Power Dispatch and Load Control with Generation Uncertainty

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***Abstract*—In this paper, we focus on the problem of joint load scheduling and generation management to better match supply and demand. We formulate an optimization problem to jointly minimize the generation cost and discomfort cost of the users subject to the voltage and power balance equations for the equivalent circuit of the power system. The optimal power flow (OPF) equations are solved using semidefinite programming (SDP) relaxation technique. In our system model, we assume that users can exploit renewable energy resources (RERs). RERs are random in nature and may cause voltage variations in different nodes of the system. To minimize the risk of having high voltage values, a barrier term is added to the objective function. This term is calculated based on the concept of conditional value-at-risk (CVaR). Simulation results show that compared to the case where there is no control over the load, our proposed algorithm reduces the generation cost by better matching the generation and demand. Moreover, the proposed algorithm reduces the voltage variations at different nodes of the system.**

I. Introduction

The increasing electricity demand requirements of the users and the emergence of new types of demand such as plug-in hybrid electric vehicles make it difficult for transmission and distribution network operators to preserve the reliability and efficiency of the power grid [1]. The process of installing new transmission and bulk generation infrastructure is lengthy and expensive. To tackle these problems and to balance generation and demand, the use of distributed generators (DGs) attracts more attention. DGs are located close to the demand. They bypass the congested transmission network and alleviate the need to install new transmission infrastructure. Moreover, DGs can provide environmental benefits by utilizing sustainable renewable energy resources (RERs) [2]–[4]. RERs such as solar and wind are non-dispatchable, since they are random in nature. In systems with high penetration of RERs, the amount of generation may exceed the demand. The reverse power flow from the DGs back to the substation can cause the voltage rise problem, which is a major challenge in integrating a large number of DGs in the distribution network [5]–[8].

To cope with the undesirable variations of voltage due to the random nature of RERs, different voltage regulation programs have been proposed in the literature [9]–[14]. The advances of power electronics make it possible for inverters, which act as an interface between the photovoltaic (PV) units and the grid, to provide ancillary services. Examples of such services include reactive power compensation or operating at unity power factor and curtailing part of active power generation [15]. The authors in [15] studied an optimal power flow (OPF) problem such that a prior risk level of PV generation surplus will not exceed a certain threshold. They adopted the concept of conditional value-at-risk (CVaR) to capture the risk of having over-voltages. In addition to minimizing the risk of having high voltage values, different objectives are considered for the OPF problem. Examples include minimizing power distribution losses and the cost of providing energy [16]. Moreover, the sufficient conditions to ensure the existence of a global optimum for the OPF problem in ac grids and ac-dc grids are examined in [17] and [18], respectively. Most of the prior works focus on determining the operation point of the generators to reduce costs. Despite its importance, the possibility of adopting demand response (DR) programs to shape the load pattern of the users in order to provide voltage regulation services has not been well-investigated. Among different techniques considered for DR [19]–[24], we focus on direct load control where the distributed network operator (DNO) can directly communicate with the users to change their power consumption and better match supply and demand.

In this paper, we consider the problems of unit commitment (i.e., determining the active and reactive power generations of DGs) and load scheduling of the users. Thereby, the voltage and power balance equations of the equivalent circuit of the power network are taken into account. We note that the random nature of RERs may result in voltage rise problem. The concept of CVaR is used to capture the risk associated with the DGs surplus power exceeding a certain threshold. The contributions of this paper are summarized as follows:

- We adopt a semidefinite programming (SDP) relaxation technique to solve an OPF problem with the objective of minimizing the generation cost and the discomfort cost of the users.
- We schedule the power consumption of the users to better match supply and demand. A quadratic cost function is exploited to quantify the discomfort level of the users when the DNO changes the power consumption of the users from their desired level.
- We use the concept of CVaR to minimize the risk of having a large reverse power flow.
- Simulation results show that the generation cost is reduced by better matching supply and demand. The voltage variations are reduced at different nodes of the grid.

