



HP Solve

Calculating solutions powered by HP

HP donates \$2.7 million in calculators

These donations will go towards the advancement of education solutions for students worldwide.

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Meet an HP calculator user

Gary Tenzer, a real estate investment banker from Los Angeles, has used HP calculators throughout his career in and outside of the office.



[Customer corner](#)

Get the buzz on HHC 2012

Richard J. Nelson

Learn about what was discussed at the 39th Hewlett-Packard Handheld Conference (HHC) dedicated to HP calculators, held in Nashville, TN on September 22-23, 2012.

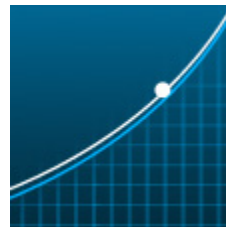


[Read more](#)

Learn about weighted power and exponential curve fitting

Palmer Hanson

By using previously published data on calculating the digits of Pi, Palmer describes how this data is fit using a power function fit, linear fit and a weighted data power function fit.

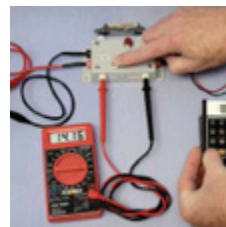


[Check it out](#)

Measure calculator current

Richard J. Nelson

Explore nine examples of measuring the current drawn by a calculator--a difficult measurement because of the requirement of inserting a meter into the power supply circuit.



[Learn more](#)

Investigate a multi-root solver

Namir Shammas

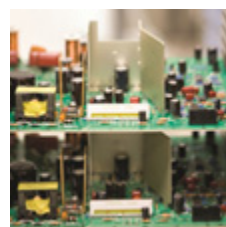
Learn about the HP models that provide solver support and the scan range method of a multi-root solver.



[Read more](#)

From the editor

Learn more about current articles and feedback from the latest Solve newsletter including a new One Minute Marvels and HP user community news.

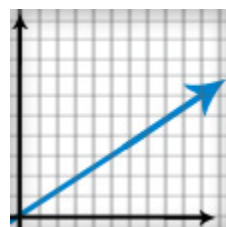


[Read more](#)

Review complex numbers

Richard J. Nelson

What do solutions of third degree equations, electrical impedance, electro-magnetic fields, light beams, and the imaginary unit have in common? Find out in this month's math review series.



[Explore now](#)

Welcome to the twenty-ninth edition of the HP Solve newsletter. Learn calculation concepts, get advice to help you succeed in the office or the classroom, and be the first to find out about new HP calculating solutions and special offers.

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HP donates \$2.7 million in calculators

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HP Donates \$2.7 Million in Calculators to Students Worldwide

HP Calculators has donated over \$2.7 million dollars in calculators to underserved education institutions in an effort to improve educational technology and resources in developing countries around the world. This donation provides thousands of advanced graphing and scientific calculators to students and professors, helping to increase their understanding of mathematics and science while improving overall standards of education in the classroom.

In conjunction with HP Sustainability and Social Innovation, HP Calculators selected Good 360, a non-profit organization, to distribute calculators through an open bid process to affiliated charities and schools in over seven countries worldwide. Products donated range from HP 30s and HP 300s scientific calculators for middle and high school students to HP 40gs and HP 50g graphing calculators designed for advanced math at high schools and universities.

This initiative marks the largest donation of calculators in HP history with units being distributed to schools in need throughout Asia, Africa and Latin America. The Philippines received HP 40gs graphing calculators for students at the Boy's Home of Liloan, which offers street children a safe environment with a variety of technical training programs to develop specialized skills (Image 4). The impact of these donations is already being observed by the director of Technical Programs who remarked, "Our programs have been dramatically enlivened in its desk exercises, that with this one-on-one personal calculation tool has enabled trainees to do speedier numeric solving." Additionally, Vietnam is implementing an educational opportunity program where their country's top-performing high school students will receive graphing calculators, with elementary school students benefiting from HP EasyCalc 300s calculators as well.



Fig. 1 - A teacher in the Philippines trains curious students to use the HP 40gs

In Africa, Ethiopia and Sierra Leone, schools that never before had access to educational technology in their classrooms will be able to advance their curricula, enhancing students' mathematics education. As Marcia Roeder of ChildFund International mentioned, "I am sure you can imagine the tremendous impact this donation will have on their ongoing education."

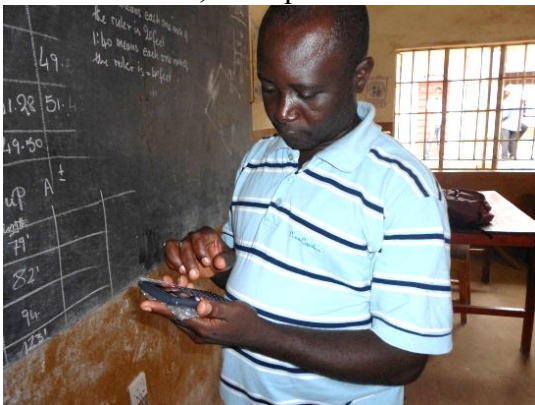


Fig. 2 - A student calculating a problem on the blackboard with the HP 30s.

Students that were previously unable to pass their exams without a calculator will now be able to take their exams and advance their skills in mathematics and science. Yusifu Mansaray of the Sierra Leone Opportunities Industrialism Centre, who received an HP 30s calculator stated, “I had started to work on someone’s farm to gather money to buy a scientific calculator for one week now and was unable to afford one. It is a great relief that these calculators have come at a time [when] we badly need them to sit our final exams at the agricultural survey department. I will now use the time I spent to look for money to buy a calculator to study for my exams.”



Fig. 3 - Students carefully reading their HP 30s user manuals.

Another student, Umiro Tarawalie of University of Makeni commented, “The calculator I got is the perfect one that I need for my calculations and graphs in the engineering department. I am so happy with this [HP 50g] calculator that I have passed half of my exams already.” Other countries receiving calculator donations include Cambodia and Chile.

With these donations, HP continues its commitment to lead in the advancement of education solutions for students worldwide. For more information on other news from HP Calculators, please visit www.hp.com/calculators.



Fig. 4 - Students of the Boy’s Home of Liloan with their new calculators.



Fig. 5 - Students of SLOIC displaying their newly acquired calculators.

Meet an HP calculator user

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Meet Gary M. Tenzer

Editor's note. Customer Corner has appeared in past issues of *HP Solve* where we interviewed the worldwide users of HP's calculators. Past interviews have been of users who live and work in the US, UK, Canada and Germany. We now go to Los Angeles, California for our next interview.



1. *HP Solve* : What is your background?

Gary: I have an undergraduate degree in Economics from UC Berkeley ('76) and two master's degrees; an MBA in Finance ('78) and MSBA in Real Estate Finance ('79) from the University of Southern California. After grad school I joined a real estate investment banking firm and when it was acquired a year later, its former president and I left to form a new company, which ultimately became George Smith Partners, the firm where I have worked for virtually my entire career. George Smith Partners structures and places multi-Billion Dollars annually in real estate debt and equity for major commercial real estate projects, investors and developers, throughout the Country.

2. *HP Solve: What did you study at school?*

Gary: As an undergrad, I majored in Economics and Urban Land Economics; I also minored in Astronomy and Psychology. At graduate school, I majored in Finance and Real Estate Finance.

3. *HP Solve: What is your occupation?*

Gary: Real Estate Investment Banker and Expert Witness. In my role as a real estate investment banker, I have had over 30 years structuring and placing both debt and equity on commercial real estate projects, nationwide. Projects include: multi-family, industrial, retail, hospitality, single-family as well as credit facilities for investment companies. For the past 20+ years I have been also working as an expert witness and litigation support consultant specializing in real estate finance issues. In that capacity, I have been served as an expert witness in over 300 cases and testified in federal and state courts over 100 times. As an expert witness, I have represented many of the most important real estate developers and investment companies in the Country.

4. *HP Solve: Do you do much traveling?*

Gary: I do some travelling for business but many of the lenders & investors that I work with have offices in Los Angeles. In my capacity as an expert witness, I do have to travel to attend depositions and trials, throughout the country. This past summer, I had three cases come to trial at the same time in San Francisco, where I spent most of my time.

5. *HP Solve: When were you first exposed to HP calculators?*

Gary: My first exposure was an ad for the HP-35 in Scientific American in 1972. When I first saw it, I was hooked and I knew that I had to have one.

6. *HP Solve: When did you first see or use an HP calculator?*

Gary: I first saw an HP-35 when my freshman chemistry professor at UC Berkeley showed it to the class. The HP-35 initially cost \$395 and after calling my father for permission, I immediately went to the student store and put my name on a list to buy two; one for me and one for my twin brother who was a pre-med at Princeton. Once I received my prized HP-35 and learned how to calculate with RPN, I wrote a "fan" letter to HP, which was based in Corvallis at the time. They invited me to visit with them and tour the factory. It was then that I became a HP aficionado for life.

7. **HP Solve: What machine did you buy after your first?**

Gary: After my HP-35 was stolen from my dorm room, I replaced it with an HP-80, HP's first financial calculator, which I purchased with the insurance money.

8. **HP Solve: What HP calculators have you used since?**

Gary: After the HP-80, I then graduated to an HP-65 programmable, then HP-67 and HP-97. I also had an HP-22, HP-12C and HP-38C. My next calculators were the HP-41C, HP-41CV and HP-41CX along with their system components like the card reader, tape drive, HP-IL printer, bar code reader, etc. I also own a HP 19b, HP 17BII, HP 19bII, HP 100/200LX, Jornada 600, 610 and 720 and probably a number of other machines which I have forgotten to list. Today I do most of my calculations with an HP-12C Platinum or HP 17BII+ (I keep one at each phone in my house, in my office and in my briefcase as well as an emulator on my Android phone). I continued my loyalty to HP by buying the first HP laptop computer, the Omnibook 300 for me as well as the other professionals in my company. We have continued to only buy HP laptops and printers for our employees and I'm sure that we have purchased hundreds of them over the ensuing years.

9. **HP Solve: What have you used your calculators for?**

Gary: I did a lot of programming on the programmables including many financial and statistical programs as well as programs to analyze real estate cash flows and proformas. However, much of that work I now do most of my modeling using Excel and I just use the calculators to do financial calculations without the need of programming.

10. **HP Solve: Have you noticed anything interesting about calculator usage during your travels?**

Gary: Yes. I brought a programmable calculator with me to China in 1979; the people there had never seen anything like it and were really amazed. I was photographed wearing an HP "Enter > =" T-Shirt on the Great Wall of China, which was featured on the cover of HP Keynotes Magazine.

11. **HP Solve: Does your calculator usage involve more than your profession?**

Gary: I use it for day-to-day arithmetic calculations; but I mostly use it now for financial calculations.

12. **HP Solve: What was the largest problem you have solved with an HP calculator?**

Gary: I used the 12C to do some calculations while structuring a \$247 Million mortgage transaction on a condominium conversion project in Los Angeles.

13. **HP Solve: What are you currently using HP calculators for?**

Gary: Compound interest calculations and loan underwriting. Many times I find that it is much quicker to use a financial calculator than to set up an Excel spreadsheet.

14. **HP Solve: What appeals to you about HP calculators?**

Gary: Primarily the quality (I'm still using my original HP-12C after 30 years), RPN, Predictability of answers, accuracy, etc. After using RPN on HP calculators for so many years, algebraic logic seems archaic and too difficult to use.

15. **HP Solve: Do you have a website?**

Gary: <http://gspartners.com/personnel/principals/gary-m-tenzer/>

16. **HP Solve: What are the important aspects of your website?**

Gary: My curriculum vitae.

17. **HP Solve:** *Do you write or post calculator articles on your website?*

Gary: I haven't written about calculators, recently; however, I used to write extensively for the *PPC Journal*.

18. **HP Solve:** *Have you had contact with HHC?*

Gary: Yes.

19. **HP Solve:** *How many HHCs have you attended?*

Gary: Several over the years.

20. **HP Solve:** *What have you gained from the HHCs?*

Gary: Knowledge, insight as well as good friends

21. **HP Solve:** *Do you have a calculator collection? How extensive is your collection?*

Gary: Yes, I have over 50 HP calculators; and, I'm sure that they all still work.

22. **HP Solve:** *Do you have any "special" HP calculators?*

Gary: Yes, my favorites are the HP 67/97 and HP-41C series and its accessories.

23. **HP Solve:** *What does your family think of your interest in calculators?*

Gary: They used to think that I was nuts but have grown to accept it as one of my hobbies and passions.

24. **HP Solve:** *What other HP calculator related projects have you worked on?*

Gary: I worked with HP in preparing the original HP-12C solutions manual and wrote some of the software and manuals for the HP-41C Real Estate Pac.

25. **HP Solve:** *Do you have any additional comments that may be of interest to HP Solve readers?*

Gary: Learning to use a finance calculator was instrumental in helping me learn the fundamentals of finance and the time value of money.

26. **HP Solve:** *Thank you for spending your time with us.*

Gary: My pleasure. I always am happy to discuss HP calculators; they have been a major interest of mine for over 30 years.

Get the buzz on HHC 2012

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Hewlett-Packard Handheld Conference #39

Richard J. Nelson

Introduction

Nashville TN – HP Calculator enthusiasts from five countries⁽¹⁾ converged in Nashville TN to present HP user projects, explore new math methodologies, explain calculator measurements, compare products, share experiences, report on trends, and indulge in personal fellowship. See details in Appendix A.

This is the 39th HHC⁽²⁾ since the first conference was held in Santa Clara CA in September 1979. HHCs follow a standard format which is reflected in the schedule at the Note 2 link. Forty one people registered and 39 of them were in the group photo shown in Fig. 1.



Photo by Joseph K. Horn

Fig. 1 – Attendees of the 2012 HHC from Canada, England, France, Germany, and the US discuss HP Calculators.

One special aspect of an HHC is the opportunity to directly discuss calculator activities with HP staff members.

How HHCs Happen

The annual HHC Conference is managed by a Committee of six long time and experienced HP Calculator users. See Fig. 2. The attendees donate from \$0 to \$50 (the range for the last ten or more years) to cover all expenses which are not known until the conferences starts. It was \$40 this year. Three meals were included – Saturday lunch and dinner and Sunday lunch. Three breaks include soft drinks and chips or crackers.

Gene Wright: Registration, genela@comcast.net
Richard J. Nelson: Speakers Schedule, Proceedings, rjnelsoncf@cox.net
Joseph Horn: Website, mail, frjoe.us
Wlodek Mier-Jedrzejowicz: European coordinator, wlodekjmj@yahoo.co.uk
Jake Schwartz: Videographer, historian, jakes@pahhc.org
Eric Rechlin: Dedicated All-Around Helper, eric@hpcalc.org

Fig. 2 – HHC 2012 Committee members with responsibilities.

The date is determined by taking the weekend that follows September 15th (the end of the summer season) that is not in conflict with a religious holiday. Rarely does the HHC occur in October. In recent years the ability to predict the location of the next Conference has been greatly reduced with the economy being what it is. West Coast and Midwest locations seem to be the most popular. Suggestions for 2013 are Chicago, San Diego, and Ft. Collins. Having the Conference in a close-airline-hub-city hotel is the ideal location because all attendees are together for the duration – Friday evening to Monday morning. Serious location discussions are started in January after CES. Interested readers may sign up for the HHC List⁽³⁾ to receive emails related to the Conference and to provide opinions and inputs related to HHC.

HHC Objectives

One of the primary objectives of an HHC is to document the progress and activities of the HP Calculator User Community, HPUC. This is accomplished by recording the presentations made during the Conference with a printed proceedings⁽⁴⁾, and videotaping⁽⁵⁾ the Conference from start to finish. The opening is with each attendee giving his name, where from, interests, etc. as described in the Conference Schedule.

Another primary objective is to have fun.

Information exchange, problem solving, and the discussion of HPUC issues are additional objectives for each HHC.

The annual HHC provides a reporting point or “mission accomplished” opportunity for the HPUC to become aware of various projects large and small. New projects are inspired by old ones, and we have been using the HHC as a stimulus for HP Calculator activity for 34 years.

Who attends an HHC?

Aside from the representatives from HP⁽⁶⁾, who attends an HHC? Obviously a strong interest in calculators and calculation technology is a major attraction to meet with and exchange ideas with the leaders of the HPUC. Everyone has their own interests and we try to provide coverage of them all. Here is one HP Museum comment.

“I felt privileged to finally attend my first HHC, more than 36 years after buying my first HP. It far exceeded my most positive expectations. It is a complete mystery to me how such a quality event can be executed for the extremely low conference fee charged (\$40).

*I just wish I hadn't needed to drive **so far** to get there (93 miles!!!) :-)”*

One attendee blogged⁽⁷⁾ about the Conference while it was happening. We had a hotel Wi-Fi connection and several attendees used it during the Conference including Joseph Horn, the HHC webmaster, who updated the Schedule page as we went. I suggest reading the blog links in Note 7 to get another perspective of who attends an HHC - and a another perspective of what happened.

See Appendix A for the details of who attended HHC 2012 and from where.

What was presented at HHC 2012?

The first speaker was Jake Schwartz describing the **WP-34S and the HP Calculator Way**. Jake discussed the two favorite topics of the HP calculator enthusiast - Wish Lists and HP Differentiators.

He described the model number vs. features in this regard with a curve. See Fig. 3. If the crossover point is the introduction of the HP 49 the time is May 1999. Jake uses HP models to describe the relationship between the HPUC and HP’s position at and past the crossover point.

HP and The Community’s Vision Diverges

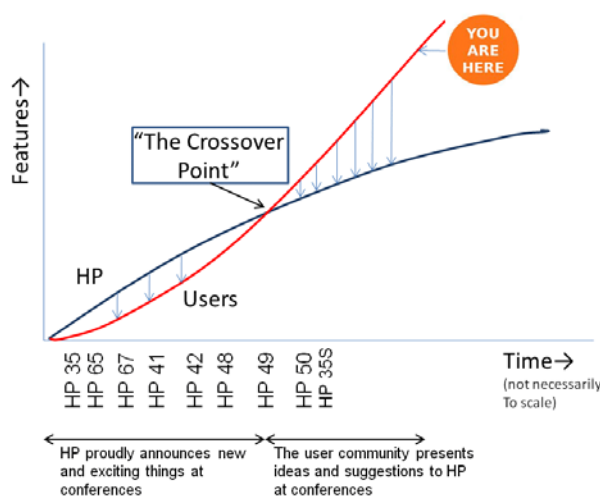


Fig. 3 – Jake’s curve of HP features over time.

