

13. Conservation of Momentum

Objectives

This lab exposes students to the concept of momentum and its conservation during common types of collisions. Students:

- ◆ Graphically understand both elastic and inelastic collisions
- ◆ Test the idea of Conservation of Momentum in different types of collisions
- ◆ Notice what happens with force, velocity, and linear momentum during the collision

Procedural Overview

Students will gain experience conducting the following procedures:

- ◆ Set up experimental trials for elastic and inelastic collisions, as well as an "explosion" type separation of two moving vehicles.
- ◆ Measure the velocity of two objects at the same time with motion sensors.

Time Requirement

- | | |
|-----------------------------------|------------|
| ◆ Preparation time | 10 minutes |
| ◆ Pre-lab discussion and activity | 15 minutes |
| ◆ Lab activity | 50 minutes |

Materials and Equipment

For each student or group:

- | | |
|--------------------------|---|
| ◆ Data collection system | ◆ Dynamics carts with magnet bumpers, Velcro [®] |
| ◆ Motion sensor (2) | bumpers, and plungers (2) |
| ◆ Dynamics track | ◆ Balance (1 per classroom) |

Concepts Students Should Already Know

Students should be familiar with the following concepts:

- ◆ Definition of momentum
- ◆ Transfer of momentum
- ◆ Position versus Time graphs and Velocity versus Time graphs

Related Labs in this Manual

Labs conceptually related to this one include:

- ◆ Position: Match Graph
- ◆ Speed and Velocity
- ◆ Acceleration
- ◆ Conservation of Energy
- ◆ Impulse Momentum

Using Your Data Collection System

Students use the following technical procedures in this activity. The instructions for them (identified by the number following the symbol: "◆") are on the storage device that accompanies this manual. Choose the file that corresponds to your PASCO data collection system. Please make copies of these instructions available for your students.

- ◆ Starting a new experiment on the data collection system ◆^(1.2)
- ◆ Connecting multiple sensors to the data collection system ◆^(2.2)
- ◆ Changing the sample rate ◆^(5.1)
- ◆ Starting and stopping data recording ◆^(6.2)
- ◆ Adjusting the scale of a graph ◆^(7.1.2)
- ◆ Selecting data points in a graph ◆^(7.1.4)
- ◆ Changing the variables on the x - or y -axis of a graph ◆^(7.1.9)
- ◆ Displaying multiple graphs simultaneously ◆^(7.1.11)
- ◆ Creating calculated data ◆^(10.3)

Background

The momentum \mathbf{p} of an object is the product of its mass and velocity:

$$\mathbf{p} = m\mathbf{v}$$

Because velocity is a vector quantity, momentum is therefore a vector quantity. For a linear system that is not influenced by outside forces, the total momentum of the system is conserved. We will use a cart and track system as the isolated system and observe three different types of momentum exchange.

Elastic collisions occur when two objects bounce off each other, like two billiards balls colliding, or a golf club hitting a golf ball. Linear momentum is transferred from one object to the next, if the two objects are the same mass, all of the momentum of the first is transferred to the second. This is the principle behind a Newton's Cradle. Inelastic collisions occur when two objects hit and stick together, moving as one object after the collision like one train car coasting into another and coupling together. "Explosions" refer to two objects pushing away from each other like two ice skaters facing each other and pushing off. Even though these situations are very different, the principles behind them are the same. As systems get more complex, two dimensional and three dimensional, the underlying behavior is the same.

Pre-Lab Discussion and Activity

Begin this lab discussion by reminding students what momentum is, namely, mass \times velocity. A great starting point would be a discussion about billiard balls: what happens when a billiards player hits the cue ball (the white one) directly at another ball of similar mass so that they hit head on? Usually, the cue ball will stop, and the other ball will move away at the same speed as the cue ball's initial speed. Ask students about this situation to check their intuitive understanding of momentum:

1. What is the momentum of the cue ball initially?

Mass (cue ball) \times Velocity (cue ball)

2. What is the momentum of the stationary ball initially?

Zero

3. What is the total momentum of the system (both balls together) before the collision?

Mass (cue ball) \times Velocity (cue ball) + Mass (other ball) \times Velocity (other ball)

