

# 3-Pole and 4-Pole Transfer Switch Switching Characteristics

## Abstract

Whether to, and how to, switch a neutral connection when transferring a load between two separate three-phase sources is a topic of frequent discussion [1][2][3][4][5][6]. Should a three-pole or four-pole switch be used? If the neutral is switched, should it be done in an “overlapping” way to insure that during the switching operation the neutral connection to the load is always maintained? Is there a way of successfully using three-pole switching devices on a separately derived (that is, source derived) system?

There are many issues to consider, and those issues have been, by and large, covered adequately in the previously referenced texts. However, what hasn't been covered in those texts is the exact quantification of two key problems.

1. How much circulating neutral current is created when using overlapping neutral switching transfer switches and,
2. How large are the transient overvoltages produced when switching a neutral in a non-overlapping manner

The first point focuses on accurate GF sensing when using overlapping neutral switching schemes. To understand this issue we first examine a system that has only one grounded source. By definition, all ground current must flow through that single ground. Placing a sensor on that grounded connection makes it easy to capture and measure all ground current. By convention, a single-point grounded system with multiple sources is called a “non-separately derived system”.

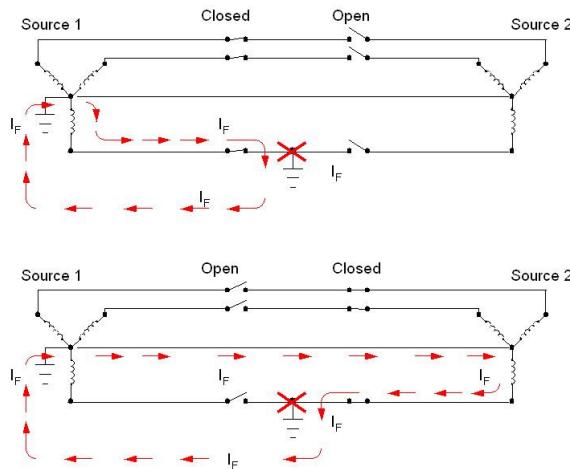


Figure 1: *Regardless of which source is energized, a ground fault on a single point grounded system always returns to the grounded source.*

GF sensing becomes more difficult when multiple sources each have their own grounded conductor and when the neutral conductor is not switched. In such a case, ground current can flow through multiple paths. This complicates the ground fault sensing scheme. Systems with multiple grounded sources are called “separately derived systems.”

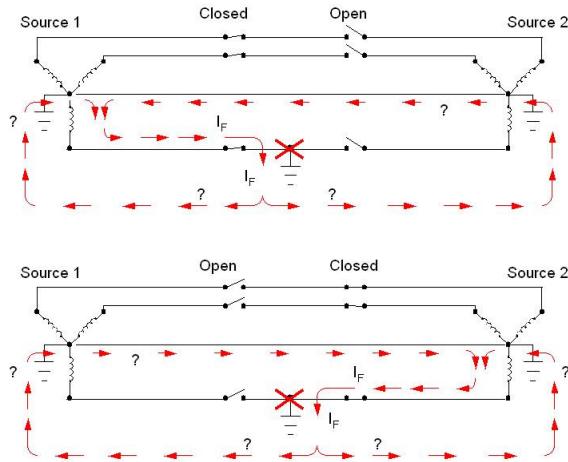


Figure 2: With multiple grounding points and an unswitched neutral, ground current can flow through multiple paths. A ground current sensor on any one source may not record the total ground current flowing into the fault, even if that source is the only source feeding the fault.

One of the solutions to prevent ground current from dividing through the neutral conductor is to switch the neutral conductor. While this insures that no neutral current can flow through a de-energized source, some have raised concerns that such a load break switching of the neutral can result in unacceptably high transient voltages. A method to reduce these reported transient voltages has been the 3-pole switch with overlapping neutral switching [1].

But a search of the available literature leaves us with an unanswered question -- is there quantifiable proof that these transients are really present, and if so, how large are they?

According to McMorrow [1] the overvoltages are minimal. He claims testing was done to prove they were minimal, but his paper did not include the results of those tests. Also, while he lists various problems with overlapping neutral switching schemes, neither McMorrow, nor any of the other referenced papers quantifies the problem of circulating ground fault currents between sources using overlapping neutral switching.

The purpose of this paper, then, is to quantify these two things:

- The maximum voltage transients across a switching neutral contact, as well as
- Quantify the magnitude of circulating neutral currents during an overlapping neutral switching operation.

We begin by discussing the test procedure used. We will perform tests on both separately and non-separately derived systems.

## Non-Separately Derived Systems

Non-separately derived systems are, by definition, those systems where only one bonding jumper between the neutral and ground exists.

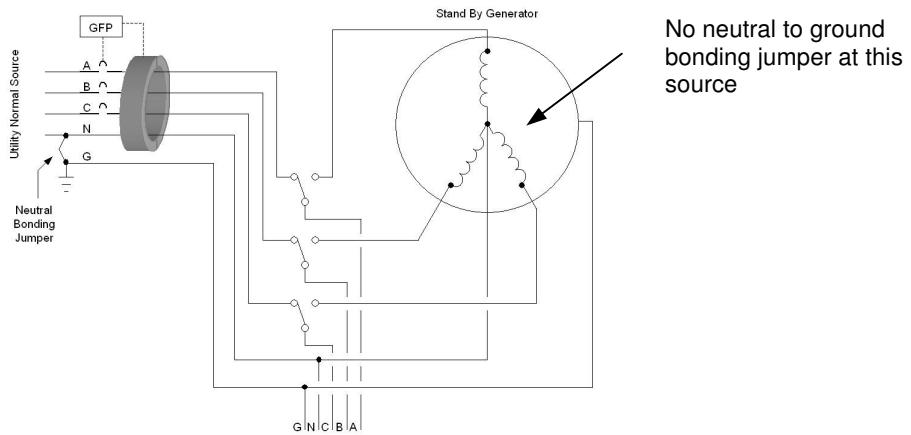


Figure 3: Neutral bonded at utility service entrance but not at stand-by generator.

Since there is only one neutral to ground connection in a non-separately derived (i.e. service-derived) system, all ground fault current flows back to this single grounding point. If the ground current flows through the de-energized source's neutral, this can cause a nuisance trip of the ground fault relay protecting that *de-energized* source. Tripping a de-energized source is obviously a nuisance, but the problem is broader than just nuisance tripping.

Additionally, on the ungrounded source, ground current is *never* detected to be flowing through that source because the same magnitude ground fault current flows in then out of that source's CT. The opposite currents result in a zero output from CT 2. Note that while we have shown a "zero sequence" CT wrapping around all phase and neutral conductors, a "residual" CT ground fault sensing scheme could also have been used. Refer to the section on system grounding in the Eaton *Consulting Application Guide* [11] for more information.

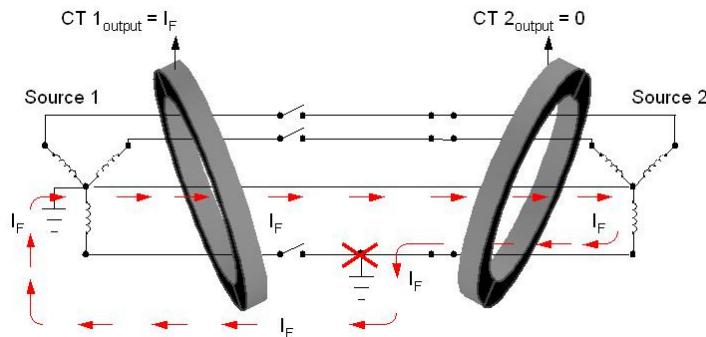


Figure 4: 3-pole devices do not switch the neutral. This leaves the possibility of improper sensing of GF conditions. In this example, a GF fed from Source 2 would not be detected by Source 2's CT since equal and opposite GF currents would cancel in that source's CT. Meanwhile, GF current returning to the ground from source 1 via the common neutral would result in a nuisance trip of the Source 1 GF relay.

While we have shown how this problem occurs with 3-pole switches, it is also an important problem for 3-pole switches with "overlapping" neutrals applied on non-separately derived systems. This is because during the time that both neutral connections are closed, the system will mimic a 3-pole switch as shown in Figure 4.

It is reasonable to ask about the likelihood of a ground fault occurring during the short time that the neutral contacts are overlapped. Certainly it is as likely as any other time, but perhaps it is slightly more likely during the period of the transition for the following reasons.

As those who perform arc flash safety audits can attest, an arc flash event is more likely to occur during movement of energized electrical conductors. As such, a fault is more likely to occur during a breaker operation such as racking or opening or closing.

So, while faults can occur at any time, initiating a change to the system (moving contacts, vibration from switching, energizing previously de-energized lines, etc.) introduces changes to the system. Changes in currents cause changes in magnetic fields through those conductors, and via eddy current coupling, changes in magnetic forces between structural elements. This can result in different mechanical forces being placed on those objects. These changes in forces pull and push equipment and cables in new and different ways. As a result, these new mechanical forces that didn't occur prior to the switching operation can create new motion within the conductor system and possibly result in a fault where none existed immediately prior to the transition.

If or when that ground fault occurs, it is important to clear it in an amount of time as specified by the system coordination study. Dividing and reducing current through a sensor can cause the relay connected to that sensor to trip more slowly or even to not trip at all. A slower tripping relay can result in higher levels of incident arc flash energy being released from the fault. It may also cause selective coordination failures resulting in wider area outages and greater downtime.

There are a variety of solutions to this problem, but one approach is to use auxiliary contacts on the source 1 and 2 switching devices to route the tripping signal to the energized source protective device only. The tripping signal at any deenergized source is simultaneously disabled. This helps insure that a GF relay only trips the source(s) powering the load at that time.

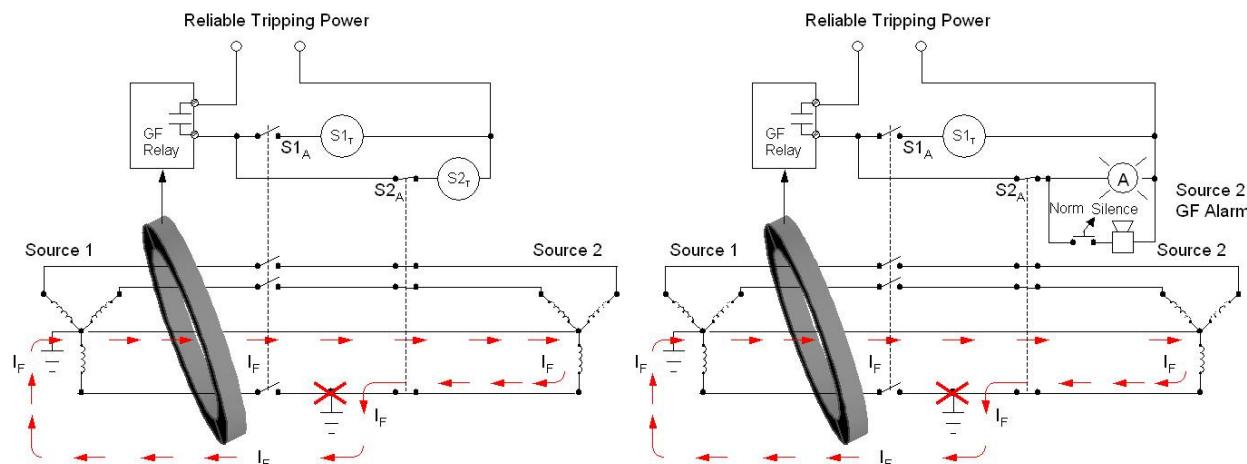


Figure 5: Source 1 is grounded and source 2 is not. Wiring the ground fault relay tripping contacts through normally open auxiliary contacts ("A" contacts) of each switching device allows a tripping signal to trip only the energized sources. Only when S1 is closed (implying that source 1 is feeding the load), can the fault clearing device protecting source 1 be tripped open by the GF relay. Likewise, only when S2 is closed can the fault clearing device protecting source 2 be tripped open by the same GF relay. If second source is an NEC emergency source, you may need to alarm rather than trip on ground fault. One possible alternative scheme for GF alarm is shown at the right.

