

## 1

# Financial mathematics

## 1

**'The best job in the world!'** Tourism Queensland once ran a world-wide advertising campaign with this phrase as the headline. They received 34 000 applications!

The position of 'Island Caretaker' was created to promote the islands of the Great Barrier Reef to the world. The list of 'duties' for the successful applicant to undertake included swimming, snorkelling, exploring and keeping an online photo diary and blog of their experiences. Anyone wishing to apply for the position had to make a 60-second video that showcased their skills and creativity. After sifting through the thousands of videos that were submitted, a shortlist of 16 applicants was formed. After the interview process, Englishman Ben Southall was announced as the successful applicant of the 6-month contract worth

\$150 000. Ben's enthusiastic blogging convinced many that the ad's headline was not an exaggeration!

### Forum

What is your idea of 'The best job in the world'? Give some reasons for your answer. Is it more important to have a well-paying job or to do something you enjoy? If you had to make a 60-second video as a job application, what would you include in your 60 seconds?

### Why learn this?

How do you decide whether that discounted pair of jeans really is a bargain? How much money is the store making by selling them to you? How can you work out whether you have been paid the right amount of money for the work you have done? What is tax? How is it calculated? What is interest? How is it paid? What is credit? How does it work? The answers to these questions, and an understanding of the mathematics involved in earning, borrowing and spending money, are essential for all of us in our daily lives.

### After completing this chapter you will be able to:

- perform calculations with percentages
- calculate profit, loss, discount, mark-up, selling price and cost price
- calculate incomes earned under a variety of payment systems
- understand how tax works and calculate basic income tax rates and deductions
- perform simple interest calculations
- understand different methods of payment (such as credit cards) and perform calculations associated with them
- calculate typical costs of using resources such as electricity, gas and water
- calculate typical costs of using communications devices such as mobile phones.



## Curriculum links



### Number and Algebra

#### Money and financial mathematics

ACMNA211/VCMNA304

Solve problems involving simple interest

- understanding that financial decisions can be assisted by mathematical calculations

#### General capabilities

- Numeracy
- Literacy
- Critical and creative thinking
- Personal and social capability
- Information and communication technology (ICT) capability
- Ethical understanding

### Big ideas

- Financial mathematics is real-world mathematics that can be helpful in everyday life.
- Percentages can be used to calculate discounts, mark-ups and other changes to prices.
- There are different ways of earning an income, paying tax and spending money.
- Interest is earned by people or companies who lend money, and is paid by those who are borrowing.
- The costs of utilities and communications are calculated in specific mathematical ways.

### notes:

Notes area with horizontal lines for writing.

Sample pages

## Recall 1

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

- Convert the following fractions to decimals.
  - $\frac{17}{100}$
  - $\frac{57}{1000}$
  - $\frac{321}{100\,000}$
- Convert the following fractions to decimals. If the decimal is a recurring decimal, use the appropriate notation.
  - $\frac{16}{25}$
  - $\frac{9}{8}$
  - $\frac{23}{40}$
  - $\frac{1}{6}$
  - $\frac{10}{11}$
  - $\frac{5}{7}$
- Convert the following decimals to fractions in simplest form.
  - 0.79
  - 0.003
  - 0.014
  - 1.065
  - 10.140
  - 0.0902
- Evaluate the following.
  - $5.8 + 2.9$
  - $12.7 - 3.6$
  - $5.2 \times 1.5$
  - $6.2 \div 0.03$
- Convert the following fractions and decimals to percentages. (Round your answers to 2 decimal places, if necessary.)
  - $\frac{14}{20}$
  - 0.91
  - 0.032
  - $\frac{23}{28}$
- Write the following percentages as fractions in simplest form.
  - 85%
  - 22%
  - 6.7%
  - $\frac{1}{2}\%$
- Write the following percentages in decimal form.
  - 49%
  - 30%
  - 2.5%
  - 370%
- Calculate the following:
  - $\frac{5}{6}$  of 90 m
  - $\frac{3}{4}$  of 105 kg
  - $\frac{11}{10}$  of \$25 600
- Find:
  - 5% of 20 m
  - 10% of 63.5 km
  - 22% of 18.1 cm

## Exploration Task

You can download this activity from the eBook or the Pearson Places website.

