PROJECT WHIRLWIND

SUMMARY REPORT NO. 35
THIRD QUARTER 1953

DIGITAL COMPUTER LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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DIGITAL COMPUTER LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Cambridge 39, Massachusetts

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FOREWORD

Project Whirlwind

Project Whirlwind at the Massachusetts Institute of Technology Digital Computer Laboratory is sponsored by the Office of Naval Research under Contract N5ori60. The objectives of the Project are the development of an electronic digital computer of large capacity and very high speed, and its application to problems in mathematics, science, engineering, simulation, and control.

The Whirlwind Computers

The Whirlwind computer is of the high-speed electronic digital type, in which quantities are represented as discrete numbers, and complex problems are solved by the repeated use of fundamental arithmetic and logical (i.e., control or selection) operations. Computations are executed by fractional-microsecond pulses in electronic circuits, of which the principal ones are (1) the flip-flop, a circuit containing two vacuum tubes so connected that one tube or the other is conducting, but not both; (2) the gate or coincidence circuit; (3) the electrostatic storage tube, which uses an electron beam for storing digits as positive or negative charges on a storage surface.

Whirlwind I (WWI) may be regarded as a prototype from which other computers will be evolved. It is being used both for a study of circuit techniques and for the study of digital computer applications and problems.

Whirlwind I uses numbers of 16 binary digits (equivalent to about 5 decimal digits). This length was selected to limit the machine to a practical size, but it permits the computation of many simulation problems. Calculations requiring greater number length are handled by the use of multiple-length numbers. Rapid-access electrostatic storage initially had a capacity of 4096 binary digits, sufficient for some actual problems and for preliminary investigations in most fields of interest. This capacity is being gradually increased toward the design figure of 32,768 digits. Present speed of the computer is 20,000 single-address operations per second, equivalent to about 6000 multiplications per second. This speed is higher than general scientific computation demands at the present state of the art, but is needed for control and simulation studies.

Reports

Quarterly reports are issued to maintain a supply of up-to-date information of the status of the Project. Detailed information on technical aspects of the Whirlwind program may be found in the R-, E-, and M-series reports and memorandums that are issued to cover the work as it progresses. Of these, the R-series are the most formal, the M-series the least. A list of the publications issued during the period covered by this Summary, together with instructions for obtaining copies of them, appears in the Appendix.

1. QUARTERLY REVIEW AND ABSTRACT

During the third quarter of 1953, the Scientific and Engineering Computation (S&EC) Group contributed much of their time to the preparation and teaching of an MIT summer session course, Digital Computers and Their Applications. Programs were written for an idealized computer, simulated on Whirlwind I. The course was taken by 106 students, representing 18 commercial and business groups, 39 industrial research groups, and 10 military and government establishments. While not working on the summer session computer, the group ran 28 diversified problems on Whirlwind. Twenty-five of these problems originated at research projects and departments within MIT.

On August 8, one bank of WWI electrostatic storage was replaced by a bank of magnetic-core storage. The replacement bank had been operating successfully as part of the Memory Test Computer. The second bank of ES was replaced by a second bank of magnetic-core storage on September 5. The access time of the new storage is 9 microseconds, compared with 25 microseconds for ES. Reliability is also greater, reducing maintenance time. Detailed information on the performance of the new system will be reported in the next quarterly.

The auxiliary magnetic drum has operated reliably; a master program for drum storage of all test programs used with marginal checking is being written. New Ferranti photoelectric tape readers are being tested and will soon replace the old ERA tape units. The Ferranti equipment does not use light rods and therefore will not require opaque tape. Standard teletype tape will be satisfactory.

The change in internal storage caused a major revision in the tube complement of Whirlwind I; corrected parts lists are unavailable at this time. Research on the problem of cathode stability continued at an intensive level during the quarter.

The annual meeting of the Association for Computing Machinery was held at MIT on September 9, 10, 11. The Digital Computer Laboratory cooperated with the Office of Statistical Services and with the Industrial Liaison Office, Dynamic Analysis and Control Laboratory, and the Departments of Physics, Mathematics and Electrical Engineering in the actual conduct of the meeting.

2. MATHEMATICS, CODING, AND APPLICATIONS

2.1 Introduction

Progress on problems being solved by or in cooperation with members of the Scientific and Engineering Computation (S&EC) Group was severely retarded during this quarter by the effort required to prepare and teach the MIT Summer Session Course, Digital Computers and Their Applications, offered at MIT from August 24 through September 4. During this period and a few of the weeks preceding it, most of the WWI time allotted to the S&EC Group was devoted to the development of the programs needed for the course. The object of these programs was to simulate an idealized computer, called the Summer Session Computer, on Whirlwind I. A detailed discussion of this Summer Session Computer is given in Section 5.4.

In September, the Scientific and Engineering Computation Group released a preliminary version of a 30-minute motion picture, in color, which depicts the various activities within the Digital Computer Laboratory with emphasis on the "evolution of the solution of a scientific problem" on the computer. This film, entitled "Making Electrons Count," is the product of the combined efforts of the Scientific and Engineering Computation Group members and the Photographic Laboratory. It is planned that an improved version of the film, incorporating many of the suggestions made by laboratory personnel who viewed the preliminary film, will be completed within the next two months.

Section 2.3 contains reports, arranged in numerical order, describing the progress on some 28 problems that have made use of computer time allotted to the S&EC Group during this last quarter. Tables 2-1 and 2-2 have been set up to provide the reader with a convenient reference to various interesting aspects of these problems. Table 2-1 lists the problems according to their fields of application. Of the 28 problems listed, 25 originated within MIT; the others were suggested by the Educational Testing Service (112), by the Retina Foundation (114), and by Dr. J. Sternberg of the Harvard Chemistry Department (139).

Of the MIT-originated problems the 5 marked with asterisks represent work being performed by members of the MIT Project on Machine Methods of Computation, another phase of the work under contract N5ori60, which is reported more extensively in a separate report ("Machine Methods of Computation and Numerical Analysis," Quarterly Progress Report No. 8, Project DIC 6915, MIT, June 15, 1953).

Field	Description	Problem Number	Source
Chemistry	Optical properties of thin metal films	101	MIT
	Calculation of nuclear magnetic resonance absorption lines	139	Harvard
Civil Engineering	Analysis of reinforced concrete walls	*113	MIT
	Study of shock waves; vibration problems in solid bodies	*142	MIT
Geology & Geophysics	Geophysical data analysis	106	MIT
Instrumentation Lab	System of nonlinear differential equations	109	MIT
	Fourier analysis and autocorrelation calculation	111	MIT
	Atmospheric turbulence; autocorrelation, crosscorrelation, and Fourier transform calculation	137	MIT
Mathematics	Educational testing studies; Lawley's method of factor analysis	112	Outside
	Spherical wave propagation	*119	MIT
	Convolution of a transformed probability density function	121	MIT
	Matrix equations; modified conjugate-gradient method	136	MIT
Mechanical Engineering	Turbine design (Aerothermopressor)	120	MIT
Physics	Design of optical instruments	114	Outside
	Eigenvalues of a real symmetric matrix	134	MIT
	Spheroidal wave functions	*138	MIT
	Vibrational frequency spectrum of a copper crystal	143	MIT
	Self-consistent molecular orbits	144	MIT
	Temperature diffuse scattering of X-rays from a crystal of zinc	145	MIT
	Largest eigenvalue of a real symmetric matrix	*146	MIT
	Energy bands in crystals	147	MIT
Servomechanisms Lab	Solution of the convolution integral equation	116	MIT
	Subroutines for the numerically controlled milling machine	132	MIT
	Data reduction program; polynomial fitting	126	MIT
Miscellaneous	Comprehensive system of service routines	100	MIT
	Staff training and demonstrations	131	MIT
	The summer session system	140	MIT
	Library of subroutines	141	MIT

Table 2-1. Current Problems Arranged According to Field of Application (* MIT Project on Machine Methods of Computation)

	Mathematical Problem	Procedure	Problem Number
1.	Matrix inversion and eigenvalue problems		
	Matrix equation	Iteration involving Hotelling's method for solving the eigenvalue problem	112
	System of fifteen linear equations	Relaxation procedure	*113
	Eigenvalues of a real symmetric matrix	Numerical diagonalization procedure	134
	Matrix inversion	Modified conjugate-gradient method	136
	Evaluation of elements and roots of a third order secular determinant	Standard analytical solution	143
	Largest eigenvalue of a real symmetric matrix	Iteration	*146
2.	Transcendental equation	Iteration	101
3.	Ordinary differential equations		
	Fourteen simultaneous non-linear	Fourth order Runge-Kutta	109
	Seven non-linear first order	Step-by-step Euler method	120
	Second order	Finite difference approximation	*142
	Roothaan's formulation of the Hartree-Fock differential equations	Iteration and matrix diagonalization	144
	Second order linear with variable coefficients	Gauss-Jackson forward integration formula	147
4.	Partial differential equations		
	Non-linear hyperbolic	Difference equations written along the characteristics and solved simultaneously by iteration	*119
	Tabulation of spheroidal wave functions (see also #113)	Iteration involving the solution of a difference equation	*138
5.	Integration		
	Fourier analysis and autocorrelation	Trapezoidal integration	111
	Inversion of the convolution integral	Iteration using Simpson's rule	116
	Convolution of a transformed probability density function	Trapezoidal integration	121
	Auto-, crosscorrelation and Fourier transform	Simpson's rule (utilizes Prob. 107 routines)	137
	Integral evaluation	Simpson's rule	139
6.	Function evaluation		
	Evaluation of a function involving ln and $\tan^{-1}$	Series approximation	145
7.	Statistics	Prediction by linear operators	106
8.	Data reduction program	Polynomial fitting, etc.	126

Table 2-2. Current Problems Arranged According to the Mathematics Involved
(* MIT Project on Machine Methods of Computation)

The mathematical problems and procedures represented by the various current problems are tabulated separately in Table 2-1 because different problems in the same field frequently involve different mathematical methods, while problems in different fields sometimes are solved by identical methods.

2.2 Problems Being Solved

100 COMPREHENSIVE SYSTEM OF SERVICE ROUTINES

The comprehensive system of service routines has been described in Summary Reports 32, 33, and 34. This system has been developed by the Digital Computer Laboratory to simplify the process of coding. There have been no changes in the logic of the system during the past quarter and none are at present contemplated. The system provides for conversion, by WWI, from Flexowriter-coded perforated tapes to tapes in pure binary-coded form. Among the many provisions included in the CS, the following two are referred to in the text. Consequently, brief descriptions of them are repeated here for the reader's convenience.

(1) Floating Address

A floating-address system enables a programmer to write his instructions so that they refer to the words of his program rather than to the location of those words in storage. The assignment of final storage locations is made by the computer as part of the conversion.

(2) Number Systems and Programmed Arithmetic

(m, n) numbers shall mean numbers which are of the form $z = x \cdot 2^y$ where x is an m-binary-digit number and y is an n-binary-digit number. For example, (24, 6) signifies a two-register floating-point system dealing with numbers of 24 significant binary digits (roughly 7 decimal digits) with magnitudes between 2^{63} and 2^{-64}.

Arithmetic involving these (m, n) numbers is carried out by means of (m, n) interpretive subroutines. These subroutines enable the programmer to write coded programs using (m, n) numbers as easily as, or even more easily than, he might write programs in the single-length fixed-point (15, 0) number system which is built into Whirlwind I.

In addition to the drum input program several utility programs have been recorded on Group 11 of the auxiliary magnetic drum which has been locked in the read mode. These programs are: (1) the paper tape input program using the photoelectric tape reader, (2) a read-in program to initiate a comprehensive system conversion, (3) a post mortem program which displays the contents of storage on an oscilloscope, and (4) a paper tape input program using the mechanical tape reader. The input program has been modified so that automatic selection of these programs can be accomplished by setting up a toggle-switch register prior to reading in. An expansion of the utility programs on Group 11 to include the post mortem programs now available is being planned. As a corollary to this the read-in program will be further modified so that automatic selection of a utility program may be obtained either by setting up a toggle-switch register (the present case) or by means of an appropriate special tag, punched at the beginning of the paper tape which is being read in.

101 OPTICAL PROPERTIES OF THIN METAL FILMS

The properties of thin metal films deposited on non-absorbing backings are examined by measuring the reflection R and transmission T by these films of radiation perpendicularly incident either on the film or on the backing. These properties are expressed by the index of refraction, n; the absorption coefficient, k; the conductivity, δ ; and the dielectric constant, $\mathcal{E}$. The equations relating these properties are so complicated that about 15 hours per set of measurements are required by desk calculator.

Therefore, a program was coded for WWI by A. L. Loeb of the MIT Chemistry Department. This program will take data in a form which can be supplied by anyone not acquainted with electronic computers; it produces and labels the results in a form which does not require intermediate interpretation in order to be understood by the "consumer". This program speeds the calculator up by a factor of 3000.