The second speaker was Gene Wright who spotlighted the Conference concern of convergence with a presentation of [Tablets vs. HP Calculators](#). Gene described how the most successful tablet, the iPad, introduced on 27 January of 2010 has become extremely popular. The idea is expressed in the cover graphic of the paper shown in Fig. 4. The HP-39gII shown is only representative of its class. The three important realities that were presented: (1) The tablet is being required by an increasing number of educational institutions, (2) The tablet has an unbelievable number of applications being written for it, and (3) The rate of adoption is accelerating.



Fig. 4 – General purpose tablet does “everything.”

Add to this the fact that most HP Calculators emulators will run on a tablet and it is clear that convergence of tablets and calculators is taking place like an explosion.

The third presentation by Namir Shammass discussed [Calculator Root Finding: Scan Range Method](#). Namir reviewed the root finding applications of previous HP calculators, i.e., HP-65, HP-55, HP-67, HP-25, HP-34C, HP-41, HP-15C, and the solvers of all current machines. The Scan Range Method was compared with traditional methods with several examples and error comparisons.

The fourth presentation, impromptu, was by Roger Hill who demonstrated the [Wrongulator](#). This strictly-for-fun calculator gives the wrong answer. Of course the challenge was to “reverse engineer” the answers to determine the algorithm it uses to give the wrong answer.

The fifth presentation was by me on [Measuring HP Calculator Current](#). See the details elsewhere in this issue.

The sixth presentation was by Palmer Hanson on [Calculating before Calculators](#). Palmer is an old time calculator user (and collector) of all kinds of machines and we discussed this topic while I was writing the Math Review Series # 4 in [HP Solve](#) issue 21, November 2010 page 27. He demonstrated and passed around various handheld mechanical calculators, e.g. slide rules and addiators.

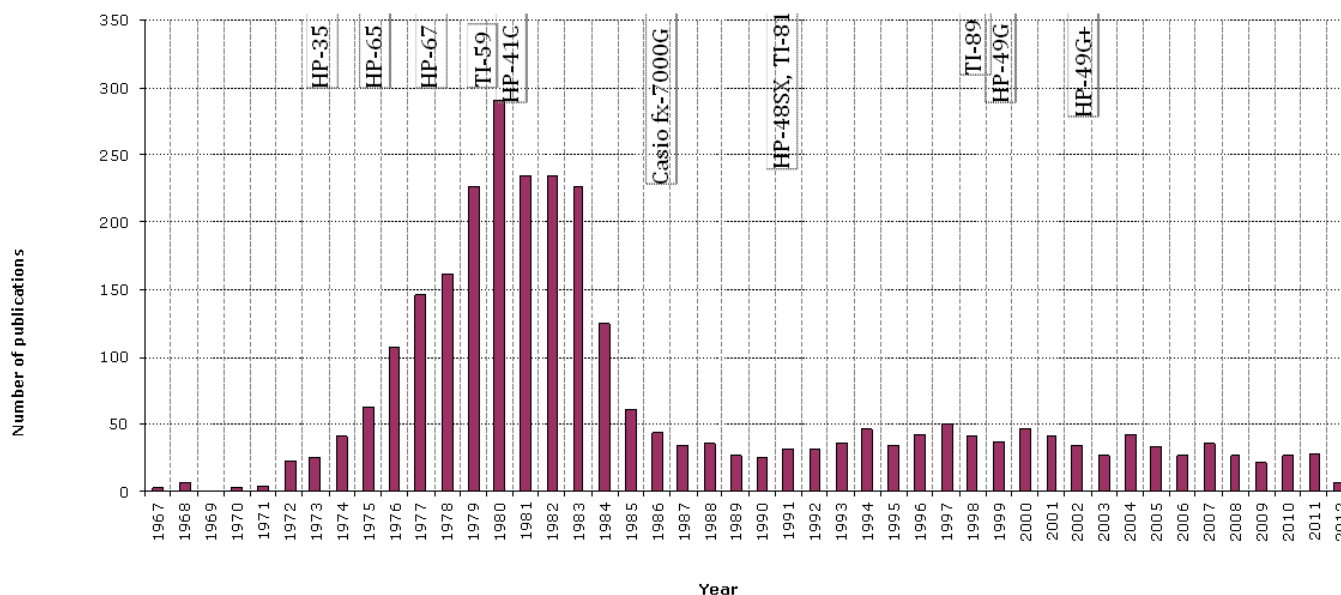
The seventh presentation was by Warren Farrow who described and answered questions about his website⁽⁸⁾ dedicated to the HP-41 reminding us that [The HP-41 is Alive and Well](#).

The eighth presentation was by Felix Gross on [Where Can you Find the History of Calculator Applications?](#) This is an example of a major project related to HP calculators. The 266 pages of all calculator references is on the Conference USB drive. Felix has identified hundreds of additional references that will be added as time allows. The plot extracted from his paper illustrates this most complete compilation of all calculator references – See Fig. 5 on the next page.

The ninth presentation was by Pavneet Arora on [Re-tooling the Calculating Tool](#). Pavneet takes the view that conversion is never complete for several reasons. The cost of failure of the general purpose device is primary. The tablet is a fragile device and special cases are required to ruggedize it.

The tenth presentation was by Katie Wessman describing a [Show and Tell – Calculator Quilt](#) for a door prize. Based on the HP-42 this work of art was enjoyed by all.

Publication Year of Calculator Publications



The distribution remained rather unchanged when the sample size of the bibliography was increased from 1000 to over 3000 references. Most new references were found in the databases mentioned in sections 2.2 and 2.3 above.

Fig. 5 – Felix’s plot of the number of worldwide calculator related publications vs. time. The peak was in 1980.

The eleventh presentation was by Gene Wright on [These are a Few of My Favorite Things](#). Gene showed his favorite non-HP and non-TI machines from his collection. The exception was a TI prototype that is one of only two known to exist. All were in working order and were passed around for all to see the various milestones in calculator development. Oh, so many batteries!

The twelfth presentation was by Jackie Woldering on [Converting HP-41C Programs to PDF - PPC barcode lives on](#). Jackie was a PPC Chapter coordinator for Cleveland. This is another example of a major project undertaken by the HPUC. He reviewed all of the bar code generating documents and software for creating HP-41 barcode. His project converted the V6N4 to V10N4 articles with HP-41 programs to PDF files – all of which are on the HHC 2012 USB drive. This project strongly contributes to the purpose of an HHC and Jackie will continue “until the end” as time allows.

The thirteenth presentation was by Eric Smith on [Even Larger Scaled Reptiles of the Nordic Countries](#). This Do It Yourself, DIY, calculator project⁽⁹⁾ is getting ever closer to a final physical and feature design.

The fourteenth presentation by Namir Shammass on [Curve Fitting with Least Squares Relative Errors](#). This “new” Algorithm is in competition to that of the classic Gauss method.

The fifteenth presentation by Wlodek Mier-Jedzejowicz on [Is it Really a Pathetic Name?](#) The alternate meaning to RPN is from a book⁽¹⁰⁾ with a section by Bill Wickes who provided “reflections on the RPN wars.” Wlodek serves on the HHC committee as the European Coordinator and as his name reflects he is Polish. Since his family is firmly rooted in Poland and the name is based on the work of Jan Łukasiewicz he has several “connections” to Jan and his historical work. All of this was covered by Wlodek during his unique HHC presentation.

The sixteenth presentation by Namir Shammass on [Variable Sampling Integration](#) reviews numerical integration and offers a new technique for obtaining a more accurate result.

The seventeenth presentation by Rick Furr on [The Making of The HPV2 Poster](#) was a fascinating mixture of describing his website and his remaking of the most useful poster available covering all of HP's calculators. See additional information on Rick below in the Competition winners section.

The eighteenth and last presentation (impromptu) by Jackie Woldering was on the [Raspberry Pi](#). This low cost working \$35 Linux computer circuit board – shown in a \$15 plastic case – provides the next generation of computer experimenters a “breadboard” to test various I/O methods and interfacing with networks. Jackie described a 64 board super computer for a cost of \$4,096. A UK engineer built the super computer using a Lego rack mounting scheme.



Fig. 6 – Raspberry Pi computer circuit board photographed at HHC 2012.

See note 11 for additional links for this state-of-the-art very small computer.

Competition winners

One of the standard activities of an HHC is voting for the best speaker on Sunday afternoon. The attendee voted best speaker winner this year was [Rick Furr](#) for his presentation on [The Making of The HPV2 Poster](#). As the winner Rick was the first to pick from the nearly 80 door prizes donated this year. The certificate was not delivered to any of the winners and Eric Rechlin snail mailed them to the three winners.

The second and third pick from the door prize tables were the two programming Contest winners. Dave Hayden won the RPN programming contest and Bill Butler won the RPL programming contest. See appendix B for more programming contest information.

Door prizes

Every year we accept door prizes from HP, dealers, and attendees. On Sunday, when we are sure that we have all of them, we separate the more expensive, unusual, and rare items which are put into a separate group we call the premium group.

The two groups, door prizes and premium door prizes are given to competition winners and the attendees by random drawing. Fig. 7 shows the two door prize tables.



Fig. 7 – Door prize tables. The premium group is in the corner behind Felix Gross.

The competition winners get first pick from the door prizes. The remainder of the door prizes is drawn at random using the tickets that are provided at registration. Winners who may have to leave early may appoint a remaining attendee to select and get the prize to the winner. Gene confirms the names when the numbers are called.

When the door prizes are given all the tickets are put back into the pot to be randomly drawn to give out the premium door prizes. In this way everyone gets a chance at these prizes as well.

Horse trading is often done during this process as well as after the Conference.






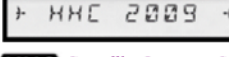



Fig. 8 – Premium door prizes. See list in appendix B.



Fig. 9 – Calculator Quilt by Katie Wessman.

HHC Websites

The HHC website is created (from scratch using raw HTML programming) and maintained by Joseph K. Horn. You may visit 14 past websites at: hhuc.us These websites also serve to record the work of the HPUC. An especially nice aspect of these websites are the annotated photos of the conferences. These are indicated by the photo file symbol at the left of each HHC logo. The white on black logo is the PPC style logo used since the HP 65 Users Club changed its name to PPC.

 HHC 2012 Nashville, Tennessee -- September 22-23, 2012 Convergence	 HHC 2005 Franklin Park, Illinois -- September 17-18, 2005 Nostalgia and the Cutting Edge!
 HHC 2011 San Diego, California -- September 24-25, 2011 Back to the Future	 HHC 2004 San Jose Airport, California -- September 25-26, 2004 The Future Beheld!
 HHC MMX Fort Collins, Colorado -- September 25-26, 2010 Getting Back to Basics	 HHC 2003 Newport Beach Orange County, California -- September 20-21, 2003 A New Hope
 HHC 2009 Fort Collins, Colorado -- October 3-4, 2009 30th Anniversary of the HP-41 System	 HPCC 2002 , Imperial College, London UK -- September 21-22, 2002 (Anybody remember what the theme was that year?)
 HHC 2008 Corvallis, Oregon -- September 27-28, 2008 HP20B Business Calculator & ARM7 Development System	 HHC 2001 Chicago, Illinois -- September 15-16, 2001 2001: A Handheld Odyssey
 HHC 2007 San Diego, California -- September 29-30, 2007 35th Anniversary of the HP-35	 HHC 2000 Los Angeles, California -- September 9-10, 2000 (Anybody remember what the theme was that year?)
 HHC San Jose Airport, California -- September 16-17, 2006 The Year of the HP-12C	 HHC 1999 , Vancouver, Washington -- August 21-22, 1999 (Anybody remember what the theme was that year?)

Freebe table

We had a freebe table this year where attendees bring their surplus items to give them to a new user who will give them a good home. This year we had a varied number of these items. Here are a few I remember. Jackie Woldering had a box of squeeze flashlights so everyone would never be in the dark, batteries not required. He also had a number of pocket Frensel lenses for reading small print. I brought extra copies of previous Conference proceedings. Vern Lindsay brought a stack of non-working HP-48's for those who needed spare parts.

Conference materials

The Conference materials are provided in a Tyvek® 12” x 15.5” envelope with a registration number on its label. The envelope contained the representative proceedings, an article by Joseph Horn, and the schedule. It also had a USB drive as shown in Fig. 10 with the Programming Contest rules. The numbered blue door prize ticket was also in the envelope



Eric Rechlin provided a CD of his website, <http://www.hpcalc.org>. This is an excellent reference of a broad range of HP calculator information. *Fig. 10 – USB Drive with all HHC 2012 related files.*

Post Conference

Eric Rechlin took videos of the individual presentations. All of Eric’s HHC videos may be found at: <http://www.youtube.com/user/hpcalcorg>. Below are the HHC 2012 links to each presentation.

- | | |
|--|---|
| 1. Jake Schwartz, WP-34S and the HP Calculatgor Way | http://www.youtube.com/watch?v=R_S-6rT5BzI |
| 2. Gene Wright, Tablet s vs. HP Calculators | http://www.youtube.com/watch?v=Zpf4u2K7Uow |
| 3. Namir Shammas, Root Finding: Scan Range Method | http://www.youtube.com/watch?v=rntYYhzBgA |
| 4. Roger Hill, The Wrongulator Calculator | http://www.youtube.com/watch?v=eSyfg8bzqow |
| 5. Richard J. Nelson, Measuring Calculator Current | http://www.youtube.com/watch?v=nPz0nFwmjTo |
| 6. Palmer Hanson, Calculating Before Calculators- | http://www.youtube.com/watch?v=bJxxrCRxhq8 |
| 7. Warren Farrow, The HP-41 is Alive and Well | http://www.youtube.com/watch?v=bylWsxftqc |
| 8. Felix Gross, Where Can you History of Calculators? | http://www.youtube.com/watch?v=sf5GaceWYK0 |
| 9. Pavneet Aora, Re-tooling the Calculating Tool | http://www.youtube.com/watch?v=_F5sr1ipTkU |
| 10. Katie Wessman, Show and Tell (her HP-42S Quilt) | http://www.youtube.com/watch?v=g3Iv2h4Lv0M |
| 11. Gene Wright, These are a few of My Favorite things | http://www.youtube.com/watch?v=kXQBV5h_SxE |
| 12. Jackie Woldering, Converting HP-41 Programs to PDF | http://www.youtube.com/watch?v=HR6t9ip35mg |
| 13. Eric Smith, Even Larger Scales Reptiles of the Nordic | http://www.youtube.com/watch?v=E0ftRUtXGrI |
| 14. Namir Shammas, Least Squared Relative Errors | http://www.youtube.com/watch?v=vt2-QWOWWks |
| 15. Wlodek Meir-Jedzejowicz, Is it a Really Pathetic Name? | http://www.youtube.com/watch?v=qRrAj-GCTQM |
| 16. Namir Shammas, Variable Sampling Integration | http://www.youtube.com/watch?v=dMz1a4JQ_oc |
| 17. Rick Furr, The Making of The HPV2 poster | http://www.youtube.com/watch?v=zJA_kPC137I |
| 18. Jackie Woldering, The Raspberry Pi | http://www.youtube.com/watch?v=ij_0FE-LRXg |

Two old combatants meet

Some years ago I published an HP-TI challenge. The two final contenders in this challenge met at HHC 2012. Here is how the TI-59 Accomplishments website describes the challenge.

Printing a year's calendar does not sound like a big deal, but the friendly competition between the TI-59 and the HP-41C owners produced masterpiece programs. The contest started in May 1978, when Jaren Weinberger and Lou Cargile wrote a TI-59 program able to print a calendar in 34 minutes. Panos Galidas managed to cut that time to 5.5 minutes, but Roger Hill's HP-41C program, published in July-August 1980 issue of PPC Journal, produced a calendar in 2 minutes 17 sec (to be honest, HP-41C printout looked better). After many iterations, Palmer Hanson wrote a fast mode calendar-printing program that seemed unbeatable - 1 minute 32 sec (read Palmer Hanson's [story](#) about those programs). Yet, Roger Hill finally won the contest with 1 minute 14 sec HP-41C program. Well, win some, lose some...

It seems that Roger’s program won because he realized that the printer speed was the limitation and he structured his program to have the printer always print. See Fig. 11 and Fig. 12. Roger is on the left, Palmer is on the right. *HP Solve* readers will also recognize Palmer as a regular contributor.



Fig. 11 – Roger and Palmer relive the “challenge.”



Fig. 12 – The winner, 1m 14s, has the biggest smile.

Observations and conclusions

The weak economy was felt more strongly this year, but we still had a respectable turn out in a part of the country that we haven't been to very often. This may be seen from the list of previous Conferences as given in Note 2. We had several attendees that we haven't seen for many years as well as five first time attendees. Several major projects were presented; Felix's worldwide Calculator publication list, Eric's DIY calculator, and Jackie's HP-41 program barcode “clean up” project of PPC Calculator Journal articles. Rick Furr's high quality HP Calculator Poster involved a number of special techniques to produce, and his presentation was voted as best speaker. Our thanks go to HP for their support of this Conference. We had a good crop of door prizes and the most common comment I heard was, “It went by so fast.” That is how it usually happens when you have so much fun.

X <> Y,

Richard

Notes - Hewlett-Packard Handheld Conference #39

- (1). An attendee list with home country may be found at <http://hhuc.us/2012/reglist.htm>
- (2). A list of past conferences may be found at: <http://hhuc.us/2012/conflist.htm> All of the most recent conference websites (1999 – 2012) may be found at: hhuc.us An updated (as it happened) schedule of the speakers and topics may be found at: <http://hhuc.us/2012/schedule.htm>
- (3). The HHC list is maintained by Eric Smith and Conference related information is (also archived) at the Brouhaha website at: <http://lists.brouhaha.com/mailman/listinfo/hhc> . Sign up to be on the list for the latest information related to HHCs. Most list activity is just before or just after a Conference.
- (4). Printing a hundred pages of proceedings is rapidly becoming counterproductive in this age of the internet and personal websites. In 2008 we used a USB drive to provide the Conference materials by writing the contents on Friday afternoon to be given in the Conference Tyvek® envelope used to contain the CD's, printed materials, badges, programming contest challenge and rules, door prize ticket, etc. We did the same this year because one document, “Where can you find the history of calculator applications” is 266 pages. Everyone brings a laptop so copying 50 USB drives doesn't take very long. Still all available materials are available at that time for several reasons. Since we do not use the USB Drive during the Conference it may be better to snail mail the drive after the Conference concludes to insure that all files are complete - Power Point Presentations, pdfs of the papers, reference materials, software, photographs, etc. A 1 GB capacity is adequate.