### Get rich quick?

In this activity, you will learn how to choose between investments with different compounding periods and dividends.



## Percentages review



The ability to work with percentages, fractions and decimals is important for many daily calculations and estimations. It is helpful to know the fraction, decimal and percentage equivalents of these common values.

Fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{20}$	$\frac{1}{100}$
Decimal	0.5	0.25	0.75	0.3	0.6	0.1	0.2	0.05	0.01
Percentage	50%	25%	75%	$33\frac{1}{3}\%$	$66\frac{2}{3}\%$	10%	20%	5%	1%

### Finding a percentage of an amount

'Per cent' means 'out of 100' or 'for every hundred'. Percentages are just fractions with a denominator of 100; for example,  $40\% = \frac{40}{100} = \frac{2}{5}$ .

If you know some common fraction and percentage equivalents and how to find them, you will be able to perform many percentage calculations mentally (see Method 1 below). Method 2 involves writing the percentage as a fraction or decimal and multiplying by the amount. This is a good 'pen and paper' method.

### Worked example 1

**W.E. 1**

Calculate the percentages of the given amounts:

- 35% of \$62
- 27.5% of \$300

### Method 1: Use basic percentages

#### Thinking

- Break down the percentage amount into basic percentages that are easy to calculate, such as 25%, 10% or 5%.
- Calculate each of these basic percentages. (Here, 10% is found by dividing by 10, and 5% by halving 10%.)
- Multiply and/or add the values of each of the basic percentages to find the required percentage value.
- Write the answer.

#### Working

- $$35\% = 30\% + 5\%$$

$$= 3 \times 10\% + 5\%$$
- |  |  |
|--|--|
| $10\% \text{ of } 62$ $= 62 \div 10$ $= 6.2$ | $5\% \text{ of } 62$ $= (10\% \text{ of } 62) \div 2$ $= 6.2 \div 2$ $= 3.1$ |
|--|--|
- $$3 \times 10\% + 5\%$$

$$= (3 \times 6.2) + 3.1$$

$$= 18.6 + 3.1$$

$$= 21.7$$
- $$35\% \text{ of } \$62 = \$21.70$$



## Resources

### Recall Worksheets

- R1.1 Converting fractions with a power of 10 as the denominator to decimals
- R1.2 Converting fractions to decimals
- R1.3 Converting decimals to fractions
- R1.4 Operations with decimals
- R1.5 Converting fractions and decimals to percentages
- R1.6 Writing percentages as fractions
- R1.7 Writing percentages in decimal form
- R1.8 Finding a fraction of an amount
- R1.9 Finding a percentage of an amount

### Exploration Task

- Get rich quick?

### Lightbook Starter

- Before you begin 1

## Answers

### Recall 1

- 1 (a) 0.17 (b) 0.057  
(c) 0.003 21
- 2 (a) 0.64 (b) 1.125  
(c) 0.575 (d) 0.1 $\dot{6}$   
(e) 0.9 $\dot{0}$  (f) 0.714 285
- 3 (a)  $\frac{79}{100}$  (b)  $\frac{3}{1000}$  (c)  $\frac{7}{500}$   
(d)  $\frac{213}{200}$  (e)  $\frac{507}{50}$  (f)  $\frac{451}{5000}$
- 4 (a) 8.7 (b) 9.1 (c) 7.8  
(d) 206. $\dot{6}$
- 5 (a) 70% (b) 91% (c) 3.2%  
(d) 82.14%
- 6 (a)  $\frac{17}{20}$  (b)  $\frac{11}{50}$  (c)  $\frac{67}{1000}$   
(d)  $\frac{1}{200}$
- 7 (a) 0.49 (b) 0.3  
(c) 0.025 (d) 3.7
- 8 (a) 75 m (b) 78.75 kg  
(c) \$28 160
- 9 (a) 1 m (b) 6.35 km  
(c) 3.982 cm