At first a program was coded to calculate R and T for a given set of n and k. This program is henceforth referred to as the "Main Program." If the observed reflection and transmission are labeled R_o and T_o, then for each pair of values of n and k, a pair of values $(R - R_o)$ and $(T - T_o)$ is calculated. The second stage in the development consisted of coding a program for determining a series of successive approximations, decreasing $(R - R_o)$ and $(T - T_o)$ with each new set of n and k.

The method of successive approximations employed consists of computing three pairs of values for R (or R') and T, corresponding to the following pairs of values of the optical constants: (n, k), $(n+\delta, k)$ and $(n, k+\delta)$, where $\delta << n, k$. From **these** pairs of values the partial derivatives of the optical constants with respect to the reflection and transmission can be found. These, together with $(R - R_o)$ and $(T - T_o)$, are sufficient to allow for a further approximation to n and k, because the values $(R - R_o)$ and $(T - T_o)$ indicate the discrepancies in R and T, while the partial derivatives indicate the changes necessary in n and k to produce $R=R_o$ and $T=T_o$.

If n, k and R, T were linearly related, the resettings in n and k would produce exactly the right n and k. However, since non-linearity is indicated, the new values of n and k, while better than the original ones, are only a new approximation. The procedure can then be repeated until both $(R - R_o)$ and $(T - T_o)$ are less than experimental error. The value of the experimental error is stored in WWI as a parameter and can be varied at will.

The "Main Program" is used three times for the computation of the partial derivatives; each calculation is followed by a particular sequence of operations - the first by resetting n, the second by resetting k, the third by the calculation of the partial derivatives. Therefore, the machine must keep track of the required operation that is to follow each performance of the "Main Program." The directing counter consists of a sequence of orders and contains two control registers whose sign digits are examined and reset each time the directing counter is consulted. The directing counter, therefore, enables the machine to obtain directions after performing the "Main Program" -- simply by consulting the control registers and performing any one of four tasks designated on the combination of signs of the two control registers.

A register is used whose sign indicates which of the two reflection measurements is under consideration. When this "switch" register contains a positive number, film incidence is indicated; when it contains a negative number, backing incidence is indicated. One register in the "Main Program" is reserved for the reported value for the reflection, whether from the film or the backing incidence. Two registers are used as buffers for information, one containing reflection from the film, the other from the backing. Either one of these reflection values is transferred from its "buffer register" to the permanent "reflection register" in the "Main Program," depending on the sign of the "switch". When no backing incidence is reported, the buffer register reserved for reflection from backing incidence should contain the information "-." (minus point, which is negative zero). Since no reflection can have a negative value, the machine can easily distinguish the lack of information in this way, and eliminate the steps involved in the calculation for backing incidence. The results are labeled "f" and "b" by WWI, depending on whether the radiation was incident on the film or backing.

When the thickness of the film decreases, the reflection and transmission become less sensitive to the properties of the metal, and the index of the backing becomes relatively more important. Therefore, the accuracy afforded in n and k by experimental accuracy decreases with decreasing film thickness, so that the partial derivatives $\frac{\partial n}{\partial R}$, $\frac{\partial n}{\partial T}$, $\frac{\partial k}{\partial R}$, and $\frac{\partial k}{\partial T}$ should be reported along with the computed n and k in order to estimate the probable error in the reflection and transmission data.

The conductivity and dielectric constants are also computed and the results printed out.

106 MIT SEISMIC PROJECT

During the past quarter the MIT Geophysical Analysis Group has used a good part of its WWI time on production runs of the General Prediction Program and Auto- and Crosscorrelation Programs, amassing valuable data. Analysis of this data has indicated that the problem (described in Geophysics "Detection of Reflection on Seismic Records By Linear Operators," Wadsworth et al, July 1953) is not completely solved and that further theoretical work and computational work are necessary. Nevertheless, many phases of the problem have been clarified and new avenues of approach have been opened. The next quarter will see a greatly expanded effort on these new approaches to the problem, with less time used on the prediction-program production runs.

A power-frequency-spectrum program, written in the spring but never completed, has recently been revived. This is not yet working.

A modification of the existing prediction program, which will give running averages overlapping by nine, has been written and is being tested.

A modification of the correlation program is being written which will give running correlations.

A flexible cross-products program used in setting up the matrix of the linear operator is under way.

A good deal of consideration has been given to the possibility of writing a matrix-inversion program for the correlation matrix which is often nearly singular.

109 FIGHTER GUNSIGHT CALIBRATION, 8th ORDER DIFFERENTIAL EQUATION

The problem of determining a two-dimensional aircraft pursuit course in a horizontal plane is being extended by M.H. Hellman of the MIT Instrumentation Laboratory to the solution of a three-dimensional aircraft pursuit course. The problem consists essentially of solving 14 simultaneous non-linear differential equations by the Runge-Kutta Method. These equations take into account aircraft dynamics and projectile ballistics.

A preliminary program for this problem using CS is now being tested, and the process of eliminating all errors from it is being carried out.

111 FOURIER ANALYSIS -- AUTOCORRELATION PROBLEM

This problem is one of harmonic analysis. A given length of gyro data is assumed to contain the first two harmonics plus some random effects. Once the statistical nature of this randomness is determined (e.g., its autocorrelation function) some steps can be taken to minimize this effect on the gyro.

A program written by C. Block of the MIT Instrumentation Laboratory subtracts the first two harmonics from the original gyro data, then computes and prints out the autocorrelation function of the remainder.

112 LAWLEY'S METHOD OF FACTOR ANALYSIS; CHARACTERISTIC VECTORS (MODIFIED)

This problem, undertaken in cooperation with Dr. F.M. Lord of the Educational Testing Service, Princeton, seeks the m x m diagonal matrix, J, whose elements are the first m latent roots of the positive definite 33 x 33 matrix $S^{-1}RS^{-1} - I$, where R is an observed matrix of correlations, and S is a diagonal matrix whose elements are themselves functions of the unknown matrix J, as defined by the following equations:

$$S^2 = [I + //\text{diagonal elements of } M'M//]^{-1} ,$$

$$M = J^{1/2}U,$$

where U is the matrix whose rows are the first m latent vectors. The problem also seeks the matrix of "factor loadings" F = MS, and to test F for statistical significance, in accordance with the Lawley maximum-likelihood method of factor analysis.

The rewritten program described in Summary Report No. 34 has been corrected. Final results have been obtained for m=4, 5, 6. The significance test indicates the need for continuing with larger values of m.

113 SHEAR WALL ANALOGY, SIMULTANEOUS LINEAR EQUATIONS

The lattice analogy has been used to analyse rectangular flat plates subjected to various planar loading conditions and restraints. This technique is now being expanded by S. Sydney of the MIT Civil Engineering Department to consider plates with a hole in the central region of the plate. The specific plate that is being studied will simulate a massive concrete structure that has been proposed to provide a reaction for hydraulic presses.

The head of the press will be attached to the underside of the roof of this structure and will react against the dead weight of this concrete mass instead of the conventional bars connecting the head of the press to the base. The program that is being prepared for this problem will use floating point numbers in order to avoid some of the difficulties encountered in the previous analyses which used single-length numbers.

114 DESIGN OF OPTICAL INSTRUMENTS

This problem involves the calculations necessary in the design of optical systems, particularly those systems which are useful in ophthalmoscopy.

Two programs have been used to obtain results applied in the construction of an improved model ophthalmoscope. The first of these is a standardized ray tracing program which determines the location of an image and direction of light ray which has passed through an optical system, given the initial direction of the ray and location of an object. The second program computes the third-order aberration terms of an optical system. Both these programs are standardized in the sense that data specifying the optical system or light rays may be readily varied, without extensive programming.

A third program, part of which was written during this quarter, will be used in the attempt to determine optimal lens systems by using better and faster methods than used before.

This work is being carried out in cooperation with the Retina Foundation at the Massachusetts Eye and Ear Infirmary.

116 TORPEDO IMPULSE RESPONSE; CONVOLUTION

This problem seeks the solution of the convolution integral equation which relates the output of a linear system to its input and impulse response. An analog computer is being used to estimate impulse responses and the corrections to these on the basis of the difference between the predicted and measured outputs. WWI is being used to convolve these impulse responses and impulse-response corrections with the measured input to determine the accuracy of the estimates.

During this quarter, the technique outlined in Summary Report No. 34 has been applied to all sets of input-output data. The error function, obtained from convolving an initial impulse response with input data, was used in conjunction with the analog computer to estimate a correction to the trial impulse response. These corrections were convolved with the input data to obtain a check upon the accuracy of the new impulse-response estimate. An average of the best estimated impulse responses for the runs is being convolved with the input data to test the ability of this average impulse response to predict the system outputs. In addition an impulse response based upon hydrodynamic model data has been convolved with the input data of three runs in order to compare results from convolutions using this model response with results from convolutions using the best estimated impulse responses of the system.

The Fourier transform developed under Problem No. 107 was used to try to determine the system impulse response. The results were meaningless, even after decreasing the computing interval by a factor of four. Inaccuracy in the higher frequency content of the basic data apparently results in meaningless transform information in this higher frequency range.

In the future, in order to check the ability of this response to predict the output of a similar system, the best impulse response will be convolved with new input data similar to that from which it was estimated.

Work on this problem is being carried out by D. Hamilton and R. Kramer of the MIT Servomechanisms Laboratory.

119 SPHERICAL WAVE PROPAGATION

Work continued during this period on a program designed to integrate numerically, along the characteristic directions, the equations representing the motion of spherical waves.

Considerable time was spent in the first part of the quarter in an attempt to eliminate failures of the magnetic tape unit to operate correctly in the rerecord mode. The tape is used as auxiliary storage for the values at the mesh points in the time-distance plane of the problem, and the amount of such storage required increases as the computation progresses.

The difficulties with tape operation were solved towards the end of the time available this summer and a couple of runs were made, giving partial results. However, the computation will have to be pushed much further before we reach the region of particular interest in which the pressure gradients in the gas increase to the point of shock wave development.

A. Ralston of the MIT Mathematics Department will carry on the work during the next quarter.

120 THERMODYNAMIC AND DYNAMIC EFFECTS OF WATER INJECTION INTO GAS STREAMS OF HIGH TEMPERATURE AND HIGH VELOCITY

This problem is connected with the development of a potential gas turbine component called the "Aerothermopressor" -- a device for increasing the stagnation pressure of a hot, high-velocity gas stream by evaporative cooling. Further description of this device may be found in Summary Reports No. 32 and 33.

The analytical aspect of this development program is concerned with obtaining a better understanding of the Aerothermopressor process and with predicting its performance under various operating conditions. The Aerothermopressor process is defined by nine parameters: 1) entrance Mach number, 2) entrance stagnation pressure, 3) entrance stagnation temperature, 4) water injection rate, 5) initial droplet diameter, 6) initial droplet temperature, 7) initial droplet velocity, 8) wall friction and 9) variation of duct cross-sectional area. For fixed values of these parameters the state at every cross-section of the Aerothermopressor is fully defined and may be calculated from a simultaneous solution of seven non-linear, first order differential equations.

The computations which have been carried out on Whirlwind I involve a step-by-step numerical solution of these equations for various combinations of the initial parameters. This analysis is accomplished by a (24, 6) program occupying approximately 2000 registers, with each execution of the program (one step) requiring about 9 seconds computational time. This program incorporated provisions for starting and stopping the calculations at any point in order to obtain flexibility in varying the cross-sectional area of the duct, as well as providing for the insertion of normal shocks.

A final report on this work is being prepared by Bruce D. Gavril as an Sc.D. thesis for the MIT Department of Mechanical Engineering. The Aerothermopressor development program is being carried out at MIT under sponsorship of the Office of Naval Research and under the guidance of Professor Ascher H. Shapiro.

121 DETERMINATION OF WEAK SIGNAL PLUS NOISE PROBABILITY FUNCTIONS

This problem computed a probability density function resulting from the linear superposition of two other identical density functions. The resultant distribution was obtained by means of the convolution integral

$$W(\tau) = \int_0^{\tau} W(B)\ W(\tau - B)\ dB.$$

The function W(B) was the familiar amplitude density function of the envelope of combined sine wave plus Gaussian Noise developed by Rice,

$$W(B) = \frac{e^{-\frac{\phi^2 + \rho}{2}} \cdot I_0(\phi\sqrt{\rho})}{5\phi^3} \quad \Bigg| \quad \phi = B^{1/5} \qquad , \ \phi > 0$$

The singularity at the origin was removed by defining $W(B) = 0$ at $\phi = 0$.

The computer was programmed by Dr. G. C. Sponsler of MIT to compute both W(B) and the convolution thereof with four different interval ranges of B and τ. Integration was performed using the trapezoidal approximation formula with the machine choosing automatically the proper interval for each of the 16 possible interval combinations.