4. What about after the collision? What's the momentum of the cue ball after collision?

Zero

5. What is the momentum of the other ball after collision?

Mass (other ball) \times Velocity (other ball)

So the total momentum after collision is the same as the total system momentum before collision. Now consider the ball that's moving after the collision. Does it keep moving forever? Of course not; it eventually slows down and comes to rest because of rolling friction with the surface of the billiards table. But if friction is minimized and accurate measurements can be taken both before and after the collision, students should be able to verify (within reasonable error ranges) that momentum is conserved in 3 different types of events.

Conservation of Momentum

6. What two quantities must be measured to calculate an object's momentum?

The object's mass and its velocity.

7. If two objects will be moving, what additional equipment will be needed to accurately measure each object's momentum?

Students will need two motion sensors to track the velocity of each cart before and after collisions.

8. If two objects collide, what must be true if their momentum is to be conserved?

No external forces must be present, so friction must be minimized to conserve momentum.

Lab Preparation

Although this activity requires no specific lab preparation, allow 10 minutes to assemble the equipment needed to conduct the lab.

Safety

Add these important safety precautions to your normal laboratory procedures:

- ◆ Make sure students catch dynamics carts before they go off the track or hit the motion sensors.

Sequencing Challenge

The steps below are part of the Procedure for this lab activity. They are not in the right order. Determine the proper order and write numbers in the circles that put the steps in the correct sequence.

3	5	1	4	2
Determine the velocity of the carts before and after the collision from your graph.	Compare the total momentum of the system before the collision to the total momentum of the system after the collision.	Set up a dynamics track with two motion sensors and two carts to engage in collisions.	Calculate the momentum of each cart before and after the collision.	Push one cart toward another stationary cart so that they bounce off each other, and gather data for an elastic collision.

Procedure with Inquiry

After you complete a step (or answer a question), place a check mark in the box (☐) next to that step.

Note: Students use the following technical procedures in this activity. The instructions for them (identified by the number following the symbol: "◆") are on the storage device that accompanies this manual. Choose the file that corresponds to your PASCO data collection system. Please make copies of these instructions available for your students.

Part 1 - Elastic Collision


Set Up

1. ☐ Start a new experiment on the data collection system. ◆^(1.2)
2. ☐ Connect the motion sensors to your data collection system. ◆^(2.2)
3. ☐ Set the track on a level surface with a motion sensor attached to each end. Be sure the sensors are positioned horizontally to best receive signals from the moving carts on the track.

Note: The motion sensors are pointed in opposite directions. Choose which sensor will be your point of reference. Remember that moving away from this sensor will be the positive direction, so the direction reported by the other sensor will need to be corrected.

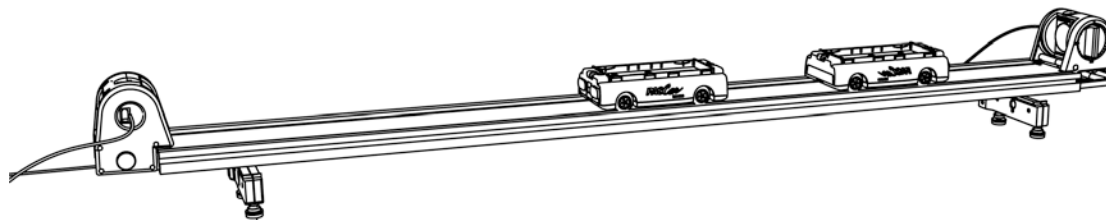
4. ☐ In the data collection system, create a calculation to reverse the sign of one of your motion sensors. ◆^(10.3)

$$\text{Velocity2} = -[\text{Velocity (m/s)}]$$

5. ☐ Display two graphs simultaneously: one graph will display Velocity on the y-axis and Time on the x-axis and the second will display Velocity2 on the y-axis and Time on the x-axis. ◆^(7.1.11)
6. ☐ Make sure that your sampling rate is set to at least 20 samples per second for each motion sensor. ◆^(5.1) If your motion sensors have a selector switch, make sure that they are in the cart or near setting. 
7. ☐ Find the mass of each cart, and record them in Table 1 in the Data Analysis section.

