



## HP Solve

Calculating solutions powered by HP

### Introducing HP Classroom Manager

Every teacher dreams of better classroom control. Manage class PCs, deploy assessment tools and maintain a collaborative environment to encourage student attention and performance.

[See how](#)

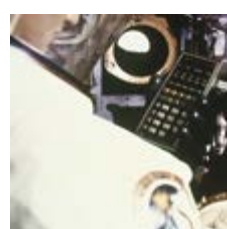


### The past, present and future of HP Solve

*Richard Nelson, Jessica Cespedes and Kevin Regardie*

HP has been supporting its calculator users with a newsletter since September 1974. The history of HP Solve puts this support in historical perspective and outlines what's expected in the future—including how it's becoming a newsletter for K-12 and HED educators.

[Read more](#)

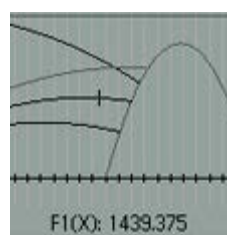


### Quadratics and rocketry

*Kevin Regardie*

Explore the real world applications of the quadratic equation in preventing avalanches, plotting a rocket trajectory and safely launching fireworks. Lesson plans, teacher aids and answers are provided.

[Explore now](#)



### STEM education moves full-“STEAM” ahead

*Laura Berlin*

"STEM" is a buzzword in the education world that most readers know stands for Science, Technology, Engineering, and Mathematics. This article explores the addition of 'A' for the Arts.

[See more](#)



### Navigating the "common core" maze

*Kevin Regardie*

The phrase "common core" has crept into the consciousness of most, if not all, of the educational community. But the who, what, where, why, and when of the Common Core State Standards (CCSS) may not be as clear.

[Learn why](#)



### STEM is dead, long live STEMx

*Jim Vanides*

In this article, Jim explains how the acronym STEM is no longer suitable to describe the focus of a technology driven education program.

[Find out why](#)



### Real-time assessment and applied business math

Launched in 2010, the HP Catalyst Initiative is working to transform STEMx teaching and learning to inspire students to use their technical ingenuity and creativity to address urgent social issues.

[Read more](#)



Check out current articles and feedback from the latest HP Solve newsletter, including a new calculator tip, what is RPN, and a technical article on HP's randomness features.

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# Introducing HP Classroom Manager

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# Introducing HP Classroom Manager:

## Classroom control, management, and collaboration at the click of a mouse

As our world becomes more and more dependent on technology, so do our classrooms. With desktops, laptops, and tablets rapidly becoming the norm for students, educators have infinite possibilities to make teaching truly engaging but are also having to address the issues technology presents in the classroom. But with HP's new software, *HP Classroom Manager*, teachers can take control of the classroom, manage activity in all class PCs, and develop unique ways to communicate with students all while making IT management cost effective and easy for school administrators.

Through PC management and an array of interactive teaching tools, HP Classroom Manager redefines the digital classroom, preventing unwanted distractions and enhancing learning to elevate student achievement levels. With this product, educators can:

### **View and manage multiple student PCs at one time**

- Easily view an entire classroom of PCs at one time
- Watch, share, or control the screen of any student PC
- Add names to student screens and arrange them to appear as they do in class
- Send and receive instant messages and alerts
- Initiate group or 1:1 chats



### **Control PCs in the classroom**

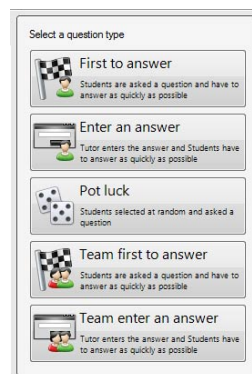
- Power on or off all classroom computers from the teacher PC.
- Lock or black out student PCs with a single click.
- Specify allowed or restricted websites to limit internet activity.
- Open or close specified programs and application on one or multiple computers.

- Nominate a student to be assigned teacher rights and act as a Group Leader whenever desired
- Remotely launch applications, specific documents or websites instantly with the Quick Launch feature



## Enrich learning

- Easily share digital content including pictures, documents, and videos
- Administer quizzes and surveys in real-time with instant reports and scoring
- Enable creative learning in “game show” style with the Question and Answer feature
- Create digital journals of a day’s lessons for student revision and lesson plan management
- Enrich language lab learning with audio monitoring and recording tools
- Design tests and exams with minimum of effort, including text, picture, audio and video questions.



## Make IT management easy and cost efficient

- Monitor all computers across the school network in a single view.
- Control printer and connected devices such as keyboards and USB flash drives.
- Power on, Power off, Reboot and Login to classroom computers remotely.
- Monitor all computers across the school network in a single view.
- Set Security Policies to identify computers without anti-virus, Windows updates or Internet protection.

- Secure teacher profiles each allowing customized levels of functionality as required.
- Use the Policy Management tool to apply restrictions permanently across the school.



HP Classroom Manager can be deployed and used across multiple Windows-based platforms, including desktops, laptops and workstations. Its features lower electricity costs, reduced printing accidents and helps ensure PC security, immediately cutting costs to schools.

And with HP Classroom Manager's easily scalable low cost per seat, it's a win-win for students, teachers and school administrations.

For more information on HP Classroom Manager and to download a free 30 day trail, visit [www.hp.com/go/hpclassroommanager](http://www.hp.com/go/hpclassroommanager).

# The Past, Present, and Future of HP Solve

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# The Past, Present, and Future of *HP Solve*

Richard Nelson, Jessica Cespedes, & Kevin Regardie

## Introduction

HP has a long history of providing calculator news letters that began in 1974 with the first HP calculator to offer a new mix of features<sup>(1)</sup> that astounded the world. The machine was the HP-65 and the first newsletter was called HP-65 Key Notes with its first issue dated Summer 1974 as V1N1. The next generation of advanced programmable calculators, the HP-67 and HP-97 required a newsletter name change and the HP calculator newsletter was renamed HP Key Notes with its first issue dated January 1974 Vol. 1 No. 1. There were other short lived newsletters between then and the first issue of *HP Solve* Volume 1 February 2008. All of the *HP Solve* issues are available online to view and download<sup>(2)</sup>.

## The history of *HP Solve*

*HP Solve* was started by the then calculator general manager Wing Kin Cheung in early 2008. One of the marketing aspects of the newsletter was to support and promote one of HP's most memorable features, RPN. Richard Nelson was "given ownership" of an RPN column called "RPN Tips" starting in Volume 1. *HP Solve* was defined as a technical newsletter dedicated<sup>(4)</sup> to the history, evaluation, explanation, and suggestion of all aspects of HP calculators. The masthead said, "Welcome to the first edition of the *HP Solve* newsletter. Learn calculation concepts, get advice to help you succeed in the office or the classroom, and be the first to find out about new HP calculating solutions and special offers." Fig. 1 & Fig. 2 illustrate a recent *HP Solve* banner change.



Fig. 1 - Initial newsletter banner starting with issue 1.



Fig. 2 – New newsletter banner starting with issue 28.

As with any electronic document its appearance will be determined by the software that generates it. Since the newsletter was sourced on the HP website it uses software that is web oriented. Richard became the editor with issue 17 and officially becoming the editor with issue 18. One of the changes that were made at that time was to format the issue as a single PDF file so the reader could download it, print it if desired, and essentially use it in the classical newsletter manner. Since it was in electronic form it could also use external links to take advantage of the World Wide Web as well. Mobile readers could either store the file locally or read it in real time from the Internet. A more consistent format was established and the nine slots that structured the newsletter were more organized according to "columns" that also followed classical newsletter formatting. The newsletter, up to issue 30, has been content driven as a technical newsletter. See note (3).

Another recent *HP Solve* improvement is a specific signup page so HP customers may more easily sign up to receive the newsletter at:

[http://h20468.www2.hp.com/external/DFBF29CC-2E13-416E-81D8-CC1868461A18/hp\\_calculator\\_newsletter/index.html](http://h20468.www2.hp.com/external/DFBF29CC-2E13-416E-81D8-CC1868461A18/hp_calculator_newsletter/index.html)

## The future of *HP Solve*

Education is constantly evolving with the introduction of new technology and teaching methodologies. As education evolves, we at HP Calculators want to help educators understand how these changes are

affecting their profession as well as way they teach in the classroom. We are very proud of the first 30 issues of *HP Solve* and the articles that have given our readers a unique look into the technical side of calculators, but feel that now is the time to make a change in our newsletter's focus.

Just like education is changing, so is HP Calculators. In the past few months we have tightened our calculator product lines and have incorporated new products like our student monitoring software, HP Classroom Manager, to help provide solutions for the larger education ecosystem. In doing so, we wanted to shift away from technical information and offer content that any educator could enjoy, from news on Common Core Standards to advances in the STEM fields and information on our new products. But we will remain true to our past as well, still offering content on calculating solutions for the revolutionary products that many have grown to love, from the sleek HP 12c to the powerful HP 50g.

Additionally, we are very excited to announce the addition of our new Editor, Kevin Regardie, who will be driving the new *HP Solve* Education Newsletter. Jessica Cespedes, who leads Marketing Communications for HP Calculators, will be taking on the role of *HP Solve's* manager and will be driving the newsletter's scope and content. Richard Nelson, who has been the foundation of *HP Solve* for the past 30 issues, will be staying on the team as well to provide interesting articles on advanced calculator functions as well as serving as our lead Document Facilitator.

All of us at HP Calculators are very excited for the bright future of *HP Solve* and look forward to providing both educators and calculator enthusiasts with the best in news and information for many more years to come.

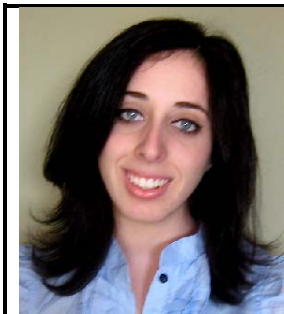
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#### **Notes for The Past, Present and Future of *HP Solve***

- (1). *The HP-65 provided off line program storage in the form of a chewing gum sized magnetic card. It was user programmable and the issue of the time was that it should be called a computer because it met all of the criteria for computers, or should it be called a calculator.*
- (2). See <http://h20331.www2.hp.com/Hpsub/cache/580500-0-0-225-121.html> for all of the *HP Solve* E-Newsletters. See the work of Jake Schwartz who has scanned all newsletters related to HP calculators onto a DVD which contains more than 24,518 pages at: <http://www.pahhc.org/ppccdrom.htm>
- (3). *Richard Nelson has written, edited, and published several HP calculator newsletters dating back to the print only days. Even before then he produced two amateur radio newsletters (in two countries) and a company (Statek) newsletter. Newsletter content may be described as covering **TIPPE**, an acronym meaning **T**hings, **I**deas, **P**eople, **P**laces, and **E**vents. In addition a newsletter may be primarily technical or non-technical. The difference is that a technical newsletter is written with an engineering or academic perspective with a specific vocabulary and lots of detail. Unless you are very interested in the subject a technical newsletter is less appealing in terms of readership numbers.*
- (4). *Actually this description is more of what it became rather than how it was initially defined. Like any newsletter it must meet the needs and interests of its readers, which quite naturally, change over time.*



## About the Author



Jessica Cespedes is the manager of HP Solve, driving the newsletter's content and overall direction. She works as the Marketing Communications Manager for HP Calculators and is also an MBA candidate at San Diego State University. In her role at HP, Jessica works alongside our channel partners and agencies to develop advertising campaigns and also handle all marketing collateral for current and future products. She is fluent in English, Spanish, French, and Italian. She is also proficient in managing multi-medium marketing campaigns, client relationships, copywriting strategies, and Social Media marketing.

## About the Author



As a retired EE and Senior Technical Writer Richard J. Nelson has five full years (including summers) of teaching electronics experience at three different schools. One of them with 3,000 students. Richard has written hundreds of articles on the subject of HP's calculators. His first article was in the first issue of *HP 65 Notes* in June 1974. He became an RPN enthusiast with his first HP Calculator, the HP-35A, he received in the mail from HP on July 31, 1972. One of his hobbies is designing, building, and writing about electronic gadgets. Other hobbies include microscopy, macrophotography, and recreational technical writing. He is a member of the Microscopical Society of Southern California and has articles published on their website. Contact Richard at: [rjnelsoncf@cox.net](mailto:rjnelsoncf@cox.net)

## About the Author



Kevin Regardie holds a B.S. in Aerospace Engineering from the University of Southern California and a MS in Education from National University. In his 10 years as a GATE certified teacher, Kevin has taught an array of high school mathematics courses including AP Calculus. As the faculty advisor for his school's Astronomy Club, Team America Rocketry Challenge (TARC) team, and Science Fair Competition, Kevin strives to increase student engagement and facilitate hands-on experience in STEM learning. His work demonstrates that technology integration and hands on learning can revive and reinvigorate mathematics education.

# Quadratics and Rocketry

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# Quadratics and Rocketry

*Kevin Regardie*

This is a multi-faceted lesson based on quadratic functions and their application to the study of rocketry. Quadratic functions have important applications in science and engineering. In this lesson, students will apply their knowledge of quadratic functions in three distinct modular themes. By considering real-world examples in the classroom, students have an opportunity to broaden their perspectives, make connections to future careers, and build excitement in the application of mathematical concepts. The focus on Science, Technology, Engineering, and Mathematics (STEM) concepts and skills provides cross-curricular opportunities and is vital to becoming a productive member of our workforce.

This themed unit studies rocket flight characteristics and their applications to quadratic equations. Using the HP 39GII calculator, students will complete an activity designed to deepen their understanding of quadratic concepts, including graphing, finding the vertex, uses of the discriminant, and quadratic inequalities. The lessons consist of an interactive demo, lesson plans and student activities. The demo, and calculator activities may be used separately or as described in the sample instructional plan. It may even be spread out into two days or extended into a homework assignment.

The avalanche component is designed as a review and extension of graphing quadratic functions as it requires some previous understanding of this process. Concepts such as extrema, roots, axis of symmetry, and sketching a graph are emphasized. The model rocket component is best applied after covering factoring, completing the square, and vertex form of a quadratic equation. Previous work with regression or lines of best fit is recommended as well. The fireworks component wraps up a chapter covering quadratic equations by covering the discriminant and transformations of quadratic graphs. It also touches on quadratic inequalities.

As a result of utilizing these lessons, students will be able to model real world problems using quadratic functions; develop depth of understanding of the interconnected nature of solutions, graphs, and representations of quadratic functions; analyze and interpret applications of quadratic functions.

It's been my experience that many existing text book examples and/or internet resources do not develop desired depth or critical thinking skills that students should be expected to demonstrate.

While you are not required to use the HP 39gII calculator, it is advantageous as it offers many features that don't exist in other graphing calculators on the market.

For more information on the HP 39gII Graphing Calculator, click [here](#) to download the data sheet.

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To download the Interactive Rocketry Demo, please click on this link: [Interactive Demo](#)

All six PDF versions of the lesson plans can be seen and printed below.

Avalanche Activity. . . . . page 15  
Avalanche Solutions. . . . . page 18

Model Rocket Activity. . . page 21  
Model Rocket Solutions. . page 24

Fireworks Activity. . . . . page 27  
Fireworks Solutions. . . . . page 29

## About the Author



Kevin Regardie holds a B.S. in Aerospace Engineering from the University of Southern California and a MS in Education from National University. In his 10 years as a GATE certified teacher, Kevin has taught an array of high school mathematics courses including AP Calculus. As a the faculty advisor for his school's Astronomy Club, Team America Rocketry Challenge (TARC) team, and Science Fair Competition, Kevin strives to increase student engagement and facilitate hands-on experience in STEM learning. His work demonstrates that technology integration and hands on learning can revive and reinvigorate mathematics education.

# Quadratics and Rocketry

## Lesson Plan

### Description:

This is a multi-faceted lesson based on quadratic functions and their application to the study of rocketry. Quadratic functions have important applications in science and engineering. In this lesson, students will apply their knowledge of quadratic functions in three distinct modular themes.

By considering real-world examples in the classroom, students have an opportunity to broaden their perspectives, make connections to future careers, and build excitement in the application of mathematical concepts. The focus on Science, Technology, Engineering, and Mathematics (STEM) concepts and skills provides cross-curricular opportunities and is vital to becoming a productive member of our workforce. This themed unit studies rocket flight characteristics and their applications to quadratic equations. Using the HP 39GII calculator, students will complete an activity designed to deepen their understanding of quadratic concepts, including graphing, finding the vertex, uses of the discriminant, and inequalities.

Grade: 10th, 11th, 12th  
 Subjects: Intermediate Algebra, Pre-Calculus, Mathematical Analysis  
 Engineering Concepts: Rocket design & modeling, aerodynamics, flight characteristics  
 Topics: Quadratic Equations, Graphing, Solving Equations  
 Time Needed: 3 50-minute class period

### Objectives:

- Students will be able to model real world problems using quadratic functions
- Students will develop depth of understanding of the interconnected nature of solutions, graphs, and representations of quadratic functions
- Students will analyze and interpret applications of quadratic functions

### Standards:

<b>Common Core Standards</b>	<b>Reasoning with Equations and Inequalities</b> A – REI	<b>Solve equations and inequalities in one variable</b> 4. Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$ ), taking square roots, completing the square, the Quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$ . <b>Solve systems of equations</b> 7. Solve a simple system consisting of a linear equation and a Quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$ .
		<b>Analyze functions using different representations</b> 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. $y$ and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$ .

	<b>Linear, Quadratic, and Exponential Models</b> F - LE	<b>Interpret expressions for functions in terms of the situation they model</b> 6. Apply quadratic equations to physical problems, such as the motion of an object under the force of gravity. (CA Standard Algebra I – 23.0)
<b>NCTM Standards</b>		<ul style="list-style-type: none"> <li>● analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior</li> <li>● understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions, using technology to perform such operations on more-complicated symbolic expressions</li> <li>● use a variety of symbolic representations, including recursive and parametric equations, for functions and relations</li> <li>● understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions, using technology to perform such operations on more complicated symbolic expressions</li> <li>● interpret representations of functions of two variables</li> <li>● draw reasonable conclusions about a situation being modeled</li> </ul>

**Instructional Plan:** 3 50-minute periods

Day 1:

Intro: 5-10 minutes

Show the ‘Avalanche flipchart’

Review concepts of quadratic graphs

Model: 10-15 minutes

Show the Avalanche portion of the interactive demo

Practice: 25-30 minutes

Have students complete the Avalanche calculator activity

Close: 5 minutes

Answer questions, check for understanding

Day 2:

Intro: 5-10 minutes

Show the ‘Model Rocket’ flipchart

Review concepts of solving quadratic equations

Model: 10-15 minutes

Show the Model Rocket portion of the interactive demo

Practice: 25-30 minutes

Have students complete the Model Rocket calculator activity

Close: 5 minutes

Answer questions, check for understanding

Day 3:

Intro: 5-10 minutes

Show the ‘Fireworks’ flipchart

Review concepts of transformations of quadratic functions and inequalities

Model: 10-15 minutes

Show the Fireworks portion of the interactive demo

Practice: 25-30 minutes

Have students complete the Fireworks calculator activity

Close: 5 minutes

Answer questions, check for understanding

### Teacher Notes:

The flipchart, demo, and calculator activity may be used separately or as described in the sample instructional plan. It may even be spread out into two days or extended into a homework assignment.

#### Avalanche:

This component is designed as a review and extension of graphing quadratic functions as it requires some previous understanding of this process. Concepts such as extrema, roots, axis of symmetry, and sketching a graph are emphasized.