This GF relay switching scenario can be extended to multiple sources. As with the example shown in Figure 5 using only two generators, extending the number of generators only requires that you add additional auxiliary contacts to trip the additional energized sources. An example of how that might be accomplished is shown schematically in Figure 6.

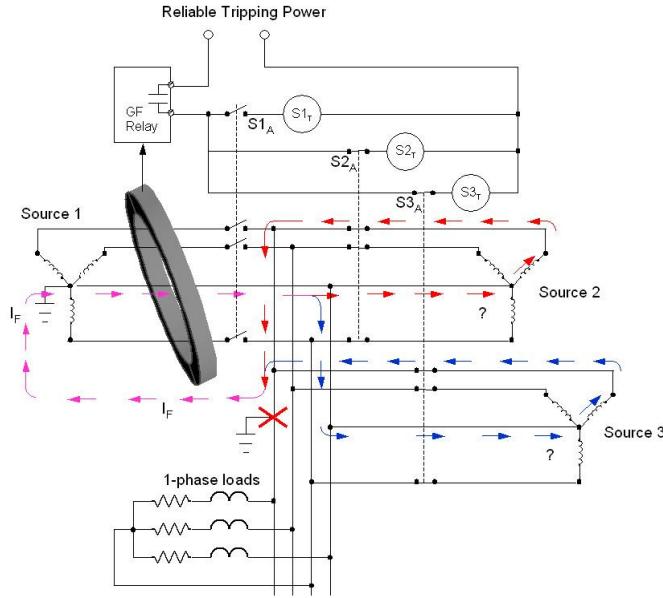


Figure 6: *Ground fault current flows through a single ground even if multiple, ungrounded sources are simultaneously feeding the fault. The GF relay detects this fault and sends tripping signals to any energized source(s), in this case , sources 2 and 3.*

A second solution would be to use a transfer switch that switches the neutral. However this would not be a good solution for non-separately derived systems. If a 4-pole device was used, the load would be ungrounded when connected to an ungrounded source. If a line-to-ground fault were to occur while the ungrounded source was powering the load, there would be no return path back to the source for the ground current.

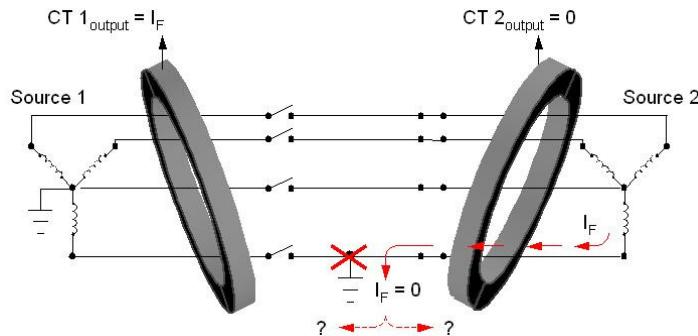


Figure 7: *Using a 4-pole switching device solves the problem of blocking circulating ground currents through neutral connections into de-energized sources (as seen in Figure 5), but when the grounded source is disconnected from the load, no ground reference to the load remains.*

While on the surface this would seem to improve the reliability of the system (since a ground fault does not result in a trip), ungrounded systems are prone to a very dangerous overvoltage condition [7] brought on by intermittent ground faults.

Because of the serious problems of overvoltages caused by intermittent ground faults, modern power system publications [8] recommend grounded systems.

So, if an ungrounded source is a problem, should we insure that all sources are grounded? When a power system includes multiple sources, and each source is separately grounded, those additional grounded sources are called separately derived systems.

## Separately Derived Systems

As defined by the National Electrical Code article 250.20, a power source with its own reference to ground is called a separately derived system.

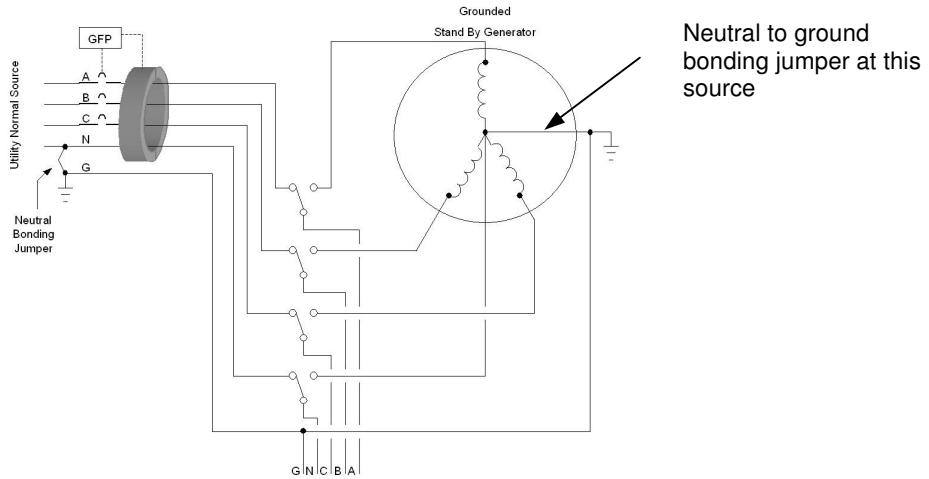


Figure 8: Each separately derived source bonds its neutral and ground at each source. Therefore to prevent currents from circulating between each source's ground during a ground fault, it is important to open all current carrying conductors at each grounded source, including the neutral conductor.

Since transfer switches that include a fully-rated fourth pole are more expensive and larger than transfer switches with only three-switched poles, or with three-switched poles that include non-fault-break rated “overlapping” neutral, frequently the question is raised “do I need a four-pole switch?”

To answer that question we examine how currents circulate between switches during a ground fault. We have already looked at how currents circulate in a non-separately derived system. Our next step is to look at how they circulate in a separately-derived system applied using a three-pole transfer switch. We will examine this layout with and without an overlapping neutral.

However, we do need to be careful with our examination of an overlapping neutral design. As we saw in Figure 2, a particular problem exists for three-pole switches with an *unswitched* neutral. But there is also a problem with a switch that includes an *overlapping* neutral. The problem is that during the transition from one source to another, an overlapping neutral switch is electrically identical to the problematic 3-pole switch. At the point of transition, the overlapping switch is electrically identical to the 3-pole switch.

This becomes evident when we install zero sequence CTs around each source's phase and neutral conductors and we model how much ground current would be detected at each CT during a transition from one source to another.

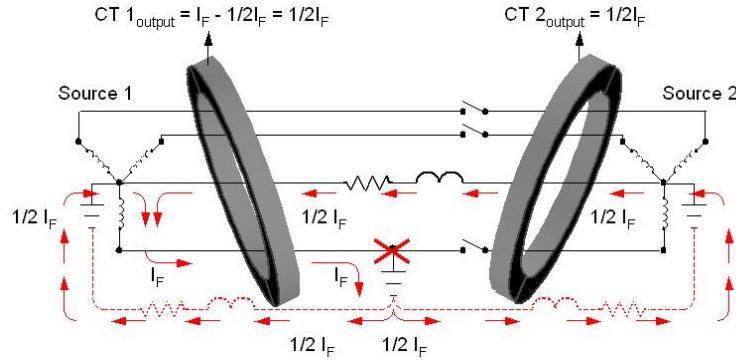


Figure 9: It is evident that two grounded sources connected through 3-pole switching or through 3-pole switching with an overlapping neutral will both have a “cheat” path for ground current to circulate through the neutral of a de-energized source, even though all phase contacts on that source are open. This can cause a nuisance trip on the GF relay protecting the de-energized source while at the same time partially reducing the ground current detected by the energized source GF relay.

Referring to the figure above, source 1 is feeding a ground fault. Because the neutral connection on source 2 is not open, there is a path for some of the ground current to flow through CT 2 as it returns to the source 1 transformer over their common (and unswitched) neutral connection. The two ground impedances and the neutral impedance path form a current divider. When this happens:

- Current is able to flow through a de-energized source CT, potentially causing a nuisance trip of that source’s protective relay while at the same time...,
- Not all of the ground current flows through the energized source CT. This potentially de-sensitizes that protective relay, resulting in slower or no tripping. This can result in higher arc flash incident energy being released during the fault

As we see, a 3-pole switch has some serious problems when applied to separately derived systems.

## Solutions

There are two common ways of overcoming the problems of switching separately derived systems.

- Switch the neutral
  - 4-pole switching devices are used. This solution opens the neutral at the same time as the phase conductors. There is no nuisance tripping and no desensitization of GF relays.
- Use 3-pole switching devices with modified residual ground fault sensing
  - This design tolerates the circulating ground current through each neutral by using auxiliary contacts to switch certain CTs along with some tripping contacts out of the circuit when the three power poles are open. We will not discuss this topic in this paper, but the interested reader is referred to the referenced section in Eaton’s *Consulting Application Guide* [11].

Note that a 3-pole switch with an overlapping neutral does not address either of these two problems (nuisance trip & desensitized trip). At the point of source transition, that type of switch appears as a conventional 3-pole switch with a solid neutral connecting the two sources. The neutral current that must flow during this transition can be enough to cause either a nuisance trip or desensitize the relay that is supposed to trip.

Since there are many solutions, each with certain advantages and disadvantages, we summarize our options for **separately derived** systems in the table below:

Method	Advantages	Disadvantages
<b>Option 1</b> 3-Pole Switching	<ul style="list-style-type: none"> <li>• Lowest Cost</li> </ul>	<ul style="list-style-type: none"> <li>• Nuisance tripping of GF relay on de-energized source</li> <li>• De-sensitizing the energized source GF relay.</li> <li>• Added complexity for GF relay switching as shown in Figure 5 to prevent nuisance tripping of de-energized source.</li> </ul>
<b>Option 2</b> 4-Pole Switching	<ul style="list-style-type: none"> <li>• No circulating current, so no possibility of desensitizing energized source GF relay and no possibility of nuisance tripping a GF relay protecting a de-energized source</li> </ul>	<ul style="list-style-type: none"> <li>• Higher cost</li> <li>• Larger footprint (size)</li> <li>• Reported neutral transients*</li> </ul>
<b>Option 3</b> 3-Pole Switching with Overlapping Neutral	<ul style="list-style-type: none"> <li>• May be less expensive than true 4-pole since overlapping neutral typically is not rated for fault duty switching</li> </ul>	<ul style="list-style-type: none"> <li>• During the time when both neutrals are connected, the same disadvantages as a 3-pole switch (nuisance tripping of GF relay on de-energized source and de-sensitizing energized source GF relay) exists</li> <li>• Added complexity and reduced reliability from an external switch controlled by levers and interlocks connecting to main switch</li> <li>• Added complexity to add GF relay switching as shown in Figure 5 to prevent nuisance tripping of de-energized source.</li> </ul>
<b>Option 4</b> 3-Pole Switching with Special GF Sensing Scheme	<ul style="list-style-type: none"> <li>• Less expensive than 4-pole or 3-pole with overlapping neutral</li> </ul>	<ul style="list-style-type: none"> <li>• More complex wiring as de-energized sources have their trip circuits de-energized and their CT circuits paralleled with the CTs of active sources [11]</li> </ul>

\* As we examine each option, we must scrutinize the claim of some that 3-pole transfer switches with overlapping neutrals offer a benefit of lower neutral switching transients when compared to 4-pole switches.

