Reaching New Heights

N°. 62

INTERMEDIATE/SENIOR

Math

LESSON SUMMARY

Students will use the basics of trigonometry to learn to measure the height of a tree.

FORESTS ONTARIO

Focus on Forests

Focus on Forests is a program of Forests Ontario.

Activity Information

Grade Level: Intermediate/Senior

Estimated Duration: Two 90 minute periods

Materials: Downloadable clinometer template (optional) (find on Forests Ontario website), inexpensive 180^0 plastic protractors, straws, fishing line or string, small washers (weights), tape, measuring tapes (the longer the better), chart paper, markers, blackboard/smartboard

Setting: Outdoor and indoor

Key Vocabulary: Trigonometry, trigonometric ratios, forestry, clinometer, angle of elevation, angle of descent

Extension

Day two could be removed if time is constraining and the word problems could be assigned for homework or review later.

You could add some distracting information to some word problems to create a harder challenge if the students are operating at a high level.

Curriculum connections:

(Grade 10 Advanced Math): Trigonometry unit.

(Grade 10 Applied Math): Measurement and Trigonometry unit.

Solve problems involving the measures of sides and angles in right triangles in real life applications (e.g., in surveying, in navigating, in determining the height of an inaccessible object around the school), using the primary trigonometric ratios and the Pythagorean theorem.

Mathematical Processes: problem solving, reflecting, representing, communicating

Teacher Background

This exercise provides students with the opportunity to use their knowledge of the basic trigonometric ratios in a real-world context. They will become familiar with the use and theory of the **clinometer**, a tool many foresters consider among their most important! This tool allows them to quickly and accurately determine the height of various trees (or other tall objects) using right angle triangles.

Students will have the opportunity to brainstorm strategies for determining the height of trees or other large objects, based on an introduction to what the clinometer does. The construction and use of homemade clinometers will allow the students to make their own measurements in a nearby field/park/forest. Students will contemplate how best to achieve accuracy, and also contemplate what uses the knowledge of tree height may have.

The second day, like the first, will largely consist of group work. This time, however, student groups are given challenging problems related to what they've learned, and are expected to solve them and explain their work. Mini-presentations to groups of their peers will help them to practice their use of mathematical language and ensure that each group has a solution to every problem. These lessons could be placed near the end of the unit as consolidation, or prior to the introduction of acute angle trigonometry.

ACTIVITY

DAY 1

Step 1 - Divide your class into groups of three to four, and have them brainstorm strategies for solving the following problem:

Problem: You are hiking in your local woods with a friend and spot what looks like an extremely old tree. It is massive both in height and width of its trunk. You get into an argument with your friend about its height; your friend says it is well over 10 meters tall, but you have your doubts. Thinking back to your trigonometry lessons, what information would you need to resolve your dispute?

- Prompt students to draw diagrams and discuss what information is feasible to acquire
- Provide hints about needing the angle of elevation. You could have pre-assembled a makeshift clinometer and ask the students to figure out how it can be relevant to the problem. Say that you only need a tape measure and this 'tool' (the clinometer).

Step 2 – On your blackboard/smartboard, craft a diagram with feedback from the class that overviews the strategy to be used. Prompt students to correct for the height of the observer (either through measuring their height or making a second measurement of angle to the base of the tree).

Step 3 – Have the class brainstorm ideas about the value of measuring tree heights. Some examples: for logging select trees, for IDing trees for conservation, to thin out a forest, to determine information about the canopy, to identify a tree's potential as habitat for various animals, etc.

Step 4 – Have each group assemble a clinometer according to the following instructions (if not using the downloadable clinometer):

1) Tape a straw along the straight edge of the protractor.

2) Cut a piece of string or fishing line at least 15 cm longer than the radius of the protractor.

3) Tie one end of the string into the hole at the center of the angles (below the 0^0). If there is no hole, tape it to the equivalent location.

4) Tie the washer to the loose end of the string. The string should move freely when the clinometers is rotated.

Step 5 – Take your students outside and have them choose a tree or other tall object on which they can test their strategy. Demonstrate how to use a clinometer. Have them record the necessary measurements from two different distances from the object. Note which trees seem tallest for later comparison with students' results.

Step 6 – Provide students with time to determine the height of their chosen object on their own (in class if time allows, or for homework). Homework problems can be assigned at the end of this day or the second day.

DAY 2

Step 1 - Have students reassemble into their groups. Provide each group with a challenging problem and give them time to create a strategy and solve the problem. Each group will be given a sheet of chart paper on which they will outline their solution, including a clear diagram.

When groups are finished with their problem they can spend time solving the others.

When all groups are finished, have each group explain their solution to their peers in turns. Depending on the class size, you could have each group present to the class, or split the class into two and have groups present to their half of the class.

Problem 1: You want to measure the height of the tree on the top of the hill. There is no level ground nearby from which to do your measurements, so you must measure from a lower elevation. The horizontal distance between the base of the tree and your body is 24 m. Using your clinometer, you measure the angle of elevation to the base of the tree as 14^{0} and the angle of elevation to the top of the tree as 32^{0}. What is the height of the tree? How far are you from the base of the tree?

Problem 2: You just used your new clinometer skills to determine the height of your treetop fortress at 6 m above the ground. You plan to run a rope from your tree fort to the ground at a point 8 m from the base of the tree. How long must your rope be (not including length to tie knots), and what at what angle if your rope descending?