- (5). Jake Schwartz video tapes the conference and makes the videos available on DVD's. He has done this since HHC 1986. The list and videos are available at: <http://www.pahhc.org/video.htm>
- (6). A list of HP managers and engineers who have attended HHCs may be found at: <http://hhuc.us/2012/hp.htm>
- (7). The blog by Eddie Shore mentioned on the HP Museum is at:
 Day 1 blog. <http://edspi31415.blogspot.com/2012/09/hhc-2012-day-1-summary-9222012.html>
 Day 2 blog. <http://edspi31415.blogspot.com/2012/09/hhc-2012-day-2-summary-9232012.html>
- (8). See his website at: <http://www.hp41.org/Intro.cfm>
- (9). Eric Smith further describes his Do It Yourself, DIY, calculator on the HP Museum at:
<http://www.hpmuseum.org/cgi-sys/cgiwrap/hpmuseum/forum.cgi?read=231748#231748>

- (10) The book is RCL 20, which may be found at:
<http://www.limov.com/rcl20/>

Bill Wickes' article may be found on page 105.

Many (22) of the HPUC leaders contributed to this book.

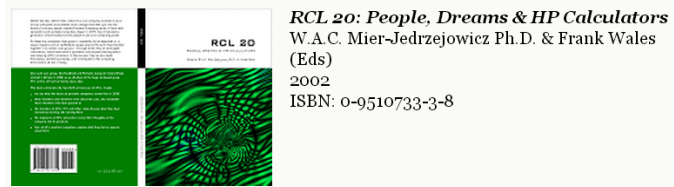


Fig. 13 – RCL 20 book records the people history of HPCC.

- (11) See the primary link at: www.raspberrypi.org. Newark and Allied carries the board: www.newark.com or www.alliedelec.com. The Supercomputer may be seen at several websites:
www.engadget.com/2012/09/13/supercomputer-built-from-raspberry-pi-and-lego/
http://news.cnet.com/8301-17938_105-57513107-1/supercomputer-clicked-together-from-legos-and-raspberry-pis/

Appendix A - HHC 2012 Attendance – Page 1 of 1

By Country:

- 1. Canada . . . 2
 - 2. England. . . 1
 - 3. France . . . 1
 - 4. Germany. . . 2
 - 5. USA. 35
- Total 41

Official Group Photo



By US State:

- AL Alabama 1
 - AZ Arizona 1
 - CA California 3
 - CO Colorado 2
 - GA Georgia 1
 - IA Iowa 1
 - ID Idaho 1
 - IL Illinois 2
 - MI Michigan 2
 - NV Nevada 2
 - NJ New Jersey 2
 - NY New York 1
 - NC North Carolina 2
 - OH Ohio 3
 - OK Oklahoma 2
 - OR Oregon 1
 - TN Tennessee 1
 - TX Texas 4
 - VA Virginia 2
 - WA Washington 1
- 20 State Total 35

Four wives attended as may be seen in the official photo.

Missing from the photo are #4 and #26 wives on the registration list at the right.

HHC 2012 Attendees

39 of these are in the official group photo.

Name (A-Z)	From	Presentation Title
1. Olivier Arbey	WA	
2. Pavneet Arora	Canada	"Re-tooling the Calculating Tool"
3. Bill Butler	Canada	
4. Brenda (Tis) Chrapkiewicz	MI	
5. Thomas Chrapkiewicz	MI	
6. Warren Furlow	GA	"The HP-41 is Alive and Well"
7. Rick Furr	VA	"The Making of The HPv2 Poster"
8. Felix Gross	Germany	"Where can you find the history of calculator applications?"
9. Palmer Hanson	NC	"Calculating Before Calculators"
10. David Hayden	NJ	
11. Tom Hearn	NC	
12. Roger Hill	IL	
13. Joseph Horn	CA	
14. Brion Keller	NY	
15. Matt Kernal	OR	
16. Thomas Kline	IA	
17. Vern Lindsay	ID	
18. Lora Marschall	OK	
19. Peter Marschall	OK	
20. Wlodek Mier-Jędrzejowicz	England	"Is It Really a Pathetic Name?"
21. Mike Morrow	AL	
22. Richard Nelson	AZ	{ "Tablets vs. HP Calculators" (w/ Gene Wright) and "Measuring Calculator Current"
23. Bob Patton	TX	
24. Cam Patton	TX	
25. David Ramsey	NV	
26. Mary Ramsey	NV	
27. Philip Reagan	TX	
28. Eric Rechlin	TX	
29. Günter Schink	Germany	
30. Jake Schwartz	NJ	"WP-34S and the HP Calculator Way"
31. Namir Shammas	VA	{ "Calculator Root Finding: Scan Range Method" and "Variable Sampling Integration"
32. Eddie Shore	CA	
33. Eric Smith	CA	"Even Larger Scaled Reptiles of the Nordic Countries"
34. Patrice Torchet	France	
35. Jeff Turner	OH	
36. Brian Walsh	IL	
37. Katie Wessman	CO	
38. Tim Wessman	CO	"Promoting Healthy Foliage Growth in Gymnosperms"
39. Jackie Woldering	OH	{ "Converting HP-41C Programs to PDF" and "The Raspberry Pi"
40. Mary Woldering	OH	
41. Gene Wright	TN	"These Are a Few of My Favorite Things"

Appendix B - HHC 2012 RPL Programming Contests – Page 1 of 3

September 22-23, 2012, Nashville

Problem Description: In the diagram below, each small circle has non-negative integer coordinates in the usual Cartesian coordinate system. You can move from one circle to another following the path denoted by the arrow symbols.

To move from (0,3) to (3,0), you have to pass through (1,2) and (2,1) then you arrive at (3,0), so this journey takes 3 “steps.”

In this problem, you must compute the minimal number of steps needed to go from a given source circle to a given destination circle.

Input: Each test case consists of three integers, x_1, y_1, x_2, y_2 where each value is less than 1,000,000. These will be loaded as follows: x_1 ENTER y_1 ENTER x_2 ENTER y_2 then R/S.

Output: Return the minimum number of steps required with the sign of the answer indicated forward steps (positive) or backward steps (negative).

Sample Cases:

- (A) Input of 1 ENTER 0 ENTER 1 ENTER 1 R/S should return 3.
- (B) Input of 4 ENTER 0 ENTER 3 ENTER 0 R/S should return -4.

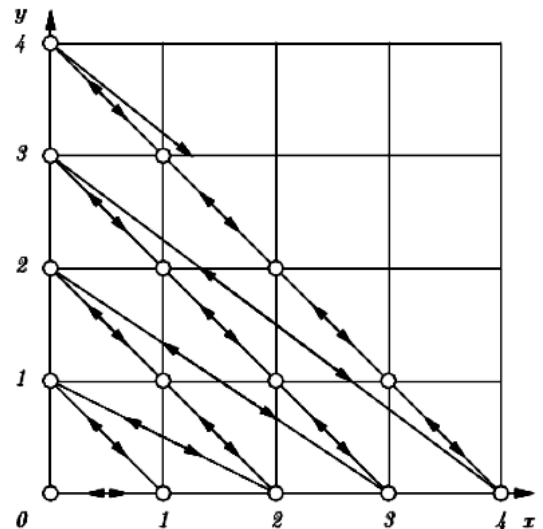


Fig. 1 – Travel path requirements.

Machines Eligible: This contest is open to any and all RPN machines:

15c, 15c+, 15c LE, 34S, 41CL, 42S, 67, 65, etc. RPL users are welcome to try the problem, but this is for RPN machines only.

The winner will be the program that a) returns correct answers, b) has the shortest number of steps x speed in seconds, or c) if the speed is in the judge’s sole opinion, nearly identical, the shortest routine.

Rules: (aka the fine print)

- 1) The decision of the judge is FINAL. No appeals are allowed to anyone or anything.
- 2) The purpose of this contest is to have fun and learn.
- 3) At least two contestants must submit an entry.
- 4) No custom built ROM or machine code can be built and used for this problem. Any already existing functionality in the machine is ok.
- 5) You must submit a machine with your program already keyed in to the judge AS WELL as a legible listing of your program with your name on the listing AND the machine. Machines with no names that are given to the judge are assumed to be gifts to the judge. Thank you!
- 6) Submission must be made by the end of the contest (Time is TBA).
- 7) Assume the program will start running with step 001 and/or a R/S.
- 8) By submitting a program, you agree to allow it to be shared with the community.
- 9) This is a contest between individuals, not teams. One submittal \leq one person.
- 10) You may not access the internet for any help in any fashion. Do not cheat in any way. Do not check the HP Museum Forum either.
- 11) You must be present to win.
- 12) If a point is unclear, ask immediately. No excuses for ignorance. Clarifications will be shared with the entire group during the conference.
- 13) Assume default machine settings. Your program must stop with the default settings in place.
- 14) Winner will be the program with the fastest times over the test cases giving correct results. If in the judge’s sole discretion, two entries are “about the same speed,” the winner will be the shortest routine. In case of a tie, the most elegant solution (according to the judge) wins.
- 15) The purpose of this contest is to learn and have fun. Happy Programming.

Appendix B - HHC 2012 RPL Programming Contest – Page 2 of 3

September 22-23, 2012, Nashville

Problem Description: The diameter of a set of points on the plain is the distance between its two most widely separated points. For example, the diameter of this set of points (1,1) (0,0) (2,3) (3,4) (1,0) is 5, which is the distance between (0,0) and (3,4). Given a set of points, compute its diameter.

Input: Each test case consists a list of up to 10 pairs of numbers. Each value in the list will be less than 10,000 in absolute value. The list will contain at least one pair of numbers and will always contain a multiple of 2 numbers, i.e., there will not be 3 values in the list.

Output: Return the diameter.

Sample Case: Input of { 1 1 0 0 2 3 3 4 1 0 } should return 5.

Machines Eligible: This contest is open to any and all RPL machines.

Rules: (aka the fine print)

- 1) The decision of the judge is FINAL. No appeals are allowed to anyone or anything.
- 2) The purpose of this contest is to have fun and learn.
- 3) At least two contestants must submit an entry.
- 4) No custom built ROM or machine code can be built and used for this problem. Any already existing functionality in the machine is ok. Sysevals, etc are allowed.
- 5) Your program must be transferred to the judge's machine under some identifying three-letter name before the announced deadline and you must also submit a legible listing of your program with your name on the listing.
- 6) Submission must be made by the end of the contest (Time is TBA).
- 7) By submitting a program, you agree to allow it to be shared with the community.
- 8) This is a contest between individuals, not teams. One submittal \diamond one person.
- 9) You may not access the internet for any help in any fashion. Do not cheat in any way. Do not check the HP Museum Forum either.
- 10) You must be present to win.
- 11) If a point is unclear, ask immediately. No excuses for ignorance. Clarifications will be shared with the entire group during the conference.
- 12) Assume default machine settings. Your program must stop with the default settings in place.
- 13) The winning program will be the one for which size*speed (bytes*sec) is least, where the speed of execution will be determined for one or more test cases chosen by the judge.
- 14) The program must be a (self-contained) single object in user code which does not call itself by name.
- 15) Default flag settings (except for flag -95) are assumed and must be restored if changed.
- 16) The stack, apart from input and output, must be left as found.
- 17) The program must not contain KILL or otherwise interfere with the programmatic testing and evaluation of submissions, i.e., you cannot delete everything on the judge's machine except your own program!
- 18) Happy Programming.

HHC Programming Contest Winners

- | | |
|----------------|--------------|
| 1. RPN program | David Hayden |
| 2. RPL program | Bill Butler |

HP Museum Programming Contest Winner

- | | |
|----------------|-----------|
| 1. RPN Program | Paul Dale |
|----------------|-----------|

HHC 2012 Premium Door Prize Winners

- | | |
|-----------------------------|----------------|
| 1. HP 48GX (new packaged) | Namir Shammass |
| 2. HP 48GX | Gene Wright |
| 3. HP-15C Clone | Jeff Turner |
| 4. Repurposed 30b (WP34S) | Oliver Arbey |
| 5. 82240B IR Printer | Brion Keller |
| 6. HP-71B | David Hayden |
| 7. Griffin Universal Remote | Pavneet Arora |
| 8. iPod Touch in belt case | Rick Furr |
| 9. Aristo Slide Rule | David Ramsey |
| 10. Calculator Quilt | Mary Woldering |

Learn about weighted power and exponential curve fitting

HP Solve #29 page 23

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Using Weighted Power and Exponential Curve Fitting

Palmer Hanson

Introduction

The June 1980 PPC Calculator Journal (V7N5P9-11) presented an HP-41 program by Ron Knapp which would calculate 1,000 digits in 11.5 hours. That result was the basis for a challenge in the so-called "friendly competition" between users of HP and TI machines. Ron provided execution times for a total of five numbers of digits: 30 digits in 2 minutes, 90 digits in 9 minutes, 200 digits in 34 minutes, 1,000 digits in 11.5 hours and 1,160 digits in 15.25 hours. In this article I will use Ron's data in an analysis of curve fitting techniques which will show that the use of weighted data can yield a significant improvement in the quality of a power function fit.

When I took a curve fitting course at the University of Minnesota back in 1949 Professor Eggers opened the course with a discussion of how to select models to test. Professor Eggers said we should always consider the physics of the problem. In a similar vein page 7 of William Kolb's book on curve fitting with programmable calculators [Reference 1] states "We should always select a model for our data on the basis of either theoretical or empirical knowledge." When I considered the nature of pi-finding programs I concluded that the time of execution versus the number of digits calculated should be square function. My reasoning was that the number of iterations and the execution time per iteration would both be proportional to the number of digits. In the olden days I would have plotted the Knapp data on log-log paper, and when the plot approximated a straight line I would have tried a power function fit as my first choice. That was before there were computer or calculator solutions such as that in the HP-48S which will solve using several different functions and select the best function based on some criteria, typically the correlation coefficient.

A Power Function Fit to the Knapp Data

If I use any of the typical multiple curve fitting methodologies and with the elapsed time in minutes I find that the "best fit" is of the form $y = Ax^B$ where with the HP-48S:

$$A = 4.87796502271E-3$$

$$B = 1.70772919979$$

$$r = 0.997876797805$$

The results seem to be roughly consistent with my expectations. But when I use the SCATR and FCN functions of my HP-48S to superimpose a plot of the function on a scatter diagram of the input data I get a plot which shows the fourth and fifth data points well above the plot of the function

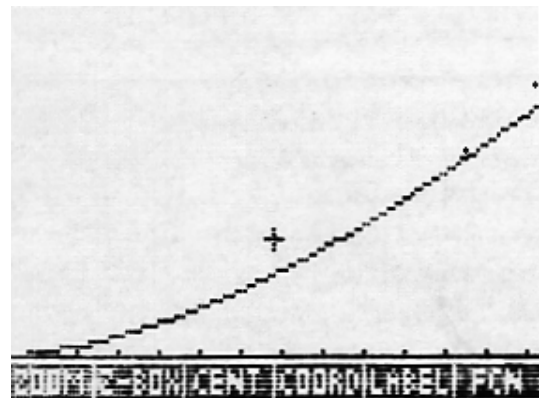


Fig. 1 - HP-48S plot on a scatter diagram.

Calculating the residuals using the PREDY function of my

Table 1 - Knapp's Pi Run Time Residuals

Digits	Time	Residuals
30	2 minutes	0.375342...
90	9 minutes	-1.606099...
200	34 minutes	-7.473892...
1000	690 minutes	42.221870...
1160	915 minutes	80.352548...

HP-48S, where residual = observed value – calculated value, yields the following results which confirm the observations from the plot. The sum of the residuals is 113.8697702 and the sum of the squares of the residuals is 8297.797888 . Those residuals suggest that there might be another power function which would yield smaller residuals. There is. Why don't the solutions from typical curve fitting programs yield that better solution? Because the typical solutions are not done with the measured values but with transformed values. The logarithm is taken of each side of the power equation yielding the transformed equation

$$\ln(y) = \ln(A) + B\ln(x)$$

which is linear in $\ln(y)$ and $\ln(x)$. When I do the problem at hand with the logarithms of the number of digits as the independent variable and the logarithms of the elapsed time as the dependent variable and solve using the linear function I get

Intercept = -5.32302714963 where the intercept from the linearized solution is the logarithm of the coefficient A of the power function. Then, $A = e^{\text{intercept}} = 0.487796502271E-3$;

Slope = B = 1.70772919479 = the exponent, and

$r = 0.997876797805$ which is the same answer as that received from the power function solution. The residuals are

Table 2 - Residuals for a Linear Fit with a Transformed Equation

ln(Digits)	ln(Time)	Residuals
3.401...	0.693...	+0.207850...
4.499...	2.197...	-0.164204...
5.298...	3.526...	-0.198703...
6.907...	6.536...	+0.063143...
7.056...	6.818...	+0.091914...

The sum of the residuals using the complete values in the machine is 1E-10, very near zero as it should be. The sum of the squares of the residuals is. 0.12208... Thus, the solution of the linearized function does appear to be a least squares solution in the linearized coordinate system. The transformation back to the original coordinate system does NOT yield a least squares solution in the original coordinate system. I suspect that we all knew that it might not be. We hoped that it would be close. In some cases such as the Knapp data it is not close at all.

