Circumcircle

Teachers Teaching with Technology™
Professional Development from Texas Instruments

TI-Nspire™   Investigation   Student   50 min

Introduction

A circumcircle passes through the each vertex of a polygon. This investigation focuses on the specific case of the circumcircle as it applies to a triangle. The concept of a circumcircle and other similar geometric problems underpin concepts associated with electronic navigation such as Global Positioning Systems (G.P.S).

Geometry

Open a New TI-Nspire Document and insert a **Geometry Application**.

Draw an acute angled triangle.

> **[menu] > Shapes > Triangle**

Once the triangle tool has been selected, a small 'triangle' icon appears in the upper left corner of the screen. The point also changes to a pencil icon with the word 'Point'.

Click on the location for the first vertex (corner), move to the next and click, and again to complete the triangle.

Press **ESC** to release the triangle tool.

Calculator Tip!

Move the pointer over any of the triangle vertices to see it change to a hand. Press Ctrl and Click to grab the vertex or click and hold for a second in the middle of the navigation pad to obtain the same effect. Once the vertex has been grabbed it can be moved to a different location.

The ESC key can be used to quickly exit any currently selected tool or menu or to release an object.

The next aim is to draw a circle that exactly passes through each of the triangle's vertices. The circle also needs to respond to any changes in the location of the vertices, so a simple 'guess' is not sufficient.

> **[menu] > Construction > Perpendicular Bisector**

Using the perpendicular bisector tool, bisect each side of the triangle. This can be done by simply pointing to each side and clicking when the word 'side' appears.

Author: P. Fox

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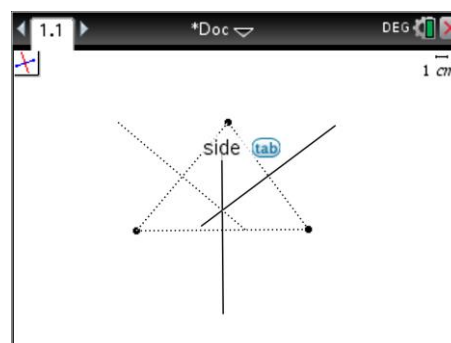
Question: 1.

What do you notice about the three perpendicular bisectors? Try moving the vertices of the triangle. Does your observation still hold?

All three perpendicular bisectors intersect at the one point. (Circumcentre).

A point of intersection needs to be established between pairs of perpendicular bisectors.

[menu] > Points and Lines > Intersection Point(s)

Two non-parallel lines will meet at one point. Three lines do not necessarily have to intersect at a single point; that is why it is important to select pairs of lines and observe where they intersect, particularly once the triangle vertices are moved.
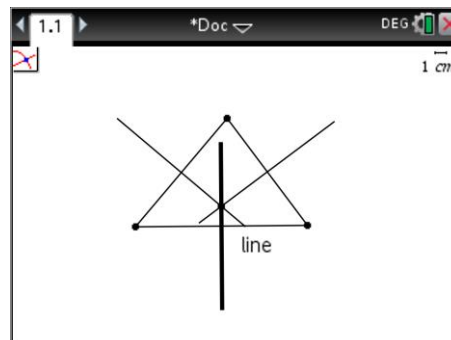
The final step is to draw the circle.

[menu] > Shapes > Circle

Place the pointer at the point of intersection of the perpendicular bisectors and click to locate the centre of the circle.

Move the pointer to one of the triangle's vertices then click to locate the edge of the circle on the vertex. (As shown opposite)

Release the circle tool and explore what happens when the triangle's vertices are moved.
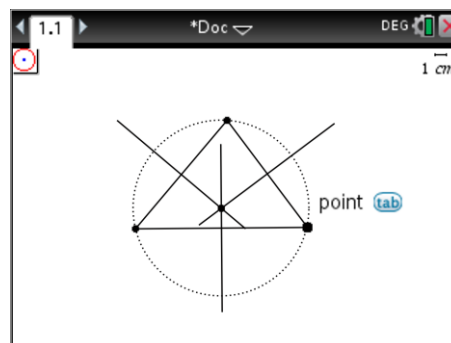
Question: 2.

Does the circle always pass through all three of the triangle's vertices? Discuss specific cases with regards to the location of the centre of the circle (circumcentre) and the type of triangle.

Students may 'hypothesis' that the circle always passes through the three vertices, however a simple proof is also available.
Consider traingles AMP and BMP.
AM = MB (Bisected)
MP is common
$\angle$AMP = $\angle$BMP (90°)
$\therefore \triangle$AMP = $\triangle$BMP (Two sides and included angle)
$\therefore$ AP = BP

Other triangles formed on the edges of the triangle with a vertex at P can also be proved congruent using the same approach.
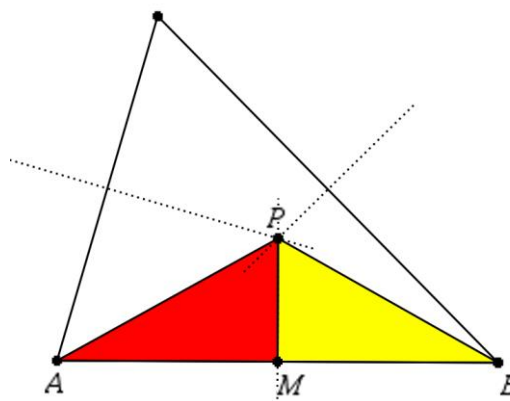
Therefore lengths: AP = BP = CP so a circle placed at P with radius AP will also pass through B and C.

