Important Note (October 2021)

The book titled *The Old Measure* (referred to at several places in the following article) is now available online:

http://www.ibiblio.org/bosak/pub/wam/the-old-measure-2010.pdf

The data presented in Appendix A of the book ("Grain weight as a natural standard") are particularly relevant to the theory presented here.

Canonical grain weights as a key to ancient systems of weights and measures

by Jon Bosak
Last revised 19 April 2014

Introduction[1]

Historians have puzzled over the principles underlying ancient systems of measurement for more than a century. Some have proposed various theories to explain what we know based on conjectured relationships between cubic volumes of specific substances (usually water or grain) and specific units of the native weight system. In other words, they posit a connection between standards of length and standards of weight based on the density or weight per volume of a particular substance.

These interpretations have been out of favor for many years, not, I think, because such definitional relationships are unlikely but because, for several reasons I will review here, the proposed theories have been next to impossible to test with any rigor.

In this paper I introduce the concept of a *canonical weight of grain* as a tool with which to resolve this problem, and then I draw on recent discoveries to propose theories for the ancient Egyptian, Sumerian, Indus, and Chinese systems of weights and measures that are both historically motivated and unprecedentedly precise.

I also use this paper as the occasion to briefly discuss subtractive weight sets and their possible role in shaping some ancient weight standards.[2]

Some considerations *a priori*

Theoretical weight/volume explanations for ancient systems of measurement make for an intriguing archaeological "cold case" because the reasons for suspecting such systems to have existed are so compelling.

We start with the explicit relationship in many traditional systems of measurement between a certain standard measure of capacity and a specified weight of water, wine, or grain. For example, in the ancient system that the British gave up almost 200 years ago but that continues to be used in the United States, the gallon is defined by the old English statute 12 Henry 7 c. 5 (1496) as a measure holding 8 troy pounds of wheat. (I have written a book on this subject and related matters under the title *The Old Measure*; the earlier work will be referred to in several places further on.) Equivalences like this are not uncommon in the traditional systems that survived into the 19th century. And in some cases, in particular, the ancient Egyptian and Sumerian systems, we know the relationship between measures of capacity and cubic volumes (thus implying a connection with lengths) from surviving texts. Therefore, the notion of an ancient system that was, like our metric system, defined conceptually as an interrelationship between volume, capacity, and weight[3] based on a standard substance (water in the case of the metric system) does not in itself seem strange or unlikely.

The case for believing that such systems *must* have existed becomes compelling when we think through what an ancient trader in grain would have needed to calculate in order to stay in business. Considerations such a person would have needed to take into account include:

- Volume of grain in a cylindrical or rectangular granary of a certain size
- The same volume of grain expressed in capacity units[4]
- That same volume (or number of capacity measures) of grain expressed in units of weight at the origin
- Weight that an animal can carry
- Percentage of the carrying capacity of a caravan that must be allocated to water and that portion of the grain that will be used as food for the animals
- Displacement of water by a sailing vessel, in cubic units, per load of grain, in capacity units
- Capacity in units of sale at the delivery location corresponding to a weight of grain, or sacks or barrels of grain, or the capacity of a hold
- Size of a granary in the receiving location necessary to contain a shipment of grain, as calculated from dimensions expressed in length units current in that location

Commercial transport and trade has always been a complicated business, and only the smart traders survive. It is a virtual certainty that the more successful traders mastered some system for calculating answers to the questions above, and it is difficult to see how they could have done this without some system of measurement that offered easy, standard (that is, generally agreed upon) conversions between units of length, weight, and capacity based on some assumed standard density of a particular trade good. We do this now based on the density of water and refer the density of everything else to that via the concept of specific gravity. I think that the ancients based their systems to some extent on the density of water, too, but (at least in the beginning) only so that weights of liquids and the volume displaced in shipping a certain weight by water could easily be calculated.

The fundamental basis of any ancient system would have been an assumed standard density for the major trade good, grain. Based purely on considerations *a priori,* therefore, we have very good reason to be looking for the foundation of these ancient systems in the *average bulk weight of the predominant grain or grains.* The question is whether we can use this approach to identify and explain the mathematical relationships upon which each ancient system was based.

To be clear: the point of the inquiry is to understand the fundamentals upon which the ancients *thought* their systems were founded; we are looking to reconstruct their notional framework, because *that* is the system. Their varying ability to actually implement their own definitions (as discussed in more detail below) is only of interest here as something to be overcome in trying to answer the main question.

This is what we have to work with:

- *We have the compelling reasons just enumerated for believing that the kind of system we are looking for had to exist in order for successful large-scale competitive ancient trade to have taken place at all.*

- *Considering especially the calculations that go into figuring the sizes and displacement of sailing vessels, we can see a strong likelihood that such systems would have been based on the density of the predominant trade grain and the density of water (or wine, which is not the same but can often be considered so).*

- *We can be certain that the numerical relationships in such a system were very simple.* We must always be careful to distinguish the abilities of the most expert users of the system—including the priests or other officials who maintained the standards—and the abilities of the everyday users of the system. I believe that the greatest experts in any era were capable of precision that archaeologists have tended to underestimate (beginning with the assumption that an ancient expert could not have been more capable than the archaeologist), but this does not mean that mathematical abilities at the other end of the spectrum went much above counting. As I have noted elsewhere,[5] the ancients were notoriously bad with fractions, so any system of relationships must have been based on the simplest of ratios that workably represented the average densities of actual trade goods while at the same time providing the simplest possible factors for conversion to the standards and systems used by major trading partners.

- *We have a good understanding of the relationships among units of a certain* kind *of measure in every major ancient system.* We are not in any real doubt about how many digits were in an Egyptian cubit or how many shekels were in a Sumerian mina; scholarship over several centuries has settled these details for most major ancient civilizations.

- *There is a fair archaeological consensus regarding the most important ancient standards of length,* including the Sumerian cubit at 49.5 cm (19.488 inches), the Egyptian cubit at 20.62–20.63 inches (52.4 cm), the Roman foot at 11.65 inches (29.59 cm), the Northern foot at 13.2 inches (33.53 cm), and the common Greek foot at 12.44–12.45 inches (31.6 cm). All of these ancient standards of length are well attested, and the last two were still going strong in national systems of weights and measures as late as the 19th and 20th centuries.[6]

- *Because they were usually made of pottery, intact ancient capacity measures are quite rare, leaving us with relatively little physical data about capacity standards.* Fortunately, however, we have in the case of the ancient Egyptians and Sumerians textual attestation of relationships between standards of capacity and cubic volumes that allow the capacity standards to be calculated from the measures of length.

- *Because they were generally made of metal or stone, we have a fair amount of physical data regarding ancient commercial weights,* i.e., the weights used to measure trade goods—several thousand examples in all, including a few very elaborately wrought weights with inscriptions (sometimes in multiple languages) stating that they are official standards. There are many more examples if we include coins as indicators of weight standards, and some researchers[7] have succeeded in deducing commercial weight standards from the evidence provided by coins, but I believe that varying monetary policies, inflation, and seignorage make coins too problematic a basis for this work. Even leaving coins out of the picture, however, there are a reasonably large number of examples of ancient commercial weights—vastly more of them than intact examples of ancient capacity measures.

It might seem that those of us interested in this question have everything we need in order to discover the conceptual relationships underlying the integrated systems of weights and measures used in ancient times. But two problems have stymied our search for this understanding by preventing us from asserting specific, testable relationships among the ancient standards: first, a general belief among historians of the subject that the bulk density of grain is too variable to serve as a precise point of reference; and second, the fact that actual examples of ancient weight standards are so variable that it is often difficult to even tell what standard a given specimen was intended to represent. These points will be taken in order.

Weight of grain as a natural standard

Weight/length theories about ancient systems of measurement hinge on the weight of grain or the weight of water per unit volume (cubic finger, cubic foot, cubic cubit, etc.), that is to say, its *density,* or sometimes when speaking of grain in particular, its *bulk density,* by which we mean to emphasize that we are referring to its weight in the mass and not to the weight of individual seeds or kernels.

Estimates of grain weight based on small samples of modern hybrid grain obtained by individual researchers are far too random and variable to enable the precision we need here. Luckily for us (though apparently not noticed by researchers), the bulk density of grain has been a longstanding and constant concern of agronomists, grain inspectors, millers, bakers, and sellers of flour from ancient times to the present day. These people call the density of grain its *test weight* and consider it the single most important measure of its quality. The test weights of wheat, barley, and other grains (expressed as pounds per bushel, lb/bu, in the United States and kilograms per hectoliter, kg/hl, everywhere else) are basic to laws concerning their grading; for example, U.S. Number 1 wheat is required to have a minimum test weight of 60 avoirdupois pounds per Winchester bushel of 2150.42 cubic inches, which is a bulk density of approximately 0.7723116 grams per cubic centimeter (g/cm^3). This definition has been in place for over 150 years. The weight of 60 pounds is a minimum; the average test weight of actual U.S. Number 1 wheat is about 61 lb/bu (about 0.79 g/cm^3).

A few sets of test weight statistics go back far enough to have captured this essential parameter for traditional varieties of wheat and barley before modern breeding techniques began to reshape these grains around the middle of the 20th century. I have published some of the best of these data in *The Old Measure.* Findings that can be drawn from the data include the following:

- The bulk density of a given kind of grain averaged over a large area of production varies from year to year by just a few percentage points from the mean—unlike yield per acre, which can fluctuate by dozens of percentage points from year to year depending on the weather.
- The longer that data from a given variety of grain is collected, the flatter the test weight curve becomes. Test weight data taken over the course of a century and plotted in ten-year intervals graphs as an almost perfect horizontal line. It is clear that data accumulated over the course of several centuries would in effect establish the average test weight of the best quality grade of a given variety of grain from a given area of production as a physical constant.
- The *ratio* of the test weights of barley and wheat is very close to $\frac{4}{5}$ (this is for classical six-row barley, not modern two-row malting barley). No other simple ratio fits the densities of real barley and wheat as well as $\frac{4}{5}$.

In *The Old Measure,* I show how the system of weights and measures still used in the United States can be explained by this ratio between the weights of barley and wheat (which has long been embedded in U.S. state laws as $\frac{48}{60}$) together with another $\frac{4}{5}$ ratio, also long noted, between the densities of wheat and wine. In addressing the oldest systems, however, I have found the most satisfying explanations by conjecturing that they were based on just barley and water.

I have, of course, no way of proving this, but I think that anyone who has studied the middle eastern civilizations of the late bronze age will acknowledge that these civilizations were originally built on barley and water and on the very basic foods that can be made from them, including porridge, bread, and beer. If the reader will grant me this conjecture, then everything else follows, because the simplest ratio that accurately describes the average density of real-world best quality traditional barley in terms of the weight of water is a matter of empirical fact. That ratio is $\frac{5}{8}$ (a measure of barley weighs $\frac{5}{8}$ as much as the same measure of water). No other simple ratio yields a calculated weight for classical six-row barley that is as close to the real thing.

The problem with ancient weights

While data about the test weight of grain narrow down the range of acceptable densities for a given grain considerably, data about ancient weights—the artifacts themselves—carry us unhappily in the opposite direction. To put it bluntly, the weight data are chaotic. Even the best "official" ancient examples, all purporting to represent the same standard, can differ from one another by a full percentage point or more. One percent on either side of a value may not sound like much, but it is a window of imprecision big enough to accommodate a number of competing theories, none of which can be ruled out based on the data. For that, we need about an order of magnitude better precision.

The "bad equipment" theory

Based on a reconstruction of an ancient Egyptian balance, F. G. Skinner concluded[8] that such an instrument had a precision of no better than two percent, and some later authors have followed him in explaining the almost random evidence of the ancient weights by the imprecision of ancient balances.

This is unlikely on the face of it—I hope that by now the Antikythera mechanism has put to rest the image of our ancestors as bereft of technical competence—and indeed an hour spent with a couple of cheap aluminum ashtrays for pans, some thread, a stick, and a pocket knife will produce an instrument whose sensitivity is better than what Skinner was getting from his Egyptian reconstruction. More to the point, surviving texts specify weights to a level of precision that proves that their users *thought* they were maintaining weights far more accurately. In accounts from Lagash, about the time of the Third Dynasty of Ur, we find entries recording goods received such as

> 1 mina and 16 shekels less 5 grains of silver. . .
>
> $\frac{1}{3}$ shekel less $7\frac{1}{2}$ grains of silver. . .
>
> Total: 2 minas 10 shekels 25 grains of silver.[9]

A Sumerian grain was $\frac{1}{10,800}$ of a mina, so specifying a weight of one mina to the nearest grain implies a precision of about 0.01 percent, not 2 percent.

While it would be a mistake to overestimate either the ability or the honesty of the average ancient trader, attributing the chaos of the surviving weight data to a fundamental limitation of the equipment assumes that the people who invented agriculture, writing, mathematics, astronomy, and the wheel were delusional when it came to their ability to weigh things. I do not find this a persuasive explanation for the confusion presented by the data. Instead, I would point to a combination of other factors.

First and most important is the physical condition of the weights. The very oldest are made of stone, typically of different shapes representing different units, systems, applications, or sets; these are generally chipped to some extent, which of course directly affects the thing in which we are most interested here, which is the weight of the object. Unless perfectly preserved (and very few of them are), the weight of an ancient stone weight today is always less than its original weight, and estimates of the loss will vary according to the researcher and the methodology.