One typographical and two programming errors were discovered before the program ran successfully. Problem 121 is now satisfactorily concluded.

126 DATA REDUCTION

Problem 126 is a very large data-reduction program for use in the Servomechanisms Laboratory. The overall problem is composed of many component sections which will be developed separately and then combined at a later date. The first stage of this development, a program to fit polynomials automatically to arbitrary empirical functions, was described in the Summary Report No. 34. During this quarter investigations were carried out on that program to determine the effects of scale factoring and round-off. At this time the results are inconclusive; but it appears that for moderate ranges of the independent variable, the program will serve its purpose well.

To aid in the tests described above and in order to shorten testing time for future programs, a general-purpose Mistake Diagnosis Routine (MDR) has been written. The purpose of the MDR is to interrupt the normal operation of any program at specified "break points." During the interruption the MDR will automatically investigate any numbers and orders which the programmer specifies, and print intermediate results on delayed or direct output before allowing the program being tested to continue. Up to 128 break points may be used, and the operation of the MDR when each break point is encountered is completely independent of all other break points. Counters are included at each break point to allow intermittent sampling. The program itself is in no way changed by the use of the MDR, and the setting up of the MDR for each case is simple and fully automatic. After testing, the MDR will be made available to all users of WWI, and intelligent use of it should greatly speed the trouble shooting of any program.

131 SPECIAL PROBLEMS (STAFF TRAINING, DEMONSTRATIONS, ETC.)

During the Association for Computing Machinery meeting at MIT, September 9-11, 1953, approximately 240 ACM members visited the Barta Building and were given a tour of Whirlwind installations, a Flexowriter demonstration and a demonstration of several programs on the computer. During each of these three days about seven staff members assisted in the tours and demonstrations. The visitors represented practically all of the large computer installations in this country and in several foreign countries.

On September 25, the Digital Computer Laboratory was host to a group of 40 students and Faculty members from the Worcester Polytechnic Institute. The group was organized and headed by Professor Francis J. Adams of the Electrical Engineering Department at Worcester Polytechnic. The group was given a one-hour lecture, a tour of Whirlwind installations, a Flexowriter demonstration, and a demonstration of several programs on the computer. This is the second consecutive year that the Laboratory has been host to Worcester Polytechnic Institute.

132 SUBROUTINE STUDY FOR THE NUMERICALLY CONTROLLED MILLING MACHINE

The writing of the basic subroutine library for Numerically Controlled Milling Machine (NCMM) computations is essentially complete (see Summary Report No. 34). Eleven of the twelve subroutines written to date have been successfully tested. Testing of the twelfth is in progress. Most of the errors detected in the successfully tested routines were in programming, with tape preparation and computer difficulties running a rather poor second and third.

The use of some of the subroutines in the near future for tape preparation for a supersonic nozzle is being contemplated. The inside of the nozzle is a surface of revolution, so the computations required are routine, but the amount of tape required is large so that manual tape punching would be very time consuming.

134 NUMERICAL DIAGONALIZATION PROCEDURE

A subroutine written by A. Meckler of the MIT Solid State and Molecular Theory Group, to obtain the eigenvalues and eigenvectors of a Hermitian matrix is now available. For the special case of a real symmetric matrix, Whirlwind I can handle up to size 32 x 32. A true Hermitian matrix, one with complex elements, must be split and a composite matrix formed by the method described below which doubles the order, so that the largest size of such a matrix is restricted to 16 x 16.

The method of diagonalization was first made known to Dr. Meckler by Dr. A. J. Perlis, formerly of the MIT Digital Computer Laboratory. Dr. Perlis has written a preliminary report for the Digital Computer Laboratory which covers the method and contains a detailed error analysis. We will present the mechanics of the routine but we will not attempt any study of error propagation. Empirically the routine is very accurate. The roots of a 12 x 12 were given to 6 or 7 decimal digits, the eigenvectors to 4 or 5. (We take this difference in accuracies to be an expression of the variation principle.) High accuracy is to be expected because only a few elements are handled at a time, cycles are largely independent of preceding cycles, and other numerically disastrous matrix operations and determinant operations are not used.

The method used is as follows: Let the real symmetric matrix be denoted by A. We are to find an orthogonal matrix U such that UAU^{-1} is diagonal. The diagonal elements are the eigenvalues of A, and the rows of U are the eigenvectors of A. The technique is to build up this orthogonal transformation through a succession of two dimensional transformations. Each such transformation is accomplished by an orthogonal matrix which has unity as every diagonal element except for, let's say, the i'th and j'th which are cosine-like elements. All the off-diagonal elements are zero except for the i, j'th which is sine-like and the j, i'th which is the negative of the i, j'th.

Now, the transformation UAU^{-1} will affect only those elements in the i'th or j'th rows (or columns). If B is the resultant matrix, c and s are determined by two conditions. The first is the trigonometric identity

$$c^2 + s^2 = 1$$

and the second is the demand that b_{ij} vanish:

$$b_{ij} = 0$$

The conservation of trace and determinant implies

$$b_{ii}^2 + b_{jj}^2 = a_{ii}^2 + a_{jj}^2 + 2a_{ij}^2$$

This is a norm preserving transformation. That is, the sum of the squares of all the elements stays the same. The key to the convergence is that the sum of the off-diagonal elements has been definitely decreased by twice the square of the extinguished i, j'th element.

If now a_{ij} is always selected as that off-diagonal element of largest absolute magnitude, the off-diagonal norm decreases by the maximum amount at each cycle. The process is convergent; it is stopped when the largest off-diagonal element is considered negligibly small.

The input to the machine consists of the following: 1) pre-set parameter which is the order of the matrix; 2) the matrix elements ordered in the manner indicated in the figure;

$$\begin{vmatrix} 0 & 1 & 3 & 6 & 10 \\ & 2 & 4 & 7 & 11 \\ & & 5 & 8 & 12 \\ & & & 9 & 13 \\ & & & & 14 \end{vmatrix}$$

and 3) a criterion for the smallness of the largest off-diagonal element. The transforming matrix, which is built up at each cycle, is stored on the magnetic drum. The two rows to be transformed are read out, processed, and re-stored. Only one channel of the drum is used and this sets the limit on the order of the matrix. To find this limit we have:

$$2n^2 = 2048$$

or n = 32. The output is on the scope: first the eigenvalues in a column, then the eigenvectors as columns four to a frame.

As for the case of complex elements. Let a Hermitian matrix be represented as

$$A + i\ B,$$

where A is real symmetric and B is real antisymmetric. The eigenvalue-eigenvector equation is

$$(A + i B) (x + i y) = \lambda (x + i y),$$

where λ is a real scalar, x and y are real vectors. Equating real and imaginary parts:

$$Ax - By = \lambda x$$
$$Bx + Ay = \lambda y$$

These simultaneous matrix equations can be expressed as one matrix equation

$$\begin{vmatrix} A & -B \\ B & A \end{vmatrix} \begin{vmatrix} x \\ y \end{vmatrix} = \lambda \begin{vmatrix} x \\ y \end{vmatrix}$$

and we are back to the real symmetric case at the expense of doubling the order.

136 MATRIX EQUATIONS

A program has been written by Dean Arden of the Digital Computer Laboratory which will solve a system of equations of the form

$$Ax = b$$

by a variation of the n-step method of conjugate gradients described by Hestenes and Stiefel in National Bureau of Standards Report 1659, March 10, 1952. The variation was developed by E. Craig of MIT and is described in the Digital Computer Laboratory Memorandum M-2229.

The routine consists of proceeding in a direction p_1 to a point as close as possible to the solution h of Ax = b, then proceeding in a new direction p_2 to a point as close as possible to h, and so forth. The directions $p_1, p_2 \ldots$ are chosen to be mutually orthogonal, so that at the nth step an exact solution should be obtained.

The program will be tested on matrices of varying condition in order to determine the limits of its applicability. The results of this investigation will be reported in the future.

137 INVESTIGATION OF ATMOSPHERIC TURBULENCE; AUTOCORRELATION, CROSSCORRELATION, AND FOURIER TRANSFORMS

When the orientation of an aircraft must be accurately controlled, as in any system for automatic guidance, a quantitative description of "large-scale" atmospheric turbulence (or "gusts") is necessary for a rational design of the guidance system. If the turbulence can be assumed to be a stationary random phenomenon, the methods of generalized harmonic analysis may be used to specify statistical properties in terms of the power spectral density.

In the present investigation, the power spectra of the three gust components that cause lateral motions of an aircraft are being determined simultaneously by flight measurements made with an automatically controlled aircraft as a "probe." Correlation and Fourier transform programs are used to determine the power spectrum matrix for three selected outputs (e.g., lateral acceleration, roll rate, yaw rate) and, by inversion of the performance matrix of the aircraft-control system, the three desired input power spectra are calculated. A suitable cross power analysis has been developed for this calculation.

Further discussion of the physics and background of the problem may be found in Summary Report No. 34.

A general crosscorrelation program, which can do autocorrelation as a special case, has been written by C. Block and N. Zierler of the MIT Instrumentation Laboratory. The program is based on a modification of Ross' autocorrelation program (see Problem 107 in Summary Report No. 33) and makes use of the magnetic drum for storage of one of the functions. Several scale factors (equal to 1/10, 1/14, and 1/28) had to be introduced as program modifications in order to prevent overflow when handling the data of the various flights. These programs were used successfully to determine three autocorrelations and three crosscorrelations for each of four flights. The results have been checked for repeatibility and appear to be satisfactory thus completing the correlation phase of the problem.

Attempts have been made to obtain the Fourier transforms (sine and cosine) of the above mentioned correlation functions using the transform program developed in Problem 107. Unfortunately, check solutions indicate that this program may produce cumulative errors in the sines and cosines for large values of the argument. These cumulative errors seem to manifest themselves as negative trends in the power spectra. If this cannot be readily corrected in the program, it may be necessary to take the transforms several times over restricted frequency bands; this would increase the required computation time several-fold.

The successful computation of the Fourier transforms will determine the matrix of output power spectra for the four test flights and will complete Problem 137 as far as Whirlwind is concerned.

This investigation is part of an Sc.D. thesis being carried out by R.A. Summers of the MIT Aeronautical Engineering Department and sponsored by the Instrumentation Laboratory.

138 SPHEROIDAL WAVE FUNCTIONS

Spheroidal wave functions are solutions of the scalar Helmholtz equation in spheroidal coordinates. They arise in physical problems involving the wave equation and having spheroidal geometry.

Because these functions have two indices and two variables, the tabulation of the functions on a fine mesh of the variables would be extremely lengthy. Instead the functions are expanded in terms of a series of Associated Legendre Functions of the First Kind. Thus, in the limit, as the spheroidal geometry reduces to a sphere, only a single non-zero expansion coefficient remains.

With this scheme, it is intended to work out the expansion coefficients of the function for 55 different pairs of indices. For each pair of indices there will be 56 values of h (the parameter describing the eccentricity of the spheroid), each of which will have associated a set of coefficients varying in number from one to approximately twenty. Although no distinction has been made of the types of coefficients, actually 4 types would be useful, namely: prolate angular, prolate radial, oblate angular, and oblate radial. The angular and radial coefficients of a given geometry are intimately related and are most efficiently calculated together; the prolate and oblate coefficients are independent but the program can calculate either type with but a trivial modification. Thus in summary there are the following number of calculations: (2 types of geometry) x (55 pairs of indices) x (56 h values) = 6160 double sets of angular and radial coefficients.

About 5 seconds will be needed to calculate each double set of coefficients and another 10 seconds to record the results on magnetic tape. Thus the total estimated machine time is 6160 double sets x 15 seconds = 27 hours, yielding a set of tables approximately 1320 pages in length.

The problem being programmed for Whirlwind I is the determination of these expansion coefficients. The ratios of these unknown coefficients must satisfy two continued fraction equations each of which contains the unknown separation constant (i. e., eigenvalue) of the original differential equation as a parameter. Thus the numerical procedure is to vary the trial eigenvalue (in a manner insuring convergence) until the two equations are consistent. Finally, by an appropriate normalization, one obtains the correct values of the coefficients.

The program for Whirlwind consists of two sections: 1) the calculation and accumulation of the normalized coefficients in sets of five and the eigenvalue, and 2) the layout program which records on magnetic tape for delayed punch-out the appropriate Flexo characters necessary to type out the results in final form. The Flexo tape punched out will subsequently be used to type out the results on special paper. A draftsman will then box the result sheets with ink lines. Finally these result sheets will be photolithographed with a 2/3 reduction and from this the final tables printed. Note that the results are never retyped manually and thus transcription errors are avoided.

At present the layout program is completed and being given a final test; the normalization and accumulation routine is written and is being tested; and the calculation of the ratios by the iterative continued fraction scheme is nearly completely written and is partially tested. This work is being carried out by F. J. Corbató and J. D. C. Little of the MIT Physics Department.

