JEE (Advanced)—2017
Mathematics Paper-II

SECTION I

- This section contains **Seven** questions.
- Each question has **Four** options (a), (b), (c) and (d). **Only One** of these options is correct.
- For each question, marks will be awarded in one of the following categories:

Full marks	:	+ 3	If only the bubble corresponding to the correct option is darkened.
Zero marks	:	0	If none of the bubbles is darkened
Negative marks	:	–1	In all other cases

1. How many 3 × 3 matrices M with entries from {0, 1, 2} are there, for which the sum of the diagonal entries of M^TM is 5?

(a) 135 (b) 198

(c) 162 (d) 126

2. Three randomly chosen nonnegative integers x, y and z are found to satisfy the equation $x + y + z = 10$. Then the probability that z is even, is

(a) $\frac{5}{11}$ (b) $\frac{6}{11}$

(c) $\frac{1}{2}$ (d) $\frac{36}{55}$

3. The equation of the plane passing through the point (1, 1, 1) and perpendicular to the planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$, is

(a) $14x + 2y - 15z = 1$

(b) $14x - 2y + 15z = 27$

(c) $-14x + 2y + 15z = 3$

(d) $14x + 2y + 15z = 31$

4. If $y = y(x)$ satisfies the differential equation

$8\sqrt{x}\left(\sqrt{9+\sqrt{x}}\right)dy = \left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right)^{-1} dx,\ x > 0$

and $y(0) = \sqrt{7}$, then $y(256) =$

(a) 3 (b) 16

(c) 9 (d) 80

5. If $f : \mathbf{R} \to \mathbf{R}$ is a twice differentiable function such that $f''(x) > 0$ for all $x \in \mathbf{R}$, and $f\left(\frac{1}{2}\right) = \frac{1}{2}$, $f(1) = 1$, then

(a) $f'(1) \le 0$

(b) $\frac{1}{2} < f'(1) \le 1$

(c) $f'(1) > 1$

(d) $0 < f'(1) \le \frac{1}{2}$

6. Let $S = \{1, 2, 3, \ldots, 9\}$. For $k = 1, 2, \ldots, 5$, let N_k be the number of subsets of S, each containing five elements out of which exactly k are odd. Then $N_1 + N_2 + N_3 + N_4 + N_5 =$

(a) 126 (b) 252

(c) 210 (d) 125

7. Let O be the origin and let PQR be an arbitrary triangle. The point S is such that

$$\mathbf{OP}\cdot\mathbf{OQ} + \mathbf{OR}\cdot\mathbf{OS} = \mathbf{OR}\cdot\mathbf{OP} + \mathbf{OQ}\cdot\mathbf{OS} = \mathbf{OQ}\cdot\mathbf{OR} + \mathbf{OP}\cdot\mathbf{OS}$$

Then triangle PQR has S as its

(a) circumcentre (b) incentre

(c) centroid (d) orthocenter

SECTION – 2

- This section contains **Seven** questions.
- Each question has **Four** options (a), (b), (c) and (d). **One or More than One** of these four options is(are) correct.
- For each question, marks will be awarded in one of the following categories:

Full marks	:	+ 4	If only the bubble(s) corresponding to all the correct option (s) is (are) darkened.
Partial marks	:	+ 1	For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened

Zero marks	:	0	If none of the bubbles is darkened
Negative marks	:	– 2	In all other cases

- For example, if (a), (c) and (d) are all the correct options for a question, darkening all these three will get +4 marks; darkening only (a) and (d) will get +2 marks; and darkening (a) and (b) will get –2 marks, as a wrong option is also darkened.

