A Graphical Look At Continuity — Teacher Notes

Topic: Continuity

NCTM Standard

- Organize and consolidate their mathematical thinking through communication; communicate their mathematical thinking coherently and clearly to peers, teacher, and others.

Objectives

The student will be able to develop a visual understanding of how limits and continuity relate and be able to understand and communicate what it means for a function to be continuous at a point.

Getting Started

This activity will have students explore the concept of continuity at a point. It will also allow them to discover that simply having a limit at a point will not guarantee that the function is also continuous. It also explores the idea that having a limit is a necessary, but not a sufficient condition to determine the continuity of a function at a point, and through all points.

Prior to using this activity:

- Students should be able to produce and manipulate graphs of functions manually and a graphing calculator.
- Students should be able to produce split-defined (or piecewise) functions.
- Students should have a basic understanding of the language of limits.

Ways students can provide evidence of learning:

- Students should be able to produce graphs of functions and communicate symbolically, graphically and verbally the relationship between having a limit and being continuous.

Common mistakes to be on the lookout for:

- Students may produce a graph on the calculator and not be able to communicate the concept of a split-defined function because the window chosen may produce the appearance of a single unbroken formula.
- Students may confuse the pixel values with the actual function values.

Definitions:

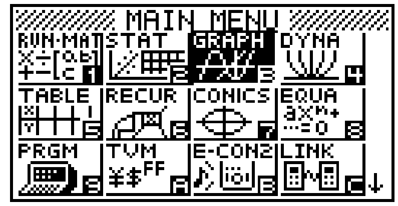
- Asymptote
- Continuity
- Discontinuity
- Limit
- Parabola
- Vertex

The following will demonstrate how to graph a function, graph a split-defined function and examine its behavior on the Casio *fx-9750GII*.

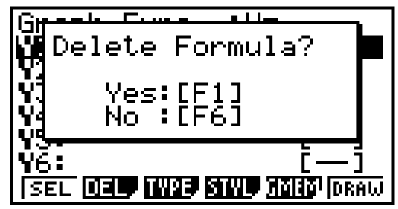
Explore the behavior of the function $f(x) = -x^2 + 3x - 5$.

To display a graph of the function:

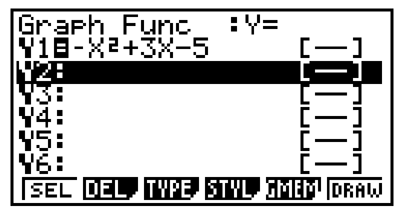
1. From the Main Menu, highlight the GRAPH icon and press [EXE] or press [3].



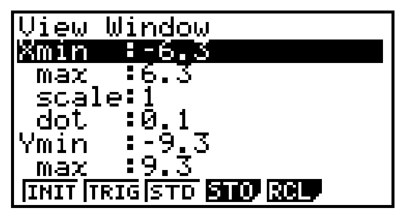
2. To delete any previous equations, highlight the equation and press [F2] (DEL) [F1] (Yes.)



3. Enter the equation in **Y1** by pressing [(−)] [X,θ,T] [x²] [+] [3] [X,θ,T] [−] [5] [EXE].

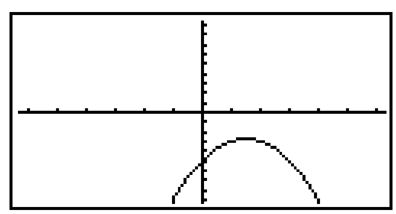


4. Set the view window to the initial screen by pressing [SHIFT] [F3] **(V-Window)** [F1] (INIT). Then, change **Ymin** to –9.3 and **Ymax** to 9.3. This view window will be used in this activity to give the best display of the quadratic functions.



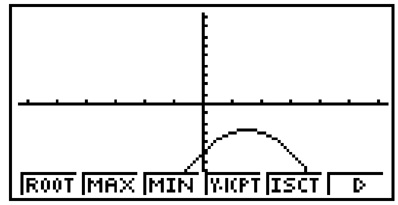
5. Press [EXIT] to return to the initial GRAPH screen.

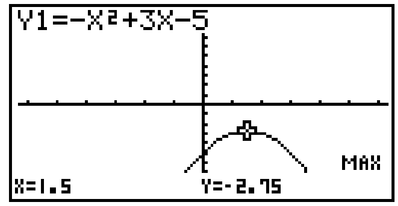
6. Press [F6] (DRAW) to view the graph of the function.



To find the vertex of the graph:

1. Press [SHIFT] [F5] **(G-Solv)** [F2] (MAX). The coordinates of the vertex will be displayed at the bottom of the screen. [Note: [F2] (MAX) was pressed since the vertex of this parabola is the highest, or maximum, point. If the graph of the parabola opened up, the vertex would be the lowest, or minimum, point and you would have chosen [F3] (MIN).]

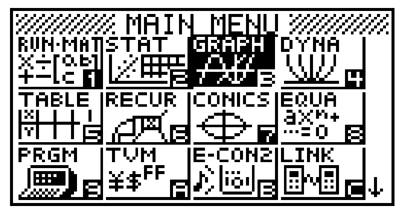




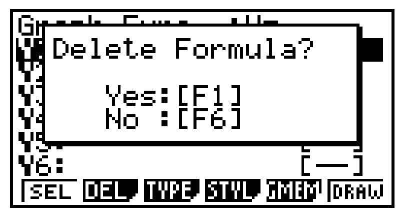
Explore the behavior of the split-defined function $g(x) = \begin{cases} -x^2+3x-5, x<1.5 \\ -4, x=1.5 \\ -x^2+3x-5, x>1.5 \end{cases}$.

To graph a split-defined function:

1. From the Main Menu, highlight the GRAPH icon and press [EXE] or press [3].



2. To delete any previous equations, highlight the equation and press [F2] (DEL) [F1] (Yes.)

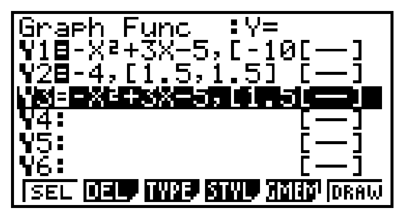


3. Enter each branch of the split-defined (piece-wise) function in its own **Y=** slot, then create the restrictions by putting the lower and upper bounds in brackets.

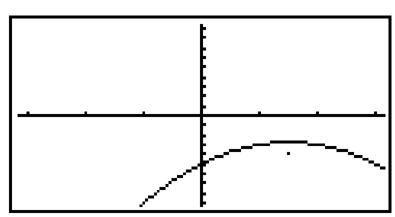
   For **Y1**, press [(−)] [X,θ,T] [x^2] [+] [3] [X,θ,T] [−] [5] [,] [SHIFT] [+] [(−)] [1] [0] [,] [1] [•] [5] [SHIFT] [−] [EXE].

   For **Y2**, press [(−)] [4] [,] [SHIFT] [+] [1] [•] [5] [,] [1] [•] [5] [SHIFT] [−] [EXE].

   For **Y3**, press [(−)] [X,θ,T] [x^2] [+] [3] [X,θ,T] [−] [5] [,] [SHIFT] [+] [1] [•] [5] [,] [1] [0] [SHIFT] [−] [EXE].



4. Press [F6] (DRAW) to view the graph of the functions.

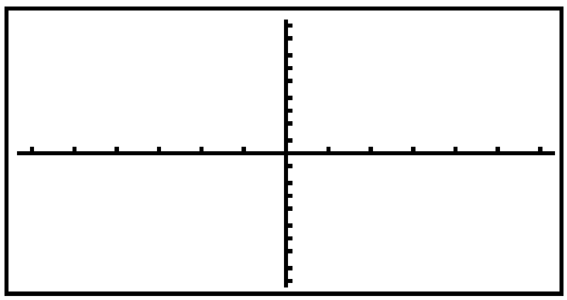


This activity will have you explore the concept of continuity at a point. It will also allow you to discover that simply having a limit at a point will not guarantee that the function is also continuous. Using the Casio *fx-9750GII*, you will be working in pairs or small groups.

Questions

Explore the behavior of the function $f(x) = x^2 - x - 6$ around the vertex.

1. Graph the function using the initial view window, changing Ymin to –9.3 and Ymax to 9.3, and copy on the axes below.



2. Find and record the vertex of the function.

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3. Trace to the vertex and zoom in, record what you see.

____

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4. What does the value of $\lim_{x \to 0.5} f(x)$ appear to be? Explain how you arrived at that answer.

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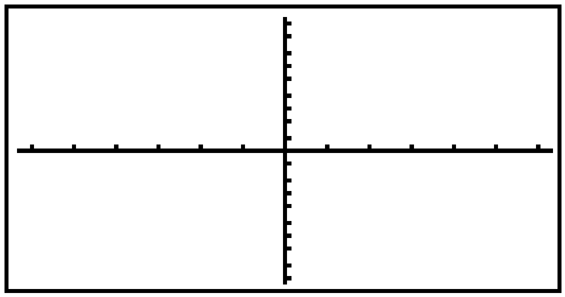
5. Explore the behavior of the following split-defined function:

$$g(x) = \begin{cases} x^2 - x - 6, x < 0.5 \\ -6, x = 0.5 \\ x^2 - x - 6, x > 0.5 \end{cases}$$

Use the same viewing window as before. Record what you see below.



6. What does the value of $\lim_{x \to 0.5} g(x)$ appear to be?

___

7. How does it compare to the $\lim_{x \to 0.5} f(x)$?

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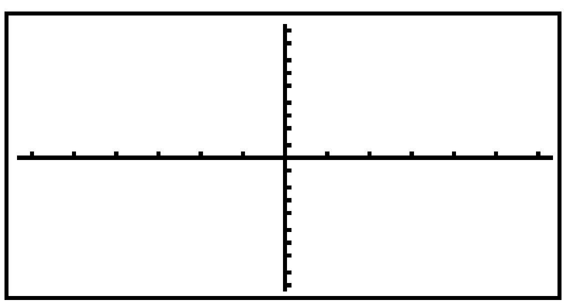
8. Now trace to a value where x = 0.5 and zoom in. Describe and record what you see.



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9. Find the following limits:

a. $\lim_{x \to 0.5^+} g(x)$ ______________________

b. $\lim_{x \to 0.5^-} g(x)$ ______________________

c. $\lim_{x \to 0.5} g(x)$ ______________________

10. Find g(0.5). How does this compare to your answer for $\lim_{x \to 0.5} g(x)$?

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11. Draw a conclusion about the relationship between limits and continuity.

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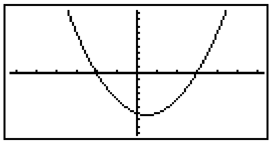
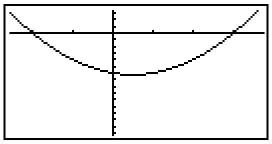
Solutions

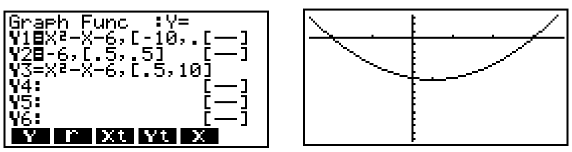
1. 

2. (0.5, –6.25)

3. 

Nothing unusual should be seen. The function is continuous.

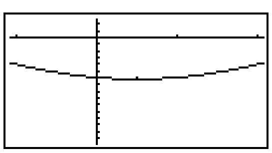
4. The limit is -6.25, the vertical value of the vertex. Answer will vary; care should be taken to point out that simply tracing to a value is not confirmation enough and can be tricky. Direct substitution is a valid explanation. A good answer might also include a mention of “passing through” or even a mention of continuity.

5. 

[Note: The discontinuity will not be immediately apparent.]

6. The limit is –6.25.

7. Answers may vary as students begin to get the idea that the change in the definition of the function may be creating some problems, although not with the limit. This is a good checkpoint for the understanding of what it means to be a “limit.”

8.  [Note: the point becomes more visible.]

9. All three limits are –6.25, although some students may try to refine the answers to longer decimals. This provides another good opportunity to stress the idea of “limit” as the value the function approaches.

10. g(0.5) = –6, a value different from the limit.

11. Answers will vary; a good answer will include the fact that the function has a gap or a hole or a jump at the point of discontinuity. The idea is to have the students begin to think about the fact that simply having a limit does not guarantee the continuity of a function.

Topic Area: Area of Regular Polygons

NCTM Standards:

- Use trigonometric relationships to determine lengths and angle measures.
- Explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them.

Objective

The student will be able to find the central angle of a regular polygon, find the length of a side of a regular polygon given its radius, find the length of the apothem of a regular polygon given its radius, find the area of an isosceles triangle formed by the central angle of a regular polygon and it's side, and apply finding the area of regular polygons to real-life problems.

Getting Started

As a class, review the meaning of a regular polygon to include the sides and angles. Relate regular polygons to a circle by showing examples of how circles are depicted on computer screens and how a circle can be thought of as a polygon with an infinite number of sides. Review the trigonometric functions used with right triangles. Ask the class to name areas where they have seen figures in the shape of a regular polygon.

Prior to using this activity:

- Students should be able to find the central angle of a regular polygon.
- Students should be able to use the sine and cosine functions in solving right triangles.
- Students should be able to find the perimeter of a regular polygon.

Ways students can provide evidence of learning:

- The student will be able to find the area of an isosceles triangle formed by the central angel of a regular polygon and it's radius.
- The student will be able find the area of a regular polygon and apply this to solving real-life problems.

Common mistakes to be on the lookout for:

- Students may use the wrong trigonometric functions in the formulas.
- Students may not divide the central angle by two for the formula.

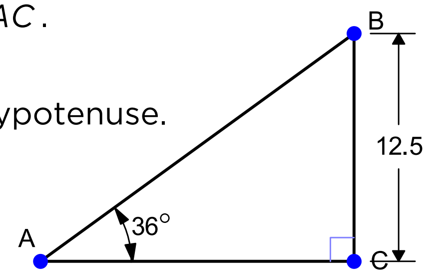
Definitions

- Regular Polygon
- Side
- Radius of a Polygon
- Apothem
- Perimeter
- Central Angle
- Sine
- Cosine

The following will demonstrate how to enter a formula into the Equation mode of the Casio *fx-9750GII* and solve for a specified unknown value.

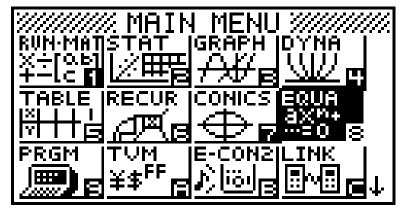
Given ΔABC, with $\angle A = 36°$, find the lengths of $\overline{AB}$ and $\overline{AC}$.

Using the equation, $\sin\theta = \dfrac{\text{Opp.}}{\text{Hyp.}}$, find the length of the hypotenuse.

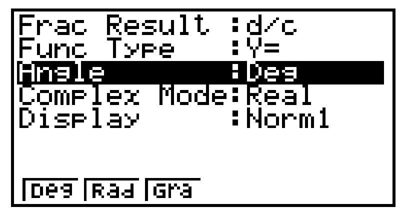


To enter the formula into the Equation Solver:

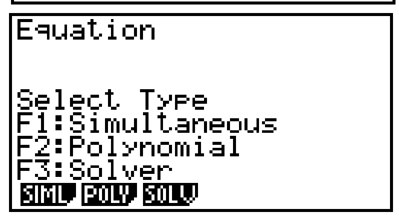
1. From the Main Menu, highlight the EQUA icon and press [EXE] or press [8].



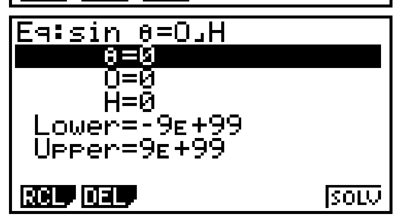
2. To set up the calculator for degree measures, press [SHIFT] [MENU] **(SET UP)**. Arrow down to highlight **Angle** and press [F1] (Deg) to change the calculator to **Degree** mode. Press [EXIT] to return to the initial Equation screen.



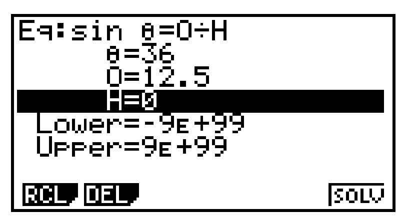
3. Press [F3] (SOLV) to enter the Equation Solver mode. If there is already an equation in the calculator, press [F2] (DEL) [F1] (Yes).



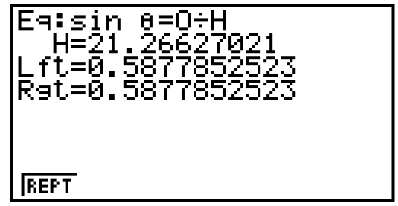
4. To enter the equation in to the calculator, press [sin] [ALPHA] [^] [SHIFT] [•] [ALPHA] [9] [a b/c] [ALPHA] [F↔D]. The calculator may display numbers next to the variables, which represent values already in the calculator. We will enter values to replace them.



5. With θ highlighted, press [3] [6] [EXE]. With O highlighted, enter [1] [2] [•] [5] [EXE].



6. Press F6 (SOLV) to see the length of the hypotenuse.



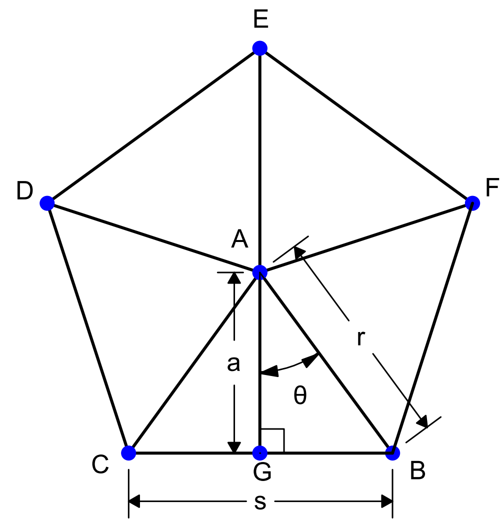
A regular polygon has all congruent sides and all congruent angles. The distance from the center to a vertex is called the radius and the distance from the center perpendicular to a side is called the apothem. The following formulas are used to find the side and the apothem if the length of the radius is known.

Length of a side: $s = 2r\sin\theta$ Length of the apothem: $a = r\cos\theta$

Area of Each Triangle: $A = \frac{1}{2}sa$ Area of a Regular Polygon: $A = \frac{1}{2}aP$

Questions

Use the diagram at the right to answer the questions.



1. a. The measure of the central angle of a regular pentagon is __________ .

   b. The $m\angle\theta$ is ____________.

2. If the radius of the pentagon is 5, find the length of s to the nearest tenth.

   ____

3. Find the perimeter to the nearest tenth.

   ____

4. If the radius of the pentagon is 2.75, find the length of s to the nearest tenth.

   ____

5. Find the perimeter to the nearest tenth.

   ____

6. A circle can be thought of as a polygon with infinite sides. Using the formulas, fill in the given table for a figure with a radius of 4.

Inscribed Polygon	Hexagon	Octagon	Decagon	25-gon	60-gon	Circle
No. of Sides						
Measure of Side						
Measure of Apothem						
Area						

Based on the table above, answer the following questions:

7. What do you notice about the length of the sides as the number of sides increases?

____

8. What do you notice about the apothem as the number of sides increases?

____

9. How does the area of a hexagon compare to that of the circle?

____

10. How does the area of a 60-gon compare to that of the circle?

____

A carpenter is making a table in the shape of six isosceles triangles that will fit together to make a regular hexagon with a 5 ft. radius. Find the indicated measures to the nearest hundredth.

11. What is the length of each side of the table?

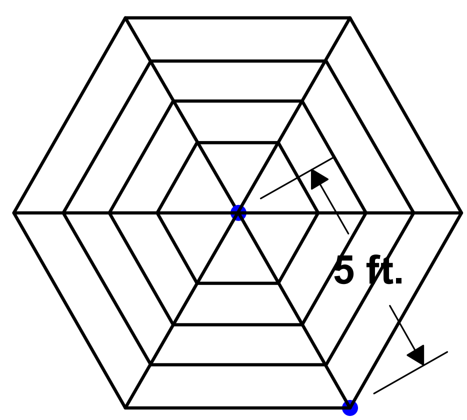
____

12. What is the length of the apothem for the table?

____

13. What will be the area of the one of the triangles?

____



14. What will be the total area of the top of the table?

____

15. If the lumber to make the table top costs $9.50 a square foot, how much would it cost for the lumber to build this table top, adding 1.5 square feet for waste?

____

The city of Dover has a metal shop make octagonal stop signs whose radii are 1.25 feet.

16. What is the total area of one stop sign to the nearest hundredth?

____

17. If sheet metal costs $12.39 a square foot, what is the cost of one blank sign? How much would it cost to make 50 signs?

____

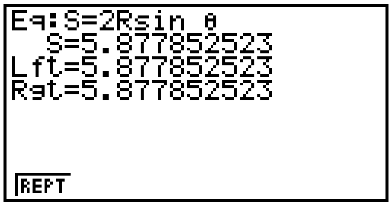
18. If painting and lettering for each sign costs a total of $3.98 each, what would be the cost of making 50 signs?

____

Solutions

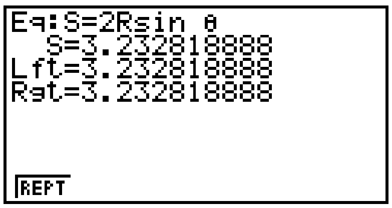
1. a. 360° ÷ 5 = 72°
   b. 72° ÷ 2 = 36°

2. s = 5.9



3. P = 5(5.9) = 29.5

4. s = 3.2

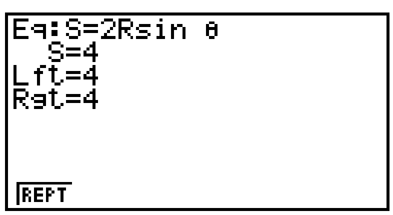


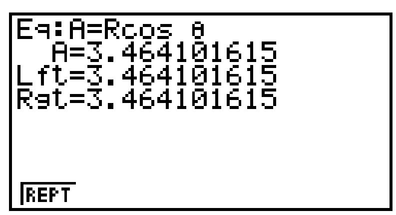
5. P = 5(3.2) = 16.0

6.

Inscribed Polygon	Hexagon	Octagon	Decagon	25-gon	60-gon	Circle
No. of Sides	6	8	10	25	60	
Measure of Side	4	3.06	2.47	1.00	0.42	
Measure of Apothem	3.46	3.70	3.80	3.97	3.99	
Area	41.52	45.29	46.93	49.63	50.27	50.27

Screen Shots for Hexagon:





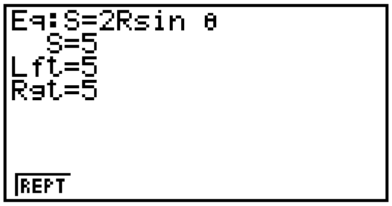
7. The length of the sides decrease and approaches 0.

8. The apothem gets longer and approaches 4 which is the radius.

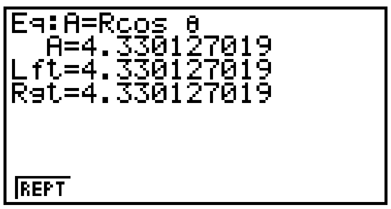
9. The difference is 50.27 – 41.52 = 8.75 which is a 17% difference.

10. The difference is 50.27 – 50.27 = 0, which is a 0% difference. This is approximately equal, since we rounded, thus a circle can be thought of as a polygon with infinite sides.

11. 5 ft.



12. 4.33 ft.

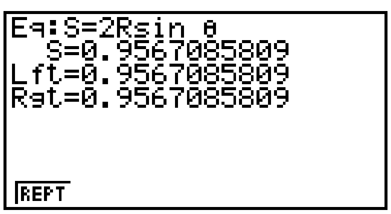


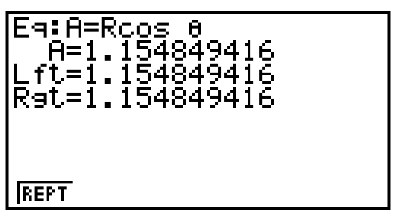
13. $A = \frac{1}{2}(5)(4.33) = 10.83 \text{ ft.}^2$

14. $A = \frac{1}{2}(4.33)(5\text{x}6) = 64.95 \text{ ft.}^2$

15. $9.50(64.95 + 1.5) = $631.56

16. $A = \frac{1}{2}(1.15)(0.96 \times 8) = 4.46 \text{ ft.}^2$





17. One sign: C = $12.39(4.46) = $55.26
50 signs: C = 50($55.26) = $2,778

18. $2,778 + $3.98(50) = $2,977

Area of a Triangle and Circular Sector Teacher

Topic Area: Trigonometric Applications

NCTM standards:

- Compute fluently by developing fluency in operations with real numbers using technology for more-complicated cases
- Understand functions by interpreting representations of functions

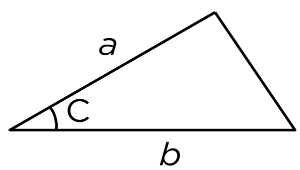
Objective

To calculate the area of a triangle and a circular sector using trigonometry.

Getting Started

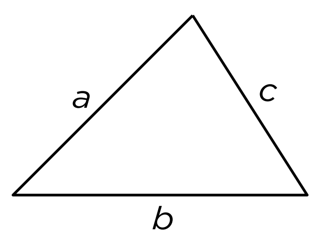
In this activity, the student will learn how to calculate the area of a triangle and a circular sector using trigonometry. The area of a triangle is defined to be one-half of the product of the lengths of the two sides (*a, b)* and the sine of angle (*C*) included between those two sides. The formula looks like:

area = $\frac{1}{2}\bullet a\bullet b\bullet \sin C$



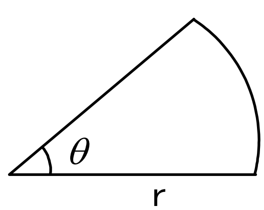
Another useful formula for finding the area of a triangle is Heron's formula In this formula you need to know the length of the three sides (*a, b, c*) to find the area of the triangle. Heron's formula is:

area = $\sqrt{s(s-a)(s-b)(s-c)}$ where s = $\frac{1}{2}$ (a + b + c)

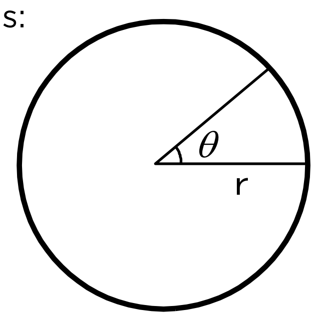


The area of a circular sector is defined to be one-half of the product of the radius (*r*) squared and the central angle. The formula is:

area = $\frac{1}{2}\bullet r^2\bullet \theta$



or



Prior to using this activity:

- Students should understand the difference between right triangles and non-right triangles.
- Students should know the difference between degrees and radians as angle measurements.

Ways students can provide evidence of learning:

- Students will be able to calculate the area of any triangle.
- Students will be able to calculate the area of a circular sector.

Common mistakes to be on the lookout for:

- Students may forget to switch between degree and radian mode on the calculator.
- When using Heron's formula, students may incorrectly calculate s (or not calculate it at all).

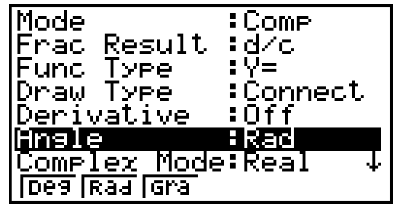
Definitions

- arc
- sector
- circular sector
- degree
- radian

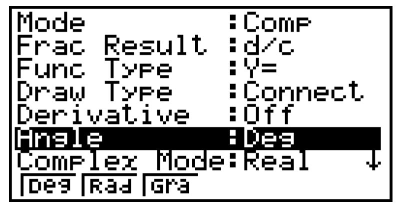
The following will demonstrate how to enter the data into the Casio *fx-9750GII* and interpret the results.

To set up the calculator to calculate in degrees:

1. From the Main Menu, highlight the RUN•MAT icon and press [EXE] or press [1].



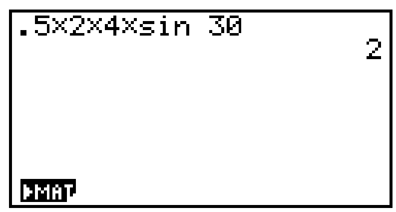
2. To set the calculator to Degrees, press [SHIFT] [MENU] **(SET UP)** and move the cursor down to **Angle**. Press [F1] (Deg) to change it into degrees Press [EXIT] to exit the setup screen.



To solve equations including trigonometric functions:

For this example, we will use the equation for area = $\frac{1}{2}a \cdot b \cdot \sin C$. Let a = 2, b = 4, and C = 30°

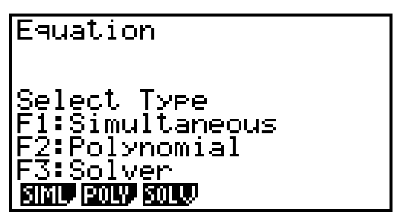
1. To calculate the area with the given parameters, press [•] [5] [×] [2] [×] [4] [×] [sin] [3] [0] [EXE]. The area is 2.



To use the Solver feature:

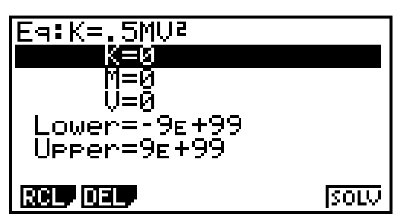
For this example, we will use the equation for kinetic energy, KE = 0.5 mv^2. Let m = 10 kg, v = 25 $\frac{m}{\sec^2}$.

1. From the Main Menu, highlight the EQUA icon and press [EXE] or press [8]. Then, press [F3] (SOLV)

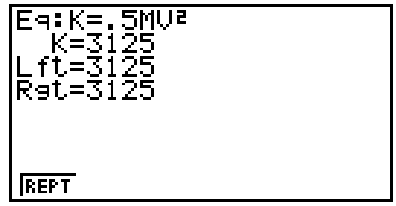


2. To input the equation, press: [ALPHA] [,] [SHIFT] [•] [•] [5] [ALPHA] [7] [ALPHA] [2] [x^2] [EXE]

Note: K is used to represent KE.



3. Enter 10 for M and 25 for V, then highlight K.



4. Press F6 (SOLV).

5. Kinetic energy for this example is 3125 kg$\bullet \frac{m}{\text{sec}^2}$.

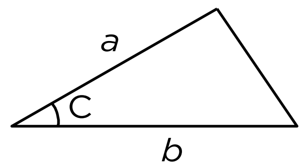
6. To use the Solver to find m, input values for K and V, highlight M and press F6 (SOLV) to find the value for M. Use this same method to find V.

Area of a Triangle and Circular Sector — Activity

Introduction

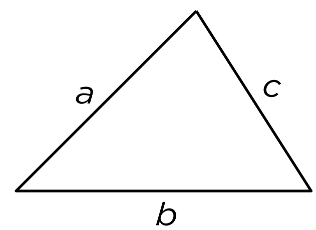
In this activity, you will learn how to calculate the area of a triangle and a circular sector using trigonometry. The area of a triangle is defined to be one-half of the product of the lengths of the two sides (*a, b*) and the sine of angle (*C*) included between those two sides. The formula looks like:

$$\text{area} = \frac{1}{2} \bullet a \bullet b \bullet \sin C$$



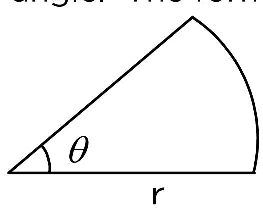
Another useful formula for finding the area of a triangle is Heron's formula In this formula you need to know the length of the three sides (*a, b, c*) to find the area of the triangle. Heron's formula is:

$$\text{area} = \sqrt{s(s-a)(s-b)(s-c)} \quad \text{where } s = \frac{1}{2}(a + b + c)$$

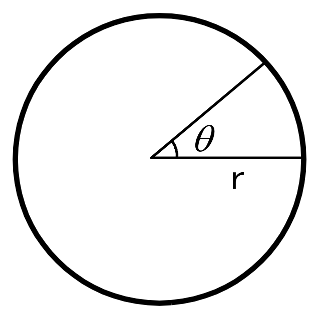


The area of a circular sector is defined to be one-half of the product of the radius ® squared and the central angle. The formula is:

$$\text{area} = \frac{1}{2} \bullet r^2 \bullet \theta$$



or



Questions

1. Find the area of the following triangles for the given values using one of the following formulas:

$$\text{area} = \frac{1}{2} \bullet a \bullet b \bullet \sin C \qquad \text{area} = \frac{1}{2} \bullet a \bullet c \bullet \sin B \qquad \text{area} = \frac{1}{2} \bullet c \bullet b \bullet \sin A$$

a. $a = 5$, $b = 7$, $C = 35^\circ$ area = ____________

b. $b = 10$, $c = 8$, $A = 28^\circ$ area = ____________

c. $a = 16$, $c = 4$, $B = 40^\circ$ area = ____________

2. Find the area of the following triangles for the given values using Heron's formula and the Solve feature.

a. $a = 5$, $b = 7$, $c = 10$ area = ____________

b. $a = 10$, $b = 8$, $c = 6$ area = ____________

c. $a = 6$, $b = 4$, $c = 8$ area = ____________

3. Find the area of the following circular sectors for the given values using the formula in the Solve feature.

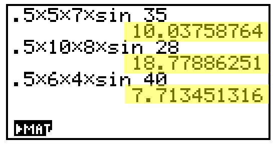
a. $r = 8$, $\theta = \frac{\pi}{5}$ area = ____________

b. $r = 10$, $\theta = \frac{2\pi}{3}$ area = ____________

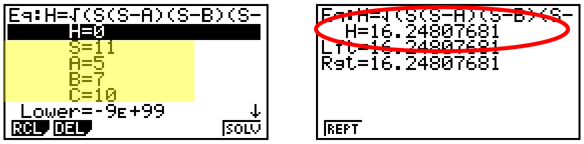
c. $r = 7$, $\theta = \frac{4\pi}{9}$ area = ____________

Solutions

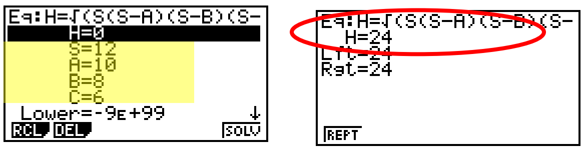
1. a. area = 10.0376 units^2
   b. area = 18.7789 units^2
   c. area = 7.7135 units^2



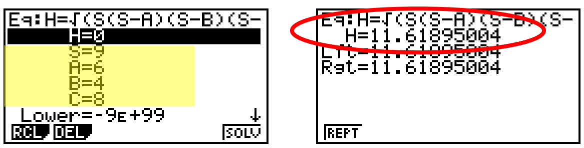
2. a. area = 16.2481 units^2



   b. area = 24 units^2

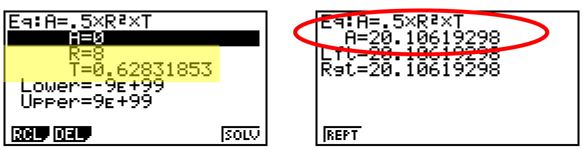


   c. area = 11.6190 units^2

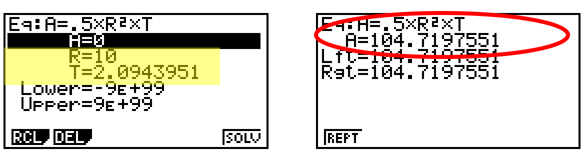


3. *Note that angles are given in radians for this question. Calculator must be changed over to radian mode before proceeding.*

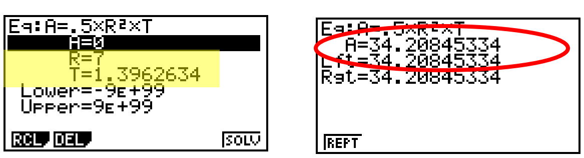
   a. area = 20.1062 units^2



   b. area = 104.7198 units^2



   c. area = 34.2085 units^2



Area Under A Curve — Teacher Notes

Topic: Integration

NCTM Standard

- Apply informal concepts of successive approximation, upper and lower bounds, and limit in measurement situations.
- Analyze precision, accuracy, and approximate error in measurement situations.

Objectives

The student will calculate area and perform integration using the graphing calculator.

Getting Started

In this activity students find the area bounded by a function and the x-axis. Using this information, students will make conjectures about the sum of integrals, multiplying integrals by a constant, and the differences between integrals above and below the x-axis. Students will sketch and calculate the area using the Casio *fx-9750GII*.

Prior to using this activity:

- Students should understand that finding the area under the curve is the same as finding the definite integral on the same interval.

Ways students can provide evidence of learning:

- Students should be able to find the value of a definite integral.
- Students should be able to sketch and find a numerical value for the area under the curve for a defined integral.

Common mistakes to be on the lookout for:

- Students may arrive at an incorrect area if they do not use the correct interval.
- Student trigonometric graphs may appear different. Make sure all students are in radian mode.

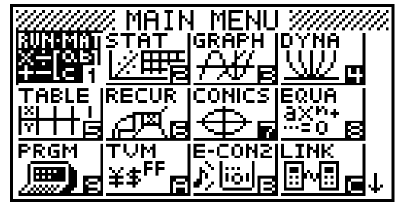
Definitions:

- Area under a curve
- Interval
- Integral
- Conjecture
- Bounded

The following will demonstrate how to calculate an integral, graph a function, set a view window, and sketch the area under a curve on the Casio *fx-9750GII*.

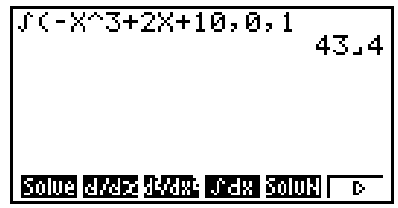
Explore the behavior of the function $f(x) = -x^3 + 2x + 10$.

To find the integral on the interval [0,1]:

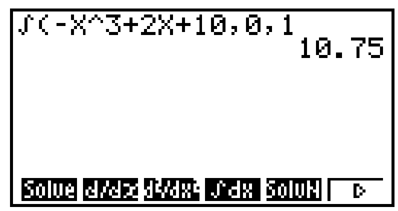


1. From the Main Menu, highlight the RUN•MAT icon and press [EXE] or press [1].

2. Press [AC/ON] to clear the screen.

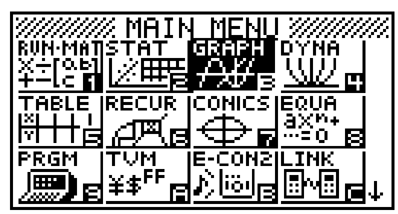


3. To calculate the integral, press [OPTN] [F4] (CALC) [F4] (∫dx) [(–)] [X,θ,T] [^] [3] [+] [2] [X,θ,T] [+] [1] [0] [,] [0] [,] [1] [EXE].

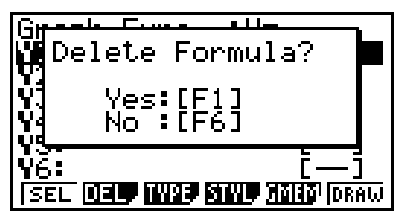


4. To convert the fraction to a decimal, press [F↔D].

To display a graph and calculate the integral:

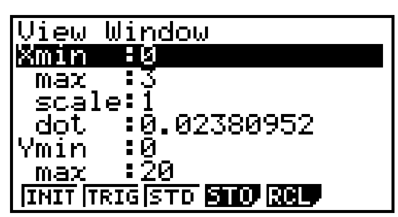


1. From the Main Menu, highlight the GRAPH icon and press [EXE] or press [3].



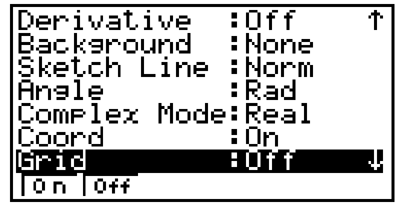
2. To delete any previous equations, highlight the equation and press [F2] (DEL) [F1] (Yes.)

3. Enter the equation in **Y1** by pressing [(–)] [X,θ,T] [^] [3] [+] [2] [X,θ,T] [+] [1] [0] [EXE].



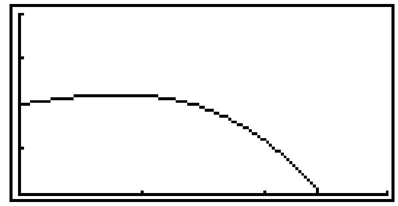
4. Set the view window by pressing [SHIFT] [F3] **(V-Window)**. To set the window for the activity, press [0] [EXE] [3] [EXE] [1] [EXE] [▼] [0] [EXE] [2] [0] [EXE] [5] [EXE].

5. Press [EXIT] to return to the initial GRAPH screen.

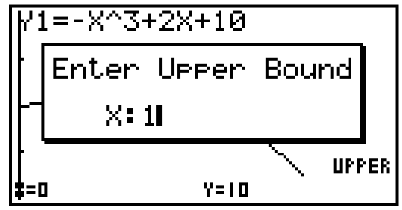


6. To turn the grid off, press [SHIFT] [MENU] **(Set Up)**, Arrow down to highlight **Grid**, and press [F2] (Off).

7. Press [EXIT] to return to the initial GRAPH screen.



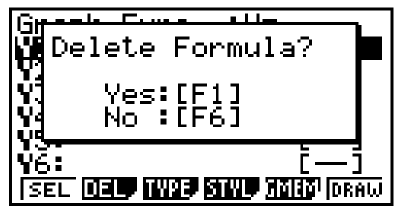
8. Press [F6] (DRAW) to view the graph of the function.



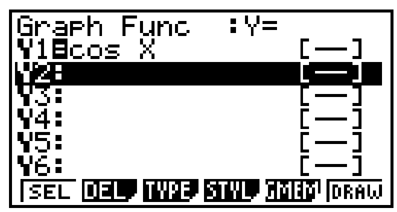
9. To find the area between the function and the x-axis, Press [F5] **(G-Solv)** [F6] (▷) [F3] (∫dx) [0] [EXE] [1] [EXE].

Explore the behavior of the function f(x) = cos(x).

To display a graph and calculate the integral:

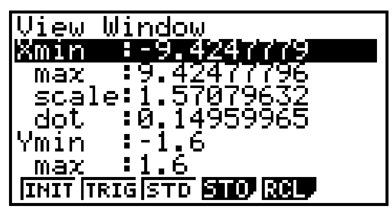


1. From the Main Menu, highlight the GRAPH icon and press [EXE] or press [3].

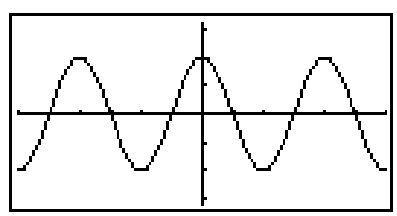


2. To delete any previous equations, highlight the equation and press [F2] (DEL) [F1] (Yes.)

3. Enter the equation in **Y1** by pressing [cos] [X,θ,T] [EXE].

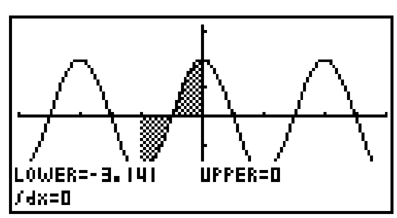


4. Set the view window to the initial trig settings press [SHIFT] [F3] **(V-Window)** [F2] (TRIG).



5. Press [EXIT] to return to the initial GRAPH screen.

6. Press [F6] (DRAW) to view the graph of the function.



7. To find the integral between the function and the x-axis on the interval [-π, 0], press [F5] **(G-Solv)** [F6] (▷) [F3] (∫dx) [(−)] [SHIFT] [EXP] [EXE] [0] [EXE].

Area Under A Curve — Activity

In this activity you will find the area bounded by a function and the x-axis. Using this information, you will make conjectures about a few properties of integrals. You will graph the area and calculate the area using the Casio *fx-9750GII*.

Questions

1. Using your calculator in Run-Matrix mode, find the area bounded by the x-axis and $f(x) = e^x$ on the interval [0, 1].

   ____

2. Graph the function $f(x) = e^x$ in window described in the "How-To" section.

3. Without using your calculator sketch the area bounded by the x-axis and $f(x) = e^x$ on the interval [0, 1] on the graph above.

4. Use the G-Solv feature on your calculator to find the area bounded by the x-axis and $f(x) = e^x$ on the interval [0, 1] to confirm you result in #3. What is the area?

   ____

5. Using any method, find the area bounded by the x-axis and $f(x) = e^x$ on the interval [1, 2]. Explain how you found your answer.

   ____

   ____

   ____

6. Find the area bounded by the x-axis and $f(x) = e^x$ on the interval [0, 2]. How does your answer compare with your answers in #1 and #5. Explain.

7. Using this information, make a conjecture for the property:

$$\int_a^c f(x)dx + \int_c^b f(x)dx =$$

8. Graph the function $f(x) = e^x$ and $f(x) = 2e^x$ in the same window.

9. Without using the calculator, shade the area bounded by the x-axis and $f(x) = 2e^x$ on the interval [0, 2] on the above graph. Predict how the area will compare to the area you found in #6.

10. Find the area bounded by the x-axis and $f(x) = 2e^x$ on the interval [0, 2]. How does this compare to your answer in #6.

11. Using this information, make a conjecture for the property:

$$\int_a^b 2f(x)dx =$$

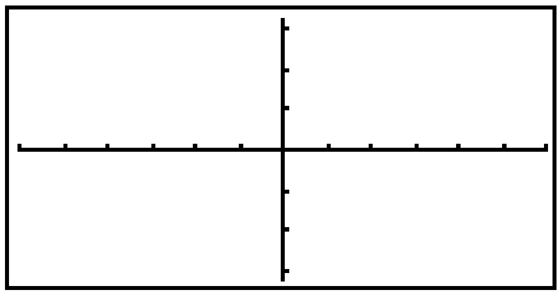
12. Do you think your conjecture in #11 works for any constant multiplied by an integral? Give some examples to support your answer.

____

____

____

13. Graph f(x) = sin x on the standard trig window.



14. Calculate $\int_{0}^{\pi} \sin(x)dx$. Shade the area on the graph above.

$\int_{0}^{\pi} \sin(x)dx =$ ____________

15. Calculate $\int_{-\pi}^{0} \sin(x)dx$. Shade the area on the graph above.

$\int_{-\pi}^{0} \sin(x)dx =$ ____________

Does this answer surprise you? Why or why not?

____

____

16. Before calculating, predict what $\int_{-\pi}^{\pi} \sin(x)dx$ will equal.

____

17. Using your calculator, find $\int_{-\pi}^{\pi} \sin(x)dx$.

$$\int_{-\pi}^{\pi} \sin(x)dx = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

18. Summarize your findings about integrals whose area is above the x-axis and those that are below the x-axis.

____

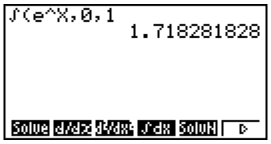
____

____

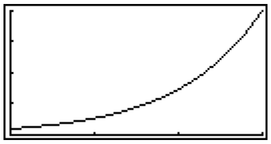
____

Solutions

1. Area = 1.71828

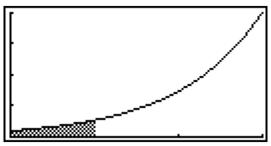


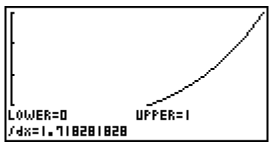
2.



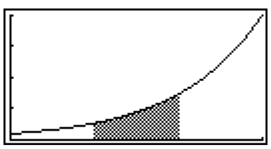
3. Sketch should look the same as #4 below.

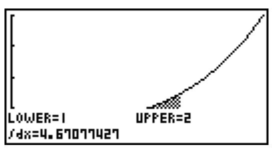
4. .Area = 1.71828

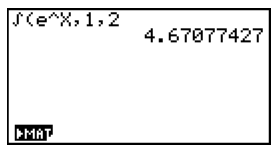




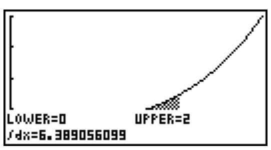
5. Area = 4.67077427

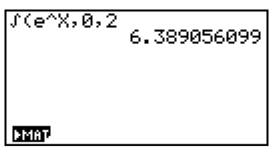






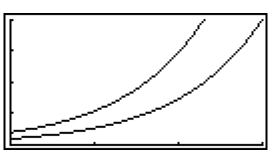
6. Area = 6.389056099. The area is the sum of the areas in #1 & #5.



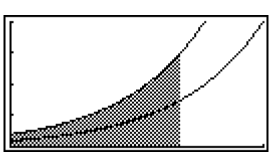


7. $\int_a^c f(x)dx + \int_c^b f(x)dx = \int_a^b f(x)dx$

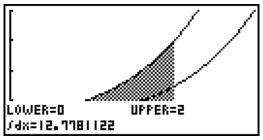
8.

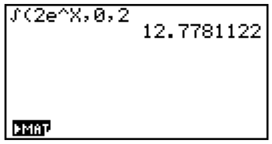


9. Predictions may vary. Students may realize that the area will be twice.



10. Area = 12.7781. The area is twice the area in #6.

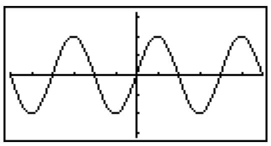




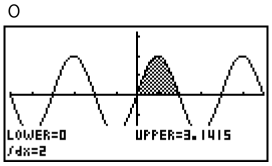
11. $\int_a^b 2f(x)dx = 2\int_a^b f(x)dx$

12. Answers will vary. Students should find that any constant multiplied by an integral will work.

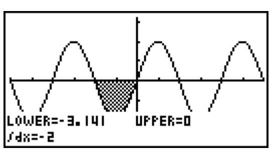
13.



14. $\int_0^{\pi} \sin(x)dx = 2$

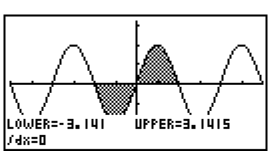


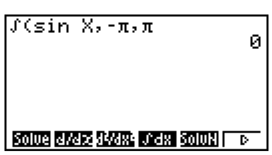
15. $\int_{-\pi}^{0} \sin(x)dx = -2$; Answers will vary.



16. Predictions will vary.

17. $\int_{-\pi}^{\pi} \sin(x)dx = 0$





18. When the integral is above the x-axis, it is positive. When the integral is below the x-axis, it is negative.

Topic Area: Medians and Centroid of a Triangle

NCTM Standards:

- Analyze properties and determine attributes of two- and three-dimensional objects.
- Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations.

Objective

The student will be able to find the coordinates of the midpoint for a given line segment, find the length of a line segment given the coordinates of its endpoints, and find the equation of a line containing two given points. The student will also be able to find the coordinates of the intersection of two lines and determine the property of the centroid of a triangle.

Getting Started

As a class, review the formulas for finding the midpoint between two pairs of coordinates and the distance between the coordinates. Discuss the term median in relation to a triangle and have an illustrated example. Create and show the students a triangle that is balanced on the centroid without showing any measurements. Ask students to give some examples where this may be useful such as architecture and sculpture.

Prior to using this activity:

- Students should be able to find the distance and midpoint between two points.
- Students should be able to line of best fit.
- Students should be able to find the ratio between two values.

Ways students can provide evidence of learning:

- The student will be able to find the midpoints for the sides of a given triangle.
- The student will be able write a conjecture about the lengths of segments that form the medians of a triangle.

Common mistakes to be on the lookout for:

- Students may use the wrong values when using the formulas.
- Students may enter values into the data lists for the linear regression incorrectly.

Definitions

- Endpoint
- Distance
- Midpoint
- Intersection
- Centroid
- Ratio
- Median of a Triangle

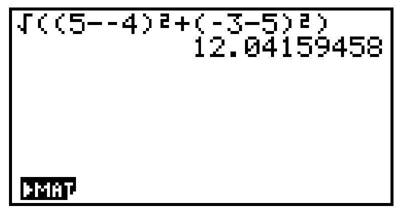
The following will demonstrate how to calculate the length and midpoint between two points by applying the distance and midpoint formulas using the Run-Mat icon of the Casio *fx-9750GII* and graph the results. Then, using a combination of the Statistics and Graph icons, you will see how to find the line of best fit, graph two lines, and find their intersection.

Given $\overline{AB}$ with endpoints at (–4, 5) and (5, –3) and $\overline{CD}$ with endpoints at (–5, –4) and (6, 5) find the length and midpoint of $\overline{AB}$. Graph $\overline{AB}$ and $\overline{CD}$ on the same plane and find the coordinates of their intersection.

Distance Formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Midpoint Formula: $\left(\frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}\right)$

To calculate the length using the distance formula:

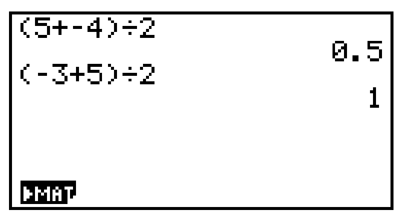
1. From the Main Menu, highlight the RUN•MAT icon and press [EXE] or press [1].



2. Enter the distance formula by pressing [SHIFT] [x²] [(] [(] [5] [−] [(−)] [4] [)] [x²] [+] [(] [(−)] [3] [−] [5] [)] [x²] [)] [EXE]; being sure to use two parentheses at the beginning and end.

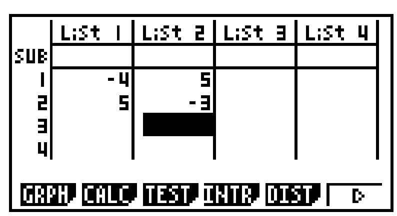
To find the coordinates of the midpoint using the midpoint formula:

1. Press [AC/ON] to clear the screen. Enter [(] [5] [+] [(−)] [4] [)] [÷] [2] [EXE] to find the x-coordinate and [(] [(−)] [3] [+] [5] [)] [÷] [2] [EXE] to find the y-coordinate.



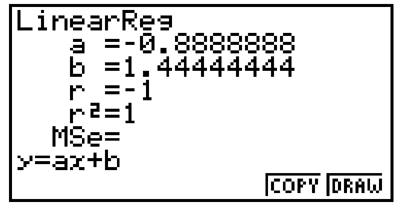
To graph two lines and find their intersection:

1. From the Main Menu, highlight the STAT icon and press [EXE] or press [2].



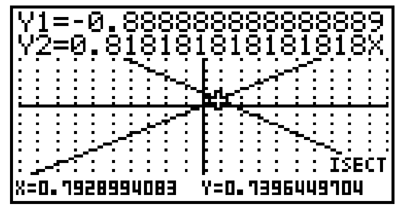
2. Enter the x-coordinates for $\overline{AB}$ into **List 1** and the y-coordinates for $\overline{AB}$ into **List 2**.

3. Press [F1] (GRPH) [F6] (SET) [▼] [F2] (xy) [EXE] [F1] (GPH1) to graph the segment.



4. Press [F1] (CALC) [F2] (X) [F1] (ax+b) to get the line of best fit. Press [F5] (COPY) [EXE] store it in the Graph Menu as **Y1**. Repeat this process to get the line of best fit for $\overline{CD}$ and store the results in **Y2**.

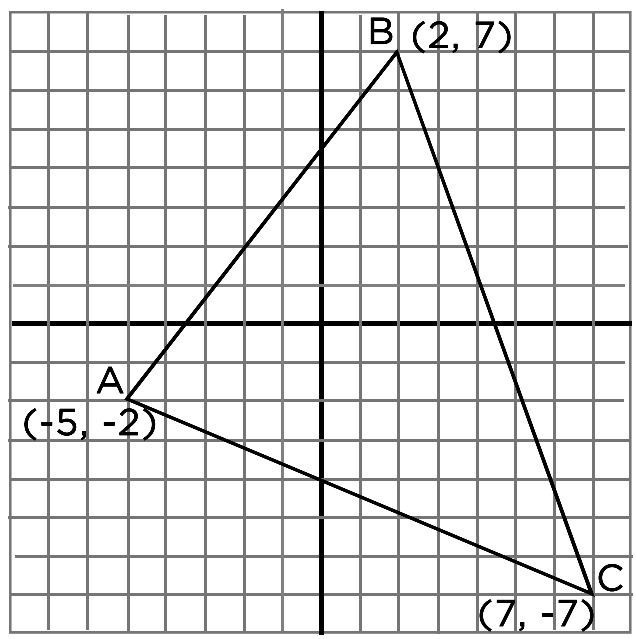
5. From the Main Menu, highlight the GRAPH icon and press [EXE]. Enter [F1] (SEL) [▼] [F1] (SEL) [F6] (DRAW) to see the graph of the two lines and then press [F5] **(G-Solv)** [F5] (ISCT) to see the coordinates of the intersection.



The medians of a triangle connect the vertex of the triangle to the midpoint of the opposite side. When all three medians are drawn, they intersect at a point called the centroid. This is the point of balance for any triangle. In this activity, you will find the coordinates of the centroid for a given triangle and then investigate how this point relates to the three vertices of the triangle.

Questions

Given the diagram below of ΔABC, answer the following questions:



1. Find the midpoints for $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$. Plot these points on the diagram above. Label these points C', A', and B' respectively.
   a. Midpoint $\overline{AB}$ ______________
   b. Midpoint $\overline{BC}$ ______________
   c. Midpoint $\overline{AC}$ ______________

2. Find the equation of the line of best fit for A and the midpoint of $\overline{BC}$. What is the equation of this line in slope-intercept form?

   ____

3. Find the equation of the line of best fit for B and the midpoint of $\overline{AC}$. What is the equation of this line in slope-intercept form?

   ____

4. What are the coordinates for the intersection of the two lines to the nearest tenth? This is the centroid of the triangle. Plot this point and label it D.

____

5. Draw all three medians in the diagram. Does this verify what you found in Question 4?

____

6. Find the distance from the centroid to each of the vertices to the nearest hundredth.
   a. $\overline{AD}$ = ______________
   b. $\overline{BD}$ = ______________
   c. $\overline{CD}$ = ______________

7. Find the distance from the centroid to each midpoint to the nearest hundredth.
   a. $\overline{A'D}$ = ______________
   b. $\overline{B'D}$ = ______________
   c. $\overline{C'D}$ = ______________

8. How does the length from the centroid to the vertex compare to the length from the centroid to the midpoint?

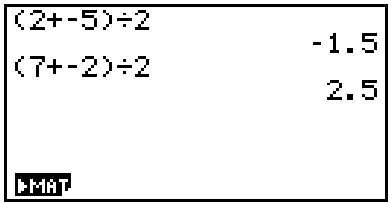
____

9. Will the centroid of a triangle always lie in the interior of a triangle? Why or why not?

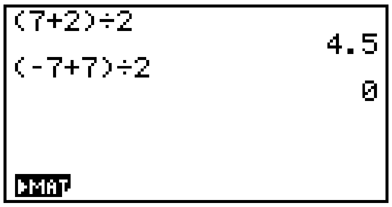
____

Solutions

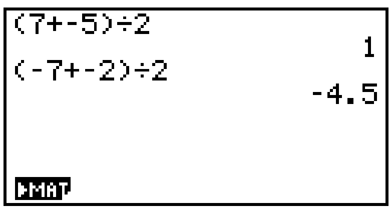
1. a. C': (–1.5, 2.5)

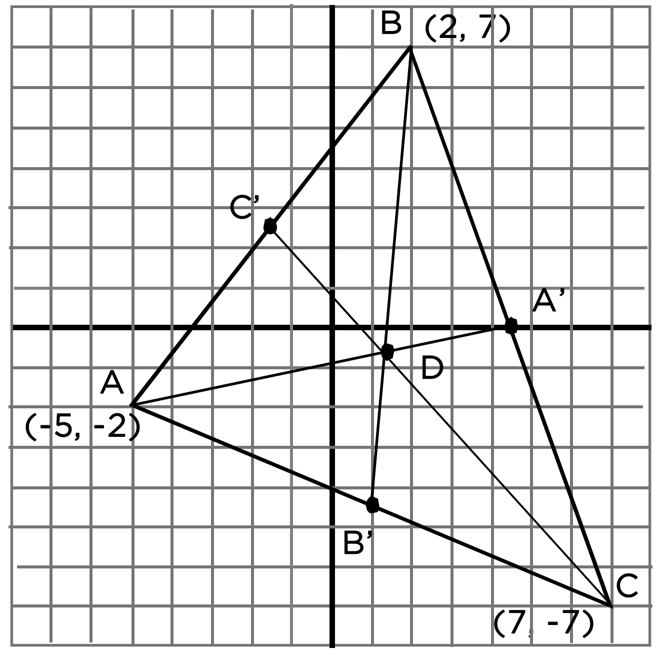


b. A': (4.5, 0)

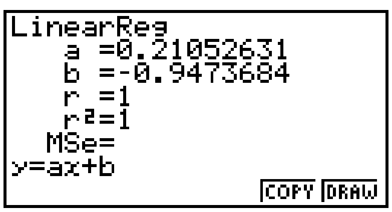


c. B': (1, –4.5)

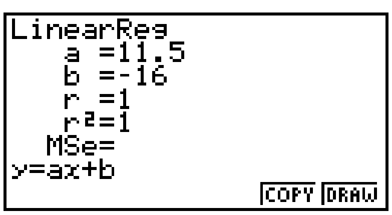




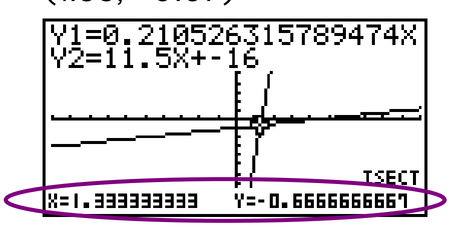
2. y = 0.21x – 0.95



3. y = 11.5x – 16



4. (1.33, –0.67)

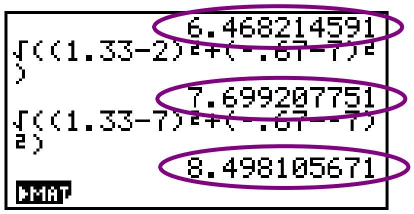


5. Yes

6. a. $\overline{AD}$ = 6.47

   b. $\overline{BD}$ = 7.70

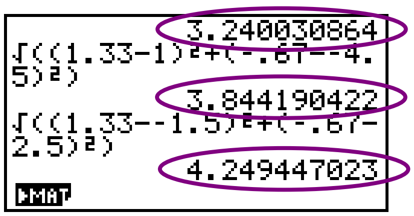
   c. $\overline{CD}$ = 8.55



7. a. $\overline{A'D}$ = 3.24

   b. $\overline{B'D}$ = 3.84

   c. $\overline{C'D}$ = 4.25



8. The distance from the vertex to the centroid is equal to twice the distance from the centroid to the midpoint.

9. Answers will vary.

Being Locally Linear Teacher Notes

Topic: Derivatives and Slope

NCTM Standards

- Approximate and interpret rates of change from graphical and numerical data.
- Make and investigate mathematical conjectures.
- Organize and consolidate their mathematical thinking through communication; communicate their mathematical thinking coherently and clearly to peers, teacher, and others.

Objectives

The student will be able to connect the much earlier concept of linear slope to the examination of the rate of change of a function and the idea of what a derivative is. The student will also be able to understand and communicate the visual and numerical ideas of linear slope and its relationship to the instantaneous rate of change of any function.

Getting Started

This activity will begin to bring home the point that as the behavior around a single point on a differentiable function is examined, the function will "flatten out" and very much resemble the behavior of a line drawn through the point of interest. The example given should motivate a discussion of what it means to be locally linear with regard to a differentiable function.

Prior to using this activity:

- Students should be able to produce and manipulate functions manually and with a graphing calculator.
- Students should have an understanding of "decimal" and "standard" window and how to easily produce them.
- Students should be able to use the Zoom features of the graphing calculator to examine specific parts of the graph.
- Students should have an understanding of slope of a line, as a rate of change.

Ways students can provide evidence of learning:

- Students should be able to produce graphs of functions and communicate changes taking place to the appearance of a function as they zoom in on a particular value.

Common mistakes to be on the lookout for:

- Students may not understand the zoom process and what is taking place.

Definitions:

- Average Rate of Change
- Derivative
- Rate of Change
- Slope

Formula:

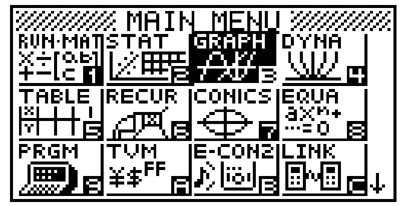
$$\text{slope} = m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

The following will demonstrate how to graph a function, set the zoom factors, examine its behavior, and find the equation of a line on the Casio *fx-9750GII*.

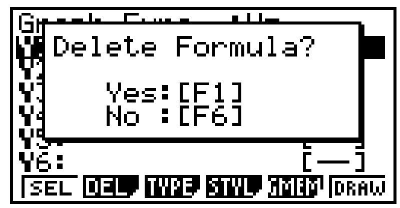
Graph the function $y = x^2 + \frac{2}{3}x - 2$.

To graph a function:

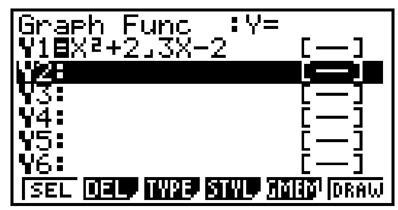
1. From the Main Menu, highlight the GRAPH icon and press [EXE] or press [3].



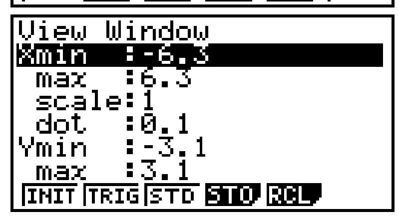
2. To clear out previous graphs, highlight the equation you want to delete and press [F2] (DEL) [F1] (Yes).



3. Enter the equation by pressing [X,θ,T] [x^2] [+] [2] [a b/c] [3] [X,θ,T] [−] [2] [EXE].

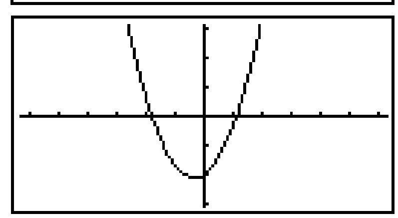


4. Set the view window, to the initial viewing window, by pressing [SHIFT] [F3] **(V-Window)** [F1] (INIT).



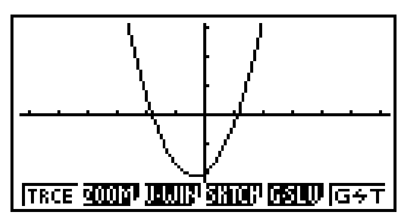
5. Press [EXIT] to return to the initial GRAPH screen.

6. Press [F6] (DRAW) to view the graph of the function.

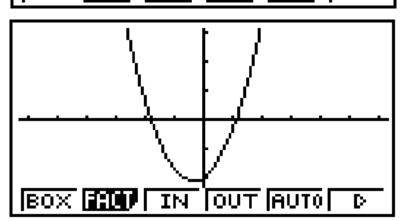


To set the zoom factors:

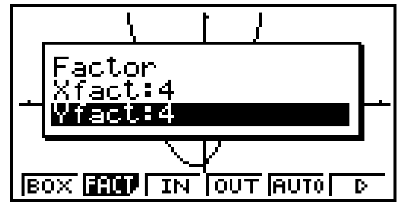
1. To set both zoom factors to 4, press [SHIFT] [F2] **(ZOOM)**.



2. Press [F2] (FACT) to change the zoom factors.

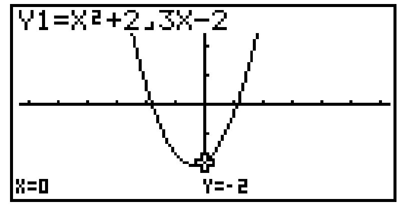


3. Press **4** **EXE** for the **x-factor** and **4** **EXE** for the **y-factor**.



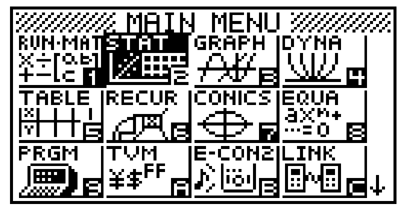
To examine the behavior of a graph using Trace:

1. To trace, press **F1** **(Trace)**. Use the Replay key pad to move the cursor left and right.



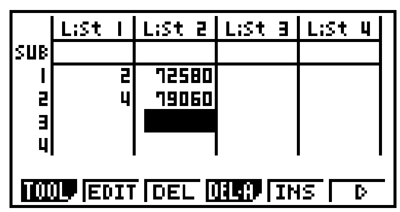
To find the equation of a line:

1. From the Main Menu, highlight the STAT icon and press **EXE** or press **2**.

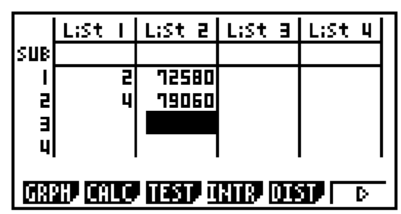


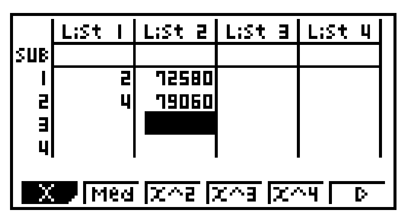
2. To delete any previous data, use the arrows to highlight the list and press **F6** (▷) **F4** (DEL-A) **F1** (Yes). Move the cursor to any other lists that may contain data and follow the same steps to delete the data.

3. Enter the points, with x-values in **List 1** and y-values in **List 2**. Be sure to press **EXE** after each value.

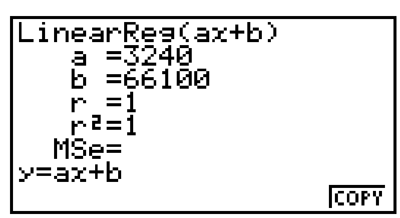


4. Press **F6** (▷) to return to the initial Stat screen. Press **F2** (CALC) **F3** (REG). The basic menu choices are linear, med-med line, quadratic, cubic, and quartic.



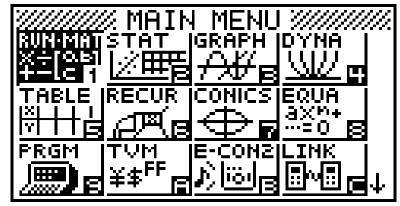


5. For this example, press **F1** (x) **F1** (ax+b). The screen displays the slope (a) and the y-intercept (b).

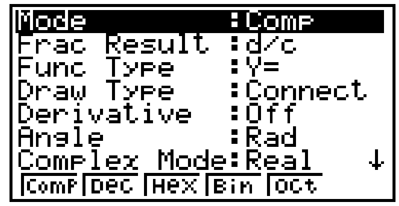


To change the angle mode:

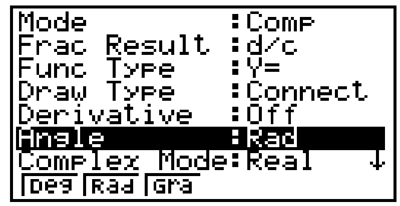
1. From the Main Menu, highlight the RUN-MAT icon and press EXE or press 1.



2. Press SHIFT MENU **(SET UP)** to enter the set up menu.



3. Using the Replay keypad, press the down arrow until Angle is highlighted.



4. Press F1 (Deg) to change to degree mode. Press F2 (Rad) to change to radian mode.

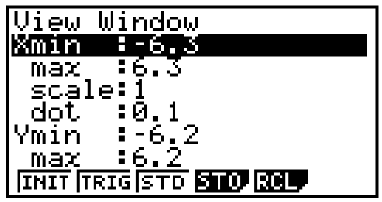
5. Press EXIT.

Being Locally Linear — Activity

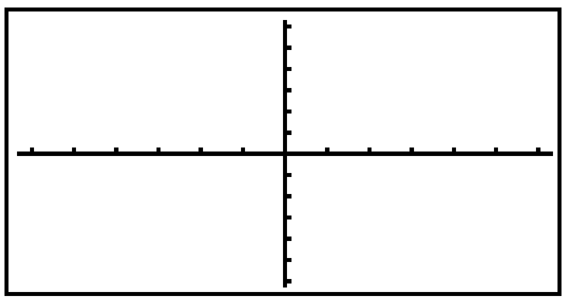
This activity will begin to bring home the point that as the behavior around a single point on a differentiable function is examined, the function will “flatten out” and very much resemble the behavior of a line drawn through the point of interest, using the Casio *fx-9750GII*.

Questions

1. Graph the function $y = x^2 - 2x - 3$ with the following view window.



Sketch the graph below.



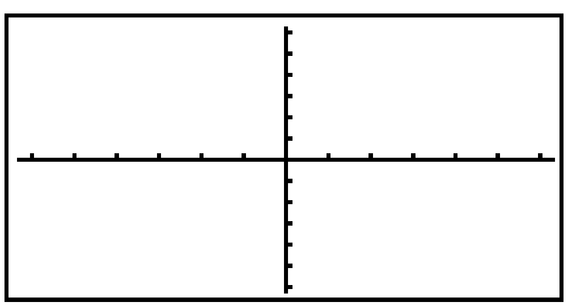
2. What can you say about the slope of the function?

____

____

____

3. Set your zoom factor to 4. Trace to x = 2 and zoom in at that point. Sketch your results below.

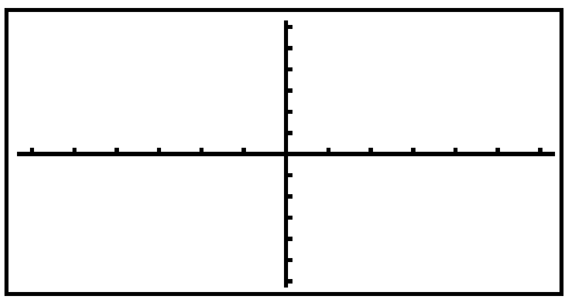


4. Using the trace function, record both the x- and y-values immediately above and below x = 2.

x	y
2	-3

5. Find the equation of the line connecting the first and third points in your table above.

6. Graph the line along with the original function in the last window you have and record the results below.

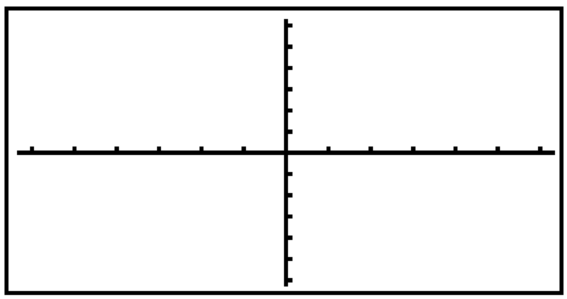


7. Zoom in on both at x = 2 and describe what you see.

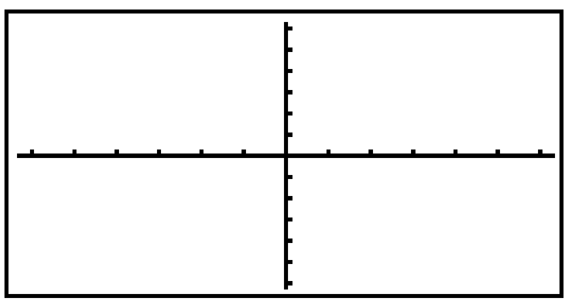
Extension

As the behavior of a function is examined closer and closer to a particular point of interest, in many cases the function begins to “flatten out”, or become approximately linear over a very small neighborhood around the particular point of interest. This behavior is called being locally linear and for this small interval can be very closely approximated by examining the behavior of the line tangent to the graph at the particular point of interest.

1. Examine the graph of y = sin(x), in radian mode, using the same window used at the beginning of the activity. Sketch what you see below.

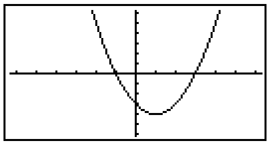


2. Change the settings to degree mode and sketch what you see below.

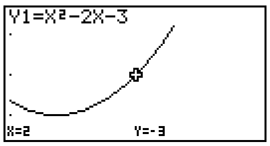


3. Explain why the results change in light of this activity.

Solutions

1. 

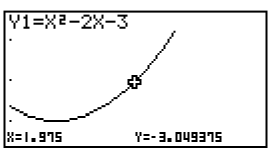
2. Answers will vary. A good answer should contain comments about the slope changing throughout the behavior of the function. A better answer should contain comments about the slope changing from negative to positive and perhaps mentioning where the slope is zero.

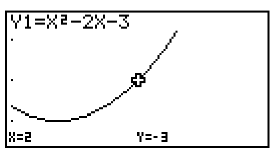
3. 

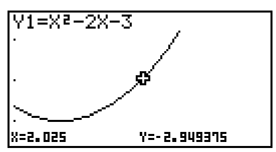
4.

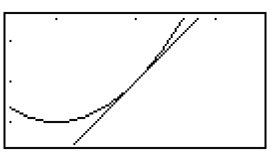
x	y
1.975	-3.049375
2	-3
2.025	-2.949375

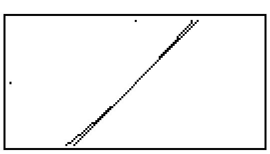
5. y = 2x – 6.999375





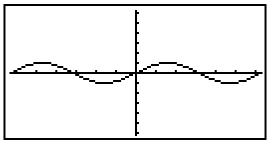


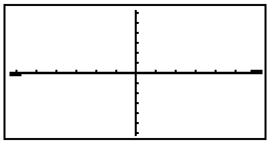
6. 

7. 

A good answer will include comments that the line and the function begin to be very close together around the point where x = 2. Some students might begin to discuss the line being very close to tangent (Care should be taken to point out that while it appears to be tangent, it is not tangent but a secant line). For some students an extra zoom or two might clarify the idea being presented.

Extension Solutions

1. 

2. 

3. The goal here is for students to realize that if the mode is changed to degree, they are looking at a graph that is being produced over the view window of ± 6.3 degrees away from sin(0) thus creating a graph that is very close to y = 0.

Bouncing Balls — Teacher Notes

Topic Area: Transformations for Achieve Linearity

NCTM standards:

- Select and use appropriate statistical methods to analyze data.
- For bivariate measurement data, be able to display a scatterplot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools.
- Identify trends in bivariate data and find functions that model the data or transform the data so that they can be modeled.

Objective

The student will be able to perform a transformation of data to linearize an exponential curve to measure the decay of the height of a bouncing ball.

Getting Started

Transformation to achieve linearity can be helpful especially when dealing with data, especially if the data displays a logarithmic or square root relationship. To do this, the student will take the logarithm or square root of both sides of the equation to give a straight line. After the transformation, the scatterplot data should be linear. The students must use the reverse transformation to interpolate and extrapolate information from the graphs.

Prior to using this activity:

- The student should be familiar with the logarithmic function and the square root function.
- The students should understand the concept of linearity.

Ways students can provide evidence of learning:

- Students will be able to linearize various types of data.
- Students will be able to find a linear regression.
- Students should be able to interpret data to determine what is necessary in order for linearization to occur.

Common mistakes to be on the lookout for:

- Students may mistake an exponential graph for a linear graph.
- Students will not be able to control all of the variables when repeated the experiment.
- Students may confuse the dependent and independent variables.

Definitions

- transformation
- linearity
- regression
- slope
- extrapolate
- interpolate

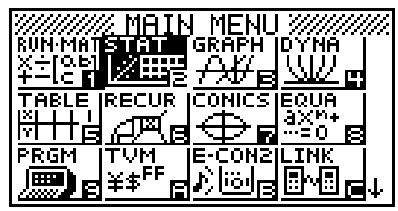
The following will demonstrate how to enter the data into the Casio *fx-9750GII* and interpret the results.

64 blue M & M's were placed in a cup. The cup was shaken and the M & M's were poured out. The number of M & M's that were face down (the M was not showing) was counted and removed from the mix. This was done three more times and the data was recorded below. The experiment was done three times.

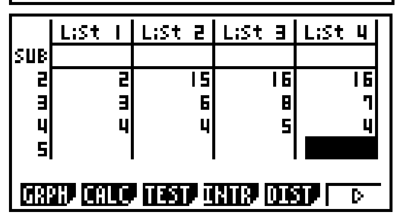
	Trial 1	Trial 2	Trial 3	Average
Toss 1	32	34	33	33
Toss 2	15	16	17	16
Toss 3	6	8	9	7
Toss 4	4	5	4	4

To find the average number of M & Ms that decayed (landed M side down) and plot a scatter plot of this data:

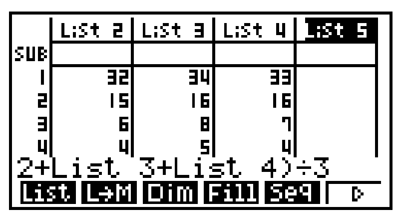
1. From the Main Menu, highlight the STAT icon and press [EXE] or press [2].

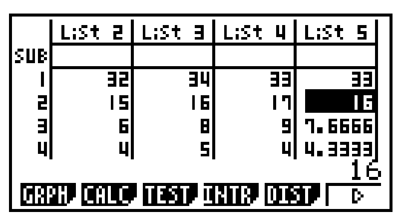


2. Enter the toss number in **List 1**, Trial 1 data into **List 2**, Trial 2 data into **List** 3 and Trial 3 data into **List 4**. Press [EXE] after each value is entered.



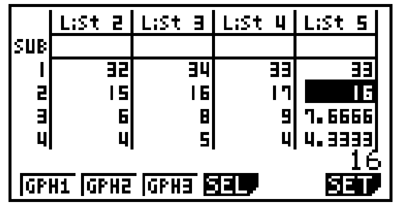
3. To list the average counts in **List 5**, highlight **List 5** and press [(] [OPTN] [F1](LIST) [F1](List) [2] [+] [F1](List) [3] [+] [F1](List) [4] [)] [÷] [3] [EXE].





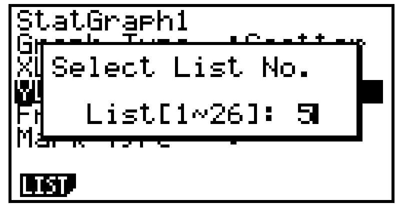
To view the graph:

1. Press F1 (GRPH) F6 (SET).

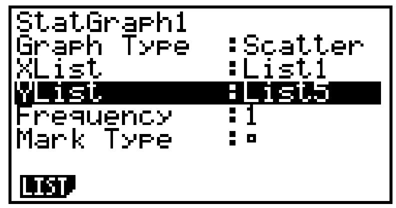


2. Arrow down to **Graph Type** and press F1 (Scat).

3. Arrow down to **XList** and press F1 (LIST) 1 EXE to select **List** 1.

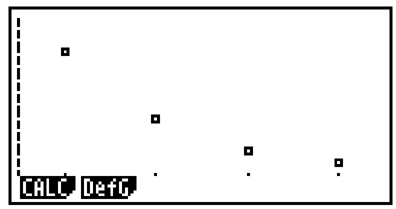


4. Arrow down to **YList** and press F1 (LIST) 5 EXE to select **List** 5.



5. Press EXIT.

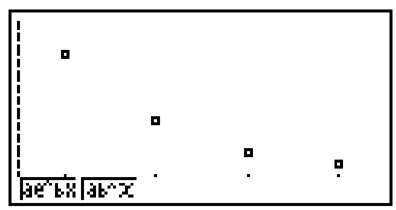
6. Press F1 (GPH1) to view the graph.



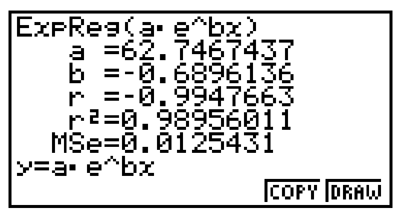
To calculate an exponential regression:

1. Press F1 (CALC).

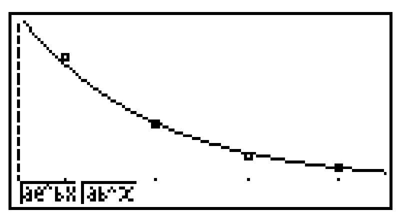
2. Press F6 (▷) and press F3 (EXP).



3. Press F1 (ae^bx).

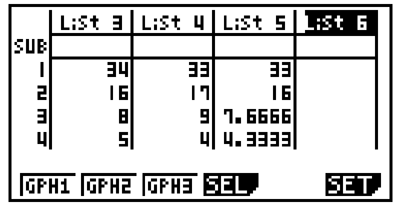


4. Press F6 (DRAW).

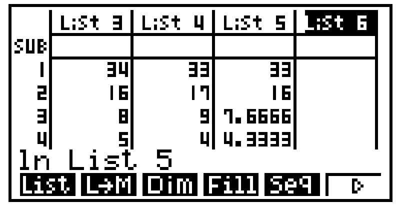


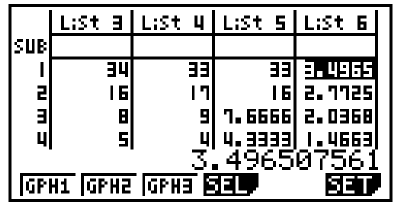
To find a table of values for linearized data:

1. Press [EXIT] until you return to the initial STAT screen.



2. With **List 6** highlighted, press [ln] [OPTN] [F1](LIST) [F1](List) [5] [EXE]. The linearized data should be in **List 6**.

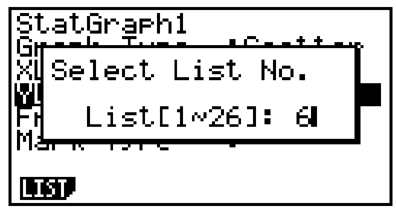


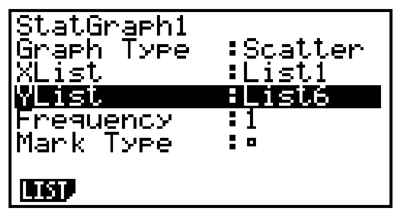


To graph a linearized scatter plot and find the linear regression:

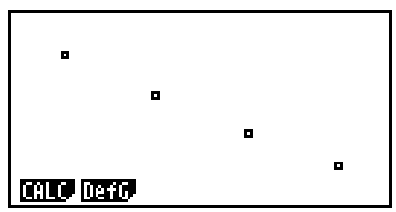
1. Press [F6](SET).

2. Change the **YList** to **List 6**, by pressing [F1](LIST) [6] [EXE].

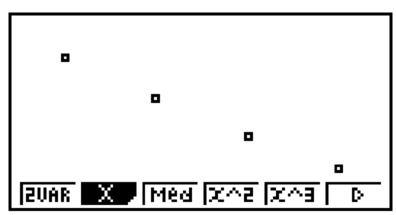


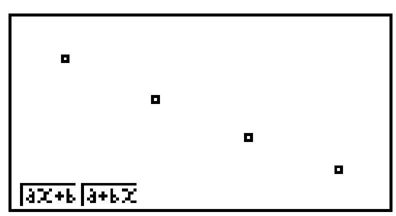


3. Press [EXIT] [F1](GRPH1) to view the scatter plot.

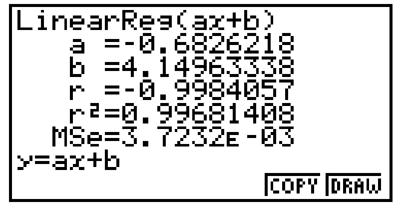


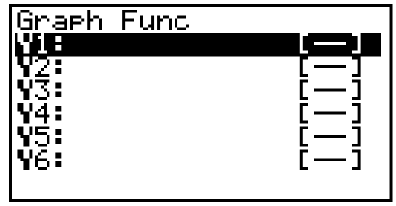
4. To calculate the linear regression, press [F1](CALC) [F2](x) [F1](ax+b).



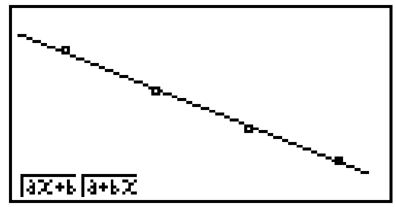


5. Press [F5] (COPY) [EXE] to store the equation for later analysis.



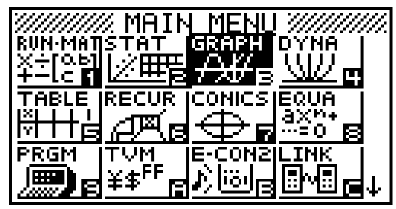


6. Press [F6] (DRAW) to view the graph.

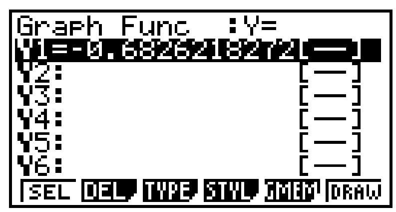


To graph and find extrapolated data:

1. From the Main Menu, highlight the GRAPH icon and press [EXE] or press [3].

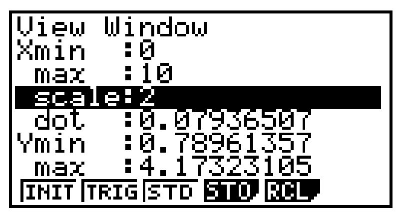


2. The equation is stored in the graph function Y1. Press [F1] (SEL) to select the equation.

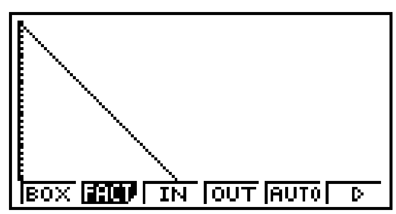


3. Set up the viewing window by pressing [SHIFT] [F3] **(V-Window)**.

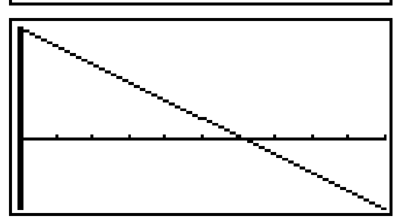
4. Set Xmin to 0 and Xmax to 10. Press [EXIT].



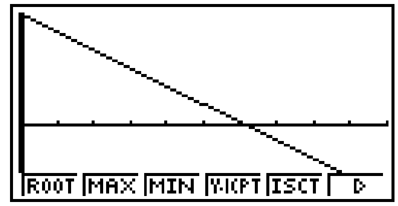
5. Press [F6] (DRAW).



6. Press [F2] **(ZOOM)** [F5] (AUTO) to have the calculator automatically set the window.

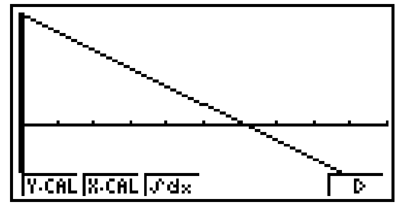


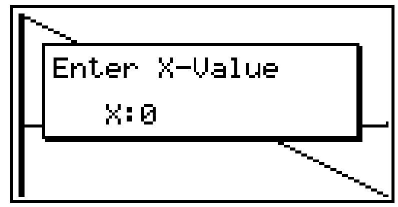
7. To find a y-value when x = 0, press F5 **(G-Solv)**.

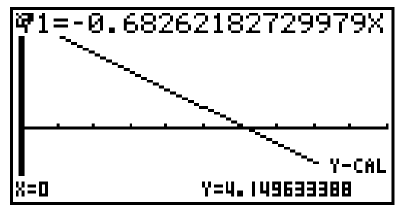


8. Press F6 (▷) F1 (Y-CAL) 0 EXE.

The solution will appear at the bottom of the screen.







Bouncing Balls — Activity

Introduction

With a partner, get a rubber ball and a meter stick or measuring tape. Line up the meter stick vertically against the wall. From about one meter off the ground, let the ball bounce close to the measuring tape. Measure how high the ball bounces on the first through sixth bounce. Repeat this three times and calculate the average height for each bounce. Record the data in the chart below.

Bounce	Trial 1 Height (m)	Trial 2 Height (m)	Trial 3 Height (m)	Average Height (m)
0				
1				
2				
3				
4				
5				
6				

Questions

1. What type of relationship is there between the bounce number and the bounce height? How do you know?

2. Plot a scatter plot of bounce height versus bounce number. Draw the scatter plot in the box below.

3. What is the non-linear regression equation for this graph? What do the values in your equation mean?

____

____

4. What transformation would you do to achieve a linear graph?

____

____

5. Complete a table of values for the linearized data.

6. Plot a scatter plot of the linearized data and draw it in the box below.

7. What is the regression equation of your graph? What do the values in your equation mean?

____

____

8. Using your linearized graph, extrapolate what the bounce height would be after 7 bounces.

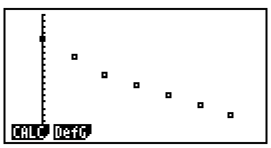
____

Solutions

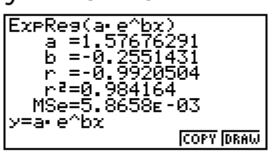
Answers will vary depending on data collected. The following solutions are based on the sample data below.

Bounce	Trial 1 Height (m)	Trial 2 Height (m)	Trial 3 Height (m)	Average Height (m)
0	1.5	1.5	1.5	1.5
1	1.18	1.25	1.21	1.21
2	0.91	0.96	0.93	0.93
3	0.75	0.79	0.79	0.78
4	0.63	0.62	0.61	0.62
5	0.48	0.46	0.44	0.46
6	0.31	0.30	0.30	0.30

1. Exponential decay

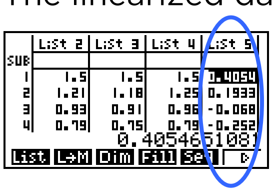
2. 

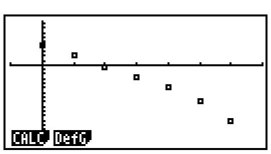
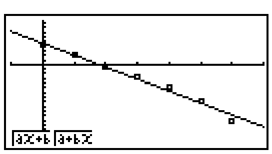
3. $y = 1.577e^{-0.255x}$



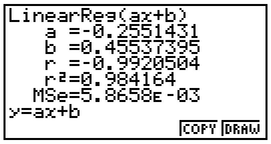
4. Take the ln of the data.

5. The linearized data is in List 5.

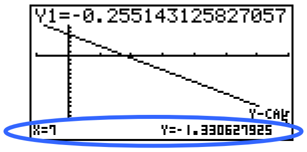


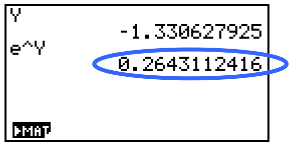
6.  

7. $y = -0.255x + 0.455$



8. After 7 bounces, the height of the ball should be 0.26 meters.





Topic Area: Area of Sectors

NCTM Standards:

- Use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume.
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.

Objective

The student will be able to use the Casio *fx-9750GII* to solve problems involving areas of sectors in a circle.

Getting Started

Many modern architects use circles as a basis for their designs on buildings, theaters, and planned communities. Being able to find the area covered by circles requires a different formula from the standard polygon. Students should be able to calculate the area of a sector and apply this to finding areas of regions within a circular pattern, such as the leftover space between a semicircle and an inscribed rectangle.

Prior to using this activity:

- Students should be able to find the radius of a circle given the diameter.
- Students should be able to find the measure of a central angle of a circle.
- Students should know that there are 360° in a circle.
- Students should be able to use the formulas for finding the area of various polygons.

Ways students can provide evidence of learning:

- Given the radius and central angle of a circle, the student will be able to find the area of a sector intercepted by the central angle.
- Given the diameter and central angle of a circle, the student will be able to find the area of a sector intercepted by the central angle.
- Given the radius and area of a sector, the student will be able to find the measure of the corresponding central angle.

Common mistakes to be on the lookout for:

- Students may use the diameter instead of the radius in the formula.
- Students may use radian measure instead of degree measure.
- Students may use the wrong angle measure in the formula.

Definitions:

- Circle
- Diameter
- Radius
- Arc
- Sector
- Central Angle
- Degrees
- Polygon

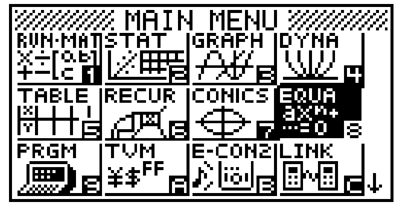
The following will demonstrate how to enter a formula into the Equation mode of the Casio *fx-9750GII* and solve for a specified unknown value.

Given that the volume of a cylinder is $V = \pi r^2 h$, find the following:

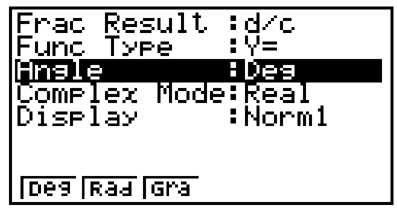
1. The height of a cylinder whose volume is 36.4 and whose radius is 1.4.
2. The volume of a cylinder whose diameter is 5 and whose height is 8.25.

To enter the formula into the Equation Solver:

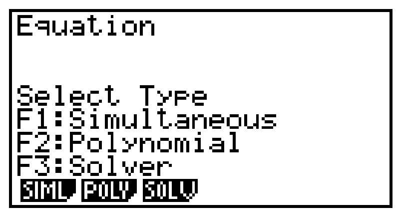
1. From the Main Menu, highlight the EQUA icon and press EXE or press 8.



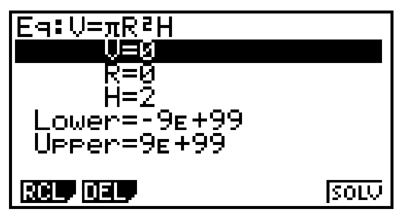
2. To set up the calculator for degree measures, press SHIFT MENU **(SET UP)**. Arrow down to highlight **Angle** and press F1 (Deg) to change the calculator to **Degree** mode. Press EXIT to return to the initial Equation screen.



3. Press F3 (SOLV) to enter the Equation Solver mode. If there is already an equation in the calculator, press F2 (DEL) F1 (Yes).

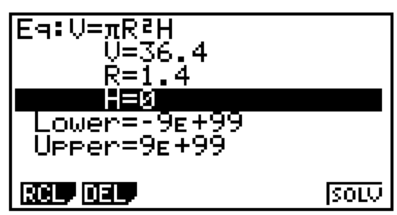


4. To enter the equation in to the calculator, press ALPHA 2 SHIFT • SHIFT EXP ALPHA 6 x^2 ALPHA F↔D EXE. The calculator may display numbers next to the variables, which represent values already in the calculator. We will enter values to replace them.

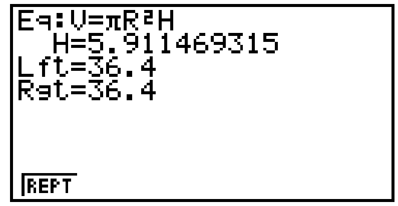


To find the height of the cylinder:

1. With V highlighted, enter 3 6 • 4 EXE.
2. With R highlighted, enter 1 • 4 EXE.



3. The H will already be highlighted. Press F6 (SOLV) to see the height.



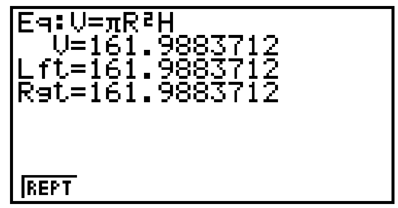
To find the volume of the cylinder:

1. To find the volume, press F1 (REPT) to return to the previous screen.

2. Move the cursor up to R and enter 2 • 5 EXE (This is half of the diameter).

3. Enter 8 • 2 5 EXE to enter the height.

4. Use the arrow keys to highlight V and press F6 (SOLV) to see the volume.



A sector is a fractional part of the total area of a circle, such as, calculating water coverage of a sprinkler system to area of visibility put out by a flashlight. The formula for finding the area of a sector is given by $A = \frac{\theta}{360}\pi r^2$. In this activity, we will use this formula to solve a variety of problems.

Questions

Many homeowners use sprinklers to water their lawns. One variety sends water out at a distance of 15 ft. and another sends out water out at a distance of 25 ft., both sent out through an angle set by the homeowner. Answer the given questions.

1. What is the area of coverage for each of the sprinklers if the angle is set at 120°?
   a. 15 ft. Sprinkler

   ____

   b. 25 ft. Sprinkler

   ____

2. At what angle would each sprinkler need to be set in order to cover 250 sq. ft.?
   a. 15 ft. Sprinkler

   ____

   b. 25 ft. Sprinkler

   ____

The Miller's front yard measures 120' by 40'. Mr. Miller bought a sprinkler that has a water range of 20 ft. and can be set to go through an angle, between 30° to 150°.

3. What is the minimum area that is covered by the sprinkler?

   ____

4. What is the maximum area that is covered by the sprinkler?

   ____

5. If he uses the maximum setting, how many times would he have to move the sprinkler in order to water the entire front yard?

   ____

Lighthouses put out a beam of light in the shape of a cone as they rotate. If sliced in half, the surface would be that of a sector whose radius is the distance from the origin to the end of the beam. The table below gives the distance of the light in various weather conditions. Use the maximum distance for all calculations. [Note: 1 Nautical Mile = 6,076 feet]

Meteorological Optimal Range Table	
Weather	Distance
Thick Fog	50 - 200 yards
Light Fog	500 – 1,000 yards
Clear	5.5 – 11.0 nautical miles
Exceptionally Clear	> 27 nautical miles

6. Calculate the area of light if there is a thick fog and the angle of light is 150°.

____

7. What would be the difference in area if the weather were clear?

____

8. Calculate the range of coverage of a light if the weather is light fog and the angle of the light is 175°.

____

Luigi's Pizza Shop wants to advertise the largest slice of pizza in the area. The pizza will be 3 ft. diameter. Currently, they sell a slice of an 18" diameter pizza for $2.75. Each pizza is cut into 8 even slices.

9. What is the area of a slice of the 18" pizza?

____

10. What will be the area of a slice of the new pizza?

____

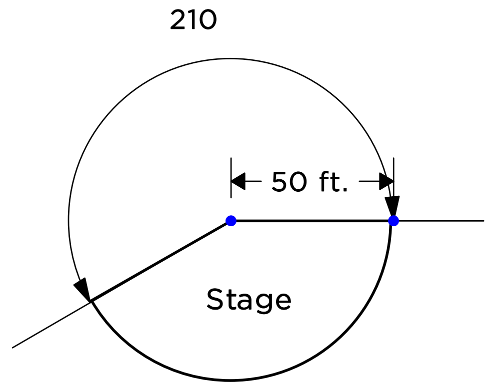
11. What is the change in the area of the slices?

____

12. What should the price be for the new slice at the same rate as the old slice? (Round to the nearest nickel.)

____

The New Haven Players are building a stage for their summer productions. They want to make it the shape of a circle which can turn. The front part will be the actual stage and the other section will be for storage and wardrobe.



13. What is the measure of the area for storage and wardrobe?

____

14. What will be the area of the stage?

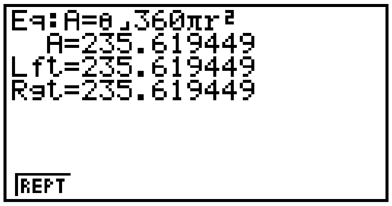
____

15. The stage will be covered in a special non-skid material to prevent accidents while it is turning. If the cost is $12.75 per square foot, how much will be spent on the stage covering?

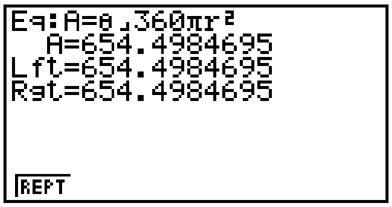
____

Solutions

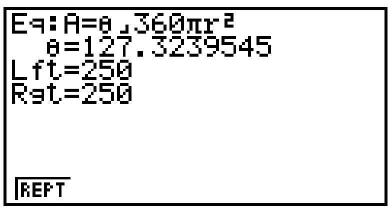
1. a. 235.6 sq. ft.



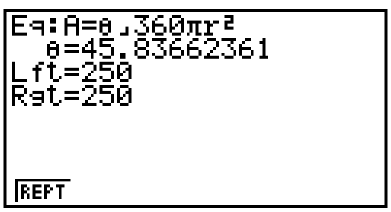
b. 654.5 sq. ft.



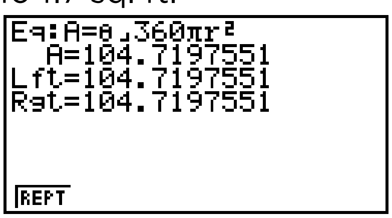
2. a. 127°



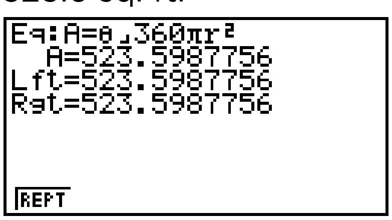
b. 46°



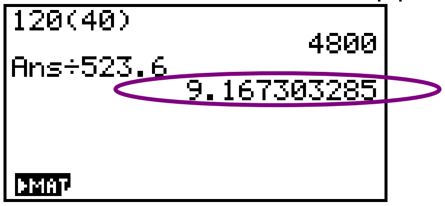
3. 104.7 sq. ft.



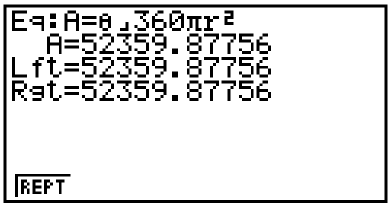
4. 523.6 sq. ft.



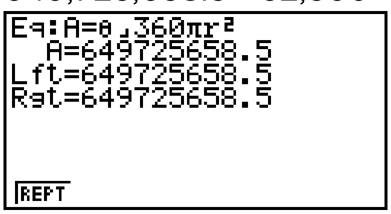
5. Area of the Yard = 120 (40) = 4800 sq. ft.
4800 ÷ 523.6 = 9.2 or approximately 9 times

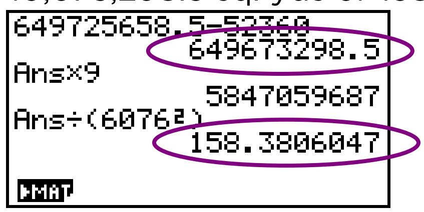


6. 52,360 sq. yd.

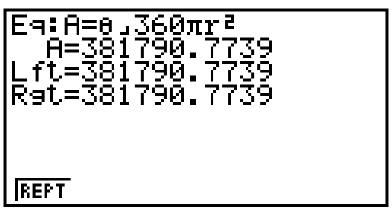


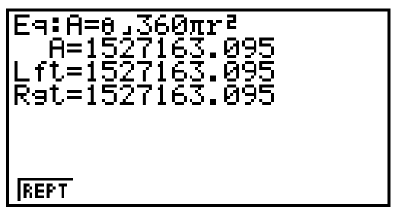
7. 649,725,658.5 - 52,360 = 649,673,298.5 sq. yds or 158.3 sq. nautical miles



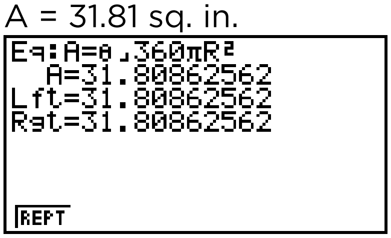


8. 381,790.8 sq. yd. to 1,527,163.1 sq. yd.

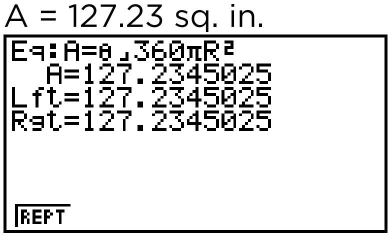




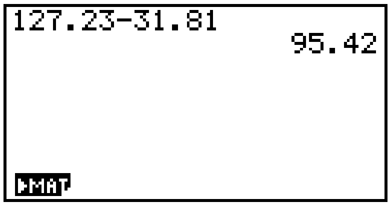
9. A = 31.81 sq. in.



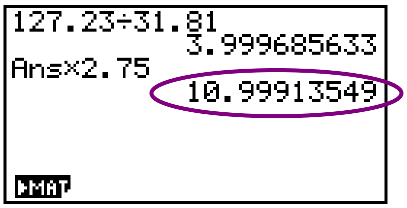
10. A = 127.23 sq. in.



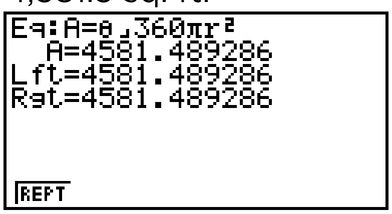
11. difference = 127.23 – 31.81 = 95.42 sq. in



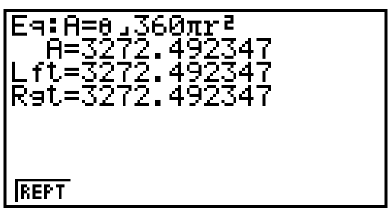
12. $11.00



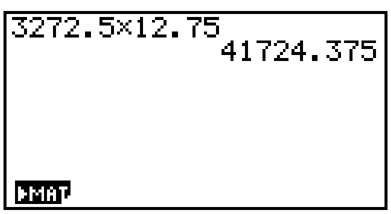
13. 4,581.5 sq. ft.



14. θ = 360 – 210 = 150° ; 3,272.5 sq. yd



15. Cost = 3,272.5($12.75) = $41,724.38



Class Height — Teacher Notes

Topic: Confidence Intervals

NCTM Standards:

- Develop and evaluate inferences and predictions that are based on data.
- Use simulations to explore the variability of sample statistics from a known population and to construct sampling distributions.
- Understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference.

Objective:

The students will be able to calculate the confidence interval for the height of the students in their class.

Getting Started

Showing students why it is important to be able to calculate confidence intervals may be difficult. This activity is chosen for its ease of data collection so that the students can concentrate on the statistics.

Use the following descriptions to go through the solutions manually:
There are two types of confidence interval tests that you can perform with the data: t-test and z-test. The t-test is performed when the mean and standard deviation for the sample are known but not for the entire population. The standard deviation of the population is estimated based on the number of degrees of freedom. The z-test requires both the mean and standard deviation for the sample and population. In this activity, the students will assume the mean of the population is the same as the sample and will now know the standard deviation for the population, thus they will be doing a t-test.

If a student wants to compare a sample population to an entire population, the student may use a t-test. There are two types of t-tests: one-tailed and two-tailed. A one-tailed test would be used if the student wanted to know if the mean of the sample smaller were larger or smaller than the mean of the population. A two-tailed test would show if the mean of the sample were smaller or larger than the mean of the population. With a certain degree of freedom value for a particular sample, the confidence value would determine how sure the results were. For example, if you wanted to be 99% sure that your hypothesis was correct, you would go with a confidence of p = 0.01 for a one-tailed test or p = 0.005 for a two-tailed test. The t-value must be found using a table of values based on degrees of freedom and the confidence value. There are also many t-value calculators on the Internet. In this activity, the students will perform a two tailed-test.
The t-test calculation is shown below:

$t = \frac{\overline{X} - \mu}{\frac{s}{\sqrt{n}}}$, where $\overline{X}$ is the mean of the sample, μ is the mean of the population, s is the standard deviation of the sample and n is the number of values in the sample. The confidence interval can be calculated using the following formula:

$$\overline{x} = \pm\left[t \bullet \frac{s}{\sqrt{n}}\right]$$

Prior to using this activity:

- The student should be able to gather data from a group.
- The student should be able to calculate basic statistical data.

Ways students can provide evidence of learning:

- The student will correctly interpret the T and Z score using a table.
- The student will be able to draw conclusions about the population, given a sample.

Common mistakes to be on the lookout for:

- The formulas for this activity is long, students need to be careful.
- The students might mix up the data for the sample and the entire population.

Definitions

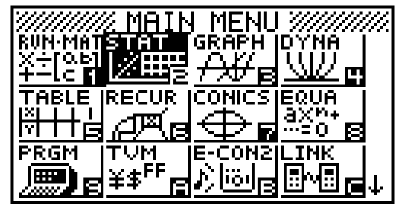
- Mean
- Standard Deviation
- Degrees of Freedom
- Confidence
- T-test
- Z-test
- Population
- Sample

Class Height "How-To"

The following will demonstrate how to enter a set of data into the Casio *fx-9750GII*, calculate the measures of central tendency, and find a % confidence interval.

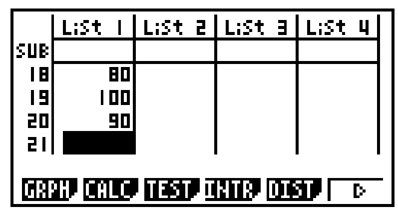
Scores on the first Algebra Test of the Year									
70	60	80	80	80	60	90	70	90	100
90	70	80	70	90	90	70	80	100	90

To enter the above set of data:



1. From the Main Menu, highlight the STAT icon and press EXE or press 2.

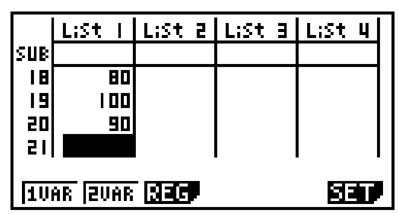
2. To clear previous data lists press F6 (▷) F4 (DEL-A) F1 (Yes).



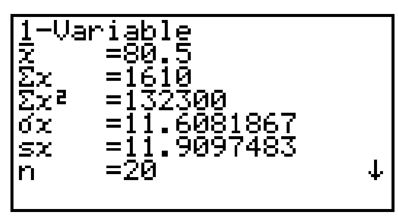
3. Enter the data by typing each number and pressing EXE after each entry.

4. The display should look like the screen shot on the right when completed.

To see 1-variable statistical calculations for this set of data:

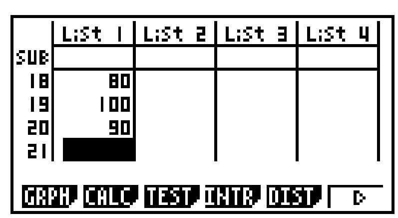


1. Press F2 (CALC) F1 (1VAR).



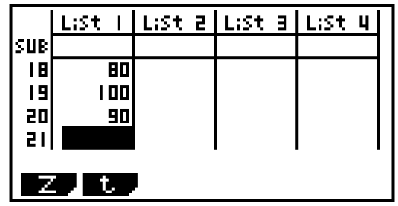
2. Use ▲ ▼ to access information like mean, median, mode, quartiles, maximum and minimum values. The beginning of the list is shown at the right.

To find the t-interval at 95% confidence:



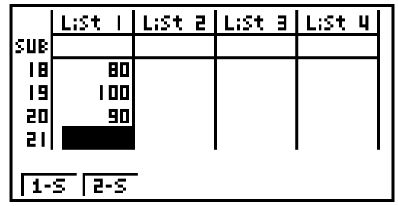
1. Press EXIT until you return to the initial STAT screen.

2. Press [F4] (INTR).

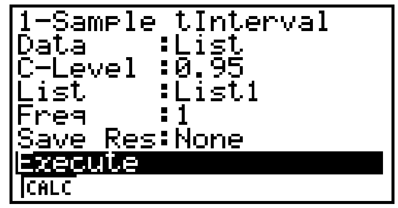


3. Press [F2] (t).

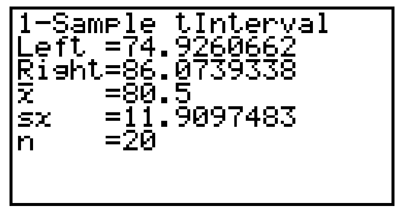
4. Press [F1] (1-S) for a 1-sample test.



5. For this problem, Data should say List, C-Level is at 0.95, for 95%, List is where the data is located, and the Frequency is 1. If everything is correct, arrow down to Execute and press [F1] (CALC).



6. The following values will be displayed on the screen. Left represents the minimum value in the interval, Right represents the maximum value in the interval, $\bar{x}$ represents the mean of the sample, *sx* represents the standard deviation of the sample, and n represents the number in the sample.



Introduction

Your class will estimate the mean height of all students in your school, using the heights for your class. This assumes that your class is a good representative sample of all the students at the school.

Questions

1. How many students do you have in your class today?

   ____

2. What is the degree of freedom value for the n-value in question 1?

   ____

3. For the 95% confidence interval, what would the confidence level be?

   ____

4. What is the mean height of all the students in your class today?

   ____

5. What is the standard deviation of the height?

   ____

6. What is the 95% confidence interval for the height of the students?

   ____

7. If the actual population height is 172 centimeters, would it fall into that 95% confidence interval? What about a 99% confidence interval? Is there a difference? Why or Why not? Is it possible that a population parameter does not fall in an interval that is based on one sample?

   ____

   ____

   ____

   ____

   ____

Use the table below to record your data from the class

Height in Centimeters	Height in Centimeters

Extension

1. Gather data on students' heights from either a basketball or volleyball team, perform the same questions in the exercise and compare the results.

Solutions

Answers will vary due to the data that is calculated. Below are the answers for a sample set of data:

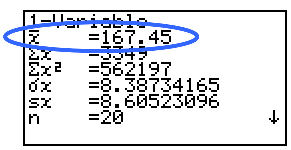
Height in centimeters	Height in centimeters
165	172
160	178
165	169
174	155
164	179
167	172
159	172
164	176
167	180
145	166

1. 20

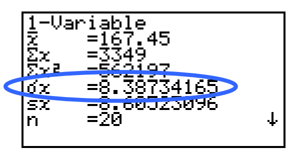
2. (20 – 1) = 19

3. 0.95

4. 167.45

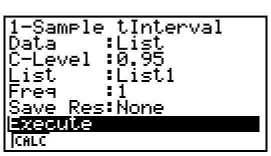


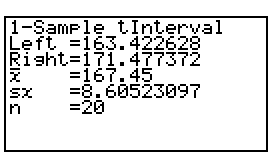
5. 8.387

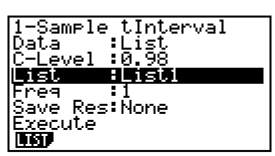


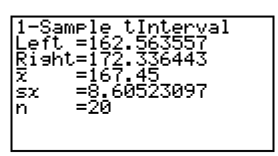
6. Heights between 163.42 cm and 171.48 cm

7. It would not fall within the 95% confidence interval, because the max height of that interval is 171.48, it would fall into a 99% confidence interval. It is possible that a confidence interval does not contain the population parameter, due to sampling variability.









Continuity Meets Differentiability — Teacher Notes

Topic: Derivatives and Continuity

NCTM Standards

- Approximate and interpret rates of change from graphical and numerical data.
- Make and investigate mathematical conjectures.
- Organize and consolidate their mathematical thinking through communication; communicate their mathematical thinking coherently and clearly to peers, teacher, and others.

Objectives

The student will be able to connect the ideas of slope, local linearity and differentiability. The student will be able to understand and communicate the idea that continuity alone does not guarantee that a function has a derivative.

Getting Started

This activity will begin to extend the idea of local linearity and derivatives. It will also connect those concepts to continuity and point out that continuity is a necessary but not sufficient condition for differentiability. The connection will be made visually using the idea of local linearity (or what happens when it's missing). Symbolic derivatives will, where appropriate, be used to support these findings.

Prior to using this activity:

- Students should be able to produce and manipulate graphs of functions manually and with a graphing calculator.
- Students should have had an introduction to basic symbolic derivatives to make an easier connection to the visuals.
- Students should have an understanding of slope of a function at a point as the visual presentation of the derivative.

Ways students can provide evidence of learning:

- Students should be able to produce graphs of function and communicate why a certain function may not have a derivative at a certain point.
- Students should be able to, where appropriate, back up their graphical presentation with a symbolic analysis.

Common mistakes to be on the lookout for:

- Students may not understand the zoom process and what is taking place.
- Students may not be able to communicate the concept of derivatives verbally.
- Students may enter the rational exponents incorrectly resulting in the calculator producing a graph different than the one desired.

Definitions:

- Continuity
- Derivative
- Differentiability
- Discontinuity
- Function
- Linear
- Rational Exponents
- Slope
- Symbolic Derivatives

Formula:

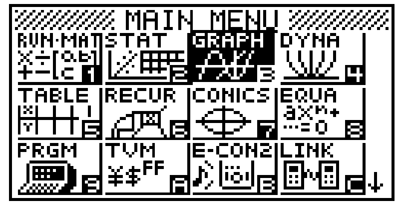
$$\text{slope} = m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

The following will demonstrate how to graph a function, set the zoom factors, examine its behavior, and find the slope of a line on the Casio *fx-9750GII*.

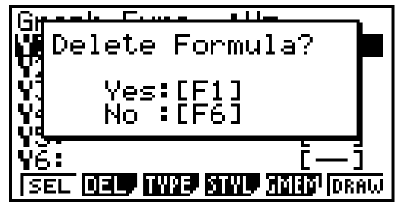
Graph the function $y = x^{\frac{2}{3}}$.

To graph a function:

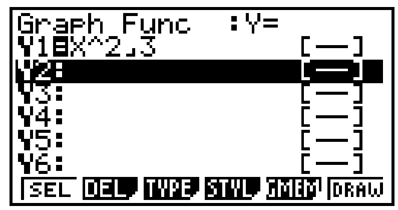
1. From the Main Menu, highlight the GRAPH icon and press [EXE] or press [3].



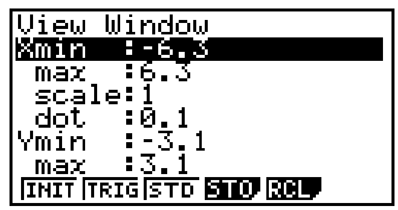
2. To clear out previous graphs, highlight the equation you want to delete and press [F2] (DEL) [F1] (Yes).



