



Lesson 10

Glencoe Geometry Chapter 4.6

Analyzing Isosceles Triangles

By the end of this lesson, you should be able to

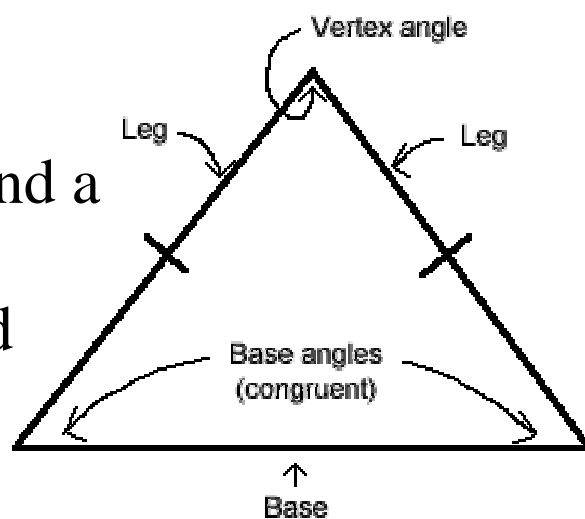
1. Use properties of isosceles and equilateral triangles.
2. Look forward to next week's show.

This week we are **AGAIN** sticking with our topic from the last two weeks: **TRIANGLES**

Today, we are going to focus primarily on the properties of isosceles triangles, which will naturally lead us into the realm of equilateral triangles as well.

To review:

An *isosceles triangle* has at least two congruent sides called *legs* and a third side called the *base*. The *vertex angle* is the angle included by the legs. The other two angles are called *base angles*. The base angles are congruent.



Isosceles Triangle

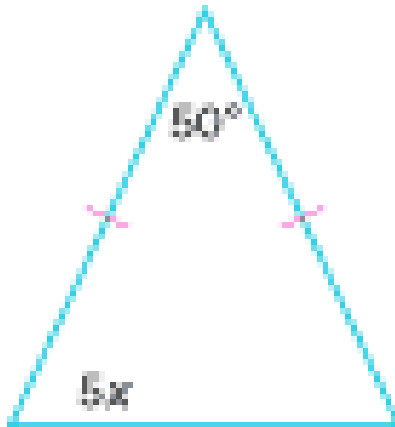
There are a few special rules to learn and remember when dealing with isosceles triangles. The following is called the **ISOSCELES TRIANGLE THEOREM**.

THEOREM: If two sides of a triangle are congruent (*like in any isosceles triangle*), then the angles opposite those sides are congruent.

Example:

Find the value of x .

- A. 20
- B. 10
- C. 26
- D. **13**

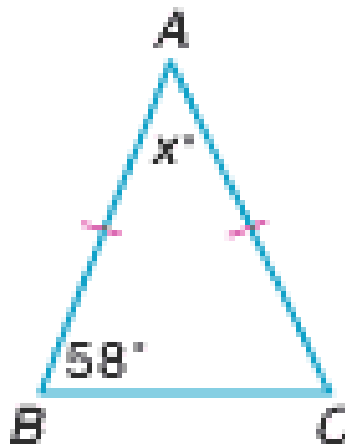


By the theorem, the missing angle equals $5x$ also. We remember the sum of all the interior angles equals 180 degrees, so . . .
 $5x + 5x + 50 = 180$
 $10x = 130$
 $x = 13$

Example:

What is the value of x ?

- A. 122°
- B. 58°
- C. 65°
- D. **64°**



By the theorem, the missing angle is 58 degrees, so . . .
 $58 + 58 + x = 180$
 $116 + x = 180$
 $x = 180 - 116$
 $x = 64$

Example:

In isosceles $\triangle PQR$, with base \overline{QR} , $PQ = 2x + 3$, and $PR = 9x - 11$. What is the value of x ?

- A. -7 B. 7 C. $\frac{8}{11}$ D. 2

$$\begin{aligned} PQ &= PR \text{ since they are the two congruent legs, so} \\ 2x + 3 &= 9x - 11 \\ 2x - 9x &= -11 - 3 \\ -7x &= -14 \\ x &= 2 \end{aligned}$$

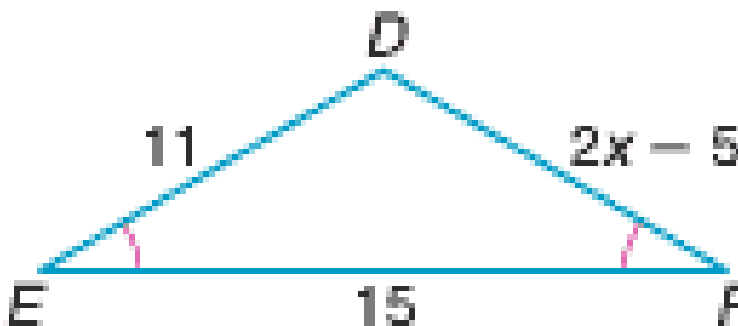
The converse of the **ISOSCELES TRIANGLE THEOREM** is also true.