#### Model Rocket:

This component is best applied after covering factoring, completing the square, and vertex form of a quadratic equation. Previous work with regression or lines of best fit is recommended as well. The solution to Part 3 follows:

$$y = ax^2 + bx + c$$

standard form

$$y = a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right) + c - a\left(\frac{b}{2a}\right)^2$$

factor a, set up to complete the square

$$y = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - a\left(\frac{b^2}{4a^2}\right)$$

complete the square

$$y = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$$

complete the square

$$y = a(x - h)^2 + k$$

vertex form

$$h = -\frac{b}{2a} \quad k = c - \frac{b^2}{4a}$$

**h,k in terms of b,a**

$$0 = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$$

solve by setting y equal to zero

$$\frac{b^2}{4a} - c = a\left(x + \frac{b}{2a}\right)^2$$

subtract

$$\frac{b^2}{4a^2} - \frac{c}{a} = \left(x + \frac{b}{2a}\right)^2$$

divide by a

$$\frac{b^2}{4a^2} - \frac{4ac}{4a^2} = \left(x + \frac{b}{2a}\right)^2$$

common denominator

$$\pm \sqrt{\frac{b^2 - 4ac}{4a^2}} = \left(x + \frac{b}{2a}\right)$$

square root

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

quadratic formula

#### Fireworks:

This component wraps up a chapter covering quadratic equations by covering the discriminant and transformations of quadratic graphs. It also touches on quadratic inequalities.

### Resources:

Interactive Demo

‘Avalanche’ flipchart, calculator activity, solution guide

‘Model Rocket’ flipchart, calculator activity, solution guide

‘Fireworks’ flipchart, calculator activity, solution guide

HP 39gII calculator or other graphing calculator

# Avalanche Calculator Activity

## INTRO

Avalanches are dangerous events that can cause loss of life and can destroy settlements, roads, railways and forests. Avalanche control uses guns to fire explosive projectiles onto slopes too distant or dangerous for patrollers to approach on skis. As a gunner for the Mile High Avalanche Mitigation Company, your job is to calculate the proper velocity to fire the rocket from an initial height of 140 meters that will hit the designated target at a height of 1380 meters. You know from experience that the rocket should be in flight for 20 seconds.


$$y = -\frac{1}{2}gt^2 + vt + h$$

where :

{ y is the height of the target (1380 m), g is the gravitational constant (9.8 m/s<sup>2</sup>), v is the initial velocity ( m/s), t is the time (20 seconds), h is the initial height (140

## Part I

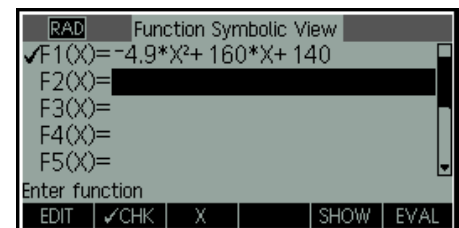
- Let's begin by solving for the initial velocity of the avalanche gun using

the Solve App. Press  , select **Solve**, and press the **Start**

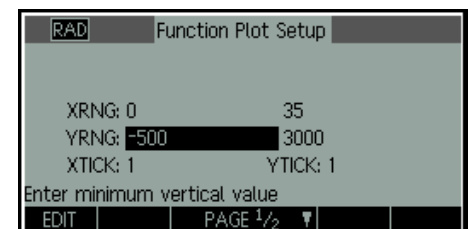
menu key(F6). Enter the equation above as well as the values given. What did you find? How do the values change if the gun is fired from ground level?



- Next, go to the Function App and enter the equation, including the initial velocity, into F1(X). Will the parabola open up or down? Explain?



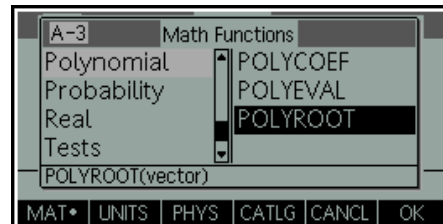
- Use the viewing window to the right.



4. Find the y-coordinate of the vertex. Is this the maximum or minimum of the function? What do the coordinates of the vertex tell you about the flight of the projectile? Explain why negative values for y and t do not make sense for this problem. Repeat this procedure for the other velocities by graphing the next two equations (into F2(X) and F3(X)) and completing the table to the right.

Velocity	Maximum Altitude
130	
160	
190	

5. When will the projectile hit the ground? How far away does it hit the ground? What method did you use to find the roots of the equation? An alternative method exists using the Math Menu in the 39gII graphing calculator. Go to **Home Modes**, then, **Math Cmds B** scroll to polynomial and select



polyroot. This calculator function returns the roots of a polynomial with specified coefficients. What values does polyroot([-4.9,160,140]) return? Complete the table for the remaining velocities. Which roots are valid for this problem? Why? Are the results from #4 and #5 consistent with your expectations? Explain.

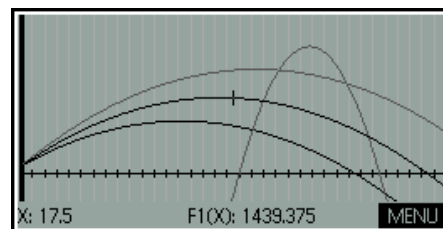
Velocity	Roots
130	
160	
190	

## Part II

6. Let's assume we have a snow covered mountain that is shaped like a parabola and is defined by the equation:

$$y = -72.5x^2 + 3445x - 38512$$

Enter this equation into F4(X).



7. Find the vertex of the mountain equation. Estimate the roots by examining the graph as well as the table of values (an example is provided to the right). Then find the exact roots using one of the methods described previously.

X	F1	F2	
16	1445.6	-1952	
17	1443.9	-899.5	
18	1432.4	8	
19	1411.1	770.5	
20	1380	1388	
20			

ZOOM      BIG\*      DEFN      WIDTH3



8. Find the points of intersection between the gun equations and the left side of the mountain equation. Complete the table.

Velocity	Points of Intersection
130	
160	
190	

### Part III

9. A sky diver is planning on jumping from a plane and landing on the mountain. His descent is modeled by the equation:

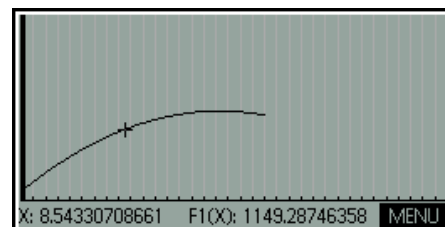
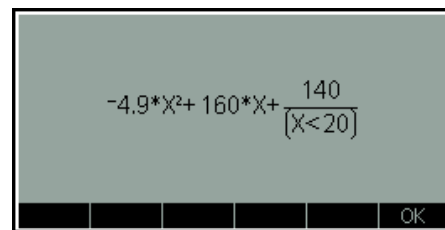
$$y = -4.9x^2 + 49x + 2777.5$$

Enter this equation in F5(X). Find the vertex. Write an equation for the plane (assume horizontal motion). Enter this equation in F6(X).

10. Find the point of intersection between the skydiver equation and the left side of the mountain equation. How long was the skydiver in the air? Is the skydiver in danger of being hit by the avalanche gun? Explain.

### Part IV

11. Finally, we want to ‘clean up’ the graph by graphing only the segments that are applicable to this activity. You can do this by dividing the equation by the domain, in inequality form, that you wish to display. An example is shown to the right. Complete this procedure for the remaining functions to create a clean pictograph of the mountain, avalanche gun, skydiver, and airplane.



# Avalanche Calculator Solutions

## INTRO

Avalanches are dangerous events that can cause loss of life and can destroy settlements, roads, railways and forests. Avalanche control uses guns to fire explosive projectiles onto slopes too distant or dangerous for patrollers to approach on skis. As a gunner for the Mile High Avalanche Mitigation Company, your job is to calculate the proper velocity to fire the rocket from an initial height of 140 meters that will hit the designated target at a height of 1380 meters. You know from experience that the rocket should be in flight for 20 seconds.


$$y = -\frac{1}{2}gt^2 + vt + h$$

where :

{ y is the height of the target (1380 m), g is the gravitational constant (9.8 m/s<sup>2</sup>), v is the initial velocity ( m/s), t is the time (20 seconds), h is the initial height (140

## Part I

- Let's begin by solving for the initial velocity of the avalanche gun using

the Solve App. Press , select **Solve**, and press the **Start**

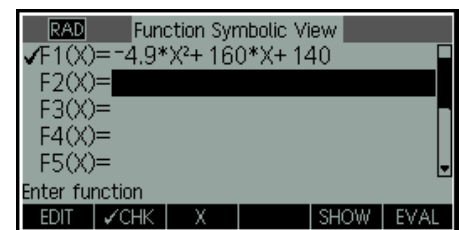
menu key(F6). Enter the equation above as well as the values given.

What did you find? **V=160 m/s** How do the values change if the gun is fired from ground level? **V=167 m/s**

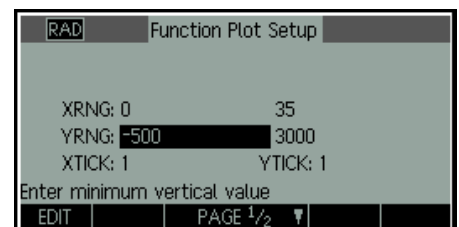


- Next, go to the Function App and enter the equation, including the initial velocity, into F1(X). Will the parabola open up or down? Explain?

**Down. The leading coefficient is negative.**

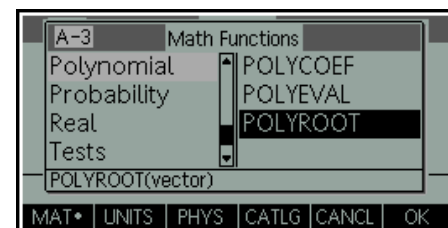


- Use the viewing window to the right.



4. Find the y-coordinate of the vertex. **1446 m**. Is this the maximum or minimum of the function? **Max**. What do the coordinates of the vertex tell you about the flight of the projectile? **The x value of the vertex represents the time when the projectile reaches the maximum height, the y value represents the maximum height**. Explain why negative values for y and t do not make sense for this problem. **Time is defined for values greater than or equal to zero. Negative values for y would indicate the projectile is below the ground**. Repeat this procedure for the other velocities by graphing the next two equations (into F2(X) and F3(X)) and completing the table to the right.

Velocity	Maximum Altitude
130	1002 m
160	1446 m
190	1982 m



5. When will the projectile hit the ground? **33.51 seconds**. What method did you use to find the roots of the equation? **MENU, FNC, Root**. An alternative method exists using the Math Menu in the 39gII graphing calculator. Go to **Home Modes**, then **Math Cmds B**, scroll to polynomial and select

Velocity	Roots
130	<b>-1.04, 27.57</b>
160	<b>-.85, 33.51</b>
190	<b>-.72, 39.5</b>

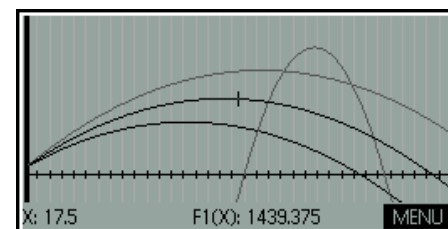
polyroot. This calculator function returns the roots of a polynomial with specified coefficients. What values does polyroot([-4.9,160,140]) return? **(33.51, -.85)**. Complete the table for the remaining velocities. Which roots are valid for this problem? **Positive**. Why? **Negative roots are outside the domain for the time**. Are the results from #4 and #5 consistent with your expectations? Explain. **Answers vary**.

## Part II

6. Let's assume we have a snow covered mountain that is shaped like a parabola and is defined by the equation:

$$y = -72.5x^2 + 3445x - 38512$$

Enter this equation into F4(X).



7. Find the vertex of the mountain equation. **2412 m**. Estimate the roots by examining the graph as well as the table of values (an example is provided to the right). **Roots are between 17 and 18 as well as between 29 and 30**. Then find the exact roots using one of the methods described previously. **17.99 and 29.53**

X	F1	F2
16	1445.6	-1952
17	1443.9	-899.5
18	1432.4	8
19	1411.1	770.5
20	1380	1388
20		

8. Find the points of intersection between the gun equations and the left side of the mountain equation. Complete the table.

Velocity	Points of Intersection
130	(19.1, 835.7)
160	(20, 1380.5)
190	(21.3, 1964.4)

### Part III

9. A sky diver is planning on jumping from a plane and landing on the mountain. His descent is modeled by the equation:

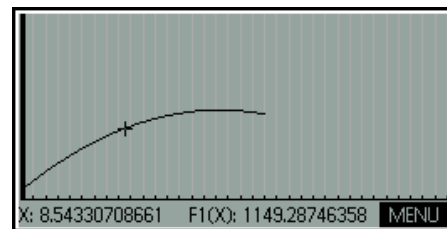
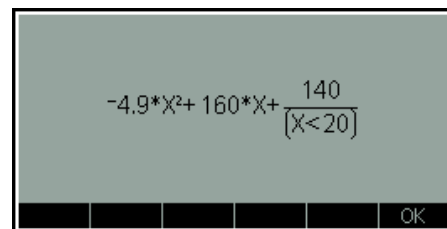
$$y = -4.9x^2 + 49x + 2777.5$$

Enter this equation in F5(X). Find the vertex. (5, 2900) Write an equation for the plane (assume horizontal motion).  $Y=2900$  Enter this equation in F6(X).

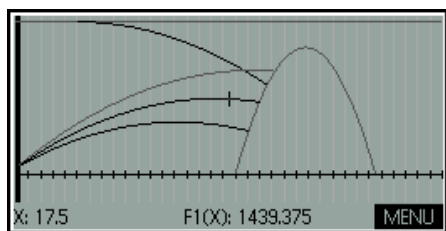
10. Find the point of intersection between the skydiver equation and the left side of the mountain equation. (20.6, 1702.9) How long was the skydiver in the air? 14.6 seconds (20.6 sec — 5 sec) Is the skydiver in danger of being hit by the avalanche gun? Explain. Only at the velocity set to 190 m/s.

### Part IV

11. Finally, we want to ‘clean up’ the graph by graphing only the segments that are applicable to this activity. You can do this by dividing the equation by the domain, in inequality form, that you wish to display. An example is shown to the right. Complete this procedure for the remaining functions to create a clean pictograph of the mountain, avalanche gun, skydiver, and airplane.



Final Pictograph:



# Model Rocket Calculator Activity

Model rockets have evolved after generations of research and experimentation from weapons of war to the modern version of this safe and widespread hobby. Model rockets utilize various recovery systems and motor performance in applications such as aerial photography, experimentation, and high powered rocketry.

## Part I

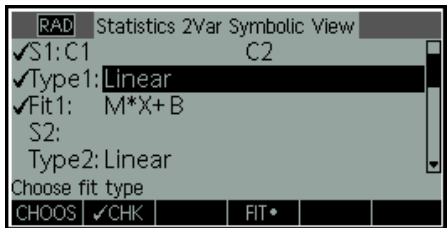
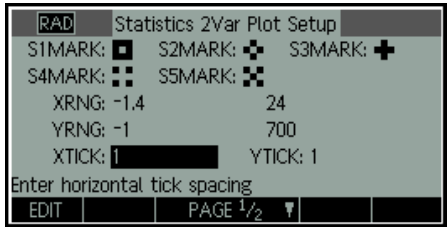
As a student at White Sands High School, you are building a model rocket for the science fair competition. The table shows the height of the rocket  $y$  measured in meters after  $x$  seconds. Find and graph a linear regression equation as well as a quadratic regression equation. Determine which is a better fit for the data.

Time (sec.)	1	2	3	4	5	6	7	8
Height (m)	105.1	200.4	286.9	361.6	427.5	483.6	529.9	566.4

1. To find the linear regression equation, start the Statistics 2Var App and enter the times into C1 and the heights into C2. Use the Plot setup shown below for a proper viewing window. Press **Symb Setup**, choose linear, and press **Plot Setup**. How many points does the line cross through? Is this a good fit for the data? Why or why not?



2. Find the quadratic regression equation. Press **Symb Setup** and choose quadratic. How many points does the line cross through? Is this a good fit for the data? Why or why not?



## Part II

As a student at White Sands High School, you are building a model rocket for the science fair competition. Your data recorder gives you the altitude for the first three seconds: (1, 75.2), (2,140.5), (3,196). Your goal is to find the maximum height to deploy the parachute and allow for a safe recovery. You will need to find the maximum height by using a quadratic regression model to fit a curve to the data points.

3. Find the quadratic regression equation. Copy the equation, open the function app, and paste into F1(X).

4. Adjust the view and find the vertex by pressing **MENU**(F6), **FCN** (F4), and then **Extremum**. What do the coordinates of the vertex tell you about the flight of the rocket? Use the graph to verify the original data points. If the parachute fails to deploy, when will the rocket hit the ground?

$$y = -\frac{1}{2}gx^2 + vx + h$$

where :

{y is the height of the rocket (in meters), g is the gravitational constant (9.8 m/s<sup>2</sup>), v is the initial velocity (in m/s), x is the time (in seconds), h is the initial height (in meters).}

## Part III

5. Given the quadratic equation in standard form to the right, solve for x by completing the square. What equation did you derive?

Standard Form:

$$y = ax^2 + bx + c$$

### Part III

6. You have been tasked to build another rocket that satisfies the following information. Write an equation for the parabola that has a vertex of (5.1, 127.5) and crosses through the point (1, 45). Use vertex form of a quadratic equation.

Vertex form:

$$y = a(x - h)^2 + k$$

What options exist to verify that the graph passes through the points? Find the initial velocity.

### Part IV

7. Use the discriminant to show that there are always two elapsed times at which the altitude is zero, assuming the initial velocity is positive. Then, find the maximum altitude of the rocket.

$$h(t) = -4.9t^2 + v_0t$$

# Model Rocket Calculator Solutions


Model rockets have evolved after generations of research and experimentation from weapons of war to the modern version of this safe and widespread hobby. Model rockets utilize various recovery systems and motor performance in applications such as aerial photography, experimentation, and high powered rocketry.

## Part I


As a student at White Sands High School, you are building a model rocket for the science fair competition. The table shows the height of the rocket  $y$  measured in meters after  $x$  seconds. Find and graph a linear regression equation as well as a quadratic regression equation. Determine which is a better fit for the data.

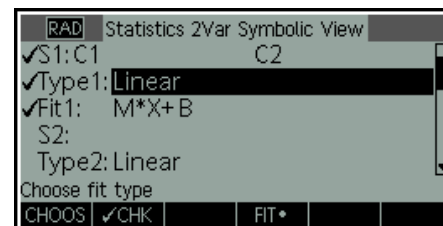
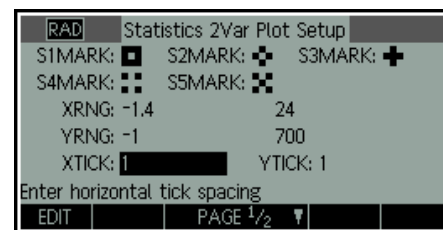
Time (sec.)	1	2	3	4	5	6	7	8
Height (m)	105.1	200.4	286.9	361.6	427.5	483.6	529.9	566.4

1. To find the linear regression equation, start the Statistics 2Var App and enter the times into C1 and the heights into C2. Use the Plot setup shown

below for a proper viewing window. Press  , choose linear, and



2. Find the quadratic regression equation. Press  and choose quad- ratic. How many points does the line cross through? **All**. Is this a good

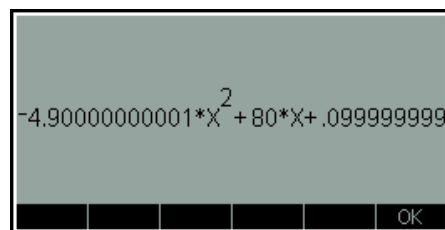




## Part II

As a student at White Sands High School, you are building a model rocket for the science fair competition. Your data recorder gives you the altitude for the first three seconds: (1, 75.2), (2,140.5), (3,196). Your goal is to find the maximum height to deploy the parachute and allow for a safe recovery. You will need to find the maximum height by using a quadratic regression model to fit a curve to the data points.