3. Enter the equation by pressing [X,θ,T] [^] [2] [a b/c] [3] [EXE].

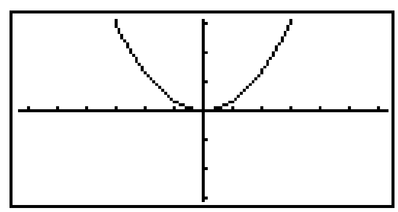


4. Set the view window, to the initial viewing window, by pressing [SHIFT] [F3] **(V-Window)** [F1] (INIT).



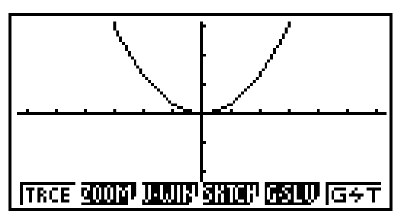
5. Press [EXIT] to return to the initial GRAPH screen.

6. Press [F6] (DRAW) to view the graph of the function.

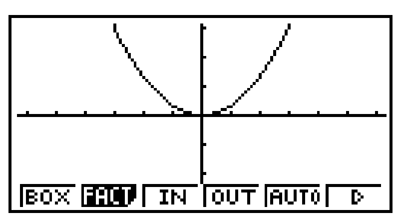


To set the zoom factors:

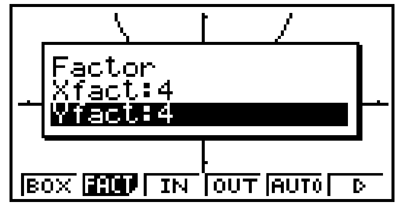
1. To set both zoom factors to 4, press [SHIFT] [F2] **(ZOOM)**.



2. Press [F2] (FACT) to change the zoom factors.

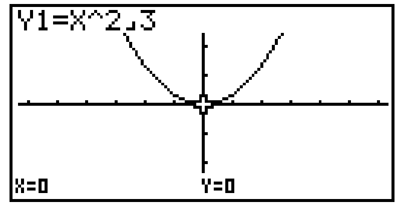


3. Press [4] [EXE] for the **x-factor** and [4] [EXE] for the **y-factor**.



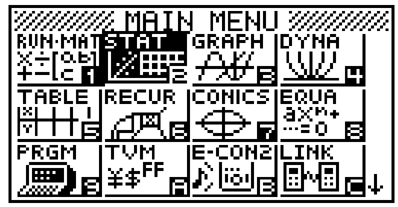
To examine the behavior of a graph using Trace:

1. To trace, press [F1] **(Trace)**. Use the Replay key pad to move the cursor left and right.



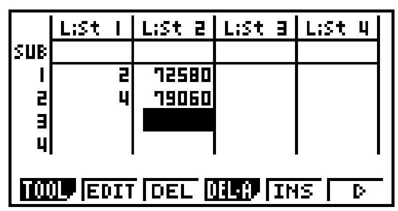
To calculate the slope between two points:

1. From the Main Menu, highlight the STAT icon and press [EXE] or press [2].

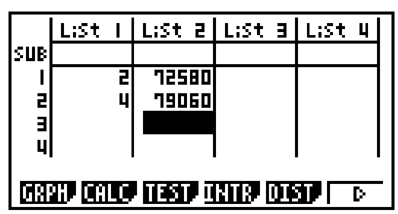


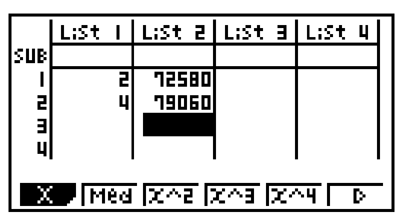
2. To delete any previous data, use the arrows to highlight the list and press [F6] (▷) [F4] (DEL-A) [F1] (Yes). Move the cursor to any other lists that may contain data and follow the same steps to delete the data.

3. Enter the points, with x-values in **List 1** and y-values in **List 2**. Be sure to press [EXE] after each value.

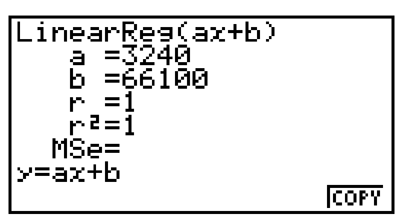


4. Press [F6] (▷) to return to the initial Stat screen. Press [F2] (CALC) [F3] (REG). The basic menu choices are linear, med-med line, quadratic, cubic, and quartic.





5. For this example, press [F1] (x) [F1] (ax+b). The screen displays the slope (a) and the y-intercept (b).



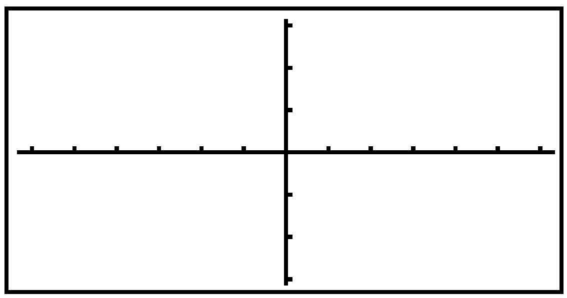
Continuity Meets Differentiability Activity

This activity will begin to extend the idea of local linearity and the derivative. It will also connect those concepts to continuity. The connection will be made visually using the idea of local linearity (or what happens when it is missing), using the Casio *fx-9750GII*.

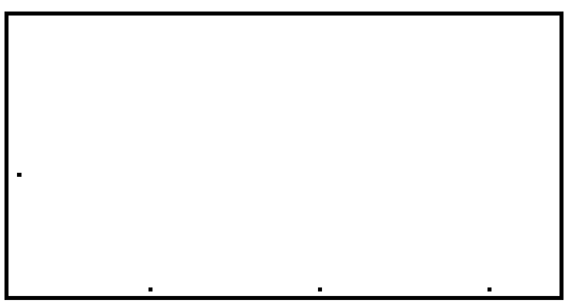
Questions

Explore the behavior of the function $y = x^{\frac{2}{3}}$ around the value $x = 2$.

1. Sketch the graph of the function $y = x^{\frac{2}{3}}$ in the initial default viewing window and describe what you see.



2. Trace to the value of $x = 2$ and with your zoom factors at 4 for both x and y, zoom in twice. Sketch what you see and explain what is going on.



3. Using trace, fill in the following values for the function accurate to 5 decimal places.

Point	x	y
A	2.0125	
B	2.00625	
C	2	
D	1.99375	
E	1.9875	

4. Calculate slopes of A & B, B & C, C & D, and D & E and record them as Slope 1, Slope 2, Slope 3, and Slope 4, respectively.

Slope 1	
Slope 2	
Slope 3	
Slope 4	

5. What do your results indicate? Explain how the graph you saw either agrees or disagrees with those results.

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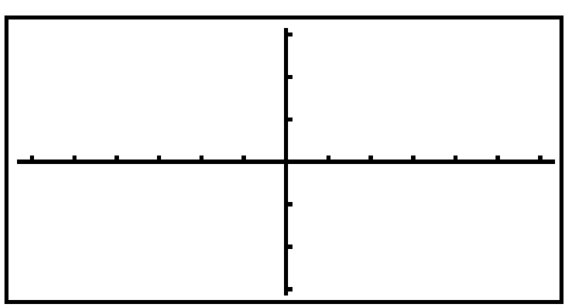
6. Examine the same function around the point x = 0 by graphing the function in the initial viewing window. Trace to x = 0 with the zoom factors still set at 4 and zoom in twice. Record the graph below.



7. Describe what you see.

8. Using trace, fill in the following values for the function accurate to 5 decimal places.

Point	x	y
F	-0.0125	
G	-0.00625	
H	0	
I	0.00625	
J	0.0125	

9. Repeat the same slope procedure as before. Calculate slopes of F & G, G & H, H & I, and I & J and record them as Slope 5, Slope 6, Slope 7, and Slope 8:

Slope 5	
Slope 6	
Slope 7	
Slope 8	

10. What do these results indicate? Compare them to the results from the exploration of the graph around x = 2.

11. While continuity is a necessary condition for a function to have a derivative at that same point, it is not a sufficient condition as these two examples indicate. The function explored is both continuous and differentiable at $x = 2$, however, it is continuous but NOT differentiable at $x = 0$. Use symbolic derivatives to support the visual evidence found in these explorations.

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Extension

1. Can you come up with some other simple functions that might provide places where the function is continuous and differentiable at one point in its domain, and continuous but NOT differentiable at another point in its domain?

____

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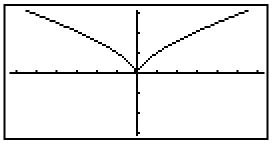
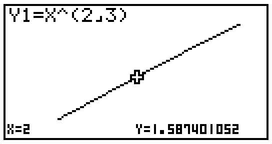
Solutions

1. 

Answers will vary. A good answer will include a statement about the hard corner at x =0.

2. 

Answers will vary. Students should comment the graph appears to be virtually linear. There should be a comment about the function straightening out and seeing a good linear approximation of the function. There should also be a comment regarding the continuity around x = 2.

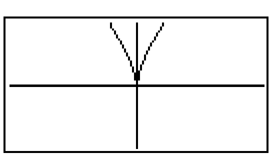
3.

Point	x	y
A	2.0125	1.59401
B	2.00625	1.59071
C	2	1.58740
D	1.99375	1.58409
E	1.9875	1.58078

4.

Slope 1	0.528
Slope 2	0.5296
Slope 3	0.5296
Slope 4	0.5296

5. Answers will vary. A good answer will include statements about the slopes being the same and the graph becoming linear around the point x = 2. The graph should show a picture that it is highly linear in the small neighborhood of x = 2.

6. 

7. Answers will vary. In stark contrast to the previous exploration, a good answer should include comments about the graph NOT straightening out or becoming locally linear. There should also be some comments about the continuity being maintained.

8.

Point	x	y
F	-0.0125	0.05386
G	-0.00625	0.03393
H	0	0
I	0.00625	0.03393
J	0.0125	0.05386

9.

Slope 5	-3.188
Slope 6	-5.4288
Slope 7	5.4288
Slope 8	3.188

10. Answers will vary. A good answer should include a direct comparison indicating that the graph is not becoming locally linear around x=0, while it did straighten out around x = 2. The idea that the graph is continuous at both x = 0 and x = 2 should be discussed.

11. $\frac{dy}{dx}\left(x^{\frac{2}{3}}\right)=\frac{2}{3}\left(x^{\frac{-1}{3}}\right)$

Answers will vary. A good answer will point out that the derivative at x = 2 exists (and is equal to 0.52913, very close to the value found in the exploration). However, the derivative at x = 0 does not exist (division by 0). In fact repeated zooming around x = 0 will continue to provide the same slope with different signs on either side of x = 0.

Extension Solutions

1. Answers will vary.

Cost Analysis — Teacher Notes

Topic Area: Angles/Unit Circle

NCTM Standard:

- Use trigonometric relationships to determine lengths and angle measures.
- Understand the meaning of equivalent forms of expressions, equations, inequalities, and relations.

Objective

Students will solve a triangle using special trigonometric ratios.

Getting Started

Have the students work in pairs or small groups and discuss the trigonometric functions of acute angles. Students should indicate for a given angle (θ), why the value of each of the six trigonometric functions is independent of the size of the right triangle that contains angle (θ).

Prior to using this activity:

- Students should be able to graph functions into the Casio *fx-9750GII* calculator.
- Students should understand how to perform an analysis of graphs.
- Students should understand how to create a table of function values.

Ways students can provide evidence of learning:

- Given any decimal angle, the student will be able to convert it into degrees-minutes-seconds.
- Given a trigonometric function, the student will be able to evaluate values of θ.
- Student will be able to write all trigonometric functions in terms of sine or cosine.

Common mistakes to be on the lookout for:

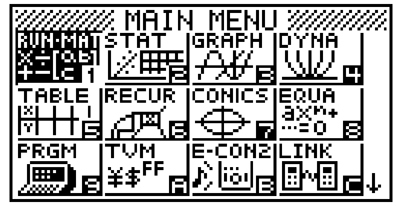
- Students may create a syntax error when entering the function to be graphed
- Students may create a condition error when entering the function
- Students may be unaware of the sexagesimal structure of degrees

Definitions

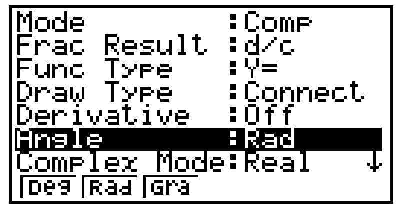
- Acute Angle
- Radian
- Right Triangle
- Quadrantal Angle
- Greatest Integer Function

The following will demonstrate how to: change calculator angle preferences, evaluate angles in degrees or radians, convert decimal degrees (DD) into degree-minute-second form (DMS), and evaluate trigonometric functions on the Casio *fx-9750GII*.

To set the default angle unit:



1. From the Main Menu, highlight the RUN•MAT icon and press EXE or press 1.

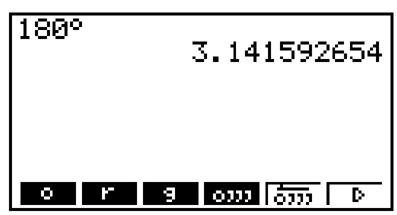


2. To set the default angle unit, press SHIFT MENU **(SET UP)** ▽ ▽ ▽ ▽ ▽. From here, you will choose either F1 (Deg) or F2 (Rad).

To convert angles in degrees to radians, when default angle unit is radians:

1. Suppose we wanted to convert 180° into radians.

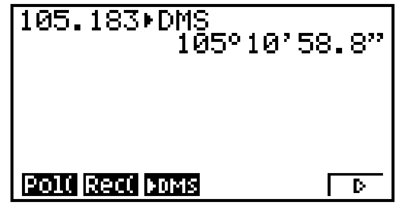
2. Press 1 8 0 OPTN F6 (▷) F5 (ANGL) F1 (°) EXE.



To convert angles in decimal degrees (DD) into degree-minute-second (DMS):

1. Suppose we wanted to convert 105.183° to DMS.

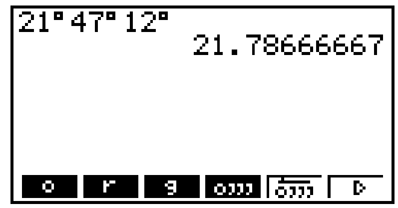
2. Press 1 0 5 • 1 8 3 OPTN F6 (▷) F5 (ANGL) F6 (▷) F3 (▶DMS) EXE.



To convert angles in degree-minute-second (DMS) into decimal degrees (DD):

1. Suppose we wanted to convert 21°47'12" to DD.

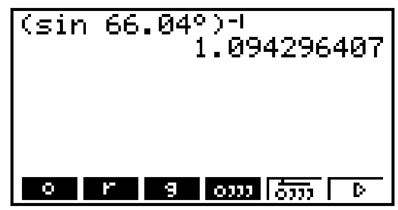
2. Press 2 1 OPTN F6 (▷) F5 (ANGL) F4 (○,,,) 4 7 F4 (○,,,) 1 2 F4 (○,,,) EXE.



To evaluate secant, cosecant, and cotangent:

1. Suppose we wanted to evaluate csc 66.04°.

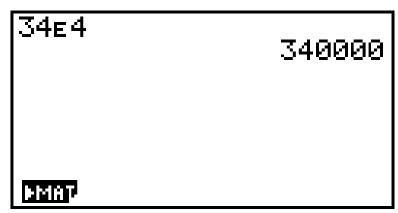
2. Press (sin 6 6 • 0 4 OPTN F6 (▷) F5 (ANGL) F1 (°)) SHIFT) EXE.



To evaluate write numbers in scientific notation:

1. Suppose we wanted to write 340,000 into scientific notation.

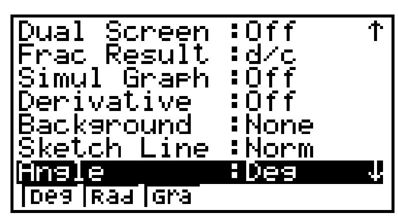
2. Press 3 4 EXP 4.



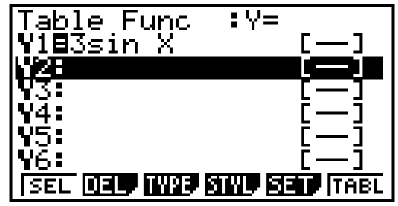
To evaluate create a table of values for a trigonometric function:

1. Suppose we wanted to evaluate 3(sin x) for 0° < x < 90°.

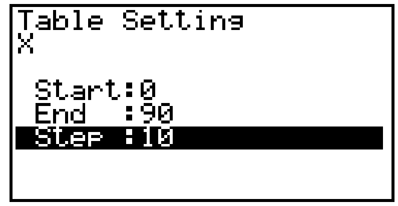
2. To change the angle unit into degrees, press SHIFT MENU **(SET UP)** ▽ ▽ ▽ ▽ ▽ F1 (Deg).



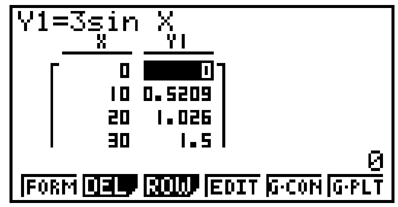
3. From the Main Menu, highlight the TABLE Icon and press EXE or press 5.



4. To enter the function, press 3 sin X,θ,T EXE.



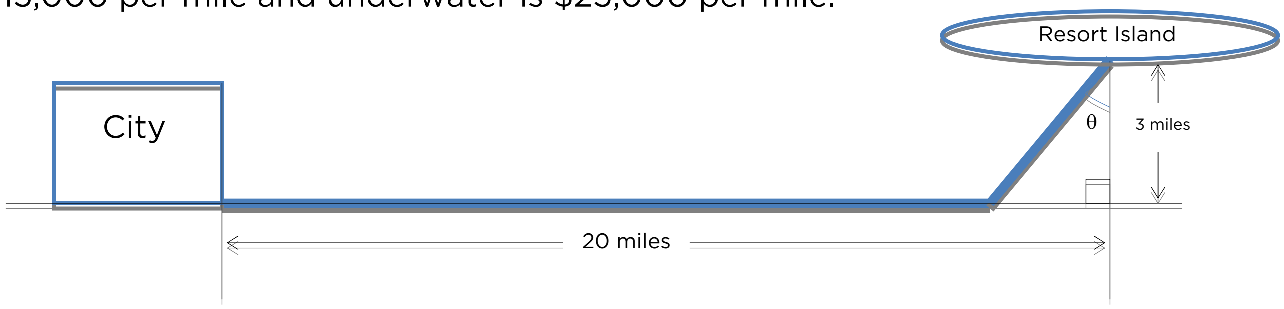
5. To set the table input values, press F5 0 EXE 9 0 EXE 1 0 EXE EXE.



6. To display the table of values, press F6 (TABL).

Cost Analysis Activity

A cable television company wishes to run a cable from a city to a resort island 3 miles offshore. The cable is to go along the shore, then to the island underwater, as indicated in the accompanying figure. The cost of running the cable along the shore is $15,000 per mile and underwater is $25,000 per mile.



Questions

1. Referring to the figure above, find the equation that would be used to find the total cost, in terms of θ.

2. Calculate a table of cost, each cost to the nearest dollar, for the following values:

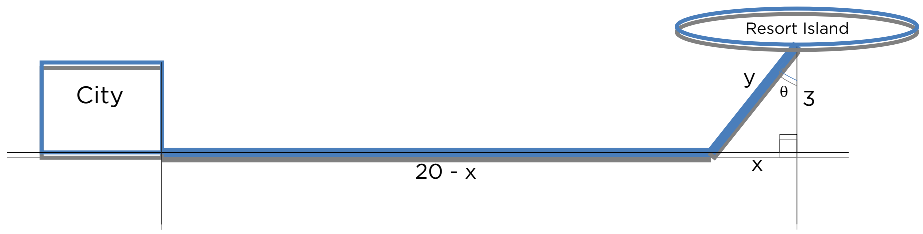
θ	C(θ)
10°	$
20°	$
30°	$
40°	$
50°	$
60°	$
70°	$
80°	$
90°	$

Extension

1. Determine the legitimate values for θ? Explain.

2. What integer value of θ will minimize the cost of running cable to the resort island?

Solutions



1. If we label the figure with the unknowns 20 – x, x & y then we may use trigonometry to define these in terms of θ.

$\tan\ \theta = \dfrac{x}{3}$, therefore, $x = 3\tan\theta$

$\sin\ \theta = \dfrac{3\tan\theta}{y}$, thus $y = \dfrac{3\tan\theta}{\sin\theta} = 3\bullet\dfrac{\sin\theta}{\cos\theta}\bullet\dfrac{1}{\sin\theta} = \dfrac{3}{\cos\theta} = 3\sec\theta$

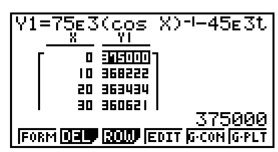
Therefore, our two quantities of interest are:
$20 - x = 20 - 3\tan\theta$
and
$y = \sec\theta$

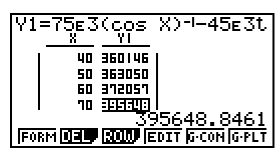
Our cost equation is
$C(\theta) =$
(Distance along shore)•(Cost/mile) + (Distance underwater)•(Cost/mile)
$C(\theta) = (20 - 3\tan\theta)\bullet 15{,}000 + (3\sec\theta)\bullet 25{,}000$

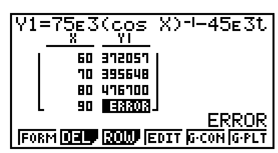
This simplifies into
$C(\theta) = 75{,}000\sec(\theta) - 45{,}000\tan(\theta) + 300{,}000$

2.

θ	$C(\theta)$
10°	$368,222.28
20°	$363,434.67
30°	$360,621.78
40°	$360,146.06
50°	$363,050.38
60°	$372,057.71
70°	$395,648.85
80°	$476,700.10
90°	$ undefined

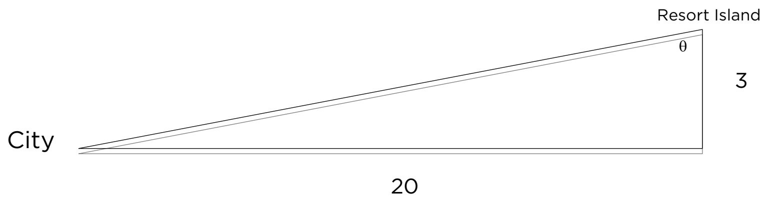






Extension Solutions

1. The two extreme cases are 100% underwater and 20 miles by shore with 3 miles underwater. Using the right triangle below you can solve for θ for the former case and use inspection for the latter case.



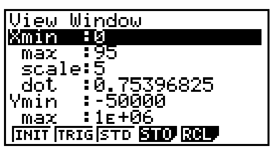
Using trigonometric ratios, you find tan θ = $\dfrac{20}{3}$, therefore

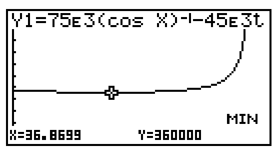
$\theta = \tan^{-1}\left(\dfrac{20}{3}\right) \approx 81.47°$

Thus legitimate values of the angle are: $0° \le \theta \le 81.47°$

Coincidently, the hypotenuse is $\sqrt{20^2 + 3^2} = \sqrt{409} \approx 20.22$.

2. Using the G-Solve capabilities to locate the minimum value in the Graph Menu yields the minimum value of 36.87°.





Topic Area: Represent and Analyze Mathematical Situations

NCTM Standard:

- Represent and analyze mathematical problems using algebraic symbols, and use mathematical models to understand relationships.

Objective

The student will be able to use the Casio *fx-9750GII* to solve a linear programming problem. The student will be able to write the objective function and the inequalities for the constraints, enter them into the Graph Module, set up the View Window and use the resulting graph to answer questions.

Getting Started

Discuss with students the use of linear programming, and how to interpret the graph. Explain what is meant by the objective function and constraint. Review how to write an inequality and how to graph the results. Discuss situations where constraints are applied to a product or service and allow the students to generate their own ideas. Discuss what is meant by revenue for a product, being sure to include the roles of supply and demand.

Prior to using this activity:

- Students should be able to enter the inequalities used in the Calculator Notes and know how to change the inequality sign.
- Students should be able to set up the view window, find the intersections of the vertices, and find the maximum for the graph

Ways students can provide evidence of learning:

- When given a series of constraints, students can discuss the results of the graph by explaining how they determined the intersections and what each intersection represents.
- When given a series of constraints, students can discuss the graph for the revenue, including how to set it up on the calculator, what each point on the graph indicates, and what the vertex point shows.
- When given a linear programming graph, students should be able to relate the vertex point to the objective equation and tell how to find the vertex algebraically.

Common calculator or content errors students might make:

- Students may have a hard time selecting the right inequality.
- Students may forget to convert the inequality to slope-intercept form to enter into the calculator.
- Students may experience difficulty converting a decimal slope to a fractional slope.

Definitions: Constraint, Inequality, Vertex

Given a linear programming problem, the students will be able to write the objective function and the inequalities for the constraints, enter them into the GRAPH Module of the Casio *fx-9750GII*, set up the View Window and use the resulting graph to answer questions.

Given a quadratic equation with constraints, the students will be able to set up the window, graph the function, and find the maximum or minimum point for the graph.

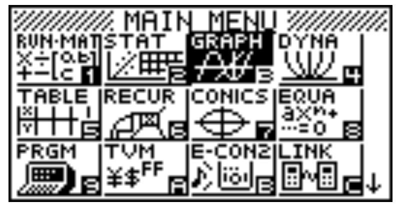
Data: Inequalities to Graph

$y \le -x + 5$; $y \le -2x + 9$; $y \le 3$; $x \ge 0$; $y \ge 0$

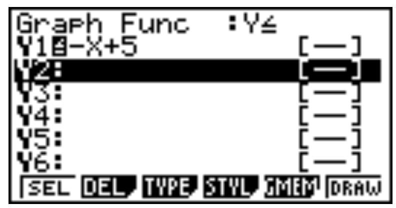
Quadratic Equation:

$y = -\frac{1}{8}x^2 + 40x \quad 0 \le x \le 320$

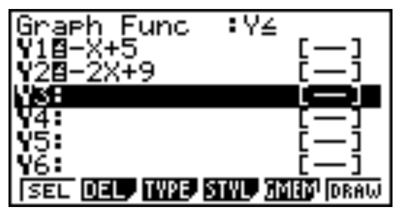
To enter the inequalities:



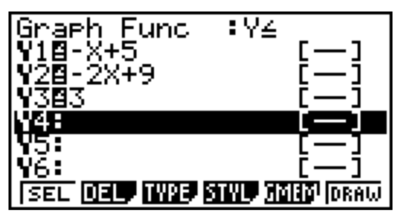
1. From the Main Menu, highlight the GRAPH icon and press [EXE] or [3].



2. Press [F3] (TYPE), [F6] for more options and then [F4] (Y≤) to set **Y1:** as a ≤ inequality.



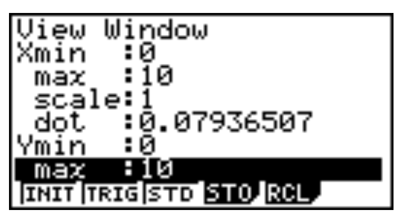
3. Enter the first inequality into **Y1:** by entering: [(–)] [X,θ,T] [+] [5] [EXE].



4. Enter the second inequality into **Y2:** by entering: [(–)] [2] [X,θ,T] [+] [9] [EXE].

5. Enter the last inequality into **Y3:** by entering: [3] [EXE].

To set up the view window:

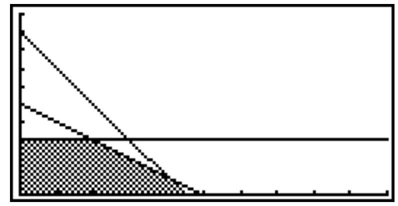


1. Press [SHIFT] [F3] **(V-WIN)**.

2. Enter [0] [EXE] for **Xmin** and enter [1] [0] [EXE] for **Xmax**.

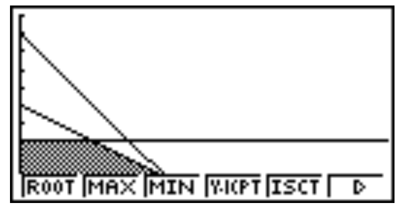
3. Press [▼] twice, enter [0] [EXE] for **Ymin**.

4. Enter [1] [0] [EXE] for **Ymax**. Press [EXE] twice to view the graph. The screen should look like the one on the right.



To find graphical intersections:

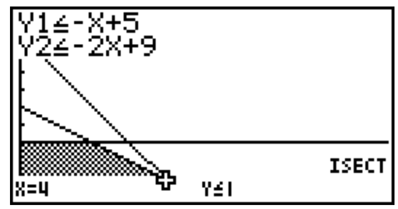
1. Press [SHIFT] [F5] **(G-Solv)** and [F5] (ISCT).



2. Press the [▼] until the desired inequality is selected and press [EXE].

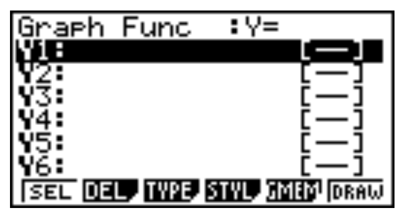
3. Press the [▼] until the desired inequality is selected and press [EXE].

4. The screen at the right shows the intersection of **Y1:** and **Y2:**.



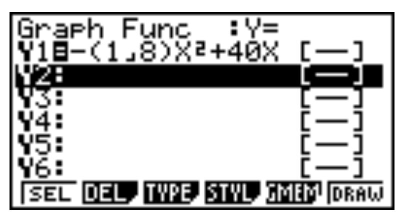
To find the maximum point for a quadratic function:

1. Press [EXIT] and [▲].



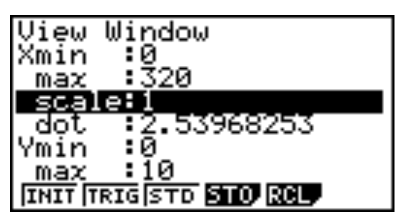
2. Press [F2] (DEL), [F1] (YES) to clear the inequality from **Y3**. Press the [▲] and repeat the process to clear out all the inequalities.

3. Press [F3] (TYPE) and [F1] to set up **Y1:** for the quadratic equation.



4. To enter the quadratic equation $y = -\frac{1}{8}x^2 + 40x$ input the following:

[(–)] [(] [1] [a b/c] [8] [)] [X,θ,T] [x²] [+] [4] [0] [X,θ,T] [EXE].

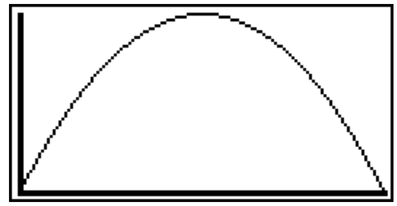


5. Press [SHIFT] [F3] to adjust the View Window.

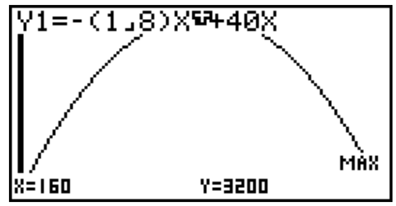
6. Enter [0] [EXE] for **Xmin** and [3] [2] [0] [EXE] for **Xmax**.

7. Press [EXE] twice to display the graph.

(It won't show on screen yet.)



8. Press [SHIFT] [F2] **(Zoom)** and [F5] (Auto) to have the calculator automatically set up the View Window.



9. To find the maximum point, press [SHIFT] [F5] **(G-Solv)**, then [F2] (Max).

Crafty Money **Activity**

Crafters of all ages work hard to create an inventory of products to sell at various shows. They must decide on a price that will cover the cost of materials and time, as well as make a profit. In this activity, a crafter sells embroidered shirts. You will be asked to determine the best combination of shirts to create an inventory that will minimize cost and maximize the revenue.

In this activity, you will write inequalities to represent the conditions for creating two types of shirts, graph the inequalities to find the best combination of shirts to create, and calculate the best price and maximum revenue.

Type of Shirt	Number of shirts	Cost per shirt	Embroidery Time
T-shirt	5x	x	0.5x hours
Polo Shirt	y	3y	0.75y hours
Maximum Allowed	256	$108	36 hours

Questions

1. Write the objective function for the profit of making the shirts if each t-shirt sells for $8.00 and each polo shirt sells for $10.50.

____

2. Write the inequality with the constraints for the number of shirts.

____

3. Write the inequality with the constraints for the cost per shirt.

____

4. Write the inequality with the constraints for the time for embroidering.

____

5. Graph the inequalities and draw a sketch of the graph.

____

6. What is the maximum combination of shirts that can be made, if only the number of shirts and the cost per shirt are considered?

____

7. What is the maximum combination of shirts made, if only the cost per shirt and the time for embroidering are considered?

____

8. What is the maximum combination of shirts made, if only the number of shirts and the time for embroidering are considered?

____

9. What combination of t-shirts and polo shirts will maximize the profit, but still stay within the constraints?

____

Extension

1. Revenue is found by multiplying the number of items times the price (R = px). There is a point at which a given number of items sold at a particular price will maximize the revenue.

   Given that the crafter has anywhere from $0 < x < 120$ t-shirts to sell and the price of the t-shirts is found using the equation $p = -\frac{1}{6}x + 20$, write an equation representing the revenue.

____

2. Graph the equation and find the maximum revenue (R) that can be expected and the number of t-shirts that must be sold in order to reach that maximum revenue. How many t-shirts must be sold to maximize revenue?

____

3. Use the answer to question 2 to determine the best price for each t-shirt.

____

4. Given that the crafter has anywhere from $0 < x < 100$ polo shirts to sell and the price of the polo shirts is found using the equation $p = -\frac{1}{4}x + 25$, write an equation representing the revenue.

____

5. Graph the equation and find the maximum revenue that can be expected (R) and the number of polo shirts that must be sold in order to reach that maximum revenue. How many polo shirts must be sold to maximize revenue?

____

6. Use the answer to question 5 to determine the best price for each polo shirt.

____

7. Re-evaluate the profit equation from the original problem using the best prices for the t-shirts and the polo shirts. Is the profit the same? If not, find the difference between the original profit and the revised profit.

____

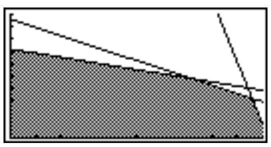
Solutions

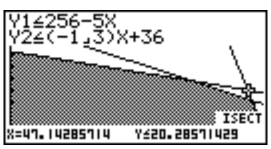
1. x = t-shirt, y = polo shirts
   C = 8x + 10.50y

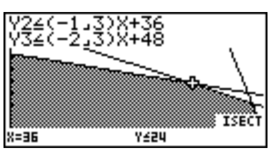
2. $5x + y \le 256$
   $y \le -5x + 256$

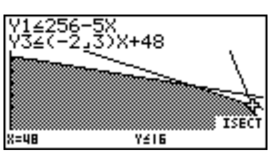
3. $x + 3y \le 108$
   $y \le -\frac{1}{3}x + 36$

4. $0.5x + 0.75y \le 36$
   $y \le -\frac{2}{3}x + 48$

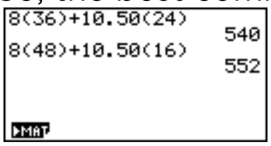
5. 

6. Intersection of Y1 & Y2: (47.14, 20.29)
   47 T-shirts and 20 Polo shirts, but it is not in the shaded region


7. Intersection of Y2 & Y3: (36, 24)
   36 T-shirts and 24 Polo shirts


8. Intersection of Y1 & Y3: (48, 16)
   48 T-shirts and 16 Polo shirts


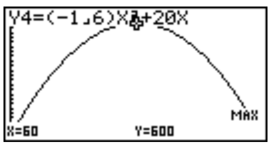
9. $8(36) + 10.50(24) = \$540$
$8(48) + 10.50(16) = \$552$
So, the best combination is 36 T-shirts and 24 Polo shirts.



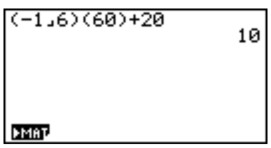
Extension Solutions:

1. $R = -\frac{1}{6}x^2 + 20x$

2. 60 t-shirts

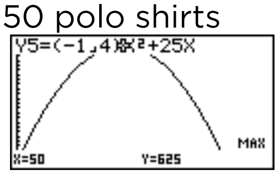


3. $p = -\frac{1}{6}(60) + 20 = \$10.00$

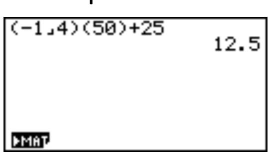


4. $R = -\frac{1}{4}x^2 + 25x$

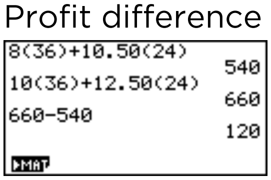
5. 50 polo shirts



6. $p = -\frac{1}{4}(50) + 25 = \$12.50$



7. Best combination: 36 t-shirts and 24 polo shirts
Original profit: $8(36) + 10.50(24) = \$540$
Revised profit: $10(36) + 12.50(24) = \$660$
Profit difference = $\$660 - \$540 = \$120$



Topic Area: Trigonometric Applications

NCTM Standard:

- Compute fluently by developing fluency in operations with real numbers using technology for more-complicated cases.
- Understand functions by interpreting representations of functions.

Objective

To evaluate and graph a model for a damped pendulum

Getting Started

Students will learn how to evaluate and graph a model for a damped pendulum.

Prior to using this activity:

- Students should understand how a pendulum works.
- Students should understand the impact of air resistance (friction) on the bob of a pendulum.

Ways students can provide evidence of learning:

- Students will be able to appropriately model the period of a damped pendulum and interpret the results.

Common mistakes to be on the lookout for:

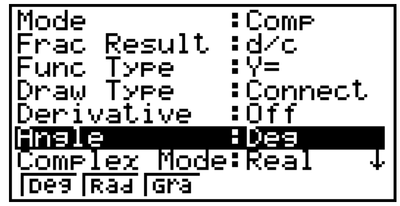
- Calculator may be in degrees instead of radians

Definitions

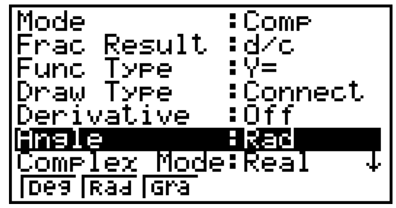
- pendulum
- bob
- period
- amplitude
- sine
- damped

The following will demonstrate how to enter data into the Casio *fx-9750GII* and interpret the results.

To set up the calculator to calculate in radians:



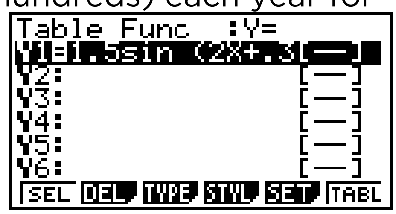
1. From the Main Menu, highlight the RUN•MAT icon and press [EXE] or press [1].



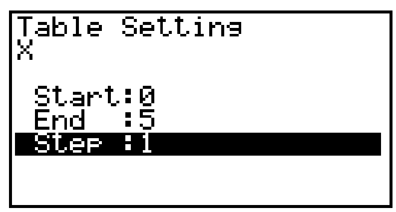
2. To set the calculator to Radians, press [SHIFT] [MENU] **(SET UP)** and move the cursor down to **Angle**. If it says **Deg**, press [F2] (RAD) to change it to Radians. Press [EXIT] to exit the setup screen.

To view a table of trigonometric values:

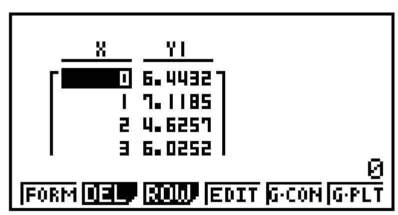
For this example we will use the equation y = 1.5sin(2x + 0.3) + 6, where *y* is the number of employees (in hundreds) and *x* is time (in years) since the analysis began. We want to calculate the number of employees (in hundreds) each year for 5 years.



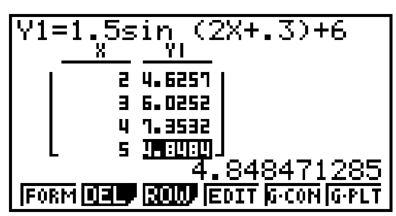
1. From the Main Menu screen, highlight the TABLE icon and press [EXE] or press [5].



2. In **Y1**, enter the equation by pressing [1] [•] [5] [sin] [(] [2] [X,θ,T] [+] [•] [3] [)] [+] [6] [EXE].

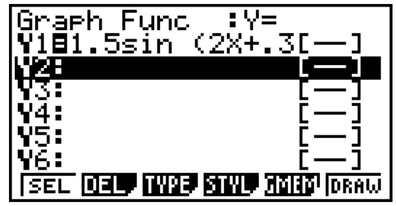


3. To set up the table so that it begins in year x = 0 and ends in year x = 5, with 1 year increments, press [F5] (SET) [0] [EXE] [5] [EXE] [1] [EXE]. Then, press [EXIT].



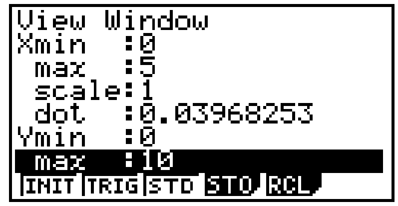
4. Press [F6] (TABL) to see the table with the appropriate data.
5. Scroll down to see additional years.

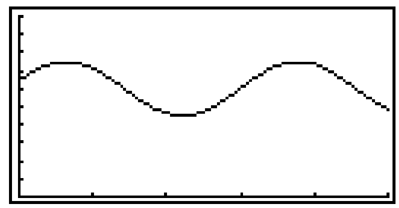
To graph a sine function:



1. Press [MENU] [3] to go to the GRAPH icon.

2. The sine function should already be entered, if you entered it in to the Table icon. If not, press [1] [·] [5] [sin] [(] [2] [X,θ,T] [+] [·] [3] [)] [+] [6] [EXE].





3. To graph the model from time = 0 to time = 5, press [SHIFT] [F3] **(V-Window)** [0] [EXE] [5] [EXE] [▼] [▼] [0] [EXE] [1] [0] [EXE]. This will give you the Xmin = 0, Xmax = 5, Ymin = 0, and Ymax = 10.

4. Press [EXIT] to exit the view window and press [F6] (DRAW) to view the graph.

Damped Pendulum — Activity

Introduction

A pendulum is a weight located on one end of a rod, wire, or string/rope that is attached on the other end to a fixed position. When pushed, the pendulum's weight will swing back and forth under the influence of gravity over the lowest or center point.

In a perfect setting, the pendulum would not be met with air resistance or friction and its motion would continue indefinitely. If the motion were tracked it would form a regular periodic curve.

However, we do not live in a perfect setting so the motion of the pendulum is damped, or slowed by friction. The motion of a damped pendulum is modeled by a sine function whose amplitude is decreasing as time increases. This amplitude decreases in proportion to an exponential function with a base less than 1. The model for a particular damped pendulum is: $y = (0.85^x) \cdot 1.03\sin(3.5x - 1.8)$, where y is the distance (in feet) and direction (negative is left, positive is right) away from the center point 0, and x is the time passes in seconds.

Questions

1. Using the given formula, find the position of the damped pendulum at the given times:

   a. x = 2 seconds y = ______________________

   b. x = 4 seconds y = ______________________

   c. x = 6 seconds y = ______________________

   d. x = 8 seconds y = ______________________

2. Graph the model for the motion of the damped pendulum from x = -0.01 seconds to 10 seconds, and from y = -2 feet to 2 feet. Draw your graph in the box below.

3. Using the solver feature, find the initial position of the pendulum. This is represented by the *y*-intercept of the curve.

The initial position is _________ feet to the left/right (choose one) of the center point.

4. Using the trace feature, find the first five peaks of the graph which represent the first five right hand swing positions of the damped pendulum.

a. peak 1 x = ________ y = ________

b. peak 2, x = ________ y = ________

c. peak 3, x = ________ y = ________

d. peak 4, x = ________ y = ________

e. peak 5, x = ________ y = ________

5. Then, find the ratio for each successive peak change by dividing the latter peak height by the previous peak height.

a. $\text{y ratio 2} = \dfrac{\text{y peak2}}{\text{y peak1}} =$ _________

b. $y \text{ ratio 3} = \dfrac{\text{y peak3}}{\text{y peak2}} =$ _________

c. $y \text{ ratio 4} = \dfrac{\text{y peak4}}{\text{y peak3}} =$ _________

d. $y \text{ ratio 5} = \dfrac{\text{y peak5}}{\text{y peak4}} =$ _________

6. What did you find?

____

____

____

7. Find the ratio for each successive peak change by dividing the previous peak time by the latter peak time.

a. x ratio 2 = $\dfrac{\text{x peak1}}{\text{x peak2}}$ = __________

b. x ratio 3 = $\dfrac{\text{x peak2}}{\text{x peak3}}$ = __________

c. x ratio 4 = $\dfrac{\text{x peak3}}{\text{x peak4}}$ = __________

d. x ratio 5 = $\dfrac{\text{x peak4}}{\text{x peak5}}$ = __________

8. What did you find?

____

____

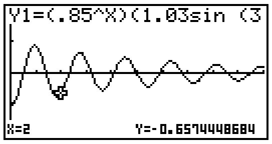
____

9. Increase the Xmax in the viewing window and determine at what elapsed time has the pendulum virtually stopped (peak is less than 0.05).

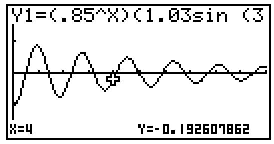
____

Solutions

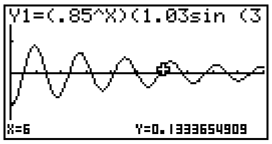
1. a. y = -0.6574



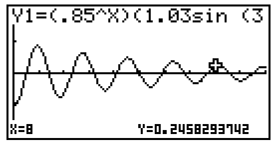
b. y = -0.1926



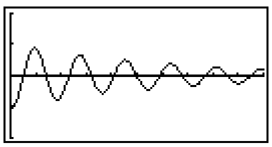
c. y = 0.1334



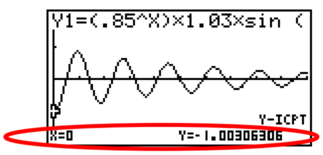
d. y = 0.2458



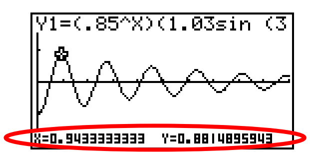
2.



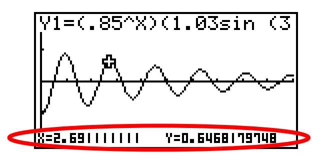
3. The initial position is <u>1.0031</u> feet to the <u>left</u> of the center point.



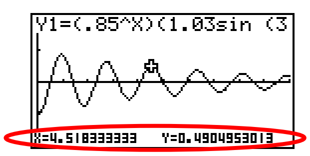
4. peak 1: x = 0.9433, y = 0.8815



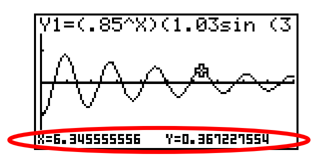
peak 2: x = 2.6911, y = 0.6468



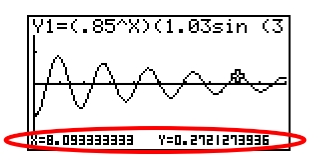
peak 3: x = 4.5183, y = 0.4905



peak 4: x = 6.3456, y = 0.3672



peak 5: x = 9.0933, y = 0.2721



5. a. y ratio 2 = 0.7337

   b. y ratio 3 = 0.7583

   c. y ratio 4 = 0.7486

   d. y ratio 5 = 0.7410

6. The resulting ratios are virtually the same, therefore the frictions is damping the pendulum in a proportional manner. In other words, the pendulum is slowing down in a constant manner.

7. a. x ratio 2 = 0.3505

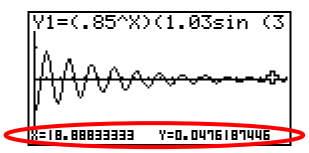
   b. x ratio 3 = 0.5956

   c. x ratio 4 = 0.6553

   d. x ratio 5 = 0.6978

8. The resulting ratios are increasing, which means the time intervals between peaks is decreasing.

9. x = 18.8883 seconds



Derivative Behavior of Common Trigonometric Functions — Teacher Notes

Topic: Derivatives

NCTM Standard
- Use symbolic algebra to represent and explain mathematical relationships.
- Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.

Objectives

The student will be able to express verbally and graphically the behavior of a sine and cosine function and its derivative. Students will express the behavior of the derivative as producing output values relative to the SLOPE of the original function.

Getting Started

This activity will lead students to making connections between the behaviors the sine and cosine functions and their derivatives. They will be asked to plot the functions and their derivatives manually and confirm the expected behavior using the CASIO *fx-9750GII*.

Prior to using this activity:
- Students should be able to produce and manipulate graphs of functions manually and with a graphing calculator.
- Students should have a basic understanding of the behavior and appearance of basic trigonometric functions.

Ways students can provide evidence of learning:
- Students should be able to graph a sine or cosine function with and without the use of a graphing calculator.
- Given a sine or cosine function, students should be able to produce a graph of its derivative.

Common mistakes to be on the lookout for:
- Students may have difficulty graphing trigonometric functions if their window is not appropriate. .
- Students may forget to change the calculator angle mode to radian and have difficulty graphing in the interval $[0,2\pi]$.

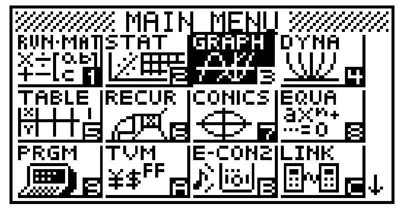
Definitions:
- Trigonometric
- Interval
- Derivative
- Slope
- Sine
- Cosine
- Radian

The following will demonstrate how to graph a function, set a view window, graph its derivative and examine the behavior on the Casio *fx-9750GII*.

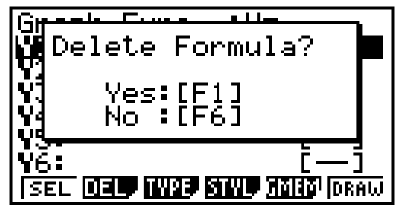
Explore the behavior of the function f(x) = sin(x) using radian mode.

To display a graph of a trigonometric function and its derivative:

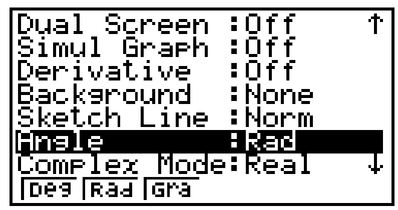
1. From the Main Menu, highlight the GRAPH icon and press [EXE] or press [3].



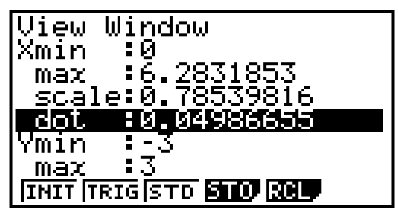
2. To delete any previous equations, highlight the equation and press [F2] (DEL) [F1] (Yes.)

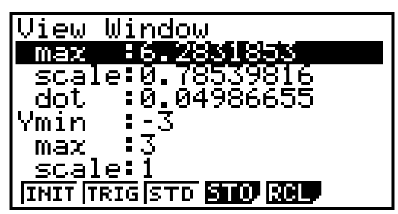


3. Make sure your calculator is in radian mode by Pressing [SHIFT] [MENU] **(SET UP)**. Arrow down [▼] to **Angle**. Press [F2] (Rad). Press [EXIT] to return to the graphing screen.



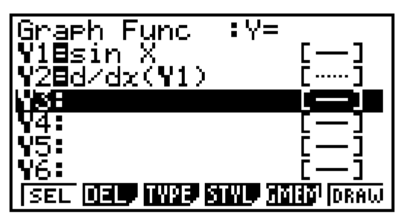
4. Press [SHIFT] [F3] **(V-Window)**. To enter the window, Press [0] [EXE] [2] [SHIFT] [EXP] [EXE] [SHIFT] [EXP] [÷] [4] [EXE] [EXE] [(−)] [3] [EXE] [3] [EXE] [1] [EXE]. Press [EXIT] to return to the initial GRAPH screen.



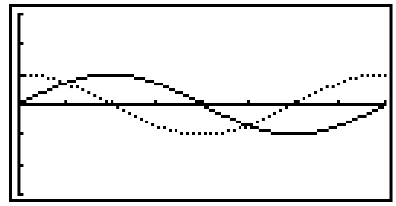


5. Enter the equation in **Y1** by pressing [sin] [X,θ,T] [EXE].

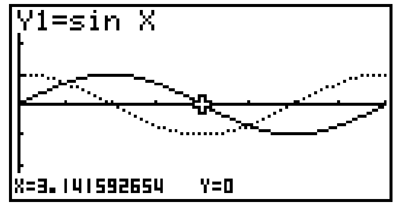
6. For **Y2**, press [OPTN] [F2] (CALC) [F1] (d/dx) [F1] (Y) [1] [)] [EXE]. To change the style of the derivative, arrow up [▲] to **Y2** and press [F4] (STYL) [F4] (.......).



7. Press F6 (DRAW) to view the graph of the function and its derivative.

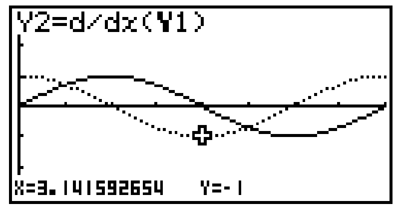


To examine the behavior of a graph using Trace:



1. To trace, press F1 **(Trace)**. Use the Replay key pad to move the cursor left and right.

2. Press ▲ and ▲ to switch between Y1 and Y2.



3. To examine the behavior of the graph, press ◀ and ▶ to view different values of x and y.

Derivative Behavior of Common Trigonometric Functions

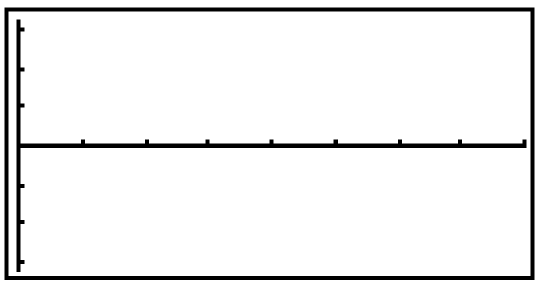
Activity

In this activity you will make connections between the behavior of a trigonometric function and its derivative. Using the sine and cosine functions, you will be asked to plot the function and its derivative manually and then confirm the expected behavior using your Casio *fx-9750GII.*

The derivative of a function represents the behavior of the slope of the function at each point along its domain. The goal of this activity is to make connections between the picture of the function and the picture of its derivative by looking at the slope.

Questions

1. Draw the graph of $y = \sin(x)$ in the trigonometric window described in the "How-To" section.



2. Describe the slope of the function over the interval $[0,2\pi]$.

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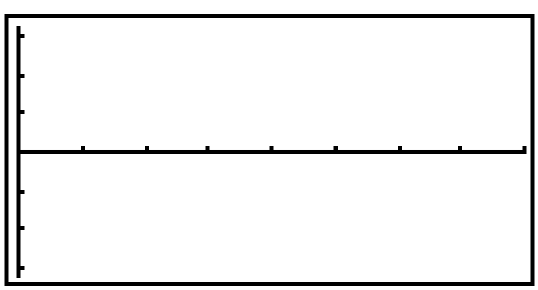
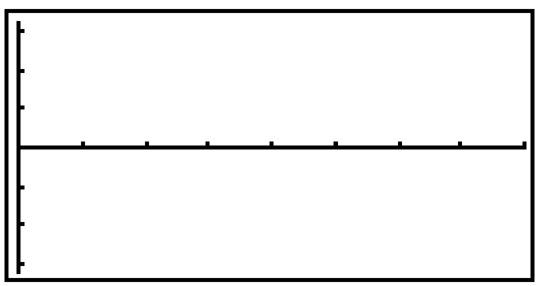
3. Using your understanding of derivative as slope, sketch the function, $y = \sin x$ and its derivative over the interval $[0,2\pi]$ without using the graphing calculator.



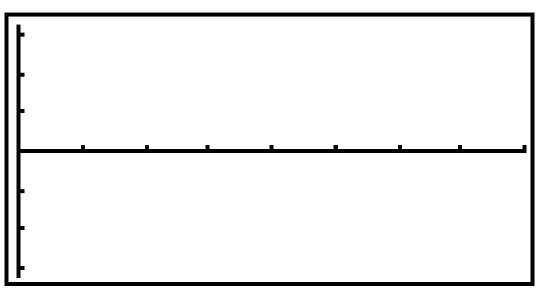
4. Using your calculator, produce the same graphs as above. Do the graphs produced agree with what you expected to see? Explain.

5. Draw the graph of $y = 2\sin(x)$, in the interval $[0,2\pi]$ without using the graphing calculator. Using your knowledge of slope, overlay the graph of the slope function (the derivative).



6. Use your graphing calculator to produce the graph of $y = 2\sin(x)$ and its derivative. Does it agree with your sketch? Explain.

7. Draw the graph of $y = \sin(2x)$, in the interval $[0,2\pi]$ without using the graphing calculator. Using your knowledge of slope, overlay the graph of the slope function (the derivative).



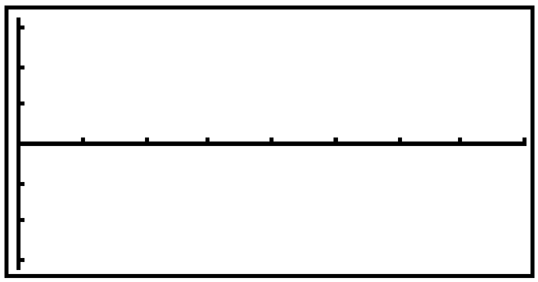
8. Use your graphing calculator to produce the graph of $y = \sin(2x)$ and its derivative. Does it agree with your sketch? Explain.

____

____

____

9. Draw the graph of $y = \cos(x)$.



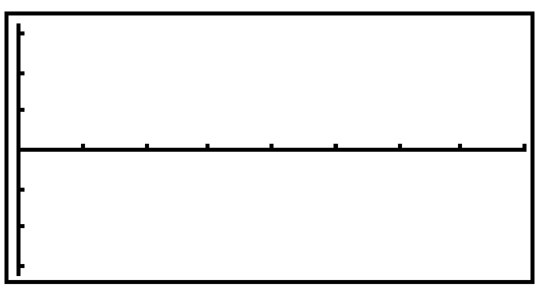
10. Describe the slope of the function over the interval $[0,2\pi]$.

____

____

____

11. Using your understanding of derivative as slope, sketch the function $y = \cos(x)$, and its derivative over the interval $[0,2\pi]$ without the graphing calculator.



12. Use your calculator to produce the graph of $y = \cos(x)$ and its derivative, Do the graphs produced agree with what you expected to see? Explain.

____

____

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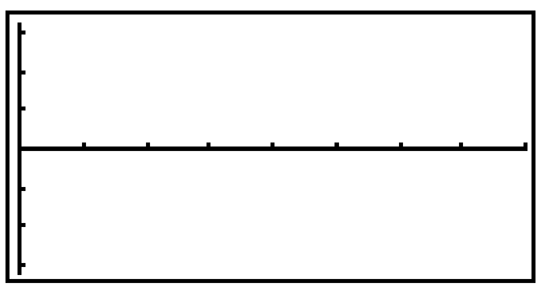
13. Draw the graph of $y = 2\cos(x)$ over the interval $[0, 2\pi]$ without the graphing calculator. Using your knowledge of slope, overlay the graph of the slope function (the derivative).

14. Use your calculator to produce the graph of $y = 2\cos(x)$ and its derivative, Do the graphs produced agree with what you expected to see? Explain.

15. Draw the graph of $y = \cos(2x)$ over the interval $[0, 2\pi]$ without the graphing calculator. Using your knowledge of slope, overlay the graph of the slope function (the derivative).

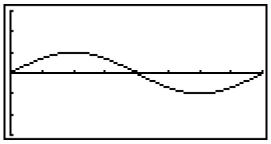


16. Use your calculator to produce the graph of $y = \cos(2x)$ and its derivative, Do the graphs produced agree with what your sketch? Explain.

17. Compare and contrast the behaviors of the derivatives of the sine and cosine functions.

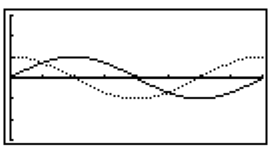
18. Given the general functions $y = A\sin(x)$ and $y = \sin(Bx)$, and using the calculator, explore their derivative behaviors for additional values of A and B. Do the same for the cosine functions and draw a general set of conclusions of the effects of A and B on the derivative behavior. Write general symbolic rules using these results.

Solutions

1. 

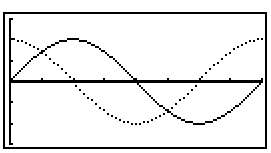
2. Answers will vary. A good answer will include statements that the slope is positive (increasing) over the intervals $\left[0,\frac{\pi}{2}\right)$ and $\left(\frac{3\pi}{2},2\pi\right]$ and negative (decreasing) over the interval $\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$. A well thought out answer should also include statements that the slope is 0 at the vertices.

3. Answers will vary but should look like the graph the calculator produces for question #4. The graph of the derivative is dotted.

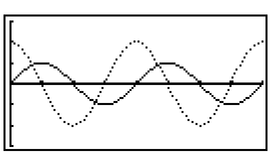
4. 

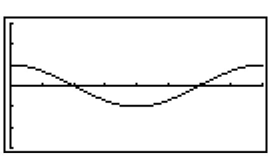
If the graphs do not agree in 3 & 4, discussion should take place regarding the differences.

5. Answers will vary but should look like the graph the calculator produces for question #6. The graph of the derivative is dotted.

6. 

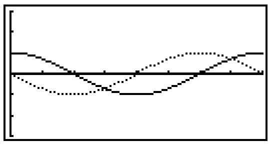
7. Answers will vary but should look like the graph the calculator produces for question #8. The graph of the derivative is dotted.

8. 

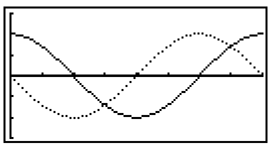
9. 

10. Good answers will be similar to the response for the sine function indicating that the slope is positive (increasing) over the interval $(\pi, 2\pi)$ and negative (decreasing) over the interval $(0, \pi)$. A well thought out answer should also include statements that the slope is 0 at the vertices.

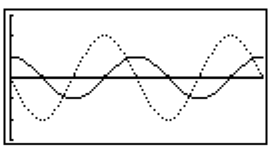
11. Answers will vary but should look like the graph the calculator produces for question #12. The graph of the derivative is dotted.

12. 

13. Answers will vary but should look like the graph the calculator produces for question #14. The graph of the derivative is dotted.

14. 

15. Answers will vary but should look like the graph the calculator produces for question #16. The graph of the derivative is dotted.

16. 

17. Answers will vary. Complete answers should include statements that the sine function derivative produces graphs that look and behave like the cosine function, while the cosine function derivative produces graphs that seem to be the opposite (negation) of the sine function.

There should also be mention that changing the amplitude of the function is consistent with the amplitude of the derivative, but changing the period of the function is consistent with the period of the derivative, but also changes the amplitude of the derivative.

18.

$$y = a\sin(x) \rightarrow \frac{dy}{dx} = a\cos(x)$$

$$y = \sin(bx) \rightarrow \frac{dy}{dx} = b\cos(bx)$$

$$y = a\cos(x) \rightarrow \frac{dy}{dx} = -a\sin(x)$$

$$y = \cos(bx) \rightarrow \frac{dy}{dx} = -b\sin(bx)$$

Derivative Behavior of Polynomials — Teacher Notes

Topic: Derivatives

NCTM Standards

- Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.
- Approximate and interpret rates of change from graphical and numerical data.

Objective

The student will be able to express verbally and graphically the behavior of a polynomial function and its derivative. Students will express the behavior of the derivative as producing output values relative to the SLOPE of the original function.

Getting Started

In this activity students will make connections between the behavior of polynomial functions and their derivatives. They will be asked to plot the functions, predict the expected behavior, and then overlay the derivative, confirming using the Casio *fx-9750GII*.

Prior to using this activity:

- Students should be able to produce and manipulate graphs and functions manually and with the graphing utility.
- Students should have an introduction to basic symbolic derivatives to make an easier connection to the visuals.
- Students should have a basic understanding of the transformations of polynomial functions.

Ways students can provide evidence of learning:

- Given a function, the student can produce a graph of that function and its derivative.
- Given the graph of a function, the student can describe the behavior of its derivative.

Common mistakes to be on the lookout for:

- Students may not realize that when asked to graph only the slope of a function, the graph will be a horizontal line and not have a variable in the equation.

Definitions:

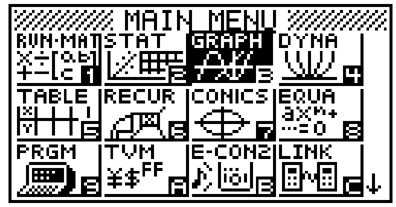
- Slope
- Local Linearity
- Continuity
- Domain
- Derivative
- Polynomial

The following will demonstrate how to graph a derivative and examine its behavior on the Casio *fx-9750GII*.

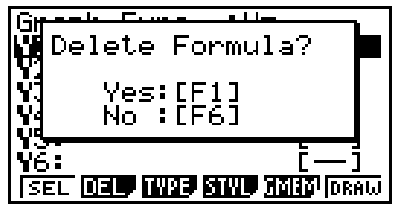
Graph f(x) = $x^4 - x^3 + 1$.

To graph a function and its derivative:

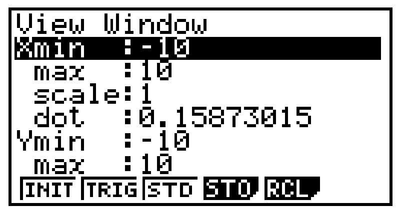
1. From the Main Menu, highlight the GRAPH icon and press [EXE] or press [3].



2. Press [F2] (DEL) and [F1] (Yes) to delete any previous functions.

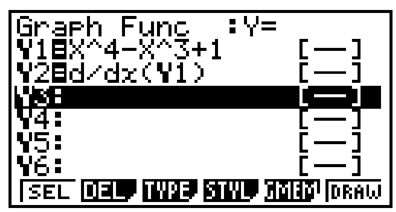


3. Press [SHIFT] [F3] **(V-Window)** [F3] (STD) to set the view window to standard. Press [EXIT] to return to the initial GRAPH screen.

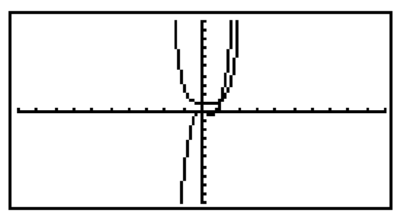


4. Enter the function into **Y1** by pressing [X,θ,T] [^] [4] [−] [X,θ,T] [^] [3] [+] [1] [EXE].

5. For **Y2**, press [OPTN] [F2] (CALC) [F1] (d/dx) [F1] (Y) [1] [EXE].

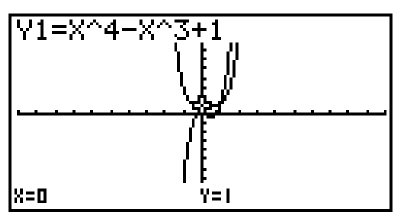


6. Press [F6] (DRAW) to view the graph of the function and its derivative.



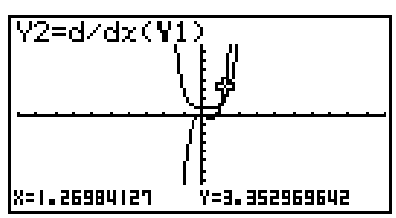
To examine the behavior of a graph using Trace:

1. To trace, press [F1] **(Trace)**. Use the Replay key pad to move the cursor left and right.



2. Press [▲] and [▲] to switch between **Y1** and **Y2**.

3. To examine the behavior of the graph, press [◀] and [▶] to view different values of x and y.



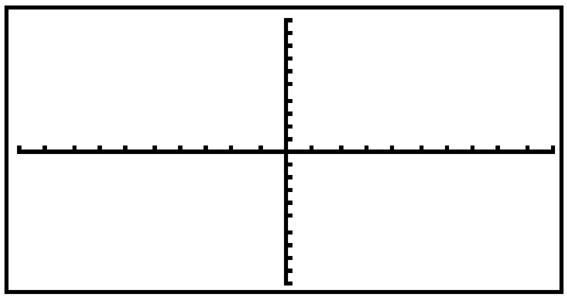
Derivative Behavior of Polynomials — Activity

In this activity you will extend the idea of local linearity and derivative and connect those concepts to continuity. The goal of this activity is to make the connections between the graph of the function and the behavior of the derivative of the function using the Casio *fx-9750GII*.

Questions

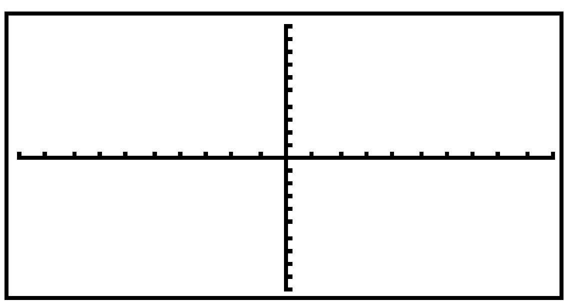
1. Draw the graphs of the following functions in the window.

   a. $y = 2x$ b. $y = 2x + 5$ c. $y = 2x - 3$



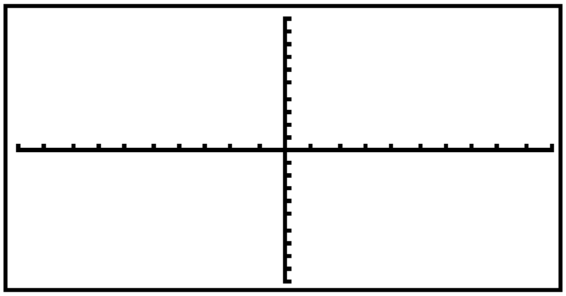
2. Describe what you see.

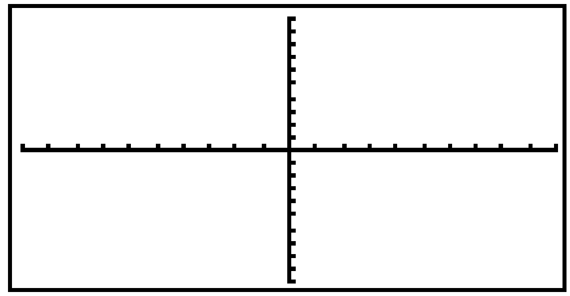
3. Draw the function $y = 2x - 3$ and the graph of its slope on the same axes. Copy it below.



4. Does this agree with what you expected to see? Explain.

5. Given the general equation of a linear function ax + by = c, generalize the relationship between the linear function and its derivative. Provide two examples to support your hypothesis.





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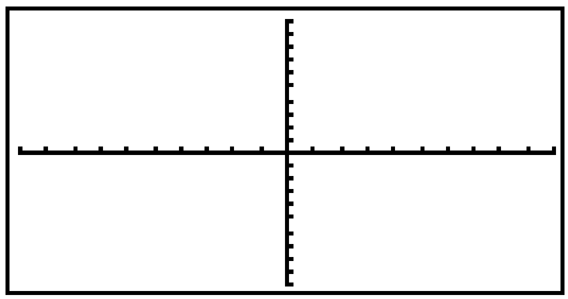
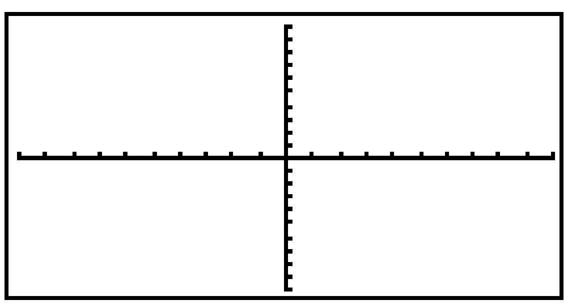


6. Using the same window as before draw the graph of $y = x^2$ on the graph below. Confirm the behavior on your graphing calculator.



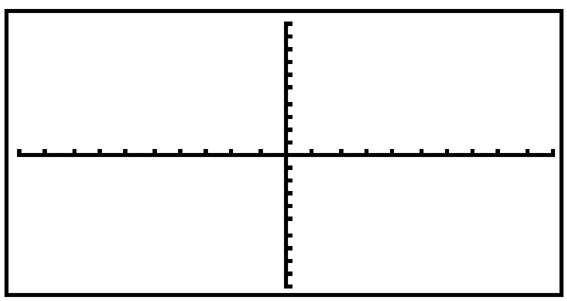
7. Describe the behavior of the slope of the function over the following intervals:

- $(-\infty,0)$

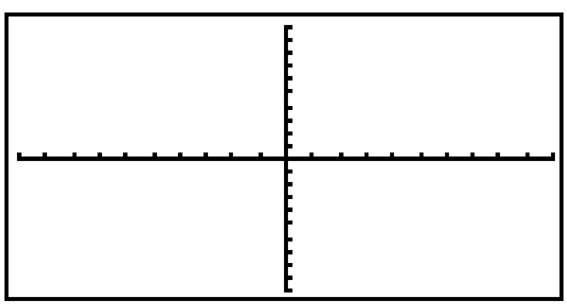
- $(0, \infty)$

8. Based upon your knowledge of what a derivative is, what would you say the derivative of the function is when x = 0? Explain.

9. Sketch the function over again on the axes provided below and then overlay what you think the behavior of the derivative would look like.

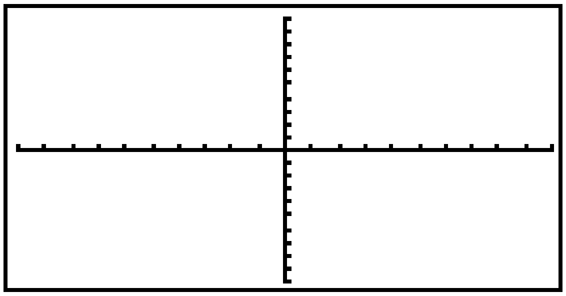


10. Use your graphing calculator to produce the picture of the actual derivative, does it agree with the graph you produced manually?

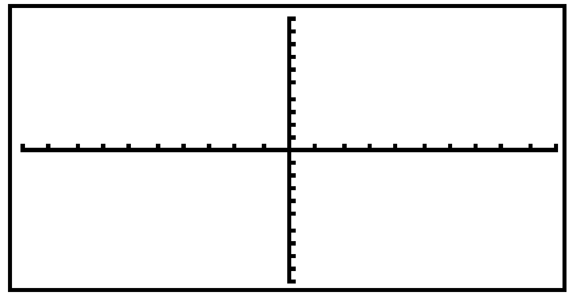


11. Now try the same procedure with the following function:
$y = -3(x-2)^2 + 2$

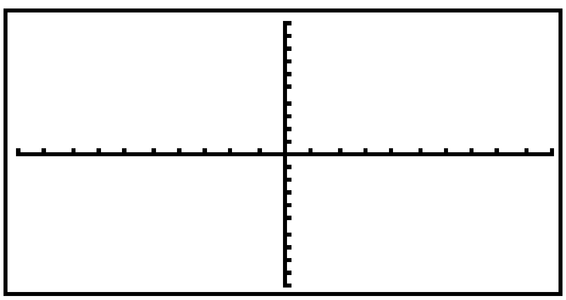
Manually:

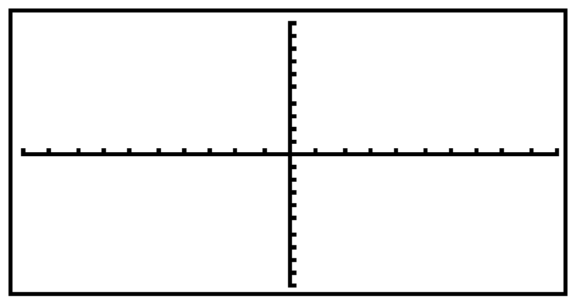


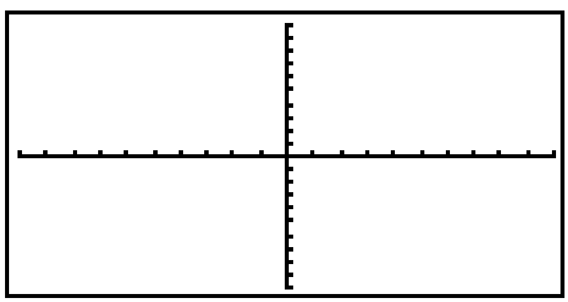
Graphing calculator:



12. Given the general equation of a quadratic function $y = -ax^2 + bx + c$, generalize the relationship between the quadratic function and its derivative. Provide some examples to support your hypothesis.

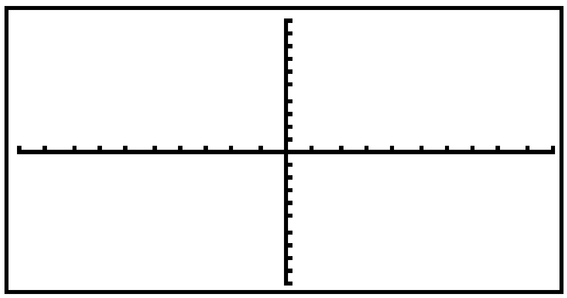




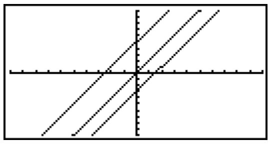


13. Given that the general form of a polynomial is $y = -a_n x^n + a_{n-1}x^{n-1} + \ldots + a_0$, make a general statement about any polynomial function and its derivative.

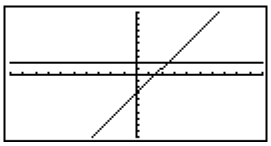
14. Provide one fourth degree example to support your conclusion.



Solutions

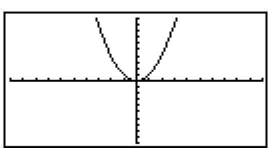
1. 

2. Answers will vary, but the goal is to have students discuss that the slopes are all the same (lines are parallel). A likely answer will also include a comment that the slope = 2 for each line.

3. 

4. Answers will vary. A complete answer should contain a statement that the slope is constant therefore the graph of the derivative should be a horizontal line.

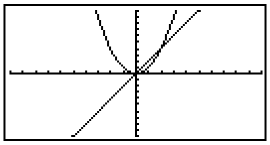
5. Answers will vary. A complete answer should contain a statement that the slope of the line will always be the horizontal line $y = -\frac{a}{b}$.
All provided examples should contain linear functions and horizontal lines as the derivative sketches. Students thinking farther ahead my start with a horizontal line as an example and then show the line y = 0 as the derivative.

6. 

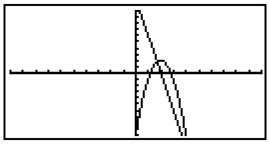
7. $(-\infty, 0)$: A complete answer should cover the fact that in this entire interval the slope is negative but changing. Some answers may include statements about the slope “slowing down” or being smaller or less as the interval approaches 0 [alternately from zero]. Care should be taken that the students are talking about the behavior of the slope relative to the values of x.

   $(0, \infty)$: A complete answer should cover the fact that in this entire interval the slope is positive but changing. Some answers may include statements about the slope “speeding up” or being larger or more as the interval moves away from zero.

8. A complete answer should include a statement that the derivative =0 @ x = 0. The explanation could use the difference quotient/limit approach using small values from the graph, or could make the connection that the tangent is horizontal at x = 0.

9. 

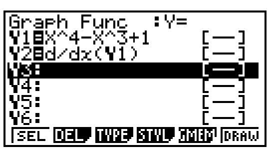
10. Same graph as above. They should agree.

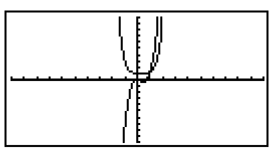
11. 

12. A complete answer should contain statements that the derivative of a quadratic function will always be linear. Students should be very clear that the line exits above the x-axis when the slope of the function is positive, has a root at the vertex of the parabola, and exits below the x-axis when the slope is negative. Examples should be consistent.

13. The goal is to have students recognize that the derivative of any polynomial will be another polynomial of one degree less. Good answers will also contain statements consistent with the fact that the derivative graph is above the x-axis when the slope of the function is positive, has a root at any vertex, and exits below the x-axis when the slope is negative. This might require further investigation. This is also a good lead into the power rule for derivatives.

14. Answers will vary. One example provided here:





Do Limits Take Sides? — Teacher Notes

Topic: Limits

NCTM Standards

- Make and investigate mathematical conjectures.
- Organize and consolidate their mathematical thinking through communication; communicate their mathematical thinking coherently and clearly to peers, teacher, and others.

Objectives

The student will be able to develop an understanding of the meaning of one sided limits and to be able to understand and communicate the idea that for a function to have a limit at a point, it must approach the same output value from either direction.

Getting Started

This activity will encourage students to use graphical and numerical representations to examine the idea of a limit needing to be the same from both directions of approach. The concept of a limit creates the framework for discussing continuity. Using split-defined functions, the goal of this activity is to put a face on the idea of one-sided limits.

Prior to using this activity:

- Students should be able to produce and manipulate graphs and tables of values manually and with a graphing calculator.
- Students should be able to produce split-defined (or piecewise) functions.
- Students should have a basic understanding of the language of functions.
- Students should be able to identify rational and exponential functions.

Ways students can provide evidence of learning:

- If given a split-defined function, the student can produce a picture of the function using the calculator.
- If given a graphical representation of a function, the student can state and explain what the limit is as it approaches an input value from the left side or the right side.

Common mistakes to be on the lookout for:

- Students may produce a graph on the calculator and not be able to communicate the concept of a split-defined function because the window chosen may produce the appearance of a single unbroken formula.

Definitions:

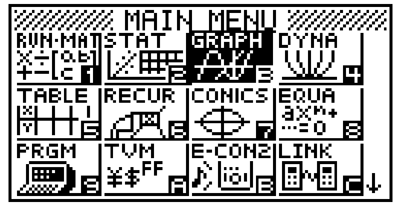
- Split-Defined (Piecewise) Functions
- Asymptote
- Discontinuity
- Input
- Limit
- Output
- Value

The following will demonstrate how to graph a split-defined function and examine its behavior on the Casio *fx-9750GII*.

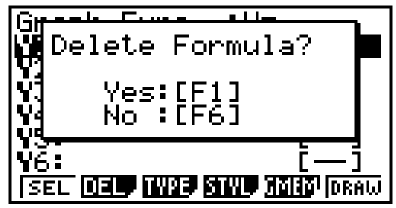
Graph $f(x) = \begin{cases} x-4, x<2 \\ x-1, x>2 \end{cases}$.

To graph a split-defined function:

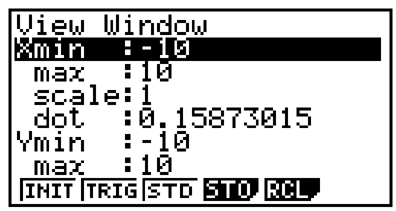
1. From the Main Menu, highlight the GRAPH icon and press [EXE] or press [3].



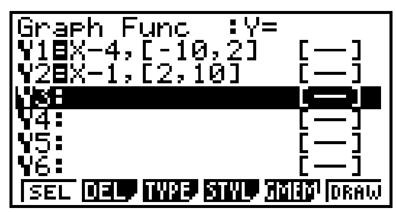
2. Press [F2] (DEL) and [F1] (Yes) to delete any previous functions.



3. Press [SHIFT] [F3] **(V-Window)** [F3] (STD) to set the view window to standard. Press [EXIT] to return to the initial GRAPH screen.



4. Enter each branch of the split-defined (or piecewise) function in its own **Y=** slot, then create the restrictions by putting the lower and upper bounds in brackets.

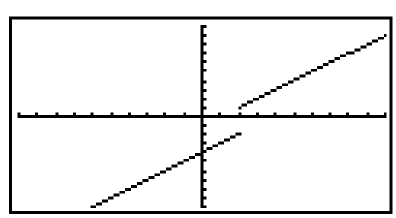


[Note: The lower and upper bounds can usually be the minimum and maximum values of the viewing window if you have two branches. When students graph it, they should be very clear to indicate that they are open circles at the endpoints of the “jump.”]

Press [X,θ,T] [−] [4] [,] [SHIFT] [+] [(−)] [1] [0] [,] [2] [SHIFT] [−] [EXE] for Y1.

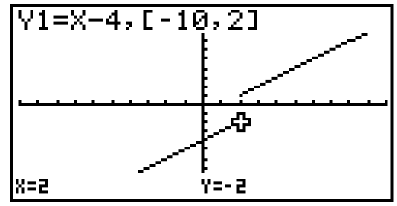
Press [X,θ,T] [−] [1] [,] [SHIFT] [+] [2] [,] [1] [0] [SHIFT] [−] [EXE] for Y2.

5. Press [F6] (DRAW) to view the graph of the function.

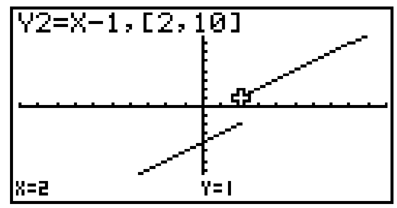


To examine the behavior of the graph:

1. Press F1 **(Trace)** and press ▶ to trace the output (y) as the input (x) gets closer and closer to 2. Remember, since $x < 2$, the x–value never actually reaches 2. The y–value gets closer and closer to –2 as the x-value gets closer and closer to 2 coming from the left.



2. Press ▲ and trace the line starting at $x = 3$ and notice that the y–value gets closer and closer to 1 as the x-value gets closer and closer to 2 from the right side.



3. This function does not have a limit as x approaches 2; since the values are different depending on the direction you approach 2.

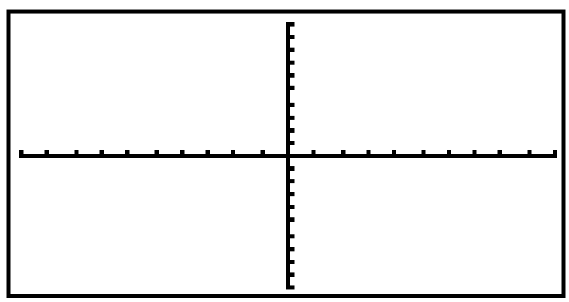
Do Limits Take Sides? Activity

This activity will encourage you to use graphical representations to examine the behavior of a function as it approaches a particular input value from the left, right or both sides, using the Casio *fx-9750GII*.

Questions

1. Graph the following split-defined (or piecewise) function. Set the view window to Standard and sketch a copy of the graph on the axes shown.

$$f(x) = \begin{cases} x+2, & x<1 \\ -2x-1, & x>1 \end{cases}$$



2. Describe what you see.

3. Using the trace function, record your observations as to what happens as you move closer and closer to the value x = 1.

4. From your knowledge of limits, and based upon what you see in this case, what is the $\lim\limits_{x\to 1} f(x)$? Explain.

5. Find $\lim_{x \to 1^+} f(x)$. Explain your answer.

____

____

____

6. Find $\lim_{x \to 1^-} f(x)$. Explain your answer.

____

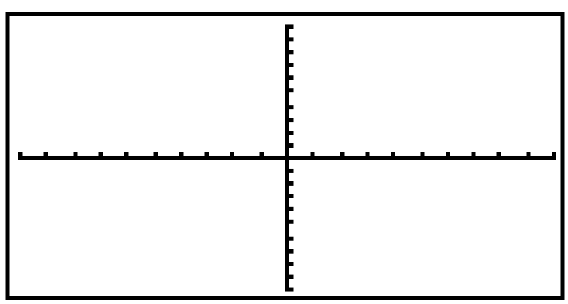
____

____

Extensions

1. Graph a sketch of the function, using a standard viewing window, on the axes below.

$$h(x) = \begin{cases} \dfrac{1}{x+3}, x < -1 \\ x^2 + 3, -1 < x < 3 \\ -x + 9, x > 3 \end{cases}$$



Find each of the following limits and explain how you arrived at your conclusion.

2. $\lim_{x \to -3} h(x)$

____

3. $\lim_{x \to -2^-} h(x)$

4. $\lim_{x \to -3^+} h(x)$

5. $\lim_{x \to -1} h(x)$

6. $\lim_{x \to -1^-} h(x)$

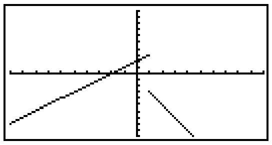
7. $\lim_{x \to -1^+} h(x)$

8. $\lim_{x \to 3} h(x)$

9. $\lim_{x \to 3^-} h(x)$

10. $\lim_{x \to 3^+} h(x)$

Solutions

1. 

2. Answers may vary; students should mention the break in the graph at x = 1.

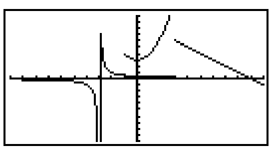
3. One section of the graph gets closer and closer to +3 and the other section gets closer and closer to –3.

4. The function does not have a limit. As *x* approaches 1, the y-values are different depending upon the direction you approach the input.

5. -3. As *x* approaches 1 from the right side, f(*x*) approaches –3.

6. 3. As *x* approaches 1 from the left side, f(*x*) approaches 3.

Extension Solutions

1. 

2. None, there are two different one-sided limits.

3. $-\infty$; none is also acceptable, but $-\infty$ is a more complete description of what is actually taking place. As x approaches –3 from the left side, h(x) approaches $-\infty$.

4. ∞; none is also acceptable, but ∞ is a more complete description of what is actually taking place. As x approaches –3 from the right side, h(x) approaches ∞.

5. None, there are two different one-sided limits.

6. 0.5. As x approaches –1 from the left side, h(x) approaches 0.5,

7. 4. As x approaches –1 from the right side, h(x) approaches 4.

8. None, there are two different one-sided limits.

9. 12. As x approaches 3 from the left side, h(x) approaches 12.

10. 6. As x approaches 3 from the right side, h(x) approaches 6.

Do You Have Tunnel Vision? Teacher Notes

Topic Area: Radical Equations

NCTM Standards:
- Understand relations and functions and select, convert flexibly among, and use various representations for them.
- Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations.

Objective

The student will be able to write the equation of a circle or ellipse given a diagram on a coordinate plane and solve the formula for a circle or ellipse for *y*. The student will be able to write the equation for finding the area and perimeter of a rectangular region under a curve, and determine the maximum and minimum values for the area and perimeter.

Getting Started

Discuss with students the various shapes of tunnels they have driven thru. Have the students discuss what shape a tunnel should be to ensure that traffic can travel along the roadway efficiently.

Prior to using this activity:
- Students should be able to solve equations for a specified variable.
- Students should have an understanding of how to find the area and perimeter of a rectangle.

Ways students can provide evidence of learning:
- The student will be able to write an equation for finding the area in terms of x-values.
- The student will be able to graph the formulas, find a range of values and find the maximum value for the area and perimeter.

Common calculator or content errors students might make:
- Students may use the wrong formula for a circle or ellipse.
- Students may use incorrect formulas for finding the area and perimeter of a rectangle.

Definitions

Area Perimeter Semicircle

Formulas

Circle: $x^2 + y^2 = r^2$ Ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

The following will demonstrate how to enter a formula into the Casio *fx-9750GII*, trace the graph, and find the value of a specified variable.

Enter the formula $Y1 = 2x\sqrt{(16 - x^2)}$ into the calculator, graph the equation and find the maximum point. Find the x-coordinate that corresponds to a given y-coordinate.

To enter a formula into the Graph Function:

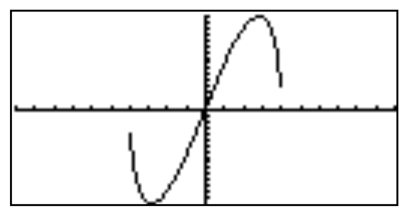
1. From the Main Menu, highlight the GRAPH icon and press [EXE] or [5].

2. Select **Y=** by pressing [F3] [F1]. To enter the formula, $2x\sqrt{(16 - x^2)}$, input the following:
   [2] [X,θ,T] [SHIFT] [x^2] [(] [1] [6] [−] [X,θ,T] [x^2] [)] [EXE].
   The screen will look like the one at the right.

To set up the values for the window:

1. Press [SHIFT] [F3] **(V-Window)**, then [F3] (STD) to display a standard 10x10 grid. Press [EXE] twice to view the graph.

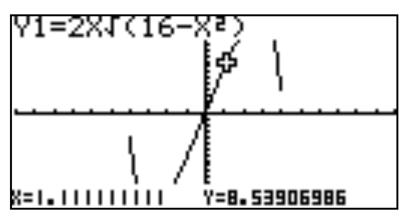
2. If you do not see the graph, press [F2] **(Zoom)** then [F5] (Auto) to view the graph seen at the right.



To trace a graph and locate values:

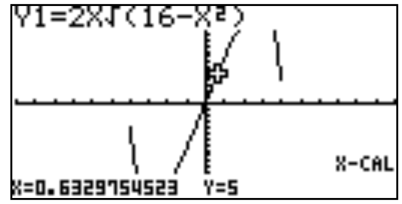
1. With the graph showing, press [F1] **(Trace)** and use the ◀ ▶ keys to move along the graph.

2. If more than one graph is visible, use the ▲ ▼ keys to move between the graphs. The graph at the right shows the value of *y* when x = 1.1.



To find the maximum point using G-Solve:

1. After graphing, press [F5] **(G-Solv)**, then [F2] (Max).



2. To find the value of x for a specified y-value, press [SHIFT] [F5] [F6] (▷) then [F2] (x-cal). Enter the specified y-value followed by [EXE].

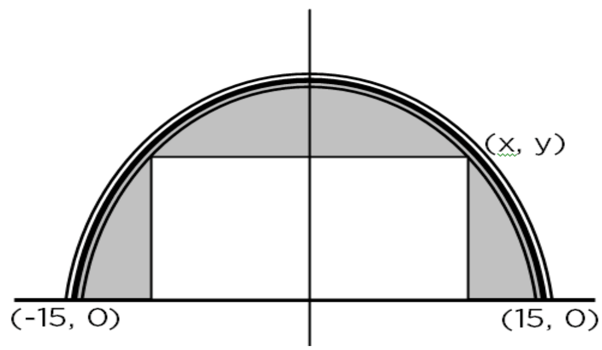
   The screen displays the value of x when y = 5.

Do You Have Tunnel Vision? Activity

Many tunnels are built using either a circular or elliptical shape in order to distribute the stress on the walls. The stress of the ground, from above, is spread along the circular or elliptical surface of the tunnel, which reduces the possibility of the tunnel collapsing. The tunnel opening must be large enough to accommodate the desired number of lanes of traffic.

Using the formula for a circle and solving for the y-value, you will explore a circular entrance for a tunnel and find its area and perimeter.

Questions



1. Given the tunnel entrance at the right, write an equation for the semicircle used in the figure.

   ____

2. What would be the length of the rectangular region in terms of *x*?

   ____

   What would be the height in terms of *x*?

   ____

3. Write the formula for finding the perimeter and area of the figure using the expressions found in question 2.

   Perimeter: ______________________ Area: _________________________

4. Graph the formula for finding the perimeter of the region. What is the range of positive values for *x*?

   ____

5. What is the maximum perimeter for the entrance to this tunnel?

   ____

6. What would be the value of *x* for a perimeter of 20? Of 30?

____

7. Graph the formula for finding the area of the region. What is the range of positive values for *x*?

____

8. What is the maximum area for the entrance to this tunnel?

____

9. What would be the value of *x* for an area of 50? Of 100?

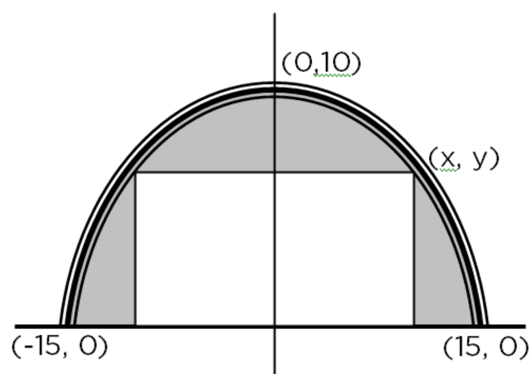
____

10. Would this be an adequate opening for a tunnel? Why or why not?

____

____

Extensions



1. Use the diagram at the right to write an equation for the elliptical entrance.

____

2. Find equations for the perimeter and area of the rectangular region.

Perimeter: ______________________

Area: ______________________

3. How does the range of x-values compare to that of the circle?

____

4. How does the maximum area compare to that of a circle?

____

5. Which shape do you feel would be better? Why?

____

____

Solutions

1. $x^2 + y^2 = 225$

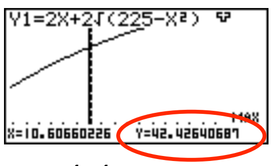
2. 2x; $\sqrt{225 - x^2}$

3. Perimeter: $P = 2x + 2\sqrt{225 - x^2}$

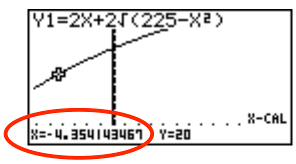
   Area: $A = 2x\sqrt{225 - x^2}$

4. 0 < x < 15

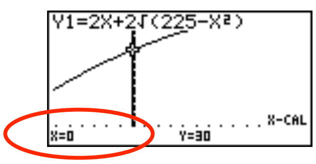
5. Pmax = 42.426



6. x = 4.4

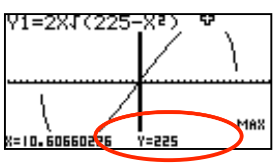


x = 0

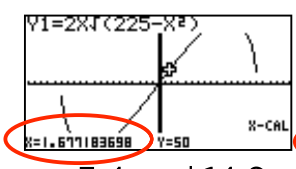


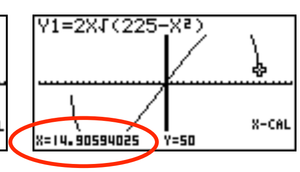
7. 0 < x < 15

8. Amax = 225

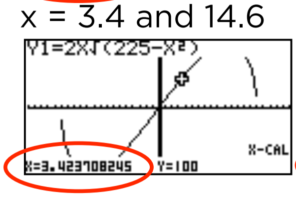


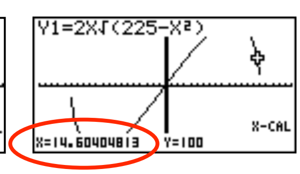
9. x = 1.7 and 14.9





x = 3.4 and 14.6





10. Answers will vary.

Extension Solutions

1. $\frac{x^2}{225} + \frac{y^2}{100} = 1$

2. Perimeter: $P = 2x + \sqrt{\left(100 - \frac{4}{9}x^2\right)}$

   Area: $A = 2x\sqrt{\left(100 - \frac{4}{9}x^2\right)}$

3. The range is the same.

4. It is less than that of the circle.

5. Answers will vary.

Employment Cycles — Teacher Notes

Topic Area: Evaluating and Graphing Trigonometric Functions

NCTM Standard:

- Compute fluently by developing fluency in operations with real numbers using technology for more complicated cases.
- Understand functions by interpreting representations of functions.

Objective

To evaluate and graph a trigonometric function

Getting Started

Students will learn how to evaluate and graph trigonometric functions.

Prior to using this activity:

- Students should understand that trigonometric functions are periodic, cyclical or circular functions with the angle measurement (θ) in degrees or radians. The sine (sin) and cosine (cos) relationships map the angle measurement (θ) to the x and y coordinates of the point on the unit circle that represents the angle.
- Students should know that the additional trigonometric relationships of tangent (tan), cotangent (cot), secant, (sec) and cosecant (csc) are defined in terms of the sine and cosine relationships
- Students should understand that the trigonometric functions are defined by letting the independent variable (x) be the angle measurement and letting the dependent variable (y) be the evaluation of the trigonometric relationship at the angle.

Ways students can provide evidence of learning:

- Graph a sine function.
- Interpret a sine function.

Common mistakes to be on the lookout for:

- Students may confuse sine and cosine.
- Student may use degrees, instead of radians.

Definitions

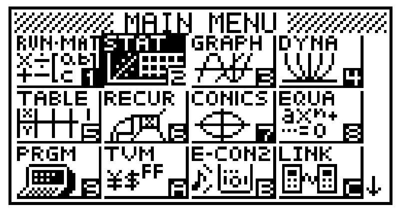
- sine
- cosine
- tangent
- cotangent
- secant
- cosecant

The following will demonstrate how to enter data into the Casio *fx-750GII*, graph the data, and interpret the data.

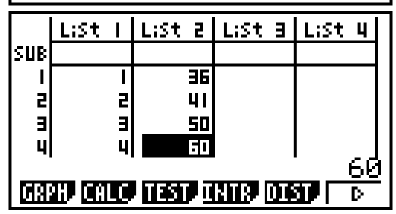
The monthly temperatures for a city are provided in the table below. You can use a trigonometric function to model this data.

m (month)	1	2	3	4	5	6	7	8	9	10	11	12
t (temp)	36	41	50	60	68	77	82	81	73	62	50	50

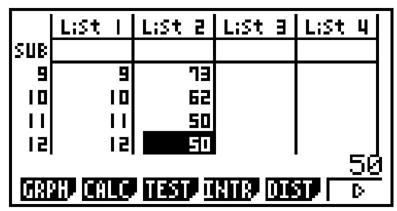
Steps for using the STAT Menu:



1. From the Main Menu, highlight the STAT icon and press [EXE] or press [2].

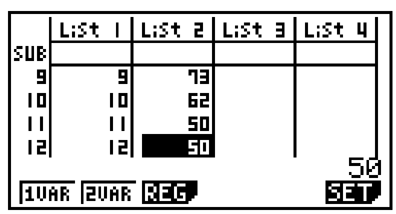


2. Enter the first row of data in the calculator under **List 1**. Remember, you will need to press [EXE] after you enter each number.



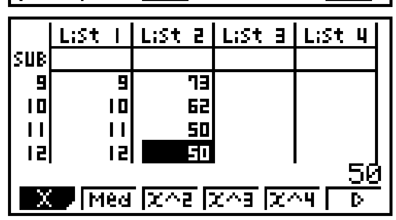
3. Enter the second row of data in the calculator under **List 2**. The screens to the right should show you what you will end up seeing.

Steps for calculating the Sine Regression:

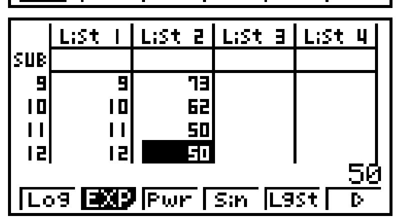


1. From the initial STAT screen, press [F2] (CALC).

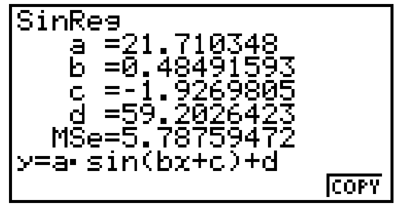
2. Then, press [F3] (REG).



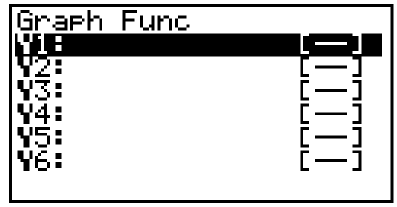
3. The Sine Regression is not one of the first functions listed across the bottom of the screen. To see it, you will need to press [F6] (▷) once.



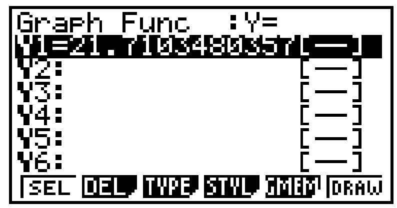
4. Press F4 (Sin). The best-fitting sine curve is approximately y = 21.71•sin(0.48x – 1.93) + 59.20.



5. The calculator also gives you the option to copy the equation in to the Graph menu. To do this, press F6 (Copy). The Graph function will show any equations you already have there. Highlight the line that you would like for the calculator to save the equation on and press EXE.

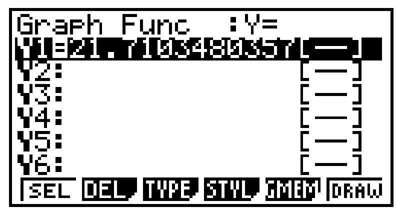


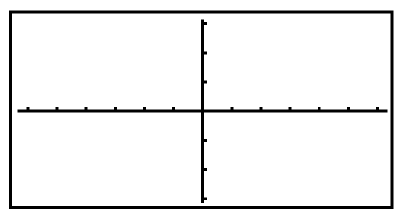
6. Now, when you go to the GRAPH Icon, by pressing MENU 3, the equation should appear in the line that you designated.



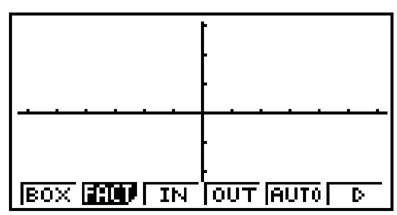
Steps to view the graph of this model:

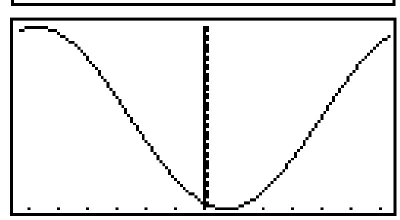
1. From the Graph menu, press F1 (SEL) to select the equation you would like the calculator to graph. Once the equal sign is highlighted, press F6 (DRAW) to view the graph.



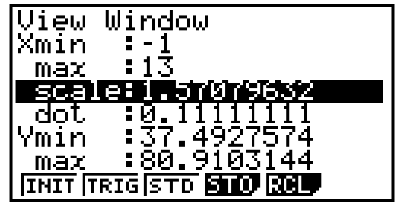


2. The graph will not show up in the initial window that is set with the calculator. Press F2 **(Zoom)** to go to the Zoom functions and press F5 (AUTO).

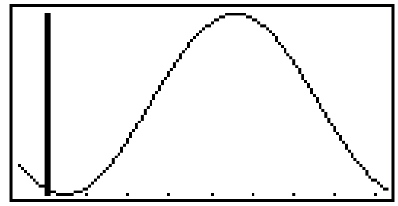




3. To view this graph in a more realistic situation, press [F3] **(V-Window)** and enter [(−)] [1] [EXE] for Xmin and [1] [3] [EXE] for Xmax.

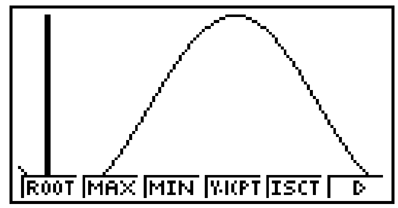


4. Press [EXE] twice to display the graph.

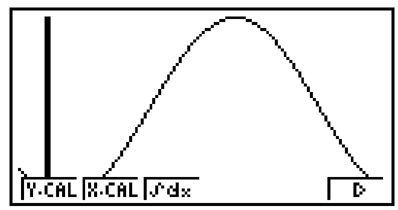


Using the G-Solv Function:

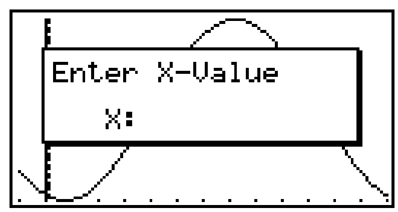
1. To find a value of y when given a value for x, press [F5] **(G-Solv)**, [F6] (▷), then [F1] (Y-Cal).



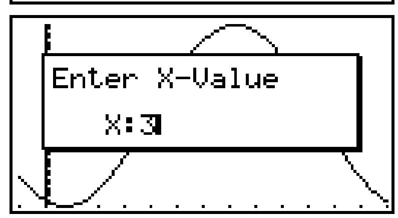
2. To find a value of x when given a value for y, press [F5] **(G-Solv)**, [F6] (▷), then [F2] (X-Cal).



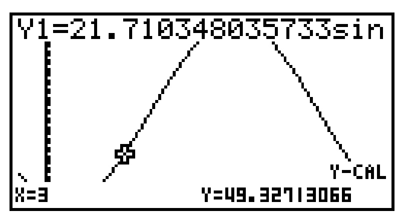
3. For example, to solve for a value of y, when x = 3. Press [F2] (Y-Cal) and the calculator will prompt you to input a value for x.



4. Press [3] [EXE].



5. The calculator displays, along the bottom of the screen, that y = 49.327.



Employment Cycles — Activity

Introduction

The sine function is often used to model cyclical or periodic behavior. There are many applications for the sine function where measured data goes up and down. One of these applications is in modeling employment cycles.

When you look at the employment of some industries, over a period of years, the number of employees will increase and decrease. In this activity, you will work with a model that represents the employment cycle for a particular industry.

In the employment model, $y = 1.5 \cdot \sin(2x + 0.3) + 6$, y is the number of employees (in hundreds) and x is time (in years) since the analysis began.

Questions

1. Using the model, estimate the number of employees for each quarter for the following. Remember that time is in terms of years; therefore, to calculate the number of employees at the end of the first quarter, you will need to enter 0.25 for your x-value. Round the number of employees to the nearest whole person.

   a. Beginning ____________________
   b. End of 1st quarter ____________________
   c. End of 2nd quarter ____________________
   d. End of 3rd quarter ____________________
   e. End of 1st year ____________________
   f. End of 5th quarter ____________________
   g. End of 6th quarter ____________________
   h. End of 2nd year ____________________
   i. End of 10th quarter ____________________
   j. End of 3rd year ____________________

2. Graph the employment model from time = 0 to time = 4 years.

3. What viewing window did you use to see the graph?

Xmin = ________________ Ymin = ________________

Xmax = ________________ Ymax = ________________

4. What appears to be the largest number of employees?

____

5. What appears to be the least number of employees?

____

6. What is the baseline or “average” number of employees?

____

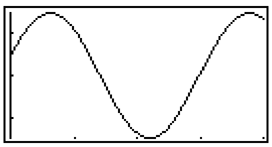
7. What is the period or length of the employment cycle?

____

Solutions:

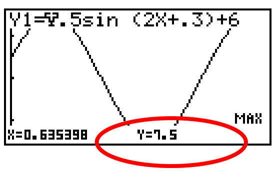
1.
a. 644 employees
b. 708 employees
c. 745 employees
d. 746 employees
e. 712 employees
f. 650 employees
g. 576 employees
h. 463 employees
i. 475 employees
j. 603 employees

2.

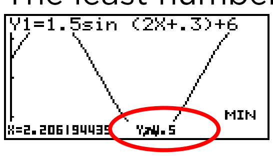


3. Xmin = 0 Xmax = 4 Ymin = 4.5 Ymax = 7.5

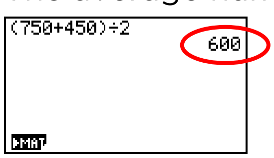
4. The largest number of employees is 750.



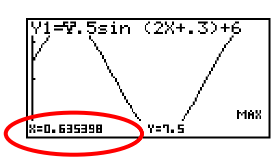
5. The least number of employees is 450.

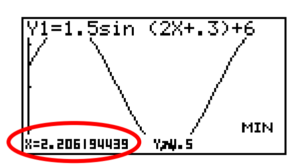


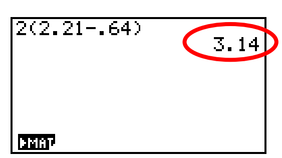
6. The average number of employees is the horizontal line of symmetry between the peaks and valleys of the graph. This value can be found by averaging the maximum and minimum values.
The average number of employees is 600.



7. The length of the employment cycle would be twice the distance from peak to valley. The peak occurred when *x* (time) had a value of 0.64. The valley occurred when *x* (time) had a value of 2.21.
The period or length of employment is 3.14 years- a little over 3 years.







Topic: Hypothesis Tests- Chi-square, Two-way Tables

NCTM Standards:

- Develop and evaluate inferences and predictions that are based on data.
- Use simulations to explore the variability of sample statics from a known population and to construct sampling distributions.
- Understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference.

Objective:

The students will be able to support or reject a null hypothesis based on given data and calculations of significance using a chi-square distribution.

Getting Started

The Chi-Squared test is used to figure out if the rows of a set of data are contingent upon the columns in that same set of data, or a test of independence. In this activity, the students are presented with a set of an exit poll showing the number of votes for three different candidates at the sophomore, junior and senior level. The data was taken by a random poll of students after the vote. The null in this case states that the class level is independent of the candidate voted for. If this null hypothesis is rejected, the students could say that there was a difference in candidate preference between the three classes of students.

Use the following descriptions to go through the chi-square test manually: In order to calculate the chi-square test, the student needs to calculate the row and column totals and calculate the expected cell frequencies using the equation:

$$E_{cell,1,A} = \frac{T_1 x T_A}{N}$$

Where $E_{cell,1,A}$, the expected frequency for the cell, is in the first column and the first row, T is the total number in the row or column, and N is the total number in the entire table.

	Candidate 1	Candidate 2	Candidate 3	Total
Sophomore	35 (30.449)	33 (31.058)	20 (26.491)	88
Junior	40 (35.986)	38 (36.705)	26 (31.307)	104
Senior	25 (33.564)	31 (34.235)	41 (29.2)	97
Total	100	102	87	289

The next step in the Chi-square calculation is the computation of the Chi-square value:

$$X^2 = \sum \frac{\left(E - O^2\right)}{E}$$

Where E is the expected value, O is the observed value. The comparison chi-square value must be found using a table of values based on degrees of freedom and the significance value. There are also many chi-square value calculators on the internet. The degrees of freedom are calculated by the number of rows minus one times the number of columns minus one, *df* = (R – 1)(C – 1). The degrees of freedom value for this activity is four. If the significance value is p = 0.01 parameter is set, the student cannot reject the null hypothesis, therefore the candidate chosen is independent of the class of the student.

Another example of when chi-square tests are used is when the results of a drug are compared against the results of a placebo.

Prior to using this activity:

- Students should be able to gather data from a group.

Ways students can provide evidence of learning:

- Students will be able to support or reject a null hypothesis based on given data.

Common mistakes to be on the lookout for:

- There are many steps in the process that all depend on the previous calculation and students may make small errors that will effect the outcome.

Definitions:

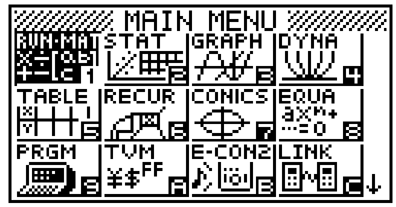
- Mean
- Standard deviation
- Degrees of Freedom
- Significance
- Population
- Sample
- Chi-square test

Use the data in the table below; follow the instructions below to find a Chi-Square value on the Casio *fx-9750GII*.

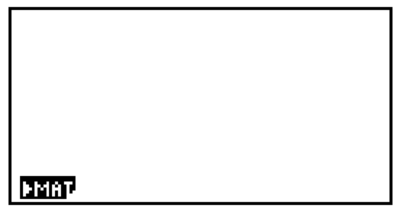
	Candidate A	Candidate B	Candidate C	Total
18-24 yrs	45	16	36	97
25-35 yrs	33	29	30	92
60-70 yrs	16	18	77	111
Total	94	63	143	254

To enter the data from a table into a matrix:

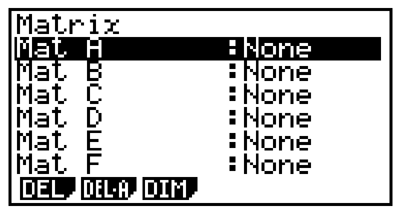
1. From the Main Menu, highlight the RUN•MAT icon and press [EXE] or press [1].

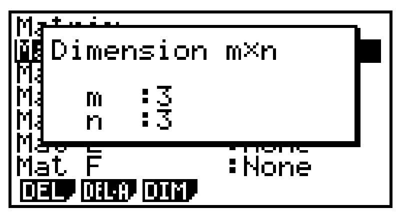


2. Press [F1] (Mat) to access Matrix mode.

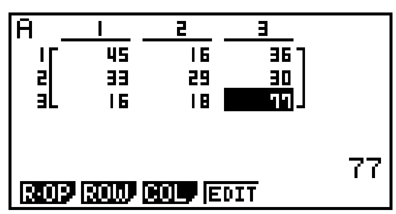


3. Press [F3] (DIM) to set the dimensions of the matrix; **m** represents the number of rows and **n** represents the number of columns in the matrix. For this example, we want a 3 x 3 matrix. Press [3] [EXE] [3] [EXE] [EXE].



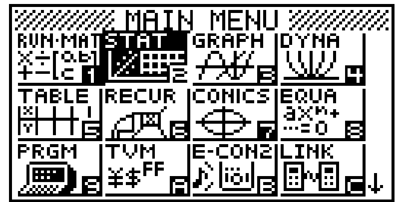


4. Input the values; remember to press [EXE] after each value to store it in the calculator.

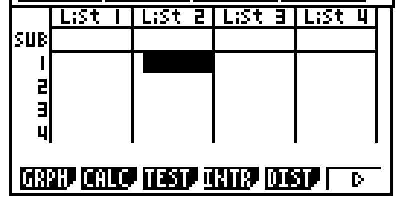


To perform a Chi-square test:

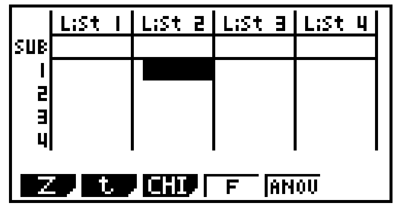
1. From the Main Menu, highlight the STAT icon and press EXE or press 2.



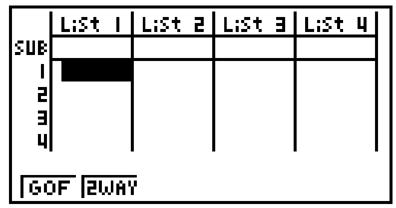
2. Press F3 (TEST).



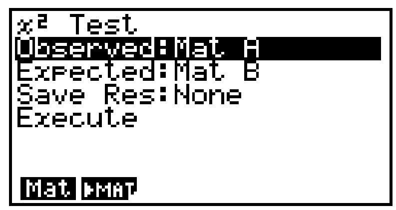
3. Press F3 (CHI).

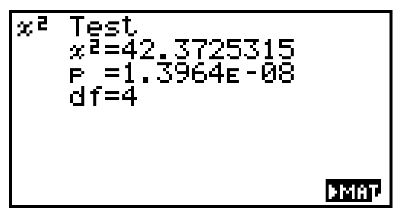


4. For this example, we will choose F2 (2WAY).



5. Since the data is in **Matrix A**, arrow down to **Execute** and press EXE.





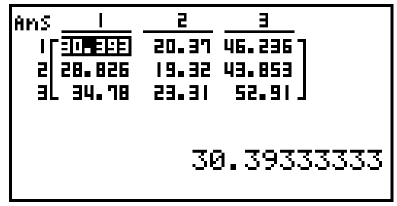
Note: If your data is in a different Matrix, press F1 (Mat), then press the button corresponding to the letter of the Matrix the data is in. You do not have to press ALPHA before.

To find the expected value for each cell:

1. From the Main Menu, highlight the RUN•MAT icon and press EXE or press 1.

2. Press OPTN, then F2 (MAT).

3. When the Chi-Square test was performed the expected values were automatically stored into **Matrix B**. To view that matrix, press F1 (Mat) ALPHA log EXE.



Introduction

Three candidates were running for student council president. After the vote, random students from each class were asked who they voted for: the results are shown in the table below.

	Candidate 1	Candidate 2	Candidate 3	Total
Sophomore	35	33	20	
Junior	40	38	26	
Senior	25	31	41	
Totals				

Questions

1. What is the total number of votes for each candidate? What is the total number of polled voters in each class? Complete this on the table above.

2. What are the expected frequencies for each cell? Compete this on the table above.

3. Establish the null and alternative hypothesis.

4. Compute the degree of freedom.

5. Does the candidate chosen depend on the class of the student? How do you know?

____

____

Extension

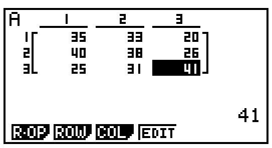
1. Create a poll at your school, perhaps poll a sample of students on their favorite teacher, food, activity, or subject. Create a table and use hypotheses testing to make predictions for your whole student body.

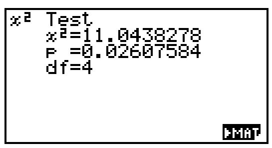
Solutions:

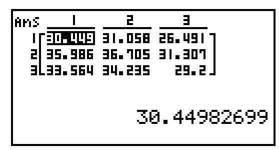
1. Totals are listed in the bottom row and last column.

2. The Expected values are in parenthesis for each cell.

	Candidate 1	Candidate 2	Candidate 3	Total
Sophomore	35 (30.449)	33 (31.058)	20 (26.491)	88
Junior	40 (35.986)	38 (36.705)	26 (31.307)	104
Senior	25 (33.564)	31 (34.235)	41 (29.2)	97
Totals	100	102	87	289



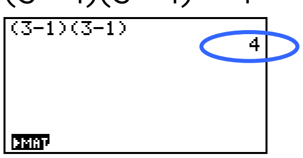




3. The null hypothesis for the data is that the class level of the students (the matrix rows) is independent of the candidates that they voted for (matrix columns).

   The alternative hypothesis is that the class level of the students has a difference in preference for the candidates.

4. Degree of freedom = 4 (number of rows = 3, number of columns = 3)
   (3 – 1)(3 – 1) = 4



5. The calculated value for the Chi square test is 11.0438278 and is only significant to the p-value of 0.02607584. Assuming you are using a 1% confidence level or p = 0.01, you cannot reject the null hypothesis. Therefore, the candidate chosen was independent of the classes of students that voted for that candidate.