The metal weights that come later in the record are even worse; they are almost always corroded, which changes their weight unpredictably to make them lighter or heavier than they originally were, depending on the alloy and the circumstances. For this reason, ancient metal weights that are not in perfect condition are almost useless for establishing ancient standards with any precision.

A second factor contributing to the confusion is the mixing together of weights belonging to different weight standards. Very few of the centers of trade in which weights are typically found used just one system of weights. There were as many as half a dozen distinct national standards in the late bronze age and early classical era, and to accomplish trade, measures from several different traditions had to be available in the marketplace. So it is rare to find a collection of any size that does not include a mixture of units based on different standards.

Yet another source of confusion, not generally acknowledged in the literature, arises from weights whose values are not derived from the set of ordinary decimal-based (5, 10, 20, 50, 100, ...), binary (2, 4, 8, ...), or sexagesimal/duodecimal-based (12, 15, 30, 60, 120, ...) factors we would expect—the kind we would call (or understand someone else calling) "round numbers," though perhaps "significant numbers" might be a better term. The job of analyzing ancient weights hinges on figuring out the appropriate "round number" factor by which to multiply or divide their measured weight in order to identify the base unit in each case. For example, based on what we know of ancient systems, it is quite reasonable to divide the weight of an object weighing about 250 g by 30 in an attempt to identify its base shekel because 30 shekels would be a very common weight (half a mina) in systems that have a mina of 60 shekels, and a mina weighs about 500 g. But it is not reasonable to divide this weight by (say) 27, because 27 shekels does not play a role as a unit of measure in any of the ancient systems for which we have textual information.

It turns out, however, that weights were used by our canny ancestors in ways that probably did create sets containing values like 27.

Subtractive weight sets

In his brilliant 1978 analysis of a collection of Viking gold ring weights from the 9th century A.D.—a much later culture than the ones we are looking at here, of course, but not more technologically advanced—Egil Bakka found that a set of seven weights could be used, not just additively against the goods, as usual, but also sometimes subtractively by being placed *in the pan with the goods being weighed.*[10] When used this way, the set of seven weights serves to measure 95 distinct values out of a range of 96, thus subdividing a range of two ounce-size units called aurar into 48 parts each for the weighing of coins. Bakka was then able to show that another set of eight Viking weights, found in a different location, can be used to accomplish the same thing for two distinct standards at once, one for buying and one for selling!

As it is highly unlikely that such astonishingly clever assemblages were the unique and isolated inventions of two Viking moneychangers, there can be little doubt that they represent a much older tradition of using the subtractive method to minimize the number of weights needed in a set. In particular, a set consisting of just the units 1, 3, 9, and 27 can in this manner weigh all values between 1 and 40, and this is the largest continuous range of subdivisions it is possible to obtain from a set of four weights.

It seems to me probable that this unique property of the number 40 was noticed early in the use of weights; if so, it would have conferred upon 40 a kind of mystical significance that we would expect to see reflected in traditions of the cultures that developed complex metrological systems. In fact, this influence can clearly be seen in ancient Hebrew scriptures and in the Christian and Muslim traditions that follow.[11]

Adding a weight of 80 units to the basic set of four weights allows the set to weigh all values between 1 and 120, and a sixth weight of 240 units extends the range to cover 1 to 360. The values 120 and 360 are very close to the largest continuous range of integer values that can be represented by any set of five or six weights; the actual maximum for a set of five weights is 121, achieved with the series 1, 3, 9, 27, and 81, and the actual maximum for a set of six weights is 362, achieved with the series 1, 3, 9, 27, 81, and 241,[12] but no ancient with any sense would have chosen to divide an interval into 121 or 362 parts. The subtractive weight set that yields the greatest *practical* continuous range of gradations with no more than five weights is the one that multiplies some unit base by 1, 3, 9, 27, and 80 and allows the weighing of any multiple of the chosen base from 1 to 120, and the set that yields the greatest practical continuous range of gradations with no more than six weights is the one that multiplies some unit base by 1, 3, 9, 27, 80, and 240 and allows the weighing of any multiple of the chosen base from 1 to 360. The unique "maximal practical" five-weight set and the unqualifiedly maximal four-weight subtractive set are both subsets of the six-weight set, which means in practice that any four-weight subtractive set can be made into the "maximal practical" five-weight set or the "maximal practical" six-weight set by adding weights (80 and 240) already common in the larger commercial environment. It also means that it is impossible to tell whether weights of 80 units and 240 units originally belonged to a subtractive set, an additive set, or both.

The accompanying chart shows how a maximal four-weight system works in practice. The right side represents the scale pan in which goods will be weighed, and the left side represents the pan opposite. The merchant begins by putting some of the four weights on the left-hand pan (as one would expect) and also, depending on the total weight desired, some on the right; for example, to weigh out 32 units of some trade good, the merchant would place the 9 and 27 weights on the left and the 1 and 3 weights on the right. He then adds quantities of the goods being weighed onto the right-hand pan till the scale balances, having in this example 36 units on either side of the scale (although this detail is irrelevant to using the method as far as I can see[13]). Learning the system was probably largely by rote, but one notes mnemonic landmarks in the paired left-pan combinations at 4 (1 and 3), 10 (1 and 9), 12 (3 and 9), 28 (1 and 27), 30 (3 and 27), and 36 (9 and 27), and also the "split" pairings at 2, 6, 8, and 24. These two-weight combinations would be the most familiar to everyone and the easiest to set up; they would therefore tend to drive the quantities of sale thought of as "normal" to assume those values. An investigation of the many other interesting patterns to be found in this system is left as an exercise for the reader.

To extend the system from 40 units to 120, one merely adds an 80-unit weight to the set and then allocates the remaining weights to add from 81 to 120 or to subtract from 80 down to 41. The range 81–120 is obvious; just place the 80 on the left and then use the system exactly as shown. The range 80–41 is accommodated by arranging the weights as shown for 1–40 but flipping the pan assignments; to weigh 48 units, for example, one would put the 80 on the left and then arrange the weights as shown in the chart for 32 (because $48 = 80-32$) but place the 1 and 3 on the left with the 80 and the 9 and 27 on the right with the goods being weighed.

Given the obvious advantages of reducing the number of heavy objects a trader in bulk would have to carry in order to weigh the goods, it is very likely that subtractive systems were used far back into antiquity, and it would appear that the five-weight (120-division) and six-weight (360-division) subtractive series just described go a long way toward explaining the choice of 60 as the basis for the Sumerian/Babylonian number system, not because these particular subtractive series are the only ones possible (the Viking sets show that other arrangements were used for more specialized purposes), but because they are the simplest. Various consequences of this finding will be discussed as we continue.

The problem subtractive weights present when they are found divorced from their sets is obvious; is a given weight a perfect 27 or a bad 30? This prob-

Weights in pan opposite goods to be weighed			
1			
	3		
	3		
1	3		
		9	
		9	
1		9	
		9	
		9	
1		9	
	3	9	
	3	9	
1	3	9	
			27
			27
1			27
			27
			27
1			27
	3		27
	3		27
1	3		27
			27
			27
1			27
			27
			27
1			27
	3		27
	3		27
1	3		27
		9	27
		9	27
1		9	27
		9	27
		9	27
1		9	27
	3	9	27
	3	9	27
1	3	9	27

Weights in pan with goods to be weighed			Weight of goods
			1
1			**2**
			3
			4
1	3		**5**
	3		**6**
	3		**7**
1			**8**
			9
			10
1			**11**
			12
			13
1	3	9	**14**
	3	9	**15**
	3	9	**16**
1		9	**17**
		9	**18**
		9	**19**
1		9	**20**
		9	**21**
		9	**22**
1	3		**23**
	3		**24**
	3		**25**
1			**26**
			27
			28
1			**29**
			30
			31
1	3		**32**
	3		**33**
	3		**34**
1			**35**
			36
			37
1			**38**
			39
			40

lem becomes even more evident when one considers sets like Bakka's in which the weight systems are realized quite precisely in factors that appear in isolation to be completely random. For example, the weights in the first of the sets studied by Bakka have values of $\frac{2}{96}$, $\frac{3}{96}$, $\frac{6}{96}$, $\frac{6}{96}$, $\frac{13}{96}$, $\frac{28}{96}$, and $\frac{38}{96}$ of 2 aurar, which is of course not itself a unit of the system. Bakka was able to unravel this set only because it was discovered as a set; it would be virtually impossible to correctly identify an unmarked weight of $\frac{19}{48}$ or $\frac{7}{24}$ of a unit (to take two examples from the series above) if encountered in a random mix of weights based on several different standards.

While none of this helps explain variation in official inscribed standard weights—these constitute a special case that I will return to later on—the factors just listed amply explain the jumble that the general run of ancient weights presents to analysis, and we are no longer required to believe that the ancients were unable to accurately weigh anything while fooling themselves into thinking that they were. But the confusion in the data remains, and the consequence for any attempt to reconstruct the original, carefully maintained ancient standards is that the most attractive approaches are brute-force statistical ones, with results that are fuzzy at best.

Underwater archaeology establishes a standard

No one tried harder to fix a particular ancient weight standard than legendary Egyptologist W. M. F. Petrie, who invested half a century of work in studying several thousand weights recovered from his digs in Egypt. Petrie, considered by many the father of modern archaeology, not only carefully reconstructed and statistically analyzed the artifacts; he also developed taxonomies based on shapes, style, materials, and provenance to help decipher the data. Not suprisingly, Petrie almost always found several weight systems present in any one place, but the one that dominated, the distinctively Egyptian one, was based on a standard shekel that Petrie named "qedet." At some points in his lifetime study of the data, Petrie put the value of the qedet as high as 146 troy grains (9.46 g), but he arrived in the end at 144 troy grains (9.33 g) as the best estimate.[14]

Petrie's huge collection was eventually taken over by the Science Museum in London, where it was studied for another 30 years by the curator of the weights and measures section, the aforementioned F. G. Skinner. Skinner also arrived at 144 troy grains (9.33 g) as the best estimate for the qedet, and some other researchers adopted this value as the weight of the main Egyptian shekel standard; but it was generally understood that this was a statistical representation of a central tendency whose physical exemplars exhibit very high individual variation. And this is where the matter stood for several decades.

Two major discoveries since 1967 have confirmed the early work of Petrie and Skinner and finally established a value for this most important of ancient weight standards precise enough to serve in an inquiry about numerical relationships. Both discoveries were made in the new field of underwater archaeology, and both involved shipwrecks that occurred off the coast of Turkey along a major ancient trade route.

The first find was published in 1967 by G. F. Bass.[15] Dating from the 15th century B.C., the wreck contained a number of scale weights used by the traders on board. As usual, the collection exhibits evidence of several contemporary regional standards, the most prominent being the qedet. Bass did not attempt to reconstruct the weights to determine their original values, simply noting a span for the value of the qedet ranging from 9.25 g (for a six-qedet stone weighing 55.50 g) to to 9.4 g (for a 20-qedet weight of 188.00 g and a 50-qedet weight of 470 g). My own analysis of the reported data identifies 11 weights belonging to a qedet standard that averages 9.328 g.

The second wreck, dating from 1305 B.C. or shortly thereafter, contained 149 objects identified as scale weights.[16] After elimination of the metal weights as too corroded to provide useful data, the weights of the remaining stone pieces were reconstructed with extraordinary care and then put through a computer analysis to eliminate any subjective judgement on the part of the researcher (C. Pulak). The collection was found to contain weights belonging to several different standards, as usual, with those belonging to the qedet standard in this case falling into two groups based on their shape. The first group consists of weights of a "sphendonoid" shape showing an average unit weight of 9.35 g and a weighted average value (favoring the heavier weights) of 9.33 g; the second group, consisting of weights having a domed shape, shows an average unit weight of 9.34 g and a weighted average of 9.32 g.

To sum up, the best information now available places the original qedet standard—the fundamental weight standard of ancient Egypt—at 9.33 ± 0.01 g. The center of the range is the same as the value arrived at by Petrie and Skinner decades ago, but now our estimate of the original standard has been narrowed to about a tenth of a percent either way. Together with the concept of a density canon, this is close enough to establish the probable weight/volume relationship notionally underlying the ancient Egyptian system of weights and measures, as I shall now demonstrate.

The Egyptian system

Let us begin with a brief review of the ancient Egyptian system and what we know about its standards.

For Egyptian length standards, we can do no better than Petrie, who approached metrology from a civil engineering angle and amassed measurements of the most important ancient monuments carried out with the skill of a professional surveyor. Based on the average cubit used in constructing the six most important 4th–6th dynasty pyramids and temples, Petrie found the royal Egyptian cubit to be 20.63 inches (52.400 cm)[17] or "most accurately 20.620 [52.375 cm] in the Great Pyramid."[18] Based on much earlier data, Sir Isaac Newton put the Egyptian cubit at 1.719 feet, which is 20.628 inches (52.395 cm). In the analysis coming up, I split the difference between Petrie's two estimates and assume a cubit of 20.625 inches (52.3875 cm).[19]

We know the Egyptian units of capacity and their basic relationships from surviving texts.[20] The most important units were the hon (about a pint, or half liter), the hekat of 10 hons, and a unit of $\frac{1}{32}$ hon called the ro. The hon was $\frac{1}{300}$ cubic cubit, putting the hekat at $\frac{1}{30}$ cubic cubit and the ro at $\frac{1}{9600}$ cubic cubit. These textually attested relationships are borne out by surviving ancient Egyptian capacity measures as estimated by Petrie purely from the physical data; he put the hon (based mostly on fragments) at 29.1 cubic inches or 477 cm^3,[21] whereas by the definition based on the cubic cubit, the hon is 29.246 in^3 (479.25 cm^3).

