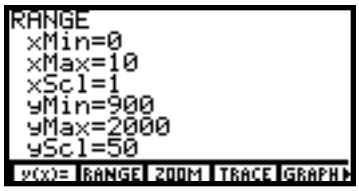
Remove the lower menu with EXIT, and press F3 (ZOOM) MORE F4 (ZDECM). Notice that the graphics screen is blank.	
TI-85 Press EXIT F2 (RANGE) to see the view set by ZDECM.	
TI-86 Press EXIT F2 (WIND) to see the view set by ZDECM.	

- **Both** xMin and xMax are the settings of the left and right edges of the viewing screen, and yMin and yMax are the settings for the lower and upper edges of the viewing screen. xScl and yScl set the spacing between the tick marks on the x- and y-axes. The view you see is $-6.3 \le x \le 6.3$, $-3.1 \le y \le 3.1$.

Follow the procedures shown in either 1.1.3 or 1.1.4 to draw a graph with your calculator. Whenever you draw a graph, you have the option of manually changing the view or having the calculator automatically find a view of the graph.

1.1.3 MANUALLY CHANGING THE VIEW OF A GRAPH

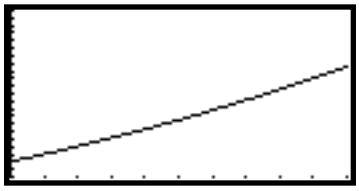
If you do not have a good view of the graph or if you do not see the graph, change the view with one of the ZOOM options or manually set the view. (We later discuss the ZOOM options.)

TI 85 Press GRAPH F2 (RANGE). Set xMin to 0, xMax to 10, leave xScl at 1, set yMin to 900, yMax to 2000, and yScl to 50.	
TI 86 Press GRAPH F2 (WIND). Set xMin to 0, xMax to 10, leave xScl at 1, set yMin to 900, yMax to 2000, and yScl to 50. Below yScl is a setting called xRes. For all applications in this *Guide*, have xRes set to 1.	

Both Press F5 (GRAPH) to draw the graph of

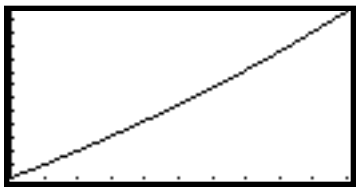
y1 = 1000(1 + 0.05^x)

in the new view. CLEAR removes the menu from the bottom of the screen if you wish to see more of the graph.

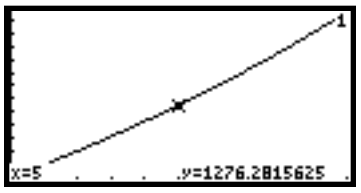
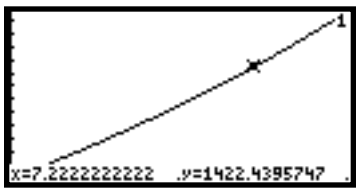


1.1.4 AUTOMATICALLY CHANGING THE VIEW OF THE GRAPH If your view of the graph is not good or if you do not see the graph, change the view using the built-in autoscaling feature of your calculator. This option will automatically find a view to see all the functions that you have turned on in the graphing list.

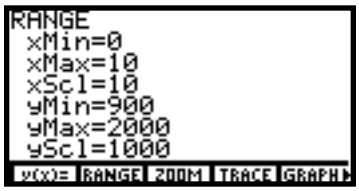
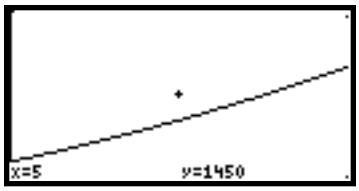
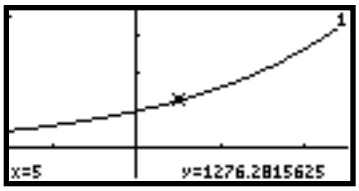
Be sure the function you are graphing, $y = 1000(1 + 0.05)^x$, is entered in the y1 location of the y(x)= list. (Delete all other functions that may be entered in other locations.) Before doing what follows, access the graphics menu with GRAPH .

To access the range on the TI-85 (called the *window* on the TI-86), press F2 . Set xMin to 0 and xMax to 10. (It does not matter what values are set in the yMin and yMax positions.) Press F3 (ZOOM) MORE F1 (ZFIT) .	
Your calculator automatically sets a vertical view (based on the xMin and xMax you set) and draws a graph of the function.	

1.1.5 TRACING You can display the coordinates of certain points on the graph by tracing. The x-values shown when you trace depend on the horizontal view that you choose, and the y-values are calculated by substituting the x-values into the equation that is being graphed.

With the graph on the screen, press F4 (TRACE) and use ▷ to move the trace cursor to the right and ◁ to move the trace cursor to the left. The number 1 in the upper right hand corner of the screen tells you that you are tracing on the equation in y1.	
Trace past the edge of the screen and notice that even though you cannot see the trace cursor, x and y values of points on the line are still displayed at the bottom of the screen. Also notice that the graph scrolls to the left or right as you move the cursor past the edge of the current viewing screen.	

1.1.6 ESTIMATING OUTPUTS You can estimate outputs from the graph using TRACE. It is important to realize that such outputs are *never* exact values unless the displayed x-value is *identically* the same as the value of the input variable.

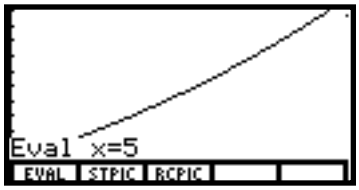
Estimate the value of A where $A = 1000(1 + 0.05)^x$ when x = 5, x = 7, and x = 10. Press EXIT F2 (RANGE) (or WINDOW) . If you do not have the settings shown to the right, reset the values.	
Press F3 (ZOOM) F3 (ZOUT) . After the graph finishes drawing, press ENTER to once more enlarge your view of the graph. (Press EXIT F2 (RANGE) and observe the values now defining the graphics screen.)	 Note that (5, 1450) is a point on the screen, *not* a point on the graph of the function.
Press F4 (TRACE) and use ▷ to move as close as you can to $x = 5$. (Your screen may look slightly different than the one shown to the right.) Continue pressing ▷ and notice that the values 7 and 10 cannot be obtained by tracing in this view. Therefore, choose values close to these numbers to obtain *estimates* such as A is approximately \$1386.80 when $x = 7$ and A is about \$1637.37 when $x = 10$.	 Because the number 5, *not* a value close to 5 is shown, $A = \$1276.28$ when $x = 5$.