II. System Model

We consider a smart power system with the set of buses or nodes $\mathcal{N}$, and the set of lines $\mathcal{L} \subseteq \mathcal{N} \times \mathcal{N}$. We divide the intended operation cycle into $T \triangleq |\mathcal{T}|$ time slots, where $\mathcal{T} \triangleq \{1, \ldots, T\}$. Let $i^2 = -1$. We denote $S_{n,t}^D \triangleq P_{n,t}^D + iQ_{n,t}^D$ as the complex power of the load connected to bus $n \in \mathcal{N}$ at time slot $t \in \mathcal{T}$. Let $S_{n,t}^G \triangleq P_{n,t}^G + iQ_{n,t}^G$ denote the complex power generated by the DG connected to bus $n \in \mathcal{N}$

at time slot t. $S_{nm,t} \triangleq P_{nm,t} + iQ_{nm,t}$ is the complex power that flows through line $(n,m) \in \mathcal{L}$ at time slot t. V_t^n is defined as the complex voltage of bus n at time slot t, and $\mathbf{V}_t \triangleq (V_t^1, \dots, V_t^N)$. Each transmission line is replaced by its equivalent Π model [25]. Let y_{nn} and y_{nm} denote the admittance of the bus n to the ground, and the mutual admittance between buses n and m, respectively. $\bar{y}_{nm}$ denotes the shunt admittance of bus n associated with the Π model of the line (n,m). The admittance matrix of the system is represented by Y. Similar to [17], we define $e_1, \dots, e_N$ as the standard basis vectors in $\mathbf{R}^N$, and the following matrices are defined for each $n \in \mathcal{N}$ and $(n,m) \in \mathcal{L}$:

$$Y_n \triangleq e_n e_n^{\mathrm{T}} Y, \tag{1a}$$

$$Y_{nm} \triangleq (\bar{y}_{nm} + y_{nm}) e_n e_n^{\mathrm{T}} - (y_{nm}) e_n e_m^{\mathrm{T}}, \tag{1b}$$

$$\mathbf{Y}_n \triangleq \frac{1}{2} \begin{bmatrix} \mathrm{Re}\{Y_n + Y_n^{\mathrm{T}}\} & \mathrm{Im}\{Y_n^{\mathrm{T}} - Y_n\} \\ \mathrm{Im}\{Y_n - Y_n^{\mathrm{T}}\} & \mathrm{Re}\{Y_n + Y_n^{\mathrm{T}}\} \end{bmatrix}, \tag{1c}$$

$$\mathbf{Y}_{nm} \triangleq \frac{1}{2} \begin{bmatrix} \mathrm{Re}\{Y_{nm} + Y_{nm}^{\mathrm{T}}\} & \mathrm{Im}\{Y_{nm}^{\mathrm{T}} - Y_{nm}\} \\ \mathrm{Im}\{Y_{nm} - Y_{nm}^{\mathrm{T}}\} & \mathrm{Re}\{Y_{nm} + Y_{nm}^{\mathrm{T}}\} \end{bmatrix}, \tag{1d}$$

$$\bar{\mathbf{Y}}_n \triangleq -\frac{1}{2} \begin{bmatrix} \mathrm{Im}\{Y_n + Y_n^{\mathrm{T}}\} & \mathrm{Re}\{Y_n - Y_n^{\mathrm{T}}\} \\ \mathrm{Re}\{Y_n^{\mathrm{T}} - Y_n\} & \mathrm{Im}\{Y_n + Y_n^{\mathrm{T}}\} \end{bmatrix}, \tag{1e}$$

$$\bar{\mathbf{Y}}_{nm} \triangleq -\frac{1}{2} \begin{bmatrix} \mathrm{Im}\{Y_{nm} + Y_{nm}^{\mathrm{T}}\} & \mathrm{Re}\{Y_{nm} - Y_{nm}^{\mathrm{T}}\} \\ \mathrm{Re}\{Y_{nm}^{\mathrm{T}} - Y_{nm}\} & \mathrm{Im}\{Y_{nm} + Y_{nm}^{\mathrm{T}}\} \end{bmatrix}, \tag{1f}$$