## 1.1

### Resources

#### eWorked examples

- Finding a percentage of an amount
- Increasing and decreasing by a given percentage
- Writing one amount as a percentage of another

#### Lessons

- Percentages (1)
- Percentages (2)

#### Lightbook Starter

- Check in 1.1

#### Appendices

- 1A Fraction, decimal and percentage equivalence cards
- 1B Ordering fractions and percentages cards

## Recap

Question	Answer
1 Write each of the following fractions as percentages by first finding equivalent fractions with a denominator of 100.	
(a) $\frac{7}{10}$	(a) $\frac{70}{100}$ , 70%
(b) $\frac{2}{5}$	(b) $\frac{40}{100}$ , 40%
(c) $\frac{11}{20}$	(c) $\frac{55}{100}$ , 55%
(d) $\frac{19}{25}$	(d) $\frac{76}{100}$ , 76%
2 Write the following fractions as percentages, using a different method. Write any remainders as decimals.	
(a) $\frac{5}{8}$	(a) 62.5%
(b) $\frac{32}{40}$	(b) 80%
(c) $\frac{10}{15}$	(c) 66. $\dot{6}$ %
(d) $\frac{100}{90}$	(d) 111. $\dot{1}$ %
3 Write the following decimals as percentages.	
(a) 0.9	(a) 90%
(b) 0.04	(b) 4%
(c) 1.052	(c) 105.2%

(cont.)

1.1

- |  |  |
|--|--|
| <p>(b) 1 Break down the percentage amount into basic percentages that are easy to calculate.</p> <p>2 Calculate each of these basic percentages. (Here, 25% is found by dividing by 4 and 2.5% by dividing 25% by 10.)</p> <p>3 Multiply and/or add the values of each of the basic percentages to find the required value.</p> <p>4 Write the answer.</p> | <p>(b) <math>27.5\% = 25\% + 2.5\%</math></p> $\begin{array}{l} 25\% \text{ of } 300 \\ = 300 \div 4 \\ = 300 \div 2 \div 2 \\ = 150 \div 2 \\ = 75 \end{array}$ $\begin{array}{l} 2.5\% = 25\% \div 10 \\ = 75 \div 10 \\ = 7.5 \end{array}$ $\begin{array}{l} 25\% + 2.5\% \\ = 75 + 7.5 \\ = 82.5 \end{array}$ <p><math>27.5\% \text{ of } \\$300 = \\$82.50</math></p> |
|--|--|

### Method 2: Fraction or decimal multiplication

#### Thinking

- (a) 1 Use the fact that 'of' is equivalent to 'x' to rewrite the sentence as a mathematical one.
- 2 Convert the percentage to a fraction by dividing by 100.
- 3 Do the multiplication. Simplifying the fractions first makes the multiplication easier. (Here, divide by common factors of 5 and then 2.)
- 4 Write the answer.

#### Working

(a)  $35\% \text{ of } 62$   
 $= 35\% \times 62$

$$= \frac{35}{100} \times 62$$

$$= \frac{7 \cancel{35}^7 \times \cancel{62}^{31}}{10 \cancel{20}^2 \times 1}$$

$$= \frac{7}{10} \times \frac{31}{1}$$

$$= \frac{217}{10}$$

$$= 21.7$$

$35\% \text{ of } \$62 = \$21.70$

- (b) 1 Use the fact that 'of' is equivalent to 'x' to rewrite the sentence as a mathematical one.
- 2 Convert the percentage to a fraction by dividing by 100.
- 3 Do the multiplication. Simplifying the fractions first makes the multiplication easier. (Here, divide by a common factor of 100.)
- 4 Write the answer.