3. Find the quadratic regression equation. See screen shot to the right. Copy the equation, open the function app, and paste into F1(X).



$-4.900000000001 * X^2 + 80 * X + .0999999999$

4. Adjust the view and find the vertex by pressing **MENU**(F6), **FCN** (F4), and then **Extremum**. (8.16, 326.63) What do the coordinates of the vertex tell you about the flight of the rocket? It reaches a maximum height of 326.63 m after 8.16 seconds. Use the graph to

verify the original data points. Go to **M** If the parachute fails

## Part III

5. Given the quadratic equation in standard form to the right, solve for x by completing the square. See teacher notes. What equation did you derive? The quadratic formula.

Standard Form:

$$y = ax^2 + bx + c$$

### Part III

6. You have been tasked to build another rocket that satisfies the following information. Write an equation for the parabola that has a vertex of (5.1, 127.5) and crosses through the point (1, 45). Use vertex form of a quadratic equation. See below (plug vertex into h,k and point into x,y and solve for a)

$$y = -4.908(x - 5.1)^2 + 127.5$$

Vertex form:

$$y = a(x - h)^2 + k$$

What options exist to verify that the graph passes through the points?

Graph or plug in points. Find the initial velocity.  $V=50$  m/s (convert vertex form into standard form. Initial velocity is b.

### Part IV

7. Use the discriminant to show that there are always two elapsed times at which the altitude is zero, assuming the initial velocity is positive. See solution to the right. Then, find the maximum altitude of the rocket.

Maximum altitude:

$$\frac{(v_0)^2}{19.6}$$

Let:

$$h(t) = -4.9t^2 + v_0t$$

$$\{a = -4.9$$

$$b = v_0$$

$c = 0\}$ . The discriminant is:

$$(v_0)^2 - 4(-4.9)(0) = (v_0)^2 > 0$$

Thus, there are two values of  $t$  for which  $h(t) = 0$ .

# Fireworks Calculator Activity

Fireworks date back to 7<sup>th</sup> century China. Today, fireworks displays are a common focal point of many celebrations around the world. City officials of San Diego want to arrange a fireworks show as part of the city's anniversary festival. Due to safety and legal constraints, and to allow for optimal viewing, they set the minimum height for each fireworks device to be set off at 300 meters. As an employee of The Garden State Fireworks Company, you have been tasked to set the fuse times of the fireworks to fit within these parameters.

We will be analyzing families of parabolas in vertex form. What conjecture can you make about the effect of changing the value of each of the constants  $a$ ,  $h$ , and  $v$  on the graph of the parabola? You can use the HP39gII calculator to analyze these relationships.

Vertex form:  

$$Y = a(X + h)^2 + v$$

Standard Form:  

$$Y = aX^2 + bX + c$$

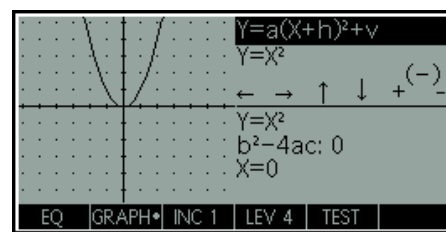
## Part I

Let's begin by exploring the capabilities of the Quadratic Explorer App.

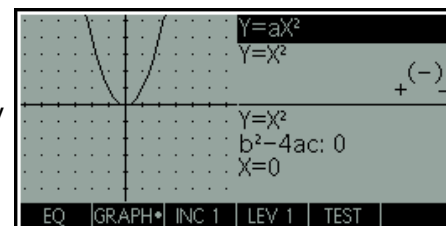
Press,  select **Quadratic Explorer**, and press the **Start** menu key(F6).



1. Notice the information that is available. Listed first is the vertex form of a quadratic equation with variables and the next line with numerical values. Next are the available active calculator keys to adjust the graph. Then are standard form of a quadratic equation the discriminant, and roots  $X_1$  and  $X_2$ , respectively.

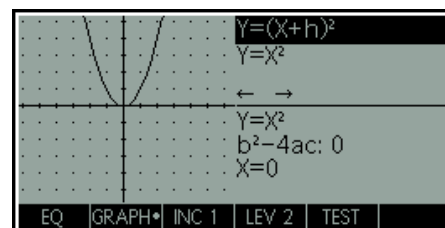


2. Press menu key F4 to access level 1. Notice the active calculator keys that are available. Try experimenting with each key. What does each key do to the graph? To the equation? Are the roots and/or the discriminant affected by these keys? Why or why not?

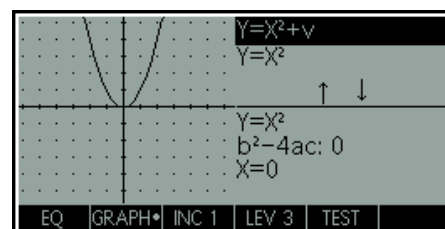


# Fireworks Calculator Activity

3. Press menu key F4 to access level 2. Notice the active calculator keys that are available. Try experimenting with each key. What does each key do to the graph? To the equation? How is  $h$  related to  $b$  and  $c$ ? Write an expression for  $b$  and  $c$  in terms of  $h$ .

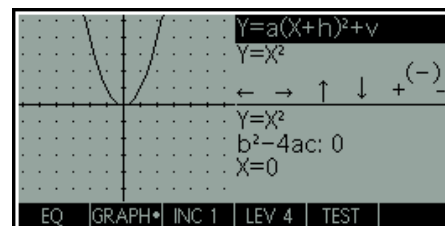


4. Press menu key F4 to access level 3. Notice the active calculator keys that are available. Try experimenting with each key. What does each key do to the graph? To the equation? What is  $a$  and  $h$  equal to? Why is the discriminant changing now? How is the discriminant related to the roots?

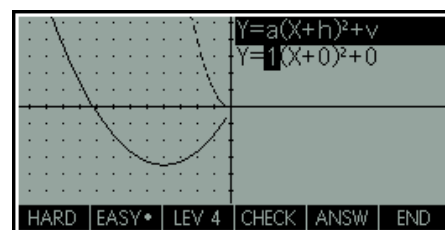


5. Press menu key F4 to access level 4.

- Click the negative sign ( $-$ ). What changes?
- Click the plus and minus signs. What changes?
- Click the left and right arrows. What changes?
- Click the up and down arrows. What changes?



6. Press menu key F5 to access the **TEST** mode. You must manipulate the equation's parameters to make the equation match the target graph. When you feel that you have correctly chosen the parameters a **CHECK** menu key evaluates the answer and provide feedback. An **ANSW** menu key is provided for those who give up! Write down each of your guesses. Try again with the **HARD** option. Can the parabola given in your test be used as a model for the trajectory of a fireworks rocket launch? Why or why not? If not, what can be changed?



# Fireworks Calculator Solutions

Fireworks date back to 7<sup>th</sup> century China. Today, fireworks displays are a common focal point of many celebrations around the world. City officials of San Diego want to arrange a fireworks show as part of the city's anniversary festival. Due to safety and legal constraints, and to allow for optimal viewing, they set the minimum height for each fireworks device to be set off at 300 meters. As an employee of The Garden State Fireworks Company, you have been tasked to set the fuse times of the fireworks to fit within these parameters.

We will be analyzing families of parabolas in vertex form. What conjecture can you make about the effect of changing the value of each of the constants  $a$ ,  $h$ , and  $v$  on the graph of the parabola? You can use the HP39gII calculator to analyze these relationships.

## Part I

Let's begin by exploring the capabilities of the Quadratic Explorer App.

Press , select **Quadratic Explorer**, and press the **Start** menu key

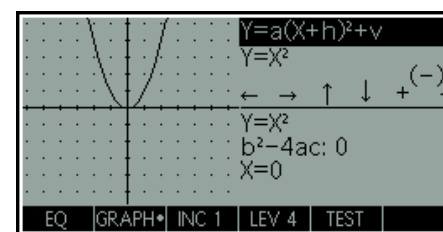
Vertex form:

$$Y = a(X + h)^2 + v$$

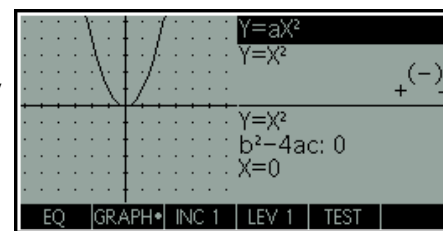
Standard Form:

$$Y = aX^2 + bX + c$$


1. Notice the information that is available. Listed first is the vertex form of a quadratic equation with variables and the next line with numerical values. Next are the available active calculator keys to adjust the graph. Then are standard form of a quadratic equation the discriminant, and roots  $X_1$  and  $X_2$ , respectfully.

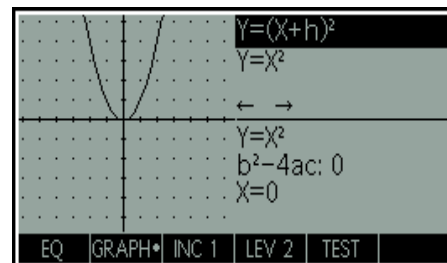


2. Press menu key F4 to access level 1. Notice the active calculator keys that are available. Try experimenting with each key. What does each key do to the graph? **+** and **-** changes the shape while **(-)** reflects the graph about the x-axis. To the equation? **+** increases  $a$ , **-** decreases  $a$ , **(-)** changes the sign. Are the roots and/or the discriminant affected by these keys? **No**. Why or why not? **Roots: double root at zero unaffected by  $a$ . Discriminant:  $b$  and  $c$  are zero.**

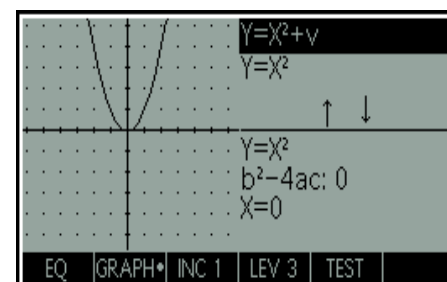


# Fireworks Calculator Solutions

3. Press menu key F4 to access level 2. Notice the active calculator keys that are available. Try experimenting with each key. What does each key do to the graph? **Arrows move graph left and right respectfully.** To the equation? **Arrows change the value of h.** How is h related to a and b? **Found by completing the square** Write an expression for h in terms of a and b.  **$h = -b/(2a)$**

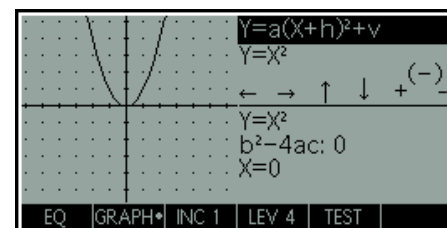


4. Press menu key F4 to access level 3. Notice the active calculator keys that are available. Try experimenting with each key. What does each key do to the graph? **Arrows move graph up and down respectfully.** To the equation? **Arrows change the value of v.** What is a and h equal to?  **$a=1, h=0$ .** Why is the discriminant changing now? **Discriminant is related to the number of roots, so as the graph moves up and down the discriminant and the number and type of roots is affected.** How is the discriminant related to the roots? **Discriminant negative: no real roots, discriminant zero: 1 root, discriminant positive: 2 roots.**

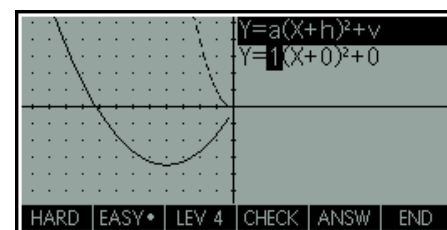


5. Press menu key F4 to access level 4.

- Click the negative sign (—). What changes? **a, discriminant, roots**
- Click the plus and minus signs. What changes? **a, b, c, discriminant, roots**
- Click the left and right arrows. What changes? **h, b, c, roots**
- Click the up and down arrows. What changes? **v, c, discriminant, roots**



6. Press menu key F5 to access the **TEST** mode. You must manipulate the equation's parameters to make the equation match the target graph. When you feel that you have correctly chosen the parameters a **CHECK** menu key evaluates the answer and provide feedback. An **ANSW** menu key is provided for those who give up! Write down each of your guesses. Try again with the **HARD** option. Can the parabola given in your test be used as a model for the trajectory of a fireworks rocket launch? **Yes** Why or why not? **When a is negative and the discriminant is positive.** If not, what can be changed? **When a is negative and the discriminant is positive.**



# S.T.E.M. Education Moves Full-"S.T.E.A.M." Ahead

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## **S.T.E.M. Education Moves Full-“S.T.E.A.M” Ahead!**

*Laura Berlin*

There is a wild idea out there that has recently struck the Science, Technology, Engineering, and Mathematics community and is being contemplated by S.T.E.M. educators across the Nation. That idea is: What would happen if “the Arts” were infused into S.T.E.M. education? What would happen if the arts-based methods that are inherent in the teaching of Music, Drama, Dance and Art were used to teach the Math and Sciences? Would the passion, motivation and understanding that seem to come so easily to students of the arts transfer to students of the Sciences? Would S.T.E.M. benefit from adding Art and becoming (S.T.E.M. + “Art”=) S.T.E.A.M? The growing school of thought is, “It’s worth a try!”

With all test data in recent years corroborating The New York Times, December 11, 2012, report, “U.S. Students Still Lag Globally in Math and Science, Tests Show”, it is past time for the United States to do something drastic and different to raise the achievement of our students in Science, Technology, Engineering and Math or fail to be competitive in the global market. Perhaps the needed “new” approach will be found within the Arts. The National Science Foundation is intrigued enough by this possibility, that it has awarded a multimillion dollar, four year grant to three U.S. cities; San Diego, Chicago, and Worcester, Massachusetts. These cities will carry out a collaborative project creating a national arts-based curriculum for science and math and research how integrating the arts into S.T.E.M. education and practices influences innovation. The project will study, specifically, the effect of integrating arts-based learning strategies such as ideation, improvisation and ensemble thinking with science, to create solutions to difficult civic challenges. The purpose of the project, as outlined by it’s director, Harvey Seifter, is “to implement and investigate the effectiveness of arts-based learning as a way to generate innovation in the field of informal science learning, strengthen the creativity and innovation skills of S.T.E.M. learners and professionals, enhance the innovativeness of cross-disciplinary S.T.E.M. teams, and foster S.T.E.M. engagement in the general public”.

The three collaborating cities have set up operation centers or “incubators for innovation” in major cultural institutions. The San Diego center is the Balboa Park Cultural Partnership. In Chicago, it is the Chicago Museum of Science and Industry and in Worcester, it is the EcoTarium, Worcester’s Science and Nature Center. Each city has, through public submissions and voting, chosen a civic challenge. Over the course of a year, solutions to these challenges will be generated by teams of volunteers from a diverse group of professions, including artists, educators, scientists, engineers and students. Other teams will be put together to work on educational materials and developing processes and problem-solving approaches.

San Diego, after narrowing the many submissions down to three choices; water, healthcare and climate change, voted to have the issue of sustaining a safe, reliable water supply as its challenge to solve. The “innovation” teams will be assembled and will begin working on this challenge in September of 2013. The entire project will culminate in 2015, exhibiting results at Balboa Park’s Centennial Celebration. Meanwhile, other organizations such as “STEM Education Economics and Equity” have developed interest in exploring the benefits of integrating the arts into STEM education. San Diego’s SEEE has launched a series of seminars, hosted by San Diego Gas & Electric’s Energy Innovation Center in Clairemont Mesa, for the purpose of debating whether integrating the arts with STEM education enhances or dilutes student learning. The question up for discussion is, “should the arts be part of science curriculum?” The last seminar, “STEM or STEAM?”, was held on October 1, 2012. The next one, will take place on April 29, 2013. These seminars allow attendees to participate in interactive stations that



showcase projects ranging from elementary through high school and college level and involve using arts-based methods to teach science, technology, engineering, and math concepts. These presentations are followed by discussions relating to integrating the arts into science and math curriculum, creating hands-on learning in different environments, and what we learn from such experiences. These discussions are led by educators who have experience using S.T.E.A.M. methods in education.

The upcoming spring, SEEE Seminar is entitled, Innovative STEM Education for a Diverse San Diego. Teachers from various public schools (preK-12) facing complex, social, economic, and cultural issues will share their work and innovations toward improving STEM education for their students.

San Diego Area colleges and universities are also utilizing STEAM power! San Diego State University, the University of San Diego, University of California, San Diego, and California State University, San Marcos all have programs that integrate the arts into the teaching of science, technology, engineering or math.

From my perspective, as a science-minded high school art teacher, I can give a glimpse into the S.T.E.A.M. related activities that I have been a part of at my school, Clairemont High. I have never actually thought to separate the disciplines of art and science and I agree with the late entrepreneur and Apple CEO, Steve Jobs, who said, "I actually think there's very little distinction between an artist and a scientist.....to me, they're people who pursue different paths headed to the same goal, which is to express what they perceive to be the truth, so others around them can benefit." At Clairemont High School, we have explored and continue to explore ways that the arts and sciences can support and reinforce each other. I have been using science and math in my art classes for quite sometime. Fractals, geometry, proportions, ratios and color theory, are just a few aspects of science and math that also apply to art. One example of this is an interdisciplinary unit on color theory created through collaboration between myself and the physics teacher. The unit combines art and physics and compares pigment theory versus light theory. This has applications not only for our science and visual arts students, but for our "theatre tech" students, as well!. What better way to engage high school students, than "hooking" them with stage and concert lighting!

For the last few years, the music, drama and visual arts teachers at Clairemont High School have worked together, as a department, at the beginning of each school year, to design a project that supports student literacy and/or success in math and the sciences. At some point during the year, based on a common prompt or theme, the teachers engage their students in creating ways that each individual discipline can be used to satisfy the prompt and in the process support the intended literacy, math or science objective. Students have been highly motivated by these projects and the results have been meaningful and insightful. In one such project, we asked students to express through, drama productions and music and art compositions, the issues that get in the way of their literacy success. They created guitar songs with lyrics, theatrical performances and visual compositions that expressed their frustrations and suggestions to facilitate improved achievement. The projects were exhibited and performed during a "Literacy Café" venue and evening event designed by the students. Another project has been so successful that we have repeated it!. We asked students to select a concept in math or science in which they struggled or experienced difficulty. Then, they worked in groups, helping each other understand the concepts and creating ways, to teach the concepts through theater, visual art, and music! Outstanding student projects have included paintings which illustrated things like how actual buildings in Taipei are structured to withstand earthquakes by flexing with movement, teaching Newton's Laws of motion through

demonstrations of stage combat, and songs with catchy rhythms and lyrics to help students remember mathematical formulas.