Are neutral switching transients a problem on a 4-pole system? As we will see, both laboratory tests and mathematical models show that 4-pole neutral switching transients are very small. This is because even in a true 4-pole switch with high neutral currents (e.g. as occurs with large single-phase or other unbalanced load, or when high triplen harmonics are present), the current interruption is never fast enough to generate a large inductive transient voltage. This is because as the contacts open, an arc maintains current flow across the open contacts. Eventually when the arc does extinguish, it does so near a current zero. This low current interruption reduces the current "chop" resulting in a smaller transient.

Some have also claimed that those arcs across the neutral contacts contribute to the premature wear of these contacts, versus with an overlapping neutral design where there is no arcing. Even with a 4-pole switch we will show that the arc generated across the neutral is very small, especially when compared to the rating of those contacts.

Note that the energy dissipated within an arc is roughly proportional to the square of the magnitude of the arcing current (i.e. Arc Energy  $\propto I^2t$ ). Assuming the neutral contact on a true 4-pole device has the same fault clearing capacity as each of the phase contacts, interrupting even full load current results in the release of arc energy several orders of magnitude below the neutral contact rating. For example, the incident energy released from a 5000 ampere arc is nearly three orders of magnitude lower than the rating of an Eaton transfer switch rated for 100 kA interrupting. A 600 ampere interruption would release nearly five orders of magnitude less arc energy than the energy released when interrupting an arc at its interrupting rating. The result is that the neutral contacts are minimally affected from switching neutral load current.

Table 1:  $I^2t$  versus load current

Amperes	$I^2t$	
600	$3.6 \times 10^5 t$	Nearly 5 orders of magnitude difference in $I^2t$ released during arcing
1200	$1.4 \times 10^6 t$	
4000	$1.7 \times 10^7 t$	
5000	$2.5 \times 10^7 t$	Nearly 3 orders of magnitude difference in $I^2t$ released during arcing
100000	$1 \times 10^{10} t$	

To confirm these claims, we created experiments to measure the magnitude of the transient voltage and currents developed across these contacts during switching.

## Neutral Switching Transients

Since electrical distribution systems contain many inductive loads, including transformers, cable in conduit or tray, busway and motors, attempting to switch current across a transfer switch almost always causes a voltage transient. This occurs because inductive loads are switched and the current attempts to continue to flow through the opening contacts. An inductive circuit acts, at least for small time intervals, like a constant current source. This constant current attempts to flow across a much higher impedance air gap, raising the voltage drop across that gap applying that higher voltage to the system.

However, what we learned from lab testing is that current is not interrupted immediately as contacts part. High speed analysis shows that as the contacts part, a conduction pathway is maintained, first through a thread of molten metal formed between the contacts and then later by the arc itself.

Because both the molten metal and the arc have a resistance higher than the contacts alone, the current through the circuit is reduced. This reduction does produce a small increase in the voltage developed across the system inductance. Typically this voltage is modeled using Eq. 1 shown below. Voltage developed across an inductor is proportional to the inductance of the system and proportional to the amount of current interrupted and inversely proportional to the time it takes to interrupt the current.

$$V_L = L \frac{di}{dt}$$

Eq. 1

Where:

- $V_L$  Voltage developed across inductance
- $L$  Inductance (Henries)
- $di$  Differential (change) current (amperes)
- $dt$  Differential time (seconds)

The values ( $L$  and  $di$ ) are electrical parameters that are relatively easy to model and to change in our experiments. The third parameter, time, is related to mechanical functioning of the switch. How fast do we set that value for our model? While laboratory tests will be used, our goal is to use those tests to verify mathematical simulations that will later be used to accurately copy real systems. This allows us to quickly expand our range of testing by changing circuit parameters on the computer, without having to connect and disconnect real hardware. To create this test (and our mathematical model), we first examine the characteristics of arcing contacts. Properly modeling these characteristics will be key to our obtaining accurate results from our mathematical computer models.

#### Progression of Arc Development

When air break contacts part they do not immediately interrupt current. As contact pressure is reduced, resistance is increased producing localized heating. This heating increases to the point where some of the contact begins to melt. As the contacts begin to part, the point of contact between the contacts has become liquid. As the contacts draw further apart this liquid metal forms a bridge or a strand between the opening contacts.

As this bridge creates an even higher path resistance than the contacts themselves, the voltage drop across these contacts increases as constant current from the inductive circuit attempts to flow through this bridge. The bridge temperature increases rapidly. As this liquid bridge increases in temperature, a point is reached where the metal is vaporized. At the point of vaporization, an arc forms between the contacts maintaining current flow. The high temperature of the arc melts more contact material, sustaining the arc until the first alternating current zero. By definition, no arcing can occur at this current zero since no current is flowing.

Once the arc extinguishes, the material begins to cool. If the contacts have parted a sufficient distance and if the plasma gas has cooled enough, the arc will not restrike as the voltage across those contacts increases from zero towards the opposite sine wave polarity.

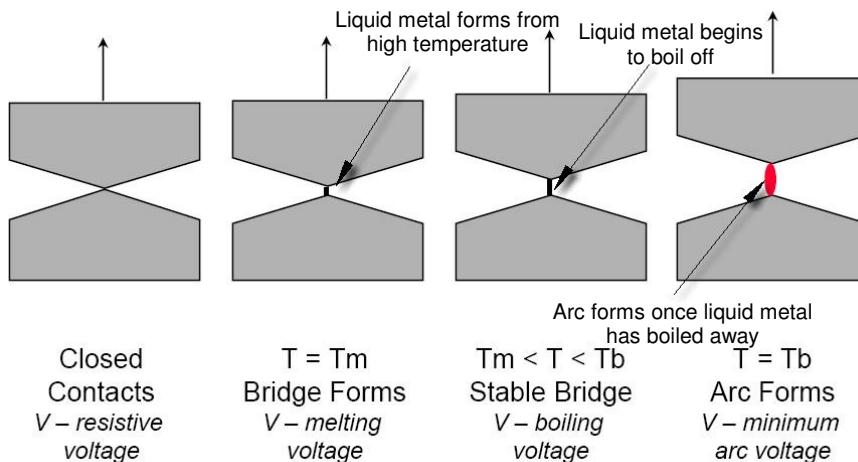


Figure 10: As contacts begin to open, the reduced surface area results in localized heating, raising the contact temperature to the melting point. As the contacts part, this molten metal maintains continuity between contacts. The temperature continues to rise until the metal reaches its boiling point, at which point an arc forms. As the current waveform approaches a current zero, the arc extinguishes.

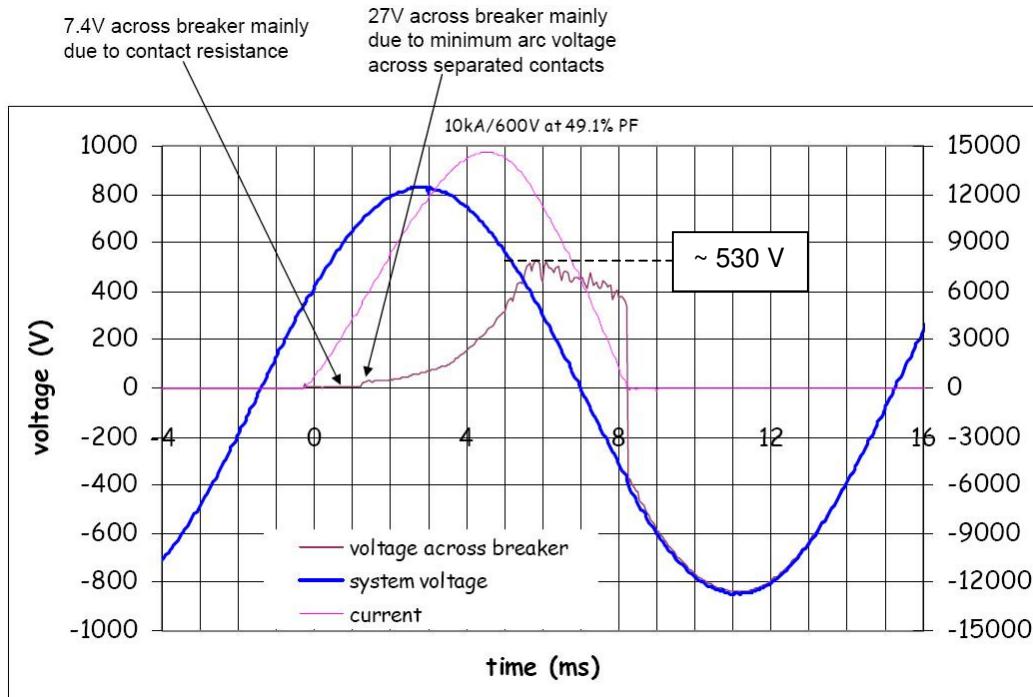


Figure 11: In our first laboratory test, even when interrupting far greater than nominal load currents (10 kA in this laboratory test), arcing voltage across the contacts never reaches even the nominal system peak voltage. This is because only a portion of the voltage is dropped across the arc (balance by the load) and because current chop is minimal, reducing inductive transient voltages.

The current chop is minimized because of the continued current flow through the melting and vaporizing contact material.

Since our lab test was performed at 10 000 amperes, for more typical load currents the ' $dI$ ' term in Eq. 1 will be lower. Our prediction is that contact arcing voltage would be much lower for our lower, normal load currents. However, to obtain evidence confirming this hypothesis, we must design and test models covering a wide range of layouts including:

- Magnitude of current switched,
- Power Factor (X/R) of source(s) and of the load,
- Number of poles switched (3 and 4), and
- Whether overlapping and non-overlapping neutral switching is used

## Model Development

For our mathematical models, our first step was to insure that our model for the arcing voltage generated across our contacts conservatively matched laboratory results. As shown in Figure 11, a 600 Vrms (848 Vpeak), 49.1% PF (X/R = 1.77) system produced a maximum arcing voltage of approximately 530 Vpeak when 10 kA flowed through those opening contacts. As mentioned earlier, to test the many combinations of circuit parameters that needed to be changed in our tests, the decision was made to use a circuit simulation program instead of performing the many separate laboratory tests.

Based on our need to calculate transient in addition to steady-state response, a SPICE circuit simulation program [9] was chosen. As those who have used SPICE simulators know, cables, motors, transformers and the like must be entered using circuit elements such as resistors, inductors and capacitors. Our first step, then, was to convert our known or stated circuit conditions into equivalent values of resistance, inductance and capacitance. We developed an calculator [10] that assisted in this process by converting current, voltage and PF (or X/R) into equivalent resistances and inductances.