(b)  $27.5\% \text{ of } 300$   
 $= 27.5\% \times 300$

$$= \frac{27.5}{100} \times \frac{300}{1}$$

$$= \frac{27.5 \times 3 \cancel{00}^3}{1 \cancel{100}^1}$$

$$= 27.5 \times 3$$

$$= 82.5$$

$27.5\% \text{ of } \$300 = \$82.50$

### Increasing or decreasing by a given percentage

A percentage can also represent a change in a quantity. You can calculate a percentage increase or decrease in a simple 'one-step' calculation if you use the fact that the original amount represents 100%.

#### Worked example 2

W.E. 2

Increase or decrease the following amounts by the given percentages. Round decimal answers to 2 decimal places.

- (a) Increase \$420 by 65%      (b) Decrease 87 kg by 13.6%

#### Thinking

- (a) 1 Add the percentage increase to 100%. Convert the new percentage to a decimal.
- 2 This decimal represents the scale factor. Multiply it by the original amount to find the new amount. (Use your calculator.)
- 3 Write the answer.
- (b) 1 Subtract the percentage decrease from 100%. Write this new percentage as a decimal.
- 2 This decimal represents the scale factor. Multiply it by the original amount to find the new amount. (Use your calculator.)
- 3 Write the answer.

#### Working

(a)  $(100 + 65)\% = 165\%$   
 $= 1.65$

$$1.65 \times 420 = 693$$

$\$420 \text{ increased by } 65\% \text{ is } \$693.$

(b)  $(100 - 13.6)\% = 86.4\%$   
 $= 0.864$

$$0.864 \times 87 = 75.168$$

$87 \text{ kg decreased by } 13.6\% \text{ is } 75.17 \text{ kg.}$

### Writing one amount as a percentage of another

To write one amount as a percentage of another:

- 1 Make sure both amounts are the same type, or measured in the same units. Convert units if necessary.
- 2 Write one amount as a fraction of the other.
- 3 Convert this fraction into a percentage by multiplying by 100%.

## Recap (cont.)

Question	Answer
<p>4 Write the following percentages as fractions in simplest form.</p> <p>(a) 35%</p> <p>(b) 56%</p> <p>(c) 43.5%</p>	<p>(a) <math>\frac{7}{20}</math></p> <p>(b) <math>\frac{14}{25}</math></p> <p>(c) <math>\frac{87}{200}</math></p>
<p>5 Write the following percentages as decimals.</p> <p>(a) 92.4%</p> <p>(b) 5.9%</p> <p>(c) 213%</p>	<p>(a) 0.924</p> <p>(b) 0.059</p> <p>(c) 2.13</p>

### Suggested examples

- 1 A \$64 pair of sunglasses is advertised at a 25% discount. What is the dollar value of this saving?

Answer:

Method 1: Use basic percentages.  $25\% = \frac{1}{4}$ , so you can find 25% by dividing by 4.

$$\begin{aligned} \$64 \div 4 &= \$64 \div 2 \div 2 \\ &= \$32 \div 2 \\ &= \$16 \end{aligned}$$

The value of the saving is \$16.

Method 2: Fraction or decimal multiplication

$$\begin{aligned} 25\% \text{ of } \$64 &= \frac{25}{100} \times \frac{64}{1} \text{ or } 0.25 \times 64 \\ &= \frac{1}{4} \times \frac{1664}{1} \\ &= 16 \end{aligned}$$

The value of the saving is \$16.

- 2 Timothy's parents decide to increase his fortnightly allowance of \$30 by 20%. Find the new amount of the allowance he will receive.

Answer:

The original amount represents 100%. Add the percentage increase to this amount  $100\% + 20\% = 120\% = 1.2$

The decimal is a scale factor. Multiply it by the original amount to find the new amount.

$$1.2 \times 30 = \$36$$

Timothy will now receive \$36 fortnightly.

- 3 Alexandra scored 11 netball goals from a total of 16 attempts during her first match of the season. She is aiming for an accuracy of 75%. What was her percentage accuracy for this match and did her first game meet her expectations?

Answer:

Create a fraction with the denominator as the 'whole amount' and the numerator is the 'part amount'. Convert to a percentage by multiplying by 100.