The connection between Egyptian measures of volume and capacity is clear; the question is whether there is a relationship between the Egyptian measures of capacity and the Egyptian measures of weight. If such a relationship exists, it will also relate weight to volume, and through volume to length.

Canonical barley

Earlier it was shown that the logistical calculations needed to conduct ancient trade would have necessitated a canonical relationship between the densities of basic trade substances. The most important ancient trade good was grain, and this leads to the principal concept I am trying to introduce here, which is the idea of a *canonical grain weight.*

Since it is clear that isolated samples of grain will vary widely in test weight, and since we have good reason to suspect the existence of a canonical density relationship, it appears that the sensible thing for the ancients to have done would have been to base the assumed average weight of grain on its canonical relationship with water. In other words, a measure might have been *defined* as holding a unit weight of barley, but in practice it would have been *tested* by seeing whether it contained $\frac{8}{5}$ of that unit weight of water. The definition would thus be understood as saying that the measure holds the unit weight of *canonical barley,* that is, barley that weighs exactly $\frac{5}{8}$ as much as water. This device would allow an ancient trader to make accurate and repeatable calculations involving the transport of grain using a simple relationship that referred the definition based on grain in practice to water, the density of which is far more constant from sample to sample. Calculation of the density of canonical barley will give us the single most important datum with which to unravel the old systems of measurement.

In northern European systems like the ones I discuss in *The Old Measure*, the most reasonable estimates of the weight of water assume well water or spring water, which has a temperature in the neighborhood of 5 to 10 degrees Celsius. In the Middle East, however, it is more likely that water would have been weighed at what we consider room temperature, so I will adopt water at 20 degrees Celsius or 68 degrees Fahrenheit (a generally accepted value for room temperature) as "standard water" for this analysis. According to the *Handbook of Chemistry and Physics,* pure water at 20 degrees Celsius weighed in air at standard pressure against brass weights has a density of 3774.653 g per U.S. gallon of 231 in^3, which works out to 0.9971578 g/cm^3. Canonical barley at $\frac{5}{8}$ the density of water would therefore weigh 0.6232236 g/cm^3 (for comparison with the agricultural data presented in *The Old Measure*, this is equivalent to about 48.42 lb/bu, which is an excellent representative value for actual good-quality six-row barley; by comparison, the bushel of barley as a unit of weight in the U.S. is defined as 48 pounds). We now have a precise enough value for the weight of barley to determine the basic weight/capacity relationship.

Taking the cubit to be 20.625 inches (52.3875 cm) as above, the hon works out to be 479.25 cm^3 and the ro ($\frac{1}{32}$ hon) to be 14.977 cm^3; and the ro of canonical barley ($\frac{1}{9600}$ cubic cubit) is therefore found to weigh 9.334 g, which is within the very narrow range for the weight of the qedet established by the shipwreck data. This very precise explanation for the relationship between volume/capacity and weight in the ancient Egyptian system is also arguably the most obvious and straightforward explanation possible: the most important small unit of capacity (the ro), when filled with the most common trade good (barley), weighs the same as the most important small unit of weight (the qedet).

Earlier I noted that the medieval system explored in *The Old Measure* was based on three substances—barley, wheat, and wine—rather than the two (barley and water) that I see as basic in the oldest systems. It is possible to imagine some transitional phase in which wheat, as it became a more important grain than barley, was first incorporated into the oldest system with a density ratio of 5 to 4 with barley (which, as already stated, is the simplest accurate ratio between the bulk weights of actual wheat and barley). In such a system, the ratio of wheat density to water density would be $\frac{25}{32}$. Adding wheat to the barley/water ratio yields a compelling set of weight equivalents for the Egyptian unit that was used for measuring grain, the hekat of 10 hons:

hekat of barley weighs	320 qedets
hekat of wheat weighs	400 qedets
hekat of water weighs	512 qedets

This interpretation fits the facts as currently known much better than the theory advanced by Petrie and later by Skinner that the hekat was intended to hold 500 qedets of water, which would put the qedet at about 9.5 g. This theory was tenable given the broad range formerly assumed for the qedet, but it cannot now be considered persuasive in light of the more precise value for the qedet established by the shipwreck data, which the relationship of 512 qedets to the hekat of water fits perfectly. The number 512 itself (2^9) would have been well known to ancient Egyptian scribes, because factors of 2 played a central role in their system of arithmetic, and a knowledge of the binary series 1, 2, 4, 8, ... 256, 512, ... was essential to their methods of multiplication and divison.[22]

The Sumerian system

The preceding exegesis of the Egyptian system gives us a lens through which to examine the ancient Sumerian system as well. Here a basis in weights of barley is, if anything, even more obvious than in the case of the Egyptian system, as the role of barley in Sumerian metrology is well attested; and the results just obtained for Egypt give us a way to look beyond the less precise values we have for the Sumerian weights. As in the preceding analysis, I will first review what we know from the surviving texts and artifacts.

The archaeology of Sumeria, Babylonia, and Assyria that gave us most of the metrological data we still have today was dominated by French and German researchers in the pre-WW1 era just as Egyptian archaeology of the period was dominated by the British. Here the names to be remembered are F. Thureau-Dangin and O. Neugebauer.

Sumerian lengths

The Sumerian/Babylonian "foot" of 20 "digits" was established by Thureau-Dangin early in the 20th century at 330 mm (12.99 inches) and the cubit of 30 "digits" at 495 mm (19.49 inches).[23] As far as I know, these estimates have persisted.

The quotation marks around "foot" and "digit" are intended as reminders that the actual Sumerian and Babylonian words for these measures have nothing to do with feet or fingers, and neither do their actual sizes. At almost exactly 13 inches, the “foot” is quite a bit longer than a human foot (the Sumerian word for the unit means something like “measure”), and the corresponding “digit” of 1.65 cm (0.65 inch) is quite a bit narrower than an adult human finger, which is reckoned in most systems at about 0.75 inch (about 1.9 cm). The Sumerian cubit, on the other hand, while belonging to a distinct standard, is an ordinary cubit.

Sumerian weights

We know the names of the Sumerian/Babylonian weight units and their numerical relationships quite well from surviving texts. According to Neugebauer,[24] and generally agreed to by other researchers, the system was:

grain	a (theoretical) grain of barley
shekel	180 grains
mina	60 shekels
talent	60 minas

Again, the generic names used here are for convenience and do not resemble the actual Sumerian and Babylonian names of these units.

While the basic aspects of the weight system itself are uncontroversial, actual instances of weights belonging to this system vary in a way that has frustrated attempts to precisely ascertain the standard for more than a century. Everyone agrees that there was a standard, or standards, somewhere in the neighborhood of 8.1 to 8.6 g for the shekel or about 486 to 516 g for the mina; that this standard is very old, existing examples dating back to before 2000 B.C.; that the original Sumerian standard was carried forward by the Babylonians; and that it was given a further lease on life when much later it was made the basis for Persian coinage under Darius the Great (521–486 B.C.), for which reason it is often called the “Daric” standard.

In my opinion, the best attempts to nail down the standard or standards involved in the weight systems of the Sumerians, Babylonians, and Assyrians were the taxonomic analyses of the early 20th century, which were based on well-preserved weights marked with numbers indicating their value. This restriction automatically screens out unrelated objects misidentified as weights and provides a starting place for estimating the base unit in each case, though the apparent variability of even the best official examples still poses a challenge.

Most of the researchers who adopted this method of analysis concluded that there are at least two related standards represented in the data, a predominant one based on a mina of about 503 g and another based on a lighter mina of about 491 g. This was also my own conclusion after a 1985 study of all the data from all of the major digs published up to that time. On the basis of 128 items I identified as belonging to the upper standard and 124 as belonging to the lower standard, I arrived at estimates of 504.59 ± 2.81 g and 492.54 ± 1.62 g, respectively. A comprehensive overview of the research in this direction was provided in 1929 by N. T. Belaiew, who noted the findings of the leading researchers regarding the upper (primary) standard as follows:[25]

Petrie	502 g
Viedebantt	502 g
Belaiew	502.2 g
Thureau-Dangin	505 g
British Museum	505 g

From this we may fairly say that the primary Sumerian weight standard as it came down through the Babylonians and Persians was between 502 and 505 g for the mina and thus 8.37 to 8.42 g for the shekel. The midpoint of this range would be 503.5 g for the mina (8.39 g for the shekel), but with a margin of error of at least a gram and a half or 0.3 percent on each side, and much greater variation in the actual examples.

I should note that it is possible to reject the reading that perceives a separate lower variety of Sumerian/Babylonian mina averaging around 491 g. If the range of acceptable values is increased, then, of course, the margin of error will be much larger. I think, however, that the earlier researchers got it right, and that there is a distinct lighter mina standard whose purpose we do not understand. Perhaps it had some function related to paying and receiving rates; we have evidence from several traditions (China being a prime example) for a practice that worked this differential through the maintenance of parallel weight standards with values about as far apart as we see here. The 19th-century Chinese government standards for paying and receiving differed by a factor of somewhat under two percent; the two Sumerian mina standards differ by about two and a half percent, which might have been intended as the ratio 125 to 128. Thus the continued existence of two closely parallel weight standards in (more or less) the same place at (more or less) the same time is neither unique nor difficult to imagine. But as we will see further along, it is possible to construct another story about the lower standard that seems to tie it to a separate tradition.

In what follows, the term “Sumerian shekel” without further qualification will refer specifically to the upper standard of about 8.4 g, because this is the one upon which the rest of the Sumerian system seems to have been based.

Sumerian capacity measures

As with the weights, the textual accounts of Sumerian capacity measures are more certain than the physical data.

The Sumerian system of capacity measures for barley was

grain	the smallest unit of *capacity* (not weight)
shekel	180 (capacity) grains
sila	60 (capacity) shekels
ban	10 silas
bariga	60 silas (6 bans)

Notice that the grain and shekel, which in this context are measures of capacity, have the same names (in Sumerian) as the smaller two units of weight and that they are related to each other and to the larger units by the same factors. This is exactly like our use of the name "ounce" for both a unit of capacity ($\frac{1}{16}$ pint) and a unit of weight ($\frac{1}{16}$ pound), and as in that case, it strongly suggests a connection between capacity and weight. In our tradition, the two uses of "ounce" have become disconnected, the capacity-ounce and the weight-ounce being descended from two different sets of standards, as I explain in *The Old Measure*; but we did originally use capacity-ounces and weight-ounces that were directly connected, and we have every reason to be looking for a connection between the identically named Sumerian units of capacity and weight as well.

Our analysis of the Egyptian measures was critically enabled by a textually attested relationship between the measure of capacity (the hon) and the measure of volume (the cubic Egyptian cubit). A similar piece of textual attestation provides a critical link in the case of the Sumerian units, too. The most common value for the capacity of the sila found in ancient clay tablets is 21,600 sila per volume-SAR, the latter being a rectangular solid measuring 12 cubits by 12 cubits by 1 cubit, i.e., 144 cubic cubits.[26] Thus the Sumerian sila was $\frac{1}{150}$ of a cubic Sumerian cubit, echoing the definition of the Egyptian hon as $\frac{1}{300}$ of the cubic Egyptian cubit and hinting at a connection between the two systems that will be investigated further below.

If the Sumerian cubit is 49.5 cm, then the calculated sila is 808.6 cm^3. Two surviving representatives of this standard[27] imply a unit averaging 808.4 cm^3, so the textually implied sila is in line with the physical evidence.

It follows from the above that a capacity-shekel is the volume of three cubic Sumerian digits and the sila therefore the volume of 180 cubic digits. The number 180 is, of course, a perfect fit with the Sumerian and Babylonian approach to numbers and measurement in general.

Explaining the Sumerian system

As just noted, the parallelism between the weight and capacity units, and the use of the same word for the weight-shekel and the capacity-shekel, suggest a connection between the two series of units, and the weight of barley provides the link, just as in the Egyptian system. I am not the first person to observe this, but I think I am the first person to provide an exact figure for the canonical weight of the barley. Using the same room-temperature water and canonical $\frac{5}{8}$ barley/water ratio applied previously to explain the Egyptian system, the sila calculated from the Sumerian cubit holds a volume of canonical barley weighing 503.93 g (60 shekels of 8.3988 g). This is an excellent fit for the weight of the Sumerian mina and the Sumerian shekel as far as they can presently be determined.

The relationship between the units of capacity and the units of weight thus becomes:

1 capacity-grain of barley weighs 1 grain
1 capacity-shekel of barley weighs 1 shekel
1 sila of barley weighs 1 mina
1 bariga of barley weighs 1 talent

This is a system even more compelling in its straightforward simplicity than the Egyptian, and it is so obvious that I can claim no credit for pointing it out. The contribution here lies in fixing the relationship precisely through use of canonical density, and it seems to me that the correspondence between the calculated theoretical weight of the mina and our best attempts to estimate the weight based on the empirical data greatly increases our confidence in the intuitively apparent reconstruction.