THEOREM: If two angles of a triangle are congruent (*like the base angles in any isosceles triangle*), then the sides opposite those angles are congruent.

Example:

Find the value of x .

- A. 9
B. 10
C. 8
D. 3

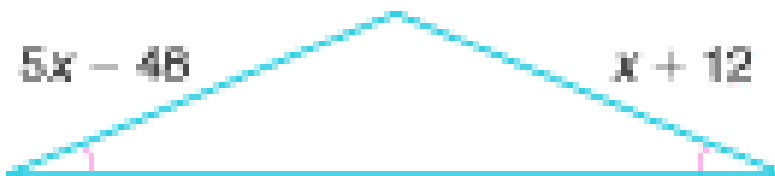


$$\begin{aligned} \text{By the theorem, the two sides opposite the congruent angles are congruent,} \\ \text{so} \\ 11 &= 2x - 5 \\ 11 + 5 &= 2x \\ 16 &= 2x \\ x &= 8 \end{aligned}$$

Example:

What is the value of each of the two sides opposite the congruent angles?

- A. 39
- B. 15
- C. 75
- D. 27



By the theorem, the two given side lengths must be congruent, so finding x first we get . . .

$$5x - 48 = x + 12$$

$$5x - x = 12 + 48$$

$$4x = 60$$

$$x = 15$$

plugging this value of x into either expression, we get

$$(15) + 12 = 27 \text{ or}$$

$$5(15) - 48 = 75 - 48 = 27$$
Example:

In isosceles $\triangle ABC$ with base \overline{AC} , $m\angle A = 3x + 9$, and $m\angle C = 5x - 13$, Find the measure of $\angle B$.

- A. 34
- B. 96
- C. 42
- D. 52

Angle A and Angle C must be the congruent base angles, so

$$3x + 9 = 5x - 13$$

$$-2x = -22$$

$$x = 11$$

so, each base angle is $3(11) + 9 = 42 = 5(11) - 13$

The sum of all angles must equal 180 . . .

This means angle B must measure $180 - 42 - 42 = 180 - 84 = 96$

Example:

If two angles of an isosceles triangle are each twice the measure of the other angle, what is the measure of this other angle?

- A. 36°
- B. 72°
- C. 64°
- D. 18°

If the two mentioned angles must be congruent, so the other angle must be the third VERTEX angle. All these angles must add to 180 degrees.

Let x be the length of the measure of the vertex angle, so we know . . .

$$(2x) + (2x) + x = 180$$

$$5x = 180$$

$$x = 36$$

The **ISOSCELES TRIANGLE THEOREM** leads to two important **corollaries**. (A *corollary* is just a natural or immediate consequence of a proven theorem that requires little or no proof).

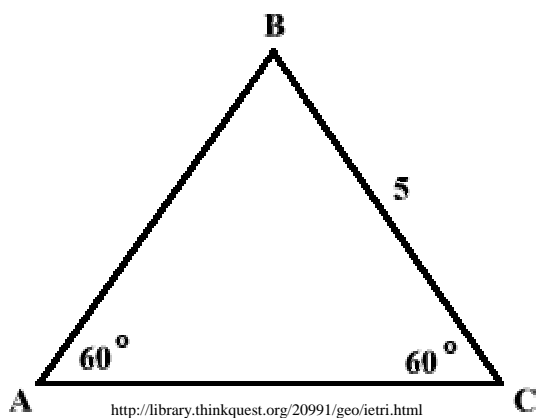
Ex) Blind jealousy is a frequent corollary of passionate love

COROLLARY: A triangle is equilateral IFF it is equiangular

COROLLARY: A triangle is equilateral IFF each angle measures 60°

Example:

Find AB and AC in the given triangle.



<http://library.thinkquest.org/20991/geo/ietri.html>

By the converse of the Isosceles Triangle Theorem, AB must be 5. However, knowing the lengths of the two legs doesn't necessarily give information about the length of the base, but look at the angles!!! Angle B must equal $180 - 60 - 60 = 60$ making the triangle Equiangular. Remember that Equiangular triangles are also Equilateral triangles, so AC must also equal 5.

RANDOM JOKE INSERTION:

Q: What did the complementary angle say to the isosceles triangle?

A: Nice legs!

Say What??!!

1. How many sides are congruent on an isosceles triangle? **2**
2. What side of an isosceles triangle is not equal to the rest? **The Base**
3. Besides the two legs, what else is congruent in an isosceles triangle? **Base Angles**
4. How many sides are equal on an equilateral triangle? **3**
5. Are the angle measures different in an isosceles triangle? **Yes, we only know at least two are congruent**
6. What is the measure of an angle in an equilateral triangle? **60 degrees**
7. You are given a triangle you know to be isosceles. You find a base angle to measure 60 degrees, what conclusion can you make? **Equilateral triangle**
8. A given triangle has 2 congruent sides and one angle measure of 70 degrees. What type of triangle is this? **Acute Isosceles**
9. Given that a triangle is equilateral and that one side is 20, what are the measures of the other sides? **20**
10. One of the base angles of an isosceles triangle measures 45 degrees, what is the vertex angle's measure? **90 degrees, a right triangle**