Using that calculator we obtained the following circuit elements to plug into our SPICE model:

Given Circuit Conditions	Equivalent SPICE Model Elements
600 V <sub>L-L</sub> rms	Voltage Source: 848.528 V <sub>L-L</sub> peak, 60 Hz
10 kA 49.1% PF	$Z = 0.06 \Omega$ $X/R = 1.77$ , therefore $R = 0.0295137 \Omega$ , $X_L = 0.0522393 \Omega$ $\therefore @ 60 \text{ Hz}, L = 0.138568957 \text{ mH}$

The equations used by the calculator [10] are explained in this paper's appendix (see page 29). Plugging these elements into our SPICE simulation program we obtain what we draw as Figure 12 below:

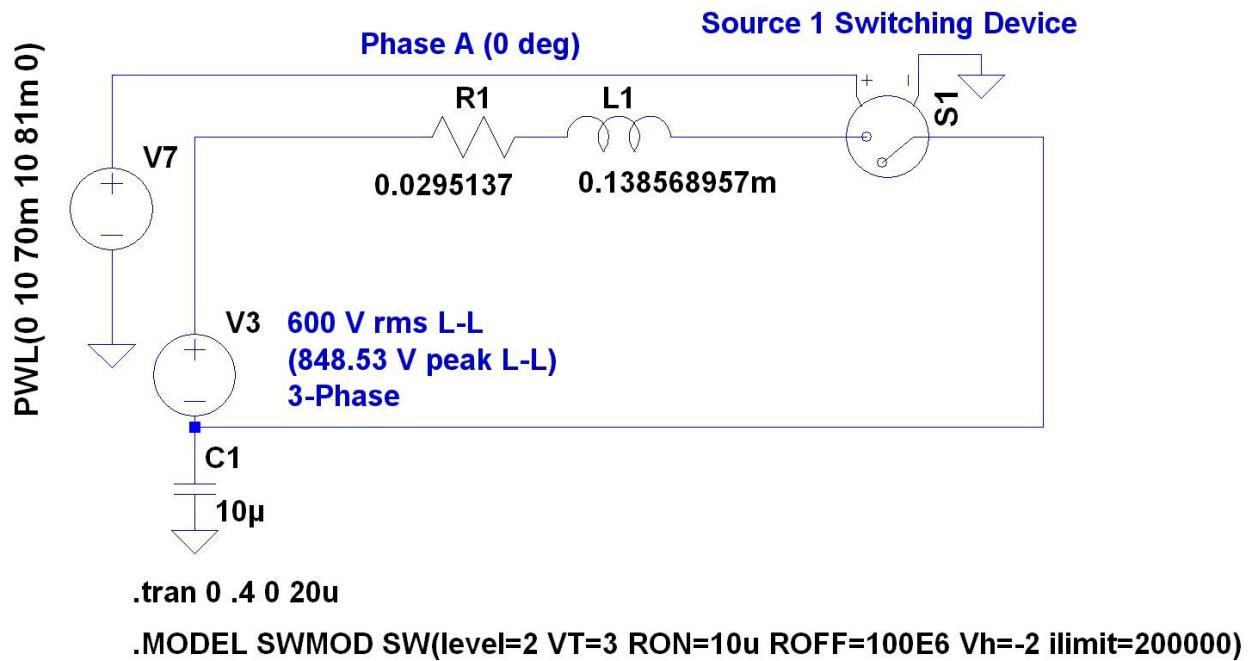


Figure 12: A single-phase of a 10 kA, 600V 3-phase power supply and switch. V7 operates our voltage controlled switch that applies and removes our source current to the load (or to the fault as in this case where our return path to the source is zero impedance).

The SPICE model for the switch (S1 – SWMOD) includes several tuned parameters to more accurately model an arcing contact. Refer to the documentation [9] for an explanations of these model parameters. Using this model we obtained the output shown on the right side of Figure 13 below. As you can see, our model predicted a conservative estimate of 801 V peak arcing voltage, while the laboratory test saw a peak of only 530 V.

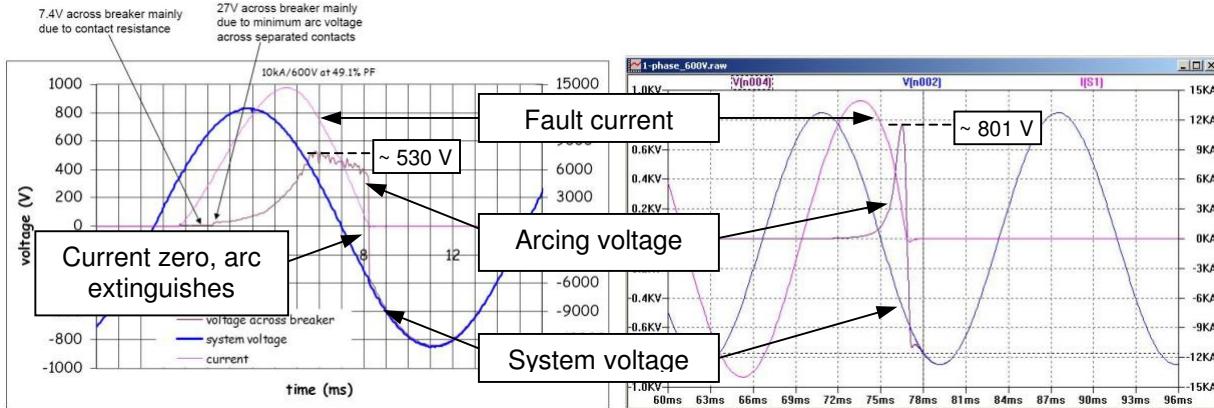


Figure 13: Conservative agreement between lab test and SPICE model.

#### Actual versus Modeled Arc Performance

As shown in the diagram on the left side of Figure 13 above, the laboratory test of a switch opening during a 10 kA fault shows the voltage across the contacts increasing slightly from 0 to 7.4 Volts as the current through the contacts increases. The contacts heat and begin to separate when, as was described for Figure 10, a liquid metal bridge forms between the separating contacts. This liquid bridge maintains a conductive path across the contact, although at a higher impedance than when the contacts were firmly pressed against each other.

As a result, the voltage drop across the contacts increases from 7.4 Volts to 27 Volts. As the metal bridge is stretched, the length increases while the diameter decreases, increasing the resistance of the path further. This caused even greater heating. Finally the temperature increases to the boiling point of the metal where the metal vaporizes and an arc forms between the contacts.

As we saw in the lab test, as the arc current increases, the temperature of the arc intensifies. This results in a lower arc impedance and therefore a lower voltage drop across the contacts. Since the impedance drops as the current increases, this negative resistance characteristic causes the voltage across the contact to stabilize (flat-top). Depending on how long it took to create this arc, the instantaneous value of the fault current would likely begin to decrease at this point. This would contribute to flat to decreasing arcing voltage until the arcing current reaches zero. At that point the arc extinguishes and the voltage across the contacts jumps to the system voltage.

On a properly designed switching device, operated within its voltage and interrupting current ratings, the contacts will be far enough apart and the plasma gas will have been cooled sufficiently to prevent a restrike as the voltage across those contacts begins to rise during the next half-cycle following the arc interruption.

As you see from the laboratory test (left side of Figure 13), the arcing voltage peaks at approximately 530 V, or about 63% ( $530\text{ V}/848\text{ V} = 63\%$ ) of the system voltage applied to those contacts. Our very conservative SPICE simulation (right side) shows the arcing voltage across the contacts as about 94% ( $801\text{ V}/848\text{ V} = 94\%$ ).

What this means is that the simulation we are using for our switching element products switching transients approximately 1.4 times as high as measured in laboratory tests. This increases our confidence that our model produces a very conservative maximum estimate of transient voltages across our neutral contacts. This is useful information since it implies that “real world” transient voltages will be less than those calculated by our model.

Armed with this data, we now proceed to model our various three phase, 3 and 4-pole transfer switches in our different configurations.

For our first model, we changed the electrical parameters from our lab test values of 10 kA short circuit current at a source X/R ratio of 1.77 (PF of 49.1%) to 30 kA at an X/R of 4.9 (PF of 20%). This simulates a closer connection to the upstream transformer. The resultant higher inductive reactance levels will cause a greater voltage offset during a circuit closing operation. This would tend to produce more pronounced arcing as this energy is dissipated, making our mathematically modeled environment more stressful on our neutral contacts than our laboratory tests. Again, our goal is to create a model that accurately predicts the maximum neutral switching voltage.

#### Test 1: Non-Separately Derived System – 3-pole Switching Test

- Source 1 – Utility 1
  - 1500 kVA transformer,  $Z = 5.75\%$ , 480Y/277 Vrms, 60 Hz
  - $X/R = 6.6$ , ( $I_{FL} = 1804$  A,  $I_{SC \text{ phase-phase}} = 31377$  A rms)
  - Sufficient conductor length is included to limit the fault current to 30 kA rms with  $X/R = 4.9$  at load (equivalent to a 20% PF)
- Source 2 – Utility 2
  - Ratings and characteristics are the same as Source 1
- Load (for both sources)
  - 600 A rms, 80% PF ( $X/R = 0.75$ )

Using our calculator [10] we convert the electrical system characteristics listed above into circuit elements (resistors and inductors) appropriately sized to provide our required system characteristics. Note that while we typically discuss voltage and current magnitudes in their root mean square (rms) values, we must program our SPICE model voltage sources with peak voltages since the model for an alternating current power supply is in the form  $V(t) = V_p \sin(\omega t)$ , where  $V_p$  is the peak sine wave magnitude and  $\omega$  is equivalent to  $2\pi f$  (i.e.  $2\pi$  times the frequency in Hertz).

## Configuring SPICE

We begin by reviewing the method used to convert our circuit parameters (V, A, PF, etc.) into circuit elements (sources, resistors, inductors). As this is a well understood procedure, readers may choose to skip ahead to the next topic of SPICE model construction beginning on page 20.

Our first step is to convert our 480 V rms source line-to-line measurement into the peak voltage:

$$V_{L-L\_peak} = V_{rms} \sqrt{2}$$

Eq. 2

$$V_{L-L\_peak} = 480\sqrt{2} = 678.8225 \text{ V}$$

Eq. 3

Our transformer secondary will be a grounded wye. Since these 480 and 678 volt values are line-to-line measurements, we must divide the line-to-line voltage by the square root of 3 to convert this value into a peak line-to-neutral voltage for each leg of our modeled wye transformer.

$$V_{L-N\_peak} = \frac{V_{L-L\_peak}}{\sqrt{3}} = \frac{678.8225}{\sqrt{3}} = 391.92 \text{ V}$$

Eq. 4

Our three-phase power supply will connect three of these voltage sources together in a wye configuration with each source phase offset by 0, 120 and 240 degrees respectively.

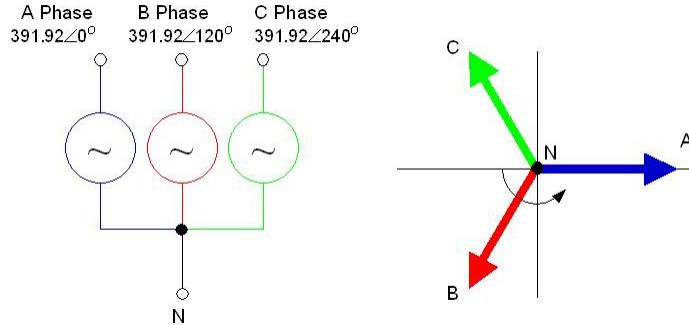


Figure 14: 3-phase power supply schematic and resultant phasor diagram showing counter-clockwise rotation around the neutral. One rotation of the phasor diagram is made around the neutral point in one ac cycle.

Our next step is to convert our fault current into impedance ( $Z$ ) in ohms, then convert this total impedance ( $Z$ ) into resistive ( $R$ ) and inductive reactive ( $jX_L$ ) ohms using our specified  $X$  over  $R$  ratio ( $X/R$ ). Since most power systems are inductive in nature (current lags voltage), we have simplified this model by not including distributed capacitive ( $-jX_C$ ) reactive elements in our circuit.

