

The Rydberg Constant

The light you see when you plug in a hydrogen gas discharge tube is a shade of lavender, with some pinkish tint at a higher current. If you observe the light through a spectroscope, you can identify four distinct lines of color in the visible light range. The history of the study of these lines dates back to the late 19th century, where we meet a high school mathematics teacher from Basel, Switzerland, named Johann Balmer. Balmer created an equation describing the wavelengths of the visible hydrogen emission lines. However, he did not support his equation with a physical explanation. In a paper written in 1885, Balmer proposed that his equation could be used to predict the entire spectrum of hydrogen, including the ultra-violet and the infrared spectral lines. The Balmer equation is shown below.

$$\lambda = \left(\frac{hm^2}{m^2 - n^2} \right)$$

where m and n were integers, and $h = 3654.6 \times 10^{-8}$ cm. When one solves the equation using $n=2$ and $m = 3, 4, 5$, or 6, the calculated wavelengths are very close to the four emission lines in the visible light range for a hydrogen gas discharge tube. Balmer apparently derived his equation by trial and error. Sadly, he would not live to see Niels Bohr and Johannes Rydberg prove the validity of his equation.

Johannes Rydberg was a mathematics teacher like Balmer (he also taught a bit of physics). In 1890, Rydberg's research of spectroscopy (inspired, it is said, by the work of Dmitri Mendeleev) led to his discovery that Balmer's equation was a specific case of a more general principle. Rydberg substituted the wavenumber, $1/\text{wavelength}$, for wavelength and by applying appropriate constants he developed a variation of Balmer's equation. The Rydberg constant bears witness to his contribution to understanding the wave behavior of particles and helped paint a clearer picture of emission spectra.

In 1913, Niels Bohr added to the description of the line spectra from the hydrogen discharge tube. Bohr postulated that electrons orbited an atom in discrete energy levels. Along with Rydberg's work, Bohr called upon Max Planck's investigation of black body radiation and Albert Einstein's determination of the energy of a photon. The combined thrust of these scientific heavyweights resulted in "proving" Johan Balmer's clever little formula. The Bohr equation takes the form shown below.

$$\frac{1}{\lambda} = R_m \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

Niels Bohr used this equation to show that each line in the hydrogen spectrum corresponded to the release of energy by an electron as it passed from a higher to a lower energy level. The energy levels are the integers in the equation, labeled n_i and n_f for initial and final levels, with R_m representing the Rydberg constant. The term $1/\lambda$ is the wavenumber, as expressed by Rydberg in his version of the Balmer equation.

OBJECTIVES

In this experiment, you will

- Use a spectrometer to measure the emission from a hydrogen gas discharge tube.
- Identify the four emission lines of hydrogen that appear in the visible light range.
- Use the emission data to calculate the Rydberg constant.

Before Starting the Experiment...**Working with the Balmer Equation**

Use the Balmer equation, as described in the introductory remarks, to calculate the four wavelengths in the visible light range for the hydrogen gas emission. Record your information in the data table below. The value of h in the Balmer equation is in centimeters, and your calculated wavelengths need to be in nanometers.

m	n	λ (nm)	color
	2		
	2		
	2		
	2		

1. In the Balmer equation, what do the terms m and n represent?
2. Choose any two hydrogen gas emission lines to calculate the Rydberg constant.
3. An accepted value of the Rydberg constant, R_m , is $1.097 \times 10^7 \text{ m}^{-1}$. Compare your values of R_m to the accepted value.

MATERIALS

Vernier Spectrometer or Ocean Optics
Red Tide Spectrometer
computer
Logger *Pro 3* software