The Clairemont High School Astronomy Club is currently integrating an arts-based activity to help astronomy students learn the history of space exploration. Students have conducted research, selected major events in space exploration and designed a mural depicting these events, starting with the Big Bang and continuing to the present day Mars Rover. This mural is in the process of being painted on the school's observatory by Astronomy Club members. In addition, a summer school program that integrates science and art has been planned for the 2013 Clairemont High School summer session.

These are exciting times for S.T.E.A.M.-based learning! With increasing activity, there is growing evidence that the arts help motivate students and help students remember and understand information. Harvey Seifter, director of the National Science Foundation's Incubators for Innovation project and author of *The Art of Science Learning*, reveals in this report that research shows high achieving scientists are more likely to practice art than other scientists, and students who study the arts outperform students who don't.

I, for one, am ready to say good bye to the old, ineffective practices of teaching math and science principles through rote memory and skill and drill. I enthusiastically embrace the idea of using arts-based and project-based learning strategies to strengthen the creativity, problem solving and innovation skills of S.T.E.M. learners. At this juncture, I can't help but bring up the familiar quote by Albert Einstein; "Imagination is more important than knowledge. For knowledge is limited to all we know now and understand, while imagination embraces the world and all there ever will be to know and understand."

Let us foster the imaginations of our students. Let us embrace the world!

## About the Author



Laura Berlin is a visual arts teacher and department head at Clairemont High School in San Diego. She holds Single Subject Teaching Credentials in Visual Art and English and a Masters Degree in Technology and Learning.

# Navigating the 'Common Core' Maze

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# Navigating the ‘Common Core’ Maze

*Kevin Regardie*

By now, the phrase ‘common core’ has crept into the consciousness of most, if not all, of the educational community. But the who, what, where, why, and when of the Common Core State Standards (CCSS) may not be as clear. The navigation through this transition can be daunting to all who have an invested stake in the state of education now and into the future.

Forty five states, the District of Columbia, and four territories have adopted the standards for English language arts and mathematics as a key component to help students learn at higher levels. They will be fully implemented during the 2014-2015 school year. The incentive for the CCSS arose from a growing awareness that the existing system of allowing each state to establish its own standards, assessments, and benchmarks for proficiency resulted in tremendous disparity in the educational experiences of students from across the country. The rigor of student curriculum, quality of assessments, and the standards themselves has varied significantly from state to state. So these standards can be seen as the first step in providing young people with a high quality education that has clear and consistent goals.

In mathematics, the common core consists of eight Standards of Mathematical Practice, which describe varieties of expertise that math educators at all levels should seek to develop in their students. Additionally, the CCSS include Standards of Mathematical content, which lay out the math content that should be learned at each level from kindergarten through grade 8 as well as high school level mathematics. The content standards are further broken down into domains and conceptual categories, depending on the grade level. Upon initial reflection, six instructional shifts in mathematics have been identified that stem from the CCSS including focus, coherence, fluency, deep understanding, application, and dual intensity. See the Table 1 for a more detailed breakdown in each category.

**Table 1 – Mathematical Instructional Shifts**

Focus	Teachers significantly narrow and deepen the scope of how time and energy are spent in the classroom. They do so to focus deeply on only the concepts that are prioritized in the standards so that students reach strong foundational knowledge and deep conceptual understanding and are able to transfer mathematical skills and understanding across concepts and grades.
Coherence	Principals and teachers carefully connect the learning with and across grades so that students can build new understanding onto foundations built in previous years. Teachers can begin to count deep conceptual understanding of core content and build on it. Each standard is not a new event but an extension of previous learning.
Fluency	Students are expected to have speed and accuracy with simple calculations; teachers structure class time for students to memorize core functions such as arithmetic operations so that they are more able to understand and manipulate more complex concepts.
Deep Understanding	Teachers support students’ abilities to access concepts from a number of perspectives so that students are able to see mathematics as more than a set of mnemonic or discrete procedures. Students demonstrate deep conceptual understanding of core mathematics concepts by applying them to new situations as well as by writing and speaking about their understanding.

**Table 1 – Mathematical Instructional Shifts Continued**

Application	Students are expected to use mathematics and choose the appropriate concept for application even when they are not prepared to do so. Teachers provide opportunities at all grade levels for students to apply mathematical concepts in real-world situations. Teachers in content areas outside of mathematics, particularly science, ensure that students are using mathematics to make meaning of and access content.
Dual Intensity	Students are practicing and understanding. There is more than a balance between these two things in the classroom – both are occurring with intensity. Teachers create opportunities for students to participate in ‘drills’ and make use of those skills through extended application of mathematical concepts. The amount of time and energy spent practicing and understanding learning environments is driven by the specific mathematical concept and, therefore, varies throughout the given school year.

These are not the only changes that will take place as a result of the shift to the CCSS. Once we define what the CCSS are, we must next turn our focus to the implementation, application, and transition of the standards to the classroom. The mathematical practices are not a checklist of teacher to-dos. Rather, they sustain an environment in which the CCSS content standards are enacted and framed by the specific expertise you help students develop in order to support their understanding and application of mathematics. Merely adopting new curriculum or standards will not, in and of itself, improve student learning. Significant issues in education still exist that must be addressed in determining how student achievement may be maximized. For example, social promotion in many school districts remains the norm, where students are passed on between grade levels and between courses without demonstrating proficiency at the existing level of mathematics. Student performance is also largely attributed to socio-economic disparities. So, the challenges remain vast, yet with each new change is an opportunity to improve.

This is an unprecedented time in terms of implementation of research informed practices within the school sites. Special education providers, administrators, parents, counselors, teachers, and business leaders should work collaboratively to ensure that students are receiving exemplary teaching, applicable training, and top-notch learning experiences. New pedagogical approaches towards meeting the standards have already begun across the country, and it is at this critical juncture that everyone needs to work together to reflect, plan, engage, and inspire our youth. If the proper connections are made between content, instruction, intervention, and assessment, then this ambitious yet attainable goal that is the Common Core State Standards can be realized. The CCSS provide a roadmap through this maze of educational enlightenment, but remember it’s only a first step towards successful navigation. It’s up to us to foster an environment to support our students while developing a culture of success and preparation for the future of this country.

### **About the Author**



Kevin Regardie holds a B.S. in Aerospace Engineering from the University of Southern California and a MS in Education from National University. In his 10 years as a GATE certified teacher, Kevin has taught an array of high school mathematics courses including AP Calculus. As the faculty advisor for his school’s Astronomy Club, Team America Rocketry Challenge (TARC) team, and Science Fair Competition, Kevin strives to increase student engagement and facilitate hands-on experience in STEM learning. His work demonstrates that technology integration and hands on learning can revive and reinvigorate mathematics education.

# STEM is Dead; Long live STEMx

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# STEM is Dead; Long live STEMx

Jim Vanides

Education systems around the world are placing a growing emphasis on the importance of Science, Technology, Engineering, and Mathematics (STEM) programs and curricula. The siren calls of “innovation” and “workforce competitiveness” are often the engine behind the drive, accompanied by the call for “21st century skills”. But are we fully considering the future into which our students will be graduating? Are we giving our students the experiences they need to be ready for the high-tech world of work and global citizenship?

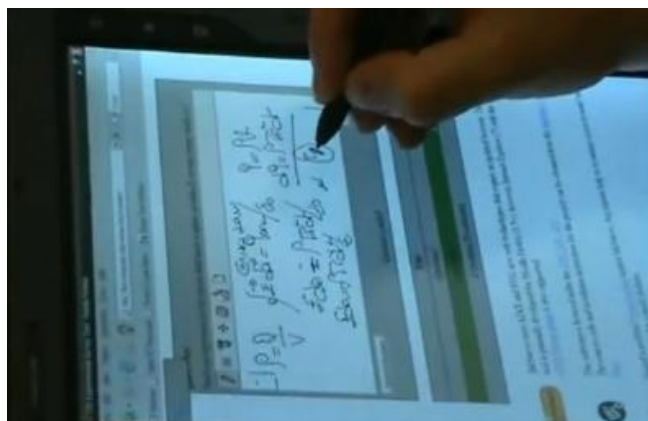


Fig. 1 – Touch screen math being used on a computer.

## The World of Work Has Changed... and is Changing

To begin with, if you don't actually graduate, your opportunity pie is likely only a sliver. Assuming then that you've managed to find the opportunity and support to obtain a STEM-related degree, then let's picture what “work” might look like – and where it might be. You may have an “office”, but it may be a space you only visit on occasion. With wireless internet and a mobile phone, your office can be anywhere you need to be. Your commute to work may in fact be a short 20 steps from your kitchen to your computer.

Functioning in a “flat world” means that your organization is likely to have your colleagues scattered across geographies and time zones. From Research to Product Development to Marketing and Sales, international teams and partnerships require that you master new forms of collaboration in a world that is utterly interdependent. It's a world where decisions and ideas move quickly, and the pace is increasing.

**It may surprise you, but you weren't just hired for what you know. You were hired because of what you're able to learn.** Your task is to tackle challenges that no one has solved – or even imagined. It is impossible to come to work knowing everything you need to know because information and knowledge are expanding exponentially. The universe isn't the only thing that is accelerating.

For the high tech world of work, this is not the future. It is the present. The future, it seems, will be this and even more so... for everyone.

## Global Citizens

Even if our students' careers take them into “non-STEM” directions, they still live in an increasingly high tech world. As a result, the bar has been raised: ALL students need a deeper understanding of S, T, E, and M if they want to thrive, participate, and shape their community and the world.

Our students are inheriting global challenges of the sort their parents have barely anticipated. Whether they are simply trying to be informed citizens, public policy leaders, or innovative engineers, finding solutions to hunger, disease, poverty, and environmental disasters will demand a keen appreciation and mastery of the disciplines we call STEM – and so much more.

## **STEM is Dead – Long Live STEMx**

Given the current reality and our simple prognostication of the future, it's clear that the STEM acronym has outlived its usefulness. If our current education systems and student assessments focus only on S, T, E, and M, our students will not be prepared for today, let alone tomorrow.

At last count, the STEM acronym is missing well over 20 letters. There are critically important disciplines that need more airtime in our “standards” and expectations. These include disciplines like Computer Science (CS), and new areas of discovery and development like Design for Sustainability (DfS), Nanoscience (NS), and BioTech (BT), that don't easily fit the classic STEM silos that students experience.

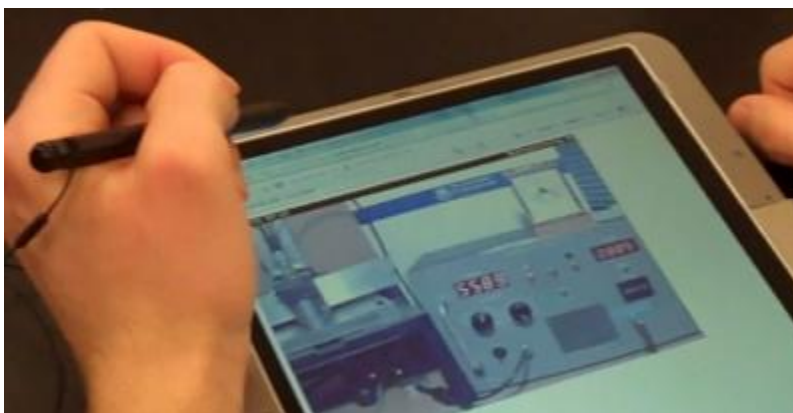
Then there's the myriad of critical skills like the three C's of collaboration (CO), creativity (CR), and communication (COM) that are essential to high performance innovation teams. Add to this the list of additional “21st century skills” like Problem Solving (PS), Inquiry (I), Computational Thinking (CT), and you can see why acronyms like STEAM are noble but not sufficient.

## **New Models of STEMx Teaching & Learning**

One thing is clear – education cannot remain where it has been. Thankfully, there are innovative educators who are creating new types of learning experiences that are relevant, engaging, and paving the way to greater levels of achievement for all students. Among them are educators who are part of the HP Catalyst Initiative, a network of 56 organizations in 15 countries who have been exploring how technology can help create these new models of teaching and learning.

What we're learning is that when you combine great teaching and the right technologies, you really can create exciting and more powerful STEMx learning experiences. Below are a few examples:

### *STEMx for Everyone*



*Fig. 2 - Tablets are becoming ever more popular in education.*

**Northwestern University's Project ACCESS** has created a network of remote iLabs to give high school students laboratory science experiences that their schools could not otherwise offer. These are not online simulations, but rather access to actual experimental equipment that is hundreds or even thousands of miles away. Interestingly, equitable access is not the only headline. By tracking student use of the remote labs, they have discovered that the lab science experience is often “flipped” into the evening, freeing up



time in-class for discussion. Furthermore, students are going well beyond what they would have done in a typical 55 minute class period, running experiments multiple times as they think about the results. Real equipment and real data leads to real scientific inquiry.



*Fig. 3 – Two room high school uses chemistry education on battery powered workstations driven to the school.*

**Masinde Muliro University** in Kenya that the same simulations being used to enhance chemistry education in their university could be used by remote one or two room high schools in rural locations that have no electricity and no internet. After downloading the simulations onto mobile workstations, the computers are driven many hours to remote locations for use in village high schools, giving their students a learning experience that was not possible before – at least as long as the batteries remain charged.

### *STEMx that's Global*



*Fig. 4 – Example of an international collaborative approach to field based science teaching community water quality testing.*

**Stamford Public Schools** in Connecticut have added a global spin to what is already a great science learning experience. Middle school students are engaged in authentic water quality testing in their community, understanding and addressing a very real challenge in their own community. The experience includes a variety of mobile testing and data collection technologies. What extends the experience beyond

their community is their sister school in China, where students who are their same age are also exploring water quality challenges in a community half way around the world. This international collaborative approach to field based science has been an eye-opener for ALL the students, and the experience more closely mirrors how scientists are collaborating today.

### STEMx Success

**Colorado School of Mines** has created a free and simple way to change lectures into conversations that reveal and address student misconceptions. The tool is a free web service called **Ink Survey**, and it provides a quick way to engage students in “graphical polling”. Students are sent to a webpage to answer an open-ended question whose answer requires a drawing. The drawings are sent to the teacher’s webpage and can be shared with the class in real-time. This simple interaction changes lectures into conversations, and reveals misconceptions during class so they can be addressed immediately. The results can be stunning. For example, in a very difficult chemical engineering course, students were using simulations to explore important concepts on their own (“free play”). Student understanding rose somewhat, from a pre-test average of 45% to 58%. But by adding graphical polling to the mix (“guided play”), student understanding rose to 78%, almost doubling again. This interaction changes the pedagogy and demonstrates the power of “knowing what your students don’t know in real time” through graphical polling.

### STEMx Everywhere

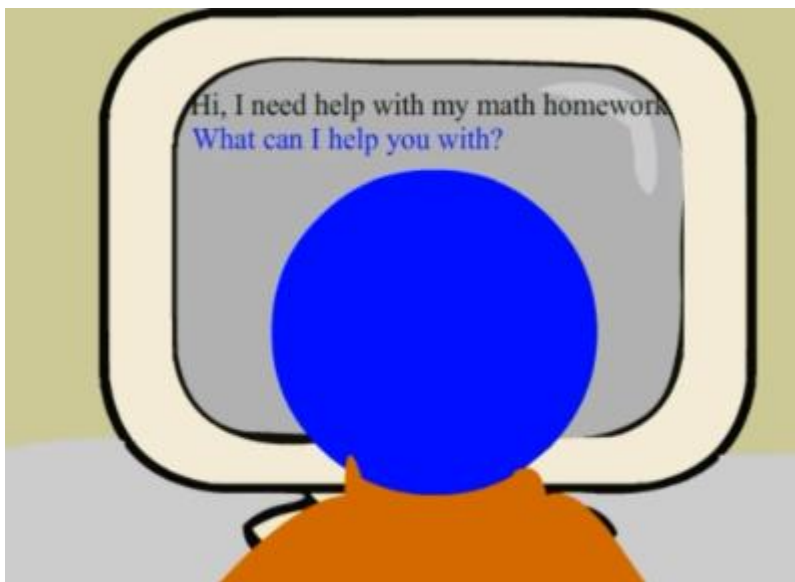


Fig. 5 – “Help desk” is run by students for students in South Africa.

**CSIR Meraka Institute** in South Africa is addressing the challenge to reach marginalized secondary and first-year tertiary students via a mobile-based service that provides personal tutoring on demand. Their Dr. Maths initiative is now expanded to include the Mathlete program, providing mathematics tutoring support to middle school (up to grade 9) learners through phone-based instant messaging tutoring by upper secondary students (grades 10 and 11). The infrastructure is essentially a text messaging based “help desk” run by students for students.

## The New Learner

The examples above are only a small selection of how innovative educators are re-imagining STEMx education. Ultimately, our goal should be to think beyond “school” – to help all of our students develop a personal learning network that allows them to knit together formal (school), informal (science centers, libraries,...) , and non-formal (home) STEMx learning experiences. In the HP Catalyst Initiative we call this vision the New Learner.

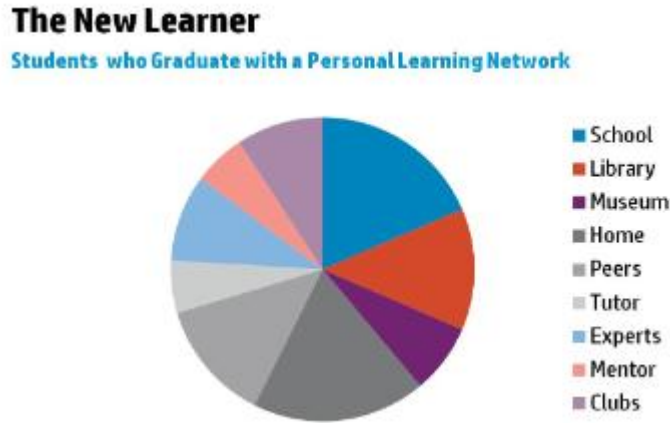


Fig. 6 – HP Catalyst Initiative for STEMx is called the new Learner.

Students that develop this kind of personal learning network will discover a vast set of resources and people who can help them explore, inquire, invent, and find meaning in their pursuit of learning. These are the students who will be well prepared for the high-tech world that we are only beginning to imagine...

For more articles by Jim Vanides please visit [www.hp.com/go/edublog](http://www.hp.com/go/edublog)

## About the Author



Jim Vanides is a member of the HP Office of Global Social Innovation, responsible for worldwide education philanthropy strategy and programs. This includes the 2010 HP Catalyst Initiative ([www.hp.com/go/hpcatalyst](http://www.hp.com/go/hpcatalyst)) and the 2009 HP Innovations in Education initiative, a \$20M investment reaching schools, colleges, and universities in 26 countries. In addition to authoring the blog, “Teaching, Learning, and Technology in Higher Education” ([www.hp.com/go/hied-blog](http://www.hp.com/go/hied-blog)) he is a contributing author on the K12 education blog, Guide to Digital Learning Environments ([www.guide2digitallearning.com/blog](http://www.guide2digitallearning.com/blog)). In his "spare" time, Jim teaches an online course for Montana State University on the Science of Sound ([www.scienceteacher.org](http://www.scienceteacher.org)), a masters-level, conceptual physics course for teachers in grades 5 through 8. Jim’s past work at HP has included engineering design, engineering management, and program management in R&D, Manufacturing, and Business Development. He holds a BS in Engineering and a MA in Education, both from Stanford University.