Here we calculate the total impedance necessary to limit our source voltage to a 30 kA fault:

$$Z_{30kA} = V_{L-N\_rms} / I_{\text{fault current rms}} = 277.13 / 30000 = 0.009238 \Omega$$

Eq. 5

A 30,000 ampere fault on a 277 source means the source impedance is about 9.2 milliohms. Using trigonometric properties, based on our 4.9  $X/R$  ratio (20%), we convert this value of  $Z$  into equivalent  $X$  and  $R$  values:

$$X_{L-30kA} = \sqrt{\frac{Z^2}{1 + \left(\frac{X}{R}\right)^2}} = \sqrt{\frac{0.009238^2}{1 + (4.9)^2}} = 0.009051 \Omega$$

Eq. 6

$$R_{30kA} = \frac{X}{\left(\frac{X}{R}\right)} = \frac{0.009051 \Omega}{(4.9)} = 0.001847 \Omega$$

Eq. 7

Both  $X_L$  and  $R$  are calculated in ohms. While we can size a resistor  $R$  in SPICE in ohms, we cannot size an inductor  $L$  in ohms since inductive reactance varies based on frequency. However, we know that at a particular frequency ( $f$ ) we can work backwards to calculate the inductor ( $L$ ) size in Henries ( $H$ ) required to provide the reactive impedance  $X_L$ . We do this using equations Eq. 8 through Eq. 10:

$$X_L = \omega L = 2\pi f L \text{ Ohms}$$

Eq. 8

Rearranging terms:

$$L = \frac{X_L}{2\pi f} \text{ Henries}$$

Eq. 9

Plugging in our value Z from Eq. 5, we solve for L in our 277 V<sub>L-N</sub> rms, 30 kA, 4.9 X/R circuit:

$$L_{30kA} = \frac{X_L}{2\pi f} = \frac{0.009051\Omega}{2\pi \cdot 60} = 24.0086\mu H$$

Eq. 10

We now have a complete model of one phase of our source. Later we will see the consequences of an unbalanced circuit, but for this first experiment, we will assume that all three phases have the same characteristics.

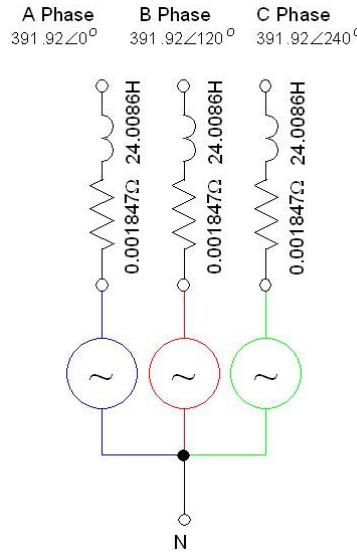


Figure 15: Model of 480V<sub>L-L</sub> rms, 30 kA, 4.9 X/R (20% PF) 3-phase power supply

Our next step is to repeat the steps to solve for the equivalent values of R and L for our 600 A, 80% PF load.

Using Eq. 5 again, we calculate the equivalent impedance needed to limit a 277.13 V<sub>L-N</sub> rms (480V<sub>L-L</sub> rms) source to our specified load current of 600 amperes:

$$Z_{Load} = \frac{V_{rms}}{I_{rms}}$$

Eq. 11

$$Z_{Load} = \frac{277.13 \text{ V}}{600 \text{ A}} = 0.462\Omega$$

Eq. 12

Using the same equations as Eq. 6 and Eq. 7, we calculate the R and  $X_L$  values for this 600A, 480 V rms circuit using the calculated Z (from Eq. 12) and the given X/R ratio (0.75).

$$X_{L-600A} = \sqrt{\frac{Z^2}{1 + \left(\frac{X}{R}\right)^2}} = \sqrt{\frac{0.462^2}{1 + (0.75)^2}} = 0.2771\Omega$$

Eq. 13

$$R_{600A} = \frac{X}{\left(\frac{X}{R}\right)} = \frac{0.2771\Omega}{(0.75)} = 0.3695\Omega$$

Eq. 14

$$L_{600A} = \frac{X_{L-600A}}{2\pi f} = \frac{0.2771\Omega}{2\pi \cdot 60} = 0.7351mH$$

Eq. 15

These would be the values of the load R and L if our load was connected to an “infinite (zero impedance) source.” We do not have a zero impedance source. Note that while the values of  $R_{600A}$  and  $L_{600A}$  represent the *total* resistance and inductance for the path from the source to the load, they are *not* the resistance and inductance values of the load alone. This is because we must account for the source impedance (Eq. 5 through Eq. 10).

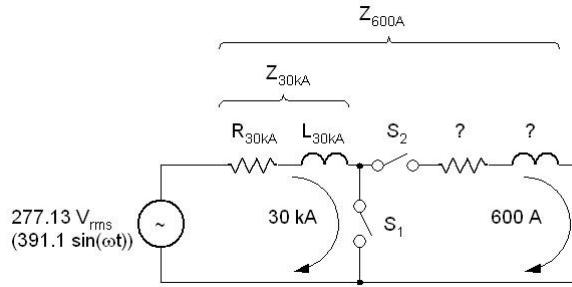


Figure 16 – Single phase model of short circuit and load current showing unknown load resistance and inductance

When switch  $S_1$  is closed, as shown in Figure 16, full fault current flows. We do know the R,  $X_L$ , and equivalent L values for that circuit and we show those values as  $R_{30kA}$  and  $L_{30kA}$ . If we open switch  $S_1$  and close switch  $S_2$ , the current drops from fault current to load current. We define the total impedance for this complete circuit from the source to the load as  $Z_{600A}$  for it includes both the known ( $Z_{30kA} = R_{30kA} + jX_{L-30kA}$ ) plus an unknown additional impedance necessary to drop the 30 kA fault down to a 600 A load. That *additional* impedance is what we need to calculate.

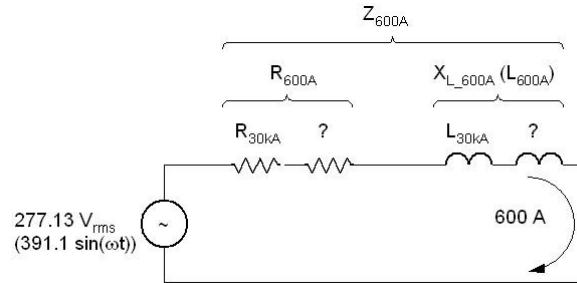


Figure 17: Grouping unknown real and reactive elements with known values for  $R_{30kA}$  and  $L_{30kA}$

To determine that additional load path resistance, what we call  $Z_{600A}$  in our Figure 17 and Figure 18, we group resistance and inductance values together into terms we call  $R_{600A}$  and  $L_{600A}$ .

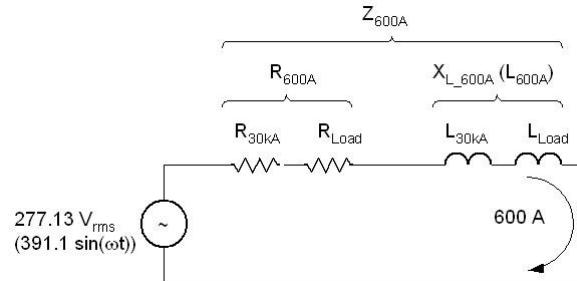


Figure 18: We call the unknown additional real resistance  $R_{Load}$  and the unknown additional inductance  $L_{Load}$ .

The load values of resistance and inductance will be the difference between the total  $Z$  (and  $R$  and  $L$ ) values for the load and the values for the source (30 kA) only. Therefore, to solve for the unknown  $R_{Load}$  and  $L_{Load}$  we use Eq. 16 and Eq. 17:

$$R_{Load} = R_{600A} - R_{30kA} = 0.3695 \Omega - 0.001847 \Omega = 0.3677 \Omega \quad \text{Eq. 16}$$

$$L_{Load} = L_{600A} - L_{30kA} = 735.1 \mu\text{H} - 24.0086 \mu\text{H} = 711.1 \mu\text{H} = 0.000711 \text{ H} \quad \text{Eq. 17}$$

We now have completed our model of the source voltage, source impedance and now load impedance. We place a switch between the source and load impedances to simulate one side of our transfer switch. For this model we will show our load as being a wye connected three phase load. A wye connected load will accurately simulate a three-phase load (such as a motor) as well as single-phase circuits connected line-to-neutral.

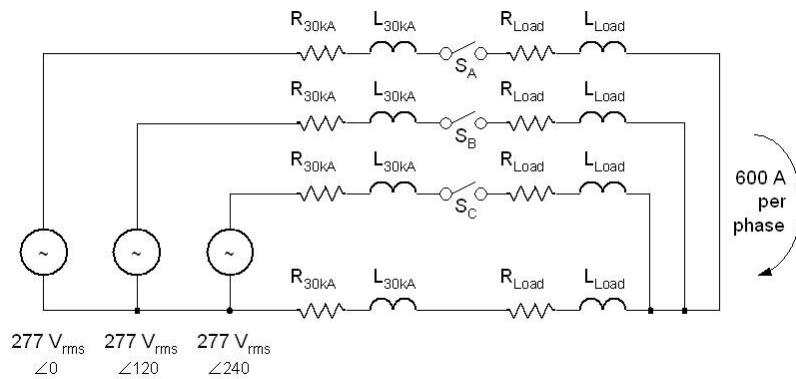


Figure 19: Wye connected three-phase source and wye connected load

Replacing our variables  $R_{30kA}$ ,  $L_{30kA}$ ,  $R_{Load}$  and  $L_{Load}$  with the actual values of Ohms and Henries, we redraw Figure 19 as shown below:

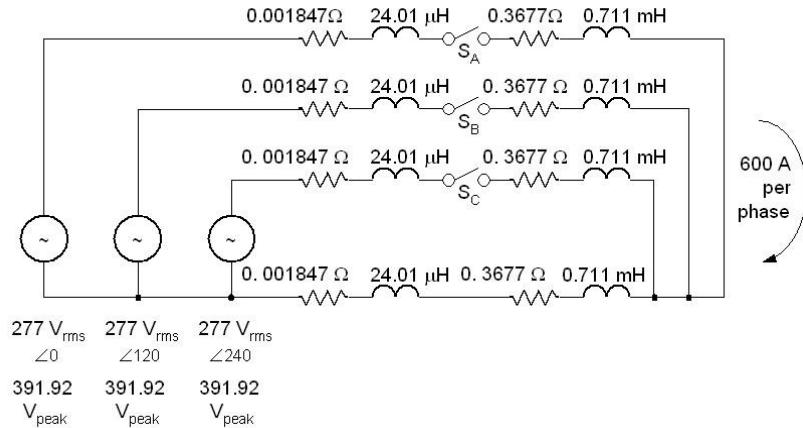


Figure 20: 30 kA, 480V<sub>rms</sub> wye source feeding a 600A, 80% PF wye connected load

Notice that for a 3-phase circuit, each circuit conductor (including the neutral) is sized the same as the single-phase conductors carrying the same amount of current. For balanced three-phase loads without harmonics, the neutral size is somewhat irrelevant since the neutral current will be zero. However, for single phase loads, or ground faults or for loads with zero-sequence (such as triplen) harmonics, the neutral current will be greater than zero and a realistic value for neutral impedance is necessary.

We are now ready to run our simulations and determine the transient voltages produced across the switching contacts. However, before we run the SPICE model, we need to clarify a few points.

Arc modeling is not an exact science

When a contact opens, the arc formed does not maintain a constant impedance. While this arc can be modeled approximately (and we have attempted to do so), the models available today typically do not account for changes due to magnetic field interaction with other current carrying conductors, turbulent convective air currents, changes in properties of the metal or due to contamination, corrosion or changes in the metallurgy of the contacts as they heat up and possibly even other effects. The result of these effects is that the modeled arc will not mimic actual arcing precisely.