fiber optic accessory
hydrogen gas discharge tube
discharge tube power supply

PROCEDURE

1. Use a USB cable to connect a Spectrometer to your computer. Connect a fiber optic cable to the spectrometer.
2. Start the Logger *Pro 3* program (version 3.8.2 or newer).
3. To prepare Logger *Pro 3* to measure light emission, open the Experiment menu and choose **Change Units** → **Spectrometer** → **Intensity**.
4. Turn on the hydrogen gas discharge tube. Aim the tip of the fiber optic cable at the tube.
5. Click  **Collect**. An emission spectrum will be graphed. There are two methods of optimizing the graph of intensity vs. wavelength.
 - a. Set the distance between the light source and the tip of the fiber optic cable so that the peak intensity on the graph stays below 1.0.
 - b. Click  **Stop** to start over. Open the Experiment menu and choose **Set Up Sensors** → **Show All Interfaces**. In the Spectrometer dialog box, increase or decrease the **Sample Time** to optimize your plot.
6. When you have a suitable emissions plot, click  **Stop**. To analyze your emission spectrum graph click the Examine icon, , on the toolbar. Identify as precisely as possible each of the four wavelengths of hydrogen's Balmer series. The third and fourth peaks are very small but they can be identified. Write down the four peaks of the graph in the data table below.
7. To store the data, open the **Experiment** menu and choose **Store Latest Run**.

NOTE: Do not exit the Logger *Pro 3* program until you have written down the necessary data to complete the calculations. You will need to examine your graph of the hydrogen emissions to complete the Data Analysis section of the experiment.

DATA TABLE

Complete the table below. Determine the wavelengths by examining the graph of the hydrogen discharge tube emission.

Wavelength (nm)	Wavenumber (m ⁻¹)	Frequency (Hz)	Photon Energy (J)	n (Balmer Series)
				3
				4
				5
				6

Calculation Guide

- Wavelength: examine the graph and write down the peak in the specified regions.
- Wavenumber = $10^9 / (\text{wavelength in nm})$
- Frequency = $(3 \times 10^8 \text{ m/s}) / (\text{wavelength in m})$ **Note:** $1 \text{ nm} = 1 \times 10^{-9} \text{ m}$
- Photon Energy = $(\text{frequency}) \times h$ **Note:** $h = 6.626 \times 10^{-34} \text{ m}^2 \cdot \text{kg} \cdot \text{s}^{-1}$

DATA ANALYSIS

1. Use the equation described in the introductory remarks to calculate the Rydberg constant for the four lines in Balmer Series that you identified in the table above. What is the average value for the Rydberg constant, based on your data?
2. A second method of determining the Rydberg constant is to analyze a graph of the values of n in the Balmer Series vs. the wavenumber. Prepare a plot of $1/n^2$ (X-values) vs. wavenumber (Y-values). Calculate the best-fit line (linear regression) equation for the plot; the slope of this line is equal to $-R_m$.
3. An accepted value of the Rydberg constant, R_m , is $1.097 \times 10^7 \text{ m}^{-1}$. Compare your value of R_m to the accepted value.
4. Use the R_m that you calculated in Question #3 to predict the wavelength of the fifth line in the Balmer Series ($n = 7$). Examine your graph of the hydrogen discharge tube emissions. Does the fifth Balmer line appear as a peak in your graph? Explain.

PRE-LAB EXERCISES: The Answer Key

Working with the Balmer Equation

m	n	λ (nm)	color
3	2	657.8	red
4	2	487.3	blue-green
5	2	435.1	blue
6	2	411.1	violet

Example calculation: $m = 3$ and $n = 2$

$$\lambda = \frac{(3^2 \times 3654.6 \times 10^{-8} \text{ cm})}{(3^2 - 2^2)}$$

$$\lambda = 6.578 \times 10^{-5} \text{ cm}$$

or 657.8 nm

1. The variables m and n in the Balmer equation appear in the Rydberg equation as n_f and n_i respectively.
2. Using the first two wavelengths, the values of R_m are shown below. **Note:** you must convert wavelength from nm to m

$$(1 \div 6.578 \times 10^{-7} \text{ m}) = R_m (1/4 - 1/9)$$

$$R = 1.095 \times 10^7 \text{ m}^{-1}$$

$$(1 \div 4.873 \times 10^{-7} \text{ m}) = R_m (1/4 - 1/16)$$

$$R = 1.094 \times 10^7 \text{ m}^{-1}$$

3. The values of R_m compare favorably to the accepted value of $1.097 \times 10^7 \text{ m}^{-1}$.