The relationship between the Egyptian and Sumerian systems

Considering the importance of the Sumerian/Babylonian and Egyptian cultures in the ancient world, and the practical need for simple relationships between units of the two traditions given the very limited mathematical abilities of the average ancient person, it would be strange indeed if there were not some fairly direct relationship between the two national standards. On investigation, the relationship turns out to be very simple:

1 cubic Egyptian cubit = 4 cubic Sumerian feet

If we take the generally accepted figure of 33.0 cm for the Sumerian foot as exact, then the value for the Egyptian cubit that results from this relationship is 52.3842 cm or 20.6237 inches; this is virtually identical to the estimate adopted earlier based on the archaeological data. I find it hard to believe that I am the first person to notice this relationship, which must have been known to every ancient middle eastern trader, but so far I have not been able to find an earlier mention of it.

Since the preceding analysis has established a relationship between the Egyptian and Sumerian measures of volume and the predominant weight standard of each place, the relationship of Egyptian cubic cubit to Sumerian cubic foot also establishes a mathematical relationship between the Egyptian and Sumerian weight standards, which is

$$9 \text{ Egyptian qedets} = 10 \text{ Sumerian shekels}$$

I am not the first person to notice the $\frac{9}{10}$ relationship between the Egyptian and Sumerian shekels,[28] but once again the canonical barley method allows the relationship to be established mathematically based on a coherent theory.

Later manifestations of the Sumerian/Egyptian standards

Based on our assumptions about the computational needs of ancient traders, all the major standards of measurement must have had relatively simple relationships with each other, and this implies a *web* of relationships held together by various ratios.

Consider, for example, the later Babylonian division of the original Sumerian cubit into 24 digits rather than 30.[29] These digits were a little more than 2 cm (0.8 inch) wide, which is much closer to the typical width of the digit in traditional systems than the Sumerian "digit" of 1.65 cm.

Using all the same assumptions as in the Egyptian and Sumerian examples, the cubic Babylonian digit of water is found to weigh 8.7487 g; and this is the unit called the Babylonian stater, which Skinner puts at 8.75 g. It later became the Attic stater or didrachma, 50 of which made the Attic mina, and from this descended a number of later European standards, some of which survived well into the 19th century. The premetric pfund of the Austro-Hungarian Empire, for example, was 560.012 g; this was divided into 32 loth and each loth into 4 quentchen, making the latter 4.37509 g. By comparison, the drachma (half stater) implied by the canonical barley analysis of the Babylonian cubic digit is 4.3744 g, and the drachma corresponding to the stater as found by Skinner is 4.375 g.

It is the Sumerian cubic foot, however, that can be considered the mother of the bronze age capacity and weight standards used by surrounding trading partners.

First, the main Egyptian qedet standard can be understood as an outgrowth of the cubic Sumerian foot used as a capacity measure. If viewed using the later barley-wheat-water canon conjectured in discussing the Egyptian system, we get the following weights for one Sumerian cubic foot ($\frac{1}{4}$ cubic Egyptian cubit) of each substance:

cubic Sumerian foot of barley	2400 qedets
cubic Sumerian foot of wheat	3000 qedets
cubic Sumerian foot of water	3840 qedets

The weight of canonical wheat (3000 qedets) is especially significant: this is the Syrian talent or talent of Carchemesh, from which descended the medieval pound of Cologne (the mina of this system), which is closely related to the troy weight system still used in some places today.[30]

Earlier we saw how the canonical barley method connects the ancient Egyptian and Sumerian standards of weight and the two primary capacity measures of each system, the hon in Egypt and the sila in Sumeria. To these may be added the xestes of ancient Greece, which Skinner puts at 540 cm^3. This volume of canonical barley weighs 336.541 g, which is 40 shekels of 8.41352 g or $\frac{2}{3}$ of a mina weighing 504.811 g. In other words, the xestes is the volume of canonical barley equal to the total weight of a maximal four-weight subtractive set based on the Sumerian shekel.

It follows from the relationships worked out earlier that the xestes is $\frac{2}{3}$ of a Sumerian sila or 120 cubic Sumerian digits, and indeed, if the Sumerian foot is taken to be 33.0 cm as before, then the theoretical xestes calculates out to 539.055 cm^3, about 0.18 percent smaller than the xestes as found if Skinner's estimate of 540 cm^3 is taken as exact.

In addition to the simple relationship between the Greek xestes and the Sumerian sila, there is an equally simple connection between the xestes and the Egyptian hon. This relationship is produced by using the ratio $\frac{9}{8}$ to express the increase of a measure of grain due to heaping rather than leveling ("striking") it in the container. Heaping is important because it is the way large quantities of grain are measured as it is being flung into a granary or hold, so some assumed heaping ratio is always required to bridge actual practice with more careful measurement using striked measure. The heaping ratio $\frac{9}{8}$ is enshrined in old English statutes relating to a unit called the faat (a variant of *vat*), and evidence of its use can also be found in several traditional premetric European systems.[31] If a xestes of barley weighs 40 Sumerian shekels, then (according to the analysis above) it also weighs 36 Egyptian qedets. The hon of barley weighs 32 qedets, so if the heaping ratio is $\frac{9}{8}$, then the Greek xestes is identical to a heaped Egyptian hon. Since, as is generally accepted, Greece inherited much of its culture from Egypt, it is easy to imagine the xestes arising this way.

The connection with troy weight

The history of troy weight (still used for weighing precious metals), its connection with the ancient systems, and its role as the basis for the system still used to measure liquids in the United States is detailed at length in *The Old Measure,* so I will just note two of the more salient points of connection here.

First is the barley weight of the cubic Sumerian foot noted above: 2400 qedets at 9.33 g is 22.392 kg, which is 60 troy pounds of 373.2 g. If we calculate this directly from the density of canonical barley and the value of 33.0 cm for the foot, this is 60 troy pounds of 373.28 g. By comparison, the current official definition of the troy pound is 373.2417216 g. In other words, since the talent in ancient times was universally the weight of 60 minas or pounds of a certain standard, the weight of a cubic Sumerian foot of canonical barley is a "troy talent" of 60 troy pounds.

Over on the Egyptian side, the relationship with troy weight is even more convincing; the troy pound is 40 qedets. In other words, it is the total weight of the maximal four-weight subtractive set based on the qedet. If we take the troy pound at its current legal weight, then $\frac{1}{40}$ pound is approximately 9.331 g.

It seems apparent that the same standard was inherited by the Greeks. According to a recent comprehensive summary of Greek weight standards, the earliest system, instituted by Pheidon, King of Argos, was based on a unit of 748.44 g; this is two troy pounds of 374.22 g or 80 qedets of 9.356 g.[32]

It can be calculated from the system of relationships already described that the weight of one Egyptian hon of canonical wheat (weighing $\frac{5}{4}$ canonical barley or $\frac{25}{32}$ water) is one troy pound. As I mentioned earlier, the old English law specifying the capacity measures still used in the United States defines the gallon as the measure containing 8 troy pounds of wheat, which means that the pint ($\frac{1}{8}$ gallon) was the measure holding one troy pound of wheat. On my reading, the difference between the U.S. pint (about 473.18 cm^3) and the Egyptian hon (calculated as 479.25 cm^3 above) is the difference between the earlier density canon, based on barley and water with wheat as a later addition, and the later wheat-centered barley-wheat-wine canon that appears to underly the old English system. Both canons yield acceptable figures for the bulk weights of good quality wheat, but in the later system, the canonical wheat is slightly heavier, 61.28 lb/bu (0.7888 g/cm^3) vs. 60.52 lb/bu (0.7790 g/cm^3) for canonical wheat calculated from the weight of water. As it happens, this is in accord with what would be seen in wheat of slightly higher quality and what would tend to be seen in wheat grown farther north.[33]

Relationships via subtractive weights

The basic 1-3-9-27 subtractive weight set explains some relationships in ancient weight data that have previously been overlooked or misunderstood. It is the role of the 27 that commands our attention here; the others are, in addition to their subtractive role, perfectly ordinary additive weights as well and are abundantly represented in the data, but the fact that some weights in the data probably represent 27 of some base unit throws a different light on certain key weight relationships.

As I noted earlier, the apparent relationship between the Egyptian cubic cubit and the Sumerian cubic foot, when combined with the concept of canonical barley and the explanations I have offered for the volume/weight relationships underlying each system, imply that 10 Sumerian shekels are equal in weight to 9 Egyptian qedets. Expressed as $\frac{9}{10}$ this relationship does not look particularly interesting, though it was no doubt very useful in trade between the two cultures, but multiply it by 3 and suddenly it becomes relevant to the subtractive use of weights:

27 Egyptian qedets = 30 Sumerian shekels

In other words, the Sumerian half mina (30 shekels) was equivalent to the 27-qedet weight that would have been included by any Egyptian in the most efficient Egyptian subtractive weight set based on the qedet (the set that counts to 40 qedets with just the weights 1, 3, 9, and 27). Or to put it another way, every Egyptian trader already had a key Sumerian weight (the Sumerian half mina) in his own subtractive weight set. This happy equivalence works for larger units, too; basing the same subtractive series on the Egyptian sep (100 qedets) puts a 27-sep weight equivalent to 50 Sumerian minas in the Egyptian's subtractive set.[34]

This example suggests that we look for other possible cross-system relationships where a subtractive 27-unit weight in one system is equivalent to a common unit of another's, and when we do, a further instance becomes immediately evident: the Sumerian subtractive 27-shekel weight is equivalent to (and very possibly the ancestor of) the avoirdupois "mark" or half pound. If we assume the Sumerian shekel to weigh 8.3988 g as calculated above from canonical barley, then 27 shekels weighs 226.77 g; under current legal definitions, an avoirdupois half pound weighs 226.796185 g.

There should be nothing particularly controversial about this interpretation, as it has long been accepted that the avoirdupois pound came to England from Florence during the medieval wool trade and that it had a long history in Italy going back at least to Roman times;[35] this last link back to the unit of the subtractive Sumerian set just helps fill out the story, though it does not really explain where avoirdupois came from in the first place.

The Romans

The connection between the Roman system of weights and measures and the Egyptian is in its way even simpler than the relationship between the cubic Sumerian foot and the cubic Egyptian cubit. As noted above, the smallest unit of capacity in the Egyptian system was the ro of $\frac{1}{300}$ hon or $\frac{1}{9600}$ cubic Egyptian cubit; and the ro thus defined is the cubic Roman inch.

The best estimate I have been able to establish for the archaeological value of the Roman foot puts it at 11.649 inches (29.588 cm).[36] If a ro is a cubic Roman inch, and the Egyptian cubit has the length assumed earlier, then the length of the calculated theoretical Roman foot is 11.645 inches (29.579 cm).

If the Romans followed a pattern anything like the ones we have been looking at, we would expect to see some relationship between the water weight or canonical barley weight of this unit of volume and the smaller units of the Roman weight system. In fact, the calculated weights do provide a very accurate value for one of the Roman ounce standards.

Not surprisingly given the way their empire was formed, the ancient Romans seem to have inherited more than one set of weight standards; Skinner identifies no fewer than nine of them based primarily on coin weights. But surely the most significant for our purpose (to reconstruct the systems the ancients *thought* they were using) must be the one that aligns with the best-known textual definition of their standard. As testified by several ancient sources, the Roman pound was defined as $\frac{1}{80}$ the weight of water in a cubic Roman foot.

The equivalence of a cubic Roman foot of water to 80 Roman pounds is of course a perfect fit for the barley/water density canon, because a cubic Roman foot of canonical barley would weigh exactly 50 Roman pounds (a ratio of $\frac{50}{80}$ or $\frac{5}{8}$). According to this definition and the value for room-temperature water used earlier, and taking the Roman foot to be 11.645 inches (29.578 cm), the Roman pound thus defined weighs 322.55 g and the ounce therefore 26.879 g. (The cubic Roman inch of water calculated on this basis weighs 14.933 g, and the cubic Roman inch of canonical barley therefore weighs 9.333 g, which, as shown earlier, is the Egyptian qedet.)

As I note in *The Old Measure*, this value for the Roman pound is in very close accord with one of the best preserved sets of Roman weights still in existence. This is a set of scripulum weights from Roman Gaul, the Roman scripulum being $\frac{1}{24}$ of a Roman ounce. The mean weight of the scripulum on which the set is based is 17.283 ± 0.01 troy grains (1.11992 ± 0.0006 g), which corresponds to an ounce of 26.878 ± 0.0144 g. Compare this with the ounce of 26.879 g found above from the length of the foot and the weight of water.

Another important data point for this ounce is provided by the best preserved example of a Roman grain measure, the Carvoran "modius." This object was gauged very precisely by the British standards department in 1948 and was found to imply a volume for the Roman trade hemina ($\frac{1}{2}$ sextarius) between 19.7171 and 19.7732 cubic inches, depending on the method of measurement.[37] This hemina is $\frac{1}{80}$ of a cube measuring 11.6407 to 11.6517 inches (29.5673 to 29.5953 cm) on a side, and it contains water at room temperature weighing one Roman pound with ounces of 26.8489 to 26.9252 g; the midpoint of this range is 26.8871 g.

An ancient problem in arithmetic highlights the connection between the Egyptian system and standards and those of both the Sumerians and the Romans. The Kahun Papyrus preserves an exercise in which one calculates the volume of 40 baskets of 90 hons each, the basket being "a reasonable load for a laborer to carry."[38] Using the same values for the Egyptian cubit and the weight of canonical barley assumed before, the basket of barley weighed 26.881 kg. This is 3200 Sumerian shekels of 8.4004 g and also, of course, one thousand Roman ounces of 26.881 g.