# Real-time assessment and applied Business Math

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## Real-time assessment and applied Business Math

Technology has profoundly transformed our lives—from the ways we communicate, to how we are entertained and even how we manage our personal finances. Yet, one of the most important aspects—education—has yet to embrace technology’s transformational benefits. This is not just a matter of providing every student with a laptop and access to the Internet; a fundamental change is under way in the approach to education as well as the knowledge needed to compete in today’s global economy.

The bar has been raised on the baseline of science, technology, engineering and math or STEM skills needed to compete and lead in the 21st century. Nations around the world are facing extreme challenges that will require creativity, collaboration, and STEM literacy to solve them, (STEMx).

These critical challenges in education cannot be solved alone. Collaboration between education leaders, institutions and governments is vital to develop classroom and ‘beyond the classroom’ innovations and to scale them for the benefit of many more teachers and students. HP has a long tradition of supporting innovation in education, beginning with our founders, Bill Hewlett and Dave Packard.

Launched in 2010, [the HP Catalyst Initiative](#), a global partner network of 56 schools, universities, nonprofits, and social enterprises from 15 countries, is working to transform STEMx teaching and learning, and inspire students to use their technical ingenuity and creativity to address urgent social issues. To date, HP Catalyst network members have had a positive impact on over 130,000 students.

This initiative is about collaboration between education leaders and institutions, governments, and other partners – and across time zones, languages, and cultures – to develop classroom and beyond-the-classroom innovations, and to scale them to benefit many more teachers and students.

Organized around 6 educational themes, each consortium is focused on a range of issues, from teacher preparation and student-driven learning models to student assessment. Each is charged with investigating how emerging technologies can lead to learning experiences that promote creative thinking, open-ended problem-solving, and cross-cultural collaboration.

In the ‘Measuring Learning’ consortium, the use of tablet PCs is helping with real-time assessment.

### **Teachers are using HP technology to provide their students with real-time assessments and feedback --- boosting achievement as a result.**

Most people believe that technology can improve teaching and learning. But how exactly?

Which methods of assessing knowledge provide immediate and relevant student feedback? Do students perform better when technology gives them immediate feedback, and does this hold true for every student despite differences of gender, socioeconomic status, and learning disabilities?

Math and science teachers at Rancocas Valley Regional High School in Mount Holly, New Jersey, USA are asking these and other essential questions as they explore real-time assessment of knowledge. Funded by the HP Catalyst Initiative, the Measuring Learning Consortium member, they are hoping to boost student performance with new ways of using technology to provide immediate feedback on classroom activities.

The aim is to go beyond basic recall of facts to develop critical thinking skills – the Rancocas Valley teachers want to see students interpreting information contained in graphs, charts, and diagrams, generating original content, using models and simulations to explore concepts, collecting and analyzing real-time experimental data, and collaborating with other learners beyond the classroom setting to solve problems.

Preliminary results from one math teacher show a near 20-point improvement between the target group using HP technology and a control group. Other science teachers have more mixed assessment results, but the overall impact is positive, and everyone is enthusiastic about using technology in classrooms. This highly collaborative project has already connected the Rancocas Valley team with teachers in other departments.

Teachers are using HP tablet PCs and netbooks, HP TouchPads, math and science kits to create simulations and online assessments, while their students use individual laptops to search the web, take surveys, and respond to assessments.

Another way HP Catalyst is affecting the lives of students is in blending two often separate fields of study, STEM and business. Here is one project from the STEM-preneur consortium, which demonstrates how technology can engage students with business and maths.

### **Math has been the bane of many a student's existence – but help is at hand.**

Students in the Business Foundations and Ontario Business programs at the Conestoga College Institute of Technology and Advanced Learning are receiving help in building their math and business skills by taking part in the college's HP Catalyst project "Business Math".

Studies have shown that a significant percentage of first semester students either fail or barely pass mathematics, which puts them at risk of not completing their chosen program.

The Conestoga Catalyst project targets these students. A business math lab, which includes 45 HP Elite Tablets, an interactive white board and use of online learning tools, videos, and collaborative software, will serve as the cornerstone of the project. Connecting math concepts to entrepreneurship and the business world helps students see the real life relevance of what they are learning.

"We're using technology and a variety of different means to improve these students' base skills in math so when they continue on they'll be more successful", says faculty member Lisa Koster. "We're also trying to connect math concepts to entrepreneurship and the business world so that students can see the real life relevance of what they are learning. For example, we regularly have entrepreneurs come in to talk to us about how they use math in their day-to-day business dealings."

With this in mind, hands-on workshops in budgeting and business planning will be offered in addition to math tutorials for struggling students. In the second phase of the project, a mini-lab consisting of ten HP Elite Tablet PCs will be deployed to support the addition of more business math courses.

Koster and her team received the HP Elite Tablet PCs late in the Fall 2011 semester, and implemented them in business math classes. The tablets have already made a positive impact in the classroom, "With the tablets in front of them the students are more involved", says Koster. One student put it perfectly. He said: "with the tablet in front of me, I can't help but pay attention."

Learn more about HP Catalyst at: [www.hp.com/go/hpcatalyst](http://www.hp.com/go/hpcatalyst)

Take a look at the [HP Catalyst Project Gallery](#)

Join the [HP Catalyst Initiative on Facebook](#)

# From The Editor

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## From The Editors – Issue 31

It is Spring and welcome to the new issue of *HP Solve*. It is new because it is a new issue, and it is new because *HP Solve* has a new direction. The *HP Solve* E-Newsletter is now dedicated to Teachers and STEM education. Advanced technical support for calculators will continue with at least one feature article in each issue to be included here.

### Here is the content of this issue

**S01 – Classroom Control at the Click of a Mouse** HP advances its classroom teacher support with this advanced method of putting the teacher in charge and in control.

**S02 – The Past, Present and Future of *HP Solve*** by Richard J. Nelson, Jessica Cespedes, & Kevin Regardie. HP has been supporting its calculator users with a newsletter since September 1974. The history of *HP Solve* puts this support in historical perspective and it outlines what expected in the future..

**S03 – Quadratics and Rocketry** by your editor, Kevin Regardie. Explore the real world applications of the quadratic equation in preventing avalanches, plot a rocket trajectory, and safely launching fireworks. Lesson plans and teacher aids/answers are provided.

**S04 – S.T.E.M. Education Moves full-“S.T.E.A.M.” Ahead!** by Laura Berlins. STEM is a buz word in the Education World. Most readers know that it is for Science, Technology, Engineering, & Mathematics. Adding an A for the Arts is advocated in this article.

**S05 – Navigating the ‘Common Core’ Maze** by Kevin Regardie provides insights as to what Common Core State Standards mean.

**S06 – STEM is DEAD; Long live STEMx** by Jim Vandies who explains how the acronym STEM (Science, Technology, Engineering, and Mathematics) is no longer suitable to describe the focus of a technology driven education program.

**S07 – HP Catalyst, Real-time assessment and applied Business Math** by HP. HP launched the HP Catalyst Initiative in 2010 as described in this article.

### **S08 – Regular/assorted Columns**

- ◆ From the editors.
- ◆ HP Calculator Tip – Use List Processing.
- ◆ What is RPN?
- ◆ Technical article on HP’s Randomness features. You won’t find this stuff in your Owner’s Manuals.

That is it for this issue. We hope you enjoy it. Write us with your ideas for future topics including being an author yourself at: [hpsolve@hp.com](mailto:hpsolve@hp.com)

Kevin Regardie – STEM Editor.

Richard J. Nelson – Technical Editor.



## HP Calculator Tip – Use List Processing

One of the many very powerful features of HP's scientific/graphing calculators is list processing. List Processing was first offered on the HP28C in June of 1986. Data (numeric or statistical) is entered into a list and the list processed by a wide range of functions. The list feature allows the data to be entered once. Copies may be made or stored for future use. You may then sum the list (most common use), sort the list, reverse the order of the list, calculate the difference between values, multiple each value in the list, etc.

The most common use of List Processing is adding a batch of numbers. Most users will key the value of each number, N values, followed by ENTER to place the values onto the stack. The + key is then pressed N-1 times to get their total. **The Calculator Tip is to train yourself to ALWAYS enter your numerical values into a list.** If the values are equal to the number of levels on the stack, or less, you may save time doing it the old fashioned way. If you have five or more values you should take the time to use List Processing. You will save time and increase the accuracy of your work because you may go back and review/edit your values without having to re-enter them.

## What is RPN?

Many teachers may not be familiar with the HP calculator term RPN. RPN is an acronym for Reverse Polish Notation. Any term starting with Reverse isn't an attractive term so RPN students often start out with a negative bias. RPN is a calculator user interface that works with an automatic stack in such a way that an equal key is meaningless to solving problems. Normally/historically an RPN calculator has a double wide ENTER key e.g. HP 35s. In terms of mathematics logic RPN is called postfix. Normal algebraic logic is called infix. These terms describe the order of the data and the operands in calculator problem solving.

In the early days of calculators the explosion of technology continually lowered the cost of the internal electronics and the technical advantages of RPN became less of a cost advantage. Calculators converted infix problems to postfix problems internally for simpler problem solving logic. When other manufacturers joined HP in the scientific/financial calculator business they decided that algebraic logic better matched the solution of problems expressed with infix notation e.g.  $1 + 2 \times 3 = ?$ .

RPN<sup>(1)</sup> has many advantages but it is a different way of problem solving that doesn't use parentheses. It is a way of thinking that must be studied/understood. Learning RPN takes but a few minutes and it really makes problem solving much easier. The idea is simple. You enter the data and then you decide what to do with it. If you want to add you press the + key. In the early days there was a "calculator-logic-system war" between HP and other manufacturers. One T-shirt, for example, sported ENTER >=. RPN is simpler and faster. RPN was different and uniquely HP. Serious problem solvers saw the advantages and they have difficulty with algebraic calculators. Algebraic logic users simply can't use an RPN calculator.

There are two points that make the RPN "controversy" especially interesting. The first one is that one system is "better" than the other. Better is a subjective term and in problem solving there is only one solution, the correct one. Is faster "better"? Is fewer keystrokes "better"? Is easy error correction "better"? Remember that we are talking about solving mathematics problems. NO calculator solves problems exactly like they are written on paper. ALL calculators have user interface convenience features that make this statement true.

Let's examine the example problem given above:  $1 + 2 \times 3 = ?$  This will illustrate the second less known point regarding RPN. A vital feature of algebraic calculators is that they follow a defined hierarchy of

operation order. If you try this example and you press the corresponding keys on various calculators proceeding left to right you will discover that you could get two answers: 9 or 7. Why is this? Some calculators have no logic and simply perform the operations as they are keyed. Machines that return 9 as an answer might be called arithmetic logic, i.e. no logic. The machines that return 7 as an answer follow an algebraic logic in that multiplication is performed before addition. What was the problem? We all agree that there can only be one correct answer.

Perhaps it is the way the problem is given. It could be  $(1 + 2) \times 3 = ?$  OR it could be  $1 + (2 \times 3) = ?$ . This is where parentheses play an important role in mathematics. Calculators that are truly algebraic will return 7. Algebraic logic Calculators always use parenthesis, but not all calculators that use parentheses are algebraic (using the hierarchy of operations). Calculators that use RPN cannot solve the problem without the parentheses. Since there are no parentheses on an RPN (only) calculator the problem without parentheses cannot be solved because it is unclear. Many calculator users won't even think about this.

Thus far three of the common four calculator user interfaces has been discussed – Arithmetic (no logic), RPN (postfix logic), and algebraic (infix logic). There is a fourth logic system that most readers use on their computers every day. It is called command line logic. The idea is that the user types in the problem with data, operators, and parentheses pretty much as they find it in their text books. The machine parses the command line and if the input follows all of the rules it returns an answer. If not it returns an error. All graphing calculators use Command Line logic.

Here is one more interesting factoid regarding HP calculators. HP has always made calculators of all four user logic interfaces and no other calculator manufacturer does that. In fact many of HP's current models give the user a choice of two or even three user interfaces.

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### Notes: From the Editors

(1) For a more technical explanation of RPN see [HP Solve](#) issue #4 page 3.

# The Randomness of HP

Richard J. Nelson & Namir Shammam

## Introduction

Randomness is a term we have all heard because it is frequently used in normal life and mathematics. We flip a coin to “let fate” make a decision. We draw numbered balls from a mixed up container for Bingo and Lotto drawings. We finger snap a spinner, shuffle a deck of cards, or roll a pair of dice for choices to be made for board games. The list is long. Exactly what is meant by, and what are the important qualities of an event or number being truly random? How may we get our own random numbers? This seven page article is supported by two appendices that provide technical details and linked resources.

## Being Random

Technically randomness means different things in various fields. When used as described above it means a lack of a pattern and an equal chance of each possible outcome. It means that a series of random events occur in such a way that the next event is not predictable. Depending on how the random event is caused, however, there are probabilities that a certain value will occur. For example, if we flip a properly made coin<sup>(1)</sup> most people will agree that when the coin lands there will be an equal chance of it showing heads or tails. It is not the purpose of this article to delve into the very deep mathematical techniques of defining and evaluating all aspects of randomness. Rather it is the purpose of this article to explore the seldom documented random features of HP calculators. Just like other features such as SOLVE, MOD, and a Calendar, HP has implemented random features that are exceptional and mathematically rigorous. The ability to produce high quality random options suitable for all of the situations described in the Introduction is a typical hall mark of HP calculators.

## Sources of Randomness

There are two sources of randomness. (1) A physical process such as rolling die or, (2) a mathematical operation such as that implemented in a computer or HP calculator. In its most basic form a series of numbers are generated mathematically by various processes such as those previously mentioned. When computers are involved the processing of numbers is very fast and vast quantities of data may be used for a particular application. In certain modeling situations billions of random numbers may be required and that is beyond the capability of our hand held calculators. The discussion of random numbers is a vast subject and the available articles, books, programs and research papers would keep any teacher or student busy for many years. When vast numbers of high quality true random numbers are required computers are teamed up with electronic devices such as a noisy resistor, noisy semiconductor, atmospheric noise, or detecting a radioactive decay process<sup>(2)</sup>.

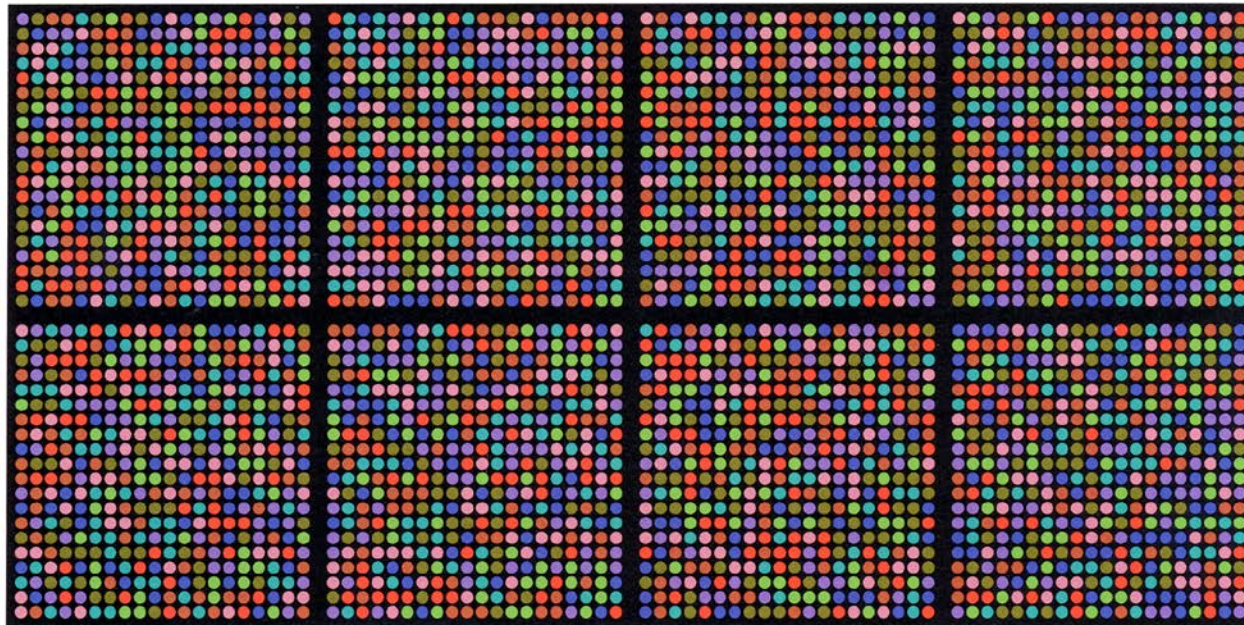
## Randomness Testing

Testing for true randomness is a challenging task<sup>(3)</sup> and is usually a college level topic that will not be covered here. One of the simplest tests is a tally of all the numbers – usually digits – and noting their frequency. For true randomness the distribution of the digits, and their odd-evenness should be uniform<sup>(4)</sup>. There are dozens of (statistical) tests that may be performed and looking for repeating sequences is but one of them. While these tests involve advanced mathematics and a computer there is one simple test that a graphing calculator may easily perform to test for randomness. Use the generated numbers two at a time as x-y coordinate values, scale them, and plot a point on the calculator screen. The screen should look similar to Fig. 1 which shows a 131 x 80 pixel plot (HP50g display) of random points. It is a simple task to examine the screen pattern as having no pattern.



Fig. 1 – 10,480 random pairs plotted. <http://www.random.org/>

If the numbers are not random vertical, horizontal, or diagonal streaks or other patterns will be obvious. Fig. 2 is from the first link in note 2. It illustrates a more complex graphical way of comparing a batch of 1,200 random numbers. Adding a third number to represent color greatly increases the information that may be derived in terms of the desired lack of patterns.



*American Scientist, July-August 2001, Volume 89, p300*

Figure 2. Eight specimens of randomness come from very different sources but yield similar patterns. The 400 colored dots in each panel represent 1,200 random bits taken three at a time. The sources of randomness are, from left to right and top to bottom: a pseudo-random generator, the 1955 Rand Corporation table, the HG202 random-bit generator (faust.irb.hr/~stipyl), the Pennsylvania Daily Number lottery (www.palottery.com), 1,200 coin flips, rapid pounding on a keyboard, an electrocardiogram of atrial fibrillation (www.physionet.org), and six Lava Lites (lavarand.sgi.com).

### Random or Pseudorandom?

Using a computer or calculator program to generate random numbers is by definition not considered possible because the same numbers may be generated by another machine using the same program and therefore is predictable. A fundamental part of being random is the inability to predict the next values in a series. It is possible to program a very well tested<sup>(3)</sup> algorithm that will produce a very long series of random numbers. The “trick” is to jump into the series at some known point - a value which may be kept secret or itself being random. These programs are called pseudorandom number generators (PRNG) compared to true random number generators<sup>(3)</sup> (TRNG) based on a physical process. All HP calculators use a PRNG. The point where you start the series is called the seed. See additional details in the next section and Appendix A. Fig. 3 shows examples of TRNG’s. Opposite side spot counts equal seven.



Fig. 3 – L to R Conventional dice, rounded corners in various colors, game dice with special markings, and assorted non-standard sided dice which are often 8, 10, 16, and 20 sided for various applications of a TRNG.

### HP Calculator PRNGs

Table 1 shows the current calculators offered by HP. Models that feature randomness are indicated by \*.

Circa late 1980's and with the HP 28C on June 1<sup>st</sup> 1986, HP revised their PRNG algorithm to improve it. All current machines will have the values shown in table 2 using the latest version of the algorithm except for the remanufactured HP-15C which uses a different (earlier) algorithm.