Conservation of Momentum

8. ☐ Place Cart 2 in the middle of the track and Cart 1 near one end of the track (just over 15 cm from the motion sensor) such that their magnetic bumpers are facing each other.



9. ☐ Make sure someone is at the other end of the track to catch the cart before it hits the motion sensor because you will be pushing the cart closet to the motion sensor toward the cart in the middle of the track.

Collect Data

10. ☐ Start data recording. ♦^(6.2)
11. ☐ While keeping hands out of the way of the motion sensors, push Cart 1 toward Cart 2, and let them collide.
12. ☐ Catch Cart 2 before it hits the second motion sensor.
13. ☐ Stop data recording. ♦^(6.2)

Analyze Data

14. ☐ Adjust the scales of your graphs so that all of the data is visible, and the Time scales are aligned. ♦^(7.1.2)
15. ☐ Find the velocity of Cart 1 on the graph just before the collision and just after the collision, and enter the values in Table 1 in the Data Analysis section. ♦^(7.1.4)
16. ☐ Find the velocity of Cart 2 on the graph just before the collision and just after the collision, and enter the values in Table 1 in the Data Analysis section. ♦^(7.1.4)
17. ☐ Sketch your Velocity versus Time graphs in the Velocity versus Time - Elastics Collision blank graph axes in the Data Analysis section.

Part 2 - inelastic Collision

Set Up

18. ☐ Put the carts in position again, but this time orient the carts so that they will hit Velcro to Velcro and stick together.

Collect Data

19. ☐ Start data recording. ♦^(6.2)
20. ☐ While keeping hands out of the way of the motion sensors, push Cart 1 toward Cart 2, and let them collide.
21. ☐ Catch Cart#1 and Cart 2 before they hit the second motion sensor.
22. ☐ Stop data recording. ♦^(6.2)

Analyze Data

23. ☐ Adjust the scales of your graphs so that all of the data is visible, and the Time scales are aligned. ♦^(7.1.2)
24. ☐ Find the velocity of Cart #1 on the graph just before the collision and just after the collision, and then enter the values in Table 2 in the Data Analysis section. ♦^(7.1.4)
25. ☐ Find the velocity of Cart 2 on the graph just before the collision and just after the collision, and then enter the values in Table 2 in the Data Analysis section. ♦^(7.1.4)
26. ☐ Sketch your Velocity versus Time graphs in the Velocity versus Time - Inelastic Collision blank graph axes in the Data Analysis section.

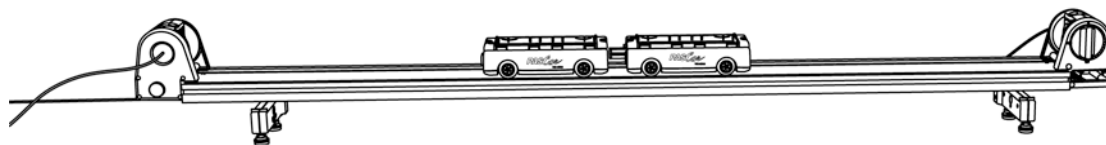
Part 3 - Explosion

Set Up

27. ☐ Push in the plunger on Cart 1 to the second position so that it is ready to push the carts away from each other in an "explosion."

Conservation of Momentum

28. ☐ Place the carts together in the middle of the track with the cocked plunger on Cart 1 just touching the flat end of Cart 2.