In addition to the precise correspondence with the well-known definition connecting the cubic Roman foot with a weight of water, this value of about 26.88 g corresponds to two of the Roman ounce standards listed by Skinner as implied by coin weights: the ounce of 26.7 g, which Skinner calls the "Roman Trade Weight Standard" beginning around 250 B.C., and the ounce of 27.02 g, corresponding to six of the gold "solidus" coins first issued in A.D. 324 by Constantine the Great and maintained by subsequent rulers of Constantinople until the end of the Eastern Empire in 1453. It is easy to see this as a single standard that became slightly heavier over the course of more than 500 years; if so, its average value would have been 26.86 g.

The German inheritance

The preceding analysis of the Roman/Egyptian systems also sheds light on an important later unit, the Rhineland foot. This was the traditional standard unit of length in German-speaking countries until its replacement by the metric system.

The 19th century Rhineland foot in Berlin was 31.38535 cm (12.35643 inches)[39] and in Amsterdam was 31.39465 cm (12.3601 inches).[40] A colonial variant of the Dutch version of the Rhineland foot survived into the mid-20th century as the Cape foot of South Africa at 31.4858 cm (12.3960 inches).[41] The

Scotch foot, which appears to come from the same tradition, is put by various sources at 12.353–12.400 inches.

It is well known that the Egyptian cubit was officially divided, rather strangely, into 28 digits. (I will return to this in a different context further on.) It is apparent from surviving cubit rods, however, that for practical work the Egyptians sometimes divided the same cubit into 20 units of a size that we would call an inch.[42] In fact, this "inch" is the inch *(zoll)* of the Rhineland foot. If the Egyptian cubit is taken to be 20.625 inches (52.3875 cm) as assumed before, then 12 "inches" of this cubit (i.e., $\frac{12}{20}$ or $\frac{3}{5}$ of the cubit) is 31.4325 cm, which is right at the middle of the range delimited by the Prussian and South African versions of the Rhineland foot.

Canonical barley analysis of the Rhineland foot demonstrates the way this unit relates to the Roman and earlier standards. Assuming the derivation of the Rhineland foot just described, and using the same values for canonical barley and the Egyptian cubit employed earlier, the weight of a cubic Rhineland foot of barley is found to be 60 Roman pounds of 322.574 g or 720 ounces of 26.8812 g. To put it another way, using the ancient definition of a talent as 60 minas, the cubic Rhineland foot holds a "Roman talent" of barley. Compare this with the "troy talent" of the same canonical barley contained in the cubic Sumerian foot, as noted earlier.

This correspondence works out well for the smaller units, too. The cubic Rhineland *zoll* on this reckoning holds 11.20049 g of canonical barley; this is 10 of the scripula upon which the Roman weight set from Gaul noted above was based.

The Indus and Chinese systems

As detailed in *The Old Measure*, we know the length standard (or one of the length standards) of the bronze age Indus culture from a very accurately engraved section of a ruler found at Mohenjo-Daro that puts the unit at 1.320 inches (the last digit of precision is real). Divisions on the ruler suggest that the segment is $\frac{1}{10}$ of a foot of 13.20 inches (about 33.53 cm). This aligns perfectly with a very old and widespread length standard called the Northern foot that still constitutes the basis of English and American land measures at a value of 13.2 inches (33.528 cm) exactly. The same standard underlay a major French land measure, the canne of Marseilles, which was the length of six Northern feet of 13.206 inches (33.543 cm). Some researchers believe that the Northern foot and the Sumerian foot are loose variants of the same standard, but according to my reading, these are two distinct standards that arose within two different traditions.

In *The Old Measure*, I demonstrate the relationships among the Indus length unit ($\frac{1}{10}$ of the Northern foot), the troy system of weight, and the traditional English system of wine capacity units. For example, the cubic Northern foot holds 10 Anglo-American wine gallons (U.S. gallons), which can contain an amount of wine weighing 10 troy pounds.[43] Pushing this unit back into the archaic barley/water system, we find that the cube $\frac{1}{10}$ of a Northern foot on a side weighs, if filled with water, 580.25 troy grains or 37.6 g. As detailed further in *The Old Measure,* this is an excellent mean value for the tael, the unit (originally Chinese) traditionally used throughout Asia for the measurement of precious metals and spices, similar to the way the troy ounce is used in the Anglo-American tradition.

The Chinese connection is completed by the fact that the Northern foot survived as a Chinese land measure with a value in Peking (Beijing) and Shanghai of 33.5261 cm or 13.1993 inches, virtually identical to the Indus, French, and English versions,[44] and this land foot was divided into tenths called t'sun that are equal to the 1.320 inch (33.528 mm) unit found at Mohenjodaro.

Turning now to the relationship between the Northern foot and the Indus weight system, the weight standard is easily explained on an alternative division of the Northern foot of 13.20 inches into 12 Northern "inches," each of which would be 1.100 inches (27.94 mm) in length. (This alternative division of the same foot into different subunits depending on the use is similar to the practice of the Romans, who divided foot rulers into 12 inches along one edge and 16 digits along the other.) We know that the Saxon version of the Northern foot (at an average value of 13.22 inches) was, in fact, divided into 12 "thumbs," so it is not a stretch to imagine that the Indus version was sometimes so divided as well. If barley canonically weighs $\frac{5}{8}$ as much as water, then a cubic Northern inch of this barley would weigh 13.59 g; this is a reasonably good fit for the unit that has been identified as the basis for the Indus weight system, which A. S. Hemmy put at 13.625 g.[45]

Variation in official standard weights

In addition to providing what I believe to be the most compelling explanation of the ancient Egyptian, Sumerian, and other late bronze age systems of weights and measures, the canonical grain theory also explains one remaining puzzle about ancient weights, which is why even the least ambiguous and best preserved examples can vary more than one percent from the mean.[46]

If these systems are (as I maintain they must be) the systems their users *intended,* then the definitions underlying them could have been considered as dependable as any artifact in establishing a weight or capacity standard. Thus, in officially re-establishing a standard after a lapse of some years, the authorities might well have decided (for example) to reconstruct a cubic measure based on still existing length measures and then weigh it full of something to re-establish the standard weight. Re-establishing the weight standard based on the weight of some substance contained by a surviving old capacity measure could have been attempted, too, as could re-establishing a weight standard based on the weight contained by a newly manufactured cylindrical measure of given dimensions; and, of course, any of these techniques could have been used to check an existing weight claiming to conform to a standard in the absence of a trustworthy copy.

If the canonical relationship with water had been lost by that time and the artisans were working from the primitive notional basis on a weight of grain by filling a volume or capacity measure with actual grain, the resulting derived weights would vary by even more than we observe in the data. Such is the practical difficulty of precisely reproducing a given volume, however, that even if the density canon were fully observed and the calculation were based on a weight of water, it would have been everything an ancient craftsman could have done to hold the resulting derived weight within the range of variation we see in the surviving examples.

This interpretation not only explains the variability of ancient official weight standards, it also explains why their average fits what we find independently from the ordinary run of commercial weights and also what we can calculate from lengths and densities. Given specific official length standards (the existence of which is not in doubt), the average of multiple best efforts by various craftsmen over the centuries to establish weights based on their canonical definitions would inevitably converge on a mean value close to what would have been found if the recreated volumes had been perfectly constructed. All we have to assume here is that water at "room temperature" represents a reasonably constant density, which of course it does, and then reversion to the mean is guaranteed even though attempts by individual artisans would have varied randomly, as seen in the data.

A second standards web

The foregoing analysis has used "canonical barley" to reveal a conceptual basis for the ancient Sumerian and Egyptian systems of weights and capacity measures and to demonstrate connections with some weights and measures of length used in ancient India, China, and Rome. These can all be considered aspects of a single, mathematically consistent interlocking web of standards. One way to sum up this standards web (I will call it W1) in terms of units of length is as follows:

1 cubic Egyptian cubit = 4 cubic Sumerian feet = 9600 cubic Roman inches

1 cubic Roman inch = $3\frac{1}{3}$ cubic Sumerian digits (Sumerian digit = $\frac{1}{30}$ Sumerian cubit)

5 cubic Babylonian digits = 2 cubic Northern inches (Babylonian digit = $\frac{1}{24}$ Sumerian cubit)

Applying the canonical barley analysis technique starting from a different base reveals another set of relationships that seems to have existed in parallel with the one that I have just discussed. I will call this second set of relationships W2. Like W1, W2 exhibits a coherent set of precise mathematical relationships between volume and weight units, but unlike W1, W2 does not present the same kind of satisfying relationships with measures of capacity. I will come back to what (if anything) to make of this after reviewing the numbers.

The analysis that yields W2 applies the same set of techniques employed in the analysis of W1, but it proceeds from a different starting point—not the Egyptian Royal cubit of 20.625 inches (52.3875 cm) but rather the digit of this cubit.

The peculiarity of Egyptian digits

As noted before, the Royal Egyptian cubit was regularly divided into 28 digits, i.e., seven palms of four digits each. This division of a linear measure by 28 (or any other factor of 7) seems to be unique among major systems of measurement, and it poses a continuing puzzle for the analyst. To put it plainly, there is no apparent advantage to the use by the Egyptians of a base-7 system of linear measurement, and such a system fits their decimal system of notation and what we know of the rest of their system of weights and measures very badly. The only sort of practical reason that suggests itself would be something based on cylindrical measurements that took advantage of the excellent working approximation $\frac{22}{7}$ for π, the ratio of a circle's circumference to its diameter (there is a longstanding theory that the angles of the Great Pyramid attest to the use by the Egyptians of this value of the constant). But no such cylindrical interpretation of the known Egyptian measures of capacity is apparent.

The Egyptian digit makes sense, however, if considered in relation to a shorter cubit also known to have been used by the Egyptians that consisted of 24 of the same digits of which 28 made the Royal cubit. This cubit—6 palms of 4 digits each—is a perfectly ordinary representative of a tradition of length units that persisted in some places right up to its replacement by the metric system. The shorter Egyptian cubit was widely enough used to have been adopted by the ancient Hebrews as their own; they called this shorter cubit of 6 palms "the cubit" and the Royal Egyptian cubit of 7 palms "the cubit and a handsbreadth" (Ezekiel 40:5 and 43:13). Using canonical barley analysis, the Egyptian digit understood in the context of the shorter Egyptian cubit provides a tool with which to reveal a number of relationships among Sumerian, Roman, Northern, and later European units of measure.

The light Sumerian standard revisited

Earlier I noted the existence of a lighter version of the Sumerian weight standard based on a shekel of about 8.2 g alongside the more dominant shekel of about 8.4 g. Using figures published by Belaiew in the article cited earlier, estimates of the mina of this standard and its corresponding shekel ($\frac{1}{60}$ mina) can be listed as follows:

	Mina	**Shekel**
Viedebantt	489.50 g	8.158 g
Lehmann-Haupt[47]	491.20 g	8.187 g
Belaiew	491.14 g	8.186 g

Canonical barley analysis immediately reveals a connection between this standard of weight and the cubic Egyptian digit; if the Royal Egyptian cubit is taken to be 20.625 inches (52.3875 cm) as before, and the digit is $\frac{1}{28}$ of this cubit, then the cubic digit of canonical barley weighs 4.0818 g, which is half a shekel of 8.1636 g and $\frac{1}{120}$ of a mina of 489.82 g. As Belaiew pointed out, the lighter Sumerian mina is the premetric French livre (489.505847 g); and as Lehmann-Haupt observed as far back as 1889,[48] two of these minas or livres line up perfectly with three Roman libras of a Roman weight standard distinct from the one discussed earlier, this libra (figured as $\frac{2}{3}$ of the theoretical mina) being 326.54 g and the Roman ounce therefore 27.212 g. This is the ounce standard of the gold aureus of Augustus, which Skinner put at 27.28 g. In the 1930s, a perfectly preserved weight of this standard with the Greek inscription "Libra 1" was discovered at the site of a Roman villa in North Linconshire and found to weigh 5051.25 troy grains, which is 12 Roman ounces of 27.2763 g.[49]

This Roman ounce of about 27.2 or 27.3 g does not fit the classical definition of the Roman pound as $\frac{1}{80}$ the weight of a cubic Roman foot of water, but it does tie in nicely with the derivation of the Indus shekel from the cubic Northern inch of canonical barley, the Roman ounce in this case simply being, as Berriman observed, two Indus shekels.[50] (Two of the Indus shekels calculated above from the canonical barley weight of the archeological Northern inch would weigh 27.18 g; two Indus shekels as statistically determined by Hemmy would weigh 27.25 g.) It also ties in very well with the Attic mina (the mina of Athens), which was 50 Babylonian shekels (see above) and is easily interpreted as equivalent to 16 Roman ounces of this standard.

The relationship implied between the Roman aureus libra and the lighter Sumerian standard is interesting with regard to the earlier discussion of equivalences to subtractive weight sets: under this interpretation, the Roman aureus libra is 40 lighter Sumerian shekels. In other words, the Roman libra of this standard is equivalent to the total weight of a four-weight subtractive set (1, 3, 9, 27) based on the lighter Sumerian shekel as the unit.