However, since what we are looking for is the *peak voltage* generated from an arc, our solution is to create a model that conservatively simulates this peak voltage, while not necessarily modeling all the other chaotic behavior or characteristics of the arc.

Transient voltage varies depending on where the arc is interrupted

As described in Eq. 1, the magnitude of the transient overvoltage is proportional to the instantaneous current flowing at the time of the interruption and inversely proportional to the amount of time needed to interrupt the current. On a three-phase system with each phase 120 degrees offset from each other, only one phase at a time can be reaching a peak value, so to create a worst case condition, we configured our model to interrupt current of all three phases when one of the phase current waveform's reaches its peak. We will assume that the current is extinguished at the next zero crossing of that phase. Since, by definition, this will occur  $\frac{1}{4}$  cycle after a current peak, we have programmed our model to insure that the arc is interrupted at least within one quarter of a cycle of our 60 Hz system, that is  $\frac{1}{4} \cdot \frac{1}{60} = \frac{1}{240} = 4.1667$  milliseconds later.

## SPICE Model

As mentioned before, the examples used in this paper are based on a SPICE simulator and schematic capture program available from Linear Technologies [9]. A copy of the model files used in the simulations shown in this paper as well as the tools to calculate the necessary circuit values can be downloaded from Eaton [10].

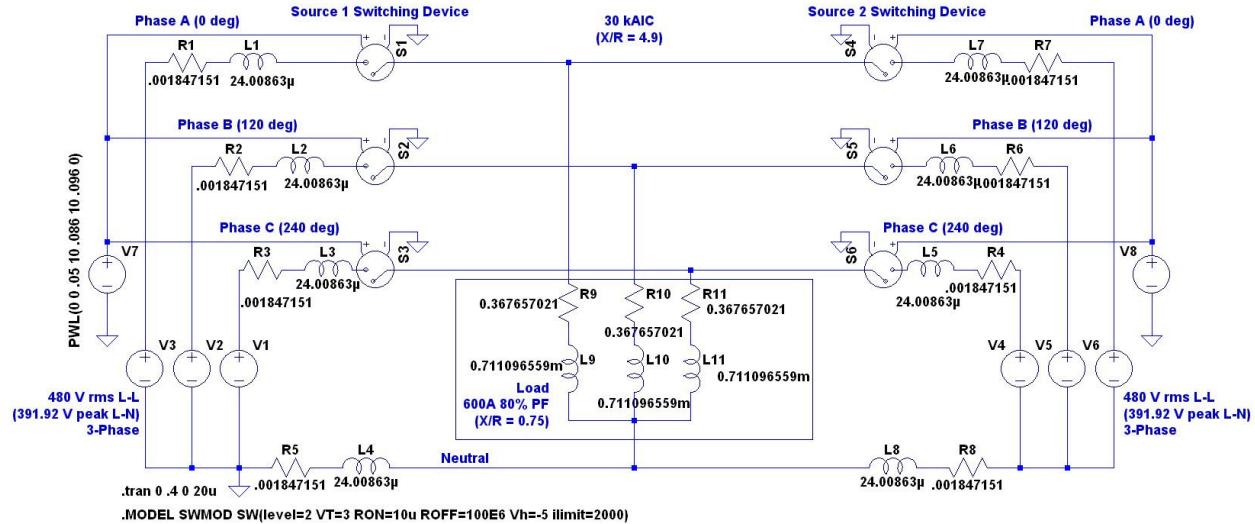


Figure 21: LTSpice model of 3-pole ATS switching between two sources, each 480Y/277 with 30 kAIC available (20% PF  $X/R = 4.9$ ). Each source feeds into a 600 ampere 80% PF ( $X/R = 0.75$ ) load. Transients were measured across the phase switching elements (this is a 3-pole model, so no neutral switching is performed) and the results compare with laboratory model. Since voltage control switches ( $S_1$ - $S_6$ ) were used, we included voltage source  $V_7$  to control when and how the “normal” side opens and closes. Likewise, we programmed a voltage source  $V_8$  to control the “emergency” side of the transfer switch. By programming how those two voltage sources operate, we can configure our transfer switch to operate either open transition or closed transition. By modifying the voltage and phase “parametrically” (i.e. under program control) on our emergency source ( $V_4$ - $V_6$ ) we could have (but have not in this model) modeled soft-load/zero-power transfer or open transition out-of-phase transfer.

## Results of Testing

For this paper, we tested several models:

- Non-separately derived sources (1<sup>st</sup> source grounded, 2<sup>nd</sup> source ungrounded)

Test 1. Open transition 3-pole switching (Figure 21)

- Separately derived (individually grounded sources separated by impedance)

Test 2. Open transition 3-pole switching

Test 3. Open transition 4-pole switching

Test 4. Open transition phase 3-pole, closed transition neutral (overlapping) switching

### Test 1: Non-Separately Derived Source: 3-Pole Switching

As we discussed earlier, for a single point grounded system without a switched neutral, there will not be a neutral transient voltage. Furthermore, we will assume that the solution outlined in Figure 5 is used to direct GF relay switching to only the source(s) feeding the load. Without a transient or GF issue, there is no need to simulate this circuit.

## Test 2: Separately Derived Source: 3-Pole Switching

Since our neutral is not switched, there will not be any neutral transient voltages in this test. However, since we have more than one ground and since our neutral connects those grounds together, we have an opportunity for circulating ground fault currents to flow through the common neutral connection during a ground fault. Since current seeks the path of least resistance, the amount of current flowing through each ground path or the neutral will be based on the relative ratio of impedances of each of those paths.

For this model, we will assume that our neutral conductor has identical electrical characteristics to our phase conductors. We will assume that the impedance between ground points is based on distance between points, with a minimum resistance of 0.5 ohms for closely space grounds and a maximum impedance of 10 ohms for longer distances. All ground impedances are assumed to be purely resistive.

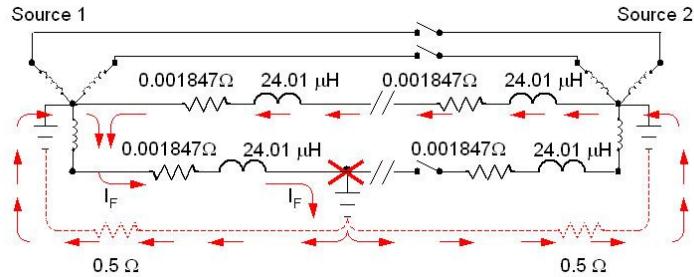


Figure 22: 3-Pole switch applied to separately derived system does not block circulating GF current in neutral

In our circuit above, we will close source 1, apply a ground fault, and measure the zero sequence current flowing through a CT surrounding the three phase and neutral conductors at each source. We construct our SPICE model as shown in Figure 23 below:

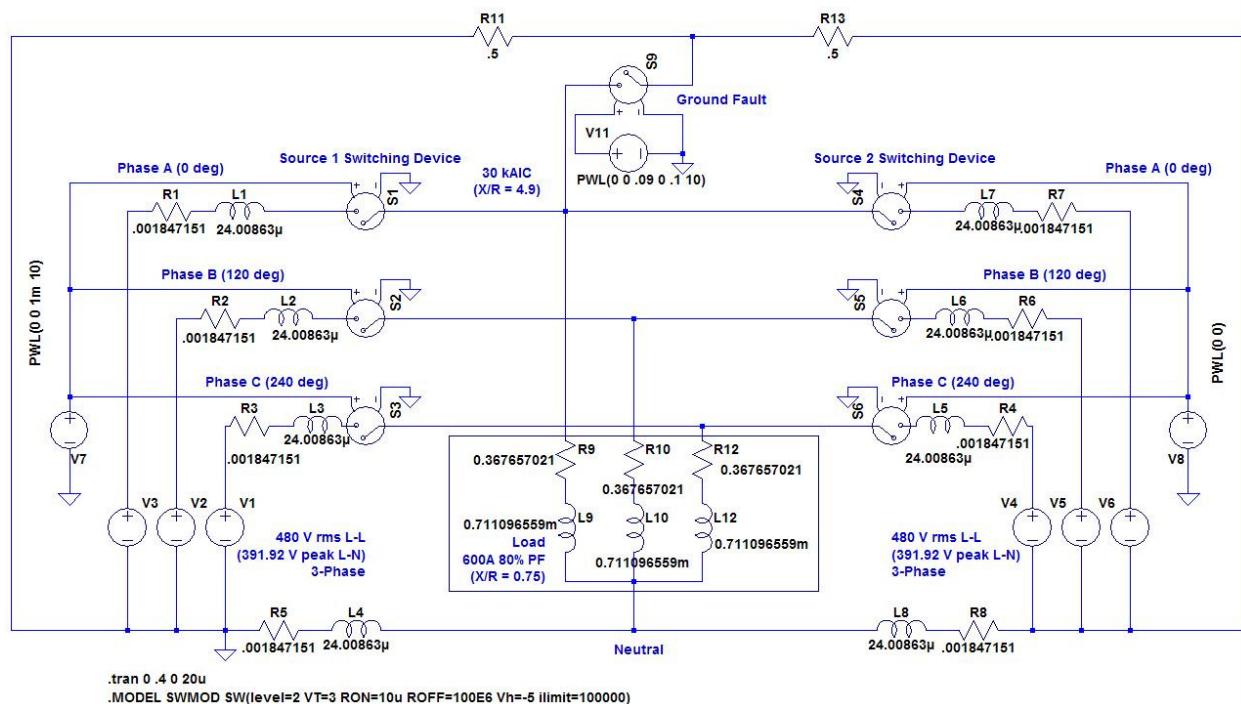


Figure 23: Two separately derived systems connected to load through a 3-pole transfer switch. Switch S9 is our programmable ground fault.

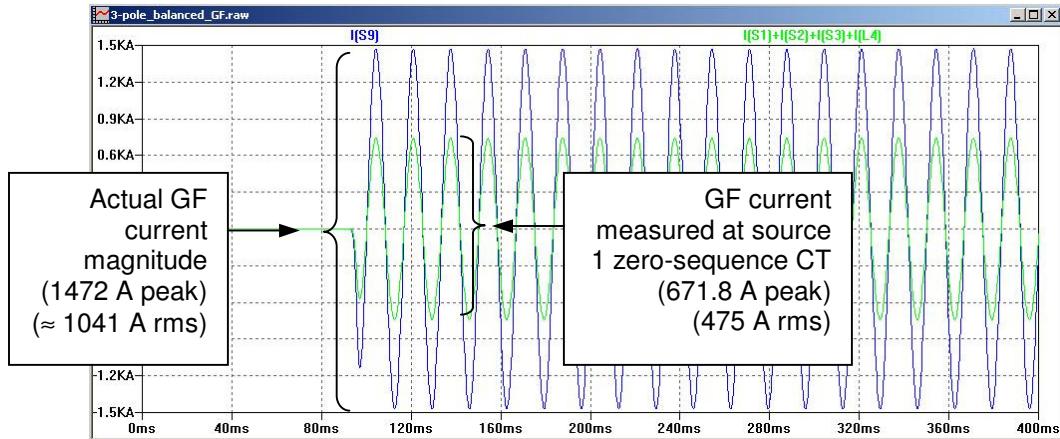


Figure 24: Actual ground fault magnitude is higher than the measured ground fault current.

If the actual ground fault is 1041 amperes rms, but the source 1 CT only reports 475 A rms, where is the “missing” current?