Collect Data

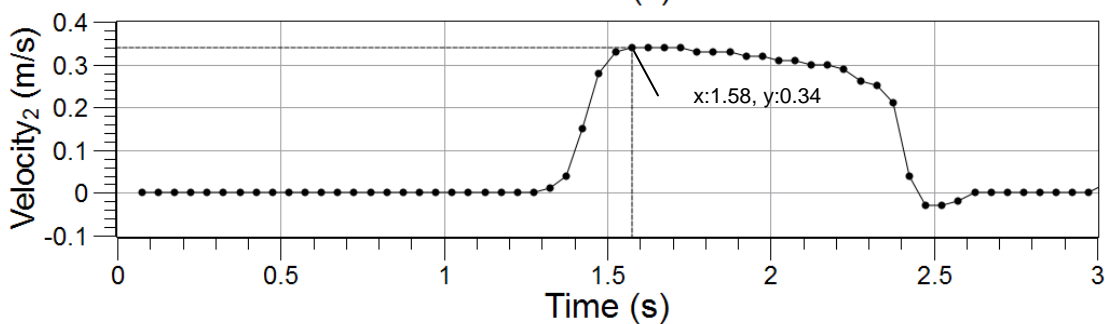
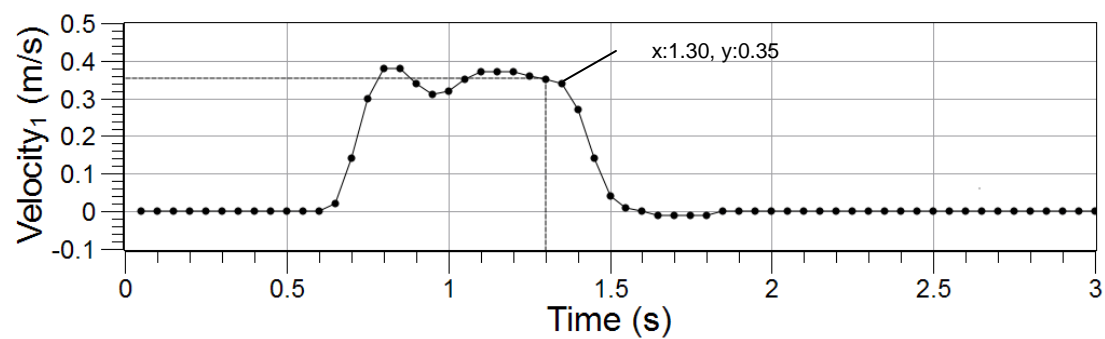
29. ☐ Start data recording. ♦^(6.2)
30. ☐ While keeping hands out of the way of the motion sensors, tap the plunger release button to send the carts in opposite directions.
31. ☐ Catch the carts before they hit the motion sensors.
32. ☐ Stop data recording. ♦^(6.2)

Analyze Data

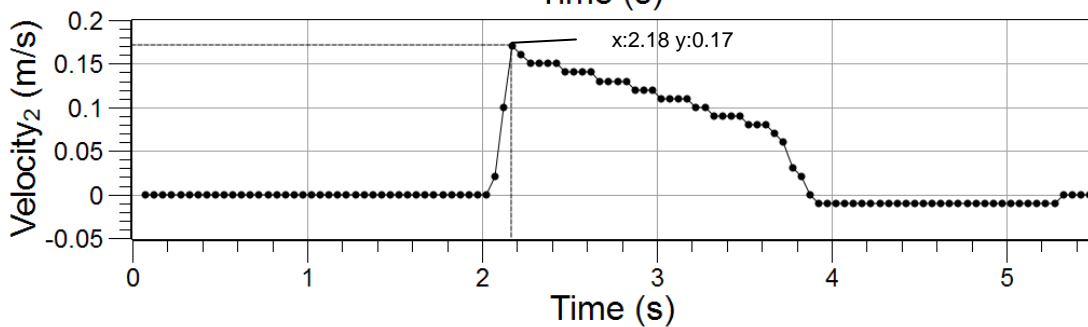
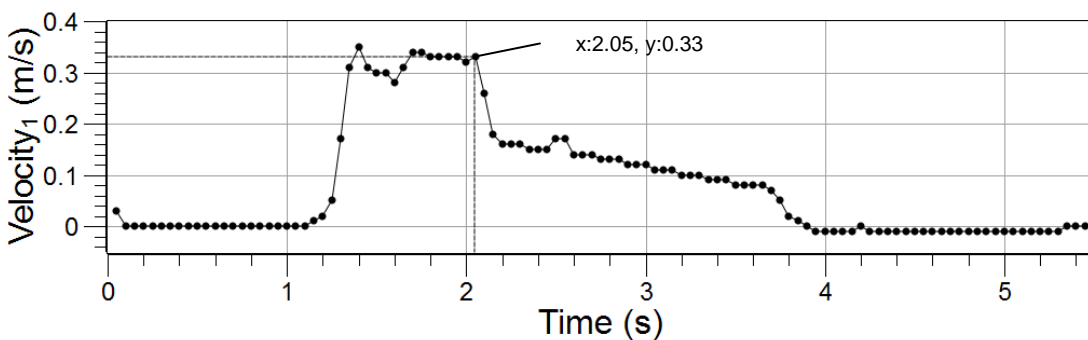
33. ☐ Adjust the scales of your graphs so that all of the data is visible and the Time scales are aligned. ♦^(7.1.2)
34. ☐ Find the velocity of Cart #1 on the graph just before the collision and just after the collision, and enter the values in Table 2 in the Data Analysis section. ♦^(7.1.4)
35. ☐ Find the velocity of Cart 2 on the graph just before the collision and just after the collision, and enter the values in Table 2 in the Data Analysis section. ♦^(7.1.4)
36. ☐ Sketch your Velocity versus Time graphs in the Velocity versus Time - Explosion blank graph axes in the Data Analysis section.