More relationships based on the Egyptian cubic digit

Analysis based on the cubic Egyptian digit and the lighter Sumerian shekel reveals several other interesting relationships among various ancient units of weight and length. Berriman noted one of the most direct: the talent of the lighter Sumerian standard (3600 shekels) is the weight of an Olympic Greek foot of water. If the mina is theoretically 489.817 g as calculated above, and "room temperature" water is estimated as before, the Olympic foot is 30.889 cm or 12.161 inches. This is an excellent fit for archaeological estimates of the Olympic foot.[51]

Several interesting equivalences fall out of the relationship between the cubic Egyptian digit and the Northern foot implied by the relationships just noted between weight standards. For example:

1 cubic Northern inch = $3\frac{1}{3}$ cubic Egyptian digits

If the Egyptian cubit is 20.625 inches (52.3875 cm) as assumed before, then according to this, the Northern foot is 13.204 inches (33.539 cm). It follows that

1 cubic Northern foot = 5760 cubic Egyptian digits

As noted earlier, the unit that was traditionally known as the wine gallon and continues in existence as the U.S. gallon is $\frac{1}{10}$ of the cubic Northern foot. This means that

3 gallons = a cube $\frac{1}{2}$ shorter Egyptian cubit (12 Egyptian digits) on a side

This unit of 3 gallons is represented by the Carvoran measure described earlier, which has a measured capacity between 690.10 and 692.06 in^3, depending on the method of measurement. If this is a cube half a shorter Egyptian cubit on a side (i.e., $\frac{1}{8}$ of a cubic cubit), then the shorter Egyptian cubit is between 17.674 and 17.691 inches (44.892 and 44.934 cm) in length, and a Royal Egyptian cubit ($\frac{7}{6}$ of the shorter cubit) is between 20.620 and 20.639 inches (52.374 and 52.423 cm) in length. The midpoint of this range is 20.629 inches (52.399 cm), which is virtually identical to the value for the length of the Royal Egyptian cubit assumed throughout this analysis.

It follows from the relationship between the gallon and the shorter Egyptian cubit that

1 gallon = 576 cubic Egyptian digits

which is significant because if $\frac{\pi}{4}$ is taken to be $\frac{64}{81}$ (an approximation that we know from papyri to have been used by the Egyptians), then the gallon is exactly the volume of a cylinder 9 Egyptian digits in height and 9 Egyptian digits in diameter.

Another Northern foot relationship can be expressed as follows:

36 cubic Northern inches of canonical barley weighs 1 lighter Sumerian mina

That is,

1 cubic Northern foot of canonical barley weighs 48 lighter Sumerian minas

Forty-eight Sumerian minas is $\frac{4}{5}$ of a talent. The special significance of this relationship lies in the theory, alluded to earlier, of a later density canon in which canonical wheat weighed $\frac{5}{4}$ as much as canonical barley; in this case, the relationship would have been

1 cubic Northern foot of canonical wheat weighs 1 lighter Sumerian talent (60 minas)

And combining this with Berriman's observation regarding the cubic Olympic foot, we get

1 cubic Northern foot of canonical wheat weighs the same as 1 cubic Olympic foot of water

Later Arabic units

The Egyptian digit and the lighter Sumerian shekel also provide an explanation for the two most important later Arabic length standards, the Hashimi cubit and the Black cubit. According to Skinner, the Hashimi cubit was 64.9 cm in length and was divided into 24 inches. Approached through a comparison of cubic measures, it is immediately apparent that

1 cubic Hashimi cubit = 3 cubic shorter Egyptian cubits

and therefore

1 cubic Hashimi inch = 3 cubic Egyptian digits

The theoretical Hashimi cubit implied by this relationship is 64.762 cm (25.497 inches), which is 0.21 percent smaller than the Hashimi cubit as found. In terms of weight, we find from the volume relationship that

40 cubic Hashimi digits of canonical barley weigh 1 light Sumerian mina

The significance of the number 40 here should be apparent from previous comments regarding subtractive weight systems: the weight of one cubic Hashimi digit of barley is the base unit of weight that produces a 1-3-9-27 subtractive set the total weight of which is the light Sumerian mina.

The other important later Arabic standard of length, the Black cubit, is put by Skinner at 54.05 cm. Canonical barley analysis shows the cubic Black cubit of barley to weigh 200 lighter Sumerian minas and the same volume of water to weigh 320 minas, the mina in this case (using Skinner's figure for the length of the Black cubit) evaluating to 492.0407 g and the corresponding shekel to 8.20068 g. It is notable that a later addition of wheat to the density canon at $\frac{5}{4}$ the weight of barley would form the series 200-250-320, which is one of the most graceful expressions of the later canon.

It follows from the weight relationships that the cubic Black cubit is equivalent to 24,000 cubic Egyptian digits; the calculated value we get for the Black cubit using this relationship (and assuming, as usual, the Royal Egyptian cubit of 28 digits to have been 20.625 inches or 52.3875 cm) is 53.9684 cm, which is 0.15 percent shorter than the Black cubit found by Skinner.

Comparing the two standards webs

For convenience, I will repeat here the basic relationships of the web of ancient standards I am calling W1. These relationships are mathematically completely consistent:

1 cubic Royal Egyptian cubit = 4 cubic Sumerian feet = 9600 cubic Roman inches

1 cubic Roman inch = 1 Egyptian ro = $3\frac{1}{3}$ cubic Sumerian digits

1 cubic Roman inch or Egyptian ro of canonical barley weighs 1 Egyptian qedet

1 Sumerian sila or $\frac{1}{150}$ cubic Sumerian cubit of canonical barley weighs one heavier Sumerian mina

1 Greek xestes of canonical barley weighs 40 heavier Sumerian shekels or 36 Egyptian qedets

If the heaping ratio is $\frac{9}{8}$, then 1 Greek xestes = 1 heaped Egyptian hon

5 cubic Babylonian digits = 2 cubic Northern inches (Babylonian digit = $\frac{1}{24}$ Sumerian cubit)

Here are the most significant relationships found above for W2. As in the case of W1, all these relationships are mathematically consistent with each other.

1 cubic Northern inch = $3\frac{1}{3}$ cubic Egyptian digits

2 cubic Egyptian digits of canonical barley weigh 1 lighter Sumerian shekel

1 lighter Sumerian mina = 1 French livre (Belaiew) = $1\frac{1}{2}$ Roman libras, aureus standard (Berriman)

1 cubic shorter Egyptian cubit (a.k.a. Hebrew cubit) = 24 wine gallons

1 cubic Hashimi cubit = 3 cubic shorter Egyptian cubits

1 cubic Black cubit of canonical barley weighs 200 lighter Sumerian minas or 300 Roman aureus libras

1 cubic Black cubit of water weighs 320 lighter Sumerian minas or 480 Roman aureus libras

Some units are common to both W1 and W2, and their existence makes it easy to show that while each of the two sets of relationships is mathematically consistent, they are not exactly consistent with each other. For example, in W1 we have the relationship

1 cubic Royal Egyptian cubit = 4 cubic Sumerian feet

whereas in W2 we have

$\frac{4}{3}$ cubic shorter Egyptian cubit = 1 cubic Sumerian cubit

If the Sumerian foot is taken to be 33.0 cm (and the cubit therefore 49.5 cm), then by the W1 relationship, the Royal Egyptian cubit is 52.384 cm (20.624 inches), whereas by the W2 relationship, the Royal Egyptian cubit ($\frac{7}{6}$ the size of the shorter Egyptian cubit) is 52.469 cm or 20.657 inches. This is a difference of about 0.16 percent.

To take another example, by W1 we have

2 cubic Northern inches = 5 cubic Bablylonian digits (digit = $\frac{1}{24}$ Sumerian cubit)

while by W2 we have

1 cubic Northern inch = $3\frac{1}{3}$ cubic Egyptian digits

Assuming the Sumerian cubit to be 49.5 cm as before, the Northern foot implied by W1 is 33.591 cm (13.224 inches), whereas the Northern foot implied by W2 is 33.539 cm (13.204 inches), a difference, again, of about 0.16 percent.

The two sets of relationships differ qualitatively as well. Relationships in W1 are anchored by an explanation of the Egyptian and Sumerian systems of weights and measures, and the relationship between them, that lends them a kind of inevitability, whereas the relationships in W2 do not have that level of logical and historical coherence. In particular, W2 lacks any clear relationships with national standards of capacity, though this could of course just be from lack of data. The wine gallon is the outstanding exception, but it increasingly appears to have belonged to no particular national standard but rather to all of them. The historical logic that generates the gallon from the Sumerian cubic cubit (32 wine gallons) and the aureus libra and French livre from the lighter Sumerian mina seems sound enough, but it is difficult to explain a Sumerian weight standard based on a cubic Egyptian digit, which is where this whole analysis begins.

Perhaps the best explanation for W2 is that there is no logic to it—or that the logic it has is akin to the organic logic of linguistics and art history, as another aspect of what happens when cultures collide. As noted earlier, because of the requirement to keep conversions simple, standards of different cultures would necessarily migrate toward values exhibiting simple whole-number ratios with the foreign standards most often encountered in trade. But the particular shape of those sets of relationships would not be logical at all but fortuitous, the consequence of the values they had upon first encounter. And that is very much the appearance of W2.

One explanation that seems possible to me in the historical case is that a very early co-evolution produced the coherent and apparently consciously designed systems of Egypt and Sumeria described earlier that formed the basis for W1, and then a later collision with the Northern foot tradition and weights from the Indus civilization produced conversion ratios along lines that were essentially random and lacking in coherent systemic relationships with units of capacity, yielding some or all of the web of possible relationships exhibited here as W2. The wine gallon and avoirdupois pound make sense as pragmatic adaptations to this collision, with the lighter Sumerian mina and cubic Egyptian digit playing a possible bridging role.

It should also be observed that the arithmetic discrepancy between the mathematically self-consistent systems of relationships W1 and W2 is so small that for all practical purposes it could have been ignored; for the ordinary run of commercial transactions, a 0.16 percent error would in most cases have been undetectable. Thus the later adaptations to W1 represented by W2 (if that is what actually happened) could for most purposes be considered part of one seamless universal set of relationships, even though strictly speaking these relationships are mathematically inconsistent. Indeed, since no individual ancient person could possibly have been aware of the entire standards web, it seems likely that traders at any given interface between W1 and W2 would have adjusted their local subset of these relationships very slightly to make a 0.16 percent difference disappear, and this may well have made its own contribution to the variation we see in the data.

The avoirdupois connection

The idea that W2 arose from a pragmatic cross-cultural adaptation to the earlier Egyptian and Sumerian systems is made stronger by some observations relating to that most mysterious of all the major historical standards of weight—the avoirdupois pound. Earlier I pointed out that the avoirdupois "mark" or half pound (8 ounces) is equivalent to the 27-shekel weight of the subtractive set based on the primary Sumerian standard. But in the cubic shorter Egyptian cubit we can see another derivation for the pound:

1 cubic shorter Egyptian cubit of canonical barley = 125 avoirdupois pounds

1 cubic shorter Egyptian cubit of water = 200 avoirdupois pounds

Here the implied pound is 451.416 g, about 0.48 percent less than the current standard. If one accepts my contention that the wine gallon is $\frac{1}{24}$ of the cubic shorter Egyptian cubit, it follows that a dozen gallons of water weighs 100 avoirdupois pounds; and as a matter of fact, based on current definitions, this relationship is exact for water weighed in air at 10.65 °C or 51.2 °F.

It is important that these are not odd or obscure relationships in the context of the avoirdupois system as it has come down to us in the U.S. from England; they mean simply that 10 cubic shorter Egyptian cubits of water weighs one avoirdupois ton (2000 pounds).

Since the cubic Hashimi cubit appears to be the same as 3 cubic shorter Egyptian cubits, a simple set of canonical weight relationships can be worked out there, too:

1 cubic Hashimi cubit of canonical barley weighs 375 avoirdupois pounds

1 cubic Hashimi cubit of water weighs 600 avoirdupois pounds

Taking Skinner's estimate of 64.9 cm for the Hashimi cubit as exact, these relationships imply a pound of 454.304 g, 0.16 percent above the current pound.

Strangely (because a $\frac{6}{7}$ ratio ordinarily makes simple relationships almost impossible), there is in addition to the fit between the avoirdupois pound and the shorter Egyptian cubit of 24 Egyptian digits a rough but workable set of canonical weight relationships between the avoirdupois pound and the Royal Egyptian cubit of 28 digits as well; if the Royal cubit has the length we usually assume here, then a cubic Royal cubit contains 200 pounds of canonical barley or 250 pounds of canonical wheat ($\frac{5}{4}$ barley) or 320 pounds of water ($\frac{8}{5}$ barley). Of course this relationship is not mathematically consistent with the much more accurate one just observed, the calculated pound in this case being 448.020 g, about 1.23 percent below the current legal unit. I will return to this rough coincidence further on.

A look around at other well established ancient middle eastern standards of length reveals another set of possible connections with the avoirdupois pound. According to Skinner, the Royal Persian cubit was 64.0 ± 0.25 cm and the Assyrian cubit was 54.9 ± 0.5 cm; and, as Skinner observes, the two are related in the same way that the Royal Egyptian cubit is related to the Hebrew or shorter Egyptian cubit, the longer unit in each case being $\frac{7}{6}$ the length of the shorter[52] (though in this case, the two units are both sensibly divided into 24 digits, rather than one being 24 digits and the other 28 of the same digits). A cubic Assyrian cubit holds 225 pounds of canonical barley and 360 pounds of water, the pound being 458.330 g or about 1.04 percent above current value; and a cubic Royal Persian cubit holds 360 pounds of canonical barley or 576 pounds of water, the implied pound in this case (453.818 g) being practically identical to the current legal standard (453.59237 g).

