

## INTERMEDIATE MATH \& GEOMETRY ${ }^{\text {" }}$ Teacher's Guide

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## Acknowledgements:

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## $\triangle$ WARNING: $\triangle$ AVERTISSEMENT:

CHOKING HAZARD - Small parts. DANGER D'ÉTOUFFEMENT - Piéces de petite taille. Not for children under 3 years. Ne pas donner aux enfants de moins de 3 ans.

## Table of Contents

Introduction ..... 4
Teacher's Notes ..... 6
NCTM Standards Alignment Chart ..... 8
I. Measurement and Geometry
Lesson 1 Point, Line, Line Segment, and Ray Conventions
Recognize, compare and sort points, lines, line segments, and rays ..... 10
Lesson 2 Angle Conventions
Define, name, measure, and compare angles ..... 12
Lesson 3 Length Conventions
Develop concepts and vocabulary related to length and measurement ..... 14
Lesson 4 Equalities and Inequalities
Compare and contrast the measurement of line segments ..... 16
Lesson 5 Can You Name that Shape?
Recognize, build, draw and compare shapes; identify angles in shapes ..... 20
Lesson 6 Perimeter Concepts
Develop concepts and vocabulary related to perimeter ..... 22
LESSON 7 Symmetry
Develop concepts and vocabulary related to symmetry ..... 26
Lesson 9 Area Concepts: Polygons
Develop concepts and vocabulary related to area ..... 34
Lesson 10 The Area and Perimeter of Polygons
Investigate the relationship between area and perimeter ..... 38
Lesson 12 Transformations: Translations (Slides) and Reflections (Flips)
Identify and create flips and slides; review congruence and symmetry ..... 46
Lesson 13 Exploring Transformations: Rotations (Turns) Identify and create rotations; compare and contrast flip, slide, and rotation ..... 50
II. Number and Operations
Lesson 8 Probability ConceptsDescribe probability as likely, unlikely, impossible, or certain30
Lesson 11 Exploring Fractions
Define, describe and explore fractions ..... 42
III. Algebra
Lesson 2 Length Conventions
Use algebraic language to describe length and measurement ..... 12
Lesson 4 Equalities and Inequalities
Use algebraic language to describe and compare measurements ..... 16
Lesson 6 Perimeter Concepts
Express perimeter in algebraic terms ..... 22
Lesson 9 Area Concepts: Polygons
Use algebraic language to describe geometric shapes ..... 34
Lesson 10 Area and Perimeter of Polygons
Use algebraic language to describe geometric shapes and the relationship between perimeter and area ..... 38
Lesson 11 Exploring Fractions
Investigate the relationship between area and perimeter ..... 42
Lesson 14 Rules Make Patterns
Identify attributes of a pattern; extend a pattern; express a pattern algebraically ..... 54
Glossary ..... 58

## Area Concepts: Polygons

## Lesson Topics: The Measurement, Computing and Reporting of the Area of Polygons Using K'NEX.

Lesson Length: 60-minutes

## Student Objectives:

## Students will:

- Describe the area of polygons using math symbols.
- Investigate and explore the concept of area.
- Determine and describe the area of polygons.


## Grouping for Instruction:

- Four groups of 3-4 students each for building models.
- Individual students for drawing models, labeling sides, and listing a representation of area measurements.
- Individual students for performance assessment and written assessment activities.


## Materials and Equipment:

- K'NEX Intermediate

Math \& Geometry set and Instructions Booklets

- Chart paper (one sheet per group)
> Pencils
- Colored pencils or crayons
> Journals and drawing paper


## Overview of Lesson:

- Students will use K'NEX materials to build, investigate, draw, and describe the area of polygons.
- Students will use K'NEX mathematics conventions as they describe the area of polygons.
- Individual students will transfer the polygons they construct to paper and describe the area of these polygons with appropriate mathematics terminology to demonstrate their individual understanding of the relationship between the K'NEX models, drawings and their areas.
- Students will include representative polygon drawings and area solutions for a variety of models they or their classmates create.


## Background Information:

- Students should have had exposure to the basics of length measurement and concepts they investigated in Lessons \#3 and \#4. They will use a non-standard measurement system to describe the area of polygons. Actual number values may be attached to the length of the various K'NEX line segments in future years after the students have internalized a working understanding of measurement and area concepts.


## A - Motivation and Introduction:

1. Ask student groups to prepare a collection of at least four two-dimensional rectangular shapes using the blue and red rods. (For this activity, students can use red, green, yellow, or white connectors.)
2. Provide time for creative exploration with the rectangles. Do students only stay with one-colored rod to make squares that are also rectangles? Do they connect blue rods to red rods to make larger rectangles that are not squares? If so, how do they sequence them? How many different rectangles have been created in the classroom?
3. Lead students to realize that one characteristic of their rectangle is area, the space within the figure
and that this space is measured by the number of square units that are needed to cover the figure.
4. Have students discuss how they might express the area of the rectangles that they have made. Ask several groups to share how they would describe the area of a given rectangle. Lead students to realize that they can express the area of each of their rectangles by how many square rectangles with four blue sides are needed to cover a given rectangle that they have made. Have students call this unit a "blue square" and the smallest rectangle they have made would be a blue square, so its area would be one blue square. (If your students are familiar with exponents, they may express one blue square using $K^{\prime} N E X$ conventions with the term $b^{2}$.)