- If you had used the original range (window) with xMax =10, and traced, you should obtain the *exact* value $A = 1500$ when $x = 10$ because 10, not a value close to 10, is shown when tracing.
- If you want "nice, friendly" values displayed for x when tracing, set xMin and xMax so that xMax–xMin is a multiple of 12.6, the width of the ZDECM viewing screen. For instance, if you set xMin = 0 and xMax = 12.6 in the example above, the *exact* values when x = 5, x = 7, and x = 10 are displayed when you trace. Another view that gives friendly values is xMin = −5 and xMax = 20.2 since 25.2 = 2(12.6). Try it!

1.1.7 EVALUATING OUTPUTS The values obtained by this evaluation process are *actual* output values of the equation, not estimated values such as those generally obtained by tracing.

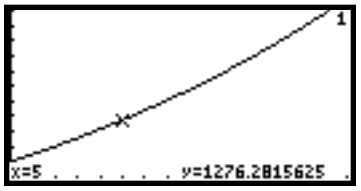
Begin by entering the equation whose output you want to evaluate in the y(x)= list. Even though you can use any of the locations, let us say for this illustration you have y1 = 1000(1 + 0.05^x).

To evaluate an output while the graph is on the screen:

Draw the graph of y1 in a view containing the input you intend to use. (If the input value is not between xMin and xMax , an error message results when you evaluate.) Press MORE MORE F1 (EVAL) , and type the input value.	

Press ENTER to display the output and see the point marked on the graph of the function.

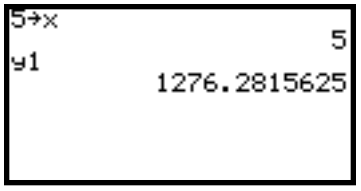
Repeat this process to find the outputs when $x = 7$ and $x = 10$.



To evaluate an output from the home screen:

First, press EXIT until you are on the home screen. Store the input value, say 5, in x by with 5 STO X-VAR ENTER .

Next, type the location in the y(x)= list in which the function is stored with 2nd ALPHA 0 (y) 1. Note that the second function key is pressed before typing the letter y in order to type lower-case y.



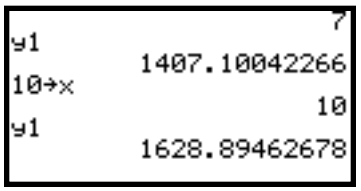
Warning: The TI-85 and the TI-86 distinguish between lower-case and upper-case variable names. Locations in which equations are stored in the graphing list are always referred to with *lower-case* letters.

If you had not pressed 2nd before ALPHA and therefore entered Y1, you would be using another variable stored in the calculator's memory. (Your values may be different from the ones shown to the right or you may get an UNDEFINED error.)

Note that on the TI-85, y1(5) *multiplies* y1 by 5.

To evaluate another output, simply store the input value in x, type y1 and press ENTER .

Evaluate y1 = 1000(1 + 0.05^x) at x = 7 and x = 10.



- The values obtained by either of these evaluation processes are *actual* output values of the equation, not *estimated* values such as those generally obtained by tracing.

TI-86: Evaluating outputs from the home screen While the evaluation process indicated above works on the TI-86, advances in the TI-86 technology give you a way to evaluate functions from the home screen using fewer keystrokes than for the TI-85.

Type 2nd ALPHA 0 (y) 1 and then press (5). Press ENTER .

On the TI-86, typing a value in parentheses following a function evaluates the function at that input value.

Evaluate y1 at $x = 7$ by recalling the previous entry with 2nd ENTER (ENTRY) , edit the 5 to 7 by pressing ◀ and typing over the 5, and press ENTER . Repeat the process to evaluate y1 at $x = 10$.	

1.2 Functions and Graphs

When you are asked to *estimate* or *approximate* an output or an input value, you can use your calculator in the following ways:

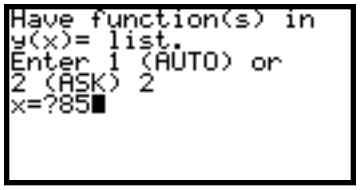
- tracing a graph (Sections 1.1.5, 1.1.6)
- close values obtained from a table of function values (End of Section 1.2.2)

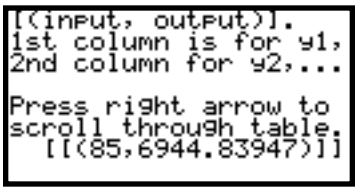
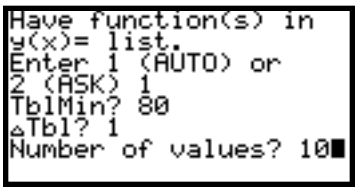
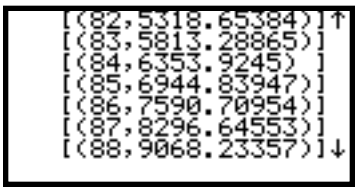
When you are asked to *find* or *determine* an output or an input value, you should use your calculator in the following ways:

- evaluating an output on the home screen (Section 1.1.7)
- find a value using the AUTO or ASK features of the table (Section 1.2.1)
- determine an input using the solver (Section 1.2.2)

1.2.1 DETERMINING OUTPUTS Function outputs can be determined by evaluating on the home screen, as discussed in 1.1.7. You can also evaluate functions using the calculator's TABLE. (On the TI-85, the table is accessed with program TABLE.) When you use the table, you can ask for specific output values corresponding to the inputs you enter or generate a list of input values that begin with TblMin and differ by ΔTbl and their corresponding outputs.

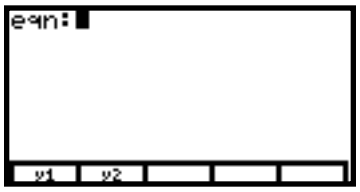
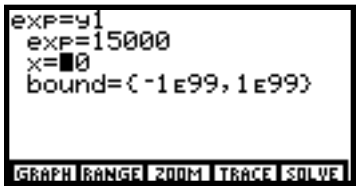
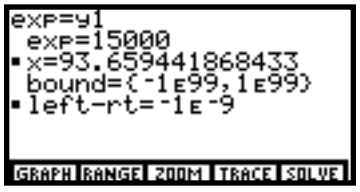
Let's use the TABLE to determine the output of the function $v(t) = 3.622(1.093)^t$ when t =85. Press GRAPH F1 (y(x)=) , clear any functions, and enter 3.622(1 .093)^X in location y1. Exit.