Over many years I had encountered difficulties such as this in my work with power function fitting, but I didn't know what to do about it. The subject hadn't been discussed by Professor Eggers during his course on curve fitting. Then back in 1983, as editor of TI PPC Notes, I received a submission of an article from my friend and mentor George Wm. Thomson. His solution to the problem is derived from a 1943 book by W. Edwards Deming [Reference 2]. That is the same Deming who became famous after World War II by his introduction of statistical process control into Japanese industry. Instead of simply solving with the transformed data, the improved solution is obtained by weighting each transformed data point by the square of the y values. Thomson's solution appears in the volume 8 number 1 issue of TI PPC Notes [Reference 3]. I could not find a similar capability in the HP48S or for any other HP calculators in my collection. I am not saying the capability does not exist but only that I could not find it. I wrote a

program which includes that capability on the HP-35s -- see the appendix. Why didn't I write a program for my HP-48S? The answer is lack of sufficient familiarity with RPL. A weighted solution for the Knapp data from the HP-35s program yields the following results:

$$A = 0.001512745\dots$$

$$B = 1.886571961\dots$$

$$r = 0.999895601\dots$$

The residuals are:

Table 3 - Improved Pi Run Time Residuals

Digits	Time	Residuals
30	2 minutes	1.074319...
90	9 minutes	1.644963...
200	34 minutes	0.823973...
1000	690 minutes	-1.005615...
1160	915 minutes	0.705279...

and the sum of the residuals is 3.242919... and the sum of the squares of the residuals is 6.0476825.... That is a much better solution than the one obtained with weighted data.

A Second Power Function Solution Using Weighted Data

My second example of the advantage of using weighted data in the solution for a power function to fit data is not from experimental data. Rather, the problem is a contrived one which provides a better example of just how much the use of a weighted solution can improve the result. To develop the problem I begin with the function $y = Ax^B$ where $A = 1.25$ and $B = 2.5$. Then I generate a table of eight data pairs as in the first two columns of the table by first calculating the exact values for y as a function of x . To introduce some noise in the data pairs to be used in the curve fitting exercise I define each y value to be the nearest integer to the exact y value. See the second and third columns in the table. The fourth column in the table labeled "Error" is the difference between the integerized y values and the exact values. One would expect that the residual errors from a least squares power function curve fit would look something like the induced errors. The weighted solution using the HP-48S or the HP-35s program is

$$A = 1.226080166$$

$$B = 2.507529704$$

$$r = 0.999994365$$

When I use the SCATR and FCN functions of my HP-48S to superimpose a plot of the function on a scatter diagram of the input data I do not see any points which are clearly not on the curve. This is not because there aren't any such points but because the scale of the plot doesn't allow the errors to appear to the eye.

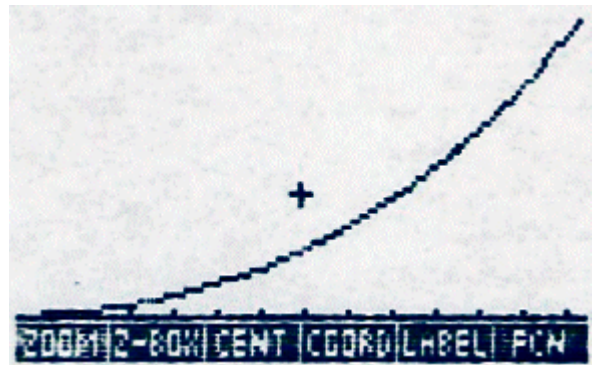


Fig. 2 - SCATR and FCN plot for the second power function problem

The residuals for the weighted solution in the fifth column are similar to the residuals of the weighted power fit to the Knapp data in the sense that the residuals for the larger y values are clearly not close the curve defined by the solution function. The

residuals show no obvious similarity to the errors in the contrived function. A linear fit with the errors in the contrived function as the independent variable and the residuals from the weighted power fit as the dependent variable yields an intercept of 0.899..., a slope of 0.386... and a correlation coefficient of 0.0346... Those numbers confirm the idea that there is no linear relationship between the residual and the input errors.

The weighted solution using the HP-35s program is

$$A = 1.250262080 \quad B = 2.499926401 \quad r = 0.999999844$$

The residuals for the weighted solution in the sixth column of the table show an obvious similarity to the errors in the contrived function. A linear fit with the errors in the contrived function as the independent variable and the residuals from the weighted power fit as the dependent variable yields an intercept of -0.004969..., a slope of 0.991604... and a correlation coefficient of 0.99864.. Those numbers confirm the idea that there is a close relationship between the residual and the input errors. The weighted solution yields a fit that is what would be expected.

Table 4 - Residuals for a Second Power Fit Problem

x	y	y exact	Error Residuals	Weighted Residuals	Weighted
2	7	7.071...	-0.071..	0.027949	-0.072189...
3	19	19.485...	-0.485...	-0.271458...	-0.488081...
5	70	69.877...	0.123...	0.624373...	0.116503...
7	162	162.052...	-0.052...	0.702622...	-0.063032...
10	395	395.284...	-0.284...	0.498579...	-0.300588...
13	762	761.672...	0.328...	0.333399..	0.311400...
16	1280	1280	0	-1.992519...	-0.007145...
20	2236	2236.067....	-0.067...	-7.314829...	-0.043737
Sum of Residuals				-7.391882....	-0.546870...
Sum of Squares of Residuals				0.58.794596..	0.450268....

A Weighted Solution for Exponential Functions

Deming's book indicates that weighting with the square of the dependent variable is also appropriate when using linearized procedures to fit the exponential function $Y = Ae^{Bx}$. The program in the appendix includes options to perform curve fitting of an exponential function without and with y^2 weighting. I do not have a set of experimental data to submit to the program. Instead, I generated a contrived set of exponential data in the same manner that I used earlier to obtain a contrived set of data for the investigation of weighting with a power function.

To develop a function to demonstrate the use of a weighted solution for an exponential fit I begin with the function $y = Ae^{Bx}$ with $A = 2$ and $B = 0.4$. Then I generate a table of eight data pairs by first calculating the exact values for y as a function of x . To introduce some noise in the data pairs to be used in the curve fitting exercise I define each y value to be the nearest integer to the exact y value. See the second and third columns in the table. The fourth column in the table labeled "Error" is the difference between the integerized y values and the exact values. One would expect that the residual errors from a least squares

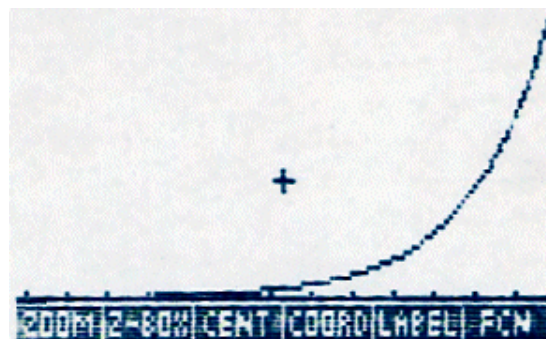


Fig. 3 - SCATR and FCN plot for the exponential function problem.

exponential function curve fit would look something like the induced errors. The weighted solution using the HP-48S or the HP-35s program is:

$$A = 1.941103046 \quad B = 0.402224248 \quad r = 0.999892737$$

When I use the SCATR and FCN functions of my HP-48S to superimpose a plot of the function on a scatter diagram of the input data I do not see any points which are clearly not on the curve. As with the situation with the second power function problem this is not because there aren't any such points but because the scale of the plot doesn't allow the errors to appear to the eye.

The residuals for the weighted solution in the fifth column of the table are similar to the residuals of the weighted power fit problems in the sense that the residuals for the larger y values are clearly not close the curve defined by the solution function. The residuals show no obvious similarity to the errors in the contrived function. A linear fit with the errors in the contrived function as the independent variable and the residuals from the weighted power fit as the dependent variable yields an intercept of -3.638..., a slope of -15.587... and a correlation coefficient of -0.316... Those numbers confirm the idea that there is no close relationship between the residual and the input errors.

The weighted solution using the HP-35s program is

$$A = 2.001474365 \quad B = 0.399961991 \quad r = 0.999999907$$

The residuals for the weighted solution in the sixth column of the table show a similarity to the errors in the contrived function but not to the same extent as with the power function solution earlier in this paper. A linear fit with the errors in the contrived function as the independent variable and the residuals from the weighted power fit as the dependent variable yields an intercept of -0.0479..., a slope of 0.833... and a correlation coefficient of 0.95772. The weighted solution does yield a substantial reduction in the residuals relative to those for the weighted solution.

Table 5 - Residuals for an Exponential Fit Problem

x	y	y exact	Error	Weighted Residuals	Weighted Residuals
1	3	2.983..	0.016...	0.097766...	0.014264...
2	4	4.451...	-0.451...	-0.339264...	-0.454024...
4	10	9.906...	0.093..	0.299733...	0.088139...
6	22	22.046...	-0.046...	0.315409...	-0.057574
8	49	49.065...	-0.065...	0.524894...	-0.086302..
11	163	162.901	0.098...	0.979460...	0.046319...
15	807	806.857...	0.142...	-2.664615...	0.007838...
18	2679	2678.861...	0.138...	-27.178713...	-0.002834...
Sum of residuals				-27.965328...	-0.444173...
Sum of residuals squared				747.331463..	0.227088.

Conclusions

The use of the weighted solutions yields a substantial improvement in the least squares solutions obtained with power and exponential functions.

There are cases in which using a weighted solution may not be appropriate. For example, Deming⁽²⁾

(page 201) suggests that weighting is not needed if the dependent variable does not change very much. A weighted solution would also not be expected to be appropriate if the error in the dependent values vary with the magnitude of the dependent values. I do not have actual data for such a case. As a possible example I suggest that if the dependent values were electrical values taken with one of those meters which change ranges as the measurements increase, then larger residuals would be expected at larger dependent variable values.

REFERENCES:

1. William Kolb, Curve Fitting for Programmable Calculators Second Edition, IMTEC, 1983.
2. W. Edward Deming, Statistical Adjustment of Data, Dover Books, 1964. This was a republication of the book published by John Wiley & Sons in 1943.
3. George W. Thomson, "Thoughts about Curve Fitting", TI PPC Notes, V8N1P28-31.

APPENDIX A: Unweighted and Weighted Curve Fitting for the Power and Exponential Functions on the HP-35s

The weighting with this program is limited to be the square of the dependent variable.

The weighted solution will yield values for N which are not integers if the dependent variables include numbers which are not integers. The built-in statistical solutions of the HP-35s will return the error message STAT ERROR if N is not an integer. The HP-33s does the same. Because of that idiosyncrasy it is necessary to solve the weighted case by directly implementing the necessary equations in the program rather than using the built-in linear solution. See program steps Z093 through Z134. Machines that will solve when N is not an integer include the HP-10B, HP-11C, and HP-12C.

Nomenclature:

Equations: $y = Ax^B$; $y = Ae^{BX}$

Flag 1 is set for a power function solution. Flag 1 is reset for an exponential solution

A = Coefficient	Y = Dependent variable	D = Residual = $Y_{\text{observed}} - Y_{\text{calculated}}$
B = Exponent	R = Correlation coefficient	S = Sum of the residuals
X = Independent variable	N = Number of data pairs	T = Sum of the Residuals squared

HP-35s Program Listing

Z001 LBL Z	Z051 STO X	Z101 n	Z151 e^x
Z002 SF 10	Z052 1	Z102 Σx^2	Z152 GTO Z154
Z003 EQN WEIGHTED CURVE FIT	Z053 STO+ I	Z103 x	Z153 y^x
Z004 CF 10	Z054 RCL (I)	Z104 Σx	Z154 RCLx A
Z005 0	Z055 STO Z	Z105 x^2	Z155 1
Z006 STO I	Z056 STOx Z	Z106 -	Z156 STO+ I
Z007 INPUT N	Z057 LN	Z107 /	Z157 R↓
Z008 STO A	Z058 STO Y	Z108 STO B	Z158 RCL- (I)
Z009 1	Z059 -27	Z109 Σx	Z159 +/-
Z010 STO+ I	Z060 STO J	Z110 x	Z160 STO+ S
Z011 INPUT X	Z061 RCL Z	Z111 Σy	Z161 ENTER
Z012 STO (I)	Z062 STO+ (J)	Z112 -	Z162 x^2
Z013 1	Z063 1	Z113 +/-	Z163 STO+ T
Z014 STO+ I	Z064 STO- J	Z114 n	Z164 R↓
Z015 INPUT Y	Z065 RCL X	Z115 /	Z165 STO D
Z016 STO(I)	Z066 RCLx Z	Z116 e^x	Z166 VIEW D
Z017 DSE A	Z067 STO+ (J)	Z117 STO A	Z167 DSE C
Z018 GTO Z009	Z068 1	Z118 RCL R	Z168 GTO Z144
Z019 SF 1	Z069 STO- J	Z119 n	Z169 VIEW S
Z020 SF 10	Z070 RCL Y	Z120 Σx^2	Z170 VIEW T
Z021 EQN 1=PWR, 2=EXP	Z071 RCLx Z	Z121 x	Z171 GTO Z031
Z022 CF 10	Z072 STO+ (J)	Z122 Σx	Z172 CL Σ
Z023 1	Z073 1	Z123 x^2	Z173 RCL N
Z024 $x=y?$	Z074 STO- J	Z124 -	Z174 STO A
Z025 GTO Z031	Z075 RCL X	Z125 n	Z175 0
Z026 $x \diamond y$	Z076 x^2	Z126 Σy^2	Z176 STO I
Z027 2	Z077 RCLx Z	Z127 x	Z177 1
Z028 $x \neq y?$	Z078 STO+ (J)	Z128 Σy	Z178 STO+ I
Z029 GTO Z019	Z079 1	Z129 x^2	Z179 RCL (I)

Z030 CF 1	Z080 STO- J	Z130 -	Z180 FS? 1
Z031 SF 10	Z081 RCL Y	Z131 x	Z181 LN
Z032 EQN 1=WITH WTS, 2=WITHOUT	Z082 x^2	Z132 SQRT x	Z182 1
Z033 CF 10	Z083 RCLx Z	Z133 /	Z183 STO+ I
Z034 2	Z084 STO+ (J)	Z134 STO R	Z184 R↓
Z035 $x=y?$	Z085 1	Z135 VIEW A	Z185 RCL (I)
Z036 GTO Z172	Z086 STO- J	Z136 VIEW B	Z186 LN
Z037 $x \leftrightarrow y$	Z087 RCL X	Z137 VIEW R	Z187 $x \leftrightarrow y$
Z038 1	Z088 RCLx Y	Z138 RCL N	Z188 $\Sigma+$
Z039 $x \neq y ?$	Z089 RCLx Z	Z139 STO C	Z189 DSE A
Z040 GTO Z031	Z090 STO+ (J)	Z140 0	Z190 GTO Z177
Z041 CL Σ	Z091 DSE A	Z141 STO S	Z191 b
Z042 RCL N	Z092 GTO Z046	Z142 STO T	Z192 e^x
Z043 STO A	Z093 n	Z143 STO I	Z193 STO A
Z044 0	Z094 Σ_{xy}	Z144 1	Z194 m
Z045 STO I	Z095 x	Z145 STO+ I	Z195 STO B
Z046 1	Z096 Σ_x	Z146 RCL (I)	Z196 r
Z047 STO+ I	Z097 Σ_y	Z147 RCL B	Z197 STO R
Z048 RCL (I)	Z098 x	Z148 FS? 1	Z198 GTO Z135
Z049 FS? 1	Z099 -	Z149 GTO Z153	Z199 STOP
Z050 LN	Z100 STO R	Z150 x	

Measure calculator current

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Measuring Calculator Current – Nine Measurement Examples

Richard J. Nelson

Introduction

In *HP Solve*, issue #28 page 30, I described the importance and problems of measuring the current that a calculator draws from its “battery⁽¹⁾.” In that article I gave an example of the HP-41 and the current it draws under a very few selected conditions. I also described how the current a calculator draws frequently changes, often quite rapidly and drastically, depending on what the calculator is doing.

Low power consumption is essential

Nearly every reader has an extremely complex battery operated device such as a cell phone, game device, tablet, etc. Since we use these battery operated devices every day the need for drawing a minimum amount of current is a critical aspect of their design. Using the lowest possible power is the mantra for every circuit designer for battery operated products these days. What is amazing is the fact that since the digital circuits are able to operate so fast, and the sloth like human operators are so slow, that the circuit may be turned “off” and back on again while the human “decides” what to do, or how to deal with the previous result. Modern circuit design utilizes this current shut down concept in as many situations as possible throughout the complex circuitry.

One of the reasons to measure the current draw – in addition to troubleshooting – is to be able to estimate the battery life of the machine. In terms of doing this you have to be careful, however, because of the many possible operating conditions, and that they may change faster than your meter can respond.

Most calculators are, however; still relatively simple so making current measurements under carefully stated conditions is still a powerful diagnostic and battery-life-estimating tool.

Operating conditions

Suppose you need the square root of 13. The calculator sitting on the table is off and it is drawing a very small amount of current for two purposes. The first is being able to recognize that a key is being pressed. The idea of having an actual on/off power switch on a calculator has long been abandoned primarily because of the second reason. The second reason for “always being on” is to keep memory alive for programs, data, and display contents. This is our first operating condition, let’s call it **OFF**.

Our next step in calculating the square root of 13 is to press the “on” key. This is our second condition; let’s call it pressing the **ON KEY**. Now the calculator is on and ready for keying in our number, let’s call this state **ON**. Press the keys 1 and 3. When we press the square root key, however, the calculator is calculating a problem and the current will be another higher value, let’s call it **RUNNING**. These four different conditions are only a few state possibilities and high end calculators will have larger displays, and I/O capabilities that will draw even more current. The I/O activity itself is performed with the same current saving process of shutting down when nothing new is happening.

2012 HP calculator models

HP has made approximately 110 different models since 1972. Many of these are of similar design in terms of being in a family of machines and these machines have similar operating systems with similar current drawing values. The list of 2012 HP models is shown in Table 1. I selected three models from each category to measure the current drawn in the four states – **OFF**, **ON KEY**, **ON**, & **RUNNING**.

Table 1 – 2012 HP Calculator Products (24)

#	Financial Calculators	Scientific & Graphing	Home & Office
1	HP 10bII*	HP 10s	HP CalcPad 100
2	HP 10bII+	HP-15C Limited Edition*	HP CalcPad 200
3	<u>HP 20b</u>	HP 33s	OfficeCalc 100
4	<u>HP 30b*</u>	HP 35s*	OfficeCalc 200*
5	HP-12C*	HP 39gs	OfficeCalc 300
6	HP-12C 30 th Anniversary	HP 39gII*	PrintCalc 100*
7	HP-17bII+	HP 40gs	QuickCalc*
8		HP 48gII	
9		HP 50g	
10		SmartCalc HP 300s	

5 of 7 = 71% programmable 8 of 10 = 80% programmable None programmable

Notes: Programmable calculators are in blue. Underlined models are at or near end of life.