## B - Development (including discussion points and feedback):

1. Instruct students to place all the rectangles that their group has constructed on a single sheet of chart paper.
2. Ask students to work as a group to transfer all of their polygons to the paper using pencils or crayons and rulers. Students should use a blue crayon/pencil to draw the blue rods and a red crayon/pencil to draw the red rods.

## CAUTION FOR STUDENTS:

Complete the drawing of each polygon before moving on to the next. It is very easy to make drawing errors when facing a sheet of 20 or 30 vertices [points] and not being able to remember which ones go with which rectangle.
3. Instruct students to discuss the area of each model and to use a pencil or possibly a blue square model to determine the area of the rectangles they have created. List the number of blue squares in each model to the side of the shape.
4. Use a round-robin-response format and ask groups to hold up one of their polygons and to describe its area orally.

## TEACHER NOTES:

Reinforce students' oral information by sketching each figure and writing the student's response on the chalk board or white board.
5. Ask the other groups to verify their understanding of the information provided by each group.
6. Continue taking responses from the various groups until you are confident that students are providing informed responses that demonstrate both an understanding of area and a proper description of area values expressed in blue squares.
7. When students seem comfortable with this activity, provide them with the area of a rectangle and see if they are able to build a rectangle to match the given area. This requires that students transfer what they have learned and practiced to a new learning situation. The ability to successfully complete these activities will provide very useful diagnostic data that you can use to determine the level of your students' understanding.
8. Here are suggested areas (with answers) that you can provide to the students when you ask them to make a model that matches the given information.
Possible Questions and Answers:

- Area $=4$ blue squares (multiple answers)
- 1 red square
- 1 rectangle with two $b$ sides and two sides of $2 r$
- Area $=6$ blue squares
- 1 rectangle with two sides of $r$ and two sides of $r+b$
- 1 rectangle with two sides of $b$ and two sides of $3 r$
- Area $=8$ blue squares
-1 rectangle with two sides of $r$ and two sides of $2 r$
- 1 rectangle with two sides of $b$ and two sides of $4 r$
Let your imagination and the skill level of your students determine any other appropriate area challenges to use.

9. Provide area challenges of irregular polygons for students to solve.

- Find the area of the L-shaped polygon in blue squares (on page 13 of the Instructions Booklet):
- Solution: $A=3 b^{2}$
- Find the area of the L -shaped polygon in blue squares (on page 14 of the Instructions Booklet):
- Solution: $A=4 b^{2}$
- Find the area of the cross-shaped polygon in white squares (on page 13 of the Instructions Booklet): - Solution: $A=5 w^{2}$

Ask each group to make models of three different figures that have an area of 9 blue squares. These figures may be either rectangles or irregular polygons. Instruct students to transfer the models to paper and show the area for each by drawing in the 9 blue squares using a blue crayon or blue pencil.

## TEACHER NOTES:

Based on the abilities that your students demonstrate, you may use all of the challenges above or you may select individual challenges for your students. You may also choose to use some of these challenges for performance assessment questions on a quiz or test.
10. Ask students to construct the model of the concave hexagon on page 13 of the Instructions Booklet. Challenge the students to see if they can discover a way that they can find the area of that shape in blue squares. Provide ample time for them to explore and move about the groups to provide encouragement.

- If students experience difficulty with the challenge, ask them if there are any rods in the shape that they can replace with blue rods.
- Yes! The red rods can be replaced by 2 blue rods with a connector in between.
- Also, is it possible to place blue rods in the shape to break it down into smaller units?
- Yes, 2 blue rods and 1 connector can be used to connect the green connectors that form two of the vertices.
- A bit more experimentation should help students to see that they can fill the shape in with either:
-8 triangles that are $1 / 2$ blue squares each
or
- 2 blue squares and 4 triangles that are $1 / 2$ of a blue square each
- In either case, the area of the shape is $4 b^{2}$.

11. Ask the students to construct the pentagon model on page 14 of the Instructions Booklet. Challenge the students to find the area of the shape in blue squares.

- They will use the same strategy that they did with the shape in \#10 above.
- Solution: $A=3 b^{2}$.
- Further challenge the students to find the area of the same shape in yellow squares.
- Solution: $A=11 / 2 y^{2}$.


## C - Summary and Closure

1. Have a collection of polygon models made up in advance, some rectangles and some irregular polygons.
2. Hold up the models one at a time. Ask students to write down the appropriate letters and symbols that could be used to describe the area of the rectangle or irregular polygons.
3. Ask students to write down three things that they learned in class today.
4. Have students share their statements with students in their group.
5. Have each group share at least one observation with the class.

## Assessment:

- Establish a series of four or five performance assessment stations around the room where students can visit during a pencil and paper test or quiz that they are working on at their seats.


## Questions may include:

- Use the pieces provided to make a polygon that has an area of 3 yellow squares. (When a student has made the model, he or she can use a piece of masking tape to place his or her name on their creation before placing the model in the box.)
- Place polygon models at several stations. Some models should be rectangles and some should be irregular polygons. Provide a question at each station. What is the area of the polygon?


## Extensions:

- Challenge the students to build models of polygons to use in the assessment scenarios.
- Student models can be placed about the room or hung from hooks above the board for other students to solve. This activity works best if the polygons are
numbered. That allows the students to hand in a sheet with the polygon numbers and their appropriate areas.
- As students develop their skills with the determination of area they will eventually be able to express the area of many shapes
in terms of green, white, blue, yellow, red, or silver squares.
- They will also discover that they can build some shapes that include angles less than 90 degrees which they will be unable to solve.