TI-85 Note that when using program TABLE, any functions you want to evaluate should be in locations y1, y2, and so forth. That is, don't leave an empty function location in the y(x)= list. (Program TABLE can be found in the TI-85/86 Appendix.) An alphabetical list of the programs that you have entered in your TI-85 appears on the bottom menu.	Press PRGM and then press F1 (NAMES) . 
Find the name TABLE (press MORE if necessary). Press the F-key corresponding to the location of the program and ENTER to "run" the program. Since we only want $v(85)$, enter 2 for ASK and type 85 at the x=? prompt.	

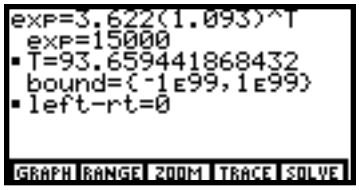
The message that appears tells you to press ENTER to see the output. The output is in the form [*t*, *v(t)*]. We see that $v(85) \approx$ \$6945.	
If you had wanted to generate a list of values, starting at a value called TblMin and differing by ΔTbl, use the AUTO option in the program. Press ENTER to re-run the program, choose 1 for AUTO, and enter the values like those shown to the right.	
Pressing ENTER and ▽ allows you to see the outputs that have been generated and their corresponding inputs. (If you had evaluated several different functions, their outputs appear in the same order (horizontally) as the functions in the graphing list.)	
TI-86 After entering the function *v(t)* in y1, choose the TABLE SETUP menu.	Press TABLE F2 (TBLST) .
To generate a list of values beginning with 80 with the table values differing by 1, enter 80 in the TblStart location, 1 in the ΔTbl location, and choose AUTO in the Indpnt: location. Remember that you "choose" a particular setting by positioning the blinking cursor over that setting and pressing ENTER .	
Press F1 (TABLE) , and observe the list of input and output values. Notice that you can scroll through the table with ▽, ▷, △, and/or ◁. The table values may be rounded in the table display. You can see more of the output by moving to the value and looking at the bottom of the screen.	
Return to the TABLE SETUP menu with F1 (TBLST) . (TblSet). To compute specific outputs rather than a list of values, choose ASK in the Indpnt: location. (When using ASK, the settings for TblMin and ΔTbl do not matter.)	

Press F1 (TABLE) , type in the x-value(s) at which the function is to be evaluated, and press ENTER . You can scroll through the table with ▽, ▷, △, and/or ◁. Unwanted input entries can be cleared with DEL .	$v(85) \approx \$6945$

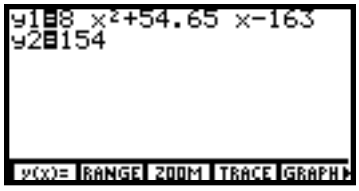
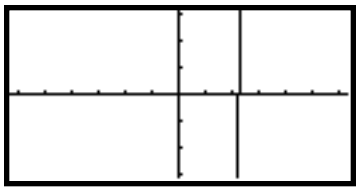
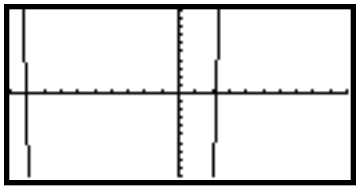
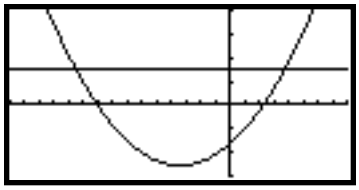
1.2.2 SOLVING FOR INPUT VALUES Your calculator will solve for input values of an equation that you enter in the SOLVER. You can use any letter you wish for the input variable when using the SOLVER. (Of course, if you are using a function that is entered in the graphing list, the input variable is x.) You can even enter an equation consisting of several variables! You should find your calculator's solver a very valuable tool.

Press 2nd GRAPH (SOLVER) to access the SOLVER. If there is already an equation in the solver, clear it with CLEAR . You may need to press △ to get to the screen shown on the right before you can clear the old expression.	
You can access a function stored in another location, such as our current y1 = 3.622(1 .093)^X, by pressing the F-key corresponding to the equation or you can type in a new equation. Suppose we want to solve $v(t) = 3.622(1.093)^t$ for t when $v = \$15{,}000$.	
Press ENTER and you will see the last values the calculator used for y1 and x. Enter 15,000 for y1. Use ▽ and/or △ to move between locations in the SOLVER.) The displayed bound gives the values between which your calculator will look for a solution to the equation -- just leave the bound[1] as it is.	
Position the blinking cursor on the row corresponding to the variable for which you want to solve and press F5 (SOLVE) . The left side of the equation, y1, evaluated at x ≈ 93.66 and the right side of the equation, 15,000, are subtracted and displayed in the *left–rt* row. When this value is 0, an *exact* solution (rather than a rounded one) was found.	

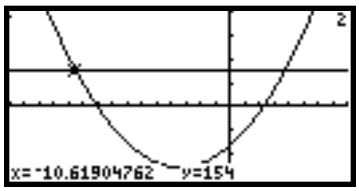
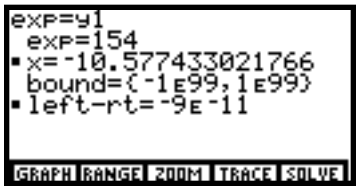
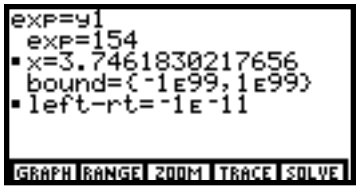
[1]It is possible to change the bound and/or the tolerance if the calculator has trouble finding a solution to a particular equation. This, however, should not usually happen. Refer to your *Owner's Manual* for details.

Notice that you could have typed the $v(t)$, with input variable t, directly in the solver. The solution is, of course, the same.	

If there is more than one solution to an equation, you need to give the solver an approximate location for each answer. Suppose you are given $q(x) = 8x^2 + 54.65x - 163$ and asked to find what input(s) correspond to an output of $q(x) = 154$. (The procedure outlined below also applies to finding where two functions are equal.)

