Area – "FOILed" Again!

MATH NSPIRED

TEACHER NOTES

Math Objectives

- Students will compute the area of rectangles with sides represented by algebraic expressions.
- Students will use the distributed property to multiply algebraic expressions
- Students will discover the FOIL method of multiplying algebraic expressions by equating it to the areas of rectangles with algebraic expressions for side lengths.

Vocabulary

- Area
- Distributed Property
- Binomial
- Rectangle
- FOIL
- Trinomial

About the Lesson

- In this lesson, students will practice finding rectangular areas with algebraic expressions for the lengths of the sides by using either the Distributive property or FOIL. Some instruction on these methods is assumed
- It is likely that students will need more "paper and pencil" practice after this activity, since the embedded assessment questions are self-check, and the interactive pieces of the document allow students to obtain answers without necessarily doing the work themselves
- Since the problems are presented as dimensions of rectangles, most of the numbers used (as coefficients and constants) are positive. This may make more sense to students than using negative numbers as part of those expressions. Emphasize, however, that those expressions simply represent the relationship between the two numbers that are the length and width of the given rectangle. For example, if a rectangle has sides of length 7 and 12, the two could be described algebraically as 4x – 5 and 6x – 6. The negative numbers in those expressions do NOT mean the rectangle has any negative lengths!
- Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.

Tech Tips:

- This lesson includes screen captures taken from the TI-Nspire CX II handheld. It is also appropriate for use with the TI-Nspire family of products, including TI-Nspire CX software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- AreaFOILedAgain_Student.pdf
- AreaFOILedAgain_Student.doc

TI-Nspire document

- Area_FOILed_Again.tns

TI-Nspire™ Navigator™ System

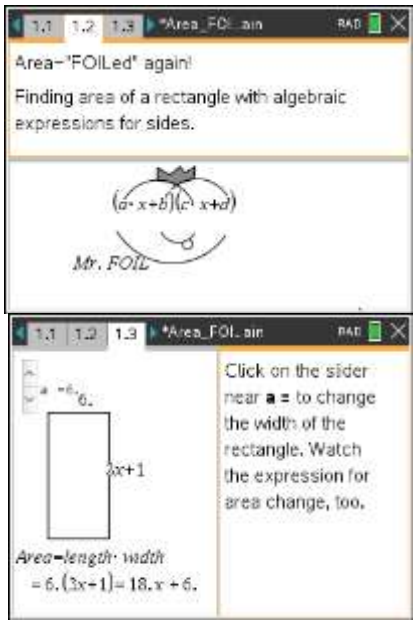
- Use Class Capture to check students' reasoning about a range of intervals and to explore multiple solutions.
- Use Quick Poll to assess student understanding of the Mean Value Theorem.

Activity Materials

Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Discussion Points and Possible Answers

Problem 1 – Introduction to area of a rectangle.



Move to page 1.2 – Meet Mr. FOIL

This page is included as an introduction to the FOIL method but is not explained at this point in the activity.

Move to page 1.3

On page 1.3, the students will interact with the width of the rectangle, and will observe the changes in the calculation for area. Several problems are provided on the student worksheet.

1 What are the lengths of the sides of the rectangle?

Answer: 6 by (3x + 1)

2. What is the area of the rectangle when a = 6?

Answer: 18x + 6

Change the length of the side by clicking on the slider.

3. What is the area of the rectangle when a = 4? When a = 11?

Answer: When a = 4 the area is 12x + 4. When a = 11 the area is 33x + 11

4. How is the expression for the area simplified?

   Answer: Students should see that when one of the sides of a rectangle is a number or a monomial, the distributive property can be used to simplify the expression for the area.

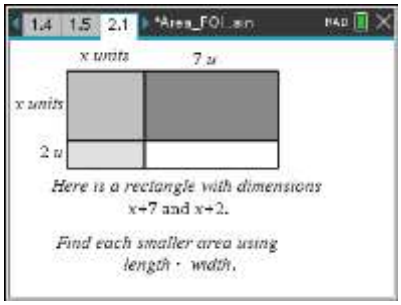
Move to 1.5

5. Choose the expression for the area of a rectangle with sides of length 5 units and x + 8 units.

   Answer: 5x + 40

TI-Nspire Navigator Opportunity: Screen Capture
You can use Screen Capture at this point and throughout the lesson to ensure students are able to manipulate the dimensions. If projected, scroll through the screen captures as the students change the value of ***a*** to show the different equations for area produced.

Problem 2 – Areas of small rectangles



Move to page 2.1

On page 2.1, you see a rectangle of dimensions (x + 7) and (x + 2). Each piece of the rectangle is a different color so that you can focus on its area.

On page 2.1, the rectangle is shaded so that the students will see the four different pieces that must be added together to find the total area. The sides of the entire rectangle are of length x + 7 and x + 2.

The visual is adapted on the next page to show the four area calculations of the small rectangles. Students are asked to add the parts together.

Page 2.3 has a self-check multiple choice question concerning the total area.

6. What is the area of each small rectangle?

   Answer: x^2, 7x, 2x, 14

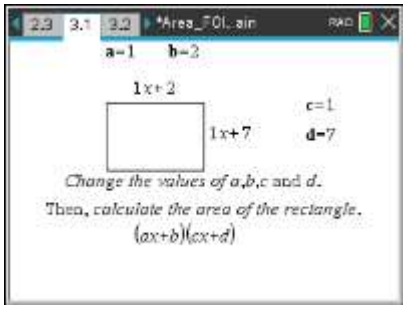
Move to page 2.2 and check your answers.

7. What is the total area of the rectangle?

   Answer: $x^2 + 9x + 14$

Move to page 2.3 and mark your answer.