139 CALCULATION OF THEORETICAL LINE SHAPES

The calculation of theoretical line shapes for nuclear magnetic resonance absorption lines has been carried out to make possible a comparison with experimentally obtained line shapes. In order to obtain a theoretical line suitable for comparison with experimental lines, it is necessary to modify the rigorously derivable ideal theoretical line shape for an isolated methyl group by applying a "broadening function." This is a symmetric function which compensates for the net accumulated effect of neighboring nuclei on those of the isolated methyl group, and which has the effect of broadening and flattening the idealized theoretical line leaving the area under the curve invariant. Both the so-called "Gaussian"

$$A \exp(-x^2/(\beta/\alpha)^2)$$

and the "Lorentz"

$$B/(1 + \frac{x^2}{(\beta/\alpha)^2})$$

will serve as broadening functions. The Gaussian had been used in this study. The value of the parameter β/α determines the extent of flattening out.

Eight values of β/α were chosen and for each value the general line shape curves in the interval (0, 3) were obtained by plotting values of μ/α against values of the function

$$F(\mu/\alpha) = \frac{1}{\mu} \frac{1}{\beta/\alpha} \int_0^1 \left\{ \exp\left(- \frac{\left[\frac{\mu}{\alpha} + (1-3x^2)\right]^2}{(\beta/\alpha)^2}\right) + \exp\left(- \frac{\left[\frac{\mu}{\alpha} - (1-3x^2)\right]^2}{(\beta/\alpha)^2}\right) \right\} dx$$

The evaluation was carried out using Simpson's rule with n = 50.

The results obtained have been described in a paper presented to the Gordon Conference on Elastomers by Dr. H. LeClair, Dr. J. Sternberg and Professor Rochow of the Chemistry Department at Harvard University. The results have also been included in LeClair's doctorate thesis and in an ONR project report.

140 SUMMER SESSION SYSTEM

This problem concerned the development of an instruction code, a conversion program, an interpretive program, and error-diagnosis routines used by the students who participated in the MIT 1953 summer session course, "Digital Computers and Their Applications."

A discussion of the system developed, together with plans for its future use are given in Section 5.4.

141 S&EC SUBROUTINE STUDY

A "Library of Subroutines" notebook which lists 12 subroutines and which contains all of the information required by the programmer is now available in the tape room. These subroutines, written for use with the Comprehensive System of Service Routines, include programs for e^x, ln x, cosh x, sinh x, sin x, cos x, $\sqrt{x}$, a Runge-Kutta routine, an auxiliary buffer routine, and five output routines which use the scope, typewriter, and/or the delayed typewriter.

Subroutines written and used by programmers in connection with regular problems will be added to the Library whenever feasible, but no routines will be written expressly for the Library.

142 A STUDY OF SHOCK WAVES

A study of shock waves is being carried on by R. Bart and S. Sydney of the MIT Civil Engineering Department in order to develop a method for the solution of vibration problems in solid bodies. The vibration of a one-dimensional bar may be obtained by replacing the bar with an equivalent system of spring-connected mass points. The equation for the acceleration of this system, a function of displacement only, is expressed by finite difference and is integrated to obtain the displacement. The acceleration is then reexpressed in terms of finite differences, integrated and a new expression obtained which is an even closer approximation for the displacement.

Three iterations are used to obtain a series expansion for the displacement and velocity of this system. With a time-spacing ratio of 100 microseconds/ft. the error in the missing terms is kept below 1.0% for the spring-mass solution.

The effects of loads applied to the system may be incorporated into the solution so that a complete time history of the vibrations can be obtained regardless of the type of loading.

These solutions agree reasonably well with the theoretical response of a distributed mass system, indicating that this method may be used to study the response of multidimensional mass systems.

A program for the two-dimensional problem has been prepared and is being tested at the present time.

143 VIBRATIONAL FREQUENCY SPECTRUM OF A COPPER CRYSTAL; THIRD ORDER POLYNOMIAL

The problem of determining a vibrational spectrum is a natural outgrowth of the physics of periodic structures, particularly that of crystals, wherein it is desired to ascertain (1) the various modes of vibration of the atomic lattice, (2) the subsequent contribution to the specific heat of the structure and to the scattering of conduction electrons, and (3) the transport phenomena in general. Such results are of direct interest to both the experimental and theoretical solid-state physicist.

In Problem 143, metallic crystal copper, having a face-centered cubic lattice structure, is being investigated by E. H. Jacobsen of the MIT Physics Department. The force constants between neighboring atoms are being determined from X-ray diffraction experiments. These constants are to be fed into a third order secular determinant, the solution of which gives the frequency of a thermal wave propagating through the lattice as a function of the wavelength and direction of propagation. Each element of the determinant consists of a finite Fourier series of approximately 12 terms. One solution for a general wave propagation vector requires about three hours on a desk calculator. It is felt than an accurate spectrum determination will require about 25,000 solutions of the secular equation (i.e., 25,000 different wave vectors).

The program for the general solution of this secular equation is being assembled and tested in parts on Whirlwind I.

144 SELF-CONSISTENT MOLECULAR ORBITALS

Roothaan has given a procedure for determining best linear combinations of atomic orbitals for the case of a closed-shell state (a single determinant many-electron wave function). His equations are the algebraic equivalent of the Hartree-Fock differential equations and are similarly characterized by the discouraging prospect of a self-consistent calculation. However, Roothaan's procedure can be mechanized and performed by Whirlwind I so that it becomes a fast, repeatable routine. Roothaan's equations can be re-expressed in a slightly modified form:

Consider the one-electron functions which are to be combined in the best possible ways. Let them be orthonormalized. Begin with a well considered first guess to the molecular orbitals, an orthonormalized set. A general linear combination of these functions is still a general linear combination of the original atomic functions.

This original set of functions is denoted as v_μ and the number in the set by N. If there are 2n electrons in the system, we are to find n linear combinations of the v_μ's

$$\phi_i = \sum_\mu c_{i\mu} v_\mu$$

such that an antisymmetrized many-electron wave function formed with each ϕ_i doubly occupied will be associated with a minimum value of the energy. The expression for the energy is

$$E = 2\sum_{\mu\nu} \rho_{\mu\nu} H_{\nu\mu} + \sum_{\mu\nu\lambda\sigma} \rho_{\mu\nu} G^{\mu\nu}_{\lambda\sigma} \rho_{\lambda\sigma}$$

where

$$H_{\mu\nu} = \int v_\mu{}^*(1) \left[-\nabla_1{}^2 + V(1)\right] v_\nu\,(1)\, d^3x_1$$

$$G^{\mu\nu}_{\lambda\sigma} = 2(\mu\nu|\lambda\sigma) - (\mu\sigma|\lambda\nu)$$

$$(\mu\nu|\lambda\sigma) = \int v_\mu{}^*(1)\, v_\nu\,(1)\, \frac{2}{r_{12}}\, v_\lambda{}^*(2)\, v_\sigma(2)\; d^3x_1\, d^3x_2$$

$$\rho_{\mu\nu} = \sum_i c^*_{i\mu} c_{iv}$$

or in matrix notation

$$\rho = C^+C \qquad (+ = \text{Hermitian adjoint})$$

and the orthonormalization of the ϕ_i's is implied by

$$CC^+ = 1$$

C is an n x N rectangular matrix. ρ is a square N x N. The n rows of C, which are the complex conjugates of the n columns of C, are the n lowest eigenvectors of the matrix

$$K_{\lambda\sigma} = H_{\lambda\sigma} + \sum_{\mu\nu} \rho_{\mu\nu} G^{\mu\nu}_{\lambda\sigma}$$

That is

$$KC^{+} = \Lambda C^{+} \qquad \Lambda \text{ diagonal}$$

We are to pick a ρ, form K, diagonalize it, select its n lowest eigenvectors, form ρ, re-cycle. H and G are the input to the machine and do not change from cycle to cycle.

The machine is to do matrix-vector multiplication (symbolized as ρG), matrix addition, matrix diagonalization, magnitude selection and comparison, and many cycles. Even with the type of factorization by symmetry discussed by Roothaan much information will be stored in the machine, but the large amount of high-speed storage and the magnetic drum in Whirlwind have made this feasible.

The matrix diagonalization procedure, which is just a sub-routine in the whole self-consistent process, has been programmed and will exist as a separate routine in the Whirlwind library. The present program can handle up to a 32 x 32 real symmetric matrix and a 16 x 16 complex Hermitian matrix. The rest of the linear combinations of atomic orbitals determination is now being programmed and will be first applied to the NH_3 molecule.

The routine for the diagonalization of K has been successfully programmed under Problem Number 134 and is described under that number. The remainder of the routine has been programmed and is being tested by Dr. A. Meckler of the MIT Solid State and Molecular Theory Group.

145 EVALUATION OF SECOND-ORDER TEMPERATURE DIFFUSE SCATTERING FROM ZINC

This problem was carried out by R. Joynson of the MIT X-ray Laboratory whose PhD Thesis involved the measurement of temperature diffuse scattering (TDS) of X-rays from a single crystal of zinc. Analysis of the measurements showed that the observed scattering was nearly twice as much as could be accounted for by first-order scattering alone. It is known that second-order TDS may, for some crystals, be as large as 50% of the first-order scattering. However, the relative magnitude of second-order TDS for zinc had not been calculated; so it was decided to use the Whirlwind Computer to see if the experimental results might not be explained, at least in part, by second-order effects.

The calculation requires primarily the evaluation of the integral

$$I = \int_{z_1}^{z_2} \left\{ \frac{1}{u^2-v^2} \left[(2u-v)\tan^{-1} \frac{1}{v} + (u-2v)\tan^{-1} \frac{1}{u} \right] + \frac{1}{u-v} \quad \frac{1}{\sqrt{2}} \,\mathrm{lu}\, \frac{v^2+1}{u^2+1} \right\} d\left[(2-t)s + 2t \right]$$

where, $u = \dfrac{[(2 - t)s + 2t]^2}{114.56}$, $v = \dfrac{[(2 - t)s - 2t]^2}{114.56}$

The integrand was evaluated at 25 values of the independent variable s in the range $0<s\leq 2.0$ for each 20 values of the parameter t ranging from 0.1 to 1.9. The program was written and the problem completed in two days. The integration was carried out by graphing the integrand and using a planimeter, which gave sufficient accuracy for the desired result.

An important result of the calculations was the vanishing of the second-order temperature diffuse scattering in zinc at reciprocal lattice points whose X-ray structure factor is zero. Since this was the region where experimental results were inconsistent with first-order TDS, the calculations showed that probably there was a serious experimental error. New measurements at more closely spaced intervals revealed that this in fact was true, the error having arisen from the introduction of impurities on the surface of the zinc crystal during the etching process.

146 LARGEST EIGENVALUE OF REAL SYMMETRIC MATRIX

The following problem arose in connection with a special problem on configuration interaction in even-even nuclei. Given a real symmetric matrix, what is the largest eigenvalue in magnitude? Hildebrand (Methods of Applied Math, pp. 68-69) has given an iterative procedure for finding this number. One starts with an initial guess $x^{(1)}$ at the eigenvector and multiplies this by the matrix A to get:

$$Ax^{(1)} = \lambda^{(1)}x^{(2)}$$

λ can be that factor which makes the first component of $Ax^{(1)}$ equal to 1. Similarly one finds

$$Ax^{(2)} = \lambda^{(2)} x^{(3)}$$

$$\cdots\cdots$$

$$Ax^{(n)} = \lambda^{(n)} x^{(n+1)}$$

It can be shown that for any matrix whose eigenvalues are real that

$$\lim_{n \to \infty} \lambda^{(n)} \to \lambda \qquad \text{(actual value of the largest eigenvalue)}$$

$$\lim_{n \to \infty} x^{(n)} \to x \qquad \text{associated eigenvector}$$

The iterations are continued until subsequent values differ by less than a predetermined amount. A program has been written and tested; some final refinements are being made to put it in convenient subroutine form.

This work is being carried out by A. Temkin of the MIT Physics Department.

147 ENERGY BANDS IN CRYSTALS

Approximate solutions P_ℓ (r, E) are required for the differential equation arising in the study of energy bands in crystal

$$\frac{d^2P_\ell}{dr^2} = P_\ell \left[\frac{\ell(\ell+1)}{r^2} - V(r) - E \right]$$

These approximate solutions will be found using two starting values near r = 0 and the Gauss-Jackson formula for forward integration

$$P_\ell = h^2 \left(\frac{1}{2} P_\ell'' + \delta^{-2} P''\right)$$

where h is the interval of r and $P_\ell'' = \frac{d^2P_\ell}{dr^2}$. The notation δ^{-2} is to be interpreted by formally multiplying the equation by δ^2 and treating δ^2 as a second difference operator. Iterations using this formula will be made without advancing the independent variable until the values of P_ℓ converge. At this time the value of the independent variable will be increased by h and a new trial value of P_ℓ obtained using the same formula. Solutions are desired for 160 values of E and 13 values of ℓ.