Data Analysis

Velocity versus Time - Elastics Collision



Velocity versus Time - Inelastic Collision



Conservation of Momentum

Velocity versus Time - Explosion

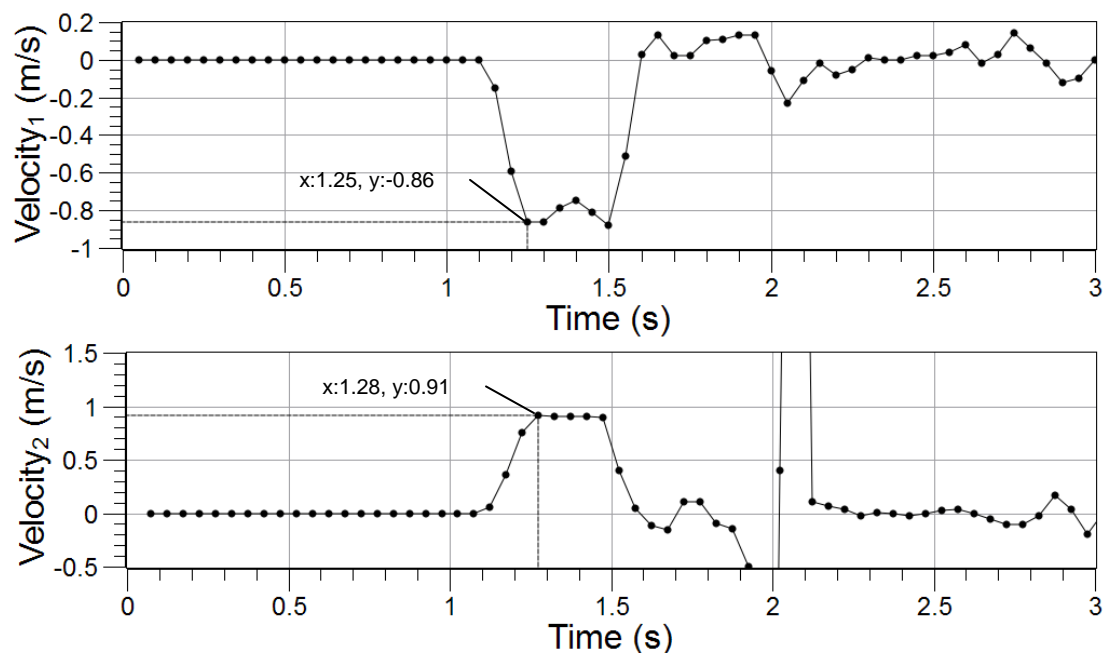


Table 1: Elastic Collision

	Cart 1	Cart 2
Mass	0.258 kg	0.249 kg
Initial Velocity	0.35 m/s	0 m/s
Final Velocity	0 m/s	0.34 m/s
Initial Momentum	0.090 kg·m/s	0 kg·m/s
Final Momentum	0 kg·m/s	0.085 kg·m/s