It is important to note that there is nothing the slightest degree strained or unusual about the factors involved in these relationships; numbers such as 125, 200, 320, 360, and 600 would have been just as obvious and familiar to any ancient accountant as they are to us—even more so in the case of 360 and 600—and even the ones that look slightly less ordinary to us, such as 225, 375, and 576, would have seemed "round numbers" to anyone working in a system of numeration based on 30, 60, or 12. The interesting thing here is that, as far as I can tell, such a set of easy canonical relationships between the avoirdupois pound and these five ancient standards of volume can be found for no other well established ancient standard of weight. Of course I cannot claim to have completely exhausted every avenue of inquiry, but I cannot find such a wide range of straightforward relationships for any of the other weight standards discussed in this paper.

One way of illustrating the breadth of the possible avoirdupois-based relationships is to use canonical barley weight to calculate what the theoretical value of each of the five ancient standards of length would be if all of these relationships just postulated were taken as real and the pound was assumed to have its current legal weight. The results are shown in the table on the next page: by using the canonical densities of barley and water adopted previously, it is possible to define all five ancient length standards in terms of one weight standard without deviating by as much as half a percent from the archaeological estimates.

The avoirdupois factors in the table also reveal the curious role of the ratio $\frac{7}{6}$, which appears twice here: as the ratio between the Royal Egyptian and shorter

Standard of length	Barley wt. of cubic cubit (lbs.)	Water wt. of cubic cubit (lbs.)	Calculated cubit	Actual cubit	Difference
Shorter Egyptian cubit	125	200	44.976 cm	$\frac{6}{7}$ Royal Eg. cubit	+0.16%
Royal Egyptian cubit	200	320	52.604 cm	52.3875 cm (Petrie)	+0.41%
Assyrian cubit	225	360	54.710 cm	54.9 cm (Skinner)	−0.35%
Royal Persian cubit	360	576	63.989 cm	64.0 cm (Skinner)	−0.02%
Hashimi cubit	375	600	64.866 cm	64.9 cm (Skinner)	−0.05%

Egyptian cubits, and again as the ratio between the Royal Persian cubit and the Assyrian cubit. The table shows that the water weight of each cubic shorter unit is (roughly) equal to the barley weight of each cubic longer unit:

1 cubic shorter Egyptian cubit of water weighs roughly the same as 1 cubic Royal Egyptian cubit of barley (200 pounds)

1 cubic Assyrian cubit of water weighs roughly the same as 1 Royal Persian cubit of barley (360 pounds)

These rough equivalences work because the actual ratio between the cubes of two lengths related as $\frac{7}{6}$—i.e., the cube of $\frac{7}{6}$, or $\frac{343}{216}$ (1.58796...)—is approximately the same as the canonical water-to-barley ratio, $\frac{8}{5}$ (1.6), the difference being about 0.75 percent. This is close enough to raise the possibility that the shorter and longer units were thought of by at least some of their users in terms of an exact canonical $\frac{8}{5}$ weight/volume ratio rather than in terms of a (relatively useless) exact $\frac{7}{6}$ length ratio.

As I said earlier, it is hard to know what to think of the apparent relationship between avoirdupois pounds and the canonical weights of the ancient volume standards. On the one hand, the avoirdupois pound fits the data in a way that can be claimed (as far as I can tell) for no other ancient standard of weight; but on the other hand, it is difficult to construct a convincing narrative that would derive the avoirdupois standard from any of the ancient systems of weights and measures. Skinner derives the avoirdupois ounce as the weight of two Phoenecian shekels, and indeed, the appearance of avoirdupois weight may make the most sense as the synthetic creation of cross-cultural traders who took advantage of what may have begun simply as a set of coincidences among the volume standards they found already in place.

Once again, the later English experience shines some light on this possibility. The later English length standard (the foot used in the United States today) can be related to the Egyptian length standards even more precisely than the other units I've been discussing, the relationship being

1 cubic inch = $2\frac{1}{2}$ cubic Egyptian digits

This usually shows up in calculations as

1 cubic Egyptian digit = 0.4 cubic inches

According to this, the Royal Egyptian cubit is 20.6306 inches (52.4017 cm), virtually identical to the value I have been using in these analyses.

Exact as this relationship is, however, the fact is that the English foot is a medieval invention, and there appears to be no evidence in the archaeological record for its use before then. Thus, there is no historical relationship between the English inch and the Egyptian digit; nevertheless, there is, or could be, a mathematical one. There is a long tradition that the English cubic foot is the volume of a weight of water equivalent to 1000 avoirdupois ounces ($62\frac{1}{2}$ pounds), a relationship so close that until the early 19th century it was thought to be exact even by scientists. As I explain in *The Old Measure,* this is one of the relationships that probably caused the creation of the English foot somewhere around the 13th century for purposes of convenience in working with the avoirdupois pound and several other units in use at that time. It could therefore be said that the English foot was *mathematically determined* or *implied* by the Egyptian digit through a shared relationship with avoirdupois weight even though there is no direct historical link between the two standards of length. Compare this with the relationship

4 cubic English feet = 3 cubic Northern feet

which puts the Northern foot at 13.2077 inches or 33.5476 cm.[53] This, too, is mathematically implied by what has come before, but here we can easily imagine a genuine historical connection, as it is generally agreed that the English foot directly succeeded the Northern foot in England for commercial purposes (the Northern foot continued to underly the land measures, as it does today). This handy volume relationship more than likely contributed to the invention of the English foot, along with the water weight of 1000 avoirdupois ounces. Just such pragmatic considerations may have led to a Phoenecian invention of the avoirdupois pound, but without further evidence, there is no way of telling which parts of the system we can see are the result of actual historical causation and which are just arithmetical implications of the rest of the standards web.

A similar case involving the relationship between avoirdupois weight and certain Chinese weights demonstrates the principle. Earlier I noted that

> 1 cubic Northern t'sun (decimal Northern inch) of water weighs 1 Chinese tael

where I am using the Chinese word t'sun to represent both the tenth of the Chinese foot for land measure and the equally sized tenth of the Northern foot found at Mohenjodaro. Since (according to the relationships spelled out above)

> 3000 cubic t'sun of water weighs 4000 avoirdupois ounces

it follows that

> 1 Chinese tael = $\frac{4}{3}$ avoirdupois ounces

and this mathematically implied relationship was in fact the working conversion factor in trade with Asia up through the 19th century[54] and the official conversion factor in China as late as 1929.[55] So here we may have a case in which the causal chain goes east from the Indus culture to China, and west via the Phoenecians and later the medieval Italians to England and then North America, until it finally reaches around the world to meet up on the other side, the ounce and the tael fitting together along lines determined by their ancient systemic relationships to the ancestral standards web.

Notes

[1]This paper represents the delayed completion of a project researched in 1984–1987 and written up in 2009–2012. I thank my daughter, Clara Bosak-Schroeder, for a number of helpful editorial suggestions and note that she is not responsible for the content.

[2]The word "standard" is generally used in this paper not to refer to a unique object like the standard kilogram (a singular actual piece of metal in France) but rather to the value of a particular unit as shared among all of its physical instantiations.

[3]"Weight" instead of "mass" is used here not just for historical reasons but also because it is the downward force exerted by gravity upon the load that we are interested in, not its resistance to acceleration.

[4]The terms volume and capacity refer to the same thing but have (apparently universally) been considered conceptually distinct—a nice demonstration of the difference between *Sinn* and *Bedeutung*. Volume is specified in terms of a number of rectangular solids, which are usually but not always cubical in form, and are always measured in units of length (for example, cubic inches, cubic centimeters). Capacity is specified in terms of a number of vessels, usually cylindrical, that are often associated with weights of a particular substance. Thus, in the metric system, the liter is a unit of *capacity* associated with a kilogram of water, and the cubic decimeter is a unit of *volume* derived from a unit of length, and we maintain them, use them, and teach them as separate units; but in fact, the liter and the cubic decimeter are by definition two names for the same thing. Unlike the metric system, most traditional systems associated a different series of factors with volumes and capacity measures to better fit each set of standards with the tasks for which they were most often used.

[5]In *The Old Measure*.

[6]The standard foot of the Austro-Hungarian Empire (historically, the foot of Vienna) survived at 31.6081 cm or 12.4441 inches until its replacement at last by the metric system, and the land measures of Britain and the U.S. (rod, furlong, and acre) still preserve the Northern foot ($\frac{1}{15}$ rod or $\frac{1}{4800}$ mile) at 13.2 inches exactly.

[7]See in particular Robert Tye, *Early World Coins and Early Weight Standards* (Early World Coins: York, 2009).

[8]Despite my disagreement with him on this point, I will be citing Skinner frequently in what follows. All mentions of Skinner refer to his book *Weights and Measures: Their Ancient Origins and Their Development in Great Britain up to AD 1855* (London: Her Majesty's Stationery Office, 1967). Skinner's work provides an appropriate source for this inquiry because in addition to his unsurpassed scholarship of the subject, he was categorically opposed to explanations of ancient systems that related volumes and weights, and his figures are therefore uncontaminated by theories of the sort presented in this paper. His book is well indexed, so I have often omitted page references.

[9]M. Lambert, "L'Usage de l'Argent-métal a Lagash au Temps de la IIIe Dynastie d'Ur," *Revue d'Assyriologie* 57 (1963) pp. 193–94.

[10]E. Bakka, "Two Aurar of Gold: Contributions to the Weight History of the Migration Period," *Antiquaries Journal* 58 (1978), pp. 279–98.

[11]A list of scriptural references to the number 40 can be found online at http://en.wikipedia.org/wiki/Number_40.

[12]Lacking the mathematical ability to prove this by analysis, I just programmed a computer to work out all the possibilities.

[13]This quantity (the total in each pan when the scale balances using this system, which is always the total of the weights on the left in the diagram) varies in a rather unintuitive way as the amount being weighed increases. The series for 1 through 40 is 1, 3, 3, 4, 9, 9, 10, 9, 9, 10, 12, 12, 13, 27, 27, 28, 27, 27, 28, 30, 30, 31, 27, 27, 28, 27, 27, 28, 30, 30, 31, 36, 36, 37, 36, 36, 37, 39, 39, 40.

[14]W. M. F. Petrie, "Measures and Weights (Ancient)," *Encyclopædia Britannica,* 14th ed.

[15]"Cape Gelidonya: A Bronze Age Shipwreck," *Transactions of the American Philosophical Society,* n.s., vol. 57, pt. 8 (1967), pp. 44–142.

[16]C. Pulak, "The balance weights from the Late Bronze Age shipwreck at Uluburun," in *Metals Make the World Go Round: The Sup-*

ply and Circulation of Metals in Bronze Age Europe, ed. C. F. E. Pare (Oxford: Oxbow Books, 2000).

[17]W. M. F. Petrie, *The Pyramids and Temples of Gizeh* (London: Field & Tuer, 1883), p. 179. These details were omitted from the second, more popular edition of the book.

[18]"Weights and Measures (Ancient Historical)," *Encyclopædia Britannica,* 11th ed., vol. 28, p. 482.

[19]Based on independent theoretical considerations having to do with area measurements, A. E. Berriman arrived at exactly the same figure for the royal Egyptian cubit ($20\frac{5}{8}$ inches). *Historical Metrology: A New Analysis of the Archaeological and the Historical Evidence Relating to Weights and Measures* (London: J. M. Dent & Sons, 1953), pp. 3, 28, 29.

[20]See, for example, R. J. Gillings, *Mathematics in the Time of the Pharaohs* (Dover Books, 1982 (originally MIT Press, 1972)).

[21]W. M. F. Petrie, "Measures and Weights (Ancient)," *Encyclopædia Britannica,* 14th ed., p. 143. "The amount of the Egyptian hon by five regular measures (of metal or stone) averages 29.2 ± .5 cu. in., from ten bronze vessels 29.0 ± .3, from eight marked vases 29.2 ± .6. So 29.1 may best be adopted." In the 11th edition of the *Britannica,* he had put it at 29.2.

[22]Gillings, *op. cit.,* pp. 18–20.

[23]F. Thureau-Dangin, "Numération et Métrologie Sumériennes," *Revue d'Assyriologie* 18 no. 3 (1921), p. 133.

[24]O. Neugebauer and A. Sachs, *Mathematical Cuneiform Texts* (New Haven: American Oriental Society, 1986 (originally published 1945)), pp. 4–6.

[25]"Au Sujet de la Valeur Probable de la Mine Sumérienne," *Revue d'Assyriologie* 26, pp. 115–132.

[26]E. Robson, "Mesopotamian Mathematics," in *The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook,* ed. V. J. Katz (Princeton: Princeton University Press, 2007), p. 123.

[27]The two examples are, first, an Old Babylonian jar implying a unit of 806.87 mL (J. N. Postgate, "An Inscribed Jar from Tell al Rimah," *Iraq* 40, pt. 1 (Spring 1978), p. 73), and second, an inscribed but fragmentary alabaster vessel from the palace of "Evil-Mérodach, roi de Babylone, fils de Nabuchodonosor" with a capacity corresponding to a unit of about 0.81 liters (F. Thureau-Dangin, "La Mesure du *qa*," *Revue d'Assyriologie et d'Archéologie Orientale* 9 (1912) pp. 24–25; also illustrated in V. Scheil and L. Legrain, *Textes Élamites-Sémitiques,* 5th ser., Memoires de la Mission Archéologique de Susiane 14 (Paris: Ernest Leroux, 1913), p. 60). The average of these two figures is 808.435.