$$\mathbf{M}_n \triangleq \begin{bmatrix} e_n e_n^{\mathrm{T}} & 0 \\ 0 & e_n e_n^{\mathrm{T}} \end{bmatrix}, \tag{1g}$$

$$\mathbf{X}_t \triangleq [\mathrm{Re}\{\mathbf{V}_t\}^{\mathrm{T}}, \mathrm{Im}\{\mathbf{V}_t\}^{\mathrm{T}}]^{\mathrm{T}}, \tag{1h}$$

$$\mathbf{W}_t \triangleq \mathbf{X}_t \mathbf{X}_t^{\mathrm{T}}, \tag{1i}$$

$$\mathrm{rank}(\mathbf{W}_t) = 1, \tag{1j}$$

where T is the transpose operation. The voltage value and the complex power injected to each bus can be represented by the matrices defined in (1). That is, the following relations hold for every $n \in \mathcal{N}$, $(n,m) \in \mathcal{L}$, and $t \in \mathcal{T}$ [17].

$$P_{n,t}^G - P_{n,t}^D = \mathrm{Tr}\{\mathbf{Y}_n \mathbf{W}_t\}, \tag{2a}$$

$$Q_{n,t}^G - Q_{n,t}^D = \mathrm{Tr}\{\bar{\mathbf{Y}}_n \mathbf{W}_t\}, \tag{2b}$$

$$P_{nm,t} = \mathrm{Tr}\{\mathbf{Y}_{n,m} \mathbf{W}_t\}, \tag{2c}$$

$$|S_{nm,t}|^2 = \mathrm{Tr}\{\mathbf{Y}_{nm} \mathbf{W}_t\}^2 + \mathrm{Tr}\{\bar{\mathbf{Y}}_{nm} \mathbf{W}_t\}^2, \tag{2d}$$

$$|V_t^n|^2 = \mathrm{Tr}\{\mathbf{M}_n \mathbf{W}_t\}. \tag{2e}$$

For the underlaying circuit of the power system, the voltage and power values are subject to the power balance equations and physical constraints at all time slots [17]. We have

$$P_{n,t}^{G,\min} - P_{n,t}^D \le \mathrm{Tr}\{\mathbf{Y}_n \mathbf{W}_t\} \le P_{n,t}^{G,\max} - P_{n,t}^D, \tag{3a}$$

$$Q_{n,t}^{G,\min} - Q_{n,t}^D \le \mathrm{Tr}\{\bar{\mathbf{Y}}_n \mathbf{W}_t\} \le Q_{n,t}^{G,\max} - Q_{n,t}^D, \tag{3b}$$

$$\mathrm{Tr}\{\mathbf{Y}_{nm} \mathbf{W}_t\} \le P_{nm}^{\max}, \tag{3c}$$

$$(V_n^{\min})^2 \le \mathrm{Tr}\{\mathbf{M}_n \mathbf{W}_t\} \le (V_n^{\max})^2, \tag{3d}$$

$$\mathrm{Tr}\{\mathbf{Y}_{nm} \mathbf{W}_t\}^2 + \mathrm{Tr}\{\bar{\mathbf{Y}}_{nm} \mathbf{W}_t\}^2 \le (S_{nm}^{\max})^2, \tag{3e}$$

where $P_{n,t}^{G,\min}$, $P_{n,t}^{G,\max}$, $Q_{n,t}^{G,\min}$, and $Q_{n,t}^{G,\max}$ are the lower bound and the upper bound for the generated active and reactive power at bus n, respectively. $V_n^{\min}$ and $V_n^{\max}$ denote the lower bound and upper bound on the acceptable range of voltage values for bus n, respectively. $S_{nm}^{\max}$ and $P_{nm}^{\max}$ determine the maximum apparent power and the maximum active power that can flow through line (n,m), respectively.