As an illustration of the limited information related to this algorithm and its application, the text of the HP33s "user's manual" is reproduced below. This example is as complete as it gets in terms of the descriptions of the two randomness functions typically called RAND and SEED. The indication of its quality is given with its Donald Knuth reference.

**Table 1 – 2013 HP Calculator Products – 24 Total**

#	Financial Calculators	Scientific & Graphing	Home & Office
1	HP 10bII	HP 10s	HP CalcPad 100
2	HP 10bII+*	<b>HP-15C Limited Edition*</b>	HP CalcPad 200
3	<u>HP 20b</u> *	<b>HP 33s*</b>	OfficeCalc 100
4	<u>HP 30b</u> *	<b>HP 35s*</b>	OfficeCalc 200
5	HP-12C	<b>HP 39gs*</b>	OfficeCalc 300
6	<b>HP-12C 30<sup>th</sup> Anniversary</b>	<b>HP 39gII*</b>	PrintCalc 100
7	<b>HP-17bII+</b>	<b>HP 40gs*</b>	QuickCalc
8		<b>HP 48gII*</b>	
9		<b>HP 50g*</b>	
10		SmartCalc HP 300s	
	<b>5 of 7 = 71% programmable</b>	<b>8 of 10 = 80% programmable</b>	<b>None programmable</b>

*Notes: Programmable calculators are in blue. Underlined models are at or near end of life.*

*\* Machines that have a PRNG function.*

**Table 2 – Random Sequences for Current Models (Seed =  $\sqrt{5}$ )**

#	Machine	Random #1	Random #2	Random #3
1	HP 17bII+ <sup>(a)</sup>	0.521548989463	0.0593946804209	0.666602695109
2	HP 10bII+ <sup>(a)</sup>	0.521548989463	0.0593946804209	0.666602695109
	HP 20b <sup>(b)</sup>	0.521548989463	0.0593946804209	0.666602695109
3	HP30b <sup>(a)</sup>	0.521548989463	0.0593946804209	0.666602695109
4	<b>HP-15C LE<sup>(a,c)</sup></b>	<b>0.635762643</b>	<b>0.5681838663</b>	<b>0.6749247476</b>
5	HP33s	0.521548989463	0.0593946804210 <sup>(d)</sup>	0.666602695109
6	HP35s	0.521548989463	0.0593946804210 <sup>(d)</sup>	0.666602695109
7	HP39gs	0.521548989463	0.0593946804209	0.666602695109
8	HP39gII	0.521548989463	0.0593946804209	0.666602695109
9	HP40gs	0.521548989463	0.0593946804209	0.666602695109
10	HP48gII	0.521548989463	0.0593946804209	0.666602695109
	HP50g	0.521548989463	0.0593946804209	0.666602695109

*Notes: (a) The seed is stored using the keys STO RAND.*


*(b) The HP 20b does not have the ability for the user to store a seed.*

*(c) Based on original ROMs of HP-15C announced on July 1<sup>st</sup> 1982.*


*(d) Last digit error because of a bug in the implementation of the algorithm wherein the rounded<sup>(7)</sup> value was used (as with most normal functions) instead of the truncated value.*

The HP33s Calculator Random number usage instructions below will serve as an example of the brevity of the typical RAND information.

**Seed**

To store the number in **x** as a new seed for the random number generator, press  **SEED**.

## Random number generator

To generate a random number in the range  $0 \leq x < 1$ , press  **RAND**. (The number is part of a uniformly-distributed pseudo-random number sequence. It passes the spectral test of D. Knuth, *The Art of Computer Programming*, vol. 2, *Seminumerical Algorithms*, London: Addison Wesley, 1981.)

The RANDOM function uses a seed to generate a random number. Each random number generated becomes the seed for the next random number. Therefore, a sequence of random numbers can be repeated by starting with the same seed. You can store a new seed with the SEED function. If memory is cleared, the seed is reset to zero. A seed of zero will result in the calculator generating its own seed.

For most applications, especially documented ones, it is most desirable to start with a specified seed for testing and so others may repeat and validate your results. Additional details of the current models RAND function may be found in Appendix A.

## PRNG and TRNG Applications

The more obvious calculator applications of RAND (as it is usually notated on an HP calculator keyboard) are described in the Introduction. Selected example RAND programs for various HP “languages” e.g. Classical RPN: HP-15C, HP35s; ENTRY RPN: HP30b; RPL: HP48gII, HP50g; and Pascal like: HP39gII may be found in Appendix B. Cryptology requires the highest quality random numbers and a TRNG is essential. Bank and Internet transactions are examples. Simulations and research applications that use Monte Carlo<sup>(5)</sup> methods require billions of high quality random numbers. In certain simulations a PRNG may be suitable.

## Scaling

HP Calculators produce pseudorandom numbers from 0 to 0.999 999 999 999. If you want other values such as 6 for a die or 52 (both integers) for a deck of cards you must scale the 0.nnn nnn nnn nnn PRNG output range to another range. The general formula to use is:

$$\text{rand integer } (x,y) = (\text{round}(\text{rand}() * (y-x))) + x$$

(x represents the minimum and y the maximum integer value).

An example is 1 to 6 for a standard die.

$$\text{Random } 0,0.\text{nnn nnn nnn nnn} = \text{ROUND} (\text{RAND} * (6-0) + 1) = \text{ROUND } 0.\text{nnn nnn nnn nnn}$$

An RPL program to do this is: `<< RAND 6 * CEIL >>` where CEIL returns the smallest integer greater than or equal to the argument. Each execution returns a random digit from 1 to 6. Using  $\sqrt{5}$  the result of 102 throws of the die are: 1=17, 2=21, 3=10, 4=15, 5=20, 6=19. A uniform distribution would be 17 of each.

## Non-uniform Rand distributions

The output of most random number generators has a uniform distribution but there are applications that require that the numbers be distributed non-uniformly<sup>(6)</sup>. The basic uniformly-distributed random number generators play a vital role in generating non-uniformly distributed random numbers. Mapping the (0,1] range of a basic uniform random number generator to any other range (A,B] is very easy and uses the following equation:

$$X = A + (B-A) U$$

The notation (x,y) signifies a range that includes the value of x but very closely approaches y. The variable U is the uniform random number in the range (0,1). The range of 0 to 1 is useful in generating non-uniformly distributed random numbers by applying that range to cumulative distribution functions.

Since any cumulative distribution function has values that range from 0 to 1, we generate a random number in that range and then calculate the value in a non-uniform distribution that has the matching cumulative value. Here is a simple example that illustrates the concept. Consider the exponential probability distribution function (PDF):

$$f(x) = \lambda e^{-\lambda x} \text{ for } x > 0 \text{ and } \lambda > 0$$

The cumulative distribution function (CDF)  $F(x)$  is:

$$F(x) = 1 - e^{-\lambda x}$$

To generate an exponentially distributed random number we use the inverse of the CDF:

$$x = F^{-1}(U) = \ln(1/U) / \lambda$$

This method is called the *inversion method*. It is a popular one, but not the only method. Table 3 shows a set of probability distributions to which we can easily apply the inversion method.

**Table 3 – Examples of Simple Probability Distributions Where the Inversion Method Can Be Easily Applied**

<b>Probability Distribution</b>	<b>Probability Distribution Function</b>	<b>Cumulative Distribution Function</b>	<b>Inverse CDF</b>
Weibull	$a x^{a-1} \exp(-x^a); x > 0, a > 0$	$1 - \exp(-x^a)$	$(\ln(1/U))^{1/a}$
Logistic	$1/(2+e^x+e^{-x})$	$1/(1+e^{-x})$	$\ln(U/(1-U))$
Cauchy	$1/(\pi (1+x^2))$	$1/2 + 1/\pi \arctan(x)$	$\tan(\pi U)$
Pareto	$a/x^{a+1}$ for $a > 0$ and $x > 1$	$1 - 1/x^a$	$1/U^{1/a}$

Unfortunately Table 3 shows the minority of probability distributions to which the inversion method can be easily applied. Other distributions, like the normal distribution, Student-t, Chi-Square, and the Fisher F distributions, all require extensive and/or iterative calculations to use the inversion method. For example,

The PDF for the normal distribution is:

$$\text{PDF}(x) = \exp(-(x-\mu)^2/2\sigma^2)/(\sigma\sqrt{2\pi})$$

The CDF for the normal distribution is:

$$\text{CDF}(x) = [1 + \text{erf}((x-\mu) / \sqrt{2\sigma^2})]/2$$

Where  $\text{erf}(x)$  is the error function. Calculating the values for this function typically involves a summation series. Solving for  $x$  in the above equation, given a uniform random number (as the value for  $\text{CDF}(x)$ ) requires an iterative root-seeking process in which each iteration has to calculate one or more values for the error function.

In the case of a simple normal distribution (with a mean of 0 and a standard deviation of 1), the PDF is:

$$\text{PDF}(x) = \exp(-x^2/2)/\sqrt{2\pi}$$

Statisticians have developed a much simpler and non-iterative algorithm to generate standard normally distributed random numbers. The method, which generates pairs of random numbers, uses the following algorithm:

1. Generate uniform random numbers  $U_1$  and  $U_2$ .
2. Calculate  $\text{Rand1} = \sqrt{\ln(1/U_1)} \cos(2\pi U_2)$
3. Calculate  $\text{Rand2} = \sqrt{\ln(1/U_1)} \sin(2\pi U_2)$

You can store the value of Rand2 and use it when you want the next normally distributed number. Alternately, you can obtain a single random number using the expression (Rand1 + Rand2) mod 1. The book *Computation Methods in Statistics and Econometrics*, by Hishashi Tanizaki offers excellent short and easy to read FORTRAN subroutines that generate non-uniform random numbers. Tanizaki uses a very clever approach by generating non-uniform random numbers based on the values of other, simpler-to-calculate, non-uniform random numbers. Thus, the author avoids complicated and CPU-intense calculations.

Another approach uses the following rational polynomial approximation to calculate x, the normally distributed random number:

$$x = t - (c_0 + c_1 t + c_2 t^2) / (1 + d_1 t + d_2 t^2 + d_3 t^3)$$

Where t is defined as:

$$t = \sqrt{\ln(1/U^2)}$$

And the constants are  $c_0= 2.515517$ ,  $c_1= 0.802853$ ,  $c_2= 0.010328$ ,  $d_1=1.432788$ ,  $d_2=0.189269$ , and  $d_3=0.001308$ .

Once you have the standard normal random number x, you can map it onto a general normal distribution with a mean  $\mu$  and standard deviation  $\sigma$  using the following equation to get y:

$$y = (\sigma x + \mu)$$

Statisticians have created similar approximations for the Student-t, Chi-Square, and Fisher F distributions.

It is worth mentioning that the HP 39gII implements functions that generate non-uniformly distributed random number for the normal, binomial, Chi-Square, Student-t, Poisson, and Fisher F distributions. Tables 4 shows these functions with sample calls. The last argument in each sample call represents a uniformly distributed random number in the range of (0,1]. These functions make generating random numbers for the above distribution a breeze!

**Table 4. The list of functions in the HP 39gII that can generate popular non-uniform random numbers**

<b>Distribution</b>	<b>Example of Function Call</b>
Normal	normald_icdf(0, 1, 0.841344746069) returns 1
Binomial	binomial_icdf(4, 0.5, 0.6875) returns 2
Chi-Square	chisquare_icdf(2, 0.952641075609) returns 6.1
Fisher F	fisher_icdf(5, 5, 0.76748868087) returns 2
Poisson	poisson_icdf(4, 0.238103305554) returns 2
Student-t	student_icdf(3, 0.0246659214813) returns 3.2

## Observations and Conclusion

The use of randomness is extensive in our high tech society. Randomness is used for bingo games, lotto drawings, coin flipping, dice rolling (for board games) and gambling in places like Las Vegas. There are two sources of randomness. (1) A physical process such as rolling dice, or (2) a mathematical operation, such as that implemented on a computer or HP calculator. Many HP calculators provide randomness features in the form of a pseudorandom number generator, PRNG, – vs. a true random generator, TRNG, such as the throwing of dice - and a SEED function. Unfortunately most calculator manuals do not provide very much information on the RAND and SEED/RDZ randomness functions and it is the purpose of this article to expand on this information. All HP calculators with these features made since the late



1980's use the same algorithm as detailed in Appendix A. HP's PRND will generate more than enough quality random numbers for any calculator program you can run in your lifetime.

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## Notes for: The Randomness of HP

- (1). *The primary assumption is that the coin is homogenous, uniform, throughout. Another assumption is that the coin is symmetrical about a center slice through the coin between the head and the tail sides. This is not true in reality but the raised portions of each side are quite close. Yet another assumption is that the nervous system of a human being is not consistently repeatable in terms of the force and direction of the thumb and fingers when making the flip and the distance traveled while turning. For an additional resource see: <http://en.wikipedia.org/wiki/Randomness>*
- (2). *For a very readable overview of randomness see an article titled Randomness As A Resource by Brian Hayes in American Scientist, July-August 2001, Volume 89, p300. See and download the article at: <http://www.americanscientist.org/issues/pub/randomness-as-a-resource> For an excellent source of true random numbers (based on atmospheric noise) and other randomness information see: <http://www.random.org/> If you only visit one randomness website this should be the one.*
- (3). *The critical issue here is being well tested and documented by multiple team efforts using the best testing tools available. The constants used in the algorithms are critical and once a good algorithm is developed it should not be altered because the results will no longer be valid. HP uses a good quality low output algorithm. Documentation (especially in the User's Manual) is sparse and this article will provide additional information and resources. See <http://www.random.org/> Especially see: <http://www.random.org/analysis/> Additional test descriptions may be found at: [http://en.wikipedia.org/wiki/Diehard\\_tests](http://en.wikipedia.org/wiki/Diehard_tests) Also see: <http://csrc.nist.gov/groups/ST/toolkit/rng/index.html>*
- (4). *A discussion of using a computer to download 10,000 digits of  $\pi$  and counting the distribution of the digits is described and illustrated in **HP Solve** # 25 page 69, Table 5. Each digit is very well distributed with an average of 1,000 occurrences of each digit 1 through 0. The Odd/Even distribution is not quite so uniform with occurrences being 1004/994. To read the article See: [http://h20331.www2.hp.com/hpsub/downloads/HP\\_Calculator\\_eNL\\_09\\_September\\_2011.pdf](http://h20331.www2.hp.com/hpsub/downloads/HP_Calculator_eNL_09_September_2011.pdf)*
- (5). *An outline of the Monte Carlo process may be found in the Introduction at: [http://en.wikipedia.org/wiki/Monte\\_Carlo\\_method](http://en.wikipedia.org/wiki/Monte_Carlo_method).*
- (6). *For an excellent discussion of how some of these distributions may be implemented on a calculator see an HHC 2011 presentation by Richard Schwartz. The Power Point version is titled To Deviate Normally (714 KB). The Conference paper, 11 pp, is titled Generating Normal Deviates (145 KB). The HHC proceedings may be obtained at: <http://www.pahhc.org/ppccdrom.htm> You may also request copies from Richard J. Nelson: [rjnelsoncf@cox.net](mailto:rjnelsoncf@cox.net)*
- (7) *Bug Hunter Joseph K. Horn explains. "Bottom Line: HP 33s and 35s programs that use random numbers cannot be relied upon to obtain precisely the same results as the other HP models that use the same RNG engine. This is critically important if the RNG is re-seeded with one of these different random numbers; in that case, the random number sequence in the 33s and 35s will diverge from the other models."*

Example of critical divergence: 11 RDZ RAND RDZ RAND RAND RAND --> (Note: RDZ is used to store the seed)  
RPL models: 0.975035362027, 0.529463266203, 0.783522353434  
33s & 35s: 0.826166290494, 0.626934236292, 0.213277976998  
As you can see, the obtained sequence is totally different.

# Appendix A – HP’s Pseudorandom Number Generator

## HP 48 Random Number Generator

*Richard J. Nelson*

The following discussion of the HP 48 Pseudorandom number generator is based on postings made by John H. Meyers on May 28, 1997 to the Newsgroups: comp.sys.hp48<sup>(A1)</sup>. John disassembled the ROM code to better understand how the algorithm works. **This PRNG is used on all HP calculators since the introduction of the RPL machines in the late 1980’s.** See Table 2 in the article text and Note (7) above.

The process used by the HP 48 to generate “random” numbers is one that has been tested by some of the best minds in the field of mathematics and it is well known for its mathematical properties of randomness. These programs are known as pseudorandom number generators and they have many uses in games, simulations, and modeling. An important aspect of pseudorandom number generators is being able to *repeat* the sequence of random numbers.

Most random number generators start with a number called the seed and apply a mathematical process to the seed value. The result is the generator output which also serves as the next value for the seed. Each random number is the result of the previous number. If 100 numbers are generated with a given seed the same sequence will be generated if the same seed is used again at the start. This aspect of random number generators is important for experiments, which must be repeated by others for verification.

A major concern is the non-repeating length of the random number sequence — cycle length. If the same value of the initial seed is used by the random number generator, the sequence will repeat. The non-repeating sequence length must be many times longer than the number of RNs needed to complete the task at hand. The HP 48 random number generator has a sequence length of 50,000,000,000,000 (5E13).

The following program generates 200 random numbers on the stack in one second. << 1 200 START RAND NEXT >>. This program will run on a fast (3.7 Mhz.) HP 48 for more than 7,922 years before the cycle repeats. This is certainly more than adequate for any HP handheld calculator program. Today’s much faster models (HP38gII) are 20 times faster and you will only have 396 years of continuous non-repeating random numbers.

The second command related to random numbers on the HP 48 is RDZ. This command accepts the level one value and stores it as the RAND seed. A common seed is generated by taking the reciprocal of ‘e’ with the key sequence: 1 e^x 1/x. Add these three commands at the beginning of the program for repeatable results. The seed used for testing the machines in Table 2 of the current machines is  $\sqrt{5} = 2.2360679775$ . (2.23606797749979)

If the machine is powered up for the first time or you use zero as the seed the system sets the seed value to 999,500,333,083,533. The HP 48 uses 15 digit arithmetic internally and truncates the answer to 12 digits. If you are interested in random number generators here are four good sources.

1. “Semi-Numerical Algorithms”, Volume 1, The Art of Computer Programming, Addison Wesley, 1969, by Donald E. Knuth. See Volume 2 page 9.
2. Numerical Recipes in C by Press; Teukolsky, Vetterling, & Flamery; Cambridge University Press. See Chapter seven.
3. HP 48 Goodies Disk No. 9 (#GD9) See Postings Directory, text file RAND.DOC.
4. PPC ROM User’s Manual . See RN routine on page 380 and GN routine on page 176.

Technically the HP random number generator is described as a multiplicative linear congruential generator. The equation (used by the HP 28C and all following models) is:

$$X_{n+1} = (a * X_n + c) \pmod m \quad \text{Where:}$$

- $X_{n+1}$  = next random number
- $a = 2,851,130,928,467.$
- $X_n$  = seed or previous random number.
- $c = 0$
- mod = modulo function.
- $m = 1E15.$

### Choosing a seed

The more digits you use for the seed the better. Rather than keying in a 12 digit number it is more convenient to use a function to generate the digits. A second consideration for the longest cycle is a seed that is *not* divisible by two (it should be odd) or five. The output is *not* “ruined” but it will not strictly conform to the randomness tests if it is not an “odd” seed for a period of 5E13 instead of 5E15. One way to verify that these conditions are met is to test the number. ‘**TS1**’, **T**est **S**eed version **1** tests an integer number not to be divisible by 2 or 5. It returns a one if the number is a “good” number.