8. If $I = \sum_{k=1}^{98} \int_k^{k+1} \frac{k+1}{x(x+1)}\, dx$, then

(a) $I < \frac{49}{50}$ (b) $I > \frac{49}{50}$

(c) $I < \log_e 99$ (d) $I > \log_e 99$

9. If the line $x = \alpha$ divides the area of region $R = \{(x, y) \in \mathbf{R}^2 : x^3 \le y \le x,\ 0 \le x \le 1\}$ into two equal parts, then

(a) $\alpha^4 + 4\alpha^2 - 1 = 0$

(b) $0 < \alpha \le \frac{1}{2}$

(c) $2\alpha^4 - 4\alpha^2 + 1 = 0$

(d) $\frac{1}{2} < \alpha < 1$

10. If $g(x) = \int_{\sin x}^{\sin(2x)} \sin^{-1}(t)\,dt$, then

(a) $g'\left(\frac{\pi}{2}\right) = 2\pi$

(b) $g'\left(-\frac{\pi}{2}\right) = 2\pi$

(c) $g'\left(\frac{\pi}{2}\right) = -2\pi$

(d) $g'\left(-\frac{\pi}{2}\right) = -2\pi$

11. If $f(x) = \begin{vmatrix} \cos(2x) & \cos(2x) & \sin(2x) \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{vmatrix}$, then

(a) $f(x)$ attains its maximum at $x = 0$

(b) $f(x)$ attains its minimum at $x = 0$

(c) $f'(x) = 0$ at exactly three points in $(-\pi, \pi)$

(d) $f'(x) = 0$ at more than three points in $(-\pi, \pi)$

12. Let α and β be nonzero real numbers such that $2(\cos\beta - \cos\alpha) + \cos\alpha\cos\beta = 1$. Then which of the following is/are true?

(a) $\tan\left(\frac{\alpha}{2}\right) + \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$

(b) $\tan\left(\frac{\alpha}{2}\right) - \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$

(c) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) - \tan\left(\frac{\beta}{2}\right) = 0$

(d) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) + \tan\left(\frac{\beta}{2}\right) = 0$

13. Let $f(x) = \frac{1-x(1\quad|1-x|)}{|1-\quad|}\cos\left(\frac{1}{1-x}\right)$, for $x \ne 1$. Then

(a) $\lim_{x\to 1^-} f(x)$ does not exist

(b) $\lim_{x\to 1^-} f(x) = 0$

(c) $\lim_{x\to 1^+} f(x) = 0$

(d) $\lim_{x\to 1^+} f(x)$ does not exist

14. If $f : \mathbf{R} \to \mathbf{R}$ is a differentiable function such that $f'(x) > 2f(x)$ for all $x \in \mathbf{R}$, and $f(0) = 1$, then

(a) $f'(x) < e^{2x}$ in $(0, \infty)$

(b) $f(x)$ is increasing in $(0, \infty)$

(c) $f(x)$ is decreasing in $(0, \infty)$

(d) $f(x) > e^{2x}$ in $(0, \infty)$

SECTION – 3

- This section contains **Two** paragraphs.
- Based on each paragraph, there are **Two** questions.
- Each questions has **Four** options (a), (b), (c) and (d). **Only One** of these four options is correct.
- For each question, marks will be awarded in one of the following categories:

Full marks	:	+3	If only the bubble corresponding to the correct option is darkened.
Zero marks	:	0	In all other cases

Paragraph-1

Let O be the origin, and **OX, OY, OZ** be three unit vectors in the directions of the sides **QR, RP, PQ** respectively, of a triangle PQR.

15. If the triangle PQR varies, then the minimum value of $\cos(P + Q) + \cos(Q + R) + \cos(R + P)$ is

(a) $\frac{3}{2}$ (b) $\frac{5}{3}$

(c) $-\frac{5}{3}$ (d) $-\frac{3}{2}$

16. $|$ **OX** $\times$ **OY**$|$ =

(a) $\sin(P + R)$ (b) $\sin(Q + R)$

(c) $\sin(P + Q)$ (d) $\sin 2R$

Paragraph-2

Let p, q be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$, where $\alpha \neq \beta$. For $n = 0, 1, 2,...,$ let $\alpha_n = p\alpha^n + q\beta^n$.

FACT : If a and b are rational numbers and $a + b\sqrt{5} = 0$, then $a = 0 = b$.