Enter $8x^2 + 54.65x - 163$ in one location, say y1, and 154 in another location, say y2, in the y(x)= list. (Remember that if the input variable in the equation is not x, you must rewrite the equation in terms of x to graph using the y(x)= list.)	
To better obtain a guess as to where y1 equals (intersects) y2, graph the equations. If you are not told where you want to view the graph, begin by pressing GRAPH F3 (ZOOM) F4 (ZSTD) or	
GRAPH F3 (ZOOM) MORE F4 (ZDECM) . You want to see a "good" graph, that is, one that shows all the important features. In this case, the important features are where y1 and y2 intersect. Neither this graph nor the previous one is a good graph for viewing the intersections.	
To improve the view, use EXIT F2 (RANGE or WIND) * and change to xMin = −15, xMax = 8, yMin = −300, and yMax = 400 . Draw the graph with F3 (GRAPH) . (There are many other windows that work just as well as the one shown to the right. Also, instead of setting a window manually, you could draw the graph with ZFIT in the ZOOM menu.)	

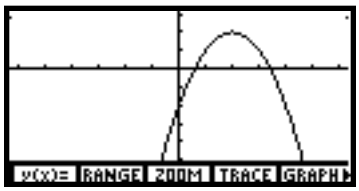
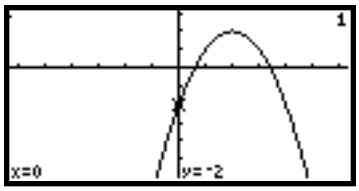
*The current settings for the graphics screen are obtained with GRAPH F2 (RANGE) on the TI-85 and with GRAPH F2 (WIND) on the TI-86. When the screen appears, the word RANGE is at the top of the TI-85 screen and the word WINDOW is at the top of the TI-86 screen. Because the only difference in the calculators in this situation is the name that is used, we from this point forward refer to this key for both calculators as F2 (RANGE or WIND) . In such cases, the screen shown in the right column will be the one from the TI-85.

Press F4 (TRACE) and use ▷ and/or ◁ to move along the graph of the equation in y1. Press ▽ to jump from the graph of y1 to the graph of y2. Tracing reveals that guesses for the input values where these two graph intersect are x ≈ ⁻10.6, x ≈3.8. Return to the home screen with 2nd EXIT (QUIT) .	
Access the SOLVER and enter the equation y1 = y2 as shown on the right. Enter the guess for the left-most intersection point and solve for x.	
Enter the guess for the rightmost intersection point and solve for x. The two solutions to the equation, reported to four decimal places, are $x = ⁻10.5774$ and $x = 3.7462$.	

1.2.3 GRAPHICALLY FINDING INTERCEPTS Finding where a function graph crosses the vertical and horizontal axis can be done graphically as well as by the methods indicated in 1.2.2 of this *Guid*e. Remember the process by which we find intercepts:

- To find the y-intercept of a function $y = f(x)$, set x=0 and solve the resulting equation.
- To find the x-intercept of a function $y = f(x)$, set y=0 and solve the resulting equation.

Also remember that an x-intercept of a function $y = f(x)$ has the same value as the root or solution of the equation $f(x) = 0$.

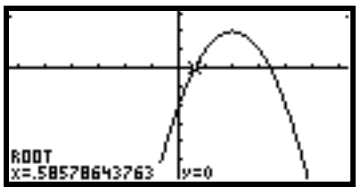
Clear all locations in the *y(x)=* list and enter in y1 $f(x) = 4x - x^2 - 2$. Draw a graph with F3 (ZOOM) MORE F4 (ZDECM) . Press F2 (RANGE or WIND) and reset yMin to ⁻6 for a good view of all intercepts. Press F5 (GRAPH) .	
Even though it is very easy to find $f(0) = ⁻2$, you can have the calculator find the y-intercept while viewing the graph by pressing **TI-85** MORE F1 (MATH) MORE F4 (YICPT) **TI-86** MORE F1 (MATH) MORE F2 (YICPT) **Both** View the y-intercept $f(0) = ⁻2$.	

To graphically find an x-intercept, i.e., a value of x at which the graph crosses the horizontal axis, first press EXIT to return the menu to the bottom of the screen.

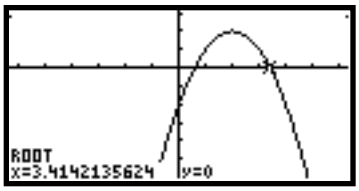
TI-85 Press F1 (MATH) F3 (ROOT)

TI-86 Press F1 (MATH) F1 (ROOT)

Both Use ▷ to move the trace cursor near the first x-intercept, and press ENTER .



To find the second x-intercept, press EXIT and the F-key under ROOT, use ▷ to move the trace cursor near the other x-intercept, and press ENTER .



1.3 Constructed Functions

Your calculator can find output values of and graph combinations of functions in the same way that you do these things for a single function. The only additional information you need is how to enter constructed functions in the graphing list. Suppose that a function $f(x)$ has been entered in y1 and a function $g(x)$ has been entered in y2.

- Enter y1 + y2 in y3 to obtain the sum function $(f+g)(x) = f(x) + g(x)$.
- Enter y1 – y2 in y4 to obtain the sum function $(f-g)(x) = f(x) - g(x)$.
- Enter y1*y2 in y5 to obtain the product function $(f \cdot g)(x) = f(x) * g(x)$.
- Enter y1/y2 in y6 to obtain the quotient function $(f \div g)(x) = \dfrac{f(x)}{g(x)}$.

On the TI-85,

- The TI-85 interprets y1(y2) as the *product* of y1 and y2. To have the calculator graph the composite function $f(g(x))$, you should substitute $g(x)$ everywhere an x appears in $f(x)$. For instance, if $f(x) = x^2 - 1$ is in y1 and $g(x) = 0.3x + 5$ is in y2, enter $y7 = y2^2 - 1$ for the composite function $(f \circ g)(x)$.

On the TI-86,

- Enter y1(y2) in y7 to obtain the composite function $(f \circ g)(x) = f(g(x))$.

Both Your calculator will evaluate and graph these constructed functions. Although it will not give you an algebraic formula for a constructed function, you can check your algebra by evaluating the calculator-constructed function and your constructed function at several different points. (You will very likely have to reset the horizontal and vertical views when graphing constructed functions.)