‘**TS1**’ << DUP 2 MOD SWAP 5 MOD AND >>

27.5 Bytes, #EA6h. Timing: 123456789012 ⇒ 0 in 9.01\_ms., 123456789011 ⇒ 1 in 8.89\_ms.

Here is how ‘**TS1**’ works. ‘**TS1**’ is similar to the built-in tests in that it returns a one for pass, or a zero for fail. If the input integer is NOT divisible by 2 or 5 it is a “good” number and the result is a one. A copy of the input number, n, is made with DUP. If n is evenly divisible by 2 the result of 2 MOD is 0, otherwise it is 1. The result of the first MOD “test” is Swapped with n and a similar “test” is made with 5 MOD. A zero results if n is evenly divisible by five and one through four otherwise. At this point level two may be either zero or one (non-zero) and level one may be either zero or non-zero. The logic operator AND compares level two with level one according to the “rules” in the table below.

AND Truth Table

Input	Input	Output
Level 2	Level 1	AND
0	0	0
0	1	0
1	0	0
1	1	1

The one in the truth table is any non-zero real according to the rules defined in the Advanced User’s Reference Manual, **AUR**. ‘**TS1**’ is great for integers, but fails for decimal numbers. 0.88 is divisible by two yet it passes the ‘**TS1**’ test. The reason for this is MOD only works for integers. Let’s solve this problem by converting any number, decimal or mixed, into an integer with ‘**TS2**’.

‘**TS2**’ << ABS MANT DUP →STR 3 OVER SIZE SUB SIZE ALOG \* >>

37.5 Bytes, #58C0h. Timing: 0.123456789012 ⇒1234567890012 in 24.0\_ms.

Here is how ‘**TS2**’ works. ABS insures that only positive numbers are used. Negative numbers may result from some math functions. MANT returns the mantissa of the number in the form N.N ...N. The test input 0.123456789012 becomes 1.23456789012. The goal is to determine how many places, digits, there are after the decimal point. A copy is made with DUP and the level one number is made a string with →STR. The first position in the string is followed by a decimal point. That means that we want the third through the total, SIZE, number of characters and OVER makes a copy of the string, SIZE gives the

total, and SUB returns the desired characters. Another SIZE provides their number and ALOG makes a power of ten equal to the SIZE value. The last command multiplies the MANT number by this number effectively moving the decimal point to the far right converting the decimal number into an integer.

Any input to 'TS2' is converted into an integer. 'TS1' tests it for meeting the non-divisible by 2 or 5 rule. The last task is to convert a "good" number back to a decimal number to use as a seed. 'TS3' performs this task. Any number may be used as a seed. It does not have to be a decimal number. Let's explore the inverse of 'TS2' for "educational/programming" purposes.

'TS3' << MANT 10 / >> 25.5 Bytes, #4DEAh. 123456789011  $\Rightarrow$  0.123456789011 in 6.5\_ms.

Does RAND generate numbers that are not recommended as *initial* seeds? We now have the tools to test numbers so let's test the RAND output just for fun.

'TS4' << DO RAND DUP 'SEED' STO TS2 1200 .05 BEEP UNTIL TS1 END >>

74.0 Bytes, #32beh. NOTE: Commands not native to the HP 48 are in **bold**.

'TS4' has great entertainment value. It produces a 'chirp' for each RAND number generated. Our 'TS1' test is applied and if it is a good seed the program ends with the RAND value stored in a variable called 'SEED'. The program generates another random number if the first one is not acceptable. It will continue to 'chirp' until a good one is found. I have heard (actually the program was modified to count them) eleven consecutive 'chirps' on one occasion. The chirps is NOT an indication of a PRNG failure.

Here is how 'TS4' works. A DO ...UNTIL ... END loop structure is used. The DO clause is the commands between DO and UNTIL. The commands between UNTIL and END is the test clause. The first command in the DO clause is RAND. A copy is made with DUP with one of the RAND numbers stored in 'SEED'. TS2 converts the decimal number to an integer and a 1,200 hertz tone of 0.05 seconds duration is sounded with BEEP. The UNTIL clause is simply our test program TS1. It returns a one if the number is acceptable. If this is the case the END is executed and the program is finished. If TS1 produces a zero, the DO clause is executed again. The DO loop repeats the DO clause until the UNTIL clause tests true.

I mentioned that I "counted" the number of 'chirps'. This is done with 'TS5' with the addition of the three underlined commands at the beginning. The operation of the program is similar to 'TS4'.

'TS5' << 0 DO 1 + RAND DUP 'SEED' STO TS2 1200 .05 BEEP UNTIL TS1 END >>

81.5 Bytes, #145Ch. NOTE: Commands not native to the HP 48 are in **bold**.

You may leave out 0, 1 + 1200 .05 and BEEP and use the modified program,

'TS6', as a create-a-seed-and-record-it program. Add an RDZ command after the END if you call this program as part of your main program.

'TS6' << DO RAND DUP 'SEED' STO TS2 UNTIL TS1 END (RDZ optional) >>

50.5 Bytes, #E57Fh. NOTE: Commands not native to the HP 48 are in **bold**.

It should be kept in mind that the basis for RAND is 15 digits internally and that the 12 digits we see are

truncated from the internal 15 and that they are NOT necessarily an accurate indication that the PRNG is not working properly. These test programs are intended as educational programming/test examples.

We now have the basic tools to create and store a seed for repeatability purposes. The above programs illustrate the modularity nature of the HP 48 programming language. It is easy to write and test short small modules and call them as needed. This also makes modifications easy. You may use a module just as you do a built-in command as illustrated above. Of course, you may key in the respective code into the program to make it a complete stand-alone program if you wish. The interactive stack makes assembling programs this way easy, and the amount of re-keying is kept to a minimum. These programs are provided as an educational exercise for those readers wanting to explore programming and RAND.

After many users have researched the performance of the HP random number generator it is not surprising to find that HP has done an excellent job in the programming of the PRNG. **The ONLY requirement the user has for using RDZ and selecting a seed is that it be as many decimal digits as possible.** All other requirements are automatically taken care of by the algorithm. ANY 12 digit number will suffice with an exponent up to 99 maximum. The not divisible by 2 or 5 is taken care of by internally shifting the exponent so it always ends in one. e.g. 362 becomes 621, 539 becomes 391, 045 becomes 451. Of course you cannot know this unless you examine the internal ROM programming.

Another aspect of the HP 48 RNG described by John H. Meyers is the details of storing zero as a seed. The HP 48G Series User's Guide mentions that 0 RDZ sets the seed according to the system time. The TICKS counter uses five nibbles as a real time counter and the over flow is "counted" to an eight nibble register. The rightmost five HEX digits are taken from the real time clock with the leftmost eight digits taken from memory. The sequence, **12 →TIME 0 RDZ RAND**, however, will not produce consistent results. The reasons for this are unclear. The intentional system clock jitter<sup>(A2)</sup> is likely the major cause. If you run the program 100 times putting the results into a list and sorting them you will find that there are a limited number of variations - usually less than a dozen with the majority of them being one or two values.

---

### Notes for The Randomness of HP Appendix A

(A1) See <http://www.hpcalc.org/search.php?query=John+Meyers+random+Numbers> for additional details.

(A2) *The HP48 system clock is not a smooth running clock. In order to ensure that the calculator's digital signals do not cause radio frequency interference the system clock has a small amount of jitter introduced. This jitter causes a small ambiguity in precise time measurements based on it and the resolution of TICKS (8192 TICKS per second, 122 microseconds) is enough to detect it.*

## Appendix B – Selected PRNG Programs

These programs illustrate the flipping of a coin, rolling a pair of dice, dealing a deck of cards (selection without replacement) the use of scaling, and other assorted RAND applications.

### Selected Random Number Generators for the HP-41 (Classical RPN<sup>B1</sup>)

Also see these at: Jean-Marc Baillard's excellent website at <http://hp41programs.yolasite.com> which contains a sizeable collection of classical RPN programs for Analytical Methods, Arithmetic, Astronomy, Calendars, Complex & Hypercomplex Numbers, Differentiation & Integration, The Earth, Geometry, Matrices, Polynomials & Rational Functions, special Functions, Spectral Analysis, Functions of Several Variables, Statistics (*random numbers*), Physics, Games, Miscellaneous, and Links to other program collections.

Five pseudo random number generators are listed here. "RNG1" "RNG2" "RNG3" work on every HP-41. "RNG4" requires a Time-Module. Finally, the last program is an attempt to play (win?) the lottery.

#### Program#1

A well known RNG is given by the formula:  $x_{n+1} = \text{FRC}(9821 x_n + 0.211327)$  which provides 1 million random numbers. The following program gives 1,000,000 random numbers  $r$  ( $0 \leq r < 1$ ). The formula  $x_{n+1} = \text{FRC}(9^8 x_n + 0.236067977)$  is used.

The coefficient  $9^8 = 43,046,721$  may be replaced by  $a$  where  $a = 1 \pmod{20}$

and  $0.236067977$  may be replaced by  $b$  where  $b \cdot 10^9$  is not divisible by 2 or 5.

01 LBL "RNG1"	06 *	11 FRC	16 *	21 FRC	26 END
02 9	07 FRC	12 *	17 FRC	22 5	
03 ENTER^	08 *	13 FRC	18 *	23 SQRT	
04 ENTER^	09 FRC	14 *	19 FRC	24 +	
05 R^	10 *	15 FRC	20 *	25 FRC	

( 35 bytes / SIZE 001 )

STACK	INPUTS	OUTPUTS
X	xn	xn+1

Example:

0.2 XEQ "RNG1" yields 0.436067977  
 R/S 0.779021394 ... etc ...

#### Program#2

"RNG2" provides 9,999,999,996 random numbers with the formula:  $x_{n+1} = (10^{59} x_n) \text{MOD } p$  where  $p = 9,999,999,967$  is the greatest prime  $< 10^{10}$ .  $x_n$  are integers between 0 and  $p$  (exclusive) which are then divided by  $p$  to be reduced to a number between 0 and 1. This routine works well because the MOD function gives exact results even when the operands are greater than  $10^{10}$ . Actually, the exponent 59 may be replaced by any integer  $m$  provided  $m$  is relatively prime to  $p-1 = 2 \cdot 3 \cdot 11 \cdot 457 \cdot 331543$ , but I don't know what is the best choice. Unlike "RNG1" and other routines based upon the same type of formulae, the least significant digits don't go through any cycle of ten, one hundred and so on. Register R00 is used to store the different  $x_n$  integers.

01 LBL "RNG2"	04 *	07 33	10 STO 00	13 END
02 RCL 00	05 10	08 -	11 LASTX	
03 E59	06 10^X	09 MOD	12 /	

( 26 bytes / SIZE 001 )

STACK	INPUTS	OUTPUTS
X	/	$0 < r < 1$

Example:

1 STO 00

XEQ "RNG2" gives 0.3129146797 (and R00 = 3129146787 = 1059 (mod p))

R/S gives 0.6904570204 (R00 = 690457018 ) ... etc ...

Actually if p is a prime,  $(\mathbb{Z}/p\mathbb{Z} - \{0\}; *)$  is a group and if a is an integer, the number of distinct elements in the subset  $\{ 1; a; a^2; \dots; a^k; \dots \} \pmod p$  divides p-1

If p-1 is the smallest positive integer q such that  $a^q = 1 \pmod p$ , then the sequence  $a; a^2; \dots; a^k; \dots; a^{p-1} \pmod p$  is a permutation of  $1; 2; \dots; p-1$

In particular, if  $p = 2p' + 1$  where p' is also a prime, and if  $a^{p'}$  is not equal to  $1 \pmod p$  then a satisfies the required property.

For instance,  $p = 7,841,296,787 = 2 * 3,920,648,393 + 1$  7,841,296,787 and 3,920,648,393 are primes and  $-1024 = 4,851,307,369 \pmod p$  satisfies  $(-1024)^{p'} = -1$  therefore the routine:

E24

\*

CHS

7841296787

MOD

gives 7,841,296,786 random integers. These ideas may be used to create your own RNG.

### Program#3

The following algorithm is given by Clifford Pickover in "Keys to Infinity" (John Wiley & Sons) ISBN 0-471-11857-5

01 LBL "RNG3"	03 E2	05 1	07 END
02 LN	04 *	06 MOD	

( 17 bytes / SIZE 001 )

1<sup>st</sup> 3 Ex .25x 50%

STACK	INPUTS	OUTPUTS
X	xn	xn+1

Example:

0.1 XEQ "RNG3" produces 0.74149070

R/S gives 0.09073404 ... etc ...

### Program#4

01 LBL "RNG4"	04 +	07 PI	10 R-D
02 DATE	05 E49	08 MOD	11 FRC
03 TIME	06 *	09 LN1+X	12 END

( 25 bytes / SIZE 000 )

STACK	INPUTS	OUTPUTS
X	/	$0 < r < 1$

I cannot give any example since the result depends on the instant you press R/S

### Program#5 - Winning the Lottery?

If you need, for instance, 7 integers between 1 and 49 , you can use 7 random numbers between 0 and 1, multiply them by 49 , add 1 and take the integer part of the result. The small routine hereafter is another possibility, if you accept to calculate the integer part in your mind:

```
01 LBL "$$$"    03 49      05 1      07 END
02 R-D          04 MOD     06 +
```

( 16 bytes / SIZE 000 )

STACK	INPUTS	OUTPUTS
X	r	$0 < N < 50$

Example:

```
41 XEQ "$$$" yields 47.1270
R/S 6.1759, R/S 1.8535, R/S 43.1567, R/S 23.6963, R/S 35.6962 R/S 37.2418
suggesting 47-06-11-43-23-35-37
```

Notes:

- 1- I'm not a statistician and I can't assure all these RNGs would stand up to sophisticated tests, but one may use his imagination in devising variations.
- 2- If you win one million dollars thanks to one of these programs, I accept to share the jackpot...

## Selected RPL RAND Application Programs (HP48/49/50)

### More meaningful random passwords (Wlodek Meir-Jedrzejowicz)

Many people like to have a random password generator rather than make up their own. The best advice is to combine upper case, lower case, digits and special symbols, but that can make for very unmemorable

passwords. It is often enough to use a string of 7 or more lower case letters - at least those make up something that can be related as a foreign word! The following program generates a string of 7 lower case characters - change 7 to another number if you wish. It uses `RAND 26 * 96.5` to generate the random letter between a and z. 96.5 is used instead of 97 because `CHR` rounds to the nearest number. Totally random letter combinations contain too few vowels, and too many letters from the end of the alphabet, so I add `SQ` after `RAND` to increase the likelihood of the early letters, which contain a higher proportion of vowels. This makes for a higher proportion of readable words, though with too many "a"s in them. Run the program repeatedly until you find a password you like!

```
'RPAS' << "" 1 7 START RAND SQ 26 * 96.5 + CHR + NEXT >>
```

13 commands, 61 bytes, F0B2h. Timing: with  $\pi$ RDZ,  $\Rightarrow$  "htabbst" in 176ms. Following: "aaekkg", "sjatauz".



**How many random numbers are required (summed) to be  $\geq$  to n?** (Detlef Müller)

This is an unusual application of START...STEP. Key n and execute 'NRN'.

```
'NRN' << 0 0 ROT START 1 + RAND STEP >>
```

8 commands, 30.0 Bytes, # 43DCh.

Reminder. In order to get the same results you must first store a value in RDZ. If RDZ starts with the  $\sqrt{5}$  the results for n = 7 are. 17, 14, 13, 15, 16, 15, 11 . . .

**Dealing a deck of cards**

Drawing a BINGO<sup>(B2)</sup> number or dealing a deck of cards<sup>(B3)</sup> are examples of using random numbers as selection without replacement. A fixed number of random numbers ordered randomly and used. This problem may be solved in many ways depending on how it will be used. Breaking the program into parts allows the reader/student to better understand how the programs work. Four HP 48 programs are used as described in Table B0.

**Table B0 – RPL Card Dealing Program Statistics**

Name	Description	Size	Calls	Chk Sum <sup>[a]</sup>
<b>DEAL</b>	Deals a card.	59.5 B	<b>DECK, &amp; CRDID</b>	# 6004h
<b>CRDID</b>	Identifies/displays the card with value & suit as a text string.	219 B	—	# 31BDh
<b>DECK</b>	A list of randomly ordered card numbers (1 – 13 is Spades, 14 – 26 is Hearts, 27 – 39 is Diamonds, 40 – 51 is clubs).	479 B	—	#3FA3h
<b>MAKE</b>	Creates a list of randomly ordered card numbers 1 to 52.	77.5 B	—	# 47B8h
<b>SEED</b>	Makes repeatable deck, not required, only used for testing.	17.5 B	—	# 7CF0h

Note: Check sum is for program only. The deck is based on a seed of  $\sqrt{5}$ . Variables not native to the HP 48 are in bold.

```
'DEAL' << DECK HEAD LASTARG TAIL 'DECK' STO CRDID MEM DROP >>
```

Note MEM DROP is not required unless memory is inadequate. This sequence clears LASTARG, etc.

```
'CRDID' << { ACD DUCE TREY FOUR FIVE SIX SEVEN EIGHT NINE TEN JACK QUEEN KING } OVER 1 – 13 MOD 1 + GET " of " + { Spades Hearts Diamonds Clubs } ROT 13 / CEIL GET + >>
```

Note that the text is entered as a name which is automatically converted to a string when it is concatenated with the " of " text string.

```
'DECK' { 17 46 37 24 36 21 41 47 23 52 44 49 32 39 7 19 1 22 12 28 9 5 26 40 34 14 10 38 25 13 2 48 11 29 29 31 42 43 8 27 35 45 15 18 39 33 6 34 51 16 59 } Note: SEED, (RDZ) IS  $\sqrt{5}$ .
```

```
'MAKE' << 1 52 FOR n n DUP RAND * CEIL ROLLD NEXT 52 →LIST 'DECK' STO >>
```

```
'SEED' << 5  $\sqrt{RDZ}$  >>
```

If all 52 cards are delt and the output is put into a list of text strings the list will be 1,006 bytes with a check sum of # 51C2h,

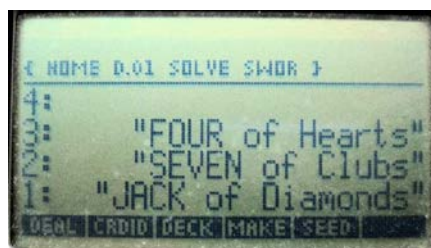


Fig B1 – 1<sup>st</sup> three cards dealt. Seed =  $\sqrt{5}$ .

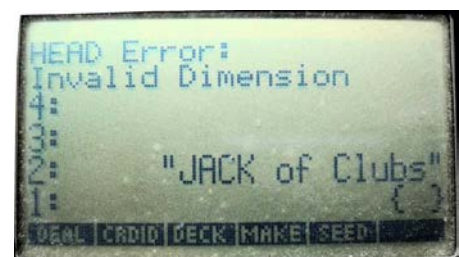


Fig. B2 – Error with 53<sup>rd</sup> card dealt.

## Selected Pascal like **RAND** programs for the HP 39gII

*Namir Shammas*

Three HP 39gII listings for:

1. A program that simulates tossing a fair coin.
2. A program that simulates tossing a pair of dice.
3. A program that simulates shuffling a deck of cards and drawing cards from that deck.