Specific Cases:

Acute angled triangles – Circumcentre is inside the triangle

Obtuse angled triangles – Circumcentre is outside the triangle

Right angled triangles – Circumcentre is on the midpoint of the hypotenuse

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Graphing

The Cartesian plane will now be used to explore this geometric problem.

Insert a **Graph Application** and a **Calculator Application**.

Three points will be plotted representing the three vertices of a triangle.

Points: A:(0, 0) B:(6, 0) and C:(4, 3)

In the calculator application the x coordinates (abscissa) will be assigned to the variable: xp

  xp := {0, 6, 4}

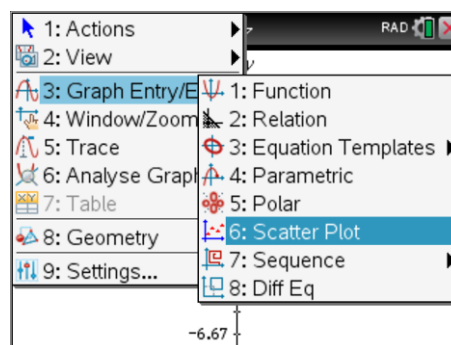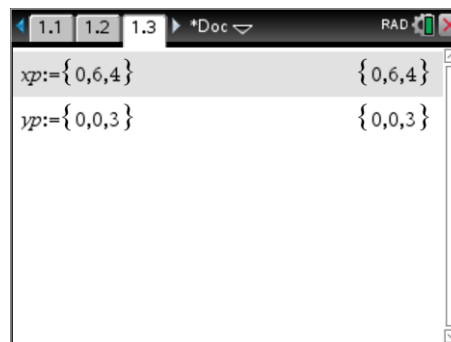The y coordinates (ordinates) will be assigned to the variable: yp

  yp := {0, 0, 3}

Navigate to the Graph Application and plot the three points (vertices) using a scatterplot.

  [menu] > Graph Entry / Edit > Scatter Plot

The x values for scatterplot 1 will be XP which can either be typed or accessed from the variable menu. [var]

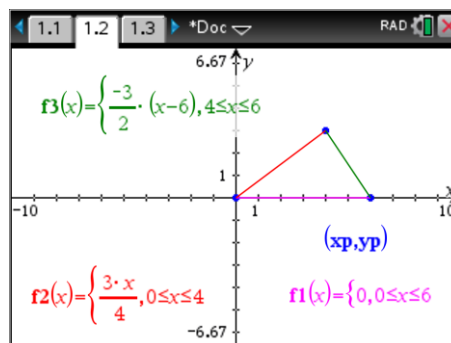The y values for scatterplot 1 will be YP.

Question: 3.

Determine the equation to the each of the three lines (triangle sides) and then graph each on the calculator.

Hint: You will need to change the Graph Entry/Edit back to function.

Equation AB: $y = 0$

Equation AC: $y = \dfrac{3}{4} x$

Equation BC: $y = -\dfrac{3}{2}(x - 6)$

Calculator Tip!

Domain restrictions can be applied to each equation so as to neatly form the sides of the triangle. Press Ctrl + = to access the constraints and inequality symbols. Domain constraints can be applied using the "|" key followed by the corresponding constraint.

Example: f(x)=x | 0≤ x ≤ 5 will graph the function between x = 0 and x = 5.

Question: 4.

Perpendicular bisectors pass through the midpoint of the vertices and at a 90° angle to the sides of the triangle. Determine the midpoint for each of the triangle's sides.

Midpoint AB = (3, 0).   Midpoint AC = (2, 3/2).   Midpoint BC = (5, 3/2)

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Question: 5.

Determine the gradient of each side of the triangle and the corresponding perpendicular gradient.

Gradient AB = 0. Gradient of Perpendicular = Undefined

Gradient AC = ¾ Gradient of Perpendicular = -4/3

Gradient BC = -3/2 Gradient of Perpendicular = 2/3

Question: 6.

Use your answers to the previous two questions to determine the equations to each of the perpendicular bisectors.

$x = 3$

$$y = \frac{-4}{3}(x-2) + \frac{3}{2}$$

$$y = \frac{2}{3}(x-5) + \frac{3}{2}$$

Note: The equations have been provided in 'translational' form making it much easier for students to complete the equation.

Question: 7.

Verify that all three perpendicular bisectors intersect at the same point.

Students may solve simultaneously, however it should be noted that the circumcentre will be located on the line x = 3, therefore substitution into the two bisectors is sufficient. Solution: (3, 1/6)

Question: 8.

Determine the equation to the circle that just passes through the three vertices.
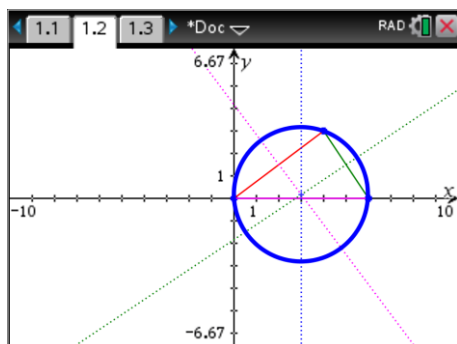
Centre: (3, 1/6)   Radius = $\dfrac{5\sqrt{13}}{6}$

Equation: $(x-3)^2 + \left(y - \dfrac{1}{6}\right)^2 = \dfrac{325}{36}$

Question: 9.

Graph the perpendicular bisectors and circumcircle on the calculator.

Hint: The circle is a relationship (not a function) and can be graphed using the Relation or Equation Template options under Graph Entry/Edit.

Question: 10.

Repeat this investigation for your own three points.

Answers will vary … however the TNS solutions file with this activity download will generate all the necessary equations via the notes application.

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