1.3.1 GRAPHING PIECEWISE CONTINUOUS FUNCTIONS

Piecewise continuous functions are used throughout the text. It is often helpful to use your calculator to graph and evaluate outputs of piecewise continuous functions. Consider the following example.

The population of West Virginia from 1985 through 1993 can be modeled by

$$P(t) = \begin{cases} -23.514t + 3903.667 \text{ thousand people when } 85 \le t < 90 \\ 9.1t + 972.6 \text{ thousand people when } 90 \le t \le 93 \end{cases}$$

where t is the number of years since 1900.

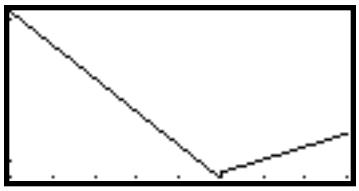
Enter the function $P(t)$, using x as the input variable, in the y1 location of the y(x)= list using the keystrokes

((–) 23.514 x-VAR + 3903.667) (x-VAR 2nd 2 (TEST) F2 (<) 90) + (9.1 x-VAR + 972.6) (x-VAR F5 (≥) 90) .

Each piece of the function and its corresponding input must be enclosed in parentheses.

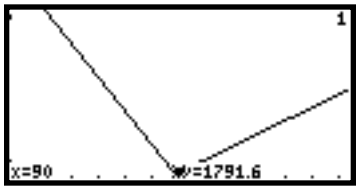
Notice that the function is defined only when the input is between 85 and 93. You could find $P(85)$ and $P(93)$ to help you set the vertical view. However, we choose to let the calculator set the vertical view.

Set the horizontal view xMin = 85 and xMax = 93. Press GRAPH F3 MORE F1 ZOOM (ZFIT) to graph $P(x)$.



If you wish to see the "break" in the function where the two pieces join, the width of the screen must be a multiple of 12.6 and include 90.

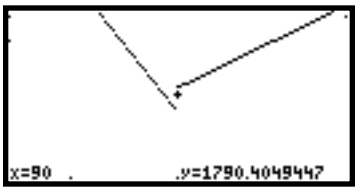
Since 90 – 0.5(12.6) = 83.7 and 90 + 0.5(12.6) = 96.3, change xMin and xMax to these values. Press F5 (GRAPH) .



Because the two pieces are close together at $x = 90$, you may need to take a closer look to see the break. However, because the calculator draws graphs by connecting function outputs wherever the function is defined, it will connect the two pieces unless you tell it not to do so by pressing MORE F5 (FORMT), use ▽ and ▷ to choose DrawDot. Press ENTER.

Now, take a closer look with F5 (GRAPH) and F3 ZOOM F2 (ZIN).

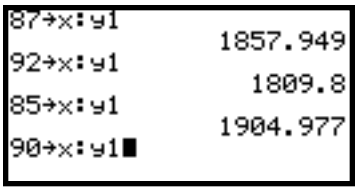
To keep the point where the functions break in view, use ▽ to move the small cursor that appears in the middle of the screen down to where the two functions join before pressing ENTER to actually zoom in.



The breaks you see in the left portion of the function are because you are in dot mode.

TI-85 You can find function values by evaluating outputs on the home screen or using the table.

Do not forget to change the calculator's FORMAT setting back to DrawLine when you finish graphing the piecewise function.



TI-86 The same procedure as given in the above steps will also work for the TI-86. However, the TI-86 has some additional features that make graphing of piecewise functions less complicated.

(See Section 6.1.2 of this *Guide* for an example showing how to fit a piecewise model to data.)

Instead of going to the FORMAT menu, use the following to choose DrawDot.

Have the cursor on the y1 line, press EXIT until you return to the graph menu, and then press F1 (y(x)=) MORE F3 (STYLE). Press F3 five more times to choose the dotted line[2] to the *left* of y1.

The dotted line you now see to the left of y1 indicates the graph will draw without joining the outputs of the function.

[2]The different "graph styles" you can draw from this location are described in more detail on page 10 in your TI-86 *Graphing Calculator Guidebook*.

After setting the proper horizontal view, you can now use ZFIT to draw a graph of the piecewise function.	
You can find function values by evaluating outputs on the home screen or using the table.	

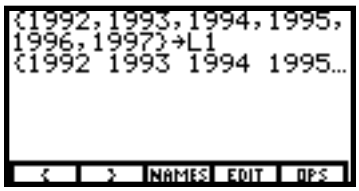
1.4 Linear Functions and Models

Actual real-world data is used throughout *Calculus Concepts*. It is necessary that you use your calculator to find a curve that models the data. Be very careful when you enter the data in your calculator because your model and all of your results depend on the values that you enter!

1.4.1 ENTERING DATA There are several ways to input data in your calculator. Two of these, entering data from the home screen and entering data using the list editor, are discussed below. If you do not see the list names L1 and L2 return to the statistical setup instructions at the beginning of this *Guide*.

We will explore data entry with the following data:

Year	1992	1993	1994	1995	1996	1997
Tax	2541	3081	3615	4157	4703	5242

To enter data from the home screen, Return to the home screen with [2nd] [EXIT] (QUIT) . Press [2nd] [–] (LIST) , press [F1] ({) to begin the list, type in each of the x-data values separated by commas, and end the list with [F2] (}) . Store this list to the name with L1 with [STO] L [ALPHA] 1 [ENTER] .	

To enter data using the list editor,

Press 2nd – (LIST) F4 (EDIT) to access the list editor. Enter the name L2 at the name prompt and press ENTER .

Enter the first *y*-value as e1, press ENTER or ▽, enter the second *y*-value as e2, etc.

If there are data values in this list, see 1.4.3 of this *Guide* and first delete the "old" data.

TI-86 Even though both of the methods discussed above are ways to enter data on the TI-86, the most convenient method is to enter the data in the TI-86's stat lists. Return to the home screen and press 2nd + (STAT) F2 (EDIT) . Go to Section 1.4.3 of this *Guide*, delete the data currently in these lists, and enter the data again using the method described next.

Position the cursor in the first location in list L1. Enter the *x*-data into list L1 by typing the numbers from top to bottom in the L1 column, pressing ENTER after each entry.

After typing the L1(5) value, 1997, use ▷ to go to the top of list L2. Enter the *y*-data into list L2 by typing the entries from top to bottom in the L2 column, pressing ENTER after each entry.

1.4.2 EDITING DATA IN THE TI-85 LIST EDITOR If you incorrectly type a data value, access the data with the list editor and use the cursor keys to move to the value you wish to correct. Type the correct value and press ENTER .