We assume that all the load subscribers in the network are equipped with an energy consumption controller (ECC) unit to control the user's power consumption. It receives control signals from the DNO. That is, based on an agreement, the DNO can remotely control the operation of some appliances of the users. We use $P_{n,t}^{D,\min}$, $P_{n,t}^{D,\max}$, $Q_{n,t}^{D,\min}$, and $Q_{n,t}^{D,\max}$ to denote the minimum and maximum acceptable values for the active and reactive power of the load connected to bus n, respectively. Let E_n denote the total energy requirement of the load connected to bus n. We have

$$P_{n,t}^{D,\min} \le P_{n,t}^D \le P_{n,t}^{D,\max}, \tag{4a}$$

$$Q_{n,t}^{D,\min} \le Q_{n,t}^D \le Q_{n,t}^{D,\max}, \tag{4b}$$

$$0 \le E_n \le \textstyle\sum_{t \in \mathcal{T}} P_{n,t}^D. \tag{4c}$$

For the generator connected to bus n, we consider a cost function $C_n(P_{n,t}^G)$ indicating the cost of generating electricity at power level $P_{n,t}^G$ at time slot t. We assume that the cost function is increasing and strictly convex in the offered energy. We consider a quadratic cost function $C_n(P_{n,t}^G) = a_n(P_{n,t}^G)^2 + b_n(P_{n,t}^G) + c_n$, where a_n, b_n, c_n are pre-determined coefficients. Substituting (2a) into $C_n(P_{n,t}^G)$, we obtain

$$\begin{aligned} C_n(\mathbf{W}_t, P_{n,t}^D) = a_n(&\mathrm{Tr}\{\mathbf{Y}_n \mathbf{W}_t\} + P_{n,t}^D)^2 \\ &+ b_n(\mathrm{Tr}\{\mathbf{Y}_n \mathbf{W}_t\} + P_{n,t}^D) + c_n. \end{aligned} \tag{5}$$

For the load connected at bus n, $L_{n,t}$ denotes the desired level of power consumption at time slot t. $L_{n,t}$ can either be inferred from the historical data or be determined by the users in a day-ahead basis. Therefore, any deviation from the $L_{n,t}$ as a result of load scheduling will incur a cost to the load. We adopt $H_{n,t}(P_{n,t}^D)$ to model the dissatisfaction cost of the load connected to bus n at time slot t. In the following, we assume a quadratic dissatisfaction cost function as

$$H_{n,t}(P_{n,t}^D) = \theta_{n,t}(P_{n,t}^D - L_{n,t})^2, \tag{6}$$

where $\theta_{n,t}$ is a non-negative constant.

III. Problem Formulation

The DNO has to determine the output generation of DGs while taking into account the uncertainty about the generation. The unforeseen variation of the output generation from RERs can cause significant changes to the voltage level of the buses. It is difficult to maintain the voltages in their acceptable range, and there are scenarios in which the OPF problem may become infeasible. In this section, we first formulate the OPF problem ignoring the possible infeasibility problem. We then explain how to tackle the possible infeasibility.

We formulate the OPF problem as a relaxed SDP problem by relaxing the constraint (1j). For $n \in \mathcal{N}$ and $t \in \mathcal{T}$, we have

$$\underset{\substack{\mathbf{W}_t,\, S_{n,t}^D, \\ \lambda_{n,t},\, \gamma_{n,t}, \\ t \in \mathcal{T},\, n \in \mathcal{N}}}{\text{minimize}} \quad \sum_{t \in \mathcal{T}} \sum_{n \in \mathcal{N}} \lambda_{n,t} + \gamma_{n,t} \tag{7a}$$

$$
\begin{aligned}
\text{subject to} \quad & \text{constraints } (3), (4), && (7b)\\
& C_n(\mathbf{W}_t, P^D_{n,t}) \le \lambda_{n,t}, && (7c)\\
& H_{n,t}(P^D_{n,t}) \le \gamma_{n,t}, && (7d)
\end{aligned}
$$

where $\lambda_{n,t}$ and $\gamma_{n,t}$ are auxiliary variables associated with each node n at time slot t. By adopting Schur's complement formula [26], constraint (3e) can be rewritten as

$$
\begin{bmatrix} (S^{\max}_{nm})^2 & \mathrm{Tr}\{\mathbf{Y}_{nm}\mathbf{W}_t\} & \mathrm{Tr}\{\bar{\mathbf{Y}}_{nm}\mathbf{W}_t\} \\ \mathrm{Tr}\{\mathbf{Y}_{nm}\mathbf{W}_t\} & -1 & 0 \\ \mathrm{Tr}\{\bar{\mathbf{Y}}_{nm}\mathbf{W}_t\} & 0 & -1 \end{bmatrix} \preceq 0, \tag{8}
$$