Extension Solutions

1. Answers will vary.

Explore & Evaluate Radical Operations Teacher Notes

Topic: Radicals

NCTM Standard:

- Understand and use the inverse relationships of addition and subtraction, multiplication and division, and squaring and finding square roots to simplify computations and solve problems.

Objective

The student will be able to use the Casio *fx-9750GII* to evaluate numbers expressed in radical form as rational or irrational numbers, express radicals in simplest radical form, and perform various operations with radicals, including addition, subtraction and multiplication.

Getting Started

Students initially experience evaluating radicals by simply pressing a key on a calculator. We understand that the exploration of radicals is much more than that. Students need to understand where the radical fits on the number line with respect to integers and non-integers. By navigating through this activity, students will gain a greater understanding of the relationship radicals play in the numerical family.

Prior to using this activity:

- Students should have a strong foundation in determining the decimal equivalent of a radical.
- Students should know how to evaluate the perfect squares from 1-400.
- Students should know how to combine like terms.
- Students should know how to use the distributive property.

Ways students can provide evidence of learning:

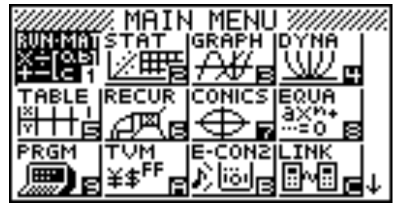
- If given a radical, students can express that radical in simplest form.
- If given a radical, students can express between which two integers that radical exists.
- Students can correctly communicate how to derive the correct sum, difference, or product when operating with radicals.

Common calculator or content errors students might make:

- Students may incorrectly enter the radical into the calculator especially concerning problems which involve expressing radicals inside parentheses.

The following will demonstrate how to enter a radical, simplify that radical and perform various mathematical operations with radicals on the Casio *fx-9750GII*.

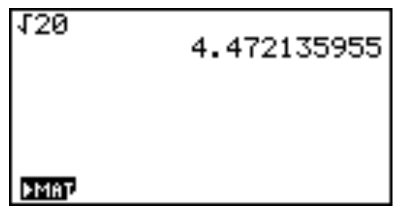
To enter a radical:



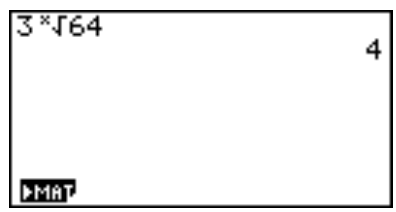
1. From the **Main Menu**, highlight the Run•Mat icon and press [EXE] or [1].

2. To display a radical, press [SHIFT] [x^2].

3. Enter the radicand after entering the radical sign. For example, to evaluate $\sqrt{64}$, input the following: [SHIFT] [x^2] [6] [4] [EXE].

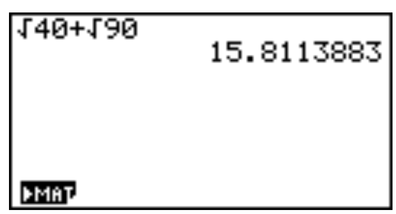


4. If the square root is not a perfect square, the answer will be displayed as decimal. For example: to evaluate $\sqrt{20}$, input: [SHIFT] [x^2] [2] [0] [EXE].



To find the nth root of a number:

1. To evaluate $\sqrt[3]{64}$, input: [3] [SHIFT] [^] [6] [4] [EXE].



To perform an operation with radicals:

1. To evaluate $\sqrt{40} + \sqrt{90}$, input: [SHIFT] [x^2] [4] [0] [+] [SHIFT] [x^2] [9] [0] [EXE].

To access the Complex mode and change the way complex numbers are displayed:

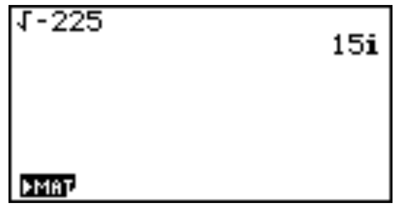
1. Press SHIFT MENU **(SET UP)**.

2. Press ▼ until Complex Mode is highlighted.

3. F1 (Real) sets the display to real numbers,

   F2 (a+bi) sets the display standard form to

   a + bi form. This will change the way answers will be displayed in all modes, including Equation Editor.



Explore & Evaluate Radical Operations **Activity**

Radicals are an important part of our number system. Being able to operate with and evaluate radicals, both in decimal and radical form, is an important number sense skill to master as you progress through your Pre-Algebra and Algebra courses. It will also allow you to explore problems in new and exciting ways.

Use your calculator to help you answer the following questions. Make sure you check to see if your answer is reasonable.

Questions

1. Evaluate $\sqrt{25} + \sqrt{225} + \sqrt{625}$.

____

2. $\sqrt{372}$ is between what two consecutive integers?

____

3. Why do $\sqrt{4}$, $\sqrt{9}$, and $\sqrt{16}$ simplify to integers, but $\sqrt{14}$, $\sqrt{19}$, and $\sqrt{26}$ do not?

____

____

4. A square has an area of 6,084 in^2. What is the length of its side?

____

5. What is $\sqrt{61}$ rounded to the nearest ten-thousandth?

____

6. Explain why $\sqrt{44}$ is closer to 7 than to 6.

____

____

7. Is $\sqrt{85} + \sqrt{131}$ greater than, less than, or equal to $\sqrt{74} + \sqrt{142}$? Explain.

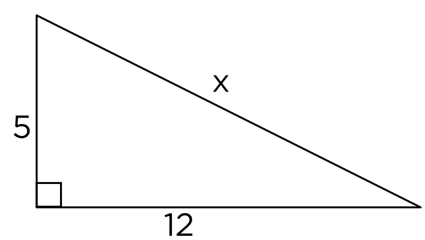
8. Evaluate $\sqrt[3]{8} + \sqrt[3]{125}$.

9. Evaluate $\sqrt{2}(\sqrt{8} + \sqrt{32}) - \sqrt{5}(\sqrt{5} - \sqrt{20})$.

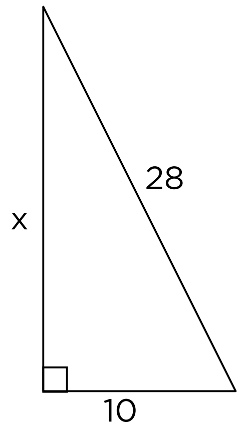
10. What value of x for $\sqrt[5]{x}$ will result in an integer between 2 and 5, inclusive?

11. Evaluate $\sqrt{-9}$.

12. Use the Pythagorean Theorem to determine the value of x. Explain how you arrived at your answer.



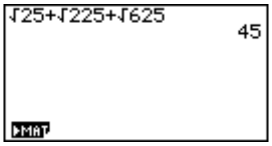
13. Use the Pythagorean Theorem to determine the value of x.



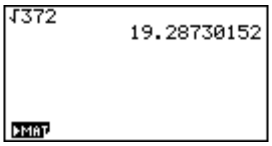
14. If a leg of a right triangle measures 7 inches and its hypotenuse measures 25 inches, what is the length of the other leg? Explain how you arrived at your answer.

Solutions

1. $\sqrt{25} + \sqrt{225} + \sqrt{625} = 45$

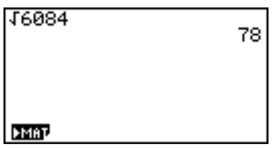


2. The $\sqrt{372}$ is between 19 and 20.

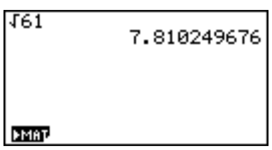


3. $\sqrt{4}$, $\sqrt{9}$, and $\sqrt{16}$ simplify to integers because the numbers under the radicals are perfect squares.

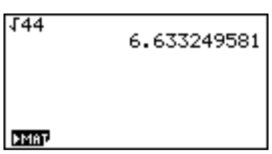
4. side = 78 inches



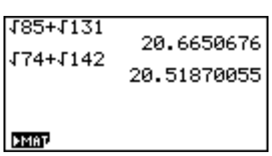
5. $\sqrt{61}$ rounded to the nearest ten-thousandth is 7.8102.



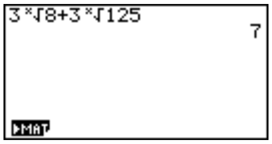
6. $\sqrt{44}$ is closer to 7 than 6 because the decimal equivalent of $\sqrt{44}$ is approximately 6.6. Also, $\sqrt{44}$ is closer to 7 because $\sqrt{44}$ is closer to $\sqrt{49}$ than $\sqrt{36}$,



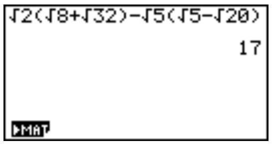
7. $\sqrt{85} + \sqrt{131}$ is greater than $\sqrt{74} + \sqrt{142}$, because the decimal equivalent of $\sqrt{85} + \sqrt{131}$ is 20.67, while the decimal equivalent of $\sqrt{74} + \sqrt{142}$ is 20.52.



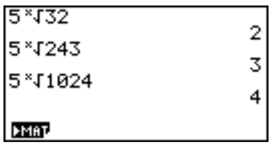
8. $\sqrt[3]{8} + \sqrt[3]{125} = 7$

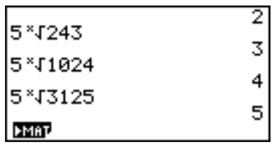


9. $\sqrt{2}(\sqrt{8} + \sqrt{32}) - \sqrt{5}(\sqrt{5} - \sqrt{20}) = 17$

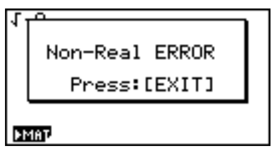


10. x = 32, 243, 1,024 or 3,125





11. When the calculator is set to real number mode, $\sqrt{-9}$ cannot be found. When the calculator is set the a + bi mode, the answer for $\sqrt{-9}$ is 3i.



12. $a^2 + b^2 = c^2$

$5^2 + 12^2 = x^2$

$25 + 144 = x^2$

$x^2 = 169$

$x = \sqrt{169}$

$x = 13$

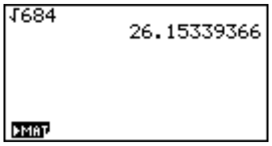
13. $x^2 + 10^2 = 28^2$

$x^2 + 100 = 784$

$x^2 = 684$

$x = \sqrt{684}$

$x = 26.15339366$



14. $7^2 + x^2 = 25^2$

$49 + x^2 = 625$

$x^2 = 576$

$x = \sqrt{576}$

$x = 24$ inches

Topic: Volume of Cylinders

NCTM Standard:

- Understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders.

Objective

The student will be able to use the Casio *fx-9750GII* to calculate the volume of a cylinder.

Getting Started

Students are expected to use formulas to solve problems. Volume of prisms, cylinders, pyramids, cones and spheres are all common formulas, and students need to recognize the dimensions of these solids. Once the dimensions have been identified, they may substitute these into the formula and make the appropriate calculation.

Prior to using this activity:

- Students should know how to find the dimensions of a solid.
- Students should know how to substitute values into a formula.

Ways students can provide evidence of learning:

- If given dimensions of a cylinder, students can calculate the volume.
- If given a volume and one other measure, students can determine the missing measure.

Common calculator or content errors students might make:

- Students may confuse the diameter and radius.
- Students may confuse surface area and volume.

Formulas

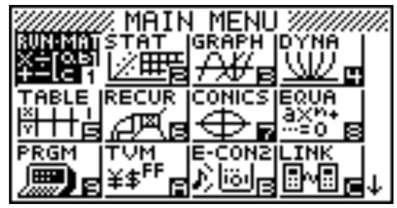
- Volume of Cylinder: $V = BH$ (Where B is the area of the base) or $V = \pi r^2 H$
- Volume of Prism: $V = LWH$
- Volume of Pyramid: $V = \frac{1}{3}BH$ or $V = \frac{1}{3}LWH$
- Volume of Cone: $V = \frac{1}{3}BH$ or $V = \frac{1}{3}\pi r^2 H$
- Volume of Sphere: $V = \frac{4}{3}\pi r^3$

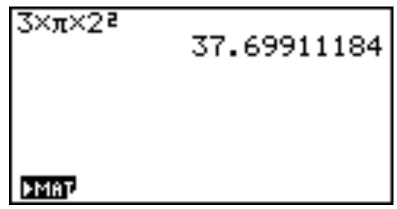
The following will demonstrate how to enter an objective function with the inequalities for the constraints, graph it, set the appropriate view window, and calculate a specific value for an objective function on the Casio *fx-9750GII*.

Evaluate the following formula to find the volume of a cylinder with a radius of 2 units and a height of 3 units: $V = 3\pi 2^2$

To solve an equation in the Run•Mat mode:

1. From the Main Menu, highlight the RUN•MAT icon and press [EXE] or [1].
2. To enter the equation, input: [3] [×] [SHIFT] [EXP] [×] [2] [x^2] [EXE].



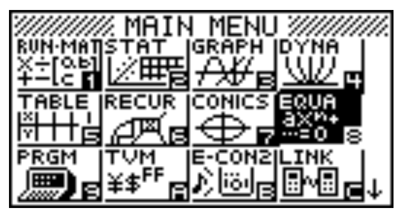


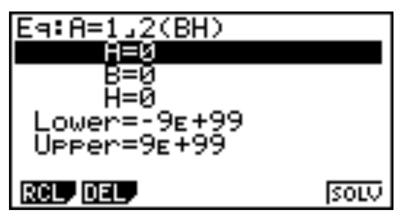
Given a triangle with an area of 60 cm^2 and a height of 12 cm, calculate the length of the base. Be sure to check your answer.

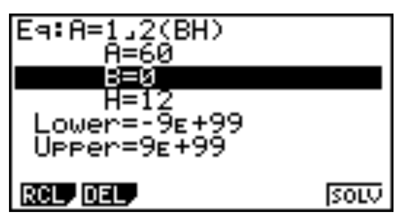
Remember, the formula for area of a triangle is $A = \frac{1}{2}bh$.

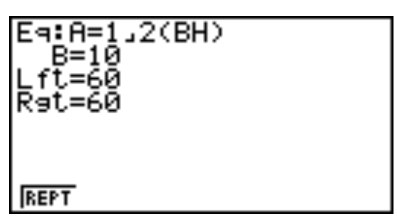
To check an answer using the Equation function:

1. Highlight the EQUA icon from the Main Menu and press [EXE]. Press [F3] to select Solver.
2. To enter the formula, input the following: [ALPHA] [X,θ,T] [SHIFT] [•] [1] [$a\frac{b}{c}$] [2] [(] [ALPHA] [log] [ALPHA] [F↔D] [)] [EXE].
3. To solve for **B**, enter in the values you know for **A** and **H**, pressing [EXE] after each entry. [▲] to **B = 0** and press [F6] (SOLV). The value for **B** will display.









Using formulas is an important part of mathematics. Choosing the correct formula and identifying the appropriate dimensions are skills that are required for success in Algebra and Geometry. In this activity, we will explore various problems using formulas for calculating the volume of a cylinder.

For each question, write the equation used and your answer in a complete sentence.

Questions

1. A local Italian restaurant purchases cans of tomatoes sauce from a food warehouse. The cans are 12 inches tall and have a diameter of 11.5 inches. About how much tomato sauce is in each can?

   ____

2. Health-E Living is selling a new type of oatmeal. They will be packaging the product in cylindrical containers that are 11 inches tall and have a radius of 1.85 inches. About how much oatmeal will be in a container?

   ____

3. A movie theater is trying to decide the best container to use to sell popcorn. They are considering using a rectangular prism and a cylinder. The dimensions of the rectangular prism are 8 inches by 2 inches by 4 inches and the cylinder is 8 inches with a diameter of 2.5 inches. Which container will hold the most popcorn?

   ____

4. Suppose you roll two identical sheets of 8.5 in. by 11 in. paper into cylinders, one that is tall and skinny and the other short and wide, will the volumes be the same? Justify your answer.

   ____

5. What is the volume of a cylinder with a radius of 12 ft and a height of 9 ft?

   ____

6. What is the volume of a cylinder with a radius of 2 m and a height of 12 m?

   ____

7. What is the volume of a cylinder with a diameter of 19.6 in and a height of 11 in?

____

8. What is the volume of a cylinder with a diameter of 9.4 ft and a height of 17 ft?

____

9. What is the volume of a cylinder with a radius of 8.1 ft and a height of 17 ft?

____

10. What is the volume of a cone with a diameter of 4 cm and a height of 7 cm?

____

11. What is the volume of a sphere with a diameter of 36 inches?

____

12. What is the volume of a sphere with a radius of 20 cm?

____

13. A cylinder has a volume of 628 units^3 and a height of 8 units. What is the radius of the cylinder?

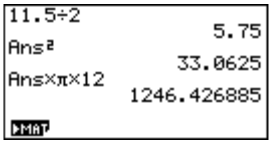
____

14. A cylinder has a volume of 254.34 units^3 and a height of 9 units. What is the diameter of the cylinder?

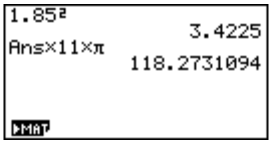
____

Solutions

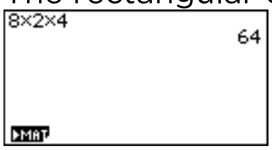
1. There are about 1,246 ounces of tomato sauce.



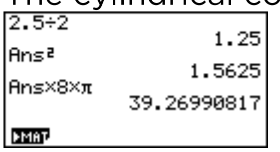
2. There is about 118 cubic inches of oatmeal in the container.



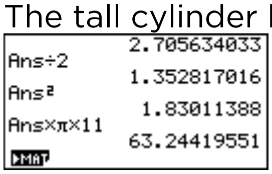
3. The rectangular container has a volume of 64 $units^3$.



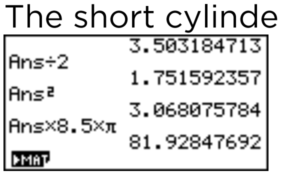
The cylindrical container has a volume of about 39 $units^3$.



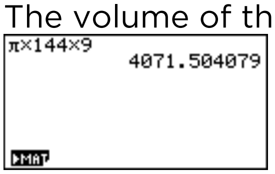
4. The tall cylinder has a volume of about 63 $units^3$.



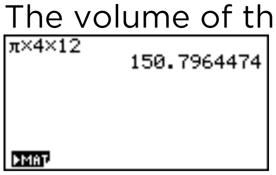
The short cylinder has a volume of about 82 $units^3$.



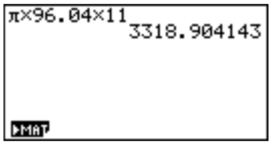
5. The volume of the cylinder is 4071 ft^3.



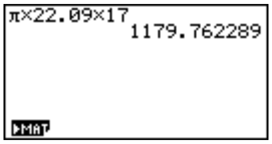
6. The volume of the cylinder is 151 m^3.



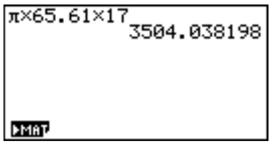
7. The volume of the cylinder is 3319 in^3.



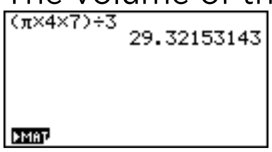
8. The volume of the cylinder is 1180 ft^3.



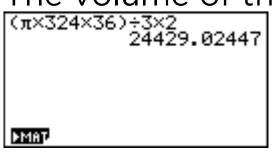
9. The volume of the cylinder is 3504 ft^3.



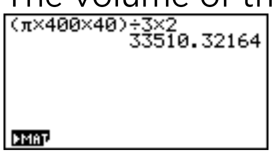
10. The volume of the cone is 29 cm^3.



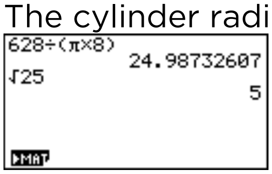
11. The volume of the sphere is 24,429 in^3.



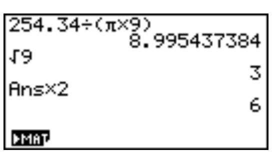
12. The volume of the sphere is 33,510 cm^3.



13. The cylinder radius is 5 units.



14. The cylinder diameter is 6 units.



Exploring Right Triangles Teacher Notes

Topic: Pythagorean Theorem

NCTM Standard:

- Create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship.

Objective

The student will be able to use the Casio *fx-9750GII* to apply the Pythagorean Theorem.

Getting Started

Students should understand the relationship between the lengths of the legs of a right triangle and the length of the hypotenuse. Knowing how the Pythagorean Theorem works will allow students to use the theorem to problem solve efficiently.

Prior to using this activity:

- Students should know how to locate the legs and hypotenuse of a right triangle.
- Students should know the formula for the Pythagorean Theorem.

Ways students can provide evidence of learning:

- If given two of the three measures of a right triangle, students can find the missing measure.

Common calculator or content errors students might make:

- Students may confuse legs and hypotenuse measurements.

Definitions

- Legs
- Hypotenuse
- Pythagorean Theorem
- Pythagorean Triple

Formula

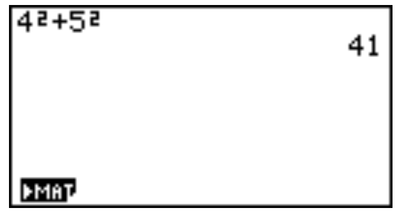
Pythagorean Theorem: $a^2 + b^2 = c^2$, where *a* and *b* are the lengths of the legs, and *c* is the length of the hypotenuse.

The following will demonstrate how to enter exponents and square roots into the Casio *fx-9750GII* to solve equations. Additionally, you will learn how to use the Equation Mode to check your answers.

Given the following equation, solve for c: $4^2 + 5^2 = c^2$

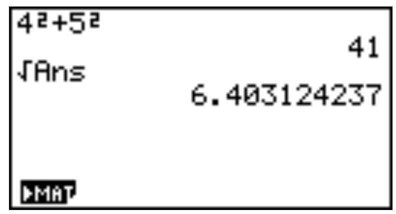
To solve an equation in the Run•Mat Mode:

1. Highlight the RUN•MAT icon the main menu and press [EXE] or [1].



2. Input: [4] [x^2] [+] [5] [x^2] [EXE].

3. To calculate the square root of your answer, press [SHIFT] [x^2] [SHIFT] [(−)] [EXE].



Given the following equation, solve for the missing leg: $6^2 + b^2 = 15^2$

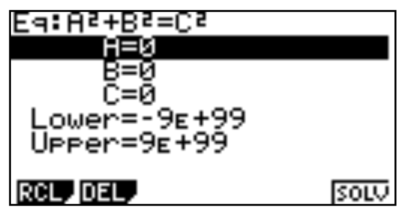
To check an answer using the Equation function:

1. Highlight the EQUA icon from the Main Menu and press [EXE]. To select Solver, press [F3]. To enter the formula input the following:

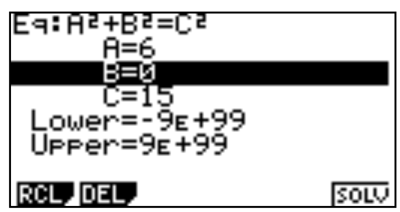
   [ALPHA] [X,θ,T] [x^2] [+] [ALPHA] [log] [x^2] [SHIFT] [•] [ALPHA] [ln] [x^2] [EXE]

   Note: To input a variable, press [ALPHA] then the key associated with the letters written in red.

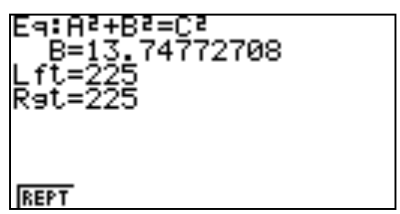
   The screen should look like the one to the right.



2. To solve for an unknown variable, enter each of the known values and press [EXE].



3. Use the arrow keys to highlight the unknown value and press [F6] (Solv). The value for B will display.



Exploring Right Triangles **Activity**

The Pythagorean Theorem is useful when working with right triangles. In this activity, we will explore various problems where using the Pythagorean Theorem.

For each question, write the equation used and your answer in a complete sentence.

Questions

1. A football field is a rectangle 160 feet wide and 360 feet long. If a player were to run along the diagonal of the field from one corner to another, how far would he run?

____

2. On a baseball diamond the bases are 90 ft apart. If you were to walk from home plate to second base, how far would you walk?

____

3. A local electronics stores is advertising a sale on 32" flat-screen televisions. If the diagonal measure is 32" and one of the sides are 25", what is the measure of the missing side?

____

4. A right triangle has leg lengths of 6 in. and 8 in. What is the length of the hypotenuse?

____

5. A right triangle has leg lengths of 12 cm. and 5 cm. What is the length of the hypotenuse?

____

6. A right triangle has leg lengths of 9 ft. and 12 ft. What is the length of the hypotenuse?

____

7. A right triangle has a leg length of 8 m and a hypotenuse length of 10 m. What is the length of the other leg?

____

8. A right triangle has a leg length of 15 units and a hypotenuse length of 17 units. What is the length of the other leg?

____

9. What is the length of the diagonal of a 10 in by 15 in rectangle?

____

10. The area of a square is 144 square centimeters. Find the length of a side. Find the length of the diagonal.

____

11. Two measures of a right triangle are 9 and 41. What is the value of the missing measure that would complete the Pythagorean Triple?

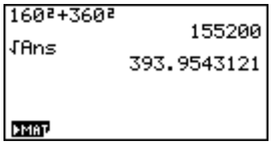
____

12. Two measures of a right triangle are 13 and 85. What is the value of the missing measure that would complete the Pythagorean Triple?

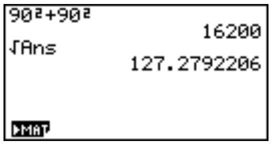
____

Solutions

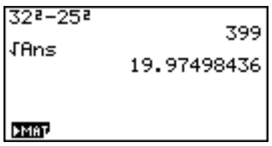
1. The player would run about 394 ft.



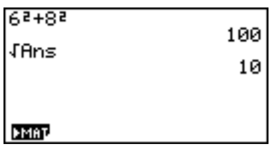
2. You would walk about 127 feet.



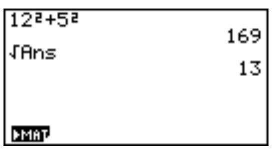
3. The missing measure is about 20 inches.



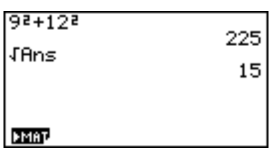
4. The hypotenuse is 10 inches.



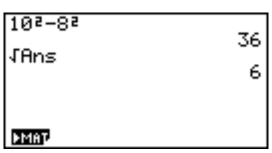
5. The hypotenuse is 13 cm.



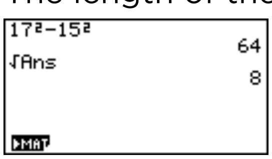
6. The hypotenuse is 15 feet.



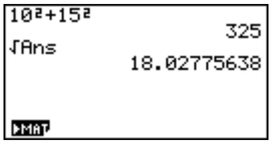
7. The missing leg is 6 m.



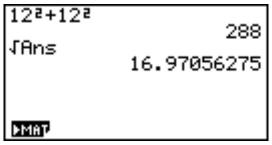
8. The length of the other leg is 8 units.



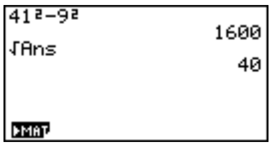
9. The diagonal measures about 18 inches.



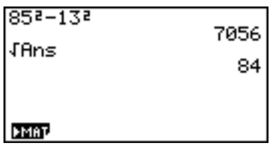
10. The length of the diagonal is about 17 cm.



11. The missing measure of the Pythagorean Triple is 40.



12. The missing measure of the Pythagorean Triple is 84.



Exploring with Expressions — Teacher Notes

Topic: Evaluating Expressions

NCTM Standard:

- Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.

Objective

The student will be able to use the Casio *fx-9750GII* to evaluate expressions.

Getting Started

Students are expected to evaluate expressions in Algebra. Making the distinction between the terms "simplify," "evaluate" and "solve" is often difficult for students. In mathematics, "simplify" is the term used when manipulating a string of numbers and operators. "Evaluate" is the term used when substituting a value into a string of numbers, variables and operators. "Solve" is the term used when there is an equation or inequality.

Prior to using this activity:

- Students should know the order of operations.
- Students should know how to translate expressions from written form to symbolic form.

Ways students can provide evidence of learning:

- Students can evaluate an expression for a given value.

Common calculator or content errors students might make:

- Students may incorrectly enter a sequence of numbers into the graphing calculator.

Note: The "How To" section for Activity 2 will be used for this activity as well.

Exploring with Expressions **Activity**

Each day we are asked to make decisions that require us to evaluate expressions. Recognizing the expression and being able to solve for a given value are real life skills that all students should have. In this activity, we will explore various problems where expressions are given and must be evaluated for a particular value.

The following is a series of questions involving expressions. For each question, provide the expression that represents the situation and then your answer in a complete sentence.

Questions

1. A trip to the A-1 Amusement Park is $22.95 per student. If there are 26 students in your third period class, what would the admission cost be for your class?

____

2. Two brothers, Ben and Jerry, were participating in a fundraising event for an after school program. At the end of the event, Ben had walked 14 miles and Jerry had walked six more miles than half of Ben's total distance. How far had Jerry walked?

____

3. Mary Anne's age is five more than a quarter of Tim's weight. If Tim weighs 168 pounds, how old is Mary Anne?

____

4. The sum of the angles of a triangle is 180°. To calculate the sum of the angles of any polygon, multiply two less than the number of sides of the polygon by 180. If you know that a polygon has 17 sides, what is the sum of the measures of the angles?

____

5. Your grandparents have opened a savings account for you and make a $50 deposit each month. After 19 months, how much money would be in your savings account?

____

Translate these expressions and evaluate for the given value. For your responses, please use complete sentences.

6. Translate the following expression and then evaluate when n = 4:

three more than five times a certain number

---

7. Translate the following expression and then evaluate when n = 17:

twenty times a certain number increased by two

---

8. Translate the following expression and then evaluate when n = 9:

one half more than a certain number

---

9. Translate the following expression and then evaluate when n = 5:

seventeen more than six times a number

---

10. Translate the following expression and then evaluate when n = 8:

five times a certain number decreased by seven

Evaluate each expression for the given value. For your responses, please use complete sentences.

11. $3x^2 - 2x - 4$ when $x = -5$

---

12. $6(4x + 2)$ when $x = -3$

---

13. $(7 - 3x)(6 + 2x)$ when $x = -2$

____

14. $\frac{4x}{6x}$ when $x = 7$

____

15. $\frac{25+2x}{2x+4}$ when $x = -2$

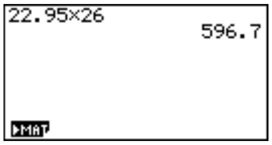
____

Explain your answer to this expression.

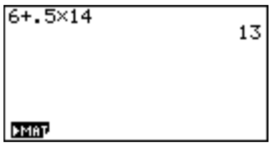
____

Solutions

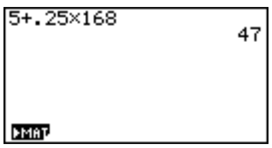
1. The expression is **22.95n**. The cost for 26 students is **$596.70**.



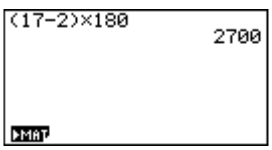
2. The expression is **6 +** $\frac{1}{2}$**n**. Jerry walked **13 miles**.



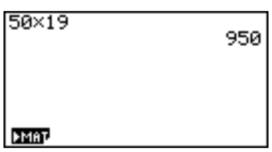
3. The expression is **5 + 0.25n**. Mary Anne is **47 years old**.



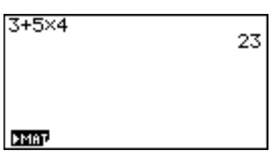
4. The expression is **(n - 2)180**. The sum of the angle measures is **2700 degrees**.



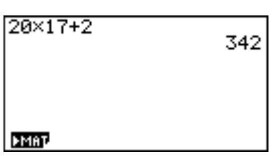
5. The expression is **50n**. There is **$950** in the savings account.



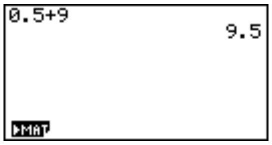
6. The expression is **3 + 5n**. When n = 4, the value is **23**.



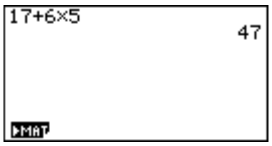
7. The expression is **20*n* + 2**. When n = 17, the value is **342**.



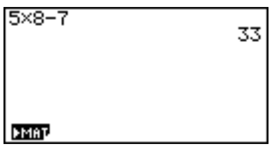
8. The expression is $\frac{1}{2}$ **+ n.** When n = 9, the value is **9.5**.



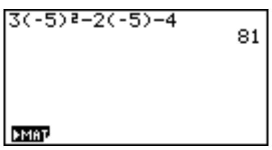
9. The expression is **17 + 6n**. When n = 5, the value is **47**.



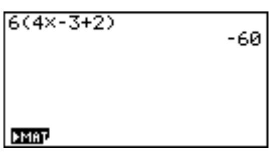
10. The expression is **5n - 7**. When n = 8, the value is **33**.



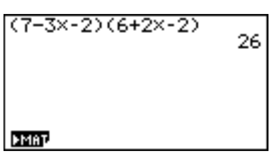
11. The value of **$3x^2$** – **2x** – **4** when x = –5 is **81**.



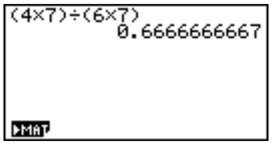
12. The value of **6(4x + 2)** when x = –3 is **–60**.



13. The value of **(7 – 3x)(6 + 2x)** when x = –2 is **26** .

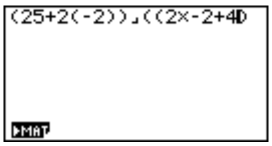


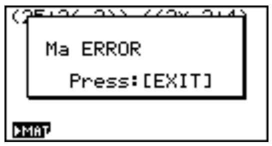
14. The value of $\frac{4x}{6x}$ when x = 0.6666667 = $\frac{2}{3}$.



15. The value of $\frac{25+2x}{2x+4}$ when x = –2 is undefined.

Student responses may vary but you are looking for "not being able to divide by 0".





Topic: Factoring Polynomials

NCTM Standard:

- Understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions.

Objective

The student will be able to use the Casio *fx-9750GII* to factor polynomials.

Getting Started:

Students are expected to factor polynomials in Algebra. Being able to understand how a polynomial is factored both algebraically, as well as graphically is extremely important. For some, this can be a difficult task as students have trouble understanding the relationship between the factors and the roots of the function when graphed.

Prior to using this activity:

- Students should know the properties of a polynomial.
- Students should know special polynomials such as a perfect square trinomial and the difference of two squares.

Ways students can provide evidence of learning:

- When given a polynomial, students can factor it completely.
- When given a polynomial, students can determine its roots graphically.
- When given a polynomial, students can determine its roots algebraically.

Common calculator or content errors students might make:

- Students may incorrectly solve for the roots of a polynomial.
- Students may fail to see the connection between a polynomial expressed as an equation and a polynomial expressed as a function.
- Students may confuse the degree of the polynomial with the number of terms.

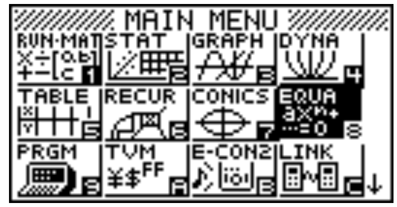
The following will demonstrate how to factor polynomials and find the roots or solutions to polynomials using the Polynomial function under the Equation Solver on the Casio *fx-9750GII*.

Factor the following polynomial and find the solution(s).

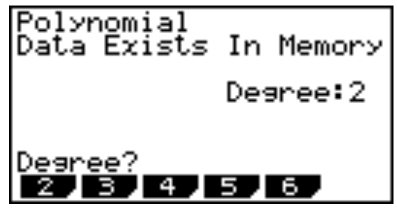
$x^2 + 2x - 35 = 0$

To factor a polynomial:

1. From the Main Menu, highlight the EQUA icon and press [EXE] or [8].

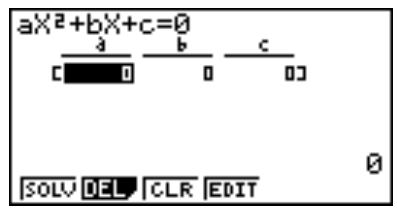


2. Press [F2] Polynomial.



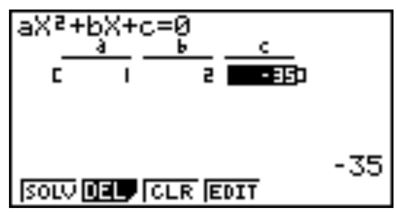
3. Press [F1] for a 2nd degree polynomial.

4. The standard form for a polynomial is displayed in the upper right corner of the screen. This polynomial is in standard form, so only the coefficients and constant need to be entered into the matrix.



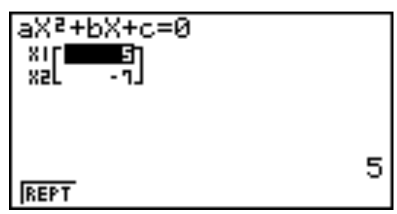
5. Since, a = 1, b = 2, and c = –35; input the following:

[1] [EXE] [2] [EXE] [(–)] [3] [5] [EXE].



6. Press [F1] to solve.

7. There are two solutions for x. They are 5 and –7. The factors of this polynomial are (x – 5)(x + 7) and the solutions are x = 5 & x = –7 .



Note: Remember, the answers on screen are the *roots* or *solutions* to the polynomial. This means if the value of **x1** = 5, the factor is (x – 5) because 5 would be the value necessary for *x* to equal zero. Make certain to remember how to correctly interpret the answers and double check your work if necessary.

8. Press [F1] (REPT) to repeat the process or press [EXIT]. To change the degree of the polynomial, press [EXIT] twice and choose the new degree.

Factoring Polynomials Activity

Polynomials are algebraic expressions composed of two or more monomials where at least one monomial contains a variable. Binomials are polynomials which consist of two terms; a special kind of binomial is a difference of two squares, where each term is a square term. Trinomials, polynomials consisting of three terms, are more frequent and are the most common to be factored. A special kind of trinomial is called a perfect square trinomial where both factors are the same.

This activity is designed to help you become more familiar with factoring with the Casio *fx-9750GII*. You will use the Equation Solver to assist you in solving these problems. However, you should also remember that these problems can also be graphed to help you obtain the solutions.

Questions

For each of the following polynomials, give the factored form.

1. $3x^2 + 18x + 27$

 ____

2. $x^2 - 81$

 ____

3. $x^3 - 2x^2 - 8x$

 ____

4. $3x^3 + 15x^2$

 ____

5. $x^3 + 3x^2 + 3x + 1$

 ____

6. $x^2 – 0.75x – 3.375$

 ____

7. The volume of a rectangular solid measures $x^3 + 10x^2 + 21x$ cubic feet. What are the dimensions of each side?

____

8. The area of a square measures $9x^2 - 81$ in^2. What is the measure of each side?

____

Extension

9. A polynomial has two real roots that are $x = -2$ and $x = 5$. What is the polynomial?

____

10. Create a 5th degree polynomial and use the Equation Solver to factor that polynomial.

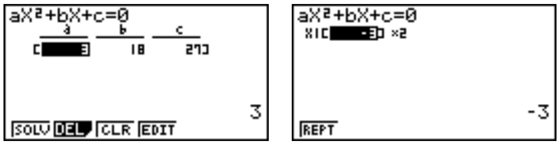
____

11. Create a 4th degree polynomial that contains a greatest common factor other than 1. Use the Equation Solver to factor that polynomial.

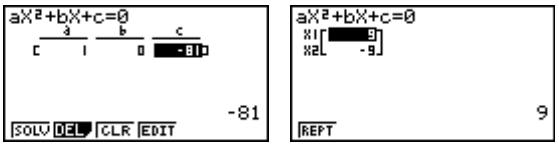
____

Solutions

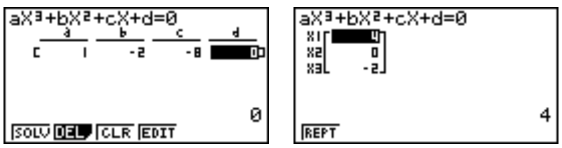
1. $(x - 3)(x - 3)$



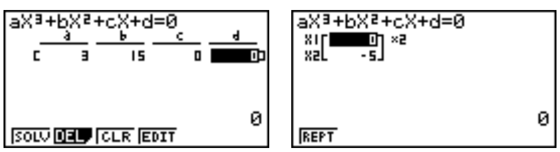
2. $(x - 9)(x + 9)$



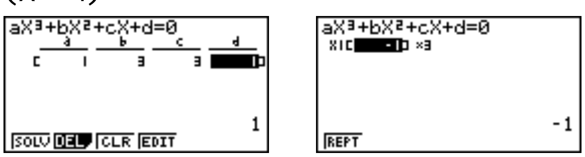
3. $x (x - 4)(x + 2)$



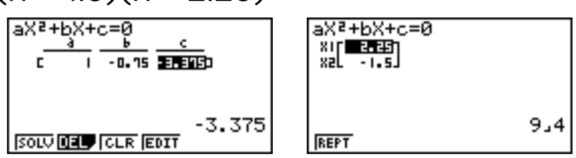
4. To factor $3x^3 + 15x^2$, you must factor out the greatest common factor first. The GCF in this polynomial is $3x^2$. This solution is shown in line **X1** of the solution matrix. Therefore, the correct solution is $3x^2(x + 5)$.



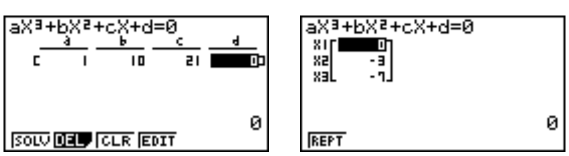
5. $(x + 1)^3$



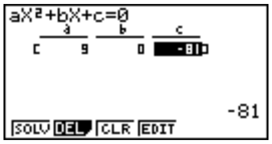
6. $(x + 1.5)(x - 2.25)$

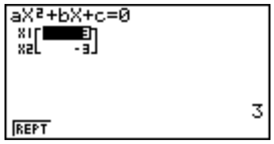


7. x feet by $(x + 3)$ feet by $(x + 7)$ feet



8. (x + 3) inches by (x – 3) inches





9. (x + 2)(x – 5) = x^2 - 3x – 10

10. Answers will vary.

11. Answers will vary.

Topic: Functions and Function Notation

NCTM Standard:

- Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules.
- Relate and compare different forms of representation for a relationship.
- Identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations.

Objective

The student will be able to use the Casio *fx-9750GII* to input data relating to functions as well as to evaluate functions using function notation.

Getting Started

Being able to understand functions in various forms empowers students to see patterns in relationships and make connections with functions to their everyday lives. Begin this activity by looking at various tables and gain an understanding of how that data relates to its graph on a coordinate plane. Students should be able to examine a table and/or a graph and be able to determine any specific trends or characteristics about that data set.

Prior to using this activity:

- Students should be able to graph points on a coordinate plane.
- Students should be able to read and interpret data presented in a table and graph.
- Students should be familiar with all key strokes involved in entering data into a table and entering a function into the graph editor window.
- Students should understand the formula for a linear function.
- Students should be able to correctly identify the independent (x) and dependent (y) variables.

Ways students can provide evidence of learning:

- When given a function, students can state and explain whether that function is increasing or decreasing.
- When given a function, students can evaluate it for a specific value of x or y.

Common calculator or content errors students might make:

- Students may incorrectly set the viewing window when showcasing a function.
- Students may enter data into a table incorrectly by switching the x- and y-coordinates.

Fun With Functions "How To"

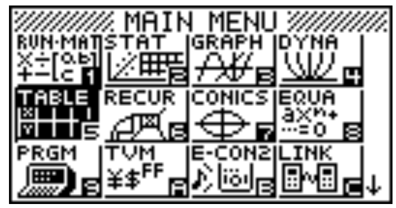
The following will demonstrate how to store a value for a variable, input a function to generate a table, and enter data into a table to construct a graph with the Casio *fx-9750 GII*.

To store a value for a variable:

1. To store 5 for the variable *x*, input:

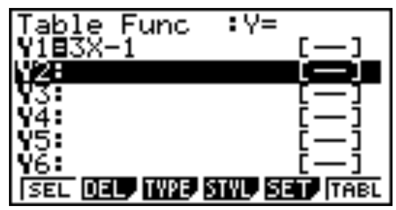
   [5] [→] [X,θ,T] [EXE].

To input a function and generate a table:



1. From the Main Menu, highlight the TABLE icon and press [EXE] or [5].

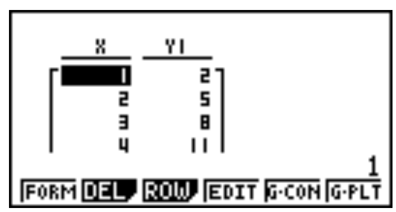
2. To enter a function such as 3x – 1, input: [3] [X,θ,T] [–] [1] [EXE].



3. The equal sign to the right of **Y1:** is highlighted to indicate this function is active.

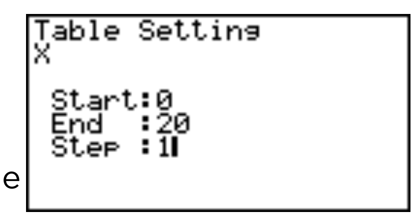
4. To generate a table of values for the selected function, press [F6] (TABL).

   The default x-values for the table menu is x starts 1, ends at 5 and increases by steps of 1.



5. To navigate through the table, use the replay pad ◀ ▶ ▲ ▼.

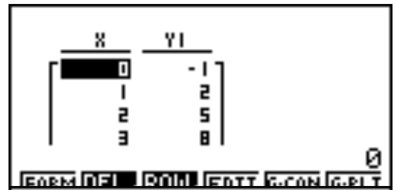
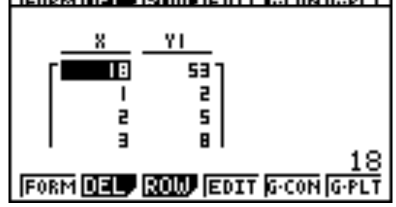
To adjust the values in a table:



1. Use [EXIT] to return to the TABLE home screen. Press [F5] (SET) to change the default table values.

   Enter [0] for the **start** value, [2] [0] for the **end** value and a **step** value of [1], pressing [EXE] after each entry.

2. Press [EXIT] then [F6] (TABL) to display the table.

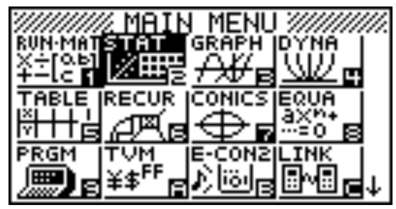



3. To display a corresponding y-value for a specific x-value, highlight any x-value and enter the desired value.

   To display the corresponding y-value when x = 18, input [1] [8] [EXE].

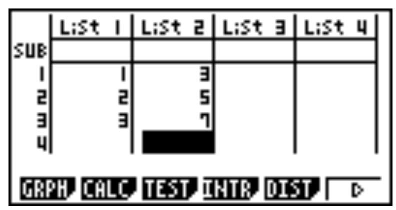
   Note: You do not need to change the tables settings; you could just enter all given x-values and create a custom table.

To enter data into a list and graph the data:



1. From the Main Menu, highlight the STAT icon and press [EXE] or [2].

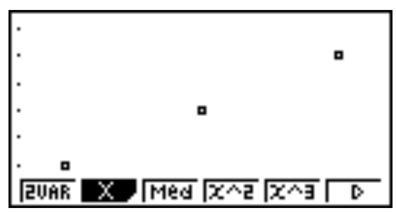
2. In **List 1**, input: [1] [EXE] [2] [EXE] [3] [EXE].



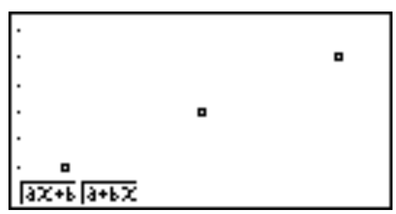
3. Press ▶ to move the cursor to **List 2**.

4. In **List 2**, input: [3] [EXE] [5] [EXE] [7] [EXE].

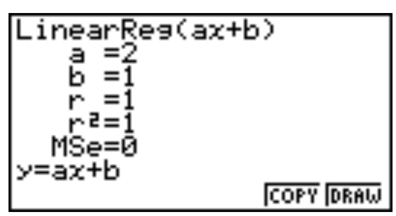
5. Press [F1] (GRPH), then [F1] to select Graph 1 (the default graph type is a scatterplot).



6. Press [F1] (CALC) for regression options.



7. Since the data appears linear, press [F2] (X) to calculate linear regression (line of best fit).



8. Press [F1] (ax +b) for slope-intercept form of a line. Substitute the a- and b-values displayed into the given formula.
   When the correlation coefficient, r, equals 1, you have a perfect regression.

Functions help establish various types of numeric patterns, based upon whether those functions are linear, quadratic, cubic, etc. Building a strong foundation in Algebra includes a comprehensive study of linear functions. Functions are a rule used to calculate values. Functions are written using a specific notation called function notation. Each function has an independent and a dependent variable. The independent variable is the value you get to choose or control. The dependent variable is the value created when the independent variable is plugged into the function. Another name for the independent variable is the "input" and for the dependent variable is the "output". We will define a series of coordinate points as a relation.

In this activity, we will explore how to assign a single value to a variable and evaluate a given function. We will also explore how to input a function and generate a table of values as well as enter points in a data set and determine the function.

Functions can be expressed in these different forms:

1. The **Slope-Intercept Form of a Line** is defined as **y = mx + b**; where ***m*** represents the slope of the line, ***b*** represents the y-intercept, ***x*** represents the independent variable and ***y*** represents the dependent variable.

2. The **Standard Form of a Line** is defined as **Ax + By = C** where **A**, **B**, and **C** are integers and **x** represents the independent variable while **y** represents the dependent variable.

3. f(x) is often described as function notation. In this example, where f(x) = 5x – 3, *x* represents the independent variable and f(x) is synonymous with *y*, representing the dependent variable.

Remember when graphing a function, it must pass the Vertical Line Test. A function is defined as a relation where for every one *x*-value, there is one and only one *y*-value. When looking at the graph, if any two points appear directly above each other, the graph fails the vertical line test and thus, the relation is not a function.

Questions

1. Given the function $f(x) = 3x + 7$, evaluate the function at $f(4)$.

---

2. Given the function $f(x) = -2x - 5$, evaluate the function where $-5 \leq x \leq -1$.
(Let x = set of integers)

---

3. Given the function $f(x) = 2x^2 + 6x - 5$, evaluate the function at $f(-2)$.

---

4. Enter the data into the calculator and determine the linear function.

List 1	List 2
1	2
2	5
3	8
4	11

---

5. Enter the data into the calculator and determine the linear function.

List 1	List 2
1	–1
2	17
5	32
8	47

---

6. Enter the data into the calculator and determine the linear function.

List 1	List 2
0	8
2	6
5	3
2	0

____

7. Lauren works as a babysitter to earn some extra money. She charges her customers seven dollars an hour. Write a function to determine the amount of money Lauren will earn if she works *x* hours. How much money will she earn if she works 4 hours? 7 hours? 11 hours?

____

____

8. A cell phone company charges its customers $30 per month for phone calls plus an additional charge of seven cents per text message after the first 50 text messages. Write a function that accurately models how much money you will spend per month with this plan. How much money will you spend if you send 50 text messages per month? 100 text messages per month? 225 text messages per month? 500 text messages per month?

____

____

____

Extension

1. Create a data set that models a function which is non-linear.

____

____

2. Write a non-linear function and evaluate that function where $-5 \leq x \leq 5$.

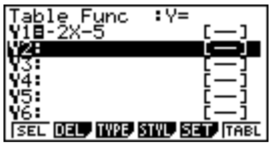
____

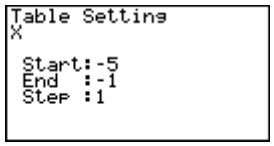
____

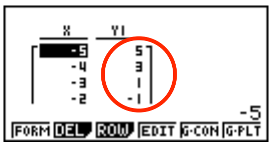
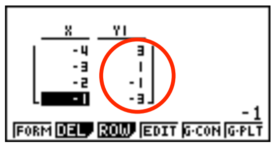
Solutions

1. f(4) = 19

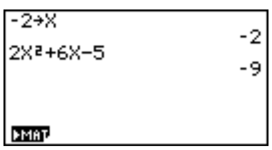
2. f(-5) = 5, f(-4) = 3, f(-3) = 1, f(-2) = -1, f(-1) = -3



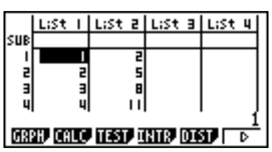
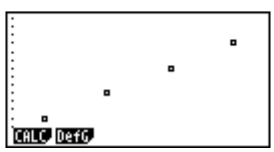
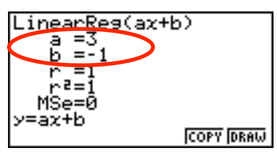


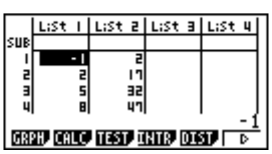
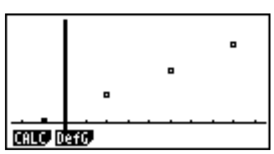
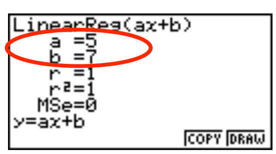
3. f(-2)= -9



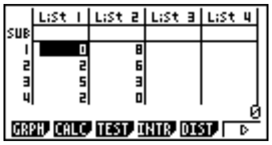
4. The function is: y = 3x – 1 or f(x) = 3x – 1

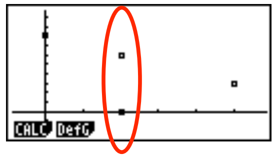




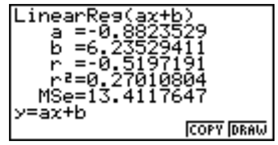
5. The function is y = 5x + 7 or f(x) = 5x + 7

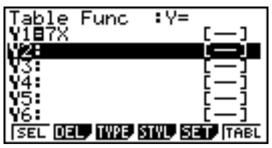
6. This relation fails the vertical line test as evident by the two coordinates directly above each other. Since the relation fails the vertical line test, the relation is not a function.

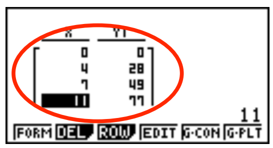




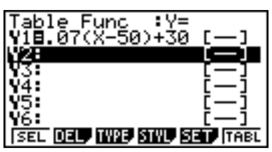


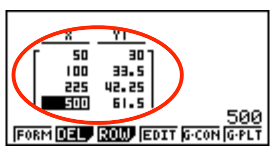
7. The function is y = 7x or f(x) = 7x. If Lauren works four hours, she will earn $28. If Lauren works seven hours, she will earn $49. If Lauren works 11 hours, she will earn $77.





8. The function is y = .07(x - 50) + 30 or f(x) = .07(x - 50). For 50 text messages per month, you will spend $30. For 100 text messages per month, you will spend $33.50. For 225 text messages per month, you will spend $42.25 and for 500 text messages per month, you will spend $61.50.





Extensions

1. Answers may vary.

2. Answers may vary.

Gas Mileage — Teacher Notes

Topic Area: One-way Analysis of Variance

NCTM Standards:

- Develop and evaluate inferences and predications that are based on data.
- Use simulations to explore the variability of sample statistics from a known population and to construct sampling distributions.
- Understand how sample statistics reflect the values of populations parameters and use sampling distributions as the basis for informal inferences.

Objective

The student will be able to complete a one-way analysis of variance using sample data.

Getting Started

For this activity you will need the students to work individually or in pairs. For data with more than one level of independent variable, students cannot perform a t-test. An Analysis of Variance can be used in this case. In this activity the students will use a one-way analysis of variance because there is only one independent variable, the model type.

Prior to using this activity:

- Students should recognize that ANOVA makes several assumptions about the data:
    - The scores are normally distributed, though it allows for violations of the assumption and can be helpful even when the data is not distributed normally.
    - The population variances of the groups are the same and as long as the group sizes are relatively the same, violations of this do not cause huge errors in the results.
    - The observations are independent of one another.
- Students should have a basic understanding of interpreting statistical data.
- Student should be able to create and interpret a box and whiskers plot.
- Students should be able to calculate the average for a data set.

Ways students can provide evidence of learning:

- Students will be able to analyze the data sets to determine if they are statistically the same.
- Given a set of data, the student will be able to accept or reject the null hypotheses.

Common mistakes to be on the lookout for:

- Students may misinterpret the results of ANOVA.
- Students may not understand the effect that outliers have on the set of data.

Definitions

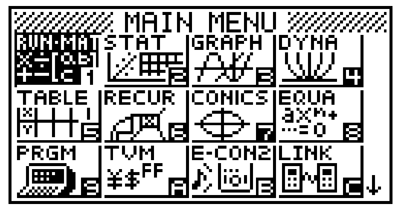
- Analysis of variance (ANOVA)
- Average

The following will demonstrate how to enter a set of data into the Casio *fx-9750GII*, find the averages, make a box and whiskers plot, and conduct a one-way analysis of variance (ANOVA).

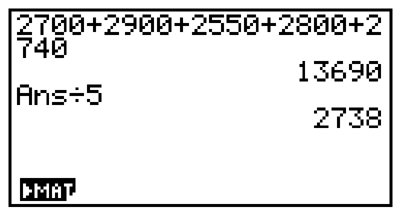
Weekly Sales for Three Sales People					
	Week 1	Week 2	Week 3	Week 4	Week 5
Person A	2,700	2,900	2,550	2,800	2,740
Person B	2,800	2,900	3,200	3,100	2,620
Person C	2,100	2,700	2,950	3,000	2,660

To find the average for each individual:

1. From the Main Menu, highlight the RUN•MAT icon and press [EXE] or press [1].



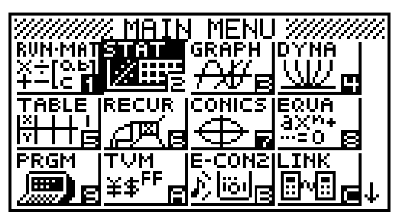
2. Input the weekly sales for Person A and press [EXE]; this will give the total sales for Person A.



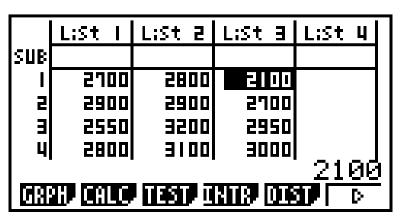
3. To calculate the average sales, divide the above answer by 5, since there were 5 weeks recorded. It is not necessary to retype the value; simply press [÷] [5] [EXE].

To enter the above set of data:

1. From the Main Menu, highlight the STAT icon and press [EXE] or press [2].

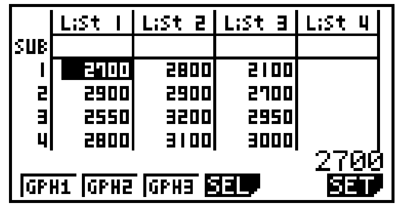


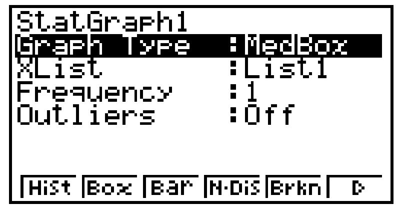
2. To clear previous data lists, press [F6] (▷) [F4] (DEL-A) [F1] (Yes). Pressing [F6] (▷) will bring you back to the initial STAT screen.



3. Type in each value and pressing [EXE] after. The display should look like the screen shot on the right when completed.

To select the type of graph for this data:





1. Press [F1] (GRPH).

2. Press [F6] (SET) to set the type of graph for **StatGraph1**.

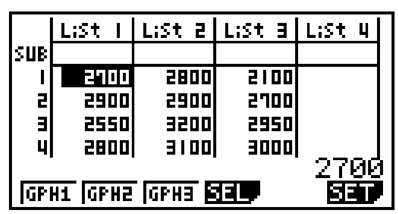
3. Press ▽ to highlight **Graph Type**.

4. There are five choices; Scat, *XY*, NPP, Pie, and (▷). Selecting [F6] (▷) will provide more graph choices.

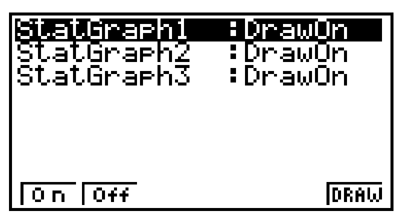
5. Select [F2] (BOX) to make **StatGraph1** a box and whiskers plot.

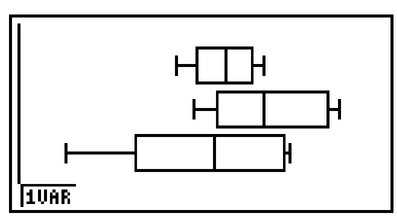
6. ▽ to select **List1** as your data list, then [EXIT].

7. Repeat this process for data sets 2 and 3, graphing them in **StatGraph 2** and **3**, respectively.



To display a multiple graphs of the data:





1. Press [F4] (SEL) [F1] (On) to turn on **StatGraph1**.

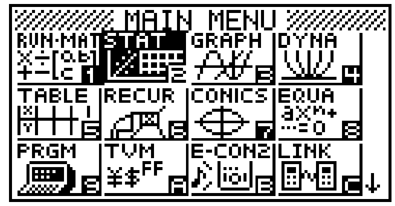
2. Arrow down to **StatGraph2** and press [F1] (On).

3. Arrow down to **StatGraph3** and press [F1] (On).

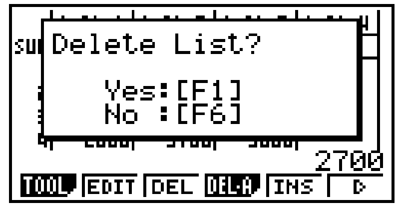
4. Press [F6] (Draw).

To conduct a one-way analysis of variance:

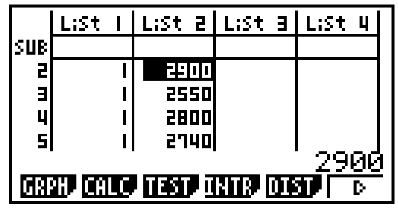
1. From the Main Menu, highlight the STAT icon and press EXE or press 2.

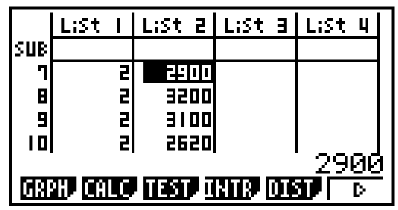


2. To clear previous data lists press F6 (▷) F4 (DEL-A) F1 (Yes). Pressing F6 (▷) will bring you back to the initial STAT screen.



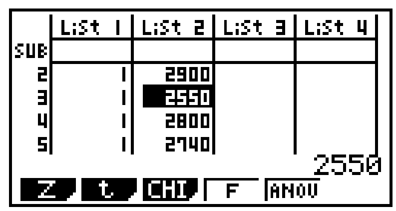
3. Each salesman is given a number 1, 2, or 3. This number is typed into **List 1**. In **List 2**, type in the sales figure for each salesman. Enter the data by typing each number, and pressing EXE after each entry. The display should look like the screen shots on the right when completed.





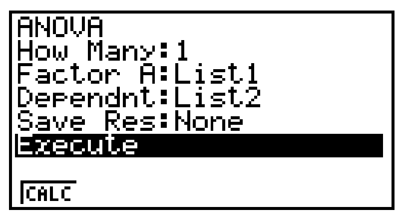
4. Press F3 (TEST).

5. Press F5 (ANOVA).

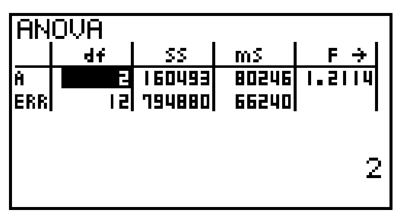


6. Set **Factor A** to **List 1**. Set **Dependnt** to **List 2**.

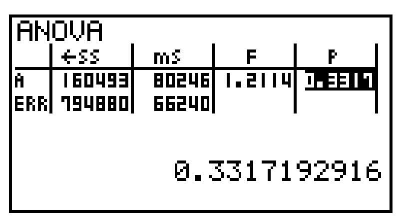
7. Highlight **Execute** and press F1 (CALC).



8. The **F** value will be visible on the screen.



9. Press ▶ to see the **P** value.



Gas Mileage Activity

In a controlled study, three popular compact SUV model's gas mileage was tested. Five cars of each model were tested and the results are reported below.

Model	Gas Mileage (over 500 miles)				
Honda CR-V	26.9	28.5	25.5	27.6	28.7
Toyota RAV-4	28.0	29.0	26.5	30.2	26.2
Saturn VUE	24.0	26.0	27.5	30.0	26.5

Questions

1. What is the average mileage for each of the models?

    ____

2. Create a parallel box and whiskers plot for each models' gas mileage. Draw and label the plots in the space below.

3. Explain any differences you see in the box plots.

    ____

    ____

4. Conduct a one-way analysis of variance. Is there evidence to conclude that there is a statistically significant difference in the average gas mileage for each model? Explain.

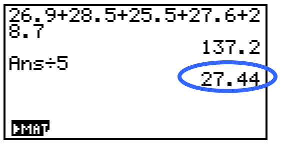
    ____

    ____

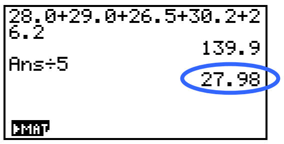
5. Which model would you choose? Why?

Solutions

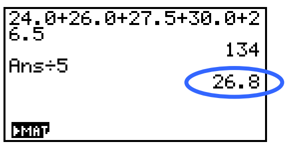
1. Honda CR-V = 27.44 mpg



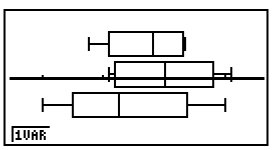
Toyota RAV-4 = 27.98 mpg



Saturn VUE = 26.8 mpg

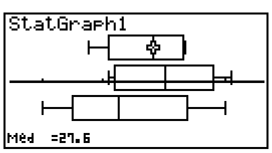


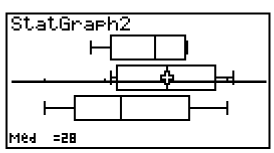
2.

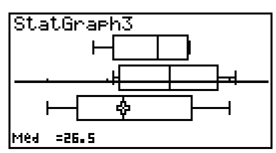


3. Answers will vary; students can discuss the statistical differences between each graph.

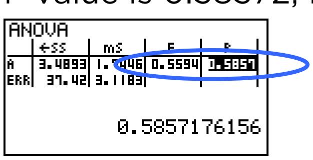
Pressing SHIFT F1 **(TRACE)** will allow students to find the median values on each graph. Press ◀ ▶ to find the data on one graph and press ▲ ▼ to switch between each graph.







4. P value is 0.58572; F value is 0.5595.



5. Answers will vary.

Germs, Germs, Everywhere — Teacher Notes

Topic: Patterns and Functions- Algebraic Thinking

NCTM Standard:

- Understand patterns, relations, and functions by interpreting representations of functions of two variables and use symbolic algebra to represent and explain mathematical relationships.

Objective

The student will be able to use the Casio *fx-9750GII* to take a set of formulas, graph them, and find solutions, specifically related to half-life.

Getting Started:

Discuss the Law of Growth and Decay with the students, including where they are used and what information they give. Demonstrate how to find the value of the constraint, k, and write an equation to represent the Law of Decay.

Prior to using this activity:

- Students should be able to enter an exponential equation, using base e, into the calculator.
- Students should be able to set up the view window and calculate x- and y-values using the calculator.

Ways students can provide evidence of learning:

- Students can determine if an equation represents the Law of Growth or Law of Decay.
- Students can explain their findings and discuss what is happening, given an equation for Law of Growth or Decay,
- Students will be able to interpret the graph of an equation and give alternative methods for calculating x and y.

Common calculator or content errors students might make:

- Students may have trouble setting up the window to the desired parameters.

Definitions

- Law of Growth
- Law of Decay
- Constant
- Natural Constant

Formula

Law of Growth and Decay: $A = A_0e^{kt}$

The following will demonstrate how to take a formula, graph it, and calculate a specific value on the Casio *fx-9750GII*.

Formula: $A = A_0e^{kt}$

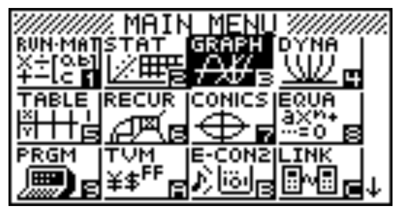
A_0: Initial amount
A: Ending amount
e: Natural constant
k: Rate of growth or decay
t: Time

If $A_0 = 500$, $A = 250$, and $t = 4$, find k.

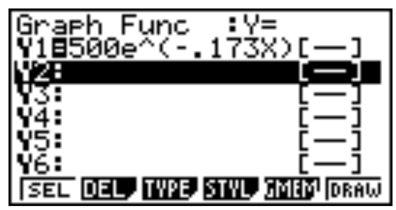
$250 = 500e^{4k}$
$0.5 = e^{4k}$
$\ln 0.5 = 4k$
$k = -.173$

If $A = 500e^{-.173t}$, solve for A when t = 3. Solve for t, when A = 50.

To graph the equation:

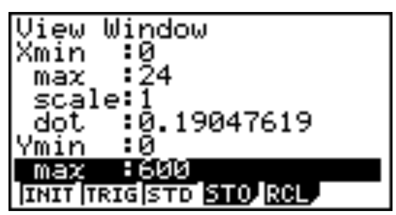


1. From the Main Menu, highlight the GRAPH icon and press EXE or press 3.



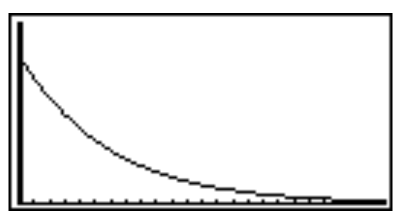
2. Enter the formula into **Y1:** by entering: 5 0 0 SHIFT ln ((−) • 1 7 3 X,θ,T) EXE.

To change the viewing window:



1. Press SHIFT F3 **(V-Window)**.

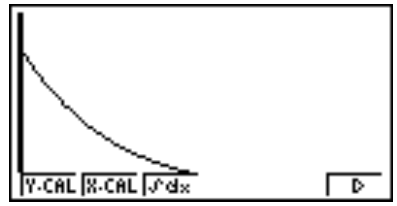
2. Enter 0 EXE for the **Xmin** and 2 4 EXE for the **Xmax**. Enter 0 EXE for the **Ymin** and 6 0 0 EXE for the **Ymax**.



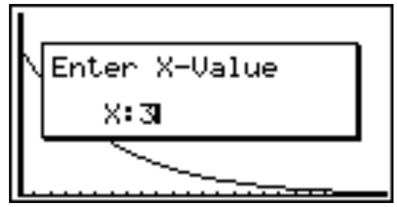
3. Press EXIT and then F6 to view the graph.

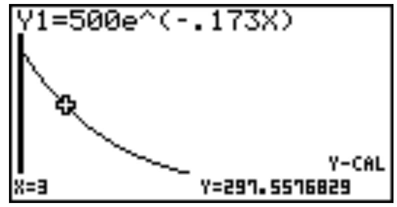
To calculate a y-value:

1. Press SHIFT F5 **(G-Solv)**. To calculate a value for *y* given "A", the x-value, press F6 (▷) for more options.



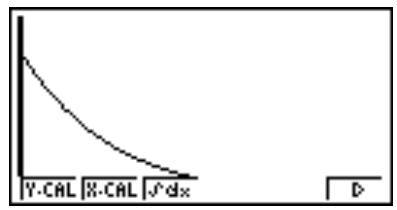
2. Select F1 for **Y-Cal**, to calculate the *y*-value. Enter 3 EXE for the x-value. The equation displays in the upper left corner, the values of *x* and *y* are displayed along the bottom along with the calculation performed .



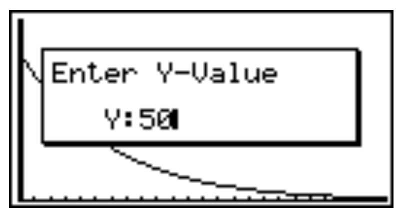


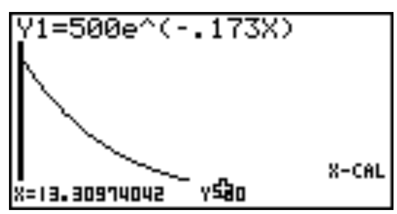
To calculate an x-value:

1. Press SHIFT F5 **(G-Solv)**. Press F6 for more options.



2. Press F2 for X-Cal, to calculate the value of "t", the x-value. Enter 5 0 EXE for the y-value.





Germs, Germs, Everywhere **Activity**

Medications are given to patients in a variety of forms including liquids, solids, and sprays. The form of the medication is directly related to the absorption rate by the body and how much of medicine actually gets into the system. Once in the system, the medication is metabolized within the system and then excreted from the body.

In this activity, you will determine the amount of medication that is actually used by the body and how much is left after a certain amount of time. You will write an exponential equation to represent the Law of Decay for a given medication and calculate the amount of medication in the body's system after a given amount of time.

Reminder: $A = A_0e^{kt}$

Questions

1. Pharmaceutical Company A is marketing a medication with a half-life of 2 hours. The medication comes in 200 mg tablets. Calculate the value of the constant for the rate of decay for this medication.

   ____

2. The specified adult dose is 600 mg every 6 hours. Write an equation to represent the amount of medication in the blood stream at a given time.

   ____

3. Find the amount of medication that is remaining in the system after 1 hour.

   ____

4. Find the amount of medication that is remaining in the system after 8 hours, assuming only one dose of medication is taken.

   ____

5. In order to be effective, at least 10% of the medication must be in the system. How many milligrams of medication is this?

   ____

6. After how many hours will the system contain this amount, assuming only one odes of medication is taken.

____

7. Pharmaceutical Company B is marketing a medication with a half-life of 3 hours. The medication comes in 400 mg tablets. Calculate the value of the constant for the rate of decay for this medication.

____

8. The specified adult dose is 400 mg every 8 hours. Write an equation to represent this situation.

____

9. Find the amount of medication that is remaining in the system after 1 hour.

____

10. Find the amount of medication that is remaining in the system after 8 hours.

____

11. In order to be effective, at least 20% of the medication must be in the system. What is the least amount of medication that must be in the system in order to be effective?

____

12. After how many hours will the system contain this amount?

____

Extensions

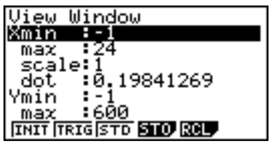
1. Which medication do you think would be more effective? Explain your answer.

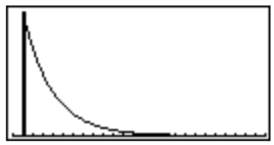
    ____

Solutions

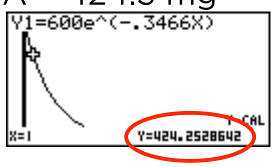
1. $100 = 200e^{k(2)}$
   $0.5 = e^{k(2)}$
   $\ln 0.5 = 2k$
   $k = \dfrac{\ln .5}{2}$
   $k = -0.3466$

2. $A = 600e^{-0.3466t}$

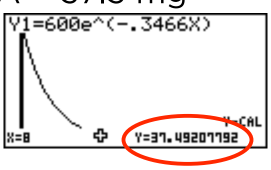




3. A = 424.3 mg

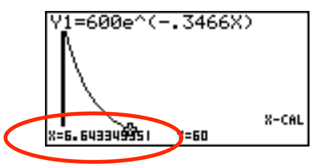


4. A = 37.5 mg



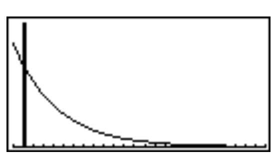
5. 600 x .1 = 60 mg

6. 6.6 hours

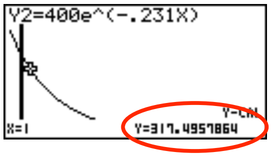


7. $200 = 400e^{3k}$
   $0.5 = e^{3k}$
   $\ln 0.5 = 3k$
   $k = \dfrac{\ln .5}{3}$
   $k = -0.2310$

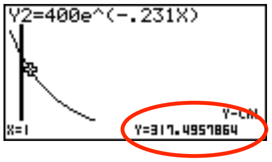
8. $A = 400e^{-0.2310t}$



9. A = 317.5 mg

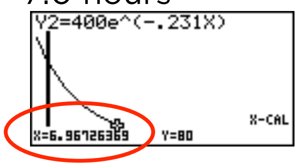


10. A = 63.0 mg



11. 400 x .2 = 80 mg

12. 7.0 hours



Extension Solutions

1. Answers will vary.

Topic Area: Slope

NCTM Standards:

- Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations.
- Draw and construct representations of two- and three-dimensional geometric objects using a variety of tools.
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.

Objective

The student will be able to use the Casio *fx-9750GII* to calculate the slope of a line segment using the line of best fit and find the values associated with a point on the line.

Getting Started

The concept of slope is found in many areas of mathematics. Students need to understand how to calculate slope and interpret its meaning. This concept will be presented in many different ways such as equations, graphs, and formulas to be used for a variety of applications, stemming from construction to aeronautics. Having a firm understanding of slope will be essential in solving many real-life problems.

Prior to using this activity:

- Students should be able to plot points on a coordinate axes.
- Students should know the slope formula.
- Students should be able to identify the slope of a line given its equation in slope-intercept form.
- Students should be able to graph the equation of a line on a graphing calculator and adjust the view window.

Ways students can provide evidence of learning:

- Given two points contained in a line, the student will be able to find the slope of the line using the slope formula.
- Given two points on a line, the student will be able to find the slope of a line using statistics and finding the line of best fit.

Common mistakes to be on the lookout for:

- Students may interchange the values of the x- and y- coordinates while using the formula.
- Students may omit the use of parentheses in inputting the formula causing an error in the order of operations.
- Students may put the values of the coordinates into the wrong lists while finding the line of best fit.

Definitions

- Slope
- Horizontal
- Vertical
- Rate of Change
- Pitch
- Rise
- Span
- Gradiant

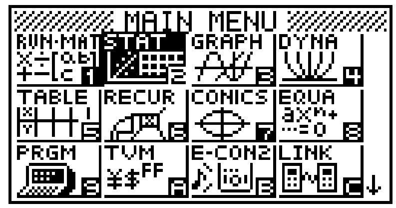
Formula

Slope/Rate of Change: $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

The following will demonstrate how to find the slope between two points on a line using a graph and a linear regression by entering a set of coordinates into two lists using the Statistics mode on the Casio *fx-9750GII*.

Given the points (–6, –3) and (2, 1), find the slope of a line containing the two points. Graph the line, find the value of y when x = 5, and the value of x if y = 4.

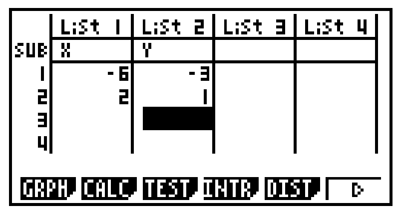
To enter values into a list and draw the line:



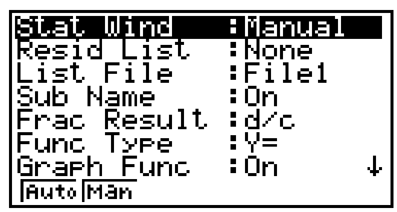
1. From the Main Menu, highlight the STAT icon and press EXE or press 2.

2. To label the first column, highlight the space below **List 1** and press ALPHA + (X) EXE.

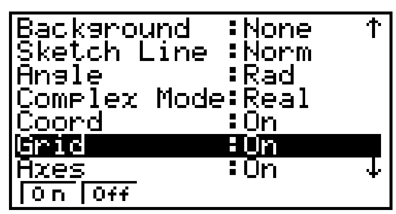
3. To label the second column, highlight the space below **List 2** and press ALPHA − (Y) EXE.



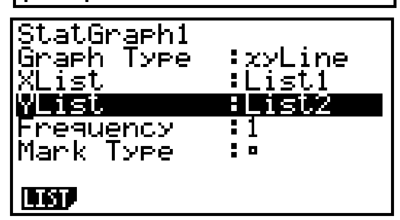
4. Enter the x-values into **List 1** and the y-values into **List 2** as shown at the right. Be sure to press EXE after each value.



5. Set up the calculator so that the axes can be set up manually by pressing SHIFT MENU **(SET UP)**, highlight **Stat Wind** and press F2 (Man).

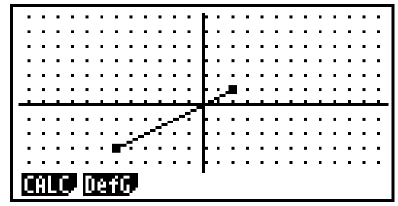


6. To show the grid, arrow down to highlight **Grid** and press F1 (On). Press EXIT when finished.



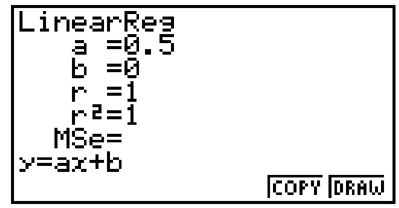
7. To view the points, press F1 (GRPH) F6 (Set) ▼ F1 (GPH1) ▼ F2 (XY) ▼ F1 (List) 1 ▼ F1 (List) 2 EXE F1 (GPH1).

8. With **StatGraph1** on, press F6 (Draw). Press SHIFT F2 **(Zoom)** F4 (Out) EXE to see the entire line.

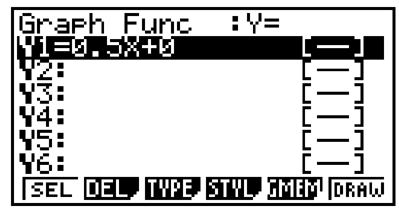


To find the equation for the linear regression and draw the graph:

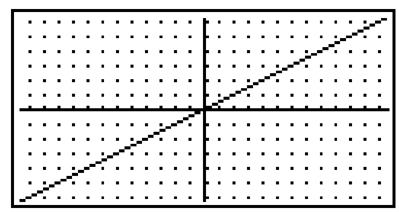
1. Press F1 (Calc) F2 (X). This will give you the equation. The equation would be y = 0.5x.



2. Press F5 (Copy) EXE to copy the equation into the graph function.

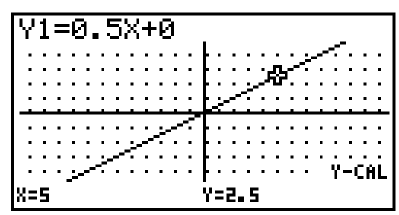


3. Now, press MENU, highlight the GRAPH icon, and press EXE. Press F1 (Sel) F6 (Draw) to draw the line.

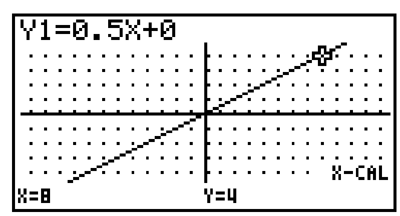


To calculate values of points on the line:

1. With the graph of the line on the screen, press F5 **(G-Solv)** F6 (▷) F1 (Y-cal) and enter the value of x which is 5 and press EXE. The value of y is 2.5.

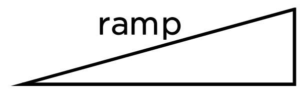


2. To find the y-value, press SHIFT F5 **(G-Solv)** F6 (▷) F2 (X-cal) and enter the value of y which is 4. Press EXE.



Getting the Slant on Construction Activity

All buildings must be constructed according to a set of codes that are specific to their area and use. When a new construction does not follow the code, builders must make corrections which, in some cases, may mean tearing down a structure and rebuilding it. In this activity, we will explore the use of slope in various aspects of construction and determine if the builder is following the code. (Hint: use a 20 by 30 view window)



Questions

Public buildings are required by code to have handicap accessible entrances. The maximum standard for a ramp is 1 ft. of rise for every 8 ft. of run.

1. Find an equation that can be used for constructing a handicap ramp for this standard.

   ____

2. If the change in height from the ground to an entrance is 3 ft. 6 in., find the required distance along the ground for the ramp?

   ____

3. If the distance in front of the building is 10 ft., will the builder be able to install a straight ramp?

   ____

4. The entrance to a public library is located 6 ft. above ground. A ramp is to be built for access in two equal parts. What will be the vertical distance for each section of the ramp?

   ____

5. The local registrar's office has a front that is 16 ft. 9 in. in length. In order to keep within code, how high would the handicap ramp be if it spanned the entire front of the building?

   ____

6. If the entrance is 2 ft. off the ground, will they need the entire length for the ramp?

   ____

Architects have their own styles with which they design a home. One of the choices is the style of roof. This will vary according to the climate and precipitation of the area. In a snowy climate, the pitch of the roof is steep in order to keep snow from piling up and causing the roof to cave in.

7. If the roof of a house in this climate rises 8 in. for every 12 in. span, what is the slope of the roof?

____

8. What would be the height of the roof with this pitch, if the width of the house was 18 ft. and the ridge line of the roof was at the center point?

____

In the United States, the pitch of a roof is given as so many inches or rise over a 12 in. span.

9. Find the slope for a shed with a 4:12 pitch.

____

10. Find the slope of a gable roof with a 9:12 pitch.

____

Stairs are another place where slope plays an important part. There are codes that are used in building stairs to prevent accidents.

11. The maximum height of a stair is given as 9 in. and the maximum depth is 10 in. What is the slope for stairs built using these measures?

____

12. Using the linear regression line, find the maximum height of a staircase with 12 steps.

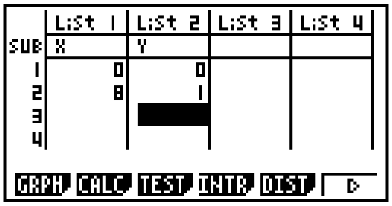
____

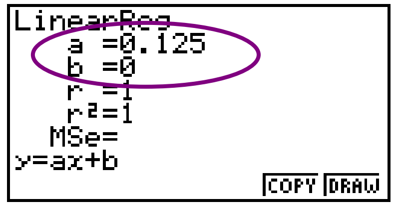
13. The minimum height of a stair is 7.375 in. and the minimum depth is 8.25 in. What is the difference between the two slopes?

____

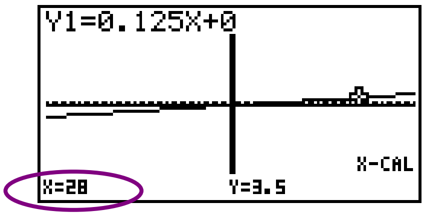
Solutions

1. $y = 0.125x$



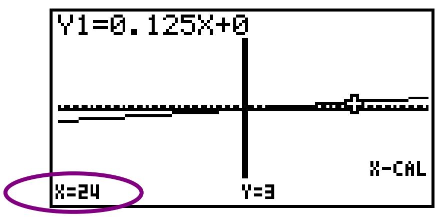


2. 28 feet

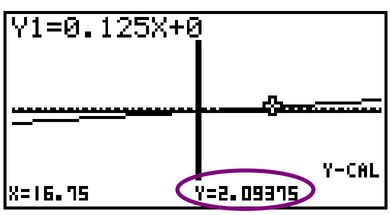


3. No

4. 24 feet

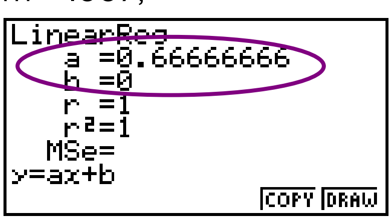


5. 2.09375 feet = 2' $1\frac{1}{8}$"

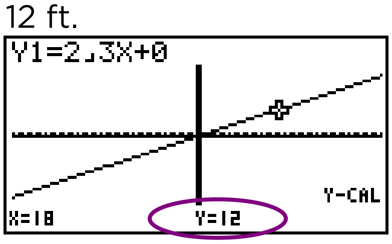


6. No

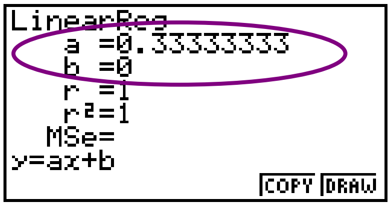
7. m ≈ .667;



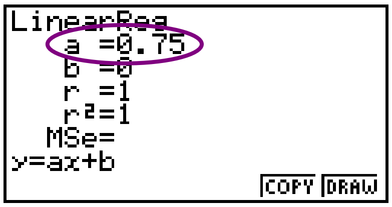
8. 12 ft.



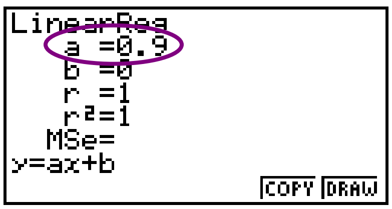
9. m ≈ 0.33



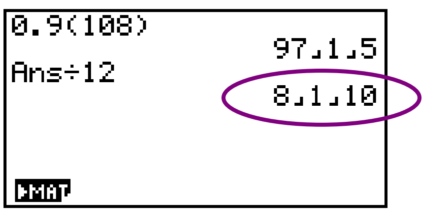
10. m = 0.75



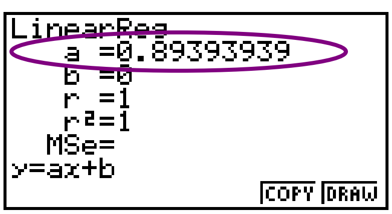
11. m = 0.9



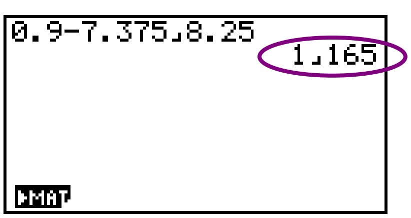
12. y = 0.9x = 0.9(108) = 97.2 in. = $8\frac{1}{10}$ ft.



13. Slope for the minimum is ≈ 0.894



Difference: 0.9 - .894 = .006 or $\frac{1}{165}$ of an inch



Topic Area: Calculating Distance

NCTM Standards:

- Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations.
- Draw and construct representations of two- and three-dimensional geometric objects using a variety of tools.
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.

Objective

The student will be able to calculate the distance between two points using the distance formula, calculate the distance between two points using the Pythagorean Theorem and solve real-life problems involving distance.

Getting Started

As a class, discuss the definition of a distance and how it is measured. Ask students to give examples of the use of calculating distance and whether direction has any bearing on the distance. Provide examples of the graph of the distance between two points on a coordinate axes. Show how this is related to the Pythagorean Theorem.

Prior to using this activity:

- Students should be able to plot points on a coordinate plane.
- Students should be able to find the square root of a number.
- Students should be able to read a topographic map and use its scale.

Ways students can provide evidence of learning:

- Given a pair of coordinates, the student will be able to calculate the distance between the two points using the distance formula.
- Given a pair of coordinates, the student will be able to calculate the distance between the two points using the Pythagorean Theorem.
- Given a problem, the student will be able to solve the problem using either the distance formula or the Pythagorean Theorem.

Common mistakes to be on the lookout for:

- Students may not understand the coordinate plane.
- Students may enter the x and y values incorrectly into the calculator.
- Students may use the incorrect formula for their calculations.
- Students may use incorrect notation when inputting the formulas into the calculator.

Definitions

- Coordinate Plane
- x-axis
- y-axis
- Distance
- Pythagorean Theorem
- Leg
- Hypotenuse
- Contour line
- Scale

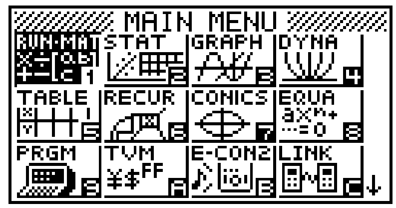
Formulas

Pythagorean Theorem $a^2 + b^2 = c^2$

The following will demonstrate how to enter a set of coordinates into two lists using the Statistics mode of the Casio *fx-9750GII* and graph the results. Then, calculate the distance between the two points using the distance formula and the Pythagorean Theorem.

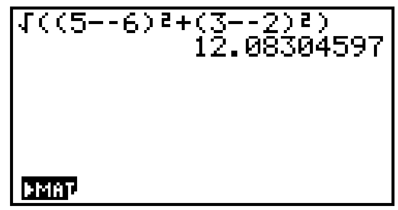
Given the coordinates (-6,-2), and (5, 3), find the distance between the two points using the distance formula and the Pythagorean Theorem.

To calculate the distance using the distance formula:

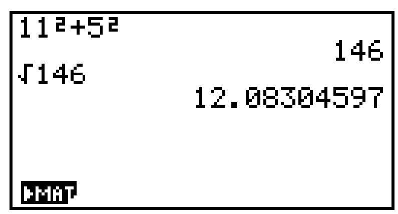


1. From the Main Menu, highlight the RUN•MAT icon and press [EXE] or press [1].

2. Enter the distance formula by pressing [SHIFT] [x^2] [(] [(] [5] [−] [(−)] [6] [)] [x^2] [+] [(] [3] [−] [(−)] [2] [)] [x^2] [)] being sure to use two parentheses at the beginning and end. Press [EXE] to see the answer.



To calculate the distance using the Pythagorean Theorem:

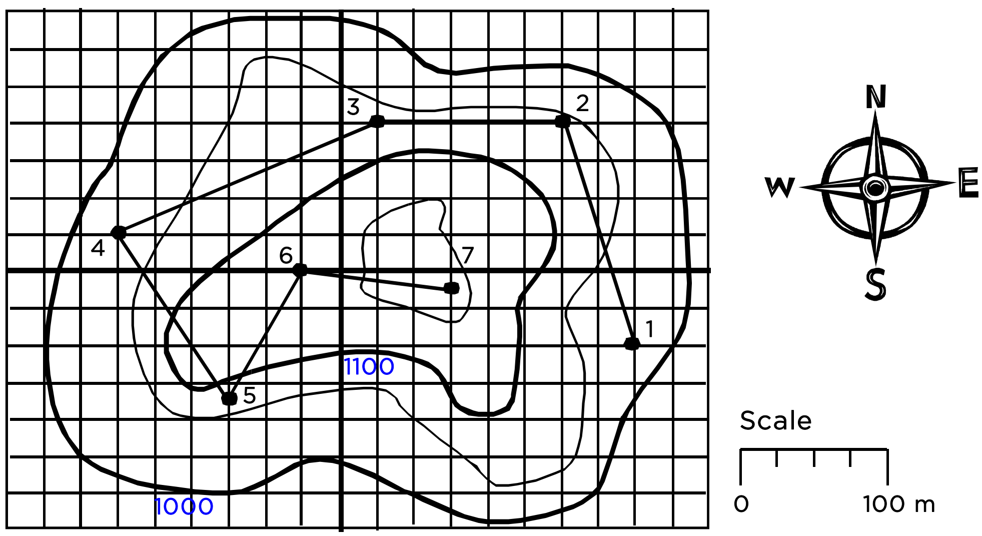


1. Press [AC/ON] to clear the calculator.

2. Using the graph of the line segment, find the change in the horizontal distance (11 units) and the change in the vertical distance (5 units). These are the legs. Using the Pythagorean Theorem enter [1] [1] [x^2] [+] [5] [x^2] [EXE].

3. Find the square root of the value to calculate the distance, by pressing [SHIFT] [x^2] [1] [4] [6] [EXE].

Being able to read a map and interpret what is given is very important in many areas such as surveying, navigation, urban planning, and sports. One of the sports that use maps is orienteering, where participants use a map and compass to travel along a given route through a particular terrain. In this activity, we will use the distance formula and the Pythagorean Theorem to find distances and heights for a given map and we will use this information to read its terrain.



Questions

1. What are the coordinates of Point 1?

2. What are the coordinates of Point 2?

3. What is the horizontal distance between the two points to the nearest tenth?

4. The elevation at Point 1 is 1,015 m and the elevation at Point 2 is 1,068 m. What is the change in elevation between the two points?

5. What is the actual distance between the two points to the nearest tenth?

6. What are the coordinates of Point 3?

7. What are the coordinates of Point 4?

8. What is the horizontal distance between the two points to the nearest tenth?

9. The elevation at Point 4 is 1,008 m and the elevation at Point 3 is 1,057 m. What is the change in elevation between the two points?

10. What is the actual distance between the two points to the nearest tenth?

11. If you were to travel along this route, would you actually travel this distance? Why or why not?

12. What are the coordinates of Point 7?

13. What is the horizontal distance between Point 1 and Point 7 to the nearest tenth?

14. The elevation at Point 7 is 1,185 m. What is the change in elevation?

15. What is the actual distance between the two points?

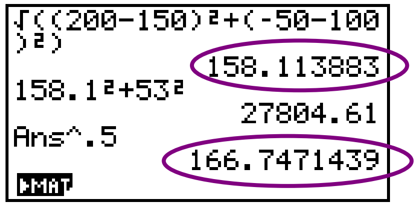
16. Although the distance is shorter, why would someone follow the route instead of going direct?

17. What is the total horizontal distance for the entire route?

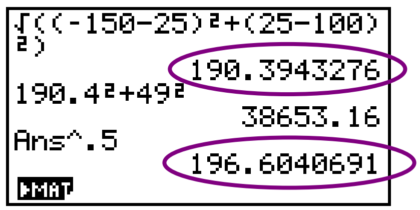
18. Which way would you travel to the top? Explain.

Solutions

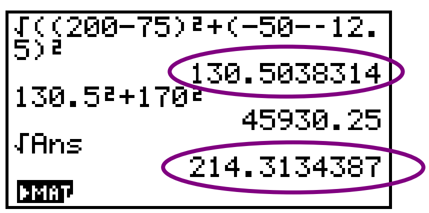
1. (200, -50)
2. (150, 100)



3. 158.1 m
4. 1068 – 1015 = 53 m
5. 166.7 m
6. (25, 100)
7. (-150, 25)
8. 190.4 m



9. 1057 – 1008 = 49 m
10. 196.6 m
11. Answers will vary.
12. (75, -12.5)
13. 130.5



14. 1,185 – 1,015 = 170 m
15. 214.3 m
16. Answers will vary; one reason may be because of the steepness of the terrain.
17. 158.1 + 125.0 + 190.4 + 97.6 + 107.8 + 100.8 = 779.7 m
18. Answers will vary. Longer route may be easier or required. Shorter route if time is an important factor such as a rescue mission or evacuation.

Grade Distribution Teacher Notes

Topic: Normal Probability

NCTM Standards:

- Develop and evaluate inferences and predictions that are based on data.
- Use simulations to explore the variability of sample statistics from a known population and to construct sampling distributions.
- Understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference.

Objective

The students will be able to determine if a set of data is normal and determine the skew of the data using a normal probability plot.

Getting Started

The students will use sample end of the year grade data to explore normal probability in a histogram and normal probability plot. If your students do not have experience with normal probability plots, you may want to review with them.

Prior to using this activity:

- The student should be able to calculate basic statistics.
- The student should be able to graph a normal probability curve.

Ways students can provide evidence of learning:

- Given data, the student should be able construct a normal curve.
- The student should be able to interpret the shape of the normal curve and draw conclusions based on the shape.

Common mistakes to be on the lookout for:

- The students might misread the stem and leaf plot to gather the end of the year grades.

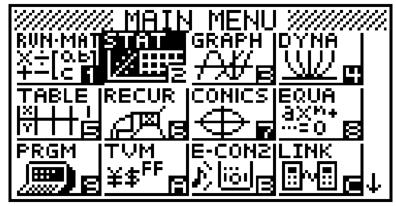
Definitions:

- Sample
- Histogram
- Normal probability
- Normal probability plot
- Conjecture

The following will demonstrate how to enter a set of data into the Casio *fx-9750GII*, graph the data using a histogram and a normal probability plot.

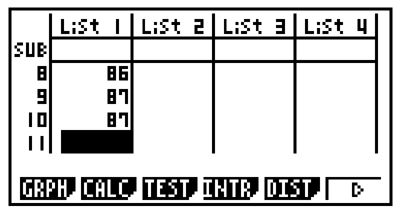
50	55	71	72	77	77	85	86	87	87

To enter the data from the table in the problem:



1. From the Main Menu, highlight the STAT icon and press [EXE] or press [2].

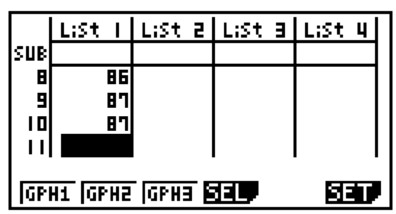
2. To clear previous data lists press [F6] (▷) [F4] (DEL-A) [F1] (Yes).



3. Enter the data by typing each number, pressing [EXE] after each entry.

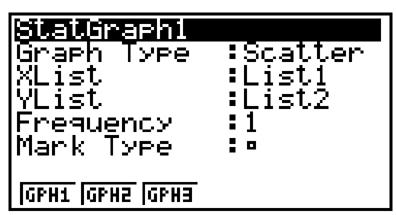
4. The display should look like the screen shot on the right when completed.

To select the type of graph for this data:

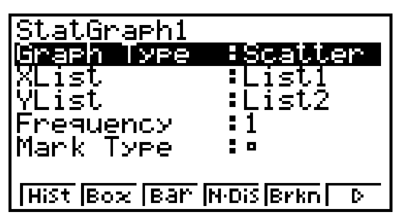


1. Press [F1] (GRPH) and [F1] (GPH1).

2. Select [F6] (SET) to set the type of graph for **StatGraph1**.

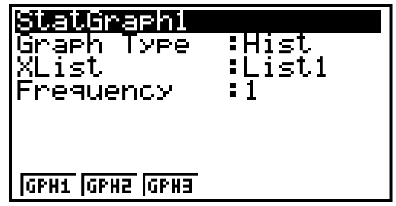


3. Press (▼) to highlight **Graph Type**.

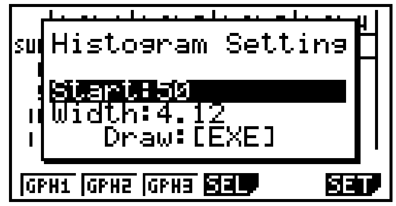


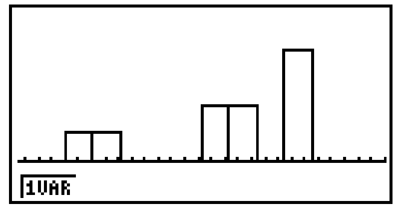
4. Choose [F6] (▷) for more options, then [F1] (Hist).

5. Make sure the correct lists are chosen, then [EXE].



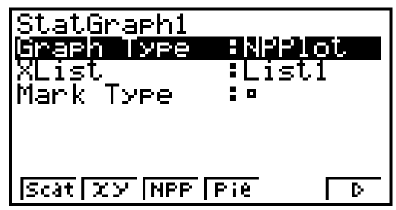
6. Press [EXIT], then [F1] (GPH1) to graph. Next, you can determine the size of your Histogram. Press [EXE].





To plot normal probability:

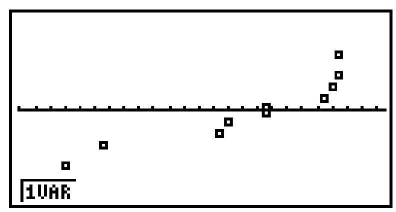
1. Press [EXIT] to return to the previous screen.



2. Select [F6] (SET) to set the type of graph for **StatGraph1**.

3. Press [▼] to highlight **Graph Type**.

4. For a normal probably plot, press [F3] (NPP). Make sure the correct lists are chosen, then [EXE].



5. Press [F1] (GPH1) to view the graph.

Introduction

The data below represents two different years of one teacher's end of course grades. Use the data to answer the following questions.

	Data Set 1	Data Set 2
1		0
2	0 3	
3		5
4	1 3	
5	2 3 3 7 9	2 3
6	0 0 1 1 2 5 5 5 9	
7	1 2 3 4 5 5 7 8	0 2 3 5 6 7 7 8 9
8	1 1 5 5 5 8 8 9 9	0 0 0 1 2 2 2 3 3 4 4 5 5 5 5 6 7 7 7 8 8 8 8 9 9
9	3 4 5 8	0 1 1 2 2 2 2 3 3 4 4 4 4 6 6 6 7 7 7 8
10	0	0 0 0 0 0

Key: 2 | 0 3 = 20, 23

Questions

1. Draw a histogram for each set of data.
   Set 1 Set 2

2. According to the histograms for the two sets of data, are they normally distributed? Why or why not?

   ___

   ___

   ___

3. Create and draw the normal probability plot for each set of data.

Set 1 Set 2

4. What does the normal probability plot tell you about the data? Does this support your answer from question 2? Why or why not?

5. If you were to make a conjecture based on this sample data (set 1) as to how future students would do in the class, what would your conjecture be?

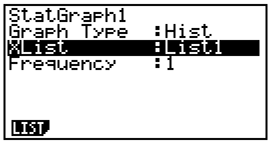
Extension

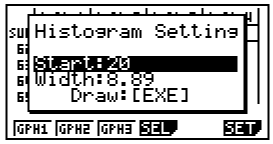
1. Gather data from your state's assessment test, and compare 10 different districts. Create a histogram and a normal probability plot.

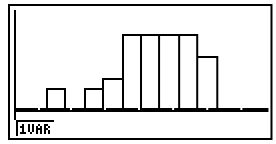
2. What conjecture can you make based on the data?

Solutions

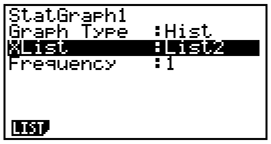
1. Data Set 1

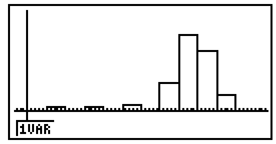






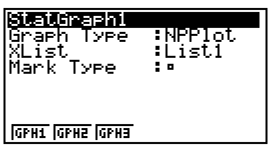
Data Set 2

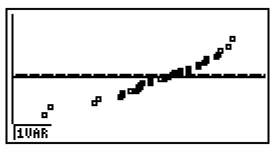




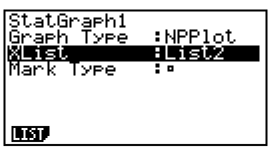
2. Both sets seem to have clusters in one area, which would imply normal distribution. Data Set 1 has more of a bell shaped curve, compared to Data Set 2. From the histograms, Data Set 2 appears skewed with a left tail.

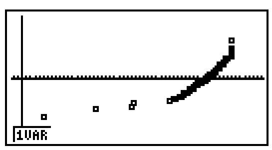
3. Data Set 1





Data Set 2





4. The first data set appears to be more linear, which implies that the data is normally distributed. Yes, Data Set 1 appeared more normally distributed from the histograms.

5. Data Set 1 has a higher rate of failures, so the students should study better.

Extension Solutions

1. Answers will vary.

2. Answers will vary.

Topic: Solving Systems of Equations by Graphing

NCTM Standards:

- Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.

Objective

The student will be able to use the Casio *fx-9750GII* to factor polynomials.

Getting Started

Solving systems of equations by graphing is an easy way for students to see a solution. Knowing the variables being compared and the value assigned to the solution allows the students to make meaning.

Prior to using this activity:

- Students should know how to graph an equation from slope-intercept form.
- Students should know how to algebraically convert an equation written in standard form into slope-intercept form.

Ways students can provide evidence of learning:

- When given a system of equations, students can determine if there is one solution, infinite solutions or no solutions for the system.

Common calculator or content errors students might make:

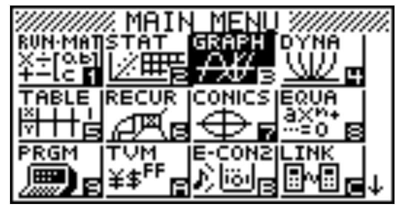
- Students may incorrectly enter the equation into the Casio *fx-9750GII*.
- Students may have trouble moving from the standard form of the equation to the slope-intercept form of an equation.

The following will demonstrate how to enter an equation, display its graph and find the point(s) of intersection on the Casio *fx-9750GII*.

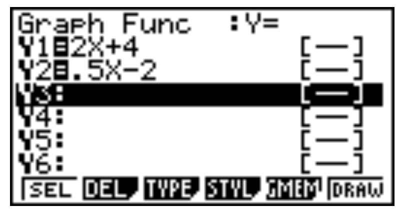
Graph the following systems of equation and give the point(s) of intersection:

$y = 2x + 4$
$y = 0.5x - 2$

To enter an equation into the Graph Function:

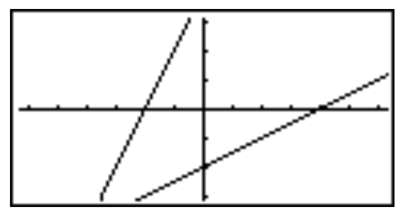


1. From the Main Menu, highlight the GRAPH icon and press [EXE] or [3].

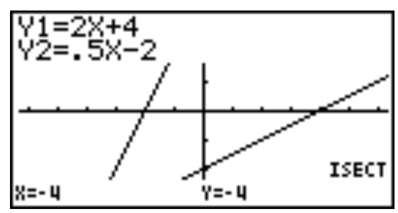


2. For **Y1:** input: [2] [X,θ,T] [+] [4] [EXE].

   For **Y2:** input: [•] [5] [X,θ,T] [−] [2] [EXE].



3. To display the graph, press [F6] (DRAW).



4. To find the point of intersection, press [F5] **(G-Solv)**, then [F5] (ISCT).

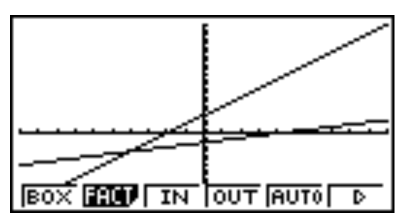
   (Note: Press [EXIT] to return to the initial Graph screen.)

To change the Viewing Window:

1. To manually move the viewing window, use [◀] [▶] [▲] [▼].

2. To display a standard 10x10 grid, press [SHIFT] [F3] **(V-Window)**, then [F3] (STD).

   Press [EXE] twice to display the graph.



3. If the graph still does not display, press [F2] **(Zoom)**, then [F5] (Auto) to automatically fit the graph to the viewing window.

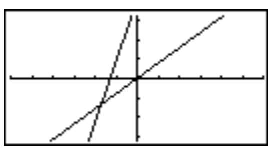
Graphing Systems **Activity**

In Algebra, finding where two lines intersect can often give you information needed to solve a problem. This is called solving systems of equations. When looking at two linear equations, one of three things can happen when you graph the lines: 1) the two lines can be parallel and have no intersection, 2) the two lines can intersect at one point, or 3) the two lines can be represented by the same line and have an infinite number of solutions.

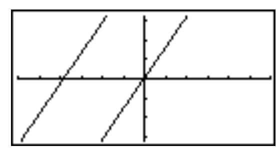
For this section, we are looking at situations where two lines may intersect. The point of intersection lets you see where the two equations would intersect on a graph and when the two equations share a common point.

Questions

1. Look at each of the graphs and determine the number of solutions that satisfy the systems of equations.

a. 

b. 

c. 

Enter the following equations and identify the point(s) of intersection and number of solutions.

2. $y = -x + 5$
   $y = x - 2$

3. $y = -x - 3$
   $y = -x + 3$

4. $y = 2x + 1$
   $y = -x - 2$

Define variables and write a system of equations to accurately model each situation. Solve each system and express your answer in a complete sentence.

5. Find the value of two numbers if their sum is 12 and their difference is 4.

6. Find the value of two numbers if their sum is 13 and their difference is 3.

For the following questions, define your variables, write your equations and give your answer in a complete sentence.

7. The equations 5x + 2y = 48 and 3x + 2y = 32 represent the money collected from a two day school fundraiser. If x represents the cost for each adult ticket and y represents the cost for each student ticket, what is the cost for each adult ticket?

8. The Math Club and the Science Club have planned two different field trips. The Math Club rented and filled 1 van and 6 buses with 372 students. The Science Club rented and filled 4 vans and 12 buses with 780 students. Each van and each bus carried the same number of students. How many students can a van carry? How many students can a bus carry?

9. Mary Ann's school is selling tickets to the spring talent show. On the first day of ticket sales the school sold 3 staff tickets and 9 student tickets for a total of $75. The school took in $67 on the second day by selling 8 staff tickets and 5 student tickets. How much does each type of ticket cost?

10. Linda spent $131 on shirts for her calculator team. The button-down shirts cost $28 and t-shirts cost $15. If she bought a total of 7 shirts, how many of each kind did she buy?

11. Old McDonald has 13 animals in the barn. Some are chickens and some are pigs. There are 40 legs in all. How many of each animal are there?

____

____

____

12. A local copy center has two types of copy machines. The Big Momma can copy 70 pages a minute and the Old Reliable can copy 55 pages per minute. The company owns a total of 14 copy machines and can print 905 pages per minute. How many of each type of press do they have?

____

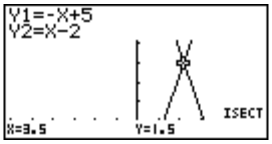
____

____

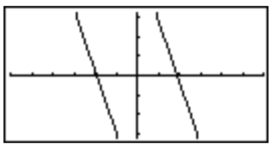
Solutions

1. a. one solution
   b. infinite solutions
   c. no solutions

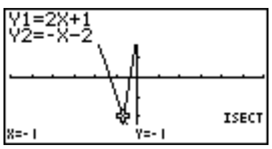
2. The two lines intersect at (3.5, 1.5).



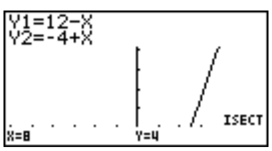
3. The two lines are parallel and do not intersect.



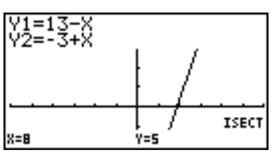
4. The two lines intersect at (–1, –1).



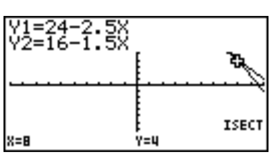
5. The two equations are: x + y = 12 and x – y = 4. The value of x is 8 and the value of y is 4.



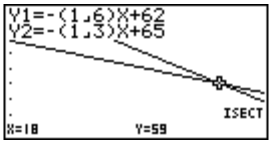
6. The two equations are: x + y = 13 and x – y = 3. The value of x is 8 and the value of y is 5.



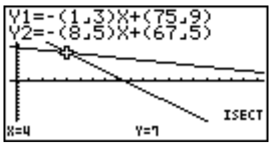
7. Let x represent the number of adult tickets and y represent the number student tickets. The cost for an adult ticket is $8 and the cost for a student ticket is $4.



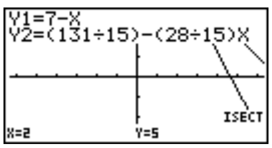
8. Let x represent the number or vans and y represent the number of buses. A van can carry 18 students and a bus can carry 59 students.



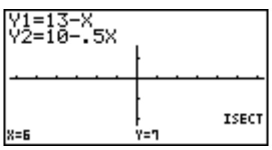
9. Let x represent the number of staff tickets and y represent the number student tickets. The cost for a staff ticket is $4 and the cost for a student ticket is $7.



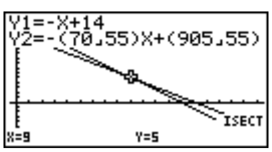
10. Let x represent the number of button-down shirts and y represent the number of t-shirts. Linda purchased 2 button-downs and 5 t-shirts.



11. Let x represent the number of chickens and y represent the number of pigs. Old McDonald has 6 chickens and 7 pigs on his farm.



12. Let x represent The Big Momma copier and y represent The Old Reliable Copier. The copy center has 9 Big Momma's and 5 Old Reliable's.



Graphing with Quadratics — Teacher Notes

Topic: Graphing Quadratic Functions

NCTM Standards:
- Identify functions as linear or non-linear and contrast their properties from tables, graphs or equations.

Objective

The student will be able to use the Casio *fx-9750GII* to graph a quadratic function, determine any roots of the function, and express the functional relationship in a table of values.

Getting Started

Provided that students have a solid understanding of linear functions, they should be able to make an easy transition to the study of quadratic functions. Quadratic functions model many situations in real life and making those connections will help students understand these concepts more easily. This is an important algebraic concept which requires students to spend time investigating and exploring to fully understand.

Prior to using this activity:
- Students should thoroughly understand function notation.
- Students should have a complete understanding of linear functions.
- Students should be able to enter a function into the graph editor window.
- Students should understand the relationship between the data table and its representation on the coordinate plane.

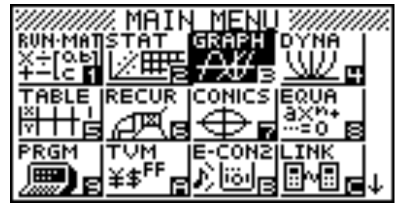
Ways students can provide evidence of learning:
- When given a quadratic function, students can enter the function into the graph editor window and successfully display its graph.
- When given a quadratic function, students can state and explain how the values of *a*, *b*, and *c* affect the shape of the graph.
- When given a quadratic function, students can evaluate the function for a specific value of *x* or *y*.

Common calculator or content errors students might make:
- Students may incorrectly set the viewing window so it doesn't showcase the function.
- Students may incorrectly set an input or table value yielding an error message regarding memory overflow.
- Students may incorrectly enter an exponent thus creating a linear function as opposed to a quadratic function.

The following will demonstrate how to enter and graph a quadratic function, set the appropriate view window, generate a table of values, and calculate a specific value for a quadratic function on the Casio *fx-9750GII*.

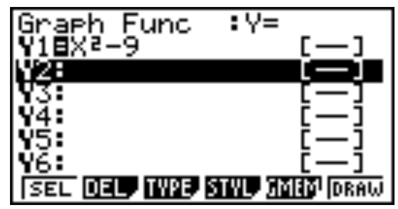
To enter a quadratic function using the Graph Menu:

1. From the Main Menu, highlight the GRAPH icon and press [EXE] or [3].



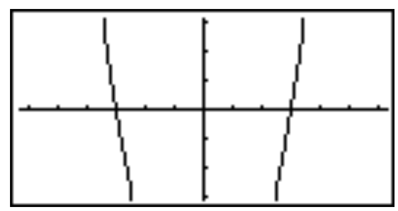
2. With the cursor to the right of **Y1**:, enter the function, $x^2 - 9$, by inputting: [X,θ,T] [x^2] [−] [9] [EXE].

   The equal sign is highlighted to indicate that this function is active.

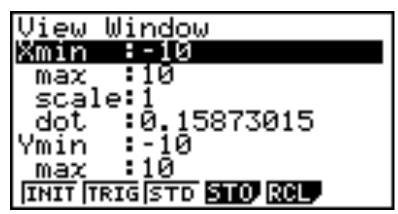


3. To display the graph, press [F6] (DRAW).

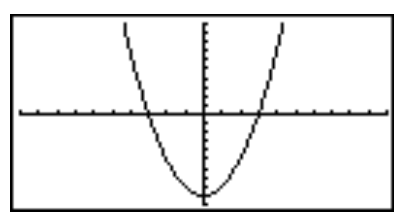
To change the viewing window:



1. The default view is a screen that is squared relative to the rectangular screen. This results in images that are displayed more accurately.
2. To see different parts of the graph, use ◀ ▶ ▲ ▼, to manually move the screen.



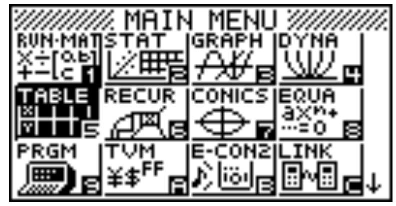
3. To display a standard 10x10 grid, press [SHIFT] [F3] **(V-Window)**, then [F3] (STD). Press [EXE] twice to view the graph.



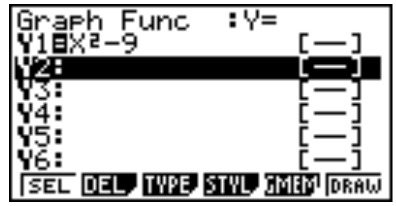
Note: [F4] (STO) allows you to store up to six personalized window settings and [F5] (RCL) allows you to recall those settings.

To input a function and generate a table:

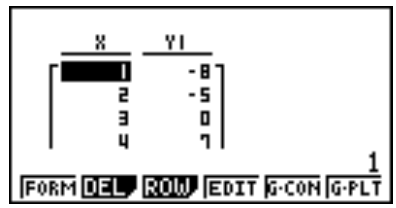
1. From the Main Menu, highlight the TABLE icon and press [EXE] or [5].



2. With the cursor to the right of **Y1**:, enter the function, $x^2 - 9$, by inputting: [X,θ,T] [x^2] [−] [9] [EXE].



3. The equal sign to the right of **Y1:** is highlighted to indicate this function is active. If a function was previously entered into the graph mode, it will automatically transfer to the table mode and vice versa. To deselect a transferred function, press [F1] (SEL).