17. If $a_4 = 28$, then $p + 2q =$

(a) 12 (b) 14

(c) 7 (d) 21

18. $a_{12} =$

(a) $a_{11} + 2a_{10}$ (b) $a_{11} - a_{10}$

(c) $2a_{11} + a_{10}$ (d) $a_{11} + a_{10}$

Answers

Section-1

1. (b) **2.** (b) **3.** (d)
4. (a) **5.** (c) **6.** (a)
7. (d)

Section-2

8. (b), (c) **9.** (c), (d)
10. None of the given answer is correct
11. (a), (c) **12.** (a), (b) **13.** (b), (d)
14. (b), (d)

Section-3

15. (d) **16.** (c) **17.** (a)
18. (d)

Hints and Solutions

1. Let $M = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$

$$M^TM = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

Sum of the diagonal elements of M^TM is

$d = a_1^2 + a_2^2 + a_3^2 + b_1^2 + b_2^2 + b_3^2 + c_1^2 + c_2^2 + c_3^2$

where $a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3 \in \{0, 1, 2\}$

Now, $d = 5$, if exactly 5 of

$a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3$ are 1's and rest 4 of them are zeros

This 5 numbers can be chosen in ${}^9C_5 = 126$ ways

Also, $d = 5$, if exactly one of

$a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3$ is 2 and exactly one of it is 1.

We can choose two numbers in ${}^9C_2$ and arrange them in two ways, that is, in $({}^9C_2)(2) = 72$ ways

Hence, the number of ways is 198.

2. Number of non-negative integral solutions of $x + y + z = 10$ is ${}^{12}C_2 = 66$.

We now count the number of ways in which z is even.

For $0 \leq k \leq 5$,

$$x + y = 10 - 2k$$

has exactly $(10 - 2k) + 1$ non-negative integral solutions

$\therefore$ number of ways in which z is even is

$$\sum(11 - 2\ ) = 6^2 = 36.$$

Thus, probability of required event

$= \frac{36}{66} = \frac{6}{11}$

3. Equation of any plane through (1, 1, 1) is

$a(x - 1) + b(y - 1) + c(z - 1) = 0$ .....(i)

As (i) is perpendicular to the given planes

$$2a + b - 2c = 0$$
$$3a - 6b - 2c = 0$$

$$\Rightarrow \quad \frac{a}{-2-12} = \frac{-b}{-4+6} = \frac{c}{-12-3}$$

or $$\frac{a}{14} = \frac{b}{2} = \frac{c}{15}$$

Thus, equation of required plane is

$$14(x - 1) + 2(y - 1) + 15(z - 1) = 0$$

or $$14x + 2y + 15z = 31$$

4. Write the differential equation as

$$\frac{dy}{dx} = \frac{1}{8\sqrt{x}\sqrt{9+\sqrt{x}}\sqrt{4+\sqrt{9+\sqrt{x}}}} = f(x) \text{ (say)}$$

$\Rightarrow \quad y = \int f(x)dx$

Put $\quad \sqrt{4+\sqrt{9+\sqrt{x}}} = t$

$\Rightarrow \quad \sqrt{9+\sqrt{x}} = t^2 - 4$

$\Rightarrow \quad 2tdt = \dfrac{1}{2\sqrt{9+\sqrt{x}}} \cdot \dfrac{1}{2\sqrt{x}} dx$

$\therefore \quad y = \int dt = t + C$

$= \sqrt{4+\sqrt{9+\sqrt{x}}} + C$

Now, $\quad \sqrt{7} = y(0) = \sqrt{4+\sqrt{9+0}} + C$

$\Rightarrow \quad C = 0.$

$\therefore \quad y(256) = \sqrt{4+\sqrt{9+\sqrt{256}}}$

$= \sqrt{4+\sqrt{9+16}}$

$= \sqrt{4+5} = 3$

5. As $f''(x) > 0 \ \forall \ x \in \mathbf{R}$,

f' is a strictly increasing function on **R**. By the first mean value theorem there exists some $\alpha \in (1/2, 1)$ such that

$\dfrac{f(1) - f(1/2)}{1 - 1/2} = f'(\alpha)$

$\Rightarrow \quad f'(\alpha) = 1$ for some $\alpha \in (1/2, 1)$

As f' is strictly increasing on **R**,

$f'(x) > f'(\alpha) \ \forall \ x > \alpha$

$\Rightarrow \quad f'(1) > 1 \qquad [\because 1 > \alpha]$

6. $N_k = ({}^5C_k)({}^4C_{5-k})$

$\Rightarrow N_1 + N_2 + N_3 + N_4 + N_5$

$= (5)(1) + (10)(4) + (10)(6) + (5)(4) + (1)(1)$

$= 126$

7. Let **OP = p, OQ = q, OR = r, OS = s**

We are given

$\mathbf{p}\cdot\mathbf{q} + \mathbf{r}\cdot\mathbf{s} = \mathbf{r}\cdot\mathbf{p} + \mathbf{q}\cdot\mathbf{s} = \mathbf{q}\cdot\mathbf{r} + \mathbf{p}\cdot\mathbf{s}$