where $\preceq$ is the matrix inequality sign in the positive semidefinite sense. Constraints (7c) and (7d) can be rewritten as

$$
\begin{bmatrix} b_n\delta_{n,t} - \lambda_{n,t} + c_n & \sqrt{a_n}\delta_{n,t} \\ \sqrt{a_n}\delta_{n,t} & -1 \end{bmatrix} \preceq 0, \tag{9a}
$$

$$
\begin{bmatrix} -2L_{n,t}P^D_{n,t} + L^2_{n,t} - \gamma_{n,t}/\theta_{n,t} & P^D_{n,t} \\ P^D_{n,t} & -1 \end{bmatrix} \preceq 0, \tag{9b}
$$

where $\delta_{n,t} \triangleq \mathrm{Tr}\{\mathbf{Y}_n\mathbf{W}_t\} + P^D_{n,t}$. Therefore, problem (7) can be reformulated as

$$
\begin{aligned}
\underset{\substack{\mathbf{W}_t, S^D_{n,t},\\ \lambda_{n,t}, \gamma_{n,t},\\ t\in\mathcal{T}, n\in\mathcal{N}}}{\text{minimize}} \quad & \sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} \lambda_{n,t} + \gamma_{n,t} && (10a)\\
\text{subject to} \quad & \text{constraints } (3a)-(3d), (4), (8), (9). && (10b)
\end{aligned}
$$

To tackle the possible infeasibility problem of (10), one option is to add a barrier term to the objective of (10) which penalizes any voltage deviation from its nominal value, and relax the constraint (3d). Examples of such barrier functions include value at risk (VaR) and conditional value at risk (CVaR). Equation (2a) implies that fluctuations in power generation at bus n can lead to changes in matrix $\mathbf{W}_t$. Therefore, the voltage variation can be indirectly related to the fluctuations in the RERs' power generation. We now use CVaR to deal with the uncertainty related to generation level from the RERs. Let $R(\mathbf{P}^G_t, \hat{\mathbf{P}}_t)$ denote a real-valued function that captures the possible excess power generation from its *presumed value* [27], where $\hat{\mathbf{P}}_t \triangleq (\hat{P}_{1,t}, \dots, \hat{P}_{N,t})$ $\cancel{\mathbf{P}^G_t \triangleq (P^G_{1,t}, \dots, P^G_{N,t})}$ is the vector of presumed values of the output power generation obtained from solving the OPF and $\mathbf{P}^G_t \triangleq (P^G_{1,t}, \dots, P^G_{N,t})$ $\cancel{\hat{\mathbf{P}}_t \triangleq (\hat{P}_{1,t}, \dots, \hat{P}_{N,t})}$ is the vector of the actual power generation at all the buses. We define $R(\cdot)$ as

$$
R(\mathbf{P}^G_t, \hat{\mathbf{P}}_t) = \sum_{n\in\mathcal{N}} \left[P^G_{n,t} - \hat{P}_{n,t}\right]^+, \tag{11}
$$

where $[\cdot]^+ \triangleq \max\{\cdot, 0\}$. The cumulative distribution function for the random variable $R(\cdot)$ is defined as

$$
\Psi(\hat{\mathbf{P}}_t, \alpha) \triangleq \Pr\{R(\mathbf{P}^G_t, \hat{\mathbf{P}}_t) \le \alpha\}. \tag{12}
$$

Based on (12), for the probability level $\beta \in (0,1)$, the corresponding VaR, α_β, is defined as the minimum threshold α for which the probability of voltage deviation from its nominal value being less than α is at least β. That is,

$$
\alpha_\beta(\hat{\mathbf{P}}_t) \triangleq \min\{\alpha : \Psi(\hat{\mathbf{P}}_t, \alpha) \ge \beta\}. \tag{13}
$$