4. To generate a table of values for the selected function, press [F6] (TABL).

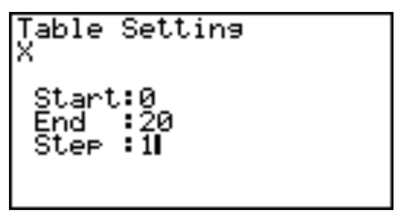
   The default x-values for the table menu is x **starts** 1, **ends** at 5 and **increases** by steps of 1.
   To navigate through the table, use the replay pad.
   ◀ ▶ ▲ ▼.

To adjust the values in a table:

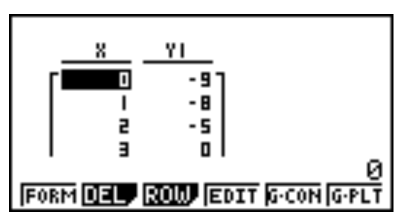
1. Use [EXIT] to return to the TABLE home screen.

   Press [F5] (SET) to change the default table values.

   Enter [0] for the **start** value, [2] [0] for the **end** value and a **step** value of [1], pressing [EXE] after each entry.

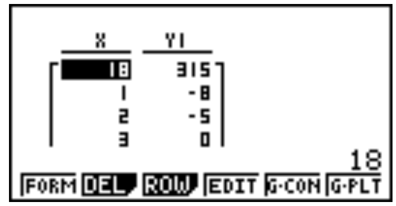


2. Press [EXIT] then [F6] (TABL) to display the table.



3. To display a corresponding y-value for an x-value, highlight any x-value and enter a value.
To display the corresponding y-value when x = 18, input [1] [8] [EXE].

Note: You do not need to change the tables settings; you could just enter all given x-values and create a custom table.

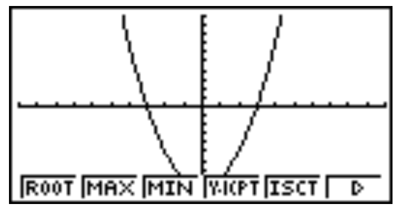


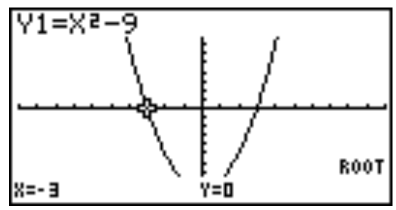
To calculate a specific value for a quadratic function:

1. With the graph displayed, press [F5] **(G-Solv)**.

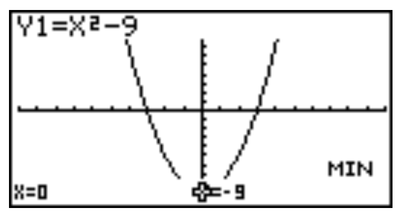
   [F1] (ROOT) will calculate the roots of the function.

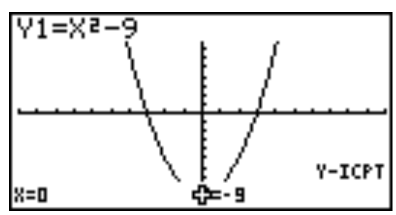
   The left-most root will be displayed first. The quadratic function appears at the top of the screen. This is an easy way to mentally substitute the root to see that the function equals 0. To display the next root, press ▶.





2. Within the [F5] **(G-Solv)** menu, [F2] (MAX) and [F3] (MIN) will display the value of the maximum or minimum value. Since the quadratic function opens up, there is no maximum value. However, press [F2] (MAX) to confirm. The “Not Found” message appears indicating that there is no maximum value. Press [F3] (MIN) to calculate the minimum value.



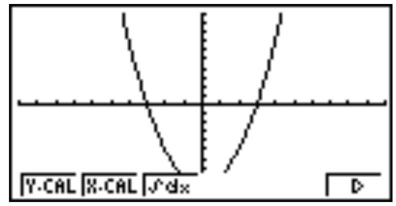


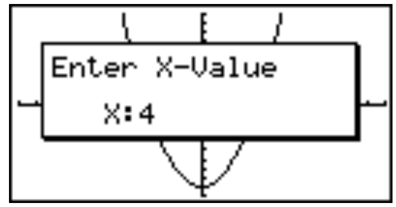
3. Within the [F5] **(G-Solv)** menu, press [F4] (Y-ICPT) to calculate the y-intercept.

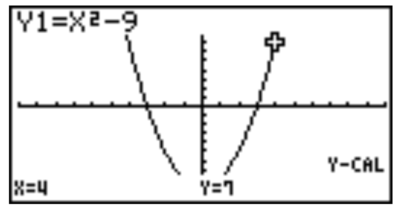
4. Pressing [F6] (▷) will display additional on screen options.

5. F1 (Y-CAL) calculates the value of the function for a specific x-value. To find the y-value when x = 4, input F1 (Y-CAL) 4 EXE.

To calculate an x-value for a specific y-value, press F2 (X-CAL).

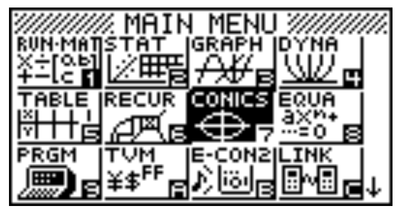


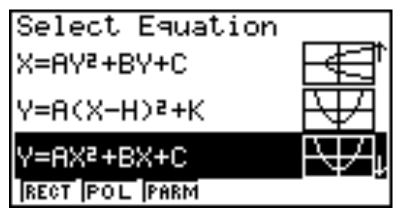


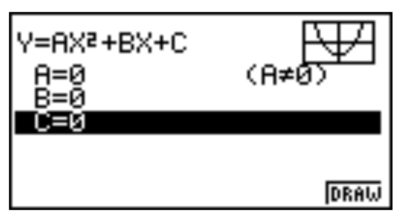


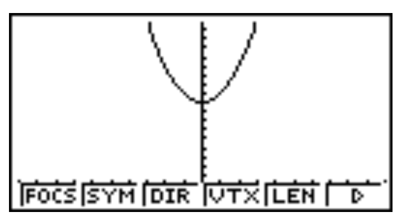
Additional Note:

The Casio *fx-9750GII* has a Conics icon on the Main Menu. This module lists a variety of conic formulas and forms, where you can assign values to the variables. In the Conics mode, F5 **(G-Solv)**, offers a more advanced series of calculations. Take some time to explore this feature as you progress through your study of quadratic functions.









Quadratic functions are written in the form $f(x) = ax^2 + bx + c$. The graph is referred to as a parabola and its graph will either open up or down depending upon the value of a. If $a > 0$, the parabola will open up. If $a < 0$, the parabola will open down. When a quadratic function opens up, its graph looks like a cereal bowl. When a quadratic function opens down, its graph looks like the path of a ball thrown in the air. Quadratic functions model many real life situations like velocity and financial calculations, calculating the height of a football when passed, or determining the maximum or minimum area of a figure.

Questions

1. Given the function $y = x^2 - 5x + 4$, determine the roots.

   ____

2. Use the function in question 1 to determine whether you need to find a maximum or minimum value. Then, determine the value

   ____

3. Use the function in question 1 to evaluate the function when $x = 5$.

   ____

4. How many roots are there in the function $y = 5x^2 + 9x + 4$?

   ____

5. Given the function $y = -x^2 + 4$, create a table of values for the function where $-4 \leq x \leq 4$ and where x represents an integer.

   ____

6. Given the function $y = 8x^2 - 14x - 8$, write the dimensions of a view window which will showcase the minimum and roots of the function.

   ____

7. Graph $y = 3x^2 - 27$. Is this a quadratic function? Why or why not? If this is a quadratic function, what are its roots?

8. Graph $y = 0x^2 + 3x - 2$. Is this a quadratic function? Why or why not? If this is a quadratic function, what are its roots?

9. The sides of a rectangle measure $(x + 6)$ inches and x inches respectively. If the area of the rectangle measures 40 square inches, what are the dimensions of each side? Create a quadratic function which models how to solve this problem by graphing.

10. The sides of a square measure $(x - 7)$ feet. If the area of the square measures 25 square feet, what is the value of x?

Extension

11. Create a quadratic function where the roots of the function are $x = -8$ and $x = 3$.

12. Create a quadratic function that has a maximum value that exists in the second quadrant.

13. Using the conics module, describe what happens when you change the a-value in a quadratic equation.

14. Using the conics module, describe what happens when you change the b-value in a quadratic equation.

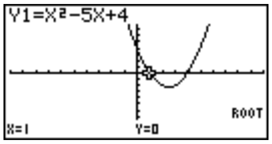
____

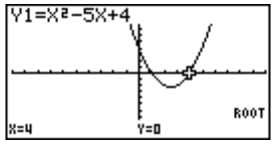
15. Using the conics module, describe what happens when you change the c-value in a quadratic equation.

____

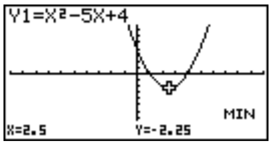
Solutions

1. The roots are x = 1 and x = 4.

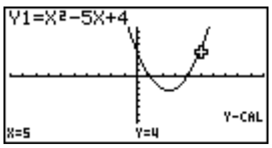




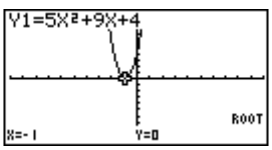
2. You will need to find a minimum value.
The minimum value is (2.5, 2.25).

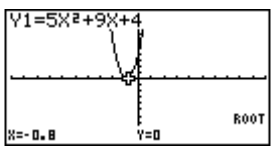


3. When x = 5, y = 4.

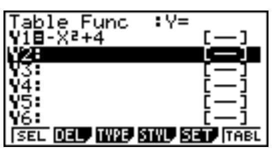


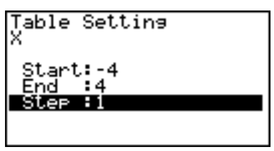
4. There are two roots. They are x = –1 and x = –0.8.

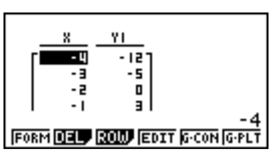


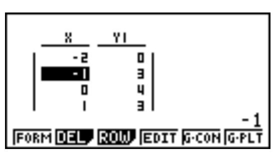


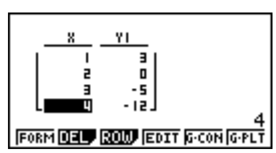
5.



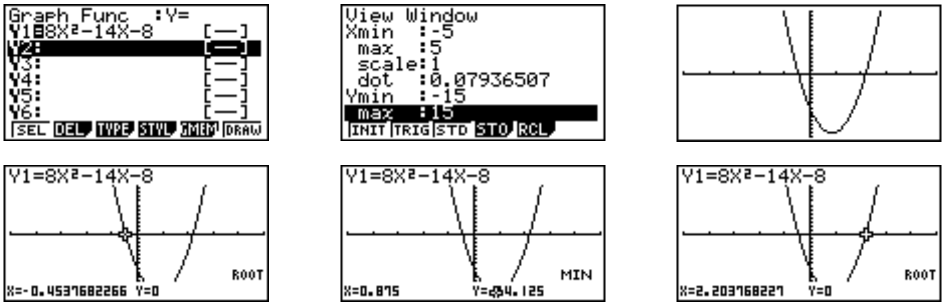




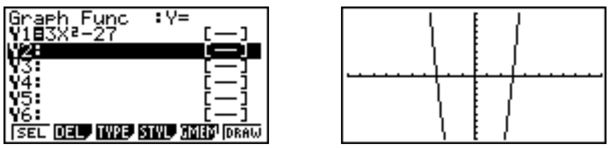




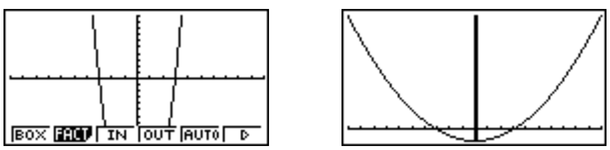
6. Answers may vary regarding the View Window.
The roots are x = 0.4537682266 and x = 2.203768227.
The minimum value is (0.875, 14.125)



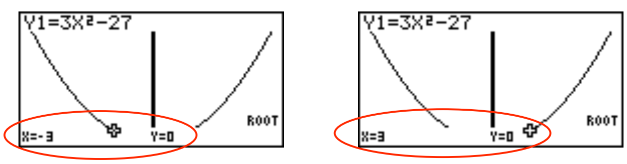
7. $y = 3x^2 - 27$ is a quadratic function because its graph is a parabola and the equation fits the correct form of a quadratic function.



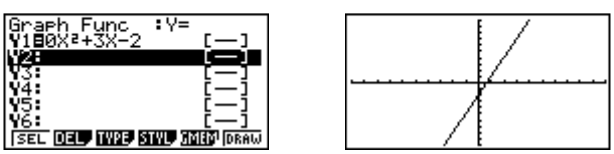
Press F2 (Zoom) once the graph is drawn to automatically have the calculator fit the graph into the View Window.



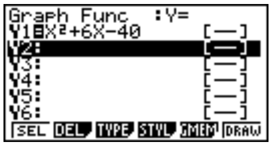
The roots of this function are x = –3 and x = 3.

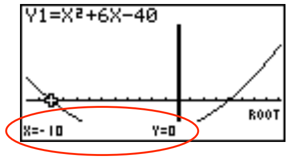


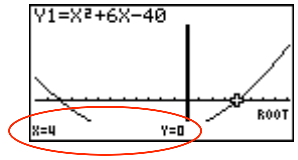
8. This is not a quadratic function because its graph is a straight line and a = 0.



9. $y = x^2 + 6x - 40$
The roots of this function are x = –10 and x = 4.
Since a rectangle's dimension cannot be negative, x = –10 will not create a solution. However, x = 4 does work and makes the dimensions 4 inches by 10 inches.

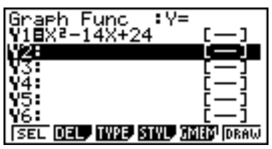


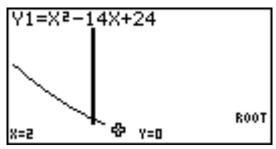


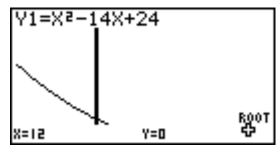


10. (x – 7) (x – 7) =25
$x^2 - 14x + 49 = 25$
$x^2 - 14x + 24 = 0$
$x^2 - 14x + 24 = y$

Since x = 2 will not yield a positive value for the dimension, we must use x = 12 making the square's side equal to 5 feet.







11. $y = x^2 + 5x - 24$

12. Answers may vary. a < 0, b > 0 , c > 0.

13. The a value determines whether the parabola opens up or opens down. If a > 0, the parabola opens up. If a < 0, the parabola opens down.

14. The b value shifts the parabola to the left or right. If b > 0, the parabola shifts to the left. If b < 0, the parabola shifts to the right.

15. The c value shifts the parabola up or down. If c > 0, the parabola shift up. If c < 0, the parabola shifts down.

Topic Area: Patterns and Functions

NCTM standards:

- Understand patterns, relations, and functions by interpreting representations of functions of two variables.
- Use symbolic algebra to represent and explain mathematical relationships.

Objective

Given a set of formulas, the students will be able to solve problems involving centripetal force and centripetal acceleration.

Getting Started

Discuss with the students what is meant by centripetal force and why it is important when objects are traveling around a circle. Include the fact that centripetal acceleration is what keeps an object on a circular path preventing it from going in a straight line. Relate this to traveling around a clover leaf on a highway. Include the meaning of g-force and its effects on an object. Compare 2 g's to someone carrying another person of equal weight on their back to give students a feel of what g-force means. Include the danger that a force of 8 g's causes most people to black out, 20 g's causes bleeding, and 40 g's is fatal.

Prior to using this activity:

Students should know the following formulas used:

- Centripetal force $F = \frac{mv^2}{R}$

  (F: centripetal force; m: mass; v: velocity; R: radius of loop)
- Centripetal Acceleration $a_c = \frac{v^2}{R}$

  (a_c: centripetal acceleration; v: velocity; R: radius of loop)
- Height of loop $h = R + R\sin\theta$

  (h: height of object; R: radius of loop)
- Final velocity $v_1 = \sqrt{v_0^2 - 2(9.8)h}$

  (v_1: final velocity; v_0: initial velocity; h: height of object
- G-Force $g\text{'s} = \frac{a_c}{9.8\, m/s^2}$

  (g's:amount of g force; a_c: centripetal acceleration

Ways students can provide evidence of learning:

Students will be able to explain how:

- different sized loops affect the force needed to travel around a loop.
- different velocities affect the force needed to travel around a loop.
- the increase in acceleration will affect the g-force felt.
- design of a loop needs to include a safety factor involving the g-force felt.

Common mistakes to be on the lookout for:

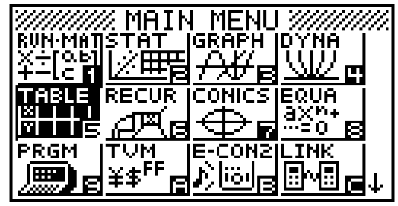
- Students may confuse acceleration and velocity.
- Students may neglect to make sure measurements are in the correct units.

Definitions

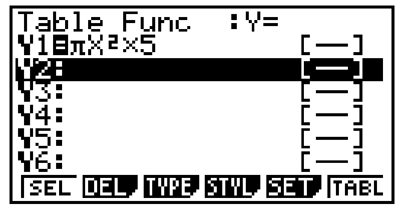
- Centripetal force (F)
- Centripetal acceleration (a)
- Velocity
- G-Force (g's)

The following will demonstrate how to enter the data into the Casio *fx-9750GII* and interpret the results.

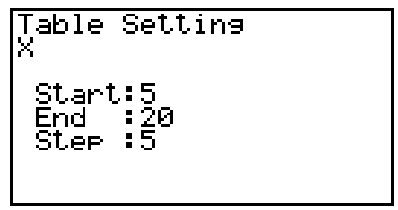
Steps for using the TABLE menu with a single formula:



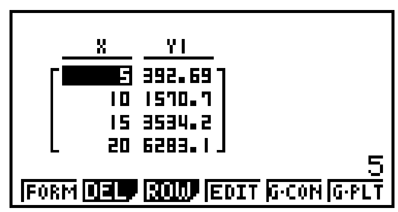
1. From the Main Menu, highlight the TABLE icon and press [EXE] or press [5].



2. For this example, we will use the formula for the volume of a cylinder, V = πr^2h, where the height is 5 cm. To enter the formula in Y1, press [SHIFT] [EXP] [X,θ,T] [x^2] [×] [5] [EXE].



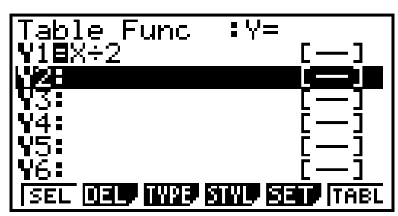
3. Press [F5] (SET) to set up the range for a set of x-values and press [5] [EXE] [2] [0] [EXE] [5] [EXE].



4. To see the table, press [EXIT] [F6] (TABL). For this example, the table shows radius and volume.

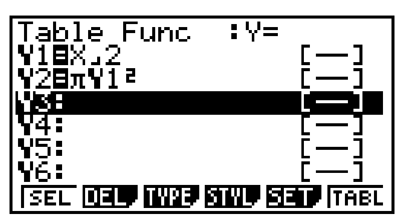
Steps for using the TABLE menu with a sequence of formulas:

1. Press [EXIT] to return to the initial Table screen.



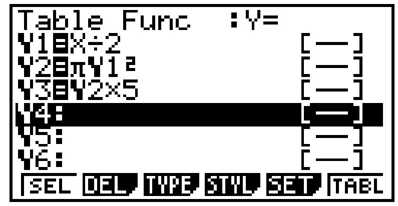
2. To delete a formula, press [F2] (Del) [F1] (Yes).

3. Enter the first formula (in this case, $r = \frac{d}{2}$) by pressing [X,θ,T] [$a\frac{b}{c}$] [2] [EXE].

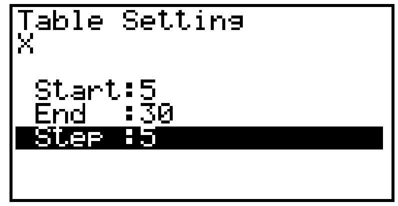


4. Use the results from Y1 (radius) and enter the second formula (A = πr^2), by pressing [SHIFT] [EXP] [F1] (Y) [1] [x^2] [EXE].

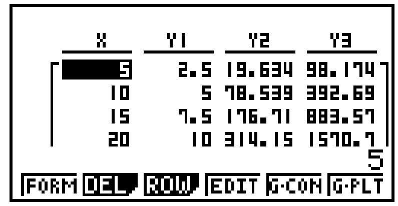
5. Using the results of **Y2** (area), enter the third formula ($V = \pi r^2 h$) to find volume by pressing VARS F4 (GRPH) F1 (Y) 2 × 5 EXE.



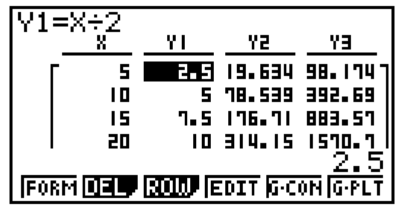
6. To set the range of diameters, press F5 (SET) 5 EXE 3 0 EXE 5 EXE.



7. To see the table, press EXIT F6 (TABL). You now have the diameters, radius, area and volume of a cylinder on one screen.



8. Moving the cursor to another column will show the formula used for that column in the upper left hand side of the screen.



Here we go Loopty Loop Activity

Introduction

Roller coasters, probably one of the most popular rides, have become bigger, faster, and longer since they were first introduced at amusement parks. One addition that has made these rides more thrilling is the use of loops, spirals, and an increase in speed. This increase in speed results in an increase in what is known as the "g" force which, for safety, is limited to no more than 5 g's. In this problem, you will calculate the force necessary to travel around a loop safely.

When a rider goes through a loop, the track applies a force of gravity known as centripetal force. This force is found by using the formula F = $\frac{mv^2}{R}$. In order for enough force to be created to make it around the loop, one of two situations must exist, either the loop must have a large radius or the velocity must be substantial.

Questions

1. Using a velocity of 10 m/sec, a mass of 450 kg, and the TABLE function, enter the formula into Y1 to find the amount of force created by a loop with each of the following radii:

   5 m F = ____________________

   10 m F = ____________________

   15 m F = ____________________

2. Using a radius of 15 m, a mass of 450 kg, and the TABLE function, enter the formula into Y2 to find the amount of force created by a loop with each of the following velocities:

   25 m/sec F = ____________________

   35 m/sec F = ____________________

   45 m/sec F = ____________________

3. Looking at the results in the Table function, what do you notice happens as the size of the radius increases?

   ____

   ____

4. What happens as the velocity increases?

____

____

In order to stay on the track, an inward force called centripetal acceleration is applied to each rider. This force feels as if it is pushing outwards, when in reality, it is preventing the rider from going straight or flying off of the track. This force is found by using the formula $a_c = \frac{v^2}{R}$.

5. Using the velocity of 30 m/sec and the Table function, enter the formula into Y1 to find the amount of centripetal acceleration for the following radii:

5 m a_c = ______________________

10 m a_c = ______________________

15 m a_c = ______________________

6. Using a radius of 15 m, enter the formula into Y2 to find the amount of centripetal acceleration for the following velocities:

25 m/sec a_c = ______________________

35 m/sec a_c = ______________________

45 m/sec a_c = ______________________

The amount of "g" force that a rider experiences at any given point on the loop is found by using a combination of formulas. Theses are height, final velocity, centripetal acceleration and "g" force, g's = $\frac{a_c}{9.8 \, m/s^2}$.

7. Given the initial velocity of 30 m/sec and the radius of the loop to be 15 m, calculate the number of g's for each of the following angles, by inputting the formulas in a sequence in the calculator. Make sure that the calculator is in degree mode.

30° g's = ______________________

45° g's = ______________________

60° g's = ______________________

90° g's = ______________________

8. The rider will experience (g – 1)’s at any point in the top half of the loop and (g + 1)’s at any point at the bottom half of the loop. Calculate the g’s actually felt by the rider while in the top portion of the loop and the bottom portion of the loop.

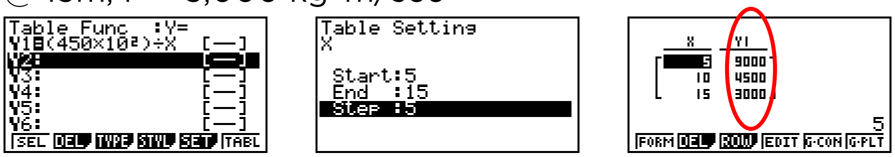
Top:	30°	g’s = ________	Bottom:	30°	g’s = ________
	45°	g’s = ________		45°	g’s = ________
	60°	g’s = ________		60°	g’s = ________
	90°	g’s = ________		90°	g’s = ________

9. Most riders experiencing 8 g’s will black out. Using the results from question 9, will a rider pass out on this ride?

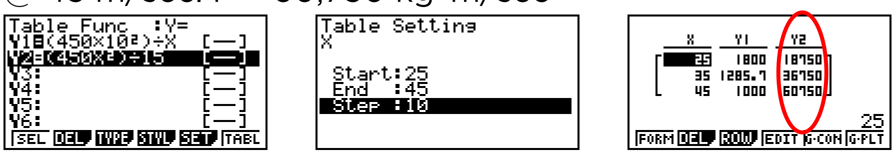
____

Solutions

1. @ 5 m, F = 9,000 kg•m/sec^2
   @ 10 m, F = 4,500 kg•m/sec^2
   @ 15m, F = 3,000 kg•m/sec^2



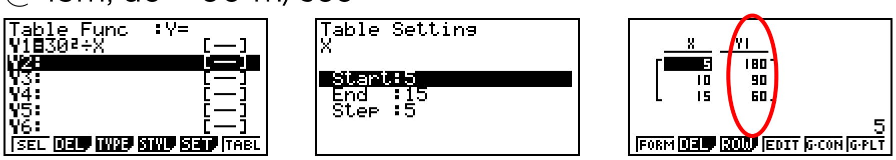
2. @ 25 m/sec, F = 18,750 kg•m/sec^2
   @ 35 m/sec. F = 36,750 kg•m/sec^2
   @ 45 m/sec. F = 60,750 kg•m/sec^2



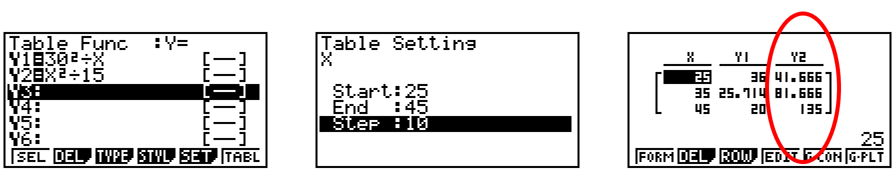
3. As the radius increases, the force decreases.

4. As the velocity increases, the force increases.

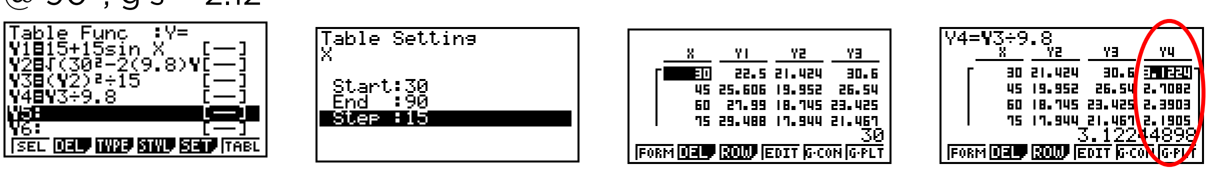
5. @ 5 m, ac = 180 m/sec^2
   @ 10m, ac = 90 m/sec^2
   @ 15m, ac = 60 m/sec^2



6. @ 25 m/sec, ac = 41.67 m/sec^2
   @ 35 m/sec. ac = 81.67 m/sec^2
   @ 45 m/sec. ac = 135.00 m/sec



7. @ 30°, g's = 3.12
   @ 45°, g's = 2.71
   @ 60°, g's = 2.39
   @ 90°, g's = 2.12



8. Top: Bottom:

Top:	Bottom:
@ 30°, g's = 2.12	@ 30°, g's = 4.12
@ 45°, g's = 1.71	@ 45°, g's = 3.71
@ 60°, g's = 1.39	@ 60°, g's = 3.39
@ 90°, g's = 1.12	@ 90°, g's = 3.12

9. No, a rider will not pass out on this ride.

Intelligence Quotient (IQ) Testing — Teacher Notes

Topic Area: Normal Distribution Calculations

NCTM Standards:

- Understand and apply basic concepts of probability.
- Understand the concepts of sample space and probability distribution and construct sample spaces and distributions in simple cases.
- Compute and interpret the expected value of random variables in simple cases.

Objective

The student will be able to calculate percentiles, ranges and outliers for a set of normally distributed data.

Getting Started

In this activity, students will explore the cumulative density function and the inverse cumulative density function for normally distributed data. Students may work in pairs or alone on this activity.

Prior to using this activity:

- Students should be familiar with the normal curve of a "bell curve" and know that it is helpful in analyzing many types of data. The normal distribution makes several assumptions. The data is: continuous, symmetric around the mean, bell-shaped, and the mean equals the mode and the median.

Ways students can provide evidence of learning:

- Given criteria, students will be able to demine the percentage of a population that falls into a specific range of IQ scores.
- Students will be able to determine the IQ score for a specific percentage of population.

Common mistakes to be on the lookout for:

- Students may not understand the impact of outliers on the data.
- Students may misinterpret the answer by not converting to percentage.

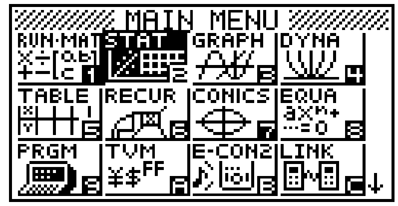
Definitions

- Mean
- Population
- Outliers
- Normal distribution
- Inverse density function
- Standard deviation
- Sample
- Percentile
- Cumulative density function

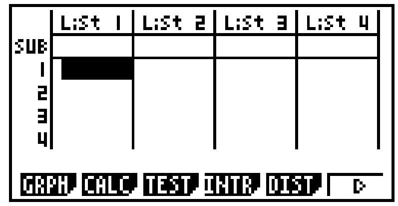
The following will demonstrate how to enter the data into the Casio *fx-9750GII* and interpret the results.

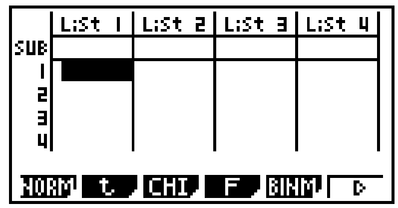
To find the normal cumulative density function:

1. From the Main Menu, highlight the STAT icon and press EXE or press 2.

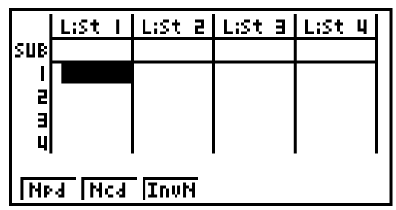


2. Press F5 (DIST), then press F1 (NORM).





3. Press F2 (NCD). The calculator gives you a choice of using a list of values, F1 (List) or inputting your own numbers, F2 (Var) for **Data**.

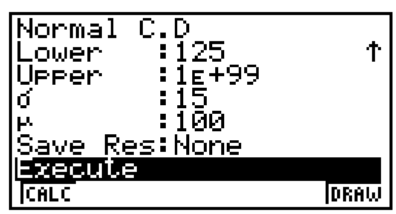


4. With **L. List** highlighted, type in the lower limit and press EXE.

5. With **U. List** highlighted, type in the upper limit and press EXE.

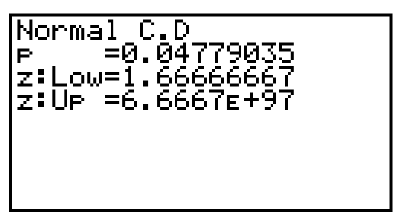
6. With **σ** highlighted, type in the standard deviation and press EXE.

7. With **μ** highlighted, type in the mean and press EXE.

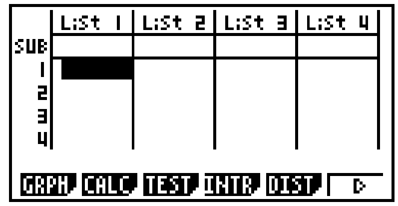


8. With **Execute** highlighted, press F1 (CALC).

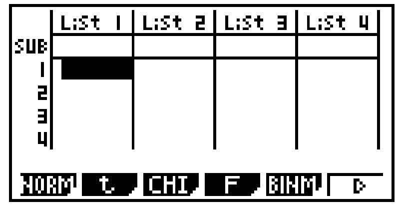
9. The solution is shown as **p =** in decimal form.



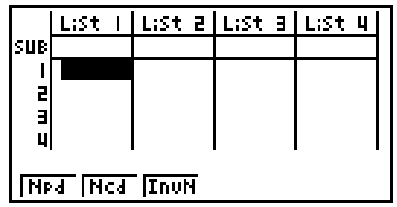
To find the inverse normal density function:



1. Press **EXIT** until you return to the initial STAT screen.

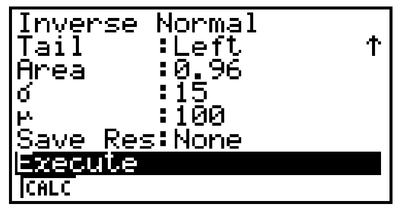


2. Press **F5** (DIST).



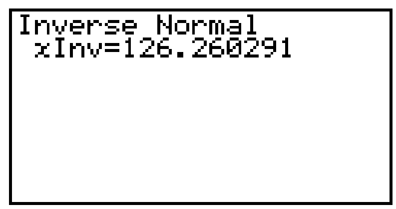
3. Press **F1** (NORM).

4. Press **F3** (InvN).



5. With **Area** highlighted, type in the area value and press **EXE**.

6. With **σ** highlighted, type in the standard deviation and press **EXE**.

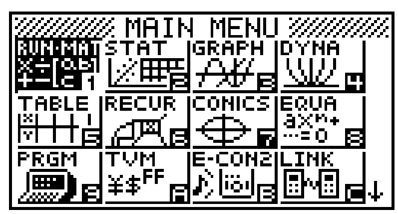


7. With **μ** highlighted, type in the mean and press **EXE**.

8. With **Execute** highlighted, press **F1** (CALC).

9. The answer is shown as *x*Inv.

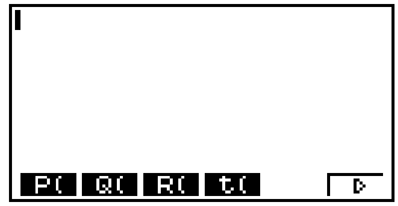
To find IQ scores of outliers:



1. From the Main Menu, highlight the RUN•MAT icon and press **EXE** or press **1**.

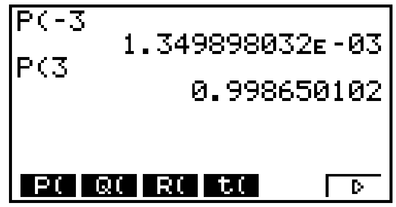
2. Press **OPTN** **F6** (▷) **F3** (PROB).

3. Press [F6] (▷) [F1] (P().



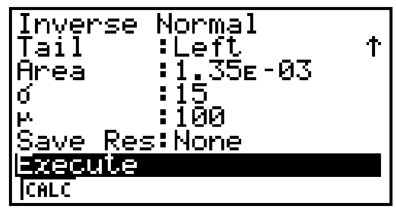
4. Press [(−)] [3] [EXE].

5. Press [F1] (P() [3] [EXE].



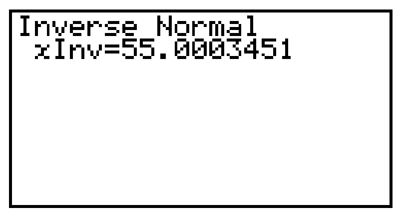
6. Press [MENU] and [2] for the STAT icon.

7. Press [F5] (DIST).



8. Press [F1] (NORM).

9. Press [F3] (InvN).



10. Type in the area value for your problem and press [EXE].

11. Arrow down (▼) to **Execute** and press [F1] (CALC).

Intelligence Quotient (IQ) Testing — Activity

Introduction

The Intelligence Quotient test measures a person's intellectual age versus their actual age. The scores reported from IQ test are distributed normally. The mean is 100 and the standard deviation is 15.

Questions

1. What percent of the population has a score above 125?

   ____

2. What percent of the population has a score below 75?

   ____

3. What percent of the population has a score between 115 and 150?

   ____

4. What IQ score would you need to be place in the 96th percentile?

   ____

5. What IQ score would you need to be placed in the 99th percentile?

   ____

6. Mensa states that to become a member your IQ must be within the top 2% of the populations on an IQ test. What is the minimum IQ score you must have in order to gain membership in Mensa?

   ____

7. What is the range of scores that fall within 2 standards deviations from the mean for IQ?

   ____

8. What would the IQ scores be of any outliers? Explain.

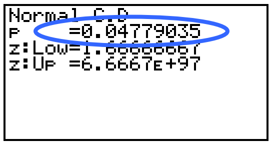
   ____

   ____

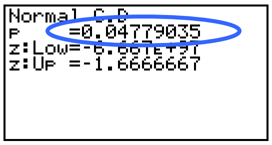
   ____

Solutions

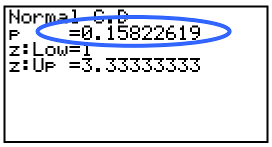
1. 4.78% shown as p in decimal form.



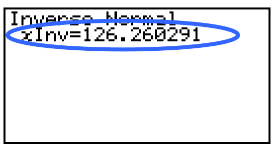
2. 4.78% shown as p in decimal form.



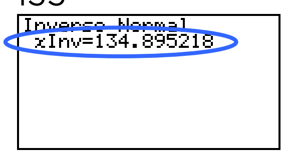
3. 15.82% shown as p in decimal form.



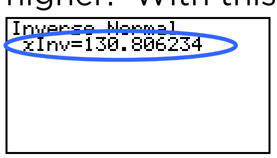
4. 126



5. 135



6. A score in the top 2% means that a person must score in the 98 percentile or higher. With this criterion, the IQ score must be at least 131.



7. 95% of the scores fall within 2 standard deviations from the mean. If the mean is 100 and the standard deviation is 15, the scores will be between (100 - 2•15) and (100 + 2•15) or 70 and 130.

8. Outliers are scores lower than 55 or higher than 145.

Topic: Derivatives

NCTM Standards

- Approximate and interpret rates of change from graphical and numerical data.
- Organize and consolidate their mathematical thinking through communication; communicate their mathematical thinking coherently and clearly to peers, teacher, and others.

Objectives

The student will be able to develop an understanding of the slope of a function that is not just linear. The student will also be able to understand and communicate the visuals connected with the average rate of change and the secant line to a function.

Getting Started

This activity will have students begin to connect the concept of slope and rate of change to the derivative. It also provides an introduction to the concept that the slope of a function extends beyond linear slope, but that using the slope of a line can foster a discussion of average vs. instantaneous rates of change.

Prior to using this activity:

- Students should be able to produce and manipulate graphs of functions manually and with a graphing calculator.
- Students should be able to use the Statistics mode to produce linear and quadratic regression models.
- Students should have a basic understanding of the language of limits.
- Students should have an understanding of what a secant line is.
- Students should have an understanding of slope, as a rate of change.

Ways students can provide evidence of learning:

- Students should be able to produce graphs of functions and communicate symbolically, graphically, numerically and verbally the relationship between the slope of a line, a function and an average rate of change.

Common mistakes to be on the lookout for:

- Students may not being able to relate slope to a real world rate of change concept.
- Students may not being able to communicate the slope as the rate of change of output over input.

Definitions:

- Average Rate of Change
- Derivative
- Linear Function
- Quadratic Regression Models
- Rate of Change
- Secant Line
- Slope
- Tangent

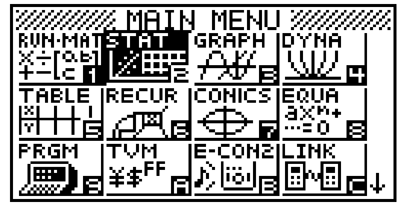
Formula:

$$\text{slope} = m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

The following will demonstrate how to plot points, find a regression, and calculate values on the Casio *fx-9750GII*.

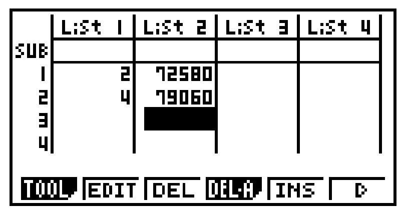
Use the following points (2, 72580) and (4, 79060).

To calculate a regression:

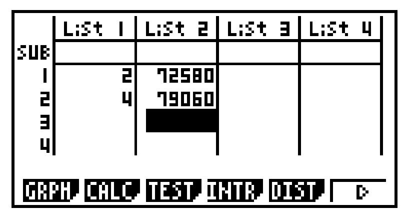


1. From the Main Menu, highlight the STAT icon and press [EXE] or press [2].

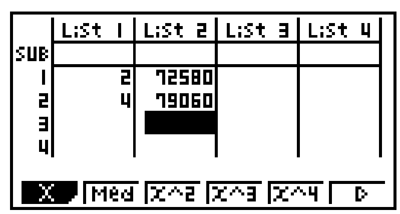
2. To delete any previous data, use the arrows to highlight **List 1** and press [F6] (▷) [F4] (DEL-A) [F1] (Yes). Move the cursor to any other lists that may contain data and follow the same steps to delete the data.



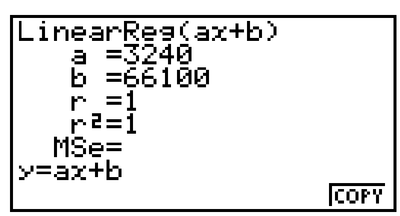
3. Input the x-values into **List 1**, and the y-values into **List 2**. Press [EXE] after each value to enter the data into the calculator.



4. Press [F6] (▷) to return to the original choices across the bottom.



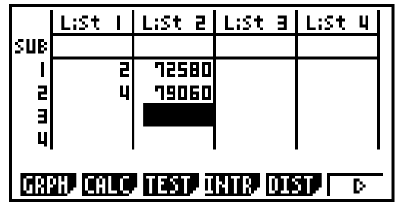
5. To fine the regression of the data, press [F2] (CALC) [F3] (REG). The basic menu choices are linear, med-med line, quadratic, cubic, and quartic. Once again, you will notice [F6] (▷), which tells you that there are more choices.



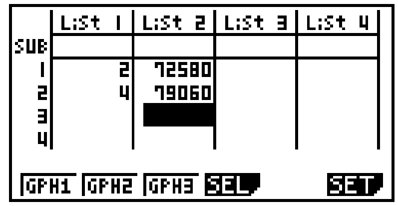
6. After choosing the model you want, the next screen will produce the values and the general model. For this example, press [F1] (X) [F1] (ax+b). The screen displays a slope (a) of 3240 and a y-intercept (b) of 66100.

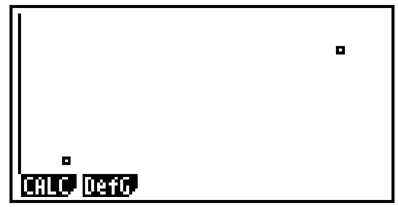
To graph the points, create, and copy the model of a regression:

1. Press EXIT until you return to the initial STAT screen.

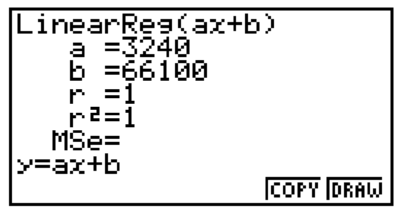


2. To plot the points, press F1 (GRPH) F1 (GRPH1). The calculator will set a proper view window and plot the points.

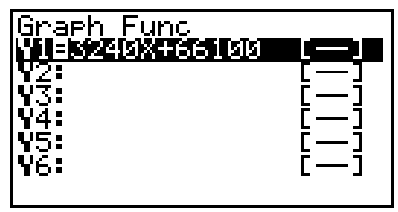




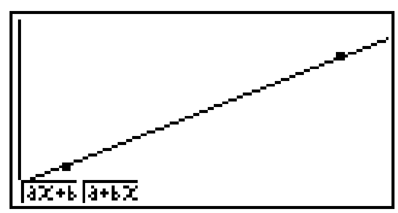
3. To calculate a linear regression, press F1 (CALC) F2 (X) F1 (ax+b). Notice that you are given the same screen as before, except that you now have two choices: F5 (COPY) and F6 (DRAW).



4. Press F5 (COPY) to copy the entire function in a Y= spot. Choose the spot you want the calculator to place the equation and press EXE. You can access the equation now by going to the GRAPH mode from the Main Menu.

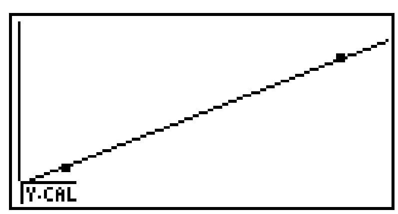


5. Press F6 (DRAW) to view the graph of the regression with the plotted points.

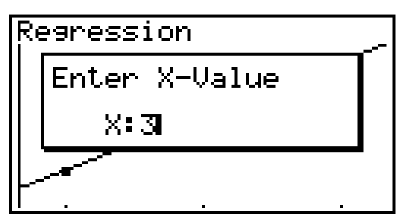


To use Y-CAL to make a prediction:

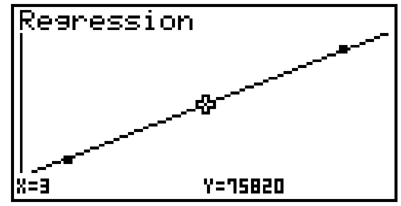
1. Start from the graph of the regression.



2. Press SHIFT F5 **(G-Solv)** F1 (Y-CAL). Enter in the desired value of x and press EXE.



3. The value of the corresponding y-coordinate is displayed in the lower right corner of the screen.



This activity provides an introduction to the concept that the slope of a function extends beyond linear slope, but that using the slope of a line can foster a discussion of average versus instantaneous rates of change, using the Casio *fx-9750GII*.

Questions

1. Calculate the slope of the line connecting the points (2, 5) and (5, 2) using the slope formula. (Remember: slope = $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$)

____

2. Describe the meaning of the slope you just found in terms of input and output.

____

____

____

3. Calculate the slope of the line connecting the points (–1, 8) and (11, –4) using the regression technique.

____

4. What conclusions, if any, can you draw about these 4 points? Explain.

____

____

____

5. Name two other points that would share the same characteristics as these points. Explain your choices.

____

____

____

6. At the end of his first year of employment, Mike's annual salary was $42,000. At the end of his third year of employment, with the same company, Mike's annual salary was $49,000. What conclusion could you draw about the growth of Mike's salary over that period of time? Explain.

____

____

____

7. Given the same data as above, if Mike were to stay with the same company for 10 years, predict what his salary should be at the end of those 10 years. Explain.

____

____

____

8. What if Mike's actual salary after 10 years was $100,000? How does that agree with your prediction from above? How does that compare to the rate of growth you used in your prediction in question 7?

____

____

____

9. Create a good model using the data at the end of the first, third and tenth year salaries. Record the result here and explain why you chose your model.

____

____

____

10. Using your model from question 9, what would you say the average change in Mike's salary was between years 4 and 10? Between years 4 and 9? Between years 4 and 6? Explain how you arrived at your answers.

____

____

____

____

11. How might you estimate the rate that Mike's salary would be growing at the end of the fifth year with the company?

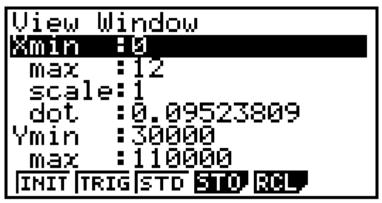
____

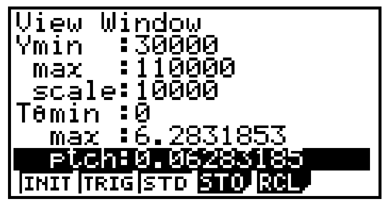
____

12. Find the equation of the secant line connecting the points (4.9, 58767) and (5.1, 59972).

____

13. Graph the model you created in question 9, and the equation of the line from question 12 using the following view window:





Sketch the graph and explain what you see.

____

____

____

14. The derivative of a function at a point (also known as the instantaneous rate of change) is the same as the slope of the line tangent to the function at that point. Based upon your exploration, what could you estimate the derivative of your salary model to be at the end of the fifth year? And how does that translate to Mike's salary growth rate during that same time period?

____

____

____

Extension

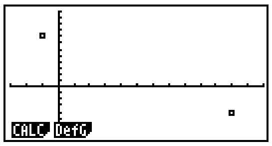
1. Given the function $f(x) = 3x^2 - 2x + 1$, find a good estimate for the equation of the line tangent to $f(x)$ at $x = 2$. Explain your process and how accurate you think you are.

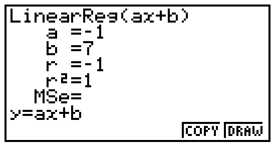
Solutions

1. $\frac{2-5}{5-2}=\frac{-3}{3}=-1$

2. Answers will vary; students should include a mention of the relative change of a decrease in output by 1 for every increase in the input of 1.

3. –1





4. Answers will vary; students should include mention that they have the same slope. Plotting the points using STAT mode will also show that they are on the same line. Care should be taken to point out that just because the slopes are the same does not mean they are on the same line.

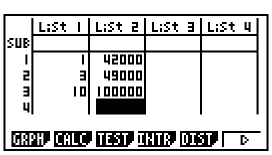
5. Answers will vary; any points that have slope of –1 will work. If the answer to question 4 includes that the points were collinear, then the points chosen should also be on the same line.

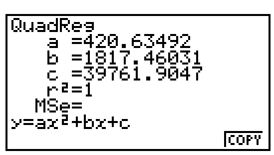
6. Answers will vary; students should mention that his salary has raised an average of $3,500 per year over the time period in question.

7. $73,500. This answer can be found by either using the slope or creating the equation of the line connecting the points (1, 42000) and (3, 49000).

8. The actual salary would be greater, thus the growth rate will have had to have been greater at some point for that to take place. If a numerical comparison of the growth rates is attempted, it must be made clear by the student what they are using to create that new comparison and they should be prompted to explain why they have made that choice.

9. A good answer should be the creation of the quadratic equation that results from using the three points (1, 42000), (3, 49000), and (10, 100000).
$y = 420.634x^2 + 1817.46x + 39761.904$

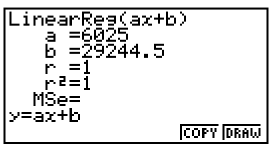




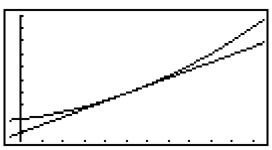
10. Answers will vary. Most students will find the values of the model associated with 4, 6, and 9 using the given value at 10 and find the slopes of the respective secant lines. Some students may begin to suggest that because of the function behavior, these secant values are not good predictors.
Between 4 and 10, the average increase is $7,706 per year.
Between 4 and 9, the average increase is $7,286 per year.
Between 4 and 6, the average increase is $6,024 per year.

11. Answers will vary. Some students might take the growth between 4 and 5 ($5,603) and then 5 and 6 ($6,445) and take the average ($6,024). Some students may begin to estimate closer, perhaps anticipating question 12. Some students may estimate over a closer slope interval. Care should be taken to make sure that the students continue to use slope and discuss rate of change and not simply plug 5 into some model and use the output for the answer.

12. y = 6025x + 29244.50



13.



While answers will vary, a good answer should point out that the parabola is the model of the actual data and the line is the secant line connecting the two given points. Some answers may mention the concept of the tangent line and its close relationship to the curve at the point of tangency.

14. The actual value of the derivative at 5, to the nearest cent is $6,023.81. This is close to the secant line slopes as the student gets closer and closer to 5 from either side. A discussion of limits as it pertains to finding the slope is a good extension.

Extension Solutions

1. Answers will vary, the actual answer is y = 10x – 11. Make sure that students don't simply use the calculator function to create the line without being able to communicate the connection between the slope of the line and the value of the function at x = 2. The student's estimation of accuracy will depend upon their process.

Topic: Graphing Linear Equations

NCTM Standards:

- Explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope.
- Use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships.
- Recognize and generate equivalent forms for simple algebraic expressions and solve linear equations.

Objective

The student will be able to use the Casio *fx-9750II* to graph linear equations.

Getting Started

Students are expected to graph equations in Algebra. Being able to understand how an equation is represented in graphic form is extremely important. For some, this can be a difficult task as students have trouble understanding the relationship between the slope and y-intercepts and how these can affect a graph.

Prior to using this activity:

- Students should know how slope affects a graph.
- Students should know how the y-intercept affects a graph.

Ways students can provide evidence of learning:

- When given an equation, students can display the correct graph.
- When given a situation, students create equations that meet the parameters.

Common calculator or content errors students might make:

- Students may incorrectly enter the equation in the calculator.

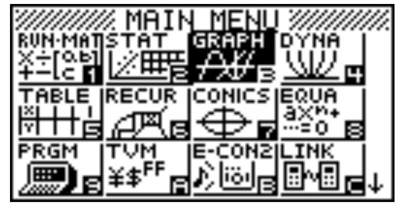
Definitions

- Parallel

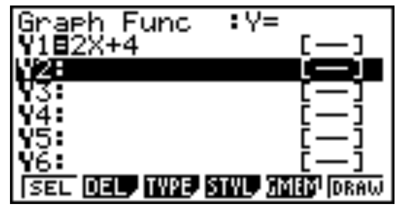
The following will demonstrate how to input an equation into the graph mode of the *Casio fx-9750GII* and display the graph of the equation.

Enter the following equation into the calculator and display the graph: $y = 2x + 4$

To graph an equation:

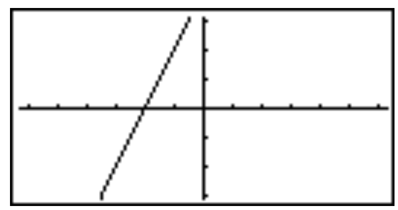


1. From the Main Menu, highlight the GRAPH icon and press EXE or 3.

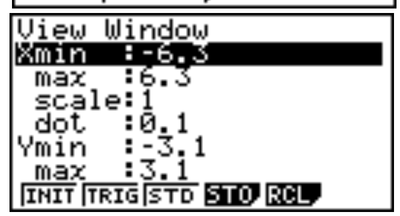


2. To enter the equation, place the cursor to the right of **Y1:** and input the following: 2 X,θ,T + 4 EXE.

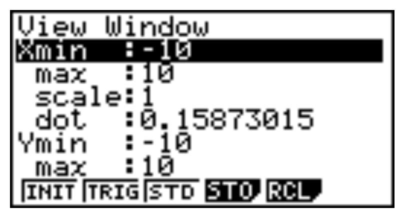
3. To display the graph, press F6 (DRAW).



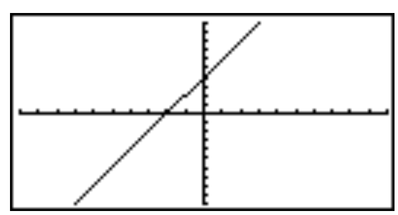
To change the viewing window:



1. The default view for the *fx-9750GII* is a screen that is squared relative to the rectangular screen. This results in images that are displayed more accurately.

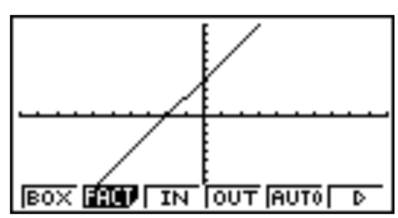


2. To see different parts of the graph, use ◀ ▶ ▲ ▼, to manually move the screen.



3. To display a standard 10x10 grid, press SHIFT F3 **(V-Window)**, then F3 (STD). Press EXE twice to view the graph.

Note: If you do not see the graph, press F2 **(Zoom)** then F5 (Auto) to view the graph.



Is It Straight? Activity

Being able to graph linear equations and knowing how they react when certain aspects are changed is a critical part of Algebra. Linear equations can be written in a variety of forms, each telling us something important about the linear relationship. In this activity, we will explore various slope-intercept equations and learn how they appear in a graph.

Questions

1. Write an equation for a line that has a slope of 4 and a y-intercept of 3, and then sketch the graph.

____

2. Write an equation for a line that has a slope of $\frac{2}{3}$ and a y-intercept of -1, and then sketch the graph.

____

3. Write an equation for a line that has a slope of 1 and a y-intercept of 0, and then sketch the graph.

____

Graph each of the following equations:

4. $y = \frac{1}{2}x - 1$

5. $y = x - 1$

6. $5y = 10x$

7. What happens to your graph when you change the slope to a value greater than 1?

____

8. What happens to your graph when you change the slope to a value less than 1?

____

9. What happens to your graph when the slope is zero?

____

10. What would be an equation that would give you a vertical line?

____

11. What happens to your graph when you add values to the equation?

____

12. What happens to your graph when you subtract values from the equation?

____

Extensions

13. Parallel lines have the same slope, but different y-intercepts. Knowing this information, draw three parallel lines that are 3 units apart in your calculator and list the equations you used.

____

____

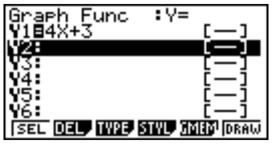
14. A parallelogram is a quadrilateral with two sets of parallel lines. Draw a parallelogram in your calculator and list the equations you used.

____

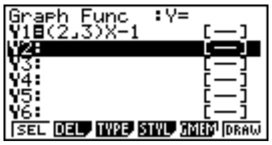
____

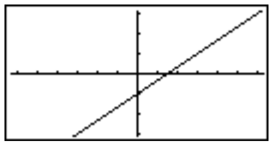
Solutions

1. The equation of the line is y = 4x + 3.

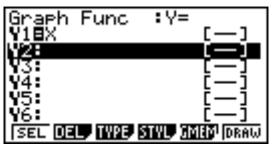


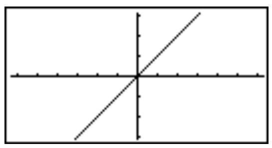
2. The equation of the line is y = $\frac{2}{3}$x – 1.



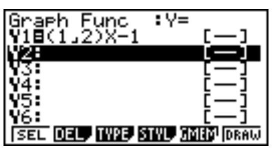


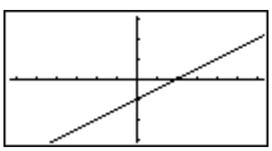
3. The equation of the line is y = x.



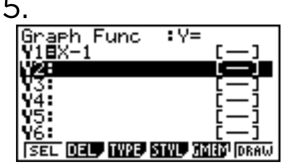


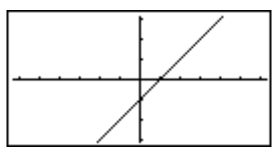
4.





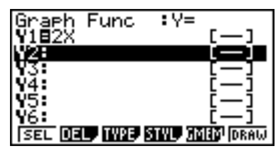
5.

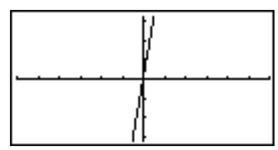




Students will need to simplify 5y = 10x to y = 2x.

6.





7. When the slope of the line is a value greater than 1, the line becomes steeper.

8. When the slope of the line is a value less than 1, the line becomes flatter or less steep.

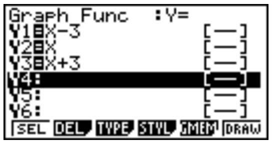
9. When the slope of a line is zero, the line is horizontal.

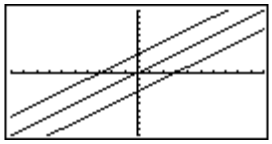
10. Any equation that begins with an “x=”.

11. Adding values to an equation will move the line up the y-axis.

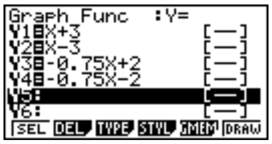
12. Subtracting values from an equation will move the line down the y-axis.

13. Possible answer:





14. Possible answer:



Is the Lottery Worth Playing? Teacher Notes

Topic Area: Data Analysis and Probability

NCTM Standard:

- Understand and apply basic concepts of probability

Objective

Given a description of lottery, calculate the possible arrangements for each winning situation, calculate the probability of winning in each situation, and calculate the expected gain or loss of playing the lottery.

Getting Started

Discuss with students what a lottery is, and how the system works. Include finding combinations of various games of chance such as dealing cards and selecting marbles out of a container. Be sure to include combinations with several possibilities for desired outcomes. Discuss what is meant by probability including what is meant by the number of ways in which an event can occur, and the total of all logical probabilities.

Prior to using this activity:

- Students should find the number of arrangements for a given set of items.
- Students should find a smaller combination out of a larger combination.

Ways students can provide evidence of learning:

- Discuss the results of the activity, and their answers to the questions.
- Discuss how the activity may relate to areas other than the lottery.

Common mistakes to be on the lookout for:

- Student misuse of parenthesis when dealing with equations containing complex numerators and/ or denominators.

Definitions

- Combination

Formulas

Combination: $_{n}C_{r}\,\dfrac{n!}{r!(n-r)!}$

Smaller Combination out of larger combination: $\dfrac{_{n}C_{r}}{_{k}C_{j}\bullet{}_{(n-k)}C_{(r-j)}}$

$$\text{Probability} = \frac{\text{P(E) Number of Ways an Event Can Occur}}{\text{P(S) Totalof AllPossiblities}}$$

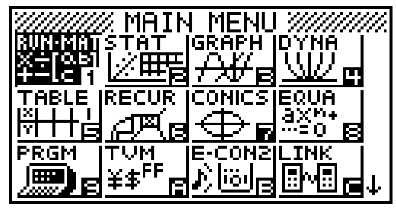
Is the Lottery Worth Playing? "How-To"

Given the formula for calculating combinations, the students will be able to use the Casio *fx-9750GII* to calculate the number of arrangements possible for a set of items and find the probability of selecting a particular item out of the total number of arrangements.

Data: Formulas for Combinations

${}_nC_r$ means $\dfrac{n!}{r!(n-r)!}$ $\qquad \dfrac{{}_nC_r}{{}_kC_j \bullet {}_{(n-k)}C_{(r-j)}}$ $\qquad P(\text{occurance}) = \dfrac{P(E)}{P(S)}$

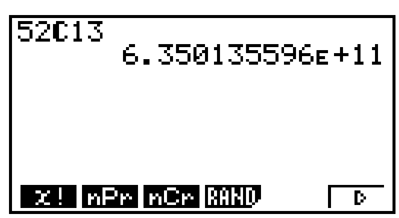
To find combinations:



1. From the Main Menu, highlight the RUN•MAT icon and press [EXE] or press [1].

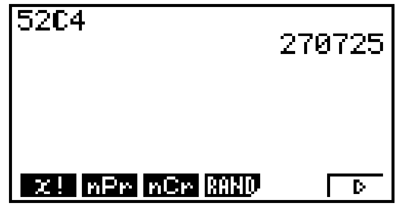
2. Press [OPTN] [F6] (▷) [F3] (PROB) to get to the Probability section on the calculator.

3. To calculate the number of possible arrangements of selecting thirteen cards from a standard deck of 52 cards without replacing the cards. The combination that will give the number of possible arrangements is ${}_{52}C_{13}$.



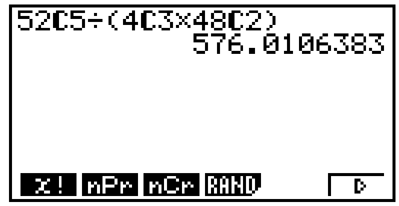
4. To calculate this, press [5] [2] [F3] (nCr) [1] [3] [EXE].

5. To calculate the combination of selecting an ace out of a standard deck of cards, press 5 2 F3 (nCr) 4 EXE.



6. The total number of items is 52 of which the subset of items is 4, since there are four aces in a deck of cards.

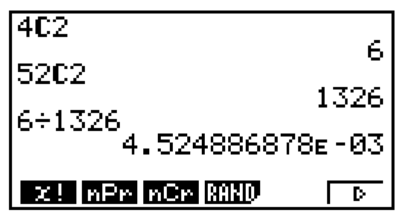
You are selecting five cards of which you want 3 of them to be aces. The formula would look like this $\frac{{}_{52}C_5}{{}_4C_3 \bullet {}_{48}C_2}$.



To enter this into the calculator, press 5 2 F3 (nCr) 5 ÷ (4 F3 (nCr) 3 × 4 8 F3 (nCr) 2) EXE.

An alternative way to find combinations:

1. Press 4 F3 (nCr) 2 EXE.



2. Press 5 2 F3 (nCr) 2 EXE.

3. The probability of selecting two aces from the deck is found by pressing 6 ÷ 1 3 2 6 EXE.

Many states now have some sort of lottery that is used to improve revenue. The games of chance usually involve the winner matching a set of randomly selected numbers from a set of numbers. The payoff is seen as a quick way to earn easy money, but does not always happen. In this activity, you will explore different types of lotteries, what the chances are of winning a lottery, and the actual cost to play the lottery.

Questions

A certain state legislature is thinking of putting in a lottery in which the winner would match six numbers out of a set of numbers correctly. They are looking at several possibilities.

The first choice is to have the winner match six numbers randomly selected without repetition from the set of given numbers in which the order does not matter.

1. Calculate the number of possible arrangements for numbers 1 to 30.

   ____

2. Calculate the number of possible arrangements for numbers 1 to 40.

   ____

The second choice is to have several opportunities to win. The winner would match all six numbers for the top prize, 5 out of 6 numbers for a second prize, and 3 out of 6 numbers for the third prize.

3. Calculate the possible arrangements for 1 to 45 for the following prizes:

   1st Prize: ____________________

   2nd Prize: ____________________

   3rd Prize: ____________________

4. Calculate the possible arrangements for 1 to 50 for the following prizes:

   1st Prize: ____________________

   2nd Prize: ____________________

   3rd Prize: ____________________

5. Which of the above choices would you select to use for the state lottery? Why?

____

____

Before making a decision, the state legislature wants to know the probability of winning for each of the choices.

6. Calculate the probability for 1 to 30.

____

7. Calculate the probability for 1 to 40.

____

8. Calculate the probability for 1 to 45 for the following prizes:

1st Prize: ______________________

2nd Prize: ______________________

3rd Prize: ______________________

9. Calculate the probability for 1 to 50 for the following prizes:

1st Prize: ______________________

2nd Prize: ______________________

3rd Prize: ______________________

The second choice was the chosen method. The 1st prize award will be $1,000,000, the 2nd prize award will be $10,000, and the 3rd prize award will be $100. The expected value or gain for each prize is the respective probability times the associated prize.

10. Calculate the expected value for the following prizes using numbers 1 to 45.

1st Prize: ______________________

2nd Prize: ______________________

3rd Prize: ______________________

11. What is the total expected gain for this choice?

____

12. Calculate the expected value for the following prizes using numbers 1 to 50.

1st Prize: ______________________

2nd Prize: ______________________

3rd Prize: ______________________

13. What is the total expected gain for this choice?

____

Extensions

The expected value of a lottery ticket is the result of subtracting the value of the ticket from the expected gain from the lottery.

1. Calculate the value of a ticket for number 1 to 45 with the following values:

$1 value: ______________________

$2 value: ______________________

$3 value: ______________________

2. Calculate the value of a ticket for number 1 to 50 with the following values:

$1 value: ______________________

$2 value: ______________________

$3 value: ______________________

3. Which game would benefit the state the most in generating revenue?

____

4. Which game would benefit the player the most?

____

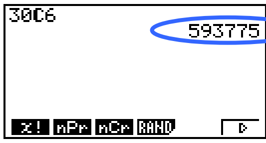
5. Which game and ticket price should be used? Why?

____

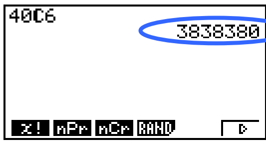
____

Solutions

1. $_{30}C_6$ = 593,775 possible arrangements



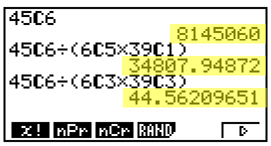
2. 40C6 = 3,838,380 possible arrangements



3. 1st Prize: 45C6 = 8,145,060

2nd Prize: $\dfrac{45C6}{6C5 \bullet 39C1} = 34{,}808$

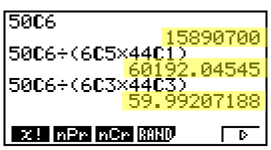
3rd Prize: $\dfrac{45C6}{6C3 \bullet 39C3} = 45$



4. 1st Prize: 50C6 = 15,890,700

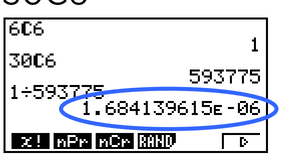
2nd Prize: $\dfrac{50C6}{6C5 \bullet 44C1} = 60{,}192$

3rd Prize: $\dfrac{50C6}{6C3 \bullet 44C3} = 60$

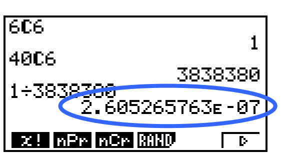


5. Answers will vary.

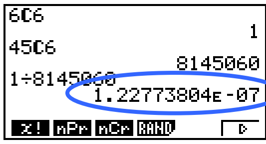
6. $\dfrac{6C6}{30C6} = 1.684 \times 10^{-6}$ = 1 in 593,775



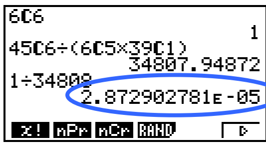
7. $\dfrac{6C6}{40C6} = 2.605 \times 10^{-7}$ = 1 in 3,838,380



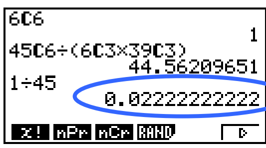
8. 1st Prize: $\frac{6C6}{45C6}$ = 1.228 x 10^{-7} = 1 in 8,145,060



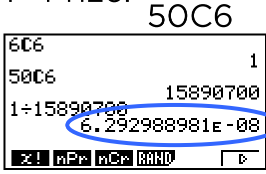
2nd Prize: $\dfrac{\frac{6C6}{45C6}}{6C5 \bullet 39C1}$ = 2.873 x 10^{-5} = 1 in 34,808



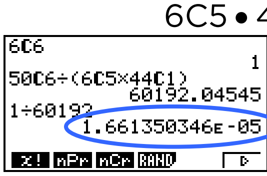
3rd Prize: $\dfrac{\frac{6C6}{45C6}}{6C3 \bullet 39C3}$ = 0.022 = 1 in 45



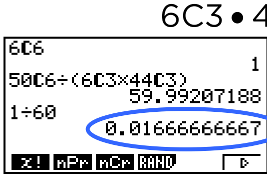
9. 1st Prize: $\frac{6C6}{50C6}$ = 6.293 x 10^{-8} = 1 in 15,890,700



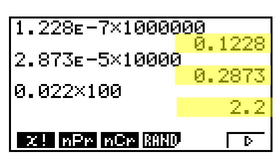
2nd Prize: $\dfrac{\frac{6C6}{50C6}}{6C5 \bullet 44C1}$ = 1.661 x 10^{-5} = 1 in 60,192



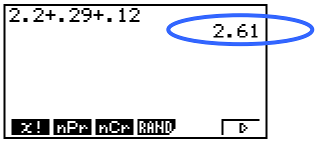
3rd Prize: $\dfrac{\frac{6C6}{50C6}}{6C3 \bullet 44C3}$ = 0.017 = 1 in 60



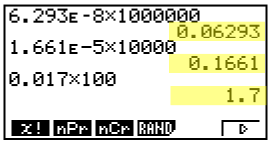
10. 1st Prize: 1.228 x 10^{-7} • \$1,000,000 = \$0.12
2nd Prize: 2.873 x 10^{-5} • \$10,000 = \$0.29
3rd Prize: 0.022 • \$100 = \$2.20



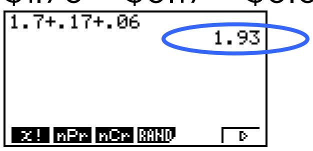
11. $2.20 + $0.29 + $0.12 = $2.61



12. 1st Prize: 6.293 x 10^{-8} • $1,000,000 = $0.06
2nd Prize: 1.661 x 10^{-5} • $10,000 = $0.17
3rd Prize: 0.017 • $100 = $1.70

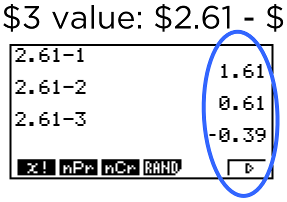


13. $1.70 + $0.17 + $0.06 = $1.93

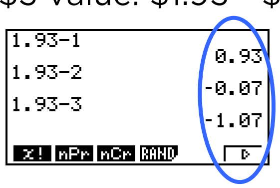


Extension Solutions

1. $1 value: $2.61 - $1 = $1.61
$2 value: $2.61 - $2 = $0.61
$3 value: $2.61 - $3 = -$0.39



2. $1 value: $1.93 - $1 = $0.93
$2 value: $1.93 - $2 = -$0.07
$3 value: $1.93 - $3 = -$1.07



3. 1 to 50 with a $3 ticket.

4. 1 to 45 with a $1 ticket.

5. Answers will vary.

Keeping up the Pressure Teacher Notes

Topic Area: Patterns and Functions – Algebraic Thinking

NCTM Standard:

- Understand patterns, relations, and functions by interpreting representations of functions of two variables, and use symbolic algebra to represent and explain mathematical relationships.

Objective

Given a set of data, the students will be able to use the GRAPH Menu and the TRACE Function to solve problems involving water pressure used by firefighters.

Getting Started

Discuss with the students what is meant by force as determined in Newton's Third Law which states that for every action there is an equal and opposite reaction. Relate this to the formulas used in the activity.

Prior to using this activity:

- Students should be able to enter various formulas into GRAPH Menu and use the TRACE Function to find specific x- and y-values.

Ways students can provide evidence of learning:

- The student will be able to discuss the results of the activity and justify their answers to the questions.
- The student will be able to discuss how the formulas relate to their corresponding graphs used to answer questions.

Common calculator or content errors students might make:

- Students may need to adjust the window prior to tracing so that they can follow the pointer using the trace function

Definitions

- Nozzle Pressure
- Nozzle Reaction
- Friction Loss
- Pump Discharge Pressure

d = nozzle bore diameter
NP = nozzle pressure
C = coefficient of friction
GMP = Gallons per Minute
L = length of hose

Formulas

Nozzle Reaction: $NR = (1.57)(d^2)(NP)$

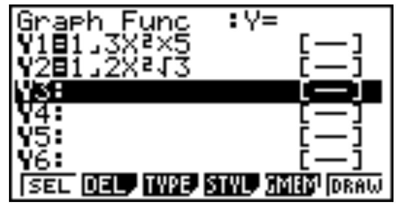
Gallons Per Minute: $GPM = (29.7)(d^2)(\sqrt{NP})$

Friction Loss: $FL = (C)(0.01 \bullet GPM)^2(0.01 \bullet L)$

The following will demonstrate how to enter a given formula into the graph menu of the Casio *fx-9750GII*, graph the data, and trace along the graph to find x- and y-values.

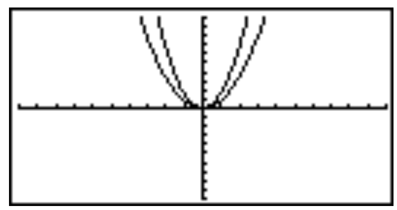
$V = \frac{1}{3}B^2H$; where H = 5 and $A = \frac{1}{2}B^2\sqrt{3}$

Steps for using the GRAPH Menu:



1. Press [MENU] to access the Main Menu, and then press [3] to select the GRAPH Icon.

2. Enter the first formula into **Y1:** by entering: [1] [a b/c] [3] [X,θ,T] [x²] [×] [5] [EXE].

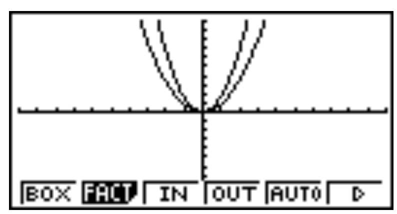


3. Enter the next formula into **Y2:** by entering: [1] [a b/c] [2] [X,θ,T] [x²] [SHIFT] [x²] [3] [EXE].

4. Notice the equal signs are highlighted, this indicates they are selected to be graphed.

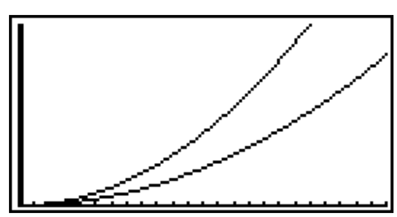
5. Press [F6], both of the graphs will display.

Steps for changing the size of the View Window:



1. If you are unable to view the graph, or if the graph appears too small; pressing [SHIFT] [F2] **(Zoom)** and [F5] (AUTO), will automatically fit the graph to the viewing window.

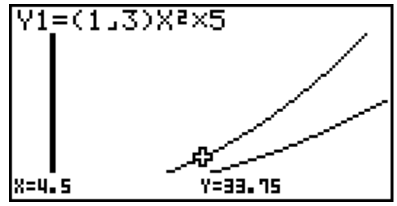
2. To change the view window so that only the first quadrant shows; press [F3] **(V-Window)** and enter [(−)] [1] [EXE] for Xmin, [▼] twice and enter [(−)] [1] [EXE] for Ymin.



3. Press [EXE] twice to display the graph.

Steps for using the Trace Function:

1. Press SHIFT F1 **(Trace)**. A small + cursor will be displayed on the graph and the x- and y-coordinates will be displayed along the bottom of the screen.



2. Press ◀ ▶ to move the cursor along the graph and ▲ ▼ to change the graph being traced.

When a building or property is burning, people call upon the fire department to put out the fire. These firefighters are a group of highly trained personnel, each having a specific duty during the incident. One job, the Driver/Operator, is responsible for initiating and maintaining the correct water pressure so that the proper amount of water is delivered to the firefighters operating the nozzle. In this activity, you will be asked to calculate the gallons per minute (GPM) for specific nozzle diameters, the amount of friction loss (FL) in pounds per square inch (psi), the nozzle reaction (NR), and the pump discharge pressure (PDP). These are all basic calculations skills that are required when a firefighter is trained as a Driver/Operator.

In this activity, you will find the NR for a smoothbore nozzle operating at both 50psi and 80 psi and the GPM for both. In addition, you will calculate the FL using given information.

Questions

Nozzle reaction is the force pushing back against a nozzle when water is forced through a nozzle tip. One such nozzle used in firefighting is called a smoothbore which may be handheld or attached to a mechanical device.

1. Handheld smoothbore nozzles generally operate at 50 psi at the nozzle (NP). Find the nozzle reaction of the nozzle with a ½” diameter.

   ____

2. Find the NR of the nozzle with a ¾” diameter.

   ____

3. Find the NR of the nozzle with a 1” diameter.

   ____

4. Find the NR of the nozzle with a 1 ¼” diameter.

   ____

5. Smoothbore nozzles that create master streams generally operate at 80 psi at the nozzle (NP). Find the nozzle reaction of the nozzle with a 1 ½” diameter.

   ____

6. Find the NR of the nozzle with a 2” diameter.

7. Find the NR of the nozzle with a 2 ½” diameter.

8. Find the NR of the nozzle with a 3” diameter.

9. Look at the graph used for questions 1 – 4. What is the shape of the graph?

10. What does the graph tell you about the amount of nozzle reaction as the diameter increases?

11. How does the graph for questions 1 – 4 compare to the graph used for questions 5 – 8?

12. How do your answers help to explain why the larger diameter nozzles are attached to a mechanical device before they are used?

The amount of water used, or gallons per minute, varies due to the diameter of the nozzle tip.

13. Calculate the amount of water discharged at 50 psi through a ½’’ diameter nozzle tip.

14. Calculate the amount of water discharged at 50 psi through a ¾'' diameter nozzle tip.

____

15. Calculate the amount of water discharged at 50 psi through a 1'' diameter nozzle tip.

____

16. Calculate the amount of water discharged at 50 psi through a 1 ¼'' diameter nozzle tip.

____

17. Calculate the amount of water discharged at 80 psi through a 1 ½'' diameter nozzle tip.

____

18. Calculate the amount of water discharged at 80 psi through a 2'' diameter nozzle tip.

____

19. Calculate the amount of water discharged at 80 psi through a 2 ½'' diameter nozzle tip.

____

20. Calculate the amount of water discharged at 80 psi through a 3'' diameter nozzle tip.

____

21. Compare the graph used in questions 13 – 16 and the graph used in questions 17 – 20. What do these graphs show about the amount of water that is being expelled through the nozzle?

____

Friction loss (FL) is based on the flow rate, hose length, and friction loss coefficient associated with the diameter of the hose. A $1\frac{3}{4}$" hose has a friction loss coefficient of 15.5 and a $2\frac{1}{2}$" hose has a friction loss coefficient of 2.

22. Calculate the FL for a 1 ¾" hose with a length of 100 feet using 100 GPM.

____

23. Calculate the FL for a 1 ¾" hose with a length of 200 feet using 100 GPM.

____

24. Calculate the FL for a 2 ½" hose with a length of 200 feet using 250 GPM.

____

25. Calculate the FL for a 2 ½" hose with a length of 500 feet using 250 GPM.

____

26. Calculate the FL for a 2 ½" hose with a length of 1000 feet using 250 GPM.

____

Extensions

1. Pump Discharge Pressure (PDP) is the "actual velocity pressure of water as it leaves the pump and enters the hose line" and is found using the formula:

   PDP = NP + FL.

   Using the results from question 22; find the PDP for a 100 foot hose using a nozzle pressure of 50 psi.

   ____

2. Using the results from question 23; find the PDP for a 200 foot hose using a nozzle pressure of 50 psi.

   ____

3. Using the results from question 24; find the PDP for a 200 foot hose using a nozzle pressure of 80 psi.

---

4. Using the results from question 25; find the PDP for a 500 foot hose using a nozzle pressure of 80 psi.

---

5. Using the results from question 26; find the PDP for a 1000 foot hose using a nozzle pressure of 80 psi.

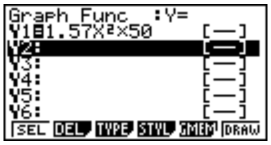
Solutions

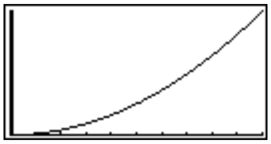
1. 19.6 lbs

2. 44.2 lbs

3. 78.5 lbs

4. 122.71 lbs



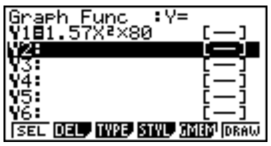


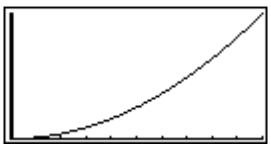
5. 282.6 lbs

6. 502.4 lbs

7. 785 lbs

8. 1130.4 lbs





9. Parabolic

10. As the diameter increases, the amount of nozzle reaction increases rapidly.

11. The second graph is narrower than the first; increases faster.

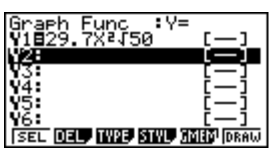
12. Answers will vary based on student experience. Students should note that the force of water through larger diameter nozzles is too much for handheld control.

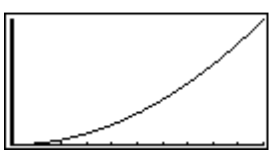
13. 52.5 gpm

14. 118.1 gpm

15. 210 gpm

16. 643.2 gpm



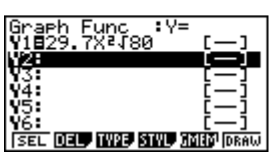


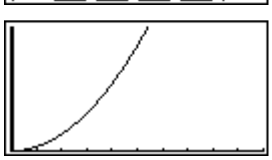
17. 597.7 gpm

18. 1062.57 gpm

19. 1660.3 gpm

20. 2390.8 gpm





21. As the nozzle diameter increases, the amount of water rapidly increases.

22. 15.5 psi

23. 31 psi

24. 25 psi

25. 62.5 psi

26. 125 psi

Extension Solutions

1. 65.5 psi

2. 81 psi

3. 105 psi

4. 142.5 psi

5. 205 psi

License Plate Numbers — Teacher Notes

Topic: Probability- Permutations

NCTM Standards:

- Understand and apply basic concepts of probability.
- Understand the concepts of conditional probability and independent events.
- Understand how to compute the probability of a compound event.

Objective

The student will be able to determine the number of possible variations of license plate numbers in their state.

Getting Started

Discuss with students how license plate numbers are created.

Prior to using this activity:

- Students should find the number of arrangements for a given set of items when placement is important.

Ways students can provide evidence of learning:

- Discuss the results of the activity, and their answers to the questions.

Common mistakes to be on the lookout for:

- Student misuse of parenthesis when dealing with equations containing complex numerators and/ or denominators.
- Students may use the formula for finding combinations instead.

Definitions

- Permutations
- Probability

Formulas

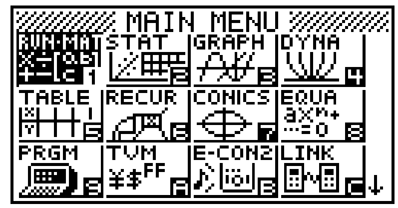
Permutations: $_{n}P_{k} = \dfrac{n!}{(n-k)!}$

Given the formula for calculating permutations, the students will be able to use the Casio *fx-9750GII* to calculate the number of arrangements possible for a set of items.

$$nPk \text{ means } \frac{n!}{(n-k)!}$$

To find permutations:

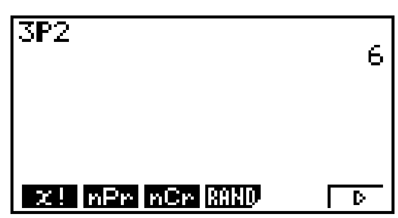
1. From the Main Menu, highlight the RUN•MAT icon and press **EXE** or press **1**.



2. Press **OPTN** **F6** (▷) **F3** (PROB) to get to the Probability section on the calculator.

3. Suppose we want to figure out the number of ways to arrange the three letters of CAT in different two-letter groups, where CA is different from AC and there are no repeated letters. The permutation that will give the number of arrangements is $_3P_2$.

4. To calculate this, press **3** **F2** (nPr) **2** **EXE**.



The state of Virginia has a total of seven places available for numbers and letters on its license plate. Usually the first three are letters and the second four are numbers, but they are not necessarily set, especially if you purchase specialty license plates.

Questions

1. How many different permutations are available if the first three places are letters?

____

2. How many different permutations are available if the last four places are numbers?

____

3. How many different permutations are available on the standard issue Virginia license plate (those that have three letters first and four numbers second)?

____

4. How many different permutations are available if you can have any combination of numbers and letters in those seven places?

____

5. How many more do they have now than they had when they only had six places on the plate?

____

6. How many more would they have if they could fit another place on the plate?

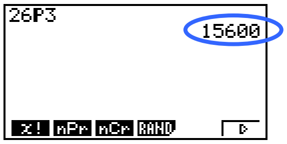
____

7. How many permutations does your state have on its standard issue license plate?

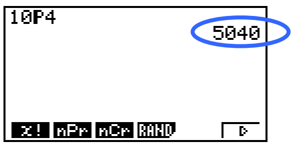
____

Solutions

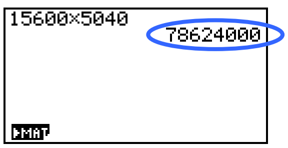
1. $_{26}P_3 = \dfrac{26!}{(26-3)!} = 15{,}600$



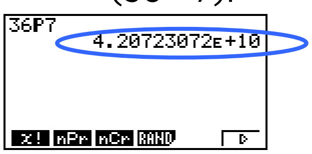
2. $_{10}P_4 = \dfrac{10!}{(10-4)!} = 5{,}040$



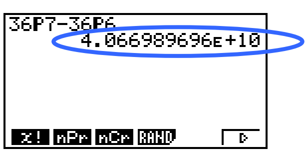
3. $_{26}P_3 \cdot {}_{10}P_4 = 15{,}600 \cdot 5{,}040 = 78{,}624{,}000$



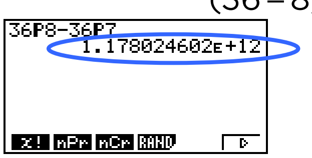
4. $_{36}P_7 = \dfrac{36!}{(36-7)!} = 2.307 \times 10^{10}$



5. $_{36}P_7 - {}_{36}P_6 = \dfrac{36!}{(36-7)!} - \dfrac{36!}{(36-6)!} = 4.067 \times 10^{10}$



6. $_{36}P_8 - {}_{36}P_7 = \dfrac{36!}{(36-8)!} - \dfrac{36!}{(36-7)!} = 1.178 \times 10^{12}$



7. Answers will vary.

Line Up!! Teacher Notes

Topic: Writing and Solving Equations

NCTM Standard:
- Explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope.
- Use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships.

Objective

The student will be able to use the Casio *fx-9750GII* to write and solve equations.

Getting Started

Students should have some experience translating written expressions to symbolic notation. Writing equations is an extension of this skill. Understanding what each variable and number means will make the process of solving equations much easier.

Prior to using this activity:
- Students should know how to translate written expressions into symbolic expressions.
- Students should know how to identify a variable and a constant from a written expression.
- Students should know inverse operations.

Ways students can provide evidence of learning:
- Given a written expression, students can translate to a symbolic form.
- Given an equation, students can solve for an unknown.
- Given an equation, students can create a context for the equation.

Common calculator or content errors students might make:
- Students may incorrectly translate the written expression into symbolic form.
- Students may incorrectly use inverse operations.

Note: The "How To" section for Activity 2 will be used for this activity as well.

Line Up!!! Activity

When solving real world problems in mathematics, there is always a context or situation containing the problem information. Knowing how to translate an equation from a written form to a symbolic form is a very practical skill. When translated correctly, we may use what we know about equations to solve for the missing values.

Please use complete sentences in your responses.

Questions

1. Translate the statement using symbols and solve.

   A number increased by 8 is 11.

   ____

2. Translate the statement using symbols and solve.

   Twice a number decreased by 4 is 20.

   ____

3. Translate the statement using symbols and solve.

   A number divided by 3 increased by 7 is 14.

   ____

4. Translate the statement using symbols and solve.

   The quotient of thirty and five more than a number is two.

   ____

5. One Super Bowl ring weighs about 3.6 ounces. If the total weight of all of the rings for a winning team is 540 ounces, how many rings were purchased?

   ____

6. Six bags contain the same amount of money. If all of the contents were combined and $28 were removed, there would be $92. How much money is in each bag?

____

7. Jeff downloaded some music using a prepaid card. He purchased 18 songs at 1.29 each and had $26.78 remaining on the card. How much was on the card before he made his purchases?

____

8. You and three of your friends win a coupon worth $10 off your next dinner bill. If the total bill is $46.84 and you split the bill evenly, what will each person owe after the discount has been applied?

____

Solve each of the following equations for the missing value.

9. $2n + 6 = 56$

____

10. $5 = -2 - \frac{n}{4}$

____

11. $\frac{n + 4}{-2} = -22$

____

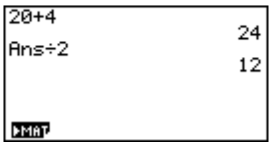
12. $\frac{3n - (-2)}{5} = 4$

____

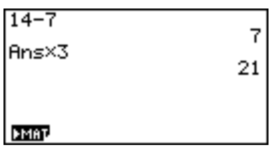
Solutions

1. The equation is n + 8 = 11 and n = 3.

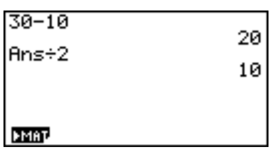
2. The equation is 2n – 4 = 20 and n = 12.



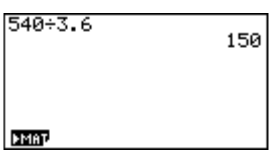
3. The equation is $\frac{n}{3} + 7 = 14$ and n = 21.



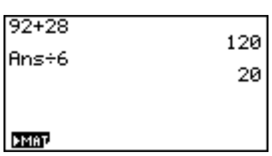
4. The equation is $\frac{30}{5 + n} = 2$ and n = 10.



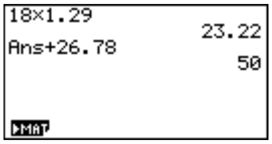
5. The equation is 540 = 3.6n and n = 150.



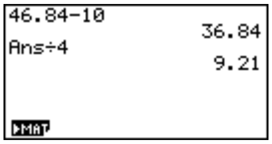
6. The equation is 6n – 28 = 92 and n = $20.



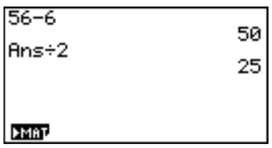
7. The equation is 26.78 = n – 18(1.29) and n = 50.



8. The equation is 4n + 10 = 46.84 and n = $9.21.



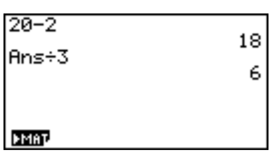
9. n = 25



10. n = -28

11. n = 40

12. n = 6



Looking At Limits Teacher Notes

Topic Area: Limits

NCTM Standards:

- Make and investigate mathematical conjectures.
- Organize and consolidate their mathematical thinking through clear communication; communicate their mathematical thinking coherently and clearly to peers, teacher, and others.

Objective

The student will be able to develop an understanding of the meaning of a "limit" and be able to estimate the value of a limit using a numerical view from a table and a graphical view.

Getting Started

This activity will encourage students to use graphical and numerical representations to examine the behavior of a function as it approaches a particular input value.

Prior to using this activity:

- Students should be able to produce and manipulate graphs and table of values manually and with a graphing calculator.
- Students should have a basic understanding of functions.
- Students should be able to identify rational and exponential functions.

Ways students can provide evidence of learning:

- The student will be able to state and explain what the limit is at a particular value given the function, a graph of the function, or a table of the function.

Common mistakes to be on the lookout for:

- Students may use viewing windows that appear to show functions being defined, when they are not.
- Students may use an input or table value with an increment too small for the calculator, which results in a rounded value that does not exist or an error message due to memory overflow.

Definitions:

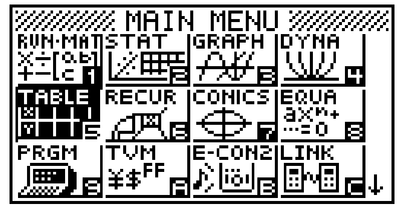
- Limit
- Hole
- Functions

The following will demonstrate how to enter a function into a table, set the table, and graph the function on the Casio *fx-9750GII*.

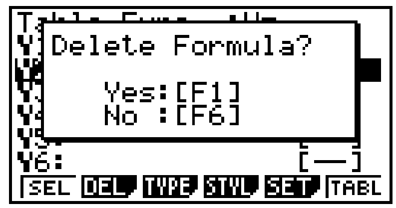
Examine the value of the function $f(x) = \dfrac{x^2 - 1}{x - 1}$, as the value of x gets close to 1.

To enter a function into the Table mode:

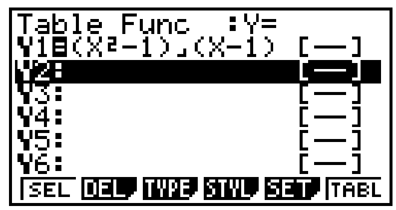
1. From the Main Menu, highlight the TABLE icon and press [EXE] or press [5].



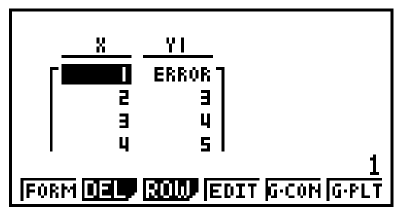
2. To delete previous functions, press [F2] (DEL) [F1] (Yes).



3. Enter the function in **Y1**. To do so, press [(] [X,θ,T] [x²] [−] [1] [)] [a b/c] [(] [X,θ,T] [−] [1] [)] [EXE].

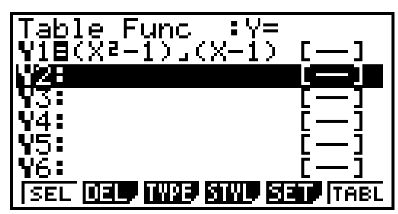


4. Press [F6] (TABL) to view the table with the default table settings.

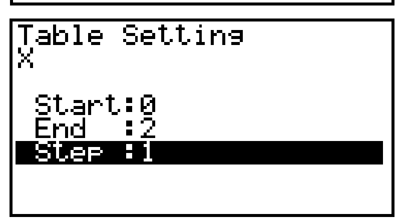


To set the table of the function:

1. Press [EXIT] to return to the initial TABLE screen.

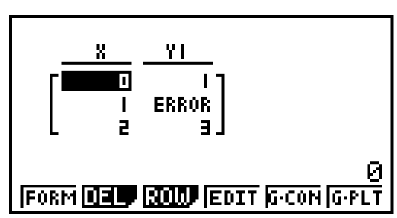


2. Press [F5] (SET) to change the table settings.

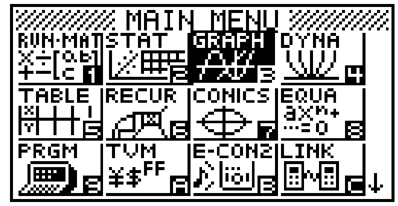


3. Press [0] [EXE] to set the **Start** value, press [2] [EXE] to set the **End** value, and press [1] [EXE] to set the **Step** value.

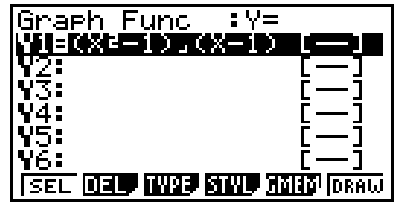
4. Press [EXIT] to return to the initial TABLE screen and press [F6] (TABL) to view the table.



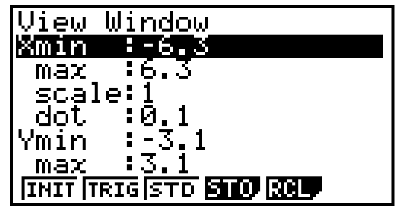
To display a graph of the data:



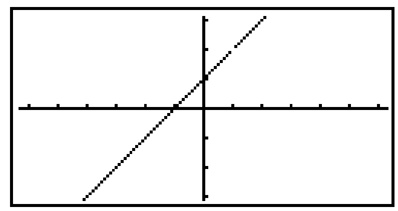
1. From the Main Menu, highlight the GRAPH icon and press [EXE] or press [3]. Notice that the formula used in the TABLE icon is automatically transferred to the GRAPH icon.



2. To set the viewing window to the initial screen, press [SHIFT] [F3] **(V-Window)** [F1] (INIT).



3. Press [EXIT] to return to the initial GRAPH screen and press [F6] (DRAW) to view the graph. A good sketch reveals a hole in the graph.



Note: The reason for setting the view window in this activity is to make sure the hole is visible. The calculator will only show the hole if it is a specific pixel in the screen that does not exist. In many other viewing windows, the point (1, 2) would not visible as a hole and the graph would appear to be continuous.

Looking At Limits Activity

This activity will encourage you to use graphical and numerical representations to examine the behavior of a function as it approaches a particular input value. Using the Casio *fx-9750GII* you will be working in pairs or small groups.

Questions

1. Examine the function $f(x) = \frac{x^2 - 4}{x - 2}$ around the value x = 2. Set up a table with starting value of 1, ending value of 3, step of 1, and record the values in the table below.

x	y
1	
2	
3	

2. Change the step of the table (table increments) to 0.5, then 0.25, then 0.1, and finally 0.01 and record the values directly above and below x = 2 in the table below.

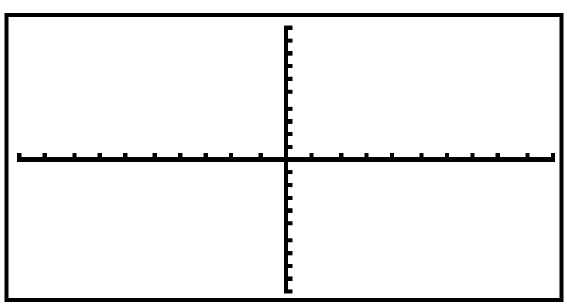
x	y
2	

x	y
2	

x	y
2	

x	y
2	

3. Using the standard screen, draw the graph of this function on the axis below:



4. Write a conjecture based upon the numerical and graphical evidence of this function.

____

____

5. Examine the function $f(x) = \dfrac{x^2 - 4}{x - 1}$ around the value x = 1. Set up a table with starting value 0, ending value of 2, step of 1, and record the values in the table below:

x	y
0	
1	
2	

6. Change the step of the table (table increments) to 0.5, then 0.25, then 0.1, and finally 0.01 and record the values directly above and below x = 1 in the table below.

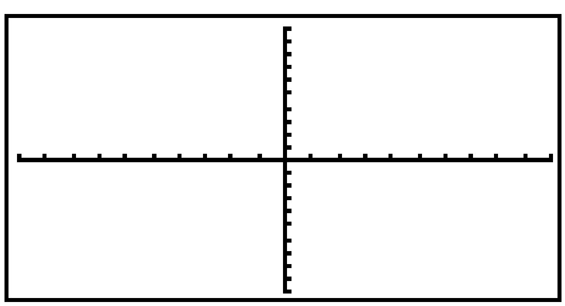
x	y
1	

x	y
1	

x	y
1	

x	y
1	

7. Using the standard screen, draw the graph of this function on the axis below:



8. Write a conjecture based upon the numerical and graphical evidence of this function.

____

____

Extensions

Find the limit, if it exists, for each of the following. If it does not exist, explain why.

1. $\lim_{x \to 5} \frac{2x^2 - 50}{3^x + 15}$

____

2. $\lim_{x \to 0} \frac{3^x - 1}{x}$

____

3. $\lim_{x \to 0} \frac{x + 2}{\sqrt{x + 4}}$

____

4. $\lim_{x \to -3} \frac{\sqrt{3x + 12}}{4 - x}$

____

Solutions

1.

x	y
1	3
2	Error
3	5

2.

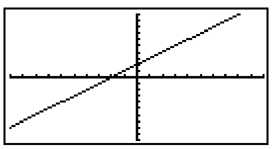
x	y
1.5	3.5
2	Error
2.5	4.5

x	y
1.75	3.75
2	Error
2.25	4.25

x	y
1.9	3.9
2	Error
2.1	4.1

x	y
1.99	3.99
2	Error
2.01	4.01

3.



Note: there should be a hole at (2, 4).

4. Answers may vary; as x approaches 2, f(x) has a limit of 4.

5.

x	y
0	4
1	Error
2	0

6.

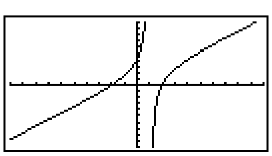
x	y
0.5	7.5
1	Error
1.5	-3.5

x	y
0.75	13.75
1	Error
1.25	-9.75

x	y
0.9	31.9
1	Error
1.1	-27.9

x	y
0.99	301.99
1	Error
1.01	-297.9

7.



8. Answers may vary; as x approaches 1, f(x) has no limit.

Extension Solutions

1. 0
2. approximately 1.099
3. 1
4. approximately 0.2474

Looking at Relationships Teacher Notes

Topic: Derivatives and Graphs

NCTM Standards
- Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.
- Organize and consolidate their mathematical thinking through communication.

Objectives
The student will be able to interpret the graph of a function based on the first and second derivatives. Given a function, students will explain the behavior of a derivative as positive when the function increases and negative when the function decreases. They will also understand that a positive second derivative means the function is concave upward and a negative second derivative means the function is concave downward.

Getting Started
This activity will provide a graphical examination of the relationships between the function, its derivative, and its second derivative. Students will analyze the behavior of the first and second derivatives of a third degree polynomial.

Prior to using this activity:
- Students should be able to take basic symbolic derivatives.
- Students should know the terms relative minimum and maximum.
- Students should be able to produce the graph of a derivative and second derivative on the graphing calculator.

Ways students can provide evidence of learning:
- Students should be able to explain how the first derivative yields information about the increasing/decreasing nature of the function.
- Students should be able to explain how the second derivative yields information about the concavity of the graph.

Common mistakes to be on the lookout for:
- Students may understand where a function is increasing or decreasing but they may misrepresent the graph of the derivative as being above or below the x-axis instead of being positive/negative.
- The speed at which the calculator shows a second derivative graph is relatively slow. Some students may conclude there is no graph being produced.

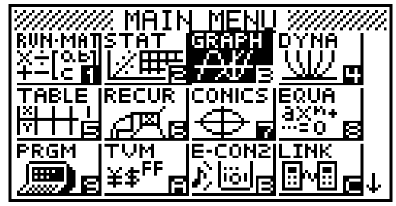
Definitions:
- Relative minimum
- Relative maximum
- Derivative
- 2nd derivative
- Concavity
- Inflection point

The following will demonstrate how to set the view window, graph a function, graph its derivative and second derivative, and find the maximum and minimums of a function on the Casio *fx-9750GII*.

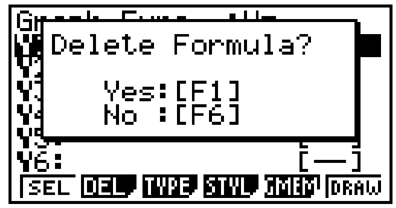
Graph the function $y = -x^3 - 3x^2 + x + 2$.

To graph a function:

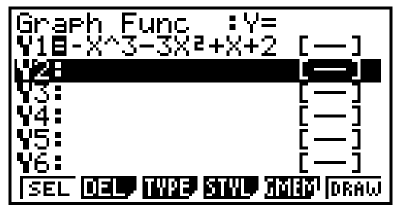
1. From the Main Menu, highlight the GRAPH icon and press EXE or press 3.



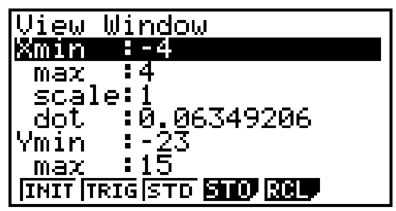
2. To clear out previous graphs, highlight the equation you want to delete and press F2 (DEL) F1 (Yes).



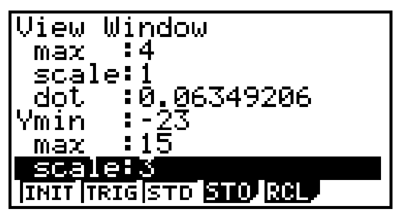
3. Enter the equation by pressing (−) X,θ,T ^ 3 − 3 X,θ,T x^2 + X,θ,T + 2



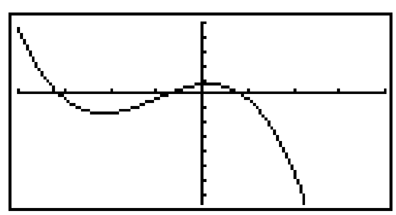
4. Set the view window, by pressing SHIFT F3 **(V-Window)** (−) 4 EXE 4 EXE 1 EXE EXE (−) 2 3 EXE 1 5 EXE 3 EXE.



5. Press EXIT to return to the initial GRAPH screen.

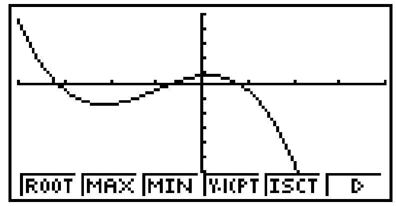


6. Press F6 (DRAW) to view the graph of the function.

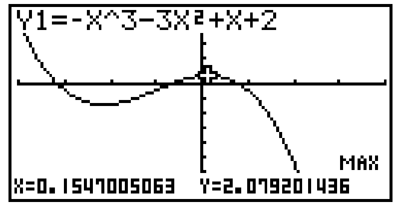


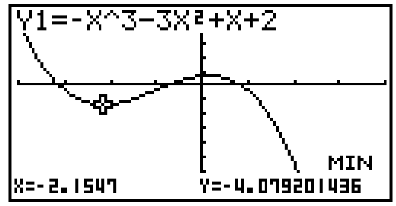
To find the minimum and maximum values:

1. To find a maximum value, while viewing the graph, press [F5] **(G-Solv)** [F2] (MAX) [EXE].



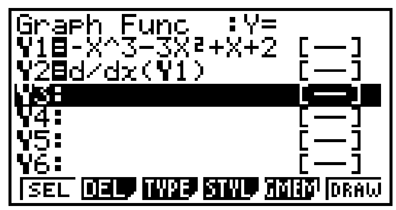
2. To find a minimum value, while viewing the graph, press [F5] **(G-Solv)** [F3] (MIN) [EXE].





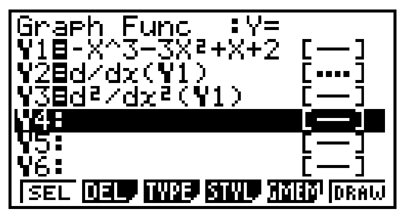
To find the first and second derivatives:

1. Press [EXIT] to return to the initial GRAPH screen.



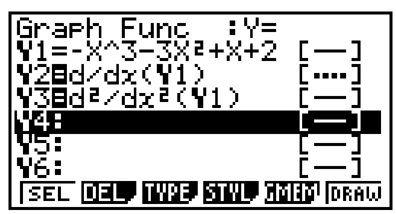
2. For the first derivative, arrow to **Y2** and press [OPTN] [F2] (CALC) [F1] (d/dx) [F1] (Y) [1] [)] [EXE].

3. To change the style of the graph, arrow up to **Y2**. Press [F4] (STYL) [F3] (•••••) [EXE].



4. For the second derivative, arrow to **Y3** and press [OPTN] [F2] (CALC) [F2] (d^2/dx^2) [F1] (Y) [1] [)] [EXE].

5. To turn on and off a graph, press [EXIT] to return to the initial GRAPH screen. With the graph highlighted, press [F1] (SEL). If the equals sign is highlighted, the graph is on. If it is not, the graph is off. In the example to the right, **Y1** is turned off. **Y2** and **Y3** will graph.

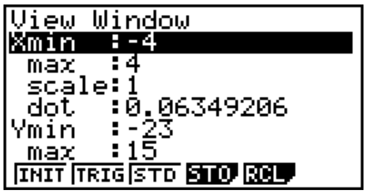


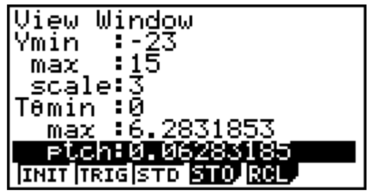
Looking at Relationships Activity

In this activity you will examine the relationships between the function and its first and second derivatives on the Casio *fx-9750GII*.

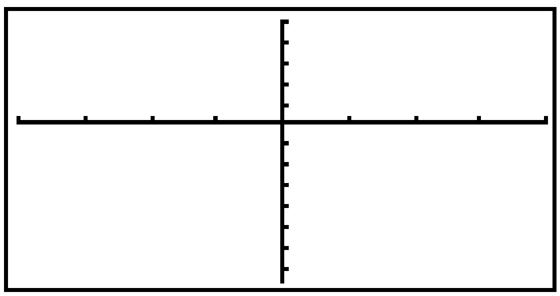
Questions

1. Graph the function $y = 2x^3 - 3x^2 - 12x + 4$ with the following view window.





2. Record the results here:



3. At what x-values does it appear the function reaches its relative minimum and maximum values?

____

4. Using your graphing calculator, find the minimum and maximum function values to confirm your solutions in #3.

5. Record the domain interval/intervals where the function increases.

____

6. Record the domain interval/intervals where the function decreases.

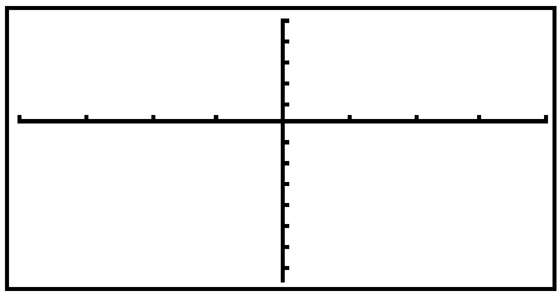
____

7. Explain what kind of values you would expect the derivative to have over the interval where the function increases.

____

____

8. Graph both the function and its derivative together on the axis below. Be sure to label which sketch is the function and which is the derivative.



9. From the graph, what are the y-values of the derivative where the original function has a relative maximum or minimum?

____

10. Explain the nature of the y-values of the derivative over the interval(s) where the original function increases.

____

____

____

11. Explain the nature of the y-values of the derivative over the interval(s) where the original function decreases.

____

____

____

12. When the derivative crosses the x-axis explain what happens to the graph of the original function?

____

____

____

13. Over what x interval(s) does the derivative increase?

____

14. Over what x interval(s) does the derivative decrease?

____

15. If the first derivative is increasing, do you expect the second derivative to be positive or negative? Explain.

____

____

____

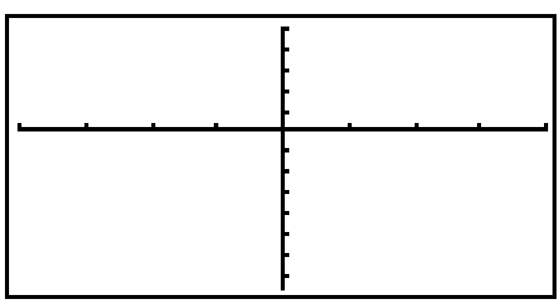
16. If the first derivative is decreasing, do you expect the second derivative to be positive or negative? Explain.

____

____

____

17. Graph the first and second derivative together on the axis below. Be sure to label which graph is the derivative and which is the second derivative.



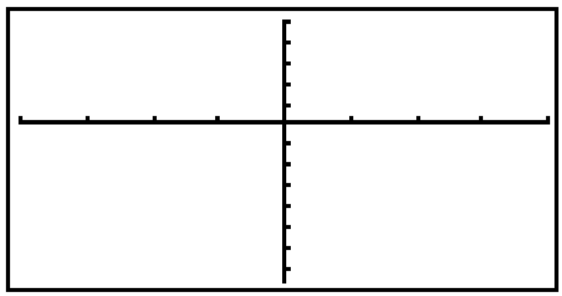
18. Do the graphs produced match your expectations? If not, explain any differences you see.

____

____

____

19. The graph of a function is concave down when the graph of the first derivative is decreasing. Sketch the portion of the original function that is concave down and record it here.



Explain what is true about both the first and second derivatives over the interval you just sketched.

____

____

____

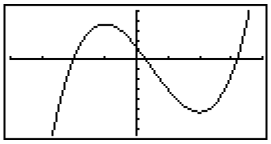
20. A point of inflection is a point where the concavity changes. Based upon your exploration, what is the point of inflection for the original graph? Explain how you know.

____

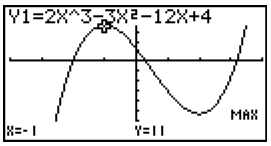
____

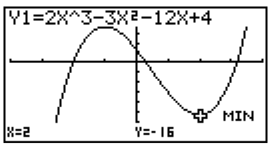
____

Solutions

2. 

3. Relative Max at x = -1, Relative Min at x = 2.

4. 

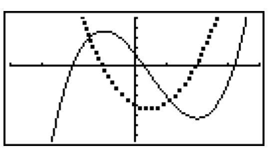


5. Increases: $(-\infty,-1)\cup(2,\infty)$

6. Decreases: (-1,2)

7. Positive values

8. The derivative is dotted.



9. Y = 0 in both cases.

10. Where the function is increasing, the y-values of the derivative are positive.

11. Where the function is decreasing, the y-values of the derivative are negative.

12. The function reaches a relative extreme point.

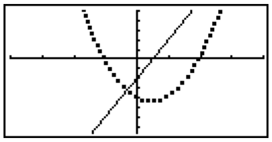
13. $(0.5,\infty)$

14. $(-\infty,0.5)$

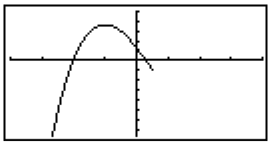
15. Positive. It should follow the same behavior as the relationship between the original function and its 1st derivative.

16. Negative, same reason as above.

17. The first derivative is dotted.



18. Answers may vary, but they should match.

19. 

20. The point of inflection is (0.5,-2.5). The concavity will change at the root of the second derivative which is x = 0.5, that produces a y-value of -2.5 in the original function.

Topic: Slope Fields

NCTM Standards

- Approximate and interpret rates of change from graphical and numerical data.
- Make and investigate mathematical conjectures.
- Organize and consolidate their mathematical thinking through communication; communicate their mathematical thinking coherently and clearly to peers, teacher, and others.

Objectives

The student will understand what a slope field represents in terms of dy/dx. Given a differential equation, the student will create and explain a slope field.

Getting Started

A slope field is a visual representation of the solution to a differential equation created by creating a series of small linear approximations to the slope at certain points. In this activity students will sketch some slope fields and then confirm their appearance using the Casio *fx-9750GII*. This can be used as a first introduction to the idea of slope fields and also as a general introduction to antidifferentiation.

Prior to using this activity:

- Students should have a working knowledge of differentiation and be conversant with the language of differential equations.
- It is not necessary for students to know any symbolic antidifferentiation methods.

Ways students can provide evidence of learning:

- Students should be able to sketch their own slope fields for a given differential equation over specific grid points using pencil and paper.

Common mistakes to be on the lookout for:

- Students may misunderstand that the graph being produced is the graph of the solution to dy/dx (the graph of the antiderivative) and be confused when the equation does not seem to fit the slope field.
- For example, the slope field of the expression dy/dx = x, correctly drawn will produce a parabolic fit, students may incorrectly expect a linear fit.

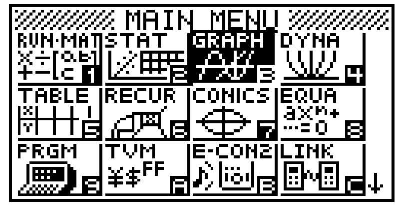
Definitions:

- Slope field
- Derivative
- Differential equation
- Family of curves

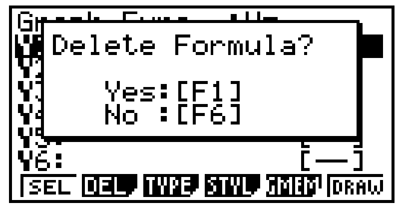
The following will demonstrate how to graph a function, set a view window, and create a slope field using the SLOPEFLD program on the Casio *fx-9750GII*.

Enter the function y = x^4.

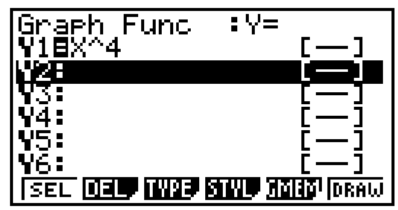
To enter a function to be used by the SLOPEFLD program:



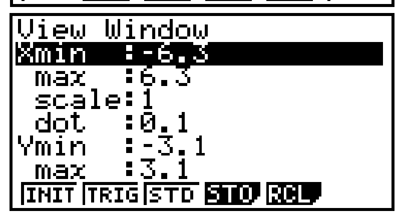
1. From the Main Menu, highlight the GRAPH icon and press EXE or press 3.



2. To clear out previous graphs, highlight the equation you want to delete and press F2 (DEL) F1 (Yes).

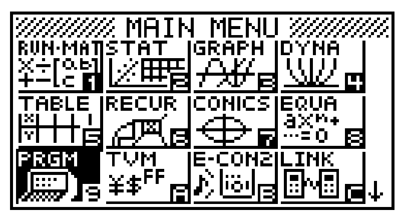


3. Enter the equation by pressing X,θ,T ^ 4 EXE.



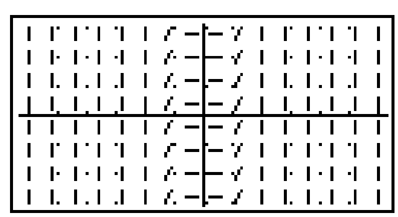
4. Set the view window, to the initial viewing window, by pressing SHIFT F3 **(V-Window)** F1 (INIT).

To use the program SLOPEFLD:



1. From the Main Menu, highlight the PROGRAM icon and press EXE or press 9.

2. Highlight the SLOPEFLD program. Press F1 (EXE) to run the program.



(**See Download Appendix for instructions on how to download program to your software or calculator.)

A **slope field** is a visual representation of the solution to a differential equation created by a series of small linear approximations to the slope at certain points. In this activity you will sketch some slope fields and then confirm their appearance using the Casio *fx-9750GII*.

An equation like dy/dx = 2x which contains a derivative is called a "differential equation". The problem becomes finding a function, y, in terms of x when we are given its derivative. Note the phrasing, "We are looking for "a solution" and not "the solution"." This is due to the fact that the slope will allow us to arrive at a family of curves missing some initial condition value. We will not be able to arrive at a single solution but should be able to make some conclusions as to the appearance of the family of solutions. If we examine the plot of the slope over a series of grid points we end up with a slope field.

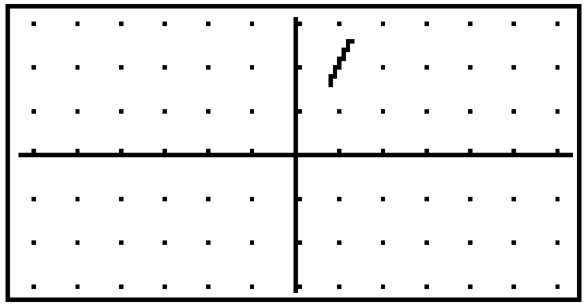
If given the differential equation dy/dx = f(x,y), a plot of short line segments with slopes f(x,y) over specific grid points produces a slope field. This slope field will give you a look at the behavior of the solution to the original differential equation.