It is flowing through the de-energized source 2 neutral. See Figure 25 below.

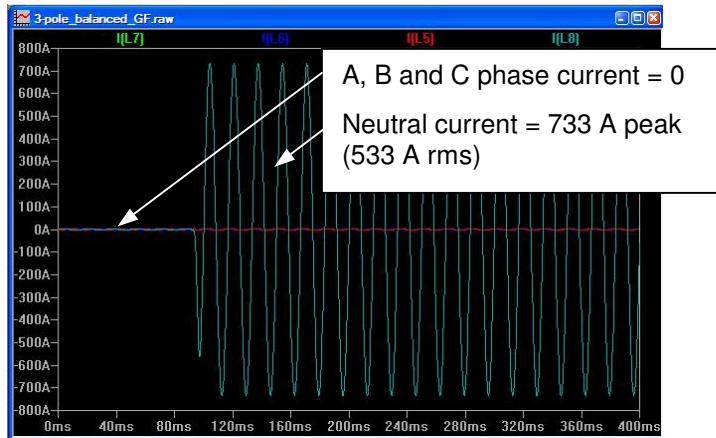


Figure 25: GF current detected flowing through the de-energized source 2 unswitched neutral.

As evident from our analysis, the use of 3-pole devices on separately derived systems causes problems because ground fault current divides between all sources in a ratio that is function of the relative impedance of each ground path. As a result, GF relays monitoring sources feeding the fault won't see the full ground fault current. Not only that, the leftover GF currents are flowing through the neutral of de-energized sources resulting in possible nuisance tripping of the GF relay protecting those sources.

How do we design a solution to this problem? One solution is to open the neutral when we open the three phase poles. We examine that solution next.

### Test 3: Separately Derived Source: 4-Pole Switching

While we understand that opening the neutral will stop leftover GF currents from flowing through neutrals of de-energized sources, will switching the neutral produce unacceptable transient overvoltages? To answer this question we conducted another series of tests:

- Transient voltages test (neutral and phase) - Test 3A
  - To verify if excessive neutral transient voltages are present on 4-pole switch

- This first test was performed without a GF occurring during transition
- Improperly circulating ground fault currents test - Test 3B
  - To verify that GF currents can be detected properly by protective relays
  - This second test was performed while a GF occurs during transition

We begin by modifying our 3-pole schematic used in Test 2 (Figure 23) and add a fourth pole to switch the neutral at the same time as the three phase power poles. Note that we kept the programmable GF crow-bar switch. For the neutral transient test (Test 3A), we programmed the GF crow-bar to remain open as would occur during a ordinary transfer of loads from one source to another. In the second test (Test 3B) we activated the S9 crow-bar to created a GF during the transition.

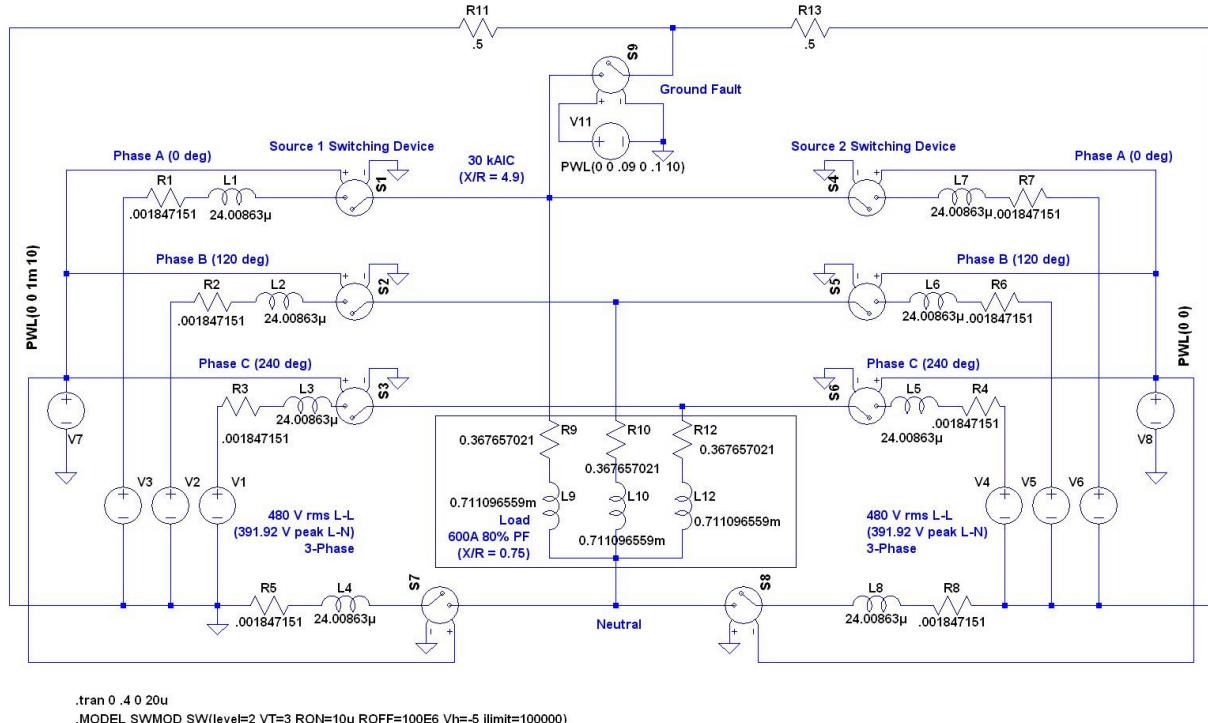


Figure 26: LTSpice model of 4-pole transfer switch. Switch S9 is GF crow-bar.

The transient voltages generated across the neutral switch are shown in Figure 27:

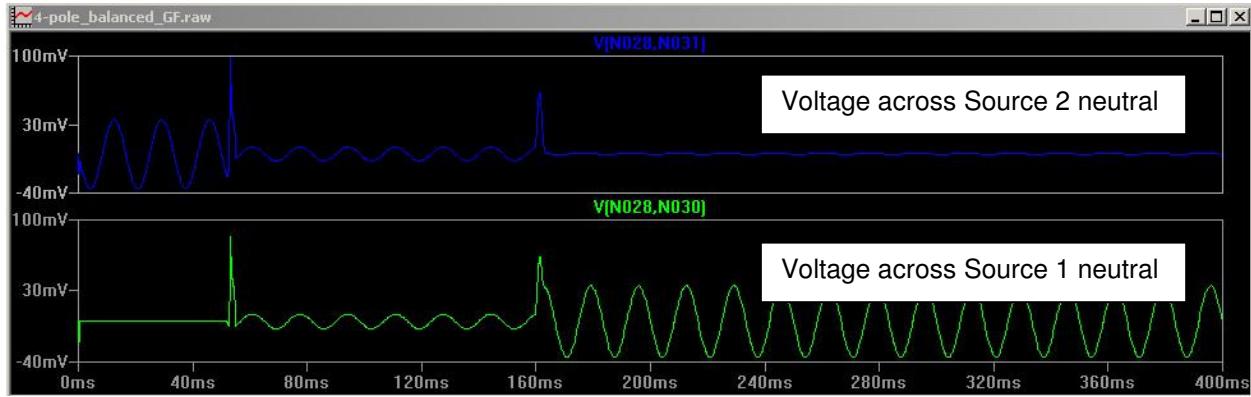


Figure 27: Voltages developed across the neutral pole while switching a balanced load using the 4-pole SPICE model shown in Figure 26.

As shown in this simulation, the maximum voltage across either neutral contact is less than 0.1 volt. Referring to the schematic shown in Figure 26, the source 2 neutral voltage (from right side of schematic in Figure 26) corresponds to the top (blue) trace. The source 1 neutral voltage (left side of schematic) corresponds to the bottom (green) trace.

We see from Figure 27 that the neutral transient voltages on either source are very low (<0.1 volt), but of course the minimal neutral current supplying a balanced three-phase load contributes to this low voltage. Our next step, then, is to increase the neutral current to a level that simulates a worst-case unbalanced load. Using an extreme case of setting the neutral current equal to the phase current (i.e. we have a single phasing condition), we see that the transient voltage generated across the contacts rises to just under 300 V. However, as we mentioned in our discussion “Actual versus Modeled Arc Performance” (see page 13), our model is very conservative. Laboratory testing indicated that actual transient voltages are less than our modeled values. We can assume that the actual transient would likely be somewhat less than 300 V.

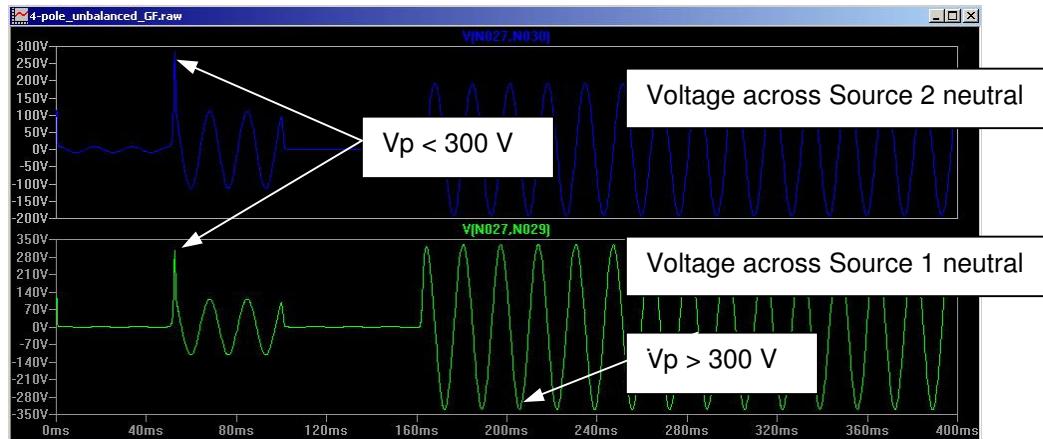


Figure 28: 100% unbalanced load (all single-phase) loads switched by 4-pole transfer switch generates a 300 V peak transient voltage across the neutral contact. However, a sustained ground fault on this system generates an even higher 350 V voltage across the neutral. The switching transient can therefore be considered trivial.

But notice that this transient voltage is approximately equal to what the *normal* line-to-ground voltage would be during a ground fault. As shown in Figure 28, the steady-state maximum voltage generated across the neutral during a sustained ground fault is even higher than the transient voltage generated during the transition itself. Obviously, since a distribution system and any loads connected to it must be designed to withstand a ground fault (which could occur at any time), a 300 V peak is tolerable.

Our next step is to confirm that the 4-pole switch correctly isolates ground currents during a ground fault. To do this, we calculated the zero-sequence ( $\mathbf{A}+\mathbf{B}+\mathbf{C}+\mathbf{N}$ ) current flowing from each source during the ground fault and confirmed that this value was non-zero only for those sources feeding current into the fault.