Furthermore, the CVaR is defined as the expected value of the surplus function $R(\cdot)$ when only the generating powers that are greater than or equal to α_β are considered. That is,

$$
\phi_\beta(\hat{\mathbf{P}}_t) = \mathbb{E}\{R(\mathbf{P}^G_t, \hat{\mathbf{P}}_t) : R(\mathbf{P}^G_t, \hat{\mathbf{P}}_t) \ge \alpha_\beta(\hat{\mathbf{P}}_t)\}, \tag{14}
$$

where $\mathbb{E}\{\cdot\}$ denotes the expectation with respect to uncertain generation output. It has been shown that the CVaR can also be represented as [27]: $\phi_\beta(\hat{\mathbf{P}}_t) = \min_{\alpha\in\mathbb{R}} \Gamma_\beta(\alpha, \hat{\mathbf{P}}_t)$, where

$$
\Gamma_\beta(\alpha, \hat{\mathbf{P}}_t) \triangleq \alpha + \frac{1}{1-\beta}\int \left[R(\mathbf{P}^G_t, \hat{\mathbf{P}}_t) - \alpha\right]^+ \rho(\mathbf{P}^G_t) d\mathbf{P}^G_t, \tag{15}
$$

and $\rho(\mathbf{P}^G_t)$ is the probability density function of random vector $\mathbf{P}^G_t$. CVaR is convex in $\hat{\mathbf{P}}_t$, and for any threshold α, it is always greater than or equal to the VaR. Thus, minimizing the CVaR results in having a low VaR as well. It is possible to estimate the CVaR by adopting sample average technique. This is useful especially in situations where it is difficult to obtain a closed-form solution for $\rho(\mathbf{P}^G_t)$. Samples of the random variable $\mathbf{P}^G_t$ can be observed from the real system. Considering the set $\mathcal{K} \triangleq \{1, \dots, K\}$ of K samples of the random vector $\mathbf{P}^G_t$, the $\Gamma_\beta(\cdot)$ in (15) can be approximated as

$$
\hat{\Gamma}_\beta(\alpha, \hat{\mathbf{P}}_t) = \alpha + \frac{1}{K(1-\beta)} \sum_{k\in\mathcal{K}} \left[R(\mathbf{P}^{G,k}_t, \hat{\mathbf{P}}_t) - \alpha\right]^+, \tag{16}
$$

where $\mathbf{P}^{G,k}_t$ denotes the kth sample of random vector $\mathbf{P}^G_t$. It has been shown in [27] that minimizing $\phi_\beta(\hat{\mathbf{P}}_t)$ over all possible values of $\hat{\mathbf{P}}_t$ is equivalent to minimize $\Gamma_\beta(\alpha, \hat{\mathbf{P}}_t)$ over all possible values of α and $\hat{\mathbf{P}}_t$. That is,

$$
\min_{\hat{\mathbf{P}}_t\in\mathbb{R}^N} \phi_\beta(\hat{\mathbf{P}}_t) = \min_{\alpha\in\mathbb{R},\ \hat{\mathbf{P}}_t\in\mathbb{R}^N} \Gamma_\beta(\alpha, \hat{\mathbf{P}}_t). \tag{17}
$$

We formulate the problem of minimizing the joint generation and dissatisfaction costs while taking into account the uncertainty about the generation as

$$
\begin{aligned}
\underset{\substack{\mathbf{W}_t, S^D_{n,t}, \lambda_{n,t},\\ \gamma_{n,t}, \alpha, \hat{\mathbf{P}}_t,\\ t\in\mathcal{T}, n\in\mathcal{N}}}{\text{minimize}} \quad & \sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} \left(\lambda_{n,t} + \gamma_{n,t}\right) + \eta_t\hat{\Gamma}_\beta(\alpha, \hat{\mathbf{P}}_t) && (18a)\\
\text{subject to} \quad & \text{constraints } (3a)-(3c), (4), (8), (9), && (18b)
\end{aligned}
$$