In addition to the above, approximations to the expressions

$$v_\ell(E, r) = \frac{r^2}{P_\ell}\frac{dP_\ell}{dr} - r$$

$$F(E, E_o, r) = \sum_\ell (2\ell+1) j_\ell^2 (\sqrt{E_o}\, r) v_\ell (E, r)$$

will be calculated for 8 values of r, 160 values of E, and 70 values of E_o. $j_\ell(x)$ is the spherical Bessel function

$$j_\ell(x) = \sqrt{\frac{\pi}{2x}}\, J_{\ell+\frac{1}{2}}(x)$$

These functions will be computed using the recurrence formula

$$j_{\ell+1} = \frac{2\ell+1}{x} j_\ell - j_{\ell-1}$$

with initial values

$$j_o = \frac{\sin x}{x} \qquad j_{-1} = \frac{\cos x}{x}$$

The routine for the approximate integration of the differential equation has been successfully programmed by David Howarth of the Solid State and Molecular Theory Group and a program for varying the values of E, ℓ and r and calculating the V_ℓ (E, r) has been written and is being tested.

3. OPERATION OF WHIRLWIND I

During the period 1 June - 30 September 1953, 1126 hours were assigned to applications groups (see Fig. 3-1). Of this time, 86 per cent yielded useful results.

Considerable effort was made to increase the reliability of operation of WWI and to reduce the required amount of maintenance time. Installation of Magnetic-Core Storage and the use of the new programmed marginal-checking system have been a substantial help in this direction. A new Ferranti photoelectric tape reader will be installed during the coming quarter and is expected to have better operating margins than our present photoelectric tape reader.

3.1 Systems Engineering

High-Speed Storage

The maintenance difficulties with Electrostatic Storage mentioned in the last quarterly (deflection shift and positive switching) continued to cause trouble in this quarter. Since permanent cures for these maintenance troubles seemed impossible to achieve and since Magnetic-Core Storage was proving itself extremely promising in the Memory Test Computer, it was decided to replace Electrostatic Storage with Magnetic-Core Storage. On August 8, the bank of Magnetic-Core Storage, Fig. 3-2, which had been part of the Memory Test Computer, was installed in place of one bank of ES. A second bank of Magnetic-Core Storage, Fig. 3-3, was then constructed, and installation took place on September 5.

		Hours Per Week																
Activity		June 1-7	June 8-14	June 15-21	June 22-28	June-July 29-5	July 6-12	July 13-19	July 20-26	July-Aug 27-2	Aug 3-9	Aug 10-16	Aug 17-23	Aug 24-30	Aug-Sept 31-6	Sept 7-13	Sept 14-20	Sept 21-27	Total Hours
Storage Maintenance	Electrostatic	20	21	22	18	19	20	20	20	20	17	18	15	8	11	11	0	0	260
	Magnetic-Core																5	5	10
Marginal Checking		7	6	5	8	5	8	7	8	10	10	10	9	6	9	9	8	8	133
Installation		13	14	13	10	9	6	6	6	18	7	7	6	8	10	0	11	8	152
Maintenance		25	18	21	20	20	19	17	26	7	20	17	25	15	26	14	27	22	339
Terminal Equipment Testing		33	35	32	35	34	36	33	21	27	23	22	16	21	17	35	33	35	488
Engineering & Scientific Computation		31	28	29	35	17	39	34	32	26	29	29	32	42	36	22	20	24	505
Other Applications		25	25	20	22	24	19	29	36	40	41	54	52	57	47	38	49	43	621
Total Hours		154	147	142	148	128	147	146	149	148	147	157	155	157	156	129	148	140	2508

Fig. 3-1. Allocation of Computer Time during Period 1 June - 30 September 1953

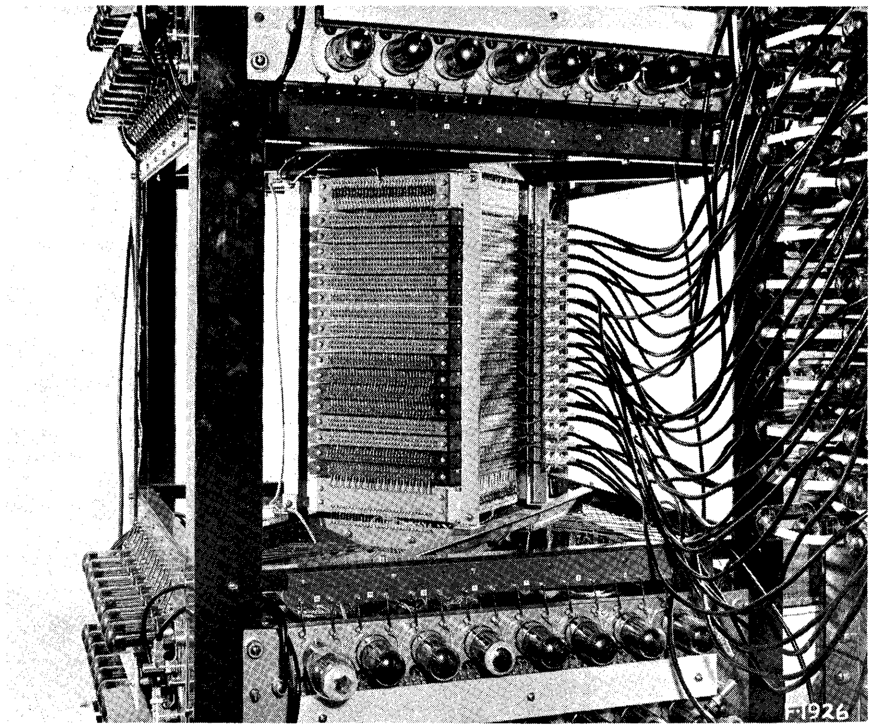

Fig. 3-2. First Bank of Magnetic-Core Storage

Thus Electrostatic Storage has been eliminated from Whirlwind I. However, it should be pointed out that the computer could not have reached its present state of development without ES. No other form of high-speed storage was available when Whirlwind I was put in operation.

Magnetic-Core Storage has two big advantages: (1) greater reliability with a consequent reduction in maintenance time devoted to storage; (2) shorter access time (core access time is 9 μsec; tube access time is approximately 25 μsec) thus increasing the speed of computer operation.

Marginal Checking

Programmed marginal checking has been in use since the last quarter. The numbers of the marginal-checking lines to be varied with the test program being performed are, at present, stored on a Flexowriter tape and are read in via the Flexowriter reader as called for by the computer program. There are thus two tapes for each test program.

We are now working on a master program for storing all our test programs on the auxiliary drum. This program will include a coded table of marginal-checking lines to be used with each test program. When programmed marginal checking is used, the computer will search through this table and decode binary marginal-checking line numbers into their binary-coded-decimal form (for use with the programmed-marginal-checking equipment) whenever the table indicates a line is to be checked. A control program, also included, will enable the operator to select a desired test program by changing the setup of a flip-flop register.

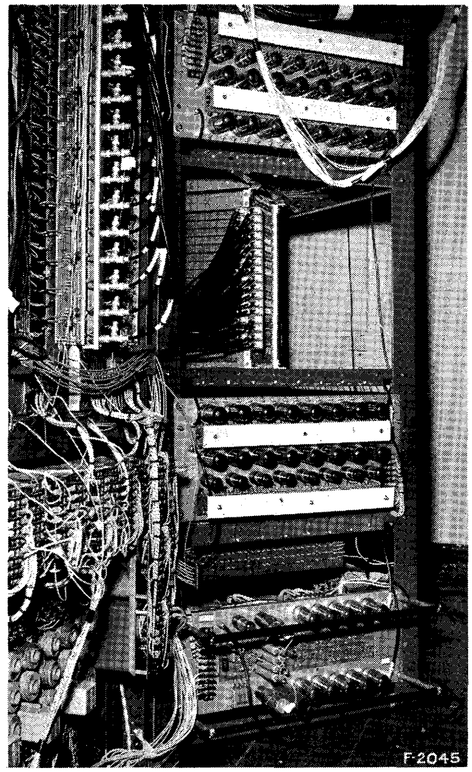

Fig. 3-3. Second Bank of Magnetic-Core Storage

The master program will be on one long tape, read in at the beginning of a marginal-checking period. After the contents of the tape are stored on the drum, no further use need be made of the photoelectric tape reader. Using the drum will be convenient and speedy both in selecting MC lines automatically and in recovering programs that may be destroyed by the margins applied during marginal checking.

3.2 Input-Output System

Magnetic Drum

The auxiliary drum system in its original form is still operating reliably with the computer. A monitor system has been installed so that the various drum pulses can be selected and observed on a scope in the computer control room. A test stand, Fig. 3-4, has been set up for the purpose of checking the individual chassis of the drum system independently of the drum. All of the reading amplifiers and some of the other chassis have been checked, and their supply voltage margins were found to be quite good.

During the next quarter we expect to replace the auxiliary drum with a new one containing improved heads. The new drum will allow the use of the additional tracks which were damaged in the old drum by some of the old heads expanding and touching the recording surface. Some of the additional tracks will be used to provide a parity digit for checking drum-information transfers.

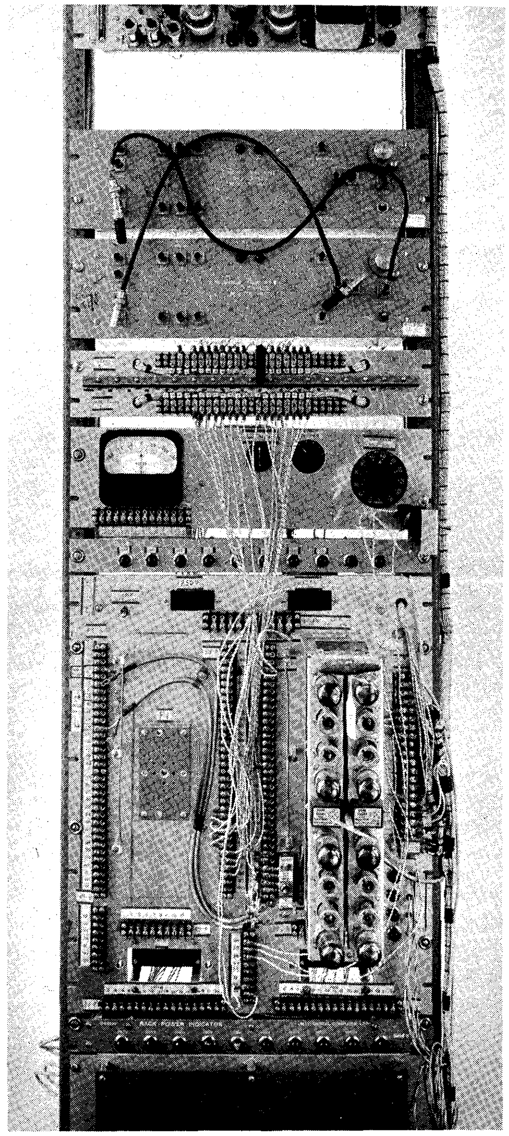

Fig. 3-4. Test Stand

Display

As was mentioned in Summary Report 34, noise on the scope deflection lines has been an annoying problem. The noise has been reduced on a test line in two ways: (1) A balanced decoder output amplifier was constructed and used to give constant current drain on the power supplies and thus reduce cross talk due to common power supply impedance. (2) The output of the amplifier was made 10 times as large as needed at the scopes so that it could be attenuated at the scopes and reduce the effects of any ground noise that existed between amplifier output and scope input. This system gave considerably less noise than the system in use, and it (or a similar system) will be installed as soon as extra power supplies become available from the electrostatic storage tube equipment.

Magnetic Tape

There have been several minor improvements in the magnetic tape print-out equipment. The typewriter is now housed in a sound-absorbing cabinet to reduce noise, and the controls for the selection of the unit to print from are on the cabinet. Operation has been further simplified by the addition of an automatic rewind feature.

Lately it was discovered that the heads of the tape units were being worn flat by the abrasive action of the oxide coating. This trouble has been corrected by regrinding the head surface to make it round again, but the reground head often will not read back correctly tape that had been previously recorded on it. In the future, semi-permanent data will be recorded on a second tape unit, and the second tape stored away as insurance in case of failure of the head of the first unit.

Photoelectric Tape Reader

The new Ferranti photoelectric tape readers, Fig. 3-5, have arrived, and after they have been tested and modified to suit our equipment, they will replace the old ERA unit we now have. The old unit has been causing trouble from time to time because of its low margins of operation. The signals appear to be getting weaker, presumably because of mechanical wear and increased loss in the light rods. The new reader does not use light rods, and it will not require the use of opaque tape. This means that standard teletype tape, much easier on the tape-punching equipment, can be used.