Table 2: Inelastic Collision

	Cart 1	Cart 2
Mass	0.258 kg	0.249 kg
Initial Velocity	0.33 m/s	0 m/s
Final Velocity	0.17 m/s	0.17 m/s
Initial Momentum	0.085 kg·m/s	0 kg·m/s
Final Momentum of Cart 1 and Cart 2	0.086 kg·m/s	

Table 3: Explosion

	Cart 1	Cart 2
Mass	0.258 kg	0.249 kg
Initial Velocity	0 m/s	0 m/s
Final Velocity	-0.86 m/s	0.91 m/s
Initial Momentum	0 kg·m/s	0 kg·m/s
Final Momentum	-0.22 kg·m/s	0.23 kg·m/s

Analysis Questions

1. **Look at your graphs for the elastic collision. How does the velocity of Cart 1 before the collision compare to the velocity of Cart 2 after the collision?**

They are about the same.

2. **Explain why this is significant, and be sure to include momentum in your explanation, not just velocity.**

The velocity of Cart 1 before the collision is very close to the velocity of Cart 2 after the collision. Because the mass of the carts is roughly the same, this quick graphical analysis shows that momentum is conserved in this collision.

3. **Consider the graph for your inelastic collision. How does the final velocity of the carts compare to the initial velocity of Cart 1? What does this tell you about the collision?**

The velocity after the collision is about half of Cart 1's initial velocity before the collision. Because both carts stick together after the collision (and they have about the same mass), the mass is doubled, so the velocity after the collision should be about half of Cart 1's velocity before collision.

4. **Look at your graph for the "explosion" trial. How do the velocities of each cart compare before and after the "explosion?"**

The velocities are the same, but in opposite directions.

5. **What is the total momentum of the system after the "explosion?"**

Zero because the carts are moving in opposite directions at about the same speed with the same mass. Therefore, momentum is equal and opposite, and total momentum is zero, as it was before the "explosion."

6. **Calculate the initial and final momentums to complete Tables 1, 2 and 3.**

See the tables in the Data Analysis section.

Conservation of Momentum

7. Calculate % Difference for the elastic collision.

$$\frac{p_{\text{cart\#1}} - p_{\text{cart\#2}}}{\left(\frac{p_{\text{cart\#1}} + p_{\text{cart\#2}}}{2}\right)} = \frac{0.090 - 0.085}{\left(\frac{0.090 + 0.085}{2}\right)} = \frac{0.005}{0.088} = 0.0571 = 5.71\%$$

8. Calculate % Difference for the inelastic collision.

$$\frac{p_{\text{cart\#1before}} - p_{\text{bothcartsafter}}}{\left(\frac{p_{\text{cart\#1before}} + p_{\text{bothcartsafter}}}{2}\right)} = \frac{0.085 - 0.086}{\left(\frac{0.085 + 0.086}{2}\right)} = \frac{0.001}{0.086} = 0.0116 = 1.16\%$$

9. What do you think is the primary cause of the differences you calculated?

Friction. Momentum is dependent on velocity, and velocity is lost to friction.

Synthesis Questions

Use available resources to help you answer the following questions.

1. Two locomotives, each weighing 100,000 kg and having a speed of 100 m/s, race toward each other and collide inelastically. What is the final momentum of the system? Explain.

Zero, the two locomotives have equal momentum in opposite directions so the vector sum is zero. However, in the real world, the catastrophic nature of this collision would likely impart momentum to large amounts of debris that would imbalance the equation.

2. A 10 kg bowling ball travelling at 3 m/s collides with a 9 kg bowling ball elastically, after the collision the 10 kg ball is travelling at 0.2 m/s, what is the speed of the 9 kg ball after the collision?

$$p_{1\text{before}} + p_{2\text{before}} = p_{1\text{after}} + p_{2\text{after}} \quad 30 + 0 = 2 + 9(v_{2\text{final}}) = \frac{28}{9} = v_{2\text{final}} = 3.11 \text{ m/s}$$

Multiple Choice Questions

Select the best answer or completion to each of the questions or incomplete statements below.