Problem 3: You are a wildlife specialist for the Ministry of Natural Resources and find yourself high in a tree investigating the health of Bald Eagle eggs while the parents are away. You spot your coworker down below who looks extremely far away. Having previously identified the height of the tree as 12 meters, you want to determine how far your coworker is from the base of the tree. You luckily have your clinometer in hand, and measure the angle of descent as 15 degrees. How far away is your coworker? If you climbed halfway down the tree, what angle of descent should your clinometers now read when viewing your coworker?

Problem 4: You are standing 13 m from a tree, and using your clinometer you determine that the angle of elevation between your eye and the top of the tree to be 42^{0}. Your friend is 5 meters behind you and is wondering about the angle of elevation between their eyes and the top of the tree. Without making further measurements, how would you answer their question using the information you've already collected.

Step 2 - Assign homework questions if you did not on Day 1. Otherwise, take them up.

Extension

Day two could be removed if time is constraining and the word problems could be assigned for homework or review later.

You could add some distracting information to some word problems to create a harder challenge if the students are operating at a high level.

Homework Sheet

Homework problems:

1) Determine the height of a tree that is 13 m away from you if you measure an angle of elevation to its top as 25^{0} and an angle of descent to its base as 4^{0}.

2) Determine the height of a tree that is 20 m away from you, if your eyes are 1.55 meters above the ground, and you measure the angle of elevation to the top of the tree as 38^{0}.

3) If a tree is 4 meters tall, what is the angle of elevation between the ground and the top of the tree:

   a. 2 meters away

   b. 3 meters away

   c. 4 meters away

4) Tree A and Tree B are 3 meters apart. Tree B and Tree C are 8 meters apart. The angle between lines connecting trees A/B and B/C is 90^{0}. What is the distance between Tree A and Tree C. What is the angle between lines connecting trees B/C and A/C?

5) Many commercial clinometers are designed for use at specific distances. This allows the manufacturer to put height readings directly onto the dial for direct reading. If you possessed a clinometer that was intended to be used only 15 m away from the object and measured an angle of 52 degrees, what should the height on the dial read?

Clinometer Lesson – Handout for Day 1

Today's focus: Applying trigonometry skills to a real world tasks in the field of forestry.

1) **Problem:** You are hiking in your local woods with a friend and spot what looks like an extremely old tree. It is massive both in height and width of its trunk. You get into an argument with your friend about its height; your friend says it is well over 10 meters tall, but you have your doubts. Thinking back to your trigonometry lessons, what information would you need to resolve your dispute?

Brainstorm Strategies:

2) Assemble a clinometer according to the following instructions:
   1) Tape a straw along the straight edge of the protractor.
   2) Cut a piece of string or fishing line at least 15 cm longer than the radius of the protractor.
   3) Tie one end of the string into the hole at the center of the angles (below the 0^0). If there is no hole, tape it to the equivalent location.
   4) Tie the washer to the loose end of the string. The string should move freely when the clinometers is rotated.

Clinometer Lesson – Handout for Day 1

3) Make a table to record the measurements of your chosen tree/object.

4) Show your calculations for tree height below:

Answer sheet

Homework problems:

1) Height to top = (tan 25) x 13 = 60.062
Height to bottom = (tan 4) x 13 = 0.909

Added = 6.97 m = height of tree

2) Height to top = (tan 38) x 20 = 15.625 m
Add height of observer (1.55 m)

Total height = 17.176 m

3) a) (tan e) = 4 / 2 = 63.4 degrees (e is angle of elevation)
b) (tan e) = 4/3 = 53.1 degrees
c) (tan e) = 4/4 = 45 degrees

4) If d is distance between A and C, then d2 = 32 + 82 . d2 = 73 . d = 8.54 m
If x is angle between BC and AC, then (tan x) = 3 / 6 . Therefore x = 20.56 degrees

5) (tan 52) x 15 = height = 19.2 meters

Word Problems:

Problem 1) height of tree = ((tan 32) x 24) – ((tan 14) x 24) = 14.99 – 5.98 = 9.01 m

Distance between tree and person is the hypotenuse of the triangle with elevation 14 degrees. Lets call the distance x. x = 24 / (cox 14) = 24.73 m. Could also be solve with Pythagoras.

Problem 2) length of rope. X2 = 62 + 82 = 36 + 64 = 100 . Therefore x = 10 m

If c is the angle between the rope and the tree trunk, and d is the angle of descent. Then (tan c) = 8/6. Therefore c = 53.1 degrees. d + c = 90 degrees, therefore d = 36.9 degrees.

Problem 3) Let a be angle of descent, let b be angle between line of vision and tree trunk. a+b = 90. Therefore b = 75. (tan 75) x 12 = d, if d is distance from tree to coworker. Therefore d = 44.8 m.

For second part. Height is 6 m and distance is still 44.8 m. Rename angles a and b, to a' and b'. Find angle b' first. (tan b') = 44.8/6 . b' = 82.3 degrees. Therefore angle of descent = 7.63 degrees.

Problem 4) First find height of tree, h. h = (tan 42) x 13 = 11.7 m

Call a the angle of elevation between friend and tree. Tan a = 11.7 / (13 + 5) = 33 degrees.