where η_t is a weight coefficient. The included CVaR objective $\hat{\Gamma}_\beta(\alpha, \hat{\mathbf{P}}_t)$ is as (16) and reflects the risk of having high values of voltage deviations. To reformulate (18) as a standard semidefinite program, auxiliary vector $\boldsymbol{\mu}_t \in \mathbb{R}^K$ is introduced to upper bound the projection term $\hat{\Gamma}_\beta(\alpha, \hat{\mathbf{P}}_t)$. The vector of auxiliary variables $\mathbf{u}^k_t \in \mathbb{R}^N$ are introduced for each sample k of random vector $\mathbf{P}^{G,k}_t$. Problem (18) can be rewritten as

$$
\begin{aligned}
\underset{\substack{\mathbf{W}_t, S^D_{n,t}, \lambda_{n,t},\\ \gamma_{n,t}, \alpha, \hat{\mathbf{P}}_t,\\ \boldsymbol{\mu}_t, \mathbf{u}^k_t, k\in\mathcal{K},\\ t\in\mathcal{T}, n\in\mathcal{N}}}{\text{minimize}} \quad & \sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} (\lambda_{n,t} + \gamma_{n,t}) + \eta_t\alpha + \frac{\eta_t}{K(1-\beta)}\mathbf{1}^{\mathrm{T}}_K\boldsymbol{\mu}_t && (19a)\\
\text{subject to} \quad & \text{constraint } (18b), && (19b)\\
& \mathbf{1}^{\mathrm{T}}_N\mathbf{u}^k_t \le \alpha + \mu^k_t, && (19c)\\
& P^{G,k}_{n,t} - \hat{P}^k_{n,t} \le u^k_{n,t}. && (19d)
\end{aligned}
$$

The solution of problem (19), $\mathbf{W}_t^{\text{opt}}$ and $S_{n,t}^{D,\text{opt}}$, can be used to determine the operating point of the generators as in (2a) and (2b). The assumptions made in [17] to determine the rank of $\mathbf{W}_t^{\text{opt}}$ for ac OPF problem are valid for problem (19) as well. Therefore, $\mathbf{W}_t^{\text{opt}}$ is at most rank two for practical ac grids such as the IEEE test systems. Algorithm 1 explains how to determine the vector of bus voltages from $\mathbf{W}_t^{\text{opt}}$.

Algorithm 1 Algorithm which determines the voltage of buses.

Solve problem (19).
if $\mathbf{W}_t^{\text{opt}}$ is rank one with eigenvalue r and eigenvector $\boldsymbol{\nu}$
    Calculate $\mathbf{X}_t^{\text{opt}} = \sqrt{r}\boldsymbol{\nu}$.
else if $\mathbf{W}_t^{\text{opt}}$ is rank two with two nonzero eigenvalues r_1 and r_2 and corresponding eigenvectors $\boldsymbol{\nu}_1$ and $\boldsymbol{\nu}_2$
    Calculate rank one matrix $\hat{\mathbf{W}}_t^{\text{opt}} = (r_1 + r_2)\boldsymbol{\nu}_1\boldsymbol{\nu}_2^{\text{T}}$.
    Calculate eigenvalue $\hat{r}$ and eigenvector $\hat{\boldsymbol{\nu}}$ of $\hat{\mathbf{W}}_t^{\text{opt}}$.
    Calculate $\mathbf{X}_t^{\text{opt}} = \sqrt{\hat{r}}\hat{\boldsymbol{\nu}}$.
end if

IV. Performance Evaluation

We present simulation results and assess the performance of our proposed load control algorithm. In our simulation setting, the operation period is divided into 3 time slots representing on-peak hours, off-peak hours, and mid-peak hours. The IEEE 30-bus distribution network is considered as a test case. The values of line impedance are derived from [28]. Some of the buses in the network are equipped with RERs. The Monte Carlo samples $\mathbf{P}_t^{G,k}$ are obtained based on a normal distribution. We consider a flexible load connected to each bus. We assume that the DNO is able to control the load. The desired load level in each bus, i.e., $L_{n,t}$, is obtained from [28]. The DNO ensures that the total energy requirements of the users are met at the end of the operation period. The generation capacity of the DGs varies between 5 MW and 60 MW. The generation cost parameters are set to $a_n = 0.01$, $b_n = 0$, and $c_n = 0$. The loads connected to different buses may require power up to 20 MW. $\theta_{n,t}$ in (6) is set to 0.5. CVX is used to solve the OPF problem. To estimate the CVaR, we use the sample average technique. $K = 100$ samples of power generation vector $\mathbf{P}_t^{G,k}$ are considered to approximate the CVaR in (14). β in (15) is set to 0.9. To have a baseline to compare with, we consider a system in which the loads connected to the buses of the system are not flexible. In this system, the DGs have to generate more power to meet the demand on peak hours. Thus, the generation cost increases. The chance of the voltage rise problem also increases.