** Models that were measured for current drawn.*

Using a convenience current measuring test fixture

The techniques of making the current measurement were described in the previous article. Fig. 1 shows

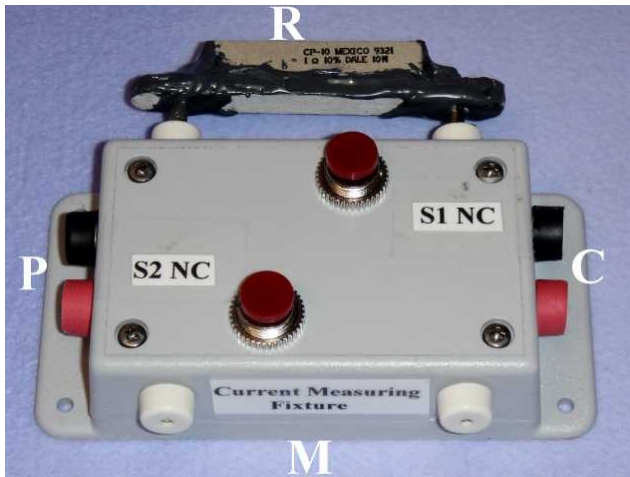


Fig. 1 – Current measurement test fixture.



Fig. 2 – Test fixture with test leads connected.

a simple current measurement test fixture. “M” is for Meter (DMM). “R” is for a series Resistor. “P” is for the power supply test leads. “C” is for the calculator test leads. These four pin jacks have heat shrink tubing added to identify polarity. The black “P” jack is connected to the closest “R” pin jack. The second “R” pin jack is connected to the black “C” jack. The S1 normally closed push button switch is connected between the two “R” pin jacks. The red “P” jacks are connected in a similar manner. The current normally flows through the test fixture and the two normally closed switches.

If, however, one of the two push button switches is pressed down the current is diverted through the respective pin jacks. As described in the previous article this arrangement allows the switching of the meter ranges without disturbing the calculator electronics. Fig. 2 shows the test fixture with the test leads that are used with calculators. The “P” leads have alligator clips on one end to connect to a suitable power source. Two “c” cells are shown. The “C” leads have micro hook connectors to be able to connect to most calculator cell terminals. A one ohm resistor is plugged into the negative “R” line to allow a

volt meter to be connected across it to measure voltage drop as described in the previous article. The second set of “M” pin jacks normally have a current meter⁽²⁾ plugged into them.

Making the measurements

Once you have a convenient test fixture it is easy to make measurements⁽³⁾. Fig. 3 shows the **OFF** (standby) current being measured for the HP-15C Limited Edition, LE. Fig. 4 shows the current drawn by pressing the **ON KEY**. Notice that the ratio of these two currents is 3,827 to 1. Fortunately turning the calculator on doesn't happen very often and this example illustrates how the current drawn changes



Fig. 3 – Off current being measured for HP-15C LE.

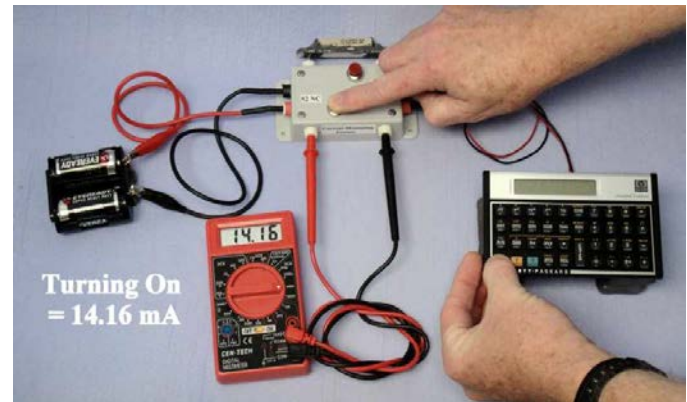


Fig. 4 – HP-15C LE turn on current being measured.

depending on what the user is doing. The student user should be aware of this because you will better understand why your calculator is DEAD when a book in your back pack was pressing the ON key. In the case of the HP-15C LE that uses a lithium CR 2032 cell with a capacity of 225 milliampere hours, $225/14.2 = 15.8$ hours of life. If you dropped your back pack at the end of the day and checked your calculator the next day the cell(s) would surely be dead. The actual life is considerably less than the calculation because of the heavy load which is not what the 225 milliampere hour specification is based on. The CR 2032 cell is rated at a 200 microampere continuous load. Actually the two cell holders of the HP-15C LE are connected in parallel so all values are doubled. More on this topic may be found in the “Cell holder comparison” section.

The HP-15C LE **OFF** current is far below this “continuous standard load” and the life would theoretically be $225/.2 = 1,125$ Hrs. (46.9 days). Since this current is less than the load assumed by the 225

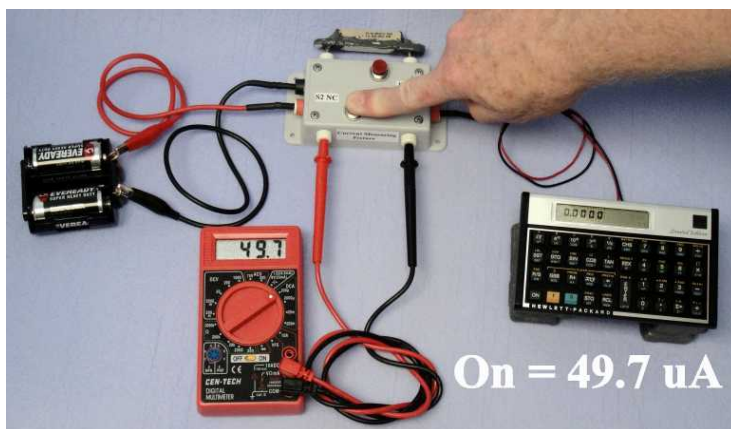


Fig. 5 – HP-15C LE ON current is being measured.

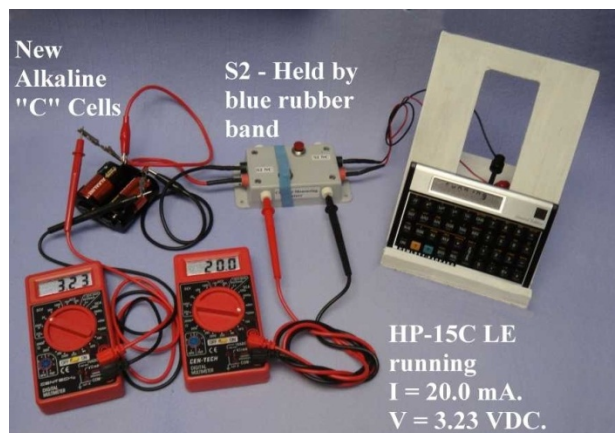


Fig. 6 – HP-15C LE running current measurement.

milliampere hour capacity the actual life will be much longer. Battery manufacturers provide load curves and from these values you can see that battery life is nearly impossible to calculate. One condition of many is the cut off voltage of the cell. The “standard” is an end of life of 2.0 VDC for the 3.0 VDC cell - 2/3 of the nominal cell voltage. And don’t forget about temperature affects, humidity effects (low humidity dries out the cells), calculator IC variances, etc. etc. It is not the intent of this article to describe how to predict calculator cell life, but to provide some perspective on why cell life is a very illusive issue.

Fig. 5 shows the calculator after it has been turned on, the **ON** state.

Fig. 6 shows the calculator in a running mode, usually running a program, calculating a function, or in a self-test mode (HP 39gII). Let’s call it the **RUNNING** state.

These four states are the ones that will serve to illustrate specific measurements made with the same equipment for three models of the three product lines shown in Figure 1.

The calculator shown in Figs 3 – 5 is sitting on four one pound lead weights to provide space for the micro hook leads to connect to the cell holder contacts. This was a klutzy situation so I built the calculator stand shown in the running measurement of Fig. 6. I also replaced the zinc carbon “C” cells with alkaline cells for a more consistent voltage for the Table 2 measurements. The display flashes running and I did catch the display at the right moment to show it. The stand provided a more convenient way of connecting the micro hooks to the delicate cell terminals. A close up view is provided by Fig. 7 which is the same as Fig. 8, *HP Solve* issue # 28 page 35, in the previous article.

The stand was designed to accommodate nearly all of the current models⁽⁴⁾. Fig 8 shows the three views of the stand. It is made of wood and the heavy base is intentional to provide stability. The height is based on the largest model the HP 39gII. The slot in the back is sized for the various Table 1 models and the location of their power cells.



Fig. 7 – Typical CR 2032 cell connections.

It works quite well except that I made one mistake. The tilt angle is 30 degrees from the vertical and I should have made it closer to 45 degrees. This would have provided better lighting for the LCD displays.

The cell holder has a third connection between the two “c” cells to provide 1.5VDC.

Measurements of selected models

As previously mentioned there are many different modes and operating conditions that affect the current that a calculator will draw from its power source. This current is ever changing and unless you are able to specify the current drawn under each possible condition, and the percentage of time it is in that state, it is impossible to calculate the cell life for any modern machine. Add to the calculator’s ever changing conditions and the changing conditions of the cell chemistry of the power source, and the problem complexity further increases. HP does not specify the minimum operating voltage of its machines and that is an essential value to include in the cell life conditions.

The four typical operational states of selected machines are measured and tabulated in Table 2.

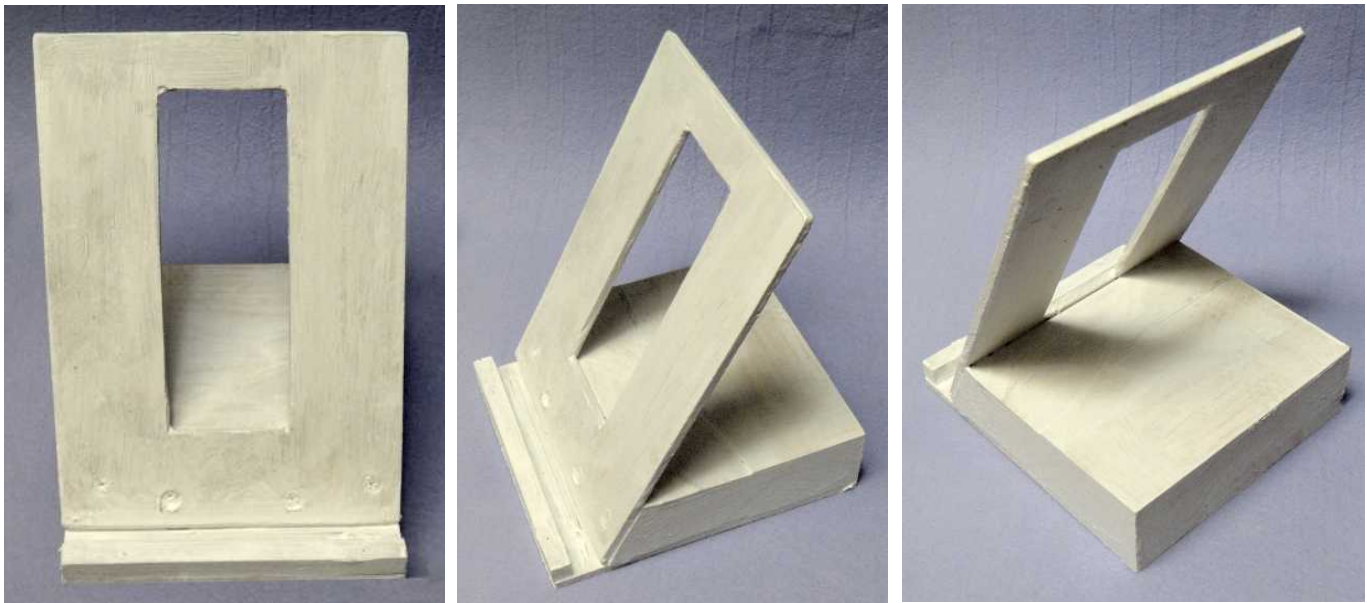


Fig 8a - Current measuring stand, front. Fig 8b - Current measuring stand, side. Fig 8c - Current measuring stand, back.

Table 2 – Selected HP Calculators and the Currents They Draw

#	Model	Voltage ¹	OFF ²	ON KEY ³	ON ⁴	RUNNING ⁵	Comments
1	HP 10bII	3.0 VDC	4.2 μ A	790 μ A	57.9 μ A	N/A	ON key \approx same as all other keys.
2	HP 30b	3.0 VDC	4.3 μ A	700 μ A	160 μ A	19,900 μ A	
3	HP-12C Platinum	3.0 VDC	0.1 μ A	98 μ A	12 μ A	1,560 μ A	Low ON KEY drain.
4	HP-15C LE	3.0 VDC	3.8 μ A	19,800 μ A	49.4 μ A	19,700 μ A	Works down to <2 VDC ⁶ .
5	HP 35s	3.0 VDC	8.4 μ A	32.0 μ A	28.2 μ A	4,590 μ A	Isolated two supplies
6	HP39gII	1.5 VDC	8.9 μ A	12,400 μ A	264 μ A	20,700/36,300 μ A ⁷	Isolated two supplies
7	OfficeCalc 200	1.5 VDC	4.2 μ A ⁸	13.5 μ A	12.0 μ A	N/A	See text.
8	PrintCalc 100	6.0 VDC	62.3 μ A	656 μ A	118 μ A	N/A, 145/185 μ A ⁹	4 AA cells.
9	QuickCalc	1.5 VDC	<0.1 μ A ¹⁰	8.8 μ A	2.7 μ A	N/A	Single cell. All 8's = 6.7 μ A.

Notes: 1 – Normal operating (cell) voltage.

2 – Calculator **OFF** current in microamperes.

3 – Turn **ON KEY** pressed current in microamperes.

4 – Machine (display) is **ON** with the current measured in microamperes.

5 – Current in microamperes **RUNNING** a loop program: LBL 1, 1 2 3 4 5 6 7 8 9 1 0, GTO 1.

6 – Variable DC supply was used to verify operation to 2/3 normal operating voltage.

7 – Measured with the hour glass annunciator on/ Self-test (ON + F4, 2, 2,) full power mode.

8 – Measured after the auto turnoff, there is no off key.

9 – Printing/paper advance.

10 – The 7 function CEN-TECH DMM has a very sensitive current range of 200 μ A.

QuickCalc is a special measurement challenge

The HP 10 QuickCalc is HP's smallest current calculator model. It runs on a single small button cell. You have to remove the 10 small front screws. Magnetize the small Philips screwdriver in order to better handle the screws. See Fig. 9. The metal cell holder has been unsoldered at "A" & "B" as shown in Fig. 10. "C" is the circuit board contact area for the negative terminal of the cell. A Popsicle stick (*cherry*) is slightly smaller than the thickness of the cell. A 14 Ga. wire is flattened at the end and held to the Popsicle stick with heat shrink tubing. It is then slid into the cell holder to make the negative cell contact.

See Fig. 11 and Fig. 12. Rubber bands hold the circuit board assembly to the front of the case so the key board may be used.

This model has the least **OFF** current and it is so low the DMM can't register it with its sensitivity of 0.1 μA (100 pico amperes). The estimated current has to be on the order of a few tens of picoamperes. It is not surprising that this machine is especially sensitive to meter ranges and meter range changes.

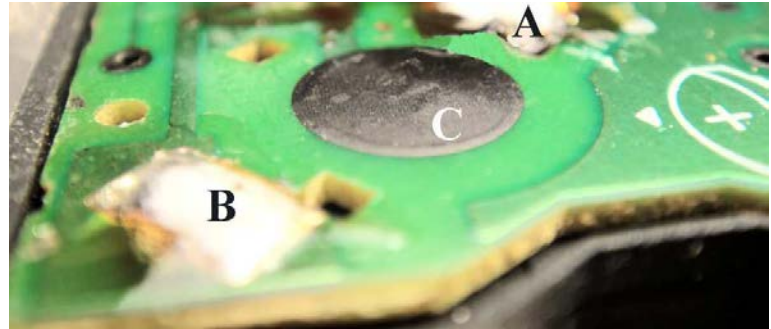


Fig. 9 – QuickCalc is HP's smallest calculator. Fig. 10 – Cell holder is unsoldered to show negative contact.

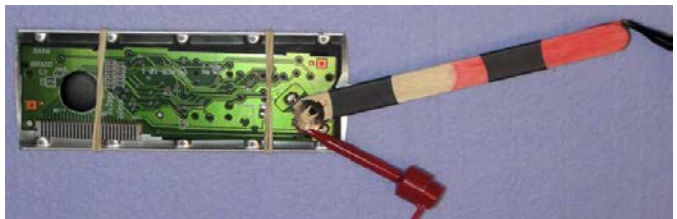
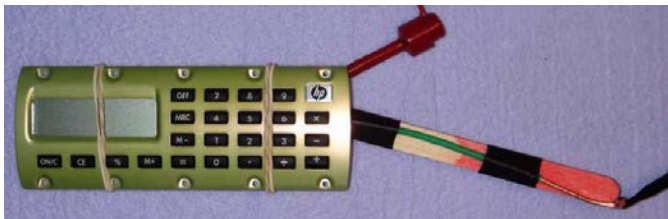


Fig. 11 – Popsicle stick replaces button cell. See text. Fig. 12 – Popsicle stick makes cell contact. See text.

Large OfficeCalc 200 display measurably affects current drawn

The OfficeCalc 100, 200 & 300 are solar cell powered with a single 1.5 VDC small button cell as a backup power source. The machine works using only the solar cell - without the button cell - in normal room light. The current measurements were made with a piece of black paper taped over the solar cell.

This “simple” machine nicely illustrates how various conditions will affect the current draw even though it is in the low tens of microamperes. The 14 digit segmented display is the primary current drain. “1”s in the display will draw less current than “8”s; 14 “1”s = 18.6 μA while 14 “8”s = 22.9 μA . Pressing any key increases the current drain a few tenths of a microampere plus the function involved plus the display result. I didn't find a **RUNNING** state to measure.

Cell holder comparison

Of the 24 current HP calculator models all but seven have two or more cell holders. The single cell models are the HP 12C, HP CalcPad 100, HP CalcPad 200, HP OfficeCalc 100, HP OfficeCalc 200, HP OfficeCalc 300, and HP 10 QuickCalc.