Questions

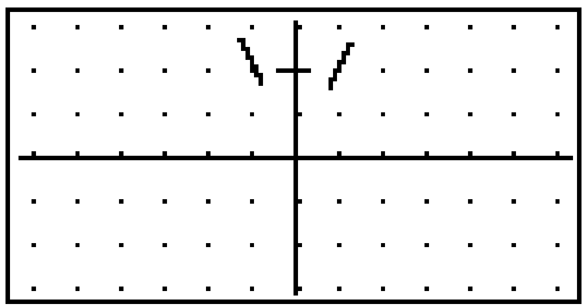
1. Fill in the accompanying table representing the slope of dy/dx = 2x for the grid points shown. For example, at the point (0,0) you should have a slope of 0, at (1,2) a slope of 2, etc.

(x, y)	dy/dx	(x, y)	dy/dx	(x, y)	dy/dx	(x, y)	dy/dx	(x, y)	dy/dx
(-6, -2)		(-6, -1)		(-6, 0)		(-6, 1)		(-6, 2)	
(-5, -2)		(-5, -1)		(-5, 0)		(-5, 1)		(-5, 2)	
(-4, -2)		(-4, -1)		(-4, 0)		(-4, 1)		(-4, 2)	
(-3, -2)		(-3, -1)		(-3, 0)		(-3, 1)		(-3, 2)	
(-2, -2)		(-2, -1)		(-2, 0)		(-2, 1)		(-2, 2)	
(-1, -2)		(-1, -1)		(-1, 0)		(-1, 1)		(-1, 2)	
(0, -2)		(0, -1)		(0, 0)		(0, 1)		(0, 2)	
(1, -2)		(1, -1)		(1, 0)		(1, 1)		(1, 2)	
(2, -2)		(2, -1)		(2, 0)		(2, 1)		(2, 2)	
(3, -2)		(3, -1)		(3, 0)		(3, 1)		(3, 2)	
(4, -2)		(4, -1)		(4, 0)		(4, 1)		(4, 2)	
(5, -2)		(5, -1)		(5, 0)		(5, 1)		(5, 2)	
(6, -2)		(6, -1)		(6, 0)		(6, 1)		(6, 2)	

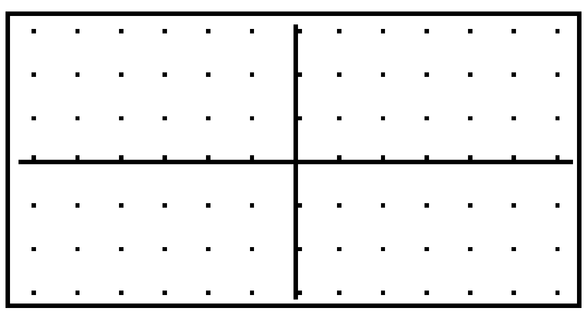
2. To create a slope field sketch a small line segment with the slope calculated, centered at a grid point. For example, at the point (1, 2), which will have a slope of 2, this is what you should sketch:



And for the points (-1, 2), (0, 2), and (1, 2) (with slopes of -2, 0, 2; respectively) you should see this:



Create a slope field. Sketch small line segments with the slopes calculated, centered at each grid point that you calculated in #1.



3. What familiar family of curves does this slope field seem to indicate is the solution to the given differential equation? Explain.

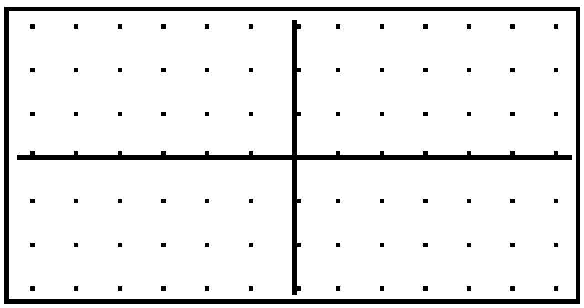
____

____

____

4. Using the Slope Field program, have the calculator produce the slope field for dy/dx = 2x. How does it compare with what you sketched manually?

5. Have the calculator draw one member of the family of curves you think is best represented by the slope field, then, have it overlay the slope field on the graph produced. Record what you see here:

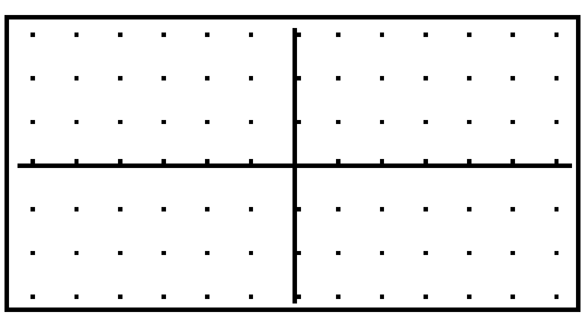


6. Look at the graph the calculator produced. How does it compare to what you expected to see?

Extension

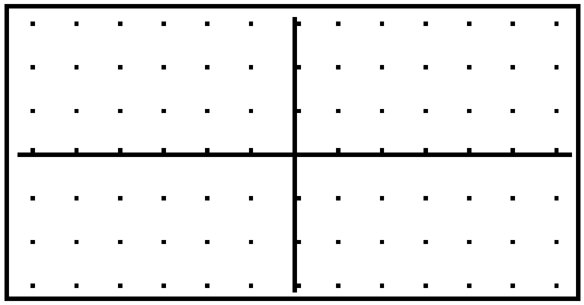
Use the following differential equations and generate a slope field for each. Have the calculator generate the slope field and draw a conclusion about the general solution to the differential equation.

1. dy/dx = x^2



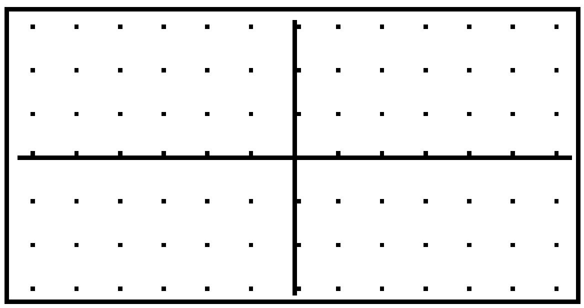
y = ____________________

2. dy/dx = sin(x)



y = ____

3. dy/dx = e^x



y = ____

Solutions

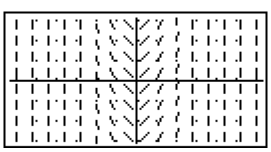
1.

(x, y)	dy/dx	(x, y)	dy/dx	(x, y)	dy/dx	(x, y)	dy/dx	(x, y)	dy/dx
(-6, -2)	-12	(-6, -1)	-12	(-6, 0)	-12	(-6, 1)	-12	(-6, 2)	-12
(-5, -2)	-10	(-5, -1)	-10	(-5, 0)	-10	(-5, 1)	-10	(-5, 2)	-10
(-4, -2)	-8	(-4, -1)	-8	(-4, 0)	-8	(-4, 1)	-8	(-4, 2)	-8
(-3, -2)	-6	(-3, -1)	-6	(-3, 0)	-6	(-3, 1)	-6	(-3, 2)	-6
(-2, -2)	-4	(-2, -1)	-4	(-2, 0)	-4	(-2, 1)	-4	(-2, 2)	-4
(-1, -2)	-2	(-1, -1)	-2	(-1, 0)	-2	(-1, 1)	-2	(-1, 2)	-2
(0, -2)	0	(0, -1)	0	(0, 0)	0	(0, 1)	0	(0, 2)	0
(1, -2)	2	(1, -1)	2	(1, 0)	2	(1, 1)	2	(1, 2)	2
(2, -2)	4	(2, -1)	4	(2, 0)	4	(2, 1)	4	(2, 2)	4
(3, -2)	6	(3, -1)	6	(3, 0)	6	(3, 1)	6	(3, 2)	6
(4, -2)	8	(4, -1)	8	(4, 0)	8	(4, 1)	8	(4, 2)	8
(5, -2)	10	(5, -1)	10	(5, 0)	10	(5, 1)	10	(5, 2)	10
(6, -2)	12	(6, -1)	12	(6, 0)	12	(6, 1)	12	(6, 2)	12

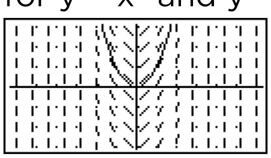
2. The slope field should reflect a parabolic shape.

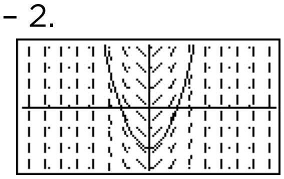
3. Students should recognize this as a family of parabolas. However, the apparent vertical nature of the horizontal extremities could throw some students off. This presents a good opportunity to discuss the nature of graphical approximations.

4. If the original field was drawn correctly, they should see a graph very similar to what they manually sketched.



5. Answers will vary based upon the chosen graph. Shown here are the graphs for $y = x^2$ and $y = x^2 - 2$.

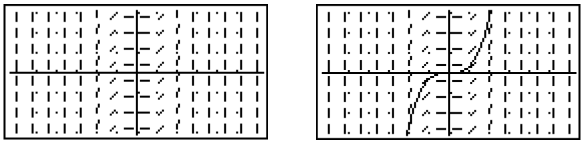




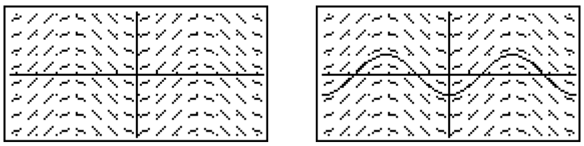
6. If done correctly they should be very similar. If not, this presents a good opportunity to discuss why their graphs are not accurate.

Extension Solutions

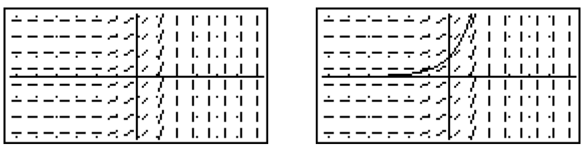
1. Cubic family. Actual family: $y = \frac{1}{3}x^2 + c$.



2. Trigonometric family. Actual family: y = -cos(x) + c.



3. Exponential family. Actual family: $y = e^x + c$



Maximize Space for Minimum Price — Teacher Notes

Topic: Represent and Analyze Mathematical Situations

NCTM Standard:

- Represent and analyze mathematical situations and structures using algebraic symbols, and use mathematical models to represent and understand quantitative relationships.

Objective

The student will be able to use the Casio *fx-9750GII* to write an objective function with inequalities, graph the function and find solutions to the problem.

Getting Started

Discuss with students the use of linear programming and how to interpret the graph. Explain what is meant by an objective functions and a constraint. Review how to write an inequality and how to graph the results. Discuss situations where constraints are put on a product or service and let the students give examples of their own.

Prior to using this activity:

- Students should be able to enter inequalities on the calculator and know how to change the inequality sign, when necessary.
- Students should be able to set up the view window, and find intersections of the vertices.

Ways students can provide evidence of learning:

- The students will be able to discuss the resulting graph, given an objective function.
- The students can determine the intersection, given an objective function.
- The students can discuss what each intersection represents, given an objective function.
- The students can discuss the outcome when given an objective function.

Common calculator or content errors students might make:

- Students may choose the wrong inequality.

Definitions

- Objective Function
- Constraints
- Inequalities
- Vertices

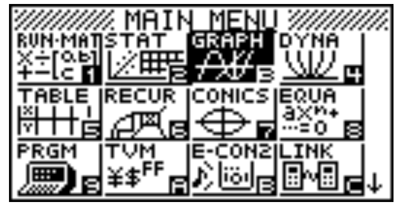
The following will demonstrate how to enter an objective function, the inequalities for the constraints, graph each of them, set the appropriate view window, and calculate the value for the objective function on the Casio *fx-9750GII*.

Inequalities to graph:

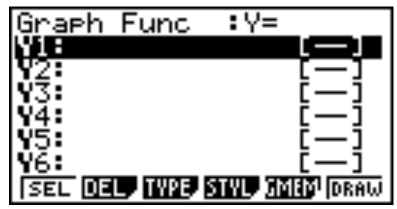
y ≥ –x + 6   y ≥ 2x – 3   y ≤ 5   x ≥ 0   y ≥ 0

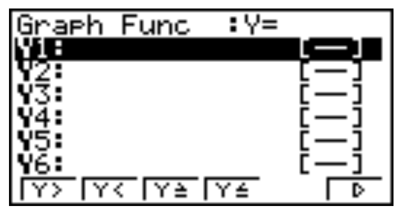
To graph an inequality:

1. From the Main Menu, highlight the GRAPH icon and press [EXE] or press [3].

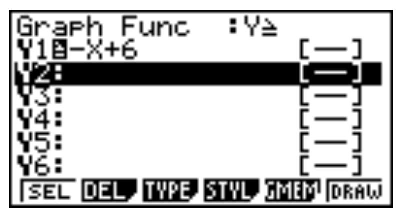


2. Press [F3] (TYPE) to select the type of graph. Press [F6] (▷) for more options. The inequality options are displayed across the bottom of the screen. To graph the first inequality, we will choose [F3].

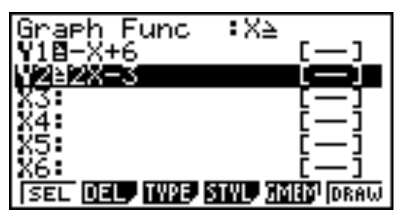




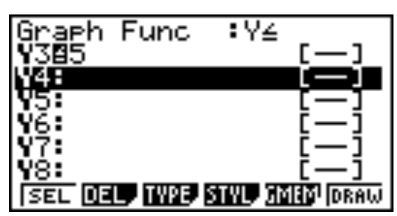
3. The calculator now shows **Y**≥ at the top of the screen. Press [(–)] [X,θ,T] [+] [6] [EXE] to enter the first inequality.



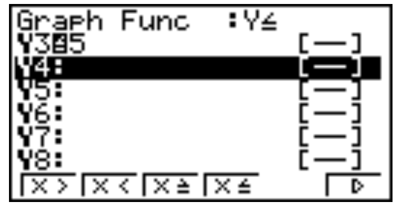
4. To enter the second inequality, input: [2] [X,θ,T] [–] [3] [EXE].

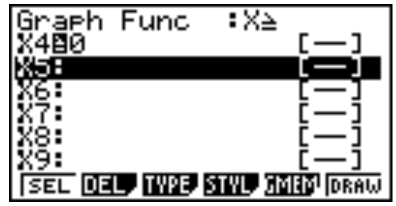


5. The third inequality does not have the same sign, so you will need to change the type of graph again. Press [F3] (TYPE), then press [F6] for more options, and finally, press [F4] for the **Y**≤ inequality followed by [5] [EXE].

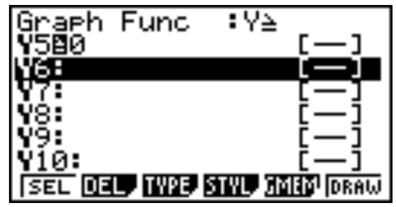


6. The fourth inequality starts with x. (The Casio *fx-9750GII* can graph this inequality).Press F3 (TYPE), F6 for more options, F6 again the choices for an x-inequality will display. Press F3 for the **X** ≥ inequality followed by 0 EXE.

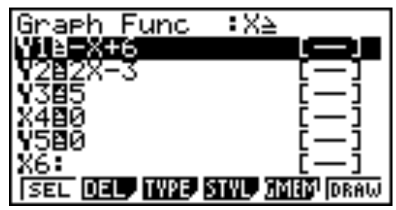




7. To enter the last inequality, press F3 (TYPE), F6 for more options, and then F3 0 EXE.

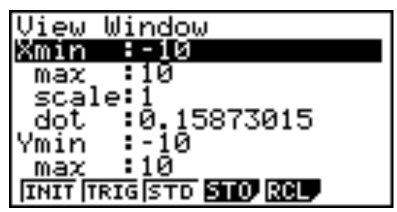


8. ▲ to see all the inequalities entered. The screen should look like the one to the right. The highlighted inequality sign indicates it will be graphed. To deselect an inequality, press F1 (SEL).



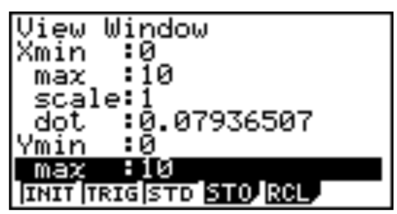
To set up the View Window and Graph:

1. Press SHIFT and then F3 **(V-Window)**.



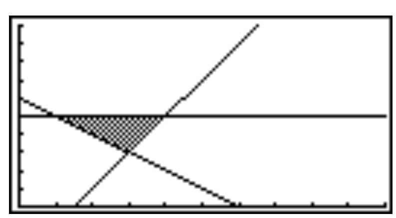
2. F3 (STD) will set the window to a Standard viewing window.

3. Since the graph will be located in the first quadrant, change the minimum x- and y-values to 0. Enter 0 for **Xmin** and press EXE then ▼ three times to change the **Ymin**. Enter 0 and press EXE.



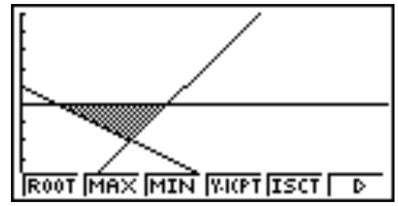
4. Press EXIT to return to the graphing home screen.

5. Press F6 (DRAW) to view the graph.

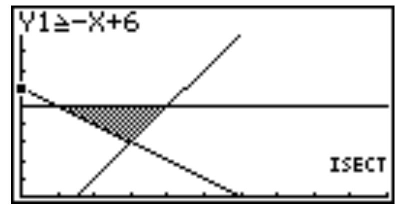


To find the intersections:

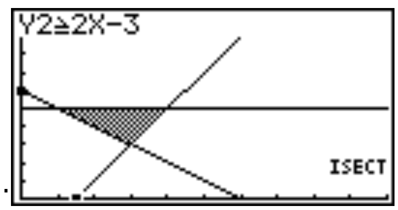
1. Press SHIFT F5 **(G-Solv)**, then F5 (ISCT) to find the intersection of two of the inequalities.



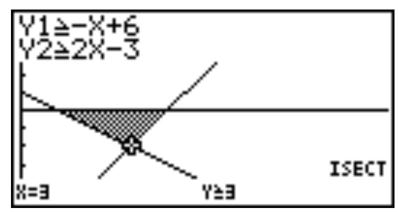
2. Press ▲ or ▼ until the desired inequality is displayed in the upper left corner and press EXE.



3. Use the same process to select the second inequality.



4. The calculator will calculate the intersection of the two inequalities. Notice the two inequalities displayed in the upper left corner. The x- and y-values will be displayed at the bottom of the screen. In the right hand corner, the calculator displays the letters ISECT to remind you that you are finding an intersection point.



5. To find the intersection of two different inequalities, repeat the steps from the beginning of this section.

Maximum Space for Minimum Price **Activity**

With the population ever increasing, the need for homes increases as well. Developers design new housing areas to maximize the number of homes, minimize costs, and increase profits. Large sections of land are divided into lots that are then sold. The size of the lot often determines the type of home that can be built on the property. In this activity, you will complete a linear programming problem that will take a piece of land, determine the number of lots to be developed, and the profit for the company according to specific requirements.

In this activity, you will write inequalities to represent the conditions for sectioning a piece of land, graph the inequalities to determine the best combination of homes to be built, and calculate the profit to be gained.

Type of Home	Number of Lots	Lot Size	Cost per Lot
One Story	x	0.75 acres	$30,000
Two Story	y	1 acres	$50,000
Maximum Number	150	130 acres	$6,200,000

Questions

1. Write the inequality with the constraint on the number of lots.

    ____

2. Write the inequality with the constraint on the size of the lots.

    ____

3. Write the inequality with the constraint on the cost per lot.

    ____

4. What is the largest combination of types of homes that can be built, if only the number of lots and the size of the lots are considered?

    ____

5. What is the largest combination of types of homes that can be built, if only the number of the lots and the cost per lot are considered?

____

6. What is the largest combination of types of homes that can be built, if only the size of the lots and the cost per lot are considered?

____

7. Write the objective function for the profit from the sales of the lots if each 0.75 acre lot will yield $3,500 and each acre lot will yield $4,375.

____

8. What will the profit be if only the number of lots and the size of the lots are considered?

____

9. What will the profit be if only the number of lots and the cost per lot are considered?

____

10. What will the profit be if only the size of the lots and the cost per lot are considered?

____

11. Which combination will give the highest profit?

____

Extensions

1. The number of one-story houses will be divided into 1,500 square feet and 2,000 square feet homes. The number of 2,000 square feet homes will be one-and-a-half times that of the 1,500 square feet homes. Write an inequality to show the total number of one-story houses. Write another inequality to show the relationship between the two sizes of one-story homes, based on your answer from question 11 in the previous section.

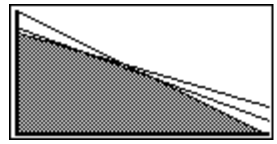
   ____

2. How many of each type of home should be built?

   ____

Solutions

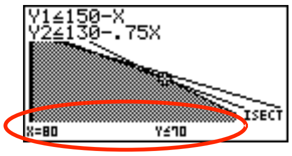
1. x + y ≤ 150 or y ≤ 150 – x



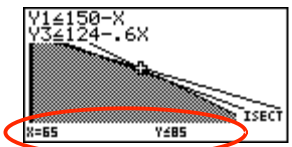
2. 0.75x + y ≤ 130 or y ≤ 130 – 0.75x

3. 30,000x + 50,000y ≤ 6,200,000 or y ≤ 124 – 0.6x

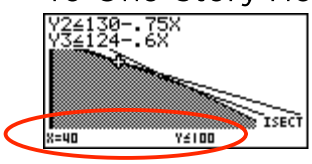
4. 80 One-Story Houses and 70 Two-Story Houses. x = 80, y = 70



5. 65 One-Story Houses and 85 Two-Story Houses. x = 65, y = 85



6. 40 One-Story Houses and 100 Two-Story Houses. X = 40, y = 100



7. Profit = 3500x + 4375y

8. Profit = $586,250. Plug x = 80 and y = 70 into the Profit equation

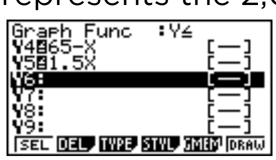
9. Profit = $599,375

10. Profit = $577,500

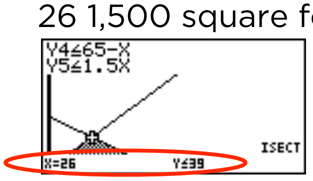
11. 65 one-story homes and 85 two-story homes

Extensions

1. x + y ≤ 65 or y ≤ 65 – x (*x* represents 1,500 square foot homes and *y* represents the 2,000 square foot homes.); y ≤ 1.5x



2. 26 1,500 square foot homes and 39 2,000 square foot homes



Minimum Wage — Teacher Notes

Topic Area: Data Analysis and Probability

NCTM Standard:

- For univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics.

Objective

The student will be able to use the Casio *fx-9750GII* to analyze a set of data. The student will graph the data using a median box-and-whisker graph and calculate the measures of central tendency.

Getting Started

Discuss with students the use of one-variable statistical data and how that data is used in real life situations. Have students give examples of how and when one-variable statistics might be used.

Prior to using this activity:

- Students should be able to use the calculator to enter the data into a list.
- Students should have a basic understanding of interpreting statistical graphs.
- Students should understand how to create a median box-and-whisker graph.
- Show the students how to find the measures of central tendency for the data.

Ways students can provide evidence of learning:

- Given a set a data, the student will be able to graph the data and identify the measures of the central tendency.

Common calculator or content errors students might make:

- Students may not know to use utilize certain functions, causing errors in their results.

Definitions

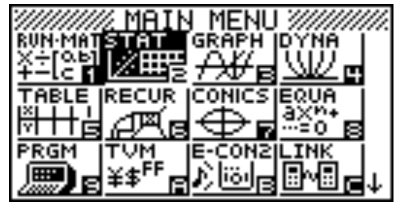
- Mean
- Median
- Range
- Quartile
- Central Tendency

Given a set of data points, the students will be able to enter the data into the statistics menu of the CASIO *fx-9750GII*, graph the data using a median box-and-whisker graph, calculate the measurement of the central tendency, and trace the graph to determine the quartiles.

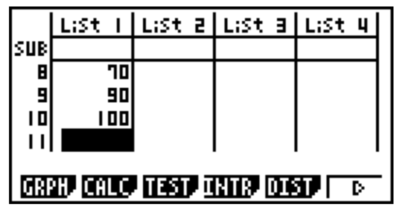
Scores on the first Math Test

75 60 85 80 80 65 95 70 90 100

To enter the above set of data:



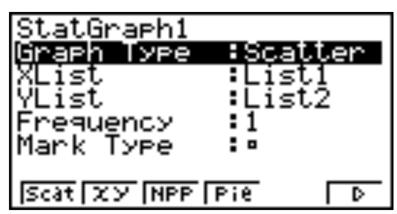
1. From the Main Menu, highlight the STAT icon and press [EXE] or [2].



2. Enter the data by inputting the numbers, pressing [EXE] after entry.

3. The screen should look like the one to the right when done.

To select the type of graph for this data:

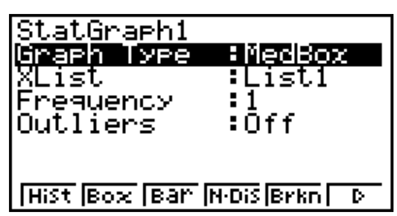


1. Press [F1] (GRPH) then [F6] (SET).

2. Press [▼] to highlight **Graph Type**.

3. Press [F6] (▷) for more graph choices.

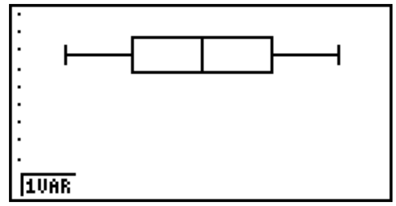
4. Select [F2] (Box) for median box-and-whisker plot.



5. Press [▼] if you need to change the **XList** to a list other than **List 1**. The calculator will ask for the List of data to be used as the **XList**. Input correct list and press [EXE].

6. Press [EXIT] to go back to the previous screen.

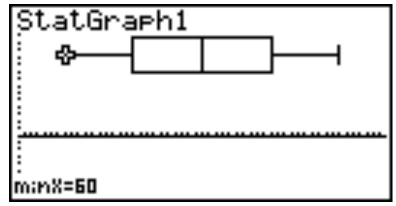
To display a graph of the data:

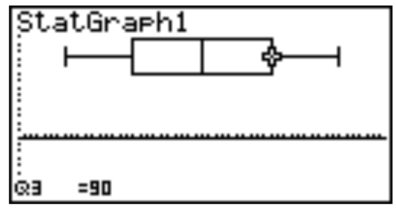


1. Press F1 (GPH1).

2. The screen should look like the one on the right.

To display data on the graph:



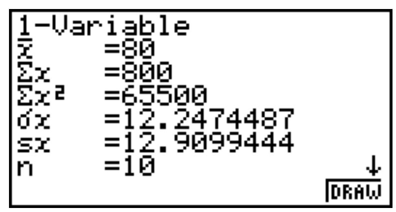


1. Press SHIFT F1 **(Trace)**, and ▶ to see the five-number summary data.

To see 1-Variable statistical calculation:

1. Press F1 (1VAR) twice.

2. Use ▲ ▼ to view information like the mean, median, mode, quartiles, maximum and minimum values. The beginning of the information is shown at the right.



While state minimum wages vary from $6.15 to $7.35 an hour, the federal minimum wage is $5.15 an hour. "In cases where an employee is subject to both the state and federal minimum wage laws, the employee is entitled to the higher of the two minimum wages. Youths under 20 years may be paid a minimum wage of no less than $4.25 an hour during the first 90 consecutive calendar days of employment with an employer."
(Fair Labor Standard Act of 1938 as amended.)

In this activity, you will compare the minimum wages between different states and with federal minimum wages. You will also graph a set of data involving minimum wages, calculate the statistics involving the set of data, and solve problems using the data and calculations.

State	Minimum Wage	State	Minimum Wage
Washington	7.35	Illinois	6.50
Oregon	7.25	Maine	6.35
Alaska	7.15	Hawaii	6.25
Connecticut	7.10	Delaware	6.15
Vermont	7.00		
Rhode Island	6.75		

Questions

1. What is the difference between the highest and lowest values for the state minimum wage data?

 ____

2. How much would the difference be for a 40 hour week?

 ____

3. How much would the difference be for a year? (Assume a 40 hour week each week and a 52-week year.)

 ____

4. What is the difference between the maximum and the mean values for state minimum wages?

 ____

5. What is the difference between the highest wage and the mean wage for the list of wages?

____

6. Are the values for Question 4 and 5 the same? Why might these values be different?

____

____

7. What is the difference between the maximum value and median value for state minimum wages?

____

8. How much would the difference be for a 40 hour work week?

____

9. How much would the difference be for one year? (Assume a 40 hour week each week.)

____

10. Compare the answers from questions 1 – 3 with the answers from questions 7 – 9. Why are they so different?

____

____

Extensions

1. Sketch a median Box-and-Whiskers Graph using List 1 as the XList. Trace the graph on the calculator and label each quartile on the graph representation.

2. Into which quartile does the state minimum wage of each of these states fall?

Alaska: ________

Vermont: ________

Delaware: ________

3. How does the mean state minimum wage compare to the federal minimum wage?

____

____

4. If an employee is entitled to the higher of the state or federal minimum wage, how much would be gained for a 40 hour work week if they are paid the average state minimum wage rather than the federal wage?

____

____

5. A 16-year old high school student has a part time job working 12 hours a week. If the job pays the federal minimum wage of $4.25, how much would the student earn in a week?

____

____

6. How would the wage in question 5 compare with wages earned using the mean state minimum wage?

____

____

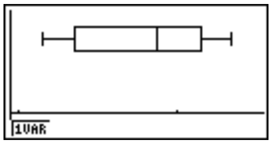
7. How would the wage in question 5 compare with wages earned using the federal minimum wage of $5.15?

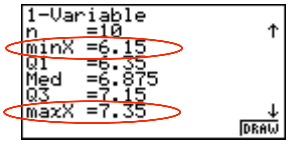
____

____

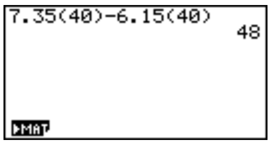
Solutions

1. Minimum value = 6.15, Maximum value = 7.35
   \$7.35 – \$6.15 = \$1.20

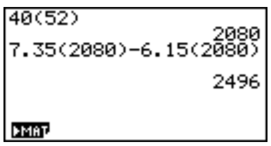




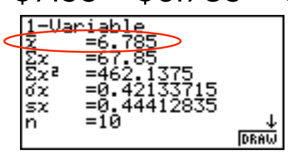
2. \$7.35(40) - \$6.15(40) = \$48



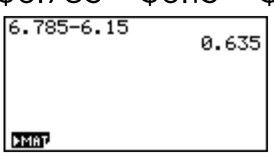
3. 40 hours per week(52 weeks in a year) = 2080 hours in a year
   \$7.35(2080) - \$6.15(2080) = \$2496



4. Maximum value = 7.35, Mean = 6.785
   \$7.35 – \$6.785 = \$0.5625

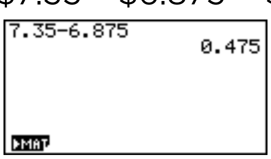


5. Minimum value = 6.15, Mean = 6.785
   \$6.785 – \$6.15 = \$0.635

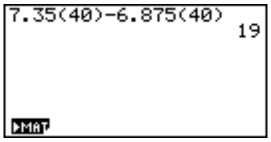


6. No, the values are not the same. The values would be different if there are more numbers above the mean and less numbers below the mean.

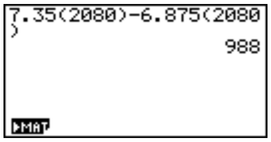
7. Maximum value = 7.35, Median value = 6.875
   \$7.35 – \$6.875 = \$0.475



8. $7.35(40) - $6.875(40) = $19



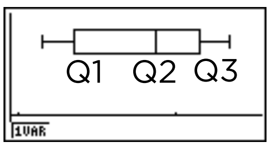
9. 40 hours per week(52 weeks in a year) = 2080 hours in a year
$7.35(2080) - $6.875(2080) = $988



10. The answers in questions 1 - 3 are more than the answers in questions 7 – 9.

Extensions

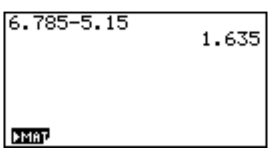
1.



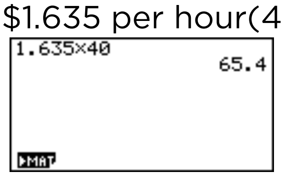
Q1: $6.35 Q2: $6.875 Q3: $7.15

2. Alaska: Q3
Vermont: Q3
Delaware: Q1

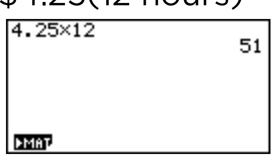
3. Mean = $6.785, Federal Wage = $5.15
Mean state minimum wage is $1.635 more than the federal minimum wage.



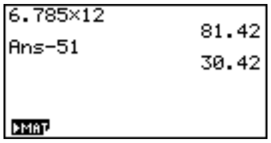
4. $1.635 per hour(40 hours per week) = $65.40 per week



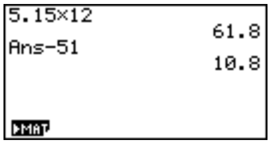
5. $4.25(12 hours) = $51.00



6. $6.785(12 hours) = $81.42
$81.42 - $51.00 = $30.42 more than the federal minimum wage for a student.



7. $5.15(12) = $61.80
$61.80 - $51.00 = $10.80 more than the federal minimum wage for a student.



Moving Along the Ground — Teacher Notes

Topic Area: Translations of Polygons

NCTM Standards:

- Understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, vectors, function notation, and matrices.
- Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations.

Objective

The student will be able to use the properties of translations to represent the vertices of a polygon, write a translation using the form $T_{a,b}(x, y)$ (x + a, y + b), find the new vertices of a polygon given the coordinates of the preimage and find the components of a vector translation and the coordinates of the preimage.

Getting Started

As a class, discuss the definition of a translation and its components. Have them relate this to the parts of a right triangle in terms of trigonometric functions. How would they represent the horizontal and vertical change in a translation? Discuss with them various career areas that use translations in their field such as engineering, aeronautics, and computer graphics.

Prior to using this activity:

- Students should have an understanding of the coordinate plane.
- Students should have an understanding of vectors and magnitude.

Ways students can provide evidence of learning:

- The student will be able to write an equation for a translation for a given problem.
- The student will be able to write a vector in component form and use in to translate a polygon on the coordinate plane.

Common mistakes to be on the lookout for:

- Students may use the wrong operation for a translation.
- Students may confuse the x and y values in the translation.
- Students may use the incorrect trigonometric functions when using vectors.
- Students may not use the magnitude correctly in writing the components.

Definitions

- Translation
- Sine
- Cosine
- Vector
- Magnitude
- Component Form

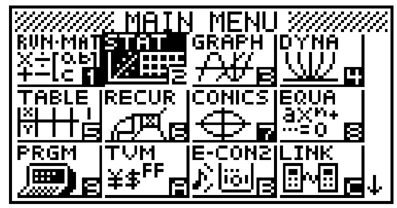
The following will demonstrate how to enter a set of coordinates into two lists using the Statistics mode on the Casio *fx-9750GII* and apply this to solving a problem. Using the vertices of (3, 1), (4, –7), and (–2, 5), find the coordinates of the image under the given translations.

a. $T_{2,-3}(x, y) = (x + 2, y - 3)$

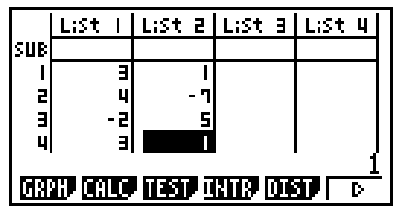
b. Translate according to $\vec{v}$ whose standard position forms a 30° angle and has a magnitude of 6.

To enter values into a list and create a polygon:

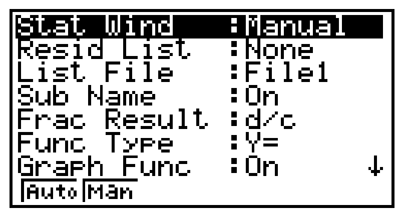
1. From the Main Menu, highlight the STAT icon and press [EXE].

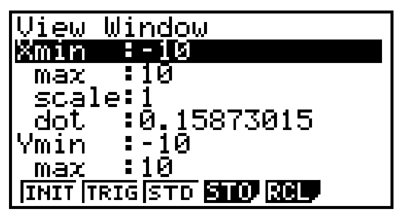


2. Enter the x-values into **List 1** and the y-values into **List 2**. [Note: the first pair of coordinates appears twice so that the line graph will form a polygon.]

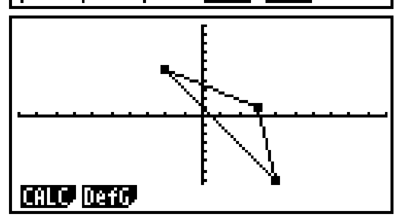


3. Set up the calculator so that the axes can be set up manually by pressing [SHIFT] [MENU] **(SET UP)** [F2] (Man) [EXE] and set up the window by pressing [SHIFT] [F3] **(V-WIN)** [F3] (STD) [EXE].





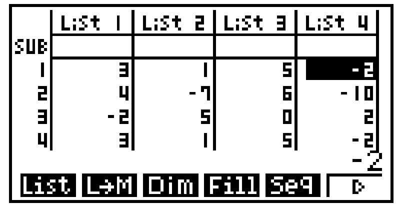
4. To draw the line graph and create the polygon, press [F1] (GRPH) [F6] (SET) [▼] [F2] (XY) [▼] [F1] (List) [1] [EXE] [▼] [F1] (List) [2] [EXE] [EXE] [F1] (GPH1).



To perform the first translation on the list:

1. Press [EXIT] twice to get back to the initial STAT screen.

2. Move the cursor to highlight **List 3** and press [OPTN] [F1](LIST) [F1](List) [1] [+] [2] [EXE]. This will add 2 to each of the x-values.

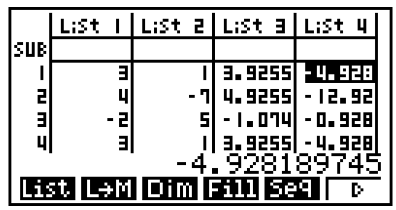


3. Highlight **List 4** and press [F1](List) [2] [−] [3] [EXE]. This will subtract 3 from each of the y-values.

The coordinates of the image are (5, –2), (6, –10), and (0, 2).

To perform the second translation:

1. To translate according to the vector, find the horizontal and vertical components and add these to the x and y values. To do this, highlight **List 3** and press [OPTN] [F1](LIST) [F1](List) [1] [+] [6] [cos] [3] [0] [EXE].



2. Highlight **List 4** and press [F1](List) [2] [+] [6] [sin] [3] [0] [EXE].

[Note: Be sure that the calculator is in the current angle unit.]

Moving Along the Ground — Activity

A landscaper must be a person of many talents, besides having a green thumb. They are artists and engineers who use a variety of mathematics to make dream areas come alive. In this activity, you will explore some of the projects that require the mathematics of translations to create that perfect outdoor paradise.

Questions

The Green Grass Landscaping service is creating several flower beds for a client. The design was drawn on a coordinate grid with the first garden located at points (–5, 4), (–5, 7), (–1, 5), and (–1, 2). Two more are to be created that will be located 5.25 units to the right and 2.5 units up from the previous garden.

1. Write a translation for finding the coordinates of the second and third garden.

   ____

2. Find the coordinates of the second garden.

   ____

3. Find the coordinates of the third garden.

   ____

4. Graph the results of both gardens on the same coordinate plane on your calculator. Sketch a sample below.

5. What is the distance between the two farthest points of the gardens?

A client for Blooms and Branches Landscapers wants to put canvas sails over his patio to provide shade. The sails will be in the form of triangles. In the design plan, the corners of the first sail will be located at points (4, 1), (2, 5), and (8, 10). If the first point is to be moved to (3, 4), answer the following questions:

6. What translation would be applied to the vertices of the first sail to obtain the second set of vertices?

7. What will be the locations of the vertices for the second sail?

8. Graph your results on your calculator and sketch an sample below.

9. Do the two sails overlap to provide a shady area?

In order to preserve a local monument, its garden must be moved 5 ft. at an angle of 40°. On a grid map, the corners are located at (–5, –2.5), (–5, 2.5), (–2.5, 2.5), and (–2.5, –2.5).

10. What would be the horizontal component for this vector to the nearest tenth?

11. What would be the vertical component for this vector to the nearest tenth?

____

12. What are the new coordinates for the four corners of this garden?

____

13. A powerline runs along the x-axis on the grid. If the new garden cannot be underneath this powerline, will this move satisfy this condition?

____

14. There is a street that is located along the y = x line when x > 0. Will this move satistfy this condition? If not, what change needs to be made to that the new garden does not cross this line?

____

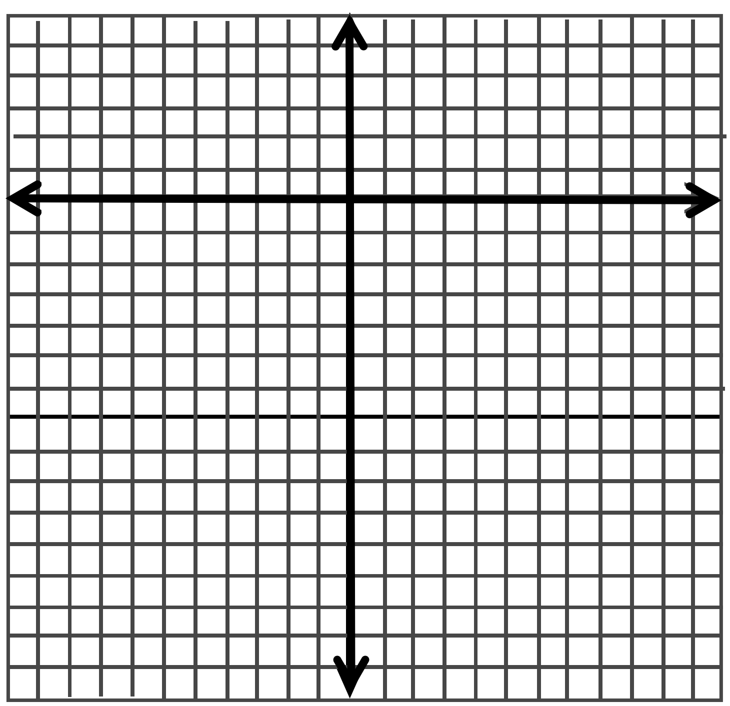
____

Trees are to be planted along a driveway to provide shade and interest. The designer's plan locates the first tree at (–2, 2). The next tree will be 3 ft. away from the first at an angle of –30°. The third tree will be 3 ft. away from the second at an angle of –150°. This pattern will be repeated for a total of six trees.

15. Find the coordinates for the locations for each of the remaining five trees to the nearest tenth.

____

16. Draw a diagram showing the placement of the trees and their coordinates.

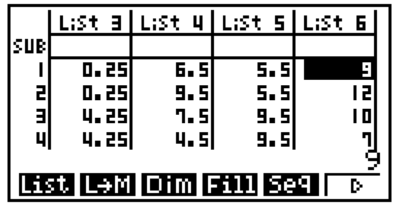


Solutions

1. $T(x, y) \rightarrow (x + 5.25, y + 2.5)$

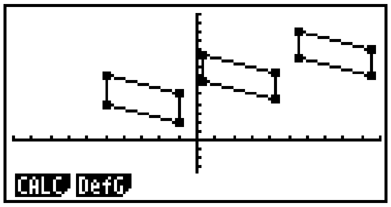
2. (0.25, 6.5), (0.25, 9.5), (4.25, 7.5), (4.25, 4.5)

3. (5.5, 9), (5.5, 12), (9.5, 10), (9.5, 7)



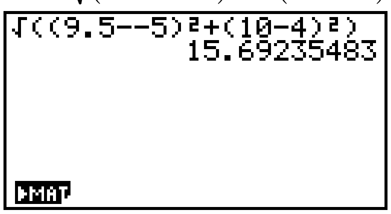
SUB	List 3	List 4	List 5	List 6
1	0.25	6.5	5.5	9
2	0.25	9.5	5.5	12
3	4.25	7.5	9.5	10
4	4.25	4.5	9.5	7

4.



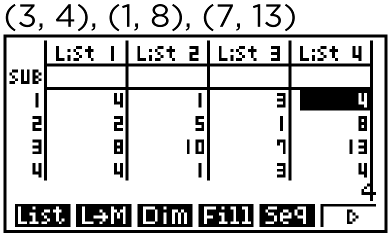
5. Lower Point is (–5, 4) Upper Point is (9.5, 10)

$d = \sqrt{(9.5 - -5)^2 + (10 - 4)^2} \approx 15.69$ units



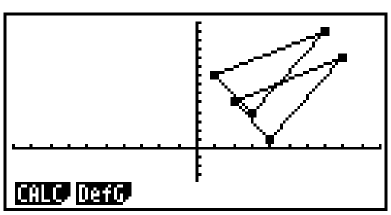
6. $T(x,y) \rightarrow (x - 1, y + 3)$

7. (3, 4), (1, 8), (7, 13)



SUB	List 1	List 2	List 3	List 4
1	4	1	3	4
2	2	5	1	8
3	8	10	7	13
4	4	1	3	4

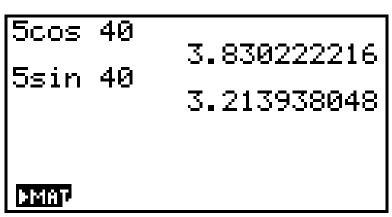
8.



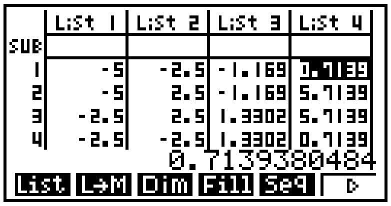
9. Yes

10. Horizontal: 3.8

11. Vertical: 3.2

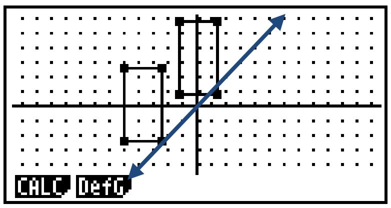


12. (–1.2, 0.7), (–1.2, 5.7), (1.3, 5.7), (1.3, 0.7)



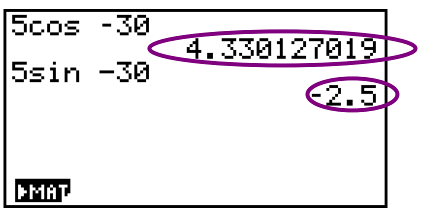
13. Yes

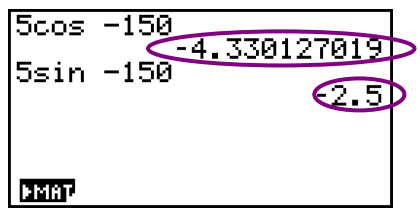
14. No; Answers Will Vary



15. Horizontal Movement (–30°): 4.3
Vertical Movement (–30°): –2.5

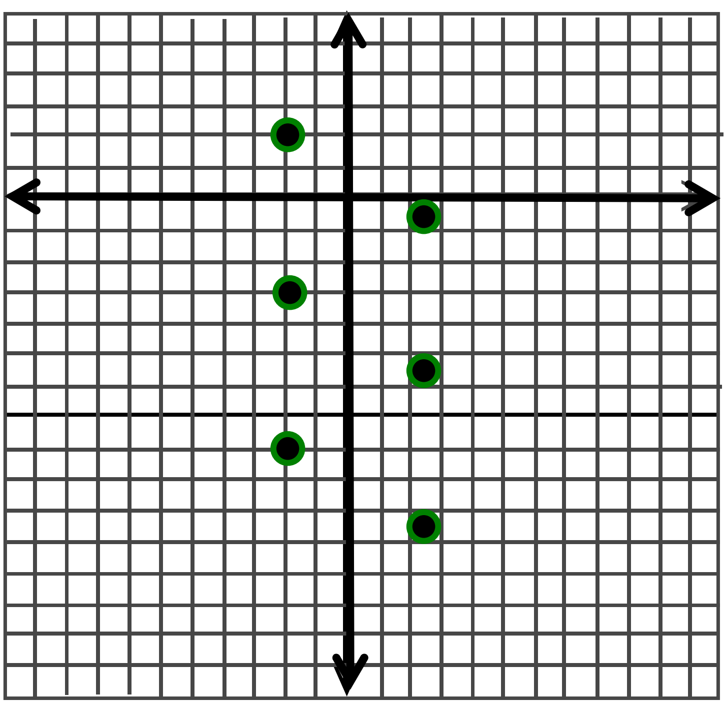
Horizontal Movement (–150°): –4.3
Vertical Movement (–150°): –2.5





Locations: (–2, 2), (2.3, –0.5), (–2, –3), (2.3, –5.5), (–2, –8), (2.3, –10.5)

16.



Topic: Fractions, Decimals, Exponents, and Order of Operations

NCTM Standard:

- All students should work flexibly with fractions, decimals, and percents to solve problems.

Objective

The student will be able to use the Casio *fx-9750GII* to solve problems involving fractions, decimals, exponents, and the order of operations.

Getting Started

Assess students' prior knowledge of fractions and decimals by asking them questions designed to see if they are familiar with simplifying fractions and converting between fractions to decimals. Assess students' prior knowledge about exponents and the order of operations.

Prior to using this activity:

- Students should know what the numerator and denominator of a fraction represent, as well as the definitions of proper fractions, improper fractions, and mixed numbers.
- Students should know the names of whole number and decimal place values.
- Students should know how to evaluate an expression involving exponents.
- Students should know how to raise a power to a power.
- Students should know how to use the order of operations when evaluating a numerical or algebraic expression.

Ways students can provide evidence of learning:

- If given a fraction, students can state its decimal equivalent.
- If given a decimal, students can state its fractional equivalent.
- If given an expression involving exponents, students can evaluate it for a particular value.
- If given an expression using the order of operations, students can determine its correct value.

Common calculator or content errors students might make:

- Students may state an incorrect equivalent value for a fraction and/or decimal.
- Students may multiply the base and the exponent when evaluating an expression involving exponents.
- Students may incorrectly evaluate an expression when using the order of operations.

Operations and Expressions "How To"

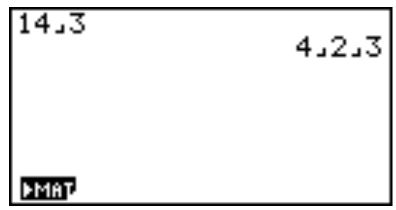
The following will demonstrate how to enter fractions, decimals, and exponents into the CASIO *fx-9750GII.* While the CASIO *fx-9750GII.* uses an algebraic operating system and automatically calculates expressions using the order of operations, the steps below will also show how to use various grouping symbols to change the values of an expression.

To enter a proper fraction:

1. From the Main Menu, highlight the RUN•MAT icon and press [EXE] or [1].

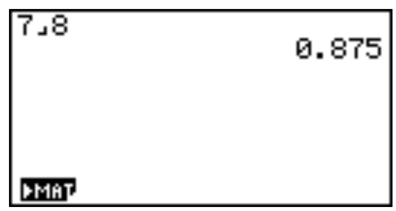
2. To enter $\frac{4}{5}$, input [4] [a b/c] [5] [EXE].

To convert an improper fraction to a mixed number:

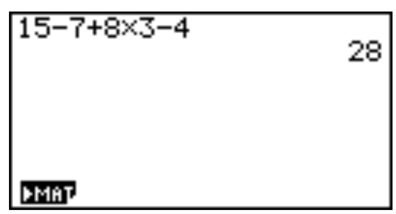


1. To change $\frac{14}{3}$ into a mixed number, input:
   [1] [4] [a b/c] [3] [EXE] [SHIFT] [F↔D].

To determine the decimal equivalent of a fraction:

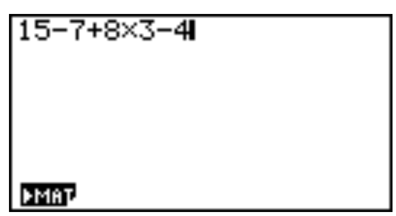


1. To find the decimal equivalent of $\frac{7}{8}$, input:
   [7] [a b/c] [8] [EXE] [F↔D].



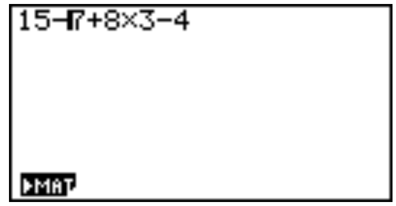
To evaluate an expression:

1. To evaluate the expression 15 – 7 + 8 • 3 – 4,
   input:
   [1] [5] [−] [7] [+] [8] [×] [3] [−] [4] [EXE].

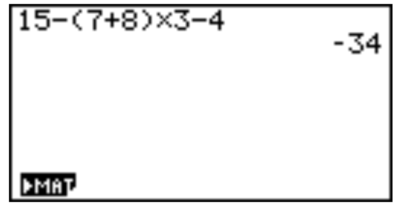


2. To insert a set of parentheses around 7 + 8, [◀] on the replay pad. The cursor appears at the end of the expression and the previous answer is no longer displayed.

3. Continue to [◀] until the cursor is located directly to the left of the 7.



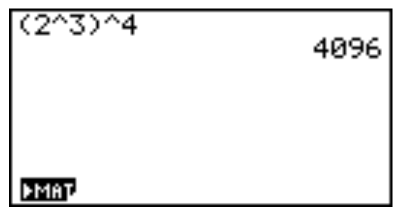
4. Press [(] and [▶] to move the cursor to the right of the 8 and press [)] then [EXE] to calculate the new answer.



To enter an expression using exponents:

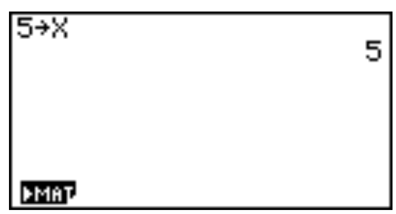
1. To evaluate 9^2, enter [9] [^] [2] then [EXE].

2. To raise a power to a power, for example: $(2^3)^4$, input: [(] [2] [^] [3] [)] [^] [4] [EXE].

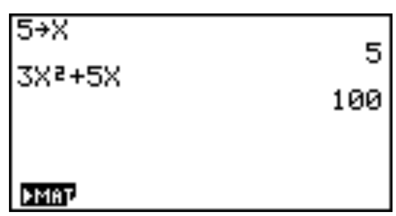


To store a value for a variable:

1. To store 5 as the x-value, input: [5] [→] [X,θ,T] [EXE].



2. To evaluate $3x^2 + 5x$, when x = 5, press [3] [X,θ,T] [x^2] [+] [5] [X,θ,T] [EXE].



Being able to express fractions and decimals equivalently is an important skill we need to master. Often times, people will use these numerical representations interchangeably based upon a given situation. In this activity, we will explore various problems where fractions and decimals are used.

Questions

1. A local electronics stores is advertising a sale on flat-screen televisions for $\frac{1}{3}$ off. If a flat-screen television normally sells for $1,290, what is the sale price of the television set? How did you arrive at your answer?

2. A cookie recipe calls for one-half cup of chocolate chips, one-fourth cup of walnuts, and one cup of flour. If you were to make this recipe six times, how many cups of each ingredient would you need? Explain how you came up with the answer.

3. A marathon measures 26.2 miles. Bobby says that he currently runs thirty-five hundredths of the distance. How far can Bobby run? How did you set up your problem?

4. Carolyn goes to the supermarket on Monday and spends $24.79. On Tuesday, she fills her car's gas tank and it costs her $18.26. On Wednesday, she goes to dinner with some friends and spends $12.99. Thursday she doesn't spend any money but on Friday, she goes to the mall and spends $84.18 on some clothes. How much does she spend in total?

5. Your digital video recorder is currently $\frac{2}{5}$ filled with various programs. If the recorder can hold a maximum of 80 hours of programming, how many hours are left to record? Explain your answer.

These questions directly relate to exponents.

6. Evaluate $1^2 + 2^2 + 3^2 + 4^2 +5^2 + 6^2 +7^2 + 8^2 +9^2 + 10^2$.

7. Which expression is greater and why? 2^{-3} or 3^{-2}

8. Choose any positive integer and raise it to the zero power. Try several different positive integers raised to the zero power. What do you notice about any positive integer raised to the zero power?

9. If $x = 4$, find the difference between $3x^2$ and $(3x)^2$.

10. If $x = 5$, what is the value of x^2 and $(x^2)^2$?

These questions directly relate to the Order of Operations.

11. Evaluate the expression: $18 + 7 – 9 • 3 + 4$.

12. Refer to question 11 and insert a set of parentheses so the expression will equal 16.

____

13. Evaluate the expression: $15 - 5^2 - 4^3 \div 2$.

____

14. Refer to question 13 and insert a set of parentheses that will not change the original answer.

____

15. Evaluate the expression: $2 + 3 - 4 \cdot 5 - 6 + 7$.

____

16. Refer to question 15 and insert a set of parentheses so the expression will equal 6.

____

17. In the blank below, describe the error made in the evaluation process for the following expression. To the right of the expression, show the correct steps.

	Correction
$(20+5^2)\div 10$	$(20+5^2)\div 10$
$(25^2) \div 10$	__________________
$625 \div 10$	__________________
62.5	__________________

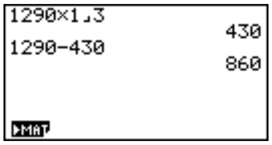
____

____

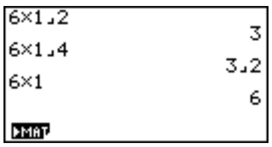
____

Solutions

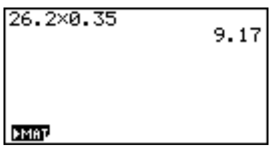
1. The price of the television set is $860.



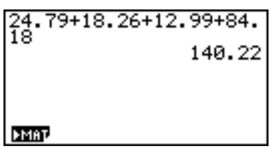
2. You will need 3 cups of chocolate chips, $\frac{3}{2}$ or $1\frac{1}{2}$ cups of walnuts, and 6 cups of flour.



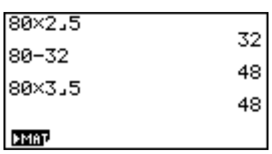
3. Bobby is currently running 9.17 miles.



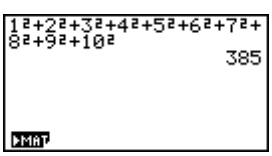
4. Carolyn spends $140.22 in total.



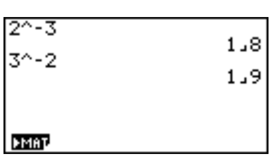
5. There are 48 hours of programming left on the digital video recorder.



6. The answer to $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2$ is 385.

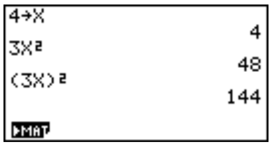


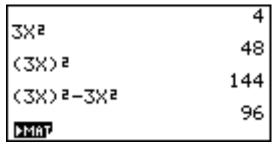
7. 2^{-3} is greater than 3^{-2} because the fractional equivalent of 2^{-3} is $\frac{1}{8}$ and the fractional equivalent of 3^{-2} is $\frac{1}{9}$. Since $\frac{1}{8}$ is greater than $\frac{1}{9}$, 2^{-3} is greater.



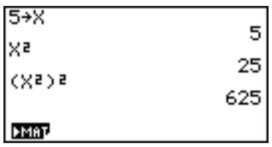
8. Any positive integer raised to the zero power is 1.

9. The difference between $3x^2$ and $(3x)^2$ is 96.

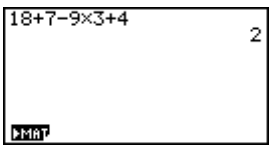




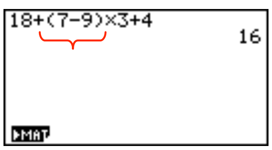
10. The value of x^2 = 25 and the value of $(x^2)^2$ = 625.



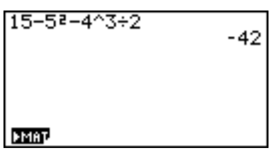
11.



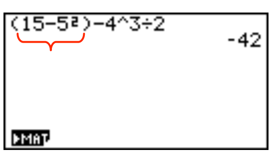
12.



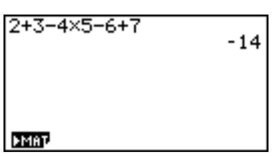
13.



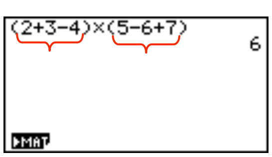
14.



15.



16.



17. 20 was added to 5 before five was multiplied by itself. Exponents should always be evaluated before addition.

Correct Steps

$(20 + 5^2) \div 10$

$(20 + 25) \div 10$

$45 \div 10$

4.5

Topic Area: Hyperbolas

NCTM Standards:
- Interpret representations of functions of two variables.
- Solve problems that arise in mathematics and in other contexts.
- Recognize and apply mathematics in contexts outside of mathematics.

Objective

The student will be able to write an equation for a hyperbola, evaluate the hyperbola for a specified location, and use the equation of the hyperbola to solve problems involving navigation.

Getting Started

Discuss with students the properties of a hyperbola. Use a diagram of a hyperbola to show the foci and vertices. Discuss how hyperbolas are used in locating ships via radio waves.

Prior to using this activity:
- Students should have an understanding of hyperbolas.
- Students should have an understanding of the Pythagorean Theorem.

Ways students can provide evidence of learning:
- The student will be able to write an equation given the foci and a point on the hyperbola.
- The student will be able to solve problems involving navigation of ships and apply this to other areas such as location of aircraft.

Common calculator or content errors students might make:
- Students may use the formula for an ellipse instead of a hyperbola.
- Students may use the wrong form of the hyperbola.

Definitions:
- LORAN

Formulas

Hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Pythagorean Theorem: $a^2 + b^2 = c^2$

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

The following will demonstrate how to enter an equation and a formula into the Equation Solver Function of the Casio *fx-9750GII* and graph the resulting equation.

Enter the equation $\sqrt{(x-8)^2+5^2} = 13$ and solve for x.

Enter the formula $a^2+b^2=c^2$ and find the value of b when a = 6 and c = 15.

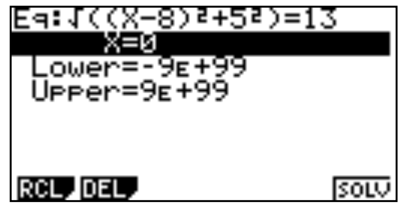
To enter an equation into the Solver function:

1. Highlight the EQUA icon from the Main Menu and press [EXE]. To select Solver, press [F3].

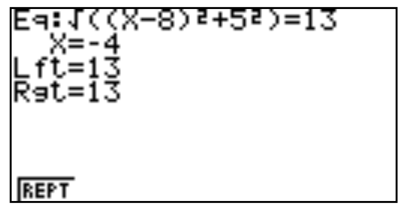
   To enter the formula, input the following:

   [SHIFT] [x^2] [(] [(] [X,θ,T] [−] [8] [)] [x^2] [+] [5] [x^2] [)] [SHIFT] [•] [1] [3] [EXE].

   The screen should look like the one to the right.



2. Press [F6] (Solv) to see the solution shown at the right.



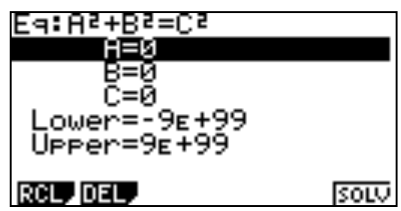
To use the Solver function with a formula:

1. Highlight the EQUA icon from the Main Menu and press [EXE]. To select Solver, press [F3]. To enter the formula input the following:

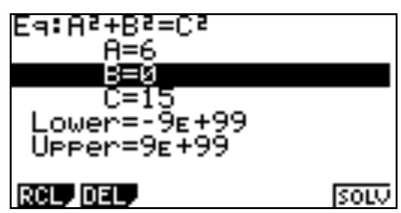
   [ALPHA] [X,θ,T] [x^2] [+] [ALPHA] [log] [x^2] [SHIFT] [•] [ALPHA] [ln] [x^2] [EXE].

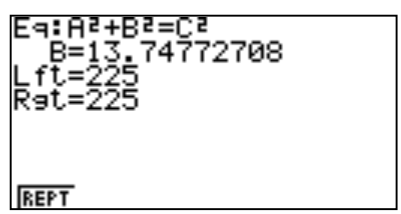
   Note: To input a variable, press [ALPHA] then the key associated with the letters written in red.

   The screen should look like the one to the right.



2. To solve for an unknown variable, enter each of the known values and press [EXE]. Use the arrow keys to highlight the unknown value and press [F6] (Solv).



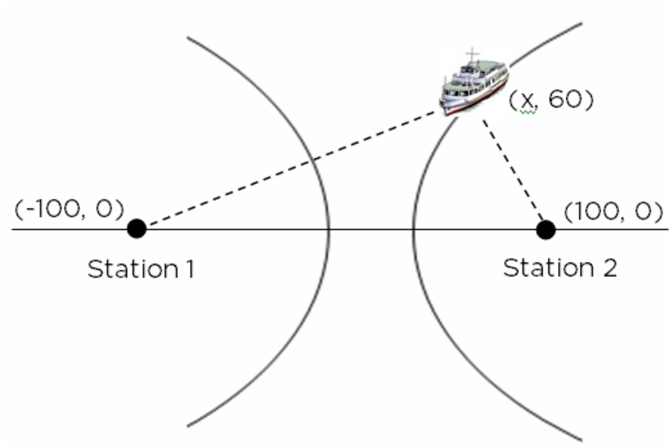


Out to Sea   Activity

To locate the position of aircraft and ships, navigators use the LORAN system, which stands for long-distance radio navigation for aircraft and ships. The system uses synchronized pulses that are sent from transmitting stations which are located at the foci of a hyperbola. These pulses travel at the speed of light (186,000 miles per second) and represent the difference in the times of arrival of an aircraft or ship which is constant on a hyperbola.

In this activity, you will be given the location of two stations on shore, 200 miles apart and positioned on a graph at (-100, 0) and (100, 0). A ship is traveling on a path with coordinates (*x*, 60). The time difference between the pulses from the two transmitting stations is 1000 microseconds (0.001 seconds) and light travels at 186,000 miles per second. Using this information, you will find the x-coordinate for the position of the ship and you will determine where the ship will dock on shore.

Questions



1. Use the information above to determine the difference in the distance the pulses travel.

   ____

2. Using the distance formula, find the x-coordinate for the position of the ship.

   ____

3. Find the values of c, a, and b for this hyperbola.

   c = ______; a = _______; b = _______

4. Find the equation for the hyperbola that represents these two transmitting stations.

   ____

5. What will the x-coordinate be if the ship is 85 miles off shore?

   ____

6. How far is the ship from Station 2 at this point?

____

7. Find the distance from shore for the given x-coordinates.

   a. x = 150 __________   b. x = 95 __________

Extensions

1. A distress call comes into Station 1 from a ship starting 45 miles off shore. What are the coordinates of the ship?

____

2. What is the distance of the ship from Station 1?

____

3. A rescue helicopter located at Station 1 can travel at 80 mph. A rescue vessel located at Station 2 can travel 75 mph. How long would it take each to reach the disabled ship?

____

Solutions

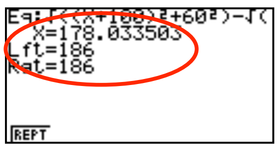
1. 186,000 (.001) = **186 miles**

2. d_1 (Station 1 to Ship) = $\sqrt{(x+100)^2+60^2}$

   d_2 (Station 2 to Ship) = $\sqrt{(x-100)^2+60^2}$

   $$\sqrt{(x+100)^2+60^2} - \sqrt{(x-100)^2+60^2} = 186$$

   x = 178

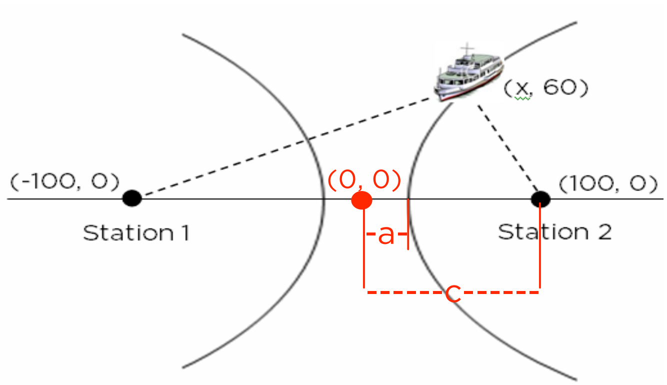


3. c = 100; a = $\frac{186}{2}$ = 93

   $c^2 - a^2 = b^2$

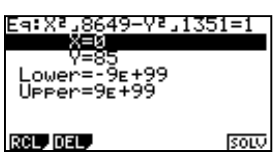
   $100^2 - 93^2 = 1351 = b^2$

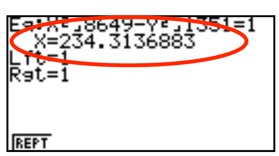
   b = 36.8



4. $$\frac{x^2}{8649} - \frac{y^2}{1351} = 1$$

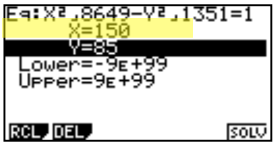
5. x = 234.3 mi.

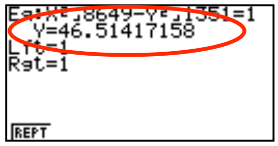




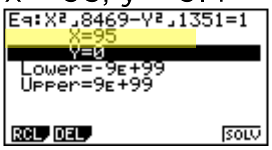
6. $d = \sqrt{(234.3-100)^2+85^2}$ = 158.9 mi.

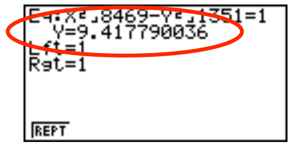
7. x = 150; y = 46.5





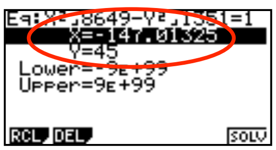
x = 95; y = 9.4



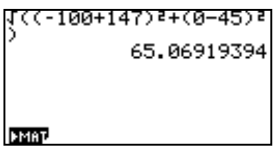


Extension Solutions

1. Coordinates: (-147, 45)



2. Station 1; 65 miles away



3. Helicopter : 48.75 min.
Rescue Vessel: 3.3 hrs.

Painting Roses — Teacher Notes

Topic Area: Rose (rhodonea) curves

NCTM Standard

- Make and investigate mathematical conjectures.

Objectives

- Predict the equation for a sinusoid with a desired number of petals.
- Determine the area of the sinusoid.

Getting Started

This activity will encourage students to use graphical and numerical representations to examine the behavior of a sinusoid plotted in polar coordinates of the form $r = a \sin(k\theta)$. The idea is to have students investigate, both numerically and graphically, the behavior of the rose curve and to determine the equation needed to construct a desired flower.

Prior to using this activity:

- Students should be able to produce and manipulate polar graphs manually and with a graphing utility.
- Students should have a basic understanding of rose curves.
- Students should be able to use formulas to determine area.

Ways students can provide evidence of learning:

- If given a number of petals, the student can state and explain what polar equation should be used.
- If given an equation of a rose curve, the student can evaluate the area of the sinusoid.
- If given the area of the sinusoid, the student will be able to determine the amount of paint needed to paint the rose curve on the plates.

Common mistakes to be on the lookout for:

- Students may choose the wrong equation.

Definitions:

- Integer
- Polar Equations
- Rhodonea curve
- Rose curve
- Sinusoids
- $r = a \sin(k\theta)$
- $A = \dfrac{\pi a^2}{2}$, if k is even
- $A = \dfrac{\pi a^2}{4}$, if k is odd

Gabriella has designed a set of 10 dinner plates. She wants to paint a flower in the shape of a sinusoid with 8 petals in the center of each plate. Gabriella wants each petal to be 2 inches and painted with expensive precious metal gold paint. The paint costs $32 for 2 grams. Two grams of paint covers 4 square inches. How many containers of paint will Gabriella need to order to complete her project? How much will the paint cost?

To determine the sinusoid formula to use:

1. If k is an integer, the curve will be rose shaped with
   - 2k petals if k is even, and
   - k petals if k is odd

   Since Gabriella needs 8 petals, k = 4.

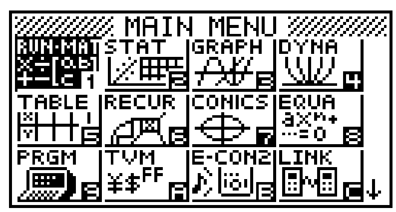
2. The length of each petal is a. Since Gabriella wants each petal to be 2 inches, a = 2.

3. Sine and cosine rose curves are identical, except that the cosine curve is rotated $\frac{\pi}{2k}$ radians. On a plate, the rotation would not matter. Therefore, you could choose either of the two trigonometric functions. We will use sine in this activity.

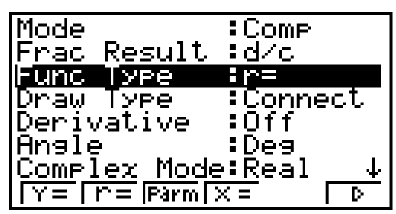
4. Using the formula r = a sin(kθ), Gabriella's formula is: r = 2 sin (4θ).

To set calculator to graph polar equations:

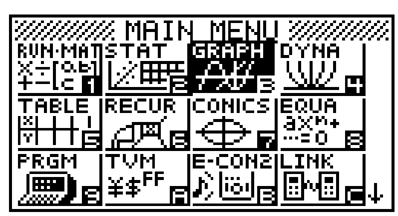
1. From the Main Menu, highlight the RUN•MAT icon and press [EXE] or press [1].



2. Press [SHIFT] [MENU] (SET UP), [▼] to **Func Type** and press [F2] (r=) to change to r=, as shown on the right.

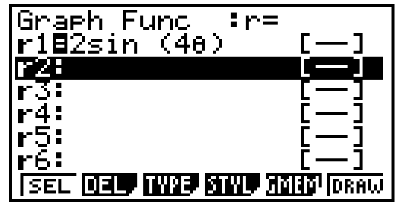


3. Return to the Main Menu, highlight the GRAPH icon and press [EXE] or press [3].

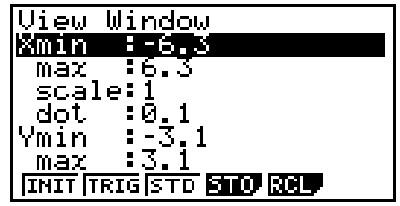


4. Enter the new equation, by pressing

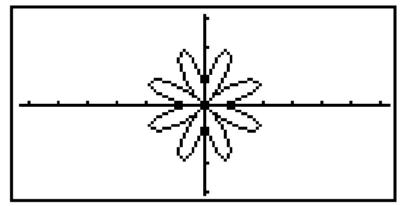
[2] [sin] [(] [4] [X,θ,T] [)] [EXE].



5. Set the viewing window by pressing [SHIFT] [F3] **(V-Window)** [F1] (INIT).



6. Press [EXIT] to return to the original screen. Press [F6] (DRAW) to view the graph of the function.



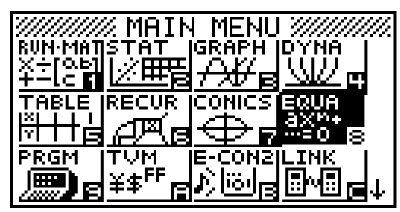
To decide which area formula to use:

1. There are two area formulas to choose from:

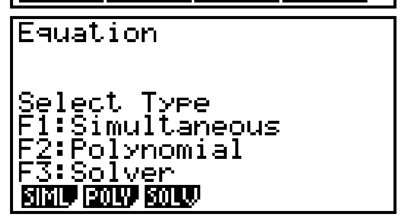
$A = \frac{\pi a^2}{2}$, if k is even or $A = \frac{\pi a^2}{4}$, if k is odd

2. Since k = 4, Gabriella will need to use the first formula.

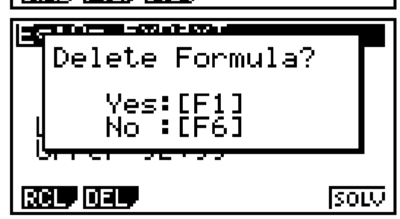
To calculate the area of the sinusoid:



1. From the Main Menu, highlight the EQUA icon and press [EXE] or press [8].



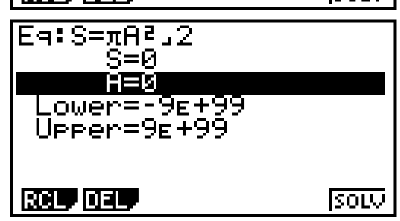
2. Press [F3] (Solver). To delete a previous equation, pressing [F2] (DEL) and [F1] (Yes)



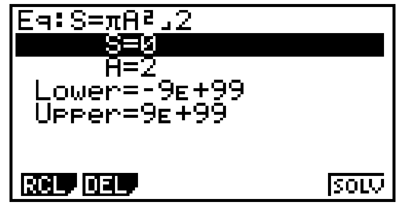
3. The equation will need to be changed from

$A = \frac{\pi a^2}{2}$ to $S = \frac{\pi a^2}{2}$ since the calculator is not able to accept two different A variables; S will represent Area. To input the equation, press:

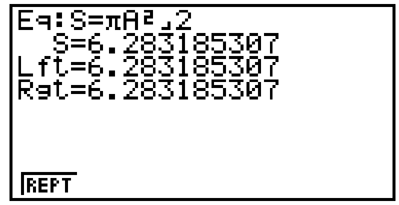
[ALPHA] [X] [SHIFT] [•] [SHIFT] [EXP] [ALPHA] [X,θ,T] [x^2] [$a\frac{b}{c}$] [2] [EXE].



4. Make sure S = 0. Press [2] [EXE] to set A = 2 inches. Highlight the line S = 0 to tell the calculator that you are solving for S.



5. Press [F6] (SOLV) to find the area of the sinusoid.



Painting Roses — Activity

Introduction

This activity will encourage you to use represent sinusoids graphically and numerically. In polar form, the sinusoid is represented by r = $a \sin(k\theta)$. If k is an integer, remember that the curve will be a rose shape. If k is even, then there will be 2k petals. If k is odd, then there will be k petals. The variable *a* represents the length of each petal.

To find the area of the rose curve, there are two formulas that you will choose from. If k is even, you will use A = $\frac{\pi a^2}{2}$. If k is odd, you will use A = $\frac{\pi a^2}{4}$.

Questions

Determine the equation of the following rose curves with the given characteristics, using the sine function. Then, draw a sketch of the graph.

1. A flower with 4 petals and the length of each petal 1 inch.

2. A flower with 9 petals and the length of each petal 3.5 inches.

Determine the area of the following sinusoids.

3. A flower with 4 petals and the length of each petal 1 inch.

____

4. A flower with 9 petals and the length of each petal 3.5 inches.

____

Alice has designed a set of 4 dinner plates. She wants to paint a flower in the shape of a sinusoid with 12 petals in the center of each plate. Alice wants each petal to be 1.5 inches and painted with expensive precious metal gold paint. The paint costs $32 for 2 grams. Two grams of paint covers 4 square inches. How many containers of paint will Alice need to order to complete her project? How much will the paint cost?

5. What equation should Alice use?

____

6. Graph your equation using the sine function.

7. Graph your equation using the cosine function.

8. Does it matter whether Alice uses the sine or cosine function? Why?

9. What is the area of the sinusoid?

10. What is the total area the flower will cover on the 4 plates?

11. How many containers of the paint will Alice need?

12. What will be the cost of the gold metal paint?

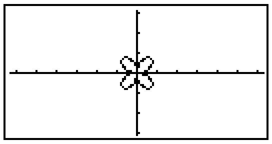
Extensions

1. Determine the equation of the rose curve that has 8 petals, each petal being 2 inches, using the cosine function. Then, draw a sketch of the graph.

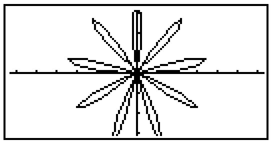
2. Determine the area of the sinusoids that has 8 petals, each petal being 2 inches, using the cosine function.

Solutions

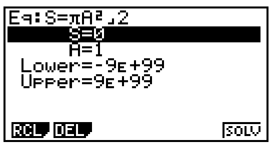
1. r = sin(2θ)

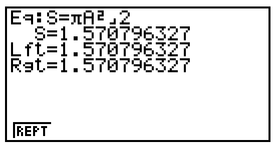


2. r = 3.5sin(9θ)

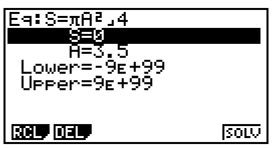


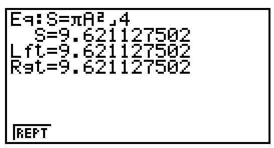
3. Area = 1.570796327 sq.in.





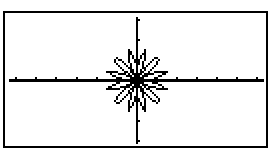
4. Area = 9.621127502 sq.in.



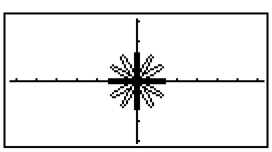


5. r = 1.5sin(6θ) or r = 1.5cos (6θ)

6.

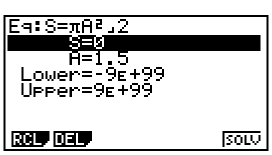


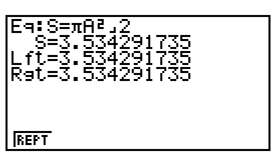
7.



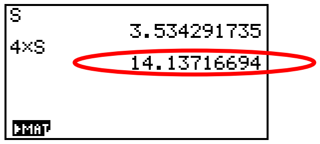
8. No. The cosine function is just a rotation of the sine function. On a round plate, the rotation will not make a difference.

9. Area = 3.534291735 sq. in.

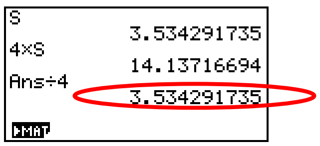




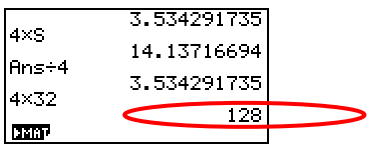
10. Area = 14.13716694 sq. in.



11. 4 containers

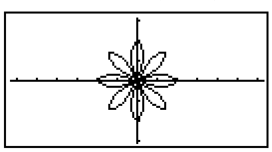


12. $128

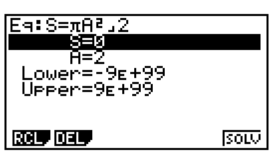


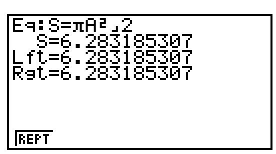
Extension Solutions

1. r = 2 cos (4θ)



2. Area = 6.283185307 sq. in.





Topic Area: Sequences and Series

NCTM Standards:

- Use a variety of symbolic representations, including recursive and parametric equations, for functions and relations.
- Use symbolic expressions, including iterative and recursive forms, to represent relationships arising from various contexts.

Objective

The student will be able to write a recursive function, graph the function using a graphing calculator and solve problems based on the graph of the recursive function.

Getting Started

Have the students work in pairs or in small groups to research the current retirement age in order to collect social security payments. Students should research and discuss various investment options and the corresponding annual interest rates. After completing the research and discussion, students should determine how much money they feel would be needed to supplement their retirement income per month.

Prior to using this activity:

- Students should have an understanding of sequences and series.
- Students should have an understanding of the compound interest formula.

Ways students can provide evidence of learning:

- The student will be able to create an equation of a sequence for a given situation.
- The student will be able to graph the equation and solve problems using the information derived from the graph.

Common calculator or content errors students might make:

- Students may use annual interest instead of monthly interest.
- Students may confuse the formulas for simple interest and compound interest.

Definitions:

- Annual Percentage Rate (APR)
- Compound Interest
- Continuous Interest

Formulas: Compound Interest: $A_1 = A_0\left(1+\frac{r}{n}\right)^{rt}$

Continuous Interest: $A_1 = A_0e^{rt}$

The following will demonstrate how to enter an equation into the Recursion Function of the Casio *fx-9750GII* and graph the resulting equation.

Enter the formula $A_{n+1} = A_n(1.5) - 200$ and set up the parameters.

Graph the formula using the calculator and find the x- and y-intercepts using the Trace Function.

To enter a formula into the Recursive Function:

1. From the Main Menu highlight the RECUR icon and press [EXE]. Press [F4] (n a_n), then [F2] (a_n).

   Now enter:

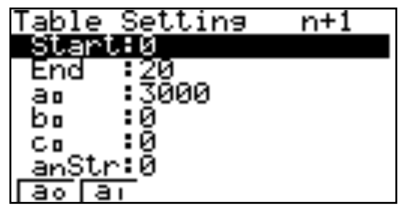
   [(] [1] [•] [0] [5] [)] [−] [5] [0] [0] [EXE].

   The screen should look like the one at the right.

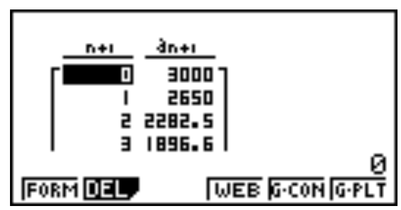
2. To set up the parameters for the formula, press [F5] (Set) then [F1] (a_0). To start the set up for the formula, enter the following:

   [0] [EXE] [2] [0] [EXE] [3] [0] [0] [0] [EXE].

   The screen should look like the one at the right

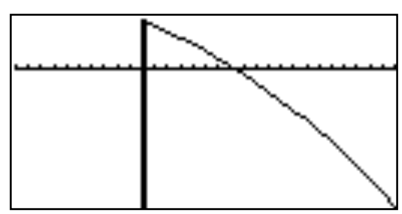


3. Press [EXE] [F6] (TABL) to see the table of values.



To graph the recursive function:

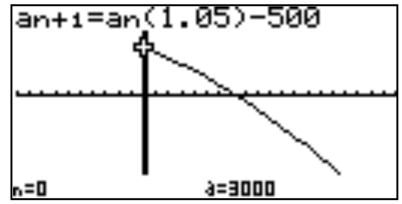
1. Press [F5] [F2] **(Zoom)**, then [F5] (Auto) to see the graph for the table of values. The screen should look like the one at the right.



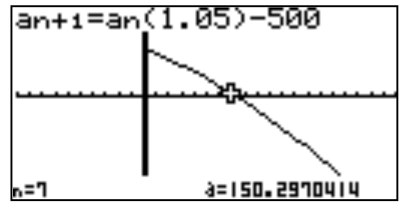
2. The [◀] [▶] [▲] [▼] keys can be used to shift the graph, if necessary.

To trace the graph:

1. To trace the graph, use [F1] **(Trace)** and the arrow keys to move along the graph.



2. Move the cursor until the bottom screen display shows n = 0; this is the beginning amount. Now move the cursor so that n = 7. This is the last positive amount, therefore a_{n+1} will equal 0 somewhere between n = 7 and n = 8. See the screen shots to the right.



When you are young and just starting out on your own, you may not be thinking about retirement. However, this is the perfect time to start thinking about how much money it will take to maintain your lifestyle. Investing in an annuity is a wise decision but just how long will it last? Investing just $2,400 each year compounded continuously for 25 years at 12% APR yields approximately $88,000. Would this be enough?

In this activity, you will discover how long it would take for a balance in an annuity to reach $0 if the rate of interest for the annuity is 7.5% APR compounded monthly and $3,000 is withdrawn each month starting with the first month of retirement. You will then determine the amount that would have to be invested at 10% APR to achieve that beginning balance if interest is compounded continuously.

Questions

1. How much money should be invested initially in order to have a balance of $125,000 in 45 years, if the investment earns 10% compounded continuously?

____

2. How much money should be invested initially in order to have a balance of $250,000 in 45 years, if the investment earns 10% compounded continuously?

____

3. How much money should be invested initially in order to have a balance of $350,000 in 45 years, if the investment earns 10% compounded continuously?

____

4. Using the formula for compound interest, $A_1 = A_0\left(1+\frac{r}{n}\right)^{nt}$, and an APR of 7.5%, calculate the value of $\left(1+\frac{r}{n}\right)^{nt}$ if the number of interest periods, *n*, is 12 and the *t* is 1 month. (Hint: What part of a year is 1 month?)

____

5. Write a recursive equation that shows an annuity that earns 7.5% APR and includes a withdrawal of $3,000 each month.

____

6. Given an annuity that has a balance of $125,000, graph the equation in Question 3 and find the number of months until the balance is near or at $0.

____

How many years does this represent?

____

7. If the same annuity has a balance of $250,000, graph the equation in Question 3 and find the number months until the balance is near or at $0.

____

How many years does this represent?

____

8. If the same annuity has a balance of $350,000, graph the equation in Question 3 and find the number of years until the balance is near or at $0.

____

9. If your monthly income were $1,400, which of the above investment amounts would be the best choice? Explain your answer choice.

____

____

10. If your monthly income were $3,300, which of the above investment amounts would be the best choice? Explain your answer choice.

____

____

11. Would it be a good idea to depend on an annuity as the only source of income for retirement? Justify your answer.

____

____

12. How does the number of years needed to accumulate the different balances relate to the number of years for available withdrawal?

____

____

Extensions

1. Using the same interest rates given in the activity, determine the amount of an annuity at which the annuity would start to gain value, even though money is being withdrawn.

____

2. Draw a graph to compare the initial annuity balances and the number of years they would be available for withdrawal. The average life expectancy is approximately 78 years or 13 years past current retirement ages. Using the graph, determine the balance needed in an annuity in order to withdraw $3,000 a month of supplemental income from the account for at least 13 years.

____

3. What factors would affect one's ability to invest money in an annuity?

____

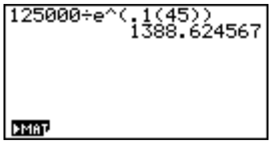
4. If an investment is not used as a supplemental retirement income, what other reasons might someone have an annuity?

____

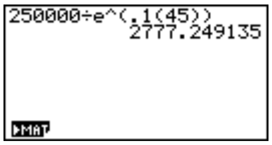
____

Solutions

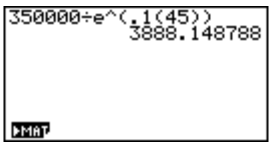
1. $1,388.63



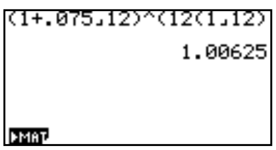
2. $2,777.25



3. $3,888.15

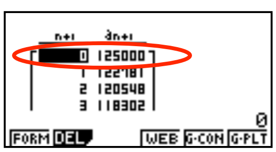


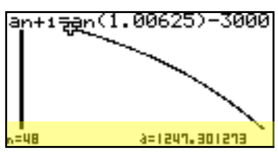
4. 1.00625



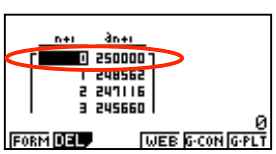
5. $A_{n+1} = A_n\ (1.00625) - 3000$

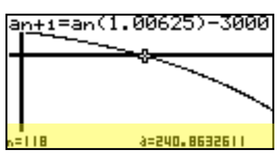
6. 48 months; 4 years



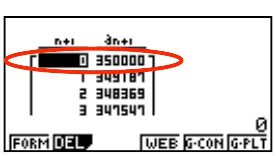


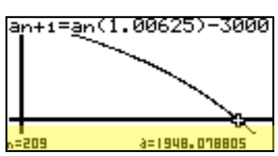
7. 118 months; 9 years 10 months





8. 17 years 5 months





9. Answers will vary according to experience.

10. Answers will vary according to experience.

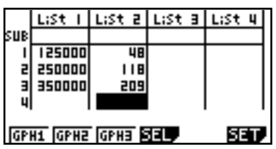
11. Answers will vary according to experience.

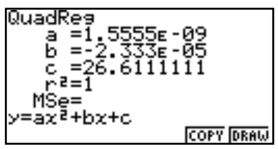
12. Answers will vary according to experience.

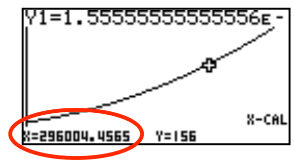
Extension Solutions

1. At approximately $500,000, the annuity would begin gaining value even though money is being withdrawn.

2. $296,004.46







3. Answers will vary according to experience.

4. Answers will vary according to experience.

Topic Area: Trigonometric Basics

NCTM Standard:

- Use mathematical models to represent and understand quantitative relationships.
- Understand functions by interpreting representations of functions.
- Compute fluently by developing fluency in operation with real numbers using technology for more complicated cases.

Objective

Given a set of data, the student will be able to create a periodic function that best fits the given data.

Getting Started

Have the students work in pairs or small groups and discuss various quantities that may be interrelated, such as: the demand for an item versus its availability, or the number of seniors attending high school prom compared to the number of seniors attending the high school. Students should indicate how the size of one quantity is related to the size of the other quantity.

Prior to using this activity:

- Students should be able to enter data and graph on a calculator.
- Students should be able to estimate features of a graph.
- Students should understand how to perform a trigonometric regression.
- Students should have a basic understanding of interpreting statistical graphs.

Ways students can provide evidence of learning:

- Given a set of data, the student will be able to interpret the y-intercept, amplitude, phase shift, and period of the resulting graph as it relates to the given situation.
- Given a set of data, the student will be able to enter the data, create a trigonometric function to model the data, and calculate the trigonometric regression.

Common mistakes to be on the lookout for:

- Students may create a typographical error when entering the set of data.
- Students may create a dimension error when entering the set of data.
- Students may pick an incorrect regression that does not best describe the situation.

Definitions

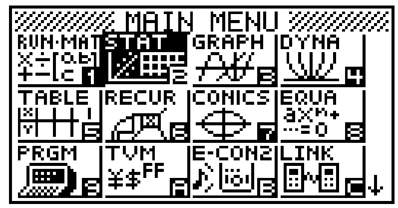
- Amplitude
- Phase shift
- Period
- Fluctuation
- Interrelated
- Regression

The following will demonstrate how to: enter data into the Casio *fx-9750GII*, configure each StatGraph, graph the data using a scatter plot, trace the graph to determine significant features of a trigonometric function, and define a function to model the data.

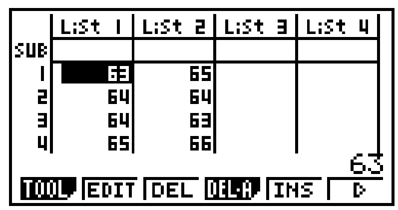
A Ferris wheel that stands 4 feet above the ground has a radius of 20 feet and makes a complete revolution in 10 seconds. The table contains data pertaining to the relationship between the height *h* of a rider above the ground and time *t*.

Time (sec)	0.00	1.25	2.50	3.75	5.00	6.25	7.50	8.75	10.00
Height (ft)	4	14	24	34	44	34	24	14	4

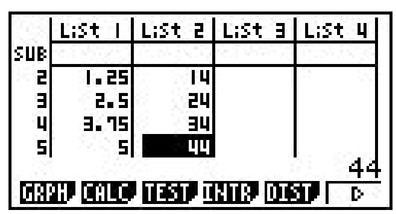
To enter the above set of data:



1. From the Main Menu, highlight the STAT icon and press [EXE] or press [2].



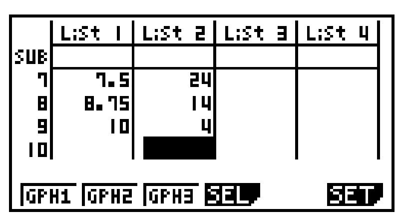
2. To clear previous data, press [F6] (▷), [F4] (DEL-A), [F1] (Yes), and [F6] (▷).



3. Enter the data by typing each time into **List 1** and each height into **List 2**, pressing [EXE] after each entry.

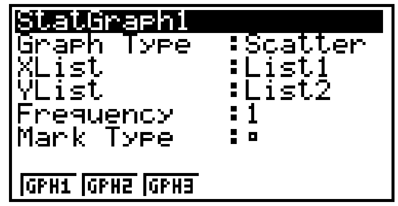
4. The display should look like the screen shot on the right when completed.

To select the type of graph for this data:



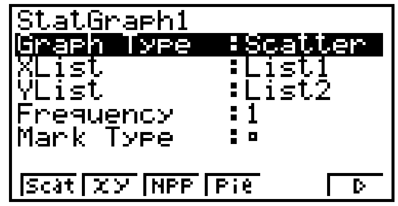
1. Press [F1] (GRPH).

2. Select F6 (SET) to set the type of graph for **StatGraph1**.



3. Press ▼ to highlight **Graph Type**.

4. There are five choices; Scat, XY, NPP, Pie, and (▷) .

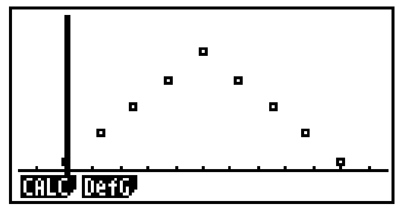


5. Select F1 (Scat) to make **StatGraph1** a scatter plot.

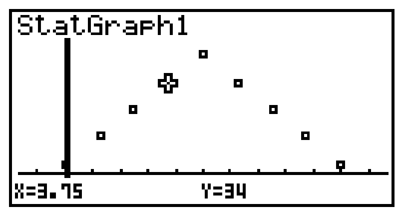
6. Press ▼ to select **List 1** as the **XList**, and ▼ again to select **List 2** as the **YList**.

To display a graph of the data:

1. Press EXIT F1 (GPH1).

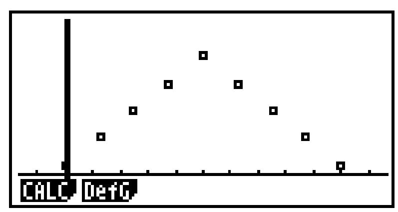


2. To explore information about this graph, press SHIFT F1 **(TRACE)** and then ◀ ▶ to move between the data points.

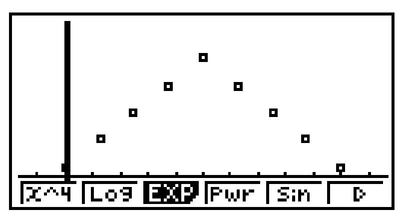


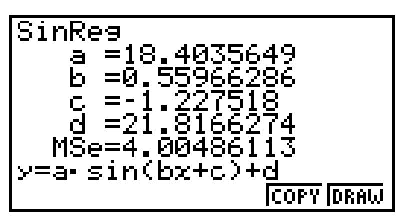
To perform a regression on the data:

1. Press EXIT F1 (CALC) .

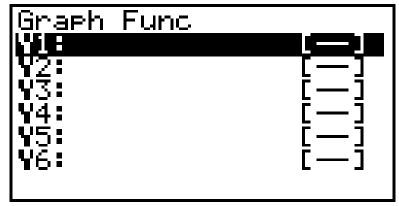


2. Press F6 (▷) F5 (Sin) to display the regression.

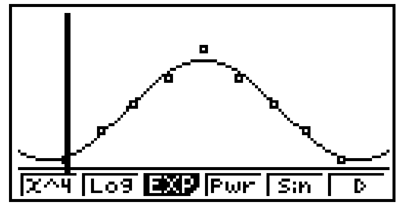




3. To copy the equation in to the Graph mode, press [F5] (COPY) then ▲ ▼ to chose an appropriate line to place the function. Then, press [EXE] to save the equation.

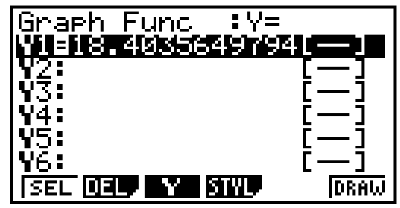


4. Press [F6] (DRAW) to sketch the regression over the data.



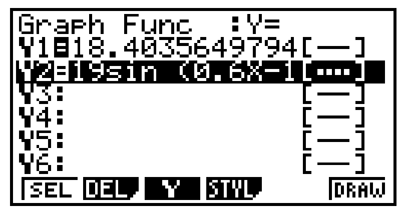
To perform a manual regression on the data:

1. Press [EXIT] once to return to the previous screen.

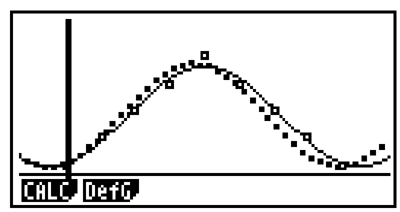


2. Press [F2] (DefG).

3. Enter the desired regression equation into any empty line.



4. Press [EXE] then [F6] (DRAW) to sketch the regression over the data. Adjust the equation accordingly.



In some western state wilderness areas, deer and mountain lion populations are interrelated, since the mountain lions rely on the deer as a food source. The population of each species goes up and down in cycles, but out of phase with each other. A wildlife management research team estimated the respective populations in a particular region every 2 years over a 16-year period, with the results shown in Table 1. In this activity, you will produce two scatter plots.

TABLE 1: Mountain Lion-Deer populations

Years	0	2	4	6	8	10	12	14	16
Deer	1,272	1,523	1,152	891	1,284	1,543	1,128	917	1,185
Mtn. Lions	39	47	63	54	37	48	60	46	40

Questions

Deer Population Analysis

1. Enter the data for the deer population for the time interval [0, 16] in the calculator and produce a scatter plot of the data.

2. A function of the form $y = a \cdot \sin(bx + c) + d$ can be used to model this data. Perform a trigonometric regression on the data. Write the regression below.

   ____

3. Plot the data from Question 1 and the equation from Question 2 in the same viewing window. Sketch the graph below.

4. Write a summary of the results, describing fluctuations and cycles of the deer population.

   ____

   ____

   ____

Mountain Lion Population Analysis

5. Enter the data for the mountain lion population for the time interval [0, 16] in the calculator and produce a scatter plot of the data.

6. A function of the form y = a•sin(bx + c) + d can be used to model this data. Perform a trigonometric regression on the data. Write the regression below.

____

7. Plot the data from question 5 and the equation from question 6 in the same viewing window. Sketch the graph below.

8. Write a summary of the results, describing fluctuations and cycles of the mountain lion population.

____

____

____

____

9. Discuss the relationship of the maximum predator populations to the maximum prey populations relative to time.

____

____

____

____

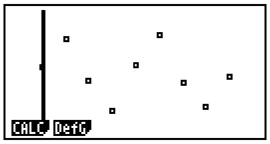
10. Discuss the relationship of the minimum predator populations to the minimum prey populations relative to time.

11. Discuss the dynamics of the fluctuations of the two interdependent populations. What causes the two populations to rise and fall, and why are they out of phase from one another?

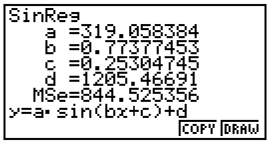
Extension

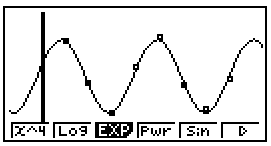
1. Draw a manual plot of both deer and mountain lion data. Use the data in Table 1 to determine *d*, *a*, and *b* for each graph. Use each graph to visually estimate *c* to one decimal place.

Solutions

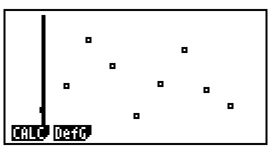
1. 

2. y = 319.06 sin(0.77x + 0.25) + 1205.47

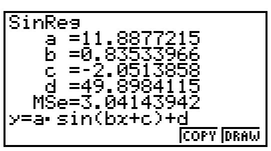


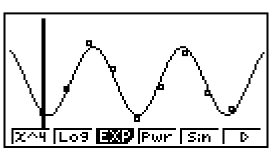
3. 

4. Answers may vary; the deer population is on an eight-year cycle having years 2 and 10 as high population years, whereas, years 6 and 14 as low population years.

5. 

6. y = 11.89 sin(0.84x – 2.05) + 49.90



7. 

8. Answers may vary; the mountain lion population is on an eight-year cycle having years 4 and 12 as high population years, whereas, years 0 and 8 as low population years.

9. Answers may vary. During maximum predator years, there is a sufficient supply of deer to support the proliferation of mountain lions, given the prey population reaching highs just two years earlier.

10. Answers may vary. During minimum predator years the prey population experiences minimums just two years prior.

11. Answers may vary. During minimum predator years, there are plenty of deer because the mountain lions have quickly depleted the deer population soon after the maximum predator years. This dramatic decrease in prey population causes a ripple effect into the predator population and thus cannot support larger numbers of mountain lions. The low number of mountain lions allows the deer population to flourish and encourages the number of mountain lions to grow.

Extension Solution

1. Answers may vary.

Topic Area: 1-Variable Statistical Calculations with Histograms

NCTM Standard:

- For univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics.

Objective

Given a set of data, the student will be able to calculate measures of central tendency, measures of variation and interpret histograms to draw conclusions about the data.

Getting Started

Have the students work in pairs or small groups and come up with examples of using one-variable data, what kind of information can be obtained from one-variable data and what types of graphs can be used to represent one-variable data.

Prior to using this activity:

- Students should be able to calculate measures of central tendency and measures of variation.
- Students should have a basic understanding of interpreting statistical graphs.
- Students should understand how to create a histogram.

Ways students can provide evidence of learning:

- Given a set of data, the student will be able to calculate the measures of central tendency and relate those measures to a histogram of that data.
- Given a set of data, the student will be able to describe the shape and defining characteristics of the histogram as it relates to the given situation.

Common mistakes to be on the lookout for:

- Students may pick a measure of central tendency that does not best describe the situation.
- Students may not understand the effect that outliers have on the set of data.

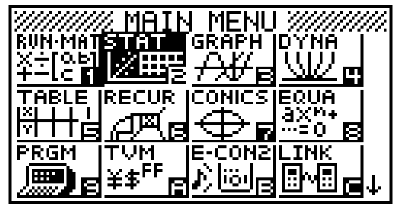
Definitions

- Mean
- Median
- Mode
- Frequency
- Range
- Interval
- Histogram

The following will demonstrate how to enter a set of data into the Casio *fx-9750GII*, graph the data using a histogram, calculate the measures of central tendency and trace the graph to determine the intervals and frequencies.

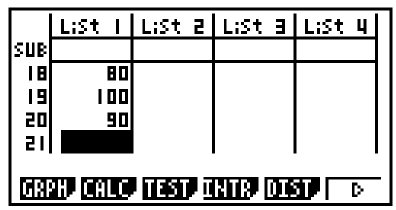
Scores on the first Algebra Test of the Year									
70	60	80	80	80	60	90	70	90	100
90	70	80	70	90	90	70	80	100	90

To enter the above set of data:



1. From the Main Menu, highlight the STAT icon and press EXE or press 2.

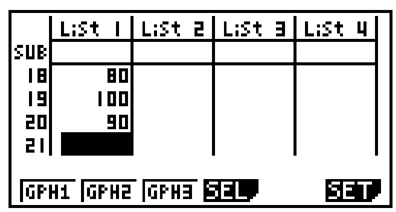
2. To clear previous data lists press F6 (▷) F4 (DEL-A) F1 (Yes).



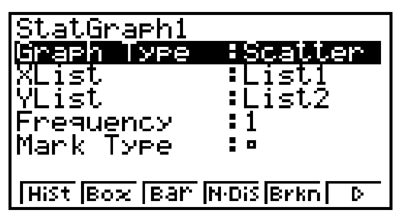
3. Enter the data by typing each number and pressing EXE after each entry.

4. The display should look like the screen shot on the right when completed.

To select the type of graph for this data:

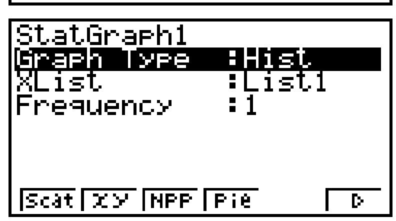


1. Press F1 (GRPH) and F6 (SET) to set the type of graph for StatGraph1.



2. Press ▼ to highlight **Graph Type**.

3. There are five choices: Scat, *XY*, NPP, Pie, and (▷). Selecting F6 (▷) will provide more graph choices.



4. Select F1 (Hist) to make **StatGraph1** a histogram.

5. Press ▼ to select **List1** as your data list, then EXIT.

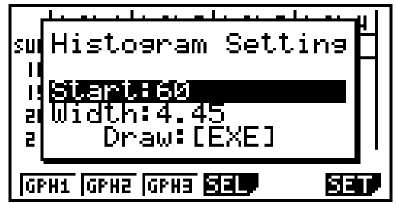
To display a graph of the data:

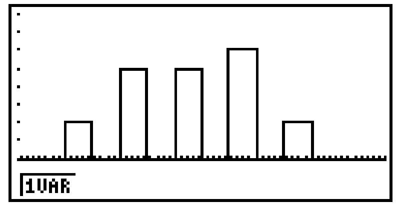
1. Press F1 (GPH1).

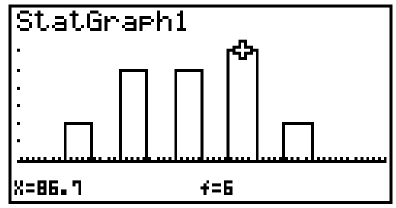
2. The calculator will display the starting value and width of each bar (the settings may be changed at this time).

3. To see the graph, press EXE.

4. To explore information about this graph, press SHIFT F1 **(Trace)** and then ◀ ▶.





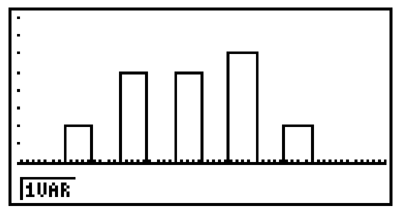


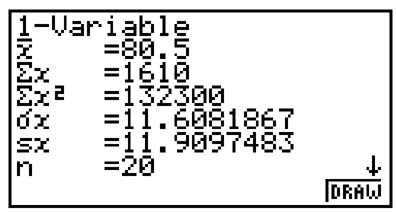
To see 1-variable statistical calculations for this set of data:

1. Press EXIT F1 (GPH1) F1 (1VAR).

2. Use ▲ ▼ to access information like mean, median, mode, quartiles, maximum and minimum values. The beginning of the list is shown at the right.

3. To return to the graph, press F6 (DRAW).





Professional Basketball Salaries — Activity

In today's society, professional athletes are often seen as role models and paid handsomely for their talents and entertainment they provide their fans. Many young athletes aspire to play professional sports for a variety of reasons; one reason is the lure of making a large salary. In this activity, you will explore the salaries of players from two NBA teams and compare the different salaries using graphs and statistics.

In this activity, you will graph a set of data involving the salaries for players with the Detroit Pistons and Los Angeles Lakers, calculate different one-variable statistical information and discuss the results of the statistics as it relates to differences in salaries.

Player Salaries for Detroit Pistons		Player Salaries for Los Angeles Lakers	
6,500,000	3,300,000	26,517,858	1,096,000
6,167,000	2,895,000	13,498,000	1,070,000
5,500,000	1,100,000	4,917,000	1,070,000
5,000,000	971,160	4,550,000	752,000
4,200,000	813,679	4,545,000	563,679
4,000,000		3,000,000	
3,900,000		1,500,000	
3,595,800		1,500,000	

Questions

1. What is the mean salary for a Detroit Piston's player? How do you calculate mean?

   ____

2. What the range of salaries for the Detroit Piston's? How do you find the range for a set of data?

   ____

   ____

3. Based on the histogram of this data, in what interval does the highest salary frequency lie?

   ____

4. How many salaries are in that interval?

   ____

5. What percentage of the team's salaries fall within this interval? Describe how you came up with your answer.

____

____

6. Describe the shape and appearance of this histogram.

____

____

7. What does your answer from question 6 tell about the salaries of Detroit Pistons players?

____

____

8. What is the mean salary for a Los Angeles Lakers player?

____

9. What the range of salaries for the Los Angeles Lakers?

____

____

10. Based on the histogram of this data, in what interval does the lowest salary frequency lie?

____

11. What percentage of the team's salaries fall within the highest frequency interval? Describe how you came up with your answer.

____

____

12. Describe the shape and appearance of this histogram.

____

____

13. What does your answer from question 12 tell about the salaries of Los Angeles Lakers players?

____

____

14. Based on the two histograms you have seen, describe some of the differences in salaries among the two teams.

____

____

____

15. Prior to drawing a histogram of your data, the calculator displays a “start” value and a “width” value; what do those values represent?

____

16. For the histogram of the Los Angeles Lakers salaries, change the “width” to the width of the Detroit Piston’s histogram. Describe what happens to your graph?

____

____

17. Would this new histogram cause you to change any of your previous answers? Why or why not?

____

____

Extensions

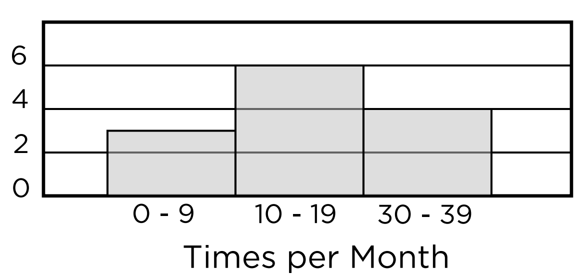
1. Explain what information goes on the horizontal axis? Vertical axis?

____

2. A group of 15 students were asked how often they ate at a fast food restaurant in one month; the frequency table is shown below:

Times per Month	0-9	10-19	20-29	30-39
Frequency	III	~~IIII~~ I		IIII

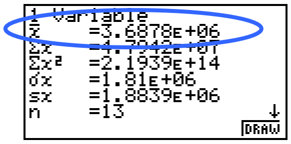
A student in the previous class period created a histogram based on the table above.



This student has made an error; in the space below, describe the error and in the blank graph below, sketch a new histogram that will correct the error.

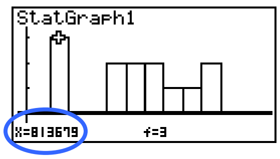
Solutions

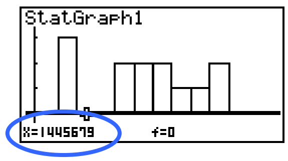
1. $3,687,800; the mean is found by taking the sum of your data and dividing it by the number of data points. Mean is usually denoted by $\overline{x}$, which is read "x-bar".



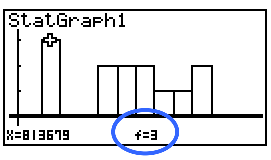
2. $5,686,321; the range is found by taking the difference of the greatest value and the least value.

3. $813,679 to $1,445,679

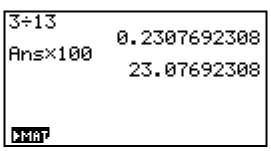




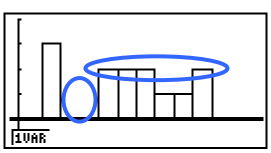
4. 3



5. 23%; 3 out of the 13 salaries listed fall in this interval.

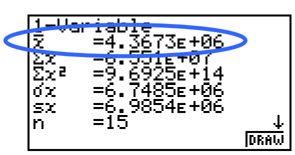


6. The answers will vary but things worth note are the interval with no salaries and four intervals contain the same number of frequencies.



7. Answers will vary depending on what observations the student makes in question 6.

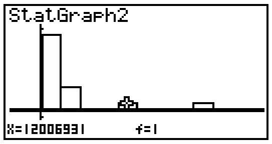
8. $4,367,300

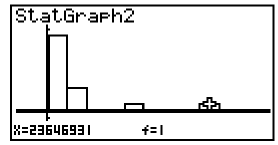


9. $26,150,927

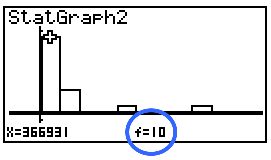
10. There are two intervals that each contains only one salary:

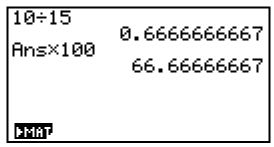
$12,006,931 to $14,916,931 $23,736,931 to $26,556,931



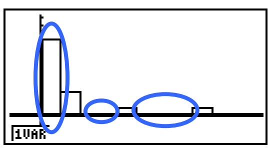


11. 67%; 10 out of the 15 salaries listed fall in this interval





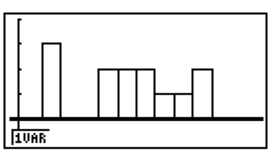
12. Answers will vary but things worth note are the number of intervals without salaries and the frequency of the first salary interval.



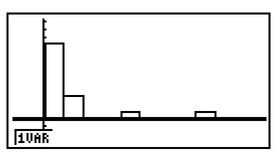
13. Answers will vary depending on what observations the student makes in question 12.

14. Answers will vary. The mean salary and salary range is much higher for the Lakers; however, the salary is more evenly distributed across the Pistons.

Pistons

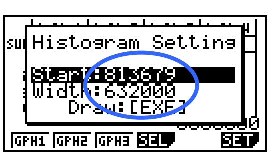


Lakers

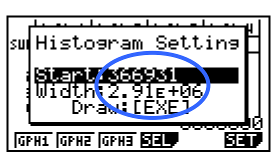


15. The starting value is the lowest salary in the data set; the width is the range for each interval.

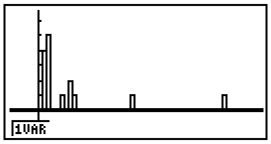
Pistons



Lakers



16. The graph develops more intervals and becomes more spread out.

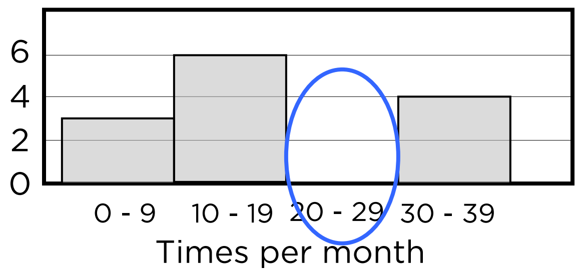


17. The answers to 10, 11 and 12 would change due to increased intervals.

Extension Solutions

1. The horizontal axis contains the interval range while the vertical axis shows the frequency of each interval.

2. The student made an error by not including the interval “20-29” in the histogram. This is a common mistake to assume that a frequency of zero correlates to not being included on the graph.



Topic Area: Parametric Equations

NCTM standards:

- Compute fluently by developing fluency in operations with real numbers using technology for more-complicated cases.
- Understand functions by interpreting representations of functions.

Objective:

To evaluate and graph a set of parametric equations that models the motion of a projectile.

Getting Started

Students will learn how to evaluate and graph a set of parametric equations that model the motion of a projectile.

Prior to using this activity:

- Students should know how to measure angle of elevation and angle of depression.
- Students should understand the difference between degrees and radians.

Ways students can provide evidence of learning:

- Students will be able to graph an object in projectile flight.
- Students will be able calculate the range of a projectile.
- Students will be able to determine the maximum height achieved by a projectile.
- Students will be able to calculate the time an object is in flight.

Common mistakes to be on the lookout for:

- Student may confuse range with altitude.
- Students may measure in degrees rather than radians.
- Students may use *g* in English units and the other measurements in metric.

Definitions

- range
- altitude
- velocity
- trajectory
- initial angle
- parabolic
- projectile
- variable
- apogee
- duration

Formulas

$x = v \bullet \cos(\theta) \bullet t$

$y = v \bullet \sin(\theta) \bullet t - (g \bullet \frac{t^2}{2})$

where v = initial velocity of the object (ft/sec),

θ = initial angle (radians) of trajectory from the ground

t = elapsed time (seconds)

g = acceleration due t0 gravity (32 ft/sec^2)

The following will demonstrate how to enter the data into the Casio *fx-9750GII* and interpret the results.

The equations for a parametric equation are as follows:

$x = v \cdot \cos(\theta) \cdot t$

$y = v \cdot \sin(\theta) \cdot t - (g \cdot \frac{t^2}{2})$

where v = initial velocity of the object (ft/sec),

θ = initial angle (radians) of trajectory from the ground

t = elapsed time (seconds)

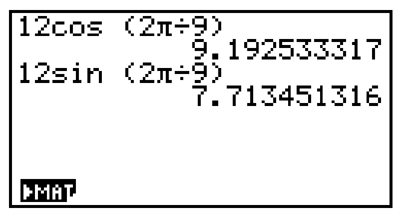
g = acceleration due t0 gravity (32 ft/sec^2)

Jamie kicks a soccer ball with an initial velocity of 12 ft/sec, at an angle of 40° ($\frac{2\pi}{9}$ radians) above the horizon.