- To *insert* a data value, put the cursor over the value that will be directly below the one you will insert, and press F1 (INSi) . The values in the list below the insertion point move down one location and a 0 is filled in at the insertion point. Type the data value to be inserted and press ENTER . The 0 is replaced with the new value.
- To *delete* a single data value, move the cursor to the value you wish to delete, and press F2 (DELi) . The values in the list below the deleted value move up one location.

1.4.2 EDITING DATA IN THE TI-86 STAT LISTS If you incorrectly type a data value, use the cursor keys to darken the value you wish to correct and type the correct value. Press ENTER .

- To *insert* a data value, put the cursor over the value that will be directly below the one you will insert, and press 2nd DEL (INS) . The values in the list below the insertion point

move down one location and a 0 is filled in at the insertion point. Type the data value to be inserted over the 0 and press ENTER . The 0 is replaced with the new value.

- To *delete* a single data value, move the cursor over the value you wish to delete, and press DEL . The values in the list below the deleted value move up one location.

1.4.3 DELETING OLD DATA Whenever you enter new data in your calculator, you should first delete any previously-entered data. There are several ways to do this, and the most convenient method is illustrated below.

TI-85 Whenever you enter new data in your calculator, you should first delete any previously-entered data using one of the following methods:

- Whenever you enter data from the home screen, previously-entered data is automatically replaced with new data. Thus, {1, 2, 3} STO L2 replaces the "old" L2.
- When you enter the list editor and there is "old" data in the list, position the cursor over the e1 value and repeatedly press F2 (DELI) until all data is deleted.

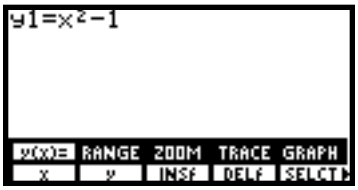
TI-86 Access the data lists with STAT 1 (EDIT) . (You probably have different values in your lists if you are deleting "old" data.) Use ▲ to move the cursor over the name L1.	
Press CLEAR ENTER .	
Use ▶ ▲ to move the cursor over the name L2. Press CLEAR ENTER . Repeat this procedure to clear data from any of the other lists you want to use.	

1.4.4 ALIGNING DATA Let's now return to the data entered in Section 1.4.1 of this *Guide*. Suppose you want L1 to contain the number of years since a certain year (here, 1992) instead of actual years. That is, you want to *align* the x-data. In this example, you are to shift all the data values 1992 units to the left of where they currently are located.

Return to the home screen. **TI-85** Replace the L1 values with L1 – 1992 values by pressing ALPHA L1 – 1992 STO L ALPHA 1 ENTER . L1 now contains the aligned x-values.	L1-1992→L1 {0 1 2 3 4 5}

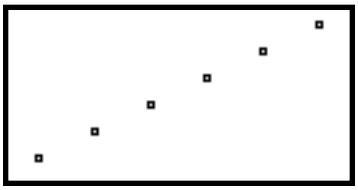
TI-86 Position the cursor over the L1 at the top of the first column. Replace the L1 values with L1 – 1992 values by pressing F3 (NAMES), the menu key under L1, and then press (–) 1992 ENTER. Instead of an actual year, the input now represents the number of years after 1992.	

1.4.5 PLOTTING DATA Any functions you have in the y(x)= list will graph when you plot data. Therefore, you should delete them or turn them off before drawing a scatter plot.

Access the y(x)= graphing list. If any entered function is no longer needed, delete it with F4 (DELf) or clear it with CLEAR. If you want the function to remain but do not want it to graph, position the cursor in that function location and press F5 (SELCT).	 A "turned off" function.

To graph data on the TI-85:

The TI-85 command Scatter L1, L2 draws a scatter plot of L1 versus L1. However, you must set an appropriate RANGE before using this command. Program STPLT found in the TI-85/TI-86 Appendix automates this task.

Press PRGM and then press F1 (NAMES). Find the name STPLT (press MORE if necessary).	Press the F-key corresponding to the location of STPLT to run the program.
Press CLEAR to remove the menu from the bottom of the screen for a better view. A scatter plot is drawn of the data with the L1 data as the input data and the L2 data as the output data.	

- Program STPLT sets the x and y-axis tick marks to 0 so they do not interfere with your view of the scatter plot.
- Because the dots the calculator uses to plot data are sometimes difficult to see when overdrawing the model of best fit, the program places a small box around each data point. (The boxes may appear a slightly different size with different screen settings.)
- Even though the TI-85 generally allows you to call lists by any names you want, *you must enter the input data in the list named L1 and the output data in the list named L2* when using program STPLT.
- Lists L1 and L2 must be of the same length or an error message results.
- It is not possible to trace a scatter plot drawn on the TI-85.

To graph data on the TI-86:

The TI-86 has a built-in command to graph data and autoscale the data window.

Press 2nd + (STAT) F3 (PLOT) to display the STAT PLOTS screen. (Your screen may not look exactly like this one.) Note: When drawing a graph from the y(x)= list, you may get an error message or see a scatter plot of "old" data as well as the function graph. If so, turn off the STAT PLOTS with F5 (PlOff) ENTER .	
On the STAT PLOTS screen, press F1 (PLOT1) to display the Plot1 screen, press ENTER to turn Plot1 "On", press ▽ F1 (SCAT) ▽. Press the menu key under list L1. Press ▽ and the menu key under list L2. Press ▽ and press the menu key corresponding to the mark. You can choose any of the three marks at the bottom of the screen.	You can enter the names of any lists. However, it is easiest to always work with L1 and L2.
Press EXIT until you return to the home screen. Go to the graph menu and clear the y(x)= list . To have the TI-86 set an autoscaled view of the data and draw the scatter plot, press GRAPH F3 (ZOOM) MORE F5 (ZDATA) . (ZDATA does not reset the *x* and *y*-axis tick marks. You should do this manually with RANGE or WIND if you want different spacing between the marks.)	You can trace the scatter plot with the TRACE key.
Press GRAPH and access the function list. Notice that "Plot1" at the top of the screen is now dark. This is because you have turned Plot1 "on". If you always put input data in list L1 and output data in list L2, you can turn the scatter plot off and on from the y(x)= screen rather than the stat plots screen from this point on. To turn Plot1 off, use △ to move the cursor to the Plot1 position, and press ENTER . Reverse the process to turn Plot1 back on.	A scatter plot is turned *on* when its name on the y(x)= screen is darkened. Remember that you will have lower-case functions.