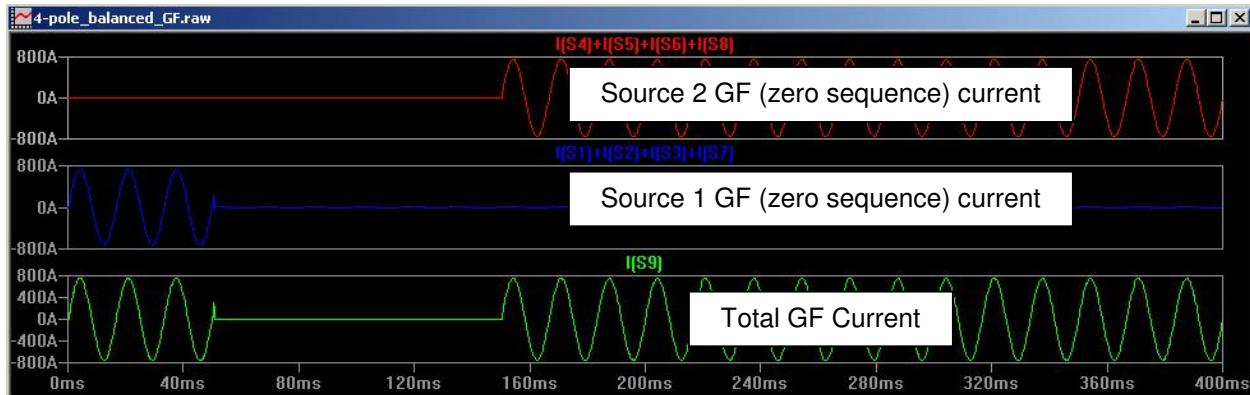


Figure 29: All GF current is sourced (and correctly measured) as coming from source 1 from  $t = 0$  to 50 ms. During the open-transition transfer from 50 to 150 ms no current (including any ground fault current) flows. From  $t = 150$  ms onward all ground fault current is sourced from source 2.

We confirm, then, that a 4-pole switching scheme prevents circulating current from returning to other than an energized separately derived source. And we confirm that any switching transients generated across the neutral pole of a 4-pole switch are minor.

### 3-Pole Overlapping Neutral – Balanced Load

We next examine a 3-pole switch with overlapping neutral. Something very interesting happens when a GF occurs during an overlapping neutral operation. During the time when both neutrals are closed, ground current will circulate between sources over this neutral connection similar to how it circulates on the neutral when using ordinary 3-pole switches. Because the de-energized source, by definition, has no phase current, the neutral current appears as ground current to a zero sequence or residual ground fault sensing scheme. This effect occurs regardless of whether the load is balanced (minimal neutral current) or unbalanced. In both cases, the ground current divides between grounded sources.

Figure 30 shows a modeled schematic of a 3-pole overlapping neutral transfer switch. Figure 31 shows the total ground current (bottom trace), the source 2 zero sequence current ( $\mathbf{A}+\mathbf{B}+\mathbf{C}-\mathbf{N}$ ) (subtract phasor N due to polarity of that switch modeled in SPICE being inverted from that of the power poles) and the source 1 zero sequence current ( $\mathbf{A}+\mathbf{B}+\mathbf{C}-\mathbf{N}$ ) all superimposed on the same oscilloscopic chart.

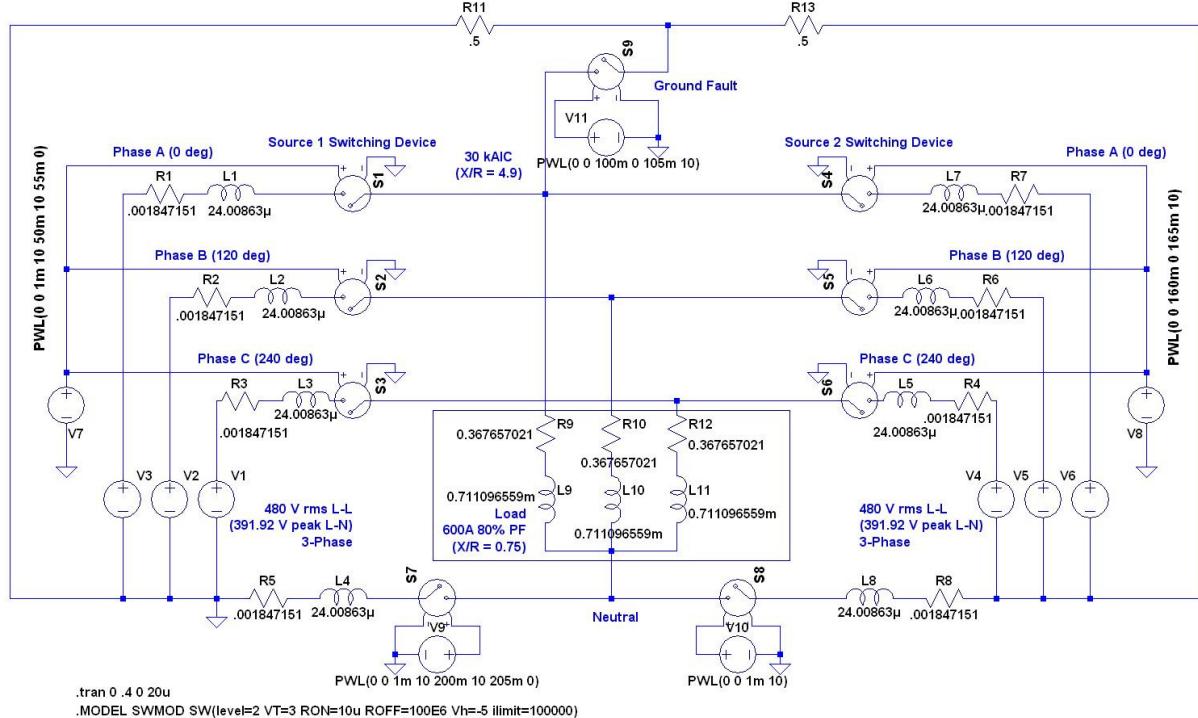


Figure 30: 3-Pole overlapping neutral, balanced 3-phase loads (minimal neutral current)

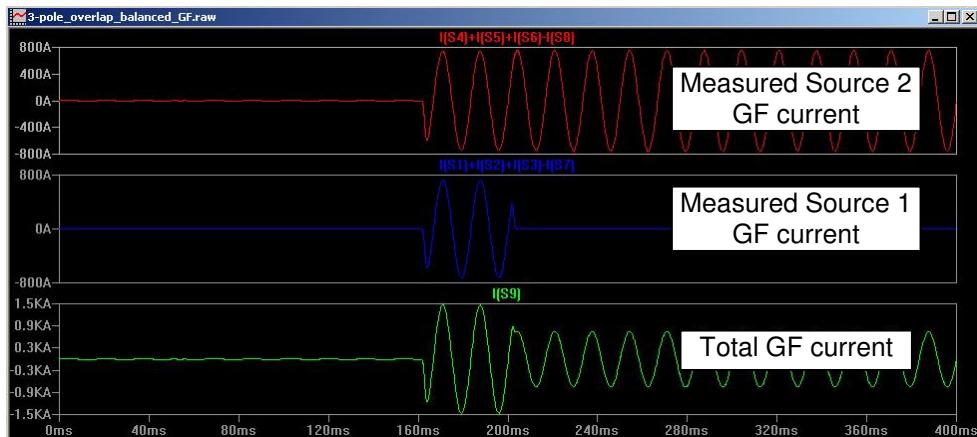


Figure 31: A GF is started at  $t=100$  ms (during overlapping neutral transition) after source 1 has been de-energized. Even though source 1 is de-energized, the zero sequence CT monitoring source 1 (middle, blue trace) shows GF current during the transition time. The source 1 GF relay continues to incorrectly detect GF current flowing through source 1 until the source 1 neutral contact opens. This false indication of GF current can cause a nuisance trip of the source 1 GF relay. Likewise, more current is flowing in the fault while both neutrals are overlapped than either GF relay detects. This can result in shower tripping than the fault current magnitude should otherwise require.

As you see, if a ground fault occurs during the time when both neutrals are connected to the load (and each other), you have the same problem as with a conventional 3-pole switch. Ground current can flow through the neutral of the de-energized source, resulting in a possible nuisance trip of the GF relay on the de-energized source.

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## Summary

While 3-pole transfer switches can be used on non-separately derived systems as long as suitable control interlocks prevent tripping of de-energized sources, those same switches are not acceptable on separately derived systems. On separately derived systems, 4-pole transfer switches are necessary since they prevent circulating neutral currents from nuisance tripping ground fault relays on de-energized sources.

Note that it is possible to construct a system of interlocked tripping circuits and switched CT inputs feeding into GF relays to create a transfer scheme that successfully switches separately derived systems using only 3-pole switches. Refer to Eaton's *Consulting Application Guide* [11] under the heading of "Dual Source System – Multiple Point Grounding" for more information.

However, to simplify the design, 4-pole transfer switches should be used on any separately derived system as they successfully inhibit circulating ground currents that would otherwise attempt to flow between sources. Ground current cannot circulate through the neutrals of de-energized sources when 4-pole switches are used. No complex GF sensing wiring is required with 4-pole switching.

Regarding the one problem leveled against 4-pole switches, namely transient voltage generation across the neutral contacts, testing (and modeling) indicates that the magnitude of transient voltages generated across the neutral switch of a 4-pole switch do not exceed the normal neutral-to-ground voltages ordinarily present on the neutral bus, even under worst case conditions of unbalanced load switching.

As a result of this testing, we endorse the position of McMorrow [1] and confirm that transient voltages generated across the neutral contacts of a 4-pole switches are not detrimental to system operation.

## Acknowledgements

The author would like to thank Dr. Xin Zhou from Eaton for the laboratory data and analysis of the progression of voltage developed across an opening air break contact.

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[www.linear.com/litspice](http://www.linear.com/litspice). An active user group providing examples, tutorials and assistance is available at: <http://tech.groups.yahoo.com/group/LTspice/>
- [10] Download the SPICE models shown in this paper along with an Excel calculator that converts X/R to PF and vice versa, calculates Z, R and X based on available short circuit current and computes per unit impedances from absolute impedances and absolute impedances from per unit impedances: <http://pps2.com/files/xfer/spice>
- [11] Refer to the Eaton Consulting Application Guide (CAG) for more information on ground fault sensing schemes. In the CAG section titled "Power Distribution Systems – System Application Considerations," there is a section titled "Grounding / Ground Fault Protection." A link to the 2006 CAG that takes you directly to this page is [http://www.eaton.com/ecm/idcplg?IdcService=GET\\_FILE&allowInterrupt=1&RevisionSelectionMethod=LatestReleased&noSaveAs=1&Rendition=Primary&&dDocName=TB08104003E#page=67](http://www.eaton.com/ecm/idcplg?IdcService=GET_FILE&allowInterrupt=1&RevisionSelectionMethod=LatestReleased&noSaveAs=1&Rendition=Primary&&dDocName=TB08104003E#page=67). If this link is no longer valid, you can navigate to [www.eaton.com/consultants](http://www.eaton.com/consultants) and click on the link for the table of contents for the most recent CAG.

## Appendix

Conversion of Power Factor into X/R Ratio:

$$\frac{X}{R} = \left( \left( \frac{1}{PF} \right)^2 - 1 \right)^{0.5} \quad \text{or} \quad \frac{X}{R} = \tan(\cos^{-1} PF)$$

Conversion of X/R Ratio into Power Factor:

$$PF = \left( \frac{1}{\left( \left( \frac{X}{R} \right)^2 + 1 \right)} \right)^{0.5} \quad \text{or} \quad PF = \cos(\tan^{-1} \left( \frac{X}{R} \right))$$

Calculation of X based on X/R ratio and Z:

$$X = \sqrt{\frac{Z^2}{1 + \left( \frac{X}{R} \right)^2}}$$

Calculation of R based on X/R ratio and X:

$$R = \frac{X}{\left( \frac{X}{R} \right)}$$

Calculation of L based on X (inductive reactance) and frequency (including 60 Hz):

$$L = \frac{X}{2\pi f} = \frac{X}{377}$$

An Excel calculator with these and other equations can be found at: <http://pps2.com/files/xfer/spice> under the “tools” folder.