$x = 12\cos(\frac{2\pi}{9})t$

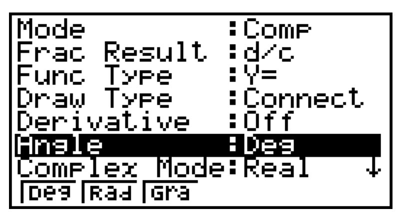
$y = 12\sin(\frac{2\pi}{9})t - (32 \cdot \frac{t^2}{2})$

The equations simplify to the following:
$x = 7.71t$
$y = 9.19t - 16t^2$

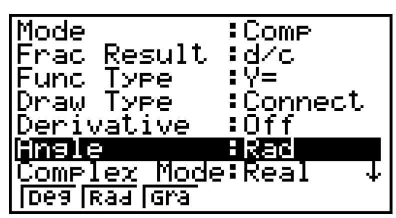


To set up the calculator to calculate in radians:

1. From the Main Menu, highlight the RUN•MAT icon and press [EXE] or press [1].

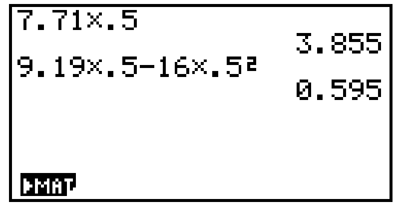


2. To change the calculator to radian mode, press [SHIFT] [MENU] **(SET UP)** and move the cursor down to **Angle**. Press [F2] (Rad) to change to radian mode, then press [EXIT] to exit the setup screen.



To find the coordinates of the ball after 0.5 seconds:

1. Substitute 0.5 in for *t* in the given equation using the RUNMAT menu.

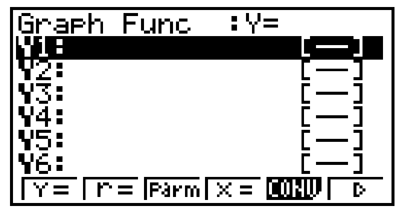


2. To calculate the *x*-coordinate, press 7 · 7 1 × · 5 EXE

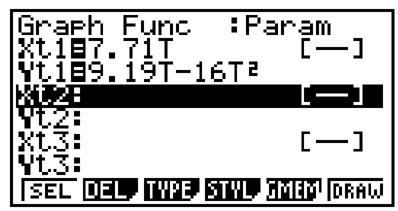
3. To calculate the *y*-coordinate, press 9 · 1 9 × · 5 − 1 6 × · 5 x^2 EXE

The coordinates are (3.855, 0.595) for the soccer ball after 0.5 seconds.

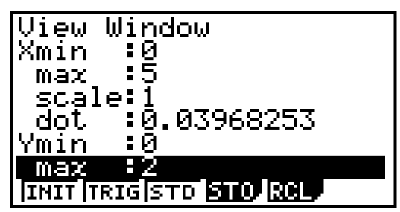
To graph a parametric equation:



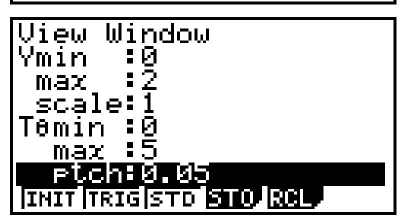
1. Press MENU 3 for the GRAPH icon.



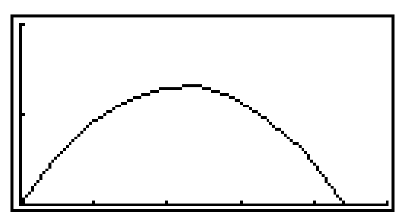
2. Press F3 (TYPE) to for type of equation, then F3 (Parm) to choose a parametric equation.



3. Enter the equations by pressing 7 · 7 1 X,θ,T EXE and 9 · 1 9 X,θ,T − 1 6 X,θ,T x^2 EXE



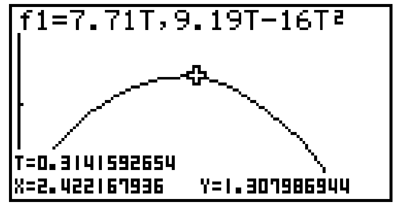
4. To graph the model from *x* = 0 to 5 feet, scale 1; from *y* = 0 to 2 feet, scale 1; and from *t* = 0 to 5 seconds, scale 0.05; press SHIFT F3 **(V-Window)** 0 EXE 5 EXE 1 EXE ▼ 0 EXE 2 EXE 1 EXE 0 EXE 5 EXE · 0 5 EXE.



5. Press EXIT to return to the graph screen

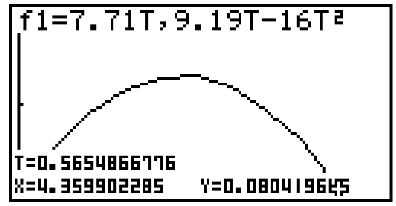
6. Press F6 (Draw).

To use the trace feature:



1. Press [F1] (Trace).

2. Press ▶ until the *y*-value reaches the maximum. For this example, the height is 1.31 feet at a time of 0.314 seconds.



3. Continue moving the cursor to the right ▶ to find the point of impact (*y* = 0).

Introduction

In this activity, you will learn how to evaluate and graph a set of parametric equations that model the motion of a projectile. In parametric equations, the *x* and *y* coordinates are generated by separate functions of a third variable *t* (usually time). Parametric equations are represented as $x = f(t)$ and $y = g(t)$.

A projectile is an object moving on a path, like a ball that is thrown or a missile that is fired. The model for projectile motion assumes no air resistance and the object is traveling on a parabolic path. The *x*-axis represents the ground and the *x*-coordinate represents the distance traveled horizontally at any time *t*. The y-coordinate represents the height of the object at time *t*. The parametric equations for projectile motion are:

$x = v \bullet \cos(\theta) \bullet t$ $\quad\quad$ $y = v \bullet \sin(\theta) \bullet t - (g \bullet \frac{t^2}{2})$

where *v* is the initial velocity (ft/sec) of the object, θ is the initial angle (radians) of trajectory from the ground, *t* is elapsed time in seconds and *g* is the acceleration due to gravity (32 ft/sec^2).

Questions

Investigate a projectile that is fired at an angle of 60° ($\frac{\pi}{3}$) to the ground with an initial velocity of 200 ft/sec.

1. Using the information provided, find the set of parametric equations.

   x = ______________________

   y = ______________________

2. Using the equations, find the coordinates of the projectile after 2 seconds.

   x = ______________________

   y = ______________________

3. Graph the parametric equations from $x = -100$ to 1500 feet, scale 100; from $y = -100$ to 500 feet, scale 100; and from $t = 0$ to 15 seconds, pitch 0.1. Draw your graph in the space below.

4. Using the trace feature, find the maximum height (apogee) of the projectile. At what time is the apogee reached?

   Maximum height = ____________________

   Time of apogee = ____________________

5. Using the trace feature, find the point of impact for the projectile. What is the range of the projectile (x value at point of impact)? What is the time of flight (duration – t value at time of impact)?

   Point of impact = ____________________

   Range = ____________________

   Duration = ____________________

6. Repeat the investigation for a projectile that has an angle of elevation of 45° ($\frac{\pi}{4}$) with an initial velocity of 100 ft/sec.

   a. Parametric equations:

      x = ____________________

      y = ____________________

   b. Coordinates of the projectile after 2 seconds:

      x = ____________________

      y = ____________________

c. Graph from x = –10 to 500 feet, scale 10; from y = –10 to 100 feet, scale 10; and from t = 0 to 10 seconds, pitch 0.1. Draw your graph in the space below.

d. Find the following:

apogee = ____________________

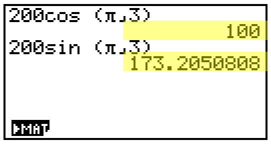
time of apogee = ____________________

range = ____________________

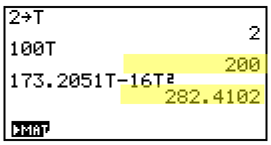
duration = ____________________

Solutions

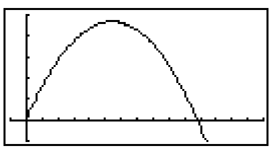
1. $x = 100t$
   $y = 173.2051t - 16t^2$



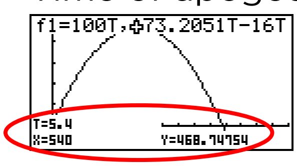
2. x = 200
   y = 282.4102



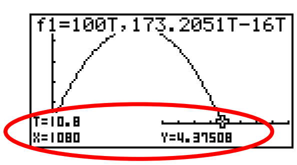
3.



4. Maximum height = 468.72 feet
   Time of apogee = 5.4 seconds

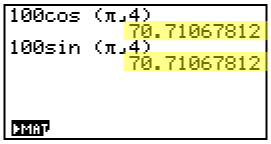


5. Point of impact = (1080, 4.37508)
   Range = 1080 feet
   Duration = 10.8 seconds



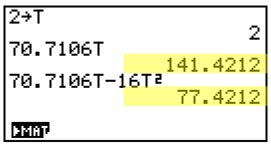
6. a. x = 70.7107t

y = 70.7107t – 16t^2

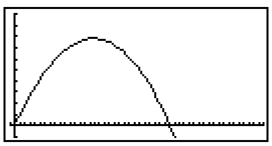


b. x = 141.4212

y = 77.4212



c.

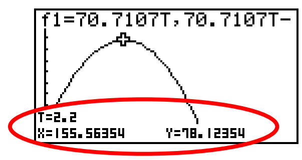


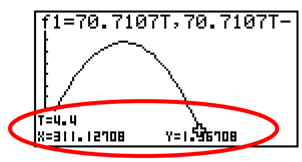
d. apogee = 78.1235 feet

time of apogee = 2.2 seconds

range = 311.1271 feet

duration = 4.4 seconds





Topic: Solving Proportions

NCTM Standard:

- Develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.

Objective

The student will be able to use the Casio *fx-9750GII* to solve proportions.

Getting Started

Students are expected to use proportions to solve problems in a variety of contexts. Knowing proportional growth is an expectation for Algebra, as well as being able to identify proportional relationships by examining a table, a graph, an equation and a statement.

Prior to using this activity:

- Students should have a strong foundation in equivalent fractions.

Ways students can provide evidence of learning:

- Students can solve a proportion by using scaling, specifically by using a table.
- Students can solve a proportion by finding equivalent ratios.

Common calculator or content errors students might make:

- Students may set up the proportion incorrectly.
- Students may be tempted to use cross multiplication to solve the proportion without understanding the algorithm.

Definitions

- Proportionality
- Ratio

The following will demonstrate how to enter fractions into the Casio *fx-9750GII* as improper fractions and mixed numbers.

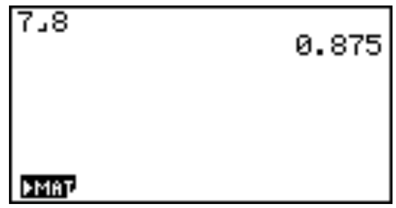
To enter a proper fraction:

1. From the Main **Menu**, highlight the RUN•MAT icon and press [EXE] or [1].

2. To enter $\frac{4}{5}$, input [4] [a b/c] [5] [EXE].

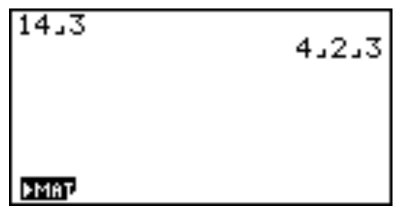
To determine the decimal equivalent of a fraction:

1. To find the decimal equivalent of $\frac{7}{8}$, input:
[7] [a b/c] [8] [EXE] [F↔D].



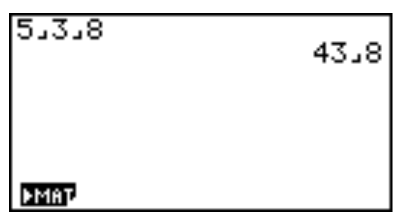
To convert an improper fraction to a mixed number:

1. To change $\frac{14}{3}$ into a mixed number, input:
[1] [4] [a b/c] [3] [EXE] [SHIFT] [F↔D].



To enter a mixed number to an improper fraction or a decimal equivalent:

1. The [a b/c] key will provide the space in between the whole number and the fractional part of the number.



2. To enter $5\frac{3}{8}$, input the following:
[5] [a b/c] [3] [a b/c] [8] [EXE].

3. The answer will be displayed as an improper fraction; to change and improper fraction to a decimal, press [F↔D].

In Algebra, patterns and relationships are everywhere. One type of relationship that occurs frequently is a proportional relationship. This means that the ratio between the variables is always the same. In this activity, we will explore various problems that are proportional.

The following is a series of questions involving proportions. Write a proportion, or equation, that represents the situation and solve. Give your answer as a complete sentence.

Questions

1. Mac took a trip to Cancun for Spring Break. When he returned to the U.S., he wanted to exchange his pesos for dollars. The current exchange rate is 11.1 pesos is $5.30. How many dollars did he have if he had 42.7 pesos?

____

____

2. To create different paint colors, several different colors are mixed together. To make purple paint, a painter mixes red paint and blue paint in the ratio of two to three. If a painter used 4 gallons of red paint, how much blue paint will he need to make the perfect purple?

____

____

3. A hummingbird's wings will beat an average of 18 times in ten seconds. How many times will it beat its wings in three and one-half minutes?

____

____

4. Amber earns 12 days of vacation every 9 months she works. How many days of vacation will she have at the end of two years?

____

____

5. Patty can read 243 pages of her book in 4.5 days. At this rate, how many pages could she read in one day?

6. A flag pole casts a shadow that is 360 m long. At the same time, a person who is 2 m tall casts a shadow that is 6 m long. How tall is the flag pole?

7. Michelle and Melody are trying to determine the distance between two particular cities by using a map. The map key indicates that 4.5 cm is equivalent to 75 km. If the cities are 12.7 cm apart on the map, what is the actual distance between the cities?

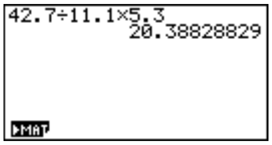
8. Mr. Turner has a garden. Of the 40 seeds he planted, 35% were vegetable seeds. How many vegetable seeds were planted?

9. Mary Ann usually makes 88% of her shots in basketball. If she shoots 200 shots, how many will she likely make?

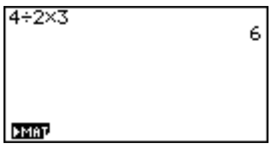
10. It costs about $90 to feed a family of 3 for one week. How much will it cost to feed a family of 5 for one week? How much will it cost to feed a family of six, seven, or eight?

Solutions

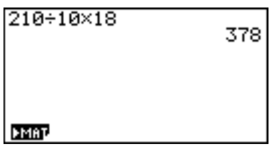
1. Mac had about $20.39.



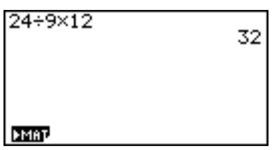
2. Six gallons of blue paint are needed.



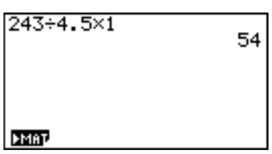
3. The bird will beat his wings 378 times. (210 = 3.5•60)



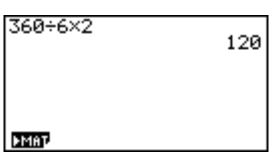
4. Amber will have earned 32 days of vacation.



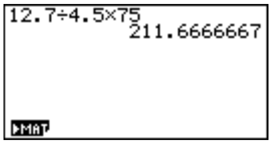
5. Patty will read 54 pages in one day.



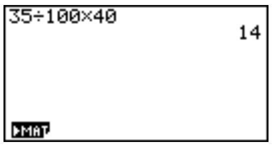
6. The flag pole is 120 m tall.



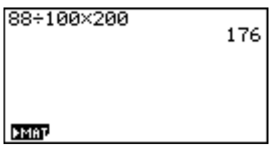
7. The two cities are about 212 km apart.



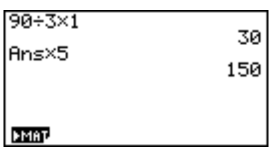
8. There were 14 vegetable seeds planted.



9. Mary Ann will make 176 shots.



10. The cost is $30 per person and it will cost $150 to feed 5 people.



It will cost $180 to feed 6 people, $210 to feed 7 people and $240 to feed 8 people.

Pure Tones — Teacher Notes

Topic Area: Amplitude and Period

NCTM Standard:

- Use mathematical models to represent and understand quantitative relationships.
- Understand functions by interpreting representations of functions.
- Compute fluently by developing fluency in operations with real numbers using technology for more complicated cases.

Objective

To graph the sine curve representing different pure tones.

Getting Started

Students will learn how to graph sine curves that represent pure tones. Students should understand that sounds are vibrations caused by a source and picked up by our ears.

Prior to using this activity:

- Students should know that sound waves are usually represented graphically by the sine wave y = Asin(Tx), where A is amplitude and T is the angular frequency. Frequency is defined as the number of wave cycles per second and amplitude is the size of the cycles.
- Students should understand that, according to Fourier theory, all sounds are made up of sine waves, at differing frequencies and amplitudes. Therefore, sound can be created by adding different sine waves together. A pure tone is a sound with a single sine wave that has a fixed frequency and amplitude.
- The frequency of a sound is in terms of Hertz (Hz). For example, a piano will play an A above middle C at 440 Hz. The sound is said to take on the form of a 440 Hz sine wave which is represented by the equation $y = \sin(2\pi \bullet 440x)$.

Ways students can provide evidence of learning:

- Graph a sine function.
- Interpret a sine function for pure tones.

Common mistakes to be on the lookout for:

- Students confuse sine and cosine.
- Student may use degrees, instead of radians.

Definitions

- sine
- cosine
- pure tone
- Hertz
- amplitude
- frequency

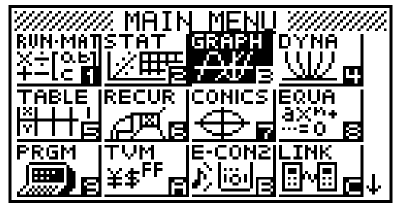
Pure Tones "How-To"

The following will demonstrate how to enter data into the Casio *fx-750GII*, graph the data, and interpret the data.

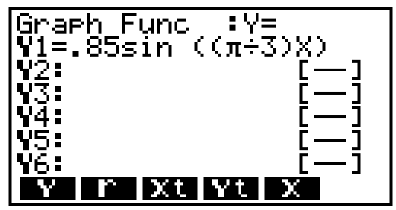
For a person at rest, the velocity *v* (in liters per second) of airflow during a respiratory cycle is $v = 0.85\sin\frac{\pi t}{3}$, where *t* is time in seconds. Inhalation occurs when $v > 0$ and exhalation occurs when $v < 0$.

Steps to graphing the above equation:

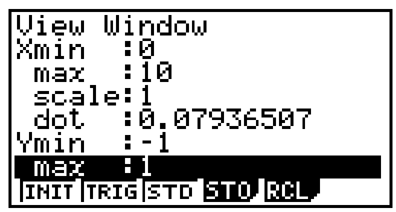
1. From the Main Menu, press [3] to select the GRAPH Icon.



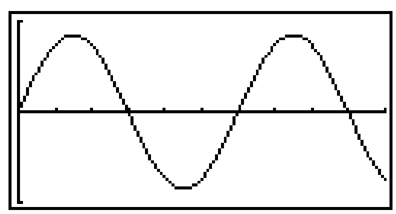
2. Enter the equation from above as follows: [•] [8] [5] [sin] [(] [(] [SHIFT] [EXP] [÷] [3] [)] [X,θ,T] [)] [EXE]



3. You will want to change your View Window, in order to see the graph properly. You can set your **Xmin** to 0, **Xmax** to 10, **Ymin** to -1, and **Ymax** to 1.

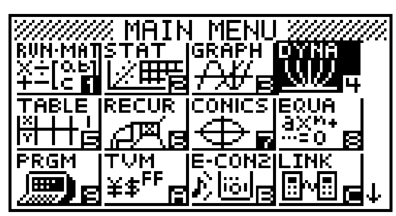


4. Press [EXE] twice to view the graph.

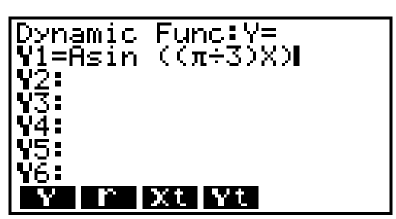


Steps to using the Dynamic Function to change the variables:

1. From the Main Menu, press [4] to select the DYNA Icon for Dynamic Graphing.

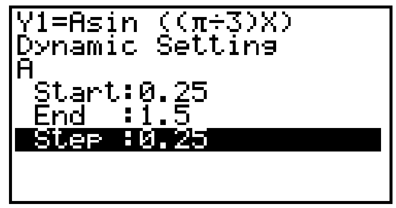


2. To enter the equation using the variable **A**, press [ALPHA] [X,θ,T] [sin] [(] [(] [SHIFT] [EXP] [÷] [3] [)] [X,θ,T] [)] [EXE].



3. To set the variable **A** to go from 0.25 to 1.5, with increments of 0.25, press F4 (Var) F2 (Set) • 2 5 EXE 1 • 5 EXE • 2 5 EXE

4. Press EXIT to take you back one screen.

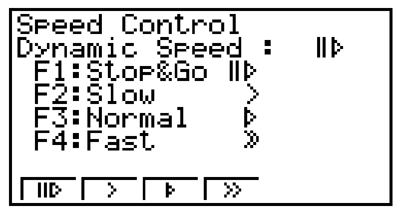


5. You can also set the speed of the graph, by going to F3 (Speed).

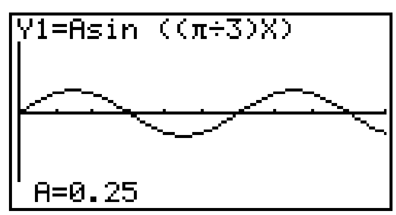
   On the screen, you will see four different options.

   F1: **Stop & Go**, requires you to hit EXE before the variable changes.

   F2: **Slow**, F3: **Normal**, & F4: **Fast** will change the variable at those varying speeds.



6. Once you have decided on a speed, press EXIT and F6 (DYNA). The calculator will process the information, then display the graph. The first graph displayed is shown below.



Pure Tones Activity

Introduction

The sine function is often used to model cyclical or periodic behavior. There are many applications for the sine function where measured data goes up and down. One of these applications is in modeling pure tones.

Sound waves are usually represented graphically by the sine wave y = Asin(Tx) where A is amplitude and T is the angular frequency. Frequency is defined to be the number of wave cycles per second and amplitude is the size of the cycles.

According to Fourier theory, all sounds are made up of sine waves at differing frequencies and amplitudes. Therefore, sound can be created by adding different sine waves together. A pure tone is a sound with a single sine was that has a fixed frequency and amplitude.

The frequency of a sound is in terms of Hertz (Hz). For example, a piano will play an A above middle C at 440 Hz. The sound is sais to take on the form of a 440 Hz sine wave which is represented by the equation y = sin($2\pi \bullet 440$x).

Questions

1. Draw the 440 HZ sine wave in the window Xmin = 0, Xmax = 0.01, scale = 1, Ymin = -1, Ymax = 1, scale = 1.

2. What amplitude and period does the 440 Hz sine have?

   a.) amplitude = ____________________________

   b.) period = ____________________________

3. Graph the 440 Hz sine wave using the Dynamic function and the equation y = sin($2\pi \bullet 440$x). Vary the amplitude A from 0.5 to 2 with an increment of 0.5. Describe what you see when the amplitude changes.

   ____

   ____

4. Graph the 440 Hz sine wave using the Dynamic Function and the equation $y = \sin(2\pi \bullet Hx)$. Vary the Hertz, H, from 440 to 470 with an increment of 10. Describe what you see when the Hertz changes.

____

____

5. The original sine wave, $f(x) = \sin(2\pi \bullet 440x)$, has a very short period. The display of the graph takes on some interesting forms in larger windows. Draw the graph in the following windows: [Xmin, Xmax, scale, dot, Ymin, Ymax]

[0, 0.1, 1, 0.003, -1, 1]

[0, 0.5, 1, 0.003, -1, 1]

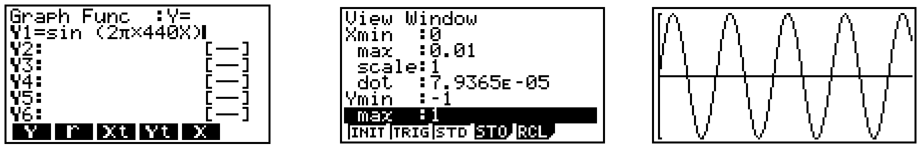
[0, 1, 1, 0.003, -1, 1]

[0, 5, 1, 0.003, -1, 1]

[0, 10, 1, 0.003, -1, 1]

Solutions:

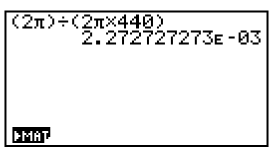
1.



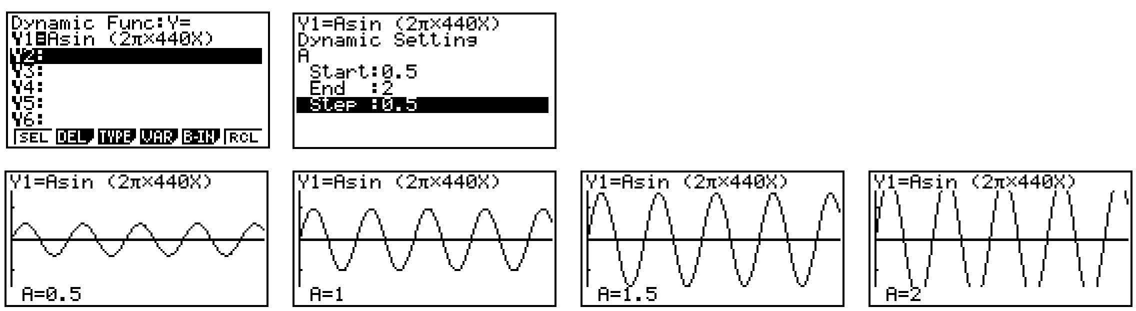
2. amplitude = 1

period = $\frac{1}{440}$ = 0.0023

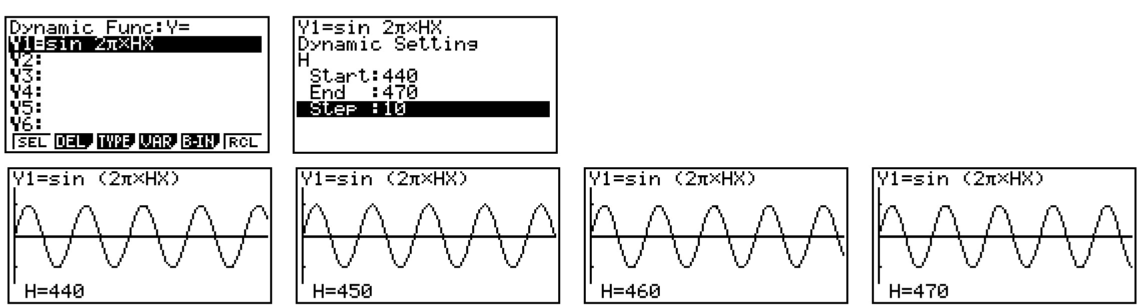
The general sine function is f(x) = *A* sin(*B*x). The amplitude of the sine curve is the coefficient *A*. In our function, it is an implied 1. The period of a sine curve is determined by the coefficient *B*. The period of a sine function is $\frac{2\pi}{\text{B}}$. In our case, you divide 2π by the coefficient of $2\pi \bullet 440$. The resulting period is $\frac{1}{440}$ or 0.0023.



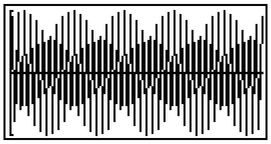
3. As the amplitude (*A*) increases, the height of the graph increases. (The **Ymin** was set to –2 and the **Ymax** was set to 2.)



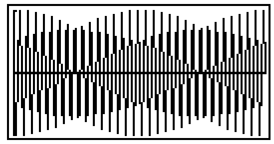
4. As the hertz (*H*) increases, the period of the sine waves decreases.



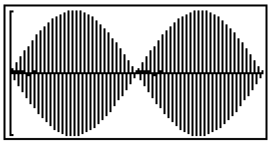
5. [0, 0.1, 1, 0.003, -1, 1]



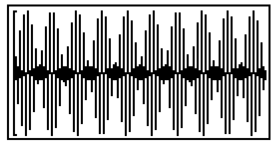
[0, 0.5, 1, 0.003, -1, 1]



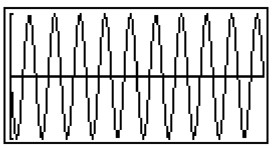
[0, 1, 1, 0.003, -1, 1]



[0, 5, 1, 0.003, -1, 1]



[0, 10, 1, 0.003, -1, 1]



Topic Area: Properties of Parallelograms

NCTM Standards:

- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.
- Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations.
- Investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates.

Objective

The student will be able to use algebra and statistics to prove that a quadrilateral is a parallelogram, demonstrate that the opposite sides are equal, demonstrate that the diagonals bisect each other, and prove that the opposite angles are equal.

Getting Started

As a class, review the meaning of slope and the slope-intercept form of an equation; include in the discussion the relationship of the slopes between parallel lines and perpendicular lines. Review methods of proving triangles congruent using the Side-Side-Side method.

Prior to using this activity:

- Students should be able to find the xy-line for a pair of coordinates using a graphing calculator.
- Students should be able to perform calculations involving square roots, ratios, and parentheses using a graphing calculator.
- Students should know the formula for finding the distance between two points.

Ways students can provide evidence of learning:

- The student will be able to write conjectures pertaining to a parallelogram.
- The student will be able apply the properties of a parallelogram to real-life problems.

Common mistakes to be on the lookout for:

- Students may confuse the x and y values in the calculations.
- Students may enter the problem incorrectly into the calculator.

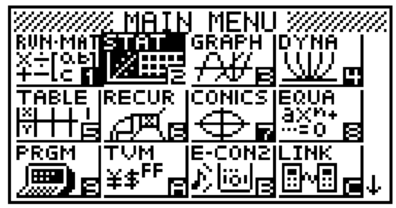
Definitions

- Parallelogram
- Perpendicular
- Endpoint
- Slope
- Diagonal
- Intersection
- Midpoint
- Congruent
- Hypotenuse
- Leg

The following will demonstrate how to enter a set of coordinates into two lists using the Statistics mode of the Casio *fx-9750GII*. After the list is set up, you will find the slope of a line containing the points, save the equation in the Graph mode, and find the intersection of two lines. You will then find the length of a segment.

Line segment AB has endpoints at (–5, -2) and (3, 6) and segment CD has endpoints at (–6, 4) and (3, –7). Find the slope for each line segment, the coordinates of their intersection, and the length of $\overline{AB}$.

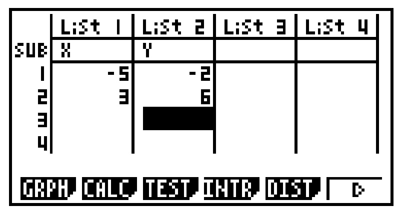
To enter values into a list and find the line of best fit:



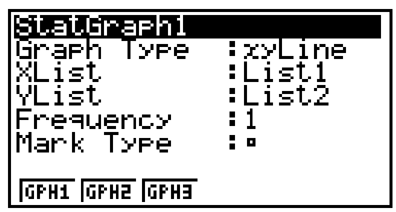
1. From the Main Menu, highlight the STAT icon and press [EXE] or press [2].

2. To label the first column, highlight the space below **List 1** and press [ALPHA] [+] (X) [EXE].

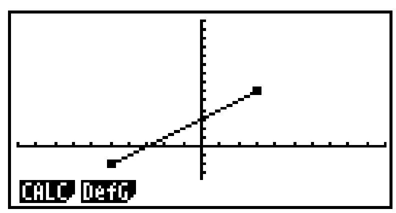
3. To label the second column, highlight the space below **List 2** and press [ALPHA] [–] (Y) [EXE].



4. Enter the x-values into **List 1** and the y-values into **List 2**. Be sure to press [EXE] after each value.

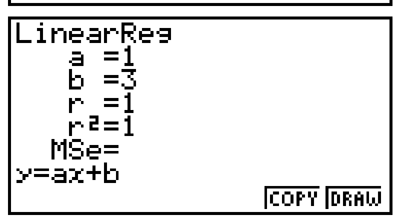


5. To view the points, press [F1] (GRPH) [F6] (Set) [▼] [F1] (GPH1) [▼] [F2] (XY) [▼] [F1] (List) [1] [▼] [F1] (List) [2] [EXE] [F1] (GPH1).



6. Press [EXIT] and [F1] (GPH1) to view the graph.

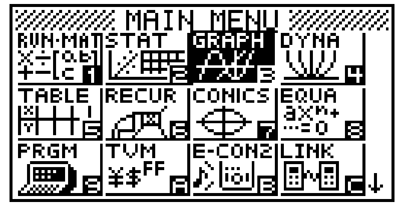
7. Press [F1] (Calc) [F2] (X) [F1] (ax+b) to find the line of best fit.



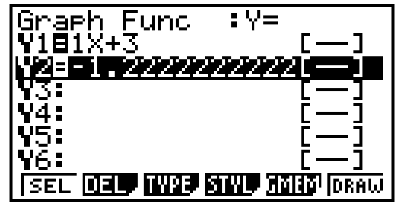
8. Press [F5] (Copy) [EXE] to copy the equation into the graph function.

9. Repeat the same steps to find the equation for the second segment.

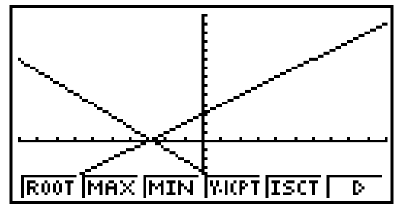
To graph the two equations and find the intersection:



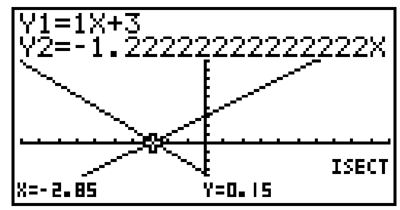
1. From the Main Menu, highlight the GRAPH icon and press [EXE] or press [3].



2. To graph the two equations, highlight each equation and press [F1] (Sel) to turn the function on; when the equal signs are highlighted, you know the equation is selected. Then press [F6] (Draw).

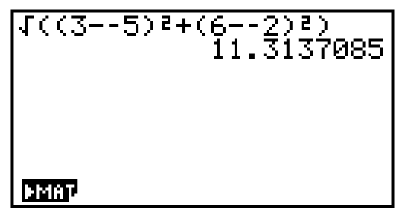


3. While viewing the graph, press [F5] **(G-Solv)** [F5] (ISCT) to find the intersection of the two equations.



4. The coordinates are displayed at the bottom of the screen.

To find the length of $\overline{AB}$:



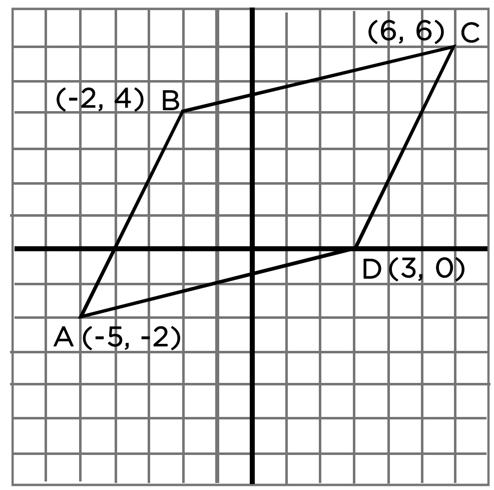
1. Using the distance formula,

   d = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, press [SHIFT] [x^2] [(] [(] [3] [−] [(−)] [5] [)] [x^2] [+] [(] [6] [−] [(−)] [2] [)] [x^2] [)] [EXE] to find the length of $\overline{AB}$.

The parallelogram is a special quadrilateral with special properties that is used in a variety of areas, especially in design. In this activity, we will explore the properties and then solve some problems using those properties.

Questions

The diagram at the right shows Quad ABCD. By definition, a parallelogram is a quadrilateral with both pairs of opposite sides parallel.



1. Find the equation of a line that contains the following points:
   a. points B and C
   ____
   b. points A and D
   ____

2. What is the slope for each line?
   ____

3. Find the equation of a line that contains the following points:
   a. points A and B
   ____
   b. points D and C
   ____

4. What is the slope for each line?
   ____

5. Are the opposite sides parallel?
   ____

Let us see what else we can find out about the sides of a parallelogram.

6. Find the length for the following segments to the nearest tenth.
   a. $\overline{BC}$: ______________________________
   b. $\overline{AD}$: ______________________________

7. Find the length for the following segments to the nearest tenth.

   a. $\overline{AB}$: ______________________________

   b. $\overline{DC}$: ______________________________

8. What can you conclude about the opposite of a parallelogram?

   ____

Draw the two diagonals for the figure. We are now going to look at their properties in relation to quadrilaterals.

9. Find the equation for the following segments.

   a. $\overline{AC}$: ______________________________

   b. $\overline{BD}$: ______________________________

10. Find the coordinates for the intersection of the two diagonals. Draw it on the diagram and label it E.

    ____

11. Find the length of the following segments to the nearest tenth.

    a. $\overline{AE}$: ______________________________

    b. $\overline{CE}$: ______________________________

12. Find the length of the following segments to the nearest tenth.

    a. $\overline{BE}$: ______________________________

    b. $\overline{DE}$: ______________________________

13. What conclusion can be made about the diagonals of a parallelogram?

    ____

14. Using the information above, determine the reason why each of the following pairs of triangles are congruent.

    a. $\Delta ABC \cong \Delta ADC$ by ______________________________

    b. $\Delta ABD \cong \Delta CDB$ by ______________________________

15. Since the two pairs of triangles are congruent, then give two pairs of angles that are equal.

    ______________________ and ______________________

16. What conclusion can be made about angles in a parallelogram?

____

One method of demonstrating vector addition is by creating a parallelogram. The sum is the coordinates of the fourth vertex of the parallelogram.

17. Given the diagram below and using the properties of parallelograms, find the sum of v_1 and v_2.

____

18. The magnitude of a vector is equal to its length. Find the magnitudes of v_1, v_2, and the resulting vector to the nearest hundredth.

a. v_1

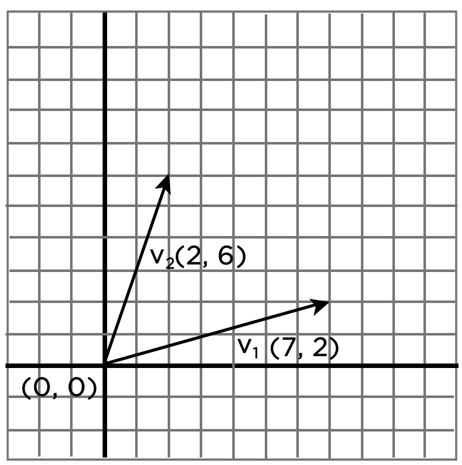
____

b. v_2

____

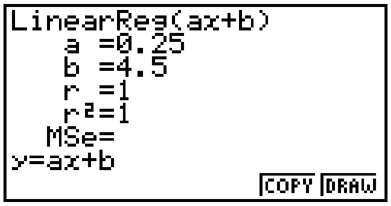
c. $v_1 + v_2$

____

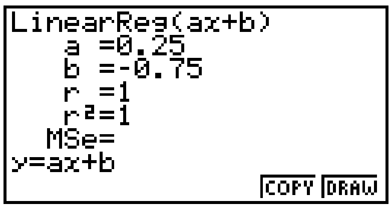


Solutions:

1. a. y = 0.25x + 4.5

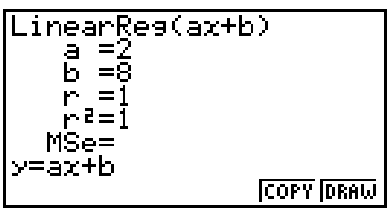


b. y = 0.25x – 0.75

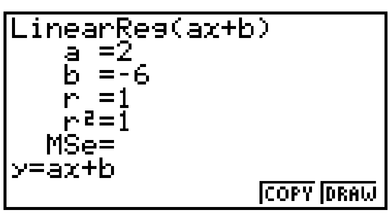


2. The slope of each line is 0.25.

3. a. y = 2x + 8;



b. y = 2x – 6



4. The slope of each line is 2.

5. The opposite sides are parallel.

6. a. $\sqrt{(6--2)^2+(6-4)^2}$ = 8.2

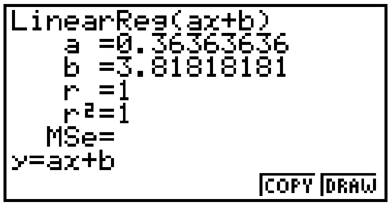
b. $\sqrt{(3--5)^2+(0--2)^2}$ = 8.2

7. a. $\sqrt{(-2--5)^2+(4--2)^2}$ = 6.7

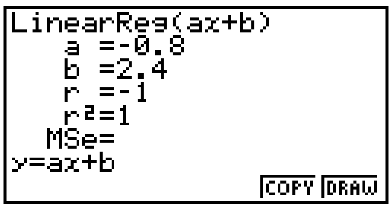
b. $\sqrt{(6-3)^2+(6-0)^2}$ = 6.7

8. Opposite sides of the parallelogram are equal.

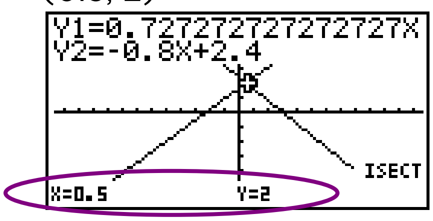
9. a. $y = 0.36x + 3.81$



b. $y = -0.8x + 2.4$



10. (0.5, 2)



11. a. $\sqrt{(0.5 - -5)^2 + (2 - -2)^2} = 6.8$

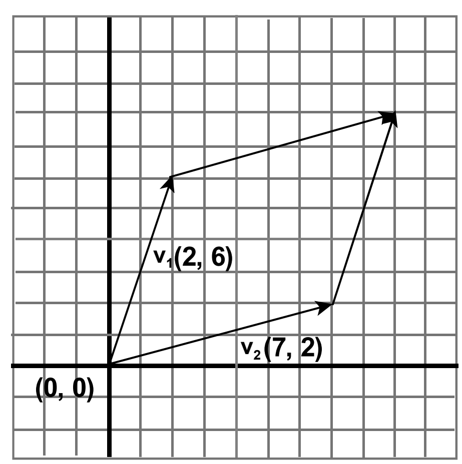
b. $\sqrt{(0.5 - 6)^2 + (2 - 6)^2} = 6.8$

12. a. $\sqrt{(0.5 - -2)^2 + (2 - 4)^2} = 3.2$

b. $\sqrt{(0.5 - -5)^2 + (2 - 4)^2} = 3.2$

13. The diagonals bisect each other.

14. a. SSS Congruency
b. SSS Congruency



15. $\angle ABC \cong \angle CDA$ & $\angle BAD \cong \angle DCB$

16 Opposite angles of a parallelogram are equal.

17. $v_{1,} + v_2 = \langle 9,8 \rangle$

18. a. $v_1 = \sqrt{2^2 + 6^2} = 6.32$ units

b. $v_2 = \sqrt{7^2 + 2^2} = 7.28$ units

c. $v_{1,} + v_2 = \sqrt{9^2 + 8^2} = 12.04$ units

Topic: Hypothesis Testing

NCTM Standards:

- Develop and evaluate inferences and predictions that are based on data.
- Use simulations to explore the variability of sample statistics from a known population and to construct sampling distributions.
- Understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference.

Objective

The students will be able to accept or reject a null hypothesis based on given data and calculations of confidence.

Getting Started

This activity should work well with the students because they get to have candy. Only one rule: they have to count before they eat.

The students will need their own personal data as well as a full class set of data. Have each student open their bag of Skittles, count and record the number of each color they have in a chart. After each student has finished counting, have each student share their data with the class. Make a class chart on the overhead, board, or flip chart. Have them follow the directions on the student worksheet to complete the activity.

Prior to using this activity:

- Students should be able to calculate the measures of central tendency and measures of variation.
- Students should have a basic understand of a Hypothesis Test.

Ways students can provide evidence of learning:

- Given a table of data, the student will be able to accept or reject a null hypothesis based on calculations of confidence.

Common mistakes to lookout for:

- Students may switch the different variables around.

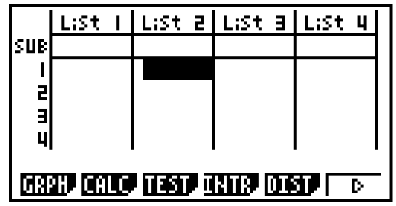
Definitions

- Sample
- Population
- Parameters
- Confidence
- Variables: Z, p, and n
- Sample distributions
- Mean Inference
- Z-Test

The following will demonstrate how to perform a probability Z-Test on the Casio *fx-9750GII,* graph the data using a standard deviation curve.

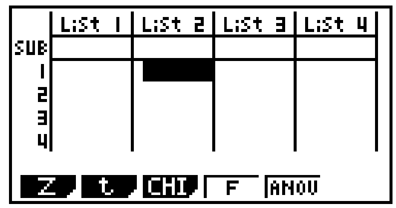
Performing a 1-probability Z-Test:

1. From the Main Menu, highlight the STAT icon and press EXE or press 2.

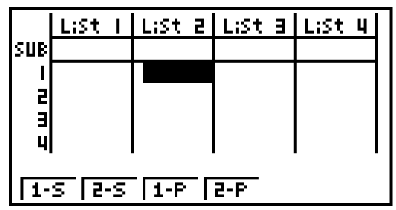


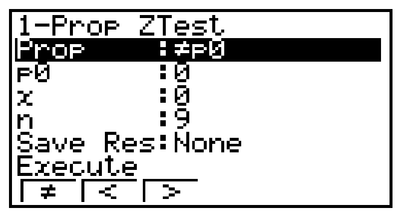
2. Press F3 (TEST).

3. Press F1 (Z) for a Z-Test.

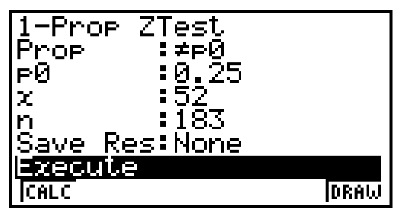


4. For this type of problem, we have one population so press F3 (1-P).

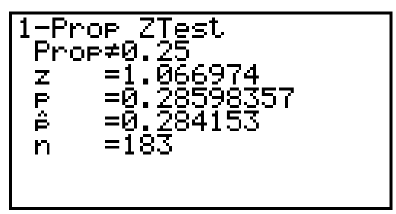




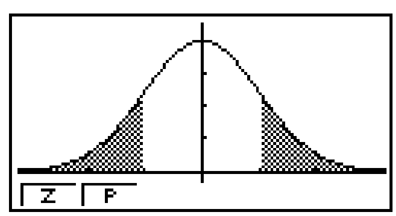
5. We want **Prop** $\neq$ p0, next fill in the information for *p0* = value of the null hypothesis, *x* = success in sample, and *n* = the total size of the sample. For this example, enter *p0* = 0.25, *x* = 52, *n* = 183, pressing EXE after each entry.



6. Scroll down to the line that says **Execute** and press EXE or F1 (CALC).



7. To view the curve, press EXIT and F6 (DRAW).



Introduction

Open your bag of Skittles and count the number of each color that is in the bag. Record those numbers in the first row of the chart below. Share your sample information with the class and record the total number of each color in the entire class in the second row of the chart (include your data in the class data row).

	Red	Orange	Yellow	Green	Blue	Purple	Total
Student							
Class							

Questions

1. If you could only have one color in the bag, which would you choose?

____

2. What percent of the total was that color in your bag?

____

3. What percent of the total is that color in the class data?

____

4. Create a null hypothesis by filling in the blanks below:

Null Hypothesis:
My sample bag of ____________ percent of ____________ color came from the same population as the class set of ____________ percent of ____________ color.

5. Perform a 1-prop Z-test of population proportion to test the null hypothesis. What is the Z value?

____

6. What is the p value?

____

7. Is the p value high or low?

____

8. Draw the proportion graph in the box below.

9. Can you reject the null hypothesis? Why or why not?

____

____

____

10. How many students have rejected the null hypothesis? Describe a couple of reasons that this can happen.

____

____

____

Extension

1. Select a student and compare your data with them, analyze the differences and similarities. Display your data in a graph and table form.

2. Using your Skittles, have a blind taste test, compare the results of the male and female students. Was one group more correct in choosing the flavor than the other? Why do you think that was? Also, collect data from teachers and students and compare your results.

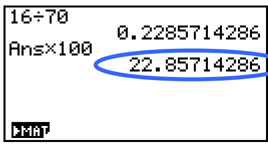
Solutions

Every class will have slightly different data. Please use this sample data as an example of how your answers might look.

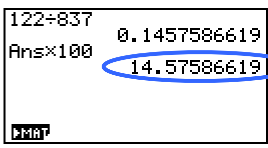
	Red	Orange	Yellow	Green	Blue	Purple	Total
Student	15	13	9	11	16	6	70
Class	167	155	100	160	122	133	837

1. I would select blue, because that color had the most in the bag.

2. Blue was 22.86% of the total bag.

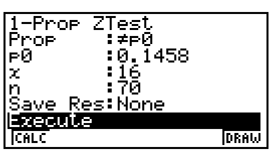


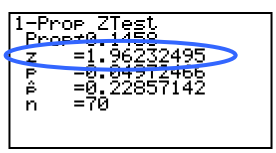
3. The class had 14.58% of its total as blue.



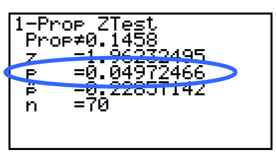
4. **Null Hypothesis:** My sample bag of <u>22.86</u> percent of <u>Blue</u> color came from the same population as the class set of <u>14.58</u> percent of <u>Blue</u> color.

5. z = 1.96232495



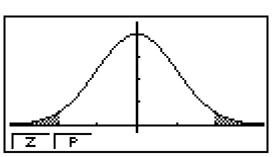


6. p = 0.04972466



7. The p value is less than 0.05.

8.



9. I would reject the hypothesis and conclude that I had a higher number of blue Skittles in my bag than my classmates.

10. Even though an individual bag came from the population, it is possible to reject the null hypothesis. Due to sampling variability, one sample may have a high or low proportion. Also, the samples were not really selected randomly. It is possible that the process used for making the individual bags does not result in a very random color mix.

Reflecting on the Rules — Teacher Notes

Topic Area: Reflections of Polygons

NCTM Standards:

- Understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, vectors, function notation, and matrices.
- Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations.

Objective

The student will be able to use the properties of reflections to use the matrix function of a calculator to represent the vertices of a polygon, write a reflection as a product of two matrices, find the reflection of a polygon across a given line, and write a translation as a series of reflections.

Getting Started

As a class, discuss the definition of a reflection. Ask students to give examples of reflections in everyday life. Provide picture examples of reflections from various areas such as nature, quilting, corporate logos, and other areas. Discuss with them various career areas that use reflections such as architecture, photography, sports, and graphic design.

Prior to using this activity:

- Students should have an understanding of the coordinate plane.
- Students should have an understanding of operations with matrices.
- Students should know the matrix rules for reflections across the x-axis, y-axis, line y = x, and the origin point.

Ways students can provide evidence of learning:

- The student will be able to write a matrix equation for a reflection for a given problem.
- The student will be able to determine the line of reflection.
- The student will be able to relate reflections to other translations.

Common mistakes to be on the lookout for:

- Students may use the wrong line for a given reflection.
- Students may confuse the x and y values in the reflection.
- Students may use the incorrect matrix rule.
- Students may enter a matrix into the calculator incorrectly.

Definitions

- Reflection
- Line of Reflection
- Point Reflection
- Matrix, Rotation
- Translation

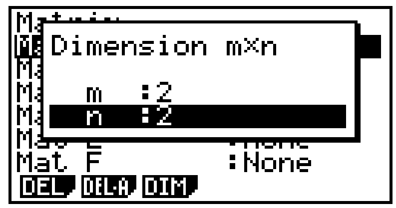
The following will demonstrate how to enter a set of coordinates into a matrix using the Run-Matrix mode on the Casio *fx-9750GII* and apply this to reflections. Using the vertices of (1, 1), (3, 4), and (6, 2), find the coordinates of the image under the given reflections.

a. Create a reflection matrix for the x-axis and a matrix for the polygon.
b. Reflect the points across the x-axis.

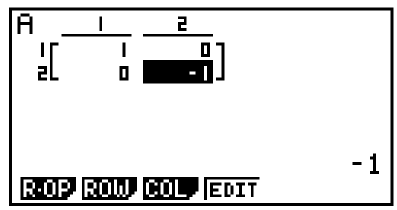
To create a reflection matrix and a matrix for the polygon:

1. From the Main Menu, highlight the RUN•MAT icon and press [EXE] or press [1].

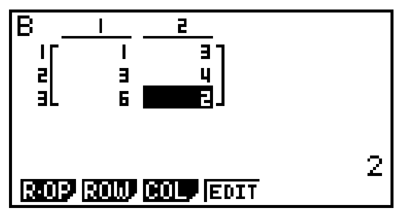
2. Press [F1] (MAT). With **Mat A** highlighted, press [EXE] and press [2] [EXE] [2] [EXE] to set up the dimensions.



3. Press [EXE] and then enter the values of the matrix to be used for a reflection across the x-axis by pressing [1] [EXE] [0] [EXE] [0] [EXE] [(–)] [1] [EXE].

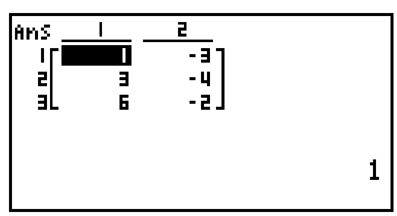


4. Press [EXIT] [▼] [EXE] to prepare to enter the matrix for the ordered pairs. Enter [3] [EXE] [2] [EXE] [EXE] and enter the x-coordinates in the first column and the y-coordinates into the second column.



To reflect the figure across the x-axis:

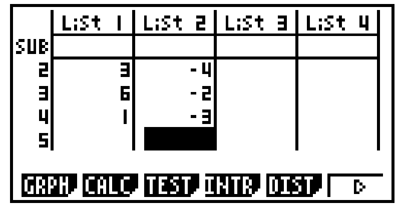
1. Press [EXIT] twice to return to the initial RUN•MAT screen. Enter [OPTN] [F2] (MAT) [F1] (Mat) [ALPHA] [log] (B) [×] [F1] [ALPHA] [X,θ,T] (A) [EXE].



2. To store this answer, press [EXE] [F1] (Mat) [SHIFT] [(–)] (Ans) [→] [F1] [ALPHA] [ln] (C) [EXE].

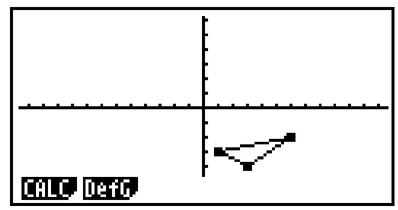
To draw the reflection:

1. Press [EXE] and enter [F2] (Mat) [F1] (M→L) [ALPHA] [ln] (C) [,] [1] [)] [→] [OPTN] [F1] (List) [1] [EXE] to store the x-values in **List 1**. Repeat this process to store the y-values in **List 2**.



2. Press [MENU] and highlight the STAT icon and press [EXE]. Enter the first pair of coordinates at the bottom of the lists so that the graph will be a polygon.

3. Enter [SHIFT] [F3] **(V-Window)** [F1] (Init) [EXE] to set up the view window. Press [F1] (GRPH) [F6] (SET) [▼] [F2] (XY) [▼] [F1] (List) [1] [EXE] [▼] [F1] (List) [2] [EXE] [EXE] [F1] (GPH1) to graph the to graph the polygon.

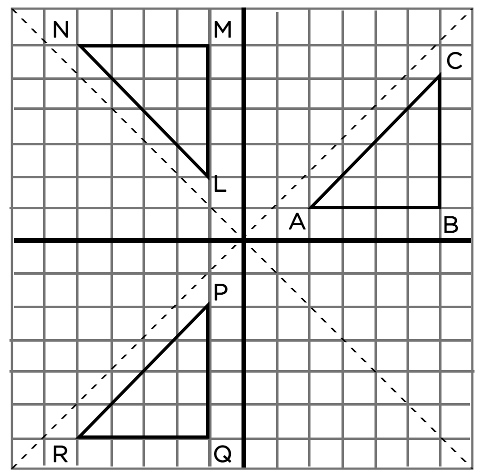


4. Press [SHIFT] [F2] **(Zoom)** [F4] (Out) [EXE] to see the graph as shown.

Many designers use reflections to create patterns and obtain symmetry. It has been shown that symmetry is very pleasing to the eye and thus it is used in many architectural designs. Take a look at some buildings that you find pleasing and see if this is not true. In this activity, we will look into the relationship between reflections and other forms of transformation. We will also take two simple polygons and create a quilt block that could be incorporated into a bed cover.

Questions

1. What are the coordinates of the vertices for ΔABC?



Enter these into a 3x2 matrix. Find coordinates for the images for each of the following reflections using the proper reflection matrix.

2. a. Across the x-axis

   b. Across the y-axis

   c. Across the origin

   d. Across the line y = x

3. Draw each image and label the vertices.

4. How many reflections of ΔABC are needed to equal a 90° rotation of the same triangle?

5. Reflections over which lines will give a 90° rotation? Graph this on the calculator.

6. Which reflection is equal to a 180° rotation?

7. What is the maximum number of reflections needed to translate ΔABC to ΔPQR? What is the minimum number?

____

8. What reflections are needed to create ΔLMN?

____

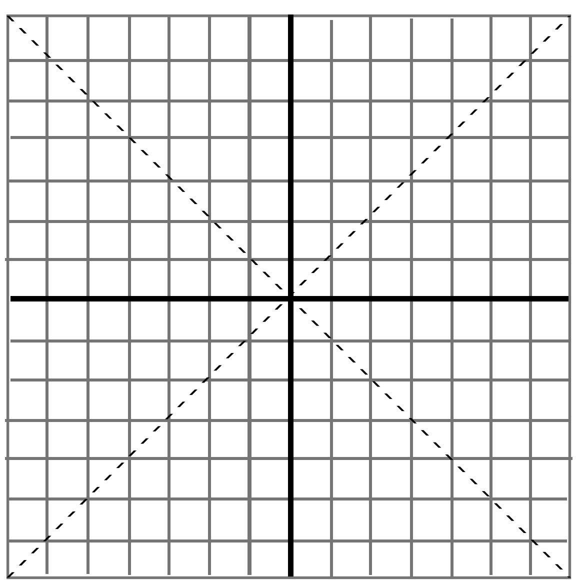
A quilt block is made of two pattern pieces. The first is a triangle with vertices at (2, 0), (6, 2), and (6, –2). The second is a parallelogram with vertices at (0, 0), 4, 2), (6, 6), and(2, 4).

9. Reflect the triangle across the origin point, the y = x line, and the line y = –x. What are the new coordinates?
   a. Origin Point: ____________________
   b. Line y = x: ____________________
   c. Line y = –x: ____________________

10. Plot all four triangles.

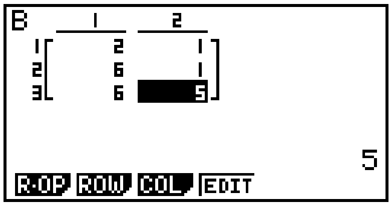
11. Reflect the parallelogram across the y-axis, x-axis, and the origin point. What are the new coordinates?
   a. Origin Point: ____________________
   b. x-axis: ____________________
   c. y-axis: ____________________

12. Plot all four parallelograms.

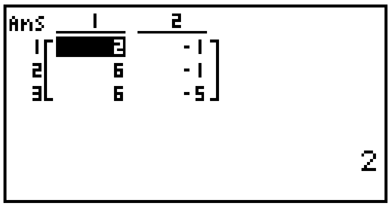


Solutions

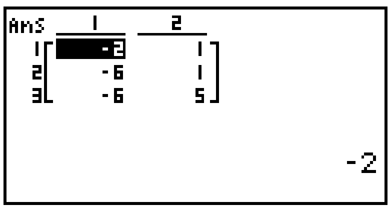
1. A(2, 1) B(6, 1) C(6, 5)



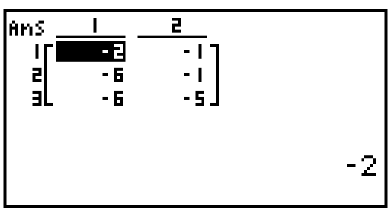
2. a. A’(2, –1); B’(6, –1); C’(6, –5)



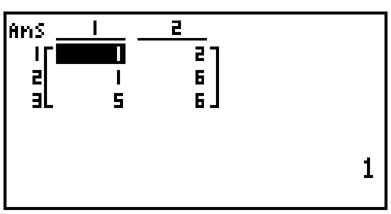
b. A”(–2, 1); B”(–6, 1); C”(–6, 5)

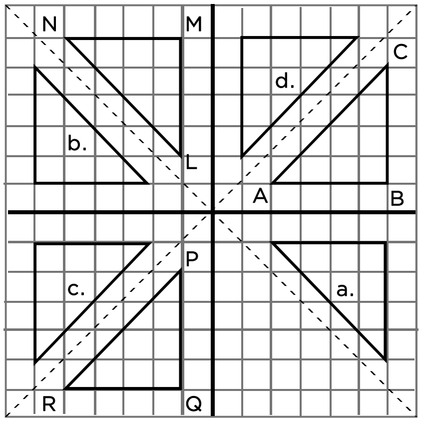


c. A”’(–2, –1); B”’(–6, –1); C”’(–6, –5)



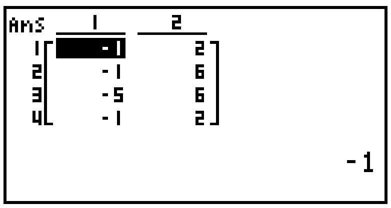
d. A””(1, 2); B””(1, 6); C””(5, 6)



3. 

4. Two reflections

5. Line y = x and y-axis



6. Reflection across the origin point.

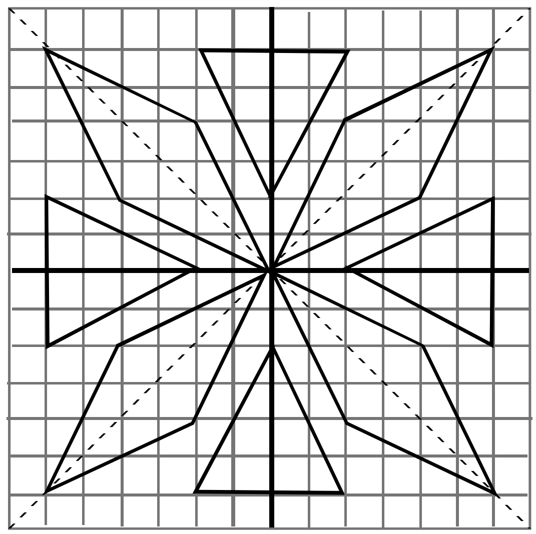
7. three; two

8. Reflection across the y-axis and then a reflection across the line y = –x.
or
Reflection across the line y = x and then a reflection across the y-axis.

9. a. Origin Point: (–2, 0), (–6, –2), (–6, 2)
   b. Line y = x: (0, 2), (–2, 6), (2, 6)
   c. Line y = -x: (0, –2), (2, –6), (–2, –6)

11. a. Origin Point: (0, 0), (–4, –2), (–6, –6), (–2, –4)
    b. x-axis: (0, 0), (2, –4), (6, –6), (4, –2)
    c. y-axis: (0, 0), (–4, 2), (–6, 6), (–2, 4)

10. & 12.



Rental Charges — Teacher Notes

Topic Area: Piecewise Functions

NCTM Standard:

- Use symbolic algebra to represent and explain mathematical relationships.
- Interpreting representations of functions of two variables.

Objective

Students will develop procedures for graphing functions formed by "piecing" together two or more functions.

Getting Started

Have the students work in pairs or small groups and discuss the absolute value as a function and analyze the graph of $y = |x|$. Students should indicate how the parent absolute value function could be perceived as two separate linear equations.

Prior to using this activity:

- Students should be able to enter functions into the Casio *fx-9750GII* calculator.
- Students should be able to estimate features of a graph.
- Students should understand how to perform an analysis of graphs.
- Students should have a basic understanding of interpreting graphs.

Ways students can provide evidence of learning:

- Given an absolute value function, the student will be able to write (i) as a piecewise defined function.
- Given a piecewise defined function, the student will be able to evaluate values of x.
- Given a piecewise defined function, the student will be able to sketch a graph.

Common mistakes to be on the lookout for:

- Students may create a syntax error when entering the function.
- Students may create a condition error when entering the function.
- Students may pick an incorrect domain.

Definitions

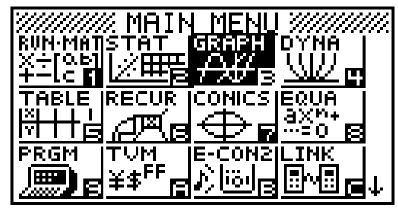
- Domain
- Parent Function
- Piecewise Defined Function
- Absolute Value Function
- Greatest Integer Function
- Step Function

The following will demonstrate how to: enter piecewise defined functions into the Casio *fx-9750GII*, define each domain, graph the function, trace the graph to determine significant features of a function, and locate pre-defined functions.

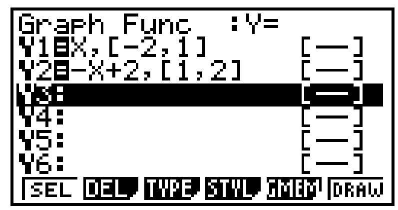
Consider the function $f(x) = \begin{cases} x, \text{ if } -2 \le x < 1 \\ -x + 2, \text{ if } 1 \le x \le 2 \end{cases}$

To enter the above function:

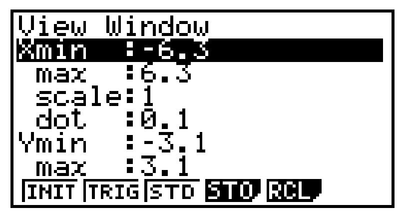
1. From the Main Menu, highlight the GRAPH icon and press [EXE] or press [3].



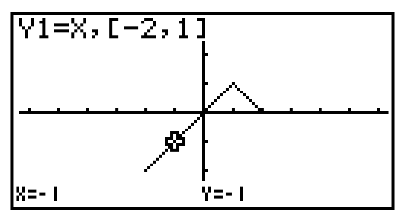
2. To clear any previous functions, press [F2] (DEL) [F1] (Yes).



3. Input the functions by typing each piece into **Y1:** and **Y2:** respectively. Each domain is formatted by using brackets.
   Press [,] [SHIFT] [+] [(–)] [2] [,] [1] [SHIFT] [–] [EXE] for **Y1:** and [,] [SHIFT] [+] [1] [,] [2] [SHIFT] [–] [EXE] for **Y2:**.

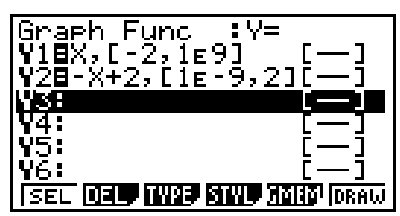


4. Change the window to Initial, by pressing [SHIFT] [F3] **(V-Window)** [F1] (INIT) [EXE].

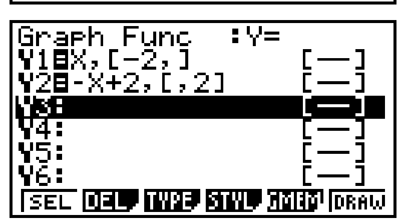


5. Press [F6] (DRAW).

To use infinity as an upper or lower bound:



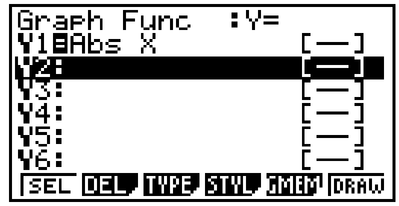
1. Use the [EXP] key to create very large or small numbers. For a large number (∞), press [1] [EXP] [9]. For a small number ($-\infty$), press [1] [EXP] [–] [9]. Or, simply leave the that spot blank. For example, for (∞), the calculator should read [-2,].

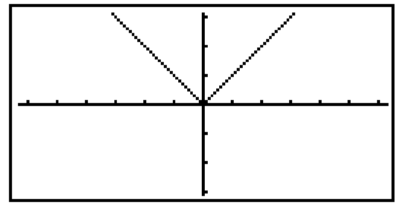


To locate the absolute value function:

1. Press OPTN F5 (NUM) F1 (Abs) X,θ,T.

2. Press F6 (DRAW).



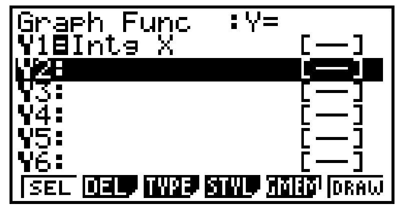


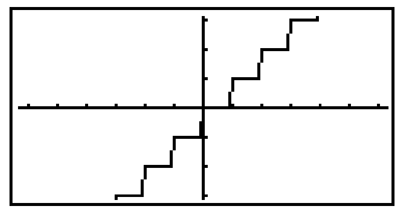
To locate the greatest integer function:

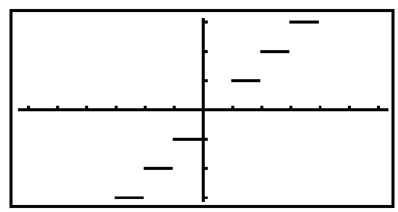
1. Press OPTN F5 (NUM) F5 (Intg) X,θ,T.

2. Press F6 (DRAW).

3. To remove the vertical lines, press SHIFT MENU **(SET UP)** F2 (Plot) EXE.







Rental Charges Activity

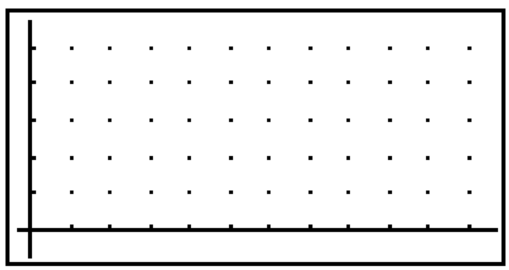
A car rental agency charges \$0.25 per mile if the total mileage does not exceed 100. If the total mileage exceeds 100, the agency charges \$0.25 per mile for the first 100 miles plus \$0.15 per mile for the additional mileage.

Questions

1. If x represents the number of miles a rented vehicle is driven, express the mileage charge C(x) as a function of x. Enter the function into the Casio *fx-9750GII* calculator.

$$C(x) = \begin{cases} \\ \end{cases}$$

2. Sketch a graph of this function.



3. What is the charge for a rented vehicle that has been driven for 50 miles?

____

4. What is the charge for a rented vehicle that has been driven for 150 miles?

____

5. Does it matter which linear function is used to determine the rental charges for 100 miles driven? Why or why not?

____

____

____

____

____

Extension

Let $y = [[x]]$ denote the greatest integer function and suppose $f(x) = \frac{[[10x + 0.5]]}{10}$.

Find:

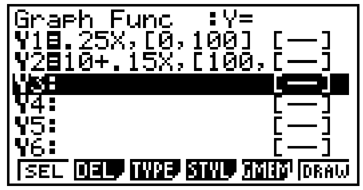
(A) f(6) (B) f(1.8) (C) f(3.24) (D) f(4.582) (E) f(-2.68)

What operation does this function perform?

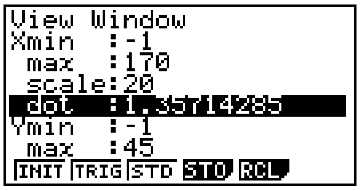
___

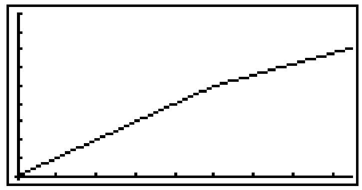
Solutions

1. $C(x) = \begin{cases} 0.25x & \text{if } 0 \le x \le 100 \\ 10 + 0.15x & \text{if } x > 100 \end{cases}$



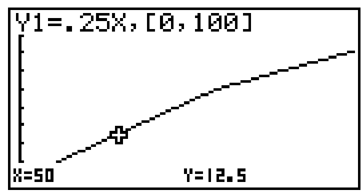
2. Answers may vary: Set an appropriate viewing window.





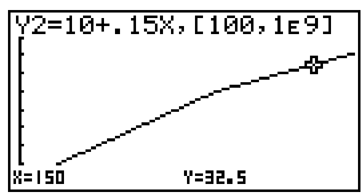
3. It is important to trace to the correct function in order to properly evaluate and avoid an ARGUMENT ERROR.

   Thus C(50) = $12.50



4. It is important to trace to the correct function in order to properly evaluate and avoid an ARGUMENT ERROR.

   Thus C(150) = $32.50



5. Answers may vary: Technically the top function should be used, however, both functions will yield the same value for 100. This is because the function is continuous. If there were any discontinuity, 100 would yield different values.

Extension Solutions

(A) f(6) = 6
(B) f(1.8) = 1.8
(C) f(3.24) = 3.2
(D) f(4.582) = 4.6
(E) f(-2.68) = -2.7

Comparing the values it is conclusive that this function rounds decimals to the nearest tenth.

Riemann Sums — Teacher Notes

Topic: Anti Derivatives, Integration

NCTM Standards
- Apply informal concepts of successive approximation, upper and lower bounds, and limit in measurement situations.
- Analyze precision, accuracy, and approximate error in measurement situations.

Objectives
The student will be able to calculate Riemann sums and develop an understanding of when a Riemann sum approximation will be over or under the actual value of a definite integral. As the number of rectangles increases, the student will observe the convergence of the upper and lower Riemann sum values.

Getting Started
This activity will present students with the tools to calculate and analyze Riemann sums. They will sketch rectangles manually and use their sketches to approximate the area under a curve. They will use the Casio *fx-9750GII* to assist with the calculations to observe the convergence of upper and lower Riemann sums with regular partitions as the size of the partitions decrease and the number of rectangles increase and compare those sums to the integral result.

Prior to using this activity:
- Students should be able to find the area of a rectangle.
- Students should understand what it means to find the area under a curve.
- Students should be able to graph a function over a specified interval.

Ways students can provide evidence of learning:
- Students should be able to sketch their own upper and lower rectangles for a given function over specified intervals.
- Students should be able to explain the difference in the upper and lower Riemann sums.

Common mistakes to be on the lookout for:
- Students may, in calculating the sums manually, forget to multiply the series sum by the base width.

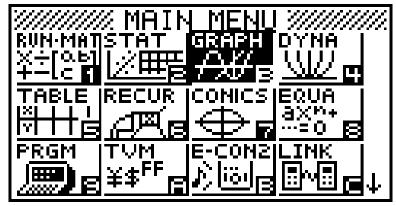
Definitions:
- Riemann sum
- Area under curve
- Upper and lower values
- Convergence

The following will demonstrate how to graph functions, specify a view window, turn on the grid, find a sum of values, and calculate the integral on the Casio *fx-9750GII*.

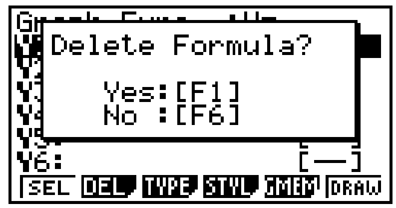
Use the following function: $y = x^3$.

To graph a function:

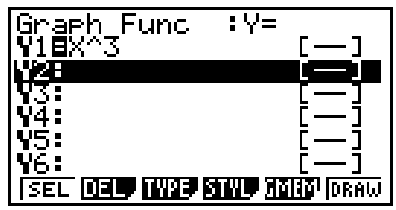
1. From the Main Menu, highlight the GRAPH icon and press [EXE] or press [3].



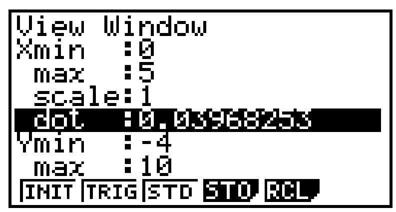
2. To clear out previous graphs, highlight the equation you want to delete and press [F2] (DEL) [F1] (Yes).



3. Enter the equation by pressing [X,θ,T] [^] [3] [EXE]

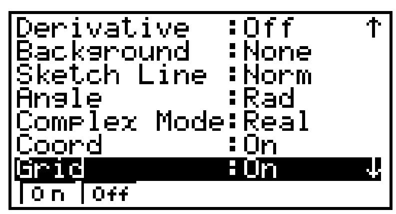


4. Set the view window, by pressing [SHIFT] [F3] **(V-Window)** [0] [EXE] [5] [EXE] [1] [EXE] [▼] [(–)] [4] [EXE] [1] [0] [EXE] [1] [EXE].



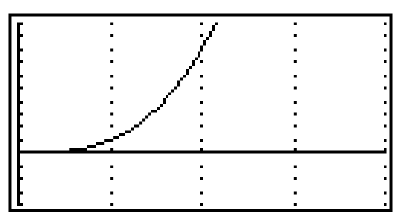
5. Press [EXIT] to return to the initial GRAPH screen.

6. To turn the grid on, press [SHIFT] [MENU] **(Set Up)**. Arrow down to highlight **Grid**, and press [F1] (On).



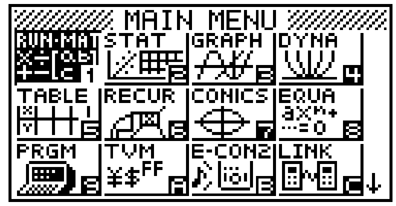
7. Press [EXIT] to return to the initial GRAPH screen.

8. Press [F6] (DRAW) to view the graph of the function.

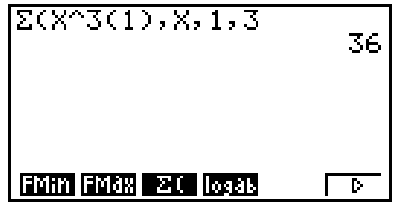


(**See Download Appendix for instructions on how to download program to your software or calculator.)

To find the sum of the values $(1)^3 \cdot (1) + (2)^3 \cdot (1) + (3)^3 \cdot (1)$:

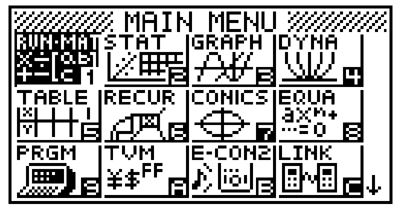


1. From the Main Menu, highlight the RUN•MAT icon and press [EXE] or press [1].

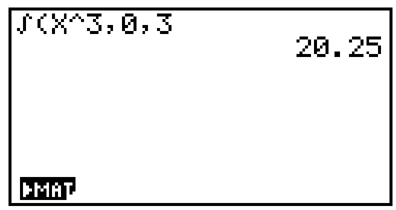


2. Press [OPTN] [F4] (CALC) [F6] (▷) [F4] (Σ() [X,θ,T] [^] [3] [(] [1] [)] [,] [X,θ,T] [,] [1] [,] [3] [EXE].

To calculate the integral $\int_{x=0}^{3} x^3 dx$:

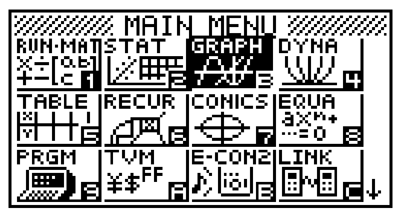


1. From the Main Menu, highlight the RUN•MAT icon and press [EXE] or press [1].

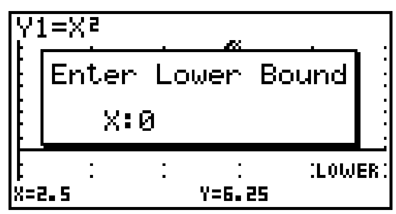


2. Press [OPTN] [F4] (CALC) [F4] (∫dx) [X,θ,T] [^] [3] [,] [0] [,] [3] [EXE].

To graph the integral $\int_{x=0}^{3} x^3 dx$:

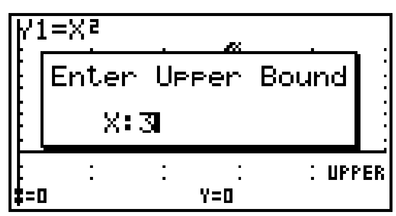


1. From the Main Menu, highlight the GRAPH icon and press [EXE] or press [3].

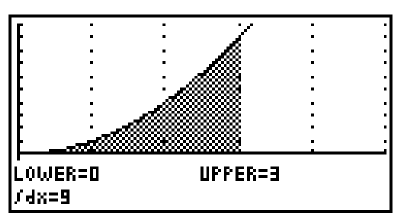


2. Press [EXIT] to return to the initial GRAPH screen.

3. Press [F6] (DRAW) to view the graph of the function.



4. Press [F5] **(G-Solv)** [F6] (▷) [F3] (∫dx). Press [EXE] to select the graph. Press [0] [EXE] to enter x = 0 as the lower bound. Press [3] [EXE] to enter x = 3 as the upper bound.



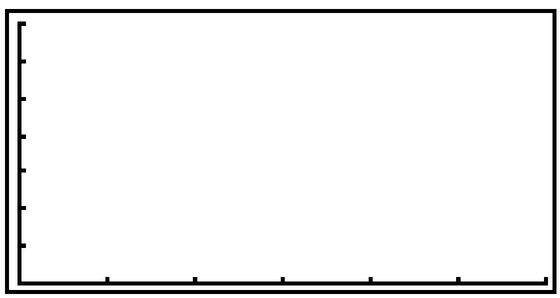
Riemann Sums Activity

A Riemann sum is a method of approximation for calculating the area under a curve. When a function represents change, the area under the curve represents the accumulation of that change. For instance, if you have curve measuring velocity over time, the sum of those velocities over specific time intervals represents the distance traveled during that time period.

This activity will present you with the tools to calculate and analyze Riemann sums. You will sketch rectangles manually and use that to approximate the area under a curve. You will use the Casio *fx-9750GII* to assist with the calculations to observe the convergence of upper and lower Riemann sums with regular partitions as the size of the partitions decrease and the number of rectangles increase.

Questions

1. Assume you are on cruise control driving down a clear highway at a constant rate of 60 miles per hour. Record the graph of your velocity over the first 5 hours on the graph below. Label the axes.



2. Given these conditions, how far would you have gone over the first hour, two hours, or three hours? Plot these points on the graph.

   1 hour: ____________________

   2 hours: ____________________

   3 hours: ____________________

3. Draw a vertical line from the point (1, 60) to the x axis. Find the area of the rectangle for this first hour. Repeat the process for the next 2 hours.

   1 hour: ____________________

   2 hours: ____________________

   3 hours: ____________________

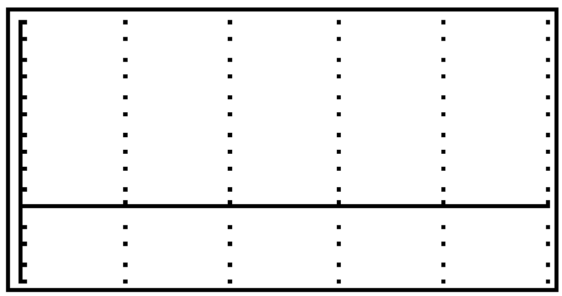
4. What does this area represent with regards to the original problem?

____

____

____

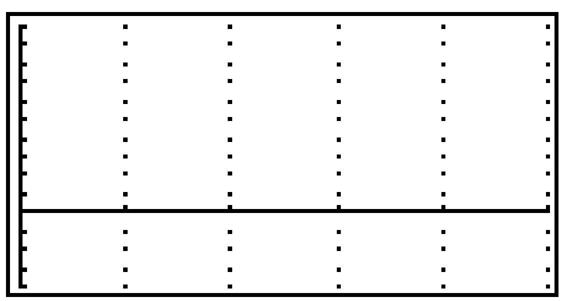
5. Now assume you have a rate of change that is being modeled by the function $y = x^2$ using the window from the “How-To” section. Sketch the function on these axes:



6. It is not as easy to find the area under the curve over the first three hours in this example because the rate of change itself is changing at each point on the interval. We can, however, approximate the area using rectangles. For the first step, find the values of y at the x = 1, x = 2, and x = 3.

(1, _______) (2, _______) (3, _______)

7. Plot those points on the graph of $y = x^2$.



8. Use those points to create three rectangles with the heights given in a. Draw the rectangles in the above graph.

9. Find the total sum of all the rectangles. This is called the Upper Riemann Sum. What does this sum represent?

____

____

____

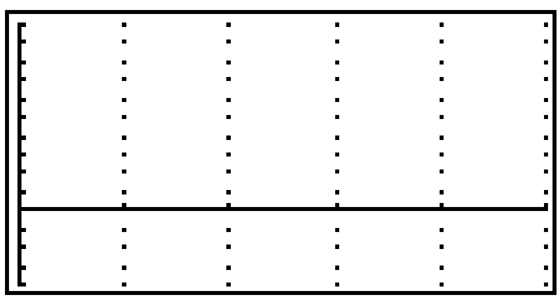
10. Does it seem that this approximation will be more or less than the actual area under the curve? Why?

____

____

____

11. Now sketch the same rectangles, this time using the Left or lower endpoints to mark the height of the triangles. (Hint: You will only have 2 visible rectangles)



12. Calculate the total area represented by these lower rectangles. This is called the Lower Riemann Sum. Does it seem that this approximation will be more or less than the actual area? Why?

____

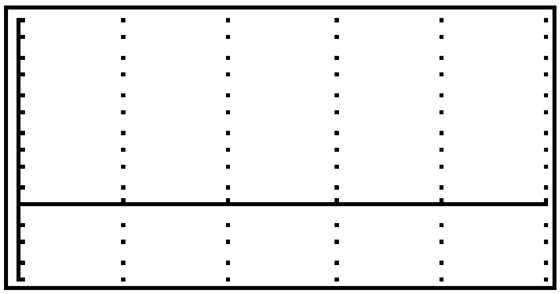
____

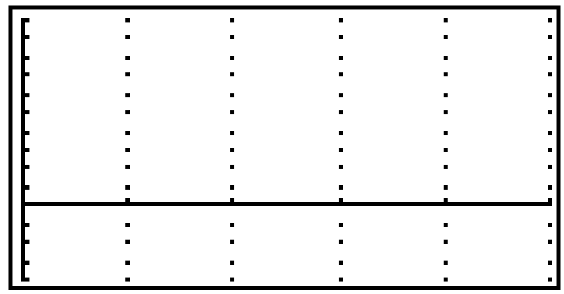
____

13. Now change from 3 to 6 equally spaced rectangles. What will happen to the width of the rectangles in this case?

____

Sketch the rectangles and record the areas first for the Right endpoint, then the Left endpoint rectangles.





Area: ____________________ Area: ____________________

14. You have now found 4 approximations for the area under the curve. Which approximation do you think best represents the area? Why?

____

____

____

15. If we were to continue our exploration with rectangles, what would happen to the width of each rectangle as the number of rectangles increase?

____

16. What would happen to the area under the curve as the number of rectangles increase? Why do you suppose this happens?

____

____

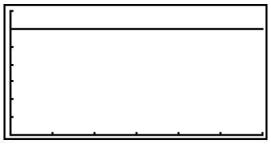
____

17. In your own words explain what these Upper and Lower Riemann Sums represent.

18. Using your calculator, calculate the integral $\int_{x=0}^{3} x^2 dx$. How does this answer compare to your approximations?

Solutions

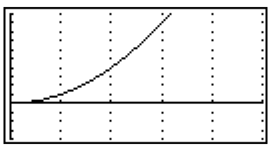
1. $y = 60$



2. 60 miles; 120 miles; 180 miles.
   Rate x time = distance

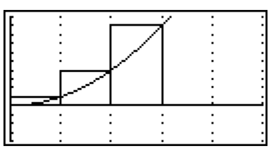
3. 60 units^2; 120 units^2; 180 units^2.

4. The distance traveled each hour. The sum of all the rectangles will equal the total distance traveled.

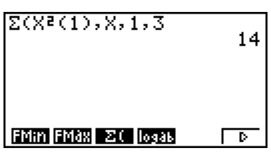
5. 

6. (1, 60) (2, 60) (3, 60)

7. See graph below.

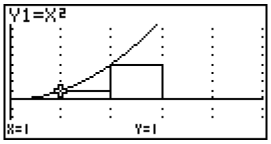
8. 

9. $(1)^2 \cdot 1 + (2)^2 \cdot 1 + (3)^2 \cdot 1 = 14$



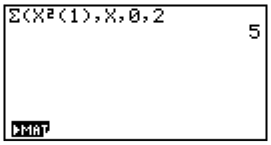
The sum of all the rectangles which represents an approximation of the total distance traveled.

10. Answers should clearly state the area is more than what is covered by the function, as the rectangles are above the function in each case.

11. 

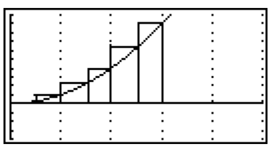
(Note: Some students may be confused in not seeing the three rectangles, they need to be reminded that the left endpoint here is at x = 0, so there will be no rectangle shown.)

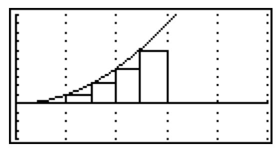
12. $(0)^2 \cdot 1 + (1)^2 \cdot 1 + (2)^2 \cdot 1 = 5$



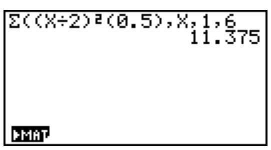
Answers should clearly state the area is less than what is covered by the function, as the rectangles are below the function in each case.

13. The width of the triangles will be half the size of the original or ½ unit.

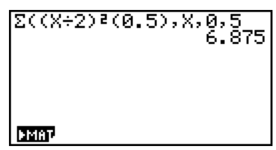




Area = 11.375



Area = 6.875



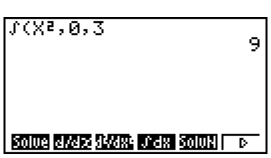
14. Answers will vary.

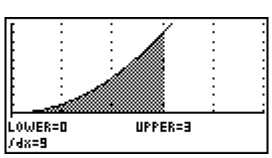
15. The widths get smaller.

16. The areas begin to converge toward a common value. The "why" answers will vary, but a good answer should contain a statement about the error decreasing as the amount of rectangles increase.

17. Answers will vary, but should indicate statements about increasingly accurate approximations of the area under a curve. Some students who may have seen the process of integration might connect the Riemann Sum to the definite integral over that same interval.

18. The area under the curve is 9. This is the number that the Upper and Lower Riemann Sums would approach as the size of the rectangles decreased.





Topic Area: Patterns and Functions – Algebraic Thinking

NCTM Standard:

- Understand patterns, relations, and functions by interpreting representations of functions of two variables using symbolic algebra to explain mathematical relationships.

Objective

Given a set of formulas, the student will be able to use the GRAPH Menu and G-Solve Function to solve problems involving the investment of money.

Getting Started

Discuss the importance of saving money and the difference between simple and compound interest.

Prior to using this activity:

- Students should have a working knowledge of using the calculator to enter various formulas and display the corresponding graph.
- Students should be able to use the G-Solve Function to find specific x- and y-values.

Ways students can provide evidence of learning:

- The students will be able to discuss the results of the activity and justify their answers to specific questions.
- The students will be able to discuss how the graphical results correlate to their answers.

Common calculator or content errors students might make:

- When there are multiple formulas used, students could utilize the wrong formula or substitute the incorrect information for a particular variable.

Definitions

- Interest & Compound Interest
- Principal
- Rate of Interest
- Future value

Formulas

Future Value of an Annuity: $A = \frac{P}{i}\left[\left(1+i\right)^{n} - 1\right]$

[Note: Variables defined in Activity]

Number of Payments: $N = \frac{\log(1 + iF \div P)}{\log(1 + i)}$

Monthly Payment for a Desired Future Value: $P = \frac{iF}{(1+i)^{n} - 1}$

The following will demonstrate how to enter a given formula into the GRAPH module the Casio *fx-9750GII*, graph the data, and use G-Solve to find x- and y-values.

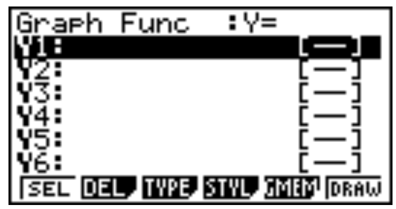
Example Formulas:

$$A = P\left(\frac{1+r}{n}\right)^n \quad \text{where } n = 4,\ P = 100, \text{ and } r = x$$

$$A = \frac{nP}{(1+r)^n} \quad \text{where } n = x,\ P = 1000, \text{ and } r = 0.05$$

Steps for using the GRAPH Menu:

1. From the Main Menu, press [3] for the GRAPH icon.



2. Enter the first formula into **Y1**: by entering:

   [1] [0] [0] [0] [(] [1] [+] [X,θ,T] [a b/c] [4] [)] [^] [4] [EXE].

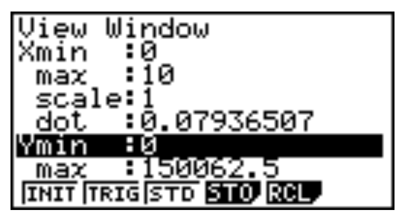
3. Enter the second formula into **Y2**: by inputting:

   [(] [X,θ,T] [×] [1] [0] [0] [0] [)] [÷] [(] [(]

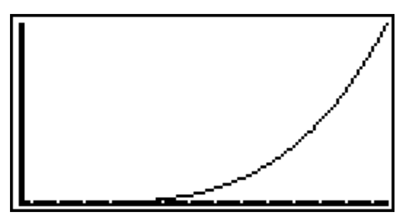
   [1] [+] [.] [0] [5] [)] [^] [X,θ,T] [)] [EXE].

To select the viewing window for the graph of this data:

1. Press the [▲] once, then press [F1] to deselect **Y2**:



2. Press [F6] to display the graph of **YI:**.

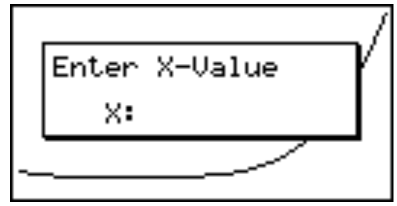


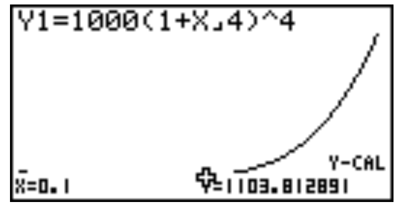
3. Press [SHIFT] [F2] **(V-Window)**, then [F5] (AUTO).

4. To see only the first quadrant, press [SHIFT] [F3] [0] [EXE]

   [▼] three times, [0] then [EXE] three times.

Steps for using G-Solve:





1. Press SHIFT F5 **(G-Solv)**, F6 (▷)and F1 (Y-CAL).

2. To find the y-value when x = 0.1, input the following: • 1 EXE.

3. Press SHIFT F5 **(G-Solv)**, F6 (▷) and F2 (X-CAL).

4. To find the x-value when y = 2000, input the following: 2 0 0 0 EXE.

Saving for a Rainy Day **Activity**

Investing for the future is usually the last thing on a person's mind when they are just entering the workforce. Paying bills, buying groceries, and purchasing a home are usually at the top of the list. However, putting money away in some form of savings should be the number one priority of every budget. Social security and retirement plans are often not enough to allow a person to continue to afford their current lifestyle.

In this activity, you will investigate how much a single investment will earn, calculate the balance of an account with a given monthly payment, and determine the investment amount that is needed to reach a specific financial goal.

Questions

The future value of an annuity can be calculated using the following formula:

$$A = \frac{P}{i}\left[(1+i)^n - 1\right]$$

Where A is the ending balance, P is the principal, i is the rate of interest, and n is the number of times the interest is calculated.

1. What would be the future value of an annuity if $1000 is invested yearly for 5 years at 2.5% APR?

____

2. What would be the future value of an annuity if $1000 is invested yearly for 5 years at 4% APR?

____

3. What would be the future value of an annuity if $1000 is invested yearly for 5 years at 10% APR?

____

4. What would be the future value of an annuity if $10,000 is invested yearly for 5 years at 2.5% APR?

____

5. What would be the future value of an annuity if $10,000 is invested yearly for 5 years at 4% APR?

____

6. What would be the future value of an annuity if $10,000 is invested yearly for 5 years at 10% APR?

____

7. For a principal amount of $10,000, what is the difference between the amount of income earned at 2.5% and the amount of income earned at 10%?

____

8. How long would it take for an investment of $500, earning 4% APR, compounded annually, to earn $1,000?

____

9. How long would it take for an investment of $500, earning 4% APR, compounded annually, to earn $2,000?

____

10. How long would it take for an investment of $500, earning 4% APR, compounded annually, to earn $3,000?

____

11. How long would take to earn $5,000?

____

The formula for finding the number of payments, at a given percent, for a particular annual investment, to reach a specified goal is:

$$N = \frac{\log(1 + iF \div P)}{\log(1 + i)}$$

Where N is the number of payments, i is the interest rate, F is the future value of the investment, and P is the monthly amount invested.

12. Calculate the future value of an investment of $100 a year, at 5% APR, for 10 years.

____

13. Calculate the future value of an investment of $100 a year, at 5% APR, for 30 years.

____

14. Calculate the difference between question 12 and question 13.

____

15. Calculate the future value of an investment of $500 a month, at 5% APR, for 10 years.

____

16. Calculate the future value of an investment of $500 a month, at 5% APR, for 30 years.

____

17. Calculate the difference between question 15 and question 16.

____

The smart move is to start investing early and put aside a set amount each year. The formula for finding the amount of money earned from an annual investment at a given rate is:

$$P = \frac{iF}{(1+i)^n - 1}$$

Where P is the annual amount invested, i is the rate of interest, n is the number of times the interest is calculated, and F is the future value of the investment.

18. Calculate the annual investment needed at 10% APR to earn $100,000 in 10 years.

____

19. What is the difference between the investment for 10 years and the investment at 30 years?

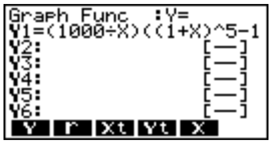
20. What is the benefit of starting to invest early?

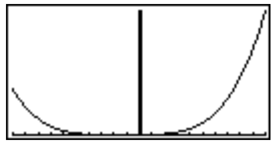
Extension

1. Credit cards charge interest, however, that interest is compounded daily. What changes would need to be made to the compound interest formula to be able to calculate credit card interest that is compounded daily? Explain your thinking.

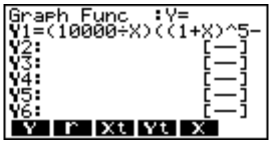
Solutions

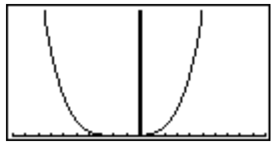
1. $5256.33
2. $5416.32
3. $6105.10



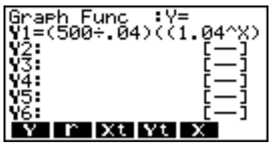


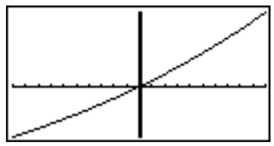
4. $52,563.28
5. $54,163.22
6. $61,051.00



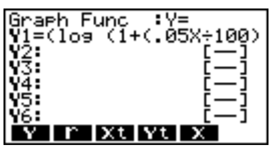


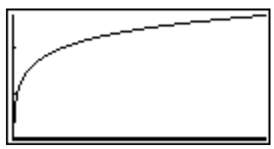
7. $8,487.72
8. 1.9 years
9. 3.8 years
10. 5.5 years
11. Answers will depend on student experience



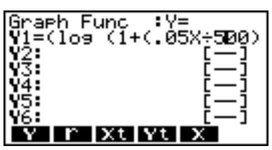


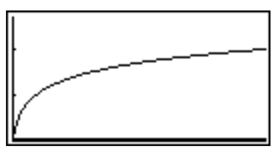
12. $1,257.79
13. $6,643.88
14. $5,386.09





15. $6,288.95
16. $33,219.42
17. $26,930.47





18. $627.45
19. $566.66
20. Answers will vary depending on student experience but should involve the fact that the longer money is invested, the more money you will have.

Extension Solution

1. Answers will vary. Things of note would be the interest rate is being charged not received and that *n* would be changed to a compounded daily rate.

Slope of Curves — Teacher Notes

Topic: Tangent Lines

NCTM Standard

- Understand functions by interpreting representation of functions.
- Compute fluently by developing fluency in operations with real numbers using technology for more-complicated cases.

Objectives

The student will be able to graph and interpret tangent lines to a curve at points of interest.

Getting Started

In this activity the students will investigate the tangent lines to a curve at a point of interest (x_1, y_1) and the slope (m_{tan}) of the curve at that point. Students will find an approximation its slope using a secant line and confirm that slope using the Casio *fx-9750GII*.

In general, it takes two points to determine a line and to calculate its slope. The problem is that a tangent line is defined to intersect the curve only at one point of interest (x_1, y_1). Therefore, an approximation of the tangent line is made by creating a secant line, where we choose a second point (x_2, y_2) that is close to the point of interest. These approximations become more accurate by choosing points closer and closer to the point of interest (x_1, y_1). In fact, the slope of the tangent line (m_{tan}) is defined to be the limit of the secant line slopes as the second point (x_2, y_2) approaches the point of interest (x_1, y_1).

Prior to using this activity:

- Students should be able to produce and manipulate graphs of functions manually and a graphing calculator.
- Students should be able to draw the tangent line to a curve.
- Students should have a basic understanding of the language of limits.

Ways students can provide evidence of learning:

- Students should be able to produce graphs of functions and find the slope of the tangent line using the Casio *fx-9750GII* and estimate the slope using the slope of a secant line.

Common mistakes to be on the lookout for:

- Students may have difficulty understanding the definition of a secant line.
- Students may not be able to find the slope of the tangent line on the calculator. If the derivative is not turned “on” in the set up menu, the equation of a tangent line will not appear.

Definitions:

- Tangent line
- Secant line
- Limit
- Point of interest

Formulas:

$$m_{sec} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \qquad m_{sec} \approx m_{tan} = \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

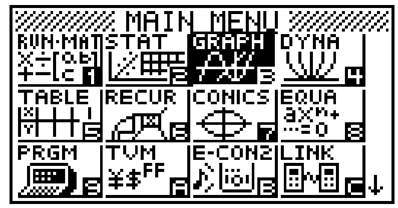
Slope of Curves "How-To"

The following will demonstrate how to graph a function, zoom in to find a second point of interest, graph the tangent line to that function, and find the equation of the tangent on the Casio *fx-9750GII*.

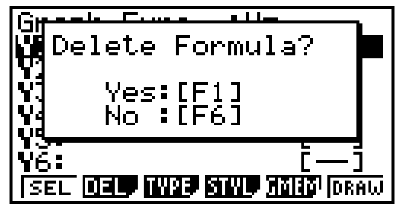
Explore the tangents of the function $f(x) = -2x^3 - 3x^2 + 1$.

To display a graph of the function:

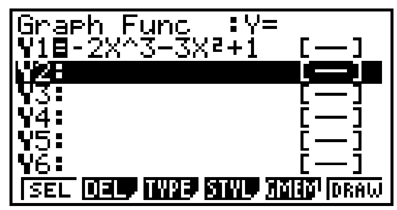
1. From the Main Menu, highlight the GRAPH icon and press [EXE] or press [3].



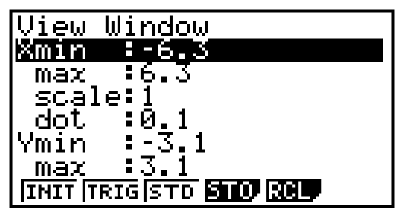
2. To delete any previous equations, highlight the equation and press [F2] (DEL) [F1] (Yes.)



3. Enter the equation in **Y1** by pressing [(−)] [2] [X,θ,T] [∧] [3] [−] [3] [x^2] [+] [1] [EXE].

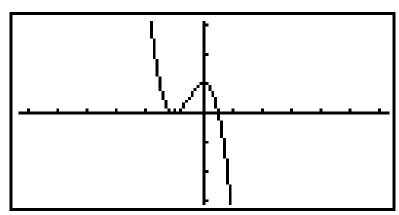


4. Set the view window to the initial screen by pressing [SHIFT] [F3] **(V-Window)** [F1] (INIT).

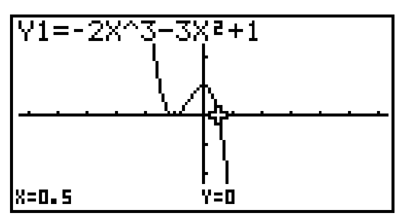


5. Press [EXIT] to return to the initial GRAPH screen.

6. Press [F6] (DRAW) to view the graph of the function.



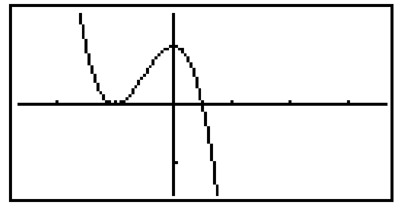
To zoom in and find a second point of interest:

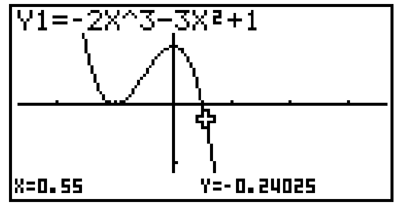


Find a secondary point near x = 0.5.

1. While viewing the graph, press [F1] **(Trace)** [0] [•] [5] [EXE] to find the value of the function at X = 0.5.

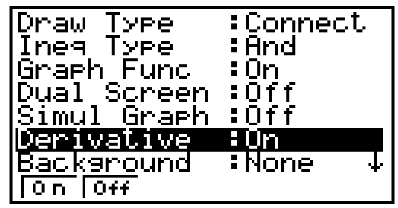
2. To find a secondary point near (0.5, 0), press [F2] **(Zoom)** [F3] (IN). Press [F1] **(Trace)**. Move the tracer once to the right by pressing the right arrow key to arrive at the secondary point (0.55, -0.24025).





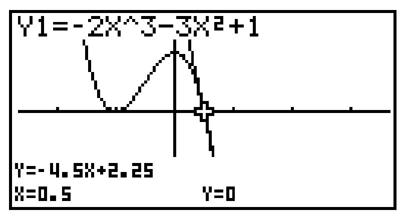
To graph the tangent line and find its equation:

1. Press [SHIFT] [MENU] **(SET UP)**. Arrow down to highlight Derivative. Press [F1] (On).



2. Press [EXIT] to return to the initial GRAPH screen. Press [F6] (DRAW) to view the graph of the function. Set the view window to the initial screen by pressing [SHIFT] [F3] **(V-Window)** [F1] (INIT).

3. Press [F4] **(Sketch)** [F2] (Tang). Move the tracer to the point (0.5, 0) and press [EXE] to draw the tangent line and get its equation. To clear the screen, press [F4] **(Sketch)** [F1] (Cls).

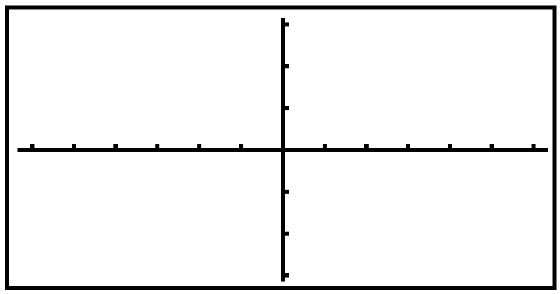


Slope of Curves — Activity

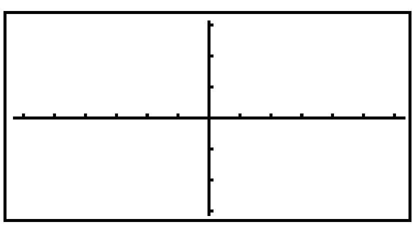
In this activity you will investigate the tangent lines to a curve at a point of interest (x_1, y_1) and the slope (m_{tan}) of the curve at that point. You will make an approximation for the slope of the tangent line and confirm that slope using the Casio *fx-9750GII*.

Questions

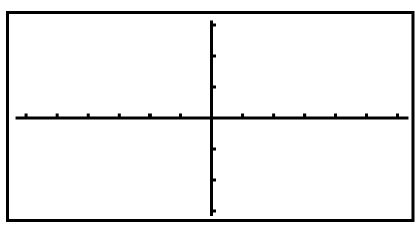
1. On the graphing calculator, graph the function $f(x) = x^3 - x^2 + 1$ using the initial view window. Sketch the results below.



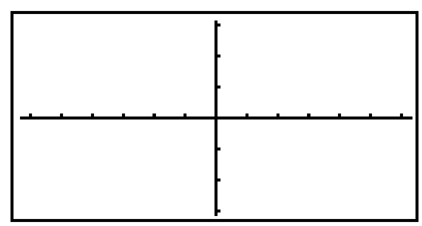
2. Without using the graphing calculator, graph the tangent lines to the function $f(x) = x^3 - x^2 + 1$ at x = -1, 0, and 1. Draw what you see in the boxes provided.



x = -1



x = 0



x = 1

3. Given the function $f(x) = x^3 - x^2 + 1$, use the zoom and trace features to zoom in on the points of interest (x = -1, 0, and 1) and approximate the slope of the tangent line using a second point of interest and the calculation of m_{sec} at these points.

   a. point of interest is (-1, ______), Second point used is (______, ______)

$$m_{sec} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \underline{\hspace{6cm}}$$

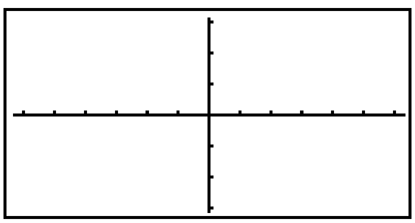
b. point of interest is (0, ______), Second point used is (______, ______)

$$m_{sec} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

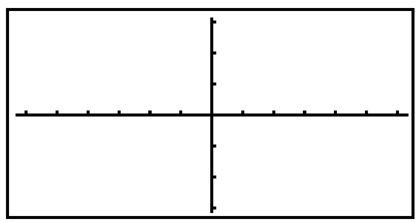
c. point of interest is (1, ______), Second point used is (______, ______)

$$m_{sec} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

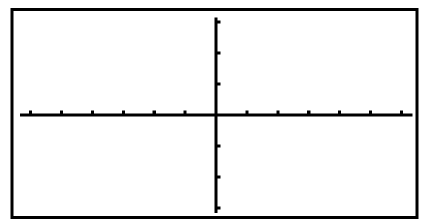
4. Using the graphing calculator, graph the tangent lines and write the equations for these lines to the function $f(x) = x^3 - x^2 + 1$ at $x = -1$, 0, and 1.



x = -1



x = 0



x = 1

Tangent Equations:

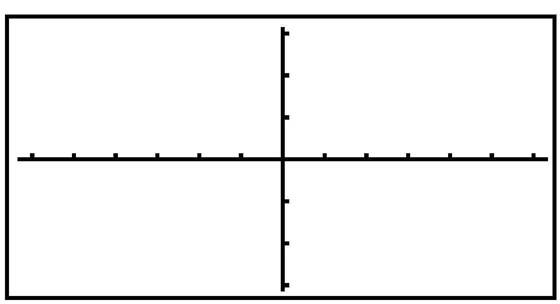
____________________________ ____________________________ ____________________________

5. Compare the slope of the tangent lines in #3 with the slopes of the tangent lines found in #4. What do you notice?

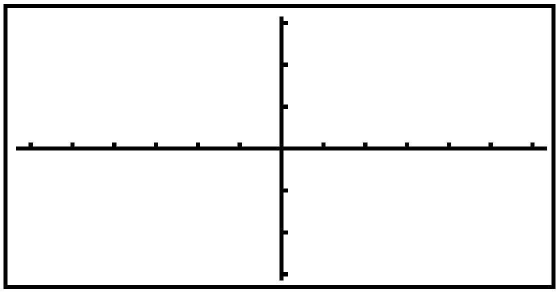
____

____

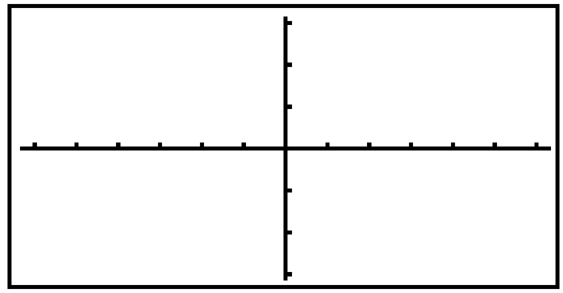
6. Graph the function $f(x) = \sqrt{x}$ in the initial view window.



7. Without using the graphing calculator, graph the tangent lines to the function $f(x) = \sqrt{x}$ at x = 1 and 2. Draw what you see in the boxes provided below.



x = 1



x = 2

8. Given the function $f(x) = \sqrt{x}$, use the zoom and trace features to zoom in on the points of interest (x = 1 and 2) and approximate the slope of the tangent line using a second point of interest and the calculation of m_{sec} at these points.

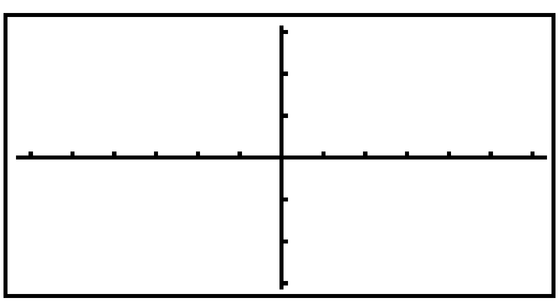
a. point of interest is (1, ______), Second point used is (______, ______)

$$m_{sec} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

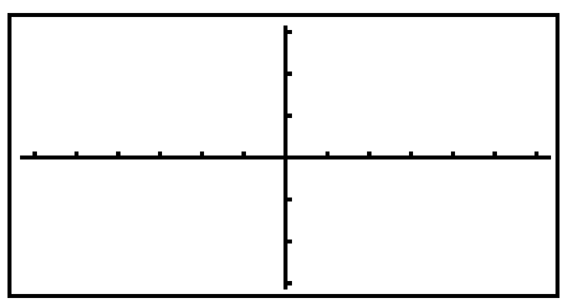
b. point of interest is (2, ______), Second point used is (______, ______)

$$m_{sec} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

9. Using the graphing calculator, graph the tangent lines and write the equations for these lines to the function $f(x) = \sqrt{x}$ at x = 1 and 2.



x = 1



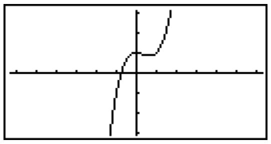
x = 2

Tangent Equations:

______________________________ ______________________________

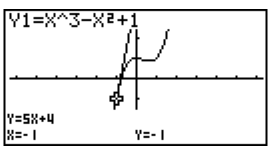
10. Compare the slope of the tangent lines in #8 with the slopes of the tangent lines found in #9. Why aren't they exactly equal? How could you make the slopes equal?

Solutions

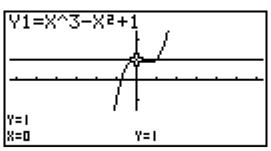
1. 

2. Graphs should look like the solutions in #4.

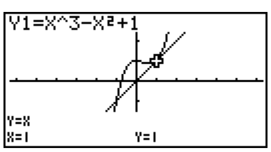
3.
   a. point of interest is (-1, -1), Second point used is (-0.95, -0.759875)
      m_{sec} = 4.8
   b. point of interest is (0, 1), Second point used is (0.05, 0.997625)
      m_{sec} = -0.0475
   c. point of interest is (1, 1), Second point used is (1.05, 1.055125)
      m_{sec} = 1.1025.

4. 

   Tangent Equation: y = 5x + 4

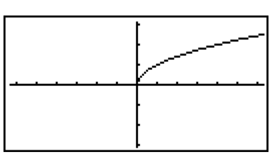


   Tangent Equation y = 1



   Tangent Equation: y = x

5. They are approximately equal.

6. 

7. Graphs should look like the solutions in #9.

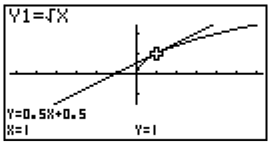
8.

   a. point of interest is (1, 1), Second point used is (1.05, 1.024695)

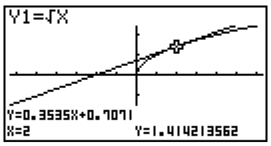
      $m_{sec} = 0.4939$

   b. point of interest is (2, 1.4142), Second point used is (2.05, 1.43178)

      $m_{sec} = 0.3516$.

9. 

   Tangent Equation: $y = 0.5x + 0.5$



   Tangent Equation: $y = 0.3435x + 0.7071$

10. Answers will vary. A good answer should include that the slope of the secant is an approximation of the slope of the tangent line. In order to get a closer approximation, a closer point could be selected to find the slope of the tangent. To actually find the slope, the limit should be found:

$$m_{sec} \approx m_{tan} = \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Topic: Quadratic Equations

NCTM Standard:

- Identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations.

Objective

The student will be able to use the Casio *fx-9750GII* to solve quadratic equations.

Getting Started

Solving quadratic equations is an integral part to studying Algebra. Being able to explore a quadratic equation as a graph will help the student determine how many solutions there are to that equation. Additionally, students need to use the quadratic formula to help determine the roots of the quadratic equation, when the roots are not easily determined by a graph. This activity will guide the student through a variety of ways to determine the solutions for a quadratic equation.

Prior to using this activity:

- Students should understand the properties of a quadratic equation.
- Students should be able to graph a quadratic equation.
- Students should be able to solve for the roots of a quadratic equation.

Ways students can provide evidence of learning:

- When given a quadratic equation, students can solve the equation by graphing it and by using the quadratic formula.
- Students can correctly model how to obtain a solution using two modules on the Casio *fx-9750GII*.

Common calculator or content errors students might make:

- Students may incorrectly enter the quadratic equation into the calculator thus leading to an error message or an incorrect solution.

Formulas

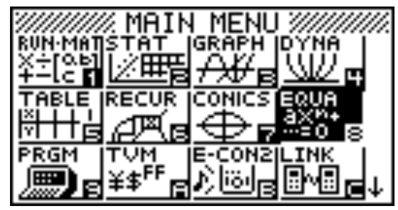
- Discriminant: $b^2 - 4ac$
- Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The following will demonstrate how to use the Equation Solver on the Casio *fx-9750GII* to find the solution(s) for a quadratic equation. It is recommended that you use the polynomial solver to find the solution(s) for a quadratic equation because there may be more than one answer to a given equation.

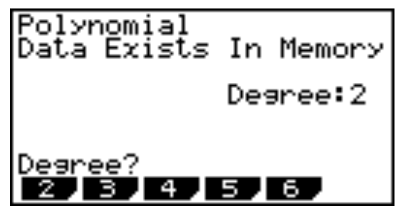
Solve the quadratic equation: $2x^2 + 7x + 6 = 0$.

To solve a quadratic equation in Polynomial Mode:

1. From the Main Menu, highlight the EQUA icon and press EXE or 8.

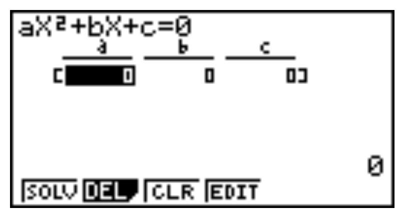


2. Press F2 Polynomial.

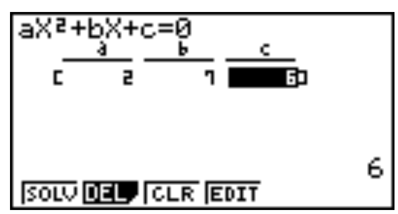


3. Press F1 for a 2^{nd} degree polynomial.

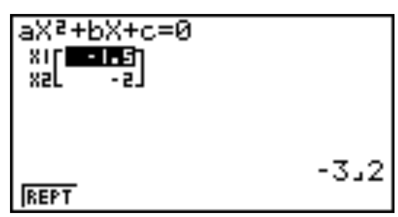
4. The standard form for a polynomial is displayed in the upper right corner of the screen. This polynomial is in standard form, so only the coefficients and constant need to be entered into matrix.



5. To solve, $2x^2 + 7x + 6 = 0$. Enter the values for a, b, and c by entering the following:
   2 EXE 7 EXE 6 EXE



6. There are two solutions for x; -1.5 and -2. The solution which is highlighted in the answer matrix also displays in the lower right corner of the display in its fractional form.



7. To return to the previous screen, press F1 or EXIT.

Solving Quadratic Equations **Activity**

Quadratic equations are written in the form $ax^2 + bx + c = 0$. Quadratic equations can be solved by graphing the equation. These solutions are called roots; when these are substituted for x in the original equation, the equation will equal zero. This activity will guide you through using the Equation Solver on the Casio *fx-9750GII*. Remember that you can use the Graph module to check your work as well.

Questions

1. Solve $x^2 - 5x - 24 = 0$.

____

2. Solve $3x^2 + 2x - 5 = 0$.

____

3. Solve $14x^2 + 6x + 7 = 0$.

____

4. Solve $2x^2 - 32 = 0$.

____

5. The width of a rectangle measures x inches and its length measures twelve more than its width. If the rectangle's area measures 589 square inches, what are the dimensions of the rectangle? Show your equation.

____

6. A ball is projected from a starting point of 5 feet. It is thrown with an initial velocity of 60 feet per second. Given the formula $f(x) = -16x^2 + v_o x + h$, where v_o is the initial velocity and h is the starting height, determine how long the ball will remain in the air. Show your equation.