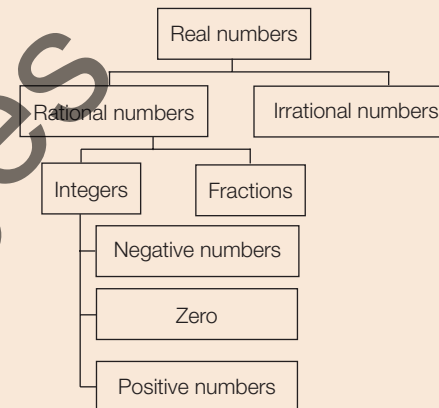
$$\begin{aligned} \frac{11}{16} \times \frac{100\%}{1} \\ &= \frac{11}{4} \times \frac{25\%}{1} \\ &= \frac{275\%}{4} \\ &= 68.75\% \end{aligned}$$

$\therefore$  Her accuracy was 68.75%, which is less than 75%.

## Teaching strategies

### Equivalent representations of fractions, decimals and percentages

- Fractions are a way to express numbers in their exact form. Decimals are often approximations of numbers. For example, the exact probability of choosing an even number from the first 7 integers is  $\frac{3}{7}$ , but a decimal approximation of 0.43 might be used.
- Rational numbers are those that can be expressed in the form  $\frac{a}{b}$  ( $b \neq 0$ ), that is, in fractional form. Integers (whole numbers), can simply be written with  $b = 1$  (i.e. with 1 as the denominator). Students can be shown this classification chart:



- Rational numbers can be expressed in decimal form by dividing the numerator by the denominator. In some cases, the division will result in a terminating decimal such as  $\frac{3}{4} = 0.75$ . Others repeat in a recurring pattern such as  $\frac{1}{3} = 0.333\dots$ . In many cases, the decimal can be rounded off to a set number of decimal places and the approximation can be used for practical purposes. Irrational numbers are non-terminating, non-recurring decimals (e.g.  $\sqrt{2} = 1.414\ 213\ 5\dots$ ).
- Percentage means 'per hundred'. Percentages can be thought of as the numerators of fractions where the denominator is 100. A whole amount is called 100 per cent, written as 100%. Percentages are easily compared because the denominator is always 100. Fractions are more difficult to compare if they have different denominators.

1.1

## Worked example 3

W.E. 3

Round your answers to the following questions to 1 decimal place.

- (a) Tyler kicked 13 goals from 18 shots at goal. Calculate his kicking accuracy as a percentage.  
 (b) A packet of 'Chocky' biscuits is on sale for \$2.35, a saving of 57c. Write the saving as a percentage of the original price.

## Method 1: Fraction multiplication

## Thinking

## Working

- |  |  |
|--|--|
| <p>(a) 1 Write the number as a fraction of the total. Multiply this fraction by 100%.</p> <p>2 Simplify the multiplication by cancelling common factors first.</p> <p>3 Write the answer, rounded to the specified number of decimal places, in a sentence.</p>  | <p>(a) <math>\frac{13}{18} \times \frac{100\%}{1}</math></p> <p><math>= \frac{13}{9} \times \frac{50 \times 100}{1}</math></p> <p><math>= \frac{650}{9}</math></p> <p><math>= 72\frac{2}{9}\%</math> or 72.2%</p> <p>Tyler's kicking accuracy is 72.2% (1 d.p.).</p> |
| <p>(b) 1 Find the original price by adding the saving back on to the sale price.</p> <p>2 Write the saving as a fraction of the original price, making sure both amounts are expressed in the same units (dollars). Multiply this fraction by 100%.</p> <p>3 Write the answer, rounded to the specified number of decimal places, in a sentence.</p> | <p>(b) <math>\\$2.35 + \\$0.57 = \\$2.92</math></p> <p><math>\frac{0.57}{2.92} \times \frac{100\%}{1}</math></p> <p><math>= \frac{57}{292}\%</math></p> <p><math>= 19.52054\dots\%</math></p> <p>The saving is 19.5% of the original price (1 d.p.).</p>             |