1. In an automotive safety test, the first car is launched at high speed at a second car that is sitting sideways across its path. The front end of the first car crushes into the side of the second car, which wraps around the first car. The cars continue spinning until they are stopped by a safety barrier. How would you characterize this collision?

- A. Elastic
- B. Inelastic
- C. Explosion
- D. A waist of automobiles

2. A mother (mass 60 kg) skates across an ice rink with negligible friction toward her child (mass 20 kg), who is standing still on the ice. If the mother moves at 4 m/s before she picks up her child, what will her new velocity be after she picks up her child and holds onto him?

- A. 4 m/s
- B. 3 m/s
- C. 2 m/s
- D. 1 m/s
- E. Can't tell; not enough info

3. A projectile of mass M is moving through the air with speed v_0 to the left when it suddenly explodes into 2 pieces. Afterwards, one piece of the projectile (mass $2/3 M$) is moving to the left with half its original velocity. What is the velocity of the other piece?

- A. $v_0/2$
- B. $v_0/3$
- C. $7v_0/5$
- D. $3v_0/2$
- E. $2v_0$

4. A gun is fired so that the bullet (mass = 0.015kg) lodges inside a ballistic pendulum (mass = 5.0 kg), which then swings up to a height after absorbing the momentum of the bullet. If the pendulum recoils with a speed of 2.5 m/s, what is the velocity of the bullet before it collides with the pendulum?

- A. 255.7 m/s
- B. 835.8 m/s
- C. 133.3 m/s
- D. 0.0075 m/s
- E. 800 m/s

Key Term Challenge

Fill in the blanks from the list of randomly ordered words in the Key Term Challenge Word Bank.

1. The momentum of an object equals its **mass** multiplied by its **velocity**. When two objects collide, the total momentum of the system is **conserved** as long as there are no outside **forces** acting on the system. This concept is known as the law of **conservation of momentum**, which states the total momentum of the system **before** an event (like a collision) is the same as the momentum **after** the event.

2. There are several types of **collisions**, which are events where **momentum** is often conserved. **Elastic** collisions occur when two objects hit and bounce away from each other, while **inelastic** collisions are when objects hit and stick together. Another type of momentum-related event is the **explosion**, where one object may split into two or more separate objects. In all 3 of these types of events **momentum** is conserved as long as there are no **external forces** acting on the objects involved in the event.

3. If momentum is conserved during a collision or an explosion, this means that the total momentum of the objects will be **the same** both **before** and **after** the event. Consider an inelastic collision with two identical cars, one which is initially moving, and the other stationary. Because the **mass** of each object is the same, the **velocity** of the two carts together after collision must be **half** the initial velocity of the cart that was moving before the collision.

Extended Inquiry Suggestions

Follow this lab with a discussion of Conservation of Energy and in which types of events energy is conserved. There is value in this discussion, as it forces students to think about internal versus external forces, and it relates well to other scientific laws of conservation (mass, momentum). Students must consider the difference between an internal force and an external force. This distinction can reinforce the drawing of boundary lines, which they must do in free body diagrams (force diagrams), Gaussian problems in Electricity and Magnetism, and now systems for momentum.

A logical extension for this lab is Conservation of Momentum in more than one dimension of motion. Demonstrate 2-D momentum principles through a couple of simple demonstrations, including air hockey tables, billiards examples, and projectiles being launched from the same height, but with different horizontal velocities after they collide and fall to the ground.

Sports analogies can make a rich classroom discussion (or outside the class). A kicked soccer ball goes up in the air if the athlete kicks the bottom of the ball instead of the middle or top half of the ball. Football players running down the field and hit from the side will move in which direction after the collision? What causes a baseball player to hit a ground ball or a pop fly instead of a line drive? When a professional golfer uses a sloped club (like a 9-iron), how will it affect the ball's trajectory after collision?