Fig. 13 shows the cell holders of the three selected finance models. Current versions of the HP 12C uses a single CR2032 coin cell.

Fig. 14 shows the cell holders of the three selected scientific and graphing models. The CR2032 coin

cell is one of the most popular cells used in today's models.

Fig. 15 shows the Home & Office models. These calculators use the largest size cell, AA, and the smallest, GP-189, cells. Table 3 provides a summary of the cells and cell holders used.

Comparing the current drain results

Which measured calculator model is the most energy efficient? Let's compare the Table 2 results using an un-weighted and a weighted value.



Fig. 13 – Finance Calculator cells.



Fig. 14 – Scientific and Graphing Calculator cells.



Fig. 15- Home and Office Calculator cells.

Table 3 – Selected HP Calculator Cell Usage Comparison

#	Model	Voltage ¹	Cells Used	Holders	Connected	Comments
1	HP 10bII	3.0 VDC	CR3032	Two	Parallel	
2	HP 30b	3.0 VDC	CR3032	Two	Parallel	
3	HP-12C Platinum	3.0 VDC	CR3032	one	N/A	
4	HP-15C LE	3.0 VDC	CR3032	Two	Parallel	Quick OFF if one cell is missing.
5	HP 35s	3.0 VDC	CR3032	Two	Diode Iso. ¹	
6	HP39gII	1.5 VDC	AAA	Four	Diode Iso. ¹	Top two and bottom two parallel ² .
7	OfficeCalc 200	1.5 VDC	GP-189	One	N/A	
8	PrintCalc 100	6.0 VDC	AA	Four	Series	
9	QuickCalc	1.5 VDC	GP-189	One	N/A	

Notes: (1) Diode isolation allows the two cells to power the circuits in such a way that as long as one cell is present memory (usually data registers and program memory) will not be erased.

(2) The calculator will run on one cell with a shorter life.

The un-weighted comparison is the sum of the microampere current four states, e.g. the HP-15C LE is $0.1 + 98 + 12 + 1,560 = 1,670.1 \mu\text{A}$. Rounded to two places = 1700. This number doesn't convey very much information. The highly desirable low **OFF** state value disappears and the high **ON KEY** dominates when it is only drawn for 1/2 second or less.

Let's assign a usage time weight for each value to more closely approximate how the calculator is being

used. Suppose a student uses a calculator three times per day for ten minutes (on average). This means that the **OFF** time is $23.5 \text{ hours}/3 = 7.7 \text{ Hr.}$ or 470 minutes per usage. Let's also assume when turning on, **ON KEY**, the calculator it takes $\frac{1}{2}$ second. Since the calculator typically turns off after ten minutes let's use the **ON** time as 10 minutes. For the final state value, **RUNNING**, let's assume 15 seconds or 0.25 minutes. A single usage cycle is **OFF** = 470 minutes, **ON KEY** = 0.008 minutes, **ON** = 10 minutes, **RUNNING** = 0.25 minutes. The total is 480.258 minutes. The ratio (weight factor) for each is: **OFF** = 0.97894, **ON KEY** = 0.00002, **ON** = 0.02082, **RUNNING** = 0.00052.

Now we can add our total weighted currents to make a better comparison. The HP-15C LE weighted value = $3.71997 + 0.39600 + 1.02851 + 0.01024 = 5.15472$. Comparatively speaking the lower this value is the better. Keep in mind that the type of calculator is also an important efficiency consideration factor. These values are tabulated in Table 4.

Table 4 – Selected HP Calculator Currents Drawn Comparison

#	Model	Voltage	Unweighted	3 Weighted	4 Weighted	3 Wt. Ratio	4 Wt. Ratio	Comments
1	HP 10bII	3.0 VDC	N/A	5.33	N/A	34.6x	N/A	ON key \approx same as all other keys.
2	HP 30b	3.0 VDC	20,746.3	7.55	17.9x	49.0x	10.5	
3	HP-12C Platinum	3.0 VDC	1,670.1	0.350	1.7x	2.27x	REF	Very low ON KEY drain.
4	HP-15C LE	3.0 VDC	19,872.9	5.14	5.1x	33.4x	3.0	Works down to $<2 \text{ VDC}$ ⁶ .
5	HP 35s	3.0 VDC	4,658.6	8.81	11.2x	57.2x	6.6	Isolated two supplies
6	HP39gII	1.5 VDC	33,372.9	14.4	25.2x	93.5x	14.8	Isolated two supplies
7	OfficeCalc 200	1.5 VDC	N/A	1.23	N/A	7.99x	N/A	See text.
8	PrintCalc 100	6.0 VDC	N/A	63.5	N/A	412x	N/A	4 AA cells.
9	QuickCalc	1.5 VDC	N/A	0.154	N/A	REF	N/A	Small button cell.

The HP-12C Platinum (and all 12C's) is the most energy efficient when all four measured values are used. The worst is the HP 39gII at 15 times more current for all four values. The second most efficient calculator is the HP 35s.

If the **RUNNING** value is not used and all nine machines are compared the most efficient machine (non-programmable) is the QuickCalc (used as the reference value) with the HP-12C a close second.

Using Table 2 values to diagnose the calculator

Every serious repair/service bench has a power source that provides the electrical power parameters – voltage, current and power – of the device under test. In the case of a calculator it is the current measurement fixture shown in Figs 1 – 6. Knowing the **OFF** current immediately tells the user that the calculator circuits are at least receiving power and drawing the normal amount. Any other value (over or under $\pm 20\%$) indicates something is wrong. Of course multi tens of milliamperes is a sure sign of excessive current due to a leaky capacitor or “cooked” IC's.

Pressing the **ON KEY** is the next step to having a clue of what might be wrong. If the current is within $\pm 20\%$ of the table 2 value the microprocessor circuitry is scanning the keyboard and responding to the key press. The newer designs (OfficeCalc 200) have less of a current increase; 3.2 times while the HP 39gII is the worst at 1,300 times more current.

If the display turns on the machine is much closer to being able to work properly. Of course physical defects such as a cracked display, black display, etc. indicates a display issue.

The primary value of seeing the current change – assuming a proper operating voltage - under various conditions is to know if something is wrong with the power source such as a broken wire or cracked circuit board.

Summary, observations, and conclusions

Measuring the current drawn by a calculator is a difficult measurement because of the requirement of inserting a meter (series connection) into the power supply circuit. This is part two of a two part series with part one, *HP Solve*, issue #28 page 30, describing the mechanical issues. This part applies the methods described in part one to measure nine representative models of the current 24 HP calculator models.

A very low cost (FREE to \$9.99) Digital Multi-meter is used to measure the four most common calculator state current values – **OFF, ON KEY, ON, & RUNNING**. The increasing number of calculator operating conditions (states) make these measurements important for diagnostic and power supply life estimates. The various power supply cell holders are described and four tables provide the technical data for each of the selected nine machines.

Notes: Measuring Calculator Current – Nine Measurement Examples

- (1). *Technically a battery is composed of two or more cells. Very often popular and marketing usage of the term battery is used in place of cell. All current HP calculators operate from one or more cells using one to four cell holders.*
- (2). *The CEN-TECH 7 Function meter shown below is a low cost but quite accurate Digital Multi-Meter, DMM, marketed by the US national chain Harbor Freight. This DMM normally ranges in cost from free to \$9.95. I have seen this model on the market for at least seven years. See Fig. 16 and Fig. 17 for additional details. First visit a Harbor Freight store and sign up to receive their snail mailers and you will be able to buy one on sale (or you may order from their website) or even get one free. The FREE coupons often appear with a single item 20% discount coupon in various magazines such as Sunday paper supplements. I have gotten two*



Fig. 16 – Seven function CEN-TECH DMM costs.



Fig. 17 – Mailed coupon gets you one for FREE.

FREE DMMs in the last month (August). A third one was obtained from a Labor Day Sale Coupon similar to the one shown in Fig. 17. This is certainly great news for students. I have over a dozen of these DMMs (FREE or \$3 sale) because I use them to measure multiple nodes of a circuit at the same time. A quick photo

“collects” the circuit values for future reference. The lowest current range, 200 μ A, is even lower than several DMMs that I have that cost many times that of the CEN-TECH. The primary negative aspect of this low cost DMM is that it does NOT have an auto turn off and it is easy to forget and leave it on which results in a DEAD 9 VDC battery. Fig. – 18 shows the specifications.

CEN-TECH 7 Function DMM

ITEM	DESCRIPTION
Frequency	45-450 Hz
DC Amps	Ranges: 200 μ A/2000 μ A/20mA/200mA, 10A
Accuracy	(@200 μ A-200mA) 1.2% \pm 2D; (@10A) 3% \pm 2D
DC Voltage	Ranges: 200mV/2000mV/20/200/1000V
Accuracy	(@200mV) 0.5% \pm 1D; (@2000mV-200V) 1.0% \pm 1D; (@1000V) 1.0% \pm 2D
AC Voltage	Ranges: 200/750V
Accuracy	(45-450 Hz) 1.2% \pm 10D
Resistance	Ranges: 200/2000/20K/200K/2000K Ohm
Accuracy	(@200-200K Ohm) 0.8% \pm 2D; (@2000K Ohm) 1.0% \pm 2D;
Sampling Rate	2.5 times/Secon
Overload Protection	Fast-Acting 500mA/250V Fuse
Operating Temperature	Range: 32° - 104° F
Display	1/2" high 3-1/2-digit LCD
Battery	One 9 V (included)
Weight	.45 lb.
Features	32" Test Leads, Transistor (NPN and PNP) Testing Function, Battery Testing Function, and Automatic Polarity and Zero Adjust

Fig. 18 – CEN-TECH 7 Function DMM specifications

- (3). Making current measurements is always a physical challenge. The hook connectors work if large enough (and you use tape to hold them in place if needed) because they tend to pop off the coin cell holders.
- (4). The OfficeCalc 100, OfficeCalc 200, and OfficeCalc 300 calculators are much larger than the stand. The PrintCalc 100 has a paper roll and is not practical to place on the stand.
- (5). The early versions of the HP-12C used three button cells in series (4.5 VDC.). For a more complete description of this calculator see [HP Solve](#), issue #25 page 10 (especially pages 11 & 15). The comparison of button cells (slightly larger than the ones used currently) of Silver or Lithium chemistry was noted in Note 5 with a table that is reproduced here.


<p>Early HP-12Cs used 3 Lithium or Silver button cells. Here is a price comparison. Silver has twice the life.</p>		<p>32¢ Li vs. 52¢ Ag in 500 piece quantity.</p>
<p>Sony Batteries LR44 Alkaline Watch Battery</p> <p>SONY LR44 (A76) Alkaline Button Cell 1.5V, Blister 1 pc Replaces - LR44H, 357A, RW82, KA, A76, 208-904, SB-F9, GPA76, L1154, AG13, G13-A, L1154, A76, V13GA, LR44, AG-13, AG 13, PX76A, 675A, 303, Duracell PX-76A Button Cell Battery, G13A, Energizer A76 Man</p>	<p>Bulk Batteries 357 Batteries Silver Oxide Watch Battery Tray Pack</p> <p>Watch Battery Tray Pack Cross Reference With - 1131SO SR44 D357/303 SP357 SR44W V357 AG13 G13 A76 PX76 675 1166A PX76A A76-1 LR44H V13GA GP76A L1154 RW82 As Low as \$0.41 Per Battery</p>	<p>\$1.00, 10 for \$6.50, 200 for \$70.00, 500 for \$160.00</p>
<p>\$0.70, 50 for \$35.00, 150 for \$88.50, 450 for \$234.00, 1500 for \$690.00, 3000 for \$1,230.00</p>		

Fig. 18 – Silver & Lithium button cell comparison. →

Investigate a multi-root server

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A Multi-Root Solver

Namir Shamma

Introduction

In 1978, HP launched the HP-34C that contained the very first built-in root-finding Solver. The Solver was a new, powerful, and convenient tool for calculating the roots of single-variable nonlinear equations. The Solver implemented a clever version of the secant method. HP implemented and also enhanced the Solver in many machines that succeeded the HP34C. The early implementations for the Solver required two initial guesses for the root and gave you an answer that was as accurate as possible for the machine. Professor Kahan, who was behind the design of the first Solver, pushed for full accuracy for the root, as opposed to getting answers based on a user-specified accuracy (a.k.a. tolerance)—the typical approach in iterative root solving. As Solver's implementations matured, they required single initial guesses. Moreover, HP launched special functions that calculated all of the roots (both real and complex) of real-coefficient polynomials, including duplicate roots. Several graphing calculators, including the HP 50g and the new HP 39gII offer the POLYROOT function that calculates the multiple roots for polynomials, given an array of polynomial coefficients. The POLYROOT implementations use the Laguerre method to calculate the polynomial roots and do not require user-supplied initial guesses for the roots.

The HP calculator models that included the Solver are the HP-34C, HP-15C, the HP-41C (using the Advantage ROM module), HP28C/S, HP 19B, HP 19BII, HP 17B, HP 17BII, HP 17BII+, HP32S, HP32SII, HP33s, HP35s, and RPL graphing calculators, and the non-RPL graphing calculators.

You may ask, “What about a multi-root solver for non-polynomial functions?” This article presents a method that obtains the multiple real roots of real functions in general. The method basically scans a range of values you specify and looks for roots in that range. This method has the added bonus of finding the minima and maxima in the scanned range. The final set of results gives you a good idea about the roots and inflection points of a mathematical function.

The Scan Range Method

The scan range method works with continuous and differentiable functions. The method scans a range of values by using a divide-and-conquer approach. The method divides the scanned range into a large number of small sub-intervals and looks in each sub-interval for the following:

- A change in the function's sign. The sub-interval in which such a change occurs contains a root. Most roots appear in such sub-intervals.
- A change in the slope's sign. The sub-interval in which such a change occurs contains a minimum, or a maximum, or a saddle point, possibly combined with a root. For example, the range $[0.9, 1.1]$ for $f(x)=(x-1)^2$ contains both a root and a function minimum at $x=1$. Keep in mind that roots that are also minima or maxima are difficult to obtain using typical root-seeking algorithms such as the Bisection, False Position, Newton, Halley, and the Ostrowski methods. It is worth to point out that the Solve application in many HP Graphing calculators can solve for the roots that are also minima or maxima. The

limitation that exists with these machines' Solve is that you only get the closest root to the guess that you supply.

The scan range method requires the following input:

- The range of values to scan. These values are needed to allow you to focus on a range that you are interested in. More importantly, such a range allows you to obtain a finite set of roots for sinusoidal functions that typically have a very large or infinite number of roots.
- The search step size. This value determines the search resolution and computational effort. Selecting the appropriate value for the search step can, for certain functions, allow the search to simply *land* on one or more roots! Such roots are usually integer roots and roots that have a few decimal places. Typical values for the search step size should be between 1/100 and 1/1000 the scanned range.
- The tolerance value used in the root-seeking algorithm. This value indicates the accuracy of the roots you obtain. Typical values can range from 1E-6 to 1E-9.
- The function tolerance value is used to examine the second derivative of the scanned function. The values for this derivative can tell us if the root is at a minimum, maximum, or saddle point (when the second derivative is zero). A typical value for this input is 1E-4. Smaller values may lead the scan range method to make the wrong conclusion about a point being a minimum, maximum, or saddle point.

The scan method is a composite method which employs two supporting algorithms:

- An algorithm to calculate the root of a function. Since the scan range method works with sub-intervals, you can use root-bracketing algorithms like the Bisection method. Using Newton's method, Halley's method, or even Ostrowski's method works very well since we supply these algorithms with good initial guesses for the roots. Small search step sizes provide excellent initial guesses.
- An algorithm that calculate the maximum or minimum by solving for the root of the function's derivative. Again, we can use root-bracketing algorithms, interpolation algorithms, or even the more advanced and popular Newton's method, or better algorithms. Once more, small search step sizes provide excellent initial guesses.

Table 1 shows the pseudo-code for the scan range algorithm:

Given the function $f(x)$, slope of function $d1(x)$, second derivative $d2(x)$, range to scan $[A, B]$, search step size (StepSize), tolerance for x (Toler), and function tolerance (FxToler).

Initialize mechanism or structure used to report the results

Xa=A

Fa = f(Xa)

Da = d1(Xa)

SFa = sign of Fa

SDa = sign of Da

NumSteps = 0

Repeat	
Increment NumSteps	
$X_b = A + \text{NumSteps} * \text{StepSize}$	
$F_b = f(X_b)$	
$D_b = d_1(X_b)$	
SignFb = sign of Fb	
SignDb = sign of Db	
// Xb landed on a root?	
If Fb=0 Then	
Report and/or store Xb, Fb	
// Second derivative	
Drv2 = d2(Xb)	
If SignDb and SignDa have opposite signs Then	
If $ \text{Drv2} > F_x \text{Toler}$ Then	
If Drv2 < 0 Then	
Report a maximum point	
Else	
Report a minimum point	
End	
Else	
Report a saddle point	
End	
End	
Else If SignFb and SignFa have opposite signs Then	
X = calculated root in $[X_a, X_b]$ with tolerance Toler	
Report and/or store X, f(x)	
Else If (SignFa and SignFb have same values) AND	
(SignDa and SignDb do not have the same values) THEN	
X = calculated minima/maxima in $[X_a, X_b]$ with tolerance Toler	
Report and/or store X, f(X)	
Drv2 = d2(X)	
If Drv2 < 0 Then	
Report a maximum point	
Else	
Report a minimum point	
End	
End	
If found a root, minimum, or maximum Then	
Increment NumSteps	
$X_a = A + \text{NumSteps} * \text{StepSize}$	
$F_a = f(X_a)$	
$D_a = d_1(X_a)$	

SignFa = sign of Fa
SignDa = sign of Da
Else
Xa = Xb
Fa = Fb
Da = Db
SignFa = SignFb
SignDa = SignDb
End
Until Xa>=B
Return accumulated information

Table 1 – The Pseudo-Code Listing.

I would like to point out that the pseudo-code in Table 1 is missing important code for error-handling. The syntax and mechanism of error-handling, if supported by a calculator, depends very much on that calculator. Using error-handling allows you to try to continue scanning the range of values or at least salvage the results obtained up to that point, when a runtime error occurs. I recommend using an error counter and a maximum number of errors, in the implemented error-handler. These variables help you prevent repeated or cascading runtime errors from breathing chaos in the implementation.