From first two expressions,

$(\mathbf{r} - \mathbf{q})\cdot\mathbf{p} + (\mathbf{q} - \mathbf{r})\cdot\mathbf{s} = 0$

$\Rightarrow \quad (\mathbf{r} - \mathbf{q}) \cdot (\mathbf{p} - \mathbf{s}) = 0$

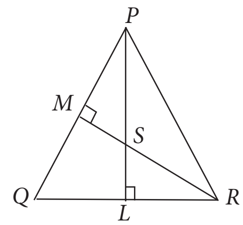
$\Rightarrow \quad (\mathbf{QR})\cdot(\mathbf{SP}) = 0 \Rightarrow \mathbf{SP} \perp \mathbf{QR} \quad ...(1)$

Similarly, from last two expressions,

$\Rightarrow \quad (\mathbf{PQ})\cdot(\mathbf{SR}) = 0 \Rightarrow \mathbf{SR} \perp \mathbf{PQ} \quad ...(2)$

From (1), (2) we get S is orthocentre of ΔPQR.

See figure in the next column.



8. For $k < x < k + 1$, $k \in \mathbf{N}$

$\dfrac{1}{x+1} < \dfrac{1}{k+1} < \dfrac{1}{x} < \dfrac{1}{k}$

$\Rightarrow \quad \dfrac{k+1}{x+1} < 1 \Rightarrow \dfrac{k+1}{(x+1)x} < \dfrac{1}{x}$

$\Rightarrow \quad \int_k^{k+1} \dfrac{k+1}{(x+1)x}dx < \int_k^{k+1} \dfrac{1}{x}dx = \ln(k+1) - \ln k$

$\Rightarrow \quad \sum_{k=1}^{98} \int_k^{k+1} \dfrac{k+1}{(x+1)x}dx < \ln(99) - \ln(1)$

$= \text{In}(99)$

Next, for $k < x < k + 1$, $1 \le k \le 98$

$\dfrac{k+1}{x(x+1)} > \dfrac{k+1}{(k+1)(x+1)} = \dfrac{1}{x+1} > \dfrac{1}{100}$

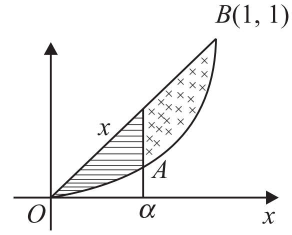
$\Rightarrow \quad \int_k^{k+1} \dfrac{k+1}{x(x+1)}dx > \dfrac{1}{100}$

$\Rightarrow \quad \sum_{k=1}^{98} \int_k^{k+1} \dfrac{k+1}{x(x+1)}dx > \dfrac{98}{100} = \dfrac{49}{50}$

9. Let $\quad A_1 = \int_0^{\alpha} (x - x^3)dx$

$A_2 = \int_{\alpha}^{1} (x - x^3)dx$

Now, $A_1 = A_2$



and $\quad A_1 + A_2 = \int_0^1 (x - x^3)dx = \dfrac{1}{2} - \dfrac{1}{4} = \dfrac{1}{4}$