____

7. How would you solve $(x + 3)(x - 9) = 0$?

____

Extension

8. A rectangle whose length measures (2x - 5) feet and whose width measures (x + 9) feet. Write the quadratic equation you would use to solve for the area.

____

9. Will finding the roots to the quadratic equation in question 8 allow you to find the area of the rectangle? Explain your answer.

____

____

10. Write a quadratic equation that has two solutions and solve it.

____

11. Write a quadratic equation that has only one solution and solve it.

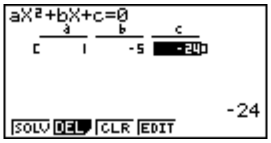
____

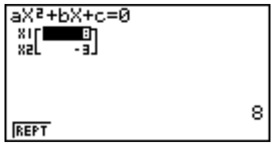
12. Write a quadratic equation that has no solutions and solve it.

____

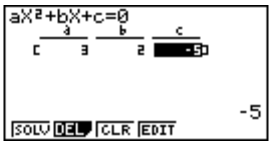
Solutions

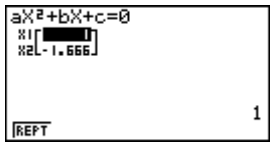
1. There are two real roots. They are x = 8 and x = –3.



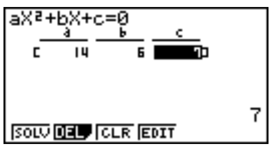


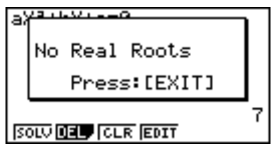
2. There are two real roots. They are: x = 1 and x = $-\frac{5}{3}$.



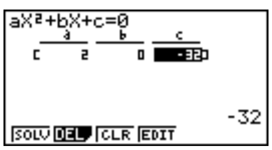


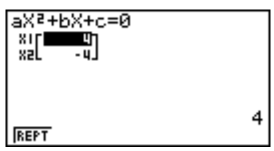
3. There are no real roots to this equation.





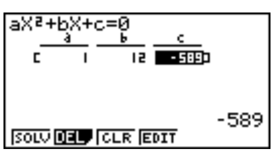
4. There are two real roots. They are x = 4 and x = –4.

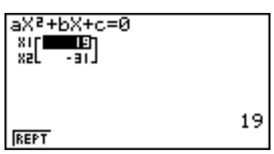




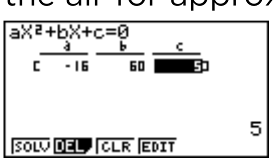
5. The area of the rectangle is x (x + 12) = 589.
$x^2 + 12x - 589 = 0$
This equation has two roots and they are x = 19 and x = -31.

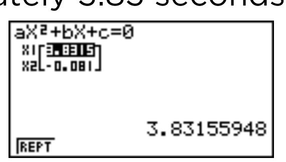
Since we cannot use a negative value for a length, we must use x = 19.
The dimensions of the rectangle are 19 inches by 31 inches.



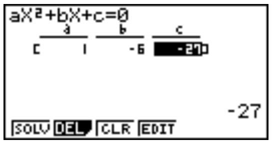


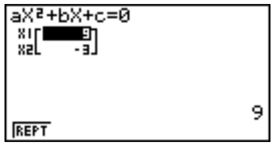
6. The equation you will use is $-16x^2 + 60x + 5 = 0$.
This equation has two roots and they are x = 3.8315 and x = -0.081.
Since x represents time in this problem, a negative value for x has no meaning. Therefore, using the second root tells us that the ball will remain in the air for approximately 3.83 seconds.





7. To solve (x + 3) (x – 9) = 0, begin by multiplying (x + 3)(x – 9) to equal $x^2 - 6x - 27$. Enter the values for a, b, and c into the Equation Solver. There are two real roots for x and they are x = 9 and x = -3.

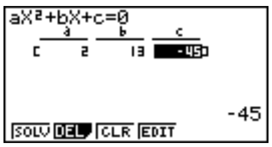


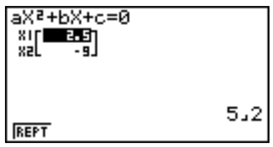


Extension Solutions

8. Area = Length x Width
Area = (2x – 5)(x + 9)
Area = $2x^2 + 13x - 45$

9. To find the solutions, you would use the equation $2x^2 + 13x - 45 = 0$.
Plug the values into the equation solver.





There are two roots to this equation. They are x = 2.5 and x = –9. Either solution you choose would result in an area of zero. So, the solutions for a quadratic equation do not help you find the dimensions of a rectangle, without knowing the actual area.

10. The discriminant of a quadratic equation is $b^2 - 4ac$. This allows you to determine the number of solutions a quadratic equation can have. Any quadratic equation that has a discriminant > 0 will have two solutions.

11. Any quadratic equation that has a discriminant = 0 will only one solution.

12. Any quadratic equation that has a discriminant < 0 will have no solutions.

Solving Systems of Equations — Teacher Notes

Topic: Solving Systems of Equations with Two Variables

NCTM Standard:

- Interpret representations of functions of two variables.

Objective

The student will be able to use the Casio *fx-9750GII* to solve a system of equations.

Getting Started

Students are expected to solve systems of equations symbolically and graphically. Being able to use both strategies will allow students some flexibility in a problem solving situation.

Prior to using this activity:

- Students should know how two solve a system of equations by graphing the two equations.
- Students should know how to simplify equations by combining like terms.

Ways students can provide evidence of learning:

- When given a system of equations, students can graph the equations to find a solution.
- When given a system of equations, students can combine like terms in the equations to find a solution.

Common calculator or content errors students might make:

- Students may incorrectly distribute a negative in an equation.
- Students may incorrectly combine like terms in an equation.

The following will demonstrate how the Casio *fx-9750GII* can be used to check a solution (point of intersection) for a system of equations that has been solved using a non-graphical method like substitution or elimination.

Solve the following system of equations using elimination; check your point of intersection using the calculator.

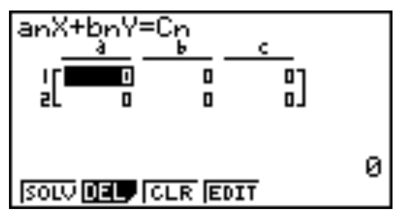
$$\begin{cases} 4x + 5y = 35 \\ -3x + 2y = -9 \end{cases}$$

To enter a system of equations into Simultaneous Mode:

Note: To use the Simultaneous mode, the system of equations must be in Standard Form.

1. From the main icon menu, highlight the EQUA icon and press [EXE] or [8].

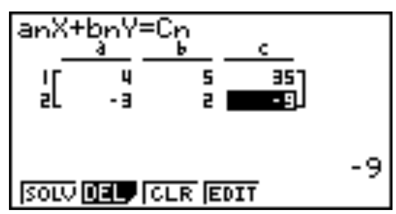
2. To solve more than one equation at a time, press [F1] for Simultaneous. Since there are two variables (unknowns), enter [F1] (2).



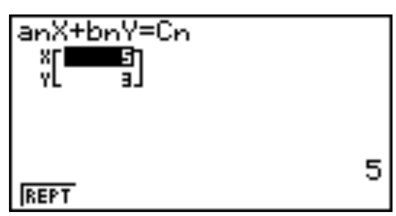
3. The calculator will display a matrix where the coefficients and constants will be entered. To enter this system of equations, input the following:

   [4] [EXE] [5] [EXE] [3] [5] [EXE]

   [(–)] [3] [EXE] [2] [EXE] [(–)] [9] [EXE] [F1] (SOLVE).



4. The solution or point of intersection of this system of equations is (5, 3).



5. To enter another system, press [EXIT] or [F1] (REPT).

   There is no need to delete the previous data, just begin typing the new information.

In the previous lesson, you solved systems of equations by graphing two equations and locating the point of intersection. You may also solve systems of equations using other methods. In this activity, we will explore various ways to solve systems of equations other than graphing.

Questions

One way to solve systems of equations is by substitution. For this strategy, you replace one of the variables with a single value or an algebraic expression. Consider the following two equations:

$y = 3x$ and $x + 2y = -21$

By "substituting" 3x for the y-variable in the second equation, you would now have the following equation:

$x + 2(3x) = -21$

If you follow the order of operations to solve for *x*, you discover that $x = -3$. Now that you have a value for x, you can substitute -3 as the x-value in the first equation, you discover the value for y is -9. So the answer for this system is (-3, -9).

Solve the following using the substitution method. Give your answer as an ordered pair.

1. $y = 4x$
   $x + y = 5$

   ____

2. $4x + 2y = 20$
   $x = 2y$

   ____

3. $3x - 2y = 8$
   $x + 5y = -3$

   ____

Another strategy for solving systems of equations is by elimination. In this strategy, you combine like terms in a system and eliminate one of the variables. Consider the following two equations:

$2r + s = 5$ and $r - s = 1$

When you combine like terms, you are left with $3r = 6$, where $r = 2$. By replacing the *r* with a 2 into either one of the equations, you can solve for *s*. In this case, $s = 1$. Therefore the answer is (2, 1)

When using the elimination method, sometimes you will be required to add, while other times you will be required to subtract. Think about this as you solve the following problems.

4. $x - 2y = 0$
   $4x + 2y = 15$

____

5. $-x + 3y = 6$
   $x + 3y = 18$

____

6. $3x + 4y = 2$
   $4x - 4y = 12$

____

7. $x - y = 4$
   $2x + y = -4$

____

8. $3x + y = 5$
   $2x + y = 10$

____

9. $2x - 5y = -6$
   $2x - 7y = -14$

____

A third strategy is an extension of the elimination strategy.

Consider the following two equations:

x + y = 14 and 9x – 9y = 36

If you multiply the first equation by 9, you have 9x + 9y = 126. If you combine the new equation with the second equation, you will find that x = 9. Complete the solution by replacing *x* with 8 in the original equation and find y = 5. Therefore, the solution is (9, 5).

Solve the following using the elimination method.

10. 2x + y = 5
3x – 2y = 4

____

11. 3x – 2y = 19
5x + 4y = 17

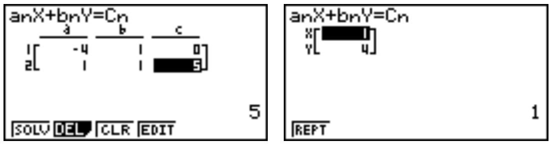
____

12. 3x + 2y = 0
x – 5y = 17

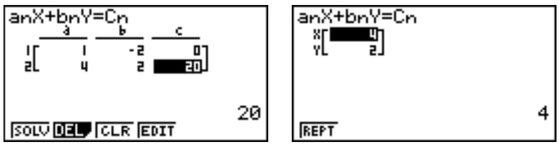
____

Solutions

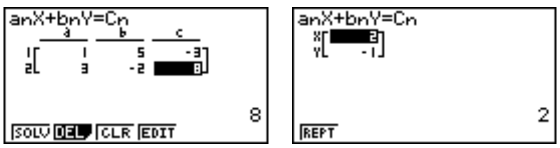
1. The solution is (1, 4).



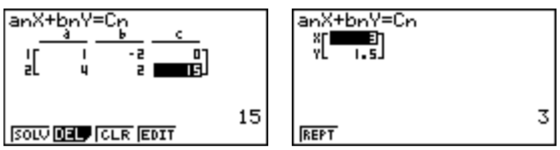
2. The solution is (4, 2).



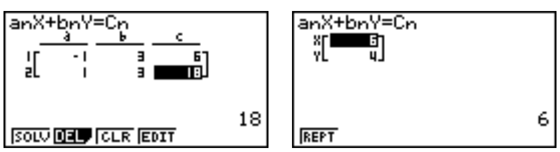
3. The solution is (2, -1).



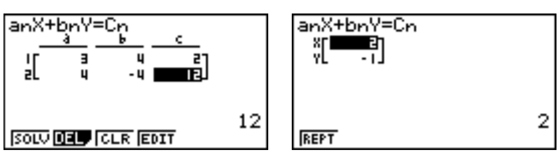
4. The solution is (3, 1.5).



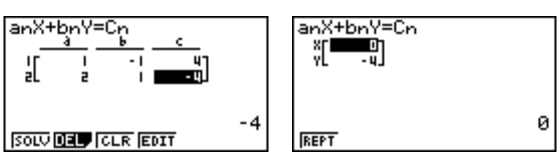
5. The solution is (6, 4).



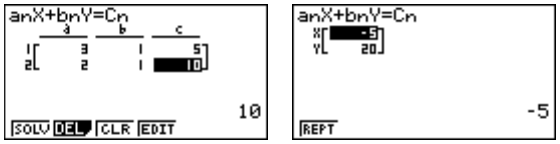
6. The solution is (2, -1).



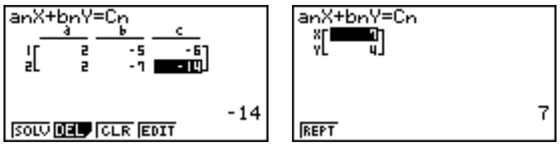
7. The solution is (0, -4).



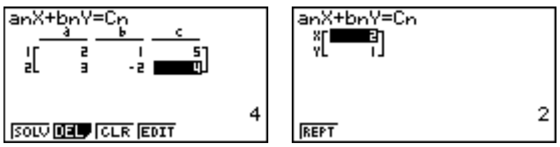
8. The solution is (-5, 20).



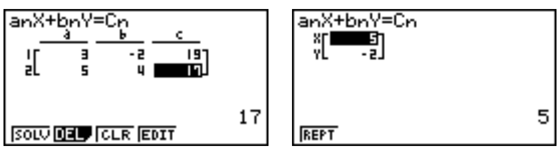
9. The solution is (7, 4).



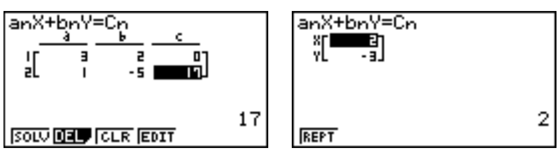
10. The solution is (2, 1).



11. The solution is (5, -2).



12. The solution is (2, -3).



Topic Area: Areas and Volume of Rectangular Solids

NCTM Standards:

- Use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume.
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.

Objective

The student will be able to determine the measures of rectangular solids to find the lateral area of a rectangular solid, find the surface area of a rectangular solid, find the volume of a rectangular solid, and apply finding areas and volume to real-life problems.

Getting Started

As a class, review the definition of a rectangular solid. Bring in examples for students to see and measure. Review the definitions of length, width, and height as well as the formulas for areas and volumes. Elicit ideas of how each of these formulas is used in real life such as painting walls of rooms, gift wrapping packages, and filling rectangular swimming pools.

Prior to using this activity:

- Students should be able to work with measures both in standard and metric systems.
- Students should know the appropriate measures used in finding area and volume.
- Students should understand how to use formulas to solve for any variable.

Ways students can provide evidence of learning:

- The student will be able to use a formula to solve for a given value.
- The student will be able to solve a problem by using a formula and applying the results to the problem.

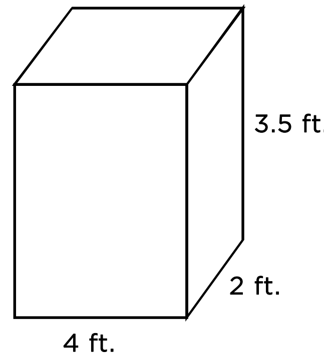
Common mistakes to be on the lookout for:

- Students may use the wrong formula.
- Students may use the wrong measurements.
- Students may solve for the wrong variable.

Definitions

- Rectangular Solid
- Lateral Area
- Surface Area
- Volume
- Length
- Width
- Height

The following will demonstrate how to enter a formula into the Equation mode of the Casio *fx-9750GII* and solve for a specified unknown value.

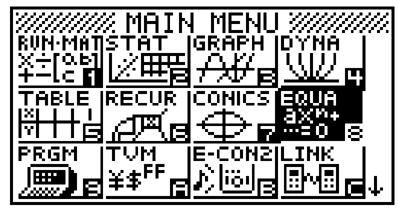


Find the surface area of the container to calculate the amount of plastic film needed and the volume of the container to determine the amount of space necessary to transport the equipment.

Surface Area = 2lw + 2lh + 2wh
Volume = lwh

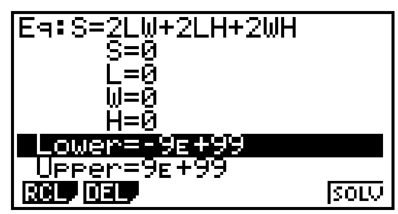
To enter the formula into the Equation Solver:

1. From the Main Menu, highlight the EQUA icon and press [EXE] or press [8].

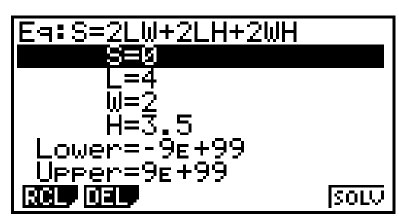


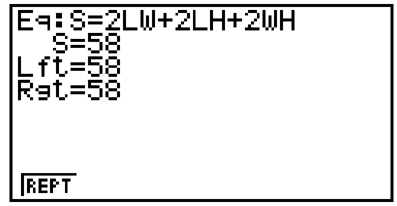
2. Press [F3] (SOLV) to access the Equation Solver. If there is already an equation in the calculator, press [F2] (DEL) [F1] (Yes) to clear the formula.

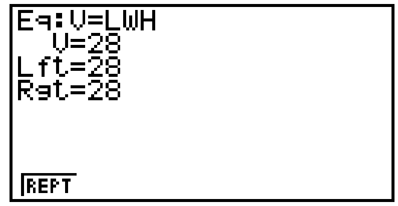
3. Enter the formula for finding the surface area by pressing [SHIFT] [ALPHA] [×] [SHIFT] [•] [2] [ALPHA] [→] [ALPHA] [3] [+] [2] [ALPHA] [→] [ALPHA] [F↔D] [+] [2] [ALPHA] [3] [ALPHA] [F↔D] [EXE]. The calculator should show the screen at the right. If numbers appear next to the variables, these represent values already in the calculator. When you enter values for the variable, the calculator will replace them.



4. With L highlighted, press [4] [EXE], with W highlighted, press [2] [EXE], and with H highlighted, press [3] [•] [5] [EXE]. Highlight S using the arrow keys and press [F6] (SOLV) to see the surface area of the container.







5. To find the volume, press [F1] (REPT) [F2] (DEL) [F1] (Yes) to clear the formula. Enter the formula for finding volume by pressing [ALPHA] [2] [SHIFT] [•] [ALPHA] [➡] [ALPHA] [3] [ALPHA] [F↔D] [EXE]. Enter Highlight V and press [EXE].

With the invention of the internet, people were able to sell and purchase goods from across the country and around the world. Many of these items are shipped in containers aboard airplanes, ships, trains, and eighteen wheeler trucks. Some of the packages must be specially sealed to prevent moisture damage contamination. In this activity, you will determine the volume of a package, how much surface area must be covered, and how many packages can be put into a given container.

Questions

The Pleasant Valley Orchard ships apples and pears to customers all over the country. They use crates that measure 1.5 feet x 2 feet x 1 foot. Each crate holds 128 pieces of fruit.

1. What is the volume for each crate used for shipping the fruit?

____

2. If a container for an eighteen wheeler has measurements of 6 feet x 8feet x 12 feet, what is the capacity of the container?

____

3. How many of the crates can be loaded into the container?

____

4. If each crate must be sealed with plastic to prevent damage, how much plastic wrap is needed for each shipment?

____

The Seafarer Restaurant imports salmon from Alaska to serve to their patrons. The fish arrives in crates that are 3 feet x 1.5 feet x 2 feet and sealed with a water proof material. Each crate is sealed in a larger crate that measures 3.25 feet x 1.75 feet x 2.25 feet, so that the shipper can pack the smaller crates in dry ice.

5. What is the volume of the smaller crate?

____

6. What is the volume of the larger crate?

____

7. How much dry ice is used to keep each crate of salmon cold?

8. How much water proof material is needed to cover one crate?

9. Calculate the cost of shipping each crate if the cost to ship is $2.25 per pound. (Each cubic foot equals 4 lbs.)

10. What is the cost to ship 50 crates of salmon to the restaurant?

The Lil Tot Toy Company is getting ready to package their new toy. The box needs a base that measures 5 inches x 5 inches.

11. If the total package needs 250 sq. in. of surface area for advertising, how tall must the package be?

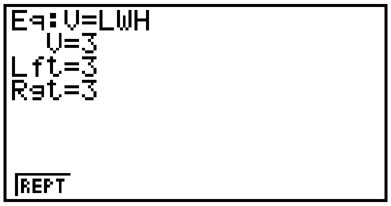
12. What is the volume of the package?

13. A local toy store has a display area with a volume of 14,400 cu. in. which will allow each row of toys to be 6 packages across and 4 packages deep. What is the length of the display area? What is the width of the display area?

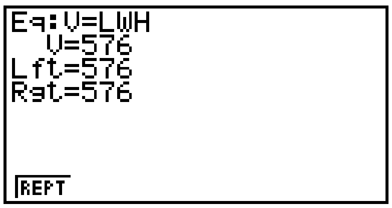
14. Find the number of rows that can be displayed at the store.

Solutions

1. V = 3 cu. ft.

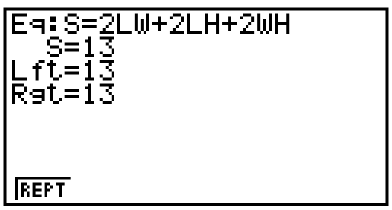


2. V = 576 cu. ft.

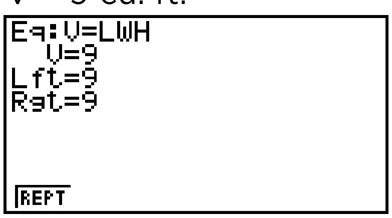


3. 576 ÷ 3 = 192 crates

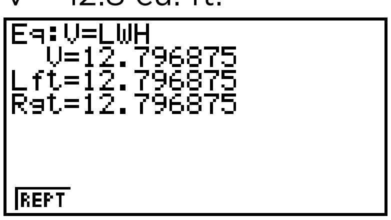
4. S = 13 sq. ft.
192 x 13 sq. ft. = 2,496 Sq. Ft.



5. V = 9 cu. ft.

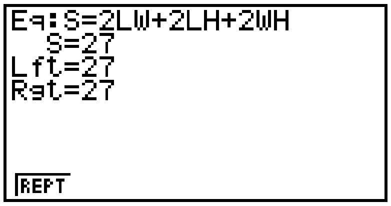


6. V = 12.8 cu. ft.



7. 12.8 – 9 = 3.8 cu. ft.

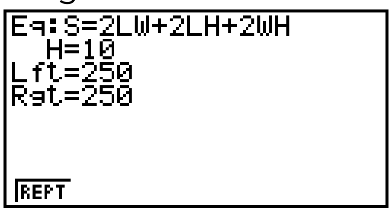
8. V= 27 cu. ft.



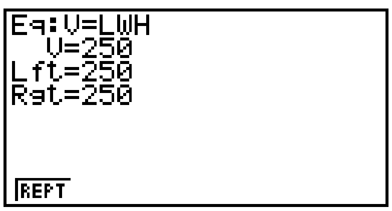
9. 12.8 x 4 x 2.25 = $115.20 per crate

10. $115.20 x 50 = $5,760.00 for 50 crates

11. Height = 10 in.

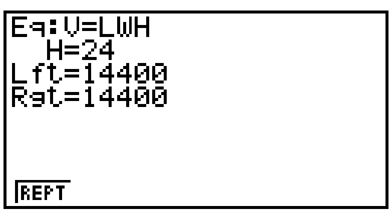


12. V = 250 cu. in.



13. Across = 6 x 5 in. = 30 in.
Depth = 4 x 5 in. = 20 in.

14. 24 in. ÷ 10 in. = 2.4 or 2 rows



Spending Too Much — Teacher Notes

Topic: Non-linear Bivariate Data

NCTM Standards:

- Understand the meaning of measurement data and categorical data, of univariate and bivariate data, and of the term variable.
- Understand histograms, parallel box plots, and scatter plots and use them to display data.
- Compute basic statistics and understand the distinction between a statistic and a parameter.
- Find, use, and interpret measures of center and spread, including mean and interquartile range.
- For bivariate measurement data, be able to display a scatter plot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools.

Objective

The student will be able to plot bivariate data in a scatter plot, make conjectures based on the plot, and create a regression equation using the graphing calculator.

Getting Started

Have the students work in pairs or small groups and come up with examples using two-variable data. Have the students also decide the best way to represent their two variable data.

Prior to using this activity:

- Students should be able to calculate basic statistics.
- The students should be able to create a histogram, scatter plot, and box and whisker plot.
- Students should be able to calculate the regression equation that best fits the data.

Ways students can provide evidence of learning:

- Given a set of data, the student should be able to create a scatter plot and calculate the regression equation.
- Given a set of data, the student should be able to create several types of graphs to display the data.

Common Mistakes to be on the lookout for:

- Students might only choose a linear regression, when another type of regression might fit better.

Definitions

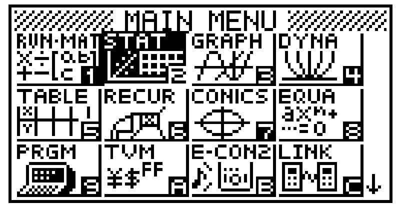
- Univariate
- Regression
- Interpolation
- Y-intercept
- R and R^2 values
- Bivariate
- Trend
- Extrapolation
- Slope

The following will demonstrate how to enter a set of data into the Casio *fx-9750GII*, graph the data using a scatter plot, calculate different types of regression, and use a graph and table to estimate a pattern from the regression equation.

x	1	2	3	4	6	8
y	10	7	4	2	-1	0

To enter the data from the table in the problem:

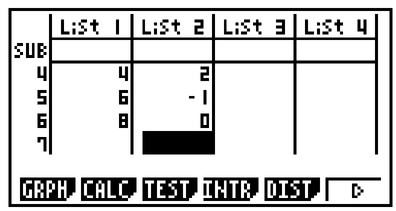
1. From the Main Menu, highlight the STAT icon and press [EXE] or press [2].



2. To clear previous data lists press [F6] (▷) [F4] (DEL-A) [F1] (Yes).

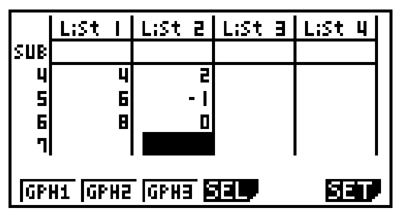
3. Enter the data by typing each number, pressing [EXE] after each entry.

4. The display should look like the screen shot on the right when completed.



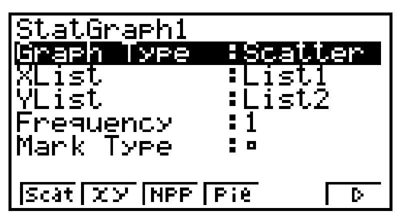
To select a scatterplot to view the data:

1. Press [F1] (GRPH).



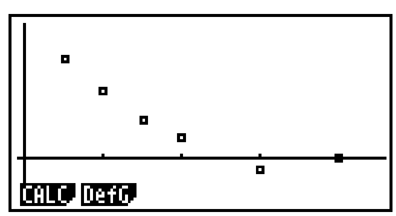
2. Select [F6] (Set) to set the type of graph for **StatGraph1**.

3. Press ▼ to highlight **Graph Type**.



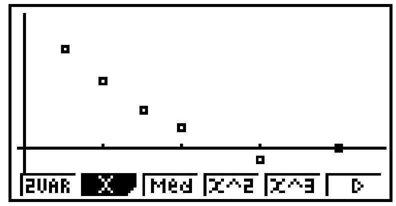
4. Choose [F1] (Scat) to select scatterplot, [EXE].

5. Press [F1] (GPH1) to display the graph.

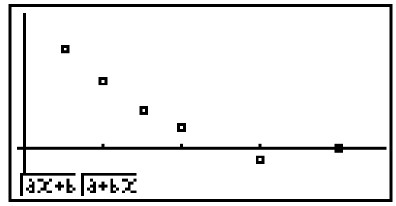


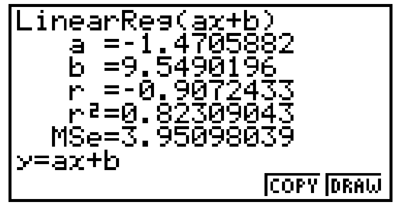
To select types of regression:

1. Press F1 (CALC).

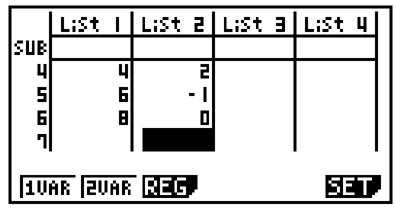


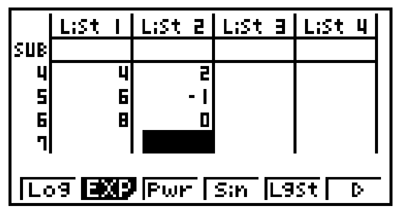
2. Choose the type of regression to be performed; for linear regression, select F1 (X). Select F1 for the regression to be displayed in ax + b form.

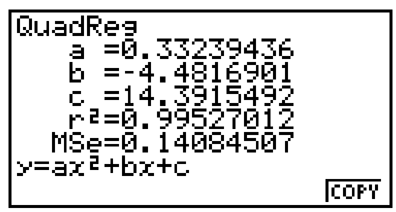




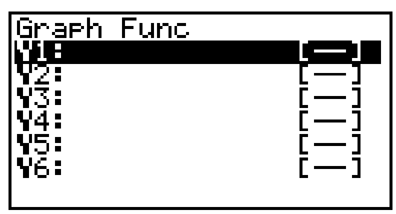
3. To calculate another type of regression, press EXIT twice to display the on-screen regression options; this time, instead of graphing the data, press F2 (CALC) to go straight to calculating a regression, then F3 (REG). Pressing F6 (▷) will provide more on-screen options. A quadratic regression is shown to right.



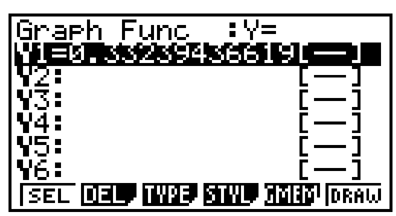




4. To copy the regression equation into the graph or table mode, select F6 (COPY), then EXE.



5. From the Main Menu, highlight the GRAPH or TABLE icon. Press F1 (SEL) to select the equation, then F6 (DRAW) to draw or view the table.



Sandy's parents loaned her some money for her to use during college. She noticed that she spent a certain percentage of that money each month. The chart below represents the amount of money left in Sandy's account at the end of each month for her first twenty months away from home.

Month	Money Left in Account	Month	Money Left in Account
1	$5200.00	11	$3887.10
2	$5050.00	12	$3776.50
3	$4904.50	13	$3669.20
4	$4763.30	14	$3565.10
5	$4626.40	15	$3464.10
6	$4493.60	16	$3366.20
7	$4364.80	17	$3271.20
8	$4239.90	18	$3179.10
9	$4118.70	19	$3089.70
10	$4001.10	20	$3003.00

Questions

1. What regression equation best represents her spending?

 ____

2. Explain what the coefficients and other numbers in the equation mean?

 ____

 ____

 ____

3. How much money will she have in her account in four years, if she continues the same spending habits?

 ____

4. Will she have enough money to spend for the four years of college? Why or why not?

 ____

 ____

 ____

5. When will she run out of money according to the regression equation? Is this realistic? Why or why not?

____

____

____

Extensions

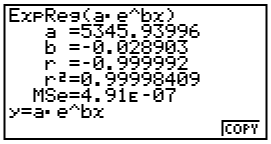
1. With your family, create a budget for your first year of college, try to estimate the amount of money you will need for all non-school related expenses.

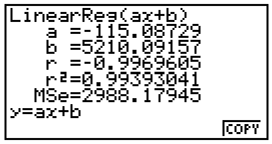
2. Create a table, similar to the table in the exercise; remember that you will probably use more money near the winter break, as well as during Spring Break.

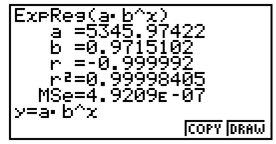
3. Compare your table with other students in your class, and also with a friend that is already in college to see if it is realistic.

Solutions

1. Students might choose linear regression which is y = -115.087x + 5210.092, but trying other types of regression, the one that fits best is the exponential regression $y = 5345.940e^{-0.029x}$.



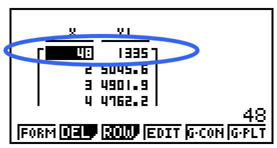




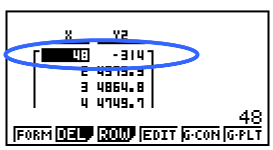
2. The coefficient in front (5345.904) represents the y-intercept. Since the table started at 1, it estimates the value of the money in her account at time zero.

   In the linear equation, the *a* represents the amount of money spent each month, and the *b* represents the amount of money at time zero, or how much money she had at the beginning.

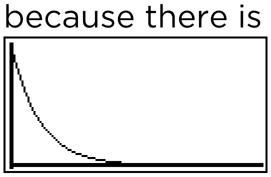
3. She will have $1335 left in her account after four years, if you use the exponential regression equation.



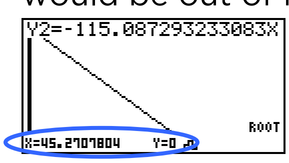
   If you use the linear regression, she will in debt $314.



4. According to the exponential regression, she will never run out of money, because there is an asymptote at y = 0.



   It would be better to use the linear regression equation, in that case, she would be out of money at her 45th month, just shy of 4 years.



5. She should run out of money at the 45th month. Answers many vary on whether this is realistic.

Topic Area: Absolute Value Equations

NCTM Standards:
- Use symbolic algebra to represent and explain mathematical relationships.
- Use trigonometric relationships to determine lengths and angle measures. Specify locations and describe spatial relationships using coordinate geometry and other representational systems

Objective

The student will be able to find the slope of a line in a three-dimensional model, the value of the tangent for an angle between two non-vertical lines, and find the measure of the angle between the lines in degrees.

Getting Started

Have the students work in pairs or in small groups to discuss how lights are used to enhance displays of objects in museums and art galleries. Provide them with a small flashlight, have them change the focus of the light, and discuss how this can be applied to displays. Have students think of other areas that would use this type of technology.

Prior to using this activity:
- Students should have an understanding of finding the slope of a line on a coordinate plane.
- Students should have an understanding of how to find the measure of an angle given the value of its tangent.

Ways students can provide evidence of learning:
- The student will be able to create a three dimensional model of a room and show how they would set up lights to enhance items displayed in the room.
- The student will be able to draw a three-dimensional drawing of a room with coordinates and write problems that can be solved by other students.

Common calculator or content errors students might make:
- Students may use the wrong formula for finding slope.
- Students may find the measure of the angle in radians instead of degrees.

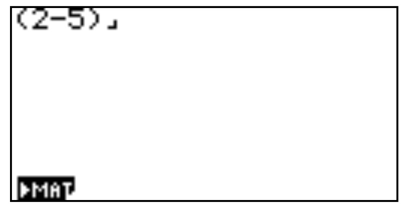
Definitions & Formulas
- Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Tangent Between Lines**:** $\tan\alpha = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$

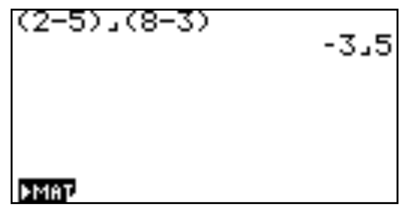
The following will demonstrate how to use the fraction key to find the slope of a line, the tangent key to find the measure of an angle, and how to enter absolute values using the Casio *fx-9750GII*.

Find the slope of a line whose coordinates are (3, 5) and (8, 2).
Find the measure of an angle whose tangent value is 5.3417.
Find the distance between the coordinates $-\frac{11}{20}$ and $\frac{3}{20}$ on a number line.

To find the slope of a line:

1. Highlight the RUN•MAT icon in the Main Menu and press [EXE]. Enter the numerator (difference of the y-coordinates) in parentheses, press [a b/c]. Using parentheses, enter the denominator (difference of the x-coordinates) and press [EXE]. Since the slope was entered as a fraction, the answer will be displayed as a fraction.

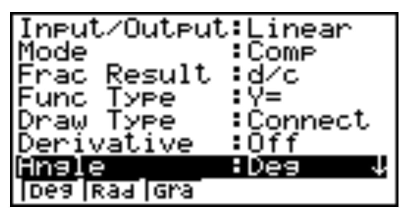




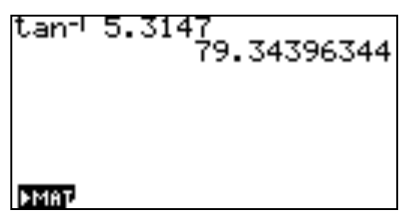
2. To see the decimal equivalent of $-\frac{3}{5}$, press [F↔D].

To find the measure of an angle given its tangent:

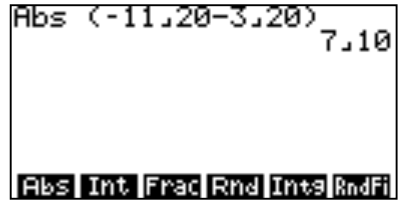
1. To find the measure of an angle, the calculator must be in Degree mode. To do this, press [SHIFT] [MENU] **(Setup)**, [▼] to highlight the word **Angle**. If **Deg** is not selected, press [F1] (Deg) and [EXE].



2. Press [EXIT] to return to the Run screen. Press [SHIFT] [tan] and enter the value for the tangent. Press [EXE] to see the measure of the angle as shown at the right.



To find the absolute value using the calculator:



1. Press **OPTN** **F6** **F4** (Num), then **F1** (Abs).

2. Enter the expression and press **EXE**.

Spotlight on Art   Activity

Many museums and galleries set up spectacular displays that are enhanced by spot lighting. The lights are set at specific angles to highlight items within the display. One such use would be to highlight a particular piece of clothing or tool that was used by an ancient culture.

In this activity, you will use the slope of a line and a formula to find the angle of a spot light beam directed at different objects.

Questions

1. A spotlight is being directed at a painting on the opposite side of a 20 ft. wide room. The spotlight is located at the top of a 12 ft. wall. The bottom of the painting is 5 ft. from the floor and the top of the painting is 8 ft. from the floor. Draw a diagram to illustrate this spot light.

   What is the slope from the spotlight to the top of the painting?

   ____

   What is the slope from the spotlight to the bottom of the painting?

   ____

2. Use the formula to find the measure of the angle for the spotlight.

   ____

3. In another room, a sculpture is being spotlighted from the floor 3 ft. from the base. The bottom of the light hits the sculpture 2 ft. above the floor and the top of the light hits the sculpture 8 ft. from the floor. What is the angle of this light?

   ____

4. In a gallery displaying ancient buildings, two spotlights are directed at a model of the Tower of London. The first light is 1 ft. off the floor and 5 ft. away from the model. It shows a spot ranging from 3 ft. to 5 ft. off the floor. The second spot is 2 ft. off the floor and 2 ft. away from the model. It shows a spot ranging from 4 ft. to 8 ft. off the floor. Determine and describe where the lights are in reference to each other- same side or opposite sides of the display? What is the angle of each light?

First Light: ____

Second Light: ____

Extension

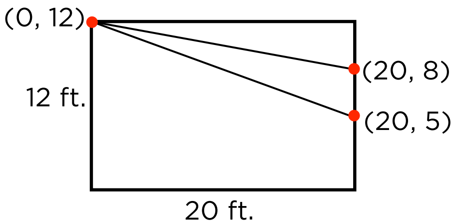
1. Take several flashlights of different sizes and find the angle of their light. Does the angle change if the flashlight is pointed at different angles? Justify your answer.

____

____

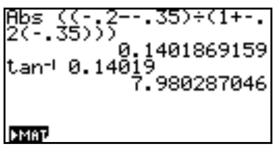
2. Create a display that illustrates the use of a spotlight on an object and show the measurements including the angle of the light.

Solutions

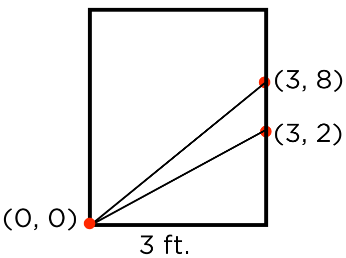
1. 

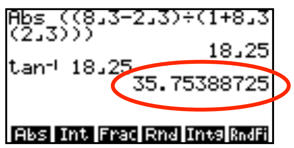
Slope to Top = –0.2
Slope to Bottom = -0.35

2. α ≈ 8°

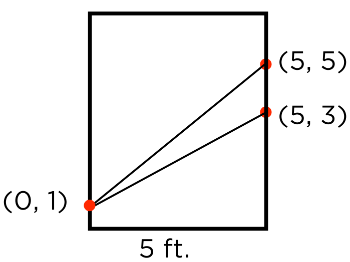


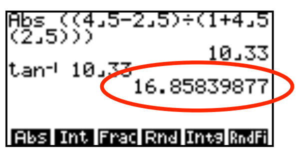
3. α ≈ 35.8°



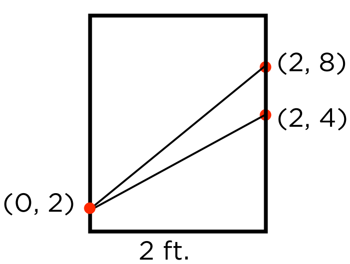


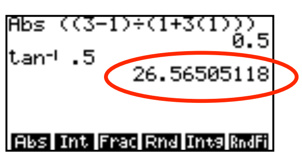
4. α ≈ 16.9°





α ≈ 26.6°





Extension Solutions

1. Answers will vary

2. Answers will vary

Topic Area: Properties of Squares

NCTM Standards:

- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.
- Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations.
- Investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates.

Objective

The student will be able to use algebra and statistics to prove that a parallelogram is a rectangle or square, demonstrate that the diagonals of a rectangle are equal, demonstrate that all four sides of a square are equal, demonstrate that the diagonals of a square are perpendicular to each other, and apply the properties of rectangles and squares to real-life problems.

Getting Started

As a class, review the meaning of slope and the slope-intercept form of an equation; include in the discussion the properties of a parallelogram. Review methods of proving triangles congruent using the Hypotenuse-Leg method.

Prior to using this activity:

- Students should be able to find the xy-line for a pair of coordinates using a graphing calculator.
- Students should be able to perform calculations involving square roots and trigonometric functions using a graphing calculator.

Ways students can provide evidence of learning:

- The student will be able to write conjectures pertaining to squares.
- The student will be able apply the properties of a parallelogram to real-life problems.

Common mistakes to be on the lookout for:

- Students may confuse the x and y values in the calculations.
- Students may enter the problem incorrectly into the calculator.

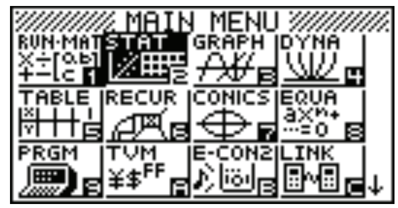
Definitions

- Parallelogram
- Rectangle
- Square
- Perpendicular
- Endpoint
- Slope
- Diagonal
- Midpoint
- Congruent
- Hypotenuse
- Leg

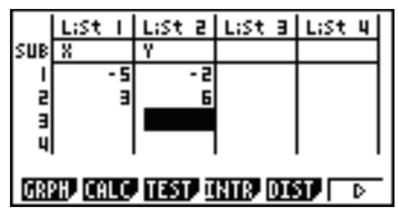
The following will demonstrate how to enter a set of coordinates into two lists using the Statistics mode of the Casio *fx-9750GII*. After the list is set up, you will find the slope of a line containing the points, save the equation in the Graph mode, and find the intersection of two lines.

Line segment AB has endpoints at (–5, –2) and (3, 6) and segment CD has endpoints at (–6, 4) and (3, –7). Find the slope for each line segment and the coordinates of their intersection.

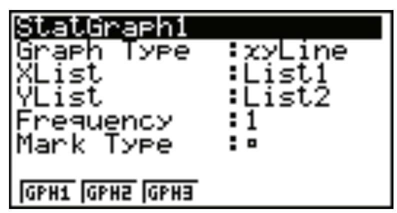
To enter values into a list and find the line of best fit:

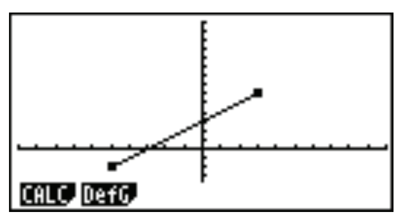


1. From the Main Menu, highlight the Statistics icon and press EXE or press 2.
2. To label the first column, highlight the space below **List 1** and press ALPHA + (X) EXE.
3. To label the second column, highlight the space below **List 2** and press ALPHA – (Y) EXE.



4. Enter the x-values into **List 1** and the y-values into **List 2**. Be sure to press EXE after each value.
5. To view the points, press F1 (GRPH) F6 (Set) ▼ F1 (GPH1) ▼ F2 (XY) ▼ F1 (List) 1 ▼ F1 (List) 2 EXE F1 (GPH1).



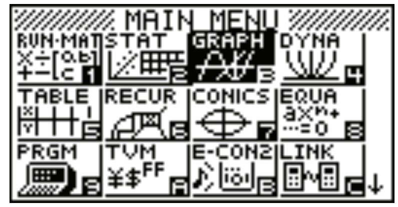


6. Press EXIT and F1 (GPH1) to view the graph.
7. Press F1 (Calc) F2 (X) F1 (ax+b) to find the line of best fit.

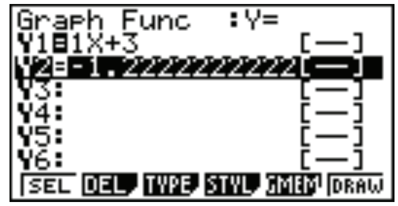
8. Press F5 (Copy) EXE to copy the equation into the graph function.
9. Repeat the same steps to find the equation for the second segment.

To graph the two equations and find the intersection:

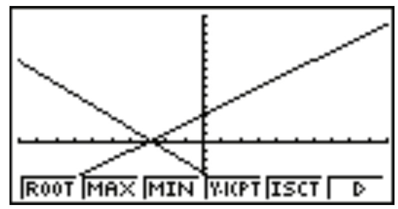
1. From the Main Menu, highlight the Graph icon and press EXE or press 3.



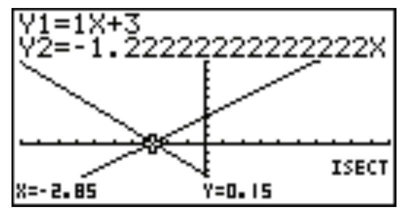
2. To graph the two equations, highlight each equation and press F1 (Sel) to turn the function on; when the equal signs are highlighted, you know the equation is selected. Then press F6 (Draw).



3. While viewing the graph, press F5 **(G-Solv)** F5 (ISCT) to find the intersection of the two equations.



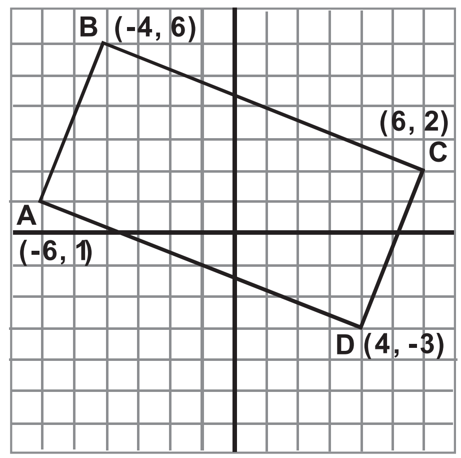
4. The coordinates are displayed at the bottom of the screen.



Two special quadrilaterals that are also parallelograms are the rectangle and the square. These are figures that are seen everywhere. In the activity, we will determine what makes them so special and solve problems that involve those special properties.

Questions

The diagram at the right shows ▱ABCD. By definition, a rectangle is a quadrilateral with four right angles.



1. Find the equation of the following line segments
   a. $\overline{AD}$

   ____

   b. $\overline{AB}$

   ____

2. Are these segments perpendicular?

   ____

3. Are all four angles equal to 90°? Explain.

   ____

   ____

4. Find the lengths of the following segments to the nearest tenth.
   a. $\overline{AC}$

   ____

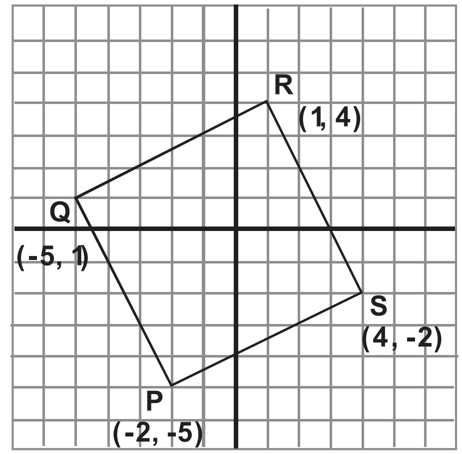
   b. $\overline{BD}$

   ____

5. What does this tell you about the diagonals of a rectangle?

   ____

The diagram below shows ▱PQRS. By definition, a square is a quadrilateral with four congruent sides and four congruent angles.



6. Find the lengths of the following segments to the nearest tenth.
   a. $\overline{PQ}$

   ____

   b. $\overline{QR}$

   ____

7. If a square is a special parallelogram, then are all four sides congruent? Explain.

   ____

   ____

8. Find the equation of the following segments.
   a. $\overline{PQ}$

   ____

   b. $\overline{QR}$

   ____

9. Are these segments perpendicular? If so, does that make this figure a square?

   ____

10. Find the equation of the following segments.
    a. $\overline{QS}$

    ____

    b. $\overline{PR}$

    ____

11. What can you conclude about the diagonals of a square?

    ____

A gazebo is being placed in the middle of a town park that is rectangular in shape. The plan was drawn on a grid showing the corners of the park at (–4, 1), (2, 5), (4, 2), and (–2, –2).

12. Is the park truly rectangular in shape? Explain.

____

____

13. Where will the center of the gazebo be located on the grid?

____

14. What is the perimeter of the park to the nearest tenth?

____

The Better Built construction company is building a deck for a customer. The deck is to measure 28 ft. by 16 ft.

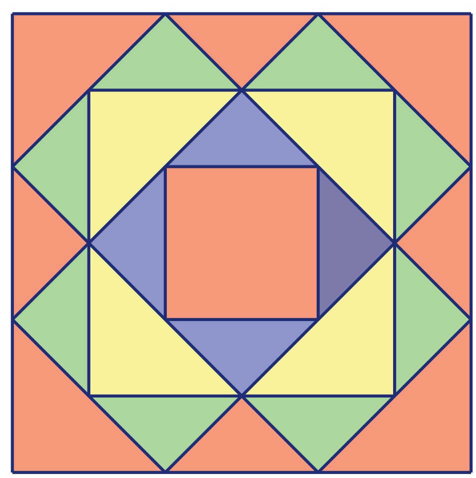
15. How would they ensure that the deck is square (has all right angles)?

____

16. What would this measure be to the nearest hundredth?

____

Mrs. Santiago is creating a quilt using the block pattern, shown below. Each block will be 8 in. on a side. In this pattern, the red square measures 4 in. on a side.



17. If the red square touches the blue square at its midpoints, what is the length of each side of the blue square?

____

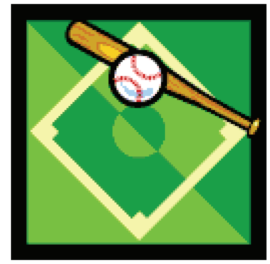
18. The blue square touches the yellow square at its midpoints. What is the length of the side of the yellow square?

____

19. The finished quilt will measure 6 blocks by 8 blocks. What will be the perimeter of the finished quilt before the binding is sewn around the quilt?

____

A local youth center is planning to build a baseball field in a nearby field. The distance between home plate and first base will be 60 ft.



20. What is the distance between home plate and second base to the nearest tenth?

____

21. If a player makes a home run, how far will they run?

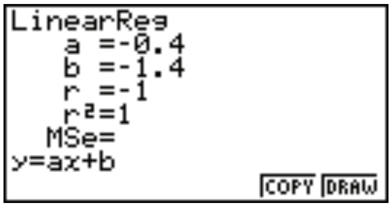
____

22. If the pitcher is 3 ft. further away from home plate than the center of the infield, how far away would the pitcher be from the batter to the nearest tenth?

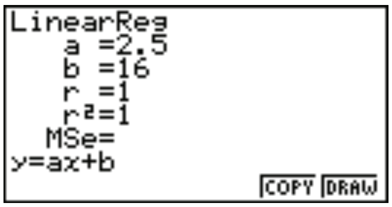
____

Solutions

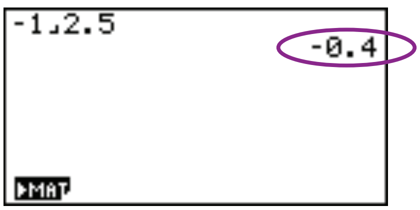
1. a. y = –0.4x – 1.4



b. y = 2.5x + 16



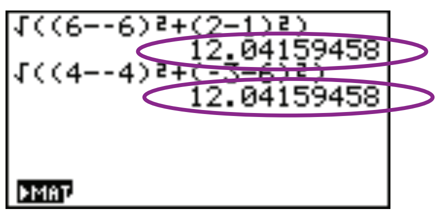
2. Yes; 2.5 is the negative reciprocal of –0.4.



3. Yes. Since the figure is a parallelogram, the opposite sides are parallel making the consecutive angles supplementary and the opposite angles are equal.

4. a. $\overline{AC}$ = 12.04

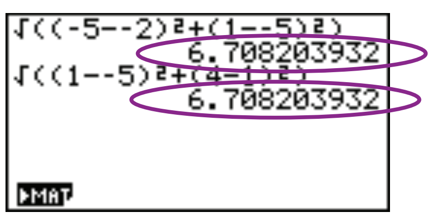
b. $\overline{BD}$ = 12.04



5. The diagonals are equal.

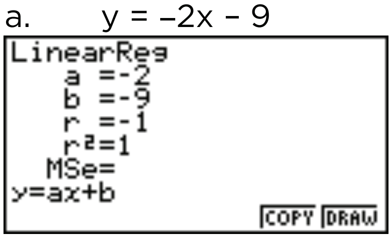
6. a. $\overline{PQ}$ = 6.7

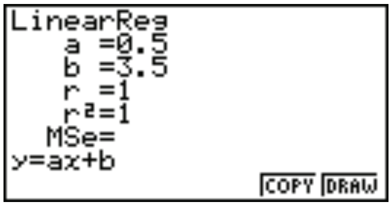
b. $\overline{QR}$ = 6.7



7. Yes; since opposite sides are equal, all the sides would equal 6.7.

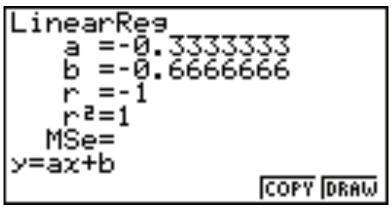
8. a. y = –2x – 9



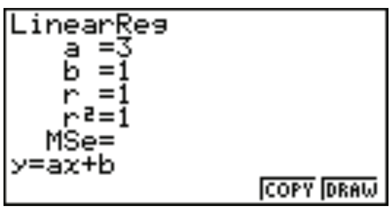


9. Yes; all four sides are congruent and all four angles equal 90°.

10. a. y = –0.33x – 0.66

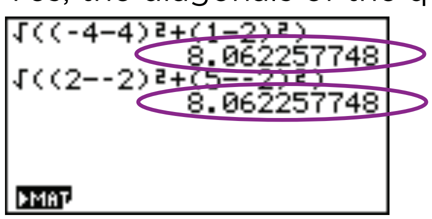


b. y = 3x + 1

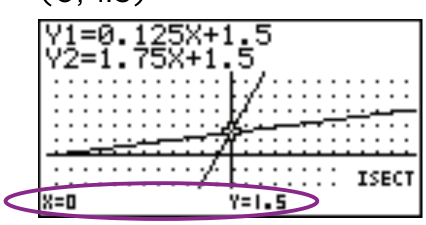


11. The diagonals are perpendicular.

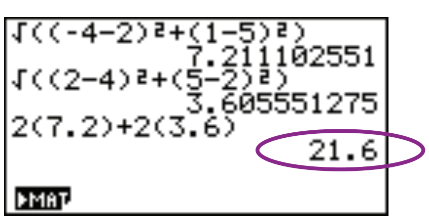
12. Yes; the diagonals of the quadrilateral are equal.



13. (0, 1.5)

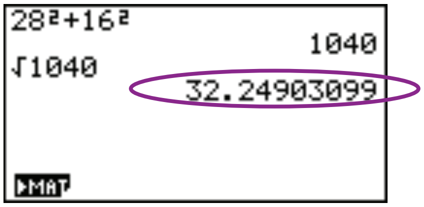


14. 21.6 units

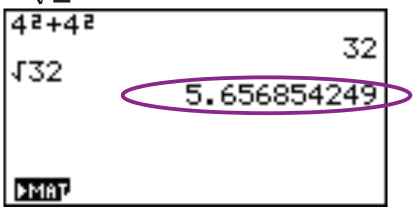


15. Measure the diagonals to see that they are equal.

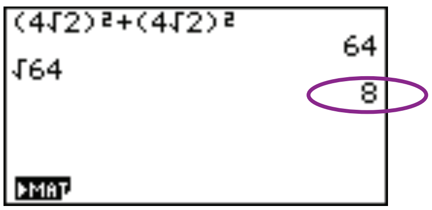
16. 32.2 feet



17. $4\sqrt{2}$

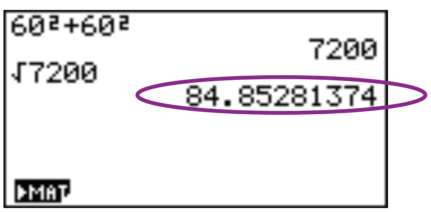


18. 8 inches



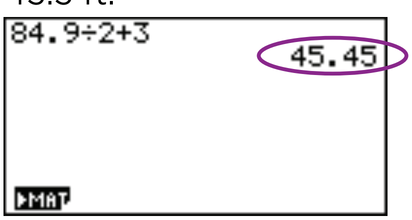
19. Length = 6(8) 48 in. Width = 8(8) = 64 in.
Perimeter = 2(48) + 2(64) = 224 in.

20. 84.9 ft.



21. 4(60) = 240 ft

22. 45.5 ft.



Topic: Limits

NCTM Standards

- Understand functions by interpreting representations of functions.

Objectives

The student will be able to estimate the limit of a function using the graphing and/or table features of the calculator.

Getting Started

In this activity, students will learn how to investigate the limit of a function. This activity provides graphical and numerical methods for evaluating limits. We can achieve very accurate approximations using these methods; however, they do not prove the existence of limits.

Using the Casio *fx-9750GII*, students will use the graph and table of a function to see whether or not a limit exists. If the limit does exist, the graph or table will be helpful in estimating the value of the limit.

Prior to using this activity:

- Students should be able to produce and manipulate functions manually and with a graphing calculator.
- Students should have a basic understanding of the meaning of a limit.

Ways students can provide evidence of learning:

- Students should be able to produce graphs of functions and be able to discuss the limits of those functions as they approach different values from the left and right.

Common mistakes to be on the lookout for:

- Students may have difficulty understanding the concept of infinity with regards to looking at limits.

Definitions:

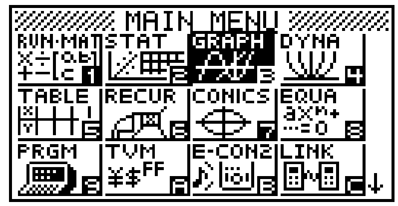
- Limit
- End behavior
- Infinite
- Negative infinite
- Approaches the limit

The following will demonstrate how to graph a function, set the view window, and examine its behavior using trace and the table on the Casio *fx-9750GII*.

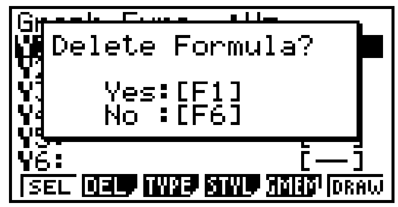
Graph the function $f(x) = \dfrac{-(x-2)}{(x^2-4)}$.

To graph a function:

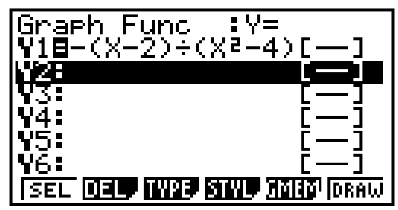
1. From the Main Menu, highlight the GRAPH icon and press [EXE] or press [3].



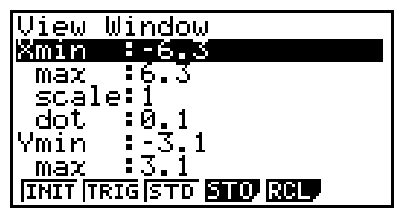
2. To clear out previous graphs, highlight the equation you want to delete and press [F2] (DEL) [F1] (Yes).



3. Enter the equation by pressing [(–)] [(] [X,θ,T] [–] [2] [)] [÷] [(] [X,θ,T] [x²] [–] [4] [)] [EXE].

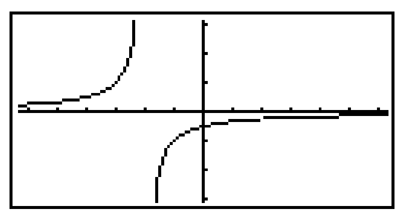


4. Set the view window, to the initial viewing window, by pressing [SHIFT] [F3] **(V-Window)** [F1] (INIT).



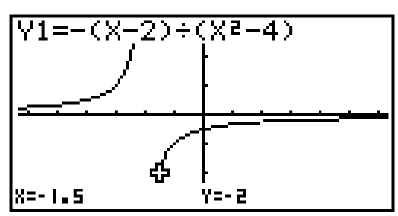
5. Press [EXIT] to return to the initial GRAPH screen.

6. Press [F6] (DRAW) to view the graph of the function.

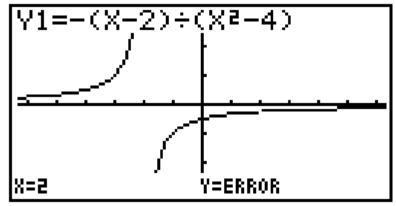


To examine the behavior of a graph using Trace:

1. To trace, press [F1] **(Trace)**. Use the Replay key pad to move the cursor left and right. The viewing window will shift to allow you to see the tracer as it moves to the right or left of the window.

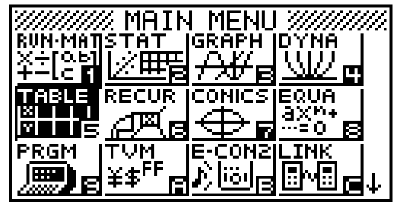


2. To examine the behavior at x = 2, while tracing press [2] [EXE]. The errors signifies that there is a hole at x = 2. If you look at x as you approach from the right or the left, f(x) approaches 0.25. Therefore the $\lim_{x \to 2} f(x) = 0.25$



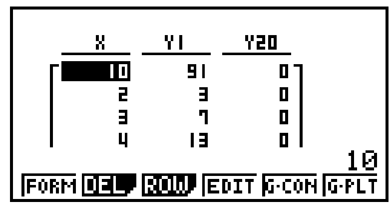
To use the table to find the limit:

1. From the Main Menu, highlight the TABLE icon and press [EXE] or press [5].



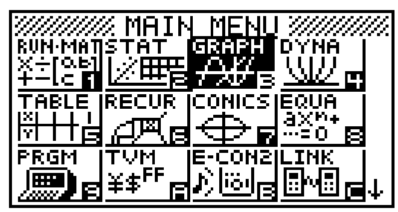
2. Using the same equation as above, press [F6] (TABL) to view the table.

3. To find the value of f(10), press [1] [0] [EXE]. Repeat for additional values of x.

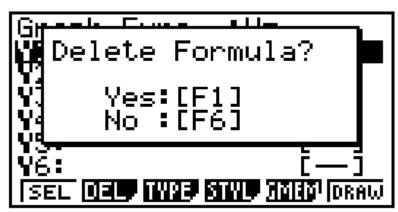


To graph a function with absolute value:

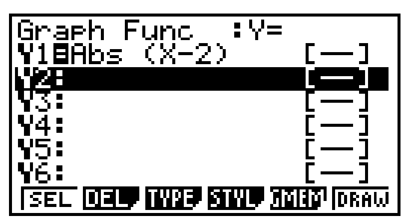
1. From the Main Menu, highlight the GRAPH icon and press [EXE] or press [3].



2. To clear out previous graphs, highlight the equation you want to delete and press [F2] (DEL) [F1] (Yes).



3. Enter the equation by pressing [OPTN] [F5] (NUM) [F1] (Abs) [(] [X,θ,T] [−] [2] [)] [EXE].

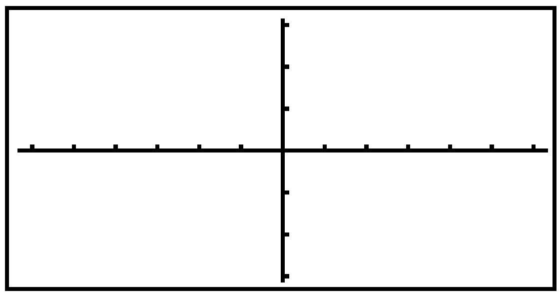


Take It To The Limit — Activity

In this activity you will investigate the limit of a function using graphical and numerical methods to approximate the limit. Limits are typically used to investigate the end behavior of a function and the behavior of a function at interesting points. Using the Casio *fx-9750GII*, you will examine these limits using a graph and a table.

Questions

1. Use a graph in the initial viewing window to investigate the following limits of $f(x)=\dfrac{(x+1)}{(x^2-1)}$. Draw the graph in window provided below:



   a. $\lim\limits_{x\to 1} f(x) =$ ______________________________

   b. $\lim\limits_{x\to -1} f(x) =$ ______________________________

   c. $\lim\limits_{x\to \infty} f(x) =$ ______________________________

   d. $\lim\limits_{x\to -\infty} f(x) =$ ______________________________

2. Use tables to investigate the following limits of $f(x)=\dfrac{(x^3+1)}{(x+1)}$

   a. $\lim\limits_{x\to -1} f(x) =$ ______________________________

x	-1.05	-1.01	-1.001	-1.0001	-1	-0.9999	-0.999	-0.99	-0.95
y									

   b. $\lim\limits_{x\to \infty} f(x) =$ ______________________________

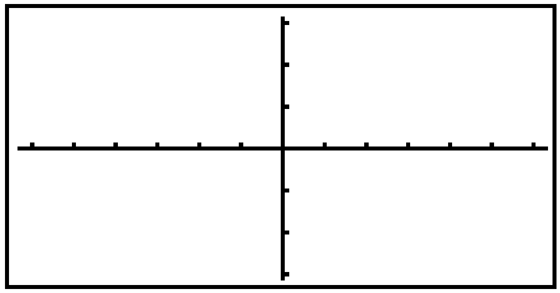
x	10	50	100	250	500	1000	10,000
y							

   c. $\lim\limits_{x\to -\infty} f(x) =$ ______________________________

x	-10	-50	-100	-250	-500	-1000	-10,000
y							

3. Use a graph in the initial viewing window to investigate $\lim_{x \to 2} f(x)$, where

$f(x) = \dfrac{(x-2)}{|2-x|}$.

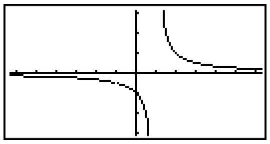


a. $\lim_{x \to 2} f(x) =$ ______________________________

b. $\lim_{x \to \infty} f(x) =$ ______________________________

c. $\lim_{x \to -\infty} f(x) =$ ______________________________

Solutions

1. 

a. $\lim_{x \to 1} f(x) =$ Does not exist.

b. $\lim_{x \to -1} f(x) =$ -0.5

c. $\lim_{x \to \infty} f(x) =$ 0

d. $\lim_{x \to -\infty} f(x) =$ 0

2. Use tables to investigate the following limits of $f(x) = \dfrac{(x^3 + 1)}{(x + 1)}$

a. $\lim_{x \to -1} f(x) =$ 3

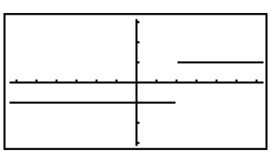
x	-1.05	-1.01	-1.001	-1.0001	-1	-0.9999	-0.999	-0.99	-0.95
y	3.15	3.03	3.003	3.0003		2.9997	2.997	2.97	2.85

b. $\lim_{x \to \infty} f(x) = \infty$

x	10	50	100	250	500	1000	10,000
y	91	2,451	9,901	62,251	249,501	999,001	9.9×10^7

c. $\lim_{x \to -\infty} f(x) = \infty$

x	-10	-50	-100	-250	-500	-1000	-10,000
y	111	2,551	10,101	62,751	250,501	1×10^6	1×10^8

3. 

a. $\lim_{x \to 2} f(x) =$ Does not exist

b. $\lim_{x \to \infty} f(x) =$ 1

c. $\lim_{x \to -\infty} f(x) =$ -1

That Shaky Feeling — Teacher Notes

Topic Area: Logarithms

NCTM Standards:
- Recognize and apply mathematics in contexts outside of mathematics.
- Use a variety of symbolic representations, including recursive and parametric equations, for functions and relations.
- Use symbolic expressions, including iterative and recursive forms, to represent relationships arising from various contexts.

Objective

Given two formulas for finding the magnitude of an earthquake, the student will be able to determine the intensity (energy) of an earthquake and its magnitude.

Getting Started

As a class, discuss what causes an earthquake and how the area around the earthquake is affected. Discuss the need for studying earthquakes. What industries would be affected by these studies? How would this benefit populations facing future earthquakes?

Prior to using this activity:
- Students should have a basic understanding of the properties of logarithms.
- Students should be able to enter and graph equations using a graphing calculator.
- Students should understand how to work with scientific notation.

Ways students can provide evidence of learning:
- The student will be able to determine the intensity of an earthquake and compare this with other energy levels, given the magnitude.
- The student will be able to determine the magnitude and types of damage associated with the earthquake, given the intensity.

Common calculator or content errors student might make:
- Students may enter the formula into the graph function of a graphing calculator incorrectly.
- Students may have difficulty correctly using scientific notation to express large numbers.

Definitions
- Richter Scale
- Intensity
- Seismic Moment
- Moment Magnitude
- Ergs

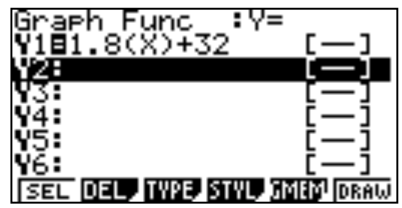
The following will demonstrate how to enter a formula into the Casio *fx-9750GII*, find a value for *y* given its corresponding x-value, and find the value for *x* given its corresponding y-value.

From a graph of the formula F = 1.8C + 32, find the temperature in degrees Celsius for a temperature of 75^0 Fahrenheit and find the temperature in degrees Fahrenheit for a temperature of 18^0 Celsius.

To enter a formula into the Graph Function:

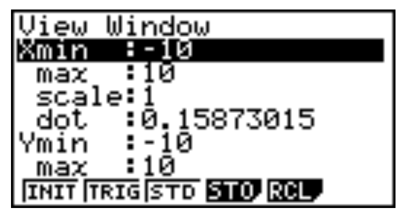
1. From the Main Menu, highlight the GRAPH icon and press [EXE] or [5].

2. Select the graph type by pressing [F3] [F1] for **Y=**. Enter the formula into the calculator by entering: [1] [•] [8] [(] [X,θ,T] [)] [+] [3] [2] [EXE]. The screen should look like the one at the right.

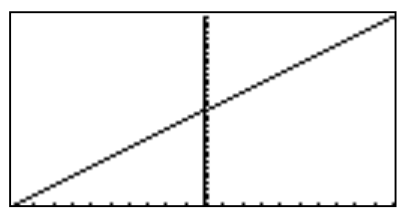


To set up the values for the window:

1. Press [SHIFT] [F3] **(V-Window)** then [F3] (STD) for a standard 10 by 10 grid. Press [EXE] twice to see the graph.

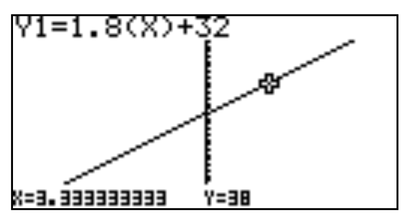


2. If you can not see the graph, press [F2] **(Zoom)** and [F5] (Auto) to see the graph as seen at the right.



To trace a graph and locate values:

1. With the graph showing, press [F1] **(Trace)** and use the [◀] [▶] to move along the graph.



2. If more than one graph is visible, use the [▲] [▼] to move between the graphs. The graph at the right shows the value of *y* when x = 3.33.

Natural changes in the Earth occur all over the world and can bring devastation to large areas. One such disaster is an earthquake. Caused by the shifting of layers of rock below the Earth's surface, these changes may or may not be felt. However, those carrying a tremendous amount of force can cause cracks in the Earth's surface resulting in collapsed structures and loss of lives. Some may even trigger other disasters such as tsunamis.

In this activity, you will explore the relationship between the intensity and magnitude of earthquakes as measured on the Richter scale and the Moment Magnitude scale. The Richter scale was developed to measure the seismic magnitude and energy of an earthquake. This was acceptable for smaller earthquakes. Later, a formula to calculate moment magnitude scale was developed to measure the area of damage created by the earthquake. This scale provided a better picture of larger earthquakes. By understanding the magnitude and intensity, scientists and engineers can work together to build safer structures and possibly lessen the effect of an earthquake in populated areas.

Questions

Using Seismic Magnitude: $M_e = \frac{2}{3}\log E - 2.9$, where M_e is the magnitude on the Richter scale, and E is the intensity of the earthquake.

1. In 1906, San Francisco experienced an earthquake with a magnitude (M_e) of 8.3 on the Richter scale. What was the intensity of this earthquake?

____

2. In 2007, San Francisco experienced another earthquake with a magnitude of 5.6 on the Richter scale. What was the intensity of this earthquake?

____

3. How many times more intense was the earthquake in 1906 than the one in 2007?

____

4. One of the largest earthquakes in the world occurred in Prince William Sound, Alaska in 1964. The magnitude of the earthquake measured 8.4 on the Richter scale. What was its intensity?

____

5. An earthquake in Canada had an intensity that was one-fourth the intensity of the Alaskan earthquake. What was the magnitude of the Canadian earthquake?

____

Using the Moment Magnitude: $M_w = \frac{2}{3}\log M_o - 10.7$, where M_w is the moment magnitude and M_o is the seismic moment.

1. Seismic moment is the measure of an earthquake according to its size rather that its energy. What is the seismic moment of the 1906 San Francisco earthquake if its moment magnitude (M_w) is 7.7?

____

2. The Alaskan earthquake of 1964 had a moment magnitude of 9.2. How much larger was this seismic moment than the one in San Francisco?

____

3. If an earthquake occurring in Peru has a seismic moment (M_o) of 5.4 and another earthquake occurring in Indonesia has a seismic moment of 6.2; what would be the difference in the magnitudes of these earthquakes?

____

4. If the seismic moment of one earthquake is 20 times that of a second, what is the difference in their magnitudes?

____

Extensions

1. One of the most active parts of the world for earthquakes is China. Make a table comparing the year with the magnitude of the earthquake for the last 10 years. Graph the results using the statistics function of a graphing calculator. What can you conclude from the graph of the data?

___

___

2. Research other magnitude scales used in measuring earthquakes. Select one of these scales and discuss what it measures. Give any formulas that are used to determine the magnitude using this scale.

___

___

3. Compare the magnitude of the Alaskan earthquake with that of a nuclear blast. What is equivalent magnitude of an earthquake compared to that of a nuclear explosion?

___

___

Solutions

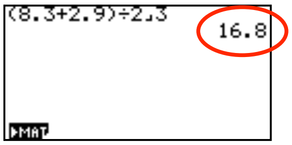
Using Seismic Magnitude:

1. $M_e = 8.3$

   $8.3 = \frac{2}{3}\log E - 2.9$

   $16.8 = \log E$

   $E = 10^{16.8}$ ergs

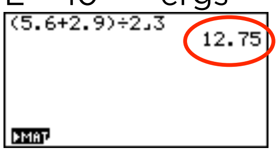


2. $M_e = 5.6$

   $5.6 = \frac{2}{3}\log E - 2.9$

   $12.75 = \log E$

   $E = 10^{12.75}$ ergs



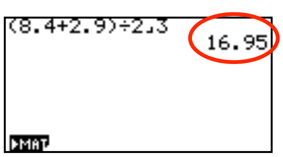
3. $10^{16.8} / 10^{12.75} = 10^{4.05}$

4. $M_e = 8.4$

   $8.4 = \frac{2}{3}\log E - 2\text{-}9$

   $16.95 = \log E$

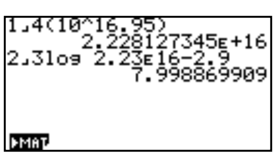
   $E = 10^{16.95}$ ergs.



5. $M_e = \frac{2}{3}\log 2.23\text{E}16 - 2.\ 9$

   $E = \frac{1}{4}(10^{16.95})$

   $E = 8.0$

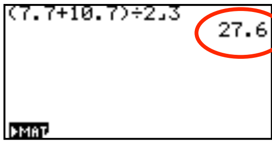


Using the Moment Magnitude:

1. $M_w = 7.7$

$7.7 = \frac{2}{3}\log M_o - 10.7$

$M_o = 10^{27.6}$

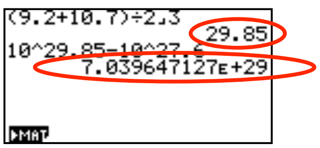


2. $M_w = 9.2$

$9.2 = \frac{2}{3}\log M_o - 10.7$

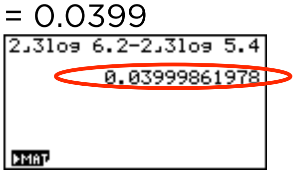
$M_o = 10^{29.85}$

$M_A - M_{SF} = 10^{29.85} - 10^{27.6} = 7.04 \text{ E } {+29}$



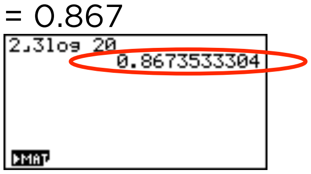
3. $M_I - M_P = \frac{2}{3}\log 6.2 - 10.7 - \frac{2}{3}\log 5.4 - 10.7$

$= 0.0399$



4. $\frac{2}{3}\log 20x - \frac{2}{3}\log x = \frac{2}{3}(\log 20x - \log x) = \frac{2}{3}\left(\frac{\log 20x}{\log x}\right) = \frac{2}{3}\log 20$

$= 0.867$



Extension Solutions

1. Answers will vary.

2. Answers will vary.

3. 90.7 gigatons

The Cost of Car Insurance — Teacher Notes

Topic: Data Analysis and Probability

NCTM Standard(s)

- For univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics (9-12)

Objective:

Given a set of data, the student will be able to enter data into the statistics menu of the Casio *fx-9750GII*, graph the data using a median box-and-whisker graph, and calculate the measures of central tendency.

Getting Started

Have the students work in pairs or small groups and come up with examples of using one-variable data, what kind of information can be obtained from one-variable data and what types of graphs can be used to represent one-variable data.

Prior to using this activity:

- The student should be able to calculate basic statistics.
- The students should be familiar with interquartile values.

Ways students can provide evidence of learning:

- Given a set of data, the student should be able to create a box and whisker plot.
- The student should be able to answer questions about the range of a set of data.

Common mistakes to be on the lookout for:

- Students may pick a measure of central tendency that does not best describe the situation.
- Students may not understand the effect that outliers have on the set of data.

Definitions:

- Mean
- Median
- Mode
- Standard Deviation
- Interquartile Range
- Central tendency

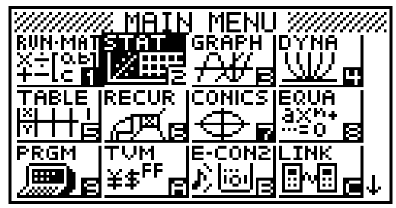
The following will demonstrate how to enter a set of data into the Casio *fx-9750GII,* graph the data using a Box and Whisker Plot and find important information from the graph.

Scores on the First Math Test									
55	60	75	80	90	65	75	60	50	80

Scores on the Second Math Test									
75	90	85	60	95	85	80	70	95	100

To enter the data from the table in the problem:

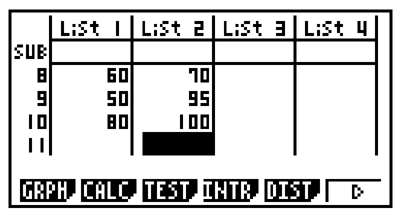
1. From the Main Menu, highlight the STAT icon and press EXE or press 2.



2. To clear previous data lists press F6 (▷) F4 (DEL-A) F1 (Yes).

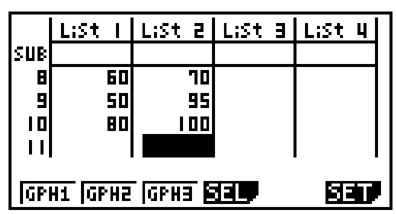
3. Enter the data by typing each number, pressing EXE after each entry.

4. The display should look like the screen shot on the right when completed.

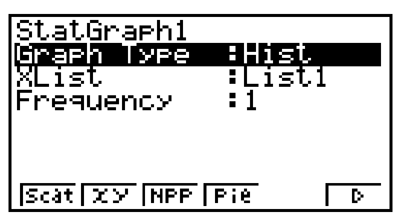


To select the type of graph for this data:

1. Press F1 (GRPH) and F6 (SET) to set the type of graph for **StatGraph1**.

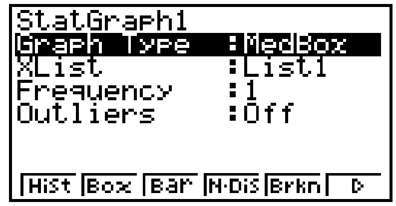


2. Press ⊙ to highlight **Graph Type**.

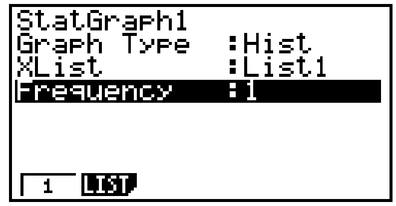


3. There are five choices: Scat, *XY*, NPP, Pie, and (▷). Selecting F6 (▷) will provide more graph choices.

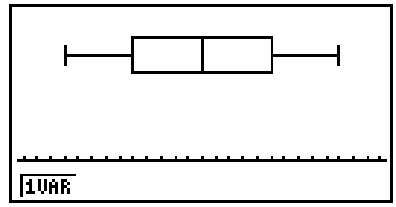
4. Press [F2] (Box) for a box-and-whisker plot.



5. Make sure that the **XList** is **List 1** and a **Frequency** of 1. If not, scroll down and press [F1] (1) to select a frequency of 1.



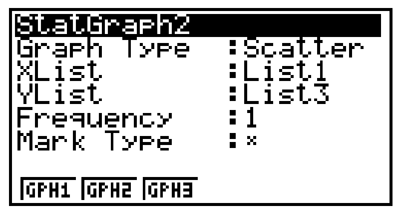
6. Press [EXIT], then [F1] (GPH1) to view the graph.



7. Pressing [F1] (1VAR) will display the statistical data from the list.

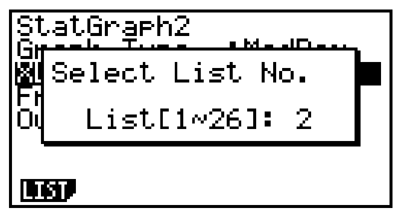
To graph multiple sets of data:

1. Press [EXIT] to go back one screen.

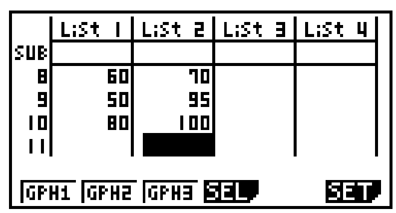


2. Press [F6] (SET) and [F2] (GPH2) to set the type of graph for **StatGraph 2**.

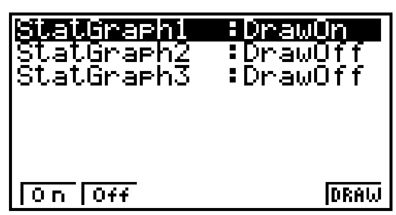
3. Press [F2] (Box) for a box-and-whisker plot, then press ▼ to change the **XList** to **List 2**.

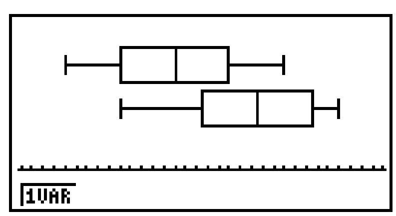


4. Press [F4] (SEL) to select the graphs to be displayed.



5. Arrow down to the graphs that you would like to see drawn and press [F1] (On). Then, press [F6] (DRAW).



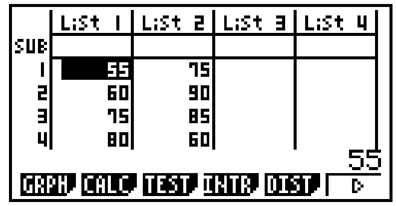


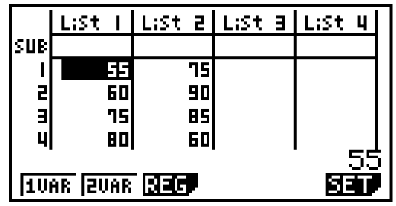
To perform a 2 variable statistic analysis of the data:

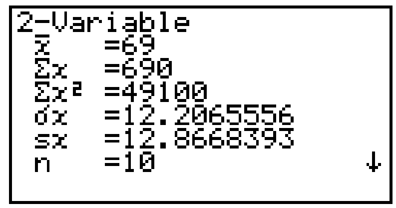
1. EXIT twice until you are at the initial STAT screen.

2. Press F2 (CALC), then F2 (2VAR) for a two-variable analysis.

3. Scroll down to see the data.







The Cost of Car Insurance — Activity

For many years, actuaries have kept track of the driving records of car insurance policy holders. These statistics compare males and females and those under or above 21 years old. This data is used to determine the amount paid for car insurance premiums. In this activity, you will compare the cost of car insurance premiums that resulted from the analysis of this data.

Insurance Co.	Female < 21	Female ≥ 21	Male < 21	Male ≥ 21
Company A	$2,046	$1,520	$3,041	$2,108
Company B	$1,825	$1,239	$2,617	$1,514
Company C	$2,152	$1,637	$2,946	$1,701
Company D	$1,773	$1,129	$2,459	$1,477
Company E	$2,381	$1,748	$3,291	$2,439

Questions

1. What is the range of costs for car insurance for a female aged < 21?

   ____

   What is the mean cost?

   ____

2. What is the range of costs for car insurance for a male aged < 21?

   ____

   What is the mean cost?

   ____

3. What is the difference between the mean costs of a female and male driver < 21?

   ____

4. Can you think of some reasons why the cost is so different for male and female drivers under the age of 21?

   ____

   ____

   ____

5. Use your Casio *fx-9750GII* to graph a box and whisker for each of the age and gender groups. Draw a sketch of each graph. Be sure to label the interquartile values for each age and gender group.

Female < 21

Female ≥ 21

Male < 21

Male ≥ 21

6. What is the range of costs for car insurance for females over 21 years old?

What is the mean cost?

7. What is the range of costs for car insurance for males over 21 years old?

What is the mean cost?

8. What is the difference between a male driver over 21, and a female driver over 21 years old?

9. Looking at the data overall, compare the mean and median costs for all the data sets, and find the best insurance company and the yearly rate each would pay for the following:

Male, 17 years old	______________	Amount paid	______________
Female, 18 years old	______________	Amount paid	______________
Male, 76 years old	______________	Amount paid	______________
Female, 35 years old	______________	Amount paid	______________

10. Do you think that this is fair? Why or why not?

____

____

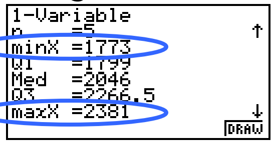
____

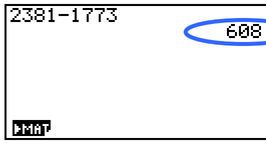
Extensions

Speak with some of your local insurance companies and get rate quotes for someone your age, but for different types of vehicles, like a sports car, a truck, a compact car or an old Caprice.

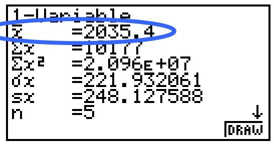
Solutions

1. Range = $2381 - $1773 = $608

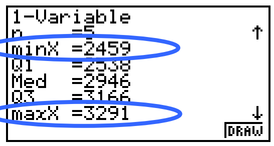


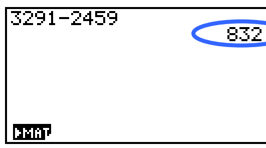


Mean = $2035.40

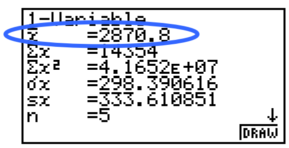


2. Range = $3291 - $2459 = $8732

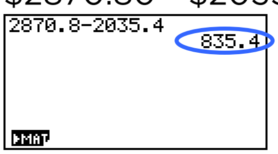




Mean = $2870.80

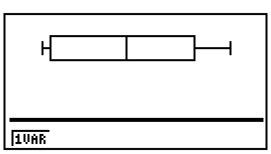


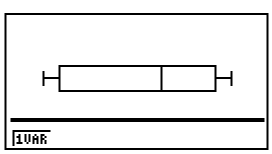
3. $2870.80 - $2035.40 = $835.40

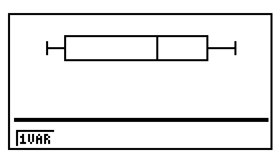


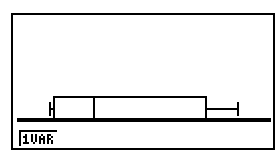
4. Answers will vary

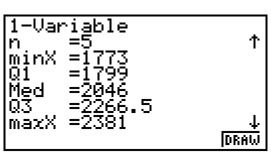
5. Female < 21 | Female ≥ 21 | Male < 21 | Male ≥ 21

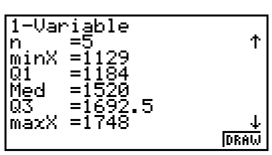


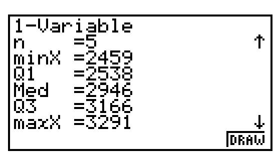


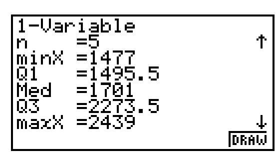




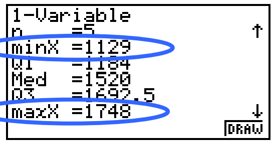


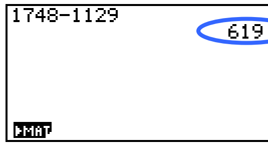




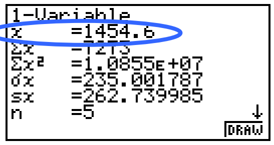


6. Range = $1748 - $1129 = $619

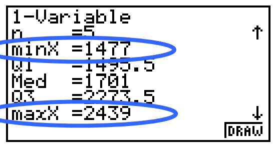


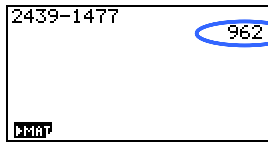


Mean = $1456.60

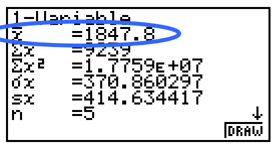


7. Range = $2439 - $1477 = $962

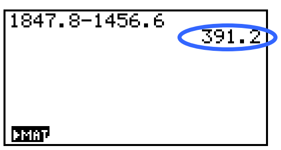




Mean = $1847.80



8. $1847.80 – $1456.60 = $391.20



9.

Male, 17 years old:	Company D	Amount paid: $2,459
Female, 18 years old:	Company D	Amount paid: $1,773
Male, 76 years old:	Company D	Amount paid: $1,477
Female, 35 years old:	Company D	Amount paid: $1,129

10. Answers will vary

The Many Sides of Pythagoras Teacher Notes

Topic Area: Pythagorean Theorem

NCTM Standards:
- Use geometric models to represent and explain numerical and algebraic relationships.
- Establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others.
- Draw geometric objects with specified properties, such as side lengths or angle measures.

Objective

The student will be able to investigate a variety of proofs of the Pythagorean Theorem, use algebraic and numeric representations to confirm the Pythagorean Theorem, and apply a variety of geometric formulas in investigating the Pythagorean Theorem.

Getting Started

As a class, discuss the characteristics of a right triangle including its parts by reviewing the definitions; include in the review a discussion of right triangle proportionality. Demonstrate the original proof of the Pythagorean Theorem.

Prior to using this activity:
- Students should be able to work with formulas.
- Students should be able to square algebraic expressions.
- Students should know the parts of a right triangle and their names.
- Students should be able to evaluate algebraic expressions.

Ways students can provide evidence of learning:
- Given a diagram for a proof of the Pythagorean Theorem, the student will be able to demonstrate the solution both geometrically and algebraically.
- Given the Pythagorean Theorem, the student will be able to apply the results to real-life problems.

Common mistakes to be on the lookout for:
- Students may not be able to multiple algebraic expressions.
- Students may use incorrect formulas.
- Students may incorrectly substitute the wrong values into the formula.

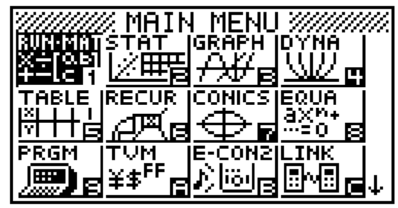
Definitions
- Pythagorean Theorem
- Leg
- Hypotenuse
- Right Angle
- Trapezoid
- Proportion

The following will demonstrate how to evaluate algebraic expressions and formulas using the Run-Matrix mode and the store function of the Casio *fx-9750GII*.

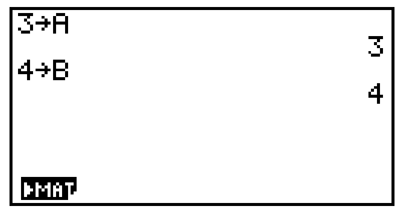
Given the values of a = 3 and b = 4, evaluate each of the given expressions.

a. $\frac{1}{2}ab$ b. $a^2 + 2ab + b^2$ c. $\frac{3}{4(b^2 - a^2)}$

To store values in variables:



1. From the Main Menu, highlight the RUN•MAT icon and press [EXE] or press [1].

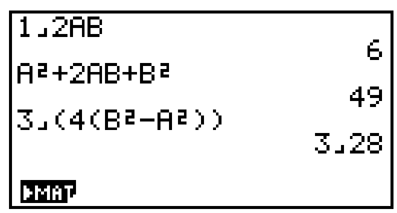


2. To store 3 as the value of A, press [3] [→] [ALPHA] [X,θ,T] [EXE].

3. To store 4 as the value of B, press [4] [→] [ALPHA] [log] [EXE].

To evaluate an expression:

1. Press [AC/ON] to clear the screen.



2. Enter the first expression by pressing [1] [a b/c] [2] [ALPHA] [X,θ,T] [ALPHA] [log] [EXE].

2. Press [ALPHA] [X,θ,T] [x^2] [+] [2] [ALPHA] [X,θ,T] [ALPHA] [log] [+] [ALPHA] [log] [x^2] [EXE] to evaluate the second expression.

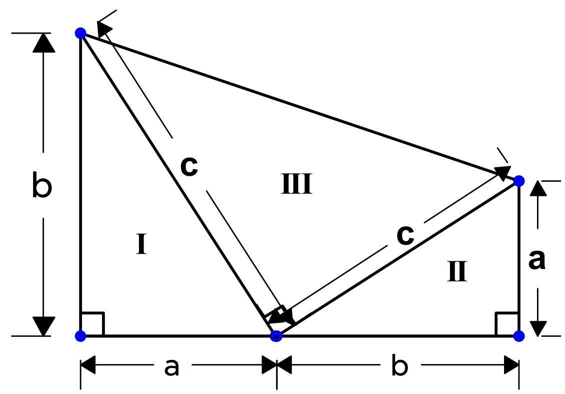
3. Press [3] [a b/c] [(] [4] [(] [ALPHA] [log] [x^2] [−] [ALPHA] [X,θ,T] [x^2] [)] [)] [EXE] to evaluate the third expression.

The Many Sides of Pythagoras — Activity

There have been many different methods used to prove the Pythagorean Theorem. The most famose proof involves using the sides of a right triangle to form three squares and then comparing the areas. In this activity, you will use the areas of triangles and a trapezoid, as well as right triangle proportionality, to show that $c^2 = a^2 + b^2$.

Questions

The first proof using triangles and a trapezoid was discovered by President James Garfield. He took two congruent right triangles and created a trapezoid with an isosceles right triangle in the middle. He then used the formula for the area of a triangle ($A = \frac{1}{2}bh$) and the area of a trapezoid ($A = \frac{1}{2}h(b_1 + b_2)$) to prove the theorem.



1. Write an expression to find the area of Triangles I and II.

   ____

2. Write an expression to find the area of Triangle III.

   ____

3. Using the values of a = 8, b = 15, and c = 17, evaluate the expressions for each triangle.
   a. Triangle I = ____________________
   b. Triangle II = ____________________
   c. Triangle III = ____________________

4. Find the sum of the areas of the three triangles.

   ____

5. Write an expression for finding the area of the trapezoid.

   ____

6. Evaluate the expression for the area of the trapezoid.

   ____

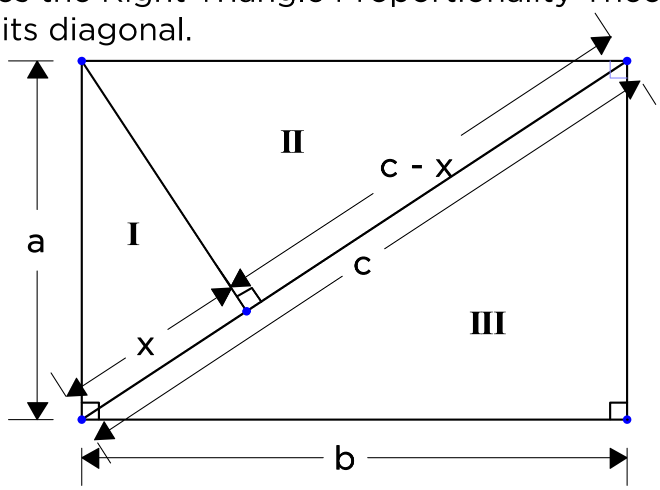
7. What do you notice about the sum of the areas of the triangles and the area of the trapezoid?

____

8. Write an equation to show this relationship and simplify the results. Did you get the formula used for the Pythagorean Theorem?

____

The second proof uses the Right Triangle Proportionality Theorem and a rectangle divided in half along its diagonal.



9. Write an expression to find the value of x using a.

____

10. Write an expression to find the value of x using b.

____

11. Using the values of a = 7, b = 24, and c = 25, evaluate both of the expressions.

What do you notice?

____

12. Write an equation to show this result using the expressions and simplify the results. Is this a form of the Pythagorean Theorem?

____

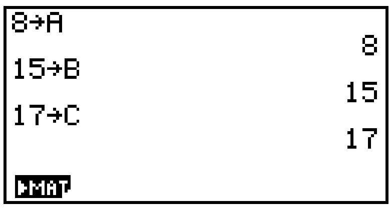
____

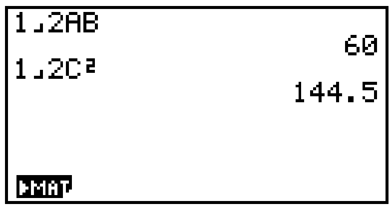
Solutions

1. $A = \frac{1}{2}ab$

2. $A = \frac{1}{2}c^2$

3. a. Area of Triangle I = 60 units^2
   b. Area of Triangle II = 60 units^2
   c. Area of Triangle III = 144.5 units^2
   (Use F↔D to convert the fraction to a decimal.)

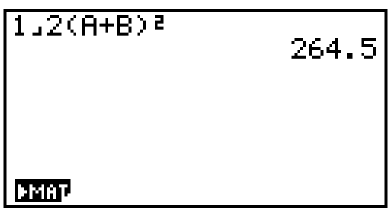




4. 60 + 60 + 144.5 = 264.5 units^2

5. $A = \frac{1}{2}(a + b)(a + b) = \frac{1}{2}(a + b)^2$

6. Area of Trapezoid = 264.5 un.^{2}



7. The areas are equal.

8.
$$\frac{1}{2}(a + b)^2 = \frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}c^2$$
$$\frac{1}{2}(a^2 + 2ab + b^2) = \frac{1}{2}(ab + ab + c^2)$$
$$a^2 + 2ab + b^2 = 2ab + c^2$$
$$a^2 + b^2 = c^2$$

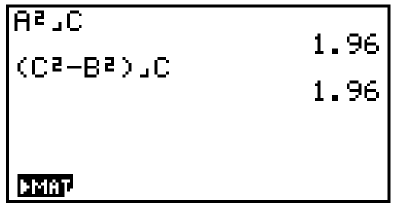
9. $x = \frac{a^2}{c}$

10. $x = \frac{c^2 - b^2}{a}$

11. $x = \dfrac{a^2}{c} = \dfrac{7^2}{25} = \dfrac{49}{25} = 1.96$

$x = \dfrac{c^2 - b^2}{a} = \dfrac{25^2 - 24^2}{25} = \dfrac{49}{25} = 1.96$

Both expressions are equal.



12. $\dfrac{a^2}{c} = \dfrac{c^2 - b^2}{a}$

$a^2 = c^2 - b^2$

$a^2 + b^2 = c^2$

Topic Area: Rotations of Polygons

NCTM Standards:

- Understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, vectors, function notation, and matrices.
- Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations.

Objective

The student will be able to use the properties of rotations to represent the vertices of a polygon, write a rotation using the form r_θ(x, y) where θ is the angle of rotation, find the new vertices of a polygon given the coordinates of the preimage and find the original vertices of a polygon given the coordinates of the image.

Getting Started

As a class, discuss the definition of a rotation and the difference between clockwise and counterclockwise. Review that the four quadrants are named in a counterclockwise direction and have them relate this to a 90°, 180°, and 270° rotation. Discuss with them various uses of rotations in real-life situations. Refer to career areas that use rotations in their field such as surveying, aeronautics, and communications.

Prior to using this activity:

- Students should have an understanding of the coordinate plane.
- Students should have a basic understanding of types of angles.
- Students should know the rules for rotations of 90°, 180°, and 270°.
- Students should know how to draw a line graph on the calculator.

Ways students can provide evidence of learning:

- The student will be able to write the coordinates for a set of points transformed according to a specified rotation.
- The student will be able to identify the angle of rotation for a figure.

Common mistakes to be on the lookout for:

- Students may use the wrong values for a given rotation.
- Students may confuse the x and y values used in the rotation.

Definitions

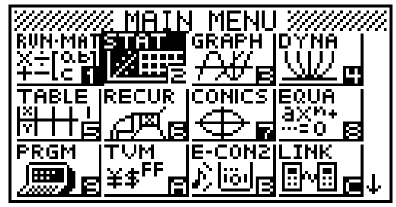
- Rotation
- Right Angle
- Straight Angle
- Counterclockwise

The following will demonstrate how to enter a set of coordinates into the Statistics mode on the Casio *fx-9750GII* and rotate the coordinates according to a given angle.

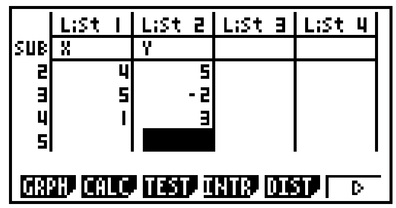
Using the vertices of (1, 3), (4, 5), and (5, –2), find the coordinates of the image for the vertices under a 90° and 270° rotation.

To enter values into a list and create a polygon:

1. From the Main Menu, highlight the STAT icon and press [EXE].

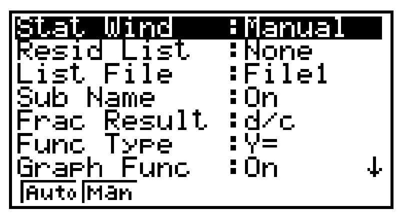


2. To label the first column, highlight the space below **List 1** and press [ALPHA] [+] (X) [EXE]. To label the other columns, highlight the space below them and press [ALPHA] and enter the label. Press [EXE].

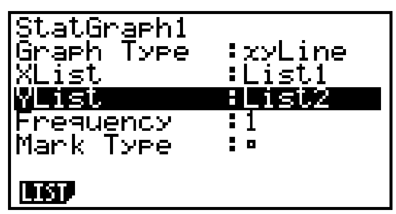


3. Enter the x-values into **List 1** and the y-values into **List 2**. [Note: the first pair of coordinates appears twice so that the line graph will form a polygon.]

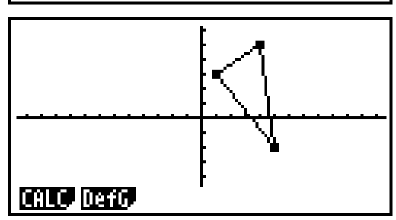
4. Set up the calculator so that the axes can be set up manually by pressing [SHIFT] [MENU] **(SET UP)**, highlight **Stat Wind** and press [F2] (Man).



5. To draw the line graph and create the polygon, press [F1] (GRPH) [F6] (Set) [▼] [F1] (GPH1) [▼] [F2] (XY) [▼] [F1] (List) [1] [▼] [F1] (List) [2] [EXE] [F1] (GPH1).



6. With **StatGraph1** on, press [F6] (Draw). Press [SHIFT] [F2] **(Zoom)** [F4] (Out) [EXE] to see the entire triangle. This is shown at the right.



To find the coordinates of the figure rotated 90°, 180°, and 270°:

1. Press [EXIT] twice to get back to the list screen.

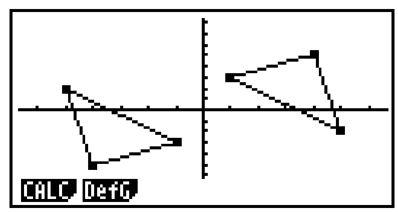
2. Move the cursor to highlight **List 3** and press [OPTN] [F1] (LIST) [F1] (List) [1] [×] [(–)] [1] [EXE].

3. Move the cursor and highlight **List 4** and press [F1] (List) [2] [×] [(–)] [1] [EXE]. Use the results of the columns to write the coordinates for each rotation.

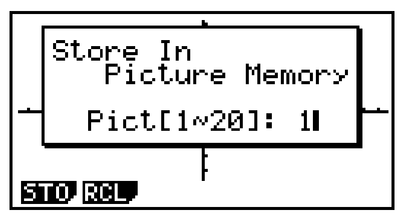
90°: (–3, 1), (–5, 4), (2, 5), (–3, 1)
180°: (–1, –3), (–4, –5), (–5, 2), (–1, –3)
270°: (3, –1), (5, –4), (–2, –5), (3, –1)

To graph the rotation of the polygon, save it as a PIC, and recall the graph:

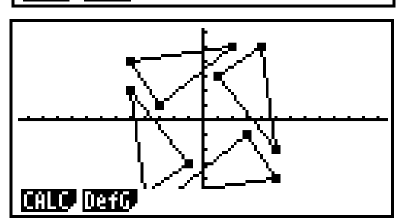
1. Press [EXIT] twice to get back to the list screen. To setup the preimage of the polygon as shown above, use **List 1** for **X-List** and **List 2** for **Y-List**. To setup the image of the polygon, use **List 4** for **X-List** and **List 1** for **Y-List**. Select and draw the graph.



2. Press [OPTN] [F1] (PICT) [F1] (STO) [1] [EXE] to save the graph as **PIC 1**.



3. Press [EXIT] and graph the 180° and 270° rotations. Recall **PIC 1** by pressing [OPTN] [F1] (PICT) [F2] (RCL) [1] [EXE].



The North Star Lodge has contracted a graphic artist to design a new logo for their business. They want to use a star motif that looks similar to a compass rose. In this activity we will create a logo by rotating polygons about the origin point, graphing the results, and adding color.

Questions

1. The first polygon has vertices at (0, 0), (9, 0), (2, 2), and (0, 9).
   a. Find the coordinates of the polygon after a rotation of 90°.

   ____

   b. Graph the preimage and the image on the calculator and store as PIC 1.

   c. Find the coordinates of the polygon after a rotation of 180° and 270°.
   180°:

   ____

   270°:

   ____

   d. Graph the two rotations on the calculator, recall PIC 1, and store as PIC 1.

2. The second polygon has vertices at (2, 2), (4, 1.5), (7, 3), (3, 3), (3, 7), and (1.5, 4).
   a. Find the coordinates of the polygon after a rotation of 90°.

   ____

   b. Graph the preimage and the image on the calculator and store as PIC 2.

   c. Find the coordinates of the polygon after a rotation of 180° and 270°.
   180°:

   ____

   270°:

   ____

   d. Graph the two rotations on the calculator, recall PIC 2, and store as PIC 2.

3. The third polygon has vertices at (3, 3), (4, 3), (6, 6), and (3, 4).

   a. Find the coordinates of the polygon after a rotation of 90°.

   ____

   b. Graph the preimage and the image on the calculator and store as PIC 3.

   c. Find the coordinates of the polygon after a rotation of 180° and 270°.
   180°:

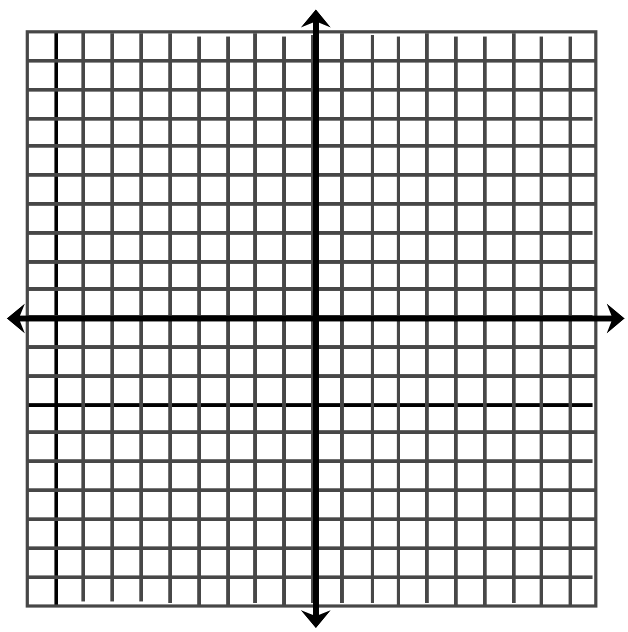
   ____

   270°:

   ____

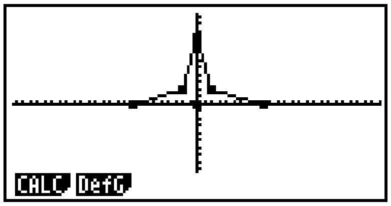
   d. Graph the two rotations on the calculator, recall PIC 3, and store as PIC 3.

4. Recall PIC 1 and PIC 2. Sketch the final logo on the given coordinate plane.

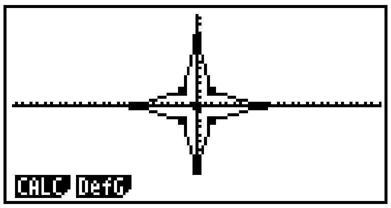


Solutions

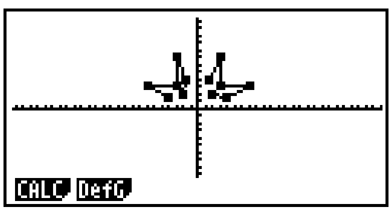
1. a. (0, 0), (0, 9), (–2, 2), (–9, 0)

   b. 

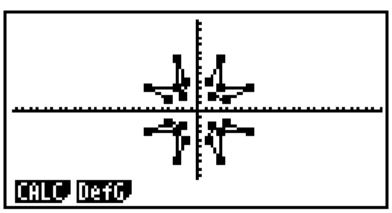
   c. 180° : (0, 0), (–9, 0), (–2, –2), (0, –9)
   270° : (0, 0), (0, –9), (2, –2), (9, 0)

   d. 

2. a. (–2, 2), (–1.5, 4), (–3, 7), (–3, 3), (–7, 3), (–4, 1.5)

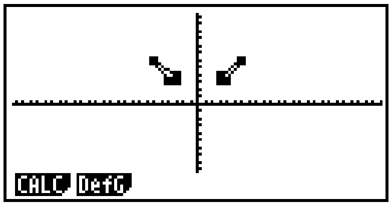
   b. 

   c. 180° : (–2, –2), (–4, –1.5), (–7, –3), (–3, –3), (–3, –7), (–1.5, –4)
   270° : (2, –2), (1.5, –4), (3, –7), (3, –3), (7, –3), (4, –1.5)

   d. 

3. a. (–3, 3) (–3, 4), (–6, 6), (–4, 3)

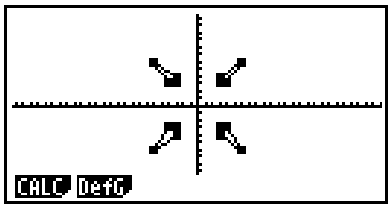
b.



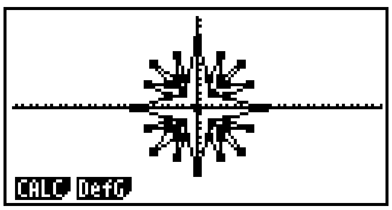
c. 180° : (–3, –3), (–4, –3), (–6, –6), (–3, –4)

270° : (3, –3), (3, –4), (6, –6), (4, –3)

d.



4.



Topic: Linear Bivariate Data

NCTM Standards:

- Understand the meaning of measurement data and categorical data, of univariate and bivariate data, and of the term variable.
- Understand histograms, parallel box plots, and scatterplots and use them to display data.
- Compute basic statistics and understand the distinction between a statistic and a parameter.
- Find, use and interpret measures of center and spread, including mean and interquartile range
- For bivariate measurement data, be able to display a scatterplot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools.

Objective:

The student will be able to plot bivariate data in a scatterplot, make conjectures based on the plot, and create a regression equation using the graphing calculator.

Getting Started

You can introduce this activity by asking your students some questions relating to their guess as to life expectancy. Ask them if they know the current life expectancy and what the trend has been in the past fifty years. Ask them to conjecture as to why they think the trend is there.

Prior to using this activity:

- Students should be able to perform linear regression.
- Students should understand the meaning of the slope and y-intercept.

Ways students can provide evidence of learning:

- Given a set of data, the student should be able to perform linear regression.
- The student should be able interpret the slope and y intercept.
- The student should understand the domain and range of the scatter plot.

Common mistakes to be on the lookout for:

- This is a very long list of data, have the students double check their entries. Student can also link calculators to make inputting the data faster.

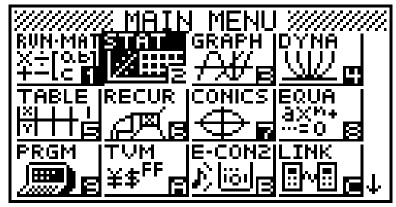
Definitions

- Univariate
- Regression
- Bivariate
- Trend
- Interpolation
- Y-Intercept
- Extrapolation
- Slope

The following will demonstrate how to enter a set of data into the Casio *fx-9750GII,* graph the data using a scatter plot, calculate the linear regression and use the table to answer questions.

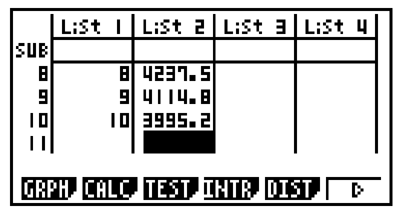
1	2	3	4	5	6	7	8	9	10
5200	5050	4904.5	4763.3	4626.2	4493	4363.5	4237.5	4114.8	3995.2

To enter the data from the table in the problem:



1. From the Main Menu, highlight the STAT icon and press EXE or press 2.

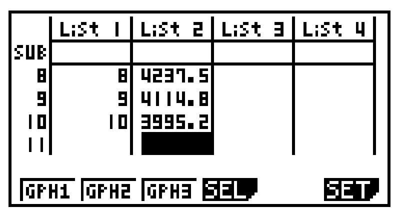
2. To clear previous data lists press F6 (▷) F4 (DEL-A) F1 (Yes).



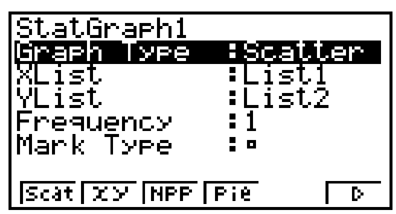
3. Enter the data by typing each number, pressing EXE after each entry.

4. The display should look like the screen shot on the right when completed.

To select a scatter plot to view the data:



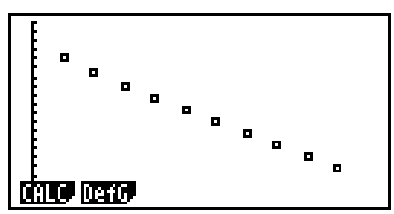
1. Press F1 (GRPH).



2. Select F6 (Set) to set the type of graph for **StatGraph1.**

3. Press ▼ to highlight **Graph Type.**

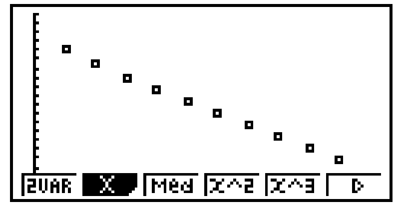
4. Choose F1 (Scat) to select scatter plot, EXE.

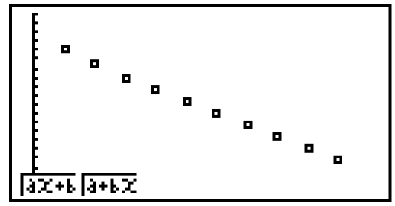


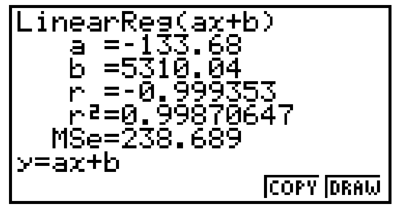
5. Press F1 (GPH1) to display the graph.

To Calculate the Linear regression from the graph:

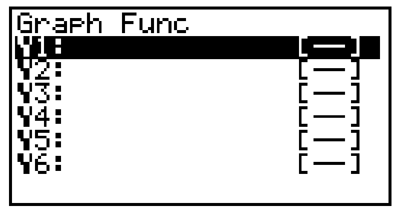
1. To find the regression line, press F1 (CALC), then choose what type of regression you want. For this problem, we want linear, so choose F2 (X) and to put it in ax + b form, press F1.



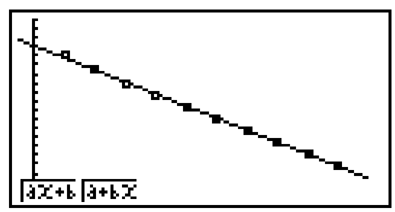




2. If you want to copy the equation to the graph or table, select F5 (COPY), then EXE.



3. To see the graph and the scatter plot, press F6 (Draw).



The Years of Your Life Activity

Introduction

Over the past fifty years, we have seen a trend in the life expectancy of humans. In this activity, you will determine what that trend is and make a conjecture as to how long this trend will continue.

Year	Life Expectancy
1900	47.3
1950	68.2
1960	69.7
1970	70.8
1975	72.6
1980	73.7
1981	74.1
1982	74.5
1983	74.6
1984	74.7

Year	Life Expectancy
1985	74.7
1986	74.7
1987	74.9
1988	74.9
1989	75.1
1990	75.4
1991	75.5
1992	75.8
1993	75.5
1994	75.7

Year	Life Expectancy
1995	75.8
1996	76.1
1997	76.5
1998	76.7
1999	76.7
2000	77
2001	77.2
2002	77.3
2003	77.9
2004	78.3

Questions

1. What is the relationship between the year and the life expectancy in years of a person?

____

____

2. In the year 2050, what would the life expectancy be if this trend continues?

____

3. In the year 1800, what was the life expectancy if this trend was seen at that time as well?

____

4. When would the life expectancy have been zero? Was this likely?

____

____

5. Will this trend continue forever into the future? Why or why not?

____

____

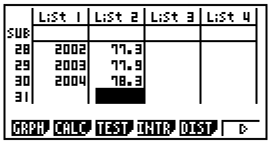
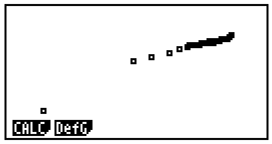
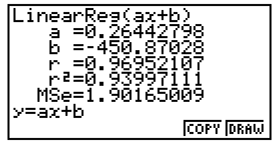
____

Extension

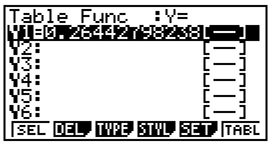
Go to the website listed below and see how the relationship between gender and life expectancy is different for males and females.
http://www.cdc.gov/nchs/data/hus/hus08.pdf#026

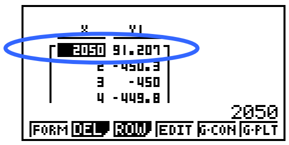
Solutions

1. The relationship is linear; the model can be represented by the equation, y = 0.2644x – 450.8703.

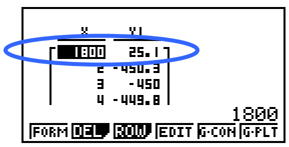




2. In the year 2050, the life expectancy should be 91.2 years.

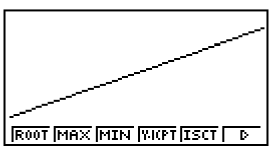
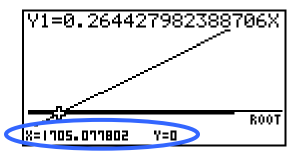




3. In the year 1800, the life expectancy would have been 25.1 years.



4. If the trend is continuous, there would have been a life expectancy of 0 in the year of 1705, we know this is not really a possibility.