Fig. 3-5. Ferranti Photoelectric Tape Reader

3.3 Electrostatic Storage

During the first half of this quarter, construction and testing facilities of the Storage Tube Group were operated at full capacity to meet requirements for 800-series tubes in the electrostatic-storage system. (This series of tubes had a conductive coating of stannic-oxide inside the envelope. Operational experience demonstrated that stannic-oxide released less gas under ion bombardment than the usual aquadag coating.)

Midway in the quarter it was decided to install magnetic-core storage in the computer. All further storage-tube research and development was halted, and personnel associated with that work were transferred to other Groups. Remaining personnel continued to direct most of their efforts toward production of 800-series tubes.

Prior to the decision to supplant electrostatic storage in WWI with magnetic-core storage, the Storage Tube Group was producing an average of four 800-series stannic-oxide-coated storage tubes and one research tube per week. The now-reduced staff is expected to turn out five 800-series tubes every two weeks until production stops in October.

4. CIRCUITS AND COMPONENTS

4.1 Vacuum Tubes

4.11 Vacuum-Tube Life

During the past quarter the WWI computer operated 1800 hours. Considering that it underwent a major change from a computer using electrostatic storage to one using magnetic-matrix storage, this is quite a great amount of time. The change in storage caused a major revision in the tube complement of the machine. However, corrected parts lists are not yet available. Since WWI was an electrostatic machine during most of the last quarter, it is probably satisfactory for the purposes of failure analysis to use the old parts list. These figures appear as the total-in-service quantities of Table 4-1, which lists all tube failures during the third quarter of 1953.

Average failure rates for the three major types in WWI have been computed for operation during the past quarter. This data, together with previous experience, is shown below:

	FAILURE RATE, PERCENT PER 1000 HOURS			
Tube Type	1952	First Quarter 1953	Second Quarter 1953	Third Quarter 1953
7AD7, SR1407, 6145	2.00	4.5	3.1	2.8
7AK7	0.26	0.7	0.4	0.4
6SN7GT	1.07	1.5	2.0	1.9

This data for the 7AK7 and 6SN7GT is quite valid and is not open to serious question. The same cannot be said for the data on the 7AD7/SR1407/6145 types. These form a well mixed population, and the numbers of each type in actual use are unknown. Because of the many tubes involved, it is a considerable task to determine the distribution by types. The 7AD7 tubes, totaling about 1000, contributed 104 of the total 169 failures for the 3 similar types. Hence, the 7AD7 failures govern in large measure the total failure rate, although the 7AD7 tubes constitute only about 1/3 of the population.

Types 6080 and 6CD6G are largely used in the magnetic-matrix memory. As noted above, these types are not listed in the tube complement since the bill of materials is not yet up to date. The large number of 6080 failures occured when these tubes were retested prior to installation of the magnetic-core memory in WWI.

The major cause of failure was a grid-to-grid short of the high-resistance types, with resistance values mostly between 1 and 10 megohms. These tubes are used with the grids tied together externally, so that the high-resistance shorts had no effect on circuit operation. The cause of the shorts was a film deposited on the top shield mica by sublimation of material from inside the cathode.

4.12 Vacuum-Tube Research

Thesis research on the problem of cathode stability has continued at an intensive level during this quarter. A total of eight special research tubes have been constructed and tested thus far. These tubes allow the measurement of cathode potential changes caused by changes in the current through the cathode. Thus far, the techniques have been evaluated and tested, but no observations of the change in cathode voltage have been made because these tubes have such low resistance cathodes that changes do not occur under the usual test conditions. Some tubes will be placed on life in the very near future to discover if life operation contributed to the phenomenon. Other modifications to the cathodes can be made by certain special test conditions; these modifications will be studied while other tubes are operating on life test.

Theoretical analysis shows that changes in the cathode resistance when the cathode is caused to draw current depend upon the amount of current drawn and the amount of cathode resistance observed under pulsed conditions. According to the analysis, there is a limit to the amount of steady current that can be drawn; this limit is reached when the voltage drop across the cathode for the same current under pulse conditions is between 0.15 to 0.34 volt at the usual cathode temperatures. Only the steady-state conditions have been analyzed thus far in detail, but the transient equations have been written for computer solution.

Life tests are now in progress for 5965, 6145, and 7AK7 tubes. The 6145 and 7AK7 life tests still appear about as reported in Summary Report 34. However, the status of the 5965 test has changed somewhat. This life test was operated to 1500 hours with one side conducting, the other side cut off. For this particular twin triode, a dissipation of about 1.2 watts seemed reasonable for the conducting side, and this condition was used for the first 1500 hours of life. However, interest developed in the possibilities for using this tube at a higher level of dissipation, and so the dissipation of the tubes on life test was increased at 1500 hours to about 2.3 watts. They have been run with the conducting side at this higher dissipation for the last 1000 hours.

During the first 1500 hours no major changes were observed in the tubes.

			Reason for failure; number failed				
Type	Total in Service	Hours at Failure	Change in Characteristics	Shorts, Opens	Breakage	Burn-out	Gassy
7AD7/6145 SR-1407	3354						
7AD7		0-1000	6	1			
		1000-2000	1	1			
		3000-4000		2			1
		4000-5000	2				
		5000-6000	1				
		6000-7000	5	1			
		8000-9000	4	6			1
		9000-10000	1				
		10000-11000	2	3			
		11000-12000	3	7			
		12000-13000	1	5			
		13000-14000	1	5			
		14000-15000	2	1			
		15000-16000	1	1			
		16000-17000	1	3			
		17000-18000		4			
		18000-19000	11	14			
		19000-20000	1	6			
		20000-21000		1			
6145		0-1000	7	36			2
		1000-2000	2	5			
		2000-3000	1	4			
		3000-4000	1	1			
SR-1407		2000-3000	4				
		3000-4000	2				
		6000-7000		1	1		
7AK7	2336	0-1000		2			
		2000-3000	3				
		4000-5000		1			
		6000-7000		1			
		9000-10000		1			
		10000-11000	1				
		12000-13000		1			
		15000-16000		1			
		17000-18000		1			
		18000-19000	1	2			
		19000-20000	1	2			
6SN7GT	436	0-1000	1	1	1		
		9000-10000	1				
		10000-11000	1	1			
		11000-12000		1	1		
		16000-17000		1			
		17000-18000		2			
		18000-19000		2			
		19000-20000	1	1			
OA2	19	0-1000			1		
		1000-2000		2			
		10000-11000		1			
		13000-14000	1				
OC3/VR-105	13	16000-17000			1		
OD3/VR-150	30	10000-11000	1				
		13000-14000			1		
2D21	96	0-1000			1		
		14000-15000		1			
2X2A	2	3000-4000	1	1			
3D21A	4	5000-6000		1			
3E29/829B	207						
3E29		0-1000		1			
		2000-3000	2				
		3000-4000		1			
		4000-5000	2				
		7000-8000	1				
		10000-11000	2	1			
		13000-14000	1				
		14000-15000	1				
		18000-19000	2	1			
		19000-20000	1				
829B		0-1000			1		
5U4G	22	2000-3000		1	1		
		8000-9000	1	1			
6AC7	7	5000-6000		1			
6AG7	126	13000-14000	1				
		18000-19000	2				
6AL5	249	10000-11000	2				
6AN5	25	1000-2000		1			
6AS6	3	7000-8000		1			

Fig. 4-1. WWI Tube Failures

Type	Total in Service	Hours at Failure	Reason for failure; number failed: Change in Characteristics	Shorts, Opens	Breakage	Burn-out	Gassy
6AS7G/6080	203						
6AS7G		3000-4000	1				
		4000-5000	1				
		8000-9000		1			
		9000-10000		2			
		11000-12000		2			
		15000-16000	1				
		16000-17000		4			
		17000-18000		2			
6080		0-1000		3	3		
		1000-2000		51	3		
6AU6/6136	80						
6AU6		0-1000		1			
		1000-2000		1			
		2000-3000		1			
6136		0-1000		1			
6BL7GT	85	0-1000			2		
6CD6G	*	0-1000			1		
6J5	6	6000-7000		1			
6L6G/5881	117						
6L6G		0-1000	1	1			
		1000-2000	2				
		4000-5000	1				
		6000-7000	2				
		11000-12000	1				
		13000-14000			1		
		16000-17000	5				
		18000-19000	11				
5881		0-1000	2	1			
		1000-2000	3				
		2000-3000	1				
6SH7	29	4000-5000	1				
		5000-6000	1				
		9000-10000	1				
6SL7GT	12	10000-11000	1				
6V6GT	31	4000-5000	1				
		9000-10000	2				
6X5GT	13	1000-2000		1			
		17000-18000	1				
6Y6G	327	1000-2000		2			
		2000-3000	1	1			
		7000-8000		1			
		13000-14000		1			1
		14000-15000		1			
		17000-18000			1		
		18000-19000		3			
		19000-20000		1			1
		20000-21000	2	2			
7X6	12	10000-11000	4	2			
715B/715C	36						
715B		0-1000	1	10	1	1	
		2000-3000		5	1		
715C		0-1000	7	6	3	1	1
5670/2C51	63						
5670		1000-2000	3				
		6000-7000		1			
		7000-8000		1			
2C51		0-1000		1			
5687	100	5000-6000		1			
		6000-7000	1	1			
5696	128	0-1000	15	41	1	2	
5963/12AU7	135	3000-4000	1				
5965	298	3000-4000		1	1		
8008	12	0-1000				2	

* Total in Service counts for this type are not up to date.
See text for details.

The plate current level shifted but slightly, and the two halves of each tube compared quite well. In the past 1000 hours two tubes have shown some considerable deterioration of the conducting side. There seems to be some cathode damage which causes "droop" of the type discussed above. There has been no opportunity for careful analysis of these tubes as yet. It is very likely that the cathodes were damaged by the higher level of dissipation during the past 1000 hours.

Only one of the ten sections of the 5965's has developed any interface impedance by 2500 hours; the value is only 2 ohms. It seems quite likely that interface impedance will not be a problem with this tube. Some electrical leakage has been observed, but the leakage currents were well under 1 microamp. Neither appreciable gas nor grid emission has been observed.

4.2 Component Replacements in WWI

Fig. 4-2 lists the replacements of components other than tubes during the third quarter of 1953 (see p. 43).

Component	Type	Total in Service	No. of Failures	Hours of Operation	Comments
Capacitors	.1 μf 600 volt oil filled bath tub	97	1	1000-2000	Faulty end seal
	220 μμf ±10% mica	52	1	0-1000	Shorted
	7-45 ceramic condenser (trimmer)	122	1	9000-10000	Open
	.01 μf 500 volt disc ceramic	3315	2	0-1000	Shorted
Crystals	D-358/1N38A	13875			
	D-358		3 5	8000-9000 19000-20000	2 low I_b; 1 drift Low I_b
	1N38A		3 1 1 1	0-1000 1000-2000 7000-8000 9000-10000	2 open; 1 high forward and low I_b Low I_b Low I_b Low I_b
	D-357/1N34A	3948			
	D-357		1 2	10000-11000 13000-14000	Low I_b and drift Low I_b and drift
	1N34A		1 4 2 1	0-1000 2000-3000 7000-8000 9000-10000	Low I_b and drift Low I_b Low I_b Shorted
Potentiometers	1000 ohm 2 watt carbon	114	1	15000-16000	Intermittent contact
	10000 ohm 2 watt carbon	103	1	9000-10000	Noisy
Resistors	5000 ohm 1 watt ±1% deposited carbon	360	2 1	9000-10000 12000-13000	Above tolerance Above tolerance
	100 ohm 1 watt ±5% carbon	734	1	10000-11000	Overheated
	1200 ohm 1 watt ±5%	260	1	18000-19000	Overheated
	220 ohm 1/2 watt 10% carbon	9773	5	0-1000	1 burned up; 3 above tolerance; 1 under tolerance
	5100 ohm 1 watt ±1% carbon	24	1	1000-2000	Above tolerance
Switch	Cutler Hammer 7501K12 SPST toggle	614	1	19000-20000	Intermittent
Transformer	Pulse 3:1 1/2:1	39	1	0-1000	Intermittent
	5:1	33	1	18000-19000	Primary open

Fig. 4-2. WWI Component Failures July 1 - September 30, 1953

5. ACADEMIC PROGRAM

5.1 Summer Session 1953

A special two-week summer program in digital computers and their applications, similar to the one given in the summer of 1952, was offered at MIT from August 24 to September 4, 1953. Much of the staff of the Scientific and Engineering Computation Group was temporarily assigned to the MIT Summer Session as lecturers and instructors. In conjunction with the course, a very elaborate conversion and interpretive routine was developed to make the Whirlwind I Computer simulate a hypothetical, idealized computer as described in Section 5.4 of this quarterly report.