$\therefore \quad 2A_1 = \dfrac{1}{4} \Rightarrow A_1 = \dfrac{1}{8}$

$\Rightarrow \quad \int_0^{\alpha} (x - x^3)dx = \dfrac{1}{8}$

$\Rightarrow \quad \dfrac{1}{2}\alpha^2 - \dfrac{1}{4}\alpha^4 = \dfrac{1}{8}$

$\Rightarrow \quad 4\alpha^2 - 2\alpha^4 = 1$

$\Rightarrow \quad 2\alpha^4 - 4\alpha^2 + 1 = 0$

Let $f(x) = 2x^4 - 4x^2 + 1$

We have

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{16}\right) - 4\left(\frac{1}{4}\right) + 1 = \frac{1}{8} > 0$$

and $\quad f(1) = 2 - 4 + 1 = -1 < 0$

$\Rightarrow \quad f(x) = 0$ has a root in $(1/2, 1)$

We have $f'(x) = 8x^3 - 8x$

$= \quad 8x(x^2 - 1) < 0 \quad$ for $\quad 0 < x < 1/2$

$\Rightarrow \quad f(x)$ decrease in $(0, 1/2]$

Since $f(1/2) = 1/4 > 0$

$\Rightarrow f(x) > 0$ for $0 < x \le 1/2$.

10. $g'(x) = 2\cos(2x)\sin^{-1}(\sin 2x) - \cos x \sin^{-1}(\sin x)$

$$g'\left(-\frac{\pi}{2}\right) = 0 \ne 2\pi, -2\pi$$

and $g'\left(\frac{\pi}{2}\right) = 0 \ne 2\pi, -2\pi$

11. Using $C_1 \to C_1 - C_2$, we get

$$f(x) = \begin{vmatrix} 0 & \cos(2x) & \sin(2x) \\ -2\cos x & \cos x & -\sin x \\ 0 & \sin x & \cos x \end{vmatrix}$$

$= 2\cos x\,(\cos 2x \cos x - \sin x \sin 2x)$

$= 2\cos x \cos 3x = \cos 2x + \cos 4x$

$f(x)$ attains its maximum value 2 at $x = 0$

$f'(x) = -2\sin 2x - 4\sin 4x$

$$f'\left(-\frac{\pi}{2}\right) = 0, f'(0) = 0, f'\left(\frac{\pi}{2}\right) = 0$$

12. $2(\cos\beta - \cos\alpha) + \cos\alpha\cos\beta = 1$

$\Rightarrow (\cos\alpha + 2)(\cos\beta - 2) = -3$

$$\Rightarrow 2 + \cos\alpha = \frac{3}{2 - \cos\beta} \qquad (1)$$

Let $a = \tan(\alpha/2)$ and $b = \tan(\beta/2)$, now (1) can be written as

$$2 + \frac{1-a^2}{1+a^2} = \frac{3(1+b^2)}{2(1+b^2) - (1-b^2)}$$

$$\Rightarrow \frac{3+a^2}{1+a^2} = \frac{3(1+b^2)}{1+3b^2}$$

$\Rightarrow (3 + a^2)(1 + 3b^2) = 3(1 + a^2)(1 + b^2)$

$\Rightarrow 3 + (a^2 + 9b^2) + 3a^2b^2$

$\qquad = 3 + 3a^2 + 3b^2 + 3a^2b^2$

$\Rightarrow a^2 = 3b^2 \ \Rightarrow a = \pm\sqrt{3}\, b$

$\Rightarrow \tan(\alpha/2) + \sqrt{3}\tan(\beta/2) = 0$

and $\tan(\alpha/2) - \sqrt{3}\tan(\beta/2) = 0$

13. $$f(x) = \frac{1 - x(1 + |1-x|)}{|1-x|}\cos\left(\frac{1}{1-x}\right), x \ne 1$$

$$= \begin{cases} \dfrac{1 - x(1 + (1-x))}{1-x}\cos\left(\dfrac{1}{1-x}\right) & \text{if } x < 1 \\ \dfrac{1 - x(1 + x - 1)}{x-1}\cos\left(\dfrac{1}{1-x}\right) & \text{if } x > 1 \end{cases}$$

$$= \begin{cases} \dfrac{(1-x)^2}{1-x}\cos\left(\dfrac{1}{1-x}\right) & \text{if } x < 1 \\ \dfrac{1-x^2}{x-1}\cos\left(\dfrac{1}{1-x}\right) & \text{if } x > 1 \end{cases}$$

$$= \begin{cases} (1-x)\cos\left(\dfrac{1}{1-x}\right) & \text{if } x < 1 \\ -(x+1)\cos\left(\dfrac{1}{1-x}\right) & \text{if } x > 1 \end{cases}$$