Problem 3 – FOIL method



Move to pages 3.1 and 3.2

The rectangle on pages 3.1 and 3.2 has the same dimensions as the rectangle from Problem 2. The FOIL method is shown on page 3.2.

Students should notice that the rectangle on page 3.1 has the same dimensions as the rectangle in Problem 2. Have them make the connection between the 4 terms shown underneath the letters FOIL on page 3.2 and the areas of the 4 smaller rectangles in Problem 2.

Page 3.1 has a rectangle with changeable sides of length ax + b and cx + d. Students may double click to change any part of the dimensions, and then calculate the area themselves.

Discourage students from entering in decimals or extremely large or small number values. Suggest whole numbers to begin with (integers will also be acceptable if students feel comfortable multiplying negative numbers.

8. How do the areas of the small rectangles in Problem 2 relate to the expression shown on page 3.2?

   Answer: The x · x is the multiplication of the **F**irst terms. The 7 · x is the multiplication of the **O**utside terms. The 2 · x is the multiplication of the **I**nside terms. The 14 is the multiplication of the **L**ast terms.

You can change the dimensions of the rectangle on page 3.1 by double-clicking on **a, b, c,** or **d.** Practice finding the area of the rectangle and then check your answers on page 3.2.

9. What is the area of a rectangle with dimensions (3x + 5) and (6x + 2)?

   Answer: $18x^2 + 6x + 30x + 10 = 18x^2 + 36x + 10$

10. What is the area of a rectangle with dimensions (4x + 1) and (3x + 9)?

    Answer: $12x^2 + 36x + 3x + 9 = 12x^2 + 39x + 9$

11. What is the area of a rectangle with dimensions (x + 8) and (7x + 3)?

    Answer: $7x^2 + 3x + 56x + 24 = 7x^2 + 59x + 24$

12. What is the area of a rectangle with dimensions (2x + (-3)) and (5x + 8)?

    Answer: $10x^2 + 16x + (-15x) + (-24) = 10x^2 + 1x - 24$

Homework/Extensions

Pages 4.1 – 4.3

On page 4.2, there is another opportunity for you to practice finding area. Record your answer to the first problem here. Show each step of your work. Advance to page 4.3 to check your answer.

1. **a.** $(4x + 2)(x + 7) =$

Answer: $4x^2 + 28x + 2x + 14 = 4x^2 + 30x + 14$

b. $(3x – 7)(2x + 4) =$

Answer: $6x^2 + 12x + (-14x) + (-28) = 6x^2 - 2x - 28$

c. $(2x + 5)(6x + 1) =$

Answer: $12x^2 + 2x + 30x + 5 = 12x^2 + 32x + 5$

d. $(5x + 3)(9x – 2) =$

Answer: $45x^2 + (-10x) + 27x + (-6) = 45x^2 + 17x - 6$

Pages 5.1 – 5.2

On pages 5.1 and 5.2, you will be multiplying a trinomial (3 terms) times a binomial (2 terms) to find the area of a rectangle.

On page 5.1, a rectangle is shown with a trinomial as the length of one side, and a binomial for the other side. The area is calculated on the next page with steps shown, as well as the total area with like terms combined. Again, the numbers are interactive and can be double clicked to change for practice problems. You may wish to have students use the Mr. FOIL model of showing their distributive steps. Underlining like terms is also a good idea.

Students are to determine the formula used to find the 6 terms of the expression for area before like terms are combined. The formula or pattern is:

$(ax^2 + bx + c)(dx + e) =$

$$(a \cdot d)x^3 + (a \cdot e)x^2 + (b \cdot d)x^2 + (b \cdot e)x + (c \cdot d)x + (c \cdot e)$$

This formula invokes the use of the distributive property.

2. What method can you use to find the simplified expression for the area?

Answer: Answers may vary but should have some combination of the explanation provided above. Terms to look for include: Distributive property and combining like terms

3. Use the letters **a**, **b**, **c**, **d**, and **e** to determine the formula used to find the 6 terms of area shown on page 5.2.

Answer: $ax^2 + bx + c)(dx + e) = (a \cdot d)x^3 + (a \cdot e)x^2 + (b \cdot d)x^2 + (b \cdot e)x + (c \cdot d)x + (c \cdot e)$

4. What is the area of the rectangle with dimensions $(1x^2 + 3x + 4)$ and $(5x + 6)$?

Answer: $5x^3 + 6x^2 + 15x^2 + 16x + 20x + 24 = 5x^3 + 21x^2 + 36x + 24$

5. Find the areas of each rectangle. Show each step of your work

a. $(2x^2 + 1x + 7)(3x + (–6)) =$

Answer: $6x^3 + (-12x^2) + 3x^2 + (-6x) + 21x + (-21) = 6x^3 - 9x^2 + 15x - 21$

b. $(4x^2 + 3x + 8)(x + 3) =$

Answer: $4x^3 + 12x^2 + 3x^2 + 9x + 8x + 24 = 4x^3 + 15x^2 + 17x + 24$

c. $(2x^2 + 6x + 4)(–3x + 9) =$

Answer: $(-6x^3) + 18x^2 + (-18x^2) + 36x + (-12x) + 36 = -6x^3 + 24x + 36$

Other Possible Extension Ideas:

A rectangular prism could be introduced with algebraic expressions for length, width and height. Other geometric figures could also be created with various algebraic expressions as dimensions.

TI-Nspire Navigator Opportunity: *Quick Poll*
You may choose to give quick polls throughout the lesson to verify student understanding of the FOIL method. The homework and extension problems can be used to create a quick poll.