The class of 106 students represented 67 different organizations, including 28 from 18 commercial and business groups, 53 from 39 industrial research groups, 25 from 10 military and government establishments. The students attended lectures each morning. During the afternoons they were divided into 9 sections (about 12 students per instructor) and worked in teams, of about 4 students each, on the preparation of actual summer session computer programs for the solution of two different problems. Each team selected one problem from each of the two groups below, the first being simple problems which were programmed, punched, and run on the Whirlwind I computer during the first 4 days of the course, the second being longer problems which were generally, but not universally, completed by the student and run on the Whirlwind I computer. The poker problem (V) was suggested for those interested in logistics and turned out to be the most difficult.

Minor Problems	I	(a)	Plot the trajectory of a ball bouncing on a horizontal plane.
		(b)	Same, making it hit a hole in the plane.
	II	(a)	Print the final value of $1.00 deposited at 3% interest compounded annually since 1626, rounded down each year.
		(b)	Same, rounded to nearest even cent each year.
Major Problems	III		Mortgage Loan Remittance Allocation.
	IV		Inventory Control.
	V		Selection of the winning hand from 5 given poker hands.
	VI		Determination of the Roots of a Polynomial.

VII	Solution of a System of Simultaneous Linear Algebraic Equations
VIII	Solution of $\partial^2 u/\partial x^2 = c\partial u/\partial t$ with $u(o, t) = u(a, t) = u_0$; $u(x, 0) = 0$
IX	Analytical Treatment of Power Series

In addition to Dr. Stanley Gill of Cambridge University, who spent the summer at the Digital Computer Laboratory, Dr. M. V. Wilkes of Cambridge and Mr. Bruse Moncrieff of Battelle Memorial Institute spent the two weeks at MIT as guests and contributed to the course as lecturers and advisers. Ten manufacturers of commercial computing equipment sent representatives to describe their equipment during four evening gatherings.

5.2 ACM Meeting at MIT

Five hundred and twenty-eight persons registered at a three-day annual meeting of the Association for Computing Machinery held at Massachusetts Institute of Technology on September 9, 10, and 11, 1953. Seventy papers were presented in two sets of parallel sessions as follows:

Wednesday	
Punched Card Techniques	Digital Computer Techniques
Punched Card Techniques	Digital Computer Mathematics
Numerical Analysis	Recent Systems Developments
Thursday	
Digital Computer Programming	Punched Card Mathematics
Digital Computer Programming	Logical Algorithms
Friday	
Numerical Solution of Partial Differential Equations	Analog Computation
Operation of a Computation Center	Analog Computation
Operation of a Computation Center	Business Data Handling

The Digital Computer Laboratory cooperated with the Office of Statistical Services and with the Industrial Liaison Office, Dynamic Analysis and Control Laboratory, and the Departments of Physics, Mathematics and Electrical Engineering in the actual conduct of the meeting.

5.3 Fall Term 1953

6.535 Introduction to Digital Computer Coding and Logic

Notes similar to those used for the two-week summer session program and featuring the so-called Summer Session Computer are being used in this regular graduate subject for which 35 students have registered. For the first time, students in 6.535 are preparing and actually executing a practice program, using the Whirlwind I Computer to simulate the Summer Session Computer.

6.567T Switching Circuits

Two sections are scheduled to handle the 38 students registered for this rapidly expanding course. An excellently equipped laboratory augments the Boolean algebra theory with practice.

6.25 Machine-Aided Analysis

This new survey course in machine techniques is intended to acquaint Electrical Engineering seniors, and others, with the potentials of analog computers, numerical analysis, punched card equipment and digital computers. Professors Linvill, Scott, and Adams, and Drs. Verzuh and Booton are collaborating in offering this course to 25 students. Each student will, among other things, solve a sample differential equation by hand and on a REAC, a CPC, and Whirlwind I.

M411 Numerical Analysis

Offered for the second time by Professor F.B. Hildebrand in the Mathematics Department and using the text by Milne and course notes, M411 has 24 registered students and four listeners.

2.215 Numerical Analysis

Professor S.H. Crandall of the Mechanical Engineering Department uses comprehensive notes of his own in this second-year-graduate course being taken by 8 students.

Digital Computer Laboratory Program Preparation Office

To facilitate the class work in 6.535 and 6.25 as well as to make the Laboratory facilities more accessible to all MIT users, an office, equipped with two

Flexowriters, a film viewer, etc., has been set up within the Department of Electrical Engineering area in Building Ten. A senior staff member is available as a consultant on new problems and to aid programmers generally. A technical assistant spends full time working for MIT users, Half of her time is spent in the office assisting users in making tapes, correcting mistakes, requesting computation time, collecting and interpreting results, and so forth; the other half of her time is spent in the Barta Building processing the tapes and performance requests and otherwise acting as an agent for the student users. This new facility opened its doors on the last day of the quarter and no definite evidence is yet available as to the success of the undertaking.

5.4 Summer Session Computer

The MIT Summer Session Computer, hereafter referred to as the SS Computer, does not exist as an assembly of electronic apparatus; rather its realization is achieved by appropriate conversion, assembly, and interpretive routines operating in the Whirlwind I Computer.

In order to allow students to write and operate programs within only a few days after their introduction to the basic concepts of digital computers, programming had to be easy to learn and teach. In addition, it was necessary to provide means for finding mistakes in programs, means which were simple to use and the results of which were easy to interpret, so that, within the very short time available, students would write one or more programs and run them successfully on the computer. The SS Computer, the development of which required about 12 man-months of work by experienced programmers, is an attempt to achieve these goals.

5.41 Description of the SS Computer

The SS Computer is a single-address, medium-speed (about 600 operations per second) digital computer with a basic word length of 28 binary digits and a 4-binary-digit tag. Each word is stored in two consecutive 16-binary-digit Whirlwind I registers. There are three kinds of words: (1) fixed-point integers, (2) floating-point numbers, and (3) instructions. Input to the computer is by means of punched Flexowriter paper tape; output equipment includes a direct printer, a "delayed" printer for later printing of information recorded at high speed on magnetic tape, and an oscilloscope, on which individual points may be plotted by the computer and the result observed and/or simultaneously automatically photographed.

Integers and Numbers

Fixed point integers are represented by 28 bits, the first being the sign, the last 27 bits representing the magnitude of the integer.* Since $2^{27} = 134,217,728$ is approximately equal to 10^8, integers may be thought of as roughly equivalent to 8 decimal digits.

An integer is written in an SS program with a + sign (may be omitted) or - sign, followed by 1 to 8 decimal digits, and is terminated by either a tab or carriage return.

Floating point numbers are numbers in the form $A \cdot 2^B$, where B is an integer and A is a fraction with $\frac{1}{2} \leq |A| < 1$. In the SS Computer the mantissa A is represented by a sign and 20 bits, the exponent B by a sign and 6 bits. Since $2^{20} = 1,048,576$ is approximately equal to 10^6, numbers in the SS computer have 6 decimal digits of precision. The 6 binary digits available for the magnitude of the exponents allow non-zero numbers to range in magnitude from 2^{-64} to 2^{63} or approximately from 10^{-19} to 10^{+19}. Zero has the special representation 0×2^{-63}, i.e., a zero mantissa, and a negative exponent of the largest permissible magnitude. A number written with a decimal point is treated as a floating-point number. Alternatively, or in addition to the decimal point, a number intended as a floating-point number may be followed by the letter x and 10 to some power; thus any of the following are treated as floating-point numbers:

$$-12.73 \qquad +.0063 \times 10^{-2} \qquad +.0 \qquad 97.6 \times 10^4$$

Arithmetic Element

The Arithmetic Element consists of principally an Accumulator (AC), which deals with either integers or floating-point numbers as the situation demands.

When integers are involved, the AC contains a sign and 54 bits, i.e., is double-length. Another register, known as the Remainder Register (RR) may be thought of as a kind of right-hand extension of the AC -- the RR holds the remainder after unrounded division of one integer by another.

* The remaining 4 bits of the 32 available comprise a so-called logical information tag. This tag contains information about the kind of word, i.e., integer, number, instruction, or undefined, and also whether the word has been altered from its original form during the operation of a program.

Sums, differences, products, or quotients may be as large as 2^{32767} without exceeding the capacity of AC, but only numbers less than $2^{63} \approx 10^{19}$ in magnitude may be copied from AC into storage. Numbers which become smaller than 10^{-19} are automatically set to zero when copied into storage.

To discourage little tricks and to help isolate real mistakes, one special restriction is that integers and numbers may not be mixed in the AC; e.g., it is not permitted to add an integer to a number. If such mixed operation is attempted the computer stops and prints out information likely to be useful to the programmer in diagnosing the mistake. This automatic print-out when a programming mistake is made is called a "computation post-mortem," and more will be said of it later.

Words in the SS Computer

Instructions are represented in the SS Computer by an operation section, an address section, and an additional "counter letter" to select one of 7 counters, or no counter at all. The counters are used for cycle-counting and address modification (like the Manchester B-box), as will be explained.

There are 35 operations, including: arithmetic operations (most of which apply equally well to either numbers or integers), operations which copy words from one place to another, "jump" operations for interrupting the normally consecutive carrying out of instructions, operations for changing the contents of the counters, and operations for controlling the in-out equipment. Operations are specified by the programmer as a mnemonic combination of three lower case letters, a tabulation of which is given on the next two pages along with the meanings of the three letters, the definition of each associated instruction, and information about what may cause a post-mortem when performing each instruction.

Addresses may be written in absolute or floating form. An absolute address is any positive integer 0 through 299. (This limitation thus restricts SS programs to a maximum length of 300 words.) A floating address is a single lower case letter (except o or l) followed by not more than 3 digits.

Floating addresses are used as part of an instruction by writing, for example:

ccf b3

The word referred to by the instruction ccf b3 must have the floating address b3 assigned to it. This is done by using a comma; thus

b3, 750

will tag the register containing the integer 750 with the floating address b3 so that all instructions in the program with b3 as their address sections will refer to this same register.

INSTRUCTION CODE OF THE MIT SUMMER SESSION COMPUTER

INSTRUCTION	MEANING	DEFINITION	POST-MORTEM*if
ccf al b	copy contents from	$C(al+i_b) \to AC$	14L
cci al b	copy contents into	$C(AC) \to al+i_b$	4A, 5A, 9U
cnf al b	copy negative from	$-C(al+i_b) \to AC$	14L, 15L
cmf al b	copy magnitude from	$\lvert C(al+i_b) \rvert \to AC$	14L, 15L
cri al b	copy remainder into	$C(RR) \to al+i_b$	
xch al b	exchange	$C(AC) \to al+i_b$, $C(al+i_b) \to AC$	4A, 5A, 14L, 9U
add al b	add	$C(AC) + C(al+i_b) \to AC$	1A, 12L, 3U
sub al b	subtract	$C(AC) - C(al+i_b) \to AC$	1A, 12L, 3U
mby al b	multiply by	$C(AC) \times C(al+i_b) \to AC$	1A, 12L, 3U
dby al b	divide by	divide $C(AC)$ by $C(al+i_b)$ rounded quotient $\to AC$	11A, 12L, 3U
dhr al b	divide holding remainder	divide $C(AC)$ by $C(al+i_b)$ quotient $\to AC$, remainder $\to RR$	11A, 13L
txi al b	transfer excess into	divide $C(AC)$ by 2^{27} quotient $\to al+i_b$, remainder $\to AC$	10U
jmp al b	jump	take next instr. from $al+i_b$	17L
jip al b	jump if positive	ditto, if $C(AC) > 0$	8L, 17L, 9U
jin al b	jump if negative	ditto, if $C(AC) < 0$	8L, 17L, 9U
jiz al b	jump if zero	ditto, if $C(AC) = 0$	8L, 17L, 9U
jir al b	jump if remainder	ditto, if $C(RR) \neq 0$	17L
jix al b	jump if excess	ditto, if $\lvert C(AC) \rvert \geq 2^{27}$	17L
sra al b	set return address**	replace address section of $C(al+i_b)$ with 1+the address of the register containing the most recent jmp or conditional jump which took effect	16L
caf al b	copy address from**	address section only (as an integer) of $C(al+i_b) \to AC$	16L
cai al b	copy address into**	$C(AC)$ becomes the new address section of $C(al+i_b)$	7A, 16L
rst m b	reset (counter b)	set $i_b = 0$, $n_b = m$	19U
jii al b	jump if incomplete	increase i_b by 1, then jump to al if $i_b < n_b$	18A, 17L
jic al b	jump if complete	increase i_b by 1, then jump to al if $i_b \geq n_b$	18A, 17L
inc m b	increase (counter b)	increase both i_b and n_b by m	18A, 19U
dec m b	decrease (counter b)	decrease both i_b and n_b by m	18A, 19U
cii al b	copy index into	i_b as an integer $\to al$	
cnv al b	convert	$C(AC)$ as an integer $\to AC$ $C(AC)$ as a number $\to al+i_b$	1A, 8L, 9U
stp 0	stop	stop the computation	

*The programming mistakes which result in a post-mortem are listed on the next page. A post-mortem results while performing an instruction if any of the programming mistakes listed with that instruction are made. A post-mortem will always occur if $(al+i_b) \geq 300$ or if $(al+i_b) < 0$.