For $x < 1$, $-1 \le \cos\left(\frac{1}{1-x}\right) \le 1$

$$\Rightarrow \quad -(1-x) \le (1-x)\cos\left(\frac{1}{1-x}\right) \le (1-x)$$

Since $\lim_{x \to 1^-}(1-x) = 0$, we get

$$\lim_{x \to 1^-}(1-x)\cos\left(\frac{1}{1-x}\right) = 0$$

or $\quad \lim_{x \to 1^-} f(x) = 0$

As $\quad \lim_{x \to 1^+}\cos\left(\frac{1}{1-x}\right)$ doesn't exist,

$\lim_{x \to 1^+}(-x-1)\cos\left(\frac{1}{1-x}\right)$ doesn't exist

14. $f'(x) > 2f(x)\ \forall\ x \in \mathbf{R}$

Let $\quad g(x) = e^{-2x} f(x)$, then

$\quad g'(x) = e^{-2x} f'(x) - 2e^{-2x} f(x)$

$\quad = e^{-2x}(f'(x) - 2f(x)) > 0\ \forall\ x \in \mathbf{R}$

$\Rightarrow \quad g$ is strictly increasing on $\mathbf{R}$

Also, $\quad g(0) = f(0) = 1$

$\Rightarrow \quad g(x) > g(0)\ \forall\ x > 0$

$\Rightarrow \quad e^{-2x} f(x) > 1\ \forall\ x > 0$

$\Rightarrow \quad f(x) > e^{2x}\ \forall\ x > 0$

As $\quad f'(x) > 2f(x) > e^{2x} > 0\ \forall\ x > 0$

$\Rightarrow \quad f(x)$ is increasing on $(0, \infty)$

Also, $f'(x) > 2f(x) > 2e^{2x} > e^{2x}\ \forall\ x \in (0, \infty)$

15. Let $E = \cos(P + Q) + \cos(Q + R) + \cos(R + P)$

$= \cos(\pi - R) + \cos(\pi - P) + \cos(\pi - Q)$

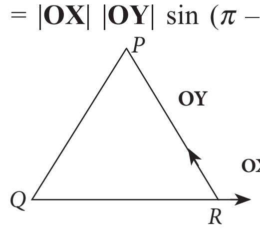
$= -(\cos P + \cos Q + \cos R)$

But in ΔPQR,

$$\cos P + \cos Q + \cos R \le 3/2$$

therefore $E \ge -3/2$

16. $|\mathbf{OX} \times \mathbf{OY}| = |\mathbf{OX}|\ |\mathbf{OY}| \sin(\pi - R)$



$= \sin(P + Q)$

17. $\alpha + \beta = 1$, $\alpha\beta = -1$,

$\alpha = \frac{1}{2}(\sqrt{5}+1)$, $\beta = \frac{1}{2}(-\sqrt{5}+1)$

$a_n = p\alpha^n + q\beta^n$ for $n = 0, 1, 2, 3, \ldots$

For $n \ge 1$

$$a_{n+1} = p\alpha^{n+1} + q\beta^{n+1}$$

$$= (\alpha + \beta)(p\alpha^{n+1} + q\beta^{n+1}) - \alpha\beta(p\alpha^{n-1} + q\beta^{n-1})$$

$\Rightarrow$ $a_{n+1} = a_{n\cdot} + a_{n-1}\ \forall\ n \ge 1$ (1)

$a_0 = p + q$

$a_1 = p\alpha + q\beta$

$a_2 = a_1 + a_0$

$a_3 = a_1 + a_2$

$a_4 = a_2 + a_3 = a_2 + (a_1 + a_2)$

$= a_1 + 2a_2$

$= a_1 + 2(a_1 + a_0)$

$= 3a_1 + 2a_0$

$\Rightarrow$ $28 = 3(p\alpha + q\beta) + 2(p + q)$

$= \frac{7}{2}(p + q) + \frac{3\sqrt{5}}{2}(p - q)$

As p, q are integers, we get

$$28 = \frac{7}{2}(p + q),\ p - q = 0$$

$\Rightarrow p + q = 8$, $p = q \Rightarrow p = q = 4$

$\therefore\ p + 2q = 12$

18. From (1)

$a_{12} = a_{11} + a_{10}$.