The HP 39gII Listing

Table 2 contains the commented listing for the HP 39gII calculator. The table shows the following functions:

- The function **MYFX** which implements the code for calculating the mathematical function $f(x)$. The mathematical function can be a transcendental function, a polynomial, or any other continuous and differentiable function.
- The function **MYSLOPE** which implements the code for calculating an approximation for the first derivative (i.e. the slope) of the mathematical function $f(x)$.
- The function **SECDRV** that implements the code for calculating an approximation for the second derivative (i.e. the curvature) of the mathematical function $f(x)$.
- The function **NWRT** that returns the refined root of a function using Newton’s method.
- The function **NWOP** which calculates a minimum or maximum value using Newton’s method.
- The function **SCAN** which implements the scan range method and returns a matrix that contains the results.

<i>Statement</i>
EXPORT MYFX(X)
BEGIN
EXP(X)-3*X*X;
END;
EXPORT MYSLOPE(X,FX)

BEGIN
0.001*(1+Abs(X))→H;
(MyF _x (X+H) - FX)/H;
END;
EXPORT SECDRV(X)
BEGIN
LOCAL FX,FP,FM;
0.001*(1+Abs(X))→H;
MYFX(X)→FX;
MYFX(X+H)→FP;
MYFX(X-H)→FM;
(FP-2*FX+FM)/H/H;
END;
EXPORT NWRT(X,TOLER)
BEGIN
LOCAL FX,DIFF;
// APPLY NEWTON'S METHOD
REPEAT
0.001*(1+Abs(X))→H;
MYFX(X)→FX;
H*FX/(MYFX(X+H)-FX)→DIFF;
X-DIFF→X;
UNTIL ABS(DIFF)<=TOLER;
X;
END;
EXPORT NWOP(X,TOLER)
BEGIN
LOCAL FX,DIFF,FP,FM,DRV1,DRV2;
// APPLY NEWTON'S METHOD TO SOLVE
// f'(X)=0
REPEAT
0.001*(1+ABS(X))→H;
MYFX(X)→FX;
MYFX(X+H)→FP;
MYFX(X-H)→FM;
(FP-FM)/2/H→DRV1;
(FP-2*FX+FM)/H/H→DRV2;
DRV1/DRV2→DIFF;
X-DIFF→X;
UNTIL Abs(DIFF) <= TOLER;
X;
END;
EXPORT SCAN(A,B,STP,TOLER,FXTOLER)

BEGIN
LOCAL XA,FA,DA,SFA,SDA;
LOCAL XB,FB,DB,SFB,SDB;
LOCAL NMSTP,MOES,RES;
LOCAL DRV1,DRV2;
LOCAL TRUE,FALSE,TAG;
LOCAL ERRCOUNT,ERRMAX;
// DEFINE LOGOCAL VALUES
0->FALSE;
1-TRUE;
// CREATE RESULTS MATRIX
MAKEMAT(0,20,3)→RES;
// CALCULATE FUNCTION & SLOPE AT XA
// WHICH IS THE START OF SUB-INTERVAL
A→XA;
MYFX(XA)→FA;
MYSLOPE(XA,FA)→DA;
SIGN(FA)→SFA;
SIGN(DA)→SDA;
0→NMSTP;
1→N;
0→ERRCOUNT;
5→ERRMAX;
// START THE MAIN LOOP
REPEAT
NMSTP+1→NMSTP;
// ACTIVATE ERROR HANDLER
IFERR
// MOES = Move One Extra Step
FALSE→MOES;
// CALCULATE END OF SUB-INTERVAL
A+NMSTP*STP→XB;
// CALCULATE FUNCTION & SLOPE AT XB
MYFX(XB)→FB;
MYSLOPE(XB,FB)→DB;
SIGN(FB)→SFB;
SIGN(DB)→SDB;
0→TAG;
// DID WE LAND ON A ROOT?
IF ABS(FB) = 0 THEN
// STORE RESULTS
XB→RES[N,1];

0→RES[N,2];
10→RES[N,3];
// CALCULATE SECOND DERIVATIVE
SECDRV(XB)→DRV2;
1→TAG;
END;
// DID SLOPE SIGN CHANGE IN [XA,XB]?
IF (SDA*SDB<=0) AND (TAG==1) THEN
// IS ROOT AT MIMIMUM ?
RES[N,3]+IFTE(DRV2 > 0 AND ABS(DRV2)>FXTOLER,2,0)→RES[N,3]
// IS ROOT AT MAXIMUM?
RES[N,3]+IFTE(DRV2 < 0 AND ABS(DRV2)>FXTOLER,3,0)→RES[N,3]
// ROOT IS AT A SADDLE POINT?
RES[N,3]+IFTE(ABS(DRV2)<=FXTOLER,1,0)→RES[N,3]
// UPDATE RESULTS COUNTER
N+1→N;
TRUE→MOES;
END;
// FOUND A RANGE THAT CONTAINS A ROOT?
IF (SFB*SFA<=0) AND (TAG==0) THEN
// CALCULATE ROOT
NWRT((XA+XB)/2,TOLER)→X;
// STORE RESULTS
X→RES[N,1];
MYFX(X)→RES[N,2];
10→RES[N,3];
// UPDATE RESULTS COUNTER
N+1→N;
TRUE→MOES;
1→TAG;
END;
// LOCATED A RANGE THAT HAS A MINIMUM/MAXIMUM/ROOT?
IF (SFA*SFB > 0) AND (SDA*SDB<=0) AND (TAG==0) THEN
// CALCULATE MINIMUM/MAXIMUM
NWOP((XA+XB)/2,TOLER)→X;
// CALCULATE SECOND DERIVATIVE
SECDRV(XB)→DRV2;
// STORE RESULTS
X→RES[N,1];

MYFX(X) →FX;
FX→RES[N,2];
TRUE→MOES;
// FOUND A ROOT?
IFTE(ABS(FX)<=FXTOLER,10,0) →RES[N,3];
// IS ROOT AT MIMIMUM OR MAXIMUM?
RES[N,3]+IFTE(DRV2>0,2,0) →RES[N,3];
RES[N,3]+IFTE(DRV2<0,3,0) →RES[N,3];
// UPDATE RESULTS COUNTER
N+1→N;
END;
// ERROR HANDLER CLAUSE
THEN
1+ERRCOUNT→ERRCOUNT;
IF ERRCOUNT>ERRMAX THEN
MSGBOX("MAX RUNTIME ERRORS: ENDING CALCULATIONS");
RETURN RES;
END;
TRUE→MOES;
END;
// MOVE ONE EXTRA POINT?
IF MOES==TRUE THEN
NMSTP+1→NMSTP;
// CALCULATE VALUE, FUNCTION, AND SLOPE FOR XA
A+NMSTP*STP→XA;
MYFX(XA)→FA;
MYSLOPE(XA,FA)→DA;
SIGN(FA)→SFA;
SIGN(DA)→SDA;
ELSE
// SHIFT SUB-INTERVAL
XB→XA;
FB→FA;
DB→DA;
SFB→SFA;
SDB→SDA;
END;
UNTIL XA >= B;
// RETURN THE RESULT MATRIX
RETURN RES;
END;

Table 2 – The HP 39gII Listing.

The code for function **MYFX** is simple and requires one or more statements to calculate the tested function. The function **MYFX** has the parameter **X** to pass the value needed to calculate the mathematical function $f(x)$.

The code for function **MYSLOPE** calculates an approximation to the first derivative for the mathematical function at **X**. The parameters for this function are **X** and **FX**--the value of $f(x)$. I chose to pass parameter **FX** instead of calculating to save computational effort, since the value for $f(x)$ is always available before the function **SCAN** needs to calculate the first derivative.

The code for function **SECDRV**, which has the parameter **X**, calculates an approximation to the second derivative for the mathematical function at **X**.

The function **NWRT** refines a given guess for the root. The function has the parameters **X** and the tolerance value **TOLER**.

The function **NWOP** calculates the minimum/maximum in a given range. The function has the parameters **A**, and **B**, which define the range $[A,B]$. In addition the function includes the parameter **TOLER** to pass arguments for the tolerance value.

The function **SCAN** is the main function that implements the scan range method as outlined in Table 1. The function parameter list include the parameters **A** and **B** which define the search range, the search step size parameter, **STP**, the tolerance parameter, **TOLER**, and the function tolerance parameter, **FXTOLER**. The function uses the latter parameter to determine if the second derivative of the mathematical function represents a minimum, a maximum, or a saddle point. The function returns a three-column matrix. Store the results in one of the ten predefined matrices so you can later examine the results with ease using the built-in matrix viewer/editor. The first column contains values for x that represent roots, minima, maxima, and saddle points. The second column contains the corresponding values for $f(x)$. The third column contains numeric codes that interpret the results. Table 3 shows the different values of the numeric codes and their meaning.

<i>Code Value</i>	<i>Meaning</i>
0	Data is meaningless.
1	Saddle point.
2	Minimum value.
3	Maximum value.
10	Root.
11	Root and saddle point.
12	Root and minimum.
13	Root and maximum.

Table 3 – The Numeric Code for the Results

Function **SCAN** creates the local matrix **RES** to have 20 rows and three columns, using the command **MAKMAT**. This command also initializes all of the elements of matrix **RES** with zeros. If the function **SCAN** returns fewer than 20 results, the extra rows will be zeros. You may ask, “What of the function **SCAN** finds more than 20 results?” The HP 39gII runtime system

automatically expands the rows of matrix **RES**. This deluxe feature absolves you from selecting the number of initial rows of matrix **RES** and monitoring the number of results.

Working with the Function **SCAN**

The function **SCAN** is a tool, and not a magic wand, for finding multiple roots. As such, you need to observe a few simple guidelines that will help you make the best of the **SCAN** function:

- Plot the function you intend to examine. This task is a relatively easy when using a graphing calculator like the HP 39gII. The plot gives you a good idea about the variation in function values and also about the number and values for the roots.
- Selecting the range to scan is easy, and is made even easier if you have a plot for the targeted mathematical function. You can always re-execute the function **SCAN** with a wider, narrower, or even different range of values.
- Selecting the search step size is very important. Choosing positive powers of 0.1 can help you land on some roots and reduce computational effort. Small search step size values can sometimes (depending on the mathematical function coded in **MYFX**) cause multiple results, for the same root value, that are very close to each other. This is where having a plot for the targeted mathematical function comes in very handy in discerning the appearance of multiple values for the same root.
- Selecting the tolerance and function tolerance values within the ranges I recommended earlier can help the function **SCAN** generate results with ease.
- You can code program function **MYFX** to calculate the values of a polynomial. If you chose to code a polynomial, then you need to hard code the polynomial's coefficients in function **MYFX**. The code in Table 2 has no way of determining if function **MYFX** evaluates a polynomial or any other type of function. If a polynomial coded in **MYFX** has duplicate roots, the function **SCAN** returns one value for that duplicate root without mentioning that it is a duplicate root or the number of duplicate roots. The lack of this kind of information is the main weak point of the scan root method.

You may ask, "What happens if the program encounters a runtime error?" The HP 39gII code uses the **IFERR** construct to trap runtime errors inside the main **REPEAT-UNTIL** loop. The function **SCAN** uses the local variables **ERRCOUNT** and **ERRMAX** to store the number of runtime errors and the maximum number of allowable runtime errors (hardcoded as 5), respectively. The clause **THEN** associated with the **IFERR** error handler increments the variable **ERRCOUNT** each time an error occurs. If the value in that variable exceeds the value in **ERRMAX**, the function **SCAN** displays an error message and returns the results obtained so far. By contrast, if the number of run time errors is still within the maximum allowable limit, the function **SCAN** sets the variable **MOES** to **TRUE** to move ahead in the search for roots, and hopefully move away from offending x values.

Let's look at a few cases that represent different kinds of mathematical functions.

Case 1

Consider the function:

$$f(x) = \exp(x) - 3*x^2$$

Which is already coded in function **MYFX** in Table 2. This function has three roots, a maximum, and minimum. Figure 1 shows a plot for the mathematical function. Let's apply the range scan method to the function. Select the range of $[-1,4]$, the search step of 0.01, tolerance value of $1E-8$, and function tolerance value of $1E-4$. Type **SCAN(-1,4,0.01,1E-8,1E-4)->M1** at the command input line. The function **SCAN** returns a three-column matrix that contains the results in about 11 seconds. Invoke the Matrix command to view the list of matrices. Select matrix **M1**. Figure 2 shows the contents of that matrix.

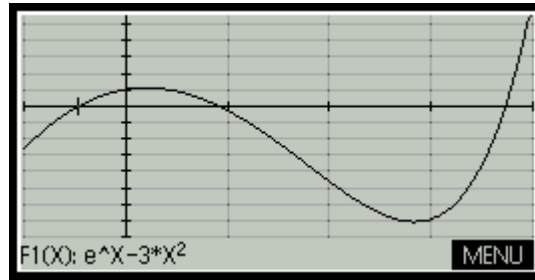


Fig. 1 – The plot for $f(x)=exp(x)-3*x^2$.

M1	1	2	3
1	-4.6E-1	0	10
2	2.04E-1	1.1015	3
3	9.10E-1	1E-11	10
4	2.8331	-7.0813	2
5	3.7331	-1E-10	10

-0.458962267537

EDIT INS BIG • GO → WIDTH4

Fig. 2 – The results for $f(x)=exp(x)-3*x^2$.

Table 4 shows the results revealing more digits for the values of x and $f(x)$.

x	$f(x)$	Comment
-0.458962267537	0	Root
0.204481512112	1.10145070666	Maximum
0.910007572487	1E-11	Root
2.83314410811	-7.0812935823	Minimum
3.73307902863	-1E-10	Root

Table 4 – Detailed results for for $f(x)=exp(x)-3*x^2$.

Case 2

Consider the function:

$$f(x) = (x - 1)^2$$

You must code the above mathematical function in **MYFX**. This function is one that has a root which doubles up as a minimum point. You can code it as $(x - 1)^2$ or as $x^2 - 2*x + 1$. Figure 3 shows the plot for the above mathematical function. Let's apply the range scan method to the above function. We select the range of $[0,2]$, the search step of 0.01, tolerance value of $1E-8$, and function tolerance value of $1E-4$. Type **SCAN(0,2,0.01,1E-8,1E-4)->M1** at the command input

line. The function **SCAN** returns a three-column matrix that contains the results in about 4 seconds. Invoke the Matrix command to view the list of matrices. Select matrix **M1**. Figure 4 shows the contents of that matrix.

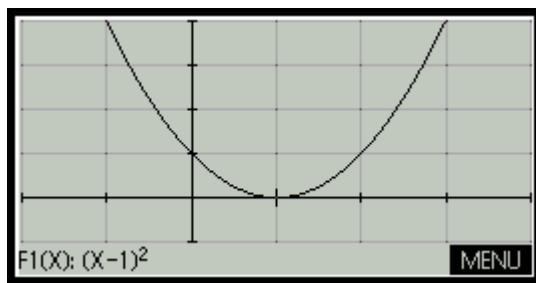


Fig. 3 – The plot for $f(x)=(x-1)^2$.

M1	1	2	3	
1	1	0	12	
2	0	0	0	
3	0	0	0	
4	0	0	0	
5	0	0	0	
1				
EDIT	INS	BIG•	GO→	WIDTH4

Fig. 4 – The results for $f(x)=(x-1)^2$.

Table 5 shows the results revealing more digits for the values of x and $f(x)$. Both figure 4 and Table 5 show a single result for a function that has two duplicate roots. The result does not indicate the presence of duplicate roots and neither does it mention the number of these duplicate roots. As stated earlier, the lack of this kind of information is the main weak point of the scan root method. If the tested function was $(x-1)$ raised to any positive integer power, you would still get the same results shown in Figure 4.

x	$f(x)$	Comment
1	0	Root & minimum

Table 5 – Detailed results for $f(x)=(x-1)^2$.

Case 3

Consider the following function that has an infinite number of roots, all being minima values:

$$f(x) = \sin(x)+1$$

You must code the above mathematical function in **MYFX**. This function represents a sinusoidal function that has periodic roots that are also minima. The function has no negative values! We'll apply the range scan method to the above function. Select the range of $[-1,20]$, the search step of 0.1, tolerance value of $1E-8$, and function tolerance value of $1E-4$. Type **SCAN(-1,20,0.1,1E-8,1E-4)->M1** at the command input line. The function **SCAN** returns a three-column matrix that contains the results in a bit under 4 seconds. Invoke the Matrix command to view the list of matrices. Select matrix **M1**. Figure 3 shows the contents of that matrix.

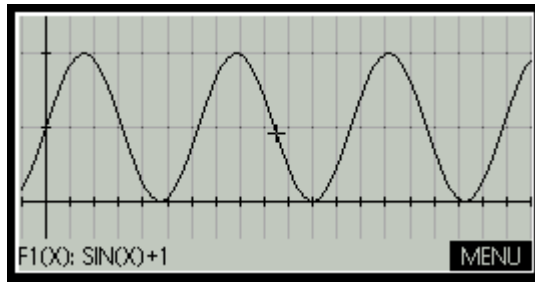


Fig. 3 – The plot for $f(x)=\sin(x)+1$.

M1	1	2	3
1	1.5708	2	3
2	4.7124	0	12
3	7.8540	2	3
4	1.100E10		12
5	1.414E12		3

1.57079632667

EDIT INS BIG• GO→ WIDTH4

Fig. 4 – The results for $f(x)=\sin(x)+1$.

Table 6 shows the results revealing more digits for the values of x and $f(x)$.

x	$f(x)$	Comment
1.57079632667	2	Maximum
4.71238898045	0	Root and minimum
7.85398163407	2	Maximum
10.9955742876	0	Root and minimum
14.1371669413	2	Maximum
17.2787595948	0	Root and minimum

Table 6 – Detailed results for $f(x)=\sin(x)+1$.

Case 4

Consider the following function:

$$f(x) = [\sin(x)+1]*[\exp(x)-3*x^2]$$

You must code the above mathematical function in **MYFX**. This function is the product of two functions that I presented earlier in this article. Figure 5 shows a plot for the above mathematical function. Let's apply the range scan method to the above function. Select the range of $[-1,20]$, the search step of 0.1, tolerance value of $1E-8$, and function tolerance value of $1E-4$. Type **SCAN(-1,20,0.1,1E-8,1E-4)->M1** at the command input line. The function **SCAN** returns a three-column matrix that contains the results in under 7 seconds. Invoke the Matrix command to view the list of matrices. Select matrix **M1**. Figure 6 shows the contents of that matrix.