**When executing this instruction, a counter letter, if any, is not considered part of the address section of the instruction in register $al+i_b$.

INSTRUCTION	MEANING	DEFINITION	POST-MORTEM*if
pat al b	plot at	plot a point on the scope at x = C(al+i_b) and y = C(AC)	6A, 8L, 14L 15L, 9U
frc 0	frame (scope) camera	move the next film frame into place and open the camera shutter if it was closed	
ric 0	read in character	read the next char. via the MTR into AC as a pos. integer≤77	(Comp. stops if no tape in MTR)
rin 0	read in numerically	read the next complete integer or number via the MTR into AC	1A, 2A, (also see ric above)
tyc m tyc m+100	type character	record on delayed printer (m), or on direct printer (m+100), the Flexo. char. specified by m	20L
tyn m tyn m+100	type numerical value	record on delayed printer (m), or on direct printer (m+100), C(AC) as specified by m (See table below)	4A, 5A, 8L 9U, 21U

Tabulation of m values for use with tyn

m	initial zeros	no. of digits printed = d	total space pos.	total space neg.	zero prints as	
0	ignored	1≤d≤9	d	d+1	0	
1-9	printed	d=m	m	m+1	m 0's	$\|C(AC)\| \geq 10^{m}$
11-19	spaced over	1≤d≤(m-10)	m-10	m-9	see examples	$\|C(AC)\| \geq 10^{m-10}$
21-29	ignored	d=m-20	m-11	m-11	0	

Examples	C(AC) = 1234	C(AC) = -789	C(AC) = .004786	C(AC) = 13.57	Direct/Delayed
tyn0	1234	-789	0	14	Delayed
tyn103	Post-Mortem	-789	000	014	Direct
tyn5	01234	-00789	00000	00014	Delayed
tyn116	**1234	***-789	*****0	****14	Direct
tyn23	*1.23x10^{3}***	-7.89x10^{2}***	*4.79x10^{-3}**	*1.36x10****	Delayed

* represents a space on the printed copy

PROGRAMMING MISTAKES which cause a POST-MORTEM

1A. Result is an integer of magnitude $\geq 2^{54}$
2A. Result is a number of magnitude $\geq 2^{63}$
3U. Result is a number of magnitude $\geq 2^{32767}$
4A. Result is a number of magnitude $\geq 2^{27}$
5A. C(AC) is a number of magnitude $\geq 2^{63}$
6A. |C(AC)| ≥ 1024 or |C(al+i_b)| ≥ 1024
7A. C(AC) is not a positive integer < 300
8L. C(AC) is an instruction
9U. C(AC) is undefined
10U. C(AC) is not an integer
11A. C(al+i_b) = 0
12L. C(AC) and C(al+i_b) are not either both integers or both numbers
13L. C(AC) and C(al+i_b) are not both integers
14L. C(al+i_b) is undefined
15L. C(al+i_b) is an instruction
16L. C(al+i_b) is not an instruction
17L. If C(al+i_b) is not an instruction and the jump takes effect, the Post-Mortem will occur after the jump is executed
18A. Resulting magnitude of $i_b \geq 512$
19U. m≥512
20L. m>77 or m corresponds to an illegal Flexo character
21U. m=10 or m=20 or m≥30

A - Arithmetic overflow   L - Logical mistake   U - Unlikely mistake

DEFINITIONS OF SYMBOLS

⟶	becomes the new contents of
AC	Accumulator
C(al)	Contents of register al. al represents any floating address, i.e., any letter except o or l, followed by any non-negative decimal integer < 1000.
C(al+i_b)	Contents of the register whose address is obtained by adding to al the value of i_b.
i_b	The index associated with counter b, where b represents any of the 7 counters a, b, c, d, e, f or g. Except for the 6 instructions rst, jii, jic, inc, dec, cii, a counter letter need not be specified at all.
n_b	The criterion associated with counter b.
RR	Remainder Register, which holds the remainder after dhr and is not changed by any other instruction.
MTR	Mechanical Tape Reader into which is inserted a punched Flexo tape to be read in under the control of the computer.

For correction purposes only, words may be assigned to registers by writing, for example,

b3|-750

Following the floating address by the vertical bar instead of a comma results in the previous contents of the register b3 being replaced, in this case, by the integer -750.*

A counter letter (a, b, c, d, e, f, or g) may be appended to the address section of an instruction, and the contents of the specified index will be added to the address section of any such instruction before the instruction is executed without changing the original form of the instruction in storage. Each of the 7 counters consists of an index and a criterion, i_a and n_a, i_b and n_b, etc., respectively. In an ordinary cyclic process i_b, for example, is set to 0 and the criterion is set to some value n_b. Then, for each step in the cycle, i_b is increased by 1 until $i_b = n_b$, at which time the cycle is complete. The counters are designed primarily for counting and for modifying addresses, although other applications are possible.

Programmed Output

There are three output devices: (1) A scope on which discrete points may be plotted. A camera is attached to the scope so that a display may be photographed if a permanent record is desired. The operation pat and frc are used in controlling the scope. (2) A "direct" typewriter on which integers and arbitrary characters may be recorded at the rate of 10 per second. (3) A "delayed" typewriter on which the same sort of characters may be recorded in Flexowriter-coded form on magnetic tape at a rate of 125 per second and later typed out at 10 per second while the computer is doing something else. The instruction tyc m will type the Flexo character whose octal equivalent is equal to the address section m. The instruction tyn m will type out the contents of AC as a series of decimal digits, the particular form of the print and the number of digits to be printed being specified by the address section m. Details of these very useful instructions are given in the tabulation of the operation code.

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* Words to which no floating address has been assigned may also be modified. For example, if instead of the word in b3, the fifth word after the one in b3 were to be changed to +625, the word assignment would be

b3+5| +625

Programmed Input

Once a program has been read into the computer more data can be supplied to it only by using the operations ric or rin. Both these operations control the Mechanical Tape Reader (MTR) into which is inserted a punched Flexo tape. The operation ric is used to read in individual characters punched in the tape; rin is used to read in an entire integer or number, the termination of which is indicated by a tab or carriage return character punched on the tape.

5.42 Program Preparation

Programs are prepared for input to the computer by typing them on a Flexowriter tape perforator. The sequence is as follows.

1. The first line of the typed program consists of at least one lower case letter s followed by an identifying program title followed by a carriage return.
2. 25 or more equal signs followed by a carriage return.
3. The program itself, consisting of integers and/or numbers, and instruction; each such word must be terminated by one or more tabs or carriage returns.
4. Any word assignments (e.g., b3|-750) which are necessary, each terminated as in 3. above.
5. The address at which the program is to start operating followed by a vertical bar, followed by the word start in lower case.

Examples are:

a7|start 127|start g6+3|start

5.43 Post-Mortems

When an error has been made by the programmer, and that error is detected either during input or operation of the program, the computer stops and prints out information about that error. Such a print-out is called a post-mortem. There are two types of post-mortems in the SS computer.

Conversion Post-Mortem

The punched program tape is read into the computer through the photoelectric tape reader. After the tape has been read in and the binary equivalents of the characters stored, the SS conversion program processes the stored data and eventually produces a sequence of words in binary form which will be correctly interpreted as a program by the SS computer interpretive routine. If, however, certain logical or typographical errors have been made which are detectable during the

the conversion process, the computer will stop the conversion and print on the direct typewriter the title of the program tape followed by a description of the mistake and its location in floating address form.

The mistakes detected by the conversion program are:

1. unassigned floating address
2. undefined instruction
3. floating address assigned to two or more registers
4. absolute address too large
5. program longer than 300 words
6. integer or number too large
7. numerical part of floating address too large
8. no counter letter specified in rst, jii, jic, inc, dec, or cii instruction.

Computation Post-Mortem

If, during the operation of a correctly converted program, a situation arises which is defined as a programming mistake (see list page 51), the computer automatically records on magnetic tape certain information and then stops. The recorded information is known as a Computation Post Mortem, and is subsequently typed out on the delayed typewriter. A typical Computation Post Mortem might appear as follows:

ss program 27, John Smith, Sept. 23, 1953
STOPPED AT dl+6 dl+6|patal7 al7|-981
AC|1034 RR|5
h4..j8 d2..dl+8 (dl..dl+8)^{21} dl..a8 z7..p97+3 q6..q9
p97+4..a5-1 h6..h7+4 r4..r6-2 t8..y37-1 d2..dl+6 stop
COUNTERS a|2,5 b|6,6 c|23,23 d|12,12 e|0,0 f|0,0 g|0,0
il|19 -601 al7|-981 p902|ccfp920d q9|jmpp97+4 etc.

The information given is as follows:

Line 1: the program title for identification purposes.

Line 2: the computer stopped while performing the patal7 instruction in dl+6. al7 contained the integer -981.

Line 3: The AC contained 1034, which is why the computer stopped on the patal7 instruction. (See programming mistake 6A in the operation code.) The Remainder Register (RR) contained 5 -- had the RR not been used, no information about it would have been printed.

Lines 4 and 5: traces the path the computer followed over the ten most recent jump instructions -- only those jumps which actually were effective are included. The example given shows that the computer performed each instruction from h4 to j8; then some kind of a jump instruction in j8 took it to d2. Each instruction from d2 to dl+8 was carried out; then the sequence dl to dl+8 was repeated 21 times. The next time through this sequence it went right on through to a8, whence some kind of jump took it to z7, etc. Each address is given in terms of the nearest floating address.

Line 6: gives the index and criterion, in that order, for each of the 7 counters -- had no counters been used this item would not appear.

Line 7, etc.: lists the address, in terms of the nearest floating address, and the contents of every register whose contents have been altered during the program. If the contents of several consecutive registers have been changed, an address is given for the first and for each one to which a floating address has been assigned.

6. APPENDIX

6.1 Reports and Publications

Project Whirlwind technical reports and memorandums are routinely distributed to only a restricted group known to have a particular interest in the Project, and to ASTIA (Armed Services Technical Information Agency) Document Service Center, Knott Building, Dayton, Ohio. Regular requests for copies of individual reports should be made to ASTIA; emergency requests, to Robert R. Rathbone, Digital Computer Laboratory, 68 Albany Street, Cambridge 39, Mass. Att: Code DCL-6.1.

The following reports and memorandums were among those issued during the third quarter of 1953.

No.	Title	No. of Pages	Date	Author
SR-34	Summary Report No. 34, Second Quarter 1953	40		
R-225	Treatment of Digital Control Systems and Numerical Processes in the Frequency Domain	95	7-1-53	Ed. by E. Craig
E-549	Basic Conversion Program	4	7-2-53	H. Briscoe
M-2184	Marginal Checking Systems Mod II, WWI	15	7-20-53	J.H. Hughes
M-2269	Use of the Magnetic Tape and Delayed Output Equipment	32	7-3-53	H. Denman
M-2316	Proposed Sense Winding for a 64 x 64 Memory Plane	2	7-23-53	W. Canty
M-2319	Procedure for Handling Cores During Testing	2	7-23-53	J. Schallerer
M-2357	An Approach to the Writing of Tube Specifications	5	8-12-53	R. Fallows
M-2381	Statistical Filtering by Inverse Probability	44	8-24-53	W. Wells
M-2383	Testing the Magnetic-Core Memory System in a Computer	7	9-18-53	B. Widrow
M-2420	Interpretation of Memory-Core Specifications	2	9-22-53	D. Brown
M-2425	Digital Techniques for Sorting by Areas in a Plane	4	9-24-53	G. Young

6.2 Professional Society Papers

At the IRE West Coast Convention, on August 19, D.R. Brown spoke on "Magnetic Materials for Digital Computers."

J. Porter presented a paper on "General Operation of Digital Computers" at the Fourteenth Annual IRE Seminar, held on August 21-22 at Emporium, Pa.

At the National Bureau of Standards in Corona, California, D.A. Buck gave a series of talks on July 6, 7, and 8. They were entitled "Magnetic and Dielectric Amplifiers," "Ferromagnetic and Ferroelectric Memory Systems," "Ferromagnetic and Ferroelectric Switches," and "Nondestructive Sensing of Magnetic Cores."

D.A. Buck also spoke on "Nondestructive Sensing of Magnetic Cores" at UCLA on July 6; and gave a series of ten lectures on "Ferromagnetic and Ferroelectric Components for Digital Computers" at Wayne University, Detroit, Michigan, August 10-21.

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