[28]Petrie, for example, notes "one obvious 10 daric weight" that has been "remarked with 9 cuts to show its value as 9 qedets." *Ancient Weights and Measures: Illustrated by the Egyptian Collection in University College, London* (London: Department of Egyptology, University College, 1926), p. 12.

[29]Neugebauer and Sachs, *op. cit.,* p. 4.

[30]I discuss this at length in *The Old Measure*. Another notable German survival was the medieval and premetric pfund of Vienna, which was 420.045 g (A. Martini, *Manuale di metrologia, ossia misure, pesi e monete in uso attualmente e anticamente presso tutti i popoli* (Torino: Loescher, 1883), p. 827) or 50 Sumerian shekels of 8.4009 g.

[31]A demonstration of this must await another paper.

[32]K. Hitzl, *Die Gewichte griechischer Zeit aus Olympia.* Deutsches Archäologisches Institut: Olympische Forschungen, Band 25 (Berlin and New York: Walter de Gruyter, 1996), p. 111.

[33]See the appendix "Grain weight as a natural standard" in *The Old Measure* for much more on this.

[34]As noted earlier, the Egyptian unit corresponding to a mina or pound was the half sep (50 qedets), which had a long history as the pound of Carchemesh and was later the medieval pound of Cologne. So we also have

> 25 Sumerian minas = 27 half seps (pounds of Carchemesh)

The ratio 25:27 can also be written as 100:108; and in this form, these two weights appear side-by-side in several premetric German city standards, the Sumerian mina being known as the "schwere [heavy] Pfund" or Pfund of commerce and the half sep as the "leichte [light] Pfund" or Pfund for silver. We have from Martini, *op. cit.,* the following 19th century German city definitions of a unit called the Centner, which in each place was divided into 100 schwere Pfunden and 108 leichte Pfunden:

- Frankfurt, Germany
  - Centner = 50.512788 kg
    - Schwere Pfund = 505.12788 g (shekel = 8.418798 g)
    - Leichte Pfund = 467.71100 g (qedet = 9.354220 g)
- Hanau, Germany
  - Centner = 50.512788 kg
    - Schwere Pfund = 505.12788 g (shekel = 8.418798 g)
    - Leichte Pfund = 467.71100 g (qedet = 9.354220 g)
- Heidelberg, Germany
  - Centner = 50.540760 kg
    - Schwere Pfund = 505.40760 g (shekel = 8.42346 g)
    - Leichte Pfund = 467.97000 g (qedet = 9.35940 g)
- Mannheim, Germany
  - Centner = 50.534712 kg
    - Schwere Pfund = 505.34712 g (shekel = 8.422452 g)
    - Leichte Pfund = 467.91400 g (qedet = 9.358280 g)
- Marburg, Germany
  - Centner = 50.534690 kg
    - Schwere Pfund = 505.34690 g (shekel = 8.4224483 g)
    - Leichte Pfund = 467.91380 g (qedet = 9.3582759 g)

[35]Skinner, p. 97, and R. Connor in *The Weights and Measures of England* (London: Her Majesty's Stationery Office, 1987), p. 132. The 19th century value of the libbra in Florence (Martini, *op. cit.,* p. 207) was 339.542 g, divided into 12 ounces of 28.2952 g, which is about 0.19 percent smaller than the current avoirdupois ounce.

[36]R. Hussey, after considering six different approaches to an estimate of the Roman foot, arrived at 0.9708 English feet, which is 11.6496 inches or 29.5900 cm (*An Essay on the Ancient Weights and Money, and the Roman and Greek Liquid Measures, with an Appendix on the Roman and Greek Foot.* Oxford (for the University), 1836, pp. 227–230). A. Böckh, in *Metrologische Untersuchungen über Gewichte, Münzfüsse und Masse des Alterthums in ihrem Zusammenhange* (Berlin: Verlag von Veit und Comp, 1838, pp. 196–198), cited Hussey's estimate alongside those of Wurm (131.15 lines of Paris, which according to the most precise figures for the line—see Martini, *op. cit.,* p. 466—is 29.58521 cm or 11.64772 inches) and Paucker (11.650 inches, which is 29.591 cm); Böckh finally adopted Wurm's figure as the best. On the other hand, W. Smith, in *A New Classical Dictionary of Greek and Roman Biography, Mythology, and Geography* (New York: Harper and Brothers, 1851) adopted Hussey's 11.6496 inches as final. It would be fair to say, therefore, that the best 19th century estimate of the mean Roman foot was between 11.6477 and 11.6496 inches (29.5852 to 29.5900 cm).

Petrie's estimate of the Roman foot, based on an analysis that tied it historically to a wide range of older instances and a theory of origin based on an Egyptian measure called the remen, put it at 11.62 inches in the Eleventh edition of the *Encyclopædia Britannica* and 11.613 inches or 294.9 mm in the Fourteenth. This estimate would be considered on the low side today. The Roman foot appears to have grown slightly longer over time, ending up as the 19th century Piede romano of Rome at a length of 29.7896 cm or 11.728 inches (Martini, *op. cit.,* p. 596).

Based on all these sources, which at this point includes surviving foot measures, measurements of Roman buildings whose dimensions were recorded, and measurements of itinerary distances compared with records of their length, Skinner gives "a generally agreed mean length for the Roman foot of 296 mm (11.65 in) ± 4 mm (0.15 in)" (p. 67), but this appears simply to accept the rounded-off metric version of Hussey's original estimate of the average value (29.59 cm rounded off to 29.6) as definitive and to calculate the English equivalent (11.65 inches) from that. There appears, therefore, to be no reason not to continue to take Hussey's and Wurm's estimates as the soundest and most precise, and the average of these two very closely aligned figures for the mean Roman foot is 29.588 cm or 11.649 inches.

[37]See Skinner for the details.

[38]Gillings, *op. cit.,* p. 163.

[39]Martini, *op. cit.,* p. 74

[40]Martini, *op. cit.,* p 30.

[41] *[UN] World Weights and Measures: Handbook for Statisticians,* Series M No. 21 (Provisional Edition), Statistical Office of the United Nations in Collaboration with the Food and Agriculture Organization of the United Nations (New York: United Nations, 1955), p 136.

[42] W. M. F. Petrie, *Ancient Weights and Measures*, p. 39.

[43] The relationship between the wine gallon and the cubic Northern foot was first noted by A. E. Berriman (*Historical Metrology,* p. 175), who referred to this gallon as "the troy gallon." Berriman identified the unit of 13.2 inches with the Sumerian foot, which I believe to be a mistake.

[44] This is the value for the land measure of Peking set by treaty between China and Italy in 1866 (Martini, *op. cit.,* p. 513). Hoang, writing in 1897, puts the kong, the "mesure traditionnelle juridiquement admise dans le territoire di Chang-hai [Shanghai]," at a metric value of 1.673 m and an English value of 66 inches; this would be 5 Northern feet of either 13.17 inches or 13.2 inches (F. Hoang, *Notions techniques sur la propriété en Chine avec un choix d'actes e de documents officiels,* Variétés sinologiques no. 11 (Chang-hai: Imprimerie de la Mission Catholique, 1897), p. 58). The *Encyclopædia Britannica* (11th Edition, s.v. "Weights and Measures") puts the "treaty kung" at 78.96 inches, which is 6 Northern feet of 13.16 inches. *Tate's Modern Cambist,* a standard reference on 19th and early 20th century measures and coinage, puts the land foot of Shanghai at either 13.2 inches or 33.5 cm, which is 13.19 inches (W. F. Spalding, ed., *Tate's Modern Cambist,* 27th Edition (London: Effingham Wilson, 1926), p. 244). The Chinese used an unrelated foot of about 14 inches for most other purposes.

[45] "The Statistical Treatment of Ancient Weights," *Ancient Egypt and the East,* December 1935, p. 92.

[46] In addition to Belaiew, *op. cit.,* see also M.-C. Soutzo *et al., Recherches Archéologiques,* 4th Series, *Délegation en Perse: Mémoires,* Vol. XII (Paris: Ernest Leroux, 1911); Belaiew, N. T., "Métrologie Élamite: Examen Préliminaire des Documents Pondéraux, Fouilles de Suse 1921–1933," in *Archéologie, Métrologie et Numismatique Susiennes; Mémoires de la Mission Archéologique de Perse,* Vol. XXV (Paris: Ernest Leroux, 1934); *Ninth Annual Report of the Warden of the Standards on the Proceedings and Business of the Standard Weights and Measures Department of the Board of Trade* (London: Her Majesty's Stationery Office, 1875); and, of course, Skinner. Other data may have come to light in the quarter century since I conducted this part of my research.

[47] The passing reference to the great 19th and early 20th century classicist Carl Friedrich Lehmann-Haupt in an earlier version of this article (20 March 2012) prompted an inquiry late in 2012 from his grandson, the author and critic Christopher Lehmann-Haupt, who was in search of details regarding his illustrious grandfather's metrological studies in order to resolve questions raised by references in the writings of Livio Stecchini (see http://www.metrum.org/measures/).

Since my intention in citing Belaiew's summary was specifically to avoid immersion in now discredited 19th century metrological theories, I had until this point held back from any exploration of Lehmann-Haupt's work or that of his German contemporaries Viedebantt and Weissbach. In attempting to respond to Christopher Lehmann-Haupt's inquiry, however, it became clear that the elder Lehmann-Haupt's half century of investigations are highly relevant to the present paper. In particular, it is clear that while Lehmann-Haupt's theoretical structure is fatally flawed due to his adoption of a figure for the weight of the qedet that cannot now be accepted, the elaborate system of ratios he found in comparing the various ancient weight standards corresponds in many respects to the systems of relationships described in this paper. A complete investigation into this subject would therefore credit Lehmann-Haupt with many of the observations independently arrived at here. Unfortunately, my dimly recollected high-school German is inadequate to anything like a complete understanding of this material, and I am reluctantly forced to leave a more thorough review to someone with a better command of the language. The two best sources for such a review appear to be Lehmann-Haupt's seminal work "Altbabylonisches Maass und Gewicht und deren Wanderung," *Verhandlungen der Berliner Gesellschaft für Anthropologie, Ethnologie und Urgeschichte* (1889), pp. 245–328, and the magisterial overview published as the article "Gewichte" in the 1918 edition of *Paulys Realencyclopädie der Classischen Altertumswissenschaft,* Supplementband III, pp. 588–654. I would be pleased to provide copies of these pieces to anyone willing to follow up on this (or to translate them!).

[48] "Altbabylonisches Maass und Gewicht und deren Wanderung," p. 257. Lehmann-Haupt based this on his figure of 491.2 g for the mina and Hultsch's 327.45 g (corresponding to an ounce of 27.288 g) for the Roman pound of this standard. Berriman, apparently independently, noticed the same relationship (*Historical Metrology,* p. 58 and elsewhere) and later expanded on this theory in "A New Approach to the Study of Ancient Metrology," *Revue d'Assyriologie* 49 (1955) pp. 193–201. Berriman based his estimate of what he called "Mina N" on a two-mina weight in the British Museum. This stone bears an elaborate inscription stating that it was made by order of Nebuchadnezzar II (605–562 B.C.) as a copy of a weight of the reign of Shulgi, a king of the Third Dynasty of Ur (around 2100 B.C.). It weighs 978.3 g, which puts the mina at 489.15 g and the Roman ounce of this derivation at 27.175 g.

[49] *Antiquaries Journal* 13, pp. 57–58.

[50] *Historical Metrology,* pp. 10 and 34ff.

[51] This foot was estimated at 12.15 inches or 30.9 cm by both Petrie and Skinner. As Berriman and others have noted, the Olympic foot is (unlike the meter) perfectly aligned with the nautical mile, which represents one minute (60 seconds) of arc along any great circle on the earth's surface (for example, the equator). By international agreement, the nautical mile is exactly 1,852 metres (approximately 6,076 feet); this is 6000 Olympic feet of 30.8667 cm or 12.1522 inches, and 100 of these feet correspond exactly to one second of arc. A more elegantly designed system for geographical measurement can scarcely be imagined. The major problem with this interpretation is that there appears to be little evidence that the Olympic foot was much used outside of the games at Olympia. Use of the common Greek foot of 12.44 inches (31.6 cm) spread much farther, being, for example (according to Petrie), the most common measure in medieval English construction.

[52] Petrie credited Oppert with this observation ("Measures and Weights (Ancient)." *Encyclopædia Britannica,* 14th ed., p. 143).

[53] The relationship appears to have first been noticed by Berriman (*Historical Metrology,* p. 66), though as noted before, he thought the Northern foot was a variant of the Sumerian foot. But Berriman also (p. 72) noted the relationship

10 English poles or rods = 96 Royal Egyptian cubits

which is the same as

25 Northern feet = 16 Royal Egyptian cubits

and implies a Northern foot of exactly 13.2 English inches and a Royal Egyptian cubit of exactly 20.625 inches.

[54] See, for example, H. T. Easton, ed., *Tate's Modern Cambist,* 24th Edition (London: Effingham Wilson, 1908), pp. 188, 205.

[55] *[UN] World Weights and Measures,* p. 48.