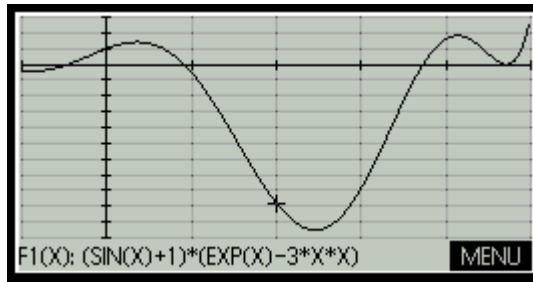


Fig. 5 – The plot for $f(x)=[\sin(x)+1]*[\exp(x)-3*x^2]$.

M1	1	2	3
1	-9.0E-1	-4.4E-1	2
2	-4.6E-1	0	10
3	3.59E-1	1.4125	3
4	9.10E-1	1.790E-10	10
5	2.4584	-1.05E1	2
-0.900078175436			
EDIT	INS	BIG•	GO→ WIDTH4

Fig. 6 – The results for $f(x)=[\sin(x)+1]*[\exp(x)-3*x^2]$.

Table 7 shows the results revealing more digits for the values of x and f(x).

x	f(x)	Comment
-0.90007817536	-0.438422918275	Minimum
-0.458962267537	0	Root
0.359355747256	1.41249185024	Maximum
0.910007572488	1.7895083873E-11	Root
2.45840730999	-10.5137016896	Minimum
3.73307902863	-4.424045264E-11	Root
4.13489742206	1.81480015417	Maximum
4.71235866759	2.05152030823E-8	Root & Minimum
9.44170203918	12126.9519043	Maximum
10.9955019812	0.00015486049	Minimum
15.7080140915	6634883.80328	Maximum
17.2785925371	0.445332022011	Minimum

Table 7 – Detailed results for $f(x)=[\sin(x)+1]*[\exp(x)-3*x^2]$.

Observations and Conclusions

The article presented the scan range method that scans for roots in a user specified range. The article also presented the source code for the HP 39gII graphing calculator. The examples given illustrate that by monitoring the values, first derivative, and second derivative for a targeted function, the scan range method can find roots, minima, and maxima. These include roots that are at the same time minima or maxima, making them hard to calculate using the traditional Newton method. I hope that the designers for the HP calculator take note of the scan range method and will enhance the Solve application to locate multiple roots.

References

1. W. Kahan, "Personal Calculator Has Key to Solve Any Equation $f(x)=0$ ", Hewlett-Packard Journal, December 1979.
2. Richard L. Burden and J. Douglas Faires, "Numerical Analysis", 9th edition, Brooks/Cole, 2011.
3. William H. Press, Saul A. Teukolsky, William T. Vetterling, and Brian P. Flannery, "Numerical Recipes: The Art of Scientific Computing", third edition, Cambridge University Press, 2007.

About the Author



Namir Shammas is a native of Baghdad, Iraq. He resides in Richmond, Virginia, USA. Namir graduated with a degree in Chemical Engineering from the University of Baghdad. He also received a master degree in Chemical Engineering from the University of Michigan, Ann Arbor. He worked for a few years in the field of water treatment before focusing for 17 years on writing programming books and articles. Later he worked in corporate technical documentation. He is a big fan of HP calculators and collects many vintage models. His hobbies also include traveling, music, movies (especially French movies), chemistry, cosmology, Jungian psychology, mythology, statistics, and math. As a former PPC and CHHU member, Namir enjoys attending the HHC conferences. Email me at: nshammas@aol.com

From the editor

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From The Editor – Issue 29

Fall is in the air and summer temperatures have abated. The HHC has happened and everyone is busy with increased energy of the upcoming holiday season.

Here is the content of this issue

S01 – HP denotes \$2.7 million in calculators to students worldwide Good 360, a non-profit, part of Gifts in Kind International, distributes goods to needy students. They worked with HP to provide calculators to schools in Asia.

S02 – Meet Gary M. Tenzer (Customer Corner) Gary Tenzer is a 30 year HP financial calculator enthusiast. As a Real Estate Investment Banker and Expert Witness he has been very involved in many Southern California George Smith Partners projects. Read how Gary has traveled the HP financial calculator history from the HP-35A forward.

S03 – Hewlett-Packard Handheld Conference #39 in Nashville TN by your editor. The down economy didn't prevent HP Calculator enthusiasts from five countries traveling to discuss the latest in the HP Calculator world. The HP User Community, HPUC, spent the weekend as described in this detailed 13 page report. A group photo is included.

S04 – Using Weighted Power and Exponential Curve Fitting by Palmer Hanson. Palmer uses previously published data for a programming competition to calculate the digits of Pi. He describes how this data (run times of a program vs. the number of digits) is fit using a Power Function Fit, Linear Fit, and a weighted data Power Function Fit. He further explores a weighted solution for an Exponential Function. The lesson to be learned is when this method applies and improves the fit.

S05 – Measuring Calculator Current (part II) - Nine Measurement Examples Part I described the basic techniques to be used and this part provides data for the current product line. “The list of 2012 HP models is shown in Table 1. I selected three models from each category to measure the current drawn in the four states – **OFF, ON KEY, ON, & RUNNING.**”

S06 – A Multi-Root Solver by Namir Shammas. Namir reviews the HP models that provide Solver support. “The HP calculator models that included the Solver are the HP-34C, HP-15C, the HP-41C (using the Advantage ROM module), HP28C/S, HP 19B, HP 19BII, HP 17B, HP 17BII, HP 17BII+, HP32S, HP32SII, HP33s, HP35s, and RPL graphing calculators, and the non-RPL graphing calculators.” He describes the Scan Range method of a multi-root solver.

S07 – Regular Columns

- ◆ From the editor.
- ◆ One Minute Marvels.
- ◆ HP User Community News.
- ◆ Identifying a voyager Series Calculator

S08 – #11 of Math Review Series – Complex Numbers The solutions of third degree equations, electrical impedance, electro-magnetic fields, light beams, and the imaginary unit are all related to a numerical representation of two quantities we call a complex number.

That is it for this issue. I hope you enjoy it. If not, tell me!

Also tell me what you liked, and what you would like to read about.

X <> Y,

Richard

Email me at: hpsolve@hp.com or rjnelsoncf@cox.net

HP 48 One Minute Marvels No. 16 – Keyboard Decoding

One Minute Marvels, OMMs, are short, efficient, unusual, and fun HP 48 programs that may be entered into your machine in a minute or less. These programs were developed on the HP 48, but they will usually run on the HP 49 and HP 50 as well. Note the HP48 byte count is for the program only.

Keyboard keycode to sequential number (Joseph Horn)

The ZERO WAIT sequence is very powerful for use with screen (DISP) prompts because the machine waits for a key press in a low battery drain state. This is unlike KEY which leaves the machine running.

The routine below is useful to convert the A through X key keycodes into sequential numbers 1 through 24 with ‘K→S’.

```
‘K→S’ << 0 WAIT 10 MOD SWAP 10 / IP 1 - 6 * + IP >>
```

14 commands, 63.5 Bytes, # D8D7h.

TABLE I2 — “A” through “X” HP48 Keycodes

Key	In	Out	Key	In	Out	Key	In	Out	Key	In	Out	Key	In	Out	Key	In	Out
A	11.1	1	E	15.1	5	I	23.1	9	M	31.1	13	Q	35.1	17	U	43.1	21
B	12.1	2	F	16.1	6	J	24.1	10	N	32.1	14	R	36.1	18	V	44.1	22
C	13.1	3	G	21.1	7	K	25.1	11	O	33.1	15	S	41.1	19	W	45.1	23
D	14.1	4	H	22.1	8	L	26.1	12	P	34.1	16	T	42.1	20	X	46.1	24

‘K→S’ works for keys past the “X” key except it “counts” the ENTER key twice. It also “skips” keys after the three shift keys. The + key returns 53. The sequential number may be used to GET something from a list, SUB a character from a string, etc.

HP User Community News

Calculators and their users have been on the forefront of the “computer world” in many ways. Having multiple programs in memory and user networking are hardware and software examples. A recent issue of the IEEE Annals of the History of Computing, Volume 34 Number 3 July-September 2012 documents this nicely. A 12 page article starting on page 55 by Dejan Risanovic and Jelica Protic titled “Once Upon a Pocket: Programmable Calculators from the Late 1970s and Early 1980s and the Social Networks Around Them” illustrates the HPUC and HHCs that continue today. <http://www.computer.org/>

Identifying a Voyager Series Calculator

The HP Logos in the upper right corner of the older Voyager Series HP calculators, see Fig. 3, often fall off and because the model number is part of the HP Logo the machine is no longer easily identifiable.

Here is how you may identify each model. An example HP Logo and model number may be seen in Fig. 1. Later models such as the HP 12c platinum, etc. had text printed below the Logo area and identifying these models is usually not an issue. See Fig. 2.



Fig. 1 HP-15C Logo

Examine the upper right corner key and note what notations are printed for this key.



Fig. 2 –Newer HP voyager machines; e.g. 12c platinum, prestige, 25th Anniversary 12c

The key may have one, two, or three notations.

A yellow notation is above the key. The only model without a yellow notation is the HP-12C.

All five models have a white notation on the top of the key. This is the divide symbol and it is the same for all five, HP-10C, HP-11C, HP-12C, HP-15C, & HP-16C models.

Three models have a blue notation on the front sloping surface of the key. The HP-10C and HP-12C do not.

See the notations below. I used this technique while providing phone technical support at EduCALC many years ago. I found that this was the easiest way to help a user to identify the Voyager machine they found in a desk drawer without a Logo/model number. The power cells usually still worked.

HP-10C

X ≤ Y	Above key
÷	On key top
none	On key slope

HP-11C

X ≤ Y	Above key
÷	On key top
X < 0	On key slope

HP-12C

none	Above key
÷	On key top
none	On key slope

HP-15C

SOLVE	Above key
÷	On key top
X < Y	On key slope

HP-16C

XOR	Above key
÷	On key top
DBL ÷	On key slope

The HP-16C keyboard layout illustrates the overall appearance of a Voyager calculator.



Fig. 3 – Voyager Keyboard.

About the Editor

	<p>Richard J. Nelson, a long time HP Calculator enthusiast, was editor and publisher of <i>HP-65 Notes</i>, <i>The PPC Journal</i>, <i>The PPC Calculator Journal</i>, and the <i>CHHU Chronicle</i>. He has also had articles published in <i>HP65 Key Note</i> and <i>HP Key Notes</i>. As an Electronics Engineer turned technical writer Richard has published hundreds of articles discussing all aspects of HP Calculators. His work may be found on the Internet and the HCC websites at: bhuc.us. He proposed and published the PPC ROM and actively contributed to the UK HPCC book, <i>RCL 20</i>. His primary calculator interest is the User Interface. Richard may be reached at: rjnelsoncf@cox.net</p>
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Review complex numbers

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Complex Numbers- Part I

Richard J. Nelson

Introduction

What do solutions of third degree equations, electrical impedance, electromagnetic fields, light beams, and the imaginary unit have in common?

Mathematicians are always looking for better and more efficient ways to express relationships. One challenge that was solved at the turn of the 19 century was a means to express the negative roots of a third degree equation. The issue was expressing square root of a negative number e.g. $\sqrt{-64}$?

We answered this question in Math Review #10⁽¹⁾. Let's review this topic.



Fig. 1 - HP35s "i" key.

Remembering that a negative multiplied by a negative results in a positive, the square root of any real number, excluding zero, has two values \pm . The term often applied to these roots is the principle or positive root.

Let's use the definitional property of i ⁽²⁾.

$$i^2 = -1$$

In practical terms: $i = \sqrt{-1}$.

The square root of -1 has two values, $+i$ and $-i$. While these two values are the negatives of each other, they are algebraically the same.

The square root of -2 is expressed as $1.41421356237i$ which is an imaginary number⁽³⁾ and a particular case of complex numbers.

Imaginary numbers may be operated on in a similar manner as real numbers. Squaring an imaginary number is a negative number (zero excluded). The HP 35s has operations to perform complex arithmetic ($+$, $-$, \times , \div), complex trigonometry (\sin , \cos , \tan) and the mathematical functions $-z$, $1/z$, Z^{Z^2} , $\ln z$, and e^z - where z and z^2 are complex numbers⁽⁴⁾.

The square root of -2 is also expressed on more advanced calculators as a complex number which is an ordered pair of numbers shown as $(0, 1.41421356237)$. The i is assumed to be attached to the second number shown either before or after. The meaning is the same either way. Most complex number functions are found on advanced algebraic and graphing calculators.

Imaginary Numbers

i is a number and not a variable. i is not found on the real number line because i is not a real number.

Dealing with powers of i , you must remember four cases that repeat themselves in a cycle. See Tble 2.

Table 2 – Cyclic 4 Values of Powers of i

Negative power	Positive 1 st cycle	Positive cycle repeats
$i^{-4} = 1$	$i^0 = 1$	$i^4 = 1$
$i^{-3} = i$	$i^1 = i$	$i^5 = i$
$i^{-2} = -1$	$i^2 = -1$	$i^6 = -1$
$i^{-1} = -i$	$i^3 = -i$	$i^7 = -i$

Imaginary numbers require care in their use. Use the same rules you normally do. Here are some examples.

$$i^2 = \sqrt{-1} \cdot \sqrt{-1} = -1$$

$$i^3 = i^2 \cdot i = -1 \cdot i = -i$$

$$\sqrt{-7} = \sqrt{1 \cdot 7} = \sqrt{1} \cdot \sqrt{7} = i\sqrt{7}$$

$$\sqrt{-a} = \sqrt{1 \cdot a} = \sqrt{1} \cdot \sqrt{a} = i\sqrt{a}$$

Here is an example of multiplying square roots that is incorrect because $\sqrt{x}\sqrt{y} = \sqrt{xy}$ is only true if at least one of the two numbers (x & y) is positive. $i^2 = \sqrt{-1}\sqrt{-1} = \sqrt{(-1)(-1)} = \sqrt{1} = 1$

Formal definition of a complex number

Wikipedia⁽⁵⁾: “A complex number is a number that can be put in the form $a + bi$, where a and b are real numbers and i is called the imaginary unit, where $i^2 = -1$. In this expression, a is called the real part and b the imaginary part of the complex number. Complex numbers extend the idea of the one-dimensional number line to the two-dimensional complex plane by using the horizontal axis for the real part and the vertical axis for the imaginary part. The complex number $a + bi$ can be identified with the point (a, b) . A complex number whose real part is zero is said to be purely imaginary, whereas a complex number whose imaginary part is zero is a real number. In this way the complex numbers contain the ordinary real numbers while extending them in order to solve problems that cannot be solved with only real numbers.”

In effect representing a quantity with a complex number combines two parameters into one representation. In electronics, for example, DC current flows according to Ohms law, $I = \frac{E}{R}$. Several assumptions are made with this representation. One is that the resistance doesn't change with changes in voltage or current. Another is that the resistance is pure and doesn't contain any capacitance or inductance. These qualities add reactance to the circuit and AC applied to the same circuit will have the effect of shifting the current to be out of phase with the voltage. Representing these quantities with complex numbers allow calculations to be made similar to DC, but using AC, $I = \frac{E}{Z}$ where Z represents electrical (AC) impedance.

Complex numbers and HP calculators

The four high stack of HP calculators nicely facilitates the stack being used for complex number

calculations. Users of the early machines, starting with the HP-35A “programmed” key stroke sequences to perform complex number operations⁽⁶⁾. The first HP calculator to include complex number operations was the HP-15C introduced on July 1, 1982. CAS and algebraic machines are more likely to support complex number operations. Virtually every real number operation may be done with complex numbers.

Observations and Conclusions

Complex numbers are specialized numbers that include two parameters of a quantity into one “number.” The i operator is a vital part of this notation and it wasn’t until the late 1800’s that this idea allowed the taking of negative roots of even powered numbers, e.g. $\sqrt{-18}$. Several advanced HP calculators include complex number operations. Electrical engineers use the j operator instead of the i operator because i , I , is used for current. Part II will explore complex number calculations and other forms of complex numbers.

Notes - Complex Numbers

- (1) See [HP Solve Issue 27, page 53](#) for a review of *Calculating Numerical Roots*.
<http://h20331.www2.hp.com/hpsub/us/en/hp-solve-2012.html>
- (2) Electrical engineers will use j as the imaginary unit because i , or I , is used for electrical current.
- (3) The use of the term *imaginary number* is often confusing. Perhaps it was named to contrast with real numbers.
- (4) HP 35s training aids related to working with complex numbers may be found at:
Part I: http://h20331.www2.hp.com/Hpsub/downloads/35_16_Complex_Numbers_1.pdf
Part II: http://h20331.www2.hp.com/Hpsub/downloads/35_17_Complex_Numbers_2.pdf
- (5) See http://en.wikipedia.org/wiki/Complex_numbers for additional details.
- (6) See the HP-35 Math Pac © 1974 by HP. A similar version is the HP-45 Math Pac. Complex number operations described are: Add, subtract, multiply, divide, reciprocal, absolute value, square, square root, natural logarithm, exponential, number to a power, integral roots of complex number, complex number to a complex power, root of a complex number, pp 24 to 35. Complex trigonometric and hyperbolic functions; sine, cosine, tangent, cotangent, cosecant, secant, arc sine, arc cosine, arc tangent, arc cotangent, arc cosecant, arc secant, hyperbolic sine, hyperbolic cosine, hyperbolic tangent, hyperbolic cotangent, hyperbolic cosecant, hyperbolic secant, and their inverses, pp 36 to 54.

About the Author



Richard J. Nelson has written hundreds of articles on the subject of HP’s calculators. His first article was in the first issue of *HP 65 Notes* in June 1974. He became an RPN enthusiast with his first HP Calculator, the HP-35A, he received in the mail from HP on July 31, 1972. He remembered the HP-35A in a recent article that included previously unpublished information on this calculator. See <http://hhuc.us/2007/Remembering%20The%20HP35A.pdf> He has also had an article published on HP’s website; HP Calculator Firsts. See <http://h20331.www2.hp.com/Hpsub/cache/392617-0-0-225-121.html>.