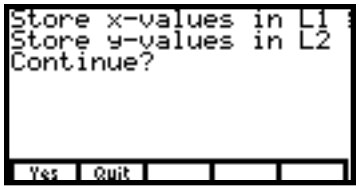
- TI-86 lists can be named and stored in the calculator's memory for later recall and use. If you do this and use the list by its stored name, you must use the name of the list in the stat plot setup or on the stat plot screen each time you change lists. Refer to your *TI-86 Guidebook* for details.

1.4.6 FINDING FIRST DIFFERENCES When the input values are evenly spaced, use program DIFF to compute first differences in the output values. If the data are perfectly linear (*i.e.*, every data point falls on the graph of the line), the first differences in the

output values are constant. If the first differences are "close" to constant, this is an indication that a linear model *may* be appropriate.

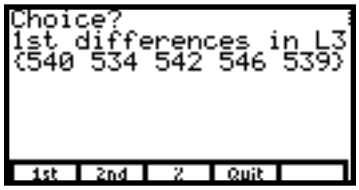
Program DIFF is given in the TI-85/TI-86 Appendix.

To run the program, press PRGM F1 (NAMES) and then the F-key under the name of the program. Press ENTER . If you have not entered the data, stop the program by pressing F2 (Quit) . To continue, press F1 (Yes) .



At the Choice? prompt, press F1 (1st) to generate the list of first differences.

Note: You may not be able to see the complete list on this screen. If not, go to the list editor to see the complete list of first differences stored in L3.



Exit program DIFF with F4 (Quit) .

TI-85 View list L3 containing the first differences in the output data by pressing 2nd – (LIST) F4 (EDIT) F3 (L3) ENTER .

TI-86 If you do not want to use program DIFF, you can use your TI-86 to compute first differences of any list.

Press 2nd – (LIST) F5 (OPS) MORE MORE F4 (Deltal() EXIT F3 (NAMES) and the menu key under L2. Press) ENTER to see the list of first differences in the output data.

If necessary, use ▷ to scroll to the right to see the remainder of the first differences. Use ◁ to scroll back to the left.

Be certain you have evenly-spaced input values if you are using this to give information about first differences and linear models.

- **Both** Program DIFF **should not** be used for data with input (L1) values that are *not* evenly spaced. First differences give no information about a possible linear fit to data with inputs that are not the same distance apart.

1.4.7 FINDING A LINEAR MODEL Use your calculator to obtain the linear model that best fits the data. Your calculator can find two different, but equivalent, forms of the linear model: $y = ax + b$ or $y = a + bx$. For convenience, we always choose the model $y = ax + b$.

TI-85 It is possible to enter data while in the statistics mode of your calculator. However, the examples in this *Guide* assume that input data is already entered in list L1 and output data is in list L2.

Press STAT F1 (CALC) and press the F-keys on the menu to enter L1 as the xlist Name and L2 as the ylist Name. Press ENTER after each entry.

Choose the linear model with F2 (LINR) .

The *y*-intercept, a, and the slope, b, of the model is displayed along with the number, n, of data points. (The corr that is shown is called the *correlation coefficient*. It is something you will learn about in a statistics course and should be ignored in this course.)

TI-86 Press 2nd + (STAT) F1 (CALC) F3 (LinR) .

To have the calculator find the linear model of best fit using L1 as the input and L2 as the output and paste the model into the y(x)= list, put the following after the LinReg instruction:

The linear model fit by the TI-86 is of the form

$$y = a + bx$$

where *a* is the *y*-intercept and *b* is the slope. This is equivalent, but not of the same form, as the model in your text.

the input and output list names either by typing them from the keyboard or by pressing 2nd – (LIST) F3 (NAMES), press the menu key under L1, press , , and press the menu key under L2. Next, type , y1. Press ENTER . (The model will be pasted into the location that you specify.)

The linear model of best fit for the aligned tax data entered in Section 1.4.4 of this *Guide* is displayed on the home screen.

Note: It is not necessary to first clear any previously-entered function from the location of the y(x)= list.

(The corr that is shown is called the *correlation coefficient*. It is something you will learn about in a statistics course and should be ignored in this course.)

Go the y(x)= list to verify that the model has been pasted into the y1 location.

1.4.8 PASTING A TI-85 MODEL INTO THE FUNCTION LIST

The coefficients of the model found by the calculator should *not* be rounded. This is not a problem because the calculator will paste the entire model into the function list!

TI-85 Press MORE F4 (STREG) . At the Name= prompt, type in y1 and press ENTER .

Remember that you must use a lower-case y to refer to functions in the y(x)= graphing list.

(Any function currently in y1 will be replaced with the linear model.)

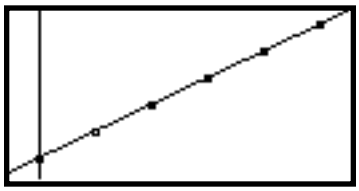
Press GRAPH F1 (*y(x)=*) to see the equation of the model in the y1 location. If you cannot see all of the equation, press the right arrow key to scroll the screen to the right.

1.4.9 GRAPHING A MODEL

After finding a model, you should always graph it on a scatter plot of the data to verify that the model provides a good fit to the data.

TI-85 After you have copied the model to the y(x)= list, run program STPLT to graph the model and the scatter plot on the same screen. The model will graph first and then the scatter plot will appear.

TI-86 After viewing the model, be sure that Plot1 at the top of the y(x)= list is darkened. Then, just press EXIT F5 (GRAPH) to see the model and scatter plot.



1.4.10 PREDICTIONS USING A MODEL

You could use one of the methods described in Sections 1.1.7 or 1.2.1 of this *Guide* to evaluate the linear model at the desired input value. Remember, if you have aligned your data, the input value at which you evaluate the model may not be the value given in the question you are asked.

However, your calculator has a feature that gives you a very easy way to predict either input or output values calculated from a model.