Fig. 1 depicts simulation results for the total generation cost of the system for different levels of load flexibility. Load flexibility is defined as the percentage of the desired level of load in each time slot that can be reduced or increased. That is, $\chi_{n,t} \triangleq \Delta P_{n,t}^D / P_{n,t}^D \times 100\%$, where $\Delta P_{n,t}^D$ is the amount of power demand that can be adjusted. We note that even with different levels of $\chi_{n,t}$ users will receive their total energy requirements. By increasing $\chi_{n,t}$, the DNO can better match supply and demand. The generation cost reduces as the DNO can shift more load from peak hours to off-peak hours.

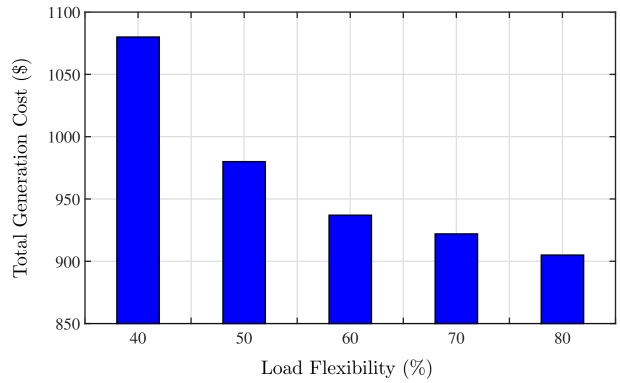

Fig. 1. Generation cost for different levels of load flexibility.

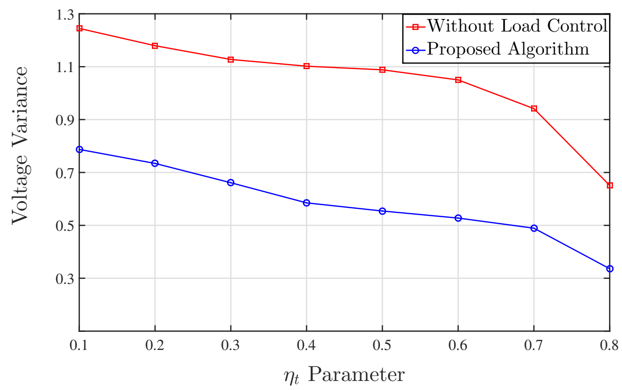

Fig. 2. Voltage variance for different values of η_t parameter.

In order to minimize the risk of having high values of voltage deviations, the term (16) is added to the objective function. To better understand the effect of weight coefficient η_t in (18), we focus on the expected voltage values for different values of parameter η_t. We consider the expected square of the voltage deviation from its desired value as a measure to evaluate the severity of voltage variations. Our proposed load control algorithm reduces the expected variations of the voltage by jointly controlling the output generation and the load level at different buses of the system. As illustrated in Fig. 2, by increasing the parameter η_t, the voltage variations are reduced as more weights are put on minimizing the risk of having high voltage values.

V. Conclusion

In this paper, we formulated an optimization problem to minimize the generation cost and the discomfort cost subject to power flow constraints for the equivalent circuit of the power system. We adopted an SDP relaxation technique to solve the OPF problem. Moreover, the risk of having high voltage values was also minimized by including a barrier term based on CVaR to the objective function. Simulation results showed that our proposed algorithm reduces the generation cost and better eliminates the mismatch between the supply and demand. The proposed algorithm can also mitigate the voltage variations at different nodes of the power network.

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