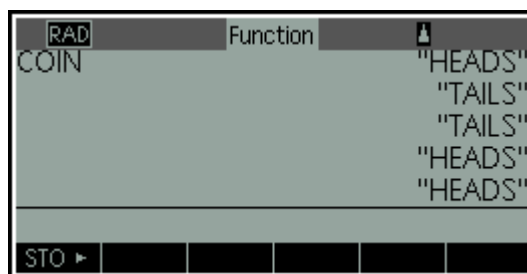
### The Coin Tossing Program

Table B1 shows the listing for the coin tossing program. The program calls the function **RANDOM(0,1)** to generate a random number between 0 and 1. The program passes the random number to function **ROUND** to round that value to either 0 or 1. Based on the result, the program **COIN** returns the text **HEADS** or **TAILS**.

**Table B1 – HP 39gII Program for Flipping a Coin**

Statement	Comment
EXPORT COIN()	
BEGIN	
IF ROUND(RANDOM(0,1),0)==0 THEN	Generate a random number between 0 and 1, and round it to either 0 or 1. Test if the result is 0.
RETURN "TAILS";	If the result is zero, return Tails
ELSE	
RETURN "HEADS";	Otherwise, return Heads.
END;	
END;	

Figure B1 shows sample sessions with the function **COIN**. The output is the result of executing the function **COIN** multiple times.



*Figure B1. Sample sessions with program COIN.*

### The Dice Tossing Program

Table B2 shows the listing of a program that generates the result of rolling a pair of dice. To calculate the value of each die, the program uses the expression **RANDOM(1,6)** rounded to 0 decimals. If you want to change the program to emulate playing with eight-face dice, then use **RANDOM(1,8)** in each of the two assignment statements.

**Table B2 – HP 39gII Program for Throwing Dice**

Statement	Comment
EXPORT DICE()	
BEGIN	
LOCAL d1,d2;	
d1:=ROUND(RANDOM(1,6),0);	Simulate rolling the first die.

Statement	Comment
d2:=ROUND(RANDOM(1,6),0);	Simulate rolling the second die.
RETURN CONCAT(d1,d2);	Return a list of the two dice.
END;	

Figure B2 shows sample sessions with the function **DICE**. The output is the result of executing the function **DICE** multiple times.



Figure B2. Sample sessions with program DICE.

Now let's look at another version of the dice throwing program—one that throws loaded dice. The next program **DICE2** is a function that heavily favors the number 6. Table B3 shows the source code for function **DICE2**. The program generates values for the dice using **RANDOM(1,8)** and stores them in variables **d1** and **d2**. The function **DICE2** examines the values in **d1** and **d2** to determine if they exceed 6. If so, the function assigns 6 to either variables. This scheme gives each of the faces 1 through 5 a 12.5% chance (down from 16.67% for a fair die), while handing the face 6 a whopping 37.5% chance! If you replace **RANDOM(1,8)** with **RANDOM(1,10)** in the program, the face 6 will have a 50% chance against a 10% chance for each of the other die faces.

**Table B3 – The cheater's Dice program**

Statement	Comment
EXPORT DICE2()	
BEGIN	
LOCAL d1,d2;	
d1:=ROUND(RANDOM(1,8),0);	Simulate rolling the first die.
IF d1>6 THEN	If the value of d1 exceeds 6, set it to 6.
d1:=6;	
END;	
d2:=ROUND(RANDOM(1,8),0);	Simulate rolling the second die.
IF d2>6 THEN	If the value of d2 exceeds 6, set it to 6.
d2:=6;	
END;	
RETURN CONCAT(d1,d2);	Return a list of the two dice.
END;	

Figure B3 shows a sample output from the function **DICE2**. Notice how many 6s appear!



Figure B3. Sample sessions with program DICE2.

## The Deck of Cards Program

Unlike the first three programs that are short and simple, the next set of programs are more elaborate. They perform the following tasks:

- Initialize a deck of cards.
- Shuffle the deck of cards.
- Draw a card out of the deck.

### Initializing the Deck

Table B4 shows the listing for the **InitDOC** program that initializes the deck of cards. The first few statements in the listing export the following variables:

- The matrix **cards** which has one row and 52 columns. This matrix stores the numeric codes that represents the cards in the deck.
- The variable **numCards** stores the number of cards in the deck.
- The variable **setNames** stores a list of card face names.
- The variable **cardNames** stores a list of card names.
- The variable **stackHeight** stores the current number of available cards to draw.

The exported matrix, **cards**, is a single-row matrix with 53 columns. To examine its contents at any time between function calls, copy the data in that matrix into one of the ten global matrices. Then use the

**Table B4 – The Card Initialization Program**

Statement	Comment
EXPORT cards,numCards;	Export variables used by the other related functions.
EXPORT setNames, cardNames;	
EXPORT stackHeight;	
EXPORT InitDOC()	
BEGIN	
LOCAL i,j,k;	
numCards:=4*13;	Calculate the number of cards in the deck.
cards:=MAKEMAT(0,1,numCards);	Create the row of data that stores the numeric codes for the cards.
k:=0;	
FOR i FROM 1 TO 4 DO	
FOR j FROM 1 TO 13 DO	
k:=k+1;	
cards(1,k):=100*i+j;	Store the numeric codes of the cards in matrix cards.
END;	
END;	
setNames:=CONCAT("Diamonds","Clubs","Hearts","Spades");	Initialize the names of the sets of cards.
cardNames:=CONCAT("Ace","2","3","4","5","6","7");	Initialize the names of the cards.
cardNames:=CONCAT(cardNames,"8","9","10","Jack","Queen","King");	
RETURN "DECK INITIALIZED";-	Return affirmation message
END;	

matrix editor to view the value in the global matrix. Table B5 shows the ranges of values used with each face.

**Table B5– The Numeric Codes for the Cards**

<i>Range</i>	<i>Meaning</i>
101 to 113	Ace of diamonds, 2 of diamonds, ..., queen of diamonds, and king of diamonds.
201 to 213	Ace of clubs, 2 of clubs, ..., queen of clubs, and king of clubs.
301 to 313	Ace of hearts, 2 of hearts, ..., queen of hearts, and king of hearts.
401 to 413	Ace of spades, 2 of spades, ..., queen of spades, and king of spades.

### Shuffling the Cards

This section presents two functions that differently shuffle the deck of cards. The first function, SHUFCARDS, shuffles the cards, computer style, as if they were an array of integers. The basic algorithm performs an in-place shuffling of cards (treated as array of integers). The leading array elements (or rows of the matrix cards) represent the cards *TO BE* shuffled. The trailing array elements represent the cards *THAT WERE* shuffled. Initially, the number of the cards to be shuffled is 52 and the number of shuffled cards is 0. The function selects an element in the range of 1 to 52 and swaps that element with the

last card. Now there are 51 cards to shuffle and one shuffle card. The function selects an element in the range of 1 to 51 and swaps that element with the second-from-last card (at index 51). The third pass selects a card in the range of 1 to 50, and so on.

Table B6 shows the listing of function SHUFCARDS.

**Table B6– The listing for the SHUFCARDS Program**

Statement	Comment
EXPORT SHUFCARDS()	
BEGIN	
LOCAL nc,nc2,i,j,k,temp;	
nc:=numCards;	Set nc to be the initial number of cards to be shuffled.
nc2:=0;	Set nc2 to be the initial number of shuffled cards.
WHILE nc>1 DO	Iterate while there are 2 or more cards to shuffle.
i:=ROUND(RANDOM(1,nc),0);	Select a card in the range of 1 to nc.
j:=numCards-nc2;	Store the index of the next shuffled card.
temp:=cards(1,j);	Swap the shuffled card(1,i) with card(1,j).
cards(1,j)=cards(1,i);	
cards(1,i)=temp;	
nc2:=nc2+1;	Increment the number of shuffled cards.
nc:=nc-1;	Decrement the number of cards to be shuffled
END;	
stackHeight:=numCards;	Initialize the number of available cards to draw.
RETURN "DECK SHUFFLED";	
END;	

It is also possible to shuffle the arrays of cards by repeatedly selecting two cards at random and then swapping them. Here is the algorithm for this approach:

1. For MAX\_SHUFFLE times perform the remaining tasks
2. Select i as a random number between 1 and 52.
3. Select j as a random number between 1 and 52.
4. If i and j are different swaps the values at element i and j.

The above algorithm is easy to implement but does not guarantee that every card is moved to a new random location in the deck. The value of MAX\_SHUFFLE should be high enough to ensure a good card shuffling.

The second card shuffling program emulates splitting the deck of cards around the middle (between 40% to 60% of the number of cards) and then merging the two halves. The function moves the cards from the split deck onto another deck of cards, by alternating card selection from each sub-deck.

Table B7 shows the listing of function **SPLITCARDS**. This function has the parameter **numSplits** which tells the function how many times to split and merge the cards.

### Drawing a Card

Table B8 shows the listing of function **GETCARD** which returns a card from the shuffled deck. The function uses the exported variable **stackHeight** to determine the next card to draw, if there are cards available to draw. The function returns a list containing the face name and the card name.

**Table B7– The listing for the SPLITCARDS program**

Statement	Comment
EXPORT SPLITCARDS(numSplits)	
BEGIN	
LOCAL ii,cardsCopy,median;	
LOCAL i,i1,i2;	
cardsCopy:=MAKEMAT(0,1,numCards);	
FOR ii FROM 1 TO numSplits DO	Repeat the shuffling of cards numSplits times.
FOR i FROM 1 TO numCards DO	Make a duplicate card deck.
cardsCopy(1,i):=cards(1,i);	
END;	
median:=ROUND(RANDOM(0.4*numCards,0.6*numCards),0);	Select a median for splitting the card.
i:=1;	Initialize the indices for copying cards from the two sub-decks.
i1:=1;	
i2:=median+1;	
REPEAT	Start merging alternating cards from the split deck.
IF i1≤median THEN	Any more cards from the first sub-deck to merge?
cards(1,i):=cardsCopy(1,i1);	Copy the card to the merged deck.
i1:=i1+1;	
i:=i+1;	
END;	
IF i2≤numCards THEN	Any more cards from the second sub-deck to merge?
cards(1,i):=cardsCopy(1,i2);	Copy the card to the merged deck.
i2:=i2+1;	
i:=i+1;	
END;	
UNTIL i1>median AND i2>numCards;	Stop when all cards have been merged.
END;	
stackHeight:=numCards;	Set the number of available cards to draw.
RETURN "DECK SPLIT SHUFFLED";	
END;	

**Table B8– The listing for the GETCARD program**

Statement	Comment
EXPORT GETCARD()	
BEGIN	
LOCAL i,j;	
IF stackHeight>0 THEN	Can we draw another card?
i:=cards(1,stackHeight);	Get the numeric code for the next card to draw.
j:=i;	
i:=INT(i/100);	Calculate card face index.
j:=j-100*i;	Calculate card value.
stackHeight:=stackHeight-1;	
RETURN CONCAT(setNames(i),cardNames(j));	Return a list that identifies the card drawn.
ELSE	
RETURN "NO CARDS AVAILABLE!";	
END;	
END;	

### Sample Sessions

Let’s use the functions **InitDOC**, **SHUFCARDS**, and **GETCARD** to initialize a deck of cards, shuffle the cards, and draw cards, respectively. Figure B4 shows the output when using the following functions:

- The function **InitDOC** which initializes the deck of cards.
- The function **SHUFCARDS** that shuffles the deck by randomly arranging the cards.
- The function **GETCARD** that draws a card from the deck. Figure B4 shows two calls to function **GETCARD**.

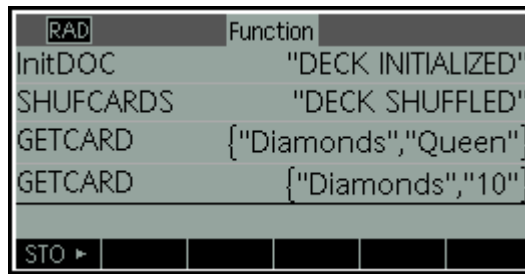


Figure B4. Using functions *InitDOC*, *SHUFCARDS*, and *GETCARD*.

Figure B5 shows additional calls to function **GETCARD**.

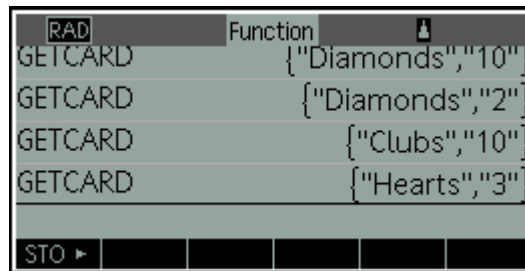


Figure B5. Additional calls to function *GETCARD*.

Now, let's use the functions **InitDOC**, **SPLITCARDS**, and **GETCARD** to initialize a deck of cards, shuffle the cards, and draw cards, respectively. Figure B6 shows the output when using the following functions:

- The function **InitDOC** which initializes the deck of cards.
- The function **SPLITCARDS(7)** that shuffles the deck by splitting and merging the cards seven times.
- The function **GETCARD** that draws a card from the deck.

```

RAD      Function
InitDOC      "DECK INITIALIZED"
SPLITCARDS(7)  "DECK SPLIT SHUFFLED"
GETCARD      {"Diamonds","4"}
STO ►
  
```

Figure B6. Using functions *InitDOC*, *SPLITCARDS*, and *GETCARD*.

Figure B7 shows additional calls to function **GETCARD**.

```

RAD      Function
GETCARD   {"Diamonds","4"}
GETCARD   {"Clubs","Jack"}
GETCARD   {"Spades","2"}
GETCARD   {"Diamonds","9"}
STO ►
  
```

Figure B7. Additional calls to function *GETCARD*.

### Bonus Program

This section presents a bonus program that allows you to check that a shuffled deck contains no duplicates or missing cards. Table B9 contains the listing for the **COMPARE** function. This function has one parameter, **myCards**, which is the single-row matrix that stores the numerical codes for the various cards. The function compares the values in the argument for **myCards** with those in an internally created standard card deck. The function returns the number of missing or duplicate cards. A zero result indicates that the argument for **myCards** stores a valid deck of cards.

Figure B8 shows a sample session for using function **COMPARE**. The output of this function shows that the shuffled cards, stored in variable **cards**, match the cards of a standard deck.

```

RAD      Function
InitDOC      "DECK INITIALIZED"
SPLITCARDS(7)  "DECK SPLIT SHUFFLED"
COMPARE(cards)  0
STO ►
  
```

Figure B8 - Additional calls to function *GETCARD*



Let's test **COMPARE**'s ability to catch errors in a corrupted deck of cards. Perform the following tasks:

- Execute the **InitDOC** function to initialize the deck of cards.
- Set cards(1,2) to 101 by typing **cards(1,2)=101**.
- Set cards(1,3) to 101 by typing **cards(1,3)=101**. Now we have two duplicates 101 and no values for 102 and 103.
- Execute **COMPARE(cards)**. The function returns 2, the number of duplicate/missing cards.

Figure B9 and B10 show the above tasks.

**Table B9 – The listing for the COMPARE Program**

Statement	Comment
EXPORT COMPARE(myCards)	
BEGIN	
LOCAL stdCards;	
LOCAL i,j,k,n;	
stdCards:=MAKEMAT(0,1,numCards);	Create matrix that will store the data for a standard deck of cards.
k:=0;	
FOR i FROM 1 TO 4 DO	Start nested loop to store numerical codes for cards in matrix stdCards.
FOR j FROM 1 TO 13 DO	
k:=k+1;	
stdCards(1,k):=100*i+j;	
END;	
END;	
n:=numCards;	Initialize number of different cards.
FOR i FROM 1 TO numCards DO	Start looping for each member of matrix stdCards.
FOR j FROM 1 TO numCards DO	Start looping for each member of matrix myCards.
IF stdCards(1,i)==myCards(1,j) THEN	If the values of stdCards(1,i) and myCards(1,j) have the same values, decrement variable n.
n:=n-1;	
j:=numCards;	Set j to last loop value for an early loop exit.
END;	
END;	
END;	
RETURN n;	Return the number of different cards.
END;	

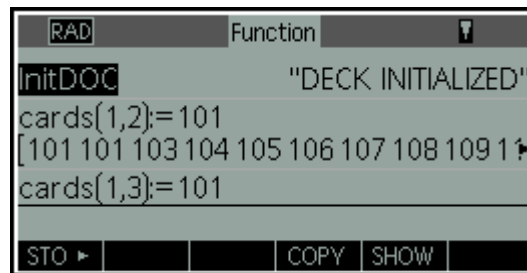


Figure B9. Testing a corrupted deck of cards, part 1.

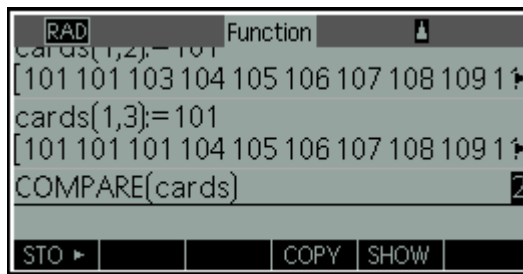


Figure B10. Testing a corrupted deck of cards, part 2.

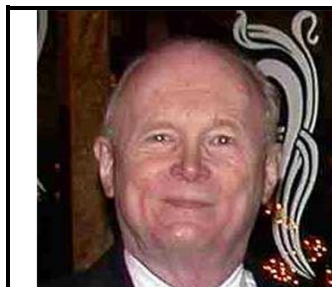
## References

1. Hishashi Tanizaki, *Computation Methods in Statistics and Econometrics*. Available from <http://stat.econ.osaka-u.ac.jp/~tanizaki/cv/books/cmse/cmse.pdf>.
2. Luc Devroye, *Non-Uniform Random Variate Generation*. Available from <http://www.eirene.de/Devroye.pdf>.
3. HP-65 STAT PAC 1 manual, page 28.
4. HP 39gII User's Guide, Edition 1, November 2011.

## Notes for The Randomness of HP Appendix B

- (B1). For a discussion on how HP's RPN has evolved see the article *HP RPN Evolves* (pdf file) at: <http://h20331.www2.hp.com/hpsub/downloads/S07%20HP%20RPN%20Evolves%20V5b.pdf>
- (B2) These programs are taken from an Educalc class handout from instructors Joe Horn & Richard Nelson titled *BINGO – Part I* (identifying the balls, 2 pp.) and part II (creating, mixing, and selecting the balls, 4 pp.) dated March 27, 1998.
- (B3) The card identification program was taken from a 7 pp. HHC 1994 HP 49 Programming Problem CHIP Meeting – June 24, 1994 titled *Describe Playing Card from its number* by Richard J. Nelson. This is written from a Programming Exercise perspective which includes multiple programs for a problem with each version making improvements towards an optimum shortest/fastest program as a student's thought process to learn programming.

## About the Author



Richard J. Nelson has written hundreds of articles on the subject of HP's calculators. His first article was in the first issue of *HP 65 Notes* in June 1974. He became an RPN enthusiast with his first HP Calculator, the HP-35A he received in the mail from HP on July 31, 1972. He remembered the HP-35A in a recent article that included previously unpublished information on this calculator. See <http://hhuc.us/2007/Remembering%20The%20HP35A.pdf>  
He has also had an article published on HP's website on HP Calculator Firsts. See <http://h20331.www2.hp.com/Hpsub/cache/392617-0-0-225-121.html>.  
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