## Method 2: Convert to a decimal, then multiply

## Thinking

## Working

- |  |   |
|--|---|
| <p>(a) 1 Write the number as a fraction of the total. Convert this fraction to a decimal by dividing.</p> <p>2 Multiply the decimal by 100%.</p> <p>3 Write your answer in a sentence (rounded).</p> | <p>(a) <math>\frac{13}{18} = 13 \div 18 = 0.7222\dots</math></p> <p><math>0.7222\dots \times 100\% = 72.2222\dots\%</math></p> <p>Tyler's kicking accuracy is 72.2% (1 d.p.).</p> |
|--|---|

- |   |   |
|---|---|
| <p>(b) 1 Find the original price by adding the saving back on to the sale price.</p> <p>2 Write the saving as a fraction of the original price, making sure both amounts are expressed in the same units (dollars).</p> <p>3 Convert this fraction to a decimal by dividing.</p> <p>4 Multiply the decimal by 100%.</p> <p>5 Write your answer in a sentence (rounded).</p> | <p>(b) <math>\\$2.35 + \\$0.57 = \\$2.92</math></p> <p><math>\frac{0.57}{2.92}</math></p> <p><math>\frac{0.57}{2.92} = 0.57 \div 2.92 = 0.1952\dots</math></p> <p><math>0.1952\dots \times 100\% = 19.52\dots\%</math></p> <p>The saving is 19.5% of the original price (1 d.p.).</p> |
|---|---|

## Rounding errors

Many financial applications of percentages use decimals instead of fractions. For practical purposes, the rounded approximation of a recurring decimal is often used in a calculation. For example,  $\frac{2}{9}$  expressed as a decimal is 0.833333... Rounded to 1 decimal place, it is 0.8. If this was rounded to 2 decimal places, it would be 0.83. A question will usually tell you how many decimal places to round your answer to.

As soon as you use a rounded value, you introduce error into your answer. To minimise this error you should round as late as possible in a calculation or use fractions for as long as you can before converting to a decimal and rounding. If you give a rounded answer, then you should state this (e.g. if you have rounded to 2 decimal places, write '2 d.p.').

## Fractions and decimals on the calculator

If you are working with decimals on your calculator, keep all the decimal places in the calculator display and only round your final answer.

Most scientific calculators can convert between fractions, decimals and mixed numbers, using a key that is usually labelled  $\frac{\square}{\square}$  or  $\frac{a}{b/c}$ .

## 1.1 Percentages review

## Navigator

- |   |  |   |                   |
|---|--|---|-------------------|
| 1 (a-f, j-l), 2 (a, b), 3 (a, c, e),<br>4 (a-d), 5, 6, 7, 9, 10, 11, 15, 17 | 1 (g-l, m-o), 2 (a, b, d, e), 3,<br>4 (a-d), 5, 6, 7, 8, 11, 12, 14, 15,<br>17, 18 | 1 (j-l, m-o), 2 (c, d, e), 4 (e-h), 7,<br>8, 12, 13, 14, 16, 17, 18 | Answers<br>p. 628 |
|---|--|---|-------------------|

## Fluency

- 1 Calculate the percentages of the given amounts. If necessary, round answers to 2 decimal places.
- |                                  |                              |                                  |
|----------------------------------|------------------------------|----------------------------------|
| (a) 50% of \$200                 | (b) 25% of \$400             | (c) 10% of \$75                  |
| (d) 20% of \$480                 | (e) 100% of \$55             | (f) 10% of \$38                  |
| (g) 25% of \$40                  | (h) 45% of \$50              | (i) 30% of \$150                 |
| (j) 11% of 60 L                  | (k) 66% of 400 m             | (l) 77% of 35 kg                 |
| (m) 0.5% of \$3500               | (n) 12.5% of 1500 m          | (o) 7.2% of 15 L                 |
| (p) $5\frac{3}{4}\%$ of \$17 000 | (q) $6\frac{1}{5}\%$ of \$90 | (r) $\frac{2}{3}\%$ of \$200 000 |

W.E. 1