After finding a model,

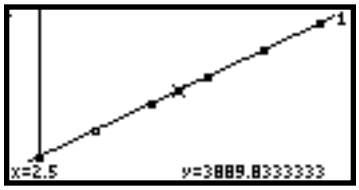
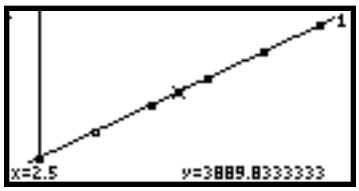
TI-85 press STAT F4 (FCST)

TI-86 press 2nd + (STAT) MORE F1 (FCST)

Both Now all you need do is enter the input value x or the output value y and press the SOLVE key to find the value of the variable you didn't enter.

Predict the tax owed in 1998 where the tax is found using the linear model computed from the data given in Section 1.4.1 of this *Guide*: Tax = 540.37143t + 2538.90476 dollars and t is the number of years since 1992. Note that 1998 is six years since 1992, so $x = 6$. Enter 6 in the x location, press ▽ to move to the y location, and press F5 (SOLVE) .	 The 1998 tax is predicted to be about \$5781.
Predict the year in which the tax will be \$6000. Enter 6000 in the y location, press △ to move to the x location, and press F5 (SOLVE) . Remember the data is aligned, so the answer is about 1992 + 6.4 or near the middle of 1999.	

1.4.11 COPYING GRAPHS TO PAPER Your instructor may ask you to copy what is on your graphics screen to paper. If so, use the following to more accurately perform this task.

TI-85 Press GRAPH to return the graph to the screen. Press F4 (TRACE) to trace the graph. Use ▷ and/or ◁ to locate several values that are as "nice" as possible and mark those points on your paper. Use a ruler to connect the linear model points.	
TI-86 Press GRAPH to return the graph to the screen. Press F4 (TRACE) to trace the graph. The P1 in the upper right-hand corner of the screen indicates that you are tracing the scatter plot of the data. Use either these trace values or the data lists to mark the data points on your paper.	
Both Press ▽ to move the trace cursor to the linear model graph. The number 1 at the top right of the screen tells you which function you are tracing (in this case, y1). Use ▷ and/or ◁ to locate values that are as "nice" as possible and mark those points on your paper. Use a ruler to connect the model points and draw the line.	

1.4.12 WHAT IS "BEST FIT"? Even though your calculator easily computes the values a and b for the best fitting linear model $y = ax + b$, it is important to understand the method of least-squares and the conditions necessary for its application if you intend to use this model. You can explore the process of finding the line of best fit with program LSLINE. (Program LSLINE is given in the TI-85/TI-86 Appendix.) For your investigations of the least-squares

process with this program, it is better to use data that is not perfectly linear and data for which you do *not* know the best-fitting line.

Before using program LSLINE, clear the y(x)= list and enter your data in lists L1 and L2. Next, draw a scatter plot. Reset xScl and yScl so that you can use the tick marks to help identify points on the graphics screen. Press GRAPH to view the scatter plot.

To activate program LSLINE, press PRGM F1 (NAMES) followed by the F-key under the program, and press ENTER . (Since your calculator "loads" each program in memory when you run it, longer programs such as this one take a moment before anything appears on the screen.) The program first displays the scatter plot you constructed and pauses for you to view the screen.

- While the program is calculating, there is a small vertical line in the upper-right hand corner of the graphics screen that is dashed and "moving". The program pauses several times during execution. Whenever this happens, the small vertical line is "still". You should press ENTER to resume execution.

The program next asks you to find the y-intercept and slope of *some* line you estimate will go "through" the data. (You should not expect to guess the best fit line on your first try!) After you enter a guess for the y-intercept and slope, your line is drawn and the errors are shown as vertical line segments on the graph. (You may have to wait just a moment to see the vertical line segments before again pressing ENTER .)

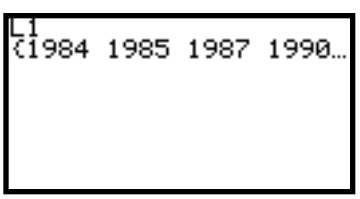
Next, the sum of squares of errors, SSE, is displayed for your line. Choose the TRY AGAIN? option by pressing 1 ENTER . Decide whether you want to move the y-intercept of the line or change its slope to improve the fit to the data. After you enter another guess for the y-intercept and/or slope, the process of viewing your line, the errors, and display of SSE is repeated. If the new value of SSE is smaller than the SSE for your first guess, you have improved the fit.

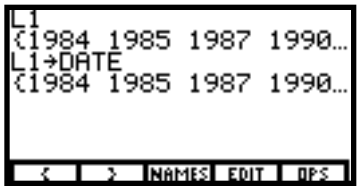
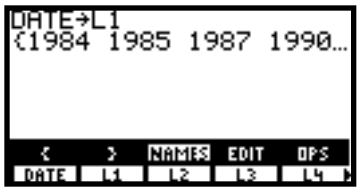
When it is felt that an SSE value close to the minimum value is found, you should press 2 at the TRY AGAIN? prompt. The program then overdraws the line of best fit on the graph for comparison with your last attempt and shows the errors for the line of best fit. The coefficients a and b of the best-fitting linear model $y = ax + b$ are then displayed along with the minimum SSE. Use program LSLINE to explore the method of least squares to find the line of best fit.

1.4.13 NAMING DATA LISTS (optional) You may or may not want to use the additional features given below for data entered on your calculator. You can name data (either input, output, or both) and store it in the calculator memory for later recall.

For instance, suppose you wanted to call the following list L1 by another name :

1984 1985 1987 1990 1992

First, enter the data in the list L1. Return to the home screen. (You can view any list from the home screen by typing its name and pressing ENTER .)	

Pressing ▷ allows you to scroll through the list to see the portion that is not displayed. Type L1, press STO, type the letters D A T E and press ENTER to store this list with the name DATE. This list should now appear in your list menu.	
If you later want to access this list, press 2nd – (LIST). Under NAMES, find DATE. Press the number corresponding to the location of the list, press STO, and type the location you wish to move the list to (say, L1). Press ENTER.	

- **TI-85** The original data remains in DATE. It is not deleted until you delete it using 2nd + (MEM) F2 (DELET) F4 (LIST), move the cursor with ▽ to the location of DATE, and press ENTER. Press 2nd EXIT (QUIT) to return to the home screen.

- **TI-86** The original data remains in DATE. It is not deleted until you delete it using 2nd 3 (MEM) F2 (DELET) F4 (LIST), move the cursor with ▽ to the location of DATE, and press ENTER. Press 2nd EXIT (QUIT) to return to the home screen.