5. Answers will vary.

Thought It Was Fixed   Teacher Notes

Topic Area: Data Analysis and Probability

NCTM Standard:

- For bivariate measurement data, be able to display a scatterplot, describe its shape, determine regression coefficients, regression equations, and correlation coefficients using technological tools.

Objective

Given a set of data, the student will be able to use the Financial Menu (TVM), the Statistical Menu, and the Table Menu to solve a problem involving calculations of future payments for a mortgage payment that includes real estate tax.

Getting Started

Discuss with students what a mortgage loan involves, including the definition of interest, principal, and escrow accounts. Include a discussion on budgets, and the effects an increase in a payment can make on a budget.

Prior to using this activity:

- Students should be able to calculate loan payments using the TVM Menu.
- Students should be able to enter data into lists using the STAT Menu, perform operations on the list, set up the calculator to draw a scatterplot, find the equation for the line of best fit, and copy the equation to the TABLE Menu.
- Students should be able to find values using the TABLE Function.

Ways students can provide evidence of learning:

- The students will be able to discuss graphs relating to the problem and explain its appearance.
- The students will be able to discuss details regarding data in statistical lists and the TABLE Function, including the meaning of the values and their importance.

Common calculator or content errors students might make:

- Students might incorrectly enter data in the table or list menus.
- When using percentages, students might have issues with decimal placement.

Definitions

- Interest
- Principal
- Escrow Accounts

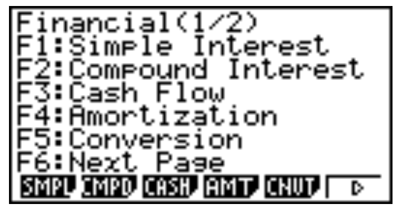
Thought it Was Fixed "How To"

The following will demonstrate how to find the payment amount for a loan given the principal, rate of interest, an number of years using the TVM Menu of the Casio *fx-9750GII*. This will also demonstrate how to find the amount of tax for a column of values using the STAT menu, find the equation for the line of best fit using the STAT Menu, use the line of best fit (linear regression) and use the TABLE Menu to make predictions of future values.

Find the payment for a $100,000 mortgage for 25 years at a rate of 8.5%.

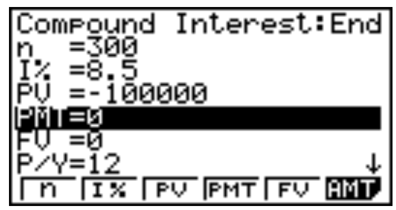
To enter the above set of data using the TVM Menu:

1. Select the TVM icon and press [EXE] or [X,θ,T].



2. Press [F2] for Compound Interest.

3. For *n* (number of payments), enter the number of years x 12 by entering: [2] [5] [×] [1] [2] [EXE].



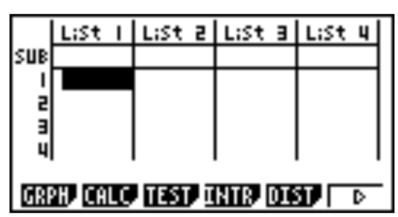
4. For I% (interest rate), enter the rate as a percent by entering: [8] [•] [5] [EXE].

5. For PV (Principal Value), enter the principal amount of the loan as a negative number by entering: [(–)] [1] [0] [0] [0] [0] [0] [EXE].

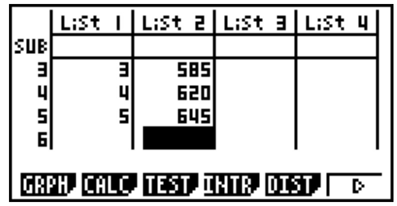
6. To calculate the payment amount, press [F4].

Steps for using the STAT Menu:

Property	1	2	3	4	5
Value	$500	$525	$585	$620	$645



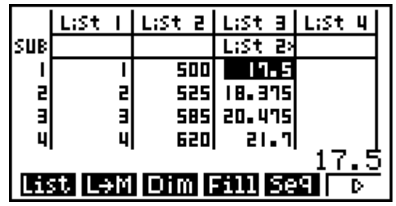
1. Press [MENU] to return to the Main **Menu**. Use the arrow keys to highlight the STAT icon and [EXE].



2. To enter the property numbers into **List 1,** move the cursor to the right of **SUB 1** and begin entering each number, pressing [EXE] after each entry.

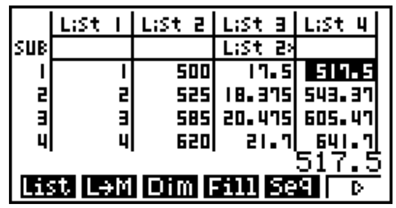
3. Press the ▶ to move to the beginning of **List 2.** Enter **List 2** values as in the previous step.

To find 3.5% tax for the property values in List 2:



1. Press ▶ once and ▲ twice to highlight **List 3**.

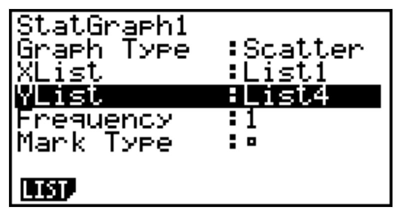
2. Press [OPTN] [F1] [F1] [2] [×] [•] [0] [3] [5] [EXE].



3. To add the tax to the property value in **List 2** to the tax in **List 3**, press ▶ once and then ▲ twice to highlight **List 4**.

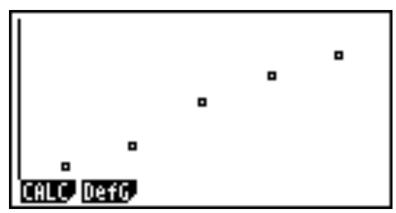
4. Press [OPTN] [F1] [F1] [2] [+] [OPTN] [F1] [F1] [3] [EXE].

To graph a scatterplot for the data:



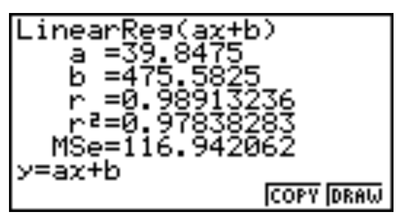
1. Press [EXIT] until the List home screen shows. Press [F1] (GRPH) then [F6] (SET).

2. To change your **YList** to **List 4**, press ▼ three times then [F1] [4] [EXE].



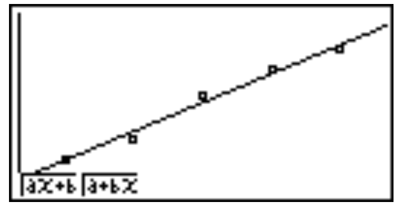
3. Press [EXIT] [F1] (GPH1) to view the scatterplot.

To find the equation for the line of best fit:



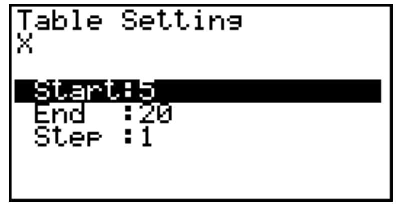
1. Press [F1] (CALC), [F2] (X), [F1] (ax+b) to view the linear regression. To copy the equation for use in the Table or Graph menu, press: [F5] (COPY) [EXE].

2. Press [F6] (DRAW) to draw.



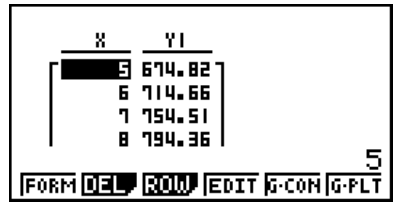
3. Press [MENU], highlight the TABLE icon and press [EXE].

4. Press [F5] (SET).



5. Enter [5] [EXE] for **Start:** and enter [2] [0] [EXE] for **End:**

6. To view the table, press [EXIT], [F1] (SEL) to select the equation and then [F6] (TABLE). Use ◀ ▶ ▲ ▼ to move through the table.



Thought it Was Fixed **Activity**

When someone takes out a mortgage on a piece of property, part of the payment may include an escrow account that will be used to pay the real estate taxes on the property. The fixed rate advertised with many mortgages applies only to the percent of interest. The monthly payment can increase with the increase in taxes. Since property is seen as an investment, it is hoped that the value of the property will increase. This increase in property value will cause an increase in the amount of taxes owed which, in turn, causes an increase in the mortgage payment. If a person is budgeting for their mortgage payment, they need to be able to calculate the increase of taxes and how this would affect the mortgage payment.

In this activity, you will calculate the total mortgage payment (principal plus interest), the amount of real estate tax due and the monthly payment. You will also calculate the increase in the mortgage payment due to an increase in property value.

The Jones' took out a 30-year mortgage in the amount of $125,000 for a home valued at $150,000 five years ago. The interest rate on the mortgage is 6.5%. Part of their payment goes into an escrow account that is used to pay their real estate tax; their current tax rate is 1.5%. The property values for the first five years are:

Year	**1**	**2**	**3**	**4**	**5**
Property Value	**$150,000**	**$152,000**	**$155,000**	**$159,000**	**$161,000**

Questions

1. Find the amount of the monthly mortgage payment that covers principal plus interest.

 ____

2. During the first year, what were the Jones' escrowing for monthly real estate taxes?

 ____

3. What was the monthly amount of real estate taxes that the Jones' family was charged during the second year?

 ____

4. How much had the monthly real estate tax amount changed in the first 5 years?

____

5. Find the total amount paid per month for both the mortgage and real estate taxes for the first year.

____

6. Find the total amount paid per month for both the mortgage and real estate taxes for the fifth year.

____

____

7. What was the yearly dollar amount needed to pay both the mortgage and the real estate taxes during the first year?

____

____

8. What was the yearly dollar amount needed to pay both the mortgage and the real estate taxes during the fifth year?

____

9. The Jones' are trying to plan ahead for their budget. They need to find an equation that could be used to estimate their monthly payment for both the mortgage and real estate taxes. What equation should they use? How did you come up with your answer?

____

____

____

10. Why would it be important for the Jones' family to estimate their future payment?

____

11. For the tenth year, how much would they need per month?

12. What would be the total amount paid by the Jones' during the final year of their mortgage?

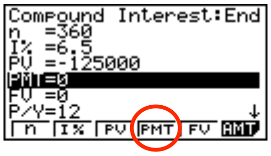
13. If their yearly income were $68,000 during the first year of the mortgage, what percent of their income went to paying the mortgage and real estate taxes?

14. If they estimate their yearly income to be $83,500 after ten years, what percent of their income would have to be budgeted during that year for the mortgage payment and real estate taxes?

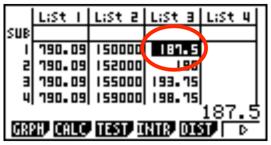
15. Describe the significance between your answers for question 13 and 14.

Solutions

1. $790.09

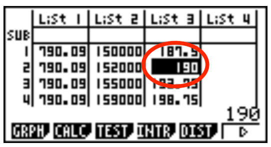


2. $187.50



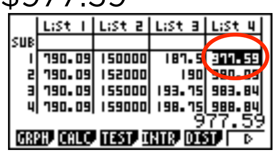
Note: List 3 = (List 2•.015)/12

3. $190.00

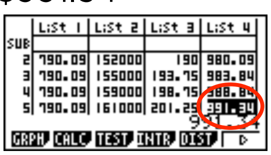


4. $201.25 - $187.50 = $13.75

5. $977.59



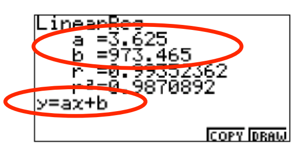
6. $991.34



7. $11,731.08

8. $11,896.08

9. y = 3.625x + 973.465



Answers will vary based on student experience.

10. $1009.72

11. $1045.90

12. $1082.20 (12) = $12,986.40

13. 17.5%

14. 14.5%

15. Answers will vary based on student experience. Hopefully, they note that after 10 years, they are spending a smaller percentage of their

Topic Area: Evaluating and Graphing Inverse Trigonometric Functions

NCTM standards:
- Compute fluently by developing fluency in operations with real numbers using technology for more-complicated cases
- Understand functions by interpreting representations of functions

Objective

To evaluate and graph an inverse trigonometric function

Getting Started

In this activity, students will learn how to evaluate and graph an inverse trigonometric function.

Several indices of refraction are given in the table below:

Medium	Index
air	1.00
ice	1.31
water	1.33
ethanol	1.36
glycerin	1.47
crown glass	1.50
polystyrene	1.59
flint glass	1.75
diamond	2.42

Prior to using this activity:
- Students should understand about reflection and refraction of light.
- Students should be able to determine the critical angel for two mediums.

Ways students can provide evidence of learning:
- Students will be able to evaluate and graph an inverse trigonometric function.
- Students will be able interpret internal reflection to determine if total internal reflection has occurred.

Common mistakes to be on the lookout for:
- Student may use radians instead of degrees in their calculations.
- Students may calculate the critical angle incorrectly.

Definitions

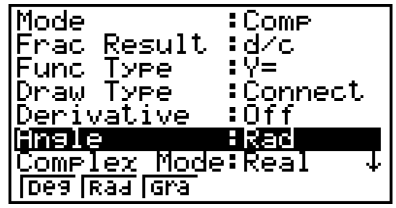
- reflection
- critical angle
- refraction
- index of refraction
- medium
- boundary

Formula

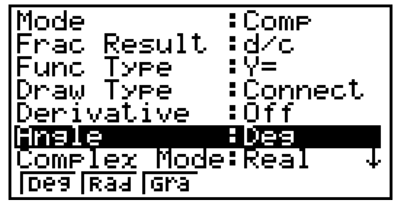
$$\theta = \sin^{-1}\left(\frac{n_r}{n_i}\right)$$

The following will demonstrate how to enter the data into the Casio *fx-9750GII* and interpret the results.

To set up the calculator to calculate in degrees:



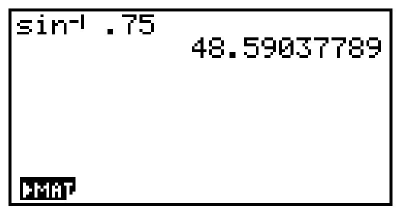
1. From the Main Menu, highlight the RUN•MAT icon and press [EXE] or press [1].



2. To change the calculator to degree mode, press [SHIFT] [MENU] **(SET UP)**, move the cursor down to Angle. Press [F1] (Deg) to change it into degrees, and press [EXIT].

To solve equations including inverse trigonometric functions:

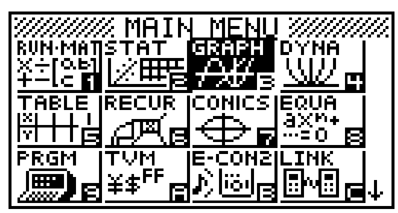
For this example we will find the $\sin^{-1}$ of 0.75.



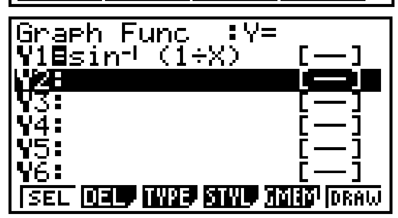
1. To find the $\sin^{-1}$, press [SHIFT] [sin] [•] [7] [5] [EXE]. The angle is 48.59°.

To graph an equation:

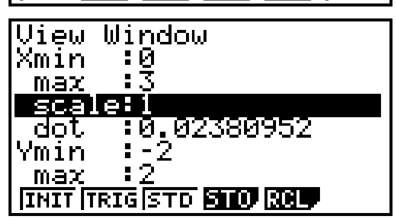
For this example, we will use the equation $y = \sin^{-1}\left(\frac{1}{x}\right)$, from x = 0 to x = 3.



1. Press [3] to select the Graph icon from the Main Menu.

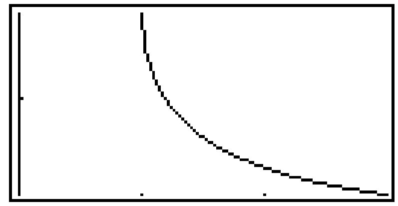


2. Enter the expression by pressing [SHIFT] [sin] [(] [1] [÷] [X,θ,T] [)] [EXE].



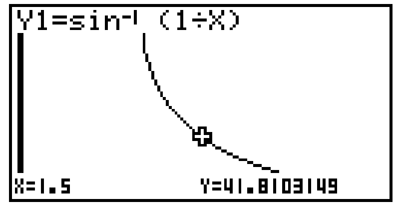
3. To set the view window, press [SHIFT] [F3] **(V-Window)** [0] [EXE] [3] [EXE] [EXIT].

4. Press F6 (DRAW), F2 **(ZOOM)**, F5 (AUTO) to view the graph.



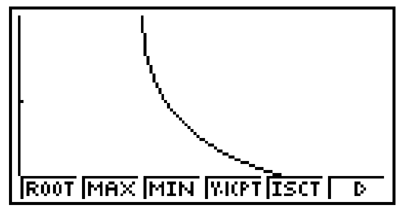
To use the trace feature:

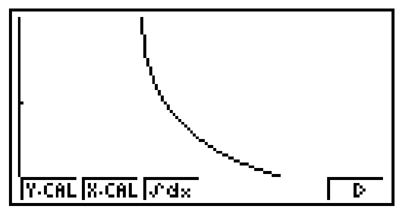
1. Press F1 **(Trace)** and a cursor will appear on the curve. Press the ◀ ▶ to move the cursor along the graph.



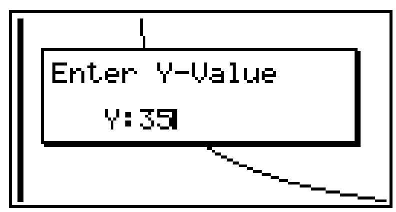
To use the Graph Solver feature:

1. Press F5 **(G-Solv)**, F6 (▷), F2 (X-CAL).

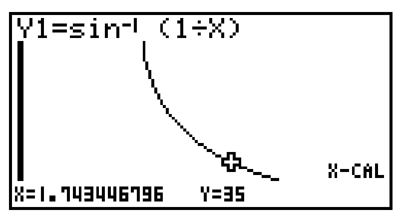




2. A window will display asking for a Y-value. For this example, we will use 35 as the y-value.



3. The x-value, y-value and equation, along with the graph at that point, will display on the screen.



Introduction

In this activity, you will learn how to evaluate and graph an inverse trigonometric function. Inverse trigonometric functions are found by swapping the range and domain of the corresponding trigonometric function. For example, the domain of the sine function becomes the range of the inverse sine function and the range of the sine function becomes the domain of the inverse trigonometric function. The inverse sine function is denoted using arcsin or $\sin^{-1}$.

Total internal reflection (TIR) occurs when all of the light of a beam within a medium strikes a boundary and is reflected inside the medium instead of exiting the medium. For TIR to occur, the light source must be in a denser medium and hit a less dense boundary. Examples of a denser medium might be water or a diamond in comparison to a less dense boundary such as air. Also for TIR to occur, the angle in which the light strikes the boundary must be greater than or equal to a value determined by the two mediums in use. This value is called the critical angle for the two mediums.

The critical angle θ for the two mediums can be determined by using the formula $\theta = \sin^{-1}\left(\frac{n_r}{n_i}\right)$ where n_r and n_i are indices of refraction, the ratio of $\left(\frac{n_r}{n_i}\right)$ is less than 1.0 and θ is in degrees. The value of n_r is the index of refraction for the less dense refractive medium and n_i is the index of refraction for the denser incident medium.

Questions

1. Using the formula, find the critical angle for the given boundaries:

   a. water (n_i = 1.33) – air (n_r = 1.00) boundary angle (θ) = __________

   b. diamond (n_i = 2.42) – water (n_r = 1.33) boundary angle (θ) = __________

   c. crowned glass (n_i = 1.50) – ice (n_r = 1.31) boundary angle (θ) = __________

2. Graph the model for the fixed refractive medium (air) (n_r = 1) and a variable incident medium (n_i = x) from $n_i = x = 1$ to $x = 3$.

3. Using the trace feature, what is the critical angle for:

   a. a polystyrene (x = 1.59) – air boundary? angle (θ) = ___________

   b. a flint glass (x = 1.75) – air boundary? angle (θ) = ___________

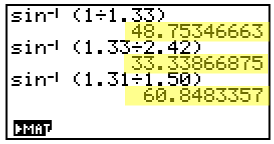
4. Use the Graph Solver feature to find the index of reflection for the incident medium, that would produce a critical angle of:

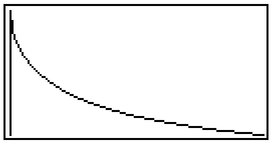
   a. 45° n_i = ___________

   b. 60° n_i = ___________

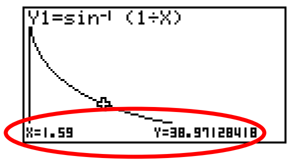
Solutions

1. a. angle (θ) = 48.7535°
   b. angle (θ) = 33.3387°
   c. angle (θ) = 60.8483°

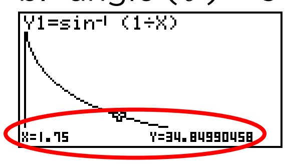


2. 

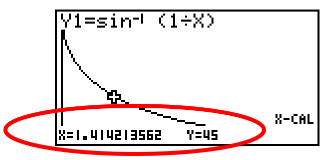
3. a. angle (θ) = 38.9713°



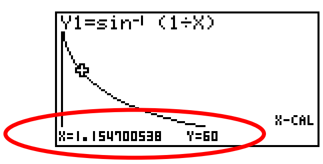
   b. angle (θ) = 34.8499°



4. a. n_i = 1.4142



   b. n_i = 1.1547



Topic Area: Arc Length

NCTM Standards:

- Use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.

Objective

The student will be able to use the Casio *fx-9750GII* to solve problems involving arc length of a circle.

Getting Started

There are many areas in which circles have a major role from calculating the linear speed of a tire to determining the radius of a plate from a broken artifact. In order to solve these problems, students must be able to find the length of the arc for a given radius and central angle.

Prior to using this activity:

- Students should be able to find the radius of a circle given the diameter.
- Students should be able to find the measure of a central angle of a circle.
- Students should know that there are 360° in a circle.

Ways students can provide evidence of learning:

- Given the radius and central angle of a circle, the student will be able to find the length of the arc intercepted by the central angle.
- Given the diameter and central angle of a circle, the student will be able to find the length of the arc intercepted by the central angle.
- Given the radius and length of an arc, the student will be able to find the measure of the corresponding central angle.

Common mistakes to be on the lookout for:

- Students may use the diameter instead of the radius in the formula.
- Students may use radian measure instead of degree measure.
- Students may use the wrong angle measure in the formula.

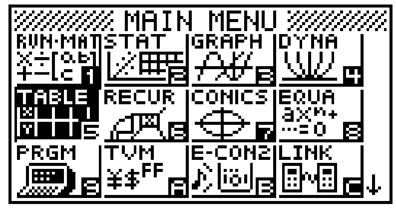
Definitions

- Circle
- Diameter
- Radius
- Arc
- Arc Length
- Central Angle
- Degrees

The following will demonstrate how to enter a formula into the Table mode and use that information to solve a problem using the Run-Matrix mode of the Casio *fx-9750GII*.

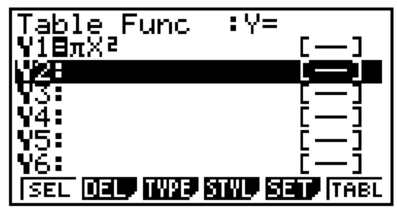
Given the formula for finding the area of a circle is $A = \pi r^2$, find the areas for a group of circles whose radii start at 4, end at 10, and increase by 2. Then, find the volume of a cylinder whose height is 7.5 and whose radius is 8.

To create a table of values:

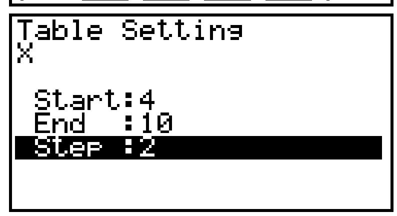


1. From the Main Menu, highlight the TABLE icon and press [EXE] or press [5].

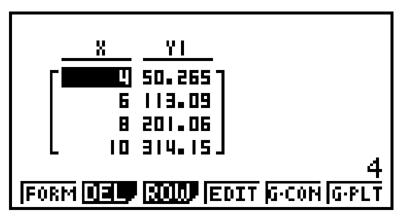
2. Highlight any functions already in the calculator and press [F2] (DEL) [F1] (Yes) to delete them.



3. With the cursor at **Y1**. enter the formula for finding the area of a circle by pressing [SHIFT] [EXP] [X,θ,T] [x^2] [EXE] using X to represent the radius.



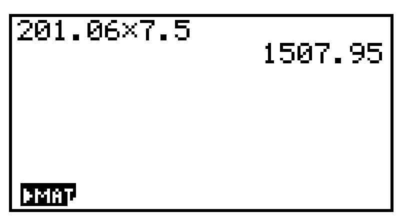
4. To set up the table, press [F5] (SET) to enter the setup menu. Enter [4] [EXE] for the **Start** value, enter [1] [0] [EXE] for the **End** value, and enter [2] [EXE] for the **Step** value to set up the conditions for the table.



5. To see the table of values, press [EXIT] [F6] (TABL).

To find the volume of a cylinder:

The formula for finding the volume of a cylinder is $V = \pi r^2 h$. From the table, we found that $\pi r^2 = 201.06$ when the radius is 8.



1. From the Main Menu, highlight the RUN•MAT icon and press [EXE]. Now, multiply the area of the base by the height by pressing [2] [0] [1] [•] [0] [6] [×] [7] [•] [5] [EXE].

An arc is a part or section of the circumference of a circle. These figures are found all around from the tires on a car to the ends of an outdoor track used in sports. In this activity, we will complete a table that will give the measure of the central angle that intercepts an arc in radian measure. We will then apply that information to solving problems involving circles.

Questions

The formula for finding the length of an arc is $S = 2\pi r \bullet \frac{\theta}{360}$, where $2\pi r$ is the circumference of the circle and $\frac{\theta}{360}$ represents the radian measure of the arc found by dividing the central angle by 360°. This can be given as a fraction or a decimal.

1. Complete the given table by finding the radian measure for each of the indicated arcs.

Central Angle	Radian Measure (Fraction Form)	Radian Measure (Decimal Form)	Central Angle	Radian Measure (Fraction Form)	Radian Measure (Decimal Form)
30°			120°		
45°			135°		
60°			180°		
90°			270°		

A truck tire has a diameter of 36”. Answer the following questions:

2. Find the radius of the truck.

____

3. Find the distance traveled through a 90° turn.

____

4. Find the distance traveled through a 120° turn.

____

5. The distance traveled through a full turn.

____

6. The distance traveled through $4\frac{3}{4}$ turns.

____

A carpenter is creating shelves using four circles as shown in the given diagram. Each shelf will have a radius that is 2" shorter than the previous shelf. A wedge with an angle of 60° will be cut out of each circle, in order to attach a pole. The outside edge of each circle will have a decorative band placed around it. If the largest circle has a radius of 20", find the amount of banding needed for each shelf, the total amount of banding needed, and the total cost of banding, if it sells for $1.25 a foot.

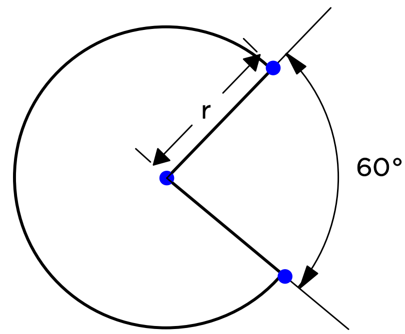
7. Find the amount of banding needed for each shelf.

a. 20" ______________

b. 18" ______________

c. 16" ______________

d. 14" ______________



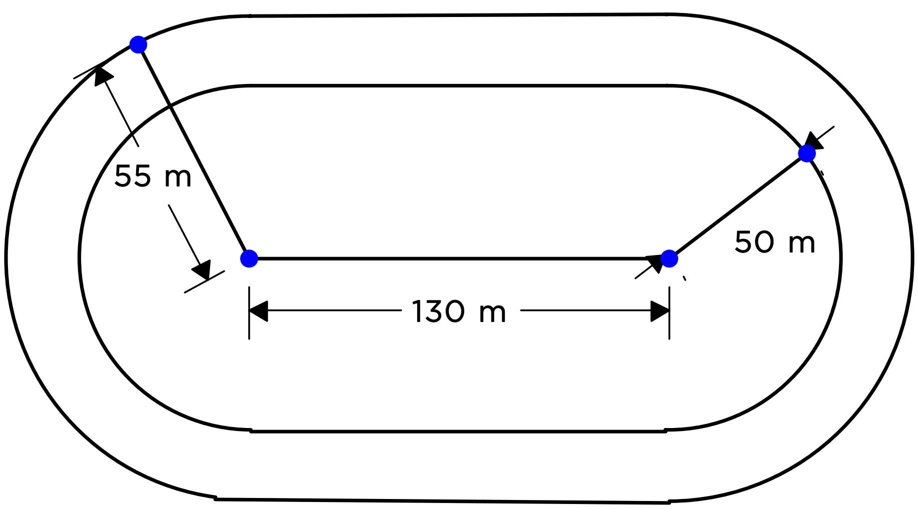
8. What is the total amount of banding needed for this project?

____

9. If the banding costs $1.25 per foot, how much would it cost for this project?

____

Mt. Pilot High School has an outdoor track that has two arcs at each end. The interior radius of each end is 50 m and the outside radius is 55 m as shown in the diagram. The straight away section is 130 m.



10. What is the perimeter for the inside lane of the track?

____

11. What is the perimeter for the outside lane of the track?

____

12. How far would a runner in the inside lane have to go around the track to complete a 500 m race?

____

13. How many times would a runner in the outside lane have to go around the track to complete a 1500 m race?

____

An observatory with a circular dome has a sliding opening that goes from 0° to 150°. The diameter of the dome is 84 ft. What is the length of the opening for each of the given degree measures?

14. What is the length of the opening when the dome is open at 45°?

____

15. What is the length of the opening when the dome is open at 60°?

____

16. What is the length of the opening when the dome is open at 90°?

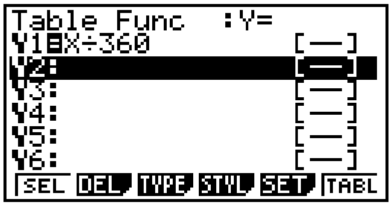
____

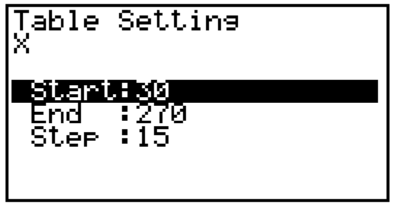
17. What is the length of the opening when the dome is open at 135°?

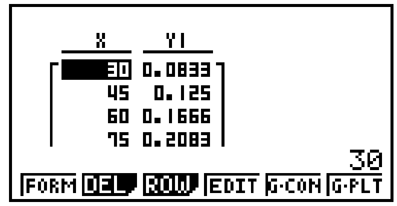
____

Solutions

1.





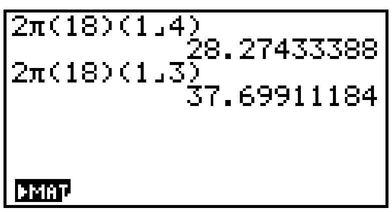


Central Angle	Radian Measure (Fraction Form)	Radian Measure (Decimal Form)	Central Angle	Radian Measure (Fraction Form)	Radian Measure (Decimal Form)
30°	$\frac{1}{12}$	0.083	120°	$\frac{1}{3}$	0.333
45°	$\frac{1}{8}$	0.125	135°	$\frac{3}{8}$	0.375
60°	$\frac{1}{6}$	0.167	180°	$\frac{1}{2}$	0.500
90°	$\frac{1}{4}$	0.250	270°	$\frac{3}{4}$	0.750

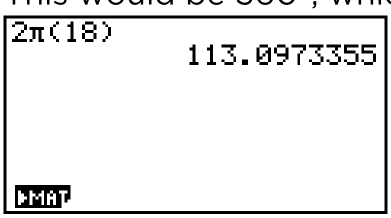
2. 18”

3. 28.27“

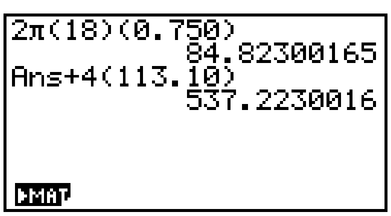
4. 37.70”



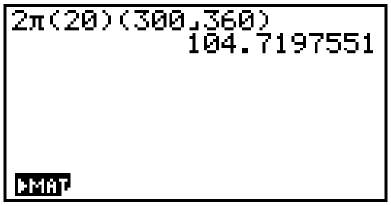
5. This would be 360°, which is 2πr = 113.10”



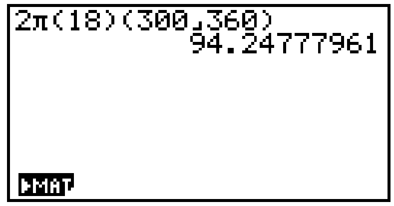
6. This would be 4 full turns and a $\frac{3}{4}$ turn = 537.22”



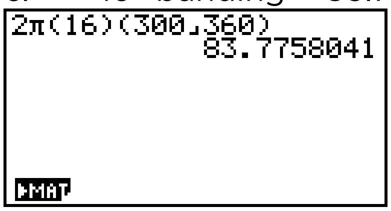
7. a. 20” shelf = 104.72”



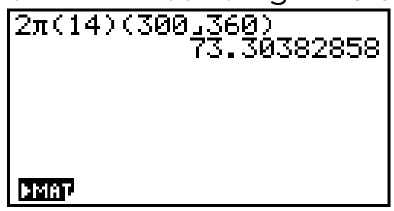
b. 18” shelf = 94.25”



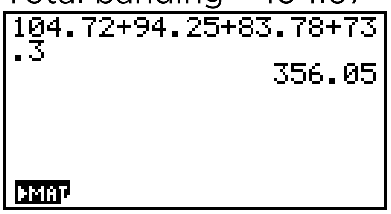
c. 16” banding = 83.78”



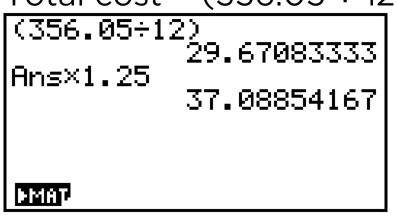
d. 14” banding = 73.30”



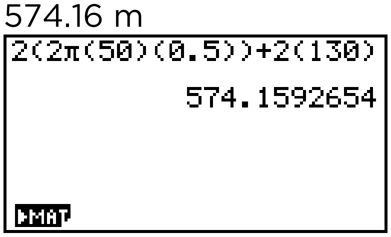
8. Total banding = 104.67 + 94.21 + 83.74 + 73.27 = 356.05”



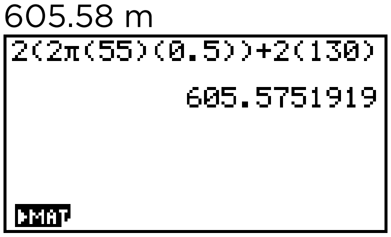
9. Total cost = (356.05 ÷ 12)•$1.25 = $37.09



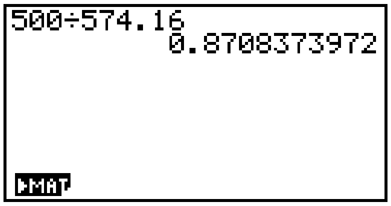
10. 574.16 m



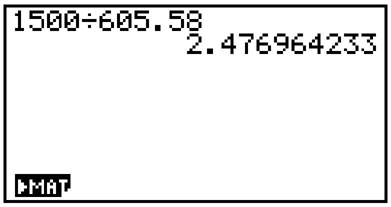
11. 605.58 m



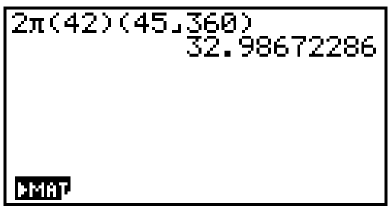
12. 0.87 times



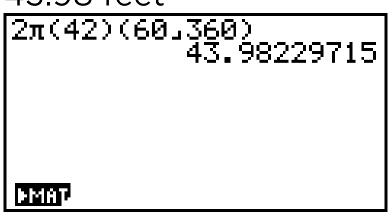
13. 2.48 times



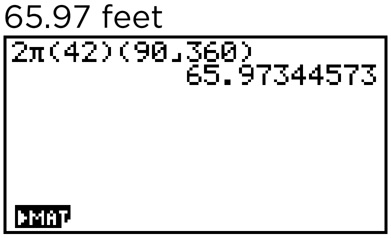
14. 32.99 feet



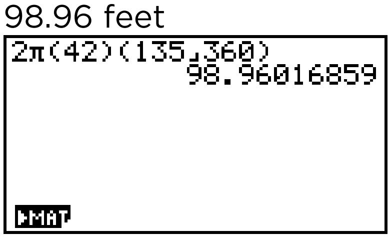
15. 43.98 feet



16. 65.97 feet



17. 98.96 feet



Trigonometry in a Circle Teacher Notes

Topic Area: Trigonometric Equations

NCTM Standard:

- Develop fluency in operations using mental computation or paper-and-pencil calculations for simple cases and technology for more-complicated cases.
- Understand the meaning of equivalent forms of expressions, equations, inequalities, and relations.

Objective

To evaluate and graph trigonometric equations.

Getting Started

Students will learn to evaluate and graph trigonometric equations.

Prior to using this activity:

- Students should be able to manipulate algebraic expressions.
- Students should be able to recognize quadratic relationships.
- Students should be able to graph functions.
- Students should understand how to analyze graphs and create a table of values.

Ways students can provide evidence of learning:

- Given a trigonometric equation, the student will be able to factor the equation.
- Given a trigonometric equation, the student will be able to evaluate angle values.
- Student will be able to solve trigonometric inequalities.

Common mistakes to be on the lookout for:

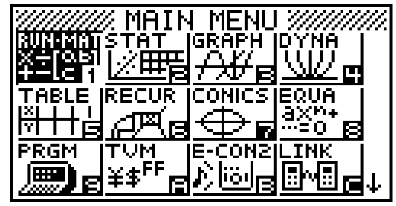
- Students may analyze graphs of functions visually and not analytically.
- Students may be unaware of domain restrictions.

Definitions

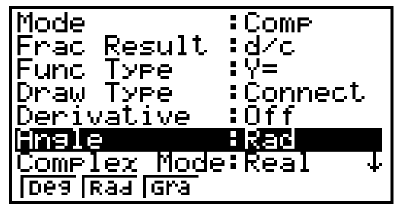
- Trigonometric Identity
- Conditional Equations
- Arc
- Chord
- Central Angle
- Subtended

The following will demonstrate how to: change calculator angle preferences, evaluate inverse trigonometric functions in degrees or radians, solve equations in quadratic form, graph trigonometric functions, and verify trigonometric identities on the Casio *fx-9750GII*.

To set the default angle unit:

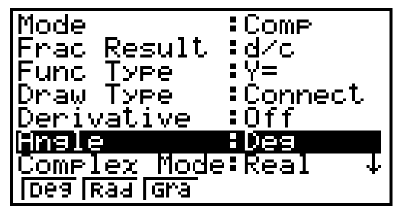


1. From the Main Menu, highlight the RUN•MAT icon and press EXE or press 1.

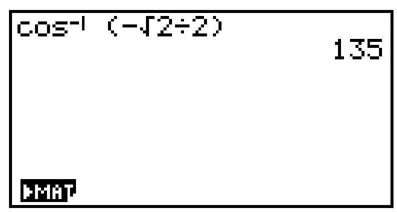


2. To set the default angle unit, press SHIFT MENU **(SET UP)** ▼ ▼ ▼ ▼ ▼. From here, you will choose either F1 (Deg) or F2 (Rad).

To evaluate inverse trigonometric functions:



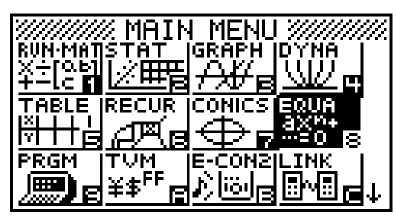
1. Suppose we wanted to solve cos x = $\frac{-\sqrt{2}}{2}$.



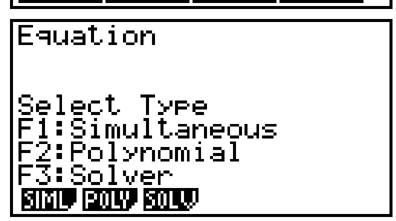
2. First, change the angle unit to degrees for easier interpretation. Press SHIFT MENU **(SET UP)** ▼ ▼ ▼ ▼ ▼ F1 (Deg).

3. To solve, press SHIFT cos ((−) SHIFT x^2 2 ÷ 2) EXE.

To solve an equation in quadratic form:

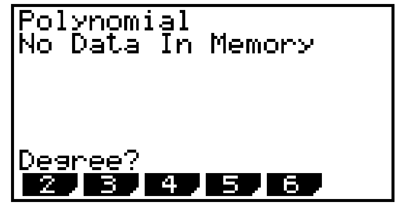


1. Suppose we wanted to solve $2\cos^2 x - 4\cos x + 1 = 0$ for cos x.

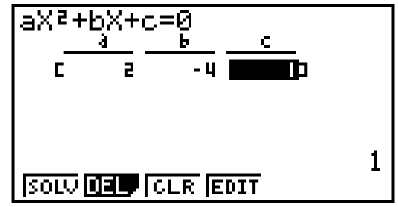


2. From the Main Menu, highlight the EQUA icon and press EXE or press 8.

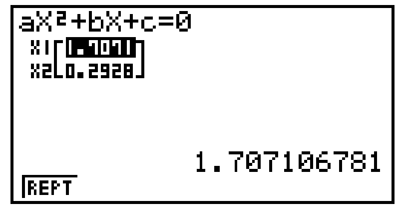
3. Press F2 (POLY) and F1 (2) for a 2nd degree polynomial.



4. Input the coefficients for the equation, by pressing 2 EXE (−) 4 EXE 1 EXE.

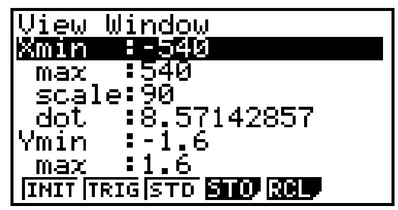


5. Press F1 (SOLV).



To graph trigonometric functions:

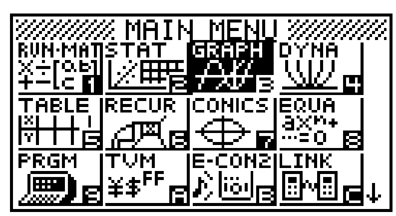
1. When working with trigonometric functions, always set the viewing window to the trigonometric default. This will auto-correct regardless of the default angle unit.



2. In the Graph icon, press SHIFT F3 (V-Window) F2 (TRIG) EXE.

To verify trigonometric identities:

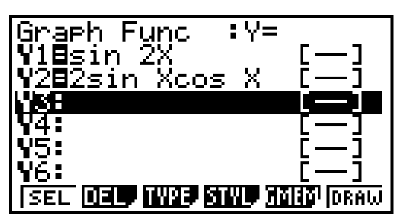
1. Suppose we wanted to verify the double-angle identity sin 2x = 2sin x•cos x.



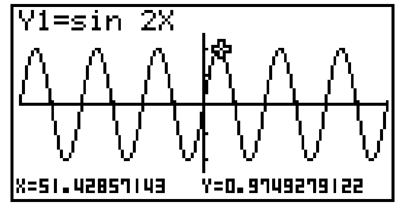
2. From the Main Menu, highlight the GRAPH icon and press EXE or press 3.

3. For Y1, press: sin 2 X,θ,T EXE.

4. For Y2, press: 2 sin X,θ,T 2 X,θ,T EXE.

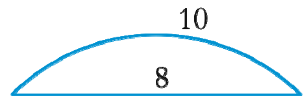


5. To graph both functions, press F6 (DRAW).



6. Press F1 **(Trace)** and use the ▲ ▼ to verify the functions are identical.

A 10-centimeter arc on a circle has an 8-centimeter chord.



Questions

1. Sketch the entire circle with auxiliary lines to represent this situation.

2. If θ is in radians, what is the radius of the circle, rounded to four decimal places?

____

3. What is the radian measure of the central angle, to four decimal places, subtended by the arc?

____

Extension

1. Solve $\cos 2x = 4\cos x - 2$ for all real x values.

____

____

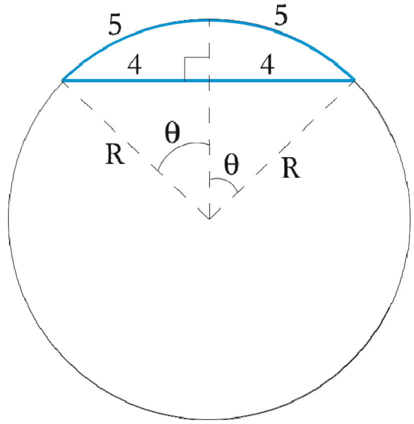
2. Compute the inverse function to four decimal places.

____

____

Solutions

1. Drawings may vary.

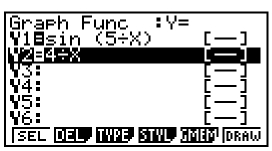


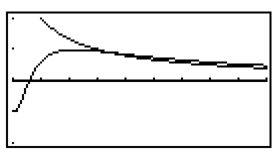
2. Solutions may vary, R = 4.4205 centimeters.

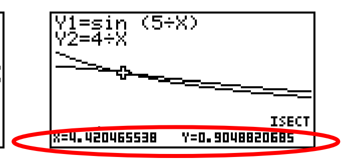
From the figure, θ can be expressed in two ways. One method uses the definition of radian measure and the other is defined as a trigonometric ratio.

Thus, $\theta = \frac{5}{R}$ and sin $\theta = \frac{4}{R}$, or sin $\frac{5}{R} = \frac{4}{R}$.

Our problem is to solve this trigonometric equation for R. Algebraic methods will not isolate R, so we will use the Casio *fx-9750GII* graphing calculator. Graph **Y1** = sin $\frac{5}{x}$ and **Y2** = $\frac{4}{x}$ in the same viewing window using the settings below. It appears that the graphs intersect for x between 4 and 5. Use **G-Solv** to find the point of intersection.







To check the values, insert R into both equations:

$$\sin \frac{5}{R} = \sin \frac{5}{4.4502} = 0.9049 \text{ or } \frac{4}{R} = \frac{4}{4.4502} = 0.9049$$

3. Having R = 4.4205, we can compute the radian measure of the central angle subtended by the 10-centimeter arc.

Central angle = $\frac{10}{R} = \frac{10}{4.4502}$ = 2.2622 radians

Extension Solutions

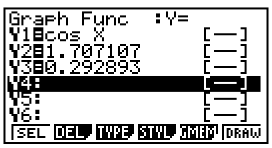
1. Use double angle identity to solve for cos x.

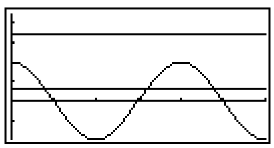
$\cos 2x = 4\cos x - 2$
$2\cos^2 x - 1 = 4\cos x - 2$
$2\cos^2 x - 4\cos x + 1 = 0$

$$\cos x = \frac{4 \pm \sqrt{16 - 4(2)(1)}}{2(2)}$$

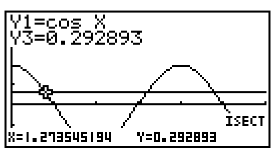
$= 1.707107$ or 0.292893

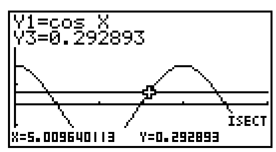
Solve each equation over one period $[0, 2\pi)$ by sketching a graph of Y1 = cos x, Y2 = 1.707107, and Y3 = 0.292893 in the same viewing window.





The equation cos x = 1.707107 has no solution since $-1 \le \cos x \le 1$. The equation cos x = 0.292893 appears to have both a first- quadrant and fourth-quadrant solution. If the reference angle is α, then $x = \alpha$ or $x = 2\pi - \alpha$.





2. $\alpha = \cos^{-1} 0.292893 = 1.2735$
$2\pi - \alpha = 2\pi - 1.2735 = 5.0096$

Since the cosine function is periodic with a period of 2π, all solutions are given by

$$x = \begin{cases} 1.2735 + 2k\pi \\ 5.0096 + 2k\pi \end{cases}, \text{ k = any integer}$$

Topic: Trigonometric Applications

NCTM Standard

- Understand functions by interpreting representations of functions.
- Compute fluently by developing fluency in operations with real numbers using technology for more-complicated cases.

Objectives

The student will calculate and graph the derivative at a given point, and graph and interpret a derivative.

Getting Started

In this activity the students will learn how to calculate the derivative at a given point, graph the derivative, and interpret a derivative graphically using the Casio *fx-9750GII*. Once students have learned the techniques, they will use this knowledge to investigate projectile motion. The derivative will be used to find the instantaneous rate of change for the motion at a given point while looking at the path of a cannon ball. Share with the students that the model assumes there is no air resistance and the object is traveling on a parabolic path.

Ways students can provide evidence of learning:

- Students should be able explain the relationship between derivative and the original function.
- Students will apply their knowledge of the derivative to exploring the behavior of projectile motion.

Common mistakes to be on the lookout for:

- When graphing the derivative, students may think there is no derivative because the calculator takes longer to graph the derivative than the function.

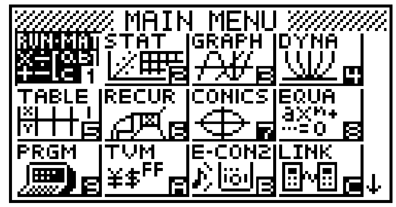
Definitions:

- Slope
- Derivative
- Increasing
- Decreasing
- Velocity
- Vertical height
- Radian
- Acceleration

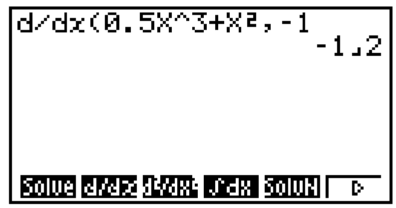
The following will demonstrate how to calculate the derivative at a given point, graph the derivative, and interpret a derivative using the Casio *fx-9750GII*.

Explore the behavior of the function $f(x) = 0.5x^3 + x^2$.

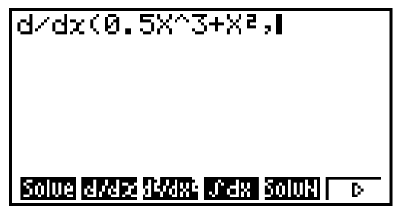
To find the derivative from the RUN-MAT menu:



1. From the Main Menu, highlight the RUN•MAT icon and press [EXE] or press [1].

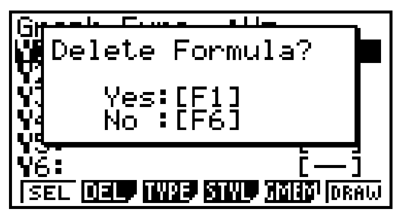


2. To find the value of the derivative at x = -1, press [OPTN] [F4] (CALC) [F2] (d/dx) [0] [•] [5] [X,θ,T] [^] [3] [+] [X,θ,T] [x²] [,] [(–)] [1] [EXE].

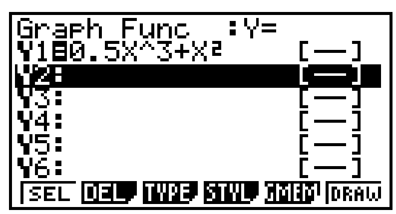


3. To find an additional value of the derivative press [◀] to return to the derivative. Press [DEL] [DEL] to delete the -1. Enter the new value and press [EXE].

To display a graph of the function:

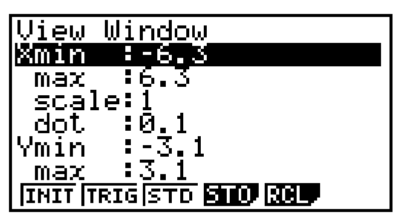


1. From the Main Menu, highlight the GRAPH icon and press [EXE] or press [3].



2. To delete any previous equations, highlight the equation and press [F2] (DEL) [F1] (Yes.)

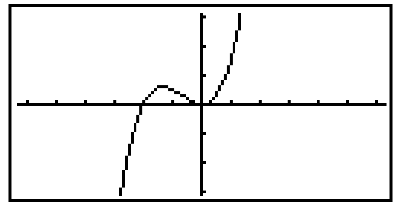
3. Enter the equation in Y1 by pressing [0] [•] [5] [X,θ,T] [^] [3] [+] [X,θ,T] [x²] [EXE].



4. Set the view window to the initial screen by pressing [SHIFT] [F3] **(V-Window)** [F1] (INIT).

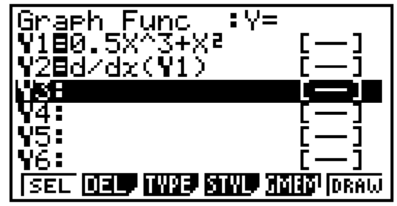
5. Press [EXIT] to return to the initial GRAPH screen.

6. Press [F6] (DRAW) to view the graph of the function.



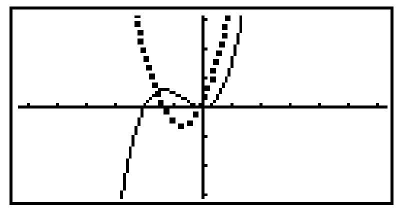
To graph the derivative of the function:

1. Press [EXIT] to return to the initial GRAPH screen.



2. In **Y2**, press [OPTN] [F2] (CALC) [F1] (d/dx) [F1] (Y) [1] [)] [EXE].

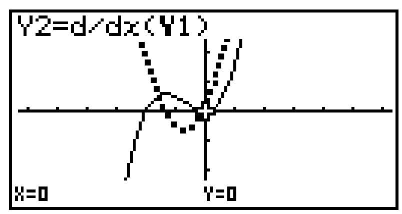
3. To change the style of and graph the derivative, arrow up to highlight **Y2**. Press [F4] (STYL) [F3] (••••) [EXE].



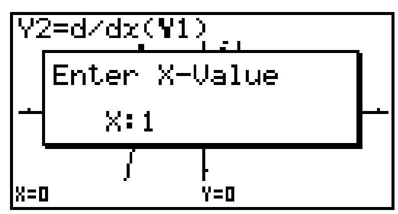
4. Press [F6] (DRAW) to view the graph of the function.

To examine the behavior of the derivative:

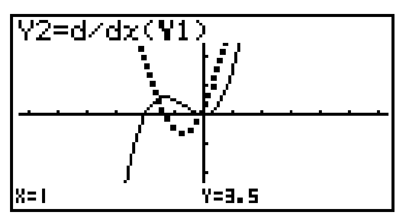
1. When viewing the graph, press [F1] **(Trace)**. Arrow down to select **Y2**. Use the arrows to trace to the right or left.

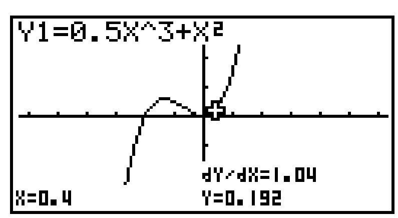


2. To find the exact value of the derivative at x = 1, while tracing, press [1] [EXE].



3. To find the value of the derivative while tracing, press [SHIFT] [MENU] **(SET UP)**. Arrow down to highlight **Derivative**. Press [F1] (On) [EXE]. Trace the original function. The derivative (dY/dX) will appear on the right side of the screen.





Up and Down — Activity

In this activity you will learn how to calculate the derivative at a given point, graph the derivative, and interpret a derivative graphically using the Casio *fx-9750GII*. You will use this knowledge to investigate projectile motion and look at the derivative to find the instantaneous rate of change for the motion at a given point while looking at the path of a cannon ball.

Questions

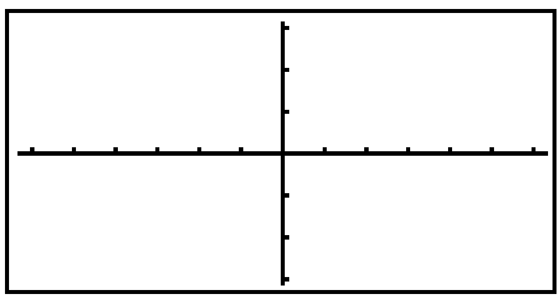
1. Using the derivative in the Run-Matrix menu, calculate the slope of $f(x) = x^3 - x^2 + 1$ at $x = -1$, 0, and 1.

   a. $f'(-1) =$ ________________________

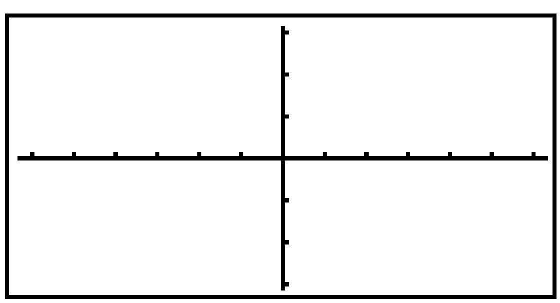
   b. $f'(0) =$ ________________________

   c. $f'(1) =$ ________________________

2. Graph the function $f(x) = x^3 - x^2 + 1$ in the initial view window.



3. For the function $f(x) = x^3 - x^2 + 1$, graph the function and its derivative $f'(x)$ in the same window below. Label the function and derivative.



4. For the function $f(x) = x^3 - x^2 + 1$, trace the graph of the derivative to find the slope of f(x) at x = -1, 0, and 1.

   a. f'(-1) = ______________________

   b. f'(0) = ______________________

   c. f'(1) = ______________________

5. Looking at the graph of the function $f(x) = x^3 - x^2 + 1$, when f'(x) is above the x-axis or positive, is the original function f(x) increasing (going up) or decreasing (going down)?

   ____

6. When the graph of the derivative f'(x) is below the x-axis or negative, is the original function f(x) increasing or decreasing?

   ____

7. When the graph of the derivative f'(x) intersects the x-axis or is zero, what is the original function f(x) doing? Where does this occur?

   ____

   ____

   ____

Extension

A cannon fires a cannon ball with an initial velocity of 700 ft/sec, at an angle of 45° $\left(\frac{\pi}{4}\text{ rad}\right)$ from the ground with the muzzle 4 feet off the ground. Use this information to answer the following questions.

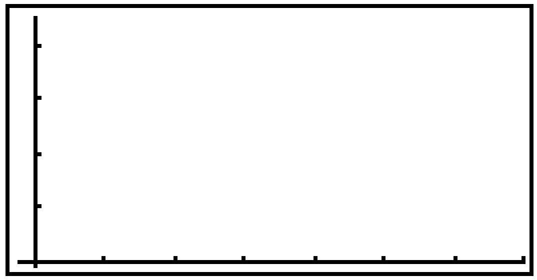
1. The equation for the vertical height of a projectile, y, is

$$y = s + (v\sin(\theta))x - 0.5gx^2$$

   where s is the initial height (ft) of the object, v is the initial velocity (ft/sec) of the object, θ is the initial angle (radians) of trajectory from the ground, x is the elapsed time (seconds), and g is the acceleration due to gravity (32 ft/sec^2). Write the equation for the vertical height of the cannon ball. Note: the model assumes there is no air resistance and the object is traveling on a parabolic path.

   ____

2. In the window [-1, 35, 5, -100, 4500, 1000], graph the cannon ball's motion.



3. What will the derivative of this function tell us about the cannon ball?

____

4. Use the derivative trace to find the velocity of the cannon ball at its highest point of travel.

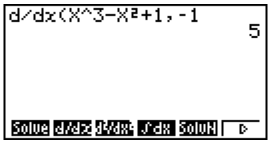
____

5. Use the derivative trace to find the velocity of the cannon ball at its point of impact.

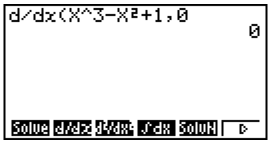
____

Solutions

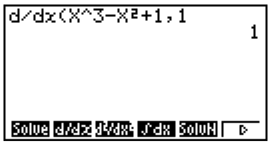
1. a. f'(-1) = 5

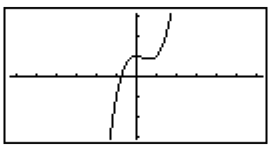


b. f'(0) = 0

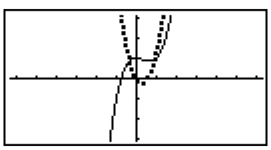


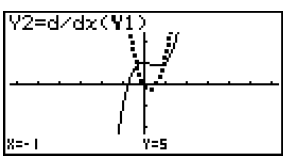
c. f'(1) = 1

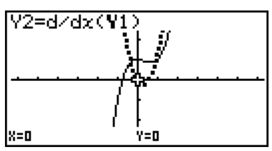


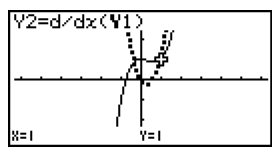
2. 

3. The derivative is dotted.



4 . 





5. The original function is increasing.

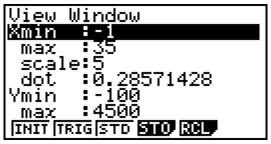
6. The original function is decreasing.

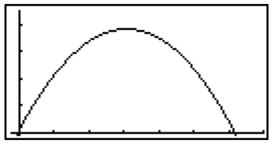
7. The original function has a slope of 0. This occurs twice where the function has a peak or relative maximum on the left and where the function has a valley or relative minimum on the right.

Extension Solutions

1. $y = 4 + (700 \sin(\frac{\pi}{4}))x - 0.5(32)x^2$

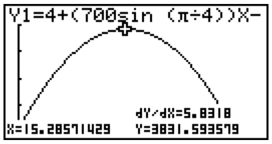
2.

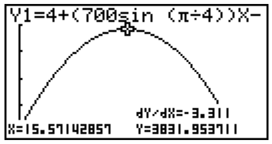




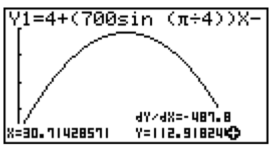
3. The derivative will give us the vertical velocity at a point.

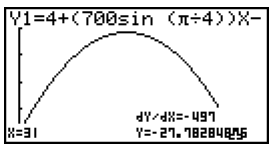
4. The vertical velocity of the cannon ball at its highest point is between 5 ft/sec and -3 ft/sec. If we zoom in on the peak, it would approach 0.





5. The velocity of the cannon ball at the point of impact is not zero. In fact it is between 487 and 497 ft/sec. If we zoom in on y = 0, it would approach 495 ft/sec.





Weighted Averages — Teacher Notes

Topic: Weighted Averages

NCTM Standard:

- Find, use, and interpret measures of center and spread, including mean and interquartile range.

Objective

The student will be able to use the Casio *fx-9750GII* to calculate a weighted average and make various conclusions about the data.

Getting Started

Students are exposed to averages in a variety of academic and athletic settings. Teachers use weighted averages to calculate a course grade based on a series of factors such as quizzes, tests, projects, and homework. This activity will help students gain a greater understanding of how these grades are calculated.

Prior to using this activity:

- Students should be able to calculate an average.
- Students should be able to read and interpret data presented in a table.

Ways students can provide evidence of learning:

- When given a data set, students can calculate an average.
- When given a data set for multiple data categories, students can calculate an average for each category and determine the overall weighted average.
- Students can communicate how an average is affected when different weights are applied to each category within that data set.

Common calculator or content errors students might make:

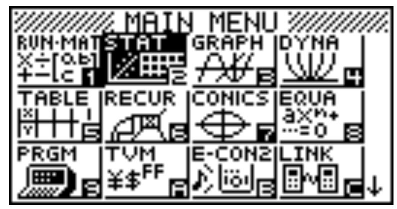
- Students may incorrectly calculate a weighted average by taking an average of an average.
- Students may incorrectly enter a percentage for the calculation of the weighted average.
- Students may incorrectly communicate the impact a particular category has on the overall weighted average.

Weighted Averages "How To"

The following will demonstrate how to use the Casio *fx-9750GII* to determine the weighted average for a set of data. In order to calculate a weighted average, you must begin by entering data into the STAT module. Once that data is entered, you will switch over to the RUN•MAT module and perform the final calculations.

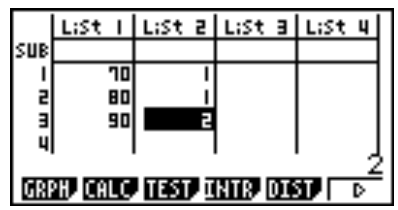
For this example, we will determine the weighted average for three test scores, where the third score is counted twice as much as the other two scores.

To enter a list of data and frequencies into STAT List screen:



1. From the Main Menu, highlight the STAT icon and press [EXE] or press [2].

2. In **List 1**, enter the three scores, by inputting: [7] [0] [EXE] [8] [0] [EXE] [9] [0] [EXE].

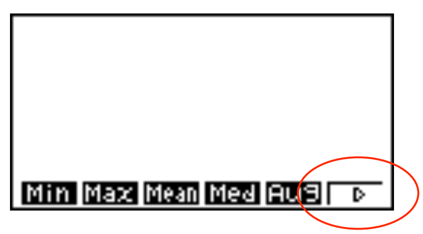


3. From the bottom of **List 1**, press ▶ to go to the top of **List 2**. In **List 2**, enter the frequency of each score by inputting: [1] [EXE] [1] [EXE] [2] [EXE].

To perform calculations using Lists from the STAT List screen:

1. Press [MENU] and highlight the RUN•MAT icon and press [EXE] or [1].

2. Press [OPTN] to display a series of calculation options.



3. Press [F1] to display the **LIST** menu; a series of statistical calculations are displayed.

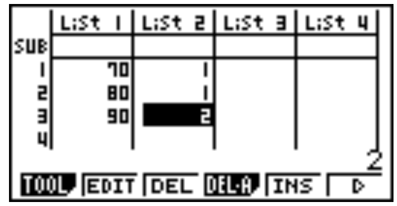
4. Press [F6] (▷) to view more on screen options.

5. Press [F3] for Mean, then [F6] [F6] for the **LIST** tab.

6. Press F1 for **LIST** then input 1 , followed by F1 for **LIST** followed by the number 2).

7. Press EXE to calculate the weighted average.

To delete data from the STAT Editor Window:



1. From the Main Menu, highlight the STAT icon and press EXE or 2 .

2. Press F6 (▷) to view more on screen options.

3. Press F4 (DEL-A) to delete all data in that list. Press F1 to confirm the deletion.

4. Press ◀ to move to **List 1** and follow the directions above to delete all the data from **List 1**.

Weighted Averages **Activity**

Weighted averages are most commonly used to calculate course grades where each component is worth a different amount. For example, your math grade may be calculated where your homework counts for 20%, quizzes count for 30% and tests represent 50% of your final average. Each category makes up a specific part of the entire grade, but it is important to remember that each category is not represented equally. Take a moment to answer these questions making certain that you are assigning the correct weight to each piece of data.

Questions

A mid-term and a final exam determine the overall grade for a math course. The mid-term is worth 1/3 of your grade while the final exam is worth 2/3 of your grade.

1. If you earned a 72 on the mid-term and an 84 on the final exam, what would be your course average?

   ____

2. If you earned a 72 on the mid-term, what is the minimum score needed on the final exam in order to receive an 85 average for the course? How did you come up with this solution?

   ____

   ____

3. If you earned a 72 on the mid-term, is there a way to earn a 90 for the course average? Explain how you found your answer.

   ____

   ____

Vickie is enrolled in a Literature course where her semester average is based upon four tests. The last test is double in comparison to the other tests. Vickie's test scores were 82, 91, 75 and 86 on the final test.

4. What is Vickie's average for the Literature course?

   ____

5. Vickie's first three test scores are: 82, 91, and 75. What is the minimum score she must earn in order to get at least a 70 average? How did you come up with this solution?

____

____

6. If Vickie earn an 80 on each of the first three tests and scored a 100 on the final test, what would be her average?

____

____

An eighth grade Algebra class determines their grade based upon three criteria: homework, projects and tests. Homework counts for 20% of the grade. Projects count for 30% of the grade. Tests count for 50% of the grade. Examine the following table to review Xavier's grades.

Homework	Projects	Tests
90	95	88
100	82	92
100		81
50		
80		

7. How much does each homework grade weigh? Explain how you came to this conclusion.

____

____

____

8. How much does each project grade weigh? Explain how you came to this conclusion.

____

____

____

9. How much does each test grade weigh? Explain how you came to this conclusion.

____

____

____

10. Determine Xavier’s average in Algebra rounded to the nearest hundredth.

____

11. If Xavier completed one more project and earned an 87, what will be his new course average now? Round the average to the nearest hundredth.

____

____

How will that score change his course average?

____

12. Leaving the newly entered project grade as entered in question 11, if homework equaled 50%, tests equaled 30% and projects equaled 20%, will Xavier’s grade be different than in question 11? If so, which grade is better? Explain your answer.

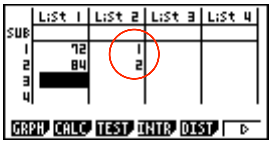
____

____

____

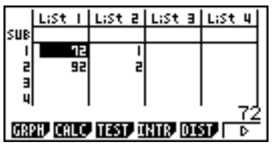
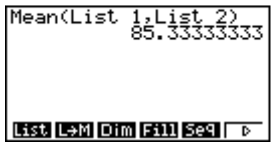
Solutions

1. Make sure to assign a frequency of 1 for the mid-term exam grade and 2 for the final exam grade.

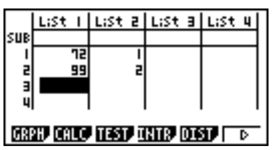



The course average is an 80.

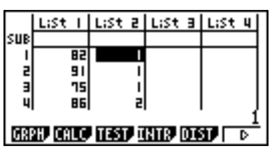
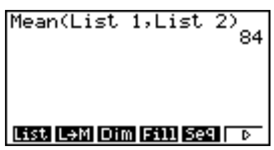
2. The minimum score needed on the final exam to obtain a course average of an 85 is a 92. Answers will vary; possible answer choices are guess and check, knowing that the final exam grade needs to be above an 84, from the last problem.

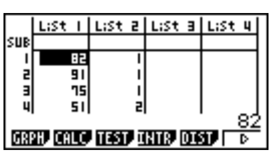
3. To earn a 90 for the course average, you would need to earn a 99 on the final exam. Answers will vary; students could have used the same process as they did in question 2.



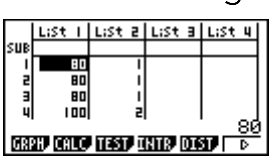

4. Vickie's average is an 84.

5. Vickie must earn a 51 in order to get at least a 70 average. Answers will vary; students could have used the same process as they did in question 2, knowing that the last test grade needs to be below an 86.




6. Vickie's average will be an 88.

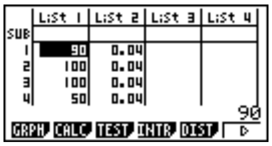


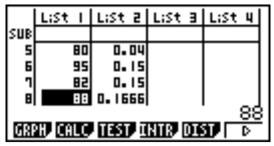

7. Homework represents 20% of the grade and there are five grades. Since 20% = 0.2, each homework grade equals 0.04 because 0.2 ÷ 5 = 0.04.

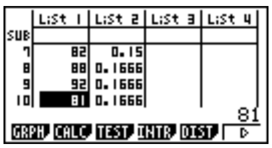
8. Projects are 30% of the grade. There are two projects so each project is worth 15% of the grade or 0.15.

9. Tests are 50% of the grade and there are three tests. Since 50% = 0.5, 0.5 ÷ 3 = 0.1666.

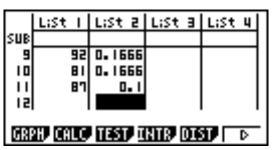
10. Each of the above values needs to be entered in List 2 directly across from the corresponding grade. Xavier's average is approximately an 86.85.





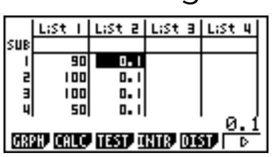


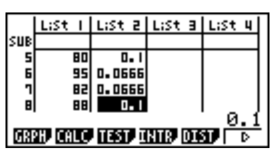
11. Xavier's new average will be approximately 86.7.
If Xavier completed one more project then each project will then account for 10% of the overall grade. (30% = 0.3 and 0.3 ÷ 3 = 0.1). Change cells 6 and 7 in List 2 to 0.1. Also, enter 87 to cell 11 in List 1 and 0.1 in cell 11 in List 2. Then, perform the commands to determine the weighted average.

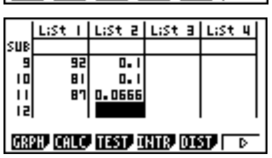


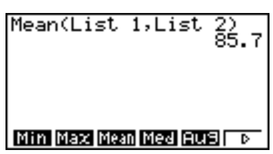
Xavier's course average will decrease by approximately 0.15 points.

12. Yes, Xavier's grade will now be an 87.18. The new percents will give Xavier the better grade.









Homework now has a frequency of 0.1, tests have a frequency of 0.1, and projects have a frequency of $\frac{0.2}{3}$ or 0.066.

What Color is Your Car? Teacher Notes

Topic: Random Sampling, Categorical Data

NCTM Standards:

- Understand the differences among various kinds of studies and which types of inferences can legitimately be drawn from each.
- Understand the meaning of measurement data and categorical data, of univariate and bivariate data, and the term variable.
- Display and discuss bivariate data where at least one variable is categorical.
- Use simulations to explore the variability of sample statistics from a known population and to construct sampling distributions.
- Understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference.

Objective:

The student will be able to create a frequency table of a random sample of car colors in categories and create a histogram representation of the data.

Getting Started:

For this activity, you will need access to a large parking area in your school or community. The students should collect color data from the cars in the parking lot. Have them count how many cars in the parking lot is each color. You may want the students to break up in the parking lot into sections and share the data once the entire parking lot has been counted.

Prior to using this activity:

- The student should be able to calculate basic statistics.
- The student should be able to create a histogram.

Ways students can provide evidence of learning:

- Students should be able to gather a large number of data in an organized manner.
- Given data, the student should be able to create a histogram.

Common mistakes to be on the lookout for:

- Students might be too specific when grouping car colors, have them choose 10 colors to categorize their data.

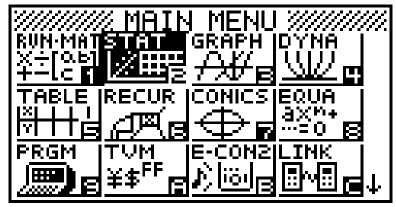
Definitions

- Categorical data
- Sampling
- Frequency
- Histogram

The following will demonstrate how to enter a set of data into the Casio *fx-9750GII* and graph the data using a histogram.

1	2	3	4	5	6	7	8	9	10
12	13	22	33	12	22	1	13	65	35

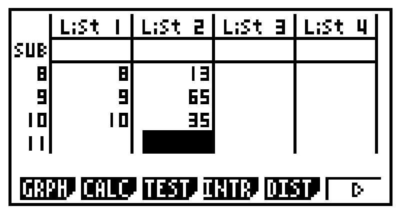
To enter the data from the table in the problem:



1. From the Main Menu, highlight the STAT icon and press EXE or press 2.

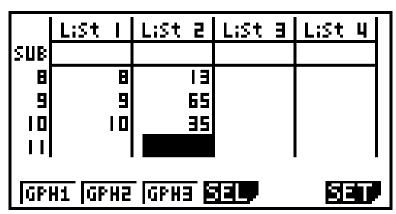
2. To clear previous data lists press F6 (▷) F4 (DEL-A) F1 (Yes).

3. Enter the data by typing each number, pressing EXE after each entry.

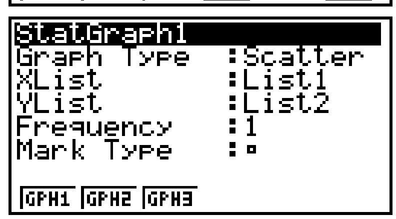


4. The display should look like the screen shot on the right when completed.

To select the type of graph for this data:

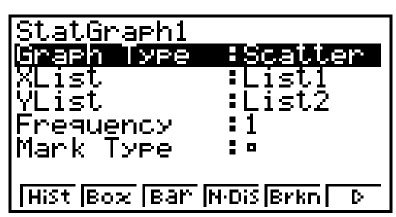


1. Press F1 (GRPH) and F1 (GPH1).



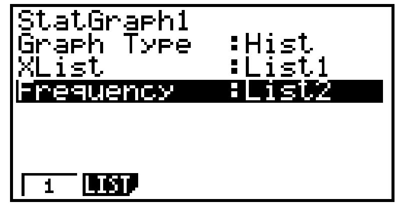
2. Select F6 (SET) to set the type of graph for **StatGraph1**.

3. Press ⊽ to highlight **Graph Type**.

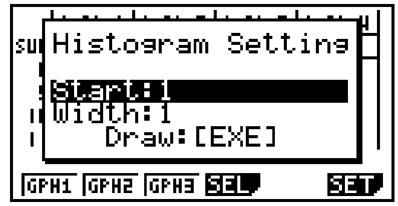


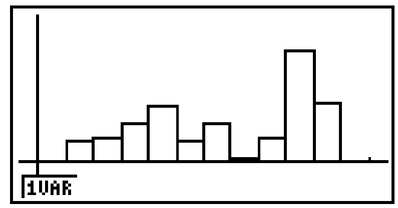
4. Choose F6 (▷) to find more choices, then F1 (Hist).

5. Make sure that the **XList** is **List 1**, and the **Frequency** is **List 2**. If not, scroll down and press F2 (LIST) to select the list. You will be prompted to select the list number, then press EXE.



6. Press EXIT, then F1 (GPH1) to graph. Next, you can determine the size of your Histogram, select the start and the width equal to one.





What Color is Your Car? Activity

Introduction

Find a partner to do this activity with. Go to the parking lot of your school or community and record the colors and the number of cars of each color there are. Follow your teacher's instructions for the data collection. Create a table with color categories in the first column and the number of cars in the second column. Use the data to answer the following questions.

The following table will be used for the solutions section:

Color	Silver	Black	Blue	White	Grey
# of Cars	238	167	132	120	103

Color	Red	Green	Beige	Gold	Brown
# of Cars	101	58	45	27	9

Questions

1. What is the most popular car color in your school or community (a sample of the whole population)?

   ____

2. What percent of the total number of cars was this color?

   ____

3. Create a histogram for the data you collected and draw it in the box below.

4. How did you go about categorizing the colors in the histogram? Explain why you chose to do it this way?

   ____

   ____

5. Based on your information, what colors do you think the next 50 cars will be?

   ____

   ____

6. Based on the JD Power and Associations data for the year 2008 fourth-quarter sales, 24% of the cars bought were silver. Do your results match this result? Make a conjecture as to why the data did or did not match the data from JD Power and Associates.

____

____

____

Extension

In groups, use the internet to find what types of cars are the most popular. Examine data based on age, gender, and geographic location.

1. Make a histogram representing all the data you gathered, focus on car type, rather than a specific model.

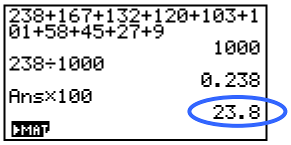
2. Select one subgroup and explain why that particular group might prefer a certain type of vehicle.

Solutions

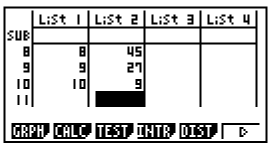
Answers on this activity will vary depending on data. The answers below are from the table at the beginning of the activity.

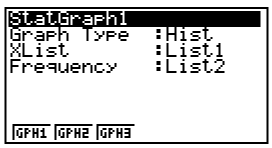
1. Silver was the most popular car color.

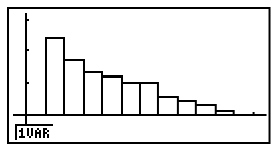
2. 23.8%



3.







4. I gave each color a numerical value, starting with number 1 for silver, 2 for black. I put the colors in List 1 and the number of cars in list 2.

5.

Color	Silver	Black	Blue	White	Grey
# of Cars	12	8	7	6	5

Color	Red	Green	Beige	Gold	Brown
# of Cars	5	3	3	1	0

6. My data had silver cars at 23.8%, which is almost 24%. The data contained a large number of cars in it, so it is close to the percent from JD Power and Associates.

Extension Solutions

1. Answers will vary depending on data collected.

2. Answers will vary.

Topic Area: Matrices

NCTM Standard:
- Develop fluency in operations with real numbers, vectors, and matrices, using mental computation or paper-and-pencil calculations for simple cases and technology for more-complicated cases.
- Develop an understanding of properties of, and representations for, the addition and multiplication of vectors and matrices.

Objective

Given several data tables, the student will be able to create a matrix to represent the data, perform operations using the matrices, and apply the results to problem solving tasks.

Getting Started

Have the students work in pairs or in small groups to determine what information would be needed to calculate the cost of manufacturing an item for retail sales and what would influence these expenses.

Prior to using this activity:
- Students should have a basic understanding of the properties of matrices.
- Students should be able to determine if two matrices can be multiplied and perform the operation.
- Students should be able to understand the meaning of the resulting matrix.
- Students should be able to understand and calculate percent in relationship to profit and loss.

Ways students can provide evidence of learning:
- The student will be able to create an appropriate matrix to represent the data, given a table of data.
- The student will be able to multiply the matrices and analyze the results, given two matrices.

Common calculator or content errors students might make:
- Students may create a matrix that does not accurately represent the data.
- Students may multiply matrices in the wrong order, resulting in a single number rather than a list of numbers.

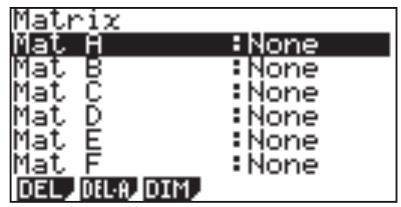
Definitions:
- Matrix
- Cell
- Percent

The following will demonstrate how to create a matrix and enter the values of the cells into the Casio *fx-9750GII*, to recall a matrix, and to perform operations with matrices.

Orchard	Apples	Pears	Peaches
Farm 1	125	110	135
Farm 2	205	95	185
Farm 3	158	82	170

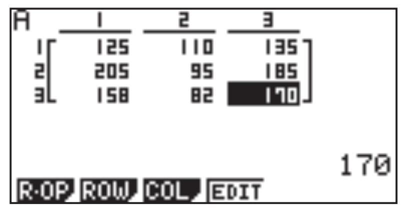
Price/Box
$29
$22
$27

To create a matrix for the table above:

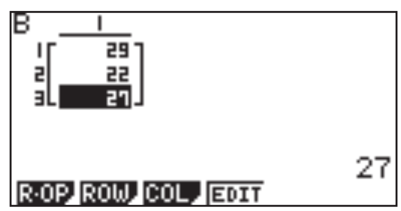


1. From the Main Menu, highlight the RUN•MAT icon and press [EXE] or press [1].
   Press [F1] to access the matrix editor.

2. To clear any data from previous matrices, press [F2] (DEL-A), then [F1] (Yes).

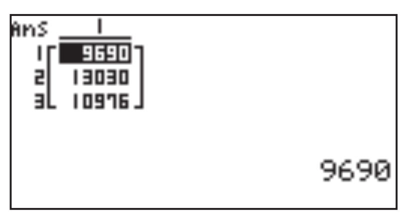


3. To create a 3 x 3 matrix, press:
   [▶] [3] [EXE] [3] [EXE] [EXE].
   Enter the values for each cell and press [EXE].
   The screen should look like the one to the right.



4. To enter the second matrix, press:
   [EXIT] [▼] [▶] [3] [EXE] [EXE] for a 3 x 1 matrix.
   Enter the values using the method described in step 3. The screen should like the one to the right.

To multiply the two matrices:



1. Press [EXIT] twice to return to the RUN-MAT screen.

2. Press [OPTN] [F2] [F1] [ALPHA] [X,θ,T] [×] [F1] [ALPHA] [log] [EXE] to multiply the two matrices. The results are shown on the screen to the right.

What Happened to the Cost? Activity

Ever wonder why it cost so much to buy your favorite sport jersey? The amount of material it would take to make the shirt does not seem to match the price. Some talented people can make a great jersey and save themselves money while others are willing to spend the money and save the time for other things. In this activity, you will explore the costs involved in manufacturing an athletic jersey and discover why it costs so much more to buy one than make it yourself.

In this activity, you will use the given data tables and matrices to calculate the cost of materials, production, labor, and advertising and apply these costs to the final price of a football, basketball, and baseball jersey. The company is manufacturing 2000 football jerseys, 1500 basketball jerseys, and 1200 baseball jerseys. The cost for the materials needed for each jersey is $5.99, $3.99, and $4.99, respectively. Using your calculations, you will then determine the selling price for each shirt in order to make a profit.

Production Costs			
Sport	**Cutting**	**Assembly**	**Packaging**
Football	$0.25/hr.	$1.75/hr.	$0.08/hr.
Basketball	0.10/hr.	0.50/hr.	0.05/hr.
Baseball	0.15/hr.	0.75/hr.	0.05/hr.

Labor Costs			
Sport	**Cutting**	**Assembly**	**Packaging**
Football	$1.88/hr.	$10.31/hr.	$0.58/hr.
Basketball	0.75/hr.	4.13/hr.	0.36/hr.
Baseball	1.13/hr.	6.19/hr.	0.36/hr.

Advertising Costs			
Sport	**Photography**	**Copy**	**Model**
Football	$2.25/hr.	$0.38/hr.	$22.50/hr.
Basketball	3.00/hr.	0.50/hr.	24.00/hr.
Baseball	3.75/hr.	0.63/hr.	20.83/hr.

Questions

1. Create a matrix for the production costs, the labor costs, the advertising costs, and the number of each type of jersey to be manufactured. Multiply each of the matrices for the various costs by the number of jerseys. Fill in the resulting matrices. What is the meaning for each of the values in the matrices?

Production Costs **Labor Costs** **Advertising**

$$\begin{bmatrix} \\ \\ \\ \end{bmatrix} \quad \begin{bmatrix} \\ \\ \\ \end{bmatrix} \quad \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

2. Calculate the total cost for material for each jersey. Explain why this cannot be done using matrices.

3. Calculate the total cost for manufacturing a football jersey. What are some reasons why production and labor costs are higher for this jersey than for the others?

4. Find the cost of manufacturing each baseball jersey; is it more or less than the football jersey? Does this seem like a reasonable cost? What would account for the difference?

5. Calculate the total cost for manufacturing a basketball jersey. What are some of the reasons why advertising costs are higher for this jersey than for the others?

____

____

6. If each jersey sells for $55.00, create a matrix that can be used to find the product of the cost per jersey and the number of jerseys to be sold. Record your answer in the space provided. Multiply this matrix by the matrix for the total number being manufactured. What is the projected total from the sales of the jerseys? What will be the percentage of profit from the sales?

[ ]

____

____

Extensions

1. Explain how the sizes of two matrices determine whether they can be multiplied. In order to multiply a 2 x 3 matrix and a 4 x 2 matrix, which one should be first? Give the dimensions of the resulting matrix.

____

____

2. The school store is selling shirts with short sleeves, shirts with long sleeves, and shirts with hoods. Short sleeve shirts sell for $15.00 each, long sleeve shirts sell for $17.50 each, and hooded shirts sell for $20.00 each. Below are the matrices that were set up to calculate the total sales.

Number of Shirts
[200 155 125]

Cost per Shirt
[15.00 17.50 20.00]

a) Explain why an error message was given when these matrices were multiplied using a calculator.

b) Explain how to work the problem in the calculator, in order to obtain a correct answer for total sales overall.

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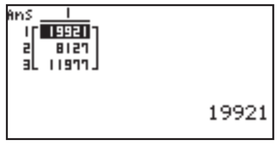
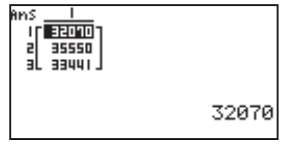
Solutions

1. **Production Cost** **Labor Cost** **Advertising Cost**





For each matrix, row 1 is the total cost for football jerseys, row 2 is the total cost for basketball jerseys, and row 3 is the total cost for baseball jerseys.

2. **Football:** 2000 ($5.99) = **$11,980.00**
Basketball: 1500 ($3.99) = **$5,985.00**
Baseball: 1200 ($4.99) = **$5,988.00**

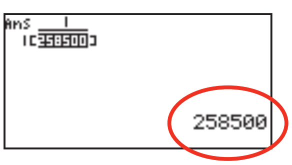
Multiplying a 3 x 1 matrix by a 1 x 3 matrix would result in a 3 x 3 matrix which would not give a correct response. Multiplying a 1 x 3 matrix by a 3 x 1 matrix would result in giving the total cost of materials for all of the jerseys.

3. -The total cost is $3,221 + $19,921 + $32,070 + $11,980 which equals **$67,192**.
-Answers may vary. Some reasons would include the cost of materials are higher, the amount of time for construction is higher, and the cost of advertising due to endorsements by athletes who are paid higher salaries.

4. -The total cost is $1,485 + 11,977 + 33,441 + 5,988 which equals **$52,891**.
-This cost is less than a football jersey.
-Answers will vary according to experience.
-Some of the difference is the number of jerseys being made and the extra materials.

5. -The total cost is $1,485 + 8,127 + 35,550 + 5,985 which equals **$51,147**.
-One answer may be that although endorsements may not cost as much due to popularity, the cost per jersey is more since there are less being manufactured.

6. **[** 55 55 55 **]**



Percentage of Profit:
[258,500 – (67,192 + 52,891 + 51,147)] /258,500 = .3376 or **33.8%**

Extension Solutions

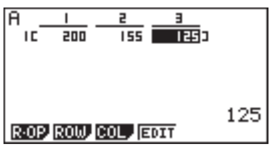
1. The dimensions of the matrices must be such that the number of columns in the first matrix is the same as the number of rows in the second matrix.

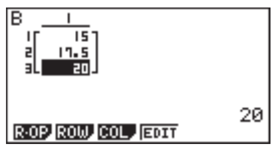
   The 4 x 2 matrix would be entered first followed by the 2 x 3 matrix. The resulting dimensions would be a 4 x 3 matrix.

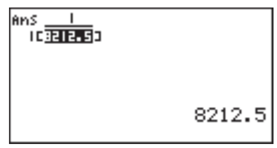
2. a) **Matrix A** = **[** 200 155 125 **]** **Matrix B** = **[** 15.00 17.50 20.00 **]**

   Matrix AB = **[** 200(15) + 155(?) 200(17.50) 200(20.00) **]**

   b) Since the first cell of AB must contain the product of $cell_{1,1}$ + $cell_{2,2}$ and there is not a second row, these matrices will not work. The second matrix would need to be rewritten as a 3 x 1 matrix in order to calculate the total.







What's for Dinner? Teacher Notes

Topic Area: Three Variable Equations

NCTM Standards:
- Use symbolic algebra to represent and explain mathematical relationships.
- Develop fluency in operations with real numbers, vectors, and matrices, using mental computation or paper-and-pencil calculations for simple cases and technology for more-complicated cases.

Objective

Given a set of equations in three variables, the student will be able to solve the equations for the unknown values using inverse matrices, then use the information to solve other problems.

Getting Started

As a class, discuss how someone in the catering business would determine the price of an item they are selling. What factors would the seller take into account when selecting the items? Could these factors change during the year? How does location affect their decision?

Prior to using this activity:
- Students should have an understanding of how to set up a matrix.
- Students should be able to solve a system of equations with two unknown values.
- Students should understand what is meant by an inverse matrix.

Ways students can provide evidence of learning:
- The student will be able to create a matrix for a set of equations with three unknowns.
- The student will be able to demonstrate how to use a calculator to solve a system of equations using inverse matrices.

Common calculator or content errors students might make:
- Students may set up the matrices using the wrong values.
- Students may interchange the order of the matrices when entering them into the calculator.

Definitions:

- System of Equations
- Matrix
- Inverse Matrix

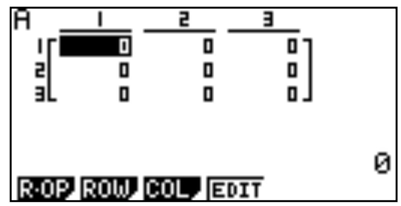
The following will demonstrate how to enter a three variable system of equations into the Casio *fx-9750GII* as two separate matrices, and then find the value of the variables using an inverse matrix.

Solve the given system for x, y, and z.

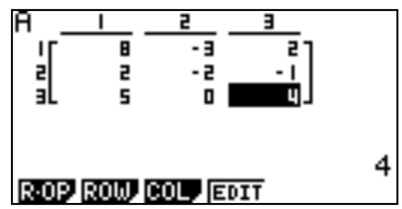
$$\begin{cases} 8x - 3y + 2z = 5 \\ 2x - 2y - z = 6 \\ 5x + 4z = 10 \end{cases}$$

To set up the matrices:

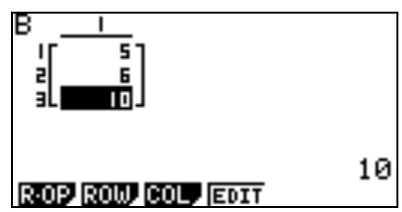
1. Highlight the RUN•MAT icon and press [EXE].
Press [F1] (Mat) to enter the matrix editor.
To set up the matrix for the variable terms, press [▶] [3] [EXE] [3] [EXE] [EXE]. The screen should look like the one at the right.



2. Enter the coefficient values for each variable into its appropriate cell followed by [EXE]. The screen at the right shows the results.

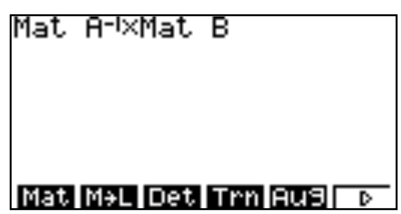


3. Press [EXIT] to set up the matrix for the constant terms. To set up a 3x1 matrix, press: [▼] [▶] [3] [EXE] [1] [EXE] [EXE]. Enter the constant values into its appropriate cell followed by [EXE].
Press [EXIT] twice to return to the RUN•MAT Menu.

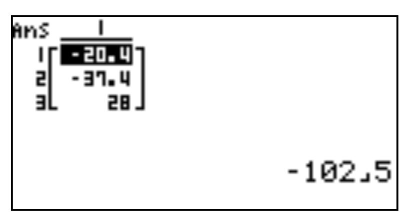


To find the unknown values:

1. To enter the inverse of **Matrix A**, press:
[OPTN] [F2] [F1] [ALPHA] [X,θ,T] [SHIFT] [)].



2. To see the values of the variables press:
[÷] [F1] [ALPHA] [log] [EXE].



What's for Dinner? Activity

The Silver Spoon Catering Company offers several different party trays. Four of the categories are listed below:

Meat	Seafood	Cheese	Veggies
Roast Beef	Shrimp	American	Peppers
Turkey	Crab	Cheddar	Carrots
Ham	Lobster	Swiss	Celery

In determining the possible pounds of each item used for the trays, the owners created three possible combinations for each type of tray based on past experience.

Questions

1. The following are three possible meat trays and their price according to cost per pound. Find the cost per pound for each of the different meat choices.

Meat Trays	Roast Beef	Turkey	Ham	Price
Large Meat Tray	5 lb.	4.5 lb.	4 lb.	$51.10
Small Meat Tray	3 lb.	2.5 lb.	3 lb.	$32.20
Small Two Meat Combo	4.5 lb.	4.5 lb.	-	$34.65

Roast Beef: ______________________________

Turkey: ______________________________

Ham: ______________________________

2. The possibilities for the seafood trays are as follows:

Seafood Trays	Shrimp	Crab	Lobster	Price
Shrimp and Crab Combo	3 lb.	4.5 lb.	-	$45.15
Seafood Medley	3 lb.	2.5 lb.	2.5 lb.	$47.75
Shrimp and Lobster Combo	4.5 lb.	3 lb.	-	$38.15

Find the cost per pound for each of the different seafood choices.

Shrimp: ______________________________

Crab: ______________________________

Lobster: ______________________________

3. For appetizers, the Silver Spoon Catering Company offers a variety of items including a cheese tray and veggie tray. Find the cost per pound for each of these items.

Cheese Trays	American	Cheddar	Swiss	Price
Large Cheese Tray	3 lb.	2.5 lb.	2 lb.	$21.85
All American	2.5 lb.	2.5 lb.	-	$14.00
Small Cheese Tray	1.5 lb.	1.5 lb.	1 lb.	$11.65

Veggie Trays	Peppers	Carrots	Celery	Price
Large Dipping Tray	2.5 lb.	3 lb.	4 lb.	$16.05
Classic Dipping Tray	-	2.5 lb.	2.5 lb.	$6.75
Small Dipping Tray	.5 lb.	2 lb.	2 lb.	$6.75

American: ______________________________

Cheddar: ______________________________

Swiss: ______________________________

Peppers: ______________________________

Carrots: ______________________________

Celery: ______________________________

4. A client is requesting a special combination of meat and cheese trays for a reception. Use the price per item found in problems 1 and 3 to determine the price of each tray.

Meat Tray: 4 lb. Roast Beef 2.5 lb. Turkey 2.5 lb. Ham

Cost: ______________________________

Cheese Tray: 2 lb. American 2 lb. Cheddar 1.5 lb. Swiss

Cost: ______________________________

5. A hostess has ordered 3 Large Meat Trays for an upcoming event. It is recommended that there be at least 0.5 pounds of meat for each guest. If the party will have 50 guests, will this be enough food for the party? Why or Why not?

____

____

6. If it is also recommended that a hostess plan for 0.25 pounds of cheese for each guest, how many Large Meat Trays and Large Cheese Trays are needed for an event with 150 guests?

   Meat Trays: ______________________ Cheese Trays: ________________________

7. If there is a 9% sales tax, how much will the total cost be for the trays in question 6?

   ____

Extensions

1. A client is planning a special party and needs two Large Meat Trays with the following amounts: 5 lb. roast beef, 3 lb. turkey, and 4 lb. of ham. Using the same prices, find the cost of each tray and the total bill if there is a 8% sales tax.

   Cost per Tray: ____

   Total Cost: ____

2. The same client needs two Seafood Medley Trays with the following amounts: 4 lb. shrimp, 3 lb. crab, and 4 lb. lobster. How much would each of these trays cost and what would be the total cost?

   Cost per Tray: ____

   Total Cost: ____

3. Mrs. Smith is in charge of planning a reception for a visiting speaker. She has a budget of $500 for food, not including the condiments. There are 75 guests expected to attend. Decide what type and how many of each tray she should order and calculate the total cost, including 6% sales tax. She may not go over budget.

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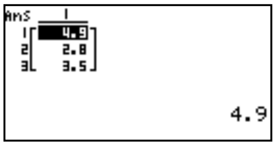
   ____

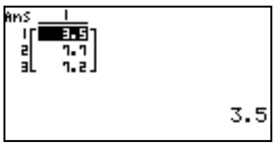
   ____

Solutions

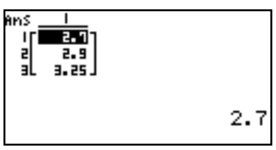
1. Roast Beef: $4.90
   Turkey: $2.80
   Ham: $3.50

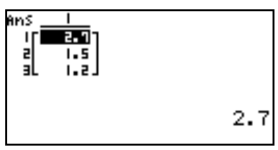


2. Shrimp: $3.50
   Crab: $7.70
   Lobster: $7.20

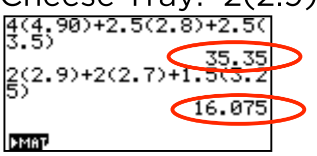


3. American: $2.90
   Cheddar: $2.70
   Swiss: $3.25
   Peppers: $2.70
   Carrots: $1.50
   Celery: $1.20

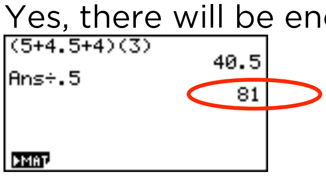




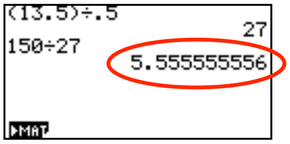
4. Meat Tray: 4(4.90) + 2.5(2.8) + 2.5(3.5) = $35.35
   Cheese Tray: 2(2.9) + 2(2.70) + 1.5(3.25) = $16.08



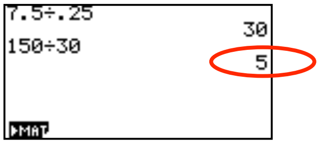
5. (13.5 lb.)(3)/.5 = 81 servings;
   Yes, there will be enough food for 31 more people.



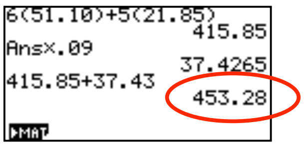
6. Meat Trays: 6



Cheese Trays: 5

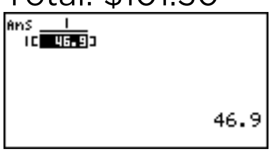


7. \$415.85 + 37.43 = \$453.28

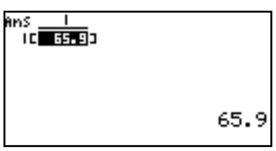


Extension Solutions

1. Cost: \$46.90
Total: \$101.30



2. Cost: \$65.90
Total: \$142.34



3. Answers will vary according to choices.

Where Did That Come From? Teacher Notes

Topic Area: Circles

NCTM Standards:
- Use symbolic algebra to represent and explain mathematical relationships.
- Use geometric models to gain insights into, and answer questions in other areas of mathematics.
- Recognize and apply mathematics in contexts outside of mathematics.

Objective

The student will be able to write the equation of a circle given the center and radius, find the intersection of two circles using a graphing calculator, and apply finding the equation and intersections of circles to problem solving.

Getting Started

Discuss with students how sound and motion waves travel. Give examples such as dropping a stone in a pond and discuss how this is related to earthquakes. Discuss with them how triangulation is used to locate a particular site. Have the students create a list of places where this would be useful.

Prior to using this activity:
- Students should have an understanding of how to write an equation for a circle.
- Students should have an understanding of how to find the intersection of two functions.

Ways students can provide evidence of learning:
- The student will be able to graph and locate the intersections of two or more circles.
- The student will be able to discuss the difference between using circles and linear functions when using triangulation.

Common calculator or content errors students might make:
- Students may use the diameter instead of the radius in the equation of the circle.
- Students may make sign errors on the center of a circle when writing the equation.

Definitions

Radius Epicenter

Formulas

Distance Formula: $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Equation of a Circle: $(x - h)^2 + (y - k)^2 = r^2$

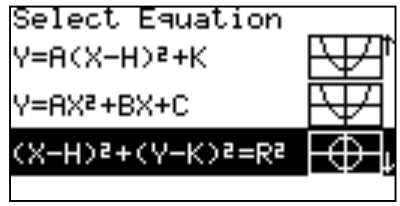
The following will demonstrate how to graph a circle using the Conics Function, save it as the background for another graph, and use the Trace function to find the intersection for the two graphs using the Casio *fx-9750GII*.

Circle 1: Center (2, 1) Radius = 3
Circle 2: Center (-1, -4) Radius = 5

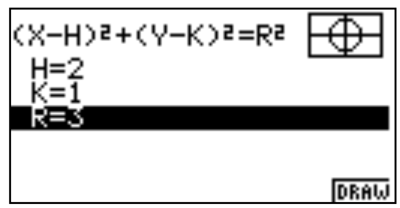
Draw the two circles and find the points of intersection.

To enter values for a circle into the Conics Function:

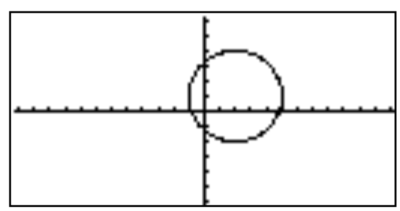
1. Highlight the CONICS icon in the Main Menu and press [EXE]. Use [▼] to scroll to the formula for a circle given the center and radius, press [EXE].



2. Enter the values of **H, K,** and **R** for **Circle 1**, pressing [EXE] after each entry. The screen should look like the one to the right. Press [F6] (Draw) to view the graph. Note: The graph may appear elliptical due to the viewing window values.



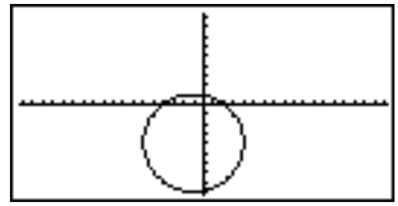
3. To change the view screen so that the circle appears round, press [F3] **(V-Window)** and enter the following values:
Xmin = -18.3; **Xmax** = 18.3
Ymin = -9.3; **Ymax** = 9.3
Press [EXIT] [F6] (Draw). The screen should look like the one to the right.



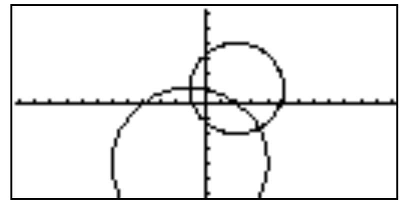
To save a graph as a background for another graph:

1. Press [OPTN] [F1] (PICT), then [F1] (Sto) [1] [EXE].
The graph can now be recalled to appear as the background of another graph.

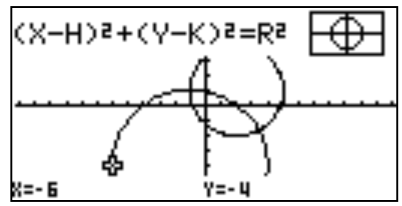
2. Press EXIT ; follow the steps above to enter the information for **Circle 2**.



3. To recall the background, press OPTN F1 (Pic), Then F2 (Rcl) 1 EXE. The screen at the right shows **Circle 2** with **Circle 1** as a background.



To trace a graph using the Trace function:

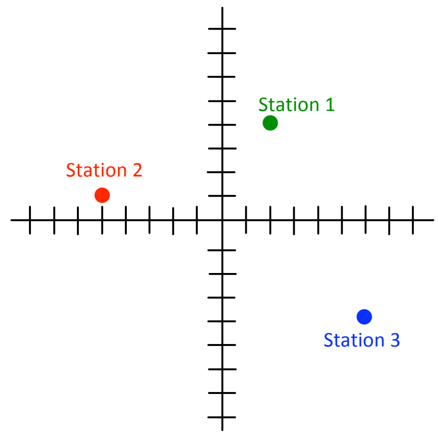


1. Press F1 **(Trace)** and use ▶ to move the cursor along the graph. Note: For tracing circles, the cursor will only move to the right.

Where Did That Come From? Activity

When an earthquake occurs, it causes waves of energy in the form of movement. These waves are picked up by devices known as seismographs. In areas where earthquakes happen frequently, scientists set up a seismic network comprised of different stations that record these waves. In order to determine the origin, or epicenter of an earthquake, a system called triangulation is used. This method uses the intersection of three or more graphs is used to locate the position of the epicenter on a map.

In this activity, you are going to calculate the distance from the epicenter of an earthquake to three different stations to determine the location of the epicenter. The grid at the right shows the location of the stations; each tick mark equals 10 km.



Questions

1. What are the coordinates for each of the stations in the grid to the right?

   Station 1: ______________________________

   Station 2: ______________________________

   Station 3: ______________________________

2. Station 1 registers an earthquake and determines that its center is 60 km from the station. Write an equation to show the possible locations of this earthquake?

   ____

3. At the same time, Station 2 registers the earthquake and determines that its epicenter is halfway between Station 1 and Station 2. How far is the epicenter from Station 2?

   ____

   Write an equation for possible locations for the earthquake from Station 2.

   ____

4. Graph the equations for questions 2 and 3. What are the intersections between Station 1 and Station 2?

   ____

5. Station 3 also registers the earthquake. The seismologist determines that from the time it started until it reached the station was 25 sec. If it is known that the waves travel at 3 km/sec in this area, how far was the epicenter from Station 3?

____

6. Write an equation representing the distance of the earthquake from Station 3.

____

7. Graph the equation in question 6 along with the other two equations. What are the intersections between Station 1 and Station 3?

____

8. Determine the possible location for the epicenter of the earthquake. (Hint: Draw lines through the intersections from questions 4 and 7.)

____

Extensions

1. Station 4 is located at (-80, -70) and registers an earthquake with an epicenter estimated to be 40 miles away. Station 1 registers the same earthquake with an epicenter that is 68 miles away after 5 min. What is the speed of the waves from the earthquake?

____

2. A new station is being established whose x-coordinate is halfway between Station 1 and Station 3 and whose y-coordinate is 50 miles north of Station 1. What will be the coordinates for the new station?

____

3. The epicenter of a minor earthquake is located at (-40, 35). If the range for registering earthquakes for each station is 80 miles, which stations will be able to register this earthquake? Justify your answer.

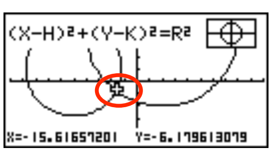
Solutions

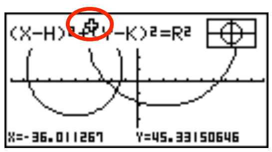
1. Station 1: (20, 40)
   Station 2: (–50, 10)
   Station 3: (60, –40)

2. Center is (20, 40)
   Radius is 60
   $(x-20)^2+(y-40)^2=3600$

3. $D=\sqrt{(-50-20)^2+(10-40)^2}\approx 76$ km
   $76 \div 2 = 38$ km
   $(x+50)^2+(y-10)^2=1444$

4. (15.6, –6.2) and (–36.0, 45.3)

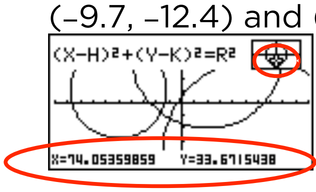


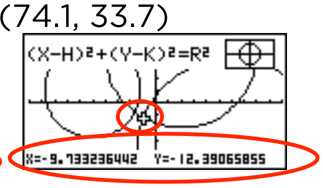


5. 75 km

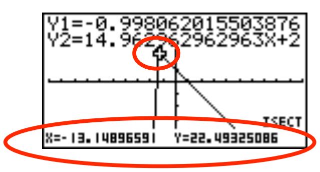
6. $(x-60)^2+(y+40)^2=5625$

7. (–9.7, –12.4) and (74.1, 33.7)





8. (–13.1, 22.5)



Extension Solutions

1. (68 – 40) mi. / 5 min. = 5.6 mi./min.

2. (40, 90)

3. Stations 1, 4, and 5.
   Answers will vary.

World Cocoa Market Teacher Notes

Topic: Two-Variable Statistics

NCTM Standards:

- Understand the meaning of measurement data and categorical data, of univariate and bivariate data, and of the term variable.
- Understand histograms, parallel box plots, and scatterplots and use them to display data.
- Compute basic statistics and understand the distinction between a statistic and a parameter.
- Find, use, and interpret measures of center and spread, including mean and interquartile range.
- For univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics.

Objective

The student will be able to calculate summary statistics for two sets of data simultaneously, make conjectures based on these statistics, and display the result in parallel box plots.

Getting Started

Have the students look online for the daily futures for Cocoa. Have the students decide how best to analyze the data.

Prior to using this activity:

- The student should be able to calculate Mean, Standard Deviation of a sample and a population.
- The students should be able to create a box and whisker plot and compare parallel box and whisker plots.

Ways students can provide evidence of learning:

- Given data, the student should be able to create a box and whisker plot.
- Given data, the student should be able compare a sample with an entire population.

Common mistakes to be on the lookout for:

- The student might have difficulty with analyzing their data, if they have outliers.

Definitions:

- Univariate
- Mean
- Conjecture
- Mode
- Parallel Box and Whisker plot
- Standard deviation of a sample
- Standard deviation of a population
- Median
- Ratio
- Bivariate

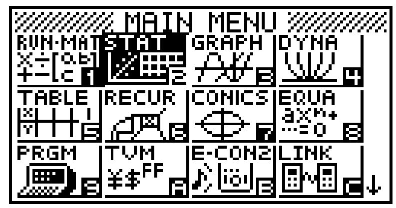
World Cocoa Market "How-To"

The following will demonstrate how to enter sets of data into the Casio *fx-9750GII*, graph the data using a box and whisker plot, and calculate important data from the graph.

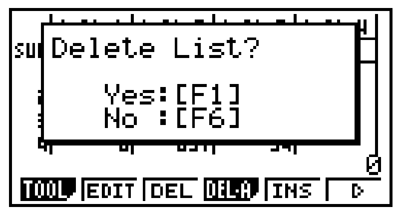
Scores on the First Math Test									
55	60	75	80	90	65	75	60	50	80

Scores on the Second Math Test									
75	90	85	60	95	85	80	70	95	100

To enter the data from the table in the problem:

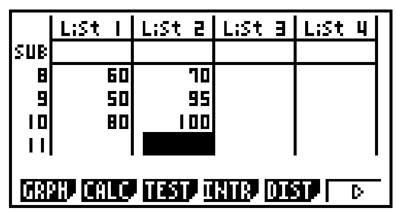


1. From the Main Menu, highlight the STAT icon and press [EXE] or press [2].



2. To clear previous data lists press [F6] (▷) [F4] (DEL-A) [F1] (Yes).

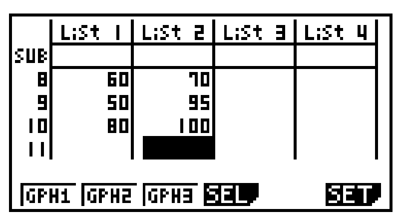
3. Enter the data by typing each number, pressing [EXE] after each entry.



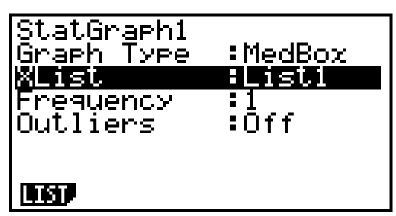
4. The display should look like the screen shot on the right when completed.

To select the type of graph for this data:

1. Press [F1] (GRPH).



2. Press [F1] (GPH1), then [F6] (SET) to set the type of graph for **StatGraph1**.

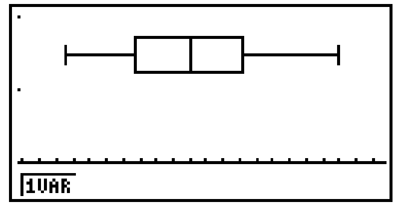


3. Press [▼] to highlight the **Graph Type**.

4. Press [F6] (▷), then [F2] (Box).

5. Make sure the correct lists are chosen, then press [EXIT].

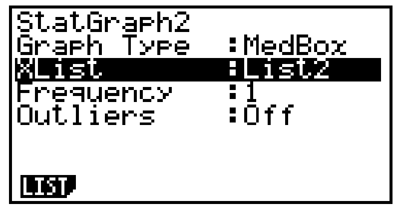
6. Press [F1] (GPH1) to display the graph.



To graph parallel box and whisker plots:

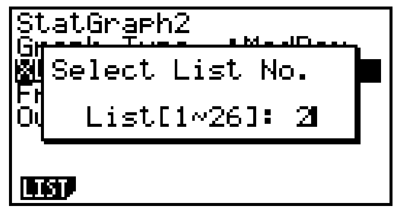
1. Press [EXIT] and press [F6] (SET) to set the type of graph for **StatGraph2**.

2. Press [F2] (GPH2), then ▼ to highlight the **Graph Type**.

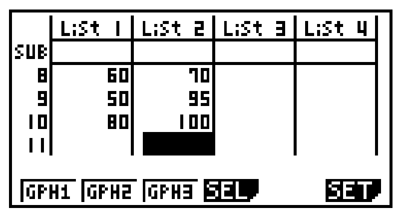


3. Press [F6] (▷), then [F2] (Box).

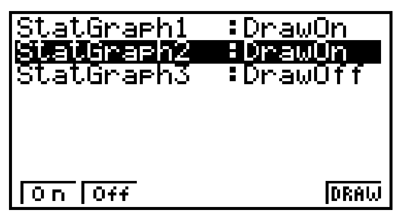
4. Make sure the correct lists are chosen. If not, press ▼ to highlight **XList:**, then [F1] (LIST) and the number of the list and [EXE].



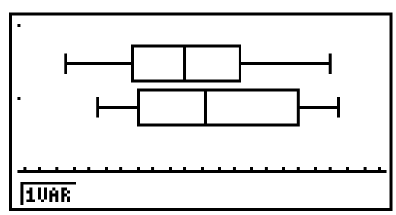
5. Press [EXIT].



6. Press [F4] (SEL) to select multiple graphs to be displayed.



7. Make sure to turn on **StatGraph1** and **StatGraph2**. Use ▲ ▼ to move between the graph choices. Press [F1] (On) for both **StatGraph1** and **StatGraph2**. Now, press [F6] (DRAW) to draw the parallel graphs.



World Cocoa Market Activity

The latest statistics on the cocoa market can be found at the following website: www.dailyfutures.com/softs. On this website, the net production and grindings statistics for each of the years is listed in a chart. For this activity, we will use the following data:

World Cocoa Market Statistics (in million metric tons)

Year	1996	1997	1998	1999	2000	2001	2002	2003	2004
Net Production	2.91	2.67	2.67	2.77	3.03	2.83	2.86	3.14	3.42
Grindings	2.66	2.75	2.75	2.78	2.96	3.04	2.88	3.05	3.18

Questions

1. Find the average net production and grindings.

   Average Net Production _______________

   Average Grindings _______________

2. What is the ratio between those two numbers?

   ____

3. What does the ratio represent?

   ____

   ____

4. Find the median for net productions and net grindings.

   Net Production _______________

   Net Grindings _______________

5. Find the standard deviation of the sample for net productions and net grindings.

   Net Production _______________

   Net Grindings _______________

6. Find the standard deviation of the population for net productions and net grindings.

   Net Production ______________

   Net Grindings ______________

7. What do the mean, standard deviation of the sample and standard deviation of the population represent for net production and grindings?

   ____

   ____

   ____

8. Create parallel box and whisker plots for the net production and grindings. Draw and label the plots in the space below.

9. Compare and contrast the two plots.

   ____

   ____

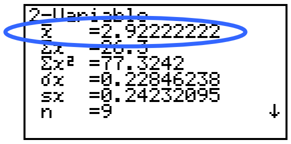
   ____

Extension

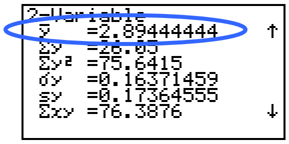
Look online and find more information about the future of a product that you are interested in. Also, talk to you Economics teacher about how futures play a role in the economy.

Solutions

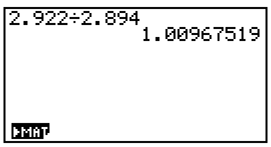
1. Average Net Production is $\overline{x} = 2.922$



Average Net Grindings is $\overline{y} = 2.894$

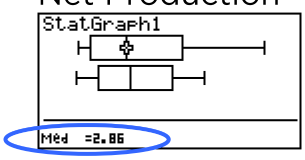


2. $\dfrac{2.922}{2.894} \approx 1.0097$

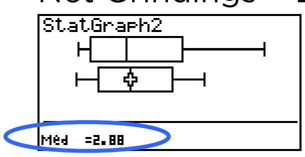


3. The average grindings are made from 1 ton of Cocoa beans.

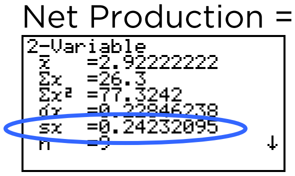
4. Net Production = 2.86



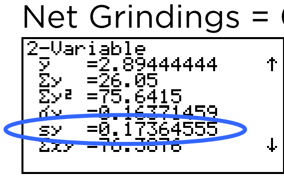
Net Grindings = 2.88



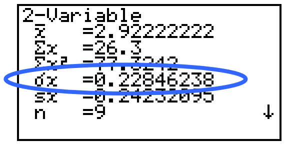
5. Net Production = 0.2423



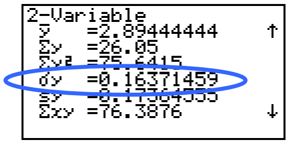
Net Grindings = 0.1736



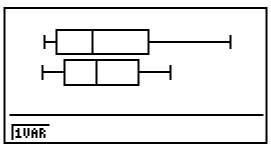
6. Net Production = 0.2285



Net Grindings = 0.1637



7. The mean is the average of the amount produced or grinded. The standard deviation is the average distance between any given number from your sample and the mean. A low standard deviation means that most of the points fall close to the mean.

8. 

9. The net production values are more evenly distributed than the grindings. The grindings have a right tail.